

Diffusive dynamics

$$\dot{x}_i = -Bx_i + Rk_i^{-1} \sum_j A_{ij}x_j$$

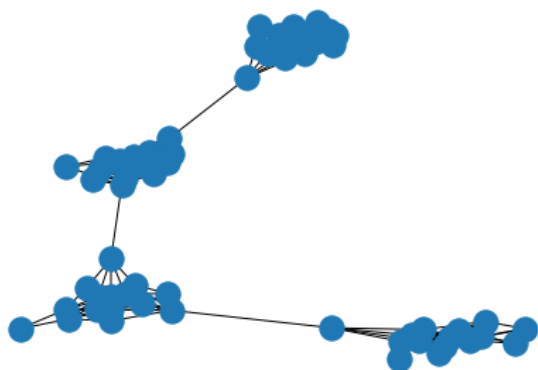
$$J_{ij} = -B * \delta_{ij} + R * k_i^{-1} * A_{ij}$$

if $B=R=1 \rightarrow J_{ij} = -L_{ij}$

$$\lambda_{max} = -B$$

$$\lambda_{min} = -(B - R)$$

stable only if $\lambda_{min} < 0 \rightarrow B > R$

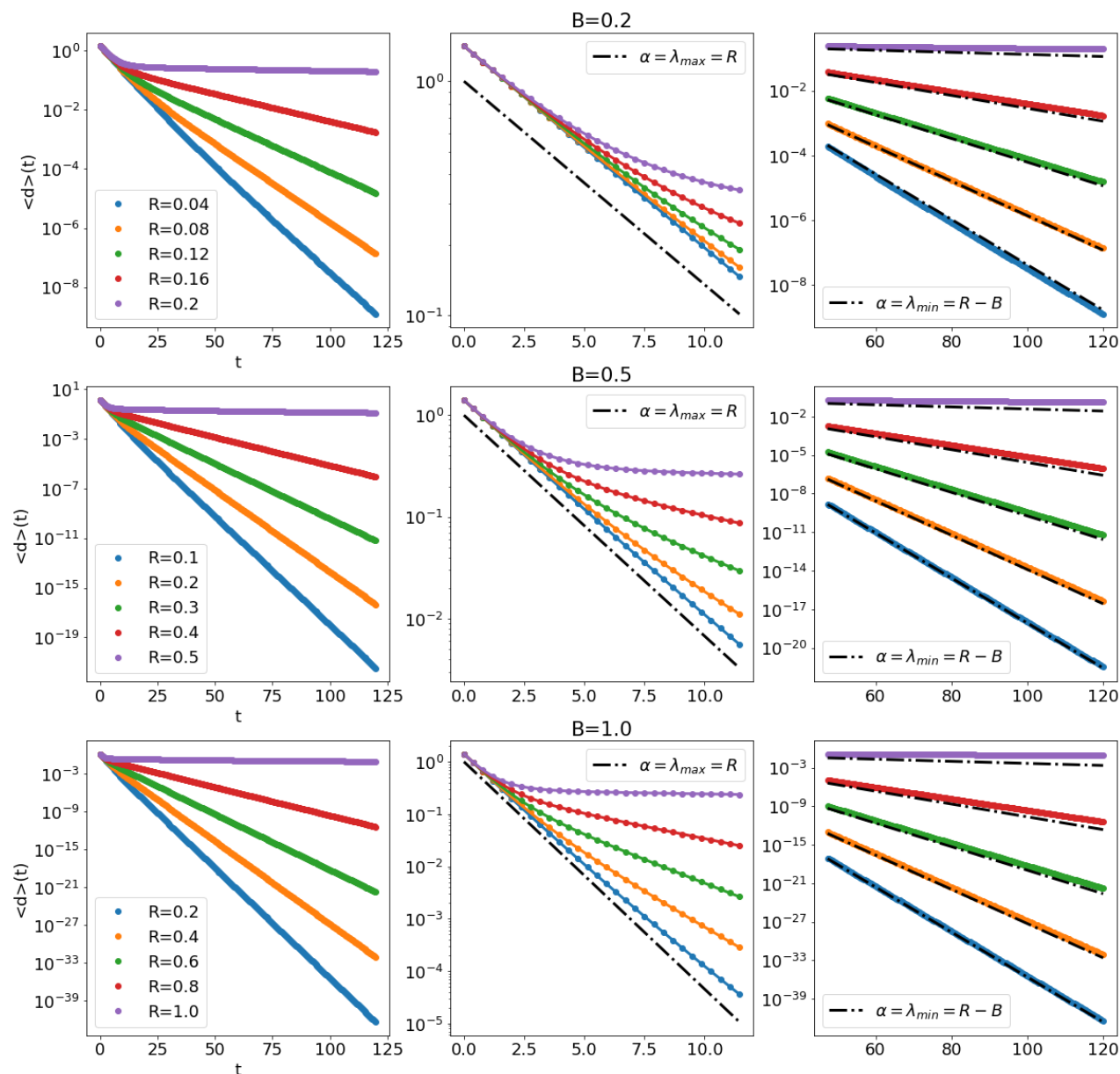


it seems that:

- $\langle d \rangle(t) \sim e^{-\lambda_{max}t}$ for small t
- $\langle d \rangle(t) \sim e^{-\lambda_{min}t}$ for large t

BUT not able to prove it analytically

Average distance vs. time
for different parameters B and R



Diffusive dynamics

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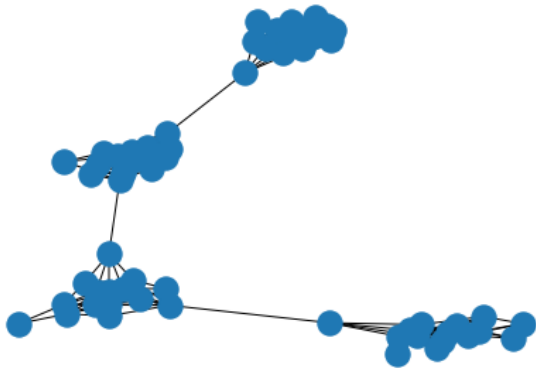
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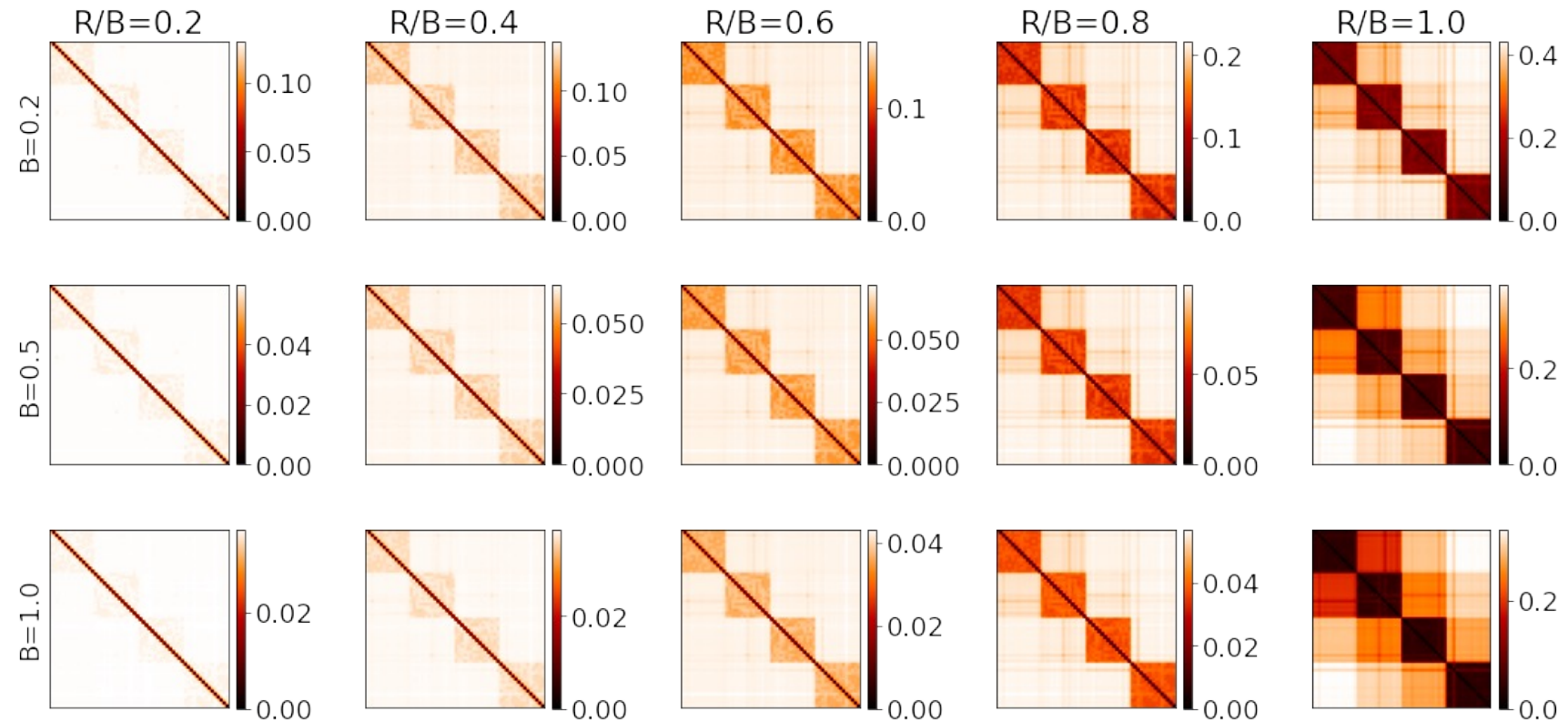
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Average distance matrix (for different parameters B and R)



** average up to $t_{max} = N$
(network size)

- large B \rightarrow more importance on exp decay \Rightarrow less effect of topology
- large R \rightarrow more importance on diffusive part \Rightarrow more effect of topology

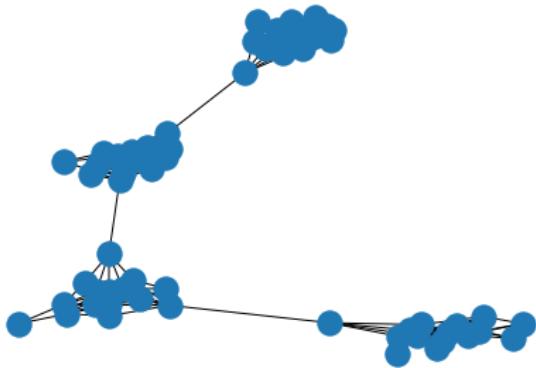
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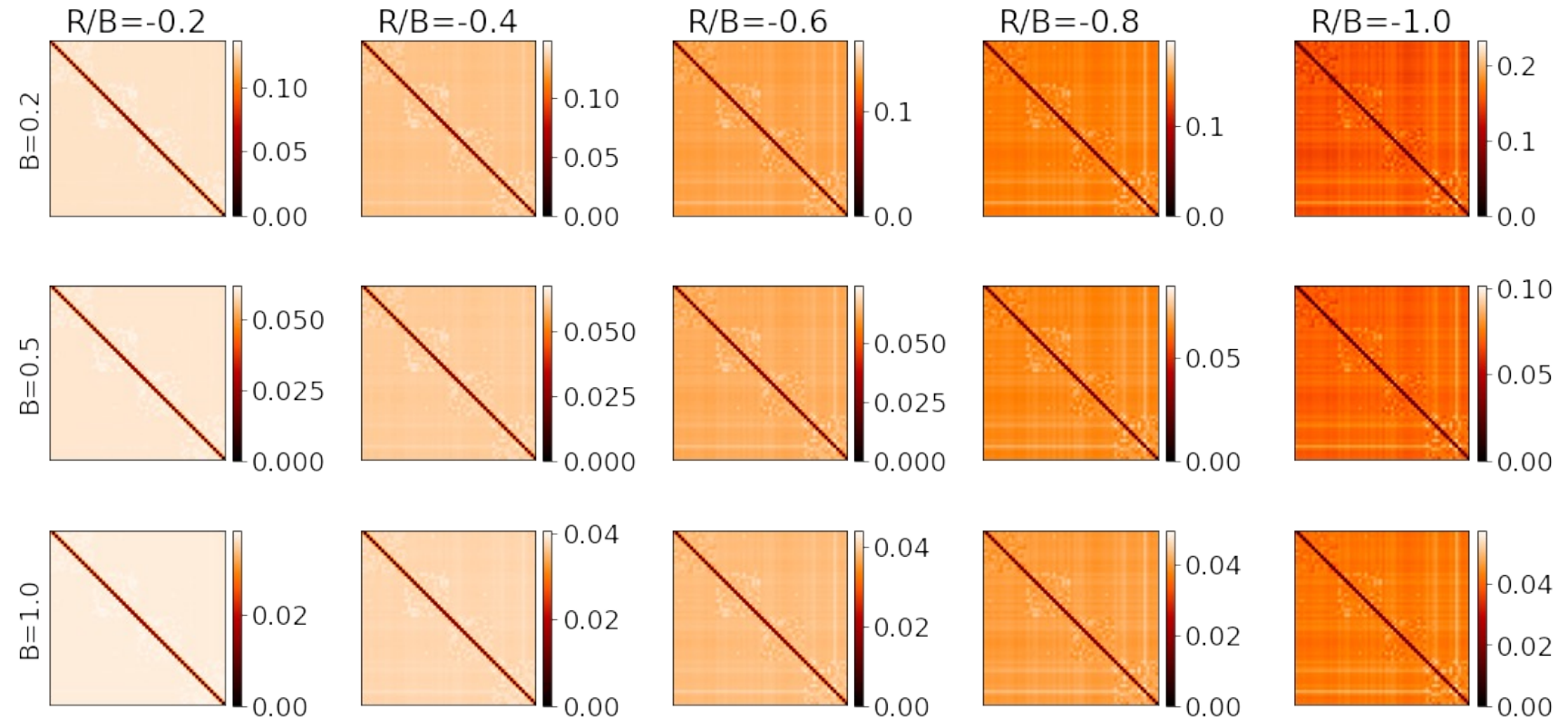
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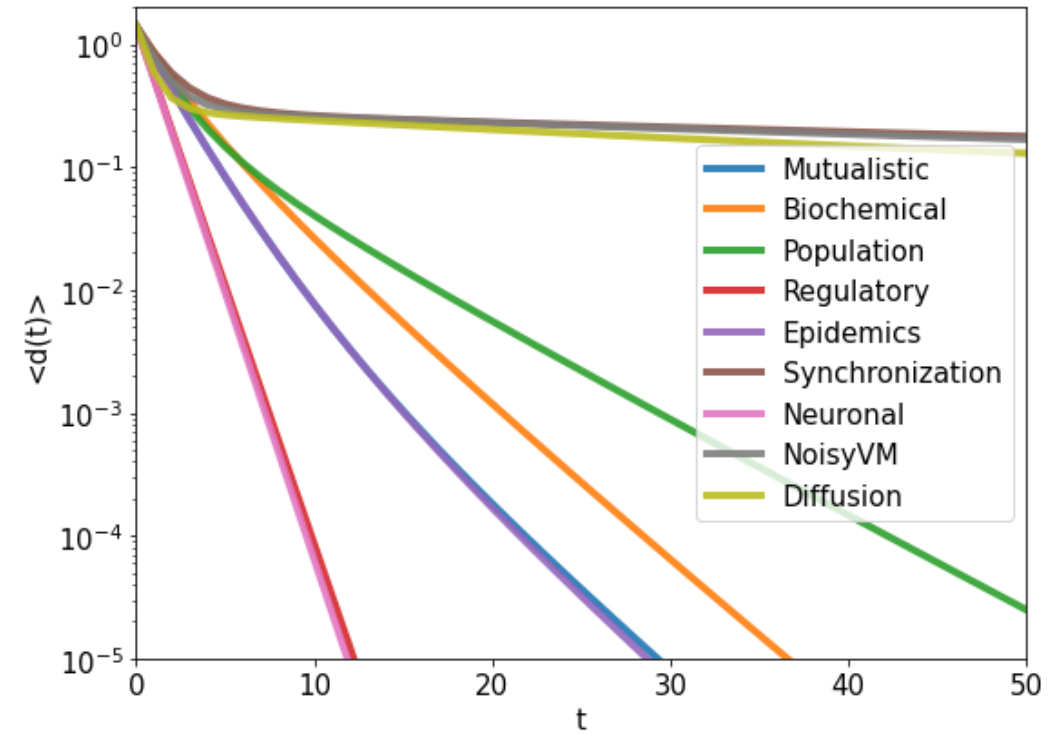
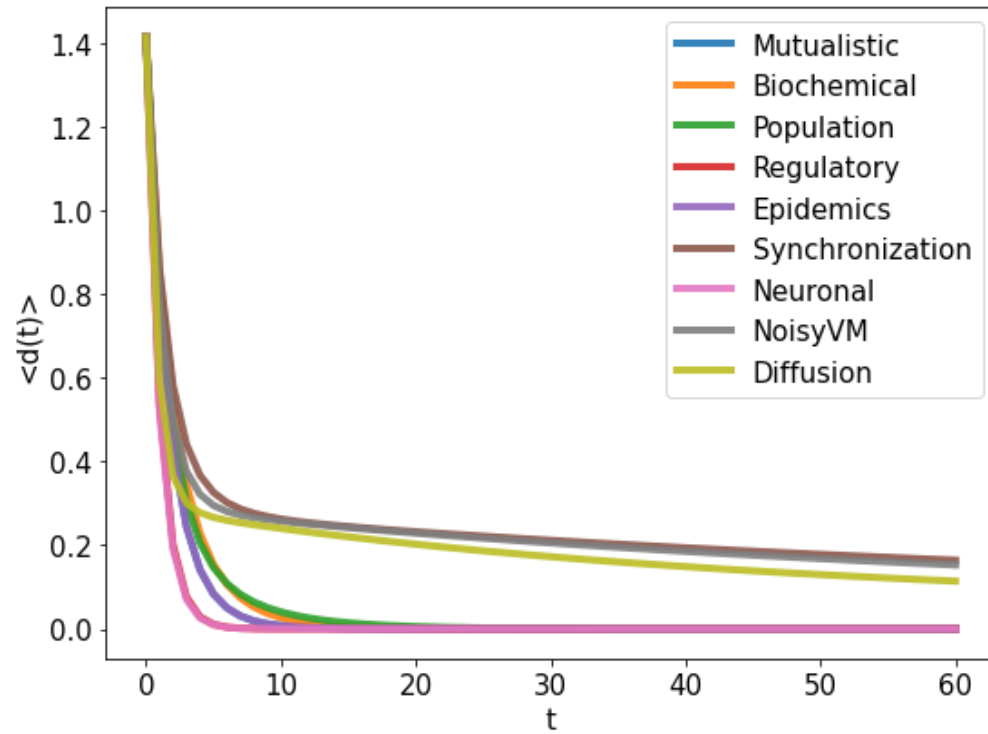
if R is negative \rightarrow even less effect of topology



Jacobian dynamics

** all the parameters are fixed to 1

** normalize $t \rightarrow t / \lambda_{\max}$

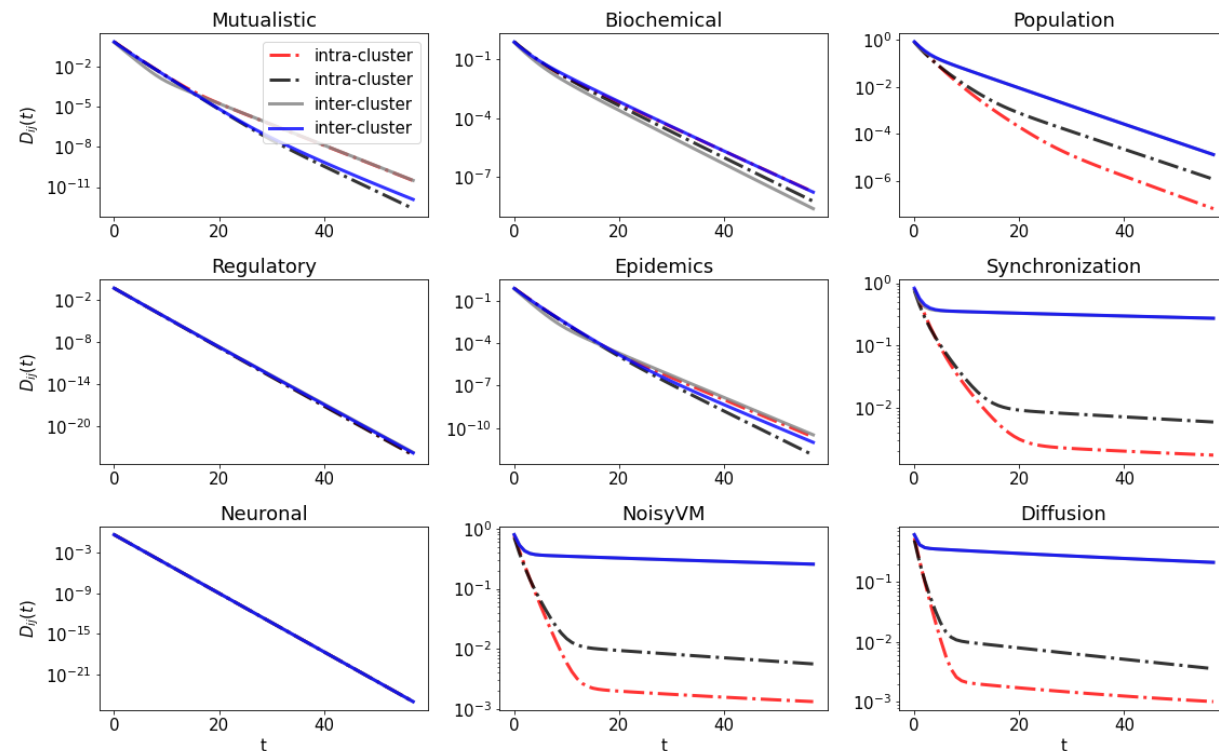
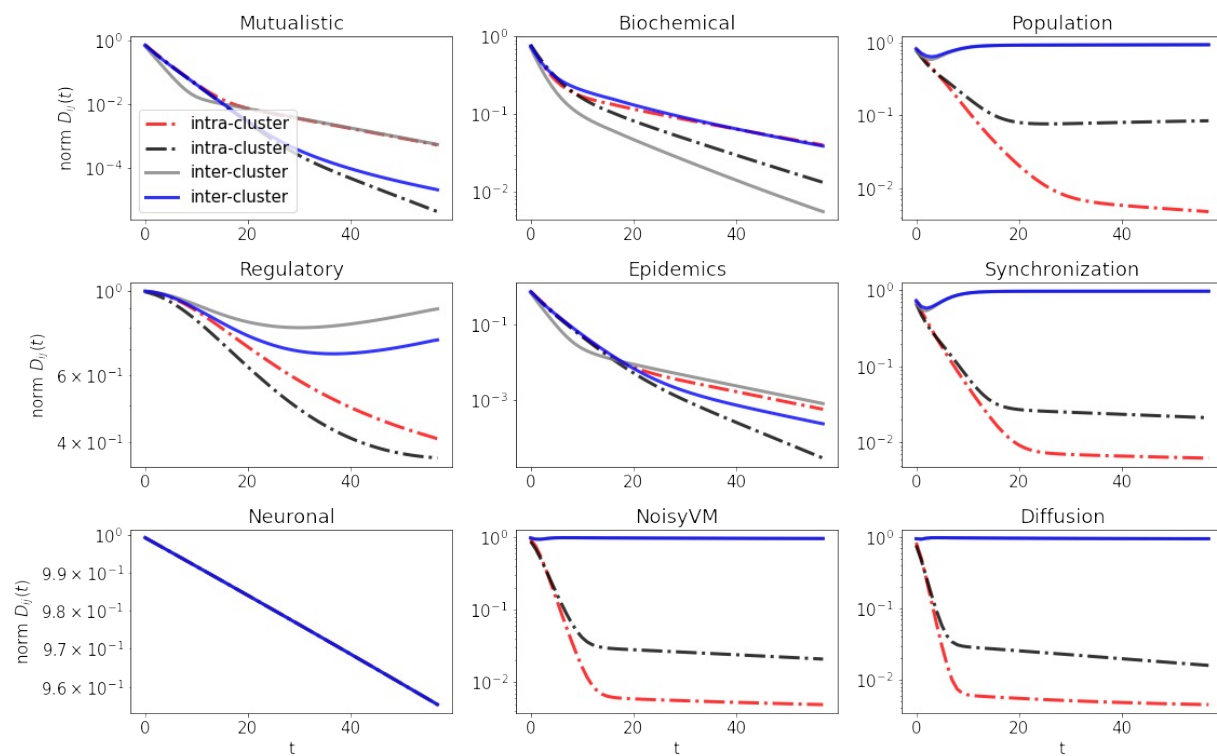


Jacobian dynamics

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if $\langle d \rangle$ decay too fast \rightarrow when we compute average distance, only the first timesteps will count

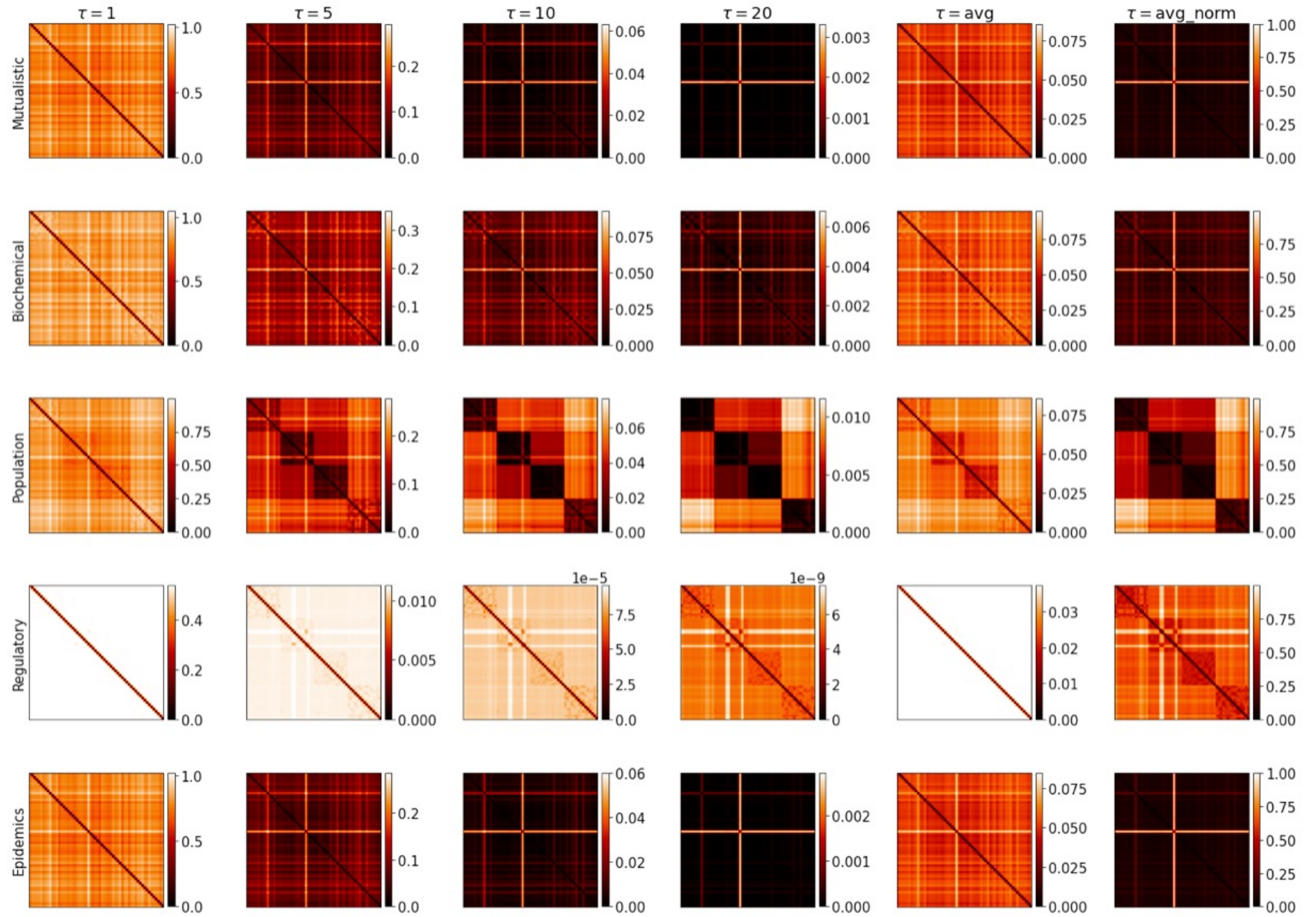
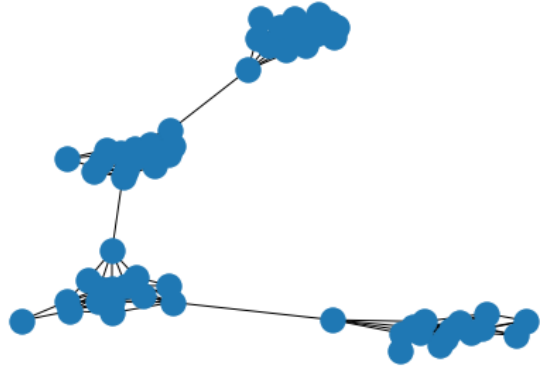


if normalize $d_{ij}(t)$ wrt $\max d_{ij}(t)$ \rightarrow all the timesteps are more evenly taken into account when computing average distance matrix

Jacobian dynamics

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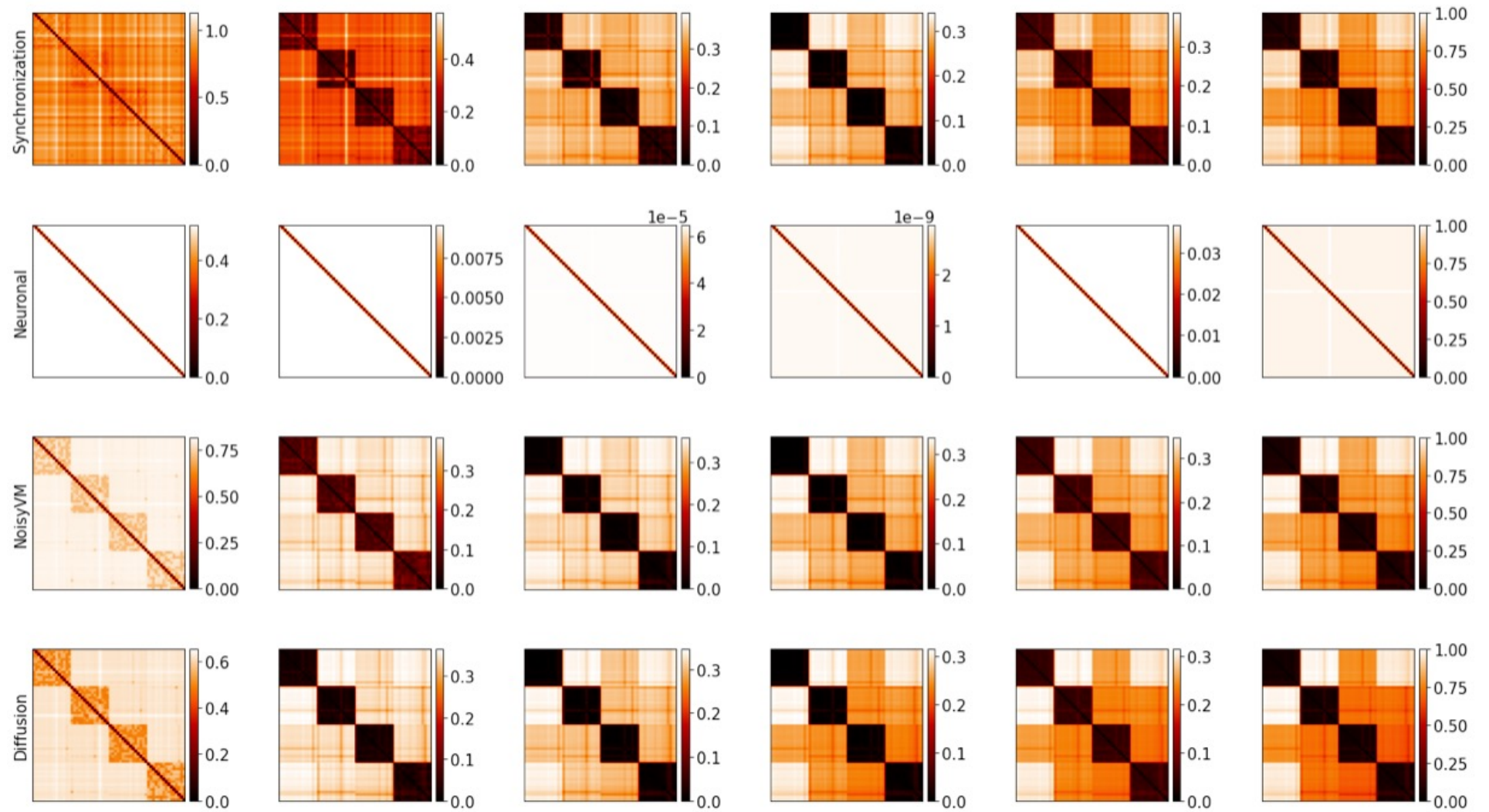
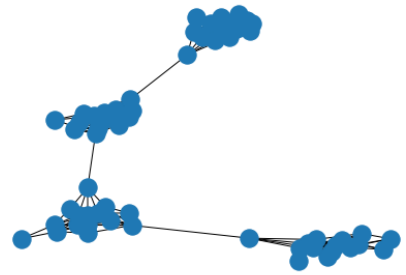
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Jacobian dynamics

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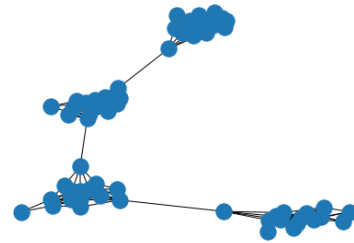
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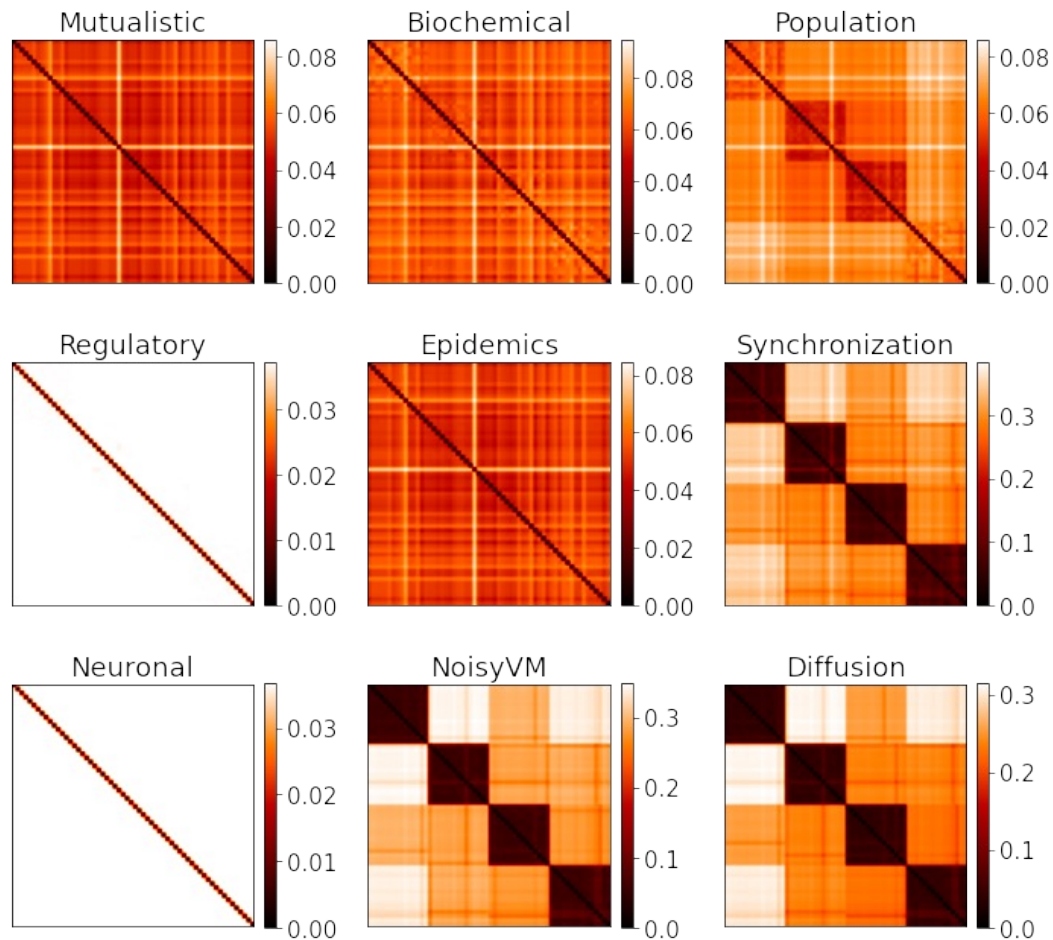
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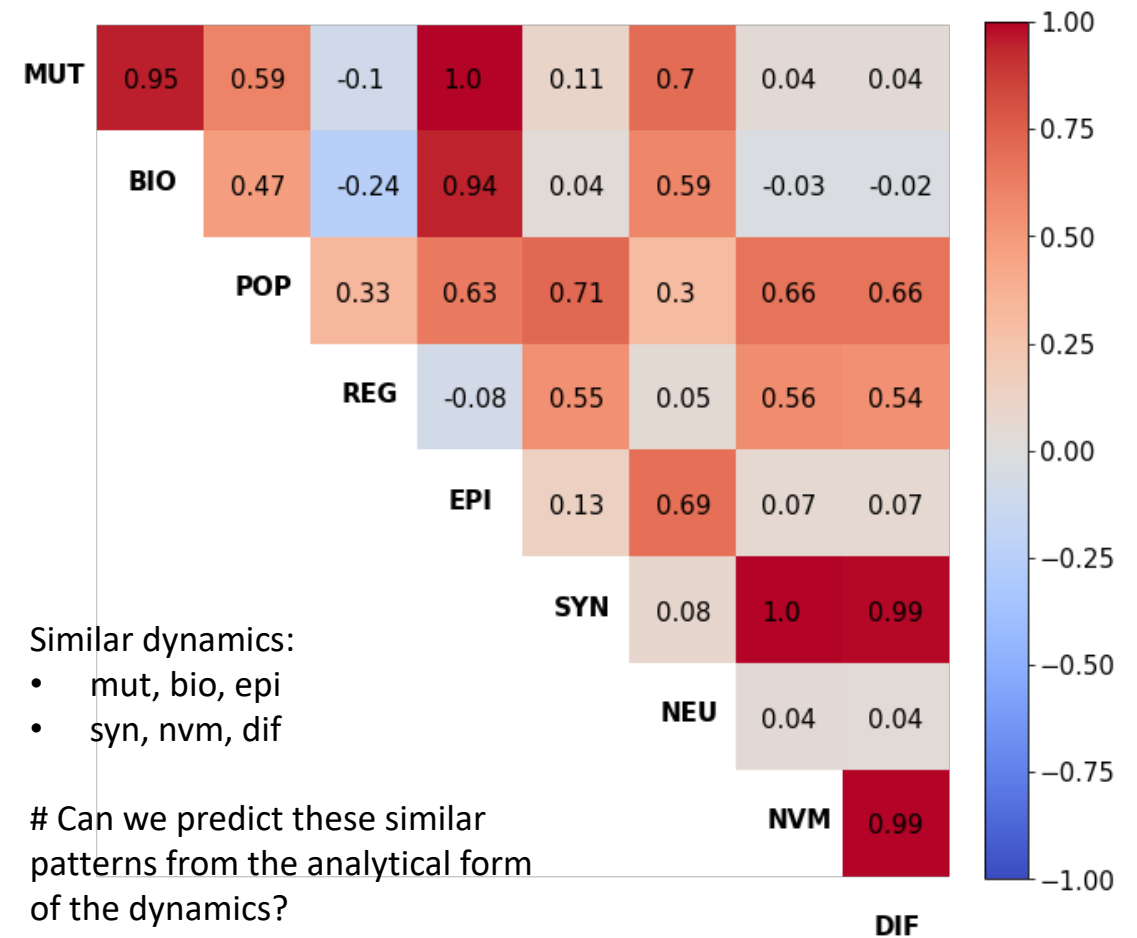
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Average



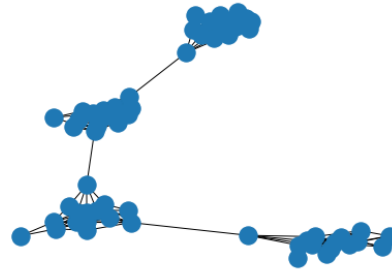
Mantel's test (to compare similarity)



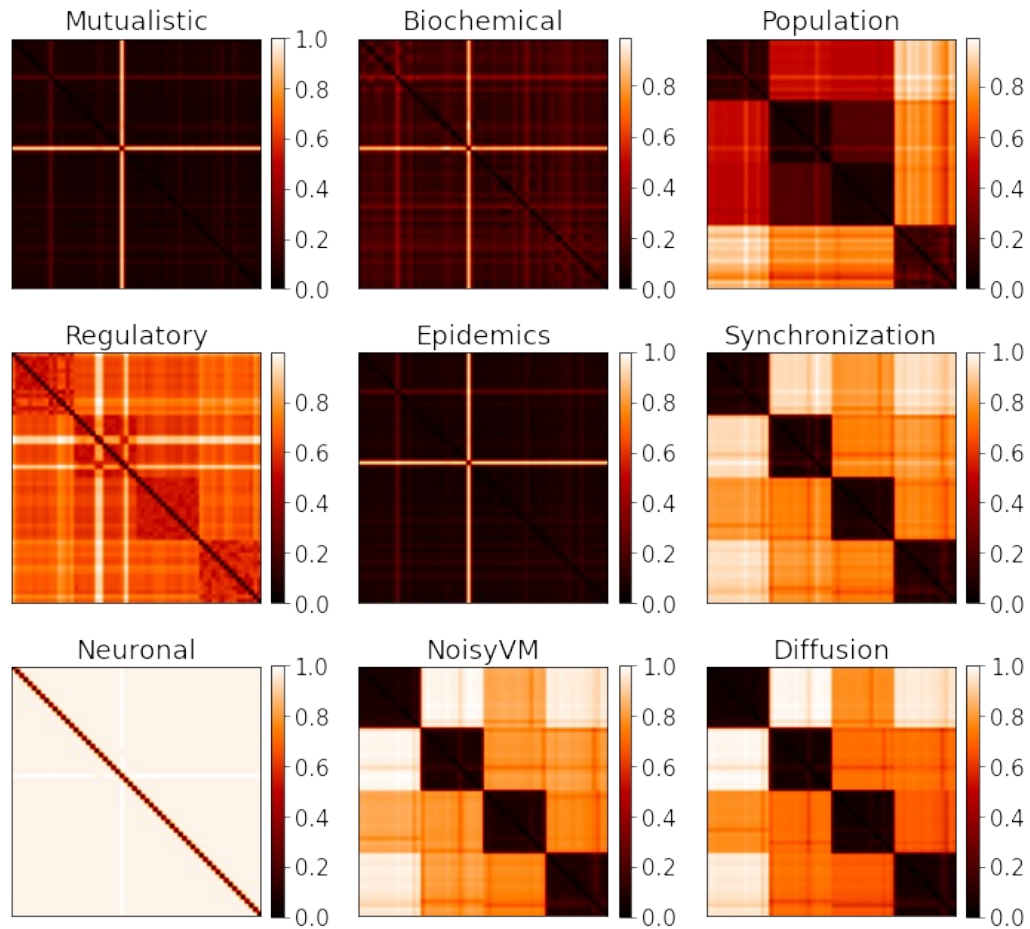
Jacobian dynamics

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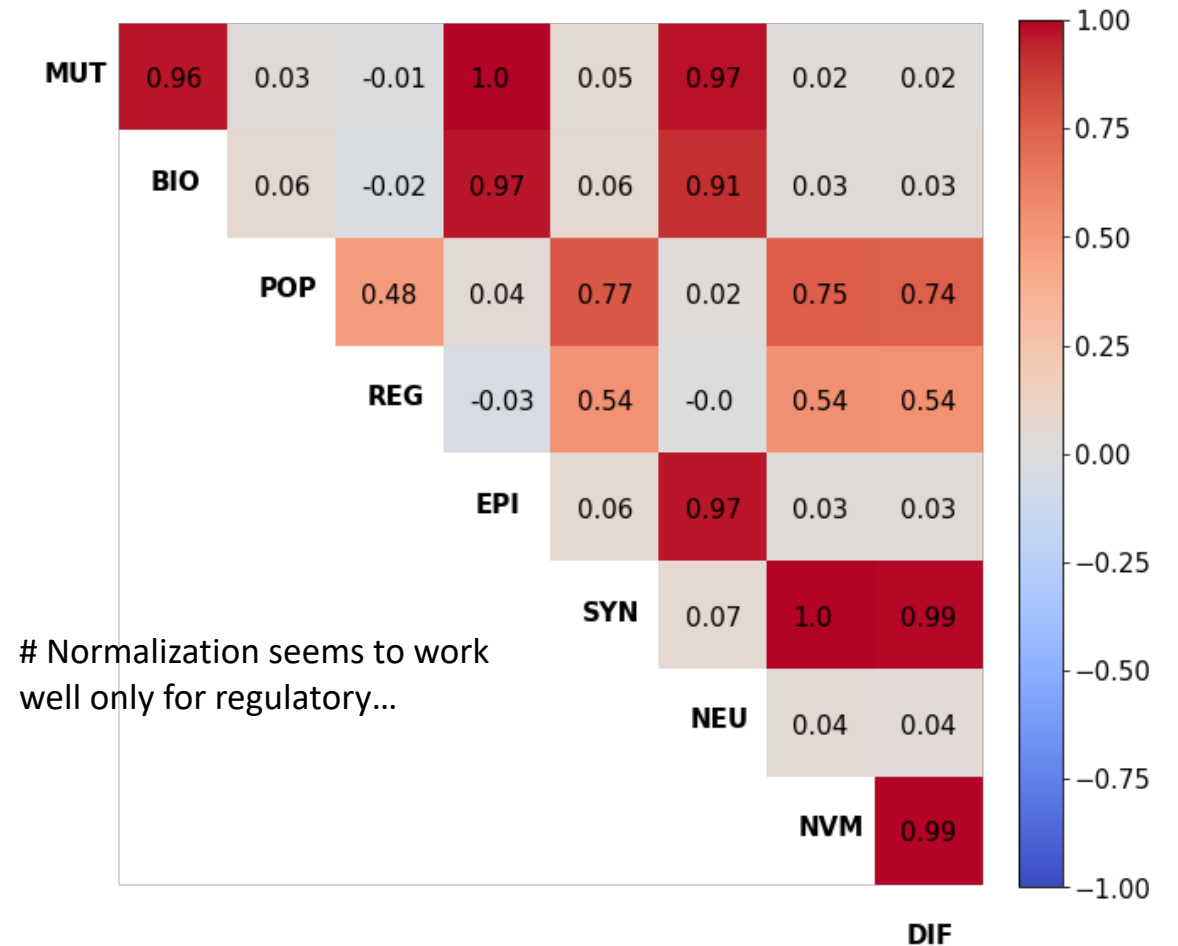
** normalize $t \rightarrow t / \lambda_{\max}$



Average Normalized



Mantel's test (to compare similarity)



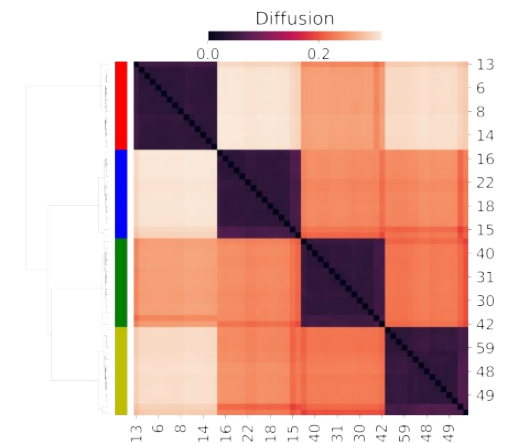
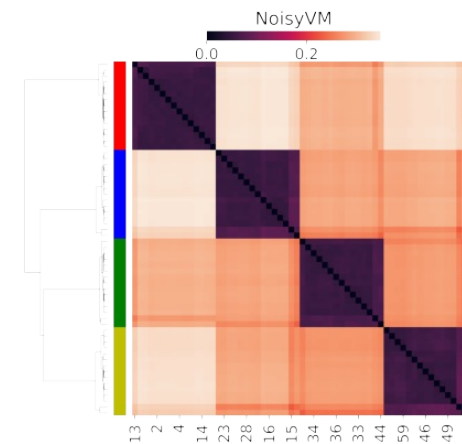
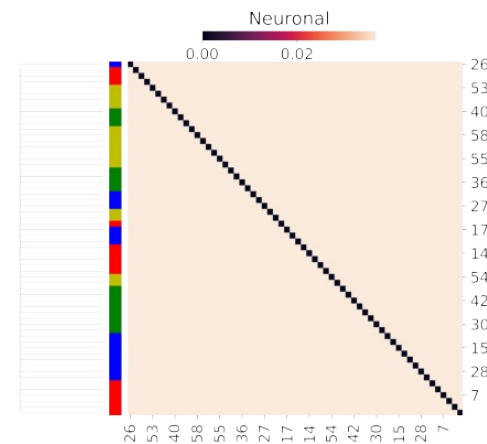
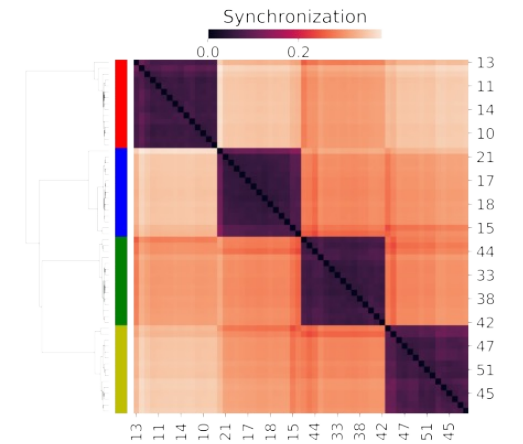
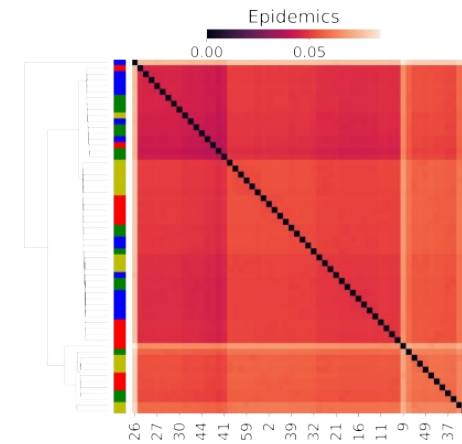
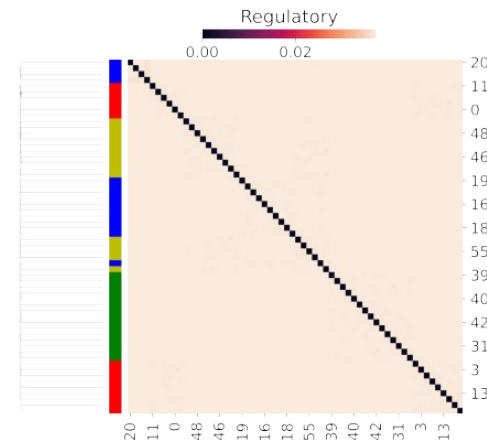
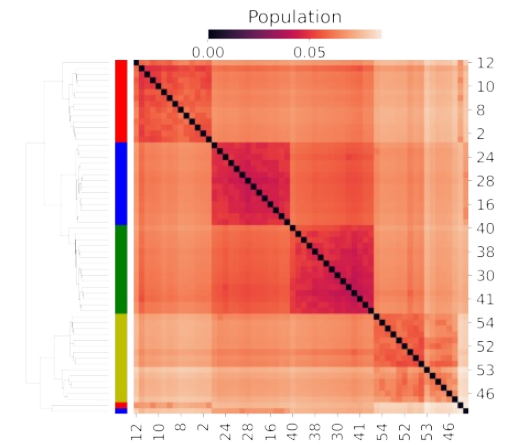
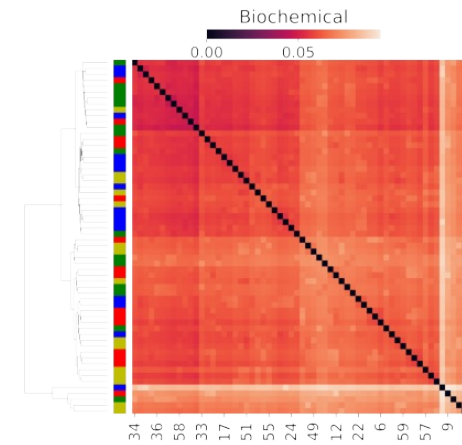
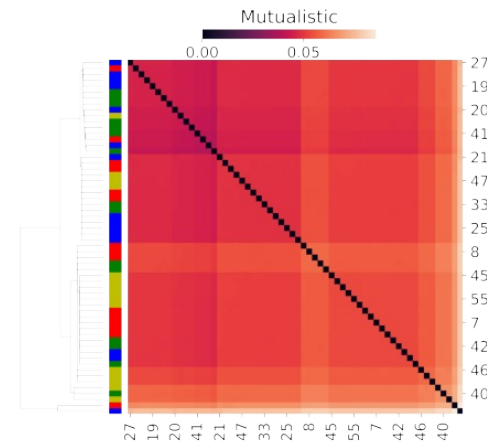
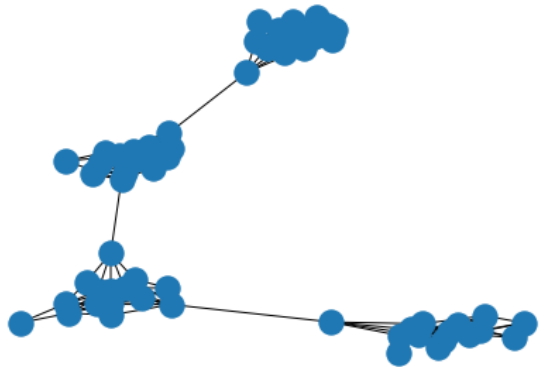
Jacobian dynamics

** all the parameters are fixed to 1

** normalize $t \rightarrow t / \lambda_{\max}$

Average distance matrix

** color in dendrogram = structural community



Jacobian dynamics

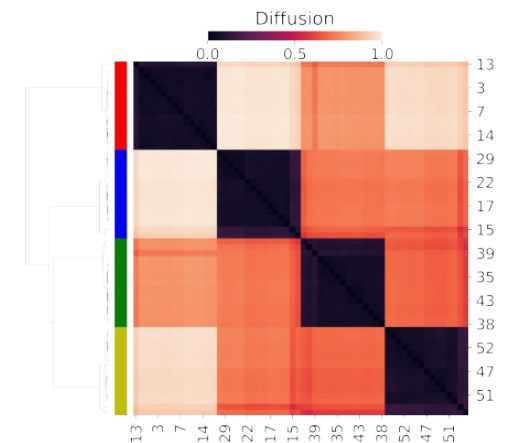
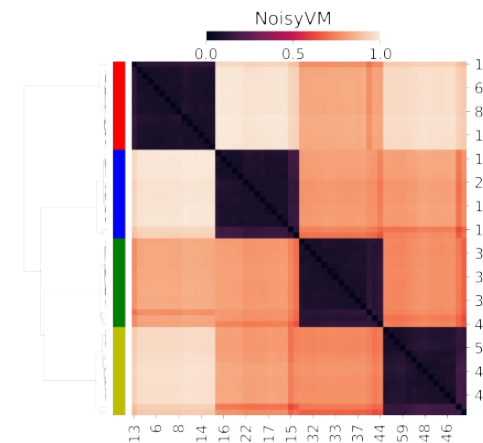
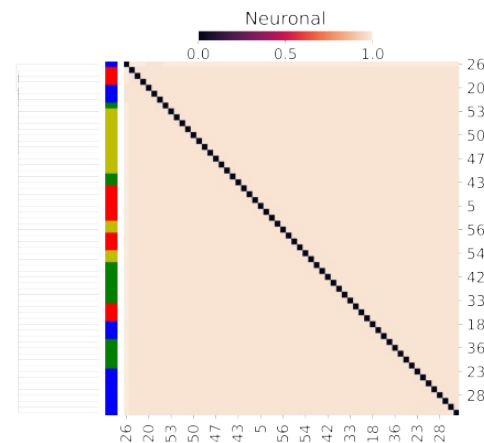
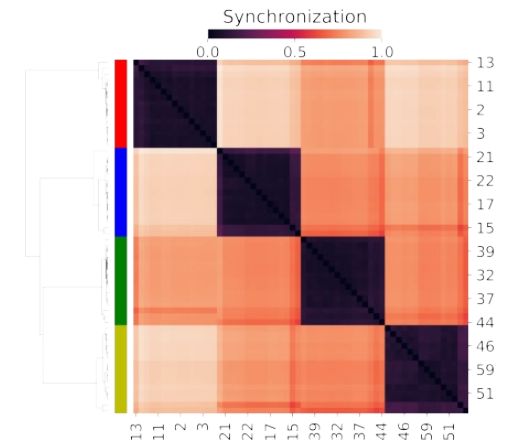
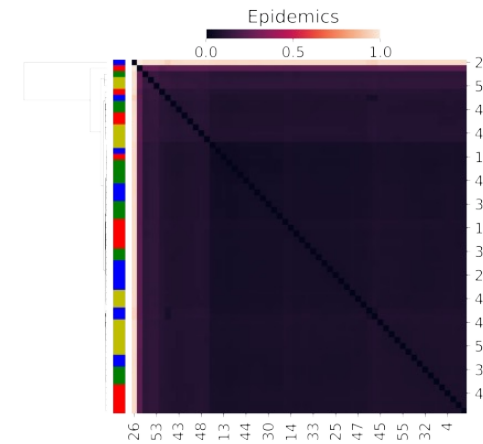
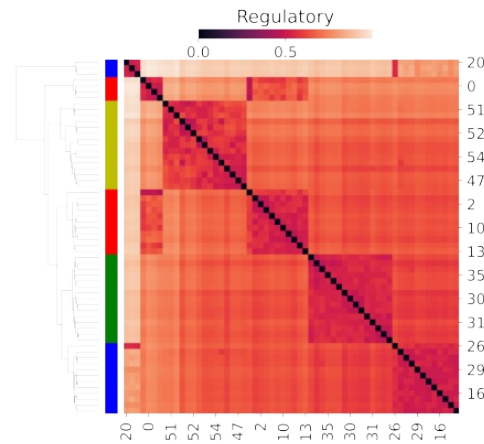
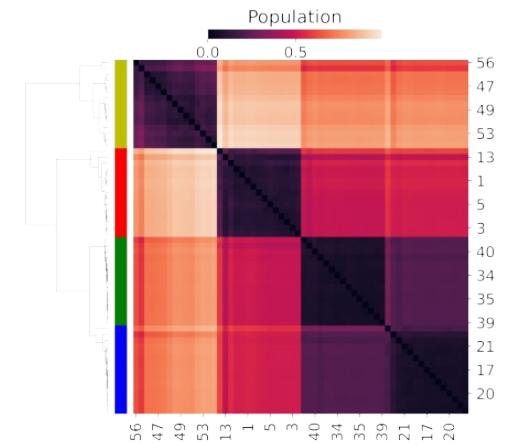
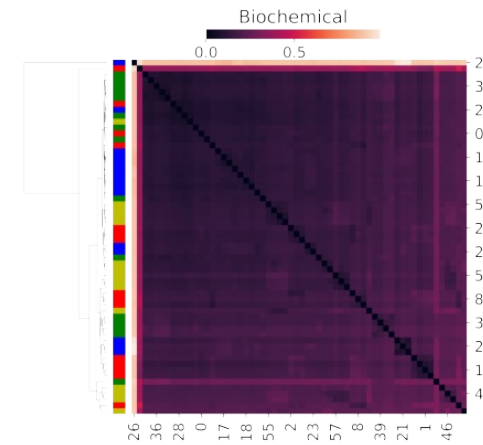
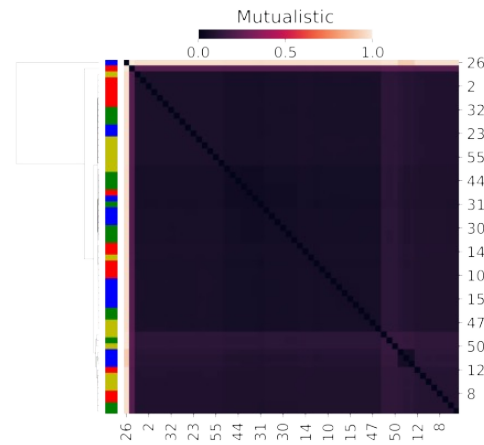
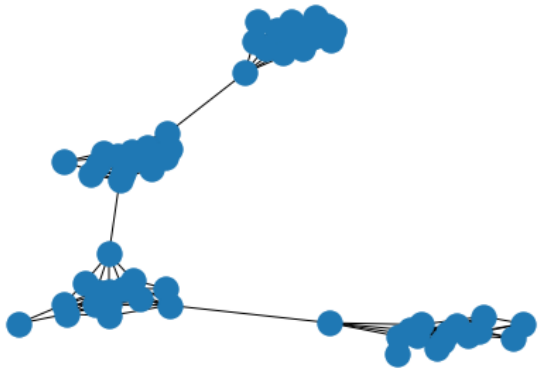
** all the parameters are fixed to 1

** normalize $t \rightarrow t / \lambda_{\max}$

Average normalized distance matrix

** color in dendrogram = structural community

from here the normalization seems not that interesting...



Jacobian dynamics – other type of networks

** all the parameters are fixed to 1

** normalize $t \rightarrow t / \lambda_{\max}$

Barabasi-Albert

Mutualistic

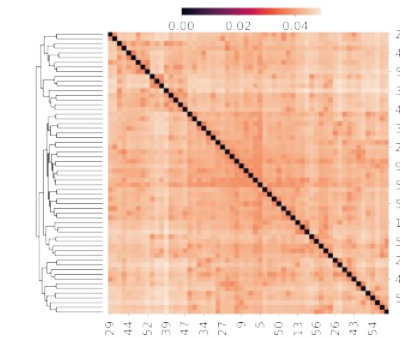
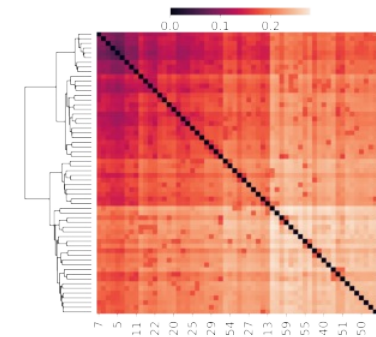
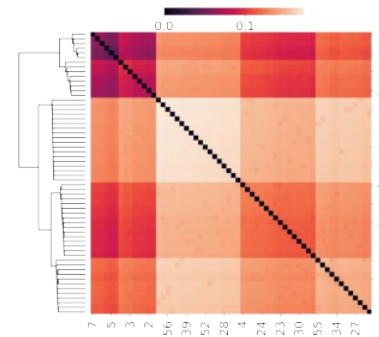
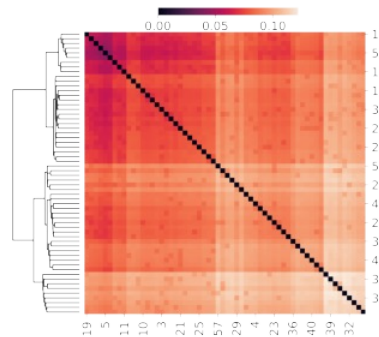
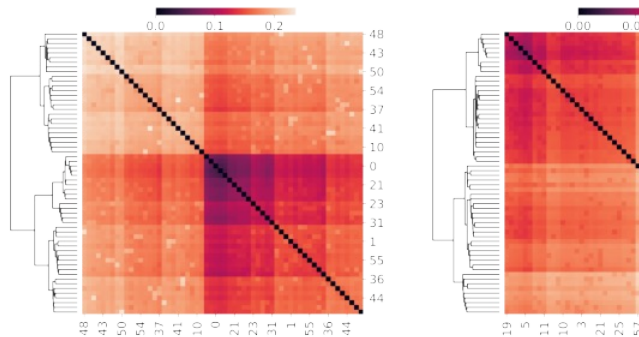
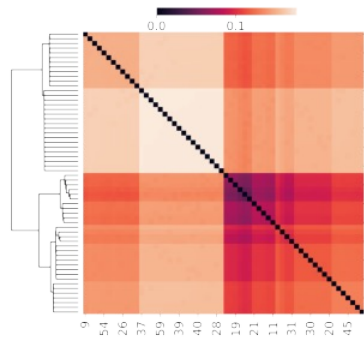
Biochemical

Population

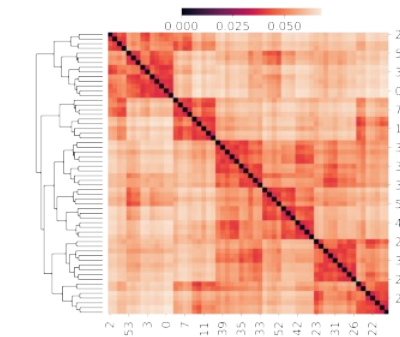
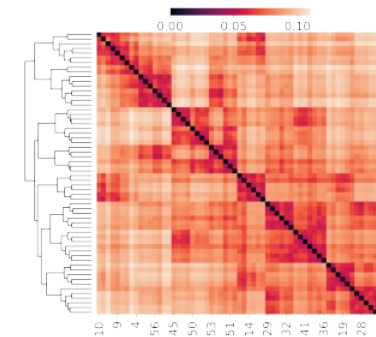
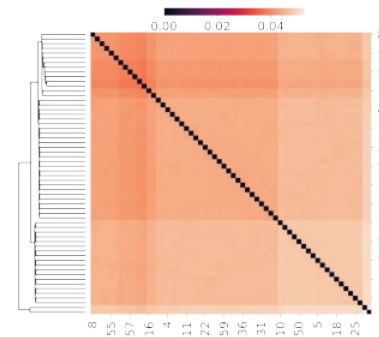
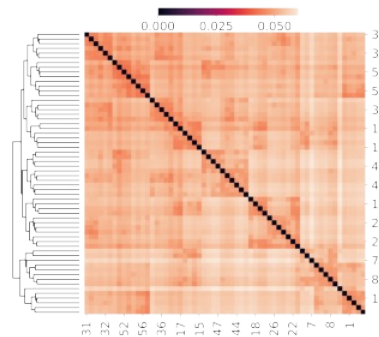
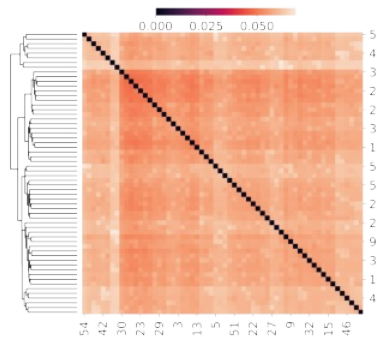
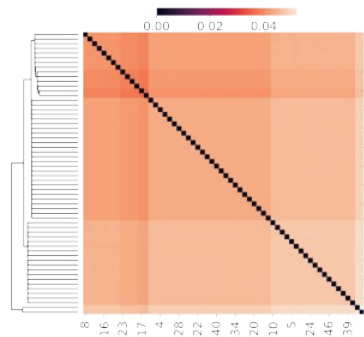
Epidemics

Synchronization

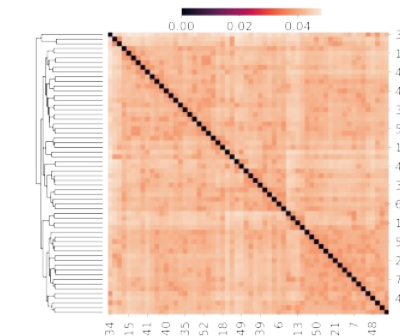
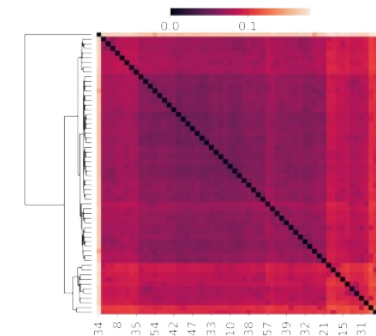
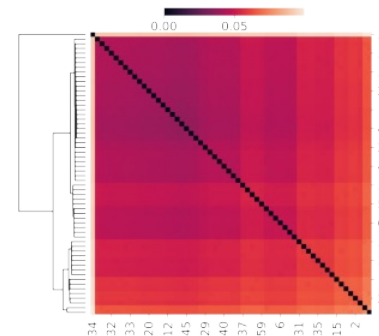
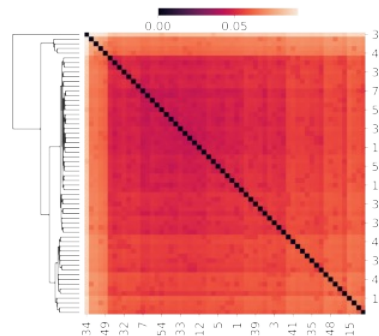
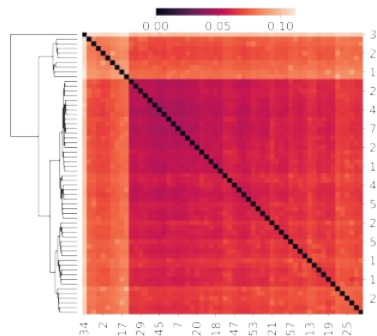
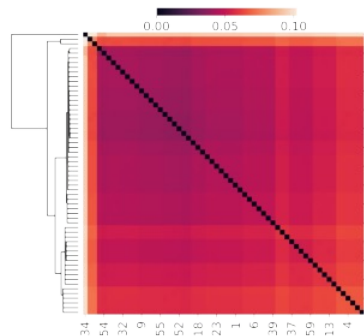
Diffusion



Watts-Strogatz



Erdos-Renyi



QUESTIONS:

- Can we prove analytically the dependence of the jacobian distance with the eigenvalues?
- Do we need the normalization?
- Mantel's test is ok? Other methods for comparing matrices?
- Apply to real networks?