Programming with Categories

Yet another attempt to teach monads

Gunnar Bastkowski

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What is Category Theory?

S. Awodey approaches the term *category theory* as follows[1]:

> *As a first approximation, one could say that category theory is the mathematical study of (abstract) algebras of functions.*

Category theory addresses the question:
How can we combine functions to create new functions?

[1] [1]

What is it good for?

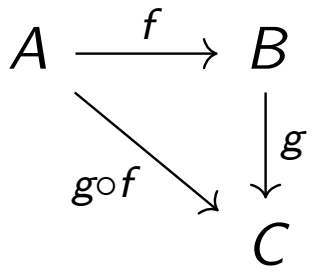
- Functional programming is about function composition
- Gives us tools to manipulate functions
- Provides a vocabulary like *Design Patterns*.
  But **much** more powerful

Vocabulary

- objects (A, B) and Arrows (f)

$$A \xrightarrow{f} B$$

- composition ($g \circ f$)



- identity (id)

$$A \xrightarrow{id} A$$

Laws

- arrows compose
  For every $f : A \to B, g : B \to C$,
  there exists a composition $g \circ f : A \to C$
- associativity
  $(A \to B) \to C = A \to (B \to C) = A \to B \to C$
- identity (id)
  For every A_1, A_2, with A_1 equal[2] to A_2,
  $f_1 : A_1 \to A_2 = f_2 : A_2 \to A_1 = id$

[2] up to isomorphism

Challenges

References

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Oxford Logic Guides. OUP Oxford, 2010.