Strategic communication of narratives

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January 8, 2025

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Abstract

We conceptualize the communication of narratives as a cheap-talk game under model uncertainty. The sender has private information about the true data generating process of publicly observable data. The receiver is uncertain about how to interpret the data, but aware of the sender's incentives to strategically provide interpretations (*"narratives"*) in her favor. We consider a general class of decision rules under ambiguity resolving the receiver's ignorance of the true data generating process, including maximum likelihood expected utility. The set of equilibria is characterized by a positive integer N: there is an equilibrium that induces n different actions for each $1 \leq n \leq N$. The diverting power of the sender is weaker than with a naïve receiver being unaware of the sender's incentives. Surprisingly, the receiver sometimes prefers to be naïve.

Keywords: Narratives, model uncertainty, strategic communication, cheap-talk game

JEL: C72, D81, D82, D83

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The authors thank Chiara Aina, Andreas Blume, Yves Breitmoser, Arthur Dolgopolov, Tilman Fries, Fabrizio Germano, Lorenz Hartmann, Ole Jann, Navin Kartik, Julian Matthes, Jurek Preker, Frank Riedel, Ina Taneva and seminar participants at Bielefeld University, the Center for Uncertainty Studies, LEMMA at Université Paris Panthéon-Assas, and Humboldt University for fruitful discussions.

An equilibrium solver based on sagemath is available at https://github.com/gbauch/narratives/ under a CC BY-NC-SA 4.0 license.

1 Introduction

Convincing others often involves the interpretation of past data. When a politician runs for office, she will try to cast her past achievements in the most favorable light. She may do so by interpreting past events, claiming a low unemployment rate for herself while blaming economic recessions on external factors like the global economy. By crafting a *narrative* that carefully arranges the arguments *seemingly* reasonably, she hopes to convince voters of her ability. However, rational voters understand that the politician makes these claims not least in order to get elected. Consequently, we expect them not to take her layout of arguments at face value. Rather, they should correct for the politician's own bias, disciplining her ability to manipulate opinions in her favor. *How and to what extent can a biased narrator convince a receiver by using narratives if they are aware of facing strategic communication?*

Our work answers this question by developing a novel cheap-talk game that captures the strategic communication of narratives between a biased sender (she) and an unbiased receiver (he). In contrast to the previous literature, (Schwartzstein and Sunderam, 2021; Aina, 2024), we assume that the receiver is aware of the sender's strategic incentives to manipulate him. This consequently limits the sender's persuasive power as she is disciplined to provide more compelling arguments if she was to convince the receiver. Surprisingly, we find that this restriction on the sender is not always in the receiver's interest.

In our model, both agents only have a prior belief about the true state of the nature θ. While the sender knows the exact underlying process that has generated observable data h from θ, the receiver faces model uncertainty in the sense that he does not know how to interpret the data. Any such interpretation is called a narrative or a *model* represented by a likelihood function, providing a probabilistic way to make sense out of observed data. The receiver has a finite set M of feasible models in mind that contains the true data generating process. The biased sender knows the true model and submits cheap-talk messages to the receiver aiming at making him interpret the data h in her favor. Any such message corresponds to the set of models under which it is sent and provides a reasoning of how to interpret the data, formalizing the strategic use of narratives. Upon observing the sender's report, the receiver narrows down the set of feasible models, resolves any remaining uncertainty by an *ambiguity rule*, and then takes an action. The most prominent example of such an ambiguity rule is the maximum likelihood rule, under which he picks the model with the best fit and maximizes the according expected utility. Communication of narratives thus becomes a game in which both agents strategically seek to maximize their own utility. An equilibrium is a stable strategy profile: The sender maps models to messages in a way that maximizes her expected payoff given the true model and the receiver's response. Upon observing a message, the receiver derives the subset of models that are consistent with the sender's strategy and applies an ambiguity rule to determine his best response.

Our running example considers a politician whose skill level θ is unknown, but partially revealed through past political participation as follows. The higher θ, the more likely the politician is able to make beneficial contributions to society, for instance stabilizing the economy or reducing unemployment. While the data on economic indicators is publicly available, it is unclear how the data is causally linked to the ability of the politician. In our concrete example, each data point may either depend on the skill level or independent of it and thus noise. A voter (the receiver), interested in forming an adequate belief on θ, thus does not know which of the data points to take into account for his belief update about the politician's ability and which ones to discard as irrelevant – he faces *model uncertainty*. One statistical approach to resolve this uncertainty is to determine that subset of data points that maximize the likelihood of the public observations. Based on this subset of data, he can apply Bayes' rule to (hopefully) infer the value of θ more accurately. A biased inside expert or member of the same party (sender) wants the citizen to hold the politician in high regards. In contrast to the citizen, the insider knows exactly which pieces of public information are causally linked to the politician's ability. However, the insider can use her knowledge to an advantage by suggesting to pay attention only to specific parts of the data, thereby fabricating a *narrative*. While the narrative necessarily must be compelling given the observable information, the receiver is suspicious about the insider's incentives, disciplining the sender and limiting her persuasive power. We formalize all these intuitions as outcomes of our mathematical set-up.

Our analysis characterizes equilibria as partitions of models into intervals, where models are ordered by the respective receiver's bliss points. Any such interval $\tilde{M}$ corresponds to a cheap-talk message the indicative meaning of which is "the true model belongs to $\tilde{M}$". By means of his ambiguity rule, the receiver assigns $\tilde{M}$ a unique optimal action. Among all equilibria in a given setting, there is a maximum number N of induced actions. We provide an algorithm that constructs an equilibrium with $n-1$ distinct actions given one with n distinct actions, thus proving that for any $1 \leq n \leq N$ there is an equilibrium inducing n distinct actions. The size of the conflict of interest between the sender and the receiver controls how much information the sender is willing to reveal. We characterize informativeness bounds that mark when the most finely granulated information revelation is full disclosure, partial disclosure or no disclosure.

We then compare the outcome of equilibrium play to the benchmark model of Schwartzstein and Sunderam (2021) in which the receiver acts in a "naïve" fashion: Being offered a narrative of how to interpret the observed data, he adopts the proposed model simply if it fits the data better than a default model. The receiver thus takes the sender's suggestion at face value and ignores her strategic incentives. As can be expected, the sender has more persuasive power when facing a naïve receiver than one engaging in equilibrium play. We characterize this formally by two observations: On the one hand, dealing with a naïve receiver reduces incentive constraints and allows the sender to reveal more information, effectively mapping models to a larger number of actions in equilibrium, and increase her expected utility. On the other hand, the set of true models under which the sender is able to convince the receiver to take an action in her favor is larger in the sense of set inclusion when facing a naïve receiver. However, the receiver may sometimes prefer to be naïve: Equilibrium play forces the sender to pool many models in one message, limiting the information flow and thus missing out on some chances to mutually increase expected utility.

Related literature. In his address, Shiller (2017) points out the necessity to study narratives from an economic point of view in order to better understand the effects of factual and non factual information on decisions. The economic literature has thus far conceptualized narratives in different ways.

Our work ties up with the approach that views narratives as likelihood functions. These establish a probabilistic link between observable data and the parameter of interest. That way, Bayesian reasoning about the desired parameter can be applied whenever facing correlated data and the likelihood function. This approach works well even if one is agnostic about the concrete data generating process. Most closely related to our work is Schwartzstein and Sunderam (2021). The authors let a biased narrator provide narratives to a receiver aiming at making plausible a data set h in her favor. The receiver initially considers a default model, which can in fact be the true data generating process, but adopts the narrative if it fits the observable data better than the true model. The receiver thus does not consider the strategic incentives of the sender. As a main result of theirs, the persuader has it easier to manipulate the receiver's beliefs if their initial model fits the data poorly. Aina (2024) considers a similar setting in which the persuader commits to a narrative before the public data h has been realized and focuses on which beliefs the persuader can induce facing a naïve receiver. This applies to the situation in which, for instance, a politician's voters believe a vote to be rigged if and only if their candidate has lost. Jain (2023) analyzes the situation where an unbiased sender strategically chooses the set of feasible models in order to limit an interloping biased narrator's possibilities to manipulate the receiver. There are instances in which the sender thus wants to restrict the narrator's model pool, thereby giving up the potential to communicate the true model to the receiver. In an experiment, Barron and Fries (2023) confirm the intuition that the likelihood of a narrative is one of its key determinants for persuasiveness. They find evidence that individuals use this fact to tailor narratives to their own benefit.

Another way of formalizing narratives is by interpreting a directed acyclic graphs as a causal model. Hereby, links indicate immediate logical consequences. Eliaz and Spiegler (2020) employ an equilibrium concept of long-run distribution over narrative-policy pairs that maximize an agent's utility. They identify multiplicity of narratives as an intrinsic property of equilibrium and that prevailing models can be misspecified causal models. Eliaz et al. (2024) provide an equilibrium model that studies how narratives are used to shape public opinion. False narratives are defined as misspecified causal models, that misrepresent causal correlation in order to raise support for a desired policy. Bénabou et al. (2020) model narratives as messages that change individual's beliefs about the externality of their action, see also Foerster and van der Weele (2021) for a closely related approach.

Our model combines the economic literature on narratives as likelihood functions with the one on cheap-talk games. Analogously to the seminal work of Crawford and Sobel (1982), the biased sender strategically partitions unknown information into intervals, relaxing the incentive constraints and thereby rendering informative communication possible. While they do this for the infinite state space itself, we do so for the discrete model space. In Colo (2024) the receiver faces model uncertainty over the prior distribution of the state of the world, parametrized by a compact interval. Equilibria are interval partitions of the parameter set. The author finds that the sender is always better off in the most informative equilibrium if the receiver faces maxmin expected utility. Relaxing the receiver's inference on cheap-talk information, Eliaz et al. (2021) design reports as messages with attached dictionaries. In equilibrium, the receiver's strategic interpretations, based on the partial information given, make the sender better off.

The importance of uncertainty when it come to narratives has thus far been acknowledged by researchers on climate change as well as psychologists, (Pedde et al., 2019; Constantino and Weber, 2021; Garcia-Lorenzo, 2010). We make ambiguity an integral part of our economic model. The uncertain belief about the set of possible explanatory models can be described by a generalization of probabilities, namely capacities. In contrast to a Bayesian world, there are many ways of updating a capacity upon the arrival of new information, the most prominent ones being the optimistic rule, Gilboa and Schmeidler (1993), the pessimistic rule (Dempster, 1968; Shafer, 1976), and the full-Bayesian rule, Fagin and Halpern (1991). Since an update may yet comprise remaining uncertainty, agents still need a decision rule to determine their optimal action. Most prominently, Gilboa and Schmeidler (1989) consider and axiomatize extremely ambiguity averse agents that always expect to face the worst possible outcome, no matter their choice. As a result, they try to maximize their minimal payoff among the uncertain parameter set. In another approach, Klibanoff et al. (2005) consider agents who have a (second-order) belief over the faced probabilistic scenario. Effectively applying the concept of expected utility twice, the second utility function over the first-order expectations characterized an individual's ambiguity attitude. A testable axiomatization of this approach has recently been given by Denti and Pomatto (2022).

The remainder of the article is organized as follows. Section 2 introduces the formal set-up of the game theoretic model, defines ambiguity rules, the equilibrium concept, and provides the running example. Section 3 characterizes equilibria as partitions of intervals of the model space up to a maximum number of partition elements. We classify the loss of persuasive power of the sender by comparing the equilibrium model to the one with a naïve receiver in Section 4. Within the running example, we calculate the informativeness bounds in Section 5. Section 6 concludes.

2 Model and notation

Two agents, a *sender* (S or she) and a *receiver* (R or he), both observe a history of past outcomes (or a public signal) $h \in H$ about the state (of the nature) $\theta \in \Theta = [0, 1]$. We assume that H is finite. The common prior over the state is a distribution F_0 on Θ with continuous and strictly positive density f_0. A *model* or *narrative* $m \in M$ is a likelihood function $\{\pi_m(\cdot|\theta)\}_{\theta \in \Theta}$, where $\pi_m(h|\theta)$ denotes the likelihood of history h given state θ under model m; M is a finite set. We assume that every history $h \in H$ has positive probability given the prior.

Nature first draws the state θ according to F_0 and the *true model* m^T according to an Ellsberg urn from M (cf. Muraviev et al. (2017)). Nature then generates the history h according to the true model m^T and the state θ. While the sender learns the true model m^T and updates her prior to F_{h,m^T} using Bayes' rule, the receiver faces model uncertainty in the sense that his initial belief about the true model $m^T \in M$ is a capacity μ_0 such that $\mu_0(\tilde{M}) = 1$ if $\tilde{M} = M$ and $\mu_0(\tilde{M}) = 0$ otherwise, i.e., he initially is willing to 'entertain' any model. Note that the corresponding mass function η_0 is such that $\eta_0(\tilde{M}) = 1$ if $\tilde{M} = M$ and $\eta_0(\tilde{M}) = 0$ otherwise, and satisfies $\mu_0(\tilde{M}) = \sum_{M' \subseteq \tilde{M}} \eta_0(M')$ for all $\tilde{M} \subseteq M$ (cf. Dempster, 1968; Shafer, 1976).

After the sender has observed the history h and the true model m^T, she submits a cheap-talk report $r \in R$, where R is a rich message space.[1] Upon observing the report, the receiver updates his belief to $\mu_1 = \mu_1^r$ using the *Dempster-Shafer updating rule* (Dempster, 1968; Shafer, 1976). That is, if the indicative meaning of message r is "$m^T \in \tilde{M}$" for some $\emptyset \neq \tilde{M} \subseteq M$, we have

$$\mu_1^r(M') = \mu_0(M' \mid \tilde{M}) = \frac{\mu_0(M' \cup \tilde{M}^c) - \mu_0(\tilde{M}^c)}{1 - \mu_0(\tilde{M}^c)} = \begin{cases} 1, & \text{if } M' \supseteq \tilde{M} \\ 0, & \text{else} \end{cases} . \tag{1}$$

Note that the corresponding posterior mass function η_1 is such that $\eta_1(M') = 1$ if $M' = \tilde{M}$ and $\eta_1(M') = 0$ else.

Definition 1 (Minimal feasible set)**.** The set $\tilde{M} = \tilde{M}(\mu_1)$ is called the *minimal feasible set* under μ_1 if the corresponding posterior mass function η_1 satisfies $\eta_1(\tilde{M}) = 1$.

The minimal feasible set is the smallest collection of models the receiver can and does not want to exclude from his considerations.

Remark 1. (i) Note that the minimal feasible set $\tilde{M}$ under μ_1 satisfies $\mu_1(M') = 1$ if and only if $M' \supseteq \tilde{M}$. In particular, if $\mu_1 = \mu_1^r$, then the indicative meaning of message r is "$m^T \in \tilde{M}$". As will become clear later, $\tilde{M}$ is the pre-image of message r under the sender's strategy.

(ii) Note that the *full Bayesian updating rule*, i.e.,

$$\mu_1(M' \mid \tilde{M}) = \frac{\mu_0(M' \cap \tilde{M})}{\mu_0(M' \cap \tilde{M}) + 1 - \mu_0(M' \cup \tilde{M}^c)},$$

[1] A message space in our setting is rich, if is has at least $\#M$ elements. Hence, communication is not artificially restricted and allows for a perfect discrimination of states.

is equivalent to the Dempster-Shafer updating rule in this case under the convention $0/0 = 1$.

Finally, the receiver takes an action $a \in A$.

2.1 Payoffs

The sender's payoff function $u_S(a, \theta, b)$ depends on the receiver's action $a \in A = \mathbb{R}$, the state θ, and the sender's bias $b > 0$, a commonly known constant that measures the conflict of interest between sender and receiver. We assume that $u_S(a, \theta, b)$ is twice continuously differentiable and strictly concave in a, with a unique maximum for fixed θ and b. Furthermore, we impose the single-crossing condition

$$\frac{\partial^2 u_S(a, \theta, b)}{\partial a \partial b} > 0. \tag{SC}$$

SC implies that the sender would like to induce a higher action with a higher bias. The receiver's payoff function is $u_R(a, \theta) \equiv u_S(a, \theta, 0)$.

The expected utility function $a \mapsto U_{m,h}(a, b) = \mathbb{E}[u_s(a, \theta, b) \mid m, h]$ after an update on the information revealed through the datum (m, h) is also strictly concave in a and its unique maximizer, denoted by $a(m, h, b)$, is strictly increasing in b. For any fixed h, the set of models can be totally pre-ordered by the bliss points of the receiver, i.e., we write $m' \leq m$ if and only if $a(m', h, 0) \leq a(m, h, 0)$. We assume *consistent ordering* across biases, meaning that $a(m', h, 0) < a(m, h, 0)$ implies $a(m', h, b) < a(m, h, b)$ for all $b > 0$.

Moreover, we assume that the expected utility functions $U_{m,h}(a, b)$ respect the *strict single-crossing differences (SSCD)* condition, cf. Kartik et al. (2024): For all m, m' the difference function

$$D \colon A \to \mathbb{R}, a \mapsto \operatorname{sgn}\left[U_{m,h}(a, b) - U_{m',h}(a, b)\right] \tag{SSCD}$$

is monotonic and $\#D^{-1}(0) \leq 1$ or $D^{-1}(0) = A$, were sgn denotes the sign function. SSCD ensures that two expected utility functions $U_{m',h}(a, b), U_{m,h}(a, b)$ either cross each other at most once or are identical, letting us identify m and m'.

An useful equivalent condition for SSCD is *interval choice*, common in many cheap talk, observational learning, and collective choice settings, cf. Kartik et al.

(2024), Kartik and Kleiner (2024): The set

$$I_a := \left\{ m \in M \mid a \in \arg\max_{a' \in A'} U_{m,h}(a', b) \right\} \tag{2}$$

is an interval for all $a \in A, h \in H, A' \subseteq A$, meaning that if $m_1, m_3 \in I_a$ and $m_1 < m_2 < m_3$ then also $m_2 \in I_a$.

Dating back to at least Crawford and Sobel (1982), the most pervasive utility function in communication games is the *quadratic loss functional*. It fulfills all the required properties, irrespective of any assumptions on M and its associated likelihood functions.

Example 1. Let $u_S(a, \theta, b) = -(\theta + b - a)^2$, which is a strictly concave function in a with bliss point $\theta + b$. Under any probability measure, the bliss point of $\mathbb{E}[u_S(a, \theta, b)]$ is $\mathbb{E}[\theta] + b$. Thus, bliss points ordered by their expected state across biases and this consistently ordered. Noting that $\mathbb{E}[u_R(a, \theta)] = -(\mathbb{E}[\theta^2] - \mathbb{E}[\theta]^2) - (\mathbb{E}[\theta] - a)^2$, (SSCD) follows from the fact that two parabolas with the same degree 2 coefficient are either equal or intersect at most once.

2.2 Ambiguity rules

Unlike in the Bayesian framework, there is no unique way how to evaluate the receiver's utility under posterior belief μ_1 with corresponding minimal feasible set $\tilde{M}$. To resolve the uncertainty, the receiver may adopt any of the utility functions in the following.

Definition 2 (Ambiguity rules)**.** An *ambiguity rule* (for the receiver) is a function $U \colon 2^M \times H \times A \to \mathbb{R}, (\tilde{M}, h, a) \mapsto U_{\tilde{M},h}(a)$, assigning a utility to each action a when facing the minimal feasible set $\tilde{M}$ under history h with the following properties:

(i) $U_{\{m\},h}(a) = \mathbb{E}[u_R(a, \theta) \mid m, h]$,

(ii) $U_{\tilde{M},h}(a)$ is strictly concave in a with unique maximizer $a(\tilde{M}, h)$ on A and the maximizers fulfill $a(M_1 \cup M_2, h) \in \operatorname{conv}(a(M_1, h), a(M_2, h))$.

A decision maker takes an ambiguity rule as a measure for the anticipated utility. We require that such a rule is based on Bayesian expected utilities (i) and retains a bliss point that hedges against increasing ambiguity by diversification (ii).

Example 2. The following functions are ambiguity rules for a belief μ_1 with minimal feasible set $\tilde{M}$ given a history h.

(i) Maximum likelihood expected utility. Let $\succ_{M,h}$ be any strict ordering on M that respects the expected fit, i.e., $m \succ_{M,h} m'$ implies $\Pr(h \mid m, F_0) \geq \Pr(h \mid m', F_0)$. The *maximum-likelihood expected utility (MLEU) (w.r.t. $\succ_{M,h}$)* is given by the expected utility

$$U_{\tilde{M},h}(a) = \mathbb{E}[u_R(a,\theta) \mid \tilde{m}, h] = \int_0^1 u_R(a,\theta) dF_{h,\tilde{m}}(\theta),$$

where $\tilde{m}$ is the largest element in $\tilde{M}$ w.r.t. $\succ_{M,h}$. Under an MLEU preference, the receiver maximizes his expected utility with respect to a narrative from the minimal feasible set $\tilde{M}$ that is most likely to explain the observed data h. The strict ordering $\succ_{M,h}$ serves as a consistent tiebreaker.[2]

(ii) Max-min expected utility. The *max-min expected utility (MEU)* of Gilboa and Schmeidler (1989) is given by

$$U_{\tilde{M},h}(a) = \min_{m \in \tilde{M}} \mathbb{E}[u_R(a,\theta) \mid m, h] = \min_{m \in \tilde{M}} \int_0^1 u_R(a,\theta) dF_{h,m}(\theta).$$

Under MEU preferences, the receiver seeks to maximize his worst-case expected utility given $\tilde{M}$.

(iii) Bayesian utility. The *Bayesian utility* weighs the expected utilities of all feasible models by a prior probability distribution υ as

$$U_{\tilde{M},h}(a) = \sum_{m \in \tilde{M}} \upsilon(m \mid \tilde{M}) \cdot \mathbb{E}[u_R(a,\theta) \mid m, h].$$

The conditional expectation computes by Bayes' rule as $\upsilon(m \mid \tilde{M}) = \frac{\upsilon(m)}{\sum_{\tilde{m} \in \tilde{M}} \upsilon(\tilde{m})}$.

(iv) Smooth model. The *smooth model* of decision making under ambiguity of Klibanoff et al. (2005) with an ambiguity index ϕ that is concave. The utility function is given by

$$U_{\tilde{M},h}(a) = \sum_{m \in \tilde{M}} \upsilon(m \mid \tilde{M}) \cdot \phi\left(\mathbb{E}[u_R(a,\theta) \mid m, h]\right),$$

[2]Using a tiebreaker is important for two reasons. First, without consistent tiebreaking, the hedging property (ii) of an ambiguity rule might fail. Second, and in contrast to the literature, we cannot resolve ties by applying a "sender-preferred" action as the receiver faces ambiguity about the true model and thus about the sender's expected utility. Note also that a choice function $C_h \colon 2^M \to M$, $C_h(\tilde{M}) \in \arg\min_{m \in \tilde{M}} \Pr(h \mid m, h)$ with the property that if $M_1 \subseteq M_2$ and $C_h(M_2) \in M_1$, then $C_h(M_1) = C_h(M_2)$ would serve the same purpose as the strict ordering.

where $\phi\colon \mathbb{R} \to \mathbb{R}$ is a function capturing the ambiguity attitude and υ is a probability measure on M, where we denote the conditional update on $\tilde{M}$ by $\upsilon(m \mid \tilde{M})$.[3]

2.3 Equilibrium concept

We now address equilibrium behavior. A strategy for the sender is a function $\sigma\colon H \times M \to R$ that assigns a report to each history and model. A strategy for the receiver is a function $\rho\colon H \times R \to A$ that assigns an action to each history and report. Note that the history h is known to both agents ex ante, i.e., before they make their move. Consequently, one can treat every h as indexing a different game. In the following, we thus omit the dependence of the agents' strategies and consider $\sigma\colon M \to R$ and $\rho\colon R \to A$ for a given history h. Likewise, we frequently drop the dependence on h from the bliss points and simply write $a(m, b), a(\tilde{M})$ instead of $a(m, h), a(\tilde{M}, h)$ whenever h is clear from the context.

We restrict attention to pure strategies and assume that the receiver takes an on-equilibrium-path action if she observes an off-equilibrium message, which implies that we can ignore such deviations.[4]

An *equilibrium* (σ, ρ) then needs to entail mutual best replies:

(i) $\sigma(m^T) \in \arg\max_r U_{m^T,h}(\rho(r), b)$ for all $m^T \in M$,

(ii) $\rho(r) \in \arg\max_a U_{\sigma^{-1}(r),h}(a)$ for all $r \in R^\sigma = \sigma(M)$, where we note that $\tilde{M}(\mu_1^r) = \sigma^{-1}(r)$.

A sender who observes the true model m^T sends a message r inducing the action most favorable for her out of $\rho(R)$. Being aware of the sender's communication strategy σ and observing a message r, the receiver first updates the considered set of models to the minimal feasible set $\tilde{M}(\mu_1^r) = \sigma^{-1}(r)$; the indicative meaning of message r is thus "$m^T \in \tilde{M}(\mu_1^r)$". Then, he chooses the action maximizing his utility w.r.t. his ambiguity rule.

An immediate consequence of the cheap-talk setting is the existence trivial, of so-called *babbling equilibria* in which no information is transmitted: For instance, let the sender's strategy be constant, i.e., $\sigma(m) = r_0$ for all m. Then the minimal feasible set upon receiving r_0 is the set M of all models. The receiver thus responds optimally by playing the *pooling action* $\rho(r) = a(M, h)$ for any received action.

[3]Using a linear ambiguity index ϕ, the smooth model becomes the Bayesian utility.

[4]Note that this restriction is without loss of generality for the receiver. At least in case of MLEU, this is also true for the sender.

Given that the receiver only plays a single action, the sender can not change his behavior and thus improving her utility by transmitting a message different than r_0. Consequently, (σ, ρ) is an equilibrium inducing solely the pooling action $a(M, h)$.

2.4 Running example

We conclude this section by introducing our running example, based on a uniformly distributed state space and quadratic loss preferences, together with the history being generated by a modified beta-binomial model.

Consider a politician (sender) who runs for office and wants to convince a representative voter (receiver) of her aptitude for political business. Her ability is summarized by a parameter $\theta \in [0, 1]$ which is thought of capturing her skill in managing economic indicators, such as unemployment rate or inflation, exigencies of a pandemic, or winning a political debate. Without further information, the ability is believed to be drawn from a uniform distribution. A record of past values of the economic indicators, R numbers, death tolls or debate outcomes linked to political activities is publicly available. The data is simplified and summarized by a vector $h \in \{0, 1\}^K$, indicating public outcomes that are categorized as positive (success) or negative (failure). The unbiased representative voter is interested in learning the true ability of the politician in question, but faces uncertainty about which pieces of the data are relevant for this assessment. A high unemployment rate might or might not be due to the general state of the world economy, the politician's past choices or the ruling government's recent passage of a law. In our example, we consider a simple case in which a model is identified with a set of relevant data points, i.e., $M = 2^{\{1,\dots,K\}}$. The biased politician on the other hand knows which of the observed data truly connected to her own ability. Since she likes to raise the receivers opinion on her aptitude as a good politician, she will interpret the data in her favor while at the same time trying to stay credible. She does so by providing a set of *narratives* to the receiver that rationalizes the observed data while also presenting her in a good light. In our framework, the voter is aware that the politician is claiming positive indicators for herself and blaming others for bad economic states. Consequently, the voter carefully trades off the plausibility of the arguments put forth by the politician and her strategic incentives by correcting for the bias. We are interested in what way and how much the biased sender can still provide credible narratives when facing equilibrium play.

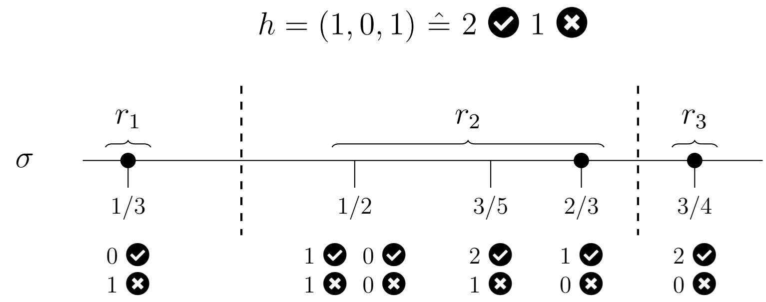

Figure 1: Depiction of an MLEU equilibrium in the setting of Example 3 where $h = (1, 0, 1)$ entails two successes and one failure, $b = \frac{1}{30}$ and $M = 2^{\{1,2,3\}}$. Messages span consecutive bliss points, each of which corresponding to a narrative claiming the relevance of certain subsets of the observations in h. Given h, subsets are more conveniently identified with a number of successes (✓) and failures (✗). If given the choice between a narrative indicating one success and no failure and the one indicating no success and no failure, the receiver breaks the tie in likelihood (both $\frac{1}{8}$) by adapting the first one. The optimal receiver response is indicated by the solid dots.

The following Example models the situation by providing a mathematical model that is anchored in our analytical framework.

Example 3 (Uniform-random-binomial model with quadratic loss)**.** Consider a politician whose negotiation power is uniformly distributed $F_0 = \mathcal{U}(0, 1)$ and $h = (h_1, h_2, \ldots, h_K) \in \{0, 1\}^K$ denotes a vector of successes and failures over the course of the politician's time in office. Let $m^T \subseteq \{1, \ldots, K\}$ indicate the instances of past decisions in which the politician's ability contributed to the outcome. At all other times, the result is independent of her skill, so that $h_1, \ldots, h_K$ are independent conditional on θ, with

$$Pr(h_k = 1 \mid \theta) = \begin{cases} \theta, & \text{if } k \in m^T \\ \frac{1}{2}, & \text{else} \end{cases} \quad \text{for all } k = 1, 2, \ldots, K. \tag{3}$$

A narrative specifies a set of decision processes, and thus a number of successes and failures, attributed to the politicians ability, i.e., $M = 2^{\{1,2,\ldots,K\}}$.
Note that the total number of successes, $h^{\Sigma} \equiv \sum_{k=1}^{K} h_k$, is a sufficient statistic for history h. Let further the sender's payoff function be $u_S(a, \theta, b) = -(\theta + b - a)^2$. Figure 1 depicts an equilibrium under the MLEU ambiguity rule in this setting.

3 General equilibrium analysis

In this section, we derive analytic results about equilibria in the general framework. In a first step, we state necessary conditions for establishing equilibrium. In equilibrium, messages span intervals of models, i.e., messages correspond to sets of narratives the respective bliss points of which are consecutive elements w.r.t. the set of all bliss points. This helps narrowing down the complexity for finding equilibria. Second, we prove that a property well-known in classic cheap talk games surprisingly carries over to a finite setting: If an equilibrium induces $n+1$ different actions, there also is an equilibrium with n different induced actions.

3.1 Necessary conditions

Our first straightforward observation establishes an upper bound for the number of actions induced in equilibrium. As the sender can at most send $\#M$ different messages, there cannot be more induced actions than that.

Lemma 1. *In any equilibrium, the number of distinct actions induced is finite and at most equal to $\#M$.*

Proof. All proofs are relegated to the appendix. □

The next lemma describes structural properties of the equilibria that also help simplify the search for equilibria.

Lemma 2 (Reduction Lemma)**.** *Let (σ, ρ) be an equilibrium.*

(i) There is an equilibrium (σ^*, ρ^*) with $\rho \circ \sigma \equiv \rho^* \circ \sigma^*$ and the property $\rho^*(\sigma^*(m)) = \rho^*(\sigma^*(m')) \Rightarrow \sigma^*(m) = \sigma^*(m')$.

(ii) We have $a(m', b) \leq a(m, b)$ if and only if $\rho(\sigma(m')) \leq \rho(\sigma(m))$.

Statement (i) lets us restrict attention to equilibria in which messages and induced actions are in a one-to-one correspondence: If an equilibrium has two different messages that induce the same action, one does not loose the equilibrium property by sending the same message for all corresponding narratives. Statement (ii) says that bliss points and induced actions of narratives must be ordered in the same way in equilibrium. Consequently, a message partitions narratives into convex sets w.r.t. the ordering of the bliss points.

3.2 Number of induced actions

In the following, we study the existence of equilibria with a certain number of induced actions.

Definition 3 (n-step equilibrium)**.** An equilibrium (σ, ρ) is called *n-step equilibrium* if it induces n distinct actions, i.e., $\#(\rho \circ \sigma)(M) = n$.

A classical result from the theory of cheap talk games states that if there is an equilibrium which induces N distinct actions, then there is an equilibrium inducing n distinct actions for all $1 \leq n \leq N$, cf. Crawford and Sobel (1982). Surprisingly, this result also holds true in our setting.

Theorem 1. *There is a natural number $N = N(h, b)$ such that there exists an n-step equilibrium for all $1 \leq n \leq N$, but none for any $n > N$.*

Together with the Reduction Lemma (Lemma 2), Theorem 1 characterizes the set of equilibria: An equilibrium is a partition of the set M into up to N intervals together with their actions induced by the considered ambiguity rule.

The proof of the theorem gives at hand an explicit algorithm to arrive at an n-step equilibrium given an $(n+1)$-step equilibrium (σ, ρ) that we explain in the following. Note first that by the Reduction Lemma (Lemma 2), the set of narratives inducing an action is an interval. Second, we can index these intervals M_i such that narratives from an interval with a higher index induce a higher action, i.e., $\rho(\sigma(m)) < \rho(\sigma(m'))$ whenever $m \in M_i$ and $m' \in M_{i+1}$. Let $m^l_{M_i}$ denote the narrative $m \in M_i$ corresponding to the lowest induced receiver action $a(m, h, 0)$.

Algorithm. Let (σ, ρ) be an $(n+1)$-step equilibrium. By the Reduction Lemma, we can assume that σ is given by a partition $M = M_1 \cup \cdots \cup M_n$ of disjoint intervals fulfilling $m' \leq m$ whenever $m' \in M_i, m \in M_j$, $i < j$.

1. Define the sender strategy

$$\sigma_a(m) = \begin{cases} r_n & , \text{ if } m \in M'_n \\ r_i & , \text{ if } m \in M'_i, i \neq n, n+1 \end{cases},$$

   where $M'_i = M_i$ for $i = 1, 2, \ldots, n-1$ and $M'_n = M_n \cup M_{n+1}$.

2. If σ_a together with the respective best reply ρ_a of the receiver is an n-step equilibrium, we are done. Otherwise go to Step 3.

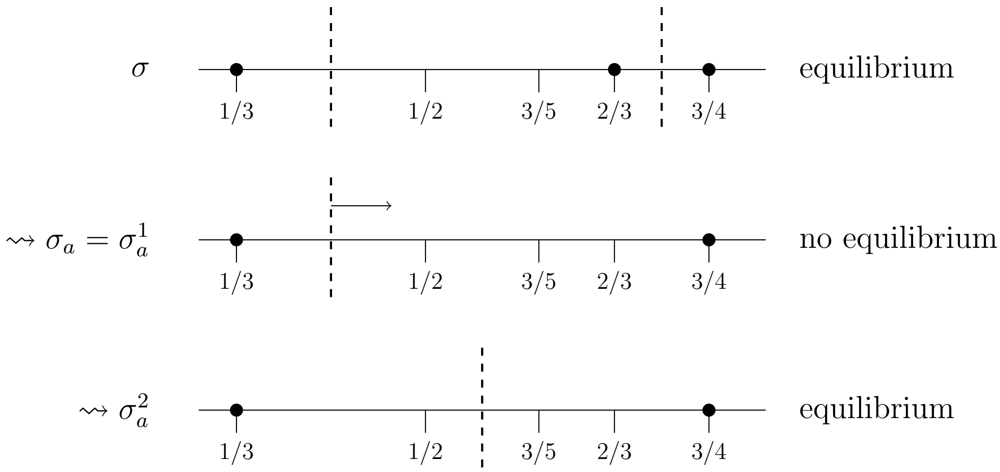

Figure 2: Sketch of the algorithm driving the proof of Theorem 1. Consider $h = (1, 0, 1), b = \frac{1}{30}, M = 2^{\{1,2,3\}}$ and the MLEU ambiguity rule with any tiebreaker that favors the narrative with no success and one failure over the one with no success and no failure. The dots mark the respective best responses of the receiver. Starting from the 3-step equilibrium σ, merge the right two words to obtain σ_a. Profitable deviations must involve shifting the boundaries to the right. By doing so, one will eventually reach equilibrium.

3. Find $i \in \{2, \dots, n\}$ such that the sender has a profitable deviation for model $m = m^l_{M'_i}$ from σ_a. Let

   - σ'_a be such that $\sigma'_a(m) = r_{i-1}$ if $m = m^l_{M'_i}$ and $\sigma'_a(m) = \sigma_a(m)$ else,
   - $M''_{i-1} = M'_{i-1} \cup \{m^l_{M'_i}\}$, $M''_i = M'_i \setminus \{m^l_{M'_i}\}$, and $M''_j = M'_j$ for $\neq i-1, i$.

   Set $\sigma_a = \sigma'_a$ and $M'_i = M''_i$ and go back to Step 2. □

In the first step of the algorithm, we merge the rightmost messages to obtain σ_a which use n messages. If σ_a together with its best reply ρ_a does not form an equilibrium, we iteratively modify σ_a: There is a profitable deviation for the sender and the proof of the theorem ensures that there is an interval M'_{i-1} that the sender can profitably enlarge by adding the leftmost model $m^l_{M'_i}$. Choosing any such deviation, check whether the deviation together with the receiver's best reply constitutes an equilibrium. If not, continue as before. If this procedure continues to produce no equilibrium, the proof ensures that σ'_a retains n distinct intervals and will eventually be equal to σ_b – the sender strategy resulting from merging the leftmost messages of σ. If the algorithm reaches that point, no further profitable deviation is possible. The procedure thus terminates at an σ'_a that forms equilibrium together with ρ'_a. The key idea is summarized in Figure 2.

3.3 Informativeness

As is common in cheap talk games, there is a plethora of different equilibria. In this section, we introduce a partial order on sender strategies and thus also equilibria, based on the (Blackwell-)informativeness of the sender's strategy, cf. Blackwell (1953). Informativeness measures how finely the sender discriminates between different models.

Definition 4 (Informativeness)**.** (i) Strategy σ is weakly more informative than σ' under history h if $\sigma(m) = \sigma(m')$ implies $\sigma'(m) = \sigma'(m')$ for all $m, m' \in M$.

(ii) Strategy σ is *fully informative (uninformative)* under history h if σ (σ') is weakly more informative than σ' (σ) under h for any strategy σ'.

(iii) Equilibrium (σ, ρ) is *most informative* under history h if for any equilibrium (σ', ρ') such that σ' is weakly more informative than σ under h, it holds that σ is weakly more informative than σ' under h.

Some remarks seem in order. First, Definition 4 defines a partial order on strategy profiles. Second, the receiver can always infer the true model m^T if σ is fully informative. Third, a fully informative equilibrium is most informative. Fourth, there might be several most informative equilibria which are not outcome equivalent, i.e., equilibria (σ, ρ) and (σ', ρ') such that $\rho(\sigma(m)) \neq \rho'(\sigma'(m))$ for some $m \in M$. The last point is illustrated in Figure 3 by two 4-step equilibria.

By the Reduction Lemma, a 2-step equilibrium consists of a partition of the set M in two intervals. Recall that for a subset $\tilde{M} \subseteq M$ the unique optimal receiver action given his ambiguity rule is denoted by $a(\tilde{M}, h)$. Define now

$$V(m^T, h, b) := U_{m^T,h}(a(\{m \mid m > m^T\}, h), b) - U_{m^T,h}(a(\{m \mid m \leq m^T\}, h), b), \tag{4}$$

we see that there is no 2-step equilibrium if $V(m^T, h, b) > 0$ for all m^T. As a consequence of Theorem 1, the only equilibrium is thus babbling.

Definition 5 (Large conflicts of interest)**.** We say that $U_{m,h}(a, b)$ *allows for large conflicts of interest*, if for any h there exists a $\hat{b}(h) \geq 0$ such that $V(m^T, h, b) > 0$ for all $b > \hat{b}(h)$ and all $m^T \in M$.

Under large conflicts of interest, the sender can not induce two different actions without having an incentive to deviate: There always will be a true model under which she likes to make the receiver pick a higher action than is designed. To illustrate this point, imagine that the bias, measuring the conflict of interest, is

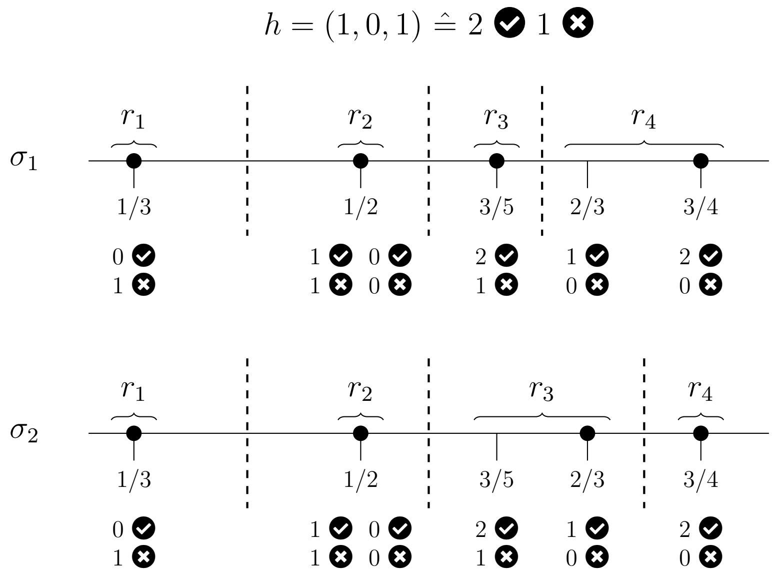

Figure 3: Depiction of two most informative MLEU equilibria in the setting of Example 3 with $h = (1, 0, 1)$, $b = \frac{1}{25}$ and $M = 2^{\{1,2,3\}}$ that induce a different set of actions.

high enough to want her make the receiver always pick the highest possible action. Then there cannot be a 2-step equilibrium. Note that under a quadratic loss the bliss points of the sender are translations of $a(m^T, h, 0)$ by exactly b, thus allowing for large conflicts of interest.

Analogously to standard cheap-talk models in which the sender is imperfectly informed (e.g., Argenziano et al. (2016); Foerster (2023)), we obtain:

Proposition 1. *For any history h, there exists*

(i) $\underline{b}(h) > 0$ *such that a most informative equilibrium under history h involves a fully informative sender strategy if and only if $b \leq \underline{b}(h)$.*

(ii) $\overline{b}(h) \geq \underline{b}(h)$ *such that any most informative equilibrium under history h is uninformative if and only if $b > \overline{b}(h)$ under $U_{m,h}(a, b)$ that allows for large conflicts of interest.*

To illustrate this result, we consider the uniform-random-binomial model with quadratic loss introduced introduced in Example 3. In particular, it may indeed be the case that the two thresholds coincide under the MLEU model (Example 2 (i)).

Example 4. Consider the setting of Example 3 where $F_0 = \mathcal{U}(0,1)$, $u_S(a,\theta,b) = -(\theta+b-a)^2$, h is generated according to (3) and let the receiver behave according to an MLEU model.

(i) If $h^\Sigma = 2$, e.g., $h = (1,0,1)$, then $\underline{b}(h) = \frac{1}{30} < \frac{5}{24} = \overline{b}(h)$.

(ii) If $h^\Sigma = 0$, i.e., $h = (0,0,0)$, then $\underline{b}(h) = \overline{b}(h) = \frac{1}{40}$ if the receiver breaks the tie between one failure and no information in favor of the first one.[5]

Since models that induce a smaller posterior belief are associated with a higher likelihood in the case of $h = (0,0,0)$ and $b > 0$, less than fully informative communication does not relax the incentive-compatibility constraints. As is the case with a fully informative strategy, the sender has incentives to deviate from the partially informative strategy associated with the weakest incentive-compatibility constraints if $b > \frac{1}{40}$, see Figure 4 for an illustration.

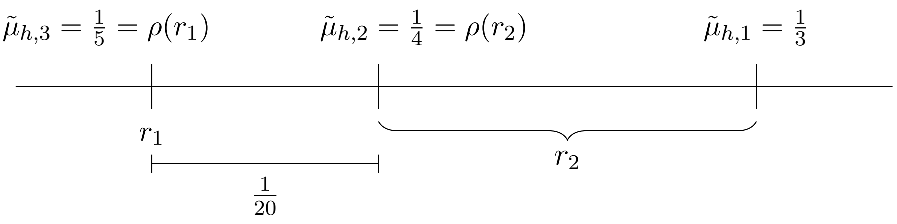

Figure 4: Partially informative strategy σ characterized by $\sigma(m^T) = r_1$ if $m^T \in \{m | \#m = 3\}$ and $\sigma(m^T) = r_2$ else and the corresponding best reply ρ for $h^\Sigma = 0$, where $\tilde{\mu}_{h,\#m} = E[\theta | h, \#m] = \frac{1}{2+\#m}$.

The following proposition establishes that the sender always prefers to play a more informative equilibrium – and thus a most informative one – if facing a receiver who applies an MLEU ambiguity rule.

Proposition 2. *Let (σ, ρ) and (σ', ρ') be equilibria under an MLEU ambiguity rule and σ weakly more informative than σ'. Then, the sender prefers the outcome of (σ, ρ) over (σ', ρ') for any m^T.*

A similar statement for the MEU rule is not true as the example of MEU equilibria in Figure 5 shows. The counterexample marks a stark contrast to Theorem 2 in Colo (2024), where the corresponding statement holds. The reason for this is that in our model the utilities are not linear in the uncertainty parameter.

[5]If, instead of $M = 2^{\{1,2,3\}}$, we consider $M = 2^{\{1,2,3\}} \setminus \{\emptyset\}$, the results of the Example hold true and are independent of tie-breaking rules.

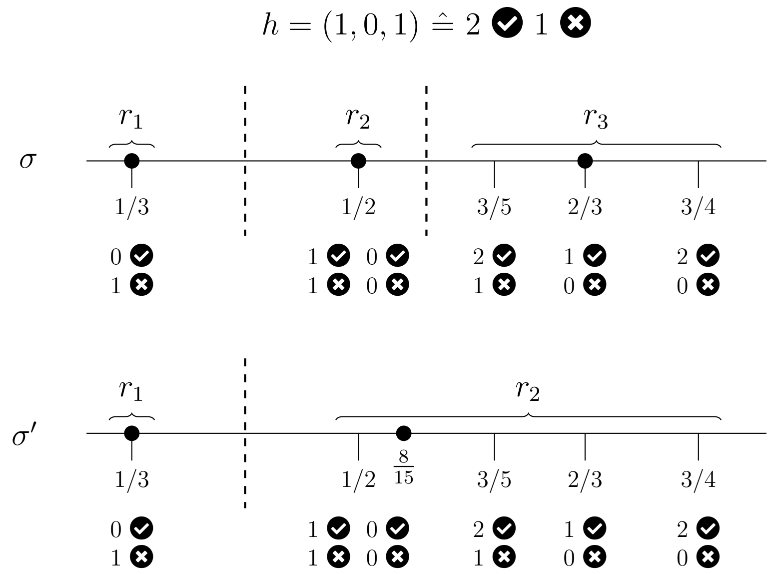

Figure 5: MEU equilibria in the framework of our running Example 3 for $h = (1, 0, 1)$ and $b = 0.04$. While σ is more informative than σ', the sender prefers the equilibrium action $\rho'(\sigma'(m^T)) = \frac{8}{15} \approx 0.53$ under σ' over $\rho(\sigma(m^T)) = \frac{1}{2}$ under σ for $a(m^T, h, 0) = \frac{1}{2}$ since her bliss point is $a(m^T, h, b) = 0.54$.

4 Comparison with a naïve receiver

In this section, we compare the equilibria of our model (in the setting of the example) with Schwartzstein and Sunderam (2021), where the receiver is naïve in the sense that he takes the sender's message m at face value and adopts the proposed model m if it has a higher likelihood than a default model m^d given the observed history h, i.e., $Pr(h|m, F_0) \geq Pr(h|m^d, F_0)$. Otherwise he sticks with the default model. The receiver thus does not take the sender's strategic incentives into account. We henceforth assume that $m^d = m^T$ and refer to the model of Schwartzstein and Sunderam (2021) as the *naïve model*. This choice is reasonable for at least two reasons. First, as Schwartzstein and Sunderam (2021) point out, it allows to ask when the wrong story wins. Second, it can be seen as a 'best case' scenario, under which the truthful communication benchmark is outcome equivalent to our model.[6]

We measure the power of the sender by the set of models m^T for which she prefers to induce an action $a(m^*, h, 0)$ to $a(m^T, h, 0)$ for another narrative m^* and succeeds to do so. These sets are defined below, implicitly fixing a set of narratives M with corresponding likelihood functions, a bias $b > 0$, a history h and an MLEU

[6]Note that the only other choice under which this is the case is the 'worst case' $m^d \in \arg\min_{m \in M} Pr(h|m, F_0)$.

equilibrium (σ, ρ) in the following.

$$\begin{aligned} M_{\text{naïve}} = \big\{ m^T \in M \mid \exists\, m^*\colon\ & \Pr(h \mid m^*, F_0) \geq \Pr(h \mid m^T, F_0) \\ & \text{and } U_{m^T,h}(a(m^*, h, 0), b) > U_{m^T,h}(a(m^T, h, 0), b) \big\}, \\ M_{(\sigma,\rho)} = \big\{ m^T \in M \mid\ & U_{m^T,h}(\rho(\sigma(m^T)), b) > U_{m^T,h}(a(m^T, h, 0), b) \big\}. \end{aligned}$$

$M_{\text{naïve}}$ contains all models m^T under which the sender wants and successfully can convince a naïve receiver to move away from the action he would choose by default by providing a narrative m^* even if the default model corresponds to the true model m^T. Likewise, $M_{(\sigma,\rho)}$ contains all models m^T under which the sender in equilibrium (σ, ρ) is able to make the receiver implement an action that she prefers over the one the receiver would like to play if he knew m^T. In other words, these sets contain the models under which the sender does better than under truthful communication. The following proposition proves that the sender can more easily convince (is always better off if facing) a naïve receiver.

Proposition 3. *For any MLEU equilibrium (σ, ρ) we have $M_{(\sigma,\rho)} \subseteq M_{naïve}$.*

Proposition 3 ensures that a sender facing a naïve receiver is never worse off than facing a rational receiver who understands equilibrium play. The preference for facing a naïve receiver is expressed by two observations: First, note that we have $U_{m^T,h}(\rho(\sigma(m^T)), b) \leq \max\{U_{m^T,h}(a(m^*, 0), b) \mid \Pr(h \mid m^*, F_0) \geq \Pr(h \mid m^T, h)\}$ for every true model $m^T \in M_{(\sigma,\rho)}$. Thus, if the sender manages to profitably delude the receiver into taking an action favorable to her, the increase in her expected utility is *higher* when facing a naïve receiver. Second, there are situations in which the inclusion can either be an equality or strict. Hence, the sender can induce favorable actions more often when facing a naïve receiver. The following Proposition characterizes generic cases in which the persuasive power of the sender is the same or distinctly different when comparing a naïve receiver to one embracing equilibrium play.

Proposition 4. *For any history h we have*

(i) $M_{(\sigma,\rho)} = M_{naïve} = \emptyset$, *if* $b \leq \underline{b}(h)$ *and*

(ii) $M_{(\sigma,\rho)} \subsetneq M_{naïve}$, *if* $b > \overline{b}(h)$ *under large conflict of interest and there are* m^T, m^* *with* $\Pr(h \mid m^*, F_0) \geq \Pr(h \mid m^T, F_0)$ *and* $a(M, h) \leq a(m^T, h, 0) < a(m^*, h, 0)$.

The intuition of the above result is as follows. On the one hand, if $b \leq \underline{b}(h)$, the best the sender can do is to reveal the true model and make the receiver take action $a(m^T, b, 0)$. On the other hand, if $b > \bar{b}(h)$ and a large conflict of interest, the only equilibrium is babbling, inducing action the pooling action $a(M, h)$ which the receiver would opt for if no communication had taken place. If a true model's best reply $a(m^T, h, 0)$ exceeds the babbling action and the sender can credibly convince a naïve receiver to adapt an even higher action, she strictly improves her expected utility in a case where equilibrium can not.

We conclude this section by providing an instance within our running example in which the sender is strictly better off when facing a naïve receiver - both, in terms of the number of models for which she can persuade the receiver into taking an action more favorable for her, as well as in the size of the gain of her expected utility.

Example 5. In the setting of our running Example 3, let $K = 3$, $h^\Sigma = 2$ and $b \in (\frac{1}{20}, \frac{3}{40}] \subseteq (\underline{b}(h), \bar{b}(h))$. A most informative MLEU equilibrium (σ, ρ) is given by $\sigma(m) = r_1$ if $a(m, 0) = \frac{1}{3}$, $\sigma(m) = r_2$ if $a(m, 0) = \frac{1}{2}$ and $\sigma(m) = r_3$ if $a(m, h, 0) \in \{\frac{3}{5}, \frac{2}{3}, \frac{3}{4}\}$ (inducing posterior and action $\frac{3}{4}$). Facing a naïve receiver, the sender wants to and can use a narrative inducing a higher action if $a(m^T, 0) \in \{\frac{3}{5}, \frac{2}{3}\}$. We thus have $M_{(\sigma,\rho)} = \{m \mid a(m, 0) \in \{\frac{3}{5}, \frac{2}{3}\}\}$, a proper subset of $M_{\text{naïve}} = \{m \mid a(m, 0) \in \{\frac{1}{2}, \frac{3}{5}, \frac{2}{3}\}\}$. Furthermore, if $a(m^T, 0) = \frac{3}{5}$, the sender prefers to induce action $\frac{2}{3}$ over $\frac{3}{4}$. She cannot do so in equilibrium, while this is possible when facing a naïve receiver. Figure 6 illustrates the stronger position the sender has when facing a naïve receiver.

Our results and examples show that the sender prefers to deal with a naïve receiver. On the other end, one might suspect that the receiver is always better off when following equilibrium play. However, this is not true as can be seen from Figure 6: On the one hand, if $a(m^T) = \frac{1}{2}$, the sender reveals the true model in equilibrium, while if he was naïve he would be persuaded to take the action $\frac{3}{5}$. On the other hand, if $a(m^T) = \frac{3}{5}$, equilibrium play enforces the action $\frac{3}{4}$, whereas both would prefer the action $\frac{2}{3}$ which is possible if the receiver was naïve.

Remark 2. Depending on the true model, the receiver is better off following equilibrium play while in others being naïve yields a higher expected payoff. Hence, it is uncertain whether it is better for the receiver to be rational or naïve.

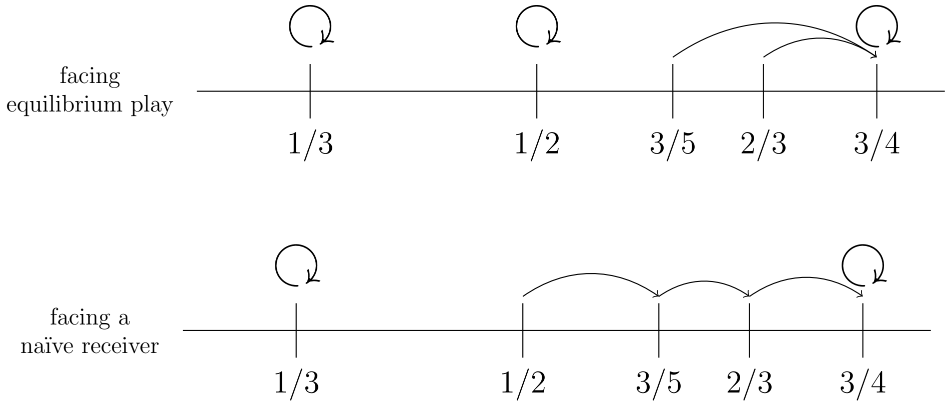

Figure 6: Illustration of the persuasive power of the sender when facing the equilibrium play in Example 5 or a naïve receiver. The arrows indicate what action the sender manages to make the receiver take given the action optimal for the receiver under the true model. The sender can induce a favorable shift in the action taken by the receiver more often than in an equilibrium play. Furthermore, if the true model fulfills $a(m^T, 0) = \frac{3}{5}$, a sender with bias $b \in (\frac{1}{20}, \frac{3}{40}]$ prefers to induce action $\frac{2}{3}$ over $\frac{3}{4}$. Consequently, facing a naïve receiver does not only allow to divert more actions of the receiver, but also to induce ones that are more favorable for the sender.

5 Analysis of the running example

In this section, we provide some more insights in the setting of the running example, characterizing the informativeness bounds under the MLEU ambiguity rule.

Recall the set-up of the running example in which an expert tries to make the receiver have a higher opinion on the ability of a politician. The ability parameter θ is drawn from $F_0 = \mathcal{U}(0,1)$ and the observable data is given by a sequence $h = (h_1, h_2, \ldots, h_K) \in \{0,1\}^K$ of successes and failures. The model space is $M = 2^{\{1,2,\ldots,K\}}$ indicating a subset of observations. The true model m^T captures that subset the data points from h of which are actually relevant for the inference on θ. All other data points are noise. Formally, the data generating process is given by independent draws of h_k according to

$$Pr(h_k = 1 \mid \theta) = \begin{cases} \theta, & \text{if } k \in m^T \\ \frac{1}{2}, & \text{else} \end{cases} \quad \text{for all } k = 1, 2, \ldots, K. \tag{5}$$

Further suppose that the utility function is a quadratic loss $u_S(a, \theta, b) = -(\theta + b - a)^2$ and $u_R(a, \theta) = -(\theta - a)^2$, where $b > 0$ is the sender's bias, a constant that measures the conflict of interest between sender and receiver that is publicly known.

Note that $h^{\Sigma} \equiv \sum_{k=1}^{K} h_k$ is a sufficient statistic for history h, counting the bare number of successes and failures. In our concrete setting, the Reduction Lemma (Lemma 2) reveals that in equilibrium, the action induced (respective the message sent) for by the message of narrative m must be the same for all models m' with $\#m' = \#m$ and $\sum_{i \in m} h_i = \sum_{i \in m'} h_i$. Consequently, they can be identified.

Recall that the expected value given m, h is the bliss point of the receiver. Under the beta-binomial model with uniform prior we find $a(m, h, 0) = \tilde{\mu}_{h,m} \equiv \mathbb{E}[\theta \mid h, m] = \frac{\sum_{i \in m} h_i + 1}{\#m + 2}$ and we can restrict our attention to the set of models given by $\mathcal{M}_1(h^{\Sigma}) = \left\{ \tilde{\mu}_{h',m} \mid \sum_{k=1}^{K} h'_k = h^{\Sigma}, m \in M \right\}$.

5.1 Informativeness bounds under MLEU

The biased sender chooses her strategy in order to induce a posterior belief on the ability of the receiver that favors her own purposes. If interests are sufficiently aligned, she is willing to reveal the true model to her while if her bias is too large, she can not credibly reveal information in a way that favors her. The bounds for the extreme behaviors above are given by the values $\underline{b}(h)$ and $\underline{b}(h)$ defined in Section 3.3. We establish that the bias threshold $\underline{b}(h^{\Sigma})$ is symmetric and that extreme histories are the most difficult to 'explain', in the sense that $\underline{b}(h^{\Sigma})$ attains its minimum.

Proposition 5. *$\underline{b}(h^{\Sigma})$ is symmetric, i.e., $\underline{b}(h^{\Sigma}) = \underline{b}(K - h^{\Sigma})$ for any h^{Σ}, with $\{h^0, h^K\} = \arg\min_{h^{\Sigma}=0,1,\ldots,K} \underline{b}(h^{\Sigma})$.*

Figure 7 illustrates our findings and shows that, somewhat surprisingly, $\underline{b}(h^{\Sigma})$ is not quasi-concave.
We now move on to partially informative equilibria, i.e., equilibria that are neither fully informative nor uninformative. As we will show, we no longer have symmetry in this case due to the receiver's uncertainty.

Proposition 6. *Suppose that $K \geq 3$ and that the receiver behaves according to the MLEU model. Then $\overline{b}(h^{\Sigma})$ is asymmetric. In particular, $\overline{b}(K) = \frac{K-1}{6(K+2)} > \overline{b}(0) = \underline{b}(0) = \frac{1}{2(K+1)(K+2)}$.*

The following example illustrates this finding.

Example 6. Suppose that $K = 3$ and $b \in (\frac{1}{40}, \frac{1}{15}]$.

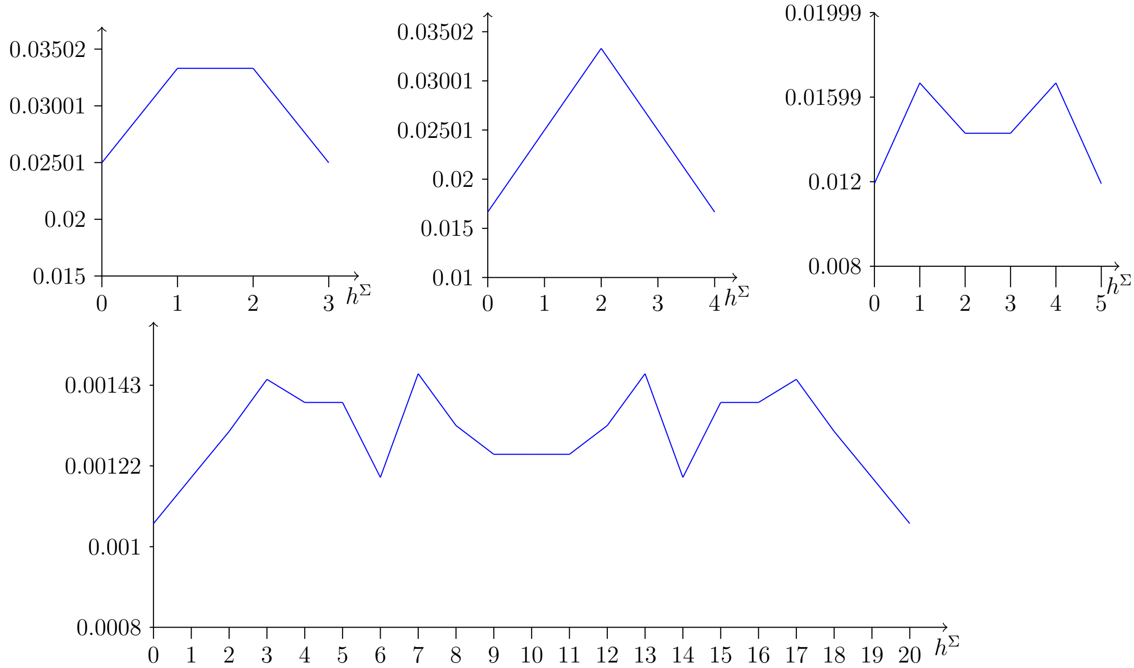

Figure 7: $\underline{b}(h^\Sigma)$ as a function of h^Σ for $K = 3, 4, 5, 20$.

(i) $h^\Sigma = 3$. Then a most informative equilibrium is partially informative, with $r = r_1$ if $m^T \in \{m | \#m = 1\}$ and $r = r_2$ else. Note that after receiving report $r = r_2$, models that induce posteriors $\frac{3}{4}$ and $\frac{4}{5}$ are feasible, with models inducing the latter having a higher likelihood.

(ii) $h^\Sigma = 0$. The the informative equilibrium is uninformative. In this case, a model's likelihood decreases in the posterior it induces, such that any partially informative communication strategy is associated with a tighter IC constraint than a fully informative strategy.

6 Conclusion

This article conceptualizes the communication of narratives as a cheap-talk game under model uncertainty. Narratives are modeled as likelihood functions, associating conditional probabilities to observable data given a state of the nature. While an unbiased receiver and a biased sender both do not know the true state of the nature, the sender is aware of the data generating process. The sender provides a set of models that we call narratives in order to rationalize the data in her favor. In contrast to the pre-existing literature, the receiver is aware of a set of data generating processes from which the true one is drawn. As a result, the receiver is knowledgeable and can play a best response to the sender's strategic communication. We provide a general formal framework to study equilibria under model uncertainty for the communication of narratives. The receiver resolves the model uncertainty by means of a class of ambiguity rules, including the benchmark maximum likelihood expected utility, as well as maxmin expected utility and the smooth model of decision making.

Ordering narratives by their respective receiver bliss points, an equilibrium is characterized by an interval partition of the set of narratives. The number of these intervals spans every positive integer from 1 (babbling equilibria) to a maximum number N, depending on the data h and the bias b. For small conflicts of interest, the preference of the receiver and the sender are sufficiently aligned to allow truthful communication while for large conflicts of interest, informative communication breaks down. We give an exact characterization of these bounds in an example. We compare the proposed equilibrium model under the maximum likelihood ambiguity rule to one in which the receiver adjusts to any narrative that explains the data better than the model he originally held. Not surprisingly, the sender can induce a profitable persuasion of the receiver more often when facing a naïve receiver, formalizing that under equilibrium considerations a receiver cannot be deluded so easily.

While this article studies the communication of narratives from an equilibrium point of view, one might ask whether individuals correct for the strategic play of a sender by adjusting their action after being exposed to a narrative or accept the narrative whenever it fits the data better than some default model. To this end, we are preparing an experimental study on Prolific along the lines of Barron and Fries (2023). Our hypotheses concern whether or not subject receivers tend towards equilibrium or naïve play and what ambiguity rule best fits the data.

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A Proofs

Proof of Lemma 1. In an equilibrium with sender strategy $\sigma \colon M \to R$, the receiver can act upon the different messages in $\sigma(M)$, which contains at most $\#M$ elements. $\square$

Proof of Lemma 2. (i) Fix any induced action $a^* \in \rho(\sigma(M))$. Define $M^* := \{m \in M \mid \rho(\sigma(m)) = a^*\}$, which is not empty and contains some element m^*. Set $r^* := \sigma(m^*)$ and consider the sender strategy $\sigma^*(m) = r^*$ if $m \in M^*$ and $\sigma^*(m) = \sigma(m)$ otherwise. Let ρ^* be the receiver's best reply to σ^*. We find

$$\rho^*(r^*) = a(M^*, h, 0) \in \operatorname{conv}(\{\underbrace{\rho(\sigma(m))}_{=a^*} \mid m \in M^*\}) = a^*$$

and $\rho^*(r) = a(\sigma^{-1}(r), h, 0) = \rho(r)$ if $r \in \sigma^*(M) \setminus \{r^*\}$. Note that σ^* is a best response to ρ^* as it induces the same actions and that the action a^* is induced solely by the message r^*. Repeating this construction for all induced actions yields the result.

(ii) Let $a_1 < \cdots < a_N$ be the distinct action induced in equilibrium. By (i) we may assume that the sender on path solely uses messages $r_1, \ldots, r_N$ with $\rho(r_i) = a_i$. Let $M_i := \sigma^{-1}(r_i) \subseteq M$. Since σ is a best reply to ρ and by interval choice each M_i is an interval. The M_i form a partition of M, thus we find $m_i \leq m_j$, i.e., $a(m_i, h, 0) \leq a(m_j, h, 0)$, whenever $i < j$ and $m_i \in M_i, m_j \in M_j$. Note that $a_i = \rho(r_i) = a(M_i, h, 0) \in \operatorname{conv}(\{a(m, h, 0) \mid m \in M_i\})$. Thus, induced actions and the receiver's bliss points are equally ordered. Consistent ordering concludes the proof.

□

Proof of Theorem 1. By Lemma 1 there is a maximum number $N \leq \#M$ such that an N-step equilibrium exists. In the following, we show that if (σ, ρ) is an $(n+1)$-step equilibrium, there exists an n-step equilibrium. The assertion is trivial for $n = 1$ as the babbling equilibrium exists. We thus assume $n \geq 2$. By the reduction lemma (Lemma 2) (i) we may assume that the equilibrium amounts to a sequence $r_1, \ldots, r_{n+1}$ of distinct messages on path, which can be identified with the pre-images $M_i := \sigma^{-1}(r_i) \subseteq M$ that form a partition of M consisting of intervals. Re-labelling the messages appropriately, statement (ii) of the reduction lemma asserts that $\rho(\sigma(m)) < \rho(\sigma(m'))$ whenever $m \in M_i$ and $m' \in M_{i+1}$. We order all models $m \in M$ by their induced receiver action $a(m, h, 0)$, respecting the partition order of the M_i in case of indifference, to obtain a sequence $m_1 \leq \cdots \leq m_{\#M}$. Especially, M_i is a set of consecutive elements $m_\ell, \ldots, m_{\ell + \#M_i - 1}$ for some $1 \leq \ell \leq \#M - \#M_i + 1$.

Consider now the two strategies σ_a and σ_b which result from merging the messages r_n, r_{n+1} respectively r_1, r_2. They are explicitly given by

$$\sigma_a(m) = \sigma_a^1(m) = \begin{cases} r_n & , \text{ if } m \in M_n \cup M_{n+1} \\ r_i & , \text{ if } m \in M_i, i \neq n, n+1 \end{cases}$$

and

$$\sigma_b(m) = \begin{cases} r_1 & , \text{ if } m \in M_1 \cup M_2 \\ r_{i-1} & , \text{ if } m \in M_i, i \neq 1, 2 \end{cases}.$$

If one of the two, together with the respective best reply $\rho_a = \rho_a^1$ or ρ_b of the receiver, is an n-step equilibrium, we are done.

Suppose now, that neither of the two amounts to an n-step equilibrium. Then, given ρ_a, there is a model m^* for which the sender has a profitable deviation from σ_a. Since ρ and ρ_a agree on the partition elements $M_1, \ldots, M_{n-2}$, the profitable deviation must concern $m^* \in M_{n-1} \cup M_n \cup M_{n+1} = M_{n-1} \cup M_n'$, where $M_n' = \sigma_a^{-1}(r_n) = M_n \cup M_{n+1}$. Note that $\rho(r_n) \leq \rho_a(r_n)$ by the hedging property of the ambiguity rule. Applying strict concavity of the utility function, we find $m^* \notin M_{n-1}$. Consequently, the profitable deviation stems from M_n'. By interval choice, we find that a profitable deviation must be possible for the element with the smallest index in M_n', denoted $m^l_{M_n'} = \min M_n'$. A reverse argument holds for σ_b which we want to recall for later: Here, profitable deviations must entail mapping a model to a message of lower index.

In the next step, consider the modified version σ_a^2 of $\sigma_a = \sigma_a^1$ in which now $\sigma_a^2(m^l_{M_n'}) = r_{n-1}$, everything else as before. More precisely, σ_a^2 sends r_{n-1} on the set $M''_{n-1} = M_{n-1} \cup \{m^l_{M_n'}\}$ and r_n' on $M''_n = M_n' \setminus \{m^l_{M_n'}\}$. Consider the best reply ρ_a^2. In comparison to $\rho_a = \rho_a^1$ both are the same except for possibly $\rho_a^1(r_{n-1}) \leq \rho_a^2(r_{n-1})$ and $\rho_a^1(r_n) \leq \rho_a^2(r_n)$. If (σ_a^2, ρ_a^2) is an equilibrium, we are done. If not, there is a profitable deviation of the sender and it necessarily must concern models in $M_{n-2}, M''_{n-1}, M''_n$. Since their actions $\rho_a^2(r_{n-1}), \rho_a^2(r_n)$ are weakly tilted upwards, a profitable must either be found in $m^l_{M''_{n-1}}$ or $m^l_{M''_n}$.

Picking any of these and modifying σ_a^2 to σ_a^3 by adding either $m^l_{M''_{n-1}}$ to M_{n-2} or $m^l_{M''_n}$ to M''_{n-1}, we again consider the respective best reply and in case they do not form an equilibrium proceed with such an operation again. Note that in every such step, a model is assigned to a message of a lower index. Also note that during this process, no message will become void in the sense that it will not be sent anymore: If, at some point, a message is sent only for a single model m^*, the receiver answers with his bliss point $a(m^*, h, 0)$ which is smaller than the sender's bliss point $a(m^*, h, b)$. Assigning m^* to a message of lower index leads to a smaller action, making no profitable deviation possible.

Assume, for the sake of the argument, that this process keeps going on, creating a sequence $(\sigma_a^\ell)_\ell$ without leading to an equilibrium. For any σ_a^ℓ, call the rightmost model of the leftmost partition element $C_1^{a,\ell}$, the rightmost model of the second leftmost partition element $C_2^{a,\ell}$ and so forth until $C_n^{a,\ell}$. Do so likewise for σ_b with $C_1^b, \dots, C_n^b$. As seen above, $C_i^{a,\ell} \le C_i^{a,\ell+1}$ for all i and ℓ, with the inequality being strict for exactly one i given any ℓ. Note that $C_i^{a,1} < C_i^b$ for all i. While progressing in $\ell \in \mathbb{N}$, at some point ℓ_0 for the first time we have an $i_0 \in \{1, \dots, n\}$ with $C_{i_0}^{a,\ell_0} = C_{i_0}^b$. There will not be any more profitable deviation by shifting $C_{i_0}^{a,\ell}$ to the right, because $\rho_a^{\ell_0}(r_{i_0}) \le \rho_b(r_{i_0})$ and there is no such profitable deviation for σ_b. Any further profitable deviation in the process of modifying σ_a^ℓ thus must concern the other boundaries $C_i^{a\ell}$, $i \ne i_0$. Proceeding this way, we will end up with $C_i^{a,\ell_*} = C_i^b$ for all i at some $\ell_* \in \mathbb{N}$. Now, however, no further profitable deviation is possible anymore. Consequently, $(\sigma_a^{\ell_*}, \rho_a^{\ell_*}) = (\sigma_b, \rho^b)$ must form an n-step equilibrium. □

Proof of Proposition 1. (i) Consider any fully informative strategy σ of the sender. Especially, σ is injective and receiving a message r, the minimal feasible set will be $\tilde{M}(\mu_1^r) = \sigma^{-1}(r) = \{m^T\}$, the singleton containing the true model. The best response ρ is thus given by $\rho(r) = a(\sigma^{-1}(r), h, 0) = \arg\max_a \mathbb{E}[u_R(a, \theta) \mid \sigma^{-1}(r), h]$.

For now, consider any $m^T \in M$ with $a^T := a(m^T, h, 0)$ and let $a^* := \min\{a = a(m, h, 0) \mid m \in M, a^T < a\}$ and define $H_{m^T,h} \colon \mathbb{R}_{\ge 0} \to \mathbb{R}, b \mapsto U_{m^T,h}(a, b) - U_{m^T,h}(a', b)$, which is a continuous function. Note that the sender has an incentive to deviate from revealing m^T if and only if $H_{m^T,h}(b) < 0$ as $\rho(\sigma(m^T)) = a(m^T, h, 0) < a(m^T, h, b)$ from (SC). Define $\underline{b}(m^T, h) := \min\{b \ge 0 \mid H_{m^T,h}(b) = 0\} \cup \{\infty\}$. Since $H_{m^T,h}(0) > 0$ we have $\underline{b}(m^T, h) > 0$. Now set $\underline{b}(h) := \min\{\underline{b}(m^T, h) \mid m^T \in M \colon a(m^T, h, 0) < \max_{m \in M} a(m, h, 0)\}$. We have $\underline{b}(h) > 0$ as M is finite and for all $b \le \underline{b}(h)$ there is no profitable deviation for the sender from the fully revealing strategy.

(ii) Note that $V(m^T, h, b)$ is increasing in b by (SC). Under a large conflict of interest, we take $\bar{b}(H)$ to be the infimum of all $\hat{b}(h)$ with $V(m^T, h, b) > 0$ for all $b > \bar{b}(h)$ and all m^T. For $b > \bar{b}(h)$, any equilibrium is babbling while for $b \le \bar{b}(h)$ there exists a 2-step equilibrium.

□

Proof of Proposition 2. Fix any h and let m^T be arbitrary. In order to show that the sender is better off, it suffices to show that the sender can induce the action $\rho'(\sigma'(m^T))$ in equilibrium (σ, ρ) as well. As σ is weakly more informative than σ', the set of models $M' := \sigma'^{-1}(\sigma'(m^T))$ decomposes into a (possibly singleton) partition $\dot{\bigcup}_{r \in \tilde{R}} \sigma^{-1}(r)$ for a subset $\tilde{R} \subseteq R$. The MLEU rule of the receiver selects some $m' \in M'$ under σ' with highest likelihood and chooses action $a(m', h, 0)$. Let $r' \in \tilde{R}$ be such that $m' \in \sigma^{-1}(r')$. By the consistent tie-breaking MLEU rule, m' is also chosen under the minimal feasible set $\sigma^{-1}(r')$ and thus $\rho(\sigma(r')) = a(m', h, 0)$. □

Proof of Proposition 3. Let $m^T \in M_{(\sigma,\rho)}$. Then $\rho(\sigma(m^T)) \neq a(m^T, h, 0)$. Since we consider an MLEU equilibrium, we have $\rho(\sigma(m^T)) = a(m^*, h, 0)$ for some $m^* \in \sigma^{-1}(\sigma(m^T))$ with $\Pr(h \mid m^*, F_0) \geq \Pr(h \mid m^T, F_0)$. Consequently, $m^T \in M_{\text{naïve}}$. □

Proof of Proposition 4. First, let $b \leq \underline{b}(h)$. At any true model m^T, the sender can not do better than inducing $a(m^T, h, 0)$. Consequently, $M_{(\sigma,\rho)} = M_{\text{naïve}} = \emptyset$.Second, if $b > \bar{b}(h)$ and we have large conflicts of interest, the only equilibrium is babbling, inducing the constant $a(M, h)$. For every true model m^T with $a(M, h) \leq a(m^T, h, 0)$, the sender's utility thus stays at $U_{m^T,h}(a(M, h), b) \leq U_{m^T,h}(a(m^T, h, 0), b)$ in equilibrium. If there exists m^* with $m^T < m^*$ and $\Pr(h \mid m^*, F_0) \geq \Pr(h \mid m^T, F_0)$, a naïve receiver can be convinced to take action $a(m^*, h, 0)$ under the true model m^T, which strictly improves the sender's expected utility. □

Proof of Proposition 5. Consider any h^Σ. Note that $\underline{b}(h^\Sigma)$ is strictly increasing in $\min_{\tilde{\mu}, \tilde{\mu}' \in \mathcal{M}_1(h^\Sigma): \tilde{\mu} \neq \tilde{\mu}'} |\tilde{\mu} - \tilde{\mu}'|$. Symmetry then follows since, for any given $k \in \{1, 2, \ldots, K\}$,

$$\begin{aligned}
\frac{s+1}{k+2} \in \mathcal{M}_1(h^\Sigma) &\Leftrightarrow \max\{0, h^\Sigma - K + k\} \leq s \leq \min\{k, h^\Sigma\} \\
&\Leftrightarrow \max\{0, K - h^\Sigma - K + k\} \leq k - s \leq \min\{k, K - h^\Sigma\} \\
&\Leftrightarrow \frac{k-s+1}{k+2} \in \mathcal{M}_1(K - h^\Sigma) \\
&\Leftrightarrow 1 - \frac{s+1}{k+2} \in \mathcal{M}_1(K - h^\Sigma).
\end{aligned}$$

For the second claim, note first that the smallest common divisor of distinct $\tilde{\mu}, \tilde{\mu}' \in$

$\mathcal{M}_1(h^\Sigma)$ is at most $(K+1)(K+2)$, which implies

$$|\tilde{\mu} - \tilde{\mu}'| \geq \frac{1}{(K+1)(K+2)}. \tag{6}$$

In particular, (6) holds with equality only if $\tilde{\mu} = \frac{s+1}{K+2}$ and $\tilde{\mu}' = \frac{s'+1}{K+1}$ (or vice versa) for $0 \leq s \leq K$ and $0 \leq s' \leq K-1$. Next, observe that

$$\begin{aligned}
|\tilde{\mu} - \tilde{\mu}'| = \frac{1}{(K+1)(K+2)} &\Leftrightarrow |(s+1)(K+1) - (s'+1)(K+2)| = 1 \\
&\Leftrightarrow |s(K+1) - s'(K+2) - 1| = 1 \\
&\Leftrightarrow s'(K+2) = s(K+1) \vee s = s' + \frac{2+s'}{K+1} \\
&\Leftrightarrow s = s' = 0 \vee (s = K \wedge s' = K-1),
\end{aligned}$$

where the last step follows from $s(K+1) > (s-1)(K+2)$ for all $s \leq K$ and $\left(\frac{2+s'}{K+1} \in \mathbb{N} \wedge 0 \leq s' \leq K-1\right) \Leftrightarrow s' = K-1$. The claim then follows since $\frac{1}{K+2} \in \mathcal{M}_1(h^\Sigma) \Leftrightarrow h^\Sigma = 0$ and $\frac{K+1}{K+2} \in \mathcal{M}_1(h^\Sigma) \Leftrightarrow h^\Sigma = K$. $\square$

Proof of Proposition 6. Fix any K and consider $h^\Sigma = K$. Note first that in this case $\tilde{\mu}_{h,m} = \frac{\#m+1}{\#m+2}$ for any m. Thus, the likelihood of model m is

$$Pr(h|m, F_0) = \int_0^1 \theta^{\#m} \left(\frac{1}{2}\right)^{K-\#m} d\theta = \left(\frac{1}{2}\right)^{K-\#m} \frac{1}{\#m+1},$$

which is strictly increasing in $\#m$. Consider the reporting strategy $r = r_1$ if $m^T \in \{m | \#m = 1\}$ and $r = r_2$ else, which thus induces posteriors $\frac{2}{3}$ and $\frac{K+1}{K+2}$, respectively. This strategy is incentive compatible iff

$$b \leq \frac{1}{2}\left(\frac{K+1}{K+2} - \frac{2}{3}\right) = \frac{K-1}{6(K+2)}. \tag{7}$$

In particular, $\left|\frac{K+1}{K+2} - \frac{2}{3}\right| = \max_{m,m' \in \mathcal{M}(K)} |\tilde{\mu}_{h,m} - \mu_{h,m'}|$ implies that informative communication is impossible if (7) does not hold.

Next, consider $h^\Sigma = 0$. Analogously, $\tilde{\mu}_{h,m} = \frac{1}{\#m+2}$ for any m and the likelihood of model m is strictly increasing in $\#m$. Thus, any equilibrium (σ, ρ) is such that $\rho(\sigma(m^T)) \leq \tilde{\mu}_{h,m^T}$ for all m^T. This implies that whenever there is an informative equilibrium (σ, ρ), then there also is a fully informative equilibrium (σ', ρ') (where $\rho'(\sigma'(m^T)) = \tilde{\mu}_{h,m^T}$ for all m^T). Finally, note that a fully informative strategy is part of an equilibrium iff

$$b \leq \frac{1}{2} \min_{m,m' \in \mathcal{M}(0):\tilde{\mu}_{h,m} \neq \mu_{h,m'}} |\tilde{\mu}_{h,m} - \mu_{h,m'}| = \frac{1}{2}\left(\frac{1}{K+1} - \frac{1}{K+2}\right)$$
$$= \frac{1}{2(K+1)(K+2)}.$$

$\square$