Design of Low-Delay Distributed Joint Source-Channel Codes Using Irregular LDPC Codes

Iqbal Shahid and Pradeepa Yahampath
Department of Electrical and Computer Engineering
University of Manitoba, Winnipeg, Canada
Email: umshahi5@cc.umanitoba.ca, pradeepa@ee.umanitoba.ca

Abstract—A practical approach to designing distributed joint source channel (DJSC) codes for correlated binary sources using LDPC codes is presented. In this approach, both distributed compression and channel error protection are achieved by optimally puncturing bits of a systematic channel code to match the source correlation and channel error probability. This requires the design of a channel code with a specific unequal error protection (UEP) property. Towards this end, we present a linear-programming based algorithm for optimizing an irregular LDPC code for a given level of source correlation and channel noise. Experimental results are presented for both binary symmetric channels and Gaussian channels, which demonstrate that the proposed DJSC codes can significantly outperform the best code found through random search, tandem source-channel codes and previously reported schemes based on turbo codes when the encoding delay is constrained.

I. INTRODUCTION

Distributed source coding (DSC) is considered as one of the enabling technologies for wireless sensor networks (WSNs) in which a set of distributed sensors pick-up correlated information (without collaborating with each other) and communicate their readings to a central decoder. The basis for lossless DSC is given by the Slepian-Wolf (SW) theorem [1] which shows that the rates achievable with the joint lossless encoding of two discrete sources can also be achieved with separate encoding, if joint decoding can be used. Recently, considerable progress has been made in practical construction of SW codes using syndromes of a linear channel code [2]. However, such SW codes have two shortcomings: (i) very long codes are required to achieve acceptable performance, and (ii) they can be extremely sensitive to channel noise [3]. Therefore, syndrome-based SW codes are unlikely to be effective when low delay/encodingcomplexity is required, such as in WSNs. An alternative (lesswidely studied) practical approach to SW coding is the paritybit method [3], [4]. This approach is more suitable for lowdelay encoding over noisy channels, as the possibility exists for implementing distributed JSC (DJSC) codes by transmitting additional parity bits to account for channel noise [5]–[8]. When codeword length is constrained, it is well known that joint source-channel (JSC) coding outperforms separate source and channel (SSC) coding. In [8] we presented a DJSC coding procedure based on the parity-bit method. However the given encoding scheme is strictly sub-optimal, as an excess rate is used to enable the use of two separate decoders. The approach in [8] is an extension of the SW coding method presented in [9]. In either work, no systematic methods for code design were

In this paper we present a practical approach to designing

a DJSC code with arbitrary (non-asymmetric) rates for transmitting two correlated binary sources over independent noisy channels. The design is based on an encoding scheme which can be shown to be asymptotically (in coding block length) optimal. The practical implementation of this idea involves the design of channel codes with a specific UEP property which is required to exploit the statistical dependence between two sources for simultaneous compression and channel error correction. To this end we develop a low-complexity linear programming algorithm based on extrinsic information chart (EXIT) analysis [10] for optimizing the degree profiles of an irregular LDPC code for given levels of source correlation and channel noise-level. As specific examples, we consider both binary-symmetric channels (BSC) and additive white Gaussian noise (AWGN) channels with binary PSK (Bi-AWGN channel).

Our simulation results confirm that DJSC codes designed by the proposed method considerably outperform the alternative SSC coding schemes which uses SW coding followed by independent channel coding, when the coding block length (or equivalently, the delay and complexity) is decreased. Furthermore, we found that our optimization algorithm always produced codes better than the best codes found by random search. It is also demonstrated that when coding block-length is increased, the proposed code designs rapidly approach the theoretical lower-bound of the achievable rate region.

II. A FRAMEWORK FOR OPTIMAL DJSC CODING

Let X_1 and X_2 be two correlated and uniformly distributed binary sources. The dependence between X_1 and X_2 is modeled by a "virtual" BSC (denoted by V) with cross-over probability $p = P(X_1 \neq X_2)$. In distributed encoding, the two sources are separately encoded and transmitted through independent noisy channels (denoted by \mathcal{C}_1 and \mathcal{C}_2 respectively), while both sources are decoded by a joint decoder. Consider encoding of k-bit sequences $\mathbf{X}_1 = (X_1^{(1)}, \dots, X_1^{(k)})$ and $\mathbf{X}_2 = (X_2^{(1)}, \dots, X_2^{(k)})$ from each source. Let the target *JSC coding* rate of X_1 and X_2 be R_{X_1} and R_{X_2} channel-uses/sourcebit respectively. In order to achieve any arbitrary rate-pair, the encoder for X_1 transmits the first t bits $(X_1^{(1)}, \ldots, X_1^{(t)})$, while the encoder for X_2 transmits the last k-t bits $(X_2^{(t+1)},\ldots,X_2^{(k)})$, where $0 \le t \le k$. Let a=t/k so that $0 \le a \le 1$. Recall that the corresponding bits in X_1 and \mathbf{X}_2 are related by a virtual BSC \mathcal{V} . Also, those bits which are explicitly transmitted by each encoder pass through noisy communication channels (C_1 or C_2). We will now establish that, if an appropriate number of parity bits generated by encoding X_1 and X_2 using a systematic channel code are transmitted

for respective sources, all bits of X_1 and X_2 can be decoded error-free by a joint decoder. Furthermore, we will prove that this encoding scheme is asymptotically optimal, i.e., rate-region given by (4)-(6) of [6] can be achieved.

Let the encoder for X_1 transmit m_1 parity bits (in addition to source bits) obtained by encoding X_1 using a rate r_1 systematic channel code and let the encoder for X_2 transmit m_1 parity bits obtained by encoding X_2 using a rate r_2 systematic channel code, where $r_i = k/(k + m_i)$, i = 1, 2. Now we observe that for error-free decoding of X_1 , m_1 parity bits transmitted by encoder for X_1 must be sufficient to correct the errors in $(X_1^{(1)}, \ldots, X_1^{(t)})$ which pass through the communication channel \mathcal{C}_1 and the errors in $(X_1^{(t+1)}, \ldots, X_1^{(k)})$ which pass through the virtual channel \mathcal{V} . From Shannon's channel coding theorem, it follows that

$$r_1 \le \frac{C_1}{1 + (1 - a)(C_1 - C_v)},\tag{1}$$

where C_1 is the capacity of C_1 and $C_v = 1 - H(p)$ is the capacity of V. Similarly it can be shown that

$$r_2 \le \frac{C_2}{1 + a(C_2 - C_n)},\tag{2}$$

where C_2 is the capacity of C_2 . In this paper, assume that $C_1 =$ $C_2 = C$. Then, noting that $R_{X_1} = (ak + m_1)/k$ and $R_{X_2} =$ $((1-a)k+m_2)/k$, we obtain the corresponding lower bounds on the JSC coding rates as

$$R_{X_1} \geq \frac{1 - (1 - a)C_v}{C},$$
 (3)
 $R_{X_2} \geq \frac{1 - aC_v}{C},$ (4)

$$R_{X_2} \geq \frac{1 - aC_v}{C},\tag{4}$$

$$R_{X_1} + R_{X_2} \ge \frac{2 - C_v}{C}.$$
 (5)

Note that $H(X_1|X_2) = H(X_2|X_1) = 1 - C_v$. Hence we observe that (3) with a = 0, (4) with a = 1, and (5) corresponds to (4)-(6) in [6] which define the achievable rate-region for DJSC coding of correlated binary sources and over independent channels with capacity C (note that in [6] $R_{c_i} = 1/R_{X_i}$, i = 1, 2, and a = 0, 1 are the corner points). Hence, the above described DJSC coding scheme is asymptotically optimal.

Before we present a practical method for implementing this scheme, observe that, by varying a from 0 to 1, we can increase r_1 from R_{\min} to R_{\max} , while r_2 decreases from R_{\max} to R_{\min} , where $R_{\min} = \frac{C}{1 - C_v + C}$ and $R_{\max} = C$. Given R_{X_1} (or R_{X_2}), p, and C, we can find the corresponding value of a by using (3) [or (4)]. Since the range of r_1 and r_2 are the same, we can design a channel code with rate R_{\min} (and m parity bits) for each source and achieve any compression level by puncturing the parity bits. Towards this end, we next present an encoding scheme based on a rate-compatible systematic block code, which can achieve the optimal JSC coding rates given by (3), (4), and (5), as the block length $k \to \infty$. This procedure forms the basis for the practical code design procedure presented in the next section.

The proposed encoding scheme is illustrated in Fig. 1. Suppose we have a systematic channel code of rate R_{\min} . Consider

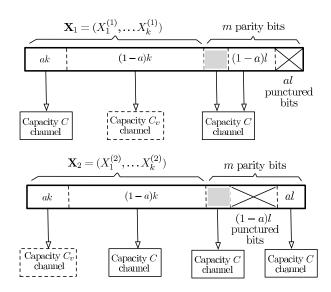


Fig. 1. A channel code structure for realizing the proposed DJSC scheme (top: source X_1 , bottom: source X_2). The shaded part indicates k(1/C-1)parity bits which are not available for puncturing.

encoding of X_1 as described above. Since a capacity achieving channel code of rate R_{\min} must have at least $m = k \frac{1 - C_v}{C}$ parity-bits and the highest achievable rate in this case is the channel capacity C, it follows that at least $k\frac{1-C}{C}$ parity bits must be transmitted for the source X_1 . In other words, only albits out of the remaining $l = ak(1 - \frac{C_v}{C})$ parity bits can be punctured (see Fig. 1). The JSC coding rate of this scheme is

$$\frac{ak + k\frac{1-C}{C} + (1-a)l}{k} = \frac{1 - (1-a)C_v}{C},$$

which is identical to (3). For X_2 , the minimum number of parity bits that must be transmitted and their locations are the same as that of X_1 . Out of the remaining l parity bits, first (1-a)lbits are punctured and only the last al bits are transmitted as shown in Fig 1. It then follows that for this encoding scheme, R_{X_2} and hence $R_{X_1} + R_{X_2}$ satisfy (4) and (5) respectively. Therefore, encoding scheme in Fig. 1 can achieve bounds (3) - (5).

III. CODE DESIGN AND OPTIMIZATION

The implementation of the DJSC coding scheme presented above requires a channel code with a specific UEP property. More specifically, we note that a fraction α of the codeword bits of X_1 passes through channel with capacity C where

$$\alpha = \frac{k}{nC} [1 - C - C_v(1 - a)], \qquad (6)$$

where n is the codeword length. We refer to these bits as *type 1* bits. The remaining $\bar{\alpha} = 1 - \alpha$ fraction of bits passes through a BSC with capacity C_v . We refer to these bits as type 2 bits. For simplicity we will consider punctured bits also as type 1. UEP codes are easy to construct using irregular LDPC codes, as the level of protection that each bit receives is proportional to the degree (number of connected edges) of the corresponding node in the bipartite graph [11]. While LDPC code design for UEP has been considered before, e.g., [12], [13], these code designs

only assign a priority to some bits compared to the others and channel capacity is not taken into account. In our case, we need to directly match the level of protection provided for each bit (type 1 or type 2) to the capacity of the channel through which the bit passes (C or C_v). In the following, we present a simple linear programming approach to designing such a UEP code. We describe the code optimization for source X_1 . For X_2 the same procedure applies with (1-a) replaced by a in (6).

An LDPC code is completely specified by a bipartite graph in which the variable and check nodes represent the columns and rows of the parity check matrix H [11]. As usual we assign degree polynomials $\lambda(x)$ and $\rho(x)$ to variable and check nodes respectively. In order to impose the required UEP property we define $\lambda_i^{(k)}$ to be the fraction of edges connected to type k variable nodes of degree i, where k=1,2. Then, our code is specified completely by $\lambda^{(1)}(x)$, $\lambda^{(2)}(x)$ and $\rho(x)$, where $\lambda^{(1)}(x)=\sum_{i=2}^{d_{vmax}}\lambda_i^{(1)}x^{i-1}$ and $\lambda^{(2)}(x)=\sum_{i=2}^{d_{vmax}}\lambda_i^{(2)}x^{i-1}$. The maximum variable node degree d_{vmax} is kept the same for nodes of both types. As observed in [14], only two coefficients with consecutive degrees are enough for $\rho(x)$. Also, in [15], the authors show that a high value of maximum check node degree tend to degrade the performance of an LDPC code. We therefore use d_{cmax} to be 20 and fix $\rho(x) = (1 - \zeta_s)x^{s-1} + \zeta_s x^s$ in our design, where $\zeta_s \in [0,1]$ and $s \leq d_{cmax}$. In our simulations, we have observed that the choice of ζ_s is not critical to the performance of the code. We have chosen $\zeta_s \in [0.3, 0.7]$. For given $\lambda^{(1)}(x)$, $\lambda^{(2)}(x)$ and $\rho(x)$, the design rate of the code is given by

$$R_{des} = 1 - \frac{\int_0^1 \rho(x)dx}{\int_0^1 \lambda^{(1)}(x)dx + \int_0^1 \lambda^{(2)}(x)dx}.$$
 (7)

The design rate equals the actual rate of the code if the parity check matrix H has full row rank. The design of the code involves finding degree distributions $\lambda^{(1)}(x)$ and $\lambda^{(2)}(x)$ which maximize R_{des} , subject to a set of constraints. Since $\rho(x)$ is fixed, maximizing R_{des} is equivalent to maximizing

$$\sum_{i} \frac{\lambda_i^{(1)}}{i} + \sum_{i} \frac{\lambda_i^{(2)}}{i}.$$
 (8)

which is the objective function in our optimization problem. We next establish the constraints to enforce the UEP property. $\sum_{n=0}^{\infty} d_n m_{n,n} \wedge \binom{n}{n} = 1$

Let $\Lambda^{(1)}(x) = \sum_{i=2}^{d_{vmax}} \Lambda_i^{(1)} x^i$ be the polynomial associated with the variable nodes of $type\ 1$, where $\Lambda_i^{(1)}$ is the number of variable nodes of $type\ 1$ having degree i. It is easy to see that $\Lambda^{(1)}(1) = \sum_{i=2}^{d_{vmax}} \Lambda_i^{(1)} = \alpha n$. Similarly we define $\Lambda^{(2)}(x) = \sum_{i=2}^{d_{vmax}} \Lambda_i^{(2)} x^i$ for $type\ 2$ variable nodes with $\Lambda^{(1)}(1) = \bar{\alpha} n$. It is known that $\lambda^{(1)}(x)$ and $\Lambda^{(1)}(x)$ are related by [11]

$$\lambda^{(1)}(x) = \frac{\frac{d}{dx}\Lambda^{(1)}(x)}{\frac{d}{dx}\Lambda^{(1)}(x)|_{x=1}},$$
(9)

and conversely,

$$\frac{\Lambda^{(1)}(x)}{\Lambda^{(1)}(1)} = \frac{\int_0^x \lambda^{(1)}(z)dz}{\int_0^1 \lambda^{(1)}(z)dz}.$$
 (10)

Similar relationships exist between $\lambda^{(2)}(x)$ and $\Lambda^{(2)}(x)$. Since $\lambda_i^{(1)}$ is the ratio between the number of edges connected to degree i variable nodes of $type\ I$ to the total number of edges,

$$\lambda_i^{(1)} = \frac{i\Lambda_i^{(1)}}{\sum_i^{d_{vmax}} i\Lambda_i^{(1)} + i\Lambda_i^{(2)}}.$$

Using this and noting that $\int_0^1 \lambda^{(1)}(z)dz = \sum_i^{d_{vmax}} \frac{\lambda_i^{(1)}}{i}$, we get

$$\int_0^1 \lambda^{(1)}(z)dz = \frac{\Lambda^{(1)}(1)}{\frac{d}{dx} \left(\Lambda^{(1)}(x) + \Lambda^{(2)}(x)\right)|_{x=1}},$$
 (11)

and

$$\int_0^1 \lambda^{(2)}(z)dz = \frac{\Lambda^{(2)}(1)}{\frac{d}{dx} \left(\Lambda^{(1)}(x) + \Lambda^{(2)}(x)\right)|_{x=1}},$$
 (12)

from which it follows that

$$\frac{\int_0^1 \lambda^{(1)}(z)dz}{\Lambda^{(1)}(1)} = \frac{\int_0^1 \lambda^{(2)}(z)dz}{\Lambda^{(2)}(1)}.$$
 (13)

Thus, we have the following constraint relating $\lambda^{(1)}(x)$ and $\lambda^{(2)}(x)$:

$$\frac{1}{\alpha} \sum_{i=2}^{d_{vmax}} \frac{\lambda_i^{(1)}}{i} = \frac{1}{\bar{\alpha}} \sum_{i=2}^{d_{vmax}} \frac{\lambda_i^{(2)}}{i}.$$
 (14)

Since $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$ represent fractions of edges connected to various variable nodes, we also require the constraint

$$\sum_{i}^{d_{vmax}} \lambda_i^{(1)} + \lambda_i^{(2)} = 1.$$
 (15)

For our code design, we will use the EXIT chart method [10] to track the mutual information of messages [in our case log-likelihood ratios (LLRs)] passed in belief propagation decoding of the LDPC code. To simplify the description, define the mutual information quantities (see [10, (12),(19)])

- I_{AC}: average a priori information coming into the check nodes.
- I^j_{EC}: output extrinsic information coming from a check node of degree j.
- $I_{EC}(I_{AC})$: average output extrinsic information into a variable node (which is a function of I_{AC}).
- Iⁱ_{EV}: output extrinsic information from a variable node of degree i.
- $I_{EV}(I_{AC})$: average output extrinsic information to a check node (which is function of I_{AC} through I_{EC}).

In order to find a relationship between I_{EV} and I_{AC} we first use the duality approximation [16], [11, pp. 236]

$$I_{EC} \approx 1 - \sum_{j=s-1}^{s} \rho_j J((j-1)J^{-1}(1-I_{AC})).$$
 (16)

where $\rho_{s-1}=1-\zeta_s$, $\rho_s=\zeta_s$, and $J(\cdot)$ is the mutual information function given by [10, (15)] (which can be computed using the approximation given in [17]). Observe that the variable nodes receive information from the check nodes as well as from the channel. In our coding scheme, the information received by $type\ I$ variable nodes from the communication channel is

 $I_{ch}^{(1)}=C$, whereas the information received by $type\ 2$ variable nodes from channel $\mathcal V$ is $I_{ch}^{(2)}=C_v$. Therefore, for a variable node of $type\ k$ having degree i

$$I_{EV}^{i,k} = J(J^{-1}(I_{ch}^k) + (i-1)J^{-1}(I_{EC})), \tag{17}$$

for k = 1, 2. By averaging over the variable node degree of each type, we obtain

$$I_{EV}(I_{AC}) = \sum_{k=1}^{2} \sum_{i=2}^{d_{v_{max}}} \lambda_i^{(k)} J(J^{-1}(I_{ch}^{(k)}) + (i-1)J^{-1}(I_{EC})),$$
(18)

where I_{EC} is given by (16). For successful decoding, we require the constraint [10]

$$I_{EV}(I_{AC}) > I_{AC},\tag{19}$$

where $I_{AC} \in (0,1]$ with a fixed point at 1. Finally, a constraint is required to enforce the decoder stability condition [11] (which ensures that error probability does not increase once it has reached zero at some intermediate iteration). From [11], it follows that for *type 1* bits (which pass through the virtual channel \mathcal{V}), it is required that

$$\lambda_2^{(1)} < \frac{\bar{\alpha}}{2\sqrt{p(1-p)}\sum_{i=s-1}^s (i-1)\rho_i}.$$
 (20)

For type 2 bits, this condition depends on the model of the communication channels C_1 and C_2 . If the channels are BSCs with error probability q, then we require

$$\lambda_2^{(2)} < \frac{\alpha}{2\sqrt{q(1-q)}\sum_{i=s-1}^s (i-1)\rho_i},$$
 (21)

whereas if the communication channels are Bi-AWGN channels with noise variance σ^2 , then we require

$$\lambda_2^{(2)} < \frac{\alpha e^{\frac{1}{\sigma^2}}}{\sum_{i=s-1}^s (i-1)\rho_i}.$$
 (22)

The objective function given in (8) as well as the constraints (14), (15), (19), (20), (21), and (22) are linear in the unknown coefficients $\lambda_i^{(k)}$'s. Therefore, it is straight forward to find the optimum solution using a linear program.

IV. SIMULATION RESULTS

In practical code design, we first obtain the desired degree distribution of the LDPC code by using the optimization procedure presented in the previous section. We then create the parity check matrix H using a random interleaver of length n, while ensuring that here are no cycles of length 2 in the bipartite graph which represents the LDPC code (we do not check for cycles of length 4). Gaussian elimination has been used to convert H into systematic form, from which the generator matrix of the LDPC code is obtained. We have implemented the joint decoder using the belief propagation algorithm on a combined factor-graph for LDPC codes of both X_1 and X_2 (details omitted for brevity), along with a target decoding error probability of 10^{-4} .

In Fig. 2, the rate-pairs (R_{X_1}, R_{X_2}) achieved by several JDSC code designs for two correlated binary sources with p=0.05 and BSCs with error probability of 0.02 are presented. The lower bound of the achievable rate region given by (3)-(5)

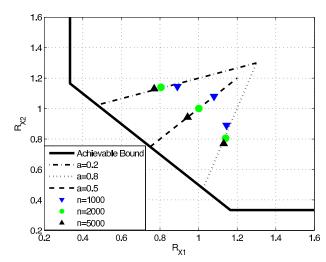


Fig. 2. JSC coding rate-pairs achieved by proposed code-designs for different codeword lengths n for correlated binary sources with p=0.05 and a BSCs of error probability 0.02). R_{X_1} and R_{X_2} are in channel-bits/source-bit.

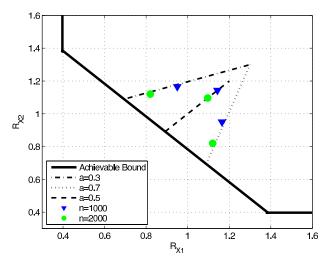


Fig. 3. JSC coding rate-pairs achieved by proposed code-designs for different codeword lengths n for correlated binary sources with p=0.05 and AWGN channels with CSNR of 3 dB). R_{X_1} and R_{X_2} are in channel-uses/source-bit.

is shown in Fig. 2 as a solid line. Achieving the lower-bound for the given source-pair and the channel requires $R_{min}=0.7$. DJSC codes approaching the lower bound can be designed by increasing the rates r_1 and r_2 in (1) and (2). However, for finite-length codes, maintaining a specified decoding error probability requires increasing the block length n as shown in Fig. 2. For a=0.2 and a=0.8, rates achieved by the code-designs are within 0.16 bits of the bound, while for a=0.5 (symmetric-rates) the gap is 0.18 bits. Fig. 3 compares the proposed DJSC code designs for the same pair of binary sources and Bi-AWGN channels with channel signal-to-noise ratio (CSNR) of 3 dB.

We next compare the performance of our DJSC codedesigns based on LDPC codes with those based on Turbocodes reported in [5, Fig. 8] for the case of transmission over BSCs with error probability 0.03 (capacity C=0.8) at $R_{X_1}=0.5, R_{X_2}=1$ (corresponds to a=0), as shown in Fig.

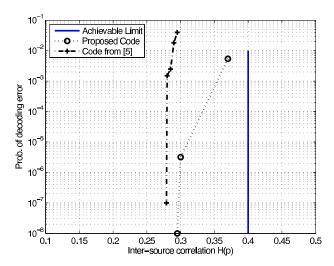


Fig. 4. Performance comparison of DJSC codes based on irregular LDPC codes and those based on Turbo codes from [5] .

??. Theoretically, error-free decoding is possible in this case if $H(X_1|X_2) \leq R_{X_1}C = 0.4$. We vary the correlation parameter p and design codes until the desired decoding error probability is reached. The codes in [5] are about 0.12 bits away from the theoretical bound with codeword lengths of the order of 10^5 bits, while the proposed DJSC codes with a length of only 5000 bits come within 0.105 bits

Finally, in Fig. 5, we compare the probability of error performance of codes optimized using the proposed algorithm with the best codes found by random-search. Also, for comparison, we show the performance of the system DSC-CC with separated source and channel coding, in which DSC is first performed by an SW code as in [9] and subsequent channel coding is performed by a separate LDPC code in such a way that the channel-uses per source-bit is equal to the DJSC coding counterpart. In this case, the bit sequence received from each source is first decoded (using a belief propagation decoder) to correct the channel errors and the output of each channel decoder is then fed to a SW source decoder. All three schemes use a=0.5 and codeword length of n=2000. The results in Fig. 5 shows that, while both DJSC schemes substantially outperform the DSC-CC approach, the optimized codes clearly are better than the best codes found by random search. For example, at a probability of error of 10^{-5} , a coding gain of about 0.4 source bits per channel use is achieved by the optimized code over the best code found by random search.

V. CONCLUDING REMARKS

An asymptotically optimal DJSC coding scheme for uniform binary sources and noisy channels, together with a practical code design procedure based on irregular LDPC codes have been introduced. Future extensions to this work include the design of DJSC codes which exploit non-uniform distribution and the memory of correlated binary sources.

REFERENCES

[1] D. Slepian and J. K. Wolf, "Noiseless coding of correlated information sources," *IEEE Trans. Inf. Theory*, vol. 19, no. 4, pp. 471–480, Jul. 1973.

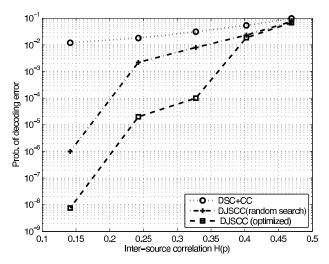


Fig. 5. Comparison of optimized codes [DJSCC (optimized)] with best codes found by random search [DJSCC (random search)] and separate SW coding and channel coding (DSC-CC) for different levels of source dependence H(p).

- [2] S. S. Pradhan and K. Ramachandran, "Distributed source coding: Symmetric rates and applications to sensor network," in *Data Comp. Conf.*, March 2000, pp. 363–372.
- [3] P. Tan and J. Li, "Enhancing the robustness of distributed compression using ideas from channel coding," in *IEEE Globecom*, 2005, pp. 2385– 2389.
- [4] J. Garcia-Frias and Y. Zhao, "Compression of correlated binary sources using turbo codes," *IEEE Commun. Lett.*, vol. 5, no. 10, pp. 417–419, Oct. 2001.
- [5] A. Aaron and B. Girod, "Compression with side information using turbo codes," in *Data Comp. Conf.*, April 2002, pp. 252–261.
- [6] J. Garcia-Frias, Y. Zhao, and W. Zhong, "Turbo-like codes for transmission of correlated sources over noisy channels," *IEEE Signal Process. Mag.*, pp. 58–66, Sept. 2007.
- [7] F. Daneshgaran, M. Laddomada, and M. Mondin, "LDPC-based channel coding of correlated sources with iterative decoding," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 577–582, Apr. 2006.
- [8] I. Shahid and P. Yahampath, "Distributed joint source-channel coding of correlated binary sources in wireless sensor networks," *IEEE 8th Intl.* Symp. Wireless Comm. Systems, pp. 236–240, Nov. 2011.
- [9] M. Sartipi and F. Fekri, "Distributed source coding using short to moderate length rate-compatible LDPC codes: The entire Slepian-Wolf region.," *IEEE Trans. Commun.*, vol. 56, no. 3, pp. 400–411, 2008.
- [10] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, pp. 1727–1737, Oct 2001.
- [11] Tom Richardson and Rudiger Urbanke, Modern Coding Theory, Cambridge University Press, 2008.
- [12] V. Kumar and O. Milenkovic, "On unequal error protection of LDPC codes based on plotkin-type constructions," *IEEE Trans. Commun.*, vol. 54, no. 6, pp. 994–1005, 2006.
- [13] N. Rahnavard, H. Pishro-Nik, and F. Fekri, "Unequal error protection using partially regular LDPC codes," *IEEE Trans. Commun.*, vol. 55, pp. 387–391, Mar 2007.
- [14] T. Richardson, A. Shokrollahi, and R. Urbanke, "Design of capacity-approaching irregular low-density-parity-check codes," *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 619–637, Feb. 2001.
- [15] M. Luby, G. Mitzenmacher, A. Shokrollahi, and D. Spielman, "Improved low-density parity-check codes using irregular graphs," *IEEE Trans. Inf. Theory*, vol. 47, pp. 585–598, Feb 2001.
- [16] Eran Sharon, Alexei Ashikhmin, and Simon Litsyn, "EXIT functions for binary input memoryless symmetric channels," *IEEE Trans. Commun.*, vol. 54, pp. 1207–1214, July 2006.
- [17] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density-parity-check codes for modulation and detection," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 670–678, April 2004.