On the interplay of sensing and erasure correction in opportunistic spectrum access

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***Abstract*—We address the interplay of sensing performance and the choice of an erasure correcting code used for the recovery of missing data occurring in opportunistic spectrum access when there is a collision with a primary user. The main result of this paper is the existence of an optimum functioning point on the receiver operating characteristic (ROC) curve [1], in terms of efficiency of the secondary spectrum reuse. This functioning point depends on the sensing performance and on code related parameters such as the code rate and the erasure correction capability of the code. It is a trade-off between using at best the opportunities (with a small false alarm probability P_{FA}), the number of collisions experienced (a small non detection probability P_{ND}) and the erasure correction capability of the code, characterized by its minimum distance d, whose cost is the necessary redundancy we must introduce to perform erasure correction (*i.e.* the code rate).**

I. Introduction

Among the new dynamic access schemes designed for a more efficient usage of the spectrum resource [2], secondary use is characterized by the absence of coordination between a primary user (PU) and one or more secondary users (SU) who are allowed to access the channel when the PU is not transmitting (opportunistic spectrum access, OSA), or to transmit with very low power (underlay schemes). Our work is mainly related to the OSA approach.

As there is no coordination between PU and SU, the latter needs to listen the channel to detect PU's idle periods; this sensing operation must be performed as quickly as possible in order to seize opportunities as they appear, but suffers from two kinds of errors:

- False alarm when there is no PU while the detection stage decides there is one; as a consequence the SU experiences a missed opportunity to use the channel.
- Conversely, it may happen that there is actually a PU but it is not detected by the SU. The result is a collision between PU and SU and a loss of data of both users.

Because of the possible loss of packets during collisions the SU's link can be modelled as an erasure channel. As a return channel would worsen the problem (the SU would have to find another opportunity to send a retransmission request) the use of forward error codes for erasure correction has already been envisioned in [3]. The solution described therein for distribution of multimedia contents is based on LT codes or their generalization [4], [5]. These long codes (some thousands of information packets, 6000 in the above papers) are very interesting for the wireless internet and broadcast applications. However, for the scenario of secondary user network using OSA, short codes may be of interest and we explore these codes [6], especially since [7] has shown that their performance can be greatly improved by the proper choice of the parity check matrix. Another difference is that we consider a single frequency band where [3] assumes that SU can opportunistically access a number of different sub-bands.

Assuming a *On/Off* model for the PU's activity, we present some results on the respective effects of sensing, *i.e.* finding a good trade-off between false alarm probability and non detection probability, and two code parameters, namely minimum distance and code rate. In particular we show there exists an optimum functioning point on the receiver operating characteristic (ROC) curve for the efficiency of secondary reuse of the spectrum.

The rest of the paper is organized as follows: The system model is described in section II. We give a short reminder of the way parity check equations can be applied on a packet basis, and how they can be used for recovery of missing packets. The metric used to compare fairly different erasure correcting codes operating in different channel occupancy rates is defined in section III, and their performance evaluation by simulations is given in section IV. Section V is devoted to concluding remarks and perspective.

II. System model

Measurements reported in [8] have shown that PU's activity as a succession of active and idle periods, as sketched in fig. 1, can be modelled with a two-state Gilbert model [9], [10] with given transition probabilities between state *On* (the PU is transmitting) and state *Off* (PU is idle) as depicted in fig. 2. We denote p_1 the transition probability from state *On* to

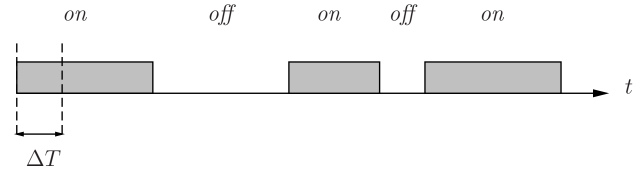

Fig. 1. Time activity of the PU

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state *Off*, conversely p_2 is the transition probability from state *Off* to state *On*. The steady state probabilities P_{on}, P_{off} of the channel to be active or idle are then given by the classical formula :

$$P_{on} = \frac{p_2}{p_1 + p_2}, \quad P_{off} = \frac{p_1}{p_1 + p_2}$$

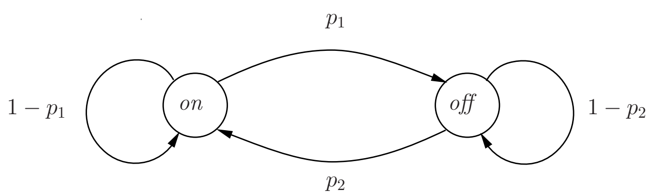

Fig. 2. Simple On/Off model

The secondary users need to listen continuously to the channel in order to detect the idle time slots, and sensing is required for this purpose. A part T_s of the time-slot is reserved for sensing the channel, the remainder $T - T_s$ will be used for SU's transmission if no PU is detected.

All packets are protected at physical layer by error correcting codes but in case of opportunistic spectrum access (OSA), some packets may be lost due to a higher level of interference during a collision. The corresponding SU's packets will be considered as erased.

In order to address the problem of missing packets we introduce an erasure correcting code $\mathcal{C}(n, k)$ operating at the application layer. The code is defined by a set of parity check equations (the rows of the parity check matrix) whose graphical representation is called a Tanner graph. Such parity check equations can be extended to packets if we assume that the SU data are encoded as packets $P_1, P_2, P_3, ...P_n$ such that all bits i of the packets constitute a codeword of the given code $\mathcal{C}$, as sketched in fig. 3.

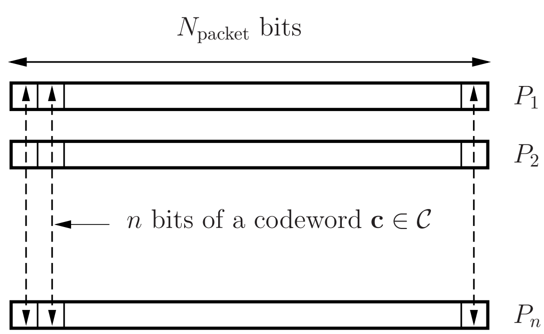

Fig. 3. Packet level coding

If we denote $\mathbf{H}$ a parity check matrix of the code $\mathcal{C}$, the parity check equation i has the form $\sum_{j=1}^{n} h_{i,j} P_j = \mathbf{0}$, so that if there is only one missing packet involved in this equation, it can be recovered by XOR-ing the other packets involved in this equation, as depicted in fig. 4 in a simple example.

We apply iteratively this operation until there is no more erasure or until we cannot find an equation with only one erased packet. The performance of the code is measured by

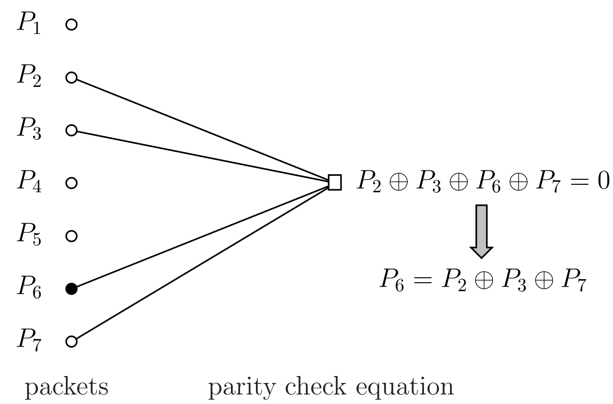

Fig. 4. Erased packet P_6 can be recovered from a parity check equation

the probability that all erasures are not completely recovered (a codeword recovery failure), denoted p_f, and the residual erasure probability after decoding, denoted p_r.

We have used short codes for erasure recovery. The preference of short codes for erasure recovery over long codes (digital fountain, LT, Raptor etc) for OSA has already been discussed in [6]. Moreover, some analytical formulas to compute the quantities p_f, p_r were derived in this reference, under the assumption that erasures were considered as uniformly distributed with a given probability p. We address here the case where erasures distribution is based upon the PU traffic model which is On/Off model.

III. The Efficiency

We have now to define some criterion to compare fairly erasure correction performance of different short codes (Golay, BCH, ...). The intuitive figure of merit would be the rate (number of data packet per unit time) of the SU, and depends on several parameters:

- It depends on the PU's activity. The greater the P_{on} probability the smaller the available periods the SU can exploit to send its packets.
- It depends on the sensing impairments: false alarm probability P_{FA} has as consequence missed opportunities and a lower number of transmitted packets; non detection probability P_{ND} will affect the collision probability with the PU and the number of erasures to be recovered. The two extreme cases are $P_{FA} = 0$ which means SU will always access the channel at the cost of high number of collisions with PU, and conversely $P_{FA} = 1$ which means no collision with PU since $P_{ND} = 0$, at the cost of no efficiency since the SU will never access the channel.
- The last set of parameters is related to the code used to recover erasures. The minimum distance d is relevant because we know that the code could potentially correct all erasure patterns of at most $d - 1$ erasures. This can be shown simply by rewriting the Singleton bound for a code with minimum distance d as $d = n - (k + i) + 1$, $i \geq 0$, with $i = 0$ when the code is minimum distance separable (MDS). If $\mathbf{x}$, $\mathbf{y} \in \mathcal{C}$ are two codewords having $k + i$

equal components then their (Hamming) distance would be $d(\mathbf{x},\mathbf{y}) \leq n-(k+i) < d$, which is impossible unless $\mathbf{x}=\mathbf{y}$. It means that $k+i$ components of a codeword is sufficient to recover the whole codeword, that is to say it is theoretically possible to recover up to $n-(k+i)=d-1$ erasures.

The code rate k/n is also important, because it is the cost (as redundancy in transmitted packets) to be paid for the possibility of erasure recovery.

- Last, the choice of the parity check matrix is also a degree of freedom that has impact on the performance of erasure correction, it impacts the decoding complexity.

It is possible to define a criterion that makes obvious the effects of these parameters. First, we introduce a reference corresponding to an ideal detector ($P_{FA}=0$ and $P_{ND}=0$). In this case the SU would be able to take advantage of all idle periods of the PU; within a number $t \gg 1$ of consecutive time-slots, an average $t \times P_{off}$ are free so that the average number of information packets correctly transmitted is given by

$$N_p(ideal) = t \times P_{off} \tag{1}$$

This is the best that can be achieved by a SU with an ideal detector as there is no redundant packet (there is no collision) and all opportunities are used (no missed opportunities). When we take into account sensing impairments, the average number of packets transmitted by SU over $t \gg 1$ time slots is now equal to $N_p(real)$, which is given by:

$$N_p(real) = t \times \Big(P_{on}\, P_{ND} + P_{off}\,\left(1-P_{FA}\right)\Big) \tag{2}$$

Part of these packets transmitted while PU is transmitting are collisions, so we have the collision probability p conditionally to SU's transmission:

$$p = \frac{P_{on}\, P_{ND}}{P_{on}\, P_{ND} + P_{off}\,\left(1-P_{FA}\right)} \tag{3}$$

The average number of transmitted codewords is then equal to $N_p(real)/n$, and $(1-p_f)$ is the fraction of these codewords which is entirely recovered after decoding. Last, we take into account the code rate, that means we have only k information packets for n transmitted packets. The average number of received information packets during $t \gg 1$ is now

$$N_p(info) = k \times \frac{N_p(real)}{n} \times (1-p_f)$$

so that the efficiency $\eta = N_p(info)/N_p(ideal)$ is given by:

$$\eta = \frac{1}{P_{off}}\,\frac{k}{n}\Big(P_{on}\, P_{ND} + P_{off} \times \left(1-P_{FA}\right)\Big)\,(1-p_f) \tag{4}$$

A first observation is the good agreement between simulated p_f and p_r with theoretical values obtained with the help of the two variable polynomial enumerator already introduced in [6], this polynomial enumerates erasure patterns that are not recovered after decoding (dead-end sets): $T(x,z) = \sum_{i,j=0}^{n} T_{i,j}\, z^j x^i$, where $T_{i,j}$ is the number of non recoverable erasure patterns of initial size i and final size j. From this polynomial we can derive the approximations:

$$p_r \approx \frac{1}{n}\sum_{i\geq 0}\Big\{\sum_{j>0} j\, T_{i,j}\Big\} p^i (1-p)^{n-i} \tag{5a}$$

$$p_f \approx \sum_{i=0}^{n} T_i\, p^i (1-p)^{n-i}, \text{ with } T_i = \sum_{j>0} T_{i,j} \tag{5b}$$

As we work with small to moderate code length, the first few coefficients of the polynomial can be obtained easily, but, in most cases, a complete computation of the enumerator would take too long because of the explosion of the number of erasure patterns to test. We have computed this enumerator for the BCH(31,21) code and obtain:

$$\begin{aligned} T(x,z) = {} & 19z^3x^3 \\ & + \left(220z^3 + 1084z^4\right)x^4 \\ & + \left(216z^3 + 7659z^4 + 19569z^5\right)x^5 \\ & + \left(7607z^4 + 79849z^5 + 183833z^6\right)x^6 \\ & + \left(2025z^4 + 69062z^5 + 425698z^6 + 1097005z^7\right)x^7 \\ & + \ldots \end{aligned}$$

Figure. 5 compare prediction using analytical formulas (5a) and (5b) with simulation results for this BCH code, for $p \leq 0.1$. The discrepancy for greater values of p comes from the fact we have only the first coefficients of the polynomial $T(x,z)$. This makes untractable an analytical evaluation of the efficiency using (4) and (5b).

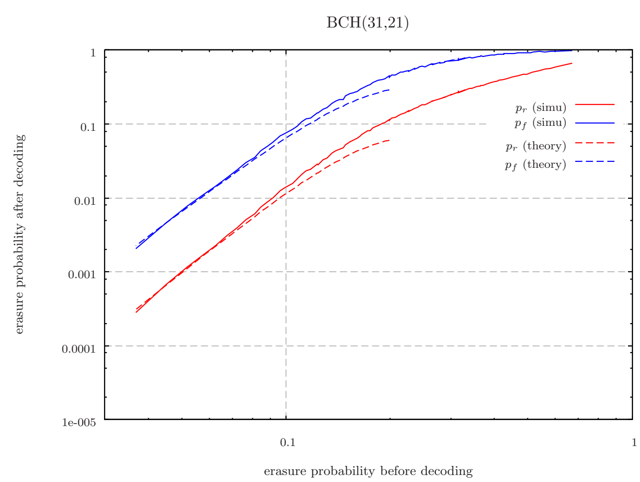

Fig. 5. Erasure recovery performance of the BCH(31,21) code. $p_1 = 0.1, p_2 = 0.2$, $SNR = -10$dB, $N_s = 500$ samples

IV. Evaluation of efficiency by simulations

Several simulations corresponding to the scenario described in the section II have been performed in order to assess the way different parameters have an effect on efficiency as defined in (4).

As regards sensing, we consider only energy detection with different number of samples N_s. Several short codes have been simulated, having different code rate, code length and minimum distance, that is to say different erasure correction capabilities. Last, different PU's activity probabilities P_{on} have been simulated.

A. Effect of sensing related parameters

We first show how the choice of a functioning point $(P_{FA}, 1 - P_{ND})$ on the ROC curve can impact the performance of the SU.

As expected the number of samples N_s has a big effect on the efficiency. This can be explained with the help of fig. 6: for a same target value of P_{ND}, the optimum false alarm probability decreases as N_s increases, that is to say the SU loses less opportunities to access the channel. As a consequence the collision probability p of (3) decreases also.

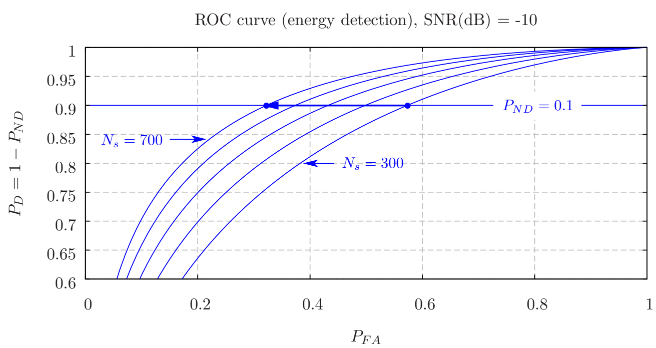

Fig. 6. Changing the number of samples: $N_s = 300, 400, \dots 700$

Figure 7 depicts simulation results for Hamming(15,11) code. We observe that efficiency η depends on P_{FA}. When

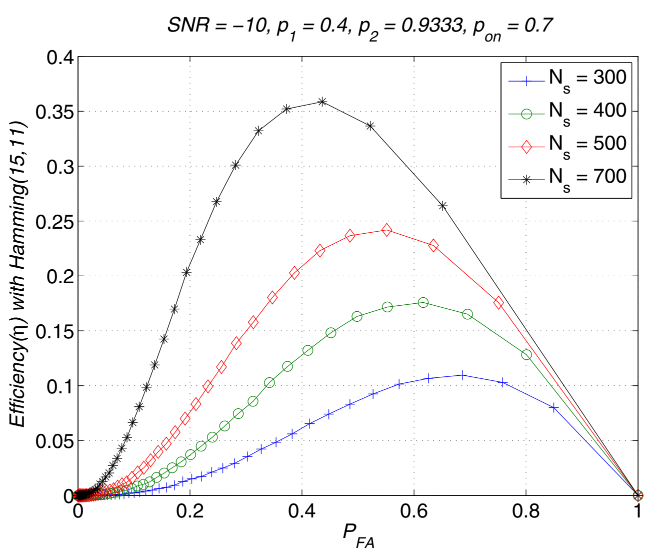

Fig. 7. Efficiency with Hamming(15,11) code, for several values of N_s

$P_{FA} \approx 0$, the SU access the channel all the time, with a collision every time the PU is active, and the code (with a modest error correction capability since its minimum distance is $d = 3$) cannot recover the too many erasures. As a result, the SU has very few successful accesses to the channel. Likewise, when $P_{FA} \approx 1$, the efficiency is close to 0, the reason being that the SU never access the channel. The maximum efficiency is the result of two contradictory effects: as P_{FA} increases some opportunities will be missed by the SU, but the number of collisions (erasures) diminishes also so that the code is able to recover erasures. If P_{FA} is too high the efficiency will suffer mainly of a reduced number of access to the channel and decreases.

B. Effect of PU's activity

Although we have defined the efficiency with respect to the PU's *Off* probability to define the maximum throughput the SU could achieve without sensing impairments, PU's activity probability P_{on} does have an effect on the SU's efficiency. This is depicted in fig. 8 which shows the very different behaviour of the efficiency for $P_{on} = 0.1, 0.3, 0.5, 0.7$, for the same Hamming(15,11) code. Best efficiency is achieved for $P_{on} = 0.1$ for $P_{FA} \simeq 0$ and decreases as P_{FA} increases, which is understandable: the SU uses all available opportunities without experiencing too many collisions as the PU is rarely active in the channel. As P_{on} increases, the trade off between accessing the channel (small value of false alarm probability) and recovering data lost in collisions (small non detection probability) is again the limiting factor on efficiency.

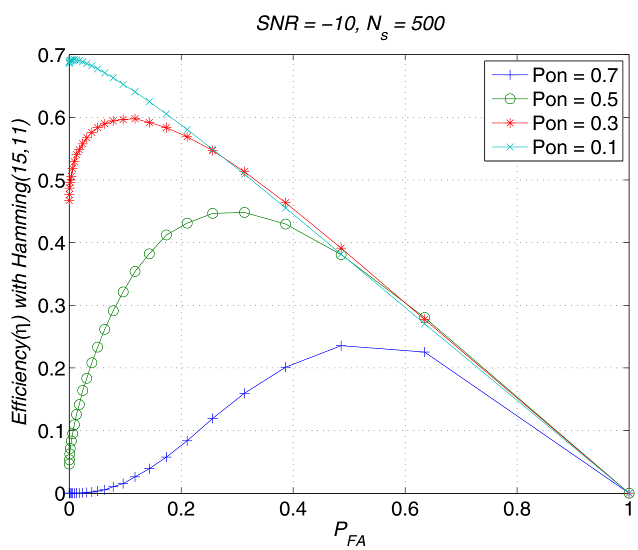

Fig. 8. Efficiency with Hamming(15,11)

C. Influence of the rate and minimum distance of the code

We now address the influence of the code used to recover the erasures. We observe two different behaviours depending on whether the PU's activity is high or not.

When PU is often active, for instance $P_{on} = 0.7$, we observe two interesting phenomena. First, there is not a great gap in efficiency between the Golay(24,12) code with ordinary parity check matrix and a Hamming(15,11) code. Both codes achieve a near 25% efficiency, despite their different erasure correction capabilities due to different minimum distances, $d = 8$ for the Golay code to be compared with $d = 3$ for the Hamming code. In fact the limiting factor is the code rate, since working with the ordinary parity check matrix of the Golay code leaves several erasure patterns of size less than $d-1 = 7$ not recovered [7], [6]. Things changes greatly when using the extended parity check matrix H_e given in [7], the efficiency raises to above 40% because we can now recover all erasure patterns of size up to $d-1$, as depicted in fig. 9.

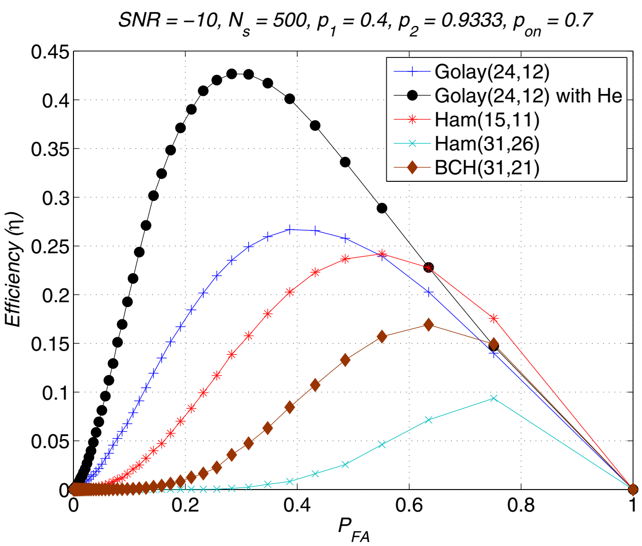

Fig. 9. Comparison of several codes, $P_{on} = 0.7$

For lower PU's transmitting rate, for instance $P_{on} = 0.3$, using the Golay code with extended parity check matrix allows a small improvement in efficiency for small values of P_{FA}. The number of collisions is higher because P_{ND} is greater but erasure recovery is more powerful. Nevertheless, the same efficiency of 60% can be achieved with a simple Hamming(15,11) code. It is worth noticing that the maximum efficiency of the Golay code is greater than 1/2 (code rate); this can be explained as follows: for small values of P_{FA} the SU access more often to the channel than what could be expected without collisions, and the erasures can be recovered because the code is powerful.

V. Conclusion

We have addressed the interplay of sensing and choice of an erasure correcting code when the PU's activity is modelled by a *On/Off* model. We have defined a metric (the efficiency) so that different codes having different rates can be compared fairly. The main result is the existence of an optimum functioning point on the ROC curve of the detector, and we explained this result as a trade-off between secondary access to the channel (depending on the parameter P_{FA}) and the collision rate experienced (the relevant parameter is P_{ND}), the choice of an erasure code having an impact on the possibility to recover a given number of erasures, at the expense of the redundancy we must introduce. Furthermore,

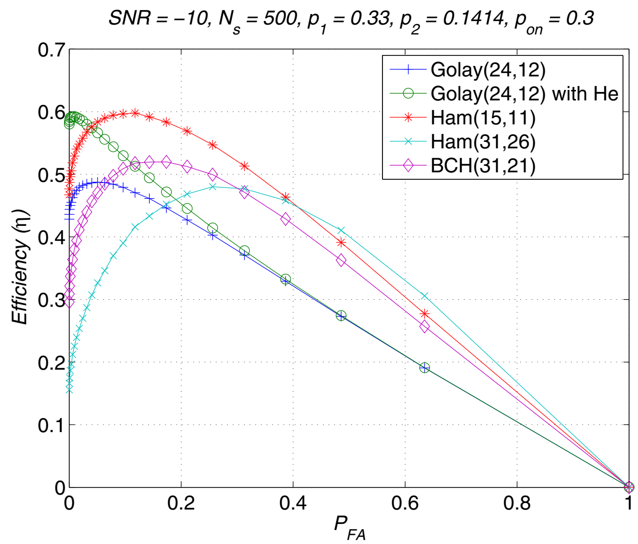

Fig. 10. Comparison of several codes, $P_{on} = 0.3$

we have seen how the erasure correcting code can be chosen with respect to PU's transmission probability P_{on}.

The results with the Golay(24,12) code confirm the importance of the choice of a good parity check matrix (*i.e.* with redundant parity check equations) for implementing the iterative erasure recovery on the associated Tanner graph. The problem of deriving the minimum set of parity checks so that the decoding on the Tanner graph achieves the recovery of all erasure patterns of size up to $d-1$ seems to be of great interest.

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