Spectrum Sharing in Cognitive Radio Systems: Ergodic and Outage Capacities

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***Abstract*—In this paper, we obtain the resource allocation policies pertaining to the ergodic and outage capacity notions in spectrum-sharing cognitive radio systems where the transmission parameters of the secondary users are adaptively changed based on the availability of channel state information (CSI) of the secondary user link, and soft-sensing information (SSI) about the activity of the licensed-band (primary) user as obtained from the sensing detector at the secondary user's transmitter. Assuming availability of SSI and CSI statistics at the secondary transmitter and considering operation under constraints on average received-interference and peak transmit-power, we investigate two different capacity notions of spectrum-sharing fading channels, namely, ergodic and Outage, and obtain their corresponding optimal power allocation policies. We also sustain our theoretical results by numerical and simulation analysis.**

I. Introduction

Sharing the under-utilized licensed spectrum band by unlicensed users is the main idea of spectrum-sharing cognitive radio (SS-CR) systems [1]. A classical SS-CR system includes several secondary users (unlicensed users) that communicate over the same spectrum band originally assigned to existing licensed users (primary users). In such system, two important issues must be addressed to avoid performance degradation for the primary users (PUs) and maximize the throughput performance of the secondary users (SUs): (*i*) the aggregate interference at the primary receivers (PRs) and (*ii*) the exactness of the sensing outcome with respect to the activity level of PUs in the shared spectrum band [1].

Regarding the performance evaluation and design of SS-CR systems, using the appropriate capacity metric has significant importance. The ergodic capacity, which considers the maximum average achievable rate over all fading states without any constraint on delay, can be used as a long-term throughput measure in these systems [2], [3]. Hence, the achievable transmission rate in this case could be very low or even zero in severe fading conditions. However, in SS-CR systems, by imposing constraints on the interference generated by the SUs while adhering to the PUs' requirements, it is obvious that outage is unavoidable. Hence, for delay-sensitive applications, outage capacity [4] is a more appropriate metric. In outage capacity (also known as delay-limited capacity), using *channel inversion* technique [5], the SU can transmit at higher power levels in weak channel states to guarantee a constant rate at the receiver all the time. In this regard, the delay-limited capacity of SS-CR systems under different types of power and interference constraints was investigated in [6], considering availability of the channel state information (CSI) pertaining to the SU link and the one corresponding to the interference channel between the secondary transmitter (ST) and PR, both at the ST. It is noteworthy that said availability of the CSI pertaining to the interference channel may not always be a practical assumption for CR systems, e.g., WRAN standard [7].

As aforementioned, CR technology provides the ability of sensing the environment in which it operates and adapts the transmission parameters such as power, according to the radio resource variations in time and space [2], [3]. The sensing ability is provided by the sensing detector, mounted at the SU's equipment, which scans the spectrum band for a specific time. Then, the activity statistics of the PU's signal in the shared spectrum band is calculated [8]. According to this soft-sensing information (SSI), if the presence of PU is not probable, this will imply a safe opportunity for SUs to occupy the licensed spectrum band. In this regard, the mentioned sensing statistics about the PU activity has been used by the authors in [2] and [8] to adaptively control the transmit power at the ST while the interference inflicted at the secondary receiver due to the PU transmission was assumed negligible.

In this work, considering that the power of the ST in a SS-CR system is adaptively controlled based on SSI about the PU's activity, and CSI pertaining to the secondary link, we investigate ergodic and outage capacities in spectrum-sharing systems. It is worth noting that a specific distribution to model the primary link interference at the SU receiver is considered in this paper. Considering the constraints on the average interference at the PR and on the peak transmit power of the ST, we first study the ergodic capacity of the SU's link in fading environments and derive the associated optimal power allocation policy. Then, we obtain the power allocation policy under outage probability constraint and investigate the achievable capacity with such transmission policy in fading environments.

The remainder of this paper is organized as follows. First, the SS-CR system and channel models are described in section II. Then, the ergodic capacity of the SU's fading channel is presented in section III. In section IV, we investigate the outage capacity of fading channels under the above-mentioned resource constraints. Numerical results followed by concluding remarks are presented in sections V and VI, respectively.

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II. SS-CR System and Channel Models

A classical spectrum-sharing cognitive radio (SS-CR) system with a pair of primary/secondary transceivers: (PT, PR) and (ST, SR), as shown in Fig. 1 [9]. The SU is allowed to use the spectrum occupied by the PU as long as it adheres to the predefined interference limit at the PR. The link between ST and SR is assumed to be a discrete-time flat fading channel with instantaneous gain $\sqrt{\gamma_{\mathrm{s}}}$. In this paper, we assume that perfect knowledge of $\sqrt{\gamma_{\mathrm{s}}}$ is available at the SR and provided to the ST through a no-delay error-free feedback channel. The channel gain between ST and PR is defined by $\sqrt{\gamma_{\mathrm{p}}}$ and the one between PT and ST by $\sqrt{\gamma_{\mathrm{m}}}$. Channel power gains, γ_{s}, γ_{p} and γ_{m} are independent. We assume γ_{s} has unit-mean distribution[1] and consider exponential distributions for γ_{p} and γ_{m} with means that depend on the distances between the associated nodes ($\frac{1}{d_{\mathrm{p}}^2}$ for γ_{p} and $\frac{1}{d_{\mathrm{m}}^2}$ for γ_{m}). Moreover, the PU's interference and the additive noise at the SR are considered as two zero-mean Gaussian random variables with different variances, δ_{p}^2 and δ_{n}^2, respectively.

Regarding the PU link, we consider a stationary block-fading channel with coherence time T_{c}. It is also assumed that the PT uses a Gaussian codebook with average transmit power P_{t}, and that the PU's activity follows a block-static model with T_{c} block period. This implies that the PT remains inactive (OFF state) with probability α or active (ON state) with probability $\bar{\alpha} = 1 - \alpha$, in T_{c} time periods.

A spectrum sensing detector (Fig. 1) is mounted on the ST to assess the PU's activity state in the shared spectrum band. The sensing detector scans the frequency band originally assigned to the PU and calculates a single sensing metric, ξ. We consider that the statistics of ξ, conditioned on the PU's activity being in ON or OFF state, are known a priori to the SU's transmitter. We define the probability density functions (PDF) of ξ given that the PT is ON or OFF by $f_1(\xi)$ and $f_0(\xi)$, respectively. Notice that conditioned on the PT being ON or OFF, ξ is a sum of independent and identically distributed random variables and distributed according to Chi-square PDF with N degrees of freedom, where N is the number of observation samples in each sensing interval [10]. Accordingly, under "PU is ON" condition, ξ follows a noncentral Chi-square distribution with variance $\delta^2 = 1$ and non-centrality parameter μ [2] [11]:

$$f_1(\xi) = \frac{1}{2}\left(\frac{\xi}{\mu}\right)^{\frac{N-2}{4}} e^{-\frac{\mu+\xi}{2}} I_{N/2-1}\left(\sqrt{\mu\xi}\right), \tag{1}$$

where $I_\nu(\cdot)$ is the ν^{th}-order modified Bessel function of the first kind [12]. Similarly, under the "PU is OFF" condition, ξ will be distributed according to central Chi-square PDF as:

$$f_0(\xi) = \frac{1}{2^{N/2}\Gamma(N/2)} \xi^{N/2-1} e^{-\frac{\xi}{2}}, \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function [12]. These sensing statistics can be used by the ST to optimally adjust its transmit power while satisfying the interference constraint at the PR. Given that transmission pertaining to the SU should not harm the communication process of the PU, we impose constraints on (i) the average interference-power inflicted at the PU's receiver when the PU is ON and (ii) the peak transmit-power of the SU. These constraints are defined as

$$\mathcal{E}_{\gamma_{\mathrm{s}},\,\xi,\,\gamma_{\mathrm{p}}}\left[S(\gamma_{\mathrm{s}},\xi)\gamma_{\mathrm{p}}\middle|\mathrm{PU\,is\,ON}\right] \leq W, \tag{3}$$

$$S(\gamma_{\mathrm{s}},\xi) \leq Q, \;\; \{\forall\, \gamma_{\mathrm{s}}, \gamma_{\mathrm{p}}, \xi\}, \tag{4}$$

where $S(\gamma_{\mathrm{s}},\xi)$ is the transmit power of the SU and W, Q denote the interference and peak power limit values, respectively. Furthermore, $\mathcal{E}_{\gamma_{\mathrm{s}},\xi,\gamma_{\mathrm{p}}}[\cdot]$ defines the expectation over the joint PDF of random variables γ_{s}, ξ and γ_{p}.

[1]The expressions derived hereafter can be applied for any fading distribution. In the numerical results section, however, we will assume $\sqrt{\gamma_{\mathrm{s}}}$ to be distributed according to Rayleigh, Nakagami and Lognormal functions.

[2]Note that μ can be obtained in terms of the ratio of PT's signal energy to noise spectral density, as detailed in [10].

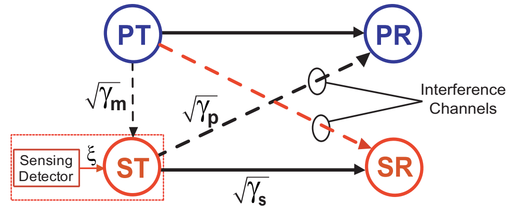

Fig. 1: Spectrum-sharing CR system model.

III. Ergodic Capacity

The ergodic capacity of single-user time-varying channel is studied in [13]. Considering the average transmit power to be constrained, the ergodic capacity of a fading channel with CSI at both the transmitter and the receiver is obtained in [5] and [14]. The corresponding optimal power allocation is a water-filling strategy over the fading states. The capacity of fading channels in a SS-CR system is limited by the interference and transmit power constraints in a dedicated channel bandwidth. In our case, the ST uses the CSI of the secondary link and SSI in order to achieve optimum channel capacity under interference (3) and peak transmit-power (4) constraints. Considering availability of SSI and CSI pertaining to the secondary link at the ST, the ergodic capacity of the SU's link in fading environment under the above-mentioned constraints represents the solution to the following problem:

$$\frac{C_{\mathrm{er}}}{B} = \max_{S(\gamma_{\mathrm{s}},\xi)} \Bigg\{ \mathcal{E}_{\gamma_{\mathrm{s}},\xi}\left[\alpha \log\left(1 + \frac{S\left(\gamma_{\mathrm{s}},\,\xi\right)\gamma_{\mathrm{s}}}{\delta_{\mathrm{n}}^2}\right)\right] + \mathcal{E}_{\gamma_{\mathrm{s}},\xi}\left[\overline{\alpha} \log\left(1 + \frac{S\left(\gamma_{\mathrm{s}},\,\xi\right)\gamma_{\mathrm{s}}}{\delta_{\mathrm{n}}^2 + \delta_{\mathrm{p}}^2}\right)\right] \Bigg\}, \tag{5}$$

$$\text{Subject to} \;\; (3) \text{ and } (4). \tag{6}$$

To find the optimal power allocation policy, we adopt the Lagrangian optimization approach [2]. Thus, the Lagrangian objective function, L_{C}, can be formed based on the maximization problem in (5), which after taking the derivative of L_{C} with respect to $S(\gamma_{\mathrm{s}},\xi)$ and setting it to zero yields (7) under the necessary Karush-Kuhn-Tucker (KKT) conditions (corresponding to the constraints (3) and (4)) given by (8)-(10). Note that λ_1^{er}, $\lambda_2^{\mathrm{er}}(\gamma_{\mathrm{s}},\xi)$ and $\lambda_3^{\mathrm{er}}(\gamma_{\mathrm{s}},\xi)$ are the Lagrangian

$$\left(\alpha\frac{\gamma_{\mathrm{s}}f_0\left(\xi\right)}{\delta_{\mathrm{n}}^2+S\left(\gamma_{\mathrm{s}},\,\xi\right)\gamma_{\mathrm{s}}}+\bar{\alpha}\frac{\gamma_{\mathrm{s}}f_1\left(\xi\right)}{\delta_{\mathrm{n}}^2+\delta_{\mathrm{p}}^2+S\left(\gamma_{\mathrm{s}},\,\xi\right)\gamma_{\mathrm{s}}}-\lambda_1^{\mathrm{er}}f_1\left(\xi\right)\right)f_{\gamma_{\mathrm{s}}}\left(\gamma_{\mathrm{s}}\right)+\lambda_2^{\mathrm{er}}\left(\gamma_{\mathrm{s}},\,\xi\right)-\lambda_3^{\mathrm{er}}\left(\gamma_{\mathrm{s}},\,\xi\right)=0. \tag{7}$$

$$\begin{aligned}B\left(Q,\lambda,\xi,\delta^2\right)\triangleq&\frac{\left(Q\lambda\left(2\delta^2+\delta_{\mathrm{p}}^2\right)-\bar{\alpha}\delta^2\right)f_1\left(\xi\right)-\alpha\left(\delta^2+\delta_{\mathrm{p}}^2\right)f_0\left(\xi\right)}{2Q\left(f_1\left(\xi\right)\left(\bar{\alpha}-Q\lambda\right)+\alpha f_0\left(\xi\right)\right)}\\&+\frac{\sqrt{\left(\alpha\left(\delta^2+\delta_{\mathrm{p}}^2\right)f_0\left(\xi\right)+\left(\bar{\alpha}\delta^2-Q\lambda\delta_{\mathrm{p}}^2\right)f_1\left(\xi\right)\right)^2+4\bar{\alpha}Q\lambda\delta^2\delta_{\mathrm{p}}^2\left(f_1\left(\xi\right)\right)^2}}{2Q\left(f_1\left(\xi\right)\left(\bar{\alpha}-Q\lambda\right)+\alpha f_0\left(\xi\right)\right)}.\end{aligned} \tag{14}$$

parameters[3].

$$\lambda_1^{\mathrm{er}}\left(\mathcal{E}_{\gamma_{\mathrm{s}},\xi|\mathrm{PU\ is\ ON}}\left[S\left(\gamma_{\mathrm{s}},\xi\right)-Wd_{\mathrm{p}}^2\right]\right)=0. \tag{8}$$

$$\lambda_2^{\mathrm{er}}S\left(\gamma_{\mathrm{s}},\xi\right)=0. \tag{9}$$

$$\lambda_3^{\mathrm{er}}\left(S\left(\gamma_{\mathrm{s}},\xi\right)-Q\right)=0. \tag{10}$$

The optimal transmit power $S\left(\gamma_{\mathrm{s}},\xi\right)$ can be 0, Q, or a value in the open interval $(0,Q)$.

1) $S(\gamma_{\mathrm{s}},\xi)=0$: Let the transmit power be 0 for some γ_{s} and ξ. In this case, equation (10) requires that $\lambda_3^{\mathrm{er}}=0$ and (9) implies $\lambda_2^{\mathrm{er}}\geq 0$. Substituting these conditions into (7) yields

$$\alpha\frac{\gamma_{\mathrm{s}}f_0\left(\xi\right)}{\delta_{\mathrm{n}}^2}+\bar{\alpha}\frac{\gamma_{\mathrm{s}}f_1\left(\xi\right)}{\delta_{\mathrm{n}}^2+\delta_{\mathrm{p}}^2}-\lambda_1^{\mathrm{er}}f_1\left(\xi\right)<0,$$

which, after further manipulation, simplifies to

$$\gamma_{\mathrm{s}}\leq A\left(\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right), \tag{11}$$

where the function $A\left(\lambda,\xi,\delta^2\right)$ is defined as

$$A\left(\lambda,\xi,\delta^2\right)\triangleq\frac{\lambda\delta^2\left(\delta^2+\delta_{\mathrm{p}}^2\right)f_1\left(\xi\right)}{\bar{\alpha}\delta^2f_1\left(\xi\right)+\alpha\left(\delta^2+\delta_{\mathrm{p}}^2\right)f_0\left(\xi\right)}. \tag{12}$$

2) $S(\gamma_{\mathrm{s}},\xi)=Q$: In this case, (9) requires that $\lambda_2^{\mathrm{er}}=0$ and (10) implies that $\lambda_3^{\mathrm{er}}\geq 0$, which when substituted into (7) yield

$$\alpha\frac{\gamma_{\mathrm{s}}f_0\left(\xi\right)}{\delta_{\mathrm{n}}^2+Q\gamma_{\mathrm{s}}}+\bar{\alpha}\frac{\gamma_{\mathrm{s}}f_1\left(\xi\right)}{\delta_{\mathrm{n}}^2+\delta_{\mathrm{p}}^2+Q\gamma_{\mathrm{s}}}-\lambda_1^{\mathrm{er}}f_1\left(\xi\right)>0,$$

which can further be simplified according to

$$\gamma_{\mathrm{s}}\geq B\left(Q,\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right), \tag{13}$$

where the function $B\left(Q,\lambda,\xi,\delta^2\right)$ is defined in (14).

3) $0<S(\gamma_{\mathrm{s}},\xi)<Q$: For such interval for $S(\gamma_{\mathrm{s}},\xi)$, from the conditions in (9) and (10), it follows that $\lambda_2^{\mathrm{er}}=\lambda_3^{\mathrm{er}}=0$. Substituting these conditions into (7) yields

$$\alpha\frac{\gamma_{\mathrm{s}}f_0\left(\xi\right)}{\delta_{\mathrm{n}}^2+S\left(\gamma_{\mathrm{s}},\xi\right)\gamma_{\mathrm{s}}}+\bar{\alpha}\frac{\gamma_{\mathrm{s}}f_1\left(\xi\right)}{\delta_{\mathrm{n}}^2+\delta_{\mathrm{p}}^2+S\left(\gamma_{\mathrm{s}},\xi\right)\gamma_{\mathrm{s}}}-\lambda_1^{\mathrm{er}}f_1\left(\xi\right)=0.$$

Then after simple manipulation, the optimal power adaptation policy for $0<S(\gamma_{\mathrm{s}},\xi)<Q$ can be expressed as,

$$S\left(\gamma_{\mathrm{s}},\xi\right)=\mathcal{P}\left(\gamma_{\mathrm{s}},\xi,\lambda_1^{\mathrm{er}},\delta_{\mathrm{n}}^2\right), \tag{15}$$

[3]Hereafter and for simplicity, we omit the random variables γ_{s} and ξ whenever it is clear from the context.

where the power function $\mathcal{P}\left(\gamma_{\mathrm{s}},\xi,\lambda,\delta^2\right)$ is defined as

$$\begin{aligned}\mathcal{P}\left(\gamma_{\mathrm{s}},\xi,\lambda,\delta^2\right)\triangleq&\frac{\alpha f_0\left(\xi\right)+\bar{\alpha}f_1\left(\xi\right)}{2\lambda f_1\left(\xi\right)}-\frac{\left(2\delta^2+\delta_{\mathrm{p}}^2\right)}{2\gamma_{\mathrm{s}}}\\&+\frac{\sqrt{\left(\left(\delta_{\mathrm{p}}^2\lambda-\bar{\alpha}\gamma_{\mathrm{s}}\right)f_1\left(\xi\right)+\alpha\gamma_{\mathrm{s}}f_0\left(\xi\right)\right)^2+4\alpha\bar{\alpha}f_0\left(\xi\right)f_1\left(\xi\right)\gamma_{\mathrm{s}}^2}}{2\lambda f_1\left(\xi\right)\gamma_{\mathrm{s}}}.\end{aligned} \tag{16}$$

According to the results in (11), (13) and (15), the optimal allocation policy for the SU's transmit power can be expressed according to (17), where the value of λ_1^{er} is such that both constraints in (6) are satisfied.

$$S\left(\gamma_{\mathrm{s}},\,\xi\right)=\begin{cases}0, & \gamma_{\mathrm{s}}<A\left(\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)\\ Q, & \gamma_{\mathrm{s}}>B\left(Q,\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)\\ \mathcal{P}\left(\gamma_{\mathrm{s}},\xi,\lambda_1^{\mathrm{er}},\delta_{\mathrm{n}}^2\right), & B\left(Q,\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)\geq\gamma_{\mathrm{s}}\\ & \quad\geq A\left(\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right).\end{cases} \tag{17}$$

As observed, the optimal power allocation, in (17), is partitioned into three regions depending on the variation of the SU channel state. In the first region, we do not use the channel as long as γ_{s} is below the threshold $T_1^{\mathrm{e}}=A\left(\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)$. In other words, transmission is suspended when the secondary channel is weak compared to threshold T_1^{e}. The second region is defined by the range $A\left(\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)\leq\gamma_{\mathrm{s}}\leq B\left(Q,\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)$, where the power allocation is related to the water-filing approach. Finally, a constant power equal to Q_{peak} is considered for the third region which corresponds to $\gamma_{\mathrm{s}}>T_2^{\mathrm{e}}=B\left(Q,\lambda_1^{\mathrm{er}},\xi,\delta_{\mathrm{n}}^2\right)$. The threshold values of the power allocation policy, T_1^{e} and T_2^{e}, are determined such that the interference constraint (3) is satisfied. Indeed, in the above transmission policy, the SU transmits with higher power levels in strong CSI, whereas it remains silent in weak CSI.

According to the power allocation in (17), the ergodic capacity expression of the secondary link under interference and peak transmit-power constraints can be expressed as shown in (18).

IV. Outage Capacity

For delay-sensitive applications, outage capacity is a more appropriate capacity notion. In outage capacity, the transmission rate is kept constant in all channel states by using channel inversion [5]. The latter technique inverts the channel fading to maintain a constant received power at the SU receiver.

$$\frac{C_{\mathrm{er}}}{B} = \underset{T_1^{\mathrm{e}} \leq \gamma_{\mathrm{s}} \leq T_2^{\mathrm{e}}}{\mathcal{E}_{\gamma_{\mathrm{s}},\xi}} \left[\alpha \log\left(1 + \frac{\mathcal{P}\left(\gamma_{\mathrm{s}}, \xi, \lambda_1^{\mathrm{er}}, \delta_{\mathrm{n}}^2\right)\gamma_{\mathrm{s}}}{\delta_{\mathrm{n}}^2}\right) + \overline{\alpha} \log\left(1 + \frac{\mathcal{P}\left(\gamma_{\mathrm{s}}, \xi, \lambda_1^{\mathrm{er}}, \delta_{\mathrm{n}}^2\right)\gamma_{\mathrm{s}}}{\delta_{\mathrm{n}}^2 + \delta_{\mathrm{p}}^2}\right)\right]$$
$$+ \underset{\gamma_{\mathrm{s}} \geq T_2^{\mathrm{e}}}{\mathcal{E}_{\gamma_{\mathrm{s}},\xi}} \left[\alpha \log\left(1 + \frac{Q\gamma_{\mathrm{s}}}{\delta_{\mathrm{n}}^2}\right) + \overline{\alpha}\log\left(1 + \frac{Q\gamma_{\mathrm{s}}}{\delta_{\mathrm{n}}^2 + \delta_{\mathrm{p}}^2}\right)\right]. \tag{18}$$

Making use of channel inversion technique, the delay of the transmission link is independent of the channel variations. However, in some fading channels, e.g., Rayleigh, the outage capacity is zero because of the severe fading conditions. Accordingly, by allowing some percentage of outage in deep fading states, called outage probability, we can achieve nonzero constant rate at the receiver. This nonzero outage capacity is referred to as truncated channel inversion with fixed-rate (*tifr*) capacity [8]. Moreover, the constant-rate that can be achieved with an outage probability less than a certain threshold is called outage capacity [13].

In SS-CR systems, the activity state of the PU can also yield outage onto the SU. Indeed, while the spectrum is occupied by the PU, the SU transmission must be suspended and, consequently, outage is experienced at the secondary link. Hence, the available information about the PU's activity can be used at the ST to control its transmit power such that a constant-rate with an outage probability less than a given threshold is provided at the SU receiver. Herein, the outage capacity of the SU when using available CSI and SSI at the ST is investigated. We consider a *tifr* policy that only suspends transmission when γ_{s} is less than a certain cutoff threshold: $\gamma_{\mathrm{s}} < A\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right)$. Accordingly, we express the power allocation policy as follows:

$$S\left(\gamma_{\mathrm{s}}, \xi\right) = \begin{cases} 0, & \gamma_{\mathrm{s}} < A\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right) \\ \dfrac{\sigma}{\gamma_{\mathrm{s}}}, & \gamma_{\mathrm{s}} \geq A\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right) \end{cases} \tag{19}$$

where λ^{out} and σ must satisfy the interference and peak transmit power constraints, (3) and (4), at equality:

$$W' = \iint\limits_{\gamma_{\mathrm{s}} \geq A\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right)} \frac{\sigma}{\gamma_{\mathrm{s}}} f_{\gamma_{\mathrm{s}}}\left(\gamma_{\mathrm{s}}\right) f_1\left(\xi\right) d\gamma_{\mathrm{s}} d\xi, \tag{20}$$

$$\frac{\sigma}{\gamma_{\mathrm{s}}} \leq Q, \quad \forall\, \gamma_{\mathrm{s}}: \quad \gamma_{\mathrm{s}} \geq A\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right), \tag{21}$$

where $W' = W\, d_{\mathrm{p}}^2$, with d_{p} denoting the distance between the ST and the PR. Moreover, from (21), the inequality $\sigma \leq QA\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right)$ must hold true.

In (19), the ST is allowed to transmit as long as γ_{s} exceeds a cutoff threshold $T_1^{\mathrm{o}} = A\left(\lambda^{\mathrm{out}}, \xi, \delta_{\mathrm{n}}^2\right)$. As observed in (19), the SU uses a higher power level in weak channel conditions, whereas in (17), the higher power strength is used in strong channel conditions.

The capacity under *tifr* transmission policy can be obtained by solving the following maximization problem:

$$\frac{C_{\mathrm{tifr}}}{B} = \max_{\lambda^{\mathrm{out}},\xi} \left\{ \left(\alpha \log\left(1 + \frac{\min\left\{\sigma,\, QT_1^{\mathrm{o}}\right\}}{\delta_{\mathrm{n}}^2}\right) + \overline{\alpha}\log\left(1 + \frac{\min\left\{\sigma,\, QT_1^{\mathrm{o}}\right\}}{\delta_{\mathrm{n}}^2 + \delta_{\mathrm{p}}^2}\right)\right) \times \Pr\left\{\gamma_{\mathrm{s}} \geq T_1^{\mathrm{o}}\right\} \right\}. \tag{22}$$

In (22), $\Pr\left\{\gamma_{\mathrm{s}} \geq T_1^{\mathrm{o}}\right\}$ is defined as $(1 - P_0)$, where P_0 denotes the percentage of time that the transmission remains in outage condition and is called outage probability. Using (19), the outage probability expression can be obtained as follows:

$$P_0 = 1 - \Pr\left\{\gamma_{\mathrm{s}} \geq T_1^{\mathrm{o}}\right\} = 1 - \iint\limits_{\gamma_{\mathrm{s}} \geq T_1^{\mathrm{o}}} f_{\gamma_{\mathrm{s}}}\left(\gamma_{\mathrm{s}}\right) f_1\left(\xi\right) d\gamma_{\mathrm{s}} d\xi. \tag{23}$$

On the other hand, to find the achievable capacity for a fixed P_0, the cutoff value λ^{out} must be determined so as to satisfy (23) and, consequently, the capacity in the case with P_0 probability of outage can be obtained by maximizing over all possible ξ and λ^{out}:

$$\frac{C_{\mathrm{out}}}{B} = \max_{\lambda^{\mathrm{out}},\xi} \left\{ \left(\alpha \log\left(1 + \frac{\min\left\{\sigma,\, QT_1^{\mathrm{o}}\right\}}{\delta_{\mathrm{n}}^2}\right) + \overline{\alpha}\log\left(1 + \frac{\min\left\{\sigma,\, QT_1^{\mathrm{o}}\right\}}{\delta_{\mathrm{n}}^2 + \delta_{\mathrm{p}}^2}\right)\right)(1 - P_0) \right\}. \tag{24}$$

V. Numerical Results

Herein, we provide numerical results for the ergodic and outage capacities studied in Sections III and IV, respectively. The SU channel variations are modeled through Nakagami (nak) with unit-mean and fading parameter $m = 2$, Rayleigh (ray) with unit-mean, and Log-normal (log) with several values for the standard deviation: $\mathcal{K} = 4, 6, 8$ dB. We assume the CSI of the secondary link to be available at the ST, through an error-free feedback channel. The interference channel gain $\sqrt{\gamma_{\mathrm{p}}}$ is also distributed according to Rayleigh PDF with unit variance, $d_{\mathrm{p}} = 1$. Furthermore, the sensing detector is assumed to calculate the sensing information metric in an observation time $N = 30$, and the non-centrality parameter in $f_1(\xi)$ is set to unity ($\mu = 1$). About the PU's activity, we consider that the PU remains active 50% of the time ($\alpha = 0.5$) and we set the PU's transmit power to $P_t = 1$. In the following, we assume $\delta_{\mathrm{p}}^2 = 0.5$ and $\delta_{\mathrm{n}}^2 = 1$.

In Figs. 2-4, we plot the ergodic, *tifr* and outage capacities (formulae (18), (22) and (24), respectively) as a function of

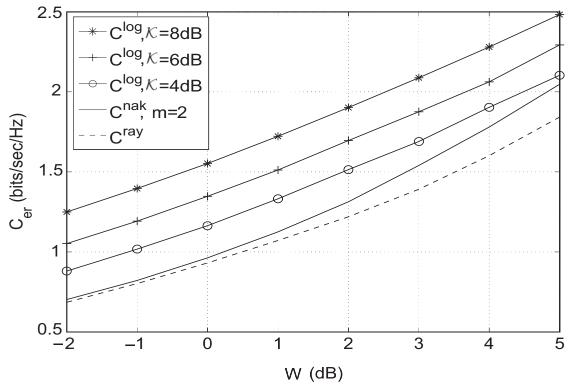

Fig. 2: Ergodic capacity in different fading channel environments for $\rho = 1.5$.

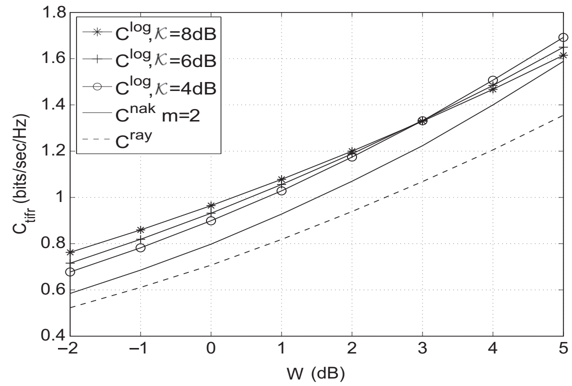

Fig. 3: Truncated channel inversion with fixed-rate (*tifr*) capacity in different fading channel environments for $\rho = 1.5$.

the average interference limit, W, with $\rho = 1.5$, where $\rho = \frac{Q}{W}$. In Fig. 4, the outage probability is given by $P_0 = 0.2$. By comparing the capacity plots in Figs. 2-4, we provide the following remarks and observations.

Considering Rayleigh and Nakagami ($m = 2$) fading channels, the capacity difference between these fading channels grows more in the *tifr* and outage capacities in comparison with the ergodic capacity. This implies that as the fading severity decreases (goes from Rayleigh to Nakagami), the capacity of the channel shows more improvement compared to adaptive channel transmission policies, i.e., *tifr* and outage. On the other hand, for the Log-normal fading case, as the standard deviation increases, the probability of being in deep fading states also increases, and consequently results in a large amount of capacity penalty for Log-normal fading channels with high $\mathcal{K}$ under *tifr* and outage transmission strategies.

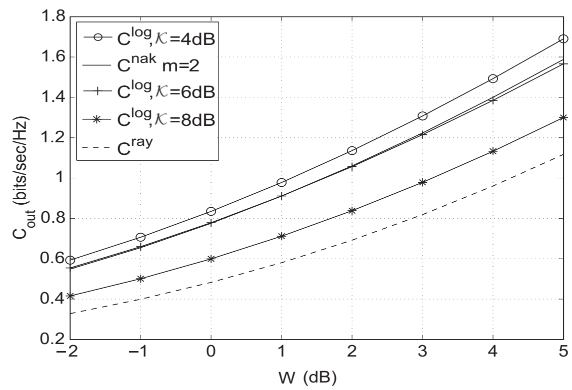

Fig. 4: Outage capacity in different fading channel environments for $\rho = 1.5$ and $P_0 = 0.2$.

VI. CONCLUSION

We investigated the ergodic and outage capacities of spectrum-sharing CR (SS-CR) systems operating under constraints on the average received-interference and peak transmit-power. We assumed that the transmission power of the secondary users (SUs) can be adapted based on availability of the SU's channel state information and soft-sensing information about the primary user's activity provided by the energy-based sensing detector at the SU transmitter. In particular, we investigated the effect of assuming delay constraint on the transmission policies pertaining to the two aforementioned capacity notions in SS-CR communication systems. Theoretical analysis besides numerical results and comparisons for different fading environments have shown that each capacity notion has some features that can be used according to the different services required in SS-CR systems.

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