MIMO Channel Model and Correlation Between Channel Matrix Elements in Multipath Channel

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Abstract- To clarify MIMO channel complex correlation for various cell types, we studied the correlation analytically and by computer simulation. We prepared a MIMO channel model consisting of base and mobile antenna configurations and a delay profile with arriving and incident wave angles (ξ_w, ζ_n) in line-of-sight (LOS) and non-line-of-sight (NLOS) paths. Then, the general correlation formula $\rho_{i-j,i'-j'}(bm)$ was derived for the model. It shows that $\rho_{i-j,i'-j'}(bm)$ depends on two factors: antenna configuration at mobile and base sites $(z_{i-i}, z_{i-j}, \psi_i)$ $_{i'}, \psi_{j-j'}$) and angle distribution $(\xi_c, \Delta \xi_n, \zeta_c, \sigma)$. It also shows that $\rho_{i-i,i'-i'}(bm)$ is expressed as the addition of a directive wave term $k \exp(j\Delta\theta_0)$ to the product of mobile site correlation $\rho_{i-i'}(m)$ and base site correlation $\rho_{i-i'}(b)$ without LOS, which are calculated independently of each other. So, the formulas for $\rho_{i-i'}(m)$ and $\rho_{i-i'}(b)$ were derived for uniform and Gaussian angular distributions. Furthermore, a computer simulation was performed and the simulated and theoretical values agreed well. Therefore, we can apply the $\rho_{i-i,i'-i'}(bm)$ to various cell types indoors and outdoors by combining them, so it is possible to calculate the correlation between MIMO channel matrix elements for a given cell and

Index Terms—MIMO channel model, MIMO channel correlation, multipath with LOS and NLOS, arriving and incident wave angles, uniform and Gaussian distributions.

I. INTRODUCTION

To support realtime multimedia communication, future mobile communications will require a high-bit-rate transmission system with high utilization of the frequency spectrum in multipath channels with line-of-sight (LOS) and non-line-of-sight (NLOS) paths [1]. Systems capable of fulfilling this requirement, with features such as multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) [2], have been studied extensively. MIMO is especially advantageous in high utilization of the frequency spectrum and has therefore been paid attention from this

viewpoint. In MIMO, transmission quality and capacity depend on the channel matrix property, which is evaluated from the correlation between the channel matrix elements, i.e., mutual channels on the path between antenna elements at base and mobile stations. Therefore, the channel matrix correlation must be clarified, and many such studies have been reported [3] [4]. As an analytical model that tries to produce the matrix with a given fixed spatial correlation between MIMO antenna elements, a stochastic MIMO channel model was analyzed on the basis of an independent and identically distributed (i.i.d) random matrix [5]. However, with the analysis method used in the model, it is difficult to interpret the channel situation directly and visualize the physical propagation in MIMO transmission studies. Moreover, MIMO can operate in a wide area such as indoors and outdoors with LOS and NLOS paths. However, there has been little analytical work on such as a situation, especially on channels with multiple paths between the base and mobile station antennas and with a LOS path. Therefore, this paper analytically clarifies MIMO channel correlation on the basis of the propagation mechanism for various cells and environments and presents confirmation of the derived formulas by computer simulation.

II. THEORY

A. MIMO channel model

MIMO systems are used in various areas, so the cells are called pico, micro, and macro cells. The MIMO channel model, which consists of a delay profile measured around the origins at both the base station and mobile station and the antenna coordinate systems are shown in Fig. 1. The delay profile has two angles (ζ_n, ξ_n) incident to multipath scattering and to the receiving point, except for path data with an ordinary delay profile. Moreover, each arriving wave expresses a representative wave, which is the peak value in each cluster. Assuming that the movement distance is within a few wavelengths, an example of a delay profile is shown in Fig. 1 with the following conditions.

i) The number of arriving waves is N+1, waves are independent of each other, and the nth-path wave is

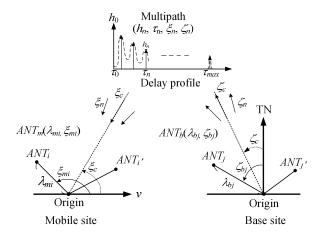


Fig. 1. MIMO channel model.

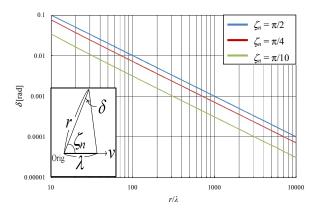


Fig. 2. Angular difference δ from ξ_n or ζ_n (for spacing λ from origin).

denoted by subscript n, where n=0 means a directive wave and n≥1 means no directive waves.

- ii) The waves have excess delay time τ_n relative to the shortest path between both origins and maximum excess delay time τ_{max} . The τ_n values are random from τ_{min} to τ_{max} .
- iii) The amplitude is h_n , the power ratio of the directive and nondirective waves is denoted by k (K in dB), and k=0 (K=- ∞ dB) means NLOS. Furthermore, the nondirective wave's power is normalized to 1.
- iv) Whether a wave's arriving angle at the mobile or base station is also the incident angle to multipath scattering from the mobile or base station depends on whether the station is receiving or transmitting. Here, the angle at the mobile station is denoted by ζ_n and that at the base station is denoted by ζ_n . The ζ_n and ζ_n are angles from the mobile station's movement direction and from true north (TN) at the two origins, respectively. The arriving wave's initial phase is ϕ_n and the values are random from $-\pi$ to π .

On the other hand, the antenna coordinates use a polar coordinate system centered at the origin of each station, as shown in Fig. 1. For the base station, the coordinates are denoted by $ANT_b(\lambda_{bj}, \zeta_{bj})$, where λ_{bj} and ζ_{bj} mean the

radius normalized by wavelength λ and counterclockwise angle from TN for the *jth* antenna. For a mobile station, the coordinates are similarly denoted by $ANT_m(\lambda_{mi}, \xi_{mi})$. The origins are at mobile and base station antenna elements with i=1 and j=1, respectively.

In this paper, we assume that all antennas of each station have the same values of ξ_n and ζ_n , but strictly ξ_n and ζ_n differ slightly among antennas by value δ . The angular difference δ from ξ_n or ζ_n for the normalized distance (r/λ) between the origin and multipath scattering when the antenna is set at spacing λ away from the origin is shown in Fig. 2. This δ is less than 0.01 rad when $r/\lambda=100$. As shown later, the correlation is sensitive to ξ_n or ζ_n when the antenna is high and far away and when ξ_n and ζ_n have Gaussian distributions, but not so sensitive to ξ_n or ζ_n when r/λ is small and ξ_n and ζ_n are spread widely, as in the case with mobile stations or indoor cells.

B. Complex received signal of MIMO channel

Under the conditions described above, the MIMO channel composed of the *jth* base station antenna and *ith* mobile station antenna is denoted by MIMO ch(i-j) and the complex received signal level $E_{i-j}(t, f_c)$ is given by

$$E_{i-j}(t, f_c) = \sum_{n=0}^{N} h_n e^{j\theta_{i-j,n}}$$
 (1)

$$\theta_{i-j,n} = 2\pi [f_c \tau_n - f_m t \cos \xi_n - \lambda_{mi} \cos(\xi_n - \xi_{mi}) - \lambda_{bj} \cos(\zeta_n - \zeta_{bj})] + \phi_n,$$
 (2)

where $\theta_{i ext{-}j ext{-}n}$ is the *n*th path phase of MIMO $ch(i ext{-}j)$, f_c is the radio frequency, and f_m is the maximum Doppler frequency. Furthermore, the third and fourth terms of $\theta_{i ext{-}j ext{-}n}$ in brackets depend on the mobile and base station antenna constructions and the phase is different from that at each origin.

C. Derivation of correlation formula

a) General formula: We start by studying the general spatial-cross correlation between MIMO ch(i-j) and ch(i'-j'), that is the correlation between MIMO channel matrix elements: the complex correlation is denoted by $\rho_{i\cdot j,i'\cdot j'}(bm)$. With variables of x meaning $E_{i\cdot j'}(t, f_c)$ and those of y meaning $E_{i'\cdot j'}(t, f_c)$ obtained by (1), $\rho_{i\cdot j,i'\cdot j'}(bm)$ is expressed by

$$\rho_{i-j,i-j}(bm) = \frac{\langle x^*y \rangle}{[\langle x^*x \rangle \langle y^*y \rangle]^{1/2}}.$$
 (3)

Here, <> and * mean ensemble average and conjugate complex, respectively. Assuming that N is a large number under the delay profile in II-A, we start to calculate the denominator in (3). Each value <x*x> and <y*y> is k+1 since the nondirective wave's power is normalized to 1. Therefore, the denominator becomes k+I.

Next, we calculate $\langle x^*y \rangle$, i.e., $\langle E_{i\cdot j}(t,f_c)^* E_{i\cdot j}(t,f_c) \rangle$, in the numerator. Since τ_n and ϕ_n are random values, the $\theta_{i\cdot j,n}$ given by (2) becomes a random value in the range from $-\pi$ to π , moreover, $\theta_{i\cdot j,n}$ is independent of h_n . As a result, the sum of the product of $E_{i\cdot j}(t,f_c)^*$ and $E_{i\cdot j\cdot j}(t,f_c)$ remains

the same for the *nth* arriving wave on ch(i-j) and ch(i'-j'), but vanishes for all other differences for that, and we get (4), considering that the directive wave is deterministic within a few wavelengths through consideration of the correlation, where $\Delta\theta_n$ expresses the nth path phase difference between MIMO ch(i'-j') and ch(i-j).

$$\langle x^* y \rangle = \langle \sum_{n=0}^{N} h_n^2 \exp(j\Delta\theta_n) \rangle$$

$$= \langle h_0^2 \exp(j\Delta\theta_0) \rangle + \langle \sum_{n=1}^{N} h_n^2 \rangle$$

$$\times \langle \sum_{n=1}^{N} [\cos(2\pi z_{i-i'}\cos(\xi_n - \psi_{i-i'})) - j\sin(2\pi z_{i-i'}\cos(\xi_n - \psi_{i-i'}))] \rangle$$

$$\times \langle \sum_{n=1}^{N} [\cos(2\pi z_{j-j'}\cos(\zeta_n - \psi_{j-j'})) - j\sin(2\pi z_{j-j'}\cos(\zeta_n - \psi_{j-j'}))] \rangle$$

$$= k \exp(j\Delta\theta_0) + \rho_{i-i'}(m) \cdot \rho_{i-j'}(b)$$
(5)

The z_{i-i} , ψ_{i-i} and z_{j-j} , ψ_{j-j} are antenna construction parameters calculated by (6) for the mobile station.

$$z_{i-i'} = [a^2 + b^2]^{1/2}, \quad \psi_{i-i'} = \tan^{-1}(b/a)$$

$$a = \lambda_{mi} \cos \xi_{mi} - \lambda_{mi} \cos \xi_{mi}, \quad b = \lambda_{mi} \sin \xi_{mi} - \lambda_{mi} \sin \xi_{mi}$$
(6)

The first and second terms in (4) are concerned with a directive wave and nondirective waves. Moreover, three ensemble averages in the second term are nondirective power, mobile station, and base station factors, respectively. So we can calculate these averages independently and denote them as in (5). From the above description, we can get $\rho_{i-j,i'-j'}(bm)$ by rewriting (3) as (7), where $\rho_{i-i'}$ (m) and $\rho_{i-i'}(b)$ are mobile and base site correlations without a directive wave, respectively.

$$\rho_{i-j,i'-j'}(bm) = [k \exp(j\Delta\theta_0) + \rho_{i-i'}(m) \cdot \rho_{j-j'}(b)]/(k+1)$$
 (7)
Though the value of the numerator depends strongly on the first term by a directive wave, it becomes $\rho_{i-i'}(m)\rho_{j-j'}(b)$ in the case without LOS. Also $i=i'$ and $j=j'$ mean $\rho_{i-i'}(m)=1$ and $\rho_{j-j'}(b)=1$, respectively. Furthermore, we expand $\rho_{i-i'}(m)$ in (7) to a Neumann expansion because the ensemble average is integrated with respect to ξ_n [6], and we get (8).

$$\rho_{i-l'}(m) = \langle \sum_{n=1}^{N} \left[\sum_{l=0}^{\infty} \varepsilon_{l}(-1)^{l} J_{2l}(2\pi z_{i-l'}) \cos(2l(\xi_{n} - \psi_{i-l'})) \right] \rangle
- j \langle \sum_{n=1}^{N} \left[2 \sum_{l=0}^{\infty} (-1)^{l} J_{2l+1}(2\pi z_{i-l'}) \cos((2l+1)(\xi_{n} - \psi_{i-l'})) \right] \rangle$$
(8)

Here, $\varepsilon_l = 1 (l=0)$, $\varepsilon_l = 2 (l > 1)$, and $J_{2l}(\cdot)$ is a Bessel function of the first order. We can get $\rho_{i-j'}(b)$ similarly to $\rho_{i-i'}(m)$. Finally, we can get the general equation of MIMO channel correlation $\rho_{i-j,i'-j'}(bm)$ by substituting (8) in (7). From (7), $\rho_{i-j,i'-j'}(bm)$ is calculated from two items: antenna configuration $(z_{i-i}, z_{j-j}, \psi_{i-i}, \psi_{j-j})$ and multipath state (k, ζ_n, ζ_n) .

b) Example of correlation $\rho_{i-j,i'-j'}(bm)$: The $\rho_{i-j,i'-j'}(bm)$ given by (7) contains the product of $\rho_{i-i'}(m)$ and $\rho_{i-i'}(b)$, which are independent of each other. Therefore, it is sufficient to prepare just one side (either transmitting or receiving) for various environments because we can get $\rho_{i-i,i'-j'}(bm)$ by combining them. Therefore, we will study the correlation with uniform distribution of ξ_n and with Gaussian distribution of ζ_n .

i) For uniform distribution of ξ_n : We first calculate $\rho_{i-i'}(m)$ by (8) when ξ_n for $n \ge 1$ has a uniform distribution centered at ξ_c over ξ_c - $\Delta \xi_n$ $<\xi_n$ $<\xi_c$ + $\Delta \xi_n$ and the probability density function of ξ_n is $pdf(\xi_n)=1/2\Delta \xi_n$. Assuming a large N, we can calculate the ensemble average in (8) by integration with respect to ξ_n and we get (9).

$$\begin{split} \rho_{i-i^{\prime}}(m) &= \sum_{l=0}^{\infty} \mathcal{E}_{l}(-1)^{l} J_{2l}(2\pi z_{i-i^{\prime}}) \frac{1}{2lA \xi_{n}} \sin(2lA \xi_{n}) \cos(2l(\xi_{c} - \psi_{i-i^{\prime}})) \\ &- j2 \sum_{l=0}^{\infty} (-1)^{l} J_{2l+1}(2\pi z_{i-i^{\prime}}) \frac{1}{(2l+1)A \xi_{n}} \sin((2l+1)A \xi_{n}) \cos((2l+1)(\xi_{c} - \psi_{i-i^{\prime}})) \ (9) \end{split}$$

ii) For Gaussian distribution of ζ_n : Next, we calculate $\rho_{i-i}(b)$ when ζ_n for $n \ge 1$ has a Gaussian distribution centered at ζ_c with deviation σ . Similarly, we get (10).

	Tabl	e 1.	Simulation par	ameters.
channel	Base/mobile	ase/mobile one-side channel		Base & n
			-	

MIMO channel		Base/mobile one-side channel		Base & mobile both-sides channel		
Area		Indoors	Outdoors	Indoors	Outdoors	
Incident	Distribution	Uniform	Gaussian	Uniuni.	UniGaussian	
/arriving	ξ_c ,($\Delta \xi_n$)	$0, (\pi, \pi/2, \pi/4)$		$0, (\pi, \pi/2, \pi/4)$	0, (π)	
angle	ζ_c , (σ)	-	$\pi/6, (\pi/90, \pi/36, \pi/18)$	$0, (\pi, \pi/2, \pi/4)$	$\pi/6, (\pi/90, \pi/36, \pi/18)$	
Delay profile	N	10				
	h_n	Exponential distribution				
	K	$-\infty$, 5dB ($\xi_0 = 5\pi/6$)		- ∞		
Antenna-	λ_m , (ξ_m)	$0 \sim 5$, (0)		0~5, 0.5, (0)	0~20, 0.5, (0)	
	λ_b , (ζ_b)		0~20, (0)	0~5, (0)	0~20, (0)	

$$\begin{split} \rho_{j-j'}(b) &= \sum_{l=0}^{\infty} \varepsilon_{i}(-1)^{l} J_{2l}(2\pi z_{j-j'}) \cos(2l(\zeta_{c} - \psi_{j-j'})) \exp(-2l^{2}\sigma^{2}) \\ &- j2 \sum_{l=0}^{\infty} (-1)^{l} J_{2l+1}(2\pi z_{j-j'}) \cos((2l+1)(\zeta_{c} - \psi_{j-j'})) \exp(-\frac{(2l+1)^{2}\sigma^{2}}{2}). \end{split}$$

III. SIMULATION

A. Simulation method

A computer simulation was performed to verify (7), (9), and (10) by using (1), (2), and (3). The simulation parameters are listed in Table 1. As suggesting by (7), we need to simulate two terms: mobile or base site one-sidechannel, i.e., $\rho_{i\cdot i'}(m)$ or $\rho_{i\cdot i'}(b)$ with and without

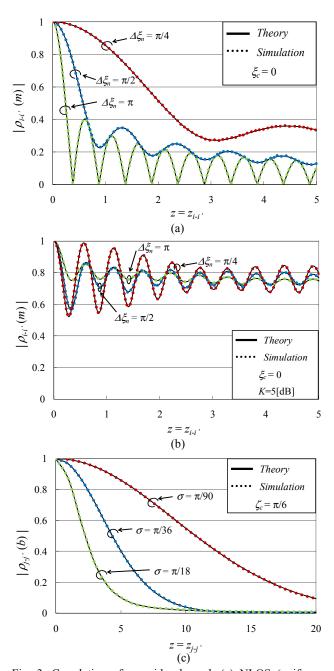


Fig. 3. Correlation of one-side channel. (a) NLOS (uniform distribution). (b) LOS (uniform distribution) . (c) NLOS (Gaussian distribution).

the addition of $k\exp(j\Delta\theta_0)$, and mobile-base channel, i.e., $\rho_{i\cdot j,i'\cdot j'}(bm)$. The radio frequency f_c was 3 GHz and the delay profile in II-A was used for uniform and Gaussian distributions. The multipath state (k, ξ_n, ζ_n) and antenna parameters $(\lambda_b, \zeta_b, \lambda_m, \xi_m)$ were set as shown in Table 1. Each simulated correlation value was calculated from an ensemble average for more than 10^6 delay profiles, when the path data such as h_n , τ_n , and ξ_n with nondirective waves were changed every delay profile, but a directive wave's data did not change because we assumed a deterministic wave in the set environment. In this way, we performed a statistical analysis.

B. Simulation results and discussion

a) Base or mobile site one-side channel: Figures 3(a) and (b) show the absolute value of the correlation simulated for arriving angle ξ_n with uniform distribution, and Fig. 3(c) shows that with Gaussian distribution, when antenna spacing $z=z_{i-i}$, $\psi_{i-i}=0$. Figures 3(a) and (b) are for NLOS and LOS with K=5 dB and $\xi_0=5\pi/6$, and they also show the theoretical value calculated by (7) with j=j'; the simulated and theoretical values agree well. Theory in Fig. 3(a) for $\Delta \xi_n = \pi$ becomes $\rho_{i-i}(m) = J_o(2\pi z_{i-i})$, as is well known. The correlation value with Gaussian distribution in Fig. 3(c) is higher than that with uniform distribution.

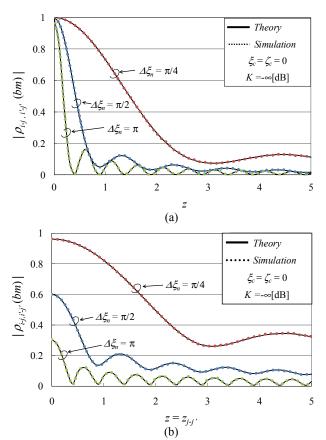
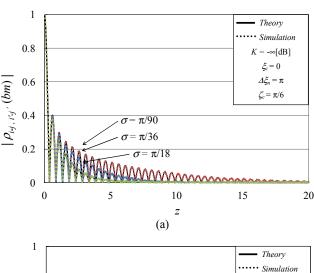


Fig. 4. Correlation between base and mobile channels (uniform-uniform distribution). (a) $z = z_{i-i} = z_{j-j}$. (b) $z_{i-i} = 0.5$.

The simulated value also agrees with the theory according to (7) with i=i and k=0.

b) Between base and mobile site channel: Figure 4 shows the simulated correlation $|\rho_{i-i,i'-j'}(bm)|$ for both incident ζ_n and arriving ξ_n with uniform distribution in NLOS supposing base and mobile sites indoors in the case of different channels between base and mobile sites, i.e., $i\neq i$ at the mobile station and $j\neq j$ at the base station. Figure 4(a) shows the simulated correlation when changing $z=z_{i-i}=z_{j-j}$ while keeping $\psi_{i-i}=\psi_{j-j}=0$. It also shows the theoretical value obtained by (7) with (9), i.e., $\rho_{i-j,i'-j'}(bm)$ products of $\rho_{i-i}(m)$ and $\rho_{j-j}(b)$, or here $\rho_{i-j,i'-j'}(bm)$ = $\rho_{i-i'}(m)^2$. The simulated and theoretical values agree well. Figure 4(b) is the simulated value when changing $z=z_{j-j}$ as in Fig. 4(a), except keeping $z_{i-i}=0.5$. The theoretical values of $\rho_{i-j,i'-j'}(bm)$ were calculated by (7) with (9) as $\rho_{i-i'}(m)$ for the mobile station is constant at $z_{i-i}=0.5$. The simulated and theoretical values agree well. Figure 5 shows the correlation simulated $|\rho_{i-i,i'-i'}(bm)|$ for arriving ξ_n with uniform distribution (ξ_c =0, $\Delta \xi_n$ = π) and incident ζ_n with Gaussian distribution ($\zeta_c = \pi/6$, σ . parameter), respectively, supposing mobile and base sites outdoors with NLOS. Figure 5(a) was simulated like Fig. 4(a). The simulated values rapidly become small with increasing z and σ since they have a site with uniform



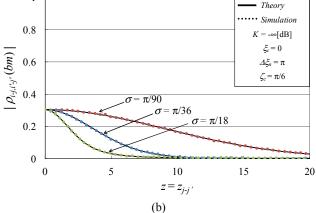


Fig. 5. Correlation between base and mobile channels. (uniform-Gaussian distribution). (a) $z = z_{i \cdot i} = z_{j \cdot j}$. (b) $z_{i \cdot i} = 0.5$.

distribution. Theoretical value $\rho_{i ext{-}i ext{-}j}(bm)$ obtained by (7) was calculated as $\rho_{i ext{-}i}(m)$ and $\rho_{j ext{-}j}(b)$, corresponding to (9) for the mobile station and (10) for the base station, respectively. The simulated and theoretical values agree well. Figure 5(b) is the correlation simulated by changing $z = z_{j ext{-}j}$ as in Fig. 5(a), except keeping $z_{i ext{-}i} = 0.5$. The simulated value is lower than 0.3 whenever z is small since $\rho_{i ext{-}i}(m)$ at $z_{i ext{-}i} = 0.5$ is small. The theoretical value was also calculated as in Fig. 4(b). The simulated and theoretical values agree well.

IV. CONCLUSION

A MIMO channel model with propagation mechanism was prepared to clarify MIMO channel complex correlation for various cell types and environments. The formula $\rho_{i-j,i'-j'}(bm)$ between MIMO channel matrix elements was derived and verified by computer simulation. It shows that $\rho_{i-j,i'-j'}(bm)$ is expressed as the addition of a directive wave term $k\exp(j\Delta\theta_0)$ to the product of mobile site correlation $\rho_{i-i}(m)$ and base site correlation $\rho_{i-i'}(b)$, which are calculated independently of each other. The $\rho_{i-i'}(m)$ and $\rho_{i-i'}(b)$ were calculated by using two factors: antenna configuration and arriving or incident wave angle distribution. The $\rho_{i-i}(m)$ with uniform angle distribution and $\rho_{j-j'}(b)$ with Gaussian angle distribution were derived. The $\rho_{i\cdot j,i'\cdot j'}(bm)$ can be applied to various cell types by combining them in indoors and outdoors: therefore, it is possible to calculate the correlation between MIMO channels for a given cell and environment.

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