

# Inter-Signal Interference Cancellation Filter for Four-Element Single Sideband Modulation

Yi Jiang<sup>1</sup>, Zhenyu Zhou<sup>1</sup>, Masahiko Nanri<sup>2</sup>, Gen-Ichiro Ohta<sup>1</sup> and Takuro Sato<sup>1</sup>

<sup>1</sup>Graduate School of Global Information and Telecommunication Studies, Waseda University  
Room 216, Bldg. No. 29-7, 1-3-10, Nishi-waseda, Shinjuku-ku, Tokyo, Japan 169-0051

<sup>2</sup>Panasonic Mobile Communications Co., Ltd. Yokohama, Japan  
600 Saedo-Cho, Tsuzuki-Ku, Yokohama, 224-8539, Japan

Email: jiangyi@toki.waseda.jp, zhenyu\_zhou@fuji.waseda.jp, nanri.masahiko@jp.panasonic.com, t-sato@waseda.jp

**Abstract**—This paper aims at improving the demodulation performances of Four-element Single Sideband signals by proposing a new receiver structure involving an Inter-Signal Interference cancellation filter. The filter makes use of the estimated symbols generated by the receiver to approximate the interference between multiplexed signals inherent to the use of low complexity Hilbert Transform filters at the transmitter. Simulation results show that the new receiver structure performs better than the one formerly proposed in early works on this topic.

**Keywords:** Single Sideband, Turbo Equalization, Orthogonal 4 SSB Elements Modulation, Interference cancellation

## I. INTRODUCTION

As wireless communications evolve toward the fourth generation (4G), throughput requirements have been drastically increased. However due to the scarce nature of the spectrum, alternative solutions to system bandwidth enlargement are required. Consequently, research aiming at increasing the resource efficiency of channel coding is nowadays of outmost importance.

An interesting research direction to achieve this goal is Single Sideband (SSB) Modulation method exploiting practical digital Hilbert transform filters. Hilbert transform filters can be used to generate the Single Sideband version of any signal, which occupies half the bandwidth of the original signal but still carries the same amount of information. Consequently, spectral efficiency is increased. However, bandwidth reduction causes Inter-Symbol Interference (ISyMI) in complex signals, such as QPSK. ISyMI removal is thus necessary in order to make possible the exploitation of SSB modulation in the current communication systems. Previous papers on the topic include a method involving a zero-interpolation alternated between In-Phase and Quadrature signals of a QPSK signal to remove the ISyMI [1]. Although the performances of the proposed scheme are shown to beat those of Double Sideband signals, it does not increase the throughput, since bandwidth reduction is obtained at the cost of doubled transmission duration. Recent work on SSB modulation for narrow-band channel has seen to be important progress thanks to the work in [2], where a QPSK signal occupying only half the bandwidth of the original signal is shown to be effectively demodulated in flat fading channel conditions by a turbo equalizer. Separately,

an interesting idea was found in [3], where Ohta has proposed a new modulation/demodulation process having the potential to double the spectral efficiency of the transmitted signal. In his paper, it is shown that two complex signals can be combined into one baseband complex signal called “Orthogonal Four-Single Sideband elements signal”, which we mention as Four-SSB signal in the remaining of this paper for the sake of brevity. A 4-SSB signal contains an Upper-Side Band (USB) modulated signal and a Lower-Side Band (LSB) modulated signal. The main challenge of its demodulation is the cancellation of inter-symbol interference arising due to the SSB modulation of complex signals. In our previous work [4], a transmission scheme of 4-SSB symbols over OFDM has been proposed. The demodulation process involved a turbo-equalizer tackling the interference inherent to SSB modulation of complex signals. However, the use of imperfect Hilbert Transform filters also results in interference between USB and LSB signals, namely, Inter-SSB Signal Interference (ISigI). This source of ISigI was not taken into account in the previous paper and limited the performance of the proposed scheme. In the current paper, we focus on the demodulation process of 4-SSB signals, without consideration for the transmission scheme employed. Since the impact of imperfect channel equalization was carried out in our previous work, we consider here that the channel equalization is ideal and that its output can be seen as the output of an equivalent AWGN channel. Focusing on adapting the demodulator to the intrinsic properties of the 4-SSB signal, we propose the addition to the 4-SSB receiver block of an interference cancellation filter. This filter uses soft-feedback information from the output of the turbo-equalizer to estimate the ISigI and cancel it. The simulation results presented highlight the effectiveness of the new receiver architecture.

The remaining of this paper is organized as follows. Firstly, we give an introduction to the Four-SSB modulation/demodulation process and emphasize the structure of the 4-SSB signal. In the second part, we present the new 4-SSB receiver block with interference cancellation filter. In the third part, simulation results for QPSK modulation in various channel conditions are shown to illustrate the performances of the receiver.

## II. 4-SSB-OFDM MODULATION AND DEMODULATION

### A. 4-SSB-OFDM modulation

This chapter presents briefly the “orthogonal 4-SSB elements technique” initially introduced in [3]. We start by defining some of the notation used in the remaining of this paper. Let the complex discrete sequence  $\mathbf{x}$  be the baseband representation of a signal and let  $\hat{\mathbf{x}}$  denotes its ideal Hilbert Transform. Then, the analytical Upper Sideband version of  $\mathbf{x}$ , denoted  $\mathbf{x}_{USB}$  is expressed as:

$$\mathbf{x}_{USB} = \frac{1}{\sqrt{2}}(\mathbf{x} - j\hat{\mathbf{x}}) \quad (1)$$

where  $j = \sqrt{-1}$ . We define in the same way the analytical Lower Sideband representation of  $\mathbf{x}$ , denoted  $\mathbf{x}_{LSB}$ , as:

$$\mathbf{x}_{LSB} = \frac{1}{\sqrt{2}}(\mathbf{x} + j\hat{\mathbf{x}}) \quad (2)$$

We may later refer to those two expressions by using the concept of Single Sideband (SSB) modulation. Now let us consider 4 independent real discrete sequences denoted  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{p}$  and  $\mathbf{r}$  with unit power. The “4-SSB signal” corresponding to those 4 real signals is given as [3]:

$$\mathbf{s}_{4SSB} = \frac{1}{\sqrt{2}}(\mathbf{s}_{4SSB,I} + j\mathbf{s}_{4SSB,Q}) \quad (3)$$

$$\mathbf{s}_{4SSB,I} = \frac{1}{2}(\mathbf{u} - \hat{\mathbf{v}} + \mathbf{p} + \hat{\mathbf{r}}) \quad (4)$$

$$\mathbf{s}_{4SSB,Q} = \frac{1}{2}(-\hat{\mathbf{u}} - \mathbf{v} + \hat{\mathbf{p}} - \mathbf{r}) \quad (5)$$

The above expression can be applied to four BPSK-modulated signals or to two complex modulated signals (e.g, QPSK). Higher order complex modulations may be considered, however, we restrict our study to the case of QPSK. Let us denote two complex signals as:

$$\mathbf{d}_1 = \frac{1}{\sqrt{2}}(\mathbf{u} + j\mathbf{v}) \quad (6)$$

$$\mathbf{d}_2 = \frac{1}{\sqrt{2}}(\mathbf{p} + j\mathbf{r}) \quad (7)$$

Denoting  $\text{Re}[\cdot]$  and  $\text{Im}[\cdot]$  as the real and imaginary parts respectively, we can re-express equations (4) and (5) as:

$$\mathbf{s}_{4SSB,I} = \text{Re}[\mathbf{d}_{1,LSB}] + \text{Re}[\mathbf{d}_{2,USB}] \quad (8)$$

$$\mathbf{s}_{4SSB,Q} = -\text{Im}[\mathbf{d}_{1,LSB}] - \text{Im}[\mathbf{d}_{2,USB}] \quad (9)$$

The signal  $\mathbf{s}_{4SSB}$  occupies the same bandwidth as  $\mathbf{d}_1$  or  $\mathbf{d}_2$  but carries twice more information. The 4-SSB modulation can be thought about as the multiplexing of two conventional complex signals into one. The spectral profile of the 4-SSB signal is given in Fig 1.

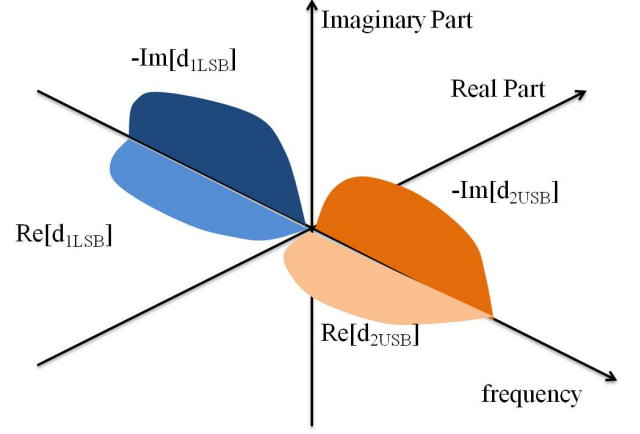


Figure.1 Spectral profile of a 4-SSB signal

### B. 4-SSB-OFDM demodulation with turbo-receiver

While it is not possible to recover the complex signals directly from the received 4-SSB signal, their SSB-modulated versions may be obtained according to the following equations [3]:

$$\mathbf{u} - \hat{\mathbf{v}} = \mathbf{s}_{4SSB,I} + \hat{\mathbf{s}}_{4SSB,Q} \quad (10)$$

$$\mathbf{v} + \hat{\mathbf{u}} = -\mathbf{s}_{4SSB,Q} + \hat{\mathbf{s}}_{4SSB,I} \quad (11)$$

$$\mathbf{p} + \hat{\mathbf{r}} = \mathbf{s}_{4SSB,I} - \hat{\mathbf{s}}_{4SSB,Q} \quad (12)$$

$$\mathbf{r} - \hat{\mathbf{p}} = -\mathbf{s}_{4SSB,Q} - \hat{\mathbf{s}}_{4SSB,I} \quad (13)$$

The above equations can be seen as a demultiplexing of the two transmitted complex signals. The quality of the operation depends on the number of taps used in the discrete Hilbert Transform filter. One can notice that this filter must be the same in the transmitter and in the receiver to avoid the generation of interference. Since 4-SSB demultiplexing operations are linear, perfect separation of the two complex signals is possible in AWGN channel. However, if the baseband equivalent of the channel impulse response is complex ( $\mathbf{h} = (\mathbf{h}_{Re} + j\mathbf{h}_{Im})$ ), its convolution with  $\mathbf{s}_{4SSB}$  yields the following received signal.

$$\mathbf{s}_{4SSB,Rx} = \frac{1}{\sqrt{2}}(\mathbf{s}_{4SSB,Rx,I} + j\mathbf{s}_{4SSB,Rx,Q}) \quad (14)$$

$$\mathbf{s}_{4SSB,Rx,I} = (\mathbf{h}_{Re} * \mathbf{s}_{4SSB,I}) - (\mathbf{h}_{Im} * \mathbf{s}_{4SSB,Q}) \quad (15)$$

$$\mathbf{s}_{4SSB,Rx,Q} = (\mathbf{h}_{Im} * \mathbf{s}_{4SSB,I}) + (\mathbf{h}_{Re} * \mathbf{s}_{4SSB,Q}) \quad (16)$$

Obviously, equations (15) and (16) used to compute equations (10) to (13) would yield on interference, hence the need for perfect channel equalization prior to the demultiplexing of the two signals. In the case channel effects are correctly cancelled and that the result can be considered as an AWGN channel, the recovery of the QPSK modulated sequences still require some interference removal technique. Indeed, we can see that, in their SSB-modulated form, QPSK

signals are interfered by their respective Hilbert Transforms as illustrated in (17) and (18).

$$\mathbf{d}_{1,LSB} = \frac{1}{2}(\mathbf{u} - \hat{\mathbf{v}} + j(\hat{\mathbf{u}} + \mathbf{v})) \quad (17)$$

$$\mathbf{d}_{2,USB} = \frac{1}{2}(\mathbf{p} + \hat{\mathbf{r}} + j(\mathbf{r} - \hat{\mathbf{p}})) \quad (18)$$

In [3], channel equalization is carried out jointly with Hilbert Transform interference removal, but it is not possible in the case of 4-SSB modulation due to the presence of the 4-SSB demultiplexing step. This problem can still be solved separately from the channel equalization by representing USB and LSB modulations as discrete-time ISyml channels (i.e as tapped-delay lines). In this case, classical equalization techniques can be applied to remove interference coming from the Hilbert Transform terms. The SSB-modulated signals are thus seen as outputs of discrete-time ISyml channels with impulse responses  $\mathbf{h}_{1,LSB}$  and  $\mathbf{h}_{1,USB}$  as in (19) and (20), respectively.

$$\mathbf{d}_{1,LSB} = \mathbf{d}_1 * \mathbf{h}_{1,LSB} \quad (19)$$

$$\mathbf{d}_{2,USB} = \mathbf{d}_2 * \mathbf{h}_{1,USB} \quad (20)$$

The ISyml induced in the complex signals due to the SSB-modulation are severe and require a powerful equalization technique. By using the turbo principle, equalization techniques can obtain significant performance gains and eventually overcome such strong ISyml [2]. Thus, we choose to employ turbo-equalizer to recover each of the complex signals from their SSB-modulated form. The “turbo” structure used in this work is inspired from the one described in [5]. Turbo equalization requires the data bits to be encoded to make them locally dependent. Thus, we have to modify the structure of the transmitter to include a coder-interleaver section prior to symbol mapping. In a 4-SSB receiver, the two transmitted complex signals are demodulated separately. Thus we perform operations related to turbo equalization at the transmitter and at the receiver independently for each signal as described in Fig 2 (see next page). The structure of our turbo-equalizer is composed of the widely linear MMSE equalizer using prior information presented in [6], coupled to a Maximum A Posteriori (MAP) decoder using the BCJR algorithm [7]. Both of them exchange likelihood information about the transmitted bits in order to improve their respective decoding performances. The justification for the choice of a MMSE Equalizer over a trellis-based method is that, although the ISyml induced by SSB modulation admit a trellis-representation, the number of states yields on a decoding whose complexity grows exponentially with the number of taps in the Hilbert Transform filter. On the other hand, the matrix inversion, most costly operation of the MMSE equalization, has a polynomial complexity with respect to the number of taps in the filter.

### III. RECEIVER WITH FEEDBACK INTERFERENCE CANCELLER

In the previous section, we introduced the principles of the modulation of 4-SSB signals and gave a brief description of the challenges of their demodulation. Although the use of turbo-equalizer enables to achieve low Bit Error Rates (BER) even in low Signal to Noise Ratio (SNR) regime, the design of the receiver should also take into account signal distortions introduced by the non-ideal Hilbert Transform filters used at the transmitter. This is especially true when the transmitter has a low-complexity constraint. In this section, we first describe the effect of the imperfectness of the filters on the 4-SSB signal and then propose a demodulation process trying to cancel their influence.

#### A. Inter-SSB signal interference in 4-SSB-modulated signals

As highlighted in Fig 1, the 4-SSB signal is composed of two SSB signals having same center frequency. In the figure, the spectra of both signals appear separated. This is an illustration of two SSB signals generated using an ideal Hilbert Transform filter (i.e with an infinite number of taps). However, practical Hilbert Transform filters have a finite, usually small, number of taps. As a consequence, the spectra of SSB signals generated by them become not zero in the suppressed side and distort the desired side. We can observe this effect by rewriting equations (10) to (13) as follows. In the equations below  $H_t(\cdot)$  denotes the Hilbert Transform filter used at the transmitter. We consider that the receiver uses exactly the same filter. Equation (10) can be rewritten as (21), (11) as (22) and so on.

$$\mathbf{s}_{4SSB,I} + H_t[\mathbf{s}_{4SSB,Q}] = \frac{1}{2}(\mathbf{u} - 2H_t[\mathbf{v}] + \mathbf{p}) \quad (21)$$

$$+ \frac{1}{2}(H_t[-H_t[\mathbf{u}]] + H_t[H_t[\mathbf{p}]]) \quad (22)$$

$$H_t[\mathbf{s}_{4SSB,I}] - \mathbf{s}_{4SSB,Q} = \frac{1}{2}(\mathbf{v} + \mathbf{r} + 2H_t[\mathbf{u}]) \quad (23)$$

$$+ \frac{1}{2}(-H_t[H_t[\mathbf{v}]] + H_t[H_t[\mathbf{r}]]) \quad (24)$$

$$\mathbf{s}_{4SSB,I} - H_t[\mathbf{s}_{4SSB,Q}] = \frac{1}{2}(\mathbf{p} + \mathbf{u} + 2H_t[\mathbf{r}]) \quad (25)$$

$$+ \frac{1}{2}(-H_t[H_t[\mathbf{p}]] + H_t[H_t[\mathbf{u}]])$$

$$-H_t[\mathbf{s}_{4SSB,I}] - \mathbf{s}_{4SSB,Q} = \frac{1}{2}(\mathbf{r} + \mathbf{v} + 2H_t[\mathbf{u}]) \quad (26)$$

$$+ \frac{1}{2}(-H_t[H_t[\mathbf{v}]] + H_t[H_t[\mathbf{r}]])$$

We notice that for non-ideal Hilbert Transform filters, two iterations of the filter do not yield on the opposite of the original input signal. The output of a finite-length Hilbert Transform filter can be seen as the combination of two filters. Let us denote  $H_t(\cdot)$  as the ideal Hilbert Transform filter. Then, we can decompose this filter as in (25) where  $H_c(\cdot)$  is the complementary of  $H_t(\cdot)$ .

$$H_t = H_i - H_c \quad (25)$$

Using (25), we can rewrite two iterations of the Hilbert Transform filter as follows.

$$(H_i - H_c) * (H_i - H_c) * \mathbf{u} = -\mathbf{u} + (-2H_i * H_c + H_c * H_c) * \mathbf{u} \quad (26)$$

The second term on the right side of the equality in (26) can be considered as a single infinite length filter, which can be approximated by a finite length filter. We investigate the design of such a filter in the following of this section.

### B. Interference canceller with soft-feedback

As it is described in the last section, use of imperfect Hilbert Transform filters yields on ISyGI. A simple idea to try to cancel this interference is to approximate the equivalent infinite impulse response (IIR) filter of the interference with a finite impulse response (FIR) filter, provided that its number of taps is larger than the number of taps of the filter used at the transmitter. This setup can be considered for a scenario where the receiver is able to have a larger complexity than the transmitter. At each iteration of the turbo-structure, we feed back symbols estimated by the turbo-equalizer to this filter and subtract its output from the outputs signals of the 4-SSB Demultiplexer, we then obtain a new input signal for the MMSE Equalizer. This process is illustrated in Figure 2. Note that the interference canceller cannot replace the MMSE Equalizer. A simple reason is that during the first iteration of the system the Interference Canceller could not be used.

For a transmitted 4-SSB symbol sequence  $\mathbf{x}$ , let  $\mathbf{y}$  be the output of the channel equalizer at the receiver. After the 4-SSB demultiplexing operation, for each signal, the equalizer of the turbo-structure feeds to the decoder a sequence  $\mathbf{L}^i$ ,  $i \in \{1, 2\}$  of likelihood values associated to the transmitted coded bit sequence  $\mathbf{c}^i$ . The Soft-Input Soft-Output MAP decoder uses this prior information to produce a sequence of likelihood values  $L[\mathbf{c}^i | \mathbf{L}^i]$  associated to the coded bits. From this likelihood values, we can derive, for any transmitted symbol, the probability that it equals a given element of the symbol alphabet. To achieve that, we first define the log likelihood as in (27).

$$L[c_k^i | \mathbf{L}^i] = \frac{P[c_k^i = 0 | \mathbf{L}^i]}{P[c_k^i = 1 | \mathbf{L}^i]}, \forall i \in \{1, 2\} \quad \forall k \in \mathbb{Z} \quad (27)$$

Let us now consider a symbol mapping of sequences of  $Q$  bits on an alphabet  $\Omega = \{d_1, d_2, \dots, d_M\}$  with  $Q = \log_2(M)$ . The mapping relation is defined as in (28).

$$d_m^i \leftrightarrow \{c_1^m, c_2^m, \dots, c_Q^m\}, c_q^m \in \{0, 1\} \quad (28)$$

$$\forall q = 1, 2, \dots, Q \quad \forall m = 1, 2, \dots, M$$

Then the probability that any transmitted symbol is equal to a given element of the symbol alphabet is given by (29).

$$P[d_n^i = d_m^i | \mathbf{L}^i] = \prod_{q=1}^Q \frac{\exp[L[c_{(n-1)Q+q}^i | \mathbf{L}^i]]}{1 + \exp[L[c_{(n-1)Q+q}^i | \mathbf{L}^i]]} \quad (29)$$

$$n = 1, 2, \dots, N \quad n = 1, 2, \dots, N \quad i \in \{1, 2\}$$

From this point it is possible to estimate for each data stream the sequence of transmitted symbols, which we note as  $\hat{\mathbf{d}}^i$ ,  $i \in \{1, 2\}$ , by choosing for each symbol the most likely alphabet element. The estimated sequences can then be input to the interference canceller, whose task is to recreate the interference and subtract it from the outputs of the 4-SSB Demultiplexer. However, it is worth considering whether each element of the estimated symbol sequences will benefit to the overall performances of the system when fed back to the Interference Canceller. Without prior knowledge about the noise variance, the Interference Canceller may simply select the symbol with highest probability, namely the Maximum Likelihood (ML) symbol estimate for a given received signal. However, since the reliability of the A Posteriori Probabilities output by the turbo-equalizer is variable depending on transmission conditions and ISyGI in the signal, a probability threshold to select symbol estimates can be implemented. The Interference Canceller may select only the symbols whose probability to be correct is higher than the threshold; while other feedback symbols are set to zero. In low SNR regions, such a threshold enables to limit the spreading of interference in the signal, by feeding back more correct symbols than incorrect ones. The derivation of such an adaptive threshold is left for future work. We assume in the following simulations that the estimated symbol sequence is simply the ML sequence for a given received signal.

## IV. SIMULATION RESULTS

In this section, we present simulation results of demodulation of 4-SSB signals with Gray-mapped QPSK. Since the proposed modification of the receiver structure is independent of the channel, we simply assume perfect channel equalization and consider the input signals to the 4SSB Demultiplexer as the outputs of an equivalent AWGN channel. The transmission parameters are summarized in Table 1. We consider two different sizes of Hilbert Transform filters at the transmitter : 7 and 21 taps.

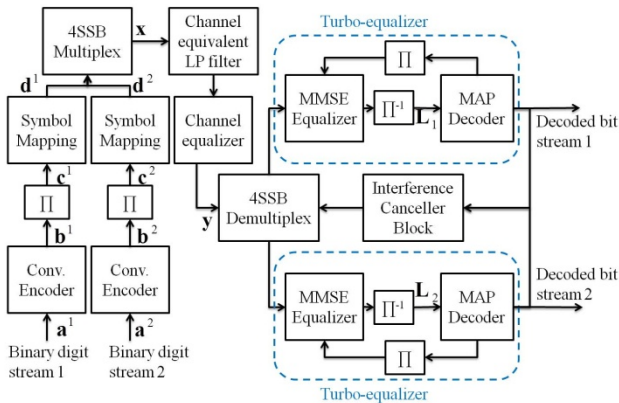


Figure.2 Position of the interference canceller block in the 4-SSB demodulation process

Table 1 : Simulation parameters

Parameters	Value
Encoder rate	1/2
Code polynomials	$G_1 = 5, G_2 = 7$
Interleaver size	1000 bits
Symbol Modulation	QPSK with Gray-coding
Tx Hilbert transform filter length	7, 21 taps
Channel Model	AWGN
Interference Canceller filter length	87, 101 taps
Feedback Probability Threshold	none
Turbo equalizer sequence length	500 symbols
Channel Decoding	Log-MAP

As one can see from Figure 3 and Figure 4, the ISigI canceller improves the BER performances in both cases, although its influence is most notable when the Hilbert Transform filter at the transmitter has a small number of taps (0.5 dB for 7 taps, instead of 0.25 dB for 21 taps). Note that the Interference Canceller does not improve the first iteration performances of the turbo-receiver. Consequently, it does not decrease the minimum SNR threshold above which the “turbo” effect starts to decrease the BER significantly. However the convergence rate of BER performances toward those of QPSK with the same convolutional code and the same MAP decoder is increased.

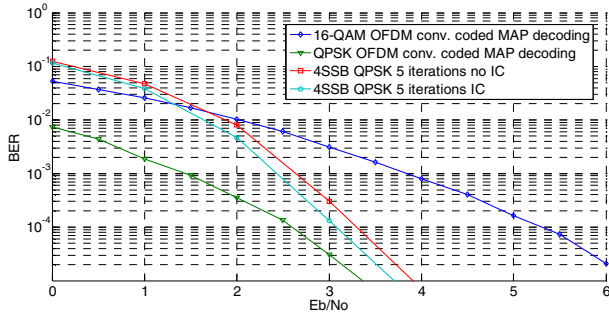


Figure.3 Influence of Interference Canceller on 4-SSB QPSK performances in AWGN channel for a Hilbert Transform filter at the transmitter of 21 taps.

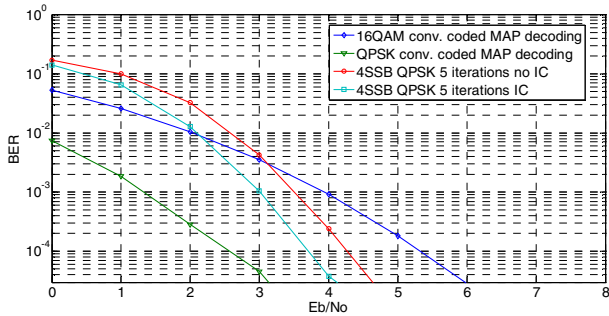


Figure.4 Influence of Interference Canceller on 4-SSB QPSK performances in AWGN channel for a Hilbert Transform filter at the transmitter of 7 taps.

## V. CONCLUSION

In this paper, we proposed a new demodulation process of 4-SSB signals. The addition of an Interference Canceller using soft-feedback from the turbo-equalizer aims at compensating for most of the performance degradation due to the use of Hilbert Transform filters with small number of taps at the transmitter. The three-stage demodulation presented in this paper appears most suited for a scenario where the receiver tries to balance the performance loss due to the low complexity of the transmitter. Future works may include the estimation of the error distribution in the Widely Linear MMSE Equalizer in the case of 4-SSB signal and the derivation of a Minimum Symbol-Error-Rate threshold for implementation in the Interference Cancellation filter proposed in this paper.

## REFERENCES

- [1] S. A. Mujtaba, “A novel scheme for transmitting QPSK as a singlesideband signal,” in *Proc. IEEE GLOBECOM*, pp. 592–597, Nov. 1998.
- [2] B. Pitakdumrongkija, *et al*, “Single sideband QPSK with turbo equalization for mobile communications,” *IEEE, Vehicular Technology Conference (VTC)- Spring 2005*, pp. 538–542 May 2005
- [3] G. Ohta, M. Nanri, M. Uesugi, T. Sato and H. Tominaga, “A Study of New Modulation Method Consisted of Orthogonal Four SSB Elements Having a Common Carrier Frequency,” *IEEE, The 11th International Symposium on Wireless Personal Multimedia Communications(WPMC)*, Lapland, Finland, Sep. 2008.
- [4] Yi Jiang, Zhenyu Zhou<sup>1</sup>, Masahiko Nanri, Gen-Ichiro Ohta and Takuro Sato, “Performance Evaluation of Four Orthogonal Single Sideband Elements Modulation Scheme in Multi-Carrier Transmission Systems,” *IEEE, Vehicular Technology Conference (VTC)- Fall 2011*, Sept. 2011
- [5] Ralf Koetter, Andrew C. Singer, and Michael Tüchler, Turbo Equalization, *IEEE Signal Processing Magazine*, vol 21, pp.67-80. 2004.
- [6] Tüchler, M., Singer, A.C., “Turbo Equalization: An Overview”, *IEEE Trans. on Signal Processing*, vol. 57, pp.920 - 952, 2002.
- [7] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Trans. Inf. Theory*, vol. IT-20, no. 2, pp. 284–287, Mar. 197