User Pairing for Capacity Maximization in Cooperative Wireless Network Coding

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***Abstract*—In this paper, we consider a network-coded cooperative wireless network, where users mutually pair among themselves to realize network coding. We consider a multi-user environment, where users transmit to a common destination in the absence of dedicated relays. Two nodes constituting a pair periodically swap the roles of the source and relay to mutually achieve spatial diversity. As such, conditioned on the successful detection of the source's packet, a network-coded packet is formed at the relay by a linear combination of its own packet and the source's packet. A single transmission of this network-coded packet therefore helps both nodes to achieve diversity gain. In this work, we address the important problem of the mutual pairing of users, which directly governs the overall network performance. We first propose an optimal user pairing algorithm in order to maximize the total network capacity. To simplify the pairing process, we subsequently propose computationally simpler, heuristic user pairing schemes. In particular, we propose max-max pairing to maximize the network capacity, and max-min pairing to minimize the outage probability. Performance analysis of the proposed optimal and heuristic user pairing schemes is performed in terms of the average capacity, average outage probability, and user-fairness.**

I. INTRODUCTION

In today's wireless networks, diversity is regarded as an efficient and established means to combat multipath fading. Moreover, user cooperation has emerged lately as an elegant technique to achieve spatial diversity over wireless channels, by exploiting the broadcast nature of the medium [1]. The notion itself stems from the classical relaying model, with intelligent antenna sharing and signal combining at the receiver for the realization of spatial diversity.

Recently, the application of network coding [2] in cooperative wireless networks has gained increasing interest with its potential to further boost the network performance, such as in terms of the achievable throughput. With network coding, the intermediate nodes are allowed to linearly combine packets from multiple sources and then forward the linearly-combined packets for better throughput and resource utilization. The application of network coding in wireless networks has been studied in a variety of settings, including the cases of two sources transmitting to a common destination [3]-[4], multi-cast [5], and two-way relay channels [6]-[7].

The performance of network-coded cooperative networks is hugely determined by proper relay selection. Owing to its importance, this problem has received significant interest from the wireless communication research community. Relay selection schemes with network coding over two-way relay channels have been considered in [6]-[7]. Various optimal and heuristic selection criteria have been proposed for choosing the relay (or set of relays) which forwards the network coded packet(s). The relay nodes are assumed to be dedicated, i.e., the relaying nodes participate in cooperation, but transmit nothing for themselves when relaying. Furthermore, the problem of relay selection for transmission to a common destination, such as a base station (BS) in a cellular environment, is considered in [3]. However, the assumption of dedicated relays is maintained, and multi-user environments have not been considered.

This motivates us to address the *problem of mutual user pairing* (i.e., partner selection) *in a multi-user environment, where users employ network coding to transmit to a common destination* (e.g., a BS in a cellular environment). In the absence of dedicated relay nodes, and as shown in Fig. 1, users mutually pair among themselves to realize network coding. Hence only those users which have data to transmit participate in cooperation, and idle users are not engaged. This is an important communication scenario, and to the best of our knowledge, the problem of mutual user pairing in such multi-user environments has not been addressed previously.

The user pairing can be performed to optimize certain system performance metrics, such as network capacity, outage probability, and/or user-fairness. Two nodes constituting a pair periodically swap the roles of source and relay for the mutual benefit of achieving diversity gain. In this paper, we first formulate and solve an optimization problem to determine the user pairing which maximizes the total network capacity. Subsequently, we propose implementation-oriented heuristic pairing algorithms to approach the optimal performance at an alleviated computational complexity.

The rest of the paper is organized as follows. In Section II, the system model of the multi-user cell-based network-coded cooperation is established. The capacity and outage analysis of this cooperation scenario is presented in Section III. User pairing algorithms (including the optimal and heuristic ones) are proposed in Section IV. The simulation results are shown in Section V, and Section VI concludes this paper.

II. SYSTEM MODEL

The system model is shown in Fig. 1. We consider a single cell with an even number of users, N_{users}. Nodes are uniformly and randomly distributed over the cell and are assumed to be equipped with single antennas. Users strategically pair, and take turns to relay the network-coded packets for their partners. We assume the inter-source

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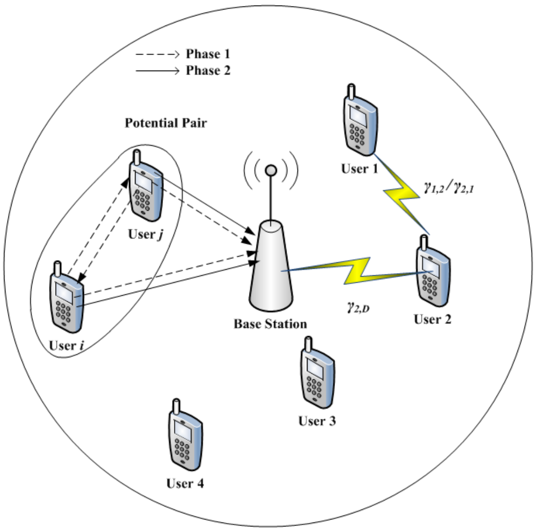

Fig. 1. The system model. Dotted and solid lines represent source- and network-coded-packet transmissions respectively.

channels to be non-ideal (noisy with Rayleigh fading). Thus, nodes may not always detect the packets of their partner, and therefore do not always forward the network-coded packets to help their partner. The network-coded packet transmission and detection of a pair of nodes follow the model which was first proposed in [4].

The communication with the common destination (BS or access point) is performed over two phases, and each phase consists of two time slots. This is depicted in Fig. 2, where it is assumed that nodes i and j constitute a pair, where $i, j \in \{1, ..., N_{users}\}$, and $i \neq j$. Node i transmits its packet to the BS in the first time slot during the first (direct transmission) phase, while node j overhears. Subsequently, j transmits its packet in the second time slot while i overhears. This is followed by the second (network coding) phase of transmission. Now, if i had decoded its partner's packet in the previous phase, it would combine its partner's packet with its own packet, and send the network-coded packet to the BS in the first time slot. Otherwise, it would send an additional packet for itself. Meanwhile, j does the same in the second time slot of the second phase.

At the BS, the two independently faded network-coded packets are combined using maximum ratio combining (MRC) to form a single packet which provides diversity. This packet is subsequently jointly decoded with the packets received in the first phase to recover the information bits. A maximum diversity order of two for each user can therefore be achieved. This concludes the two phase of communication with the BS.

Time and energy resources are split equally between the two phases, and also within the two time slots constituting each phase. Moreover, the success of decoding at the partner nodes is assumed to be determined by cyclic redundancy checks, whereas incorporating an additional flag bit in the packets transmitted in the second phase helps the BS determine the success of inter-source transmissions, and hence the nature of the packets received in the second phase.

Noteworthy is the fact that in our system model, users

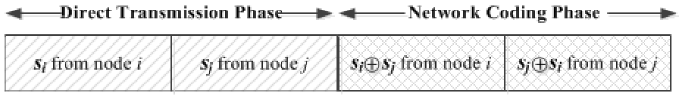

Fig. 2. Packets transmitted by the paired nodes i and j in the two phases. In case of inter-user transmission failure, an individual packet is transmitted by the relaying node in the network coding phase.

having data to transmit mutually pair and relay the network coded packets for their partners, whilst not engaging into cooperation the nodes which are otherwise idle. Moreover, the users transmit over orthogonal channels; hence, there is no same-cell interference. All channels, i.e., inter-source and source-destination, are assumed to be spatially independent, frequency flat Rayleigh fading, with additive white Gaussian noise (AWGN). We assume block fading, such that all channels remain constant during the two phases, and change independently afterwards; this accommodates for relatively low-medium mobility. In addition, the inter-source channels are assumed to be symmetric but non-reciprocal, i.e., having equal average signal-to-noise ratio (SNR) in both directions, but not necessarily the same instantaneous SNR.

The received symbols at the destination nodes are given as

$$y = hx + n, \tag{1}$$

where x represents the transmitted symbols, h is the channel coefficient, which captures the effect of path loss and Rayleigh fading, and n is the AWGN at the receiver. Perfect channel state information is assumed at all receiving nodes.

III. Capacity and Outage Performance Analysis

The inter-source, and source-destination channel capacities for pairing nodes i and j are functions of the corresponding channel coefficients. Moreover, an outage over a link is defined as the event of throughput falling below a target information rate. We use the outage probability at a certain rate as a metric of the packet error rate for the block-based transmissions under consideration. The inter-source channels are modeled as non-ideal (due to noise and fading), and successful decoding at the relay node is not guaranteed. This translates to the fact that the relay node forwards a network-coded packet in the second phase only if it decodes its partner's packet correctly. Otherwise, it transmits its own packet only. The average throughput for the pair therefore depends on the success of inter-source transmissions, which must first be determined.

A. Direct Transmission Phase

In the direct transmission phase, i and j sequentially broadcast their respective packets to the BS and also overhear each other's transmissions. The inter-source information theoretic channel capacity for i is $C_{i,j} = \log_2(1+\gamma_{i,j})$, where $\gamma_{i,j} = |h_{i,j}|^2 P_{tx} / N_0$ is the instantaneous SNR of the inter-source link, with P_{tx} as the transmit power, and N_0 as the noise power spectral density. An outage occurs whenever $C_{i,j} < 2R$, where R is the packet information rate in the case of point-to-point transmission. For Rayleigh fading, the inter-source link outage probability for i is given as [4]

$$P_{i,j} = 1 - \exp(-\frac{2^{2R}-1}{\Gamma_{i,j}}), \quad (2)$$

where $\Gamma_{i,j}$ is the average SNR. The inter-source outage probability for j can subsequently be calculated by replacing $\Gamma_{i,j}$ by $\Gamma_{j,i}$ in (2).

B. Network Coding Phase

The success of inter-source packet transmissions can lead to four distinct cases: **(a)** when both nodes i and j in the pair decode each other, **(b)** when none of them decode each other, **(c)** when only j decodes i, and **(d)** when only i decodes j [4]. In the case of decoding failure, an individual packet is transmitted. In this subsection, we perform the capacity and outage analysis for i only. A similar approach holds for j.

For node i, the source-destination channel capacities and the corresponding outage events for the four possible cases (a)-(d) are given in order in (3) [4]. For the channel capacity, the first and the second terms in the equations represent contributions from the direct transmission and the network coding phases, respectively.

$$C_{i,D} = \begin{cases} 0.5\log_2(1+\gamma_{i,D}) + 0.5\log_2(1+(\gamma_{i,D}+\gamma_{j,D})) < 2R. & \text{(a)} \\ 0.5\log_2(1+\gamma_{i,D}) + 0.5\log_2(1+\gamma_{i,D}) < R. & \text{(b)} \\ 0.5\log_2(1+\gamma_{i,D}) + 0.5\log_2[(1+\gamma_{i,D})(1+\gamma_{j,D}) & \\ \qquad < 4R/3. & \text{(c)} \\ 0.5\log_2(1+\gamma_{i,D}) + 0.5\log_2(1+\gamma_{i,D}) < 2R. & \text{(d)} \end{cases} \quad (3)$$

IV. User Pairing and Capacity Maximization

In this section, we address the problem of user pairing which directly governs the overall network performance. We first solve the problem of determining the optimal user pairing $\mathcal{P}^*$ which maximizes the total network capacity. To facilitate user pairing, we then propose implementation-oriented heuristic algorithms which are designed to address the throughput and outage performance.

A. Optimal User Pairing $\mathcal{P}^*$

We have the set of all possible pairing sets Π, such that every set $\mathcal{P} \in \Pi$ is the pairing containing $N_{users}/2$ disjoint user pairs. Each pairing $\mathcal{P}$ is therefore a symmetric mapping of elements from the set $\mathbb{X} \in \{1,2,...,N_{users}\}$ to the set $\mathbb{Y} \in \{1,2,...,N_{users}\}$, with the restriction of an element from $\mathbb{X}$ not being mapped to the same element in $\mathbb{Y}$. The goal is to find the optimal pairing $\mathcal{P}^*$ that maximizes the total network capacity $C_{sum} = \sum_i C_i$. Therefore,

$$\mathcal{P}^* = \arg\max_{\mathcal{P}\in\Pi} C_{sum}(\mathcal{P}). \quad (4)$$

At first glance, this looks like the problem of maximum weighted matching (i.e., pairing) in bipartite graphs, and any of the assignment algorithms such as the well-known Hungarian algorithm [8] seems as a candidate solution. However, as it was observed, a weight matrix $\mathbf{W}$, with zeros on the main diagonal and symmetric entries, $[\mathbf{W}]_{i,j} = [\mathbf{W}]_{j,i} = C_{i,D} + C_{j,D}$, describing the weight of the assignment of node i to j, and node j to i (where i and j constitute a potential pair) did not always lead to a symmetric assignment. To find the optimal solution, we therefore model this problem as maximum weighted matching in general graphs.

We construct a weighted, undirected graph $\mathcal{G} = (V, E)$, where the vertices V are the users to be paired, connected by the set of edges E. Furthermore, $|V| = N_{users}$ and $|E| = N_{users}(N_{users}-1)/2$ (as the graph is fully connected), where $|.|$ denotes the cardinality of the set. Each edge (i, j) has an associated weight $w_{i,j} = C_{i,D} + C_{j,D}$. The goal is to find the matching (i.e., pairing) with the maximum total weight. This maximum weighted matching covers all the vertices in the graph, and each vertex is connected only to a single edge. Moreover, each edge in the matching connects two distinct vertices.

When the number of users to be paired is large, the problem of finding the optimal pairing (i.e., the matching with the maximum total weight) is clearly far from trivial, whereas an exhaustive search is prohibitively expensive. To solve this pairing problem, we use Jack Edmond's maximum weighted matching algorithm for general graphs as described in [9] to find the optimal pairing. In the following, we present a succinct description of the algorithm, and the reader is referred to [9] for more details.

The idea is to start with an empty pairing, and then during each stage to find an augmenting path in the graph which yields the maximum increase in weight. The blossoms method is used for finding the augmenting paths, and the primal-dual method for finding the pairing with maximum weight. The problem is defined as a linear program. Considering the dual problem, we use complementary slackness to convert the optimization problem to that of solving a set of inequalities or "constraints". A pair of feasible solutions for the primal and dual problems are both optimal if, for every positive variable in one of these problems, the corresponding inequality in the other one is satisfied as an equality.

We describe this linear program as an integer program. The integrality constraints $x_{i,j} \in \{0,1\}$ (which indicate that the edge (i, j) may not or may belong to the final pairing, respectively) are replaced by $x_{i,j} \geq 0$. Moreover, additional constraints, i.e., $\sum_{i,j\in V_o} x_{i,j} \leq \lfloor |V_o|/2 \rfloor$ are added for all odd subset of vertices V_o. We have a primal solution, a pairing $\mathcal{P}$, and a dual solution which is the assignment of the dual variables u_i (for all vertices), and z_k for all odd subset of vertices V_{o_k}. The slack variables are defined as $\pi_{i,j} = u_i + u_j - w_{i,j} + \sum_{i,j\in V_{o_k}} z_k$. Moreover $\pi_{i,j} \geq 0$ are the constraints of the dual problem. By duality, we find the optimal pairing $\mathcal{P}^*$ when all of the following conditions hold true, as formally proven in [9]:

(a) For all i, j, and k, u_i, $\pi_{i,j}$, $z_k \geq 0$.
(b) The edge (i, j) is matched $\Rightarrow \pi_{i,j} = 0$.
(c) $z_k > 0 \Rightarrow V_{o_k}$ is full.

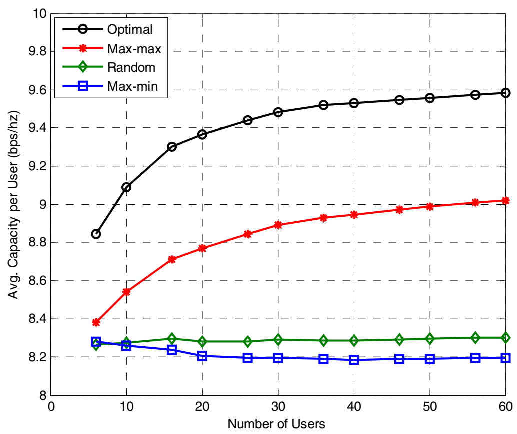

Fig. 3. Average capacity per user versus the number of paired users in the cell for the proposed pairing algorithms.

The algorithm solves the pairing problem in $O(N^3)$ time, where an exhaustive search would require $(N-1)!!$ calculations. Moreover, the set of users to be paired can be split into randomly chosen smaller groups to reduce the complexity of the algorithm, while however compromising the performance.

B. Heuristic Pairing Algorithms

1) *Max-max Pairing*: This algorithm pairs users with the objective of approaching the optimal capacity at a much reduced computational complexity. A weight matrix $\mathbf{W}$ with zeros on main diagonal, and symmetric entries $[\mathbf{W}]_{i,j}=[\mathbf{W}]_{j,i}=C_{i,D}+C_{j,D}$ is established, where i and j are potential pairs. The algorithm is formally presented in the following:

1.a) Initialize an empty pairing $\wp$,
1.b) Select the largest element from $\mathbf{W}$, for instance $[\mathbf{W}]_{i,j}$, and form the pair by augmenting $\wp$ with i and j,
1.c) Update $\mathbf{W}$ by removing the rows and columns corresponding to the pair formed in 1.b,
1.d) Continue from 1.b until $\wp$ is complete and all nodes have been paired.

The max-max pairing is significantly computationally simpler than the optimal pairing, as also reflected by the simulation times which are referred to in Section V.

2) *Max-min Pairing*: This heuristic algorithm is designed to address the system outage probability. We start with the weakest user (in terms of the SNR to the BS) in the cell and pair it with the user having the strongest of the weaker of source-relay and relay-destination links, since the outage performance is always determined by the weaker of the two links [10], and continue so on for other users. The algorithm is formally presented as follows:

2.a) Initialize an empty pairing $\wp$,
2.b) Select a node i with the lowest $\gamma_{i,D}$ and pair it with j with $\max[\min(\gamma_{i,j},\gamma_{j,D})]$,
2.c) Augment the pairing $\wp$ with the pair formed in 2.b, and update the set of eligible nodes.
2.d) Continue from 2.b until $\wp$ is complete and all nodes have been paired.

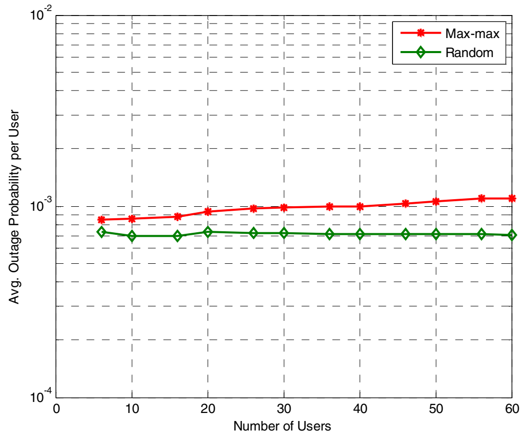

Fig. 4. Average outage probability per user versus the number of paired users in the cell for the proposed pairing algorithms.

As apparent, max-min pairing is computationally efficient; this is also reflected in the average simulation times as stated in Section V.

3) Random User Pairing: Pairing users randomly is the most straight-forward strategy, and is the simplest to implement in practice. From the set of eligible users, two randomly chosen nodes are paired. $\wp$ is augmented, the set of eligible users is updated, and the algorithm repeats until all users have been paired. Although random selection is not an effective way of pairing, we include it here for comparison purposes.

V. SIMULATION RESULTS

In this section we present the performance analysis of the proposed optimal and heuristic user pairing schemes in terms of different system performance metrics, namely, average capacity per user, average outage probability per user, and fairness among users.

A. Simulation Setup

For simulations, we use the exponential path-loss model with a break-point distance of 1 m, and a path loss exponent of 3.5. The inter-source and uplink channel bandwidth is 10 MHz. The antennas at the mobile stations and the BS are modeled as having absolute gains of 4 and 100 respectively. The information rate $R=0.25$ bps/Hz, and the results are averaged over 10^3 location sets, and 10^3 Rayleigh channel samples for each location. The users are uniformly, randomly distributed over a cell of radius 1 km, and use a transmit power of 0 dBW.

B. Results and Performance Analysis

In Fig. 3, the average capacity per user is shown versus the number of users for the four pairing schemes. As expected, the optimal pairing yields the maximum throughput per user for all values of N_{users}, and is therefore used as the benchmark for the heuristic schemes. The optimality of the algorithm was also verified through extensive comparisons with the exhaustive search pairing. From the proposed heuristic algorithms, max-max pairing

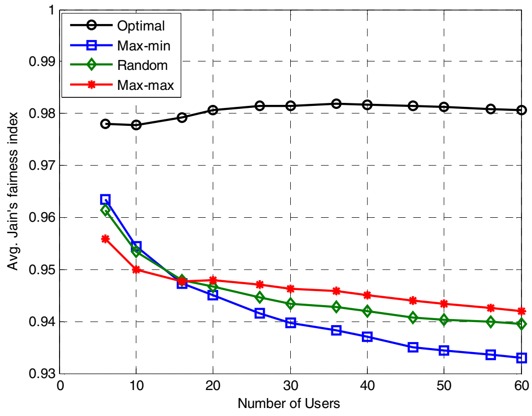

Fig. 5. Jain's fairness index versus the number of paired users in the cell for the proposed pairing algorithms.

achieves the closest capacity to the optimal pairing. For $N_{users} = 30$ and 40 for instance, max-max pairing is shy of the optimal pairing by 6.03 and 6.12 percent, respectively. This performance is achieved approximately four times faster when compared with the optimal pairing in terms of the average simulation times. Weighing the performance degradation against the relative complexities of the two algorithms, max-max pairing emerges as a very good choice for practical implementation. On the other hand, the max-min pairing algorithm, designed to address the system outage performance is significantly inferior to max-max pairing, as anticipated, and performs worse than the random pairing in terms of the average capacity per user.

Though the optimal pairing scheme is designed to maximize the network throughput, it also achieves the best outage performance. Moreover, the outage performance oriented max-min pairing algorithm matches the optimal algorithm in terms of the average outage probability per user, as they both demonstrate zero outage for all values of N_{users}. When compared with the optimal pairing, the max-min pairing achieves this performance approximately forty times faster, as reflected by the average simulation times.

Results for the average outage probability per user for the max-max pairing and random pairing are depicted in Fig. 4. Max-max pairing is observed to perform worse than random pairing for all N_{users}. This is owing to the aggressive nature of max-max pairing, which leads to a greater variance and spread within pairs (in terms of throughput), and therefore results in relatively high average outage probability per user.

Fairness performance, measured in terms of the average Jain's fairness index[1], is depicted in Fig. 5. The optimal pairing demonstrates the best fairness performance and achieves the maximum Jain's fairness index which is around 0.98. From the heuristic algorithms, the max-max pairing yields the best average fairness index (averaged over all location sets), for all $N_{users} > 16$. Moreover, the max-min pairing algorithm, which pairs users to minimize the *outage probability* performs worse than the random pairing, except at smaller values of N_{users}.

[1]defined as $\mathcal{J} = \left(\left(\sum_{i=1}^{N_{users}} C_{i,D} \right)^2 \Big/ N_{users} \sum_{i=1}^{N_{users}} C^2_{i,D} \right)$

VI. Conclusions

The important problem of the mutual pairing of users in cooperative wireless network coding is addressed in this paper. Contrary to prior works in this area, we consider a multi-user environment, and the absence of dedicated relay nodes. Users strategically pair among themselves, and swap the roles of source and relay to mutually achieve spatial diversity, transmitting to a common destination. Inter-source channels are non-ideal, which accurately reflects real world settings. We propose an optimal pairing algorithm, which exhibits the maximum achievable network throughput, lowest outage probability, and highest fairness among all the proposed schemes. For networks with smaller number of users and where pairing complexity is not the foremost concern, the optimal pairing is most favorable. To facilitate user pairing we propose computationally simpler heuristic pairing algorithms. It was demonstrated that max-max pairing exhibits good capacity and fairness performance at a significantly reduced complexity. Moreover, the max-min pairing matches the optimal pairing in terms of the average outage probability per user, with a very low complexity. Max-max pairing is therefore an excellent choice when high throughput and fairness are desirable, whereas max-min pairing is preferable for scenarios where the average outage probability is of vital concern with a reduced complexity. The joint optimization of transmission power and capacity, as well as power and outage probability is underway in this line of research.

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