SNR-Adaptive Input Quantization for Turbo Decoding

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***Abstract*— This paper presents how to control the input bit-width of a turbo decoder according to the signal-to-noise ratio (SNR) adaptively. It is crucial to minimize the input bit-width while maintaining the error-correcting performance. Several quantization schemes have been presented to properly decide the input resolution of turbo decoding, but they all assume a fixed bit-width irrespective of the SNR. This paper proposes a new method to adaptively change the quantization bit-width with respect to the SNR. In addition, this paper suggests a novel method to estimate the frame-error rate (FER) performance of a turbo decoder dealing with quantized channel outputs. As a result, the whole SNR range is divided into a number of regions each of which can be processed with a certain quantization bit-width.**

Keywords- SNR adaptive, turbo codes, turbo decoders, quantization, quantization noise.

I. INTRODUCTION

The turbo code has been known as one of the most powerful error correction codes, as its bit-error rate (BER) performance is truly close to the Shannon limit [1]. Since it was first introduced in 1993, a number of constituent decoding algorithms have been studied. The optimal decoding algorithm includes the maximum *a posteriori* (MAP) algorithm [2], [3] and the typical sub-optimal algorithm is the *a priori* soft output Viterbi algorithm (APRI-SOVA) [4].

Although the soft-input soft-output (SISO) decoding algorithms have assumed infinite-precision or floating-point arithmetic, it is desired to use fixed-point arithmetic in practical implementations. Consequently, a number of methods have been presented in literature to quantize floating-point values. A method to optimally quantize the channel output was presented in [5], and three quantizer types, such as uniform, non-uniform, and logarithmic quantizers, were analyzed in [6]. In addition, the input bit-width and the internal bit-width were optimized in a combined fashion in [7]. However, all the previous works have focused on finding a fixed bit-width that leads to almost the same BER performance as the floating-point model does. For example, 8-bit quantization is concluded to have almost no BER degradation [5], and 6-bit quantization and 8-bit quantization are sufficient for the channel output and extrinsic information, respectively [6].

The purpose of this paper is to unveil the relationship between the quantization resolution, or bit-width, of input data and the channel signal-to-noise ratio (SNR), and thus to minimize the quantization bit-width while maintaining the error-correcting performance. Based on the optimal quantization presented in [5], this paper suggests a new method to adaptively determine the input resolution according to the SNR. Though the quantization noise is dependent on the resolution and the SNR, it is shown that the difference is ignorable when the SNR is considerably high.

As the input data of less bit-width can be decoded with less hardware resources, the proposed adaptive control of input quantization can save the energy consumed in turbo decoding. This possibility of energy reduction is crucial in implementing a turbo decoder to be used for mobile devices adopting Mobile WiMAX (IEEE 802.16e) and 3GPP-LTE, as the devices are severely suffering from limited energy.

The rest of this paper is organized as follows. The system model is briefly addressed in Section II, and the quantization-noise variance is analyzed in qualitative and quantitative ways in Section III. Section IV describes the relationship between the error-correcting performance and the quantization noise. Simulation results are summarized in Section V, and conclusion remarks are made in Section VI.

II. SYSTEM MODEL

A typical channel coding system is depicted in Fig. 1. The turbo encoder and decoder in the system are compliant with a mobile communication standard, 3GPP-LTE [8]. The information input d_k to the turbo encoder is so random that the probability of d_k = 0 is equal to that of d_k = 1. The turbo encoder consists of two recursive systematic convolutional (RSC) encoders separated by a quadratic polynomial permutation (QPP) interleaver. The code rate is 1/3 as specified

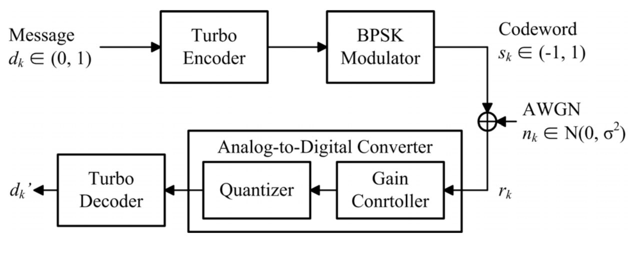

Figure 1. System model

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in the standard, and the frame length of every codeword is fixed to the maximum length, *i.e.,* N = 6144, for the simulations conducted in this work. Six tail bits are appended to each frame in order to make the two RSC encoders end with the all-zero state. The codeword is transmitted through the additive white Gaussian noise (AWGN) channel. Since the binary input s_k to the channel is disturbed by the Gaussian noise $n_k \in N(0, \sigma^2)$, the channel output r_k is soft.

As in [5], we assume that an optimal gain controller is located in front of the quantizer. The channel output signal r_k is scaled by the gain controller and then quantized to a certain resolution. Depending on the channel SNR and the quantization resolution, the gain is properly set to one of pre-calculated values so as to minimize distortion caused by the quantization. The way to calculate the optimal gain [5] will briefly be addressed in the next section.

Let the gain of the gain controller be g. The quantizer digitizes the scaled channel output gr_k to a digital value represented in a number of bits. Let n be the resolution. In this case, the quantized value is one of 2^n values spaced evenly. Suppose that gr_k is continuous in an interval, $(-Q, Q)$. The interval is evenly divided into 2^n steps each of which is associated with a width of

$$\delta = \frac{Q}{2^{n-1}} . \tag{1}$$

The boundaries between two adjacent steps are at

$$x_k = \left(-\infty, -Q+\frac{\delta}{2}, -Q+\frac{3\delta}{2}, \cdots, Q-\frac{3\delta}{2}, +\infty\right), k=0,1,\cdots,2^n. \tag{2}$$

The output of the quantizer is the value of a step closest to the scaled channel output.

The quantized channel output is fed to the turbo decoder. In this paper, the turbo decoder is assumed to employ the Max-log-MAP algorithm. The algorithm is sub-optimal but widely adopted for practical implementations, as it can be implemented with simple operations such as addition, multiplication and comparison.

III. QUANTIZATION NOISE VARIANCE

The performance metrics that should be preserved while reducing the quantization bit-width are the error-correction capability and the average number of decoding iterations. The latter is proved by simulations in Section V while the former needs complicated analysis grounded on the quantization noise and the channel SNR.

The quantization described in the previous section adds an additional noise to the channel output signal. The difference between the original and quantized values is regarded as quantization noise. When the channel output signal is scaled and quantized to a 3-bit number, the quantization noise can be analyzed by understanding Fig. 2, where the raw signal ranges from -1 to 1 approximately. The probability density function (pdf) of the channel output signal is plotted for three different SNRs: 5 dB, 10 dB and 15 dB in Fig. 2(a). Only the right half side of the pdf is plotted because the pdf curve is symmetrical

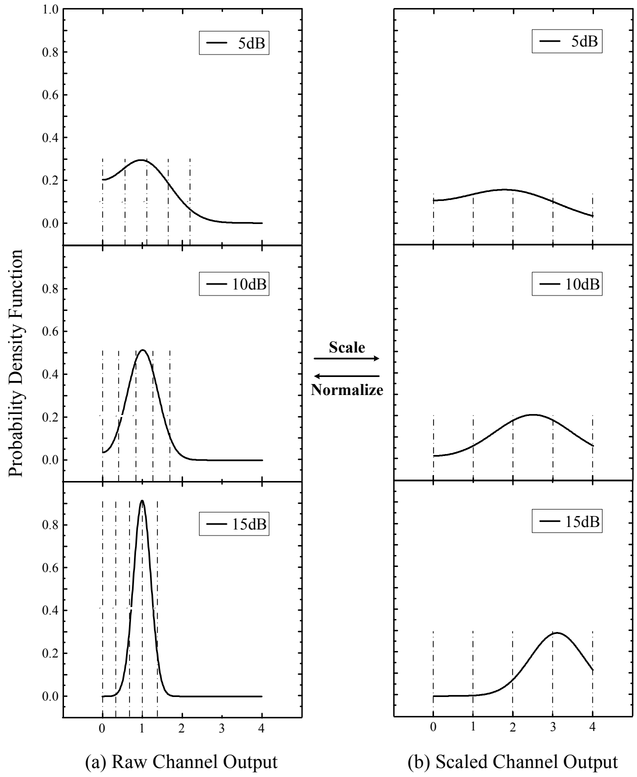

Figure 2. The pdf of AWGN channel output for various SNR

with respect to 0. Assuming that the signal is transmitted through an AWGN channel associated with the Gaussian noise $n_k \in N(0, \sigma^2)$, where σ^2 is the noise variance, the pdf of the received signal r is achieved by performing the convolution of two pdfs corresponding to the original signal $p_s(x)$ and the Gaussian noise $p_n(x)$ of the channel. It is explicitly expressed as

$$\begin{aligned} p_r(x) &= \int_{-\infty}^{\infty} p_n(x-y)p_s(y)\,dy \\ &= \frac{1}{2\sqrt{2\pi}\sigma}\left\{\exp\left(-\frac{(x-1)^2}{2\sigma^2}\right)+\exp\left(-\frac{(x+1)^2}{2\sigma^2}\right)\right\} . \end{aligned} \tag{3}$$

The raw channel output signal needs to be scaled and quantized for turbo decoding. The gain g, which is used as a factor of scaling, is an important parameter. Fig. 2(a) indicates that the pdf shape of the raw signal becomes sharper, as the SNR is getting higher, which means that there are fewer samples exceeding the range from -1 to 1. Therefore, the raw signal can be amplified larger if the channel SNR is higher, meaning that the gain can be increased according to the SNR increase. The pdf of the scaled channel output signal is plotted in Fig. 2(b), where only the right half side is shown because of the same reason described above. The pdf of the signal scaled by g is

$$\begin{aligned} p(x) &= \frac{1}{|g|} p_r\left(\frac{x}{g}\right) \qquad (g>0) \\ &= \frac{1}{2\sqrt{2\pi}g\sigma}\left\{\exp\left(-\frac{(x-g)^2}{2g^2\sigma^2}\right)+\exp\left(-\frac{(x+g)^2}{2g^2\sigma^2}\right)\right\} . \end{aligned} \tag{4}$$

Dashed vertical lines drawn in Fig. 2(b) represent the 3-bit quantization steps. The quantization is conducted for the scaled signal, and the higher channel SNR leads to the larger scaling factor. The quantization steps reflected to the raw signal are drawn with dashed vertical lines in Fig. 2(a). Looking at the dashed lines in Fig. 2(a), we see that the distance between adjacent quantization steps is getting narrower if the SNR becomes higher. The step distance represents the precision of quantization, thus the quantization is more accurate if the distance is denser. For a receiver that has the gain controller in front of the quantizer, therefore, the power of quantization noise decreases as the channel SNR increases.

To analyze the relationship furthermore, we need to understand the variance or power of the quantization noise quantitatively. A function that calculates how much a signal is distorted by the quantization was derived in [5]. In fact, the function calculates the variance of the quantization noise in effect. The equation calculating the variance is

$$\sigma_q^2 = \sum_{k=0}^{2^n-1} \int_{x_k}^{x_{k+1}} (x-\dot{x}_k)^2 \, p(x)\, dx \qquad (5)$$

where $p(x)$ is given in (4), $\dot{x}_k$ is the value of the k-th quantization step, namely, $-Q+\delta k$, and (x_k, x_{k+1}) is an interval between adjacent steps. Expanding (5),

$$\sigma_q^2 = A + \sum_{k=0}^{2^n-1} \left(-2\dot{x}_k B_k + \dot{x}_k^2 C_k \right) \qquad (6)$$

where

$$\begin{aligned}
A &= g^2(\sigma^2+1) \\
B_k &= \frac{g\sigma}{2\sqrt{2\pi}} \left\{ \exp\left(\frac{-(x_k-g)^2}{2g^2\sigma^2} \right) - \exp\left(\frac{-(x_{k+1}-g)^2}{2g^2\sigma^2} \right) \right. \\
&\quad \left. + \exp\left(\frac{-(x_k+g)^2}{2g^2\sigma^2} \right) - \exp\left(\frac{-(x_{k+1}+g)^2}{2g^2\sigma^2} \right) \right\} \\
&\quad + \frac{g}{2} \left\{ Q\left(\frac{x_k-g}{g\sigma} \right) - Q\left(\frac{x_{k+1}-g}{g\sigma} \right) \right. \\
&\quad \left. - Q\left(\frac{x_k-g}{g\sigma} \right) + Q\left(\frac{x_{k+1}-g}{g\sigma} \right) \right\} \\
C_k &= \frac{1}{2} \left\{ Q\left(\frac{x_k-g}{g\sigma} \right) - Q\left(\frac{x_{k+1}-g}{g\sigma} \right) \right. \\
&\quad \left. + Q\left(\frac{x_k+g}{g\sigma} \right) - Q\left(\frac{x_{k+1}+g}{g\sigma} \right) \right\}
\end{aligned} \qquad . \qquad (7)$$

In (7), $Q(.)$ stands for the tail probability of the standard normal distribution. In other words, $Q(x)$ is the probability that a standard normal random variable will obtain a value larger than x. Given the resolution and the AWGN variance σ, the gain g is decided to maximize the ratio between the power of the raw signal and the variance of the quantization noise. The AWGN variance can be obtained if the channel SNR is known.

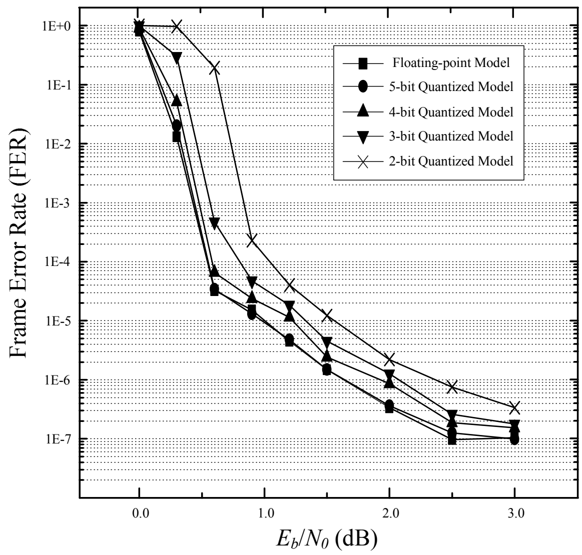

Figure 3. The FER of floating-point and fixed-point turbo decoders

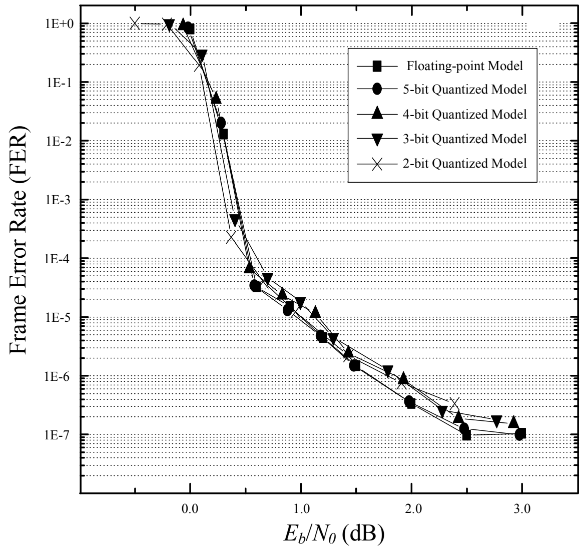

Figure 4. The FER obtained by removing the effects of quantization noise

IV. EFFECTS OF THE QUANTIZATION NOISE

The error-correction capability of a turbo decoder can be derived by measuring the bit-error rate (BER) or the frame-error rate (FER), and the FER is more proper in mobile communication applications where a codeword is a separate data frame. Fig. 3 plots how the FER is related to the channel SNR (E_b/N_0) and the quantization resolution. The FER obtained from the 3-bit quantized model is worse than those from the 4-bit and 5-bit models, as the former suffers from the stronger quantization noise. In other words, a point on the FER curve of a quantized model is a horizontally shifted version of the corresponding point on the curve of the floating-point model. The amount of shift is proportional to the power of the quantization noise. Compensated properly,

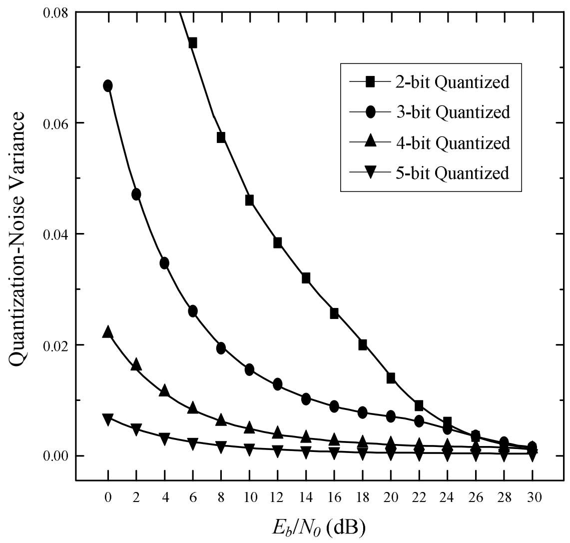

Figure 5. Quantization noise variances versus channel SNR

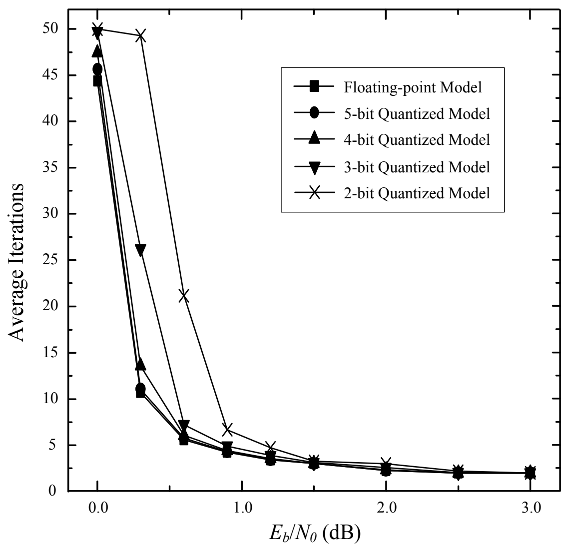

Figure 6. The average number of turbo decoding iterations versus SNR

therefore, the points will shift left and overlap onto the curve of the floating-point model.

As described in Section III, the quantization-noise variance σ_q^2 can be calculated explicitly. Suppose that σ_q^2 is obtained for n-bit quantization. In this case, the total noise power spectral density of the quantized channel output is

$$N_t = N_0 + N_q = 2\left(\sigma^2 + \frac{\sigma_q^2}{g^2}\right) \tag{8}$$

where N_0 is the noise power spectral density of the AWGN. As the quantized model suffers from the quantization noise as well as the AWGN channel noise, it is possible to estimate the FER curve of the n-bit quantized model from that of the floating-point model. To estimate the curve, each point on the FER curve of the floating-point model is shifted horizontally by replacing the x-axis value (E_b/N_0) with E_b/N_t. On the other hand, we can apply the shifting process inversely to infer the FER curve of the floating-point model from that of the n-bit quantized model. In the inverse process, the x-axis value (E_b/N_0) of the quantized model is replaced with E_b/N_0', where

$$N_0' = N_0 - N_q = 2\left(\sigma^2 - \frac{\sigma_q^2}{g^2}\right). \tag{9}$$

The inverse process is to compensate the effects resulting from the quantization noise. The FER curves generated by compensating the quantization noise are plotted in Fig. 4. If the effects due to the quantization noise are removed, several FER curves which are generated by different models are superimposed well on the FER curve of the floating-point model. The floating-point model is simulated by excluding the gain controller and the quantizer. In the simulation, the channel output and all the internal values have a resolution of 64 bits, which can be regarded as being almost infinite.

From this analysis, it is possible to reach a remark. Let us suppose two decoders in which the input values are differently quantized. If the quantization-noise variances are similar, the FER values of the two decoders will also be similar to each other, meaning that the error-correcting capability of one decoder is almost equal to that of the other.

V. Simulation Results

Combining the results from Section III and Section IV, we can preserve the error-correcting capability while controlling the quantization bit-width according to the SNR. The most important point is that the quantization-noise variance reduces and converges to a small value as the SNR becomes higher. Fig. 5 plots how the normalized quantization-noise variances (σ_q^2/g^2) are related to the input resolutions and the SNR. The solid line curves are theoretically calculated from equation (6), and discrete dots on the curves are obtained from simulations. In the simulations, the variances are calculated from the quantized values of more than 18,000 samples of the AWGN-channel output. All the dots coincide exactly with the corresponding lines for the entire SNR range, which means that equation (6) is absolutely correct. Note that the variances decrease fast as the SNR increases. As addressed in Section IV, if the quantization-noise variances of the decoder input quantized to different resolutions are similar for a given SNR, the resulting BER or FER performances will also be similar. Moreover, in the high SNR region, the BER or FER curves tend to be flat due to the phenomenon called error flooring [9], which gives an additional remark that the input resolution does not affect the error-correcting capability seriously if the SNR is sufficiently high.

Furthermore, it is confirmed by simulations that the number of decoding iterations also converges while reducing the resolution according to the SNR. Fig. 6 shows how the average number of iterations is related to the quantization bit-width. If the SNR is extremely low, less than 1.0 dB for

TABLE I
SNR BOUNDARIES WHERE THE NUMBER OF QUANTIZATION BITS CAN BE REDUCED

Resolution	Constraints		
	QNSR < 1%	QNSR < 3%	QNSR < 5%
5-bit	0.0 dB ~ 5.0 dB	-	-
4-bit	5.0 dB ~ 15.0 dB	0.0 dB ~ 4.5 dB	0.0 dB ~ 1.5 dB
3-bit	15.0 dB ~ 22.0 dB	4.5 dB ~ 15.0 dB	1.5 dB ~ 9.0 dB
2-bit	22.0 dB ~	15.0 dB ~	9.0 dB ~

example, the low-resolution model necessitates more iteration. However, the number of iterations is not increased noticeably if the SNR is beyond it, which means that there is no additional power consumption or decoding latency even though the channel output is quantized with lower resolution. To investigate the average number of iterations, a stopping criterion is applied to turbo decoding. As explained in Section II, the system is based on the 3GPP-LTE standard. As the cyclic redundancy check (CRC) code is mandatory, each codeword includes the CRC code. The CRC code is tested whenever every iteration is completed. If the test is passed, the decoding is terminated. The number of iterations is obtained by applying this precise stopping mechanism.

Finally, an analysis was performed by using equation (6) and the FER performance. Some criteria are required to decide whether the input resolution can be reduced or not for a given SNR. An example in Table 1 assures that the quantization-noise-to-signal ratio (QNSR), namely, the amplitude ratio of the quantization-noise variance over the original message signal is less than a constant such as 1%, 3% and 5%. One can decide the QNSR constraint by considering channel environment and power sources. Table 1 summarizes the resolutions resulting from the criterion. In other words, if the constant equals to 1%, 5-bit quantization can be relaxed to 4-bit quantization when the SNR ≥ 5dB, and it can be reduced to 3-bit quantization if the SNR ≥ 15dB. When the SNR increases to 22dB, the bit-width can be reduced to 2 bits.

VI. CONCLUSIONS

This paper has presented a new method to adaptively control the input resolution of a turbo decoder according to the SNR. Based on the equation to calculate the quantization-noise variance, the error-correcting performance of a quantized model is estimated from that of a floating-point model and vice versa. If different models have almost the same quantization-noise variations for a given SNR, it is shown that their BER or FER performances are similar to each other. Finally, we investigated how the quantization-noise variances are related to quantization resolution. The difference of quantization-noise variances is ignorable if the SNR is sufficiently high. Hence, the input quantization for turbo decoding can be relaxed while maintaining the error-correcting performance, when the channel SNR is high enough. Applying a simple criterion to decide the quantization bit-width properly, we can reduce the bit-width to 4 bits if the SNR ≥ 5dB and to 3 bits at 15dB. When the resolution of the decoder input is adaptively controlled according to the SNR, the power consumed in turbo decoding can be lowered, which might be significant for mobile communication devices. Implementation details are remaining as future works.

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