Efficient and Fair Resource Allocation Scheme for OFDMA Networks Based on Auction Game

Seyed Mohamad Alavi†, Chi Zhou†, Wan Wang Gen‡
†Department of Electrical and Computer Engineering,
Illinois Institute of Technology, Chicago, IL, USA
‡School of Communication and Information Engineering, Shanghai University, Shanghai, China
Emails: †{salavi1, zhou}@iit.edu, ‡wanwg@staff.shu.edu.cn

***Abstract*—A distributed allocation of resources in the uplink of OFDMA networks is studied through auction theory. A combinatorial auction is formulated and the solutions are provided. Moreover, the users' utility, which is a function of minimum rate requirement and channel gain is defined to enforce truthful resource demands. Since the original problem is NP hard, a method based on simulated annealing applied to find near-optimum results. In a competitive scenario the user valuation is sent to the Resource Allocation Unit (RAU) as the proposed bid, and then the highest bidder wins the auction and pays a value. The algorithm is shown to provide fair distribution of resources among users. Simulation results are presented to illustrate the convergence of the algorithm, the truth-telling behavior of the users, and performance utilization of the network.**

I. Introduction

The Orthogonal Frequency Division Multiple Access (OFDMA) is a promising solution for next generation wireless communication systems. The combination of OFDMA with adaptive modulation and coding (AMC) and power allocation is of great prominence in the design of future broadband radio systems. Essentially, resource allocation schemes play important role in the design of OFDMA system. An efficient resource allocation, which involves bit loading, power allocation and subcarrier assignment to improve system performance seems necessary. Especially, in an environment that all users are subjected to time-varying channel.

While an efficient resource allocation takes into account the users' Quality of Service (QoS) requirements, the crucial notion of fairness arises and must be dealt with [1]. Even though proportional fairness can provide a balance between efficiency and fairness, but the provided fairness is not the result of users' competition. A proposed solution is that users are allowed to compete for available resources. In this way, each user is responsible for his/her own action. This kind of fairness is called competitive fairness, in which users are receiving a fraction of resources through their competition. In comparison, the application of pricing functions as in [2] [3] imposes a central utility function upon all users, which has been criticized as artificial fairness [4].

Auctions have recently been introduced into several areas of wireless communications to handle the problem of resource competition among selfish users. Auction, as a subset of game theory, is discussed in [5] for centralized downlink power allocation in CDMA systems. An iterative power allocation by relays in half-duplex AF protocol is proposed by [6]. Resource allocation for downlink of OFDMA networks is investigated in [7]. [4] studies downlink bandwidth allocation in wireless fading channels through auction. The proposed auction approach in [4] is an example of so-called competitive fairness, since the users compete to receive the desired subcarriers. Although [4] studies the wireless channel allocation using an auction algorithm, the proposed approach is appropriate for single unit auction. In multicarrier environment such as OFDMA systems, we need a multi unit auction [8]. Moreover, the assumed fictitious money does not reflect the QoS requirement of the users. It is just a simple approach to resolve the resource allocation problem and there is no notion of QoS requirements such as rate or delay.

To implement an auction approach, some challenging problems must be tackled [9] [10]. In combinatorial auction, the challenges come from the computational complexity along with the problem of incentive compatibility where the users reveal their true valuation for the demands. Moreover, there is a need for a clear and expressive bidding language. To tackle the computational complexity of the problem, we propose a method based on simulated annealing to find the near-optimum results. Since rational users are selfish, they may overstate or not truthfully report their resource demand. We define a utility function that motivates users to reveal their true valuation. Users' utility depends on the channel quality and minimum rate requirement which leads to an efficient and practical bidding process.

In this paper, we propose a distributed resource allocation in OFDMA network through auction theory. A combinatorial auction is formulated and the solutions are provided. Since the original problem is NP hard, a method based on simulated annealing applied to find near-optimum results. Moreover, we define a utility function that motivates users to reveal their true valuation for their demands. The organization of this paper is as follows. In section II, system model and proposed auction solution are described. Then, performance evaluation is provided through simulation results in section III. Finally, the presented work is summarized and the conclusions are drawn in section IV.

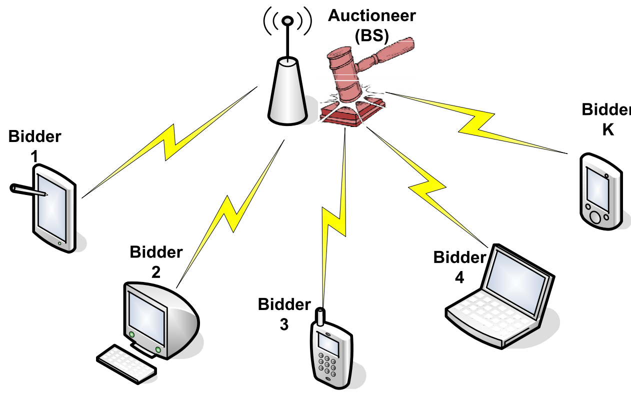

Fig. 1. Resource allocation through auction.

II. System Model

Consider a single-cell OFDMA system in Fig. 1, in which K users bid for available resources. Here resources mean the subcarriers to be allocated through auction process. There are total number of N subcarriers in the network. Each user is assigned a subset of subcarriers to send data over the wireless media. It is assumed that subcarriers are not shared by different users. This condition is referred to subcarrier occupation exclusiveness. We assume that the subcarrier separation is narrower than the coherent bandwidth. Each subcarrier can be considered as a slowly time-varying, frequency flat Rayleigh channel with bandwidth *B/N* assuming the total bandwidth is B. In our work, the Base Station (BS) has perfect channel estimation which is made known to the transmitter via a dedicated feedback channel without any delay. These channel estimations are used by users to offer a bid. The subcarrier allocation is made known to all users through a control channel, therefore each user needs only to decode the bits on their assigned subcarrier.

Problem formulation of resource allocation as combinatorial auction with budget constraint is as follows. The auction consists of K users and N subcarriers. Let P_k denote the power constraint of user k. Let vector A_k with items $a_{k,n}$ denote the allocation of subcarrier n which belongs to the bundle $\mathcal{N}_k$. $\mathcal{N}_k$ is the set of subcarriers which user k requests. $\alpha_{k,n} = 1$ if user k receives subcarrier n, and $\alpha_{k,n} = 0$, otherwise. The objective of allocation is maximizing the total utility of all users.

During each scheduling period, users transmit their sealed bid to the BS, and subcarriers are allocated to the winners of the auction. Note that, other users are not aware of this bidding. BS chooses the user with highest bid as the auction winner. Generally, this problem can be modeled as a game with K players, in which bidding functions are the strategies of players, and throughput of users are the payoff functions. Particularly, game can be formulated as $\Gamma = (K, \{S_k\}, \{u_k\})$ with K players, where $\{S_k\}$ is the strategy set, and $\{u_k\}$ is the players' payoff when choosing strategy set of s_k.

The bit rate of user k on allocated subcarrier n would be

$$r_{k,n} = \log_2\left(1 + \frac{\gamma_{k,n}}{\Gamma}\right) \quad (1)$$

where, $\Gamma = -\ln(5\mathrm{BER})/1.6$, is constant SNR gap, BER is the bit error rate, and corresponding Signal to Noise Ratio (SNR) for the k^{th} user in n^{th} subcarrier, $\gamma_{k,n}$, can be expressed as

$$\gamma_{k,n} = p_{k,n} h_{k,n}^2 / \sigma_{k,n}^2 \quad (2)$$

where $p_{k,n}$ is the power of user k on subcarrier n and $\sigma_{k,n}$ is the variance of Additive White Gaussian Noise (AWGN).

The objective of each user is to maximize its own utility, subject to power constraint. We modeled the utility function of user k as

$$U(r^k, r^k_{\min}) = \exp\left(-\frac{\left(r^k - r^k_{\min}\right)^2}{\left(r^k_{\min}\right)^2}\right) \quad (3)$$

where r^k is the allocated data rate, and $r^k_{\min}$ is the minimum required data rate.

$U(r^k, r^k_{\min})$ is a function of difference between actual allocated rate and minimum rate requirement. So, it reaches the maximum point when resource allocation meets user requirement exactly. User satisfaction decreases when allocated resources exceed the user requirement. This is because the increase in cost due to power consumption is more than the increase in achieved payoff. It means that users do not favor extra resources as they must use up their budget i.e. power, while they do not need to.

When allocated resource cannot meet the user request, satisfaction will decrease from maximum point. Some utility functions drop an incoming connection when the available resources are not enough to satisfy the user requirement. Applications can stand some QoS degradation, so system can still accept a new connection.

One of the major problems is the computational difficulty of determining an optimal allocation for a given set of bids. In the following, we propose a computationally efficient heuristic approach which finds the allocations that are close to optimum. First, the number of subcarriers a user may apply for is determined according to the rate requirements ratio of users, particularly

$$r^k_{\min} : r^v_{\min} = \beta_k : \beta_v \quad \forall k, v \in \mathcal{K} \quad (4)$$

where $\mathcal{K}$ is the user set, and β is the normalized proportionality constants, where $\sum_{k=1}^{K} \beta_k = 1$.

The number of subcarriers N_k, which user k requests, is determined to satisfy

$$N_1 : N_2 : \ldots : N_K = \beta_1 : \beta_2 : \ldots : \beta_K \quad (5)$$

This step is based on the reasonable assumption also made in [7] that the proportion of subcarriers assigned to each user is approximately the same as their eventual rates after power allocation. This is accomplished by $N_k = \lfloor \beta_k N \rfloor$, in which $\lfloor . \rfloor$ denotes the floor function.

The budget of each user is the available power. Each user knows its own budget and tends to spend it in an efficient approach. The selected approach must take into account the followings

a) Maximum available power of each user

b) The channel gain of users on different subcarriers, as bad quality channel needs more power for bit loading and vice versa. As we mentioned, each user has a power budget, P_k. Since the number of subcarriers allocated to the user k is determined to be N_k, the power level at each subcarrier for this user is then P_k/N_k. The subcarrier set $\mathcal{N}_k$ which user k requests is the N_k best available subcarriers, as each user acts rational.

We assume that value function of user k is in the form of

$$z_k = \frac{\sum_{n \in \mathcal{N}_k} \log\left(1 + \gamma_{k,n}\right)}{r_{\min}^k} \tag{6}$$

in which $\gamma_{k,n}$ is the SNR of user k on subcarrier n, and $r_{\min}^k$ is the minimum rate requirement. Note that, the value function depends on the subcarrier gain directly through SNR. The value function shows the willingness of users for a bundle of subcarriers which provides more bit loading with respect to rate requirement.

Algorithm 1 Simulated annealing algorithm for resource allocation

Initialization
$iteration = 0$, $\mathcal{K}_w \leftarrow \emptyset$, $K_{in} \leftarrow \emptyset$ $K_{out} \leftarrow \emptyset$
while $iteration \leq MaxIteration$ and $T \leq T_{th}$ **do**
  with probability p_1
  select a user i from $\mathcal{K} \backslash K_w$ randomly as K_{in}
  with probability p_2
  select a user randomly from $\mathcal{K}_w$ as K_{out}
  if $U(K_w \cup (K_{in} \backslash K_{out})) > U(K_w)$
  $K_w = (K_w \backslash K_{out}) \cup K_{in}$
  if $\mid (K_w \cup K_{in}) \backslash K_{out} \mid > N$
  if $i \equiv 0 \pmod{20}$
  decrease temperature
  $iteration = iteration + 1$
  $K_{in} \leftarrow \emptyset$ $K_{out} \leftarrow \emptyset$
end while

Combinatorial auction problems are usually formulated as packing problems and are known to be NP-hard problem to solve, or even to approximate. Usually three approaches are taken: heuristic approaches to improve the running time of finding the solutions, heuristic approaches to improve the quality of algorithms, and special cases that can be optimally solved efficiently. In this paper, we apply the second approach that finds the allocation that are near-optimum. In solving optimization problems, greedy approach is a candidate, but the efficiency of solutions are not satisfactory. One possibility to enhance the efficiency of the greedy strategy is to apply a stochastic improvement process on the initial allocation, trying to remove suboptimal bids and to replace them by bids which results in a higher reward for the auctioneer.

simulated annealing is a heuristic method that differs from the hill-climbing search in the sense that it may accept a down-hill move which can decrease the objective function value with a certain probability related to the temperature variable [6]. The algorithm is a combination of the hill-climbing approach and a random search based on the standard simulated annealing algorithm. Each step of the simulated annealing algorithm replaces the current solution by a random nearby solution, chosen with a probability that depends on both the difference between the corresponding function values and also on a global parameter T (called the temperature), that is gradually decreased during the process. The dependency is such that the current solution changes almost randomly when T is large, but increasingly uphill as T goes to zero. The allowance for downhill moves potentially saves the method from becoming stuck at local optima.

In the following, we apply a heuristic approach based on simulated annealing to solve the combinatorial problem of resource allocation. Starting with an empty set, the auctioneer tries to add a bid which is submitted by a user that has none of its bids accepted. If the resulting allocation violates the resource allocation constraint, the new allocation is discarded and another bid is tried. The fitness of an allocation is the auctioneer's utility. Besides adding and removing bids, the simulated annealing algorithm can also handle both operations simultaneously to obtain a new solution. The temperature is set at high value at the beginning and continuously decreased. For each cycle, a neighbor is randomly selected. The utility of the auctioneer at that neighbor may be less than the current value in some cases. Even in those cases, if a probability value based on the temperature is larger than 0, the state is moved to the new allocation that has less value. This could make us to get out from local optimum. Also the algorithm automatically restarts when it reaches a local optimum. Then, it repeats until the highest results are not updated in the last 5 restarts.

With a probability p_2 a random bid is chosen from the set of bids which has not yet accepted. If the new bid results in an improved value of the objective function ($\Delta U > 0$), the auctioneer accept the new bid. If the change results in a worse value ($\Delta U < 0$), the auctioneer may choose that bid with probability p_{acc}. The probability of accepting a new solution p_{acc} is determined by the metropolis probability, which depends on the temperature T

$$p_{acc}\left(\Delta U\right) = e^{\frac{\Delta U}{T}} \tag{7}$$

ΔU denotes the change in the auctioneers utility due to the insertion/removal step. Algorithm 1 shows the details of the approach we used to get the solutions of the resource allocation problem. The iteration continues until it reaches the maximum iteration number $MaxIteration$, and the temperature T is less than the threshold. $\mathcal{K}_w$ denotes the winners' set, K_{in} is the selected user to be added to the winners' set, and K_{out}

is the user to be removed from the winners' set. In each iteration a new user is chosen to be included in the winners' set with probability p_1, and to be removed by probability p_2. Probability p_1 is chosen as 0.25, and p_2 as 0.33 [11]. The condition $\mid (K_w \cup K_{in}) \backslash K_{out} \mid > N$ denotes that the total number of subcarriers to be allocated is N.

Note that, the starting temperature and cooling rate will affect the performance of the algorithm. We choose the starting temperature such that the algorithm accepts about %80 of iterations which leads to a deterioration of the utility function.

$$\sum_{i=1}^{N} e^{\frac{\Delta U}{T}} = 0.8 \tag{8}$$

As T decreases, it lowers the probability of acceptance of the bids which results in worse utility. T is controlled by the cooling function as follows

$$T_{\text{iteration}+1} = \mu T_{\text{iteration}} \tag{9}$$

where $\mu \in (0, 1)$ is the temperature decay rate.

In our simulations, The temperature between successive annealing stages is decreased by $\mu = 0.99$. Note that, there is a trade off between the efficiency and convergence rate of the proposed iterative approach. We perform our simulation with $\mu = 0.99$. In situation where convergence speed is crucial, smaller value of μ could be chosen, while maintaining a certain level of efficiency. The annealing process can be stopped if lowering the temperature does not significantly change the average utility difference.

III. Performance Evaluation

In this section, by providing simulation results, we evaluate the performance of our proposed resource allocation scheme for OFDMA systems. We consider a single cell OFDMA in which the BS locates at the cell center serving users in a point to multi-point manner. A frequency selective multipath channel is modeled consisting of six independent Rayleigh multipaths, with an exponentially decaying profile. A BER of 10^{-3} has been assumed for all users, which gives an SNR gap, $\Gamma = 3.3$. Maximum Doppler frequency is 30 Hz. Total bandwidth is divided equally among subcarriers. Total number of 32 subcarriers are considered in the network. The utility function of users is a bell shaped utility as expressed in (4).

The first numerical result evaluates the convergence performance of the proposed algorithm. Fig. 2 shows the increasing average utility during the annealing process. Obtained line has the typical shape of an annealing process. Initially, the temperature is high. Therefore, there is a random walk and no increase in the utility. Then the utility is increasing and finally, when the temperature is low enough the utility is fixed in an optimum. Note that, for large T the probability of accepting new steps which results in higher utility is small.

The utility of the users versus the revealed rate requirement in the bidding strategy, is shown in Fig. 3. Three users with different rate requirements i.e., $r^1_{\min} = 40$Kbps, $r^2_{\min} = 60$Kbps,

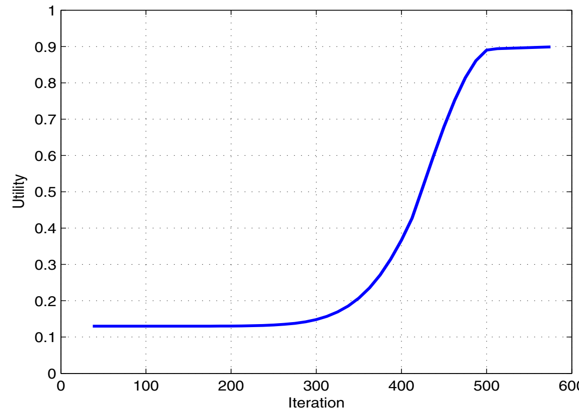

Fig. 2. Utility vs. number of iterations.

$r^3_{\min} = 90$Kbps, are considered. As the revealed rate in the biding function increases, the utility of users improves until it reaches its maximum value at the actual rate requirement. (i.e. truthful demand), beyond which the utility degrades. Hence, the proposed algorithm enforces truth telling and each user must report its true rate demand to maximize its utility. Note that, the defined utility function enforce users to reveal their truthful demands. The offered rate versus the requested rate is shown in Fig. 4. Intuitively the Hungarian approach has better performance, a result which is expected. The proposed solution based on auction has slightly lower performance compare to the central one. While the auction algorithm shows close performance to the central approach, the FFDS [12] allocation does not perform good. FFDS uses a distributed allocation of subcarriers based on Nash bargaining solutions. As Fig. 4 depicts the offered rate increases as the requested rate goes up, and then due to the fact that network resources becomes limited at some point, the curve will reach an upper bound. Note that the point of establishing a constant value is different for three approaches. Especially FFDS method curve indicates the inability of dealing with high network loads.

In order to compare fairness, we need to quantify the fairness first. We compare the fairness index of three algorithms. A commonly used measure of fairness is Jain fair index [13].

$$\text{Jain Index } = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2} \tag{10}$$

The Jain fairness index varies between 0 and 1. A rate allocation is perfectly fair if Jain Index $= 1$. Higher Jain Index indicates higher fairness among the users. Fig. 5 compares the Jain fairness index of auction algorithm with Hungarian and FFDS allocation. The results show that the auction game can provide better fairness for different rate requirements. Due to the limited radio channel resource, as the rate requirements of users increases the fairness index decreases. Since users are allowed to express their willingness for available resources, auction allocates the resources in a fair approach.

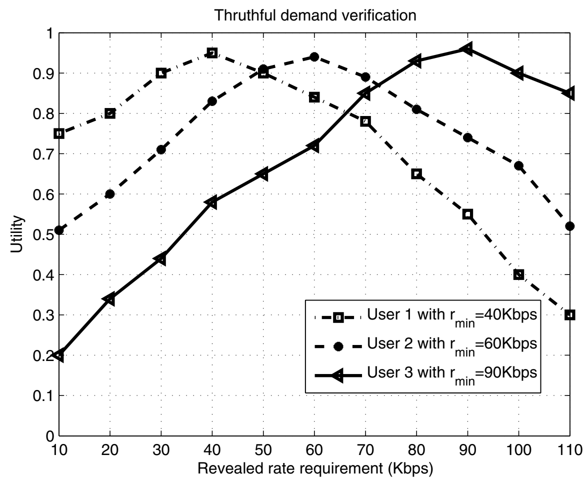

Fig. 3. Truthful demand verification.

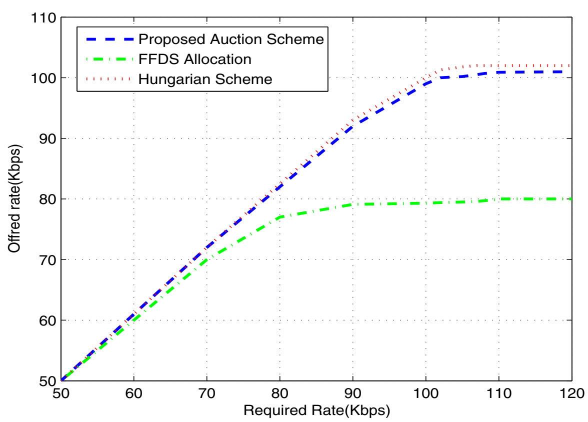

Fig. 4. Offered rate vs. required rate

IV. Conclusion

An auction-based algorithm has been used as a model to analysis the competitive resource allocation problem in OFDMA systems. In a competition between users for available resources, the valuation of each user for resources is sent as a bid to the BS. In this manner, the users are responsible for their own actions. valuation function of users depends on the channel gain and the minimum rate requirement. Since the original problem is NP hard, a method based on simulated annealing applied to find near-optimum solutions. Moreover, we define a utility function that motivates users to reveal their true valuation. The algorithm is shown to provide the fair distribution of resources while the performance utilization of the network maintained.

V. Acknowledgment

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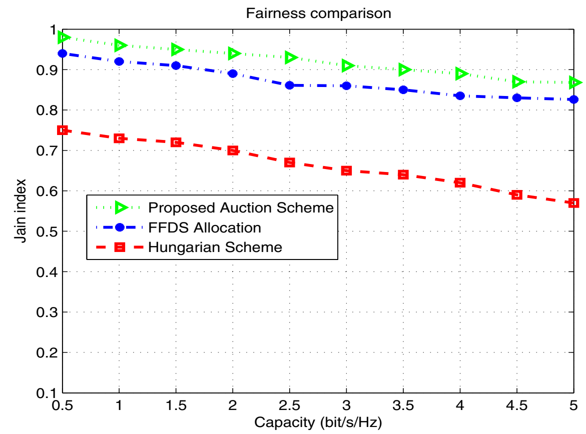

Fig. 5. Jain Fairness index for different schemes.

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