

Rate Control for Energy Minimization of Delay Constrained Cellular Transmissions

Nof Abuzainab*, André F. dos Santos** and Anthony Ephremides*

*Department of Electrical and Computer Engineering
University of Maryland, College Park, USA
Email:{nof,etony}@umd.edu

**Bell Labs, Alcatel-Lucent, Stuttgart-Germany
(A.Santos@alcatel-lucent.com)

Abstract—In cellular systems, base stations are considered the components that mostly drain energy. Hence, recent attempts have been made to reduce the power consumed by base stations by several methods such as "micro" sleep modes, where some parts of the transceiver can be temporarily switched off in short intervals where no information is transmitted. The objective of this study is to exploit the sleep mode feature of base stations and to use rate allocation techniques in order to minimize the energy spent to satisfy the demands of its users and their QoS requirements such as delay. In order to understand the behavior of the rate allocation method, the case where there is only a single active user in the cell is first considered. Then, the problem is extended to the case when several active users are present.

I. INTRODUCTION

The concern about the energy efficiency of cellular systems has vastly grown during the past decade due to the worldwide concern to reduce CO_2 emissions to combat the climate change. Further, it is anticipated that the amount of cellular traffic will tremendously grow in the coming years (by a factor of 1000) as shown in [1]. However, most current cellular systems are designed to mainly achieve the Quality of Service (QoS) requirements such as high data rates and low delays while not taking the energy consumed into consideration. Hence, it is becoming crucial to design schemes that meet the increasing demands of cellular traffic while maintaining low CO_2 emissions.

In cellular systems, base stations are considered the main components that consume energy. Besides, current base stations are designed to be always turned on, which consequently results in a considerable amount of energy wasted in the case of bursty traffic or when no users are active in the cells. In [2], it is addressed the potential in significantly reducing the energy spent in base stations when no users are active in the network. This is achieved by switching the base station to a "sleep mode" where the base station operates at a lower power than when it is fully active.

There are two types of sleep modes that are currently used in base stations. These are the "Micro" sleep mode and the "Deep" sleep mode [3]. In "Micro" sleep mode, only a fraction of the base station is switched off, and hence the base station can be turned on in a relatively fast time (in the order of milliseconds). In "Deep" sleep mode, however, most of the base station is turned off, and hence it takes a long time for

the base station to be turned on. The "Micro" sleep mode is beneficial in the case of "bursty traffic" where the base station can adapt its power efficiently based on the cell traffic, while the "Deep" sleep mode is more useful in situations when the base station will not be serving the users for a long period of time. In this paper, we will focus on the "Micro" sleep mode. Differently from the current work in the literature, e.g. [2] and [3], this paper does not discuss different frame formats of existing standards to allow the exploitation of sleep modes. Here, we allow a flexible system where any level of granularity for the sleep modes is allowed. Furthermore, theoretical simplifications, like disregarding pilot transmission, are applied here. Therefore, the results and methods presented here can be interpreted as follows: the obtained gains should be faced as upper bounds for practical systems; and the optimization framework is a first step in the direction of deriving a more "real-world" approach.

Although reducing energy consumption nowadays is of paramount importance; maintaining the users' QoS requirements is still the principal objective in the design of cellular systems.

However, the effect of turning off the base stations on other QoS requirements have not been extensively addressed. For example, it is not yet addressed in this context how to deliver the users target load with a certain delay constraint.

Also, it is not clear what rate value the base station should use in order to satisfy the user's quality of service requirements. Hence, this arises the importance of designing rate allocation techniques to minimize the energy consumed to achieve the user's QoS requirements while exploiting the "sleep mode" feature of the base station.

In this paper, we propose a rate allocation algorithm to minimize the energy spent to deliver a user's demand within a certain specified delay. The algorithm takes into account the "sleep mode" feature of the base station by using a piecewise linear model for the power consumption. Also, we propose a rate allocation algorithm for the case when multiple users are active in the network. The algorithm is based on Time Division Multiplexing where the base station allocates a fraction of the time to each user in which it transmits to that user with full bandwidth and finds the optimal rate allocation to each user that minimizes the total energy consumed in the cell.

II. SINGLE USER

A. Problem Formulation

In this case, we consider a Macro cell in which there is only one active user. The user has a demand of B bits to be delivered within T seconds. It is assumed that the base station can reduce its consumed power and switch to a sleep mode when the user is not active. In the active mode, the consumed power P_C of the base station is a linear function of the transmission power P_T . Measurements done in [4] on various base station models show that a linear function of the transmission power is a good approximation for the consumed power. Hence, the consumed power at the base station follows a piecewise linear model and is given by the following expression:

$$P_C = \begin{cases} s, & P_T = 0 \\ aP_T + b, & 0 < P_T \leq P_{max} \end{cases} \quad (1)$$

where P_{max} is the maximum transmission power and $s \leq b$. The received power at the user follows the pathloss power model given by:

$$P_R = AP_T|d|^{-\alpha} \quad (2)$$

where α is the pathloss exponent, d is the distance from the user to the base station, and A is a constant which accounts for system losses¹. Further, it is assumed the presence of receiver noise of power spectral density N_R at the user. In addition, external interference I from neighbouring cells is present². The base station transmits to the user over a bandwidth of w Hz at a rate of R bits/sec. It is assumed that the rate follows Shanon's capacity formula and hence is given by:

$$R = w \log \left(1 + \frac{AP_T|d|^{-\alpha}}{N_R w + I} \right) \quad (3)$$

The energy spent by the base station to satisfy the user's demand of B bits within the specified duration of T seconds is given by the following expression:

$$E = ((aP_T + b)\tau + s(1 - \tau))T \quad (4)$$

where τ is the fraction of time the base station is in the active mode³. The values of τ and R are related as follows:

$$\tau = \frac{B}{RT} \quad (5)$$

Combining equations 3, 4, and 5, we have:

¹No frequency selective fading is assumed here since we are interested in obtaining an upper bound on the performance of "real-world" schedulers.

²Note that the allocation problem here refer to a single-cell scenario. Therefore, no interference is taken into account in the optimization framework problem. The value of interference here is only included for completeness of the framework description. Furthermore, a background level of interference is assumed in the simulation in order the base station operates in its typical range of transmitted power.

³Here no discretization of the values of τ is made. Therefore, the obtained performance can be considered upper bounds of the performance of algorithms operating on discrete values of τ .

$$E(R) = (a \frac{2^{R/w} - 1}{\varphi(d)} + b) \frac{B}{R} + s(T - \frac{B}{R}) \quad (6)$$

where

$$\varphi(d) = \frac{A|d|^{-\alpha}}{N_R w + I} \quad (7)$$

The objective is to find the optimal rate value that minimizes the consumed energy.

Note that since $0 < \tau \leq 1$, we have $R \geq \frac{B}{T}$.

Also since $0 \leq P_T \leq P_{max}$ and by using equation 3, we obtain:

$$0 \leq R \leq w \log \left(1 + \frac{AP_{max}|d|^{-\alpha}}{N_R w + I} \right)$$

Hence, the objective can be stated as:

$$\begin{aligned} \min_R E(R) \\ \text{s.t. } \frac{B}{T} \leq R \leq w \log \left(1 + \frac{AP_{max}|d|^{-\alpha}}{N_R w + I} \right) \end{aligned}$$

B. Solution

It can be easily seen that the energy $E(R)$ given in equation 6 is a convex function of R (Since it is differentiable, it suffices to show that the second derivative with respect to R is nonnegative for every value of R in the constraint set) and the constraints are only bound constraints. Hence, any local minimizer is a global minimizer. However, the function is nonlinear in R . Hence, the optimal value is obtained by numerical nonlinear optimization methods. In this case, the standard "Interior Point" method is used to solve the nonlinear constrained optimization problem. The details of the "Interior Point" method can be found in [5]. The full proof of the convexity of $E(R)$ is shown in part A of the appendix.

C. Numerical Results

To evaluate the rate control algorithm, the following values of the parameters for the Macro base station given by [6] are used: $P_{max} = 20W$, $w = 10MHz$

The values of the user's demand and the delay constraint are: $T = 10sec$, $B = 15Mbits$

The values of the pathloss model parameters are taken from 3GPP simulation scenarios and are given by: $A = 0.03$, $\alpha = 3.76$

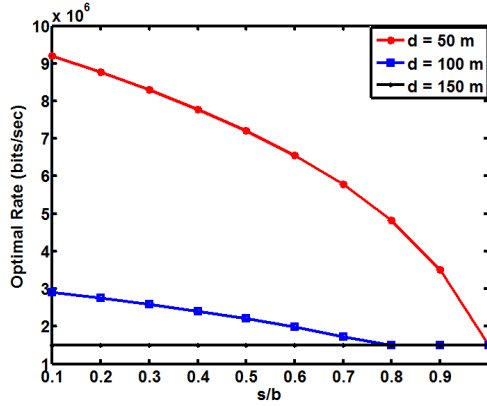
The values of the receiver's noise and the external interference used are: $N_R = 8.1875 \times 10^{-15}W/MHz$ and $I = 1.77 \times 10^{-11}W$

In order to investigate the effect of system parameters (such as the distance of the user, the values of the parameters of the power model used...), we compute the minimum energy consumed using the rate control algorithm for the following two cases:

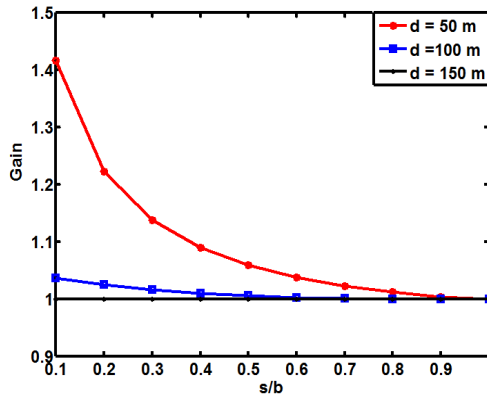
Case 1: The value of the sleep mode power value s is varied between $0.1b$ and b while the values of a and b are kept fixed at: $a = 5$ and $b = 118.7$.

The minimum energy is computed for each value of s . Also, the minimum energy value is compared to the energy spent using the non optimal rate allocation method that uses the lowest feasible rate to deliver the required load to the user. In the non optimal rate allocation method, the energy is computed by substituting the value of the lowest feasible rate in equation 6. Figure 1(a) plots the optimal rate value obtained versus the value of the sleep mode power value used for the cases when the distance between the user and the base station is given by: $d = 50, 100$ and 150 meters respectively. Figure 1(b) plots the gain of using the optimal rate allocation algorithm over the "Lowest Feasible Rate" allocation algorithm versus the sleep mode power value. The gain is defined as:

$$Gain = \frac{Energy_{Nonoptimal}}{Energy_{Optimal}} \quad (8)$$



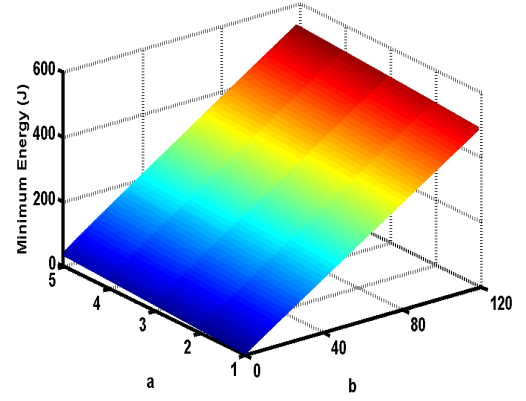
(a) Optimal Rate Value



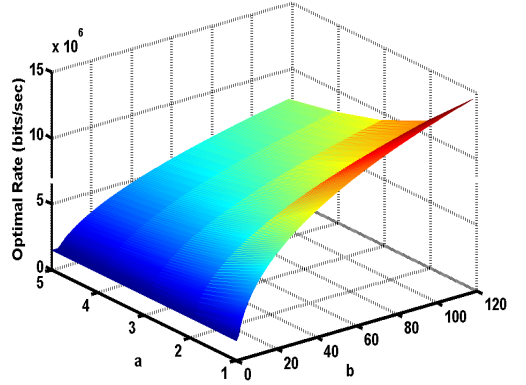
(b) The gain of the optimal rate allocation method

Fig. 1. The optimal rate values and the gain of the optimal rate allocation respectively versus the sleep mode power value s

Case 2: The value of a is varied from 1 to 5 while the value of b is varied from 0 to 119. Both values are varied in steps of one. For every value of b , the value of s is kept fixed at $0.4b$, and the minimum energy is computed for every pair of



(a) Minimum Energy Consumed



(b) Optimal Rate Value

Fig. 2. The minimum energy consumed and the optimal rate values respectively versus the base station parameters a and b

values of a and b . Figures 2(a) and 2(b) plots the minimum energy consumed (in Joules) and the optimal rate (in bits/sec) respectively versus the different values of the pair (a, b) . In this case, the value of the distance of the user from the base station used is: $d = 50m$

Figure 1(a) shows that as the distance between the user and the base station increases, the gain of the optimal rate allocation method decreases. This is because as the distance between the user and the base station increases, the optimal rate decreases and this is shown in figure 1(b). Also, figures 1(a) and 1(b) show that the gain of using the optimal rate allocation decreases as the sleep mode power value increases until it reaches unity for the case when the value of the sleep mode power is equal to the power when the base station is active (i.e. $s = b$). The reason the gain decreases is that the optimal rate decreases with increasing sleep mode power value as shown in figure 1(b) until the optimal rate value is equal to the lowest feasible rate.

Further, figure 2(a) shows that the minimum energy decreases slightly with decreasing the value of the parameter 'a' and decreases considerably with decreasing the value of the parameter 'b'. Also, figure 2(b) shows for low values of b the base station will transmit with lowest rate ; however as the value of b increases, the base station will transmit with higher

rate and hence try to maximize the duration of the sleep mode.

Similar conclusions are drawn when considering other values of system parameters.

III. MULTIPLE USERS

A. Problem Formulation

Now, we consider the case when multiple users are active in the Macro cell. Let M to be the number of users in the cell. Each user u_i is located at a distance d_i meters from the base station and has a demand of B_i bits. The base station should satisfy the demands of every user within time T seconds. Also, the base station transmits to each user with power value P_{iT} . The consumed power P_C by the base station follows the same piecewise linear model as in the preceeding section. Also, the received power at each user follows the pathloss model and it is assumed at each user the presence of receiver noise of power spectral density N_R and external interference I .

In this case, the base station uses a time division scheme in order to deliver the demands of the users. The time division scheme is explained as follows: the base station transmits using the total bandwidth w with rate R_i to user u_i during a fraction τ_i of the time duration T .

In this case, the fraction of time that the base station is transmitting to user u_i is given by:

$$\tau_i = \frac{B_i}{R_i T} \quad (9)$$

Hence, the energy spent by the base station is given by:

$$E = \left(\sum_{i=1}^M (aP_{iT} + b)\tau_i + s(1 - \sum_{i=1}^M \tau_i) \right) T \quad (10)$$

Also by substituting the value of the time fractions τ_i in equation 9 and the power P_{iT} as given by equation 3, we get:

$$E(R_1, \dots, R_M) = \sum_{i=1}^M \left(a \frac{2^{\frac{R_i}{w}} - 1}{\varphi(d_i)} + b \right) \frac{B_i}{R_i} + s \left(T - \sum_{i=1}^M \frac{B_i}{R_i} \right) \quad (11)$$

However, the rate values R_1, \dots, R_M are constrained by:

$$\sum_{i=1}^M \frac{B_i}{R_i T} \leq 1 \text{ since } 0 \leq \sum_{i=1}^M \tau_i \leq 1$$

Also, since $0 \leq P_{iT} \leq P_{max}$ and by equation 3, we obtain the following constraint on R_i at each user u_i :

$$0 \leq R_i \leq w \log \left(1 + \frac{AP_{max}|d_i|^{-\alpha}}{N_R w + I} \right)$$

Hence, the objective can be stated as:

$$\begin{aligned} & \min_{R_1, \dots, R_M} E(R_1, \dots, R_M) \\ & \text{s.t. } \sum_{i=1}^M \frac{B_i}{R_i T} \leq 1 \\ & 0 < R_i \leq w \log \left(1 + \frac{AP_{max}|d_i|^{-\alpha}}{N_R w + I} \right) \quad \forall i = 1, 2, \dots, M \end{aligned}$$

B. Solution

Similar to the case of single user, it can be shown that the function is convex. Also, the constraints are either bound constraints or convex function of R_1, R_2, \dots, R_M . Hence, a local minimizer is a global minimizer. The full proof for the convexity of the energy function and the constraint function are shown in part B of the appendix. But since the function is nonlinear, again the standard "Interior Point" numerical optimization method [5] is used to obtain the optimal solution.

C. Numerical Results

In this part, we investigate the performance improvement achieved by the optimal rate allocation algorithm. The same values of the parameters of the base station and the power consumption model used in section II.C. are used in this section. For evaluation, we assume that there are two users in the cell. The values considered for the distances of users u_1 and u_2 from the base station are: $d_1 = 100m$ and $d_2 = 30m$.

Also similar to the single user case, the performance of the optimal rate allocation algorithm is compared to the time division non optimal rate allocation method in which the time duration is divided equally between the users, and the base station transmits to each user with the lowest rate that satisfies the load within the allocated time period. Hence, the rate allocated in this case is given by:

$$R_i = \frac{B_i M}{T} \quad (12)$$

where M is the number of users.

Also for the non optimal rate allocation method, the energy is computed by substituting the rate values given by equation 12 in equation 11.

The gain achieved by the optimal rate allocation strategy over the non optimal strategy is computed and plotted in figure 3 respectively for the two cases when the sleep mode power value is given by: $s = 0.4b$ and $s = 0.5b$ respectively and both cases when user 1 load is kept fixed at 10 and 40 Mbits respectively while user 2 load is varied between 10 to 40 Mbits in steps of 10 Mbits. The gain in this case is also computed according to equation 8.

Figure 3 shows that the gain of the optimal rate allocation decreases as the users' loads increase. This is because as the users' loads increase, the sleep mode duration decreases which consequently reduces the gain.

Also, figure 3 show that as the sleep mode power value decreases, the gain achieved by using the optimal rate allocation increases. This is because the optimal rate allocation method exploits the lower power value used during sleep mode by transmitting with higher rates, which results in considerable energy savings.

IV. CONCLUSION

In this paper, we have proposed an optimal rate allocation algorithm to minimize the energy spent by a macro base station to deliver users' loads within a delay constraint. Also, we have used a piecewise linear power model in order to take into

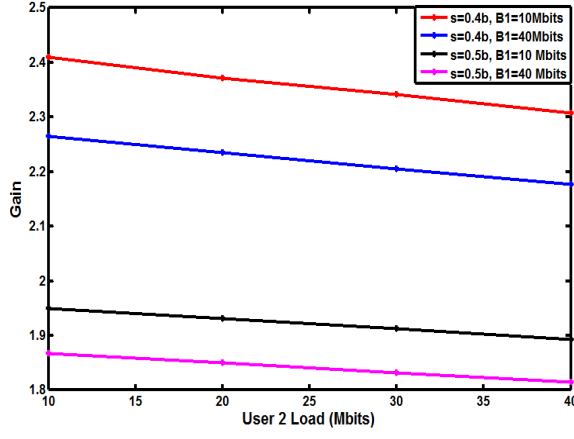


Fig. 3. The gain of the optimal rate allocation method versus user 2 load

account the reduced value of the power when the base station is in the sleep mode. Our results show that optimal rate value is dependent on the users' loads as well as their distances from the base station, and that the minimum energy decreases considerably with lower values of the power consumption model parameters. Hence, the results emphasize the need to reduce the values of the parameters of the power consumption model in order to reach the least energy spending.

APPENDIX

PROOF OF THE CONVEXITY OF THE ENERGY FUNCTIONS

A. Single User Case

For the case of a single user, the second derivative of the energy function E given by equation 6 with respect to the rate R is given by:

$$\frac{\partial^2 E}{\partial R^2} = \frac{aB((R^2 \ln(2)^2 + 2w^2 - 2Rw \ln(2))2^{R/w} - 2w^2)}{R^3 w^2 \varphi(d)} + \frac{2B}{R^3}(b - s) \quad (13)$$

where $\ln(\cdot)$ is the natural logarithm

Now since $b \geq s$, this implies $\frac{2B}{R^3}(b - s) \geq 0$. Also, the denominator $R^3 w^2 \varphi(d)$ is positive since R is positive. Hence, there remains to check the sign of the function $N(R) = (R^2 \ln(2)^2 + 2w^2 - 2Rw \ln(2))2^{R/w} - 2w^2$. Since the function $N(R)$ is zero when R is zero, it suffices to show that $N(R)$ is increasing function of R in order to prove that $N(R)$ is positive. By computing the derivative of $N(R)$ with respect to R , we get the following expression:

$$w \frac{\partial N}{\partial R} = (\ln(2)^2 R^2 + 2w \ln(2)(\ln(2) - 1)R + 2w^2(1 - \ln(2)))2^{R/w} \quad (14)$$

Since the exponential function $2^{R/w}$ is positive, there remains to check the sign the quadratic function: $\ln(2)^2 R^2 +$

$2w \ln(2)(\ln(2) - 1)R + 2w^2(1 - \ln(2))$. The discriminant Δ of the quadratic function is given by:

$$\Delta = (4\ln(2)^4 - 4\ln(2)^2)w^2 < 0 \quad (15)$$

Hence since the discriminant is negative, the roots of the quadratic equation are complex, which implies that for real values of R , the function is either positive or negative. By selecting $R = 0$, the value of the quadratic function at R is equal to $2w^2(1 - \ln(2))$, and hence positive.

Thus, the second derivative is nonnegative, and the consumed energy is a convex function of R .

B. Multiple Users

By arranging the terms of the energy function given by equation 11, it can be expressed as:

$$E(R_1, \dots, R_M) = \sum_{i=1}^M \left(\left(a \frac{2^{R_i/w} - 1}{\varphi(d_i)} + b \right) \frac{B_i}{R_i} - s \frac{B_i}{R_i} \right) + sT \quad (16)$$

In the above expression, each term i of the summation is a convex function of R_i . This can be verified since the second derivative of each term i with respect to R_i has the same expression as in equation 13. The only difference is that the term is a function of R_i instead of R .

Hence, the hessian H_E of the energy function is an $M \times M$ diagonal matrix where each entry $H_E(i, i)$ is nonnegative. Hence, the function is convex.

$H_E(i, i)$ is nonnegative. Hence, the function is convex.

As for the constraint function, the hessian matrix H_C is also diagonal matrix where each entry $H_C(i, i)$ has the following expression:

$$H_C(i, i) = \frac{2B_i}{R_i^3 T} \quad (17)$$

In the above expression, $H_C(i, i)$ is positive since the rate R_i is positive, and hence the constraint is convex.

REFERENCES

- [1] Cisco, "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2009-2014," White Paper, 2010.
- [2] P. Frenger, P. Moberg, Y. Jading, I. Godor, "Reducing energy consumption in LTE with cell DTX", in Proc. IEEE Vehicular Technology Conference (VTC 2011), 2011, Budapest, Hungary, May 2011.
- [3] L. Correia, D. Zeller, O. Blume, and D. Ferling, Y. Jading and I. Godor, and G. Auer, "Challenges and enabling technologies for energy aware mobile radio networks", IEEE Communications Magazine, vol. 48, no. 11, pp. 66-72, 2010.
- [4] G. Auer, Oliver Blume, V. Giannini, I. Godor, M. Imran, Y. Jading, E. Katranaras, M. Olsson, D. Sabella, P. Skillermark, W. Wajda, "Energy efficiency analysis of the reference systems, areas of improvements and target breakdown", 2010.
- [5] S. Wright, Primal-Dual Interior Point Method. Philadelphia, PA: SIAM, 1997.
- [6] G. Auer, V. Giannini, C. Desset, I. Godor, P. Skillermark, M. Olsson, M.A. Imran, D. Sabella, M.J. Gonzalez, O. Blume, A. Fehske, "How much energy is needed to run a wireless network?", IEEE Wireless Communications, vol. 18, no. 5, pp. 40-49, 2011.