The Smearing Filter Design Techniques for Data Transmission

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***Abstract*—Reducing Impulse noise is a very active research area in communication systems. This paper presents a digital smear-desmear technique (SDT) applied to data transmission over band limited channels. A generalized set of filter design criteria based on minimizing the average bit error probability is introduced. The design criteria were applied to a practical digital filter implementing SDT techniques. The SDT is simulated and combined with coded communication systems for high data transmission rate. Simulation results show that the SDT yields a significant improvement in bit error rates for coded systems subject to impulse noise, relative to the systems with no SDT. The technique also completely removes the error floor caused by the impulse noise.**

***Index Terms*—Smear, Desmear, Sequences, Impulse noise, Intersymbol Interference.**

I. Introduction

Most of the advances in theory and implementation of digital transmission over band limited channels have been made with respect to additive white Gaussian noise (AWGN), as the ultimate reliability limitation. With improved equalization, transmission rates achieved over these channels are close to theoretical limits [1]. However, the required error probabilities for reliable data transmission have not been achieved. One of the main impairments on these channels, causing burst errors is impulse noise (IN). Burst error correcting codes [2], matched to the channel error statistics could be effective in mitigating the effect of the burst errors. However, the fact that all known burst error correcting codes required additional bandwidth. A possible way to combat impulse noise is the smear-desmear technique (SDT) [3]. The SDT in [3] has been implemented in analog technology and the results were not satisfactory due to insufficient quality of analog devices. A digital SDT technique that applied to binary sequences of limited length was described in [4]. The design of SDT filter in [4] is based on minimizing intersymbol interference (ISI) and maximizing filter power efficiency. In the design, losses in signal to noise ratio (SNR) can be significant since transmit and receive filter are not matched. The purpose of this paper is to derive a more general set of filter design criteria based on minimizing bit error probability and derive methods for practical filter design. As a result, another necessary requirement, minimization of the signal-to-noise ratio (SNR) loss due to mismatching filters is added to the design criteria [4]. In Design 1, the smearing filters form a pair of matched filters. The filter sequences are required to have constant amplitude and good autocorrelation properties. The proposed polyphase sequences possess significantly better autocorrelation properties, measured by the merit factor F_2, than the binary sequences. A communication system with smearing filters is evaluated in the presence of gaussian and impulse noise. Simulation results show that the SDT based on polyphase sequences yields significant improvement in bit error rates compared to SDT based on binary sequences of the same length. The SDT is attractive on bandwidth limited channels since it does not require bandwidth expansion. The performance improvement is obtained at the cost of an additional delay in the system which can be tolerated in applications of interest. The paper is organized as follows. In section II, we introduce the model of a digital transmission system with smear-desmear filters and discuss the concept of the digital smear-desmear processing. Section III describes the digital SDT in more detail. Section IV defines essential criteria and parameters for SDT design. Section V presents simulation results. Finally, conclusions are summarized in section VI.

II. SYSTEM TRANSMISSION MODEL WITH SDT

A digital communication system with the SDT is modelled in Figure 1. A binary sequence generated by the digital source is mapped into a coded modulated signal. The modulator output symbols are processed by the digital smear filter. In the receiver, the desmear filter performs an inverse operation to the one in the smear filter and thus removes the ISI introduced in the transmitter. Both the smear and desmear filtering are performed in the baseband. After processing by the desmear filter the impulse noise energy is spread out over the filter impulse response length. That results in a significant reduction of the impulse noise effect on the signal. Let b denote a complex signal sequence at the output of the modulator: **b** = [b(0), b(1),· · ·,b(n),· · ·,b(m)], where b(n) is the modulated symbol transmitted at time n and m is the sequence length. We assume that sequence **b** is an independent identically distributed (iid) sequence. The smear filter is represented by a sequence of tap coefficients, denoted by s = [s(0), s(1),· · ·,s(i),· · ·s(N)], where s(i) is the ith tap coefficient and (N+1) is the number of taps. The output sequence is c = [c(0), c(1),···,c(n),···, c(m+N)]. At the time instant n output symbol denote by c(n), is give by s = [s(0), s(1),· · ·,s(i),· · · s(N)], where s(i) is the ith tap coefficient and (N+1) is the number of taps. The output sequence is c = [c(0), c(1),· · ·,c(n),· · ·, c(m+N)]. At the time instant *n*,the output symbol denote by c(n), is given as

$$c(n) = \sum_{j=0}^{N} b(n-j)s(j) \tag{1}$$

We assume that the filter gain denoted by $\mathbf{A}_s$ is normalized to unity

$$\mathbf{A}_s = \sum_{j=0}^{N} s(j)s^*(j) = 1 \tag{2}$$

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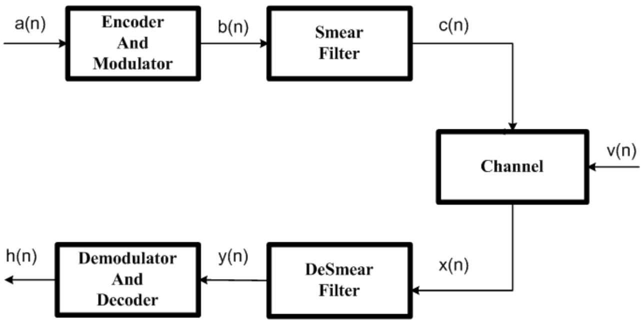

Fig. 1. System Transmission model with SDT

The total channel distortion at time *n* is given as the sum of the residual ISI, b_{ISI}(n), the additive white gaussian noise after desmearing filter v(n), with a zero mean and the variance σ_v^2 = $A_d\sigma^2$ and the impulse noise v_s(n), smeared over N symbol intervals. The impulse noise has a gaussian distribution with zero mean and the variance in the $(n+j)^{th}$ symbol interval is given by

$$\sigma_s^2 = E(v_s(n+j)v_i^*(n+j) = \sigma_i^2(n)|d(j)|^2 \tag{3}$$

where j = 0,1,$\cdots$N The residual ISI, b_{ISI}(n), is given by the sum of N independent random variables. Typically, N is larger than 10 and according to the central limit theorem b_{ISI} has a gaussian distribution. The sum of the three independent gaussian variables b_{ISI}, v and v_i is another gaussian variable given as

$$e(n) = y(n) - b(n) = b_{ISI}(n) + v(n) + v_s(n) \tag{4}$$

with the variance

$$\sigma_e^2 = E(|b_{ISI}(n)|^2) + E(|v(n)|^2) + E(|v_s(n)|^2) \tag{5}$$

where σ_e^2 can be upperbounded by

$$\sigma_e^2 = \sigma_v^2 + \sigma_{ISI}^2 + \sigma_s^2(n+j) \leq \sigma_v^2 + \sigma_{ISI}^2 + max\sigma_i^2 \tag{6}$$

where σ_{ISI}^2 is the variance of the ISI, and maxσ_i^2 is the maximum impulse noise variance.Thus the total channel distortion can be considered as an equivalent gaussian process with the variance σ_e^2. The measure of the real system performance loss relative to the ideal system is defined as the ratio of the ideal and the real system signal to noise ratios given by

$$L = 10\log_{10}\frac{\sigma_e^2}{\sigma^2}[dB] \tag{7}$$

III. THE SMEARING FILTER DESIGN CRITERIA

The smear and desmear filter are implemented as digital filters. The initial criterion in the filter design is minimization of the performance loss given by equation (8). The performance loss is required to be below a specified threshold T.

$$L \leq T \tag{8}$$

The performance loss directly depends on the equivalent gaussian process variance, σ_e^2 expressed in (6). The minimization of σ_e^2 can be done by independent minimization of each of the three components in the sum. The smearing filter design criteria are:

A. Minimize the AWGN variance

In order to minimize the AWGN variance the smear and the desmear filters should satisfy the matching condition in form of

$$s(j) = d^*(N-j), wherej = 0,1,2,\cdots,N \tag{9}$$

If the matching condition expressed by (9) cannot be satisfied, we define a measure of mismatching between the smear and desmear filter, called the mismatching loss [5]

$$L_m = 10\log_{10}\frac{\sigma_v^2}{\sigma^{\ 2}} = 10\log \mathbf{A}_d[dB] \tag{10}$$

In filter design we required that the mismatching loss, L_m, is less than a specified threshold T_m.

$$L_m \leq T_m \tag{11}$$

The mismatching loss is caused by an increase of AWGN variance relative to the ideal system with matched filters. In practical filter design we require that the mismatching loss is 0.3dB

$$L_m \leq 0.3dB \tag{12}$$

B. Minimize the residual ISI variance

To obtain the minimum ISI variance, the overall transfer function of the smear filter, the channel and the desmear filter should be flat. Since we assumed that the channel does not introduce ISI, the above condition is satisfied if the convolution of the sequences **s** and **d**, denoted by C(k), has the property where convolution C(k) is defined as

$$C(k) = \sum_{j=0}^{N} s(j)d(k+N-j) \tag{13}$$

$k = -N+1,\cdots,-1,0,1,\cdots,N-1$, if condition (9) is satisfied then (13) simplifies to

$$C(k) = R(k) = \sum_{j=0}^{N} d^*(j)d(k+j) \tag{14}$$

where R(k) is the autocorrelation function of the sequence **d**. Practical difficulties makes zero ISI unattainable objectives in filter design. Typically, a certain amount of residual ISI after the desmearing filter is tolerated. It is measured by the variance of the residual ISI given by

$$\sigma_{ISI}^2 = \sum_{k=-N+1}^{N-1}|C(k)|^2 - |C(0)|^2 \tag{15}$$

The signal to noise ratio loss caused by the ISI is defined as

$$L_s = 10\log\frac{\sigma^2 + \sigma_{ISI}^2}{\sigma^2}[dB] \tag{16}$$

Equivalently,

$$L_s = 10\log(1 + \frac{SNR}{F_2}) \qquad [dB] \tag{17}$$

where SNR is the signal to AWGN power ratio defined as $SNR = \frac{|C(0)|^2}{\sigma^2}$ and $F_2 = \frac{|C(0)|^2}{\sigma_{ISI}^2}$ is the merit factor defined in [4] . From a practical point of view it is much easier to use a quantity called normalized ISI level, denoted as L_{ISI}, and defined as

$$L_{ISI} = -10 \log F_2 \qquad [dB] \tag{18}$$

The residual ISI is considered as an additional Gaussian process introducing a certain SNR loss at the receiver. This loss should be minimized below a specified threshold Ts. For systems employing multilevel modulation schemes require the minimum of at least 20dB

C. Minimize the Impulse noise variance

This consists of minimization of the maximum smear impulse noise variance. To be consistent with the already criteria, we introduce the merit factor defined as the ratio of the maximum impulse noise variance and the maximum smeared impulse noise variance in a single symbol interval

$$F_2 = \min_j \frac{\sigma_i^2(n)}{\sigma_{vi}^2(n+j)} = \frac{1}{\max_j |d(j)|^2} \tag{19}$$

$j = 1, \cdot\cdot\cdot, N$ or in dB

$$L_{F_2} = 10 \log F_2 \qquad [dB] \tag{20}$$

In practical filter design we required that

$$L_{F_2} \leq T_{F_2} = 20 \qquad [dB] \tag{21}$$

The impulse noise variance before spreading can be as large as signal variance [6]. Impulse noise spreading should reduce the impulse noise variance to a level of the AWGN variance. The merit factor F_2 in (19) should be as large as possible. It shows how much the impulse noise variance in a single symbol interval has been reduced by smearing. A filter with a large length can produce a large merit factor F_2. It is convenient to introduce a measure for filter smearing efficiency which does not depend on filter length in [7], [8]. The power efficiency of the sequence **d**, denoted by η is defined as

$$\eta = \frac{\sum_{j=0}^{N} |d_j|^2}{(N+1)\max_j |d_j|^2} \qquad j = 0, \cdot\cdot\cdot, N \tag{22}$$

The power efficiency has its maximum value of 1, for constant amplitude sequences, while less than 1 for nonconstant amplitude sequences. Combining equation (19) and (22) we obtain the following expression for the merit factor F_2 as

$$F_2 = \frac{\eta(N+1)}{\mathbf{A}_d} \tag{23}$$

Equation (23) shows that in order to maximize the merit factor F_2, sequences should have the power efficiency as large as possible. The equality is satisfied if and only if

$$|d(j)|^2 = \frac{1}{N+1} \quad and \quad s(j) = d^*(N-j), \quad j = 0, 1, \cdot\cdot, N \tag{24}$$

That is, the optimum merit factor F_2 is achieved only when the smearing filters form a matched filter pair and both of them are represented by sequences with constant amplitude.

IV. Practical Filter Design

The optimum values for the three filter design criteria cannot be achieved simultaneously. In this paper we focus on constant amplitude sequences with good autocorrelation properties. These sequences are of general interest in digital communications and radar applications. The most common constant amplitude sequences are the binary maximum linear feedback shift register sequences referred to a M-sequences in [9]. Polyphase sequences with constant amplitude, known as Frank and P1-P4 sequences in [10], [11], [12], have better autocorrelation properties than M-sequences. It is important to note that polyphase sequences are resilient to carrier phase and timing instabilities [13] .We will discuss the properties of these sequences with respect to SDT applications.

A. Constant Amplitude Polyphase Sequences

Polyphase sequences are digital versions of Chirp signal [7] with an improved ratio between the main lobe and side lobes relative to their analog forms. For a polyphase Frank sequence of length $N = L^2$, the phase of the sequence element is

$$\phi(k,l) = \frac{2\pi}{L}(k-1)(l-1) \tag{25}$$

and sequence elements are

$$d[k + L(l-1)] = \exp(j\phi(k,l)) \tag{26}$$

where $k = 1, \cdot\cdot\cdot, L$ and $l = 1, \cdot\cdot\cdot, L$ The phases of P1 and P2 sequence elements are given by the following expressions,

$$\phi(k,l) = \frac{-\pi}{L}[L-(2k-1)][(k-1)L+(l-1)] \tag{27}$$

and

$$\phi(k,l) = \frac{-\pi}{2L}[L+1-2k)(l-1) \tag{28}$$

It is important to observe that both P1 and P2 sequences are available only for square integer lengths, i.e $N = \cdot\cdot\cdot 36, 49, 64, \cdot\cdot\cdot P2$ sequences are further restricted to even lengths only. Odd length P2 sequences possess rather bad autocorrelation properties [12]. Sequences P3 and P4 are defined for any integer length. Phases of their elements are

$$\phi(k) = \frac{\pi}{N}(k-1)^2 \tag{29}$$

and

$$\phi(k) = \frac{\pi}{4N}(2k-1)^2 - \frac{\pi}{4}(2k-1) \tag{30}$$

for $1 \leq k \leq N$ The most important property of constant amplitude polyphase sequences, relevant to SDT applications, is that the mainlobe to sidelobe power ratio is a monotonically increasing function of the sequence length. This property makes them much more effective in suppressing ISI then binary sequences [4], [14], [15], [16]. In addition, these sequences have constant amplitude and consequently, the optimum Criterion in (24). On the other hand, polyphase sequences are generated analytically.

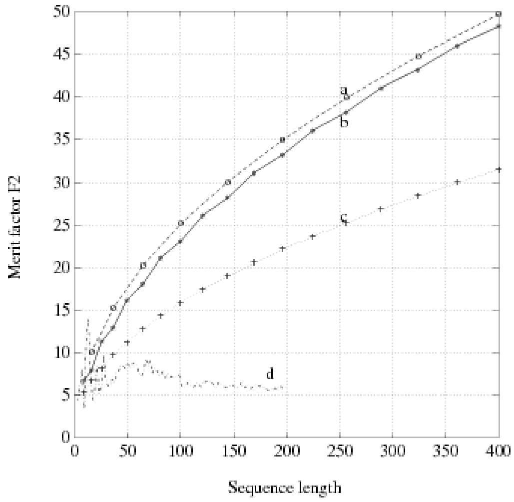

Fig. 2. Merit factor f2 for sequences with constant amplitude. a) P2 sequence, b) Frank and P1 sequence, c) P3 and P4 sequences, d) Binary sequence

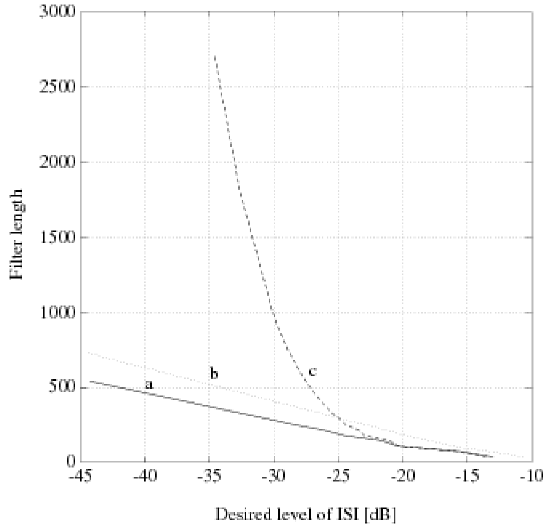

Fig. 4. The smearing filter length, K + 1 for systems with short sequence equalization: a) Frank sequence of length 36, b)Binary sequence of length 33, c)P2 sequence of length 36.

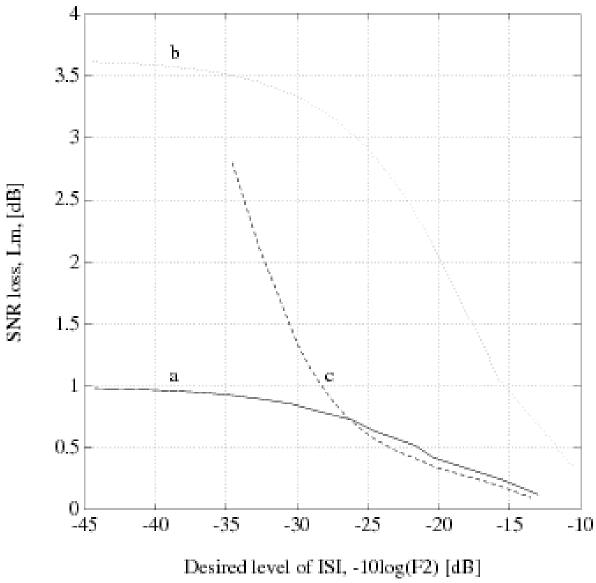

Fig. 3. SNR loss LM due to sequence equalization for short sequences with constant amplitude: a) Frank sequences of length 36, b)Binary sequence of length 33, c)P2 sequence of length 36

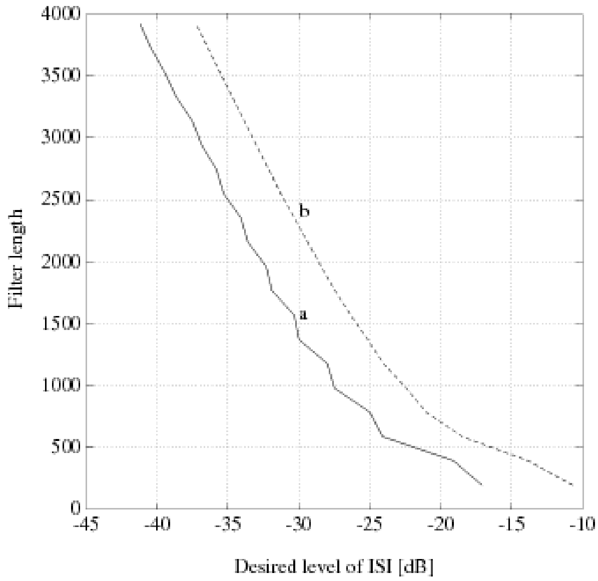

Fig. 5. The smearing filter length, K + 1 for systems with long sequence equalization: a) Frank sequence of length 196, b) Binary sequence of length 195

V. SIMULATION RESULTS

The simulation results are presented in the form of the bit error rates as functions of SNR for data communication system. Figure 2 show the performance of the merit factor F_2 binary sequences listed in [38] and polyphase sequence of length 200 and 400 respectively. The main lobe to side lobe power ratio has a floor for binary sequences, while it increases monotonically with the sequence length for polyphase sequences. It has been observed that this ratio is proportional to the square root of the sequence length. In figure 3, the sequence performance is measured by the SNR loss relative to the matched filter pair, denoted by L_m. Figure 4 shows the required filter length for the P2 sequence. Although this sequence has very good autocorrelation properties. It required very long filters, relative to binary and Frank sequences in order to achieve a low level of ISI suppression. Figure 5 illustrate the smearing filter lengths needed to obtain a specified level of ISI suppression using long Frank and Binary sequence of comparative length. Figure 6 and 7 show simulation results for coded and uncoded systems in the presence of impulse noise (IN). Parameters of impulse noise (IN) are: $\lambda = 10^{-3}$ events/s,, $SNR_{in} = 0$ [dB] The average impulse noise length is 0.0008s (two signal intervals). In a communication system subject to impulse noise, it is likely that all affected symbols will be incorrect, resulting in error bursts with the bit error

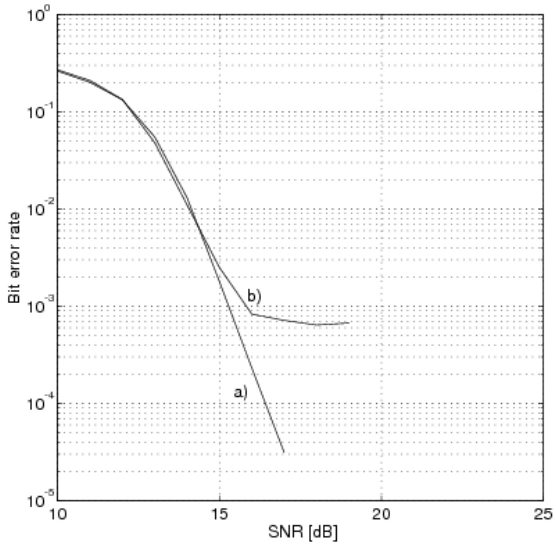

Fig. 6. Bit error rate for coded system in the presence of impulse noise: a) with SDT, b) without SDT.

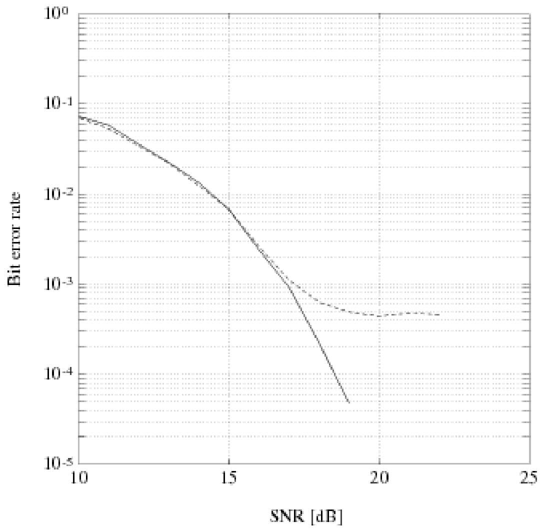

Fig. 7. Bit error rate for uncoded system in the presence of impulse noise: Solid line: with SDT, Dash line: without SDT

rates close to 0.5. Typical error rate curves exhibit an error floor which cannot be eliminated by increasing SNR. Clearly, there is an error floor in the error rate curves for both coded and uncoded systems with no SDT. The results indicate that the SDT offers a significant reduction in the SNR required to achieve the same bit error rate as in a system with no SDT for both coded and uncoded systems. The coding gain of the coded system relative to the reference uncoded system is 2.5 dB at the BER at 10^{-5}, which is almost the same as the coding gain on gaussian channels. Also, the SDT completely removes the error floor in both systems. In the above example for the filter impulse response of length N=256 and the power efficiency $\eta = 0.54$, the theoretical SD gain is F_2=22 dB. In most cases a gain of this order is sufficient to suppress the influence of IN on the bit error rate. The real SD gain is reduced due to error clustering caused by impulse noise spreading.

VI. CONCLUSIONS

The paper describes a digital SDT based on polyphase multilevel sequences of unlimited length with good autocorrelation properties. A design procedure for digital implementation of SDT is defined and new sequences with power efficiency higher than 50% are generated. These sequences are applied to design of digital smear/desmear filters and combined with uncoded and coded communication systems for high data transmission. The impulse noise is modeled as a sequence of Poisson arriving delta functions with gaussian amplitudes. The impulse noise parameters are computed from experimental data. Simulation results show that the SDT filter design method yields a significant improvement in bit error rates for both systems subject to impulse noise, relative to systems with no SDT. The technique also completely removes the error floor caused by impulse noise.

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