

# IIR Lattice-Based Blind Equalization Algorithms

Hsiao-Fu Lee<sup>1</sup>, Jenq-Tay Yuan<sup>2</sup>, *Senior Member, IEEE*, and Tzu-Chao Lin<sup>3</sup>

<sup>1</sup>Department of Electrical Engineering, <sup>2,3</sup>Graduate Institute of Applied Science and Engineering,  
Fu Jen Catholic University, Taipei 24205, Taiwan, R.O.C.

E-mail: <sup>1</sup>[497506063@mail.fju.edu.tw](mailto:497506063@mail.fju.edu.tw), <sup>2</sup>[yuan@ee.fju.edu.tw](mailto:yuan@ee.fju.edu.tw), <sup>3</sup>[tc.lin@ee.fju.edu.tw](mailto:tc.lin@ee.fju.edu.tw)

**Abstract** - This paper develops infinite impulse response (IIR) lattice structures based on two widely known blind equalization algorithms, which are the multi-modulus algorithm (MMA) and the constant modulus algorithm (CMA). The motivation for doing so is that generally for algorithms to achieve the same performance, an IIR filter requires fewer coefficients than its corresponding finite impulse response (FIR) counterpart. Moreover, the monitoring of the stability of an IIR lattice filter is simpler and less computationally complicated than that of the direct-form IIR filter. Computer simulations that compare the IIR lattice-based blind equalization algorithm with the corresponding FIR transversal algorithms and FIR-based lattice algorithms demonstrate that the former is especially suitable for equalizing channels, yielding inter-symbol interference (ISI) that mostly results from the post-cursor term.

**Keyword:** blind equalization, constant modulus algorithm (CMA), infinite impulse response (IIR), lattice filters, multi-modulus algorithm (MMA)

## I. INTRODUCTION

Linear filters may be classified into *finite impulse response (FIR)* and *infinite impulse response (IIR)* filters, which are characterized by finite memory and infinitely long memory, respectively. In this paper, the tap inputs (or observations)  $x(n)$  and tap weights  $w_m(n)$  are complex valued for  $m = 0, 1, \dots, N$ . The transversal filter produces an overall filter output  $y(n)$ , which is an estimate of the transmitted signal  $s(n)$ ,

$$y(n) = \sum_{m=0}^N w_m^*(n) x(n-m) \quad (1)$$

where the asterisk denotes complex conjugation;  $(N+1)$  is the filter length and the tap weights,  $w_m(n)$ , will be adjusted to minimize the mean-squared error (MSE)  $E[|e(n)|^2] = E[|s(n) - y(n)|^2]$ , where  $s(n)$  is the transmitted signal. In some applications, an equalizer must be capable of "blind" adaptation since  $s(n)$  is unknown to the receiver. Blind adaptation algorithms are well known to exploit some known statistical property of the transmitted signal  $s(n)$  to eliminate channel distortion.

This paper develops two novel blind equalization algorithms. They are an IIR lattice-based multi-modulus algorithm (MMA) (referred to as the *IIR lattice MMA*) and an IIR lattice-based constant modulus algorithm (CMA) (referred to as the *IIR lattice CMA*). A primary advantage of the IIR lattice filter is that the monitoring of its stability is simpler and less computationally complicated than that of the direct-form IIR filter, because the IIR lattice filter is stable provided

that the reflection coefficients  $k_m(n)$  satisfy the condition  $|k_m(n)| < 1$ ,  $1 < m < N$  [1], [2]. Computer simulations are also conducted to compare the proposed IIR lattice MMA with its FIR transversal and FIR lattice counterparts.

## II. BLIND EQUALIZATION USING FIR LATTICE MMA AND FIR LATTICE CMA

The FIR lattice structure of prediction-error filter in Fig. 1 may be implemented by using [3]

$$f_m(n) = f_{m-1}(n) + k_m^*(n) \cdot b_{m-1}(n-1) \quad (2)$$

$$b_m(n) = b_{m-1}(n-1) + k_m(n) \cdot f_{m-1}(n) \quad (3)$$

where  $k_m(n)$  is the  $m$ -th stage *reflection coefficient*;  $f_m(n)$  and  $b_m(n)$  are the  $m$ -th order forward and backward prediction errors, respectively, for  $m = 1, 2, \dots, N$ , and  $N$  is the final prediction order. In the special case in which  $m = 0$ , the initial conditions are set to  $b_0(n) = f_0(n) = x(n)$ . The  $m$ -th order *reflection coefficient* may be updated using [3]

$$k_m(n+1) = k_m(n) - (\mu / \varepsilon_{m-1}(n)) \cdot [f_{m-1}^*(n) b_m(n) + b_{m-1}(n-1) f_m^*(n)] \quad (4)$$

where  $\mu$  is the step size and  $\varepsilon_{m-1}(n) = \rho \cdot \varepsilon_{m-1}(n-1) + (1-\rho) \cdot [|f_{m-1}(n)|^2 + |b_{m-1}(n-1)|^2]$  is an estimate of the total prediction error power that enters the  $m$ -th stage, in which  $\rho$  is a constant in the range  $0 < \rho < 1$ . Lattice realizations have been demonstrated to be able to accelerate the rate of convergence in equalizers because a lattice structure transforms the correlated tap inputs  $x(n), x(n-1), \dots, x(n-N)$  into uncorrelated backward prediction errors  $b_0(n), b_1(n), \dots, b_N(n)$  [3], [4], owing to the removal of the redundancies that are associated with  $x(n)$ . The filter output  $y(n)$  in (1) may also be expressed as

$$y(n) = \sum_{m=0}^N g_m^*(n) b_m(n) \quad (5)$$

where  $g_m(n)$  are the *regression coefficients*. The regression coefficients  $g_m(n)$  of the *FIR lattice MMA* and the *FIR lattice CMA* that are presented in Fig. 1 may be updated according to the stochastic gradient descent (SGD) method by minimizing the MMA and the CMA cost functions (derivations have been omitted owing to space limitations), respectively, as follows

$$g_m(n+1) = g_m(n) - (\mu / \delta_m^2(n)) \cdot \{y_R(n) \cdot [y_R^2(n) - \rho_R] - jy_I(n) [y_I^2(n) - \rho_I]\} \cdot b_m(n) \quad (6)$$

and

$$g_m(n+1) = g_m(n) - (\mu / \delta_m^2(n)) \cdot \{y^*(n) \cdot (|y(n)|^2 - R_2)\} \cdot b_m(n) \quad (7)$$

where  $R_2 \triangleq E[|s(n)|^4] / E[|s(n)|^2]^2$ ,  $\rho_R \triangleq E[s_R^4(n)] / E[s_R^2(n)]$  and  $\rho_I \triangleq E[s_I^4(n)] / E[s_I^2(n)]$  in which  $s_R(n)$  and  $s_I(n)$  are the real and imaginary parts of  $s(n)$ , respectively;  $\delta_m^2(n)$  in (6) and (7) is power estimate for the  $m$ -th order backward prediction error, which may be updated according to

$$\delta_m^2(n) = \rho \cdot \delta_m^2(n-1) + (1-\rho) \cdot b_m^2(n). \quad (8)$$

### III. BLIND EQUALIZATION USING IIR LATTICE MMA

The IIR lattice filter may be obtained by rewiring the forward prediction error of the FIR lattice in (2) such that the  $m$ -th order IIR lattice realization of the prediction-error filter at time  $n$ , displayed in Fig. 2, may be written as [1], [5], [6]

$$f_{m-1}(n) = f_m(n) - k_m^*(n) \cdot b_{m-1}(n-1) \quad (9)$$

$$b_m(n) = b_{m-1}(n-1) + k_m(n) \cdot f_{m-1}(n) \quad (10)$$

where

$$b_0(n) = f_0(n) \quad (11)$$

$$f_N(n) = x(n) \quad (12)$$

The overall filter output  $y(n)$  is the sum of the backward prediction errors weighted by the regression coefficients  $v_m(n)$  for  $m = 0, 1, \dots, N$ ,

$$y(n) = \sum_{m=0}^N v_m^*(n) b_m(n) \quad (13)$$

If  $y_R(n)$  and  $y_I(n)$  denote the real and imaginary parts of  $y(n)$ , respectively, then the MMA cost function may be expressed as [7]-[9]

$$J_{\text{MMA}}(n) \triangleq E[(y_R^2(n) - \rho_R)^2 + (y_I^2(n) - \rho_I)^2] \quad (14)$$

The optimum reflection coefficients  $k_m(n)$ ,  $1 \leq m \leq N$ , and the optimum regression coefficients,  $v_m(n)$ ,  $0 \leq m \leq N$ , presented in Fig. 2 using the MMA, may be updated according to the SGD method by minimizing the MMA cost function in (14). The SGD adjustment for the reflection coefficients can be developed as  $k_m(n+1) = k_m(n) - \mu \cdot \partial J_{\text{MMA}}(n) / \partial k_m^*(n)$ , where

$$\begin{aligned} \frac{\partial J_{\text{MMA}}(n)}{\partial k_m^*(n)} &= E\{[y_R^2(n) - \rho_R] \cdot [2 \cdot y_R(n)] \cdot [\frac{\partial y^*(n)}{\partial k_m^*(n)} + \frac{\partial y(n)}{\partial k_m^*(n)}] \\ &\quad - [y_I^2(n) - \rho_I] \cdot [-2j \cdot y_I(n)] \cdot [\frac{\partial y^*(n)}{\partial k_m^*(n)} - \frac{\partial y(n)}{\partial k_m^*(n)}]\} \end{aligned}$$

Defining  $\psi_m(n) \triangleq \partial y^*(n) / \partial k_m^*(n)$  and  $\phi_m(n) \triangleq \partial y(n) / \partial k_m^*(n)$  yields

$$\begin{aligned} k_m(n+1) &= k_m(n) - \mu \cdot \{y_R(n) \cdot [y_R^2(n) - \rho_R] \cdot [\psi_m(n) + \phi_m(n)] \\ &\quad + j \cdot y_I(n) [y_I^2(n) - \rho_I] \cdot [\psi_m(n) - \phi_m(n)]\} \end{aligned} \quad (15)$$

Notably, the expected value,  $E\{\cdot\}$ , was ignored in the above derivation. To compute (15), both  $\psi_m(n)$  and  $\phi_m(n)$  need to be firstly computed as follows

$$\psi_m(n) \triangleq \frac{\partial y^*(n)}{\partial k_m^*(n)} = \sum_{i=0}^N v_i(n) \beta_{m,i}^+(n); \quad m = 1, \dots, N \quad (16)$$

where  $\beta_{m,i}^+(n) \triangleq \partial b_i^*(n) / \partial k_m^*(n)$ . If the filter is assumed to adapt at a slow enough rate for a small value of  $N$ , then the

following approximation may be justified [3]:  $\partial b_{i-1}^*(n-1) / \partial k_m^*(n) \approx \partial b_{i-1}^*(n-1) / \partial k_m^*(n-1) \triangleq \beta_{m,i-1}^+(n-1)$ , which may be expressed as

$$\beta_{m,i}^+(n) = \begin{cases} \beta_{m,i-1}^+(n-1) + k_i^*(n) \alpha_{m,i-1}^+(n); & m \neq i \\ \beta_{m,i-1}^+(n-1) + k_i^*(n) \alpha_{m,i-1}^+(n) + f_{i-1}^*(n); & m = i, \\ \alpha_{m,i}^+(n); & i = 0 \end{cases}$$

in which

$$\alpha_{m,i}^+(n) \triangleq \frac{\partial f_i^*(n)}{\partial k_m^*(n)} = \alpha_{m,i+1}^+(n) - k_{i+1}(n) \beta_{m,i}^+(n-1).$$

Similarly,  $\phi_m(n)$  may be computed by

$$\phi_m(n) \triangleq \frac{\partial y(n)}{\partial k_m^*(n)} = \sum_{i=0}^N v_i^*(n) \beta_{m,i}^-(n); \quad m = 1, \dots, N \quad (17)$$

where

$$\beta_{m,i}^-(n) \triangleq \frac{\partial b_i(n)}{\partial k_m^*(n)} = \begin{cases} \beta_{m,i-1}^-(n-1) + k_i(n) \alpha_{m,i-1}^-(n); & i \neq 0 \\ \alpha_{m,i}^-(n); & i = 0 \end{cases}$$

in which

$$\begin{aligned} \alpha_{m,i}^-(n) &\triangleq \frac{\partial f_i(n)}{\partial k_m^*(n)} \\ &= \begin{cases} \alpha_{m,i+1}^-(n) - k_{i+1}^*(n) \beta_{m,i}^-(n-1); & m \neq i+1 \\ \alpha_{m,i+1}^-(n) - k_{i+1}^*(n) \beta_{m,i}^-(n-1) - b_i(n-1); & m = i+1 \end{cases} \end{aligned}$$

To the best of the authors' knowledge, (16) and (17) in complex form have not been developed before, and [5], [10] only derived real versions of (16) and (17). Equation (15) can therefore be computed by using  $\psi_m(n)$  and  $\phi_m(n)$ .

The MMA cost function is now used to compute the gradient of each regression coefficients whose SGD parameterization adjustment can be developed as  $v_m(n+1) = v_m(n) - \mu \cdot \partial J_{\text{MMA}}(n) / \partial v_m^*(n)$ , where  $\partial J_{\text{MMA}}(n) / \partial v_m^*(n) = E[2y_R(n)(y_R^2(n) - \rho_R) \cdot b_m(n)] - E[2j \cdot y_I(n)(y_I^2(n) - \rho_I) \cdot b_m(n)]$

Consequently, we obtain

$$\begin{aligned} v_m(n+1) &= v_m(n) - \mu \cdot \{y_R(n) \cdot [y_R^2(n) - \rho_R] \\ &\quad - jy_I(n) [y_I^2(n) - \rho_I]\} \cdot b_m(n) \end{aligned} \quad (18)$$

To maintain the same adaptive time constant and misadjustment across all stages in the lattice structure, the step size  $\mu$  must be normalized by the power level in each stage [5], [11]. The normalized version of (15) and (18) may be written, respectively, as

$$\begin{aligned} k_m(n+1) &= k_m(n) - \frac{\mu}{\gamma_m^2(n)} \cdot \{y_R(n) \cdot [y_R^2(n) - \rho_R] \cdot [\psi_m(n) + \phi_m(n)] \\ &\quad + j \cdot y_I(n) [y_I^2(n) - \rho_I] \cdot [\psi_m(n) - \phi_m(n)]\}, \quad m = 1, \dots, N \end{aligned} \quad (19)$$

$$\begin{aligned} v_m(n+1) &= v_m(n) - \frac{\mu}{\sigma_m^2(n)} \cdot \{y_R(n) \cdot [y_R^2(n) - \rho_R] \\ &\quad - jy_I(n) [y_I^2(n) - \rho_I]\} \cdot b_m(n), \quad m = 0, \dots, N \end{aligned} \quad (20)$$

where  $\gamma_m^2(n)$  is an estimate of the *total* power (sum of both forward and backward prediction error powers) in the  $m$ -th stage for  $k_m(n)$ , while  $\sigma_m^2(n)$  is merely the backward prediction error power estimated in the  $m$ -th stage for  $v_m(n)$ .

Both  $\gamma_m^2(n)$  and  $\sigma_m^2(n)$  may be updated, respectively, as follows

$$\gamma_m^2(n) = \rho \cdot \gamma_m^2(n-1) + (1-\rho)[b_m^2(n-1) + f_m^2(n)] \quad (21)$$

$$\sigma_m^2(n) = \rho \cdot \sigma_m^2(n-1) + (1-\rho)b_m^2(n) \quad (22)$$

#### IV. BLIND EQUALIZATION USING IIR LATTICE CMA

The optimum reflection coefficients and the optimum regression coefficients obtained using the CMA, presented in Fig. 2, may be similarly obtained using the SGD method to minimize the CMA cost function,  $J_{\text{CMA}} \triangleq (1/2) \cdot E[(|y(n)|^2 - R_2)^2]$ , as follows [12]-[14]. First, an SGD parameterization adjustment for the reflection coefficients can be developed as  $k_m(n+1) = k_m(n) - \mu \cdot \partial J_{\text{CMA}}(n) / \partial k_m^*(n)$ , where  $\partial J_{\text{CMA}}(n) / \partial k_m^*(n) = E[(|y(n)|^2 - R_2)(y(n)\psi_m(n) + y^*(n)\phi_m(n))]$ . Accordingly, the following time-updated reflection coefficients holds

$$k_m(n+1) = k_m(n) - \mu \cdot (|y(n)|^2 - R_2) \cdot [y(n)\psi_m(n) + y^*(n)\phi_m(n)] \quad (23)$$

Notably,  $\psi_m(n)$  and  $\phi_m(n)$  in (23) can also be computed using (16) and (17), respectively. Similarly, the CMA cost function can be utilized to compute the gradient for the regression coefficients. An SGD adjustment can be developed as  $v_m(n+1) = v_m(n) - \mu \cdot \partial J_{\text{CMA}}(n) / \partial v_m^*(n)$  in which  $\partial J_{\text{CMA}}(n) / \partial v_m^*(n) = E[(|y(n)|^2 - R_2) \cdot b_m(n) \cdot y^*(n)]$ .

Accordingly, we have

$$v_m(n+1) = v_m(n) - \mu \cdot \{y^*(n)(|y(n)|^2 - R_2)\} \cdot b_m(n) \quad (24)$$

The normalized version of (23) and (24) may be written, respectively, as

$$k_m(n+1) = k_m(n) - \frac{\mu}{\gamma_m^2(n)} \cdot (|y(n)|^2 - R_2) \cdot [y(n)\psi_m(n) + y^*(n)\phi_m(n)] \quad (25)$$

$$v_m(n+1) = v_m(n) - \frac{\mu}{\sigma_m^2(n)} \cdot \{y^*(n)(|y(n)|^2 - R_2)\} \cdot b_m(n) \quad (26)$$

#### V. COMPUTER SIMULATIONS

The blind equalization performances of the conventional FIR transversal MMA, the FIR lattice MMA and the proposed IIR lattice MMA were compared using three channels. The source symbols were taken from the 16-QAM constellation and white Gaussian noise was also added so that the final signal-to-noise ratio (SNR) was 30dB. The tap weights of the FIR transversal MMA and the regression coefficients (RCs) of both the FIR lattice MMA and the IIR lattice MMA were initialized by setting the central tap weight (or central regression coefficient) to unity and the other weights (regression coefficients) to zero. The initial *total* power (sum of both forward and backward prediction error powers) for both the IIR lattice MMA and the FIR lattice MMA were set to  $\gamma_m^2(0) = \varepsilon_{m-1}^2(0) = 20$  for  $m=1, \dots, N$ , while the initial power for both IIR lattice and FIR lattice backward prediction errors were set to  $\sigma_m^2(0) = \delta_m^2(0) = 10$  for  $m=0, \dots, N$ . The ensemble-

averaged mean-squared error (MSE) in dB from 100 independent runs, approximated as  $10 \cdot \log E[|y(n) - s(n)|^2]$  (dB), was used as the index of the blind equalization performance of each of the three algorithms.

Channel 1 is defined as  $x(n) = s(n) + 0.9s(n-1)$ , as proposed by [15]. Figure 3 displays the ensemble-averaged MSE of the three algorithms when Channel 1 was used. The IIR lattice MMA, with only three RCs yielded a faster convergence and smaller steady-state MSE than either the FIR transversal MMA (with 25 taps) or the FIR lattice MMA (with 31 RCs). Notably, the FIR transversal MMA was not even able sufficiently to open eye with 25 taps.

Channel 2 is a three-ray multi-path channel that was proposed by [16], given by

$$c_2(n) = p(n, \beta) \cdot W(n) + 0.8 \cdot p(n-0.25T, \beta) \cdot W(n-0.25T) - 0.4 \cdot p(n-2T, \beta) \cdot W(n-2T)$$

where  $p(n, \beta)$  is a raised roll-off cosine pulse with the roll-off factor  $\beta = 0.11$ , and  $W(t)$  is a rectangular truncation window spanning  $[-3T, 3T]$ . The sampling frequency was chosen to double the baud rate  $1/T$  and the oversampled channel may be divided into two discrete FIR sub-channels, Sub-channel 1 (odd) and Sub-channel 2 (even), as displayed in Fig. 4. In the computer simulations, the outputs of both sub-channels were then rotated by  $40^\circ$  before being processed by the three blind equalizers because the MMA is known to allow simultaneous joint blind equalization and carrier phase recovery (with a multiple of  $90^\circ$  phase ambiguity) [9]. Figure 5 presents the ensemble-averaged MSE of the three algorithms when Sub-channel 1 was used. For a given same steady-state MSE, the IIR lattice MMA using five RCs converges faster than either the FIR transversal MMA (with 11 taps) or the FIR lattice MMA (with 11 RCs).

Figure 6 displays the ensemble-averaged MSE obtained using the three algorithms when Sub-channel 2 was used. All three algorithms used 21 taps. In contrast to results of the simulation presented in Fig. 5, Fig. 6 reveals that the IIR lattice MMA underperformed the other two FIR algorithms, probably because the IIR lattice structure, which may be used as a feedback filter in a decision feedback equalization (DFE) structure, is especially effective at equalizing a combined channel-equalizer impulse response with large post-cursor terms. In a conventional DFE, the combined channel-forward equalizer impulse response is given by  $\mathbf{h} = [h_0, \dots, h_{d-1}, h_d, h_{d+1}, \dots, h_{N_h}]$ , where  $d$  denotes the system delay and  $N_h$  is combined channel-equalizer length;  $[h_0, \dots, h_{d-1}]$  are referred to as the pre-cursor terms (designed to be cancelled by the feedforward filter) and  $[h_{d+1}, \dots, h_{N_h}]$  are the post-cursor terms (designed to be cancelled by the feedback filter) [17]. The DFE structure has been established to be suited to channels for which most part of the ISI results from the post-cursor of the impulse response, since the feedback filter in the DFE is chosen to cancel the post-cursor

term [18], [19]. From the simulation results, the IIR lattice MMA, which may be used as a feedback filter in a DFE structure, appears to be better suited to channels that yield ISI that is dominated by the post-cursor term. However, since Sub-channel 2 yields ISI that is dominated by the pre-cursor term, the IIR lattice MMA no longer outperforms both the FIR transversal MMA and FIR lattice MMA in terms of MSE. This distinct feature of using the IIR structure explains why the IIR lattice MMA needed only three RCs to achieve both faster convergence and a smaller steady-state MSE than achieved using either the FIR transversal MMA (with 25 taps) or the FIR lattice MMA (with 31 RCs) when Channel 1 was used. Notably, Channel 1 represents an extreme scenario in which the pre-cursor term is zero.

Let  $c_3(n)$  be a modified version of  $c_2(n)$  that is a 4-ray multi-path channel that is given by

$$c_3(n) = p(n, \beta) \cdot W(n) + 0.8 \cdot p(n - 0.25T, \beta) \cdot W(n - 0.25T) \\ + 0.6 \cdot p(n - T, \beta) \cdot W(n - T) - 0.4 \cdot p(n - 2T, \beta) \cdot W(n - 2T)$$

Channel 3 results from  $c_3(n)$  followed by a rotation of  $40^\circ$ . Figure 7 plots the magnitude of its impulse response. Clearly, the amount of ISI that results from the post-cursor term when Channel 3 is adopted exceeds that when Sub-channel 1 is adopted, as shown in Fig. 4. Consequently, the IIR lattice MMA easily outperformed both the FIR transversal MMA and the FIR lattice MMA, both of which required the use of more taps (or RCs) to reduce the MSE. The simulation results that are displayed in Fig. 8 indicate that the IIR lattice MMA with only five RCs converged faster than the other two algorithms with 31 taps (or 31RCs).

## VI. CONCLUSION

This paper presents two novel IIR lattice-based blind equalization algorithms. Simulation results reveal that the IIR lattice MMA is highly suited to equalizing channels, yielding ISI that is dominated by the post-cursor term. As a result, the IIR lattice MMA with fewer regression coefficients may achieve faster convergence as well as a smaller steady-state MSE than those achieved by either the FIR lattice MMA or FIR transversal MMA. This advantage also pertains to the use of a feedback filter in a DFE structure to cancel the post-cursor term of the combined impulse response of the channel and the feed-forward equalizer.

## REFERENCES

- [1] A. H. Gray, Jr. and J. D. Markel, "Digital Lattice and Ladder Filter Synthesis," *IEEE Trans. Audio Electroacoust.*, vol. 21, pp. 491-500, Dec. 1973.
- [2] J. J. Shynk, "Adaptive IIR filtering," *IEEE Acoust. Speech Signal Processing Mag.*, vol. 6, pp. 4-21, Apr. 1989.
- [3] S. Haykin, *Adaptive Filter Theory*, 4th ed: Prentice-Hall, 2002.
- [4] S. J. Orfanidis, *Optimum Signal Processing*, New York: Macmillan, 1985.
- [5] D. Parikh, N. Ahmed, and S. D. Stearns, "An Adaptive Lattice Algorithm for Recursive Filters," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. 28, no.1, pp. 110-111, Feb. 1980.

- [6] J. J. Shynk, "On Lattice-form Algorithms for Adaptive IIR Filtering," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, vol. 3, pp. 1554-1557, Apr. 1988.
- [7] K. Wesolowski, "Analysis and Properties of the Modified Constant Modulus Algorithm for Blind Equalization," *European Trans. Telecommun.*, vol. 3, pp. 225-230, May-June 1992.
- [8] J. Yang, J.-J. Werner, and G. A. Dumont, "The Multimodulus Blind Equalization and its Generalized Algorithms," *IEEE J. Sel. Areas in Commun.*, vol. 20, pp. 997-1015, June 2002.
- [9] J.-T. Yuan and K.-D. Tsai, "Analysis of the Multimodulus Blind Equalization Algorithm in QAM Communication Systems," *IEEE Trans. Commun.*, vol. 53, pp. 1427-1431, Sept. 2005.
- [10] S. C. Ng, S. H. Leung, C. Y. Chung, A. Luk, and W. H. Lau, "The Genetic Search Approach: a New Learning Algorithm for IIR Filtering," *IEEE Signal Processing Mag.*, vol. 13, no. 6, pp. 38-46, 1996.
- [11] S. A. White, "An Adaptive Recursive Digital Filter," *Proc. 9th Asilomar Conf. Circuits Syst. Comput.*, pp. 21-25, Nov. 1975.
- [12] D. N. Godard, "Self-recovering Equalization and Carrier Tracking in Two-dimensional Data Communication System," *IEEE Trans. Commun.*, vol. 28, pp.1867-1875, Nov. 1980.
- [13] J. R. Treichler and M. G. Larimore, "New Processing Techniques Based on the Constant Modulus Algorithm," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 33, pp.420-431, Apr. 1985.
- [14] C. R. Johnson, Jr., P. Schniter, T. J. Endres, J. D. Behm, D. R. Brown, and R. A. Casas, "Blind Equalization Using the Constant Modulus Criterion: A Review," *Proc. IEEE*, vol. 86, pp.1927-1950, Oct. 1998.
- [15] F. Ueng and Y. T. Su, "Adaptive IIR Blind Algorithms," *Electron. Lett.*, vol. 31, no. 12, pp. 942-943, 1995.
- [16] Y. Li and Z. Ding, "Global Convergence of Fractionally Spaced Godard (CMA) Adaptive Equalizers," *IEEE Trans. Signal Process.*, vol. 44, no. 4, pp. 818-826, Apr. 1996.
- [17] W. Chung, T. J. Endres, C. D. Long, J. Walsh, and C. R. Johnson Jr., "Soft Decision Approaches for Blind Adaptive Decision Feedback Equalizers," in *IEEE Signal Processing Workshop on Signal Processing Advanced in Wireless Communications*, Rome, Italy, pp. 447-451, May 2003.
- [18] A. Rontogiannis and K. Berberidis, "Efficient Decision Feedback Equalization for Sparse Wireless Channels," *IEEE Trans. Wireless Communications*, vol. 2, no. 3, pp. 570-581, May 2003.
- [19] K. Berberidis and J. Palicot, "A Frequency-domain Decision Feedback Equalizer for Multipath Echo Cancellation," in *Proc. IEEE GLOBECOM*, Singapore, vol. 1, pp. 98-102, Nov. 1995.

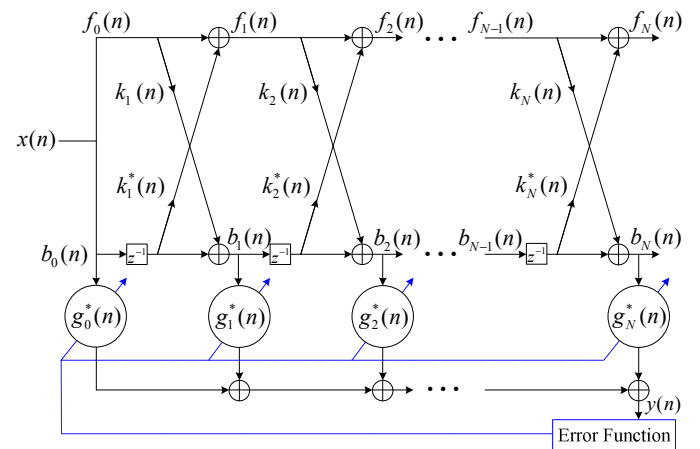


Fig. 1. FIR lattice-based structure for MMA and CMA

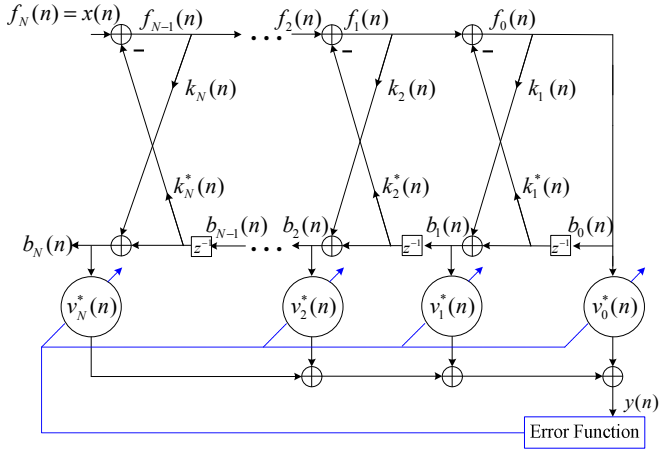


Fig. 2. IIR lattice-based structure for MMA and CMA

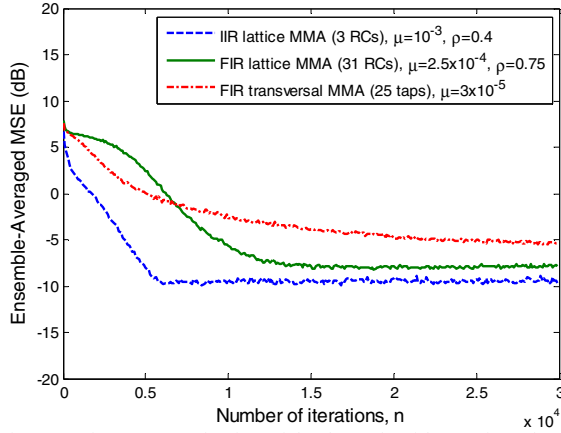


Fig. 3. Performance evaluation of the three algorithms using Channel 1

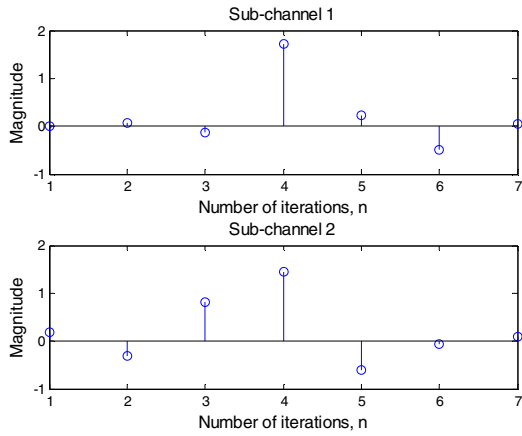


Fig. 4. Magnitude of the impulse response of Sub-channel 1 and Sub-channel 2

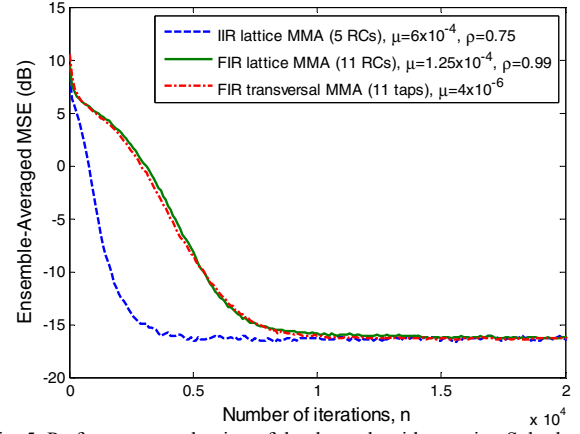


Fig. 5. Performance evaluation of the three algorithms using Sub-channel 1

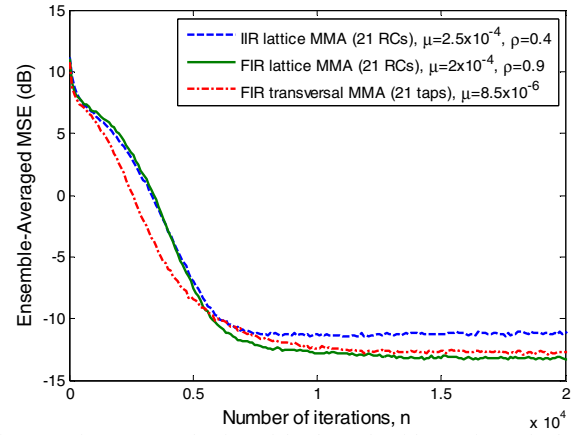


Fig. 6. Performance evaluation of the three algorithms using Sub-channel 2

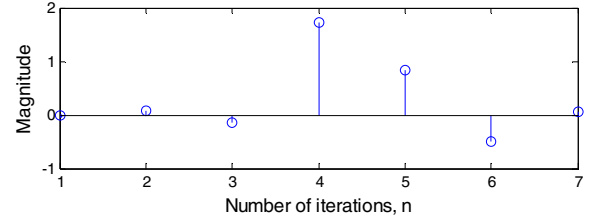


Fig. 7. Magnitude of the impulse response of Channel 3

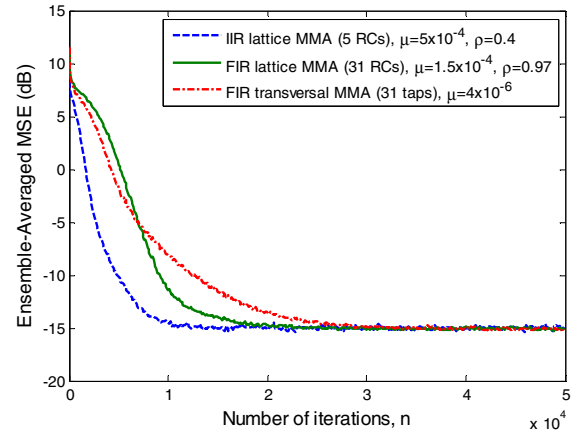


Fig. 8. Performance evaluation of the three algorithms using Channel 3