Collaborative Relay-based Multiuser Beamforming in Cellular Systems

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***Abstract*—The performance of downlink transmissions can be improved by using relays that can form a distributed virtual multiple-input multiple-output (MIMO) beamforming system. In this paper, we study a collaborative downlink transmission scheme for a cellular system which consists of a base station (BS), multiple amplify-and-forward (AF) relay nodes, and multiple users. We propose a new algorithm to determine the signal forwarding weights for both transmit power and signal phase with an objective of minimizing power consumption of the relay nodes while guaranteeing the receive signal-to-noise ratio (SNR) at multiple users for the collaborative relay beamforming. We prove that the beamforming solution obtained by a semidefinite relaxation (SDR) technique is optimal with the proposed algorithm. Numerical results validate our theoretical analysis and demonstrate that the proposed optimal scheme can effectively reduce the power consumption of relay nodes compared with other beamforming approaches.**

I. Introduction

Since multiple-input multiple-output (MIMO) systems can increase the spectral efficiency for given transmit power and bandwidth in cellular systems [1], [2], various approaches to implement MIMO systems have been investigated. As opposed to central antenna array based systems where base stations (BSs) are equipped with multiple antennas, multiple collaborative relay nodes can be used to form a distributed multi-antenna system for the next-generation cellular systems [3], [4], [5], [6], [7], [8], [9], [10], [11].

Among various relay protocols, the amplify-and-forward (AF) protocol can be promising due to its low implementation complexity. In the AF scheme, relay nodes receive and amplify the signal transmitted from a source node and forward them to destination nodes. With the aid of channel state information (CSI), the relay nodes can work collaboratively as a MIMO system to construct a virtual beam towards the destinations [4]. A design of collaborative beamforming scheme in a two-way relay network has been proposed in [5] for reciprocal and non-reciprocal channels. Havary-Nassab *et al.* [6] proposed two different beamforming approaches with the second-order statistics of the CSI, which minimize the total transmit power and maximize the SNR at the users. Similar problems are studied with the aid of perfect CSI [7] and imperfect CSI [8]. With the AF protocol, distributed beamforming is investigated when a consensus algorithm is employed for cooperative beamforming in [9] and MMSE criteria are used in [10]. Quek *et al.* [11] developed relay power allocation algorithms for noncoherent and coherent AF relay networks. It is noteworthy that most existing approaches have focused on the optimal or sub-optimal beamforming schemes for single user.

To support a group of mobile terminals or users in a cell, we consider a relay assisted communication system where a group of relay nodes forward the received signals from a BS to a group of receivers and formulate a multiuser beamforming problem with signal-to-noise ratio (SNR) constraints at users in this paper. We propose an algorithm to decide the weights for distributed beamforming using a semidefinite relaxation (SDR) approach [12]. As we can show that the solution to a semidefinite programming (SDP) relaxation problem is identical to that to the original optimization problem, it is guaranteed that the proposed algorithm can provide an optimal rank-one collaborative relay beamforming solution. With this algorithm, the collaborative AF relay nodes can construct optimal (virtual) beams for multiple users. It can be seen that the approach in this paper is a generalization of the approach in [7] or [6] to multiple users.

The rest of the paper is organized as follows. Section II presents the system model and the optimization problem. In Section III, we find weights of relay nodes using an SDR technique to solve an optimization problem and prove that the resulting solution is identical to that of the original problem. Then, a collaborative relay beamforming algorithm is proposed. Simulations results are presented and discussed in Section IV. Conclusions are drawn in Section V.

Notation: Vectors are written in boldface lowercase letters, e.g., $\mathbf{x}$, while matrices are denoted by boldface uppercase letters e.g., $\mathbf{X}$. $\mathrm{Tr}(\mathbf{S})$ represents the trace of a matrix $\mathbf{S}$ and $\mathrm{diag}(s_1, s_2, \ldots, s_k)$ denotes a diagonal square matrix with diagonal elements $s_1, s_2, \ldots, s_k$. The superscript T stands for transposition of a vector or matrix. The complex number fields are denoted by $\mathbb{C}$. The statistical expectation of a random entity z is denoted by $\mathbb{E}[z]$. The notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{D})$ means that $\mathbf{x}$ is a circularly symmetric complex Gaussian (CSCG) random vector with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{D}$.

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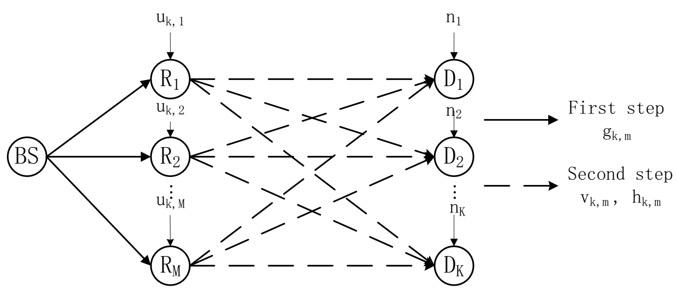


Fig. 1. System model consists of multiple relay nodes and multiple users.

II. System Model and Problem Formulation

Suppose that a single-cell system consists of a BS, M relay nodes, and K users as shown in Fig. 1. We do not consider the direct link between the BS and users. It is assumed that each user can receive his/her signal over a dedicated orthogonal channel through relay nodes. The relay nodes are synchronized to relay signals from the BS to K users using the AF protocol. We assume that in our cellular system the coherence bandwidth of an orthogonal channel is larger than the bandwidth of the signal. In this case, each channel can be considered as a flat fading channel and channel gains from the BS to the mth relay and from the mth relay to user k are denoted by $g_{k,m} \in \mathbb{C}$ and $h_{k,m} \in \mathbb{C}$, respectively. It is assumed that all the channel gains or coefficients are independent. Let $\mathbf{g}_k = [g_{k,1}, g_{k,2}, \ldots, g_{k,M}]^T$ and $\mathbf{h}_k = [h_{k,1}, h_{k,2}, \ldots, h_{k,M}]^T$. For convenience, the term of propagation loss is absorbed in $g_{k,m}$ and $h_{k,m}$ to reflect the relative locations of relay nodes and users.

Throughout the paper we consider the following two-hop AF relay protocol. During the first step, the signal $r_{k,m}$ received at relay node m is given by

$$r_{k,m} = g_{k,m} s_k + u_{k,m}, \ k = 1, 2, \ldots, K, \tag{1}$$

where s_k is the signal to user k and $u_{k,m} \sim \mathcal{CN}(0, N_{k,m})$ is independent CSCG background noise at relay node m. We assume that the s_k's are independent with $\mathbb{E}[s_k] = 0$ and $\mathbb{E}[|s_k|^2] = P_k$.

During the second step, relay node m forwards the following scaled version of the received signal to user k over sub-channel k (it is assumed that sub-channel k is assigned to user k throughout the paper):

$$x_{k,m} = v_{k,m} r_{k,m}, \tag{2}$$

where $v_{k,m}$ is the complex-valued beamforming weight of relay node m to form a virtual beam towards the kth user. Due to poor channel conditions between some relay nodes and user k, the weights of such relay nodes could be zero to make them inactive (this is a consequence of the water-filling theorem in general).

The total transmission power at relay node m conditioned on $g_{k,m}$ becomes

$$\begin{aligned} P_m(v_{1,m}, ..., v_{K,m}) &= \sum_{k=1}^{K} \mathbb{E}[|x_{k,m}|^2] \\ &= \sum_{k=1}^{K} |v_{k,m}|^2 \left(|g_{k,m}|^2 P_k + N_{k,m} \right). \end{aligned} \tag{3}$$

User k receives a superpositions of the transmitted signals by active relay nodes as follows:

$$\begin{aligned} y_k &= \sum_{m=1}^{M} h_{k,m} v_{k,m} r_{k,m} + n_k \\ &= \sum_{m=1}^{M} h_{k,m} g_{k,m} v_{k,m} s_k + w_k, \end{aligned} \tag{4}$$

where $n_k \sim \mathcal{CN}(0, N_k)$ is independent CSCG background noise at user k, and

$$w_k = \sum_{m=1}^{M} h_{k,m} v_{k,m} u_{k,m} + n_k.$$

Thus, $w_k \sim \mathcal{CN}(0, W_k)$, where $W_k = \sum_{m=1}^{M} |h_{k,m}|^2 |v_{k,m}|^2 N_{k,m} + N_k$. The receive SNR at user k, denoted by γ_k, is given by

$$\gamma_k = \frac{P_k |\sum_{m=1}^{M} h_{k,m} g_{k,m} v_{k,m}|^2}{\sum_{m=1}^{M} |h_{k,m}|^2 |v_{k,m}|^2 N_{k,m} + N_k}. \tag{5}$$

For coherent combining, the phase of $v_{k,m}$ at relay node m should be the conjugate of the phase of $h_{k,m} g_{k,m}$. Thus, we only need to find the optimal amplitude of weights, i.e. the power $|v_{k,m}|^2$. Define the transmit power at relay node as $\bar{v}_{k,m} \triangleq |v_{k,m}|$ and define the equivalent (real-valued) channel as $\bar{h}_{k,m} \triangleq |h_{k,m}|$ and $\bar{g}_{k,m} \triangleq |g_{k,m}|$. Then, (5) becomes

$$\begin{aligned} \gamma_k &= \frac{P_k (\sum_{m=1}^{M} \bar{h}_{k,m} \bar{g}_{k,m} \bar{v}_{k,m})^2}{\sum_{m=1}^{M} (\bar{h}_{k,m})^2 (\bar{v}_{k,m})^2 N_{k,m} + N_k} \\ &= \frac{P_k (\mathbf{a}_k^T \mathbf{x}_k)^2}{\mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} &\mathbf{x}_k = [\bar{v}_{k,1} \ , \ \bar{v}_{k,2} \ , \ \ldots, \ \bar{v}_{k,M}]^T, \\ &\mathbf{a}_k = [\bar{h}_{k,1} \bar{g}_{k,1} \ , \ \bar{h}_{k,2} \bar{g}_{k,2} \ , \ \ldots, \ \bar{h}_{k,M} \bar{g}_{k,M}]^T, \\ &\mathbf{B}_k = \mathrm{diag}\left((\bar{h}_{k,1})^2 N_{k,1} \ , \ (\bar{h}_{k,2})^2 N_{k,2} \ , \ \ldots, \ (\bar{h}_{k,M})^2 N_{k,M} \right). \end{aligned}$$

With SNR constraints to guarantee certain quality of service (QoS), we can formulate an optimization problem as follows:

$$\begin{aligned} \min_{\{\mathbf{x}_k\}} \quad & \sum_{m=1}^{M} P_m \\ \text{subject to} \quad & \gamma_k \geq \gamma_k^*, \ k = 1, 2, \ldots, K, \end{aligned} \tag{7}$$

where γ_k^* is the SNR threshold for the kth user. The aim of Problem (7) is to find an optimal power allocation, $\mathbf{x}_k$, to minimize the total relay power $\sum_{m=1}^{M} P_m$ with the SNR constraints $\gamma_k^*, k = 1, 2, \ldots, K$.

III. Optimal Collaborative Beamforming Scheme

With the beamforming gain $\bar{v}_{k,m} \triangleq |v_{k,m}|$ and the (real-valued) channel coefficient $\bar{g}_{k,m} \triangleq |g_{k,m}|$, the problem in (7) can be rewritten as

$$\begin{array}{cl} \min\limits_{\{\mathbf{x}_k\}} & \sum_{m=1}^{M} \sum_{k=1}^{K} (\bar{v}_{k,m})^2 \left((\bar{g}_{k,m})^2 P_k + N_{k,m} \right) \\ \text{subject to} & \gamma_k \geq \gamma_k^*, \; k = 1, 2, \ldots, K, \end{array} \tag{8}$$

or

$$\begin{array}{cl} \min\limits_{\{\mathbf{x}_k\}} & \sum_{k=1}^{K} \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} & \gamma_k \geq \gamma_k^*, \; k = 1, 2, \ldots, K, \end{array} \tag{9}$$

where

$$\mathbf{C}_k = \text{diag}\left(((\bar{g}_{k,1})^2 P_k + N_{k,1}), \; \ldots, \; ((\bar{g}_{k,M})^2 P_k + N_{k,M}) \right).$$

The problem in (9) can be decomposed into K independent sub-problems, where the kth problem is expressed as

$$\begin{array}{cl} \min\limits_{\mathbf{x}_k} & \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} & \gamma_k \geq \gamma_k^*. \end{array} \tag{10}$$

With the receive SNR at user k in (6), we can obtain the following sub-problem:

$$\begin{array}{cl} \min\limits_{\mathbf{x}_k} & \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} & \frac{P_k}{\gamma_k^*} (\mathbf{a}_k^T \mathbf{x}_k)^2 \geq \mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k. \end{array} \tag{11}$$

Letting $\mathbf{X}_k \triangleq \mathbf{x}_k \mathbf{x}_k^T$, the optimization problem (11) can be transformed into

$$\begin{array}{cl} \min\limits_{\mathbf{X}_k} & \text{Tr}(\mathbf{C}_k \mathbf{X}_k) \\ \text{subject to} & \begin{cases} \text{Tr}((\frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T - \mathbf{B}_k) \mathbf{X}_k) \geq N_k \\ \mathbf{X}_k \succeq 0 \\ \text{rank}(\mathbf{X}_k) = 1, \end{cases} \end{array} \tag{12}$$

where $\mathbf{X}_k \succeq 0$ means that $\mathbf{X}_k$ is constrained to be a symmetric positive semidefinite matrix, and the last constraint $\text{rank}(\mathbf{X}_k) = 1$ results from $\mathbf{X}_k \triangleq \mathbf{x}_k \mathbf{x}_k^T$. Note that the rank-one constraint is not convex. As a result, the optimization problem (12) is not convex. Hence, SDR techniques can be applied in order to relax Problem (12) into a semidefinite programming (SDP) problem.

We drop the rank-one constraint and obtain the following SDP problem:

$$\begin{array}{cl} \min\limits_{\mathbf{X}_k} & \text{Tr}(\mathbf{C}_k \mathbf{X}_k) \\ \text{subject to} & \begin{cases} \text{Tr}((\frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T - \mathbf{B}_k) \mathbf{X}_k) \geq N_k \\ \mathbf{X}_k \succeq 0 \; . \end{cases} \end{array} \tag{13}$$

This SDP problem can be solved using interior-point methods, whose computational complexities are polynomial [13].

The solution of the SDP problem in (13) could provide a lower bound on the objective function in the original problem in (12) due to excluding the rank-one constraint. Therefore, it is important to know whether the solution of the SDP problem in (13) is rank-one. In the following theorem, we prove that the rank-one solution can be always obtained by solving the SDP relaxation problem (13). This guarantees that the solution of (13) is that of the original problem in (12).

Theorem 1: The SDP relaxation problem in (13) always returns a rank-one solution $\mathbf{X}_{k\text{opt}}$, which is the optimal solution of Problem (12).

Proof: We introduce the Lagrangian multipliers λ and $\mathbf{Z}$. The Lagrangian $\mathcal{L}$ associated with the SDP problem (13) is given by

$$\begin{aligned} \mathcal{L}(\mathbf{X}_k, \mathbf{Z}, \lambda) =& \text{Tr}(\mathbf{C}_k \mathbf{X}_k) + \text{Tr}(\mathbf{X}_k \mathbf{Z}) \\ & + \lambda \left(N_k - \text{Tr}((\frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T - \mathbf{B}_k) \mathbf{X}_k) \right) \end{aligned} \tag{14}$$

with its dual problem

$$\begin{array}{cl} \max\limits_{\lambda \geq 0} & \lambda N_k \\ \text{subject to} & \mathbf{Z} \triangleq \mathbf{C}_k - \lambda (\frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T - \mathbf{B}_k) \succeq \mathbf{0}. \end{array} \tag{15}$$

According to the Karush-Kuhn-Tucker (KKT) conditions [13], we have

$$\begin{cases} \lambda \left(N_k - \text{Tr}((\frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T - \mathbf{B}_k) \mathbf{X}_k) \right) = 0 \\ \text{Tr}(\mathbf{X}_k \mathbf{Z}) = 0. \end{cases} \tag{16}$$

Without loss of generality, let the optimal solution of the SDP problem (13) be $\mathbf{X}_{k\text{opt}} = \widetilde{\mathbf{X}}_k \widetilde{\mathbf{X}}_k^T$, with $\widetilde{\mathbf{X}}_k = [\widetilde{\mathbf{x}}_{k,1}, \; \widetilde{\mathbf{x}}_{k,2}, \ldots, \widetilde{\mathbf{x}}_{k,M}]$, where $\widetilde{\mathbf{x}}_{k,m}, \; m = 1, 2, \ldots, M$ are the column elements of the matric $\widetilde{\mathbf{X}}_k$. From the KKT condition in (16), each vector $\widetilde{\mathbf{x}}_{k,m}$ should lie in the null space of $\mathbf{Z}$.

Now, we show that any vector, say $\mathbf{y}$, which lies in the null space of $\mathbf{Z}$ is unique up to its norm. Since $\mathbf{y}$ lies in the null space of $\mathbf{Z}$, we have $\mathbf{Z}\mathbf{y} = \mathbf{0}$, which implies

$$\begin{aligned} & (\mathbf{C}_k + \lambda \mathbf{B}_k) \mathbf{y} = \lambda \; \frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T \; \mathbf{y} \\ & \Leftrightarrow \mathbf{y} = (\mathbf{C}_k + \lambda \mathbf{B}_k)^{-1} \lambda \; \frac{P_k}{\gamma_k^*} \mathbf{a}_k \mathbf{a}_k^T \; \mathbf{y} \; \equiv \; \mathbf{\Lambda} \mathbf{y}. \end{aligned} \tag{17}$$

Note that since $\mathbf{C}_k$ and $\mathbf{B}_k$ are all positive diagonal matrices and $\mathbf{a}_k$ is a positive vector, $\mathbf{\Lambda}$ is a positive matrix. Then, $\mathbf{y}$ becomes the eigenvector of $\mathbf{\Lambda}$ associated with the positive eigenvalue 1. According to the Perron-Frobenius theorem [15], for any positive matrix, there is a unique positive eigenvalue which is associated with a unique positive eigenvector. This implies that the vector $\mathbf{y}$ is unique up to its norm.

Consequently, each vector $\widetilde{\mathbf{x}}_{k,m}$ is unique up to its norm. This further implies that the solution of the SDP relaxation problem (13) must be a positive rank-one solution, which is also the optimal solution of its original optimization problem (12). Thus, Problem (13) is equivalent to the original optimization problem in (12) and the optimal power allocation vector $\mathbf{x}_k$ can be obtained by solving (12). ■

Based on the results obtained in the previous subsection, we propose an efficient algorithm (see Algorithm 1) to solve our primal problem (7) for collaborative relay beamforming in multiuser systems. Due to the quasi-convexity of Problem (7), the optimal power allocation can be obtained by solving the SDP relaxation problem in (13) for all K users.

Algorithm 1 Optimal Algorithm

Initialize $\bar{h}_{k,m}, \bar{g}_{k,m}$ and $\gamma_k^*, k = 1, 2, \ldots, K, m = 1, 2, \ldots, M$
for all $k = 1, 2, \ldots, K$ **do**
  Using interior-point methods to solve the SDP problem (13)
  if the problem (13) is feasible **then**
    Get the solution $\mathbf{X}_{k\text{opt}}$ and obtain the vector $\mathbf{x}_k$ from $\mathbf{X}_{k\text{opt}}$ using standard matrix decomposition
  end if
  Obtain the relay power $\bar{v}_{k,m}$ from $\mathbf{x}_k$
  Adjust the phase of weight $v_{k,m}$ for the mth relay to match the conjugate of the phase of the channel $h_{k,m}g_{k,m}$
end for
Obtain the power allocation of M relay nodes, i.e., P_m
return the P_m and the complex-valued beamforming weight $v_{k,m}, \forall k, m$

With the proposed algorithm, the relay nodes can produce virtual beams towards the K users. The relay nodes can work collaboratively to guarantee that the receive SNRs of multiple users can reach their own target SNR thresholds with the least total power consumption of relay nodes.

Note that the full CSI at relay nodes is essential. This becomes a drawback of the proposed relay-aided transmission schemes in practice. Furthermore, the optimization problem is centralized, while relay nodes are distributed. In order to overcome these problems, we can use distributed consensus algorithms as in [9] or other distributed algorithms, which is a further study issue.

IV. Simulation Results

In this section, we present simulation results to see the performance of our proposed relay-aided transmission scheme. The channels are assumed to be independent Rayleigh flat fading channels, i.e. $\mathbf{g}_k, \mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{I})$ in particular. In addition, we assume that the noises at all the nodes, including the relays and the users, have the same power density, i.e. $\mathbf{N}_{k,m} = \mathbf{N}_k = \mathbf{N}_0$. For convenience, we set the BS transmission SNR for each user, denoted by $\mathcal{T}_{SNR}(k)$, as $\mathcal{T}_{SNR}(k) = P_k/\mathbf{N}_0, k = 1, 2, \ldots, K$. In the simulations, we assume that $\mathcal{T}_{SNR}(k)$ is the same for different users, i.e., $\mathcal{T}_{SNR}(k) = P_s/\mathbf{N}_0, k = 1, 2, \ldots, K$, where P_s is a fixed transmission power. The phase for the mth AF relay node is adjusted to be the conjugate of the phase of the composite channel coefficient $h_{k,j}g_{k,j}$, $j = m_1, m_2, \ldots, m_J$. Then, we can use the total relay power, which is expressed in decibels relative to one watt (dBW), to measure the performance of the collaborative relay beamforming in multiuser systems. We use CVX [14] to numerically solve the SDP problems. We present simulation results for the following schemes: a) the proposed optimal algorithm for collaborative relay beamforming in multiuser systems in Section III, b) the beamforming algorithm for single-user systems in [6]. In each scheme, the optimal relay beamforming vector is implemented to minimize the total power consumption for the given users' SNR constraints.

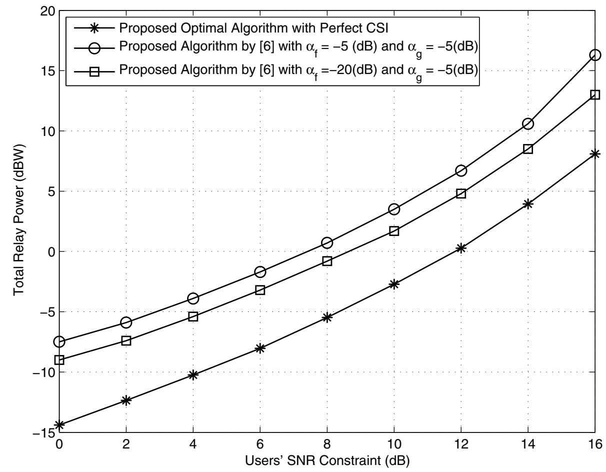
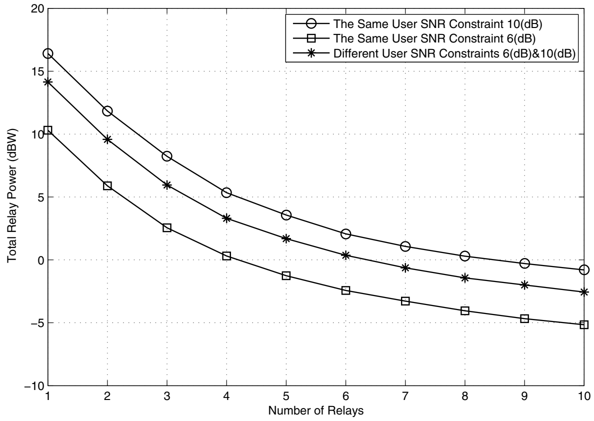


Fig. 2. Compare the proposed relay beamforming algorithm with the algorithm in [6] for 20 relays and 1 user.

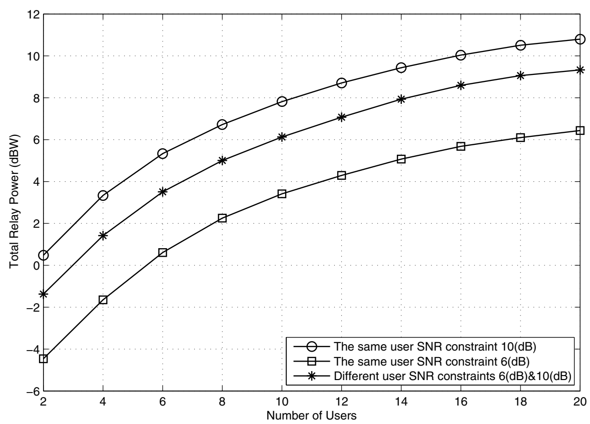
In Fig. 2, we compare the performance of our proposed algorithm with the beamforming algorithm stated in [6]. Note that the advantage of our work is that our proposed algorithm is applicable for multiple users, while Havary-Nassab *et al.* [6] only consider the single-user case. The single-user system can be seen as a special case of our proposed scheme, hence we may validate the effectiveness of our proposed algorithm by comparing the two algorithms.

In Fig. 2, we assume that there are 20 relays and single user ($K = 1$), and the power of the base station satisfies $P_s/\mathbf{N}_0 = 0$ dB. In [6], the optimal design of distributed relay beamforming is based on second-order statistics of the CSI in flat fading channels. α_f and α_g are two channel coefficients in [6] to determine the level of uncertainty in the channel from the base station to the ith relay and the channel from the ith relay to the receiver, respectively. When α_f or α_g increases, the level of the uncertainty in the channel increases as well. Similar to the simulation in [6], we choose $\alpha_f = \alpha_g = -5$ dB (the curve with circle symbols in Fig. 2) and $\alpha_f = -20$ dB and $\alpha_g = -5$ dB (the curve with square symbols in Fig. 2). The results in Fig. 2 reveal that our proposed optimal algorithm (the curve with star symbols in Fig. 2) has about $5 \sim 6$ dBW power saving compared to the algorithm stated in [6]. The reason is that when the uncertainty in the channel increases, the relays need to take more power to ensure that the SNR is above the given users' SNR constraints. Thus, the performance of our proposed scheme with the aid of perfect CSI is better than the scheme proposed in [6] where only second-order statistics of the CSI are considered. Moreover, our proposed algorithm can be applied in multiuser systems. In the following figures, we will present the simulation results in various multiuser scenarios for our proposed algorithm.

In Fig. 3, we demonstrate the expected total relay power of



(a) Performance of the proposed algorithm for fixed 4 users



(b) Performance of the proposed algorithm for fixed 5 relays

Fig. 3. Expected total relay power against the number of relays and users with different SNR constraints for the proposed optimal relay beamforming algorithm.

our proposed optimal algorithm with different number of users and relays, respectively. In both of Fig. 3(a) and Fig. 3(b), we assume that $P_s/\mathbf{N}_0 = 10$ dB. The curve with circle symbols represents that all the users have the same SNR constraint of 10 dB, i.e., $\gamma_k^* = 10$ dB, $k = 1, 2, \ldots, K$; the curve with square symbols represents that all the users have the same SNR constraint of 6 dB, i.e., $\gamma_k^* = 6$ dB, $k = 1, 2, \ldots, K$; the curve with star symbols represents that the users have different constraints, i.e., $\gamma_k^* = 6$ dB, when k is odd, and $\gamma_k^* = 10$ dB, when k is even, respectively. The results in Fig. 3 confirm that the total relay power consumption is proportional to the number of users, while it is inversely proportional to the number of relay nodes. In addition, the curve with star symbols (for different user SNR constraints 6 and 10 dB) lies between the curve with circle symbols (for the same user SNR constraint 10 dB) and the curve with square symbols (for the same user SNR constraint 6 dB), which is also consistent with the intuitive performance of optimal collaborative AF relay protocols.

V. Concluding Remarks

This paper studied a collaborative AF relay scheme for multiuser beamforming using multiple relays in cellular systems, where the relay nodes work collaboratively to construct virtual beams toward multiple users. With the assumption of full CSI at relay nodes, the optimal power allocation algorithm was proposed to decide the collaborative relay beamforming weights with users' target SNR constraints using an SDR technique.

While simulation results showed that the proposed approach can save the relays' power consumption with SNR constraints compared to other schemes, there could various practical issues to be addressed. The proposed optimization problem should be solved by a distributed algorithm as relay nodes are distributed. In building distributed algorithms, we also need to consider efficient ways to share CSI for distributed beamforming. These issues will be addressed as further study topics.

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