Reliable Communication in Wireless Meshed Networks using Network Coding

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***Abstract*—The advantages of network coding have been extensively studied in the field of wireless networks. Integrating network coding with existing IEEE 802.11 MAC layer is a challenging problem. The IEEE 802.11 MAC does not provide any reliability mechanisms for overheard packets. This paper addresses this problem and suggests different mechanisms to support reliability as part of the MAC protocol. Analytical expressions to this problem are given to qualify the performance of the modified network coding. These expressions are confirmed by numerical result. While the suggested reliability mechanisms introduce some signaling overhead, the results show that the performance is yet improved.**

I. Introduction

Network coding was first introduced in 2000 [1], [2]. We can broadly define network coding as allowing intermediate nodes in a network to either forward or combine the incoming independent flows.

Besides the theoretical work, implementation of network coding in wireless meshed networks was already carried out. At MIT, COPE [3] was demonstrated for 15 nodes. At Aalborg University CATWOMAN [4] has been invented and is currently used in the Open-Mesh community.

COPE is the first network coding approach for wireless mesh networks. The implementation of COPE with TCP and UDP protocol was proposed in [5]. COPE exploits the shared nature of the wireless medium and broadcasts each packet.

CATWOMAN implements network coding on top of an existing routing scheme known as BATMAN (Better Approach To Wireless On Mobile Ad-hoc Network) [6] which has some inherent advantages to support network coding. CATWOMAN uses unicast instead of broadcast in the MAC layer. Consider the following example which explains the Alice and Bob topology (Fig. 1) using CATWOMAN. In this example the relay receives one packet from Alice and one from Bob and combines them using XOR. Then it chooses Bob as the destination and unicasts the combined packet. The relay will receive an acknowledgment when Bob receives this packet. Alice overhears this packet and extracts its own packet. In this case Alice will not be able to send an acknowledgment because the relays destination is Bob. The lack of acknowledgments for broadcast packets and also for overhearing packets in the unicast implies that the MAC does not support reliability for network coding. The lack of reliability in the MAC layer is more serious even though they might get repaired by higher layers. TCP interprets data loss as signs of network congestion and then reduces window size which seriously degrades the throughput [7].

Hence, the use of broadcast in the MAC increases the number of collisions, and decreases the throughput of the system because there is no collision avoidance mechanism in the broadcast.

There have been some research and publications regarding the benefit of reliability in network coding in wireless networks [8], [9]. ARQ based reliability for the network coding is also discussed in [10]. However, our work is based on how the reliability is implemented into the MAC layer to support network coding. Furthermore, this work considers the loss probability of acknowledgment packets which is not considered in ARQ based network coding [10]. This paper investigates reliability in network coding. The effect of different reliability approaches in network coding is discussed and some analytical models for loss probability and the expected number of transmissions are introduced in this paper.

The paper is organized as follows. An overview of the system model and assumptions are given in Section 2. Section 3 presents the analytical model for loss probability and expected number of transmissions for different network coding approaches. In order to validate the analytical models, the numerical result is given in Section 4. Finally, discussion and conclusion remarks follow in Section 5.

II. System Model and Notation

The system setup and assumptions which are basic for the analytical model will be explained in this section. This model is based on the simple Alice-Relay-Bob topology. In this scenario it is assumed that Alice and Bob send the packets to the relay and then the relay forwards the packets to the corresponding receiver. This setup is illustrated in Fig. 1. Alice and Bob are not able to exchange packets directly and therefore all traffic must pass through the relay node. In order to derive the analytical model some definitions and assumptions have to be made for the selected scenario. These are given in the following sections.

A. Notation

- Definition 1: The erasure probability is the probability that the receiver is not able to receive the packet successfully. As illustrated in Fig. 1 the erasure probability is given as ϵ for all links.

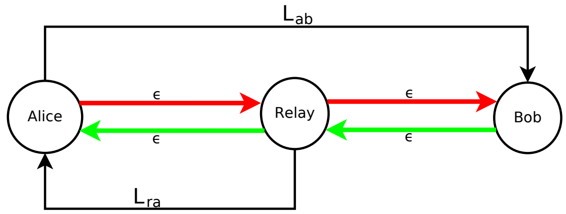

Fig. 1. Alice and Bob topology.

- Definition 2: E_{ij} is the expected number of transmissions of one packets from i to j.
- Definition 3: The loss probability is the probability that destination does not receive the packet successfully in all retransmissions of the packet from the source. The loss probability from node i to node j is given as L_{ij}.
- Definition 4: The packet loss probability from i to j when k is the destinations is given as $L_{ij,k}$.
- Definition 5: $P_{ab',r}$ is the probability that Alice receives a packet from the relay successfully but Bob does not receive the packet.

B. Assumptions

- The traffic between Alice and Bob is symmetric.
- The links are assumed to be homogeneous. It means that $\epsilon_{ar} = \epsilon_{ra} = \epsilon$.
- Because of the small packets size, the probability of packet loss for acknowledgments in the MAC is assumed to be zero [11].
- The probability of packet loss for piggyback-acknowledgment in the MAC is assumed to be non-zero.
- The number of the transmissions of one packet until sender removes it from sending queue is given as n.
- The cost of the acknowledgment in IEEE 802.11 is ignored because ACK packets are very small.
- The cost of the piggybacked-acknowledgment is ignored because it is included in other data packets.

III. Reliability Approaches

In this section four different approaches for the MAC layer will be discussed. These are IEEE unicast, CATWOMAN, RUNC (Reliable Unicast Network Coding), RBNC (Reliable Broadcast Network Coding). Each of these approaches has a different level of reliability in the MAC layer.

A. Unicast

IEEE 802.11 unicast is used for a basic approach. Alice sends a packet to the relay which in turn will reply with an acknowledgment upon a successful receiving. When Alice failed to receive the acknowledgment, it indicates that the relay neither receives the packet or the acknowledgment is lost, hence Alice will retransmit the packet again. This procedure continues until the relay receives the packet successfully. The relay then does the same procedure to send the packet to Bob.

B. CATWOMAN

CATWOMAN utilizes the unicast method to send coded packets via the relay. When Alice wants to send a packet to Bob, at first Alice uses the unicast to send the packet to the relay. Bob also sends a packet to the relay. The relay combines two packets, one from Alice and one from Bob. Then it selects Alice or Bob as the destination using a link quality estimator that is randomly weighted. (In this paper we do not include the link quality estimation.) It uses unicast to send the combined packet. Both Alice and Bob may receive the combined packet but only the destination sends an acknowledgment.

C. RUNC: Reliable Unicast Network Coding

When a node overhears a packet using CATWOMAM it does not send any acknowledgment. RUNC tries to fix this problem and is similar to CATWOMAN because it uses the network coding in the relay. In this approach when the relay receives two packets from Alice and Bob, it combines them and chooses Alice or Bob as the destination (for example the relay chooses Alice as the destination), then the relay unicasts the coded packet. The destination sends an acknowledgment similarly as with the unicast method. The difference between RUNC and CATWOMAN is that the overhearing node (Bob) sends a specific acknowledgment to the relay. This acknowledgment is piggybacked with other data packets and increases the reliability. This acknowledgment is included in the data packet, therefore, it does not impose very much overhead to the network.

D. Reliable Broadcast network coding (RBNC)

The only difference between RBNC and RUNC is that the relay broadcasts each packet. Therefore the relay does not choose any destination. In this approach when two packets are received by the relay, it combines them and broadcasts coded packet. Both destinations must acknowledge the coded packets. The relay does not use the MAC layer unicast therefore, the receiver does not send any unicast acknowledgment. The acknowledgment is piggybacked with other data packets. It is assumed that this introduces negligible overhead.

IV. Analytical Model

In this section the four different approaches introduced in the pervious section will be compared regarding packet loss probability and expected number of transmitted packets from Alice to Bob.

A. Unicast

- *Loss Probablity*

The probability that the receiver receives a packet successfully is given as p_i, where i is the number of times that the packet has been transmitted. This probability is:

$$p_i = \epsilon^{i-1} \cdot (1 - \epsilon) \quad (1)$$

The packet from Alice to Bob can be lost in two cases. The first case is when the packet is lost during the transmission from Alice to the relay and the second case is when the packet is lost during the transmission from the relay to Bob. This is also valid for other approaches. A loss of a packet is considered whenever the receiver fails to receive the transmitted packet after n transmissions from a sender. The probability of the loss for a packet from Alice to the relay is below:

$$L_{ar} = \epsilon^n \tag{2}$$

The probability of the loss for a packet from the relay to Bob is below:

$$L_{rb} = \epsilon^n \tag{3}$$

The probability of the loss for sending a packet from Alice to Bob is given as:

$$L_{ab} = L_{ar} + (1 - L_{ar})L_{rb} \tag{4}$$

$$L_{ab} = \epsilon_{ar}^n + (1 - \epsilon_{ar}^n) \cdot \epsilon_{br}^n \tag{5}$$

- *The expected number of transmissions*

If X is a discrete random variable having a probability function $p(x)$, then the expected value of X is defined by [12]:

$$E[X] = \sum_{x:p(x)>0} x \cdot p(x) \tag{6}$$

In these four approaches, unicast is used to send a packet from Alice to the relay. Therefore it is assumed that $E_{ar} = E$. The difference between these approaches is only in the relay node. The expected number of transmissions from Alice to the relay is given as:

$$E = 1 \cdot p_1 + 2 \cdot p_2 + ... + n - 1 \cdot p_{n-1} + n \cdot \epsilon^{n-1} \tag{7}$$

$$E = (1 - \epsilon) \cdot \sum_{i=1}^{n-1} i \cdot \epsilon^{i-1} + n \cdot \epsilon^{n-1} \tag{8}$$

Eq. 8 can be rewritten as:

$$E = \frac{(1 - \epsilon^{n-1})}{(1 - \epsilon)} - (n-1) \cdot \epsilon^{n-1} + n \cdot \epsilon^{n-1} \tag{9}$$

$$E = \frac{(1 - \epsilon^{n-1})}{(1 - \epsilon)} + \epsilon^{n-1} \tag{10}$$

The expected number of transmission from Alice to the relay is E and the expected number of transmission from the relay to Bob is E because in this approach Alice and the relay use the unicast. Hence the overall expected number of transmission from Alice to Bob is:

$$E_{ab} = 2 \cdot E \tag{11}$$

B. CATWOMAN

- *Loss Probablity*

The probability of loss from Alice to the relay is the same as in eq. 2. Consider that the relay wants to send a packet to Bob. When Bob is chosen as the destination, the relay tries n times until Bob receives the packet successfully. Therefore the probability of the loss from the relay to Bob is:

$$L_{rb,b} = \epsilon^n \tag{12}$$

When Alice is the destination, Bob looses the packet when it does not receive the coded packet and Alice receives it correctly (because if both of them loose the packet, the relay would send the packet again). This probability equals to:

$$P_{ab',r} = \epsilon \cdot (1 - \epsilon) \tag{13}$$

The probability of the loss of a combined packet for both Alice and Bob is:

$$P_{a'b',r} = \epsilon \cdot \epsilon \tag{14}$$

When Alice does not receive the packet, the relay sends it again. Therefore Bob would have another chance to receive the coded packet. The probability of the loss for Bob in the second try is given as $P_{a'b',r} \cdot P_{ab',r}$. In case Bob looses the packet, this procedure must continue until Bob looses the packet and Alice receives the packet within n tries or in the all n tries Alice and Bob do not receive anything. The equation for this probability is given as:

$$\begin{aligned} L_{rb,a} = &(P_{ab',r} + P_{ab',r} \cdot P_{a'b',r} + ...+ \\ &P_{ab',r} \cdot {P_{a'b',r}}^{n-1} + {P_{a'b',r}}^{n}) \end{aligned} \tag{15}$$

$$L_{rb,a} = \sum_{i=0}^{n-1} (P_{ab',r} \cdot {P_{a'b',r}}^{i}) + {P_{a'b',r}}^{n} \tag{16}$$

Eq. 16 can be rewritten as:

$$L_{rb,a} = \epsilon \cdot (1 - \epsilon) \frac{1 - (\epsilon^2)^n}{1 - (\epsilon^2)} + (\epsilon^2)^n \tag{17}$$

The relay receives the packet successfully from Alice with the probability $(1 - \epsilon^n)$. The probability of choosing Bob or Alice as the destination is $\frac{1}{2}$. When the relay chooses Bob as the destination, the probability of the loss for a packet from the relay to Bob is the same as in eq. 12. The loss probability from the relay to Bob is:

$$L_{rb} = \frac{1}{2} \cdot L_{rb,a} + \frac{1}{2} \cdot L_{rb,b} \tag{18}$$

Overall loss probability is:

$$L_{ab} = L_{ar} + (1 - L_{ar}) \cdot L_{rb} \tag{19}$$

$$L_{ab} = \epsilon^n + \frac{1}{2}(1 - \epsilon^n)(\epsilon^n + \epsilon \cdot (1 - \epsilon) \frac{1 - (\epsilon^2)^n}{1 - (\epsilon^2)} + (\epsilon^2)^n) \tag{20}$$

- *The expected number of transmissions*

The expected number of transmissions from Alice to the relay is equal to unicast. The expected number of transmissions from the relay is half than in case of unicast, because the relay uses network coding and in each transmission it sends two packets. Therefore the expected number of transmissions from Alice to Bob is given as:

$$E_{ab} = \frac{3}{2} \cdot E \tag{21}$$

C. RUNC: Reliable Unicast Network Coding

- *Loss Probability*

The loss probability in this approach is the same as with the unicast method, because for each individual packet there is an individual acknowledgment. Therefore the probability of the loss for RUNC is the same as for unicast:

$$L_{ab} = \epsilon^n + (1 - \epsilon^n) \cdot \epsilon^n \tag{22}$$

- *The expected number of transmissions*

In half of the cases the relay chooses Alice as the destination and in half of the cases it chooses Bob. Assuming that the relay chooses Bob as the destination, the relay transmits the packet for E times until Bob receives the packet correctly. When Bob receives the packet, it sends an acknowledgment similar as with unicast. In the case that the relay chooses Alice as the destination, the relay retransmits the packet when Bob does not receive the packet or the relay does not receive the acknowledgment. Therefore the expected number for transmissions of a packet from the relay to Bob not only depends on the erasure probability for the data packet from the relay to Bob but also depends on the erasure probability for the acknowledgment packet from Bob to the relay. Hence in this case the erasure probability of sending a packet from the relay to Bob is given as:

$$\gamma = \epsilon + (1 - \epsilon) \cdot \epsilon \tag{23}$$

It is needed to calculate the expected number of transmissions with a new erasure probability when the relay chooses Alice as the destination which is called Γ (Similar as with eq. 8.). Therefore in the case that the relay chooses Alice as the destination, the number of transmissions is Γ and in the case that the relay chooses Bob as the destination the number of transmissions is E. Regarding that the relay uses network coding, the expected number of transmissions from the relay to Bob is:

$$E_{rb} = \frac{1}{2} \cdot (\frac{1}{2} \cdot \Gamma + \frac{1}{2} \cdot E) \tag{24}$$

Meanwhile there are E transmissions from Alice to the relay. Expected number of transmission from Alice to Bob is:

$$E_{ab} = \frac{1}{4} \cdot (\Gamma + E) + E \tag{25}$$

D. Reliable Broadcast network coding (RBNC)

- *Loss Probability*

The probability of the loss for RBNC is the same as with unicast, because both receivers send an acknowledgment for each packet:

$$L_{ab} = {\epsilon_{ar}}^n + (1 - {\epsilon_{ar}}^n) \cdot {\epsilon_{br}}^n \tag{26}$$

- *The expected number of transmission*

Similar as with RBNC the probability of sending a packet from the relay to Bob again not only depends on erasure probability for data packet from the relay to Bob but also depends on erasure probability for acknowledgment packet from Bob to the relay. Hence erasure probability from the relay to Bob is:

$$\gamma = \epsilon + (1 - \epsilon) \cdot \epsilon \tag{27}$$

Therefore the expected number of transmissions with new erasure probability is called Γ. Also there are E transmissions from Alice to the relay. The expected number of transmissions is:

$$E_{ab} = \frac{1}{2} \cdot \Gamma + E \tag{28}$$

V. Numerical Result

We have numerically evaluated the expected number of transmissions of packets from Alice to Bob with different erasure probability. Also the simulation results confirm the analytical model of the previous section. A summary of our numerical results and simulation results are presented in this section. In the simulation and analytical result, the number of the transmissions of one packet n is assumed to be 8.

The simulation was written in Java and is based on the Alice and Bob topology. In this topology Alice and Bob both transmit 1000 packets in one simulation. Furthermore, it is assumed that there are no collisions and delays. The results of this simulation are given in Fig. 2 and Fig. 3. Fig. 2 shows the number of transmissions of each approache and Fig. 3 similarly shows the number of lost packets.

The analytical results are given in Fig. 4 and Fig. 5. Fig. 4 shows the expected number of transmissions of one packet. It can be seen that CATWOMAN is the best approach and RUNC is close to CATWOMAN. Also RBNC approach has low transmissions when $\epsilon > 0.7$ or $\epsilon < 0.4$. All the network coding approaches are better than unicast because they combine two packets in the relay.

In Fig. 2 it can be seen that the number of transmission of the CATWOMAN is lower than other approaches. However RUNC and RBNC approaches have lower transmission than unicast because they use network coding in the relay.

Fig. 5 shows the loss probability for the Alice and Bob topology. In the Fig. 3 and Fig. 5 it can be seen that CATWOMAN has more loss probability in the simulation and the analytical model, because CATWOMAN does not use acknowledgments for overhearing packets.

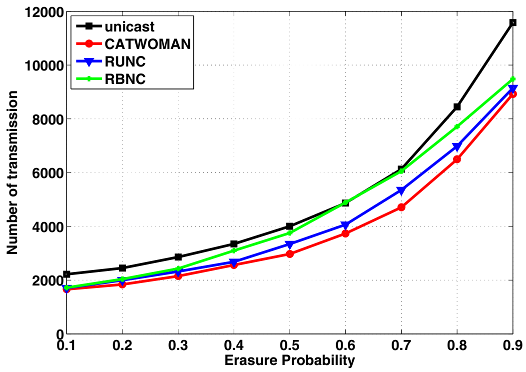

Fig. 2. Simulation result for the expected number of transmission.

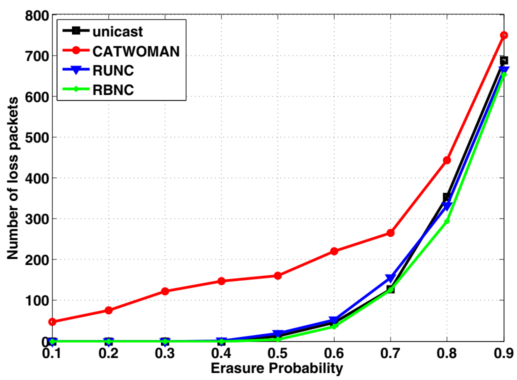

Fig. 3. The simulation result of the number of lost packets.

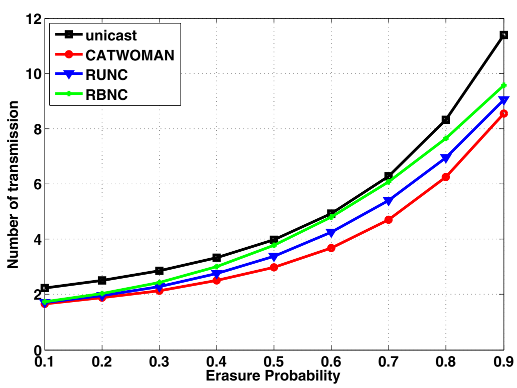

Fig. 4. The analytical result for the number of transmission.

VI. Conclusion

In this paper it was shown that network coding decreases the number of transmissions even if it supports reliability. However the reliability increases the overhead in the MAC layer while it decreases the packet loss. Besides it increases the system throughput and avoids the congestion. It was shown that RUNC has better performance than RBNC because it uses the IEEE MAC reliability so that it has less overhead. Also using unicast in RUNC would avoid the interferences in wireless networks.

The reliability in the MAC layer will increase the TCP

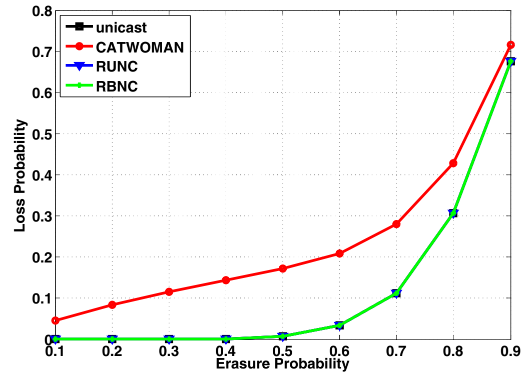

Fig. 5. The analytical result of loss probability.

performance. TCP can not differentiate packet loss caused by congestion from link errors and it considers packet loss to be the result of network congestion so that the sending rate will decrease dramatically as a precaution. Studies of measuring TCP throughput and MAC layer throughput in appearance of interference have been considered as the future works.

VII. Acknowledgement

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