A Novel Antenna Assignment Algorithm For Spectrum Underlay in Cognitive MIMO Networks

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***Abstract*—We consider a point-to-multipoint cognitive network sharing the same frequency band with a primary network assuming a spectrum underlay model. We investigate the scenario where a cognitive base station equipped with multiple antennas attempts to serve secondary users through an antenna assignment scheme. We consider quality of service constraints for secondary users and an interference constraint for the primary receiver. Hence, the cognitive base station performs both antenna assignment and optimal power allocation for the selected secondary users. Due to the high computational complexity of this problem, we propose a heuristic algorithm that separates the two tasks and tries to maximize the number of served secondary users with respect to the system constraints. The antenna assignment phase is performed using an efficient selection criterion followed by an optimal power allocation as a second phase. We show that the proposed algorithm has a very low computational complexity compared to the brute force algorithm. Furthermore, simulation results show that the proposed heuristic algorithm is able to achieve performance very close to that of the optimal solution.**

I. Introduction

The cognitive radio technology allows a dynamic spectrum sharing where unlicensed secondary users (SUs) can share frequency bands (FBs) owned by licensed primary users (PUs). Two approaches of spectrum sharing were proposed and discussed in the literature [1] [2]: overlay and underlay. In overlay systems, SUs can access an unused FB but should stop immediately their activities if a PU wants to use this band. On the other hand in underlay systems, the same band can be used simultaneously by SUs and PUs. Anyhow, resource allocation to SUs should be such that secondary transmissions are not harmful to primary ones.

Many works have focused on proposing underlay spectrum sharing algorithms for cognitive networks. In [1], based on removal algorithms proposed for cellular systems, the authors developed a resource allocation framework. Their proposed algorithms perform admission control by removing the secondary links causing or experiencing high interference. Another algorithm is proposed in [3] with the objective of maximizing the throughput of a point-to-multipoint cognitive network by performing two separated phases: (i) a distributed power control and (ii) a centralized channel assignment.

Recently, spectrum sharing for cognitive multi-input multi-output (MIMO) multiuser systems is becoming an area of active research. In [4], the authors proposed a user selection algorithm to be implemented in a cognitive multi-antenna base station (BS). The paper assumes the use of transmit beamforming which requires a high amount of accurate feedback from the secondary and primary receivers.

In this paper, we consider a cognitive network where a multi antenna BS attempts to serve SUs operating in the same spectrum band as one PU. The BS tries to maximize the number of served SUs with a limited power budget while limiting the interference perceived by PUs. In addition to the primary interference constraint, each SU cannot be scheduled unless its requirement in terms of signal to interference plus noise ratio SINR (or equivalent bit error rate BER) is fulfilled. We propose an antenna assignment algorithm that performs both user selection and power allocation. We show that the proposed algorithm has very low computational complexity compared to the brute force algorithm which performs an exhaustive search over all the possible combinations. We also show through simulations that our algorithm provides near optimal performances and hence resolves efficiently the complexity/performance tradeoff.

Antenna assignment has been investigated in conventional MIMO wireless networks. An antenna assignment scheme exploiting both multiuser and spatial diversity was proposed in [5]. The proposed algorithm assumes limited feedback and hence its performance is penalized since it cannot perform optimal scheduling and power allocation. In [6], the authors formulate the assignment problem as a bipartite graph and solve it using the Hungarian algorithm. The proposed scheme is optimal only when the number of users is equal to the number of antennas. Furthermore, even that this work assumes perfect knowledge of the channel gains at the BS, it assumes an equal power allocation scheme which penalizes the performances. Therefore, in this paper we resolve both antenna assignment and power allocation problems in order to maximize the system performances.

The remainder of this paper is organized as follows. The system model and the problem formulation are presented in Section II. Section III details the proposed algorithm and evaluates its complexity. Section IV presents an adapted round robin algorithm to solve the same problem. Simulation results are presented in Section V and conclusions in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a cognitive radio network consisting of a single M-antenna cognitive base station (CBS) which communicates with K N-antenna SUs. We consider the downlink of the cognitive network in which the transmission is from the

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CBS to the scheduled SUs. A primary network consisting of one communication link exists within the same area of the secondary network. The two networks transmits simultaneously on the same band assuming a spectrum underlay model. Extending our work to multiple non-interfered primary transmissions is straightforward.

We assume that time is divided into slots and that all the channels' states remain invariant during an entire time slot (TS). At the beginning of each TS, The BS chooses the users to be served and assigns at most one antenna to each scheduled SU. We denote by S the set of pairs (k^*, m_{k^*}) where antenna $m_{k^*} \in \{1, \ldots, M\}$ is assigned to user $k^* \in \{1, \ldots, K\}$. Since the FB is shared between the two networks, the scheduled SUs will suffer from interference coming from the primary transmitter. Similarly, the primary signals are corrupted by the interference caused by the CBS transmission.

The received signal at antenna n of SU k is given by

$$\begin{aligned} y_k^{(n)} &= \sqrt{P_{m_k}} h_k^{(n)}(m_k) x_k + \sum_{\substack{j \in S \\ j \neq k}} \sqrt{P_{m_j}} h_k^{(n)}(m_j) \\ &\quad + \sqrt{P_p} f_k^{(n)} + z_k^{(n)}, \end{aligned} \tag{1}$$

where P_{m_k} is the power allocated to user k, $h_k^{(n)}(m_k)$ is the channel coefficient between the n-th antenna of user k and the BS antenna m_k assigned to this user, x_k is the transmitted data symbol to user k, $f_k^{(n)}$ is the channel coefficient between the primary transmitter and the n-th antenna of user k, P_p is the primary transmitted power and $z_k^{(n)}$ is the additive white gaussian noise which is assumed independent and identically-distributed (i.i.d.) with zero mean and variance Z_0.

We assume that each scheduled SU applies maximum ratio combining (MRC). Therefore, the SINR of user k is [7]

$$\gamma_k = \frac{P_{m_k} \|\mathbf{h}_k(m_k)\|^2}{Z_0 + \sum\limits_{\substack{j \in S \\ j \neq k}} P_{m_j} \|\mathbf{h}_k(m_j)\|^2 + P_p \|\mathbf{f}_k\|^2}, \tag{2}$$

where $\mathbf{h}_k(m_k)$ and $\mathbf{f}_k$ are the $1 \times N$ vectors containing the coefficients $h_k^{(n)}(m_k)$ and $f_k^{(n)}$ respectively, $n = 1, \ldots, N$.

Similarly, the SINR experienced by the primary receiver is

$$\gamma_p = \frac{P_p |e|^2}{Z_p + \sum_{k \in S} P_{m_k} |g_p(m_k)|^2}, \tag{3}$$

where e is the channel coefficient of the primary transmission, Z_p is the noise variance for the primary transmission and $g_p(m_k)$ denotes the channel coefficient between BS antenna m_k and the primary receiver antenna.

The channel coefficients are modeled as: $h_k^{(n)}(m_j) = (d_{k,m_j}^{(n)})^{-\frac{\alpha}{2}}.\beta_{k,m_j}^{(n)}$, $f_k^{(n)} = (d_k'^{(n)})^{-\frac{\alpha}{2}}.\beta_k'^{(n)}$, $g_p(m_k) = (d_k'')^{-\frac{\alpha}{2}}.\beta_k''$ and $e = (d^*)^{-\frac{\alpha}{2}} \beta^*$, where $d_{k,m_j}^{(n)}$, $d_k'^{(n)}$, d_k'' and d^* are the corresponding distances, $\beta_{k,m_j}^{(n)}$, $\beta_k'^{(n)}$, β_k'' and β^* are i.i.d random Gaussian variables with zero mean and unit variance and α is the path loss exponent. We assume that the CBS knows perfectly the channel gains between its antennas and those of the K users and the primary receiver via a perfect feedback. Also, each SU transmits to the BS the channel gain between its antennas and the primary transmitter.

B. Problem Formulation

We assume that the CBS tries to maximize the total number of served users (i.e. used transmit antennas) in each TS. Since we are assuming a spectrum underlay model, the CBS has to protect the primary transmission from excessive interference. Therefore, we assume that the primary receiver tolerates only a certain interference level, denoted Γ_{th}. Furthermore, a SU can be scheduled if and only if its achievable SINR is larger than a specified minimum value, denoted γ_{th}. Hence, the CBS has to perform an appropriate antenna assignment coupled with an efficient power allocation in order to serve a large number of SUs while respecting the contraints. Our problem can be written as:

$$\begin{aligned} &\text{Maximize} && |S| \\ &\text{subject to} && \gamma_k \geq \gamma_{th}, \quad k \in S && (4) \\ &&& \sum_{k \in S} P_{m_k} \leq P_{max}, && (5) \\ &&& \sum_{k \in S} P_{m_k} |g_p(m_k)|^2 \leq \Gamma_{th}, && (6) \end{aligned}$$

where $|S|$ is cardinality of S and P_{max} is the CBS total power.

In [8], the authors prove the NP-completeness of scheduling problems assuming geometric SINR model (the same as our scheduling problem) under a uniform power allocation. They also stated that the question whether the scheduling problem with optimal power allocation is NP-complete is a hard question and remains an open research area.

III. The Cognitive Antenna Assignment Algorithm (C3A)

Due to the high computational complexity of the cognitive antenna assignment problem formulated in Section II, we propose to solve it in two phases. In the first phase, the proposed algorithm chooses the best SUs and assigns appropriately the antennas. In the second phase, the algorithm allocates power to the selected users according to (4). If such an allocation is found, then the algorithm verifies the power constraint (5) and the interference one (6). If the algorithm cannot perform such an allocation or if the constraints are violated, then it will come back to the first phase to choose a smaller set of SUs.

A. Phase 1: User Selection and Antenna Assignment

The algorithm starts by constructing a $K \times M$ matrix, denoted $\mathbf{Q}$, whose coefficients are given by

$$q_{kj} = \frac{\|\mathbf{h}_k(m_k)\|^2}{\sum\limits_{l=1; l \neq k}^{M} \|\mathbf{h}_k(m_l)\|^2}. \tag{7}$$

The numerator of q_{kj} shows how good is the channel between user k and its assigned antenna whereas the denominator represents the interference caused by the use of the other antennas (different from k). Then, the algorithm selects the user having the maximum coefficient in matrix $\mathbf{Q}$ and assign the corresponding antenna to that user. Before selecting a second user, all the coefficients corresponding to the selected user and assigned antenna are reduced to zero in order to prevent them from being used for a second time. The same selection process is repeated in a greedy fashion until choosing M users and assigning M antennas. At the end of this phase, $\mathbf{Q}$ should be a null matrix. Note that the proposed design of matrix $\mathbf{Q}$ coefficients balances between high channel gains and low mutual interference in order to choose users with high SINR.

B. Phase 2: Optimal Power Allocation

This phase performs a power allocation with respect to (4), (5) and (6). Therefore, the algorithm starts by checking if it can allocate power to the selected users without violating their QoS constraints. Constraint (4) can be rewritten as:

$$P_{m_k} - \gamma_{th} \frac{\sum\limits_{j \in S, j \neq k} P_{m_j} \|\mathbf{h}_k(m_j)\|^2}{\|\mathbf{h}_k(m_k)\|^2} \geq \gamma_{th} \frac{Z_0 + P_p |f_k|^2}{\|\mathbf{h}_k(m_k)\|^2}. \quad (8)$$

Using the matrix form, (8) can be written as the following componentwise inequality

$$(\mathbf{I} - \mathbf{B})\ \mathbf{P} \succeq \mathbf{u}, \quad (9)$$

where

$$b_{kj} = \begin{cases} \gamma_{th} \dfrac{\|\mathbf{h}_k(m_j)\|^2}{\|\mathbf{h}_k(m_k)\|^2} & k \neq j, \\ 0 & k = j, \end{cases} \quad (10)$$

and

$$u_k = \gamma_{th} \frac{Z_0 + P_p \|\mathbf{f}_k|^2}{\|\mathbf{h}_k(m_k)\|^2}, \quad (11)$$

where $\mathbf{I}$ is the $M \times M$ identity matrix.

According to Perron-Frobenius theorem [9] [10], if system (9) has a positive solution $\mathbf{P}$, then there is a unique solution $\mathbf{P}^* \preceq \mathbf{P}$ satisfying $(\mathbf{I} - \mathbf{B})\mathbf{P}^* = \mathbf{u}$. Moreover, for any given $\gamma > 0$, system (9) has a positive solution $\mathbf{P}$ if and only if the maximum eigenvalue of $\mathbf{B}$ is less than one (i.e. the spectral radius $\rho(\mathbf{B})$ is inside unit circle), and in that case, the solution is unique and called the Pareto-optimal transmit power vector denoted by

$$\mathbf{P} = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{u}. \quad (12)$$

This second phase starts by computing the eigenvalues of $\mathbf{B}$ and verifying if they are all less than one. If it is the case, then there exists a vector $\mathbf{P}$ such that (4) is satisfied. The algorithm then verifies the other two constraints. If at least one of these constraints is violated, then the algorithm starts the second iteration (i.e., $l = 2$) and returns to phase 1. The new user selection and assignment starts by constructing $C_l = \binom{M}{M-l+1}$ matrices similar to matrix $\mathbf{Q}$ and denoted by $\mathbf{Q}^{(l,i)}$ where $i = 1, \ldots, C_l$ is the index of the matrix. For each matrix $\mathbf{Q}^{(l,i)}$, we consider only $M - l + 1$ different antennas (the remaining antennas are silent). A different user selection is then performed on each matrix and the algorithm runs phase 2. The C3A is summarized in Algorithm 1.

Algorithm 1: The cognitive antenna assignment algorithm

Initialization: $S \leftarrow \emptyset$; $l \leftarrow 1$; $i \leftarrow 1$;

Step 0:
Construct matrix $\mathbf{Q}^{(l,i)}$ using (7);

Step 1 - User selection and antenna assignment:
while $\mathbf{Q}^{(l,i)} \neq$ *null matrix* **do**
  Select maximum element (k, j) on $\mathbf{Q}$;
  $S \leftarrow (k, j)$;
  Reduce elements on row k and column j to zero;
end

Step 2 - Power allocation:
Construct $\mathbf{B}$ using (10) and compute eigenvalues $\lambda(\mathbf{B})$;
if $\max(\lambda(\mathbf{B})) \geq 1$ **then**
  $S \leftarrow \emptyset$;
  if $i < \binom{M}{M-l+1}$ **then** $i \leftarrow i + 1$;
  else $i \leftarrow 1$; $l \leftarrow l + 1$;
  Go to step 0;
else
  Compute P using (12);
  if *constraints 5 and 6 are violated* **then**
    $S \leftarrow \emptyset$;
    if $i < \binom{M}{M-l+1}$ **then** $i \leftarrow i + 1$;
    else $i \leftarrow 1$; $l \leftarrow l + 1$;
    Go to step 0;
  else
    Antenna assignment found and C3A terminates;
  end
end

C. Complexity Analysis

In the following, we present the worst case complexity of the C3A, i.e. the complexity of solving the cognitive antenna assignment problem assuming the worst input. The complexity of step 0 is $O(K(M-l+1))$. In step 1, the algorithm performs $(M-l+1)$ iterations . In each iteration, finding the maximum value in matrix $\mathbf{Q}$ can be obtained in $O(K(M-l+1))$. The most complex operation in step 2 consists in computing the eigenvalues of matrix $\mathbf{B}$ which can be done in $O((M-l+1)^3)$.

If step 2 fails to find a feasible power allocation then the algorithm must return to step 0. Therefore, the C3A returns to step 0 at most M times. In each time, it tests $C_l = \binom{M}{M-l+1}$ matrices $\mathbf{Q}$. Hence, The complexity of the C3A is

$$\Psi_{C3A} = O\left(\sum_{l=1}^{M} \left(K(M-l+1)^2 + (M-l+1)^3\right) \cdot C_l\right). \quad (13)$$

The brute force algorithm (BFA) performs an exhaustive search over all the possible combinations of antenna assignment. Hence, its worst case complexity is

$$\Psi_{BFA} = O\left(\sum_{l=1}^{M} \frac{K!}{(K-M+l-1)!} \cdot (M-l+1)^3 \cdot C_l\right). \quad (14)$$

Table I presents a comparison between the complexities of the C3A and BFA by evaluating equations (13) and (14).

TABLE I
COMPARISON OF WORST COMPUTATIONAL COMPLEXITIES

$[K, M]$	C3A	BFA
$[20, 2]$	130	3080
$[20, 4]$	1824	$> 8, 1.10^6$
$[40, 4]$	3424	$> 1, 4.10^8$

IV. THE ROUND ROBIN ASSIGNMENT ALGORITHM

We propose an adapted round robin (RR) scheme that tries to maximize the number of scheduled users in each TS while maintaining the fair nature of conventional RR. Note that more fairness analysis will be considered in future works.

We denote by S_{RR} the set of SUs that can be scheduled in the current TS. The RR assignment algorithm:

1) constructs S_{RR} by adding at most M users from the users not yet scheduled and having the smallest indices;
2) runs the brute force assignment algorithm by performing an exhaustive search over the possible combinations in S_{RR} until it finds a feasible power allocation (trying at most $\sum_{l=1}^{N} \frac{K!}{(K-M+l-1)!}$ combinations).

When the RR schedules all the SUs in a given number of TSs, it restarts from the beginning. However, if one user stays twice in S_{RR} without being chosen, then the algorithm schedules it immediately in the following TS.

Let us consider the following example with $K = 6$ and $M = 3$. The set of SUs that can be scheduled in the first TS is $S_{RR} = \{1, 2, 3\}$. Assume that when performing the power allocation, only users 2 and 3 can be scheduled in the first TS, i.e. $S = \{(1, n_1), (3, n_3)\}$. Therefore, for the second TS, S_{RR} contains necessarily user 1, i.e. $S_{RR} = \{1, 4, 5\}$. If the three users are scheduled in the second slot then S_{RR} for the third TS will be $S_{RR} = \{6, 1, 2\}$ and so forth. Otherwise, if only users 4 and 5 are scheduled in the second TS, then the BS schedules only user 1 in the third TS and S_{RR} for the forth TS will be $\{6, 1, 2\}$.

V. SIMULATION RESULTS

In this section, we provide some simulation results to illustrate the performance and the near optimality of the proposed algorithm. The performance of the C3A is analyzed in terms of the number of served SUs. The CBS is located at the center (0,0) of a (1000×1000) rectangular area and the locations of the SUs are randomly generated according to a uniform distribution. The coordinates of the primary transmitter and receiver are (125,500) and (-125,500) respectively. The measure of interest is obtained by averaging over 10^4 channel realizations (TSs). The locations of the SUs are randomly regenerated for each TS. We assume that the noise variances are equal to unity, $\alpha = 4$ and $N = 2$. We assume that the primary transmission requires a SINR equal to $\gamma_p^* = 10$dB. Therefore, the primary transmitter adapts its transmission power P_p to achieve this requirement, i.e. $P_p = (1 + \Gamma_{th}).\gamma_p/|e|^2$.

Fig. 1 shows the number of users as a function of the SINR requirement γ_{th} for different numbers of antennas ($M = 3$, 4, 5 and 6). The other parameters are set as $\Gamma_{th} = 10$dB and $K = 20$. We notice that as M increases, the number of served users increases. we conclude that the

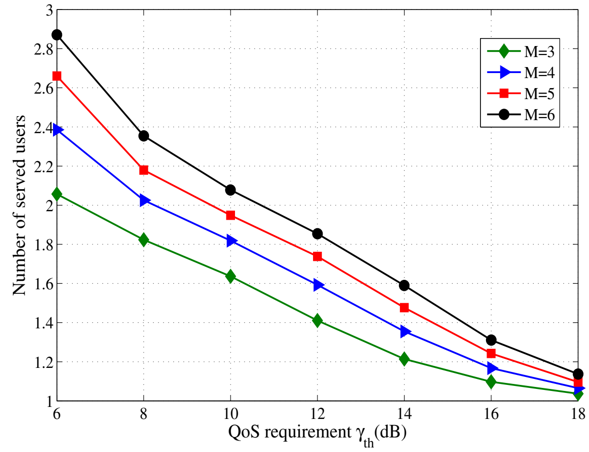

Fig. 1. Number of served users vs. γ_{th} for different number of antennas.

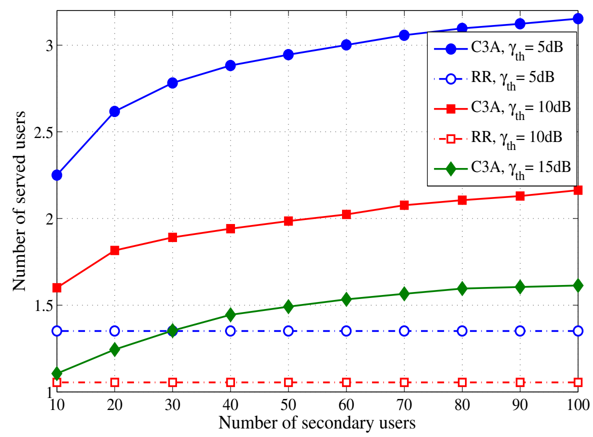

Fig. 2. Number of served users vs. the number of SUs for C3A and RRA.

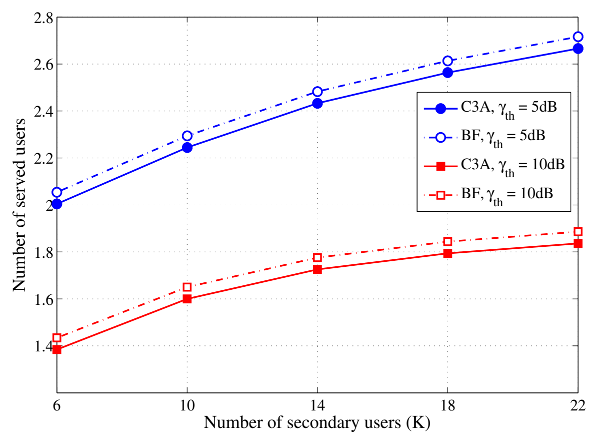

Fig. 3. Number of served users vs. the number of SUs for C3A and BFA.

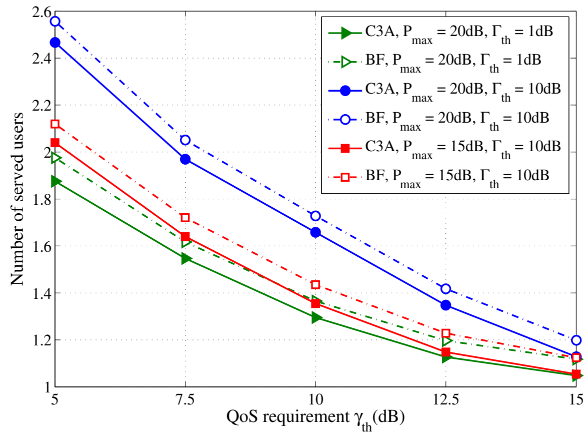

Fig. 4. Number of served users vs. γ_{th} while variying P_{max} and Γ_{th}.

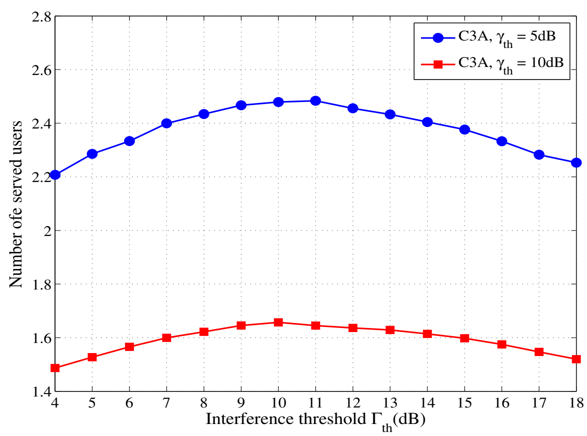

Fig. 5. Number of served users vs. Γ_{th} for different thresholds γ_{th}.

C3A is able to take benefit from the space dimension inherent in MIMO systems to serve more users.

For the remaining simulations, we assume $M = 4$. In Figs. 2 and 3, we plot the number of served users as a function of K while varying the QoS requirement. We assume that there is no limitation on the amount of transmit power available in the BS and $\Gamma_{th} = 10$dB. Fig. 2 illustrates the effect of multiuser diversity. In fact, it can be observed that as K increases, the number of served users using C3A increases. We notice that the C3A outperforms largely the RR algorithm. In fact, the RR scheduler does not take benefit from multiuser diversity since its prime objective is to achieve perfect fairness. Furthermore, we notice that the proposed C3A achieves multiuser diversity gain even for high load networks. Fig. 3 compares the performances of the C3A to those of the BF algorithm. We observe that the proposed algorithm achieves performances very close to the optimal results obtained by the highly complex BFA. We also notice that the gap between the two algorithms is almost the same for different γ_{th}.

The number of served users as a function of the QoS requirement for different values of P_{max} and Γ_{th} is illustrated in Fig. 4 ($K = 20$). We notice, once again, that the performances of C3A are very close to the optimal results. In fact, the proposed algorithm chooses almost always the set of users that minimizes the used transmit power. Therefore, when the power constraint is included in the antenna assignment process, the algorithm performance remains close to the optimal one.

Fig. 5 presents the number of served users as a function of Γ_{th} for different values of the secondary QoS requirement. It is observed that for small values of Γ_{th}, the number of served users grows with the increase of Γ_{th} until reaching a peak value. Then the performances decreases after this value. In fact, when Γ_{th} is small, the CBS can not schedule a big number of users in order to respect the interference constraint. On the other hand, when Γ_{th} has a relatively large value, the primary transmitter has to increase its transmit power and hence causes higher interference to the scheduled SUs. Therefore, the BS has to reduce the number of served SUs in order to decrease the secondary mutual interference.

VI. Conclusion

In this paper, we have considered the problem of antenna assignment in a cognitive radio network while protecting a primary transmission. We have proposed a heuristic algorithm in order to maximize the number of SUs served by a CBS equipped with multiple antennas. Particularly, the proposed algorithm operates in two phases, a user selection and antenna assignment phase based on a new efficient criterion, and a power allocation phase based in Perron-Frobenius theory. We have compared the performances and the complexity of the proposed algorithm with both the optimal highly complex brute force algorithm and an adapted RR scheduler. Simulation results demonstrate that the algorithm is capable of achieving high performance close to the one of the optimal BFA and is capable of using eficiently the space dimension.

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