Wireless Network-Coded Accumulate-Compute and Forward Two-Way Relaying

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***Abstract*—For the design of modulation schemes for the physical layer network-coded two way wireless relaying, it was observed by Koike-Akino et al. [4] that adaptively changing the network coding map used at the relay according to the channel conditions greatly reduces the impact of multiple access interference which occurs at the relay during the MA Phase and all these network coding maps should satisfy a requirement called *exclusive law*. We extend this approach to an Accumulate-Compute and Forward protocol which employs two phases: Multiple Access (MA) phase consisting of two channel uses with independent messages in each channel use, and Broadcast (BC) phase having one channel use. Assuming that the two users transmit points from the same 4-PSK constellation, every such network coding map that satisfies the exclusive law can be represented by a Latin Square with side 16, and conversely, this relationship can be used to get the network coding maps satisfying the exclusive law. Two methods of obtaining this network coding map to be used at the relay are discussed. Using the structural properties of the Latin Squares for a given set of parameters, the problem of finding all the required maps is reduced to finding a small set of maps. Having obtained all the Latin Squares, the set of all possible channel realizations is quantized, depending on which one of the Latin Squares obtained optimizes the performance. The quantization thus obtained, is shown to be the same as the one obtained in [7] for the 2-stage bidirectional relaying.**

I. Background

The concept of physical layer network coding has attracted a lot of attention in recent times. The idea of physical layer network coding for the two way relay channel was first introduced in [1]. Information theoretic studies for the physical layer network coding scenario were reported in [2], [3]. The design principles governing the choice of modulation schemes to be used at the nodes for uncoded transmission were studied in [4]. An extension for the case when the nodes use convolutional codes was done in [5]. A multi-level coding scheme for the two-way relaying was proposed in [6].

We consider the two-way wireless relaying scenario shown in Fig. 1, where two-way data transfer takes place among the nodes A and B with the help of the relay R. It is assumed that the two nodes operate in half-duplex mode, i.e., they cannot transmit and receive at the same time in the same frequency band. The relaying protocol consists of two phases, *multiple access* (MA) phase, consisting of two channel uses during which A and B transmit to R twice, two independent messages in the two channel uses, with points from 4-PSK constellation, and *broadcast* (BC) phase, in which R transmits to A and B.

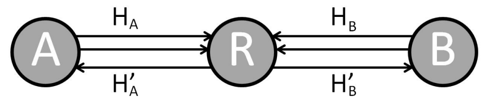

Fig. 1. A two-way ACF relay channel

The relay node R accumulates the information sent by the user nodes in the first and second channel use of the MA phase, and transmits in the BC phase a message that contains information about all the four messages received by it in the MA phase. Network Coding is employed at R in such a way that A(/B) can decode the two messages transmitted by B(/A), given that A(/B) knows its own messages. We call this strategy accumulate-compute and forward (ACF) protocol.

It was observed in [4] that for uncoded transmission, the network coding map used at the relay needs to be changed adaptively according to the channel fade coefficient, in order to minimize the impact of multiple access interference. It is shown in [8] for any choice of signal sets of equal cardinality used at the two users, that every such network coding map that satisfies the *exclusive law* is representable as a Latin Square and conversely, this relationship can be used to get the network coding maps satisfying the exclusive law. A Latin Square of order M is an $M \times M$ array in which each cell contains a symbol from a set of t different symbols such that each symbol occurs at most once in each row and column [9].

Similar to the ACF protocol, a store-and-forward protocol has been earlier studied in [10], for the two-way relaying channel information theoretically. However, explicit modulation and physical layer network coding have not been addressed in [10].

II. Accumulate-Compute and Forward Relaying

Let $\mathcal{S} = \left\{\frac{\pm 1 \pm j}{\sqrt{2}}\right\}$ denote the symmetric 4-PSK constellation, used at A and B. Assume that A(/B) wants to send two 2-bit binary tuples to B(/A). Let $\mu : \mathbb{F}_2^2 \rightarrow \mathcal{S}$ denote the mapping from bits to complex symbols used at A and B where $\mathbb{F}_2 = \{0, 1\}$. Let $x_{A_1} = \mu(s_{A_1}), x_{B_1} = \mu(s_{B_1}) \in \mathcal{S}$ denote the complex symbols transmitted by A and B at the first channel use respectively, and $x_{A_2} = \mu(s_{A_2}), x_{B_2} = \mu(s_{B_2}) \in \mathcal{S}$ denote the complex symbols transmitted by A and B at the second channel use respectively, where $s_{A_1}, s_{B_1}, s_{A_2}, s_{B_2} \in \mathbb{F}_2^2$.

Multiple Access (MA) Phase: It is assumed that the channel state information is not available at the transmitting nodes A and B during the MA phase. The received signal at R at the first and the second channel uses are given by

$$Y_{R_1} = H_A x_{A_1} + H_B x_{B_1} + Z_{R_1};\ \ Y_{R_2} = H_A x_{A_2} + H_B x_{B_2} + Z_{R_2} \quad (1)$$

where H_A and H_B are the fading coefficients associated with the A-R and B-R link respectively. Note that we are taking H_A and H_B to be the same for the two channel uses. The additive noise Z_{R_1} and Z_{R_2} are assumed to be $\mathcal{CN}\left(0,\sigma^2\right)$, where $\mathcal{CN}\left(0,\sigma^2\right)$ denotes the circularly symmetric complex Gaussian random variable with variance σ^2. We assume a block fading scenario, with $z = \gamma e^{j\theta} = H_B/H_A$, where $\gamma \in \mathbb{R}^+$ and $-\pi \leq \theta \leq \pi$, is referred to as the *fade state* for the first and second transmission by A and B at the first and second channel use, and for simplicity can also be denoted by (γ, θ). Also, it is assumed that z is distributed according to a continuous probability distribution.

Let $\mathcal{S}_R(\gamma,\theta)$ denote the effective constellation seen at the relay during the MA phase, i.e.,

$$\mathcal{S}_R(\gamma,\theta) = \left\{(x_i + \gamma e^{j\theta} y_i, x_j + \gamma e^{j\theta} y_j) | x_i, y_i, x_j, y_j \in \mathcal{S}\right\}.$$

The effective constellation remains the same over the two channel uses, since we assume H_A and H_B and hence the ratio $H_B/H_A = \gamma e^{j\theta}$ to be the same during the two channel uses.

Let $d_{min}\left(\gamma e^{j\theta}\right)$ denote the minimum distance between the points in the constellation $\mathcal{S}_R(\gamma,\theta)$ during MA phase, as given by (2) on the next page. From (2), it is clear that there exist values of $\gamma e^{j\theta}$, for which $d_{min}\left(\gamma e^{j\theta}\right) = 0$. Let, $\mathcal{H} = \left\{\gamma e^{j\theta} \in \mathbb{C} | d_{min}\left(\gamma e^{j\theta}\right) = 0\right\}$. The elements of $\mathcal{H}$ are called singular fade states. For singular fade states, $|\mathcal{S}_R(\gamma,\theta)| < 4^4$.

Definition 1: A fade state $\gamma e^{j\theta}$ is defined to be a *singular fade state* for the *ACF two-way relaying*, if the cardinality of the signal set $\mathcal{S}_R(\gamma,\theta)$ is less than 4^4. Let $\mathcal{H}$ denote the set of singular fade states for the two-way ACF relaying.

Let $(\hat{x}_{A_1}, \hat{x}_{B_1})$ and $(\hat{x}_{A_2}, \hat{x}_{B_2}) \in \mathcal{S}^2$ denote the Maximum Likelihood (ML) estimate of (x_{A_1}, x_{B_1}) and (x_{A_2}, x_{B_2}) at R based on the received complex numbers Y_{R_1} and Y_{R_2} at the two channel uses, as given in (3).

Broadcast (BC) Phase:

Depending on the value of $\gamma e^{j\theta}$, R chooses a map $\mathcal{M}^{\gamma,\theta}: \mathcal{S}^4 \rightarrow \mathcal{S}'$ where $\mathcal{S}'$ is a complex signal set of size between 4^2 and 4^4 used by R during the *BC* phase.

The received signals at A and B during the BC phase are respectively given by,

$$Y_A = H'_A X_R + Z_A \text{ and } Y_B = H'_B X_R + Z_B \quad (9)$$

where $X_R = \mathcal{M}^{\gamma,\theta}\left((\hat{x}_{A_1}, \hat{x}_{B_1}), (\hat{x}_{A_2}, \hat{x}_{B_2})\right) \in \mathcal{S}'$ is the complex number transmitted by R. The fading coefficients corresponding to the R-A and R-B links are given by H'_A and H'_B respectively and the additive noises Z_A and Z_B are $\mathcal{CN}\left(0,\sigma^2\right)$.

The elements in $\mathcal{S}^4$ which are mapped to the same signal point in $\mathcal{S}'$ by the map $\mathcal{M}^{\gamma,\theta}$ are said to form a cluster. Let $\{\mathcal{L}_1, \mathcal{L}_2, .., \mathcal{L}_l\}$ denote the set of all such clusters. The formation of clusters is called clustering, denoted by $\mathcal{C}^{\gamma e^{j\theta}}$.

In order to ensure that A(/B) is able to decode B's(/A's) messages, the clustering $\mathcal{C}^{\gamma e^{j\theta}}$ should satisfy the exclusive law, as given in (4), (5) in the next page.

Definition 2: The cluster distance between a pair of clusters $\mathcal{L}_i$ and $\mathcal{L}_j$ is the minimum among all the distances calculated between the points $((x_{A_1}, x_{B_1}), (x_{A_2}, x_{B_2})) \in \mathcal{L}_i$ and $\left((x'_{A_1}, x'_{B_1}), (x'_{A_2}, x'_{B_2})\right) \in \mathcal{L}_j$ in the effective constellation used by the relay node R, as given in (6) in the next page.

Definition 3: The *minimum cluster distance* of the clustering $\mathcal{C}^{\gamma e^{j\theta}}$ is the minimum among all the cluster distances, as given in (7) in the next page.

The minimum cluster distance determines the performance during the MA phase of relaying. The performance during the BC phase is determined by the minimum distance of the signal set $\mathcal{S}'$. For values of $\gamma e^{j\theta}$ in the neighborhood of the singular fade states, the value of $d_{min}\left(\mathcal{C}^{\gamma e^{j\theta}}\right)$ is greatly reduced, a phenomenon referred to as *distance shortening* [4]. To avoid distance shortening, for each singular fade state, a clustering needs to be chosen such that the minimum cluster distance is non zero. A clustering $\mathcal{C}^h$ is said to remove singular fade state $h \in \mathcal{H}$, if $d_{min}\left(\mathcal{C}^h\right) > 0$. For a singular fade state $h \in \mathcal{H}$, let $\mathcal{C}^h$ denote the clustering which removes the singular fade state h (if there are multiple clusterings which remove the same singular fade state h, choose any of the clusterings). Let $\mathcal{C}^{\mathcal{H}} = \left\{\mathcal{C}^h : h \in \mathcal{H}\right\}$ denote the set of all such clusterings.

Definition 4: The minimum cluster distance of the clustering $\mathcal{C}^h, h \in \mathcal{H}$ at the fade state $\gamma e^{j\theta}$ which is not necessarily a singular fade state, denoted by $d_{min}\left(\mathcal{C}^h, \gamma e^{j\theta}\right)$, is as given in (8). Note that if $\gamma e^{j\theta} = h \in \mathcal{H}$, $d_{min}\left(\mathcal{C}^h, h\right)$, reduces to $d_{min}\left(\mathcal{C}^h\right)$ given in (7) in the next page.

In general, the channel fade state $\gamma e^{j\theta}$ need not be a singular fade state. In such a scenario, among all the clusterings which remove the singular fade states, the one which has the maximum value of the minimum cluster distance at $\gamma e^{j\theta}$ is chosen by the relay R. In other words, for $\gamma e^{j\theta} \notin \mathcal{H}$, the clustering $\mathcal{C}^{\gamma,\theta}$ is chosen to be $\mathcal{C}^h$ by the relay R, which satisfies $d_{min}\left(\mathcal{C}^h, \gamma e^{j\theta}\right) \geq d_{min}\left(\mathcal{C}^{h'}, \gamma e^{j\theta}\right), \forall h \neq h' \in \mathcal{H}$. Since the clusterings which remove the singular fade states are known to all the three nodes and are finite in number, the clustering used for a particular realization of the fade state can be indicated by R to A and B using overhead bits.

The contributions and organization of this paper are as follows:

- It is shown that if the users A and B transmit points from the same 4-PSK constellation, the clusterings proposed in [8] for two-stage relaying can be utilized to get clusterings for this case for removing the singular fade states containing either 16 or 25 points by introducing the notion of Cartesian Product of Clusters (Section III).
- Another clustering is proposed for the ACF protocol in the two-way relay channel called Direct Clustering. This clustering also removes the singular fade states and reduces the number of clusters for some cases. Using this clustering, the size of the resulting constellation used by the relay node R in the BC phase is reduced to 20 for a

$$d^2_{min}\left(\gamma e^{j\theta}\right) = \min_{\substack{((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2})),((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))\in\mathcal{S}^4,\\ ((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2}))\neq((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))}} \left\{\left|\left(x_{A_1}-x'_{A_1}\right)+\gamma e^{j\theta}\left(x_{B_1}-x'_{B_1}\right)\right|^2+\left|\left(x_{A_2}-x'_{A_2}\right)+\gamma e^{j\theta}\left(x_{B_2}-x'_{B_2}\right)\right|^2\right\} \quad (2)$$

$$\left((\hat{x}_{A_1},\hat{x}_{B_1}),(\hat{x}_{A_2},\hat{x}_{B_2})\right) = \arg\min_{((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))\in\mathcal{S}^4} \left\{\left|Y_{R_1}-H_Ax'_{A_1}-H_Bx'_{B_1}\right|^2+\left|Y_{R_2}-H_Ax_{A'_2}-H_Bx'_{B_2}\right|^2\right\} \quad (3)$$

$$\mathcal{M}^{\gamma,\theta}\left(\left(x_{A_1},x_{A_2}\right),\left(x_{B_1},x_{B_2}\right)\right)\neq\mathcal{M}^{\gamma,\theta}\left(\left(x'_{A_1},x'_{A_2}\right),\left(x_{B_1},x_{B_2}\right)\right),\ whenever\left(x_{A_1},x_{A_2}\right)\neq\left(x'_{A_1},x'_{A_2}\right)\ \ \forall x_{B_1},x_{B_2}\in\mathcal{S} \quad (4)$$

$$\mathcal{M}^{\gamma,\theta}\left(\left(x_{A_1},x_{A_2}\right),\left(x_{B_1},x_{B_2}\right)\right)\neq\mathcal{M}^{\gamma,\theta}\left(\left(x_{A_1},x_{A_2}\right),\left(x'_{B_1},x'_{B_2}\right)\right),\ whenever\left(x_{B_1},x_{B_2}\right)\neq\left(x'_{B_1},x'_{B_2}\right)\ \ \forall x_{A_1},x_{A_2}\in\mathcal{S} \quad (5)$$

$$\left(d^{\mathcal{L}_i,\mathcal{L}_j}_{min}\left(\gamma e^{j\theta}\right)\right)^2 = \min_{\substack{((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2}))\in\mathcal{L}_i\\ ((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))\in\mathcal{L}_j}} \left\{\left|\left(x_{A_1}-x'_{A_1}\right)+\gamma e^{j\theta}\left(x_{B_1}-x'_{B_1}\right)\right|^2+\left|\left(x_{A_2}-x'_{A_2}\right)+\gamma e^{j\theta}\left(x_{B_2}-x'_{B_2}\right)\right|^2\right\} \quad (6)$$

$$d^2_{min}\left(\mathcal{C}^{\gamma,\theta}\right) = \min_{\substack{((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2})),((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))\in\mathcal{S}^4\\ \mathcal{M}^{\gamma,\theta}((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2}))\neq\mathcal{M}^{\gamma,\theta}((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))}} \left\{\left|\left(x_{A_1}-x'_{A_1}\right)+\gamma e^{j\theta}\left(x_{B_1}-x'_{B_1}\right)\right|^2+\left|\left(x_{A_2}-x'_{A_2}\right)+\gamma e^{j\theta}\left(x_{B_2}-x'_{B_2}\right)\right|^2\right\} \quad (7)$$

$$d^2_{min}\left(\mathcal{C}^h,\gamma e^{j\theta}\right) = \min_{\substack{((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2})),((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))\in\mathcal{S}^4\\ \mathcal{M}^{h}((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2}))\neq\mathcal{M}^{h}((x'_{A_1},x'_{B_1}),(x'_{A_2},x'_{B_2}))}} \left\{\left|\left(x_{A_1}-x'_{A_1}\right)+\gamma e^{j\theta}\left(x_{B_1}-x'_{B_1}\right)\right|^2+\left|\left(x_{A_2}-x'_{A_2}\right)+\gamma e^{j\theta}\left(x_{B_2}-x'_{B_2}\right)\right|^2\right\}. \quad (8)$$

category of cases, as compared to the Cartesian Product approach which results in the constellation size being 25 for these cases (Section IV).

- The quantization of the complex plane that contains all the possible fade states, depending on which one of the obtained clusterings maximizes the minimum cluster distance, is proven to be the same as for the two-way 2-stage relaying scenario as done in [7] (Section V).
- Simulation results indicate that at high SNR, the schemes based on the ACF protocol performs better than the schemes proposed in [4], [8] based on two-stage two way relaying. With 4-PSK signal set used at the end nodes, the ACF protocol achieves a maximum sum throughput of 8/3 bits/s/Hz, whereas it is 2 bits/s/Hz for the schemes based on 2-stage two way relaying (Section VI).

The proofs of Theorems, Lemmas and other claims are omitted due to lack of space but are available in [11], with additional illustrative examples and simulation results.

III. Exclusive Law and Latin Squares

The nodes A and B transmit symbols from the same constellation, viz., 4-PSK. Our aim is to find the map that the relay node R should use in order to cluster the 4^4 possibilities of $\left(\left(x_{A_1},x_{B_1}\right),\left(x_{A_2},x_{B_2}\right)\right)$ such that the exclusive law given by (4), (5) is satisfied. Consider the 16×16 array consisting of the 16 possibilities of (x_{A_1},x_{A_2}) along the rows and the 16 possibilities of (x_{B_1},x_{B_2}) along the columns. We fill this array with elements from $\mathcal{L}=\{\mathcal{L}_1,\mathcal{L}_2,...,\mathcal{L}_t\}$ where each symbol denotes a unique cluster. All the relay clusterings that satisfy the mutually exclusive law forms Latin Squares of order 16 with entries from $\mathcal{L}$ with $t\geq16$, when the end nodes use PSK constellations of size 4 [11]. It therefore suffices to consider the network code used by the relay node in the BC phase to be a 16×16 array with rows(/columns) indexed by the 2-tuple consisting of the symbols sent by A(/B) during the first and second channel use. The cells of the array must be filled with elements of $\mathcal{L}$ in such a way, that the resulting array is a Latin Square of order 16 and $t\geq16$.

Removing Singular fade states and Constrained Latin Squares

The relay can manage with constellations of size 16 in BC phase, but it is observed that in some cases relay may not be able to remove the singular fade states and results in severe performance degradation in the MA phase. As stated in Section II, that a clustering $\mathcal{C}^h$ is said to remove singular fade state $h\in\mathcal{H}$, if $d_{min}\left(\mathcal{C}^h\right)>0$. Removing singular fade states for a two-way ACF relay channel can also be defined as follows:

Definition 5: A clustering $\mathcal{C}^h$ is said to *remove the singular fade state* $h\in\mathcal{H}$, if any two possibilities of the messages sent by the users $\left(\left(x_{A_1},x_{B_1}\right),\left(x_{A_2},x_{B_2}\right)\right),\left(\left(x'_{A_1},x'_{B_1}\right),\left(x'_{A_2},x'_{B_2}\right)\right)\in\mathcal{S}^4$ that satisfy $h=\frac{x'_{A_1}-x_{A_1}}{x_{B_1}-x'_{B_1}}=\frac{x'_{A_2}-x_{A_2}}{x_{B_2}-x'_{B_2}}$, are placed together in the same cluster by $\mathcal{C}^h$.

Definition 6: A set $\{\left(\left(x_{A_1},x_{B_1}\right),\left(x_{A_2},x_{B_2}\right)\right)\}\subseteq\mathcal{S}^4$ consisting of all the possibilities of $((x_{A_1},x_{B_1}),(x_{A_2},x_{B_2}))$ that must be placed in the same cluster of the clustering used at relay node R in the BC phase in order to remove the singular fade state h is referred to as a *Singularity Removal Constraint* for the singular fade state h in two-way ACF relaying scenario.

Lemma 1: The singular fade states for the ACF two-way relaying scenario are the same as the 12 singular fade states for two-way 2-stage relaying scenario as computed in [4].

Let $\gamma e^{j\theta}$ be a fade state for the two-way ACF relaying scenario. Then $\gamma e^{j\theta}$ can be viewed as a fade state for the first and second channel use in the MA phase as shown in Lemma 1. In [4] and [8], it is shown that for the two-way 2-stage relaying, the 4^2 possible pairs of symbols from 4-PSK constellation sent by the two users in the MA phase, can be clustered into a clustering dependent on a singular fade coefficient, of size 4 or 5 in a manner so as to remove this singular fade coefficient. In the case of two-way ACF relaying, at the end of MA phase, relay receives two complex numbers, given by (1). Instead of R transmitting a point from the 4^4 point constellation resulting from all the possibilities of $\left(\left(x_{A_1},x_{B_1}\right),\left(x_{A_2},x_{B_2}\right)\right)$ the relay R can choose to group these possibilities into clusters represented by a smaller constellation. One such clustering for the case when $\gamma e^{j\theta}$ can be obtained by utilizing the clustering provided in [8] for the two-way 2-stage problem in order to remove this fade state.

Let $\mathcal{C}^{[h]}$ denote the clustering for the physical network coded two-way relaying scenario that removes the singular fade state $h \in \mathbb{C}$ for the two-way 2-stage relaying case as given in [8].

Definition 7: We define the *Cartesian Product* of a clustering $\mathcal{C}^{[h]} = \{l_1, l_2, ..., l_m\}$ with itself denoted by $\mathcal{D}^{[h]}$, where for $i = 1, 2, ..., m$;

$$l_i = \left\{(x_{i_1}, y_{i_1}), (x_{i_2}, y_{i_2}), ..., (x_{i_{s_i}}, y_{i_{s_i}})\right\}$$

with $x_{i_p}, y_{i_p} \in \mathbb{Z}_4 \ \forall \ p = 1, 2, ..., s_i$ as follows:

$$\mathcal{D}^{[h]} = \left\{\mathcal{C}^{\{l_1,l_1\}}, ..., \mathcal{C}^{\{l_1,l_m\}}, ..., \mathcal{C}^{\{l_m,l_1\}}, ..., \mathcal{C}^{\{l_m,l_m\}}\right\}, \text{ where}$$

$$\mathcal{C}^{\{l_i,l_j\}} = \left\{((x_{i_p}, y_{i_p}), (x_{j_q}, y_{j_q})) \mid p = 1, 2, .., s_i \text{ and } q = 1, 2, .., s_j\right\}.$$

Lemma 2: Let $\gamma e^{j\theta} \in \mathcal{H}$. The clustering obtained by taking the Cartesian Product $\mathcal{D}^{[\gamma e^{j\theta}]}$ of $\mathcal{C}^{[\gamma e^{j\theta}]}$ with itself removes the singular fade state $\gamma e^{j\theta}$ for the two-way ACF relaying scenario.

There are three classes of singular fade states depending on the radius of the circle it lies on *(Case 1:)* $\gamma e^{j\theta}$ lies on the unit circle, *(Case 2:)* $\gamma e^{j\theta}$ lies on the circle of radius $1/\sqrt{2}$ and *(Case 3:)* $\gamma e^{j\theta}$ lies on the circle of radius $\sqrt{2}$. The number of clusters in the clustering utilized by relay node R during BC phase obtained using Cartesian Product in the three cases is 16, 25 and 25 respectively.

Lemma 3: Consider the two-way ACF relaying with 2^λ-PSK signal set used at nodes A and B. The Latin Square L'' of order $2^{2\lambda}$ which removes the singular fade state (γ, θ'), can be obtained from the Latin Square L of order $2^{2\lambda}$ which removes the singular fade state (γ, θ), where $\theta' - \theta = k\frac{2\pi}{2^\lambda}$, as follows: Cyclic shift the columns of each one the $2^{2\lambda}$ Latin Squares $L_{i,j}, 0 \leq i, j \leq 2^\lambda - 1$, k times to the left to get the Latin Square L'. Cyclic shift the columns of the Square L'_B associated with L', k times to the left, to get the Square L''_B associated with the Latin Square L''.
For examples illustrating the usefulness of Lemma 3, see [11].

IV. Direct Clustering

It is observed that, if instead of taking the Cartesian Product of the clusterings given in [8], the Cartesian Product of the *Singularity Removal Constraints* corresponding to each fade state are used to fill a 16×16 array, and the resulting incomplete array so obtained is completed, so as to form a Latin Square of side 16, then the number of clusters of the resulting clustering corresponding the to this Latin Square can be reduced from 25 to a lesser number in both *Case 2* and *Case 3*. We call this the Direct Clustering. In *Case 1*, the minimum number of clusters required, i.e., 16, can be achieved using Cartesian Product Clustering as shown in Section III.

Lemma 4: When $\gamma e^{j\theta}$ lies on the circle of radius $1/\sqrt{2}$, there are a total of 80 singularity removal constraints.

The 16×16 Latin Square representing these constraints can be completed using 20 symbols, as we show in the following example.

Example 1: Consider the case for which $\gamma e^{j\theta} = -0.5 + 0.5j$. The singularity removal constraints for the case $\gamma e^{j\theta} = -0.5 + 0.5j$ in two-way 2-stage relaying as given in [8] are:
$\{(0,0),(1,3)\}, \{(1,1),(3,2)\}, \{(0,1),(2,2)\}$ and $\{(2,0),(3,3)\}$.

As a result, the singularity removal constraints for the two-way ACF relaying are the cells filled with bold lettered entries in Fig. 2. The 16×16 Latin Square representing these constraints can be completed using 20 symbols, as can be seen in Fig.2.

The Latin Squares which remove the singular fade states on the circle of radius $\sqrt{2}$, can be obtained from the Latin Squares which remove the singular fade states on the circle with radius $\frac{1}{\sqrt{2}}$ by taking transpose.

V. Quantization of the Fade State Plane

In practice, $\gamma e^{j\theta}$ can take any value in the complex plane (it takes a value equal to one of the singular fade states with zero probability). As explained in Section II, one of the Latin Squares obtained, which remove the singular fade states needs to be chosen, depending on the value of $\gamma e^{j\theta}$. For a $\gamma e^{j\theta}$ which is not a singular fade state, among all the Latin Squares which remove the singular fade states, the Latin Square $\mathcal{C}^h, h \in \mathcal{H}$ which has the maximum value of the minimum cluster distance at $\gamma e^{j\theta}$ is chosen. In other words, for a given $\gamma e^{j\theta} \notin \mathcal{H}$, the clustering is chosen to be the one which removes the singular fade state $h \in \mathcal{H}$ which maximizes the metric $d^2_{min}\left(\mathcal{C}^h, \gamma e^{j\theta}\right)$ given in (8). In this way, the $\gamma e^{j\theta}-$plane is quantized into $|\mathcal{H}|$ point set, depending on which one of the obtained Latin Squares is chosen.

For $(x_A, x_B) \neq (x'_A, x'_B) \in \mathcal{S}^2$, let $\mathcal{D}(\gamma, \theta, x_A, x_B, x'_A, x'_B)$ be defined as,

$$\mathcal{D}(\gamma, \theta, x_A, x_B, x'_A, x'_B) = |(x_A - x'_A) + \gamma e^{j\theta}(x_B - x'_B)|. \quad (10)$$

In the following lemma, it is shown that for a given $\gamma e^{j\theta}$, choosing the clustering $\mathcal{C}^h \in \mathcal{C}^{\mathcal{H}}$, where $h \in \mathcal{H}$, that maximizes $d^2_{min}\left(\mathcal{C}^h, \gamma e^{j\theta}\right)$ given in (8), is the same as choosing the clustering $\mathcal{C}^{\left[-\frac{\mathsf{x_A} - \mathsf{x'_A}}{\mathsf{x_B} - \mathsf{x'_B}}\right]}$, where $(\mathsf{x_A}, \mathsf{x_B}) \neq (\mathsf{x'_A}, \mathsf{x'_B}) \in \mathcal{S}^2$, that minimizes the simpler metric given in (10).

Lemma 5: If the complex fade state $\gamma e^{j\theta}$ and the clustering $\mathcal{C}^{\left[-\frac{\mathsf{x_A} - \mathsf{x'_A}}{\mathsf{x_B} - \mathsf{x'_B}}\right]} \in \mathcal{C}^{\mathcal{H}}$ are such that,

$$\arg \min_{(x_A,x_B) \neq (x'_A,x'_B) \in \mathcal{S}^2} \mathcal{D}(\gamma, \theta, x_A, x_B, x'_A, x'_B) = (\mathsf{x_A}, \mathsf{x_B}, \mathsf{x'_A}, \mathsf{x'_B}),$$

then $\left[-\frac{\mathsf{x_A} - \mathsf{x'_A}}{\mathsf{x_B} - \mathsf{x'_B}}\right] \in \mathcal{H}$, maximizes the metric $d^2_{min}\left(\mathcal{C}^h, \gamma e^{j\theta}\right)$ given in (8), among all $h \in \mathcal{H}$.

The decision criterion in Lemma 5 based on which R chooses one of the Latin Squares obtained, is the same as the decision criterion for the two-way 2-stage relaying in [7]. Hence, the quantization of the complex fade state plane for the ACF relaying is same as that of the two-way 2-stage relaying obtained in [7].

VI. Simulation Results

The simulation results presented are for the case when H_A, H_B, H'_A and H'_B are distributed according to Rayleigh

	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(1, 0)	(1, 1)	(1, 2)	(1, 3)	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(3, 0)	(3, 1)	(3, 2)	(3, 3)
(0, 0)	$\mathcal{L}_{1}$	$\mathcal{L}_{3}$	$\mathcal{L}_{6}$	$\mathcal{L}_{9}$	$\mathcal{L}_{4}$	$\mathcal{L}_{2}$	$\mathcal{L}_{5}$	$\mathcal{L}_{7}$	$\mathcal{L}_{10}$	$\mathcal{L}_{12}$	$\boldsymbol{\mathcal{L}_{8}}$	$\boldsymbol{\mathcal{L}_{14}}$	$\mathcal{L}_{11}$	$\mathcal{L}_{13}$	$\boldsymbol{\mathcal{L}_{17}}$	$\boldsymbol{\mathcal{L}_{18}}$
(0, 1)	$\mathcal{L}_{7}$	$\mathcal{L}_{2}$	$\mathcal{L}_{8}$	$\mathcal{L}_{1}$	$\mathcal{L}_{9}$	$\mathcal{L}_{6}$	$\mathcal{L}_{13}$	$\mathcal{L}_{4}$	$\boldsymbol{\mathcal{L}_{3}}$	$\mathcal{L}_{16}$	$\boldsymbol{\mathcal{L}_{15}}$	$\mathcal{L}_{10}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{12}$	$\boldsymbol{\mathcal{L}_{14}}$	$\mathcal{L}_{11}$
(0, 2)	$\mathcal{L}_{5}$	$\mathcal{L}_{4}$	$\mathcal{L}_{3}$	$\mathcal{L}_{10}$	$\mathcal{L}_{1}$	$\mathcal{L}_{8}$	$\mathcal{L}_{2}$	$\mathcal{L}_{11}$	$\mathcal{L}_{17}$	$\boldsymbol{\mathcal{L}_{9}}$	$\mathcal{L}_{12}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{15}$	$\boldsymbol{\mathcal{L}_{6}}$	$\mathcal{L}_{13}$	$\boldsymbol{\mathcal{L}_{7}}$
(0, 3)	$\mathcal{L}_{9}$	$\mathcal{L}_{8}$	$\mathcal{L}_{2}$	$\mathcal{L}_{5}$	$\mathcal{L}_{11}$	$\mathcal{L}_{10}$	$\mathcal{L}_{6}$	$\mathcal{L}_{1}$	$\boldsymbol{\mathcal{L}_{7}}$	$\boldsymbol{\mathcal{L}_{4}}$	$\mathcal{L}_{16}$	$\mathcal{L}_{17}$	$\boldsymbol{\mathcal{L}_{3}}$	$\boldsymbol{\mathcal{L}_{14}}$	$\mathcal{L}_{12}$	$\mathcal{L}_{15}$
(1, 0)	$\mathcal{L}_{11}$	$\mathcal{L}_{13}$	$\boldsymbol{\mathcal{L}_{17}}$	$\boldsymbol{\mathcal{L}_{16}}$	$\mathcal{L}_{5}$	$\mathcal{L}_{7}$	$\mathcal{L}_{4}$	$\mathcal{L}_{15}$	$\mathcal{L}_{12}$	$\mathcal{L}_{14}$	$\boldsymbol{\mathcal{L}_{10}}$	$\boldsymbol{\mathcal{L}_{8}}$	$\mathcal{L}_{1}$	$\mathcal{L}_{3}$	$\mathcal{L}_{6}$	$\mathcal{L}_{9}$
(1, 1)	$\boldsymbol{\mathcal{L}_{6}}$	$\mathcal{L}_{10}$	$\boldsymbol{\mathcal{L}_{4}}$	$\mathcal{L}_{11}$	$\mathcal{L}_{13}$	$\mathcal{L}_{3}$	$\mathcal{L}_{9}$	$\mathcal{L}_{5}$	$\boldsymbol{\mathcal{L}_{15}}$	$\mathcal{L}_{17}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{12}$	$\mathcal{L}_{7}$	$\mathcal{L}_{2}$	$\mathcal{L}_{8}$	$\mathcal{L}_{1}$
(1, 2)	$\mathcal{L}_{12}$	$\boldsymbol{\mathcal{L}_{17}}$	$\mathcal{L}_{13}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{6}$	$\mathcal{L}_{15}$	$\mathcal{L}_{7}$	$\mathcal{L}_{2}$	$\mathcal{L}_{16}$	$\boldsymbol{\mathcal{L}_{11}}$	$\mathcal{L}_{14}$	$\boldsymbol{\mathcal{L}_{9}}$	$\mathcal{L}_{5}$	$\mathcal{L}_{4}$	$\mathcal{L}_{3}$	$\mathcal{L}_{10}$
(1, 3)	$\boldsymbol{\mathcal{L}_{15}}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{10}$	$\mathcal{L}_{12}$	$\mathcal{L}_{14}$	$\mathcal{L}_{1}$	$\mathcal{L}_{3}$	$\mathcal{L}_{6}$	$\boldsymbol{\mathcal{L}_{19}}$	$\boldsymbol{\mathcal{L}_{13}}$	$\mathcal{L}_{17}$	$\mathcal{L}_{16}$	$\mathcal{L}_{9}$	$\mathcal{L}_{8}$	$\mathcal{L}_{2}$	$\mathcal{L}_{5}$
(2, 0)	$\mathcal{L}_{8}$	$\mathcal{L}_{9}$	$\mathcal{L}_{16}$	$\mathcal{L}_{6}$	$\mathcal{L}_{3}$	$\mathcal{L}_{14}$	$\boldsymbol{\mathcal{L}_{1}}$	$\boldsymbol{\mathcal{L}_{13}}$	$\mathcal{L}_{4}$	$\mathcal{L}_{2}$	$\mathcal{L}_{5}$	$\mathcal{L}_{7}$	$\mathcal{L}_{12}$	$\mathcal{L}_{10}$	$\boldsymbol{\mathcal{L}_{11}}$	$\boldsymbol{\mathcal{L}_{19}}$
(2, 1)	$\mathcal{L}_{14}$	$\mathcal{L}_{7}$	$\mathcal{L}_{15}$	$\mathcal{L}_{8}$	$\boldsymbol{\mathcal{L}_{2}}$	$\mathcal{L}_{16}$	$\boldsymbol{\mathcal{L}_{10}}$	$\mathcal{L}_{3}$	$\mathcal{L}_{9}$	$\mathcal{L}_{6}$	$\mathcal{L}_{13}$	$\mathcal{L}_{4}$	$\boldsymbol{\mathcal{L}_{19}}$	$\mathcal{L}_{5}$	$\boldsymbol{\mathcal{L}_{1}}$	$\mathcal{L}_{12}$
(2, 2)	$\mathcal{L}_{4}$	$\mathcal{L}_{16}$	$\mathcal{L}_{9}$	$\mathcal{L}_{3}$	$\mathcal{L}_{12}$	$\boldsymbol{\mathcal{L}_{5}}$	$\mathcal{L}_{14}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{1}$	$\mathcal{L}_{8}$	$\mathcal{L}_{2}$	$\mathcal{L}_{11}$	$\mathcal{L}_{17}$	$\boldsymbol{\mathcal{L}_{19}}$	$\mathcal{L}_{10}$	$\boldsymbol{\mathcal{L}_{13}}$
(2, 3)	$\mathcal{L}_{13}$	$\mathcal{L}_{15}$	$\mathcal{L}_{7}$	$\mathcal{L}_{4}$	$\boldsymbol{\mathcal{L}_{18}}$	$\boldsymbol{\mathcal{L}_{9}}$	$\mathcal{L}_{16}$	$\mathcal{L}_{12}$	$\mathcal{L}_{11}$	$\mathcal{L}_{10}$	$\mathcal{L}_{6}$	$\mathcal{L}_{1}$	$\boldsymbol{\mathcal{L}_{2}}$	$\boldsymbol{\mathcal{L}_{20}}$	$\mathcal{L}_{5}$	$\mathcal{L}_{17}$
(3, 0)	$\mathcal{L}_{2}$	$\mathcal{L}_{1}$	$\boldsymbol{\mathcal{L}_{12}}$	$\boldsymbol{\mathcal{L}_{13}}$	$\mathcal{L}_{10}$	$\mathcal{L}_{11}$	$\boldsymbol{\mathcal{L}_{17}}$	$\boldsymbol{\mathcal{L}_{14}}$	$\mathcal{L}_{5}$	$\mathcal{L}_{7}$	$\mathcal{L}_{4}$	$\mathcal{L}_{15}$	$\mathcal{L}_{8}$	$\mathcal{L}_{9}$	$\mathcal{L}_{16}$	$\mathcal{L}_{6}$
(3, 1)	$\boldsymbol{\mathcal{L}_{16}}$	$\mathcal{L}_{11}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{2}$	$\boldsymbol{\mathcal{L}_{17}}$	$\mathcal{L}_{12}$	$\boldsymbol{\mathcal{L}_{19}}$	$\mathcal{L}_{10}$	$\mathcal{L}_{13}$	$\mathcal{L}_{3}$	$\mathcal{L}_{9}$	$\mathcal{L}_{5}$	$\mathcal{L}_{14}$	$\mathcal{L}_{7}$	$\mathcal{L}_{15}$	$\mathcal{L}_{8}$
(3, 2)	$\mathcal{L}_{17}$	$\boldsymbol{\mathcal{L}_{5}}$	$\mathcal{L}_{1}$	$\boldsymbol{\mathcal{L}_{14}}$	$\mathcal{L}_{8}$	$\boldsymbol{\mathcal{L}_{13}}$	$\mathcal{L}_{11}$	$\boldsymbol{\mathcal{L}_{19}}$	$\mathcal{L}_{6}$	$\mathcal{L}_{5}$	$\mathcal{L}_{7}$	$\mathcal{L}_{2}$	$\mathcal{L}_{4}$	$\mathcal{L}_{16}$	$\mathcal{L}_{9}$	$\mathcal{L}_{3}$
(3, 3)	$\boldsymbol{\mathcal{L}_{10}}$	$\boldsymbol{\mathcal{L}_{19}}$	$\mathcal{L}_{11}$	$\mathcal{L}_{17}$	$\boldsymbol{\mathcal{L}_{16}}$	$\boldsymbol{\mathcal{L}_{18}}$	$\mathcal{L}_{12}$	$\mathcal{L}_{8}$	$\mathcal{L}_{14}$	$\mathcal{L}_{1}$	$\mathcal{L}_{3}$	$\mathcal{L}_{6}$	$\mathcal{L}_{13}$	$\mathcal{L}_{15}$	$\mathcal{L}_{7}$	$\mathcal{L}_{4}$

Fig. 2. Latin Square representing the clustering at the relay for the case $\gamma e^{j\theta}$ obtained using Direct Clustering, with the 4-PSK symbols that A(B) sent in the first and second channel use along the rows(columns)

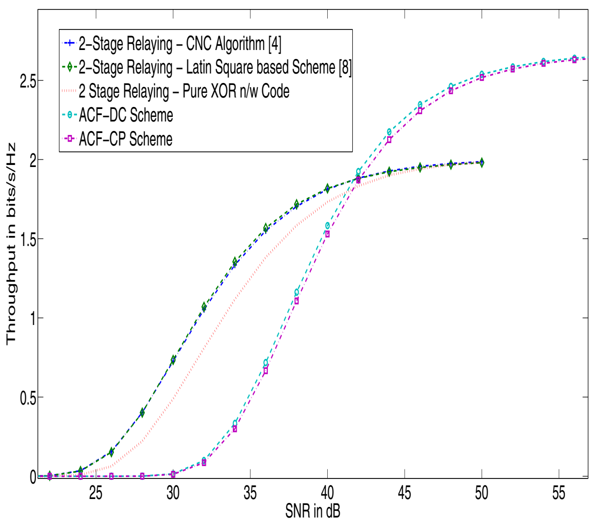

Fig. 3. SNR vs throughput for different schemes for 4-PSK signal set

distribution, with the variances of all the fading links equal to 0 dB. It is assumed that the AWGN noises at the three nodes are of variance 0 dB. By SNR, we mean the average energies of the signal set used at the three nodes A, B and R, which are assumed to be equal. The frame length of a transmission is taken to be 256 bits.

Consider the case when 4-PSK signal set is used at A and B. Fig. 3 shows the SNR vs end-to-end sum throughput curves for the following schemes: Closest-Neighbour Clustering (CNC) Algorithm based scheme for the two-way 2-stage relaying [4], the Scheme based on Latin Squares for two-way 2-stage relaying [8], the scheme in which XOR network code is used irrespective of the channel condition, the Cartesian Product based scheme for ACF (ACF-CP) relaying and the Direct Clustering based scheme for ACF (ACF-DC) relaying. From Fig. 3, it follows that at high SNR, the ACF-DC scheme outperforms all other schemes. The maximum throughput achieved by the ACF relaying schemes is 8/3 bits/s/Hz, whereas it is 2 bits/s/Hz for the 2-stage two-way relaying schemes. Also, as seen from Fig. 3, the ACF-DC scheme performs better than the ACF-CP scheme. The reason for this is that the maximum cardinality of the signal set used during the BC phase is 25 for the ACF-CP scheme whereas it is 20 for the ACF-DC scheme. Simulation results showing the SNR vs throughput for 8-PSK signal set are omitted for lack of space, but are available in [11].

VII. Conclusion

Based on the ACF protocol, two methods of obtaining the clusterings satisfying the exclusive law were proposed: Cartesian Product method and Direct Clustering. At high SNR, the schemes based on the ACF protocol were shown to outperform the schemes proposed in [4] and [8].

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