# BER Analysis with an Appropriate Friis Formula for Multi-hop ALOHA Dense Ad Hoc Networks

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*Abstract*—**This paper investigates multi-hop wireless ALOHA ad hoc networks evaluating the bit error rate (BER). It is proposed an alteration of the Friis propagation model which allows its application to any node density with random network topology and obeys the law of conservation of energy. The results show that the adapted model appropriately describes the BER performance for dense networks. It was also verified that the increase of transmission power in dense networks without augment in transmission band does not improve the BER performance and it has importance to sensor networks in which energy consumption is crucial.**

*Index Terms*—**Ad hoc networks, ALOHA, BER.**

## I. Introduction

Wireless ad hoc networks are characterized by absence of infrastructure. This feature makes them versatile and capable of establishing communication among many nodes, either employing a single hop or in a multi-hop fashion in order to send information from source to destination without a central control entity [1]. Routing, signaling and all other functionalities that the network must provide are therefore distributed among the nodes such that the whole system does not depend crucially on a single node. In such way, the network can adapt to changes in case of link breaks due to mobility, node failure, etc. Accordingly, ad hoc networks can be utilized in many situations like battle fields, rescue operations, environmental monitoring, or any other application that prescinds from an infrastructured communication system.

A related and very important issue is to know the behavior of BER in ad hoc networks when they become dense. In such case, the communicating nodes, *i.e.*, transmitter and receiver can be located very close to each other. Consequently, the channel propagation model should take care to describe more accurately the realistic case. Models that consider received power as a function of the distance between the nodes must account for the situation of dense networks in order to prevent the received power not to be greater than the transmitted power due to the law of conservation of energy.

On the other hand, the BER is related to quality measurement of data transmission in a wireless link from transmitter to receiver, *i.e.*, a hop [2]. In ad hoc networks, due to its multi-hop fashion, the BER accumulates along the path from source to destination and it impacts on other network metrics [3], [4], [5], [6]. Therefore, it is important to conveniently compute this measure.

This paper investigates the end-to-end BER performance in ad hoc networks in which the nodes are uniformly random distributed in the network area and use the ALOHA protocol [7] as the medium access control (MAC) protocol. The analysis employs a channel propagation model that can be applied to any node density and obeys the law of conservation of energy. The signal-to-noise and interference ratio (SNIR) is calculated along with the BER in order obtain the network performance as a function of important network parameters like node density, transmission power and transmission rate. The ALOHA protocol was chosen due to its simplicity and because it is totally decentralized. Future work can extend the present analysis to consider other MAC protocols.

Results are computed and compared with related works [3], [4], and it was found difference of BER values on the order of magnitude of five times depending on the used parameters showing that the proposed propagation model is more appropriate to describe the channel communication for dense ad hoc networks. Another important finding was to observe that the increase in power transmission does not improve the BER in dense networks and it is a very important information for sensor networks in which power consumption is a critical issue.

The remainder of this paper is organized as follows. Section II describes the network model and the related work. Section III delineates the propagation model alteration and the development of the new formulas for SNIR and BER. The results are presented and discussed in Section IV and Section V concludes the work.

## II. Network Model and Related Work

The network model assumed here is the one introduced by Tonguz *et al.* [3], [4]. The network consists of a square area with side length $2R$ in which static nodes are randomly placed according to a uniform distribution.[1] Two channel communication models are considered in the present work. The first is the Friis model [8], [3], [4], [5], which is widely utilized to model cellular networks [2]. The second model suggested in this paper is called adapted model which we show to be more suitable to any node density as it will be described later and, therefore, can be used to better describe the performance of ad hoc networks.

In [3], an approach to simplify the BER computation was proposed. Accordingly, the nodes were assumed to be distributed on the surface of a torus by connecting the top edge with the bottom edge and the right edge with the left edge of the plane squared area, resulting in spatial invariance which

[1] Although the analysis is performed here for static nodes, the study can be carried out for mobile scenarios as well.

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prevents edge effects. This assumption will be employed here since it permits, without loss of generality, to handle any node in the network in the same way.

### A. Friis Model

The Friis space equation relates the received power in a node as function of the transmitted power of the transmitter and other important parameters. Accordingly, from [8] and [2]

$$P_{r_L} = \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2 r_L^{\alpha}}, \tag{1}$$

in which $P_t$ is the transmitted power, $G_t$ and $G_r$ are the transmission and reception antenna gains, respectively. $f_c$ is the carrier frequency, $c$ is the speed of light, $\alpha$ is the path loss exponent and $r_L$ is the link distance between transmitter and receiver nodes.

### B. BER Analysis

In order to calculate the BER in a link, it is first necessary to obtain the SNIR for a receiver node. The noise source is consequence of thermal noise power $P_{thermal} = FkT_oB$ where $F$ is the noise figure, $B$ is the transmission band in Hertz (Hz), $k$ is the Boltzmann constant and $T_o = 300$ Kelvins is the ambient temperature. The interference in the receiver node has power $P_{INT}^{total}$ and accounts for other transmitting nodes in the network, which is computed according to the considered MAC protocol. Therefore, the signal-to-noise and interference ratio in a link is given by $SNIR_L = \frac{P_{r_L}}{P_{thermal}+P_{INT}^{total}}$. Assuming that each node utilizes binary phase shift keying (BPSK) modulation, the link BER can be computed by [2]

$$BER_L = Q\left(\sqrt{2SNIR_L}\right) = Q\left(\sqrt{\frac{2P_tG_tG_r(c/4\pi f_c)^2}{W^{\alpha}\left(P_{thermal}+P_{INT}^{total}\right)}}\right), \tag{2}$$

in which $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$ and $W$ is a random variable representing the link distance, *i.e.*, the hop length.

Because it is assumed spatial invariance, each node measures the same average interference $E[P_{INT}^{total}]$. Furthermore, it is adopted that the links on a route are independent and that the errors occurring in each hop accumulate until the destination. In such case, the end-to-end bit error rate on a route is obtained by

$$BER_R = 1 - \prod_{j=1}^{\overline{n}_h} \left(1 - BER_{L_j}\right), \tag{3}$$

in which $BER_{L_j}$ is the error rate for the $j^{th}$ hop and $\overline{n}_h$ is the average number of hops on the route.

In order to obtain the $BER_R$, it is necessary to determine the probability distribution of the distance between hops (*i.e.*, the distribution of $W$), the average number of hops ($\overline{n}_h$) and the average total interference power ($E[P_{INT}^{total}]$).

The distance between two nodes on a hop is strongly related to the routing strategy adopted. The routing scheme can be designed aiming to attain the smallest number of hops, low energy consumption or any other advantageous strategy. Here, it was employed that the route is chosen such that each hop along the path on the direction to the destination has the smallest possible length so as to reduce the end-to-end BER. Fig. 1 illustrates this situation. A line is created from source to destination which functions as a reference to the path, such

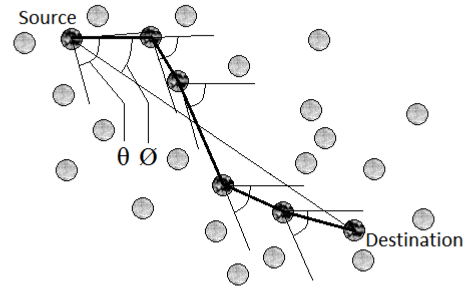


Fig. 1. Routing strategy adopted. $W$ is the hop length (distance) between two communicating nodes along the path route, $\theta$ is the maximum deviation angle and $\phi$ is the deviation angle from the referential line connecting transmitter node to the destination.

that the transmission of information to the next hop does not deviate more than a predefined angle $\theta$. Accordingly, the greater is the angle, more the followed route will deviate from the referential line and lower will be the hop length of each hop. However, if $\theta$ is lower, then the route will be closer to the referential line, but the hop length tend to be longer. Considering such routing approach, it can be shown that the cumulative probability density function (CDF) for the random variable $W$ is given by [3]

$$F_W(w) = 1 - e^{-\rho_s \frac{\theta}{2} w^2}, \tag{4}$$

in which $\rho_s = N/A$ is the network spatial node density in units of node/m$^2$, where $N$ is the total number of nodes and $A$ is the network area size.

On the other hand, the average number of hops on a route must be inversely proportional to the average size of a hop on this route. The larger is the hop size, the smaller is total amount of hops and vice-versa. When the routing strategy is the one previously described, it is possible to project each hop over the referential line and to approximate the average number of hops as [3]

$$\overline{n}_h \approx \frac{R}{3}\left[\sqrt{2} + \ln\left(1+\sqrt{2}\right)\right]\left(\sqrt{\frac{\pi}{2\rho_s\theta}}\frac{2}{\theta}\sin\frac{\theta}{2}\right)^{-1}. \tag{5}$$

Because the radio communication medium is shared among all nodes in the network, there exists the possibility that two or more nodes try to transmit simultaneously, causing interference. The average interference power $E\left[P_{INT}^{(i)}\right]$ caused by a node concurrently transmitting with another node can be calculated by the Friis formula (see Eq. (1)) considering the free space case, *i.e.*, $\alpha = 2$. However, due to the random topology, the distance ($r_L$) among nodes is random and it can be represented by the random variable $Z$. Accordingly,

$$E\left[P_{INT}^{(i)}\right] = \frac{G_tG_rP_tc^2}{(4\pi)^2 f_c^2} E\left[\frac{1}{Z^2}\right]. \tag{6}$$

It is important now to obtain the probability density function describing the random variable $Z$. Tonguz *et al.* [3] showed that

$$f_Z(z) = \begin{cases} \frac{\pi z}{2R^2}, & 0 \le z < R \\ \frac{\pi z}{2R^2} - \frac{2z\arccos(R/z)}{R^2}, & R \le z < \sqrt{2}R. \end{cases} \tag{7}$$

Because the Friis formula is restricted to the far field region [8], [2], the authors in [3] and [4] assume that, when a node is transmitting from a distance smaller than a close-in reference distance ($d_0$), the received power is equal to the transmitted power, multiplied by the gain factor $\frac{G_t G_r c^2}{(4\pi)^2 f_c^2}$. Accordingly, it can be shown that the average interference power caused by a sole concurrent node is given by [3]

$$E\left[P_{INT}^{(i)}\right] = \frac{G_t G_r P_t c^2}{(4\pi)^2 f_c^2}\left[\frac{\pi d_0^2}{4R^2} + \frac{\pi \ln(R/d_0)}{2R^2} + \frac{\pi \ln(\sqrt{2})}{2R^2} + \frac{0.37157}{R^2}\right]. \tag{8}$$

The probability of a node to be concurrently transmitting with another node is related to the MAC protocol employed. In the case of ALOHA without retransmission, the average total interference experimented by a node in a network is given by [3], [4]

$$E\left[P_{INT}^{total}\right] = \left(1 - e^{\frac{-\lambda L}{R_b}}\right)\sum_{i=1}^{N-2} E\left[P_{INT}^{(i)}\right], \tag{9}$$

in which $\lambda$ is the average packet transmission rate of a node, $L$ is the packet length in bits, $R_b$ is the data transmission rate in bits/s and $P_{INT}^{(i)}$ is the interference power generated by a node $i$. The factor $1 - e^{\frac{-\lambda L}{R_b}}$ corresponds to the probability of a given node to cause interference in another transmission, independently of its location [3].

The final result can be obtained by taking several realizations of the random variable $W$, calculating the bit error rate along the entire route ($BER_R$) and computing the average from the many measured $BER_R$ values.

However, in ad hoc and sensor networks in which high node density is possible, *i.e.*, for the cases that transmitter and receiver can be very close to each other, the modeling described above needs to be changed in order to prevent that the received power results improperly evaluated. The next section extends the analysis done for random topology, altering the Friis model so as to consider dense ad hoc networks.

## III. Adapted Model and Development

The proposal here presented aims to obtain a description that better characterizes ad hoc and sensor networks. Accordingly, the modified model developed in this section is more generic in the sense that it can be applied with no restriction in either dense or sparse networks without the need to refer to a close-in reference distance $d_0$.

As previously observed, the Friis formula is suitable for the far field region. Thus, it is very common to assume a close-in reference distance $d_0$ in order to help the computation of propagation losses [2]. Although this approach fits well to describe other networks, like the cellular case, for ad hoc networks additional care is necessary because $d_0$ must be chosen such that two communicating nodes will not be closer to each other than $d_0$. The model presented in [3] and [4] can be more or less realistic depending on the decrease or increase of node density, respectively, since the density changes the average distance among nodes; however, such modeling is not valid for all densities.

The limitation of the model presented in Section II-B results that every time a node transmits, the neighbors located at a distance smaller than $d_0$ receives exactly the total power transmitted multiplied by the gains, independently if they are closer or not from the transmitter. In other words, such nearby neighbors behave like positioned at exactly the same location as the transmitter with their antennas united. Therefore, a node never can be located at a distance greater than zero and less than $d_0$ from the transmitter. To by pass this limitation, we propose the following modification.

The adaptation used here for the Friis formula consists of exchanging the term related to the distance between transmitter and receiver from $r_L$ to $r_L + 1$. Similar change in power decaying law has been already used in studies of network scalability as in [9], [10], [11] and [12]. The adapted formula results

$$P_{r_L+1} = \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2 (r_L+1)^\alpha}. \tag{10}$$

In order to verify the validity of the above model, we can make the following comparison. First, by considering that when transmitter and receiver are very far away from each other, in the limit as $r_L \to \infty$, it results that

$$\lim_{r_L\to\infty} P_{r_L+1} = \lim_{r_L\to\infty} \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2 (r_L+1)^\alpha} = 0 \tag{11}$$

and

$$\lim_{r_L\to\infty} P_{r_L} = \lim_{r_L\to\infty} \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2 r_L^\alpha} = 0. \tag{12}$$

Second, when both nodes are very close to one another, *i.e.*, when $r_L \to 0$ we have that

$$\lim_{r_L\to 0} P_{r_L+1} = \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2}, \tag{13}$$

which is equal to the previous model proposed in [3] and [4]. That is, if the distance between the communicating nodes is sufficiently large ($r_L \to \infty$) or small ($r_L \to 0$), the two models are equivalent.

After all, according to the law of conservation of energy, one cannot receive more than it was transmitted. Our modification allows to use the adapted Friis formula to any network density, including those cases in which $r_L$ tends to zero.

Employing the modified Friis model for the free space case ($\alpha = 2$), the interference power of a transmitting node is given by

$$E^*\left[P_{INT}^{(i)}\right] = \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2} E\left[\frac{1}{(z+1)^2}\right], \tag{14}$$

in which $Z$ continues to represent the random variable for the distance between the interferer and the receiver node.

From Eqs. (7) and (14), the average interference power is obtained by

$$\begin{aligned} E^*\left[P_{INT}^{(i)}\right] &= \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2}\left\{\int_0^R \frac{\pi z}{2R^2(z+1)^2}dz + \int_R^{R\sqrt{2}}\left[\frac{\pi z}{2R(z+1)^2} - \frac{2z\cos^{-1}\left(\frac{R}{z}\right)}{R^2(1+z)^2}\right]dz\right\} \\ &= \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2}\left\{\int_0^{R\sqrt{2}} \frac{\pi z}{2R^2(z+1)^2}dz - \int_R^{R\sqrt{2}} \frac{2z \arccos(R/z)}{R^2(1+z)^2}dz\right\}. \end{aligned} \tag{15}$$

The first integral in Eq. (15) can be solved using the following identity [13]

$$\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C. \tag{16}$$

Nevertheless, there is no closed formula for the second integral in Eq. (15). Accordingly, the average interference power generated by a transmitting node is

$$E^*\left[P_{INT}^{(i)}\right] = \frac{G_t G_r c^2 P_t}{(4\pi)^2 f_c^2}\left\{\frac{\pi}{2R^2}\left[\frac{1}{1+R\sqrt{2}} + \ln\left(1+R\sqrt{2}\right)\right.\right.$$
$$\left.\left. -1\right] - \frac{2}{R^2}\int_R^{R\sqrt{2}} \frac{z}{(1+z)^2}\arccos(R/z)dz\right\}. \quad (17)$$

By employing Eq. (9) and the interference caused by each transmitter according to Eq. (17), the average total received power by a receiver node for the ALOHA protocol without retransmission is given by

$$E^*\left[P_{INT}^{total}\right] = \left(1 - e^{\frac{-\lambda L}{R_b}}\right)\sum_{i=1}^{N-2} E^*\left[P_{INT}^{(i)}\right]. \quad (18)$$

This result is now utilized to obtain the BER for a link $j$ along the route, but now using $W+1$ instead of $W$, *i.e.,*

$$BER_{L_j}^* = Q\left(\sqrt{\frac{2P_t G_t G_r (c/4\pi f_c)^2}{(W_j+1)^2\left(P_{thermal} + E^*\left[P_{INT}^{total}\right]\right)}}\right). \quad (19)$$

Finally, the BER for the entire route is obtained by substituting Eq. (19) in Eq. (3).

## IV. Results and Discussion

Each point on every curve presented in this section was established from 10000 realizations of the random variable $W$, according to the cumulative distribution function given by Eq. (4), computing the bit error rate for the entire route, and finally calculating the average over $BER_R$. For all presented figures, the results were obtained as a function of the network node density in which the total number of node was $N = 1000$ and we varied the network area in order to vary the density. The transmitting power was set to $P_t = 1\mu$W, the data transmission rate was $R_b = 1$Mbps, the transmission bandwidth was $B = 1$MHz, the transmission and reception gains were equal to one ($G_r = G_t = 1$), the close-in reference distance was $d_0 = 1$m, the carrier frequency was $f_c = 2.4$GHz, the speed of light was $c = 3\times 10^8$ m/s, the packet length was $L = 1000$ bits and the average packet transmission rate was $\lambda = 1$ packet/sec.

In Fig. 2 the results from [3] are reproduced and those from the modified model proposed in Section III are presented and compared for $\theta = \pi$, $\theta = \pi/2$ and $\theta = \pi/10$.

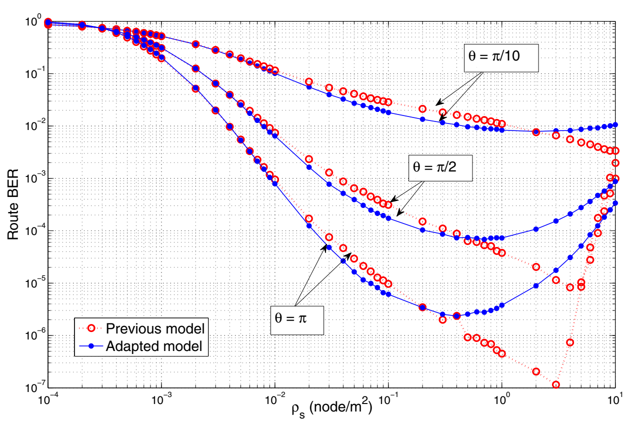


Fig. 2. Route bit error rate as a function of node density. Comparison between the previous and the adapted models for three different values of the angle theta ($\theta = \pi$, $\theta = \pi/2$ and $\theta = \pi/10$), with $d_0 = 1$m for the previous model.

Fig. 2 illustrates that the BER diminishes with the increase of node density until a point in which the total interference is greater than the utile received power. For high node densities, the analysis in [3] works as an upper bound for the route bit error rate since it overestimates the received power on those nodes on the route which have neighbors located at distance smaller than $d_0$. Accordingly, as the average utile received power is overestimated in relation to the interference, the BER decays to small values which does not describe accurately the reality. After a certain density value (around 1 node/m$^2$), the average distance between a pair of nodes become either comparable or in most of cases smaller than $d_0 = 1$m, and after this point both models begin to strongly diverge because the received power in the previous model does not represent precisely the real case, while our model behaves smoothly which is more appropriated. From all curves, our adapted model indicates that the $BER_R$ begins to degrade as a function of node density well before the previous model. Consequently, the previous model behavior gives the wrong impression that the route BER has good performance for very high node densities.

To further investigate the effect of adopting a close-in reference distance, Fig. 3 reproduces results employing the previous model described in Section II-B for $d_0 = 1$m, 10m and 100m for $\theta = \pi/2$ comparing with our adapted model. Note that for the frequency band around 2GHz $d_0$ is typically chosen to be 1m for indoor environment and 100m or 1km for outdoor environments [2]. This figure shows that the previous model does not work when the node density implies on average distances among nodes smaller than $d_0$. The curve for $d_0 = 100$m is valid up to approximately the node density of $2\times 10^{-4}$ node/m$^2$. For $d_0 = 10$m, the BER results are reasonable up to around $10^{-3}$ node/m$^2$. The case in which the previous model better approximates to our model is for $d_0 = 1$m; however, it does not behaves well for node densities above 1 node/m$^2$. Situations in which node densities surpass 1 node/m$^2$ or even higher can be easily found in reality, for example, big events like music shows, football or soccer games, crowded conferences or shopping centers, etc.

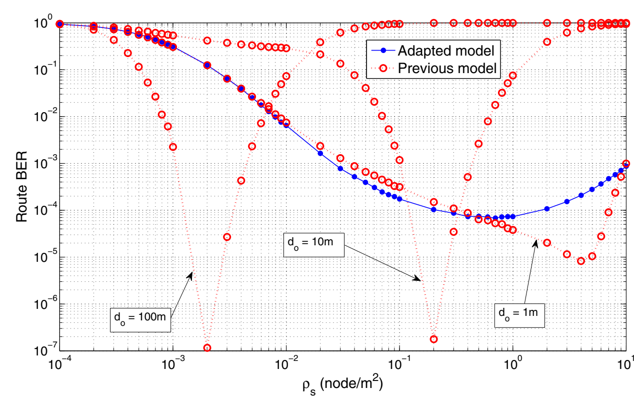


Fig. 3. Route bit error rate as a function of node density. Comparison between adapted model and previous model for $d_0 = 1$m, 10m and 100m with $\theta = \pi/2$.

Fig. 4 compares both models for two values of transmission power, *i.e.*, 1mW e 1$\mu$W. We observe that for densities smaller than $10^{-1}$ node/m$^2$, to transmit with higher power reduces the route BER. Nevertheless, if the node density is higher than $10^{-1}$ node/m$^2$, increasing the transmission power does not improve (indeed it degrades) the BER performance as the curves for each model stay practically overlapped. This behavior is a consequence of the fact that, for high densities, the increase in power causes interference to be dominant. This is a very important information for the case in which power efficiency is crucial, like in sensor networks [1].

Also, Fig. 4 illustrates that the deviation between the models is accentuated for the higher transmission power. That is, for $P_t = 1$mW, from $10^{-4}$ node/m$^2$ up to $10^{-1}$ node/m$^2$ the BER value difference is approximately of five times, since the increase in power worsens the approximation in the previous model due to the fact that the effect of overestimated interference power caused by the nodes located at a distance smaller than $d_0$ from the receiver tends to dominates the SNIR.

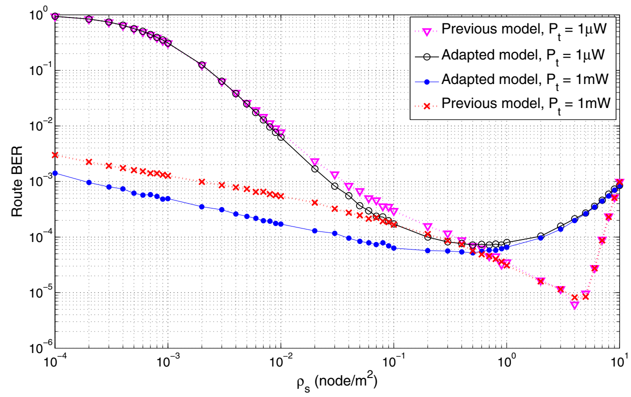


Fig. 4. Route bit error rate as a function of node density. Comparison between adapted model and previous model for two values of transmission power $P_t$ ($1\mu$W and 1mW) with $\theta = \pi/2$ and $d_0 = 1$m for the previous model.

Fig. 5 presents curves for the previous and adapted models as a function of node density for a transmission rate of $R_b = 11$Mbps employing transmission power of $1\mu$W and 1mW. The transmission rate is increased by augmenting the transmission band of the communication channel [3]. The results indicate that increasing the transmission band not always improve the BER performance because more background noise noise is captured by the receiver which reduces the SNIR. However, if both transmission band and power are increased then the improvement in performance is attained.

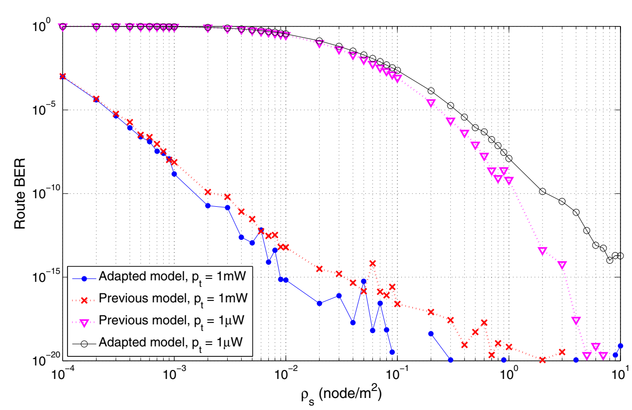


Fig. 5. Route bit error rate for previous and adapted models as a function of node density for $R_b = 11$ Mbps and $\theta = \pi/2$, employing $P_t = 1\mu$W or 1mW and $d_0 = 1$m for the previous model.

Moreover, Fig. 5 illustrates substantial reduction in BER even for high node densities, since with elevated transmission rate the time requested to transmit a packet is shortened which reduces the amount of collisions in the shared communication medium.

Finally, analogous results to those presented in this section were observed for the case that the path loss exponent is equal to four. However, due to space limitation we omit its analysis and figures.

## V. Conclusions

This paper developed a modified model to obtain the bit error rate on a route from source to destination across multiple hops in ALOHA ad hoc networks. The modification aimed to exempt the approximation employing the close-in reference distance $d_0$ adapting the Friis formula in order to prevent that a received power from a close neighbor is not greater than the transmitted power. The modified model allowed that the BER analysis could be extended to dense networks. The results indicates that the adapted model is more appropriate to describe multi-hop ad hoc networks. Our analysis also observed that the increase in transmission power in dense ad hoc networks does not improve BER performance due to augment of interference.

Future work can consider fading to the analysis and other node distributions, as well as other MAC protocols and the BER impact over upper network layers.


## Acknowledgments

This work was supported in part by DPP-UnB, by FAP-DF, by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazil.


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