Interference Evaluation in Ad-Hoc Cognitive Radio Networks

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***Abstract*—In this paper, we evaluate the total ergodic capacity of channels in an ad-hoc cognitive radio network under interference constraint at the primary users. It is assumed that all links associated with primary and secondary networks experience Rayleigh fading. First, interference from only primary transmitters is discussed. In such case, the optimum power allocation scheme and the ergodic capacity are derived under average interference power constraint. This paper is also extended to include the impact of interference from both primary transmitters and other secondary transmitters. Then, we propose the iterative algorithm to maximize the total capacity due to non-convexity of the objective function. The effect of different types of interference is investigated via numerical analysis.**

***Index Terms*—Ad-Hoc Cognitive Radio, Spectrum Sharing, Capacity, Interference**

I. Introduction

In recent years, demands for radio spectrum in wireless communication systems have significantly increased. According to the Federal Communications Commission (FCC), around 70% of the allocated radio spectrum is not utilized and the usage varies temporally and geographically [1]. The inefficiency in spectrum utilization has led to an increasing interest in development of new spectrum usage models.

Cognitive radio can adjust its transmission power and frequency band, based on the environment. In a cognitive radio network, unlicensed users or secondary users allow to operate in the frequency bands originally allocated to primary users, as long as the transmission of the unlicensed system does not harmfully affect the quality of service of the primary users [2]. This idea which is intelligently sharing the resources has significantly improved the utilization of the spectrum. Cognitive radio networks have two main modes of operation. In first mode, known as opportunistic spectrum access, secondary users in cognitive radio network are allowed to access the spectrum of primary network when the spectrum is not utilized by primary users. In second mode, known as spectrum sharing, secondary users simultaneously transmit with primary users on the same spectrum when the maximum interference offered to the primary receivers is below a predefined threshold [3], [4]. The presented work is focussed on the spectrum sharing.

In cognitive radio networks, the main issue is how to guarantee quality-of-service (QoS) in different applications. Capacity, varying as a function of the channel quality, is one of the major QoS requirements and is interference-limited in mobile communication systems.

In [5]-[6], the ergodic capacity of cognitive radio link under different constraints was evaluated, but only secondary transmitter-secondary receiver and secondary transmitter-primary receiver links were assumed. Therefore, the capacity is calculated according to signal-to-noise ratio (SNR) and the interferences from primary transmitters and other secondary transmitters to the secondary receiver has been ignored in [5]-[6].

In this paper, we consider a cellular network where primary users communicate with the base station through the uplink transmission. The secondary users existing within the coverage area of the base station share the radio spectrum with the primary users and communicate with each other in an ad-hoc fashion. The same scenario has been employed in [7], and the sum throughput as a non-convex problem was solved by a suboptimal algorithm based on sequential geometric programming. Our main objective here is to maximize the ergodic capacity in a wireless ad-hoc cognitive radio network when the average interference at primary users is less than a predefined threshold, while assuming all channels experience Rayleigh fading.

First, we propose the special case where the interference from only primary transmitters exists. In this case, we obtain the closed-form expressions for evaluating the ergodic capacity.

Considering the interference from primary and other secondary transmitters makes the optimization problem a non-convex [8] [9], consequently, Lagrangian technique cannot be applied as the duality gap may not be zero. In this case, we use two theorems found in [10] which prove that the duality gap of this optimization problem is zero. Thereafter, we form the Lagrangian dual problem and then present gradient method as an iterative algorithm to solve the dual problem.

The rest of this paper is organized as follows. We will introduce the system model in section II. The ergodic capacity is studied in section III. The next section presents the numerical results. Finally, summarizing conclusions are given in Section V.

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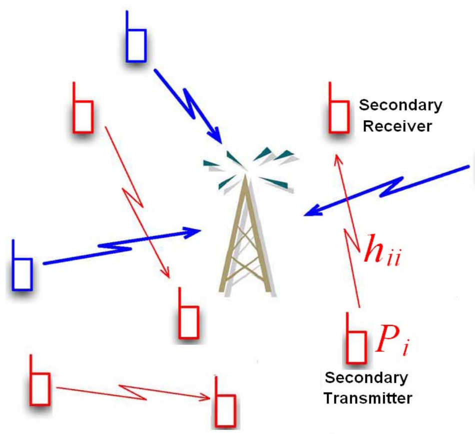

Fig. 1. Coexistence of a cognitive radio ad hoc network (red pairs) with primary users in cellular system (in blue)

II. System Model

As illustrated in Fig. 1, we introduce a scenario composed of a primary network with K users and a secondary ad-hoc network with M links, coexisting in the same area and sharing the spectrum. In the secondary network, no centralized authority is assumed to manage the network access for users. A point-to-point link is considered to communicate between a secondary transmitter and a secondary receiver. In order to avoid causing harmful interference to the primary users, secondary transmitters must control their transmit power. Active primary transmitters send information with constant power ρ and secondary transmitter i send signal with power P_i which is less than or equal to ρ.

In this system, the instantaneous received signal-to-interference-plus-noise ratio (SINR) at the secondary receiver i can be obtained using the following formula:

$$SINR_i = \frac{P_i h_{ii}}{N_0 + \sum_{k=1}^{K} \rho g_{ki} + \sum_{j=1, j\neq i}^{M} P_j h_{ji}} \tag{1}$$

where h_{ii} represents instantaneous channel power gain of the link between the secondary transmitter i and the secondary receiver i, which is assumed to be flat fading with additive white Gaussian noise (AWGN) n_0. Channel power gain h_{ji} denotes an interference channel between other secondary transmitters j and the secondary receiver i. g_{ki} denotes an interference channel power gain between an active primary transmitter k and the secondary receiver i. Furthermore, the noise n_0 is assumed to be independent random variable (RV) with the distribution $CN(0, N_0)$ (zero-mean circularly symmetric complex Gaussian noise with variance N_0).

III. Ergodic Capacity

We consider a fading environment with the received power constraint at a third party's receiver on average value. Channel capacity can be found by optimal utilization of the transmitted power over time, in which the received power constraint is met. Thus, the ergodic capacity of secondary users can be obtained by solving the following optimization problem.

$$\max_{P_i} \quad \sum_{i=1}^{M} E\left[\ln(1 + SINR_i)\right] \tag{2a}$$

$$s.t. \quad \sum_{i=1}^{M} E[P_i g_{ik}] \leq Q \tag{2b}$$

where Q denotes the average interference power threshold on a primary user. In addition, g_{ik} denotes the channel power gain of the link between the secondary transmitter i and the kth primary receiver.

A. Special Case: Interference from only Primary Transmitters

In order to discuss about the effect of interference from only primary transmitters on secondary receivers, each two secondary transmitters are assumed to be far away from each other. Therefore, the optimization problem (2a) subject to (2b) can be simplified as following

$$\max_{P_i} \quad \sum_{i=1}^{M} E\left[\ln\left(1 + \frac{P_i h_{ii}}{N_0 + \sum_{k=1}^{K} \rho g_{ki}}\right)\right] \tag{3a}$$

$$s.t. \quad \sum_{i=1}^{M} E[P_i g_{ik}] \leq Q \tag{3b}$$

The above optimization problem is equivalent to solve the following Lagrangian approach.

$$L(P_i, \mu) = \sum_{i=1}^{M} E\left[\ln\left(1 + \frac{P_i h_{ii}}{N_0 + \sum_{k=1}^{K} \rho g_{ki}}\right)\right] - \mu\left(\sum_{i=1}^{M} E[P_i g_{ik}] - Q\right) \tag{4}$$

where μ is the nonnegative dual variable corresponding to the constraint (3b). We take the derivative of (4) with respect to P_i for $i = 1, 2, ..M$, and let the derivative equal zero, it gives

$$E\left[\frac{h_{ii}}{P_i h_{ii} + N_0 + \sum_{k=1}^{K} \rho g_{ki}}\right] = \mu(E[g_{ik}]) \tag{5}$$

From equation (5), we can have

$$P_i = \left(\frac{1}{\mu g_{ik}} - \frac{N_0 + \sum_{k=1}^{K} \rho g_{ki}}{h_{ii}}\right)^{+} \tag{6}$$

where $(.)^{+}$ denotes $\max(.,0)$. Thus, the equation (6) gives

$$\frac{g_{ik}(N_0 + \sum_{k=1}^{K} \rho g_{ki})}{h_{ii}} < \frac{1}{\mu} \tag{7}$$

The parameter μ is determined such that satisfying the following Complementary Slackness Conditions [11] i.e.,

$$Q = \sum_{i=1}^{M} \int_0^{\infty} \int_{x < \frac{1}{\mu y}} \left(\frac{1}{\mu} - xy\right) f_x(x) f_y(y)\, dx dy \tag{8}$$

where $x = \frac{g_{ik}}{h_{ii}}$ and $y = (N_0 + \sum_{k=1}^{K} \rho g_{ki})$. In the case of Rayleigh fading, the channel power gains g_{ik}, h_{ii} and

g_{ki} follow exponential distribution. Furthermore, we assume that g_{ik}, h_{ii} and g_{ki} are unit-mean and mutually independent. Therefore, the probability density function (PDF) of $\frac{g_{ik}}{h_{ii}}$ can be given as [12]

$$f_{\frac{g_{ik}}{h_{ii}}}(x) = \frac{1}{(1+x)^2} \tag{9}$$

In addition, when $g_{1i}, g_{2i}, ..., g_{Ki}$ are independent and identically distributed random variables, the PDF of $(\sum_{k=1}^{K} g_{ki})$ is distributed as Gamma with parameter K. Accordingly, the PDF of $N_0 + \sum_{k=1}^{K} \rho g_{ki}$ is (see Appendix A)

$$f_{(N_0+\sum_{k=1}^{K}\rho g_{ki})}(y) = \frac{1}{\rho\Gamma(k)} e^{-\frac{y-N_0}{\rho}} \left(\frac{y-N_0}{\rho}\right)^{k-1} \tag{10}$$

Using the fact that the random variables $\{g_{ki}, h_{ii}, g_{ik}, k = 1, ..., K, i = 1, ..., M\}$ are mutually independents [13], and utilizing (9) and (10) in (8), it gives

$$Q = \frac{M}{\rho\Gamma(k)} \int_{N_0}^{\infty} \int_{0}^{\frac{1}{\mu y}} \left(\frac{1}{\mu} - xy\right) \frac{1}{(1+x)^2} \times \left(e^{-\frac{y-N_0}{\rho}} \left(\frac{y-N_0}{\rho}\right)^{k-1}\right) dxdy \tag{11}$$

Accordingly, the total channel capacity can be found as

$$\begin{aligned} C &= \sum_{i=1}^{M} \int_{0}^{\infty} \int_{x<\frac{1}{\mu y}} \ln\left(\frac{1}{\mu}\frac{1}{xy}\right) f_x(x) f_y(y) dxdy \\ &= \frac{M}{\rho\Gamma(k)} \int_{N_0}^{\infty} \int_{0}^{\frac{1}{\mu y}} \ln(\frac{1}{\mu}\frac{1}{xy}) \frac{e^{-\frac{y-N_0}{\rho}} \left(\frac{y-N_0}{\rho}\right)^{k-1}}{(1+x)^2} dxdy \end{aligned} \tag{12}$$

For general value of K, the equations (11) and (12) do not admit closed-form expressions and they need to be calculated numerically. However, for $K = 1$ and upon invoking [14, eq. (2.113), (4.222.8) and (4.331.2)], the equation (11) changes to the following form

$$\begin{aligned} Q =& \frac{M}{\mu} \left[1 + e^{\frac{1+\mu N_0}{\mu P}} (\mu\rho - 1) E_i\left(-\frac{1+\mu N_0}{\mu\rho}\right) \right. \\ & \left. - e^{\frac{N_0}{\rho}} \mu\rho E_i\left(-\frac{N_0}{\rho}\right) - \mu(N_0+\rho)\ln\left(1+\frac{1}{\mu N_0}\right) \right] \end{aligned} \tag{13}$$

where $\mathrm{Ei}(.)$ is the exponential integral function defined as $\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$. We can find μ for a given Q from the equation (13). It is worth noting that determining the μ from (13) needs to use numerical integration.

By changing the variable $t = \frac{1}{x}$ and using [14, eq. (2.727.3), (4.337.1) and (4.331.2)] in (12), the ergodic capacity can be expressed as

$$\begin{aligned} C = M &\left[e^{\frac{N_0}{p}} E_i\left(-\frac{N_0}{\rho}\right) - e^{\frac{1+\mu N_0}{\mu\rho}} E_i\left(-\frac{1+\mu N_0}{\mu\rho}\right) \right. \\ & \left. + \ln\left(1+\frac{1}{\mu N_0}\right) \right] \end{aligned} \tag{14}$$

B. General Case: Interference from Primary Transmitters and Other Secondary Transmitters

In what follows, we study the general case where the impact of the interference from both primary and other secondary transmitters on the total ergodic capacity of secondary users are considered. In this case, the optimization problem can be expressed as follows

$$\max_{P_i} \sum_{i=1}^{M} E[\ln(1+\frac{P_i h_{ii}}{N_0 + \sum_{k=1}^{K}\rho g_{ki} + \sum_{j=1, j\neq i}^{M} P_j h_{ji}})] \tag{15a}$$

$$s.t. \sum_{i=1}^{M} E[P_i g_{ik}] \leq Q \tag{15b}$$

When the problem such as (15a) subjected to (15b) is a convex problem, we can solve the dual problem by forming the Lagrangian dual. Meanwhile, the convex structure guarantees that the solutions of the primal problem and dual problem are the same and the duality gap is zero. The main challenge in solving (15a) subjected to (15b) is that the objective function (15a) is not concave in P_i, however, the concavity of the objective function is not a necessary condition for zero duality gap.

Here, we employ two theorems found in [10] which prove the duality gap of such optimization problem is zero, although the objective function (15a) is not concave.

The first theorem proves that the solution to the problem (15a) occurs at a point on the boundary of the feasible set created by the power constraint (15b). The second one shows that the solution is a concave function in the power constraint. Numerical results also confirm the concavity of (15a) in the power constraint (15b).

Using these two theorems, we conclude that the duality gap is zero. Then, we can form the Lagrangian as following

$$\begin{aligned} L(P_i,\mu) = \sum_{i=1}^{M} E[\ln(1+\frac{P_i h_{ii}}{N_0 + \sum_{k=1}^{K}\rho g_{ki} + \sum_{j=1, j\neq i}^{M} P_j h_{ji}})] \\ - \mu\left(\sum_{i=1}^{M} E\left[P_i g_{ik}\right] - Q\right) \end{aligned} \tag{16}$$

where μ is a Lagrangian dual variable. Consider $D(\mu)$ as the dual objective function and unconstrained maximization of the Lagrangian given by

$$D(\mu) = \max_{P_i} L(P_i, \mu) \tag{17}$$

The dual optimization problem is to find μ which is

$$\mu^* = \arg\min_{\mu \geq 0} D(\mu) \tag{18}$$

The Lagrange dual problem (18) can be solved by iterative algorithm such as gradient method, where either ellipsoid or subgradient method can iteratively update μ until the convergence criteria is met. The speed of convergence in subgradient method highly depends on the step size, while the convergence in ellipsoid method happens very fast [9]. The computational costs in each iteration of both methods are the same. In this paper, due to simplicity we use subgradient method to update μ.

In gradient method, it needs to design a positive step size α for updating P_i and μ. Hence, the following iterations can be implemented:

$$P_i^{(n+1)} = P_i^{(n)} + \alpha \left(\frac{\partial}{\partial P_i^{(n)}} \sum_{i=1}^{M} E[\ln(1 + \frac{P_i^{(n)} h_{ii}}{N_0 + \sum_{k=1}^{K} \rho g_{ki} + \sum_{j=1, j\neq i}^{M} P_j^{(n)} h_{ji}})] - \mu^{(n)} \right) \quad (19)$$

$$\mu^{(n+1)} = \mu^{(n)} + \alpha \left(Q - \sum_{i=1}^{M} E\left[P_i^{(n)} g_{ik} \right] \right) \quad (20)$$

where $P_i^{(n)}$and $\mu^{(n)}$ are the values of P_i and μ at stage n, respectively.

The detail of the power control algorithm is given in Table.1, which $P_i^{(n+1)}$ and $\mu^{(n+1)}$ are updated to maximize capacity.

Table. 1: Power control algorithm

Algorithm
1) Initialization n, α, $P_i^{(n)}$ and $\mu^{(n)}$
2) While not converged do
3) update $P_i^{(n+1)}$ by (19)
4) update $\mu^{(n+1)}$ by (20)
5) calculate the Ergodic Capacity
6) n=n+1
7) End While

Note that the complexity of this algorithm is the square of the number of the secondary transmitter [10].

IV. Numerical Results

In this section, we numerically present the results for ergodic capacity of the Rayleigh fading channels in the ad-hoc cognitive radio under interference constraint. In this section, we assume that $N_0 = 1$.

A. Special Case: Interference from only Primary Transmitters

Fig. 2 shows the plots for the ergodic capacity of secondary links against the average interference power threshold (Q) computed via (11) and (12) for Rayleigh fading with $M = 1$ and $\rho = 5\ dB$. This figure also shows the effect of different number of primary interferers on the capacity. Moreover, the ergodic capacity is plotted against the average interference power threshold (Q) for different value of ρ with $M = 1$ and $K = 2$ in Fig. 3. Evidently, as the number of primary interferers increases or the value of ρ increases, the total channel capacity decreases. Consequently, this indicates that the number of primary interferers and the value of ρ are dominant constraints in achieved capacity.

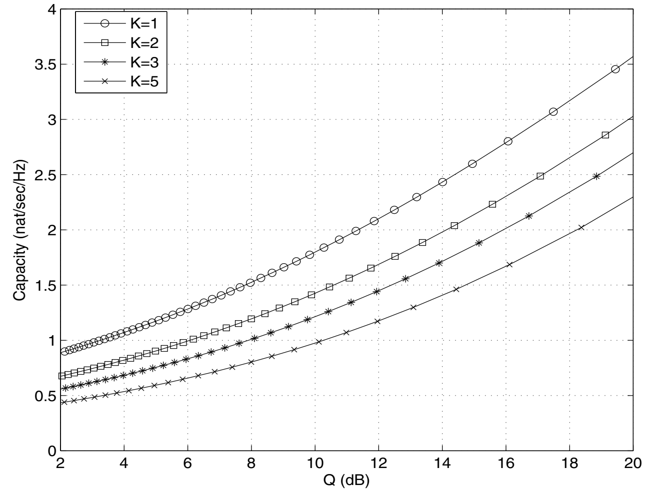

Fig. 2. The effect of different number of the primary interferers on the channel capacity of secondary users with $M = 1$ and $\rho = 5\ dB$.

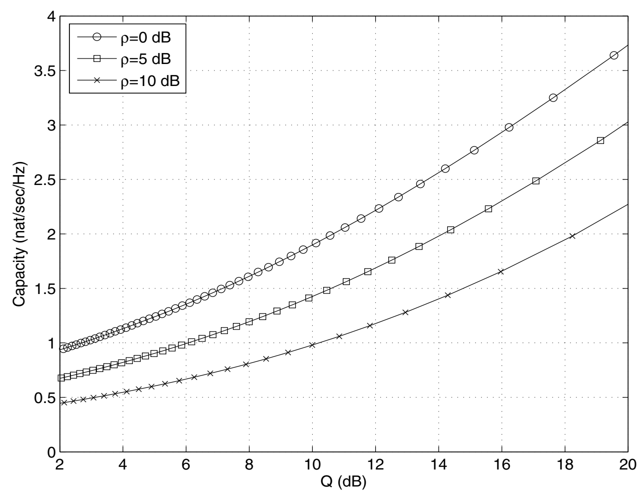

Fig. 3. The effect of different value of ρ on the channel capacity of secondary users with $M = 1$ and $K = 2$.

B. General Case: Interference from Primary Transmitters and Other Secondary Transmitters

In Fig. 4, the behavior of ergodic capacity versus the number of iterations in power control algorithm proposed in Table1 is studied for $M = 2$, $K = 1$ and $\rho = 5dB$. This figure indicates the convergence of proposed algorithm for a step size $\alpha = 0.05$. We observe that almost the same iterations are required to get convergence for each value of Q.

Fig. 5 provides the plots for the ergodic capacity versus the average interference power threshold (Q) with $K = 1$ and $\rho = 5dB$. Fig. 5 shows that the problem (15a) is concave in constraint (15b) as discussed in the duality gap. This figure also indicates that increasing the number of secondary transmitters leads to decreasing total throughput.

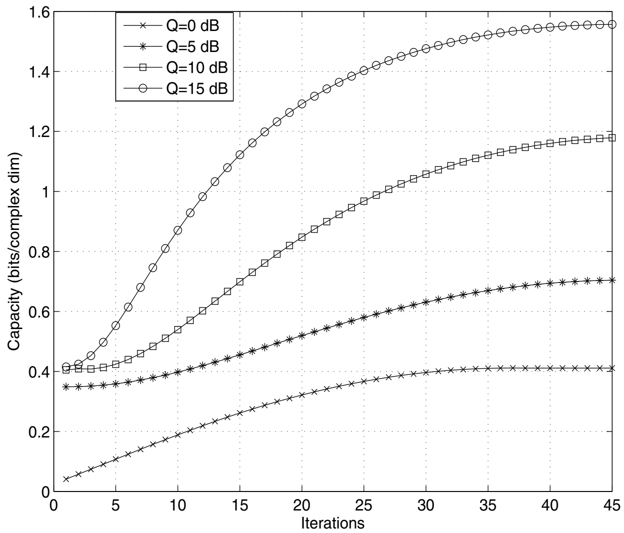

Fig. 4. The capacity against number of iterations in power control algorithm

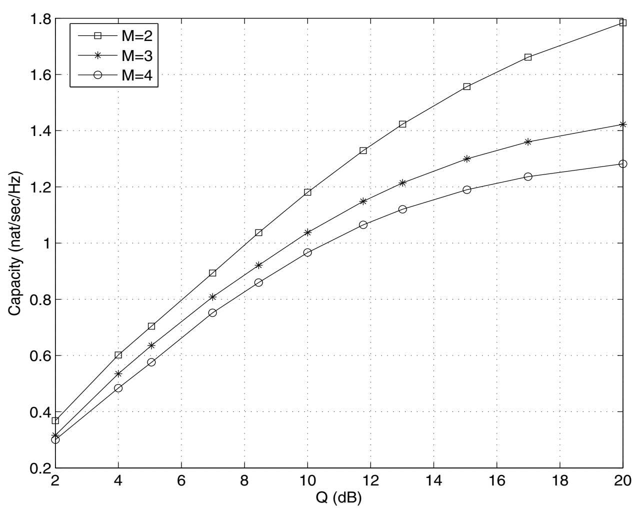

Fig. 5. The effect of different number of the secondary transmitters on the channel capacity of secondary users with $K = 1$ and $\rho = 5dB$.

Comparing Fig. 2 and Fig. 5, the secondary interferers significantly decrease the total channel capacity reducing the performance of the secondary network. For instance, when K=1 and Q=10dB, the capacity of only one link (M=1) is almost 1.8 but the total capacity of three links (M=3) is nearly 1.05.

V. CONCLUSION

We studied the channel capacity offered by spectrum sharing in time varying channels with received power constraint at a third party's receiver on average value. A primary network coexisting with an ad-hoc cognitive radio network is considered. We particularly investigated the ergodic capacity of an ad-hoc cognitive radio subjected to average interference power constraint at the primary users. The interference from primary and other secondary transmitters were discussed. Using some results from the numerical analysis, the total capacity of the secondary network highly depends on the number of secondary interferers, however the primary transmitters have a negative impact.

APPENDIX A

Here, we obtain the PDF of y ($y = N_0 + \rho \sum_{k=1}^{K} g_{ki}$). The cumulative distribution function of the random variable Y can be expressed as

$$F_Y\left(\frac{y - N_0}{\rho}\right) = \int_0^{\frac{y-N_0}{\rho}} \frac{x^{k-1}}{\Gamma(k)} e^{-x} dx \tag{21}$$

from (21) and using [14, eq. (2.33.10)], we have

$$F_Y\left(\frac{y - N_0}{\rho}\right) = \frac{1}{\Gamma(k)}\left(\Gamma(k) - \Gamma\left(k, \frac{y - N_0}{\rho}\right)\right) \tag{22}$$

where $\Gamma(.)$ is Euler gamma function and $\Gamma(.,.)$ is incomplete gamma function. Thus, the PDF $f_Y(.)$ is given by taking the derivative of (22).

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