AF MIMO Wireless Relay Networks Under Received Power Constraint

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***Abstract*—This paper considers an amplify-and-forward (AF) relay scheme for M-source-M-destination pairs and N relay nodes. Cooperative minimum mean square error (MMSE) strategies for wireless relay networks under both a jamming environment and channel uncertainty with received power constraints at the destination nodes are studied. The main contribution of this paper is the derivation of the MMSE-based amplifying relay matrices (ARMs) under both a jamming environment and channel uncertainty. With the proposed ARMs, the system performance under study is evaluated by observing bit error rate (BER) using Monte Carlo simulations. In addition, it is proven that the proposed ARM is the global optimal ARM.**

***Index Terms*—AF, MMSE, ARM, jamming, channel uncertainty, power constraint.**

I. INTRODUCTION

The distributed relay nodes in wireless cooperative networks can play a big role in improving data delivery between multiple sources and multiple destinations. Due to the spatial distribution of relay nodes, relay schemes can achieve a space diversity gain. Based on their roles, various relay schemes exist in wireless cooperative networks. For instance, relay nodes in an amplify-and-forward (AF) relay scheme, which is applied in this paper, receive signals from the source nodes in stage I and retransmit an amplified version of their received signals to destination nodes in stage II [1]-[11]. In contrast, the decode-and-forward relay scheme involves a complete decode in the first stage and transmission in the second stage. Using the decode-and-forward relay scheme, a multiple-input-multiple-output (MIMO) wireless distributed network for space-time-coded cooperative strategies has been investigated in [12].

Communication over a wireless channel can be at risk of interference from jamming signals. To evaluate these effects, various jamming signals have been analyzed in [13], [14] for wireless networks without relays. In addition, due to channel estimation errors in wireless cooperative systems, relay nodes can have imperfect channel state information in either source-relay links or relay-destination links. The authors in [5], [6] investigated distributed minimum mean square error (MMSE) relay schemes for an AF MIMO system and a single-input-single-output system under only channel uncertainty, respectively. In addition, the authors of this current paper have not studied the issue of AF MIMO wireless relay networks under both a jamming environment and channel uncertainty in their previous work [9]-[11].

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In general, like [4], [5], power was constrained at the transmitters during data transmission in wireless relay networks. However, it has been a recent trend that the power constraint at the destination is more favorable for an energy efficient and throughput efficient network system [6]-[8], [15], [16]. This is because a destination node having relays is likely to be located at the periphery, thus limiting interference for users in neighboring cells by employing the power constraint at the destination; hence, the outage probability of neighboring cells can be reduced. On the other hand, if the power constraint at the relays is used, the interference level for users in neighboring cells will keep increasing as the number of relay nodes increases.

As stated earlier, this current paper focuses on AF MIMO relay schemes under both a jamming environment and channel uncertainty with received power constraints at the destination nodes to derive the optimal amplifying relay matrix (ARM) based on MMSE criterion. In addition, it is proven that the derived ARM is the global optimal ARM. Based on the MSE cost function behavior, theoretical and simulation results will be analyzed to evaluate system performance.

The remainder of this paper is organized into four sections. Section II describes the system model and data transmission strategies applied. Section III considers cooperative MMSE relay schemes for AF MIMO wireless communication systems under both jamming and channel uncertainty with received power constraints at the destination nodes. Section IV shows the simulation BER results. Finally, Section V concludes the paper.

Notation: Matrices and vectors are denoted, respectively, by uppercase and lowercase boldface characters (e.g., $\mathbf{A}$ and

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a). The transpose, complex conjugate, inverse, trace, pseudo-inverse, and Hermitian of $\mathbf{A}$ are denoted, respectively, by $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^{-1}$, $tr(\mathbf{A})$, $\mathbf{A}^\dagger$, and $\mathbf{A}^H$. An $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. Notations $|a|$, $||\mathbf{a}||$, and $||\mathbf{A}||_F$ denote the absolute value of a for any scalar, 2-norm of $\mathbf{a}$, and Frobenius-norm of $\mathbf{A}$, respectively. The real and expectation operators are denoted by $\text{Re}\{\mathbf{A}\} = 1/2(\mathbf{A} + \mathbf{A}^*)$ and $E[\,\cdot\,]$, respectively.

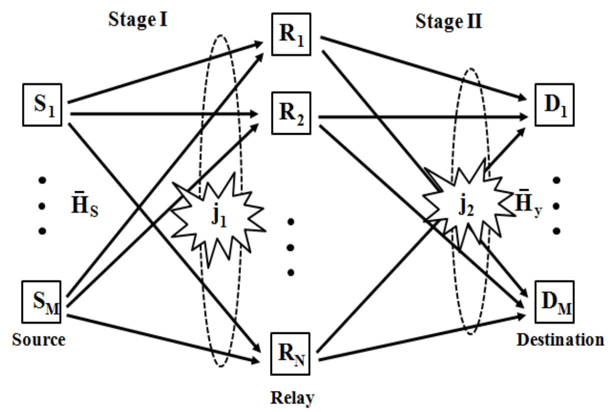

Fig. 1. AF MIMO wireless relay network under jamming environment and channel uncertainty with received power constraints at destination nodes.

II. System Models and Data Transmission

Figure 1 shows a wireless relay network under a jamming environment and channel uncertainty with received power constraints at destination nodes consisting of distributed N relay nodes, M source nodes, and M destination nodes ($N \geq M$). For data transmission under no adverse wireless communication environment, there are two stages: Stage I, where source nodes broadcast the signal vector $\mathbf{s} \in \mathbf{C}^{M\times 1} = [s_1, \cdots, s_M]^T$, and Stage II, where all relay nodes retransmit their signals to destination nodes.

Broadband noise jamming (BNJ) is an effective strategy when a jamming signal power is sufficient [13]. Hence, this paper assumes BNJ for a jamming environment. In addition, BNJ is modeled as a complex Gaussian noise with zero-mean and power $N_J BW$, where BW is the bandwidth. In this paper, the BW is normalized to 1. Namely, BNJ $\mathbf{j}_1 \in \mathbf{C}^{N\times 1}$ and $\mathbf{j}_2 \in \mathbf{C}^{M\times 1}$ are zero-mean complex additive white Gaussian noise (AWGN) vectors with covariance matrices $N_J \mathbf{I}_N$ and $N_J \mathbf{I}_M$, respectively.

Let $\mathbf{H}_s \in \mathbf{C}^{N\times M}$ denote the complex channel matrix from source nodes to relay nodes as

$$\mathbf{H}_s = [\mathbf{h}_{s,1}, \mathbf{h}_{s,2}, \ \cdots, \ \mathbf{h}_{s,M}] \tag{1}$$

where $\mathbf{h}_{s,m} \in \mathbf{C}^{N\times 1} = [h_{s,m,1}, \ \cdots, \ h_{s,m,N}]^T$, $m = 1, \cdots, M$, is a column vector representing the channel coefficients from the m-th source node to all relay nodes. It is assumed that each channel $h_{s,m,n}$ is independent identically distributed (i.i.d.) with a zero-mean and unit-variance circular complex Gaussian and quasi-static Rayleigh fading. $\mathbf{H}_y \in \mathbf{C}^{M\times N}$ denotes the complex channel matrix from relay nodes to destination nodes as

$$\mathbf{H}_y = [\mathbf{h}_{y,1}, \mathbf{h}_{y,2}, \ \cdots, \ \mathbf{h}_{y,M}]^T \tag{2}$$

where $\mathbf{h}_{y,m} = \left[h_{y,m,1}, \ \cdots, \ h_{y,m,N}\right]$, $m = 1, \cdots, M$, is a row vector representing the channel coefficient from all relay nodes to the m-th destination node. It is also assumed that each channel coefficient $h_{y,m,n}$ is i.i.d. with a zero-mean and unit-variance circular complex Gaussian and quasi-static Rayleigh fading.

In reality, due to channel estimation errors, the estimates of complex channel matrices $\bar{\mathbf{H}}_s \in \mathbf{C}^{N\times M}$ and $\bar{\mathbf{H}}_y \in \mathbf{C}^{M\times N}$ must be used instead of true channel complex matrices $\mathbf{H}_s$ and $\mathbf{H}_y$. Let $\mathbf{\Phi}_s \in \mathbf{C}^{N\times M}$ and $\mathbf{\Phi}_y \in \mathbf{C}^{M\times N}$ denote the corresponding channel-estimation error matrices consisting of complex i.i.d. zero-mean Gaussian random variables with covariance matrices $\sigma^2_{\phi_s}\mathbf{I}_N$ and $\sigma^2_{\phi_y}\mathbf{I}_M$, respectively. Therefore, the estimated complex channel matrices can be represented, respectively, as

$$\bar{\mathbf{H}}_s = \mathbf{H}_s - \mathbf{\Phi}_s \quad \text{and} \quad \bar{\mathbf{H}}_y = \mathbf{H}_y - \mathbf{\Phi}_y. \tag{3}$$

The received signal complex column vector $\mathbf{r} \in \mathbf{C}^{N\times 1}$ at the relay nodes under BNJ and channel uncertainty can be written as

$$\mathbf{r} = \bar{\mathbf{H}}_s\mathbf{s} + \mathbf{\Phi}_s\mathbf{s} + \mathbf{v}_s + \mathbf{j}_1 \tag{4}$$

where $\mathbf{v}_s \in \mathbf{C}^{N\times 1}$ is a zero-mean complex AWGN vector with covariance matrix $\sigma^2_{v_s}\mathbf{I}_N$. The amplified signal complex column vector $\mathbf{x} \in \mathbf{C}^{N\times 1}$ at the relay node outputs can be expressed as

$$\mathbf{x} = \mathbf{F}\mathbf{r} \tag{5}$$

where $\mathbf{F} \in \mathbf{C}^{N\times N}$ is an ARM employed by the relay nodes to minimize the mean square error (MSE) between the received signals at the destination nodes and the originally transmitted signal from the source nodes. Finally, the received complex signal column vector $\mathbf{y} \in \mathbf{C}^{M\times 1}$ at the destination nodes under BNJ and channel uncertainty can be written as

$$\begin{aligned}\mathbf{y} = \ &\bar{\mathbf{H}}_y\mathbf{F}\bar{\mathbf{H}}_s\mathbf{s} + \bar{\mathbf{H}}_y\mathbf{F}\mathbf{\Phi}_s\mathbf{s} + \bar{\mathbf{H}}_y\mathbf{F}\mathbf{v}_s + \bar{\mathbf{H}}_y\mathbf{F}\mathbf{j}_1 + \mathbf{\Phi}_y\mathbf{F}\mathbf{v}_s \\ &+ \mathbf{\Phi}_y\mathbf{F}\bar{\mathbf{H}}_s\mathbf{s} + \mathbf{\Phi}_y\mathbf{F}\mathbf{\Phi}_s\mathbf{s} + \mathbf{\Phi}_y\mathbf{F}\mathbf{j}_1 + \mathbf{v}_y + \mathbf{j}_2\end{aligned} \tag{6}$$

where $\mathbf{v}_y \in \mathbf{C}^{M\times 1}$ is a zero-mean complex AWGN vector with covariance matrix $\sigma^2_{v_y}\mathbf{I}_N$. It is assumed that channel estimation error is not correlated with jamming signals because channel estimation is done before transmission while jamming is applied during symbol transmission.

III. MMSE Relay Scheme

In this section, an optimum ARM $\mathbf{F}$ is presented under a jamming environment and channel certainty by applying the power constraint at the destination nodes, as done in [6]-[8], [15], [16]. This applies the MMSE criterion between the originally transmitted signal vector $\mathbf{s}$ from the source nodes and the received signal vector $\mathbf{y}$ at the destination nodes. Therefore, the optimization can be written as

$$\mathbf{F}^{\star} = \arg\min_{\mathbf{F}} J(\mathbf{F}) \quad\quad (7)$$
$$\text{s.t.} \quad E\big[||\mathbf{H}_y\mathbf{x}||^2\big] = \mathrm{P}.$$

where the superscript $\star$ means the optimum, P refers to the signal component power of the received signals at the destination nodes under BNJ and channel uncertainty, and the cost function $J(\mathbf{F}) \triangleq E[||\mathbf{y}-\mathbf{s}||^2]$ is written as

$$\begin{aligned} J(\mathbf{F}) &= \sigma_s^2\|\bar{\mathbf{H}}_y\mathbf{F}\bar{\mathbf{H}}_s\|_F^2 + \sigma_s^2\sigma_{\phi_y}^2\|\mathbf{F}\bar{\mathbf{H}}_s\|_F^2 + \eta\|\bar{\mathbf{H}}_y\mathbf{F}\|_F^2 \\ &+ \eta\sigma_{\phi_y}^2\|\mathbf{F}\|_F^2 - 2\sigma_s^2\mathrm{Re}\big[tr\big(\bar{\mathbf{H}}_y\mathbf{F}\bar{\mathbf{H}}_s\big)\big] + M\sigma_{j_2}^2 \\ &+ M\sigma_s^2 + M\sigma_{v_y}^2 \end{aligned} \quad (8)$$

where $E[|s_i|^2] = \sigma_{s_i}^2$, $\sigma_{s_1}^2 = \cdots = \sigma_{s_M}^2 = \sigma_s^2$, $\sigma_{v_{s_1}}^2 = \cdots = \sigma_{v_{s_N}}^2 = \sigma_{v_s}^2$, $\sigma_{v_{y_1}}^2 = \cdots = \sigma_{v_{y_M}}^2 = \sigma_{v_y}^2$, $E[\mathbf{v}_s] = E[\mathbf{v}_y] = E[\boldsymbol{\phi}_y] = E[\boldsymbol{\phi}_s] = \mathbf{0}$, and $\eta = \sigma_s^2\sigma_{\phi_s}^2 + \sigma_{v_s}^2 + \sigma_{j_1}^2$. As shown in (8), since the cost function $J(\mathbf{F})$ is defined as the MSE, the smaller the cost function value, the smaller the MSE, like [6]. Namely, the system performance improves as the MSE decreases. This will be verified through the simulation in Section IV. The total amount of signal component power at the destination nodes can be written as

$$\mathrm{P}{=}\sigma_s^2\|\bar{\mathbf{H}}_y\mathbf{F}\bar{\mathbf{H}}_s\|_F^2{+}\sigma_s^2\sigma_{\phi_y}^2\|\mathbf{F}\bar{\mathbf{H}}_s\|_F^2{+}\eta\|\bar{\mathbf{H}}_y\mathbf{F}\|_F^2{+}\eta\sigma_{\phi_y}^2\|\mathbf{F}\|_F^2. \quad (9)$$

To determine the constrained optimization problem, the Lagrangian multiplier λ in [17] is applied as

$$L(\mathbf{F},\lambda) = J(\mathbf{F}) + \lambda\big(E[||\mathbf{H}_y\mathbf{x}||^2 - \mathrm{P}\big). \quad (10)$$

Differentiating $L(\mathbf{F},\lambda)$ with respect to $\mathbf{F}^*$ using the cyclic properties of the trace function, i.e., $tr\big(\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3\big) = tr\big(\mathbf{A}_2\mathbf{A}_3\mathbf{A}_1\big)$, and the linear and nonlinear properties of the complex derivative matrix [18], the optimal ARM $\mathbf{F}^{\star}$ under BNJ and channel uncertainty can be obtained as

$$\mathbf{F}^{\star} = \frac{\mathbf{H}\bar{\mathbf{H}}_y^H\bar{\mathbf{H}}_s^H\mathbf{H}_d\sqrt{\mathrm{P}}}{\sqrt{tr\Big(\bar{\mathbf{H}}_y^H\bar{\mathbf{H}}_s^H\mathbf{H}_d\bar{\mathbf{H}}_s\bar{\mathbf{H}}_y\mathbf{H}^H\Big)}} \quad (11)$$

where $\mathbf{H}{=}\big(\bar{\mathbf{H}}_y^H\bar{\mathbf{H}}_y{+}\sigma_{\phi_y}^2\mathbf{I}_N\big)^{-1}$ and $\mathbf{H}_d{=}\big(\sigma_s^2\bar{\mathbf{H}}_s\bar{\mathbf{H}}_s^H{+}\eta\mathbf{I}_N\big)^{-1}$.

- *Remark 1*: From (11), various special cases for AF MIMO wireless relay networks under BNJ and channel uncertainty with the received power constraint at the destination nodes can be derived. In other words, with $\sigma_{\phi_s}^2 = \sigma_{\phi_y}^2 = 0$ in (11), the optimal ARM $\mathbf{F}_{JAM}^{\star}$ under only a jamming environment can be written as

$$\mathbf{F}_{JAM}^{\star} = \frac{\mathbf{H}_y^{\dagger}\mathbf{H}_s^H\mathbf{H}_t\sqrt{\mathrm{P}}}{\sqrt{\sigma_s^2\|\mathbf{H}_s^H\mathbf{H}_t\mathbf{H}_s\|_F^2 + \big(\sigma_{v_s}^2+\sigma_{j_1}^2\big)\|\mathbf{H}_s^H\mathbf{H}_t\|_F^2}} \quad (12)$$

where $\mathbf{H}_t{=}\big(\sigma_s^2\mathbf{H}_s\mathbf{H}_s^H{+}\sigma_{v_s}^2\mathbf{I}_N{+}\sigma_{j_1}^2\mathbf{I}_N\big)^{-1}$.

- *Remark 2*: With $\sigma_{j_1}^2 = 0$ in (11), the optimal ARM $\mathbf{F}_{CU}^{\star}$ under channel uncertainty in both the source-relay links and the relay-destination links can be written as

$$\mathbf{F}_{CU}^{\star} = \frac{\mathbf{H}\bar{\mathbf{H}}_y^H\bar{\mathbf{H}}_s^H\mathbf{H}_o\sqrt{\mathrm{P}}}{\sqrt{tr\Big(\bar{\mathbf{H}}_y\mathbf{H}\bar{\mathbf{H}}_y^H\bar{\mathbf{H}}_s^H\mathbf{H}_o\bar{\mathbf{H}}_s\Big)}} \quad (13)$$

where $\mathbf{H}_o = \big(\sigma_s^2\bar{\mathbf{H}}_s\bar{\mathbf{H}}_s^H + \sigma_{\phi_s}^2\sigma_s^2\mathbf{I}_N + \sigma_{v_s}^2\mathbf{I}_N\big)^{-1}$.

- *Remark 3*: With $\sigma_{\phi_s}^2 = \sigma_{\phi_y}^2 = \sigma_{j_1}^2 = 0$ in (11), the optimal ARM $\mathbf{F}_{NO}^{\star}$ under no adverse wireless communication environment can be written as

$$\mathbf{F}_{NO}^{\star} = \frac{\mathbf{H}_y^{\dagger}\mathbf{H}_s^H\mathbf{H}_q\sqrt{\mathrm{P}}}{\sqrt{\sigma_s^2\|\mathbf{H}_s^H\mathbf{H}_q\mathbf{H}_s\|_F^2{+}\sigma_{v_s}^2\|\mathbf{H}_s^H\mathbf{H}_q\|_F^2}} \quad (14)$$

where $\mathbf{H}_q = \big(\sigma_s^2\mathbf{H}_s\mathbf{H}_s^H + \sigma_{v_s}^2\mathbf{I}_N\big)^{-1}$.

It can be shown that the cost function $J(\mathbf{F})$ in (7) is a convex cone with respect to $\mathbf{F}$.

In addition, using the optimal ARM $\mathbf{F}_{NO}^{\star}$ in (14), the cost function $J(\mathbf{F}_{NO}^{\star})$ can be obtained as

$$J(\mathbf{F}_{NO}^{\star}) = \mathrm{P} + M\sigma_s^2 + M\sigma_{v_y}^2 - 2\sigma_s^2\left(\mathrm{P}\sum_{i=1}^{M}\omega_i\right)^{1/2} \quad (15)$$

where ω_i is the i-th positive eigenvalue of $\mathbf{H}_s\mathbf{H}_s^H\mathbf{H}_q$. To provide a better insight into the results in (16), it is assumed that the noise power level at both the relay nodes and the destination nodes can be negligible, i.e., $\sigma_{v_s}^2 = \sigma_{v_y}^2 = 0$, at a high input SNR. Hence, the cost function can be approximated as $(\sqrt{\mathrm{P}} - \sqrt{\sigma_s^2})^2 + (M-1)\sigma_s^2$. It is observed that the cost function converges to a steady-state value, i.e., $(\sqrt{\mathrm{P}} - \sqrt{\sigma_s^2})^2 + (M-1)\sigma_s^2$, as the number of relay nodes increases. As a result, the BER performance will merge as N increases. This analytical result will be verified through the simulation in Section VI.

IV. Simulation Results

Monte Carlo simulation results are obtained for cooperative MMSE relay schemes under both BNJ and channel uncertainty with received power constraints at the destination nodes. The complex channel matrices $\mathbf{H}_s$ and $\mathbf{H}_y$ are generated from zero-mean and unit-variance independent Gaussian random variables. All nodes have the same noise power, i.e., $\sigma_{v_{s_1}}^2 = \cdots = \sigma_{v_{s_N}}^2 = \sigma_{v_s}^2$ and $\sigma_{v_{y_1}}^2 = \cdots = \sigma_{v_{y_M}}^2 = \sigma_{v_y}^2$. The originally transmitted signals at the source nodes are assumed to be modulated by quadrature phase shift keying with unit power.

For two different cases of BNJ, 2.5% and 5% of the desired signal bit energy and channel uncertainty, respectively, are modeled as a complex AWGN that is added to the original received signal, i.e., variances of the jamming signals and channel estimation errors, respectively, are chosen to satisfy $10\log_{10}(E_b/N_J) = 16$ dB and 13 dB, where $E_b = \sigma_s^2$, $N_J =$

$\sigma^2_{j_k}, k = 1, 2$, and $10\log_{10}(\sigma^2_\phi/\sigma^2_h) =$ –16 dB and -13 dB, where $\sigma^2_\phi/\sigma^2_h = \sigma^2_{\phi_s}/\sigma^2_{h_s} = \sigma^2_{\phi_y}/\sigma^2_{h_y}$.

Figure 2 shows the MMSE cost function comparison of AF MIMO wireless relay networks with different numbers of relay nodes $N = 4$, 8, 16, 32, and $M = 2$. Simulation results agree well with the analysis. As analyzed in (16), it is observed that the cost function decreases as N increases. However, the cost function converges to the steady-state value for N to be large enough under the power constraints at the destinations. In addition, these results are related to BER because the cost function is defined as the MMSE, which will be explained in Fig. 3.

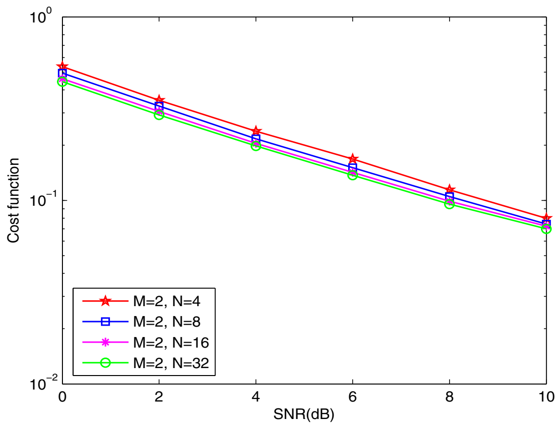

Fig. 2. MMSE cost function comparison of AF MIMO wireless relay networks with different numbers of relay nodes $N = 4$, 8, 16, 32, and $M = 2$.

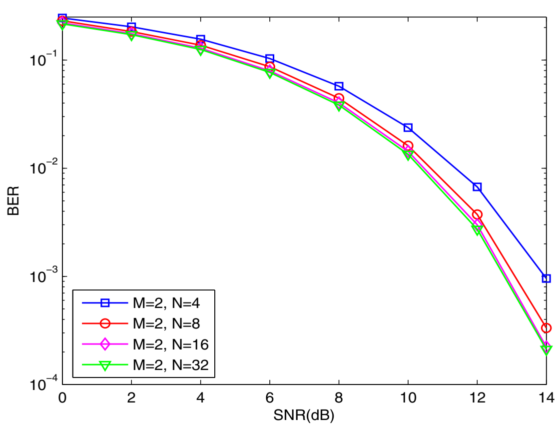

Fig. 3. BER performance of $M = 2$ AF MIMO wireless relay networks under no adverse wireless communication environment with different numbers of relay nodes $N = 4$, 8, 16, 32.

Figure 3 provides the BER performance of $M = 2$ AF MIMO wireless relay networks under no adverse wireless communication environment with a different number of relay nodes $N = 4$, 8, 16, 32. As expected, it is observed that the BER improves but converges as N increases because the cost function merges as N increases.

Figure 4 presents the BER performance of $M = 2$ AF MIMO wireless relay networks under BNJ using two different jamming conditions, i.e., 13 dB, 16 dB, with $N = 4$, but no channel uncertainty. As stated in Section II, the smaller cost function value produces better system performance. Hence, using the cost function, it can be predicted which case will yield a better BER than the others. For example, the cost function $J(\mathbf{F}^\star_{JAM})$ for BNJ can be written as $\mathrm{P} + M\sigma^2_s + M\sigma^2_{v_y} + M\sigma^2_{j_1} - 2\sigma^2_s\sqrt{\mathrm{P}\sum_{i=1}^M \gamma_i}$, where γ_i is the i-th positive eigenvalue of $\mathbf{H}_s^H\mathbf{H}_t\mathbf{H}_s$. In addition, it is observed that $J(\mathbf{F}^\star_{JAM}) - J(\mathbf{F}^\star_{NO}) = M\sigma^2_{j_1} + 2\sigma^2_s\sqrt{\mathrm{P}\sum_{i=1}^M \omega_i}\Big(1 - \sqrt{\sum_{i=1}^M \gamma_i/\sum_{i=1}^M \omega_i}\Big)$ is always greater than 0. Therefore, BER performance under BNJ is theoretically worse than the one under no adverse wireless communication environment. From the cost function $J(\mathbf{F}^\star_{JAM})$, it can be clearly shown that $J_{5\%}(\mathbf{F}^\dagger_{\text{BNJ}}) > J_{2.5\%}(\mathbf{F}^\dagger_{\text{BNJ}}) > J(\mathbf{F}^\star_{NO})$. That is, BER performance gets worse as the jamming signal power increases. This can be clearly observed in the simulation, as shown in Fig. 4. In addition, theoretically, it can be shown that $J(\mathbf{F}^\dagger_{\text{BNJ}}) > J(\mathbf{F}^\dagger_{\text{BNJ-RD}}) > J(\mathbf{F}^\dagger_{\text{BNJ-SR}}) > J(\mathbf{F}^\star_{NO})$. These are also clearly seen in the simulation, as shown in Fig. 4.

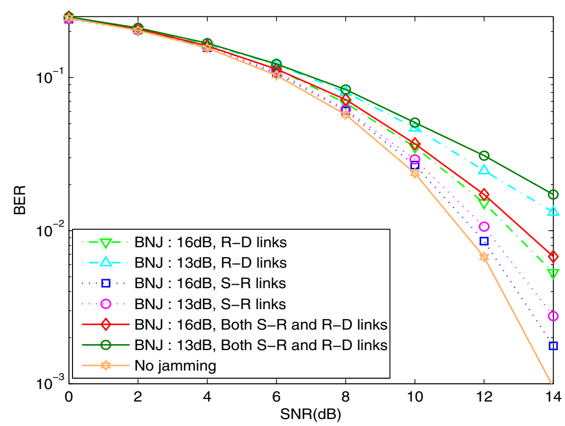

Fig. 4. BER performance of $M = 2$ AF MIMO wireless relay networks under BNJ using two different conditions, i.e., 13 dB, 16 dB, with $N = 4$, but no channel uncertainty.

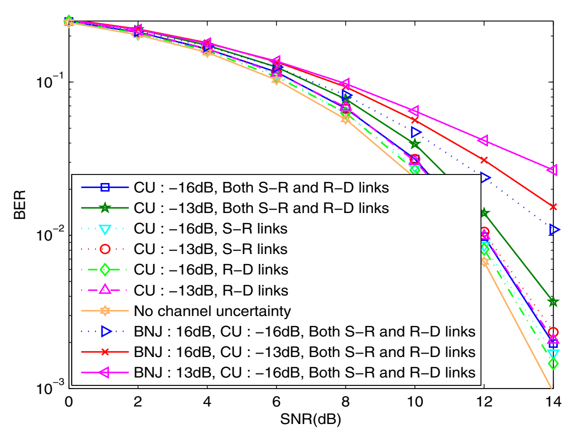

Fig. 5. BER performance of $M = 2$ AF MIMO wireless relay networks under either channel uncertainty or BNJ using two different conditions, i.e., -13 dB, -16 dB, with $N = 4$.

Figure 5 shows the BER performance of $M = 2$ AF MIMO wireless relay networks under either channel uncertainty or BNJ using two different conditions, i.e., -13 dB, -16 dB, with $N = 4$. The cost function $J(\mathbf{F}^{\star}_{CU})$ for channel uncertainty can be written as $\mathrm{P} + M\sigma_s^2 + M\sigma_{v_y}^2 - 2\sigma_s^2\sqrt{\mathrm{P}\sum_{i=1}^{M}\beta_i}$, where β_i is the i-th positive eigenvalue of $\bar{\mathbf{H}}_y\mathbf{H}\bar{\mathbf{H}}_y^H\bar{\mathbf{H}}_s^H\mathbf{H}_o\bar{\mathbf{H}}_s$. In addition, it is observed that $J(\mathbf{F}^{\star}_{JAM}) - J(\mathbf{F}^{\star}_{CU}) = M\sigma_{j_1}^2 + 2\sigma_s^2\sqrt{\mathrm{P}}\left(\sqrt{\sum_{i=1}^{M}\beta_i} - \sqrt{\sum_{i=1}^{M}\gamma_i}\right)$ is always greater than 0. Therefore, BER performance under BNJ is theoretically worse than BER performance under channel uncertainty. As shown in Fig. 4 and Fig. 5, it can be observed that BER performance under BNJ is worse than that under channel uncertainty through simulation. In addition, from the cost function $J(\mathbf{F}^{\star}_{CU})$, it can be observed that $J_{5\%}(\mathbf{F}^{\dagger}_{\text{CU}}) > J_{2.5\%}(\mathbf{F}^{\dagger}_{\text{CU}}) > J(\mathbf{F}^{\star}_{NO})$. Namely, BER performance gets worse as the channel estimation error power increases. These are displayed clearly in the simulation in Fig. 5. Similar to the case of BNJ, it can be theoretically shown that $J(\mathbf{F}^{\dagger}_{\text{CU}}) > J(\mathbf{F}^{\dagger}_{\text{CU-SR}}) > J(\mathbf{F}^{\dagger}_{\text{BNJ-RD}}) > J(\mathbf{F}^{\star}_{NO})$. These are also clearly seen in the simulation in Fig. 5. In addition, it can be observed that the lower the BNJ and channel estimation error powers, the better the BER performance. The best BER is observed in the case of the combination case $\left(10\log_{10}(\sigma_\phi^2/\sigma_h^2) = –16 \text{ dB}, 10\log_{10}(E_b/N_J) = 16\text{dB}\right)$ because the lowest power levels of both jamming and channel uncertainty are applied. As can be seen, the jamming signal power more negatively affects system performance compared to the channel estimation error power. As shown in Fig. 5, the combination case $\left(10\log_{10}(\sigma_\phi^2/\sigma_h^2) = –16 \text{ dB}, 10\log_{10}(E_b/N_J) = 13 \text{ dB}\right)$ shows a worse BER performance compared to the case $\left(10\log_{10}(\sigma_\phi^2/\sigma_h^2) = –13 \text{ dB}, 10\log_{10}(E_b/N_J) = 16 \text{ dB}\right)$.

V. Conclusion

Cooperative AF MIMO wireless relay networks were studied under both a jamming environment and channel uncertainty. The optimal ARMs were obtained by minimizing the MSE between the originally transmitted signals from the source nodes and the received signals at the destination nodes under received power constraints at the destination nodes.

The cost function is a convex cone. Therefore, the propose ARM is global optimum. It was observed that the proposed cost function decreases as the number of relay nodes increases. However, for N to be large enough, it converges to a steady-state value. In addition, the proposed cost function is related to the MSE. Hence, the MSE gets smaller as the proposed cost function decreases. Therefore, the BER gets smaller as the cost function decreases within a certain N. However, if N is large enough, the BER merges regardless of increasing N, like the cost function. This is because the signal component of the received signal is constrained to a constant.

It was also observed through both theoretical and simulation results that the loss of diversity order can occur as the jamming signal and the channel estimation error variances increase. Even low power levels of jamming and channel uncertainty can significantly affect BER performance in a negative way. As proven by the good agreement between the theoretical and simulation results, it was observed that jamming is more critical than channel uncertainty.

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