Joint Channel and Doppler Spread Estimation over Time-Varying Flat-Fading Channels

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Abstract—**In time-varying flat-fading environments, time domain channel correlations are required for accurate channel estimation and interpolation. However, the correlation property, which depends on the Doppler spread, is hard to be extracted from corrupted channel responses within a short channel estimation interval. In this work, based on an approximate channel model via the Taylor expansion, we propose a joint channel and Doppler spread estimation scheme over time-varying flat-fading channels. It employs the expectation-maximization (EM) algorithm to iteratively attain the maximum-likelihood (ML) channel and Doppler spread estimates. Simulation results show that the proposed scheme achieves accurate performance of both channel estimation and Doppler spread estimation within a short estimation interval.**

Keywords-channel estimation; Doppler spread; expectation-maximization (EM); maximum-likelihood (ML); time-varying channel

I. Introduction

In both narrowband and wideband wireless communication systems, channel estimation plays a very important role and has attracted many research interests as it directly affects the data detection performance at receivers. One of the main challenges of channel estimation is to combat the time-varying effect of wireless channels, and several well-known methods, such as minimum mean square error (MMSE) and maximum likelihood (ML) estimators as well as linear/nonlinear interpolation schemes [1], [2] have been proposed. To attain the best estimation performance, nevertheless, the time-varying characteristics, such as the correlation property between channel samples, must be obtained in advance. It then yields another important issue, the Doppler spread estimation over time-varying channels.

Several approaches have been proposed for Doppler spread estimation over Rayleigh fading channels. One frequently used concept extracts the Doppler spread by matching the ensemble autocorrelation function (ACF) produced by observations with the ideal ACF [3], [4]. Other methods deal with various Doppler spread-dependent channel characteristics, such as the level-crossing rate [5], the envelope distribution [6], etc. These estimators, however, require a long observation period, as well as a large number of channel samples, in order to obtain acceptable estimation performance. Due to limited data detection latency, channel estimation is usually performed in a short period which may be too short for the aforementioned Doppler estimators to provide accurate Doppler information, particularly for burst-mode data reception. On the other hand, ML-based Doppler estimators in [7] and [8] attain accurate performance by using a small number of samples. However, they suffer from the non-convex optimization problem whose optimal solution requires exhaustive search. To have in mind the interdependence of the channel and Doppler estimates, they should be properly combined in order to achieve better performance for both sides within short estimation duration.

Techniques combining channel estimation with other channel-related processes, e.g., data detection [9], multi-user detection [10], carrier-frequency-offset (CFO) estimation [11] etc, have attracted many research attentions. There are few researches investigate joint estimation of channel and Doppler spread [12], [13]. Based on a complex autoregressive model of the received signal in a multipath Rayleigh fading channel, a recursive method is presented in [13] for joint estimation of the channel, the channel parameters (including the Doppler spread and the power profile), and the CFO.

In this work, based on an approximate channel model via the Taylor expansion, we propose a joint ML channel and Doppler spread estimation scheme over flat Rayleigh fading channels. This scheme employs the expectation-maximization (EM) algorithm to iteratively solve the ML optimization problem. Simulation results show that the proposed algorithm provides accurate channel and Doppler estimates within a short estimation interval, and is able to be performed solely without preceding channel or Doppler spread information, especially suitable for burst-mode data reception. In addition, the ML Doppler estimation of the proposed scheme becomes a univariate polynomial optimization problem and can be solved by common optimization approaches.

The remainder of this paper is organized as follows. Section II describes the system and channel models of this work. A Taylor-based model for flat Rayleigh fading channels is introduced in Section III. In Section IV, we present the EM-based joint channel and Doppler spread estimation scheme. Section V provides simulation results of channel estimation and Doppler estimation performance. The conclusion of this work is drawn in Section VI.

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II. SYSTEM AND CHANNEL MODELS

It is assumed that the propagation channel experienced by a mobile receiver is a wide-sense stationary (WSS) and frequency-nonselective Rayleigh fading channel. Therefore, a received discrete-time signal can be represented as

$$r[n] = h[n] \times s[n] + w[n] \tag{1}$$

where $s[n]$ is the transmitted symbol, $h[n]$ is the complex-valued channel gain, $w[n]$ is the zero-mean additive white Gaussian noise (AWGN) with variance σ_w^2, and the sampling interval is the symbol duration T_s. The continuous-time version of $h[n]$, namely the low-pass equivalent channel response $h(t)$ with $h(t)|_{t=nT_s} = h[n]$, can be modeled as a sum of M sinusoidal waveforms corresponding to M different scattering paths [14], i.e.

$$h(t) = \sum_{m=1}^{M} \alpha_m \exp[j(2\pi f_d \cos\theta_m t + \phi_m)] \tag{2}$$

where α_m, θ_m and ϕ_m are random scattering parameters, respectively representing the amplitude, the incoming angle and the random phase of the m-th scattering path; f_d represents the maximum Doppler spread, and M is the number of paths. The ACF of $h[n]$ corresponds to the normalized ACF and the channel's average scattering power σ_h^2. Specifically, for an isotropic-scattering environment, the ACF is $\sigma_h^2 J_0(2\pi f_d m T_s)$ [14], where $J_0(\bullet)$ is the zero-order Bessel function of the first kind.

In this work, a collection of N received time samples $\mathbf{r} = [r[0], \cdots, r[N-1]]^T$, corresponding to a transmitted training sequence $\mathbf{s} = [s[0], \cdots, s[N-1]]^T$, is used for data-aided channel and Doppler spread estimation. Without taking into account the effect of the training sequence on the estimation performance, we simply assume $s[n]=1$ for all n. The signal-to-noise ratio (SNR) at the input of the estimator is defined as σ_h^2 / σ_w^2. Furthermore, it is assumed that the Doppler spread is fixed within an estimation period NT_s.

III. TAYLOR-BASED FLAT-FADING CHANNEL MODEL

Consider a flat-fading channel realization $h(t)$ within a domain of interest $t \in \mathcal{T}$, where $\mathcal{T}$ is a continuous interval; and assume that the scattering parameters, α_m, θ_m and ϕ_m for all m, are invariant during $\mathcal{T}$. It is well known that one can properly select an order K to accurately approximate $h(t)$ by a Taylor-based expansion at $t=0$, such that

$$h(t) \approx \sum_{k=0}^{K} (2\pi f_d t)^k \frac{j^k}{k!} \sum_{m=1}^{M} \alpha_m \exp(j\phi_m)(\cos\theta_m)^k, \quad t \in \mathcal{T} \tag{3}$$

According to (3), the N-length discrete-time channel vector $\mathbf{h}$, where $NT_s \in \mathcal{T}$, then has an approximate vector representation given as follow.

$$\begin{aligned} \mathbf{h} &= [h[0], h[1], \cdots, h[N-1]]^T \\ &= [\mathbf{v}_0, \mathbf{v}_1, \cdots, \mathbf{v}_K] \cdot \mathrm{diag}([f_d^0, f_d^1, \cdots, f_d^K]^T) \cdot [\eta_0, \eta_1, \cdots, \eta_K]^T \\ &= \mathbf{V}\mathbf{D}_f \boldsymbol{\eta} \end{aligned} \tag{4}$$

where $\mathbf{V}$ is a Vandermonde matrix composed of the column vectors $\mathbf{v}_k = [(2\pi T_s \cdot 0)^k, (2\pi T_s \cdot 1)^k, \cdots, (2\pi T_s (N-1))^k]^T$ for $0 \le k \le K$, and $\mathbf{D}_f$ is the diagonal matrix of size $(K+1)$ with the diagonal elements $\{1, f_d, f_d^2, \cdots, f_d^K\}$. And the elements of the $(K+1)$-length vector $\boldsymbol{\eta} = [\eta_0, \eta_1, \cdots, \eta_K]^T$ denote the channel coefficients corresponding to the $K+1$ orders of the Doppler spread, which are given by

$$\eta_k = \frac{j^k}{k!} \sum_{m=1}^{M} \alpha_m \exp(j\phi_m)(\cos\theta_m)^k, \quad k = 0, \cdots, K \tag{5}$$

Since the random scattering parameters related to different scatterers (indices m) are independently and identically distributed (i.i.d.), η_k converges to a complex Gaussian random variable as the number of scatterers M increases according to the central limit theorem. It is also verified that for M large enough, $\boldsymbol{\eta}$ converges to a circularly symmetric complex Gaussian random vector with zero mean and a covariance matrix $\mathbf{R}_\eta = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$. The probability density function (pdf) is expressed as

$$p(\boldsymbol{\eta}) = \frac{1}{\pi^{K+1} \det(\mathbf{R}_\eta)} \exp(-\boldsymbol{\eta}^H \mathbf{R}_\eta^{-1} \boldsymbol{\eta}) \tag{6}$$

In general, $\mathbf{R}_\eta$ depends on the shape of the Doppler spectrum, equivalently on the joint distribution of the amplitudes, the incoming angles, and the random phases of the arrival paths. Because the random phases $\{\phi_m\}_{m=1}^M$ are i.i.d. uniform random variables $U(0, 2\pi)$ and are independent of $\{\alpha_m, \theta_m\}_{m=1}^M$, the entries of $\mathbf{R}_\eta$ can be derived as

$$\begin{aligned} \left[\mathbf{R}_\eta\right]_{k+1,l+1} &= E[\eta_k \eta_l^*] \\ &= \frac{j^k(-j)^l}{k!\,l!} \sum_{m=1}^{M} E[|\alpha_m|^2 (\cos\theta_m)^{k+l}], \; 0 \le k, l \le K \end{aligned} \tag{7}$$

It should be noted that although $\mathbf{R}_\eta$ is related to the shape of the Doppler spectrum, the covariance matrix, as well as the distribution of $\boldsymbol{\eta}$ is independent of the Doppler spread f_d.

In isotropic scattering environments, specifically, the statistical model with constant amplitudes $\alpha_m = \sigma_h^2 / M$ and i.i.d. incoming angles $\theta_m \sim U(0, 2\pi)$ can be adopted for substituting the joint distribution of $\{\alpha_m, \theta_m\}_{m=1}^M$. Accordingly, the autocorrelation matrix can be further obtained as

$$\left[\mathbf{R}_\eta\right]_{k,l} = \begin{cases} \dfrac{(-1)^l j^{k+l}}{k!\,l!} \dfrac{\sigma_h^2}{2^{k+l}} \dbinom{k+l}{(k+l)/2}, & \text{if } k+l \text{ is even} \\ 0, & \text{if } k+l \text{ is odd} \end{cases} \tag{8}$$

IV. EM-Based Joint Channel and Doppler Spread Estimation

Given the value of the Doppler spread f_d and conditioned on the channel coefficient vector $\boldsymbol{\eta}$, $\mathbf{r}$ is a joint Gaussian random vector whose probability density function (pdf) is expressed as

$$p(\mathbf{r} \mid f_d, \boldsymbol{\eta}) = \frac{1}{(\pi\sigma_w^2)^N} \exp(-\frac{1}{\sigma_w^2} \| \mathbf{r} - \mathbf{V}\mathbf{D}_f \boldsymbol{\eta} \|^2) \tag{9}$$

which is also the likelihood function of $\boldsymbol{\eta}$ and f_d. Those $\boldsymbol{\eta}$ and f_d that maximize (9) then becomes the joint ML channel and Doppler spread estimates. However, the search for the optimal $\boldsymbol{\eta}$ and f_d yields very high computational complexity. We then propose an iterative method to attain the jointly optimal solution based on the EM algorithm.

To employ the EM algorithm, we first treat $\{\mathbf{r}, \boldsymbol{\eta}\}$ as the complete data, where $\mathbf{r}$ is the incomplete observation and $\boldsymbol{\eta}$ is the missing information; and f_d is the unknown parameter. An EM iteration consists of two steps: the *E-Step* and the *M-Step*, and the convergence of the two steps terminates the EM algorithm. In the following we show in detail the *E-Step* and the *M-Step* corresponding to the *i*-th iteration of the proposed joint channel and Doppler spread estimation algorithm.

A. E-Step—The Channel Estimation Step

Given the estimated Doppler spread of the previous iteration, $\hat{f}_d^{(i-1)}$, the *E-Step* evaluates the expected log-likelihood function (LLF) of f_d based on the complete data, where the expectation is taken over the conditional pdf of the missing information, $p(\boldsymbol{\eta} \mid \mathbf{r}, \hat{f}_d^{(i-1)})$. The expected LLF is expressed as

$$Q(f_d \mid \hat{f}_d^{(i-1)}) = \int \log(p(\mathbf{r}, \boldsymbol{\eta} \mid f_d)) p(\boldsymbol{\eta} \mid \mathbf{r}, \hat{f}_d^{(i-1)}) d\boldsymbol{\eta} \tag{10}$$

where $p(\mathbf{r}, \boldsymbol{\eta} \mid f_d) = p(\mathbf{r} \mid f_d, \boldsymbol{\eta}) p(\boldsymbol{\eta})$ due to the independence between f_d and $\boldsymbol{\eta}$, and $p(\boldsymbol{\eta} \mid \mathbf{r}, \hat{f}_d^{(i-1)})$ can be derived as [9], [15],

$$\begin{aligned} & p(\boldsymbol{\eta} \mid \mathbf{r}, \hat{f}_d^{(i-1)}) \\ & = \frac{\mathcal{C}_1}{\pi^{(K+1)} \det(\sigma_w^2 (\mathbf{G}^{(i)})^{-1})} \exp\{-\frac{1}{\sigma_w^2} (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}^{(i)})^H (\mathbf{G}^{(i)})^{-1} (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}^{(i)})\} \end{aligned} \tag{11}$$

In (11), $\mathbf{G}^{(i)} = \mathbf{D}_f^{(i-1)} \mathbf{V}^T \mathbf{V} \mathbf{D}_f^{(i-1)} + \sigma_w^2 \mathbf{R}_\eta^{-1}$ is a derivative covariance matrix of size $K+1$, where $\mathbf{D}_f^{(i-1)}$ is the diagonal matrix with diagonal elements $\{1, \hat{f}_d^{(i-1)}, \cdots, (\hat{f}_d^{(i-1)})^K\}$. In addition,

$$\hat{\boldsymbol{\eta}}^{(i)} = (\mathbf{G}^{(i)})^{-1} \mathbf{D}_f^{(i-1)} \mathbf{V}^T \mathbf{r} \tag{12}$$

can be regarded as the *i*-th tentative estimate of the vector of the channel coefficients. In addition, $\mathcal{C}_1$ is a constant independent of $\boldsymbol{\eta}$ and f_d. Inserting (11) into (10) and dealing with the integration of $\boldsymbol{\eta}$, the expected LLF is then rewritten as follow [15].

$$\begin{aligned} Q(f_d \mid \hat{f}_d^{(i-1)}) = & \frac{\mathcal{C}_2}{\sigma_n^2} \big(2\Re\{\mathbf{r}^H \mathbf{V}\mathbf{D}_f \hat{\boldsymbol{\eta}}^{(i)}\} \\ & - \mathrm{Tr}\{\mathbf{D}_f \mathbf{V}^T \mathbf{V} \mathbf{D}_f [\sigma_w^2 (\mathbf{G}^{(i)})^{-1} + \hat{\boldsymbol{\eta}}^{(i)} (\hat{\boldsymbol{\eta}}^{(i)})^H]\} \big) + \mathcal{C}_3 \end{aligned} \tag{13}$$

where $\mathcal{C}_2$ and $\mathcal{C}_3$ are independent of f_d.

B. M-Step—The Doppler Spread Estimation Step

The *M-Step* then aims to find the *i*-th tentative Doppler spread estimate

$$\begin{aligned} \hat{f}_d^{(i)} & = \arg\max_{f_d} Q(f_d \mid \hat{f}_d^{(i-1)}) \\ & = \arg\max_{f_d} \big(2\Re\{\mathbf{r}^H \mathbf{V}\mathbf{D}_f \hat{\boldsymbol{\eta}}^{(i)}\} \\ & \quad - \mathrm{Tr}\{\mathbf{D}_f \mathbf{V}^T \mathbf{V} \mathbf{D}_f [\sigma_w^2 (\mathbf{G}^{(i)})^{-1} + \hat{\boldsymbol{\eta}}^{(i)} (\hat{\boldsymbol{\eta}}^{(i)})^H]\} \big) \end{aligned} \tag{14}$$

It is found that $Q(f_d \mid \hat{f}_d^{(i-1)})$ is an even-order univariate polynomial with respect to f_d. The optimization problem of (14), i.e. $\max Q(f_d \mid \hat{f}_d^{(i-1)})$, therefore can be solved by the sum-of-squares (SOS) decomposition [16]. The maximizer $\hat{f}_d^{(i)}$ can then be obtained by polynomial root-finding algorithms. As mentioned above, for the existing ML-based Doppler estimation schemes, the maximization of the likelihood functions of the Doppler spread is a non-convex problem due to the log function and the matrix inverse, such that it requires heuristic or exhaustive search for the solution. In the M-step, with the help of the taylor expansion, we transform the ML Doppler estimation to a univariate polynomial optimization problem.

Assumed that the EM algorithm converges in the i^*-th iteration, the channel and Doppler spread estimation is then completed by

$$\begin{cases} \hat{\mathbf{h}} = \mathbf{V}\mathbf{D}_f^{(i^*)} \hat{\boldsymbol{\eta}}^{(i^*)} \\ \hat{f}_d = \hat{f}_d^{(i*)} \end{cases} \tag{15}$$

C. Initialization of the Proposed Algorithm

The convergence rate, equivalently the number of iterations, of the proposed algorithm highly depends on the initial value of the Doppler spread $f_d^{(0)}$. Although the ML-based Doppler spread estimation schemes in [7] and [8] outperform other approaches, they are not suitable for initialization due to much higher computational complexity. In this work, we adopt a simple but effective Doppler estimation scheme proposed in [3], namely the sample-covariance-based (SC) scheme, to acquire the initial Doppler spread estimate. In addition, in most mobile scenarios, user/devices mobility, as well as Doppler spread, mildly changes. Therefore, for successive channel and Doppler spread estimation, the Doppler spread estimate of the previously received sequence can be used as the initial Doppler spread for the EM estimation algorithm of the presently received sequence.

V. SIMULATION RESULTS

In the simulations, a small number of symbols, i.e. $N = 50$, is used for channel and Doppler spread estimation to support a short-delay estimation outcome. This corresponds to an observation interval 3.33 ms in WCDMA systems with sample duration T_s=66.67 μs. The target region of f_d is set as 30Hz $\leq f_d \leq$ 278Hz, corresponding to a maximal velocity about 150km/hr at the 2GHz band; and the approximation order for the Taylor-based channel modeling is set as $K = 10$. We compare the performance with that of the ideal MMSE channel estimator [2] and the ML Doppler spread estimator [7].

Fig. 1 shows the mean square error (MSE) of the channel estimation outcome $\hat{\mathbf{h}}$ versus the exact Doppler spread f_d^* under SNR=5dB and 15dB. We compare the performance of the proposed estimator with that of the suboptimal MMSE estimator which directly uses the Doppler spread estimate provided by the SC scheme, and is denoted as the MMSE-SC scheme. We also evaluate the MSE of the ideal MMSE [2] channel estimator as a performance baseline. For the both cases with SNR=5dB and 15dB, the proposed scheme obviously yields lower MSE values than the MMSE-SC scheme does. The performance gain comes from the EM-based iterative channel and Doppler spread estimation. In addition, Fig. 1 shows that the proposed scheme provides channel estimation performance very close to that of the ideal MMSE scheme.

Fig. 2 and Fig. 3 show the mean value and the normalized MSE (NMSE), respectively, of the Doppler estimation output of the proposed scheme, the ML Doppler estimator [7], and the SC scheme for SNR=5dB. It is noted that the NMSE for a Doppler spread estimator $\hat{f}_d$ is defined as

$$\mathrm{NMSE} \triangleq \frac{E\{|\hat{f}_d - f_d^*|^2\}}{(f_d^*)^2} \tag{16}$$

As shown in Fig. 2, we find that the proposed scheme yields small biased estimation results in low Doppler spread region; however, as the value of f_d^* increases, the degree of biasness decreases. It is also found in Fig. 3 that the NMSE of the proposed scheme approaches that of the ML estimator for larger values of f_d^*. In addition, compared to the SC estimator, the proposed scheme provides more accurate results for Doppler spread estimation.

VI. CONCLUSIONS

In this work, we have developed a joint ML channel and Doppler spread estimation scheme over time-varying flat-fading channels based on an approximate channel model and the EM algorithm. The proposed scheme alternates between the E-step and the M-step to attain both channel estimates and a Doppler spread estimate to jointly maximize the LLF. Different from the existing ML-based Doppler estimation schemes which face non-convex optimization problems, the proposed Doppler spread estimator corresponds to a univariate polynomial optimization problem and can be solved by the SOS decomposition. Simulation results show that the proposed scheme achieves accurate performance of both channel estimation and Doppler spread estimation within a short estimation interval. The joint estimation method thus is suitable to mobile receivers for burst-mode data reception.

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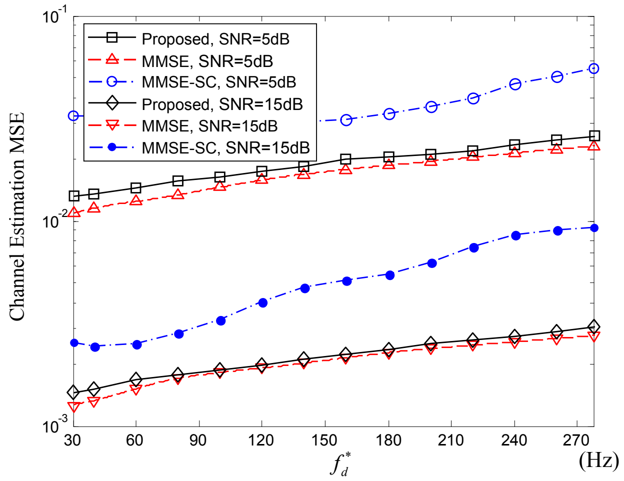

Figure 1. Performance comparison in terms of the channel estimation MSE versus the exact f_d^* for N=50 under SNR=5dB and 15dB, respectively.

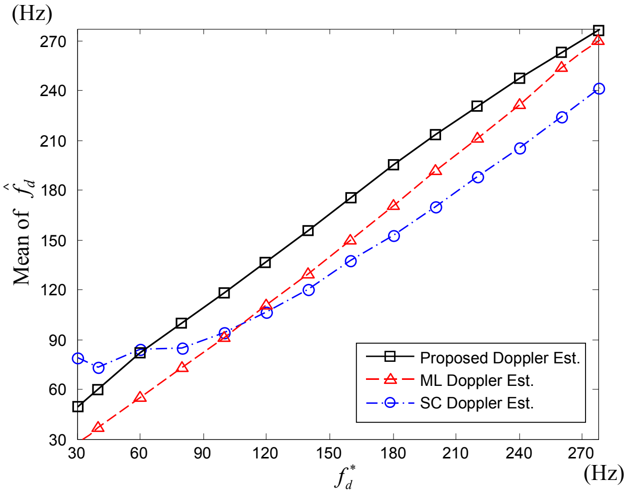

Figure 2. Performance comparison in terms of the mean of Doppler spread estimation versus the exact f_d^* for N=50 and SNR=5dB.

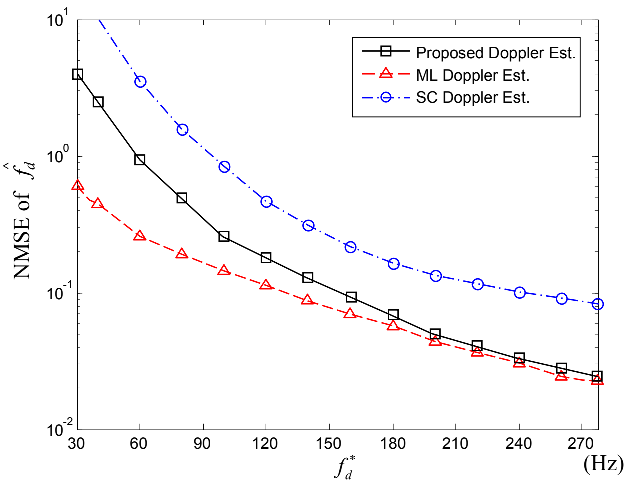

Figure 3. Performance comparison in terms of the NMSE of Doppler spread estimation versus the exact f_d^* for N=50 and SNR=5dB.