Opportunistic Spectrum Access with Hopping Transmission Strategy: A Game Theoretic Approach

Mahsa Derakhshani, Tho Le-Ngoc
Department of Electrical and Computer Engineering, McGill University, Montreal, Quebec, Canada H3A 2A7
E-mail: mahsa.derakhshani@mail.mcgill.ca; tho.le-ngoc@ mcgill.ca

***Abstract* – This paper presents a study on opportunistic spectrum access for secondary users (SUs) from a game-theoretic learning perspective. In consideration of the random return of primary users, it is assumed that a SU dynamically hops over multiple idle frequency-slots of a licensed frequency band, each with an adaptive activity factor. The problem of finding optimal activity factors of SUs is cast in a game-theoretic framework and is formulated as a potential game. Subsequently, the existence, feasibility and optimality of Nash Equilibrium (NE) are investigated analytically. Furthermore, an algorithm is developed in which each SU independently adjusts its activity factors based on the best response dynamics by learning other SUs' behavior from locally available information. Aiming to establish stability for the proposed algorithm, the convergence with probability 1 to an arbitrarily small neighborhood of the globally optimal solution is investigated with analysis and simulation.**

I. INTRODUCTION

It has been confirmed that conventional fixed spectrum allocation strategies lead to spectrum under-utilization. This motivates the idea of opportunistic spectrum access (OSA) to exploit instantaneous spectrum availabilities of licensed spectrum. Thus, for improving spectrum utilization, OSA would allow SUs to identify and utilize temporal spectrum opportunities while limiting the level of interference to the high-priority licensed users (i.e., PUs).

Recently, the design of OSA schemes has received considerable attention [1]-[2]. There are several OSA strategies presented in the literature that allow SUs to choose an idle frequency-slot (or channel) to sense and access for an entire transmission duration assuming that both PUs and SUs have the same transmission time-slot structure [3]-[5]. The assumption of synchronous slotted transmission structure between PU and SU networks is not a sensible assumption since it needs well coordination in time between PU and SU networks.

Without synchronization between PUs and SUs, PU activity in a channel with respect to SUs can be represented as a continuous ON/OFF random process. As a result, even if an idle channel is perfectly detected and used by a SU, a collision can still occur since the PU may reoccupy that idle channel at any time during the SU transmission. In [6], the collision probability caused by SUs to the PU is kept limited in the proposed OSA scheme assuming no time coordination between SUs and PUs. However, the effects of collision caused by the PU return on the SU performance are not considered.

In [7], we propose an adaptive hopping transmission strategy for SUs, aiming to reduce the effects of collision between PU and SU due to PU return. In the proposed scheme, the SU accesses multiple idle channels, each with a different sojourn time (called activity factor). Taking into account the PU activity and channel characteristics, the SU activity factor optimization problem for maximizing the overall SU throughput is formulated to develop suitable OSA algorithms for SUs in a central manner. Subsequently, in [8], we present a learning-based algorithm in which each SU independently adjusts its activity factors by learning other SUs' behavior from locally available information. It is shown that the proposed algorithm converges to the global optimum although the convergence time is not quite short.

In a highly dynamic environment such as cognitive radio networks, a key challenge is to design a proper OSA algorithm for SUs, which converges to the globally or near optimal solution fast enough in a distributed manner. To this end, in this paper, the OSA design is studied in a game-theoretic framework which enables distributed implementation with fast convergence. More specifically, the activity factor optimization problem is formulated as an exact potential game.

Via potential game framework, it is proved that the formulated game admits at least one pure strategy NE, i.e., the stable operating point of the system. Subsequently, the feasibility and efficiency of NE are investigated. Under assumption of having perfect information, we establish the convergence of the best response iterations to a pure NE which is not essentially Pareto optimal. However, in a learning-based noisy game, it is shown that the best response iterations will finally stay in a neighborhood of the Pareto optimal NE with probability one.

The remainder of this paper is organized as follows. Section II presents an overview of the system modeling and introduces the activity factor optimization problem from [7]. In Section III, the OSA design is formulated as an exact potential game. Then, the existence, feasibility and efficiency of NE of the formulated game are analyzed. Section IV investigates the convergence of the formulated game based on the best response iterations in the presence of perfect information and also noisy estimations. Finally, Section V presents the conclusions.

II. SYSTEM MODEL

Consider a frequency band licensed to PUs which is divided into N_p non-overlapping frequency-slots (or channels), each with bandwidth B, and an ad-hoc secondary network with N_s SUs looking for temporal spectrum opportunities in these N_p channels.

SUs are assumed to follow a slotted transmission scheme. Each time-slot of equal duration T consists of two periods:

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sensing of duration τ and *transmission* of duration $(T-\tau)$. In this paper, we assume sufficiently accurate sensing with negligible PU miss-detection. Let $\mathcal{N}_a := \{1, \dots, N_a\}$ denote the set of N_a channels that are detected idle at the beginning of each time-slot, and hence, can be utilized by N_s SUs. Let $g_{k,k}^i$ denote the power gain of the SU link k in the channel i. Consider an assumed block flat fading situation in which $g_{k,k}^i$ remains unchanged during a given time-slot but independently varies from one time-slot to another. The transmission capacity of SU k in channel i is $C_k^i = Blog\left(1 + P_k^i g_{k,k}^i / n_k^i\right)$ where P_k^i and n_k^i represent the signal power and noise power for the SU k in the channel i, respectively.

In general, the PU can use a time frame different from that of SUs. The PU activity in a given channel is independent and, from the SU viewpoint, it can be modeled as a two-state continuous random process with the OFF (0) and ON (1) states representing the *idle* and *busy* periods of the PU. Hence, there will be a non-zero probability of PU return (i.e., α_i) during transmission period to the channel i while it was detected idle by SUs in the sensing period.

On one hand, PU return causes collision for the PU. In order to protect the PU transmission quality, α_i can be kept smaller than a required level by designing proper transmission duration (i.e., $T-\tau$). On the other hand, the PU return may destroy the entire on-going SU transmission in the channel i. To avoid such a serious data loss due to PU return, we assume an adaptive transmission strategy for SUs in which a SU dynamically hops over *multiple idle* channels, each with an unequal sojourn time (called activity factor) to be determined, so that possible PU return in a channel may destroy only a small fraction of the SU transmission that can be recovered by erasure-correction coding [7]. Let $\beta_k^i (0 \le \beta_k^i \le 1)$ denote the activity factor of SU k in channel $i \in \mathcal{N}_a$ during a transmission slot.

SUs share idle channels using a modified carrier sensing multiple access (CSMA) scheme based on their activity factors. In the modified CSMA scheme, each transmission slot is divided into S equal sub-slots with length $\frac{T-\tau}{S}$, labeled t_1,..,t_S. For every sub-slot t_j in channel i, the SU k performs the following steps:

1-Generate a Bernoulli random variable $x_k^i(t_j)$ with success probability β_k^i to be determined. If $x_k^i(t_j) = 0$, the SU k will not transmit in the subslot t_j. If $x_k^i(t_j) = 1$, the SU k will proceed to the next step.

2-Generate a back-off time $\delta_k^i(t_j)$ according to a uniform distribution in the interval (0, δ_{max}).

3-After expiry of back-off time, sense the channel i, if it is idle, transmit.

In the proposed CSMA scheme, one SU with the smallest back-off time among the SUs who compete for the same sub-slot (i.e., $x_k^i(t_j) = 1$) will succeed and transmit in this sub-slot. Let $y_k^i(t_j)$ be a binary random variable representing the capturing status: $y_k^i(t_j) = 1$ if the SU k captures channel i in sub-slot t_j; otherwise, $y_k^i(t_j) = 0$.

In order to manage competition and control contention among SUs in the modified CSMA scheme, the total activity factors of different SUs in each idle channel is kept smaller than 1. Thus, $\sum_{k=1}^{N_s} \beta_k^i \le 1,\ i = 1, \dots, N_a$. Furthermore, the sum of all activity factors of each SU k over all idle channel is set smaller or equal to one, i.e., $\sum_{i=1}^{N_a} \beta_k^i \le 1$. In other words, the maximum total access probability of each user is one.

The normalized transmission rate of SU k in channel i is $\beta_k^i C_k^i$ since it transmits partially with activity factor β_k^i. Taking into account the possible loss in SU transmission due to PU return in idle channel i with probability α_i, the throughput of SU k in the idle channel i, is defined as its *successful* transmission rate $f_k^i = \beta_k^i C_k^i \left(1 - \beta_k^i \alpha_i\right)$ where $\beta_k^i \alpha_i$ expresses the probability that the SU k experiences transmission loss due to PU return.

In [7], aiming to determine the optimal activity factors, the optimization problem is formulated to maximize the overall throughput of all SUs under constraints of $\sum_{k=1}^{N_s} \beta_k^i \le 1$ in each idle channel and $\sum_{i=1}^{N_a} \beta_k^i \le 1$ for all SUs. More specifically,

$$max_{\boldsymbol{\beta}} \sum_{k=1}^{N_s} \sum_{i=1}^{N_a} \beta_k^i C_k^i \left(1 - \beta_k^i \alpha_i\right) \tag{1a}$$

subject to

$$\sum_{k=1}^{N_s} \beta_k^i \le 1, \quad i = 1, \dots, N_a \tag{1b}$$

$$\sum_{i=1}^{N_a} \beta_k^i \le 1, \ \ k = 1, \dots, N_s \tag{1c}$$

$$0 \le \beta_k^i \le 1, \quad i = 1, \dots, N_a, k = 1, \dots, N_s \tag{1d}$$

Then, in [8], we present a learning-based algorithm to find the optimal activity factors in a distributed manner, although the convergence time is not quite short.

In a highly dynamic environment such as cognitive radio networks, it is practically essential to find a reasonably good solution which can be obtained fast enough. To this end, in this paper, the OSA design is studied in a game-theoretic framework which enables distributed implementation and fast convergence to a reasonably good solution.

III. GAME-THEORETIC OSA

In this section, we are interested in formulating the OSA design from a game theoretical perspective aiming to present a distributed scheme. More specifically, we consider a strategic non-cooperative game in which the players are SUs.

According to the optimization problem in (1), each SU could simply maximize its transmission rate (i.e., $\sum_{i=1}^{N_a} \beta_k^i C_k^i \left(1 - \beta_k^i \alpha_i\right)$). However, SUs cannot select activity factors which violate coupled constraints in (1b). Since it is difficult for SUs to identify feasible activity factors in advance, we construct an alternative payoff function of SU k as

$$u_k = \sum_{i=1}^{N_a} \beta_k^i C_k^i \left(1 - \beta_k^i \alpha_i\right) - \sum_{i=1}^{N_a} \mu_i \theta\left(\sum_{j=1}^{N_s} \beta_j^i - 1\right) \qquad (2)$$

where $\theta(x) = \begin{cases} x & x \ge 0 \\ 0 & x < 0 \end{cases}$ and μ_i's are positive scalars, which are chosen sufficiently large to assure $u_k < 0, \forall k \in \mathcal{N}_s$ if only $\sum_{j=1}^{N_s} \beta_j^i > 1$. The second term of (2) represents coupled constraints in (1b) by severely punishing the SU who violates each of them.

Let define the activity factor vector $\boldsymbol{\beta}_k = \left[\beta_k^1, \ldots, \beta_k^{N_a}\right]$ as the strategy at SU k and $\boldsymbol{\beta}_{-k}$ as the strategy of all SUs excluding the SU k. Furthermore, the admissible strategies of SU k is defined as

$$\mathcal{B}_k = \left\{\boldsymbol{\beta}_k : \beta_k^i \in \left\{0, \tfrac{1}{S}, \tfrac{2}{S}, \ldots, 1\right\}, \forall i \in \mathcal{N}_a, \textstyle\sum_{i=1}^{N_a} \beta_k^i \le 1\right\} \qquad (3)$$

Note that β_k^i takes discrete values in the proposed CSMA-based algorithm. Then, we can define the non-cooperative game for OSA in cognitive radio networks as

$$\mathcal{G} = \left[\mathcal{N}_s, \{\mathcal{B}_k\}_{k \in \mathcal{N}_s}, \{u_k\}_{k \in \mathcal{N}_s}\right] \qquad (4)$$

where $\mathcal{N}_s \coloneqq \{1, \ldots, N_s\}$ is the set of players of the game (i.e. SUs), $\mathcal{B}_k$ is the activity factor strategy set of the SU k and u_k is the corresponding payoff function of the SU k.

An exact potential game is a strategic game in which the incentive of all players to change their strategies can be expressed in a global function so-called the potential function. The potential games are easy to analyze since improving each player's utility also increases the value of a potential function [9]. In the following theorem, we demonstrate that the game $\mathcal{G}$ falls into the framework of exact potential games.

Theorem 1: $\mathcal{G}$ is an exact potential game with the potential function,

$$\Phi = \sum_{j=1}^{N_s} \sum_{i=1}^{N_a} \beta_j^i C_j^i \left(1 - \beta_j^i \alpha_i\right) - \sum_{i=1}^{N_a} \mu_i \theta\left(\sum_{j=1}^{N_s} \beta_j^i - 1\right) \qquad (5)$$

Proof: It is clear that the game $\mathcal{G}$ satisfies the exact potential game definition [9],

$$\begin{aligned} &u_k(\boldsymbol{\beta}_k, \boldsymbol{\beta}_{-k}) - u_k(\boldsymbol{\beta}'_k, \boldsymbol{\beta}_{-k}) \\ &= \Phi(\boldsymbol{\beta}_k, \boldsymbol{\beta}_{-k}) - \Phi(\boldsymbol{\beta}'_k, \boldsymbol{\beta}_{-k}), \forall \boldsymbol{\beta}_k, \boldsymbol{\beta}'_k \in \mathcal{B}_k, \forall k \in \mathcal{N}_s \end{aligned} \qquad (6)$$

Thus, $\mathcal{G}$ is an exact potential function and Φ is the potential function of $\mathcal{G}$. ■

The fundamental solution concept in a strategic game is called Nash Equilibrium (NE) from which no player can improve its utility by changing its own strategy unilaterally [10]. In other words, Nash Equilibria represent steady states of the game. Mathematically, a strategy profile $\boldsymbol{\beta}^* = \{\boldsymbol{\beta}_k^*\}_{k=1}^{N_s}$, is a NE if and only if

$$u_k(\boldsymbol{\beta}_k^*, \boldsymbol{\beta}_{-k}^*) \ge u_k(\boldsymbol{\beta}'_k, \boldsymbol{\beta}_{-k}^*), \forall \boldsymbol{\beta}'_k \in \mathcal{B}_k, \forall k \in \mathcal{N}_s \qquad (7)$$

We are interested to investigate the existence and characteristics including feasibility and efficiency of NE of the game $\mathcal{G}$.

A. Existence

First of all, the existence of NE of the game $\mathcal{G}$ is studied in the following theorem based on the properties of the potential games.

Theorem 2: The game $\mathcal{G}$ admits at least one pure strategy NE.

Proof: This Theorem comes directly from Corollary 4 in [11], which states every finite potential game $\mathcal{G}$ has at least one pure strategy NE. ■

Remark 1: In general, the pure strategy NE of game $\mathcal{G}$ may not be unique.

B. Feasibility

Since the optimization problem in (1) has coupled constraints in (1b) which are merged in the payoff functions in the formulated game $\mathcal{G}$, it is required to verify if the pure strategy NEs are feasible. Thus, the following Theorem presents conditions that assure the feasibility of pure strategy NEs.

Theorem 3: All pure strategy NEs of the game $\mathcal{G}$ must be feasible if

$$\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a \qquad (8)$$

where $\mu_{th} = \max_{i \in \mathcal{N}_a, k \in \mathcal{N}_s, \boldsymbol{\beta}_k \in \mathcal{B}_k, \beta_k^i \neq 0} \left(\frac{\sum_{l=1}^{N_a} \beta_k^l C_k^l \left(1 - \beta_k^l \alpha_l\right)}{\beta_k^i}\right)$.

Proof: See APPENDIX A for the proof.

This Theorem assures that, by properly designing μ_i's, the payoff functions in (2) can guarantee the feasibility of the steady states of the system.

C. Efficiency

The other aspect that we are interested to study is how efficient the NE of game $\mathcal{G}$ is in comparison with the optimal solution of (1). The following Theorem specify the relation of the optimal solution and the NE of the game $\mathcal{G}$.

Theorem 4: The optimal solution of (1) is a Pareto optimal pure strategy NE of $\mathcal{G}$ if $\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a$.

Proof: See APPENDIX B for the proof.

Remark 2: In the next section, a learning-based iterative algorithm is proposed which enables the convergence to the Pareto optimal pure strategy NE.

IV. BEST RESPONSE DYNAMICS

In this section, an iterative game-theoretic algorithm for activity factor selection is presented to reach an equilibrium of the game $\mathcal{G}$, based on the best response dynamics. In particular, in a round robin fashion, SUs iteratively update their activity factors based on the best response dynamics defined as

$$\boldsymbol{\beta}_k[t+1] = \max_{\boldsymbol{\beta}'_k \in \mathcal{B}_k} u_k(\boldsymbol{\beta}'_k, \boldsymbol{\beta}_{-k}[t]) \qquad (9)$$

Our goal is to study the convergence of the proposed game-theoretic algorithm under knowledge of perfect information of current strategies of other SUs (i.e., $\boldsymbol{\beta}_{-k}[t]$) and also noisy information.

A. *Perfect Information*

From (2) and (9), it is obvious that each SU needs to know the sum of activity factors of all N_s SUs in the idle channel i, $\sum_{k=1}^{N_s} \beta_k^i(t)$, to update its activity factor. First, to study the convergence, we assume that SUs have the perfect information of other SUs' activity factors.

Theorem 5: The iterative game-theoretic algorithm under best response dynamics converges to a pure strategy NE of the game $\mathcal{G}$ from any initial strategy point if $\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a$.

Proof: Based on Theorem 19 in [10], the finite exact potential game will converge to a pure strategy NE in finite steps. Accordingly, the best response iterations will converge to a pure strategy NE. ■

According to Theorem 5, the best response iterations will converge to a pure strategy NE with perfect knowledge of the sum of activity factors of all N_s SUs in the idle channel i, $\sum_{k=1}^{N_s} \beta_k^i(t)$. Such information can be obtained with the aid of a central coordinator or heavy exchange of overhead information which causes high complexity and results in an un-scalable system. It is thus crucial that SUs learn this information to adjust their activity factors.

B. *Noisy Information*

In [8], we study how to use the capturing status feedbacks of CSMA scheme, $y_k^i(t_j)$, to estimate the sum of activity factors of all SUs in each channel. It is shown that $\beta^i = \sum_{k=1}^{N_s} \beta_k^i$ can be updated after each window of S' sub-slots as $\hat{\beta}^i \simeq \beta_k^i + \left(S'.\beta_k^i / \left(\sum_{l=f+1}^{f+S'} y_k^i(t_l)\right)\right) - 1$. Since estimation with limited samples suffers from random errors, it is shown that $\hat{\beta}^i = \beta^i + w$ where $\mathrm{E}[w]$ and $\mathrm{var}[w]$ are of $\mathrm{O}((S')^{-1})$ [8].

From (2), it is clear that the estimation noise of β^i will cause a bias (i.e., $b = \max(0, w)$) in u_k, and hence, best response iterations in (9) will also involve random errors. Since the first derivative of u_k is finite, the bias and variance of the random noise in best response iterations should be also of $\mathrm{O}((S')^{-1})$.

As shown by Theorem 3 of [12], in potential games, a bounded noise will asymptotically ensure the convergence of the best response iterations to a neighborhood of the globally optimal solution even if a potential game has suboptimal NE. That is because suboptimal NE points are less stable than the Pareto optimal NE (i.e., global optimum) in a sense that a small noise can cause the best response iterations diverge from the suboptimal NE while moving in the direction toward the Pareto optimal NE.

Similarly, in the proposed algorithm, best response iterations involve errors although they are random with bounded bias and variance. With a sufficiently large or an increasing estimation window (i.e., S'), the random noise can be approximated as a bounded noise. Therefore, it is expected that the best response iterations converge to the global optimum. Mathematically, this can be presented as the following claim.

Claim 1: $\forall \varepsilon > 0$, an estimation window size can be selected (i.e., $\exists S' > 0$) such that $\lim_{t\to\infty} \inf \Phi(\boldsymbol{\beta}[t]) \geq \Phi_{\max} - \varepsilon$ with probability of 1.

This claim declares that, by properly designing an estimation window size (i.e. S'), best response iterations can get arbitrarily close to the globally optimal solution of (1) which is also the maximizer of the potential function (i.e., Φ) assuming $\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a$.
To confirm this claim, numerical results are also provided to verify the convergence of noisy best response iterations to the global optimum. In these examples, we assume independent channels with the same bandwidth $B = 1$ and the same α_i. We set the similar $SNR = P_k^i / n_k^i = 10\ dB$ for individual SUs. Furthermore, we assume $N_s = 3$, $N_a = 3$ and $\alpha_i = 0.1$.

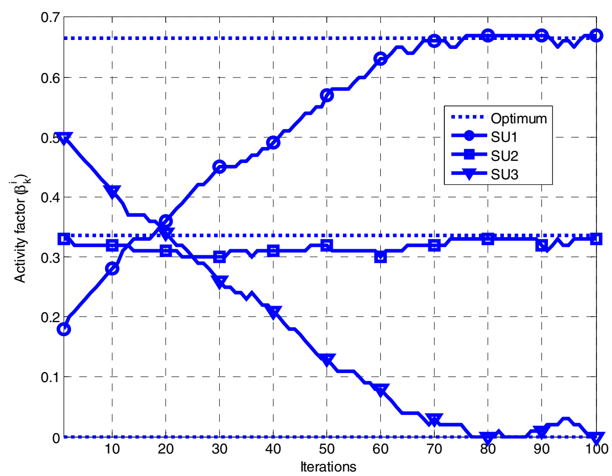

Fig. 1. Convergence of the SU activity factors of SUs.

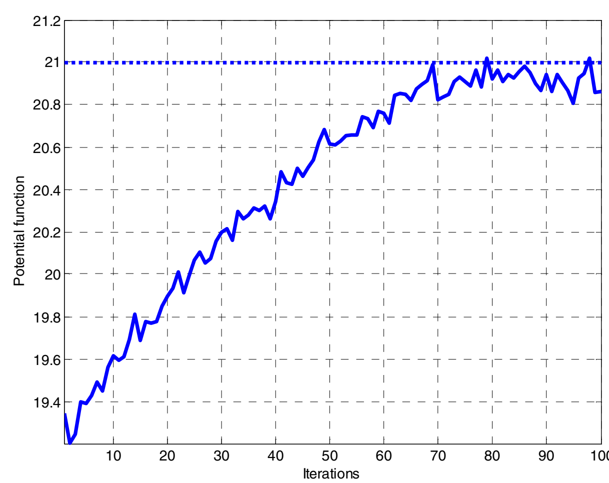

Fig. 2. Convergence of the potential function.

Figure 1 demonstrates the convergence process of the activity factors of three SUs in one of the idle licensed channels. Furthermore, Figure 2 shows the convergence of the potential function. They confirm that the best response iterations will stay in a neighborhood of the global optimum. As can be observed, the OSA scheme takes merely around 100 iterations to quickly converge to the optimum solutions. Note that each iteration corresponds to a complete round-robin update by all the SUs.

V. CONCLUSIONS

In this paper, we studied OSA for SUs transmitting over multiple idle channels, each with an adaptive activity factor using a potential game model. Via potential game framework, it is established that the formulated game admits at least one pure strategy NE. In consideration of coupled constraints among SUs, sufficient conditions are presented to assure feasibility of the pure strategy NE. In addition, it is proved that the globally optimal solution is the Pareto optimal NE. Furthermore, the convergence properties of the best response iterations of the formulated game are investigated. Assuming perfect knowledge of moves previously made by all SUs, it is proved that best response iterations converge to the pure strategy NE which is not essentially the global solution. However, in a noisy game by learning other SUs' behavior, best response iterations converge with probability one to a neighborhood of the global optimum.

APPENDIX

A. *Proof of Theorem 3*

Let assume that $\boldsymbol{\beta} = \{\boldsymbol{\beta}_k\}_{k=1}^{N_s}$ is a NE of game $\mathcal{G}$, but it is not feasible, i.e., it violates at least one of the constraints in (1b). Further suppose that m is the index of channel which has the most severe violation, i.e., $m = \text{argmax}_{i\in\mathcal{N}_a}\left(\sum_{j=1}^{N_s}\beta_j^i - 1\right)$.

Let consider $k \in \mathcal{N}_s$ such that $\beta_k^m \neq 0$. Then, $\{\boldsymbol{\beta}_k\}_{k=1}^{N_s} \neq \{\mathbf{0}\}_{k=1}^{N_s}$. By definition,

$$u_k(\boldsymbol{\beta}_k, \boldsymbol{\beta}_{-k}) = \sum_{i=1}^{N_a}\beta_k^i C_k^i\left(1-\beta_k^i\alpha_i\right) \\ -\mu_m\left(\sum_{j=1}^{N_s}\beta_j^m - 1\right) - \sum_{i=1,i\neq m}^{N_a}\mu_i\theta\left(\sum_{j=1}^{N_s}\beta_j^i - 1\right) \tag{10}$$

and

$$u_k(\mathbf{0}, \boldsymbol{\beta}_{-k}) = -\sum_{i=1}^{N_a}\mu_i\theta\left(\sum_{j=1,j\neq k}^{N_s}\beta_j^i - 1\right) \tag{11}$$

From $\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a$., we have

$$\sum_{i=1}^{N_a}\beta_k^i C_k^i\left(1-\beta_k^i\alpha_i\right) - \mu_m\beta_k^m < 0 \tag{12}$$

Subsequently,

$$\sum_{i=1}^{N_a}\beta_k^i C_k^i\left(1-\beta_k^i\alpha_i\right) - \mu_m\left(\sum_{j=1}^{N_s}\beta_j^m - 1\right) + \\ \mu_m\theta\left(\sum_{j=1,j\neq k}^{N_s}\beta_j^m - 1\right) < 0 \tag{13}$$

Furthermore, $\forall i \in \mathcal{N}_a$,

$$\theta\left(\sum_{j=1,j\neq k}^{N_s}\beta_j^i - 1\right) \leq \theta\left(\sum_{j=1}^{N_s}\beta_j^i - 1\right) \tag{14}$$

Thus, from (13) and (14),

$$u_k(\boldsymbol{\beta}_k, \boldsymbol{\beta}_{-k}) - u_k(\mathbf{0}, \boldsymbol{\beta}_{-k}) < 0 \tag{15}$$

This contradicts the assumption that $\boldsymbol{\beta} = \{\boldsymbol{\beta}_k\}_{k=1}^{N_s}$ is a NE of game $\mathcal{G}$. Hence, all pure strategy NE must be feasible if $\mu_i > \mu_{th}$. ■

B. *Proof of Theorem 4*

Let assume that $\boldsymbol{\beta} = \{\boldsymbol{\beta}_k\}_{k=1}^{N_s}$ is the optimal solution of (1). Assuming $\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a$, $\boldsymbol{\beta}$ is the maximizer of the potential function Φ. Based on the Theorem 2 in [9], the maximizer of the potential function is the NE of the potential game. Hence, $\boldsymbol{\beta}$ is the NE of the game $\mathcal{G}$.

Subsequently, we need to establish that $\boldsymbol{\beta}$ is the Pareto optimal NE. Let assume that $\boldsymbol{\beta}$ is not Pareto optimal, and then, there exists an arbitrary strategy profile $\boldsymbol{\beta}' = \{\boldsymbol{\beta}'_k\}_{k=1}^{N_s}$ such that

$$u_k(\boldsymbol{\beta}'_k, \boldsymbol{\beta}'_{-k}) \geq u_k(\boldsymbol{\beta}_k, \boldsymbol{\beta}_{-k}), \forall k \in \mathcal{N}_s, k \neq j, \tag{16}$$

and, for some j,

$$u_j\left(\boldsymbol{\beta}'_j, \boldsymbol{\beta}'_{-j}\right) > u_j\left(\boldsymbol{\beta}_j, \boldsymbol{\beta}_{-j}\right), \tag{17}$$

As a result,

$$\sum_{k=1}^{N_s}u_k(\boldsymbol{\beta}') > \sum_{k=1}^{N_s}u_k(\boldsymbol{\beta}), \tag{18}$$

Since $\boldsymbol{\beta}$ is a NE of the game $\mathcal{G}$, it is feasible based on Theorem 3 (i.e., $\sum_{i=1}^{N_a}\mu_i\theta\left(\sum_{j=1}^{N_s}\beta_j^i - 1\right) = 0$). Then, $\Phi(\boldsymbol{\beta}) = \sum_{k=1}^{N_s}u_k(\boldsymbol{\beta})$. Furthermore, Assuming $\mu_i > \mu_{th}, \forall i \in \mathcal{N}_a$, from (2) and (5), for an arbitrary $\boldsymbol{\beta}'$, we have $\Phi(\boldsymbol{\beta}') \geq \sum_{k=1}^{N_s}u_k(\boldsymbol{\beta}')$. Consequently, based on (18),

$$\Phi(\boldsymbol{\beta}') > \Phi(\boldsymbol{\beta}), \tag{19}$$

This contradicts the fact that $\boldsymbol{\beta}$ is the maximizer of the potential function Φ. Thus, the optimal solution of (1) is the Pareto optimal pure strategy NE of the game $\mathcal{G}$.■

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