Low Complexity Detectors for Cooperative Wireless Sensor Networks

Qasim Zeeshan Ahmed[†], Mohamed-Slim Alouini[†], and Sonia Aissa[‡]

Electrical Engineering Program

KAUST, Thuwal, Makkah Province, Saudi Arabia[†].

INRS, University of Quebec, Montreal, QC, Canada[‡].

Email: {qasim.ahmed,slim.alouini}@kaust.edu.sa[†]; sonia.aissa@ieee.org[‡]

Abstract—This paper investigates and compares the performance of wireless sensor networks (WSN) when sensors operate on the principles of cooperative communications. We consider a scenario where the source transmits signals to the destination with the help of L sensors. As the destination has the capacity of processing only U out of these L signals, U strongest signals are selected while the remaining (L-U) signals are suppressed. A preprocessing block similar to channel-shortening (CS) is proposed in this contribution. However, this preprocessing block employs rank-reduction technique instead of CS. This detector operates on the principles of principal components (PC). From our simulations it can be observed that this detector is capable of achieving a similar bit error rate (BER) performance as the full-rank MMSE detector with significantly lower complexity. It outperforms the CS-based detector in terms of BER performance when using fixed amplification factor. However, for variable gain amplification factor a tradeoff between the diversity gain and the receiver complexity can be observed. From the simulations it can be concluded that the BER performance of the PC-based detector when using variable gain amplification factor are better than that of the CS-based detector for lower signal to noise ratio.

I. INTRODUCTION

In wireless sensor networks (WSN) the fundamental task is to broadcast data from the origin sensor to the destination. However due to small size, low-power and low-cost of these sensors, a low-power signal is transmitted to the destination [1–3]. This low power signal can be attenuated by the propagation loss. So the best way to combat this problem is to broadcast this signal to as many sensors as possible [1,2]. These sensors will now form a distributed cooperative sensor network, enabling these devices to achieve spatial diversity which will help combat fading effects and improve network coverage [4].

Low-complexity cooperative diversity protocols have been developed and analyzed for cooperative communications in different operating conditions and environments. According to [4], the family of fixed relaying arrangements which have the lowest complexity as compared to all the other families consists of decode-and-forward (DF) and amplify-and-forward (AF) protocols. Furthermore, in [4], it has been proved that the low complexity AF protocols have the ability to achieve similar BER performance as the more complicated DF protocol. Therefore, in our contribution only AF protocol is considered.

Battery life in WSN is very important, therefore, low complexity detectors play a significant role for WSN [5].

Maximum likelihood (ML) detector is the optimal detector in terms of bit error rate (BER) [6]. However, due to the high complexity of ML detector, sub-optimal linear detectors are considered [6,7]. In sub-optimal linear detectors, minimum mean square error (MMSE) detection is preferred over other detectors, due to its improved BER performance [7]. It can be observed that as the number of sensors increases, the complexity of the MMSE detector becomes more extreme [7]. Therefore, in order to decrease the complexity and improve the spectral efficiency best sensor selection is proposed [8]. The best relay selection improves the spectral efficiency and decreases the computational complexity. In WSNs the main aim is to improve the BER performance while keeping the complexity as low as possible [9]. Therefore, best sensor selection will not be the optimal solution as it decreases the complexity but it offers a BER performance that is always worse than that of all participating sensors [9].

In this contribution, we consider the reduced-rank detection for WSNs, in order to reduce the detection complexity. Reduced-rank techniques have been widely applied to array processing [10], radar signal processing, direct-sequence code division multiple access(DS-CDMA) [7], and ultrawide bandwidth (UWB) systems [11]. It can be shown that the rank reduction techniques are capable of providing the flexibility for compromise between the computational complexity and the achievable BER performance. Therefore, in this contribution we investigate the BER performance of the WSNs using reduced-rank detection. Specifically, in this contribution principal component (PC)-based rank reduction techniques is employed.

Our study and simulation results show that the PC-based reduced-rank MMSE technique is capable of achieving a BER performance that is comparable to BER performance achieved by the full-rank MMSE, while at a significantly lower complexity. For sensors which employ fixed gain amplification factor, PC-based scheme outperform the channel shortening (CS)-based scheme with an equivalent computational complexity. For sensors which employ variable gain amplification factor, this technique perform better for low signal to noise ratio (SNR) as compared to CS-based scheme with a similar computational complexity. However, at higher SNR the performance is worse than the CS-based scheme. By increasing the rank of this technique, it can be observed that this loss in

BER performance can be avoided.

Remainder of this paper is organized as follows. In Sec. II, a detailed explanation of the WSN model and the basic assumptions are elaborated. Sec. III investigates the conventional MMSE detectors for a WSN environment. Furthermore, in Sec. IV the design of transformation matrix is presented. The performance of all the detectors are compared in Sec. V. Finally, the paper is concluded in Sec. VI.

II. WSN SYSTEM MODEL

The basic WSN system model considered in this work is shown in Fig. 1. As illustrated, the source sensor S transmits data to the destination D with the assistance of L sensors. These sensors operate on the principle of cooperative communications, therefore, each sensors amplifies and forwards the data to the destination. There exists no direct link between the source sensor and the destination. The channel gains for the source-sensor to lth relay and lth relay to destination are denoted as h_{SR_I} and h_{R_ID} , and are assumed to be mutually independent and follow the Rayleigh fading distribution model. The variance of these channels is $\sigma_{SR_l}^2$ and $\sigma_{R_lD}^2$, respectively. The data transmission takes place in two phases as shown in Fig. 1. S transmits data to the sensors in phase-I, while the data is amplified and forwarded to D through the sensors in phase-II. In order to minimize interference between the sensors, orthogonality is assumed and this can be achieved in the frequency or time domains [13–15].

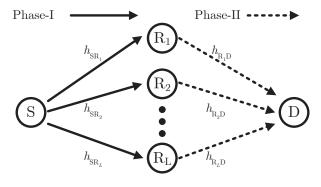


Figure 1. Block diagram of a wireless sensor network when communicating with the assistance of L sensors using amplify and forward (AF) principles.

A. Phase-I: Transmission From Source Sensor

The sensor S broadcasts a complex data symbol b to all the relays, $R_1, R_2, \cdots R_L$. The received signals at the relays can be represented as

$$y_{R_l} = \sqrt{E_S} h_{SR_l} b + n_{R_l}, \quad l = 1, 2, \cdots, L,$$
 (1)

where E_S is the average signal energy transmitted by the source and n_{R_l} is complex additive white Gaussian noise (AWGN) with mean zero and variance $\sigma_{R_l}^2$.

B. Phase-II: Transmission From Relay Sensor to Destination

During the phase-II, the l-th sensor amplifies the received signal y_{R_l} by ζ_{R_l} and forwards to the destination terminal D. The received signal at D from the l-th sensor is given by

$$y_{l} = h_{R_{l}D}\zeta_{R_{l}}y_{R_{l}} + n_{D_{l}}, \quad l = 1, 2, \cdots, L$$

$$= \sqrt{E_{S}}\zeta_{R_{l}}h_{R_{l}D}h_{SR_{l}}b + \zeta_{R_{l}}h_{R_{l}D}n_{R_{l}} + n_{D_{l}}. \quad (2)$$

Depending upon the type of sensors, the amplifying factor ζ_{R_l} can be either

$$\zeta_{R_l} = \sqrt{\frac{E_{R_l D}}{E_S \sigma_{SR_l}^2 + \sigma_{R_l}^2}},\tag{3}$$

or

$$\zeta_{R_l} = \sqrt{\frac{E_{R_l D}}{E_S |h_{SR_l}|^2 + \sigma_{R_l}^2}},\tag{4}$$

as proposed in [13]. (3) or (4) are called as fixed or variable gain amplification factors, respectively. In the fixed gain amplification factor, the sensor ensures that the an average or long-term power constraint is maintained, but allows the instantaneous transmit power to be much larger than the average [13–15]. However, in the variable gain amplification factor, each sensor uses the receive CSI from the source-sensor link to ensure that an average output energy per symbol is maintained for each realization [13–15]. This operation is performed at all the sensors.

C. Receiver Structure

As the desired signal b arrives at the destination with the assistance of L sensors, L copies of the desired signal need to be collected. The vector form of the received signal can be represented as

$$\mathbf{y} = \mathbf{h}b + \mathbf{n}, \tag{5}$$

where h and n are defined as

$$h = [\zeta_{R_1} h_{R_1 D} h_{SR_1}, \cdots, \zeta_{R_L} h_{R_L D} h_{SR_L}]^T,$$

$$= [h_1, h_2, \cdots, h_L]^T, \qquad (6)$$

$$n = [\zeta_{R_1} h_{R_1 D} n_{R_1} + n_{D_1}, \cdots, \zeta_{R_L} h_{R_L D} n_{R_L} + n_{D_l}]^T,$$

$$= [n_1, \cdots, n_L]^T. \qquad (7)$$

If the channel knowledge is available, the noise-part can be approximated as complex Gaussian noise with zero mean and variance given by

$$\sigma_l^2 = \zeta_{R_l}^2 |h_{R_l D}|^2 \sigma_{R_l}^2 + \sigma_{D_l}^2.$$
 (8)

Therefore, n is a complex Gaussian with mean zero and variance Σ . The variance Σ , will be a diagonal matrix of size L and can be expressed as

$$\Sigma = \operatorname{diag}[\sigma_1^2, \sigma_2^2, \cdots, \sigma_L^2]. \tag{9}$$

III. MMSE DETECTION FOR WSN

To estimate the desired data bit the receiver consists of a linear filter characterized by

$$z = \boldsymbol{w}^{H}\boldsymbol{y} = \boldsymbol{w}^{H}\boldsymbol{h}b + \boldsymbol{w}^{H}\boldsymbol{n}, \tag{10}$$

where $\mathbf{w} = [w_1, \cdots, w_L]^T$ and w_l is the l-th tap complex valued filter coefficient. The linear detector minimizes the mean-square error (MSE) cost function, i.e.,

$$J(\boldsymbol{w}) = E[|\boldsymbol{b} - \boldsymbol{w}^H \boldsymbol{y}|^2]$$

= $\sigma_b^2 - \boldsymbol{w}^H E[b^* \boldsymbol{y}] - E[\boldsymbol{y}^H b] \boldsymbol{w} + \boldsymbol{w}^H E[\boldsymbol{y} \boldsymbol{y}^H] \boldsymbol{w}, (11)$

where $E[\cdot]$ represents the expected value and σ_b^2 is the variance of the desired signal. The optimal weights for a MMSE detector can be easily obtained by derivation of (11) with respect to \boldsymbol{w} and setting to zero. Therefore, the optimal weights can be determined as [16]

$$\boldsymbol{w} = \boldsymbol{R}^{-1} \boldsymbol{\rho}, \tag{12}$$

where $\rho = E[\mathbf{y}b^*]$ is the cross-correlation vector between \mathbf{y} and b^* , and $(\cdot)^*$ denotes the complex conjugate. $\mathbf{R} = E[\mathbf{y}\mathbf{y}^H]$ is the auto-correlation of \mathbf{y} . By substituting (12) in (11), the cost function can be expressed as

$$J = \sigma_b^2 - \boldsymbol{\rho}^H \boldsymbol{R}^{-1} \boldsymbol{\rho}. \tag{13}$$

From (13) it can be observed that in order to minimize the MSE we need to maximize $\rho^H R^{-1} \rho$. It can be observed from (12) that the complexity of the MMSE detector is determined by the inverse of R which is a $(L \times L)$ dimensional matrix. Inverting a matrix of this size requires a complexity of $\mathcal{O}(L^3)$. In sensor networks the size of L is usually very large, therefore, the complexity of MMSE detector is very severe.

IV. DESIGN OF PREPROCESSING MATRIX

The aim of designing the preprocessing matrix will be to process U signals out of L signals such that the loss in BER performance is minimal. The designing of preprocessing matrix P will operate in two modes. In the first mode a preprocessing matrix P is designed so that the given received data which is of length L is reduced to U where U < L. Therefore, for a received vector y, the U-dimensional received vector will now be given by

$$\bar{\boldsymbol{y}} = \boldsymbol{P}^H \boldsymbol{y},\tag{14}$$

where bar indicates that the vector is now reduced to size U instead of L. In the second mode this $\bar{\boldsymbol{y}}$ is passed through a U dimensional filter. The modified cost function can now be given as

$$J(\bar{\boldsymbol{w}}) = E[|b - \bar{\boldsymbol{w}}^H \bar{\boldsymbol{y}}|^2]. \tag{15}$$

Similarly, solving (15) for \bar{w} , as mentioned in (11), the optimal weight vector can be given as

$$\bar{\boldsymbol{w}} = \bar{\boldsymbol{R}}^{-1} \bar{\boldsymbol{\rho}},\tag{16}$$

where \bar{R} is the autocorrelation matrix of \bar{y} , which is reduced to dimension $U \times U$ as compared with $L \times L$. The reduced-complexity scheme requires $\mathcal{O}(U^3)$ operations which will be significantly lower than that of the MMSE detector having a complexity of $\mathcal{O}(L^3)$. Let us now proceed to designing an optimal or an efficient preprocessing matrix P where intially, we visit the concept of channel shortening (CS) technique as presented in [9] and then follow up with our proposed rank-reduction techniques.

A. Previous Design Through Channel Shortening

CS-based technique for cooperative communication has been proposed in [9]. For time orthogonality, we assume that there is a preprocessing block p such that

$$\boldsymbol{p} = [p_1, p_2, \cdots p_U]^T, \tag{17}$$

where U is the length of the filter. The received signal \boldsymbol{y} will be convolved with \boldsymbol{p} to generate the output out of which U will be selected to be processed by the reduced optimal weight vector $\bar{\boldsymbol{w}}$. The output of the convolution can be written as

$$\boldsymbol{a} = \boldsymbol{y} * \boldsymbol{p} = (\boldsymbol{A}\boldsymbol{b} + \boldsymbol{N})\boldsymbol{p}, \tag{18}$$

where \boldsymbol{A} and \boldsymbol{N} are the convolution matrix of \boldsymbol{h} and \boldsymbol{n} , having a dimension of $(L+U-1)\times U$ respectively. The "*" sign in (18) represents convolution. The size of the output vector \boldsymbol{a} will be of (L+U-1). As we can only process U elements of \boldsymbol{a} we will require U elements to be non-zero and the other (L-1) elements to be zero ideally. The location of these U non-zero elements may be anywhere within \boldsymbol{a} , but for simpler processing they should be consecutively placed

$$\mathbf{a} = [0, 0, \cdots, 0, a_i, a_{i+1}, \cdots, a_{i+U}, 0, 0, \cdots, 0]^T,$$
 (19)

where i is an arbitrary number such that $i = 1, 2, \dots, L$. The channel-shortened received signal will now be represented as

$$\bar{\mathbf{y}} = [a_i, a_{i+1}, \cdots, a_{i+U}]^T.$$
 (20)

The matrix \boldsymbol{A} as represented in (18) will now consist of two submatrices \boldsymbol{A}_U and \boldsymbol{A}_{L-U} , where the desired signals will be placed in \boldsymbol{A}_U while the signals to be compressed will be placed in \boldsymbol{A}_{L-U} . The Rayleigh Quotient can now be employed to determine the optimized \boldsymbol{p} which can be given as

$$\boldsymbol{p} = \max_{\boldsymbol{t}} \frac{\boldsymbol{t}^{H} (\boldsymbol{A}_{U}^{H} \boldsymbol{A}_{U}) \boldsymbol{t}}{\boldsymbol{t}^{H} (\boldsymbol{A}_{L-U}^{H} \boldsymbol{A}_{L-U} + \boldsymbol{N}^{H} \boldsymbol{N}) \boldsymbol{t}}.$$
 (21)

Since the aim is to maximize this Rayleigh Quotient, a well known solution is to constraint the the denominator to be equal to 1 and maximize the numerator as mentioned in [9]. Letting $C = A_U^H A_U$ and $B = A_{L-U}^H A_{L-U} + N^H N$, the optimal value of p can be evaluated as

$$\boldsymbol{p} = (\boldsymbol{F})^{-1} \boldsymbol{v},\tag{22}$$

where F is the Cholesky factor of B, such that $B = F^H F$, and v is the eigenvector corresponding to the maximum eigenvalue of $(F)^{-1}C(F^H)^{-1}$.

B. Proposed Design Through Principal Component

In this section, we propose the designing of the preprocessing matrix ${\bf P}$ with the assistance of reduced-rank techniques. In these techniques instead of suppressing (L-U) signals we will utilize all the L signals to form a transformation matrix. In PC-based technique, the autocorrelation matrix ${\bf R}$ is decomposed in terms of eigen-values and eigen-vectors. A number of principal eigen-vectors are chosen to form a detection subspace. The decomposed autocorrelation matrix ${\bf R}$ can be given as

$$\mathbf{R} = \mathbf{\Phi} \mathbf{\Lambda} \mathbf{\Phi}^H = \sum_{i=1}^L \lambda_i \boldsymbol{\phi}_i \boldsymbol{\phi}_i^H, \tag{23}$$

where the matrix Φ contains the eigen-vectors of R and Λ corresponds to the matrix of eigen-values. As the auto-correlation matrix R has distinct eigen-values the eigen-vectors will be orthonormal, and the PC will be equivalent to maximization of a Rayleigh Quotient instead we have made use of all L signals instead of U. If these eigen-values can be arranged in a descending order such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L$ then the first U eigen-vectors corresponding to U eigen-values are retained to form the preprocessing matrix P given as

$$\boldsymbol{P} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots, \boldsymbol{\phi}_U]. \tag{24}$$

V. SIMULATION RESULTS AND DISCUSSION

In this section, the BER performance of the proposed WSN system with L=10 sensors is investigated. In our simulations, the channel gains were assumed to obey the Rayleigh distribution. The transmitted signal is assumed to have unit power and the destination and all the relays have the same noise power.

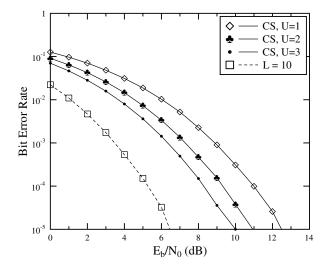


Figure 2. **Channel Shortening:** BER performance of wireless sensor networks when sensor employs variable gain amplification factor.

Fig. 2 shows the BER performance as a function of the SNR per bit, when communicating over Rayleigh fading channels.

The BER performance of all participating sensors is shown as a bench mark. It can be observed that for U=1 the CS-based technique performs equivalent to the best sensor selection scheme. However, there is a difference of 6 dB as compared to all participating sensors for U=1. The performance can be improved by increasing the size of U which will reuslt in higher complexity. It can also be observed that the CS-based algorithms have a similar slope as the all participating sensors. Therefore, the diversity order of CS-based scheme will be equal to the diversity order of all participating sensors which is 10 for this simulation.

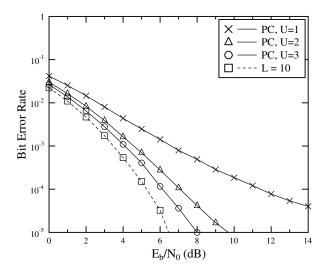


Figure 3. **Principal Component Analysis:** BER performance of wireless sensor networks when sensor employs variable gain amplification factor.

Fig. 3 shows the BER performance as a function of the SNR per bit, when communicating over Rayleigh fading channels. It can be observed that as the rank U increases there is an improvement in BER performance. However, this improved BER performance results in higher receiver complexity. It can be observed that for U=1 the diversity order is 1 as the slope of the curve will be equivalent to a single order diversity system. Furthermore, by increasing U the diversity order of the system will increase. It can also be observed that for a rank of U=1 the PC-technique performs better than the CS-based technique for lower SNR. However, for higher SNR and U=1 the performance is worse as compared to CS-based scheme.

Fig. 4 compares the BER performance of the cooperative communication system when communicating with variable gain relays using the CS- and PC-assisted transformation matrix. It can be observed that, for a given size of U, the PC-based technique significantly outperforms the CS-assisted techniques at lower SNR. However, for higher SNR region CS-based scheme outperform the PC-based scheme.

Finally, in Fig. 5, we compare the BER performance of WSNs when using fixed gain amplification factor and communicating over Rayleigh fading channels. It can be observed that for a U=1, the PC-based scheme is very close to

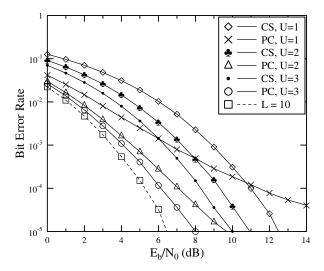


Figure 4. **Comparison:** BER performance of wireless sensor networks when sensor employs variable gain amplification factor.

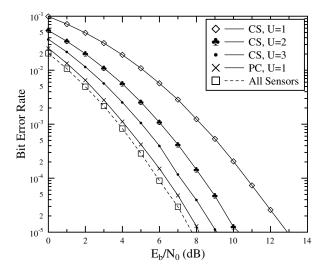


Figure 5. BER performance of wireless sensor networks when using fixed gain amplification factor.

all participating sensors. It can be observed that the BER performance of the proposed scheme is much superior to the CS-assisted scheme. It can also be observed that the slope of all schemes are the same, therefore, the system will have a diversity order of L=10.

VI. CONCLUSIONS

From our analysis and simulation results, it can be concluded that the channel shortening and principal component anlysis provide a flexible trade-off between the achievable BER performance and the receiver complexity. CS- and PC-based technique require eigen-decomposition for forming the

preprocessing matrix. The PC-based scheme outperforms a CS-based scheme at lower SNR for a given U when variable gain amplification is employed at the sensors. However, for higher SNR CS-based scheme are superior to the PC-based scheme as they achieve full diversity order when variable gain amplification is employed at the sensors. However, when fixed gain relays are employed PC-based scheme are much superior to the CS-based scheme. Finally, it can be concluded that for a wireless sensor network where there are a large number of sensors, these reduced rank techniques can be employed which can achieve a reasonable BER performance but with low complexity.

ACKNOWLEDGEMENT

This work is supported by a KAUST Global Cooperative Research (GCR) fund.

REFERENCES

- N. Khajehnouri and A. H. Sayed, "Distributed MMSE relay strategies for wireless sensor networks," *IEEE Trans. on Signal Process.*, vol. 55, no. 7, pp. 3336–3348, Jul. 2007.
- [2] R. Krishna, Z. Xiong, and S. Lambotharan, "A cooperative MMSE relay strategy for wireless sensor networks," *IEEE Signal Process. Letters.*, vol. 15, pp. 549–552, Jul. 2008.
- [3] Y. T. Hou, Y. Shi, H. D. Sherali, and S. F. Midkiff, "On energy provisioning and relay node placement for wireless sensor networks," *IEEE Trans. Wireless Comm.*, vol. 4, pp. 2579–2590, Sept. 2005.
- [4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [5] Y. Chen and Q. Zhao, "On the lifetime of wireless sensor networks," IEEE Commun. Letters, vol. 9, no. 11, pp. 976–978, Nov. 2005.
- [6] S. Verdu, Multiuser Detection. Cambridge University Press, 1998.
- [7] M. Honig and M. K. Tsatsanis, "Adaptive techniques for multiuser CDMA receivers," *IEEE Signal Process. Magazine*, vol. 17, pp. 49–61, May 2000.
- [8] A. Bletsas, A. Khisti, D. P. Reed and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE Journal on Selected Areas in Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.
- [9] S. I. Hussain, M.-S. Alouini and M. O. Hasna, "A diversity and combining technique based on channel shortening for cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 659-667, Feb. 2012.
- [10] J. S. Goldstein and I. S. Reed, "Subspace selection for partially adaptive sensor array processing," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 33, no. 2, pp. 539–544, Apr. 1997.
- [11] Q.-Z. Ahmed and L.-L. Yang, "Reduced-rank adaptive multiuser detection in hybrid direct-sequence time-hopping ultrawide bandwidth systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 156–167, Jan. 2010.
- [12] M.-S. Alouini, A. Scaglione, and G. B. Giannakis, "PCC: Principal components combining for dense correlated multipath fading environments," *IEEE 52nd Vehicular Technology Conference, VTC-Fall*, (Boston-USA), pp. 2510–2517, Fall. 2000.
- [13] C. S. Patel and G. L. Stuber, "Channel estimation for amplify and forward relay based cooperation diversity systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2348–2355, Jun. 2007.
- [14] Y. Zhu, P-Y. Kam, and Y. Xin, "Non-Coherent Detection form Amplifyand-Forward Relay Systems in a Rayleigh Fading Environment," *IEEE Global Communications Conference*, GLOBECOM, (Washington, DC-USA), pp. 1658–1662, Nov. 2007.
- [15] D. Chen, and J. N. Laneman, "Cooperative diversity for wireless fading channels without channel state information," *IEEE 38th Asilomar Conference on Signals, Systems and Computers*, ACSSC, (Pacific Grove-USA), pp. 1307–1312, Nov. 2004.
- [16] S. Haykin, Adaptive Filter Theory. Prentice Hall, 4th ed., 2002.