Censoring for Type-Based Multiple Access Scheme in Wireless Sensor Networks

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***Abstract*—In this paper, we consider binary hypothesis testing for distributed detection in Wireless Sensor Networks. Sensor nodes individually take a decision upon which hypothesis is currently present. Communication between sensor nodes and the Fusion Center is done through a Type-Based Multiple Access (TBMA) scheme, and the Fusion Center gives a global decision about the hypothesis under consideration. We consider the case where each sensor has the ability to "censor" transmission, meaning that a sensor node can locally withhold transmission if local observation is unreliable. The major contribution in this paper is to show that for the TBMA scheme with sensors sending binary decisions to the Fusion Center, censoring can achieve lower probability of decision error even if sufficient energy and/or rate of transmission is available.**

I. Introduction

Wireless Sensor Networks (WSNs) have gained a lot of interest recently. The magnificent advances in wireless communications and electronics enabled developing low-cost, low-power, multifunctional sensor nodes, which levitates the wide use of sensor networks consisting of large number of sensor nodes. The flexibility, fault tolerance, high sensing fidelity, low-cost, and rapid deployment characteristics of sensor networks are nowadays utilized in a variety of application areas such as health, military, and home [1].

In this paper, we consider the use of WSN in Binary Hypothesis Testing, which was studied in [2] under the name of Decentralized Detection Problem. Each sensor receives a sequence of observation samples about a state of nature, which takes one of two possible values. Local decisions at each sensor are made, and a global decision about which hypothesis is made at the Fusion Center (FC). Communication between sensor nodes and FC is done via a *Type-Based Multiple Access (TBMA)* scheme. Each sensor sends one of two orthonormal waveforms corresponding to each hypothesis that is locally decided. The motivation behind using TBMA is made clear in the special case (observations are conditionally i.i.d, equal channel gains) that detection of the current hypothesis is done at FC just by observing a noisy version of the histogram of sensors observations [3].

Multiple Access as a communication scheme was studied and overall system performance is proven to have improved when enabling censoring [4]. A sensor is allowed to censor transmission if its local LLR falls between two thresholds, in contrast to the conventional binary hypothesis testing where a local LLR is compared to a single threshold. The authors show that this approach embodies the quality of the local observations in the transmission, which will enhance the overall performance of the detection system in the Bayesian sense.

One major bottleneck in the utility of WSNs is the fact that the sensors are battery-powered, which can render the sensor nonfunctional in a reasonably small time if this factor is not taken into consideration during the design stage of the network. Various techniques and design approaches have been innovated in order to increase the lifetime of the sensors, e.g in [5]. One approach, which was first introduced by [6], is to reduce the number of active sensors based on the relative reliability of their observations. i.e., each sensor computes the local *log-likelihood ratio (LLR)*, and a sensor node has the ability to "censor" transmission. The idea of censoring sensors has been extensively researched, for example in [7], [8]. However, we revisit the concept of censoring from a different but rather interesting point of view.

The major contribution of this paper is to show that enabling sensor nodes to censor transmission in a certain censoring interval can also enhance the overall system performance in terms of probability of error, in addition to increasing overall system efficiency in terms of energy conservation, which was stated in [7]. A system employing Time Division Multiple Access as a communication scheme was studied and the overall system performance is proven to have improved when enabling censoring [4]. A sensor is allowed to censor transmission if its local LLR falls between two thresholds, in contrast to the conventional binary hypothesis testing where a local LLR is compared to a single threshold. This paper extends the work in [4] and shows that for a TBMA scheme, this approach embodies the quality of the local observations in the transmission, which will enhance the overall performance of the detection system (in the Bayesian sense).

The rest of the paper is organized as follows. In Section II, the system and data models are introduced. In Section III, we study the nontrivial case where a network of only two sensors is available. We find the optimum probability of error in both conventional and censoring schemes. We repeat for large sensor networks in IV. In Section V, simulations and

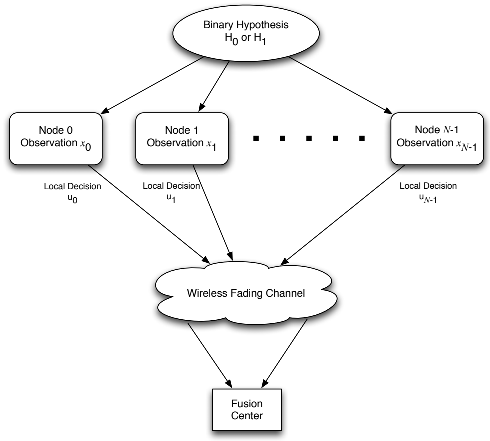

Fig. 1. Wireless Sensor Network, parallel topology with Fusion Center

asymptotic results are shown. Section VI concludes the paper.

II. System and Data Models

Consider the system model shown in Figure 1. We assume the presence of N sensors in a WSN deployed in a parallel topology with a FC [9]. Each sensor takes a decision upon which a certain phenomenon has occurred or not. We assume a binary hypothesis testing problem. i.e., the FC decides between two hypotheses, H_0 and H_1.

The *n*-th sensor measurement is $x_n, n = 0, 1, 2, ..., N-1$. These measurements are assumed to be mutually independent under each hypothesis. The data model under each hypothesis is assumed to be given by[1]

$$\begin{aligned} H_0 &: x_n \sim \mathcal{N}\left(0, \sigma_0^2\right) \\ H_1 &: x_n \sim \mathcal{N}\left(0, \sigma_1^2\right). \end{aligned} \tag{1}$$

Sensors send their local decision $u_n = S_{j_n}$, $n = 0, 1, 2, ..., N-1, j_n = 0$ or 1 to the FC based on a TBMA scheme. Each sensor sends one of two orthonormal waveforms, namely S_0 and S_1, based on which hypothesis was decided, H_0 or H_1 respectively. The FC receives the transmissions of all sensors. Due to the additive nature of the wireless medium, the FC receives

$$y = \sum_{n=0}^{N-1} h_{j_n} S_{j_n} + w, \tag{2}$$

where the channel coefficient h_{j_n} between sensor n sending waveform S_{j_n} and the FC is assumed to be real, and follows the shifted-Gaussian distribution $\mathcal{N} \sim (a, \sigma_h^2)$ where $a > 0$ [3], and w is AWGN with zero mean and variance σ_N^2. Using the inner product operation with S_0 and S_1, the FC obtains

$$\begin{aligned} \mathbf{y} &= [<y, S_0>, <y, S_1>]^T \quad \text{where} \\ &<y, S_0> = \sum_{m \in \mathcal{S}_0} h_{0_m} + w_0 \\ &<y, S_1> = \sum_{k \in \mathcal{S}_1} h_{1_k} + w_1, \end{aligned} \tag{3}$$

where m is a counter that counts over the sensors which have decided hypothesis H_0, k is a counter that counts over the sensors which have decided hypothesis H_1, $\mathcal{S}_0$ is the set of all sensors sending S_0, and $\mathcal{S}_1$ is the set of all sensors sending S_1. The receiver takes a decision for which hypothesis by computing the difference between the two elements of the vector $\mathbf{y}$; consider the following

$$\begin{aligned} D &= \mathbf{y}[2] - \mathbf{y}[1] \\ &= \sum_{k \in \mathcal{S}_1} h_{1_k} - \sum_{m \in \mathcal{S}_0} h_{0_m} + \grave{w}, \end{aligned} \tag{4}$$

where $\grave{w}$ is a Gaussian distributed noise with zero mean and variance $2\sigma_N^2$. The FC decides hypothesis H_1 if $D > 0$ and decides hypothesis H_0 if $D < 0$. In the following two sections, we consider the common one-threshold case, and the two-threshold case where censoring is enabled.

III. Two-Sensor Network

A. Conventional (One-threshold) Case

We consider a simple WSN which consists of two sensors only. Each sensor makes a decision based on local LLR. The LLR at the *n*th sensor is given by

$$\begin{aligned} LLR_n &= \log\left(\frac{P\left(x_n|H_1\right)}{P\left(x_n|H_0\right)}\right) \\ &= \log\left(\frac{\sigma_0}{\sigma_1}\right) + \frac{x_n^2}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right). \end{aligned} \tag{6}$$

The LLR is compared to a local threshold η, and a sensor takes a decision for which hypothesis is present. The local threshold is determined based upon certain required confidence levels. Based on the noisy nature of the observations, the probability that the *n*th sensor decides to send S_1 given hypothesis H_0 is

$$\begin{aligned} &\Pr(u_n = S_1|H_0) \\ &\quad = \Pr\left(\log\left(\frac{\sigma_0}{\sigma_1}\right) + \frac{x_n^2}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right) > \eta|H_0\right) \\ &\quad = \Pr\left(x_n^2 > \gamma|H_0\right) \\ &\quad = 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right) \end{aligned} \tag{7}$$

where $\gamma = \frac{2}{\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)}\left(\eta - \log\left(\frac{\sigma_0}{\sigma_1}\right)\right)$.

The probability that the sensor node decides to send S_0 under the same hypothesis is

$$\Pr(u_n = S_0|H_0) = 1 - 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right). \tag{8}$$

[1] Note that this work can be readily extended to any data model. We need only to find Type-I error (False Alarm) and Type-II error (Misdetection) probabilities. We only assume the aforementioned data model as a common example.

$$
\begin{aligned}
P_{\mathcal{E}/H_1} &= \Pr(\, U_0 = 0 \,|H_1) = \Pr(\, D < 0 \,|H_1) = \Pr\left(h_{1_0} + h_{1_1} + \grave{w} < 0 \mid H_1, u_0 = S_1, u_1 = S_1\right) \times \Pr\left(u_1 = S_1, u_2 = S_1|H_1\right) \\
&+ \Pr\left(h_{1_0} - h_{0_1} + \grave{w} < 0 \mid H_1, u_0 = S_1, u_1 = S_0\right) \times \Pr\left(u_1 = S_1, u_2 = S_0|H_1\right) \\
&+ \Pr\left(-h_{0_0} + h_{1_1} + \grave{w} < 0 \,|H_1, u_0 = S_0, u_1 = S_1\right) \times \Pr\left(u_1 = S_0, u_2 = S_1|H_1\right) \\
&+ \Pr\left(h_{0_0} + h_{0_1} + \grave{w} > 0 \mid H_1, u_0 = S_0, u_1 = S_0\right) \times \Pr\left(u_1 = S_0, u_2 = S_0|H_1\right).
\end{aligned}
\tag{5}
$$

Similarly, we can obtain the probabilities of sending both waveforms given the hypothesis H_1 by

$$
\Pr(u_n = S_1|H_1) = 2Q\left(\frac{\sqrt{\gamma}}{\sigma_1}\right), \tag{9}
$$

$$
\Pr(u_n = S_0|H_1) = 1 - 2Q\left(\frac{\sqrt{\gamma}}{\sigma_1}\right). \tag{10}
$$

The FC computes the vector $\mathbf{y}$ and based on the difference between the two elements of the vector, a global decision is made. By examining the possible outcomes of the vector $\mathbf{y}$, the probability of global miss detection can be easily formulated as in (5) shown on top of the this page.

Taking into account that channel coefficients are Gaussian distributed, and the noise component at the FC is circularly-symmetric Gaussian distributed, the probability of global miss detection can be found to be

$$
\begin{aligned}
P_{\mathcal{E}/H_1} &= \Pr(\, U_0 = 0 \,|H_1) \\
&= (1 - M_a)\left(1 - Q_{\gamma,\sigma_1}\right)^2 \\
&+\ Q_{\gamma,\sigma_1}\left(1 - Q_{\gamma,\sigma_1}\right) \ +\ M_a \,.\, Q^2_{\gamma,\sigma_1} \\
&= (1 - M_a) \ +\ Q_{\gamma,\sigma_1}\left[1 \ -\ 2\left(1 - M_a\right)\right]
\end{aligned}
\tag{11}
$$

where $M_a = Q\left(\frac{2a}{\sigma_\alpha}\right)$, $Q_{\gamma,\sigma_1} = Q\left(\frac{\sqrt{\gamma}}{\sigma_1}\right)$ and $\sigma_\alpha^2 = 2\sigma_h^2 + 2\sigma_N^2$. Similarly, the expression for the probability of false alarm is

$$
\begin{aligned}
P_{\mathcal{E}/H_0} &= \Pr(\, U_0 = 1 \,|H_0) \\
&= (1 - M_a)\, Q^2_{\gamma,\sigma_0} \ +\ Q_{\gamma,\sigma_0}\left(1 - Q_{\gamma,\sigma_0}\right) \\
&+\ M_a\left(1 \ -\ Q^2_{\gamma,\sigma_0}\right) = \ M_a \ +\ Q_{\gamma,\sigma_0}\left(1 \ -\ 2M_a\right)
\end{aligned}
\tag{12}
$$

where $Q_{\gamma,\sigma_0} = Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right)$. Finally, the probability of error can be expressed as

$$
P_e \ =\ \pi_0 P_{\mathcal{E}/H_0} \ +\ \pi_1 P_{\mathcal{E}/H_1}. \tag{13}
$$

In order to find the optimum threshold at the sensors, we differentiate P_e w.r.t the sensor threshold and we equate to zero. The optimum threshold can be found in a very straightforward manner.

B. Censoring (Two-Threshold) Case

We now begin to introduce censoring to the network. Each sensor compares its locally computed LLR to an upper and a lower thresholds, symbolized by η_1 and η_0 respectively. A sensor takes a local decision that hypothesis H_1 is present and informs the FC if local LLR value is above η_1, takes a local decision that hypothesis H_0 is present and informs the FC if local LLR value is below η_0, or censor local decision making if local LLR value falls between η_1 and η_0. A clairvoyant view of the problem would suggest that such a scheme does not necessarily improve the overall performance of the system, which we will show that it does in the examined setup.

Under each hypothesis H_0 and H_1, the probability of each local outcome is

$$
\begin{aligned}
&\Pr(u_n = S_1|H_0) \\
&\qquad = \Pr\left(\log\left(\frac{\sigma_0}{\sigma_1}\right) + \frac{x^2}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right) \ > \eta_1|H_0\right) \\
&\qquad = \Pr\left(x^2 > \gamma_1|H_0\right) = 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_0}\right) \\
&\Pr(u_n = 0|H_0) = 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_0}\right) - 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_0}\right) \\
&\Pr(u_n = S_0|H_0) = 1 - 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_0}\right) \\
&\Pr(u_n = S_0|H_1) = 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_1}\right) \\
&\Pr(u_n = 0|H_1) = 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_1}\right) - 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_1}\right) \\
&\Pr(u_n = S_0|H_1) = 1 - 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_1}\right)
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
\gamma_1 &= \frac{2}{\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)}\left(\eta_1 - \log\left(\frac{\sigma_0}{\sigma_1}\right)\right) \\
\gamma_0 &= \frac{2}{\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)}\left(\eta_0 - \log\left(\frac{\sigma_0}{\sigma_1}\right)\right).
\end{aligned}
$$

Using the same reasoning from section (III-A), we enumerate the different cases for the received vector $\mathbf{y}$. The probability of global false alarm can be formulated in the same manner as the conventional case, but we will omit the detailed, yet straightforward derivation due to space limitation. The probability of global false alarm can be expressed as in (15) where $L_a = Q\left(\frac{a}{\sigma_\beta}\right)$ and $\sigma_\beta^2 = \sigma_h^2 + 2\sigma_N^2$. Similarly, the probability of global miss detection can be expressed as in (16), and the probability of error is expressed in (13). By differentiating w.r.t γ_1 and γ_0 and equating both to zero, we can obtain the optimum thresholds.

IV. Large Sensor Network

In this section, we focus our attention on sensor networks which consist of a very large number of sensor nodes

$$P_{\mathcal{E}/H_0} = \Pr(\, U_0 = 1\, |H_0) = (1 - M_a)\, Q^2_{\gamma_1,\sigma_0} + \, 2\,(1 - L_a)\, Q_{\gamma_1,\sigma_0}\, (Q_{\gamma_0,\sigma_0} - Q_{\gamma_1,\sigma_0})$$
$$+ \, Q_{\gamma_1,\sigma_0}\,(1 - Q_{\gamma_0,\sigma_0}) \, + \, \frac{1}{2}\,(Q_{\gamma_0,\sigma_0} - Q_{\gamma_1,\sigma_0})^2 + \, 2L_a\,(Q_{\gamma_0,\sigma_0} - Q_{\gamma_1,\sigma_0})\,(1 - Q_{\gamma_0,\sigma_0}) + \, M_a\,(1 - Q_{\gamma_0,\sigma_0})^2 \tag{15}$$

$$P_{\mathcal{E}/H_1} = \Pr(\, U_0 = 0\, |H_1) = M_a Q^2_{\gamma_1,\sigma_1} \, + \, 2L_a Q_{\gamma_1,\sigma_1}\,(Q_{\gamma_0,\sigma_1} - Q_{\gamma_1,\sigma_1})$$
$$+ \, Q_{\gamma_1,\sigma_1}\,(1 - Q_{\gamma_0,\sigma_1}) \, + \, \frac{1}{2}\,(Q_{\gamma_0,\sigma_1} - Q_{\gamma_1,\sigma_1})^2 + \, 2\,(1 - L_a)\,(Q_{\gamma_0,\sigma_1} - Q_{\gamma_1,\sigma_1})\,(1 - Q_{\gamma_0,\sigma_1}) + \, (1 - M_a)\,(1 - Q_{\gamma_0,\sigma_1})^2 \tag{16}$$

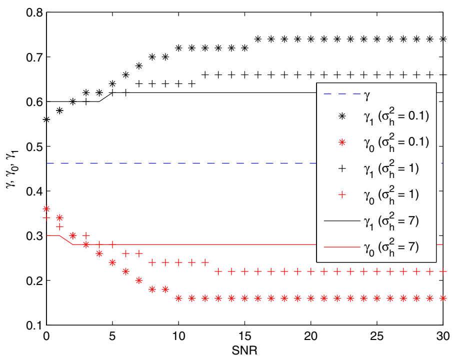

Fig. 2. Optimal Thresholds versus SNR for a two-sensors WSN in conventional (one threshold) and censoring (two threshold) schemes with $\sigma_0^2 = 1$, $\sigma_1^2 = 0.25$ and $a = 1$. Various curves are plotted for different values of σ_h^2

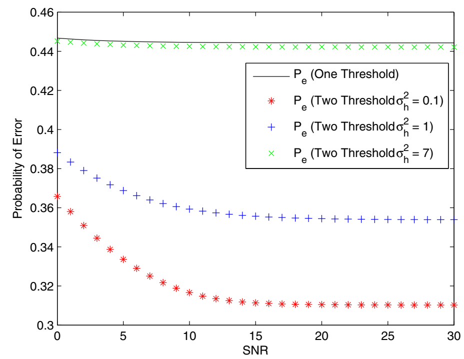

Fig. 3. Probability of Error versus SNR for a two-sensor WSN in conventional (one threshold) and censoring (two threshold) schemes with $\sigma_0^2 = 1$, $\sigma_1^2 = 0.25$ and $a = 1$. Various curves are plotted for different values of σ_h^2

$(N \to \infty)$. Hence, we are interested in understanding the asymptotics of the probability of error. We will employ Large Deviation Theory [10]. We compute the error exponent for the conventional and censoring networks, and we compare exponents in both cases.

A. Conventional (One-Threshold) Case

Now we focus on the probabilities of miss-detection and false alarm at the FC. We can assume the variable D in (4) to be

$$D = \sum_{n=0}^{N-1} Z_n, \tag{17}$$

and assuming that H_0 is present, we can write

$$Z_n = \begin{cases} Z_n^+ = h_{1_n} + \hat{w}_n & \text{w.p. } P_0 = 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right) \\ Z_n^- = -h_{0_n} + \hat{w}_n & \text{w.p. } 1 - P_0 = 1 - 2Q\left(\frac{\sqrt{\gamma}}{\sigma_0}\right) \end{cases} \tag{18}$$

where $Z_n^+ \sim \mathcal{N}\left(a, \sigma_z^2\right)$, $Z_n^- \sim \mathcal{N}\left(-a, \sigma_z^2\right)$ and $\sigma_z^2 = \sigma_h^2 + \frac{2\sigma_N^2}{N}$. Note that $\hat{w}_n \sim \mathcal{N}\left(0, \frac{2\sigma_N^2}{N}\right)$ is the noise component in equation (4) split into N terms so as to match the distributions of all Z_n variables.

Using Chernoff's formula, error probability is defined as[2]

$$P_{\mathcal{E}/H_0} = P\left(D > 0|H_0\right) \doteq e^{-N.\max_\theta[-\lambda(\theta)]}, \tag{19}$$

[2]The symbol $\doteq$ is used to denote equality in the exponential decay rate, that is $f(N) \doteq g(N)$ means that $\lim_{N\to\infty} \frac{1}{N} \log \frac{f(N)}{g(N)} = 0$.

where

$$\lambda(\theta) = \ln E\left[e^{\theta Z_n}\right] \tag{20}$$

is the Cumulant Generating Function (CGF). Knowing the probability distribution function of the variable Z_n, we can calculate

$$E\left[e^{\theta Z_n}\right] = P_0 E\left[e^{\theta Z_n^+}\right] + (1 - P_0) E\left[e^{\theta Z_n^-}\right] \tag{21}$$

$$E\left[e^{\theta Z_n^+}\right] = \frac{1}{\sqrt{2\pi\sigma_z^2}} \int_{-\infty}^{\infty} e^{\theta Z}.e^{-\frac{(Z\, -\, a)^2}{2\,\sigma_z^2}} dZ = e^{a\theta\, +\, \frac{\sigma_z^2\theta^2}{2}} \tag{22}$$

and likewise

$$E\left[e^{\theta Z_n^-}\right] = \frac{1}{\sqrt{2\pi\sigma_z^2}} \int_{-\infty}^{\infty} e^{\theta Z}.e^{-\frac{(Z\, +\, a)^2}{2\,\sigma_z^2}} dZ$$
$$= e^{-a\theta\, +\, \frac{\sigma_z^2\theta^2}{2}}. \tag{23}$$

So we finally get

$$E\left[e^{\theta Z_n}\right] = e^{\frac{\sigma_z^2\,\theta^2}{2}} \left[(1 \, - \, P_0)\, e^{-a\,\theta} \, + \, P_0 e^{a\,\theta}\right]. \tag{24}$$

The error exponent, under the presence of hypothesis H_0, $\Delta_0(\gamma)$, is defined as

$$\Delta_0(\gamma) = -\lim_{N\to\infty} \frac{1}{N} \log P_{\mathcal{E}/H_0}$$
$$\doteq -\log \inf_{\theta>0} \left(e^{\frac{\sigma_h^2\,\theta^2}{2}} \left(P_0\left(e^{a\,\theta} \, - \, e^{-a\,\theta}\right) \, + e^{-a\,\theta}\right)\right). \tag{25}$$

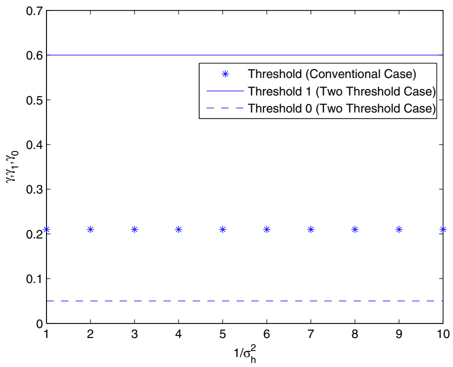

Fig. 4. Optimum Thresholds versus the inverse of channel coefficients variance for a large WSN in both conventional (one threshold) and two threshold schemes, with $\sigma_0^2 = 1$, $\sigma_1^2 = 0.25$ and $a = 1$

The error exponent, under the presence of hypothesis H_1, $\Delta_1(\gamma)$, can be found similarly to be

$$\begin{aligned}\Delta_1(\gamma) &= -\lim_{N\to\infty} \frac{1}{N} \log P_{\mathcal{E}/H_1} \\ &\doteq -\log \inf_{\theta>0} \left(e^{\frac{\sigma_h^2\,\theta^2}{2}} \left(P_1 \left(e^{-a\,\theta} \;-\; e^{a\,\theta} \right) \;+ e^{a\,\theta} \right) \right)\end{aligned} \tag{26}$$

where $P_1 = 2Q\left(\frac{\sqrt{\gamma}}{\sigma_1}\right)$. The error exponent is limited by the minimum of the two error exponents $\Delta_0(\gamma)$ and $\Delta_1(\gamma)$. Assuming that the priors of H_0 and H_1 are π_0 and π_1 respectively, the probability of error at the FC is

$$P_e = \pi_0 P_{\mathcal{E}/H_0} + \pi_1 P_{\mathcal{E}/H_1} \;\doteq\; e^{-N.\Delta_0(\gamma)} + e^{-N.\Delta_1(\gamma)}. \tag{27}$$

B. Censoring (Two-Threshold) Case

We consider the case if each sensor has the ability to censor transmission. Under hypothesis H_0, the variable Z_n in (18) in this case becomes

$$Z_n = \begin{cases} Z_n^+ = h_{1_n} + \hat{w}_n & \text{w.p. } P_{10} \\ Z_n^C = \hat{w}_n & \text{w.p. } P_{C0} \\ Z_n^- = -h_{0_n} + \hat{w}_n & \text{w.p. } P_{00} \end{cases} \tag{28}$$

where

$$\begin{aligned} P_{10} &= 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_0}\right) \\ P_{C0} &= 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_0}\right) - 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_0}\right) \\ P_{00} &= 1 - 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_0}\right). \end{aligned}$$

Similar to Section IV-A, we compute the CGF in this case

$$E\left[e^{\theta Z_n^+}\right] = -e^{a\theta + \frac{\sigma_z^2\theta^2}{2}} \tag{29}$$

$$E\left[e^{\theta Z_n^C}\right] = -e^{\sigma_z^2\theta^2} \tag{30}$$

$$E\left[e^{\theta Z_n^-}\right] = -e^{-a\theta + \frac{\sigma_z^2\theta^2}{2}} \tag{31}$$

$$E\left[e^{\theta Z_n}\right] = P_{10}E\left[e^{\theta Z_n^+}\right] + P_{C0}E\left[e^{\theta Z_n^C}\right] + P_{00}E\left[e^{\theta Z_n^-}\right]. \tag{32}$$

The error exponent under hypothesis H_0 is given by

$$\begin{aligned}\Delta_0(\gamma) = -\log \inf_{\theta>0} \Bigg(& P_{10} \left(e^{a\theta + \frac{\sigma_h^2\,\theta^2}{2}} \right) \\ & + \; P_{00} \left(e^{-a\theta + \frac{\sigma_h^2\,\theta^2}{2}} \right) \; + \; P_{C0} \Bigg). \end{aligned} \tag{33}$$

Using the same procedure, it can be easily shown that the error exponent in case of hypothesis H_1 is given by

$$\begin{aligned}\Delta_1(\gamma) = -\log \inf_{\theta>0} \Bigg(& P_{11} \left(e^{-a\theta + \frac{\sigma_h^2\,\theta^2}{2}} \right) \\ & + \; P_{01} \left(e^{a\theta + \frac{\sigma_h^2\,\theta^2}{2}} \right) \; + \; P_{C1} \Bigg) \end{aligned} \tag{34}$$

where

$$\begin{aligned} P_{11} &= 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_1}\right) \\ P_{C1} &= 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_1}\right) - 2Q\left(\frac{\sqrt{\gamma_1}}{\sigma_1}\right) \\ P_{01} &= 1 - 2Q\left(\frac{\sqrt{\gamma_0}}{\sigma_1}\right). \end{aligned}$$

V. SIMULATIONS AND RESULTS

Two-Sensor Case: In this section, computer simulations are performed which illustrate the advantage of censoring introduction in the system in terms of enhancing probability of error. In Figures 2 and 3, a small network with two sensors is considered, with $\sigma_0^2 = 1$, $\sigma_1^2 = 0.25$ and $a = 1$. Figure 2 shows the optimum upper and lower thresholds for each sensor against Signal to Noise Ratio SNR where $SNR = \frac{1}{\sigma_N^2}$. Multiple curves are presented for different values of σ_h^2. Figure 3 shows the probability of error against SNR for different values of σ_h^2. We note from Figure 2 that the optimum thresholds for minimum probability of error are less apart for better channel statistics (lower σ_h^2), which means that sensors tend to censor transmission more likely for more reliable channels. It is clear that the noise component at the FC has little to no effect on the system after certain SNR value. The idea here is that a combination of the noise variance and the channel coefficients variance are the key factor that affects the performance of the system and the values of optimum thresholds. This is obvious when observing the terms in equations (11) and (15). In Figure 3, it is clear that censoring leads to better probability of error. We also note that censoring can achieve lower probability of error for better channel statistics, unlike the conventional case, where the overall performance of the system nearly does not depend on channel statistics. Observing the curve for $\sigma_h^2 = 7$, we note that little enhancement in system performance is achieved over the conventional case. But since this achievement comes as an extra gain in performance in addition to the savings in battery

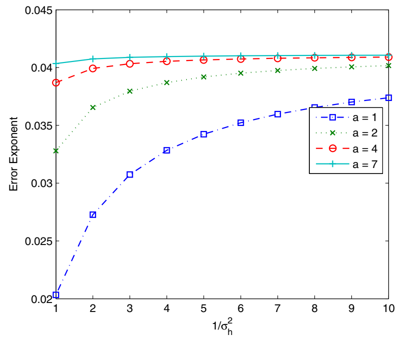

Fig. 5. Error exponent versus the inverse of channel coefficients variance for a large WSN in conventional (one threshold) scheme. Various curves are plotted for different values of a. $\sigma_0^2 = 1$ and $\sigma_1^2 = 0.25$

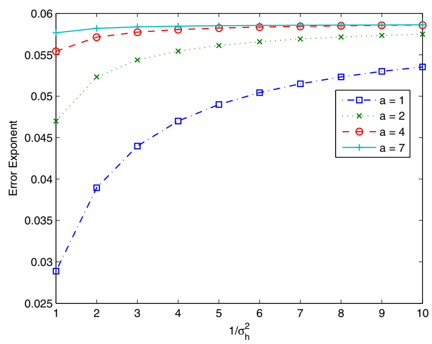

Fig. 6. Error Exponent vs the inverse of channel coefficients variance for the case of two thresholds in a large WSN. Various curves are presented for different values of a. $\sigma_0^2 = 1$ and $\sigma_1^2 = 0.25$

lifetime due to withholding transmission, adopting censoring would seem to be always a sane option.

Large-Sensor Case: Now we move to the large network case. We first consider the conventional case studied in Section IV-A. Note that equations (25) and (26) do not depend on the variance of the AWGN added to the transmitted signal through the multipath channel. This is due to the infinitesimal increase in the number of available sensors ($N \to \infty$), which overcomes the error introduced in each of the received signals due to noise added at the receiver. This is also elaborated in the Two-Threshold case in Section IV and is clear in equations (33) and (34). In Figure 4, the optimum thresholds are plotted against different values of the inverse of variance of channel coefficients for both Conventional and Two Threshold cases. It shows that it is optimum in terms of probability of detection that sensors do censor transmission for any SNR. In Figure 5 and Figure 6, the error exponents are plotted against the inverse of variance of channel coefficients, for different values of shifts for the mean of the channel coefficients a. We notice that due to the noncentral nature of the channel coefficients, the variable a plays an important role in the performance of the system. As a increases, the channel coefficients are more likely to be positive, and received signals from different sensors are more likely to add up, thus increasing reliability of the global decision taken at the FC. It is clear from (25), (26), (33) and (34) that the case of $a = 0$ leads to the complete failure of the

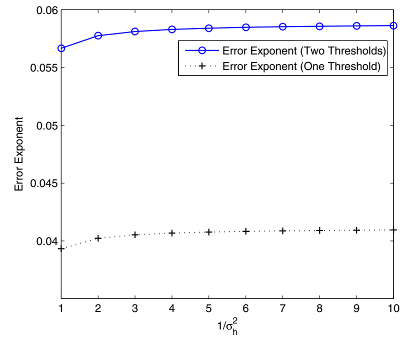

Fig. 7. Error exponents for both conventional (one threshold) and two threshold cases in a large WSN vs the inverse of channel coefficients variance. $\sigma_0^2 = 1$, $\sigma_1^2 = 0.25$ and $a = 7$

detection system. In Figure 7, we compare the error exponents for both the Conventional Case and the Two Threshold Case, with constant $a = 7$ as an example. It is clear that censoring leads to higher error exponent and lower probability of error.

VI. Conclusion

We have studied the introduction of a censoring scheme to a WSN using a TBMA communication method. In this scheme, each sensor decides whether its local observation and decision are reliable enough for transmission, by comparing its locally computed LLR to a certain censoring interval. We showed that censoring in this type of WSNs results in lower probability of error at the FC and thus better performance even if enough energy is available for all sensors to transmit.

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