Throughput-maximising link configuration for mutually interfering data terminals

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***Abstract*—A wireless communication system can be more efficient if link-layer parameters such as modulation order, symbol rate and packet size, are (adaptively) optimised. Previously, we obtained a simple and robust characterisation of the link configuration that maximises bits per second or bits per Joule for data traffic: a set of possible link configurations can be ranked by the slope of a tangent line from zero to the graph of a scaled version of the packet-success rate function (PSRF): the steeper the tangent the better the configuration. However, this applies to a single communication link. We now *analytically* find for many mutually-interfering data-uploading terminals link configurations that satisfy the necessary conditions for throughput maximisation. A key observation: the tangent line common to 2 (scaled) PSRF tells us when to switch between them. We present a simple semi-decentralised algorithm implementable through pricing, and report very encouraging numerical results.**

I. Introduction

The importance of (adaptively) optimising the link layer configuration of a wireless communication systems has long been recognised [1], [2], [3], [4]. Recently, [5] points out that for systems equipped with a selective packet re-transmission mechanism, a higher-layer link configuration criterion, such as "goodput", is appropriate. However, [5] does not yield a specific result that can be readily interpreted, and — most importantly – generalised to other systems. Building upon [5], [6] shows that an arbitrary set of possible link configurations can be evaluated by the simple procedure of drawing a tangent line from the origin to the graph of a scaled version of the packet-success rate function: the steeper the tangent, the better the configuration. Then, [7] shows that a flexible-symbol-rate terminal that applies [6]'s result enjoys a substantial performance advantage over a traditional adaptive (BER-limited) terminal with a fixed symbol rate, even if the flexible symbol rate can never exceed the fixed rate.

But [5], [6], [7] focus on a single transmitter-receiver pair, with a single channel, and [8] moves toward extending [6], [7] to multiple channels but still for a single pair. For many mutually interfering data terminals with a common receiver that treats interference as random noise, the present work finds link configurations that satisfy the Karush-Kuhn-Tucker (KKT) conditions ([9], [10]) for throughput maximisation. The analysis employs a change of coordinates from [11], [12] in which a terminal's share of the total received power plays a key role. Our experimental model follows [13], [14].

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II. Physical Model

Index m identifies one of M terminals uploading data to a common receiver. z, x may be used as generic variables.

- W : available bandwidth
- $p_0 = N_0 W$: aver. Gaussian noise power
- H_m : transmitter-receiver channel gain
- $P_m \le \hat{P}_m$: transm. power; $p_m = H_m P_m$; $\bar{p} := \sum_{i=1}^{M} p_i$
- $R_m \le \hat{R}_m$: symbol rate; $\Gamma_m := W/R_m$
- L_m : bits per packet, including C_m "overhead" bits
- b_m : bits per symbol, and $\bar{b}_m := b_m(L_m - C_m)/L_m$
- With interference treated as Gaussian noise, the per-symbol signal-to-noise-plus-interference ratio (SNIR) is defined as $(W/R_m)\kappa_m$ where κ_m is the carrier-to-noise-plus-interference ratio (CNIR):

$$\kappa_m := \frac{H_m P_m}{\sum_{\substack{n=1 \\ n\neq m}}^{M} H_n P_n + p_0} \equiv \frac{p_m}{\sum_{\substack{n=1 \\ n\neq m}}^{M} p_n + p_0} \tag{1}$$

- $F(x;\mathbf{a})$ is the packet-success rate function (PSRF) as a function of the *per-symbol* SNIR, given the link parameters, $\mathbf{a}$ (bold-face $\implies$ vector).
- For any $\mathbf{a}$, the graph $F(x;\mathbf{a})$ is assumed to be an "S" curve (see [15] and Fig. 1).
- $f(x;\mathbf{a}) := F(x;\mathbf{a}) - F(0;\mathbf{a})$ replaces F for some technical reasons ([16]). E. g., in the notation of [5], $f(x;\mathbf{a}) = [1 - P_b(x,b)]^{L/b} - [1/2]^L$ with P_b the symbol error probability.
- $\mathscr{A} := \{\mathbf{a^1}, \cdots, \mathbf{a^K}\}$: available link configuration vectors (excluding power and symbol rate).

III. Problem formulation

A. Problem Statement

$$\max_{\mathbf{a_i} \in \mathscr{A}} \quad \sum_{i=1}^{M} w_i \bar{b}_i R_i f(x_i;\mathbf{a}_i) \tag{2a}$$

subject to,

$$0 \le P_i \le \hat{P}_i \tag{2b}$$

$$0 \le R_i \le \hat{R}_i \tag{2c}$$

with

$$x_i = \frac{W}{R_i} \frac{H_i P_i}{\sum_{\substack{n=1 \\ n\neq m}}^{M} H_n P_n + p_0} \tag{2d}$$

$w_i > 0$ is a "weight" (e.g. economic value of one information bit) and $\bar{b}_i R_i f(x_i;\mathbf{a}_i)$ is i's throughput (standard definition).

Optimising over $\mathscr{A}$ appears analytically intractable. But one can apply KKT analysis ([9], [10]) to the *sub-problem* of finding the optimal P_i and R_i for *given* $\mathbf{a_i}$. This eventually leads to the optimal $\mathbf{a_i}$.

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B. Optimisation re-formulation

Following Appendix A, we re-state the problem in terms of π_i, i's fraction of the total power at the receiver.

For notational convenience, we let

$$B(x_m;\mathbf{a_m}) := \frac{\bar{b}_m f(x_m;\mathbf{a_m})}{x_m} \tag{3}$$

with $B(0;\mathbf{a_m}) := \lim_{x\downarrow 0} B(x;\mathbf{a_m}) \equiv \bar{b}_m f'(0;\mathbf{a_m})$ (to avoid technicalities when $x=0$).

By hypothesis, (3) has the form $S(x)/x$ with S an S-curve (Definition B.2), and by Lemma B.1, $S(x)/x$ has a unique maximum at the genu of the S-curve $\bar{b}_m f(\cdot;\mathbf{a_m})$ (see Fig. 1).

Problem (2) is now formulated through (3):

$$\text{maximise:} \quad \textstyle\sum_{i=1}^{M} w_i \kappa(\pi_i) B(x_i;\mathbf{a_i}) \tag{4a}$$

subject to,

$$\textstyle\sum_{i=1}^{N} \pi_i \le 1-\varepsilon \tag{4b}$$
$$W\kappa(\pi_i) - \hat{R}_i x_i \le 0 \tag{4c}$$
$$\pi_i \ge 0 \tag{4d}$$
$$x_i \ge 0 \tag{4e}$$

IV. Technical results

A. KKT conditions

Proposition IV.1. *If the vectors $(x_1,\cdots,x_M)$ and $(\pi_1,\cdots,\pi_M)$ form a (local) optimiser pair for Problem (4), then there are non-negative real numbers $\lambda_0,\lambda_1,\cdots,\lambda_M$, $\mu_1,\cdots.\mu_M,\nu_1\cdots,\nu_M$ such that, $\forall i$,*

$$\kappa'(\pi_i)\left(w_i B(x_i;\mathbf{a_i}) - \mu_i W\right) - \lambda_0 = -\lambda_i \tag{5a}$$
$$w_i\kappa(\pi_i)B'(x_i;\mathbf{a_i}) + \hat{R}_i\mu_i = -\nu_i \tag{5b}$$
$$\lambda_0\left(\sum_{j=1}^{N}\pi_j - (1-\varepsilon)\right) = 0 \tag{5c}$$
$$\mu_i\left(W\kappa(\pi_i) - \hat{R}_i x_i\right) = 0 \tag{5d}$$
$$\lambda_i\pi_i = 0 \tag{5e}$$
$$\nu_i x_i = 0 \tag{5f}$$

Proof: KKT necessary conditions [9], [10]. ■

Remark IV.1. If terminal i is "active", that is, $\pi_i > 0$ (which $\implies x_i > 0$), then by (5e) and (5f), $\lambda_i = 0 = \nu_i$.

B. Analysis of KKT conditions

It can be shown that, by (5), with dissimilar weights, at the optimum at most one terminal (which uses "left-over" power) operates below the highest available symbol rate. To characterise the optimal resource share for the maxed-out terminals, we need several auxiliary results.

Proposition IV.2. *If $\pi_i > 0$ is part of a (local) optimiser pair for Problem (4) , then*

$$-W\mu_i = w_i(W/\hat{R}_i)\kappa(\pi_i)B'(x_i;\mathbf{a_i}) \tag{6}$$

Proof: By Remark IV.1, $\pi_i > 0 \implies \nu_i = 0$. With $\nu_i = 0$, (5b) yields: $-\mu_i = (w_i/\hat{R}_i)\kappa(\pi_i)B'(x_i;\mathbf{a_i})$ ■

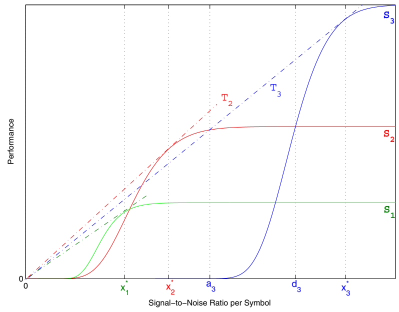


Figure 1: The tangent from the origin to a curve is its tangenu and the tangency point its genu ("knee"). With a single link, the configuration that yields S_2 is best, because it has the steepest tangenu[6].

Lemma IV.1. *If $\pi_i > 0$, and $R_i = \hat{R}_i$, then (5) yields*

$$w_i\bar{b}_i f'(x_i;\mathbf{a_i})\kappa'(\pi_i) = \lambda_0 \tag{7}$$

Proof: $R_i = \hat{R}_i \implies (W/\hat{R}_i)\kappa(\pi_i) = x_i$, which by (5d), $\implies \mu_i > 0$. Replacing x_i into (6) yields $-W\mu_i = w_i x_i B'(x_i;\mathbf{a_i})$ which replaced into (5a) — with $\lambda_i = 0$ by Remark IV.1— yields

$$w_i\kappa'(\pi_i)\left(B(x_i;\mathbf{a_i}) + x_i B'(x_i;\mathbf{a_i})\right) = \lambda_0 \tag{8a}$$

But

$$B(x;\mathbf{a_i}) + xB'(x;\mathbf{a_i}) \equiv \frac{d}{dx}\left(xB(x;\mathbf{a_i})\right) \tag{8b}$$

and by (3), $x_i B(x_i;\mathbf{a_i}) \equiv \bar{b}_i f(x_i;\mathbf{a_i})$

Thus (8b) is equivalent to $\bar{b}_i f'(x_i;\mathbf{a_i})$, which together with (8a) implies (7) . ■

Lemma IV.2. *With $\mu_i > 0$,*

$$\kappa'(\pi_i) = \left(1 + \frac{\hat{R}_i}{W}x_i\right)^2 \tag{9}$$

Proof: By (A.4), for $\pi_i \in [0,1)$, $\kappa'(\pi_i) = (\kappa(\pi_i)/\pi_i)^2$

With $\mu_i > 0$, (5d) implies that $\kappa(\pi_i) = \hat{R}_i x_i/W$

By definition of π_i, $1/\pi_i = (W/\hat{R}_i + x_i)/x_i$

$$\therefore \left(\frac{\kappa(\pi_i)}{\pi_i}\right)^2 = \left(1 + \frac{\hat{R}_i}{W}x_i\right)^2$$

■

Theorem IV.1. *If $(\pi_1,\cdots,\pi_N)$ is a (local) optimiser for Problem (4), and $\pi_i > 0$ with $\mu_i > 0$, then $\exists\lambda_0 > 0$ s. t.*

$$w_i\bar{b}_i\left(1 + \frac{x_i}{W/\hat{R}_i}\right)^2 f'(x_i;\mathbf{a_m}) = \lambda_0 \tag{10}$$

Proof: Direct from Lemmas IV.1 and IV.2. ■

A significant issue remains: how to determine λ_0 in (10).

V. Finding λ_0 and the optimal link configuration

A. An economic interpretation of FONOC

Suppose that i operates at the maximal symbol rate, and is allowed to purchase any desired power fraction, π_i, in exchange for a payment $\lambda\pi_i$. To choose optimally, it must maximise the difference between benefit and cost, that is

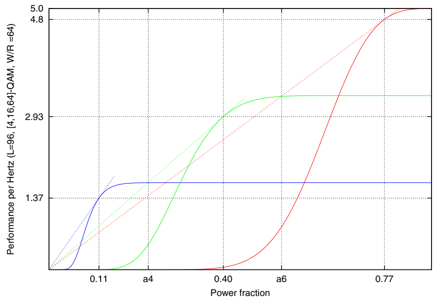


Figure 2: $S(z;b,L,C) := \bar{b}\left(f(\Gamma\kappa(z);b,L,C) - 2^{-L}\right)$ with $\Gamma := W/\hat{R} = 64$, $L = 96$, $C = 16$, for QAM modulation with $b = 2$ (blue), $b = 4$ (green) and $b = 6$ as function of the power fraction z.

$$w_i \bar{b}_i \hat{R}_i f(\Gamma_i \kappa(z); \mathbf{a}_i) - \lambda \pi_i \equiv \tag{11}$$

$$w_i \bar{b}_i \hat{R}_i f(x_i; \mathbf{a}_i) - \lambda \frac{x_i}{x_i + W/\hat{R}_i} \tag{12}$$

Setting the derivative of (12) w. r. t. to x_i to zero yields:

$$w_i \bar{b}_i \hat{R}_i f'(x_i; \mathbf{a}_i) - \lambda \frac{W/\hat{R}_i}{\left(x_i + W/\hat{R}_i\right)^2} = 0 \tag{13}$$

It is trivial to re-write (13) as (10) with $\lambda_0 = \lambda/W$. Thus, pricing can yield the optimal λ_0 (see below).

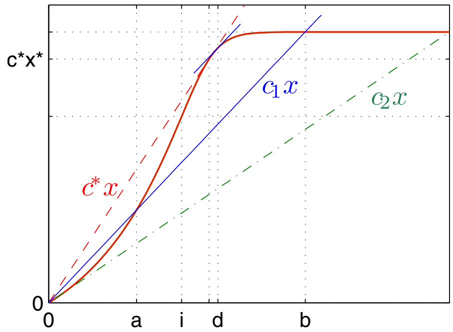
B. Maximising $S(x) - cx$ and choosing S

The composite function $\bar{b}_i \hat{R}_i f(\Gamma_i \kappa(z); \mathbf{a}_i)$ retains the S-shape (see Figure 2 for QAM modulation, and Assumption (B.1)); therefore, (12) has the form $S(z) - c_i z$ with S an S-curve and $c_i = \lambda/w_i$.

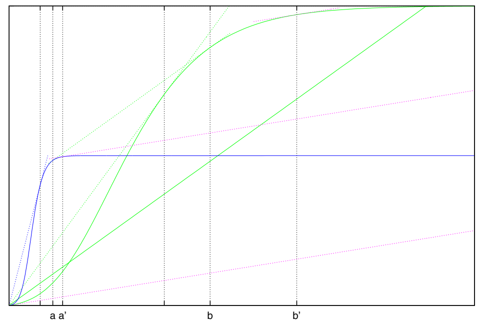
Fig. 3a illustrates the general solution to max $S(x) - c_1 x$ subject to $x \leq X$, where S is an S-curve, and x is a generic variable. If $c_1 > c^*$ (with c^* the slope of the tangenu), $c_1 x > S(x)\ \forall x > 0 \therefore x = 0$ is optimal. Likewise, if the constraint X is less than a (the abscissa where the cost line first intercepts S), then for any $x \leq X < a$ $c_1 x > S(x)$ and again $x = 0$ is optimal. Otherwise the solution is $\min(X, x_1^*)$ with x_1^* such that $S'(x_1^*) = c_1$, that is, the curve's tangent at x_1^* is parallel to the cost line $c_1 x$.

Consider now Fig. 3b and suppose that one can additionally choose either of S_1 (blue) or S_2 (green) with $c_1^* > c_2^*$ the slopes of the tangenus. By the previous paragraph, if $c > c_2^*$, $cx > S_2(x)\,\forall x > 0$, thus S_2 is ignored, and the solution is that already found for a single S. However if $c_1^* > c_2^* \geq c$, both curves should be considered.

In Fig. 3b, the green ray and the abscissas a and b are key. When the cost line cx lies exactly on the green ray, then a and b are the respective optimal choices for curves S_1 and S_2, that is, $S_1'(a) = c = S_2'(b)$. Furthermore, $S_1(a) - ca = S_2(b) - cb$ (observe the distance between the the green ray and the dotted green line that is tangent to both curves). After the cost line moves to the right of the green ray, S_2 becomes preferable to S_1, because then $S_i(x) - cx$ ("benefit minus cost"), is larger with S_2 than with S_1 (with each S_i operating at its respective optimal point for c). For example, if the lower ray (magenta, dotted) corresponds to the cost line $c'x$, then a' and b' are the



(a) To max $S(x) - c_1 x$ s.t. x≤X, if $c_1 > c^*$ or $X < a$ set $x = 0$. Else, choose $\min(X, x_1^*)$.



(b) Cost line below green ray $\implies$ S_2 is better

Figure 3: Maximising $S(x) - cx$ while possibly choosing S

agent's respective optimal choices for S_1 and S_2, (i.e., $S_1'(a') = c' = S_2'(b')$), and it is evident that $S_1(a') - c'a' \ll S_2(b') - c'b'$.

To find the transition line, one simply draws the tangent common to both curves (dotted green line): the transition cost line is the ray parallel to that common tangent (compare both green lines). This yields a *criterion* for *link configuration*, as discussed further below.

C. Optimal price and link configuration

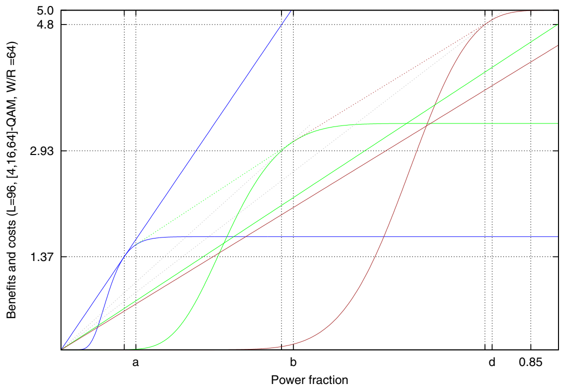


Figure 4: Optimal price λ when terminal i (of weight w_i) maximises $S(z; \mathbf{a}_i) - c_i z$ with $c_i = \lambda/w_i$ and can choose any of S_1 (lower, blue), S_3 (higher, brown) or S_2 (green) through its link configuration. If the line $c_i z$ is between the green and the brown rays, i chooses S_2.

Consider Figures 2 and 4. Following subs. V-B, the manager can find the right λ by "sweeping" a "cost line" λz from the (almost) vertical to the horizontal ($\lambda = 0$) position.

Notice that, with a common rate constraint $\hat{R}_i = \hat{R}$, a terminal can choose z by maximising $\hat{R}w_i\left(\bar{b}_i f(\Gamma\kappa(z);\mathbf{a}_i) - (\lambda/(\hat{R}w_i))z\right)$, or just the expression within the bracket: $\bar{b}_i f(\Gamma\kappa(z);\mathbf{a}_i) - c_i z$ with $c_i = \lambda/(\hat{R}w_i)$. $\bar{b}_i f(\Gamma\kappa(z);\mathbf{a}_i)$ yield the curves on Fig. 4 which are the same for all terminals. But for any given λ, the terminals with the highest w_i will "see" the lowest "net" price $c_i = \lambda/(\hat{R}w_i)$, and hence will find the price to be acceptable sooner than the others, and will ultimately choose a larger z and possibly a higher curve (which is intuitively appealing). With distinct $\hat{R}_i$, the highest $\hat{w}_i := \hat{R}_i w_i$ "sees" the lowest price.

Focus now on terminal i and let $c_i = \lambda/(\hat{R}w_i)$. If $c_i z$ lies to the left of the tangenu of S_1 (blue), then the terminal would choose zero. When $c_i z$ is between the tangenus of the blue and green curve, the terminal chooses a z such that $S_1'(z) = c_i$. Once $c_i z$ moves beneath the tangenu of the green curve, both S_1 and S_2 are in play. When $c_i z$ moves just past the green ray, then S_2 becomes preferable to S_1 (greater "benefit minus cost"). The brown line plays an identical role between the green and brown lines.

As the price line rotates clockwise, each active terminal chooses progressively larger z values and possibly higher modulations. Evidently the process must stop when the sum of the z_i chosen by all terminals equals $1-\varepsilon$.

VI. Numerical experiments

A. Experimental framework

We consider $L = 96$, $C = 16$ and {4,16,64}-QAM in a CDMA system (as in [13]). $W/\hat{R} = \Gamma = 64$. As shown by Fig. 2, the critical slopes are respectively $\rho_1^* = 12.0$, $\rho_2^* = 7.325$ and $\rho_3^* = 6.24$, and the transition slopes (slopes of green and brown lines) are $\lambda_{2,4} = 5.33$, $\lambda_{4,6} = 4.99$.

The resource is $1-\varepsilon$ with $0.1 \le \varepsilon \le 0.4$. With $M \in \{4, 10\}$, weights are taken as $w_m = \alpha^{m-1}$ with $\alpha \in (0,1)$. Thus, with $\alpha \approx 1$, the weights are also ≈ 1, but with small α the disparity between weights is very large.

We compare against 2 heuristics: egalitarian: $\hat{\pi}_m = (1-\varepsilon)/M$ and proportionate: $\hat{\pi}_m = (1-\varepsilon)\left(w_m/\sum_{j=1}^{M} w_j\right)$ (resource divided according to the weights). For the heuristic rules, the link configuration used was the "ideal" (steepest tangenu), which is 4-QAM ($b = 2$) in our case[7].

If a terminal that receives π_i wants SNIR x^* it attempts to set $R_i^* = W\kappa(\pi_i)/x^*$. If $R_i^* \le \hat{R}$ — that is, $\kappa(\pi_i) \le x^*/\hat{\Gamma}$ — this terminal achieves normalised goodput $T_i/\hat{R} = (W/\hat{R})\kappa(\pi_i)\bar{b}^* f(x^*;\mathbf{a}^*)/x^* \equiv \hat{\Gamma}\rho^*\kappa(\pi_i)$ where $\rho^* := \bar{b}^* f(x^*;\mathbf{a}^*)/x^*$ = 0.166 for 4-QAM[7] .

If $R_i^* = W\kappa(\pi_i)/x^* > \hat{R}$, i sets $R_i = \hat{R}$ and keeps 4-QAM. If a terminal that operates with $R_m = \hat{R} \equiv W/\hat{\Gamma}$ receives π_m, its normalised goodput is $T_m/\hat{R} = \bar{b}_m^* f(\hat{\Gamma}\kappa(\pi_m);\mathbf{a_m})$.

System performance is $\sum_j w_j T_j/\hat{R}$.

B. Numerical results

The results below come from the free and open-source Maxima computer algebra system (http://maxima.sourceforge.net/).

Table I: KKT vs. 2 heuristics for 1-$\varepsilon = 0.85$, $M = 10$, $w_m = \alpha^{m-1}$

α	λ^*	KKT	prop	egal	KKT/egal
7/8	5.28	6.83	6.58	5.81	1.18
5/8	4.92	5.37	3.64	2.61	2.1
1/2	4.92	5.26	2.91	1.97	2.7
3/8	4.44	5.16	2.43	1.58	3.3
1/8	1.44	5.01	1.80	1.13	4.4

Table II: KKT solution details with $M = 10$

$\alpha = 7/8$				$\alpha = 1/2$			
w_i	z_i^*	b_i^*	$\bar{T}_i$	w_i	z_i^*	b_i^*	$\bar{T}_i$
1	0.42	4	3.07	1	0.78	6	4.85
7/8	0.13	2	1.53	1/2	0.07	<2	0.82
0.77	0.13	2	1.51	1/4	0	—	0
0.67	0.13	2	1.48	1/8	0	—	0
0.59	0.04	<2	0.46	1/16	0	—	0
0.51	0	—	0	1/32	0	—	0

Table III: KKT vs. 2 heuristics for 1-$\varepsilon = 0.85$, $M = 4$, $w_m = \alpha^{m-1}$

α	λ^*	KKT	prop	egal	KKT/egal
7/8	4.92	5.57	5.50	5.51	1.01
5/8	4.92	5.37	3.59	3.76	1.4
1/2	4.92	5.26	2.92	3.12	1.7
3/8	4.44	5.16	2.43	2.61	2.0
1/8	1.44	5.01	1.80	1.90	2.6

Table IV: KKT solution details with $M = 4$

$\alpha = 7/8$				$\alpha = 1/2$			
w_i	z_i^*	b_i^*	$\bar{T}_i$	w_i	z_i^*	b_i^*	$\bar{T}_i$
1	0.78	6	4.85	1	0.78	6	4.85
7/8	0.07	<2	0.82	1/2	0.07	<2	0.82
0.77	0	—	0	1/4	0	—	0
0.67	0	—	0	1/8	0	—	0

Table I shows for 10 terminals with $\varepsilon = 0.15$, and various α values, the optimal price and performance values. Table II indicates a terminal's weight (w_i), allocated resource z_i^*, modulation order b_i^*, (<2 means operation below the highest symbol rate), and unweighted normalised goodput. The KKT advantage over egalitarianism goes from 18% to over 4-to-1, the greater the inequality among the weights (smaller α), the greater the margin. Tables III and IV give similar information, for the case $M = 4$. The egalitarian heuristic does much better with $M = 4$, and almost catches up with KKT when weights are commensurate, but KKT pulls away (over 2-to-1) as inequality grows (all of which makes sense). To establish definite performance margins, more numerical work is needed.

VII. Conclusion

For many mutually-interfering data terminals, we have jointly optimised their link parameters, which is a major challenge even for a single transmitter-receiver pair. KKT analysis has led to a surprisingly simple solution: the key is to sweep a "cost line" from near vertical to horizontal. The result applies whenever each considered parameter combination yields a packet success rate that is (at least approximately) an S-curve. We applied the procedure for QAM, and obtained encouraging results. The obtained configuration satisfies the *necessary* optimising conditions, and is eminently a maximiser, but not necessarily the global one. Further numerical work may be instructive. Consideration of more general network topologies is desirable.

APPENDIX A
CHANGE OF VARIABLES

A. Feasibility of power and rates

The next statement has been proven elsewhere, e. g. in [12]:

Fact A.1. *There exist non-negative (received) power levels p_i that produce a desired set of M (x_i, R_i) SINR/symbol-rate pairs provided that, for $0 < \varepsilon < 1$,*

$$\sum_{i=1}^{M} \frac{x_i}{x_i + W/R_i} := \sum_{i=1}^{M} \pi_i \leq 1 - \varepsilon \quad \text{(A.1)}$$

The corresponding power levels are given by

$$\frac{p_i}{p_0} = \frac{\pi_i}{1 - \sum_{j=1}^{M} \pi_j} \quad \text{(A.2)}$$

In (A.1), $\varepsilon < 1$ represents the resource allocated to a (e.g. video) terminal with a hard quality-of-service constraint.

The largest power level that terminal i may need occurs when the entire resource $1 - \varepsilon$ is allocated to i. In this case,

$$\frac{p_i/p_0}{p_i/p_0 + 1} = 1 - \varepsilon \implies \frac{p_i}{p_0} = \frac{1 - \varepsilon}{\varepsilon} \quad \text{(A.3)}$$

We assume $H_i \hat{P}_i / p_0 \geq (1 - \varepsilon)/\varepsilon \; \forall i$. Thus, if (A.1) is satisfied, we need *not* worry about the power constraints.

B. Fraction of total received power

It is simple to show ([12]) that $\pi_i := x_i/(x_i + W/R_i)$ yields i's fraction of the total power at the receiver, that is, $p_i / \left(p_0 + \sum_{i=1}^{M} p_i\right)$. From π_i the CNIR, κ_i is directly obtained:

Proposition A.1. *There is a one-to-one correspondence between κ_i and π_i given by $\kappa_i = \kappa(\pi_i)$, where*

$$\kappa(z) := \frac{z}{1 - z} \text{ for } z \in [0, 1) \quad \text{(A.4)}$$

κ is strictly increasing and convex.

Proof: Omitted (see [12]). ■

For a feasible vector of symbol rates $(R_1, \cdots, R_M)$, the corresponding SNIR are obtained as $\kappa_i W/R_i$. Likewise, for given SNIR x_i and power ratio π_i, the corresponding symbol rate R_i satisfies $x_i = W\kappa(\pi_i)/R_i$ or $R_i = W\kappa(\pi_i)/x_i$. That is,

Lemma A.1. *For given $(\pi_1, \cdots, \pi_M)$, if $x_m > 0$ and $W\kappa(\pi_m) - \hat{R}_m x_m \leq 0$, and $\kappa(z)$ given by (A.4), m's throughput is*

$$W\kappa(\pi_m) \frac{\bar{b}_m f(x_m; \mathbf{a_m})}{x_m} \quad \text{(A.5)}$$

Proof: Omitted (see [12]). ■

APPENDIX B
MATHEMATICAL ISSUES

Definition B.1. $S : \Re_+ \to [0, Y]$, is an S-curve with unique inflexion at x_f if (i) $S(0) = 0$, S is (ii) continuously differentiable, (iii) strictly increasing, (iv) convex over $[0, x_f)$ and concave over (x_f, ∞), and (v) surjective.

Assumption B.1. *If S satisfies Definition B.1, so does $S(\Gamma z/(1 - z))$ for $z \in [0, 1)$ and $\Gamma \geq 1$.*

Definition B.2. A function $h : \Re_+ \to [0, Y]$ is single-peaked over $\Re_+$ if h is continuous, and is such that: $h(X) = Y$, and $0 \leq x_1 < x_2 \leq X \implies h(x_2) > h(x_1)$ and $X \leq x_1 < x_2 \implies h(x_2) < h(x_1)$.

Lemma B.1. *Suppose $S : \Re^+ \to [0, d]$ is an S-curve (Definition B.1) with inflexion at z_f. Let $\mathscr{B}(z) := S(z)/z$ with $\mathscr{B}(0) := \lim_{z \downarrow 0} \mathscr{B}(z) \equiv S'(0)$. Then, (i) there is a unique tangent line from the origin to $S(z)$, denoted as $c^* z$ and called the tangenu, with tangency point, genu, $(z^*, S(z^*))$, where $z^* > z_f$. (ii) $\mathscr{B}$ is strictly quasi-concave, and its unique maximiser in the interval $[0, Z]$ is $\min(z^*, Z)$.[15]*

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