Training Sequence Design for Channel Estimation with Nonlinear OQPSK-Type Modulations

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***Abstract*—Nonlinear OQPSK-type (Offset Quadrature Phase Shift Keying) modulations can be designed to have almost constant envelope, making it suitable for systems requiring highly-efficient and low-cost grossly nonlinear power amplifiers. In this paper we consider the use of pseudo-random spreading sequences for channel estimation purposes in systems employing nonlinear OQPSK-type modulations. We consider Maximum-Length sequences, Kasami sequences, Gold sequences and Tomlinson, Cercas, Hughes sequences, all known to have good auto-correlation properties, making them appropriate for channel estimation purposes. The cross-correlation between different OQPSK components that constitute nonlinear OQPSK-type signals is obtained and its impact on the channel estimation performance is evaluated. It is shown that Tomlinson, Cercas, Hughes sequences outperform the other sequences in the channel estimation context.**

***Index Terms*—Channel Estimation, Spreading Sequences, MSE, OQPSK, Nonlinear**

I. Introduction

Pseudo-random sequences are widely employed in communication systems. This type of spreading sequences are commonly used to separate users in Code Division Multiple Access (CDMA) or for the synchronization process. However, due to the correlation properties of this type of sequences they can also be used for the purpose of channel estimation. Channel estimation with pseudo-random sequences has been well covered in works such as [1][2][3], but none assume nonlinear modulations. That is the main motivation behind this work.

To reduce both the complexity and cost of mobile terminals, while still providing high transmission rates, it is advantageous to use nonlinear power amplifiers. These amplifiers are simpler to implement and have higher amplification efficiency and output power linear ones. However, nonlinear power amplification should only be employed with signals with constant or quasi-constant envelope, to avoid nonlinear distortion effects. CPM (Continuous Phase Modulation) [4] schemes are constant-envelope modulations that include as special cases MSK (Minimum Shift Keying) [5] and GMSK (Gaussian MSK) [6], among others. These modulations can be denoted as OQPSK-type modulations (Offset QPSK) since they can be decomposed as the sum of OQPSK components [7][8]. OQPSK-type schemes are particularly important in the context of nonlinear amplification since an OQPSK-type signal retains its OQPSK-type structure when submitted to bandpass memoryless nonlinear devices, the usual model for power amplifiers [9], which simplifies the performance evaluation and receiver design [10].

In this paper we consider the training sequence design for nonlinear OQPSK-type modulations. CAZAC (Constant Amplitude, Zero AutoCorrelation) sequences such as Zadoff-Chu [11] sequences are usually recommended for channel estimation purposes. However, although these sequences have constant envelope, the analog signal associated to them can have strong envelope fluctuations, namely with passages close to zero. This means that these sequences are not appropriate for grossly nonlinear amplifiers. As an alternative, we can employ pseudo-random binary spreading sequences with good (but not ideal) auto-correlation properties. For this reason, we consider Maximum-Length sequences, Kasami sequences, Gold sequences and Tomlinson, Cercas, Hughes sequences, all known to have good auto-correlation properties. The cross-correlation between different OQPSK components that constitute nonlinear OQPSK-type signals is obtained and its impact on the channel estimation performance is evaluated.

This paper is organized as follows. The fundamentals of each pseudo-random code family are described in Section II. Section III presents the analytical conversion from parallel to serial representation of the OQPSK schemes along with the effect of nonlinearity applied to the OQPSK-type modulations. Section IV presents the channel characterization as well as the specification of performance measure for this work. Section V covers the simulations results, with summary and conclusion marks found in Section VI.

II. Pseudo-random Sequences

In this section we present the studied code families along with their correlation properties.

A. Maximum Length

Maximum-Length Sequences (MLS) are generated through a linear feedback shift register (LFSR). A LFSR of a given size m (number of registers) is able to produce a sequence of

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$n = 2^m - 1$ elements [12]. This type of codes present good auto-correlation properties, close to a dirac function, relevant in the process of channel estimation. Autocorrelation for this type of sequences presents the following values:

$$\phi_x(i) = \begin{cases} n & , i = 0 \\ -1 & , i \neq 0 \end{cases} \tag{1}$$

B. Kasami

Kasami sequences are binary sequences of length $n = 2^m - 1$ with m being an integer and were first proposed in [13]. For an MLS a, a' is obtained by taking every q^{th} bit of a denoted by $a[q]$. a' is called a decimated sequence of a. By choosing $q = 2^{m/2} + 1$, where m is the degree of sequence a, a' is periodic with period $2^{m/2} - 1$. By repeating a' q times, a new sequence b is obtained. Kasami sequences are formed by performing a modulo-two addition between a and a time shifted version of b. Since the decimated sequence is itself an MLS of order $m/2$, Kasami sequences can be generated using a preferred pair of polynomials. The values for their auto-correlation are:

$$\phi_x(i) = \begin{cases} n & , i = 0 \\ -1, -2^{\frac{m}{2}} + 1, 2^{\frac{m}{2}} - 1 & , i \neq 0 \end{cases} \tag{2}$$

C. Gold

Gold sequences are another family of well known pseudo-random sequences. A set of Gold code sequences consists of $2^m - 1$ sequences each one with a period of $n = 2^m - 1$. In order to generate a set of Gold sequences pick two MLS of the same length. For certain well-chosen pairs, the cross correlation only takes on three possible values, with these pairs being commonly referred to as preferred pairs or preferred polynomials. The set of $2^m - 1$ exclusive-ors of these two sequences in their various phases (relative positions) is a set of Gold Codes. According to [14], the values for the auto-correlation are:

$$\phi_x(i) = \begin{cases} n & , i = 0 \\ -1, -2^{\frac{m+2}{2}} + 1, 2^{\frac{m+2}{2}} - 1 & , i \neq 0 \end{cases} \tag{3}$$

D. Tomlinson, Cercas, Hughes

Tomlinson, Cercas, Hughes (TCH) codes [15] are a class of nonlinear block codes that were primarily studied to simplify the radio interface of a receiver, since their properties allow the use of a very efficient correlator implemented in the frequency domain. These codes are a class of binary, nonlinear, non-systematic and cyclic block codes with length $n = 2^m$ (m being any positive integer).It is possible to obtain codes analytically with lengths n of $2, 4, 16, 256$ and 65536, which are also named as Basic TCH codes or B-TCH. These code sets can be expanded for other lengths, however their correlation properties are not as good as those of B-TCH. As usual, the polynomials used to generate these codes can be used on channel coding, however their codewords can also be used as pseudo-random sequences, due to their correlation properties. The auto-correlation value of any B-TCH polynomial is not dependent from the code length presenting always only three possible values:

$$\phi_x(i) = \begin{cases} n & , i = 0 \\ -4, 0 & , i \neq 0 \end{cases} \tag{4}$$

III. Nonlinear OQPSK

A. Parallel and serial schemes

In a QPSK scheme, consider a signal to be transmitted as $x(t) = Re\{x_p(t)\exp(j2\pi f_c t)\}$ where f_c is the carrier frequency and $x_p(t)$ is the complex envelope given by

$$x_p(t) = \sum_n a_n r_p(t - 2nT) \tag{5}$$

where $a_n = a_n^I + ja_n^Q$ with $a_n^I = \pm 1$ and $a_n^Q = \pm 1$ are the in-phase and quadrature bits of the data block to be transmitted, T is the bit duration and $r_p(t)$ is the adopted pulse shape. This signal can then be viewed as a sum of PAM (Pulse-Amplitude Modulation) signals with complex symbols separated by $2T$, since we are transmitting 2 bits per symbol. For an OQPSK scheme we have the same $x(t) = Re\{x_p(t)\exp(j2\pi f_c t)\}$ signal to be transmitted but in this case the OQPSK signal can be regarded as a sum of two PAM signals with symbols separated by $2T$ and an offset T between them. This corresponds to the parallel representation of an OQPSK signal given by

$$x_p(t) = \sum_n a_n^I r_p(t - 2nT) + \sum_n a_n^Q r_p(t - 2nT - T) \tag{6}$$

An alternative way of writing the OQPSK signal is

$$x_p(t) = \sum_n a_n^p r_p(t - nT) \tag{7}$$

In this case, a_n^p represents both in-phase and quadrature components, appearing alternately assuming the following values

$$a_n^p = \begin{cases} a_{n/2}^I = \pm 1 & , n \text{ even} \\ ja_{(n+1)/2}^Q = \pm j & , n \text{ odd} \end{cases} \tag{8}$$

In both representations of the OQPSK signal, we are assuming the complex envelope in reference to f_c. By shifting the reference carrier from f_c to $f_1 = f_c + \frac{1}{4T}$, the signal can be represented in a serial (BPSK-type) format. In fact, we have $x(t) = Re\{x_s(t)\exp(j2\pi f_1)\}$, with the complex envelope

$$x_s(t) = x_p(t)\exp^{-j\pi\frac{t}{2T}} \tag{9}$$

Since the parallel signal in (7) can be represented as

$$x_p(t) = \left(\sum_n a_n^p \delta(t - nT)\right) \times r_p(t) \tag{10}$$

the following can be deducted from (9) and (10)

$$\begin{aligned} x_s(t) &= \left[\left(\sum_n a_n^p \delta(t - nT)\right) e^{-j\pi\frac{t}{2T}}\right] \times \left[r_p(t) e^{-j\pi\frac{t}{2T}}\right] = \\ &= \sum_n a_n^p e^{-j\pi\frac{n}{2}} \; r_p(t)\, e^{-j\pi\frac{t}{2T}} \end{aligned} \tag{11}$$

With the complex envelope of the OQPSK signal in the serial format given by

$$x_s(t) = \sum_n a_n^s r_s(t - nT) \tag{12}$$

we can write

$$r_s(t) = r_p(t)\ e^{-j\pi\frac{t}{2T}} \tag{13}$$

$$a_n^s = a_n^p\ e^{-j\pi\frac{n}{2}} = a_n^p(-j)^n \tag{14}$$

which means that, in the serial representation of a OQPSK-type signal, a_n^s always presents the same values $a_n^s = \pm 1$, which is an advantage when compared with the alternating real and imaginary values used in the parallel representation, since only a real sequence of symbol coefficients is needed. This characteristic allows both the modulator and demodulator to have a single-branch structure [8].

B. Nonlinear modulations

Let us now consider the transmission of an OQPSK-type signal over nonlinear bandpass systems, using a nonlinear bandpass memoryless amplifier. The signal at the amplifier input is given by

$$x_{in}(t) = R\ e^{j\arg(x(t))} \tag{15}$$

where $R = |x_{in}(t)|$ is the envelope. At the amplifier output, the complex envelope is given by

$$x_{out}(t) = A(R)\ \ e^{j\,(\Theta(R)+\arg(x_{in}(t)))} \tag{16}$$

where $A(R)$ and $\Theta(R)$ represent the AM-to-AM and AM-to-PM conversion functions of the nonlinear amplifier, respectively, for the input envelope $R = |x_{in}(t)|$. The worst case scenario in terms of nonlinear distortion, corresponds to the case where the amplifier can be modeled as an ideal bandpass hard limiter (BPHL) with $A(R) = 1$ and $\Theta(R) = 0$. With this in mind, the complex envelope at the high power amplifier (HPA) output can be written as

$$x_{out}(t) = e^{j\,(\arg(x_{in}(t)))} \tag{17}$$

Assuming a signal at the input of the HPA defined by

$$x_{in}(t) = \sum_n a_n r\,(t - nT) \tag{18}$$

where $r(t)$ is the pulse of the used modulation represented by (13), T is the bit interval duration and using a serial representation of the input signal, i.e. $a_n = \pm 1$. If the pulse $r(t)$ has duration $(\mu + 2)T$, the signal at the output of HPA is given by the general equation [8]

$$x_{out}(t) = \sum_{m=0}^{M-1} x^{(m)}(t) \tag{19}$$

where$M = 2^\mu$ and

$$x^{(m)}(t) = \sum_n a_n^{(m)} r^{(m)}(t - nT) \tag{20}$$

with

$$\begin{cases} a_n^{(0)} & = a_n = \pm 1 \\ a_n^{(m)} & = a_n \prod_{l=1}^{\mu} \left(a_{n-l}\ a_{n-l-1}\right)^{\alpha_{m,l}} \end{cases} \tag{21}$$

where $\alpha_{m,l}$ is the l^{th} bit in the binary representation of $m = [\alpha_{m,\mu}, \alpha_{m,\mu-1}, ..., \alpha_{m,l}]$. The output signal can be decomposed as a sum of M linear OQPSK components, where the pulse $r^{(m)}(t)$ is given by

$$r^{(m)}(t) = j^\gamma r_p^{(m)}(t)\ e^{-j\pi\frac{t}{2T}} \tag{22}$$

with γ representing the number of digits 1 in the binary representation of (m). Each of these $r^{(m)}(t)$ pulses relates to a coefficient $a_n^{(m)}$ that is represented by a product of coefficients, and occupies the time intervals intersection associated to each of the coefficients present in that product. For instance, pulse $r^{(1)}(t)$ is associated to coefficient $a_n^{(1)} = a_n\ a_{n-1}\ a_{n-2} = \pm 1$, and occupies the time interval $[0, \mu T]$, which comes from the intersection of the time intervals corresponding to a_n, a_{n-1} and a_{n-2}, i.e., $[0, (\mu+2)T]$, $[-2T, \mu T]$ and $[-T, (\mu+1)T]$ respectively. In the same way, the pulses $r^{(2)}(t)$ and $r^{(3)}(t)$ relate to the coefficients $a_n^{(2)}$ and $a_n^{(3)}$ accordingly. Both pulses are defined in the interval $[0, (\mu-1)T]$. In this paper pulses $r^0(t)$, $r^1(t)$, $r^2(t)$, $r^3(t)$ and their respective coefficients $a_n^{(0)}$, $a_n^{(1)}$, $a_n^{(2)}$, $a_n^{(3)}$ were used, The pulses are depicted in Fig. 1, with Fig. 2 and Fig. 3 representing the auto-correlation of coefficient $a_n^{(0)}$ and the cross-correlation of coefficients $a_n^{(0)} a_n^{(1)}$ respectively.

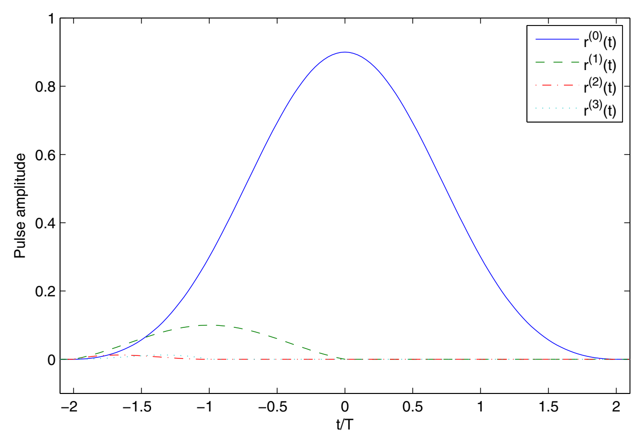

Fig. 1. Pulse shapes $r^0(t)$, $r^1(t)$, $r^2(t)$ and $r^3(t)$

IV. Channel estimation

The channel is modeled to behave as multi-path linear time-invariant channel [16] with an impulse response $h(t)$ and addictive white gaussian noise (AWGN). Fig. 4 shows the system characterization.

To estimate the system impulse response, the cross-correlation function between the output, $y(t)$ and input, $x(t)$ is estimated. Since the input-output relationship is a convolution, the relationship in terms of the cross-correlation function is given as follows

$$\phi_{yx}(t) = h(t) * \phi_{xx}(t) \tag{23}$$

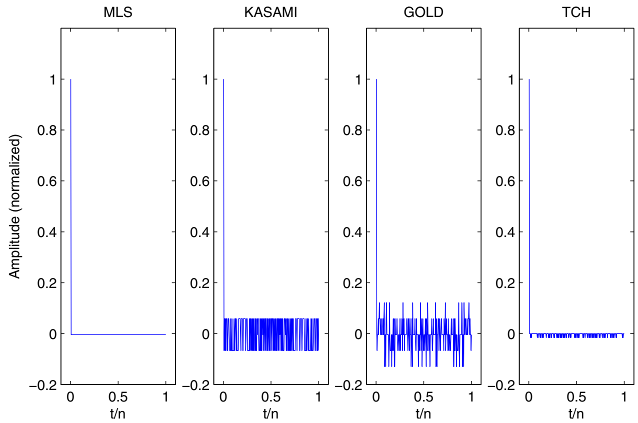

Fig. 2. Auto-correlation values for $a_n^{(0)}$ coefficients

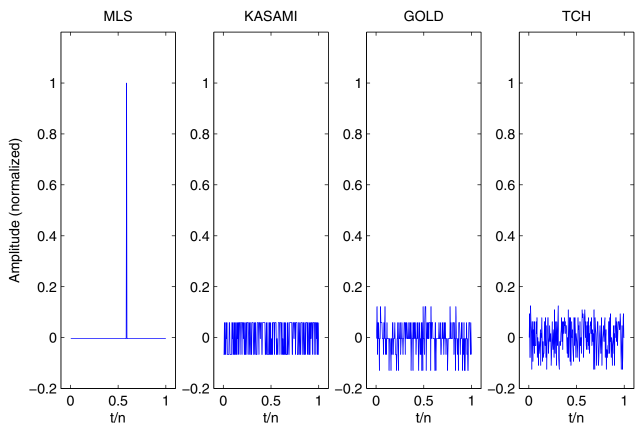

Fig. 3. Cross-correlation values for $a_n^{(0)}$ $a_n^{(1)}$ coefficients

with $*$ denoting convolution, ϕ denoting correlation result and the system channel impulse response (CIR) $h(t)$ defined as

$$h(t) = \sum_i \alpha_i \delta(t - \tau_i) \tag{24}$$

where α_i represents the attenuation factor and τ_i the corresponding time delay. The actual CIR used in this work is depicted in Fig. 5.

Since convolution in time domain is equivalent to multiplication in frequency domain, (23) becomes

$$\Phi_{yx}(f) = H(f)\Phi_{xx}(f) \tag{25}$$

where $\Phi_{yx}(f)$ denotes the Fourier transform of $\phi_{yx}(t)$, $\Phi_{xx}(f)$ denotes the Fourier transform of $\phi_{xx}(t)$ and $H(f)$ is the frequency response of the channel. Hence, the channel impulse response $h(t)$ is estimated via the inverse Fourier transform of its frequency response $H(f)$.

V. Performance Results

This section presents a set of results concerning the channel estimation performance for MLS, Kasami, Gold and TCH sequences employed with nonlinear OQPSK-type modulations. For each code family, a set of sequences was generated with length 255 or 256, depending on the type of sequence

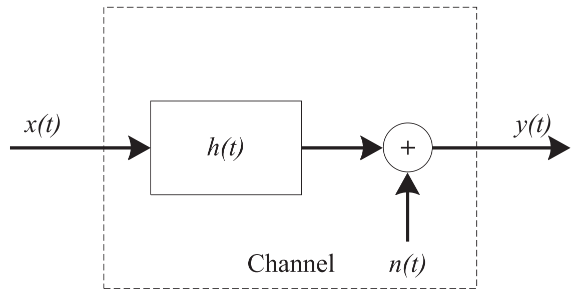

Fig. 4. Linear time-invariant channel with additive noise

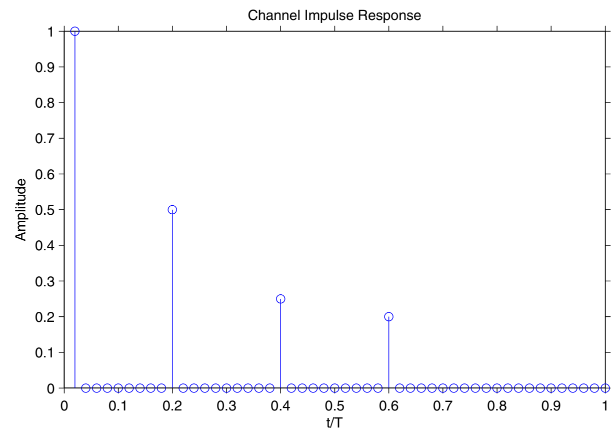

Fig. 5. Channel Impulse Response

(corresponding to $n = 8$) and modulated using the nonlinear OQPSK-type signal characterized by the pulses refereed in Section III. The resulting signal is defined in the system schematic (Fig. 4) as $x(t)$ being the signal at the output of the HPA and at the input of the channel. After creating $x(t)$, a noise component,in this case AWGN, was added. Performing convolution against $h(t)$ yields the signal at the output of the channel, $y(t)$ and at the input of the receiver. At the receiver this signal is correlated against a local replica providing us with the channel estimates. The normalized Mean square error (MSE), defined as

$$MSE = \frac{\int |\hat{h}(t) - h(t)|^2 dt}{\int h(t)^2 dt} \tag{26}$$

where $\hat{h}(t)$ is the estimated CIR and $h(t)$ is the actual impulse response, is used to quantify the channel estimation performance. This is approximated given by

$$MSE = \frac{\sum\limits_{n=0}^{N-1} (\hat{h}(n) - h(n))^2}{\sum_{n=0}^{N-1} h(n)^2} \tag{27}$$

The MSE is averaged over 1000 channel realizations for each value of E_b/N_0, with E_b denoting the average bit energy and N_0 the one-sided power spectral density of the channel noise.

Fig. 6 shows the MSE performance for different sequences. Gold sequences and Kasami sequences have the worst performance due to the higher auto-correlation values for $t \neq 0$. MLS have better auto-correlation but the cross-correlation between different OQPSK components that constitute the nonlinear OQPSK-type signal can lead to significant secondary auto-correlation lines (Fig. 3) that degrade the channel estimation performance. Therefore, although the MSE is lower than with Gold or Kasami sequences, it can still be relatively high. TCH sequences present both good auto-correlation values and small cross-correlation between different OQPSK components, making them especially interesting for nonlinear OQSPK-type modulations, with substantially lower MSE values that with other sequences.

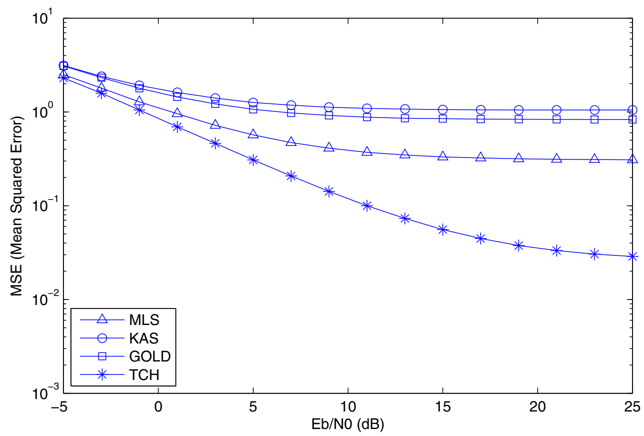

Fig. 6. MSE for the 4 different spreading sequences

The auto-correlation values for $t \neq 0$ can be regarded as a corrupting noise that degrade the channel estimation performance. To improve the channel estimation, further simulations were made, this time employing a threshold at the receiver so we could eliminate these components. Several values of threshold were tested between 0 and 0.1, taking in account the normalized auto-correlation results. The channel estimation performance based on this approach is shown in Fig. 7.

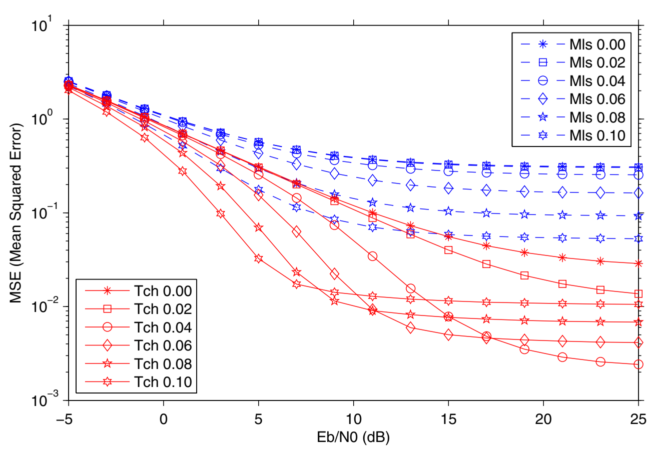

Fig. 7. Varying threshold values for MLS and TCH sequences

From Fig. 7 it is noticeable that higher values of threshold do not automatically yield the best results. This is explainable due to the fact that with a high threshold one not only eliminates the cross-correlation results deemed noise, but also important correlation values that constitute the CIR estimate. High values of threshold are appropriate only for low E_b/N_0. In situations where the relationship E_b/N_0 is high, a much lower threshold value works best since higher E_b/N_0 values result in higher peak signal-to-noise ratios.

VI. Conclusions

In this paper we considered the use of pseudo-random spreading sequences for channel estimation purposes in systems employing nonlinear OQPSK-type modulations. Sequences from the Maximum-Length, Kasami, Gold and TCH were considered for this purpose. The cross-correlation between different OQPSK components that constitute nonlinear OQPSK-type signals is obtained and its impact on the channel estimation performance is evaluated. It is shown that TCH sequences outperform the other sequences in the channel estimation context.

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