LOW-COMPLEXITY CARRIER FREQUENCY OFFSET ESTIMATION FOR MULTIUSER OFFSET QAM FILTER BANK MULTICARRIER SYSTEMS UPLINK

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ABSTRACT

In this paper, we study carrier frequency offset (CFO) estimation in the uplink of multiuser offset QAM filter bank multicarrier (FBMC) communication systems. A low-complexity frequency-domain CFO estimator using periodical training sequences is proposed. We derive the theoretical mean square error (MSE) for the proposed estimator and computer simulations show that the derived MSE matches the simulated MSE closely. Compared with state-of-the-art time-domain estimators, the proposed estimator achieves better performance with a much lower computational complexity.

I. INTRODUCTION

Multicarrier modulation is an efficient technique for implementing broadband communication systems. The Filter Bank MultiCarrier (FBMC) communication system uses an effective signaling method that achieves high spectral efficiency over broadband channels. Orthogonal Frequency Division Multiplexing (OFDM) system is a special FBMC system with a rectangular prototype filter [1]. Besides OFDM, other types of FBMC systems have recently attracted more attention in the research community. In this paper, we consider a type of FBMC system often referred to as OFDM/OQAM, where OQAM stands for offset quadrature amplitude modulation . In [2] the term *staggered* QAM has also been used to refer to OQAM and a more concise name *staggered modulated multitone* (SMT) has been suggested. For consistency, in the rest of this paper, we will use the term SMT and accordingly use the name SMTMA (SMT multiple access) to refer to SMT-based multiple access systems.

Similar to OFDM systems, SMT systems are also sensitive to carrier frequency offset (CFO), which leads to severe performance degradation [3]. The CFO is usually caused by the difference between local oscillator frequencies at the receiver and the transmitter and/or the Doppler frequency shift in the channel. Therefore, the accurate CFO estimation is essential to guarantee satisfactory performance in SMT systems.

Data-aided CFO estimation in SMT systems has been studied in [4]-[7]. In [4] and [5], data-aided CFO and timing offset estimation with robust acquisition properties in multipath channels was proposed. This algorithm exploits a periodical training sequence made up of identical blocks. In [6], a joint maximum likelihood (ML) CFO and timing offset estimator using a non-periodical training sequence in multipath channel was developed. Moreover, under the assumption that the CFO is sufficiently small, a closed-form approximate ML (AML) CFO estimator was derived. However, the performance of the CFO estimation is reduced when the CFO value is relatively large. Moreover, the performance of the CFO estimation is dependent on the training sequence used [6]. Finally, in [7] authors estimated CFO using spectral modeling. Interestingly, the CFO estimation is independent on the timing offset and also the training sequence used. However, the CFO is estimated using a maximization procedure through a 1-dimensional search, which leads to heavy computational complexity.

In this paper, we study the data-aided CFO estimation in the uplink of an SMTMA system. In the uplink, the base station (BS) needs to estimate multiple CFO's from multiple users. The choice of CFO estimation methods in the BS is closely related to the adopted subcarrier allocation scheme [8]. However, the current trend in the industry, as reflected in the 3GPP LTE documents, is more towards block allocation, where a block of contiguous subcarriers are allocated to each user [9]. The goal of this paper is to propose a low-complexity data-aided CFO estimation for SMTMA with block allocation scheme. Some studies have been performed to estimate the CFO and timing offset in the uplink of SMTMA systems. In [10] the joint ML phase, CFO, and timing offset estimation using a non-periodical training sequence in an additive white Gaussian noise (AWGN) was considered. Under the assumption that the CFO of each user is sufficiently small, an AML estimator was developed, using which the joint phase offset, CFO and timing offset estimation for different users can be carried out independently. The AML phase and CFO estimators are in closed-form, while the AML timing offset estimator requires a one-dimensional maximization procedure. By following an approach similar to that considered in [10], the estimator proposed in [6] for single-user SMT systems can be easily

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modified for SMTMA uplink in multipath channels. To distinguish it from the single user estimator proposed in [6], in the rest of the paper, we refer to the modified estimator as Fusco's multiuser CFO estimator. There are a couple of disadvantages in this estimator. Firstly, due to an approximation used in the derivation of the AML estimator, Fusco's multiuser CFO estimator works only for small CFO values. The MSE of CFO estimation will have an error floor when the CFO is relatively large. Secondly, the complexity of the estimator increases linearly with increasing the number of users and can be high when there are many users. Thirdly, the performance is sensitive to training sequence used.

In this paper, by following the approach in [5][11], a CFO estimation using frequency-domain periodical training sequences is proposed. Different from [5], the proposed algorithm estimates CFO after DFT. By inserting some guard subcarriers between each pair of adjacent user's bands, this estimator can be used for uplink scenario with block subcarrier allocation scheme. We show that the performance of the proposed estimator is better than the time-domain estimator [6] for the uplink and requires much lower computational complexity. Moreover, the estimator in [6] requires special constraints on the real and imaginary parts of the training sequences for good performance. Using the proposed estimator, the training sequence can be freely chosen to meet specific system requirements, such as to minimize the peak to average power ratio (PAPR) of the training sequence.

The rest of this paper is organized as follows. In Section II, we present the system model for SMTMA systems. Using this model, we derive the uplink CFO estimator in Section III. Also, the computational complexity of the proposed estimator and Fusco's multiuser CFO estimator [6] is compared. Section IV gives numerical results and finally the conclusions are drawn in Section V.

II. SYSTEM SETUP

We consider the uplink of a SMTMA system where P active users are communicating with a BS. It is assumed that there are $N = P \cdot Q$ subcarriers, where Q is the number of subcarriers allocated to each user, including both active and guard (null) subcarriers. The set of Q subcarriers assigned to the pth user is denoted by S_p, and we assume $\bigcup_{p=0}^{P-1} S_p = \{0, 1, \cdots, N-1\}$ and $S_p \cap S_q = \emptyset$, for $\forall\, p \neq q$. The transmitted signal from the pth user is represented by $x_p[n]$, and the channel response between the pth user and the BS is denoted by the sequence $c_p[n]$. It is assumed that $c_p[n]$ is nonzero only for $n = 0, 1, \cdots, N_c - 1$, where N_c is the maximum channel delay spread. Accordingly, the received signal at the BS can be written as

$$r[n] = y[n] + \nu[n] = \sum_{p=0}^{P-1} y_p[n] + \nu[n], \tag{1}$$

where $\nu[n]$ is an AWGN with variance σ_n^2 and $y_p[n]$ is the received signal from the pth user as

$$y_p[n] = (x_p[n] \star c_p[n])\, e^{j2\pi\varepsilon_p n/N}, \tag{2}$$

where $\star$ denotes linear convolution and $\varepsilon_p,\ p = 0, \cdots, P-1$ is the normalized CFO (with respect to the subcarrier spacing) of the pth user.

II-A. Polyphase Structure of the SMT receiver

Several polyphase structures have been proposed for efficient implementations of SMT systems. For the purpose of our study in this paper, we have chosen the polyphase structure of [12] as shown in Fig. 1. The components $E_0(z)$ through $E_{N-1}(z)$ are the polyphase components of a prototype filter $H(z)$, based on which the SMT is implemented. We assume that $H(z)$ has a length of $\beta \cdot N$, thus, each polyphase component has a length of β. The DFT block performs the demodulation at the receiver side of the system.

It is apparent that Fig. 1 is different from a conventional analysis filter bank in the literature, where the polyphase components are often followed by an IDFT [12]. This difference arises, simply, because here we have chosen to feed the input signal from the bottom of a tapped delay line, i.e., opposite to the common practice where the tapped delay line is fed from the top. This puts the input samples in a reversed order and accordingly the IDFT has to be replaced by DFT. This arrangement is chosen, here, because it matches the common practice in the presentation of an OFDM receiver.

The $N \times 1$ input vector to the DFT block at the time instant l as shown in Fig. 1 can be written as [13]

$$\mathbf{u}[l] = \sum_{i=1}^{\beta} \mathbf{u}^{(i)}[l], \tag{3}$$

where

$$\mathbf{u}^{(i)}[l] = \mathbf{r}[l+i] \odot \mathbf{h}[\beta-i] = (\mathbf{y}[l+i] + \boldsymbol{\nu}[l+i]) \odot \mathbf{h}[\beta-i], \tag{4}$$

$$\mathbf{h}[n] = \begin{bmatrix} h[nN+N-1] \\ h[nN+N-2] \\ \vdots \\ h[nN] \end{bmatrix}, \mathbf{y}[n] = \begin{bmatrix} y[nN-N+1] \\ y[nN-N+2] \\ \vdots \\ y[nN] \end{bmatrix},$$

$$\boldsymbol{\nu}[n] = \begin{bmatrix} \nu[nN-N+1] \\ \nu[nN-N+2] \\ \vdots \\ \nu[nN] \end{bmatrix},$$

and $\odot$ denotes point-wise multiplication of vectors. Furthermore, the output vector of the DFT block is given by

$$\mathbf{U}(l) = \boldsymbol{\mathcal{F}}\{\mathbf{u}(l)\}, \tag{5}$$

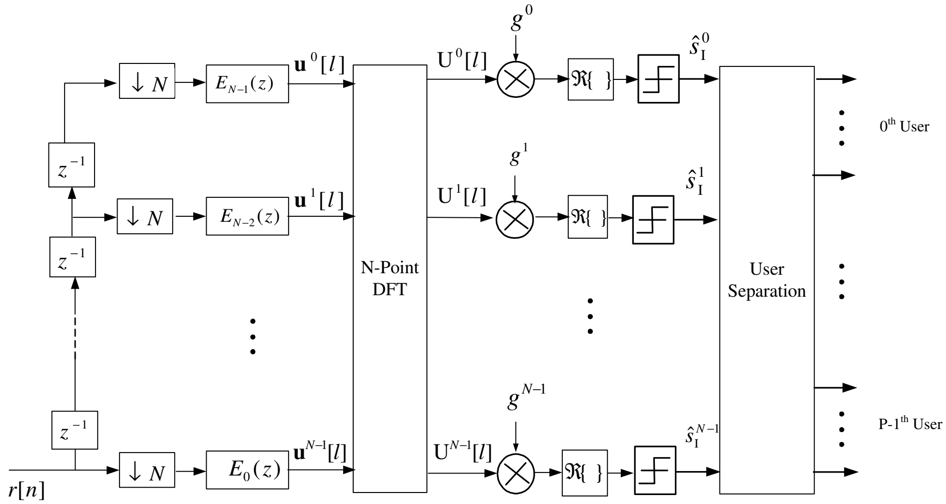

Fig. 1. The in-phase part of an SMT receiver. The quadrature part has a similar structure with the following minor differences: (i) The input is backward delayed by one half of a symbol interval ($N/2$ samples), and (ii) the $\Re\{\cdot\}$ blocks are replaced by $\Im\{\cdot\}$ blocks. $\Re\{\cdot\}$ denotes the real-part of and $\Im\{\cdot\}$ means the imaginary-part of.

where $\boldsymbol{\mathcal{F}}$ denotes the DFT operation. The output vector $\mathbf{U}(l)$ is then multiplied with a column vector $\boldsymbol{g}$ containing the one-tap equalizer coefficients $\boldsymbol{g}= \begin{bmatrix} g^0 & g^1 & \cdots & g^{N-1} \end{bmatrix}^T$. The equalizer output is then sent to a slicer and decisions on the transmitted data symbols $\hat{\mathrm{s}}_I$ are obtained as shown in Fig. 1. Data from different users are then separated through user separation.

III. CFO ESTIMATION

III-A. Fusco's multiuser CFO estimator

In [6], Fusco *et al* proposed a joint CFO and timing offset estimator in the uplink using non-periodical training sequences. The CFO estimator for the pth user may be simplified to [6][10]

$$\hat{\varepsilon}_p = \frac{1}{\pi}\angle\left\{\sum_{i=0}^{N_c-1} A_i^* B_i\right\}, \tag{6}$$

where $\angle\{\bullet\}$ denotes the angle of a complex number and the parameters A_i and B_i are equal to

$$A_i = \sum_{n=0}^{\beta N-1}\left[r[n+i]\sum_{k\in S_p} x_\mathrm{I}^{k*}[n]\right],$$
$$B_i = \sum_{n=0}^{\beta N-1}\left[r[n+i+\frac{N}{2}]\sum_{k\in S_p} x_\mathrm{Q}^{k*}[n]\right] \tag{7}$$

and $x_\mathrm{I}^k[n]$ and $x_\mathrm{Q}^k[n]$ are transmitted signals produced by the in-phase s_I^k and quadrature phase s_Q^k of the training symbol on the kth subcarrier. It should be noted that an underlying assumption in the derivation of (6) is that $\exp(-j\frac{2\pi}{N}\varepsilon_p\Delta Q) \approx 1$, where ΔQ is comparable with the length of the prototype filter. As a result, this estimator only works well for small CFO values. Also to achieve an accurate CFO estimation on the pth user, the constraint $\sum_{k=0}^{N-1} s_\mathrm{I}^k s_\mathrm{Q}^k = 0$ should be satisfied [6].

From (6) and (7), it can be seen that the CFO for each user is estimated before the DFT. Since the CFO for different users can be estimated independently, the complexity of CFO estimation grows linearly with the number of users. However, as can be seen from (7), the computational complexity for the CFO estimation for each user is dependent on the total number of subcarriers N, rather than the number of subcarriers Q allocated to each user. This is inefficient, especially when the number of users $P = N/Q$ is large.

III-B. The proposed estimator

In [5], a time-domain CFO estimation algorithm using periodical training sequences was proposed for single user SMT systems. It was shown that a least square CFO can be obtained by estimating the phase difference between the two periods of the received signal. However this may not be extended to multiuser case, where signals from P users have been received with different CFO's at BS which can not be easily separated in the time domain. To overcome this problem, in this section, we derive a frequency-domain CFO estimator for SMTMA uplink using periodical training sequences. To reduce the computational complexity of the estimator, we uses only real training sequences such that the estimator only uses the in-phase part of an SMT receiver. In most practical design of prototype filter $h[n]$ takes significantly non-zero values only when $-N \leq n \leq N$. Then it can be shown easily that if the number of identical blocks in the training sequence of the pth user are chosen as $\beta+2$, the transmitted training signal $x_p[n]$ satisfies the following condition approximately

$$x_p[n+N] \approx x_p[n], \quad n = N_1, \cdots, N_1 + \beta N - 1 \qquad (8)$$

where N_1 is the transient in the time domain. By calling (2), we have two $\beta N \times 1$ successive overlapped signal vector from the pth user called $\mathbf{y}_{p,1}$ and $\mathbf{y}_{p,2}$ such that

$$\mathbf{y}_{p,2} = e^{j2\pi\varepsilon_p}\mathbf{y}_{p,1}, \quad p = 0, 1, \cdots, P-1 \qquad (9)$$

Accordingly $N \times 1$ vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ are the received vectors at the input of DFT block due to $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively as

$$\begin{cases} \mathbf{u}_1 = \sum_{p=0}^{P-1} \sum_{i=1}^{\beta} \mathbf{y}_{p,1}[l+i] \odot \mathbf{h}[\beta - i] + \mathbf{w}_1, \\ \mathbf{u}_2 = \sum_{p=0}^{P-1} \sum_{i=1}^{\beta} e^{j2\pi\varepsilon_p}\mathbf{y}_{p,1}[l+i] \odot \mathbf{h}[\beta - i] + \mathbf{w}_2, \end{cases} \qquad (10)$$

If we put some subcarriers as a guard between each pair of adjacent users's bands for multiuser case, the overlapping between adjacent users may be neglected, thanks to the excellent frequency localized filters used in the realization of FBMC systems. The CFO estimation may be performed after DFT. Consequently the CFO for each user may be estimated separately by following the approach for OFDM in [11]

$$\hat{\varepsilon}_p = \frac{1}{2\pi}\angle\left(\sum_{k \in S_p} \mathbf{U}_1^{k*}\mathbf{U}_2^k\right) \qquad (11)$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are the DFT outputs of vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ respectively. While the estimation error depends only on total symbol energy, the algorithm works well in multipath channels. However it is required that the CFO as well as the channel impulse response remains constant during estimation. Moreover the training sequence may be chosen randomly so that to minimize its PAPR.

Using a similar approach as in [11], in the high SNR region, the mean square error (MSE) of the CFO estimation of the pth user can be approximated by

$$\mathrm{MSE}(\hat{\varepsilon}_p) = \mathrm{E}\left\{(\varepsilon_p - \hat{\varepsilon}_p)^2\right\} = \frac{1}{4\pi^2.\mathrm{SNR}.\sum_{k \in S_p}\left|H^k\right|^2} \qquad (12)$$

where H^k is the channel frequency response on the kth subcarrier and $\mathrm{SNR} \triangleq \frac{\sigma_s^2}{\sigma_n^2}$, where σ_s^2 is the power of the transmitted signal.

III-C. Computational complexity comparison

In this section, we compare the computational complexity for the proposed CFO estimator with Fusco's multiuser CFO estimator [6]. The computational complexity of the two CFO estimators in term of number of complex multiplications (CMs) required are summarized in Table I [1]. It is assumed that there are P users, N subcarriers and $\frac{N}{P}$ subcarriers are assigned to each user. The computational complexity of Fusco's multiuser CFO estimator (6) is dependent on the length of channel impulse response [6]. In this estimator, A_i and B_i need to be calculated N_c times and therefore $\beta N N_c$ CMs are performed. Also, it requires N_c CMs to estimate CFO. Accordingly, $N_c(2\beta N + 1)$ CMs are necessary for the CFO estimation for the single user case and $PN_c(2\beta N + 1)$ for the multiuser case. In the proposed estimator, the calculation of $\mathbf{u}_1$ and $\mathbf{u}_2$ requires βN CMs and $2 \times \left(\frac{N}{2}\log_2 N\right)$ are required to calculate $\mathbf{U}_1$ and $\mathbf{U}_2$. The estimation of all the CFO values for P users requires $P \times \frac{N}{P} = N$ CMs . Also, to give a sense of reduction in computational complexity in practical systems, the comparison of two estimators for two typical N values is presented in Table II. As seen from Table II, the proposed estimator has the lower complexity than that of Fusco's multiuser CFO estimator [6]. Because the period of the training sequence is N, the acquisition range of the CFO value using the proposed estimator is $\epsilon \in [-0.5, 0.5]$.

Table I. Computational complexity of different CFO estimation algorithms. N_c: Channel impulse response length, P: Number of users, βN: Prototype filter length

CFO estimator	single user	multiuser
Fusco's Est. [6]	$N_c(2\beta N+1)$	$PN_c(2\beta N+1)$
Proposed Est.	$N(\beta + \log_2 N + 1)$	$N(\beta + \log_2 N + 1)$

Table II. Computational complexity of different CFO estimation algorithms. N_c: 4, β: 3

CFO Est.	Fusco's	[6]	Proposed	
	$P=4$	$P=16$	$P=4$	$P=16$
$N=256$	24,592	98,368	3,072	3,072
$N=2048$	196,624	786,496	30,710	30,710

IV. SIMULATION RESULTS

Computer simulations were performed to compare the performance of the proposed estimator and Fusco's multiuser CFO estimator [6] for SMTMA uplink. We let $N = 256$ and assume that there are $P = 4$ users in the network, hence, $Q = \frac{N}{P} = 64$ subcarriers per user. The multipath channel with delay 0, 0.31, 0.71, 1.09, 1.73, and 2.51 microseconds and relative powers 0, -3, -9, -10, -15, and -20dB is considered [6]. The CFO values are chosen randomly and independently for each user from a uniform distribution in the interval $-0.5 < \varepsilon < 0.5$. In all cases, one guard subcarrier is inserted between each pair of adjacent users's bands. Perfect power control is assumed. The prototype filter used in the SMTMA system is an isotropic orthogonal transform algorithm (IOTA) prototype filter with a length of $3N$, designed accordingly to [14].

The MSE of the proposed estimator and Fusco's multiuser CFO estimator are shown in Fig. 2. Due to the relatively large CFO values used, we can see that the MSE of Fusco's

[1]As the computational complexity for complex multiplication is much higher than that of complex addition/subtraction, we use this as a gauge of the overall computational complexity.

multiuser CFO estimator reaches an error floor for SNR larger than 20dB. The MSE for the proposed estimator outperforms than Fusco's multiuser CFO estimator and it can be accurately approximated by the theoretical MSE derived with (12) in high SNR regions. Fig. 3 presents bit error rate (BER) for both estimators, for different E_b/N_0 values. The channel code is a rate $1/2$ convolutional code with constraint length of 5 and data symbols are from a 16-QAM constellation. We can see that the BER with the proposed estimator is much better than that with Fusco's estimator and is very close to the BER with perfect frequency synchronization.

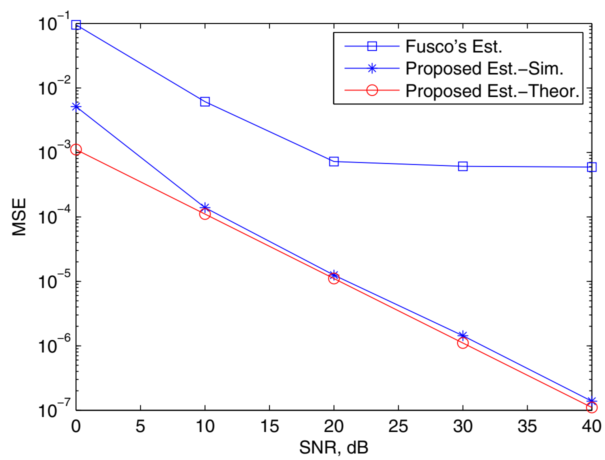

Fig. 2. MSE of the proposed estimator and Fusco's multiuser CFO estimator.

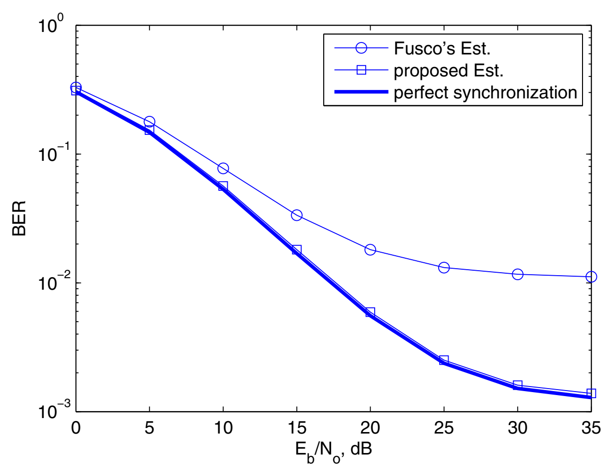

Fig. 3. BER using the proposed estimator and Fusco's multiuser CFO estimator.

V. CONCLUSION

In this paper, we proposed a low-complexity frequency-domain CFO estimator for the uplink of an SMTMA system using periodical training sequences. It was shown that the CFO estimation may be performed after DFT in the SMTMA receiver when signals from different users are separated. This results in significantly lower computational complexity compared to state-of-the-art time-domain estimators, in which signals from different users are mixed up. Simulation results showed that the performance of the proposed CFO estimator, in terms of both estimation MSE and resulted BER, is better than state-of-the-art estimators.

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