Resource Allocation for Downlink OFDMA Relay Networks with Imperfect CSI

Jaeho Lee*, Hanmok Shin†, and Jae Hong Lee†
*Smart Vehicle Tech. R&D Department, Convergence Lab.
LG Electronics, Inc.
†Department of Electrical Engineering and Computer Science, INMC
Seoul National University
Seoul 151-744, Korea
E-mail: jaeho2@snu.ac.kr

Abstract—We propose a new resource allocation scheme for a downlink orthogonal frequency division multiple access (OFDMA) relay network with imperfect channel state information (CSI). In the proposed scheme, resources are allocated to the channels between a base station, relays, and users such that the weighted sum of goodput is maximized under power constraints of the base station and relays. Simulation results show that the proposed scheme achieves much higher weighted sum of goodput than a conventional scheme.

Index terms — Relay networks, OFDMA, Imperfect CSI, Resource allocation

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) achieves high data rate in wireless networks [1]. In OFDMA, multiple users are allowed to share subcarriers in an OFDM symbol. It is known that the multiuser diversity is obtained by adaptive resource allocation which requires perfect channel state information (CSI) [2], [3].

Recently, the effect of imperfect CSI for OFDMA networks has been studied [4], [5]. In [4], while the power and subcarriers are allocated to the channels between a base station (BS) and users in order to maximize the capacity, rate allocation has not been considered. The packet outage can occurs if the transmission rate exceeds the capacity. So, for OFDMA networks with imperfect CSI, rate allocation is important because it can decrease packet outage. In [5], the power, subcarriers, and rates are allocated to the channels between the BS, relays, and users in order to maximize the goodput under the total power constraint and fixed target packet outage probability constraint.

In this paper, we propose a new resource allocation scheme for a downlink OFDMA decode-and-forward (DF) relay network with imperfect CSI. In the proposed scheme, we select the target packet outage probability and control power, subcarriers, and rates in order to maximize the weighted sum of goodput under the individual power constraints of the BS and relays.

This paper is organized as follows. In section II the system model is introduced. In section III an optimization problem is formulated. In section IV a new resource allocation scheme

is proposed. Simulation results are presented in section IV. In section V we conclude this paper.

II. SYSTEM MODEL

Consider a downlink OFDMA relay network which consists of one BS, J relays, and K mobile users with N subcarriers. Assume that each subcarrier has flat fading and an additive white Gaussian noise (AWGN). Assume that there is no direct link between the BS and the users.

Suppose that the BS has the scheduler which controls the system resources based on the CSI of all links. Assume that the BS knows perfect CSI for all BS-relay links and the BS knows outdated, not perfect, CSI for all relay-user links due to user mobility and feedback delay. For the channel of the subcarrier n between the BS and the relay R_j, assume that its fading coefficient $a_{R_j}^{BS,(n)}$ is a Rician random variable with Rician factor κ. For the channel of the subcarrier n between the relay R_j and the user U_k, assume that its fading coefficient is a Rayleigh random variable which is modeled as [6]

$$b_{U_k}^{R_j,(n)} = \rho \hat{b}_{U_k}^{R_j,(n)} + e_{U_k}^{R_j,(n)} \tag{1}$$

where $\hat{b}_{U_k}^{R_j,(n)} \sim \mathcal{CN}(0,1)$ is the outdated fading coefficient and $e_{U_k}^{R_j,(n)} \sim \mathcal{CN}(0, 1-\rho^2)$ is the coefficient error. ρ is the correlation coefficient between $b_{U_k}^{R_j,(n)}$ and $\hat{b}_{U_k}^{R_j,(n)}$ which is given by $\rho = J_0(2\pi f_d T_D)$ by Jakes' model where f_d is the Doppler frequency, T_D is the feedback delay, and J_0 is the zeroth-order Bessel function of the first kind.

Assume that the DF protocol is used for relaying, in which the transmission frame is divided into two time slots. Suppose that the subcarrier n is allocated to the channel between the relay R_j and the user U_k. In the first time slot, the BS transmits its signal $x^{(n)}$ to the relay R_j with transmit power $p_{R_j}^{BS,(n)}$ on the subcarrier n. Then, the received signal at the relay R_j on the subcarrier n is given by

$$y_{R_j}^{(n)} = \sqrt{p_{R_j}^{BS,(n)} l_{R_j}^{BS}} \, a_{R_j}^{BS,(n)} x^{(n)} + z_{R_j}^{(n)} \tag{2}$$

where $l_{R_j}^{BS}$ is the path loss between the BS and the relay R_j and $z_{R_j}^{(n)}$ is the AWGN with zero-mean and variance σ_n^2 on the subcarrier n at the relay R_j.

In the second time slot, the relay R_j decodes the received signal and forwards it to the user U_k with transmit power $p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}$ on the subcarrier n. Then, the received signal at the user U_k on the subcarrier n is given by

$$y_{\mathrm{U}_k}^{(n)} = \sqrt{p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} l_{\mathrm{U}_k}^{\mathrm{R}_j} } b_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} x^{(n)} + z_{\mathrm{U}_k}^{(n)} \tag{3}$$

where $l_{\mathrm{U}_k}^{\mathrm{R}_j}$ is the path loss between the relay R_j and the user U_k and $z_{\mathrm{U}_k}^{(n)}$ is the AWGN with zero-mean and variance σ_n^2 on the subcarrier n at the user U_k.

The capacity of the subcarrier n between the BS and the user U_k via the relay R_j is given by [7]

$$C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} = \frac{1}{2} \min \Big\{ \log_2(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}),$$
$$\log_2(1 + p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}) \Big\} \tag{4}$$

where $\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)} = l_{\mathrm{R}_j}^{\mathrm{BS}} |a_{\mathrm{R}_j}^{\mathrm{BS},(n)}|^2 / \sigma_n^2$ is the channel-to-noise ratio (CNR) of the subcarrier n from the BS to the relay R_j and $\beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}$ is the CNR of the subcarrier n from the relay R_j to the user U_k.

The probability density function of $\beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}$ conditioned on the outdated CNR, $\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} = l_{\mathrm{U}_k}^{\mathrm{R}_j} |\hat{b}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}|^2 / \sigma_n^2$, is given by [4]

$$f_{\beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} | \hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}}(\beta | \hat{\beta}) = \frac{1}{\bar{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}(1-\rho^2)} e^{-\frac{\rho^2 \hat{\beta} + \beta}{\bar{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}(1-\rho^2)}}$$
$$\times I_0 \left(\frac{2}{\bar{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}(1-\rho^2)} \sqrt{\beta \rho^2 \hat{\beta}} \right) \tag{5}$$

where $\bar{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} = l_{\mathrm{U}_k}^{\mathrm{R}_j} / \sigma_n^2$ is the long term average CNR of the subcarrier n from the relay R_j to the user U_k and I_0 is the zeroth-order modified Bessel function of the first kind.

III. PROBLEM FORMULATION

Let $R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$ denote the transmission rate of the subcarrier n from the BS to the user U_k via the relay R_j. Then, the goodput of the subcarrier n from the BS to the user U_k via the relay R_j is defined as [5]

$$G_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} = R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} 1(R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} \le C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}) \tag{6}$$

where $1(R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} \le C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)})$ is an indicator function which equals one if $R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} \le C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$, and zero otherwise.

Let $s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$ denote the subcarrier assignment indicator of the subcarrier n for the relay R_j and the user U_k. $s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$ is one if the subcarrier n is allocated to the channel between the relay R_j and the user U_k, and zero otherwise. To maximize the weighted sum of the conditional expectation of goodput given $\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}$ and $\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}$, the optimization problem is formulated as

$$G^* = \max_{\mathbf{P},\boldsymbol{S},\mathbf{R}} \sum_{k=1}^{K} w_k \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} E\Big[G_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} | \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}, \hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \Big] \tag{7}$$

$$\mathrm{s.t.} \quad \sum_{k=1}^{K} \sum_{j=1}^{J} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} = 1, \ \forall n \tag{8a}$$

$$s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} \in \{0,1\}, \ \forall j,k,n \tag{8b}$$

$$\sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \le p_{\max}^{\mathrm{BS}} \tag{8c}$$

$$\sum_{k=1}^{K} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \le p_{\max}^{\mathrm{R}_j}, \ \forall j \tag{8d}$$

where w_k is the weighting factor of the user U_k [8], $p_{\max}^{\mathrm{BS}}$ and $p_{\max}^{\mathrm{R}_j}$ are the maximum transmit power of the BS and the relay R_j, respectively, $\mathbf{P}$ is a vector with components $p_{\mathrm{R}_j}^{\mathrm{BS},(n)}$ and $p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}$, $\boldsymbol{S}$ is a vector with components $s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$, and $\mathbf{R}$ is a vector with components $R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$, $j = 1, \cdots, J$, $k = 1, \cdots, K$, $n = 1, \cdots, N$. (8a) and (8b) state that a subcarrier is allocated to only one relay and one user. (8c) and (8d) represent the transmit power constraints on the BS and the relays, respectively.

To reduce excessive complexity required to solve the problem in (7), we propose a suboptimal resource allocation scheme which consists of two steps. In the first step, power and subcarriers are allocated to channels between the BS, relays, and users. In the second step, the rates are allocated to users for a given power and subcarrier allocation.

IV. PROPOSED RESOURCE ALLOCATION SCHEME

A. Power and Subcarrier Allocation

Suppose that the rates of the users are fixed. Then, maximizing the weighted sum of goodput is equivalent to maximizing the weighted sum of capacity. The optimization problem in (7) is reformulated as

$$G_1^* = \max_{\mathbf{P},\boldsymbol{S}} \sum_{k=1}^{K} w_k \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} E\Big[C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} | \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}, \hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \Big] \tag{9}$$

s.t. (8a), (8b), (8c), and (8d).

The capacity between the BS and the user U_k via the relay R_j is maximized if the capacity between the BS and the relay R_j is equal to the capacity between the relay R_j and the user U_k [9]. The problem in (9) and its constraints become

$$G_1^* = \max_{\mathbf{P},\boldsymbol{S}} \sum_{k=1}^{K} w_k \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} \log_2(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}) \tag{10}$$

s.t. (8a), (8b), (8c), (8d), and

$$\log_2(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)})$$
$$= E\Big[\log_2(1 + p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}) | \hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \Big], \ \forall j,k,n. \tag{11}$$

As the problem in (10) involves discrete and continuous variables, it requires high computational complexity to solve. To make the problem tractable, $s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$ is relaxed to be a real

$$L(\mathbf{P}, \boldsymbol{S}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}) = \sum_{k=1}^{K} w_k \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j, \mathrm{U}_k}^{(n)} \log_2\left(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}\right) + \lambda\left(p_{\max}^{\mathrm{BS}} - \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j, \mathrm{U}_k}^{(n)} p_{\mathrm{R}_j}^{\mathrm{BS},(n)}\right)$$
$$+ \sum_{n=1}^{N} \mu_n\left(1 - \sum_{k=1}^{K} \sum_{j=1}^{J} s_{\mathrm{R}_j, \mathrm{U}_k}^{(n)}\right) + \sum_{j=1}^{J} \nu_j\left(p_{\max}^{\mathrm{R}_j} - \sum_{k=1}^{K} \sum_{n=1}^{N} s_{\mathrm{R}_j, \mathrm{U}_k}^{(n)} p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right) \tag{12}$$
$$+ \sum_{n=1}^{N} \eta_n\left[\sum_{k=1}^{K} \sum_{j=1}^{J} s_{\mathrm{R}_j, \mathrm{U}_k}^{(n)} E\left[\log_2\left(1 + p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right)|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right] - \sum_{k=1}^{K} \sum_{j=1}^{J} s_{\mathrm{R}_j, \mathrm{U}_k}^{(n)} \log_2\left(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}\right)\right]$$

$$\eta_n^l = \left[\eta_n^{l-1} - \tau_3^{l-1}\left(\sum_{k=1}^{K} \sum_{j=1}^{J} s_{\mathrm{R}_{j^*}, \mathrm{U}_{k^*}}^{(n)} E\left[\log_2(1 + p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)*} \beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)})|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right] - \sum_{k=1}^{K} \sum_{j=1}^{J} s_{\mathrm{R}_{j^*}, \mathrm{U}_{k^*}}^{(n)} \log_2(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)*} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)})\right)\right]^+, \forall n \tag{22}$$

---

value within a interval $[0, 1]$ [10]. Then, the Lagrangian of the optimization problem is given by (12) at the top of this page where λ, μ_n, ν_j, and η_n are nonnegative Lagrange multipliers.

By the *Karush-Kuhn-Tucker* (KKT) conditions [11], the optimal power transmitted from the BS to the relay R_j on the subcarrier n is given by

$$p_{\mathrm{R}_j}^{\mathrm{BS},(n)*} = \left(\frac{w_k - \eta_n}{\lambda \ln 2} - \frac{1}{\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}}\right)^+. \tag{13}$$

The optimal power transmitted from the relay R_j to the user U_k on the subcarrier n, $p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)*}$, is such that

$$E\left[\frac{\beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}}{1 + p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)*} \beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}}|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right] = \frac{\nu_j \ln 2}{\eta_n} \tag{14}$$

if $E\left[\beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right] \geq \frac{\nu_j \ln 2}{\eta_n}$. And $p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)*}$ is equal to zero otherwise.

The Lagrange dual problem is formulated as

$$d^* = \min_{\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}} g(\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}) \tag{15}$$

where

$$g(\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}) = \max_{\mathbf{P}, \mathbf{S}} L(\mathbf{P}, \mathbf{S}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}). \tag{16}$$

To solve the Lagrange dual problem, maximize $L(\mathbf{P}, \mathbf{S}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta})$ first. Suppose that the subcarrier n is allocated to the channel between the relay R_j and the user U_k. From (12) and (16), the Lagrangian dual function is given by

$$g(\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}) = \lambda p_{\max}^{\mathrm{BS}} + \sum_{j=1}^{J} \nu_j p_{\max}^{\mathrm{R}_j}$$
$$+ \max_{\mathbf{P}} \sum_{n=1}^{N} g_n(p_{\mathrm{R}_j}^{\mathrm{BS},(n)}, p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}, \lambda, \nu_j, \eta_n) \tag{17}$$

where

$$g_n(p_{\mathrm{R}_j}^{\mathrm{BS},(n)}, p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}, \lambda, \nu_j, \eta_n) = w_k \log_2(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)})$$
$$+ \eta_n\left[E\left[\log_2(1 + p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)} \beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)})|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right]\right.$$
$$\left. - \log_2(1 + p_{\mathrm{R}_j}^{\mathrm{BS},(n)} \alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)})\right] - \lambda p_{\mathrm{R}_j}^{\mathrm{BS},(n)} - \nu_j p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}. \tag{18}$$

Then, the optimal subcarrier assignment indicator is obtained as

$$s_{\mathrm{R}_{j^*}, \mathrm{U}_{k^*}}^{(n)} = \begin{cases} 1, & (j^*, k^*) = \\ & \arg\max_{j,k} g_n(p_{\mathrm{R}_j}^{\mathrm{BS},(n)}, p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}, \lambda, \nu_j, \eta_n), \\ 0, & \text{otherwise,} \end{cases} \tag{19}$$

for all n. The optimal solution of the Lagrange dual problem is obtained using the subgradient method by iterative search [12]. The Lagrange multipliers λ^l and ν_j^l are updated as

$$\lambda^l = \left[\lambda^{l-1} - \tau_1^{l-1}(p_{\max}^{\mathrm{BS}} - \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_{j^*}, \mathrm{U}_{k^*}}^{(n)} p_{\mathrm{R}_j}^{\mathrm{BS},(n)*})\right]^+, \tag{20}$$

$$\nu_j^l = \left[\nu_j^{l-1} - \tau_2^{l-1}(p_{\max}^{\mathrm{R}_j} - \sum_{k=1}^{K} \sum_{n=1}^{N} s_{\mathrm{R}_{j^*}, \mathrm{U}_{k^*}}^{(n)} p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)*})\right]^+, \forall j. \tag{21}$$

The Lagrange multiplier η_n^l is updated as (22) at the second top of the page where l is the number of iterations, τ_1^l, τ_2^l, and τ_3^l are the step sizes.

B. Rate Allocation

After power and subcarriers are allocated as in the previous section, the conditional expectation of goodput in (7) is given

by

$$E\left[G_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}|\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)},\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right]$$
$$= E\left[R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}1(R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}\le C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)})|\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)},\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right]$$
$$= R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}\left(1-\Pr\left[R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}>C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}|\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)},\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right]\right) \tag{23}$$

where $\Pr[R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} > C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}|\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)},\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}]$ is the conditional packet outage probability. Then, the optimization problem in (7) is reformulated as

$$G_2^* = \max_{\mathbf{R},\boldsymbol{\varepsilon}} \sum_{k=1}^{K} w_k \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$$
$$\times \left(1-\Pr\left[R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}>C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}|\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)},\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right]\right) \tag{24}$$

s.t. $\Pr\left[R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}>C_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}|\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)},\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right]=\varepsilon^{(n)},\forall j,k,n,$ $\tag{25}$

where $\varepsilon^{(n)}$ is the target packet outage probability on the subcarrier n and $\boldsymbol{\varepsilon}$ is a vector with components $\varepsilon^{(n)}$, $n = 1,\cdots,N$. From (4) and (25), the transmission rate of the subcarrier n from the BS to the user U_k via the relay R_j is given by

$$R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} = \frac{1}{2}\min\left\{\log_2\left(1+p_{\mathrm{R}_j}^{\mathrm{BS},(n)}\alpha_{\mathrm{R}_j}^{\mathrm{BS},(n)}\right),\right.$$
$$\left.\log_2\left(1+p_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}F_{\beta_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}}^{-1}\left(\varepsilon^{(n)}|\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}\right)\right)\right\}. \tag{26}$$

Substituting (26) into (24), the problem in (24) is solved with respect to the single variable $\varepsilon^{(n)}$. If the subcarrier n is allocated to the channel between the relay R_j and the user U_k, the target packet outage probability $\varepsilon^{(n)}$ is selected within the interval $[0, 1]$ such that $w_k R_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}(1-\varepsilon^{(n)})$ is maximized.

The performance of the proposed algorithm is compared with a conventional scheme in [13] in terms of the weighted sum of goodput, which is given by $\sum_{k=1}^{K} w_k \sum_{j=1}^{J} \sum_{n=1}^{N} s_{\mathrm{R}_j,\mathrm{U}_k}^{(n)} G_{\mathrm{R}_j,\mathrm{U}_k}^{(n)}$, versus WSNR and the number of users. WSNR is defined as the worst possible average SNR of a user on the cell edge [14]. In the conventional scheme, resources are allocated to the channels between the BS, relay, and users based only on the outdated CNR; $\hat{\beta}_{\mathrm{U}_k}^{\mathrm{R}_j,(n)}$.

V. SIMULATION RESULTS

Consider a single cell with a 500 meter radius and the BS at its center. Suppose that J relays are symmetrically located on a 250 meter radius circle centered at the BS and K users are uniformly distributed between 250 and 500 meter radii in the cell. The WINNER II path loss model is used [15]. Suppose that $p_{\max}^{\mathrm{BS}} = 10 p_{\max}^{\mathrm{R}_j}$ for all j, $w_k = 1$ for all k, and κ is 10 dB.

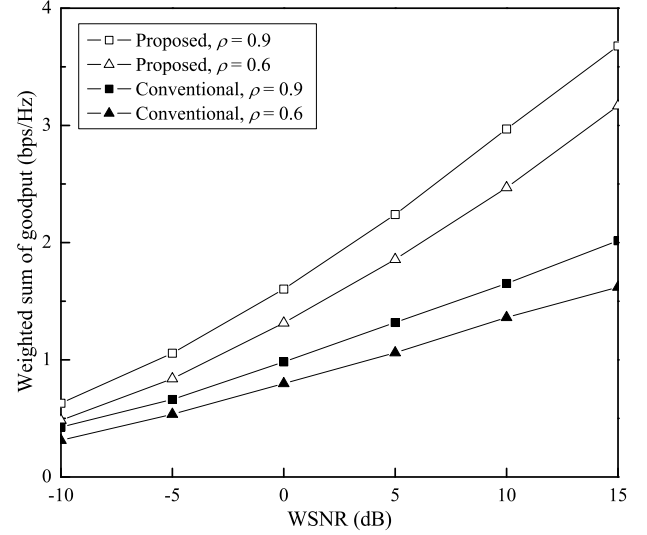Fig. 1 shows the weighted sum of goodput versus WSNR for $J = 3$, $K = 15$, and $N = 64$ with different values of

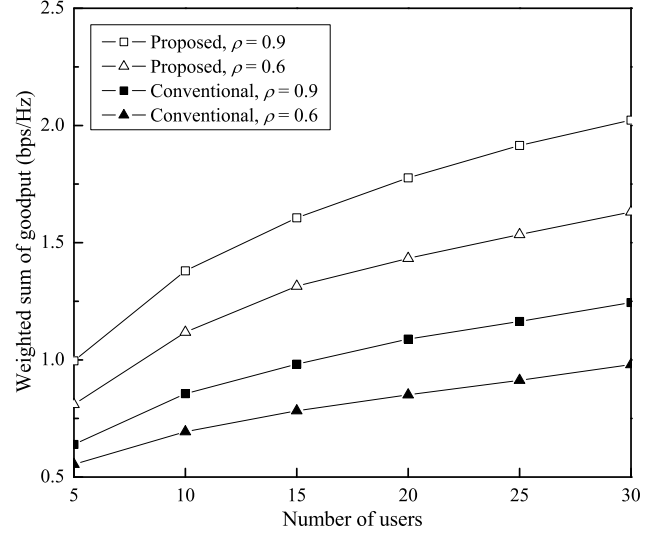Fig. 1. Weighted sum of goodput versus WSNR. $J = 3$, $K = 15$, and $N = 64$.

Fig. 2. Weighted sum of goodput versus the number of users. $J = 3$, $N = 64$, and WSNR $= 0$ dB.

correlation coefficient. It is shown that the proposed scheme achieves about 0.5 bps/Hz higher weighted sum of goodput than the conventional scheme at WSNR $= 0$ dB. It is also shown that improvement on the weighted sum of goodput increases as the correlation coefficient, ρ, increases.

Fig. 2 shows the weighted sum of goodput versus the number of users for $J = 3$, $N = 64$, and WSNR $= 0$ dB with different values of correlation coefficient. It is shown that the weighted sum of goodput increases as the number of users increases.

VI. CONCLUSION

In this paper, a new resource allocation scheme is proposed for a downlink OFDMA relay network with imperfect CSI.

we propose a suboptimal resource allocation scheme which consists of two steps. In the first step, power and subcarriers are allocated to channels between the BS, relays, and users. In the second step, the rates are allocated to users for a given power and subcarrier allocation. Simulation results show that proposed scheme achieves much higher weighted sum of goodput than the conventional scheme. It is also shown that the weighted sum of goodput increases as the correlation coefficient increases.

REFERENCES

[1] Y. Li and G. L. Stüber, *Orthogonal Frequency Division Multiplexing for Wireless Communications*. Springer, 2006.

[2] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747-1758, Oct. 1999.

[3] H. Jeong, J. H. Lee, and H. Seo, "Resource allocation for uplink multiuser OFDM relay networks with fairness constraints," in *Proc. IEEE VTC 2009-Spring*, Barcelona, Spain, Apr. 2009.

[4] I. C. Wong and L. Evans, "Optimal resource allocation in the OFDMA downlink with imperfect channel knowledge," *IEEE Trans. Commun.*, vol. 57, no. 1, pp. 232-241, Jan. 2009.

[5] W. K. Ng and R. Schober, "Cross-layer scheduling for OFDMA amplify-and-forward relay networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 3, pp. 1443-1458, Mar. 2010.

[6] J. Vicario, A. Bel, J. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2872-2876, June 2009.

[7] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[8] I. C. Wong and L. Evans, "Optimal downlink OFDMA resource allocation with linear complexity to maximized ergodic rates," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 962-971, Mar. 2008.

[9] W. Ying, Q. Xin-chun, W. Tong, and L. Bao-ling, "Power allocation and subcarrier pairing algorithm for regenerative OFDM relay system," in *Proc. IEEE ICC 2006*, Istanbul, Turkey, June 2006.

[10] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310-1322, July 2006.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[12] S. Boyd, L. Xiao, and A. Mutapcic, *Subgradient Methods*. Stanford University Press, 2003.

[13] F. Brah, L. Vandendorpe, and J. Louveaux, "Constrained resource allocation in OFDMA downlink systems with partial CSIT," in *Proc. IEEE ICC 2008*, Beijing, China, May 2008.

[14] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE VTC 2000-Spring*, Tokyo, Japan, May 2000.

[15] P. Kyosti *et al.*, "WINNER II Channel Models," IST-WINNER II D1.1.2, Nov. 2007.