Effect of Outdated CSI on the Performance of Opportunistic Relaying with ARQ

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***Abstract*—In this paper, we analyze the outage probability and average throughput of opportunistic relaying with ARQ when the outdated channel state information (CSI) is available for relay selection. Its diversity order is analyzed to show that it is determined by the number of relays and the maximum number of ARQ rounds. Numerical results verify our analysis by comparison with computer simulation and show the effect of outdated CSI on its outage probability, diversity order, and average throughput.**

***Index terms* — ARQ, opportunistic relaying, outdated CSI.**

I. Introduction

Cooperative diversity is one of the effective techniques to improve the reliability of communications over wireless channels by using one or multiple relays to aid the communication from the source to the destination [1]. For multi-relay systems, opportunistic relaying is shown to be an efficient cooperative diversity scheme as it achieves full diversity with low system complexity [2]. In opportunistic relaying, the relay with the best channel to destination is selected among available relay candidates [3].

Automatic repeat request (ARQ) can be adopted in opportunistic relaying when a delay constraint is not strict. In [4], it is shown that allowing retransmissions in opportunistic relaying improves the diversity-multiplexing tradeoff (DMT) performance. In [5], several hybrid-ARQ protocols with opportunistic relaying are studied and their delay performance are analyzed.

In opportunistic relaying, perfect channel state information (CSI) is required to select the best relay. In practice the CSI is outdated due to channel variations caused by mobility and the delay between the moments of relay selection and the transmission by the selected relay. With outdated CSI, the outage performance of opportunistic relaying is severely degraded as the diversity order reduces to one [6], and its channel capacity is affected [7]. As ARQ is utilized for opportunistic relaying in high mobility environment to improve reliability [8], effect of outdated CSI on opportunistic relaying with ARQ needs to be considered in practice, which is not investigated yet.

In this paper, we analyze the outage probability, diversity order, and average throughput of opportunistic relaying with ARQ when outdated CSI is available. Also, the outage probability is approximated at high SNR. By the analysis and approximation, we can quantitatively evaluate the effect of outdated CSI on its outage probability and average throughput.

This paper is organized as follows. In section II, the system model is described. In section III, the outage probability of opportunistic relaying with ARQ is derived and approximated when outdated CSI is available. In section IV, diversity order and average throughput are obtained. In section V, the numerical results are provided. Conclusions are drawn in section VI.

II. System Model

Consider a half-duplex DF dual-hop relay network which consists of a source, s, a destination, d, and K relays, r_k, $k = 1, 2, \cdots, K$, each with a single antenna. Assume that there is no direct path between the source and the destination.

The source transmits information to the destination through two phases. In the first phase, the source broadcasts an encoded data x to K relays with transmit power P and rate R. The received signal at the relay r_k, $k = 1, 2, \cdots, K$, is given by

$$y_{r_k} = \sqrt{P} h_{s,r_k} x + n_{r_k} \tag{1}$$

where $h_{s,r_k} \sim \mathcal{CN}(0,1)$ is the channel coefficient between the source and the relay r_k, and $n_{r_k} \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at the relay r_k. The set of relays which successfully decode the received signals is called the decoding set $\mathcal{D}$.

In the second phase, a relay $r^* \in \mathcal{D}$ having the best channel to the destination is selected to re-encode and transmit the data to the destination with transmit power P and rate R. If the destination successfully decodes the received signal from the selected relay, it broadcasts an ACK to all relays. Otherwise, the destination broadcasts a NACK and the process of the second phase repeats until the destination successfully decodes the received signal or the number of ARQ rounds reaches N. All received signals at the destination are combined by MRC at each ARQ round to reduce the number of retransmissions. Assume that each ACK/NACK is error-free.

Assume that each of the channel from each relay to the destination has independent fading at each ARQ round. The received signal at the destination at the nth ARQ round by the transmission of the relay r_k, $k = 1, 2, \cdots, K$, is given by

$$y_d^{(n)} = \sqrt{P} h_{r_k,d}^{(n)} \hat{x} + n_d^{(n)} \tag{2}$$

where $\hat{x}$ is the transmit signal from the relay r_k, $h_{r_k,d}^{(n)} \sim$

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$\mathcal{CN}(0,1)$ is the channel coefficient between the relay r_k and the destination at the nth ARQ round, and $n_d^{(n)} \sim \mathcal{CN}(0, N_0)$ is the AWGN at the destination at the nth ARQ round. The received SNR at the destination at the nth ARQ round is given by $\Gamma_{r_k,d}^{(n)} = |h_{r_k,d}^{(n)}|^2 P/N_0$ and its average is given by $\bar{\Gamma} = P/N_0$.

Assume that the CSI for relay selection is outdated due to the delay between the moments of relay selection and transmission by the selected relay [6]. Assume that the outdated channel coefficient $\hat{h}_{r_k,d}^{(n)}$, $k = 1, 2, \cdots, K$, $n = 1, 2, \cdots, N$, has the same distribution as $h_{r_k,d}^{(n)}$. The outdated SNR for relay selection at the nth ARQ round is given by $\hat{\Gamma}_{r_k,d}^{(n)} = |\hat{h}_{r_k,d}^{(n)}|^2 P/N_0$ and its average is denoted by $\bar{\Gamma}$. Then, the selected relay at the nth ARQ round, r^*, is given by

$$r^* = \arg\max_{r_k \in \mathcal{D}} \hat{\Gamma}_{r_k,d}^{(n)}. \tag{3}$$

The correlation between the SNR $\Gamma_{r^*,d}^{(n)}$ and its outdated SNR $\hat{\Gamma}_{r^*,d}^{(n)}$ is obtained by the Jakes' model as $\rho = J_0^2(2\pi f_D \tau)$ where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, f_D is the maximum Doppler frequency, and τ is the delay between the moments of relay selection by the destination and the transmission by the selected relay [9].

III. Outage Probability

We assume that if the mutual information at each relay or the destination is larger than the rate R, the relay or the destination successfully decodes its received signal. The probability that l out of K relays successfully decode the received signal from the source is given by [6]

$$\Pr[|\mathcal{D}| = l] = \binom{K}{l} e^{-\frac{l\gamma_0}{\bar{\Gamma}}} \left(1 - e^{-\frac{\gamma_0}{\bar{\Gamma}}}\right)^{K-l} \tag{4}$$

where $|\mathcal{D}|$ is the cardinality of $\mathcal{D}$, $\gamma_0 = 2^R - 1$, and $\bar{\Gamma} = P/N_0$. Then, the outage probability is defined as

$$P_{out} = \Pr[|\mathcal{D}| = 0] + P_{fail}^{(N)} \tag{5}$$

where $P_{fail}^{(N)}$ is the probability of decoding failure at the destination at the Nth ARQ round. To obtain $P_{fail}^{(N)}$, we derive the cdf of the SNR of the combined signal at the destination at the Nth ARQ round.

The relay r^* selected by the destination has the largest outdated SNR among $r_k \in \mathcal{D}$. Its cdf conditioned on $|\mathcal{D}|$ is given by [10]

$$F_{\hat{\Gamma}_{r^*,d}^{(n)} \big| |\mathcal{D}|}(\hat{\gamma}|l) = \left(1 - e^{-\frac{\hat{\gamma}}{\bar{\Gamma}}}\right)^l. \tag{6}$$

By differentiating (6) with respect to $\hat{\gamma}$ and applying binomial expansion, the pdf of $\hat{\Gamma}_{r^*,d}^{(n)}$ conditioned on $|\mathcal{D}|$ is given by

$$f_{\hat{\Gamma}_{r^*,d}^{(n)} \big| |\mathcal{D}|}(\hat{\gamma}|l) = \sum_{k=1}^{l} \binom{l}{k} \frac{k(-1)^{k-1}}{\bar{\Gamma}} e^{-\frac{k\hat{\gamma}}{\bar{\Gamma}}}. \tag{7}$$

The received SNR at the destination at nth ARQ round $\Gamma_{r^*,d}^{(n)}$ conditioned on its outdated SNR $\hat{\Gamma}_{r^*,d}^{(n)}$ follows a noncentral chi-square random variable with 2 degrees of freedom, whose pdf is given by [7]

$$f_{\Gamma_{r^*,d}^{(n)} | \hat{\Gamma}_{r^*,d}^{(n)}}(\gamma|\hat{\gamma}) = \frac{e^{-\frac{\rho\hat{\gamma}+\gamma}{(1-\rho)\bar{\Gamma}}}}{(1-\rho)\bar{\Gamma}} I_0\left(\frac{2\sqrt{\rho\gamma\hat{\gamma}}}{(1-\rho)\bar{\Gamma}}\right) \tag{8}$$

where $I_0(\cdot)$ is the modified zeroth-order Bessel function of the first kind. From (7) and (8), the pdf of $\Gamma_{r^*,d}^{(n)}$ conditioned on $|\mathcal{D}|$ is given by [7]

$$\begin{aligned} f_{\Gamma_{r^*,d}^{(n)} \big| |\mathcal{D}|}(\gamma|l) &= \int_0^\infty f_{\Gamma_{r^*,d}^{(n)} | \hat{\Gamma}_{r^*,d}^{(n)}}(\gamma|\hat{\gamma}) \cdot f_{\hat{\Gamma}_{r^*,d}^{(n)} \big| |\mathcal{D}|}(\hat{\gamma}|l) d\hat{\gamma} \\ &= \sum_{k=1}^{l} \binom{l}{k} \frac{k(-1)^{k-1} e^{-\frac{k\gamma}{\bar{\Gamma}\{k(1-\rho)+\rho\}}}}{\{k(1-\rho)+\rho\}\bar{\Gamma}}. \end{aligned} \tag{9}$$

The SNR of the combined signal at the destination at the nth ARQ round, $n = 1, 2, \cdots, N$, is given by

$$\Gamma_d^{(n)} = \sum_{i=1}^{n} \Gamma_{r^*,d}^{(i)}. \tag{10}$$

Since $\Gamma_d^{(n)}$ is the sum of independent random variables, its MGF conditioned on $|\mathcal{D}|$ is given by

$$\begin{aligned} \mathcal{M}_{\Gamma_d^{(n)} \big| |\mathcal{D}|}(s|l) &= \prod_{i=1}^{n} \mathcal{M}_{\Gamma_{r^*,d}^{(i)} \big| |\mathcal{D}|}(s|l) \\ &= \left[\sum_{k=1}^{l} \binom{l}{k} \frac{k(-1)^k / [\{k(1-\rho)+\rho\}\bar{\Gamma}]}{s - k/[\{k(1-\rho)+\rho\}\bar{\Gamma}]}\right]^n \\ &= \left[\sum_{k=1}^{l} \binom{l}{k} \frac{(-1)^k g(k)}{s - g(k)}\right]^n \end{aligned} \tag{11}$$

where $\mathcal{M}_{\Gamma_{r^*,d}^{(i)} \big| |\mathcal{D}|}(s|l)$ is the MGF of $\Gamma_{r^*,d}^{(i)}$ conditioned on $|\mathcal{D}|$ and $g(k) = k/[\{k(1-\rho)+\rho\}\bar{\Gamma}]$. By the integration property of Laplace transform [11], the cdf of $\Gamma_d^{(n)}$ conditioned on $|\mathcal{D}|$ is given by

$$F_{\Gamma_d^{(n)} \big| |\mathcal{D}|}(\gamma|l) = \mathcal{L}^{-1}\left[\frac{\mathcal{M}_{\Gamma_d^{(n)} \big| |\mathcal{D}|}(-s|l)}{s}\right]. \tag{12}$$

By multinomial expansion and partial fraction expansion, (12) becomes

$$\begin{aligned} &F_{\Gamma_d^{(n)} \big| |\mathcal{D}|}(\gamma|l) = \\ &\sum_{j=1}^{\binom{l+n-1}{n}} \frac{n!}{u_1^{(j)}!\, u_2^{(j)}! \cdots u_l^{(j)}!} \left[A_0^{(j)} + \sum_{k=1}^{l} \sum_{i=1}^{u_k^{(j)}} \frac{A_{k,i}^{(j)} \gamma^{i-1} e^{-g(k)\gamma}}{(i-1)!}\right] \end{aligned} \tag{13}$$

of which details are provided in Appendix.

From (4) and (13), the probability of decoding failure at the destination at the nth ARQ round, $n = 1, 2, \cdots, N$, is given by

$$\begin{aligned} P_{fail}^{(n)} &= \sum_{l=1}^{K} \Pr[|\mathcal{D}| = l] \Pr\left[\log_2\left(1 + \sum_{i=1}^{n} \Gamma_{r^*,d}^{(i)}\right) < R \,\middle|\, |\mathcal{D}| = l\right] \\ &= \sum_{l=1}^{K} \binom{K}{l} e^{-\frac{l\gamma_0}{\bar{\Gamma}}} \left(1 - e^{-\frac{\gamma_0}{\bar{\Gamma}}}\right)^{K-l} F_{\Gamma_d^{(n)} \big| |\mathcal{D}|}(\gamma_0|l) \end{aligned} \tag{14}$$

where $\gamma_0 = 2^R - 1$. From (4), (5), and (14), the outage probability is given by

$$P_{out} = \left(1 - e^{-\frac{\gamma_0}{\bar{\Gamma}}}\right)^K + \sum_{l=1}^{K} \binom{K}{l} e^{-\frac{l\gamma_0}{\bar{\Gamma}}} \left(1 - e^{-\frac{\gamma_0}{\bar{\Gamma}}}\right)^{K-l} F_{\Gamma_d^{(N)}||\mathcal{D}|}(\gamma_0|l). \tag{15}$$

We approximate the outage probability in (15) to a simple form which is valid at high SNR. At high SNR, the conditional MGF of $\Gamma_d^{(n)}$ in (11) is approximated as

$$\begin{aligned}\mathcal{M}_{\Gamma_d^{(n)}||\mathcal{D}|}(s|l) &= \left[\sum_{k=1}^{l} \binom{l}{k} \frac{k(-1)^k}{\{k(1-\rho)+\rho\}\bar{\Gamma}s - k}\right]^n \\ &\approx \left[\sum_{k=1}^{l} \binom{l}{k} \frac{1}{s} \frac{k(-1)^k}{\{(k(1-\rho)+\rho)\}\bar{\Gamma}}\right]^n \\ &= \frac{C(l,n)}{(\bar{\Gamma}s)^n}\end{aligned} \tag{16}$$

where

$$C(l,n) = \left[\sum_{k=1}^{l} \binom{l}{k} \frac{k(-1)^k}{k(1-\rho)+\rho}\right]^n. \tag{17}$$

By substituting the conditional MGF of $\Gamma_d^{(n)}$ in (16) into (12) and applying [11, eq. (17.13.2)], the conditional cdf of $\Gamma_d^{(n)}$ in (13) is approximated as

$$F_{\Gamma_d^{(n)}||\mathcal{D}|}(\gamma|l) \approx \frac{(-1)^n C(l,n)\gamma^n}{n!\bar{\Gamma}^n}. \tag{18}$$

By the first-order Taylor series of the exponential function $e^{-x} \approx 1 - x$ and taking only the highest order term of $\bar{\Gamma}$, (4) is approximated as

$$\begin{aligned}\Pr[|\mathcal{D}| = l] &\approx \binom{K}{l} \left\{\left(\frac{\gamma_0}{\bar{\Gamma}}\right)^{K-l} - l\left(\frac{\gamma_0}{\bar{\Gamma}}\right)^{K+1-l}\right\} \\ &\approx \binom{K}{l} \left(\frac{\gamma_0}{\bar{\Gamma}}\right)^{K-l}.\end{aligned} \tag{19}$$

From (5), (14), (18), and (19), the outage probability is approximated as

$$P_{out} \approx \gamma_0^K \bar{\Gamma}^{-K} + \sum_{l=1}^{K} \binom{K}{l} \frac{(-1)^N C(l,N)\gamma_0^{K-l+N}\bar{\Gamma}^{l-K-N}}{N!}. \tag{20}$$

IV. Diversity Order and Average Throughput

A. Diversity order

Define the diversity order of opportunistic relaying with ARQ as [6]

$$d = \lim_{\bar{\Gamma}\to\infty} -\frac{\log(P_{out})}{\log(\bar{\Gamma})}. \tag{21}$$

As the outage probability in (20) is dominated by the highest order term of $\bar{\Gamma}$ at high SNR, it is further approximated as

$$P_{out} \approx \begin{cases} \bar{\Gamma}^{-N} \frac{(-1)^K C(K,N)\gamma_0^N}{N!}, & N < K, \\ \bar{\Gamma}^{-K}\gamma_0^K \left(1 + \frac{(-1)^K C(K,N)}{K!}\right), & N = K, \\ \bar{\Gamma}^{-K}\gamma_0^K, & N > K. \end{cases} \tag{22}$$

From (21) and (22), the diversity order of opportunistic relaying with ARQ for outdated CSI is given by a function of N as

$$d(N) = \begin{cases} N, & N \le K, \\ K, & N > K, \end{cases} \tag{23}$$

which implies that additional diversity gain is obtained by increasing N when $N \le K$.

When perfect CSI is available, which is $\rho = 1$, $C(N)$ in (20) becomes 0 by [11, eq. (0.153.2)] and the outage probability is approximated as

$$P_{out} \approx \gamma_0^K \bar{\Gamma}^{-K}. \tag{24}$$

From (21) and (24), the diversity order of opportunistic relaying with ARQ for perfect CSI is given by

$$d(N) = K \tag{25}$$

which implies that there is no additional diversity gain by increasing N.

B. Average Throughput

We define the average number of ARQ rounds for relay-to-destination transmissions as

$$\bar{\mathcal{T}} = \sum_{n=1}^{N} n\left(P_{fail}^{(n-1)} - P_{fail}^{(n)}\right) + NP_{fail}^{(N)} \tag{26}$$

where $P_{fail}^{(0)} = 1$. Then, the average throughput is given by [12]

$$\eta = \frac{(1 - P_{out})R}{1 + \bar{\mathcal{T}}}. \tag{27}$$

V. Numerical Results

Suppose that $R = 1$ bps/Hz. For a given correlation ρ, the channel is simulated according to the algorithm in [13]. Perfect CSI is available for relay selection if $\rho = 1$. Otherwise, outdated CSI is available.

Fig. 1 shows the outage probability of opportunistic relaying with ARQ versus ρ for the average received SNR $\bar{\Gamma} = 16$ dB and $K = 3$. It is shown that the outage probability by computer simulation and its analysis coincide, and its approximation is close to its analysis. It is also shown that the outage probability becomes less sensitive to ρ as N increases, which implies that the outage probability becomes less affected by the level of the CSI accuracy as N increases.

Fig. 2 shows the outage probability of opportunistic relaying with ARQ versus $\bar{\Gamma}$, for $\rho = 0.5$ and $K = 3$. It is shown that the outage probability by computer simulation matches with its analysis, and its approximation is close to its analysis at high SNR. It is shown that for $\bar{\Gamma} \ge 10$ dB the slope of outage probability decreases as N increases. It implies that additional diversity gain is obtained by increasing N, which agrees with the analysis of diversity order.

Fig. 3 shows the outage probability of opportunistic relaying with ARQ for $N = 2$, $K = 3$, and various ρ. It is shown that the slope of outage probability with $\rho = 1$ is steeper than those with $\rho = 0.1$, 0.4, and 0.7. It is also shown that the slope of

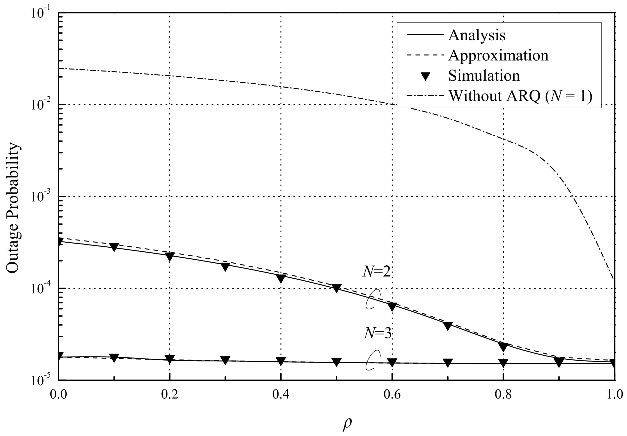

Fig. 1. Outage probability versus ρ. $\bar{\Gamma} = 16$ dB and $K = 3$.

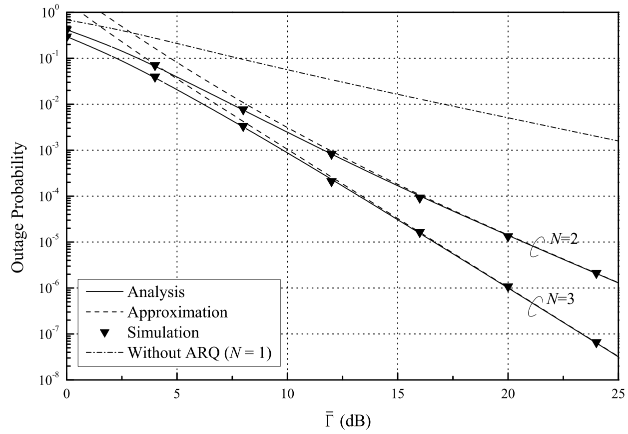

Fig. 2. Outage probability versus $\bar{\Gamma}$. $\rho = 0.5$ and $K = 3$.

outage probability is almost same for $\rho = 0.1$, 0.4, and 0.7. It implies that the diversity order for $\rho = 1$ is different from those for $\rho = 0.1$, 0.4, and 0.7, which agrees with the analysis of diversity order.

Fig. 4 shows the average throughput of opportunistic relaying with ARQ for $K = 3$. It is shown that the average throughput is decreased by outdated CSI both for $N = 1$ and $N = 2$. It is also shown that the average throughput with $N = 2$ and $\rho = 0.5$ is higher than that with $N = 1$ and $\rho = 1$ for $\bar{\Gamma} \leq 3$ dB, which implies that the average throughput is more dependent on N than the level of the CSI accuracy for low SNR.

VI. Conclusion

In this paper, we analyze the effect of outdated CSI on outage probability and average throughput of opportunistic relaying with ARQ. The outage probability is derived by the cdf of SNR of combined signal at the destination. The diversity order is derived as a function of the maximum number of ARQ rounds. Also, the average throughput is obtained. Numerical results verify the analysis and show the effect of outdated CSI on the outage probability and the average throughput over different maximum number of ARQ rounds.

Appendix

In this section we derive the cdf of $\Gamma_d^{(n)}$ conditioned on $|\mathcal{D}|$. By multinomial expansion, (11) becomes

$$\mathcal{M}_{\Gamma_d^{(n)}\big||\mathcal{D}|}(s|l) = \sum_{\substack{u_1+u_2+\cdots+u_l=n\\ u_1,\,u_2,\,\cdots,\,u_l\geq 0}} \frac{n!}{u_1!\,u_2!\,\cdots\,u_l!} \times \prod_{k=1}^{l}\left\{\binom{l}{k}\frac{(-1)^k g(k)}{s-g(k)}\right\}^{u_k} \tag{28}$$

where u_1, u_2, $\ldots$, u_l are integers.

There are $\binom{l+n-1}{n}$ possible l-tuples $(u_1, u_2, \cdots, u_l)$ which satisfy $u_1 + u_2 + \cdots + u_l = n$ and u_1, u_2, $\cdots$, $u_l \geq 0$ [14]. Let $(u_1^{(j)}, u_2^{(j)}, \cdots, u_l^{(j)})$ denote the jth l-tuple of the summation indices, $j = 1$, 2, $\cdots$, $\binom{l+n-1}{n}$. Then (28) becomes

$$\mathcal{M}_{\Gamma_d^{(n)}\big||\mathcal{D}|}(s|l) = \sum_{j=1}^{J}\frac{n!}{u_1^{(j)}!\,u_2^{(j)}!\,\cdots\,u_l^{(j)}!}\prod_{k=1}^{l}\left[\binom{l}{k}\frac{(-1)^k g(k)}{s-g(k)}\right]^{u_k^{(j)}} \tag{29}$$

where $J = \binom{l+n-1}{n}$. By the integration property of Laplace transform [11], the cdf of $\Gamma_d^{(n)}$ conditioned on $|\mathcal{D}|$ is given by

$$\begin{aligned} F_{\Gamma_d^{(n)}\big||\mathcal{D}|}(\gamma|l) &= \mathcal{L}^{-1}\left[\frac{\mathcal{M}_{\Gamma_d^{(n)}\big||\mathcal{D}|}(-s|l)}{s}\right] \\ &= \mathcal{L}^{-1}\left[\sum_{j=1}^{J}\frac{n!}{u_1^{(j)}!\,u_2^{(j)}!\,\cdots\,u_l^{(j)}!}\frac{1}{s}\prod_{k=1}^{l}\left\{\binom{l}{k}\frac{(-1)^{k-1} g(k)}{s+g(k)}\right\}^{u_k^{(j)}}\right] \end{aligned} \tag{30}$$

where $\mathcal{L}^{-1}[\cdot]$ stands for inverse Laplace transform. Expanding the term $\frac{1}{s}\prod_{k=1}^{l}\left\{\binom{l}{k}\frac{(-1)^{k-1}g(k)}{s+g(k)}\right\}^{u_k^{(j)}}$ into partial fractions, (30)

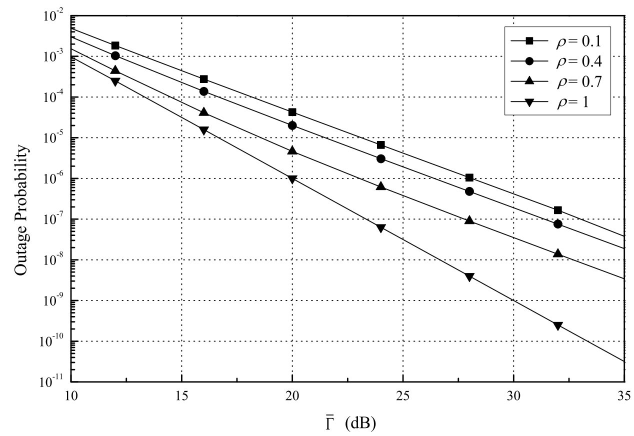

Fig. 3. Outage probability for various ρ. $N = 2$ and $K = 3$.

$$A_{k,u_k^{(j)}}^{(j)} = \frac{(-1)^{(k-1)u_k^{(j)}+1}}{g(k)} \left\{ \binom{l}{k} g(k) \right\}^{u_k^{(j)}} \prod_{\substack{r=1 \\ r \neq k}}^{l} \left\{ \binom{l}{r} \frac{(-1)^{r-1} g(r)}{g(r) - g(k)} \right\}^{u_r^{(j)}} \tag{33}$$

$$A_{k,u_k^{(j)}-m}^{(j)} = \left[\left[\frac{1}{s} \prod_{r=1}^{l} \left\{ \binom{l}{r} \frac{(-1)^{r-1} g(r)}{s + g(r)} \right\}^{u_r^{(j)}} - \sum_{r=0}^{m-1} \frac{A_{k,u_k^{(j)}-r}^{(j)}}{\big(s + g(k)\big)^{u_k^{(j)}-r}} \right] \big(s + g(k)\big)^{u_k^{(j)}-m} \right] \Bigg|_{s=-g(k)} \tag{34}$$

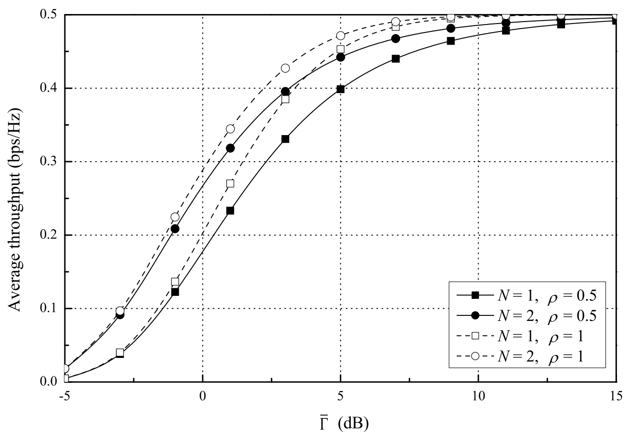

Fig. 4. Average throughput. $K = 3$.

becomes

$$F_{\Gamma_d^{(n)}}\Big|_{|\mathcal{D}|}(\gamma|l) = \mathcal{L}^{-1}\left[\sum_{j=1}^{J} \frac{n!}{u_1^{(j)}!\, u_2^{(j)}! \cdots u_l^{(j)}!} \left\{ \frac{A_0^{(j)}}{s} + \sum_{k=1}^{l} \sum_{i=1}^{u_k^{(j)}} \frac{A_{k,i}^{(j)}}{(s + g(k))^{i}} \right\} \right] \tag{31}$$

where $A_0^{(j)}$ and $A_{k,i}^{(j)}$ are coefficients of partial fractions.

By the Heaviside cover-up method [15], the coefficient $A_0^{(j)}$ in (31) is obtained as

$$A_0^{(j)} = \prod_{k=1}^{l} \left[(-1)^{k-1} \binom{l}{k} \right]^{u_k^{(j)}}. \tag{32}$$

The coefficients $A_{k,i}^{(j)}, i = 1, 2, \cdots, u_k^{(j)}$, in (31) are obtained by computing $A_{k,u_k^{(j)}}^{(j)}$ first using (33) shown at the top of the page, and then computing $A_{k,u_k^{(j)}-m}^{(j)}$, $m = 1, 2, \cdots, u_k^{(j)} - 1$ using (34) shown at the second top of the page [16].

By using [11, eq. (17.13.17)], (31) becomes

$$F_{\Gamma_d^{(n)}}\Big|_{|\mathcal{D}|}(\gamma|l) = \sum_{j=1}^{J} \frac{n!}{u_1^{(j)}!\, u_2^{(j)}! \cdots u_l^{(j)}!} \times \left[A_0^{(j)} + \sum_{k=1}^{l} \sum_{i=1}^{u_k^{(j)}} \frac{A_{k,i}^{(j)} \gamma^{i-1} e^{-g(k)\gamma}}{(i-1)!} \right]. \tag{35}$$

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