A Pragmatic Design of Frequency-Domain Equalizers for Offset Modulations

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Abstract - Offset modulations such as OQPSK (Offset Quaternary Phase Shift Keying) and OQAM (Offset Quadrature Amplitude Modulation), combined with SC-FDE (Single-Carrier with Frequency-Domain Equalization) are an interesting choice for broadband wireless systems, that require low-complexity transmitters and efficient power amplification. However, the FDE performance can be rather poor due to interference between the in-phase and quadrature components at the sampling instants, especially when large constellations are employed. In this paper we propose pragmatic iterative and non-iterative FDE designs for offset modulations. Our performance results show that both iterative and non-iterative pragmatic FDE receivers have excellent performance, even for large OQAM constellations.

Keywords: OQAM modulations, SC-FDE, I-Q interference, Iterative receivers.

I. INTRODUCTION

The signals associated to offset modulations such as OQPSK (Offset Quaternary Phase Shift Keying) and OQAM (Offset Quadrature Amplitude Modulation) have lower envelope fluctuations and lower dynamic range than the corresponding non-offset signals. For this reason, offset signals are particularly interesting when an efficient power amplification is intended. In fact, OQPSK signals can have constant or quasi-constant envelope, making them suitable for grossly, nonlinear power amplifiers; although OQAM signals always have envelope fluctuations, they can be written as the sum of OQPSK components with very low envelope fluctuations that can be separately amplified by grossly nonlinear amplifiers [2]. Furthermore if a MSK (Minimum Shift Keying) support pulse is used, the PAPR (peak to average power ratio) can be further reduced obtaining signals with quasi-constant envelope.

By combining offset modulations with SC-FDE schemes (Single-Carrier with Frequency-Domain Equalization) [3] we obtain excellent transmission schemes for broadband wireless systems with low cost transmitters and efficient power amplification. However, the performance is very poor when we employ conventional FDE receivers (designed for non-offset modulations) with offset modulations. This is due to the residual interference between the in-phase and quadrature components at the sampling instants. For this reason, FDE receivers especially designed for offset modulations were proposed in [4]. The basic idea behind these schemes is to design the FDE in such a way that the overall impulse response at its output (including channel and transmit and receive filters) become real, avoiding IQI (In-phase/Quadrature Interference). Since these schemes can have very high residual ISI (Inter-Symbol Interference), modified FDE receivers were proposed in [5] that minimize the overall residual ISI plus IQI levels, allowing improved performance. Unfortunately, even the best FDE receivers for offset modulations have a somewhat disappointing performance when large offset constellations are employed [6].

In this paper it is considered a different approach to design FDE receivers for offset modulations. A pragmatic receiver design that takes advantage from the fact that the performance of offset modulations with conventional FDE receivers can be much better when we employ raised-cosine pulses with zero roll-off (i.e., with the minimum Nyquist bandwidth) is presented. Both iterative and non-iterative FDE designs are considered. The iterative receivers are designed to take full advantage of the multipath diversity and combine an iterative FDE with the estimation and cancellation of IQI.

This paper is organized as follows: section II explains the OQAM signals and section III states the proposed receivers' designs. Some performance results are shown in section IV and section V briefly states some conclusions.

II. OQAM SIGNALS

Considering an SC-FDE scheme where the data is transmitted in blocks and a suitable cyclic prefix with N_{CP} samples is appended to each block, the length-N data block to be transmitted is $\{a_n; n = 0, 1, ..., N-1\}$, where $a_n = a_n^I + ja_n^Q$ is the nth data symbol. For a M-OQAM constellation, a_n^I and a_n^Q, take the values ±1, ±3, $\ldots$,$\pm\sqrt{M}-1$. The complex envelope of these transmitted signal is

$$x(t) = \sum_{n=-N_{CP}}^{N-1} a_n^I r(t - nT_s) + \sum_{n=-N_{CP}}^{N-1} a_n^Q r(t - nT_s - T_o), \tag{1}$$

where $r(t)$ is the adopted pulse shape, T_o is the time offset between both I and Q components (usually $T_s/2$) and N_{CP} is the length of the cyclic prefix required for an efficient FDE

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implementation [7]. The block $\{a_n; n = 0, 1, ..., N-1\}$ is periodic with period N, therefore, $a_{-n} = a_{N-n}$.

At the receiver side, if a given block is sampled with rate J/T_s, with J samples per symbol, then the samples associated to the useful part of the block (without cyclic prefix) are $\{x_n^{(J)}; n = 0, 1, \ldots, JN-1\}$, with $x_n^{(J)} \triangleq x(nT_s/J)$ (J is assumed to be large enough to avoid aliasing effects). Since $x(t)$ is cyclostationary [8] ($E[x(t)x(t-\tau)]$ is periodic in t, with period T_s), the frequency-domain block associated to $\{x_n^{(J)}; n = 0, 1, \ldots, JN-1\}$ is $\{X_k^{(J)}; k = 0, 1, \ldots, JN-1\}$ = DFT $\{x_n^{(J)}; n = 0, 1, \ldots, JN-1\}$, with

$$X_k^{(J)} = A_k^{(J)} R_k, \tag{2}$$

where $\{R_k; k = 0, 1, \ldots, JN-1\}$ is the DFT of $\{r_n \triangleq r(nT_s/J); n = 0, 1, \ldots, JN-1\}$, and $\{A_k^{(J)}; k = 0, 1, \ldots, JN-1\}$ is the DFT of $\{a_n^{(J)}; n = 0, 1, \ldots, JN-1\}$, with

$$a_n^{(J)} = \begin{cases} a_{n'}, & n = Jn' \\ 0, & \text{otherwise} \end{cases}. \tag{3}$$

It can easily be shown that

$$A_k^{(J)} = \frac{1}{J} A_k, \tag{4}$$

where $\{A_k; k = 0, 1, \ldots, N-1\}$ is the DFT of $\{a_n; n = 0, 1, \ldots, N-1\}$. This means that an implicit multiplicity exists in the frequency-domain block when the adopted pulse shape has a bandwidth higher than the Nyquist band (R_k is not restricted to N non-zero samples) [9]. Therefore, the frequency-domain sample A_k can be repeated in several $X_k^{(J)}$ samples, which are separated by multiples of N (see Fig. 1).

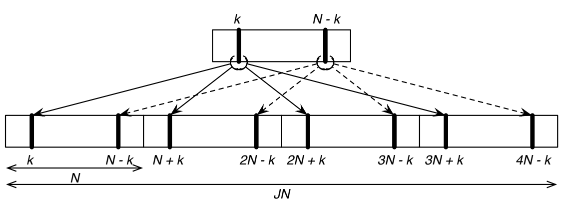

Fig. 1. Frequency Multiplicity.

In an OQAM scheme, the quadrature component has a T_o offset from the in-phase component which results in a phase deviation, $\Theta_k = \exp(-j\pi k/N)$, in the frequency domain. Clearly, we have $\Theta_{k+N} = -\Theta_k$, and $X_k^{(J)} = \left(A_k^{(J)I} + A_k^{(J)Q}\Theta_k\right) R_k$ as we can see in Fig. 2.

N JN

A_k^I	A_k^I	A_k^I	A_k^I
$A_k^Q\Theta_k$	$-A_k^Q\Theta_k$	$A_k^Q\Theta_k$	$-A_k^Q\Theta_k$

Fig. 2. Frequency Multiplicity for OQAM.

The OQPSK signal can be regarded as a special case of M-OQAM where $M = 1$.

III. FDE Design

In an FDE, the received block is sampled at the rate J/T_s, the cyclic prefix is removed and the resulting block $\{y_n^{(J)}; n = 0, 1, \ldots, JN-1\}$ is passed to the frequency domain, leading to the block $\{Y_k^{(J)}; k = 0, 1, \ldots, JN-1\}$. If the cyclic prefix is longer than the overall channel impulse response length then

$$Y_k^{(J)} = A_k^{(J)} H_k^{(J)} + N_k^{(J)}, \tag{5}$$

where $H_k^{(J)}$ is the overall channel impulse response associated to the kth subcarrier, which includes the adopted pulse shape, the channel and the $1/J$ factor inherent to oversampling (4) and $N_k^{(J)}$ is the corresponding noise component. Figure 3 shows the receiver structure presented in the following section III-A and III-B.

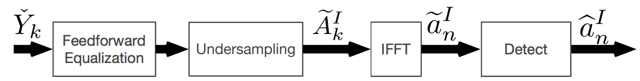

Fig. 3. Linear FDE Receiver Structure.

A. Linear FDE for offset modulations with minimization of residual IQI plus ISI

The high level of interference between both I and Q components of OQAM signals is due to its inherent time offset T_o (which need to be sampled at different instants at the receiver). When the received pulses are real, there would be no interference between the I and Q components. Nevertheless, as a consequence of channel effects, specially in extremely selective ones, the pulse at the equalizer's output will often become complex and the interference between in-phase and quadrature components will rise, hindering correct symbol detection. The symbol detection, for a linear FDE can be expressed as:

$$\widetilde{A}_k = \sum_{q=0}^{J-1} F_{k'+qN}^{(J)} Y_{k'+qN}^{(J)}. \tag{6}$$

From [5] we can obtain the optimum values of $F_{k'}^{(J)}$, under de MMSE criteria

$$F_{k'}^{(J)} = \begin{cases} \frac{(1-\lambda_k^{clip})H_{k'}^{(J)*}}{\alpha+\sum_{k'\in\left(\Psi_k^{(1)}\ \cup\ \Psi_k^{(3)}\right)}\left|H_{k'}^{(J)}\right|^2} & , \quad k' \in \Psi_k^{(1)} \\ \frac{(1+\lambda_k^{clip})H_{k'}^{(J)*}}{\alpha+\sum_{k'\in\left(\Psi_k^{(1)}\ \cup\ \Psi_k^{(3)}\right)}\left|H_{k'}^{(J)}\right|^2} & , \quad k' \in \Psi_k^{(3)} \\ \frac{(1-\lambda_k^{clip})H_{k'}^{(J)*}}{\alpha+\sum_{k'\in\left(\Psi_k^{(2)}\ \cup\ \Psi_k^{(4)}\right)}\left|H_{k'}^{(J)}\right|^2} & , \quad k' \in \Psi_k^{(2)} \\ \frac{(1+\lambda_k^{clip})H_{k'}^{(J)*}}{\alpha+\sum_{k'\in\left(\Psi_k^{(2)}\ \cup\ \Psi_k^{(4)}\right)}\left|H_{k'}^{(J)}\right|^2} & , \quad k' \in \Psi_k^{(4)} \end{cases}, \tag{7}$$

with α as the inverse of the signal to noise ratio,

$$\lambda_k^{clip} = \begin{cases} \lambda_k & , \quad |\lambda_k| < 0.5 \\ \frac{\lambda_k}{|\lambda_k|}\, 0.5 & , \quad |\lambda_k| > 0.5 \end{cases} \tag{8}$$

where,

$$\lambda_k = \frac{(A-B)(\alpha+C+D)+(C-D)(\alpha+A+B)}{(A+B)(\alpha+C+D)+(C+D)(\alpha+A+B)}, \tag{9}$$

with

$$A=\sum_{k'\in\Psi_k^{(1)}}\left|H_{k'}^{(J)}\right|^2,\quad B=\sum_{k'\in\Psi_k^{(3)}}\left|H_{k'}^{(J)}\right|^2,$$

$$C=\sum_{k'\in\Psi_k^{(2)}}\left|H_{k'}^{(J)}\right|^2,\quad D=\sum_{k'\in\Psi_k^{(4)}}\left|H_{k'}^{(J)}\right|^2,$$

$$\begin{aligned}\Psi_k^{(1)} &= \{k+2qN; q=0,1,...,J/2-1\},\\ \Psi_k^{(2)} &= \{N-k+2qN; q=0,1,...,J/2-1\},\\ \Psi_k^{(3)} &= \{N+k+2qN; q=0,1,...,J/2-1\},\end{aligned}$$

and $\Psi_k^{(4)}=\{2N-k+2qN; q=0,1,...,J/2-1\}$.

B. Pragmatic FDE receiver for offset modulations

This method's motivation started of with the comparison between the BER (Bit Error Rate) curves of raised cosine support pulses with different roll-off factors. As it is possible to observe in Fig. 4, using the conventional linear FDE equalization, the best performance is achieved with raised cosine support pulses with zero roll-off factor. From both figure 4 and 5 we can perceive that the IQI levels are lower when the support pulse bandwidth shrinks.

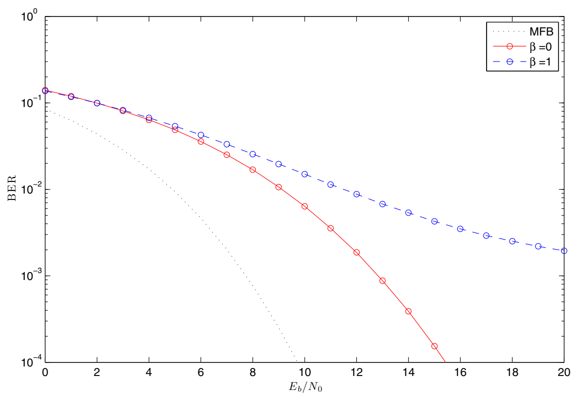

Fig. 4. BER performance vs. E_b/N_0 for conventional FDE using raised cosine support pulses with different roll-off (β) factors.

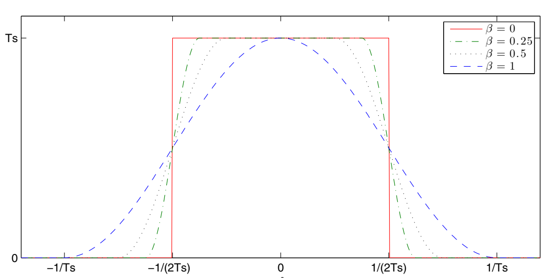

Fig. 5. Frequency spectrum of a raised cosine support pulse with different roll-off (β) factors.

From this knowledge, a very simple method was derived. This method filters the desired signal, removing all frequency multiplicity (Fig. 6). This can be mathematically expressed as

$$H_k^u=\begin{cases}H_k & , \quad -\frac{N}{2}<k<\frac{N}{2}\\ 0 & , \quad \text{otherwise}\end{cases}. \tag{10}$$

Further on, the equalizer coefficients are obtained with the traditional MMSE criteria,

$$F_k=\frac{H_k^{u*}}{\alpha+|H_k^u|^2}, \tag{11}$$

but take into account the filtered channel response, H_k^u, instead of the estimated channel H_k. As we can perceive, this method

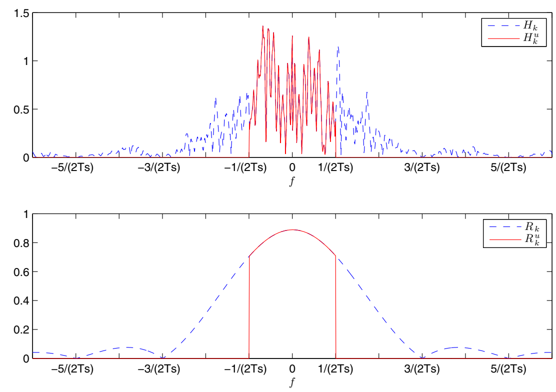

Fig. 6. Frequency spectrum of H_k, H_k^u (up) and R_k, R_k^u (down) for an MSK support pulse.

neglects all the power sent from the transmitter outside of the cut region, in fact, the total power loss, considering an MSK support pulse is $\sum_{k=0}^{JN}R_k^u/\sum_{k=0}^{JN}R_k\approx-1.5$dB, with R_k^u defined in the same way as H_k^u. Therefore, this methods starts off hindered, compared to the previous one.

C. Iterative FDE for OQAM

It is known that further improvements can be attained on the performance of a linear equalization through an iterative equalizer. According to [6], the output samples for a given iteration of the iterative FDE for OQAM can be defined as

$$\widetilde{A}_k^I=\sum_{q=0}^{J/2-1}\left[F_{k+qN}^{(J)}\left(Y_{k+qN}^{(J)}-\overline{Y}_{k+qN}^{(J)Q}\right)-B_{k+qN}^{(J)}\overline{A}_{k+qN}^{(J)I}\right] \tag{12}$$

$$\widetilde{A}_k^Q=\sum_{q=0}^{J/2-1}\left[F_{k+qN}^{(J)}\left(Y_{k+qN}^{(J)}-\overline{Y}_{k+qN}^{(J)I}\right)-B_{k+qN}^{(J)}\overline{A}_{k+qN}^{(J)Q}\right] \tag{13}$$

for the quadrature component, where $\{k=0,\dots,2N-1\}$. For the in-phase block, $\{\widetilde{A}_k^I;k=0,\dots,2N-1\}=\text{DFT}\{\widetilde{a}_n^I;n=0,\dots,2N-1\}$ only the even data bits are relevant, whereas for the quadrature block $\{\widetilde{A}_k^Q;k=0,\dots,2N-1\}=\text{DFT}\{\widetilde{a}_n^Q;n=0,\dots,2N-1\}$ the odd data bits are relevant for detection.[1] Note as well that the feedback data bits $\{\overline{A}_k^{(J)I};k=0,\dots,JN-1\}=\text{DFT}\{\overline{a}_n^{(J)I};n=0,\dots,JN-1\}$ and $\{\overline{A}_k^{(J)Q};k=0,\dots,2N-1\}=\text{DFT}\{\overline{a}_n^{(J)Q};n=0,\dots,2N-1\}$ are from soft bit decisions, and therefore more precise [10].

The first iteration of this method is, in fact, a linear equalization, where $\overline{A}_k$ are non existent. Therefore, we can

[1] This is due to the time offset between in-phase and quadrature components in OQAM.

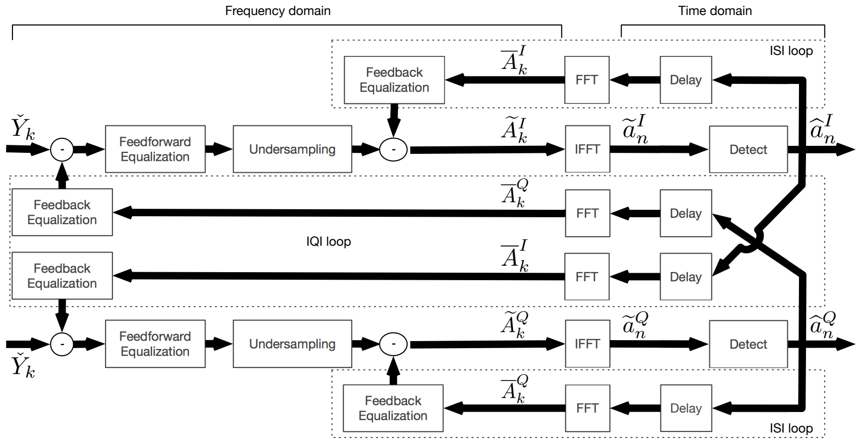

Fig. 7. Proposed Iterative Receiver Structure.

use any linear method (namely those from subsection III-A and III-B) as the first iteration of the IB-DFE.

For all the following iterations, the values of the feedforward and feedback coefficients will be calculated in the following way [6]:

$$F_k^{(J)} = \frac{\kappa H_k^{(J)*}}{\alpha + (1-\rho^2)\sum_{q=0}^{J/2-1}\left|H^{(J)}_{(k\ mod\ 2N)+qN}\right|^2}, \quad (14)$$

$$B_k^{(J)} = \left(F_k^{(J)} H_k^{(J)} - 1\right), \quad (15)$$

where κ is selected to ensure $\frac{1}{N}\sum_{k=0}^{N-1}\sum_{q=0}^{J/2-1} F^J_{k+qN} H^{(J)}_{k+qN} = 1$, and ρ is defined in (20).

The values of the overall average received frequency values that cancel IQI over the successive iterations are for In-phase component $\overline{Y}_k^{(J)I} = H_k^{(J)}\overline{A}_k^{(J)I}$ and for the Quadrature component $\overline{Y}_k^{(J)Q} = H_k^{(J)}\Theta_k\overline{A}_k^{(J)Q}$.

To obtain $\{\overline{a}_n^{(J)I}; n = 0,\ldots,JN-1\} = \text{IDFT}\{\overline{A}_k^{(J)I}; k = 0,\ldots,JN-1\}$ and $\{\overline{a}_n^{(J)Q}; n = 0,\ldots,JN-1\} = \text{IDFT}\{\overline{A}_k^{(J)Q}; k = 0,\ldots,JN-1\}$ we will need a general mapping for M-OQAM scheme. The log-likelihood ratio of the mth bit of the nth transmitted symbol component is given by

$$LLR_n^{(m)} = log\left(\frac{\sum_{a\in\Phi_1^{(m)}}\exp\left(-\frac{|\widetilde{a}_n - a|^2}{2\sigma^2}\right)}{\sum_{a\in\Phi_0^{(m)}}\exp\left(-\frac{|\widetilde{a}_n - a|^2}{2\sigma^2}\right)}\right), \quad (16)$$

where $\Phi_1^{(m)}$ and $\Phi_0^{(m)}$ are the constellation's subsets associated to the symbols with the mth bit at 1 or 0, respectively and

$$\sigma^2 = \frac{1}{2N}\sum_{n=0}^{N-1} E[|\widetilde{a}_n - a_n|^2]. \quad (17)$$

To obtain the average symbol values conditioned to the FDE output, $\overline{a}_n$, we need the average bit values conditioned to the FDE output, $\overline{b}_n^{(m)}$. They can be related as

$$\overline{b}_n^{(m)} = \tanh\left(\frac{LLR_n^{(m)}}{2}\right). \quad (18)$$

From [11], and assuming uncorrelated bits due to the usage of a suitable interleaver, we have

$$\overline{a}_n = \sum_{m=1}^{\varphi} 2^{\varphi-m}\prod_{q=1}^{m}\overline{b}_n^{(q)} \quad (19)$$

where $\overline{a}_n^{(J)I}$ is the oversampled version of $\overline{a}_n^I$ and $\overline{a}_n^{(J)Q}$ of $\overline{a}_n^Q$, according to (3).

Finally, the value of the estimate reliability to be used in the feedback loop is obtained by

$$\rho = \frac{E\left[\overline{a}_n a_n^*\right]}{E\left[|a_n|^2\right]} \approx \frac{E\left[\overline{a}_n \widehat{a}_n^*\right]}{E\left[|\widehat{a}_n|^2\right]}, \quad (20)$$

with $\widehat{a}_n$ being the hard-decisions related to $\widetilde{a}_n$.

The block diagram of the proposed iterative receiver is shown in Fig. 7.

IV. Performance Results

In this section we present a set of performance results concerning both linear and iterative receivers for OQPSK and 16-OQAM signal constellation with blocks of $N = 256$ data symbols, MSK (Minimum Shift Keying) support pulses and an oversampling factor of $J = 8$. As an example we considered a severely time dispersive propagation channel characterized by the power delay profile type C for High Performance Local Area Lan [12], with uncorrelated Rayleigh fading in different taps (similar results were observed for other severely time-dispersive channels with rich multipath propagation).

The duration of the useful part of the data block (N symbols) is $4\mu s$. It is also assumed perfect synchronization and channel estimation.

Figures 8 and 9 show the BER performance of three different linear FDE designs: Conventional MMSE FDE, $F_k = {H_k}^*/(\alpha + |H_k|^2)$, applied on OQPSK/OQAM constellations for comparison purposes, IQI+ISI minimization and pragmatic for OQPSK and 16-OQAM modulations, respectively. Both approaches have better performance than the conventional method and the pragmatic approach outperforms the IQI+ISI minimization for higher values of E_b/N_0.

Figure 10 shows the corresponding BER performance of the two different linear FDE designs: IQI+ISI minimization (from section III-A) and pragmatic (from section III-B). These results show that, despite the better performance of the IQI+ISI

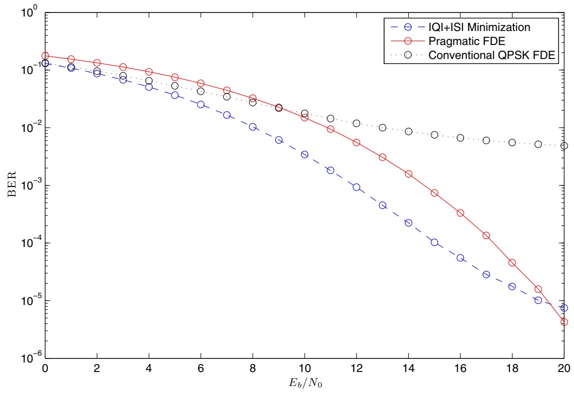

Fig. 8. BER performance versus E_b/N_0 for the linear IQI+ISI minimization and pragmatic equalization for a OQPSK signal.

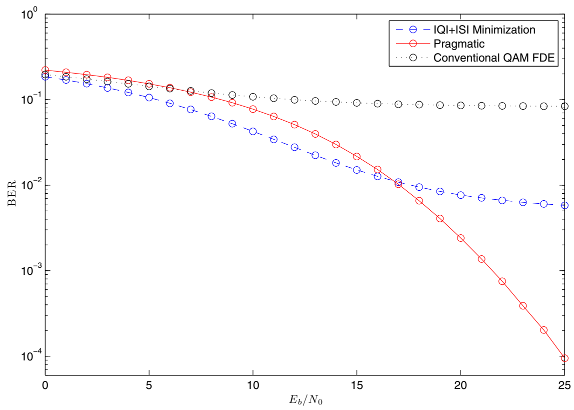

Fig. 9. BER performance versus E_b/N_0 for the linear IQI+ISI minimization and pragmatic equalization for a 16-OQAM signal.

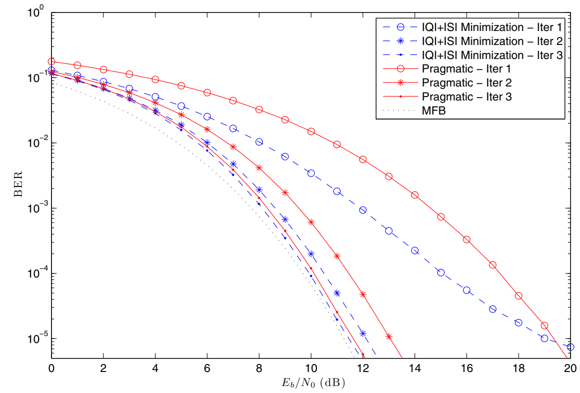

Fig. 10. BER performance versus E_b/N_0 for the iterative IQI+ISI minimization and pragmatic methods for a OQPSK signal.

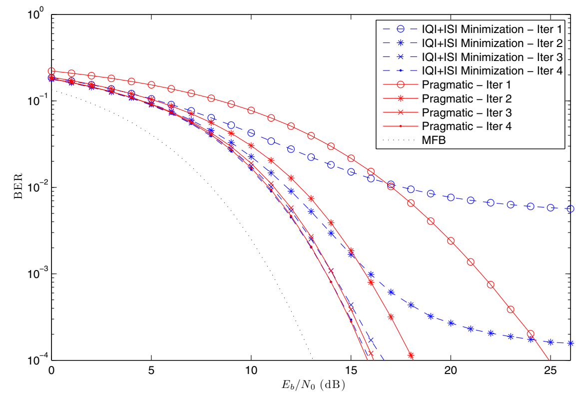

Fig. 11. BER performance versus E_b/N_0 for the iterative IQI+ISI minimization and pragmatic methods for a 16-OQAM signal.

minimization method over the pragmatic method, this simplistic approach obtains similar results over the iterations for a lower receiver's complexity. From figure 11 it is possible to observe a different behavior for both methods. As IQI and ISI increases from the added signal complexity (from 4 data symbols to 16) the IQI+ISI minimization method has reduced efficiency compared to the pragmatic one, being surpassed after $E_b/N_0 = 18$dB. This implies that the pragmatic method has better IQI and ISI immunity and is suited for high order constellations.

V. Conclusions

In this paper we considered the use of offset modulations with SC-FDE schemes and we presented a pragmatic receiver design suitable for signals with bandwidth above the Nyquist band. Both iterative and non-iterative FDE designs were presented.

Our performance results show that the pragmatic FDE receivers have excellent performance and are a promising method for offset modulations with high order constellations.

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