

# A Novel Link Scheduling Algorithm for Spatial Reuse in Wireless Networks

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**Abstract**—In this paper, we study the problem of one-slot scheduling under the exact SINR interference model, which aims to find a maximum subset of links to be scheduled simultaneously in one slot. We present a novel link scheduling algorithm (DIVISION) to give an effect solution for link scheduling. Algorithm DIVISION seeks to schedule the wireless links by their geometric locations, which provides a new angle to explore the spatial reuse in wireless networks. Combined with numerical results, it is pointed out that the proposed algorithm has the same complexity as the typical algorithm GOW\* which is also based on space scheduling, but yields potential better performance of the network throughput.

## I. INTRODUCTION

With the development of wireless communication technology, such as Femtocell[1], WLAN[2], Device-to-Device(D2D)[3], the transmission distance of wireless links becomes shorter than ever before. Short-distance communication can sustain the same transmission rate with lower transmit power. The wireless signal covers a smaller region and interference can be well controlled. Therefore, it is possible to hold more links in a specific area to enhance the spatial reuse in short-distance wireless networks.

Consider a densely deployed network where at least hundreds of nodes are randomly distributed in the network region. Communication link can be established if the SNR at the receiver is larger than a threshold  $\beta$ . Assume that each link has only one packet to transfer and should be scheduled one time during the scheduling process. Time-division multiple access (TDMA) is adopted and some links which cause low interference to each other can coexist in one slot to enhance the spatial reuse. One-slot scheduling aims to find a maximum subset of feasible links to be scheduled simultaneously in one slot. We also assume that there is a centrally coordinated scheduler. The scheduler can acquire the information of geometric locations of nodes, such as through GPS and feedback. Hence, geometric locations of nodes can be utilized during the scheduling process.

In general, there are two interference models when studying the problem of link scheduling in wireless networks. One is graph model[4]. Exclusive region is proposed as a piece of region that is around the receiver of the transmission link. Any link whose transmitter falls into the exclusive region is deemed to be conflict with the original link, while the links that are out of the exclusive region cause no interference. Graph model is simple, easy constructed, but far from accuracy since

it neglects the remote interference. The other model is SINR model. In this model, the SINR is calculated at the receiver, which is the ratio of the received signal strength and the aggregate interference caused by the concurrent transmission links (include noise). SINR model is exact to depict the interference relationship for a wireless network. However, it has been proved that one-slot scheduling under SINR model is NP-complete[5], which leaves a challenge to design an effective algorithm with low complexity for link scheduling in wireless networks.

A few recent works have been done to investigate the problem of link scheduling under the SINR model. In [6], SINR Graph Link Scheduling(SGLS) was proposed based on communication graph with SINR conditions. SGLS algorithm embeds interference conditions between pairs of nodes into edge weights and normalized noise powers at receiver nodes into vertex weights of the SINR graph. SGLS can achieve good network throughput with high computational complexity. To reduce the computational complexity, a first known approximation algorithm GOW\* based on space locations of nodes was proposed in [7]. Space partition was presented in GOW\*, and links are classified to be scheduled by SINR. GOW\* algorithm greatly reduces the computational complexity, while it has poor network throughput as it demands strict space separation between the neighbor concurrent transmission links.

In this paper, we propose a novel algorithm (Division) for one-slot scheduling under SINR model. With the information of node locations, we aims to schedule the links by their geometric locations. On one hand, compared with the classical algorithm SGLS, Division reduces the computational complexity without deteriorating the network throughput. On the other hand, compared with space-based algorithm GOW\*, Division reduces the space distance between neighbor links, which provides potential better network throughput without increasing extra computational complexity. To sum up, Division is more competitive for the densely-deployed network, where both computational complexity and network performance should be considered both.

The rest of paper is organized as follows. Section II outlines the interference model, describes the proposed scheduling algorithm in detail, and analyzes the corresponding conditions and parameters. Section III presents the simulation results, and the work concludes in IV.

## II. METHODOLOGY

### A. Interference Model

In this paper, we use the SINR interference model. The successful reception of a packet sent by node  $s$  and destined to node  $r$  depends on the SINR at  $r$ . To be specific, let  $P_r(s)$  denote the received power at node  $r$  of the signal transmitted by node  $s$ , a packet along link  $(s, r)$  is correctly received iff

$$\frac{P_r(s)}{N + \sum_{w \in v' - \{s\}} P_r(w)} \geq \beta \quad (1)$$

where constant  $N$  is the thermal noise,  $v'$  the subset of nodes to be scheduled simultaneously, and  $\beta > 0$  is a constant SINR threshold. Note that in (1), we compute all the interference from concurrent transmission links in the network deployment region.

We adopt the classical model for radio signal propagation in wireless networks, which is referred to as the log-distance path loss model. In this model, the radio signal power at a distance  $d(s, r)$  from the transmitter given by

$$P_r(s) = \frac{P}{d(s, r)^\alpha} \quad (2)$$

where  $P$  is the fixed transmit power and  $\alpha > 2$  is the path loss coefficient.

### B. Space Partitions

In this paper, we seek to study the link scheduling from a new angle of geometric location of nodes. Interference is the key factor to hinder two close links to be scheduled simultaneously. As low interference is the necessary condition for link coexistence, it is required that the concurrent links should be separated in geometric space. As we know, the traditional link scheduling algorithms schedule the links by inspecting the interference relationships among the total links, which leads to NP-complete problems and brings high complexity. Instead of inspecting directly the interference relationships, space scheduling guarantees the geometric space separation at the beginning of the link scheduling.

The main idea of this paper is space partition. We partition the network deployment region into numbers of blocks. As the blocks are separated in geometric space, links from different blocks are separated in geometric space and cause low interference to each other. For simplicity, we take square block and choose one link out of each block in this paper. Since we only take one link in one block, the other links in the same blocks are excluded automatically. This is reasonable since the links in the same blocks have high probability in causing serious interference to the chosen link. Hence, By space partition, unnecessary judgement is neglected and the computational complexity will be reduced.

The size of block is critical since it is related to SINR requirements and the number of blocks in the network deployment region. The partition operations are shown in Fig.1. To some extent, by refining the size of the block, spatial reuse can reach the number of blocks, which can be well controlled in the scheduling process.

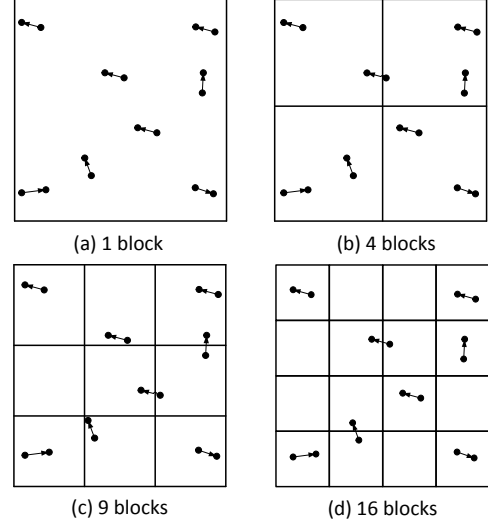


Fig. 1: Space partition with different number of blocks

TABLE I: The DIVISION Algorithm

<b>DIVISION Algorithm</b>	
<b>Input:</b>	Given a set of links $L = \{l_1, \dots, l_k\}$
<b>Output:</b>	A valid schedule $colorSet = \{S_1, S_2, \dots, S_t\}$ .
1:	$t = 1$ .
2:	Partition network deployment region into $n^2$ blocks of width $l$ .
3:	links are classified into four cases $\{P_1, P_2, P_3, P_4\}$ (Fig.2).
4:	<b>for</b> each $P_k$ ( $k = 1, 2, 3, 4$ )
5:	<b>repeat</b>
6:	for each block $C_i$ of $P_k$ , choose a link $l_i \in C_i$ with sender and receiver in $C_i$ and $SINR(l_i) > \beta; S_t = S_t \cup \{l_i\}$ .
7:	$t = t + 1$ .
8:	<b>until</b> all links of $P_k$ in selected blocks are scheduled.
9:	<b>return</b> $colorSet = \{S_1, S_2, \dots, S_t\}$ .

### C. Division Algorithm

In this section, we discuss the proposed algorithm DIVISION, which is shown in Table I. Firstly, we partition the network deployment region into numbers of blocks. We cluster the links in a block by space locations of nodes, while links in different blocks are separated in space. Secondly, we choose one link from each block randomly with the condition of SINR threshold. Then a subset of links is scheduled in one slot by gathering the chosen links from different blocks. We will discuss the algorithm parameters in detail below.

Link coexistence conditions are the first point to be further discussed. It is mentioned above that correct transmission along a link depends on the SINR. Given a fixed set of link, SINR of each link is calculated and the set is feasible only when each link meets its SINR requirement. However, during the process of link scheduling, the link set is unsure in advance. To judge whether a link can be added into the current scheduled link set, there are two aspects to consider: 1) The previous transmission links should not be corrupted due to the addition of new coming link. 2) The new coming link should not be corrupted by the previous transmission links. Hence, the SINR should be calculated for each scheduled link and the new coming link.

Another ambiguous problem is how to determine which block a link belongs to. Since a transmission link consists of both sender and receiver, the belonging of a link may be decided by: a) the sender; b) the receiver; c) both sender and receiver. In GOW\*, since four-coloring is applied to allocate different slots for the neighbor links, concurrent links are separated at least one block in geometric space. Hence, case b) is adopted. In this paper, four-coloring is taken away and neighbor blocks share one slot. To increase the space separation of links of neighbor blocks, we adopt case c) to determine the belonging of a link. Note that although both sender and receiver of a link are considered, it is still possible that two links from neighbor blocks may be close to each other. Fortunately, the link coexistence conditions will avoid scheduling two close links during the process of link scheduling.

During the process of link scheduling, we choose one link from each block in order. The link is randomly chosen in a block. If the conditions of link coexistence are not satisfied, the link will be excluded and another link will be picked randomly. The process of link picking in a block will be continue until one link is picked or there is none link to be scheduled. Note that, it is expected that the number of link to be scheduled in one slot can be equivalent to the number of blocks. However, with the conditions of link coexistence, it is possible that none link can be picked in a block, so the number of concurrent transmission links would not always sustain the maximum value.

#### D. Number of Partition

Given a fixed partition of the network deployment region, it is possible that one link belongs to no block, for the sender and receiver of a link may fall into different blocks. Therefore, more than one kind of space partition is needed in our algorithm. We discuss in this subsection that how much kinds of partitions is needed at least.

Let  $l$  denote the width of block and  $d_{max}$  the maximum length of a link. When  $l \geq 2d_{max}$ , it can be proved that only four kinds of partitions are needed to cover all of the links.

**Theorem 1:** For any 2D network, when  $l \geq 2d_{max}$ , there exists 4 kind of partitions to cover all links.

**Proof:** We give a specific example for partitions as shown in Fig.2, and prove that any link will be covered by at least one of the partition. Assume that  $(x_1, y_1)$  and  $(x_2, y_2)$  are the coordinates of the sender and receiver of a link. When block is shifted, we can write the corresponding coordinates as:

$$\left\{ \begin{array}{ll} (x_1, y_1), (x_2, y_2) & \\ (x_1 - \frac{l}{2}, y_1), (x_2 - \frac{l}{2}, y_2) & \text{right } \frac{l}{2} \\ (x_1, y_1 - \frac{l}{2}), (x_2, y_2 - \frac{l}{2}) & \text{down } \frac{l}{2} \\ (x_1 - \frac{l}{2}, y_1 - \frac{l}{2}), (x_2 - \frac{l}{2}, y_2 - \frac{l}{2}) & \text{lower right } \frac{l}{2} \end{array} \right. \quad (3)$$

Considering X-axis, we can get the node pairs  $(x_1 - \frac{l}{2}, x_2 - \frac{l}{2})$  and  $(x_1, x_2)$ . It will be proved that at least one node pair falls in a block. Then note it as  $(x'_1, x'_2)$ . There will be two cases that follow  $x_1 = x'_1, x_2 = x'_2$  out of the four kinds of partitions, with different Y-Coordinates. Similarly, there is at

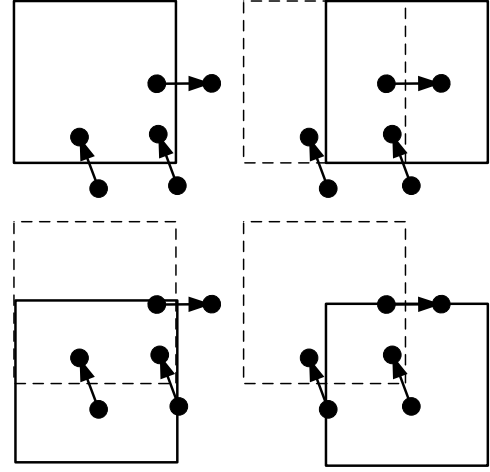


Fig. 2: Shift of the blocks in DIVISION Algorithm

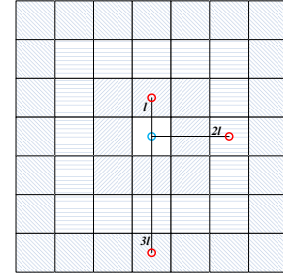


Fig. 3: interference in the center of the block

least one node pair  $(y'_1, y'_2)$  which falls into a block. Therefore, node  $(x'_1, y'_1)$  and  $(x'_2, y'_2)$  fall into a same block.

Next, we prove the conclusion that at least one of the pairs  $(x_1 - \frac{l}{2}, x_2 - \frac{l}{2})$  and  $(x_1, x_2)$  will be in a block. If none of the pairs fall into a block, we can find  $x_0 = k_0 l$  between  $x_1 - \frac{l}{2}$  and  $x_2 - \frac{l}{2}$ , and find  $x_1 = k_1 l$  between  $x_1$  and  $x_2$ . We have  $x_2 - (x_1 - \frac{l}{2}) \geq k_1 l - k_0 l \geq 0 \Rightarrow x_2 - x_1 \geq \frac{l}{2} > d_{max}$ . Since the maximum length of a link is  $d_{max}$ ,  $x_2 - x_1 \leq d_{max}$ . Contradiction occurs, so the result follows.

#### E. Size of the Block

The size of the block is a critical parameter in algorithm DIVISION, for it determines the number of blocks in a deployment region, which is related to the number of concurrent links in a slot. If it is too small, interference will be serious between neighbor blocks; and if it is too large, number of concurrent links will be reduced, which conducts inefficient space utilization and low network throughput. We give a proposal value for the width of block below.

According to Fig.3, the node in the center is the receiver  $r$ , then we will calculate the interference from the other blocks. The inner frame contains  $3^2 - 1^2 = 8$  blocks, the second frame contains  $5^2 - 3^2 = 16$  blocks, and the  $k$ th frame will contain  $(2k + 1)^2 - (2k - 1)^2 = 8k$  blocks. The generic receiver contained in the  $k$ th frame will be  $kl$  uniformly apart from  $r$ .

Assume that the number of blocks is  $n^2$ . The aggregate

interference on the centric receiver can be written as:

$$\begin{aligned} \sum IR &= 8 * \frac{p}{l^\alpha} + 8 * 2 * \frac{p}{(2l)^\alpha} + \dots + 8 * \left(\frac{n}{2} - 1\right) * \frac{p}{((\frac{n}{2}-1)*l)^\alpha} \\ &= \sum_{k=1}^{\frac{n}{2}-1} 8k * \frac{p}{(kl)^\alpha} = \frac{8p}{l^\alpha} \sum_{k=1}^{\frac{n}{2}-1} \frac{1}{k^{\alpha-1}} \end{aligned} \quad (4)$$

If the link can communicate successfully, then the interference must satisfy the next requirement:

$$\frac{\frac{p}{d(s,r)^\alpha}}{N + \sum IR} \geq \beta \Leftrightarrow \sum IR \leq \frac{p}{\beta d(s,r)^\alpha} - N \quad (5)$$

When  $n \rightarrow \infty$ , the interference can conduct this approximation:

$$\sum IR = \frac{8p}{l^\alpha} \sum_{k=1}^{\frac{n}{2}-1} \frac{1}{k^{\alpha-1}} \leq \frac{8p}{l^\alpha} \sum_{k=1}^{\infty} \frac{1}{k^{\alpha-1}} \leq \frac{8p}{l^\alpha} \frac{\alpha-1}{\alpha-2} \quad (6)$$

where the second inequality follows from a known bound on Riemann's zeta function.

Combining (5) and (6), if and only if  $\frac{8p}{l^\alpha} \frac{\alpha-1}{\alpha-2} \leq \frac{p}{\beta d(s,r)^\alpha} - N$ , the link can communicate successfully. Then we can get the minimum side length as:

$$l \geq \left[ \frac{8p}{\beta d(s,r)^\alpha - N} \frac{\alpha-1}{\alpha-2} \right]^{\frac{1}{\alpha}} \quad (7)$$

Note that the width of blocks we proposed is not optimal. Actually it is obvious that the optimal value is variable according to the network topology. However, the length  $l$  is close to the optimal when links are uniformly distributed or when the network is densely deployed.

#### F. Time Complexity

Assume that

$e$  = number of links,

$k$  = kinds of partitions,

$\#C$  = number of blocks,

$\Delta_{max}$  = maximal number of links in a block.

Theorem 2: Algorithm DIVISION has  $O(ek\#C\Delta_{max})$  time complexity.

Proof: The outer **for** is executed  $k$  times. At each iteration, all the blocks are scanned ( $O(\#C)$  operations), and possibly a link is selected in each block ( $\Delta_{max}$  operations). The **repeat-until** cycle is repeated at most  $O(e)$  time. Hence, the time complexity is  $O(ek\#C\Delta_{max})$ .

Note that when  $l > 2d_{max}$ ,  $k = 4$ .  $\#C$  is bounded by  $(l_0/l)^2$ , where  $l_0$  is the width of the deployment region.  $\Delta_{max} < e$ , hence, time complexity of DIVISION is no more than  $O(e^2)$ .

The computational complexity of algorithm SGLS, GOW\* and DIVISION are shown in Table II.

TABLE II: Representative Computational Complexity

Wireless Network Model	Algorithm	Computational Complexity
SINR graph	SGLS	$O(e^3)$
SINR	GOW*	$O(ek\#C\Delta_{max})$
SINR	DIVISION	$O(ek\#C\Delta_{max})$

### III. NUMERICAL RESULTS

In this section, we compare the performance of one-slot scheduling algorithm among DIVISION, GOW\* and SGLS. The locations of nodes are generated randomly on a  $1000 * 1000$  rectangular deployment area. Communication links are generated when the distance of any two nodes is within maximum transmission range. Detailed parameter settings are shown in Table III. In addition, for a given number of nodes, we evaluate the network performance over 500 randomly generated networks.

We use spatial reuse factor  $\sigma$  and average throughput  $\xi$  to evaluate the performance of scheduling algorithms. Spatial reuse factor  $\sigma$  indicates the average number of concurrent links in a slot, and average throughput  $\xi$  indicates the average total transmission capacity of concurrent links in a slot. The formulas are written as:

$$\sigma = \frac{\sum_{i=1}^C \sum_{j \in S_i} I(\text{SINR}_{i,j} \geq \beta)}{C} \quad (8)$$

$$\xi = \frac{\sum_{i=1}^C \sum_{j \in S_i} R_{i,j}}{C} \quad (9)$$

where  $C$  denotes the number of slots,  $S_i$  the scheduled link set of slot  $i$ ,  $I(\cdot)$  the indication function of SINR condition, and  $R_{i,j}$  the transmission rate of link  $j$  in slot  $i$ , which is computed using Shannon's capacity formula with normalized bandwidth.

Fig.4 gives the evaluation of spatial reuse factor among the algorithms. Compared with SGLS, DIVISION yields 20-30% lower spatial reuse. The SGLS calculates the interference accurately between nodes. It can make full use of the interference margin during the scheduling process. Algorithm DIVISION randomly selects one link in each block, as long as the current link can coexist with the previous scheduled links. It is worthy of note that the spatial reuse of DIVISION is 2-2.5 times of GOW\*. Compared with GOW\*, DIVISION reduces the space separation of concurrent links and makes better use of the space resource. Thus, it yields better performance of spatial reuse than GOW\*.

Fig.5 shows comparison of average throughput among the algorithms. From the simulation results, it is observed that DIVISION can still achieve 90% of SGLS and is 2-2.5 time of GOW\* network throughput. As four-coloring is taken in algorithm GOW\*, links from neighbor blocks are assigned with different slots, which is beneficial to anticipate the interference but reduces the number of concurrent links in a slot. Algorithm DIVISION shares one slot in neighbor blocks, hence, more links may be scheduled in a slot and the total

TABLE III: Parameter Settings

Area of deployment	1000m*1000m
Number of nodes	100-220
Node location	Uniform random distribution
Transmit power $P$	10 mW
Path Loss Factor $\alpha$	4
Thermal noise $N_0$	-90 dBm
SINR threshold	20 dB

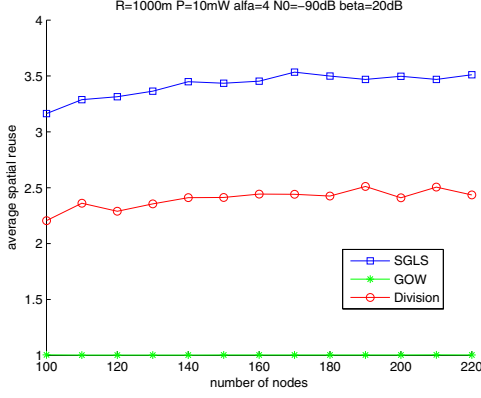


Fig. 4: The evaluation of spatial reuse among the algorithms

length of time is reduced, which leads to the improvement of network throughput.

Fig.6 shows the time complexity among the algorithms. It is observed that algorithm DIVISION and GOW\* have the same order of computational complexity, while both have lower complexity than SGLS. The gap is obvious especially when the number of nodes increases. Considering the complexity, algorithm DIVISION and GOW\*, which are based on the space scheduling, are more practical than SGLS.

To sum up, the simulation results show the trade-off between performance and complexity. Compared with GOW\*, DIVISION improves the spatial reuse and network throughput significantly without increasing computational complexity. Considering the characteristic of low complexity, DIVISION is more competent for the dense wireless networks.

#### IV. CONCLUSION

In this paper, we have investigated the problem of wireless link scheduling under SINR interference model. A novel link scheduling algorithm - DIVISION has been proposed to inspect the space scheduling. The proposed algorithm shows an effect way to use the geometric locations of nodes in wireless networks. Compared with the classical algorithms, it has a lower complexity while sustains better network throughput and spatial reuse. However, It remains a challenge to design an effect scheduling scheme without explicitly using the geometric space locations of links.

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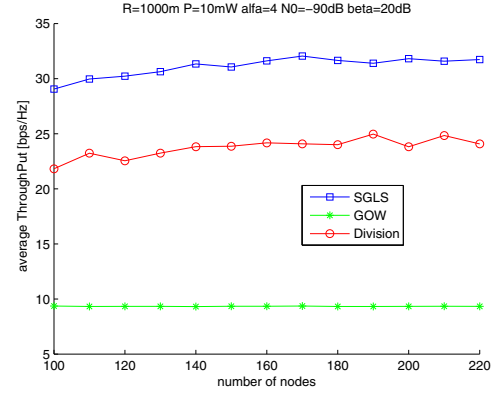


Fig. 5: The evaluation of network throughput among the algorithms

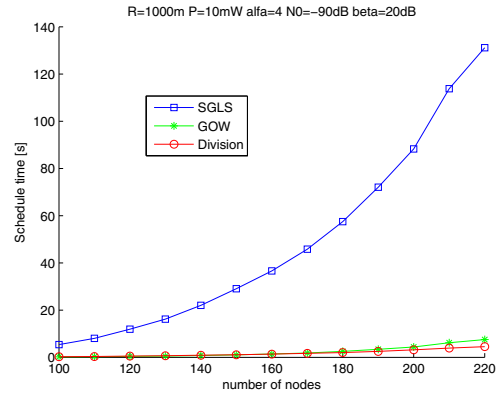


Fig. 6: The evaluation of time complexity among the algorithms

Base Station in LTE-FDD, Supported by MIIT of China, No.2010ZX03002-010-02, and MAC in Asymmetrical Wireless Network, Supported by Education Bureau of Anhui Province, No.KJ2010A333.

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