

# Iterative Blind OFDM Parameter Estimation and Synchronization for Cognitive Radio Systems

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**Abstract**—An iterative design method for Orthogonal Frequency Division Multiplexing (OFDM) system parameter estimation and synchronization under a blind scenario for cognitive radio systems is proposed in this paper. A novel envelope spectrum-based arbitrary oversampling ratio estimator is presented first, based on which the algorithms are then developed to provide the identification of other OFDM parameters (number of subcarriers, cyclic prefix (CP) length). Carrier frequency offset (CFO) and timing offset are estimated for the purpose of synchronization with the help of the identified parameters. An iterative scheme is employed to increase the estimation accuracy. To validate the proposed design, the performance is evaluated under an experimental propagation environment and the results show that the proposed design is capable of adapting blind parameter estimation and synchronization for cognitive radio with improved performances.

## I. INTRODUCTION

With the recent rapid growth in wireless applications and systems, the problem of spectrum utilization has become more critical than ever before. As an emerging solution, Cognitive Radio (CR) systems aim to improve the efficiency of spectrum usage with the principle of sharing the available spectrum resources adaptively. Orthogonal frequency division multiplexing, which has been known to be one of the most effective multicarrier techniques, has attracted significant attention in the development of CR systems due to its high spectral efficiency and flexibility in allocating transmission resources in dynamic environments.

However, the existence of dissimilar wireless transmission schemes poses a challenge to the design of CR receivers that can operate with the multi-waveform signals. Therefore, blind system parameter estimation is of significant importance for reliable communication in CR environments. Furthermore, blind estimation is also helpful to reduce signaling overhead in the case of adaptive transmission where the system parameters change depending on the environmental characteristics or the spectrum availability. The capability of identifying system parameters is necessary for spectrum survey with the purpose of monitoring the systems to discover illegal transmissions as well. In the literature, various blind estimation schemes for CR systems have been presented. These can be classified into two primary categories: cyclostationary characteristics-based [1]–[3] and nonparametric spectrum information-based [4]. Specifically, [1] presented a blind parameter estimation system through the statistical  $\chi^2$  test for the cyclo-period. However, the computational load is comparatively high because of the

exploration of entire cyclic spectrum. Moreover, [2] enhanced the work in [1] by introducing a Sliding Discrete Fourier Transform (SDFT) [3] implementation and [4] proposed an iterative cyclostationary analysis for parameter estimation. Both worked on reducing the computational complexity without considering synchronization offset. [5] investigated the non-parametric characteristics for blind estimation. However, this method does not work well under Nyquist pulse shaping which is closer to a practical implementation. Furthermore, neither of the above provided a performance evaluation under real transmission environments where the propagation behavior is more complex and thus the system may fail to maintain performance due to unknown interferences.

In this paper, a blind parameter estimation and synchronization design is proposed to identify the fundamental parameters of an OFDM system and reduce the interference from the carrier frequency offset (CFO) and timing offset. In order to guarantee that the information from the transmitter is completely obtained, an oversampling technique is carried out. Therefore, the decisive step of the proposed approach is the estimation of this oversampling ratio, for the purpose of capturing the air interface signal accurately. However, given the lack of the prior information and the existence of synchronization offsets, the oversampling ratio can be arbitrary, which makes the design difficult. Hence, an iterative blind parameter estimation and synchronization offset cancellation scheme is proposed in this paper to perform accurate signal sampling and parameters identification. Specifically, the arbitrary oversampling ratio is estimated at first through time-domain envelope spectrum information, based on which the other system parameters, including the number of subcarriers and the cyclic prefix (CP) length, are calculated sequentially. Synchronization is obtained from the estimated parameters and an iterative algorithm is employed to refine the results until a certain threshold is reached. In order to test the effectiveness of the proposed approach, an experimental environment is established. The OFDM signal is generated from a signal generator with different system parameters as well as including CFO and timing offsets. The received signal is then captured by a high speed digitizer for processing by the proposed iterative blind parameter estimation and synchronization algorithm. Test results are provided to evaluate the performance of the proposed approach under various scenarios of system interferences.

The remaining of the paper is organized as follows. Section II introduces the oversampled OFDM system model and Section III explains the arbitrary oversampling ratio estimation through the envelope spectrum. The iterative blind parameter estimation and synchronization approach is proposed in section IV. In Section V, experimental tests are conducted to evaluate the performance of the estimation and synchronization in various scenarios and finally, the paper is concluded in Section VI.

## II. SYSTEM MODEL

Consider an OFDM system with  $N_s$  transmission subcarriers, denoted by  $\{e^{j2\pi \frac{t}{T_s} n}\}_{n=0}^{N_s-1}$ , and is an OFDM symbol period  $T_s$ . Assuming that an appended sequence of length of  $N_g$  ( $N_g < N_s$ ) is inserted at the beginning of each block as a cyclic prefix (CP), the transmitted OFDM signal over  $(0, T_s)$  can be represented as

$$s(t) = \frac{1}{\sqrt{N_s}} \sum_{n=0}^{N_s-1} d_n e^{j \frac{2\pi n t}{T_s}} g_c(t) \quad (1)$$

where  $d_n$  is the complex data transmitted on the  $n$ th subcarrier with  $E[d_n d_n^*] = 1$ , and  $g_c(t)$  is a Nyquist pulse. A raised cosine waveform is taken into consideration in this paper due to its ability to minimize intersymbol interference (ISI) and smooth the sharp edges of baseband signals, which is defined on  $(0, T_s)$  and expressed as

$$g_c(t) = \text{sinc}\left(\frac{t}{T_s}\right) \frac{\cos\left(\pi \alpha \frac{t}{T_s}\right)}{1 - 4\alpha^2 \left(\frac{t}{T_s}\right)^2} \quad (2)$$

where  $\alpha$  stands for the roll-off factor. Denoting  $h(t)$  to be an unknown frequency selective multipath channel, the received time-domain signal is

$$x(t) = \sum_{\lambda} s_{\lambda} h(t - \lambda T_s) + w(t) \quad (3)$$

where  $\lambda$  is the time-domain index for received samples,  $w(t)$  is additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_w^2$ .

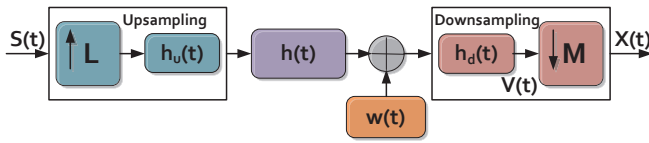


Fig. 1. Illustration of oversampling structure for blind parameter estimation.

As shown in Fig.1, upsampling with a factor  $L$  is used at the transmitter to shape the discrete data into a continuous waveform to be transmitted. An anti-image filter with impulse response  $h_u(t)$  is employed. At the receiver, the signal is downsampled by a factor  $M$  and an anti-aliasing filter,  $h_d(t)$ , is used to reduce distortion. Normally  $M < L$  for the consideration of oversampling and the oversampling ratio  $q$

is defined as  $q = L/M$ . Thus, the corresponding oversampled OFDM signal at the receiver can be denoted as [1]:

$$x(n) = \sum_{\lambda} s_{\lambda} h(n - \lambda q) + w(n) \quad (4)$$

## III. ENVELOPE SPECTRUM-BASED OVERSAMPLING RATIO ESTIMATION

Without considering the interference from the multipath fading and AWGN channels in Fig. 1, we can denote  $h_T(t) = h_u(t) * h_d(t)$  as the combination of the anti-aliasing and anti-image filters, with  $*$  being the convolution operation. Then, according to multirate signal processing theory [6], we have,

$$H_T(f) = \begin{cases} 1, & |f| \leq \min\left(\frac{1}{L}, \frac{1}{M}\right) \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where  $H_T(f)$  is the Fourier transform of  $h_T(t)$ . The time-domain relationship between  $v(t)$  and  $s(t)$  after upsampling can be expressed as:

$$v(k) = \sum_{n=-\infty}^{\infty} h_T(nL + k \oplus L) s\left(\left\lfloor \frac{k}{L} \right\rfloor - n\right) \quad (6)$$

where  $\oplus$  and  $\lfloor \cdot \rfloor$  denote modulo  $L$  and floor operations, respectively. Let  $V(f_1)$  and  $S(f_1)$  be the Fourier transform of  $v(t)$  and  $s(t)$ , we have,

$$V(f_1) = H_T(f_1) S(f_1 L) \quad (7)$$

After the downsampling by a factor  $M$ , the relationship between  $s(t)$  and  $x(t)$  is:

$$\begin{aligned} x(n) &= v(Mk) \\ &= \sum_{n=-\infty}^{\infty} h_T(nL + Mk \oplus L) s\left(\left\lfloor \frac{Mk}{L} \right\rfloor - n\right) \end{aligned} \quad (8)$$

and denoting  $X(f_2)$  as the Fourier transform of  $x(t)$ , the frequency-domain relationship then becomes

$$\begin{aligned} X(f_2) &= \frac{1}{M} \sum_{m=0}^{M-1} V\left(\frac{f_2 - m}{M}\right) \\ &= \frac{1}{M} \sum_{m=0}^{M-1} H_T\left(\frac{f_2 - m}{M}\right) S\left(\frac{(f_2 - m)L}{M}\right) \end{aligned} \quad (9)$$

Since  $f_2 = Mf_1$ , we have,

$$X(f_2) = \frac{1}{M} \sum_{m=0}^{M-1} H_T\left(f_1 - \frac{m}{M}\right) S\left(f_1 L - \frac{mL}{M}\right) \quad (10)$$

From the derivation in [6], after analyzing the properties of  $H_T(f_1)$ , it can be found that when  $m \neq 0$ , we have,

$$H_T\left(f_1 - \frac{m}{M}\right) = 0 \quad (11)$$

hence, when  $|f_2| = M|f_1| \leq \min\left\{\frac{1}{2}, \frac{1}{2q}\right\}$ ,

$$X(f_2) = \frac{1}{M} S(f_1 L) = \frac{1}{M} S\left(f_2 \frac{L}{M}\right) = \frac{1}{M} S(qf_2) \quad (12)$$

Based on the above relationship, a peak can be observed at frequency component  $1/2q$  of  $X(f_2)$ . In order to separate this peak from the frequency band of OFDM signal, the time-domain envelope spectrum is taken into consideration which can be described as follows.

Consider the time-domain relationship:  $y(n) = |x(n)|^2$ . Then the Fourier transform of  $y(n)$  is the envelope spectrum and thus, the frequency-domain relationship is:

$$Y(f_2) = X(f_2) * X(-f_2) \quad (13)$$

and the desired peak can be observed at the frequency component  $1/q$  of  $Y(f_2)$ . If the Fast Fourier Transform (FFT) is applied, this relation becomes,

$$Y(k) = X(k) * X^*(N - k) \quad (14)$$

where  $x^*(n)$  and  $X^*(k)$  represent the complex conjugate operation. The desired frequency component is located at index  $k = N/q$  with  $N$  being the number of FFT points.

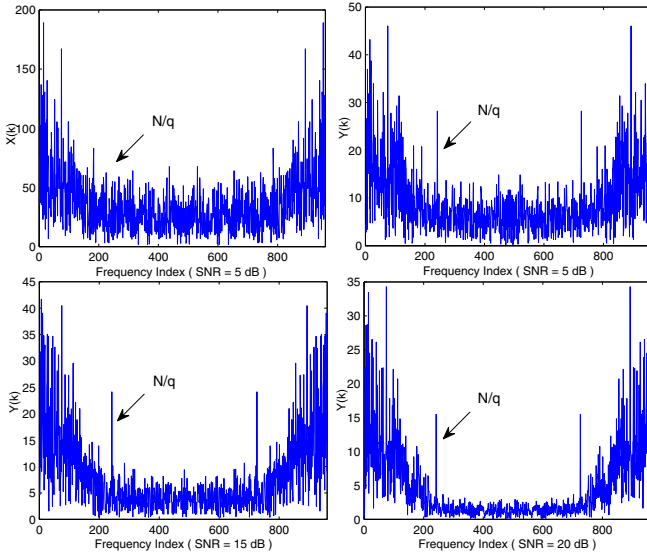


Fig. 2. Spectrum information of oversampled OFDM signal ( $q = 4$ ): (a)  $X(k)$  under 2 multipath fading channels and SNR = 5 dB; (b)  $Y(k)$  under 2 multipath fading channels and SNR = 5 dB; (c)  $Y(k)$  under 2 multipath fading channels and SNR = 15 dB; and (d)  $Y(k)$  under 2 multipath fading channels and SNR = 20 dB.

Fig. 2 shows the envelope spectrum of  $x(n)$  as  $Y(k)$ . From the above analysis, index  $k$  after FFT, to indicate oversampling ratio  $q$ , is located at  $N/q$  as depicted for the different Signal-to-Noise Ratio (SNR) scenarios. Since  $Y(k)$  has a symmetric pattern with a series of peaks, the peak detection algorithm is employed to find the peak closest to the spectrum center. If it is  $\tilde{k}$ , then the estimated oversampling factor  $\tilde{q}$  is:

$$\tilde{q} = \frac{N}{\tilde{k}} \quad (15)$$

#### IV. JOINT ITERATIVE BLIND PARAMETER ESTIMATION AND SYNCHRONIZATION

Through (15), the oversampling frequency at the transmitter is obtained and the received signal is sampled accordingly.

The next step is to estimate the other OFDM parameters and synchronize the received signals. The number of subcarriers and the CP length are estimated in this paper based on the approaches proposed by [4].

##### A. Estimation of the Number of Subcarriers and CP Length

For an OFDM symbol with  $N_s$  subcarriers, the estimated autocorrelation function  $\hat{R}_x(n, \tau)$  of the oversampled signal can be written as

$$\hat{R}_x(n, \tau) = \frac{1}{P} \sum_{n=0}^{P-1} x(n)x^*(n - \tau) \quad (16)$$

where the delay  $\tau \in [0, 1, \dots, P - 1]$  with  $P$  being the length of oversampled incoming signal. The absolute value of  $\hat{R}_x(\tau)$ , over the oversampled signal, is

$$|\hat{R}_x(\tau)| = \begin{cases} \sum_{n=0}^P |h(n)|^2 + \sigma_w^2, & \tau = 0 \\ \sum_{n=0}^P |h(n)|^2 \frac{N_g}{N_f} + \sigma_w^2, & \tau = N_b \end{cases} \quad (17)$$

where  $h(n)$  is the channel impulse response,  $N_g$  is the CP length,  $N_b$  is the OFDM symbol length of the oversampled signal with  $N_b = qN_s$  and  $N_f = N_s + N_g$ .  $|\hat{R}_x(\tau)|$  will have the most significant peak when  $\tau = N_b$ . Therefore, through peak detection, the estimated number of subcarriers can be obtained as:

$$\tilde{N}_s = \frac{N_b}{\tilde{q}} \quad (18)$$

The estimation of the CP length  $N_g$ , is accomplished by the cyclic autocorrelation which is given by

$$R_x^k(\tau) = \frac{1}{P} \sum_{n=0}^{P-1} R_x(n, \tau) e^{-j \frac{2\pi}{P} kn} \quad (19)$$

Using the estimated number of subcarriers  $\tilde{N}_s$ , we can obtain the cyclic autocorrelation with delay  $\tau = \tilde{N}_s$ . The cyclic prefix length,  $N_g$ , can be determined through peak detection on  $R_x^k$  at  $\tau = \tilde{N}_s$ . If we assume that  $k_{opt}$  represents the first peak with the smallest index above the threshold from the cyclic autocorrelation then, the estimated length  $\tilde{N}_f$  can be identified as,

$$\tilde{N}_f = \frac{2P}{k_{opt}} \quad (20)$$

and the CP length  $\tilde{N}_g$  is determined as:  $\tilde{N}_g = \tilde{N}_f - \tilde{N}_s$ .

##### B. Estimation of Carrier Frequency Offset and Timing Offset

For synchronization, the problem can be modeled as follows (no oversampling):

$$x(n) = e^{j(2\pi f_e n)} s(n - n_e) * h(n) + w(n) \quad (21)$$

where  $f_e$  is the CFO and  $n_e$  is the timing offset, both of which have an impact on blind OFDM parameter estimation. If  $N_s$  is provided as prior information and the received signal is perfectly sampled, the estimation of  $n_e$  and  $f_e$  can be solved through the Maximum Likelihood (ML) algorithm proposed by [7]. Therefore, in our case, we consider  $N_b$  as the number

of subcarriers for the oversampled OFDM symbol and  $f'_e$  and  $n'_e$  are assumed to be the CFO and timing offset respectively. The Log Likelihood estimator is given by [7],

$$\lambda(n'_e, f'_e) = |\hat{R}_x(n'_e, N_b)| \cos \left( \frac{2\pi f'_e}{N_b} + \angle(\hat{R}_x(n'_e, N_b)) \right) - \rho \Phi(n'_e) \quad (22)$$

where  $\hat{R}_x(n, N_b)$  is the autocorrelation operation in (16) when  $\tau = N_b$ ,  $\angle$  represents the argument of a complex sequence, and

$$\Phi(n) = \frac{1}{2} \sum_{k=n}^{n+P-1} [|x(k)|^2 + |x(k + N_b)|^2] \quad (23)$$

$$\rho = \frac{|E\{x(n)x^*(n + N_b)\}|}{E\{|x(n)|^2\}E\{|x(n + N_b)|^2\}}$$

The ML estimation of  $n'_e$  and  $f'_e$  is given by

$$\hat{n}'_e = \arg_{max} \{ |\hat{R}_x(n'_e, N_b)| - \rho \Phi(n'_e) \} \quad (24)$$

$$\hat{f}'_e = -\frac{\angle \hat{R}_x(\hat{n}'_e, N_b)}{2\pi} + n$$

The above estimator works well under the flat fading channel scenario, accurate sampling as well as the prior information of OFDM system parameters. However, under the blind estimation scenario and when the multipath fading channel is considered, the estimator is not optimum. Nevertheless, the above results can be utilized as initial guesses for both the carrier frequency and timing offsets to refine the estimation accuracy of the OFDM parameters in the previous steps. Therefore, an iterative scheme is employed to improve the performance of parameter estimation and synchronization jointly, which is described below:

- 1) Obtain the initial oversampling ratio  $\tilde{q}$  by setting a downsampling factor  $M$ .  $\tilde{q}$  is calculated through (15).
- 2) Estimate the number of subcarriers  $\tilde{N}_s$  and CP length  $\tilde{N}_g$  through (18) and (20) based on the oversampling ratio  $\tilde{q}$ .
- 3) Realize the synchronization from the estimation of the CFO  $\hat{f}'_e$  and  $\hat{n}'_e$  based on (24).
- 4) Recover the oversampled signal  $x(n)$  with the estimated  $\hat{f}'_e$  and  $\hat{n}'_e$ . Repeat Steps 1), 2) and 3) to refine the estimation performance in each step and improve the accuracy. If the estimated oversampling ratio at  $j - 1$ ,  $j$  and  $j + 1$  steps are  $\tilde{q}^{(j-1)}$ ,  $\tilde{q}^{(j)}$  and  $\tilde{q}^{(j+1)}$ , the final estimation result is achieved if the following conditions are met

$$|\tilde{q}^{(j+1)} - \tilde{q}^{(j)}| < |\tilde{q}^{(j)} - \tilde{q}^{(j-1)}| < \gamma \quad (25)$$

where  $\gamma$  is the level of acceptance.

## V. EXPERIMENTAL RESULTS

### A. Lab Testing Platform

To validate the algorithm, experimental testing has been done through a lab testing platform which consists of a vector signal generator, a FSP spectrum analyzer and a high speed

digitizer. In our experiments, a R&S<sup>®</sup> SMJ100A vector signal generator is used to generate the OFDM signals, as defined by the proposed blind parameter estimation and synchronization approach. The SMJ100A supports a wide range of possible input values ranging from 50 mV to 30 V as well as software-selectable 50  $\Omega$  or 1 M $\Omega$  input impedance. To intercept the transmitted OFDM signals, a NI PXI 5105 high speed digitizer is used, which is capable of capturing 8 simultaneous channels of data at a rate of 60 Msamples per second with a resolution of 12 bits. This high speed digitizer is inserted into a NI PXI-1031 4-Slot 3U Chassis, a high-power PXI chassis with reduced acoustic noise emission and enhanced cooling capacity. In the experiments, the output of the SMJ100A is wired into the input of Channel 0 of the PXI 5105 digitizer. The generated signal is captured, sampled and saved at the PXI 5105 digitizer through the NI LABVIEW interface. The blind system parameter estimation and synchronization is performed in MATLAB after loading the saved data from the PXI 5105.

TABLE I  
SYSTEM PARAMETERS FOR THE GENERATED OFDM SIGNALS.

Data Source	PRBS
Modulation Type	16QAM
Symbol Rate	0.25 Mbps
PSDU Rate	24 Mbps
Sequence Length	1024 bytes
Number of Data Subcarriers	48
Number of Pilot Subcarriers	4
Subcarriers Frequency Spacing	0.3125 MHz
Impulse Filter	Raised-Cosine Filter
Impulse Length	32
Roll-off Factor	0.5

The parameters employed for OFDM signal generation are listed in Table I. The impairments are also introduced using the SMJ100A with a Rayleigh fading channel scenario and different SNRs, CFO and timing offset levels.

### B. Experimental Results

The MSE of the iterative oversampling ratio estimation is exhibited in Fig.3 using the lab testing platform. The Mean-Square Error (MSE) is defined as

$$\text{MSE} = |\tilde{x} - x|^2, \quad (26)$$

where  $\tilde{x}$  is the estimated value and  $x$  is the original one. The parameters of the OFDM system are taken from Table I. The CFO is set to  $f_e = 20$  ppm and the timing offset is  $n_e = 10T_s$ . A multipath channel is assumed with Rayleigh fading, where the randomly generated channel coefficients are  $\{h(n)\}_{n=0}^{N_L-1}$  with length  $N_L$ . The channels have an exponentially decaying phase delay profile, comprised of four complex Gaussian distributed taps with average power  $\sigma_h^2 = E[|\sum_n h_n|^2]$ . From the experimental results, it can be seen that low MSE is achieved with the proposed algorithms and that iterations can improve the estimation accuracy efficiently. Moreover, the experimental performance demonstrates the estimation ability of the proposed approach on both integer and rational oversampling ratio scenarios.

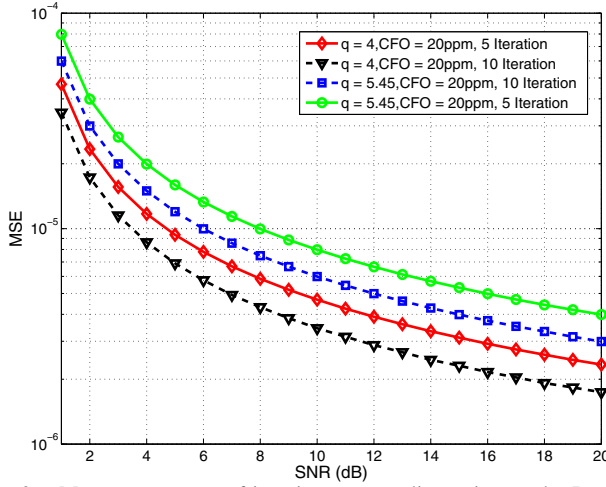


Fig. 3. Mean square error of iterative oversampling ratio  $q$  under Rayleigh multipath fading channel through lab testing platform under different SNR levels.

Fig.4 illustrates the performance of the introduced iterative approach for the estimation of the number of subcarriers and the CP length. The oversampling ratio used in the simulation is  $q = 5.45$ , and  $f_e = 20 \text{ ppm}$ ,  $n_e = 10T_s$ . The performance is analyzed under the same multipath channel environment as above for different SNR levels. From these results, it is noted that the performance is satisfying and that iterations is able to increase the estimation accuracy of both parameters.

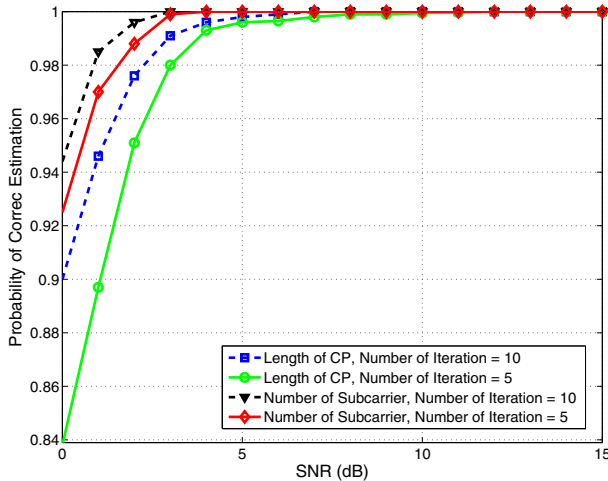


Fig. 4. Probability of correct estimation of number of subcarriers and CP length under Rayleigh multipath fading channel through lab testing platform for different SNR levels.

In order to validate the effectiveness of the proposed approach, the bit error rate (BER) of the received signal is also evaluated. We assume that the receiver has perfect knowledge of the multipath fading channel and the modulation scheme. The OFDM parameters are the same as above. Three levels of CFO are considered:  $f_e = 12 \text{ ppm}$ ,  $20 \text{ ppm}$ , and  $28 \text{ ppm}$ . The results are evaluated separately for 5 and for 10 iterations. Fig. 5, gives the performance results of proposed approach for data recovery and validates the effectiveness of the iterative scheme.

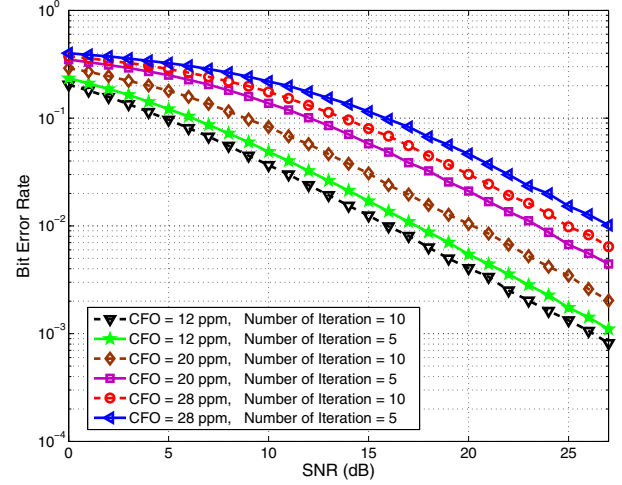


Fig. 5. Bit error rate of the proposed joint blind parameter estimation and synchronization approach under various SNR levels through lab testing platform.

## VI. CONCLUSION

In this paper, we propose a joint iterative blind OFDM parameter estimation and synchronization approach for cognitive radio (CR) systems to operate with multi-waveform signals. The paper begins with introducing the system model for over-sampled OFDM signals and proposes the envelope spectrum-based arbitrary oversampling ratio estimation. According to the oversampling ratio, the number of subcarriers and the cyclic prefix (CP) length are identified as well, all of which are used to provide the necessary information for carrier frequency offset and timing offset estimation. The iterative scheme is then explained: it refines the estimation results and increase the accuracy at every step. The proposed approach is evaluated using a lab testing platform. From the experimental results, we can see good performance on blind parameter estimation and synchronization under the presence of a Rayleigh multipath fading channel and additive noise. Moreover, the bit error rate (BER) for data recovery is satisfying, assuming prior information on the multipath fading channel and the modulation scheme.

## REFERENCES

- [1] M. Shi, Y. Bar-Ness and W. Su, "Blind OFDM systems parameters estimation for software defined radio," in *Proc. of IEEE Intl. Symp. on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Apr. 2007, pp. 119 - 122.
- [2] Q. Chen, X. Wang, P. Ho and Y. Wu, "Progressive automatic detection of OFDM system parameters for universal mobile DTV receiver," in *IEEE Vehi. Tech. Conf. Fall (VTC 2010-Fall)*, Oct. 2010, pp.1 - 5.
- [3] E. Jacobsen, R. Lyons, "The Sliding DFT," *IEEE Signal Processing Magazine*, vol.20, no. 2, pp. 74-80, Mar. 2003.
- [4] J.-G. Liu, X. Wang, J. Nadeau and P. Ho, "Blind parameter estimation for OFDM interception receiver with iterative cyclostationary analysis," in *Proc. IEEE Intl. Conf. for Military Conf. (MILCOM)*, Nov. 2011, pp. 2211 - 2215.
- [5] V. L. Nir, T. V. Waterschoot, M. Moonen and J. Duplacy, "Blind CP-OFDM and ZP-OFDM parameter estimation in frequency selective channels," *EURASIP J. on Wireless Commu. and Networking*, Vol. 2009, Jan. 2009.
- [6] F.-J. Harris, *Multirate Signal Processing*. Upper Saddle River, NJ: Prentice-Hall, 2004.
- [7] J.-J. van de Beek, M. Sandell, P.-O. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. on Signal Processing*, vol. 45, issue 7, pp. 1800 - 1805, July 1997.