Design of Hierarchical Modulation for Wireless Relay Networks

Tung T. Pham and Ha H. Nguyen
Department of Electrical & Computer Engineering, University of Saskatchewan
57 Campus Dr., Saskatoon, SK, S7N 5A9, Canada, tung.pham@usask.ca, ha.nguyen@usask.ca

***Abstract*—Two designs of un-uniformed constellations for wireless relay networks are considered. The first design is concerned with the unequal error protection issue, in which two data streams to be transmitted from a source to a destination are protected at two different levels. Based on a simple approximation of the average bit-error-rate (BER), a design parameter is obtained in a closed form. The second design focuses on minimizing the average BER of the combined data stream at the destination. Developed are two near optimal designs for $4/16$-QAM and $4/64$-QAM hierarchical modulation schemes. Simulation results confirm the superiority of the proposed designs as well as the advantages of the relay-assisted transmission with a hierarchical modulation over the conventional point-to-point transmission.**

I. Introduction

The concept of hierarchical modulation (HM) has been introduced in [1] in which information messages are divided into two or more streams with different degrees of protection. In such an application, the most important information (also called base stream) is retrieved by all receivers while the less important information (or refinement stream) can be received by some receivers with better reception conditions (e.g., near the transmitter, having good channels). In wireless cellular networks, the quality of signal reception also depends on the channels between the base station and the mobile terminals it serves. While receiving the same information from the base station, some mobile terminals can help to improve the signal quality at other mobile terminals located far away from the base station by forwarding a version of the signal of interest to those mobile terminals. As such, a relay network is formed [2]. Naturally, the similar properties of hierarchical modulation and relay network can be combined in order to further improve the quality of transmission.

Indeed, cooperative communications with HM has recently gained a lot of research interests (see e.g., [3]–[7]). Such a combination has been shown to provide a significant performance improvement [3], [7]. Focusing on unequal error protection for the two data streams, the authors in [5] propose an adaptive relaying scheme in order to improve the bit-error-rate (BER) of the less-protected data stream while maintaining the required BER of the other. Investigating the optimal non-uniform modulation schemes has not been considered in [3], [5], [7]. For a relay network with one source, one relay, and one destination, the authors in [4] derive the BERs for both the base and refinement streams when both source and relay employ HM. However, due to the complexity of the BER expressions, the optimal constellations can only be numerically obtained. Furthermore, since no benchmark model was compared, it is not clear whether and how the relay-assisted transmission with HM can always offer performance advantages over the conventional point-to-point transmission.

This paper also considers applying hierarchical modulation in wireless relay networks. Instead of forwarding both the base and refinement streams as in [4], we simplify the design by allowing the relay to forward only the refinement stream. Two design problems are formulated, which are unequal error protection and BER minimization. Based on a tight approximation of the BER provided in [8], the optimal designs of the HM constellations for the first problem is obtained. The feasibility of the problem is also analyzed, which suggests how to choose the suitable location for the relay or select the best relay when multiple relays are available, as well as to accommodate the data into the base and refinement streams to realize the performance advantage of relay-assisted transmission. In the second problem, we analyze the average BER of the combined stream and compare our designs with the conventional point-to-point transmission model while maintaining the same total transmit power and throughput (i.e., the number of bits per channel use) of the two transmission models under consideration. Especially, the "region of advantageous relaying" is identified, which clearly determines when relay-assisted transmission is advantageous over the conventional piont-to-point transmission. Based on the properties of the average BER curves of the base and refinement streams, a near optimal solution is proposed, which performs very close to the optimal design.

II. System Model and the average BER

Consider a relay-assisted transmission model consisting of a source node A, a destination node B, and a relay node R. To fulfill the two main purposes of such a model, namely, to provide different protection degrees to the base and refinement streams, or to improve the error performance of the combined data stream, $4/M$-QAM hierarchical modulation (HM) schemes with $M = 16$ and 64 are employed. As shown in Fig. 1, node A transmits a combined data stream using $4/M$-QAM HM to nodes B and R in the first phase. Specifically, the combined stream consists of a 4-QAM base stream and a $\frac{M}{4}$-QAM refinement stream. In the second phase, node R decodes and forwards only the refinement stream with a conventional $\frac{M}{4}$-QAM to node B. Node B decodes the base stream received

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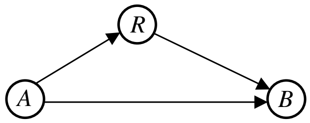

Fig. 1. A relay-assisted transmission model.

in the first phase and the refinement stream in the second phase in order to retrieve the complete information.

An example of $4/16$-QAM HM is illustrated in Fig. 2. The base bits can be viewed as the ones to be modulated to the virtual 4-QAM symbols at the centers of the four symbols in four quadrants. The refinement bits can be viewed as the ones to be modulated to the virtual 4-QAM symbols in one quadrant. Such a constellation can be defined by three distance parameters: d_1, d_2 and d_3. These parameters are related to the so-called the constellation priority parameter $\lambda = d_2/d_3 = d_2/(d_1 - d_2)$. Then finding the optimal HM constellation is equivalent to finding the optimal value of λ.

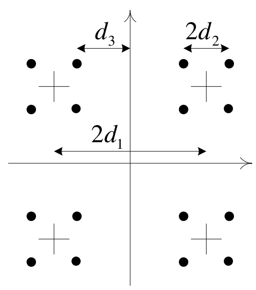

Fig. 2. $4/16$-QAM hierarchical modulation.

Each node in Fig. 1 is equipped with $n_T = 2$ transmit antennas and n_R receive antennas, and it uses the Alamouti space-time block code (STBC) [9]. Let $N = n_T n_R$.[1] All channels are assumed to undergo Rayleigh flat fading with unit variance while their attenuations follow the free-space model with path-loss exponent ν [9]. Channels between node i and node j are denoted by $\mathbf{H}_{i,j}$ where $i, j = \{A, B, R\}$. The average received signal-to-noise ratio (SNR) per symbol associated with channel $\mathbf{H}_{i,j}$ is $\gamma_{i,j} = P_i \sigma_{i,j}^2 / n_T \sigma_n^2$, where P_i is the transmit power of each symbol at the the ith node, $\sigma_{i,j}^2 = d_{i,j}^{-\nu}$ is the attenuation of the $i-j$ channel with distance $d_{i,j}$, and σ_n^2 is the variance of the complex additive white Gaussian noise, which, for simplicity, is assumed to be the same at all receivers.

Thanks to the orthogonality of the Alamouti STBC, the instantaneous SNR per symbol received over the $i-j$ channel is $\gamma_{i,j}||\mathbf{H}_{i,j}||_F^2$. Since the elements of $\mathbf{H}_{i,j}$ are i.i.d. circularly symmetric complex Gaussian random variables of unit variance, the pdf of the squared Frobenius norm $||\mathbf{H}_{i,j}||_F^2$ can be expressed as [9]:

$$f(x) = \frac{x^{N-1}}{(N-1)!} \mathrm{e}^{-x}, \ x > 0. \tag{1}$$

[1]The analysis and designs in this paper are also directly applicable for a single-antenna system, i.e., $N = n_T = n_R = 1$.

For a $4/16$-QAM HM scheme, the average BERs of the base (b) and refinement (r) streams received by the jth node in the first phase are [10]:

$$p_{A,j}^{(b)}(\lambda) = \tag{2}$$
$$\frac{1}{2}\int_0^\infty \left[Q\left(\sqrt{t_1(\lambda)\gamma_{A,j}x}\right) + Q\left(\sqrt{t_2(\lambda)\gamma_{A,j}x}\right)\right] f(x)\mathrm{d}x$$

$$p_{A,j}^{(r)}(\lambda) = \frac{1}{2}\int_0^\infty \left[2Q\left(\sqrt{t_3(\lambda)\gamma_{A,j}x}\right)\right. \tag{3}$$
$$\left. +Q\left(\sqrt{t_4(\lambda)\gamma_{A,j}x}\right) - Q\left(\sqrt{t_5(\lambda)\gamma_{A,j}x}\right)\right] f(x)\mathrm{d}x$$

where

$$t_1(\lambda) = \tfrac{1}{(1+\lambda)^2+\lambda^2}, \ t_2(\lambda) = \tfrac{(1+2\lambda)^2}{(1+\lambda)^2+\lambda^2},$$
$$t_3(\lambda) = \tfrac{\lambda^2}{(1+\lambda)^2+\lambda^2}, \ t_4(\lambda) = \tfrac{(2+\lambda)^2}{(1+\lambda)^2+\lambda^2}, \ t_5(\lambda) = \tfrac{(2+3\lambda)^2}{(1+\lambda)^2+\lambda^2}.$$

For the second phase, the average BER of the refinement stream (now using a conventional 4-QAM) received by node B is [9]:

$$p_{R,B}^{(r)} = \left(\frac{1-\beta_{R,B}}{2}\right)^N \sum_{j=0}^{N-1} \begin{pmatrix} 3+j \\ j \end{pmatrix} \left(\frac{1+\beta_{R,B}}{2}\right)^j \tag{4}$$

where $\beta_{R,B} = \sqrt{\frac{\gamma_{R,B}}{1+\gamma_{R,B}}}$.

Therefore, the average BERs of the two streams at node B can be computed as [2]

$$p^{(b)}(\lambda) = p_{A,B}^{(b)}(\lambda), \tag{5}$$
$$\begin{aligned} p^{(r)}(\lambda) &= p_{A,R}^{(r)}(\lambda)\left[1 - p_{R,B}^{(r)}\right] + p_{R,B}^{(r)}\left[1 - p_{A,R}^{(r)}(\lambda)\right] \\ &= p_{A,R}^{(r)}(\lambda)\left[1 - 2p_{R,B}^{(r)}\right] + p_{R,B}^{(r)} \end{aligned} \tag{6}$$

Moreover, when the M_1/M_2-QAM HM is employed, one can calculate the average BER of the combined stream as follows:

$$f(\lambda) = \log_{M_2}(M_1) p^{(b)}(\lambda) + \log_{M_2}\left(\frac{M_2}{M_1}\right) p^{(r)}(\lambda). \tag{7}$$

It is noted that the average BER of the base stream is not necessarily better than that of the refinement stream. The overall performance of the two streams depends on the transmit power at nodes A and R as well as the qualities of the $A-B$, $A-R$ and $R-B$ channels. In this paper, we aim to improve the BER performance without any spectral loss. This shall be illustrated by comparing the BER of the model under consideration with the BER of the conventional transmission model. In particular, the relay system using $4/16$-QAM HM is compared with the conventional model using 4-QAM (i.e, 2 bits per channel use). Similarly, the relay system with $4/64$-QAM HM is compared with the conventional 8-QAM (i.e., 3 bits per channel use).

In order to minimize the BER of each stream and/or the average BER of the combined stream, it is obvious that both node A and node R have to transmit at their maximum power levels. Therefore, the two BER minimization problems can be formally stated as follows:

$$(\mathcal{P}1): \lambda_{\rm opt} = \arg\min_{\lambda \geq 0} \; p^{(i)}(\lambda) \quad \text{s.t. } p^{(i')}(\lambda) \leq \bar{P}^{(i')}, \; \{i, i'\} = \{b, r\} \tag{8}$$

$$(\mathcal{P}2): \lambda_{\rm opt} = \arg\min_{\lambda \geq 0} \; f(\lambda) \text{ given in (7).} \tag{9}$$

Inspecting (2), (3), (5) and (6) shows that the optimal $\lambda_{\rm opt}$ can only be computed numerically. In the following, with a simple approximation, the optimal value $\lambda_{\rm opt}$ shall be obtained in a closed form. Such a closed-form solution allows us to easily obtain the system design parameters or to perform operations such as relay selection in dynamic relay networks.

III. Problem ($\mathcal{P}1$) - Unequal Error Protection

The procedures to find closed-form solutions to problem ($\mathcal{P}1$) with both $4/16$ and $4/64$-QAM HM are similar. In the following, we proceed with $4/16$-QAM HM.

The average BER of M-ary modulated data transmission over a MIMO Rayleigh fading channel can be computed with the following classical expression [11]:

$$I_N(\gamma_{i,j}) \triangleq \int_0^\infty Q\left(\sqrt{\gamma_{i,j} x}\right) f(x) \mathrm{d}x = \frac{1}{\pi} \int_0^{\pi/2} \frac{\sin^{2N}\theta \mathrm{d}\theta}{(\sin^2\theta + \gamma_{i,j})^N}$$

which can be tightly lower bounded by [8]

$$I_N\left(\gamma_{i,j}\right) \gtrsim \frac{S_N}{\left(C_N + \gamma_{i,j}\right)^N} \tag{10}$$

where $S_N = \frac{1}{2}\frac{(2N-1)!!}{(2N)!!}$, and $C_N = \frac{S_{N+1}}{S_N} = \frac{2N+1}{2N+2}$.

Using (10), the average BERs (2) and (3) can be approximated by their leading terms as

$$p_{A,j}^{(b)}(\lambda) \gtrsim \frac{S_N}{2\left[C_N + \frac{\gamma_{A,j}}{(1+\lambda)^2+\lambda^2}\right]^N} \triangleq \tilde{p}_{A,j}^{(b)}(\lambda), \tag{11}$$

$$p_{A,j}^{(r)}(\lambda) \gtrsim \frac{S_N}{\left[C_N + \frac{\lambda^2\gamma_{A,j}}{(1+\lambda)^2+\lambda^2}\right]^N} \triangleq \tilde{p}_{A,j}^{(r)}(\lambda). \tag{12}$$

Since[2] $\nabla_\lambda \tilde{p}_{A,j}^{(b)}(\lambda) > 0$ and $\nabla_\lambda \tilde{p}_{A,j}^{(r)}(\lambda) < 0$, $\forall\lambda > 0$, $\tilde{p}_{A,j}^{(b)}(\lambda)$ and $\tilde{p}_{A,j}^{(r)}(\lambda)$ are monotonically increasing and decreasing with $\lambda \geq 0$, respectively. As a result, problem ($\mathcal{P}1$) reduces to finding the optimal $\lambda_{\rm opt}$ such that $p^{(i')}(\lambda_{\rm opt}) = \bar{P}^{(i')}$. Even though obtaining $\lambda_{\rm opt}$ can be done by simply calculating the roots of quadratic equations, a detailed analysis will clearly show the feasible region of $\bar{P}^{(i')}$. Such information helps to obtain a more accurate approximation of the average BER of the base stream, which facilitates to find a near-optimal solution to problem ($\mathcal{P}2$).

[2]The notation ∇_λ means derivative of a function with respect to λ.

<u>Case 1</u>: $p^{(b)}(\lambda) = \bar{P}^{(b)}$. The optimal value $\lambda_{\rm opt}$ can be found to be the solution of the quadratic equation $\tilde{p}_{A,B}^{(b)}(\lambda_{\rm opt}) = \bar{P}^{(b)}$. Equivalently,

$$2\lambda_{\rm opt}^2 + 2\lambda_{\rm opt} + 1 - \frac{\gamma_{A,B}}{\left[S_N/\left(2\bar{P}^{(b)}\right)\right]^{1/N} - C_N} = 0. \tag{13}$$

Since $\Delta' = \frac{2\gamma_{A,B}}{\left[S_N/\left(2\bar{P}^{(b)}\right)\right]^{1/N} - C_N} - 1 \geq 0$ when $\bar{P}^{(b)} \geq \frac{S_N}{2(C_N + 2\gamma_{A,B})^N}$, it follows that

$$\lambda_{\rm opt} = \frac{1}{2}\left(\sqrt{\frac{2\gamma_{A,B}}{\left[S_N/\left(2\bar{P}^{(b)}\right)\right]^{1/N} - C_N} - 1} - 1\right). \tag{14}$$

It is noted that when $\lambda \to 0$, the base stream becomes a conventional 4-QAM transmitted over $A-B$ channel with the following average BER:

$$p_{A,B}^{(4-\mathrm{QAM})} = \left(\frac{1-\beta_{A,B}}{2}\right)^N \sum_{j=0}^{N-1} \binom{3+j}{j} \left(\frac{1+\beta_{A,B}}{2}\right)^j \gtrsim \frac{S_N}{2(C_N+\gamma_{A,B})^N} \geq \frac{S_N}{2(C_N + 2\gamma_{A,B})^N} \tag{15}$$

where $\beta_{A,B} = \sqrt{\frac{\gamma_{A,B}}{1+\gamma_{A,B}}}$. As a result, the feasible region of $\bar{P}^{(b)}$ is

$$\bar{P}^{(b)} \geq p_{A,B}^{(4-\mathrm{QAM})}. \tag{16}$$

Since $\tilde{p}_{A,B}^{(b)}(\lambda_{\rm opt}) \leq p_{A,B}^{(4-\mathrm{QAM})}$ when

$$\lambda \leq \lambda_c = \frac{1}{2}\left(\sqrt{\frac{2\gamma_{A,B}}{\left[\frac{S_N}{2p_{A,B}^{(4-\mathrm{QAM})}}\right]^{1/N} - C_N} - 1} - 1\right), \tag{17}$$

a more accurate approximation of $p^{(b)}(\lambda)$ is

$$p^{(b)}(\lambda) \approx \begin{cases} p_{A,B}^{(4-\mathrm{QAM})}, & \lambda < \lambda_c \\ \tilde{p}_{A,B}^{(b)}(\lambda), & \lambda \geq \lambda_c \end{cases} \tag{18}$$

As mentioned, λ_c will be used to find a near-optimal solution to the second problem ($\mathcal{P}2$).

<u>Case 2</u>: $p^{(r)}(\lambda) = \bar{P}^{(r)}$. Following the same way, the optimal value $\lambda_{\rm opt}$ can be found to be

$$\lambda_{\rm opt} = \frac{1}{\sqrt{\gamma_{A,R}/\beta - 1} + 1}. \tag{19}$$

where $\beta = \left[\frac{\left(1-2p_{R,B}^{(r)}\right)S_N}{\bar{P}^{(r)} - p_{R,B}^{(r)}}\right]^{1/N} - C_N$ and the feasible region of $\bar{P}^{(r)}$ in this case is

$$\frac{\left(1-2p_{RB}^{(r)}\right)S_N}{\left(C_N + \frac{\gamma_{A,R}}{2}\right)^N} + p_{R,B}^{(r)} \leq \bar{P}^{(r)} \leq \frac{\left(1-2p_{R,B}^{(r)}\right)S_N}{\left(C_N\right)^N} + p_{R,B}^{(r)}.$$

It should be emphasized that different ways of accommodating data into two streams result in different BERs. Clearly determining the feasible regions of both $\bar{P}^{(b)}$ and $\bar{P}^{(r)}$ is, therefore, needed in order to achieve the largest improvement in such a design problem.

IV. PROBLEM ($\mathcal{P}2$) - BER MINIMIZATION

First, we set up a simulation scenario in which the location of relay R lies in the line between nodes A and B in order to illustrate the dependency of the average BER of the combined stream on the associated channels amongst A, B, and R. The distance between A and B is normalized to 1 while the distance between A and R is set to κ. Note that κ affects the average SNRs, $\gamma_{A,R}$ and $\gamma_{R,B}$. The contours of $\mathrm{BER}(\lambda, \kappa)$ are plotted in Fig. 3 for $4/16$-QAM HM with different values of $\mathrm{SNR} = \frac{P_A}{\sigma_n^2} = \frac{P_R}{\sigma_n^2}$. Inside each curve is the area in which the relay-assisted transmission yields a lower BER than the direct transmission. Given a value of κ, the function $f(\lambda)$ has only one minimum point due to the monotonicity of $p^{(b)}(\lambda)$ and $p^{(r)}(\lambda)$ with respect to λ. Thus, any one-dimensional searching method can be implemented to numerically find the optimal point. As an alternative, we shall compute a near-optimal point so that the performance of the system can be quickly and conveniently evaluated.

The form of $f(\lambda)$ given in (9) suggests that the intersection point, λ_i, which sets $\log_2 M_1 p^{(b)}(\lambda_i) = \log_2\left(\frac{M_2}{M_1}\right) p^{(r)}(\lambda_i)$, can be the optimal point if the slopes of the two curves, $\log_2 M_1 p^{(b)}(\lambda)$ and $\log_2\left(\frac{M_2}{M_1}\right) p^{(r)}(\lambda)$, are opposite at that point. However, finding λ such that $\nabla_\lambda f(\lambda) = 0$ is a difficult task even with $N = 1$. For the $4/16$-QAM scheme, we observe that the intersection point can be a lower bound of the optimal point since around this point the absolute value of the slope of the BER curve of the refinement stream is always larger that that of the base stream. To find λ_i, set[3]

$$p^{(b)}(\lambda_i) = p^{(r)}(\lambda_i). \tag{20}$$

where λ_i is the intersection point to be determined. With $p_{R,B}^{(r)} < p_{A,B}^{(b)}$, using (5), (6), (11) and (12), (20) can be approximated as

$$\frac{1}{2\left[C_N + \frac{\gamma_{A,B}}{(1+\lambda_i)^2+\lambda_i^2}\right]^N} \approx \frac{\left[1 - 2p_{R,B}^{(r)}\right]}{\left[C_N + \frac{\lambda_i^2\gamma_{A,R}}{(1+\lambda_i)^2+\lambda_i^2}\right]^N} \tag{21}$$

λ_i can be found by solving the quadratic equation (21), which is omitted here due to the lack of space.

When $\lambda_i < \lambda_c$ given in (17), the average BER of the combined stream is lower than that of the direct transmission. In contrast, when $\lambda_i \geq \lambda_c$, an insignificant improvement (or even worse performance) can be observed. Since when $\lambda < \lambda_c$ one has $p^{(b)}(\lambda) \approx p_{A,B}^{(4-\mathrm{QAM})}$, the near-optimal point can be found to be

$$\lambda_n = \min\{\tilde{\lambda}_{\mathrm{opt}}, \lambda_c\}, \quad \lambda_n \geq \lambda_i \tag{22}$$

[3]Through extensive simulations, we find that λ_i can be considered as a near-optimal point for the $4/64$-QAM scheme. The derivation of λ_i for the $4/64$-QAM scheme can be carried out in a similar way by using the average BERs of the two streams given in [10, Eqs. (22)-(24)].

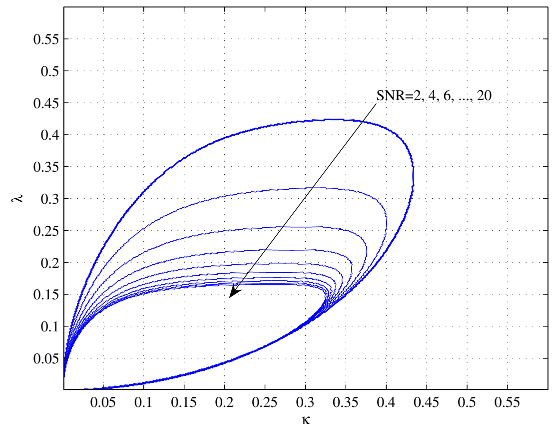

Fig. 3. Contours of $\mathrm{BER}(\lambda, \kappa) = \mathrm{BER}_{4-\mathrm{QAM}}$ with different SNR values. $4/16$-QAM hierarchical modulation with Alamouti STBC.

with $\tilde{\lambda}_{\mathrm{opt}}$ chosen such that

$$\frac{\left(1 - 2p_{R,B}^{(r)}\right) S_N}{\left[C_N + \frac{\tilde{\lambda}_{\mathrm{opt}}^2\gamma_{A,R}}{(1+\tilde{\lambda}_{\mathrm{opt}})^2+\tilde{\lambda}_{\mathrm{opt}}^2}\right]^N} = p_{R,B}^{(r)}. \tag{23}$$

Obviously, the point satisfying (22) is approximately the true optimal point if $p^{(b)}(\lambda)$ stays constant for $\lambda \leq \lambda_c$ as implicated in (18). However, taking into account the fact that $p^{(b)}(\lambda)$ still increases in the range of (λ_i, λ_c), a "back-off" amount in the $p^{(r)}(\lambda)$ is needed. By setting the average BER of the $A - R$ link equal to that of the $R - B$ link, $p^{(r)}(\lambda)$ is a little bit higher than the minimum value, which partially accounts for that "back-off". Then using (19), the candidate point is found to be

$$\tilde{\lambda}_{\mathrm{opt}} = \frac{1}{\sqrt{\frac{\gamma_{A,R}}{\left[\left(1-2p_{R,B}^{(r)}\right)S_N/p_{R,B}^{(r)}\right]^{1/N} - C_N} - 1} + 1}. \tag{24}$$

For a further explanation of how we choose the near-optimal point, please refer to Fig. 4 and Fig. 5 and the corresponding discussion given in Section V.

V. ILLUSTRATIVE RESULTS

In this section, we first validate the accuracy of the approximations used in designing the optimal HM constellations. Finding the near-optimal point λ_n will then be illustrated. Performance comparison between the two scenarios, with optimal and near-optimal points, will also be carried out.

Figs. 4 and 5 show the exact and approximated average BERs of the two streams corresponding to different values of κ and λ. As can be seen in Fig. 4, with $\kappa = 0.159$ the relay-assisted transmission can provide an improvement in the BER of the combined data stream compared to the direct transmission between nodes A and B. With $\kappa = 0.40$, no improvement can be observed in Fig. 5. Interestingly, a significant difference between the error floors of the refinement stream with different values of κ presented in Figs. 4 and 5 suggests a strategy to choose a suitable location for the relay or to select the best relay when multiple relays are available.

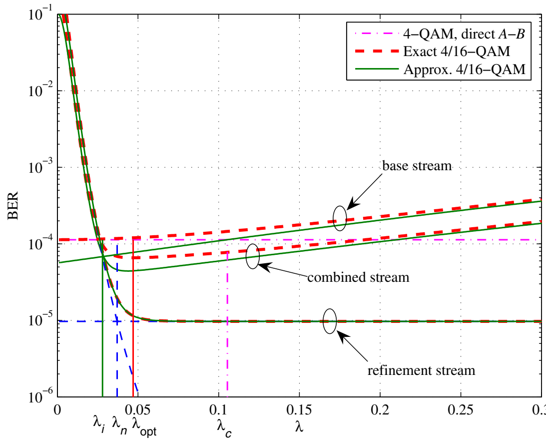

Fig. 4. BER of the two streams in 4/16-QAM hierarchical modulation with $\kappa = 0.159$. SNR $= 10$dB. $n_R = 2$.

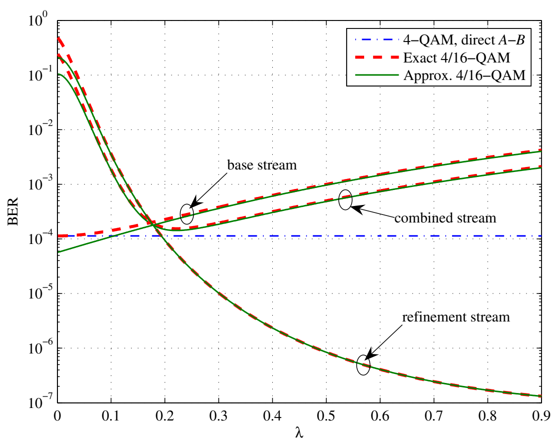

Fig. 5. BER of the two streams in 4/16-QAM hierarchical modulation with $\kappa = 0.400$. SNR $= 10$dB. $n_R = 2$.

For problem $(\mathcal{P}2)$, performance improvement can only be achieved in the optimal region as shown in Fig. 3. With a fixed value of κ, finding three points λ_c, λ_i and λ_n helps to decide whether relay-assisted transmission can offer any performance advantage. These points together with the optimal point $\lambda_{\rm opt}$ are illustrated in Fig. 4 for the $4/16$-QAM scheme.

Finally, the average BER of the combined stream provided by the two HM schemes are plotted in Fig. 6. A larger performance improvement can be seen for the case of $4/64$-QAM HM scheme. The $4/16$-QAM HM scheme is suitable for the unequal error protection problem while it only offers a moderate improvement in terms of the BER of the combined stream when being compared to the conventional 4-QAM. Fig. 6 also shows that the performance with the near optimal point is very close to the optimal solution.

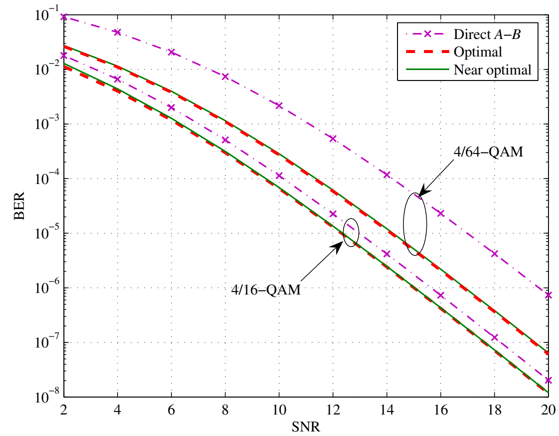

Fig. 6. BER of the combined stream in 4/16-QAM and 4/64-QAM hierarchical modulation with $\kappa = 0.159$ and 0.300, respectively. $n_R = 2$.

VI. Conclusions

This paper has considered the design of hierarchical modulation schemes in wireless relay networks. Provided are closed-form solutions to the two design problems concerning unequal error protection and BER minimization. Based on our thorough analysis, several useful design guidelines that allow to maximize the performance improvement have been given. In the first problem, based on the feasible regions, one can find the best way to accommodate data into the base and refinement streams. In the second problem, it was demonstrated that modulation orders of both base and refinement streams can strongly affect the error performance of the combined data stream.

References

[1] K. Ramchandran, A. Ortega, K. Uz, and M. Vetterli, "Multiresolution broadcast for digital HDTV using joint source/channel coding," *IEEE J. Select. Areas in Commun.*, vol. 11, pp. 6 –23, Jan. 1993.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[3] C. Hausl and J. Hagenauer, "Relay communication with hierarchical modulation," *IEEE Commun. Letters*, vol. 11, pp. 64 –66, Jan. 2007.

[4] M.-K. Chang and S.-Y. Lee, "Performance analysis of cooperative communication system with hierarchical modulation over rayleigh fading channel," *IEEE Trans. on Wireless Commun.*, vol. 8, pp. 2848 –2852, Jun. 2009.

[5] H. X. Nguyen, H. H. Nguyen, and T. Le-Ngoc, "Signal transmission with unequal error protection in wireless relay networks," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 2166 –2178, Jun. 2010.

[6] J. M. Park, S.-L. Kim, and J. Choi, "Hierarchically modulated network coding for asymmetric two-way relay systems," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 2179 –2184, Jun. 2010.

[7] C.-H. Choi, U.-K. Kwon, Y.-J. Kim, and G.-H. Im, "Spectral efficient cooperative diversity technique with multi-layered modulation," *IEEE Trans. Commun.*, vol. 58, pp. 3480 –3490, Dec. 2010.

[8] A. Conti, M. Win, and M. Chiani, "On the inverse symbol-error probability for diversity reception," *IEEE Trans. Commun.*, vol. 51, pp. 753 – 756, May 2003.

[9] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[10] P. Vitthaladevuni and M.-S. Alouini, "BER computation of 4/M-QAM hierarchical constellations," *IEEE Trans. on Broadcast.*, vol. 47, pp. 228 –239, Sep. 2001.

[11] M. K. Simon and M.-S. Alouini, *Digital Communication over fading channels*. 2nd edition. Wiley-Interscience, 2004.