$\mathcal{H}_\infty$ Filter with Adaptive Robustness Level for Space-Time Equalization

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***Abstract*—In this work, we propose the combination of mean performance and robustness in the context of adaptive space-time channel equalization. We first briefly discuss $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimization criteria, which are respectively associated to mean performance and robustness. As an alternative to the convex combination of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filters to join the desirable properties, we propose an adaptation scheme for the robustness level of the $\mathcal{H}_\infty$ filter for combining mean performance and robustness through a unique filter.**

I. Introduction

Modern communication systems are characterized by increasingly quality requirements, such as high transmission rates. The unfavorable combination of high transmission rates and temporal dispersion of channels results in intersymbol interference (ISI), which besides co-channel interference (CCI) is a major performance limitation of such systems. Linear space-time adaptive equalization is a classical technique to mitigate CCI and ISI. It is characterized by the use of a transversal filter as a linear equalizer at each reception sensor in order to exploit both spatial and temporal diversity of the received signal [1].

Adaptive algorithms for the update of equalizer coefficients are conventionally obtained through mean performance criteria, e.g. the minimization of the mean square error between a reference signal and the equalizer output. However, the transmit signal recovery may still be affected by unmodeled uncertainties or disturbances, like non-gaussian noises. In this case, robust approaches are suitable.

The representation of the adaptive filtering problem in the state space makes easier a general modeling that supports both the $\mathcal{H}_2$ filtering, based on the mean performance optimization, and the $\mathcal{H}_\infty$ filtering, which optimizes the "worst case" performance and for this reason is associated to robustness [2]–[5]. Kalman filter is the optimum solution for the $\mathcal{H}_2$ filtering, under the condition that noises present in the model are gaussian and with known statistics. Classical adaptive algorithms, as the recursive least squares (RLS), are recognized as a special case of the Kalman filter, or the $\mathcal{H}_2$ filter [2]. Kalman filter is optimal on the condition that the involved noises are Gaussian with known statistical properties. Alternatively to the conventional $\mathcal{H}_2$ filtering, the $\mathcal{H}_\infty$ filtering minimizes the maximum transfer from disturbances to the filtered error [3]–[5]. Then, the $\mathcal{H}_\infty$ filtering is suitable to deal with significant model uncertainties arising, for instance, because of difficult modeling phenomena, such as the asynchronous interference in wireless multi-access systems [6], [7]. However, $\mathcal{H}_\infty$ filters are rather conservative with respect to mean performance [3], [4].

The complementary properties of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filters motivates the study of their combination. There exist different trade-off strategies between $\mathcal{H}_2$ and $\mathcal{H}_\infty$ criteria in the context of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering, as in [8], [9]. However, the linear adaptive mixed $\mathcal{H}_2/\mathcal{H}_\infty$ channel equalization remains an open problem, since the mentioned solutions are not applicable. On the other hand, the notion of combined filtering has been widely explored. Convex and affine combinations of two adaptive filters have been proposed as conceptually simple ways of exploiting the best individual characteristics of each filter [10], [11]. The work developed in [12] proposes a convex combination of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filters for adaptive space-time equalization.

In this paper, we propose an alternative solution to the combination of the complementary properties of mean performance and robustness in supervised adaptive space-time equalization. This solution is based on a scheme of adaptation of the robustness level of the $\mathcal{H}_\infty$ filter. It allows the adoption of robust configurations as well as configurations that reduce the $\mathcal{H}_\infty$ filter to the $\mathcal{H}_2$ equalizer according to the inferred need for robustness. Simulation results are provided for scenarios characterized by asynchronous interference in wireless systems, highlighting the advantages of the proposed solution with respect to the convex combination of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filters, besides the advantage of using only one $\mathcal{H}_\infty$ filter.

In order to facilitate the proposal understanding, Section II presents first the state space representation of the adaptive equalization problem, as well as notions of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filtering. The $\mathcal{H}_\infty$ filter with adaptive robustness level is proposed in Section III, whereas simulation results are discussed in Section IV. Conclusions are given in Section V.

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II. State-space representation of adaptive space-time equalization

The space-time equalization problem is illustrated in Fig. 1 by a simplified baseband discrete-time model, where M sequences are transmitted, $\mathbf{d}_1, \ldots, \mathbf{d}_M$, and the receiver is composed of N reception sensors. The MIMO channel is composed of finite impulse response (FIR) subchannels $\mathbf{h}_{ij}$ between transmission sensor j and reception sensor i, where $i = \{1, \ldots, N\}$ and $j = \{1, \ldots, M\}$. The receiver input signals $u_i(k)$ are given by:

$$u_i(k) = \sum_{j=1}^{M} \mathbf{d}_j(k) \star \mathbf{h}_{ij}(k) + \eta_i(k), \tag{1}$$

where "$\star$" stands for discrete convolution and $\eta_i(k)$ is the white Gaussian noise. Then, the space-time equalizer input can be expressed by the concatenated vector:

$$\mathbf{u}(k) = \left[\mathbf{u}_1^T(k), \ldots, \mathbf{u}_N^T(k)\right]^T \in \mathbb{R}^{N \cdot P \times 1}, \tag{2}$$

where $\mathbf{u}_i(k) = [u_i(k), \ldots, u_i(k-P+1)]^T$ and P is the number of samples to be taken into account for equalization.

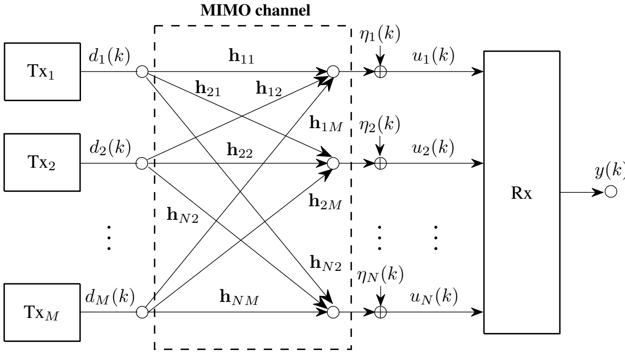


Fig. 1. Baseband MIMO communication system.

The equalizer output $y(k)$ must provide an estimate of the desired transmitted signal, represented for simplicity by $d(k)$. Also for simplicity, equalization delay is neglected in the model, and $y(k)$ can be expressed as:

$$y(k) = \mathbf{u}^H(k)\mathbf{w}(k), \tag{3}$$

where $\mathbf{w}(k) = \left[\mathbf{w}_1^T(k), \ldots, \mathbf{w}_N^T(k)\right]^T \in \mathbb{R}^{N \cdot P \times 1}$ is the space-time equalizer concatenated vector, and $\mathbf{w}_i(k) = [w_i(k), \ldots, w_i(k-P+1)]^T$, for $i = \{1, \ldots, N\}$.

In our state space representation, the state to be estimated is the coefficient vector of the optimal equalizer, $\mathbf{w}_o(k)$, while the measure is the training (desired) signal $d(k)$ available at the receiver. Their dynamic models are given by [3]:

$$\begin{aligned} \mathbf{w}_o(k+1) &= \mathbf{w}_o(k), \quad \mathbf{w}_o(0) = \overline{\mathbf{w}}_o & (4)\\ d(k) &= \mathbf{u}^H(k)\mathbf{w}_o(k) + r(k), & (5) \end{aligned}$$

where $\mathbf{w}_o$ is considered constant because of its slow variation compared to the symbol time scale; $r(k)$ is the measurement noise, which comes from the additive noise and the model uncertainty. In the supervised adaptive mode, $d(k)$ in (5) is used to generate the error signal $e(k) = d(k) - y(k)$, employed to update the equalizer coefficients.

A. $\mathcal{H}_2$ filtering

In $\mathcal{H}_2$ filtering, $\mathbf{w}_o(0)$ and $r(k)$ are assumed independent Gaussian random variables with zero mean and known second-order statistics, given by $\mathbb{E}\{\mathbf{w}_o(0)\mathbf{w}_o^H(0)\} = \mathbf{\Pi}_0$ and $\mathbb{E}\{r(k)r^*(k)\} = R(k)$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. This means that the involved disturbances are fully characterized in statistical terms. The problem consists in searching for a linear estimator that minimizes the mean square error (MSE) criterion, as follows:

$$\min_{\mathbf{w}(k)} \mathbb{E}\left\{\sum_{i=0}^{k} [d(i) - y(i)]^H [d(i) - y(i)]\right\}, \tag{6}$$

where $d(k)$ is a linear combination of the optimal equalizer (see (5)), and its estimate is given by $y(k)$, expressed in (3).

The solution for this problem is the Kalman filter, which is reduced to the RLS algorithm if $R(k) = 1$. The $\mathcal{H}_2$ equalizer considered in this work is updated by the RLS algorithm [2], [3].

B. $\mathcal{H}_\infty$ filtering

By optimizing a "worst case" performance, the $\mathcal{H}_\infty$ filtering is closely related to the notion of robustness. As already mentioned, the aim in the $\mathcal{H}_\infty$ filtering is the minimization of the maximum energy transfer from the disturbances to the estimation error. However, only in some cases it is possible to derive the optimal solution. Then, it is common to consider the following suboptimum problem:

$$\max_{\mathbf{w}_o(0), \{r\}} \frac{\displaystyle\sum_{i=0}^{k} ||d(i) - y(i)||^2}{||\mathbf{w}_o(0) - \mathbf{w}(0)||^2 + \displaystyle\sum_{i=0}^{k} r^2(i)} < \gamma^2, \tag{7}$$

where the parameter $\gamma > 0$, called disturbance attenuation level, limits the maximum energy gain from disturbances to the filtered error. In other words, γ stands for the level of robustness of the $\mathcal{H}_\infty$ filter, with low values of γ corresponding to more restrictive (robust) filters, and vice versa. The relaxation of the robustness constraint by increasing γ approximates the resultant $\mathcal{H}_\infty$ filter from the $\mathcal{H}_2$ filter [4].

For the specific case of supervised adaptive equalization, the filter output $y(k)$ is the estimate of the desired signal, and the calculation of the $\mathcal{H}_\infty$ filter requires the inversion of matrix $\mathbf{\Omega}(k)$ given below [13]:

$$\mathbf{\Omega}(k) = \mathbf{I} + (1 - \gamma^{-2})\mathbf{u}(k)\mathbf{u}^H(k)\mathbf{K}(k), \tag{8}$$

where $\mathbf{K}(k)$ is a positive definite matrix satisfying a Riccati recursion [3], [4]. To be invertible, matrix $\mathbf{\Omega}(k)$ must be definite positive for every time instant k. Then, a sufficient condition for the existence of the $\mathcal{H}_\infty$ filter is obtained from (8) as a function of the disturbance attenuation level γ [13]:

$$\gamma \geq 1 \quad \Rightarrow \quad \mathbf{\Omega}(k) > \mathbf{0}, \quad \forall k. \tag{9}$$

Therefore, the existence of the $\mathcal{H}_\infty$ equalizer that avoids amplification of disturbance energy to the filtered error ($\gamma = 1$) is guaranteed. In fact, this filter setting corresponds to a variant of the normalized least mean square (NLMS) algorithm [3].

III. Adaptive $\mathcal{H}_\infty$ space-time equalizer

As discussed in Section II-B, the robustness of the $\mathcal{H}_\infty$ filter is determined by the disturbance attenuation level γ, which corresponds to the value of the $\mathcal{H}_\infty$-norm of the $\mathcal{H}_\infty$ filter. Low values of γ are associated to more robust filters, while the relaxation of γ produces less restrictive filters, which tend to the $\mathcal{H}_2$ filter when γ assumes higher values [4].

Fig. 2 illustrates the relationship between parameter γ and the robustness of the $\mathcal{H}_\infty$ filter. The minimum $\mathcal{H}_\infty$-norm of the $\mathcal{H}_\infty$ filter, i.e. the minimum γ defines the most robust filter. Then, $\gamma = \gamma_{\mathcal{H}_\infty}$ corresponds to this filter. On the other hand, $\gamma = \gamma_{\mathcal{H}_2}$ corresponds to an $\mathcal{H}_\infty$ filter that is equivalent to the $\mathcal{H}_2$ filter, which is the optimum one in the sense of the MSE. Therefore, it is possible to join the two desirable characteristics, high mean performance and robustness, through the adaptation of parameter γ of the $\mathcal{H}_\infty$ filter. Intermediary values of γ represent a trade-off between the robustness of the $\mathcal{H}_\infty$ filter with $\gamma = \gamma_{\mathcal{H}_\infty}$ and the optimum mean performance of the $\mathcal{H}_2$ filter, obtained as the $\mathcal{H}_\infty$ filter with $\gamma = \gamma_{\mathcal{H}_2}$.

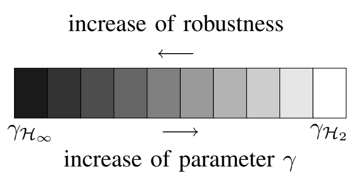


Fig. 2. Robustness level of the $\mathcal{H}_\infty$ filter.

First step towards the adaptation of parameter γ is to define its interval of variation. In general, the optimum value of γ in the sense of the minimization of the maximum $\mathcal{H}_\infty$-norm, i.e., $\gamma_{\mathcal{H}_\infty}$, is unknown. However, in the context of adaptive equalization, $\gamma \geq 1$ is a sufficient condition for the existence of the $\mathcal{H}_\infty$ filter (see Section II-B). Then, our reference of robustness is the $\mathcal{H}_\infty$ filter with $\gamma = 1$. On the other side, it worths to consider the following result established in [14]: the $\mathcal{H}_2$ filter presents an explicit upper bound for the energy gain from disturbances to the filtered error, it is 4. In other words, the $\mathcal{H}_\infty$-norm of the $\mathcal{H}_2$ filter is lower than or equal 2. Then, at least with respect to robustness, the $\mathcal{H}_\infty$ filter with $\gamma = 2$ is equivalent to the $\mathcal{H}_2$ filter. Since the filters may present similar performances in terms of robustness while performing differently with respect to the MSE, it is convenient to adopt values higher than 2 for $\gamma_{\mathcal{H}_2}$. We verify by simulations that with $\gamma = 5$ the $\mathcal{H}_\infty$ filter is numerically reduced to the $\mathcal{H}_2$ filter.

In order to guarantee that $\gamma_{\mathcal{H}_\infty} \leq \gamma(k) \leq \gamma_{\mathcal{H}_2}$, it is proposed the use of the following sigmoid function of an auxiliary variable $\alpha(k)$:

$$\gamma(k) = \frac{\gamma_{\mathcal{H}2} - \gamma_{\mathcal{H}\infty}}{1 + e^{-\alpha(k)}} + \gamma_{\mathcal{H}\infty}. \tag{10}$$

The sigmoid function determines a more dynamic adaptation for intermediary values of $\gamma(k)$, and smooth adaptation when $\gamma(k)$ approaches the extreme values $\gamma_{\mathcal{H}_\infty}$ and $\gamma_{\mathcal{H}_2}$, as shown in Fig. 3.

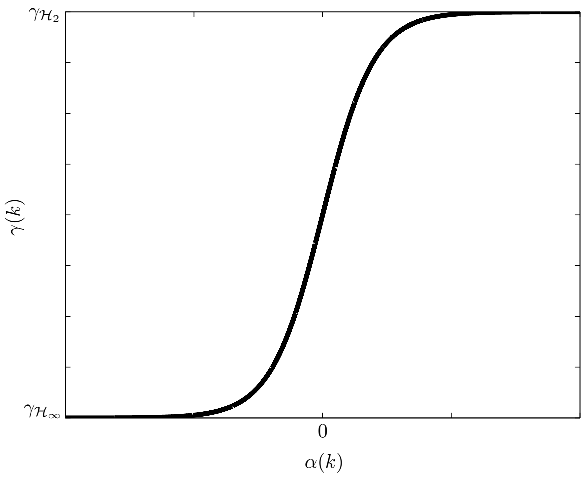


Fig. 3. Sigmoid function of an auxiliary variable $\alpha(k)$ to adapt parameter $\gamma(k)$ of the $\mathcal{H}_\infty$ filter.

The adaptation of $\gamma(k+1)$ is carried out by adjusting parameter $\alpha(k+1)$. The idea is to use high values of $\alpha(k+1)$, and consequently high values of $\gamma(k+1)$, to privilege the mean performance of the $\mathcal{H}_\infty$ filter. Decreasing $\alpha(k+1)$ (and thus $\gamma(k+1)$ too) must be a response to the need for robustness. Robustness is important in situations where model uncertainties become significant. In channel equalization, uncertainties may result from abrupt changes in system conditions, which are not represented by the adopted models, as those originated from non-Gaussian noise or asynchronous interference. These disturbances may be detected, in general, by the abrupt energy variations of signals at receiver input. Then, we design the adaptation of parameter $\alpha(k+1)$ according to the energy variation of the equalizer input signal, $\mathbf{u}(k)$, as described below:

$$\alpha(k+1) = \alpha(k) - \mu \frac{||\Delta \mathbf{u}(k)||^2}{||\mathbf{u}(k)||^2} + \varepsilon, \tag{11}$$

where $\Delta \mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$, μ is a step-size, and ε is a positive parameter. In steady-state operation, $\Delta \mathbf{u}(k)$ assumes low values and parameter $\alpha(k+1)$ is increased. This emphasizes the mean performance of the $\mathcal{H}_\infty$ filter, which will tend to the $\mathcal{H}_2$ filter. Abrupt changes make $\Delta \mathbf{u}(k)$ become higher. According to (11), if $\mu||\Delta \mathbf{u}(k)||^2/||\mathbf{u}(k)||^2 > \varepsilon$, then $\alpha(k+1)$ decreases, as well as $\gamma(k+1)$. Therefore, the $\mathcal{H}_\infty$ filter becomes more robust. The range of values for $\alpha(k)$ has a direct correspondence with the predetermined range of $\gamma(k)$ values, i.e., $\gamma_{\mathcal{H}\infty} \leq \gamma(k) \leq \gamma_{\mathcal{H}2}$, via (10). Table I summarizes the $\mathcal{H}_\infty$ equalizer with adaptive parameter γ.

TABLE I

$\mathcal{H}_\infty$ EQUALIZER WITH ADAPTIVE PARAMETER γ.

$\mathbf{G}(k) = \mathbf{K}(k)\mathbf{u}(k)\left[\mathbf{I} + \mathbf{u}^H(k)\mathbf{K}(k)\mathbf{u}(k)\right]^{-1}$
$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{G}(k)\left[d(k) - y(k)\right], \ \mathbf{w}(0) = \mathbf{0}$
$\gamma(k) = \dfrac{\gamma_{\mathcal{H}2} - \gamma_{\mathcal{H}\infty}}{1 + e^{-\alpha(k)}} + \gamma_{\mathcal{H}\infty}$
$\mathbf{\Omega}(k) = \mathbf{I} + (1 - \gamma^{-2}(k))\mathbf{u}(k)\mathbf{u}^H(k)\mathbf{K}(k)$
$\mathbf{K}(k+1) = \mathbf{K}(k)\mathbf{\Omega}^{-1}(k), \ \mathbf{K}(0) = \mathbf{\Pi}_0$
$\alpha(k+1) = \alpha(k) - \mu\|\|\Delta\mathbf{u}(k)\|\|^2/\|\|\mathbf{u}(k)\|\|^2 + \varepsilon$

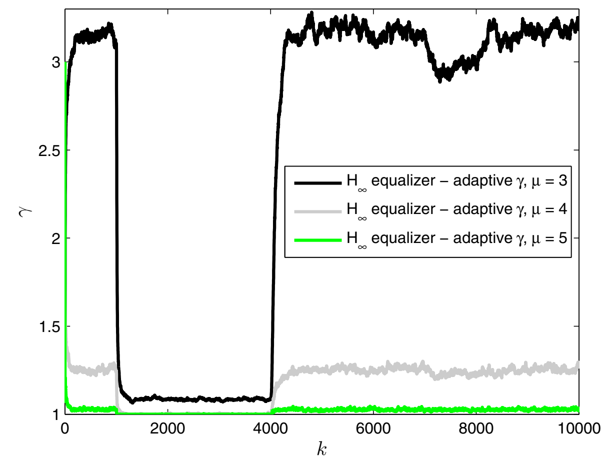
IV. SIMULATIONS

In this section, we evaluate by simulations the performance of the proposed $\mathcal{H}_\infty$ space-time equalizer with adaptive γ. For the sake of comparison, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ equalizers, and the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination proposed in [12] are also evaluated. The equalizers are compared in situations where the adopted model, presented in Section II, fails in representing the dynamics of the involved variables because of abrupt changes in transmission conditions.

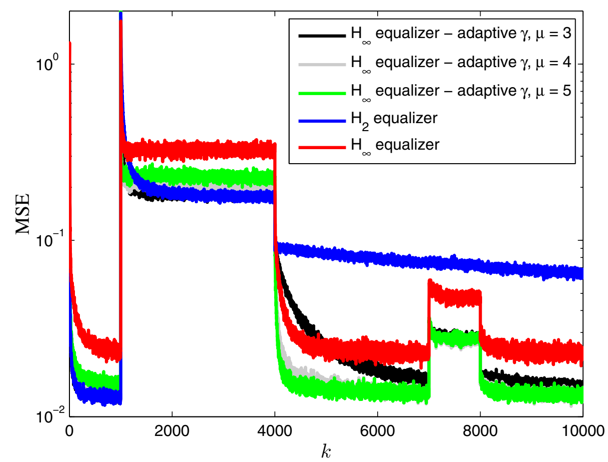
The simulated environment is the uplink transmission in a time division multiple access (TDMA) wireless system with 64-QAM (quadrature amplitude modulation) modulation, where the random time utilization of the channel produces asynchronous multi-user interference. Channel model follows the GSM/EDGE (*Global System for Mobile Communications/Enhanced Data rates for GSM Evolution*) specification for typically urban environment [15], with 5 uncorrelated multipaths spaced at the symbol time (3.692 μs). Spatial and temporal diversities are explored at the receiver with 2 antennas, each one equipped with a linear equalizer composed of 7 coefficients.

Each realization of the simulation comprises the transmission of 10,000 training symbols. The random nature of the asynchronous interference requires stochastic models for both its time intervals and intensities. For simplicity, we consider two interference time intervals randomly chosen for each realization, and two interference intensities corresponding to the following signal-to-interference ratios (SIRs): SIR = 0 dB and SIR = 20 dB. Results presented below are obtained from the average over 1,000 realizations.

We first observe the behavior of the proposed $\mathcal{H}_\infty$ space-time equalizer with adaptive γ for different settings. The proposed $\mathcal{H}_\infty$ equalizer has the disturbance attenuation level γ varying between $\gamma_{H_\infty} = 1$ and $\gamma_{H_2} = 5$ according to (10) and (11). The variation of its robustness level depends on parameters μ and ε in (11). Since it is the ratio between μ and ε that is important to define the adaptation of α in (11), we set $\varepsilon = 1$ and verify the equalizer performance for different values of μ. Fig. 4 shows the evolution in time of the disturbance attenuation level γ and the MSE at the equalizer output. MSE curves of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ equalizers are also shown. Interference is present in $1000 \leq k \leq 4000$, with SIR = 0 dB, and in $7000 \leq k \leq 8000$, with SIR = 20 dB.



(a) Disturbance attenuation level of the $\mathcal{H}_\infty$ equalizer.



(b) Mean square error.

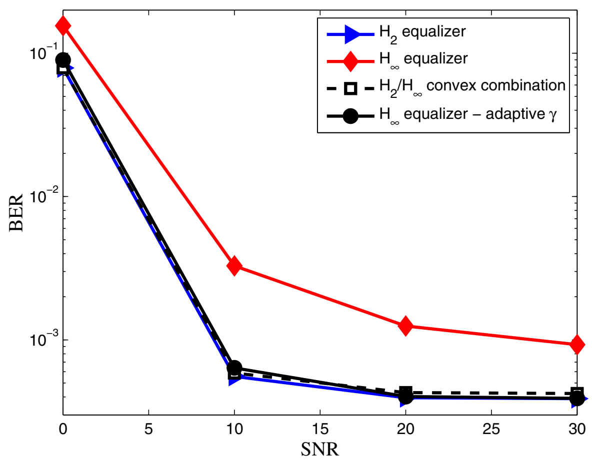
Fig. 4. Disturbance attenuation level γ and MSE of the proposed $\mathcal{H}_\infty$ equalizer under asynchronous multi-user interference for different values of the step size μ in (11).

First of all, we identify the complementarity of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ equalizers. In behaved conditions of transmission, as for $0 \leq k \leq 1000$ and also during the steady-state until $k = 4000$, the $\mathcal{H}_2$ equalizer, whose coefficients are adapted by the RLS algorithm performs well in the sense of MSE. At the same time, the conservatism of the robust $\mathcal{H}_\infty$ equalizer with $\gamma = 1$ is exposed by the worst MSE levels in the absence of abrupt changes. On the other hand, it is clear that the $\mathcal{H}_2$ equalizer has its MSE dramatically degraded from the time instant $k = 4.000$, when the interference disappearance causes an important disturbance. The $\mathcal{H}_\infty$ equalizer, however, reacts well to this event, since it is less sensitive to disturbances.

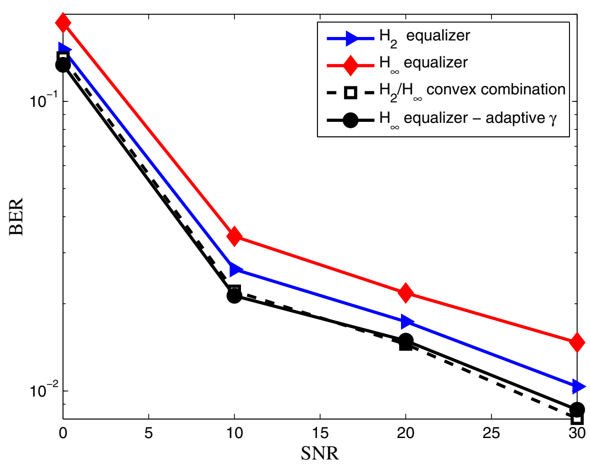
Curves shown in Figs. 4(a) and 4(b) for $\mu = 3, 4, 5$ are coherent with equations (10) and (11), and with the objective of combining mean performance and robustness. For any value of μ, intense variations on the energy of the received signal are taken as indications of disturbances on transmitted signals.

In this case, γ is decreased and the $\mathcal{H}_\infty$ equalizer assumes a robust configuration. As observed in Fig. 4(a), μ determines the range of values of parameter γ, and as a consequence, the flexibility of the $\mathcal{H}_\infty$ equalizer to operate under configurations more robust (low γ) or more close to the $\mathcal{H}_2$ equalizer.

We now confront the equalizes in terms of bit error rate (BER) performance. In addition to the $\mathcal{H}_2$ and the $\mathcal{H}_\infty$ equalizers, the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination proposed in [12] is also considered. Fig. 5 illustrates the BER curves of mentioned space-time equalizers in the absence (Fig. 5(a)) and in the presence (Fig. 5(a)) of asynchronous multi-user interference for different signal-to-noise ratios (SNRs).



(a) Absence of multi-user interference.



(b) Presence of asynchronous multi-user interference.

Fig. 5. BER performance of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ equalizers, the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination, and the $\mathcal{H}_\infty$ equalizer with adaptive γ.

Note that in the absence of interference, the $\mathcal{H}_2$ equalizer, the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination and the proposed $\mathcal{H}_\infty$ equalizer with adaptive γ present similar BER performances, while the $\mathcal{H}_\infty$ equalizer has worse performance. In the presence of interference, as expected, performance of all equalizers is degraded. However, one observes that the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination and the $\mathcal{H}_\infty$ equalizer with adaptive γ outperform the simple $\mathcal{H}_2$ and $\mathcal{H}_\infty$ equalizers. As an important remark: the proposed scheme of adaptation of the $\mathcal{H}_\infty$ equalizer robustness achieves BERs similar to the ones of the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination, where two filters operate in parallel.

V. Conclusions

The complementarity of $\mathcal{H}_2$ filtering and $\mathcal{H}_\infty$ filtering, respectively, optimal mean performance and robustness, motivates the combination of their best individual properties. In this work, we propose an alternative to the convex combination of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filters to achieve such a combination. The $\mathcal{H}_\infty$ filter with adaptive disturbance attenuation level offers an operational flexibility that goes from the most robust $\mathcal{H}_\infty$ filter to the $\mathcal{H}_2$ filter. It performs as well as the $\mathcal{H}_2/\mathcal{H}_\infty$ convex combination, but using a unique filter.

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