Frequency Offset and Channel Estimation in Co-Relay Cooperative OFDM Systems

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***Abstract*— Frequency offset and channel estimation in cooperative orthogonal frequency division multiplexing (OFDM) systems is studied in this paper. We consider the scenario of two or more source nodes sharing the same relay, i.e., co-relay cooperative communications, and a new preamble, which is central-symmetric in time-domain, is proposed to perform the frequency offset and channel estimation. The non-zero samples in the proposed preamble are sparsely distributed with two neighboring non-zero samples being separated by $\mu > 1$ zeros. As long as $\mu > 2L - 1$ is satisfied, the multipath interference can be effectively eliminated, where L stands for the channel order. Unlike [1], the proposed preamble has a much lower Peak-to-Average Power Ratio (PAPR). The interference among the multiple source nodes can also be eliminated by using a backoff modulation scheme on the proposed preamble in each source node, and the mean-square error (MSE) of the proposed Least-Square (LS) channel estimator can be minimized by ensuring the orthogonality among the source nodes. The Pairwise Error Probability (PEP) performance of the proposed system by considering both the frequency offset and channel estimation errors is also derived in this paper. For a given Signal-to-Noise-Ratio (SNR), by keeping the total power consumption to the source nodes and the relay to be constant, the PEP can be minimized by adjusting the ratio between the power allocated to the source nodes and the total power.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) provides a flexible and effective way of handling multipath fading. However, the performance of OFDM can be significantly impacted by the frequency offset, which induces inter-carrier-interference (ICI) [2]. Frequency and timing estimation issues for OFDM have widely been investigated in the literature [3]–[9]. Channel estimation errors can also degrade the OFDM system performance. Some more recent work have been done on the channel estimation in multiple-input multiple-output (MIMO) OFDM systems [10]–[12], where the effect of frequency offset on the channel estimation is also considered.

Cooperative relaying, which uses multiple cooperating nodes to form a virtual multiple-antenna array, has been studied intensively [13]–[16]. Cooperative relays can forward packets using either amplify-and-forward (AF) or decode-and-forward (DF) mode [14]. Frequency offset estimation in cooperative orthogonal frequency division multiple access (OFDMA) is studied in [15]. Channel estimation for cooperative OFDM in the presence of the frequency offset is studied in [16], where the optimal pilot design is proposed to eliminate the inter-relay-interference. However, the preamble proposed in [16] suffers a relatively high Peak-to-Average-Power-Ratio (PAPR).

In this paper, frequency offset estimation and channel estimation for cooperative OFDM is proposed. A scenario of multiple source nodes sharing a relay is considered. Similar to [1], a central-symmetric and comb-shaped preamble with a low PAPR is proposed to combat the interference introduced by multipath fading. When multiple source nodes transmit simultaneously, a backoff modulation is applied to ensure orthogonality among different sources. In both the frequency offset and channel estimation, the estimation errors can be optimized by adaptively adjusting the ratio α between the power allocated to the sources and that allocated to the relay. The Pairwise Error Probability (PEP) for a cooperative co-relay OFDM system with orthogonal space-time coding is also derived in this paper by considering the effect of both frequency offset and channel estimation errors.

This paper is organized as follows. Section II introduces the signal model of co-relay cooperative OFDM system. Frequency offset estimation based on the proposed preamble is studied in Section III, followed by channel estimation being developed in Section IV. Section V discusses the numerical results. Finally, Section VI concludes the paper.

Notation: $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ and $(\cdot)^*$ denote the transpose, conjugate transpose, inverse and complex conjugate, respectively. The imaginary unit is $\jmath = \sqrt{-1}$. A circularly symmetric complex Gaussian variable with mean a and variance σ^2 is denoted by $z \sim \mathcal{CN}(a, \sigma^2)$. The $N \times N$ identity matrix is $\mathbf{I}_N$. The $M \times N$ all-zero matrix is $\mathbf{O}_{M\times N}$, and the $N \times 1$ all-zero vector is $\mathbf{0}_N$. The diagonal matrix diag$\{\mathbf{x}\}$ has the n-th diagonal element $\mathbf{x}[n]$. The mean and the variance are represented as $\mathbb{E}\{\bullet\}$ and Var$\{\bullet\}$, respectively.

II. System Fundamentals

A. Signal Model

We consider an OFDM system with N subcarriers, where $N = 2^Q$ with Q being an integer. In each transmitter, the input bits are mapped to N complex symbols chosen from a complex signal constellation such as phase-shift keying (PSK) or quadrature amplitude modulation (QAM). Let $X_{i,k}$

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$(i=0,1,\cdots;k=0,1,\cdots,N-1)$ be the symbols of the i-th source with a zero mean and variance $\mathbb{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1}|X_{i,k}|^2\right\}=\sigma_s^2$. After performing Inverse Discrete Fourier Transform (IDFT) operation on $\{X_{i,k}\}$, the time-domain symbol is generated as $d_i(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_{i,k}\cdot e^{\frac{\jmath 2\pi nk}{N}}$, $n=0,1,\cdots,N-1$. A length-L_{CP} cyclic prefix (CP) is appended to the symbol to combat the inter-symbol-interference (ISI).

B. Preamble Construction

After performing IDFT on the frequency-domain sequence

$$X_{i,k}=\frac{A}{\sqrt{N}}\cdot\frac{\sin\frac{(m_i-k)\pi}{2}}{\sin\frac{(m_i-k)\mu\pi}{N}}\cdot e^{\frac{\jmath(m_i-k)(N-2\mu)\pi}{2N}} + \frac{A}{\sqrt{N}}\cdot\frac{\sin\frac{(m_i+k)\pi}{2}}{\sin\frac{(m_i+k)\mu\pi}{N}}\cdot e^{\frac{\jmath((m_i+k)(N-2\mu)+4k\mu)\pi}{2N}}, \tag{1}$$

a new time-domain preamble with multiple-access interference (MAI) combating capability is obtained as

$$s_i(n)=\mathcal{IDFT}\{X_{i,k}\}=\frac{\sum_{l=0}^{\mu-1}e^{\frac{\jmath 2\pi nl}{\mu}}}{\sqrt{N}}\sum_{k=0}^{N/\mu-1}X_{i,k}\cdot e^{\frac{\jmath 2\pi nk}{N}}, \tag{2}$$

where $1\le\mu\le N$, $A=\sqrt{\mu\sigma_s^2}$, $n=0,1,\cdots,N-1$ and $m_i=0,1,2,\cdots$. We can readily prove that $s_i(n)=s_i(N-\mu-n)$ for each $0\le n\le N/2-1$. We also know that $s_i(n)=0$ for $n\neq 0,\mu,2\mu,\cdots$ and $\mathbb{E}\left\{\frac{\mu}{N}\sum_{n=0}^{N/\mu-1}|s_i(n\mu)|^2\right\}=\mu\sigma_s^2$.

C. Multipath Channel

For a cooperative relaying network with sources $(S_0,S_1,\cdots)$, relay (R) and destination (D), the time-invariant composite channel impulse response between any pair of nodes a and b is modelled as $h_{a,b}(\tau)=\sum_{l=0}^{L-1}h_{a,b}[l]\delta(\tau-lT_s)$, where $h_{a,b}[l]$ is the l-th channel gain, $\delta(x)$ is the unit impulse function, L stands for the maximum channel order, and $T_s=1/B$ with B representing the total bandwidth. In this paper, we define the channel vector as $\mathbf{h}_{a,b}=[h_{a,b}(0),h_{a,b}(1),\cdots,h_{a,b}(L-1)]^T$. In this paper, we assume that the $S_i\to D$ and $R\to D$ channels suffer path-loss[1], but only small-scale fading is happened for each $S_i\to R$ channel.

D. Cooperative Transmission

The proposed cooperative transmission is performed in two time slots. Following the source nodes broadcast their packets in the first time slot, the relay R will forward the received packet in the second time slot. The total power allocated to the source and relay nodes is a const, i.e., $N\bar{P}$, and that allocated to the source nodes is $\alpha N\bar{P}$, where $0\le\alpha\le 1$ stands for a power allocation ratio. We also assume that each source node is allocated a power of $\alpha N\bar{P}/M$, where M is the total number of source nodes.

1) First Time Slot: The received signal at D and R can be represented as

$$y_{d,1}(n)=\sqrt{\frac{\alpha\bar{P}}{M}}\sum_{i=0}^{M-1}\sum_{l=0}^{L-1}s_i(n-l)h_{d,S_i}[l]e^{\jmath\left(\frac{2\pi n\varepsilon_{d,S_i}}{N}+\psi_{d,S_i}\right)} + w_{d,1}(n), \tag{3}$$

where $d\in\{R,D\}$, ε_{D,S_i} and ε_{R,S_i} are the frequency offsets for $S_i\to D$ and $S_i\to R$ links, respectively, ψ_{D,S_i} and ψ_{R,S_i} represent the initial phases for the $S_i\to D$ and $S_i\to R$ links. In this paper, we assume that the initial phases have already been corrected and, therefore, $\psi_{D,S_i},\psi_{R,S_i}=0$. $w_{D,1}(n)$ and $w_{R,1}(n)$ represent the n-th sample of additive white Gaussian noise (AWGN) random process at D and R, respectively, with variance σ_w^2.

2) Second Time Slot: The relay forwards the received signal to D with either AF or DF relaying mode.

a) AF Mode: By defining the amplifying coefficient as $\rho_R=\left(\alpha\bar{P}\sigma_s^2+\frac{L\cdot\sigma_w^2}{\mu}\right)^{-1}$, the received samples at D are

$$\begin{aligned}y_{D,2}(n)=&\rho_R\sqrt{(1-\alpha)\bar{P}}\sum_{i=0}^{M-1}\sum_{l=0}^{2L-1}s_i(n-l)h_{D,R,S_i}[l]\\&\cdot e^{\frac{\jmath 2\pi n\varepsilon_{D,R,S_i}}{N}}\\&+\rho_R\sqrt{(1-\alpha)\bar{P}}\sum_{l'=0}^{L-1}\tilde{w}_{R,1}(n-l')h_{D,R}[l']e^{\frac{\jmath 2\pi n\varepsilon_{D,R}}{N}}\\&+w_{D,2}(n),\end{aligned} \tag{4}$$

where $h_{D,R,S_i}[l]$ is the n-th sample of $\left(\mathbf{h}_{R,S_i}^T\otimes\mathbf{h}_{D,R}^T\right)^T$ with $\otimes$ denoting convolution operation [16], ε_{D,R,S_i} represents the frequency offset in the $S_i\to R\to D$ link, and $\tilde{w}_{R,1}(n)$ is defined as

$$\tilde{w}_{R,1}(n)=\begin{cases}w_{R,1}(n), & n=g\cdot\mu+c\\ & g=0,1,\cdots,N/\mu-1\\ & c=0,1,\cdots,L-1,\\ 0, & \text{otherwise}.\end{cases}.$$

$w_{D,2}(n)$ stands for the n-th sample of AWGN at node D.

b) DF Mode: Each DF relay decodes and re-encodes the received signal. The received signal at node D is

$$y_{D,2}(n)=\rho_R\sqrt{(1-\alpha)\bar{P}}\sum_{i=0}^{M-1}\sum_{l=0}^{L-1}s_i(n-l)h_{D,R}[l]e^{\frac{\jmath 2\pi n\varepsilon_{D,R}}{N}} + w_{D,2}(n). \tag{5}$$

[1]Like in [16], the path-loss coefficient $\mathcal{L}_u$ can be approximated as $\mathcal{L}_u=d_{a,b}^{-q}/2$, where $d_{a,b}$ represents the distance between nodes a and b and $2\le q\le 6$.

E. Backoff Transmission in Multipath Channel

When the source nodes transmit simultaneously, In order to eliminate the inter-source-interference, a backoff transmission scheme is performed by simply modulating the preamble as

$$\tilde{s}_i(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{i,k} \cdot e^{\frac{\jmath 2\pi(-\nu_i)k}{N}} \cdot e^{\frac{\jmath 2\pi nk}{N}},\ n = 0, 1, \cdots, N-1, \tag{6}$$

where ν_i is the backoff time of the i-th source node.

III. Frequency Offset Estimation

Frequency offset will be estimated in both time slots.

A. First Time Slot

As in [1], we can estimate ε_{D,S_i} as

$$\hat{\varepsilon}^{(1)}_{D,S_i} = \frac{N \sum\limits_{\theta=0}^{N/2\mu-1} \sum\limits_{l=0}^{L-1} |\mathbf{\Pi}^{(1)}_{\theta,l,\nu_i}| \cdot (2\theta-1) \cdot \arg\left\{\mathbf{\Pi}^{(1)}_{\theta,l,\nu_i}\right\}}{2\pi \sum\limits_{\theta=0}^{N/2\mu-1} \sum\limits_{l=0}^{L-1} |\mathbf{\Pi}^{(1)}_{\theta,l,\nu_i}| \cdot (2\theta-1)^2\mu}, \tag{7}$$

where $\mathbf{\Pi}^{(1)}_{\theta,l,\nu_i} = y_{D,1}\left(\nu_i + \frac{N}{2} + \theta\mu + l\right) \times y^*_{D,1}\left(\nu_i + \frac{N}{2} - \mu - \theta\mu + l\right)$. From [1], the variance error of (7) satisfies

$$\mathrm{Var}\left\{\hat{\varepsilon}^{(1)}_{D,S_i}\right\} \geq \frac{3N}{2\pi^2(N^2-\mu^2)\cdot\alpha\mathcal{L}_u\cdot\mathrm{SNR}} \tag{8}$$

with the Signal-to-Noise-Ratio (SNR) being defined as $\mathrm{SNR} = \bar{P}\sigma_s^2/\sigma_w^2$.

B. Second Time Slot

In this section, frequency offset estimation in only AF mode is discussed. From [15], the frequency offset ε_{D,R,S_i} is identical to ε_{D,S_i} in the AF relaying mode without considering Doppler Shift, and we have

$$\hat{\varepsilon}^{(2)}_{D,R,S_i} = \frac{N \sum\limits_{\theta=0}^{N/2\mu-1} \sum\limits_{l=0}^{2L-1} |\mathbf{\Pi}^{(2)}_{\theta,l,\nu_i}| \cdot (2\theta-1) \cdot \arg\left\{\mathbf{\Pi}^{(2)}_{\theta,l,\nu_i}\right\}}{2\pi \sum\limits_{\theta=0}^{N/2\mu-1} \sum\limits_{l=0}^{L-1} |\mathbf{\Pi}^{(2)}_{\theta,l,\nu_i}| \cdot (2\theta-1)^2\mu}, \tag{9}$$

where $\mathbf{\Pi}^{(2)}_{\theta,l,\nu_i} = y_{D,2}\left(\nu_i + \frac{N}{2} + \theta\mu + l\right) \times y^*_{D,2}\left(\nu_i + \frac{N}{2} - \mu - \theta\mu + l\right)$. Similarly, the variance error of $\hat{\varepsilon}^{(2)}_{D,R,S_i}$ satisfies

$$\mathrm{Var}\left\{\hat{\varepsilon}^{(2)}_{D,R,S_i}\right\} \geq \frac{3N}{2\pi^2(N^2-\mu^2)\cdot\alpha\cdot\mathscr{C}(\alpha,\mu,\mathrm{SNR})}, \tag{10}$$

where $h_{D,R|w}[n] = \sum_{l'=0}^{L-1} \tilde{w}_{R,1}(n - l')h_{D,R}[l']$ and $\mathscr{C}(\alpha,\mu,\mathrm{SNR}) = \frac{\mu(1-\alpha)\cdot\mathcal{L}_u\cdot\mathrm{SNR}^2}{(1-\alpha)ML\cdot\mathcal{L}_u\cdot\mathrm{SNR} + \mu(2L-1)\left(\alpha\cdot\mathrm{SNR}+\frac{ML}{\mu}\right)}$.

C. Combining Estimation

Like in [15], the estimation of ε_{D,S_i} can be represented as

$$\hat{\varepsilon}_{D,S_i} = \lambda_1\hat{\varepsilon}^{(1)}_{D,R,S_i} + \lambda_2\hat{\varepsilon}^{(2)}_{D,R,S_i}, \tag{11}$$

where λ_1 and λ_2 are two non-negative coefficients with $\lambda_1 + \lambda_2 = 1$. By solving λ_1 as $\lambda_1 = \frac{\mathcal{L}_u\cdot\mathrm{SNR}}{\mathcal{L}_u\cdot\mathrm{SNR}+\mathscr{C}(\alpha,\mu,\mathrm{SNR})}$, the variance of (11) is minimized, as given by

$$\mathrm{Var}\left\{\hat{\varepsilon}_{D,S_i}\right\} \geq \frac{3N}{2\pi^2\alpha(N^2-\mu^2)\left(\mathcal{L}_u\cdot\mathrm{SNR}+\mathscr{C}(\alpha,\mu,\mathrm{SNR})\right)}. \tag{12}$$

IV. Channel Estimation

Least-Square (LS) channel estimation by using the proposed preamble is developed.

A. LS Channel Estimation

The received vector in the second time slot is

$$\begin{aligned}\mathbf{y}_{D,2} = {} & \rho_R\sqrt{\frac{\alpha(1-\alpha)\bar{P}^2\sigma_s^4}{M}} \sum_{i=0}^{M-1} e^{\frac{\jmath 2\pi\nu_i\varepsilon_{D,S_i}}{N}} \mathbf{E}_{D,R,S_i}\mathbf{S}_i\mathbf{h}_{D,R,S_i} \\ & + \rho_R\sqrt{\frac{(1-\alpha)L\bar{P}}{\mu}} \sum_{i=0}^{M-1} e^{\frac{\jmath 2\pi\nu_i\varepsilon_{D,R}}{N}} \mathbf{E}_{D,R}\mathbf{W}_R^{(i)}\mathbf{h}_{D,R} \\ & + \mathbf{w}_{D,2},\end{aligned} \tag{13}$$

where $\mathbf{E}_{D,R,S_i}$, $\mathbf{E}_{D,R}$, $\mathbf{\Lambda}_{D,R,S_i}$, $\mathbf{\Lambda}_{D,R}$, $\boldsymbol{S}_i$, $\boldsymbol{W}_R^{(i)}$ and $\mathbf{\Omega}_R(d)$ are defined in (14), $\rho_R = \left(\frac{\alpha\bar{P}\sigma_s^2}{M} + \frac{L\cdot\sigma_w^2}{\mu}\right)^{-1}$, $\mathbf{w}_{D,2} = [\mathbf{w}^T_{D,2,0}, \mathbf{w}^T_{D,2,1}, \cdots, \mathbf{w}^T_{D,2,N/\mu-1}]^T$, $\mathbf{w}_{D,2,i} = [w_{D,2}(i\cdot\mu), w_{D,2}(i\cdot\mu+1), \cdots, w_{D,2}(i\cdot\mu+2L-1)$, and $\mathbf{h}_{D,R,S_i} = \left(\mathbf{h}^T_{R,S_i}\otimes\mathbf{h}^T_{D,R}\right)^T$.

Defining $\mathbf{P}_{D,R,S_i} = \rho_R e^{\frac{\jmath 2\pi\nu_i\varepsilon_{D,S_i}}{N}}\sqrt{(1-\alpha)\bar{P}}\mathbf{E}_{D,R,S_i}\mathbf{S}_i$, an LS channel estimator can be designed as

$$\hat{\mathbf{h}}^{\mathrm{LS}}_{D,R,S_i} = \left(\mathbf{P}^H_{D,R,S_i}\mathbf{P}_{D,R,S_i}\right)^{-1}\mathbf{P}^H_{D,R,S_i}\mathbf{y}_{D,2}, \tag{15}$$

and the mean-square error (MSE) of (15) is obtained as

$$\mathrm{MSE}\left(\hat{\mathbf{h}}^{\mathrm{LS}}_{D,R,S_i}\right) = \frac{1}{2L-1}\mathbb{E}\left\{\left\|\hat{\mathbf{h}}^{\mathrm{LS}}_{D,R,S_i} - \mathbf{h}_{D,R,S_i}\right\|_2^2\right\}. \tag{16}$$

where $\|\mathbf{x}\|_2^2$ represents the sum of square of each entry of vector $\mathbf{x}$.

B. PEP Analysis

Like in [16], an $N\times T$ orthogonal block coding matrix $\bar{\mathbf{X}}_S$ is assumed here. In AF relaying, the probability that $\bar{\mathbf{X}}_S$ will be mistaken for another code $\bar{\mathbf{L}}_S$ is upper bounded by:

$$\begin{aligned}& \mathrm{P}^{\mathrm{AF}}_{\mathrm{r}}\left\{\bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0<\alpha<1\right\} \\ & \leq \left(\prod_{n=0}^{2L-2}\frac{1}{1+\frac{\bar{\gamma}_{DRS,n}\ell_n}{4}}\right)\left(\prod_{n=0}^{L-1}\frac{1}{1+\frac{\bar{\gamma}_{DS,n}\ell_n}{4}}\right),\end{aligned} \tag{17}$$

where $\bar{\gamma}_{DS,n}$ and $\bar{\gamma}_{DRS,n}$ represent the SINR of the $S\to D$ and $S\to R\to D$ channels, respectively, in the

$$
\mathbf{E}_{D,R,S_i} = \mathrm{diag}\left\{ \mathbf{\Lambda}_{D,R,S_i}, e^{\frac{j2\pi\mu\varepsilon_{D,S_i}}{N}} \mathbf{\Lambda}_{D,R,S_i}, e^{\frac{j4\pi\mu\varepsilon_{D,S_i}}{N}} \mathbf{\Lambda}_{D,R,S_i}, \cdots, e^{\frac{j2\pi(N-\mu)\varepsilon_{D,S_i}}{N}} \mathbf{\Lambda}_{D,R,S_i} \right\}, \tag{14a}
$$

$$
\mathbf{E}_{D,R} = \mathrm{diag}\left\{ \mathbf{\Lambda}_{D,R}, e^{\frac{j2\pi\mu\varepsilon_{D,R}}{N}} \mathbf{\Lambda}_{D,R}, e^{\frac{j4\pi\mu\varepsilon_{D,R}}{N}} \mathbf{\Lambda}_{D,R}, \cdots, e^{\frac{j2\pi(N-\mu)\varepsilon_{D,R}}{N}} \mathbf{\Lambda}_{D,R} \right\}, \tag{14b}
$$

$$
\mathbf{\Lambda}_{D,R,S_i} = \mathrm{diag}\left\{ 1, e^{\frac{j2\pi\varepsilon_{D,S_i}}{N}}, e^{\frac{j4\pi\varepsilon_{D,S_i}}{N}}, \cdots, e^{\frac{j2(2L-2)\pi\varepsilon_{D,S_i}}{N}} \right\}, \quad \mathbf{\Lambda}_{D,R} = \mathrm{diag}\left\{ 1, e^{\frac{j2\pi\varepsilon_{D,R}}{N}}, e^{\frac{j4\pi\varepsilon_{D,R}}{N}}, \cdots, e^{\frac{j2(2L-2)\pi\varepsilon_{D,R}}{N}} \right\}, \tag{14c}
$$

$$
\boldsymbol{S}_i = \left[s_i(0)\mathbf{I}_{2L-1}, s_i(\mu)\mathbf{I}_{2L-1}, s_i(2\mu)\mathbf{I}_{2L-1}, \cdots, s_i(N-\mu)\mathbf{I}_{2L-1} \right]^T, \tag{14d}
$$

$$
\boldsymbol{W}_R^{(i)} = \left[\mathbf{\Omega}_R^T(\nu_i), \mathbf{\Omega}_R^T(\nu_i+\mu), \mathbf{\Omega}_R^T(\nu_i+2\mu), \cdots, \mathbf{\Omega}_R^T(\nu_i+N-\mu) \right]^T, \tag{14e}
$$

$$
\mathbf{\Omega}_R(d) = \begin{bmatrix}
w_{R,1}(d) & 0 & 0 & \dots & 0 & 0 \\
w_{R,1}(d+1) & w_{R,1}(d) & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
w_{R,1}(d+L-1) & w_{R,1}(d+L-2) & w_{R,1}(d+L-3) & \dots & w_{R,1}(d+1) & w_{R,1}(d) \\
0 & w_{R,1}(d+L-1) & w_{R,1}(d+L-2) & \dots & w_{R,1}(d+2) & w_{R,1}(d+1) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \dots & 0 & w_{R,1}(d+L-1)
\end{bmatrix}. \tag{14f}
$$

n-th multipath tap, and ℓ_n is the n-th eigenvalue of $\left(\bar{\mathbf{X}}_S - \bar{\mathbf{L}}_S\right)\left(\bar{\mathbf{X}}_S - \bar{\mathbf{L}}_S\right)^H$. In the high SINR regime, (17) can be rewritten as

$$
\begin{aligned}
&\lim_{\mathrm{SINR}\to\infty} \mathrm{P}_{\mathrm{r}}^{\mathrm{AF}} \left\{ \bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0 < \alpha < 1 \right\} \\
&\le \underbrace{\left(\frac{4\left[\frac{2M\mathcal{L}_u(1-\alpha)}{\alpha} + 1\right]}{\alpha(1-\alpha)\mathrm{SNR}} \right)^{2L-1} \left(\frac{4M}{\alpha \mathrm{SNR}} \right)^{L}}_{\text{multipath diversity gain}} \\
&\quad \times \left(\prod_{n=0}^{2L-2} \frac{1}{\left|\mathbf{h}_{D,R,S_i}[n]\right|^2 \ell_n} \right) \left(\prod_{n=0}^{L-1} \frac{1}{\left|\mathbf{h}_{D,S}[n]\right|^2 \ell_n} \right).
\end{aligned} \tag{18}
$$

V. Simulation Results

In this section, the performance of the proposed preamble in both frequency offset and channel estimation is evaluated. An outdoor dispersive, fading environment is considered, and an exponentially decaying power delay profile with a root mean square (RMS) width of $8T_s$ is assigned for the channel.

Fig. 1 shows the variance of the frequency offset estimation in the proposed AF relaying mode as a function of power allocation ratio α. For a given SNR, the variance error with $L = 2$ outperforms that with $L = 1$ (flat-fading channel). However, with L being larger than 2, the noise accumulated in the relay will degrade the frequency offset estimation, and the variance error is always worse than that with $L = 1, 2$ until $\alpha > 0.3$.

Fig. 2 illustrates the variance of the frequency offset estimations in both the $S \to D$ and $S \to R \to D$ links for the first and second time slots, respectively, with Doppler Shift being considered. The variance error of the $S \to D$ link is a monotonically decreasing function of α. However, that for the $S \to R \to D$ link is a convex function of α, and there is always an optimal α to minimize the variance error.

The PEP performance of the AF relaying as a function of power allocation ratio α is illustrated in Fig. 3. Since the multipath diversity gain is improved as L increases, a larger L implies a better PEP performance.

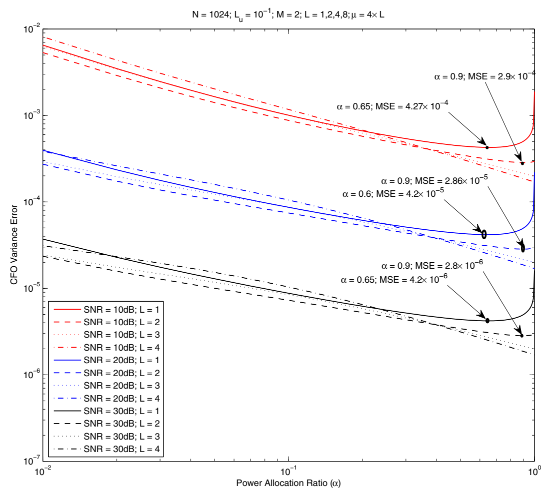

Fig. 1. Frequency offset variance error for the AF mode with ($N = 1024$, $\mathcal{L}_u = 10^{-1}$, $M = 2$, $L = 1, 2, 4, 8$, $\mu = 4L$) and without Doppler Shift.

VI. Conclusions

Cooperative OFDM frequency offset and channel estimation was treated. A new preamble with its pilot subcarriers being sparsely distributed in the frequency-domain was proposed, and its time-domain structure appears a central-symmetric and constant modulus property. This structure enabled the proposed preamble a high multipath interference combating capability. In a co-relay scenario with multiple access interference existing among source nodes, a backoff modulation can be performed in each source to employ an orthogonal transmission of different source nodes and, as a result, the interference among source nodes can be mitigated. In the proposed frequency offset and channel estimation algorithms, the total transmit power allocated on the source and relay

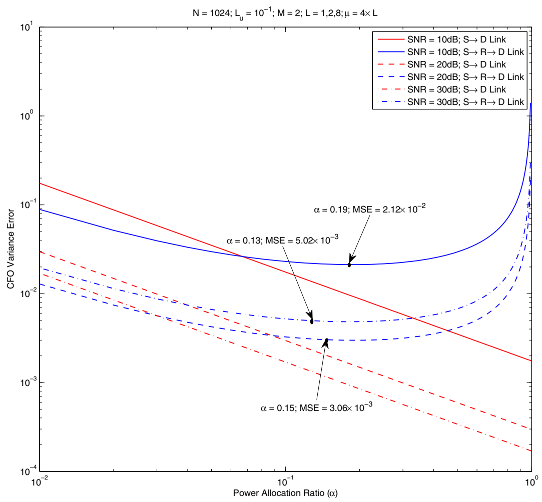

Fig. 2. Frequency offset variance errors for the AF mode for both the $S \rightarrow D$ and $S \rightarrow R \rightarrow D$ links with ($N = 1024$, $\mathcal{L}_u = 10^{-1}$, $M = 2$, $L = 1, 2, 8$, $\mu = 4L$) and with Doppler Shift.

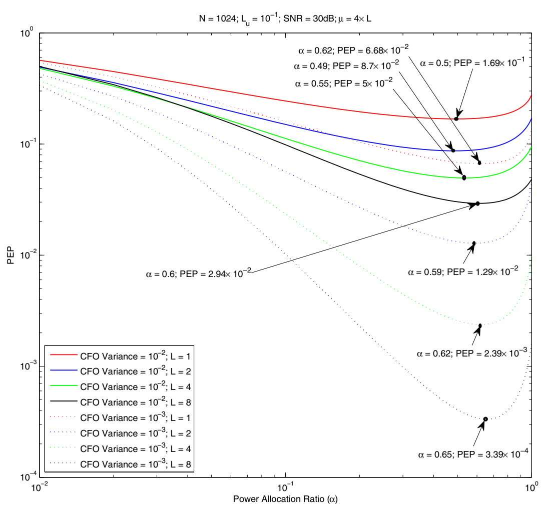

Fig. 3. PEP for the AF mode with ($N = 1024$, $\mathcal{L}_u = 10^{-1}$, $M = 2$, $L = 1, 2, 4, 8$, $\mu = 4L$, SNR=30 dB).

nodes was kept constant, but the power allocation ratio α can be adaptively adjusted to optimize the proposed estimators. With an orthogonal block coding matrix being transmitted in each source node, the PEP of a co-relay cooperative communications was also derived by considering both the frequency offset and channel estimation errors. Numerical results showed that the PEP for an AF co-relay communication system is always a convex function in terms of α, and for different multipath scenarios, an optimal α can always be found to minimize the PEP.

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