Joint Base-Station Association, Channel Assignment, Beamforming and Power Control in Heterogeneous Networks

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***Abstract*—Heterogeneous cellular networks (HetNets), where low-power base-stations are overlaid with conventional macro base-stations, is a promising technique to improve coverage and capacity of the macro-only networks. To realize the potential benefits of HetNets, it is crucial to jointly optimize the user association, channel assignment, beamforming and power control to ensure that the inter- and intra-cell interference will not overwhelm the cell-splitting gains. This paper presents an iterative algorithm to solve the joint optimization problem with an objective of maximizing the network sum rate and simultaneously guaranteeing the individual user quality-of-service. The proposed algorithm is built on the so-called convex-concave procedure, and the feasibility issue is handled by l_1-norm heuristic. Numerical results demonstrate the large gains over currently used methods for cellular networks.**

I. Introduction

Most recently, Third Generation Partnership Project (3GPP) Long Term Evolution (LTE)-Advance has started a new study item to investigate heterogeneous network (HetNet) deployment as an efficient way to improve system capacity as well as effectively enhance network coverage. Unlike traditional heterogeneous networks that deal with the interworking of wireless local area networks and cellular networks, in this new paradigm in the cellular network domain, a HetNet is a network where low-power low-complexity base stations are embedded in the conventional macro-cellular network. The low-power "pico" nodes and high-power macro nodes can be maintained by the same operator and share the same frequency band, provided by the operator. This cell splitting can provide large gains in overall system performance both in the physical layer, since the average transmitter-receiver distance is significantly reduced, and the MAC layer, since the average number of served users per base-station is also lowered.

However, the benefits of the HetNet is not possible without the well-designed resources allocation strategy. Base-station association can be regarded as one of the degrees of freedom in resource allocation, and is of crucial importance in a HetNet. In a marco-only cellular network, the base-station association is based on the received power at the mobile on the downlink. In a HetNet, however, this association will lead to the case where the macro base-stations become resource constrained while the pico base-stations serve very few users, due to the much stronger transmit power of macro base-stations. To balance the load, a mobile may be connected to a closer pico base-station even through the received power from a macro base-station could be higher. This may cause severe interference if the radio resources are not carefully partitioned. Hence, the base-station association and radio resources allocation should be optimized *jointly* to mitigate the interference and achieve the promised performance gains [1], [2].

In this paper, we study a joint base-station association, beamforming, channel assignment and power control problem in HetNets. The objective is to maximize the system sum rate, while ensuring individual user's quality-of-service (QoS). In the literature, only parts of design variables have been considered. For example, [3] focus on spectrum channel assignment in HetNets . Joint cell association, channel selection and power allocation has been studied in [1], [2], [4], but without beamforming. In particular, it is still unclear the impact of multiple antennas of the macro and pico base stations on the joint resource allocation.

In general, the joint optimization is a difficult problem. Even in the simplified case that only power allocation is optimized to maximize the sum rate in multiple interfering links, the problem turns out to be NP-hard. Thus, the global solution is prohibitive for reasonably sized network. The main contribution of this paper is an iterative algorithm derived via transforming the initial mixed-integer non-convex problem into an inner-outer formulation with continuous variables only. The algorithm requires only few inner-outer iterations to converge to (local) optimal solution. In the inner problem, we solve a successive set of convex optimization problems to optimize power allocation for the fixed base-station association. Beamforming vectors and channel assignment are implicitly optimized. In the outer problem, base-station association is updated with fixed other resources in a greedy fashion. Besides, to handle the feasibility issue, we incorporate the l_1-norm heuristic [5] into our algorithm. If the original set of user rate constraints is infeasible, the l_1-norm heuristic can make as many users being served as possible with the prescribed QoS constraints. Besides, the l_1-norm heuristic can also avoid the possible infeasibility of the inner problem during the inner-outer iterations.

II. Problem formulation

We consider a uplink cellular network consisting of coordinated B macro base-stations and $M-B$ pico base-stations with universal frequency reuse. Open access is assumed for the picocell. The mth base-station is equipped with Q_m antennas while each user has a single antenna. There are total K users in the network and they simultaneously transmit on N orthogonal (in the time or frequency domain) resource slots (channels). We assume that each user is served by only one base-station and the coordinated base-stations only exchange channel quality measurements. The received signal at base-station m on channel n is expressed as

$$\mathbf{y}_m(n) = \sum_{k=1}^{K} \sqrt{p_k(n)}\mathbf{h}_{k,m}(n)s_k(n) + \mathbf{z}_m(n), \tag{1}$$

where $p_k(n)$ is the transmit power of user k over channel n, $\mathbf{h}_{k,m}(n) \in \mathbb{C}^{Q_m \times 1}$ is the complex channel from user k to base-station m on channel n (which includes small-scale fading, large-scale fading and path attenuation), $s_k(n)$ is the complex symbol

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sent by user k on channel n with average power normalized to 1, and $\mathbf{z}_m(n)$ is the additive circular symmetric white Gaussian noise (AWGN) vector at base-station m with distribution $\mathcal{CN}(0, \sigma_m^2(n))$ on each receive antenna.

If user k is served by base-station π_k, a set of linear beamforming vectors $\mathbf{w}_{k,\pi_k}(n) \in \mathbb{C}^{Q_{\pi_k} \times 1}$, with $\|\mathbf{w}_{k,\pi_k}(n)\|^2 = 1$, are used to extract its signals, resulting in

$$\begin{aligned}\hat{s}_k(n) &= \mathbf{w}_{k,\pi_k}^H(n)\mathbf{y}_{\pi_k}(n) \\ &= \sqrt{p_k(n)}\mathbf{w}_{k,\pi_k}^H(n)\mathbf{h}_{k,\pi_k}(n)s_k(n) \\ &+ \sum_{l\neq k}^{K} \sqrt{p_l(n)}\mathbf{w}_{k,\pi_k}^H(n)\mathbf{h}_{l,\pi_k}(n)s_l(n) + \mathbf{w}_{k,\pi_k}^H(n)\mathbf{z}_{\pi_k}(n). \end{aligned} \tag{2}$$

From (2), the signal-to-interference-plus-noise ratio (SINR) for user k on channel n can be expressed as

$$\mathrm{SINR}_{k,\pi_k}(n) = \frac{p_k(n)\mathbf{w}_{k,\pi_k}^H(n)\mathbf{G}_{k,\pi_k}(n)\mathbf{w}_{k,\pi_k}(n)}{\sum_{l\neq k} p_l(n)\mathbf{w}_{k,\pi_k}^H(n)\mathbf{G}_{l,\pi_k}(n)\mathbf{w}_{k,\pi_k}(n) + \sigma_{\pi_k}^2(n)\|\mathbf{w}_{k,\pi_k}(n)\|^2}, \tag{3}$$

where $\mathbf{G}_{l,\pi_k}(n) = \mathbf{h}_{l,\pi_k}(n)\mathbf{h}_{l,\pi_k}^H(n) \in \mathbb{C}^{Q_{\pi_k} \times Q_{\pi_k}}$.

In this work, we jointly optimize the base-station association π_k, beamforming vector $\mathbf{w}_{k,\pi_k}(n)$, and the power allocation $p_k(n)$ for all K users over N channels. The objective is to maximize the sum rate of the system while satisfying the individual rate constraints. Note that channel assignments are implicitly conducted by adjusting the power. The case $p_k(n) = 0$ corresponds to an unused channel. Each user can be allocated multiple channels as long as the transmit power is constrained as

$$\sum_{n=1}^{N} p_k(n) \leq p_{\max,k}, \forall k. \tag{4}$$

We stack all powers into a power vector as $\mathbf{p} \triangleq [p_1(1), \ldots, p_1(N), \ldots, p_K(1), \ldots, p_K(N)]^T$. Thus, $\mathbf{p}$ is constrained to a convex set $\mathcal{P} = \{\mathbf{p} \in \mathbb{R}^{NK} : p_k(n) \geq 0, \forall k, \forall n, \sum_n p_k(n) \leq p_{\max,k}, \forall k\}$.

We observe from (3) that each user's SINR is independent of other users' base-station associations. This motivates us to introduce an association matrix as

$$[\boldsymbol{\alpha}]_{k,m} = \begin{cases} 1 & \text{if user } k \text{ is associated with base-station } m \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

and formulate the optimization problem as

$$\begin{aligned} \underset{\boldsymbol{\alpha},\mathbf{p},\mathbf{w}_{k,m}(n)}{\text{maximize}} \quad & \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m}\log_2(1 + \mathrm{SINR}_{k,m}(n)) && \text{(6a)} \\ \text{s.t.} \quad & \sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m}\log_2(1 + \mathrm{SINR}_{k,m}(n)) \geq \gamma_k, \forall k, && \text{(6b)} \\ & \sum_{m=1}^{M} \alpha_{k,m} = 1, \forall k, && \text{(6c)} \\ & \alpha_{k,m} \in \{0,1\}, \forall k, \forall m, && \text{(6d)} \\ & \|\mathbf{w}_{k,m}(n)\|^2 = 1, \forall k, \forall m, \forall n, && \text{(6e)} \\ & \mathbf{p} \in \mathcal{P} && \text{(6f)} \end{aligned}$$

where $\mathrm{SINR}_{k,m}(n)$ is expressed as (3) by replacing π_k with m, γ_k is the minimum rate requirement for the user k, and (6c) ensures that each user is only served by one base-station.

The above formulation is a mixed-integer nonlinear non-convex problem. However, the following theorem allows us to drop the integer constraint (6d), and instead deal with continuous variables without sacrificing the optimality.

Theorem 1: The mixed-integer problem (6) is equivalent to the one where $\alpha_{k,m} \in \{0,1\}$ is replaced by $0 \leq \alpha_{k,m} \leq 1$.

Proof: We show that although we relax $\alpha_{k,m}, \forall k, \forall m$ to continuous values between 0 and 1, the optimal solution will arrive at $\alpha_{k,m} \in \{0,1\}, \forall k, \forall m$ by contradiction. Suppose at the optimum, for some particular user $\bar{k} \in \{1, \ldots, K\}$, we have $0 \leq \alpha_{\bar{k},m}^\star < 1, \forall m$ and $\sum_{m=1}^{M} \alpha_{\bar{k},m}^\star = 1$. Let us define $R_{\bar{k},m} = \sum_{n=1}^{N} \log_2(1 + \mathrm{SINR}_{\bar{k},m}^\star(n)), \forall m$, where $\mathrm{SINR}_{\bar{k},m}^\star(n)$ is obtained by substituting optimal $\mathbf{p}^\star$ and $w_{\bar{k},m}^\star(n)$ into (3). Then we can increase the left-hand side of (6b) by increasing the $\alpha_{\bar{k},\tilde{m}}^\star$ and decreasing other $\alpha_{\bar{k},m}^\star, \forall m \neq \tilde{m}$ where $\tilde{m} = \arg\max_m\{R_{\bar{k},m}\}$, until $\alpha_{\bar{k},\tilde{m}} = 1$. Note that the maximum rate $R_{\bar{k},\tilde{m}}$ is unique with probability 1, since the channels are independent random variables. The resulting new $\{\alpha_{\bar{k},m}\}$ are still feasible and increase the objective function (6a) accordingly, which is contradictory to the optimality of $\{\alpha_{\bar{k},m}^\star\}$. ■

A. Optimal beamforming vector

If we assume $\mathbf{p}$ and $\boldsymbol{\alpha}$ are given, it is clear that $\mathrm{SINR}_{k,m}(n)$ only depends on its own beamforming vector $\mathbf{w}_{k,m}(n)$ from (3). In other words, all the beamforming vectors are decoupled in the objective function of (6a). Hence, considering log function is monotonically increasing, the optimal beamforming vectors can be obtained by solving

$$\mathbf{w}_{k,m}^\star(n) = \arg\max_{\|\mathbf{w}_{k,m}(n)\|^2=1} \mathrm{SINR}_{k,m}(n) \tag{7}$$

This problem has a well-known solution [6], expressed as

$$\mathbf{w}_{k,m}^\star(n) = \beta_{k,m}(n)\mathbf{T}_{k,m}^{-1}(n)\mathbf{h}_{k,m}(n) \tag{8}$$

where $\mathbf{T}_{k,m}(n) = \sum_{l\neq k} p_l(n)\mathbf{G}_{l,m}(n) + \sigma_m^2(n)\mathbf{I}_{Q_m}$, and $\beta_{k,m}(n)$ is a normalized factor.

With the optimal receive beamforming vector (8) and Theorem 1, the problem (6) can now be rewritten as

$$\begin{aligned} \underset{\mathbf{p}\in\mathcal{P},\boldsymbol{\alpha}}{\text{minimize}} \quad & -\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m}\log_2\left(1 + \frac{p_k(n)}{I_{k,m}(n)}\right) && \text{(9a)} \\ \text{s.t.} \quad & \gamma_k - \sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m}\log_2\left(1 + \frac{p_k(n)}{I_{k,m}(n)}\right) \leq 0, \forall k, && \text{(9b)} \\ & \sum_{m=1}^{M} \alpha_{k,m} = 1, \forall k, 0 \leq \alpha_{k,m} \leq 1, \forall k, \forall m, && \text{(9c)} \end{aligned}$$

where the objective is the negated sum rate, and $I_{k,m}(n) = \left[\mathbf{h}_{k,m}^H(n)\mathbf{T}_{k,m}^{-1}(n)\mathbf{h}_{k,m}(n)\right]^{-1}$ which can be shown as a concave function of $\mathbf{p}$.

III. The Proposed Algorithm

To solve problem (9), we propose an iterative algorithm which optimizes the base association and power allocation through outer and inner loop alternatively. Specifically, this algorithm starts with calculating the initial base association $\boldsymbol{\alpha}^0$ and power allocation $\mathbf{p}^0$. We will discuss the initialization methods in Section IV-A. At the beginning of i-th iteration ($i \geq 1$), we try to find the optimal $\mathbf{p}$ for (9) with $\boldsymbol{\alpha} = \boldsymbol{\alpha}^i$ (inner loop). However, since the problem is non-convex with no known convex reformulation, the globally optimal solution is prohibitive for reasonably sized network. Our approach is to solve sequential convex approximations by exploiting the structure of the difference-of-convex (DC) functions, which will be elaborated in Section III-A. Although global optimality is not guaranteed, this approach works efficiently and brings significant improvement to existing systems. Then we take the obtained (locally) optimal solution as $\mathbf{p}^{i+1}$.

Next we compute the optimal $\boldsymbol{\alpha}$ for (9) with $\mathbf{p} = \mathbf{p}^{i+1}$ (outer loop). We will show in Section III-B that the optimal solution can be found by a simple algorithm: for each user, the algorithm finds the base-station from which this user benefits the most. We denote this optimal solution by $\boldsymbol{\alpha}^{i+1}$ and then the $(i+1)$-th iteration starts.

Note that the proposed algorithm belongs to the class of coordinate descent methods [7], yielding non-increasing objective values as the iterations continue. Since the objective function is lower bounded by a finite value for the considered system, this algorithm will converge after a finite number of iterations. In our numerical experiments given in Section IV-A, it takes very few iterations for the algorithm to converge.

A. Inner power allocation optimization

When $\boldsymbol{\alpha} = \boldsymbol{\alpha}^i$, we first rewrite the inner optimization problem using DC functions as:

$$\begin{aligned} \underset{\mathbf{p}\in\mathcal{P}}{\text{minimize}} \quad & f_0(\mathbf{p}) \triangleq u_0(\mathbf{p}) - v_0(\mathbf{p}) \\ \text{s.t.} \quad & f_k(\mathbf{p}) \triangleq u_k(\mathbf{p}) - v_k(\mathbf{p}) \leq 0, \forall k, \end{aligned} \tag{10}$$

where

$$u_0(\mathbf{p}) = -\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m} \log_2\left(p_k(n) + I_{k,m}(n)\right)$$

$$v_0(\mathbf{p}) = -\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m} \log_2\left(I_{k,m}(n)\right)$$

$$u_k(\mathbf{p}) = \gamma_k - \sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m} \log_2\left(p_k(n) + I_{k,m}(n)\right)$$

$$v_k(\mathbf{p}) = -\sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_{k,m} \log_2\left(I_{k,m}(n)\right)$$

are all convex functions of $\mathbf{p}$. However, (10) is a non-convex problem, hence is difficult to solve.

DC programming in global optimization has attracted a lot of attention in recent years [8]. However, the global algorithms often have prohibitive complexity to solve real life DC programs in their true dimension. Thus, local methods based on convex analysis approach is more plausible in practice [9], [10]. The DC approach tackling the similar power allocation problem has been adopted in [11], [12]. However, the user rate constraints were not considered in their formulation. To solve our problem (10), we resort to solving a sequence of convex problems given as (11). We describe the whole algorithm in Algorithm 1, which is known as convex-concave procedure generally [10].

$$\begin{aligned} \underset{\mathbf{p}\in\mathcal{P}}{\text{minimize}} \quad & \hat{f}_0(\mathbf{p}) \triangleq u_0(\mathbf{p}) - v_0(\bar{\mathbf{p}}) - \nabla v_0(\bar{\mathbf{p}})^T(\mathbf{p}-\bar{\mathbf{p}}) \\ \text{s.t.} \quad & \hat{f}_k(\mathbf{p}) \triangleq u_k(\mathbf{p}) - v_k(\bar{\mathbf{p}}) - \nabla v_k(\bar{\mathbf{p}})^T(\mathbf{p}-\bar{\mathbf{p}}) \leq 0, \forall k, \end{aligned} \tag{11}$$

where $\nabla v_k(\bar{\mathbf{p}})^T$ is the gradient at $\bar{\mathbf{p}}$.

As shown in (11), we use first-order Taylor expansion to linearize the nonconvex portions of the objective function and the constraint functions, resulting in a convex optimization problem. It can be solved efficiently using interior-point methods [13]. Due to the fact that the first-order Taylor expansion is a global underestimate of the convex function $v_k(\mathbf{p})$, we have

Algorithm 1

1: **Initialize**: $t = 0$, $\mathbf{p}^{i,0} = \mathbf{p}^i$;
2: **repeat**
3: Solve (11) with $\bar{\mathbf{p}} = \mathbf{p}^{i,t}$ to obtain the optimal solution $\mathbf{p}^\star$;
4: $\mathbf{p}^{i,t+1} = \mathbf{p}^\star$;
5: $t = t+1$;
6: **until** $\|\mathbf{p}^{i,t+1} - \mathbf{p}^{i,t}\| \leq \epsilon$ with a given tolerance $\epsilon > 0$
7: **Output**: $\mathbf{p}^{i+1} = \mathbf{p}^{i,t}$.

$$f_k(\mathbf{p}) \leq \hat{f}_k(\mathbf{p}), \forall \mathbf{p}. \tag{12}$$

Thus, the solution to (11) is always feasible in (10). We solve (11) sequentially with updated $\bar{\mathbf{p}}$ as described in Algorithm 1. Actually, Algorithm 1 is a decent method, producing non-increasing objective values of (10) because

$$f_0(\mathbf{p}^{i,t}) = \hat{f}_0(\mathbf{p}^{i,t}) \geq \hat{f}_0(\mathbf{p}^{i,t+1}) \geq f_0(\mathbf{p}^{i,t+1}). \tag{13}$$

It can be shown that Algorithm 1 converges to a local minimum solution of the original non-convex problem of (10) [14].

B. Outer base-station association optimization

When $\mathbf{p} = \mathbf{p}^{i+1}$, the base-station association problem can be rewritten as

$$\underset{\alpha_{k,m}}{\text{minimize}} \quad -\sum_{k=1}^{K}\sum_{m=1}^{M} \alpha_{k,m} R_{k,m}^{i+1} \tag{14a}$$

$$\text{s.t.} \quad \gamma_k - \sum_{m=1}^{M} \alpha_{k,m} R_{k,m}^{i+1} \leq 0, \quad \forall k, \tag{14b}$$

$$\sum_{m=1}^{M} \alpha_{k,m} = 1, \forall k, 0 \leq \alpha_{k,m} \leq 1, \forall k, \forall m \tag{14c}$$

where $R_{k,m}^{i+1} = \sum_{n=1}^{N} \log_2\left(1 + \frac{p_k(n)}{I_{k,m}(n)}\right)$, whose value has been obtained through the inner power optimization. This problem can be decomposed into K subproblems, the k-th of which is to find the optimal base-station association for user k:

$$\underset{\alpha_{k,m}}{\text{maximize}} \quad \sum_{m=1}^{M} \alpha_{k,m} R_{k,m}^{i+1} \tag{15a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \alpha_{k,m} = 1, 0 \leq \alpha_{k,m} \leq 1, \forall m \tag{15b}$$

In (15), the constraint of (14b) is dropped because it is redundant. Suppose the inner power optimization problem is feasible for given $\boldsymbol{\alpha}^i$, meaning that (14b) is satisfied, the optimal $\boldsymbol{\alpha}^{i+1}$ can only reduce the value of the left-hand side of (14b). The case where the inner problem is infeasible will be discussed in Section III-C. The linear problem of (15) can be solved by setting all $\{\alpha_{k,m}\}_{m=1}^M$ to zeros, except that $\alpha_{k,\tilde{m}} = 1$, where $\tilde{m} = \arg\max_m \{R_{k,m}^{i+1}\}$. We solve all the above K subproblems independently and finally obtain the solution to (14).

The above algorithm has an interesting interpretation, stating that for each user, we actually choose the base-station from which this user benefits the most and we perform the user association individually. This is possible because the interference produced by each user to both its serving base-station and neighboring base-stations is a function of its transmit power only, and is independent of the user association. Once the power allocation is fixed to $\mathbf{p}^{i+1}$, user assignment at each base-station does not affect the interference elsewhere in the network.

C. feasibility issues

Feasibility issues arise due to the following three reasons. First, the original problem (9) could be infeasible, meaning that some users rate constraints can not be satisfied. Second, during the inner-outer iteration, some $\boldsymbol{\alpha}^i$ might result in an empty feasible set for (10). Once such $\boldsymbol{\alpha}^i$ is fed into (10), no meaningful $\mathbf{p}^{i+1}$ can be obtained and the algorithm can not proceed. Third, the convex approximation formulation of (11) could be infeasible even if the original problem (10) is feasible, since the feasible region might be shrunk due to (12). To handle the feasibility issues, we reformulate (9) into (16) by using the l_1-norm heuristic [5]:

$$\begin{aligned} \underset{\mathbf{p}\in\mathcal{P},\boldsymbol{\alpha},\mathbf{s}}{\text{minimize}} \quad & -\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N}\alpha_{k,m}\log_2\left(1+\frac{p_k(n)}{I_{k,m}(n)}\right)+\mu\|\mathbf{s}\|_1 \\ \text{s.t.} \quad & \gamma_k-\sum_{m=1}^{M}\sum_{n=1}^{N}\alpha_{k,m}\log_2\left(1+\frac{p_k(n)}{I_{k,m}(n)}\right)\le s_k, \forall k, \\ & \sum_{m=1}^{M}\alpha_{k,m}=1, 0\le\alpha_{k,m}\le 1, \forall k,\forall m, \\ & s_k\ge 0,\forall k, \end{aligned} \tag{16}$$

where $\mu > 0$ is a tuneable parameter, $\|\mathbf{x}\|_1 = \sum_i |x_i|$. The basic idea of the above formulation is that if the original problem (9) is feasible, (16) with a sufficiently large μ will give exactly the same solution as (9), resulting in $\mathbf{s} = \mathbf{0}$. On the other hand, if (9) is infeasible, (16) will give a solution which minimizes the number of user rate violations, due to the desired sparsity that l_1-norm will produce [5]. This makes sense in practice because instead of claiming the problem is infeasible, we want to serve as many users as possible with the prescribed quality-of-service constraints.

Furthermore, l_1-norm heuristic in (16) handles the other foresaid feasibility issues during the inner-outer iteration gracefully. Specifically, in the inner optimization, we replace (11) correspondingly with

$$\begin{aligned} \underset{\mathbf{p}\in\mathcal{P},\ \mathbf{s}}{\text{minimize}} \quad & \hat{f}_0(\mathbf{p})+\mu\|\mathbf{s}\|_1 \\ \text{s.t.} \quad & \hat{f}_k(\mathbf{p})\le s_k,\forall k, \\ & s_k\ge 0,\forall k. \end{aligned} \tag{17}$$

Note that (17) is always feasible. A variant of Algorithm 1 is described as follows:

Algorithm 2

1: **Initialize**: $t = 0$, $\mathbf{p}^{i,0} = \mathbf{p}^i$;
2: **repeat**
3: Solve (17) with $\bar{\mathbf{p}} = \mathbf{p}^{i,t}$ to obtain the optimal solution $\mathbf{p}^\star$ and $\mathbf{s}^\star$;
4: $\mathbf{p}^{i,t+1} = \mathbf{p}^\star$, $\mathbf{s}^{i,t+1} = \mathbf{s}^\star$;
5: $t = t + 1$;
6: **until** $\|\mathbf{p}^{i,t+1} - \mathbf{p}^{i,t}\| \le \epsilon$ and $\|\mathbf{s}^{i,t+1} - \mathbf{s}^{i,t}\| \le \epsilon$ with a given tolerance $\epsilon > 0$
7: **Output**: $\mathbf{p}^{i+1} = \mathbf{p}^{i,t}$, $\mathbf{s}^{i+1} = \mathbf{s}^{i,t}$.

The base-station association algorithm given in Section III-B remains the same, because we have exactly the same K subproblems as given in (15).

IV. Numerical results

In the section, we study the performance of the proposed algorithm in heterogeneous networks. We consider a two-tier network consisting of 1 macro base-station (with maximum transmit power of 43 dBm) and 2 pico base-stations (with maximum transmit power of 30 dBm). The base-stations are placed on a line with 100 m separation such that the pico base-stations are on either side of the macro base-station [1]. Users are dropped uniformly at random in the 300m $\times$ 200m area centered on the macro base-station, except that user-base-station distances should exceed 20 m. The maximum transmit power of the mobile users is 23 dBm. We assume identical noise figures at each receiver, and the AWGN power is found as kT_0B, where k is Boltzmann's constant, $T_0 = 290$ Kelvin is the ambient temperature, and $B = 1$ MHz is the equivalent noise bandwidth, i.e., $\sigma_m^2 = 4.0039 \times 10^{-15}$ W. The channel vector $\mathbf{h}_{k,m}(n)$ is modeled as $\mathbf{h}_{k,m}(n) \triangleq L_{k,m}\tilde{\mathbf{h}}_{k,m}(n)$, where $\tilde{\mathbf{h}}_{k,m}(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{Q_m})$ is a circularly symmetric complex Gaussian random vector accounting for Rayleigh fast fading, and $L_{k,m}$ is the large-scale path loss, given as $10\log_{10}(L_{k,m}) = -39 - 20\log_{10}(d_{k,m})$, with $d_{k,m}$ denoting the distance in metre between user k and base-station m. Shadow fading is not considered. In the simulation results, the average sum rate is obtained by averaging the sum rate over 20 independent random locations of the users; for each location, path loss is kept fixed and performance is averaged over 5 independent realizations of the fast fading coefficients. We adopt the method described in Section III-C to handle the user rate requirements. If any user rate requirement is not satisfied after the algorithm converges, we claim the set of user rate requirements is not feasible for the current channel realization, and the sum rate will be set to zero for this particular channel realization.

A. Convergence behavior

We first analyze the convergence of the proposed algorithm for different stating points. We assume $N = 2$ and $K = 5$. The macro base-station has 3 antennas and each pico base-station has a single antenna, which we denote as $\mathbf{Q} = (1, 3, 1)$. All K users have a minimum rate requirements of 0.01 bits/s/Hz. The left subfigure of Fig.1 shows the sum rate versus the number of iterations of the outer loop (l_{out}) for a randomly chosen channel realization. We compare 10 random initializations with the initialization using uniform power allocation and path-loss base association (UPA-PLBA), where $\forall k, \forall n : p_k^0(n) = (p_{\max,k}/N)$, and $\forall k : \alpha_{k,\tilde{m}}^0 = 1$, for $\tilde{m} = \arg\min_m d_{k,m}$; $\alpha_{k,m}^0 = 0, \forall m \ne \tilde{m}$. As can be seen, different initialization may result in local optimal points since the proposed algorithm does not guarantee the global optimality. However, the UPA-PLBA initialization achieves the highest sum rate. This motivates us to use UPA-PLBA for initialization. Another reason for adopting UPA-PLBA as the starting point is that our proposed algorithm should be able to find a better (or at least, no worse) solution than UPA-PLBA, which is often used by default the resource allocation method in practice.

The right subfigure of Fig.1 compares the average sum rate achieved by the random and UPA-PLBA initializations. As shown, the proposed algorithm requires only 3 iterations to converge in this experiment, and the UPA-PLBA initialization performs better than the random one. Hence, we adopt UPA-PLBA in the following simulation.

B. Performance comparison

We compare the proposed algorithm with three simplified strategies as listed in Table I. We provide the average sum rate comparison and the feasibility comparison in Table II, where we assume $N = 2$,

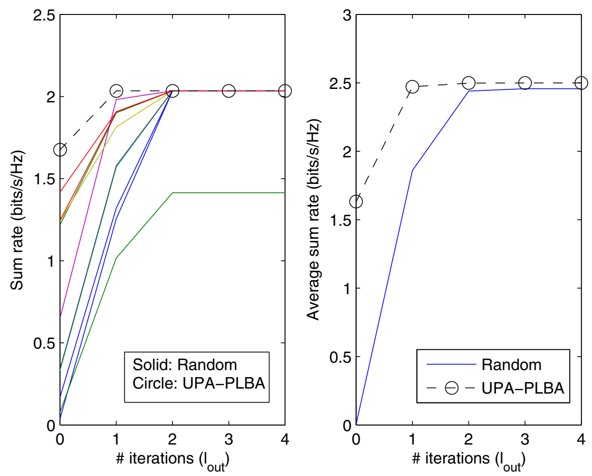

Fig. 1. Convergence behavior of the proposed algorithm, with $N = 2$, $K = 5$, $\mathbf{Q} = (1, 3, 1)$, $\gamma_k = 0.01, \forall k$. Left subfigure shows the sum rate over a randomly chosen channel realization, and right subfigure shows the average sum rate.

TABLE I
METHODS FOR COMPARISON

	Base asso.	BF	Channel allo.	Power allo.
Proposed	Jointly optimized			
Method 1	Path-loss	Jointly optimized		
Method 2	Path-loss	Optimal	Uniform	
Method 3	DL power	Jointly optimized		

TABLE II
AVERAGE SUM RATE AND PROBABILITY OF FEASIBLE QOS, SHOWN IN (PROPOSED, METHOD1,METHOD2,METHOD3). SYSTEM PARAMETER: $N = 2$, $\mathbf{Q} = (1, 3, 1)$, $\gamma_k = 0.01, \forall k$.

K	Average sum rate (bits/s/Hz)	Probability of feasible QoS
1	(0.35, 0.34, 0.30, 0.28)	(0.98, 0.93, 0.91, 0.91)
3	(1.62, 1.26, 1.07, 0.54)	(0.94, 0.82, 0.76, 0.59)
5	(2.50, 2.16, 1.63, 0.87)	(0.80, 0.71, 0.57, 0.27)
7	(2.17, 1.60, 0.91, 0.64)	(0.80, 0.68, 0.48, 0.31)
9	(3.29, 2.18, 1.15, 0.45)	(0.78, 0.51, 0.32, 0.12)

$\gamma_k = 0.01, \forall k$, $\mathbf{Q} = (1, 3, 1)$, and report the performance versus number of users. From Table II, significant performance gains are observed if we jointly optimize all system resources for $K \geq 3$. For example, compared to Method 1 and Method 3 where the degree of freedom of base-station association is restricted, the proposed algorithm achieves 51% and 631% improvement in the average sum rate for $K = 9$, respectively, meanwhile 53% and 550% improvement in feasible probability, respectively. Method 3 performs extremely unacceptable because the traditional downlink (DL) received power association will assign majority of the users to the macro base station due to its relatively high transmit power, even the user is much closer to the pico base-stations, causing strong interference in uplink transmission. Method 2 is actually the initialization of the proposed algorithm. As seen, the proposed algorithm improve the average sum rate of Method 2 by 186% for $K = 9$.

We next investigate the impact of multiple antennas of the pico base-station. As shown in Table III, jointly optimizing the base-station association with other resources always achieves the highest average sum rate. However, the performance gain over Method 1 becomes small if the pico base-station has more than 1 antenna. Thus, if Method 1 is adopted to simplify the resource allocation, installing two antennas at the pico base-stations is a worthwhile investment in the sense of improving the average sum rate.

TABLE III
AVERAGE SUM RATE (BITS/S/HZ) ACHIEVED BY DIFFERENT ALGORITHMS. SYSTEM PARAMETER: $N = 2$, $K = 5$, $\gamma_k = 0.01, \forall k$. THE TRIPLETS (X,Y,Z) IN THE FIRST ROW DENOTES THE ANTENNA NUMBER OF THE PICO, MACRO, PICO BASE-STATION, RESPECTIVELY.

	(1,2,1)	(2,4,2)	(1,3,1)	(2,3,2)	(3,3,3)
Proposed	2.08	3.94	2.50	3.83	4.42
Method 1	1.78	3.92	2.16	3.80	4.41
Method 2	1.01	3.46	1.62	3.02	4.14
Method 3	0.64	1.73	0.87	0.85	1.06

V. CONCLUSION

In this paper, we studied the joint base-station association and radio resources allocation problem in heterogeneous networks. We proposed an efficient iterative algorithm to maximize the uplink system sum rate, while satisfying the individual rate constraints. Numerical results show that adopting conventional downlink received power association or uplink path-loss association will cause considerable performance degradation in heterogeneous networks, compared to the proposed joint optimization. However, the performance gap between the joint optimization and path-loss association narrows down if pico base-stations are also equipped with multiple antennas.

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