Interference Alignment for Multi-User Multi-Way Relaying X Networks

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***Abstract*—In this paper, we consider a multi-way relaying channel where $2K$ users are divided into two groups averagely and each of them exchanges messages with every user of the other group via an intermediate relay. We term it multi-user multi-way relaying X network. We design the beamforming vectors at the users and the relay to achieve an interference alignment (IA) solution. Meanwhile, we investigate the feasibility conditions on the required amount of antennas for each node. Brief theoretical analysis and numerical simulations have been provided to demonstrate that the degrees of freedom (DOF) of $2K^2$ is obtained.**

I. Introduction

Many efforts have been devoted to analyzing the capacity region of interference channels over decades [1]–[3]. However, this is still an open problem [4]. For the complexity of expressing exact capacity of interference channels, degrees of freedom (DOF) is often utilized to characterize the capacity behavior in high signal-to-noise power ratio (SNR) regimes. It is defined as

$$d = \lim_{\rho \to \infty} \frac{C(\rho)}{\log(\rho)}, \tag{1}$$

where d is the number of DOF, ρ denotes the SNR and $C(\rho)$ stands for the sum capacity with respect to ρ.

Recently, a promising transmission technique termed interference alignment (IA) is proposed to exploit high DOF in multi-user interference channels [5], [6]. The basic idea of IA is to align the interferences at the receiver into the fewest possible signal space dimensions to maximize the number of interference-free dimensions for desired signals. IA can be achieved in time, frequency and spatial dimentions. In [5], an IA scheme in spatial domain is introduced for a multi-input multi-output (MIMO) X network involving two transmitters and two receivers, where each node is equipped with multiple antennas and each transmitter has independent messages for each receiver. The X network is a combination of interference, multiple access, and broadcast channels, it can therefore be viewed as a fundamental building block for the wireless networks with multi-user interference.

By constructing transmit precoding vectors, IA aligns the interferences into the same subspace of the receive space. Nevertheless, IA in spatial domain is only feasible if certain spatial dimensionality constraints are satified [7]. In other words, a sufficient number of transmit and receive antennas are required to enable the IA scheme for a certain number of users. The feasibility conditions for IA has been extensively studied in varieties of scenarios [7]–[9].

Relaying enables the communication when the direct link from transmitter to receiver suffers from severe fading or large path loss. However, traditional one-way relaying leads to a loss in spectral efficiency due to the half-duplex constraint [10]. Two-way relaying (TWR) has been considered as an efficient technique to improve the spectral efficiency for relay-based systems [11]. The TWR has been generalized as multi-user bi-directional relaying [12], [13] and multi-user multi-way relaying [14]–[16] recently.

The combination of IA and TWR has attracted growing interests. In [14], a so called MIMO Y channel has been focused on, where there are three users and a single relay. A network coded IA scheme named signal space alignment for network coding (SSA-NC) has been proposed to enable all the users to finish information exchange with each other via the relay in two time slots. The SSA-NC has evolved in [15]–[17] to efficiently handle co-channel interference in multi-user TWR environments.

In this paper, we propose a novel scenario based on the X networks, where there are $2K$ users and a single relay, $2K$ users are divided into two groups averagely and each of the users exchanges messages with every user in the other group via the relay. We call it multi-user multi-way relaying X networks. The motivation originates from a practical environment in cluster-based wireless sensor networks [18], where the sensors are divided into clusters and all the information in one cluster is collected by the cluster head. Sensors in one cluster may need to communicate with the ones in another cluster, then the cluster head can be used as the relay. Inspired by the methodology of SSA-NC, we combine signal space alignment and zero-forcing beamforming to obtain an IA solution for the multi-user multi-way relaying X networks. We also derive the premises on the required amount of antennas for each node to guarantee the feasibility of the proposed IA scheme. Based on our brief analysis on DOF and the simulations, we argue that the DOF of $2K^2$ is achieved. To our best knowledge, no results have been reported on the IA for this channel so far.

The rest of this paper is organized as follows: Section II introduces the system model of multi-user multi-way relaying

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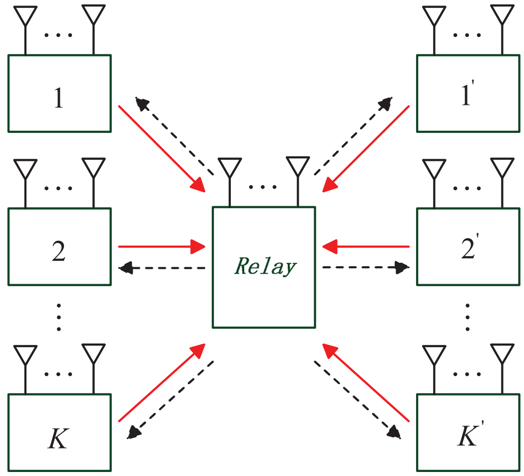

Figure 1. Multi-user multi-way relaying X networks

X networks. Section III presents a close-form IA solution, the feasibility conditions and a brief analysis on DOF. Numerical simulations are shown in Section IV to verify the derived results. Finally, conclusions are provided in Section V.

Notations: We describe matrices and vectors by bold upper and lower case letters, respectively. For any matrix $\mathbf{A}$, $\mathrm{tr}(\mathbf{A})$, $(\mathbf{A})^H$, $(\mathbf{A})^T$ and $(\mathbf{A})^{-1}$ denote the trace, conjugate transpose, transpose and pseudo inverse of $\mathbf{A}$, respectively. $\mathbb{E}\{\cdot\}$ is the expectation operator. We indicate $\|\cdot\|_2$ as the 2-norm operator for vectors. For notational convenience, we assume that $i^{'}$ belongs to $\left\{1^{'}, 2^{'}, \dots, K^{'}\right\}$ if i belongs to $\{1, 2, \dots, K\}$.

II. System Model

We consider a scenario consisting of $2K$ users and a single relay, as shown in Fig. 1. The users are divided into two groups and each group includes K users. We denote the user at the left side as user i with M_i antennas and that at the right side as user $j^{'}$ with $M_{j^{'}}$ antennas, for $i, j \in \{1, 2, \dots, K\}$. Every user intends to send independent messages to each of the K users in the other group via an intermediate relay with R antennas. We assume that all the channels undergo Rayleigh fading, whose entries are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, i.e., $\mathcal{CN}(0,1)$. The amplify-and-forward (AF) relaying strategy will be adopted for the purpose of simple implementation. The proposed transmission protocol consists of two phases: multiple access channel (MAC) phase and broadcasting channel (BC) phase. During the MAC phase, all the users transmit to the relay simultaneously, and during the BC phase, the relay broadcasts the linearly processed signal back to the end nodes.

During the MAC phase, the $R \times 1$ received signal at the relay is observed as

$$\mathbf{y}_r = \sum_{i=1}^{K} \mathbf{H}_{r,i}\mathbf{x}_i + \sum_{j=1}^{K} \mathbf{H}_{r,j^{'}}\mathbf{x}_{j^{'}} + \mathbf{n}_r, \tag{2}$$

where $\mathbf{H}_{r,i}$ is the $R \times M_i$ channel matrix for the link from user i to the relay, and $\mathbf{n}_r$ denotes the additive white Gaussian noise (AWGN) vector whose elements are assumed to be i.i.d. complex Gaussian random variables following $\mathcal{CN}\left(0, \sigma^2\right)$. The $M_i \times 1$ transmit vector $\mathbf{x}_i$ at user i is given by

$$\mathbf{x}_i = \sum_{j=1}^{K} \mathbf{v}_{j^{'},i} s_{j^{'},i}, \tag{3}$$

where $s_{j^{'},i}$ represents the symbol with unit variance transmitted from user i to $j^{'}$, and $\mathbf{v}_{j^{'},i}$ stands for the $M_i \times 1$ precoding vector for $s_{j^{'},i}$. We assume $\mathbf{x}_i$ satisfies the average power constraint of $\mathbb{E}\left\{\mathbf{tr}\left[\mathbf{x}_i\mathbf{x}_i^H\right]\right\} \leqq P_i$, where P_i is the total transmit power of user i. $\mathbf{H}_{r,j^{'}}$, $\mathbf{x}_{j^{'}}$, $\mathbf{v}_{i,j^{'}}$ and $P_{j^{'}}$ are defined similarly.

Before the BC phase, the relay multiplies the received signal with an $R \times R$ beamforming matrix $\mathbf{G}$. The signal transmitted by the relay in the second time slot is

$$\mathbf{x}_r = \beta \mathbf{G}\mathbf{y}_r, \tag{4}$$

where β is the power normalizing coefficient to satisfy the relay power constraint of $\mathbb{E}\left\{\mathbf{tr}\left[\mathbf{x}_r\mathbf{x}_r^H\right]\right\} \leqq P_r$, given by

$$\beta = \sqrt{\frac{P_r}{\mathbb{E}\{\mathbf{tr}[\mathbf{G}\mathbf{y}_r\mathbf{y}_r^H\mathbf{G}^H]\}}}.$$

Then the received signal at user $j^{'}$ is represented by

$$\mathbf{y}_{j^{'}} = \mathbf{H}_{j^{'},r}\mathbf{x}_r + \mathbf{n}_{j^{'}}, \tag{5}$$

where $\mathbf{H}_{j^{'},r}$ is the $M_{j^{'}} \times R$ channel matrix from the relay to user $j^{'}$ and $\mathbf{n}_{j^{'}}$ denotes the AWGN vector at user $j^{'}$ whose elements are assumed to be i.i.d. complex Gaussian random variables following $\mathcal{CN}\left(0, \sigma^2\right)$.

After applying a $M_{j^{'}} \times 1$ decoding vector $\mathbf{u}_{j^{'},i}$, the estimated signal from user i can be represented as

$$\begin{aligned} \hat{y}_{j^{'},i} &= \mathbf{u}_{j^{'},i}^H \mathbf{y}_{j^{'}} \\ &= \mathbf{u}_{j^{'},i}^H \mathbf{H}_{j^{'},r}\mathbf{x}_r + \mathbf{u}_{j^{'},i}^H \mathbf{n}_{j^{'}}. \end{aligned} \tag{6}$$

Denote $\mathbf{U}_{j^{'}}$ as the total decoding matrix with size $M_{j^{'}} \times K$ and $\hat{\mathbf{y}}_{j^{'}}$ as the total estimated data with size $K \times 1$ at user $j^{'}$. Then we have

$$\hat{\mathbf{y}}_{j^{'}} = \mathbf{U}_{j^{'}}^H \mathbf{H}_{j^{'},r}\mathbf{x}_r + \mathbf{U}_{j^{'}}^H \mathbf{n}_{j^{'}}, \tag{7}$$

where $\hat{\mathbf{y}}_{j^{'}}$ is given by $\hat{\mathbf{y}}_{j^{'}} = \left[\hat{y}_{j^{'},1}, \hat{y}_{j^{'},2}, \cdots, \hat{y}_{j^{'},K}\right]^T$ and $\mathbf{U}_{j^{'}}$ is defined as $\mathbf{U}_{j^{'}} = \left[\mathbf{u}_{j^{'},1}, \mathbf{u}_{j^{'},2}, \cdots, \mathbf{u}_{j^{'},K}\right]$. $\mathbf{H}_{i,r}$, $\mathbf{n}_i$, $\mathbf{U}_i$ and $\hat{\mathbf{y}}_i$ are defined similarly.

III. Interference Alignment Scheme for Multi-User Multi-Way Relaying X Networks

In this section, we first present our main results on the achievable DOF and feasibility conditions for the proposed IA scheme as follows.

Main results: The DOF of $2K^2$ can be achieved in the multi-user multi-way relaying X networks with the prerequisites that

$R \geqq K^2$, $M_i, M_{j'} \geqq K$ and $\min\left\{M_i + M_{j'}\right\} > R$ for all $i, j \in \{1, 2, ..., K\}$.

Then we will describe our proposed scheme to prove the results above. We also derive the constraints that should be satisfied to carry out the scheme. Finally, a brief theoretical analysis is provided to confirm the achievable DOF.

A. Proposed IA Scheme and Feasibility Conditions

The proposed IA scheme involves two steps: signal space alignment for both of the MAC and BC phases and zero-forcing based filter design at the relay.

1) Signal space alignment: During the MAC phase, each user transmits K independent messages so that user i should have $M_i \geqq K$ antennas, for $i \in \{1, 2, ..., K\}$. Similarly, $M_{j'}$ should be no less than K. Totally, there are $2K^2$ independent messages arriving at the relay simultaneously. In order to exploit the signal dimension efficiently, the precoding vectors are carefully designed to align each pair of desired signal vectors into the same spatial dimensions at the relay. To be more specific, we design the precoding vectors $\mathbf{v}_{j',i}$ for user i and $\mathbf{v}_{i,j'}$ for user j' so that the signal vectors $\mathbf{H}_{r,i}\mathbf{v}_{j',i}$ and $\mathbf{H}_{r,j'}\mathbf{v}_{i,j'}$ are aligned, which means

$$\text{span}\left(\mathbf{H}_{r,i}\mathbf{v}_{j',i}\right) = \text{span}\left(\mathbf{H}_{r,j'}\mathbf{v}_{i,j'}\right), \tag{8}$$

where $\text{span}\left(\mathbf{A}\right) = \text{span}\left(\mathbf{B}\right)$ denotes that the subpace spanned by two matrices $\mathbf{A}$ and $\mathbf{B}$ are equivalent. As a result, the dimension of the signal space at the relay is reduced from $2K^2$ to K^2. The relay should be equipped with $R \geqq K^2$ antennas to support the transmission of K^2 data streams. In order to satisfy equation (8), we need to find the intersection between the space spanned by the columns of $\mathbf{H}_{r,i}$ and that spanned by the columns of $\mathbf{H}_{r,j'}$. We define $\text{INT}_{(i,j')}$ as the intersection subspace between the column space of $\mathbf{H}_{r,i}$ and $\mathbf{H}_{r,j'}$. The subscript $\left(i, j'\right)$ is an index for the pair of user i and j', hence $\left(i, j'\right)$ and $\left(j', i\right)$ stand for the same pair. We further denote $\widetilde{\mathbf{v}}_{j',i}$ as the vector to represent the direction of $\mathbf{v}_{j',i}$, i.e., $\mathbf{v}_{j',i} = \alpha_{j',i}\widetilde{\mathbf{v}}_{j',i}$ where $\alpha_{j',i}$ is the power constraint factor to satisfy the transmit power constraint of user i and computed as

$$\alpha_{j',i} = \sqrt{\frac{P_i}{K\mathbf{tr}\left[\widetilde{\mathbf{v}}_{j',i}\widetilde{\mathbf{v}}_{j',i}^H\right]}}.$$

$\widetilde{\mathbf{v}}_{i,j'}$ and $\alpha_{i,j'}$ are defined similarly. Then we have

$$\mathbf{H}_{r,i}\widetilde{\mathbf{v}}_{j',i} = \mathbf{H}_{r,j'}\widetilde{\mathbf{v}}_{i,j'} = \mathbf{v}'_{(i,j')}, \tag{9}$$

where $R \times 1$ vector $\mathbf{v}'_{(i,j')}$ belongs to $\text{INT}_{(i,j')}$. According to (9), $\mathbf{v}'_{(i,j')}$, $\widetilde{\mathbf{v}}_{j',i}$ and $\widetilde{\mathbf{v}}_{i,j'}$ can be derived via solving

$$\begin{bmatrix} \mathbf{I}_R & -\mathbf{H}_{r,i} & \mathbf{0} \\ \mathbf{I}_R & \mathbf{0} & -\mathbf{H}_{r,j'} \end{bmatrix} \begin{bmatrix} \mathbf{v}'_{(i,j')} \\ \widetilde{\mathbf{v}}_{j',i} \\ \widetilde{\mathbf{v}}_{i,j'} \end{bmatrix} = \mathbf{0}, \tag{10}$$

where $\mathbf{I}_R$ denotes the $R \times R$ identity matrix. Since the size of the matrix in (10) is $2R \times \left(R + M_i + M_{j'}\right)$, $2R < R + M_i + M_{j'}$ should be satisfied to ensure the existence of the nullspace, i.e., $R < min\left\{M_i + M_{j'}\right\}$ for all $i, j \in \{1, 2, \ldots, K\}$.

Then, the observations at the relay in (2) can be rewritten as

$$\begin{aligned} \mathbf{y}_r &= \sum_{i=1}^{K}\mathbf{H}_{r,i}\sum_{j=1}^{K}\mathbf{v}_{j',i}s_{j',i} + \sum_{j=1}^{K}\mathbf{H}_{r,j'}\sum_{i=1}^{K}\mathbf{v}_{i,j'}s_{i,j'} + \mathbf{n}_r \\ &= \mathbf{V}'\mathbf{s}_{\oplus} + \mathbf{n}_r, \end{aligned} \tag{11}$$

where the total effective MAC channel matrix with size $R \times K^2$ is defined as

$$\mathbf{V}' = \begin{bmatrix} \mathbf{v}'_{(1,1')} & \mathbf{v}'_{(1,2')} \cdots & \mathbf{v}'_{(2,1')} & \cdots & \mathbf{v}'_{(K,K')} \end{bmatrix},$$

the $K^2 \times 1$ vector $\mathbf{s}_{\oplus}$ consists of K^2 sum signals is represented as

$$\mathbf{s}_{\oplus} = \begin{bmatrix} s_{\oplus_{(1,1')}} & s_{\oplus_{(1,2')}} \cdots & s_{\oplus_{(2,1')}} & \cdots & s_{\oplus_{(K,K')}} \end{bmatrix}^T,$$

and $s_{\oplus_{(i,j')}} = \alpha_{j',i}s_{j',i} + \alpha_{i,j'}s_{i,j'}$ can be viewed as the AF network coded message.

Similarly, we also align each user pair's received signal space during the BC phase. We design the decoding vectors $\mathbf{u}_{i,j'}$ and $\mathbf{u}_{j',i}$ to align $\mathbf{u}_{i,j'}^H\mathbf{H}_{i,r}$ and $\mathbf{u}_{j',i}^H\mathbf{H}_{j',r}$, i.e., the following condition should be satisfied:

$$\text{span}\left(\mathbf{u}_{i,j'}^H\mathbf{H}_{i,r}\right) = \text{span}\left(\mathbf{u}_{j',i}^H\mathbf{H}_{j',r}\right). \tag{12}$$

Similar with (10), we compute $\mathbf{u}'_{(i,j')}$, $\mathbf{u}_{i,j'}$ and $\mathbf{u}_{j',i}$ by solving the equation

$$\begin{bmatrix} \mathbf{u}'^H_{(i,j')} & \mathbf{u}_{i,j'}^H & \mathbf{u}_{j',i}^H \end{bmatrix} \begin{bmatrix} \mathbf{I}_R & \mathbf{I}_R \\ -\mathbf{H}_{i,r} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}_{j',r} \end{bmatrix} = \mathbf{0}, \tag{13}$$

where $\mathbf{u}'^H_{(i,j')}$ indicates a vector in the intersection subspace between $\mathbf{H}_{i,r}$ and $\mathbf{H}_{j',r}$. Since we have $R < \min\left\{M_i + M_{j'}\right\}$, the left nullspace of the matrix in (13) always exists. Then the received signal at user j' in (7) can be rewritten as

$$\begin{aligned} \hat{\mathbf{y}}_{j'} &= \mathbf{U}_{j'}^H\mathbf{H}_{j',r}\mathbf{x}_r + \mathbf{U}_{j'}^H\mathbf{n}_{j'} \\ &= \mathbf{U}'^H_{j'}\mathbf{x}_r + \mathbf{U}_{j'}^H\mathbf{n}_{j'}, \end{aligned} \tag{14}$$

where $\mathbf{U}'_{j'} = \begin{bmatrix} \mathbf{u}'_{(1,j')} & \mathbf{u}'_{(2,j')} & \cdots & \mathbf{u}'_{(K,j')} \end{bmatrix}$ with size $R \times K$ is the effective channel from the relay to user j'.

2) Zero-forcing based relay filter: To eliminate all the co-channel interferences, we adopt a conventional zero-forcing based filter at the relay as

$$\mathbf{G} = \left(\mathbf{U}'^H\right)^{-1}\left(\mathbf{V}'\right)^{-1}, \tag{15}$$

where

$$\mathbf{U}^{'}=\left[\begin{array}{cccccc}\mathbf{u}^{'}_{(1,1^{'})} & \mathbf{u}^{'}_{(1,2^{'})}\cdots & \mathbf{u}^{'}_{(2,1^{'})} & \cdots & \mathbf{u}^{'}_{(K,K^{'})}\end{array}\right]$$

with size $R\times K^2$ represents the total effective channel during the BC phase. Now, we need to verify the existence of the pseudo inverses of $\mathbf{V}^{'}$ and $\mathbf{U}^{'H}$ respectively. Since the alignment of the signal space of one desired signal pair is independent of that of other pairs [17], the column vectors of $\mathbf{V}^{'}$, i.e., $\left\{\begin{array}{cccccc}\mathbf{v}^{'}_{(1,1^{'})} & \mathbf{v}^{'}_{(1,2^{'})}\cdots & \mathbf{v}^{'}_{(2,1^{'})} & \cdots & \mathbf{v}^{'}_{(K,K^{'})}\end{array}\right\}$, are mutually independent. Thus, $\mathbf{V}^{'}$ has full column rank. Similarly $\mathbf{U}^{'H}$ has full row rank. Therefore, both of the pseudo inverses of $\mathbf{V}^{'}$ and $\mathbf{U}^{'H}$ exist.

Consequently, we have the observations of user $j^{'}$ as

$$\begin{aligned}\hat{\mathbf{y}}_{j^{'}} &= \mathbf{U}^{'H}_{j^{'}}\mathbf{x}_r+\mathbf{U}^{H}_{j^{'}}\mathbf{n}_{j^{'}}\\ &= \beta\mathbf{U}^{'H}_{j^{'}}\left(\mathbf{U}^{'H}\right)^{-1}\left(\mathbf{V}^{'}\right)^{-1}\left(\mathbf{V}^{'}\mathbf{s}_{\oplus}+\mathbf{n}_r\right)+\mathbf{U}^{H}_{j^{'}}\mathbf{n}_{j^{'}}\\ &= \beta\mathbf{s}_{\oplus_{j^{'}}}+\beta\mathbf{U}^{'H}_{j^{'}}\mathbf{G}\mathbf{n}_r+\mathbf{U}^{H}_{j^{'}}\mathbf{n}_{j^{'}}, \qquad (16)\end{aligned}$$

where

$$\mathbf{s}_{\oplus_{j^{'}}}=\left[\begin{array}{cccc}s_{\oplus_{(1,j^{'})}} & s_{\oplus_{(2,j^{'})}} & \cdots & s_{\oplus_{(K,j^{'})}}\end{array}\right]^T$$

with size $K\times 1$ consists of the network coded symbols desired by user $j^{'}$. Then the estimated data from user i can be rewritten as

$$\hat{y}_{j^{'},i}=\beta s_{\oplus_{(i,j^{'})}}+\beta\mathbf{u}^{'H}_{(i,j^{'})}\mathbf{G}\mathbf{n}_r+\mathbf{u}^{'H}_{(i,j^{'})}\mathbf{n}_{j^{'}}. \qquad (17)$$

Subsequently, user $j^{'}$ can substract the self-interference $s_{i,j^{'}}$ to get the estimated signal from user i as

$$\begin{aligned}\hat{y}^{'}_{j^{'},i} &= \hat{y}_{j^{'},i}-\beta\alpha_{i,j^{'}}s_{i,j^{'}}\\ &= \beta\alpha_{j^{'},i}s_{j^{'},i}+\beta\mathbf{u}^{'H}_{(j^{'},i)}\mathbf{G}\mathbf{n}_r+\mathbf{u}^{'H}_{(j^{'},i)}\mathbf{n}_{j^{'}}. \qquad (18)\end{aligned}$$

B. Brief Analysis on DOF

The total achievable DOF for this network can be presented as the sum of DOF for each link, i.e.,

$$\begin{aligned}d_{tot} &= \lim_{\rho\to\infty}\sum_{i=1}^{K}\sum_{j=1}^{K}\left(d_{j^{'},i}+d_{i,j^{'}}\right)\\ &= \lim_{\rho\to\infty}\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{R_{j^{'},i}(\rho)+R_{i,j^{'}}(\rho)}{\log(\rho)}, \qquad (19)\end{aligned}$$

where $d_{j^{'},i}$ denotes the DOF for the transmission from user i to $j^{'}$, ρ is the transmit SNR and $R_{j^{'},i}(\rho)$ stands for the achievable rate for the data from user i to $j^{'}$ [16]. For a given set of beamforming vectors $\mathbf{v}^{'}_{(i,j^{'})}$ and $\mathbf{u}^{'}_{(i,j^{'})}$, $R_{j^{'},i}(\rho)$ is defined as

$$R_{j^{'},i}(\rho)=\log\left(1+\rho_{j^{'},i}\right), \qquad (20)$$

where $\rho_{j^{'},i}$ is the received SNR from user i to $j^{'}$ given by

$$\rho_{j^{'},i}=\frac{(\alpha_{j^{'},i}\beta)^2}{(\sigma\beta)^2\left\|\mathbf{u}^{'H}_{(j^{'},i)}\mathbf{G}\right\|_2^2+\sigma^2\left\|\mathbf{u}^{'H}_{(j^{'},i)}\right\|_2^2}. \qquad (21)$$

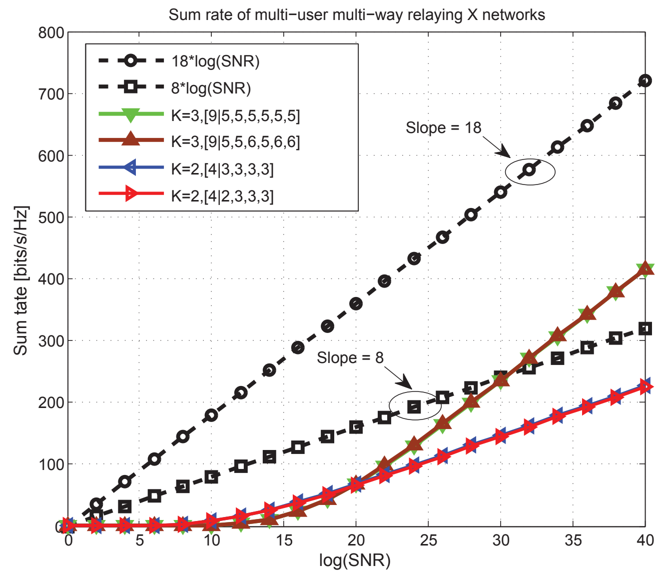

Figure 2. Sum rate performance of multi-user multi-way relaying *X* networks

$d_{i,j^{'}}$ and $R_{i,j^{'}}(\rho)$ are defined similarly.

We assume unit transmit power at each node, i.e., $P_i=P_{j^{'}}=P_r=1$, for $i,j\in\{1,2,\ldots,K\}$, then the transmit SNR can be represented as $\rho=1/\sigma^2$. The received SNR in equation (21) can be rewritten as

$$\rho_{j^{'},i} = \rho * M_{j^{'},i}, \qquad (22)$$

where the finite number $M_{j^{'},i}$ is given by

$$M_{j^{'},i}=\frac{(\alpha_{j^{'},i}\beta)^2}{\beta^2\left\|\mathbf{u}^{'H}_{(j^{'},i)}\mathbf{G}\right\|_2^2+\left\|\mathbf{u}^{'H}_{(j^{'},i)}\right\|_2^2}.$$

By substituting equation (20) and (22) into (19), the total achievable DOF is rewritten as

$$d_{tot}=\lim_{\rho\to\infty}\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{\log\left(1+\rho*M_{j^{'},i}\right)+\log\left(1+\rho*M_{i,j^{'}}\right)}{\log(\rho)}. \qquad (23)$$

Under the condition ρ goes to infinity, $\frac{\log\left(1+\rho*M_{j^{'},i}\right)}{\log(\rho)}$ and $\frac{\log\left(1+\rho*M_{i,j^{'}}\right)}{\log(\rho)}$ both tend to 1. Thereby the total achievable DOF d_{tot} equals $2K^2$.

IV. Simulation Results

In this section, by using numerical simulations, we provide the sum rate performance of the proposed scheme. In our simulation, we use the notation $[R|M_1,M_2,\ldots,M_K,M_{1^{'}},M_{2^{'}},\ldots,M_{K^{'}}]$ to denote the model shown in Fig. 1 with R antennas at the relay, M_i antennas at user i and $M_{j^{'}}$ antennas at user $j^{'}$, for $i,j\in\{1,2,\ldots,K\}$. We assume the flat Rayleigh fading environment and set the variance of the complex channel coefficient to 1. The transmit power is normalized to be 1 at each node, i.e., $P_i = P_{j'} = P_r = 1$, for $i, j \in \{1, 2, \ldots, K\}$, and the AWGN power at all the nodes is assumed to be σ^2. Thus the transmit SNR is defined as $1/\sigma^2$. The achievable rate for each link is calculated according to (20).

In Fig. 2, we plot the average sum rate of our proposed scheme under various configurations versus $\log(\mathrm{SNR})$. The slope of the sum rate curve represents the achievable DOF at high SNR. We also plot two straight lines with slope of 8 and 18 as references to verify the DOF results. As seen in Fig. 2, both of the curves under symmetric user antenna case $[4|3, 3, 3, 3]$ and asymmetric user antenna case $[4|2, 3, 3, 3]$ become parallel with the straight line with slope of 8 as SNR increases. It means that if only the feasibility conditions, i.e., $R \geqq 4, M_i, M_{j'} \geqq 2$ and $\min\left\{M_i + M_{j'}\right\} > R$ for $i, j \in \{1, 2\}$, are satisfied, the DOF of 8 is achieved with $K = 2$. Also, the slope of the curves for the symmetric configuration $[9|5, 5, 6, 5, 6, 6]$ and asymmetric configuration $[9|5, 5, 5, 5, 5, 5]$ confirm that the DOF of 18 is obtained with the prerequisites that $R \geqq 9, M_i, M_{j'} \geqq 3$ and $\min\left\{M_i + M_{j'}\right\} > R$ for $i, j \in \{1, 2, 3\}$.

V. Conclusion

In this paper, we introduce a novel scenario called multi-user multi-way relaying X networks where $2K$ users are divided into two groups averagely and each of them exchanges messages with all the users of the other group. The beamforming vectors are constructed to obtain a close-form IA solution for this channel. We also investigate the feasibility conditions on the required number of antennas for each node. DOF analysis and simulation results prove that the DOF of $2K^2$ is achieved if $R \geqq K^2, M_i, M_{j'} \geqq K$ and $\min\left\{M_i + M_{j'}\right\} > R$ for all $i, j \in \{1, 2, ..., K\}$.

Since not only the spatial but also the time and frequency dimensions can be used to achieve the IA scheme, more practical algorithm requiring fewer antennas will be our future work.

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