Block Diagonal Inversion Precoding for MIMO Broadcast Channels

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***Abstract*—The sum capacity of a broadcast MIMO channel can be achieved sub-optimally using linear precoding techniques. Block diagonalization (BD) is a linear precoding technique that achieves near to the sum capacity with low complexity by nullifying the inter-user interference. However, one limitation of BD is that the equivalent channel after block diagonalization has to be communicated to the users using additional pilot symbols or a limited feedforward link. The former method increases the system overhead and the latter suffers from quantization error. Transmitter zero forcing may be an alternative but it has a high power enhancement. Besides, transmitter zero forcing lacks flexibility for use in multi-antenna users. In this paper, a new linear precoding technique called Block Diagonalization Inversion (BDI) is proposed. The proposed scheme avoids the need to communicate the equivalent channel by using a combination of block diagonalization and partial inversion of the equivalent channel at the transmitter. BDI enables per-stream power allocation and adaptive modulation like BD; besides, BDI has a lower power enhancement than transmitter zero forcing. It is shown using simulations that BDI achieves a sum rate performance in-between an ideal BD and transmitter zero forcing. When a limited feedforward link is used with BD, it is demonstrated that BDI provides higher sum rate than BD.**

I. Introduction

MIMO broadcast is a downlink multiple-access scheme used to transmit one or more data streams to multiple users scheduled on the same resource block. Downlink precoding is crucial in multi-user MIMO systems to nullify the inter-user interference.

It is known that dirty paper coding (DPC) achieves the capacity region of MIMO broadcast channels [1]. However, DPC is a theoretical scheme that is complex to implement in practice. As a result, sub-optimal linear precoding techniques such as transmitter zero forcing [2] and block diagonalization [3] are used to achieve near to optimal performance with a lower complexity. Transmitter zero forcing is used in MIMO broadcast channels when the users have a single antenna [2]. A possibility to extend that to multi-antenna users is also discussed in [4]. However, transmitter zero forcing in multi-antenna users lacks the flexibility to vary the number of streams per user and the advantage that can be gained by combining signals from receiver antennas is not utilized. Besides, transmitter zero forcing causes a large power enhancement resulting in a large performance gap as compared to DPC. Better sum rate performance can be achieved using block diagonalization precoding [3] which has been proposed as a generalization of transmitter zero forcing in multi-antenna users.

Block diagonalization supports multiple data streams per user by using precoders orthogonal to the channel of other users such that inter-user interference is cancelled. This linear precoding scheme also achieves near to the sum capacity of MIMO broadcast channels [3]. Besides, block diagonalization allows single user precoding methods to be applied on top of the multi-user precoder. However, one limitation of block diagonalization and other similar multiuser precoding schemes is coordination of information. That means, the equivalent channel after block diagonalization or the multi-user precoder used at the transmitter must be forwarded to each user in order to allow data decoding [5]. One way to communicate the equivalent channel to the users is using additional downlink pilot symbols [6][7]. Even though, user-specific precoded pilot symbols are considered in LTE-Advanced; such pilot symbols do not comply with MIMO standards that use a common pilot channel. Besides, such additional pilot symbols increase the system overhead and design complexity [8]. The other method to deliver precoder information to users is using a limited feedforward link. In [9], feedforward link is used to jointly optimize receive and transmit beamformers. In [10], precoder information is forwarded to users to aid in decoding. Feed forwarding requires a dedicated link; moreover, performance degradation due to quantization error and delay is inevitable when using a feed forward. Therefore, the practical performance of block diagonalization is lower than the performance theoretically claimed as a result of these limitations.

Considering this problem, we propose a modified block diagonalization scheme called ”Block diagonalization Inversion” (BDI). The proposed technique avoids the need to communicate precoder information by combining block diagonalization with a ”partial inversion” of the equivalent channel. As a result, block diagonalization inversion behaves similar to block diagonalization in many aspects. For instance, power allocation and adaptive modulation can be applied similar to block diagonalization.

The paper is organized as follows. Section II outlines the system model for a multiuser MIMO downlink for users with one or more antennas. Section III describes the block diagonalization scheme on which this paper builds on. In Section IV the block diagonalization inversion precoding scheme proposed in this paper is discussed. The performance of the

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proposed scheme is compared to block diagonalization and transmitter zero forcing by using simulations in Section V. Finally, Section VI concludes the paper.

II. System model

In this paper, a time division duplex (TDD) MIMO system, in which the downlink channel state information is available at the transmitter is assumed. A MIMO broadcast system consisting of a base station with N antennas and K mobile users each with n_k antennas. The total number of receive antennas is denoted by M where $M := \sum_{k=1}^{K} n_k$. The channel from the base station to each user k is denoted by a matrix $\mathbf{H}_k \in \mathbb{C}^{n_k \times N}$.

Let the precoding matrix, the diagonal power loading matrix and the vector of symbols to be sent to user k be denoted by $\mathbf{W}_k \in \mathbb{C}^{N \times n_k}$, $\mathbf{D}_k \in \mathbb{C}^{n_k \times n_k}$ and $\mathbf{x}_k \in \mathbb{C}^{n_k \times 1}$ respectively where $\mathbb{E}\left\{\mathbf{x}_k \mathbf{x}_k^H\right\} = \mathbf{I}$ (identity matrix). In this paper, it is assumed that the number of streams to each user is equal to n_k; the analysis can be extended to the general case. In such a setting, the signal vector $\mathbf{t}$ transmitted by the base station is a sum of vectors given by $\mathbf{t} := \sum_{i=1}^{K} \mathbf{W}_i \mathbf{D}_i \mathbf{x}_i$. As a result, the signal received by user k, $\mathbf{y}_k \in \mathbb{C}^{n_k \times 1}$ in the downlink is given by:

$$
\begin{aligned}
\mathbf{y}_k &= \mathbf{H}_k \mathbf{t} + \mathbf{n}_k \\
&= \mathbf{H}_k \mathbf{W}_k \mathbf{D}_k \mathbf{x}_k + \mathbf{H}_k \sum_{i=1, i \neq k}^{K} \mathbf{W}_i \mathbf{D}_i \mathbf{x}_i + \mathbf{n}_k
\end{aligned} \tag{1}
$$

where $\mathbf{n}_k \in \mathbb{C}^{n_k \times 1}$ is a zero-mean complex Gaussian noise vector of covariance $\sigma^2 \mathbf{I}$. The first term in (1) is user $k's$ signal; the second term is the interference from other users. The precoder $\mathbf{W}_k$ for $1 \leq k \leq K$ is designed so that the inter-user interference is minimized.

Let the combined transmit vector $\mathbf{x}$, precoder and channel $\mathbf{H}$ of all users be given by $\mathbf{x} = \left[\mathbf{x}_1^T \ \mathbf{x}_2^T \cdots \mathbf{x}_K^T\right]^T \in \mathbb{C}^{M \times 1}$, $\mathbf{W} = [\mathbf{W}_1 \ \mathbf{W}_2 \cdots \mathbf{W}_K] \in \mathbb{C}^{N \times M}$ and $\mathbf{H} = \left[\mathbf{H}_1^T \ \mathbf{H}_2^T \cdots \mathbf{H}_K^T\right]^T \in \mathbb{C}^{M \times N}$ respectively. The received vector across the users is then expressed as,

$$
\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{D}\mathbf{x} + \mathbf{n} \tag{2}
$$

where $\mathbf{D}$ is a diagonal power allocation matrix which is optimized over all spatial streams so that a total maximum transmit power constraint, P_T is satisfied. Thus, the constraint given by $tr\left(\mathbf{W}^H \mathbf{W} \mathbf{D}^2\right) \leq P_T$ must be fulfilled.

III. Block Diagonalization

In block diagonalization (BD), the multiuser precoder is designed such that inter-user interference is completely cancelled [3][5]. Lets assume that the total number of receive antennas, M is less than or equal to the total number of transmit antennas, i.e. $M \leq N$. Such a dimensional constraint can be fulfilled by user selection.

In order to nullify inter-user interference, the precoding matrices $\mathbf{W}_k$ shall be chosen such that,

$$
\mathbf{H}_j \mathbf{W}_k = \mathbf{0} \text{ for } j \neq k \tag{3}
$$

Hence, $\mathbf{W}_k$ shall be selected such that it lies in the null space spanned by the matrix $\tilde{\mathbf{H}}_k \in \mathbb{C}^{(M-n_k) \times N}$ which is a combined matrix consisting of all channel matrices except $\mathbf{H}_k$.

$$
\tilde{\mathbf{H}}_k = \left[\mathbf{H}_1^T \cdots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \cdots \mathbf{H}_K^T\right]^T
$$

Therefore, $\mathbf{W}_k$ can be obtained from the singular value decomposition of $\tilde{\mathbf{H}}_k$. Let the orthogonal basis for the null space of $\tilde{\mathbf{H}}_k$ be denoted by $\tilde{\mathbf{F}}_k$, this null space basis has dimension $N \times b_k$ where, $b_k \geq N - M + n_k \geq n_k$. In block diagonalization, the precoder matrix $\mathbf{W}_k$ is selected to be this null space basis, $\tilde{\mathbf{F}}_k$.

Using such a precoder, the equivalent channel of the users is block diagonalized to parallel single user transmissions free of inter-user interference.

$$
\mathbf{H}\mathbf{W} = \begin{bmatrix} \mathbf{H}_1 \tilde{\mathbf{F}}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{H}_K \tilde{\mathbf{F}}_K \end{bmatrix} \tag{4}
$$

As a result, the signal received by user k in (1) reduces to:

$$
\mathbf{y}_k = \mathbf{H}_k \tilde{\mathbf{F}}_k \mathbf{D}_k \mathbf{x}_k + \mathbf{n}_k
$$

This is equivalent to a single user transmissions to each user k with equivalent channel $\mathbf{H}_k \tilde{\mathbf{F}}_k$. Thus, single user MIMO precoding and decoding schemes can be further applied on this channel to optimize sum rate. One way to achieve this is using singular value decomposition of the equivalent channel [3]. Let $\mathbf{H}_k \tilde{\mathbf{F}}_k = \mathbf{U}_{eq,k} \mathbf{\Sigma}_{eq,k} \mathbf{V}_{eq,k}^H$ be the reduced SVD of the equivalent channel. Using $\mathbf{V}_{eq,k}$ as precoder and $\mathbf{U}_{eq,k}^H$ as decoder, the channel of user k can be diagonalized into parallel eigen modes. As a result, both the inter-user and inter-stream interference can be nullified in block diagonalization by using the precoder $\mathbf{W}_k = \tilde{\mathbf{F}}_k \mathbf{V}_{eq,k}$ resulting in a transmit vector given by:

$$
\mathbf{t} = \sum_{i=1}^{K} \tilde{\mathbf{F}}_i \mathbf{V}_{eq,i} \mathbf{D}_i \mathbf{x}_i \tag{5}
$$

However, each user k needs to know $\mathbf{U}_{eq,k}$ or the equivalent channel $\mathbf{H}_k \tilde{\mathbf{F}}_k$ to decode its data. Each user knows its own channel $\mathbf{H}_k$ but not the channel of other users as long as there is no co-operation between users. Thus, the equivalent channel $\mathbf{H}_k \tilde{\mathbf{F}}_k$ or precoder/decoder information must be communicated to the users. The equivalent channel can be communicated by using extra downlink pilot symbols unique for each user [7] or using a feedforward link [9]. However, neither scheme is suitable from design, complexity and signaling overhead point of view as discussed in section I. Besides, the performance of block diagonalization is degraded due to quantization error in feedforward or estimation error in downlink pilots.

An alternative linear precoding technique that can avoid this limitation of block diagonalization is transmitter zero forcing (TZF) or channel inversion. In channel inversion, the base station transmits all users data using the channel inverse as precoder. The transmit vector in this case is given by,

$$\mathbf{t} = \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathbf{D}\mathbf{x} \tag{6}$$

An inversion of the equivalent channel of each user after block diagonalization is also studied in [5]. Such scheme has however similar performance as direct channel inversion of the combined channel. However, transmitter zero forcing causes a large power enhancement. As a result, applying transmit power constraint on $\mathbf{t}$ in (6) results in a lower sum rate as compared to block diagonalization. Moreover, inversion of a large channel matrix as in (6) results in high complexity.

Considering the problem of communicating equivalent channel inherent in block diagonalization and the large power enhancement of transmitter zero forcing, the following precoding scheme is proposed.

IV. Block Diagonalization Inversion

Let the reduced singular value decomposition of user k's channel be denoted by $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$ where $\mathbf{U}_k \in \mathbb{C}^{n_k \times n_k}$ is the left singular vector matrix, $\mathbf{\Sigma}_k \in \mathbb{C}^{n_k \times n_k}$ is a diagonal singular value matrix and $\mathbf{V}_k \in \mathbb{C}^{N \times n_k}$ is the right singular vector matrix. Consider again the equivalent channel of user k, $\mathbf{H}_k \tilde{\mathbf{F}}_k \in \mathbb{C}^{n_k \times b}$ in block diagonalization. The channels of all users are assumed to be independent; thus, $\mathbf{H}_k$ is independent of $\tilde{\mathbf{F}}_k$. As a result, $\mathbf{H}_k \tilde{\mathbf{F}}_k$ can be assumed to have full rank and to be invertible; such a property is also assumed in block diagonalization. Lets assume the number of null space vectors $b_k = n_k$ for simplicity (the analysis can be extended for the general case $b_k \geq n_k$). Using this assumption, $\text{rank}\left(\mathbf{H}_k \tilde{\mathbf{F}}_k\right) = \text{rank}\left(\mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H \tilde{\mathbf{F}}_k\right) \leq \text{rank}\left(\mathbf{V}_k^H \tilde{\mathbf{F}}_k\right)$. This shows that $\mathbf{V}_k^H \tilde{\mathbf{F}}_k \in \mathbb{C}^{n_k \times b_k}$ has also full rank and is thus invertible.

As shown in (3), $\mathbf{W}_k$ lies in the null space of $\tilde{\mathbf{H}}_k$; thus, any matrix that multiplies $\mathbf{W}_k$ from the right also lies in the null space of $\tilde{\mathbf{H}}_k$. In block diagonalization, the need to convey the equivalent channel to the users arise due to the fact that $\tilde{\mathbf{F}}_k$ is unknown at the users. Therefore, we propose to invert and eliminate the effect of $\tilde{\mathbf{F}}_k$ such that $\mathbf{W}_k$ is given by,

$$\mathbf{W}_k = \tilde{\mathbf{F}}_k \left(\mathbf{V}_k^H \tilde{\mathbf{F}}_k\right)^{-1} \tag{7}$$

We call this precoding technique ”Block Diagonal Inversion” (BDI), since it involves a combination of block diagonalization and ”partial” inversion of the equivalent channel to compromise the limitation of block diagonalization and the lower performance of channel inversion.

The transmit vector in block diagonalization inversion is thus given by:

$$\mathbf{t} = \sum_{i=1}^{K} \tilde{\mathbf{F}}_i \left(\mathbf{V}_i^H \tilde{\mathbf{F}}_i\right)^{-1} \mathbf{D}_i \mathbf{x}_i \tag{8}$$

Using this precoder results in a block diagonalized channel consisting of orthogonal columns,

$$\mathbf{H}\mathbf{W} = \begin{bmatrix} \mathbf{U}_1\mathbf{\Sigma}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{U}_K\mathbf{\Sigma}_K \end{bmatrix} \tag{9}$$

The equivalent channel of user k is now given by $\mathbf{U}_k\mathbf{\Sigma}_k$, a part of the SVD of the channel $\mathbf{H}_k$, which is known at user k. Therefore, the proposed precoder cancels the effect of the multi-user precoder unknown at the users while nullifying the inter-user interference. As a result, the signal received by user k in (1) reduces to:

$$\mathbf{y}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{D}_k \mathbf{x}_k + \mathbf{n}_k$$

Similar to block diagonalization precoding in Section III, each user k can use the unitary decoder $\mathbf{U}_k^H$ to decode its data without amplifying the noise.

The problem with precoding techniques that involve inversion of non-unitary matrices such as transmit zero forcing and BDI is the power enhancement. In block diagonalization inversion, the matrix $\mathbf{W}_k \triangleq \tilde{\mathbf{F}}_k \left(\mathbf{V}_k^H \tilde{\mathbf{F}}_k\right)^{-1}$ has non-unit norm columns. When equal power allocation is used for instance, *i.e.* $\mathbf{D} = \sqrt{\frac{P_T}{M}}\mathbf{I}$, the matrix $\mathbf{W}$ must be normalized by $\frac{\|\mathbf{W}\|_F}{\sqrt{M}}$ to satisfy the total power constraint. Hence, both BDI and transmitter zero forcing suffer from a power enhancement due to the inversion operations involved. However, BDI has lower power enhancement than transmitter zero forcing as demonstrated in the following section using simulations. Therefore, BDI enjoys the useful properties of BD while avoiding the feedforward link used in BD and reducing the power enhancement of transmitter zero forcing.

V. Results

The performance of the proposed BDI precoder is compared to BD and TZF using the following simulations. To evaluate the performance of the precoding schemes as a function of signal-to-noise ratio (SNR), a $\{2,2\} \times 4$ MIMO broadcast channel is considered. That is, 2 users with 2 antennas each are assumed to be scheduled for transmission from a 4 antenna base station. Three precoding schemes are considered. The first is ideal BD in which the equivalent channel is assumed to be communicated to the users without error. The second scheme considered is TZF. The third scheme considered is the BDI precoding proposed in this paper.

The sum rate performance of BDI as compared to BD and TZF for a $\{2,2\} \times 4$ i.i.d Gaussian MIMO channel using equal power allocation is shown in Fig.1. As the figure shows, the rate penalty of BDI as compared to ideal BD is small (≈ 2.5 bps/Hz at high SNRs). On the other hand, BDI performs better than TZF (by ≈ 1.5 bps/Hz at high SNR).

In Fig.2, the sum rate of BD, BDI and TZF is compared as the number of dual antenna users increases. The number of transmit antennas is increased proportionally, that is $N = 2K$. A SNR of $20\,dB$ is assumed and equal power allocation is used. The performance penalty of BDI and TZF as compared to ideal BD shows some increment with the number of users. On the other hand, the performance improvement of BDI as compared to TZF remains almost the same as the number of users increases.

The performance improvement of BDI as compared to the trivial TZF is more evident when non-equal power allocation is used. Optimal power allocation is more beneficial when the ratio of eigenvalues of the channel is large, which is the case in spatially correlated MIMO channels. In Fig.3, a spatially correlated MIMO channel is simulated. The spatial correlation is introduced by assuming compact antenna arrays (the two antennas of each user being separated by 0.2λ) at each user terminal. On the other hand, the channels of the users are assumed independent and uniform power azimuth spectrum is assumed. The performance of the precoding schemes in such a MIMO broadcast channel is compared with water-filling power allocation. Fig.3 shows the sum rate comparison as a function of SNR. The rate improvement of BDI with respect to TZF increases (up to 3 bps/Hz at high SNR) in the case of such a spatial correlation. On the other hand, the performance penalty of BDI as compared to the ideal BD is the same as in the case of an iid MIMO channel. The sum rates in Fig.3 are slightly lower than the corresponding sum rates in Figure 1 due to the spatial correlation.

It is discussed in Section III that the practical implementation of BD requires using downlink pilots or a feedforward link. There are channel estimation errors and quantization errors associated with these methods respectively. Here, the effect of feedforward is demonstrated.

The sum rate achieved by BD with a feedforward is compared with the sum rate achieved by BDI in Fig.4 for a $\{2, 2\} \times 4$ MIMO broadcast channel. A 6-bit random vector quantization codebook is used to feedforward the multiuser precoder in BD. Comparing Fig.4 and 1, it can be observed that the performance of BD is significantly degraded as SNR increases when a limited feedforward is used. However, the performance of BDI remains the same since no feedforward is needed. Therefore, the proposed BDI scheme is a good alternative to BD and achieves a better sum performance than BD in practice.

VI. Conclusions

In this paper, a modified diagonalization scheme called Block Diagonalization Inversion (BDI) is proposed. The proposed method avoids the need to forward the equivalent channel to the users by combining block diagonalization (BD) with partial inversion of the equivalent channel. It is observed that the sum rate loss of BDI as compared to an ideal BD is small and does not increase with signal-to-noise-ratio. When

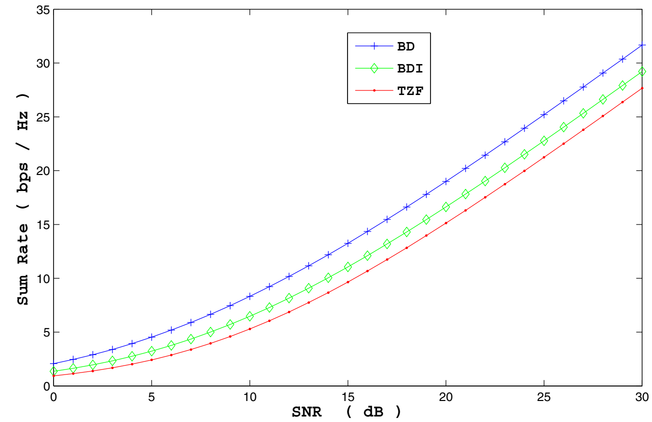

Fig. 1. Sum rate vs. SNR with equal power allocation in a $\{2, 2\} \times 4$ iid channel

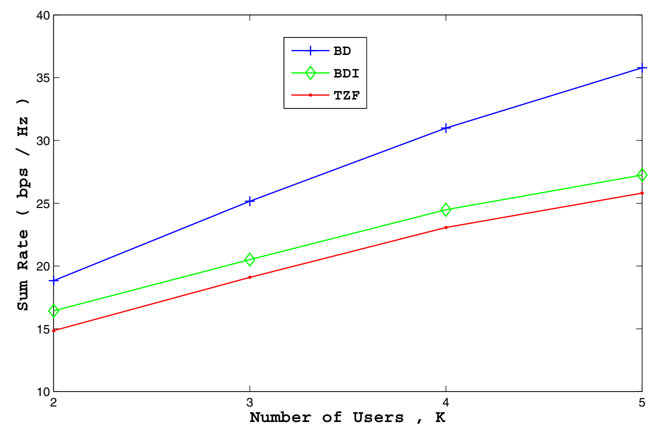

Fig. 2. Sum rate vs. number of users with equal power allocation at SNR of $20\,dB$, each user has $2 \times 2K$ iid channel

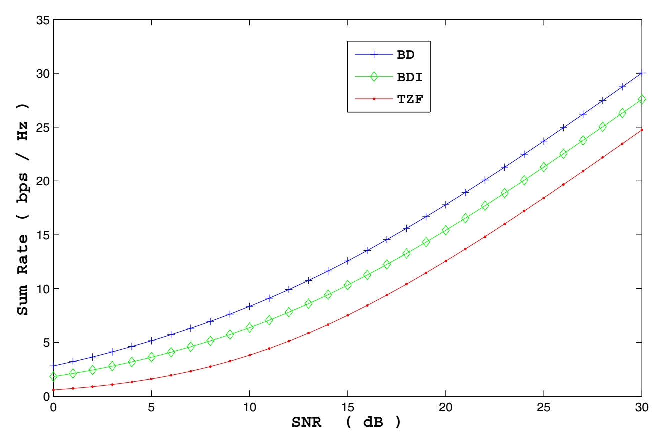

Fig. 3. Sum rate vs. SNR with optimal power allocation in a $\{2, 2\} \times 4$ channel with receiver antenna correlation

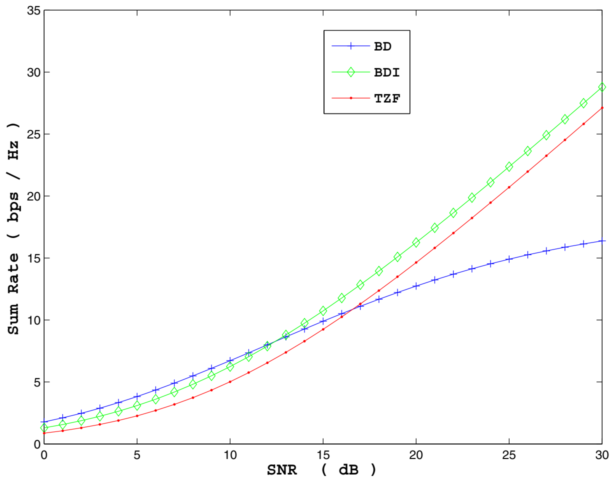

Fig. 4. Sum rate vs. SNR with equal power allocation in a $\{2,2\} \times 4$ iid channel with a 6-bit limited feedforward

a practical BD precoding that uses a limited feedforward is used, the sum rate achieved by BDI is found to be much higher than that of BD. The performance loss decreases as the quality of quantization in feedforward increases. Even though BDI involves an inversion operation, it has a low power enhancement as compared to transmitter zero forcing. As a result, it is observed that BDI performs better than transmitter zero forcing; the performance difference is higher when optimal power allocation is used. In general, the proposed BDI precoding is a practical, low overhead precoding scheme that offers good performance in MIMO broadcast without the draw backs of equivalent channel forwarding inherent in BD precoding.

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