Improved Iterative Decoders for Turbo-Coded Decode-and-Forward Relay Channels

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***Abstract*—An Improved Maximum *A Posteriori* (IMAP) decoder for turbo-coded decode-and-forward (DF) relay channels which takes into account the decoding errors at the relay is analyzed. This decoder is implemented in an iterative manner similar to the traditional iterative decoder (TID) used for turbo codes. The performance of this IMAP decoder is compared with the performance of an iterative decoder normally used in the literature which does not take into account the decoding errors at the relay, by simulation. The comparison shows that although the proposed IMAP decoder provides a better performance, especially when many decoding errors occur at the relay, the improvement is not significant. Then, another heuristic modification for the iterative decoder is proposed. In spite of the lack of theoretical analysis, the numerical results show that the proposed heuristically modified iterative decoder (HMID) gives significantly better performance than the traditional one.**

***Index Terms*—maximum *a posteriori*, iterative decoder, decode-and-forward, relay channels**

I. Introduction

Cooperative communication has been a very active research topic since the pioneering work of Van der Meulen [1] in 1971. In this work, a three-node relay channel which consists of a source node, a relay and a destinataion node was first introduced together with the capacity's inner bound and outer bound for the discrete-memoryless relay channel. Later on, the work of Cover and El Gamal [2] produced the fundamental cooperative strategies and capacity bounds for Additive White Gaussian Noise (AWGN) single-relay channels, for deterministic relay channels as well as relay channels with feedback. Since then, theoretical work on the capacity bounds for different types of relay channels has been introduced in [3], [4], [5].

Besides the theoretical work, practical coding schemes for DF relaying strategy, one of the most well-known cooperative strategies, have also been proposed for relay networks. In [6] a rate-compatible punctured convolutional code was implemented. More recently, low-density parity-check (LDPC) codes were introduced for relay channels in [7], [8], [9]. Turbo codes were also presented for relay channels in [10], [11], [12], [13], [14] and most recently turbo trellis coded modulation for cooperative communications was introduced in [15].

Besides the design of efficient practical channel codes, the design of an efficient decoder for a specific channel code is also an important research topic. The maximum likelihood decoder is the optimum decoder but usually very cumbersome or impossible to implement. Therefore, suboptimal decoders are used. For turbo-coded or LDPC-coded DF relay channels, iterative decoders which are first proposed for turbo codes [16] are usually utilized. However, unlike turbo codes whose input information bits of the two constituent encoders are always the same, the convolutional encoders of the source and the relay may not have the same input information bits due to erroneous decoding at the relay. Since the TID is not designed to take these decoding errors into consideration, these errors will be propagated between the constituent decoders of the iterative decoder which will degrade the system's performance.

In this paper, we consider a turbo-coded DF relay system with the same recursive systematic convolutional encoder used at the source and the relay. Based on the analysis of the maximum *a posteriori* decoder at the destination, we propose two new iterative decoders which are based on the TID but with some modifications which take into account the decoding errors at the relay. Simulated results show that our proposed modified iterative decoders give better performance in comparison with the TID.

The remainder of this paper is organized as follows. In Section II the system model is introduced. In Section III, the detailed analysis of the IMAP decoder for the turbo-coded DF relay system is provided. In Section IV, the performance of the IMAP decoder is by simulation compared with the performance of the iterative decoder. In Section V, the HMID is introduced. In Section VI, simulated performance of the HMID is given. Moreover, numerical analysis on the probability density function (PDF) of the input and output log-likelihood ratios (LLRs) of the HMID's constituent decoder gives some more insight into the improved performance of the HMID and provides an efficient way to find the optimum modification coefficient. Finally, conclusions are made in Section VII.

II. System model

The system model is shown in Fig. 1. The relay system consists of three nodes: a source, a relay, and a destination. The channels between the source and destination, source and relay, relay and destination are AWGN channels with signal-to-noise ratios (SNRs) denoted as γ_{sd}, γ_{sr} and γ_{rd}, respectively.

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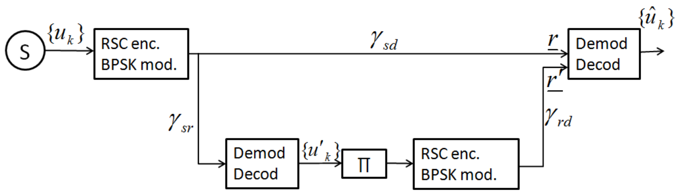


Fig. 1: System model.

Two Recursive Systematic Convolutional (RSC) encoders and Binary Phase Shift Keying (BPSK) modulators are used at the source node and the relay node. The information bit sequence is denoted by $\{u_k\}$.

The system operates in a time-division manner. In the first time-slot of the transmission, the source transmits the encoded and modulated symbols to the destination. Because of the broadcast nature of the wireless channels, the relay can also receive the noisy versions of the transmitted symbols. The relay decodes, demodulates and generates the detected $\{u'_k\}$ of the information bit sequence $\{u_k\}$ which may be different from $\{u_k\}$ due to decoding errors at the relay. The decoder at the relay can be either a Viterbi decoder [17] or a BCJR decoder [18]. The detected sequence is then re-encoded, re-modulated and transmitted to the destination in the second time-slot. The destination receives two noisy observation sequences denoted by $\underline{r}$ and $\underline{r}'$, which are sent from the source and the relay, respectively. By using these two received sequences and a modified iterative decoder (IMAP or HMID), the destination generates the detected $\{\hat{u}_k\}$ of the source's originally transmitted sequence $\{u_k\}$.

III. Improved Maximum *A Posteriori* Decoder

The maximum *a posteriori* (MAP) decoding rule at the destination is given by

$$\begin{cases} u_k = 1 & \text{if } L_k \geq 0 \\ u_k = 0 & \text{if } L_k < 0, \end{cases} \tag{1}$$

where L_k is the LLR defined by

$$L_k = \ln \frac{\sum_{u'_k \in \{0,1\}} P(u_k = 1, u'_k | \underline{r}, \underline{r}')}{\sum_{u'_k \in \{0,1\}} P(u_k = 0, u'_k | \underline{r}, \underline{r}')}. \tag{2}$$

and k is the bit interval index.

L_k can be developed further as

$$L_k = \ln \frac{P(u_k = 1, \underline{r})}{P(u_k = 0, \underline{r})} + \ln \frac{P(u'_k = 1, \underline{r}')}{P(u'_k = 0, \underline{r}')} + \ln \frac{P(u'_k = 1)}{P(u'_k = 0)} + \\ + \ln \frac{\frac{P(u'_k=0,\underline{r}')}{P(u'_k=1,\underline{r}')} P(0|1) + \frac{P(u'_k=0)}{P(u'_k=1)} P(1|1)}{\frac{P(u'_k=1,\underline{r}')}{P(u'_k=0,\underline{r}')} P(1|0) + \frac{P(u'_k=1)}{P(u'_k=0)} P(0|0)}. \tag{3}$$

The IMAP decoder is implemented in an iterative manner similar to the iterative decoder used for turbo codes introduced in [16]. It consists of two BCJR decoders, one for the source's RSC encoder (decoder 1) and the other for the relay's RSC encoder (decoder 2). The first and second term in (3) are the outputs of the decoder 1 and the decoder 2 in each decoding iteration, respectively. The third term can be approximated by the extrinsic information sent from decoder 1 to decoder 2. After a number of iterations, the LLR L_k is computed by (3), whose last term includes two conditional probabilities $P(0|1) = P(u'_k = 0|u_k = 1)$ and $P(1|0) = (u'_k = 1|u_k = 0)$ which are the bit error probabilities at the relay and can be estimated by $BERr$ which is the bit error rate of the relay's decoder. The other two conditional probabilities appearing in the last term of the right hand side of (3) are the probabilities of correct decoding at the relay and can be estimated by $1 - BERr$.

Therefore, the IMAP decoder must be able to approximate the $BERr$ at the relay in order to calculate L_k. This can be done if the destination knows the source-relay (SR) channel's SNR.

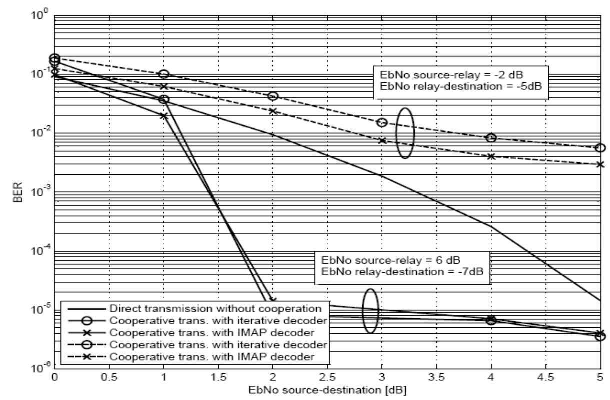
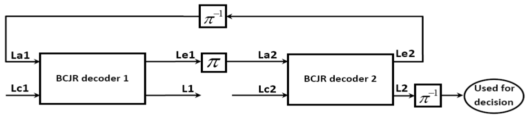


Fig. 2: Comparison between IMAP decoder and normal iterative decoder.

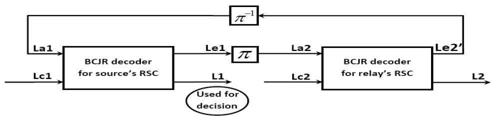
IV. Experimental results of the IMAP decoder

The simulations were made with the same 16-state RSC encoder implemented at the source and the relay whose generator matrix [19] is $G = [1, G_1/G_r]$ with $G_1 = [21]_8$ and $G_r = [37]_8$. The information block length is $N = 65536$ bits and the iteration number is 6 iterations. Two cases were considered, one is when $\gamma_{sr} = 6dB$ and $\gamma_{rd} = -7dB$, the other is when $\gamma_{sr} = -2dB$ and $\gamma_{rd} = -5dB$. The simulated performance of the IMAP decoder in terms of BER vs SNR was compared with the TID. Moreover, the performance of the cooperative systems were compared with the direct transmission system. The numerical results are shown in Fig. (2). In the first case, the SR channel condition is very good, so there are just a few errors at the relay. In this case the IMAP decoder does not provide any better performance in comparison with the TID as shown by the solid curves. In the second case, the SR channel is very bad yielding lots of errors at the relay. In this case, as shown by the dashed curves, the IMAP decoder gives a performance better than the TID.

In conclusion, we can see that although the IMAP decoder provides a better performance, the improvement is not significant, especially when there are not many errors at the relay. In the next section, another modified iterative decoder is introduced which offers a performance significantly better than the traditional one.



(a) Traditional turbo code's iterative decoder



(b) Heuristically modified iterative decoder

Fig. 3: Iterative decoders.

V. Heuristically modified iterative decoder

Before getting into the details of the HMID, the comparison between a traditional turbo code's iterative decoder and a turbo-coded DF relay system's iterative decoder is studied. The two decoders are illustrated in Fig. (3).

A. *Traditional turbo code's iterative decoder*

The traditional turbo code's iterative decoder is shown in Fig. (3a). There are two BCJR decoders, one for each of the RSCs [16]. In each decoding iteration, the inputs of each BCJR decoder are the *a priori* LLR Lai $(i = 1, 2)$ and the channel LLR Lci. The outputs of each decoder are the *a posteriori* LLR Li and the extrinsic information LLR Lei which will be sent to the other decoder as the *a priori* information Laj $(j = 1, 2; j \neq i)$ of the other decoder. In Fig. (3), π and π^{-1} denote the bitwise interleaver and deinterleaver, respectively. The output *a posteriori* LLR of either decoder 1 or decoder 2 can be used for decision after a number of iterations.

B. *Heuristically modified iterative decoder*

The HMID for the turbo-coded DF relay channel is shown in Fig. (3b). It is very similar to the turbo code's iterative decoder. The outer BCJR decoder is for the source's RSC and the inner one is for the relay's RSC. The turbo-coded DF relay system differs from the turbo code in that the input information bits of the source's RSC and the relay's RSC may not be the same due to the decoding errors at the relay, yielding the fact that the input *a priori* LLRs of the two BCJR decoders may not be for the same information bit. The errors at the relay are propagated from one iteration to the other and from the BCJR decoder 2 to the BCJR decoder 1 by the extrinsic information $Le2'$. Moreover, the *a posteriori* information $L1$ carries correct (or "reliable") information while $L2$ may carry erroneous (or "unreliable") information. Therefore, $L2$ should not be used for decision but $L1$ and there should be a modification for $Le2'$ so that it carries more reliable information and the effect of the propagated errors is decreased. It is extremely difficult to have theoretical analysis of the modification for $Le2'$. Therefore, we propose a heuristic solution to the problem.

Since $L1$ is known to carry correct information, it could help to make $Le2'$ look more reliable if being added to $Le2'$ in each decoding iteration. In this way, the erroneous information carried by $Le2'$ would be reduced in every iteration, hence not being propagated to the BCJR decoder 1 and not being propagated to the following iterations. This proposed modification can be modeled by the following formula

$$Le2' := Le2' + \alpha\pi(L1), \quad \alpha > 0 \tag{4}$$

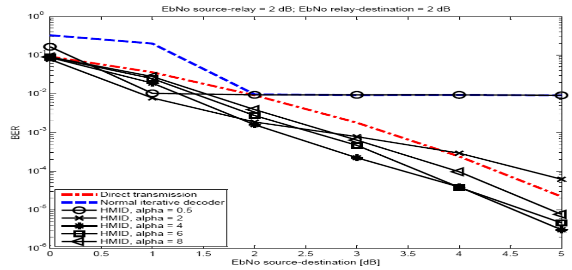
where $\pi(L1)$ denotes the interleaved value of $L1$ and α is a coefficient defining the amount of $L1$ that should be added to $Le2'$.

A question raised is how much of $L1$ should be added to $Le2'$, i.e. which value of α should be chosen. If α is too large, $L1$ will dominate the modified value of $Le2'$, then the extrinsic information transferred from the decoder 2 to the decoder 1 is in fact a scaled version of the decoder 1's output $L1$, and the effect of information evolution of an iterative decoder will be diminished. In contrast, if α is too small, the effect of $L1$ to the modified $Le2'$ will be ignorant and the HMID will be almost the same as the normal iterative decoder. We have no analytical solution to the problem of choosing the optimum value of α. However, a numerical investigation on the PDFs of the input and output LLRs can shed some light on the behavior of the HMID and provide an efficient way to find the optimum value of α for a certain condition of the SR and relay-destination (RD) channels.

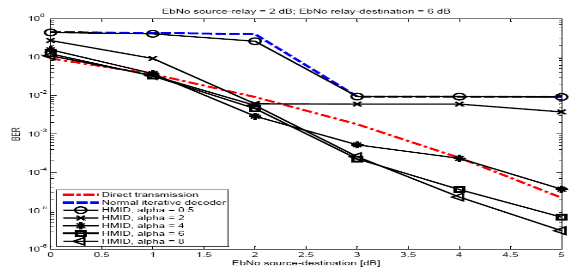
VI. Experimental results of the HMID

In this section, the performance of a turbo-coded DF relay channel using both an HMID and an iterative decoder is studied by simulation. The simulation parameters are the same as in Section IV, with different values of α of the set $\{0.5, 2, 4, 6, 8\}$ are chosen. Four cases corresponding to different conditions of the SR and RD channels are considered. The first two cases given in Fig. (4a) and Fig. (4b) are when the SR channel is bad with a low SNR $\gamma_{sr} = 2dB$ and the RD channel is either bad with a low SNR $\gamma_{rd} = 2dB$ or good with a high SNR $\gamma_{rd} = 6dB$, respectively. The last two cases shown in Fig. (4c) and Fig. (4d) are when the SR channel is good with a high SNR $\gamma_{sr} = 6dB$ and the RD channel is either bad or good similarly to the first two cases. The performance of the two decoders is measured in terms of the average BER. In Fig. (4), the performance of the direct transmission scheme and the relay-aided scheme using the TID is illustrated by the dash-dotted curve and the dashed curve, respectively.

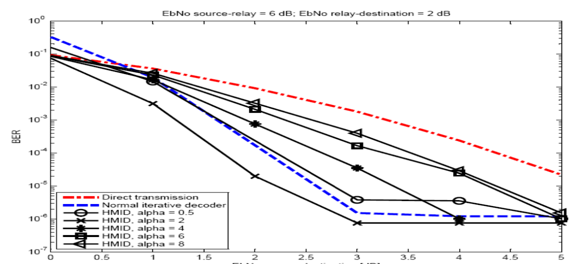
In the first two cases when the channel between the source and the relay is bad yielding lots of decoding errors at the relay, the DF relay system using the TID gives even a worse performance than the direct transmission, as shown in Fig. (4a) and Fig. (4b). Now the error propagation has a severe effect on the performance of the TID. Furthermore, the better the RD channel, the more severe the error propagation effect becomes and the worse the relay-aided scheme's performance gets. The performance of the relay system using the HMID with different values of α are illustrated by the solid curves. As expected, the performance of the HMID with $\alpha = 0.5$, shown by the circle-marked solid curve, is very close to the performance of the TID since α is too small to have any effect on $Le2'$. When α is increased, the HMID's performance is significantly improved and eventually becomes much better



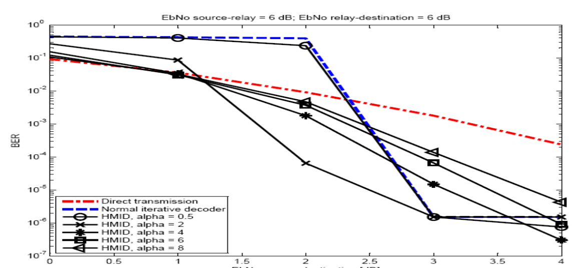
(a) $\gamma_{sr} = 2dB$, $\gamma_{rd} = 2dB$



(b) $\gamma_{sr} = 2dB$, $\gamma_{rd} = 6dB$

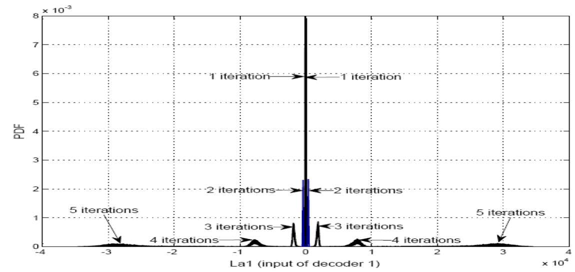


(c) $\gamma_{sr} = 6dB$, $\gamma_{rd} = 2dB$

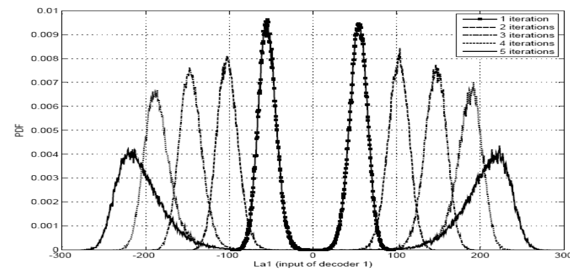


(d) $\gamma_{sr} = 6dB$, $\gamma_{rd} = 6dB$

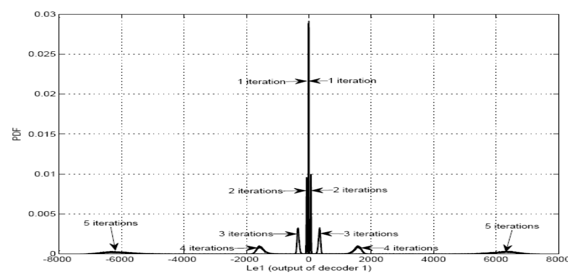
Fig. 4: Comparison between HMID and TID.



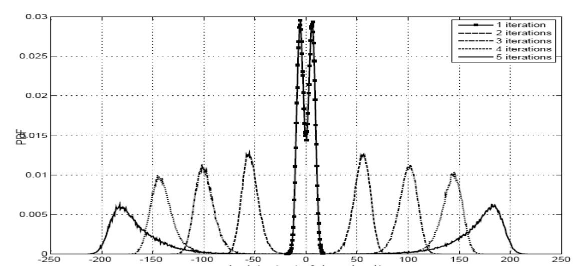
(a) $La1$, $\alpha = 2$, HMID



(b) $La1$, TID



(c) $Le1$, $\alpha = 2$, HMID

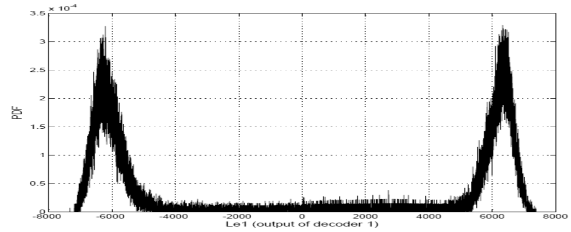


(d) $Le1$, TID

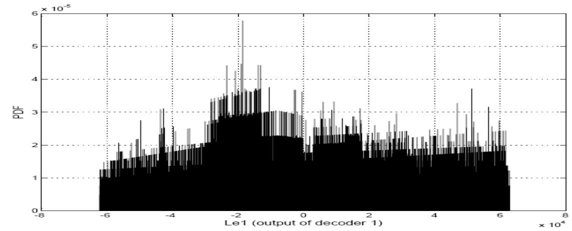
Fig. 5: PDFs of $La1$ and $Le1$, $\gamma_{sr} = 6dB$, $\gamma_{rd} = 6dB$, $\gamma_{sd} = 1.5dB$.

than the performance of the direct transmission scheme. This shows that the modification for $Le2'$ does actually help to reduce the error propagation encountered by the TID. In the first case shown in Fig. (4a), the most appropriate value of α which offers the best performance of the HMID is $\alpha = 4$. If we choose α to be larger than 4, the HMID's performance will be degraded consequently because the information evolution effect is faded out. On the other hand, when the error propagation becomes more serious, i.e., $Le2'$ carries mostly erroneous information, we need a larger value of α so that the correct information in $L1$ can compensate for the "unreliability" of $Le2'$. Therefore, in the second case, as demonstrated by Fig. (4b), the optimum value of α should be $\alpha = 8$.

In the last two cases with a good SR channel, there are just a few errors at the relay and the error propagation has less effect on the iterative decoders. The DF relay scheme is shown to provide significantly better performance than the direct transmission scheme, either with the TID or with the HMID. However, with $\alpha > 0.5$, the HMID gains a much better performance than the TID. Moreover, since the error propagation effect is less severe in these cases, the optimum value of α is not necessarily large. In fact, as demonstrated by Fig. (4c) and Fig. (4d), the optimum value of α in both cases is $\alpha = 2$. We will get similar results to the first two cases if we increase α above the optimum value, that is the HMID's performance is degraded consequently because the effect of information evolution is faded out.



(a) $\alpha = 2$



(b) $\alpha = 8$

Fig. 6: Comparison of the PDF of $Le1$ of the HMID with different values of α at the $5th$ iteration.

In order to shed some light on the behavior of the HMID, the PDFs of $La1$ and $Le1$ at the input and output of the HMID's outer decoder, respectively, are analyzed numerically in 5 decoding iterations for the last case in Fig. (4d) when $\gamma_{sr} = 6dB$, $\gamma_{rd} = 6dB$, $\gamma_{sd} = 1.5dB$, $\alpha = 2$, then compared to those of the TID as given by Fig. (5). It is shown that in every iteration, for both $La1$ and $Le1$, the separation between two "hills" of the PDFs using the HMID is much larger than those using the TID. Since the LLR used for making decision $L1$ is the sum of $La1$, $Le1$ and $Lc1$, and the latter is the same for both HMID and TID, the separation between the "hills" of the PDF of $L1$ in the HMID case is also much larger than the TID case, which will give better decoding result.

We notice that $\alpha = 2$ is the optimum value of α in this case, as demonstrated by Fig. (4d). By using the above analysis, this optimum value can also be found quite quickly without having to run the whole long simulation. A comparison between the PDF of $Le1$ of the HMID with $\alpha = 2$ and $\alpha = 8$ at the $5th$ decoding iteration is given in Fig. (6). With the optimum value $\alpha = 2$, the PDF of $Le1$ is well divided into two distinct "hills", while with $\alpha = 8$, the PDF is almost flat with no distinction between two "hills". This explains the superior performance of the case $\alpha = 2$ in comparison with the case $\alpha = 8$, which is also supported by the simulation in Fig. (4d).

VII. Conclusions

We have analyzed the maximum *a posteriori* decoder for a turbo-coded DF relay channel and made some modifications to improve the performance of the iterative decoder normally used for the considered relay channel, the traditional iterative decoder. This decoder does not take into account the decoding errors at the relay. Therefore, when the SR channel's SNR is low, the iterative decoder performance is very bad due to the error propagation between the constituent decoders. One of the improved iterative decoders we proposed is called IMAP decoder that takes into account the relay's errors by adding the probability of error at the relay to the *a posteriori* LLR used for making decision. This modification is shown to provide an improvement in the system's performance when the SR channel's SNR is low but provide almost no improvement in the opposite case. Another proposed modification is called HMID which aims to decrease the effect of error propagation in every decoding iteration. Although heuristic and very simple, the modification is shown by simulation to provide significantly better performance to the relay channel in all conditions of the SR channel. The numerical results are very promising. A theoretical analysis will be considered in the future. Fading is to be also studied.

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