Robust Distributed Cognitive Relay Beamforming

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Abstract—**In this paper, we present a distributed relay beamformer design for a cognitive radio network in which a cognitive (or secondary) transmit node communicates with a secondary receive node assisted by a set of cognitive non-regenerative relays. The secondary nodes share the spectrum with a licensed primary user (PU) node, and each node is assumed to be equipped with a single transmit/receive antenna. The interference to the PU resulting from the transmission from the cognitive nodes is kept below a specified limit. The proposed robust cognitive relay beamformer design seeks to minimize the total relay transmit power while ensuring that the transceiver signal-to-interference-plus-noise ratio and PU interference constraints are satisfied. The proposed design takes into account a parameter of the error in the channel state information (CSI) to render the performance of the beamformer robust in the presence of imperfect CSI. Though the original problem is non-convex, we show that the proposed design can be reformulated as a tractable convex optimization problem that can be solved efficiently. Numerical results are provided and illustrate the performance of the proposed designs for different network operating conditions and parameters.**

I. INTRODUCTION

Using cognitive radio (CR) as a promising technology to enhance the spectrum utilization [1]–[3], different schemes for spectrum sharing among the cognitive or secondary users (SUs) and primary users (PUs) have been proposed. Notable among these, are schemes which involve the concept of cooperation to increase the transmission opportunities among SUs and/or to enhance the performance of the PUs' communication processes. Indeed, the SUs in a cognitive radio network often operating with low transmit power in order to limit the interference to the PUs, collaborative and relay-assisted transmission techniques will be beneficial in improving the rate and range of communication among the SUs while keeping the interference to the PUs minimal [4]–[7]. Relay-assisted wireless communication techniques can provide benefits like improvement in link quality and transmission reliability, and increase in coverage [8]–[11]. Various relaying schemes have been proposed in the literature. Among them, regenerative and non-regenerative schemes have been studied widely [8], [9], [12], [13].

In this paper, we propose a distributed relay beamformer design for CR networks where a set of cognitive (secondary) non-regenerative relay nodes assist a cognitive (secondary) transmit node communicating with a secondary receive node while sharing the spectrum with a PU. The proposed beamformer design is based on minimization of the total relay transmit power with a constraint on the signal-to-interference-plus-noise ratio (SINR) at the SU transceivers and a constraint on the PU tolerable interference. When the available channel state information (CSI) is imperfect, beamformer designs that assume perfect CSI result in performance degradation. Moreover, in a cognitive radio network, such non-robust designs can lead to interference to the PUs in excess of the specified limit [14]. The robust beamformer design proposed in this paper takes into account the upper-bound on errors in the CSI and ensures that the interference to the PU is below the limit even in the presence of imperfect CSI. We show that the proposed relay beamformer design can be reformulated as convex optimization problems that can be solved efficiently.

The rest of the paper is organized as follows. The system model is detailed in Section II. The proposed designs with perfect CSI and imperfect CSI are presented in Section III. Section V provides simulation results and comparisons. Conclusions are presented in Section VI.

II. SYSTEM MODEL

We consider a cognitive radio network consisting of a SU transmit node communicating with a SU receive node assisted by M relay nodes. The spectrum band available for the secondary communication process is shared with a PU transmit and receive nodes, and all the nodes are equipped with single antennas. We assume that there is no direct link between the secondary source and destination nodes, and that non-regenerative relaying is employed for the communication between these nodes. The relays operate according to the half-duplex mode. Hence, during the first time slot, the SU source node transmits the symbol[1] $s \in \mathbb{C}$ with $\mathbb{E}\{|s|^2\} = 1$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. The signal received by the k^{th} relay is given by

$$y_k \quad = \quad g_k\sqrt{p}s + u_k + \mu_k, \quad 1 \leq k \leq M, \tag{1}$$

where p is the transmit power of the SU transmit node, $g_k \in \mathbb{C}$ is the channel gain from the SU transmit node to the k^{th} relay, $u_k \in \mathbb{C}$ is the interference to the k^{th} relay due to transmission from the PU transmitter, and $\mu_k \in \mathbb{C}$ is an independent and identically distributed (i.i.d) complex Gaussian random variable with zero mean and variance σ_μ^2 representing the additive noise at the k^{th} relay node. During the second time slot, each relay transmits the signal received in the previous time slot after multiplying it by a complex scale (or weight)

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$ denotes transpose and $\|\cdot\|_F$ denotes the Frobenius norm. $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A}-\mathbf{B}$ is positive semi-definite, and $\mathbf{A} \succ \mathbf{B}$ implies $\mathbf{A}-\mathbf{B}$ is positive definite. $\Re(\cdot)$ and $\Im(\cdot)$ denote real part and imaginary part of the argument respectively.

factor. Then, the signal received by the destination node can be written as

$$
\begin{aligned}
z &= \sum_{k=1}^{M} h_k w_k y_k + v + \nu & (2)\\
&= \sum_{k=1}^{M} h_k w_k (g_k \sqrt{p} s + u_k + \mu_k) + v + \nu, & (3)
\end{aligned}
$$

where $w_k \in \mathbb{C}$ is the scaling factor at the k^{th} relay, $h_k \in \mathbb{C}$ is the channel gain from the kth relay node to the SU receive node, $v \in \mathbb{C}$ is the interference to the SU receive node due to transmission from the PU transmitter, and $\nu \in \mathbb{C}$ is the noise at the SU receive node.

Let $\mathbf{g} = [g_1\ g_2\ \cdots\ g_M]^T$, $\mathbf{h} = [h_1\ h_2\ \cdots\ h_M]^T$, $\mathbf{w} = [w_1\ w_2\ \cdots\ w_M]^T$, $\mathbf{u} = [u_1\ u_2\ \cdots\ u_M]^T$, $\mathbf{\Gamma} = \text{diag}(\mathbf{h})$, and $\boldsymbol{\mu} = [\mu_1\ \mu_2\ \cdots\ \mu_M]^T$, where the diag$(\cdot)$ operator constructs a diagonal matrix from the components of the argument vector. Using the above-defined variables, we can represent the received signal as

$$
z = \mathbf{w}^T \boldsymbol{\alpha} \sqrt{p} s + \mathbf{w}^T \mathbf{\Gamma}(\boldsymbol{\mu} + \mathbf{u}) + v + \nu, \qquad (4)
$$

where $\boldsymbol{\alpha} = \mathbf{g} \odot \mathbf{h}$.

In the first time slot, the transmission from the SU source results in interference to the PU receiver given by

$$
I_t = |\bar{g}|^2 p, \qquad (5)
$$

where $\bar{g}$ is the channel gain from the SU transmitter to the PU receiver. In the second time slot, the signal transmitted from the relays leads to interference to the PU receiver. This interference can be written as

$$
I_r = \mathbb{E}\left\{ \sum_{k=1}^{M} |f_k w_k y_k|^2 \right\}, \qquad (6)
$$

where f_k is the channel gain from the k^{th} relay to the PU receiver.

We consider CSI uncertainties that can be modeled as

$$
\begin{aligned}
\mathbf{g} &= \widehat{\mathbf{g}} + \mathbf{e}_g, & (7)\\
\mathbf{h} &= \widehat{\mathbf{h}} + \mathbf{e}_h, & (8)\\
\mathbf{f} &= \widehat{\mathbf{f}} + \mathbf{e}_f, & (9)
\end{aligned}
$$

where $\mathbf{g}$, $\mathbf{h}$ and $\mathbf{f}$ are the true CSI, $\widehat{\mathbf{g}}$, $\widehat{\mathbf{h}}$ and $\widehat{\mathbf{f}}$ are the imperfect CSI available at the relays, and $\mathbf{e}_g$, $\mathbf{e}_h$ and $\mathbf{e}_f$ represent the error in the CSI. Further, we assume that $\|\mathbf{e}_g\| \leq \delta_g$, $\|\mathbf{e}_h\| \leq \delta_h$ and $\|\mathbf{e}_f\| \leq \delta_f$. Equivalently, $\mathbf{g}$ belongs to the uncertainty set $\mathcal{R}_g$, $\mathbf{h}$ belongs to the uncertainty set $\mathcal{R}_h$ and $\mathbf{f}$ belongs to the uncertainty set $\mathcal{R}_f$, where

$$
\mathcal{R}_g = \{\zeta \big| \zeta = \widehat{\mathbf{g}} + \mathbf{e}_g, \|\mathbf{e}_g\| \leq \delta_g\}, \qquad (10)
$$

$$
\mathcal{R}_h = \{\zeta \big| \zeta = \widehat{\mathbf{h}} + \mathbf{e}_h, \|\mathbf{e}_h\| \leq \delta_h\} \qquad (11)
$$

and

$$
\mathcal{R}_f = \{\zeta \big| \zeta = \widehat{\mathbf{f}} + \mathbf{e}_f, \|\mathbf{e}_f\| \leq \delta_f\}. \qquad (12)
$$

III. Robust Cognitive Beamforming with SINR Constraints

Design of distributed cognitive beamformer that minimizes the total transmit power at the relays while meeting SINR and PU interference constraints is of practical interest. In this section, we present the proposed robust design of such a beamformer when the available CSI ($\mathbf{g}, \mathbf{h}$ and $\mathbf{f}$) is imperfect. Before proceeding with the robust design, we briefly describe the beamforming assuming availability of perfect CSI.

A. Beamforming under Perfect CSI

When perfect CSI is available at the relays, the problem of cognitive beamformer design that minimizes the total relay transmit power under QoS and PU interference constraints can be stated as

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & P\\
\text{subject to} \quad & \text{SINR} \geq \eta',\\
& I_r \leq \theta, \qquad (13)
\end{aligned}
$$

where P is the total relay transmit power, η' is the minimum SINR required at the receive node and θ is the maximum allowed interference to the PU receiver. Based on (4), the total signal power received by the SU destination node is given by

$$
\begin{aligned}
P_s &= \mathbb{E}\{|\mathbf{w}^T \boldsymbol{\alpha} \sqrt{p} s|^2\}\\
&= p|\mathbf{w}^T \boldsymbol{\alpha}|^2. \qquad (14)
\end{aligned}
$$

Let $\underline{\mathbf{\Gamma}} = \text{diag}(\{(\pi_i + \sigma_\mu^2)^{1/2} h_i\}_{i=1}^M)$, where $\pi_i = \mathbb{E}\{|u_i|^2\}$. The undesired component of the total receive power at the SU destination consists of the interference from the PU transmitter, both received directly and via relays, and the noise power at the receive node. Thus, the total interference-plus-noise power can be expressed as

$$
\begin{aligned}
P_n &= \sum_{k=1}^{M} \left(\mathbb{E}\{|w_k h_k \mu_k|^2\} + \mathbb{E}\{|w_k h_k u_k|^2\}\right) + \mathbb{E}\{|v|^2 + |\nu|^2\}\\
&= \|\underline{\mathbf{\Gamma}} \mathbf{w}\|^2 + \phi + \sigma_\nu^2, \qquad (15)
\end{aligned}
$$

where $\phi = \mathbb{E}\{|v|^2\}$ and where we have assumed that μ_k, $k = 1, \cdots, M$, are i.i.d. and are independent of ν. The total relay transmit power can be expressed as

$$
\begin{aligned}
P &= \sum_{k=1}^{M} \mathbb{E}\left\{ |w_k y_k|^2 \Big| g_k \right\}\\
&= p \sum_{k=1}^{M} |w_k g_k|^2 + \sum_{k=1}^{M} \mathbb{E}\{|w_k u_k|^2\} + \sum_{k=1}^{M} |w_k|^2 \sigma_\mu^2\\
&= p\|\mathbf{\Lambda} \mathbf{w}\|^2 + \|\bar{\mathbf{\Lambda}} \mathbf{w}\|^2 + \|\mathbf{w}\|^2 \sigma_\mu^2, \qquad (16)
\end{aligned}
$$

where $\mathbf{\Lambda} = \text{diag}(\mathbf{g})$ and $\bar{\mathbf{\Lambda}} = \text{diag}(\{\sqrt{\pi_i}\})$. Defining SINR $= P_s/P_n$, the problem in (13) can be rewritten as follows:

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & p\|\mathbf{\Lambda} \mathbf{w}\|^2 + \|\bar{\mathbf{\Lambda}} \mathbf{w}\|^2 + \|\mathbf{w}\|^2 \sigma_\mu^2 & \text{(17a)}\\
\text{subject to:} \quad & \frac{p|\mathbf{w}^T \boldsymbol{\alpha}|^2}{\|\mathbf{\Gamma} \mathbf{w}\|^2 + \|\underline{\mathbf{\Gamma}} \mathbf{w}\|^2 + \phi + \sigma_\nu^2} \geq \eta, & \text{(17b)}\\
& \|\mathbf{\Lambda}_1 \mathbf{w}\|^2 + \|\bar{\mathbf{\Lambda}}_1 \mathbf{w}\|^2 + \frac{\sigma_\mu^2}{p}\|\mathbf{w}\|^2 \leq \frac{\theta}{p}, & \text{(17c)}
\end{aligned}
$$

where $\eta = \frac{\eta'}{p}$, $\mathbf{\Lambda}_1 = \text{diag}(\boldsymbol{\beta})$ and $\bar{\mathbf{\Lambda}}_1 = \frac{1}{\sqrt{p}}\text{diag}(\boldsymbol{\pi} \odot \mathbf{f})$ with $\boldsymbol{\beta} = \mathbf{g} \odot \mathbf{f}$. We observe that the beamformer design problem as stated above is not convex. However, it can be reformulated as a convex optimization program as follows. The constraint in (17b) can be rewritten as

$$\|\underline{\mathbf{\Gamma}}\mathbf{w}\|^2 + \phi + \sigma_\nu^2 \leq \frac{1}{\eta}|\mathbf{w}^T\boldsymbol{\alpha}|^2 \tag{18a}$$

$$\Rightarrow \|\underline{\mathbf{\Gamma}}\mathbf{w}\|^2 + \phi + \sigma_\nu^2 \leq \frac{1}{\eta}|\mathbf{w}^T\boldsymbol{\alpha}|^2 \tag{18b}$$

$$\Rightarrow \|\mathbf{M}\| \leq \frac{1}{\sqrt{\eta}}\mathbf{w}^T\boldsymbol{\alpha}, \tag{18c}$$

where

$$\mathbf{M} = \left[(\underline{\mathbf{\Gamma}}\mathbf{w})^T \quad \sqrt{\phi} \quad \sigma_\nu\right]^T. \tag{19}$$

Further, in (18c), we have assumed $\Re(\mathbf{w}^T\boldsymbol{\alpha}) \geq 0$ and $\Im(\mathbf{w}^T\boldsymbol{\alpha}) = 0$. We note that we can indeed restrict our search to those $\mathbf{w}$ which satisfy the above-mentioned conditions, since an arbitrary phase rotation of $\mathbf{w}$ does not change the objective or the constraint in (17).

In the rest of this paper, we will consider the optimization problem in (17) in terms of real variables. For this purpose, any complex matrix $\mathbf{Z}$ is mapped into the corresponding real matrix as

$$\begin{bmatrix} \Re(\mathbf{Z}) & \Im(\mathbf{Z}) \\ -\Im(\mathbf{Z}) & \Re(\mathbf{Z}) \end{bmatrix}. \tag{20}$$

The complex vector $\boldsymbol{\alpha} = \mathbf{g} \odot \mathbf{h}$ is mapped into the corresponding real vector as $[\Re(\boldsymbol{\alpha}) \ \Im(\boldsymbol{\alpha})]^T$. Any other complex vector $\mathbf{x}$ is mapped into the corresponding real vector as $[\Re(\mathbf{x}) \ \ -\Im(\mathbf{x})]^T$. In what follows, we will use the same symbols for the real variables as well in order to avoid notational complexity. The optimization problem in (17) can be reformulated as the following convex optimization problem

$$\begin{aligned} \min_{\mathbf{w},r_1,r_2} \quad & r_1 + r_2 \qquad\qquad (21) \\ \text{subject to:} \quad & \mathbf{w}^T\mathbf{\Lambda}^T\mathbf{\Lambda}\mathbf{w} \leq \frac{r_1}{p}, \\ & \|\bar{\mathbf{\Lambda}}\mathbf{w}\|^2 + \sigma_\mu^2\|\mathbf{w}\|^2 \leq r_2, \\ & \|\mathbf{M}\| \leq \frac{\mathbf{w}^T\mathbf{h}}{\sqrt{\eta}}, \\ & \mathbf{w}^T\mathbf{\Lambda}_1^T\mathbf{\Lambda}_1\mathbf{w} \leq r_3, \\ & \|\bar{\mathbf{\Lambda}}_1\mathbf{w}\|^2 + \frac{\sigma_\mu}{p}\|\mathbf{w}\|^2 \leq r_4, \\ & r_3 + r_4 \leq \frac{\theta}{p}, \end{aligned}$$

where r_i $(i = 1, \cdots, 4)$ are slack variables. The objective function and the first two constraints in the above problem are obviously convex. The third constraint is a second-order cone constraint, and hence convex. Thus, the robust power minimization problem formulated as above is a convex optimization program which can be solved efficiently by interior point methods [15].

B. Robust Beamforming under Imperfect CSI

When the CSI available at the relays nodes is imperfect, we adopt a worst-case design approach in order to make the performance of the beamformer robust to the CSI errors. The beamforming vector is designed so as to meet the SINR targets for all channel vectors in the CSI uncertainty region of given size. Mathematically, this robust beamforming design can be expressed as

$$\min_{\mathbf{b},r_1,r_2} \quad r_1 + \sigma_\mu^2 r_2 \tag{22a}$$

$$\text{subject to:} \quad \mathbf{w}^T\mathbf{\Lambda}^T\mathbf{\Lambda}\mathbf{w} \leq r_1, \tag{22b}$$

$$\|\bar{\mathbf{\Lambda}}\mathbf{w}\|^2 + \sigma_\mu^2\|\mathbf{w}\|^2 \leq r_2, \tag{22c}$$

$$\|\mathbf{M}\| \leq \tfrac{\mathbf{w}^T\boldsymbol{\alpha}}{\sigma_\mu\sqrt{\eta}}, \tag{22d}$$

$$\mathbf{w}^T\mathbf{\Lambda}_1^T\mathbf{\Lambda}_1\mathbf{w} \leq t_3, \tag{22e}$$

$$\|\bar{\mathbf{\Lambda}}_1\mathbf{w}\|^2 \leq t_4, \tag{22f}$$

$$t_3 + t_4 + \tfrac{\sigma_\mu}{p}\|\mathbf{w}\|^2 \leq \tfrac{\theta}{p}, \tag{22g}$$

$$\forall \mathbf{g} \in \mathcal{R}_g, \ \forall \mathbf{h} \in \mathcal{R}_h, \ \forall \mathbf{f} \in \mathcal{R}_f. \tag{22h}$$

The last constraint in the problem given above ensures that the SINR requirement is satisfied for all realizations of the CSI in the corresponding uncertainty regions. Due to the presence of such a constraint, the problem in (22) belongs to the class of semi-infinite optimization problems, which are in general intractable [16]. However, we show in the following that this problem can be transformed into a tractable optimization program. Towards this end, first we consider the characterization of the uncertainty sets relevant to the aforementioned problem.

Considering the CSI error model, we have

$$\begin{aligned} \boldsymbol{\alpha} &= (\widehat{\mathbf{g}} + \widehat{\mathbf{e}}_g) \odot (\widehat{\mathbf{h}} + \mathbf{e}_h) \\ &\approx \underbrace{\widehat{\mathbf{g}} \odot \widehat{\mathbf{h}}}_{\widehat{\boldsymbol{\alpha}}} + \underbrace{\widehat{\mathbf{g}} \odot \mathbf{e}_h + \mathbf{e}_g \odot \widehat{\mathbf{h}} + \mathbf{e}_g \odot \mathbf{e}_h}_{\mathbf{e}_\alpha} \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{\beta} &= (\widehat{\mathbf{g}} + \widehat{\mathbf{e}}_g) \odot (\widehat{\mathbf{f}} + \mathbf{e}_f) \\ &\approx \underbrace{\widehat{\mathbf{g}} \odot \widehat{\mathbf{f}}}_{\widehat{\boldsymbol{\beta}}} + \underbrace{\widehat{\mathbf{g}} \odot \mathbf{e}_f + \mathbf{e}_g \odot \widehat{\mathbf{f}} + \mathbf{e}_g \odot \mathbf{e}_f}_{\mathbf{e}_\beta}. \end{aligned}$$

Then,

$$\mathbf{g} \in \mathcal{R}_g, \mathbf{h} \in \mathcal{R}_h \Longrightarrow \boldsymbol{\alpha} \in \mathcal{R}_\alpha \stackrel{\triangle}{=} \{\widehat{\boldsymbol{\alpha}} + \mathbf{e}_\alpha \,|\, \|\mathbf{e}_\alpha\| \leq \delta_\alpha\},$$

where

$$\delta_\alpha = \Bigg[\delta_h^2\Big(\|\mathbf{g}\|^2 - \sum_{i=1}^{M-1}\min(|g_i|^2, |g_{i+1}|^2)\Big) + \delta_g^2\Big(\|\mathbf{h}\|^2 - \sum_{i=1}^{M-1}\min(|h_i|^2, |h_{i+1}|^2) + \delta_g^2\delta_h^2\Big)\Bigg]^{1/2}.$$

Similarly,

$$\mathbf{g} \in \mathcal{R}_g, \mathbf{f} \in \mathcal{R}_f \Longrightarrow \boldsymbol{\beta} \in \mathcal{R}_\beta \stackrel{\triangle}{=} \{\widehat{\boldsymbol{\beta}} + \mathbf{e}_\beta \,|\, \|\mathbf{e}_\beta\| \leq \delta_\beta\},$$

where

$$\delta_\beta = \Bigg[\delta_f^2\Big(\|\mathbf{g}\|^2 - \sum_{i=1}^{M-1}\min(|g_i|^2, |g_{i+1}|^2)\Big) + \delta_g^2\Big(\|\mathbf{f}\|^2 - \sum_{i=1}^{M-1}\min(|f_i|^2, |f_{i+1}|^2) + \delta_g^2\delta_f^2\Big)\Bigg]^{1/2}.$$

Now, we represent the uncertainty sets in terms of $\mathbf{\Gamma}, \mathbf{\Lambda}, \mathbf{\Gamma}_1$. Let $\mathbf{\Lambda} = \widehat{\mathbf{\Lambda}} + \mathbf{E}_\Lambda$ where $\widehat{\mathbf{\Lambda}} = \text{diag}(\widehat{\mathbf{g}})$ and $\mathbf{E}_\Lambda = \text{diag}(\mathbf{e}_g)$, $\mathbf{\Gamma} = \widehat{\mathbf{\Gamma}} + \mathbf{E}_\Gamma$ where $\widehat{\mathbf{\Gamma}} = \text{diag}(\widehat{\mathbf{h}})$ and $\mathbf{E}_\Gamma = \text{diag}(\mathbf{e}_h)$, and let $\mathbf{\Lambda}_1 = \widehat{\mathbf{\Lambda}}_1 + \mathbf{E}_{\Lambda_1}$ where $\widehat{\mathbf{\Lambda}}_1 = \text{diag}(\widehat{\boldsymbol{\beta}})$ and $\mathbf{E}_{\Lambda_1} = \text{diag}(\mathbf{e}_\beta)$. Then, $\mathbf{g} \in \mathcal{R}_g \Longrightarrow \mathbf{\Lambda} \in \mathcal{R}_\Lambda = \{\mathbf{X} = \widehat{\mathbf{\Lambda}} + \mathbf{E}_\Lambda \,\big|\, \|\mathbf{E}_\Lambda\|_F \le \delta_g\}$, $\mathbf{h} \in \mathcal{R}_h \Longrightarrow \mathbf{\Gamma} \in \mathcal{R}_\Gamma = \{\mathbf{X} = \widehat{\mathbf{\Gamma}} + \mathbf{E}_\Gamma \,\big|\, \|\mathbf{E}_\Gamma\|_F \le \delta_h\}$, and $\boldsymbol{\beta} \in \mathcal{R}_\beta \Longrightarrow \mathbf{\Lambda}_1 \in \mathcal{R}_{\Lambda_1} = \{\mathbf{X} = \widehat{\mathbf{\Lambda}}_1 + \mathbf{E}_{\Lambda_1} \,\big|\, \|\mathbf{E}_{\Lambda_1}\|_F \le \delta_\beta\}$.

In order to recast the semi-infinite problem into tractable optimization program, we make use of the following result.

Lemma 1 [17]: Consider the uncertain quadratically constrained convex quadratic program (QCQP)

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{c}^T\mathbf{x} \qquad (23)\\ \text{s.t.:} \quad & \mathbf{x}^T\mathbf{A}^T\mathbf{A}\mathbf{x} - 2\mathbf{p}^T\mathbf{x} \le \gamma,\ \forall(\mathbf{A}, \mathbf{p}, \gamma) \in \mathcal{U}, \end{aligned}$$

where $\mathcal{U} = \{(\mathbf{A}, \mathbf{b}, \gamma) = (\mathbf{A}_0, \mathbf{b}_0, \gamma_0) + \sum_{k=1}^N v_k(\mathbf{A}_k, \mathbf{b}_k, \gamma_k) \big| \|\mathbf{v}\| \le 1\}$. The robust counterpart of this uncertain QCQP is equivalent to the following semi-definite problem (SDP):

$$\min_{\mathbf{x},\lambda} \quad \mathbf{c}^T\mathbf{x} \qquad (24)$$

$$\text{subject to:} \quad \begin{bmatrix} \gamma_0 + 2\mathbf{x}^T\mathbf{p}_0 - \lambda & \mathbf{N}_1 & \mathbf{A}_0\mathbf{x}^T \\ \mathbf{N}_1^T & \lambda\mathbf{I} & \mathbf{N}_2^T \\ \mathbf{A}_0\mathbf{x} & \mathbf{N}_2 & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \qquad (25)$$

where $\mathbf{N}_2 = [\mathbf{A}_1\mathbf{x} \ \cdots \ \mathbf{A}_N\mathbf{x}]$ and $\mathbf{N}_1 = \left[\gamma_1/2 + \mathbf{x}^T\mathbf{p}_1 \ \cdots \ \gamma_N/2 + \mathbf{x}^T\mathbf{p}_N\right]$. ■

Back to the optimization problem, consider the first constraint (22b). We have

$$\mathbf{w}^T\mathbf{\Lambda}^T\mathbf{\Lambda}\mathbf{w} \quad \le \quad \frac{r_1}{p} \qquad \forall \mathbf{g} \in \mathcal{R}_g. \qquad (26)$$

It is easy to see that the uncertainty in $\mathbf{\Lambda}$ can be represented by

$$\mathbf{\Lambda} \quad = \quad \widehat{\mathbf{\Lambda}} + \delta_g\text{diag}(\mathbf{v}), \qquad \|\mathbf{v}\| \le 1. \qquad (27)$$

Application of *Lemma 1* to (22b) leads to the following linear matrix inequality (LMI):

$$\mathbf{R} \equiv \begin{bmatrix} \frac{r_1}{p} - \zeta & \mathbf{0} & \widehat{\mathbf{\Lambda}}\mathbf{w}^T \\ \mathbf{0} & \zeta\mathbf{I} & \delta_g\mathbf{w}^T \\ \widehat{\mathbf{\Lambda}}\mathbf{w} & \delta_g\mathbf{w} & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \qquad (28)$$

Next, consider the constraint (22d). We can represent the uncertainty in $\underline{\mathbf{\Gamma}}$ by $\underline{\mathbf{\Gamma}} = \widehat{\underline{\mathbf{\Gamma}}} + \delta_\Gamma\text{diag}(\mathbf{v}), \|\mathbf{v}\| \le 1$, where $\widehat{\underline{\mathbf{\Gamma}}} = \text{diag}\left(\left\{\sqrt{\sigma_\mu^2 + \pi_i\widehat{h}_i}\right\}\right)$ and

$$\delta_\Gamma = \delta_h\left(\sigma_\mu^2 + \sum_{i=1}^M \pi_i - \sum_{i=1}^{M-1}\min(\pi_i, \pi_{i+1})\right)^{\frac{1}{2}}. \qquad (29)$$

The uncertainty in $\boldsymbol{\alpha}$ can be represented by $\boldsymbol{\alpha} = \widehat{\boldsymbol{\alpha}} + \delta_\alpha\mathbf{v}$, $\|\mathbf{v}\| \le 1$. Applying a result on uncertain conic program [17, 3.3], (22d) can be reformulated as the following LMIs:

$$\mathbf{S} \equiv \begin{bmatrix} \lambda - \mu & \mathbf{0} & \mathbf{\Psi}^T \\ \mathbf{0} & \mu\mathbf{I} & \begin{bmatrix}\delta_\Gamma\mathbf{w}^T & 0\end{bmatrix} \\ \mathbf{\Psi} & \begin{bmatrix}\delta_\Gamma\mathbf{w} \\ 0\end{bmatrix} & \lambda\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \qquad (30)$$

$$\mathbf{T} \equiv \begin{bmatrix} \frac{1}{\sqrt{\eta}}\widehat{\boldsymbol{\alpha}}^T\mathbf{w} - \lambda & \frac{1}{\sqrt{\eta}}\delta_\alpha\mathbf{w} \\ \frac{1}{\sqrt{\eta}}\delta_\alpha\mathbf{w}^T & \left(\frac{1}{\sqrt{\eta}}\widehat{\boldsymbol{\alpha}}^T\mathbf{w} - \lambda\right)\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \qquad (31)$$

where $\mathbf{\Psi} = \left[(\widehat{\underline{\mathbf{\Gamma}}}\mathbf{w})^T \quad \sqrt{\phi} \quad \sigma_\nu\right]^T$. In order to reduce the computational complexity, it is possible to reformulate the LMI, $\mathbf{T} \succeq \mathbf{0}$, as a second order cone (SOC) constraint. Making use of the Schur complement lemma [15], [18], the LMI in (31) can reformulated as the equivalent SOC constraint:

$$\delta_\alpha\|\mathbf{w}\| \quad \le \quad \widehat{\boldsymbol{\alpha}}^T\mathbf{w} - \frac{1}{\sqrt{\eta}}\lambda. \qquad (32)$$

Next, consider the constraint (22e). We can represent the uncertainty in $\mathbf{\Lambda}_1$ by $\mathbf{\Lambda}_1 = \widehat{\mathbf{\Lambda}}_1 + \delta_\beta\text{diag}(\mathbf{u}), \|\mathbf{u}\| \le 1$. Application of *Lemma 1* to (22e) leads to the following LMI:

$$\mathbf{Z} \equiv \begin{bmatrix} r_3 - \kappa & \mathbf{0} & \widehat{\mathbf{\Lambda}}_1\mathbf{w}^T \\ \mathbf{0} & \kappa\mathbf{I} & \delta_\beta\mathbf{w}^T \\ \widehat{\mathbf{\Lambda}}_1\mathbf{w} & \delta_\beta\mathbf{w} & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \qquad (33)$$

Similarly, it can be shown that the constraint in (22f) can be restated as the following LMI:

$$\bar{\mathbf{Z}} \equiv \begin{bmatrix} r_4 - \epsilon & \mathbf{0} & \widehat{\bar{\mathbf{\Lambda}}}_1\mathbf{w}^T \\ \mathbf{0} & \epsilon\mathbf{I} & \delta_f\mathbf{w}^T \\ \widehat{\bar{\mathbf{\Lambda}}}_1\mathbf{w} & \delta_f\mathbf{w} & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \qquad (34)$$

Based on the above development, the robust design of the relay weight vector $\mathbf{w}$ for minimizing the total relay transmit power under SINR constraint can be written as

$$\begin{aligned} \min_{\mathbf{w},\{r_i\},\zeta,\lambda,\mu,\kappa,\epsilon} \quad & r_1 + \sigma_\mu^2 r_2 && (35a)\\ \text{subject to:} \quad & r_3 + r_4 \le \frac{\theta}{p}, && (35b)\\ & \delta_\alpha\|\mathbf{w}\| \le \widehat{\boldsymbol{\alpha}}^T\mathbf{w} - \frac{1}{\sqrt{\eta}}\lambda, && (35c)\\ & \mathbf{R}(r_1, \zeta) \succeq \mathbf{0},\ \mathbf{S}(\lambda, \mu) \succeq \mathbf{0}, && (35d)\\ & \mathbf{Z}(r_3, \kappa) \succeq \mathbf{0},\ \bar{\mathbf{Z}}(r_4, \epsilon) \succeq \mathbf{0}. && (35e) \end{aligned}$$

The reformulation of the robust cognitive beamformer as given above is a semi-definite program (SDP), which can be solved efficiently.

IV. Simulation Results

In this section, we illustrate the performance of the proposed robust distributed beamforming design, evaluated through simulations. We compare the performance of the proposed robust design with the non-robust design described. The channel fading is modeled as Rayleigh, with the channel gain coefficients

g_k, h_k, f_k, $1 \leq k \leq M$, comprised of i.i.d. samples of a complex Gaussian process with zero mean and unit variance. The noise at each node is assumed to be zero-mean complex Gaussian random variable. In all the simulations, we have assumed $\delta_g = \delta_h = \delta_f$.

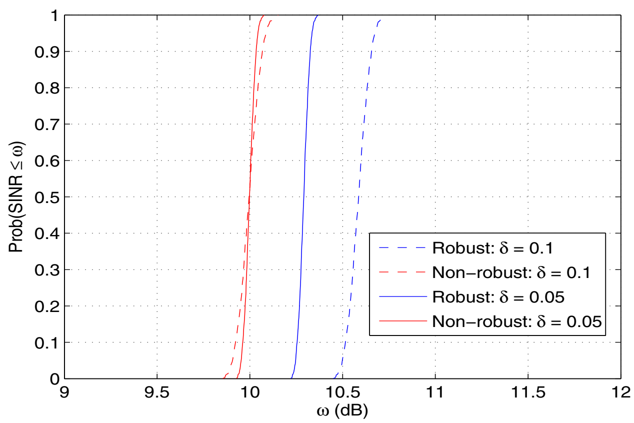

Fig. 1. Cumulative distribution of achieved SINR. Target SINR $\eta = 10$dB, $\delta_\alpha = \delta_\beta = 0.05$, 0.1, and $M = 10$ relays.

First, we compare the performance of the proposed robust design with the non-robust design in terms of the cumulative distribution of achieved SINR in the presence of CSI errors. For this comparison, we consider a system with one SU source node, one SU receive node and $M = 10$ relays. The target SINR is set to 10dB. The results are shown in Fig. 1. The non-robust design fails to achieve the SINR target. Furthermore, in the presence of CSI error, the larger the values of the CSI error bounds are, the higher is the probability of the SINR falling below the target. The robust design is found to achieve the target SINR in the presence of CSI errors.

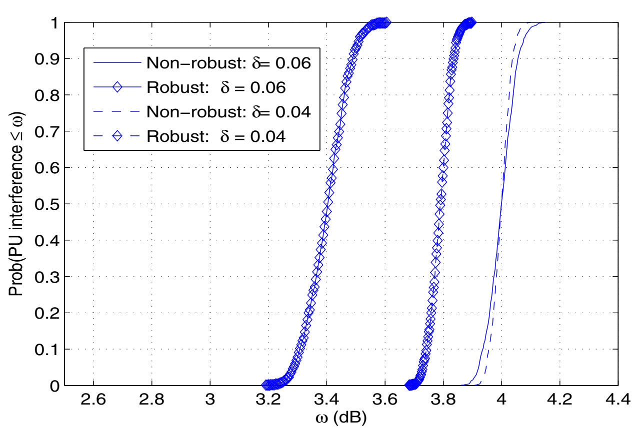

Fig. 2. Cumulative distribution of PU interference. Target SINR $\eta = 10$ dB, $\delta = \delta_g = \delta_h = \delta_f = 0.04$, 0.06, and $M = 10$ relays.

Next, we consider the performance in terms of the interference to the PU receiver. We consider a SU network with $M = 10$ relays. The PU interference threshold is set to $\theta = 4$dB. The cumulative distribution of the interference to the PU is shown in Fig. 2. The non-robust beamformer design results in PU interference beyond the permissible limit in the presence of CSI error. However, the robust design always guarantees the interference to the PU receiver remains below the threshold.

V. Conclusions

We presented a robust cognitive distributed beamforming design with SINR and PU interference constraints for a non-regenerative wireless relay network in the presence of imperfect CSI. The proposed beamformer design was based on the minimization of the total relay transmit power. We showed that the robust design can be formulated as a convex optimization problem that can be solved efficiently. Through simulation results, we illustrated the superior performance of the proposed robust design compared to the non-robust design in the presence of CSI imperfections.

References

[1] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, pp. 201–220, Feb. 2005.

[2] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, pp. 894–914, May 2009.

[3] L. Musavian and S. Aissa, "Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3472–3480, Nov. 2009.

[4] ——, "Cross-layer analysis of cognitive radio relay networks under quality of service constraints," in *Proc. IEEE Vehicular Technology Conference (VTC-Spring'09)*, Barcelona, Spain, Apr. 2009, pp. 1–5.

[5] K. B. Fredj, L. Musavian, and S. Aissa, "Closed-form expressions for the capacity of spectrum-sharing constrained relaying systems," in *Proc. Intnl. Conf. on Telecom. (ICT'10)*, Doha, Qatar, Apr. 2010.

[6] J. Lee, H. Wang, J. G. Andrews, and D. Hong, "Outage probability of cognitive relay networks with interference constraints," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 390–395, Feb. 2011.

[7] V. Asghari and S. Aissa, "Cooperative relay communication performance under spectrum-sharing resource requirements," in *Proc. IEEE Intnl. Conf. on Commun. (ICC'10)*, Cape Town, South Africa, May 2010, pp. 1–6.

[8] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity. Part I. System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[9] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[10] Y. W. Hong, W. J. Huang, F. H. Chiu, and C. C. J. Kuo, "Cooperative communications in resource-constrained wireless networks," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 47–57, May 2007.

[11] D. B. da Costa and S. Aissa, "Cooperative dual-hop relaying systems with beamforming over Nakagami-m fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3950–3954, Aug. 2009.

[12] ——, "End-to-end performance of dual-hop semi-blind relaying systems with partial relay selection," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4306–4315, Aug. 2009.

[13] S. S. Ikki and S. Aissa, "Performance analysis of dual-hop relaying systems in the presence of co-channel interference," in *Proc. IEEE GLOBECOM'10*, Miami, FL, Dec. 2010, pp. 1–5.

[14] E. A. Gharavol, Y.-C. Liang, and K. Mouthaan, "Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information," *IEEE Trans. Veh. Technol.*, vol. 59, no. 6, pp. 2852–2860, Jul. 2010.

[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[16] D. Bertsimas and M. Sim, "Tractable approximations to robust conic optimization problems," *Math. Program.*, vol. 107, pp. 5–36, Jun. 2006.

[17] A. Ben-Tal and A. Nemirovsky, "Robust convex optimization," *Mathematics of Operations Research*, vol. 23, no. 4, pp. 769–805, Nov. 1998.

[18] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.