Tone Interference Estimation for OFDM Systems Using a Frequency Domain DFT

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***Abstract*—This paper investigates the problem of reliable tone interference estimation from noisy observations. Performing the DFT of time domain observations is a commonly used tone estimation technique. However, the use of one DFT block of frequency domain samples may not yield reliable estimation of tone interference. To solve this problem, we propose to combine multiple DFT blocks of frequency domain observations which are naturally provided in OFDM systems, improving the reliability of tone interference estimation. To this end, a frequency domain DFT is used to process multiple blocks of observations. It is shown that our estimator can achieve the performance close to the Cramér-Rao bound and the conventional DFT-based estimation with much higher complexity.**

I. INTRODUCTION

Tone interference of minimal power can degrade the performance of orthogonal frequency-division multiplexing (OFDM) systems since the data are transmitted using subcarriers which are aligned in frequency. Reliable estimation of the tone interference is important because the performance of interference mitigation depends on the quality of estimation. To cancel the tone interference, we may want to use a notch filter whose bandwidth is as narrow as possible to minimize the distortion of an OFDM signal. This bandwidth can be reduced only if the tone frequency is accurately estimated. Moreover, the notch filter only requires the knowledge of the tone frequency regardless of the tone power. For this reason, we focus on reliable frequency estimation of tone interference for OFDM systems.

The problem of tone estimation has been addressed by performing the discrete Fourier transform (DFT) on a time domain signal [1]–[4] or using average autocorrelation [5]–[8]. DFT-based estimators have shown good performance for considerably noisy observations. Thus, one of the most common estimation schemes is to perform the DFT first and then interpolate the peak and its neighbors. Most of studies on DFT-based estimators have focused on improving the interpolation performance providing a fine frequency estimation [2]–[4]. However, this improvement on the interpolation performance has been marginal because the gap between the performance of existing DFT-based estimators and the Cramér-Rao (CR) bound is already small. The performance of existing DFT-based methods using only one block of DFT output is inevitably limited by the CR bound of this DFT size. On the contrary, autocorrelation-based estimators have been studied because the computational complexity of DFT-based estimators is considered to be significant even with the use of the fast Fourier transform (FFT). However, autocorrelation-based estimators have either limited estimation ranges or performance losses in the region of moderate noise power [9]. In [7], [8], authors have extended the estimation ranges of autocorrelation-based estimators while maintaining their performance but these attempts have ended up estimators that are computationally more complex than DFT-based estimators.

In OFDM systems, the tone estimation performance needs to be improved mostly when the power of the tone interference is comparable to that of the OFDM signal in frequency domain. The challenge lies in estimating weak tone interference under the presence of the strong OFDM signal. In this case, the use of one OFDM symbol may not yield reliable estimation of tone interference even if the estimator can achieve the CR bound. Only way to improve the estimation performance already close to the CR bound is to increase the sample size. Thus, we propose to *combine multiple blocks of observations already in frequency domain* (i.e., multiple OFDM symbols) to improve the quality of estimation, which effectively lowers the CR bound by increasing the sample size. Moreover, our scheme can achieve the performance close to the CR bound that is already reduced.

Our proposed method of coarse frequency estimation and interpolation is significantly different from those of conventional tone estimators because our estimator processes *frequency domain observations* instead of *time domain observations*. To the best of our knowledge, the problem of processing *multiple blocks* of frequency domain observations for tone estimation has not been studied yet. To process a series of frequency domain observations, we propose to perform a *frequency domain DFT*[1] and use a *two-dimensional (2D) interpolator*. Our scheme is most suitable for OFDM systems, where multiple blocks of frequency domain observations are provided naturally. However, we can apply our algorithm to the system that only provides a time domain signal if an additional time domain DFT[2] of small size is performed to produce frequency domain samples. This still reduces the computational complexity substantially while saving a huge processing delay for streaming data compared with conventional DFT-based

[1] A DFT is called a frequency domain DFT if its input is in frequency domain.

[2] A DFT is called a time domain DFT if its input is in time domain.

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estimation algorithms.

It is commonly misunderstood that the resolution of tone frequency estimation will be restricted by the resolution of time domain DFT (i.e., OFDM subcarrier spacing). However, this is not true if either a good interpolator is used or multiple blocks of frequency domain observations are utilized. The proposed tone estimation algorithm combines both ingredients together providing the resolution only limited by the CR bound, e.g., the resolution less than 1Hz can be obtained even with the OFDM subcarrier spacing of 15kHz, *regardless of the tone location with respect to subcarrier locations*.

The remainder of this paper is organized as follows. In Section II, we present our time domain model. In Section III, a tone frequency estimation algorithm is designed to utilize multiple blocks of frequency domain observations. In Section IV, simulation results show that the proposed algorithm can achieve the performance close to the CR bound. Finally, we provide our conclusions in Section V.

II. The Time Domain Model

We assume a pure complex tone, which can be modeled by

$$x_n = a\exp(j(\omega_T n + \theta)) \tag{1}$$

where $a > 0$, ω_T, and θ are its unknown magnitude, frequency, and phase, respectively. This tone signal is assumed to be stationary and deterministic meaning that its parameters, i.e., a, ω_T, and θ, do not change over time. Let us assume that we have L noisy observations of this signal at time $n = 0, 1, ..., L-1$,

$$y_n = x_n + w_n, \tag{2}$$

where w_n is modeled as an additive zero-mean circularly symmetric independent and identically distributed (i.i.d.) complex Gaussian random vector. In our system model, x_n represents tone interference while w_n includes all other signals except tone interference. In the case of OFDM systems, w_n *becomes the OFDM signal plus background noise*. It will be explained later in Section IV why the OFDM signal plus background noise can be treated as i.i.d. Gaussian.

In [1], it is shown that the maximum likelihood (ML) frequency estimate $\hat{\omega}_T$ is given by

$$\hat{\omega}_T = \arg\max_{\omega} |Y(\omega)|, \tag{3}$$

where $Y(\omega)$ is the normalized Fourier transform of y_n

$$Y(\omega) = \frac{1}{\sqrt{L}}\sum_{n=0}^{L-1} y_n \exp(-j\omega n). \tag{4}$$

The CR bound of frequency estimation $\hat{\omega}_T$ is given in [1],

$$\mathrm{Var}(\hat{\omega_T}) \geq \frac{6\sigma^2}{a^2 L(L^2-1)}, \tag{5}$$

where $\sigma^2 = E[|w_n|^2]$.

III. Frequency Estimation Using Frequency Domain Samples

To take advantage of a large number of samples (i.e., large L), we have to use a L-point DFT. However, any implementation of the large size DFT requires the system to wait for L samples to be collected even before processing the input, not to mention its computational complexity. This is not as an option for many cases. Moreover, the size of the available DFT can be much smaller than the sample size. To solve this, we propose to combine K blocks of N-point DFT (i.e., N-point FFT already provided by the OFDM system) results for $L = KN$.

A. Frequency Domain DFT Processing

Assume that we have K blocks of time domain observations $y_{kN+n} = x_{kN+n} + w_{kN+n}$, $k = 0, ..., K-1$, $n = 0, ..., N-1$, where k is the block index. The frequency domain signals $Y_{n,k}$, $X_{n,k}$, and $W_{n,k}$ denote the kth block outputs corresponding to the N-point DFT inputs y_{kN+n}, x_{kN+n}, and w_{kN+n}, respectively, for the DFT bin index $n = 0, ..., N-1$. For OFDM systems, $Y_{n,k}$ corresponds to the kth OFDM symbol. Note that x_{kN+n} and $X_{n,k}$ can represent time and frequency domain additive *tone interference* of our interest, respectively. We propose to perform the K-point DFT for the nth frequency domain output of the N-point DFT over K blocks as

$$\begin{aligned}
\widetilde{Y}_{n,k} &= \frac{1}{\sqrt{K}}\sum_{k'=0}^{K-1} \exp\left(-j\frac{2\pi k'}{K}k\right) \\
&\quad \times \frac{1}{\sqrt{N}}\sum_{n'=0}^{N-1} \exp\left(-j\frac{2\pi n'}{N}n\right) y_{k'N+n'} \qquad (6)\\
&= \frac{1}{\sqrt{K}}\sum_{k'=0}^{K-1} Y_{n,k'} \exp\left(-j\frac{2\pi k'}{K}k\right) \\
&= \frac{1}{\sqrt{K}}\sum_{k'=0}^{K-1} (X_{n,k'} + W_{n,k'}) \exp\left(-j\frac{2\pi k'}{K}k\right) \\
&= \widetilde{X}_{n,k} + \widetilde{W}_{n,k}. \qquad (7)
\end{aligned}$$

The N-point DFT included in the OFDM system is a *time domain* DFT while the additional K-point DFT is a *frequency domain* DFT. In (6), it can be seen that $\widetilde{Y}_{n,k}$ is the 2D DFT of y_{kN+n}. Thus, the time domain DFT plus the frequency domain DFT effectively creates the 2D DFT. The properties of 2D DFT are well known for 2D signal [10]. However, these properties cannot be directly applied here because the 2D DFT is used to process one-dimensional (1D) tone signal in our setup instead of 2D signal as in most of literatures. To the best of our knowledge, the 2D DFT of 1D tone signal has not been characterized yet.

Note that this 2D DFT preserve the same noise statistics $\widetilde{W}_{n,k}$ as w_{kN+n}. Note that it is not required to have multiple frequency domain DFTs since a single K-point DFT can be used sequentially with the memory of size KN.

Investigating the relationship between $X_{n,k}$ and $X_{n,k-1}$, the following property is discovered.

Property 1: We have

$$X_{n,k}/X_{n,k-1} = \exp(j\omega_T N), \quad (8)$$

for any $k = 1, ..., K-1$, $n = 0, ..., N-1$. $\square$

The phase of the nth frequency bin output $X_{n,k}$ rotates as k increases. The amount of this phase rotation $\exp(j\omega_T N)$ is fixed and does not change over time k as long as parameters of the tone interference remain unchanged. Moreover, in (8), it can be seen that the phase rotation $\exp(j\omega_T N)$ does not change over the frequency bin index n. Consequently, $X_{n,k}$ behaves like a tone signal with a given frequency as we observe $X_{n,k}$ over time and this frequency does not change over n. This is the rationale behind performing the frequency domain K-point DFT. Moreover, the tone energy is mostly concentrated at $X_{n,k}$ such that

$$\omega_T \approx 2\pi \frac{n}{N}. \quad (9)$$

Since the noisy observation $Y_{n,k}$ is given to the estimator instead of $X_{n,k}$, it is advantageous to process $Y_{n,k}$ around n satisfying (9) where the tone power is strongest with respect to the constant noise power of $W_{n,k}$.

Now, our next questions would be what this phase rotation means and how this information can be used to obtain the final frequency estimation. To answer these questions, let us split ω_T into a coarse frequency ($\omega_C = 2\pi\frac{n}{N}$) and a fine frequency ($\omega_F$) as follows.

$$\omega_T = \omega_C + \omega_F. \quad (10)$$

Then, (8) is given by

$$X_{n,k} = X_{n,0}\exp(jkN\omega_F), \quad (11)$$

since $N\omega_T$ and $N\omega_F$ are equal in mod 2π, i.e., $N\omega_T = N\omega_F$ (mod 2π). Even if we can estimate this phase rotation $N\omega_T$, it does not containing any information regarding the coarse frequency ω_C. However, the phase rotation $N\omega_T$ can be used to estimate the fine frequency ω_F with the limited range $|\omega_F| \leq \frac{\pi}{N}$.

Moreover, it can be shown that

$$X_{n,0} = ae^{j\theta}\frac{1}{\sqrt{N}}\exp(j(N-1)\omega_d/2)\frac{\sin(N\omega_d/2)}{\sin(\omega_d/2)}, \quad (12)$$

where $\omega_d \triangleq \omega_T - 2\pi\frac{n}{N}$ and $|X_{n,0}|$ is maximized at $\omega_d = 0$. We have the following 2D structure of $\widetilde{X}_{n,k}$.

$$\begin{aligned} \widetilde{X}_{n,k} &= X_{n,0}\frac{1}{\sqrt{K}}\sum_{k'=0}^{K-1}\exp(jk'N\omega_F)\exp\left(-j\frac{2\pi k'}{K}k\right) \\ &= ae^{j\theta}\frac{1}{\sqrt{N}}\exp(j(N-1)\omega_d/2)\frac{\sin(N\omega_d/2)}{\sin(\omega_d/2)} \\ &\times \frac{1}{\sqrt{K}}\exp(j(K-1)\omega'_d/2)\frac{\sin(K\omega'_d/2)}{\sin(\omega'_d/2)} \end{aligned} \quad (13)$$

where $\omega'_d \triangleq N\omega_F - 2\pi\frac{k}{K}$.

We can see that $|\widetilde{X}_{n,k}|$ is maximized around $\omega_d \approx 0$ and $\omega'_d \approx 0$. Thus, we have $\omega_C \approx \omega_C + \omega_F = \omega_T \approx 2\pi\frac{n}{N}$ and $\omega_F \approx 2\pi\frac{k}{KN}$ (mod $2\pi/N$) for n and k maximizing $|\widetilde{X}_{n,k}|$.

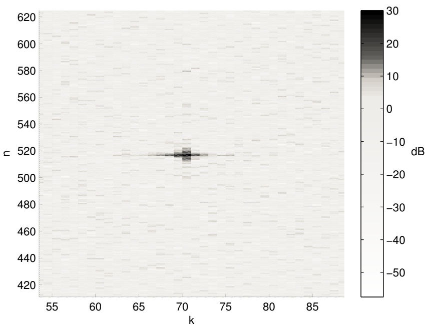

Fig. 1. An exemplary $|\widetilde{Y}_{n,k}|$ in (7).

Since $\widetilde{Y}_{n,k}$ is just a noisy observation of $\widetilde{X}_{n,k}$, we can devise the following naive algorithm. Find

$$(\hat{n}, \hat{k}) = \arg\max_{n,k} |\widetilde{Y}_{n,k}| \quad (14)$$

and estimate

$$\hat{\omega}_T = \begin{cases} 2\pi\frac{\hat{n}}{N} + 2\pi\frac{\hat{k}}{KN}, & \text{if } \hat{k} \leq \text{round}(K/2), \\ 2\pi\frac{\hat{n}}{N} + 2\pi\frac{\hat{k}-K}{KN}, & \text{if } \hat{k} \geq \text{round}(K/2), \end{cases} \quad (15)$$

assuming that $\hat{\omega}_C = 2\pi\frac{\hat{n}}{N}$ is always accurate and can limit the residual coarse frequency error $|\omega_F| \leq \frac{\pi}{N}$. However, this is not always the case. The estimate of $\hat{n}$ becomes inaccurate as $|\omega_F| \to \frac{\pi}{N}$, and thus it is likely that the frequency estimation error of $2\pi/N$ occurs while we want the resolution of $2\pi/KN$ or less.

We propose the following improvement using interpolation techniques. The idea behind this approach is that since $\widetilde{X}_{n,k}$ has the most of tone energy around $(\hat{n}, \hat{k})$, we can combine $\widetilde{Y}_{\hat{n}-1,\hat{k}}$, $\widetilde{Y}_{\hat{n},\hat{k}}$, and $\widetilde{Y}_{\hat{n}+1,\hat{k}}$ to have a rough estimate of $\hat{\omega}_F$ with respect to $2\pi\frac{\hat{n}}{N}$. Moreover, the triplet $(X_{\hat{n}-1,k}, X_{\hat{n},k}, X_{\hat{n}+1,k})$ has the same relative magnitude and phase for all k. Thus, we can apply any interpolation techniques in [2] or other literatures for any k, where the use of relative phase information is the key ingredient for accurate interpolation. However, we will obtain the most reliable estimate if we interpolate the least noisy samples of $k = \hat{k}$. We can also determine whether $\omega_T > 2\pi\frac{\hat{n}}{N}$ or $\omega_T < 2\pi\frac{\hat{n}}{N}$ even when $|\omega_F| \approx \frac{\pi}{KN}$. We will explain the detail algorithm in the following subsection.

Fig. 1 shows an exemplary $|\widetilde{Y}_{n,k}|$ taken from the simulation in Section IV. We can confirm that the tone energy is not only mostly concentrated around $(\hat{n}, \hat{k})$ but it also spreads vertically and horizontally.

B. Estimation with Cyclic Prefix

Consider the case that we have a small number of samples ($N_{CP} < N$) discarded before the N-point DFT is performed while we have the same number of samples $L = KN$ left. This can be a cyclic prefix in OFDM systems. Now, the only difference in this case is that the phase rotation in (11) has changed as follows.

$$X_{n,k} = X_{n,0}\exp(jk(N + N_{CP})\omega_F), \quad (16)$$

where

$$\omega_F = \omega_T - 2\pi \frac{n}{N + N_{CP}}. \tag{17}$$

ω_d' in (13) is now given by

$$\omega_d' = (N + N_{CP})\omega_F - 2\pi \frac{k}{K}. \tag{18}$$

Once we have $\hat{k}$,

$$\hat{\omega}_F = 2\pi \frac{\hat{k}}{K(N + N_{CP})} \left(\bmod \frac{2\pi}{N + N_{CP}} \right). \tag{19}$$

Thus, the range of this acquisition is now $2\pi/(N + N_{CP})$ reduced from $2\pi/N$ from the result of previous subsection. However, we can overcome this limitation by using interpolation techniques as explained in the previous subsection, which may need to be employed anyway even if $N_{CP} = 0$. Now, we propose the following algorithm.

DFT-Based Tone Frequency Estimation Algorithm:

1) Compute $\widetilde{Y}_{n,k}$ in (7).
2) Find $(\hat{n}, \hat{k}) = \arg\max_{n,k} |\widetilde{Y}_{n,k}|$.
3) Interpolate ($\widetilde{Y}_{\hat{n}-1,\hat{k}}$, $\widetilde{Y}_{\hat{n},\hat{k}}$, $\widetilde{Y}_{\hat{n}+1,\hat{k}}$) to obtain a coarse estimate $\hat{\omega}_{F,1}$ using the following interpolation technique [2].
   - Compute $R[m] = \mathrm{Re}\left(\widetilde{Y}_{\hat{n}+m,\hat{k}} \widetilde{Y}^*_{\hat{n},\hat{k}}\right)$.
   - Compute $\gamma = \frac{R[-1]-R[1]}{2R[0]+R[-1]+R[1]}$.
   - Obtain $\hat{\omega}_{F,1} = \frac{2\pi}{N} \frac{\sqrt{1+8\gamma^2}-1}{4\gamma}$.
4) Estimate $\hat{\omega}_{T,1} = 2\pi \frac{\hat{n}}{N} + \hat{\omega}_{F,1}$.
5) Compute $\hat{n}_{CP} = \text{floor}\left(\hat{\omega}_{T,1} \frac{N+N_{CP}}{2\pi}\right)$ and $\hat{\omega}_{F,CP,1} = \hat{\omega}_{T,1} - 2\pi \frac{\hat{n}_{CP}}{N+N_{CP}}$.
6) Find $\hat{\omega}_{F,CP,2} = \arg\min_{\omega \in B} |\omega - \hat{\omega}_{F,CP,1}|$, where $B = \left\{ 2\pi \frac{\hat{k}-K}{K(N+N_{CP})}, 2\pi \frac{\hat{k}}{K(N+N_{CP})}, 2\pi \frac{\hat{k}+K}{K(N+N_{CP})} \right\}$.
7) Substitute $\hat{\omega}_{F,CP,2}$ for $\hat{\omega}_{F,CP,1}$. The estimate is given by $\hat{\omega}_{T,2} = 2\pi \frac{\hat{n}_{CP}}{N+N_{CP}} + \hat{\omega}_{F,CP,2}$
8) Interpolate ($\widetilde{Y}_{\hat{n},\hat{k}-1}$, $\widetilde{Y}_{\hat{n},\hat{k}}$, $\widetilde{Y}_{\hat{n},\hat{k}+1}$) around $\hat{k}$.
   - Compute $R'[m] = \mathrm{Re}\left(\widetilde{Y}_{\hat{n},\hat{k}+m} \widetilde{Y}^*_{\hat{n},\hat{k}}\right)$.
   - Compute $\gamma' = \frac{R'[-1]-R'[1]}{2R'[0]+R'[-1]+R'[1]}$.
   - Obtain $\hat{\omega}_{F,CP,3} = \frac{2\pi}{K(N+N_{CP})} \frac{\sqrt{1+8\gamma'^2}-1}{4\gamma'}$.
9) Estimate $\hat{\omega}_{T,3} = \hat{\omega}_{T,2} + \hat{\omega}_{F,CP,3}$.

In Step 2, the algorithm finds the peak among 2D DFT output from Step 1. In Step 3, the algorithm vertically interpolates three samples around the peak along n. The interpolated estimate is produced in Step 4. In Steps 5-7, using the improved frequency estimation from Step 4, the coarse estimation error of $\hat{n}$ and the ambiguity of combining $\hat{\omega}_C = 2\pi \frac{\hat{n}}{N}$ and $\hat{\omega}_F$ in (19) are removed, which are the main issues with the naive estimation as explained above. Now, the root mean square (RMS) resolution of proposed algorithm is only limited by the 2D DFT RMS resolution given by

$$\frac{1}{2\sqrt{3}} \frac{2\pi}{K(N + N_{CP})}. \tag{20}$$

Thus, the resolution of the algorithm improves by the factor of $1/K$ compared with the case when only $\hat{n}$ is used, e.g., $\hat{\omega} = 2\pi \frac{\hat{n}}{N}$. If the tone signal is strong enough, the worst case estimation error will be given by the half of OFDM subcarrier spacing divided by K. However, our algorithm can further refine the estimate in Steps 8-9 because the information horizontally leaked along k around the peak are not utilized yet. The final estimation in Step 9 has the RMS error close to the CR bound.

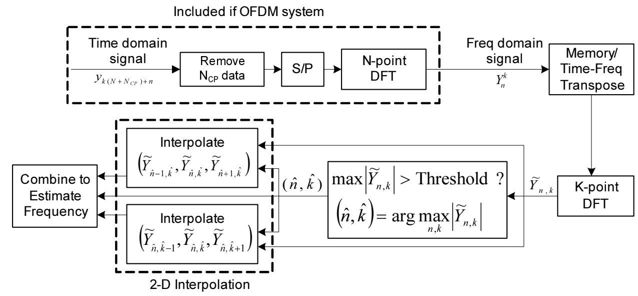

Fig. 2. Tone estimator block diagram.

TABLE I
SIMULATION PARAMETERS.

Sample size, L=NK	286,720
OFDM FFT size, N	2,048
Number of occupied subcarriers	1,200
Frequency domain DFT size, K	140
Subcarrier spacing	15 kHz
Carrier to noise ratio (CNR)	20 dB
Modulation	16-QAM
Channel	Additive white Gaussian noise

This algorithm can be used no matter whether $N_{CP} = 0$ or $N_{CP} > 0$. Note that the size of candidate set B should be adjusted according to N_{CP} length while the suggested candidate set will cover most cases with high probability for $N_{CP} \ll N$. Fig. 2 shows a block diagram of the proposed tone frequency estimator. The first part of blocks providing the frequency domain signal $Y_{n,k}$ is already included in OFDM systems. In this case, our estimator only needs to deploy one additional frequency domain K-point DFT and this K-point DFT can run sequentially to process frequency domain data in memory.

While we only focus on estimation of a single tone, this technique can be easily applied to multiple tone estimation if multiple tones are separated enough in frequency.

IV. SIMULATION RESULTS

We run computer simulation using parameters in Table I. Note that the system bandwidth is 30.72 MHz while the occupied bandwidth of 18 MHz (1,200 subcarriers) is always fully loaded. The OFDM subcarrier spacing is 15kHz following Long Term Evolution (LTE) standard [11]. The tone location is randomly generated within this occupied bandwidth. Note that the relative tone location with respect to OFDM subcarrier location is also random, and thus the probability that tone is placed exactly on the one of subcarriers is zero. For the OFDM systems where the subcarrier spacing is fixed regardless of the system bandwidth like LTE, the estimation performance only marginally depends on the system bandwidth. The frequency

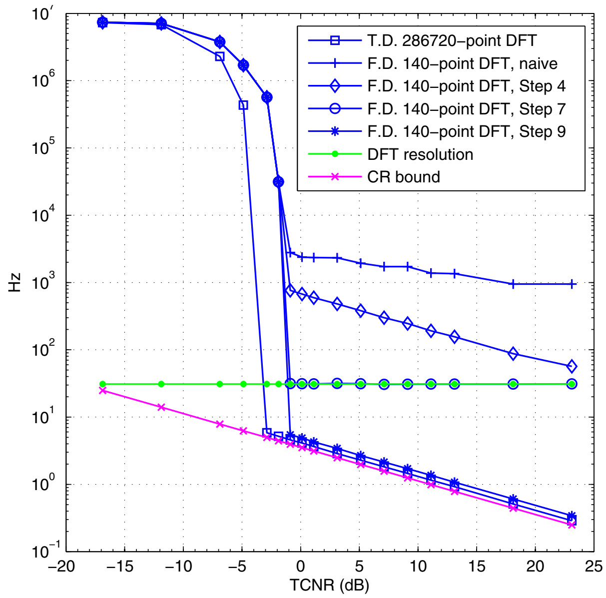

Fig. 3. RMS frequency error when $N_{CP} = 0$.

domain DFT size, K is more crucial factor determining the estimation performance beyond the subcarrier spacing. While our choice of $K = 140$ corresponds to one radio frame in LTE systems, this number in general, can be chosen to balance estimation delay and reliability.

Note that our performance measure of estimation is the RMS error. This can be compare with the CR bound in (5), which is given under the i.i.d. Gaussian noise assumption. In this simulation, the zero mean i.i.d. 16-QAM (quadrature amplitude modulation) OFDM signal is transformed into the approximate i.i.d. Gaussian signal, when included in w_{kN+n} and $\widetilde{W}_{n,k}$, by the central limit theorem. Since the transformed subcarrier plus the background noise, w_{kN+n} is approximately i.i.d. Gaussian under the assumption of their mutual independence, the CR bounds in (5) can be considered as the fundamental performance limit in our simulation setup. While CNR is fixed to 20dB in our setup, only the total power of OFDM signal plus noise determines the statistics regardless their ratio.

Fig. 3 shows the RMS frequency error of our proposed algorithm and the time domain DFT-based algorithm, the 2D DFT resolution of 28.8Hz from (20), and the CR bound in (5) when there is no cyclic prefix. Note that for $N = 2048$, the tone power can increase up to 33 dB when it interferes with the closest subcarrier. For the subcarrier overlaid with the tone interference, the tone to OFDM signal plus noise power ratio is N times higher at this subcarrier than the raw tone to OFDM signal plus noise power ratio in time domain. We name this ratio as tone to carrier plus noise ratio (TCNR). The simulation results are shown for TCNR from -17 dB to 23 dB. However, this is translated to the raw tone to OFDM signal power ratio in time domain from -50 dB to -10 dB. The tone power of our interest is very small, which is the reason why we need a huge number of samples $L = 286,720$. In this figure, we compare the estimation performance of the proposed algorithm using a frequency domain 140-point DFT sequentially, and the conventional time domain estimator using a L-point DFT ($L = 286,720$) and an interpolator proposed in [2]. Both process the same amount of samples $L = 286,720$. In Fig. 3, it is shown that the performance gap between these two estimators is small even with huge difference in computational complexity because our proposed estimator is designed to take advantage of frequency domain observations already given by OFDM systems. Both estimators can achieve to the CR bound with small gaps at moderate and high TCNR. It can be seen that the naive estimation in (15) has a high error floor while the proposed frequency estimation algorithm in Subsection III-B improves the estimation performance as we add more steps. Note that similar results are obtained for $N_{CP} > 0$.

V. Conclusion

We proposed a tone frequency estimation algorithm using a frequency domain DFT. While conventional DFT-based estimation is limited by the size of the DFT, our algorithm can combine a large number of samples only using two DFTs of much smaller sizes. In OFDM systems, our proposed estimator can utilize the OFDM signal already in frequency domain. This approach effectively lowers the CR bound, which is the fundamental performance limit of estimators. As a result, the proposed scheme can reliably estimate the tone interference even when its power is small in time domain. Simulation results showed that the performance of our estimator can approach the CR bound even at moderate TCNR.

References

[1] D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. 20, no. 5, pp. 591 – 598, Sep. 1974.

[2] M. Macleod, "Fast nearly ML estimation of the parameters of real or complex single tones or resolved multiple tones," *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 141 –148, Jan. 1998.

[3] E. Aboutanios and B. Mulgrew, "Iterative frequency estimation by interpolation on fourier coefficients," *IEEE Trans. Signal Process.*, vol. 53, no. 4, pp. 1237 – 1242, Apr. 2005.

[4] E. Jacobsen and P. Kootsookos, "Fast, accurate frequency estimators [DSP tips tricks]," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 123 –125, May 2007.

[5] G. Lank, I. Reed, and G. Pollon, "A semicoherent detection and Doppler estimation statistic," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-9, no. 2, pp. 151 –165, Mar. 1973.

[6] S. Kay, "A fast and accurate single frequency estimator," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 12, pp. 1987 –1990, Dec. 1989.

[7] S. Leung, Y. Xiong, and W. Lau, "Modified Kay's method with improved frequency estimation," *Electronics Letters*, vol. 36, no. 10, pp. 918 –920, May 2000.

[8] A. Awoseyila, C. Kasparis, and B. Evans, "Improved single frequency estimation with wide acquisition range," *Electronics Letters*, vol. 44, no. 3, pp. 245 –247, Jan. 2008.

[9] C. Yang, G. Wei, and F. Chen, "An estimation-range extended autocorrelation-based frequency estimator," *EURASIP Journal on Advances in Signal Processing*, 2009.

[10] H. Krishna, *Digital Signal Processing Algorithms: Number Theory, Convolution, Fast Fourier Transforms, and Applications*, 1st ed. CRC Press, 1998.

[11] D. Bai, C. Park, J. Lee, H. Nguyen, J. Singh, A. Gupta, Z. Pi, T. Kim, C. Lim, M. Kim, and I. Kang, "LTE-Advanced modem design: challenges and perspectives," *IEEE Commun. Mag.*, Feb. 2012.