Asymptotic Outage Probability for Amplify-and-Forward CDMA Systems over Nakagami-*m* Fading Channels

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***Abstract*—In this paper, we address the performance of cooperative code-division multiple-access (CDMA) systems using amplify-and-forward (AF) relaying over independent non-identical (i.ni.d) Nakagami-*m* fading channels. The outage probability of the system is investigated using the moment generating function (MGF) of the total signal-to-noise ratio (SNR) at the base station. Since it is complicated to derive a closed form expression of the outage probability, we derive an approximation for the probability density function (PDF) of the total SNR which in turn enables us to derive the asymptomatic outage probability for any value of the fading parameter m. Simulation results are presented to assess the accuracy of our analytical results. Furthermore, using the derived outage probability, we investigate the diversity advantage of the cooperative system under different system settings.**

***Index Terms*—Code-division multiple-access (CDMA), cooperative diversity, Nakagami-m fading, outage probability.**

I. Introduction

In wireless networks, cooperative communications offer a great capability in implementing spatial diversity. It improves the performance over fading channels without the need for multiple antennas at the transmitter and receiver. It has been shown that user cooperative diversity with multi-relay, a new form of space diversity, has the potential to enhance the channel capacity and the reliability of wireless communication systems [1], [2]. Within the research of user cooperative diversity, amplify-and-forward (AF) is a simple cooperative scheme in which the relay (i.e., cooperative user) transmits an amplified version of the received partner's signal to the base station [3].

The recent multiuser systems such as the multi-carrier code-division multiple-access (MC-CDMA) and the optical CDMA are implemented based on the direct-sequence CDMA (DS-CDMA) approach. Cooperative DS-CDMA diversity was commonly investigated using non-orthogonal spreading codes for synchronous networks over flat Rayleigh fading channels [4]. However, CDMA systems suffer from a major problem due to the multiple-access interference (MAI) effect arising from the non-orthogonality between the user's spreading codes. To eliminate the effect of MAI, multiuser detectors such as minimum-mean-square error (MMSE) and decorrelator detectors have been proposed [4].

The performance of multiuser AF relay networks has been investigated in [5] where the authors derived an approximation for the outage probability over Rayleigh fading channels. In [6], a performance analysis of incremental-relaying cooperative-diversity networks over Rayleigh fading channels is presented where the authors introduced a complete analytical method to obtain closed-form expressions for the error rate, outage probability and the average achievable rate using decode-and-forward (DF) and AF over independent non-identical (i.ni.d.) Rayleigh fading channels. In [7], a performance analysis of time-division multiple-access (TDMA) relay protocol over independent and identically distributed (i.i.d.) Nakagami-*m* fading channels is studied. The authors, in their work, focused on Alamouti-coded system with two-stage protocols and fixed gain AF. The joint iterative power allocation and the interference cancellation for DS-CDMA systems using AF relaying was presented in [8]. In their work, the authors determined the required power level across the relays. All the above works, however, did not consider the multiuser scenario as in cooperative CDMA systems in an AF relaying system over i.ni.d. Nakagami-*m* fading channel.

In CDMA systems due to the non-orthogonality between spreading codes assigned to different users and the asynchronous transmission, the MAI and inter-symbol interference (ISI) will affect the performance of the cooperative system. To solve this problem, we employ a decorrelator detector (DD) at both the relay and the base station sides. In this paper, we consider cooperative CDMA systems employing AF relaying over i.ni.d. Nakagami-*m* fading channels as a multiuser scenario. To the best of our knowledge, no results have been reported on the outage analysis of cooperative AF relaying in CDMA systems when considering Nakagami-*m* fading model. In that, we introduce an asymptotic outage probability analysis at high signal-to-noise ratio (SNR) by using an approximate expression for the probability density function (PDF) of the total SNR at the base station. Our method is simple yet it provides accurate results for different relay scenarios.

The rest of the paper is organized as follow. The system model for cooperative DS-CDMA over Nakagami-*m* fading channel is presented in section II. In section III, we analyze the outage probability for multi-relay cooperation. Section IV provides some simulation and numerical results and finally, conclusions are given in section VI.

II. System Model

In our model, we consider a set of cooperating users $s \in \{1, ..., K\}$, $\ell \in \{1, ..., L\}$ represents the number of relays (cooperating users) transmitting to the base station, b, where $L \in \{1, ..., K-1\}$. All channels are modeled as i.ni.d. Nakagami-m fading ones. In this model, we consider an uplink with K users for an asynchronous DS-CDMA transmission with non-orthogonal spreading codes with spreading gain N, where each user in the model can have L relays. Each user is assumed to be equipped with single antenna and half-duplex transmission. Subsequently, cooperation involves two phases; in the first phase, each user transmits its own modulated data to the base station and to L relays. During this phase, the received signal at the base station can be written as [9]

$$r_b^I(t) = \sum_{i=0}^{f-1} \sum_{s=1}^{K} x_s(i) C_s(t-\tau_s - iT_b) h_{sb} + n_b^I(t) \quad (1)$$

where f is the frame length, $x_s(i) \in \{1, -1\}$ is the i^{th} data symbol of user s, $C_s(t)$ is the spreading code of user k with spreading gain $N = \frac{T_b}{T_c}$, T_b is the bit period, T_c is the chip period, and τ_k is the random transmit delay of the k^{th} user which is assumed to be uniformly distributed along the symbol period. $n_b^I(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2 = N_o$. In (1), h_{sb} denotes complex channel coefficient between user s and the base station, b.

The received signal at the relay r_ℓ can be written as

$$r_{r_\ell}(t) = \sum_{i=0}^{f-1} \sum_{s=1, s\neq r}^{K} x_s(i) C_s(t-\tau_s - iT_b) h_{sr_\ell} + n_{r_\ell}(t) \quad (2)$$

where h_{sr_ℓ} is the channel coefficient between user s and the relay r_ℓ, $n_{r_\ell}(t)$ is an AWGN with zero mean and variance $\sigma_n^2 = N_o$.

In the second phase, each cooperating user transmits the amplified version of the received signal to the base station, which is expressed as

$$r_b^{II}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^{K} \sum_{\ell=1}^{L} r_{r_\ell}(t - D_{s,\ell} - \tau_\ell) h_{r_\ell b} \cdot \beta + n_b^{II}(t) \quad (3)$$

where r_ℓ is the ℓ^{th} relay cooperating with user s, $n_b^{II}(t)$ is an AWGN with zero mean and variance $\sigma_n^2 = N_o$, and $D_{s,\ell}$ is the transmission delay during the second transmission period. $h_{r_\ell b}$ is the channel coefficient between user r_ℓ and the base station, modeled as a Nakagami$-m$ random variable (RV), β is the amplification factor for the AF scheme at the relay node, expressed as [3]

$$\beta = \sqrt{\frac{E_s}{E_s |h_{sr_\ell}|^2 + N_o}}, \quad (4)$$

where E_s is the transmitted signal energy and fixed gain AF is assumed.

The independent fading channel coefficients between users and base station are as follows, source-relay h_{sr_ℓ}, source-base station h_{sb}, and relay-base station $h_{r_\ell b}$, all are modeled as independent non-identical Nakagami-m random variables. The corresponding instantaneous SNRs are represented as $\gamma_{sr_\ell} = |h_{sr_\ell}|^2 \frac{E_s}{N_o}$, $\gamma_{sb} = |h_{sb}|^2 \frac{E_s}{N_o}$, $\gamma_{r_\ell b} = |h_{r_\ell b}|^2 \frac{E_s}{N_o}$ where $|h_{sr_\ell}|^2$, $|h_{sb}|^2$, and $|h_{r_\ell b}|^2$ are gamma distributed. Their corresponding PDF is given by

$$p_{\gamma_{ij}}(\gamma) = \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} \gamma^{m_{ij}-1} \exp\left(-\gamma B_{ij}\right) \quad (5)$$

where $m_{ij} > 0.5$ is the Nakagami-m fading parameter, $\Gamma(.)$ is the gamma function [10, eq. (8.310,1)], $B_{ij} = \frac{m_{ij}}{\bar{\gamma_{ij}}}$ with $\bar{\gamma_{ij}} = E\langle |h_{ij}|^2 \rangle E_s/N_o$ is the average SNR and $E\langle . \rangle$ denote expectation. The cumulative distribution function (CDF) of γ_{ij} is given by

$$F_{\gamma_{ij}}(\gamma) = \frac{\gamma\left(m_{ij}, \gamma B_{ij}\right)}{\Gamma(m_{ij})} \quad (6)$$

where $\gamma(.,.)$ is the lower incomplete gamma function [10, eq. (8.350,1)].

The output of the bank of matched filters at the base station receiver and relays, and by dropping the time index from the module (1) and (3), can be defined in vector form for the first phase I as

$$\mathbf{y}_{b1} = \mathbf{R}_{s,s} \mathbf{H}_{s,b} \mathbf{x} + \mathbf{n}_{b1}, \quad (7)$$

where $\mathbf{x}$ is the $(Kf \times 1)$ user data vector of the K total users, defined as $\mathbf{x} = [x_1(1), \ldots, x_1(f), \ldots, x_K(1), \ldots, x_K(f)]^T$, $H_{s,b}$ being an $(Kf \times Kf)$ channel coefficient matrix between the source and base station, defined as $\mathbf{H}_{s,b} = diag(h_{1,b}, h_{2,b}, \ldots, h_{s,b})$ and $\mathbf{R}_{s,s}$ is the $(Kf \times Kf)$ base station cross-correlation matrix where $R_{ij} = \rho_{ij}$ represents the cross correlation between spreading codes C_i and C_j.

Then, the output of the bank of matched filters at the relay (first phase) and base station (second phase) are respectively expressed as,

$$\mathbf{y}_{r_\ell} = \mathbf{R}_{r_\ell, r_\ell} \mathbf{H}_{s, r_\ell} \mathbf{x} + \mathbf{n}_{r_\ell}, \quad (8)$$

$$\mathbf{y}_{b2} = \mathbf{R}_{r_\ell, r_\ell} \mathbf{H}_{r_\ell, b} (\mathbf{y}_{r_\ell}) \beta + \mathbf{n}_{b2}, \quad (9)$$

where $\mathbf{R}_{r_\ell, r_\ell}$ is $((Kf-1) \times (Kf-1))$ relay cross-correlation matrix and H_{s,r_ℓ} is the $((Kf-1) \times (Kf-1))$ channel coefficient matrix between source and relays, given as $\mathbf{H}_{s,r_\ell} = diag(h_{1,r_1}, h_{2,r_2}, \ldots, h_{s,r_L})$. $\mathbf{H}_{r_\ell,b}$ is the $((Kf-1) \times (Kf-1))$ channel coefficient matrix between relays and base station, defined as $\mathbf{H}_{r_\ell,b} = diag(h_{r_1,b}, h_{r_2,b}, \ldots, h_{r_L,b})$. Applying multi-user detector (MUD) at both the relay and the base station, we get

$$z_{b1} = (\mathbf{R}_{s,s}^{-1}) \mathbf{y}_{b1} = \mathbf{H}_{s,b} \mathbf{x} + \mathbf{R}_{s,s}^{-1} \mathbf{n}_{b1}, \quad (10)$$

Note that, we consider AF scheme in the sense that no hard decisions are made at the relay. However, to mitigate the effect of interference at the relay, we employ MUD using the decorrelator detector where the soft output of the detector is amplified before retransmission. Therefore, the soft output of the decorrelator detector, before amplification, is given by

$$\mathbf{z}_{r_\ell} = (\mathbf{R}^{-1}_{r_\ell,r_\ell})\mathbf{y}_{r_\ell} = \mathbf{H}_{s,r_\ell}\mathbf{x} + \mathbf{R}^{-1}_{r_\ell,r_\ell}\mathbf{n}_{r_\ell}, \quad (11)$$

Thus, the received signal at the base station in the second phase of relaying is given by

$$\mathbf{z}_{b2} = (\mathbf{R}^{-1}_{s,s})\mathbf{y}_{b2} = \mathbf{H}_{r_\ell,b}\mathbf{H}_{s,r_\ell}\beta\mathbf{x} + \mathbf{H}_{r_\ell,b}\beta\mathbf{R}^{-1}_{r_\ell,r_\ell}\mathbf{n}_{r_\ell} + \mathbf{R}^{-1}_{s,s}\mathbf{n}_{b2}. \quad (12)$$

III. Outage Probability

The outage probability has been widely used as an important performance analysis that measures and evaluates the system quality. The outage probability, P_{out}, defines the probability that the transmission data rate, $\Re$, exceeds the mutual information between the source and the base station. Here, the outage probability of the cooperative diversity protocol under diversity combining is studied. Let us first consider the direct link between the source s and the base station b where the mutual information is written as [11]

$$I_{sb} = \frac{K}{2N}\log\left(1 + \frac{2N\gamma_{sb}}{K^2[R^{-1}]_{s,s}}\right) \quad (13)$$

where $\frac{K}{2N}$ is the available degrees of freedom, a function of the total number of users K, the available spreading code N. $[R^{-1}]_{ij}$ represents the i*th* row and j*th* column element of the inverse of the cross-correlation matrix $\mathbf{R}^{-1}$.

The factor of 1/2 stands for the bandwidth expansion needed for relaying due to the half-duplex constraint, and $\frac{2N}{K^2}$ is the normalized discrete-power constraint [11]. The outage event occurs when I_{sb} fails to achieve a target data rate of $\Re$, which is given by

$$P_{out} = P_r[I_{sb} < \Re]. \quad (14)$$

From (13) and (14) we get

$$\begin{aligned} P_{out} &= P_r\left[\frac{K}{2N}\log\left(1 + \frac{2N\gamma_{sb}}{K^2[R^{-1}]_{s,s}}\right) < \Re\right] \\ &= P_r\left[\gamma_{sb} < \gamma_{th-sb}\right] \end{aligned} \quad (15)$$

where $\gamma_{th-sb} = \frac{2^{\frac{2N\Re}{K}}-1}{\frac{2N}{K^2[R^{-1}]_{s,s}}}$.

Finally, the outage probability of the direct link can be expressed as

$$\begin{aligned} P_{out-noncoop} &= P_r\left[\gamma_{sb} < \gamma_{th-sb}\right] = F\left[\gamma_{sb} < \gamma_{th-sb}\right] \\ &= \frac{\gamma\left(m_{sb}, B_{sb}\gamma_{th-sb}\right)}{\Gamma(m_{sb})}. \end{aligned} \quad (16)$$

In what follows, the mutual information of the AF protocol for the link $s \to r \to b$ and after many algebraic manipulations can be written as in [3]

$$I_{AF} = \frac{K}{2N}\log\left(1 + X_{sb} + \sum_{\ell=1}^{L}\frac{X_{sr_\ell}X_{r_\ell b}}{X_{sr_\ell} + X_{r_\ell b} + 1}\right). \quad (17)$$

For simplicity we define

$X_{sb} = \frac{2N\gamma_{sb}}{K^2[R^{-1}]_{s,s}}$, $X_{sr} = \frac{2N\gamma_{sr_\ell}}{K^2[R^{-1}]_{r_\ell,r_\ell}}$, and $X_{r_\ell b} = \frac{2N\gamma_{r_\ell b}}{K^2[R^{-1}]_{r_\ell,r_\ell}}$.

Therefore, the total SNR at the base station can be written as

$$X_{total} = X_{sb} + \sum_{\ell=1}^{L}\frac{X_{sr_\ell}X_{r_\ell b}}{X_{sr_\ell} + X_{r_\ell b} + 1}. \quad (18)$$

Numerical Bound

Eq. (18) should be reformulated in a more mathematically compatible form in order to ease the computation of the PDF of the total SNR. Using the upper bound introduced in [12], we have

$$X_{total} \le X_{up} = X_{sb} + X_\ell, \quad (19)$$

where $X_\ell = min(X_{sr_\ell}, X_{r_\ell b})$, and the approximated SNR value X_{up} is analytically more tractable than the exact expression in (18). This approximation introduced in [12] is shown to be accurate in medium and high SNRs. Also it will show to simplify the derivation of the outage probability. Given the CDF of X_{rb} defined in (6), the CDF of X_ℓ is given by

$$\begin{aligned} F_{X_\ell}(\gamma_{th}) &= 1 - P[X_{sr_\ell} > \gamma_{th}\ and\ X_{r_\ell lb} > \gamma_{th}] \\ &= 1 - \frac{\Gamma\left(m_{sr_\ell}, B_{sr_\ell}\gamma_{th}\right)}{\Gamma(m_{sr_\ell})}\frac{\Gamma\left(m_{r_\ell b}, B_{r_\ell b}\gamma_{th}\right)}{\Gamma(m_{r_\ell b})}. \\ &. \end{aligned} \quad (20)$$

The lower bound for the outage probability can be obtained as in [12] using the MGF to give

$$P_{out} \ge \mathcal{L}^{-1}\left\{\frac{M_{X_{total}}(s)}{s}; t\right\}_{|t=\gamma_{th}}. \quad (21)$$

Finally, the P_{out} for the AF scheme can be numerically determined by solving (21) using mathematical packages such as Maple and Mathematica. While in [12] only a lower bound on the outage probability is provided, in what follows we derive a closed form expression for the asymptotic outage probability.

Closed-Form Expression using Approximate Distributions for the SNR

The asymptotic outage probability is conventionally evaluated using the PDF of the $s \to r$ and $r \to b$ links. Due to the intractability of the PDF of SNR, X_{total}, given in (18), it becomes difficult to obtain a closed-form expression for the outage probability in the AF case. Alternatively, we introduce an approximation for the PDF of X_{total} at high SNRs. This approximation will enable us to find a closed-form expression for the outage probability of the underlying system. In the following section, the accuracy of our analytical results will be assessed through some simulation examples.

At high SNR, (5) can be approximated as

$$p_{X_{ij}}(\gamma) \approx \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})}\gamma^{m_{ij}-1} + H.O., \quad (22)$$

where H.O. stands for high order terms. Applying Laplace transform to (22) and with the help of [13], we have

$$M_{X_{ij}}(s) = \frac{B_{ij}^{m_{ij}}}{s^{m_{ij}}}. \tag{23}$$

In the case of indirect link, let $\Psi_\ell, \ell = 1, 2, ..., L$ be non-negative RV and let the PDF of Ψ_ℓ be approximated as in (22). Let us also define $Y = \sum_{\ell=1}^{L} \Psi_\ell$, then

$$M_Y(s) = \prod_{\ell=1}^{L} M_{\Psi_\ell}(s). \tag{24}$$

Using (24), we have

$$M_Y(s) = \frac{\prod_{\ell=1}^{L} B_\ell^{m_\ell}}{s^{\sum_{\ell=1}^{L} m_\ell}} \tag{25}$$

and by applying the inverse Laplace transform, $\mathcal{L}^{-1}\{M_Y(s); y\}$, the PDF of Y can be written as

$$f_Y(y) = \frac{y^{(\sum_{\ell=1}^{L} m_\ell - 1)}}{\Gamma(\sum_{\ell=1}^{L} m_\ell)} \prod_{\ell=1}^{L} B_\ell^{m_\ell}. \tag{26}$$

Using the method in [12], the PDF of the approximate distribution for the $s \to r_\ell \to b$ link can be written as

$$f_{X_{s,r_\ell,b}}(\gamma) \approx f_{X_{sr_\ell}}(\gamma) + f_{X_{r_\ell b}}(\gamma), \tag{27}$$

where $f_{X_{sr_\ell}}(\gamma)$, and $f_{X_{r_\ell b}}(\gamma)$ can be approximated at high SNR as in (22), yielding to

$$f_{X_{sr_\ell}}(\gamma) \approx \frac{B_{sr_\ell}^{m_{sr_\ell}}}{\Gamma(m_{sr_\ell})} \gamma^{m_{sr_\ell} - 1} + H.O., \tag{28}$$

$$f_{X_{r_\ell b}}(\gamma) \approx \frac{B_{r_\ell b}^{m_{r_\ell b}}}{\Gamma(m_{r_\ell b})} \gamma^{m_{r_\ell b} - 1} + H.O. \tag{29}$$

Examining (27) when $E_s/N_o \to \infty$, $f_{X_{s,r,b}}(\gamma)$ in (27) can be simplified to

$$f_{X_{s,r_\ell,b}}(\gamma) \approx \Omega_\ell \gamma^{m_{r_\ell} - 1} + H, \ell = 1, 2, ...L, \tag{30}$$

where $m_{r_\ell} = min(m_{sr_\ell}, m_{r_\ell b})$ and

$$\Omega_\ell = \begin{cases} \frac{B_{sr_\ell}^{m_{sr_\ell}}}{\Gamma(m_{r_\ell b})} & m_{sr_\ell} < m_{r_\ell b} \\ \frac{B_{r_\ell b}^{m_{r_\ell b}}}{\Gamma(m_{r_\ell b})} & m_{sr_\ell} > m_{r_\ell b} \\ \frac{1}{\Gamma(m_{r_\ell})}(B_{sr_\ell}^{m_{r_\ell}} + B_{r_\ell b}^{m_{r_\ell}}) & m_{sr_\ell} = m_{r_\ell b} = m_{r_\ell}. \end{cases} \tag{31}$$

Thus, the total PDF of the approximated SNR is given by

$$f_{X_{AF}}(\gamma) \approx f_{X_{s,b}}(\gamma) + f_{X_{s,r_\ell,b}}(\gamma), \tag{32}$$

and the corresponding MGF is given by

$$M_{X_{AF}}(s) = M_{X_{sb}}(s) \cdot M_{X_{s,r_\ell,b}}(s). \tag{33}$$

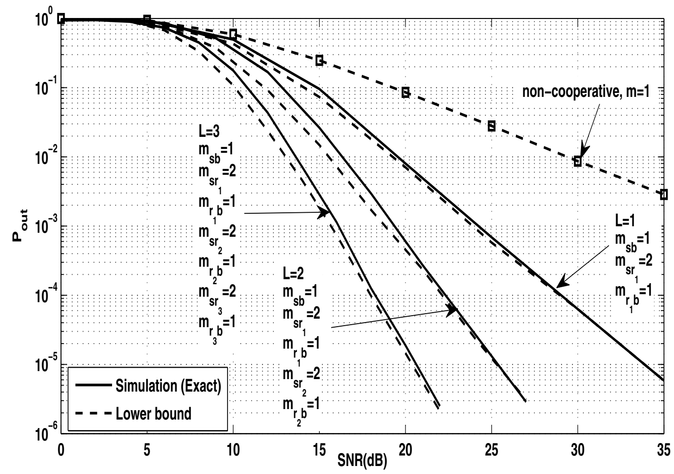

Fig. 1. Performance of multi-relay cooperative system with different L over independent non identical Nakagami-m fading channels, analytical results based on the lower bound (21).

Now, using (23) and (25), we have

$$M_{X_{AF}}(s) = \frac{B_{sb}^{m_{sb}} \prod_{\ell=1}^{L} \Omega_\ell \Gamma(m_{r_\ell})}{s^{(m_{sb} + \sum_{\ell=1}^{L} m_{r_\ell})}}. \tag{34}$$

Finally, the asymptotic outage probability P_{out} of the cooperative CDMA system using AF relaying is given by

$$P_{out} = \mathcal{L}^{-1}\left\{\frac{M_{X_{AF}}(s)}{s}; t\right\}_{|t=\gamma_{th}} \tag{35}$$

$$P_{out} = B_{sb}^{m_{sb}} \prod_{\ell=1}^{L} \Omega_\ell \Gamma(m_{r_\ell}) \cdot \frac{\gamma_{th}^{(m_{sb} + \sum_{\ell=1}^{L} m_{r_\ell})}}{\Gamma(m_{sb} + \sum_{\ell=1}^{L} m_{r_\ell} + 1)}. \tag{36}$$

Examining (36), it is easy to see that the achieved diversity order is $(m_{sb} + \sum_{\ell=1}^{L} m_{r_\ell})$.

IV. Numerical Results

We present some numerical results for the outage probability of multi-relay cooperation system. We build a Mont-Carlo link-level simulation to verify these results with the analytical model derived. We assume asynchronous cooperative DS-CDMA where every user data are spread using non-orthogonal gold codes of length $N = 31$ chips and number of users $K = 16$ with different number of relays $L = 0, 1, 2, 3$ over independent non-identical Nakagami-m fading channels and BPSK transmission. The channels are modeled as block fading channels where the fading coefficients are considered fixed for the duration of one frame $f = 100$ and change independently from one frame to another. Without loss of generality, we assume that the spectral efficiency $\Re = 1 bit/sec/Hz$. Unless otherwise mentioned, a decorrelator detector is used to mitigate the effect of MAI at both the relay and base station receivers.

Let us first assess the accuracy of the outage probability bound obtained in (21). Fig. 1 shows the outage probability for the cooperative DS-CDMA system using AF relaying over i.ni.d. Nakagami-m fading channel with different number of relays L =0,1,2,3. The matching between the analytical (eq. (21)) and simulation results at medium and high SNR is obvious from this figure. At low SNRs, the derived outage probability tends to deviate from the exact results when the number of relays is larger than one.

Now we present results for our approximate closed-form expression of the outage probability derived in (36). Figs. 2 and 3 show the asymptotic outage probability of the AF system over i.i.d. (i.e., $m_{sb} = m_{sr_\ell} = m_{r_\ell b}$) and i.in.d. (i.e., $m_{sb} \neq m_{sr_\ell} \neq m_{r_\ell b}$) Nakagami-$m$ fading channels, respectively. The system performance is examined as a function of the fading parameter m_{ij} and number of cooperating relays L. It is noticeable from the results that the asymptotic P_{out} and simulation results are in excellent agreement at medium and high SNRs. Also from these results one can notice the improvement in diversity order as a function of the number of relays L and fading parameter m_{ij}. We also noticed that the cooperative system guarantees large diversity advantages provided that interference cancellation is performed at both the relay and base station sides.

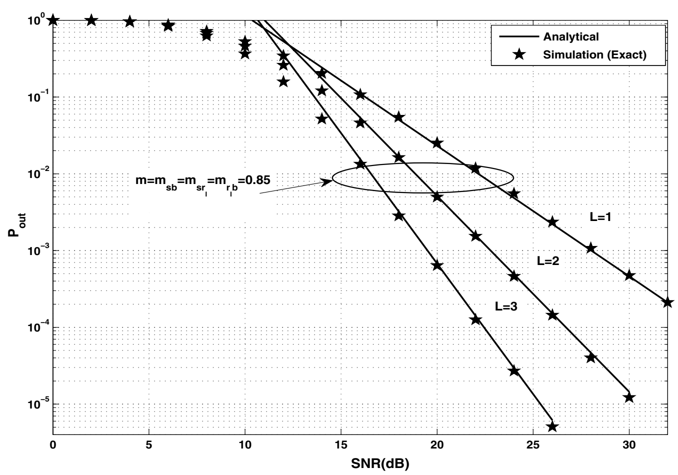

Fig. 2. Outage probability for cooperative AF DS-CDMA with different L over i.i.d. Nakagami-m fading channels, analytical based on the approximate outage probability in (36).

V. Conclusion

We obtained an accurate asymptotic outage probability expression for cooperative diversity in a DS-CDMA system using AF relaying under the diversity combining of the relayed information at the base station over Nakagami-m fading channels. The derived expression can be used for arbitrary number of relays and different fading parameters. The expression is tractable and generally can be used for different channel environments and different modulation schemes. Our cooperative system employed decorrelator detector to suppress the multi-user interference at both the base station and the relay sides. It was shown that the system is able to achieve full diversity gain by increasing the number of participating

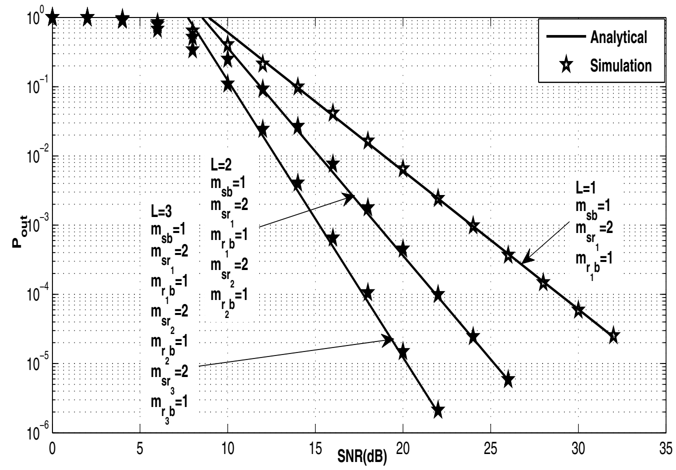

Fig. 3. Outage probability for cooperative AF DS-CDMA with different L over independent non-identical Nakagami-m fading channels.

relays and by combating the effect of MAI and ISI through the decorrelator detector.

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