Achievable Net-Rates in Multi-User OFDMA with Partial CSI and Finite Channel Coherence

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Abstract— **This paper derives an analytical framework to evaluate achievable rates in a multi-user OFDMA system. Opportunistic schedulers and OFDMA allow for exploiting multi-user diversity using a flexible and simplified resource assignment. However, the granularity of resource assignments determines the gains in spectral efficiency and requires to consider signaling overhead and finite channel coherence. Additionally to the signaling and pilot overhead, practical systems suffer from partial CSI, i. e., neither scheduler nor receiver have exact knowledge of the channel state. This paper investigates the trade-off between achievable net-rates and channel characteristics as well as system parameterization. An analytical framework for the expected net-rates in a multi-user system with partial CSI at scheduler and receiver is derived.**

I. Introduction

In broadcast channels, a central access point serves multiple terminals within its cell area. Independent channel fading among these terminals yields multi-user diversity in addition to time-, frequency-, and space-diversity. OFDMA proved to be a promising physical layer access method, which exploits multi-user diversity through a flexible resource assignment. Prominent standards applying OFDMA are 3GPP LTE [1] and WiMAX (IEEE 802.16) [2].

A. Problem Description

An opportunistic scheduler assigns resources to terminals based on channel knowledge at the transmitter and is therefore able to exploit multi-user diversity. The benefits of opportunistic scheduling highly depend on the quality of channel state information (CSI), the amount of signaling required to communicate the resource assignments, and the channel coherence which determines the resource block-size. These factors significantly impact the system performance. Hence, there is the need to find an optimal and flexible resource partitioning and assignment method that requires low overhead.

This paper tackles this problem by deriving an analytical framework to evaluate the performance of opportunistic scheduling in a Rayleigh-fading environment. In addition, we consider exponential path-loss, partial CSI at transmitter and receiver, signaling overhead, and finite channel coherence in time and frequency. Our framework may be used to adaptively optimize the resource granularity and to adjust system parameters depending on load and channel characteristics, e. g. pilot density, number of admitted users, and resource block-size. Results obtained with our framework illustrate how these parameters interact with the actual net-rates. We show how the chosen parameters of both mobile communication standards 3GPP LTE and WiMAX relate to the optimal values.

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B. Related Work

As one of the first, [3] analyzed a low-feedback scheduler which only serves users above a certain signal-to-noise ratio (SNR) threshold. The authors derived achievable rates and bit-error rates for finite constellations, and evaluated the impact of the scheduling delay. However, [3] does not consider partial CSI, signaling, and pilot overhead. The authors in [4] investigate a scenario in which rate-adaptation at the transmitter fails. Hence, the transmitter assumes an SNR higher than the actual SNR, which implies that the achievable rate is 0. The scheduler may apply an SNR back-off to reduce the probability for such a scenario and to increase the effective throughput. Similarly, [5] analyzed the probability that an allocated rate is not supported by the actual channel. However, they assumed block-static fading and perfect CSI at the receiver. In [4], the authors analyzed achievable rates in broadcast channels and took into account both partial CSI and pilot overhead. An energy-budget models pilot overhead under the assumption of infinite channel coherence. Signaling structures and communication of the resource assignment map have been investigated in [6], [7]. In [6], different assignment strategies are compared and the system overhead is quantified. By contrast, [7] investigates how time- and frequency-correlation may be exploited to bundle assignments and compress resource assignments.

C. Contribution and Outline

We derive a closed-form framework for the analysis of achievable net-rates in a multi-user OFDMA system applying an opportunistic scheduler. The framework considers exponential path-loss, Rayleigh-fading, pilot and signaling overhead, and finite channel coherence. We apply the framework to evaluate the achievable net-rates depending on selected system and channel parameters.

The paper is structured as follows: the system and channel model as well as the analyzed scheduler are introduced in

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Section II. We derive an expression for achievable rates in Section III and define the net-rate optimization problem in Section IV. Exemplary results are discussed in Section V and the paper is concluded in Section VI.

Throughout this paper, $\mathrm{x} \sim \mathcal{CN}(0, \sigma_x^2)$ denotes a circularly symmetric i.i.d. Gaussian random process with each element having zero mean and variance σ_x^2. We use italic letters to indicate scalars (N and n), upper-case non-italic letters X to indicate random variables (r.v.s), and calligraphic letters $\mathcal{B}$ to indicate sets. The function $p_{x|y}(x|y)$ denotes the probability density function (pdf) of X conditioned on Y (the subscript is omitted if it is unambiguous). The corresponding cumulative distribution function (cdf) is denoted by $F_{x|y}(x|y)$. We express the expectation value of $f(\cdot)$ with respect to $p_x(\cdot)$ as $\mathrm{E}_x\{f(\cdot)\}$.

II. System, Channel and Scheduler Model

A. System Model

This paper considers a TDD-OFDMA system with a central transmitter (base-station) and K user terminals connected to the base station (BS). Each terminal and the BS are equipped with a single antenna. The system uses a bandwidth of $B\,\mathrm{MHz}$ divided into $N\,\mathrm{subcarriers}$. Resources are organized in blocks of N_f subcarriers and N_t OFDM-symbols. One such block is the smallest assignable resource unit. In each block, N_p pilots are uniformly distributed in time and frequency. We assume channel-reciprocity, i. e., the channel from BS to terminal is equivalent to the channel from terminal to BS. Hence, the BS estimates the uplink channel based on pilot signals and uses this CSI for downlink closed-loop algorithms. We do not consider outdated CSI due to processing delay although it can be integrated in our framework.

B. Channel Model

Users are uniformly distributed in a circular area with radius R. Let $r_k, k \in [1; K]$, be the distance of user k from the BS. The pdf of each r_k is given by $p_r(r) = 2r/R^2$. Without any loss of generality we assume $R = 1$ and let $r_k \in [0, 1]$. The downlink transmission on an individual subcarrier f and OFDM-symbol t received by user k is described by

$$\mathrm{y}_{f,t}^k = r^{-\eta/2}\mathrm{h}_{f,t}^k \cdot \mathrm{x}_{f,t}^k + \mathrm{n}_{f,t}^k, \tag{1}$$

with $\mathrm{x}_{f,t}^k \sim \mathcal{CN}(0, \sigma_x^2)$, additive white Gaussian noise $\mathrm{n}_{f,t}^k \sim \mathcal{CN}(0, \sigma_n^2)$, and path-loss exponent $\eta \geq 2$.

Each channel coefficient $\mathrm{h}_{f,t}^k$ follows a Rayleigh fading distribution such that the instantaneous channel gain $\gamma_{f,t}^k = |\mathrm{h}_{f,t}^k|^2$ of user k on subcarrier (f, t) is distributed as

$$p_{\gamma_{f,t}^k}(\gamma_{f,t}^k) = \lambda_k e^{-\lambda_k \gamma_{f,t}^k}, \text{ and } F_{\gamma_{f,t}^k}(\gamma_{f,t}^k) = 1 - e^{-\lambda_k \gamma_{f,t}^k} \tag{2}$$

with $\lambda_k = 1$ throughout this work. We further use $\overline{\gamma}_k = r_k^{-\eta}\sigma_x^2/\sigma_n^2$ to denote the average SNR of user k.

We apply the model introduced in [8], i. e. rays arrive with a uniform angle-distribution and equal power but an exponentially distributed delay τ (with delay-spread σ_τ):

$$p_\tau(\tau) = \frac{1}{\sigma_\tau} \exp\left(-\frac{\tau}{\sigma_\tau}\right).$$

Assume that two subcarriers are separated in frequency by Δf and in time by Δt, then the envelope correlation of both subcarriers is given by [8, eq. (1.5-19)]

$$\rho(\Delta f, \Delta t) = \sqrt{\frac{J_0^2\left(2\pi \frac{v}{c} \Delta t\right)}{1 + (2\pi\Delta f)^2\sigma_\tau^2}},$$

where $J_0(\cdot)$ is the zero order Bessel function of the first kind, v is the relative velocity, and c is the velocity of light.

Both transmitter and receiver have only partial CSI. The estimation of the actual channel $\mathrm{h}_{f,t}^k$ is given by $\tilde{\mathrm{h}}_{f,t}^k = \mathrm{h}_{f,t}^k + \mathrm{e}_{f,t}^k$. This model has also been used in [9], [10] and considers an error independent of the channel realization. By contrast, [11], [12] applies a model where $\mathrm{h}_{f,t}^k = \tilde{\mathrm{h}}_{f,t}^k + \mathrm{e}_{f,t}^k$ with a channel estimation error independent of the estimated channel. Both models are equivalent and lead to the same capacity expressions. However, the former model significantly simplifies our derivations because the resulting rate expression involves the actual SNR $\gamma_{f,t}^k$. By contrast, the latter model uses the estimated SNR $\tilde{\gamma}_{f,t}^k$ which makes it more difficult to incorporate the user selection. Both models assume that the channel estimation error is Gaussian i.i.d. on each subcarrier. This leads to a lower bound on the capacity because in a practical system the channel estimation error at different subcarriers is correlated. Based on this assumption a lower bound on the capacity is given by [9, eq. (11-13)]

$$C \geq R = \log\left(1 + \sigma_{\text{eff}}^2 \gamma_{f,t}^k\right), \sigma_{\text{eff}}^2 = \frac{N_p}{N_p + 1 + 1/\overline{\gamma}} \overline{\gamma} < \overline{\gamma}. \tag{3}$$

This expression follows from the effective transmission equation given in [9] and equals the lower bound given in [11, eq. (17)]. Both hold for an MMSE estimator with uniformly distributed pilots. The resulting channel estimation error variance equals the Cramer-Rao lower bound (CRLB). According to the CRLB, the error variance is lower bounded by $\sigma_e^2 = (N_p\overline{\gamma})^{-1}$.

C. Scheduler Model

This work applies a normalized-SNR-based scheduler [13] which is asymptotically fair and achieves a similar performance as a proportional fair scheduler using the instantaneous and average rate [14]. Let $(f_r, t_r) \in \mathcal{B}(b_f, b_t)$ be the central reference-subcarrier of an arbitrary resource block (b_f, b_t) with subcarriers $\mathcal{B}(b_f, b_t)$. The scheduler normalizes the estimated channel with $\overline{\gamma}_k$ and uses $\tilde{\gamma}_{f_r,t_r}^k = |\tilde{h}_{f_r,t_r}^k|^2$ as decision variable. The served user is given by $k_{f_r,t_r}^{\max} = \arg \max_{k \in [1;K]} \tilde{\gamma}_{f_r,t_r}^k$ and served on all subcarriers $(f, t) \in \mathcal{B}(b_f, b_t)$.

Due to finite channel coherence, the selected user may not have the maximum normalized-SNR on all subcarriers. In the following, we denote the instantaneous normalized channel gain of the selected user on subcarrier (f, t) with $\tilde{\gamma}_{f,t}$. The cdf and pdf of $\tilde{\gamma}_{f_r,t_r}$ are given by [15]

$$F_{\tilde{\gamma}_{f_r,t_r}}(\tilde{\gamma}_{f_r,t_r}) = \left(F_{\tilde{\gamma}_{f,t}^k}(\tilde{\gamma}_{f_r,t_r})\right)^K \tag{4}$$

$$p_{\tilde{\gamma}_{f_r,t_r}}(\tilde{\gamma}_{f_r,t_r}) = K\left(1 - e^{-\tilde{\gamma}_{f_r,t_r}}\right)^{K-1} e^{-\tilde{\gamma}_{f_r,t_r}}. \tag{5}$$

Let $\tilde{\gamma}'_{f_r,t_r}$ be the maximum estimated SNR of all K users, i.e., $K-1$ users have an estimated SNR smaller than $\tilde{\gamma}'_{f_r,t_r}$. The probability for this event is $(F_{\tilde{\gamma}^k_{f,t}}(\tilde{\gamma}'_{f_r,t_r}))^{K-1}$. The probability that the selected user satisfies an estimated SNR of at least $\tilde{\gamma}'_{f_r,t_r}$ is also $F_{\tilde{\gamma}^k_{f,t}}(\tilde{\gamma}'_{f_r,t_r})$, which hence gives (4).

III. Achievable Rates

In the following, we are interested in the expected cell spectral-efficiency as defined by

$$R^*_{\text{cell}} = \arg\max_{N_f,N_t,N_p} R_{\text{cell}}(N_f,N_t,N_p)$$

$$R_{\text{cell}}(N_f,N_t,N_p) = \mathrm{E}_\gamma\left\{\frac{1}{NN_t}\sum_{(f,t)\in N_t\times N} R_{N_f,N_t,N_p}(f,t)\right\}$$
$$\approx \frac{1}{N_fN_t}\sum_{(f,t)\in\mathcal{B}(0,0)} \mathrm{E}_{\gamma_{f,t}}\{R_{N_f,N_t,N_p}(f,t)\} \quad (6)$$

The approximation in (6) results from the assumption that the channel gain of the selected user in two different resource blocks is independent. Only for very large channel coherence, very small resource blocks and very small channel estimation error this approximation diverges from the actual results. However, this paper considers practical scenarios with delay spread $\sigma_\tau \approx 2\,\mu$s and N_p in the order of 4 pilots. Within those practical scenarios the approximation error is negligibly small.

Equation (6) will be further refined in Section IV where we define the achievable net-rate. We derive the achievable rates in two steps. At first, we derive the rates for normalized channel gains, which is then extended to uniformly distributed users and taking exponential path-loss into account.

A. Solution for Normalized SNR

In the first part, we derive the achievable rates for the normalized channel gain $\gamma_{f,t}$ and normalized estimated channel gain $\tilde{\gamma}_{f,t}$ on subcarrier (f,t). The expected rate per subcarrier in (6) and considering the normalized SNR can be given by

$$\mathrm{E}_{\gamma_{f,t}}\left\{R_{N_f,N_t,N_p}(f,t)\right\} = \iiint \log\left(1+\sigma^2_{\text{eff}}\gamma_{f,t}\right)$$
$$\cdot p\left(\gamma_{f,t},\gamma_{f_r,t_r},\tilde{\gamma}_{f_r,t_r}\right) d\left(\gamma_{f,t},\gamma_{f_r,t_r},\tilde{\gamma}_{f_r,t_r}\right). \quad (7)$$

We break up the joint pdf as follows

$$p\left(\gamma_{f,t}\gamma_{f_r,t_r}\tilde{\gamma}_{f_r,t_r}\right) = p\left(\gamma_{f,t}|\gamma_{f_r,t_r}\right)p\left(\gamma_{f_r,t_r}|\tilde{\gamma}_{f_r,t_r}\right)p\left(\tilde{\gamma}_{f_r,t_r}\right).$$

In words, if the actual channel is known, the estimated channel on the reference subcarrier (f_t,t_r) cannot provide additional information about any other subcarrier (f,t).

At first we derive the pdf of the normalized SNR on the reference subcarrier. Using Bayes' rule we can express it by

$$p(\gamma_{f_r,t_r}) = \int_0^\infty \frac{p_{\tilde{\gamma}^k_{f,t},\gamma^k_{f,t}}(\tilde{\gamma}_{f_r,t_r},\gamma_{f_r,t_r})}{p_{\tilde{\gamma}^k_{f,t}}(\tilde{\gamma}_{f_r,t_r})} p_{\tilde{\gamma}_{f_r,t_r}}(\tilde{\gamma}_{f_r,t_r}) d\tilde{\gamma}_{f_r,t_r}.$$

For the case of Rayleigh fading, the joint pdf $p_{\tilde{\gamma}^k_{f,t},\gamma^k_{f,t}}(\cdot,\cdot)$ is given by [16, eq. (3.14)]

$$p(\tilde{\gamma},\gamma) = \frac{\lambda^2}{(1-\tilde{\rho}^2)}\exp\left[\frac{-\lambda}{(1-\tilde{\rho}^2)}(\tilde{\gamma}+\gamma)\right]\times I_0\left(\frac{2\lambda\sqrt{\gamma\tilde{\gamma}}\,|\tilde{\rho}|}{1-\tilde{\rho}^2}\right)$$

with the modified Bessel function of zero-order and first kind $I_0(\cdot)$ and the correlation coefficient $\tilde{\rho} = \sqrt{1/(1+\sigma^2_e)}$ reflecting the correlation of actual and estimated channel. The pdf $p_{\tilde{\gamma}^k_{f,t}}(\cdot)$ is given in (2) and $p_{\tilde{\gamma}_{f_r,t_r}}(\cdot)$ is given in (5). Hence, we can show that (see also [17] for a similar derivation)

$$p_{\gamma_{f_r,t_r}}(\gamma_{f_r,t_r}) =$$
$$\sum_{n=0}^{K-1}\binom{K-1}{n}(-1)^n\frac{K}{1+n(1-\tilde{\rho}^2)}\exp\left(-\frac{(n+1)\gamma_{f_r,t_r}}{1+n(1-\tilde{\rho}^2)}\right). \quad (8)$$

Now that we have the pdf of the actual SNR on the reference subcarrier, $p_{\gamma_{f_r,t_r}}(\cdot)$, we need to derive the pdf of any subcarrier (f,t) conditioned on γ_{f_r,t_r}, $p_{\gamma_{f,t}|\gamma_{f_r,t_r}}(\cdot|\cdot)$. Using $p_{\gamma_{f,t}|\gamma_{f_r,t_r}}(\cdot|\cdot)$, we can finally derive the pdf of the SNR on any subcarrier (f,t), $p_{\gamma_{f,t}}(\cdot)$. Using the same derivation as before for the estimated channel, we can show that $p_{\gamma_{f,t}}(\cdot)$ has the same structure as (8) but a different correlation coefficient:

$$\rho_{f,t}(\Delta f,\Delta t) = \sqrt{\frac{1}{1+\sigma^2_e}}\times\sqrt{\frac{J_0^2\left(2\pi\frac{v}{c}\Delta t\right)}{1+(2\pi\Delta f)^2\sigma^2_\tau}} \quad (9)$$

with $\Delta f = |f_r - f|$, $\Delta t = |t_r - t|$. This allows us to express the expected rate based on the normalized channel gain as

$$(7) = \int_0^\infty \log\left(1+\sigma^2_{\text{eff}}\gamma_{f,t}\right)p_{\gamma_{f,t}}\left(\gamma_{f,t}\right)d\gamma_{f,t}$$
$$= \frac{K}{\log(2)}\sum_{n=0}^{K-1}\underbrace{\left[\binom{K-1}{n}\frac{(-1)^n}{1+n}\cdot\exp\left(\frac{\sigma^{-2}_{\text{eff}}}{1-\frac{n}{n+1}\rho^2_{f,t}}\right)\Gamma\left(\frac{\sigma^{-2}_{\text{eff}}}{1-\frac{n}{n+1}\rho^2_{f,t}}\right)\right]}_{=R'(\overline{\gamma})} \quad (10)$$

with the gamma-function $\Gamma(x) = \int_x^\infty \frac{e^{-t}}{t}dt$. $R'(\overline{\gamma})$ depends on the average SNR $\overline{\gamma}$ and will be refined in the next part.

B. Solution for Uniformly Distributed Users

The previous part derived an exact solution to (7) for the case that all users are located at $r_k = R$ such that $\overline{\gamma}_k = \sigma^2_x/\sigma^2_n$. We extend this and derive an approximation to (7) for uniformly distributed users within a circular area. The effective SNR in (3) and the channel estimation error involve the average SNR $\overline{\gamma}_k = r_k^{-\eta}\sigma^2_x/\sigma^2_n$ with path-loss exponent η. Hence, eq. (10) depends on distance r_k. In the following, we focus on $\eta = 2$. We do not treat the case of $\eta > 2$ which involves the series expansion of e^x and is significantly more intricate. Let us define the following four parameters

$$\alpha_1 = \frac{N_p+1}{N_p\sigma^2_x/\sigma^2_n} \qquad \alpha_2 = \frac{1}{N_p(\sigma^2_x/\sigma^2_n)^2}$$
$$\alpha_3 = \frac{k}{k+1}\cdot\frac{J_0^2\left(2\pi\frac{v}{c}\Delta t\right)}{1+(2\pi\Delta f)^2\sigma^2_\tau} \qquad \alpha_4 = \frac{\sigma^2_n}{N_p\sigma^2_x}.$$

Function $R'(\overline{\gamma})$ can now be expressed by

$$R'(\overline{\gamma}) = \int_0^1 2r\exp\left(\frac{\alpha_1 r^\eta+\alpha_2 r^{2\eta}}{1-\frac{\alpha_3}{1+\alpha_4 r^\eta}}\right)\Gamma\left(\frac{\alpha_1 r^\eta+\alpha_2 r^{2\eta}}{1-\frac{\alpha_3}{1+\alpha_4 r^\eta}}\right)dr.$$

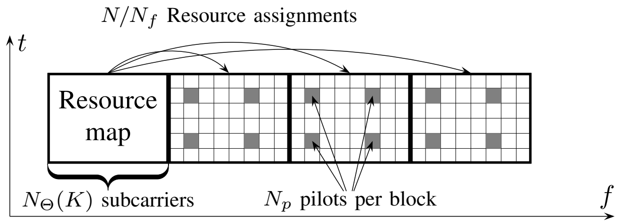

Fig. 1. Sketch of the considered overhead for the net-rate optimization

We can approximate this function for $\eta = 2$ and under the assumption that the SNR is sufficiently large ($\sigma_x^2/\sigma_n^2 \gg 1$):

$$R'(\eta=2) \approx \int_0^1 2r \exp\left(\beta r^2\right) \Gamma\left(\beta r^2\right) dr, \qquad \beta = \frac{\alpha_1}{1-\frac{\alpha_3}{1+\alpha_4}}$$
$$= \frac{1}{\beta}\left(\gamma_e - \log\left(\frac{1}{\beta}\right) + e^{\beta}\Gamma\left(\beta\right)\right) \qquad (11)$$

where γ_e is Euler's constant (≈ 0.58). Parameter α_4 is the worst-case channel estimation error which needs to be included in the variable β. This approximation may now be used in (7) to compute the achievable rates. In Section V, we evaluate this approximation and show that it provides very small approximation errors if the cell-edge SNR is $\geq 5\,\mathrm{dB}$.

IV. Net-Rate Optimization

So far, we described the achievable rates without taking into account the resource signaling overhead and the pilot overhead, which is required to achieve a certain quality of CSI. In order to reflect this overhead we extend (6) as follows

$$R_{\text{cell}}^{\text{net}}(N_f,N_t,N_p) = \Theta(N_f,N_t,N_p,K) R_{\text{cell}}(N_f,N_t,N_p)$$
$$\approx \frac{\Theta}{N_f N_t} \sum_{(f,t)\in\mathcal{B}(0,0)} \mathrm{E}_{\gamma_{f,t}}\{R_{N_f,N_t,N_p}(f,t)\} \qquad (12)$$

where $\Theta(N_f,N_t,N_p,K)$ is the relative signaling overhead depending on the resource size and number of users (the argument list has been omitted for the sake of brevity). The considered overhead is illustrated in Fig. 1. In the following, we consider a worst case scenario in which each resource is assigned individually without any compression. More intricate and realistic approaches will be considered in our future work but have to be omitted here to keep the page limit. If each resource is individually assigned, we need N/N_f assignments with $\log K$ bits each (in order to uniquely identify the selected user). In a practical system, this map is transmitted in a predefined region which underlies the same Rayleigh fading as all other resources. Therefore, the expected rate with which this map is communicated can be approximated by (for instance when using distributed resources)

$$\mathrm{E}_\gamma(R) = \int_0^1 2r \int_0^\infty \log\left(1+\sigma_{\text{eff}}^2\gamma\right) e^{-\gamma} d\gamma dr$$
$$\approx \frac{1}{\alpha_1}\left(\gamma_e - \log\left(\frac{1}{\alpha_1}\right) + e^{\alpha_1}\Gamma\left(\alpha_1\right)\right) \qquad (13)$$

using the same derivation as for (11). This result assumes that each user is able to extract its own resource map. Hence, the user needs not to decode all resource assignments which would require considering a broadcast channel. We need on average

$$N_\Theta = \underbrace{\frac{N\log(K)}{N_f}}_{\text{Required overhead data}} \cdot \frac{1}{\mathrm{E}_\gamma(R)}$$

subcarriers to communicate the resource maps. We assume that every N_t OFDM symbols a new resource assignment is defined. Furthermore, with uniformly distributed pilots, the relative signaling and pilot-overhead scaling is given by

$$\Theta = \left(1 - \underbrace{\frac{\frac{N}{N_f}\frac{\log(K)}{\mathrm{E}_\gamma(R)}}{N N_t}}_{\text{Relative signaling overhead}} - \underbrace{\frac{N_p}{N_f N_t}}_{\text{Relative pilot overhead}}\right)^+$$
$$= \left(1 - \frac{1}{N_f N_t}\left(\frac{\log(K)}{\mathrm{E}_\gamma(R)} + N_p\right)\right)^+,$$

where $(x)^+ = x$ if $x \geq 0$ and $(x)^+ = 0$ otherwise. The multi-user gain in a system using opportunistic scheduling is in the order of $\log\log K$ [18]. By contrast, the overhead increases with a slope in the order of $\log K$. Hence, eventually the overhead will outweigh the multi-user gain. However, this effect is only visible for K in the order of 10^8.

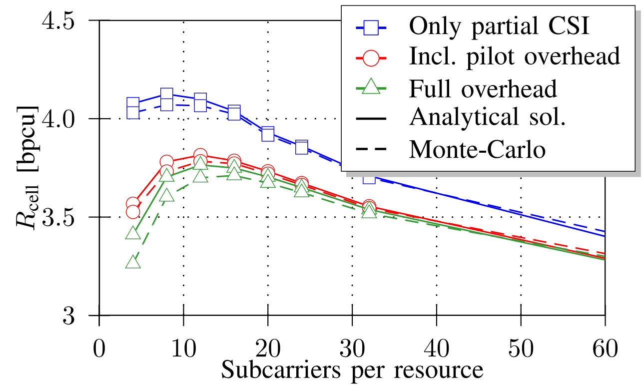

Fig. 2. Achievable net-rate R_{cell}^* as a function of the number of subcarriers N_f per resource block and assuming $N_t = 6$, $\sigma_\tau = 2\mu s$, $\eta = 2$, $v = 0$, $\sigma_x^2/\sigma_n^2 = 5\,\mathrm{dB}$, and $K = 10$. The figure shows the results of the analytical solution (solid lines) and a Monte-Carlo simulation (dashed lines).

V. Results

This section presents exemplary results using the previously derived framework. For all results, we assumed $B = 10\,\mathrm{MHz}$, $N = 1024$, and $\sigma_\tau = 2\,\mu s$. For comparison, in WiMAX [2] the cyclic prefix may range from $6.4\,\mu s$ to $25.6\,\mu s$. We further fix $v = 0$ and $N_t = 6$ OFDM-symbols as the effects are similarly in frequency and time-domain.

Fig. 2 shows the achievable net-rates (12) for different resource-block-sizes and optimal N_p. It compares the results of the solution in (11), using solid lines, and the results of a Monte-Carlo simulation, using dashed lines. More specifically, the figure shows the achievable rate without any overhead, only with pilot overhead, and with pilot and signaling overhead. All three cases have a maximum in N_f, i.e. there exists an optimum block-size. The loss due to overhead increases to up

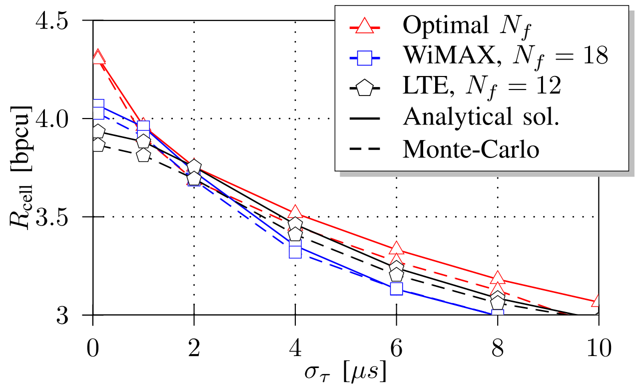

Fig. 3. Achievable net-rate R^*_{cell} as a function of σ_τ and assuming $N_t = 6$, $v = 0$, $\eta = 2$, $\sigma_x^2/\sigma_n^2 = 5\,\text{dB}$, and $K = 10$. The figure shows the results of the analytical solution (solid lines) and Monte-Carlo results (dashed lines).

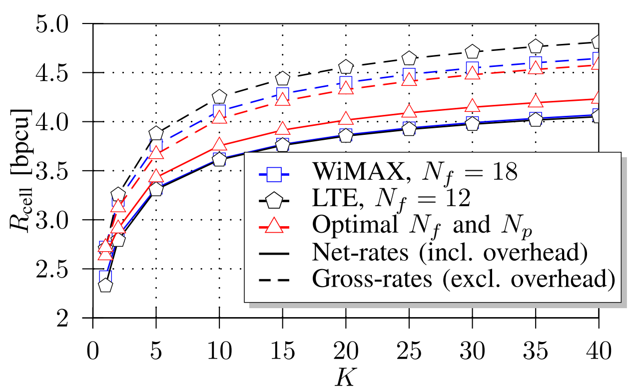

Fig. 4. Achievable net-rate R^*_{cell} as a function of the number of users K and assuming $N_t = 6$, $\sigma_\tau = 2\mu s$, $v = 0$, $\eta = 2$, $\sigma_x^2/\sigma_n^2 = 5\,\text{dB}$, and $K = 10$. Dashed lines indicate the gross-rates without overhead and solid lines show the net-rate taking signaling and pilot overhead into account.

to 16% and is about 10% for the highest net-rate value at $N_f = 12$. The results show a small approximation error for the solution in (11). Within the considered parameterization, the error does not exceed 4% and is in most cases $\leq 1\%$.

Fig. 3 illustrates how the achievable net-rates depend on the delay-spread in three different configurations, i. e. optimal N_f, $N_f = 18$ as used in WiMAX, and $N_f = 12$ as used in 3GPP LTE. For small delay-spread a large block-size provides higher net-rates, hence, $N_f = 18$ outperforms $N_f = 12$. At higher delay-spread the coherence bandwidth becomes smaller and therefore smaller N_f become preferable. Only for very small delay spread the achievable net-rates using the optimal N_f are significantly higher. However, in this case very large resource-blocks are used, which eventually results in a TDMA system where all resources are assigned to one user at a time.

The number of admitted users has an impact on the system performance which scales with $\log\log K$. In Fig. 4, this dependency is shown for $N_f = 18$ (WiMAX), $N_f = 12$ (LTE), and net-rate maximizing N_f and N_p. It compares the achievable gross-rates without overhead (dashed lines) and the achievable net-rates taking full overhead into account (solid lines). The figure shows that the achievable rates scale with $\log\log K$. For a moderate number of users as in this scenario, the overhead does not outweigh the multi-user diversity. The figure further shows that the optimal choice of N_f and N_p provides smaller gross-rates than the other two systems (about 5% compared to $N_f = 12$) but provides higher net-rates (about $5 - 13\%$) due to its optimization with respect to net-rates.

VI. Final Remarks

This paper described an analytical framework which may provide online-support for schedulers to decide on the resource granularity and user assignment. Even though it considers very practical phenomena such as partial CSI at transmitter and receiver, finite channel coherence, exponential path-loss, and Rayleigh fading, it does not incorporate QoS constraints (e. g. delay and minimum-throughput). Furthermore, this work only considers a path-loss exponent of $\eta = 2$. Higher values may also be evaluated analytically as outlined before. These topics will be part of our future work.

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