Iterative Frequency Domain Equalization for Single Carrier Signals with Magnitude Modulation Techniques

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***Abstract*—In this paper we consider broadband wireless systems with high power efficiency. For this purpose we employ MM (Magnitude Modulation) techniques to reduce the PAPR (Peak-to-Average Power Ratio) of the transmitted signals, allowing efficient power amplification. Designed transceivers combine MM with SC-FDE (Single-Carrier with Frequency-Domain Equalization) methods upon iterative-block decision feedback equalization (IB-DFE). Since IB-DFE schemes designed for conventional SC signals are not suitable when we employ PAPR-reduction techniques, the effect of MM is taken into account on the estimation reliability factor that drives IB-DFE convergence. Results for the new proposed MM-IBDFE transceiver present gains above 1.5dB in the overall power efficiency, even when considering severely time-dispersive channels.**

***Index Terms*—IB-DFE, Magnitude Modulation, PAPR**

I. Introduction

The design of broadband wireless systems is a considerable challenge because the high bandwidth of the signals means that the multipath propagation channel is very selective in the frequency, leading to significant time distortion effects and the need for power equalization schemes. Moreover, the power efficiency requirements of these systems are very demanding since the power requirements increase with the bit rate and we need to cope with propagation losses. The power requirements are especially exigent in the uplink transmission of cellular systems (to preclude low battery autonomy) and in satellite systems (due to the significant propagation losses inherent to the high transmission distances).

The overall power efficiency of a given transmission technique takes into account two effects. The first effect is the detection power efficiency which essentially can be measured as the required E_b/N_0 for a given tagged BER (Bit Error Rate) or BLER (BLock Error Rate), with E_b denoting the average bit energy and N_0 the one-sided power spectral density of the channel noise. Clearly, the detection efficiency depends on the adopted modulation scheme and the receiver implementation. The second aspect that influences the overall power efficiency is the amplification efficiency. This includes the efficiency of the power amplifier and the required amplifier back-off, which is lower-bounded by the PAPR (Peak-to-Average Power Ratio) of the adopted signals, i.e., signals with higher PAPR need amplifiers with higher back-off and, consequently, lower amplification efficiency [1].

It is widely accepted that block transmission techniques combined with FFT-based (Fast Fourier Transform) frequency domain (FD) receivers are appropriate for levelly time-dispersive channels [2], [3]. The most popular block transmission techniques are OFDM schemes (Orthogonal Frequency Division Multiplexing) [4] and SC-FDE schemes (Single-Carrier with Frequency Domain Equalization) [5]. The overall signal processing requirements for OFDM and SC-FDE are similar, as well as the achievable performances[1]. However, due to the large envelope fluctuations and PAPR of OFDM signals, SC-FDE schemes are clearly preferable when a high power efficiency is intended. To further improve the performance of SC-FDE, the conventional linear FDE can be replaced by a nonlinear FDE such as the IB-DFE (Iterative Block Decision Feedback Equalizer) [7], which can be regarded as an iterative DFE implemented in the FD.

We can design single-carrier signals with constant envelope such as MSK and GMSK. However, these signals have a bandwidth significantly higher than the minimum Nyquist band, precluding their use when we have significant bandwidth limitations (which is the case of most broadband wireless scenarios). For this reason, band-limited signals such as those based on raised-cosine pulses are usually employed. However, the PAPR of those signals can be relatively high, especially when we are close to the minimum Nyquist band. For this reason, PAPR-reduction techniques are usually employed. Post-filtering techniques (i.e. that perform PAPR-reduction after pulse shaping) such as peak windowing and error shaping [1] typically used in OFDM, can also be employed in SC. However, these usually cause spreading of the SC signal spectrum, requiring a further stage of filtering in order to put the spectrum back within allocated limits which may cause some peak regrowth. An alternative consists into making signal adjustments for PAPR-reduction prior to pulse shaping by taking in account the knowledge of the filter response. The MM (Magnitude Modulation) concept [8]–[12] is one of such solutions with two efficient algorithms known: LUT-MM [9], [10] a lookup table base implementation of the MM concept; and the recently proposed MPMM (multistage polyphase magnitude modulation) technique [11], [12].

Although adoption of these MM techniques can lead to substantial PAPR reductions, they were only proposed for ideal non-dispersive channels.

This work was supported by FCT project PEst-OE/EEI/LA0008/2011.

[1] Actually, the performance of OFDM is only comparable with the performance of SC-FDE with the same constellation when suitable channel coding schemes are employed, since the uncoded OFDM performance is very poor [2], [6].

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In this paper we consider high power-efficient broadband wireless systems. For this purpose we consider single-carrier signals with MM PAPR-reduction techniques based on [11], [12] that are combined with SC-FDE schemes and iterative FDE receivers based on the IB-DFE concept. It is shown that conventional iterative FDE schemes are not suitable when we employ PAPR-reduction techniques. Therefore, we develop and evaluate iterative FD receivers appropriate for single-carrier signals with PAPR-reduction techniques.

Paper outline is as follows. Section 2 introduces the MM concepts. In section 3, the new MM-IBDFE transceiver is developed starting with IB-DFE basics. Finally, section 4 reports achieved results and main conclusions are drawn in section 5.

II. Magnitude Modulation Techniques

Magnitude modulation (MM) [8]–[12] is a suitable solution for controlling the envelope's power peak, while minimizing the PAPR, of band-limited SC signals, allowing to maximize transmitter's HPA (High Power Amplifier) efficiency. The underlying principles of data magnitude modulation are presented in Fig. 1. The concept proposed (in the nineties) by Miller *et al.* [8] consists into the adjustment of the magnitude of SC modulator's output symbols, i.e. $s_n \in \mathbb{C}$, prior to Nyquist pulse shaping, with the aim of controlling envelope's power and prevent peak regrowth at filter's output thus guaranteeing $|x_n|^2 \leq P_{\text{in\_sat}}$, with $P_{\text{in\_sat}}$ denoting the input power that saturates the DAC+HPA block. The pulse-shaped signal is given by

$$x_n = \sum_k m_k s_k h_{n-k} \ , \tag{1}$$

where h_n is the Nyquist filter impulse response (to avoid phase distortion the MM factors m_n are usually taken in $\mathbb{R}^+$).

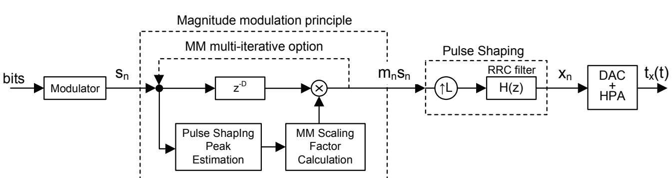

Fig. 1. Underlying principles of MM processing applied to bandwidth-limited SC transmission [8]–[12].

The main contribution to PAPR in SC systems is due to Nyquist filtering, which gives rise to undesirable envelope variations in transmitted signal x_n (this is especially true for constant-amplitude constellations such as PSK).

By adding a short delay memory to the transmitter, it is possible to predict undesirable peak excursions that would arise at filter's output x_n, and, as so, compute proper MM factors m_n at each symbol interval T_{SYMB}, that guarantees $|x_n|^2 \leq P_{\text{in\_sat}}$ while minimizing the PAPR for a given pulse shape h_n.

Tomlinson *et al.* [9], [10] proposed a LUT-based implementation of the MM system with MM factors being computed *a priori*. However, this approach is constrained by the number of states to be analyzed, making it only suitable to small, constant-amplitude constellations.

A multistage polyphase magnitude modulation (MPMM) technique was proposed recently where the MM coefficients are computed in real-time by a low complexity polyphase filter system [11], [12]. The proposed technique is independent of the modulation being used and any number of MPMM based blocks can be added in a serial cascade, for providing a better control of the signal excursion. It has been shown that MPMM is extremely efficient in reducing the PAPR of x_n (better than LUT-MM approaches), even for strict bandwidth limiting and higher-order constellations, with considerable efficiency power gains being reported for transmission on non-dispersive channels [11]–[13].

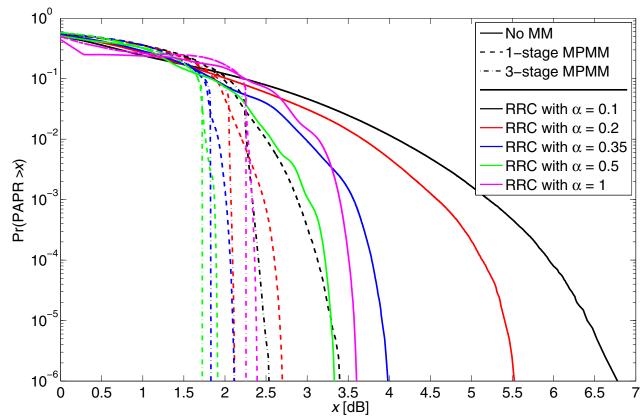

Fig. 2. CCDF of the PAPR for bandwidth limited QPSK transmission.

Fig. 2 presents the complementary cumulative distribution function (CCDF) of the PAPR of a QPSK signal, considering root-raised cosine (RRC) bandwidth limiting with different roll-offs. The potential margin for gain in power efficiency is clear, provided that the detection techniques can compensate MM distortion introduced at the transmitter.

III. Receiver Design

A. IB-DFE Basics

In this section we describe the IB-DFE system [7] for cyclic prefix (CP) based SC-FDE transmissions. The data is transmitted in blocks of N useful modulation symbols plus an appropriate CP of N_G symbols, leading to the block to be transmitted $s_n = [s_{-N_G}, \cdots, s_0, \cdots, s_{N-1}]$. If the duration of the CP is longer than the duration of the overall channel impulse response (which also includes the transmit and receive filters), the linear channel convolution is equivalent to a cyclic convolution in regard to the useful part of the block. This means that the Discrete Fourier Transform (DFT) of the received signal (after CP removal) is $Y_k = [Y_0, \cdots, Y_{N-1}]$, with

$$Y_k = S_k H_k + N_k \ , \tag{2}$$

where H_k denotes the overall channel frequency response for the k^{th} frequency, $[S_0, \cdots, S_{N-1}]$ = DFT $[s_0, \cdots, s_{N-1}]$ and N_k represents the channel noise term in the FD.

Although we could employ a linear FDE to compensate the channel effects, the performance is significantly better if we employ an IB-DFE [7], which can be regarded as an iterative DFE where the feedforward (FF) $\{F_k\}_{k=0,\cdots,N-1}$ and feedback (FB) filters $\{B_k\}_{k=0,\cdots,N-1}$ are implemented in the FD. For the i-th iteration the FD block at the output of the equalizer is $\{\tilde{S}_k\}_{k=0,\cdots,N-1}$, with

$$\tilde{S}_k^{(i)} = F_k^{(i)} Y_k - B_k^{(i)} \hat{S}_k^{(i-1)} \ , \tag{3}$$

where FB filter $B_k^{(i)}$ uses the FD version of the data estimates from the previous iteration[2] to cancel the residual ISI (inter-symbol interference) at the output of the FF filter $F_k^{(i)}$. The optimum FF and FB coefficients are given by [7]

$$F_k^{(i)} = \frac{\kappa H_k^*}{\alpha + \left(1 - \left(\rho^{(i-1)}\right)^2\right) |H_k|^2} \ , \tag{4}$$

and

$$B_k^{(i)} = \rho^{(i-1)} \left(F_k^{(i)} H_k - 1\right) \ , \tag{5}$$

where $\alpha = E\left[|N_k|^2\right] \Big/ E\left[|S_k|^2\right]$ (inverse SNR) and κ is selected to guarantee that $1/N \sum_{k=0}^{N-1} F_k^{(i)} H_k = 1$. The parameter $\rho^{(i-1)}$ denotes the reliability of the FB symbols and it is defined as the expectation of the normalized correlation between the block of hard detected symbols at the previous iteration $\{\hat{s}_n^{(i-1)}\}_{n=0,\cdots,N-1}$ and the block of transmitted symbols $\{s_n\}_{n=0,\cdots,N-1}$, i.e.,

$$\rho^{(i-1)} = \frac{E\left[\hat{s}_n^{(i-1)} s_n^*\right]}{E\left[|s_n|^2\right]} = \frac{E\left[\hat{S}_k^{(i-1)} S_k^*\right]}{E\left[|S_k|^2\right]} \ . \tag{6}$$

At the first iteration $\rho^{(0)} = 0$, since we do not have any data estimate, and the FF coefficients $F_k^{(0)}$ are those of a linear FDE [5].

B. IB-DFE with MM estimation

Conventional iterative FDE detection schemes are not suitable when we employ PAPR-reduction techniques such as MM, since neither the FB symbols nor the correlation factor $\rho^{(i-1)}$ to measure their reliability, take into account the distortion added to the transmitted symbols through the PAPR-reduction procedure.

We concentrate on the case of block-based SC transmission where MM PAPR-reduction is used in the transmitter as presented in Fig. 1, and we will denote by $\bar{s}_n$ the symbols at the output of the MM, i.e. $\bar{s}_n = m_n s_n$. In order to cope with MM distortion upon reception, we propose a change to the IB-DFE receiver by including a MM estimation block on the FB path as sketched in Fig. 3 that we will refer to as MM-IBDFE. On IB-DFE reception hard detected symbols $\{\hat{s}_n\}_{n=1,\cdots,N-1}$ are in some degree reliable, and so it is possible to infer with some accuracy MM coefficients used

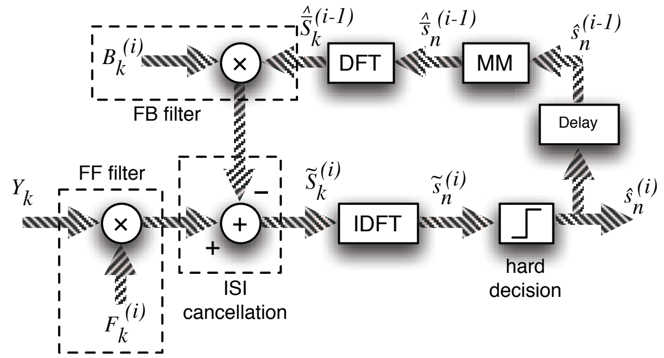

Fig. 3. General MM-IBDFE (IB-DFE receiver using MM estimation) for block-based MM SC transmission.

upon transmission. This happens because for SC, as it is known [12], the influence on the MM factor that applies to a symbol is limited to its very closer neighbors (typically 2 or 3 symbols on each side) given that it depends mainly on the time duration for which the pulse shaping filter maintains significant non-zero time-domain response. Hence, the influence of an erroneous detected symbol among $\{\hat{s}_n\}_{n=1,\ldots,N-1}$ will propagate in very limited extend on estimated MM symbols, i.e. on $\{\hat{\bar{s}}_n\}_{n=1,\ldots,N-1}$. By the same reasoning, although MM techniques have memory, as long as SC transmitted blocks are long enough, the lack of neighboring information on block boundaries upon memoryless IB-DFE block-based detection, almost does not influence estimates $\{\hat{\bar{s}}_n\}_{n=1,\ldots,N-1}$ provided by the MM module of Fig. 3.

This means that we can have better ISI cancelation if we use the estimated MM symbols in the FB loop. Their reliability $\bar{\rho}$ (used to obtain FB filter coefficients according to (5) where $\rho^{(i-1)}$ replaced by $\bar{\rho}^{(i-1)}$) is

$$\bar{\rho}^{(i-1)} = \frac{E\left[\hat{\bar{s}}_n^{(i-1)} \bar{s}_n^*\right]}{E\left[|\bar{s}_n|^2\right]} = \frac{E\left[\hat{\bar{S}}_k^{(i-1)} \bar{S}_k^*\right]}{E\left[|\bar{S}_k|^2\right]} \ , \tag{7}$$

where $\{\bar{S}_k\}_{k=0,\cdots,N-1}$ and $\{\hat{\bar{S}}_k^{(i-1)}\}_{k=0,\cdots,N-1}$ denote the N-point DFT of $\{\bar{s}_n\}_{n=0,\cdots,N-1}$ and $\{\hat{\bar{s}}_n^{(i-1)}\}_{n=0,\cdots,N-1}$ respectively.

C. Estimation of the MM symbols' reliability $\bar{\rho}$

The reliability of the MM symbol estimates used in the FB loop is a key parameter to optimize the performance of IB-DFE. To compute (7) or (6) a training sequence can be sent [7], [14], but this reduces the spectral efficiency. By using a BER estimate we can obtain ρ without the need for training sequences [14], [15]. A similar approach can be followed for estimating (7) when MM is used.

For transmission over a severe time dispersive channel with rich multi-path propagation we can assume that the noise at the equalizer IB-DFE is Gaussian-distributed. In that case, the reliability of feedback symbols can be written in terms of the BER at IB-DFE output. For the QPSK case considered in this paper we have $\rho = 1 - 2P_e$ [14], [15], with P_e denoting the BER of the QPSK transmission, given by $P_e = Q\left(\sqrt{\gamma}_s\right)$,

[2] It is assumed that $\{\hat{S}_k^{(i-1)}\}_{k=0,\cdots,N-1}$ denotes the DFT of the block of hard estimates $\{\hat{s}_n^{(i-1)}\}_{s=0,\cdots,N-1}$.

where $Q(x) = \frac{1}{\sqrt{2}} \int_x^\infty e^{-t^2/2} dt$, and γ_s is the SNR at the IB-DFE output, i.e. at the output of the hard detector block.

For the MM-IBDFE receiver of Fig. 3, a better estimation for $\bar{\rho}^{(i-1)}$ can be obtained by considering the SNR at the output of the MM block, that can be estimated via

$$\bar{\gamma}_s^{(i-1)} = \frac{\bar{E}_s}{\frac{1}{N}\sum_{n=0}^{N-1} \left| \tilde{x}^{(i-1)} - \hat{\tilde{x}}_n^{i-1} \right|} , \tag{8}$$

with $\bar{E}_s$ denoting the average energy sent per MM symbol. A good $\bar{\rho}^{(i-1)}$ estimation results to be

$$\bar{\rho}^{(i-1)} = 1 - 2Q\left(\sqrt{\bar{\gamma}_s^{(i-1)}}\right) . \tag{9}$$

IV. Results and Discussion

This section presents a set of performance results concerning the proposed receivers for SC-FDE with MM techniques. We consider uncoded transmission and FFT-blocks with $N = 512$ data symbols, selected from a QPSK constellation under a Gray mapping rule. Nyquist pulse shaping is performed using a RRC filter (unless otherwise specified, the roll-off $\alpha = 0.2$ is assumed) and matched n-stage ($n = 1$ or 3) MPMM PAPR-reduction [11], [12] employed in the transmitter. We have a severely time dispersive channel with 32 symbol-spaced multipath components with uncorrelated Rayleigh fading. Perfect synchronization and channel estimation are assumed in all cases.

In Fig. 4 we compare the performance of the standard IB-DFE receiver with the enhanced MM-IBDFE receiver of Fig. 3, with perfect knowledge of the reliability factors (clearly, the first iteration corresponds to the linear FDE). We have a 1-stage MPMM at both the transmitter and the MM-IBDFE receiver. Fig. 4 shows that the conventional IB-DFE is not suitable when we employ PAPR-reduction (in fact, it is better to employ a simple linear FDE since there is a loss of performance as we increase the number of iterations). On the other hand, the MM-IBDFE allows significant improvement relatively to the conventional linear FDE (first iteration) with the maximum achievable performance approaching a limit after just 4 iterations (fast convergence) and only about 2dB away from the MFB (Matched Filter Bound).

The theoretical and estimated values of the reliability coefficient $\bar{\rho}$ are presented in Fig. 5. It can be observed that the estimates deviate from the theoretical values for small E_b/N_0, corresponding to an optimistic P_e. However, the BER performance degradation is not significant, as shown in Fig. 6. Given the good MM-IBDFE performance when we employ the estimated $\bar{\rho}$ (9), it will be employed in rest of the paper.

The performance of MM-IBDFE under different MM configurations at the transmitter and the receiver is presented in Fig. 7. Due to its lower PAPR-reduction capacity, the LUT-MM technique (with memory size [9], [10] of $M \in \{3, 5, 7\}$ symbols) is not used in the transmitter. For sake of comparison we also include the IB-DFE performance when no PAPR-reduction technique is employed. As expected, there is some

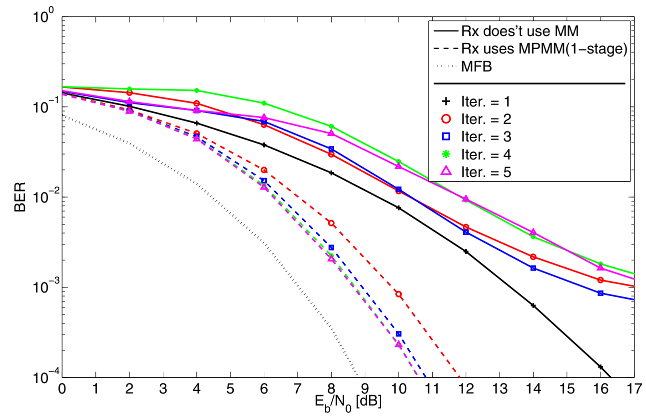

Fig. 4. IB-DFE performance with and without MM factors estimation, for a block based QPSK transmission using 1-stage MPMM PAPR-reduction and $\alpha = 0.2$ RRC pulse shaping.

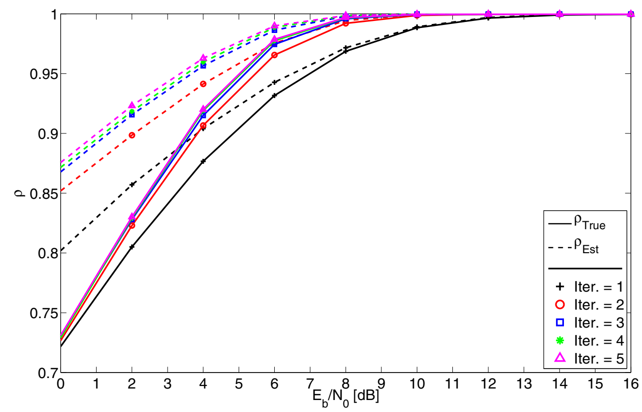

Fig. 5. Evolution of $\bar{\rho}$ of the MM-IBDFE for the exact computation (7) and estimation (9) cases as function of E_b/N_0.

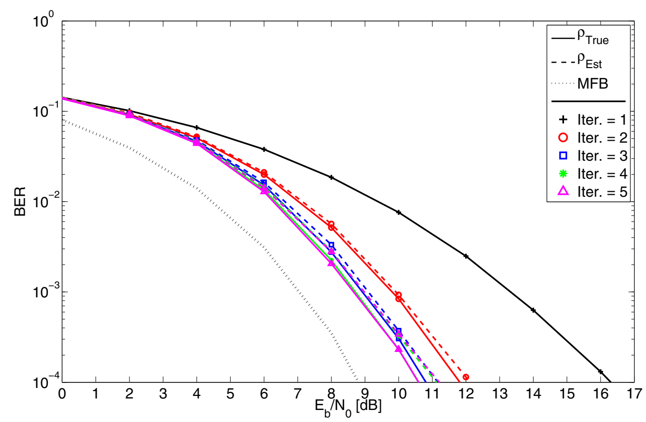

Fig. 6. MM-IBDFE performance considering exact computation and estimation of the reliability $\bar{\rho}$.

performance degradation when we use PAPR-reduction techniques. However, we should have in mind the attained PAPR values for the different techniques. A simple way of doing this is by computing the performance as a function of the 'peak bit energy' $E_b^{(p)}$ instead of the 'average bit energy' E_b, where the peak bit energy is defined as

$$E_b^{(p)} = E_b + PAPR \quad (dB) , \tag{10}$$

with *PAPR* values being obtained from Fig. 2 at 10^{-6}.

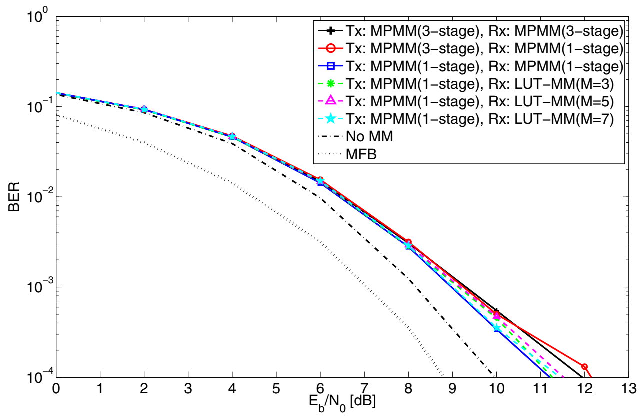

Fig. 7. BER performance after 5 iterations for different PAPR-reducing techniques, as a function of E_b/N_0 when considering RRC with $\alpha = 0.2$.

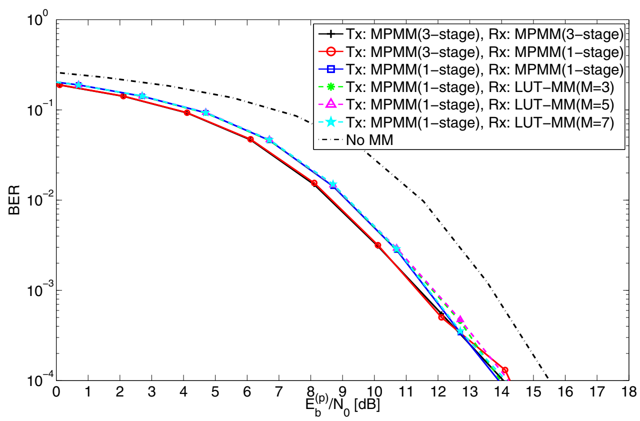

Fig. 8. BER performance after 5 iterations for different PAPR-reducing techniques, as a function of $E_b^{(p)}/N_0$ when considering RRC with $\alpha = 0.2$.

From Fig. 8 it is clear that there is a significant gain in the overall power efficiency when employing PAPR-reducing techniques combined with the proposed receiver.

Finally, Fig. 9 presents a comparison in terms of BER performance vs. $E_b^{(p)}/N_0$ of the studied iterative DFE detection for SC-FDE schemes when using or not MM PAPR-reduction, considering different scale of bandwidth limitation. Clearly, the new proposed SC-FDE scheme with MM PAPR reduction can provide a gain of at least 1.5dB.

V. Conclusion

In this paper we have considered the use of MM techniques to reduce the PAPR of transmitted signals so as to allow an efficient power amplification-reduction. These techniques were combined with SC-FDE schemes employing iterative receivers based on the IB-DFE concept. It was shown that iterative FDE schemes designed for conventional SC signals are not suitable upon the use of PAPR-reduction methods. Therefore, we developed iterative FD receivers appropriate for magnitude modulated SC signals where MM distortion is taken into account. Performance results have shown significant gains in the overall power efficiency compared to conventional block-based SC transceivers using IB-DFE.

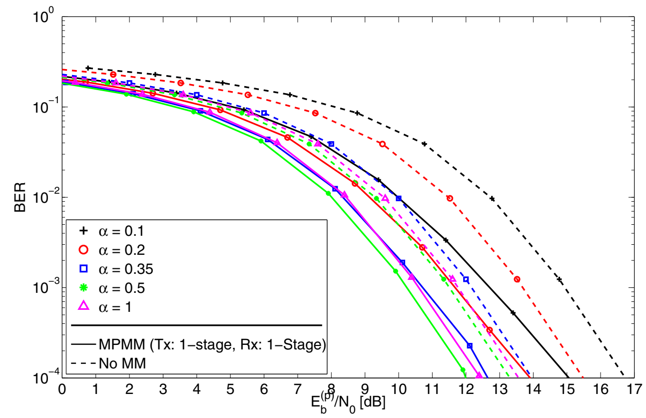

Fig. 9. BER performance as a function of $E_b^{(p)}/N_0$ for RRC pulses with different roll-off factors and receivers with 5 iterations.

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