Outage Performance and DMT Analysis of DF Parallel Relaying in FSO IM/DD Communications

Sahar Molla Aghajanzadeh[1] and Murat Uysal[2,1]

[1]Department of Electrical and Computer Engineering
University of Waterloo, Waterloo, Ontario
Email: smollaag@uwaterloo.ca

[2]Faculty of Engineering
Özyeğin University, Istanbul, Turkey
Email: murat.uysal@ozyegin.edu.tr

***Abstract*—In this paper, we investigate the performance of cooperative diversity transmission in free-space optical (FSO) communications. We consider an FSO cooperative diversity system with a single decode-and-forward relay and a line-of-sight link between the source and the destination. Focusing on high signal-to-noise ratio (SNR) regime, we develop performance characterizations in terms of outage probability, outage diversity and coding gains, and diversity-multiplexing tradeoff (DMT). Our results demonstrate that cooperative diversity improves the performance of FSO communications by bringing diversity advantages.**

I. Introduction

With its appealing features, free-space optical (FSO) communication has attracted a growing attention for a large number of applications [1]. A major performance degrading factor for practical FSO links is atmospheric fading. This has motivated many researchers to investigate temporal and spatial diversity techniques for performance improvements against the degrading effects of fading; see e.g., [2] and the references therein. The idea of cooperative transmission has been originally introduced for radio-frequency (RF) wireless communications; see e.g. [3]. Via the use of relays, cooperative transmission extends signal coverage and extracts spatial diversity gains.

Cooperative transmission has been also investigated in the context of FSO communications; see e.g. [4] and [5] and the references therein. Particularly, in [4], two configurations for cooperative transmission are considered, namely serial and parallel relaying. Serial (multi-hop) transmission is used in terrestrial wireless RF systems to extend coverage, but does not provide diversity gains. It has been shown in [4] that relay-assisted FSO systems exploit the fact that turbulence fading variance is distance dependent (a major difference from RF counterparts) and extract diversity gains by taking advantage of the resulting shorter hops. Parallel relaying [4] is another alternative to implement the concept of cooperative transmission in FSO systems. Due to the line-of-sight nature of FSO transmission, an artificial broadcasting is created through the use of multiple transmit-apertures directed to relay nodes.

The outage analysis of parallel FSO relaying presented in [4] has been obtained for log-normal turbulence fading channels which are limited for weak turbulence conditions. This analysis has been extended to Gamma-Gamma ($\Gamma\Gamma$) fading channels in [5]. Both of these works assume that there is no line-of-sight link between the source and the destination. In this work, we investigate parallel relaying over fading channel assuming the presence of a line-of-sight link between the source and the destination. Particularly, we consider an intensity modulation/direct-detection (IM/DD) FSO system with a single decode-and-forward relay. It should be also emphasized that different from [4] and [5] which assume a single receiver with large aperture, we employ two separate receivers with smaller apertures and perform maximum ratio combining. Under these assumptions, we derive an accurate closed-form approximation for the outage probability at high-SNR regime and characterize it by diversity and coding gains. Furthermore, we present diversity-multiplexing trade off (DMT) expressions. Our results demonstrate that the diversity gain in relay-assisted FSO transmission increases as much as the minimum of the diversities of source-relay and relay-destination links compared to direct transmission. In addition, we demonstrate that the outage performance at high-SNR regime is dominated by either source-relay or relay-destination link that has the worse diversity gain.

The rest of the paper is organized as follows. In Section II, we describe the system model and the $\Gamma\Gamma$ fading model adopted in our analysis. In Sections III, we present the outage performance analysis. In Section IV, we present numerical results and finally we conclude in Section V.

Notation: We use the notation $x \sim \Gamma\Gamma(\alpha, \beta, \mu)$ if the random variable x follows $\Gamma\Gamma$ distribution with mean μ and parameters α and β. We say the random variable y follows $\Gamma\Gamma^2(\alpha, \beta, \mu)$ distribution if $y = x^2$ and $x \sim \Gamma\Gamma(\alpha, \beta, \mu)$. $f(x) \doteq x^a$ means $\lim_{x\to\infty} \log f(x)/\log x = a$. We write $f(x) = o(g(x))$ as $x \to$

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x_0 if $\lim_{x\to x_0} f(x)/g(x) = 0$ and $f(x) \approx g(x)$ as $x \to x_0$ if $\lim_{x\to x_0} f(x)/g(x) = 1$.

II. System Model

Consider the parallel FSO relaying scheme in Fig. 1. We assume full-duplex (bi-directional) transceivers which are the common choice in commercial FSO links. The relay node deploys decode-and-forward (DF) relaying strategy, i.e., it first decodes the received signal after direct detection and then repeats the source transmission. We assume that the relay is allowed to forward only if it has decoded correctly[1]. Otherwise it remains silent. The destination makes its final decision by maximum ration combining [6] the received signals from the direct link and the indirect link (if the relay is active).

Let P denote the average transmitted optical power in a direct transmission system that we will use as a benchmark scheme. For a fair comparison, we assume that each of three active transmitters ($\mathrm{TRX}_{S,1}$, $\mathrm{TRX}_{S,2}$, and $\mathrm{TRX}_{R,2}$) in each cooperation cycle uses an average transmitted optical power of $P/3$. Let $y_{D,1}$ and $y_{D,2}$ respectively denote the detected electrical signals at the $\mathrm{TRX}_{D,1}$ and $\mathrm{TRX}_{D,2}$ terminals of the destination, and let y_R be the detected electrical signal at $\mathrm{TRX}_{R,1}$ terminal of the relay. These signals are consequently given by

$$y_{D,1} = \eta h_{SD} x_S + n_{D,1}, \tag{1}$$

$$y_{D,2} = \begin{cases} \eta h_{RD} x_S + n_{D,2}, & \text{if relay is on} \\ 0, & \text{if relay is off} \end{cases}, \tag{2}$$

$$y_R = \eta h_{SR} x_S + n_R, \tag{3}$$

where η is the optoelectronic conversion factor [7], and x_S is the intensity modulated source signal with $E[x_s] = P/3$. In (1)-(3), $n_{D,1}$, $n_{D,2}$, and n_R are the noise terms in $\mathrm{TRX}_{D,1}$, $\mathrm{TRX}_{D,2}$, and $\mathrm{TRX}_{R,1}$ terminals, respectively. Each noise term is the superposition of thermal noise and background light induced shot noise, and is modeled as zero-mean signal-independent Gaussian noise [7] with variance N_0. The noise terms are assumed to be mutually independent. In (1)-(3), h_{SR}, h_{RD}, and h_{SD} are, respectively, the fading channel coefficients of source-relay (S-R), relay-destination (R-D), and source-destination (S-D) links. These are assumed to follow $\Gamma\Gamma$ distribution [8]. Dropping the subscripts for the sake of presentation, the probability density function (pdf) of h is given by

$$f_h(h) = \frac{2(\alpha\beta/\mu)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h/\mu}\right), \tag{4}$$

where $K_\nu(\cdot)$ is the modified Bessel function of the second kind of order ν, and μ is the mean of the random variable h. α and β are the distribution shaping parameters which can be expressed as functions of Rytov variance [8]. In our work, it is assumed that $h_{SR} \sim \Gamma\Gamma(\alpha_1, \beta_1, l_1)$, $h_{RD} \sim \Gamma\Gamma(\alpha_2, \beta_2, l_2)$, and $h_{SD} \sim \Gamma\Gamma(\alpha_3, \beta_3, 1)$ where l_1 and l_2 are, respectively, the path loss of S-R and R-D links normalized to the path loss of S-D link.

[1]In practice, this can be implemented through the use of cyclic redundancy check (CRC).

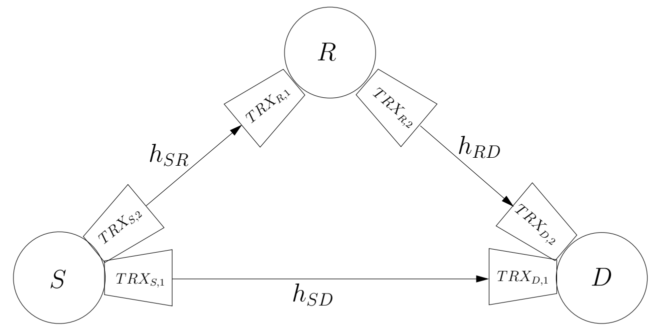

Fig. 1. FSO relay-assisted transmission.

III. Outage Performance Analysis

A. Basic Definitions

1) Outage probability: At a given data rate R (bits/s/Hz), the outage probability is defined as [6]

$$P_{out} = \Pr\left(C(\gamma) < R\right), \tag{5}$$

where $C(\gamma)$ is the instantaneous capacity of the underlying channel, and is a function of instantaneous electrical SNR γ. Since $C(\cdot)$ increases monotonically with respect to γ, the outage probability can be obtained by

$$P_{out} = \Pr\left(\gamma < \gamma_{th}\right), \tag{6}$$

where $\gamma_{th} = C^{-1}(R)$ is the threshold SNR that is required to support the data rate R.

The exact capacity of an FSO IM/DD channel corrupted by additive white Gaussian noise (AWGN) is still an open problem. However, at high-SNR regime, the capacity of this channel can be approximated by using the results of [9] as

$$C(\gamma) \approx \frac{1}{2}\log(\gamma). \tag{7}$$

2) Diversity and coding gains: At high-SNR regime, the outage probability in most cases can be approximated as [10]

$$P_{out} \approx (O \cdot \rho)^{-d}, \tag{8}$$

where ρ is the average SNR of the system in the absence of fading, and d and O are respectively inferred as outage diversity gain and outage coding gain. The diversity gain indicates the slope of the outage probability curve versus average SNR on a log-log scale while the coding gain determines the relative horizontal shift of this curve.

3) Diversity-multiplexing tradeoff: If the data rate R increases linearly with respect to $\log\rho$ for large values of ρ, i.e., $R \approx r\log\rho$, then the outage probability at high-SNR regime can be approximated as [11]

$$P_{out} \doteq \rho^{-d^*(r)}, \tag{9}$$

where r is called multiplexing gain and $d^*(r)$ denotes the optimum diversity gain that can be achieved at a given multiplexing gain. It is shown in [11] that there is a tradeoff between the diversity gain (which quantifies the link reliability) and the multiplexing gain (which quantifies the data rate); higher multiplexing gain comes at the price of sacrificing diversity. The curve of $d^*(r)$ versus r is called optimal diversity-multiplexing tradeoff (DMT). Note that $d^*(r=0)=d$, where d is the diversity gain defined in (8).

B. Direct Transmission (DT)

As a benchmark scheme, we first investigate the outage performance of direct transmission (i.e., no relay is used). In this case, the instantaneous electrical SNR at the destination can be obtained by using (1) as $\gamma_D^{DT} = \rho h_{SD}^2$, where $\rho = (\eta P)^2/N_0$ is the average electrical SNR in the absence of atmospheric channel effect. Substituting γ_D^{DT} in (6) and using [12, p. 1035, Eq. 9.34.3] and [12, p. 851, Eq. 7.811.2], the outage probability for direct transmission can be obtained as

$$P_{out}^{DT} = \frac{1}{\Gamma(\alpha_3)\Gamma(\beta_3)} G_{1,3}^{2,1}\left(\alpha_3\beta_3\sqrt{\gamma_{th}/\rho}\,\middle|\, \begin{matrix} 1 \\ \alpha_3, \beta_3, 0 \end{matrix}\right), \quad (10)$$

where $G(\cdot)$ is the Meijer G-function [12]. By using Corollary 1 of Appendix, P_{out}^{DT} can be approximated as $P_{out}^{DT} \approx (O_{DT}\rho)^{-d_{DT}}$ at high-SNR regime, wherein the diversity gain (d_{DT}) and the coding gain (O_{DT}) are respectively given by

$$d_{DT} = \sigma_3/2, \quad (11)$$

$$O_{DT} = \frac{1}{\gamma_{th}(\lambda_3\sigma_3)^2}\left(\frac{\Gamma(\lambda_3-\sigma_3)}{\Gamma(\lambda_3)\Gamma(\sigma_3+1)}\right)^{-2/\sigma_3}, \quad (12)$$

with $\sigma_3 = \min\{\alpha_3, \beta_3\}$, $\lambda_3 = \max\{\alpha_3, \beta_3\}$.

When the data rate of the system increases with respect to SNR as $R \approx r\log\rho$, utilizing (7) in (5) results in

$$\begin{aligned} P_{out}^{DT} &\doteq \Pr\left(\frac{1}{2}\log(\gamma_{DT}) < r\log\rho\right) \quad (13)\\ &\doteq \Pr\left(h_{SD}^2 < \rho^{-(1-2r)}\right) \\ &\doteq \rho^{-\frac{\sigma_3}{2}(1-2r)}, \quad \text{for } 0 \le r < 1/2 \end{aligned}$$

where we have used Corollary 1 of Appendix in the last exponential equality. Hence, from (9), the optimal DMT of the direct transmission is given by

$$d_{DT}^*(r) = \frac{\sigma_3}{2}(1-2r), \quad 0 \le r < 1/2 \quad (14)$$

It is observed that the multiplexing gain in a point-to-point IM/DD FSO channel goes only up to $1/2$ instead of one. This is in contrast to the fact that the maximum multiplexing gain achieved in a point-to-point RF channel is one [11]. Such a different result is due to the square-law operation of the direct detection on the received optical power in IM/DD FSO channels.

C. Relay-Assisted Transmission (RT)

For parallel relaying, using (1)-(3), we obtain the instantaneous electrical SNR at the destination as

$$\gamma_D^{RT} = \begin{cases} h_{SD}^2\rho/9, & \gamma_R < \gamma_{th} \\ (h_{RD}^2 + h_{SD}^2)\rho/9, & \gamma_R \ge \gamma_{th} \end{cases} \quad (15)$$

where $\gamma_R = h_{SR}^2\rho/9$ is the instantaneous electrical SNR at the relay. The first condition in (15), i.e., $\gamma_R < \gamma_{th}$, indicates that the relay was not able to decode the source signal and therefore remained silent. On the other hand, $\gamma_R \ge \gamma_{th}$ means that the relay successfully decoded the source signal and retransmitted it to the destination. The outage probability can be therefore expressed as

$$\begin{aligned} P_{out}^{RT} &= \Pr(\text{relay cannot decode}) \quad (16)\\ &\quad\times \Pr(\text{outage}\,|\text{relay cannot decode}) \\ &\quad+ \Pr(\text{relay decodes}) \times \Pr(\text{outage}\,|\text{relay decodes}) \\ &= \Pr(v < 9\gamma_{th}/\rho) \times \Pr(u < 9\gamma_{th}/\rho) \\ &\quad+ \Pr(v \ge 9\gamma_{th}/\rho) \times \Pr(q < 9\gamma_{th}/\rho), \end{aligned}$$

where $u = h_{SD}^2$, $v = h_{SR}^2$, $w = h_{RD}^2$, and $q = u + w$ were defined for simplicity in notation. Derivation of a closed form expression for P_{out}^{RT} is very difficult, if not impossible. However, an asymptotic approximation can be obtained as follows.

Theorem 1: The outage probability of relay-assisted FSO transmission at high-SNR regime can be approximated as $P_{out}^{RT} \approx (O_{RT}\cdot\rho)^{-d_{RT}}$ where the diversity gain (d_{RT}) and the coding gain (O_{RT}) are respectively given by

$$d_{RT} = (\sigma_n + \sigma_3)/2, \quad (17)$$

$$O_{RT} = \begin{cases} O_n & \sigma_1 \ne \sigma_2 \\ \left(O_1^{-d_{RT}} + O_2^{-d_{RT}}\right)^{-1/d_{RT}} & \sigma_1 = \sigma_2 \end{cases}, \quad (18)$$

with $\sigma_i = \min\{\alpha_i, \beta_i\}$ $\forall i \in \{1,2,3\}$, and $n = \arg\min_{i\in\{1,2\}}\{\sigma_i\}$. In (18), O_1 and O_2 are respectively given by

$$\begin{aligned} O_1 &= \frac{1}{9\gamma_{th}} \quad (19)\\ &\times\left(\frac{\Gamma(\lambda_1-\sigma_1)(\lambda_1\sigma_1/l_1)^{\sigma_1}}{\Gamma(\lambda_1)\Gamma(\sigma_1+1)}\frac{\Gamma(\lambda_3-\sigma_3)(\lambda_3\sigma_3)^{\sigma_3}}{\Gamma(\lambda_3)\Gamma(\sigma_3+1)}\right)^{-\frac{2}{\sigma_1+\sigma_3}}, \end{aligned}$$

$$\begin{aligned} O_2 &= \frac{1}{9\gamma_{th}}\left(\frac{B(\sigma_2/2, \sigma_3/2)}{2(\sigma_2+\sigma_3)}\right)^{-\frac{2}{\sigma_2+\sigma_3}} \quad (20)\\ &\times\left(\frac{\Gamma(\lambda_2-\sigma_2)(\lambda_2\sigma_2/l_2)^{\sigma_2}}{\Gamma(\lambda_2)\Gamma(\sigma_2)}\frac{\Gamma(\lambda_3-\sigma_3)(\lambda_3\sigma_3)^{\sigma_3}}{\Gamma(\lambda_3)\Gamma(\sigma_3)}\right)^{-\frac{2}{\sigma_2+\sigma_3}}, \end{aligned}$$

where $\lambda_i = \max\{\alpha_i, \beta_i\}$ $\forall i \in \{1,2,3\}$, and $B(x,y)$ is the Beta function [12].

Proof: See Appendix.

Using Corollary 1 of Appendix, it can be shown that the diversity orders of S-R and R-D channels are respectively given by $\sigma_1/2$ and $\sigma_2/2$. Hence, comparing (17) with (11), we observe that relay-assisted transmission increases the diversity gain of direct transmission as much as the minimum of the diversities of S-R and R-D channels.

In addition, as observed in (19), O_1 is just a function of S-R and S-D links' parameters, and is independent of the R-D link's parameters. Hence, if $n=1$, i.e., $\sigma_1 < \sigma_2$, which means that S-R link has lower diversity order than R-D link, we have $P_{out}^{RT} \approx (O_1\cdot\rho)^{-(\sigma_1+\sigma_3)/2}$ and therefore the asymptotic outage probability is independent of R-D link's characteristics.

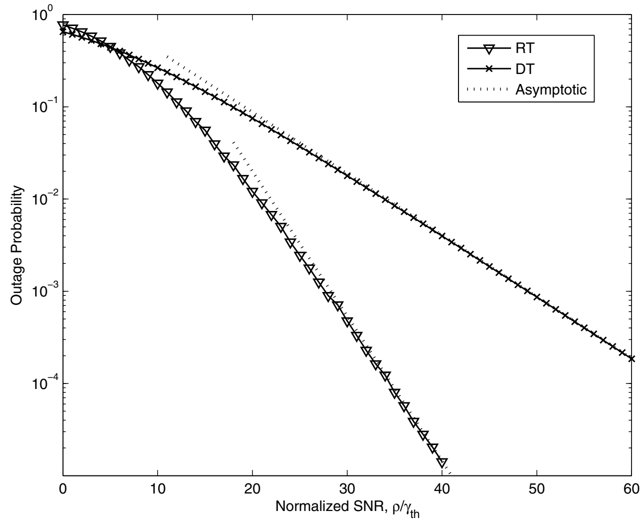

Fig. 2. Outage probability versus normalized SNR.

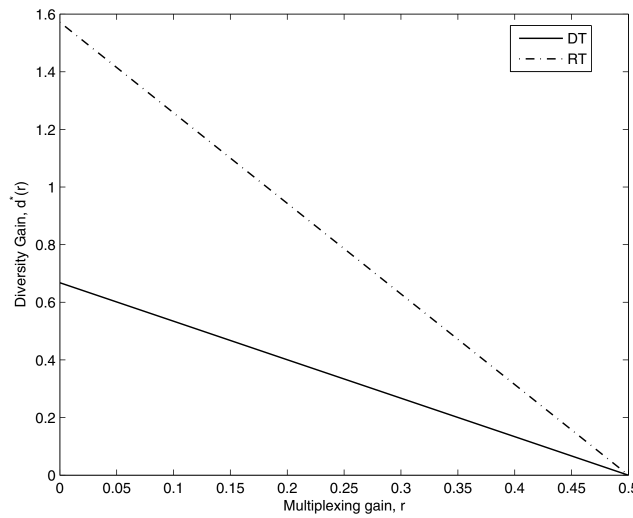

Fig. 3. DMT curves for DT and RT.

Similarly, if $n = 2$, i.e., R-D link has lower diversity order than S-R link, we have $P_{out}^{RT} \approx (O_2 \cdot \rho)^{-(\sigma_2+\sigma_3)/2}$ and therefore the asymptotic outage probability is independent of S-R link's characteristics. Consequently, the outage performance at high-SNR regime is dominated by either S-R or R-D link that has the worse diversity gain. Note that if S-R and R-D channels have the same diversity order ($\sigma_1 = \sigma_2$), both channels affect the high-SNR outage performance.

When the transmission rate R increases with respect to SNR as $R \approx r \log \rho$, using (5), (7), and (15), we have

$$\begin{aligned} P_{out}^{RT} \doteq & \Pr\left(h_{SD}^2 < \rho^{-(1-2r)}\right) . \Pr\left(h_{SR}^2 < \rho^{-(1-2r)}\right) \\ & + \Pr\left(h_{SD}^2 + h_{RD}^2 < \rho^{-(1-2r)}\right) . \Pr\left(h_{SR}^2 \geq \rho^{-(1-2r)}\right). \end{aligned} \tag{21}$$

Consequently, for $0 \leq r < 1/2$, P_{out}^{RT} at high-SNR regime satisfies

$$P_{out}^{RT} \doteq \rho^{-\frac{1}{2}(\sigma_n+\sigma_3)(1-2r)}, \tag{22}$$

where we have used the results of Appendix. Hence, the optimal DMT of the cooperative system is given by

$$d_{RT}^*(r) = \frac{1}{2}\left(\sigma_n + \sigma_3\right)\left(1 - 2r\right), \quad 0 \leq r < 1/2. \tag{23}$$

It is observed that since the underlying cooperative system is full-duplex, the maximum multiplexing gain of $1/2$ (see (14)) is achieved.

IV. Numerical Results

In this section, we provide numerical results for an FSO system with wavelength $\lambda = 1.55\,\mu$m. The lengths of S-R, R-D, and S-D links are respectively given by $L_{SR} = 1.5$ km, $L_{RD} =$ 2km, and $L_{SD} =$ 3km. We assume plane-wave propagation in turbulence conditions with structure constant of $C_n^2 = 2.5 \times 10^{-14}\mathrm{m}^{-2/3}$ and attenuation of 0.44 dB/km [8].

Fig. 2 depicts the outage probabilities of RT and DT schemes versus normalized average SNR (ρ/γ_{th}). Both exact outage probability and derived asymptotic approximation are displayed. The exact one for RT is obtained from (16) via Monte Carlo simulation. The diversity gain of RT (i.e., the slope of the outage probability curve) is 1.57 confirming the observation of $(\sigma_2 + \sigma_3)/2$ from (17). This is a significant increase over the diversity gain of $\sigma_3/2 = 0.67$ obtained from direct transmission. For example, for a targeted outage probability of 10^{-3}, cooperation brings 21 dB energy saving compared to direct transmission. We note that, in our specific simulation scenario, we have $\sigma_2 < \sigma_1$, therefore the asymptotic outage performance of RT becomes independent of S-R link.

Fig. 3 demonstrates the DMT curves of direct and cooperative transmission schemes. It is observed that RT improves the diversity gain throughout the range of the multiplexing gain. As mentioned earlier, due to the square-law operation of the direct detection on the received optical power, the multiplexing gain goes only up to $1/2$. The maximum achievable diversity gains of the direct and cooperative schemes are obtained at $r = 0$ which are obviously 0.67 and 1.57, respectively.

V. Conclusion

In this paper, we have investigated cooperative FSO transmission. Particularly, we have derived outage probability and DMT expressions for relay-assisted IM/DD FSO communications over Gamma-Gamma fading channels. Our results show that the diversity gain of relay-assisted transmission improves as much as the minimum of the diversities of S-R and R-D channels compared to the direct transmission. In addition, the outage performance at high-SNR regime is dominated by either S-R or R-D link that has the worse diversity gain.

Appendix

In this appendix, we prove Theorem 1. First, we provide a lemma and corollary that will be used in the proof.

Lemma 1: Let I be a random variable with $\Gamma\Gamma^2(\alpha,\beta,\mu)$ distribution and the pdf $f_I(I)$. Assuming [2] that $\alpha-\beta\notin\mathbb{Z}$, where $\mathbb{Z}$ denotes the set of integers [13], the Laplace transform of $f_I(I)$ satisfies

$$F_I(s)=\frac{a(\sigma,\lambda,\mu)}{s^{\sigma/2}}+o\left((1/s)^{\sigma/2}\right), \tag{24}$$

where $\sigma=\min\{\alpha,\beta\}$, $\lambda=\max\{\alpha,\beta\}$, and

$$a(x,y,z)=\frac{\Gamma(y-x)\Gamma(x/2)}{2\Gamma(y)\Gamma(x)}(xy/z)^x. \tag{25}$$

Proof: See journal version of this paper [14].

Corollary 1: The pdf of the random variable $I\sim\Gamma\Gamma^2(\alpha,\beta,\mu)$ can be approximated by a single polynomial term for small values of I ($I\to 0$), i.e.,

$$f_I(I)=\frac{a(\sigma,\lambda,\mu)}{\Gamma(\sigma/2)}I^{\sigma/2-1}+o\left(I^{\sigma/2-1}\right). \tag{26}$$

This corollary can be simply proved by taking the inverse Laplace transform of $F_I(s)$ in (24).

Noting that $v\sim\Gamma\Gamma^2(\alpha_1,\beta_1,l_1)$, $u\sim\Gamma\Gamma^2(\alpha_3,\beta_3,1)$, and using Corollary 1, we obtain

$$\Pr(v<9\gamma_{th}/\rho)\approx\frac{a(\sigma_1,\lambda_1,l_1)}{\Gamma(\sigma_1/2+1)}\left(9\gamma_{th}/\rho\right)^{\sigma_1/2}. \tag{27}$$

$$\Pr(u<9\gamma_{th}/\rho)\approx\frac{a(\sigma_3,\lambda_3,1)}{\Gamma(\sigma_3/2+1)}\left(9\gamma_{th}/\rho\right)^{\sigma_3/2}. \tag{28}$$

To derive the high SNR approximation of $\Pr(q<9\gamma_{th}/\rho)$, we should find the smallest exponent of q in the power series expansion of $f_q(q)$ (i.e. the pdf of q). To obtain this term, we first evaluate the Laplace transform of $f_q(q)$ as

$$F_q(s)=\int_0^\infty \exp(-sq)f_q(q)dq=F_u(s)\cdot F_w(s), \tag{29}$$

where $F_u(s)$ and $F_w(s)$ are the Laplace transforms of $f_u(u)$ (pdf of u) and $f_w(w)$ (pdf of w), respectively. Using Lemma 1, we can replace $F_u(s)$ and $F_w(s)$ by their polynomial expansions, which yields

$$F_q(s)=\frac{a(\sigma_3,\lambda_3,1)a(\sigma_2,\lambda_2,l_2)}{s^{(\sigma_2+\sigma_3)/2}}+o\left((1/s)^{(\sigma_2+\sigma_3)/2}\right). \tag{30}$$

By taking the inverse Laplace transform of (30), $f_q(q)$ can be written as

$$f_q(q)=\frac{a(\sigma_3,\lambda_3,1)a(\sigma_2,\lambda_2,l_2)}{\Gamma\left((\sigma_2+\sigma_3)/2\right)}q^{\frac{(\sigma_2+\sigma_3)}{2}-1}+o\left(q^{\frac{(\sigma_2+\sigma_3)}{2}-1}\right). \tag{31}$$

Consequently, for large values of ρ, $\Pr(q<9\gamma_{th}/\rho)$ satisfies

$$\Pr(q<9\gamma_{th}/\rho)\approx\frac{a(\sigma_3,\lambda_3,1)a(\sigma_2,\lambda_2,l_2)}{\Gamma\left((\sigma_2+\sigma_3)/2+1\right)}\left(9\gamma_{th}/\rho\right)^{(\sigma_2+\sigma_3)/2}. \tag{32}$$

Substituting $a(x,y,z)$ from (25) in (27), (28), and (32), and noting that $\Pr(v\geq 9\gamma_{th}/\rho)\to 1$ as $\rho\to\infty$, the theorem is proved.

[2]This assumption is valid for most practical channels under consideration.

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