Fast Baseband Polynomial Inverse Algorithm for Nonlinear System Compensation

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***Abstract*— Digital predistortion based on nonlinear system modeling and consecutively inverse is an important technique for linearization of radio frequency power amplifiers (PAs). Compensation for the nonlinear systems represented by polynomials involves polynomial inverses. In this paper, we propose a fast algorithm to invert a given baseband memoryless polynomial based on the predefined orthogonal basis. The orthogonal basis is predefined based on the distribution of the signal, which is known *a priori*. Compared with the well-established *p*th-order inverse method, the proposed algorithm can find the optimal coefficients of the inverse polynomial. Both numerical simulation and experiment with an actual PA demonstrate that the proposed algorithm can provide very good performance for compensating the nonlinear systems represented by baseband polynomial.**

Keywords- nonliner system, baseband polynomial, digital predistortion, polynomial inverses

I. INTRODUCTION

In the recent years, many research works have been carried out on identification, behavioral modeling and compensation of nonlinear systems. Especially, nonlinear system modeling and compensation of the radio frequency (RF) power amplifiers (PAs) is an important subject from the viewpoint of practical use. As an effective technique to compensate for the nonlinear distortion in PAs, digital predistortion (DPD) attracts much attention. It always involves inverse modeling of the PA, whether directly or indirectly. Most of the predistortion architectures for PA linearization are based on indirect learning which is more flexible and robust than the direct learning architecture [1]. As noticed by some researchers, however, this method suffers from noisy output of PAs [2]. They suggest more complicated architectures to solve this problem.

In [2] and [3], the PA is first modeled using a polynomial model and the predistortion function is calculated based on the inverse of the model. Such scheme is shown in Fig.1, which is also known as “PA modeling and inverse”. In their methods, both modeling and inverse estimation employs adaptive algorithm or least square fitting which are based on the measured data of the PA characteristics, resulting in heavy computation and long executing time. In order to reduce the computation load, more efficient methods for inverting the polynomial model need to be developed.

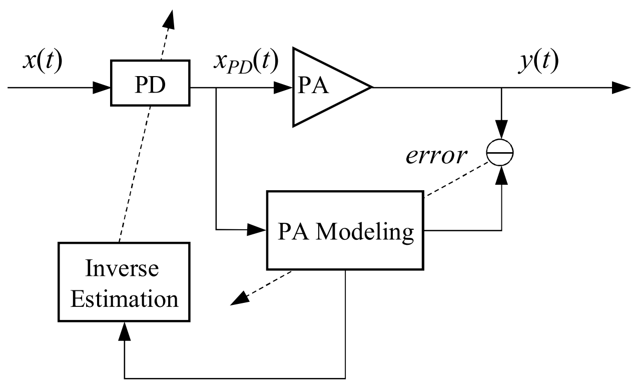

Fig. 1. Digital predistortion based on direct learning

There are several techniques for inverting the polynomials. Schetzen [4] proposed a “*p*th-order inverse” method to invert the Volterra series which is known as the most general function for representing a weak nonlinear system with memory. In order to increase the speed and decrease the complexity for constructing the Volterra kernels, Sarti et al. [5] proposed the recursive technology for the *p*th-order inverse. The limitation of the *p*th-order inverse is that it can only remove the nonlinear distortion introduced by the first *p*th-order term and leaves residual higher order distortion. It results in ineffective inverses in some circumstances. Tsimbinos [6] derived the method to invert the power series based on Chebyshev polynomial and Hermite polynomial, which provides better performance than *p*th-order inverse. Tsimbinos's method, however, is difficult to be extended to baseband, and is restricted to sine signal and the signal with Gaussian distribution.

In this paper, we propose a fast algorithm that can find the optimum inverse of a given baseband polynomial when the distribution of the signal is known *a priori*. The appealing hallmark of our approach, compared with the aforementioned methods, is that the higher order distortion that is to be suppressed is tractable with a tunable truncation factor. This results in high flexibility in terms of practical implementation.

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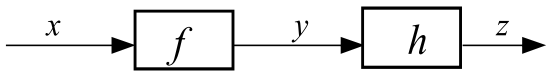

Fig. 2 Composition of two polynomials in tandem

II. BASEBAND POLYNOMIAL INVERSE

A. Composition of two baseband polynomials in tandem

We consider two cascaded circuits or systems [6]. One of them is the target system with undesired nonlinearity and the other system is added to compensate the nonlinearity of the former one. Suppose that the input / output relations for the systems can be expressed by their own polynomials.

Given two tandem baseband polynomials as depicted in Fig.2, and the first polynomial is given as

$$y = f(x) = \sum_{i=1}^{N} a_i \,|x|^{i-1}\, x \tag{1}$$

which is the baseband expression of the power series in passband, where even order terms are deliberately added [7]. x is complex signal (complex envelop) in baseband, and $|x|$ is the absolute value (magnitude) of x. This equation can also be expressed by a more compact form of y=$\mathbf{XA}$, where $\mathbf{X}$=[x, $|x|x$,... , $|x|^{N-1}x$] and $\mathbf{A}$=[a_1 a_2 ... a_N]^T.

It is followed by another Mth order polynomial h

$$z = h(y) = \sum_{i=1}^{M} b_i \,|y|^{i-1}\, y \tag{2}$$

Substituting (1) in (2) results in

$$z = \sum_{j=1}^{M} b_j \left| \sum_{i=1}^{N} a_i \,|x|^{i-1}\, x \right|^{j-1} \sum_{i=1}^{N} a_i \,|x|^{i-1}\, x \tag{3}$$

which is heuristic, since it reveals the fact that composition of the two polynomials of M and N orders, respectively, produces a polynomial of MNth order. Therefore, a polynomial cannot totally remove the distortion caused by another polynomial.

The mth-order monomial in (2) is

$$\left| \sum_{i=1}^{N} a_i \,|x|^{i-1}\, x \right|^{m-1} \sum_{i=1}^{N} a_i \,|x|^{i-1}\, x \tag{4}$$

whose expanded coefficients are

$$\mathbf{A}^{(m)} = \begin{cases} \underbrace{\mathbf{A} * \mathbf{A}^* * \cdots \mathbf{A} * \mathbf{A}^*}_{m-1 \ \ terms} * \mathbf{A} & m \text{ is odd} \\ \underbrace{\mathbf{A} * \mathbf{A}^* * \cdots \mathbf{A} * \mathbf{A}^*}_{m-2 \ \ terms} * \mathbf{A}_{abs} * \mathbf{A} & m \text{ is even} \end{cases} \tag{5}$$

where

$$\mathbf{A}_{abs} = \frac{\mathbf{A}^* a_1 + \mathbf{A} a_1^*}{2|a_1|} \tag{6}$$

and $\mathbf{A}^*$ denotes the conjugation of $\mathbf{A}$. Note that the even-order and odd-order terms are handled separately in (5), because the even order terms cannot be expanded explicitly and (6) is an approximation.

Composition of polynomials is straightforward in the light of matrix notation. We write the polynomial of (1) as y=$\mathbf{XA}$ and (2) as, in the similar way, z=$\mathbf{YB}$. We stack the coefficients for (5) in a vector and zeros are paddled for alignment

$$\mathbf{p}_m = [\underbrace{0,0,...,0}_{m-1 \ \ zeros}, (\mathbf{A}^{(m)})^T]^T \tag{7}$$

which is a column vector with length of mN+m-2. Arraying them into a matrix and it gives

$$\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_3, \cdots, \mathbf{p}_M] \tag{8}$$

which is a (MN+M-2) $\times$ M matrix. Then the composite polynomial in (3) can be expressed as

$$z = \mathbf{X}_p \mathbf{P}_{(MN \times M)} \mathbf{B} \tag{9}$$

where $\mathbf{X}_p$=[x, x^2, ... , x^{MN}], and $\mathbf{P}$ is truncated to $MN \times M$, because there are only MN terms in the composite polynomial.

B. Orthogonal Basis

In order to derive the optimum inverses, it is necessary to simultaneously suppress all the nonlinear terms in the composite polynomial. However, since the functionals in a polynomial are correlated to each other, it's natural to introduce *a priori* information to rebuild the polynomial structure, whereby each functional of the polynomial are uncorrelated.

If the signal is wide sense stationary, and its distribution is known *a priori*, then we can find a set of coefficients to orthogonalize the polynomial. Define the covariance matrix

$$\mathbf{R} = E[\mathbf{X}^H \mathbf{X}] \tag{10}$$

where the superscript ^{H} denotes complex conjugate transpose and E[ ] is the expectation value. It can be transformed to a unit matrix $\mathbf{I}$, if a transform matrix is introduced,

$$E[(\mathbf{XU})^H \mathbf{XU}] = \mathbf{I} \tag{11}$$

The orthogonal transform matrix $\mathbf{U}$ recasts the polynomial basis $\mathbf{X}$ in a space that all the subspaces are orthogonal to each other, and we can enjoy a lot of merits from the orthogonal basis.

A method to calculate the transform matrix $\mathbf{U}$ for a baseband polynomial is introduced in [8]. However, the method in [8] involves an overhead of solving of multivariate quadratic equations, which is undesirable in many respects, such as high computational complexity and poor numeric stability. A simple and robust way to synthesize $\mathbf{U}$ is the Gram-Schmidt procedure. A modified Gram-Schmidt procedure [9] can be numerically superior to the classic version.

With $\mathbf{U}$ in hand, the polynomial in (1) can be expressed by

$$y = \mathbf{X}_{orth} \mathbf{A}_{orth} = \mathbf{XUA}_{orth} \tag{12}$$

where $\mathbf{X}_{orth}$=$\mathbf{XU}$ contains the orthogonal polynomial bases, and $\mathbf{A}_{orth}$ is the corresponding coefficient vector with the same size of $\mathbf{A}$. Eq. (12) is, in essence, equivalent to (1), except that the bases in (1) are monomials. Since the distribution of the signal is known *a priori*, $\mathbf{U}$ can be seen as a constant parameter.

C. Polynomial Inverses

If (2) is the inverse of (1), the output of the (2) should be equal to the input of the (2), i.e., $z=x$. Compensating (1) with (2) amounts to linearizing the composite polynomial of (9), which yields

$$\mathbf{PB} = \mathbf{G} \tag{13}$$

where $\mathbf{G}=[1, 0, 0\ldots, 0]^T$ is a column vector with MN elements. Calculating the inverse polynomial coefficients needs to solve the variables $\mathbf{B}$ in (13). Note that there are M elements in $\mathbf{B}$, so $\mathbf{P}$ needs to be truncated for conformability.

If the coefficients of the polynomial $\mathbf{B}$ is solved as

$$\mathbf{B} = (\mathbf{P}_{(M\times M)})^{-1}\mathbf{G}_{(M\times 1)} \tag{14}$$

where $\mathbf{P}$ is truncated to $M\times M$, implying that the distortion components of Mth order and less are removed. Then, Eq. (14) is equivalent to the traditional pth-order inverse ($p=M$).

To find the optimal coefficient $\mathbf{B}$, we define the cost function which is the mean square error

$$\begin{aligned}\xi &= E[|x-z|^2] = E[|\mathbf{XG}-\mathbf{XPB}|^2] \\ &= E[\mathbf{B}^H\mathbf{P}^H\mathbf{X}^H\mathbf{XPB} + \mathbf{G}^H\mathbf{X}^H\mathbf{XG} - \\ &\quad \mathbf{G}^H\mathbf{X}^H\mathbf{XPB} - \mathbf{B}^H\mathbf{P}^H\mathbf{X}^H\mathbf{XG}]\end{aligned} \tag{15}$$

By applying the orthogonal transform to the composite polynomial, and if the transform matrix is already in hand, then (15) yields

$$\begin{aligned}\xi &= E[\mathbf{B}^H\mathbf{P}_{orth}{}^H(\mathbf{XU})^H\mathbf{XUP}_{orth}\mathbf{B} + \mathbf{G}^H(\mathbf{XU})^H\mathbf{XUG} \\ &- \mathbf{B}^H\mathbf{P}_{orth}{}^H(\mathbf{XU})^H\mathbf{XUG} - \mathbf{G}^H(\mathbf{XU})^H\mathbf{XUP}_{orth}\mathbf{B}]\end{aligned} \tag{16}$$

Note that

$$E[(\mathbf{XU})^H(\mathbf{XU})] = \mathbf{I} \tag{17}$$

is a unit matrix according to (11). Thus

$$\begin{aligned}\xi &= (\mathbf{B}^H\mathbf{P}_{orth}{}^H\mathbf{P}_{orth}\mathbf{B} + \mathbf{G}^H\mathbf{G} - \mathbf{B}^H\mathbf{P}_{orth}{}^H\mathbf{G} - \mathbf{G}^H\mathbf{P}_{orth}\mathbf{B}) \\ &= |\mathbf{P}_{orth}\mathbf{B}-\mathbf{G}|^2 = |\mathbf{U}^{-1}\mathbf{PB}-\mathbf{G}|^2\end{aligned} \tag{18}$$

which is signal-independent. The least square solution [10] for minimizing ξ is

$$\mathbf{B} = (\mathbf{\Omega}^H\mathbf{\Omega})^{-1}\mathbf{\Omega}^H\mathbf{G}_{(MN\times 1)} \tag{19}$$

where

$$\mathbf{\Omega} = (\mathbf{U}_{(MN\times MN)})^{-1}\mathbf{P}_{(MN\times M)} \tag{20}$$

In (20) all the distortion components are suppressed simultaneously because the composite coefficients matrix $\mathbf{P}$ contains all the MN terms.

We define a truncation factor K to further decrease the computation complexity, which yields

$$\mathbf{B} = (\mathbf{\Omega}^H\mathbf{\Omega})^{-1}\mathbf{\Omega}^H\mathbf{G}_{(K\times 1)} \tag{21}$$

where

$$\mathbf{\Omega} = (\mathbf{U}_{(K\times K)})^{-1}\mathbf{P}_{(K\times M)} \tag{22}$$

The truncation factor K, which is in the range of M~MN, denotes the maximum order of distortion that can be suppressed. Unlike the pth-order inverse which can only remove the first pth-order terms, the algorithm proposed in (21) and (22) can suppress any order distortion by adjusting K. Performing the proposed algorithm requires the transform matrix $\mathbf{U}$, hence the distribution of the signal should be known *a priori*. If this algorithm is applied in a real time circumstance, e.g. a digital predistortion, an offline procedure can be done to generate $\mathbf{U}$ before running the system.

III. Numeric Example

In order to clarify the proposed method, we compensate a nonlinear system in baseband as an example, which has the following polynomial

$$y = x - 0.1|x|x - 0.1|x|^2 x \tag{23}$$

It generates distortions of infinite orders because of the discontinuity nature of second order term $|x|x$.

Let x be a two-carrier wideband code division multiple access (WCDMA) signal with 10MHz frequency separation, and it has been normalized by dividing its peak amplitude for simplicity. 2000 samples with Rayleigh distribution were first exploited to calculate the transform matrix $\mathbf{U}$, and another 2^{14}=16384 WCDMA samples were generated to validate the proposed algorithm. The contents of $\mathbf{U}$ are given in Table 1.

We write the coefficient vector for (23) as $\mathbf{A}$= [1, -0.1, -0.1]^T, and there is no doubt that N=3. By applying (5), we have the vectors

$$\begin{aligned}\mathbf{A}^{(1)} &= [1\ \ -0.1\ \ -0.1]^T \\ \mathbf{A}^{(2)} &= [1\ \ -0.2\ \ -0.19\ \ 0.02\ \ 0.01]^T \\ \mathbf{A}^{(3)} &= [1\ \ -0.3\ \ -0.27\ \ 0.059\ \ 0.027\ \ 0.003\ \ 0.001]^T\end{aligned} \tag{24}$$

If we choose the truncation factor K=9, there yields $\mathbf{P}$ shown in (25), which is truncated to 9×3

$$\mathbf{P}_{(9\times 3)} = \begin{bmatrix} 1 & 0 & 0 \\ -0.1 & 1 & 0 \\ -0.1 & -0.2 & 1 \\ 0 & -0.19 & -0.3 \\ 0 & 0.02 & -0.27 \\ 0 & 0.01 & 0.059 \\ 0 & 0 & 0.027 \\ 0 & 0 & 0.003 \\ 0 & 0 & 0.001 \end{bmatrix} \tag{25}$$

By invoking (21) and (22) the coefficients of the post-inverse polynomial are calculated as

$$\mathbf{B} = [0.8072\ \ -0.0165\ \ 0.2364]^T \tag{26}$$

Therefore, the inverse polynomial is given by

$$z = 0.8074y - 0.0165|y|y + 0.2364|y|^2 y \tag{27}$$

The output spectrums are plotted in Fig. 3. Without compensation, the nonlinear system outputs a signal spectrum with out of band radiation level of around -40 dB in

TABLE I. ELEMENTS OF **U**

1.	-2.70785	5.52326	-9.64509	15.5213	-23.5103	35.1845	-50.2327	67.3251
0.	6.98841	-30.2630	83.7891	-190.050	380.561	-726.594	1292.62	-2105.10
0.	0.	36.3567	-211.728	757.004	-2127.32	5371.35	-12206.1	24579.0
0.	0.	0.	159.977	-1196.56	5291.68	-18781.6	56665.2	-144802
0.	0.	0.	0.	646.049	-5980.16	33476.2	-142816.	481700
0.	0.	0.	0.	0.	2493.32	-29293.	198567.	-941974
0.	0.	0.	0.	0.	0.	9947.93	-142726.	107067
0.	0.	0.	0.	0.	0.	0.	41297.9	-652991
0.	0.	0.	0.	0.	0.	0.	0.	164870

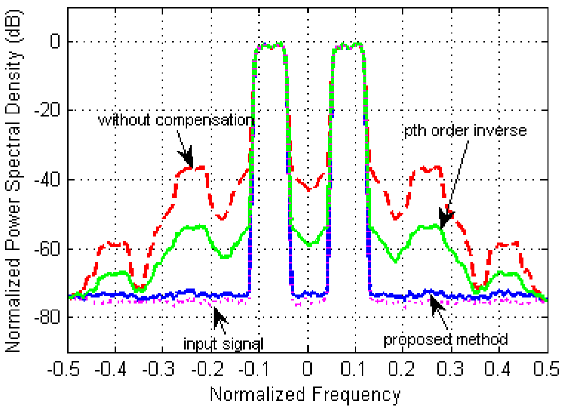

Fig. 3. Compensation with 3rd order polynomials. From up to down they are output spectra of nonlinear system, compensated by *p*th-order inverse, compensated by proposed method and input signal

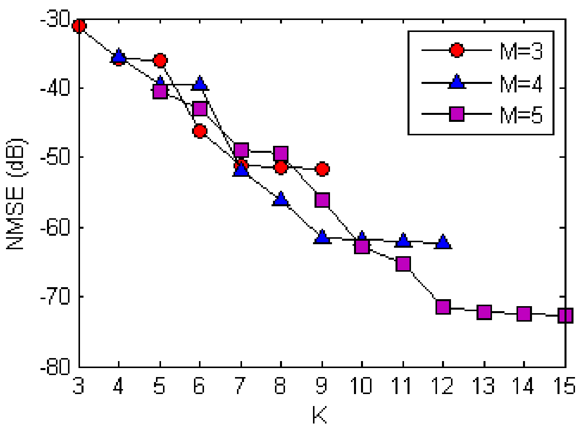

Fig. 4. Normalized mean square error for the nonlinear system compensated by 3rd, 4th and 5th order polynomials with different truncation factor.

normalized power spectrum density. It is remarkably suppressed with compensation by the proposed inverting method to the level of -70 dB or less, which shows that the proposed method can compensate the system with high accuracy. For comparison, we also apply *p*th-order inverse method with maximum 3rd order (M=3) to compensate for the same system. The result shows

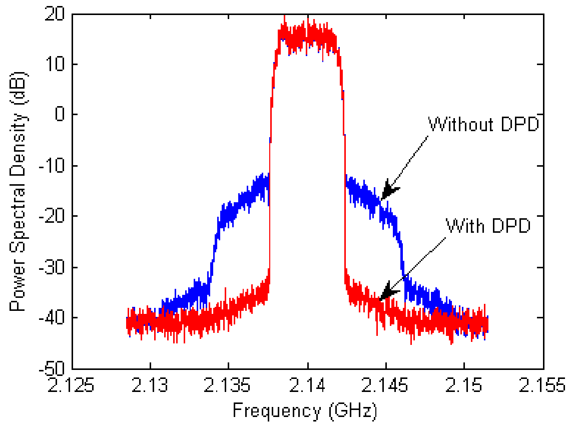

Fig. 5. The measured power spectra of the output of the PA.

inferior performance with out-of-band radiation level of around -52 dB.

In order to demonstrate the effects of truncation factor K, the normalized mean square errors (NMSE) are evaluated which is defined as

$$NMSE = \frac{mean\{(x-z)^2\}}{mean\{x^2\}} \tag{28}$$

For compensation of the same nonlinear system, 3rd, 4th and 5th order polynomials are calculated for different K. Their NMSE values are shown in Fig.4. It is not surprising that higher K and higher order polynomial allows lower NMSE. Note that when K exceeds a certain number, performance improvement will be saturated. It suggests that it is not necessary to set K to its upper limit.

IV. EXPERIMENTAL VALIDATION

An LDMOS class-AB power amplifier with 10W output power at 2.14 GHz was used to assess the proposed technique for actual PA linearization. This PA is excited by a WCDMA signal with bandwidth of 5 MHz and chip rate of 3.84 Mcps. The PAPR (peak-to-average power ratio) of the signal was reduced to 5.7dB utilizing clipping and filtering. SMJ 100A and FSP7, both from ROHDE & SCHWARZ, were used as vector signal generator (VSG) and vector signal analyzer (VSA), respectively. Since the maximum bandwidth of the resolution filter in FSP7 was

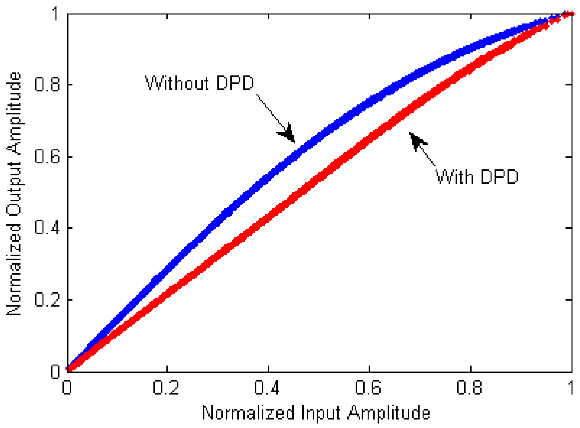

Fig. 6. The AM-AM plots of the PA under test

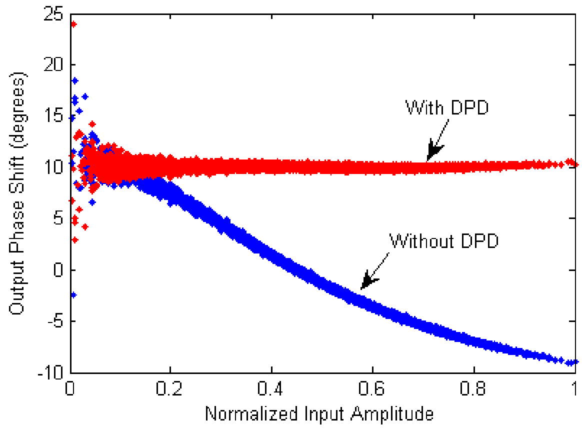

Fig. 7. The AM-PM plots of the PA under test

only 10 MHz, some post-processing techniques were necessary to extend the measurement bandwidth. Therefore a stitching method was applied to concatenate multiple measurements at different center frequencies [11]. Before 'frequency stitching', a multi-sine test was performed to characterize the linear distortion appearing in the instruments. Then the signals measured at different frequencies were equalized and stitched in frequency domain to extend the measurement bandwidth to 30.72MHz.

A 5th order baseband polynomial was assumed to be the PA model. 2000 samples of the PA output were collected by the VSA, and least square fitting was applied to extract the model's coefficients using the samples. The DPD has another 5th order polynomial, and its coefficients were calculated utilizing the proposed technique in which the truncation factor K was set to 15.

10000 predistorted samples were generated randomly, and fed to the PA. The measured power spectra output from the PA, both with and without the DPD, are illustrated in Fig. 5. The measured adjacent channel power ratio (ACPR) was reduced by around 22dB. The AM-AM and AM-PM plots with and without the DPD are presented in Fig. 6 and Fig. 7, respectively. They show that the nonlinearity of the PA has been successfully compensated by the proposed algorithm.

V. CONCLUSION

In this paper, we focus on the inverse of memoryless baseband polynomials and proposed a method based on orthogonal polynomials. Being different from traditional methods, our method can compensate higher order distortion caused by polynomial composition. It is shown from simulations and experiments that the proposed method can provide superior performance for nonlinear system compensation. Since this method requires no training signals, it is easy to be implemented with a digital signal processor (DSP), with much lower complexity than training-based methods. Therefore, our method is promising for digital baseband predistortion applications.

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REFERENCE

[1] L. Ding, G. T. Zhou, D. R. Morgan, Z. Ma, J. S. Kenney, J. Kim and C. R. Giardian, "Memory polynomial predistorter based on the indirect learning architecture," *Proc. IEEE GLOBECOM* '02, vol. 1 pp.967-971, 2002.

[2] Y. Qian and T. Yao, "Structure for adaptive predistortion suitable for efficient adaptive algorithm application," *Electron. Lett.*, vol. 38, pp. 1282–1283, Oct. 2002.

[3] D. Zhou and V. E. DeBrunner, "Novel adaptive nonlinear predistorters based on the direct learning algorithm" *IEEE Trans. Signal Process*. vol. 55, no.1 pp. 120–133, Jan. 2007

[4] M. Schetzen, "Theory of pth-order inverses of nonlinear systems," *IEEE Trans. Circuits Syst.*, vol. CAS-23, no. 5, pp.285-291, May. 1976.

[5] A. Sarti, S. Pupolin, "Recursive techniques for the synthesis of a pth-order inverse of a volterra system," *European Trans. Telecommun.* vol. 3, Issue 4, pp.315-322, Aug. 1991.

[6] J. Tsimbinos, K. V. Lever, "Nonlinear system compensation based on orthogonal polynomial inverses," *IEEE Trans. Circuits Syst*., vol. 48, no. 4, pp.406-417, April. 2001.

[7] G. T. Zhou, H. Qian, L. Ding, and R. Raich, "On the baseband representation of a bandpass nonlinearity," *IEEE. Trans. Signal Process.*, vol. 53, no. 8, pp. 2953-2957, Aug. 2005.

[8] S. Koike, "A set of orthogonal polynomials for use in approximation of nonlinearities in digital QAM systems," *IEICE Trans. Fundamentals*, vol. E86-A, no. 3, Mar. 2003

[9] A. Bjőrck, "Solving linear least squares problems by Gram-Schmidt orthogonalization," Nordisk Tidshrift for Informations behadlung, 7, 1-21, 1967.

[10] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory. pp.225, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[11] Y. Ma, S. He, Y. Akaiwa and Y. Yamao "An open-loop digital predistorter based on memory polynomial inverses for linearization of RF power amplifier," International Journal of RF and Microwave Computer-Aided Engineering, vol. 21, issue. 5, pp. 589–595, Sep. 2011.