Outage Analysis of Correlated Source Transmission in Block Rayleigh Fading Channels

Meng Cheng*, Khoirul Anwar* and Tad Matsumoto*,‡
*Japan Advanced Institute of Science and Technology (JAIST)
1-1 Asahidai, Nomi, Ishikawa, 923-1292 Japan
Email:{chengmeng, anwar-k, matumoto}@jaist.ac.jp
‡Center for Wireless Communications, FI-90014 University of Oulu, Finland
Email: tadashi.matsumoto@ee.oulu.fi

***Abstract*—A goal of this paper is to theoretically derive the outage probability of the correlated source transmission in block Rayleigh fading channels. The correlation between the two information sources is assumed to be expressed by the bit flipping model, where the information bits transmitted from the second transmitter are the $flipped$ version of the information bits transmitted from the first one, with a probability p_e. The source sequences are independently channel-encoded, and then transmitted to the destination block-by-block via different time- or frequency-slots. The channels are assumed to be suffering from independent block Rayleigh fading. This paper shows that the outage probability of this system can be expressed by double integrals with respect to the probability density functions (pdf) of the instantaneous signal-to-noise power ratios (SNRs) of those channels, where the range of the integration is determined by the Slepian-Wolf theorem. The most significant finding made by this paper is that the asymptotic diversity order is one so far as p_e is non-zero, and the 2nd order diversity can be achieved only if $p_e = 0$. The major applications of this paper's results include outage evaluation of extract-and-forward (EF) relay systems allowing intra-link (source-relay link) errors, sensor networks, and wireless mesh networks. The latter half of this paper provides results of outage probability calculations for one-way EF relay scenario utilizing the concept of the technique presented in this work.**

I. INTRODUCTION

The Slepian-Wolf theorem is well known as an efficient technique for the lossless compression of correlated sources. In the distributed source coding models, as shown in Fig. 1, each of the data streams is separately encoded, and the two encoded data streams are jointly decoded by a single decoder. According to the fundamental contribution of Slepian and Wolf in [1], it has been proven that by exploiting the correlation knowledge of data streams at the destination, the distributed source coding can achieve the same compression rate as the optimum single encoder which compresses the sources jointly.

A goal of this paper is to derive theoretical outage probability of the correlated source transmission in block Rayleigh fading channels. The source correlation can be assumed to be expressed by a bit flipping model [2], as $b_2 = b_1 \oplus e$ and $P(e = 1) = p_e$, where p_e is the bit flipping probability. The correlated information streams are separately channel-encoded for error protection, modulated and sent to a common receiver through the respective slot. This paper assumes the separation of the source-channel coding, supported by Shannon's separation theorem [3], and hence providing practical channel coding and modulation schemes is out of the scope.

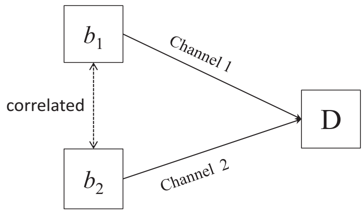

Fig. 1. Block diagram for Slepian-Wolf coding system: independent encoding of two correlated sources and joint decoding

In block fading channels, the channel gain changes, frame-by-frame. Therefore, given the channel coding and modulation schemes, sometimes the gain of either one of the two or the both channels are faded below the transmission chain requirement. This paper theoretically derives the outage probability of the correlated source transmission based on the assumption described above. It is shown in this paper that the outage probability can be expressed by double integrals with respect to the probability density function (pdf) of the instantaneous signal-to-noise ratio (SNR) of the channels, where the range of the integration is determined by the Slepian-Wolf theorem. The most significant finding of this paper is that the 2nd order diversity can be achieved only if $p_e = 0$. Otherwise, even though the decay of the outage probability curve is approximately equivalent to the 2nd order diversity with small average SNRs, it gradually changes and asymptotically the decay becomes equivalent to the case of no diversity. The latter half of this paper focuses on a one-way extract-and-forward (EF) relay as an application of the system investigated in the first half of this paper.

Unlike the conventional decode-and-forward (DF) relay system in [4], EF only extracts the information part from the received signal, re-encodes the erroneous data, and transmits the channel coded data to the destination. Performance comparison between the theoretical outage and the frame-error-rate (FER) of the proposed EF relay system using bit interleaved coded modulation with iterative detection (BICM-ID), is briefly discussed in this paper. Moreover, the impact of

978-1-4673-1881-5/12/$31.00 ©2012 IEEE

relay locations on the outage probability is also investigated. In the extreme case when $p_e = 0$, the outage probability of the proposed system is compared with that using maximum ratio combining (MRC) technique, where the signals from the two sources are first MRC-combined, and then the processing specified by the transmission chain is performed.

This paper is organized as follows. First of all, the system model based on the Slepian-Wolf theorem is briefly introduced in Section II. The outage probability is then derived in Section III. Furthermore, the proposed EF relay system is detailed in IV. Finally, we present the numerical results and provide performance analysis in Section V. The conclusion is given in Section VI.

II. System Model

The system model assumed in this paper is shown in Fig. 1, where b_1 is the source information bit to be transmitted from the first transmitter, and the correlated bit stream b_2 is to be transmitted from the second one. b_1 and b_2 are separately encoded and transmitted to a common receiver in different phases. According to the Slepian-Wolf theorem, the admissible rate region is constituted as an unbounded polygon, comprised of Parts 3, 4, 5 and 6 shown in Fig. 2. The error-free transmission can be guaranteed only if the rate pair falls into this area. For instance, if b_1 is transmitted at the rate R_1 which is equal to its entropy $H(b_1)$, then b_2 can be transmitted at the rate R_2 which is less than its entropy $H(b_2)$, but must be greater than their conditional entropy $H(b_2 \mid b_1)$, or vice versa. Specifically, R_1 and R_2 should satisfy three equations [1]:

$$R_1 \geqslant H(b_1 \mid b_2), \tag{1}$$

$$R_2 \geqslant H(b_2 \mid b_1), \tag{2}$$

$$R_1 + R_2 \geqslant H(b_1, b_2). \tag{3}$$

where $H(b_1, b_2)$ denotes the joint entropy of the correlated bit streams b_1 and b_2. For the binary symmetric sources ($P(1) = P(0) = 0.5$) adopted in our paper, we have $H(b_1) = H(b_2) = 1$, $H(b_1 \mid b_2) = H(b_2 \mid b_1) = H(p_e)$, $H(b_1, b_2) = 1 + H(p_e)$ with $H(p_e) = -p_e \log_2(p_e) - (1-p_e)\log_2(1-p_e)$. The threshold SNR can be expressed as

$$\gamma[H] = 2^{R_c \cdot H} - 1. \tag{4}$$

where R_c represents the rate taking into account of the channel coding scheme and the modulation format, which is assumed to be identical for both the first and second transmitters.

III. Outage Derivation

As shown in Fig. 2, the entire Slepian-Wolf rate region considering two correlated sources can be divided into 7 parts. It is well known that the admissible Slepian-Wolf rate region is the sum of Parts 3, 4, 5 and 6. However, in this paper, Part 7 also has to be included as the admissible region, because the decoder at the receiver only aims to retrieve b_1, such as in the relay systems, where b_2 is regarded as the recovered

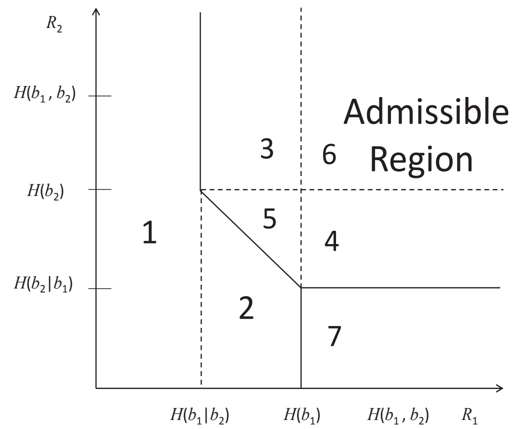

Fig. 2. Admissible Slepian-Wolf rate region

information bits containing some errors. In other word, Part 1 and 2 represent the inadmissible rate region as shown in Fig. 2. By using equation Eq. (4), the entropy values can be transformed to the SNR domain, which helps to constitute the admissible SNR region. Basically, when the instantaneous SNR pair γ_1 and γ_2 fall outside the admissible region, the outage event will happen and the error-free communication can not be guaranteed even with arbitrary transmission rate. Consequently, the definition of the outage probability of the Slepian-Wolf transmission system is given by

$$p_{o,sw} = P_1 + P_2, \tag{5}$$

where P_1 and P_2 are the probabilities that the instantaneous SNR pair γ_1 and γ_2 fall into the inadmissible areas 1 and 2, respectively. Mathematically, P_1 is defined as

$$P_1 = \int_{\gamma_1=\gamma[0]}^{\gamma[H(b_1|b_2)]} \int_{\gamma_2=\gamma[0]}^{\gamma[\infty]} p(\gamma_1, \gamma_2)\, d\gamma_1 d\gamma_2, \tag{6}$$

where $p(\gamma_1, \gamma_2)$ are the joint *pdf* of γ_1 and γ_2 as the instantaneous SNRs of the two channels, with the assumption that γ_1 and γ_2 are statistically independent. With $p(\gamma_1)$ and $p(\gamma_2)$ denoting the *pdf* of the instantaneous SNRs of the independent channels 1 and 2, respectively, Eq. (6) can be expressed as

$$P_1 = \int_{\gamma_1=\gamma[0]}^{\gamma[H(b_1|b_2)]} p(\gamma_1) d\gamma_1 \int_{\gamma_2=\gamma[0]}^{\gamma[\infty]} p(\gamma_2) d\gamma_2. \tag{7}$$

Moreover, due to the fact that the integration of $p(\gamma_2)$ from 0 to infinity equals to 1, Eq. (7) can be reduced as

$$\begin{aligned} P_1 &= \int_{\gamma_1=\gamma[0]}^{\gamma[H(b_1|b_2)]} p(\gamma_1)\, d\gamma_1 \\ &= \int_{\gamma_1=\gamma[0]}^{\gamma[H(b_1|b_2)]} \frac{1}{\Gamma_1} \exp(-\frac{\gamma_1}{\Gamma_1}) d\gamma_1 \\ &= 1 - \exp\left(-\frac{2^{R_c H(p_e)} - 1}{\Gamma_1}\right). \end{aligned} \tag{8}$$

Similarly, P_2 can be defined as follows

$$P_2 = \int_{\gamma_1=\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \int_{\gamma_2=\gamma[0]}^{\gamma[H(b_1,b_2)-H(\gamma_1)]} p(\gamma_1,\gamma_2)\, d\gamma_1 d\gamma_2, \tag{9}$$

where $H(\gamma_1) = \frac{1}{R_c}\log_2(1+\gamma_1)$, which transforms the instantaneous SNR to its corresponding entropy as an inverse of Eq. (4). Then P_2 can be expressed as

$$\begin{aligned} P_2 = & \int_{\gamma_1=\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} p(\gamma_1)\, d\gamma_1 \\ & \cdot \left[-\exp\left(-\frac{\gamma_2}{\Gamma_2}\right)\right]_{\gamma_2=0}^{2^{R_c[H(b_2,b_1)-\frac{1}{R_c}\log_2(1+\gamma_1)]}-1} \\ = & \int_{\gamma_1=\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \frac{1}{\Gamma_1}\exp(-\frac{\gamma_1}{\Gamma_1}) d\gamma_1 \\ & \cdot \left[1-\exp\left(-\frac{2^{R_c[H(b_2,b_1)-\frac{1}{R_c}\log_2(1+\gamma_1)]}-1}{\Gamma_2}\right)\right] \\ = & \frac{1}{\Gamma_1}\int_{\gamma_1=\gamma[H(b_1|b_2)]}^{\gamma[H(b_1)]} \left\{\exp\left(-\frac{\gamma_1}{\Gamma_1}\right)\right. \\ & \left. -\exp\left(-\frac{\gamma_1}{\Gamma_1}-\frac{2^{R_c[H(b_2,b_1)-\frac{1}{R_c}\log_2(1+\gamma_1)]}-1}{\Gamma_2}\right)\right\} d\gamma_1. \end{aligned} \tag{10}$$

To calculate Eq. (10) with respect to γ_1, the trapezoidal numerical integration method is used, with high enough accuracy. For comparison, the outage probability of the MRC scheme is also presented below, assuming that $p_e = 0$ and the average SNRs of the channels are equal ($\Gamma = \Gamma_1 = \Gamma_2$). The output of the MRC combiner is a weighted sum of all branches, and the *pdf* of the instantaneous SNR after MRC-combining in the block Rayleigh fading channel can be defined as [5]

$$p_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1}\exp\left(-\frac{\gamma}{\Gamma}\right)}{\Gamma^M (M-1)!}, \tag{11}$$

where M represents the diversity order. The outage probability of MRC is defined as the probability that the instantaneous SNR after combining is less than a given threshold (it is set at $\gamma[H(b_1)]$). Consequently, the outage probability of the MRC scheme with diversity M can be expressed as follows:

$$\begin{aligned} P_{o,\mathrm{MRC}} = & \int_{\gamma=0}^{\gamma[H(b_1)]} p_{\gamma_\Sigma}(\gamma) d\gamma_1 \\ = & \int_{\gamma=0}^{\gamma[H(b_1)]} \frac{\gamma^{M-1}\exp\left(-\frac{\gamma}{\Gamma}\right)}{\Gamma^M (M-1)!} d\gamma \\ = & 1-\exp(\frac{1-2^{R_c H(b_1)}}{\Gamma})\sum_{k=1}^{M}\frac{[(2^{R_c H(b_1)}-1)/\Gamma]^{k-1}}{(k-1)!}. \end{aligned} \tag{12}$$

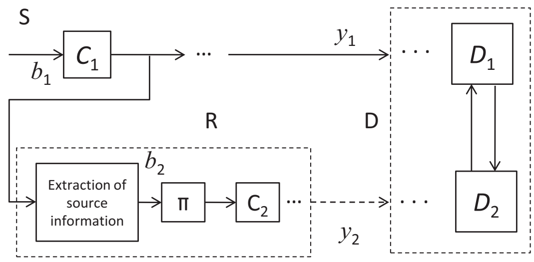

Fig. 3. Potential application schematic of the Slepian-Wolf relay system, where Π denotes the random interleaver

IV. Extract-and-Forward Relay System

This section applies the Slepian-Wolf theorem to a DF relay system, where the relay does not aim to perfectly recover the original information transmitted by the source, but it only "extracts" the source information, even though the relay knows that extracted sequence may contain some errors. In this sense, the proposed technique is referred to as "Extract-and-Forward" (EF) system. As shown in Fig. 3, the extracted sequence representing an estimate of the original information sequence, which is then interleaved and transmitted to the common destination. Obviously, the original and extracted sequences are correlated, where in this paper it is assumed that the errors caused in the source-relay (SR) channel can be expressed by the bit flipping model. This is reasonable because we assume block fading and no heavy decoding of the channel code is performed at the relay. Hence, we can apply the results of the previous sections when evaluating the outage probability of the proposed EF system.

As shown in Fig. 3, the original information stream b_1 at the source is broadcasted to both the relay and the destination using the first time slot. The channel gain G_1 of the source-destination (SD) link is normalized to 1, and the relay-destination (RD) link gain G_2, relative to G_1, is given by

$$G_2 = \left(\frac{d_1}{d_2}\right)^{\alpha}, \tag{13}$$

where d_1 and d_2 denote the distances from the source and the relay to the common destination node, respectively. Without losing the generality, d_1 is also normalized to 1. α represents the path loss exponent, which is set to 3.52 according to [6]. Therefore, the received signals from the source node (y_1) and the relay node (y_2) can be expressed as:

$$y_1 = \sqrt{G_1} h_1 s_1 + n_1, \tag{14}$$
$$y_2 = \sqrt{G_2} h_2 s_2 + n_2, \tag{15}$$

where s_1 and s_2 are the modulated symbols to be transmitted at the source and the relay. n_1 and n_2 denote the zero-mean additive white Gaussian noises (AWGN), having the same variance σ^2. h_1 and h_2 are the complex channel gains of the SD and RD channels, respectively, and are constant over each block because of the block fading assumption. Consequently, the instantaneous SNR $\gamma_i = G_i |h_i|^2 E_{s,i}/N_{0,i}$ $(i = 1, 2)$,

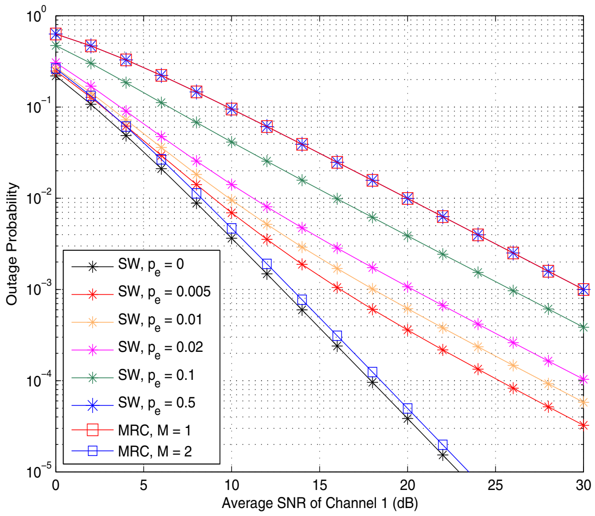

Fig. 4. Comparison of outage probabilities between the Slepian-Wolf system and the MRC scheme

where $E_{s,i}$ and $N_{0,i}$ represent the per-symbol signal power and the noise spectrum power density, both of the i-th channel, respectively. The pdf of γ_i is given by

$$p(\gamma_i) = \frac{1}{\Gamma_i}\exp(-\frac{\gamma_i}{\Gamma_i}), \tag{16}$$

where $\Gamma_i = G_i E_{s,i}/N_{0,i}$, denoting the normalized average SNR of the i-th channel. As designed before, we assume that the transmission channels 1 and 2 are statistically independent, and suffer from Rayleigh block fading.

Substituting Eq. (16) into Eq. (8) and Eq. (9), we obtain P_1 and P_2, respectively, for the proposed EF relay system.

V. Numerical Results

A. Same Distances of SD and RD

In this sub-section, the relay and source nodes are assumed to keep the same distance to the destination node $(d_1 = d_2)$, and therefore the average SNRs Γ_1 and Γ_2 are always equal for the both channels. Fig. 4 plots the theoretical outage probability of the Slepian-Wolf relay system with p_e value as a parameter. As a comparison, the outage performance of the MRC scheme with diversity $M = 1, 2$ is also presented assuming $p_e = 0$.

Note that when calculating the outage probability, the R_c value that taken into account of the channel coding and modulation scheme used in practice, was set at one as a parameter. This is because we assume source-channel separation for each channel, supported by Shannon's separation theorem [3]. However, when making the performance comparison assuming practical transmission chain, such as that provided in sub-section V-C, R_c should be set depending on the actual channel code rate and modulation format used in the system.

It is found from Fig. 4 that even though the EF system forwards the extracted systematic bits with some errors, the destination can recover the original information by utilizing the correlation knowledge. Hence, the outage probability with the proposed EF relay system is better than the conventional DF system where the relay does not forward the data when detecting errors $(p_e \neq 0)$. In fact, the line with the index MRC, $M = 1$ is equivalent to the line with the index SW, $p_e = 0.5$, which indicates the case when the two bit streams transmitted from the source and the relay are completely uncorrelated. As the p_e value decreases, the outage probability also decreases. Interestingly, the decay of the outage probability approximately follows the 2nd order diversity when the average SNR is low, but it gradually change to diversity order 1 as the average SNR increases. It can be obviously seen in Eq. (10) that P_2 goes towards 0 as Γ_1 and Γ_2 becomes large. In this case, the outage probability $P_{o,sw}$ is only dominated by P_1. It has been proven that the exponential function can be Maclaurin-expanded as:

$$\exp(-x) = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} \approx 1 - x, \tag{17}$$

With $x = \gamma\left[H(p_e)\right]/\Gamma_1 = (2^{R_c H(p_e)} - 1)/\Gamma_1$, P_1 can be estimated according to Eq. (8) as follows

$$P_1 = 1 - \exp(-x) \approx x. \tag{18}$$

Obviously, with Γ_1 being large values, the probability P_1 is proportional to x and hence the diversity order asymmetrically converges into 1.

The second order diversity can be achieved only if $p_e = 0$ over entire range of the average SNR. It is found from Fig. 4 that with $p_e = 0$, the outage probability of the proposed EF scheme is smaller than that with the MRC combining technique. However, the EF's superiority over MRC, shown in Fig. 4, is only the result of numerical calculation, and providing mathematical proof for the superiority of EF over MRC with $p_e = 0$ is still left as an open question.

B. Different Relay Locations

In our discussion so far, the source and relay are assumed to keep the same distance to the destination. While in this sub-section, we will further discuss the different cases by changing the relay's location of the Slepian-Wolf relay system. We consider the following 4 cases: (A), $d_1 = d_2$; (B), $d_1 = \frac{4}{3}d_2$; (C), $d_1 = 2d_2$; (D), $d_1 = 4d_2$.

Given the path loss parameter α equal to 3.52 [6], the average SNRs Γ_1 and Γ_2 of the channels 1 and 2 at each location scenario become as follows: $\Gamma_1 = \Gamma_2$ in the location A; $\Gamma_1 = \Gamma_2 + 4.4$ dB in location B; $\Gamma_1 = \Gamma_2 + 10.6$ dB in location C; $\Gamma_1 = \Gamma_2 + 21.19$ dB in location D.

The Fig. 5 shows the outage comparison for the different relay scenarios when $p_e = 0.005$, with the x-axis representing the average SNR of Channel 1. The differences of the decay can be found when the average SNRs take small values. More specifically, when the relay moves closer to the destination, the decay of the curve tends to be equivalent to no-diversity. This is because when the relay moves towards the destination,

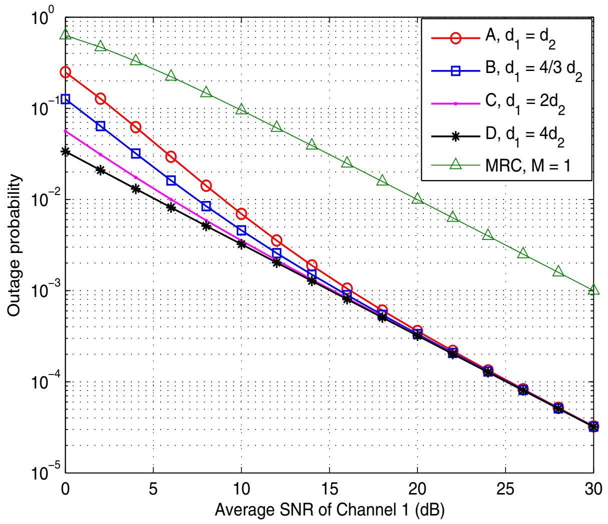

Fig. 5. Outage probability comparison at different locations, p_e = 0.005

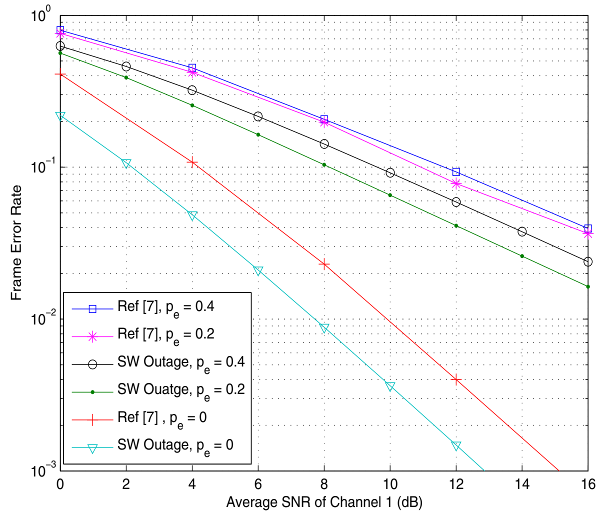

Fig. 6. Comparison between the theoretical outage probability and the FER of the practical application

Γ_2 increases significantly and P_2 rapidly becomes less dominating, and therefore for large SNR values, P_1 completely dominates the outage performance. The outage curves for different location scenarios finally overlap.

C. Practical Applications

In [7], an EF relay system using BICM-ID technique is presented as a practical application of the proposed system, where the correlation knowledge is utilized via the *vertical* iterations at the destination. Both of the source and relay are assumed to have the same distance towards to destination. Readers may look at [7] for more detailed about this technique. The FER performance of the system based on [7] is compared with the outage probability calculated using the method shown in this paper. The results are shown in Fig. 6 in terms of FER and outage probability versus average SNR of the SD channel, where $R_c = 1$ for the technique shown in [7]. The FER curve of the BICM-ID based EF relay system is roughly 2 dB away from the outage curve. This is because the extrinsic information transfer (EXIT) curve [8] of the channel code used in the BICM-ID is not exactly matched to the demapper EXIT curve, and hence there is a loss in information rate. The rate loss appears in the form of the 2 dB SNR loss from the theoretical outage.

VI. Conclusion

In this work, the outage probability of a correlated source transmission system based on the Slepian-Wolf theorem has been analyzed over the block Rayleigh fading channel. The bit-flipping model was used to express the correlation model between the two sources, and the outage probability was theoretically calculated assuming that the receiver aims to decode the information sequence transmitted from the first transmitter. We then applied the results of the outage probability analysis to an extract-and-forward (EF) strategy, where the relay only extracts the erroneous information sequence, interleavers, channel-encodes and transmits it to the destination. It has been shown that the proposed Slepian-Wolf EF technique achieves better outage performances than the conventional decode-and-forward (DF) technique using MRC, in the case that the error probability p_e of the source-relay link is zero.

Furthermore, the impact of various relay scenarios were evaluated, where the average SNR of the two channels are different. It has been found that the influence of the relay location gradually gets small as the average SNRs increase.

References

[1] D. Slepian and J. Wolf, "Noiseless coding of correlated information sources," *Information Theory, IEEE Transactions on*, vol. 19, no. 4, pp. 471 – 480, Jul. 1973.

[2] J. Garcia-Frias and Y. Zhao, "Near-shannon/slepian-wolf performance for unknown correlated sources over awgn channels," *Communications, IEEE Transactions on*, vol. 53, no. 4, pp. 555 – 559, april 2005.

[3] C. E. Shannon, "A mathematical theory of communication," vol. 27, pp. 397–423, Jul. 1948.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *Information Theory, IEEE Transactions on*, vol. 51, no. 9, pp. 3037 – 3063, Sep. 2005.

[5] A. Goldsmith, *Wireless Communications*. Stanford University, USA: Cambridge University Press, 2005.

[6] R. Youssef and A. Graell i Amat, "Distributed serially concatenated codes for multi-source cooperative relay networks," *Wireless Communications, IEEE Transactions on*, vol. 10, no. 1, pp. 253 –263, Jan. 2011.

[7] M. Cheng, A. Irawan, K. Anwar, and T. Matsumoto, "BICM-ID for relay system allowing intra-link errors and a similarity constellation to ARQ schemes," in *Progress In Electromagnetics Research Symposium (PIERS), Malaysia*, Mar. 2012, pp. 281 –286.

[8] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," vol. 49, no. 10, pp. 1727–1737, 2001.