

Impact of Path Loss Exponents on Antenna Location Design for GDAS

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Abstract—In this paper, the impact of path loss exponents on antenna location design in generalized distributed antenna system (GDAS) is investigated, with the goal of maximizing a lower bound of the average uplink capacity. A simple case when antenna ports (AP) are uniformly placed along a circle is studied first. Both the upper and lower bounds for the optimal placement radius are given. An approximate expression of the optimal radius with sufficient accuracy is derived using Newton-Cotes integration formula with a degree of 5. The given expression indicates that with different path loss exponents the optimal locations of APs differ. The interesting thing is that the optimal radius decreases as the path loss exponent grows when the number of APs is large, while it is the opposite when the number of APs is small. Analytical results are then extended to more general scenarios and a new criterion is proposed for antenna location design. Simulation results show that the given criterion outperforms those in previous literature, and the achieved capacity gain can be as much as 10%.

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) is a key technology in future wireless communication systems. By employing multiple antennas, MIMO technology can greatly increase the frequency efficiency [1], [2]. In contrast to Centralized MIMO (CMIMO) system, distributed MIMO (DMIMO) system can decrease the average access distance, reduce energy cost and increase capacity [3], [4]. Recently, DMIMO has become one of the key technologies in the Long Term Evolution Advanced (LTE-A) system, in the form of Coordinated Multi-Point (CoMP) transmission technology [5]. A CoMP system can be regarded as a type of generalized distributed antenna system (GDAS). In GDAS, the traditional base station (BS) is separated into two parts, i.e., the central unit (CU) and the distributed antenna port (AP). An AP connects to the central unit (CU) via optical fiber or coax cable, and there can be multiple antennas at each AP.

Recent research concerning antenna location of GDAS mainly focus on coverage performance analysis and antenna location design criteria [6]–[12]. A multi-cell scenario where 6 distributed antennas are placed along a circle in each cell is considered in [6]. In [7], the effect of the randomness of antennas' locations on outage probability is studied. Results in [6] and [7] show that DMIMO has better performance over CMIMO with respect to capacity and outage probability no matter antennas are fixed or randomly placed. In [8]–[12], the authors analyzed the optimal antenna locations in terms of capacity, outage probability and power efficiency.

The optimal antenna location in a linear cell is studied in [8]. In [9], the impact of shadow fading on the optimal antenna locations in a circular cell is discussed, and it is concluded that antennas should be placed symmetrically with respect to the cell center. Reference [10], [11] and [12] are based on different optimization objectives, yet derived similar criterion related to minimizing average access distance. It should be mentioned that existing researches seldom investigate the path loss exponent's impact on antenna location design, and there are few analytical results. In fact, the optimal antenna locations vary with different path loss exponents, especially when the number of APs is large.

In this paper, the optimization problem on antenna location design considering path loss exponent's impact is investigated. A special case when APs are placed along a circle is studied for convenience. An approximated expression of the optimal radius for antenna deployment with circular layout is derived using Newton-Cotes integration formula, which provides sufficient accuracy. Besides, a new antenna location design criterion is proposed for more general scenarios.

The rest of the paper is organized as follows. The system model is described in section II. In section III, analytical results of path loss exponent's impact on optimal antenna locations and a design criterion are given. The simulation and numerical results are shown in section IV, and the conclusions are drawn in section V.

Notations in this paper: $(\cdot)^T$ and $(\cdot)^H$ denotes the transpose and the Hermitian transpose of (\cdot) , respectively; $E_x(\cdot)$ denotes the expectation of (\cdot) taken over x ; $\det(\cdot)$ denotes the determinant of (\cdot) ; $f'(x)$, $f''(x)$, $f'''(x)$ denote the first, second and third order derivative, respectively.

II. SYSTEM MODEL

Consider the uplink of a *single user* GDAS in which APs are *uniformly* placed along a *circle* centered at the cell center (i.e. the circular layout) as shown in Fig. 1.

Assume that the radius of the circular cell and antenna deployment are R and r , respectively. Denote the number of antennas at mobile station (MS) as N_T , the number of APs as N and the number of antennas at each AP as N_R . Suppose that the MS does not know the uplink channel state information (CSI), and the total transmission power P is equally shared by the N_T antennas.

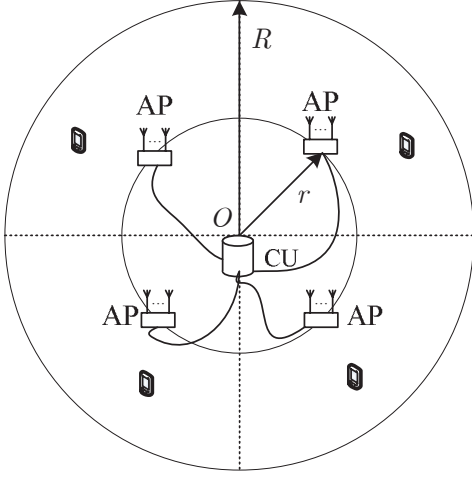


Fig. 1. Circular layout of APs with deployment radius r and cell radius R .

Let \mathbf{x} denote the transmitted signal vector in the uplink and \mathbf{n} be the Gaussian noise at the receiver, thus the received signal \mathbf{y} can be represented as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where \mathbf{H} is the uplink channel coefficients matrix, considering path loss, shadow fading and fast fading. \mathbf{H} can be represented as

$$\mathbf{H} = [\mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \dots \quad \mathbf{H}_N^T]^T, \quad (1)$$

where $\mathbf{H}_n (n = 1, \dots, N)$ is the channel matrix from the MS to the n -th AP, modeled as

$$\mathbf{H}_n = \sqrt{\mathbf{L}_n s_n} \mathbf{F}_n. \quad (2)$$

Here, \mathbf{L}_n , s_n , \mathbf{F}_n are the path loss, the log-normal shadow fading, and the Rayleigh fast fading matrix between the MS and the n -th AP, respectively. Path loss \mathbf{L}_n is a function of the distance d_n between the MS and the n -th AP, modeled as [13]

$$\mathbf{L}_n(d_n) = \frac{d_0^\alpha}{10^{A/10}} d_n^{-\alpha} \triangleq cd_n^\alpha, \quad (3)$$

where α is the path loss exponent, typically within the range of 2.0 to 6.0; d_0 is a reference distance between MS and the AP, and A is the path loss in dB at d_0 .

III. OPTIMAL ANTENNA DEPLOYMENT CONSIDERING PATH LOSS EXPONENT

A. Optimization Problem Formulation with Circular Layout

Without loss of generality, some assumptions are made for convenience in the paper as follows. *Assumption 1*: selective transmission is used in the uplink, where the MS only communicate with the nearest AP [10], [11]; *Assumption 2*: the MS's location \mathbf{l} is uniformly distributed in the cell.

By *Assumption 1*, each AP in circular layout is responsible for an area shown in Fig. 2. The MS communicates with a specified AP in the uplink only when it is in the AP's coverage area, denoted by D . Thus the circular cell is split into N equal sectors and optimization of antennas location can be taken over any sector, which is similar to the conventional cell planning

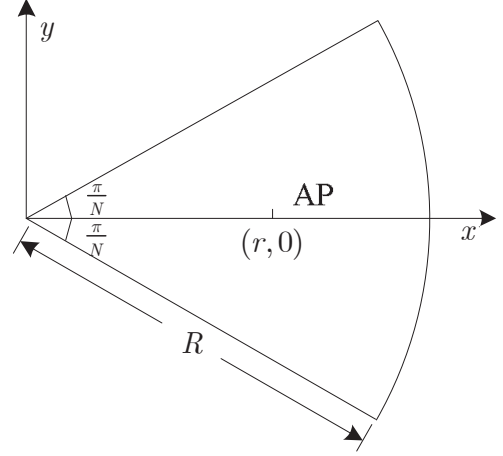


Fig. 2. An AP's coverage area D

(see e.g., [14]). The AP is placed on the horizontal axis due to symmetry.

The single user uplink instantaneous capacity C_{ins} is given by

$$C_{\text{ins}} = \log \det \left[\mathbf{I}_{N_T} + \frac{P}{N_T \sigma^2} \mathbf{H}^H \mathbf{H} \right], \quad (4)$$

where σ^2 is the Gaussian noise power, and $\mathbf{H}^H \mathbf{H}$ can be expanded as

$$\begin{aligned} \mathbf{H}^H \mathbf{H} &= [\mathbf{H}_1^H \quad \dots \quad \mathbf{H}_N^H] [\mathbf{H}_1^T \quad \dots \quad \mathbf{H}_N^T]^T \\ &= \sum_{i=1}^N \mathbf{H}_i^H \mathbf{H}_i = \sum_{i=1}^N L_i s_i \mathbf{F}_i^H \mathbf{F}_i. \end{aligned} \quad (5)$$

Substituting (5) into (4) gives

$$C_{\text{ins}} = \log \det \left[\mathbf{I}_{N_T} + \frac{P}{N_T \sigma^2} \sum_{i=1}^N L_i s_i \mathbf{F}_i^H \mathbf{F}_i \right]. \quad (6)$$

(6) can be further simplified according to *Assumption 1* as

$$C_{\text{ins}} = \log \det \left[\mathbf{I}_{N_T} + \frac{PL(r, \mathbf{l})s}{N_T \sigma^2} \mathbf{F}^H \mathbf{F} \right], \quad (7)$$

where L , s , \mathbf{F} are the path loss, the shadow fading, and the fast fading matrix between the MS and the nearest AP, respectively. The path loss L is determined by the deployment radius r of the APs and the MS's location \mathbf{l} .

Averaging C_{ins} over shadow fading and fast fading, and considering MS's distribution in the cell, we define the uplink cell average ergodic capacity C_1 as

$$C_1 = E_{\mathbf{l}} E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{PL(r, \mathbf{l})s}{N_T \sigma^2} \mathbf{F}^H \mathbf{F} \right] \right\}, \quad (8)$$

which is the average capacity that the MS experiences over the entire cell.

In this paper, we choose C_1 as the optimization objective, and then the antenna location design problem with circular

layout can be formulated as the following optimization problem

$$\max_{r \in [0, R]} E_{\mathbf{l}} E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{PL(r, \mathbf{l})s}{N_T \sigma^2} \mathbf{F}^H \mathbf{F} \right] \right\}. \quad (9)$$

B. Optimization Over A Lower Bound of C_1

Direct solution of (9) is difficult due to the expectation taken over both channel and MS's location. A lower bound of C_1 is derived first as

$$\begin{aligned} C_1 &= E_{\mathbf{l}} E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{PsL(r, \mathbf{l})}{N_T \sigma^2} \mathbf{F}^H \mathbf{F} \right] \right\} \\ &= E_{\mathbf{l}} E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{Psc \mathbf{F}^H \mathbf{F}}{d(r, \mathbf{l})^\alpha N_T \sigma^2} \right] \right\} \\ &\geq E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{Psc \mathbf{F}^H \mathbf{F}}{E_{\mathbf{l}} \{d(r, \mathbf{l})^\alpha\} N_T \sigma^2} \right] \right\}, \end{aligned} \quad (10)$$

where $d(r, \mathbf{l})$ is the distance between the MS and nearest AP.

Proof: Since $\mathbf{F}^H \mathbf{F}$ is Hermitian, it can be diagonalized as $\mathbf{F}^H \mathbf{F} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$ with unitary \mathbf{U} and non-negative diagonal $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{N_T})$. Substituting this into (10) gives

$$\begin{aligned} E_{\mathbf{l}} E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{Psc}{d(r, \mathbf{l})^\alpha N_T \sigma^2} \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H \right] \right\} \\ = E_{s, \mathbf{F}} \left\{ \sum_{i=1}^{N_T} E_{\mathbf{l}} \left[\log \left(1 + \frac{Psc}{d(r, \mathbf{l})^\alpha N_T \sigma^2} \lambda_i \right) \right] \right\}. \end{aligned} \quad (11)$$

Considering the convexity of $f(x) = \log(a + b/x)$, and applying Jensen's inequality to (11), we get

$$\begin{aligned} E_{s, \mathbf{F}} \left\{ \sum_{i=1}^{N_T} E_{\mathbf{l}} \left[\log \left(1 + \frac{Psc}{d(r, \mathbf{l})^\alpha N_T \sigma^2} \lambda_i \right) \right] \right\} \\ \geq E_{s, \mathbf{F}} \left\{ \sum_{i=1}^{N_T} \log \left(1 + \frac{Psc}{E_{\mathbf{l}} \{d(r, \mathbf{l})^\alpha\} N_T \sigma^2} \lambda_i \right) \right\} \\ = E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{Psc \mathbf{F}^H \mathbf{F}}{E_{\mathbf{l}} \{d(r, \mathbf{l})^\alpha\} N_T \sigma^2} \right] \right\}. \end{aligned} \quad (12)$$

Thus completes the proof of (10). \blacksquare

Taking optimization over the lower bound in (10), we can get

$$\begin{aligned} \max_{r \in [0, R]} E_{s, \mathbf{F}} \left\{ \log \det \left[\mathbf{I}_{N_T} + \frac{Psc \mathbf{F}^H \mathbf{F}}{E_{\mathbf{l}} \{d(r, \mathbf{l})^\alpha\} N_T \sigma^2} \right] \right\} \\ \Leftrightarrow \min_{r \in [0, R]} E_{\mathbf{l}} \{d(r, \mathbf{l})^\alpha\} \\ \Leftrightarrow \min_{r \in [0, R]} \iint_D p(x, y) [(x - r)^2 + y^2]^{\alpha/2} dx dy, \end{aligned} \quad (13)$$

in which, $p(x, y)$ denotes the probability density function of MS's location \mathbf{l} , and D is the integral area shown in Fig. 2. Hence, the optimal r here is to minimize the average path loss from the MS to the nearest AP.

According to *Assumption 2*, (13) is changed to

$$\min_{r \in [0, R]} \iint_D [(x - r)^2 + y^2]^{\alpha/2} dx dy. \quad (14)$$

It's easy to find out that (14) is a convex optimization problem, so that only one optimal r exists. Making the derivation of the objective function in (14) equal to 0, it gives

$$\iint_D [(x - r)^2 + y^2]^{\frac{\alpha}{2}-1} (r - x) dx dy = 0, \quad (15)$$

Normalizing x, y, r in (15) to the cell radius R , and taking the integration in (15) over x first, we get

$$\begin{aligned} \int_0^{\sin \frac{\pi}{N}} \left(1 - 2\sqrt{1 - y^2} r + r^2 \right)^{\frac{\alpha}{2}} dy \\ = \int_0^{\sin \frac{\pi}{N}} \left[\left(y \cot \frac{\pi}{N} - r \right)^2 + y^2 \right]^{\frac{\alpha}{2}} dy, \end{aligned} \quad (16)$$

It can be seen that (16) has a form of nonlinear integration, which is difficult to solve though the expression seems simple. The Newton-Cotes integration formula with a degree of 5 is used in this paper to approximate the integrations in (16) with sufficient accuracy [15]. Let $\tilde{f}_l(r)$ and $\tilde{f}_r(r)$ be the approximations to the left- and right-hand side of (16) respectively, and then remove the equal parts of both sides, we finally get

$$\tilde{f}_l(r) = \tilde{f}_r(r), \quad (17)$$

where

$$\begin{aligned} \tilde{f}_l(r) &= 7(1 - r)^\alpha + 12 \left[1 - r \sqrt{4 - \sin^2 \left(\frac{\pi}{N} \right) + r^2} \right]^{\alpha/2} \\ &\quad + 32 \left[1 - \frac{r}{2} \sqrt{16 - 9 \sin^2 \left(\frac{\pi}{N} \right) + r^2} \right]^{\alpha/2} \\ &\quad + 32 \left[1 - \frac{r}{2} \sqrt{16 - \sin^2 \left(\frac{\pi}{N} \right) + r^2} \right]^{\alpha/2}, \end{aligned} \quad (18)$$

$$\begin{aligned} \tilde{f}_r(r) &= 7r^\alpha + 12 \left[\frac{1}{4} - \cos \left(\frac{\pi}{N} \right) r + r^2 \right]^{\alpha/2} \\ &\quad + 32 \left[\frac{1}{16} - \frac{1}{2} \cos \left(\frac{\pi}{N} \right) r + r^2 \right]^{\alpha/2} \\ &\quad + 32 \left[\frac{9}{16} - \frac{3}{2} \cos \left(\frac{\pi}{N} \right) r + r^2 \right]^{\alpha/2}. \end{aligned} \quad (19)$$

The above expressions of $\tilde{f}_l(r)$ and $\tilde{f}_r(r)$ implies that the optimal r , i.e. the solution to (17), is a function of α and N . If α varies, the optimal antenna locations change.

Since (17) is a nonlinear equation, the solution is complicated. To solve the problem, a conclusion is given first.

Conclusion 1: The solution to (17) is bounded as follows.

$$\begin{cases} \frac{\sqrt{7}}{8} < r < \frac{1}{2} & N = 2 \\ \frac{1}{2} < r < \frac{1}{8} + \frac{\sqrt{13}}{8} & N = 3 \\ \frac{1}{2} < r < \frac{3}{8} \cos \left(\frac{\pi}{N} \right) + \frac{1}{8} \sqrt{16 - \sin^2 \left(\frac{\pi}{N} \right)} & N \geq 4 \end{cases} \quad (20)$$

Proof: From (18) and (19), it is easy to see that when r is smaller than the lower bound in (20), $\tilde{f}_l(r) > \tilde{f}_r(r)$, and when r is larger than the upper bound in (20), $\tilde{f}_l(r) < \tilde{f}_r(r)$. For the practical scenario, there must be a solution, and the solution has to be bounded as (20). \blacksquare

Using *Conclusion I*, (17) can be approximated by Taylor series with adequate accuracy as

$$a(r - 0.5)^3 + b(r - 0.5)^2 + c(r - 0.5) + d = 0, \quad (21)$$

where

$$\begin{cases} a = \frac{1}{6} [f_1'''(0.5) - f_r'''(0.5)], \\ b = \frac{1}{2} [f_1''(0.5) - f_r''(0.5)], \\ c = f_1'(0.5) - f_r'(0.5), \\ d = f_1(0.5) - f_r(0.5). \end{cases}$$

(21) is easy to solve, and when $\alpha > 2$, the solution can be formulated as

$$r = \frac{1}{2} - \frac{1}{3a} \left(b + \sqrt[3]{K_1} + \sqrt[3]{K_2} \right), \quad (22)$$

where

$$\begin{cases} K = \frac{3a}{2} \sqrt{-3b^2c^2 - 54abcd + 81a^2d^2 + 12b^3d + 12ac^3}, \\ K_1 = b^3 - \frac{9}{2}abc + \frac{27}{2}a^2d + K, \\ K_2 = b^3 - \frac{9}{2}abc + \frac{27}{2}a^2d - K. \end{cases}$$

It is easy to see that for $\alpha = 2$ the optimal r is given by

$$r = \frac{2N}{3\pi} \sin\left(\frac{\pi}{N}\right), \quad (23)$$

which is exactly the result given in [10] and [11], as a special case.

C. A Design Criterion for General Cases

The above discussion focuses on circular layout and shows that the optimal antenna location changes with α . For more general case when APs are not constrained to be placed along a circle, similar conclusion holds apparently. Let d be the minimum distance between a MS and the APs, the antenna location design criterions of minimizing $E\{d^2\}$ given in [10], [12] did not take the path loss exponent α into consideration. In this paper, we propose minimizing $E\{d^\alpha\}$ as the design criterion and the K-means algorithm proposed in [10]–[12] for antenna location design can still be used.

IV. NUMERICAL RESULTS

A. Validation of (22) and the Bounds in (20)

Fig. 3 shows the bounds of the optimal r in (20). The bounds are not very tight as shown in the figure. However, the bounds are accurate, and guarantee the approximation made in (21).

Fig. 4 illustrates the accurate value of the optimal deployment radius r in circular layout in 17 and its approximation in (22), for $N = 4, 8, 20$ respectively. As shown in Fig. 3, the approximations are very close to the accurate results. The difference is no more than 1%, and becomes even smaller as α decrease. The optimal r changes greatly with α , and the range of variation can be as much as 10%. The interesting thing is that, in Fig. 4, when $N = 4$, the optimal r increases slightly as α grows, while it is the opposite when N becomes larger. The reason is that for circular layout, when N is large, the inner area of the deployment circle near the cell center becomes more important than the outer part (as a result of the design criterion of minimizing $E\{d^\alpha\}$), and contributes more to the average capacity, so the optimal r decreases as α grows to give a better coverage for the inner area.

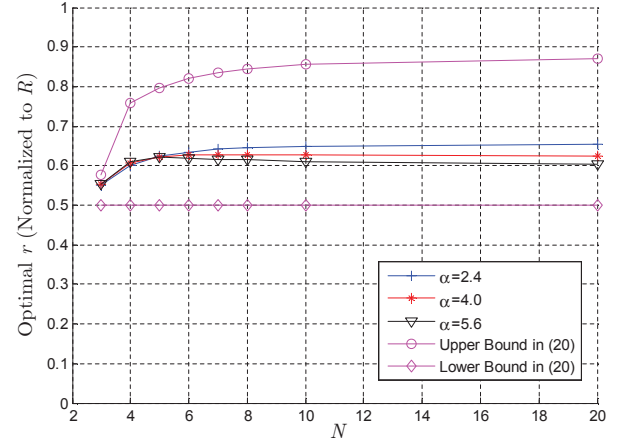


Fig. 3. Bounds of optimal deployment radius in (20), with respect to different number of APs ($N \geq 3$).

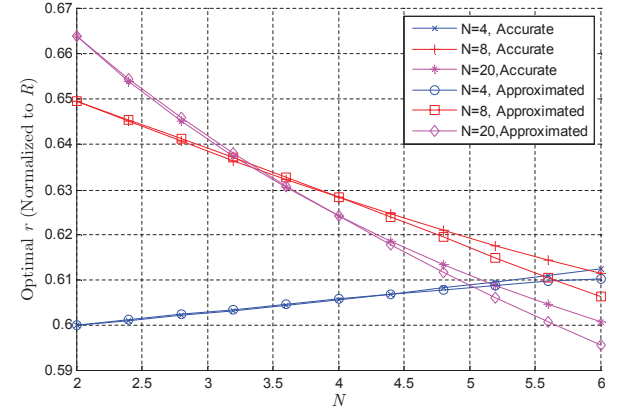


Fig. 4. Accurate value of the optimal deployment radius r in (17) and its approximation in (22).

B. Performance of Circular Layout

To evaluate the performance of circular layout, the following simulation parameters are used: $N_T = 2$, $N_R = 4$, and the standard deviate of the log-normal shadowing fading is 4 dB. C_1 is compared between circular layout with optimal deployment radius and computer searched optimal antenna layout. To avoid the influence of different transmit power and noise power, we normalized C_1 to the maximum value. Simulation results are shown in Fig. 5. For small N , the circular layout performs as well as the computer searched results. Nevertheless, when N is large, there is a significant capacity gap, and the circular layout performs worse. This is because the coverage of the central area in the cell becomes worse with large N , due to the constraint that APs are placed along a circle. Thus for large N , circular layout is not recommended.

C. Performance of the Modified Criterion

The gain in C_1 of minimizing $E\{d^\alpha\}$ over the criterions in [12] is shown in Table I. Simulation parameters are the same as above. It can be seen that when α is large, the new criterion

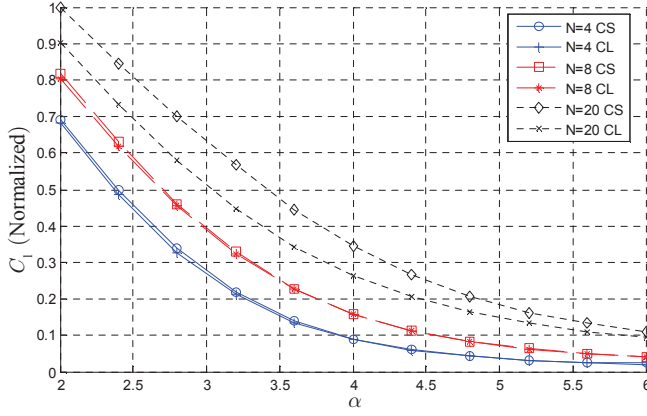


Fig. 5. Capacity comparison between circular layout (CL) and computer searched results (CS).

TABLE I
CELL AVERAGE ERGODIC CAPACITY GAIN OF MODIFIED CRITERION
OVER THAT IN [12]

α	$N = 4$	$N = 8$	$N = 20$
3.2	0.73%	0.29%	0.87%
4.8	1.21%	5.60%	1.08%
5.6	4.83%	10.98%	3.61%

performs much better, with a capacity gain up to around 10% when $N = 8$. This is due to the convexity of $f(x) = x^\alpha$ when $\alpha > 1$, and the criterions in [10]–[12] can just achieve a lower bound of the capacity achieved by the modified criterion. Since minimizing $E\{d^\alpha\}$ is a convex problem, many efficient algorithms exist, and there is limited increase in computational complexity. A gain in capacity can be converted to the saved power, and hence the modified criterion can also results in better power efficiency.

V. CONCLUSION

In this paper, the impact of path loss exponent on antenna location design in GDAS is analyzed. For circular layout of APs, an accurate approximate expression of the optimal deployment radius is given. A new criterion for antenna location design is

proposed for more general scenarios. Simulation results show that the path loss exponent has great influence on antenna location design, and the proposed criterion performs better than those in former literature.

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