

A Robust Communication System Based on Joint-Source Channel Coding for a Uniform Source

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Abstract—A communication system based on the joint source-channel coding principle is proposed where a fixed-rate source encoder using neither a codebook nor an entropy encoder is exploited to avoid the error propagation effect and thus gain in system robustness. An explicit expression of the mean square error (MSE) distortion of the system for a uniform source is derived. Based on the MSE distortion, the optimal design of the communication system under the total transmission rate constraint is formulated as a mixed integer nonlinear optimization problem. We provide an algorithm to achieve the optimal solution via a convex optimization solver. The numerical result shows that the overall performance of the proposed system is close to the performance of the entropy coding scalar quantizer (ECSQ) system established by A. György and T. Linder [1] for almost all rate regions.

Index Terms—Scalar quantizer, uniformly distributed source, joint source-channel coding, mean square error distortion, fixed-length source encoder, subchannel.

I. INTRODUCTION

A communication system with a well-structured architecture based on the binary interface was introduced by Shannon in 1948 [2]. This system is divided into two parts: source encoder/decoder and channel encoder/decoder. Shannon proved the so-called separation theorem implying that the separation architecture is optimal for point-to-point communication. So far the separation principle has been used in most practical communication systems.

Apart from the separation principle, the source encoder/decoder and channel encoder/decoder which are jointly designed, termed joint source-channel coding/decoding, have also captured attention from the research community [3]. The most integrated joint source-channel coding/decoding class is the one that the source encoder and channel encoder are combined in one entity, and the transmitted source sequence is restored based on the channel sequence by a joint source-channel decoder. This is termed “analog mapping” [4]. The other source-channel coding class is the one where the source encoder/decoder and the channel encoder/decoder are two separate entities, but there is a tight connection between them [3]. The joint source-channel coding communication system in this paper belongs to this second class.

Although the source encoder using a vector quantizer has become popular due to its asymptotically optimal performance [5, Chapter 10], the source encoder using a scalar quantizer still has its own applications in many communi-

cation systems where the implementation complexity is a decisive factor.

The optimal design of the scalar quantizer was first introduced by Max [6] and Lloyd [7]. In their work, a real-valued input signal is converted into a finite number of levels by partitioning the range of the input signal into a number of disjoint intervals and each interval has its own representation point. Both Lloyd and Max derived explicit necessary conditions and thus algorithms, known as the Lloyd-Max algorithm, for optimal designs of disjoint intervals and representation points for the preassigned number of quantization levels.

The optimal scalar quantizer subject to the transmission (or entropy) rate constraint has been studied for a wide class of memoryless sources [1], [8]. In this framework, a source sample, denoted S , is first quantized into N levels $\{S_1, S_2, \dots, S_N\}$ with their corresponding probabilities $\{p_1, p_2, \dots, p_N\}$ by a quantizer Q . Then, an entropy encoder, such as Huffman encoder [9], is used to achieve the minimum number of bits per sample as close as the entropy of the quantized outputs of the quantizer Q , $H(Q) = -\sum_{i=1}^N p_i \log_2 p_i$. The scalar quantizer is optimally designed under the transmission rate constraint, that is $H(Q) < R$, where R is a reliable transmission rate supported by the channel.

In [8], the necessary conditions have been developed for the general distortion measure in coupling with recursive algorithms (i.e., the generalized Lloyd-Max algorithm) to obtain the optimal transmission rate constrained quantizer with the predefined number of quantization levels N . The optimal transmission rate constrained scalar quantizer of a uniform source in [1] is designed where the optimal number of quantization levels N is found as a function of transmission rate $N = \lceil 2^R \rceil$, $\lceil x \rceil$ denotes the smallest integer that is greater than x . The minimum achievable distortion is explicitly represented in a parametric form.

The entropy encoder in [1], [8] achieves the minimum rate by using variable-length codes (where codewords are distinguished via the prefix [9]). The performance of the variable-length encoder is sensitive to the transmission error due to the error propagation effect (i.e., if one codeword is decoded incorrectly due to the transmission error, a large number of subsequent codewords cannot be correctly decoded since their prefixes are broken down) [10], [11], [12]. To reduce

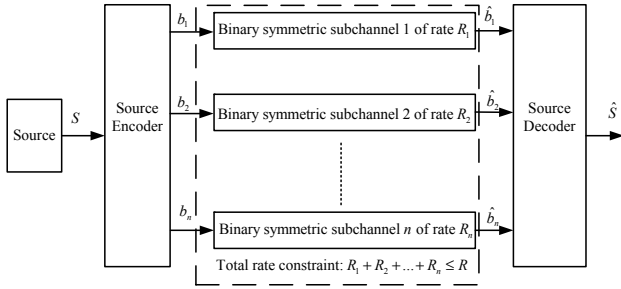


Fig. 1. The proposed communication system.

the effect of error propagation, synchronization codewords are needed. As a result, the compression efficiency of the source encoder is degraded [13]. Moreover, to cooperate between a variable-length source encoder and fixed-rate channel encoder, dealing with the issue of the buffer overflow control is raised [14].

Based on above observations, we propose a communication system based on the joint source-channel coding principle where a simple, fixed-rate source encoder is used to avoid the error propagation effect and thus to obtain system robustness.

The detailed description of the proposed system is presented in Section II. In Section III, the MSE distortion of the proposed system for a uniform source is derived. The derivation results are then employed to formulate an optimal design of the proposed system as a mixed integer nonlinear optimization problem (MINLP). An algorithm based on a convex optimization solver to achieve the optimal solution is also included. Numerical results are provided in Section IV. Section V concludes the paper.

II. DESCRIPTION OF PROPOSED SYSTEM

A communication system based on the joint source-channel coding principle depicted in Fig. 1 has following attributes:

The source block generates a sequence of i.i.d random variables which are assumed to be uniformly distributed as $\mathcal{U}(-1, 1)$. The uniform source is used in this correspondence since the MSE distortion can be found in an explicit form which facilitates the analytical evaluation of the system performance. For the non-uniform source class, especially the source with infinite support such as the Gaussian source, the overload distortion needs considering together with quantization and channel distortions.

The next block is the fixed-rate source encoder which is one of the main considerations of the paper. The source encoder consists of only a scalar quantizer and produces a sequence of binary bits directly using neither a codebook nor an entropy encoder. In particular, the n -bit quantizer converts a real-input sample, say S , into a finite number of levels which are explicitly represented via a linear combination of binary outputs as

$$S_Q = \sum_{i=1}^n 2^{-i} b_i, \quad (1)$$

where $b_i = \pm 1, i = 1, 2, \dots, n$ and S_Q denotes the quantized sample.

For the uniform source class, the scalar quantizer (1) has been proved to be optimal first by Crimmins [15] and later by McLaughlin [16]. It is asymptotically optimal for other source classes since the quantizer turns out to be uniform.

An observation is that such a quantizer is well-structured and simple while it still provides a certain degree of flexibility to vary the number of quantization bits n (i.e., the change in the number of quantization bits does not make any significant change in hardware since no codebook and partition sets are needed). Its flexibility is very well suited for fading environment where channel quality and thus the transmission rate vary in time.

The third block is the transmission channel which is assumed to have n independent multiple memoryless binary symmetric subchannels with the total rate constraint:

$$R_1 + R_2 + \dots + R_n \leq R. \quad (2)$$

This means that our proposed system transfers the variable-length source coding problem of the entropy source coder to the multiple subchannel transmission. The transferring has two advantages: (1) The error propagation effect is avoided; (2) We have more means to implement the multiple subchannel transmission scheme since the assumption of the multiple subchannel transmission is practically reasonable for today's communication systems such as MIMO and/or OFDM systems. For the MIMO system, the channel can be decomposed into K parallel subchannels [17, Chapter 8], where K is the rank of the channel matrix, provided that the channel state information (CSI) is available at both transmitter and receiver sides. The fact that the number of parallel channels can vary from time to time for the wireless fading channel. With the CSI at the transmitter side, the conventional optimal solution is either to adjust the transmit power or to adapt the transmission rate [17], [18]. However, as the low end-to-end distortion is the ultimate goal, the source coding rate can be adapted to the channel coding rate. The source rate adaptation can be easily fulfilled by the proposed source coder with the least modification. For the OFDM system, the multiple subchannels with desired reliable transmission rates can be obtained by both sub-carrier partitioning and/or power allocation [17, Chapter 3].

Turbo codes and LDPC codes [19] both offer the very close performance to the theoretical bound; so-called capacity-approaching codes. Hence, we do not explicitly use any specific channel encoder. Instead, we assume that capacity-approaching codes are used and that we can establish the performance bound for the proposed system.

Finally, the source decoder will reproduce an estimate version of the transmitted sample as

$$\hat{S}_Q = \sum_{i=1}^n 2^{-i} \hat{b}_i, \quad (3)$$

where $\hat{b}_i, i = 1, 2, \dots, n$ are the decoded bits at the receiver side.

The scalar, fixed-rate source coder is certainly suboptimal, in general, since redundancy still exists. However, the minimum end-to-end distortion performance for a given total rate constraint is our concern for the real-valued sources. Therefore, the minimum MSE distortion for a uniform source will be derived in the next section to evaluate the performance of the proposed system.

III. END-TO-END DISTORTION

It is certainly impossible to perfectly deliver a real-valued signal from a source to a destination over a finite rate channel. Hence, a distortion measure can be used to evaluate the system performance. There exists several definitions on the distortion measure, however the conventional MSE distortion is chosen due to its mathematical tractability. The MSE distortion is defined as

$$D = E[(S - \hat{S}_Q)^2], \quad (4)$$

where $E[x]$ denotes statistical expectation.

For a given n -bit quantizer and the total rate constraint of R bits/sample, the MSE distortion $D(R, n)$ for a uniformly distributed source is given by

$$D(R, n) = \sum_{i=1}^n 4^{(1-i)} D_i + \frac{4^{-n}}{3}, \quad (5)$$

where D_i is the average Hamming distortion caused by transmission errors of the i th subchannel (The derivation is in Appendix A) and the last term represents the quantization distortion.

We are thus going to present a relation between the average Hamming distortion D_i and the rate R_i of the subchannel i . From information theory [20, *Theorem 10.2.1*], we have

$$R_i(D_i) = \min_{p(\hat{x}|x): E(d(x, \hat{x})) \leq D_i} I(X; \hat{X}), \quad (6)$$

where X, \hat{X} are the source sample at encoder and decoder, respectively. $I(X; \hat{X})$ is the mutual information between the input and the output, respectively. And $p(\hat{x}|x)$ is the conditional probability.

Based on (6), the rate distortion is found for a Bernoulli source as [20, *Theorem 10.3.1*]

$$R_i(D_i) = \begin{cases} H(p_i) - H(D_i) & 0 \leq D_i \leq \min\{p_i, 1 - p_i\} \\ 0 & D_i > \min\{p_i, 1 - p_i\}, \end{cases} \quad (7)$$

where $H(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ and p_i is the probability distribution of the binary input. For a uniform source, it is easy to verify that $p_i = \Pr[b_i = -1] = \Pr[b_i = 1] = 1/2$ which yields $H(p_i) = 1, \forall i = 1, 2, \dots, n$. Combining (2) and (7), the total rate constraint is given by

$$\sum_{i=1}^n [1 - H(D_i)] \leq R. \quad (8)$$

In general, for a communication system where the channel can support the total rate of R bits/sample, the ultimate goal is to design the source encoder and the channel encoder to minimize the end-to-end distortion. For the proposed system,

this design goal is interpreted as to determine the optimal number of quantization bits per sample n and the optimal reliable transmission rate allocation among the n subchannels so that the distortion $D(R, n)$ in (5) is minimized. Combining (5) and (8), the design problem for the proposed system can be stated as:

$$\begin{aligned} & \text{minimize} && D(R, n) = \sum_{i=1}^n 4^{1-i} D_i + \frac{4^{-n}}{3} \\ & \text{subject to} && \left\{ \begin{aligned} & \sum_{i=1}^n [1 - H(D_i)] \leq R \\ & 0 \leq D_i \leq 0.5 \\ & n \geq 1. \end{aligned} \right. \end{aligned} \quad (9)$$

The optimization problem (9) contains two types of variables: the variable n is an integer/discrete variable and the other variables $D_i, i = 1, 2, \dots, n$ can take nonnegative real values. Furthermore, the rate constraint function is nonlinear. Such an optimization problem is so-called a mixed integer nonlinear programming which is known to be difficult to solve in general [21].

Fortunately, for a given $n = \hat{n}$, the optimization problem is a convex problem. This is true because the objective function $D(R, \hat{n})$ is a linear function of $D_1, D_2, \dots, D_{\hat{n}}$, and it is thus a convex function. In addition, $\sum_{i=1}^{\hat{n}} [1 - H(D_i)]$ is a convex function since $H(D_i)$ is known to be concave, which means that the domain of the optimization problem is a convex set [22]. As a result, the considered optimization problem is a convex optimization problem for each fixed value of n . This observation leads to the idea that one can use the grid search algorithm for the integer variable n , then use the well-developed convex optimization algorithms [22] to solve for the real-valued variables $D_i, i = 1, 2, \dots, n$.

For a given n , the algorithm to solve our convex problem based on convex optimization packet by S. Boyd and *et al.* [23] using MATLAB as follow.

Algorithm I:

```

begin_cvx
  variable  $D(n)$ ; % Distortion vector
  expression  $W(n)$ ; % Weight vector
  for  $i = 1:1:n$ 
     $W(i) = 4^{(1-i)}$ ;
  end
  minimize (sum( $W * D$ ));
  subject to
     $D \geq 0$ 
     $D \leq 0.5$ 
     $n \cdot \text{sum}(\text{entropy}(D)) \leq R$ 
end_cvx

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When adopting the grid search with the integer variable n , the current concern is the lack of the upper limits for the optimal value of n . Fortunately, we have the following limits

$$\begin{aligned} \lim_{n \rightarrow 1^+} D_q(n) &= 1/12 \\ \lim_{n \rightarrow 1^+} D_c(R, n) &= 0, \forall R \geq 1, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \lim_{n \rightarrow \infty} D_q(n) &= 0 \\ \lim_{n \rightarrow \infty} D_c(R, n) &= 2/3. \end{aligned} \quad (11)$$

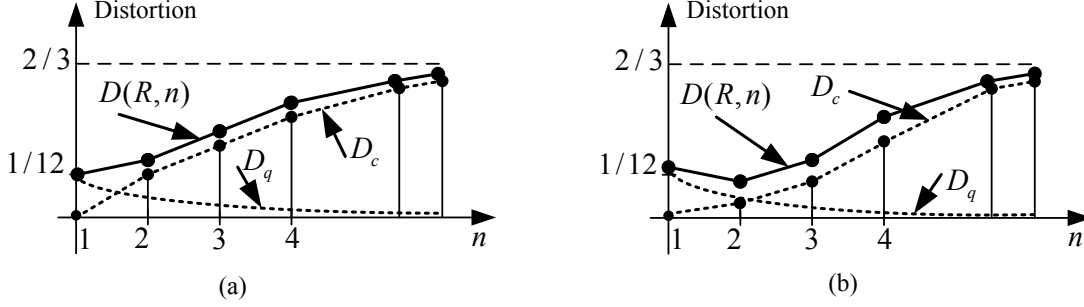


Fig. 2. Sketch of distortion function $D(R, n)$.

where $D_q(n)$ and $D_c(R, n)$ are the quantization distortion and channel distortion, respectively, and they are defined in Appendix A.

Those limits and the fact that $D_q(n)$ is a decreasing function of n and $D_c(R, n)$ is non-decreasing function of n suggest that there are two possibilities for the overall distortion function $D(R, n)$ for the fixed rate $R \geq 1$ as illustrated in Fig. 2: One possibility is that $D(R, n)$ is monotonic increasing function of n . And the other possibility is there exists n_0 such that $D(R, n)$ is a decreasing function of n for $n \leq n_0$ and is an increasing function of n otherwise. This implies that the optimal value of n always exists. Note that there will be no possibility that $D(R, n)$ is a monotonic decreasing function of n since, if possible, then we can deliver real-valued signals with arbitrary small distortion over the finite capacity channel, which is contradicted to what has been established in rate-distortion theory [20, Chapter 10].

The above observations suggest the upper limit value of n . That is if n^* is the optimal number of quantization bits, then $D(R, n)$ is an increasing function of n for all $n > n^*$, and thus $D(R, n^* + 1) > D(R, n^*)$ must be satisfied. This condition implies that the grid search algorithm to find the optimal value of n should stop once the distortion function is detected increasing as n increases. Therefore, $n^* + 1$ is used as the upper limit of the optimal value n since the increasing of $D(R, n)$ can be first detected at the value $n = n^* + 1$.

Based on the upper bound of the optimal value of n , we propose the following algorithm.

Algorithm II:

- (1) Given R , initialize $n = 1$ and $D_{min} = +\infty$.
- (2) Compute the minimum distortion, denote $D^*(R, n)$, for the given R and n by **Algorithm I**.
- (3) If $D^*(R, n) \leq D_{min}$, set $D_{min} = D^*(R, n)$ and $n = n + 1$, then repeat step 2. Otherwise, go to step 4.
- (4) The algorithm stops and the optimal number of quantization bits $n^* = n - 1$.

IV. NUMERICAL RESULTS

The minimum MSE distortion versus the total transmission rate for the proposed system is presented in Fig. 3. The optimal ECSQ performance established in [1] for a uniform

source is also included and plotted by the solid curve as the reference system. It is observed that the performance of the proposed communication system is fairly close to the optimal ECSQ system [1] in almost all rate regions. The poorer performance of our proposed system compared to the ECSQ system in [1] is incurred because the number of the quantized levels is restricted to the form of $N = 2^n$ whereas the number of quantized levels in [1] can take any arbitrary integer value that obeys the rule $N = \lceil 2^R \rceil$. This shows the trade-off between system robustness and the compression efficiency.

At the point that the transmission rate R takes integer values, the proposed system achieves the same performance as that of the ECSQ system. This behavior can be explained from the following engineering point of view: When the rate takes on an integer value, the optimal number of subchannels (number of quantization bits) is equal to R and each subchannel is exactly allocated one bit. This means that, the system operates in the error-free transmission state which coincides with the previous conventional communication systems (i.e., the systems use optimum quantizers which are designed under error-free transmission assumption such as [1], [8]). As indicated in [1], when the number of quantized level is in the form of 2^n , the entropy constraint scalar quantizer is uniform. Hence, the probabilities of the quantized levels are equally likely for a uniform source. As a result, the entropy encoder become useless in this case (i.e., no further rate reduction is achieved by an entropy encoder). In other words, the ECSQ system collapses to our proposed system at the considered rates. This explains why both our proposed system and the ECSQ system [1] have the identical performance at the integer-valued rates.

It is observed from Fig. 3 that there are regions in which the overall distortion is kept constant while the total rate increases. This is because the rate increasing amount is not large enough for the channel to admit a new source bit with lower channel distortion than the quantization distortion. Hence, the optimal number of quantization bits remains the same. This leads to the fact that both the channel distortion and quantization distortion are constants. As a result, the overall distortion remains constant in those rate regions.

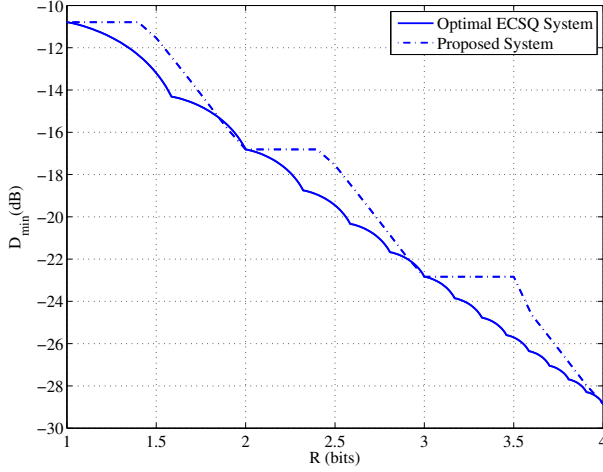


Fig. 3. End-to-end distortion for uniform source.

V. CONCLUSION AND FUTURE WORK

This paper proposes a communication system based on the joint source-channel coding principle. Instead of using a scalar quantizer coupled with an entropy encoder as usual, a simple and fixed-rate source encoder is employed to avoid the error propagation effect and thus gain in system robustness. The MSE distortion for the uniform source class is derived for the proposed system. The performance evaluation shows that the proposed system offers a performance close to the optimal for almost all rate regions. When the transmission rate takes integer values, the performance of the proposed system is identical to that of the ECSQ system.

We dig deeper into the considered problem to get more insights via investigating the Karush-Kuhn-Tucker condition. Furthermore, our analysis has been confined to the uniform source class. In future work, we would like to extend this work for other types of sources such as Gaussian and Laplacian and compare their performances of the proposed system with the previous established results in the standard systems.

VI. ACKNOWLEDGEMENT

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APPENDIX A

END-TO-END DISTORTION DERIVATION

From (4), we can rewrite the the MSE distortion as

$$D = E[(S - S_Q) + (S_Q - \hat{S}_Q)]^2, \quad (12)$$

where the first term is the quantization error and the second term is the transmission error, the end-to-end distortion is decomposed into two components as follows

$$D = E[(S - S_Q)^2] + E[(S_Q - \hat{S}_Q)^2] = D_q + D_c, \quad (13)$$

due to their statistical independence.

Consider the quantization distortion term which is denoted D_q . The quantizer in (1) is uniform. As proved in [15] such a quantizer is the optimal scalar quantizer for the uniform source class. Hence, for an n -bit quantizer, there will be 2^n levels and thus the quantization step size is $\Delta = \frac{2}{2^n} = 2^{1-n}$. As a result, the quantization distortion is

$$D_q = \frac{\Delta^2}{12} = \frac{4^{-n}}{3}. \quad (14)$$

Now, consider the transmission distortion which is denoted D_c :

$$D_c = E[(S_Q - \hat{S}_Q)^2] = E\left[\left(\sum_{i=1}^n 2^{-i}(b_i - \hat{b}_i)\right)^2\right]. \quad (15)$$

The transmission error $e_i = (b_i - \hat{b}_i)$ is a random variable whose realization takes values in the set $\{-2, 0, 2\}$. For a memoryless, binary, symmetric channel with transition probability of D , the transmission error has following distribution

$$Pr[e_i = 0] = 1 - D, Pr[e_i = -2] = Pr[e_i = 2] = \frac{D}{2}, \quad (16)$$

and thus $E[e_i] = 0$.

Quantization bits belonging to the same source sample are transmitted over independent subchannels. The error events are independent across the subchannels. The channel distortion is therefore given by

$$\begin{aligned} D_c &= E\left[\left(\sum_{i=1}^n 2^{-i}(b_i - \hat{b}_i)\right)^2\right] = E\left[\sum_{i=1}^n \sum_{j=1}^n 2^{-(i+j)} e_i e_j\right] \\ &= \sum_{i=1}^n \sum_{j=1}^n 2^{-(i+j)} E[e_i e_j] \stackrel{(a)}{=} \sum_{i=1}^n 4^{-i} E[e_i^2], \end{aligned} \quad (17)$$

where (a) holds since $E[e_i e_j] = E[e_i]E[e_j] = 0, \forall i \neq j$.

We define the Hamming distortion measure as

$$d(x, \hat{x}) = \begin{cases} 0 & \text{if } x = \hat{x} \\ 1 & \text{if } x \neq \hat{x} \end{cases} \quad (18)$$

and its average Hamming distortion is $E[d(x, \hat{x})]$, where x, \hat{x} are the source sample and restored sample, respectively. We can express $E[e_i^2]$ in terms of the average Hamming distortion as

$$E[e_i^2] = E[4|x - \hat{x}|^2] = 4D_i, \quad (19)$$

where D_i is the average Hamming distortion or the transmission error probability.

Substituting (19) into (17), the channel distortion is given by

$$D_c = \sum_{i=1}^n 4^{1-i} D_i, \quad (20)$$

where D_i is the average Hamming distortion and it is a function of the transmission rate R_i .

Combing (13), (14), and (20) the MSE distortion is finally given as

$$D(R, n) = \sum_{i=1}^n 4^{1-i} D_i(R, n) + \frac{4^{(-n)}}{3}. \quad (21)$$

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