Diversity Analysis of Minimum Distance Based Relay Selection Schemes for Two-way Relaying Systems with Physical Network Coding

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***Abstract*—A relay selection method is one of popular approaches to overcome fading effects in wireless relay channel due to its simplicity. However, previous works were concentrated on the case of analog network coding systems. In this paper, we consider a relay selection scheme in two-way relaying systems with physical network coding. Since conventional schemes based on analog network coding does not provide effective performance in physical network coded systems, we propose new selection criteria which maximize the minimum distance at the multiple access phase to achieve full diversity. Moreover, through asymptotic error rate analysis, it is confirmed that the proposed scheme is able to attain full diversity. Simulations results are also provided to verify the analysis of the proposed scheme.**

I. Introduction

Recently, a wireless relaying transmission system has been intensively studied since it can extend cell coverage and improve link performance. In practice, it is generally assumed that relays operate in a half-duplex mode where data transmission and reception do not occur simultaneously. To overcome such a loss in special efficiency, two-way relay systems have attracted much attention. The two-way relay system consists of two phases which are the multiple access (MAC) phase and the broadcast (BC) phase.

During the MAC phase, two users transmit their message to a relay at the same time, and then the relay broadcasts the information received from the users in the BC phase. Since each user knows its own information, self-interference of each user can be removed from the received signal. For the relay operation, amplify-and-forward (AF) and decode-and-forward (DF) protocol have been widely applied [1]–[4]. Two-way relay systems with the AF protocol and the DF protocol are referred to as analog network coding (ANC) [5] and physical network coding (PNC) [6], respectively, in the context of network coding [7].

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One major problem in wireless channels is a signal fluctuation due to fadings. To mitigate this fading effect, various diversity schemes have been proposed in relaying systems in the literature [8]–[10]. A relay selection method selects the best relay among multiple relays to improve the performance. Many studies have been reported related with relay selection to achieve the diversity gain [11]–[17]. In one-way relaying systems, various full diversity schemes such as maximum signal-to-noise (SNR) selection, best worse channel selection, and best harmonic mean selection have been investigated [11]–[13]. Recently, It is proven that above mentioned selection schemes also achieve full diversity for the case of ANC in two-way relaying systems [14]–[16]. Moreover, the authors in [17], [18] proposed a relay selection strategy for PNC which minimizes the outage probability of the mutual information. However, it is observed that these relay selection schemes are not able to obtain a diversity gain for two-way relay systems with PNC.

In this paper, we first propose a new relay selection algorithm for PNC. Since the overall probability of error is dominated by the minimum distance at the MAC phase, our scheme selects the relay which maximizes the minimum distance. Then, it is shown that our proposed scheme provides a significant diversity gain in contrast to conventional relay selection methods [11]–[18]. Next, we analytically derive the diversity order of the proposed scheme. Instead of the conventional pairwise error probability (PEP) based approach depending on the effective channel gain whose distribution is hard to obtain within our design strategy, we find an upper bound of the average error probability using the distribution of the minimum distance. Then, we observe that the upper bound exhibits a full diversity order. Thereby, we establish that the minimum distance based relay selection strategy achieves the full diversity with PNC. Finally, simulation results will be presented to demonstrate the efficiency of the proposed scheme and verify the accuracy of the analysis.

Throughout this paper, the superscript $(\cdot)^{\mathsf{T}}$, $(\cdot)^{\mathsf{H}}$ and $(\cdot)^*$ stand for transpose, conjugate transpose and element-wise conjugate, respectively. Also, $f_X(\cdot)$ and $F_X(\cdot)$ indicate the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable X, respectively.

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The organization of this paper is as follows: In Section II, we describe a two-way relay channel model. Section III proposes new relay selection criteria for the relay selection. Section IV presents the analytical result of the proposed scheme. In Section V, we provide numerical results to verify our analysis. Finally, this paper is concluded in Section VI.

II. System Model

In this paper, we consider two-way relay systems with PNC, where two users denoted by U_k $(k = A, B)$ exchange their information with the help of N relays represented as R_i $(i = 1, 2, \cdots, N)$ as shown in Fig. 1. All nodes are equipped with a single antenna. We assume that there is no direct communication link between two users due to large path loss. Also, it is assumed that perfect channel estimation is available at the receiver side. Let us define $s_k \in \mathbb{Z}_L$ and $x_k = \mathcal{M}(s_k)$ as the transmitted data and the modulated signal from U_k $(k = A, B)$, respectively, where $\mathbb{Z}_L$ is denoted by $\mathbb{Z}_L = \{0, 1, 2, \cdots, L-1\}$ with L being the modulation level and $\mathcal{M}$ designates the modulator. The transmitted symbols are assumed to have unit energy as $\mathbb{E}[|x_k|^2] = 1$.

During the MAC phase as shown in Fig. 1(a), U_A and U_B transmit its own symbol to N relays simultaneously. Then, the received signal at the ith relay is given by

$$y_R^{(i)} = \sqrt{P} h_A^{(i)} x_A + \sqrt{P} h_B^{(i)} x_B + n_R \tag{1}$$

where P indicates the transmit power, the channel gain $h_k^{(i)} \sim \mathcal{CN}(0, 1)$ for $k = A, B$ represents an independent complex Gaussian random variable, and $n_R \sim \mathcal{CN}(0, \sigma^2)$ stands for the complex Gaussian noise. Denoting $\mathbf{h}_i = [h_A^{(i)} \ h_B^{(i)}]^\mathsf{T}$, $\mathbf{s} = [s_A \ s_B]^\mathsf{T}$ and $\mathbf{x} = [x_A \ x_B]^\mathsf{T}$, equation (1) is rephrased as

$$y_R^{(i)} = \sqrt{P} \mathbf{h}_i^\mathsf{T} \mathbf{x} + n_R.$$

Then, among N relays, one relay is selected based on a proper relay selection strategy. Denoting $i^\star$ as the index of the selected relay, the $i^\star$th relay decodes $y_R^{(i^\star)}$ by employing maximum-likelihood (ML) detection

$$\hat{\mathbf{s}} = \arg \min_{\tilde{\mathbf{s}} \in \mathbb{Z}_L^2} \left| y_R^{(i^\star)} - \sqrt{P} \mathbf{h}_{i^\star}^\mathsf{T} \tilde{\mathbf{x}} \right|^2. \tag{2}$$

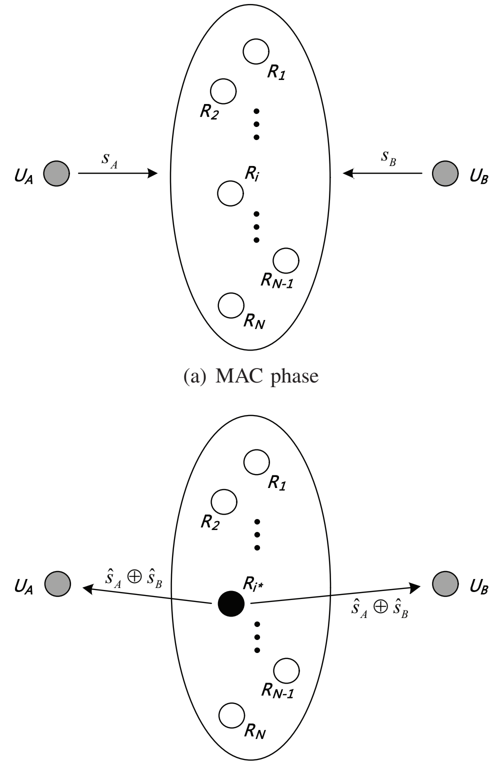
Then, the relay generates the network coded data $s_R = \hat{s}_A \oplus \hat{s}_B$ where $\oplus$ indicates the bit-wise *exclusive-or* (XOR) operation.

Next in the BC phase as described in Fig. 1(b), the $i^\star$th relay broadcasts $x_R = \mathcal{M}(s_R)$ to U_A and U_B simultaneously. Due to reciprocity between the MAC and the BC phase, we assume that the channels from the relay to the users are the same as those in the reverse direction. Also, it is assume that the relays and the users consume equal power. Then, the received signal at U_k is given by

$$y_k = \sqrt{P} h_k^{(i^\star)} x_R + n_k \quad \text{for} \quad k = A, B$$

where $n_k \sim \mathcal{CN}(0, \sigma^2)$ represents the complex Gaussian noise. Based on the received signal y_A, U_A estimates s_R as

$$\hat{s}_{R,A} = \arg \min_{\tilde{s}_R \in \mathbb{Z}_L} \left| y_A - \sqrt{P} h_A^{(i^\star)} \tilde{x}_R \right|^2. \tag{3}$$



(a) MAC phase

(b) BC phase

Fig. 1. Relay selection systems with PNC

Since U_A knows its own transmitted symbol s_A, the desired signal is detected as $\hat{s}'_B = \hat{s}_{R,A} \oplus s_A$. Similarly, U_B detects the symbol s_A.

III. Minimum Distance Based Relay Selection Criteria

In this section, we propose a new relay selection criterion for two-way relay systems with PNC which maximizes the minimum distance in the MAC phase. Note that compared to conventional schemes such as best worse channel selection and the harmonic mean selection, the proposed selection algorithm is more effective for the PNC in terms of the diversity order as will be shown later.

Let us define the probability of error given the channel $\mathbf{h}_i$ in the MAC phase and the BC phase by $P_{e,R|\mathbf{h}_i} = \Pr\{s_R \neq s_A \oplus s_B | \mathbf{h}_i\}$ and $P_{e,k|\mathbf{h}_i} = \Pr\{s_R \neq \hat{s}_{R,k} | \mathbf{h}_i\}$ for $k = A, B$, respectively, assuming that the ith relay is selected. Note that $P_{e,R|\mathbf{h}_i}$ is not equal to $\Pr\{\mathbf{s} \neq \hat{\mathbf{s}} | \mathbf{h}_i\}$. Then the probability of correct decisions given the channel $\mathbf{h}_i$ is lower-bounded by

$$P_{c|\mathbf{h}_i} > \frac{1}{2} \sum_{k=A,B} \left(1 - P_{e,R|\mathbf{h}_i}\right) \left(1 - P_{e,k|\mathbf{h}_i}\right), \tag{4}$$

which results from the fact that even if an error occurs in both the MAC and the BC phase, a correct decision might be made when the error of the MAC phase is compensated by the error of the BC phase. However since such a probability may be negligible, this bound is very tight. From (4), The instantaneous error rate is upper-bounded by

$$\begin{aligned}
P_{e|\mathbf{h}_i} &= 1 - P_{c|\mathbf{h}_i} \\
&< P_{e,R|\mathbf{h}_i} + \frac{1}{2}\left(P_{e,A|\mathbf{h}_i} + P_{e,B|\mathbf{h}_i}\right) \\
&+ \frac{1}{2} P_{e,R|\mathbf{h}_i} P_{e,A|\mathbf{h}_i} + \frac{1}{2} P_{e,R|\mathbf{h}_i} P_{e,B|\mathbf{h}_i}. \qquad (5)
\end{aligned}$$

Meanwhile, we define the minimum distance at the MAC phase as

$$d_{\min,R}^{(i)} = \min_{\substack{\mathbf{s},\tilde{\mathbf{s}} \in \mathbb{Z}_L^2 \\ s_A \oplus s_B \neq \tilde{s}_A \oplus \tilde{s}_B}} \left|\mathbf{h}_i^{\mathsf{T}}(\mathbf{x} - \tilde{\mathbf{x}})\right|. \qquad (6)$$

Since the events $\mathbf{s} \to \tilde{\mathbf{s}}$ with $s_A \oplus s_B = \tilde{s}_A \oplus \tilde{s}_B$ are not counted as an error by definition of $P_{e,R|\mathbf{h}_i}$, the corresponding pairs $(\mathbf{s}, \tilde{\mathbf{s}})$ are excluded. Also, the minimum distance at the BC phase is defined as

$$d_{\min,k}^{(i)} = \min_{s_R, \tilde{s}_R \in \mathbb{Z}_L} \left|h_k^{(i)}(x_R - \tilde{x}_R)\right| \quad \text{for} \quad k = A, B.$$

Due to multiple access interference [19], $d_{\min,R}^{(i)}$ cannot exceed the minimum of $d_{\min,A}^{(i)}$ and $d_{\min,B}^{(i)}$ [20]. Thus we have

$$d_{\min,R}^{(i)} \leq \min\left(d_{\min,A}^{(i)}, d_{\min,B}^{(i)}\right).$$

It is well known that minimum distance based approach provides an adequately tight prediction of the error probability of ML detection [21]. Hence, the error probability $P_{e,R|\mathbf{h}_i}$ and $P_{e,k|\mathbf{h}_i}$ $(k = A, B)$ of (2) and (3) is dominated by the minimum distance $d_{\min,R}^{(i)}$ and $d_{\min,k}^{(i)}$ $(k = A, B)$, respectively. Then, the following relation with respect to the error rate of the MAC phase and the BC phase is established as

$$P_{e,R|\mathbf{h}_i} > \max\left(P_{e,A|\mathbf{h}_i}, P_{e,B|\mathbf{h}_i}\right). \qquad (7)$$

From the inequality (5) and (7), We deduce the conditional error probability upper bound in (5) is determined by the probability $P_{e,R|\mathbf{h}_i}$.

Consequently, by choosing the relay with the maximum minimum distance $d_{\min,R}^{(i)}$, we can improve the overall error performance. Finally, the minimum distance selection criterion is given as

$$i^\star = \arg\max_{i \in \mathcal{R}} d_{\min,R}^{(i)}$$

where $\mathcal{R}$ is defined as $\{R_i\}_{i=1,2,\cdots,N}$. Generally, the search size of $\frac{1}{2}L^2(L^2-1)$ is required to find the minimum distance $d_{\min,R}^{(i)}$. However, a method to compute the minimum distance with a small search size is introduced in [22]. For example, for 4QAM, we can reduce the required search size from 120 to 3 using the method in [22]. Similarly, for 16QAM, the search size decreases to 55 from 32640. In the next section, we derive the diversity order of the proposed algorithm.

IV. Diversity Analysis

In this section, we show that the minimum distance based relay selection algorithm proposed in the previous section achieves full diversity. Since the conventional PEP method which is difficult to apply in our system, we derive an upper bound of the average error probability based on the distribution of the minimum distance at the selected relay and identify the diversity order lower bound.

Now, let us evaluate upper bound of the average error probability. From the inequality (5) and (7), we can see that its conditional error probability $P_{e|\mathbf{h}_i}$ is upper-bounded by $P_{e|\mathbf{h}_i} < 3P_{e,R|\mathbf{h}_i}$. Using the union bound [23], $P_{e,R|\mathbf{h}_{i^\star}}$ follows

$$P_{e,R|\mathbf{h}_{i^\star}} < (L^2-1)Q\left(\sqrt{\frac{\{d_{\min,R}^{(i^\star)}\}^2 P}{2\sigma^2}}\right) \qquad (8)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$.

Let us denote the squared minimum distance at the ith relay as $\Delta_i = \{d_{\min,R}^{(i)}\}^2$, then the average error probability of (8) denoted by $P_{e,R}$ is given by

$$P_{e,R} < \int_0^\infty (L^2-1)Q\left(\sqrt{\frac{\delta P}{2\sigma^2}}\right) f_{\Delta_{i^\star}}(\delta) d\delta. \qquad (9)$$

Applying the Chernoff bound $Q(x) \leq \exp(-x^2/2)$, (9) can be written as

$$P_{e,R} < \int_0^\infty (L^2-1)\exp\left(-\frac{\delta P}{4\sigma^2}\right) f_{\Delta_{i^\star}}(\delta) d\delta. \qquad (10)$$

Now, we obtain the PDF of $f_{\Delta_{i^\star}}(\delta)$ of $\Delta_{i^\star}$ to solve the integral in (10). From (6), Δ_i is rephrased as

$$\Delta_i = \min_{\mathbf{e} \in \mathcal{E}} |\mathbf{h}_i^{\mathsf{T}}\mathbf{e}|^2 = \min_{\mathbf{e} \in \mathcal{E}} \mathbf{h}_i^{\mathsf{T}}\mathbf{e}\mathbf{e}^{\mathsf{H}}\mathbf{h}_i^*$$

where $\mathbf{e}$ is denoted by $\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}}$ and $\mathcal{E}$ equals the set of all possible $\mathbf{e}$.

Since $\mathbf{e}\mathbf{e}^{\mathsf{H}}$ is a non-negative definite Hermitian matrix, we can apply eigenvalue decomposition as $\mathbf{e}\mathbf{e}^{\mathsf{H}} = \mathbf{P_e}\mathbf{\Lambda_e}\mathbf{P_e}^{\mathsf{H}}$ where $\mathbf{P_e}$ is a unitary matrix and $\mathbf{\Lambda_e}$ represents a diagonal matrix. Then, Δ_i is given by

$$\Delta_i = \min_{\mathbf{e} \in \mathcal{E}} \mathbf{h}_i^{\mathsf{T}}\mathbf{P_e}\mathbf{\Lambda_e}\mathbf{P_e}^{\mathsf{H}}\mathbf{h}_i^* = \min_{\mathbf{e} \in \mathcal{E}} \tilde{\mathbf{h}}_{i,\mathbf{e}}^{\mathsf{T}}\mathbf{\Lambda_e}\tilde{\mathbf{h}}_{i,\mathbf{e}}^*. \qquad (11)$$

From the fact that the rank of $\mathbf{e}\mathbf{e}^{\mathsf{H}}$ is equal to 1, equation (11) can be rewritten as

$$\Delta_i = \min_{\mathbf{e} \in \mathcal{E}} \lambda_{\mathbf{e}} |\tilde{h}_{i,\mathbf{e}}|^2 \triangleq \min_{\mathbf{e} \in \mathcal{E}} \Omega_{i,\mathbf{e}}$$

where $\tilde{\mathbf{h}}_{i,\mathbf{e}}$ is defined by $\tilde{\mathbf{h}}_{i,\mathbf{e}}^{\mathsf{T}} = \mathbf{h}_{\mathbf{e}}^{\mathsf{T}}\mathbf{P_e}$, and $\lambda_{\mathbf{e}}$ and $\tilde{h}_{i,\mathbf{e}}$ indicate the first element of $\mathbf{\Lambda_e}$ and $\tilde{\mathbf{h}}_{i,\mathbf{e}}$, respectively. Since $\mathbf{P_e}$ is unitary, $\tilde{h}_{i,\mathbf{e}}$ has a also complex Gaussian distribution. Then $\Omega_{i,\mathbf{e}}$ follows an exponential distribution and its PDF is given by

$$f_{\Omega_{i,\mathbf{e}}}(\omega) = \frac{1}{\lambda_{\mathbf{e}}} \exp\left(-\frac{\omega}{\lambda_{\mathbf{e}}}\right).$$

Because $\Omega_{i,\mathbf{e}}$ and $\Omega_{i,\tilde{\mathbf{e}}}$ are correlated with $\mathbf{e} \neq \tilde{\mathbf{e}}$, it is not easy to obtain an exact distribution of Δ_i. As shown in [24], the average error performance at high SNR depends only on the near zero behavior of the PDF $f_{\Delta_{i^\star}}(\delta)$, i.e., $\delta \to 0^+$. Also, it is proven in [25] that for K positive random variables X_k ($k = 1, 2, \cdots, K$), the PDF $f_Y(y)$ of $Y = \min(X_1, X_2, \cdots, X_K)$ near zero is computed by a sum of the PDFs $f_{X_k}(y)$ of x_k, i.e., $f_Y(y) = \sum_{k=1}^{K} f_{X_k}(y)$ for $y \to 0^+$. By utilizing these, the approximated PDF $\tilde{f}_{\Delta_i}(\delta)$ of Δ_i is simply calculated by a sum of the PDFs $f_{\Omega_{i,\mathbf{e}}}(\delta)$ of $\Omega_{i,\mathbf{e}}$ as

$$\tilde{f}_{\Delta_i}(\delta) = \sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right) \quad \text{for } \delta \to 0^+.$$

Finally, using order statistics [26], the approximated CDF $\tilde{F}_{\Delta_{i^\star}}(\delta)$ of $\Delta_{i^\star} = \max_{i\in\mathcal{R}} \Delta_i$ is given as

$$\begin{aligned} \tilde{F}_{\Delta_{i^\star}}(\delta) &= \Pr\Big[\max_{i\in\mathcal{R}} \Delta_i < \delta\Big] \\ &= \Pr\Big[\Delta_1 < \delta, \Delta_2 < \delta, \cdots, \Delta_N < \delta\Big] \\ &= F_{\Delta_1}(\delta) F_{\Delta_2}(\delta) \cdots F_{\Delta_N}(\delta) \\ &= \left[\sum_{\mathbf{e}\in\mathcal{E}} \left\{1 - \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\}\right]^N. \end{aligned} \quad (12)$$

Taking derivatives of (12), we obtain $\tilde{f}_{\Delta_{i^\star}}(\delta)$ as

$$\begin{aligned} \tilde{f}_{\Delta_{i^\star}}(\delta) &= \frac{d}{d\delta} \tilde{F}_{\Delta_{i^\star}}(\delta) \\ &= N\left[\sum_{\mathbf{e}\in\mathcal{E}} \left\{1 - \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\}\right]^{N-1} \left\{\sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\}. \end{aligned}$$

By exploiting the derived PDF $\tilde{f}_{\Delta_{i^\star}}(\delta)$, $P_{e,R}$ in (10) at high SNR can be represented as

$$\begin{aligned} P_{e,R} < \int_0^\infty (L^2-1) \exp\left(-\frac{\delta P}{4\sigma^2}\right) N \left[\sum_{\mathbf{e}\in\mathcal{E}} \left\{1 - \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\}\right]^{N-1} \\ \times \left\{\sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\} d\delta. \end{aligned} \quad (13)$$

Denoting $\lambda_{\min} = \min_{\mathbf{e}\in\mathcal{E}} \lambda_{\mathbf{e}}$, (13) can be upper-bounded as

$$\begin{aligned} P_{e,R} < \int_0^\infty (L^2-1) \exp\left(-\frac{\delta P}{4\sigma^2}\right) N E^{N-1} \left\{1 - \exp\left(-\frac{\delta}{\lambda_{\min}}\right)\right\}^{N-1} \\ \times \left\{\sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\} d\delta \end{aligned} \quad (14)$$

where E indicates the size of the set $\mathcal{E}$. To make the problem tractable, we apply a Taylor series approximation $\exp(x) \simeq 1 + x$ for $x \to 0^+$ to (14). Then, it can be approximately expressed as

$$\begin{aligned} P_{e,R} < C \int_0^\infty \exp\left(-\frac{\delta P}{4\sigma^2}\right) \left(\frac{\delta}{\lambda_{\min}}\right)^{N-1} \\ \times \left\{\sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} \exp\left(-\frac{\delta}{\lambda_{\mathbf{e}}}\right)\right\} d\delta \end{aligned} \quad (15)$$

where $C = (L^2 - 1) N E^{N-1}$.

After some manipulations, (15) can be rewritten as

$$P_{e,R} < C \sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} \int_0^\infty \exp\left(-\left(\frac{\delta P}{4\sigma^2} + \frac{1}{\lambda_{\mathbf{e}}}\right)\right) \left(\frac{\delta}{\lambda_{\min}}\right)^{N-1} d\delta \quad (16)$$

Finally after solving the integration, (16) can be given as

$$\begin{aligned} P_{e,R} &< C \sum_{\mathbf{e}\in\mathcal{E}} \frac{(N-1)!}{\lambda_{\mathbf{e}} \lambda_{\min}^{N-1}} \left(\frac{P}{4\sigma^2} + \frac{1}{\lambda_{\mathbf{e}}}\right)^{-N} \\ &< \left(\frac{P}{\sigma^2}\right)^{-N} \frac{C(N-1)!}{4^{-N} \lambda_{\min}^{N-1}} \sum_{\mathbf{e}\in\mathcal{E}} \frac{1}{\lambda_{\mathbf{e}}} = \gamma \left(\frac{P}{\sigma^2}\right)^{-N} \end{aligned} \quad (17)$$

where γ designates a constant.

It is well known that the maximum diversity order of two-way relay channels with N-relays equals N. Therefore, we now see from the result in (17) that the proposed relay selection method with PNC which maximizes the minimum distance at the relay achieves a full diversity order.

V. Simulation Results

In this section, we present the simulation results of the bit error rate (BER) performance for various relay selection schemes to demonstrate the efficiency of the proposed scheme and verify our analysis. We assume uncoded systems and the average SNR is defined as P/σ^2. To compute the average BER, we perform Monte-carlo simulations on 10^8 channel realizations per each average SNR. We compare the following two conventional selection schemes with the proposed method. The first one is best worse channel selection [11] which chooses the relay with the worse channel, $\min(|h_A^{(i)}|, |h_B^{(i)}|)$. The second scheme is best harmonic mean selection [12] where the relay which maximizes the harmonic mean of two channels, $(|h_A^{(i)}|^{-2} + |h_B^{(i)}|^{-2})^{-1}$, is selected.

Fig. 2 shows the BER performance with 4QAM. We see that the harmonic mean selection scheme exhibits a slightly better performance compared to the best worse channel selection scheme. We observe that two conventional schemes do not provide any diversity gain regardless of the number of relays. Moreover, it is interesting to see that a performance gain is saturated as the number of relays increases. In contrast, we find that the proposed minimum distance selection scheme has a diversity gain as expected from the analysis.

Fig. 3 illustrates the BER performance of the minimum distance criterion for different modulations. It is clear from the plot that the proposed criterion is able to achieve the full diversity order N for both 4QAM and 16QAM. These results are also consistent with our diversity analysis. Throughout the simulations, we confirm that our proposed scheme is very effective for relay selection with two-way PNC systems.

VI. Conclusions

In this paper, we have investigated a relay selection system in two-way relay channels with physical network coding. Since conventional schemes for analog network coding cannot yield a diversity gain in physical network coding systems, we

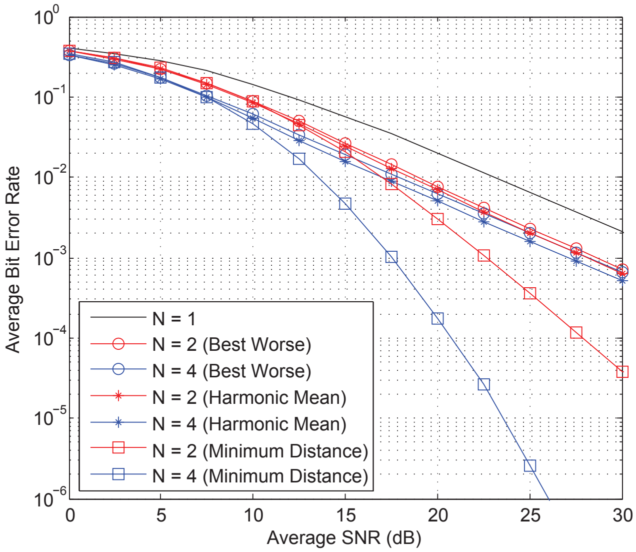


Fig. 2. Bit error rate comparison with 4QAM

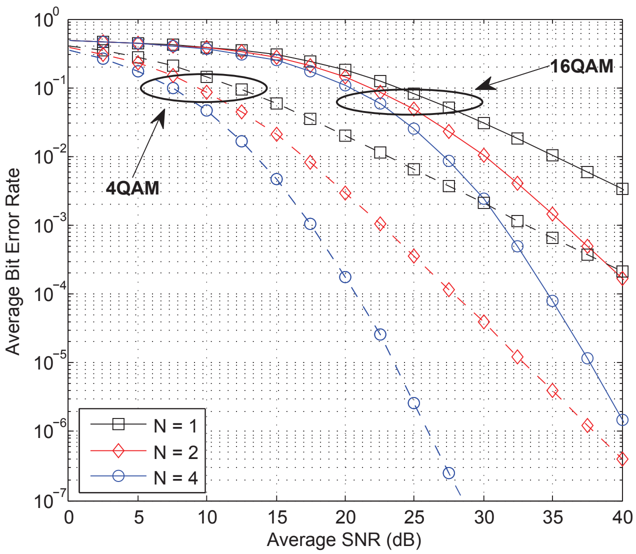


Fig. 3. Bit error rate of the proposed relay selection scheme for 4QAM and 16QAM

have proposed a new selection criterion based on the minimum distance. In addition, with an asymptotic approach, we have analytically proved that our proposed selection scheme achieves full diversity. Simulation results have been presented to confirm the superiority of our scheme and the accuracy of the analysis.

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