Analysis of Multiband Sensing-Time Joint Detection Framework for Cognitive Radio Systems

Salma Zainab Farooq[*†] and Abdul Ghafoor[*]
[*] Department of Electrical Engineering, Military College of Signals
National University of Sciences and Technology (NUST)
Islamabad, Pakistan
[†] Faculty of Electrical Engineering,
Institute of Space Technology (IST)
Islamabad, Pakistan
Email: zaineb.farooq@nust.edu.pk, abdulghafoor-mcs@nust.edu.pk

Abstract—An optimal multiband spectrum sensing framework is explored which jointly detects a wideband orthogonal frequency-division multiple access primary signal over multiple non-contiguous narrowband channels. The spectrum sensing problem optimizes the aggregate opportunistic throughput of a cognitive radio network restraining the interference to the primary user keeping the individual energy detector thresholds and sensing slot duration as optimization variables. The problem is solved as convex optimization under certain practical constraints for any input signal and channel noise conditions. Simulated results for BPSK modulated signal and real-valued Gaussian noise show the increase in secondary throughput when both subchannel thresholds and sensing time is adaptively chosen. The effect of sub-channel parameter variation on the overall problem is also shown.

Keywords- multiband; spectrum sensing; sensing-time; oppurtunistic throughput, convex optimization

I. INTRODUCTION

Radio frequency spectrum is a valuable natural resource. It is allocated statically to the end users by various government agencies. This static spectrum allocation suffers from spectrum allocation congestion and underutilized spectrum usage in some frequency bands. A promising solution appears in the form of Cognitive Radio (CR) – a technology that dynamically senses unused spectrum segments in a target spectrum pool and communicates using the unused spectrum segments in ways that cause no harmful interference to the primary users (PU) of the spectrum [1]. Usually a CR network needs to make opportunistic access simultaneously over a wideband spectrum for efficient service to the secondary users (SU) [2]. However, when the bandwidth to be monitored is large, sequential individual sensing of a large number of primary channels poses significant challenges to the receiver's analog front-end design [3]. To alleviate this problem, the bandwidth of interest is divided into subbands, which are then downconverted, digitized and can be analyzed sequentially [3-6] or in parallel [2, 7-11] though to provide reliable and efficient CR services, parallel sensing infrastructure is advisable [2].

The multiband joint detection algorithm (MJD) devised in [8] considers sensing multiple narrowband channels simultaneously so as to maximize the aggregate opportunistic throughput of a CR system while limiting the interference to the primary communication system maintaining a certain spectral utilization level. The MJD framework suffers from not sensing the channel periodically, as the spectrum must be vacated on the reappearance of the PU [13]. Based on MJD, [10] proposed a multiband sensing-time joint detection (MSJD) framework having the same constraints as the MJD. This optimized unified framework minimizes the interference to PU through an adaptive selection of sensing-time and the threshold of the narrowband detector bank. It is shown that the MSJD framework though generally non-convex, can be solved as a convex optimization problem for a complex PSK modulated primary signal and a circularly symmetric complex Gaussian (CSCG) channel under certain practical conditions.

In this paper, the MSJD framework is analyzed for generalized input signal and noise conditions. The framework is further evaluated for some specific conditions: BPSK modulated primary signal received with real valued Gaussian noise; CSCG condition for both input signal and noise and real valued Gaussian condition for both input signal and noise. The latter two cases might represent signals with rich inter-symbol interference, precoded signals such as orthogonal frequency division multiplexing (OFDM) signals or OFDM signals with linear precoding [12]. The convexity ranges under which the framework appears convex are determined. In addition, the effect of change in either SU dependent parameters or received signal parameters on the overall problem is demonstrated.

II. SYSTEM MODEL

Consider an orthogonal frequency-division multiple access based primary communication system over a wideband spectrum [2]. This spectrum is divided into K non-overlapping narrowband channels. It is assumed that J number of PUs share the spectrum where each PU occupies a subset S_j of these subbands. Some of these subbands might not be used by PUs all the time and are available for opportunistic access. To track the signal occupancy, the signal is transformed to frequency domain. The signal occupancy in the k[th] subband can be tested by binary hypothesis testing with $\mathcal{H}_{0,k}$ and $\mathcal{H}_{1,k}$ representing the PU is idle and busy respectively:

$$\mathcal{H}_{0,k} : R_k = W_k \qquad\qquad k = 1,2,...K$$
$$\mathcal{H}_{1,k} : R_k = H_k S_k + W_k$$

where W_k, H_k and S_k represent the K-point fast Fourier transformations of the noise, the discrete time channel impulse response, and the primary signal [8]. The noise is assumed to be Gaussian, independent and identically distributed with mean zero and variance σ_w^2.

The frame length T used for periodic sensing consists of one sensing slot τ and one data transmission slot T- τ. Within the duration τ of the sensing slot, each user collects $M= \tau f_s$ samples of the received signal for all the K channels, where f_s is the sampling frequency.

Energy detection is performed for each subband during the sensing slot τ. The decision statistic T_k for each of the subchannels can be written as

$$T_k = \frac{1}{M}\sum_{m=1}^{M} | R_k(m) |^2$$

The decision rule can thus be given by

$$T_k \underset{\mathcal{H}_{0,k}}{\overset{\mathcal{H}_{1,k}}{\gtrless}} \varepsilon_k$$

where ε_k is the decision threshold in subband k. Subsequently $\boldsymbol{\varepsilon}$ is the threshold vector and $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, ..., \varepsilon_K]^T$. The associated probabilities are:

$$\boldsymbol{P}_f(\boldsymbol{\varepsilon},\tau) = \left[P_f^{(1)}(\varepsilon_1,\tau),.......P_f^{(K)}(\varepsilon_K,\tau)\right]^T$$
$$\boldsymbol{P}_m(\boldsymbol{\varepsilon},\tau) = \left[P_m^{(1)}(\varepsilon_1,\tau),.......P_m^{(K)}(\varepsilon_K,\tau)\right]^T$$
$$\boldsymbol{P}_d(\boldsymbol{\varepsilon},\tau) = \left[P_d^{(1)}(\varepsilon_1,\tau),.......P_d^{(K)}(\varepsilon_K,\tau)\right]^T$$

where $\boldsymbol{P}_d$ is the probability of detection. $\boldsymbol{P}_m$ can also be expressed as $\boldsymbol{P}_m(\varepsilon,\tau)= \mathbf{1}\text{-}\boldsymbol{P}_d(\varepsilon,\tau)$, where $\mathbf{1}$ denotes the all ones vector. For a good detection algorithm, $\boldsymbol{P}_d$ should be as high as possible while $\boldsymbol{P}_f$ should be as low as possible. . For a given frame duration T, larger the sensing time τ, smaller the data transmission time T- τ. On the other hand, for a given probability of detection, the longer the sensing time, the lower the probability of false alarm.

For a large number of samples (e.g., M>40), the central limit theorem will be used to approximate the probability distribution function of T_k as a normal distribution under both hypothesis with means [13]

$$\mathbb{E}(R_k) \sim \begin{cases} \sigma_w^2 & \mathcal{H}_{0,k} \\ (\gamma_k+1)\sigma_w^2 & \mathcal{H}_{1,k} \end{cases}$$

and variances

$$\mathrm{var}(R_k) \sim \begin{cases} \dfrac{\left[E\,|W_k|^4 - \sigma_w^4\right]}{M} & \mathcal{H}_{0,k} \\ \dfrac{\left[E\,|S_k|^4 + E\,|W_k|^4 - (\sigma_s^2 - \sigma_w^2)^2\right]}{M} & \mathcal{H}_{1,k} \end{cases}$$

The probability of false alarm $P_f^{(k)}(\varepsilon_k,\tau)$ and the probability of detection $P_d^{(k)}(\varepsilon_k,\tau)$ for the k[th] subchannel would thus be approximated as

$$P_f^{(k)}(\varepsilon_k,\tau) = \mathrm{Q}\left(\frac{\sqrt{\tau f_s}\left(\varepsilon_k - \sigma_w^2\right)}{\sqrt{\left[E\,|W_k|^4 - \sigma_w^4\right]}}\right) \tag{1}$$

$$P_d^{(k)}(\varepsilon_k,\tau) = \mathrm{Q}\left(\frac{\sqrt{\tau f_s}\left(\varepsilon_k - (\gamma_k+1)\sigma_w^2\right)}{\sqrt{\left[E\,|S_k|^4 + E\,|W_k|^4 - (\sigma_s^2 - \sigma_w^2)^2\right]}}\right) \tag{2}$$

where $\gamma_k = \sigma_s^2 / \sigma_w^2$ is the received signal-to-noise ratio (SNR) of the PU for the kth subchannel measured at the secondary receiver, under the hypothesis $\mathcal{H}_1$ [13]. Q(x) is tail probability of the standard Gaussian distribution and is a monotonically decreasing function.

III. Multiband Sensing Time Joint Detection Framework

Let o_k be the opportunistic throughput of the SU, at subchannel k, when it operates in the absence of the PUs and $\boldsymbol{o} = [o_1, o_2, ..., o_K]^T$. The aggregate oppurtunistic throughput can be defined as [10,11]

$$R(\boldsymbol{\varepsilon},\tau) = \left(\frac{\mathrm{T}-\tau}{\mathrm{T}}\right)\boldsymbol{o}^T\left(\mathbf{1} - \boldsymbol{P}_f(\boldsymbol{\varepsilon},\tau)\right)$$

where $1 - P_f^{(k)}$ represents the percentage of spectrum vacancies detected by the cognitive user, (T-τ)/T represents the portion of frame duration available for opportunistic transmission.

Let ς be the cost of interfering with a PU in the k[th] subchannel and $\boldsymbol{\varsigma} = [\varsigma_1, \varsigma_2, ..., \varsigma_K]^T$. Accordingly, the aggregate interference the primary network can be defined as

$$I_j(\varepsilon,\tau) = \sum_{j \in S_j} \varsigma_j P_m^{(j)}(\varepsilon_j,\tau) \qquad\qquad j = 1,2,....J$$

For a single-user primary network, all subbands are used by one PU and J=1. The MSJD framework [10] jointly optimizes the threshold vector $\boldsymbol{\varepsilon}$ and sensing time τ, so as to maximize the available throughput of the SU while keeping the weighted interference with PUs below a desired level. Mathematically the optimization problem is stated as

$$\max_{(\varepsilon,\tau)} \quad R(\boldsymbol{\varepsilon},\tau) \qquad \text{(P1)}$$

$$\text{s.t.} \quad I(\boldsymbol{\varepsilon},\tau) \le \xi \qquad \text{(C1)}$$

$$\boldsymbol{P_m}(\boldsymbol{\varepsilon},\tau) \preceq \boldsymbol{\alpha} \qquad \text{(C2)}$$

$$\boldsymbol{P_f}(\boldsymbol{\varepsilon},\tau) \preceq \boldsymbol{\beta} \qquad \text{(C3)}$$

where ξ denotes the maximum aggregate interference tolerated by the primary network, $\boldsymbol{\alpha}=[\alpha_1, \alpha_2, \ldots\alpha_K]$ and $\boldsymbol{\beta}=[\beta_1, \beta_2, \ldots\beta_K]$ are the minimum requirements of each subband.

To analyze the problem as a convex optimization one, it is reformulated as [8,10]

$$\min_{(\boldsymbol{\varepsilon},\tau)} \quad R_{loss}(\boldsymbol{\varepsilon},\tau) \qquad \text{(P2)}$$

Here R_{loss} is the opportunistic throughput loss due to the limitations in sensing. This loss is contributed both by the false alarm and sensing time τ. i.e.,

$$R_{loss}(\boldsymbol{\varepsilon},\tau) = \boldsymbol{o}^T\left[\boldsymbol{P_f}(\boldsymbol{\varepsilon},\tau)\left(1-\frac{\tau}{T}\right)+\boldsymbol{1}\frac{\tau}{T}\right]$$

The constraints are the same as Problem (P1)

Theorem 1: The function $P_f^{(k)}(\varepsilon_k,\tau)$ *in* (1) *is convex in* ε_k *and* τ *if* $P_f^{(k)}(\varepsilon_k,\tau) \le \mathrm{Q}(1/\sqrt{3})$

Proof: The Hessian of the function can be calculated as

$$C_1 \times \begin{vmatrix} \dfrac{2\tau^2 f_s}{A}(\varepsilon_k-\sigma_w^2) & \dfrac{\tau f_s}{A}(\varepsilon_k-\sigma_w^2)^2-1 \\ \dfrac{\tau f_s}{A}(\varepsilon_k-\sigma_w^2)^2-1 & \dfrac{(\varepsilon_k-\sigma_w^2)}{2\tau}+\dfrac{f_s}{2A}(\varepsilon_k-\sigma_w^2)^3 \end{vmatrix}$$

where $A=\left[E\,|W_k|^4-\sigma_w^4\right]$ and $C_1=\dfrac{1}{4\pi A^{1/2}}\sqrt{\dfrac{f_s}{\tau}}$

The determinant of the equation is

$$Det(.)=C_1^2\times\left(\frac{3\tau f_s}{A}(\varepsilon_k-\sigma_w^2)^2-1\right)$$

For the matrix to be positive semi-definite its determinant should be $\sqrt{\dfrac{\tau f_s}{A}}(\varepsilon_k-\sigma_w^2)\ge\sqrt{\dfrac{1}{3}}$. Consequently, $P_f^{(k)}(\varepsilon_k,\tau)$ is convex under the stated condition. Exploiting the monotonically decreasing nature of Q-function proves the result. ■

Theorem 2: The function $P_m^{(k)}(\varepsilon_k,\tau)$ *in* (2) *is convex in* ε_k *and* τ *if* $P_m^{(k)}(\varepsilon_k,\tau) \le \mathrm{Q}(1/\sqrt{3})$

Proof: Following a procedure similar to the proof of Theorem 1, it can be shown that $P_d^{(k)}(\varepsilon_k,\tau)$ is concave for values less than or equal to $\mathrm{Q}(1/\sqrt{3})$, hence $P_m(\varepsilon,\tau)=1-P_d(\varepsilon,\tau)$ is a convex function under the given condition. ■

These ranges were separately determined for the cases when: input signal and noise are both CSCG; input signal is BPSK modulated received with real valued Gaussian noise; real valued Gaussian condition for both input signal and noise. The ranges under which $\boldsymbol{P_m}$ and $\boldsymbol{P_f}$ are convex are exactly the same. The proof for the first case is given in [14]. Rest of the results, omitted for brevity, reinforce the ranges obtained for the general model.

Theorem 3: The function $R_{loss}(\boldsymbol{\varepsilon},\tau)$ *in* (3) *is convex in* ε_k *and* τ *if* $P_f^{(k)}(\varepsilon_k,\tau) \le \mathrm{Q}(1/\sqrt{3})$ *and* $\tau/T \le 0.5$

Proof: The Hessian of the function can be calculated as

$$\begin{bmatrix} \left(1-\dfrac{\tau}{T}\right)\dfrac{\partial^2 P_f^{(k)}}{\partial\varepsilon_k^2} & \left(1-\dfrac{\tau}{T}\right)\dfrac{\partial P_f^{(k)}}{\partial\varepsilon_k\partial\tau}-\dfrac{1}{T}\dfrac{\partial P_f^{(k)}}{\partial\varepsilon_k} \\ \left(1-\dfrac{\tau}{T}\right)\dfrac{\partial P_f^{(k)}}{\partial\varepsilon_k\partial\tau}-\dfrac{1}{T}\dfrac{\partial P_f^{(k)}}{\partial\varepsilon_k} & \left(1-\dfrac{\tau}{T}\right)\dfrac{\partial^2 P_f^{(k)}}{\partial\tau^2}-\dfrac{2}{T}\dfrac{\partial P_f^{(k)}}{\partial\tau} \end{bmatrix}$$

It is positive definite for $P_f^{(k)}(\varepsilon_k,\tau)\le\mathrm{Q}(1/\sqrt{3})$ and $\tau/T \le 0.5$ as proven in [10,11] ■

As a non-negative weighted sum of convex functions is a convex function, under Theorems (1-3), both the objective and constraint functions are convex. The constraints for all the three cases under consideration can be summarized as

$$\left.\begin{array}{ll} 0\le\alpha_k\le\mathrm{Q}\left(1/\sqrt{3}\right), & k=1,2,\ldots K \\ 0\le\beta_k\le\mathrm{Q}\left(1/\sqrt{3}\right), & k=1,2,\ldots K \\ 0\le\tau_{\max}\le 0.5\mathrm{T} & \end{array}\right\}$$

The problem is categorized as a convex optimization one and can be efficiently solved by convex optimization algorithms.

The rest of the paper only considers the case where the primary signal is BPSK modulated and noise is real Gaussian. Accordingly the equation for $\boldsymbol{P_f}$ modifies as

$$P_f^{(k)}(\varepsilon_k,\tau)=\mathrm{Q}\left(\left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)\sqrt{\frac{\tau f_s}{2}}\right) \qquad (3)$$

and $\boldsymbol{P_d}$ becomes

$$P_d^{(k)}(\varepsilon_k, \tau) = \mathrm{Q}\left(\left(\frac{\varepsilon_k}{\sigma_w^2} - \gamma_k - 1\right)\sqrt{\frac{\tau f_s}{2(2\gamma_k + 1)}}\right) \tag{4}$$

IV. Simulation Results

In this section computer simulation results are presented to evaluate the MSJD framework. Consider a 48-MHz primary system where the wideband channel is equally divided into eight subbands. Associated with each subband is a received signal SNR γ_k, an achievable throughput rate o_k if used by CRs and cost coefficient ς_k indicating the penalty if the primary signal is interfered with by SUs. It is expected that the opportunistic spectrum utilization is at least 50 %(i.e., $\beta = 0.5$) and the probability that the PU is interfered with is at most 20% (i.e. $\alpha = 0.2$). The maximum time for which the SU is unaware of the PU's activity is chosen such that $M = f_S \mathrm{T} = 3000$.

Considering the two stage optimization framework in which first the threshold vector $\boldsymbol{\varepsilon}$ is optimized assuming a constant τ [10,11].Based on the threshold, the value for R_{loss} is determined as R_{loss}^{old}. As the second stage of the problem the optimum sensing slot duration τ is determined. This variation in τ changes R_{loss} to R_{loss}^{new} as well as the constraints of the first optimization problem. The algorithm is repeated iteratively based on updated values of $\boldsymbol{\varepsilon}$ and τ, till the value of $|R_{loss}^{old} - R_{loss}^{new}| < \delta$, where δ is the tolerance.

TABLE I. Typical System Parameter Set Used For Simulation

k	1	2	3	4	5	6	7	8
γ_k	0.38	1.37	0.32	0.24	0.35	0.27	0.39	0.38
o_k (kbps)	612	524	623	139	451	409	909	401
ς_k	1.91	8.17	4.23	3.86	7.16	6.05	0.82	1.30

The performance of the MSJD framework is compared with that of the MJD framework [8] and a uniform-threshold approach. The MJD framework optimizes the threshold vector only and the uniform threshold has constant values of the threshold vector and sensing-time. All the three approaches maximize the aggregate opportunistic throughput of the CR system subject to the same constraints on the interference. As illustrated in Fig. 1, the MSJD framework with heterogeneous thresholds and adaptable sensing-time outperforms the other two approaches as it dynamically balances the spectral utilization and interference reduction. In addition, it is observed that the aggregate opportunistic rate increases as the constraint on the aggregate interference is relaxed.

Fig. 2 is a graph of the number of samples M is versus the SNR increment above the values listed in Table 1. An improvement in the channel condition results in a decrease in the optimal number of samples required for sensing the spectrum strengthening the need of dynamically assigning the sensing time. The effect of SNR variation on the subchannel thresholds is analyzed in Fig. 3 which plots the thresholds for all the subbands when the SNR is the same as in Table I, incremented 0.5 dB and 1 dB above the values mentioned in Table I. It is seen that as γ_k increases, under the same interference constraints the thresholds increase. As a higher threshold will result in a smaller probability of false alarm, and a larger probability of miss, the secondary throughput will also increase.

Fig. 4 analyzes the effect of increasing the cost of interference with the PU on the threshold for first subchannel. As ς_1 increases, the corresponding threshold ε_1 decreases, so as to decrease the first subband's interference with the PU causing the throughput to decrease. Fig. 5 is an analysis of the effect of increasing the opportunistic throughput for the first subchannel. As expected, an increase in o_1 results in a corresponding increase in ε_1 such that the overall opportunistic throughput increases.

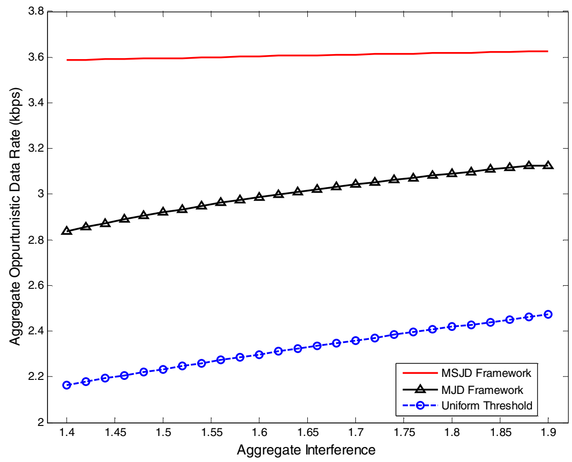
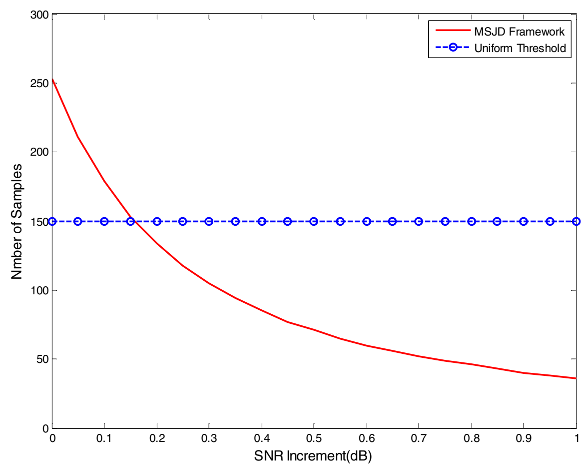


Figure 1. The available opportunistic throughput vs. the aggregate interference to the primary network



The number of samples vs. the SNR increment (dB) above γ_k listed in Table I

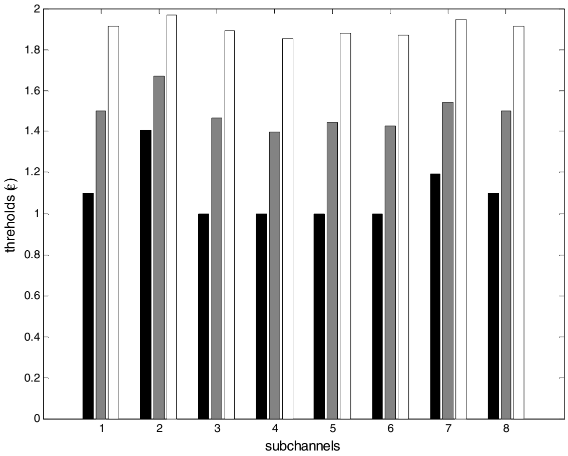


Figure 2. The subchannel threshold variation vs .the SNR increment (dB) above γ_k listed in Table I

V. CONCLUSION

This paper discusses the optimal multiband sensing-time joint detection framework for wideband spectrum sensing proposed in [10, 11]. This framework optimizes the dynamic spectrum access by the SUs over multiple contiguous frequency bands, considering the sensing slot duration and individual narrowband detector thresholds as optimization variables. The ranges are determined under which the problem can be solved as convex for a general input signal and noise conditions. Furthermore, some particular primary signals received in certain specific channels conditions are also analyzed for the MSJD framework. It can be concluded that the MSJD framework can be considered as convex for any primary signal and noise situation under the conditions $\boldsymbol{P}_m(\boldsymbol{\varepsilon},\tau) \preceq \mathrm{Q}\left(1/\sqrt{3}\right)$, $\boldsymbol{P}_f(\boldsymbol{\varepsilon},\tau) \preceq \mathrm{Q}\left(1/\sqrt{3}\right)$ and $\tau_{\max} \le 0.5\mathrm{T}$. Additionally the effect of system parameters' variation on the secondary throughput is discussed.

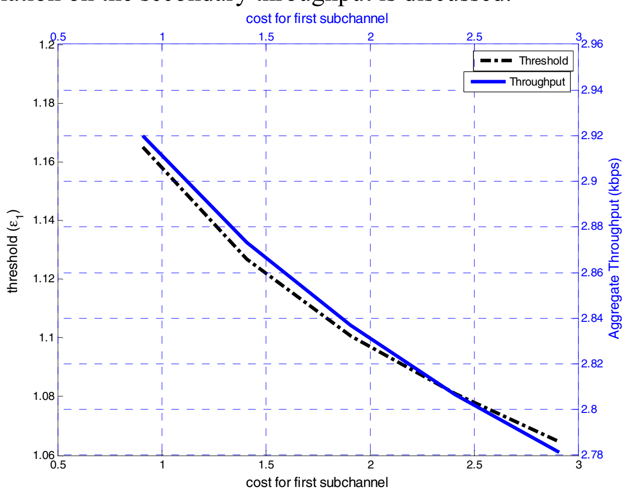


Figure 3. The effect of variation of ς_1 on ε_1

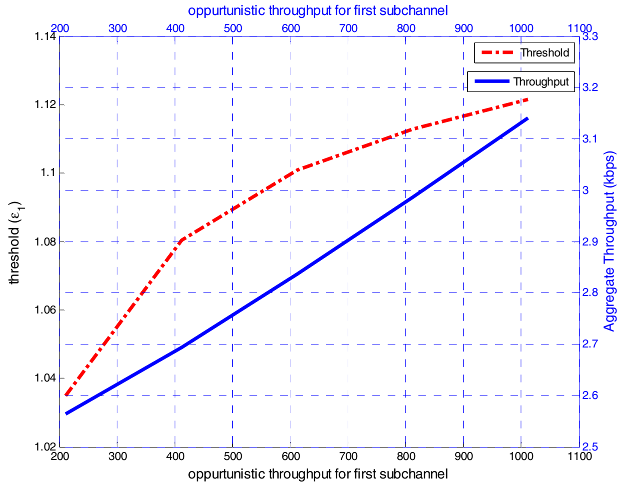


Figure 4. The effect of variation of o_1 on ε_1

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