Iterative Joint Source and Relay Optimization for Multiuser MIMO Relay Systems

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***Abstract*—In this paper, a joint source and relay optimization problem is studied for a multiuser multiple-input multiple-output (MIMO) relay system. Assuming that the channel state information (CSI) at the source and relay is available, two amplify and forward (AF) relaying schemes are proposed under the criterion of maximizing the sum-rate. First, a scheme which iteratively searches the optimal source and relay matrices by deriving the partial derivatives of the sum-rate and applying the gradient search algorithm is proposed. Next, in order to reduce the computational complexity, an alternating method utilizing the equivalent channel method is developed. This method also resorts to a so called maximum-signal-leakage-and-noise-ratio (SLNR) that can suppress the co-channel interference (CCI) and noise at the users effectively. Theoretical analysis and Monte Carlo simulation illustrate the performance of the both schemes.**

I. INTRODUCTION

Multi-antenna relaying has become a promising technique in next generation wireless communication systems including 3Gpp LTE-Advanced and IEEE 802.16m. It would provide an excellent information rate service with high data rate and ubiquitous coverage. For a three-node two-hop linear non-regenerative MIMO relay system, the achievable rate and capacity upper bound have been studied in [1]. In [2], the optimal relay amplifying matrix which maximizes the mutual information(MI) between source and user is derived assuming that the source covariance matrix is an identity matrix. Independent from [2], the authors of [3] also study a similar problem and arrived at the same optimal relay amplifying matrix. In [4], both the source covariance matrix and the relay amplifying matrix are jointly designed to maximize the source-destination MI. Minimal arithmetic mean-square error (MS-MSE)-based approaches for MIMO relay systems are developed in [5] . It has been shown in [6] that the optimal source, relay and receiving matrices jointly diagonalize the source-relay-destination channel.

The above researches focus on a relay system with a single user. In practical systems, however, each relay needs to support multiple users. For users with a single antenna, the researches are presented in [7]-[9]. In [7], the optimal design of non-regenerative relay with different structure for multiuser MIMO relay systems based on sum-rate is investigated. Assuming zero-forcing dirty paper coding at the base station and linear operations at the relay station, it proposes upper and lower bounds on the achievable sum-rate. Aiming at transmit power minimization under quality-of-service (QoS) requirements, an iterative joint precoding algorithm is proposed in [8]. With an equivalent channel method, a downlink multiuser MIMO AF relay network is investigated in [9]. For users with multiple antennas, prior efforts are reported in [10]-[11]. An efficient source and relay precoding design strategy by diagonalizing the equivalent channel of the system is proposed in [11].

In this paper, we study the design of the source and relay matrices by considering the maximization of system throughput subject to power constraints. The problem is non-convex and apparently has no simple solutions. So we propose two schemes to solve it. First, a joint source and relay gradient algorithm by deriving the partial derivatives of the sum-rate is proposed. Second, a convergence-ensured alternating algorithm is derived to successively optimize the beamforming matrix at one node (source or relay) with those at the other node being fixed in turn. In order to suppress the inter-user interference, we utilize the equivalent channel method and formulate the problem based on maximum SLNR that can effectively suppress the CCI and noise at the users to design the source matrices, where the relay matrix is designed by the gradient algorithm. Simulation results demonstrate that the two proposed algorithms can obtain a good performance.

In this paper, one contribution lies in deriving the partial derivatives of the sum-rate and getting a nearly optimal solution of the source and relay matrices. The other is we proposing an alternating algorithm to jointly utilize the criterion of maximum SLNR and gradient algorithm to design the source matrix and relay matix for the first time.

This paper is organized as follows. In section II, the system model of a MIMO multiuser relay system is introduced. In section III, we propose two schemes to maximize the sum-rate. Simulation results about the system performance are presented in section IV. Finally, conclusion is drawn in section V.

II. SYSTEM MODEL

We consider a MIMO multiuser relay system where there are $K+2$ terminal nodes with K users, one source and one relay as shown in Fig.1. In this system, the source transmits

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independent information to the different users with the aid of the relay. The source, relay and the ith user, $i=1,\cdots,K$ are equipped with N_s, N_r, N_i antennas, respectively. We assume that a direct link between the source and users can be ignored due to a large pass loss, and the relay operates in time-division duplex (TDD) mode. Also, it is assumed that the source and the relay have full channel state information (CSI) while the users have no CSI. To efficiently exploit the system hardware, the relay node uses the same antennas to transmit and receive signals.

The transmitted data symbol vector $\mathbf{S}$ is defined as $\mathbf{S}=\left[\mathbf{s}_1^T\ \ \mathbf{s}_2^T\cdots\mathbf{s}_K^T\ \right]^T$, the vector $\mathbf{s}_i$ denotes the desired signal of the ith user, source matrix $\mathbf{W}_i$ is a $N_s\times N_i$ matrix for the ith user. Here, $\mathbf{s}_i$ satisfies $E\left(\mathbf{s}_i\mathbf{s}_i^H\right)=\mathbf{I}_{N_s}$, $E\left(\right)$ denotes the expectation operator. We can get the transmitted signal vector at the source antennas as follow

$$\mathbf{x}1=\left[\mathbf{W}_1,\ \ \mathbf{W}_2,\cdots\mathbf{W}_K\ \right]\left[\mathbf{s}_1^T\ \ \mathbf{s}_2^T\cdots\mathbf{s}_K^T\ \right]^T=\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_i \tag{1}$$

To satisfy the power constraint at the source, we have

$$\mathbf{x}=\eta_1\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_i \tag{2}$$

η_1 is given as

$$\eta_1=\sqrt{\frac{P_s}{tr\left(\sum\limits_{i=1}^{K}\mathbf{W}_i\mathbf{W}_i^H\right)}}$$

P_s is the power at the source.

At the first time slot, the transmitted signal is sent by the source, the received signal $\mathbf{y}$ at relay can be written as

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{n}=\mathbf{H}\eta_1\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_i+\mathbf{n} \tag{3}$$

Where $\mathbf{H}$ denotes the channel matrix of the source to the relay link, while $\mathbf{n}$ is additive white Gaussian noise (AWGN) at the relay. Then, at the second time slot, the relay station amplifies and forwards the received signal to the users. The signal $\mathbf{r}$ transmitted from the relay and the signal $\mathbf{y}_i$ received by the user i can be expressed as follows

$$\mathbf{r}=\eta_2\mathbf{F}\mathbf{y}=\eta_2\mathbf{F}\mathbf{H}\eta_1\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_i+\eta_2\mathbf{F}\mathbf{n} \tag{4}$$

$$\begin{aligned}\mathbf{y}_i=&\eta_2\mathbf{H}_i\mathbf{F}\mathbf{H}\eta_1\mathbf{W}_i\mathbf{s}_i+\eta_2\mathbf{H}_i\mathbf{F}\mathbf{H}\eta_1\sum_{j\neq i}\mathbf{W}_j\mathbf{s}_j\\&+\eta_2\mathbf{H}_i\mathbf{F}\mathbf{n}+\mathbf{z}_i\end{aligned} \tag{5}$$

Where $\mathbf{H}_i$ and $\mathbf{z}_i$ denote the channel coefficient and complex noise variable from the relay to the ith user, respectively. η_2 is the power normalizing coefficient, in order to satisfy the relay power constraint $E\left(\|\mathbf{r}\|_F^2\right)=P_r$, η_2 is given as

$$\eta_2=\sqrt{\frac{P_r}{tr\left(\mathbf{F}\left(\eta_1^2\sum\limits_{i=1}^{K}\mathbf{H}\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}^H+\mathbf{I}\right)\mathbf{F}^H\right)}}$$

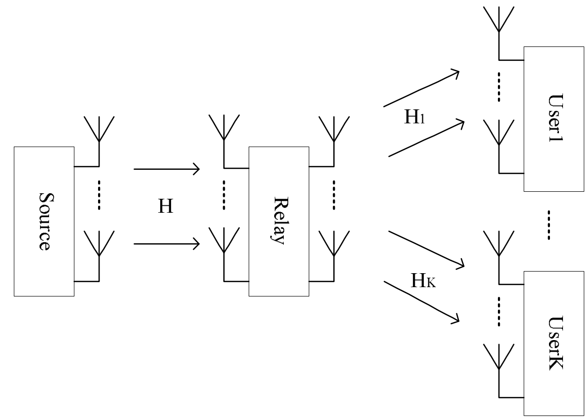

Fig. 1. system model

III. SUM-RATE MAXIMIZATION

In this section, we investigate the sum-rate maximization for the MIMO multiuser relay system. The sum-rate of the MIMO multiuser relay system model, denoted by R_{sum}, can be expressed as

$$R_{sum}=\sum_{i=1}^{K}R_i \tag{6}$$

Where R_i is the information rate of the ith user which can be given by

$$R_i=\log_2\frac{\left|\sum\limits_{j=1}^{K}\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H+\mathbf{K}_i\right|}{\left|\sum\limits_{j=1,j\neq i}^{K}\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H+\mathbf{K}_i\right|} \tag{7}$$

Where $\mathbf{K}_i=\frac{1}{\eta_1^2}\mathbf{H}_i\mathbf{F}\mathbf{F}^H\mathbf{H}_i^H+\frac{1}{\eta_1^2\eta_2^2}\mathbf{I}_{N_i}$. In order to obtain source and relay matrices to maximize the sum-rate, our problem can be formulated as

$$\mathbf{W}_{1,opt},\cdots,\mathbf{W}_{K,opt},\mathbf{F}_{opt}=\arg\max_{\mathbf{W}_1,\cdots,\mathbf{W}_K,\mathbf{F}}R_{sum} \tag{8}$$

Since the optimization problem is non-convex and a closed-form solution to the problem is intractable. Hence, we firstly derive a joint source and relay gradient algorithm by deriving the partial derivatives of the sum-rate and also develop an alternating scheme to reduce the computational complexity based on equivalent channel method and the criterion of maximmizing SLNR.

A. Iterative scheme by using partial derivatives

In this subsection, an iterative scheme by using partial derivatives and gradient descent algorithm which jointly optimize the source and relay matrices is developed. The gradient descent algorithm exploits the fact that a real-valued function $f\left(X\right)$ increases the fastest from an initial point X_0 if X_0 moves in the direction of the gradient of $f\left(X\right)$. Therefore, we derive the partial derivatives of R_{sum}. The partial derivatives of the sum-rate is given as $\nabla R_{sum}\left(\mathbf{F}\right)=\partial R_{sum}/\partial\mathbf{F}$, $\nabla R_{sum}\left(\mathbf{W}_1\right)=$

$\partial R_{sum}/\partial \mathbf{W}_1, \cdots, \nabla R_{sum}(\mathbf{W}_K) = \partial R_{sum}/\partial \mathbf{W}_K$. We firstly define $\mathbf{X}_i$ and $\mathbf{Y}_i$ according to (7) such that $R_i = \log_2|\mathbf{X}_i|/|\mathbf{Y}_i|$. Then using $\partial \log|\mathbf{X}| = tr\left\{\mathbf{X}^{-1}\partial \mathbf{X}\right\}$, we have $\partial R_i = (\log_2 e)\, tr\left\{\mathbf{X}_i^{-1}\partial \mathbf{X}_i - \mathbf{Y}_i^{-1}\partial \mathbf{Y}_i\right\}$. Also, there is the character that if $\partial J = \mathrm{Re}Tr\left(A\partial \mathbf{X}^H\right)$, then $\partial J/\partial \mathbf{X} = \mathbf{A}$. Therefore, it is easy to derive the partial derivatives of $\mathbf{X}_i$ and $\mathbf{Y}_i$ respect to matrices $\mathbf{F}$, $\mathbf{W}_1, \cdots, \mathbf{W}_K$. Above all, we can get partial derivative of $\mathbf{F}$ in (9).

$$\begin{aligned}\nabla R_{sum}(\mathbf{F}) &= 2(\log_2 e)\sum_{i=1}^{K}\left(\mathbf{H}_i^H\mathbf{X}_i^{-1}\mathbf{P}_i - \mathbf{H}_i^H\mathbf{Y}_i^{-1}\mathbf{T}_i\right)\\ &\quad + 2(\log_2 e)\sum_{i=1}^{K}\tfrac{1}{P_R}tr\left(\mathbf{X}_i^{-1} - \mathbf{Y}_i^{-1}\right)\mathbf{N}_i\end{aligned} \tag{9}$$

Where $\mathbf{P}_i = \mathbf{H}_i\mathbf{F}\mathbf{H}\sum_{j=1}^{K}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H + \frac{1}{\eta_1^2}\mathbf{H}_i\mathbf{F}$, $\mathbf{T}_i = \mathbf{H}_i\mathbf{F}\mathbf{H}\sum_{j\neq i}^{K}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H + \frac{1}{\eta_1^2}\mathbf{H}_i\mathbf{F}$, and $\mathbf{N}_i = \mathbf{F}\left(\mathbf{H}\sum_{j=1}^{K}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H + \frac{1}{\eta_1^2}\mathbf{I}_{Nr}\right)$. Similarly, we can verify that for $j \neq i$, the partial derivative of $\mathbf{W}_j$ can be expressed in (10).

$$\begin{aligned}\nabla R_{sum}(\mathbf{W}_j) &= 2(\log_2 e)\sum_{i=1}^{K}\left\{\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H\left(\mathbf{X}_i^{-1} - \mathbf{Y}_i^{-1}\right)\cdot\right.\\ &\quad \left.\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_j\right\} + 2(\log_2 e)\sum_{i=1}^{K}\left\{tr\left(\mathbf{X}_i^{-1} - \mathbf{Y}_i^{-1}\right)\cdot \mathbf{\Omega}_i\right\}\end{aligned} \tag{10}$$

and for $j = i$, the partial derivative of $\mathbf{W}_j$ can be written in (11).

$$\begin{aligned}\nabla R_{sum}(\mathbf{W}_j) &= 2(\log_2 e)\sum_{i=1}^{K}\left\{\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H\mathbf{X}_i^{-1}\cdot\right.\\ &\quad \left.\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_j\right\} + 2(\log_2 e)\sum_{i=1}^{K}\left\{tr\left(\mathbf{X}_i^{-1} - \mathbf{Y}_i^{-1}\right)\cdot \mathbf{\Omega}_i\right\}\end{aligned} \tag{11}$$

Where $\mathbf{\Omega}_i = \frac{1}{P_s}\mathbf{W}_j\mathbf{H}_i\mathbf{F}\mathbf{F}^H\mathbf{H}_i^H + \frac{1}{P_r}\mathbf{H}^H\mathbf{F}^H\mathbf{F}\mathbf{H}\mathbf{W}_j + \frac{1}{P_sP_r}tr\left(\mathbf{F}\mathbf{F}^H\right)\mathbf{W}_j$.

With the derived partial derivatives expressions, we can solve the optimization problem (8) as the following table I:

TABLE I
PROCEDURE OF SOLVING THE PROBLEM BY THE GRADIENT ALGORITHM

Given 1. Set $\mathbf{F}, \mathbf{W}_1, \cdots, \mathbf{W}_K$ as randomly feasible matrices
Repeat 2. Compute the partial derivatives $\nabla R_{sum}(\mathbf{F}) = \partial R_{sum}/\partial \mathbf{F}$, $\nabla R_{sum}(\mathbf{W}_1) = \partial R_{sum}/\partial \mathbf{W}_1, ..., \nabla R_{sum}(\mathbf{W}_K) = \partial R_{sum}/\partial \mathbf{W}_K$ 3. Update $\mathbf{F}^{(k+1)} \Leftarrow \mathbf{F}^{(k)} + \beta^m\nabla R_{sum}(\mathbf{F}), \mathbf{W}_1^{(k+1)} \Leftarrow \mathbf{W}_1^{(k)} + \beta^m\nabla R_{sum}(\mathbf{W}_1), ..., \mathbf{W}_K^{(k+1)} \Leftarrow \mathbf{W}_K^{(k)} + \beta^m\nabla R_{sum}(\mathbf{W}_K)$ 4. If $\max(\|\nabla R_{sum}(\mathbf{F})\|_F^2, \|\nabla R_{sum}(\mathbf{W}_1)\|_F^2 ... \|\nabla R_{sum}(\mathbf{W}_K)\|_F^2)$ $< \varepsilon$, stop the loop. Otherwise, go back to step 2

In table I, ε is the tolerance factor for terminating the iteration. To find a proper step size, we employ Armijo's Rule [13] which guarantees a non-decreasing sum-rate value in an iteration loop.

B. Alternating scheme based on SLNR

The iterative algorithm in subsection A needs to derive $K+1$ partial derivatives of R_{sum}, the most complexity lies in computing K partial derivatives of R_{sum} respect to source matrices. Therefore, in order to reduce the computational complexity, an easier design of the source matrices is proposed in this subsection. Maximizing SLNR is a good criterion to choose for the design of the source matrices by utilizing the equivalent channel method. Since it has close-form solutions. What's more, it makes a balance between suppressing the CCI and noise and has no constraints on the number of the transmit antennas like zero-forcing(ZF).

The proposed scheme firstly fixes the source matrices $\mathbf{W}_1, \cdots, \mathbf{W}_K$ to obtain the sub-optimal relay matrix $\widehat{\mathbf{F}}$ by using the gradient descent algorithm. Then, with the known $\widehat{\mathbf{F}}$, we design the source matrices based on SLNR and obtain $\widehat{\mathbf{W}}_1, \cdots, \widehat{\mathbf{W}}_K$. Finally, we use the received source matrices $\widehat{\mathbf{W}}_1, \cdots, \widehat{\mathbf{W}}_K$ to update the relay matrix $\mathbf{F}$, the updating process will be terminated once sum-rate converges.

For the given source matrices, $\mathbf{W}_1, \cdots, \mathbf{W}_K$, satisfying $tr\left(\mathbf{W}_i\mathbf{W}_i^H\right) = \frac{P_s}{K}\mathbf{I}_{N_s}$, we optimize the relay matrix by solving the following problem

$$\mathbf{F}_{opt} = \arg\max_{\mathbf{F}} R_{sum} \tag{12}$$

R_{sum} is given in (6), where R_i is given as

$$R_i = \log_2\frac{\left|\sum_{i=1}^{K}\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H + \mathbf{\Psi}_i\right|}{\left|\sum_{j\neq i}^{K}\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H + \mathbf{\Psi}_i\right|} \tag{13}$$

Where $\mathbf{\Psi}_i = \mathbf{H}_i\mathbf{F}\mathbf{F}^H\mathbf{H}_i^H + \frac{1}{\eta^2}\mathbf{I}$,

$$\eta = \sqrt{\frac{P_r}{tr\left(\mathbf{F}\left(\mathbf{H}\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_i\mathbf{s}_i^H\mathbf{W}_i^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_M\right)\mathbf{F}^H\right)}}$$

.

Similar to the subsection A, we can get the gradient of sum-rate respect to $\mathbf{F}$, expressed as

$$\partial R_{sum}/\partial \mathbf{F} = 2(\log_2 e)\sum_{i=1}^{K}\left(\mathbf{H}_i^H\mathbf{X}_i^{-1}\mathbf{P}'_i - \mathbf{H}_i^H\mathbf{Y}_i^{-1}\mathbf{T}'_i\right) \tag{14}$$

Where $\mathbf{P}'_i = \mathbf{H}_i\mathbf{F}\mathbf{H}\sum_{j=1}^{K}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H + \mathbf{H}_i\mathbf{F}$ and $\mathbf{T}'_i = \mathbf{H}_i\mathbf{F}\mathbf{H}\sum_{j\neq i}^{K}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}^H + \mathbf{H}_i\mathbf{F}$.

Then, using the gradient descent algorithm, the sub-optimal relay matrix $\widehat{\mathbf{F}}$ can be computed.

Once the sub-optimal $\widehat{\mathbf{F}}$ is calculated, the received signal at user i can be rewritten as

$$\mathbf{y}_i = \eta\tilde{\mathbf{H}}_i\mathbf{H}\mathbf{W}_i\mathbf{s}_i + \eta\tilde{\mathbf{H}}_i\mathbf{H}\sum_{j\neq i}^{K}\mathbf{W}_j\mathbf{s}_j + \eta\tilde{\mathbf{H}}_i\mathbf{n} + \mathbf{z}_i \qquad (15)$$

Where $\tilde{\mathbf{H}}_i = \mathbf{H}_i\mathbf{F}$, the second term in (15) is CCI. From (15), the interference made by the ith user can be viewed as the power leakage from this user to all the other users. For the ith user, the effective signal power is $E_i^e = \left\|\tilde{\mathbf{H}}_i\mathbf{H}\mathbf{W}_i\right\|^2$, while the equivalent sum-power of the leakage and the noise is $E_i^{l+n} = \sum\limits_{j=1, j\neq i}^{K}\left\|\tilde{\mathbf{H}}_j\mathbf{H}\mathbf{W}_i\right\|^2 + \|\mathbf{n}_i\|^2$, $\mathbf{n}_i = \eta\mathbf{H}_i\mathbf{F}\mathbf{n} + \mathbf{z}_i$. Therefore, we can get the source matrix for each user according to solving the following optimization problem

$$\arg\max_{\mathbf{W}_i} SLNR_i = E_i^e / E_i^{l+n} \qquad (16)$$

The $SLNR_i$ can be written as

$$\begin{aligned} SLNR_i &= \frac{\left\|\tilde{\mathbf{H}}_i\mathbf{H}\mathbf{W}_i\right\|^2}{\sum\limits_{j=1, j\neq i}^{K}\left\|\tilde{\mathbf{H}}_j\mathbf{H}\mathbf{W}_i\right\|^2 + \|\mathbf{n}_j\|^2} \\ &= \frac{\mathbf{W}_i^H\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{F}\mathbf{H}\mathbf{W}_i}{\mathbf{W}_i^H\left(\mathbf{H}^H\mathbf{F}^H\bar{\mathbf{H}}_i^H\bar{\mathbf{H}}_i\mathbf{F}\mathbf{H} + n_j^H n_j\mathbf{I}_{N_i}\right)\mathbf{W}_i} \end{aligned} \qquad (17)$$

Where $\bar{\mathbf{H}}_i = \left[\mathbf{H}_1^H \cdots \mathbf{H}_{i-1}^H\mathbf{H}_{i+1}^H \cdots \mathbf{H}_K^H\right]^H$ is assumed channel matrix that exclude $\mathbf{H}_i$ only. It was shown that in [12] that the generalized eigenvectors corresponding to the maximum N_i generalized engenvalues of the matrix $\left(\mathbf{H}^H\mathbf{F}^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{F}\mathbf{H}, \mathbf{H}^H\mathbf{F}^H\bar{\mathbf{H}}_i^H\bar{\mathbf{H}}_i\mathbf{F}\mathbf{H} + n_j^H n_j\mathbf{I}_{N_i}\right)$ are the desired source matrix for the ith user. Repeat the process K times and we can get $\widehat{\mathbf{W}}_1, \cdots, \widehat{\mathbf{W}}_K$. The norm of $\mathbf{W}_i$ is adjusted to $tr\left(\widehat{\mathbf{W}}_i\widehat{\mathbf{W}}_i^H\right) = \frac{P_s}{K}I_s$.

According to the analysis, the alternating algorithm can be presented in the following TABLE II:

TABLE II
PROCEDURE OF SOLVING THE PROBLEM BY THE ALTERNATING ALGORITHM

1. Initialize the source matrix $\mathbf{W}$
Repeat
2. Solving the optimization problem (12) with fixed $\mathbf{W}$ to obtain the updated $\mathbf{F}_{opt}$
3. Solving the optimization problem (16) with fixed $\mathbf{F}_{opt}$ to obtain the updated $\mathbf{W}$
4. go to step2,until the sum-rate converges

IV. SIMULATION RESULTS

In this section, we present the performance of the two proposed schemes compared with existing schemes introduced in [9] and [11]. In the simulation, we assume that there are three users $K = 3$. Each user is equipped with two antennas $N_i = 2$, the relay and source are both equipped with four antennas $N_s = N_r = 4$. Each of the channel parameters is realized independently using a complex Gaussian distribution with zero mean and unit variance. Every entry of the noise vectors has zero mean and unit variance. The performance is based on an average over 20000 channel realizations. For comparison, we implement four strategies as follows:

i). By using the initialized source matrices, we apply the gradient algorithm to calculate the relay matrix just as in [11]

ii). Based on the initialized relay matrix, we apply criterion of maximum SLNR to obtain the source matrices which has been introduced in [9]

iii). Alternating scheme by combing the criterion of maximum SLNR with the gradient algorithm which has been developed in subsection B.

iv). Iterative joint optimization by using partial derivatives just as in subsection A, the process in table I would be terminated by ε=10^{-3}.

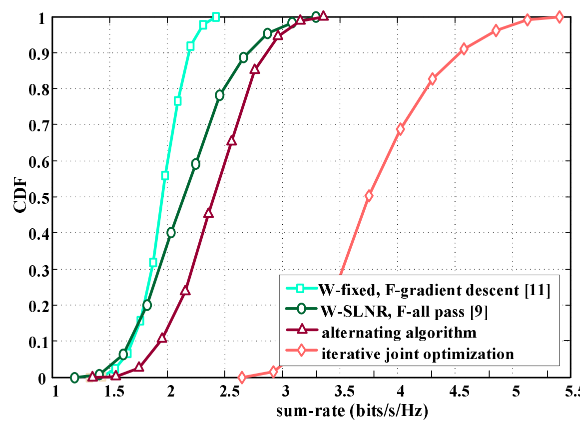

Fig. 2. CDF comparisons of four schemes

Fig.2 shows the cumulative distribution function (CDF) of the ergodic sum-rate for various schemes proposed for the MIMO multiuser relay systems with P_s= 2W and P_r= 5W. From this plot, it can be observed that our proposed two schemes outperform the conventional schemes i) and ii). From the curves of scheme i) to iii), we found that precoding at the source is more effective than precoding at the relay in the MIMO multiuser relay system.

Fig.3 compares the averaged sum-rate achieved by the four algorithms versus the relay power P_r, the power at the source is fixed as P_s= 2W. We initialize the source covariance matrices according to $tr\left(\mathbf{W}_i\mathbf{W}_i^H\right) = \frac{P_s}{K}I_{N_s}$.

From the Fig.3, we observe that the proposed scheme iv) achieves the highest sum-rate among the four strategies over the whole P_r range at the cost of relatively high computational complexity. Its achievable sum capacity can serve as an upper bound for the performance of other effectively-designed source and relay matrices of the MIMO multiuser relay system. The strategy iii) also obtains higher sum-rate compared to the existing schemes(strategy i) and strategy ii)) since it alternates to apply the gradient algorithm and SLNR criterion to design the relay and source matrix. Although it doesn't achieve as good performance as the strategy iv), it has the closed-form expression of the source matrices and reduces the computational complexity.

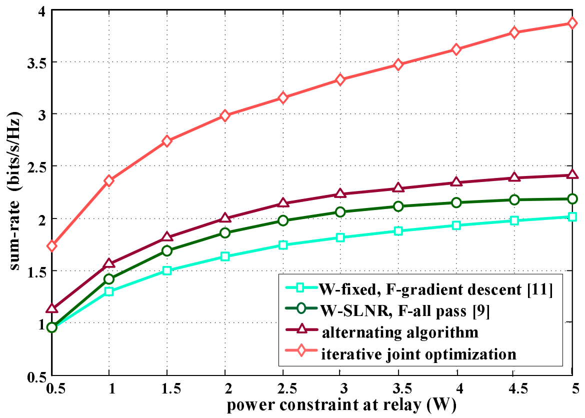

Fig. 3. sum-rate comparisons of four schemes

V. Conclusion

In this paper, two schemes are derived in a MIMO multiuser relay system to maximize the sum-rate. First, a joint source and relay optimization by deriving the partial derivatives of the sum-rate is proposed. The scheme has a remarkable performance gain compared to the other three schemes but with high computational complexity. In order to reduce the complexity, an easier scheme based on maximizing SLNR to design source matrices is developed. This scheme successively optimize the beamforming matrix at one node with the other node being fixed. Simulation results demonstrate that both of the proposed algorithms perform much better than the existing algorithms.

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