Reliable Data Aided Sparsity-Aware Approaches to Clipping Noise Estimation in OFDM Systems

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***Abstract*— In this paper, we propose reliable data aided sparsity-aware approaches to estimate and cancel the clipping noise in OFDM systems. Those are motivated by the fact that reliable data can be exploited to estimate the clipping noise in a successive interference cancellation (SIC) manner. When the clipping noise has relatively large support, a data non-aided method is not class enough to estimate the clipping noise well due to the compressed sensing (CS) based measurement shortages. Simulation results demonstrate the effectiveness of our proposed methods in estimating the clipping noise and approaching the performance with no clipping noise.**

***Keywords*— Compressed sensing, clipping noise, OFDM systems, reliable data.**

I. INTRODUCTION

One of the major drawbacks for orthogonal frequency division multiplexing (OFDM) systems is the high peak to average power ratio (PAPR). The variance of OFDM signal amplitudes with high PAPR leads to the detection efficiency degradation due to nonlinear characteristics of both high power amplifier (HPA) and digital-to-analog converter (DAC). Plenty of signal processing approaches have been proposed to reduce the high PAPR [1]. The simplest and most widely used method is to clip the peak power of transmitted signals at the transmitter. The clipping method, however, introduces both in-band signal distortion and out-of band radiation [2].

To compensate the performance degradation induced by clipping, the receiver needs to estimate both locations and amplitudes of the signals clipped at the transmitter. Recent work based on compressed sensing (CS) has been proposed to estimate the clipping noise and restore to the original OFDM signals, which exploits the fact that the clipping noise can be sparsely approximated in time domain [3]. The CS enables the recovery of sparse signals from a small number of linear measurements [4]. Thus, for estimating the clipping noise, partial measurements of frequency is sufficient. Thus, some frequencies are allocated by pilot symbols, and those are used to estimate clipped components in [3]. However, it is easy to be pushed for measurements when the number of clipped components is large. This is because accuracy of the clipping noise to be estimated is limited by available pilot tones, but the number of those only depends on channel characteristics. One system using pilot tones more than needs leads to overutilization of scarce communication resources and data rate loss.

In this paper, we propose reliable data aided reconstruction of clipped OFDM signals when the number of pilot symbols is insufficient enough for stably estimating the clipping noise. The reliable data can be also used in company with pilot tones, conditioned on the data is correctly decoded. This is consistent with successive interference cancellation (SIC) since the clipping noise is usually much weaker than OFDM signals. For the construction of a reliable data set, three different kinds of selection criteria are presented. In first criterion, the selection ordering is based on the log-likelihood (LLR) that provides the reliability information on the maximum a posteriori (MAP) probability. Next, in second criterion, the fading characteristics of the channel are used to select reliable data which naturally has higher clipping noise to noise ratio (CNR). This is because the clipping noise also fades along with the original OFDM signals. Thus, we can obtain measurements less contaminated by AWGN noise. Finally, the distance based criterion is used which first choose the data symbol having the shortest distance from the modulation symbols.

It is shown that the bit error rate (BER) performance becomes worse when pilot tones are only used for estimating the clipping noise having the large number of support. This is because the CS decoder fails in an attempt to estimate the accurate clipping noise. The proposed methods, on the other hand, improve the BER performance with the help of reliable data.

This paper is organized as follows. Section Ⅱ describes the system model and CS background. In section Ⅲ, formulation to the CS problem and approaches to reliable data selection are proposed, and section Ⅳ presents reliable data aided clipping noise estimation and cancellation. Simulation results and conclusion are presented in sections Ⅴ and Ⅵ, respectively.

Notation: A bold face letter denotes a vector or a matrix; a uppercase and lowercase letter denote frequency and time domain variables, respectively; $\mathbf{a}^T\left(\mathbf{A}^T\right)$ is the transpose of a vector (a matrix); $\mathbf{A}^{-1}$ denotes the inverse of a square matrix. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$, and N-dimensional zero vector is denoted by $\mathbf{0}_N$.

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II. System Model

A. OFDM Signal Model

Let a block of N symbols $\mathbf{X} = \left[X_0, X_1, \ldots, X_{N-1}\right]^T$ is formed, where N is the number of subcarriers. The discrete-time transmitted OFDM signals can be written as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(\frac{j2\pi kn}{LN}\right), \quad 0 \le n \le LN-1 \tag{1}$$

where L is an oversampling factor.

The PAPR computed from the L -times oversampled time domain OFDM signals can be defined as

$$\mathrm{PAPR}\left(x[n]\right) = \frac{\max\limits_{0 \le n \le NL-1} \left|x[n]\right|^2}{\mathbb{E}\left|x[n]\right|^2} \tag{2}$$

where $\mathbb{E}(\cdot)$ denotes the expectation operator. To avoid nonlinear characteristics of HPA and DAC, and power inefficiency due to the high PAPR of OFDM signals, the amplitude clipping is widely employed at the transmitter due to its simplicity. For soft limiter, that is given as

$$x_c[n] = \begin{cases} x[n], & \left|x[n]\right| \le A \\ A \cdot \exp\left(j\measuredangle x[n]\right), & \left|x[n]\right| > A \end{cases} \tag{3}$$

where $\measuredangle x[n]$ is the phase component of $x[n]$ and A is preset clipping level which is determined by clipping ratio (CR), $\mathrm{CR} = A / \sqrt{\mathbb{E}\left|x[n]\right|^2}$. As a result, the OFDM signals are transmitted with the clipping noise $c[n]$, $x_c[n] = x[n] + c[n]$.

At the receiver, the received OFDM signals in the frequency domain can be written by the matrix form as

$$\mathbf{Y} = \mathbf{H}\left(\mathbf{X} + \mathbf{C}\right) + \mathbf{Z} \tag{4}$$

where $\mathbf{H} = \mathrm{diag}\left\{H_0, H_1, \ldots, H_{N-1}\right\}$ is the $N \times N$ effective channel matrix neglecting the inter-carrier interference (ICI), the $N \times 1$ clipping noise vector is defined by $\mathbf{C} = \left[C_0, C_1, \ldots, C_{N-1}\right]^T$, and the additive white Gaussian noise is $\mathbf{Z} \sim \mathcal{CN}\left(0, \sigma_Z^2 \mathbf{I}_N\right)$. Through zero-forcing (ZF) equalizer, the received signals on the $k-$th subcarrier is given by

$$\bar{Y}_k = H_k^{-1} Y_k = X_k + C_k + H_k^{-1} Z_k \tag{5}$$

B. Compressed Sensing Background

It is well studied to find spars solution of underdetermined linear system [5]. If the $\mathbf{A}$ is $M \times N$, $M \ll N$, matrix with entries being i.i.d. complex Gaussian random variables with zero mean and M^{-1} variance and the vector $\mathbf{b}$ can be considered K -sparse vector, following inequality called restricted isometry property (RIP) of order $2K$ is allowed for small $\delta_{2K} > 0$ with only $M = O(K \log(N/K))$,

$$\left| \frac{\left\|\mathbf{Ab}\right\|_2^2 - \left\|\mathbf{b}\right\|_2^2}{\left\|\mathbf{b}\right\|_2^2} \right| \le \delta_{2K}. \tag{6}$$

That implies the matrix $\mathbf{A}$ preserves the distance between any K -sparse vectors. For the sufficient number of rows, RIP constant δ_{2K} gets smaller. In case of a random row submatrix of the discrete Fourier transform (DFT) matrix, a similar number of observations suffice in practice [6].

Besides, there is mutual coherence property which is defined by [7]

$$\mathrm{M}(\mathbf{A}) = \max_{1 \le i, j \le N \text{ and } i \ne j} \frac{\left|\mathbf{a}_i^T \mathbf{a}_j\right|}{\left\|\mathbf{a}_i\right\|_2 \cdot \left\|\mathbf{a}_j\right\|_2}. \tag{7}$$

This guarantees that the solution exists at most one (uniqueness), if the number of support K is satisfied following inequality,

$$K < \frac{1}{2}\left(1 + \frac{1}{\mathrm{M}(\mathbf{A})}\right). \tag{8}$$

These mentioned properties guarantee that matrix $\mathbf{A}$ allows exact recovery $\mathbf{x}$ to be possible by solving a convex optimization. Our proposed scheme relies on these properties.

In following section, we focus on how the clipping noise is estimated and our proposed scheme restores the clipped OFDM signals by using CS approaches with the aid of both known pilot tones and selected reliable data.

III. Formulation to the CS Problem and Approaches to Reliable Data Selection

Our proposed methods exploit the tones corresponding to the location of both pilot symbols and reliable data. Let $\mathbf{S}_p$ and $\mathbf{S}_d$ denote the selection matrices which consist of selected rows of the identity matrix $\mathbf{I}_N$ corresponding to the index set of pilot tones, $\mathcal{I}_p = \left\{i_1, \ldots, i_{M_p}\right\}$, and reliable data, $\mathcal{I}_d = \left\{j_1, \ldots, j_{M_d}\right\}$, and the compound selection matrix is defined as $\mathbf{S}_{p,d} = \left[\mathbf{S}_p; \mathbf{S}_d\right]$.

Multiplying $\bar{\mathbf{Y}}$ by $\mathbf{S}_{p,d}$, we get

$$\begin{aligned} \mathbf{Y}_{p,d} \left(= \mathbf{S}_{p,d} \bar{\mathbf{Y}}\right) &= \mathbf{S}_{p,d}\mathbf{X} + \mathbf{S}_{p,d}\mathbf{Fc} + \mathbf{S}_{p,d}\mathbf{H}^{-1}\mathbf{Z} \\ &= \mathbf{S}_{p,d}\mathbf{Fc} + \underbrace{\mathbf{S}_{p,d}\mathbf{X} + \mathbf{S}_{p,d}\mathbf{H}^{-1}\mathbf{Z}}_{\text{Noise Signals } (\mathbf{W})} \\ &= \mathbf{S}_{p,d}\mathbf{Fc} + \mathbf{W} \end{aligned} \tag{9}$$

where $\bar{\mathbf{Y}} = \left[\bar{Y}_0, \bar{Y}_1, \ldots, \bar{Y}_{N-1}\right]$, $\mathbf{F}$ denotes the $N \times LN$ DFT matrix and the $LN \times 1$ sparse vector $\mathbf{c}$ denotes the clipping noise in time domain.

From (9), the sparse clipping noise $\mathbf{c}$ can be estimated using convex relaxation. However, noise $\mathbf{W}$ reduces the accuracy of the clipping noise estimation. By subtracting the contributions of known pilot tones and reliable data, to this end, we get noise-reduced measurements as follows

$$
\begin{aligned}
\mathbf{V}_{p,d} &= \mathbf{Y}_{p,d} - \mathbf{S}_{p,d}\mathbf{X} \\
&= \begin{bmatrix} \mathbf{S}_p \\ \mathbf{S}_d \end{bmatrix} \mathbf{Fc} + \underbrace{\begin{bmatrix} \mathbf{0}_{M_p} \\ \mathbf{S}_d\mathbf{X} - D\{\mathbf{Y}_d\} \end{bmatrix} + \begin{bmatrix} \mathbf{S}_p\mathbf{H}^{-1}\mathbf{Z} \\ \mathbf{S}_d\mathbf{H}^{-1}\mathbf{Z} \end{bmatrix}}_{\text{Noise Signals } (\mathbf{W}')} \\
&= \mathbf{S}_{p,d}\mathbf{Fc} + \mathbf{W}'
\end{aligned} \tag{10}
$$

where $\mathbf{V}_{p,d} = \left[\mathbf{V}_p ; \mathbf{V}_d\right]$, $\mathbf{Y}_d = \mathbf{S}_d\bar{\mathbf{Y}}$, $\|\mathbf{W}'\|_2^2 \ll \|\mathbf{W}\|_2^2$, and $D\{\cdot\}$ denotes the hard decision operator. When the number of selected reliable data $(M_d = 0)$, Eq. (10) is equal to the CS formulation of [3] which only exploits known pilot tones. In turn, the essential work is how we construct the index set of reliable data to minimize the noise signal $\mathbf{W}'$. In this paper, we consider three kinds of criteria for selecting reliable data below: 1) LLR, 2) signal to noise ratio (SNR), 3) distance ordering based selection.

A. Criterion 1: LLR Ordering Based Selection

In this criterion, the pairwise LLR is used to choose reliable data minimizing the quantity of $\mathbf{S}_d\mathbf{X} - D\{\mathbf{Y}_d\}$. That can be written as

$$
\mathcal{L}_{k,m} = \ln \frac{\Pr\left(X_k = \hat{X}_k \mid Y_k\right)}{\Pr\left(X_k = S_m \mid Y_k\right)} \stackrel{(a)}{=} \ln \frac{\Pr\left(Y_k \mid X_k = \hat{X}_k\right)}{\Pr\left(Y_k \mid X_k = S_m\right)} \tag{11}
$$

where the MAP decision for the $k-$th tone is $\hat{X}_k = \arg\max_{S_m \in \{S_1,\ldots S_{\bar{M}}\}} \Pr\left(X_k = S_m \mid Y_k\right)$ for $\bar{M}$ -ary modulation and (a) is by equal probability that each symbol is transmitted.

The clipping noise with the large clipping components K can be assumed as Gaussian distributed with $\sim \mathcal{CN}\left(0, \sigma_C^2\right)$ but that still remains sparse $K \ll LN$. Thus, the clipping noise plus AWGN noise $W_k \left(= C_k + H_k^{-1} Z_k\right)$ can be characterized as $\sim \mathcal{CN}\left(0, \sigma_C^2 + \left|H_k^{-1}\right|^2 \sigma_Z^2\right)$ along with mutually independent characteristics [8], and the pairwise LLR is given as

$$
\mathcal{L}_{k,m} = \frac{\left|Y_k - S_m\right|^2 - \left|Y_k - \hat{X}_k\right|^2}{\sigma_C^2 + \left|H_k^{-1}\right|^2 \sigma_Z^2}. \tag{12}
$$

We select the data which minimizes $\sum_m \exp\left(-\mathcal{L}_{k,m}\right)$, since $\Pr\left(X_k \neq \hat{X}_k \mid Y_k\right)$ decreases along with decreasing $\sum_m \exp\left(-\mathcal{L}_{k,m}\right)$.

B. Criterion 2: SNR Ordering Based Selection

The ordering is determined based on the SNR in order to minimize $\Pr\left(X_k \neq \hat{X}_k\right)$. Increasing the SNR leads to decreasing $\Pr\left(X_k \neq \hat{X}_k\right)$, and the SNR for each $k-$th data tones is given by

$$
\mathrm{SNR}_k = \frac{\left|X_k\right|^2}{\sigma_C^2 + \left|H_k^{-1}\right|^2 \sigma_Z^2}. \tag{13}
$$

For $\left|X_k\right|^2 = E_s$, the ordering can be simply determined by using the channel gain $\left|H_k^{-1}\right|^2$ and thereby the reliable data is selected with the high channel gain. Also, since the clipping noise fades along with the original OFDM signals, there is naturally a thread of connections between the channel gain ordering based selection and the maximizing CNR as follows,

$$
\begin{aligned}
\mathcal{I}_d &= \arg\max_{|\mathcal{I}_d| = M_d} \left(\frac{\mathbb{E}\left\|\mathbf{S}_d\mathbf{Fc}\right\|_2^2}{\sigma_Z^2 \left\|\mathbf{S}_d\mathbf{H}^{-1}\right\|_{\mathrm{F}}^2} \stackrel{(b)}{=} \frac{\frac{M_d}{N}\left\|\mathbf{c}\right\|_2^2}{\sigma_Z^2 \left\|\mathbf{S}_d\mathbf{H}^{-1}\right\|_{\mathrm{F}}^2} \right) \\
&= \arg\max_{|\mathcal{I}_d| = M_d} \sum_{\mathcal{I}_d} \left|H_j\right|^2
\end{aligned} \tag{14}
$$

where $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix, and (b) is by $\mathbb{E}\left(\mathbf{S}_d\mathbf{F}\right)^H \mathbf{S}_d\mathbf{F} = \frac{M_d}{N}\mathbf{I}_N$. Maximizing CNR is a desired direction for achieving the estimate of the clipping noise.

C. Criterion 3: Distance Ordering Based Selection

This criterion is similar to the maximum likelihood detection (MLD). The minimum distance between the received component among $\bar{M}$ -ary symbols used as

$$
d_{\min}^{(k)} = \min_{S_m \in \{S_1,\ldots S_{\bar{M}}\}} \left|Y_k - S_m\right| \tag{15}
$$

Like criterion 1, it selects reliable data minimizing $\Pr\left(X_k \neq \hat{X}_k \mid Y_k\right)$ with equal transmission probability for each symbol, but there is some different. For BPSK, pairwise LLR is given by $\mathcal{L}_k = \frac{4\sqrt{E_s}}{\sigma_C^2 + \left|H_k^{-1}\right|^2 \sigma_Z^2} \left|\Re\left(Y_k\right)\right|$. In case of having equal channel gains, the reliable data is preferentially selected having the instantaneous large noise contributions by triangle inequality $\left|\Re\left(Y_k\right)\right| \leq \left|X_k\right| + \left|\Re\left(Z_k\right)\right|$, which results in decreasing the CNR. In contrast, the distance based selection method gives more priority to the data tone by considering both more reliability and less instantaneous noise contributions.

To sum up, both the criterion 1 and 3 select reliable data to be used to estimate the clipping noise, aiming at minimizing the probability of $\Pr\left(X_k \neq D\{Y_k\}\right)$. In order to minimizing the noise power $\left\|\mathbf{S}_d\mathbf{H}^{-1}\mathbf{Z}\right\|_2^2$, the criterion 2 is considered for stable performance of CS decoder which is susceptible to noise. All criteria eventually aim to remove contributions of noise $\mathbf{W}'$ in (10).

IV. Reliable Data Aided Clipping Noise Estimation and Cancellation

From (10) with the criteria of selecting reliable data, the sparse clipping noise is intuitively obtained by solving the following optimization problem:

$$\text{minimize}\|\mathbf{c}\|_0 \quad \text{subject to } \mathbf{V}_{p,d} = \mathbf{S}_{p,d}\mathbf{F}\mathbf{c}, \tag{16}$$

where $\|\mathbf{c}\|_0$ denotes the number of nonzero components for the vector $\mathbf{c}$. But, solving (16) is based on exhaustive search and has the complexity of combinatorial computation. In addition, it ignores the contribution of noise $\mathbf{W}'$. An alternative method is convex relaxation with inequality constraints as given by

$$\text{minimize}\|\mathbf{c}\|_1 \text{ subject to } \left\|\mathbf{S}_{p,d}\mathbf{F}\mathbf{c} - \mathbf{V}_{p,d}\right\|_2 \le \varepsilon \tag{17}$$

where the $\mathcal{L}_1$ norm is simply the sum of the amplitudes of the coefficient $\mathbf{c}$, i.e. $\sum_k |c_k|$, and the threshold parameter ε is chosen high enough to bound an error vector with high probability. Thanks to convexity of $\mathcal{L}_1$ norm, the (17) optimization problem is a quadratic program and can be solved with polynomial complexity [9].

Following solving (17), the estimated clipping noise is used to obtain the near clipping noise-free received OFDM signals as follows,

$$\begin{aligned}\hat{\mathbf{Y}} &= \mathbf{X} + \mathbf{C} - \mathbf{F}\hat{\mathbf{c}} + \mathbf{H}^{-1}\mathbf{Z} \\ &= \mathbf{X} + \mathbf{F}\left(\mathbf{c} - \hat{\mathbf{c}}\right) + \mathbf{H}^{-1}\mathbf{Z}\end{aligned} \tag{18}$$

The quantity of the estimation error, $\mathbf{c} - \hat{\mathbf{c}}$, depends on the number of clipped components, the length of the measurement vector $\mathbf{V}_{p,d}$, and noise power $\|\mathbf{W}'\|_2^2$, since those affect the performance of CS decoder. Furthermore, the cardinality of reliable data set may be determined adaptively according to SNR regions, since there is trade-off between the benefits of using the large number of data tones M_d and increased noise power $\|\mathbf{W}'\|_2^2$ due to the $\mathbf{S}_d\mathbf{X} \ne D\{\mathbf{Y}_d\}$ in the low SNR region. We defer optimizing the size of reliable data set to future work. Thus, the low SNR case is outside our interests in the following simulations.

V. Simulation Results

We evaluate the performance of the proposed reconstruction of clipped OFDM signals. The bandwidth of 1MHz is divided into N sub-channels and the oversampling factor L=1 is set, and BPSK subcarrier modulation is used. Exponential power-decaying and Rayleigh fading generated by Jakes' fading model are assumed. The normalized root mean square (RMS) delay spread of the channel is set to 0.5*us*.

We first confirm the validity of the reliable data aided estimation of the clipping noise. Referring to Fig. 1, the proposed methods estimate the locations and amplitudes of the clipping noise well while the traditional method which only exploits known pilot tones fails in an attempt to estimate those. Its sham estimation may introduce additional noise in (18). This is due to the measurements shortage which results in the failure of CS decoder.

Figure 2 shows the average BER performance. The reliable data aided methods outperform the data non-aided method. Our proposed methods closely approach the performance with no clipping noise at SNR=28dB. The data non-aided approach using only pilot tones is incapable of improving the BER performance as expected in Fig. 1.

Figure 3 shows the average BER performance as a function of the number of data tones used to estimate clipping noise. As the number of used data tones increases, the proposed methods allow more BER improvements. As explained in CS background, the more rows the sub-matrix has, the smaller RIP constant becomes [5]. The small RIP constant leads to guaranteeing that it is possible to stably estimate the clipping noise by solving (17) from partial Fourier measurements.

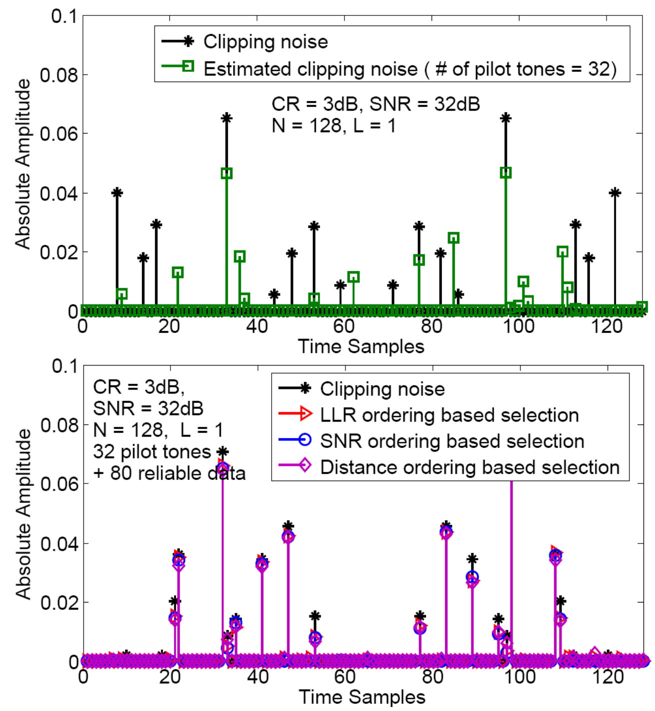

Fig. 1. The estimate of the clipping noise with the reliable data non-aided method (top) and the data aided methods (bottom).

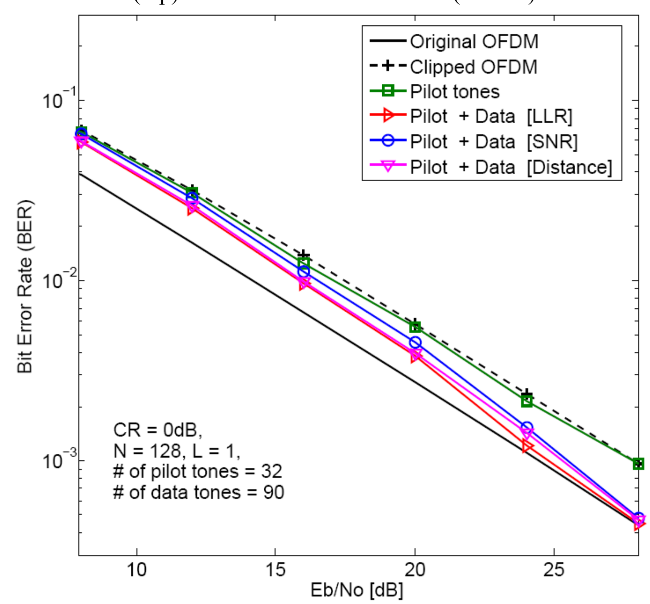

Fig. 2. The average BER comparison between the proposed methods and the traditional method, CR=0dB.

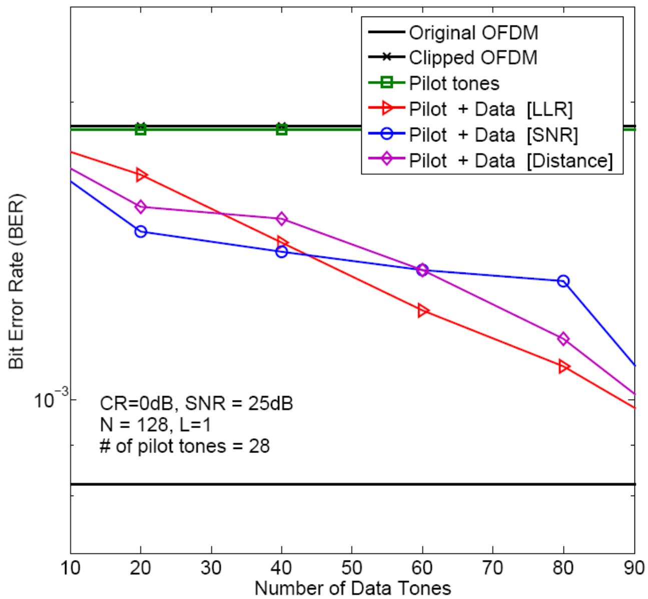

Fig. 3. The average BER comparison as a function of the number of data tones used for the estimate of the clipping noise, CR=0dB.

VI. Conclusion and Future Work

The data aided methods were derived to mitigate the clipping noise in OFDM systems. Our approaches are closely related to a strategy of SIC since the clipping noise is usually much weaker than reliable data. The proposed methods alleviate the problem with measurements shortages in the approach using only pilot tones as the compressed measurements when the support of clipping noise is large. We used reliable data tones in order to minimize the noise which impacts on the performance of the relaxation based estimator. To select more reliable data tones on the preferential basis, several ordering methods are presented. Among criteria on reliable data selection, the SNR ordering criterion is preferred in terms of computational complexity. This is because the SNR ordering criterion can be simplified into the ordering according to channel gains, when QAM is employed. Numerical results show the reliable data aided methods outperform the data non-aided method. In addition, it is shown that the cardinality of reliable data set can be increased for achieving better BER performance.

Future research direction may include exploring the trade-off between the benefits of using the large number of data tones and increased noise power in the low SNR region. Furthermore, it may include giving a best point to maximize the estimation performance and providing corresponding theoretical analysis.

References

[1] T. Jiang, Y. Wu, "An overview: Peak-to-average power ratio reduction techniques for OFDM signals," *IEEE Trans. Broad.* vol. 54, no. 2, pp. 257-268, Jun. 2008.

[2] R. O'Neill and L. B. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals," in *Proc. IEEE PIMRC'95*, Toronto, Canada, pp. 71–75, Sept. 1995.

[3] M. Mohammadnia-Avval, A. Ghassemi, and L. Lampe, "Compressed sensing based recovery of nonlinearly distorted OFDM signals," *in Proc. IEEE Int. Conf. Commun*., Jun. 2011.

[4] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1289-1306, Apr. 2006.

[5] E. J. Candes and T. Tao, "Near optimal signal recovery from random projections: Universal encoding strategies," *IEEE Trans. Inform. Theory*, vol. 52, no. 12, pp. 5406-5425, Dec. 2006.

[6] E. J. Candés, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[7] D. L. Donoho and M. Elad, "Optimally sparse representation in general (non-orthogonal) dictionaries via ℓ_1 minimization," *Proc. Nat. Acad. Sci*., vol. 100, pp. 2197-2202, 2002.

[8] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Trans. Commun.,* vol. 50, no. 1, pp. 89-101, Jan. 2002.

[9] S. Boyd and L. Vandenberghe, *Convex Opimization*. New York, NY: Cambridge Univ. Press, 2004.