

# Study of TOA Positioning using UWB Reflected Waves

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**Abstract**—With the global spread of mobile terminals, positioning technologies have become an important research issue. The recent main challenges in positioning technologies are the non-line-of sight (NLOS) and reflected wave problems. This paper presents a positioning method that utilizes reflected waves in a positive manner. This method realizes accurate positioning with few base stations (BSs) (one BS at minimum). NLOS positioning is possible if more than three reflected waves can be received at the BS. The proposed method was verified by fundamental experiments, and the estimation errors were evaluated theoretically and by computer simulation.

## I. INTRODUCTION

As the use of cellular phones, tablet terminals, radio-frequency (RF) tags, and other various mobile terminals is rapidly spreading around the world, positioning technologies are an important research issue in terms of accuracy, complexity, positioning speed, etc.

In positioning technologies using propagation time such as time of arrival (TOA) and time difference of arrival (TDOA), some studies on high-accuracy positioning with ultra wide band (UWB) signals have recently attracted attention[1]. Hamaguchi *et al.*[2] realized a standard deviation of 11.6 cm for TDOA positioning using a UWB signal with a bandwidth of 600 MHz in 4 GHz band.

The main challenges in positioning technologies using radio waves are the NLOS and reflecting wave problems. Currently positioning accuracy is decreased drastically in a NLOS environment. In addition, estimation accuracy deteriorates when waves are reflected and diffracted by obstacles around the field even if the line of sight (LOS) path is secured.

Studies mitigating these problems have recently become an active area of research. Evennou *et al.*[3] proposed a positioning algorithm that separates direct (LOS) waves from many reflected and diffracted waves and estimates the position using only the direct waves. Chao *et al.*[4] proposed a scheme that distinguishes measurements with reflected and diffracted waves based on the measurement errors. Xu *et al.*[5] proposed a positioning method that minimizes the root mean square error (RMSE) by using the measurement data of many BSs and MSs. These methods curb the effects of reflected and diffracted waves but do not work in environments without a direct path.

Kaemarungsi *et al.*[6] and other researchers are developing a location fingerprinting method. This method can be divided into two phases. First, in the offline phase, a preliminary large database of fingerprints (received signal strength (RSS) from multiple BSs) from the entire area is created. Second, in the online phase, the position is estimated by pattern matching between the measured RSSs and the RSSs in the databases. NLOS positioning can be realized by the fingerprinting method. However large databases are needed first, and the databases is necessary to be recreated when the environment changes (buildings are erected/demolished, etc.).

Kurosaki *et al.*[7] proposed an angle-of arrival (AOA) positioning technique with the fingerprinting method that is robust against changes in the environment through the use of ray-trace simulation. Kietlinski *et al.*[8] proposed a TOA positioning method that needs only two BSs and utilizes reflected waves.

As described above, research on accurate positioning methods that separates direct waves from reflected and diffracted waves and estimate position using only direct waves, used to be the main area of study[3]-[5]; subsequently, research on NLOS positioning methods that use multipath information efficiently[6]-[8] has become a recent focus.

In this paper, we propose a positioning method that expands upon Kietlinski *et al.*'s work[8]; we evaluated the effectiveness of our proposed method through fundamental experiments and computer simulations. Kietlinski *et al.* realized a three dimensional (3D) TOA positioning using only two BSs by taking advantage of reflected waves from big flat surfaces such as ceiling and walls. We count all the reflected waves from the main obstacles in and around a field and realize 3D positioning with only one BS in the minimum case. NLOS positioning is possible if more than three reflected waves can be received at the base station. The proposed method does not need large databases, and the calculation complexity is comparable to that of normal TOA estimation.

We describe the proposed positioning algorithm in section II. Section III presents the fundamental experiments in an ideal environment (two-dimensional exact square field). The estimation errors were evaluated theoretically and by computer simulation in section IV. Section V presents the conclusion and future research directions.

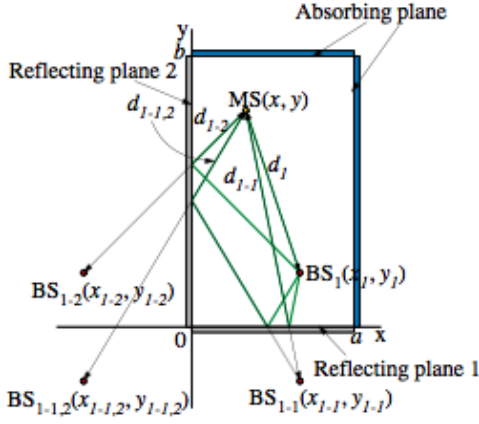


Fig. 1. 2 dimension model

## II. PROPOSED POSITIONING ALGORITHM

The position and configuration of all the obstacles in and around the measurement field were assumed to be exactly as given in this paper. We describe the algorithm in two dimensions (2D) to avoid unnecessary complexity, but extending the algorithm to three dimensions (3D) is straightforward.

Consider base stations  $BS_1, BS_2, \dots, BS_n$  in the 2D measurement field. The coordinates of  $BS_i$  are  $(x_i, y_i) (i = 1, 2, \dots, n)$ . The coordinates of the mobile station (MS) are  $(x, y)$ . The MS receives the transmitted signal from each BS and transmits the response signal synchronized to each BS's signal. Each BS estimates the distance to the MS by calculating the propagation time from the time lag between the transmitted and received times of the response signal from the MS.

In general, a BS receives many reflected waves from obstacles in and around the field. The distance to the MS is calculated by using the direct waves that is received first, and reflected waves are ignored in current TOA approaches. The proposed method positively utilizes the information from all of the principal reflected waves in order to realize the positioning using few BSs (one BS in the minimum case).

Fig.1 is a 2D model whose perimeter is  $a$  m  $\times$   $b$  m rectangle. There is one base station  $BS_1(x_1, y_1)$ , and one mobile station MS  $(x, y)$  in the rectangle. The surfaces  $y = 0$  and  $x = 0$  are reflecting planes referred to surfaces 1 and 2, respectively. The surfaces  $x = a$  and  $y = b$  are absorbing planes that do not reflect radio waves absolutely.  $BS_{1-1}(x_{1-1}, y_{1-1})$  is the mirror image of  $BS_1$  with respect to surface 1.  $BS_{1-2}(x_{1-2}, y_{1-2})$  is the mirror image of  $BS_1$  with respect to surface 2.  $BS_{1-1,2}(x_{1-1,2}, y_{1-1,2})$  is the mirror image of  $BS_{1-1}$  with respect to surface 2. Denote  $d_a$  as the distance between  $BS_a$  and MS ( $a = 1, 1-1, 1-2, 1-1,2$ ). Then  $d_a = \sqrt{(x - x_a)^2 + (y - y_a)^2}$ .

Fig.2 is the model of the delay profile of the signal received by  $BS_1$ .  $BS_1$  receives the direct wave that has a propagation time of  $\tau_1$ . The reflected wave that reflects at surface 1 has a propagation time of  $\tau_{1-1}$ , the other reflected wave that

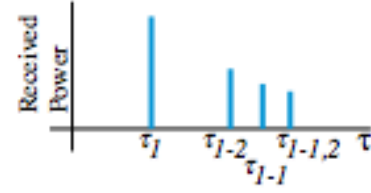


Fig. 2. Delay profile

reflects at surface 2 has a propagation time of  $\tau_{1-2}$ , and the third reflected wave that reflects at surfaces 1 and 2 has a propagation time of  $\tau_{1-1,2}$ .  $c$  is the speed of light; then,  $d_a = c\tau_a$ .  $\tau_a = \frac{1}{2}(t_a - t_m)$ , where  $t_a$  is the time lag between the transmitted and received times of the appropriate reflected (or direct) signal from the MS.  $t_m$  is the processing time at the BS and MS; this is known in advance.  $t_m$  is a constant in this case as only one BS is considered. The goal of this study was to estimate the position of the MS  $(x, y)$  when a delay profile similar to that shown in Fig.2 is measured. This is the same principle used in the TOA method to estimate position by measuring the propagation times at four base stations  $BS_1, BS_{1-1}, BS_{1-2}, BS_{1-1,2}$ .

More specifically, the proposed positioning method identifies the main obstacles in and around the field and calculates the positions of mirror-image BSs beforehand. BSs receive all of the signals from the MS, including the reflected waves, and measure the propagation times of the direct wave and main reflected waves. The TOA algorithm is then applied assuming reflected waves are received signals at the mirror-image BSs. The distinctions about which reflected waves correspond to which mirror-image BSs are not clear. So there is a possibility that plural candidate positions are estimated. The estimated position that is in the measurement field is adopted. We can narrow the candidates by changing the number of mirror-image BSs. Several TOA estimation algorithms have been proposed. The linear least square method[9] was adopted in this study to reduce the calculation complexity.

As described above, TOA positioning with one BS is possible when the position and configuration of the obstacles in and around the measurement area are given. This method is also applicable to TDOA. NLOS positioning by the proposed method is possible if the BS can receive more than three reflected waves. The errors due to individual differences among many BSs can be reduced as positioning is possible with few BSs (one BS in the minimum case).

But the number of mirror-image BSs increases by power law of the number of reflecting planes, and distinguishing which received signal reflects off which reflecting plane is difficult. This ambiguity decrease the positioning accuracy. In addition, it is usually difficult to get the exact position and configuration of all the obstacles and calculate all the mirror-image BSs. Thus, the positioning accuracy should be estimated when we can identify obstacles with realistic accuracy. In this paper we verified the proposed method by first conducting fundamental experiments in an ideal environment, and then analyzing the

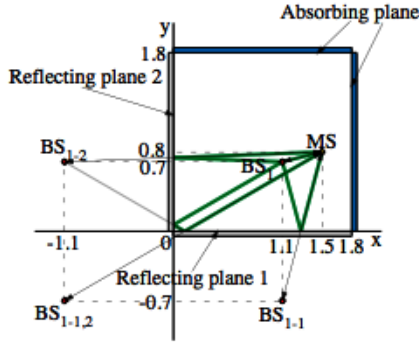


Fig. 3. Experiment layout

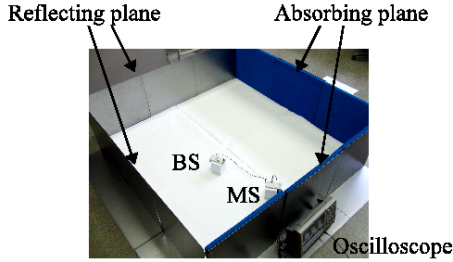


Fig. 4. Actual Experiment layout

estimation error.

### III. EXPERIMENT

#### A. Experiment Setup

Fig.3 shows the layout of the experiment field: it is a box with dimensions of 1.8 m  $\times$  1.8 m, where each surface at  $y = 0$  and  $x = 0$  is reflecting planes referred to as surfaces 1 and 2, respectively. The other two surfaces are absorbing planes. The transmitter is set as the MS at (1.5, 0.8), and the receiver is set as BS<sub>1</sub> at (1.1, 0.7). The transmitter radiated a narrow pulse at 32 ns/period. The received signal is observed with an oscilloscope. The high-band (7.25 - 10.25 GHz) UWB wireless application(UWB Locator, GIT Japan) was used with biconical antennas that have horizontal omnidirectional character. Fig.4 shows a photo of the experiment field. Authors conducted a pilot-study to evaluate the distance-accuracy of this UWB wireless system, and found the standard deviation error to be 1.06 cm[10].

The true value of the direct wave's propagation distance  $d_1$  was 41.23 cm, and the propagation time  $\tau_1$  was 1.37 ns. The propagation distance  $d_{1-1}$  of the reflected-wave from surface 1 was 155.24 cm; it was the distance between BS<sub>1-1</sub> and MS, and the propagation time  $\tau_{1-1}$  was 5.15 ns. The propagation distance  $d_{1-2}$  of the reflected wave from surface 2 was 260.19 cm, and the propagation time  $\tau_{1-2}$  was 8.67 ns. The propagation distance  $d_{1-1,2}$  of the twice-reflected-wave off surfaces 1 and 2 was 305.29 cm, and the propagation time  $\tau_{1-1,2}$  was 10.17 ns.

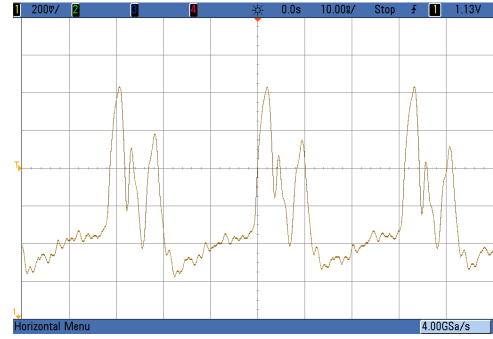


Fig. 5. A received signal

#### B. Experiment Result

Fig.5 shows a sample of the received signal on the receiver. The direct wave (the pulse whose amplitude is the highest) had a period of 32 ns. The first reflected-wave was received after the direct wave. The first reflected-wave was reflected by the surface 1. The second reflected-wave (from surface 2) and the third reflected-wave (the twice-reflected-wave off surfaces 1 and 2) each had small differences in propagation time. Thus, we could not observe the third reflected-wave. In addition, the amplitude of the third pulse was bigger than that of the first reflected-wave (the second pulse) because the former was the sum of the second and third reflected waves.

By defining the measured value of  $\tau_a$  as  $\hat{\tau}_a$ , we can measure  $\hat{\tau}_{1-1} - \hat{\tau}_1 = 3.97$  ns,  $\hat{\tau}_{1-2} - \hat{\tau}_1 = 7.87$  ns as shown in Fig.5. In this experiment, we could not determine the absolute propagation time; because of the one-way propagation experiment, we used the transmitter for MS, and receiver for BS. Thus we made our estimations under the assumption that we could measure the propagation time of the direct wave( $\hat{\tau}_1 = \tau_1$ ). Consequently, we can calculate  $\hat{\tau}_1 = 1.37$  ns,  $\hat{\tau}_{1-1} = 5.34$  ns.,  $\hat{\tau}_{1-2} = 9.24$  ns. Finally, the estimated location of MS from these measured values was (1.71, 0.86) using the linear least-square method, and the estimation error was 0.11 m.

The estimation error was caused by the widened pulse that was due to the impedance mismatch and stray capacitance caused by using a probe line from the analog circuit of the receiver, as well as the geometric arrangement error of the experiment field and thermal noise. In this experiment, we could not separate the second and third reflected-waves. The proposed method can not separate the propagation paths of received signals with almost the same propagation distances because these signals overlap.

### IV. ERROR EVALUATION

#### A. Theoretical Analysis

All BSs, including mirror-image BSs, are put into numerical order: BS<sub>1</sub>( $x_1, y_1$ ), BS<sub>2</sub>( $x_2, y_2$ ),  $\dots$ , BS <sub>$n$</sub> ( $x_n, y_n$ ). Each BS measures the propagation time to MS  $\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_n$ , and the position of MS is estimated by using TOA with the linear least square method. The real MS position is denoted as ( $x, y$ ), and

the estimated MS position is denoted as  $(\hat{x}, \hat{y})$ .

$$\hat{\tau}_i = \tau_i + \epsilon_i (i = 1, 2, \dots, n) \quad (1)$$

where  $\tau_i$  is the propagation time between  $BS_i$  and MS, and  $\epsilon_i$  is the additive white Gaussian noise (AWGN) with a mean of 0 and a variance of  $\sigma^2$ . The estimated propagation distance  $\hat{d}_i$  between  $BS_i$  and MS is then

$$\hat{d}_i = c\hat{\tau}_i = d_i + e_i \quad (2)$$

where  $d_i = c\tau_i$  is the real propagation distance between  $BS_i$  and MS, and  $e_i = c\epsilon_i$  is the propagation distance estimation error, which is AWGN with a mean of 0 and a variance of  $c^2\sigma^2$ .

By applying the linear least square method[9] and setting  $BS_1$  as the reference BS, we have

$$\mathbf{A}\hat{\mathbf{z}} = \frac{1}{2}\mathbf{p} \quad (3)$$

where

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ \vdots & \vdots \\ x_n - x_1 & y_n - y_1 \end{pmatrix} \quad (4)$$

$$\mathbf{p} = \begin{pmatrix} k_2 - k_1 + \hat{d}_1^2 - \hat{d}_2^2 \\ k_3 - k_1 + \hat{d}_1^2 - \hat{d}_3^2 \\ \vdots \\ k_n - k_1 + \hat{d}_1^2 - \hat{d}_n^2 \end{pmatrix} \quad (5)$$

$$\hat{\mathbf{z}} = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} \quad (6)$$

$$k_i = x_i^2 + y_i^2 \quad (7)$$

Equation (3) has an LS solution given by

$$\hat{\mathbf{z}} = \frac{1}{2}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{p} = \mathbf{z} + \frac{1}{2(su - t^2)} \begin{pmatrix} uv_e - tw_e \\ sw_e - tv_e \end{pmatrix} \quad (8)$$

where

$$s = \sum_{i=2}^n (x_i - x_1)^2 \quad (9)$$

$$t = \sum_{i=2}^n (x_i - x_1)(y_i - y_1) \quad (10)$$

$$u = \sum_{i=2}^n (y_i - y_1)^2 \quad (11)$$

$$v_e = \sum_{i=2}^n [2(x_i - x_1)(d_1 e_1 - d_i e_i) + e_1^2 - e_i^2] \quad (12)$$

$$w_e = \sum_{i=2}^n [2(y_i - y_1)(d_1 e_1 - d_i e_i) + e_1^2 - e_i^2] \quad (13)$$

The first term of equation (8) is the real position of MS  $(x, y)$ , and the second term is the error component. In general,  $e_i$  is relatively small compared to  $x_i, y_i$ . By ignoring  $e_i^2$ , the error component becomes the linear summation of  $e_i$ , and its mean is 0. As the variance of the error component becomes complex, we do not write down the formula, but the variance becomes small when MS is near the center of all BSs and mirror-image BSs.

### B. Numerical Simulation

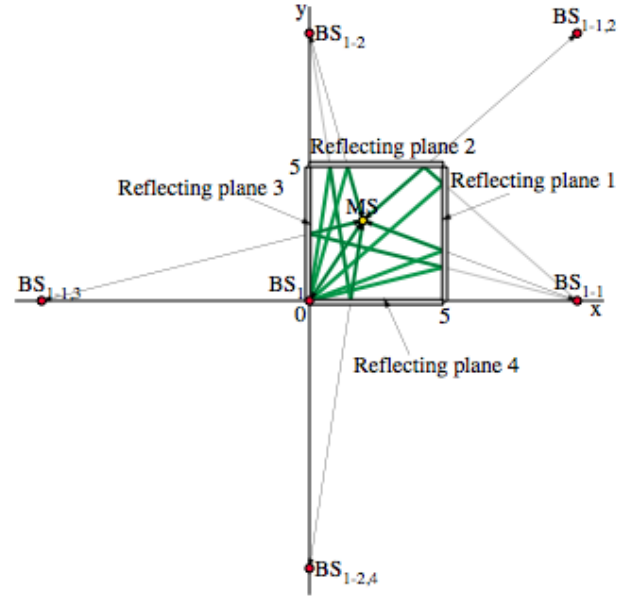


Fig. 6. Simulation model

Fig.6 shows the computation model for the numerical simulation. The field was a square, with dimensions of  $5\text{ m} \times 5\text{ m}$ . The four surfaces were reflecting planes referred to as surfaces 1 - 4. The base station  $BS_1$  was at the origin of the coordinate system, and the MS moved freely in the square field. The mirror images of the BS whose propagation waves reflected off one surface were  $BS_{1-1}$  and  $BS_{1-2}$ . The mirror images whose propagation waves reflected off two surfaces were  $BS_{1-1,2}$ ,  $BS_{1-1,3}$  and  $BS_{1-2,4}$ . Considering that the propagation waves reflected off surfaces up to two times, there were one BS and five mirror images of BSs; each BS received one direct wave and five reflected waves.

The standard deviation of the error of the distance measurement was assumed to be 20 cm in this simulation. Fig.7 (a) shows the distribution of the time-averaged error of the position estimation when the linear least square method was applied with three waves: the direct wave and two once-time-reflected-waves. Fig.7 (b) shows the error distribution estimated with four waves: the three waves noted above and

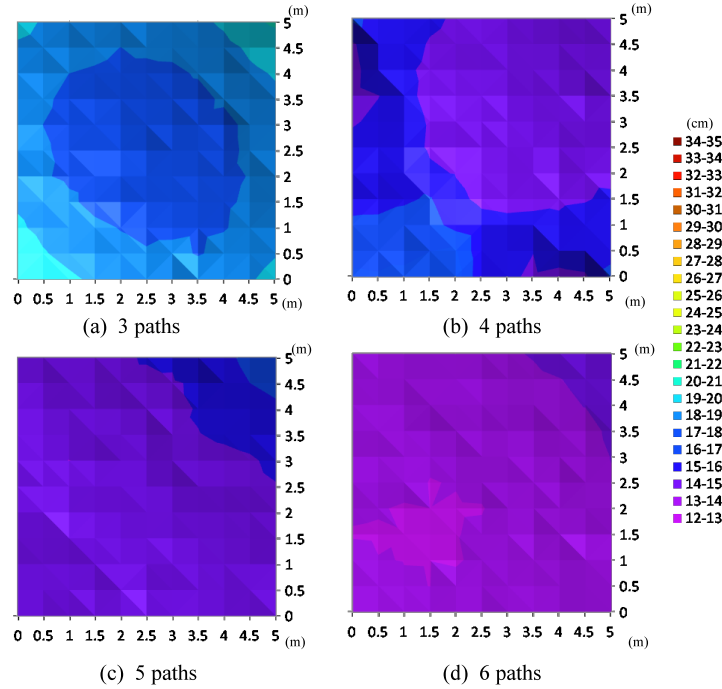


Fig. 7. Simulation of the 2-D positioning error

the twice-reflected-wave from  $BS_{1-1,2}$ . Fig.7 (c) shows the error distribution estimated with five waves: the four waves noted above and the wave from  $BS_{1-1,3}$ . Fig.7 (d) shows the error distribution estimated with all six waves. The errors became small near the center of all BSs including mirror images in all cases. These results agree with the above theoretical analysis. The space-averaged errors for the field were 18.3, 15.1, 14.6 and 13.4 cm for the cases of three, four, five, and six waves, respectively. The estimation accuracy improved when the number of measured waves was increased. However the improvement ratio was small when more than four measurements were used. As the mirror-image BSs were distributed around the measurement field, the difference in errors caused by the MS position became small when the number of images was increased.

## V. CONCLUSION

In this paper, we propose a positioning method that utilizes waves reflected from main obstacles in and around the measurement field. We verified the proposed method's efficacy through fundamental experiments. The estimation errors were evaluated theoretically and by computer simulation. This method realizes accurate positioning using few BSs (one BS in the minimum case). In this method, NLOS positioning is possible if more than three reflected waves can be received by the BS.

A Simple two-dimensional model was used for the evaluation in this study, but real environments are complex, and it is difficult to get the exact position and configuration of all the obstacles and to calculate all the mirror-image BSs. We confirmed that accurate position estimation is possible with

four or five mirror-image BSs off the main reflecting planes by the simple model. The study making clear the required accuracy of the position and configuration of obstacles is necessary and will be the subject of future study.

## ACKNOWLEDGMENT

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## REFERENCES

- [1] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, Z. Sahinoglu, "Localization via Ultra-Wideband Radios," IEEE Signal Processing Mag., vol.39, pp.70-84, Jul. 2005.
- [2] K. Hamaguchi, R. Kohno, "Development of Experimental TDOA System Test-Bed for Indoor Applications," Proc. ICUWB, pp.201-204, Oct. 2008.
- [3] F. Evrennou, F. Marx, S. Nacivet, "An Experimental TDOA UWB Location System for NLOS Environment," Proc. IEEE VTC, pp.420-423, Sep. 2005.
- [4] W-K. Chao, K-T. Lay, "NLOS Measurement Identification for Mobile Positioning in Wireless Cellular Systems," Proc. IEEE VTC, pp.1965-1969, Oct. 2007.
- [5] J. Xu, M. Ma, C. L. Law, "Position Estimation Using UWB TDOA Measurements," Proc. IEEE UWB, pp.605-610, Sep. 2006.
- [6] K. Kaemarungsri, P. Krishnamurthy, "Modeling of Indoor Positioning Systems Based on Location Fingerprinting," Proc. IEEE Comp. and Commun., vol.2, pp.1012-1022, Mar. 2004.
- [7] Y. Kurosaki, H. Yamada, Y. Yamaguchi, "Estimation of Indoor Radio Terminal Location by Extended Fingerprinting Techniques with Multi-Dimensional Signal Subspace," IEICE Trans., vol. J93-B, no. 2, pp.322-331, Feb. 2010.
- [8] J. Kietlinski-Zalenski, *et al.*, "Experimental Validation of TOA UWB Positioning with Two Receivers Using Known Indoor Features," Proc. IEEE PLANS, pp.505-509, May 2010.
- [9] I. Guvenc, C-C. Chong, F. Watanabe, "Analysis of a Linear Least-Square Localization Technique in LOS and NLOS Environment," Proc. IEEE VTC-Spring, pp.1886-1890, Apr. 2007.
- [10] M. Shimizu, T. Fujiwara, S. Uebayashi, "A Study for TOA Positioning Applying UWB Reflected Waves," Proc. IEICE General Conf., pp.596, Mar. 2011.