Receiver Design for Variable Gain Amplify-Forward Two-Way Relay with Channel Estimation Errors

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***Abstract*—In this paper, we address a two-way relay network (TWRN) where two source nodes exchange their information through a relay node (RN) in a bi-directional manner and propose a receiver design method at RN and source nodes under variable-gain amplify-and-forward (VG-AF) relay system. A two-period transmission protocol is employed: in the training period, source node and RN will send their own training signals to obtain channel state information (CSI); in the transmission period, two source nodes exchange their information based on the CSI obtained in the training period. We develop the Maximum-Likelihood (ML) estimator and Linear-Minimum-Mean-Square-Errors (LMMSE) estimator and derive the expression of channel estimation errors. Numerical results show that VG relay system outperforms fixed gain (FG) system in terms of mean square errors (MSE) of channel estimation and bit error rate (BER) performance. Besides, in contrast to the high Peak-to-Average-Power-Ratio (PAPR) in FG relay system, simulation results show that VG relay system has a more stable transmission power.**

I. Introduction

Wireless relay networks have been intensively studied due to their capability of enhancing the system capacity and providing the spatial diversity for single-antenna wireless transceivers by employing the relay nodes as "virtual" antennas [1], [2], [3]. We can achieve the cooperative diversity by using *amplify-and-forward* (AF) or *decode-and-forward* (DF) relaying scheme. Now, most of the proposed strategies can be classify into one-way relay network (OWRN), because all these works assume a unidirectional transmission.

Shannon first explored two-way transmission in his early work [4]. Compared to OWRN, two-way relay network (TWRN) shows us another insight about the modern communication where both terminals simultaneously send their information to the other one. It should be noted that TWRN has received much interest, due to its capability to improve the spectral efficiency compared to OWRN. In [5], the authors studied the optimal channel estimation and training design for fixed gain (FG) TWRN, then the optimal power allocation was derived. Network coding was utilized in [6] to improve the maximum sum-rate of both users, where each one is able to apply some algebraic operation on the received signals instead of simply amplify and forward them. Channel estimation for Orthogonal Frequency Division Multiplexing (OFDM) modulated TWRN has been deeply discussed in [7]. In [8], the authors analyzed the precoder design for two-way relay multiple-input and multiple-output (MIMO) with channel state information (CSI). Information-theoretic results on TWRN were also presented in [9].

Although there are lots of works for TWRN, most of them assumed perfect CSI and fixed gain relaying scheme. However, imperfect factors such as channel estimation errors and variable amplify factor at RN should been taken into consideration for further system improvement. In this paper, we consider a typical time-division-duplex (TDD) TWRN and employ a two-period transmission protocol. In the training period, source nodes and RN transmit orthogonal training signals to estimate each link. In this period, RN also needs to calculate the amplify factor which will be used in next period. During data transmission period, source nodes exchange their information to the other one through the variable gain (VG) RN who only amplifies the received signals and broadcasts them. Based on the above assumption, we derive Maximum-Likelihood (ML) estimator and Linear-Minimum-Mean-Square-Errors (LMMSE) estimator and obtain the expression of channel estimation error. Comparison between the two proposed estimators in FG mode and VG mode is presented. Finally, we also prove that VG relay system has low Peak-to-Average-Power-Ratio (PAPR) which is a critical character for industrial applications.

The rest of the paper is organized as follows. Section II presents the system model and transmission structure within coherent time. The channel estimation as well as the estimation errors for each link at both RN and source node are proposed in Section III. In Section IV, we provide simulation results to verify the proposed studies. Finally, conclusions are drawn in Section V.

Notations: Vectors and matrices are boldface small and capital letters, respectively; the transpose, complex conjugate, Hermitian, and inverse of $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^H$ and $\mathbf{A}^{-1}$, respectively; $|\mathbf{a}|$ denotes the two-norm of the vector $\mathbf{a}$; $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real and imaginary part of the complex argument inside, respectively; $[\mathbf{A}]_{ij}$ represents the $(i,j)-th$ element of $\mathbf{A}$ and $\mathbf{I}$ is the identity matrix; $E\{\cdot\}$ denotes the statistical expectation.

II. System Model

Consider a TWRN consisting of two sources $\mathbf{S}1$, $\mathbf{S}2$ and a relay node $\mathbf{RN}$, each of which is equipped with single antenna as shown in Fig. 1. Quasi-static flat fading relay channels are assumed and channel fading remains unchanged within the

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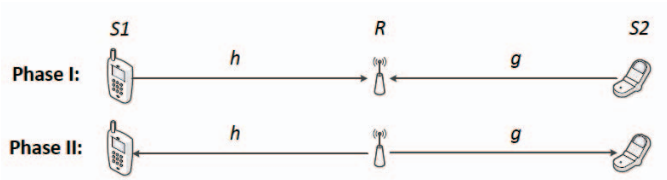

Fig. 1. A typical two-way relay network

coherent time.We denote the channel between $\mathbf{S}1$ and $\mathbf{RN}$, the channel between $\mathbf{S}2$ and $\mathbf{RN}$ as h, g, which are assumed as zero-mean circularly symmetric complex Gaussian random variables with variance σ_h^2 and σ_g^2, respectively.

The transmission structure within one coherent time is illustrated in Fig. 2. Each period has two phases which are defined in Fig. 1. We assume that there are M data symbols to send in each block and m blocks in one coherence interval (In Fig. 2, $m = 3$). During training period, both source nodes transmit N training symbols which are orthogonal to each other. Relay then amplifies the received signals with a fixed factor α_{FG} (4) and sends a private pilots $\mathbf{t}_r$ (the length is N). Based on the training signals, each node estimates every channel and we denote the estimated channels at $\mathbf{S}1$ and $\mathbf{RN}$ as $\hat{h}_1$, $\hat{g}_1$, $\hat{h}_R$ and $\hat{g}_R$, respectively. During data transmission period, the transmission signals for $\mathbf{S}1$ and $\mathbf{S}2$ are d_1 and d_2, respectively. $\mathbf{RN}$ receives

$$r = hd_1 + gd_2 + n_r, \tag{1}$$

where $E\{|d_i|^2\} = P_i$ $(i = 1, 2)$, and n_r is the complex additive white Gaussian noise (AWGN) with variance σ_n^2. Then, the relay amplifies the received signal by variable factor α_{VG-R} and broadcasts it. The variable amplify factor and the fixed amplify factor are given as

$$\alpha_{VG-R} = \sqrt{\frac{\mathrm{P}_r}{\left|\hat{h}_R\right|^2 P_1 + |\hat{g}_R|^2 P_2 + \sigma_n^2}}, \tag{2}$$

$$\alpha_{VG-1} = \sqrt{\frac{\mathrm{P}_r}{\left|\hat{h}_1\right|^2 P_1 + |\hat{g}_1|^2 P_2 + \sigma_n^2}}, \tag{3}$$

$$\alpha_{FG} = \sqrt{\frac{\mathrm{P}_r}{\sigma_h^2 P_1 + \sigma_g^2 P_2 + \sigma_n^2}}, \tag{4}$$

where P_r is the transmission power at RN, α_{VG-1} and α_{VG-R} are the amplify factor estimated at $\mathbf{S}1$ and RN, respectively. In Phase II, the two source nodes receive the signal as follows:

$$y_1 = \alpha_{VG-R} h^2 d_1 + \alpha_{VG-R} ghd_2 + \alpha_{VG-R} hn_r + n_1, \tag{5}$$

$$y_2 = \alpha_{VG-R} g^2 d_2 + \alpha_{VG-R} ghd_1 + \alpha_{VG-R} gn_r + n_2. \tag{6}$$

For simplicity, the noise variances n_1, n_2 have the same distribution as n_r, i.e., $n_1, n_2, n_r \in CN(0, \sigma_n^2)$. Note that we have reciprocity for the same link during two phases due to time-division-duplex. Because of the symmetry between $\mathbf{S}1$ and $\mathbf{S}2$, we will only discuss the process in $\mathbf{S}1$ while $\mathbf{S}2$ can be made correspondingly.

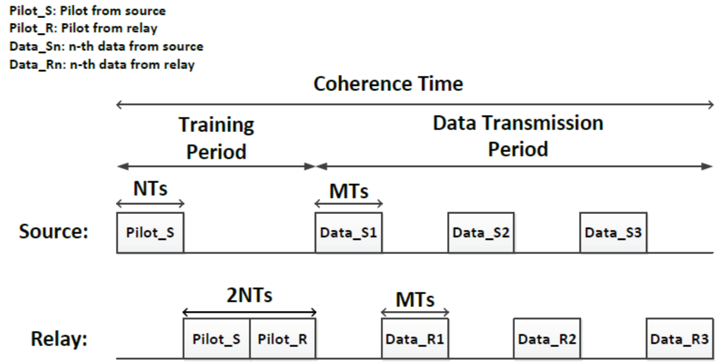

Fig. 2. Transmission structure within coherence time

III. Channel Estimation

In this Section, we explore the receiver design method in VG-AF TWRN. The coherent ML receiver at $\mathbf{S}1$ is

$$\begin{aligned} \hat{d}_2 &= \arg\max_{d_2} p(y_1 | d_2) \\ &= \arg\min_{d_2} \left| y_1 - \alpha_{VG-1} \hat{h}_1^2 d_1 - \alpha_{VG-1} \hat{g}_1 \hat{h}_1 d_2 \right| . \end{aligned} \tag{7}$$

Different from the FG relay system, RN also needs to estimate each link to get the variable factor α_{VG-R} in VG mode. For data detection, the estimated channels at $\mathbf{S}1$ can be also used to derive α_{VG-1} (3). As a result, channel estimation scheme is the key point of the receiver design. In order to find a solution to the above problem, we now focus our works on the training period.

First, we denote the training vector from $\mathbf{S}1$ and $\mathbf{S}2$ as $\mathbf{t}_1$ and $\mathbf{t}_2$, respectively. Then, the received signal at RN can be written as

$$\mathbf{y}_r = h\mathbf{t}_1 + g\mathbf{t}_2 + \mathbf{n}_r. \tag{8}$$

A. ML Channel Estimation at RN

By definition, the ML channel estimation assumes the deterministic channels, which gives the PDF of $\mathbf{y}_r$ as:

$$p(\mathbf{y}_r | h, g) = \frac{1}{(\pi\sigma_n^2)^N} \times \exp\{-\frac{|\mathbf{y}_r - h\mathbf{t}_1 - g\mathbf{t}_2|^2}{\sigma_n^2}\}. \tag{9}$$

$\hat{h}_R$ and $\hat{g}_R$ can be obtained by maximizing the likelihood function of $\mathbf{y}_r$. Hence, for a given h,

$$\hat{g}_R = \arg\min_g |\mathbf{y}_r - h\mathbf{t}_1 - g\mathbf{t}_2|^2. \tag{10}$$

From (10), we attain

$$\hat{g}_R = \frac{\mathbf{t}_2{}^H}{\|\mathbf{t}_2\|^2}(\mathbf{y}_r - h\mathbf{t}_1). \tag{11}$$

Substituting (11) into (9), we obtain

$$\begin{aligned}\hat{h}_R &= \arg\min_h \frac{\left\|(\mathbf{I}-\frac{\mathbf{t}_2\mathbf{t}_2{}^H}{\|\mathbf{t}_2\|^2})(\mathbf{y}_r - h\mathbf{t}_1)\right\|^2}{\sigma_n^2} + N\log(\pi\sigma_n^2) \\ &= \arg\min_h \|\mathbf{A}(\mathbf{y}_r - h\mathbf{t}_1)\|^2 \\ &= \arg\min_h (\mathbf{y}_r{}^H\mathbf{A}\mathbf{y}_r - h\mathbf{y}_r{}^H\mathbf{A}\mathbf{t}_1 - h^H\mathbf{t}_1{}^H\mathbf{A}\mathbf{y}_r \\ &\qquad + h^2\mathbf{t_1}{}^H\mathbf{A}\mathbf{t_1})\end{aligned} \tag{12}$$

where $\mathbf{A} = \mathbf{I} - \frac{\mathbf{t}_2\mathbf{t}_2{}^H}{\|\mathbf{t}_2\|^2}$ is a projection matrix. Note that $\angle\hat{h}_R$ can be independently estimated as $\angle\hat{h}_R = -\angle\mathbf{y}_r{}^H\mathbf{A}\mathbf{t}_1$. Finally, $\left|\hat{h}_R\right|$ is estimated as

$$\begin{aligned}\left|\hat{h}_R\right| = \arg\min_x (&\mathbf{y}_r{}^H\mathbf{A}\mathbf{y}_r - x\left|\mathbf{y}_r{}^H\mathbf{A}\mathbf{t}_1\right| - x\left|\mathbf{t}_1{}^H\mathbf{A}\mathbf{y}_r\right| \\ &+ x^2\mathbf{t}_1{}^H\mathbf{A}\mathbf{t}_1)\end{aligned} \tag{13}$$

where we define

$$f(x) = \mathbf{y}_r{}^H\mathbf{A}\mathbf{y}_r - x\left|\mathbf{y}_r{}^H\mathbf{A}\mathbf{t}_1\right| - x\left|\mathbf{t}_1{}^H\mathbf{A}\mathbf{y}_r\right| + x^2\mathbf{t}_1{}^H\mathbf{A}\mathbf{t}_1.$$

Taking derivative of $f(x)$ with respect to x, finally we arrive at

$$\hat{h}_R = \frac{(\mathbf{y}_r{}^H\mathbf{A}\mathbf{t}_1)^*}{\left|\mathbf{t}_1{}^H\mathbf{A}\mathbf{t}_1\right|}, \tag{14}$$

$$\hat{g}_R = \frac{\mathbf{t}_2{}^H}{\|\mathbf{t}_2\|^2}(\mathbf{y}_r - \frac{(\mathbf{y}_r{}^H\mathbf{A}\mathbf{t}_1)^*}{\left|\mathbf{t}_1{}^H\mathbf{A}\mathbf{t}_1\right|}\mathbf{t}_1). \tag{15}$$

The estimation errors are

$$\Delta h_R = \frac{\mathbf{n}_r{}^T\mathbf{t}_1{}^*}{\left|\mathbf{t}_1{}^H\mathbf{t}_1\right|}, \qquad \Delta g_R = \frac{\mathbf{t}_2{}^H\mathbf{n}_r}{\left|\mathbf{t}_2{}^H\mathbf{t}_2\right|}. \tag{16}$$

Observing (16), both of them are caused by the additional noise. In addition, we can also calculate the factor α_{VG-R} by (14) and (15).

B. ML Channel Estimation at $\mathbf{S}1$

The received training signals at $\mathbf{S}1$ during the second phase are

$$\mathbf{y}_{1t} = \alpha_{FG}h^2\mathbf{t}_1 + \alpha_{FG}gh\mathbf{t}_2 + \alpha_{FG}h\mathbf{n}_r + \mathbf{n}_1, \tag{17}$$

$$\mathbf{y}_{1r} = h\mathbf{t}_r + \mathbf{n}_{1r} \tag{18}$$

where $\mathbf{n}_1$, $\mathbf{n}_{1r}$ are the corresponding noise vector. The PDF of $\mathbf{y}_{1t}$ and $\mathbf{y}_{1r}$ are derived as follows

$$\begin{aligned}p(\mathbf{y}_{1t}\,|g,h) = &\frac{1}{(\pi\sigma_n^2(\alpha_{FG}^2|h|^2+1))^N} \\ &\times \exp\{-\frac{\left|\mathbf{y}_{1t} - \alpha_{FG}h^2\mathbf{t}_1 - \alpha_{FG}gh\mathbf{t}_2\right|^2}{\sigma_n^2(\alpha_{FG}^2|h|^2+1)}\},\end{aligned} \tag{19}$$

$$p(\mathbf{y}_{1r}\,|h) = \frac{1}{(\pi\sigma_n^2)^N} \times \exp\{-\frac{|\mathbf{y}_{1r} - h\mathbf{t}_r|^2}{\sigma_n^2}\}. \tag{20}$$

Using the maximum likelihood criterion, the estimated channel at $\mathbf{S}1$ are

$$\hat{h}_1 = \frac{(\mathbf{y}_{1r}{}^H\mathbf{t}_r)^*}{\mathbf{t}_r{}^H\mathbf{t}_r}, \tag{21}$$

$$\hat{g}_1 = \frac{(\hat{h}_1\mathbf{y}_{1t}{}^H\mathbf{t}_2)^*}{\alpha_{FG}\left|\hat{h}_1\right|^2\mathbf{t}_2{}^H\mathbf{t}_2}. \tag{22}$$

(see appendix for more details)

The estimation error is

$$\Delta h_1 = \frac{\mathbf{n}_{1r}{}^T\mathbf{t}_r{}^*}{\left|\mathbf{t}_r{}^H\mathbf{t}_r\right|}, \tag{23}$$

$$\Delta g_1 = \frac{h\mathbf{n}_r{}^T\mathbf{t}_2{}^*}{(h+\Delta h_1)\|\mathbf{t}_2\|^2} + \frac{\mathbf{n}_1{}^T\mathbf{t}_2{}^*}{\alpha_{FG}(h+\Delta h_1)\|\mathbf{t}_2\|^2} - \frac{g\Delta h_1}{h+\Delta h_1}. \tag{24}$$

The estimation error of $\hat{h}_1$ is caused by the additional noise at $\mathbf{S}1$ while the estimation error for $\hat{g}_1$ is complicated. The first term of (24) presents the noise in relay and the pilot from $\mathbf{S}2$ affect the estimated accuracy through h with estimation error Δh_1; the second term presents both of the pilot from $\mathbf{S}2$ which go through h in the second phase and the noise in $\mathbf{S}1$ affect the accuracy for $\hat{g}_1$. The third term stands for the cross-effect on $\hat{g}_1$ between h and g.

C. LMMSE Channel Estimation at $\mathbf{S}1$

ML estimator is unbiased in Gaussian distributional noisy environment without considering the noise statistics. Just likes Least Square estimator, ML amplifies not only our signals but also the noise part, so it's not the best solution to our problem. We resort to LMMSE method to minimize the channel estimation mean square errors in the same system model. The received signals at $\mathbf{S}1$ are (17) and (18). Suppose the LMMSE estimation of h and g are $\hat{h}_{1M} = \mathbf{u}^H\mathbf{y}_{1r}$ and $\hat{g}_{1M} = \mathbf{v}^H(\mathbf{y}_{1t} - \alpha_{FG}\hat{h}_{1M}^2\mathbf{t}_1)$, where $\mathbf{u}$ and $\mathbf{v}$ should meet the following constraints

$$\mathbf{u} = \arg\min_{\mathbf{u}} E\{\left|h - \mathbf{u}^H\mathbf{y}_{1r}\right|^2\}, \tag{25}$$

$$\mathbf{v} = \arg\min_{\mathbf{v}} E\left\{\left|g - \mathbf{v}^H(\mathbf{y}_{1t} - \alpha_{FG}\hat{h}_{1M}^2\mathbf{t}_1)\right|^2\right\}. \tag{26}$$

Substituting (18) into (25), we obtain

$$\begin{aligned}\mathbf{u} &= \arg\min_{\mathbf{u}} E\{[h^H - (h^H\mathbf{t}_r{}^H + \mathbf{n}_{1r}{}^H)\mathbf{u}] \\ &\qquad \times [h - \mathbf{u}^H(h\mathbf{t}_r + \mathbf{n}_{1r})]\} \\ &= \arg\min_{\mathbf{u}} (\sigma_h^2 - \sigma_h^2\mathbf{t}_r{}^H\mathbf{u} - \sigma_h^2\mathbf{u}^H\mathbf{t}_r \\ &\qquad + \sigma_h^2\mathbf{u}^H\mathbf{R}_{\mathbf{t}_r}\mathbf{u} + \mathbf{u}^H\mathbf{R}_{\mathbf{n}_{1r}}\mathbf{u}).\end{aligned} \tag{27}$$

Taking the derivative of the right hand of (27) with the respect of complex vector $\mathbf{u}$ and force the equation to be zero, we get the optimal solution $\mathbf{u}$

$$\mathbf{u} = (\sigma_h^2(\sigma_h^2\mathbf{R}_{\mathbf{t}_r}{}^T + \mathbf{R}_{\mathbf{n}_{1r}}{}^T)^{-1}\mathbf{t}_r{}^*)^*. \tag{28}$$

Thus, we have

$$\hat{h}_{1M} = (\sigma_h^2(\sigma_h^2\mathbf{R}_{\mathbf{t}_r}{}^T + \mathbf{R}_{\mathbf{n}_{1r}}{}^T)^{-1}\mathbf{t}_r{}^*)^T\mathbf{y}_{1r}. \tag{29}$$

Before deriving the value of $\mathbf{v}$, we assume that we have a perfect channel estimation for h, i.e., $\hat{h}_{1M} = h$. This

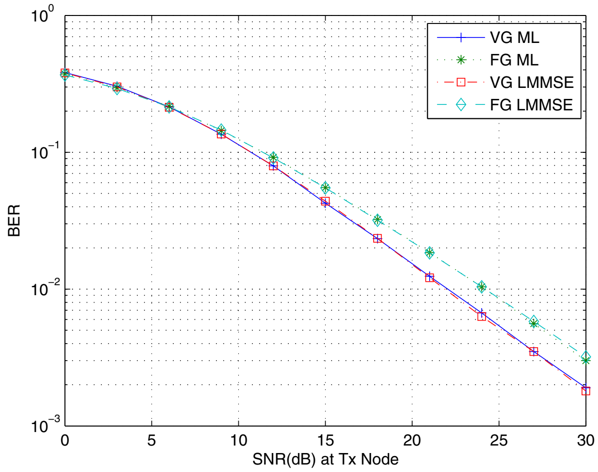

Fig. 3. BER versus SNR for different estimators and different relaying schemes

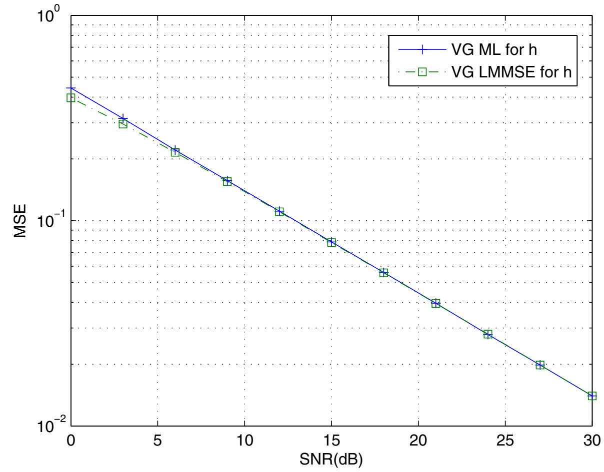

Fig. 4. Channel estimation MSEs versus SNR for $\hat{h}$ at $\mathbf{S}1$

assumption is reasonable in high SNR region. Substituting (29) and (17) back into (26), we obtain

$$\begin{aligned}\mathbf{v} = \arg\min_{\mathbf{v}}\{&\sigma_g^2 - \alpha_{FG}\sigma_g^2\hat{h}_{1M}^H{\mathbf{t}_2}^H\mathbf{v} + {\alpha_{FG}}^2\left|\hat{h}_{1M}\right|^4\mathbf{v}^H\mathbf{R}_{\mathbf{t}_1}\mathbf{v} \\ &- \alpha_{FG}\sigma_g^2\hat{h}_{1M}\mathbf{v}^H\mathbf{t}_2 + {\alpha_{FG}}^2\sigma_g^2\left|\hat{h}_{1M}\right|^2\mathbf{v}^H\mathbf{R}_{\mathbf{t}_2}\mathbf{v} \\ &+ {\alpha_{FG}}^2\left|\hat{h}_{1M}\right|^2\mathbf{v}^H\mathbf{R}_{\mathbf{n}_r}\mathbf{v} + \mathbf{v}^H\mathbf{R}_{\mathbf{n}_1}\mathbf{v}\}\end{aligned} \tag{30}$$

Taking the derivative of the right hand of (30) with respect to complex vector $\mathbf{u}$, similarly as vector $\mathbf{u}$, the expression of $\mathbf{v}^*$ can be written as

$$\begin{aligned}\mathbf{v}^* = &({\alpha_{FG}}^2\left|\hat{h}_{1M}\right|^4{\mathbf{R}_{\mathbf{t}_1}}^T + {\alpha_{FG}}^2\sigma_g^2\left|\hat{h}_{1M}\right|^2{\mathbf{R}_{\mathbf{t}_2}}^T \\ &+ {\alpha_{FG}}^2\left|\hat{h}_{1M}\right|^2{\mathbf{R}_{\mathbf{n}_r}}^T + {\mathbf{R}_{\mathbf{n}_1}}^T)^{-1}\alpha_{FG}\sigma_g^2\hat{h}_{1M}^H{\mathbf{t}_2}^*\end{aligned}. \tag{31}$$

Finally, $\hat{g}_{1M}$ is estimated as

$$\hat{g}_{1M} = \mathbf{v}^H\mathbf{y}_{1t}. \tag{32}$$

IV. Simulation Results

In this Section, we numerically study the performance of our proposed channel estimation schemes. The channels h, g are assumed as circularly symmetric complex Gaussian random variables with zero means and unit variances. We assume BPSK modulation with unit transmission power at $\mathbf{S}1$, $\mathbf{S}2$. Furthermore, transmission power at RN is $\mathrm{P}_r = 2 \times P_1$. In the simulation settings, we set the parameter N, M, m as 2, 8, 1, respectively. The signal to noise ratio (SNR) is defined as $SNR = 10 \times \log_{10}(P_1/\sigma_n^2)$. Totally 10^5 Monte-Carlo runs are adopted for average.

Fig. 3 compares the BER performance of our proposed estimator in variable gain mode with fixed gain mode in the same scenario. We see that the variable gain mode outperforms the fixed gain mode in high SNR region over the Rayleigh fading channel. It is due to the fact that the channel gain is random and there is a significant probability that the channel

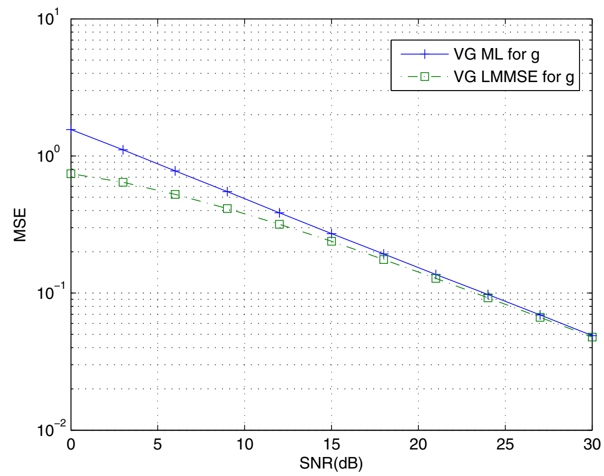

Fig. 5. Channel estimation MSEs versus SNR for $\hat{g}$ at $\mathbf{S}1$

is in a "deep fade". Deep fading channel can dramatically damage the received signals and cause data detection errors. Compared to the fixed gain relay mode, variable gain relay mode can protect our signal by multiplying an adaptive factor which is designed according to the channel estimation. As a results, VG mode achieves better performance. However, in low SNR region, FG mode wins VG mode a little because of its accuracy knowledge on factor α. In VG mode, terminal has to estimate not only h and g, but also the factor α, this adds more uncertainty compared to FG mode.

Fig. 4, Fig. 5 show the channel estimation MSE of $\hat{h}$ and $\hat{g}$ with different estimator, respectively. We can see that LMMSE method outperforms ML in all region, because LMMSE estimator takes the noise statistics into consideration. This contributes a lot to its good performance in low SNR region but low effect in high SNR region, where noise can be neglected. In other hand, we also can see that the MSE of $\hat{g}$ is bigger than $\hat{h}$. It's caused by the error propagation from $\hat{h}$

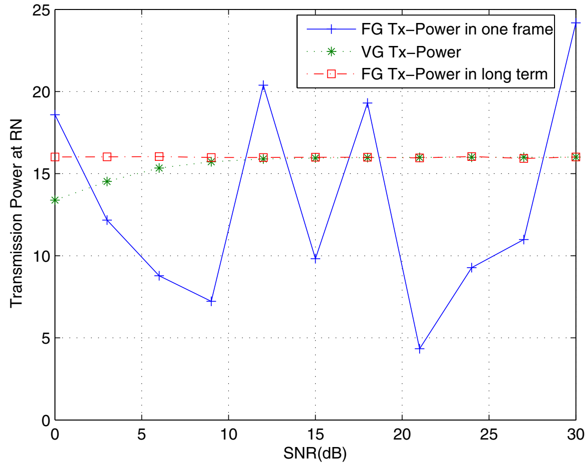

Fig. 6. Transmission power at Relay Node for VG and FG in different observation periods

at relay node.

Finally, we focus our attention on the transmission power (Tx-power) at RN. Fig. 6 shows the transmission power in different mode applying LMMSE estimator. We can see that the Tx-power in VG mode is stable despite of the random channel fading, because the variable amplify factor has compensated the fading effect. The Tx-power in FG mode is stable as VG mode if we take a long term observation, either. However, when we look into one frame, the Tx-power fluctuates sharply and causes high PAPR easily just like the blue line shown in Fig. 6. From the angle of industrial designs, high PAPR will translate into a larger bias in the power amplifier settings and a correspondingly lower average energy efficiency. From the perspective of green energy, VG relay is more promising and environmentally friendly.

V. Conclusion

In this paper, we proposed a transmission scheme and compared different estimators for AF-VG TWRN. It is seen that VG mode has almost the same BER performance as FG mode in low SNR region but does better in high SNR region. Besides, VG-AF relay has low PAPR which is more attractive to industry. We also studied the MSE performance between LMMSE and ML estimator. Apparently, ML estimator has worse performance in low SNR region but is asymptotically close to LMMSE in high SNR region.

Appendix

Derivation of ML Channel Estimation Results

Since we have got the PDF of $\mathbf{y}_{1t}$ and $\mathbf{y}_{1r}$, observing (19) and (20), we need to find the solution to

$$\hat{h}_1 = \arg\min_h |\mathbf{y}_{1r} - h\mathbf{t}_r|^2, \tag{33}$$

$$\hat{g}_1 = \arg\min_g \left|\mathbf{y}_{1t} - \alpha_{FG}\hat{h}_1^2\mathbf{t}_1 - \alpha_{FG} g \hat{h}_1 \mathbf{t}_2\right|^2. \tag{34}$$

Taking advantage of the orthogonality between $\mathbf{t_1}$ and $\mathbf{t_2}$, $\hat{h}_1$ can be given as

$$\begin{aligned}\hat{h}_1 &= \arg\min_h \{({\mathbf{y}_{1r}}^H - h^H {\mathbf{t}_r}^H)(\mathbf{y}_{1r} - h\mathbf{t}_r)\} \\ &= \arg\min_h \{{\mathbf{y}_{1r}}^H\mathbf{y}_{1r} - h{\mathbf{y}_{1r}}^H\mathbf{t}_r - h^H{\mathbf{t}_r}^H\mathbf{y}_{1r} + |h|^2{\mathbf{t}_r}^H\mathbf{t}_r\}.\end{aligned} \tag{35}$$

Therefore

$$\hat{h}_1 = \frac{({\mathbf{y}_{1r}}^H\mathbf{t}_r)^*}{{\mathbf{t}_r}^H\mathbf{t}_r}. \tag{36}$$

Substituting (36) into (34) and define $|g| = x$, similar as $\hat{g}_R$ in Section III Part A, we obtain

$$\hat{g}_1 = \frac{(\hat{h}_1{\mathbf{y}_{1t}}^H\mathbf{t}_2)^*}{\alpha_{FG}\left|\hat{h}_1\right|^2{\mathbf{t}_2}^H\mathbf{t}_2}. \tag{37}$$

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