

# Frequency-Domain Scrambling Differential Detection and Equalization for DFT Scrambling Vector OFDM System

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**Abstract**—Although V-OFDM system and DFT scrambling vector OFDM system are suitable for frequency-selective fading channels, channel estimation error may significantly degrade the achievable BER performance. In this paper, a new frequency-domain scrambling differential detection and equalization is proposed for DFT-SV-OFDM system, where a scrambling decision feedback is used at the receiver to provide a reliable noise-reference signal, thus reducing the error propagation. Compared with the two existing systems, the new scheme shows a comparable performance but more robust against the channel estimation error.

**Keywords**—V-OFDM; DFT-SV-OFDM; Differential Detection and Equalization;

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing(OFDM) is widely used as a promising method for high-speed digital wireless applications. Whereas, one of the limitations is the spectral nulls of the channel (or deep fades) on some subcarriers. As a result, the information symbols on those subcarriers will be lost. To improve the performance, Xia proposes a single antenna vector OFDM(V-OFDM), which is robust to channel spectral nulls by blocking each  $K$  consecutive information symbols together with a  $K \times 1$  vector sequence [1]. Independently, Suehiro presents a similar SD-OFDM system by a Kronecker product modulation scheme [2] and Han gives a further analysis on the system performance and proposes constellation-rotated vector OFDM(CRV-OFDM) to achieve full diversity order through constellation rotation [3]. Moreover, recently, we proposed a DFT scrambling vector OFDM scheme(DFT-SV-OFDM), which inherits the advantages of V-OFDM system, and reduces the receiver complexity and exhibits even lower peak-to-average power ratio(PAPR) as well [4].

To achieve better performance in fading channels, all the systems require equalization at the receiver, which is generally based on the minimum mean-square error(MMSE) criterion or zero forcing(ZF) and so on. With accurate channel estimation, equalizers can alleviate inter-channel interference(ICI) caused by the fading channel. However, in practical wireless communication systems, it is difficult to obtain accurate channel estimation, thus the BER performance normally suffers from channel estimation errors.

In order to acquire better tracking ability against fading, the transmission rate of the pilot-assisted channel estimation scheme has to be increased. But, this generally reduces transmission efficiency. In this case, differential and feedback detection schemes have been proposed [5][6]. Though with a delay, differential detection can be a suboptimal option for its robustness to channel estimation errors, especially in a fast-fading channel. In general, these schemes are based on the relationship between signals transmitted in two or three time instants, suitable for systems with flat noise. However, if an error occurs in the former estimation bit, the feedback structure will spread the offset to consecutive signals, which is so-called error propagation. To improve the performance, the traditional scheme in [5] requires a complex optimal equalization, say, the maximum likelihood differential detection(ML-DD). Or another scheme in [6], the detection is based on a chip-level representation. And all these algorithms are designed for single carrier systems which are not suitable for the DFT-SV-OFDM system.

To reduce the effect of the channel estimation errors and the error propagation, in this paper, we propose a new frequency-domain scrambling differential detection and equalization(FDSDDE) scheme for the DFT-SV-OFDM system, where a scrambling decision feedback is used at the receiver to provide a reliable noise-reference signal and reduce the error propagation. It is shown that the proposed scheme exhibits comparable performance to the DFT-SV-OFDM system, while more robust against the channel estimation error.

The rest of this paper is organized as follows: in Section II, the transmission model of DFT-SV-OFDM system is presented based on Kronecker product using the FDSDDE scheme; in Section III, MMSE and ZF equalization weights and error propagation reduction scheme are proposed and analyzes the system performance; in Section IV, simulation results are presented; and Section V concludes the paper.

## II. SYSTEM MODEL

### A. Kronecker Modulation/Demodulation

For the convenience of the mathematical discussion, the concept and construction algorithm are reviewed by kroneck-

er product in DFT-SV-OFDM system, instead of IFFT and FFT in traditional OFDM system.

Let  $N$ -dimensional inverse DFT (IDFT) matrix as:

$$f_n = \frac{1}{N}(W_N^0, W_N^{-n}, W_N^{-2n}, \dots, W_N^{-(N-1)n}) \quad (1)$$

where  $W_N = \exp(-\frac{2\pi j}{N})$ , then

$$\mathbf{F}_N^{-1} = (f_0, f_1, \dots, f_n, \dots, f_{N-1})^T \quad (2)$$

Assume  $N$  data vectors  $\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{N-1}$  of length  $M$  ( $M \leq N$ ) as:

$$\begin{aligned} \mathbf{x}_0 &= (x_{00}, x_{01}, \dots, x_{0(M-1)}) \\ \mathbf{x}_1 &= (x_{10}, x_{11}, \dots, x_{1(M-1)}) \\ &\vdots \\ \mathbf{x}_{N-1} &= (x_{(N-1)0}, x_{(N-1)1}, \dots, x_{(N-1)(M-1)}) \end{aligned} \quad (3)$$

And let the kronecker product between  $f_n$  and  $x_n$  be  $X_n$ , then:

$$\begin{aligned} X_n &= f_n \otimes \mathbf{x}_n \\ &= \frac{1}{N}(x_{n0}W_N^0, x_{n1}W_N^0, \dots, x_{n(M-1)}W_N^0, \\ &\quad x_{n0}W_N^{-n}, x_{n1}W_N^{-n}, \dots, x_{n(M-1)}W_N^{-n}, \\ &\quad \vdots \\ &\quad x_{n0}W_N^{-(N-1)n}, \dots, x_{n(M-1)}W_N^{-(N-1)n}) \end{aligned} \quad (4)$$

Thus  $X_n$  is a line of numbers of length  $NM$ .

Therefore,  $N$  vectors of length  $NM$ ,  $\{X_0, X_1, \dots, X_{N-1}\}$ , have the property that any two vectors of  $\{X_0, X_1, \dots, X_{N-1}\}$  are cyclic orthogonal. Then the transmitter sends the sum of the vectors  $S$  after adding a cyclic prefix larger than the order of the channel impulse response,  $L$ .

In the receiver, define a vector of length  $M$ ,  $I_M$  as:

$$I_M = \{N, 0, \dots, 0\} \quad (5)$$

In  $I_M$ , one  $N$  and  $(M-1)$  of 0s are included.

When the signal  $X_n$  passes through a matched filter denoted as:

$$f_n^{-1} \otimes I_M = (W_N^0, 0, \dots, 0, W_N^n, 0, \dots, 0, W_N^{(N-1)n}, 0, \dots, 0) \quad (6)$$

Assume path number  $L \leq M$ , then the central part of the vector (length  $M$ ) is  $\mathbf{x}_n$ .

Whereas, when  $n \neq k$ ,  $0 \leq n \leq N-1$ ,  $0 \leq k \leq N-1$ , If the received  $X_n$  is input into the matched filter of  $f_n^{-1} \otimes I_M$ , then the central part of length  $M$  is always zero.

### B. DFT-SV-OFDM System Model

From the analysis about kronecker product signal characteristics, over the multipath rayleigh fading channel, the  $n$ th received signal vector with additive white Gaussian noise(AWGN) should be:

$$\mathbf{r}_n = \mathbf{x}_n \mathbf{H}_n + \sigma_n \quad (7)$$

where  $\mathbf{H}_n$  is a  $M \times M$   $r_-$  circulation matrix as:

$$\mathbf{H}_n = \begin{pmatrix} h_0 & h_1 & \dots & h_{M-1} \\ W_N^n h_{M-1} & h_0 & \dots & h_{M-2} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^n h_1 & W_N^n h_2 & \dots & h_0 \end{pmatrix} \quad (8)$$

and  $\sigma_n$  is the  $n$ th row vector of the zero-mean AWGN with variance  $\sigma^2 = N_0/2$ .

As we know,  $r_-$  circulation matrix can be rewritten as:

$$\begin{aligned} \mathbf{H}_n &= \mathbf{\Lambda} \bar{\mathbf{H}}_n \mathbf{\Lambda}^{-1} \\ &= \mathbf{\Lambda} \begin{pmatrix} h_0 & dh_1 & \dots & d^{M-1} h_{M-1} \\ d^{M-1} h_{M-1} & h_0 & \dots & d^{M-2} h_{M-2} \\ \vdots & \vdots & \ddots & \vdots \\ d^1 h_1 & d^2 h_2 & \dots & h_0 \end{pmatrix} \mathbf{\Lambda}^{-1} \end{aligned} \quad (9)$$

where  $\mathbf{\Lambda} = \text{diag}[1, d, d^2, \dots, d^{M-1}]$ ,  $d = W_N^{n/M}$ . And  $\bar{\mathbf{H}}_n$ , a circulation matrix, is a typical multipath fading channel matrix in single carrier system and can be rewritten as:

$$\bar{\mathbf{H}}_n = \mathbf{F}_M^{-1} \bar{\mathbf{h}} \mathbf{F}_M \quad (10)$$

where

$$\bar{\mathbf{h}} = \text{diag} \left( \sum_{i=0}^{M-1} d^i h_i W_M^0, \sum_{i=0}^{M-1} d^i h_i W_M^i, \dots, \sum_{i=0}^{M-1} d^i h_i W_M^{i \times (M-1)} \right) \quad (11)$$

and  $\mathbf{F}_M$  is a  $M$  points DFT matrix. So, the Eq.(7) can be rewritten as:

$$\mathbf{r}_n = \mathbf{x}_n \mathbf{\Lambda} \mathbf{F}_M^{-1} \bar{\mathbf{h}} \mathbf{F}_M \mathbf{\Lambda}^{-1} + \sigma_n \quad (12)$$

Based on the analysis above, the DFT-SV-OFDM system is shown in Fig.1. Let  $\hat{\mathbf{x}}_n$  be the  $n$ th symbol vector taken  $M$ -points FFT,  $\mathbf{F}_M$ , and next scrambled by  $\mathbf{\Lambda}^{-1}$ , as:

$$\hat{\mathbf{x}}_n = \mathbf{x}_n \mathbf{F}_M \mathbf{\Lambda}^{-1} \quad (13)$$

Then, in the receiving side, the signals, which are perturbed by the fading channel and the AWGN, are sampled

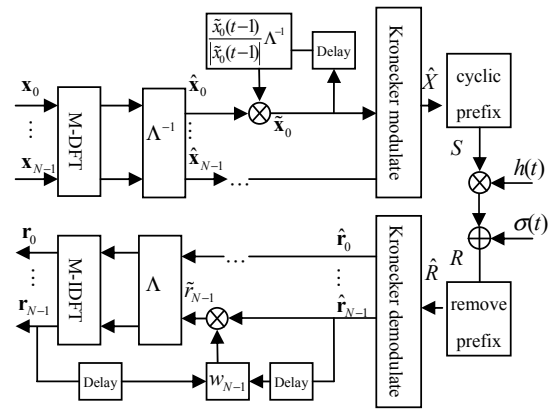


Figure 1. DFT-SV-OFDM System with FDSDD

by the ideal sampling timing, then removed the cyclic prefix, multiplied the scrambling code  $\Lambda$ , and taken  $M$ -point IFFT,  $\mathbf{F}_M^{-1}$ , as shown in Fig. 1. So, the hard decision form is produced as:

$$\mathbf{r}_n = \mathbf{x}_n \bar{\mathbf{h}} + \hat{\sigma}_n \quad (14)$$

where  $\hat{\sigma}_n = \sigma_n \Lambda \mathbf{F}_M^{-1}$  is a vector consisting of mutually independent zero-mean AWGN with variance  $N_0/2$ .

### C. Frequency-Domain Scrambling Differential Detection

Based on the analysis about the whole DFT-SV-OFDM system, the frequency-domain scrambling differential detection and equalization with a decision feedback scheme is shown in Fig.1. Different with the traditional differential detection, more simple equalization, such as MMSE or ZF, is chosen in the system while scrambling is introduced to improve the bit-level performance in the receiver.

As shown in Fig.1, after scrambling by  $\Lambda^{-1}$ , frequency-domain differential encoding at the  $n$ th subcarrier is performed as follow:

$$\tilde{\mathbf{x}}_n(t) = \hat{\mathbf{x}}_n(t) \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \Lambda^{-1} \quad (15)$$

where  $t \geq 1$ ,  $\tilde{\mathbf{x}}_n(t)$  is the differential encode vector with  $|\tilde{\mathbf{x}}_n(t)| = |\mathbf{x}_n(t)|$ . During  $t = 0$ , a pilot signal vector should be transmitted to the receiver. Then the signal vectors are modulated by kronecker product as usual before transmitted.

In the receiver, the sampled signals are removed the cyclic prefix and demodulated as:

$$\hat{\mathbf{r}}_n(t) = \tilde{\mathbf{x}}_n(t) \Lambda \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t) \mathbf{F}_M \Lambda^{-1} + \sigma_n(t) \quad (16)$$

Then frequency-domain differential equalization is given by:

$$\tilde{\mathbf{r}}_n(t) = \tilde{\mathbf{x}}_n(t) \Lambda \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t) \mathbf{F}_M \Lambda^{-1} \mathbf{w}_n(t) + \sigma_n(t) \mathbf{w}_n(t) \quad (17)$$

where  $\mathbf{w}_n(t)$  is the equalization weight vector of  $n$ th subcarrier in time  $t$ . So, the hard decision form is produced as:

$$\mathbf{r}_n(t) = \mathbf{x}_n(t) \mathbf{F}_M \Lambda^{-1} \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t) \mathbf{F}_M \Lambda^{-1} \mathbf{w}_n(t) \Lambda \mathbf{F}_M^{-1} + \sigma_n(t) \mathbf{w}_n(t) \Lambda \mathbf{F}_M^{-1} \quad (18)$$

## III. DETECTION SCHEME AND PERFORMANCE ANALYSIS

### A. Equalization Weights

Generally, ZF equalization and MMSE equalization are the main methods to achieve diversity because they are simpler than the traditional ML detection.

For ZF equalization, if the equivalent channel gains are made unity for all subcarriers and the signals transmitted always should be  $\pm 1$ , then weight for ZF can be reduced to:

$$\mathbf{w}_n^{ZF}(t) = \frac{\Lambda \mathbf{F}_M^{-1} \bar{\mathbf{h}}^T(t) \mathbf{F}_M \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|}}{\left| \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t) \mathbf{F}_M \Lambda^{-1} \right|^2} \quad (19)$$

where  $T$  denote the complex conjugate transpose matrix.

If the equalizer is based on minimum square error (MSE) criterion, that is, the error vector is orthogonal to the signal:

$$E[(\tilde{\mathbf{x}}_n - \mathbf{x}_n) \mathbf{r}_n^T] = \text{tr}\{E[\mathbf{r}_n^T (\tilde{\mathbf{x}}_n - \mathbf{x}_n)]\} = 0 \quad (20)$$

and because:

$$E[\mathbf{r}_n^T (\tilde{\mathbf{x}}_n - \mathbf{x}_n)] = E\left[\left(\tilde{\mathbf{h}} \mathbf{x}_n \mathbf{x}_n^T \tilde{\mathbf{h}}^T + N_0^2 \mathbf{I}_M\right) \mathbf{w}_n^{MMSE} - \tilde{\mathbf{h}}^T \mathbf{x}_n \mathbf{x}_n^T\right] \quad (21)$$

where  $\tilde{\mathbf{h}}(t) = \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t) \mathbf{F}_M \Lambda^{-1}$ .

Therefore, the equalizer for MMSE is:

$$\mathbf{w}_n^{MMSE}(t) = \frac{\Lambda \mathbf{F}_M^{-1} \bar{\mathbf{h}}^T(t) \mathbf{F}_M \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|}}{\left| \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t) \mathbf{F}_M \Lambda^{-1} \right|^2 + \frac{N_0}{E_s} \mathbf{I}_M} \quad (22)$$

However,  $\bar{\mathbf{h}}(t)$  and  $\frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|}$  are not known to the receiver. To realize equalization, assume  $|\tilde{\mathbf{x}}_n(t)| = |\mathbf{r}_n(t)|$ , and the channel fading is almost stationary, that is,  $\bar{\mathbf{h}}(t) \approx \bar{\mathbf{h}}(t-1)$ . From Eq.(16), the received differential signals can be written as:

$$\frac{\hat{\mathbf{r}}_n(t-1)}{|\mathbf{r}_n(t-1)|} = \frac{\tilde{\mathbf{x}}_n(t-1) \Lambda \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t-1) \mathbf{F}_M \Lambda^{-1} + \sigma_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \quad (23)$$

If ignoring additive white Gaussian noise, the equalization weights can be rewritten as:

$$\mathbf{w}_n(t) = \begin{cases} \frac{\mathbf{z}_n^T(t-1) \Lambda}{|\mathbf{z}_n(t-1) \Lambda^{-1}|^2} & \text{for ZF} \\ \frac{\mathbf{z}_n^T(t-1) \Lambda}{|\mathbf{z}_n(t-1) \Lambda^{-1}|^2 + \frac{N_0}{E_s} \mathbf{I}_M} & \text{for MMSE} \end{cases} \quad (24)$$

where

$$\mathbf{z}_n(t-1) = \frac{\hat{\mathbf{r}}_n(t-1)}{|\mathbf{r}_n(t-1)|} \quad (25)$$

When  $t = 0$ , we define  $\mathbf{z}_n(0) = \frac{\hat{\mathbf{r}}_n(0)}{|\mathbf{x}_n(0)|}$ , and  $\mathbf{x}_n(0)$  is the known reference signal vector in  $n$ th subcarrier.

Obviously  $\mathbf{z}_n(t-1)$  is noisy. From Eq.(23), using

$$\mathbf{z}_n(t-1) = \frac{\hat{\mathbf{r}}_n(t-1)}{|\mathbf{r}_n(t-1)|} \approx \delta_n(t-1) + \frac{\sigma_n(t-1) \Lambda^{-1}}{|\tilde{\mathbf{x}}_n(t-1)|} \quad (26)$$

where

$$\delta_n(t-1) = \frac{\tilde{\mathbf{x}}_n(t-1) \mathbf{F}_M^{-1} \bar{\mathbf{h}}(t-1) \mathbf{F}_M \Lambda^{-1}}{|\tilde{\mathbf{x}}_n(t-1)|} \quad (27)$$

and from Eq.(15),  $\delta_n(t-1)$  can be rewritten as:

$$\delta_n(t-1) = \frac{\tilde{\mathbf{x}}_n(t-1)}{|\tilde{\mathbf{x}}_n(t-1)|} \delta_n(t-2) \quad (28)$$

Since  $\delta_n(t-1)$  is the part infected by channel coefficients, weights will cumulate offsets. So, if erroneous decisions are made, owing to interference, such bit-level detector is prone to error propagation in case of feeding back previous error symbols as references. Considering error is occurred stochastically, scrambling code  $\Lambda^{-1}$  will improve the system performance.

### B. Performance Analysis

In this section, we first theoretically derive the conditional SINR based on Gaussian approximation of ISI, and then, numerically evaluate the theoretical average BER performance.

As stated in last section, in Eq.(18), define:

$$\begin{cases} \mathbf{\Omega}_n(t) = \mathbf{F}_M \mathbf{\Lambda}^{-1} \frac{\tilde{\mathbf{x}}_n(t-1)}{[\tilde{\mathbf{x}}_n(t-1)]} \mathbf{\Lambda} \mathbf{H}_n(t) \mathbf{w}_n(t) \mathbf{\Lambda} \mathbf{F}_M^{-1} \\ \mathbf{\Phi}_n(t) = \hat{\sigma}_n(t) \mathbf{w}_n(t) \mathbf{\Lambda} \mathbf{F}_M^{-1} \end{cases} \quad (29)$$

The output for decision on  $x_{n,k}$  is given by:

$$\begin{aligned} \Psi_{n,k} &= \sqrt{\frac{2E_s}{T_s}} x_{n,k} \Omega_{k,k} + \sqrt{\frac{2E_s}{T_s}} \sum_{\substack{i=0 \\ i \neq k}}^{M-1} x_{n,i} \Omega_{i,k} + \Phi_{n,k} \\ &= \sqrt{\frac{2E_s}{T_s}} x_{n,k} \Omega_{k,k} + \mu_{ISI} + \mu_{noise} \end{aligned} \quad (30)$$

where the first term represents the desired signal component, the second is the inter-symbol interference (ISI) and the third is the noise component, respectively. By approximating  $\mu_{ISI}$  as a zero-mean complex-valued Gaussian variable, the sum of  $\mu_{ISI}$  and  $\mu_{noise}$  can be treated as a new zero-mean complex-valued Gaussian noise, and the variance is:

$$\theta_\mu^2 = \frac{1}{2} E[|\mu_{ISI}|^2] + \frac{1}{2} E[|\mu_{noise}|^2] \quad (31)$$

For a feedback system, it's difficult to deduce an exact BER formula owing to the feedback error propagation, if not impossible. To derive an approximate conditional BER, we assume no noise contribution in the reference signal  $\mathbf{z}_n(t-1)$  like Eq.(25). Then, assume quaternary phase shift keying(QPSK) data modulation and all "1" transmission, without loss of generality, ISI can be assumed to be circularly symmetric. The conditional SINR for the given set of  $\mathbf{H} = \sum_{i=0}^{L-1} h_i$ , can be expressed as:

$$\gamma\left(\frac{E_s}{N_0}, \mathbf{H}\right) = \frac{2E_s |\Omega_{k,k}|^2}{\theta_\mu^2 T_s} \quad (32)$$

And the conditional BER for the given set of path gain  $\mathbf{H}$  can be expressed as:

$$\begin{aligned} p_b\left(\frac{E_s}{N_0}, \mathbf{H}\right) &= \frac{1}{2} \text{Prob}[\text{Re}[x_{n,k}] < 0 | \mathbf{H}] \\ &+ \frac{1}{2} \text{Prob}[\text{Im}[x_{n,k}] < 0 | \mathbf{H}] \\ &= \frac{1}{2} \text{erfc}\left[\sqrt{\frac{1}{4} \gamma\left(\frac{E_s}{N_0}, \mathbf{H}\right)}\right] \end{aligned} \quad (33)$$

where  $\text{erfc}[x] = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) dt$  is the complementary error function. The theoretical can be numerically evaluated by averaging Eq.(33) over  $\mathbf{H}$ .

To simplification, assuming ZF equalization without additive white Gaussian noise like Eq.(22), thus, the achievable BER performance improves, we have:

$$\Psi_{n,k} \approx \sqrt{\frac{2E_s}{T_s}} x_{n,k} + \Phi_{n,k} \quad (34)$$

From Eq.(29), it can be understood that first term of the output  $\Psi_{n,k}$  is a random variable with mean  $\sqrt{\frac{2E_s}{T_s}}$ . And the variance of is:

$$\theta_\Phi^2 = \frac{1}{2} E[|\Phi_{n,k}|^2] = \frac{N_0}{T_s \sum_{i=0}^{L-1} |h_i|^2} \quad (35)$$

Then, the improved conditional SINR, given by Eq.(32), can be rewritten as:

$$\gamma\left(\frac{E_s}{N_0}, \mathbf{H}\right) \approx \frac{2E_s}{N_0} \sum_{i=0}^{L-1} |h_i|^2 \quad (36)$$

The result also can be viewed as a bound. However, due to ISI effect and error propagation, the achievable BER performance degrades.

### IV. SIMULATION RESULTS

For simplification, we only consider uncoded system in frequency-selective Ricean fading multipath channels.

The simulation results of BER performance for V-OFDM system, DFT-SV-OFDM system, and DFT-SV-OFDM with FDSDD system are all presented with  $N = 8$ ,  $M = 6$ , path= 3.

As illustrated in the Fig.2, the influence of the scrambling code  $\mathbf{\Lambda}^{-1}$  on different ZF and MMSE with FDSDD on DFT-SV-OFDM system performance is investigated. With  $\mathbf{\Lambda}^{-1}$ , the system performance is improved, which means the error is reduced more effectively. From Eq.(27), it is clear that  $\mathbf{\Lambda}^{-1}$  change the channel coefficient from  $r_{\text{circulation}}$  matrix to circulation matrix, improving the symmetric of the channel. So stochastic error feedback is restrain, which shows error propagation effects system performance more directly.

Also, here, we find that MMSE outperforms ZF with FDSDD. A main reason linked to this result is that MMSE restrain noise so that the entire noise, including feedback

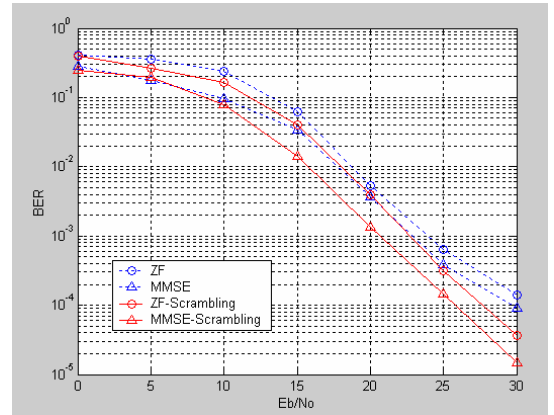


Figure 2. BER comparison for ZF or MMSE with or without scrambling code in DFT-SV-OFDM

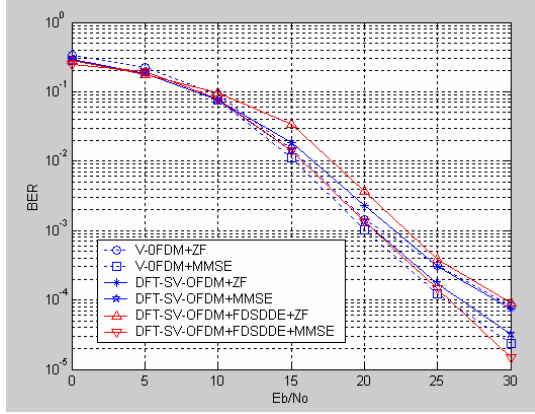


Figure 3. BER comparison for different systems with ideal channel estimation

noise, is weakened, which has also been indicated in Fig.3 and Fig.4.

In Fig.3 and Fig.4, we further examine the performance of different systems under different conditions.

Fig.3 compares the performance of different systems with the same equalization and ideal channel estimation. From this figure, if channel estimation is ideal, V-OFDM system and DFT-SV-OFDM system all achieve better performance than the new DFT-SV-OFDM FDSDD system with the same equalization receivers. With the increase in  $E_b/N_0$ , say, 25 dB, the new system converges together gradually. Two kinds of equalizers are used in the simulation, and MMSE equalizer is better than ZF equalizer at higher  $E_b/N_0$  level.

In Fig.4, more practical pilot-assisted channel estimations are introduced with a corresponding channel estimation error  $b$ . Obviously, the performance of V-OFDM system and DFT-SV-OFDM system drop quickly without ideal channel estimation, clearly outperformed by the DFT-SV-OFDM FDSDD system. For V-OFDM system and DFT-SV-OFDM system, with only 5 percent estimation error, the BER drops significantly (more than 5 dB). Other factors, such as different equalizers, only cause slight difference in these systems compared with channel estimation error  $b$ , indicating that the new DFT-SV-OFDM FDSDD system is more practical than other two existing systems.

## V. CONCLUSION

In this paper, a new frequency-domain scrambling differential detection and equalization(FDSDD) scheme in DFT-SV-OFDM system is proposed. Equalization weights for frequency-domain scrambling differential detection and equalization based on zero-forcing(ZF), and minimum mean square error(MMSE) were derived, together with its performance analysis. The new system inherits the merits of V-OFDM and DFT-SV-OFDM systems, suitable for frequency-selective fading channels, and shows very robust against the

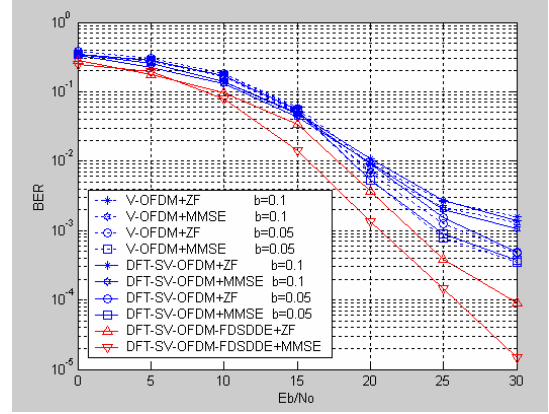


Figure 4. BER comparison for three different systems with channel estimation error  $b$

channel estimation error.

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## REFERENCES

- [1] Xiang-Gen Xia, *Precoded and Vector OFDM Robust to Channel Spectral Nulls and with Reduced Cyclic Prefix Length in Single Transmit Antenna Systems*, IEEE Trans. On Communications. August 2001 Vol.49, Page(s):1363-1374.
- [2] Naoki. Suehiro, Rongzhen JIN, *Performance of Very Efficient Wireless Frequency Usage System Using Kronecker Product with Rows of DFT Matrix*, Proceedings of 2006 IEEE Information Theory Workshop, June. 2006 Page(s):526-529.
- [3] Chenggao Han, Hashimoto. T, Suehiro. N, *Constellation-rotated vector OFDM and its performance analysis over rayleigh fading channels*, IEEE Trans. On Wireless Communications, Vol. 58, Issue 3, March 2010, Page(s):828-838.
- [4] Gao Zhou, Pinzhi Fan, Li Hao, Naoki. Suehiro, *DFT Scrambling Vector OFDM and Its Performance Analysis*, Proceedings of 2011 Third International Conference on Communications and Mobile Computing, April 2011, Page(s):381-384.
- [5] Franz Edbauer, *Bit error rate of binary and quaternary DPSK signals with multiple differential feedback detection*, IEEE Trans. On Communications, COM-40, No.3, March 1992, Page(s):457-460.
- [6] Le Liu, Fumiyuki Adachi, *Joint Frequency-Domain Differential Detection and Equalization for DS-CDMA Signal Transmissions in a Frequency- Selective Fading Channel*, IEEE Journal on Selected Areas in Communications, Vol. 24, No. 3, March 2006, Page(s): 649-658.