Compressed Channel Estimation for Sparse Multipath Non-orthogonal Amplify-and-forward Cooperative Networks

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***Abstract*—Coherent detection and demodulation at the receiver requires channel state information (CSI). We investigate channel estimation problem in sparse multipath non-orthogonal amplify-and-forward (NAF) cooperative networks. Traditional linear estimation methods can obtain lower bound at the cost of spectrum efficiency which is becoming more and more scarcity. In this paper, system model is described from sparse representation perspective. Based on the compressed sensing theory, we propose several compressed channel estimation methods to exploit sparsity of the cooperative channels. Simulation results confirm the superiority of proposed methods than LS-based linear estimation method.**

***Index Terms*—Compressed channel estimation, non-orthogonal amplify-to-forward (NAF), cooperative networks, sparse representation.**

I. Introduction

Relay-based cooperation communication networks [1]–[4] have been studied in the last decades due to its capability of enhancing the transmission capacity and providing the spatial diversity for single-antenna receivers by employing the relay nodes as virtual antennas [4]. It is well known that utilizing multiple input multiple output (MIMO) transmission can boost the channel capacity [5], [6] in broadband communication systems. In addition, diversity techniques in MIMO system could mitigate selective fading and hence enhance the quality of service (QoS) [7], [8]. However, it is very hard to integrate multiple antennas onto a small handheld terminal. To resolve the contradiction between then, one could choose relay-based cooperation networks which have been investigated in last decade years [1], [4]. The main reason is that the diversity from relay nodes existing in the network could be exploited, relay nodes can either be provided by telecommunication agencies or be obtained from cooperating terminals of other users.

In the relay-based cooperative network, data transmission is usually divided into two phases. During phase I, the source broadcasts its own information to relay. During Phase II, the relay could select different protocols and then transmit signal to the destination. Usually, there has two kinds of protocols in cooperation networks, one is purely amplify the received signal at relay and forward it to desination, which is termed as amplify-and-forward (AF); and the second is to coded the received signal and modulated again and retransmit to destination, which is often termed as decode-and-forward (DF). Due to coherent detection in these networks, accurate channel state information (CSI) is required at the destination (for AF) or at both relay and destination (for DF). About DF cooperation networks, the channel estimation methods from point-to-point (P2P) communication systems could be applied. However, extra channel estimation will increase the computational burden at relay and broadcasting the estimated channel information will result in further interference at destination. Based on the different protocols, cooperative networks correspond to three categories: (1) $\mathbb{S}$ transmits signal to $\mathbb{R}$ and $\mathbb{D}$ during the first time slot. In the second time slot, both $\mathbb{S}$ and $\mathbb{R}$ send signal to $\mathbb{D}$. This protocol realizes maximum degrees of broadcasting and receives collision; (2) $\mathbb{S}$ transmits signal to $\mathbb{R}$ and $\mathbb{D}$ during the first time slot. In the second time slot, only the $\mathbb{R}$ sends signal to $\mathbb{D}$. This protocol realizes the maximum degree of broadcasting and avoids possible receive collision; (3) $\mathbb{S}$ transmits signal to $\mathbb{R}$, assuming that $\mathbb{D}$ cannot receive the direct signal from $\mathbb{S}$ during the first time slot. At the second time slot, the amplified signal is broadcast to destination $\mathbb{D}$.

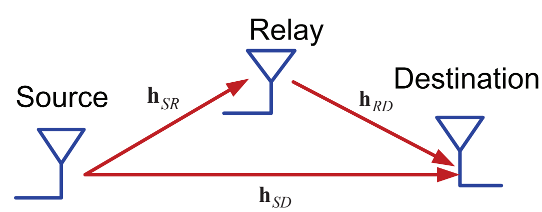

Fig. 1. An example of non-orthogonal amplify-and-forward cooperative network with source $\mathbb{S}$, relay $\mathbb{R}$ and destination $\mathbb{D}$.

Based on the theory of compressed sensing [9], [10], sparse channel estimation methods [11]–[13] have been proposed for P2P communication systems. However, channel estimation is also one of the key challenges in cooperative communication systems. In this paper, we focus our attention on category (2), relay based AF cooperative networks which is shown in Fig. 1. Linear channel estimation for the relay-based AF cooperative networks has been proposed [4] which is based on the assumption of rich multipath. Hence, low spectral efficieny is unavoidable due to that lager space is allocated

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to transmit training sequence and relatively small space is left to carry user data. As the channel measurement technique improves, wireless channels have been confirmed to often exhibit inherent sparse or cluster-sparse structure in delay-spread domain. In order to take advantages of channel sparsity, we propose a novel compressed channel estimation scheme by using compressed sensing [9], [10]. Simulation results confirm the effectiveness of the proposed methods.

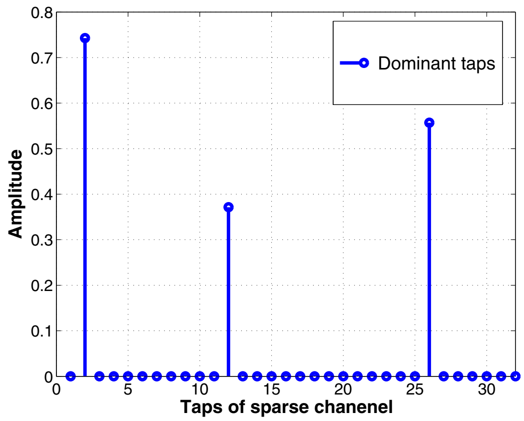

Fig. 2. A typical sparse multipath channel.

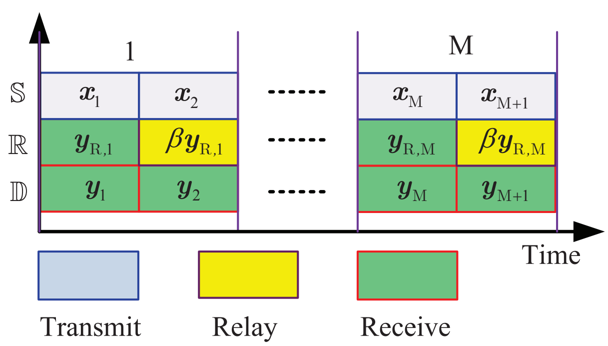

Fig. 3. M-block signal transmission in a NAF cooperative networks.

Section II introduces the NAF cooperative channel model and the sparseness measure of convoluted channel. Section III discusses compressed channel estimation of convoluted channel and presents various CCE methods. In section IV, we give various simulation results and discuss the performance of the estimators. Concluding remarks are presented in Section V.

Notations: In this paper, we use boldface lower case letters $\boldsymbol{x}$ to denote vectors, boldface capital letters $\boldsymbol{X}$ to denote matrices. x represents the complex Gaussian random variable. $E[.]$ stands for the expectation operation and $\boldsymbol{X}^T$, $\boldsymbol{X}^\dagger$ denote the matrix $\boldsymbol{X}$ transposition and conjugated transposition operations. $\|\boldsymbol{x}\|_0$ accounts the nonzero number of $\boldsymbol{x}$ and $\|\boldsymbol{x}\|_2$ is the Euclidean norm of $\boldsymbol{x}$. $Diag(\boldsymbol{x})$ is a diagonal matrix whose diagonal entries are from vector $\boldsymbol{x}$. The convolution between vectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ is denoted as $(\boldsymbol{x}_1 * \boldsymbol{x}_2)$.

II. System Model and Problem Formulation

Consider a multipath cooperative network where the source terminal $\mathbb{S}$ sends data to destination terminal $\mathbb{D}$ with partial help of relay terminal $\mathbb{R}$ which is shown in Fig. 1. The three nodes are assumed to equip single antenna each. $\boldsymbol{h}_{SR}$, $\boldsymbol{h}_{SD}$ and $\boldsymbol{h}_{RD}$ denote the impulse response of the frequency-selective fading channel vectors between three links $\mathbb{S} \to \mathbb{R}$, $\mathbb{R} \to \mathbb{D}$ and $\mathbb{S} \to \mathbb{D}$, respectively. Due to the same fading property of the channel vectors, we assume that all of channels have same length L and number of nonzero taps is K. Such as the channel model of $\boldsymbol{h}_{SD}$ which is given by [14], [15]

$$\boldsymbol{h}_{SD} = \sum\nolimits_{l=0}^{L-1} h_{SD,l}\delta(t - \tau_{SD,l}), \tag{1}$$

where $h_{SD,l}$ and $\tau_{SD,l}$ denote the complex-valued path gain with $E[\sum_{l=0}^{L-1} |h_{SD,l}|^2] = 1$ and symbol-spaced time delay of the lth path, respectively.

Fig. 3 shows M-block signal transmission over the NAF cooperative network. In one block, data symbols and training signals are assumed independent and always keeping orthogonal. The training signal vectors transmitted by the source $\mathbb{S}$ during the first and second time slots are denoted as $\boldsymbol{x}_1 = [x_1(0), x_1(1), ..., x_1(N-1)]^T$ and $\boldsymbol{x}_2 = [x_2(0), x_2(1), ..., x_2(N-1)]^T$, respectively. The power constraint of the transmission power is $E\{\boldsymbol{x}_i^H \boldsymbol{x}_i\} = NP$ for $i = 1, 2$, where P is the transmitting power from source $\mathbb{S}$.

According to the property of cooperative transmission, the transmission is divided into two phases. At the first phase, a $N-$dimensional (complex) signal $\boldsymbol{x}_1$ transmitted in equivlent complex baseband channel in the first time slot leads to a received signal at the $\mathbb{D}$ which is given by

$$\boldsymbol{y}_{D,1} = \boldsymbol{H}_{SD}\boldsymbol{x}_1 + \boldsymbol{z}_{D,1}, \tag{2}$$

where $\boldsymbol{H}_{SD}$ is an N complex circulant channel matrix with $[\boldsymbol{h}_{SD}^T \; \boldsymbol{0}_{1\times(N-L)}]^T$ as its first column; $\boldsymbol{z}_{D,1}$ is a realization of a complex additive Gaussian white noise vector with zero mean and covariance matrix $E\{\boldsymbol{z}_{D,1}\boldsymbol{z}_{D,1}^H\} = \sigma_n^2 \boldsymbol{I}_N$. At the same time, accoring to the cooperative network, the signal received at the $\mathbb{R}$ during the first time slot is given by

$$\boldsymbol{y}_R = \boldsymbol{H}_{SR}\boldsymbol{x}_1 + \boldsymbol{z}_{R,1}, \tag{3}$$

where $\boldsymbol{H}_{SR}$ is an N complex circulant channel matrix with $[\boldsymbol{h}_{SR}^T \; \boldsymbol{0}_{1\times(N-L)}]^T$ as its first column; $\boldsymbol{z}_{R,1}$ is a realization of a complex additive Gaussian white noise vector with zero mean and covariance matrix $E\{\boldsymbol{z}_{R,1}\boldsymbol{z}_{R,1}^H\} = \sigma_n^2 \boldsymbol{I}_N$. The relay $\mathbb{R}$ amplifies the received signal $\boldsymbol{y}_{SR}$ and retransmits the signal during the second time slot. The destination $\mathbb{D}$ receives a superposition of the relay transmission and the source transmission during the second time slot according to

$$\begin{aligned}\boldsymbol{y}_{D,2} &= \boldsymbol{H}_{SD}\boldsymbol{x}_2 + \beta\boldsymbol{H}_{RD}\boldsymbol{y}_R + \boldsymbol{z}_{D,2} \\ &= \boldsymbol{H}_{SD}\boldsymbol{x}_2 + \beta\boldsymbol{H}_{RD}(\boldsymbol{H}_{SR}\boldsymbol{x}_1 + \boldsymbol{z}_{R,1}) + \boldsymbol{z}_{D,2} \\ &= \boldsymbol{H}_{SD}\boldsymbol{x}_2 + \beta\boldsymbol{H}_{RD}\boldsymbol{H}_{SR}\boldsymbol{x}_1 + \tilde{\boldsymbol{z}}_{D,2},\end{aligned} \tag{4}$$

where $\boldsymbol{H}_{RD}$ is a circulant channel matrix with $[\boldsymbol{h}_{RD}^T \; \boldsymbol{0}_{1\times(N-L)}]^T$ as its first column; $\tilde{\boldsymbol{z}}_{D,2} =$

$\beta \boldsymbol{H}_{RD}\boldsymbol{z}_{R,1}+\boldsymbol{z}_{D,2}$ is a composite AWGN with zero mean and covariance matrix $E\{\tilde{\boldsymbol{z}}_{D,2}\tilde{\boldsymbol{z}}_{D,2}^{H}\} = (\beta^2|\boldsymbol{H}_{RD}|^2+\boldsymbol{I}_N)\sigma_n^2$, where $\boldsymbol{z}_{D,2}$ is a realization of a complex additive Gaussian white noise vector with zero mean and covariance matrix $E\{\boldsymbol{z}_{D,2}\boldsymbol{z}_{D,2}^{H}\} = \sigma_n^2\boldsymbol{I}_N$. Becasue of the random variant of the estimate $\boldsymbol{h}_{SR}$, the average relay power could not keep exactly as P_{SR}. If the amplify factor β could be caculated from long time point of view, hence, in this paper, by utilizing the channel variance of $\boldsymbol{h}_{SR}$, we choose the β as

$$\beta = \sqrt{\frac{P_R}{\sigma_{h_{SR}}^2 P_S + \sigma_n^2}}. \tag{5}$$

Using Eq. 2 and Eq. 4, the effective input-output relation in the AF model can be summarized as

$$\widetilde{\boldsymbol{y}} = \begin{bmatrix} \boldsymbol{H}_{SD} & \boldsymbol{0} \\ \beta\boldsymbol{H}_{RD}\boldsymbol{H}_{SR} & \boldsymbol{H}_{SD} \end{bmatrix}\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_{D,1} \\ \tilde{\boldsymbol{z}}_{D,2} \end{bmatrix} \tag{6}$$

According to matrix theory [16], all circulant matrices can share the same eigenvectors, the same matrix works for all circulant matrices. Hence, the separated circulant channel matrices $\boldsymbol{H}_{SR}$, $\boldsymbol{H}_{RD}$ and $\boldsymbol{H}_{SD}$ in Eq. (6) are de-composited as $\boldsymbol{H}_{SR} = \boldsymbol{W}^H\boldsymbol{\Lambda}_{SR}\boldsymbol{W}$, $\boldsymbol{H}_{RD} = \boldsymbol{W}^H\boldsymbol{\Lambda}_{RD}\boldsymbol{W}$, and $\boldsymbol{H}_{SD} = \boldsymbol{W}^H\boldsymbol{\Lambda}_{SD}\boldsymbol{W}$, respectively, where $\boldsymbol{W}$ is the unitary discrete Fourier transform (DFT) matrix with $[\boldsymbol{W}]_{m,n} = 1/\sqrt{N}e^{-j2\pi mn/N},\ m,n = 1,2,...,N$. Hence, cooperative channel matrix $\boldsymbol{H}_{SR}\boldsymbol{H}_{RD}$ is easily given by

$$\boldsymbol{H}_{SR}\boldsymbol{H}_{RD} = \boldsymbol{W}^H\boldsymbol{\Lambda}_{SR}\boldsymbol{\Lambda}_{RD}\boldsymbol{W} = \boldsymbol{W}^H\boldsymbol{\Lambda}_{SRD}\boldsymbol{W}, \tag{7}$$

where $\boldsymbol{\Lambda}_{SRD} = \boldsymbol{\Lambda}_{SR}\boldsymbol{\Lambda}_{RD}$.

Based on the DFT theory, $\boldsymbol{W}^H\boldsymbol{\Lambda}_{SRD}\boldsymbol{W}$ is the decomposition of a circulant matrix which is constructed from a composite channel impulse response $\boldsymbol{h}_{SRD} \triangleq (\boldsymbol{h}_{SR} * \boldsymbol{h}_{RD})$; $\boldsymbol{\Lambda}_{SR}$, $\boldsymbol{\Lambda}_{RD}$, and $\boldsymbol{\Lambda}_{SD}$ and $\boldsymbol{\Lambda}_{SRD}$ are diagonal matrices matrices which are expressed by

$$\begin{aligned} \boldsymbol{\Lambda}_{SD} &= Diag\left\{H_{SD}(0), H_{SD}(1), ..., H_{SD}(N-1)\right\}, \\ \boldsymbol{\Lambda}_{SRD} &= Diag\left\{H_{SRD}(0), H_{SRD}(1), ..., H_{SRD}(N-1)\right\}, \end{aligned} \tag{8}$$

where $H_{SD}(k)$ and $H_{SRD}(k)$, $k = 0, ..., N-1$ denote as

$$\begin{aligned} H_{SD}(k) &= \sum\nolimits_{l=0}^{L-1} h_{SD}(l)e^{-j2\pi kl/N}, \\ H_{SRD}(k) &= \sum\nolimits_{l=0}^{2L-1} h_{SRD}(l)e^{-j2\pi kl/N}, \end{aligned} \tag{9}$$

respectively.

Based on the above analysis, the input-output relation Eq. (6) can be rewritten as

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{z}, \tag{10}$$

where $\boldsymbol{y} = \left[(\boldsymbol{W}\boldsymbol{y}_{D,1})^T, (\boldsymbol{W}\boldsymbol{y}_{D,2})^T\right]^T$ denotes $2N$-length composite complex received signal vector;

$$\boldsymbol{X} = \begin{bmatrix} Diag(\boldsymbol{W}\boldsymbol{x}_1)\boldsymbol{F}_{SD} & \boldsymbol{0} \\ Diag(\boldsymbol{W}\boldsymbol{x}_2)\boldsymbol{F}_{SD} & Diag(\boldsymbol{W}\boldsymbol{x}_1)\boldsymbol{F}_{SRD} \end{bmatrix},$$

denotes $2N \times (3L-1)$-dimensional equivalent training signal matrix; $\boldsymbol{h} = [\ \boldsymbol{h}_{SD}^T \quad \boldsymbol{h}_{SRD}^T\ ]^T$ represents $(2L-1)$-length composited channel vector, based on the DFT theory; $\boldsymbol{z} = [\ (\boldsymbol{W}\boldsymbol{z}_1)^T \quad (\boldsymbol{W}\boldsymbol{z}_2)^T\ ]^T$ denotes $2N$-length (complex) AWGN vector. $\boldsymbol{F}_{SD}$ and $\boldsymbol{F}_{SRD}$ are matrices taking the first L and $(2L-1)$ columns of $\boldsymbol{W}$, respectively. And the noise interference $\boldsymbol{z}$ is a realization of a complex Gaussian random vector with zero mean and covariance matrix of $E\left\{\tilde{\boldsymbol{z}}\tilde{\boldsymbol{z}}^H\right\} = (\beta^2|\boldsymbol{\Lambda}_{RD}|^2+\boldsymbol{I}_N)\sigma_n^2$.

III. Sparse Measure of Cooperative Channels

In this part, we investigate sparse measure on two P2P channels $\boldsymbol{h}_{SR}$ and $\boldsymbol{h}_{RD}$ and its cooperative channel $\boldsymbol{h}_{SRD}$. We introduce a measure function [17] based on the relationship between the ℓ_1-norm and the ℓ_2-norm to calculate the sparseness of channel vectors. Consider a $L-$length sparse channel vector $\boldsymbol{h}_{SR}$, its sparseness is calculated by

$$Sparsness\left\{\boldsymbol{h}_{SR}\right\} = \frac{\sqrt{L} - \|\boldsymbol{h}_{SR}\|_1/\|\boldsymbol{h}_{SR}\|_2}{\sqrt{L}-1}. \tag{11}$$

This function equals to unity if and only if $\boldsymbol{h}$ contains only a single non-zero component. Due to the same sparse property as $\boldsymbol{h}_{RD}$ and $\boldsymbol{h}_{SRD}$, its sparse measure can also be calculated by Eq. (11).

To compare the sparseness between P2P channel and cooperative channels, Fig. 4 shows the sparseness of channel vectors $\boldsymbol{h}_{SR}$, $\boldsymbol{h}_{RD}$, and its cooperative channel vector $\boldsymbol{h}_{SRD}$. In Fig. 4, we assume that the length of P2P channels are $L_{SR} = 32$ and $L_{RD} = 42$. Hence, the length of cooperative channel $\boldsymbol{h}_{SRD}$ is $L_{SRD} = 73$. In real communication system, some channel degrees of freedom (DoF) are very small result in the data transmission is ineffective while dominant DoFs are active [14]. Hereby, we compute sparseness measure of a channel under different noise floors which related to signal-to-noise ratio (SNR). Here, the noise floor is defined as $10^{-SNR/10}$, where SNR is set to 10dB, 15dB and noiseless, respectively. In other words, only nonzero taps over noise floor are considered. As shown in the Fig. 4, it can be found that the sparseness of cooperative channel $\boldsymbol{h}_{SRD}$ approximates to $\boldsymbol{h}_{SR}$ and $\boldsymbol{h}_{RD}$, especial in low SNR case (SNR=10dB). Hence, compressed channel estimation methods can also be utilized to exploit channel sparsity in a sparse multipath cooperative netwroks.

IV. Compressed Estimation of NAF Coopertive Channels

For the problem in Eq. (10), mathmatically, the optimal compressed channel estimator $\tilde{\boldsymbol{h}}_{opt}$ is given by

$$\tilde{\boldsymbol{h}}_{opt} = \arg\min_{\boldsymbol{h}}\left\{\frac{1}{2}\|\boldsymbol{y}-\boldsymbol{X}\boldsymbol{h}\|_2^2 + \lambda\|\boldsymbol{h}\|_0\right\}, \tag{12}$$

where λ is a regulazied parameter which tradeoffs the estimation error and channel sparsity of channel vector $\boldsymbol{h}$. Unfortunately, Eq. (12) is nonconvex optimization problem and solving Eq. (12) is NP-hard [10]. In other words, optimal compressed channel estimators are unlikely to be calculated efficiently. However, numerous practical alternative algorithms can acquire a suboptimal solution for the cooperative channel $\boldsymbol{h}$ if the training measurement matrix $\boldsymbol{X}$ satisfies the restricted

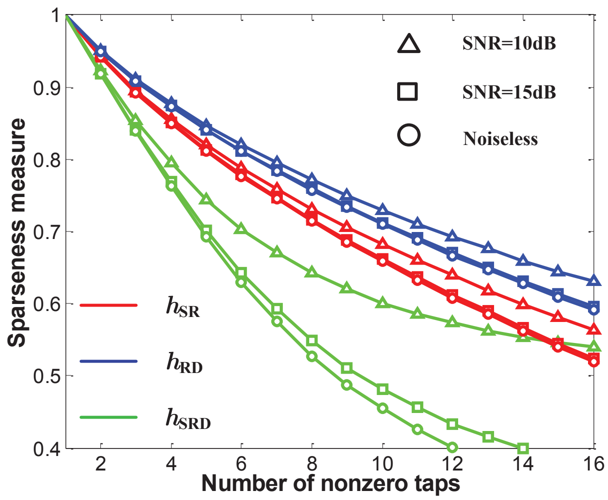

Fig. 4. Sparse measure of different channel vectors over the noise floor.

Isometry property (RIP) [18]. Here, we utilize two geedy algorithms, i.e., orthogonal matching pursuit (OMP) [19] and compressive sampling matching pursuit (CoSaMP) [20], which select each dominant coefficient in channel by iteration. Here we also give the LS channel estimator (known position of channel $\boldsymbol{h}$) for comparison. The LS estimator (known position) is given by

$$\tilde{\boldsymbol{h}} = \begin{cases} \boldsymbol{X}_T^{\dagger}\boldsymbol{y}, & T \subseteq \mathrm{supp}(\boldsymbol{h}) \\ 0, & others \end{cases}, \tag{13}$$

where $\mathrm{supp}(\boldsymbol{h})$ denotes the nonzero taps support of the channel vector $\boldsymbol{h}$, $\boldsymbol{X}_T$ is the submatrix constructed from the columns of $\boldsymbol{X}$; and T denotes the selected subcolumns corresponding to the nonzero index set of the convoluted channel vector $\boldsymbol{h}$.

The MSE of LS estimator $\tilde{\boldsymbol{h}}$ is given by

$$MSE(\tilde{\boldsymbol{h}}) = \sigma_n^2 \mathrm{Tr}\left\{ \left(\boldsymbol{X}_T^{\dagger}\boldsymbol{X}_T\right)^{-1} \right\}. \tag{14}$$

By utilizeing CS recovery algorithms for compressed channel estimation (CCE), which termed as CCE-OMP and CCE-CoSaMP. The two estimate methods for NAF cooperative channels are discribed as follows.

A. CCE-OMP estimator $\tilde{\boldsymbol{h}}_{OMP}$

Given the received signal $\boldsymbol{y}$, $\boldsymbol{W}$ and $\boldsymbol{F}$, and $\boldsymbol{x}$, CCE-OMP estimator run as follows:

Initialization. Set the nonzero coefficient index set $T_0 = \emptyset$, the residual estimation error $\boldsymbol{r}_0 = \boldsymbol{y}$ and put the initialize iteration counter as $k = 1$.

Identification. Select a column index n_k of $\boldsymbol{X}$ that is most correlated with the residual:

$$n_k = \left|\langle \boldsymbol{r}_{k-1}, \boldsymbol{X}_n \rangle\right|, and\ T_k = T_{k-1} \cup n_k. \tag{15}$$

Estimation. Compute the best coefficient for approximating the channel vector with chosen columns.

$$\boldsymbol{h}_k = \arg\min_{\boldsymbol{h}} \left\| \boldsymbol{y} - \boldsymbol{X}_{T_k}\boldsymbol{h} \right\|_2. \tag{16}$$

Iteration. Update the estimation error:

$$\boldsymbol{r}_k = \boldsymbol{y} - \boldsymbol{X}_{T_k}\boldsymbol{h}_k. \tag{17}$$

Increment the iteration counter k. Repeat (15)-(17) until stopping criterion holds and then set $\tilde{\boldsymbol{h}}_{OMP} = \boldsymbol{h}_k$.

B. CCE-CoSaMP estimator $\tilde{\boldsymbol{h}}_{CoSaMP}$

Given $\boldsymbol{y}$, $\boldsymbol{W}$ and $\boldsymbol{F}$, and $\boldsymbol{x}$ (training signal matrix $\boldsymbol{X} = diag(\boldsymbol{W}\boldsymbol{x})\boldsymbol{F}$), the maximum number of dominant channel coefficients is assumed as S. The CCE-CoSaMP runs as follows:

Initialization. Set the nonzero coefficient index set $T_0 = \emptyset$, the residual estimation error $\boldsymbol{r}_0 = \boldsymbol{y}$ and put the initialize iteration counter as $k = 1$.

Identification. Select a column index n_k of $\boldsymbol{X}$ that is most correlated with the residual:

$$n_k = \left|\langle \boldsymbol{r}_{k-1}, \boldsymbol{X}_n \rangle\right|, and\ T_k = T_{k-1} \cup n_k. \tag{18}$$

Using LS method to calculate a channel estimator as $T_{LS} = argmin \left\|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{h}\right\|$, and select T maximum dominant taps $\boldsymbol{h}_{LS}$. The positions of the selected dominant taps in this substep are denoted by T_{LS}.

Merge. The positions of dominant taps are merged by $T_k = T_{LS} \cup T_k$.

Estimation. Compute the best coefficient for approximating the channel vector with chosen columns,

$$\boldsymbol{h}_k = \arg\min_{\boldsymbol{h}} \left\| \boldsymbol{y} - \boldsymbol{X}_{T_k}\boldsymbol{h} \right\|_2. \tag{19}$$

Pruning. Select the T_k largest channel coefficients

$$\boldsymbol{h}_k = [\boldsymbol{h}]_S, \tag{20}$$

and replace the left taps $T \backslash T_k$ by zero.

Iteration. Update the estimation error:

$$\boldsymbol{r}_k = \boldsymbol{y} - \boldsymbol{X}_{T_k}\boldsymbol{h}_k. \tag{21}$$

Increment the iteration counter k. Repeat (18)-(21) until stopping criterion holds and then set $\tilde{\boldsymbol{h}}_{CoSaMP} = \boldsymbol{h}_k$.

V. Simulation Results

In this section, we will compare the performance of the proposed estimators with LS-based linear estimator and adopt 10000 independent Monte-Carlo runs for average. The length of training sequence is $N = 64$. All of K-sparse channel vectors $\boldsymbol{h}_{SR}$, $\boldsymbol{h}_{RD}$ and $\boldsymbol{h}_{SD}$ are generated following Guassian distribution and subject to $\|\boldsymbol{h}_{SR}\|_2^2 = \|\boldsymbol{h}_{RD}\|_2^2 = \|\boldsymbol{h}_{SD}\|_2^2 = 1$. And also all of the channels have same length $L = 32$, and its positions of nonzero channel taps are randomly generated. Transmit power is set as $P_S = NP$ and AF relay power is set as $P_R = 2NP$, where P is a unit transmitted power. The received SNR is defined as $10\log(P/\sigma_n^2)$. Channel estimators $\hat{\boldsymbol{h}}$ are evaluated by average mean square error (AMSE) which is defined by

$$AMSE(\hat{\boldsymbol{h}}) = \frac{\|\boldsymbol{h} - \hat{\boldsymbol{h}}\|_2^2}{M(3L-1)}, \tag{22}$$

where $\boldsymbol{h}$ and $\hat{\boldsymbol{h}}$ denote composite channel vector and its estimator, respectively, M is the number of Monte Carlo runs and $(3L-1)$ is the overall length of channel vector $\boldsymbol{h}$. In Fig. 5 shows that performance of the proposed CCE estimators (CCE-OMP and CCE-CoSaMP) are much better than LS estimator when $K=4$. In Fig. 6 shows the small performance gap between these channel estimators when $K=8$. From the Fig. 5 and Fig. 6, it is also easy found that the proposed estimators will be effected by channel sparseness. To estimate a cooperative channel will be obtained better estimation performance by exploiting more sparser information and vice versa. It is worth noting that if channels are dense rather than sparse, all of the proposed estimators will have the same performance as LS estimator.

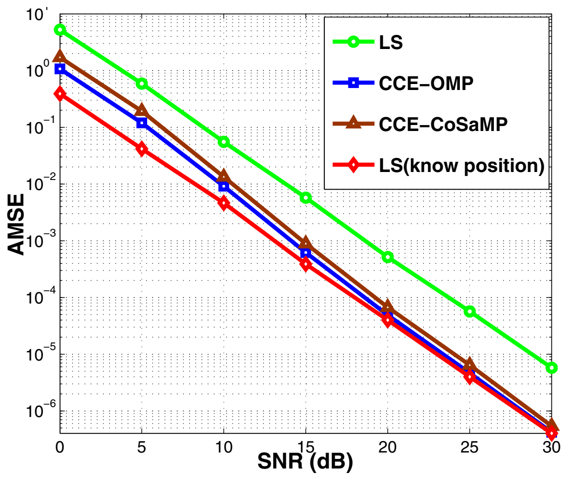

Fig. 5. AMSE performance as a function of different SNR ($K=4$).

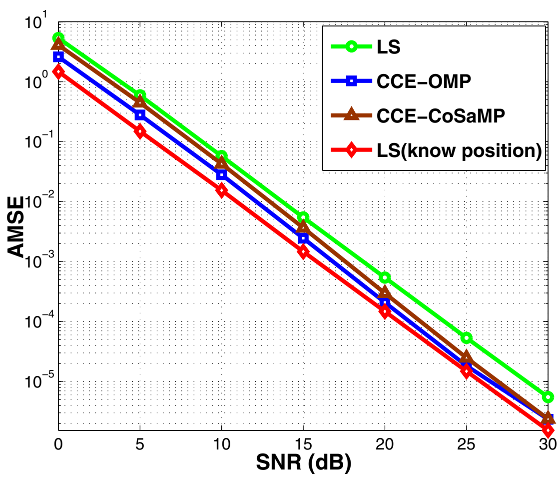

Fig. 6. AMSE performance as a function of SNR ($K=8$).

VI. Conclusion

Compressed channel estimation problem for sparse multipath non-orthogonal amplify-and-forward (NAF) cooperative networks has been inverstigated in this paper. We introduced a measure function on sparse channel vectors and compared sparsity between P2P channels and cooperative channels. From the CS perspective, CCE exploited the sparsity in multipath channel and hereby improved the estimation performance. Simulation results have confirmed the performance superiority of the proposed method to the conventional linear LS method. The proposed methods can also be extended to sparse multipath multi-antenna cooperative networks.

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