Investigation on Data Identification Problem for Data-Dependent Superimposed Training

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***Abstract* —In data-dependent superimposed training (DDST) scheme, the data-induced interference in channel estimation is eliminated at the sacrifice of data distortion. Unfortunately, data distortion causes data identification problem (DIP) for DDST scheme, which results in error floor phenomenon in bit error rate (BER). Although some literatures have analyzed the DIP, the DIP is still an open problem without solution. In this work, we firstly review the data identification problem from the view of sub-space. The analysis result inspires us to introduce a precoding matrix for resolving the DIP in DDST scheme. In order to prevent the advantages in DDST scheme being reduced by the precoding matrix, we introduce several constraints on the precoding matrix. Subsequently, we derive the requirement of the precoding matrix for solving the DIP in DDST system by using singular value decomposition. Furthermore, an appropriate precoding matrix is developed based on Zadoff-Chu sequence, which is shown to satisfy all conditions derived in this work. Finally, simulation results are conducted to verify that the precoding matrix removes the error floor in BER for DDST scheme.**

***Index Terms* —Cyclic prefixed single carrier system, Data dependent superimposed training (DDST), data identification problem.**

I. INTRODUCTION

Cyclic prefixed single carrier (CP-SC) system has emerged as a promising technique for wideband wireless communication. The performance of CP-SC is fundamentally dependent on the accuracy of channel estimation. In general, channel estimation is commonly achieved by using pilot symbols. Many literatures have investigated on pilot allocation in CP-SC system [1-3]. In [1], the authors proposed a pilot cyclic prefixed single carrier system, where a known block is added as a suffix to each symbol block before performing the cyclic prefixing. In specific, the data symbols and pilot symbols are transmitted on different time slots. For the sake of simplicity, this method is termed as time-division multiplexing (TDM) scheme. In [2-4], the superimposed training (ST) scheme is proposed, where the pilot symbols are added directly onto the data symbols.

The major advantage of ST method is the higher bandwidth efficiency than TDM scheme. However, the unknown data symbols induce extra interference in performing channel estimation, which degrades the accuracy of channel estimation. In [5], the authors proposed the data dependent superimposed training (DDST) scheme, where the data-induced interference in channel estimation is eliminated at the cost of distorting data sequences. Different methods have been proposed to mitigate the effect of data distortion [5-7]. Unfortunately, it is possible that the data distortion cannot be recovered at the receiver end, which is termed as data identification problem (DIP), resulting in that the BER is lower bounded. In [6-7], the DIP is investigated and analyzed. However, for the best of our knowledge, the solution for DIP is not provided in the existing literature. In this paper, we will therefore investigate the DIP in DDST scheme.

This work commences by reviewing the DIP in DDST scheme. It is mathematically demonstrated that the DDST scheme induces a null subspace in transmitted block. Therefore, the data sequence cannot be uniquely determined at the receiver end, which leads to the DIP in DDST scheme. The analytical results motivate us to introduce a precoding matrix for solving the DIP in DDST scheme. In order to prevent the precoding matrix reducing the advantages in DDST scheme, we introduce several constraints on the precoding matrix. And moreover, we derive the requirement which guarantees the data can be uniquely determined at the receiver end. Subsequently, we develop a precoding matrix satisfying the derived conditions. Finally, the simulation results are conducted to verify that the precoding matrix is able to solve the DIP in DDST scheme.

This paper is organized as follows. In Section II, the system model of CP-SC system with DDST scheme is described. The DIP in DDST scheme is mathematically analyzed in Section III. Subsequently, we derive in Section IV the requirements of the precoding matrix for resolving DIP in DDST scheme. We also propose in Section IV a precoding matrix that fulfills the derived conditions. Section V gives the simulation results, which demonstrate that the proposed precoding matrix solves the DIP in DDST schemes. Finally, the conclusions are drawn in Section VI.

II. SYSTEM MODEL

Consider a single carrier system with frequency domain equalization, where the sufficient cyclic prefix (CP) is added to prevent inter block interference (IBI). In addition, the pilot sequence is transmitted in accordance to data dependent superimposed training scheme (DDST). Fig. 1 displays the regard block diagram of transceiver. An assumption is made in this work that the channel is quasi-stationary, i.e. the channel remains static over an entire block. The discrete-time channel impulse response is given by

$$\mathbf{h} = [h(0), h(1), ..., h(L-1)]^T, \tag{1}$$

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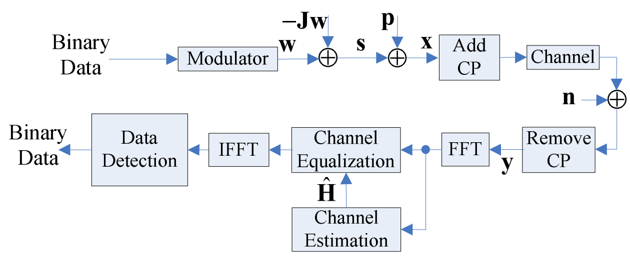

Fig. 1. Architecture of CP-SC system using DDST

where L is the total number of resolvable paths. Each channel tap $h(l)$, $0 \le l \le L-1$, is assumed to be an independent complex Gaussian distributed random variable with zero mean and a variance $\sigma^2_{h(l)}$. And moreover, we define $\mathbf{H}$ as an $N \times N$ circulant matrix with the first column being $\begin{bmatrix} \mathbf{h}^T & \mathbf{0}^T_{(N-L)} \end{bmatrix}^T$, where $\mathbf{0}_{(N-L)}$ is a zero column vector of length $N-L$.

We assume in this study that the input data stream is mapped into a finite constellation $\mathbf{\Phi}$, e.g., M-PSK or M-QAM. Note that M is used to indicate the modulation order. Subsequently, a series of N modulated symbols are collected to form an SC block, i.e.,

$$\mathbf{w} = \left[w(0), w(1), ..., w(N-1) \right]^T \tag{2}$$

The modulated data symbols are assumed to be zero mean and mutually independent.

In accordance to [5], the transmitted block is given by

$$\mathbf{x} = (\mathbf{I}_N - \mathbf{J})\mathbf{w} + \mathbf{p} \tag{3}$$

where $\mathbf{I}_N$ is identity matrix of size N, and the $N \times N$ matrix $\mathbf{J}$ is defined as

$$\mathbf{J} \equiv \left(\frac{1}{Q} \cdot \mathbf{1}_{(Q\times Q)} \right) \otimes \mathbf{I}_K = \frac{1}{Q} \begin{bmatrix} \mathbf{I}_K & \cdots & \cdots & \mathbf{I}_K \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{I}_K & \cdots & \cdots & \mathbf{I}_K \end{bmatrix} \tag{4}$$

where $K \cdot Q = N$, $\mathbf{1}_{(Q\times Q)}$ is an $Q \times Q$ matrix with all elements 1, and $\otimes$ stands for Kronnecker product. In addition, $\mathbf{p}$ is a vector of $N \times 1$, which stands for the periodic pilot sequence with period K and its energy is evenly distributed only at the K equispaced frequency bins $n = tQ$ with $t = 0, 1, ..., K-1$. As a result, it is equivalent to insert K pilot symbols on frequency domain [8]. For performing maximum likelihood channel estimation, we note that K must be larger than the channel order L. The superimposed training technique given in (3) is known as the data-dependent superimposed training (DDST) scheme where the training sequence is the sum of a known pilot sequence $\mathbf{c}$ and an unknown data dependent sequence $\mathbf{Jw}$.

At the receiver end, it is supposed that perfect synchronization is achieved. After removing CP, the received signal block can be expressed as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{n} \\ &= \mathbf{H}\left[(\mathbf{I}_N - \mathbf{J})\mathbf{w} + \mathbf{p} \right] + \mathbf{n} \\ &= \mathbf{H}(\mathbf{I}_N - \mathbf{J})\mathbf{w} + \mathbf{H}\mathbf{p} + \mathbf{n} \end{aligned} \tag{5}$$

where $\mathbf{n}$ is an $N \times 1$ vector denoting additive white Gaussian noise (AWGN) with zero mean and a variance σ^2_n. It is important to note that the DFT of $(\mathbf{I}_N - \mathbf{J})\mathbf{w}$ is zero at the frequency bins $n = tQ$ with $t = 0, 1, ..., K-1$. Therefore, the unknown data vector $\mathbf{s}$ has no impact on channel estimation. In other words, the channel estimation can be accomplished in the absence of data-induced interference.

Unfortunately, since the matrix $(\mathbf{I}_N - \mathbf{J})$ is singular, it is essentially a transformation that cannot be recovered linearly [5]. And moreover, the matrix $(\mathbf{I}_N - \mathbf{J})$ also induces the data identification problem (DIP), which results in that the data cannot be detected correctly even if the noise is absence. DIP occurs when various modulated data sequences are mapped into the same transmitted block, i.e., $(\mathbf{I}_N - \mathbf{J})\mathbf{w} = (\mathbf{I}_N - \mathbf{J})\mathbf{w}'$, where $\mathbf{w} \ne \mathbf{w}'$. Under such a scenario, the data sequence cannot be uniquely identified at the receiver. As a result, in DDST scheme, DIP induces the error floor in bit error rate (BER) performance. In [6-7], the DIP is analyzed and the lower bound on BER is provided by conducting simulation results. To our best knowledge, however, the technique to solve DIP has not been investigated. We will therefore revisit in this work the DIP of DDST scheme.

III. Analysis on Data Identification Problem in DDST

It is firstly noted that the matrix $(\mathbf{I}_N - \mathbf{J})$ can be expressed as $\mathbf{\Theta} \otimes \mathbf{I}_K$, i.e., $\mathbf{I}_N - \mathbf{J} = \mathbf{\Theta} \otimes \mathbf{I}_K$. Therefore, the transmitted block $\mathbf{x}$ given in (3) can be rewritten as

$$\mathbf{x} = (\mathbf{I}_N - \mathbf{J})\mathbf{w} + \mathbf{p} = (\mathbf{\Theta} \otimes \mathbf{I}_K) \cdot \mathbf{w} + \mathbf{p} = \mathbf{s} + \mathbf{p}, \tag{6}$$

where

$$\mathbf{s} \equiv (\mathbf{\Theta} \otimes \mathbf{I}_K) \cdot \mathbf{w} \tag{7}$$

and

$$\mathbf{\Theta} = \frac{1}{Q} \begin{bmatrix} Q-1 & -1 & \cdots & -1 \\ -1 & Q-1 & & \vdots \\ \vdots & & \ddots & -1 \\ -1 & \cdots & -1 & Q-1 \end{bmatrix} \tag{8}$$

is a principle submatrix of the matrix $(\mathbf{I}_N - \mathbf{J})$ [9]. And moreover, we can obtain from (7) that

$$\tilde{\mathbf{s}}_k = \mathbf{\Theta} \tilde{\mathbf{w}}_k \text{ for } k = 0, 1, ..., K-1, \tag{9}$$

where $\tilde{\mathbf{w}}_k$ and $\tilde{\mathbf{s}}_k$ are sub-vectors of $\mathbf{w}$ and $\mathbf{s}$, which are defined as follows,

$$\tilde{\mathbf{w}}_k \equiv \left[w(k),\ w(k+K), \cdots,\ w(k+(Q-1)K) \right]^T \tag{10}$$

and

$$\tilde{\mathbf{s}}_k \equiv \left[s(k),\ s(k+K), \cdots,\ s(k+(Q-1)K) \right]^T. \tag{11}$$

Therefore, we note that data identification problem occurs if two arbitrary possible vectors $\tilde{\mathbf{w}}_k$ and $\tilde{\mathbf{w}}'_k$ of the kth subgroup have the same output as

$$\mathbf{\Theta}\tilde{\mathbf{w}}_k = \mathbf{\Theta}\tilde{\mathbf{w}}'_k, \tag{12}$$

or equivalently,

$$\mathbf{\Theta d} \equiv \mathbf{\Theta}\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) = \mathbf{0}_{(Q)}, \tag{13}$$

where $\mathbf{d}$ is defined as $\mathbf{d} \equiv \left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right)$ and $\mathbf{0}_{(Q)}$ is a $Q\times 1$ vector with all elements 0. Equation (13) indicates that the DIP occurs when the vector $\mathbf{d}$ belongs to the null space of $\mathbf{\Theta}$, i.e.,

$$\mathbf{d} \in \mathrm{N}(\mathbf{\Theta}), \tag{14}$$

where $\mathrm{N}(\mathbf{\Theta})$ is the null space of the matrix $\mathbf{\Theta}$.

It is easily shown that the rank of the matrix $\mathbf{\Theta}$ is $Q-1$. By observing the structure of $\mathbf{\Theta}$, we further point it out that

$$\mathbf{\Theta}\mathbf{1}_{(Q)} = \mathbf{0}_{(Q)}, \tag{15}$$

which indicates that $\mathbf{1}_{(Q)}$ is the null vector of $\mathbf{\Theta}$, i.e.,

$$\mathrm{N}(\mathbf{\Theta}) = \mathrm{Span}\left\{\mathbf{1}_{(Q)}\right\} = \alpha \cdot \mathbf{1}_{(Q)}. \tag{16}$$

where α is an arbitrary complex value and and $\mathbf{1}_{(Q)}$ is a $Q\times 1$ vector with all elements 1. Consequently, the DIP exists if

$$\mathbf{d} \in \mathrm{Span}\left\{\mathbf{1}_{(Q)}\right\}. \tag{17}$$

or equivalently,

$$\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) = \alpha \cdot \mathbf{1}_{(Q)} \tag{18}$$

in DDST scheme. This observation inspires us to introduce a precoding matrix $\mathbf{E}$ of $Q\times Q$, which guarantees $\left(\mathbf{E}\tilde{\mathbf{w}}_k - \mathbf{E}\tilde{\mathbf{w}}'_k\right) \neq \alpha \cdot \mathbf{1}_{(Q)}$ for any possible $\tilde{\mathbf{w}}_k$ and $\tilde{\mathbf{w}}'_k$. Thus, the transmitted block $\mathbf{z}$ is given by

$$\mathbf{z} = \left(\mathbf{\Theta E} \otimes \mathbf{I}_K\right)\cdot \mathbf{w} + \mathbf{p}. \tag{19}$$

Equation (19) can be rewritten as

$$\begin{aligned} \mathbf{z} &= \left(\mathbf{\Theta E} \otimes \mathbf{I}_K\right)\cdot \mathbf{w} + \mathbf{p} \\ &= \left(\mathbf{\Theta} \otimes \mathbf{I}_K\right)\cdot\left(\mathbf{E} \otimes \mathbf{I}_K\right)\mathbf{w} + \mathbf{p} \\ &= \left(\mathbf{I}_N - \mathbf{J}\right)\cdot \mathbf{G}\cdot \mathbf{w} + \mathbf{p} \end{aligned} \tag{20}$$

where the precoding matrix $\mathbf{G}$ is defined as $\mathbf{G} \equiv \mathbf{E} \otimes \mathbf{I}_K$. The developed system architecture is shown in Fig. 2. The construction of the precoding matrix $\mathbf{G} \equiv \mathbf{E} \otimes \mathbf{I}_K$ will be further investigated in the next section.

IV. Precoding Matrix for Resolving DIP in DDST

This section commences by providing some basic conditions for the procoding matrix in DDST scheme. These basic conditions are beneficial to hold the energy of every data symbol and have frequency diversity. Subsequently, we derive the sufficient conditions of the precoding matrix to prevent the DIP. Finally, we develop a novel precoding matrix and show that the proposed precoding matrix satisfies all conditions derived in this section.

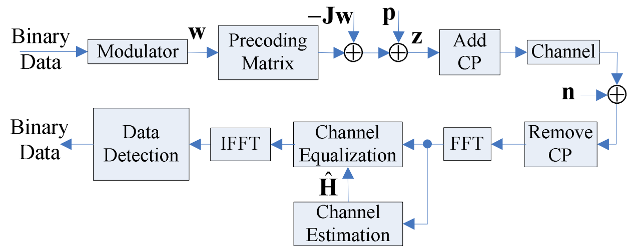

Fig. 2. Architecture of precoded CP-SC system using DDST

A. Basic Condition of Precoding Matrix

Firstly, the precoding matrix $\mathbf{G}$ should be a unitary matrix such that the average energy of every data symbol is equivalent after precoding. Therefore, the precoding matrix $\mathbf{G}$ is suggested to be a unitary matrix and has the following property,

$$\mathbf{G}^H\mathbf{G} = \mathbf{G}\mathbf{G}^H = \mathbf{I},\ \mathbf{G}^{-1} = \mathbf{G}^H, \tag{21}$$

which is equivalent to

$$\textbf{(C1)} \qquad \mathbf{E}^H\mathbf{E} = \mathbf{E}\mathbf{E}^H = \mathbf{I},\ \mathbf{E}^{-1} = \mathbf{E}^H. \tag{22}$$

Secondly, we remind that single carrier transmission has the advantage of frequency diversity [10], i.e., the energy of every data symbol is equally distributed in frequency domain. In DDST scheme, data component is null on pilot subcarriers and has equal amplitude on the other subcarriers. It is said that DDST scheme almost achieves the same frequency diversity as single carrier systems. Therefore, the precoding matrix $\mathbf{G}$ will be designed to preserve this property in DDST scheme.

To facilitate the analysis, we define the discrete Fourier transformation (DFT) of $\mathbf{G}$ as $\mathbf{\Psi}$, i.e., $\mathbf{\Psi} \equiv \mathbf{FG}$, where $\mathbf{F}$ is the normalized DFT matrix of $N\times N$. The frequency diversity is achieved when each column of $\mathbf{\Psi}$ has constant amplitude, i.e., $\left|\Psi_m(n)\right| = 1/\sqrt{N}$ for $n = 0,1,\ldots,N\text{-}1$ and $m = 0,1,\ldots,N\text{-}1$, where $\Psi_m(n)$ denotes the nth element of the mth column of $\mathbf{\Psi}$. It is equivalent that each column of $\mathbf{G}$ has the autocorrelation function of impulse. Therefore, the precoding matrix $\mathbf{G}$ is necessary to satisfy

$$\textbf{(C2)} \qquad R_m(\tau) = \sum_{n=0}^{N-1}[\mathbf{G}]_{m,n}[\mathbf{G}]^*_{m,(n+\tau)_N} = \begin{cases} 1, & \tau = 0 \\ 0, & \tau \neq 0 \end{cases} \tag{23}$$

for $m = 0,1,..N-1$, where $[\bullet]_{m,\,\mathrm{n}}$ means the (m,n) th element of the matrix and $(\bullet)_N$ denotes the operation of modulo N.

B. Condition of Precoding Matrix for Avoiding DIP

We will derive in this sub-section the requirement of precoding matrix for resolving the DIP in DDST scheme. To facilitate analysis, we firstly show that the matrix $\mathbf{\Theta}$ has the form of singular value decomposition (SVD) as follows,

$$\mathbf{\Theta} = \mathbf{U\Sigma V}^H, \tag{24}$$

where

$$\mathbf{U} = \left[\mathbf{u}_0,\ \cdots,\ \mathbf{u}_{Q-1}\right],\quad \mathbf{V} = \left[\mathbf{v}_0,\ \cdots,\ \mathbf{v}_{Q-1}\right] \tag{25}$$

and

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_0 & 0 & \cdots & 0 \\ 0 & \sigma_1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{Q-1} \end{bmatrix}. \quad (26)$$

Since $\mathbf{\Theta}$ is a symmetric matrix, we have $\mathbf{U} = \mathbf{V}$. In addition, remind that $\mathbf{\Theta}$ has rank of $Q-1$ and the associating null vector is $\mathbf{1}_{(Q)}$, i.e.,

$$\mathbf{v}_{Q-1} = \mathbf{1}_{(Q)}. \quad (27)$$

In applying the precoding matrix, the transmitted block given in (19) can be rewritten as

$$\mathbf{z} = \left(\mathbf{\Theta E} \otimes \mathbf{I}_K\right) \cdot \mathbf{w} + \mathbf{p} = \left(\tilde{\mathbf{\Theta}} \otimes \mathbf{I}_K\right) \cdot \mathbf{w} + \mathbf{p}, \quad (28)$$

where $\tilde{\mathbf{\Theta}}$ is defined as $\tilde{\mathbf{\Theta}} \equiv \mathbf{\Theta E}$. In order to solve the DIP in DDST scheme, the precoding matrix $\mathbf{E}$ must be designed to satisfy $\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) \notin \mathrm{N}\left(\mathbf{\Theta E}\right) = \mathrm{N}\left(\tilde{\mathbf{\Theta}}\right)$ for any possible $\tilde{\mathbf{w}}_k$ and $\tilde{\mathbf{w}}'_k$. Performing SVD on $\mathbf{\Theta}$, we obtain that

$$\tilde{\mathbf{\Theta}} = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H, \quad (29)$$

where $\tilde{\mathbf{U}}$, $\tilde{\mathbf{\Sigma}}$, and $\tilde{\mathbf{V}}$ have similar definition as $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}$ respectively. Subsequently, it is noted that $\tilde{\mathbf{\Theta}}^H\tilde{\mathbf{\Theta}}$ has the following form,

$$\tilde{\mathbf{\Theta}}^H\tilde{\mathbf{\Theta}} = \tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^H\tilde{\mathbf{U}}^H\tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H = \tilde{\mathbf{V}}\tilde{\mathbf{D}}\tilde{\mathbf{V}}^H, \quad (30)$$

where $\tilde{\mathbf{D}} \equiv \tilde{\mathbf{\Sigma}}^H\tilde{\mathbf{\Sigma}}$ is a diagonal matrix. On the other hand, since $\tilde{\mathbf{\Theta}} \equiv \mathbf{\Theta E}$, it is important to note that $\tilde{\mathbf{\Theta}}^H\tilde{\mathbf{\Theta}}$ can also be written as

$$\begin{aligned} \tilde{\mathbf{\Theta}}^H\tilde{\mathbf{\Theta}} &= \mathbf{E}^H\mathbf{\Theta}^H\mathbf{\Theta E} \\ &= \mathbf{E}^H\mathbf{\Theta E} \\ &= \mathbf{E}^H\mathbf{U\Sigma V}^H\mathbf{E} \\ &= \left(\mathbf{E}^H\mathbf{V}\right)\mathbf{\Sigma}\left(\mathbf{E}^H\mathbf{V}\right)^H, \end{aligned} \quad (31)$$

where the second equality holds since $\mathbf{\Theta\Theta} = \mathbf{\Theta}$ and $\mathbf{\Theta}^H = \mathbf{\Theta}$ such that $\mathbf{\Theta}^H\mathbf{\Theta} = \mathbf{\Theta}$ [5]. Inspecting (30) and (31), we obtain the following result,

$$\tilde{\mathbf{D}} = \mathbf{\Sigma},\ \tilde{\mathbf{V}} = \mathbf{E}^H\mathbf{V}. \quad (32)$$

We therefore obtain that $\tilde{D}\left(Q-1,Q-1\right) = \sigma_{Q-1} = 0$ and the basis of $\mathrm{N}\left(\tilde{\mathbf{\Theta}}\right)$ is $\tilde{\mathbf{v}}_{Q-1} = \mathbf{e}^H_{Q-1}\mathbf{v}_{Q-1}$. Hence, the pth element of $\tilde{\mathbf{v}}_{Q-1}$ is given by

$$\tilde{v}_{Q-1}\left(p\right) = \sum_{q=0}^{Q-1}\left[\mathbf{E}^H\right]_{p,q},\ \text{for}\ p = 0,\ 1,\ \cdots,\ Q-1. \quad (33)$$

As a result, to design the precoding matrix that satisfies $\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) \notin \mathrm{N}\left(\mathbf{\Theta E}\right) = \mathrm{N}\left(\tilde{\mathbf{\Theta}}\right)$ is equivalent to design the matrix such that

(C3) $$\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) \notin span\left(\sum_{q=0}^{Q-1}\left[\mathbf{E}^H\right]_{p,q}\right). \quad (34)$$

Satisfying (34) guarantees that the precoded data sequences being unique, which prevents the DIP in DDST scheme.

C. The Proposed Scheme

In this investigation, we propose the precoding matrix,

$$\mathbf{G} = \mathbf{E}_{ZC} \otimes \mathbf{I}_K, \quad (35)$$

and

$$\mathbf{E}_{ZC} = \frac{1}{\sqrt{Q}}\mathbf{D}_{ZC}\mathbf{F}^H, \quad (36)$$

where $\mathbf{D_{ZC}}$ is a diagonal matrix whose diagonal elements consist of the Zadoff-Chu sequence [11], i.e.,

$$\gamma_M\left[n\right] = \exp\left(\frac{-j\pi Mn\left(n+2g\right)}{Q}\right) \quad (37)$$

with $g = 0$, $M = 1$ and $n = 0,\ 1,\ \cdots,\ Q-1$. In present investigation, we suppose Q as an even integer. However, the results can be easily extended to the case of odd Q.

It is easily confirmed that $\mathbf{E}_{\mathrm{ZC}}$ is a unitary matrix, i.e., $\mathbf{E}^H_{\mathrm{ZC}}\mathbf{E}_{\mathrm{ZC}} = \mathbf{I}$, which fulfills the condition given in (22). And moreover, according to the result shown in [11], we have the columns of $\mathbf{E}_{\mathrm{ZC}}$ are perfect sequences, whose autocorrelation is an impulse function. As a result, the proposed matrix also satisfies the condition given in (23).

In order to verify that the proposed matrix fulfills the condition shown in (34) for any possible data sequences, we derive the expression of $\sum_{q=0}^{Q-1}\left[\mathbf{E}^H_{\mathrm{ZC}}\right]_{p,q}$, i.e.,

$$\sum_{q=0}^{Q-1}\left[\mathbf{E}^H_{\mathrm{ZC}}\right]_{p,q} = \sum_{q=0}^{Q-1}\left[\mathbf{FD}^H\right]_{p,q} = \sum_{q=0}^{Q-1}\exp\left(\frac{-j2\pi pq}{Q}\right)\cdot\exp\left(\frac{j\pi q^2}{Q}\right) \quad (38)$$

Thus, for a given Q, the value of $\sum_{q=0}^{Q-1}\left[\mathbf{E}^H_{\mathrm{ZC}}\right]_{p,q}$ can be calculated from (38). Also, the possible value of $\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k$ can be obtained for a given constellation. Taking the BPSK constellation as an example, each element of $\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right)$ has possible values of $\left\{0,2,-2\right\}$, i.e., $\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) \in \left\{0,2,-2\right\}$.Therefore, it is easy to verify that the proposed precoding matrix $\mathbf{E}_{\mathrm{ZC}}$ fulfills (34) when $Q \geq 2$ for the BPSK constellation. When the 4-QAM constellation is regarded, the possible values of $\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right)$ are given by

$$\sqrt{2}\left(\tilde{\mathbf{w}}_k - \tilde{\mathbf{w}}'_k\right) \in \left\{0,2,-2,2j,-2j,2+2j,2-2j,-2+2j,-2-2j\right\} \quad (39)$$

Subsequently, it can be verified that the proposed precoding matrix $\mathbf{E}_{ZC}$ fulfills (34) when $Q \geq 4$ for the 4-QAM constellation. By using a similar procedure above, we can show that the proposed precoding matrix satisfies (34) for other finite constellations with appropriate value of Q.

V. Simulation Results

In this section, the performance of the proposed scheme is

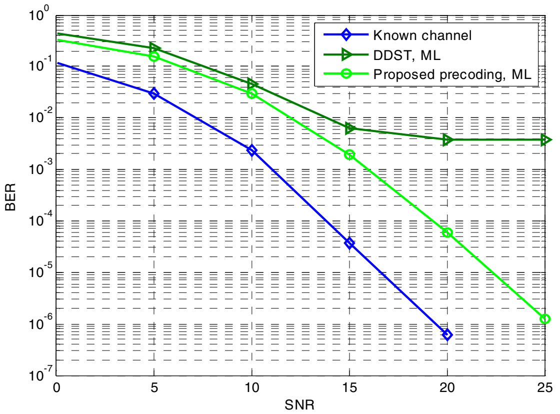

Fig. 3. Performance comparison in BER for BPSK modulation

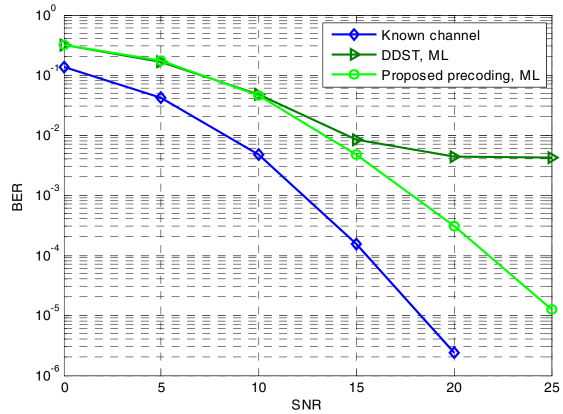

Fig. 4. Performance comparison in BER for 4-QAM modulation

evaluated by conducting simulation results. We consider a CP-SC system, where the block length and CP length are $N = 64$ and 8. The periodic pilot sequence $\mathbf{p}$ has the period $K = 8$ and the precoding matrix is obtained according to (35) and (36) with $Q = 8$. The simulations are evaluated in multipath Rayleigh fading channel with an exponential power decay profile and decay coefficient $\mu = -0.2$. In addition, the channel length L is assumed to be 8 and the MMSE channel equalizer is utilized at the receiver. Since this paper focuses on the data identification problem, it is supposed that the receiver achieves perfect synchronization.

Fig. 3 compares the BER performance of the proposed method and conventional DDST method for BPSK modulation. In both schemes, the data detection is performed in accordance to maximum likelihood (ML) principle, i.e.,

$$\hat{\tilde{\mathbf{w}}}_k = \arg\min_{\tilde{\mathbf{w}}_k \in \Omega} \left\| \tilde{\mathbf{r}}_k - \mathbf{\Theta} \mathbf{E}_{ZC} \tilde{\mathbf{w}}_k \right\|_2 , \qquad (40)$$

where $\tilde{\mathbf{r}}_k$ is the kth subgroup of received signal after MMSE channel equalization. Simulation results presented in Fig. 3 demonstrate that the proposed method outperforms the conventional DDST method in BER; particularly at higher values of the SNR. In specific, the conventional DDST method has an error floor in BER at 2^{-Q} due to the DIP in case of BPSK modulation [6]. And moreover, it is important to remark that the proposed precoding scheme eliminates the error floor in BER. Finally, the case of known channel is provided as a benchmark. In Fig. 4, the BER performance for 4-QAM modulation is evaluated, where we can obtain similar observation as from Fig. 3.

VI. CONCLUSION

In this paper, we have investigated the DIP for CP-SC system using DDST. The DIP is analyzed from the view of sub-space. It is shown that the DDST scheme induces a null sub-space in the transmitted signal block, resulting in the DIP. The analysis results demonstrate that the DIP can be solved by utilizing the precoding technique. Subsequently, we derived two conditions of the precoding matrix, which guarantees a) the average energy of every symbol is equivalent after precoding and b) the precoded signal block achieves the frequency diversity as well as DDST schemes. And moreover, the requirement of the precoding matrix for resolving DIP is obtained by using the SVD method. Finally, we developed the precoding matrix which satisfies the three conditions derived in this work. Simulation results show that the proposed precoding scheme solves the DIP, and eliminates the error floor phenomenon in BER for CP-SC system using DDST scheme.

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