Dynamic Spectrum Auction with Time Optimization in Cognitive Radio Networks

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***Abstract*—Dynamic spectrum auction has been considered as one of potential approaches on spectrum allocation in cognitive radio networks. As an modified version of traditional static or quasi-static spectrum auction, dynamic spectrum auction should not only increase the auction revenue of the owner of spectrum, but also improve spectrum utilization on fine time granularity. We propose a dynamic spectrum auction algorithm with time optimization (DSA-TO) in an 802.22 network. First, the effects of auction period on auction revenue and spectrum utilization are discussed and optimized. Then we give a complete spectrum auction algorithm where an adaptive reserve price is used to balance the revenue and utilization of spectrum auction. Performance analyses show that the DSA-TO algorithm can resist collusion effectively and has low complexity as well as good revenue and utilization. Simulation results show that the DSA-TO algorithm is reasonable in the optimization of auction period and can keep a fine spectrum utilization and bring more revenue for both single unit spectrum auction and multi-unit spectrum auction.**

I. Introduction

With the continuing increase of wireless communication, the spectrum which are applicable to wireless services have become crowded. It is well accepted that traditional command and control spectrum management should be complemented by dynamic spectrum sharing (DSS) based on cognitive radios. The secondary users (SUs) in DSS networks have no spectrum licenses and can access the unused band of the primary users (PUs) without bringing intolerable interference, which will improve spectrum utilization. Therefore, a great deal of substantial efforts on DSS have been taken in recent years [1].

There are two ways to share the PUs'spectrum for the SUs, paid-sharing and free sharing. In the second way, the SUs can use the PUs' idle spectrum bands opportunistically or occupy the PUs' bands with very low power spectrum density without payments although the PUs have licensees cost. On the other hand, the paid-sharing ways such as pricing and auction, have drawn more and more attention because they can provide the PUs with economic incentive. As an effective price discovery mechanism, auction has been used in spectrum allocation for a long time [2]. In conventional spectrum auctions the bidders who are telecommunication corporations compete for spectrum licenses in years, so the auctions are static. However, a well designed dynamic spectrum auction may be carried out frequently (such as in minutes) and a finer spectrum auction granularity such as KHz will attract many individual users. As in standard auctions, dynamic spectrum auction can be one side auction or a double auction, and can be carried out sequentially or concurrently. Although it is natural that many SUs compete for the bands from one PU in a one side auction [3], [4], many PUs may compete the spectrum purchase of one SU [5]. In a double auction, the auctioneer must deal with the asking price from the PUs and the bidding price from the SUs, and seek a clearing price to balance the demand and supply [6], [7]. Sengupta et al. show that if the SUs are constrained to at most single unit allocation, sequential auction in which bands are auctioned one by another brings the auctioneer more revenue than concurrent auctions where all the bids for all the bands are submitted simultaneously [8], [9]. Moreover, a truthful auction will give the PUs more incentive to share or lease their spectrum because the SUs will be enforced to bid according to their real value estimation of spectrum [6], [10], [11], [12]. On the other side, dynamic spectrum auction should not interfere with the PUs [14], or degrade the QoS performance of the PUs [15]. The interference conflicts among SUs [16], [17] should also be avoided when a single band is awarded to multi-winners (for frequency reuse).

However, the existing efforts on dynamic spectrum auction mainly maximize the auction revenue like conventional auctions, which is not always consistent with the original intention of dynamic spectrum sharing. In fact, both revenue and spectrum utilization should be improved in DSS networks. At the same time, the auction period has important influence on revenue and spectrum utilization. For example, a longer period not only means that more users will arrive and join auction so that severer competition results in more revenue, but also means more spectrum will remain unused for short-term wireless services. We construct a dynamic spectrum auction model in an 802.22 network where the base station allocates its spectrum opportunities to some customer-premises equipments (CPEs). First, the auction period is optimized

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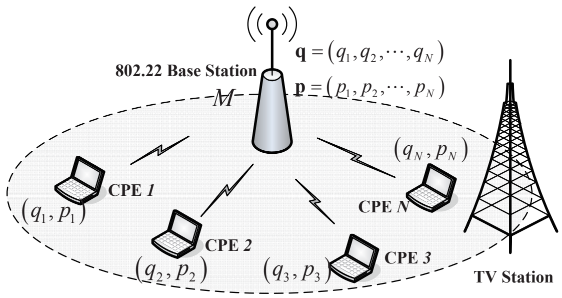

Fig. 1. Dynamic spectrum auction model in 802.22 networks.

based on the arrival distribution and services time type of the CPEs. Furthermore, an adaptive reserve price is introduced into the spectrum auction procedure to balance the spectrum utilization and revenue of the base station in next auction round. Performance analyses and simulation results show that the DSA-TO algorithm can be implemented with low complexity and has a wonderful performance.

The rest of the paper is organized as follows. In Section II, we describe the system model. The optimization of auction period is presented in Section III and the adaptive reserve price is introduced in Section IV. Theoretical analyses and simulation results are presented in Section V and Section VI, respectively. The paper concludes with Section VII.

II. System Model

Consider an 802.22 network where some CPEs access to a base station (BS) via wireless links [18], which is shown in Fig.1. With cooperative spectrum sensing [19] or spectrum leasing [20], the BS has M units non-overlapped, indivisible and homogeneous spectrum opportunities from TV station to be auctioned among the CPEs [21]. As the head of cognitive access networks, the BS wants not only to improve its auction revenue, but also to increase spectrum utilization which is disregarded by the traditional services providers such as TV broadcast stations.

Assume auction period is T. During the period of T, there are N CPEs arriving and joining the auction. Obviously N is a monotonically increasing function of T, denoted $N(T)$. Then the average arrival rate of bidders is $\lambda = N(T)/T$. CPE i, $(i = 1, 2, \cdots, N)$, submits its sealed bid (q_i, p_i) to the BS through a dedicated control channel. A sophisticated control channel design can be found in [21]. $q_i \in N^+$ denotes the spectrum demand quantity and p_i is the bidding price per unit spectrum. According to the law of diminishing marginal utility [23], p_i is a monotonically decreasing function of q_i. Without loss of generality, a brief price-demand function $p_i = -k_i q_i + b_i\ , k_i \geq 0, b_i > 0$ is used here, in which k_i denotes the price-demand sensitivity of user i and b_i is the maximum unit price of user i. If CPE i wins, it needs to pay the BS $c_i = p_i q_i$ and uses q_i units spectrum in T. The auction revenue R_r is formulated as follows.

$$\begin{array}{ll} R_r = \max\limits_{\mathbf{x}} \sum\limits_{i=1}^{N} p_i q_i x_i & \\ s.t. \quad x_i \in \{0,1\} & \\ \qquad\ \ p_i \geq p_r & , \\ \qquad\ \ \sum\limits_{i=1}^{N} q_i x_i \leq M & \end{array} \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_N), x_i \in \{0,1\}$ is the clearing vector. $x_i = 1$ means CPE i wins and $x_i = 0$ means it's a loser. p_r is the reserve price.

At the same time, CPE i has services duration t_i. Because the spectrum prizes will be withdrawn after T, the real spectrum utilization time of user i is,

$$t^* = \begin{cases} t_i & t_i \leq T \\ T & t_i > T \end{cases}. \tag{2}$$

If there are W winners in the auction, the average spectrum utilization ratio is,

$$R_u(T) = \frac{\sum\limits_{i=1}^{W} t_i^*}{WT}. \tag{3}$$

Denote the auction revenue with $N(T)$ CPEs as $R_u(T)$, the BS selects a proper T to achieve the following double objectives,

$$\max_T R_u(T) R_r(T). \tag{4}$$

The problem in (1) is a NP problem, not to mention (4). So we first use a heuristic approach to study the effects of auction period on spectrum utilization and average auction revenue in Section III. Then the total DSA-TO algorithm is given in Section IV.

III. Optimization of Auction Period

A. Effects on Spectrum Utilization Ratio

Assume the communication duration t of every CPE depends on negative exponent distribution with μ. Given auction period T, the probability that the CPEs finish their services before T is $1 - e^{-\mu T}$ and the expectation of idle time is,

$$E\{T-t\} = \int_0^T (T-t)\,\mu e^{-\mu t} dt = T + \frac{1}{\mu} e^{-\mu T} - \frac{1}{\mu}. \tag{5}$$

We can get the utilization time expectation T_u,

$$T_u = T - E\{T-t\} = \frac{1}{\mu} - \frac{1}{\mu} e^{-\mu T}. \tag{6}$$

The above computation avoids to deal with the piecewise function in (2). According to the definition in (3), the expectation of the average spectrum utilization ratio is,

$$E[R_u(T)] = E\left\{ \frac{\sum\limits_{i=1}^{W} t_i^*}{WT} \right\} = \frac{T_u}{T} = \frac{1}{\mu T}\left(1 - e^{-\mu T}\right). \tag{7}$$

Because $dE[R_u(T)]/dT < 0$, then $R_u(T)$ is the monotonically decreasing function of T and the BS will select a small T in order to keep a high expectation of the average spectrum utilization ratio.

B. Effects on Auction Revenue

If each winner can only get one unit spectrum, the winners can be determined according to the bidding price from high to low, which is called the *high price rule*. Assume p_i is uniformly distributed in $[0, V_{\max}]$, in which $V_{\max}$ is the maximum that CPE i values one unit spectrum. Because spectrum is a kind of common commodity, the maximum value should be the same for all CPEs. On the other side, if one does not need spectrum, its minimum value is zero. CPE i wins if and only if p_i is higher than the price p_j of other $(N - M)$ users. The probability that any price $p_j < p_i$ is $p_i/V_{\max}$.

For the purpose of simplification, we analyze the champion's cost. The probability of being champion of CPE i, $P_{i\ No.1}$ is,

$$P_{i\ No.1} = \left(\frac{p_i}{V_{\max}}\right)^{N-1}. \tag{8}$$

It is obvious that $P_{i\ No.1}$ decreases with the increase of N if CPE i keeps its bidding price. The expectation of the champion's cost is,

$$Cost_{i,\ No.1} = V_{\max}\left(1 - \frac{1}{N}\right). \tag{9}$$

Given the CPEs' arrival rate λ, a larger auction period T means a bigger N. It is clear that given a bigger N, the champion's cost is higher, which brings more revenue for the BS.

When each winner can get multiple units spectrum, the auction is a multi-unit award auction and a heuristic conclusion can be drawn. That is to say, if the competition is severe ($N(T)$ is big) and $\max q_i/M$ is small enough, the strategy that selecting winners with *high price rule* will be close to the optimal strategy to maximize the auction revenue. So a big T also causes the increase of the auction revenue in multi-unit award auction.

C. Optimization of Auction Period toward Application

By far we have know the effects of the auction period T exactly. However, we find it is difficult in solving the problem (4) because the function $R_r(T)$ is not clear in real multi-unit award auction. Even in single unit award auction, it is hard to get a brief formula of the expected revenue for big N and M [21].

On the other hand, it is not reasonable to maximize the $R_u(T)$ blindly. A bigger $R_u(T)$ implies that more CPEs will be interrupted before their services are finished. In fact, a more realistic way is to transform the optimization of spectrum utilization rate into a threshold constraint,

$$\max_T R_r(T) \quad s.t.\ \ R_u(T) > \eta, \tag{10}$$

where η is the lower threshold of spectrum utilization rate.

IV. Adaptive reserve Price

Given the optimal auction period T, the *high price rule* is the core of clearing algorithm if each winner can get only one unit spectrum. For multi-unit award auction, the optimal and exact solution can be gotten in acceptable time with dynamic programming [24]. Furthermore, the BS can improve the spectrum utilization and its revenue with an adaptive reserve price.

In fact, the spectrum demand will change with time. When the demand exceeds the supply, the BS will naturally raise the price and can get more revenue. While the supply exceeds the demand, it has to reduce the price to encourage CPEs to bid spectrum. A corresponding updated procedure for reserve price is given.

$$\begin{array}{l} if \quad \sum\limits_{i:\ p_i>0} q_i \geq M(1+\beta_1) \\ \quad p_r = p_r + \Delta p \ \ till\ p_r = b \\ else\, if \quad \sum\limits_{i:\ p_i>0} q_i < M(1+\beta_2) \\ \quad p_r = p_r - \Delta p \ \ till\ p_r = 0 \\ else \\ \quad p_r = p_r \end{array}, \tag{11}$$

where Δp is the step length of adjustment. β_1 and β_2 is the allocation redundancy index. For example, $\beta_2 = 0$ means that the BS will reduce the price when the total demand is less than M.

On the other hand, bidders' strategies change with time, too. A loser will reduce the price-demand sensitivity k_i to raise its price in the next auction, while a winner will raise k_i to reduce its price and winning cost in the next auction. The adjustment step length of k_i is Δk.

Now we can get the complete procedure of the DSA-TO algorithm.

Step 1. The BS broadcasts its spectrum retainment M and allocation redundancy index β_1 and β_2.

Step 2. N CPEs submit their sealed bid.

Step 3. The BS gets the allocation vector $\mathbf{x}$ with dynamic programming or *high price rule*.

Step 4. The BS adjusts its reserve price p_r. The CPEs adjust their k_i based on auction results.

After period T, a new auction starts with step 1.

V. Performance Analysis

A. Optimization of DSA-TO

The DSA-TO algorithm consists of two parts: the optimization of auction period and spectrum auction with the adjustment of reserve price. In Section III we have interpreted how to transform the optimization of spectrum utilization rate into a threshold constraint and the corresponding rationality. With the monotonic property, the maximum of $R_r(T)$ can be gotten just at the breakpoint of the constraint of utilization rate. Then the solution in the first stage is optimal.

At the second part, the IF-ELSE sentences in (11) show that the reserve price of spectrum will decrease if the total demand is less than a threshold. A lower p_r will make CPEs win auction more easily and CPEs will decrease their k_i. As a result, the auction revenue will decrease. At the same time, CPEs will be willing to use spectrum for lower price. Then the two parts of DSA-TO algorithm also fall into the BS' double objects.

B. Complexity of DSA-TO

The complexity of auction is the key factor which determines its application value in the frequent spectrum allocation facing terminal users. According to the procedure in Section IV, the complexity of the DSA-TO algorithm is made up of two parts. One part is the auction operation and the followed price adjustment. Another part is the optimization operation of auction period T. And the latter part is a regular and common operation. The update of λ needs a multiplication and one division, while the update of μ needs the detection results of M bands and compute the R_u. If the system is stationary, the update can be carried out once for several auction rounds in order to reduce the complexity.

For the single award auction, the winners is determined easily by the *high price rule*. Here we focus on the multi-award auction.

The total computation complexity of solving the clear vector in (1) with dynamic programming is,

$$\left\{ \begin{array}{ll} \times & N \\ + & 2MN + N \end{array} \right. . \tag{12}$$

If we use enumeration to look for the optimal vector, Then the total computation complexity for enumeration is,

$$\left\{ \begin{array}{ll} \times & N \\ + & 2^N N \end{array} \right. . \tag{13}$$

It is obvious that $2^N > 2M + 1$ holds in many cases, so the auction procedure in Section IV has a lower complexity compared with the NP situation in (4).

C. Resistance to Collusion

Firstly, the low price strategy of the collusion users is restrained. If collusion CPEs low bidding price, the BS will find that the number of potential winners whose bidding price is higher than p_r is small, while the total number of users is large. As a rational agent, the BS will know that a collusion has taken place and keep the reserve price, even increase it. Then the collusion CPEs will get nothing because the collusion price can not be less than the reserve price.

In order to avoid a high reserve price, the collusion CPEs may use circuity strategy. That is to say, the collusion CPEs do not join the current auction and make the demand decrease abruptly. As a response, the BS will has to reduce the conservation price p_r. Finally the collusion user will win the spectrum with a low price in new auction. In our scenario, the CPEs are assumed to be secondary users with data services and their maximum spectrum demand can be estimated. So a demand upper bound for every bidder can be settled and the exit threat of the collusion users is weaken besides a big delay.

VI. Simulation Results

In this section we conduct plenty of simulations to examine the performance of the DSA-TO algorithm. Assume that the BS has 20 units spectrum (or channels), i.e. $M = 20$. In fact, it is a bad situation of spectrum unused for a traditional wireless services provider such as a cell base station or TV station. A number of CPEs join the auction, and they are supposed to be active in auctions round by round so that we can trace the change of auction situation with time.

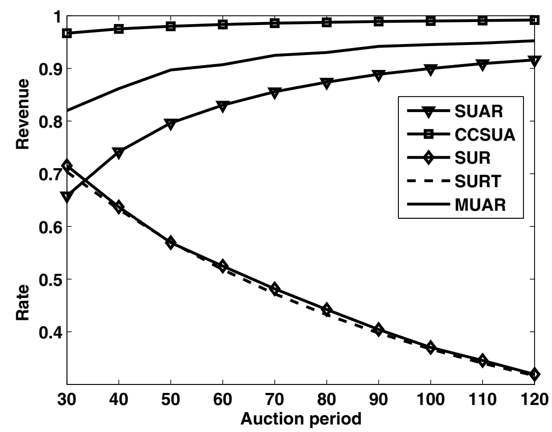

Fig. 2. the effects of auction period on spectrum utilization and auction revenue.

A. Effects of Auction Period

The system is stationary enough and the BS knows that $\lambda = 1\ user/s$ and $\mu = 40s$. For CPE i in multi-unit auction, its bidding spectrum quantity q_i is selected from $\{1, 2, 3\}$ with equal probability, and its bidding price p_i depends on uniform distribution in $(0, 1)$.

Fig.2 gives the effects of auction period on spectrum utilization and auction revenue. All the results are averaged with 100 rounds. In order to compare the primary auction situations, the reserve price adjustment is not used here. When the auction period T increases from $30s$ to $120s$, the spectrum utilization rate (SUR) gotten decreases and almost superposes the theoretic value (SURT). It is clear that a longer period will cause more idle spectrum and decrease the spectrum utilization rate. However, when N increases from 30 to 120, severer competition cause the increase of auction revenue. In Fig.2 we compare three kinds of revenue, the revenue per unit spectrum in single unit auction (SUAR), the revenue per unit spectrum in multi-unit auction (MUAR) and the champion's cost in single unit auction (CCSUA). It can be seen that they all increase. The revenue difference between multi-unit auction and single unit auction decreases with the increase of N, which stands for the heuristic conclusion in Section III.

B. Effects of Adaptive reserve Price

Consider three cases, that is to say, N, are 15, 25, and 40, respectively. $\beta_1 = 2$, $\beta_2 = 0.5$. The initial $k_i,\ i = 1, \cdots, N$ depends on uniform distribution in $(0, 1)$ and initial $p_r = 0.5$. The adjust step length of k_i and p_r are 0.1 and 0.05, respectively. The auction revenue with and without reserve price are compared in Fig.3 (The legend "Y" denotes the results with reservation price and "N" denotes the results without

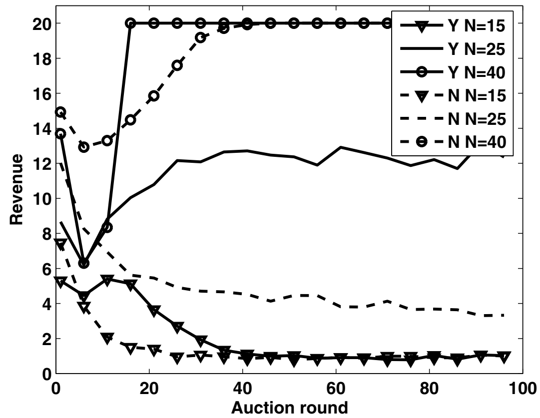

Fig. 3. the effects of reserve price on auction revenue.

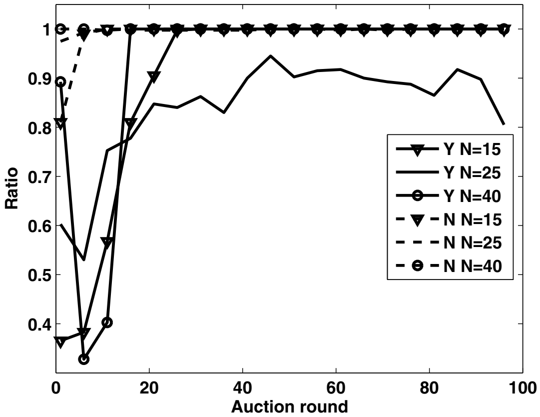

Fig. 4. the effects of reserve price on spectrum utilization rate.

reservation price), from which we can see that using adaptive reserve price improves the revenue with mediate N while it has no damage with big or small N. At the same time, from Fig.4 (The legend "Y" denotes the results with reservation price and "N" denotes the results without reservation price) we can see that the reserve price has no effect on the spectrum utilization ratio with weak or heavy demand. Of course, the spectrum utilization ratio with reserve price is a little lower than that without reserve price, which is the cost of improving revenue.

VII. Conclusion

In this paper, we present a DSA-TO algorithm for 802.22 networks. We first analyze the effects of auction period on the BS' double objects. Then we propose a complete auction procedure with adaptive reserve price which can balance revenue and spectrum utilization. Performance analysis and simulation results show that the DSA-TO algorithm has low complexity and has the ability of resisting auction collusion. On the other hand, it keeps a fine spectrum utilization and brings the BS more revenue.

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