BEP and Throughput Analysis of Incremental Selective Relaying in DS-CDMA Systems

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***Abstract*—In this paper, we derive exact form expressions for the Bit Error Probability (BEP) and throughput of cooperative Direct Sequence-Code Division Multiple Access (DS-CDMA) systems using incremental selective relaying which combines selective relaying with incremental relaying protocols in the presence of multipath propagation. The derived results are valid for any multipath intensity profile of the channel and any path delays. They also consider the correlation of the multipath gains. Simulation results along with analytical studies of BEP and throughput prove that the combination of incremental relaying with selective relaying in cooperative DS-CDMA systems improve significantly the throughput performance and can achieve the maximum possible spatial diversity when it is required by destination.**

I. INTRODUCTION

Cooperative Communication is an innovative technique to create spatial diversity without any reliance on multiple antennas. It consists in introducing relay node(s) to increase diversity order by relaying the transmitter signal to the receiver [1]–[3]. This paper focuses on digital relaying mode where the relay decodes the received message from the source and then forwards the decoded message to the destination. In incremental relaying (IR), the process of relay selection and cooperative transmission is done only if the signal-to-Noise Ratio (SNR) in source-destination link is below a predetermined threshold γ_t. Otherwise, no cooperative transmission will be performed and the source transmits the next data [7]. In case where the SNR in source-destination link is below γ_t, the system uses the selective relaying protocol (SR) which consists in building a set of "reliable" relays, defined as the relays having an SNR in source-relay link beyond γ_t. Only one "reliable" relay is selected to cooperate. We consider the selection of the relay having the largest SNR in the relay-destination link. The motivation behind the use of SR is to prevent relays which receive the source signal with an SNR below γ_t to participate in cooperation process since they are likely not able to correctly decode the source signal and thus they can not forward a correct information to destination. The selection of one relay among "reliable" relays avoid that the system squanders many time slots (a time slot for each "reliable" relay) to transmit one symbol. The combination of incremental relaying and several relaying schemes has been studied in earlier works and their performances have been analyzed over Rayleigh fading environments in the presence of a single path propagation [4]-[7]. Namely in [4], Chen *et. all* have derived closed-form expressions for error probability of incremental-selective digital relaying scheme which combines the incremental digital relaying and the selective digital relaying. [5] proposes a new incremental relaying transmission technique in conjunction with selective digital relaying, and provides its performance in terms of outage probability and bit error probability. In [6], the authors have derived a closed-form expression for the end-to-end (e2e) bit-error rate (BER) of incremental opportunistic relaying scheme which combines incremental relaying scheme and opportunistic relaying scheme. The performance of the incremental-relay-selection decode-and-forward technique over independent non-identical Rayleigh fading channels is derived in terms of average bit error probability (BEP), outage probability and average channel capacity in [7]. In [8], the author derived the BEP for selective digital relaying assuming multipath propagation. However, the combination with incremental relaying is not considered and throughput performance is not studied. Besides, the relay selection in [8] is different from the one considered in our work.

All previous works dealing with the combination of incremental relaying with other relaying schemes consider Rayleigh fading environments and single path channels. In this paper, we consider cooperative DS-CDMA systems and multipath propagation channels. We analyze e2e BEP and throughput performances of incremental selective relaying (ISR). The considered cooperative system uses incremental relaying protocol in conjunction with selective relaying. We assume that the destination knows the SNR of $R_\Theta - D$ links through training sequences, where Θ is the set of "reliable" relays and R_Θ denotes the relays in the set Θ. Based on the collected SNR information, the destination selects one relay among Θ. The derived results are valid for any multipath intensity profile of the channel, any path delays and take into account the correlation between path gains.

The remainder of this paper is organized as follows. In section II, we present the system model. In sections III and IV, we derive e2e BEP and throughput expressions of DS-CDMA cooperative system using ISR in the presence of multipath propagation. In section V, we present simulation and theoretical results. Section VI draws concluding remarks and a summary of our findings.

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II. System Model

A. Scenario of the combined digital relaying protocol

We consider a source *(S)*, a destination *(D)* and M potential relays R_i. We assume that the destination knows the SNR of $R_\Theta - D$ links, where Θ is the set of R_Θ, the "reliable" relays. Based on the collected SNR information, the destination selects one relay belonging to Θ. The derived results are valid for any multipath intensity profile of the channel, any path delays and take into account the correlation between path gains.

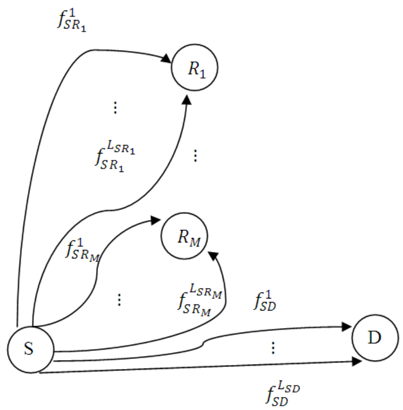

Fig. 1. Phase 1: S broadcasts a signal to D while relays listen

A two-phase digital relaying protocol is considered. In the first phase (1^{st} time slot) the source broadcasts its signal as shown in Fig. 1. If the destination has a received SNR above a threshold value γ_t, it feeds back this information informing the potential relays that cooperation is not required and then the source transmits the next data during the 2^{nd} time slot. Otherwise, the destination selects the relay belonging to Θ having the largest SNR in $R_\Theta - D$ links. In the second phase (2^{nd} time slot), the selected relay forwards data to the destination. Then, the destination uses a Rake receiver to estimate the transmitted symbol from the source and the selected relay. Finally, it combines these estimates using maximum ratio combining (MRC).

Let L_{XY}, f_{XY}^l and τ_{XY}^l the number of paths, the complex gain and the delay of the path l of the *X-Y* link. The noise at Y is an additive white gaussian noise with variance N_{XY}.

The m^{th} correlation of the Rake receiver at Y during the k^{th} symbol period can be written as [8]

$$z_{XY}^m(k) = s_k \sum_{l=1}^{L_{XY}} f_{XY}^l(kT_s) q(\tau_{XY}^m - \tau_{XY}^l) + n^m(k), \quad (1)$$

where s_k is the *k-th* transmitted symbol, $n^m(k)$ is a term due to noise, $q(\tau) = (g \otimes g)(\tau)$, $\otimes$ denotes the convolution operation and $g(t)$ is the shaping filter.

III. E2E BEP Analysis of the System

In this section, we derive the e2e BEP at D for Binary Phase Shift Keying (BPSK) modulation and Rayleigh fading channels. The e2e BEP at D can be written as

$$\begin{aligned} P_{e,D} &= P(\gamma_{SD} < \gamma_t) P_{ndiv}(e|\gamma_{SD} < \gamma_t) + P(\gamma_{SD} \geq \gamma_t) \\ &\quad \times P_{direct}(e|\gamma_{SD} \geq \gamma_t), \end{aligned} \quad (2)$$

where $P_{ndiv}(e|\gamma_{SD} \geq \gamma_t)$ is the average conditional BEP at the destination given that $\gamma_{SD} \geq \gamma_t$, i.e., the destination needs a cooperative diversity and $P_{direct}(e|\gamma_{SD} < \gamma_t)$ is the average BEP at the destination given that $\gamma_{SD} < \gamma_t$, i.e., the destination relies only on the direct transmission. Next, we derive each term of (2). The probability density function (PDF) of γ_{XY} is [8]:

$$p_{\gamma_{XY}}(x) = \sum_{j=1}^{L_{XY}} \frac{\pi_{XY}^{(j)}}{\beta_{XY}^{(j)}} \exp(-\frac{x}{\beta_{XY}^{(j)}}), \; if \; x \geq 0, \quad (3)$$

where

$$\beta_{XY}^{(j)} = \lambda_{XY}^{(j)} \frac{E_X}{N_{XY}}, \quad (4)$$

and

$$\pi_{XY}^{(j)} = \prod_{\substack{1 \leq k \leq L_{XY} \\ k \neq j}} \frac{\lambda_{XY}^{(j)}}{\lambda_{XY}^{(j)} - \lambda_{XY}^{(k)}}, \quad (5)$$

where $\lambda_{XY}^{(j)}$ is the j^{th} eigenvalue of the matrix $\sqrt{\mathbf{Q}_{XY}} E(\mathbf{f}_{XY} \mathbf{f}_{XY}^\dagger) \sqrt{\mathbf{Q}_{XY}}$. The probability that $\gamma_{XY} < \gamma_t$ is given by

$$P(\gamma_{XY} < \gamma_t) = \sum_{j=1}^{L_{XY}} \pi_{XY}^{(j)} [1 - \exp(-\frac{\gamma_t}{\beta_{XY}^{(j)}})]. \quad (6)$$

The conditional probability $P_{direct}(e|\gamma_{SD} \geq \gamma_t)$ can be written as

$$P_{direct}(e|\gamma_{SD} \geq \gamma_t) = \int_{\gamma_t}^{\infty} Q(\sqrt{2x}) p_{\gamma_{SD}|\gamma_{SD} \geq \gamma_t} dx, \quad (7)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$ and

$$p_{\gamma_{SD}|\gamma_{SD} \geq \gamma_t} = \begin{cases} \frac{p_{\gamma_{SD}}(x)}{\Lambda_{SD}(\gamma_t)}, & \text{if } x \geq \gamma_t \\ 0, & \text{o.w.} \end{cases} \quad (8)$$

where $\Lambda_{XY}(\gamma_t) = P(\gamma_{XY} \geq \gamma_t)$. Using integration by parts and an adequate variable substitution we obtain

$$\begin{aligned} P_{direct}(e|\gamma_{SD} \geq \gamma_t) &= \sum_{j=1}^{L_{SD}} \frac{\pi_{SD}^{(j)}}{\Lambda_{SD}(\gamma_t)} \Bigg[Q(\sqrt{2\gamma_t}) e^{-\frac{\gamma_t}{\beta_{SD}^{(j)}}} \\ &\quad - \sqrt{\frac{\beta_{SD}^{(j)}}{1 + \beta_{SD}^{(j)}}} Q\left(\sqrt{2\gamma_t(1 + \frac{1}{\beta_{SD}^{(j)}})}\right) \Bigg]. \end{aligned} \quad (9)$$

On the other hand, $P_{ndiv}(e|\gamma_{SD} < \gamma_t)$ can be written as

$$P_{ndiv}(e|\gamma_{SD} < \gamma_t) = \sum_{\Theta} P(\Theta) P_{ndiv}(e|\gamma_{SD} < \gamma_t, \Theta), \quad (10)$$

where

$$\begin{aligned} P(\Theta) &= \prod_{i \in \Theta} \sum_{j=1}^{L_{SR_i}} \pi_{SR_i}^{(j)} \exp(-\frac{\gamma_t}{\beta_{SR_i}^{(j)}}) \prod_{k \in \overline{\Theta}} \sum_{j=1}^{L_{SR_k}} \pi_{SR_k}^{(j)} \\ &\quad \times [1 - \exp(-\frac{\gamma_t}{\beta_{SR_k}^{(j)}})]. \end{aligned} \quad (11)$$

If Θ contains more than two relays, we have

$$P_{ndiv}(e|\gamma_{SD} < \gamma_t, \Theta) = \sum_{q\in\Theta} P_{ndiv}(e|I) P(R_{Sel\Theta} = q), \quad (12)$$

where $I = \{\gamma_{SD} < \gamma_t, \Theta, R_{Sel\Theta} = q\}$ and $R_{Sel\Theta}$ is the activated relay in Θ. The probability $P(R_{Sel\Theta} = q)$ is given by

$$P(R_{Sel\Theta} = q) = \prod_{\substack{k\in\Theta \\ k\neq q}} P(\gamma_{R_q D} > \gamma_{R_k D}). \quad (13)$$

The obtained expression of $P(R_{Sel\Theta} = q)$ is given by

$$P(R_{Sel\Theta} = q) = \prod_{\substack{k\in\Theta \\ k\neq q}} \sum_{j=1}^{L_{R_q D}} \pi_{R_q D}^{(j)} \sum_{l=1}^{L_{R_k D}} \frac{\pi_{R_k D}^{(l)}}{\beta_{R_k D}^{(l)}} \left[\beta_{R_k D}^{(l)} - \frac{\beta_{R_q D}^{(j)} \beta_{R_k D}^{(l)}}{\beta_{R_q D}^{(j)} + \beta_{R_k D}^{(l)}} \right] \quad (14)$$

If the selected relay belonging to Θ ($R_{Sel\Theta}$) decodes incorrectly the received signal, it forwards an erroneous signal leading to *error propagation* event. The BEP at the destination due to *error propagation* is denoted $P_{prop,D}(e|I)$ whereas the BEP at the destination given that $R_{Sel\Theta}$ forwarded a correctly decoded signal is $P_{coop,D}(e|I)$.

$$\begin{aligned} P_{ndiv}(e|I) &= P_{R_{Sel\Theta}}(e|I) P_{prop,D}(e|I) \\ &+ (1 - P_{R_{Sel\Theta}}(e|I)) P_{coop,D}(e|I), \end{aligned} \quad (15)$$

where $P_{R_{Sel\Theta}}(e|I)$ is the BEP at $R_{Sel\Theta}$. $P_{prop,D}(e|I)$ can be bounded by the worst case value i.e., $P_{prop,D}(e|I) \approx \frac{1}{2}$ as in [13] and [14].

Following the same methodology to obtain the conditional probability $P_{direct}(e|\gamma_{SD} < \gamma_t)$ in (9), we obtain the BEP at $R_{Sel\Theta}$,

$$\begin{aligned} P_{R_{Sel\Theta}}(e|I) &= \sum_{j=1}^{L_{SR_{Sel\Theta}}} \frac{\pi_{SR_{Sel\Theta}}^{(j)}}{\Lambda_{SR_{Sel\Theta}}(\gamma_t)} \left[Q(\sqrt{2\gamma_t}) e^{-\frac{\gamma_t}{\beta_{SR_{Sel\Theta}}^{(j)}}} \right. \\ & \left. - \sqrt{\frac{\beta_{SR_{Sel\Theta}}^{(j)}}{1 + \beta_{SR_{Sel\Theta}}^{(j)}}} Q\left(\sqrt{2\gamma_t (1 + \frac{1}{\beta_{SR_{Sel\Theta}}^{(j)}})} \right) \right] \end{aligned} \quad (18)$$

The conditional probability $P_{coop,D}(e|I)$ is given by

$$\begin{aligned} P_{coop,D}(e|I) &= \int_0^\infty \int_0^{\gamma_t} Q(\sqrt{2(x+u)}) p_{\gamma_{SD}|\gamma_{SD}<\gamma_t}(x) \\ & \times p_{\gamma_{R_{Sel\Theta}D}}(u) dx du. \end{aligned} \quad (19)$$

The conditional PDF $p_{\gamma_{SD}|\gamma_{SD}<\gamma_t}$ is given by

$$p_{\gamma_{SD}|\gamma_{SD}<\gamma_t}(x) = \begin{cases} \frac{p_{\gamma_{SD}}(x)}{\Upsilon_{SD}(\gamma_t)}, & \text{if } 0 \le x < \gamma_t \\ 0, & \text{o.w.} \end{cases} \quad (20)$$

where $\Upsilon_{SD}(\gamma_t) = P(\gamma_{SD} < \gamma_t)$. Let F denotes the integral

$$F = \int_0^{\gamma_t} Q(\sqrt{2(x+u)}) p_{\gamma_{SD}|\gamma_{SD}<\gamma_t}(x) dx. \quad (21)$$

Using integration by parts, we obtain the expression of F in (16). The SNR of $R_{Sel\Theta} - D$ is given by

$$\gamma_{R_{Sel\Theta}D} = \max_{i\in\Theta} \{\gamma_{R_i D}\}. \quad (22)$$

Let $\{l(\Theta, i, p)\}_{p=1}^{|\Theta|-1}$ be the set of relays indices which belong to Θ and different from i. The PDF of $\gamma_{R_{Sel\Theta D}}$ is given in [8, eq. (36)] where $\epsilon_{n,|\Theta|} = (\epsilon_{n,|\Theta|}(1), \dots \epsilon_{n,|\Theta|}(|\Theta| - 1))$ is the binary representation of $0 \le n \le 2^{|\Theta|-1} - 1$, $\xi(n) = \sum_{p=1}^{|\Theta|-1} \epsilon_{n,|\Theta|}(p)$ and $\frac{1}{\alpha_{nikm_1 \dots m_{|\Theta|-1}}} = \frac{1}{\beta_{R_i D}^{(k)}} + \sum_{p=1}^{|\Theta|-1} \frac{\epsilon_{n,|\Theta|}(p)}{\beta_{R_{l(\Theta,i,p)}D}^{(m_p)}}$. To determine the expression of $P_{coop,D}(e|I)$ in (19), we use the following result which can be proved using integration by parts

$$\begin{aligned} \Psi(a, b, \alpha) &= \int_0^\infty Q(\sqrt{au + b}) \frac{1}{\alpha} \exp(-\frac{u}{\alpha}) du = \\ & Q(\sqrt{b}) - \sqrt{\frac{1}{1 + \frac{2}{a\alpha}}} \exp(\frac{b}{a\alpha}) Q\left(\sqrt{b(1 + \frac{2}{a\alpha})} \right). \end{aligned} \quad (23)$$

Using (16) and (23), we find the expression of $P_{coop,D}(e|I)$ given in (17), where $\frac{1}{\alpha'_{nijkm_1 \dots m_{|\Theta|-1}}} = \frac{1}{\alpha_{nikm_1 \dots m_{|\Theta|-1}}} - \frac{1}{\beta_{SD}^{(j)}}$. By substituting equations (17) and (18) in (15), we obtain the expression of $P_{ndiv(e|I)}$. By substituting the result equation and (14) in (12), we obtain the expression of $P_{ndiv}(e|\gamma_{SD<\gamma_t,\Theta})$. Finally, by substituting the result equation and (11) in (10), we obtain the expression of $P_{ndiv}(e|\gamma_{SD<\gamma_t})$. The expressions of the conditional probability $P_{ndiv}(e|\gamma_{SD<\gamma_t,\Theta})$ for the cases where Θ contains a single relay and Θ is an empty set can be straightforwardly obtained using similar derivations. By substituting the obtained expressions of $P_{ndiv}(e|\gamma_{SD<\gamma_t})$ and $P_{direct}(e|\gamma_{SD} \ge \gamma_t)$ in (2), we obtain the expression of $P_{e,D}$.

IV. Throughput Analysis of the System

Throughput is the amount of data successfully delivered per time unit. The throughput of the three protocols is given by

$$Th^x = \frac{R(1 - P_{e,D}^x)}{E(T_x)}, \quad (24)$$

where R (bits/s/Hz) is the target transmission rate and $E(T_x)$, $x \in \{IR, SR, ISR\}$ is the expected number of time slots and $P_{e,D}^x$, $x \in \{IR, SR, ISR\}$ is the average e2e BEP at D for IR, SR and ISR, respectively.

1) Throughput Analysis of IR: For IR, the average e2e BEP $P_{e,D}^{IR}$ is given by

$$\begin{aligned} P_{e,D}^{IR} &= P(\gamma_{SD} \ge \gamma_t)(1 - P_{direct}(e|\gamma_{SD} \ge \gamma_t)) \\ &+ P(\gamma_{SD} < \gamma_t)(1 - P_{coop,D}(e|\gamma_{SD} < \gamma_t)), \end{aligned} \quad (25)$$

where $P_{direct}(e|\gamma_{SD} \ge \gamma_t)$ is given in (9), $P(\gamma_{SD} \ge \gamma_t) = 1 - P(\gamma_{SD} < \gamma_t)$, $P(\gamma_{SD} < \gamma_t)$ is given by (6) and $P_{coop,D}(e|\gamma_{SD} < \gamma_t)$ is given by (17).

The expected number of time slots is given by

$$E(T_{IR}) = P(\gamma_{SD} < \gamma_t) \times 2 + P(\gamma_{SD} \ge \gamma_t) \times 1. \quad (26)$$

2) Throughput Analysis of SR: For SR, the average e2e BEP $P_{e,D}^{SR}$ is given by

$$P_{e,D}^{SR} = \Gamma(\gamma_t)(1 - P_{direct}(e)) + (1 - \Gamma(\gamma_t))(1 - P_{coop,D}(e)), \quad (27)$$

$$F = \sum_{j=1}^{L_{SD}} \frac{\pi_{SD}^{(j)}}{1-\Lambda_{SD}(\gamma_t)} \left[Q(\sqrt{2u}) - Q(\sqrt{2(\gamma_t+u)}) e^{\frac{-\gamma_t}{\beta_{SD}^{(j)}}} - \frac{\exp(\frac{u}{\beta_{SD}^{(j)}})}{\sqrt{1+\frac{1}{\beta_{SD}^{(j)}}}} \left(Q\left(\sqrt{2(1+\frac{1}{\beta_{SD}^{(j)}})u}\right) - Q\left(\sqrt{2(1+\frac{1}{\beta_{SD}^{(j)}})(\gamma_t+u)}\right)\right)\right]. \tag{16}$$

$$\begin{aligned} P_{coop,D}(e|I) &= \sum_{i\in\Theta} \sum_{k=1}^{L_{R_{iD}}} \frac{\pi_{R_{iD}}^{(k)}}{\beta_{R_{iD}}^{(k)}} \sum_{m_1=1}^{L_{R_{l(\Theta,i,1)D}}} \pi_{R_{l(\Theta,i,1)D}}^{(m_1)} \cdots \sum_{m_{|\Theta|-1}=1}^{L_{R_{l(\Theta,i,|\Theta|-1)D}}} \pi_{R_{l(\Theta,i,|\Theta|-1)D}}^{(m_{|\Theta|-1})} \sum_{n=0}^{2^{|\Theta|-1}-1} (-1)^{\xi(n)} \alpha_{nikm_1\ldots m_{|\Theta|-1}} \sum_{j=1}^{L_{SD}} \frac{\pi_{SD}^{(j)}}{1-\Lambda_{SD}(\gamma_t)} \\ &\times \Bigg[\Psi(2,0,\alpha_{nikm_1\ldots m_{|\Theta|-1}}) - e^{\frac{-\gamma_t}{\beta_{SD}^{(j)}}} \Psi(2,2\gamma_t,\alpha_{nikm_1\ldots m_{|\Theta|-1}}) - \frac{\alpha'_{nijkm_1\ldots m_{|\Theta|-1}}}{\alpha_{nikm_1\ldots m_{|\Theta|-1}} \sqrt{\frac{\beta_{SD}^{(j)}+1}{\beta_{SD}^{(j)}}}} \Bigg[\Psi\left(2(1+\frac{1}{\beta_{SD}^{(j)}}),0,\alpha'_{nijkm_1\ldots m_{|\Theta|-1}}\right) \\ &\quad -\Psi\left(2(1+\frac{1}{\beta_{SD}^{(j)}}),2(1+\frac{1}{\beta_{SD}^{(j)}})\gamma_t,\alpha'_{nijkm_1\ldots m_{|\Theta|-1}}\right)\Bigg]\Bigg]. \end{aligned} \tag{17}$$

where $\Gamma(\gamma_t) = \prod_{i=1}^{M} P(\gamma_{SR_i} < \gamma_t)$ and is given by

$$\Gamma(\gamma_t) = \prod_{i=1}^{M} \left[\sum_{j=1}^{L_{SR_i}} \pi_{SR_i}^{(j)} [1 - exp(-\frac{\gamma_t}{\beta_{SR_i}^{(j)}})] \right]. \tag{28}$$

$P_{direct}(e)$ is given by [8, eq. (18)]. $P_{coop,D}(e)$ can be written as

$$P_{coop,D}(e) = \sum_{\Theta} P(\Theta) P_{coop,D}(e|\Theta), \tag{29}$$

where $P(\Theta)$ is given in equation (11). If Θ contains more than two relays, we have

$$\begin{aligned} P_{coop,D}(e|\Theta, R_{Sel\Theta}) &= \int_0^\infty \int_0^\infty Q(\sqrt{2(\gamma+\beta)}) p_{\gamma_{SD}}(\gamma) \\ &\quad \times p_{\gamma_{R_{Sel\Theta D}}}(\beta) d\gamma d\beta. \end{aligned} \tag{30}$$

The expression of $P_{coop,D}(e|\Theta)$ is given in [8, (30)], where

$$\begin{aligned} \Delta_{nijkm_1\ldots m_{|\Theta|-1}} &= \Omega(\beta_{SD}^{(j)}) \frac{\beta_{SD}^{(j)} \alpha_{nikm_1\ldots m_{|\Theta|-1}}}{\beta_{SD}^{(j)} - \alpha_{nikm_1\ldots m_{|\Theta|-1}}} \\ &+ \Omega(\alpha_{nikm_1\ldots m_{|\Theta|-1}}) \frac{\alpha_{nikm_1\ldots m_{|\Theta|-1}}^2}{\alpha_{nikm_1\ldots m_{|\Theta|-1}} - \beta_{SD}^{(j)}}, \end{aligned} \tag{32}$$

where $\Omega(x) = \frac{1}{2}\left[1 - \sqrt{\frac{x}{x+1}}\right]$.
The expected number of time slots is given by

$$E(T_{SR}) = \Gamma(\gamma_t) \times 1 + (1 - \Gamma(\gamma_t)) \times 2. \tag{33}$$

3) Throughput Analysis of ISR: For ISR, the average e2e BEP $P_{e,D}^{ISR}$ is given by

$$\begin{aligned} P_{e,D}^{ISR} &= P(\gamma_{SD} \geq \gamma_t)(1 - P_{direct}(e|\gamma_{SD} \geq \gamma_t)) \\ &+ P(\gamma_{SD} < \gamma_t)\Gamma(\gamma_t)(1 - P_{direct}(e|\gamma_{SD} < \gamma_t)) \\ &+ (1 - \Gamma(\gamma_t))(1 - P_{coop,D}(e|\gamma_{SD} < \gamma_t)) P(\gamma_{SD} < \gamma_t) \end{aligned} \tag{34}$$

The expected number of time slots is given by

$$\begin{aligned} E(T_{ISR}) &= [P(\gamma_{SD} \geq \gamma_t) + P(\gamma_{SD} < \gamma_t)\Gamma(\gamma_t)] \times 1 \\ & P(\gamma_{SD} < \gamma_t)(1 - \Gamma(\gamma_t)) \times 2. \end{aligned} \tag{35}$$

V. NUMERICAL AND SIMULATION RESULTS

In this section, we provide numerical and simulation results in terms of BER and throughput performances for BPSK modulation. We allocate the same power to the source and the activated relay, i.e. $E_X = E_b/2$, where E_b is the transmitted energy per bit. All the paths of a given link have equal average power and i.i.d gains.

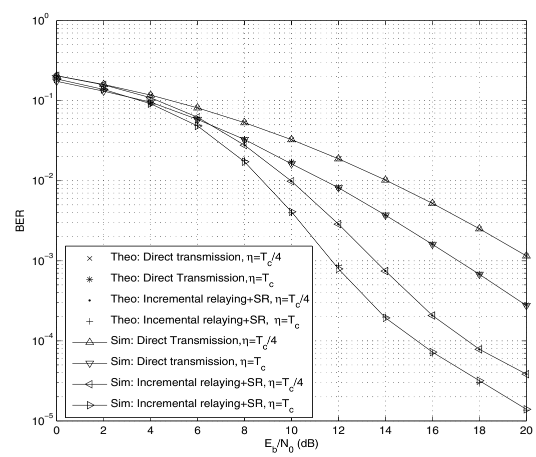

Fig. 2. Effect of time delay spacing on BER, L=2, M=2, $\gamma_t = 6$ dB

For a link *X-Y*, let L_{XY} and τ_{XY}^l be the number of paths and the delay of the path l, respectively. Fig. 2 studies the effect of time delay spacing $\eta = \tau_{XY}^2 - \tau_{XY}^1$ on the BEP for $L_{XY} = L = 2$, $M = 2$ and $\gamma_t = 6$ dB. We observe that the diversity decreases as path delays decrease. At high SNR, the BER performance tends to have a diversity order of 1. This

$$P_{coop,D}(e|\Theta)=\sum_{j=1}^{L_{SD}} \pi_{SD}^{(j)} \sum_{i\in\Theta}\sum_{k=1}^{L_{R_iD}} \frac{\pi_{R_iD}^{(k)}}{\beta_{R_iD}^{(k)}} \sum_{m_1=1}^{L_{R_{l(\Theta,i,1)D}}} \pi^{(m_1)}_{R_{l(\Theta,i,1)D}} \cdots \sum_{m_{|\Theta|-1}=1}^{L_{R_{l(\Theta,i,|\Theta|-1)D}}} \pi^{(m_{|\Theta|-1})}_{R_{l(\Theta,i,|\Theta|-1)D}} \sum_{n=0}^{2^{|\Theta|-1}-1} (-1)^{\xi(n)} \Delta_{nijkm_1\ldots m_{|\Theta|-1}} \quad (31)$$

is because at high SNR, the destination will rarely need any retransmission from the relay, thus no cooperative transmission will be performed and hence the system will have a diversity order equal to 1. Finally, we observe a perfect agreement between analytical BEP results and simulations curves.

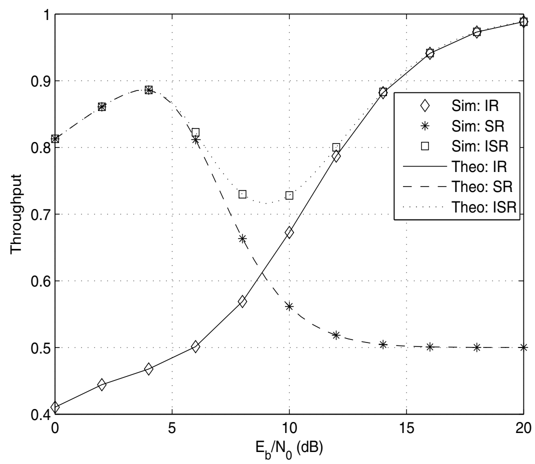

Fig. 3. Throughput comparison for $\eta = Tc$, L=2, M=2 and $\gamma_t = 6$ dB

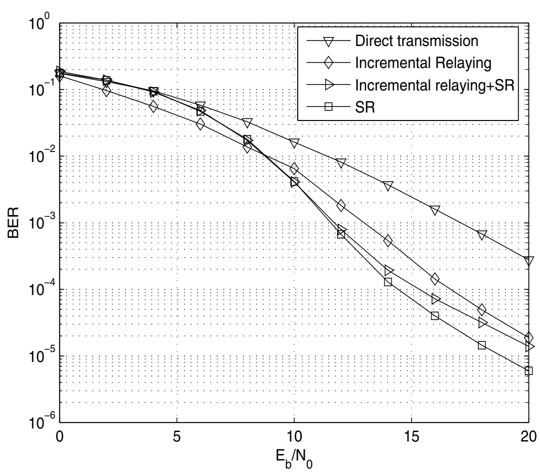

Fig. 4. BER comparison for $\eta = Tc$, L=2, M=2 and $\gamma_t = 6$ dB

Fig. 3 and Fig. 4 compare throughput and BER performances of IR, SR and the combined ISR protocol, respectively, for $R = 1\ bit/s/Hz$. We observe that ISR provides significantly higher throughput compared to SR or IR exclusively without deteriorating the BER performance mainly at medium SNR. At low SNR, performances of the combined protocol in terms of BER is confused with those of SR, while at high SNR they tend to those of IR. This is because at low SNR, the SNR of *S-D* link is often below γ_t so relaying process is controlled by the SR protocol only. At high SNR, the SNR of the links between *S* and relays are often beyond γ_t, hence, only IR protocol controls the system.

VI. Conclusion

In this paper, we studied BEP and throughput performances of cooperative DS-CDMA systems using incremental relaying in conjunction with best relay selection in the presence of multipath propagation. The derived results are valid for any multipath intensity profile, any path delays, and take into account the correlation between path gains. Throughput performance analysis shows that the combination between selective and incremental relaying significantly improves the system throughput without deteriorating BER performance.

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