Degrees of Freedom of Signal Alignment for Generalized MIMO Y Channel with General Signal Demands

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***Abstract*—In this paper, the original Multiple Input Multiple Output (MIMO) Y channel is extended into a more generalized circumstance, called a generalized MIMO Y channel with general signal demands. The model consists of $K(K \geq 2)$ nodes each equipped with M_k antennas and an intermediate relay equipped with N antennas. There is no direct link between the nodes. Therefore, supposing that node i and node j $(i \neq j \in \{1, 2, \cdots, K\})$ exchange n_{ij} signals, each node has to transmit $m_k(m_k = \sum_{i \neq k}^{K} n_{ki}, m_k \leq M_k)$ signals for other nodes in the MAC phase, and then receive m_k signals in the BC phase, via the relay. Then, it's proved that $\sum_{k=1}^{K} m_k$ degrees of freedom (DoF) can be achieved by making use of signal space alignment (SSA) and interference nulling beamforming in this model, when $N \geq 1/2(\sum_{k=1}^{K} m_k)$ and $(M_i + M_j) \geq (N + n_{ij})$.**

I. Introduction

In wireless communication system, because of its broadcast attribute, signals from various transmitters sharing the wireless medium in the same frequency create interferences to each others. Therefore, kinds of signaling schemes have appeared to successfully handle the interference problem so as to enhance the performance of network transmission rates. Recently, increasing attentions are paid to interference alignment (IA) and network coding. IA becomes an active area, because of its high efficiency to suppress interferences at high signal to noise ratio (SNR) and superior capability to increase the throughput of multi-user wireless communication system. V. R. Cadambe and S. A. Jafar introduced IA in [1], and utilized it in a interference channel of K-user with single antenna to obtain $K/2$ spatial DoF, contrary to the previous common belief that the interference channel with any number of users has only one DoF. The concept of IA is to divide the time, frequency or spatial dimensions into two halves. The transmissions of desired signals employs a half, and another half accommodates all the interferences. In [2], L. Ke, A. Ramamoorthy, Z. Wang and H. Yin utilized IA to acquire the DoF region for a general K-user interference channel, which had K transmitters, each of which transmitted an independent message, and J receivers, each of which interested in an arbitrary subset of K messages.

Another attractive scheme is network coding, which provides a new perspective about interferences, i.e., to utilize the interference. Physical-layer network coding (PNC) and analog-network coding (ANC) were implied in [3], [4] and [5], [6] respectively, to boost the two-way relay (TWR) channel [7], [8]. These schemes allow two independent signals to be transmitted in the same dimension, and let the relay jointly processes the sum of the two signals interfering with each other. In the multiple accessing(MAC) phase, the relay receives and takes advantage of the interfering signals, then broadcasts them to the desired receivers which utilize the side information to cancel interferences in the broadcast(BC) phase. A so called MIMO Y channel with three users and an intermediate relay was studied by N. Lee, J. Lim and J. Chun in [9], where every user exchanged one independent signal with other two respectively via the relay, due to no direct link between the users. The authors investigated the DoF by making use of SSA to characterize the Gaussian channel capacity, and then extended the channel model into a 4-user condition. In [10], K. Lee, N. Lee and I. Lee investigated the SSA network coding (SSA-NC) in a K-user MIMO Y channel, where each user exchanged $(K-1)$ signals with others to figured out the DoF of the channel. In [11], a simple mapping function was proposed to reduce the complexity at the relay, and the exact expressions of symbol error rate (SER) were then developed on a K-user MIMO Y channel. Also SSA-NC in multi-user TWR (MU-TWR) network can be seen in [12].

In this paper, an information exchange process is researched in the generalized MIMO Y channel with general signal demands. The channel model is composed of K nodes and an intermediate relay, and n_{ij} signals are exchanged between node j and node i. The relay is equipped with N antennas. Since each node transmits $m_k(m_k = \sum_{i=1}^{K} n_{ik})$ independent signals, it is considered that $M_k \geq m_k$. In such an information exchange process, the number of antennas on nodes must satisfy $(M_i + M_j) \geq (N + n_{ij})$. And the constraint on the amount of antennas at the relay is considered, which is $N \geq 1/2(\sum_{k=1}^{K} m_k)$. The information exchange process is studied in MAC phase and BC phase, respectively. SSA-NC is used to align the paired signals for the relay to jointly process, and interference nulling beamforming scheme is used to suppress the interference between aligned signals in the BC phase. Then $\sum_{k=1}^{K} m_k$ DoF can be achieved.

The organization of this paper is as follows. In section II, the modeling of the generalized MIMO Y channel with general signal demands is introduced. In section III, the DoF under the channel model with SSA-NC is given and proved. In section IV, simulation results are provided to verify the derived DoF. Finally, this paper is concluded in section V.

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Throughout this paper, the upper case letter is used for the matrix and the column vector. $(\cdot)^T$ and $(\cdot)^H$ demonstrate the transpose and the conjugate transpose, respectively. The nullspaces is implied by $null(\cdot)$. The rank value is demonstrated by $r(\cdot)$. $E(\cdot)$ and $Tr(\cdot)$ denote the expectation and the trace operator of a matrix.

II. SYSTEM MODEL

In this section, the system model is depicted in details. Then, the constraint on the number of antennas at the relay is described.

A. system model

Above all, a 3-user generalized MIMO Y channel with general signal demands is described to make a better understand of the difference form the original MIMO Y channel in [9], [10].

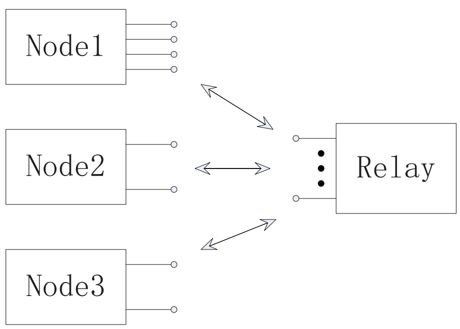

Fig. 1. K=3

As illustrated in Fig.1, node 1 has 4 antennas, and node 2 and node 3 have 2 antennas. The relay has N antennas. We consider the model as follows: node 1 exchanges two signals with node 2 and node 3, while no signal is exchanged between node 2 and node 3. There are then eight signals being transmitted in a single time slot. From the figure, we can also consider that node 2 and node 3 exchange one signal with each other, and one signal with node1. There are a total of six signals being transmitted in a single time slot, and 2 antennas at node 1 can provide the transmit diversity gain, and this model turns into the original MIMO Y channel described in [9]. These two conditions are different, and the DoF achieved is also different. However, they are both within the range of consideration. We intend to research a more generalized model. The generalized MIMO Y channel with general signal demands is obtained as depicted in Figure 2, by extending the model in Figure 1.

The model is basically composed of K nodes each equipped with M_k antennas and a relay equipped with N antennas. Each node exchanges m_k independent signals with the other nodes via the relay. If each node only exchanges one signal with all the others, then $m_k = (K-1)$, it appears like the model proposed in [11]. If K is even, and node $l (1 \leq l \leq K/2)$ only exchanges $d(d>0)$ signals with node $(l+K/2)$, it becomes the model in [12].

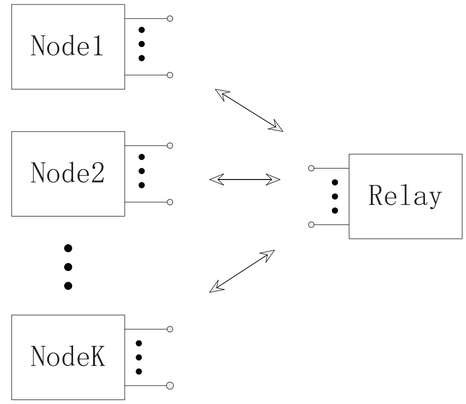

Fig. 2. various K

The whole information exchange process is divided into 2 time slots. In the first slot, every node transmits its signals to the relay simultaneously, called the MAC phase. The received signals at the relay can be expressed as

$$Y^{[r]} = H^{[r,1]}X^{[1]} + \cdots + H^{[r,K]}X^{[K]} + \eta^{[r]}, \qquad (1)$$

where $Y^{[r]}$ and $\eta^{[r]}$ are the $N \times 1$ received signal vector and additive white Gaussian noise (AWGN) vector at the relay, respectively. $X^{[k]}$ is the $M_k \times 1$ transmit vector at node k with the transmit power constraint, $E[Tr(X^{[k]}(X^{[k]})^H)] \leq P_k$. $H^{[r,k]}$ is the $N \times M_k$ transmit channel matrix from node k to the relay. The channel matrix $H^{[r,k]}$ are generated so that all the entries in the matrix are independently and identically distributed zero mean complex Gaussian random variables with unit variance, i.e., $CN(0,1)$. That almost certainly guarantees all the channel matrices have full rank, i.e.,

$$r(H^{[r,k]}) = min\{M_k, N\}. \qquad (2)$$

The relay employs a simple amplify-and-forward strategy. In the second time slot, the relay broadcasts signals to all the nodes, called the BC phase. The received signal vector at the kth node is

$$Y^{[k]} = H^{[k,r]}X^{[r]} + \eta^{[k]} \quad k \in \{1, 2, \cdots, K\}, \qquad (3)$$

where $Y^{[k]}$ and $\eta^{[k]}$ are the $M_k \times 1$ received signal vector and AWGN vector at the kth node, respectively. $X^{[r]}$ denotes the $N \times 1$ transmitted signal at the relay, with the transmit power constraint, $E[Tr(X^{[r]}(X^{[r]})^H)] \leq P_r$. $H^{[k,r]}$ is the $M_k \times N$ transmit channel matrix from the relay to node k. It is assumed that the system work in time-division duplex (TDD) mode, and the $H^{[k,r]}$ can be realized as $(H^{[r,k]})^H$ by using the reciprocal channel attribute. Yet generally, $H^{[k,r]}$ is presumably different from $(H^{[r,k]})^H$. In this paper, it is considered that perfect channel state information (CSI) is available at all nodes and the relay in both time slots.

B. the constraint on N

The key idea of SSA-NC is that in the information exchange process, signal $s_{i,j}$ transmitted from node j to node i, can always find another signal $s_{j,i}$ transmitted from node i to node

j to be paired together. Beamforming vectors can be chosen so specially that the two paired signal streams from two nodes are aligned to jointly perform detection and encoding for network coding at the relay. Therefore, every signal transmitted from each node can always find another corresponding signal to be aligned in one space-dimension when arriving at the relay by using SSA-NC. The total signal spatial dimensions is halved. Put it in another way, $\sum_{k=1}^{K} m_k$ signals are transmitted to the relay; each signal is aligned to another signal. As a result, $1/2(\sum_{k=1}^{K} m_k)$ spatial dimensions are required to accommodate all the signals. To receive and detect the aligned signals, the relay requires $N \geq 1/2(\sum_{k=1}^{K} m_k)$ antennas. To make use of the spatial dimensions to the largest extent by employing SSA-NC, the case where

$$N = 1/2(\sum_{k=1}^{K} m_k) \tag{4}$$

is considered in rest of this paper .

III. Degrees of Freedom of the generalized MIMO Y channel

Theorem 1: In the AWGN generalized MIMO Y channel with general signal demands, composed of K nodes equipped with M_k antennas and a common access relay equipped with N antennas, node $k(1 \leq k \leq K)$ exchanges $m_k(m_k = \sum_{i=1}^{K} n_{ik})$ signals with other nodes. When $N \geq 1/2(\sum_{k=1}^{K} m_k)$, $M_k \geq m_k$ and $(M_i + M_j) \geq (N + n_{ij})$, by employing SSA-NC, thus DoF

$$D = \sum_{k=1}^{K} m_k \tag{5}$$

are achieved.

Proof: The proof includes two steps: the MAC phase, and the BC phase.

A. MAC phase: signal space alignment

Considering SSA-NC in the proposed model like this, node j transmits $n_{ij}(0 \leq n_{ij} \leq M_j)$ independent signals, denoted by $n_{ij} \times 1$ vector $S_n^{[i,j]}$, to node $i(i \neq j)$ through the relay and then receives $n_{ji}(n_{ji} = n_{ij})$ independent signals, denoted by $n_{ji} \times 1$ vector $S_n^{[j,i]}$, from node i in the BC phase. Before any transmission, node j and node i precode the signal vectors by multiplying them with $V_n^{[i,j]}$ and $V_n^{[j,i]}$, the size of which are $M_j \times n_{ij}$ and $M_i \times n_{ji}$, so that $H^{[r,j]} V_n^{[i,j]} S_n^{[i,j]}$ and $H^{[r,i]} V_n^{[j,i]} S_n^{[j,i]}$ lay in the same signal spaces at the relay, and this leads to

$$span(H^{[r,j]} V_n^{[i,j]}) = span(H^{[r,i]} V_n^{[j,i]}), \tag{6}$$

where $span(A) = span(B)$ denotes that A and B span the same subspaces. A scheme to design precoding-matrices is shown as

$$[H^{[r,i]} \quad -H^{[r,j]}] \cdot [\begin{array}{c} P^{[r,i]} \\ P^{[r,j]} \end{array}] = 0. \tag{7}$$

Let $H_{ij} = [H^{[r,i]} \quad -H^{[r,j]}]$, then (7) becomes

$$[\begin{array}{c} P^{[r,i]} \\ P^{[r,j]} \end{array}] = null(H_{ij}). \tag{8}$$

$V_n^{[i,j]}$ and $V_n^{[j,i]}$ can be chosen from spaces spanned by $P^{[r,j]}$ and $P^{[r,i]}$, respectively. Assuming that the intersection subspaces is signified by Q_{ij}^{is}, $||Q_{ij}^{is}||^2 = 1$, $P^{[r,j]}$ and $P^{[r,i]}$ can be obtained by solving

$$\left[\begin{array}{ccc} I_N & -H^{[r,j]} & 0 \\ I_N & 0 & -H^{[r,i]} \end{array}\right] \cdot \left[\begin{array}{c} Q_{ij}^{is} \\ P_n^{[i,j]} \\ P_n^{[j,i]} \end{array}\right] = 0. \tag{9}$$

$P_n^{[i,j]}$ and $P_n^{[i,j]}$ meeting equation (9) are solutions for linear equation (7). Checking nullspaces of the left matrix in (9) to verify whether the solution for (9) exists. The size of the matrix is $2N \times (N + M_i + M_j)$. As a result, in the case of $(M_i + M_j) \geq N + n_{ij}$, the solution exists because the rank of nullspaces is greater than n_{ij}. Developing that restriction to the whole channel system, for any paired node i and j, the number of antennas must satisfy

$$(M_i + M_j) \geq N + n_{ij}. \tag{10}$$

When (6) is fulfilled, $2n_{ij}$ independent signals are aligned into n_{ij} signals space dimensions at the relay. And the relay can jointly perform detection and broadcast them in the BC phase. Therefore, the received signal vector at the relay is expressed as

$$Y^{[r]} = \sum_{j=1}^{K} H^{[r,j]} \sum_{i=1, i \neq j}^{K} V_n^{[i,j]} S_n^{[i,j]} + \eta^{[r]}. \tag{11}$$

Then, the relay obtains $N \times 1$ aligned signal vector $W = [w_1 w_2 \cdots w_N]^T$.

B. BC phase: interference nulling beamforming

In the BC phase, the relay transmits $N \times 1$ aligned signals vector W to the relevant nodes. Assuming

$$S_n^{[i,j]} \oplus S_n^{[j,i]} = W_n^{[ij]}, \tag{12}$$

$W_n^{[ij]}$ and $W_n^{[ji]}$ are the same, which means i, j are convertible. The aligned signals $W_n^{[ij]}$ are interferences for the rest aligned signals. Therefore, the relay employs beamforming to suppress the interference. Supposed that T_n^{ij} is the beamforming matrix for $W_n^{[ij]}$, the size of which is $N \times n_{ij}$. For the purpose of interference elimination, T_n^{ij} must lie in the nullspaces of channel matrices between other nodes and the relay. Before that, the receive beamforming is designed

$$span(U_n^{i,j} H_n^{i,r}) = span(U_n^{j,i} H_n^{j,r}), \tag{13}$$

so that two different nodes could receive the same signals $W_n^{[ij]}$ from the same space dimension, and T_n^{ij} could zero-force the effective paired channels. Here the alignment scheme can be reused. And because of $(M_i + M_j) \geq N + n_{ij}$, $U_n^{i,r}$ and

$U_n^{j,r}$ exist. UH is denoted by H'. As a result, T_n^{ij} is designed like this

$$\begin{bmatrix} H^{[1,r]'} \\ \vdots \\ H^{[i-1,r]'} \\ H^{[i,r]'}_{M_i-n_{ij}} \\ H^{[i+1,r]'} \\ \vdots \\ H^{[j-1,r]'} \\ H^{[j,r]'}_{M_j-n_{ij}} \\ H^{[j+1,r]'} \\ \vdots \\ H^{[K,r]'} \end{bmatrix} \cdot T_n^{ij} = 0, \tag{14}$$

where $H^{[k,r]'}$ denotes the $M_k \times N$ effective channel matrix from the relay to node $k(k \neq i \neq j)$. $H^{[i,r]'}_{M_i-n_{ij}}$ and $H^{[j,r]'}_{M_j-n_{ij}}$ are the $(M_i - n_{ij}) \times N$ and $(M_j - n_{ij}) \times N$ effective channel space matrices for other aligned signals at node i and j, respectively.

And after receive beamforming,

$$r\left(\begin{bmatrix} H^{[1,r]'} \\ \vdots \\ H^{[i-1,r]'} \\ H^{[i,r]'}_{M_i-n_{ij}} \\ H^{[i+1,r]'} \\ \vdots \\ H^{[j-1,r]'} \\ H^{[j,r]'}_{M_j-n_{ij}} \\ H^{[j+1,r]'} \\ \vdots \\ H^{[K,r]'} \end{bmatrix}\right) = (N - n_{ij}), \tag{15}$$

so T_n^{ij} exists. In a similar way, the beamforming matrices for all the aligned signals are obtained. The received signals at node j are

$$Y^{[j]'} = H^{[j,r]'} (\sum_{p=1}^{K} \sum_{q>p}^{K} T_n^{pq} W_n^{pq}) + \eta^{[j]'}. \tag{16}$$

The beamforming matrices of other aligned signals for other nodes lay in the nullspaces of effective channel matrix $H^{[j,r]'}$, and node j utilizes side information to cancel self-interference. Then node j only receives the desired signals vectors S_{m_j}. So in the same manner, all the nodes can receive the desired signals from the relay. ■

IV. Simulation Results

In this section, the simulation results are provided, which assesses the sum rate performance of the proposed channel model by using SSA-NC described in section III. Assuming the total transmit power and noise variance in the MAC phase and BC phase are the same, i.e., $\sum_{k=1}^{K} P_k = P_r = P$, and $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_K^2 = \sigma_r^2 = \sigma_n^2$. In addition, equal power allocation is employed for each signal. The simulation results are illustrated with respect to the ratio of the total transmitted signal power to the noise variance $(SNR = P/\sigma_n^2)$.

In the numerical results of [9], when comparing the sum rate performance with the two traditional schemes like TDMA and MU-MIMO, SSA-NC reveals better system performance, and the enhancement is significant. The improvement of performance mainly comes from the efficient utilization of signal spaces, because self-interferences don't affect the information exchange process and can be canceled at the receiver.

In this paper, the focus is concentrated on the DoF of the generalized MIMO Y channel with general signal demands. Firstly, some 3-user channel models are configured in TABLE I. Three pairs are chosen here, Config.1&2, Config.3&4, Config.5&6. Each pair has equal DoF, but the number of the exchanged signals at each node is different.

Config	m_1	m_2	m_3	N
1	3	3	2	4
2	4	2	2	4
3	6	6	4	8
4	8	4	4	8
5	8	8	8	12
6	11	11	2	12

TABLE I
NUMBER OF SIGNALS AT EACH NODE

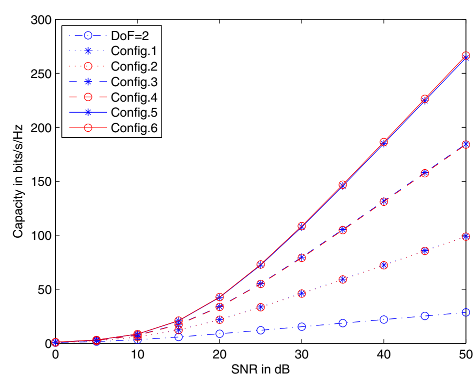

Fig. 3. capacities of K=3

Fig.3 shows the simulation results. For the purpose of easy comparison, the sum rate performance curve of $DoF = 2$ is drawn, where there are two nodes, each of which is equipped with one antenna, and a relay which is equipped with one antenna too. From the derived DoF, two things are intimated, the DoF is equal to the total number of signals being exchanged. Moreover, the number of signals at each node varies, but once $\sum_{k=1}^{K} m_k$ is constant, the achieved DoF is stationary. From the figure, these two things are verified, firstly, every sum rate curve almost grows linearly with the slope of $\sum_{k=1}^{3} m_k$. The figure also implies that once the total number of signals $\sum_{k=1}^{3} m_k$ is equivalent, the performance is equal, taking the pairs for example.

Config.	m_1	m_2	m_3	m_4	m_5	N
1	4	4	4	0	0	6
2	3	3	3	3	0	6
3	5	5	5	5	0	10
4	4	4	4	4	4	10
5	10	10	10	0	0	15
6	6	6	6	6	6	15

TABLE II
NUMBER OF SIGNALS AT EACH NODE

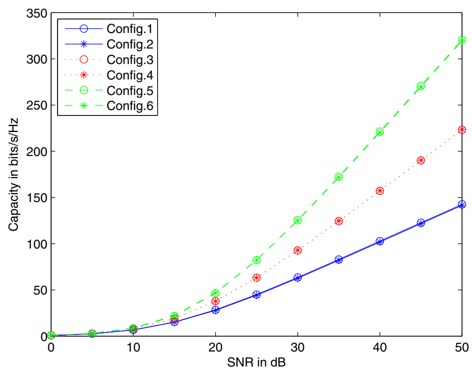

Fig. 4. capacities of different K

Fig4 shows the simulation results of different values of K. The configuration is shown in TABLE II. It is illustrated that every sum rate curve increases linearly with the slope of $\sum_{k=1}^{K} m_k$. And comparisons can be seen in the three pairs Config.1&2, Config.3&4 and Config.5&6. When the value of $\sum_{k=1}^{K} m_k$ is equivalent, each sum rate performance in every pair overlap with another. Thus from this figure, as expected, it is verified again that in the generalized Y channel, even though there are various number of nodes and each node has different number of signals to be exchanged, yet once $\sum_{k=1}^{K} m_k$ is constant, then the achieved DoF is $\sum_{k=1}^{K} m_k$, which coincidences with Theorem 1.

V. CONCLUSION

In this paper, we extend the original MIMO Y channel with 3 or 4 nodes to K nodes, each of which exchanges arbitrary number of signals with others via an intermediate relay. Then we study the proposed channel, and obtain its DoF by using SSA-NC. As a result, the DoF is related to the total number of signals being exchanged between nodes, $D = \sum_{k=1}^{K} m_k$, when the constraint on the number of antennas at nodes and the relay is satisfied.

ACKNOWLEDGMENT

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