New Algorithms for Peak-to-mean Envelope Power Reduction of OFDM Systems Through Sign Selection

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***Abstract*—It has been shown that for multi-carrier signals with n subcarriers, the peak-to-mean envelope power ratio (PMEPR) of a random codeword generated from a symmetric spherical, QAM or PSK constellation is $\log(n)$ asymptotically. Motivated by this result, recently a coding scheme with a rate of $1-\log_q(2)$ over a symmetric $q-$ary constellation has been proposed that achieves a PMEPR less than $c\log(n)$, where c is a constant. The idea of this coding scheme is to adjust the sign of subcarriers using so-called Chernoff bound-based derandomization algorithm. In this paper, using Chernoff bound and second order exponential Markov bound in conjunction with Gaussian approximation, two new variations of the derandomization algorithm are presented that yield roughly the same statistical PMEPR at the same rate. Moreover, it is rigorously established that the asymptotic PMEPR of both these algorithms is exactly the same as that of the original derandomization algorithm. Given a fixed amount of memory, our new algorithms can reduce the complexity up to one order, i.e. from $O(n^3)$ to $O(n^2)$. On the other hand, given a fixed computational complexity, our algorithms can reduce the required memory down to half.**

***Index Terms*—Coding, multi-carrier modulation, OFDM, peak-to-average power ratio (PAPR), peak-to-mean envelope power ratio (PMEPR).**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is of great importance in current wireless standards such as WiMAX, xDSL, DAB and DVB-T, see e.g [1][2]. It is also a key technology for next generation wireless communication systems. The notable advantages of OFDM include enhanced channel capacity, efficient hardware/software implementation, effective bandwidth utilization and robustness against frequency selective fading of wireless communication channels [3]. It also enables the system to dynamically estimate the Channel Impulse Response (CIR) for both slow and fast fading scenarios and by exploiting some subcarriers as pilot tones [4][5]. Note that the dynamic channel estimation allows employing coherent demodulation at the receiver side which, in turn, increases the throughput of the OFDM system at a given Signal to Noise Ratio (SNR). However, the major drawback of OFDM is its high values of peak-to-average power ratio (PAPR) or peak-to-mean envelope power ratio (PMEPR) which requires highly linear power amplifiers. One of the promising approaches proposed for alleviating high PAPR in OFDM is coding [6], see e.g. [7]-[8]. The main idea of PAPR reduction using coding is to map the data blocks into codewords with some redundancy that have small peaks.

Although the PMEPR of an OFDM system is theoretically in the order of number of subcarriers n, in [9] it is shown that with probability approaching one, the PMEPR of a random codeword generated from a symmetric spherical, QAM or PSK constellation is $\log(n)$ asymptotically. Motivated by this result, in [10], a coding scheme based on sign selection is proposed that limits the PMEPR to $c\log(n)$, where c is a constant factor. This scheme utilizes the so-called Chernoff bound-based derandomization algorithm and has a rate of $1-\log_q(2)$ over a symmetric $q-$ary constellation. In this correspondence, based on Chernoff bound and second order exponential Markov bound along with Gaussian approximation, two new variations of the sign selection coding with derandomization algorithm are proposed. These coding schemes have the same rate as that of the original derandomization algorithm, i.e. $1-\log_q(2)$. It is then proved that both our algorithms achieve a PMEPR less than $c\log(n)$, asymptotically. This performance is exactly the same as that of the original derandomization algorithm. Given a fixed amount of memory, our algorithms can also reduce the encoding complexity of the derandomization algorithm up to one order, that is from $O(n^3)$ to $O(n^2)$, while providing approximately the same PMEPR reduction. On the other hand, with the same computational complexity, our algorithms can reduce the required memory down to half.

The remainder of the paper is organized as follows. Section II presents definitions. Section III gives a mathematical formulation of sign selection code design using derandomization algorithm. Section IV proposes our algorithms and analyzes their asymptotic behavior. Section V provides simulation results and finally section VI concludes the paper.

II. Definitions

Let $\mathcal{C}(n,|\mathcal{C}|)$ be a code of length n consisting of $|\mathcal{C}|$ codewords. The elements of each codeword $\boldsymbol{c}=(c_1,c_2...,c_n)$ are taken from a symmetric $q-$ary complex constellation $\mathcal{Q}$. By symmetric constellation we mean a constellation that if an element $\mathcal{E}$ belongs to it, then $-\mathcal{E}$ also belongs to it. The complex envelope of a multicarrier signal generated by a codeword $\boldsymbol{c}$ of the code $\mathcal{C}$ is given by

$$e_{\boldsymbol{c}}(\theta) = \sum_{i=1}^{n} c_i e^{j\theta i} \tag{1}$$

where $0 \leq \theta < 2\pi$ denotes normalized time. The PMEPR of the code $\mathcal{C}$ is defined as

$$\text{PMEPR}_{\mathcal{C}} = \max_{\boldsymbol{c}\in\mathcal{C}} \max_{0\leq\theta<2\pi} \frac{|e_{\boldsymbol{c}}(\theta)|^2}{E\{\|\boldsymbol{c}\|^2\}}. \tag{2}$$

We also define the rate of the code $\mathcal{C}$ over a $q-$ary constellation as

$$R = \frac{1}{n}\log_q|\mathcal{C}|. \tag{3}$$

III. Sign Selection Using Derandomization Algorithm

In this section we briefly review the derandomization algorithm proposed in [10] for subcarrier sign selection.

Let $\mathcal{Q}$ be a symmetric $q-$ary constellation with maximum power P_{max} and average power P_{avg}. Also consider a code $\mathcal{C}$ to be constructed over this constellation. If the sign of data symbols in each subcarrier is adjusted for reducing PMEPR, then the number of codewords in $\mathcal{C}$ will be $(\frac{q}{2})^n$ and the rate of the code will be $R = 1 - \log_q(2)$. The design of such a code involves solving the following optimization problem

$$\min_{\boldsymbol{\epsilon}\in\{\pm1\}^n} \max_{0\leq\theta<2\pi} \Big|\sum_{i=1}^{n} \epsilon_i c_i e^{j\theta i}\Big| \tag{4}$$

where $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, ..., \ ilon_n)$ is the vector of signs to be determined. Note that these signs are simply discarded at the receiver side and hence, we have no decoding complexity. As a result, the only computational complexity of this coding scheme is its encoding complexity.

To simplify the problem, it is possible to convert it into a discrete optimization using the following useful Lemma.

Lemma 1 ([11]): Let $e^R_{\boldsymbol{c}}(\theta)$ and $e^I_{\boldsymbol{c}}(\theta)$ be the real and imaginary parts of $e_{\boldsymbol{c}}(\theta)$, respectively. Also for each oversampling factor $k > 1$ such that kn is an integer, let $\theta_p = \frac{2\pi p}{kn}$ for $p = 1, 2, ..., kn$. Then,

$$\max_{0\leq\theta<2\pi}|e_{\boldsymbol{c}}(\theta)| \leq \frac{1}{\cos(\frac{\pi}{2k})}\sqrt{\max_{1\leq p\leq kn}|e^R_{\boldsymbol{c}}(\theta_p)|^2 + \max_{1\leq p\leq kn}|e^I_{\boldsymbol{c}}(\theta_p)|^2}.$$

Now define $2kn$ coefficients related to the ith element of the codeword $\boldsymbol{c} = (c_1, c_2 ..., c_n)$ as

$$a_{pi} = \begin{cases} \text{Re}\{c_i e^{j\theta_p i}\}, & 1 \leq p \leq kn \\ \text{Im}\{c_i e^{j\theta_p i}\}, & kn+1 \leq p \leq 2kn \end{cases} \tag{5}$$

where $\theta_p = \frac{2\pi p}{kn}$ and $i = 1, 2, ..., n$. Using Lemma 1 we can rewrite the optimization problem (4) as

$$\min_{\boldsymbol{\epsilon}\in\{\pm1\}^n} \max_{1\leq p\leq 2kn} \Big|\sum_{i=1}^{n} \epsilon_i a_{pi}\Big|. \tag{6}$$

In fact, problem (4) and problem (6) are equivalent for large values of k.

We can find a solution for (6) among the set of all equiprobable vectors $\boldsymbol{\epsilon} \in \{\pm1\}^n$ by exploiting a recursive algorithm. Specifically, at the jth step we choose ϵ^*_j such that

$$\begin{aligned} &\sum_{p=1}^{2kn} \text{Pr.}\{A^\lambda_p | \epsilon^*_1, \epsilon^*_2, ..., \epsilon^*_j\} \\ &\quad = \min\Bigg\{\sum_{p=1}^{2kn}\text{Pr.}\{A^\lambda_p|\epsilon^*_1, \epsilon^*_2, ..., \epsilon^*_{j-1}, \epsilon_j = +1\}, \\ &\qquad\qquad \sum_{p=1}^{2kn}\text{Pr.}\{A^\lambda_p|\epsilon^*_1, \epsilon^*_2, ..., \epsilon^*_{j-1}, \epsilon_j = -1\}\Bigg\} \end{aligned} \tag{7}$$

where A^λ_p is the event that $|\sum_{i=1}^n \epsilon_i a_{pi}|$ is greater than λ and λ is chosen such that $\sum_{p=1}^{2kn}\text{Pr.}\{A^\lambda_p\} < 1$. This is called *derandomization algorithm* and it is easy to show that recursively choosing the signs according to this algorithm results in $\sum_{p=1}^{2kn}\text{Pr.}\{A^\lambda_p|\epsilon^*_1, \epsilon^*_2, ..., \epsilon^*_n\} < 1$, which guarantees that, with probability one, none of the bad events A^λ_p occur. The main task in the derandomization algorithm is to calculate the conditional probabilities which is quite cumbersome. However, one can exploit appropriate upper bounds on the conditional probabilities, instead. In particular, we can use

$$\text{Pr.}\{A^\lambda_p|\epsilon_1, \epsilon_2, ..., \epsilon_j\} \leq F^\lambda_p(\epsilon_1, \epsilon_2, ..., \epsilon_j) \tag{8}$$

provided that these upper bounds satisfy the following two conditions

1) $\min_{\epsilon_j\in\{\pm1\}} F^\lambda_p(\epsilon_1, \epsilon_2, ..., \epsilon_j) \leq F^\lambda_p(\epsilon_1, \epsilon_2, ..., \epsilon_{j-1})$
2) $\sum_{p=1}^{2kn} F^\lambda_p < 1.$ (9)

In [10], the authors showed that Chernoff bound satisfies both conditions of (9) and used it to find the signs. Specifically, in their algorithm, the signs are recursively determined using the following operation

$$\begin{aligned} \epsilon_j = -\text{sgn}\Bigg\{&\sum_{p=1}^{2kn}\sinh\left(\beta\sum_{i=1}^{j-1}a_{pi}\epsilon^*_i\right)\sinh(\beta a_{pj}) \\ &\times\prod_{r=j+1}^{n}\cosh(\beta a_{pr})\Bigg\} \end{aligned} \tag{10}$$

where $\beta = \sqrt{2\log(4kn)/nP_{max}}$. The complexity of this algorithm is $O(n^3)$ multiplications and $O(n^3)$ hyperbolic functions [12] and the memory needed for storing $\{a_{pi}\}$ is $2kn^2$ float-point real numbers. Since the complexity of hyperbolic functions is much more than multiplication [12], from now on, we focus our attention to hyperbolic functions only. One may calculate and store $\prod_{r=j+1}^{n}\cosh(\beta a_{pr})$ for all r before optimization. In this case, the complexity will be $4kn^2$ hyperbolic functions while the required memory will increase to $4kn^2$.

IV. The Proposed Algorithms

In this section we propose two new variations of the derandomization algorithm which result in the same asymptotic PMEPR and similar statistical PMEPR behavior. These algorithms can also reduce the complexity or the required memory of the derandomization algorithm.

A. Chernoff Bound-based Gaussian Approximated Derandomization Algorithm

As stated before, the main task of derandomization algorithm is calculation of conditional probabilities. Alternatively, one may use appropriate upper bounds on these probabilities. In this section, we use Chernoff bound along with Gaussian approximation to find a coding scheme with considerably reduced complexity.

Recall that at the jth step of the derandomization algorithm we need to calculate Pr.$\left\{\left|\sum_{i=1}^{n} a_{pi}\epsilon_i\right| \geq \lambda | \epsilon_1^*, ..., \epsilon_j^*\right\}$. Mathematically, we may write

$$\begin{aligned}
&\text{Pr.}\left\{\left|\sum_{i=1}^{n} a_{pi}\epsilon_i\right| \geq \lambda | \epsilon_1^*, ..., \epsilon_j^*\right\} \\
&\quad \leq e^{-\gamma\lambda}\Big\{e^{\gamma \sum_{i=1}^{j} a_{pi}\epsilon_i} \times E\big\{e^{\gamma \sum_{i=j+1}^{n} a_{pi}\epsilon_i}\big\} \qquad (11)\\
&\qquad + e^{-\gamma \sum_{i=1}^{j} a_{pi}\epsilon_i} \times E\big\{e^{-\gamma \sum_{i=j+1}^{n} a_{pi}\epsilon_i}\big\}\Big\} \qquad (12)\\
&\quad \approx e^{-\gamma\lambda} e^{\frac{\gamma^2}{2}\sigma_{pj}^2} \times \left\{e^{\gamma\mu_{pj}} + e^{-\gamma\mu_{pj}}\right\} \qquad (13)\\
&\quad = 2e^{-\gamma\lambda \frac{\gamma^2}{2}\sigma_{pj}^2} \cosh(\gamma\mu_{pj}) \\
&\quad \triangleq F_p^{\lambda}(\epsilon_1, ..., \epsilon_j) \qquad (14)
\end{aligned}$$

where $\mu_{pj} \triangleq \sum_{i=1}^{j} a_{pi}\epsilon_i^*$, $\sigma_{pj}^2 \triangleq \sum_{i=j+1}^{n} a_{pi}^2$ and $\gamma > 0$ is a to-be-optimized constant. Note that inequality (12) follows from Chernoff bound [10] while (13) is Gaussian approximation due to Central Limit Theorem (CLT). That is, the distribution of random variable $X_{pj} \triangleq \sum_{i=j+1}^{n} a_{pi}\epsilon_i$ has been approximated by $\mathcal{N}(0, \sigma_{pj}^2)$ and hence the corresponding characteristic function would be $e^{\frac{\gamma^2}{2}\sigma_{pj}^2}$.

Having obtained an approximate expression for conditional probabilities, we find the signs according to the following rule [10]

$$\epsilon_j^* = -\text{sgn}\Big\{\sum_{p=1}^{2kn} F_p^{\lambda}(\epsilon_1^*, ..., \epsilon_j = +1) - \sum_{p=1}^{2kn} F_p^{\lambda}(\epsilon_1^*, ..., \epsilon_j = -1)\Big\}. \qquad (15)$$

Substituting (14) into (15), we obtain

$$\begin{aligned}
\epsilon_j^* &= -\text{sgn}\Big\{2e^{-\gamma\lambda} \sum_{p=1}^{2kn} e^{\frac{\gamma^2}{2}\sigma_{pj}^2} \Big\{\cosh(\gamma\mu_{p(j-1)} + \gamma a_{pj}) \\
&\qquad - \cosh(\gamma\mu_{p(j-1)} - \gamma a_{pj})\Big\}\Big\} \\
&= -\text{sgn}\Big\{\sum_{p=1}^{2kn} e^{\frac{\gamma^2}{2}\sigma_{pj}^2} \sinh(\gamma\mu_{p(j-1)}) \sinh(\gamma a_{pj})\Big\}
\end{aligned}$$

The complexity of this algorithm is $4kn^2$ hyperbolic functions and $2kn^2$ exponential functions, which is in contrast with that of the original derandomization algorithm, namely $O(n^3)$. If we are allowed to use more memory, we can compute and store all $e^{\frac{\gamma^2}{2}\sigma_{pj}^2} \sinh(\gamma a_{pj})$. In this case, the complexity is reduced to $2kn^2$, which is half of the original derandomization algorithm with the same amount of memory.

It is easy to verify that the set of upper bounds $F_p^{\lambda}(\epsilon_1, ..., \epsilon_j)$ in (14) satisfy the conditions of (9). To verify the first condition, we can write

$$\begin{aligned}
&\min_{\epsilon_j \in \{\pm 1\}} F_p^{\lambda}(\epsilon_1, \epsilon_2, ..., \epsilon_j) \\
&\leq \frac{F_p^{\lambda}(\epsilon_1, \epsilon_2, ..., \epsilon_j = +1) + F_p^{\lambda}(\epsilon_1, \epsilon_2, ..., \epsilon_j = -1)}{2} \\
&= 2e^{-\gamma\lambda + \frac{\gamma^2}{2}\sigma_{pj}^2} \frac{\cosh(\mu_{p(j-1)} + \gamma a_{pj}) + \cosh(\mu_{p(j-1)} - \gamma a_{pj})}{2} \\
&= 2e^{\gamma\lambda + \frac{\gamma^2}{2}\sigma_{pj}^2} \cosh(\mu_{p(j-1)}) \cosh(\gamma a_{pj}) \qquad (16)\\
&\leq 2e^{-\gamma\lambda + \frac{\gamma^2}{2}\sigma_{pj}^2} \cosh(\mu_{p(j-1)}) \times e^{\frac{\gamma^2}{2}a_{pj}^2} \\
&= 2e^{-\gamma\lambda + \frac{\gamma^2}{2}\sigma_{p(j-1)}^2} \cosh(\mu_{p(j-1)}) \\
&= F_p^{\lambda}(\epsilon_1, \epsilon_2, ..., \epsilon_{j-1}) \qquad (17)
\end{aligned}$$

where we have used the inequality $\cosh(x) \leq e^{\frac{x^2}{2}}$.

We can also find the minimum λ that satisfies the second condition of (9). We have

$$\begin{aligned}
\sum_{p=1}^{2kn} F_p^{\lambda} &= 2e^{-\gamma\lambda} \sum_{p=1}^{2kn} e^{\frac{\gamma^2}{2}\sigma_{p0}^2} \\
&\leq 2e^{-\gamma\lambda} \sum_{p=1}^{2kn} e^{\frac{\gamma^2}{2}nP_{max}} \qquad (18)\\
&= (4kn)e^{-\gamma\lambda} e^{\frac{\gamma^2}{2}nP_{max}} \qquad (19)
\end{aligned}$$

Optimizing over γ we obtain $\gamma_{opt} = \lambda/nP_{max}$. Substituting γ_{opt} into (19), we obtain

$$\sum_{p=1}^{2kn} F_p^{\lambda} \leq (4kn)e^{-\frac{\lambda^2}{2nP_{max}}} \qquad (20)$$

Let us assume $\lambda^2 = \alpha nP_{max}\log(n)$. Substituting into (19), we obtain

$$\sum_{p=1}^{2kn} F_p^{\lambda} \leq 4kn^{1-\frac{\alpha}{2}} \qquad (21)$$

which vanishes asymptotically for any $\alpha > 2$, hence ensuring that the second condition of (9) is also satisfied. Consequently, according to Lemma 1, the asymptotic PMEPR of this algorithm will be less than $\frac{2\alpha}{\cos^2(\pi/2k)}\left(\frac{P_{max}}{P_{avg}}\right)\log(n)$.

B. Markov Bound-based Gaussian Approximated Derandomization Algorithm

In this section and using Markov bound in conjunction with Gaussian approximation we present another new variation of the derandomization algorithm. First, let us state the Markov bound.

Lemma 2: Let W be a real random variable and $h(\cdot)$ be a nonnegative and non-decreasing function over the support set of W. If the expectation of $h(W)$ exists, then

$$\text{Pr.}\{W \geq \lambda\} \leq \frac{E\{h(W)\}}{h(\lambda)}. \qquad (22)$$

The proof is simple (see e.g. [13]).

From Lemma 2 and using the function $h(w) = e^{\gamma w^2}$ that is a non-negative and non-decreasing function for $\gamma, w > 0$, we have

$$\text{Pr.}\{W \geq \lambda\} \leq e^{-\gamma\lambda^2} E\left\{e^{\gamma W^2}\right\}, \quad \gamma, W \geq 0 \qquad (23)$$

for some nonnegative valued random variable W. Now suppose that at the jth step we have chosen $(\epsilon_1^*, \epsilon_2^*, ..., \epsilon_j^*)$. Define

$$W_{pj} \triangleq \Big| \sum_{i=1}^{n} a_{pi}\epsilon_i \Big| = |\mu_{pj} + X_{pj}| \tag{24}$$

which is clearly a positive-valued random variable. Here, μ_{pj} and X_{pj} are as defined in section IV-A.

Using (23) we can write

$$\begin{aligned} &\text{Pr.}\Big\{ \Big| \sum_{i=1}^{n} a_{pi}\epsilon_i \Big| \geq \lambda | \epsilon_1^*, ..., \epsilon_j^* \Big\} \\ &\leq e^{-\gamma\lambda^2} E\Big\{ e^{\gamma|\mu_p + X_p|^2} | \epsilon_1^*, ..., \epsilon_j^* \Big\} \end{aligned} \tag{25}$$

where the non-negative value γ is to be optimized. The conditional expectation in equation (27) cannot be expressed in closed form. However, we can exploit CLT to approximate X_{pj} as a Gaussian random variable and calculate the expectation. The following Lemma gives the result.

Lemma 3: Let Y be a Gaussian random variable with mean η and variance Σ. Then, for any $s < \frac{1}{2\Sigma}$

$$E\Big\{ e^{sY^2} \Big\} = \frac{1}{\sqrt{1-2s\Sigma}} e^{\frac{s\eta^2}{1-2s\Sigma}} \tag{26}$$

Proof: Lemma follows from explicit integration over normal distribution. ■

From Lemma 3 and equation (25), we can write

$$\begin{aligned} &\text{Pr.}\Big\{ \Big| \sum_{i=1}^{n} a_{pi}\epsilon_i \Big| \geq \lambda | \epsilon_1^*, ..., \epsilon_j^* \Big\} \\ &\leq e^{-\gamma\lambda^2} E\Big\{ e^{\gamma(\mu_p + X_p)^2} | \epsilon_1^*, ..., \epsilon_j^* \Big\} &(27) \\ &\approx \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{pj}^2}} e^{\frac{\gamma\mu_{pj}^2}{1-2\gamma\sigma_{pj}^2}} &(28) \\ &\triangleq F_p^{\lambda}(\epsilon_1, ..., \epsilon_j). &(29) \end{aligned}$$

where (28) is Gaussian approximation. From general rule of (15), the second algorithm assumes the form

$$\begin{aligned} \epsilon_j^* &= -\text{sgn}\Big\{ e^{-\gamma\lambda^2} \sum_{p=1}^{2kn} \frac{1}{\sqrt{1-2\gamma\sigma_{pj}^2}} \Big\{ e^{\frac{\gamma(\mu_{p(j-1)} + a_{pj})^2}{1-2\gamma\sigma_{pj}^2}} - e^{\frac{\gamma(\mu_{p(j-1)} - a_{pj})^2}{1-2\gamma\sigma_{pj}^2}} \Big\} \Big\} \\ &= -\text{sgn}\Big\{ \sum_{p=1}^{2kn} \frac{1}{\sqrt{1-2\gamma\sigma_{pj}^2}} \Big\{ e^{\frac{\gamma(\mu_{p(j-1)}^2 + a_{pj}^2)}{1-2\gamma\sigma_{pj}^2}} \times \sinh\left(\frac{2\gamma\mu_{p(j-1)}a_{pj}}{1-2\gamma\sigma_{pj}^2} \right) \Big\} \Big\} \end{aligned} \tag{30}$$

The complexity of this algorithm is $2kn^2$ hyperbolic functions and $2kn^2$ exponential functions with the memory of $2kn^2$ float-point real numbers. Therefore, at this amount of memory, this algorithm would be preferable to the Chernoff bound-based Gaussian approximated derandomization algorithm as well as the original derandomization algorithm. However, if the amount of memory is increased, the Chernoff bound-based Gaussian approximated derandomization would still be the best. On the other hand, with the complexity of $4kn^2$ hyperbolic functions, the Markov bound-based Gaussian approximated derandomization algorithm halves the required memory, as compared to the original derandomization algorithm.

In what follows, we show that the upper bounds of (29) satisfy the conditions of (9), asymptotically. Let $B_{pj} = \frac{\gamma}{1-2\gamma\sigma_{pj}^2}$. Note that $\sigma_{p(j-1)}^2 = \sigma_{pj}^2 + a_{pj}^2$ and hence $B_{p(j-1)} > B_{pj} \geq 0$. We shall have

$$\begin{aligned} &\min_{\epsilon_j \in \{\pm 1\}} F_p^{\lambda}(\epsilon_1, \epsilon_2, ..., \epsilon_j) \\ &= \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{pj}^2}} e^{B_{pj}(\mu_{p(j-1)}^2 + a_{pj}^2 - 2|\mu_{p(j-1)}a_{pj}|)} \\ &\leq \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{pj}^2}} e^{B_{pj}a_{pj}^2} e^{B_{pj}\mu_{p(j-1)}^2} \\ &< \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{pj}^2}} e^{B_{pj}a_{pj}^2} e^{B_{p(j-1)}\mu_{p(j-1)}^2} \\ &= \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{p(j-1)}^2}} \sqrt{\frac{1-2\gamma\sigma_{p(j-1)}^2}{1-2\gamma\sigma_{pj}^2}} e^{B_{pj}a_{pj}^2} e^{B_{p(j-1)}\mu_{p(j-1)}^2} \\ &= \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{p(j-1)}^2}} \sqrt{1-2B_{pj}a_{pj}^2} e^{B_{pj}a_{pj}^2} e^{B_{p(j-1)}\mu_{p(j-1)}^2} \\ &\leq \frac{e^{-\gamma\lambda^2}}{\sqrt{1-2\gamma\sigma_{p(j-1)}^2}} e^{B_{p(j-1)}\mu_{p(j-1)}^2} &(31) \\ &= F_p^{\lambda}(\epsilon_1, \epsilon_2, ..., \epsilon_{j-1}) &(32) \end{aligned}$$

where (31) follows from the fact that $\sqrt{1-2x}e^x \leq 1$ for all $x \leq \frac{1}{2}$.

We can also obtain the minimum λ that satisfies the second condition of (9). We have

$$\begin{aligned} \sum_{p=1}^{2kn} F_p^{\lambda} &= e^{-\gamma\lambda^2} \sum_{p=1}^{2kn} \frac{1}{\sqrt{1-2\gamma\sigma_{p0}^2}} \\ &\leq e^{-\gamma\lambda^2} \sum_{p=1}^{2kn} \frac{1}{\sqrt{1-2\gamma n P_{max}}} &(33) \\ &= (2kn) e^{-\gamma\lambda^2} \frac{1}{\sqrt{1-2\gamma n P_{max}}}. &(34) \end{aligned}$$

where we have used the fact that the maximum power of the constellation is P_{max}. Optimizing over γ we obtain

$$\gamma_{opt} = \frac{1}{2} \left(\frac{1}{nP_{max}} - \frac{1}{\lambda^2} \right) \tag{35}$$

Note that

$$\gamma_{opt} < \frac{1}{2nP_{max}} \leq \frac{1}{2\sigma_{pj}^2} \qquad \forall p, j, \tag{36}$$

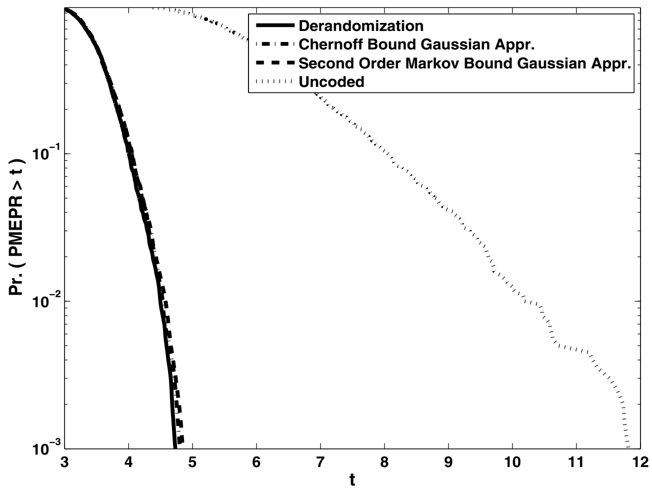

Fig. 1. CCDF of PMEPR for $64-$QAM constellation: Chernoff bound-based Gaussian approximation scheme, Markov bound-based Gaussian approximation scheme, original derandomization algorithm and uncoded system.

which ensures that the condition of Lemma 3 is met. Substituting γ_{opt} into (34) we end up with

$$\sum_{p=1}^{2kn} F_p^{\lambda} \leq (2kn) \frac{\lambda}{\sqrt{nP_{max}}} e^{\frac{1}{2}\left(1-\frac{\lambda^2}{nP_{max}}\right)}. \tag{37}$$

We shall show that for any $\alpha > 2$, $\lambda^2 = \alpha n P_{max} \log(n)$ is sufficient to satisfy the second condition of (9), asymptotically. Note that with this expression for λ^2 we have $\gamma_{opt} > 0$, thus ensuring that (23) holds. Substituting into (37) for λ^2, we have

$$\sum_{p=1}^{2kn} F_p^{\lambda} \leq 2k\sqrt{e \cdot \alpha} \frac{\sqrt{\log(n)}}{n^{\frac{\alpha}{2}-1}} \tag{38}$$

which vanishes asymptotically for any $\alpha > 2$. Therefore, it follows from Lemma 1 that the PMEPR of the resulting code will be less than $\frac{2\alpha}{\cos^2(\pi/2k)} \left(\frac{P_{max}}{Pavg}\right) \log(n)$, asymptotically.

V. Numerical Results

Figure 1 shows the Complementary Cumulative Distribution Function (CCDF) of PMEPR for Markov bound-based Gaussian approximation scheme, Chernoff bound-based Gaussian approximation scheme, original derandomization algorithm and uncoded system. These curves are obtained by simulating 6000 codewords of length $n = 128$ over a $64-$QAM constellation. Here, the rate of all three codes is $R = \frac{5}{6}$. It is seen that our coding schemes have improved the CCDF of PMEPR significantly, compared to the uncoded system. Moreover, the figure shows that the CCDF of PMEPR of all coding schemes are quite similar. Specifically, at clip probability of Pr.$\{\text{PMEPR} > t\} = 10^{-3}$, the Chernoff and Markov bound-based Gaussian approximation schemes provide reduced PMEPRs that are only 0.11 dB and 0.13 dB more than the original derandomization algorithm, respectively.

VI. Conclusion

In this paper, we proposed new algorithms for PMEPR reduction of OFDM systems through sign selection. In particular, using Gaussian approximation, we presented two variations of the derandomization algorithm that result in the same asymptotic behavior as that of the original derandomization algorithm. Furthermore, our simulations showed that the statistical behavior of the PMEPR of our algorithms are quite similar to that of the original derandomization algorithm even for finite values of n. We also showed that our algorithms can reduce the computational complexity of the derandomization algorithm up to one order or reduce the required memory down to half.

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