Optimal Resource Allocation Scheme in OFDM-Based Cognitive Radio Networks

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Abstract—**In Cognitive Radio (CR) networks, there could be a spectrum market which operates in real time with primary users (PU) as manager, where the secondary users (SU) pay the PUs for spectrum resource usage. Multi-carrier systems such as OFDM are best candidates for applying in CR networks because of the spectrum shaping and high adaptive capabilities. In this paper, an optimal resource allocation scheme aims at maximizing PU's reward in OFDM-based CR networks is proposed. Both the interference limit and BER requirements of SUs are considered by the PU to allocate its spectrum resources. The scheme is modeled as restless bandits problem, which can dramatically simplify the computation and implementation. Furthermore, extensive simulation results show that our proposed scheme can improve the reward of PU significantly compared to the existing random scheme and greedy scheme.**

Keywords-cognitive radio; OFDM; resource allocation; restless bandits; spectrum market

I. INTRODUCTION

Nowadays, the demand for using the radio spectrum is increasing, and it seems that the fixed spectrum assignment method is not so suitable to meet the demand. The limitation on spectrum resources and their improper usage, make the change of the frequency assignment policy inevitable. The concept of cognitive radio (CR) has been proposed as a smart and agile technology allowing non-legitimate users to utilize licensed bands opportunistically [1]. By detecting particular spectrum holes and jumping into them rapidly, the CR devices could improve the spectrum utilization significantly [2].

In a CR network, radio spectrum can be shared between primary (PU) and secondary (SU) users to improve spectrum utilization. This creates an opportunity for the primary service providers to generate some revenue by selling the spectrum resources to the SUs. This is referred to as a spectrum market which operates in real time involving spectrum selling and buying processes [3]. Pricing is a key issue which impacts the incentive of the PUs in selling the spectrum and the satisfaction of the SUs in buying the spectrum. Therefore, spectrum price scheme must be properly chosen based on the preferences and objectives of both PUs and SUs.

Among many possible technologies for SUs' transmission in the spectrum pooling systems, orthogonal frequency division multiplexing (OFDM) has already been widely recognized as a highly promising candidate [4]. This is mainly due to its great flexibility in dynamically allocating the unused spectrum among SUs as well as its ability to monitor the spectral activities of PUs at no extra cost [5]. Since different users in this structure use subcarriers adjacent to each other, issues of interference suppression and total capacity improvement become important. Many papers focus on these issues [6][7].

However, most existing work on OFDM-based CR networks ignores the SU selection scheme when the spectrum holes cannot satisfy the requirements of all SUs. With the increasing demand for spectrum, large number of SUs may compete for the same resource at the same time. The spectrum utilization may become disordered and the efficiency may stay low. In this paper, an optimal resource allocation scheme in OFDM-based CR networks is proposed. The distinct features of the proposed scheme is the utility function defined from PU's perspective to choose proper SUs to use the spectrum, taking into account the rent paid by the secondary users. The throughput affected by interference limit between adjacent subcarriers and the bit error rate of each SU are considered as part of the spectrum price. And SUs are divided into several states according to the amount of their data cache as another part of the spectrum price. The main contributions of this paper are as follows.

- SUs share the spectrum dynamically under the control of PU. PU makes price rules and the potential SUs can only access the spectrum holes when they are admitted by PU.
- Interference limit and BER requirement are considered to guarantee the quality of service (QoS) and maximize throughputs of SUs [8].
- The scheme is proposed from the perspective of PU and the objective of the optimization is to maximize the reward of PU.
- The resource allocation scheme is formulated as a restless bandits system [9], which can dramatically simplifies the implementation and computation. It is illustrated that the proposed scheme increases the rent reward compared to the existing ones.

The rest of this paper is organized as follows. Section II describes the system model of the resource allocation problem. In Section III formulates the problem as a restless bandit system. Section IV brings forth the whole resource allocation process. Simulation results and discussions of our proposed

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scheme are presented in Section V. This paper is concluded in Section VI.

II. SYSTEM MODEL

In this section, we have a discussion on the cognitive radio network architecture, the QoS consideration, data arrival model of SUs and optimization objective of PU.

A. Network Architecture

The network architecture here could be a complement in hot spot areas with a high demand for bandwidth (airports, convention centers, etc.). One PU and lots of SUs coexist in the same geographical location showed in Fig.1. CR network has individual base-stations that identifies the spectrum holes after collecting information about spectrum; then ask PU for spectrum with their cache information. Assume that there are N SUs who send request messages, agree to obey the rules and pay corresponding rent to PU. Then one of the SUs is presented as n , $n \in \{1,2,...,N\}$.

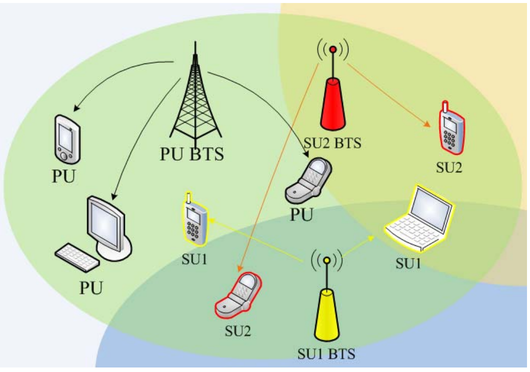

Fig. 1. Network architecture

It is assumed that a public signaling channel is used to transmit the users' states and control information. The spectrum pooling duration is divided into T equal-length time slots. One of the slots is $t_k \in \mathscr{T} = \{t_0, t_1, \ldots, t_{T-1}\}$, where $\mathscr{T}$ is the set of the time slots. Note that it's necessary in an OFDM system that the coherence time of the channel be greater than the duration of one OFDM symbol T_s (i.e., the channel can be considered constant during every time slot). For the infinite time horizon case, we choose T large enough for approximation.

B. Constrains of Subcarriers

We assume the available spectrum in the pool is M ($M < N$) subcarriers with frequency interval B . The subcarriers are allocated in the unit of one subcarrier every time slot, which means that SU n can only use one subcarrier each time. PU needs to choose M SUs among N spectrum applicants. The allocation of resources occurs before adaptive modulation, and SUs do subcarrier modulation and power control according to PU's control information.

When doing Fourier transform of the signals, power emitted from one rectangular modulated subcarrier into the spectral range of the adjacent subcarrier. In a pure OFDM system this power does not cause any interference to the adjacent subcarriers as their signals are orthogonal and can be separated in the OFDM receiver. In CR networks it's difficult to make signals from different users completely orthogonal and synchronized, so interference suppression is a must.

Assume the transmit signal of SU n on subcarrier i' ($1 \le i' \le M$) is a rectangular non-return-to-zero signal**.** The power spectral density (PSD) of this signal can be modeled as

$$\Phi(f) = P_{i'}\left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 \tag{1}$$

Where $P_{i'}$ is the total transmit power of SU n on subcarrier i' and T_s is the symbol duration. The resulting interference power spilling into the adjacent band is given by[10]

$$\dot{I}_{i'j'}(d_{i'j'}, P_{i'j'}) = \left|h_{i'j'}\right|^2 P_{i'} T_s \int_{d_{i'j'}-B/2}^{d_{i'j'}+B/2} \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df \tag{2}$$

$h_{i'j'}$ denotes channel gain from the subcarrier i' to subcarrier j' ($1 \le j' \le M$) or PU, $d_{i'j'}$ represents the frequency distance and B represents each subcarrier's bandwidth. Here we only consider the interference between adjacent subcarriers (i.e. $d_{i'j'} = B$). Set an upper boundary of $\dot{I}_{i'j'}$ as $\dot{I}_{spec}$ to guarantee QoS, which means the transmit power will be restricted.

Different types of services, different BER requests. BER_{nspec} is the max BER allowed by SU n and $BER_n(i')$ is SU n 's BER on subcarrier i' when it's transmission rate is V_n [8]:

$$BER_n(i') = 0.2\exp\left[\frac{-1.5 P_{i'} h_{i'}}{(\sigma^2 + \sum I)(2^{V_n} - 1)}\right] \tag{3}$$

σ^2 is the PSD of noise and $\sum I$ is total interference caused by subcarriers other than i' . SU n calculate max $P_{i'}$ under $I_{i'j'}(d_{i'j'}, P_{i'j'})$ constraint and SUs do modulation adaptively to get different V_n under BER constraint. SUs always try to maximize its throughput by getting the max value of V_n .

C. Data Transmission Model

Assume that more data to transmit, more urgent the spectrum request would be. SUs with different amounts of data would like to pay differently. SUs can be divided into several states according to the amount of their cache. SU n in state i_n pay corresponding rent r_{i_n} if it is allowed to use the spectrum resource. If there is too much data in the cache that SU's spectrum requests cannot be met, this SU would no longer stay but turn to other PUs' networks.

For SUs, the arriving of the data into the cache is assumed to be Poisson process, the expected data arrival rate of SU n is represented by $\lambda(n)$. According to $\lambda(n)$, we can calculate the state transition probability matrix of the SU under different PU actions. The system state transits as a Markov process according to the state transition probability matrix.

D. Action And Objective

The action of SU n in time slot t is represented by $a_n(t) \in \mathscr{A} = \{0,1\}$. $a_n(t) = 1$ means the SU n is active in time slot t, and $a_n(t)=0$ means the SU n is passive in time slot t. The action decision is made by PU according to the system state and SUs obey the spectrum accessing decisions. SUs pay rent corresponding with their states, and the objective of PU is to maximize the total rent reward in the whole spectrum pooling duration, through decisions of choosing M SUs from N SUs at each time slot. The spectrum price we proposed in our paper is composed of two parts: actual data flow price R_v (v for velocity) and state price R_s (s for state). The total reward gained during whole T time slots can be represented by

$$R=\sum_{k=0}^{T-1}(R_v^a(t_k)+R_s^a(t_k)) \tag{4}$$

The optimal objective in the proposed scheme is to maximize R through proper actions decision of PU and u is the admissible policy.

$$\begin{aligned} R^* &= \max_{u\in\mathscr{U}} R(u)=\sum_{k=0}^{T-1}(R_v^a(t_k)+R_s^a(t_k)) \\ s.t.\quad & I_{i'j'}(d_{i'j'},P_{i'j'})<I_{spec} \\ & BER_n < BER_{nspec} \\ \forall.\quad & i',j'\in\{1,2,...,M\},\ n\in\{1,2,...,N\} \end{aligned} \tag{5}$$

III. RESTLESS BANDITS FORMULATION

In this section, the resource allocation problem mentioned above is modeled as a restless bandits problem. Then primal dual index heuristic algorithm is used to solve this problem.

A. System States And Transition Probability Matrix

The state of SU n in time slot t is determined by the quantity of SU's data cache. The set of the state of each SU can be presented by $\mathscr{I}=\{I_0,I_1,\dots,I_{G-1},I_G\}$, where G+1 is the total number of cache states and i_n is $I_n(t)$ for SU n in time slot t. I_0 means no data to transmit, and I_G means too much data that SU turns to other PU's spectrum holes. It is also assumed that SUs will send their cache messages to PU at the beginning of each time slot only when $i_n\in\{I_1,\dots,I_{G-1}\}$.

We use ε_g to respect the upper bound of state I_g, and ε is the actual amount of data cache. i_n can be described by

$$i_n=\begin{cases} I_1, & 0<\varepsilon<\varepsilon_1 \\ \vdots & \vdots \\ I_g, & \varepsilon_{g-1}<\varepsilon<\varepsilon_g \\ \vdots & \vdots \\ I_G, & \varepsilon>\varepsilon_{G-1} \end{cases} \tag{6}$$

The upper bound of state I_G should be $\varepsilon_G=\infty$. Then the set $\{\varepsilon_1,\varepsilon_2,\cdots\varepsilon_G\}$ can describe SUs' state division thresholds.

The state transition probability matrix of the SU taking action a is denoted as $P_n^a=[p_{i_nj_n}(t)]_{(G+1)\times(G+1)}$, where

$$p^a_{i_nj_n}(t)=\Pr\left(I_n(t+1)=j_n \mid I_n(t)=i_n,a_n(t)=a\right) \quad i_n,j_n\in\mathscr{I},a\in\mathscr{A} \tag{7}$$

P_n^a can be also written as

$$P_n^a=\begin{bmatrix} p_{EE} & p_{01}^a & p_{02}^a\cdots p_{0G}^a \\ p_{10}^a & p_{11}^a & \cdots\ p_{1G}^a \\ \vdots & \ddots & \cdots\ \vdots \\ p_{BK} & 0 & \cdots\ p_{OT} \end{bmatrix} \tag{8}$$

where p_{EE} denotes the probability the cache of SU is empty during the time slot, we have $\sum_{g=1}^{G}p_{0g}^a=1-p_{EE}$. p_{BK} denotes the probability SU comes back to the PU's network after turning to other networks because of too much data, and p_{OT} denotes the probability SU is still in other networks, $p_{OT}+p_{BK}=1$.

The other elements in P_n^a depend on SUs' actions, the expected data arrival rate $\lambda(n)$ and the max transmission rate V_n of SU n_i. Define $\dot{P}$ the Poisson probability distribution with expected data arrival rate $\lambda(n)$ and X is quantity of data arrived in current time slot. The probability for SU transfer from state i_n to state j_n under action a can be presented by

$$p_{ij}^a=\begin{cases} \dot{P}\left\{\varepsilon_{j-1}-\dfrac{\varepsilon_i+\varepsilon_{i-1}}{2}\le X\le\varepsilon_j-\dfrac{\varepsilon_i+\varepsilon_{i-1}}{2}\right\}, & a=0 \\ \dot{P}\left\{\varepsilon_{j-1}-\dfrac{\varepsilon_i+\varepsilon_{i-1}}{2}+V_n\le X\le\varepsilon_j-\dfrac{\varepsilon_i+\varepsilon_{i-1}}{2}+V_n\right\}, & a=1 \end{cases} \tag{9}$$

where $i,j\in\{1,2,...,G\}$.

B. System Reward

Define the state price set as $\mathscr{R}=\{r_{sI_0},r_{sI_1},\dots,r_{sI_{G-1}},r_{sI_G}\}$, corresponding to the state set $\mathscr{I}$ of SUs, where $r_{sI_0}=r_{sI_G}=0$ and $r_{sI_1}<r_{sI_2}<\ldots<r_{sI_{G-1}}$. An instantaneous reward $R^a_{si_n}(t)$ is accrued in each time slot t when SU n is in state i_n and takes action a.

$$R^a_{si_n}(t)=\begin{cases}0, & a=0\\ r_{si_n}, & a=1\end{cases},\ r_{si_n}\in\mathscr{R} \tag{10}$$

Then the instantaneous reward $R_s^a(t_k)$ accrued in each time slot for PU can be presented as $R_s^a(t)=\sum_{n=1}^{N}R^a_{si_n}(t)$.

Define data flow price as r_{v_0} for per unit of data flow. According to $\dot{I}_{i'j'}(d_{i'j'},P_{i'j'})$ and BER_n, SU has its own transmission rate and the max value can be estimated by PU. An instantaneous reward $R^a_{vi_n}(t)$ is accrued

$$R^a_{vi_n}(t)=\begin{cases}0, & a=0\\ \eta_{i_n}r_{vi_n}, & a=1\end{cases} \tag{11}$$

η_{i_n} is the factor that denotes the inefficiency of spectrum utilization, which is affected by SUs' states and actual channel fading, $0\le\eta_{i_n}\le1$. Then the instantaneous reward $R_v^a(t)$ for PU can be presented as $R_v^a(t)=\sum_{n=1}^{N}\eta_{i_n}R^a_{vi_n}(t)$.

The optimal objective can be written as

$$\begin{aligned} R&=\sum_{k=0}^{T-1}\beta^{T-t_k}(R_v^a(t_k)+R_s^a(t_k)) \\ &=\sum_{k=0}^{T-1}\beta^{T-t_k}\left(\sum_{n=1}^{N}\eta_{i_n}R^a_{vi_n}(t_k)+\sum_{n=1}^{N}R^a_{si_n}(t_k)\right) \end{aligned} \tag{12}$$

Where β is the discount factor, which indicates how much the reward obtained before contributions to the total reward. $\beta = 0$ means the total reward only includes the current reward, $\beta = 1$ means the reward obtained before is of equal importance to the total reward. When $0 < \beta < 1$, it means the earlier the reward obtained, the less the weighted.

C. Indices And Polices

The index for SU n in state i_n is represented as δ_{i_n}. The optimal policy has an index rule: The M SUs with the smallest indices in a given time slot t act as the active SUs. That is, assuming $\left\{\delta_{k_1}, \delta_{k_2}, \ldots, \delta_{k_N}\right\}$ to be the set of indices arranged from the smallest value to the largest value in time slot t, the SU 's action should be

$$a_n(t) = \begin{cases} 1, \text{ if } n \in \left\{k_1, k_2, \ldots, k_M\right\} \\ 0, \text{ other.} \end{cases} \quad (13)$$

Let $\mathscr{U}$ denote the class of all admissible policies. The admissible policy $u \in \mathscr{U}$ is a $T \times N$ matrix, whose element of the t th row and the n th column is $a_n(t)$, representing the action on SU n in time slot t. In each time slot, the number of active SUs is equal to M.

According to (5), the optimal policy u is the policy that achieves the optimization objective.

$$u^* = \arg\max_{u \in \mathscr{U}} R(u) \quad (14)$$

D. Solving the Restless Bandits Problem

To solve the restless bandits problem, a primal-dual index heuristic algorithm method is developed based on the first order relaxation. The primal-dual index heuristic algorithm first deduces the first order relaxation of the restless bandits problem and its corresponding dual problem, where details can be found in [11]. As both primary and dual problems are linear program problem, they can be solved by the classical algorithm such as simplex search method.

Denote $\left\{\overline{x}_{i_n}^{a_n}\right\}$ and $\left\{\overline{\pi}_{i_n}, \overline{\pi}\right\}$ as the optimal solution of primal and dual problems respectively. Therefore, the corresponding optimal reduced cost coefficients can be represented as

$$\begin{aligned} \overline{\pi}_{i_n}^0 &= \overline{\pi}_{i_n} - \beta \sum_{j_n \in \Omega} p_{i_n j_n}^0 \overline{\pi}_{j_n} - R_{i_n}^0 \\ \overline{\pi}_{i_n}^1 &= \overline{\pi}_{i_n} - \beta \sum_{j_n \in \Omega} p_{i_n j_n}^1 \overline{\pi}_{j_n} + \overline{\pi} - R_{i_n}^1 \end{aligned} \quad (15)$$

where $\overline{\pi}_{i_n}^0, \overline{\pi}_{i_n}^0 \geq 0$, $\overline{\pi}_{i_n}^0$ and $\overline{\pi}_{i_n}^1$ mean the rates of decrease in the objective value of the primal problem per unit increase in the value of the variable $x_{i_n}^0$ and $x_{i_n}^1$ respectively. δ_{i_n} here can be calculated as

$$\delta_{i_n} = \overline{\pi}_{i_n}^1 - \overline{\pi}_{i_n}^0 \quad (16)$$

The primal-dual heuristic can be interpreted as a priority-index heuristic as well. The priority-index rule is to select the M senders that have the smallest indices to be active. At each time slot, PU looks up the table for the current $(i_1, i_2, ..., i_N)$ and chooses SUs with lowest M indices.

IV. Resource Allocation Process

The resource allocation process is based on the indices of SUs, which can be computed off-line for each available state of each SU network, and stored in a table. In the network initialization procedure, interference limitation is given before transmission and PU gets data arrival rate $\lambda(i)$ and BER_i through SUs' spectrum request messages. The state transition probability matrix can be calculated before real-time transmission. PU can calculate the finite set of the indices according to all possibilities of $(i_1, i_2, ..., i_N)$ under the restless bandit framework mentioned above. These indices are then stored in a table at PU.

In the on-line stage, it is only needed to lookup the table to decide the action according to the state. The proposed scheme works as follows.

1) Potential secondary users sense the wireless environment for spectrum holes and send spectrum request messages to primary user, with their cache, λ and BER request information in the messages.

2) PU collects request messages, negotiats with SUs on states division, prices and announces specific subcarriers's information.

3) PU collects acceptance messages from SUs and knows the number N. PU sets up indices table off-line and SU determine modulation mode according to related limitation.

4) PU looks up the table to find M SUs with smallest indices according to current states, which are allowed to use a certain subcarrier in the next time slot.

5) At the end of the time slot, SUs send states and data flow messages to PU if they still have data to transmit and the data cached doesn't exceed its tolerance.

6) Go to step 4 until PU has to use the spectrum. PU sends messages to announce transmission ending and make confirmation of each SU's cost.

V. Simulations Results and Analysis

In this section, the simulation results of the proposed optimal resource allocation scheme are presented and compared to the existing random scheme and greedy scheme.

Suppose the number of cache state levels G+1=5 and the length of each time slot to be $1s$. We use values for T_s and B of $4\mu s$ and $0.3125MHz$. Additive white Gaussian noise of variance 10^{-7} is assumed and channel gains h is assumed to be Rayleigh fading with an average power gain of $10dB$. Assume the discount factor $\beta = 0.9$, the probabilities $p_{EE} = 0.3$, $p_{BK} = 0.1$, the prices $r_{v_0} = 2$, $\mathscr{R} = \{0, 2, 4, 6, 0\}$, and BER of SUs is different from 10^{-8} to 10^{-3}.

The interference could affect the performance dramatically. In fig.2, for $-90dB < \dot{I}_{spec} < -60dB$, we present the simulation results on the total reward. There are 6 subcarriers for 10 SUs and average data arrival rate of SUs is $1.1Mbps$. We can see from the figure that the proposed optimal decision scheme improves the PU's reward. At the same time, greater the interference allowed, greater the transmit power of SUs is, and the reward of PU would increase in response. It shows that the interest of PU is contradictory to guaranteeing QoS of SUs, which is easily understood.

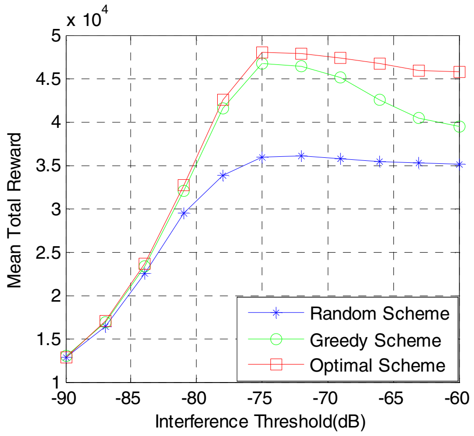

Fig. 2. Mean total reward with different interference thresholds

With the increasing of the number of spare subcarriers, we can see the improvement of the optimal scheme in Fig.3. There are 10 SUs in PU's network, more spare subcarriers means more SUs would be selected. The optimal scheme can choose the most proper SUs to maximizing the PU's total reward. We can estimate from the figure that the proposed scheme is much better than others when $N/2 < M < N$.

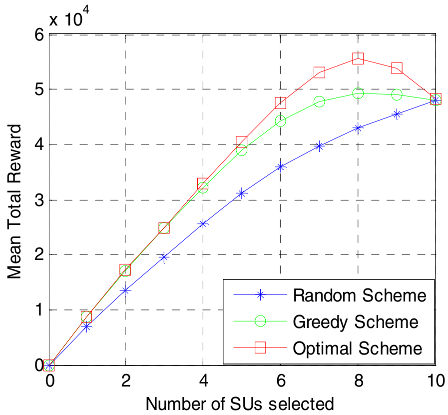

Fig. 3. Mean total reward with different M

While the upper bound of state set $\mathscr{I}$ and the corresponding price set are static, the date arrival rate may change according to different SUs. Fig.4 shows the effect of different λ, from which we can see that there is a highest point at which PU would get the maximum reward under that data arrival rate. When data arrival rate is large, current PU's network may not be able to satisfy SUs' spectrum request, so consequently, more SUs leave the current network and the mean total reward for PU decreases. This implies that it is important for PUs to arrange proper state levels.

VI. Conclusions

In this paper, we have proposed an optimal recourses allocation scheme for primary users in OFDM-based cognitive networks. Both Interference limit and BER requests are considered to guarantee QoS and maximize reward of primary users. The problem was then formulated as a restless bandit problem which can simply select the SUs with the lowest indices. The maximized total discounted reward could be obtained by solving this problem. Extensive simulation results were also presented to illustrate that the proposed optimal action decision scheme can improve the PU's reward compared with the existing scheme.

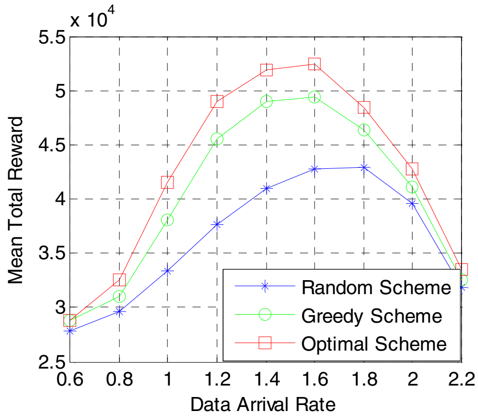

Fig. 4. Mean total reward with different λ

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