Theoretical Study of the Impact of Channel Estimation Errors on the Performance of IDMA Detectors

Manouane Caza-Szoka[1], Adel Omar Dahmane[1], Claude D'Amours[2] and Frédéric Domingue[1]

[1] Electrical and Computer Engineering Department, Université du Québec à Trois-Rivières, Trois-Rivières, Canada

[2] School of Electrical Engineering and Computer Science, University of Ottawa Ottawa, Canada

***Abstract*—In this paper, the performance of an Interleave-Division-Multiple-Access (IDMA) detector is considered when channel estimation errors are present. This detector can be quite effective under severe channel estimation errors when channel estimator statistics are considered. The detector is generalized for QPSK signaling. It is shown that the overall effect of the CSI mismatch can be seen as an additive noise term, without approximation. Moreover, the term is constant for all users, paths and for in-phase and quadrature components. The performances of the detector are studied by Monte-Carlo simulations. From the results, it is shown that the improvement depends on the severity of the mismatch, the number of paths and the number of users. A 10% capacity increase can be obtained with respect to the number of users.**

***Index Terms*—IDMA, multi-user detection, iterative decoding, channel estimation error, channel state information.**

I. Introduction

Interleave Division Multiple Access (IDMA) is a promising wideband multiuser method for future wireless communication [1]. The method separates the users by assigning them different independent interleavers. IDMA systems have high spectral efficiency, are resistant to and exploit multipath and delays and is suited for very large number of users. Also, IDMA has been combined with OFDM [2] and MIMO systems [3]. Moreover, it allows simple chip-by chip detection and parallel iterative interference cancellation (PIC). However, knowledge of the channel state information (CSI) is required. In [4]-[6], channel estimation for IDMA has been considered.

Detection of IDMA signals has been widely studied, mostly assuming perfect CSI. However, this hypothesis is not generally true. Hence, [5,7,8] have considered the effects of imperfect CSI. For this, channel error statistics are needed and can be obtained from the estimator. However, the method proposed in [7] needs long-term channel statistics which are not readily available. Otherwise, the approach in [8] does not require these statistics but is incomplete. A more appropriated method is shown in [5] that does not need long-term channel statistics as in [7], is more precise than [8] and is less complex than both. The CSI estimation errors of all users and all paths can be treated as additive noise. However, the performance of this detector has not been reported since the objective of [5] was to present a channel estimation method. In this paper we propose to generalize this technique to QPSK signaling and analytically determine the performance of this detector.

The rest of this paper is organized as follows. In Section 2, a brief description of a conventional IDMA system is given. In Section 3, the generalized IDMA receiver is developed. In Section 4, numerical results are shown. Finally, Section 5 draws conclusions and discusses other implications of the results.

II. IDMA System Description

A. Transmitter Structure

Transmitters used in IDMA systems are shown in Fig. 1. First, a low rate forward error correction (FEC) code is applied to the data sequence. Typically,data is convolutionally encoded and then spread by direct sequence spreading. The spread sequence is interleaved at the chip level and QPSK modulated. If the error control code is identical for every user, then the interleavers must be different in order to allow user separation. Independent interleavers produce signals which appear random. Implementing the interleaver before modulation makes the interleaver twice as effective, as the inphase and quadrature components of the sequence become independent. The transmitter output of user k is hence

$$x_k(j) = x_K^{Re}(j) + i x_K^{Im}(j) \quad (1)$$

where $\boldsymbol{i} = \sqrt{-1}$ and where $\boldsymbol{x_K^{Re}(j)}$ is independent of $\boldsymbol{x_K^{Im}(j)}$.

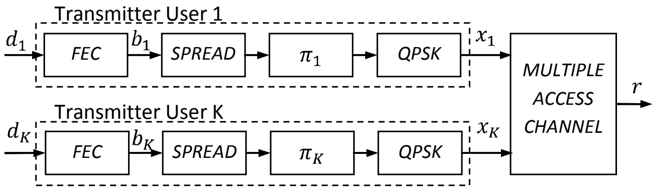

Fig. 1. IDMA transmitters for K users and Multiple Access Channel Model

B. Complex Multiple-Access Channel

The outputs of the transmitters are convoluted and summed by a complex, multipath, multiple-access channel. It is assumed for simplicity that signals are fully synchronized. However, different delays between users can be included in the impulse response of the channels. Hence, the signal at the receiver is:

$$r^{Re}(j) = \sum_{k,l}\left(h^{Re}_{k,l}x^{Re}_k(j-l) - h^{Im}_{k,l}x^{Im}_k(j-l)\right) + n^{Re}(j) \quad (2)$$

$$r^{Im}(j) = \sum_{k,l}\left(h^{Re}_{k,l}x^{Im}_k(j-l) + h^{Im}_{k,l}x^{Re}_k(j-l)\right) + n^{Im}(j) \quad (3)$$

where $h^{Re}_{k,l}$ is the real part of the channel gain for l^{th} path of the k^{th} user. The real part of the noise is $n^{Re}(j)$. The additive noise is considered white, Gaussian with the following statistics:

$$E[n^{Re}(j)] = E[n^{Im}(j)] = 0 \quad (4)$$

$$V[n^{Re}(j)] = V[n^{Im}(j)] = \sigma_n^2/2 \quad (5)$$

C. Conventional IDMA Receiver

A conventional IDMA receiver uses chip-by-chip detection and parallel interference cancellation (PIC) (Fig. 2.) For QPSK signaling, it also separates the detection of the real and imaginary parts of the chip. First, the Elementary Signal Estimator (ESE) gives the estimation of the chips. It makes use of the channel statistics to produce log-likelihood ratios of the signals (LLRs.) The LLRs of different paths are resolved independently and then combined.

To produce better estimates, interference statistics are generated. Next, these LLRs are demultiplexed for the In-Phase and Quadrature components, deinterleaved and despread (RAKE) for each user. Then, a soft-input soft-output decoder is used to produce the extrinsic LLRs of the coded bits. Afterwards, a spreader, interleaver and multiplexer for QPSK signaling are used. The results are fed to the PIC. After some iterations of PIC, the decoders produce the uncoded data estimates.

For a more detailed treatment of the conventional IDMA receiver, see [1].

III. Channel Estimation Error Resistant Elementary Signal Estimator

This section shows the calculation of the signal statistics when the channel estimation errors are considered, without prior knowledge of the channel distribution.

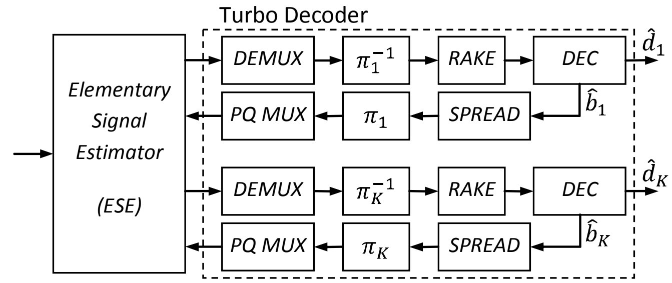

Fig. 2. Conventional IDMA receiver with multi user detection and parallel interference cancellation.

A. Channel Estimator Performances

A channel estimator can generally give an error variance of its estimates. When complex multipath is considered, an estimate of each path is given with associated error variance for both real and imaginary components. These are not necessarily equal. Hence, we have the channel estimates:

$$\hat{h}^{Re}_{k,l}(j) = h^{Re}_{k,l}(j) + e^{Re}_{k,l}(j),\ \ \hat{h}^{Im}_{k,l}(j) = h^{Im}_{k,l}(j) + e^{Im}_{k,l}(j) \quad (6)$$

where $\hat{h}^{Re}_{k,l}$ is the channel estimate of the real part of the l^{th} path of the k^{th} user and $e^{Re}_{k,l}$ is its corresponding error with the following statistics:

$$E\left(e^{Re}_{k,l}\right) = E\left(e^{Im}_{k,l}\right) = 0 \quad (7)$$

$$Var\left(e^{Re}_{k,l}\right) = \sigma^2_{\hat{h},l,Re}\ \ \ Var\left(e^{Im}_{k,l}\right) = \sigma^2_{\hat{h},l,Im} \quad (8)$$

B. Channel Error Effect

Here, it is shown that the overall effect of the combination of every user, every path and chip component errors can be treated as a single equivalent noise term. This noise term can hence be included in the thermal additive noise.

First, the channel impulse response in (2) is substituted by its estimated version:

$$r^{Re}(j) = \sum_{k,l}\left(\left(\hat{h}^{Re}_{k,l} - e^{Re}_{k,l}\right)x^{Re}_k(j-l) - \left(\hat{h}^{Im}_{k,l} - e^{Im}_{k,l}\right)x^{Im}_k(j-l)\right) + n^{Re}(j). \quad (9)$$

Next, it is reformulated in order to include the channel error into the noise term:

$$r^{Re}(j) = \sum_{k,l}\left(\hat{h}^{Re}_{k,l}x^{Re}_k(j-l) - \hat{h}^{Im}_{k,l}x^{Im}_k(j-l)\right) + n^{Re'}(j) \quad (10)$$

where:

$$n^{Re'}(j) = -\sum_{k,l}\left(e^{Re}_{k,l}x^{Re}_k(j-l) - e^{Im}_{k,l}x^{Im}_k(j-l)\right) + n^{Re}(j). \quad (11)$$

Equivalent equations can be found for the imaginary components of the signal. The equivalent noise has the following statistics:

$$E\left[n^{Re'}(j)\right] = E\left[n^{Im'}(j)\right] = 0 \quad (12)$$

$$V[n^{Re'}(j)] = V[n^{Im'}(j)] = \sum_{k,l} (\sigma^2_{\tilde{h},l,Re} + \sigma^2_{\tilde{h},l,Im}) + \sigma_n^2 \tag{13}$$

$$E[n^{Re'}(j)] = E[n^{Im'}(j)] = 0, \tag{14}$$

$$Cov\left(n^{Re'}(j), n^{Im'}(j)\right) = 0. \tag{15}$$

Hence, the modified noise term is identical for every user, every path and chip component.

C. Statistics of the Received Signal

Statistics of the received signals with consideration of channel estimator performances have the same structure as the one shown in [5]. The only differences are that the estimated multipath components are used instead of the exact value and that the additive noise term is included.

$$E[r^{Re}(j)] = \sum_{k,l} (\hat{h}^{Re}_{k,l}(j) E[x_k^{Re}(j-l)] - \hat{h}^{Im}_{k,l}(j) E[x_k^{Im}(j-l)]) \tag{16}$$

$$E[r^{Im}(j)] = \sum_{k,l} (\hat{h}^{Re}_{k,l}(j) E[x_k^{Im}(j-l)] + \hat{h}^{Im}_{k,l}(j) E[x_k^{Re}(j-l)]) \tag{17}$$

$$V[r^{Re}(j)] = \sum_{k,l} \left(\hat{h}^{Re^2}_{k,l}(j) V[x_k^{Re}(j-l)] + \hat{h}^{Im^2}_{k,l}(j) V[x_k^{Im}(j-l)]\right) + V[n^{Re'}(j)] \tag{18}$$

$$Cov[r^{Re}(j), r^{Im}(j)] = \sum_{k,l} \left(\hat{h}^{Re}_{k,l}(j) \hat{h}^{Im}_{k,l}(j) (V[x_k^{Re}(j-l)] - V[x_k^{Im}(j-l)])\right) \tag{19}$$

D. Statistics of Real and Imaginary Parts of Chip Signals

Each component of the chip signals is independent of each other due to the interleavers. This independence allows the receiver to resolve each component individually and use interference cancellation for the other component. The component statistics are computed by removing the corresponding contribution to the received signal statistics:

$$E[\zeta^{Re}_{k,l,Re}(j)] = E[r^{Re}(j)] - \hat{h}^{Re}_{k,l}(j) E[x_k^{Re}(j-l)] \tag{20}$$

$$V[\zeta^{Re}_{k,l,Re}(j)] = V[r^{Re}(j)] - \hat{h}^{Re^2}_{k,l}(j) V[x_k^{Re}(j-l)] \tag{21}$$

$$Cov[\zeta^{Re}_{k,l,Re}, \zeta^{Im}_{k,l,Re}] = Cov[r^{Re}(j), r^{Im}(j)] - \left(\hat{h}^{Re}_{k,l}(j) \hat{h}^{Im}_{k,l}(j) V[x_k^{Re}(j-l)]\right) \tag{22}$$

where $\zeta^{Re}_{k,l,Re}(j)$ is the real part of the interference of the real component of the l^{th} path of the k^{th} user. The other statistics calculated in the same manner are: $[r^{Im}(j)]$, $V[r^{Im}(j)]$, $E[\zeta^{Re}_{k,l,Im}(j)]$, $E[\zeta^{Im}_{k,l,Re}(j)]$, $E[\zeta^{Im}_{k,l,Im}(j)]$, $V[\zeta^{Re}_{k,l,Im}(j)]$, $V[\zeta^{Im}_{k,l,Re}(j)]$, $V[\zeta^{Im}_{k,l,Im}(j)]$, $Cov[\zeta^{Re}_{k,l,Im}, \zeta^{Im}_{k,l,Im}]$.

E. Complex Elementary Signal Estimator

The elementary signal estimators are calculated using the statistics above. First, the interference is removed for each user, path and component:

$$s^{Re}_{k,l,Re}(j) = r^{Re}(j-l) - E[\zeta^{Re}_{k,l,Re}(j)] \tag{23}$$

The expression of the LLRs for each path is given in (26) and simplified in (27). The matrix $\Sigma_{k,l,Re}$ used in (24) is the covariance matrix of the interference the desired signal.

$$\Sigma_{k,l,Re} = \begin{bmatrix} E[\zeta^{Re}_{k,l,Re}(j)] & Cov[\zeta^{Re}_{k,l,Re}, \zeta^{Im}_{k,l,Re}] \\ Cov[\zeta^{Re}_{k,l,Re}, \zeta^{Im}_{k,l,Re}] & E[\zeta^{Im}_{k,l,Re}(j)] \end{bmatrix} \tag{24}$$

The LLRs of the different paths of each signal are combined:

$$e_{ESE}(x_k^{Re}(j)) = \sum_l e_{ESE}(x_k^{Re}(j))_l \tag{25}$$

Similar equations are developed for imaginary components. Finally, the chip signals statistics can be computed:

$$E[x_k^{Re}(j)] = tanh(e_{ESE}(x_k^{Re}(j))/2) \tag{28}$$

$$V[x_k^{Re}(j)] = 1 - E[x_k^{Re}(j-l)]^2 \tag{29}$$

IV. Simulation Results

We present the performance of IDMA systems with severe channel estimation errors. The simulations are also done to show the impact of considering the channel estimation error on the detector performance. The received power of all users is normalized and the interleavers are generated randomly as well as the channels. A convolutional code of type (2,1,3) is used.

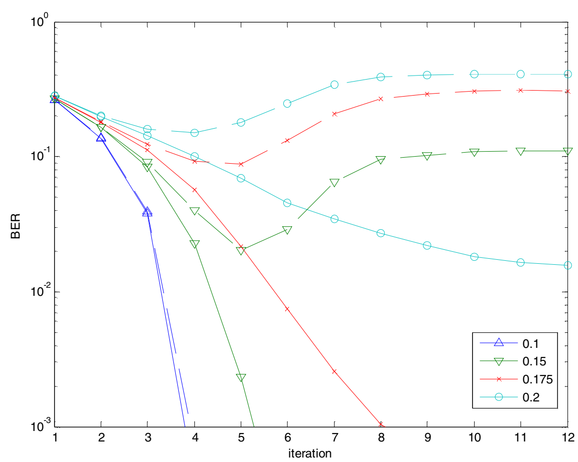

Fig. 3. BER vs number of iteration for channel error standard deviation from 0.1 to 0.2, single path, without noise, 24 users. Solid lines include channel estimation error treatment while dashed lines omit it.

$$e_{ESE}(x_k^{Re}(j))_l = log \left[\frac{exp\left(-\frac{1}{2}\begin{bmatrix} s^{Re}_{k,l,Re} - \hat{h}^{Re}_{k,l} & s^{Im}_{k,l,Re} - \hat{h}^{Im}_{k,l} \end{bmatrix} \Sigma_{k,l,Re}^{-1} \begin{bmatrix} s^{Re}_{k,l,Re} - \hat{h}^{Re}_{k,l} \\ s^{Im}_{k,l,Re} - \hat{h}^{Im}_{k,l} \end{bmatrix}\right)}{exp\left(-\frac{1}{2}\begin{bmatrix} s^{Re}_{k,l,Re} + \hat{h}^{Re}_{k,l} & s^{Im}_{k,l,Re} + \hat{h}^{Im}_{k,l} \end{bmatrix} \Sigma_{k,l,Re}^{-1} \begin{bmatrix} s^{Re}_{k,l,Re} + \hat{h}^{Re}_{k,l} \\ s^{Im}_{k,l,Re} + \hat{h}^{Im}_{k,l} \end{bmatrix}\right)} \right] \tag{26}$$

$$e_{ESE}(x_k^{Re}(j))_l = 2\frac{h^{Re}_{k,l} s^{Re}_{k,l,Re} V[\zeta^{Im}_{k,l,Re}] + h^{Im}_{k,l} s^{Im}_{k,l,Re} V[\zeta^{Re}_{k,l,Re}] - (h^{Re}_{k,l} s^{Im}_{k,l,Re} + h^{Im}_{k,l} s^{Re}_{k,l,Re}) Cov[\zeta^{Re}_{k,l,Re}, \zeta^{Im}_{k,l,Re}]}{V[\zeta^{Re}_{k,l,Re}] V[\zeta^{Im}_{k,l,Re}] - Cov[\zeta^{Re}_{k,l,Re}, \zeta^{Im}_{k,l,Re}]^2} \tag{27}$$

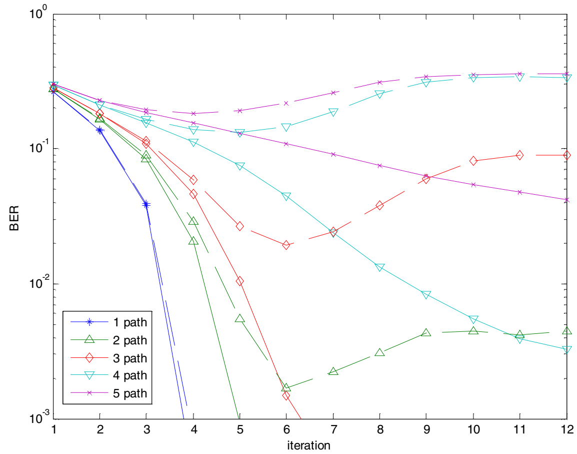

Fig. 4. BER vs number of iteration for different number of paths, channel error standard deviation of 0.1, without noise, 24 users. Solid lines include channel estimation error treatment while dashed lines omit it.

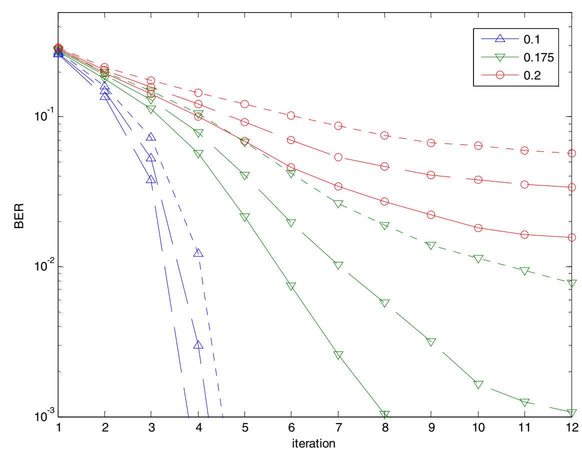

Fig. 6. BER vs number of iteration for different SNR and channel error standard deviation, 24 users, single path. Solid lines are at 20 dB, large dashed lines at 3dB and small dashed lines at 0dB.

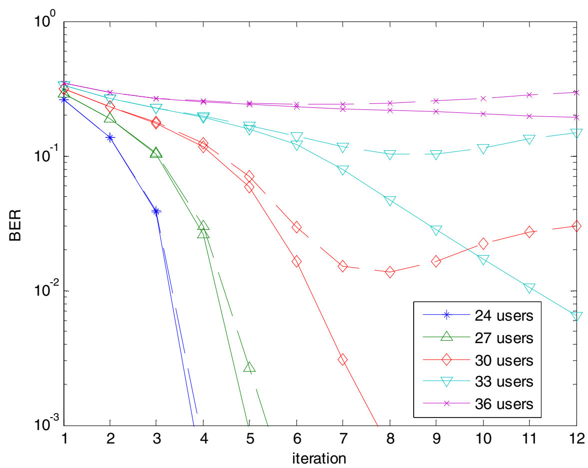

Fig. 5. BER vs number of iteration for different number of users, channel error standard deviation of 0.1, without noise, single path. Solid lines include channel estimation error treatment while dashed lines omit it.

Fig. 3 compares the performances of IDMA whether or not the channel estimation error is considered for different level of errors. It can be seen that IDMA performs much better when channel estimation errors are considered. Not only is the BER systematically lower, the BER decreases monotonically with the number of iterations.

Fig. 4 shows the effect of channel estimation error for multipath channels. The estimation error standard deviation is 0.1 for every path considered. The decrease in the BER when the receiver considers channel estimation errors is greater for a multipath channel.

Fig. 5 shows the performance of the decoder when the number of users is increased. It can be seen that the performance in case of 33 users considering the estimation error are better than a system of 30 users that does not take channel estimation errors into account. We hence have found a gain in the number of users capacity of approximately 10%.

Fig. 6 shows the performance of IDMA with channel estimation error in the presence of noise. The algorithm is shown to converge when noise is added.

V. CONCLUSION

IDMA detector in presence of channel estimation errors and QPSK signaling has been proposed. This detector can tolerate high channel estimation error levels. The advantages of considering the error have been shown. The modification needed for this consideration has been shown to be equivalent to additive noise. This term is equivalent for every user, every path and for in-phase and quadrature components.

REFERENCES

[1] Li Ping; Lihai Liu; Keying Wu; Leung, W.K.; , "Interleave division multiple-access," *Wireless Communications, IEEE Transactions on* , vol.5, no.4, pp. 938- 947, April 2006

[2] Li Ping; Qinghua Guo; Jun Tong; , "The OFDM-IDMA approach to wireless communication systems," *Wireless Communications, IEEE* , vol.14, no.3, pp.18-24, June 2007

[3] Novak, C.; Hlawatsch, F.; Matz, G.; , "MIMO-IDMA: Uplink Multiuser MIMO Communications using Interleave-Division Multiple Access and Low-Complexity Iterative Receivers," *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on* , vol.3, no.,

[4] L. Ping, L. Liu, K. Wu, and W. Leung, "Interleave division multiple access (IDMA) communication systems," in Proc. 3rd International Symposium on Turbo Codes & Related Topics, (Brest, France), pp. 173-180, 2003.

[5] Schoeneich, H.; Hoeher, P.A.; , "Semi-blind pilot-layer aided channel estimation with emphasis on interleave-division multiple access systems," Global Telecommunications Conference, 2005. GLOBECOM '05. IEEE , vol.6, no., pp.5 pp.-3517, 2-2 Dec. 2005

[6] Xiangyun Zhou; Zhenning Shi; Reed, M.C.; , "Iterative Channel Estimation for IDMA Systems in Time-Varying Channels," Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE , vol., no., pp.4020-4024, 26-30 Nov. 2007

[7] Chulhee Jang; Hyunwoo Choi; Jae Hong Lee; , "Novel Detection Algorithm of IDMA System under Channel Estimation Error," Vehicular Technology Conference, 2008. VTC 2008-Fall. IEEE 68th , vol., no., pp.1-5, 21-24 Sept. 2008

[8] Xingzhong Xiong; Jianhao Hu; Feng Yang; Xiang Ling; , "Effect of channel estimation error on the performance of Interleave-Division Multiple Access systems," Advanced Communication Technology, 2009. ICACT 2009. 11th International Conference on , vol.03, no., pp.1538-1542, 15-18 Feb. 2009