Proportional Fair Scheduling for Single-Carrier FDMA in LTE Uplink System

Jeongchan Kim*, Donggeun Kim†, and Youngnam Han*
*Department of Electrical Engineering and Computer Science
Korea Advanced Institute of Science and Technology (KAIST)
Daejeon, Republic of Korea
Email: {monirer, ynhan}@kaist.ac.kr

†Pantech co., ltd
Seoul, Republic of Korea
E-mail: kim.donggeun@pantech.com

***Abstract*—In uplink 3GPP LTE, SC-FDMA has been regarded as an attractive alternative to OFDMA due to lower Peak-to-Average Power Ratio (PAPR) characteristic. However, in order to get the benefit of PAPR, it requires a contiguous assignment of resource blocks to a single user. In this paper, we suggest heuristic algorithm for proportional fair (PF) scheduling which is known to be NP-hard problem. Under the contiguity constraint, conventional heuristic algorithms assign resource block to a user who has the highest metric and keep on doing assignment according to greedy selection but without any examination of prior selected choice. Our work differs from the conventional works in considering user selection based on PF metric difference and comparison of marginal scheduling metric for examination of selected choice. In simulation results, we demonstrated that the proposed scheme shows better fairness in terms of CDF of normalized throughput and Jain's fairness index.**

***Index Terms*—LTE uplink, SC-FDMA, proportional fair scheduling, fairness**

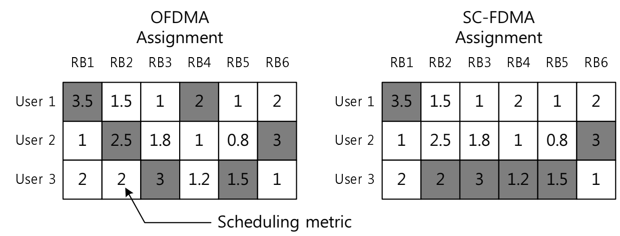

Fig. 1. OFDMA and SC-FDMA Assignment Examples

I. Introduction

Single-carrier Frequency Division Multiple Accessing (SC-FDMA), a modified technology of Orthogonal Frequency Division Multiple Accessing (OFDMA), is a promising technique for higher uplink (UL) data rates. It supports similar throughput and it requires same overall complexity as OFDMA. The important contribution of SC-FDMA is peak-to-average-power ratio (PAPR) characteristic, which is lower than that of OFDMA. Since radio frequency power variation of OFDMA is dramatically high, destructive nature of high envelope fluctuation in time domain makes large PAPR causing energy inefficiency [1]. Thus, 3GPP-Long Term Evolution (LTE) has adopted SC-FDMA as its UL scheme mainly because of the power savings in the mobile handsets.

In 3GPP LTE UL, slot structure is described based on resource grid which consists of resource blocks (RB) and time slots [2], where RB is a basic element for resource scheduling in LTE system and various RB assignment for users can be applied based on scheduling strategies. In OFDMA-based strategy, RBs are allocated to users experiencing better scheduling metric. However, in SC-FDMA, there is a constraint in RB assignment to users, which means that users can not be allocated to separated RBs because of its single carrier property. Therefore, conventional scheduling algorithms for OFDMA can not be directly applicable to SC-FDMA due to its contiguity constraint. For the purpose of maximizing the sum of selected metric in scheduling, as seen in Fig. 1, the optimal solution of OFDMA assignment is to select a user having the highest metric in each RB. However, it is not easy to find an optimal solution for SC-FDMA assignment because of contiguity constraint. This has already been proved as a NP-hard problem [3].

Many heuristic algorithms have been addressed to provide near optimal solution under the contiguity constraint [4]-[7]. Conventional algorithms are proposed based on a well-known greedy allocation strategy, by which they find the best RB-to-user assignment among all pairs of one user and one of legitimate sets of RBs. Concerning a greedy based algorithm with contiguity constraint, [4] and [5] proposed heuristic algorithms. They select pair of a user and a RB having the highest metric value and then look nearby RBs over for next RB assignment. H. Yang et al. investigate frequency-domain packet scheduling (FDPS) problem, where they suggest a universal objective function for the FDPS problem and approximation algorithms based on greedy strategy [6]. Using legitimate sets of contiguous RBs, they divide the FDPS problem into several subproblems according to their profit. Then, they apply a greedy method to each subproblem followed by selecting the best subproblem showing largest total profit. However, above works do not compare the marginal scheduling metric

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of selected user with others as the selected user takes nearby RBs contiguously. So, this implies the possibility of a loss caused by transmit power degradation. In [7], they find a RB which has the highest channel gain and then find a user who can maximize the marginal scheduling metric. However, they do not consider the contiguity constraint in RB allocation.

Our works differ from the conventional works in considering user selection based on PF metric difference and comparison of marginal scheduling metric for examination of selected choice. Our objective to maximize the fairness of SC-FDMA starts from selecting a user who has the best PF metric difference in chunk-to-user domain under contiguity constraint. Then, scheduler examines whether marginal scheduling metric of selected user is still high than others at the chunk. It is observed that the usage of PF metric difference helps to takes right path in greedy selection before the comparison of marginal scheduling metric. Note that the PF metric difference is used as candidate set of greedy algorithm. In simulation results, we demonstrated that proposed schemes shows better fairness in terms of CDF of normalized throughput and Jain's fairness index.

The remainder of this paper is organized as follows. Detailed descriptions of the system model and problem formulation are presented in Section II. Section III describes proposed algorithms. Simulation results are described in Section IV. Concluding Remarks are viewed in Section V.

II. System Model & Problem Formulation

In this paper, SC-FDMA based 3GPP-LTE UL System is considered. We assumed that there are $F \times K$ RBs in frequency domain and K active users in a single cell environment. We assumed that there are K chunks and one chunk consists of F RBs, especially by making the number of chunks same with the number of users. We define the set of all users by N $(N = \{1, 2, ..., K\})$ and the set of all chunks by M $(N = \{1, 2, ..., K\})$. Base station (BS) listens sounding reference signal (SRS) to measure the channel condition from users. It is assumed to be error free, so that scheduling controller performs chunk-to-user assignment efficiently in order to maximize an objective function.

The objective function of PF scheduling for SC-FDMA and basic OFDMA systems are very similar except assignment constraints. In general, the PF algorithm aims to maximize logarithmic utility function $\sum_i \log R_i$, where R_i is the long-term service rate of user i. In order to maximize $\sum_i \log R_i$, as proven by other researches, $\sum_i d_i(t)/R_i(t)$ should be maximized where $d_i(t)$ is instant total data rate of user i at time slot t. $d_i(t)/R_i(t)$ is called as PF metric of user i at time slot t.

Based on the PF criteria, the objective function of SC-FDMA can be built to make it briefly and focusing on scheduling criteria as follows:

Maximize

$$\sum_{(i,a)\in N\times A} w_{i,a} \sum_{n\in a} \frac{d_{i,n}(t)}{R_i(t)} \tag{1}$$

subject to

$$\sum_{i\in N, a:c\in a} w_{i,a} \leq 1 \text{ for each chunk c } \in M,$$

$$\sum_{a\in A} w_{i,a} \leq 1 \text{ for each user } i \in N$$

$$w_{i,a} \in 0, 1 \text{ for } i \in N, a \in A$$

where A is the collection of all sets of contiguous chunks, which means that A is a subset of power set of M, where $\forall a \in A$, $a = \{n, n+1, ..., n+l\}, 1 \leq n \leq n+l \leq K$. The first constraint is that no more than one user is allowed to transmit on the same chunk, and the second constraint ensure that no more than one set of contiguous chunks is assigned to each user. For the assigned users, $R_i(t)$ is updated every time slot using following equation.

$$R_i(t) = (1-\alpha)R_i(t-1) + \alpha D_i(t) \tag{2}$$

On the contrary, $R_i(t)$ of not-assigned users is updated as follow:

$$R_i(t) = (1-\alpha)R_i(t-1) \tag{3}$$

where α is the window length of PF and $D_i(t)$ is the received data rate of user i through the assigned chunks at time slot t, which could be described using the shannon's capacity as follow:

$$D_i(t) = \sum_{n\in I_i} B_n \log_2 \left(1 + \frac{|H_{n,i}(t)|^2}{B_n N_0} \frac{P_{\max}}{|I_i|}\right) \tag{4}$$

where we define the set of assigned contiguous chunks a of user i as I_i and $|\cdot|$ means cardinality. B_n is bandwidth of nth chunk, $H_{n,i}$ is the channel of user i at nth chunk, N_0 is noise power density and $P_{\max}$ is maximum UL power for data transmission. We assumed that total UL power for data transmission is distributed equally only to the set of assigned chunks I_i.

III. Proposed Algorithms

In SC-FDMA, the optimal solution of PF scheduling with contiguous assignment constraint is known as a NP-hard problem [3]. It requires exhaustive search and it is not effective for practical systems. Also, conventional heuristic approaches based on the greedy algorithm inherited a critical problem of the greedy algorithm in getting optimal solution. They select certain choice too early without examination of prior path which prevent them from finding the best later [8].

In this section, we introduce a heuristic algorithm which keeps track of PF metric differences at a chunk to obtain the set of assigned chunks I_i in (4). As seen Fig. 2-(a), conventional resource allocation based on greedy algorithm peaks up a pair of user and chunk having the highest metric in chunk-to-user domain. Then, to ride peak metric value contiguously, scheduler expands chunk assignment in neighbor chunk which has next high metric value. However, our proposed algorithm selects a pair of user and chunk having the highest PF metric difference as Fig. 2-(b).

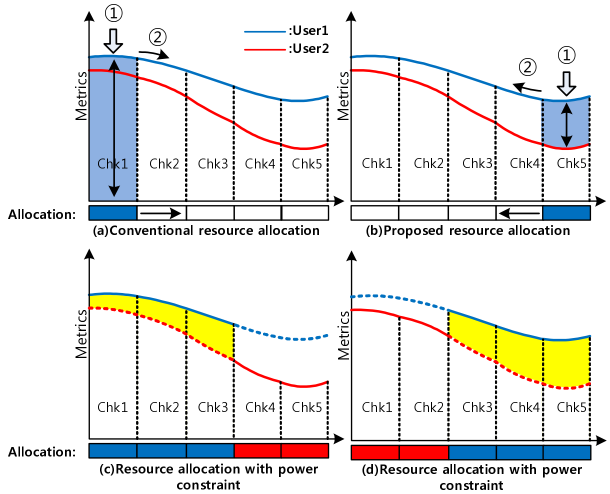

Fig. 2. OFDMA and SC-FDMA Assignment Examples

Because PF scheduler calculates the expected data rates per chunk in PF metric by using predefined power (in this paper, the transmission power of SRS is used). It can differ with actual data rate with transmission power per assigned chunk (refer to (4)). It means that as one user takes several chunk contiguously, the actual transmission power per assigned chunk could be lower than the predefined power because of its channel condition. So, PF scheduler should consider comparison of marginal scheduling metric to fulfill expected data rate per chunk in doing chunk assignment. The usage of PF metric difference helps to take a right path in greedy selection while comparing marginal scheduling metric. For example, let's assume that user1 take no more than three chunks among five chunks. It comes from the fact that actual transmission power involved in more three chunks can not fulfill the requirement power for expected rates. Also, if user2 takes remaining two chunks, there are two assignment cases as described in Fig. 2-(c) and (d). The result (c) is obtained by applying conventional greedy algorithm as shown in Fig. 2-(a). It is found that (d) shows the best performance in terms of sum of metric. The results (d) can be obtained by assigning chk5 to user1 first and keep assignment going as described in Fig. 2-(b).

The SRS power at nth chunk of ith user is described as follows:

$$P^{\text{SRS}}_{n,i} = \frac{P^{\text{SRS}}_{\max}}{K}, \forall n \ (n \in 0, 1, .., K) \tag{5}$$

where $P_{\max}$ denotes maximum SRS power and (5) means $\sum_n P^{\text{SRS}}_{n,i} = P_{\max}$. The PF metric of a chunk is obtained under the assumption of equal bit equal power allocation to all chunks. Using Shannon's capacity formula, the PF metric of user i at nth chunk at time t can be derived from (1) as follows:

$$\eta_{n,i}(t) = \frac{B_n}{R_i(t)} \log_2 \left(1 + \frac{|H_{n,i}(t)|^2 P^{\text{SRS}}_{n,i}}{B_n N_0}\right) \tag{6}$$

Then, the PF metric difference at nth chunk is described as follows:

$$\Delta_{n,\mu(n,p_n)}(t) = \eta_{n,\mu(n,p_n)}(t) - \eta_{n,\mu(n,p_n+1)}(t) \tag{7}$$

where $\mu(n, p_n)$ is a user having p_nth PF metric at nth chunk. So, $\Delta_{n,\mu(n,p_n)}$ means relative gain over user $\mu(n, p_{n+1})$ in terms of PF metric. At first, the scheduler chooses a user i^* who has the highest PF metric difference involving $p_n = 1$, $\forall n$ among all users as follows,

$$i^* = \mu(n^*, p_n) \tag{8}$$

where

$$n^* = \arg\max_n \Delta_{n,\mu(n,p_n)}(t),$$
$$\mu(n, p_n) \in U$$

where U denotes the set of active users. If the assigned set of user i^* is empty, then the user i^* is allocated to n^*th chunk without any comparison (i.e $w_{i^*,n^*} = 1$). In the next scheduling step, if another chunk is assigned to the same user with contiguity constraint violation (i.e. the selected chunk is separated from already assigned chunks), scheduler updates the priority p_{n^*} of n^*th chunk so that the same chunk assignment to the user could be ignored in next assignment step. On the other hand, if the user is allocated to n^*th chunk adjacent to I_{i^*}, especially I_{i^*} is not empty, scheduler compares marginal scheduling metric of the user with others at the chunk.

The reason why we consider of the comparison of marginal scheduling metric is because the actual transmission power per chunk could be lower than the SRS power for PF metric value. As shown in (4), as new assigned chunk is included into the assigned set I_{i^*}, actual transmission power per chunk is decreased by the size of I_i. Therefore, the new assigned chunk to user i^* requires to be compared with others in terms of marginal scheduling metric. Our proposed algorithm checks whether the marginal scheduling metric of selected user is still higher than that of others at each scheduling step.

We define the set of available chunks $C_i(t)$ as follow:

$$C_i = \begin{cases} n^* \bigcup I_i & \text{if } I_i \neq \phi \text{ and } \varphi_i = 1 \\ \phi & \text{if } I_i \neq \phi \text{ and } \varphi_i = 0 \\ n^* & \text{if } I_i = \phi \end{cases} \tag{9}$$

where φ_i is the contiguity indicator when $I_i \neq \phi$, which is described as follow:

$$\varphi_i = \begin{cases} \prod_{j=\{n^*, n^*+1, \ldots, I_i^{(t)}\}} w_{i,j} & \text{if } n^* < I_i^{(h)} \\ \prod_{j=\{I_i^{(h)}, I_i^{(h)}+1, \ldots, n^*\}} w_{i,j} & \text{if } I_i^{(t)} < n^* \end{cases} \tag{10}$$

where $I_i^{(h)}$ is the head value of I_i and $I_i^{(t)}$ is the tail value of I_i. For example, if $I_i = \{3, 6\}$, then $I_i^{(h)} = \{3\}$ and $I_i^{(t)} = \{6\}$. Using (9), the PF metric of user i for comparison of marginal scheduling metric becomes as follow:

$$\hat{\eta}_{n^*,i}(t) = \frac{B_{n^*}}{R_i(t)} \log_2 \left(1 + \frac{|H_{n^*,i}(t)|^2 \hat{P}_i}{B_{n^*} N_0}\right) \tag{11}$$

where,

$$\hat{P}_i = \begin{cases} \frac{P_{\max}}{|C_i|} & \text{if } C_i \neq \phi \\ 0 & \text{if } C_i = \phi \end{cases}$$

Then, scheduler compares $\hat{\eta}_{n^*,i^*}(t)$ with $\hat{\eta}_{n^*,j}(t),\ j \neq i^*$ to identify that assigning n^* to i^* is still adaptable even after comparing marginal scheduling metric. If $\hat{\eta}_{n^*,i^*}(t) \geq \hat{\eta}_{n^*,j}(t), \forall j$ but $j \neq i^*$, then assigning n^* to i^* is acceptable. Detailed procedures are illustrated in Table I.

TABLE I
PROPOSED SCHEDULING ALGORITHM.

Let S be the set of not-yet-assigned chunks.
Let U be the set of users.
Let I_i be the set of assigned chunks of user i, $i \in U$.
Set $I_i = \phi,\ \forall i$ and $p_n \leftarrow 1, \forall n$.
while $S \neq \phi$ **do**
  $flag = 0$.
  while $flag = 1$ **do**
    obtain n^* and i^* using Eq. (8).
      if $I_{i^*} = \phi$ **then**
        $w_{i^*,n^*} = 1$ (i.e. n^* become a element of I_{i^*})
        $S = S - \{n^*\}$, $p_n \leftarrow 1$, $flag = 1$
      else if $I_{i^*} \neq \phi$ and $\varphi_{i^*} = 1$ **then**
        calculate $\hat{\eta}_{n^*,i^*}(t)$ and $\hat{\eta}_{n^*,j}(t),\ j \neq i^*$ using (9), (11)
        if $\hat{\eta}_{n^*,i^*}(t) \geq \hat{\eta}_{n^*,j}(t),\ j \neq i^*, \forall j$ **then**
          $w_{i^*,n^*} = 1$ (i.e. n^* become a element of I_{i^*})
          $S = S - \{n^*\}$, $p_n \leftarrow 1$, $flag = 1$
        else
          $p_{n^*} \leftarrow p_{n^*} + 1$
        end if
      else if $I_{i^*} \neq \phi$ and $\varphi_{i^*} = 0$ **then**
        $p_{n^*} \leftarrow p_{n^*} + 1$
      end if
  end while
end while

IV. Simulation and Results

The Monte-Carlo simulation is considered to get simulation results from 10 drop trials. User distribution is changed every drop and 3,000 sub-frame is generated per drop. The results from each drop is assumed to be independent. The coverage of a BS is 1 km radius. For the channel generation, we refer to 3GPP TR 25.814 for pathloss. A lognormal distribution with zero mean and 8dB standard deviation is assumed for shadowing and $\mathcal{CN}(0,1)$ is assumed for the Rayleigh fading. Full instantaneous channel conditions(i.e. perfect channel estimation) are assumed at the scheduler. The window length α is assumed 1000 in the simulation. We assumed that SRS is transmitted from all users every time slots. Table II includes the details of the simulation parameters.

We use the normalized cumulative distributive function (CDF) of throughput per user, which is described in WiMAX standard [9], to analyze the fairness of heuristic algorithms. The normalized throughput of user i is defined as follow:

$$N_i = \frac{D_i}{\frac{1}{K}\sum_{i=1}^{K} D_i} \tag{12}$$

where D_i is the average of $D_i(t)$ denoting the received data rate of user i during long service time. Another method of

TABLE II
SIMULATION PARAMETERS

Parameter	Description/Assumption
Target System	Uplink SC-FDMA
Number of Cell	1
Transmission bandwidth(MHz)	10
Mobile Velocity	3km/h
Sub-frame Duration (ms)	1 ms
Number of Resource Block	96
Cell-level Uuser Distribution	Uniform
Number of Active Users in Cell	4, 8, 12, 16, 24, 32
Traffic Model	Full buffer
RB Mapping Type	Localized Type
Antenna Configurations	single antenna transmission and receiver
Pathloss Model	128.1 + 37.6*log10(R in km)
Shadowing Model	lognormal distribution
Fading Model	Rayleigh fading
Channel Estimation	Perfect channel estimation

fairness assessment is the Jain's fairness index proposed in [10] where the fairness index is given by:

$$F = \frac{\left|\sum_{i=1}^{K} D_i\right|^2}{K \cdot \sum_{i=1}^{K} D_i^2} \tag{13}$$

In the simulation results, optimal solution is not included. For the performance comparison, we adopt the schemes proposed in [5] as conventional algorithms. Basically, the first one is the carrier-by-carrier in turn algorithm (C-by-C) and the second one is riding-peaks algorithm.

In the C-by-C algorithm, the first chunk is assigned to a user who has the highest PF metric. In next scheduling step, scheduler find a user having the highest PF metric at next chunk. This process keeps going on until last chunk is assigned. In the riding-peaks algorithm, all PF metrics are sorted in descending order. Then, given that the contiguity constraint is satisfied, a user having highest PF metric is allocated to the involved chunk. If not satisfied, the user index is removed from user list for assignments. This process keeps going on until all chunks are assigned.

First, we show Fig. 3 to compare the proposed algorithm with others in terms of system throughput. In general, all heuristic algorithms exploit multiuser diversity gain as the number of user increases. It is observed that proposed algorithm and riding peaks are so fine-tuned enough to exploit multiuser diversity effectively. In [5], the author demonstrated that the riding peaks seek to take advantage of each users' peak so that it could be best in the system throughput. However, proposed algorithm shows the best system data-rate at less than 8 user while the riding-peaks achieves the best performance over 12 user.

Fig. 4 describes CDF of normalized throughput in order to compare the fairness performance. The proposed algorithm shows the best fairness among all heuristic algorithms. This means that the proposed algorithm improves the average data

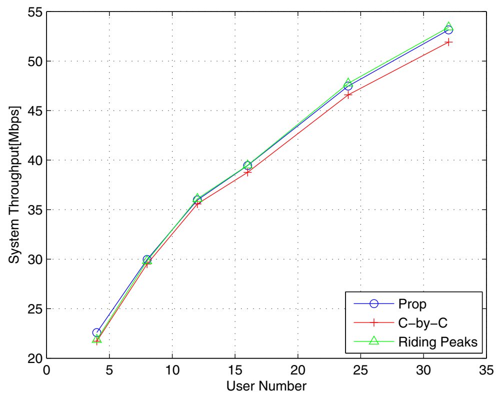

Fig. 3. System Throughput

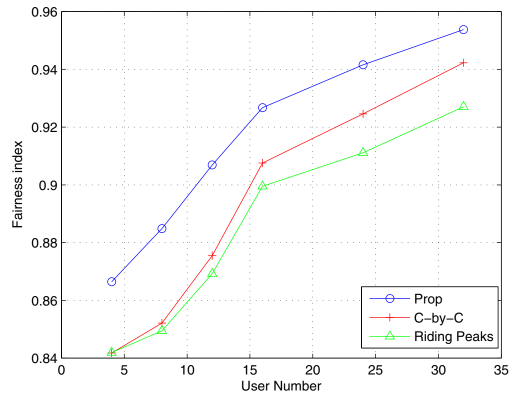

Fig. 5. Jain's fairness index

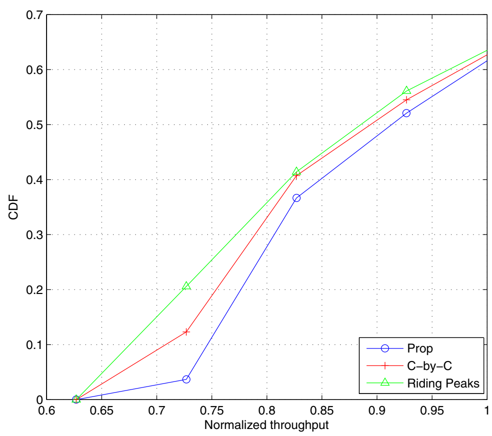

Fig. 4. CDF of normalized throughput with K=24 (zoomed in)

rate of users under weak channel condition, to upward. The performance improvement comes from the fact that the proposed scheduler takes near optimal choice in greedy selection by adopting PF metric difference and comparing of marginal scheduling metric. Interestingly, C-by-C shows better fairness than riding-peaks on the contrary of results in [5]. Actually, [5] does not apply the chunk-based assignment to C-by-C. When adopting the chunk-based assignment, C-by-C shows more favorable performance than riding-peaks. In Fig. 5, the difference between the proposed algorithm and conventional algorithms is found in terms of Jain's fairness index. Moreover, C-by-C shows higher fairness than riding-peaks as shown in Fig. 4. Specifically, as the number of user increases, it is observed that the performance gap between C-by-C and riding-peaks increases. From the results in Fig. 4 and 5, we concluded that the proposed algorithm shows the best fairness performance.

V. Conclusion

In this paper, we proposed PF scheduling algorithm in SC-FDMA based on user selection with PF metric difference and comparison of marginal scheduling metric. Our objective to maximize the fairness of SC-FDMA starts from selecting a user who has best PF metric difference in chunk-to-user domain under contiguity constraint. Then, scheduler examines whether the marginal scheduling metric of selected user at a chunk is still high than others. From simulation results, it is observed that proposed algorithm is well-suited for fairness scheduling in SC-FDMA.

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