Exact Analytical Solution for Dual-Hop and Opportunistic Dual-Hop AF Relaying Systems

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***Abstract*—Novel exact closed-form expressions are derived for the probability density function (PDF) and the cumulative distribution function (CDF) of the instantaneous received end-to-end signal-to-noise ratio (SNR) of dual-hop amplify-and-forward (AF) relaying systems operating over Rayleigh, Nakagami-*m* and Rician fading channels. New exact closed-form expressions are also obtained for opportunistic dual-hop AF relaying systems, with maximum relay-to-destination SNR relay selection. The average symbol error probability, ergodic capacity and outage probability are calculated using the derived PDF and CDF expressions. It is found that the opportunistic dual-hop AF system, with relay selection pool size $M = 2$, has at least $2.84\,dB$ power advantage over the dual-hop AF system without relay selection for average error probabilities less than 10^{-2} in the case of Rayleigh fading links. It is shown that although the performance of a dual-hop AF system with maximum relay-to-destination SNR relay selection is improved by increasing the selection pool size, the improvement has diminishing returns and a relay selection pool of more than 6 relays is not of practical benefit.**

***Index Terms*—Amplify-and-forward, average symbol error probability, cooperative networks, dual-hop relaying, ergodic capacity, opportunistic relaying, outage probability, relay selection.**

I. INTRODUCTION

Cooperative relaying networks are receiving intense study for their significant capabilities in enhancing system capacity and in providing diversity gain in wireless networks [1]–[9]. The most common relaying techniques are amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying. Much research has focused on the performance of AF relaying systems because of its lower complexity. The average error probability and outage probability of dual-hop transmission were studied in [1] for the case of Rayleigh fading channels. The authors proposed the harmonic mean bound of the end-to-end signal-to-noise ratio (SNR). In [2], the harmonic mean bound was used to obtain an approximate expression for the moment generating function (MGF) of the reciprocal of the end-to-end SNR for multihop transmission over Nakagami-*m* fading channels, which in turn was used to obtain an approximate outage probability of the system. However, it has been found that performance results based on the direct harmonic mean bound of the individual per hop instantaneous SNRs, which was studied in many publications, such as [3], [4], are neither tight for small-to-moderate values of SNR, nor for Nakagami-*m* fading channels for large values of *m*. In [5], [6], a new analytical approach, the generalized transformed characteristic function (GTCF), was proposed to obtain the first exact results for the average error probability, outage probability and ergodic capacity for multihop AF relaying systems operating over general fading channels. In related work in [7], the dual-hop AF system with selection diversity was studied and an exact closed-form expression for the outage probability was derived for the case of Nakagami-*m* fading links with integer values of m, yet that expression was not used to obtain other performance metrics.

In [8], [9], the authors studied relay selection to enhance the performance of AF relaying systems. They used a distributed relay selection method based on measurements of the relay-to-destination channel conditions, yet exact results for the performance metrics of such systems operating over Nakagami-*m* fading channels were not derived.

In this paper, we derive exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of dual-hop AF relaying systems operating over Rayleigh, Nakagami-*m* and Rician fading channels. We then use these expressions to obtain exact closed-form solutions for the outage probability and exact single integral solutions for the average symbol error probability and ergodic capacity. These results represent the first exact results for dual-hop AF relaying systems. The most important differences between the work presented here and that presented in [7], [10] are: 1) In [7], [10], the authors did not consider the important case of Rician fading channels, although the Rician fading model is one of the most practical fading models. 2) In [7], [10], the authors did not obtain exact results for the average symbol error probability nor for the ergodic capacity.

In this paper, we also derive exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of opportunistic dual-hop AF relaying systems with maximum relay-to-destination SNR relay selection for the case of Nakagami-*m* fading links. Using these expressions, we obtain the first exact results for the average error probability, outage probability and ergodic capacity of these systems. These results represent major contributions because "exact" results for the performance metrics of opportunistic systems operating over Nakagami-*m* fading links have never been reported in the literature.

Another important contribution of the paper is that we compare systems employing relay selection to systems where no relay selection is adopted. This comparison is important

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because it shows the benefits gained from exhausting some of the communication resources in relay selection. We also study quantitatively the effect of increasing the number of relays in the relay selection pool on the performance metrics in opportunistic relaying systems.

II. System And Channel Models

In dual-hop AF relaying systems, the source node, S, and the destination node, D, communicate through one intermediate relay node, R. In the variable gain AF relaying technique, the relay amplifies the received signal using amplification factor, $A=\sqrt{\frac{P_1}{P_0|\alpha_1|^2+N_{0_1}}}$ where P_0 and P_1, are the transmitter powers at the source and the relay, respectively, and where $\alpha_i,\ i=1,2$, and $N_{0i},\ i=1,2$, are respectively the fading gain of the i^{th} link, and the noise power at the i^{th} node. It is assumed that orthogonal half-duplex operation is implemented to avoid inter-signal interference. The exact instantaneous end-to-end SNR at the destination, γ_t, is given by [1, eq. (5)] as,

$$\gamma_t=\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}. \tag{1}$$

where $\gamma_i=\frac{P_{i-1}}{N_{0_i}}|\alpha_i|^2$ represents the instantaneous received SNR on the i^{th} link. When the PDF of the instantaneous end-to-end SNR, γ_t, is available, different system performance metrics can be evaluated using standard methods found in [11].

III. Dual-Hop AF Relaying

In this section, we study the performance of a dual-hop AF relaying system without relay selection over general fading links. The instantaneous end-to-end received SNR (1) can be used for direct exact integral evaluation of the performance metrics of dual-hop AF systems. To see this, let $g_{\gamma_t}(\gamma_t)$ represent a performance metric; for example, $g_{\gamma_t}(\gamma_t)$ may represent the ergodic capacity for which $g_{\gamma_t}(\gamma_t)=\frac{1}{2}\log_2(1+\gamma_t)$ in [11, eq. (15.21)], or it may represent the average error probability for which $g_{\gamma_t}(\gamma_t)=b\,Q\left(\sqrt{a\,\gamma_t}\right)$ in [11, eq. (5.1)]. Then any end-to-end performance measure can be evaluated as

$$\int_0^\infty\int_0^\infty g_{\gamma_t}\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}\right)f_{\gamma_1}(\gamma_1)\,f_{\gamma_2}(\gamma_2)\,d\gamma_1\,d\gamma_2. \tag{2}$$

The two-fold integral in (2) can be used with any arbitrary $f_{\gamma_1}(\gamma_1)$ and $f_{\gamma_2}(\gamma_2)$ to obtain the ergodic capacity and the average error probability. To the best of the authors' knowledge, this evaluation method, although simple and direct, is not addressed in any of the related literature. In the worst case, the evaluation of the two-fold integral can be performed numerically using common software packages. Note that numerical calculation of a two-fold integral is widely practised in the literature [11]. Although the expression in (2) can be used as a "brute force" method to get the performance metrics of the dual-hop AF relaying system, further analytical solution is possible and in the following, we develop a general framework to find the PDF, $f_{\gamma_t}(\gamma_t)$, of the instantaneous end-to-end SNR.

We start by obtaining the PDF of the instantaneous end-to-end SNR conditioned on the second link's SNR. With some mathematical manipulation, and according to a theorem on transformations of a random variable [12, Ch. 7], the PDF of $\gamma_t|\gamma_2$ can be obtained in terms of the PDF of γ_1 as,

$$f_{\gamma_t|\gamma_2}(r)=\frac{\gamma_2(\gamma_2+1)}{(\gamma_2-r)^2}\,f_{\gamma_1}\left(\frac{(\gamma_2+1)r}{\gamma_2-r}\right),\ 0\leqslant r\leqslant\gamma_2. \tag{3}$$

The PDF of γ_t can be then found as

$$f_{\gamma_t}(r)=\int_r^\infty\frac{\gamma_2(\gamma_2+1)}{(\gamma_2-r)^2}f_{\gamma_1}\left(\frac{(\gamma_2+1)r}{\gamma_2-r}\right)f_{\gamma_2}(\gamma_2)\,d\gamma_2. \tag{4}$$

We can also find the outage probability by substituting (4) into [11, eq. (15.6)] to obtain, after some mathematical manipulation,

$$\begin{aligned}P_{out}=F_{\gamma_t}(\gamma_{th})&=F_{\gamma_2}(\gamma_{th})\\&+\int_{\gamma_2=\gamma_{th}}^\infty F_{\gamma_1}\left(\frac{(\gamma_2+1)\gamma_{th}}{\gamma_2-\gamma_{th}}\right)f_{\gamma_2}(\gamma_2)\,d\gamma_2.\end{aligned} \tag{5}$$

The expressions in (4) and (5) can be used with any arbitrary $f_{\gamma_1}(\gamma_1)$ and $f_{\gamma_2}(\gamma_2)$ to obtain, respectively, the PDF and the CDF of the instantaneous end-to-end SNR.

For Nakagami-*m* fading links, the PDF of γ_t is obtained by substituting the PDF of the Nakagami-*m* fading distribution [11, eq. (2.21)] into (4) resulting in

$$f_{\gamma_t}(r)=C_g\,I_1(r;m_1-1,m_2-1,\frac{m_1}{\overline{\gamma}_1},\frac{m_2}{\overline{\gamma}_2}) \tag{6a}$$

where $C_g=\left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1}\frac{1}{\Gamma(m_1)}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2}\frac{1}{\Gamma(m_2)}$, and where $I_1(r;\alpha,\beta,\theta,\phi)$ is given as

$$\begin{aligned}I_1(r;\alpha,\beta,\theta,\phi)&=2\,\exp\left[-(\theta+\phi)\,r\right]\,r^{(\alpha+\beta)}\\&\times\sum_{k=0}^{\alpha}\sum_{j=0}^{\beta}{}^{\alpha}C_k{}^{\beta}C_j\left(\frac{\theta}{\phi}\right)^{\frac{j-k}{2}}\left(\frac{r+1}{r}\right)^{\frac{j+k}{2}}\\&\times\Bigg[\sqrt{\frac{\theta}{\phi}\,r(r+1)}\,K_{j-k+1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right)\\&+(2r+1)\,K_{j-k}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right)\\&+\sqrt{\frac{\phi}{\theta}\,r(r+1)}\,K_{j-k-1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right)\Bigg]\end{aligned} \tag{6b}$$

where $K_\nu(\cdot)$ is the ν^{th}- order modified Bessel function of the second kind [13, eq. (9.6.2)] and where ${}^{\alpha}C_\beta$ is the binomial coefficient. The full derivation of $I_1(r;\alpha,\beta,\theta,\phi)$ must be omitted here due to space limitations.

For Rician fading links, we use the infinite series representation [13, eq. (9.6.10)] for the Bessel function $I_0(\cdot)$, namely, $I_0\left(2\sqrt{\frac{K_i(1+K_i)}{\overline{\gamma}_i}x}\right)=\sum_{n=0}^\infty a_i(n)x^n$ where $a_i(n)=\frac{1}{(n!)^2}\left(\frac{K_i(1+K_i)}{\overline{\gamma}_i}\right)^n$. Substituting this representation with the Rician PDF into (4), it can be proved that the PDF of the end-to-end SNR in the case of Rician fading links is obtained as

$$\begin{aligned}f_{\gamma_t}(r)=C_r\sum_{n=0}^\infty\sum_{m=0}^\infty a_1(n)a_2(m)\\\times I_1\left(r;n,m,\frac{1+K_1}{\overline{\gamma}_1},\frac{1+K_2}{\overline{\gamma}_2}\right)\end{aligned} \tag{7}$$

where $C_r = \left(\frac{1+K_1}{\overline{\gamma}_1}\right)\left(\frac{1+K_2}{\overline{\gamma}_2}\right) e^{-(K_1+K_2)}$. Note that although the expression in (7) involves double nested infinite summations, the summands decay (slightly faster than) exponentially with increase of n and m, because of the $\frac{1}{(n!)^2}$ and $\frac{1}{(m!)^2}$ factors. Note that Stirling's approximation specifies that $n!$ grows as $e^{n\ln n}$, and hence $\frac{1}{(n!)^2}$ decays as $e^{-2n\ln n}$. As a result, a truncated summation with a finite number of terms will achieve a required accuracy. Owing to this rapid rate of convergence, a careful empirical test for convergence is allowable and also for truncation error estimation. Estimation of the truncation errors will be shown through the numerical examples.

Note that the PDF of the end-to-end SNR in the case of Rayleigh fading channels can be obtained by substituting $m_1 = m_2 = 1$ into (6) or by substituting $K_1 = K_2 = 0$ into (7). Note also that the same procedure can be followed as well for the case when the two link fadings are distributed according to different fading families, Rayleigh, Nakagami-*m* and Rician, because the PDF of the instantaneous end-to-end SNR can be expressed in terms of $I_1(r;\alpha,\beta,\theta,\phi)$, and hence can be obtained in closed-form.

Next, we use the single-fold integral (5) to obtain the CDF of the instantaneous end-to-end SNR of dual-hop AF relaying systems. For Nakagami-*m* fading links, we substitute the PDF and the CDF of Nakagami-*m* fading channels [11] into (5). We use the finite series representation of the lower incomplete gamma function, namely, $\frac{\Gamma_{inc}\left(m_1, \frac{m_1}{\overline{\gamma}_1} r\right)}{\Gamma(m_1)} = 1 - \exp\left[-\frac{m_1}{\overline{\gamma}_1} r\right] \sum_{n=0}^{m_1-1} \frac{1}{n!}\left(\frac{m_1}{\overline{\gamma}_1} r\right)^n$, and after some mathematical manipulation we get

$$F_{\gamma_t}(r) = 1 - \sum_{n=0}^{m_1-1} C_g(n) I_2(r; n, m_2-1, \frac{m_1}{\overline{\gamma}_1}, \frac{m_2}{\overline{\gamma}_2}) \quad \text{(8a)}$$

where $C_g(n) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^n \frac{1}{n!}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)}$, and where $I_2(r;\alpha,\beta,\theta,\phi)$ is given as

$$\begin{aligned} I_2(r;\alpha,\beta,\theta,\phi) &= 2\,\exp\left[-(\theta+\phi)\,r\right]\, r^{(\alpha+\beta+1)} \\ &\times \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} {}^{\alpha}C_k\, {}^{\beta}C_j \left(\frac{\theta}{\phi}\right)^{\frac{j-k+1}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}} \\ &\times K_{j-k+1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right). \quad \text{(8b)} \end{aligned}$$

A similar expression was presented in [7, eq. (12)] to obtain the outage probability of dual-hop AF systems with path selection diversity. However, this expression was not used in [7] to obtain the system performance metrics of the variable-gain AF relaying system.

For Rician fading links, we use the infinite series representation of the first-order Marcum Q-function in [11, eq.(4.35)], in addition to the infinite series representation of the modified Bessel function of the first kind in [13, eq.(9.6.10)], and after some mathematical manipulation, we get the CDF of the end-to-end SNR as

$$\begin{aligned} F_{\gamma_t}(r) = 1 - e^{-(K_1+K_2)} &\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} C_r(l,n,m) \\ &\times I_2(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}) \quad (9) \end{aligned}$$

where $C_r(l,n,m) = \frac{{K_1}^{l+n}\, {K_2}^{m}}{(l+n)!\, n!\, (m!)^2}\left(\frac{1+K_1}{\overline{\gamma}_1}\right)^n\left(\frac{1+K_2}{\overline{\gamma}_2}\right)^{m+1}$. Although the expression in (9) involves nested infinite summations, the summands decay rapidly with increase of l, n and m, because of the $\frac{1}{(l+n)!\, n!\, (m!)^2}$ factor. The rate of decay is $e^{-(l+n)\ln(l+n)-n\ln n-m\ln m}$ and hence a truncated summation with a finite number of terms will achieve required accuracy. Note that the Rayleigh fading distribution can be considered a special case of either the Nakagami-*m* fading distribution or the Rician fading distribution.

IV. Relay selection based on Maximum Relay-to-destination SNR

In this section, we study the opportunistic dual-hop AF relaying system, in which the intermediate relaying node is selected from a pool of M available relays. We assume that the relay selection follows a maximum SNR policy such that the relay which maximizes the instantaneous relay-to-destination SNR is considered the best node to relay the data signal [8], [9], i.e.

$$\max_{R_m \in \{R_1, \cdots, R_M\}} \gamma_{R_m - D} \Rightarrow \gamma_2 \quad (10)$$

where γ_{R_m-D} is the instantaneous SNR of the $R_m - D$ link. Accordingly, the PDF of the second hop can be obtained using order statistics [12, p. 192] as

$$f_{\gamma_2}(r) = M f_{\gamma_n}(r)\left[F_{\gamma_n}(r)\right]^{M-1}. \quad (11)$$

In order to get the PDF of the instantaneous end-to-end SNR of the opportunistic dual-hop AF relaying system, we substitute (11) and the Nakagami-*m* fading statistics into (4). After some mathematical manipulation, we obtain the PDF of γ_t, in the case of Nakagami-*m* fading links for integer values of m_1 and m_2, as

$$\begin{aligned} f_{\gamma_t}(r) &= M C_g \sum_{l=0}^{M-1} {}^{M-1}C_l\,(-1)^l \sum_{n_1=0}^{m_2-1}\cdots\sum_{n_l=0}^{m_2-1} h_g(n_i) \\ &\times I_1\left(r; m_1-1,\, m_2-1+\sum_{i=1}^{l} n_i,\, \frac{m_1}{\overline{\gamma}_1},\, \frac{m_2}{\overline{\gamma}_2}(1+l)\right) \quad (12) \end{aligned}$$

where $h_g(n_i) = \left(\frac{m_2}{\overline{\gamma}_2}\right)^{\sum_{i=1}^{l} n_i} \frac{1}{\prod_{i=1}^{l} n_i!}$ and where C_g and $I_1(r;\alpha,\beta,\theta,\phi)$ were defined in the previous section. To the best of the authors' knowledge, (12) represents the first exact closed-form solution for the end-to-end SNR of dual-hop AF relaying with relay selection over Nakagami-*m* fading links.

For the CDF of the instantaneous end-to-end SNR of the opportunistic dual-hop AF relaying systems, we follow similar steps as those leading to (8) in order to get

$$F_{\gamma_t}(r) = 1 - M \sum_{n=0}^{m_1-1} C_g(n) \sum_{l=0}^{M-1} {}^{M-1}C_l\,(-1)^l \times \sum_{n_1=0}^{m_2-1} \cdots \sum_{n_l=0}^{m_2-1} h_g(n_i) \times I_2\left(r; n,\, m_2 - 1 + \sum_{i=1}^{l} n_i,\, \frac{m_1}{\overline{\gamma}_1},\, \frac{m_2}{\overline{\gamma}_2}(1+l)\right) \quad (13)$$

for integer values of m_1 and m_2, where $C_g(n)$ and $I_2(r;\alpha,\beta,\theta,\phi)$ were defined in the previous section. The expression in (13) represents the first exact closed-form solution for the outage probability of opportunistic dual-hop AF relaying over Nakagami-*m* fading links.

V. Numerical Examples

In this section, we study the behaviours of dual-hop AF relaying systems without relay selection, and opportunistic dual-hop AF relaying systems with relay selection. Figs. 1 and 2 show, respectively, the average symbol error probability versus the link average SNR, $\overline{\gamma}$, and outage probability versus the threshold SNR, γ_{th} for dual-hop AF relaying systems without relay selection. Different cases of identically distributed and non-identically distributed Nakagami-*m* and Rician fading channels are considered. Binary phase shift keying (BPSK) is assumed. We assume also an uniform power allocation policy, that is the total available power, P, is evenly allocated to the source and the relays. Without loss of generality, we assume equal noise powers, N_0, at all the nodes. Note that for the cases of Rician fading links, each of the infinite summations in (7) and (9) was truncated at the 20^{th} term. It was found that adding more terms does not affect the result in the 10^{th} decimal figure.

The figures show precise agreement between the analytical results and simulation results at all values of SNR, for all the different performance measures. The outage probability is obtained directly using (8) and (9) for Nakagami-*m* and Rician fadings, respectively. The ergodic capacity and the average symbol error probability are obtained by substituting (6) and (7) into [11, eq. (15.21)] and [11, eq. (5.1)], respectively. Note that most of the state-of-the-art results report performance bounds and approximations, rather than exact results, and comparisons with those results are made in the full paper. It can be noted first that the system performance improves for the less severe Rician and Nakagami-*m* fading channels, as expected. It is observed also that the limiting slopes of the average error probability curves and the outage probability curves in the case of Rician fading are the same as the limiting slopes for Rayleigh fading links, irrespective of the value of the fading parameter K. On the other hand, the limiting slopes of the average symbol error probability and outage probability curves are proportional to the value of the fading parameter m in the case of Nakagami-*m* fading links. This results in a considerable SNR gain for systems subject to Nakagami-*m* fading links over those subject to Rician or Rayleigh fading links for sufficiently large SNR. For example, an average

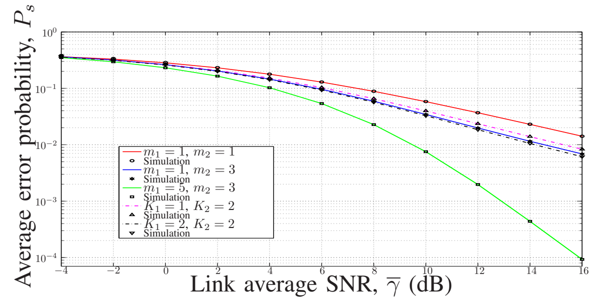

Fig. 1. The average error probability for a dual-hop AF relaying systems with BPSK modulation.

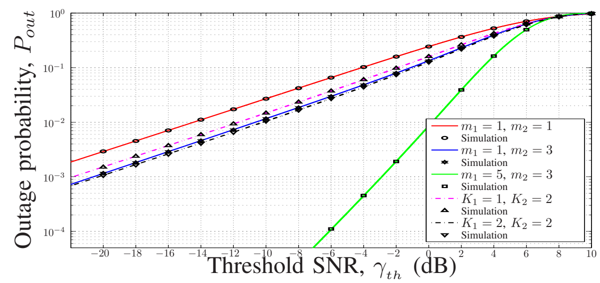

Fig. 2. Outage probability for a dual-hop AF systems with $\overline{\gamma} = 10\,dB$.

error probability of 10^{-2} is achieved by Rician fading links ($K_1 = 1,\ K_2 = 2$) for $\overline{\gamma} = 15.28\,dB$. The same average error probability occurs at $\overline{\gamma} = 14.19\,dB$ and $\overline{\gamma} = 9.48\,dB$ for Rician fading links ($K_1 = K_2 = 2$) and for Nakagami-*m* fading links ($m_1 = 5,\ m_2 = 3$), respectively. However, the limiting slope is affected only by the minimum value of the fading parameters m_1 and m_2. For example, It can be seen in Fig. 2 that the limiting slope of the outage probability curves in the case of $m_1 = m_2 = 1$ is the same as in the case of $m_1 = 1$ and $m_2 = 3$. The same observation can be drawn from Fig. 1. This indicates that the system performance is more affected by the "weakest" link in the dual-hop path.

The next example set compares the dual-hop AF systems without relay selection to an opportunistic dual-hop AF system where the intermediate relay is selected from $M = 2$ relays based on a maximum relay-to-destination SNR relay selection policy. Figs. 3 and 4 show the performance metrics for the mentioned AF systems. It is observed that the opportunistic

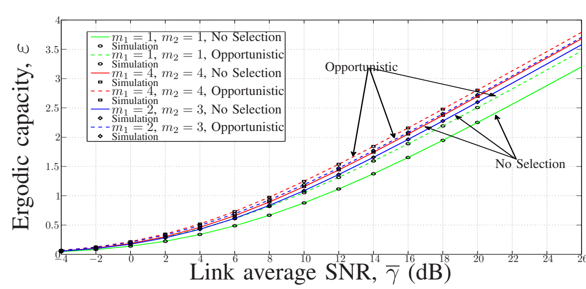

Fig. 3. The ergodic capacity for a dual-hop (without relay selection) and opportunistic dual-hop AF relaying systems.

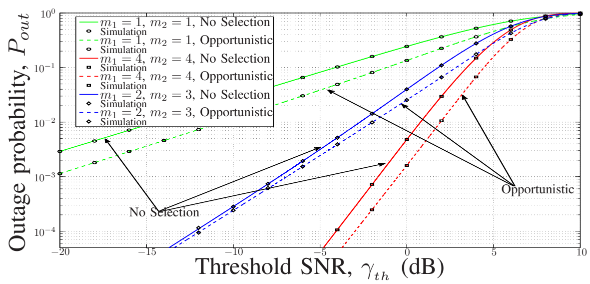

Fig. 4. Outage probability for a dual-hop (without relay selection) and opportunistic dual-hop AF relaying systems with $\overline{\gamma} = 10\,dB$.

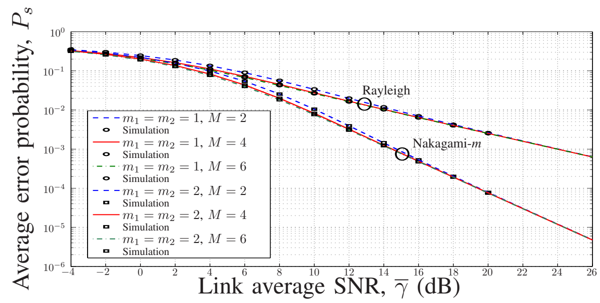

Fig. 5. The average error probability for opportunistic dual-hop AF relaying systems with BPSK modulation.

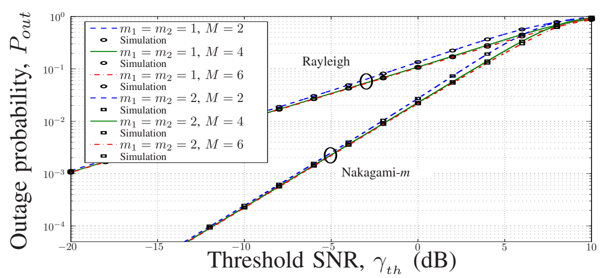

Fig. 6. Outage probability for opportunistic dual-hop AF relaying systems with $\overline{\gamma} = 10\,dB$.

dual-hop AF system, with a selection pool size as small as $M = 2$ relays, has higher ergodic capacity than a dual-hop AF system without relay selection. Note that for both systems the bandwidth/time resource is divided into 2 orthogonal subchannels to avoid inter-signal interference. Fig. 4 also shows interesting observations in the comparison between both systems. As expected, the opportunistic dual-hop AF system has better outage probability performance than the dual-hop AF systems without relay selection, for all the different cases of Nakagami-*m* fading channels. For example, in the case of Rayleigh fading channels, an outage probability of 10^{-2} occurs at $\gamma_{th} = -14.5\,dB$ in the case of the system with no relay selection, while it occurs at $\gamma_{th} = -10.7\,dB$ in the case of the relay selection system. For Nakagami-*m* fading channels, with $m_1 = m_2 = 4$, an outage probability of 10^{-4} occurs at $\gamma_{th} = -2.8\,dB$ in the case of the relay selection system, compared to $\gamma_{th} = -4.1\,dB$ in the case of no relay selection. However, it is noted that the limiting slopes of the average error probability and the outage probability curves are the same for both the dual-hop AF system without relay selection and the opportunistic dual-hop AF system. This means that the gain of the opportunistic dual-hop AF system over the dual-hop AF system without relay selection is limited to a SNR gain and no diversity gain is obtained.

In the last set of examples we study the effect of increasing the size of the relay selection pool, M, in opportunistic dual-hop AF relaying. Figs. 5 and 6 show respectively the average error probability and the outage probability of opportunistic dual-hop AF systems with $M = 2, 4$ and 6. Identical Rayleigh fading channels and Nakagami-*m* fading channels with $m = 2$ are considered. It is observed that increasing the pool size, M, improves the performance metrics, as expected. However, this improvement is subject to diminishing returns even when additional independent links are available in the relay pool. For example, at $\overline{\gamma} = 10\,dB$, the average error probability for Rayleigh fading links is 3.37×10^{-2} for $M = 2$, and 2.63×10^{-2} for $M = 6$. The same occurs for the outage probability. For example, 10^{-3} outage probability, in case of the Nakagami-*m* fading channels, occurs at $\gamma_{th} = -6.97\,dB$ for $M = 2$, while it occurs at $\gamma_{th} = -6.74\,dB$ for $M = 6$. It can be concluded that there is a limited practical benefit from increasing the selection pool size beyond, say $M = 6$.

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