Quadratic Estimation of Success Probability of Greedy Geographic Forwarding in Unmanned Aeronautical Ad-hoc Networks

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***Abstract*—Due to the availability of location information in unmanned aerial vehicles (UAVs), we propose to use geographic routing mechanisms as a core forwarding protocol in unmanned aeronautical ad-hoc networks (UAANETs) for the purpose of reducing routing overhead. As a result, this paper investigates the performance of the core forwarding mechanism i.e. the greedy geographic part. Since the forwarding mechanism for dynamic UAANETs with many statistical inter-dependencies is complex, a closed-form model does not exist. Therefore, a quadratic polynomial estimation is proposed for computing the success probability of greedy geographic forwarding based on the results of a set of realistic Monte Carlo simulations. This mathematical model can later be used to predict and evaluate the performance of other greedy-based geographic routing protocols for UAV applications.**

Keywords–Geographic routing, greedy forwarding, Mobile Ad-hoc Network (MANET), quadratic modeling, source-destination (SD) communication, Unmanned Aerial Vehicle (UAV), Unmanned Aeronautical Ad-hoc Network (UAANET).

I. INTRODUCTION

An UAANET consists of a number of cooperative UAVs (typically in the range from 10 to 25). The UAVs collaborate with each other with the goal of decreasing mission delay and increasing reliability in highly critical aerial operations [1]. Keeping a team of UAVs connected while retaining certain formation imposes some specific characteristics on the trajectories of each individual UAV. This mobility characteristic is referred to as swarming in the literature [2]. In this paper, a group of UAVs participating in a swarming application forms what we call a flocking UAANET. In a flock of UAVs, all nodes need to follow certain rules to maintain the overall network connectivity among all UAVs.

Most UAVs nowadays are equipped with a global positioning system (GPS) for acquiring their current geographic information (i.e. coordinates, velocity, etc.). In particular, since UAVs have a good line-of-sight to the GPS satellite signals, they have access to location information more conveniently, compared to some terrestrial applications. The availability of accurate location information makes it possible to exploit the geographic routing mechanisms as the core communication protocol for UAANETs, i.e., taking advantage of the availability of location information of UAV in designing routing protocols for UAANETs [3]. As in terrestrial applications, the potential advantages of geographic routing in UAANETs may include scalable packet forwarding and a low latency in routing.

Recently, many geographical routing techniques have been developed for ad-hoc network applications. They use greedy geographic forwarding when a neighbor node can be found that is closer to the final destination [4]. In general, when the average number of neighbors is large, successfully establishing greedy routes from each source to different destinations is highly probable. In other words, if the average number of neighbors is high enough, it is highly likely to find a greedily better (i.e. geographically closer) neighbor node to a destination node. If no closer neighbor node can be found, then a fallback mechanism such as face routing needs to be deployed. In this paper, we only focus on the greedy geographic forwarding of geographical routing. We investigate the performance of geographic routing for UAANETs, in particular, how greedy geographic forwarding performs for sparse UAANETs. The dynamic nature of UAANETs imposes a lot of complexity on the analysis of the routing protocols. In a previous paper [5], we used Monte Carlo simulations to examine the performance of greedy geographic forwarding in sparse UAANETs. Simulation results showed that greedy geographic forwarding is suitable for non-critical UAANET applications, where UAVs can tolerate a certain level of packet dropouts. However, for applications that require guaranteed delivery, greedy geographic forwarding alone cannot provide the paths and alternative mechanisms need to be combined.

The rest of the paper is organized as follows. In Section II, related work on geographic routing protocols and challenges in employing such routing protocols are discussed. Section III describes the simulation environment and scenarios. In Section IV, we discuss the topological features of the UAANETs to motivate the necessity of an estimation model. The quadratic model for the success probability of greedy geographic forwarding is then derived in Section V. Finally, conclusions are presented in Section VI.

II. Related Work

Geographic routing was introduced to provide a scalable routing capability even in highly dynamic mobile ad-hoc networks (MANETs) by exploiting location information of mobile nodes [4]. Many of the papers published in the literature on geographic routing protocols use deterministic graph theory. The most commonly used graph models in geographical routing are: Gabriel graph, relative neighborhood graph, and circular neighborhood graph [6]. In general, there are two types of geographic routing approaches: beacon-based and beacon-less. In beacon-based geographic approaches, each node periodically broadcasts beacons containing its location information to its neighbors. Beacon-less routing protocols, on the other hand, do not require a periodic exchange between neighboring nodes, resulting in less control overhead [6]. Instead of sending periodic beacon messages, a node only acquires neighbors' location information reactively (when a node has a data packet to send). Thus, only those nodes that participate in a routing task consume resources [7].

The main research topic in beacon-less geographic routing has been focused on the problem of providing guaranteed delivery. Two different solutions are available in the literature, which are based on the analysis of message complexity of beacon-less face routing. For example, in [6], the use of a circular neighbor graph is proposed as an alternative to the Gabriel and the relative neighborhood graph. However, the problem of guaranteed delivery for beacon-less routing is beyond the scope of this paper, and will be explored in separate studies in the context of UAANETs. In this paper, we will focus on the statistical characteristics of the network architecture, which is an important aspect that deterministic graph models have been unable to address.

Geographic routing protocols have also been proposed for sparsely connected networks. In particular, geographic routing has been proposed to work together with the Store-Carry-Forward (SCF) procedures for delay tolerant networks [8]. In [8], a geographic routing algorithm, called LAROD (Location Aware Routing for Opportunistic Delay Tolerant Networks), was introduced for intermittently connected MANETs. LAROD is a geographic beacon-less routing algorithm based on the SCF principle. The UAV that holds the packet (i.e. custodian) uses greedy packet forwarding if there are other UAVs nearby. The custodian will have to know that the forwarded packet has been received by other UAVs. If several nodes in the forwarding area receive the packet, then the first expired-timer node is selected as the next forwarder to rebroadcast the packet. Custody of the packet is relinquished when the custodian overhears the transmission by other UAVs.

In [9], a statistical method is proposed for analyzing the geographic random forwarding (GeRaF) technique. In [9], nodes are assumed to be distributed in a region according to a Poisson distribution. It follows that the remaining distance to the destination would have an exponential distribution. Based on the analysis, a multi-hop mechanism is proposed to ensure that the best node is always chosen in the ideal case. However, the results in [9] are specific to the GeRaF forwarding mechanism. In this paper, we propose a quadratic estimation of success probability of greedy geographic forwarding in UAANETs. The proposed estimation method is a general solution and can be applied to all greedy-based geographic routing mechanisms.

A statistical evaluation of the properties of greedy geographic forwarding in spontaneous wireless mesh networks was presented in [10]. In the proposed architecture, node distribution is assumed to follow a Poisson distribution. Similar to the analysis in [9] the authors evaluated the amount of progress in each step. Although the use of the Poisson assumption provides tractable analytical formulations, the results do not seem to be realistic.

In [5], we evaluated the performance of greedy geographic forwarding in UAANETs by using Monte Carlo simulation. In this paper, we expand the work in [5] by proposing a mathematical formulation that can be used to predict the success probability of greedy geographic forwarding.

III. Monte Carlo simulation

The scenarios considered in this paper are similar to those in [5]. UAVs are assumed to have some degree of randomness to wander around while the flocking concept is satisfied. Such a flocking UAANET can be considered as a semi-random sparse network.

The Monte Carlo simulations proposed in [5] for evaluating the performance of greedy geographic forwarding in UAANETs can be described by the flow chart. In the simulations, one snapshot of the network is assumed containing a source UAV (S) and a destination UAV (D). The network operation area is assumed to be a square with length $L = 100$. Each UAV is assumed to have a fixed transmission range denoted by T_r. Note that L and T_r are both unitless. It will be shown later that this unit-less environment does not affect the generality of the results which only involve the ratio of T_r/L. Also, please note that we consider a unit disc model for the transmission range. Since the focus of the paper is on the forwarding strategy, we assume that lower layers handle channel impairments.

The algorithm generates a flock of UAVs with a constant average number of neighbors, and runs the greedy geographic forwarding on the generated network. In the first step, N UAVs are uniformly distributed in the simulation environment. In the second step, the transmission range for each UAV is selected based on the distance between each pair of UAVs. Since our goal is to evaluate the performance of the network as a function of the average number of neighbors, we need to select the right value of transmission range to have the desired average number of neighbors. To achieve that, we first find the M shortest links for each UAV. A distance matrix D is defined, in which the ijth element, $D(i, j)$, is the Euclidean distance between UAV i and j. Then, all the distances from UAV i to all other UAVs

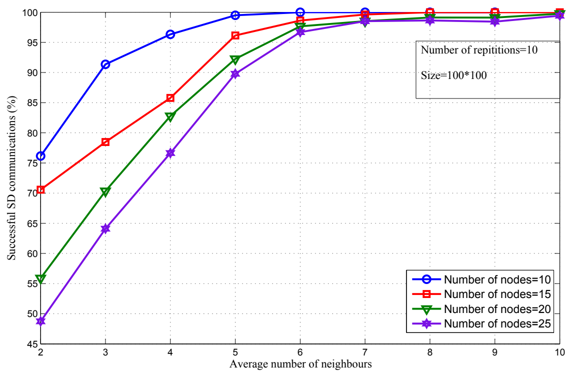

Fig. 1. Successful SD communications vs the average number of neighbors [5]

are sorted in an ascending order in an array called the distance vector. In the next step, the first M entries of the vector are selected as the M shortest links in the network. Since each link is connecting 2 UAVs, the corresponding average number of neighbors per UAV is equal to $2M/N$. After generating a network with the desired average number of neighbors per UAV, we check if the network is connected in the third step of the algorithm, i.e., no UAVs are isolated from the UAANET. In the fourth step, the greedy geographic forwarding is applied to the connected UAANET. By applying the greedy geographic forwarding, we evaluate how many source-destination (SD) communication pairs would be successful. Finally, in the fifth step, the percentage of successful SD communications is plotted.

In order to obtain averaged results, 10 different networks are simulated. The average number of neighbors is initially set to 2, which is close to the minimal average number of neighbors that is required to have a connected UAANET. Figure 1 shows the variation of the percentage of SD pairs for which greedy geographic forwarding would successfully allow communication versus the average number of neighbors using a Monte Carlo simulation. In the simulation, the average number of neighbors increases from 2 to 10 for scenarios in which $10, 15, 20$, and 25 UAVs are used respectively. For the network of 10 UAVs, the percentage of successful SD pairs increases from almost 75% to almost 100% when the average number of neighbors increases from 2 to 9 (note that when $N = 10$, the maximum number of neighbors is 9). As the number of neighbors increases, the percentage of successful SD communications also increases. Another observation from Figure 1 is that when the average number of neighbors is small, the percentage of SD communications can go as low as 48% (in the case of 25 UAVs). Note that current UAANET operation concepts are conceived to have relative small connectivity level with the average number of neighbors typically less than 5. As a result, in the next section, we will focus on UAANETs containing an average number of neighbors between 2 and 5.

IV. Discussion on the Topological Features of the UAANET

In this section, we discuss why evaluating the performance of greedy geographic forwarding in UAANETs requires an estimation technique. Trajectory design and relative position of UAVs in an UAANET are important facts for mission optimization of aeronautical applications. From the point of view of wireless networking among UAVs, the probability density function of instantaneous UAV positions is important in evaluating the delivery ratio of the routing protocol. For example, if the UAVs are normally distributed around the central point of the area, the network will be more likely to have a connected cluster in the central area, compared to the case where the nodes are uniformly distributed in the whole area. Therefore, considering the right distribution of nodes in the area is a crucial task in modelling UAANETs. The other important factor in modelling UAANETs is the concept of flocking. As explained in Section I, a flocking UAANET consists of several moving UAVs. Each UAV has some degree of freedom to move around as long as the flock is kept connected. Inspired by flocking mobility in nature, many state-of-the-art UAV applications require a network with such an architecture.

In this paper, UAVs' locations are derived via a uniform distribution, as explained in Section III. However, we select some specific samples of the distribution to generate a connected UAANET. Hence, the probability distribution of the final random variable for representing a flock of UAVs is not uniform anymore. Due to the fact that we drop some of the samples of the originally uniform distribution, the probability distribution function for instantaneous UAV positions as a flock is neither uniform nor independent[1]. Despite the fact that the performance of greedy geographic forwarding has been formulated in [11] for independently distributed uniform nodes, the results cannot be expanded for the case of an UAANET, because of the non-uniform interdependencies available in a flock of UAVs. Likewise, deriving a closed-form mathematical formula for success probability is not easy due to dependent non-uniform effects among UAVs. As a result of the complexity in deriving a closed-form formula for success probability, in the rest of this paper, our approach is to estimate the success probability of greedy geographic forwarding based on the results of several sets of Monte Carlo simulations.

V. Quadratic Estimation of Success Probability

In this section, based on the simulation data, a quadratic function for computing the probability of success is proposed. Denote $P_{success}$ as the success probability of greedy geographic forwarding and h as the number of hops. We use quadratic polynomials to fit the simulation data. The coefficients of the fitting quadratic polynomials are determined by minimizing the sum of the squared errors between the

[1]Please note that dependent UAV position is the direct result of keeping the UAANET connected. In order to have a connected flock, an UAV cannot independently go towards a direction without considering other UAVs' mobility.

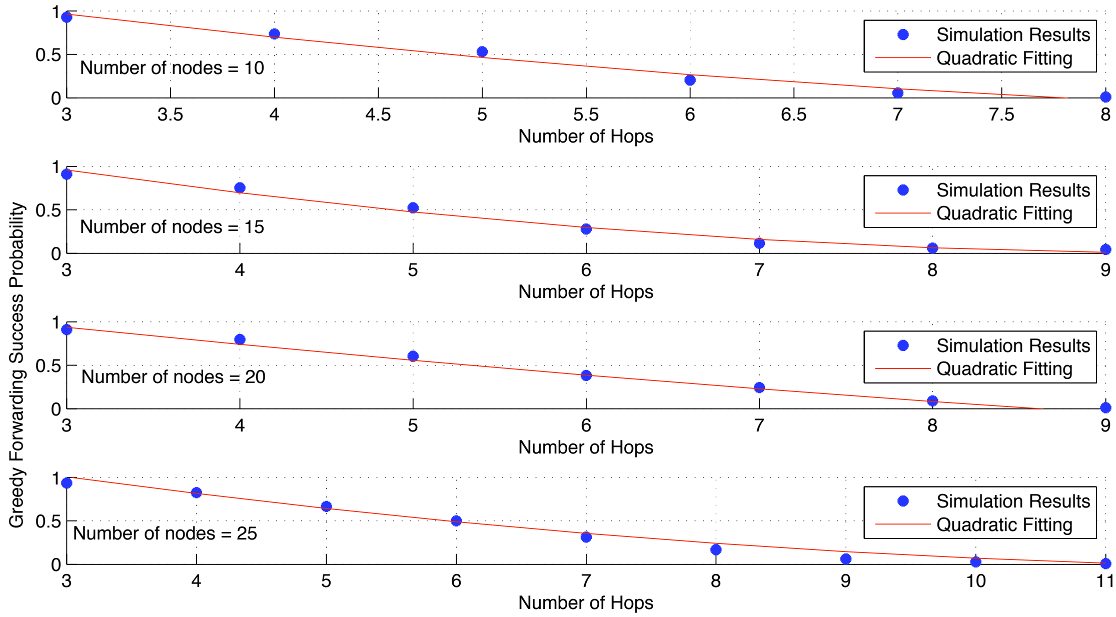

Fig. 2. Greedy geographic forwarding success probability (for number of UAVs=10, 15, 20, and 25)

function and the Monte Carlo simulation data. Goodness of fit of each of the estimated functions, which has a value in the interval [0,1], is also calculated based on the definition in [12].

When the number of nodes is 10, the greedy forwarding success probability can be fitted to the quadratic function shown in Equation (1) and plotted in the top curve of Figure 2. All curves in Figure 2 were generated by averaging the probability of success for 4 different simulations in which the average number of neighbors was varied from 2 to 5.

$$P_{success} \approx 0.0172h^2 - 0.387h + 1.97 \qquad 3 \leq h \leq 7 \quad (1)$$

The goodness of fit for Equation (1) is 0.9924. In order to have a wider observation of the greedy geographic success probability for different number of hops, the following approximation function can be used:

$$P_{success} \approx \begin{cases} 1 & h < 3 \\ 0.0172h^2 - 0.387h + 1.97 & 3 \leq h \leq 7 \\ 0 & h > 7 \end{cases} \quad (2)$$

In Equation (2), please note that for number of hops equal to 1 or 2, the probability of success is 1 due to the fact that in a connected network, greedy geographic forwarding is always successful for one and two hop neighbors.

The polynomial estimation for the success probability of greedy geographic forwarding for number of UAVs equal to 15 is shown in Equation (3). The goodness of fit for this curve is 0.9947. Monte Carlo simulation results and equivalent fitting function are depicted in the second curve of Figure 2.

$$P_{success} \approx 0.021h^2 - 0.41h + 2 \qquad 3 \leq h \leq 9 \quad (3)$$

Equivalently, we can come up with the following piecewise function:

$$P_{success} \approx \begin{cases} 1 & h < 3 \\ 0.021h^2 - 0.41h + 2 & 3 \leq h \leq 9 \\ 0 & h > 9 \end{cases} \quad (4)$$

The greedy geographic success probability for a network of 20 UAVs can be estimated by the quadratic polynomial shown in Equation (5). The goodness of fit for the curve is also 0.9972. Monte Carlo simulation results and values of the quadratic fitting function are plotted in the third curve of Figure 2.

$$P_{success} \approx 0.0063h^2 - 0.24h + 1.6 \qquad 3 \leq h \leq 8 \quad (5)$$

Similarly, Equation (5) can be expanded to include the number of hops from 3 to 8 as:

$$P_{success} \approx \begin{cases} 1 & h < 3 \\ 0.0063h^2 - 0.24h + 1.6 & 3 \leq h \leq 8 \\ 0 & h > 8 \end{cases} \quad (6)$$

Finally, for a UAANET containing 25 UAVs, the quadratic fitting function, with a goodness of fit equal to 0.996, is derived as:

$$P_{success} \approx 0.0097h^2 - 0.26h + 1.7 \qquad 3 \leq h \leq 11 \quad (7)$$

The Monte Carlo simulation results and values computed using Equation (7) are plotted in the bottom curve of Figure 2.

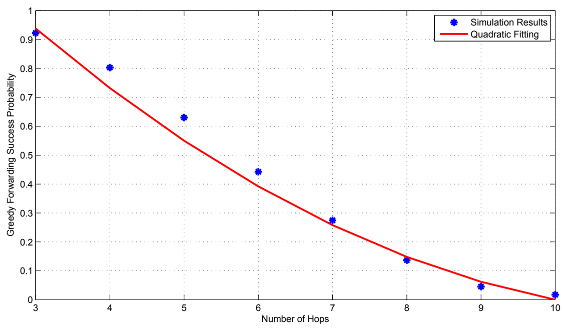

Fig. 3. Average success probability of greedy geographic forwarding

Similarly, for an arbitrary h, we have the following piecewise approximation:

$$P_{success} \approx \begin{cases} 1 & h < 3 \\ 0.0097h^2 - 0.26h + 1.7 & 3 \leq h \leq 11 \\ 0 & h > 11 \end{cases} \quad (8)$$

In order to have a good understanding of the behaviour of the success probability of greedy geographic forwarding, we compute the overall probability of success versus the number of hops and average them over the number of hops.

$$P_{success} \approx 0.012h^2 - 0.29h + 1.7 \qquad 3 \leq h \leq 10 \quad (9)$$

Equation (9) is the estimated function for the overall value for the probability of success versus the average number of hops. The goodness of fit for this function is 0.9958. In Equation (10), the approximated $P_{success}$ is shown, and its values versus the number of hops are plotted in Figure 3.

$$P_{success} \approx \begin{cases} 1 & h < 3 \\ 0.012h^2 - 0.29h + 1.7 & 3 \leq h \leq 10 \\ 0 & h > 10 \end{cases} \quad (10)$$

It is worth-noting that $P_{success}$ in Equation (10) is estimated for number of hops from 3 to 10 for UAANETs that contain either 10, 15, 20, or 25 UAVs. These are typical values that are used in UAV missions that require a team of UAVs.

As indicated in Equation 10, the success probability of greedy geographic forwarding is 1 for the one and two hop neighbors. For $h \geq 3$, it is shown that the constant term of the quadratic equation is larger than the other coefficients. In other words, the constant term is dominant in shorter hop routes. As h increases, other terms play a more significant role. This is also shown in Figure 3, where the success probability degrades to less than 50% for 6-hop neighbors. One idea that arise here is to use the greedy geographic forwarding in shorter routes while the destination is not more than 2-3 hops away. In such a case, the success probability of greedy geographic forwarding is either 1 or at least more than 0.9.

VI. Conclusions and Future Work

In this paper, based on the Monte Carlo simulations, quadratic functions were proposed to approximate the probability of successful SD communication pairs when greedy geographic forwarding is used in UAANETs. It was observed that deriving closed-form solutions for such a probability is not easy due to the degree of dependence in a flock of UAVs participating in a cooperative UAANET mission. As a result, a Monte Carlo methodology was used that involves approximation using quadratic function for the success probability.

For applications requiring guaranteed delivery, greedy geographic method alone may not be a preferred solution. However, the method could potentially be used as a core forwarding mechanism and combined with other routing/forwarding strategies. Future work will be focused on combining the greedy geographic forwarding with a reactive routing procedure, which aims at reducing the routing delay and improving the overall network throughput for UAANETs.

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References

[1] E. Frew and T. Brown, "Airborne communication networks for small unmanned aircraft systems," *Proceedings of the IEEE*, vol. 96, no. 12, pp. 2008–2027, Dec. 2008.

[2] J. Li, Y. Zhou, and L. Lamont, "Analysis of the impact of swarm mobility on performance of routing protocols in manets," in *Performance Computing and Communications Conference (IPCCC), 2009 IEEE 28th International*, Dec. 2009, pp. 193–199.

[3] E. W. Frew and T. X. Brown, "Networking issues for small unmanned aircraft systems," *J. Intell. Robotics Syst.*, vol. 54, no. 1-3, pp. 21–37, 2009.

[4] M. Mauve, A. Widmer, and H. Hartenstein, "A survey on position-based routing in mobile ad hoc networks," *Network, IEEE*, vol. 15, no. 6, pp. 30–39, 2001.

[5] R. Shirani, M. St-Hilaire, T. Kunz, Y. Zhou, J. Li, and L. Lamont, "The performance of greedy geographic forwarding in unmanned aeronautical ad-hoc networks," in *Communication Networks and Services Research Conference (CNSR), 2011 Ninth Annual*, May 2011, pp. 161–166.

[6] S. Ruhrup, H. Kalosha, A. Nayak, and I. Stojmenovic, "Message-efficient beaconless georouting with guaranteed delivery in wireless sensor, ad hoc, and actuator networks," *Networking, IEEE/ACM Transactions on*, vol. 18, no. 1, pp. 95–108, 2010.

[7] J. Sanchez, P. Ruiz, and R. Marin-Perez, "Beacon-less geographic routing made practical: challenges, design guidelines, and protocols," *Communications Magazine, IEEE*, vol. 47, no. 8, pp. 85–91, Aug. 2009.

[8] E. Kuiper and S. Nadjm-Tehrani, "Geographical routing in intermittently connected ad hoc networks," in *Advanced Information Networking and Applications - Workshops, 2008. AINAW 2008. 22nd International Conference on*, 2008, pp. 1690–1695.

[9] M. Zorzi and R. Rao, "Multihop performance of geographic random forwarding for ad hoc and sensor networks," in *Global Telecommunications Conference, 2003. GLOBECOM '03. IEEE*, vol. 7, 2003, pp. 3948–3952.

[10] E. Schiller, P. Starzetz, F. Theoleyre, and A. Duda, "Properties of greedy geographical routing in spontaneous wireless mesh networks," in *Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE*, 2007, pp. 4941–4945.

[11] R. Shah, A. Wolisz, and J. Rabaey, "On the performance of geographical routing in the presence of localization errors," in *IEEE International Conference on Communications, ICC 2005*, vol. 5, May 2005, pp. 2979–2985.

[12] "Matlab curve fitting toolbox," [Accessed: August 2011]. [Online]. Available: http://www.mathworks.com/help/toolbox/curvefit/