Game Theory Based Power Allocation Algorithm in High-Speed Mobile Environment

Lina Mao[1,2], Shaoyi Xu[1,2], Tianhang Fu[1,2], Qing Huang[1]

[1] Research Institute of Broadband Wireless Mobile Communications, School of Electronics and Information Engineering
Beijing Jiaotong University, Beijing, China

[2] State Key Laboratory of Integrated Services Networks, Xidian University

E-mail: 10120132@bjtu.edu.cn, shyxu@bjtu.edu.cn, 07274004@bjtu.edu.cn, qhuang1@bjtu.edu.cn

***Abstract*——Recently, the high-speed railway has developed rapidly. In a wireless fading channel, sharing the same physical resources, user equipments (UEs) with high speed (HUE) and UEs in a macro cell (MUE) interfere with each other. A power allocation scheme in high-speed mobile environment is considered to suppress such interference. Specifically, our power allocation scheme aims at maximizing the effective capacity of users. Besides, each user tries to compete for these resources to satisfy its own quality of service (QoS) guarantee. In order to handle the selfish behaviors of two users, the power allocation problem for the MUE and the HUE is modeled as a non-cooperative game. Then the existence of the Nash Equilibrium of the game is established, as well as its uniqueness. Simulation results show our power allocation scheme has a better performance.**

Keywords- effective capacity; game theory; quality of service (QoS) guarantees; power allocation

I. INTRODUCTION

More and more people are choosing to travel by high-speed railway systems due to their low carbon footprint. With the development of railway, the velocities of high-speed trains will reach 350~580km/h. In order to develop the communication system under high-speed mobile environment, various system architectures are proposed. Moreover, distinct network structures corresponding to different architectures cause different resource allocation problems. In this paper, we focus on the system based on the mobile relay (MR) as shown in Fig.1. In detail, high-speed user equipments (HUE) communicate with the base station (BS) by the help of the mobile relay (MR). Meanwhile, user equipments (UEs) in a macro cell (MUE) communicate directly with BS. In addition, MR is a Type 1 relay which is part of Long Term Evolution (LTE)-Advanced system [1]. A Type 1 relay is characterized by the following properties: (1) a Type 1 relay shall have its own Physical Cell Identity and the relay node shall transmit its own synchronization channels, reference symbols. (2) In the context of single-cell operation, the UE shall receive scheduling information and Hybrid Automatic Repeat Request (HARQ) feedback directly from the relay node and send its control channels to the relay node. Moreover, the MR can be considered as a base station. However, radio resources are scarce so that it is necessary to share the same resources for different links. Therefore, such links will interfere with each other.

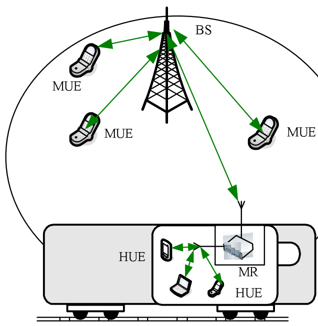


Fig.1. Illustrative example of high-speed mobile system architecture

Quality-of-service (QoS) guarantee plays an important role in mobile wireless networks, especially in high-speed mobile environment. Depending on their distinct QoS requirements, differentiated mobile users satisfy their own QoS constrains so as to tolerate different levels of delay. Moreover, coexisting wireless communication links often compete for these resources to guarantee their own reliable communication in a selfish way for their own QoS requirements. In addition, the non-cooperative game has emerged as a very powerful tool because it provides a convenient framework to study interactions among self-interested individuals. In a fading multiple-access channel (MAC), increasing the transmit power can improve the system performance, but also has the undesirable effect like increasing the interference to other users. Therefore it is rational and effective to realize power allocation using non-cooperative game to obtain a suitable tradeoff between the performance and the interference. In this paper, we adopt the non-cooperative game to handle the interference.

Most of the previous papers are based on information theory, with the purpose of allocating power correctly for maximizing the channel capacity. In [3], the author investigated MAC capacity through a non-cooperative water-filling game theoretic approach. Moreover, the authors of [5] showed that the optimal power and rate control policy which can maximize spectral efficiency is the so-called water-filling

This work was supported in part by the National Natural Science Foundation of China with Grant 61101138 and 61032002, Important National Science & Technology Specific Projects of China with Grant 2010ZX03003-001-02 and the Fundamental Research Funds for the Central Universities with Grant 2012JBM007.

978-1-4673-1881-5/12/$31.00 ©2012 IEEE

algorithm. According to a mass of related literatures, water-filling algorithm is a better algorithm for power allocation from the information-theoretic viewpoint. However, it is important to note that Shannon theory does not place any restrictions on the delay QoS constraints. In terms of QoS guarantees [8], water-filling algorithm is not always optimal one. Consequently, it is necessary to take the delay QoS constraints into account when applying the information theory.

Here, we focus on the power allocation in high-speed mobile environment such as Fig.1. As a result, the effective capacity, which characterizes the capability of the wireless channels to support data transmission subject to the statistical delay QoS constraints, is brought into the high-speed mobile system. In a MAC, users sharing the same physical resource compete for these resources to guarantee their own reliable communication in a selfish way. Based on the effective capacity, we formulate the problem as a non-cooperative game. And then we identify the Nash Equilibrium (NE) and obtain the corresponding power-allocation policy.

The rest of this paper is organized as follows. The system model is described in Section II. In Section III, we give a brief introduction about effective capacity and statistical QoS guarantees. Section IV formulates the problem as a non-cooperative game and proves the existence and uniqueness of the NE. Numerical and simulation evaluations are presented in Section V and conclusions are given in Section VI.

II. System Model

The system model is illustrated in Fig.2. According to the introduction in Section I, an HUE communicates with the BS by the help of the MR, while an MUE communicates directly with the BS. In our work, we focus on the uplink system. Two uplinks sharing the same physical resource will interfere with each other. Specifically, when an MUE sends signals to the BS, the MR can also receive the MUE's signals. We consider that the base station does not use any joint decoding strategies (e.g., successive interference cancelation), thus the MR will simply treat the MUE's signals as the background noise, which cannot be eliminated at the base station through techniques such as successive interference cancelation. Similarly, when an HUE sends signals to the MR, the BS can also receive an HUE's signals.

Generally, the upper-protocol-layer packets are divided into frames at the data link layer, and we assume that the frames have the same time duration, which is denoted by T. The discrete-time channel fading process is assumed to be stationary and ergodic. It means that the channel fading process is invariant within a frame's time duration T, but varies from one frame to another. Moreover, the wireless channel is assumed to be flat-fading with its envelope following Rayleigh distribution. We denote the transmit power of an HUE within a time-frame by p_1, the transmit power of an MUE within a time-frame by p_2, and corresponding power gain by h_1 and h_2.The power spectrum density of Gaussian noise is σ^2. Actually, it's also assumed that the transmit power of two types of users are limited in their corresponding constraints. We assume that the channel state information (CSI) is perfectly estimated at receiver and reliably fed back to the transmitter without any

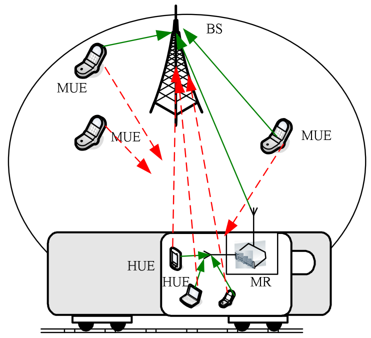


Fig.2. system model

delay. That is to say, the base station can estimate the power gain $\mathbf{h}=(h_1, h_2)$ perfectly.

Under the above assumption, when the transmit power and power gain are given, the maximum achievable rate of an HUE is determined by

$$R_1 = \log\left(1+\frac{p_1 h_1}{\sigma^2+p_2 h_2}\right). \tag{1}$$

Similarly, the maximum achievable rate of an MUE is determined by

$$R_2 = \log\left(1+\frac{p_2 h_2}{\sigma^2+p_1 h_1}\right). \tag{2}$$

III. Statistical QoS Guarantees And Effective Capacity

A. Statistical QoS Guarntees

From Section II, we know that the upper-protocol-layer packets are divided into frames at the data link layer stored in the buffer, queuing for being processed. So here is a queuing system. The asymptotic results in [6] show that for stationary and ergodic arrival and service processes under sufficient conditions, the probability that queue size Q exceeds a certain threshold x (i.e., the buffer overflow probability) decays exponentially as the threshold x increases. That is

$$-\lim_{x\to\infty}\frac{\log(\Pr\{Q(\infty)\}>X)}{x} = \theta, \tag{3}$$

where θ is a certain positive constant called the QoS exponent [7].The parameter θ plays an important role for statistical QoS guarantees. In detail, a smaller θ corresponds to a slower delaying rate, which implies that the system can only provide a looser QoS requirement. On the other hand, a larger θ leads to a faster delaying rate, which represents a more stringent QoS constraint. That is to say, when θ is close to ∞, the system can't tolerate any delay.

B. Effective Capacity

In [7], the authors give the concept of effective capacity, which is the dual of the original effective bandwidth. Effective capacity in [7] is proven to be an effective approach to characterize statistical delay QoS provisioning for data transmission in time-varying wireless channels. To be more specific, the effective capacity is defined as the maximum rate in order to guarantee a QoS requirement specified by θ. In [8], the authors provide a simple derivation for effective capacity.

The effective capacity of an HUE can be mathematically written as [8]

$$I_1 = -\frac{1}{\theta_1}\log\left(E_h\{e^{-\theta_1 TBR_1}\}\right). \quad (4)$$

In the same way, the effective capacity of an MUE can be written as

$$I_2 = -\frac{1}{\theta_2}\log\left(E_h\{e^{-\theta_2 TBR_2}\}\right). \quad (5)$$

We denote the QoS exponent of the HUE and the MUE by θ_1 and θ_2, respectively. Here, B represents the signal bandwidth, and $E_h\{\cdot\}$ is the expectation over h.

IV. Power-Allocation Game for Guarantees

A. Problem Formulation

In our work, in order to guarantee better performance, we will try to maximize the throughput subject to a given delay QoS constraint, i.e. I_1 and I_2. For the case of the HUE, we suppose that the power of the MUE is given. So the HUE's power policy is the solution to the following maximization problem

$$\max_{p_1} -\frac{1}{\theta_1}\log\left(E_h\left\{e^{-\theta_1 TB\log\left(1+\frac{p_1 h_1}{\sigma^2+p_2 h_2}\right)}\right\}\right), \quad (6)$$

$$\text{s.t. } 0 \le E_h\{p_1\} \le P_{1max}. \quad (7)$$

According to (6) and (7), our original maximization problem is converted into a minimization problem. In detail, we can formulate the problem as

$$\min_{p_1} E_h\left(1+\frac{p_1 h_1}{\sigma^2+p_2 h_2}\right)^{-\beta_1}, \quad (8)$$

$$\text{s.t. } 0 \le E_h\{p_1\} \le P_{1max}, \quad (9)$$

where $\beta_i = \theta_i TB$ is termed normalized QoS exponent. In order to maximize the effective capacity of an MUE and an HUE, the power allocation problem is modeled as the following pure strategic non-cooperative game.

Players: an HUE and an MUE. Define an HUE as user 1 and an MUE as user 2.

Utility function:

$$u_i(p_i, p_{-i}) = E_h\left(1+\frac{p_i h_i}{\sigma^2+p_j h_j}\right)^{-\beta_i} + \alpha_i p_i, \quad (10)$$

where $\alpha_i > 0$, is the cost factor, $\forall i, j \in \{1,2\}, j \ne i$. $u_i(p_i, p_{-i})$ is the utility function of the user i, p_i is the strategy adopted by the user i, p_{-i} denotes the strategy adopted by the other user.

Strategy space: $P=P_1\times P_2$, where $P_i=\{ p_i, 0\le E_h\{ p_i\}\le P_{imax}\}$, $i \in \{1,2\}$.

Mathematically, the game can be expressed as

$$G: \min u_i(p_i, p_{-i}), \quad (11)$$

$$\text{s.t. } p_i \in P_i. \quad (12)$$

In (10), α_i is the cost factor [4], which shows the impact when increasing the transmit power. Specifically, when one player increases its transmit power unreasoningly, he will impose much interference to the other player. In G, each game player is selfish and attempts to choose an appropriate power to maximize its own effective capacity. Moreover, both game players are assumed to be rational, and they also trust that the other player is rational. Consequently, their strategies should aim at reaching a stable equilibrium of the power-allocation game. Next, we will introduce the definition of Nash Equilibrium [9] as follows.

Definition 1: the Nash Equilibrium is an admissible power allocation policy pair, denoted by $P^*= (p_i^*, p_{-i}^*)$, for each player, which satisfies

$$u_i(p_i^*, p_{-i}^*) \ge u_i(\tilde{p_i}, p_{-i}^*), \ \forall \tilde{p_i} \in P_i, \ \forall i \in \{1,2\}. \quad (13)$$

NE can be regarded as a stable solution, which may not be the optimal for each single player. At NE, any player has no incentive to change its power.

B. Existence and Uniqueness of the Nash Equilibrium

Theorem 1 [4]: Game G admits at least one NE, if

1) The set P_i is a nonempty, convex, and compact subset of some Euclidean space for all i.

2) The utility function $u_i(p_i, p_{-i})$ is continuous on P and quasi-concave (quasi-convex) on P_i.

Proof: In G, it is clear that its strategy space is nonempty, convex, and compact [2], and its payoff function is continuous. So we only need to prove that its payoff function is quasi-concave.

By taking the second derivative of $u_i(p_i, p_{-i})$ with respect to p_i, we obtain

$$\frac{\partial^2 u_i}{\partial {p_i}^2} = \beta_i(\beta_i+1)\left(1+\frac{p_i h_i}{\sigma^2+p_j h_j}\right)^{-\beta_i-2}\left(\frac{h_i}{\sigma^2+p_j h_j}\right)^2 \ge 0, \quad (14)$$

where $j \ne i$. From (14), we can confirm the payoff function of the ith user is quasi-convex. Under the above description, the conditions of *Theorem* 1 are satisfied, which means game G has at least one NE. Meanwhile, we can express the derivative of payoff function with respect to p_i as

$$\frac{\partial u_i}{\partial p_i} = -\beta_i\left(1+\frac{p_i h_i}{\sigma^2+p_j h_j}\right)^{-\beta_i-1}\left(\frac{p_i h_i}{\sigma^2+p_j h_j}\right) + \alpha_i. \quad (15)$$

The necessary condition for minimizing the payoff function is $\frac{\partial u_i}{\partial p_i} = 0$. Therefore, for the user 1, we can get

$$p_1 = \left(\frac{\sigma^2+p_2h_2}{h_1}\right)^{\frac{\beta_1}{1+\beta_1}}\left[\mu_1 - \left(\frac{\sigma^2+p_2h_2}{h_1}\right)^{\frac{1}{1+\beta_1}}\right]^+, \qquad (16)$$

where $[\cdot]^+ = \max\{\cdot\,,\ 0\}$, $\mu_1 = \left(\frac{\alpha_1}{\beta_1}\right)^{-\left(\frac{1}{1+\beta_1}\right)}$. Moreover, it is clear to find that the payoff function of one player is similar to Lagrangian function of (8). According to Lagrangian method in [8], the parameter μ_1 can be obtained by $E_h\{p_1\} = P_{1max}$. In order to realize the power policy for NE, we apply the best response strategy. Specifically, given the other user's strategy p_{-i}, we obtain the optimal strategy of the ith user. Therefore, equation (16) is the optimal strategy of the user 1.

Similarly, the optimal strategy of the user 2 is

$$p_2 = \left(\frac{\sigma^2+p_1h_1}{h_2}\right)^{\frac{\beta_2}{1+\beta_2}}\left[\mu_2 - \left(\frac{\sigma^2+p_1h_1}{h_2}\right)^{\frac{1}{1+\beta_2}}\right]^+, \qquad (17)$$

where μ_2 is the unique positive constant such as $E_h\{p_2\} = P_{2max}$ holds.

Now we have proven the existence of Nash Equilibrium at the beginning of this section. Clearly, the Nash Equilibrium needs to satisfy (16) an (17). That is to say, we only need to prove the solution of (16) an (17) is unique.

Theorem 2:Game G possesses a unique NE.

Proof: From the above description, there exists the case that the transmit power is 0. When at least one user's power is equal to 0, it is clear that the power pair (p_1, p_2) is unique. However, when p_1 and p_2 are both larger than 0, we need to further prove the power pair (p_1, p_2) is unique.

Given $p_1>0$ and $p_2>0$, (16) and (17) reduce to

$$p_1 = \left(\frac{\sigma^2+p_2h_2}{h_1}\right)^{\frac{\beta_1}{1+\beta_1}}\mu_1 - \left(\frac{\sigma^2+p_2h_2}{h_1}\right), \qquad (18)$$

$$p_2 = \left(\frac{\sigma^2+p_1h_1}{h_2}\right)^{\frac{\beta_2}{1+\beta_2}}\mu_2 - \left(\frac{\sigma^2+p_1h_1}{h_2}\right). \qquad (19)$$

Simplifying (18) and (19), we will find

$$h_1\left(\frac{\sigma^2+p_2h_2}{h_1}\right)^{\frac{\beta_1}{1+\beta_1}}\mu_1 = h_2\left(\frac{\sigma^2+p_1h_1}{h_2}\right)^{\frac{\beta_2}{1+\beta_2}}\mu_2. \qquad (20)$$

We denote p_1 of (18) and (20) by $f(p_2)$ and $g(p_2)$, respectively. Because game G admits at least one NE which has been proven above, (18) and (20) at least have one solution. In other words, there must exist such p_2 that $f(p_2)$-$g(p_2)$=0. Assume that $\widetilde{p_2}$ and $\overline{p_2}$ are two solutions to (18) and (20), where $\widetilde{p_2} < \overline{p_2}$ .Then $f(\widetilde{p_2}) - g(\widetilde{p_2}) = 0$ and $f(\overline{p_2}) - g(\overline{p_2}) = 0$ hold. We assume $\beta_1 \geq \beta_2$, thus $f(p_2)$ and $g(p_2)$ are strictly concave and convex functions of p_2. Then, we get $f(0) > g(0)$.

Under the above setup, we obtain

$$\begin{cases} f(\widetilde{p_2}) = f(0) + \int_0^{\widetilde{p_2}} \frac{\partial f}{\partial p}dp \\ g(\widetilde{p_2}) = g(0) + \int_0^{\widetilde{p_2}} \frac{\partial g}{\partial p}dp\,. \end{cases} \qquad (21)$$

Because of $f(\widetilde{p_2}) = g(\widetilde{p_2})$, we obtain the inequality

$$\int_0^{\widetilde{p_2}} \frac{\partial f}{\partial p}dp < \int_0^{\widetilde{p_2}} \frac{\partial g}{\partial p}dp. \qquad (22)$$

Furthermore, $f(p_2)$ and $g(p_2)$ are strictly concave and convex functions of p_2, respectively. We derive

$$\begin{cases} \int_0^{\widetilde{p_2}} \frac{\partial f}{\partial p}dp > \widetilde{p_2} \times \left(\frac{\partial f}{\partial p}|p = \widetilde{p_2}\right) \\ \int_0^{\widetilde{p_2}} \frac{\partial g}{\partial p}dp < \widetilde{p_2} \times \left(\frac{\partial g}{\partial p}|p = \widetilde{p_2}\right). \end{cases} \qquad (23)$$

According to the above conditions, we can get

$$\frac{\partial f}{\partial p} < \frac{\partial g}{\partial p}, \forall p > \widetilde{p_2}. \qquad (24)$$

According to (24), we can get

$$f(\overline{p_2}) = f(\widetilde{p_2}) + \int_{\widetilde{p_2}}^{\overline{p_2}} \frac{\partial f}{\partial p}dp < g(\widetilde{p_2}) + \int_{\widetilde{p_2}}^{\overline{p_2}} \frac{\partial g}{\partial p}dp = g(\overline{p_2}),$$

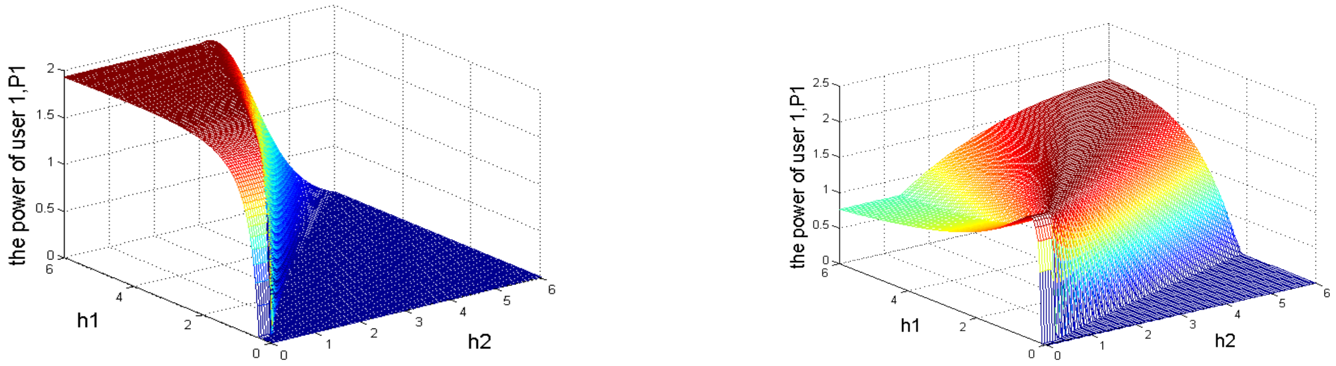
resulting in a contradiction. By derivation, we get that solution of (18) and (20) is unique. From the above analyses, we conclude that game G possesses a unique NE. In summary, we prove that Nash Equilibrium is existent and unique.

V. NUMERICAL RESULTS

In this section, some simulations results are presented to evaluate our algorithm. We compare power allocation game subject to QoS guarantee with power allocation game based on the concept of Shannon capacity. Throughout the simulation, the frame duration T=2ms and the signal bandwidth B=10^4Hz. Moreover, the average power limits of two users are set by P_{1max}=P_{2max}=1.

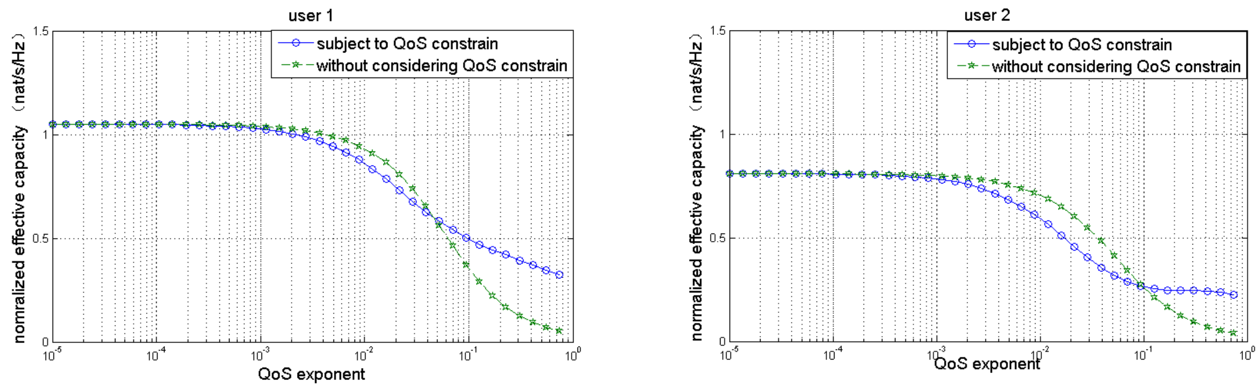
To evaluate our power allocation scheme, we plot the instantaneous power of user 1 against the channel gain **h** in Fig.3. Fig.3 (a) and Fig.3 (b) show the instantaneous power corresponding to a loose delay QoS constrain (θ is small) and a stringent delay QoS constrain (θ is large), respectively. In Fig.3 (b), the QoS exponent of the HUE is set to be larger than that of the MUE. Because HUE has the same speed as the train, Fig.3 (b) can explain the power allocation scheme in high-speed mobile environment subject to a stringent delay QoS constrain. From these two figures, we find the instantaneous power of the first user is a continuous function of **h**. Moreover, as shown in Fig.3 (a) and Fig.3 (b), the system has different power allocation strategies under the different levels of delay QoS requirements, while the communication in high-speed mobile environment is subject to a stringent delay QoS constrain. That is to say, it is necessary to take delay QoS constrains into our power allocation scheme in high-speed mobile environment.

The normalized effective capacity (which is defined as



(a) (b)

Fig.3. The instantaneous power of the first user against the channel gain **h**, $E_h\{h_1\} = 2, E_h\{h_2\} = 1$. (a) power allocation game subject to a loose delay QoS constrain, where $\beta_1 = \beta_2 = 0.125$. (b) power allocation game subject to a stringent delay QoS constrain, where $\beta_1 = 1.25$, $\beta_2 = 0.125$.



(a) (b)

Fig.4. The normalized effective capacity against the QoS exponent.

the effective capacity divided by B and T) comparisons are shown in Fig.4. As shown in Fig. 4 (a) and Fig. 4 (b), when θ is small, especially approaching zero, our power allocation game converges to the performance of the power allocation scheme which is based on Shannon capacity without considering delay QoS constrains. In contrast, when θ is large, our power allocation policy can obtain better effective capacity performance. In the high-speed mobile environment, the system is delay-sensitive, which means that a more stringent QoS requirement can be guaranteed.

VI. CONCLUSIONS

In this paper, we study the power allocation for interference suppression in high-speed mobile environment. Considering QoS guarantee, a non-cooperative game is proposed to formulate the effective-capacity maximization problem. We derive the Nash Equilibrium for the above game and obtain the corresponding the power-allocation policy. By comparing the simulations with the power allocation game without considering QoS guarantee, we conclude the proposed algorithm obtain the better performance in the high-speed mobile environment.

REFERENCES

[1] 3GPP TR 36.814." Evolved Universal Terrestrial Radio Access (E-UTRA);Further advancements for E-UTRA physical layer aspects", V9.0.0.

[2] Qinghe Du and Xi Zhang,"QoS-driven power allocation game over fading Multiple-Access Channels " *IEEE Globecom 2009,* Hawaii, USA, November 30 - December 4, 2009

[3] L. Lai and E. Gamal, "TheWater-Filling Game in Fading Multiple-AccessChannels," *IEEE Trans. Inform. Theory*, vol. 54, no. 5, May 2008, pp. 2110-2122

[4] Dan Wu and Yueming Cai, "Power Allocation in Interference Relay channels based on non-cooperative game theroy",WCSP, September 2009.

[5] A. J. Goldsmith and P. Varaiya, "Capacity of fading channels with channelside information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.

[6] C.-S. Chang, "Stability, queue length, and delay of deterministic andstochastic queueing networks," *IEEE Trans. Automatic Control*, vol. 39, no. 5, pp. 913–931, May 1994.

[7] D. Wu and R. Negi, "Effective capacity: a wireless link model forsupport of quality of service," *IEEE Trans. Wireless Commun*., vol. 2, no. 4, pp. 630–643, July 2003.

[8] J. Tang and X. Zhang, "Quality-of-service driven power and rate adaptationover wireless links," *IEEE Trans. on Wireless Commun.*, vol. 6,no.8,pp.3058-3068,August2007.

[9] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*. Philadelphia, PA: SIAM, 1998.