The Idle Period Distribution for CSMA/CA Networks for Spectrum Sensing Applications

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Abstract— **The CSMA/CA protocol leaves a idle/busy period signature in its band of operation that can easily be profiled by energy detectors. In this paper, we present a hypothesis on the protocol's idle period distribution and verify its exactness through simulation. This distribution may be useful in spectrum sensing applications where there is a need to fingerprint such networks, or to infer the number of nodes actively participating in the system using simple, protocol-independent circuitry.**

Index Terms— **CSMA/CA, spectrum sensing, idle period, analytical model.**

I. Introduction

CSMA/CA is a slotted random access protocol that is widely used in indoor wireless systems such as the IEEE 802.11 [1] and Zigbee. It is also used in wired systems such as HomePlug. In CSMA/CA, each node assigns itself a random backoff counter value uniformly polled from the range [0, *CW* – 1] when it has a new frame to be transmitted. This backoff value represents the number of idle slots it must wait before transmitting. At the start of each slot, the node will sense the state of the channel. If the channel is sensed idle, the node will decrement its counter. If the channel is sensed busy, the counter is suspended. When a slot is entered with the counter value equals to zero, the node transmits the frame. If the transmission collides with other nodes' transmissions, the procedure is repeated again, with the backoff counter reset again to a random value from a range which is either twice wider than the last employed range if collision-based binary exponential backoff is employed, or the same range if fixed contention windows is used.

In this paper, we formulate upon a characteristic of the CSMA/CA protocol: its idle period distribution, in terms of its backoff slots. In some spectrum sensing or fingerprinting applications, it may be useful to have the functionality of determining with some level of certainty if a channel is currently being occupied by some wired or wireless protocol, or to determine the number of transmitters actively communicating in that channel using that protocol, without actually implementing the full physical layer (PHY) for that protocol. This may be the case when the device performing this function may be required to do so across a wide range of protocols while using limited computing resources (e.g. FPGA gates). Also, the cost of obtaining and integrating multiple transceiver designs maybe prohibitive. In this case, fingerprinting schemes relying on basic signal power detection may be favored. By taking the statistics of the idle periods sensed using an energy detector, it should be possible to devise a best-of-fit test to determine if the statistics corresponds to the idle period distribution that we will derive and validate in this paper.

II. Related Work

The contributions of this paper are of an analytical nature, serving to add to the understanding of the theoretical basis of the CSMA/CA protocol to which many authors have contributed over the years.

Early analytical studies have provided us with models for the protocol's transmission attempt probability, channel state and state transition probabilities, and throughput under saturation loads. For these characteristics, Bianchi [2,3] made the most significant headway by introducing the Markov chain modeling method. In his model, the backoff counter is decremented right after a busy period. However, in actual implementation, it is only decremented at the end of an idle slot following that period. In [4], Ziouva and Antonakopoulos tried to correct this assumption, but did not take into account that only nodes that had transmitted in the previous busy period could start transmitting immediately after that period. In [5] and [6] done by Foh et al. and Hu et al., respectively, this shortcoming was addressed independently. Building on the results of these early works, two more characteristics - transmission delay and delay jitter - were derived in [7] by Chatzimisios et al., in [8] by Li et al., and in [9] by T. Sakurai and Hai L. Vu.

In this paper, we analyze the distribution of the idle period of the protocol. However, due to the complexity involved, we limit our discussion to a specific case of the protocol whereby the contention window remains constant despite collisions. While this limitation may seem to narrow down the applicability of our contribution, it should be noted that CSMA/CA with fixed contention provides much better performance than the exponential scheme when the contention window length is set optimally [10]. This is because the exponentially increasing contention window significantly degrades access delay. Furthermore, it is a mode that can be easily configured into at least one popular chipset [11], either for specific applications, or to outperform other standards-compliant implementations. The rest of this paper will be presented as follows: in section III, we recapitulate the Markov model for CSMA/CA, which we use as the starting point for our work. In section IV, we derive the suspended backoff counter distribution. In section V, we use this backoff counter distribution to derive the protocol's idle period distribution. In section VI, we validate the idle period distribution using statistics obtained via simulation. Finally, in section VII, we conclude with a brief review of the contributions of this paper.

III. MARKOV CHANNEL STATE MODEL

A. Markov Model for the Channel State

The method we use to derive the idle period requires, as starting point, the channel state distribution for CSMA/CA. Let $C \in [0, N]$ be the discrete random variable representing the channel state for a cell, with $C = 0$ representing idle state and $C = n$, the channel being busy with n simultaneous transmissions. For CSMA/CA with fixed contention windows, the transition probabilities for the Markov chain model of the channel state is given in [5] as follows:

$$P_{CT}(c_1,c_0) = \begin{cases} \binom{N}{c_1}\left(\frac{2}{CW}\right)^{c_1}\left(\frac{CW-2}{CW}\right)^{N-c_1} & c_o = 0,\ \ 0 \le c_1 \le N \\ \binom{c_o}{c_1}\left(\frac{1}{CW}\right)^{c_1}\left(\frac{CW-1}{CW}\right)^{c_o-c_1} & 1 \le c_0 \le N,\ \ c_1 \le c_0 \\ 0 & 1 \le c_0 \le N,\ \ c_1 > c_0 \end{cases} \quad (1)$$

Here, $P_{CT}(c_1, c_0) \equiv P\{C_i = c_1 \mid C_{i-1} = c_0\}$ with c_1 and c_0 being the next and current states respectively. Using a transition matrix $\underline{P}$ with (i, j)th elements equal to $P_{CT}(j, i)$, the channel state probability mass function $P_C(c) \equiv P\{C = c\}$ can be obtained via the steady-state equation,

$$\underline{\pi} = \underline{\pi}\underline{P} \quad (2)$$

with $P_C(c)$ given by the (c)th element of $\underline{\pi}$.

IV. SUSPENDED BACKOFF COUNTER DISTRIBUTION

Before formulating the idle period distribution, the distribution of the backoff counter value when it is suspended must first be determined. To this end, we do the following statistical study:

A. Observation Period and Samples

We know that the counter is suspended in relation to some transitions in the channel state as given by Eq.(1). Each of these specific transitions generates a sample of the distribution we are interested in. To narrow down the span of our analysis, we need to identify a recurring period in the indefinite sequence of channel state transitions which will give us a sample distribution that is also representative of samples throughout the entire, indefinite transition sequence.

Concerning the samples, we observe that the counter is suspended on two separate and mutually unrelated types of events. The first type of events is when the channel state changes from idle to busy, i.e., from $(C = 0)$ to $(C > 0)$. For example, in a cell of 5 nodes, a transition in the channel state from $(C = 0)$ to $(C = 2)$ would result in (5-2) suspended backoff counter samples, contributed by nodes that will not participate in the transmission in the subsequent busy state. Since these samples are generated by the channel changing from an idle state to a busy state, we shall call these samples "idle-to-busy" samples. The second type of events is when the channel state drops from $(C = c_i)$ to $(C = c_{i+1})$ with $2 \le c_i \le N$ and $1 \le c_{i+1} < c_i - 1$. For instance, a transition from the channel state $(C = 4)$ to $(C = 1)$ would yield (4 - 1) instances of suspended backoff counter samples, contributed by nodes which, having transmitted in a prior busy state, selects a new counter value that is non-zero. We shall call these "busy-to-busy" samples.

Taking into account how these two types of events occur, the observation period which will serve us in our study is therefore the sequence of channel state transitions $\{0, c_0, c_1, ..c_i, ..1\}$, for all possible values of $1 \le c_i \le N$ and $c_{i+1} \le c_i$. Without knowing the actual values of either of these two types of samples, we can at least work out the average number of such samples generated in the observation period as follows.

B. Average Number of Idle-to-Busy Samples

Let $Q(c_0)$ be the average number of idle-to-busy samples generated by a non-transmitting node in the observation period $\{0, c_0, c_1, ..c_i, ..1\}$. With $c_0 = 1$, the corresponding channel state transitions $\{0, 1, 1, ..1\}$ yields,

$$\begin{aligned} Q(1) &= (1 + P_{CT}(1,1) + P_{CT}(1,1)^2 + P_{CT}(1,1)^3 + ..) \\ &= \frac{1}{1 - P_{CT}(1,1)} \end{aligned} \quad (3)$$

With c_0=2, we have,

$$Q(2) = \frac{1}{1 - P_{CT}(2,2)}\left(1 + P_{CT}(1,2).Q(1)\right) \quad (4)$$

Working with upward values of c_0, we deduce that $Q(c_0)$ is solvable with the recursive formula,

$$Q(c_0) = \begin{cases} \frac{1}{1 - P_{CT}(1,1)} & c_0 = 1 \\ \frac{1}{1 - P_{CT}(c_0,c_0)}\left(1 + \sum_{i=1}^{c_o-1}\{P_{CT}(i,c_o).Q(i)\}\right) & c_0 > 1 \end{cases} \quad (5)$$

C. Average Number of Busy-to-Busy Samples

By a similar method of recursive deduction, we define the following function that gives the average number of busy-to-busy samples in the observation period $\{0, c_0, c_1, ..c_i, ..1\}$:

$$R(c_i,c_0) = \begin{cases} \frac{1}{1-P_{CT}(2,2)} P_{CT}(1,2) \frac{1}{1-P_{CT}(1,1)} (c_0-1) & c_i = 2 \\ \frac{1}{1-P_{CT}(c_i,c_i)} \left(\begin{array}{l} \sum_{i=1}^{c_i-1}\left\{P_{CT}(i,c_i) \frac{1}{1-P_{CT}(i,i)} (c_0-i)\right\} \\ + \sum_{i=2}^{c_i-1}\{P_{CT}(i,c_i) R(i,c_0)\} \end{array} \right) & 2 < c_i \le c_0 \end{cases} \quad (6)$$

To get an intuition for how this equation is formulated, the reader is urged to attempt its verification with upward values of $c_i = c_0$. For example, with $c_i = c_0 = 2$, Eq.(6) gives us the average number of samples when the sequences are of the form {2,..,2,1,..,1}. By substituting $c_i = c_0 = 3$ and expanding it recursively, it can be confirmed to yield the number of samples when the sequence is of the patterns of {3,..,3}, {3,.3,1,..,1}, {3,..,3,2,..,2}, and {3,..,3,2,..,2,1,..,1}.

D. Probability for Busy-to-Busy Samples

Let $F \in [1, CW - 1]$ be the discrete random variable representing the suspended backoff counter value, and $W' \in [1, CW - 1]$ the variable representing the initial non-zero value that the backoff counter is assigned after a transmission attempt. Since busy-to-busy samples occur as a result of nodes suspending their counters upon selecting a new non-zero backoff value just after transmitting in the previous channel state, their values are equal to this new non-zero values. So, if we divide the average number of busy-to-busy samples over the average number of samples of both types within the observed space, we get the aggregate probability for $\{F = w \mid W' = w\}$ over all possible values for w:

$$\sum_{w=1}^{CW-1} P_F(w)\,|_{W'=w} = \frac{\sum_{c_0=2}^{N}\left\{P_{CT}(c_0,0)\cdot R(c_0,c_0)\right\}}{\left(\begin{array}{l}\sum_{c_0=1}^{N-1}\left\{P_{CT}(c_0,0)\cdot Q(c_0)\cdot(N-c_0)\right\}\\ +\sum_{c_0=2}^{N}\left\{P_{CT}(c_0,0)\cdot R(c_0,c_0)\right\}\end{array}\right)} \quad (7)$$

E. Probability for Idle-to-Busy Samples

As for the idle-to-busy samples, these have to be at least one less than the newly assigned backoff counter values. This should be apparent given that the backoff counter is decremented at every idle state, and the idle-to-busy samples are generated by idle to busy channel state transitions. Using the same approach as for the busy-to-busy samples, we get to express the aggregate probability for $\{F < w \mid W' = w\}$ over all possible values of w (bearing in mind that $F \geq 1$) as:

$$\sum_{w=2}^{CW-1} P\{F < w \mid W' = w\} = \frac{\sum_{c_0=1}^{N-1}\left\{P_{CT}(c_0,0)\cdot Q(c_0).(N-c_0)\right\}}{\left(\begin{array}{l}\sum_{c_0=1}^{N-1}\left\{P_{CT}(c_0,0)\cdot Q(c_0)\cdot(N-c_0)\right\}\\ +\sum_{c_0=2}^{N}\left\{P_{CT}(c_0,0)\cdot R(c_0,c_0)\right\}\end{array}\right)} \quad (8)$$

Note that for each variate w of W', $P\{F < w \mid W' = w\}$ represents the sum of $(w - 1)$ component terms, one for each variate f of F satisfying the condition $f < w$. For example, $P\{F < w \mid W' = 3\}$ is the sum of $P_F(1)|_{W'=3}$ and $P_F(2)|_{W'=3}$. Furthermore, since the event of a node having its backoff counter suspended is independent of the event of it selecting a variate of W', it is just as probable for $\{F = 1 \mid W' = 2\}$ to occur as $\{F = 2 \mid W' = CW - 1\}$. In other words, all the component probabilities of $P\{F < w \mid W' = w\}$ are equal to one another regardless of the value of w. As such, we can relate the probability of the suspended backoff counter taking on a variate of F from idle-to-busy samples to the component probability term $P_F(f)|_{W'=w,\, w>f}$ as follows:

$$\begin{aligned} &P_F(f)\,|_{W'\neq f} \\ &= \sum_{w=f+1}^{CW-1} P_F(f)\,|_{W'=w} \\ &= P_F(f)\,|_{W'=f+1} + P_F(f)\,|_{W'=f+2} + \ldots + P_F(f)\,|_{W'=CW-1} \\ &= (CW-1-f).P_F(f)\,|_{W'=w,w>f} \end{aligned} \quad (9)$$

Since the probability of occurrence of the idle-to-busy backoff samples is equivalent to the sum of all the component probability terms across all possible values of f and w restricted to the condition $f < w$, we get the following expression relating it to the component probability term:

$$\begin{aligned} &\sum_{w=2}^{CW-1} P\{F < w \mid W' = w\} \\ &= \sum_{f=1}^{CW-2} P_F(f)\,|_{W'\neq f} \\ &= \sum_{f=1}^{CW-2} (CW-1-f).P_F(f)\,|_{W'=w,w>f} \\ &= \{(CW-2)+(CW-3)+..+1\}.P_F(f)\,|_{W'=w,w>f} \\ &= \frac{(CW-1)(CW-2)}{2}.P_F(f)\,|_{W'=w,w>f} \end{aligned} \quad (10)$$

Eliminating the component probability term from Eqs.(9,10), we get:

$$P_F(f)|_{W'\neq f} = \frac{2(CW-1-f)}{(CW-1)(CW-2)}\sum_{w=2}^{CW-1} P\{F < w \mid W' = w\} \quad (11)$$

F. Full Expression for the Suspended Backoff Counter Value Distribution

Combining Eqs.(7,11), we obtain the probability mass function for the suspended backoff values:

$$\begin{aligned} P_F(f) &= P_F(f)|_{W'=f} + P_F(f)|_{W'\neq f} \\ &= P_{W'}(w)\sum_{w=1}^{CW-1} P_F(w)\,|_{W''=w} \\ &\quad + \frac{2(CW-1-f)}{(CW-1)(CW-2)}\sum_{w=2}^{CW-1} P\{F < w \mid W' = w\} \\ &= \frac{1}{CW-1}\sum_{w=1}^{CW-1} P_F(w)\,|_{W''=w} \\ &\quad + \frac{2(CW-1-f)}{(CW-1)(CW-2)}\sum_{w=2}^{CW-1} P\{F < w \mid W' = w\} \end{aligned} \quad (12)$$

Note that due to the second term involving a sum from $w = 2$ to $w = CW - 1$, Eq.(12) is not valid for $CW < 3$. This is consistent with the fact that the idle-to-busy variety of suspended backoff samples cannot occur with $CW = 2$ since only $\{F = w \mid W' = w\}$, i.e., the busy-to-busy variety, is possible in this case. Taking this into consideration, and incorporating Eqs.(7, 8), we finalize the expression for $P_F(f)$ thus:

$$P_F(f)=\begin{cases}\dfrac{\sum_{c=2}^{N}\{P_{CT}(c,0)R(c,c)\}}{\sum_{c=1}^{N-1}\{P_{CT}(c,0)Q(c)(N-c)\}+\sum_{c=2}^{N}\{P_{CT}(c,0)R(c,c)\}} & CW=2\\ \dfrac{\left(\begin{array}{l}\frac{2(CW-1-f)}{(CW-1)(CW-2)}\sum_{c=1}^{N-1}\{P_{CT}(c,0)Q(c)(N-c)\}\\ +\frac{1}{CW-1}\sum_{c=2}^{N}\{P_{CT}(c,0)R(c,c)\}\end{array}\right)}{\sum_{c=1}^{N-1}\{P_{CT}(c,0)Q(c)(N-c)\}+\sum_{c=2}^{N}\{P_{CT}(c,0)R(c,c)\}} & CW>2\end{cases} \tag{13}$$

V. Idle Period Distribution

Having derived the suspended backoff counter distribution, we now apply it to calculate the distribution of the idle periods for the protocol. For comparison, we will also derive the idle period based on the idle state probabilities given by the Markov channel state model.

A. Derivation using Suspended Backoff Distribution

We can express the probability mass function for the idle period given the preceding channel state, $P_I(i)|_{C=c}$ as the following permutation:

$$\begin{aligned}P_I(i)|_{C=c} &= P\{W\geq i\}^c.P\{F\geq i\}^{N-c}\times\\ &\quad (1-(1-P_W(i)|_{W\geq i})^c(1-P_F(i)|_{F\geq i})^{N-c})\end{aligned} \tag{14}$$

A bit of explanation may be necessary to induct the reader to how the above equation is derived: Note that for the idle period to be equal to i, both the new and suspended backoff counters must be equal to or higher than i, with at least one of them equalling i. So the first two factors involving the terms $P\{W\geq i\}$ and $P\{F\geq i\}$ ensures the former condition, while the remaining factor ensure the latter.

For the complete idle period distribution, we perform a weighted summation of Eq.(14) over all channel states:

$$P_I(i)=\frac{\sum_{c=1}^{N}\{P_C(c).P_I(i)|_{C=c}\}}{\sum_{c=1}^{N}P_C(c)} \tag{15}$$

B. Derivation using Markov Channel State Model

By way of Eq.(1), each variate i of the distribution can be approximated as having the probability of a string of idle states of length i. Though this method is flawed since it ignores that the string length is bounded to $CW-1$, to the best of our knowledge, it is the only one available currently:

$$P_I(i)=\begin{cases}\dfrac{\sum_{c=1}^{N}\{P_C(c)P\{C>0|C'=c\}\}}{\sum_{c=1}^{N}P_C(c)} & i=0\\ \dfrac{\left(\begin{array}{l}\sum_{c=1}^{N}\{P_C(c)P\{C>0|C'=c\}\}\\ \times P_{CT}(0,0)^{i-1}P\{C>0|C'=0\}\end{array}\right)}{\sum_{c=1}^{N}P_C(c)} & 0<i<CW\end{cases} \tag{16}$$

VI. Validation

The ns-2 [12] IEEE 802.11b program was used to obtain the simulation results of the idle period statistics of the protocol. Default transceiver settings (e.g. transmit power, receive threshold, carrier sense threshold, etc.) were used, and nodes were placed arbitrarily within a 10m by 10m area. Changes were made to the IEEE 802.11 C source code to ensure that the contention window is fixed.

The idle period measurement is carried thus: in a tagged node, after a transmission ends, a DIFS period triggers the start of the measurement. On sensing the channel busy at the start of a backoff slot, or if the node transmits, the measurement is ended. The period is counted in term of the backoff slot, and recorded into frequency bins.

For each combination of CW and N, the program was run 30 times, with each run spanning 5000 transmissions. For each of these 30 runs, the frequency bin is normalized and tested against the analytical distribution given by Eq.(15) using the one sample T tests. In all cases, the analytical values passed the tests.

We present some graphical results in Fig.1. Here, the idle period distribution computed using Eq.(15) is seen to fall within the 95% CI of the simulation results for $CW \in \{4, 8, 16, 32\}$ and $N \in \{2, 3, 5, 10\}$. Meanwhile, the Markov approximation given by Eq.(16) seems to be decently accurate when the number of nodes is large. But for smaller number of nodes, it visibly deviates from the simulation results.

VII. Conclusion

We have presented an analytical model for the idle period distribution for CSMA/CA, backing it with simulation results that confirm its correctness. As an intermediate step, we also formulated the distribution of the backoff counter when it is suspended. The derivation of both of these distributions are not found elsewhere in the literature, and may be useful in spectrum sensing applications where there is a need to determine, with some level of certainty, if a particular channel is being used by the CSMA/CA protocol, or the number of active transmitters involved. Not only have we shown that our idle period distribution found to be very accurate, but we have also demonstrate that the only method prior to the availability our model – i.e., the one derived from the Markov model, is inaccurate where small number of nodes are involved.

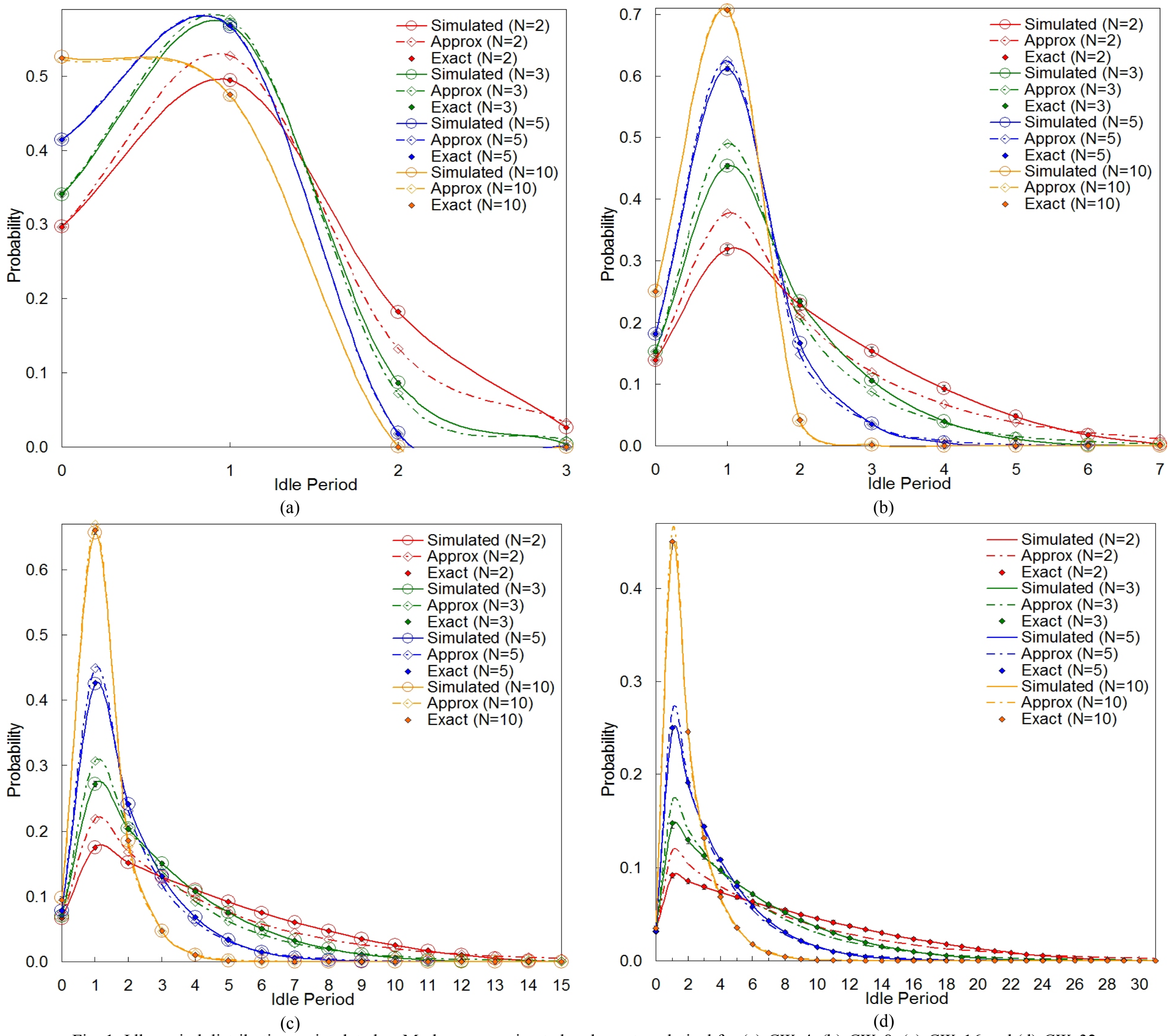

Fig. 1: Idle period distribution – simulated vs Markov-approximated and exact analytical for (a) *CW*=4, (b) *CW*=8, (c) *CW*=16 and (d) *CW*=32.

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