

Modulation optimization for achieving energy efficient communications over fading channels

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Abstract—It is commonly assumed that the energy consumption of wireless communications is minimized when low-order modulations such as BPSK are used. Nevertheless, the literature provides some evidence that low-order modulations are suboptimal for short transmission distances. A thorough analysis on how the modulation scheme and transmission power must be chosen as a function of distance in order to achieve energy-efficient communications over fading channels has not been reported yet. In this paper we provide this analysis by presenting a model that determines the energy consumed per payload bit transferred without error over correlated or uncorrelated random channels.

We find that each modulation scheme has a single optimal signal-to-noise ratio (SNR) at which the energy consumption is minimized. We also find that if all modulations are operated at their optimal SNR, BPSK and QPSK are the optimal choices for long transmission distances, but as the transmission distance shortens the optimal modulation size grows to 16-QAM and even to 64-QAM. This result leads to showing that for short-range communications the lifetime of a typical low-power transceiver can be increased by up to 600% by selecting the optimal constellation rather than BPSK.

I. INTRODUCTION

Attaining high energy efficiency is a key condition that wireless communications technologies like wireless sensor networks (WSN) must satisfy in order for the technology to prosper into large-scale, autonomous networks. Requirements on size and cost of the nodes pose vital constraints to the problem. In fact, battery depletion has been identified as one of the primary causes of lifetime limitation of these networks, and replacing them regularly is impractical in large networks or may even be impossible in hostile environments [1].

The communications energy budget depends on choices such as the modulation scheme, packet structure and transmission power. When the communication system is power-limited (as in WSN), the common notion is to choose low-order modulations such as BFSK or BPSK, which has a low SNR requirement for achieving a desired bit error rate [2]. These modulations are, in fact, the ones used in commercially available low-power transceivers like the TI CC1000 [3] or CC2420 [4], often used for WSN applications. Nevertheless it has been shown that the above notion leads to suboptimal operation for short link distances [5]–[7].

The rules by which the modulation scheme and transmission power shall be chosen to attain energy-efficient communi-

cations through random channels have not yet been clearly established. Most of the work reported so far focuses on the additive white Gaussian noise channel (AWGN) [5]–[12]. In [13], energy consumption of block fading Rayleigh channels is studied, but channel fading is included by its outage probability rather than by taking into account the actual symbol error rate (SER) degradation due to the fading process.

In this paper, we present an energy consumption model which allows for optimizing the modulation scheme and transmission power for communications over correlated or uncorrelated random channels.

Furthermore, we establish rules for choosing the modulation size that achieves highest energy efficiency as a function of link distance on fast fading channels. For large transmission distances, our model confirms the common notion discussed above for power-limited systems, while for short transmission distances it coincides with the results reported in [14] and [7] for AWGN, extending them to fast-fading random channels.

Our model also shows that a single optimal SNR level exists at which the least amount of energy is consumed per data bit transferred without error, and reveals how it depends on the packet frame length, modulation size and channel statistics. Many existing energy consumption models such as the ones reported in [5], [6], [8]–[11] share the assumption that the bit error rate is a given constant, which is determined by upper-layer requirements independently of physical layer parameters such as the modulation type or the power consumption of electronic components. The idea that the bit error rate should be a parameter to be optimized can be found in [7] and [15], but those results are only valid for AWGN channels.

The paper is organized as follows: Section II presents the energy consumption model, Section III uses this model for optimizing the SNR to achieve energy efficiency, and Section IV presents the principles for selecting modulation scheme and SNR. Finally, Section V summarizes our conclusions.

II. ENERGY CONSUMPTION MODEL

Our goal is to determine the total energy that is necessary for transferring one bit of data successfully, without error, in a point-to-point packet-switched wireless communication link (e.g. between two sensor nodes). We assume that every frame transmitted in the *forward* direction is matched by a feedback

frame in the *reverse* direction, which acknowledges correct reception or requests a re-transmission. We also assume that the irradiated power is determined by the transmitter based upon knowledge of the statistics of the signal-to-noise ratio (SNR) at the decision stage of the intended receiver. We further assume that all frames in both directions are always detected and that all feedback frames are decoded without error.

Transmissions in both directions cause energy expenses at respective transmitters and receivers. In short range communications, the energy consumption for receiving a frame is known to be on the same order as the consumption for transmitting it [5] and must hence be accounted for.

In the sequel, we first analyze the components of energy consumption of a transceiver from the standpoint of a node that transmits one payload frame and receives the corresponding feedback frame (the reverse case—a transceiver that receives one payload frame and transmits the corresponding feedback frame—follows by analogy). We continue by analyzing the statistics of re-transmissions and finally we present of our total energy consumption model.

A. Total Energy Consumption per Successfully Transferred Bit

The energy consumed by the transmitter of forward frames per error-free transferred bit, and for also decoding the corresponding feedback frames, is given by

$$\mathcal{E}_T = \mathcal{E}_{st} + \left[(P_{el,tx} + P_{PA})T_b + P_{el,rx} \frac{T_{fb}}{L} \right] \tau . \quad (1)$$

Here \mathcal{E}_{st} is the energy needed to wake up the transmitter from a low power consumption (sleep) mode, divided by the number of payload bits that are going to be transmitted before the transceiver goes again into low power consumption mode. P_{PA} is the power consumed by the power amplifier (PA), and $P_{el,tx}$ (respectively $P_{el,rx}$) is the power consumed by the remaining baseband and radio-frequency electronic components that perform the forward transmission (respectively the feedback frame reception). T_b is the average air time per payload bit on a forward frame, which includes acquisition, synchronization and frame overhead. $T_b = R^{-1}(1 + O/L)$, where R is the physical layer bit-rate, L is the number of payload bits per frame and O is a measurement of the overhead in bits. $T_{fb} = F/R$ is the air time of the feedback frame, where F is the feedback frame length. Finally τ is the number of trials until the frame that contains the considered bit is decoded without errors in the receiver.

By analogy, the total energy used by the receiver of forward frames for demodulating τ forward transmissions, and for transmitting the corresponding τ feedback frames, is

$$\mathcal{E}_R = \mathcal{E}_{st} + \left[P_{el,rx}T_b + (P_{el,tx} + P_{PA}) \frac{T_{fb}}{L} \right] \tau . \quad (2)$$

The total energy consumption per bit transmitted without error is the sum of (1) and (2):

$$\mathcal{E}_b = 2\mathcal{E}_{st} + (P_{el,tx} + P_{PA} + P_{el,rx}) \left(T_b + \frac{T_{fb}}{L} \right) \tau \quad (3)$$

$$= S + (P_{el} + P_{PA})T\bar{\tau} , \quad (4)$$

where we have defined $S = 2\mathcal{E}_{st}$, $P_{el} = P_{el,tx} + P_{el,rx}$ and $T = T_b + T_{fb}/L$.

It is to be noted that because of τ , \mathcal{E}_b is a random variable that depends on the realizations of the channel and of the thermal noise. Its mean value is

$$\bar{\mathcal{E}}_b = \mathbb{E} \{ \mathcal{E}_b \} = S + (P_{el} + P_{PA}) T \bar{\tau} . \quad (5)$$

Expressions for $\bar{\tau}$ are discussed in the sequel.

B. Re-transmission Statistics

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors at the receiver. The probability of frame error (and hence the probability of re-transmission) depend on the mean received SNR, $\bar{\gamma}$, and on the statistics of the wireless channel. Therefore, the number of trials (τ) until a frame is decoded without error is a random variable. It can be shown that its mean value, $\bar{\tau} = \mathbb{E} \{ \tau \}$, where $\mathbb{E} \{ \cdot \}$ denotes the expectation operator, can be expressed as

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E} \left\{ \prod_{j=1}^n P_j \right\} , \quad (6)$$

where P_j is the probability of decoding the frame with error during the j -th transmission trial. For lack of space, the complete derivation will appear in a future journal article.

The value of $\bar{\tau}$ depends on the joint distribution of the probabilities $\{P_j\}_{j=1}^{\infty}$. In effect, consider first a static channel where $P_j = P_1 \forall j \in \mathbb{N}$. In this case, (6) becomes

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \mathbb{E} \{ P_1^n \} = \mathbb{E} \left\{ \frac{1}{1 - P_1} \right\} \triangleq \bar{\tau}_{static} . \quad (7)$$

Consider now a fading channel in which the SNR levels of any two frame transmission trials are statistically independent. Then (6) can be re-written as

$$\bar{\tau} = 1 + \sum_{n=1}^{\infty} \prod_{j=1}^n \mathbb{E} \{ P_j \} = \frac{1}{1 - \mathbb{E} \{ P_1 \}} \triangleq \bar{\tau}_f , \quad (8)$$

where P_j and P_k are i.i.d. random variables whenever $j \neq k$.

Using the Jensen inequality for the convex function $\Phi(x) = (1 - x)^{-1}$ with $x \in [0, 1]$, it can be shown that

$$\bar{\tau}_f \leq \bar{\tau}_{static} , \quad (9)$$

where the equality is attained by the AGWN channel. This result shows that transferring successfully one frame across uncorrelated channels takes, on average, fewer transmission attempts than doing it over fully correlated channels. An intuitive explanation for this is that unfavorable (initial) realizations of static channels have a permanent low SNR level, and require therefore a large number of trials until a frame is received without error. However unlikely, the poor performance of these unfavorable cases raise the mean number of trials enough to spoil the average performance beyond the case of uncorrelated channels.

III. OPTIMAL MEAN SNR LEVEL FOR ENERGY EFFICIENCY

In this section, we seek to determine the mean SNR for which a communication with M-QAM modulation uses, on the average, the least amount of energy per bit transferred without error.

Consider rewriting (5) so that the terms that depend on the mean SNR, $\bar{\gamma}$, observed at the decision stage of the receiver, become explicit:

$$\bar{\mathcal{E}}_b(\bar{\gamma}) = S + [P_{\text{el}} + P_{\text{PA}}(\bar{\gamma})] T \bar{\tau}(\bar{\gamma}) . \quad (10)$$

Above, $P_{\text{PA}}(\bar{\gamma})$ is a linear function of $\bar{\gamma}$. In effect, the PA's power consumption is related with the transmission power as $P_{\text{tx}} = (\eta/\xi)P_{\text{PA}}$ where ξ is the peak-to-average power ratio of the transmitted signal and η is the drain efficiency of the PA [6]. The transmitted power attenuates over the air with path loss and arrives at the receiver with a mean power given by $P_{\text{rx}} = P_{\text{tx}}/(Ad^\alpha)$, where A is a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength, d is the distance between transmitter and receiver and α is the path loss exponent [16]. At the input of the decision device of the receiver, $\bar{\gamma}$ is related to P_{rx} as $\bar{\gamma} = P_{\text{rx}}/(N_0 W N_f M_1)$, where N_0 is the power spectral density of the baseband-equivalent additive white Gaussian noise (AWGN), W is the transmission bandwidth, N_f is the noise figure of the receiver's front end and M_1 is a link margin term which represents any other additive noise or interference [6]. Putting all these relationships together, we find that

$$P_{\text{PA}}(\bar{\gamma}) = \frac{\xi A d^\alpha N_0 W N_f M_1}{\eta} \bar{\gamma} = A_{\text{total}} \bar{\gamma} , \quad (11)$$

with A_{total} a constant.

Assume that an uncoded M -ary modulation is used for these transmissions. Then, there are $\lambda = (L+O)/\log_2(M)$ symbols per frame, where L is the number of payload bits per frame and O is the overhead (c.f. Section II-A). The frame error rate P_f can be written in terms of the symbol error rate $P_s(\gamma)$ as $P_f = 1 - \prod_{k=1}^{\lambda} [1 - P_s(\gamma_k)]$, where γ_k is the SNR in effect during the k -th symbol. Using this in (8) and assuming that that all γ_k are i.i.d. random variables with mean $\bar{\gamma}$ (i.e. fast fading) we obtain:

$$\bar{\tau}_f(\bar{\gamma}) = \frac{1}{\prod_{k=1}^{\lambda} [1 - \mathbb{E}\{P_s(\gamma_k)\}]} = \frac{1}{[1 - \mathbb{E}\{P_s(\gamma_1)\}]^\lambda} , \quad (12)$$

where the first equality follows from the assumption that each symbol is decoded independently from all others.

Under fast fading conditions, replacing (11) and (12) into (10), we find

$$\mathcal{E}_b(\bar{\gamma}) = S + \frac{(P_{\text{el}} + A_{\text{total}} \bar{\gamma}) T}{[1 - \bar{P}_s(\bar{\gamma})]^\lambda} , \quad (13)$$

where we are using the shorthand notation $\bar{P}_s(\bar{\gamma})$ for $\mathbb{E}\{P_s(\gamma)\}$. In general, $\bar{P}_s(\bar{\gamma})$ is a strictly decreasing function of $\bar{\gamma}$ that satisfies $\lim_{\bar{\gamma} \rightarrow \infty} \bar{P}_s(\bar{\gamma}) = 0$. Therefore, the average number of transmissions needed to successfully transfer one

frame under fast fading conditions, given by (12), is also a strictly decreasing function of $\bar{\gamma}$ and satisfies $\lim_{\bar{\gamma} \rightarrow \infty} \bar{\tau}(\bar{\gamma}) = 1$ (this reflects the intuitive fact that the average number of retransmissions drops as the SNR grows). By construction, (13) is the product of the decreasing function $\bar{\tau}(\bar{\gamma})$ and the increasing linear function $P_{\text{PA}}(\bar{\gamma})$. Such a product attains a unique minimum at the optimal SNR level $\bar{\gamma}_0$. Lower SNR levels are suboptimal because they force the system into too many retransmissions, and higher SNR levels are also suboptimal because the overall irradiated power is excessive.

The minimization of (13) over $\bar{\gamma}$ is straightforward by taking derivative and equating the result to zero. This leads to the following implicit expression for $\bar{\gamma}_0$:

$$\lambda \left(\frac{P_{\text{el}}}{A_{\text{total}}} + \bar{\gamma}_0 \right) \frac{d\bar{P}_s}{d\bar{\gamma}}(\bar{\gamma}_0) - \bar{P}_s(\bar{\gamma}_0) + 1 = 0 . \quad (14)$$

It is to be noted that the only parameters that influence $\bar{\gamma}_0$ are the mean symbol error rate, $\bar{P}_s(\bar{\gamma})$, the number of payload symbols per frame, λ , and the ratio between the power consumption of electronic components, P_{el} , and the coefficient A_{total} , which is proportional to the irradiated power.

Equation (14) can be used to find the optimal SNR for different random channel models. The corresponding analysis for AWGN, Rayleigh and Nakagami- m channels is currently work in progress.

IV. OPTIMAL MODULATION AS A FUNCTION OF DISTANCE

We wish to understand how the energy consumption of a given modulation varies with the transmission distance. Our investigation will be focused on the case of fast fading channels as defined in Section II-B. We will study the energy consumption of M-QAM modulations compared against BPSK and BFSK, motivated by the popularity of these modulations among available of-the-shelf low-power transceiver components [3], [4].

A. Energy consumption analysis

Numerical evaluations of (13) using the parameters presented in Table I show that BPSK, BFSK and various M-QAM modulations attains its minimum energy consumption at a different SNR. As can be seen in Figure 1, the SNR at which these minima occur varies with transmission distance (curves are plotted against E_b/N_0 to compare the results against an equal amount of energy per bit.).

The mean energy consumption (13) evaluated at the optimal mean SNR as function of link distance ($\bar{\gamma}\{d\}$) gives the minimal consumption for that modulation at a given distance, which we denote as $\bar{\mathcal{E}}_b(d)$ (Figure 2). Analytically, $\bar{\mathcal{E}}_b(d)$ can be expressed from (10) as

$$\bar{\mathcal{E}}_b(d) = S + [P_{\text{el}} + P_{\text{PA}}\{d, \bar{\gamma}(d)\}] T \bar{\tau}\{\bar{\gamma}(d)\} . \quad (15)$$

In long range communications, the power consumed by the power amplifier (P_{PA}) dominates over the power consumed by the electronic components (P_{el}). Under these conditions, the

[‡]Source: [6]

TABLE I
GENERIC LOW-POWER DEVICE PARAMETERS

Parameter	Description	Value
R_s	Symbol rate	10 kBaud [‡]
L	Frame Payload	98 bits
O	Overhead	30 bits
F	Feedback frame length	11 bits
\mathcal{E}_{st}	Start-up energy	0.125 nJ [‡]
α	Path-loss coefficient	3.5 [‡]
A	Channel loss	30 dB [‡]
η	PA efficiency	0.35% [‡]
$P_{el,tx}$	Tx electric power consumption	98.2 mW [‡]
$P_{el,rx}$	Rx electric power consumption	112.5 mW [‡]
N_f	Receiver noise figure	10 dB [‡]
M_l	Link margin	40 dB [‡]

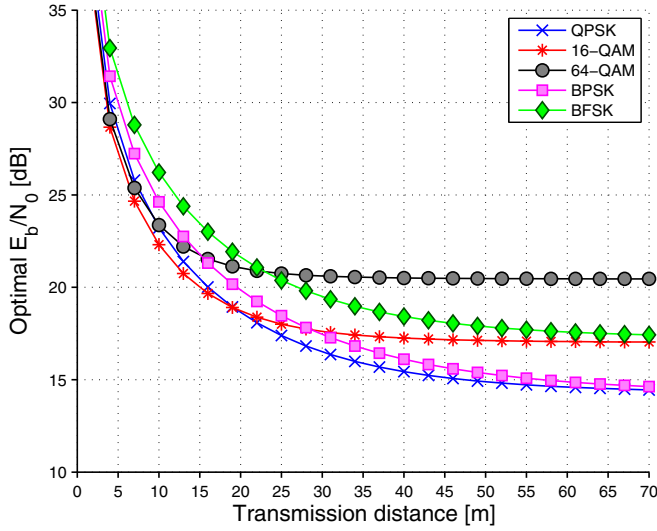


Fig. 1. Optimal SNR for achieving energy efficiency as function of link distance for a fast fading Rayleigh channel.

curves in Figure 2 confirm that the energy-optimal modulations are the ones with lowest spectral efficiency (low M -ary number).

However, for short transmissions distances (less than 15 meters in Figure 2) the power consumed by electronic components (P_{el}) dominates over the irradiated power and therefore also over the consumption of the power amplifier (P_{PA}). The energy consumption for this case can therefore be approximated as $\mathcal{E}_b(d) \approx S + P_{el}T\bar{\tau}\{\bar{\gamma}(d)\}$. This shows that for these conditions the average time per bit, T , becomes a relevant parameter in the total energy budget. This compels to pack more bits into each symbol in order to reduce the transmission time T of each bit.

It can further be seen in Figure 2 that for long range transmissions BPSK and QPSK are optimal among the studied modulations and nearly equally energy-efficient. At short distances, on the other hand, when P_{el} is relevant, BPSK almost doubles the energy consumption per bit of QPSK because of the doubly long time per bit.

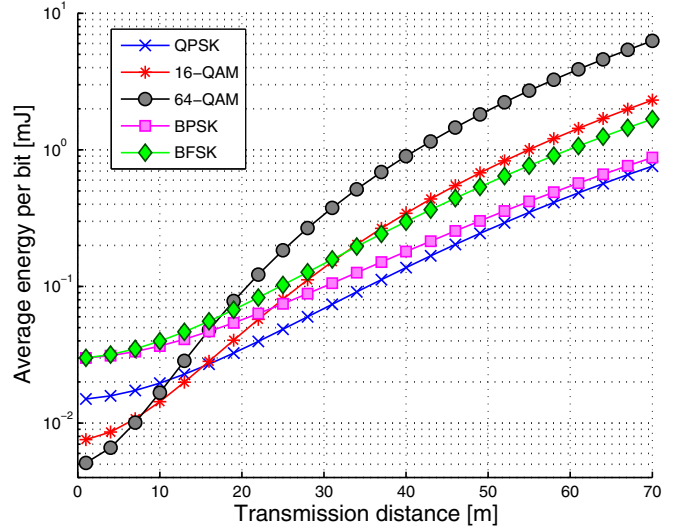


Fig. 2. Energy consumption per successfully transmitted bit of various modulations over a fast fading Rayleigh channel as a function of link distance. Each modulation is operated at its own optimal SNR. As distances decreases, modulations with higher spectral efficiency become optimal.

BFSK is never an optimal modulation (Figure 2). For long range transmissions it suffers the classic 3dB SNR gap with respect to BPSK and is hence suboptimal in the energy sense. For short range, it performs similarly to BPSK because of their equal spectral efficiency, but worse than larger size modulations.

B. Transceiver lifetime analysis

The results presented so far allow for studying the lifetime of networks with finite energy supply. For illustration, consider a simple network composed by two wireless sensor nodes with parameters as given in Table I. The nodes exchange 10 kbits of data every 5 minutes. Each node is powered by an ideal 1.2 Volt AA battery with a 2000 mAh initial energy charge. This charge is used exclusively for the communications tasks described in Section II.

Using this model, the average lifetime of the batteries of these two nodes was calculated for BPSK, BFSK and M-QAM transmissions over different channel models as a function of link distance, with each modulation operated at its optimal SNR. It was found that as distance decreases, the longest network lifetime is achieved by more spectrally efficient modulations (Figure 3 for the AWGN channel and Figure 4 for fast fading Rayleigh channel). It is apparent that, regardless of the channel type, lifetime extensions up to 600% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

V. CONCLUSIONS

We studied the optimization of the SNR and modulation size in order to minimize the energy consumed by a transceiver for delivering one bit of data without error. In our study we considered different transmission distances and various

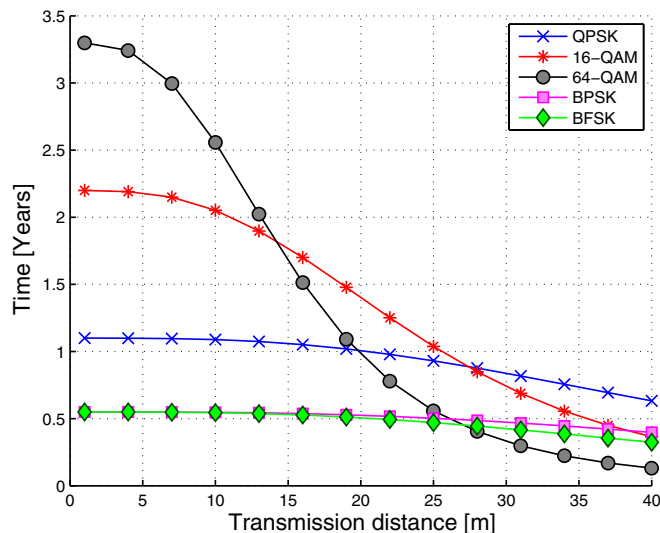


Fig. 3. Lifetime of two wireless sensor nodes that exchange 10 kbits of payload data every five minutes over an AWGN channel. At short transmission distances, BPSK yields a shorter lifetime than higher order modulations.

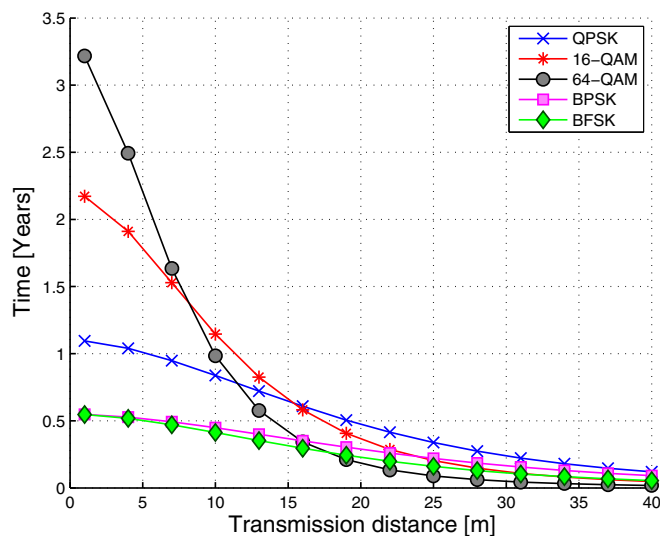


Fig. 4. Same as in Figure 3 but for the fast fading Rayleigh channel case. It is to be noted that the optimality of more spectrally efficient modulations at short distances holds.

channel statistics, as well as the energy cost of retransmissions, feedback frames and the consumption of electronic components.

We found that for a given modulation scheme the average energy consumed per bit by transmissions over a fast fading channel as function of the SNR has a unique minimum value, which is obtained at an SNR which is optimal in the energy consumption sense. The parameters that influence this optimal SNR are the mean symbol error rate, the number of payload symbols per transmission frame and the ratio between the power consumption of electronic components versus the irradiated power.

We prove that transferring successfully one frame of data across a fading channel in which the SNR levels of any two frame transmission trials are statistically independent takes, on the average, fewer transmission attempts than doing it over static channels.

We also found that for long transmission distances, low bandwidth efficiency modulations (small M -ary number, like BPSK) are optimal in the energy consumption sense. As the transmission distance shortens the optimal modulation size grows. In short range communications the power consumed by electronic components dominates over the irradiated power, and hence also does so over the energy consumption of the power amplifier. Under these conditions the average air time spent per data bit becomes a relevant parameter in the total energy budget. This makes optimal to pack more bits into each symbol and thereby to choose a larger modulation size. Finally, our results show that lifetime extensions up to 600% can be gained in short range networks by selecting modulations with larger constellations than BPSK.

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