Non-regenerative Multi-way Relaying: Ordered MMSE-SIC Receivers Exploiting Temporal Diversity

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***Abstract*—We consider the multi-way relaying (MWR) network, where each user communicate with the other users through a relay. The single-antenna users transmit to the multi-antenna relay simultaneously. Then the relay randomly precodes the received or estimated signal, and broadcasts it back to all the users. Focusing on a specific user, we set up the equivalent space-time channel from the other users to it. Afterwards, for both *amplify-and-forward* and *estimate-and-forward* MWR, we design the ordered minimum mean square error (MMSE) receivers with successive interference cancellation (SIC) at users. In particular, three types of metrics determining the order of SIC are derived by approximating the interference-plus-noise term as Gaussian. Simulation results on error performance evidence that our proposed receivers are able to harvest the inherent temporal diversity gains within the MWR networks. Moreover, the proposed receivers perform even well in the overloaded scenarios.**

I. Introduction

In wireless applications, such as emergence and disaster relief, multi-player gaming and so forth, users desire to exchange their data with each other. Motivated by two-way relaying [1], the multi-way relaying (MWR) [2, 3] has been proposed to fulfill the data exchange among users with the aid of a relay. To illustrate the multi-way communication explicitly, we consider it as two consecutive phases, i.e., the multiple access phase (uplink slot) and the broadcast phase (multiple downlink slots). In uplink slot, all the users transmit to the relay simultaneously. Afterwards, the relay broadcasts the post-processing signals to all the users during multiple downlink slots.

For non-regenerative MWR, [2, 4] proposed the linear receiver, switcher and precoder at the relay to simplify the receiver structure of users. With side information, i.e., the self-interference and previously detected symbols, the physical-layer network coding can thus be applied to enhance the throughput [5].

However, in this paper, we pay more attention to the broadcast phase than previous works. It is clear that using the precoder at the relay per downlink slot may eliminate the co-channel interference (CCI) for a specific user, but these CCI in other downlink slots are desired signals to that user. Hence, there is temporal diversity which can be exploited by the single-antenna users. With this observation, [3] designed the space-time MWR protocols, and [6] found the unified optimal precoder at the relay and its corresponding receivers at users.

In this paper, for both the *amplify-and-forward* MWR (AF MWR) and *estimate-and-forward* MWR (EF MWR), we aim to exploit the temporal diversity gains within multiple downlink slots. To this end, we employ the ordered minimum mean square error (MMSE) receiver with successive interference cancellation (SIC) at the users. Correspondingly, we use the pseudo random precoding at the relay to separate the levels of detection reliability as much as possible.

Particularly, we first set up the equivalent space-time channel from the other users to a specific user, and then carry out the initial MMSE estimate. By approximating the interference-plus-noise term as Gaussian, we derive three types of performance metrics on detection reliability for the equivalent MWR channel, whereas the reliability metrics in [7] are obtained for the multiple access channel. Regarding to the derived metrics on detection reliability, the ordered MMSE-SIC receivers are implemented from the most reliable symbol to the most unreliable one. Simulation results on error performance evidence that the proposed receivers achieve the temporal diversity gains over MMSE-based precoder and other SIC scheme.

The following notations are adopted in this paper. Italic boldface lowercase and uppercase letters denote vectors and matrices; the transpose, conjugate, conjugate transpose, (m, n)th element and trace of the matrix $\boldsymbol{A}$ are denoted by $\boldsymbol{A}^T$, $\boldsymbol{A}^*$, $\boldsymbol{A}^H$, $\boldsymbol{A}(m, n)$ and $\mathrm{Tr}(\boldsymbol{A})$, respectively; $\boldsymbol{I}_M$ is the $M \times M$ identity matrix; $\mathbf{0}_M$ is the $M \times 1$ zero vector; diag$(\cdot)$ represents the diagonal operation on vector and $\mathbb{E}\{\cdot\}$ is statistical expectation.

II. System Description

Consider a multi-way relaying network shown in Fig. 1, where K single-antenna users exchange data through a N-antenna relay. We assume no direct link between any two of the users, as well as half-duplex transmission within this time-slotted system.

A. Multiple Access Process

In the uplink slot, all the users transmit to the relay simultaneously. Then the received signal at the relay is

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{v}, \tag{1}$$

where $\boldsymbol{s} = [s_1, s_2, ..., s_K]^T$ is the transmit symbol vector with independent and identically distributed (i.i.d.) random elements satisfying $\mathbb{E}\{|s_k|^2\} = 1$ for $\forall k$, $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, ..., \boldsymbol{h}_K] \in \mathbb{C}^{N\times K}$ whose column vector $\boldsymbol{h}_k = [h_{1k}, h_{2k}, ..., h_{Nk}]^T$ represents channel impulse response (CIR) from the kth user to the

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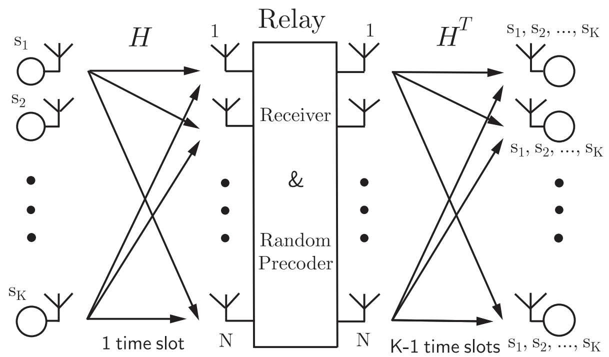

Fig. 1. Paradigm of non-regenerative MWR where K single-antenna users are exchanging information via a N-antenna relay.

relay, $\boldsymbol{v} \in \mathbb{C}^{N\times 1}$ is a zero-mean Gaussian noise vector at the relay with covariance matrix $\mathbb{E}\{\boldsymbol{v}\boldsymbol{v}^H\} = \sigma_v^2 \boldsymbol{I}_N$, and σ_v^2 is the noise variance at the relay. For simplicity, the BPSK signalling is applied in this work and more general signalling schemes, such as M-QAM, M-PSK, can be extended to the receivers proposed in the forthcoming sections.

B. Relay Processing

In this work, the relay either amplifies or estimates the received signal in (1). Now, we prepare the vector $\boldsymbol{x}_i$ for the broadcast phase, where the subscript $i \in \{AF, EF\}$ indicates the AF or EF MWR scheme we employ.

1) Amplify-and-Forward: For AF MWR, it is straightforward that $\boldsymbol{x}_{AF} = 1/\sqrt{\beta_{AF}}\boldsymbol{r}$ where β_{AF} is the power control parameter at the relay.

2) Estimate-and-Forward: For EF MWR, the relay employs the linear MMSE filter [8] to estimate all the symbols. The estimated vector $\boldsymbol{z} = [z_1, z_2, ..., z_K]^T$ can be expressed as

$$\boldsymbol{z} = \boldsymbol{W}^H \boldsymbol{r}, \tag{2}$$

where $\boldsymbol{W} = \left(\boldsymbol{H}\boldsymbol{H}^H + \sigma_v^2 \boldsymbol{I}_N\right)^{-1} \boldsymbol{H}$ is the MMSE weighting matrix.

Thus, we denote $\boldsymbol{x}_{EF} = 1/\sqrt{\beta_{EF}}\boldsymbol{z}$ where β_{EF} is the power control parameter as well.

C. Broadcast Phase

The relay randomly precodes $\boldsymbol{x}_i, i \in \{AF, EF\}$, and then broadcasts it back to all the users during $K-1$ downlink time slots. We assume the quasi-static fading channel with reciprocal property, implying that the channel matrix from the relay to the users remains $\boldsymbol{H}^T$ within $K-1$ downlink time slots.

Particularly, for either AF or EF MWR, the received signal at the kth user of downlink slot t is

$$\begin{aligned} \boldsymbol{y}_{k,i}[t] &= \boldsymbol{h}_k^T \boldsymbol{G}_i[t]\boldsymbol{x}_i + n_{k,i}[t], \\ & t \in [1, ..., K-1], k \in [1, ..., K], i \in [AF, EF], \end{aligned} \tag{3}$$

where $\boldsymbol{G}_i[t]$ is the random precoding matrix of the tth time slot, $n_{k,i}[t]$ is the zero-mean Gaussian noise satisfying $\mathbb{E}\{|n_{k,i}[t]|^2\} = \sigma_k^2$, and σ_k^2 is the noise variance at the kth user.

During the $K-1$ downlink time slots, the received signal vector $\boldsymbol{y}_{k,i} = [\boldsymbol{y}_{k,i}[1], \boldsymbol{y}_{k,i}[2], ..., \boldsymbol{y}_{k,i}[K-1]]^T$ at the kth user can be expressed as

$$\begin{aligned} \boldsymbol{y}_{k,i} &= \boldsymbol{H}_k \boldsymbol{G}_i \boldsymbol{x}_i + \boldsymbol{n}_{k,i}, \\ & k \in [1, ..., K], i \in [AF, EF], \end{aligned} \tag{4}$$

where $\boldsymbol{G}_i = \left[\boldsymbol{G}_i[1]^T, \boldsymbol{G}_i[2]^T, ..., \boldsymbol{G}_i[K-1]^T\right]^T$, $\boldsymbol{H}_k = \text{diag}\left(\boldsymbol{h}_k^T, \boldsymbol{h}_k^T, ..., \boldsymbol{h}_k^T\right) \in \mathbb{C}^{K-1\times(K-1)N}$ and $\boldsymbol{n}_{k,i} = [n_{k,i}[1], n_{k,i}[2], ..., n_{k,i}[K-1]]^T \in \mathbb{C}^{(K-1)\times 1}$ satisfying $\mathbb{E}\{\boldsymbol{n}_{k,i}\boldsymbol{n}_{k,i}^H\} = \sigma_k^2 \boldsymbol{I}_{K-1}$.

For the ordered MMSE-SIC receivers applied at the users, we expect the levels of detection reliability to be separated as much as possible [9]. Therefore, in (4), for both AF and EF MWR, we employ the pseudo random precoding [10] at the relay to facilitate such receivers.

Specifically, for downlink slot $t, t \in [1, ..., K-1]$, we first generate the random matrix $\boldsymbol{\Psi}[t] \in \mathbb{R}^{N\times K}$ whose elements are i.i.d. Gaussian random variables with zero mean and unit variance. By using the QR-decomposition, we obtain $\boldsymbol{\Psi}[t] = \boldsymbol{G}_{AF}[t]\boldsymbol{S}_{AF}[t]$ where $\boldsymbol{G}_{AF}[t] \in \mathbb{R}^{N\times N}$ is used for AF MWR. Similarly, we decompose $\boldsymbol{\Psi}[t]^T = \boldsymbol{G}_{EF}^{all}[t]\boldsymbol{S}_{EF}[t]$ and select the first N rows of $\boldsymbol{G}_{EF}^{all}[t]$ to construct $\boldsymbol{G}_{EF}[t] \in \mathbb{R}^{N\times K}$. Consequently, we achieve the orthonormal random precoding matrix $\boldsymbol{G}_i, i \in \{AF, EF\}$ in (4).

Furthermore, we also consider the average power constraint P_R per slot at the relay. Correspondingly, we have

$$\beta_{AF} = \frac{\mathbb{E}\{(\boldsymbol{G}_{AF}[t]\boldsymbol{r})^H \boldsymbol{G}_{AF}[t]\boldsymbol{r}\}}{P_R} = \frac{\text{Tr}\left(\boldsymbol{H}\boldsymbol{H}^H + \sigma_v^2 \boldsymbol{I}_N\right)}{P_R}, \tag{5}$$

$$\beta_{EF} = \frac{\mathbb{E}\{(\boldsymbol{G}_{EF}[t]\boldsymbol{z})^H \boldsymbol{G}_{EF}[t]\boldsymbol{z}\}}{P_R} = \frac{\text{Tr}\left(\boldsymbol{z}\boldsymbol{z}^H\right)}{P_R}. \tag{6}$$

III. MMSE-SIC RECEIVER FOR AF MWR

To exploit the temporal diversity gains at users, we propose three types of ordered MMSE-SIC receivers for AF MWR in this section. Without loss of generality, we next focus on user 1, and establish the equivalent channel from the other users to it. After the self-interference cancellation, user 1 determines the order of SIC by using the initial MMSE estimate.

Specifically, by substituting (1) into (4) and considering the relay transmit power, we attain the channel model for user 1 as

$$\boldsymbol{y}_{1,AF} = \frac{1}{\sqrt{\beta_{AF}}}\boldsymbol{H}_1 \boldsymbol{G}_{AF}\boldsymbol{H}\boldsymbol{s} + \frac{1}{\sqrt{\beta_{AF}}}\boldsymbol{H}_1 \boldsymbol{G}_{AF}\boldsymbol{v} + \boldsymbol{n}_{1,AF}. \tag{7}$$

For notional simplicity, we henceforth drop the subscript "1" of user 1 and then rewrite (7) as

$$\boldsymbol{y}_{AF} = \boldsymbol{H}_{AF}\boldsymbol{s} + \tilde{\boldsymbol{n}}_{AF}, \tag{8}$$

where $\boldsymbol{H}_{AF} = 1/\sqrt{\beta_{AF}}\boldsymbol{H}_1\boldsymbol{G}_{AF}\boldsymbol{H}$ denotes the equivalent channel from all the users to user 1 (including the self-interference) and $\tilde{\boldsymbol{n}}_{AF} = 1/\sqrt{\beta_{AF}}\boldsymbol{H}_1\boldsymbol{G}\boldsymbol{v} + \boldsymbol{n}_{1,AF}$ is the equivalent noise vector with correlation matrix $\boldsymbol{R}_{\tilde{n}_{AF}} = \mathbb{E}\{\tilde{\boldsymbol{n}}_{AF}\tilde{\boldsymbol{n}}_{AF}^H\} = 1/\sqrt{\beta_{AF}}\sigma_v^2\boldsymbol{H}_1\boldsymbol{H}_1^H + \sigma_1^2\boldsymbol{I}_{K-1}$. In addition, we can also express the equivalent channel in (8) as $\boldsymbol{H}_{AF} = [\boldsymbol{h}_{AF,1}, \boldsymbol{h}_{AF,2}, ..., \boldsymbol{h}_{AF,K}] \in \mathbb{C}^{(K-1)\times K}$ where the column vector $\boldsymbol{h}_{AF,k}, k \in [2, ..., K]$ represents the equivalent CIR from user k to user 1.

With the side information s_1, user 1 subtracts the self-interference from (8). Then we denote $\bar{\boldsymbol{y}}_{AF} = \boldsymbol{y}_{AF} - \boldsymbol{h}_{AF,1}s_1$ and $\bar{\boldsymbol{H}}_{AF} = [\boldsymbol{0}_{K-1}, \boldsymbol{h}_{AF,2}, ..., \boldsymbol{h}_{AF,K}]$, respectively.

Afterwards, to prepare the metrics indicating the detection reliability, we carry out the initial MMSE estimate as

$$\boldsymbol{u}_{AF} = \Re\{\boldsymbol{W}_{AF}^H \bar{\boldsymbol{y}}_{AF}\}, \tag{9}$$

where $\boldsymbol{W}_{AF} = \left(\bar{\boldsymbol{H}}_{AF}\bar{\boldsymbol{H}}_{AF}^H + \boldsymbol{R}_{\tilde{n}_{AF}}\right)^{-1}\bar{\boldsymbol{H}}_{AF}$ is MMSE weighting matrix for AF MWR. To derive the metrics of user $k, k \in [2, ..., K]$ individually, we express the kth column of $\boldsymbol{W}_{AF}$ as

$$\boldsymbol{\omega}_{AF,k} = \frac{\boldsymbol{R}_{AF,k}^{-1}\boldsymbol{h}_{AF,k}}{1+\boldsymbol{h}_{AF,k}^H\boldsymbol{R}_{AF,k}^{-1}\boldsymbol{h}_{AF,k}}, \tag{10}$$

where we denote $\boldsymbol{R}_{AF,k} \triangleq \sum_{l\neq k, l\neq 1}\boldsymbol{h}_{AF,l}\boldsymbol{h}_{AF,l}^H + \sigma_1^2\boldsymbol{I}_{K-1}$. Then, we obtain $u_{AF,k} = \Re\{\boldsymbol{\omega}_{AF,k}^H\bar{\boldsymbol{y}}_{AF}\}$.

A. SINR-based Receiver for AF

As conventional MMSE-SIC receiver [9], we next derive $\text{SINR}_{AF,k}, k \in [2, ..., K]$ for determining the order of SIC as

$$\gamma_{_{AF,k}} = \boldsymbol{h}_{AF,k}^H\boldsymbol{R}_{AF,k}^{-1}\boldsymbol{h}_{AF,k} = \frac{\boldsymbol{h}_{AF,k}^H\boldsymbol{R}_{\bar{y}_{AF}}^{-1}\boldsymbol{h}_{AF,k}}{1-\boldsymbol{h}_{AF,k}^H\boldsymbol{R}_{\bar{y}_{AF}}^{-1}\boldsymbol{h}_{AF,k}}, \tag{11}$$

in which we denote $\boldsymbol{R}_{\bar{y}_{AF}} \triangleq \bar{\boldsymbol{H}}_{AF}\bar{\boldsymbol{H}}_{AF}^H + \boldsymbol{R}_{\tilde{n}_{AF}}$.

B. Magnitude of MMSE Estimate-based Receiver for AF

To be explicit, we rewrite $u_{AF,k}$ as

$$u_{AF,k} = \frac{\gamma_{_{AF,k}}}{1+\gamma_{_{AF,k}}}s_k + \underbrace{\Re\left\{\frac{\boldsymbol{h}_{AF,k}^H\boldsymbol{R}_{AF,k}^{-1}}{1+\gamma_{_{AF,k}}}\left(\sum_{l\neq k, l\neq 1}\boldsymbol{h}_{AF,l}s_l + \tilde{\boldsymbol{n}}_{AF}\right)\right\}}_{G_{AF,k}}. \tag{12}$$

Then straightforwardly, in the case of BPSK modulation, the magnitude of the linear MMSE estimate's output, i.e., $|u_{AF,k}|, k \in [2, ..., K]$ can be viewed as the metric implying the detection reliability. Specifically speaking, the higher value $|u_{AF,k}|$ has, the higher detection reliability is with symbol s_k.

C. ML-based Receiver for AF

Alternatively, we may measure the detection reliability in the ML criteria [7]. Owing to the random precoding applied at the relay, the interference-plus-noise term $G_{AF,k}$ in (12) can be well approximated by Gaussian random variable [11] with mean zero and variance $\sigma_{AF,k}^2$. Hence given s_k, $u_{AF,k}$ can be considered as a Gaussian random variable with mean $m_{AF,k} = \gamma_{_{AF,k}}/(1+\gamma_{_{AF,k}})s_k$ and variance $\sigma_{AF,k}^2 = \gamma_{_{AF,k}}/(2(1+\gamma_{_{AF,k}})^2)$.

In particular, assuming the entries in $\boldsymbol{u}_{AF}$ are mutually independent, we thus evaluate the detection reliability using the ML criteria in the form of log-likelihood ratio (LLR) as

$$L_{AF,k}^{ml} = \left|\ln\left[\frac{f(u_{AF,k}|s_k=+1)}{f(u_{AF,k}|s_k=-1)}\right]\right|, \tag{13}$$

where the conditional PDF is listed below

$$f(u_{AF,k}|s_k) = \frac{1}{\sqrt{2\pi}\sigma_{AF,k}}\exp\left[-\frac{(u_{AF,k}-m_{AF,k})^2}{2\sigma_{AF,k}^2}\right]. \tag{14}$$

By substituting (14) into (13), we consequently obtain the detection reliability of $s_k, k \in [2, ..., K]$ as

$$L_{AF,k}^{ml} = 4\left(1+\gamma_{_{AF,k}}\right)|u_{AF,k}|, \tag{15}$$

where the metric of detection reliability in (15) involves both the $\gamma_{_{AF,k}}$ and $|u_{AF,k}|$ to arrange the order of SIC.

For AF MWR, we have derived three metrics implying the detection reliability, according to which the ordered MMSE-SIC receivers detects from the most reliable symbol to the most unreliable one. Simulation results on error performance are provided and compared later.

IV. MMSE-SIC Receiver for EF MWR

For EF MWR, we propose three types of ordered MMSE-SIC receivers to harvest the temporal diversity gains at users. Unlink the receivers in Section III, we aim to minimize the mean square error between the estimated vector $\boldsymbol{z}$ and the symbol vector $\boldsymbol{s}$. But similarly, we also concentrate on user 1, and derive three types of metrics determining the order of SIC.

Particularly, by substituting the estimated vector $\boldsymbol{z}$ into (4), we obtain the equivalent channel for user 1 as

$$\boldsymbol{y}_{1,EF} = \frac{1}{\sqrt{\beta_{EF}}}\boldsymbol{H}_1\boldsymbol{G}_{EF}\boldsymbol{z} + \boldsymbol{n}_{1,EF}. \tag{16}$$

For notional simplicity, we drop the subscript "1" of user 1 and rewrite (16) as

$$\boldsymbol{y}_{EF} = \boldsymbol{H}_{EF}\boldsymbol{z} + \boldsymbol{n}_{EF}, \tag{17}$$

where $\boldsymbol{H}_{EF} = 1/\sqrt{\beta_{EF}}\boldsymbol{H}_1\boldsymbol{G}_{EF}$ denotes the equivalent channel from the other users to user 1. Additionally, we also express $\boldsymbol{H}_{EF} = [\boldsymbol{h}_{EF,1}, \boldsymbol{h}_{EF,2}, ..., \boldsymbol{h}_{EF,K}] \in \mathbb{C}^{K-1\times K}$ whose column vector $\boldsymbol{h}_{EF,k}, k \in [1, .., K]$ represents the equivalent CIR from user k to user 1.

Given the equivalent channel in (17), user 1 first subtracts the self-interference to obtain $\bar{\boldsymbol{y}}_{EF} = \boldsymbol{y}_{EF} - \boldsymbol{h}_{EF,1}s_1$. Afterwards, we denote $\bar{\boldsymbol{H}}_{EF} = [\boldsymbol{0}_{K-1}, \boldsymbol{h}_{EF,2}, ..., \boldsymbol{h}_{EF,K}]$, and implement the initial MMSE estimate as

$$\boldsymbol{u}_{_{EF}} = \Re\{\boldsymbol{W}_{EF}^H\bar{\boldsymbol{y}}_{EF}\}, \tag{18}$$

where $\boldsymbol{W}_{EF} = \left(\bar{\boldsymbol{H}}_{EF}\boldsymbol{R}_{zz}\bar{\boldsymbol{H}}_{EF}^H + \sigma_1^2\boldsymbol{I}_{K-1}\right)^{-1}\bar{\boldsymbol{H}}_{EF}\boldsymbol{R}_{zs}$ is MMSE weighting matrix.

Now, let us define and derive the correlation matrices $\boldsymbol{R}_{zz}$ and $\boldsymbol{R}_{zs}$ in $\boldsymbol{W}_{EF}$. Specifically, recall the linear MMSE estimator we employed at the relay in Section II and express the individual entry z_k as

$$z_k = \boldsymbol{\omega}_k^H\boldsymbol{r}, \;\; k \in [1, ..., K], \tag{19}$$

where $\boldsymbol{\omega}_k = \boldsymbol{R}_k^{-1}\boldsymbol{h}_k/(1+\boldsymbol{h}_k^H\boldsymbol{R}_k^{-1}\boldsymbol{h}_k)$ is the kth column vector of $\boldsymbol{W}$ with $\boldsymbol{R}_k \triangleq \sum_{l\neq k}^K\boldsymbol{h}_l\boldsymbol{h}_l^H + \sigma_v^2\boldsymbol{I}_N$. Then, we may rewrite (19) as

$$z_k = \frac{\gamma_k}{1+\gamma_k}s_k + \underbrace{\Re\left\{\frac{\boldsymbol{h}_k^H\boldsymbol{R}_k^{-1}}{1+\gamma_k}\left(\sum_{l\neq k}^K\boldsymbol{h}_l s_l + \boldsymbol{v}\right)\right\}}_{G_k}, \tag{20}$$

where $\gamma_k = \boldsymbol{h}_k^H\boldsymbol{R}_k^{-1}\boldsymbol{h}_k$ is the SINR for user k and the interference-plus-noise term G_k in (20) can be well approximated as Gaussian random variable [11] with mean zero and variance $\gamma_k/(2(1+\gamma_k)^2)$.

Therefore, by using (20), we calculate the correlation matrix $\boldsymbol{R}_{zz} \triangleq \mathbb{E}\{\boldsymbol{z}\boldsymbol{z}^H\} = \text{diag}\left(\frac{2\gamma_1^2+\gamma_1}{2(1+\gamma_1)^2}, \frac{2\gamma_2^2+\gamma_2}{2(1+\gamma_2)^2}, ..., \frac{2\gamma_K^2+\gamma_K}{2(1+\gamma_K)^2}\right)$ and $\boldsymbol{R}_{zs} \triangleq \mathbb{E}\{\boldsymbol{z}\boldsymbol{s}^H\} = \text{diag}\left(\frac{\gamma_1}{1+\gamma_1}, \frac{\gamma_2}{1+\gamma_2}, ..., \frac{\gamma_K}{1+\gamma_K}\right)$.

Now let $\hat{\boldsymbol{H}}_{EF} \triangleq \bar{\boldsymbol{H}}_{EF}\boldsymbol{R}_{zz}^{\frac{1}{2}}$ in (18), then we may rewrite (18) as

$$\boldsymbol{W}_{EF} = \left(\hat{\boldsymbol{H}}_{EF}\hat{\boldsymbol{H}}_{EF}^H + \sigma^2\boldsymbol{I}_{K-1}\right)^{-1}\hat{\boldsymbol{H}}_{EF}\boldsymbol{V}_z, \tag{21}$$

where $\boldsymbol{V}_z \triangleq \boldsymbol{R}_{zz}^{-\frac{1}{2}}\boldsymbol{R}_{zs}$ is a diagonal matrix. By denoting $\hat{\boldsymbol{H}}_{EF} \triangleq [\boldsymbol{0}, \hat{\boldsymbol{h}}_{EF,2}, ..., \hat{\boldsymbol{h}}_{EF,K}] \in \mathbb{C}^{(K-1)\times K}$, we express the kth column of $\boldsymbol{W}_{EF}$ as

$$\boldsymbol{\omega}_{EF,k} = \frac{\hat{\boldsymbol{R}}_{EF,k}^{-1}\hat{\boldsymbol{h}}_{EF,k}\boldsymbol{V}_z(k,k)}{1+\hat{\boldsymbol{h}}_{EF,k}^H\hat{\boldsymbol{R}}_{EF,k}^{-1}\hat{\boldsymbol{h}}_{EF,k}}, \tag{22}$$

where we have $\hat{\boldsymbol{R}}_{EF,k} \triangleq \sum_{l\neq k, l\neq 1}\hat{\boldsymbol{h}}_{EF,l}\hat{\boldsymbol{h}}_{EF,l}^H + \sigma_1^2\boldsymbol{I}_{K-1}$.

A. SINR-based Receiver for EF

Similarly with AF MWR, we first measure the detection reliability via $\text{SINR}_{EF,k}, k \in [2, ..., K]$ which is derived as

$$\gamma_{EF,k} = \hat{\boldsymbol{h}}_{EF,k}^H\hat{\boldsymbol{R}}_{EF,k}^{-1}\hat{\boldsymbol{h}}_{EF,k} = \frac{\hat{\boldsymbol{h}}_{EF,k}^H\boldsymbol{R}_{\bar{y}_{EF}}^{-1}\hat{\boldsymbol{h}}_{EF,k}}{1-\hat{\boldsymbol{h}}_{EF,k}^H\boldsymbol{R}_{\bar{y}_{EF}}^{-1}\hat{\boldsymbol{h}}_{EF,k}}, \tag{23}$$

where we denote $\boldsymbol{R}_{\bar{y}_{EF}} \triangleq \bar{\boldsymbol{H}}_{EF}\boldsymbol{R}_{zz}\bar{\boldsymbol{H}}_{EF}^H + \sigma_1^2\boldsymbol{I}_{K-1}$.

B. Magnitude of MMSE Estimate-based Receiver for EF

Then, the output of the initial MMSE estimate is

$$u_{EF,k} = \Re\left\{\boldsymbol{\omega}_{EF,k}^H\bar{\boldsymbol{y}}_{EF}\right\}, \quad k \in [2, ..., K]. \tag{24}$$

Remind the discussion on (12), the magnitude of $u_{EF,k}, k \in [2, ..., K]$ implies the detection reliability of user k.

C. ML-based Receiver for EF

Alternatively, to measure the reliability of each symbol in the ML criteria, we can express (24) as

$$u_{EF,k} = \frac{\gamma_{EF,k}}{1+\gamma_{EF,k}}z_k + \frac{\boldsymbol{V}_z(k,k)}{1+\gamma_{EF,k}} \times \Re\left\{\hat{\boldsymbol{h}}_{EF,k}^H\hat{\boldsymbol{R}}_{EF,k}^{-1}\left(\sum_{l\neq k, l\neq 1}^{K}\hat{\boldsymbol{h}}_{EF,l}z_l + \boldsymbol{n}_{EF}\right)\right\}. \tag{25}$$

By substituting (20) into (25), we rewrite (25) as

$$u_{EF,k} = \frac{\gamma_{EF,k}}{1+\gamma_{EF,k}}\frac{\gamma_k}{1+\gamma_k}s_k + \underbrace{\frac{\gamma_{EF,k}}{1+\gamma_{EF,k}}\Re\left\{\frac{\hat{\boldsymbol{h}}_{EF,k}^H\hat{\boldsymbol{R}}_{EF,k}^{-1}}{1+\gamma_k}\left(\sum_{l\neq k}^{K}\boldsymbol{h}_l s_l + \boldsymbol{v}\right)\right\}}_{G_{EF,k}^a} + \underbrace{\Re\left\{\frac{\boldsymbol{V}_z(k,k)\hat{\boldsymbol{h}}_{EF,k}^H\hat{\boldsymbol{R}}_{EF,k}^{-1}}{1+\gamma_{EF,k}}\left(\sum_{l\neq k}^{K}\hat{\boldsymbol{h}}_{EF,l}z_l + \boldsymbol{n}\right)\right\}}_{G_{EF,k}^b}, \tag{26}$$

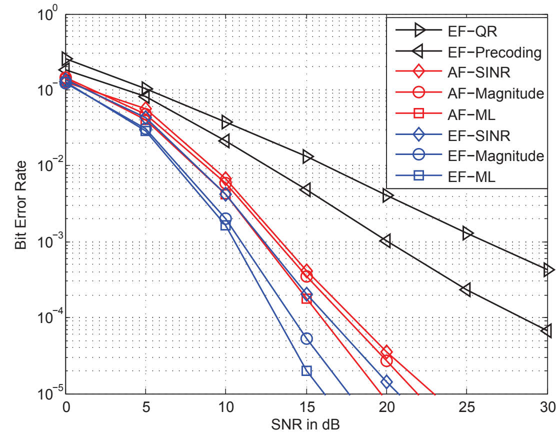

Fig. 2. Comparisons on BER when $K = N = 10$ and $\text{SNR}_{up} = \text{SNR}_{dl} = \text{SNR}$ for all the precoder and receivers.

where the second and third term of (26) can be well approximated by zero mean Gaussian random variables $G_{EF,k}^a$ and $G_{EF,k}^b$, respectively. Then, we calculate the variance of those two random variables in (26) as $\mathbb{E}\{|G_{EF,k}^a|^2\} = \frac{\gamma_{EF,k}^2}{(1+\gamma_{EF,k})^2}\frac{\gamma_k}{2(1+\gamma_k)^2}$ and $\mathbb{E}\{|G_{EF,k}^b|^2\} = \frac{\gamma_{EF,k}}{2(1+\gamma_{EF,k})^2}\frac{2\gamma_k^2+\gamma_k}{2(1+\gamma_k)^2}$.

Recall the fact that the sum of two Gaussian random variables also obeys Gaussian distribution. Therefore, given s_k for $\forall k$, $u_{EF,k}$ can be closely approximated as a Gaussian random variable with mean $m_{EF,k}$ and variance $\sigma_{EF,k}^2$ which are

$$m_{EF,k} = \frac{\gamma_{EF,k}}{1+\gamma_{EF,k}}\frac{\gamma_k}{1+\gamma_k}s_k, \quad \sigma_{EF,k}^2 = \frac{\gamma_{EF,k}\gamma_k(2\gamma_{EF,k}+2\gamma_k+1)}{4(1+\gamma_{EF,k})^2(1+\gamma_k)^2}. \tag{27}$$

Now, let us restate how we measure the detection reliability in the ML criteria. In particular, we derive the LLR of user k as

$$L_{EF,k}^{ml} = \left|\ln\left[\frac{f(u_{EF,k}|s_k=+1)}{f(u_{EF,k}|s_k=-1)}\right]\right|, \quad k \in [2, ..., K], \tag{28}$$

where the conditional PDF is given as

$$f(u_{EF,k}|s_k) = \frac{1}{\sqrt{2\pi}\sigma_{EF,k}}\exp\left[-\frac{(u_{EF,k}-m_{EF,k})^2}{2\sigma_{EF,k}^2}\right]. \tag{29}$$

Thus, by substituting (27) and (29) into (28), we achieve the ML-based detection reliability of $s_k, k \in [2, ..., K]$ as

$$L_{EF,k}^{ml} = \frac{8(1+\gamma_k)(1+\gamma_{EF,k})}{2\gamma_k+2\gamma_{EF,k}+1}|u_{EF,k}|. \tag{30}$$

Regarding these reliability metrics derived in the section, the ordered MMSE-SIC receivers for EF MWR can be implemented. Simulation results on error performance are also given and compared below.

V. Simulation Results

In this section, we provide simulation results on average bit error rate (BER) to clearly illustrate the temporal diversity gains achieved by the proposed MMSE-SIC receivers. To make comparisons, we choose the MMSE-based linear precoder [2]

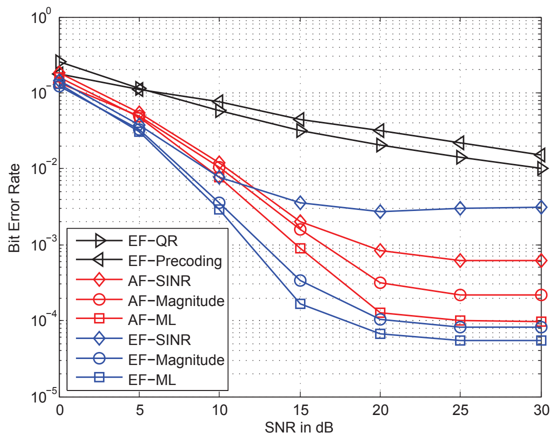

Fig. 3. Comparisons on BER when $K = N = 10$ and $\text{SNR}_{up} = 0$dB for all the precoder and receivers.

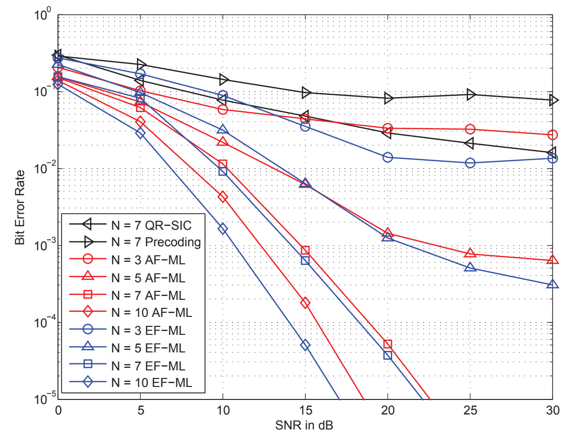

Fig. 4. Comparison on BER when $K = 10$ and $N = [3, 5, 7, 10]$ for the ML-based MMSE-SIC receivers.

at the relay and QR-decomposition based SIC receiver [12] at the users as benchmarks.

Without other statement, we specify the following setting to simulate the error performance. First, we assume independent quasi-static Rayleigh fading channel between each antenna pair, BPSK signalling between relay and users, and $K = 10$ users are involved in both AF and EF MWR. Second, we denote the uplink SNR as $\text{SNR}_{up} \triangleq P_S/\sigma_v^2$ where P_S is the transmit power per user, whereas $\text{SNR}_{dl} \triangleq P_R/\sigma_k^2$ is the downlink SNR.

In Fig. 2, we compare the error performance of the proposed receivers with the selected baseline schemes, when we set $\text{SNR}_{up} = \text{SNR}_{dl} = \text{SNR}$ and $N = 10$ antennas at the relay. It is clear that the proposed MMSE-SIC receivers greatly outperform the baseline schemes due to the temporal diversity gains achieved at the users. One may also observe that the receivers with EF MWR are slightly superior to those of AF MWR in this scenario. The reason lies in that for EF MWR, the relay estimates the symbol vector by minimizing the mean square error. Finally, we find that the ML-based receiver is advantageous to the magnitude-based receiver, where the latter outperforms the conventional SINR-based receiver.

We next compare the error performance when we set $\text{SNR}_{up} = 0$dB in Fig. 3. Within this scenario, there exist different error floors associated with the proposed MMSE-SIC receivers. In general, the error floors reflect the error performance achieved by corresponding receivers. Particularly, we find that the ML-based receiver of EF MWR achieves the lowest error floor, whereas the SINR-based receiver of EF MWR falls into the highest error floor among all the proposed receivers.

Finally, in Fig. 4, we compare the error performance of the ML-based receivers in the overloaded scenarios where $K > N$. As expected, the baseline schemes provide unacceptable error performance. But, for either AF or EF MWR, our proposed ML-based receivers yield practically acceptable error performance in the slightly overloaded scenario, e.g., $K = 10$ and $N = 7$.

VI. Conclusions

For both AF and EF MWR, we established the equivalent channel from the other users to an arbitrary user. After that, three types of reliability metrics determining the order of SIC were derived with the aid of Gaussian approximation. Then, the ordered MMSE-SIC receivers were implemented from the most reliable symbol to the most unreliable one. Therefore, as expected, the single-antenna users can harvest the temporal diversity gains within multiple downlink slots. Simulation results verified that our proposed receivers perform even well in the overloaded scenarios.

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