Joint Design of Linear Relay and Destination Processing for Two-hop MIMO Multi-relay Networks

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Abstract—This paper presents a joint linear processing scheme for two-hop and half-duplex distributed amplify-and-forward (AF) relaying networks with one source, one destination and multiple relays, each having multiple antennas. Under the perfect CSI (channel state information) assumption, a joint relay and destination design is proposed to minimize the mean-square error (MSE) of the output signal at the destination. By the Wiener filter principle, it is formulated as an optimization problem with respect to the relay precoding matrix under the constraint of a total relay transmit power. And then, the constrained optimization with an objective to design the relaying block-diagonal matrix is simplified to an equivalent problem with scalar optimization variables. Moreover, it is revealed that the scalar-version optimization is convex when the total relay power or the second-hop SNR (signal to noise ratio) is above a certain threshold. The underlying optimization problem, which is non-convex in general, is solved by the complementary geometric programming (CGP). Simulation results show that the proposed scheme outperforms the existing relaying method in the case of relatively large second-hop SNR.

Keywords-Joint Optimization; amplify-and-forward(AF); mean-square-error(MSE); MIMO relay networks

I. INTRODUCTION

Applying multiple-input multiple-output (MIMO) technology into cooperative relaying network has recently gained considerable attention since it can offer significant improvement in spectral efficiency and diversity gain [1,2]. As to the cooperation strategies, the amplify-and-forward (AF) relaying is relatively simple, where the relay just scales the signal from the source without decoding and then forwards it to the destination. There have been a lot of studies on the cooperative MIMO AF relay networks. The design of multiple relays in MIMO relay networks is much more complex since the relays are of distributive nature and need to process the multiple signal streams from the source. Some prior works on MIMO relay networks have been found in [3-5]. In [3], some simple matrix operation based relaying schemes have been proposed for MIMO relay networks without using optimization. The authors of [4] have designed the optimal joint processing to minimize the MSE, and maximize the defined SNR or the transmission rate in the relay networks, where the optimization problems are solved under a power constraint at the receiver. It should be mentioned that the power constraint at the receiver results in vague transmit power requirement due to the randomness of the second-hop channel. In [5], a relaying scheme is proposed to approximately satisfy the SNR requirements by the hypothesis that the overall channel matrix had been diagonalized. But the power efficiency of this scheme is not guaranteed when the exact SNR requirements need to be satisfied.

In this paper, we consider the minimum MSE (MMSE) criterion for the optimization design in the MIMO AF relay network with a source, a destination and multiple relays, all equipped with multiple antennas. The joint optimal linear processing for the relays and the destination is investigated under the perfect CSI assumptions. After obtaining the optimum destination receive filter, we design the relaying precoder under a transmit power constraint. Then, by the MMSE criterion, the constrained optimization problem with respect to (w.r.t.) the block-diagonal relay precoding matrix is converted to an optimization problem w.r.t. scalar variables. The simplified problem is finally solved via the complementary GP method. Monte Carlo simulations are performed to confirm the efficacy of the proposed optimization technique.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A two-hop MIMO AF relay network is considered where the source and destination terminals, each equipped with N antennas, communicate through K relay terminals each with L transmit/receive antennas (assuming $N \le L$). It is assumed that there is no direct link between the source and destination, and the transmission from the source to the relays and that from the relays to the destination are completed over two consecutive time slots in half-duplex mode. Also, we assume frequency-flat block fading for the two-hop channels.

Let $\mathbf{t}_k$ ($1 \le k \le K$) be an L-element vector denoting the signal transmitted by the k-th relay. So the signal transmitted by all the relays can then be expressed using the following LK-element vector as

$$\mathbf{t} = \left[\mathbf{t}_1^T, \ \cdots, \ \mathbf{t}_K^T\right]^T = \mathbf{FHx} + \mathbf{Fn} \qquad (1)$$

where $\mathbf{F}$ is the relay precoder matrix, the $LK\times N$ matrix $\mathbf{H} = \left[\mathbf{H}_1^T, \ \cdots, \ \mathbf{H}_K^T\right]^T$ represents the first-hop channel with $\mathbf{H}_k \in \mathbb{C}^{L\times N}$ being the random channel matrix between the source and the k-th relay, $\mathbf{x}$ is the $N\times 1$ transmitted signal vector with its correlation matrix $\mathbb{E}\left\{\mathbf{xx}^H\right\} = \sigma_x^2 \mathbf{I}_N$, and $\mathbf{n}$ is an $LK\times 1$ additive white Gaussian noise vector at the relays with $\mathbb{E}\left\{\mathbf{nn}^H\right\} = \sigma_n^2 \mathbf{I}_{LK}$. Due to the distribuitive relays, $\mathbf{F}$ in (1) is a block-diagonal matrix, namely, $\mathbf{F} = diag\left(\mathbf{F}_1, \ \cdots, \ \mathbf{F}_K\right)$, where the k-th diagonal partition $\mathbf{F}_k \in \mathbb{C}^{L\times L}$ is the precoding matrix of the k-th relay. Then, the received signal at the destination can be written as

$$\mathbf{y} = \mathbf{Gt} + \mathbf{z} = \mathbf{GFHx} + \mathbf{GFn} + \mathbf{z} \qquad (2)$$

where $\mathbf{G} = \left[\mathbf{G}_1, \ \cdots, \ \mathbf{G}_K\right]$ denotes the $N\times LK$ matrix of the second-hop channel with $\mathbf{G}_k \in \mathbb{C}^{N\times L}$ being the random channel matrix between the k-th relay and the destination, and $\mathbf{z}$ is an

$N\times 1$ additive white Gaussian noise vector at the destination with $\mathbb{E}\{\mathbf{z}\mathbf{z}^H\}=\sigma_z^2\mathbf{I}_N$.

At the destination, a linear decoder $\mathbf{W}\in\mathbb{C}^{N\times N}$ is then employed to detect the received signal, giving an estimate of the transmitted signal as

$$\hat{\mathbf{x}}=\mathbf{W}\mathbf{y}=\mathbf{W}\mathbf{G}\mathbf{F}\mathbf{H}\mathbf{x}+\mathbf{W}\mathbf{G}\mathbf{F}\mathbf{n}+\mathbf{W}\mathbf{z} \tag{3}$$

From the above discussion, the system MSE matrix is given by

$$\begin{aligned} MSE(\mathbf{F},\mathbf{W}) &= \mathbb{E}\left\{(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^H\right\} \\ &= \sigma_x^2(\mathbf{WGFH}-\mathbf{I})(\mathbf{WGFH}-\mathbf{I})^H \\ &\quad +\mathbf{W}\left(\sigma_n^2\mathbf{GFF}^H\mathbf{G}^H+\sigma_z^2\mathbf{I}\right)\mathbf{W}^H \end{aligned} \tag{4}$$

In this paper, the destination receive matrix $\mathbf{W}$ and the relay precoder matrix $\mathbf{F}$ are jointly designed to minimize the MSE of the transmitted symbol $\mathbf{x}$.

Note that when the relay block-diagonal matrix $\mathbf{F}$ is known, the optimum $\mathbf{W}$ can be found by setting the derivative of the trace of the MSE in (4) to zero, which yields the well-known Wiener filter [7] of $\mathbf{W}$ as given below

$$\mathbf{W}=\sigma_x^2\mathbf{H}^H\mathbf{F}^H\mathbf{G}^H\left(\sigma_x^2\mathbf{GFHH}^H\mathbf{F}^H\mathbf{G}^H+\sigma_n^2\mathbf{GFF}^H\mathbf{G}^H+\sigma_z^2\mathbf{I}\right)^{-1} \tag{5}$$

By substituting (5) into (4), and using the matrix inversion lemma $\mathbf{P}-\mathbf{PM}^H\left(\mathbf{MPM}^H+\mathbf{Q}\right)^{-1}\mathbf{MP}=\left(\mathbf{P}^{-1}+\mathbf{M}^H\mathbf{Q}^{-1}\mathbf{M}\right)^{-1}$, the minimum MSE can be obtained as

$$MSE(\mathbf{F})=\sigma_x^2\left\{\mathbf{I}+\frac{\sigma_x^2}{\sigma_n^2}\mathbf{H}^H\left[\mathbf{I}-\left(\mathbf{I}+\frac{\sigma_n^2}{\sigma_z^2}\mathbf{F}^H\mathbf{G}^H\mathbf{GF}\right)^{-1}\right]\mathbf{H}\right\}^{-1} \tag{6}$$

Thus, the joint design problem of relay and destination is converted to the MSE minimization problem with respect to the relay block-diagonal matrix $\mathbf{F}$. It is observed that without any constraint such as the relay transmitting power limit, the minimization of $MSE(\mathbf{F})$ in (6) would lead to an extreme solution of $\mathbf{F}$ whose norm tends to infinity. As such, we introduce the total relay power P_R as a constraint for the MSE minimization,

$$tr\left\{\mathbf{t}\mathbf{t}^H\right\}=tr\left\{\mathbf{F}\left(\sigma_x^2\mathbf{HH}^H+\sigma_n^2\mathbf{I}_{LK}\right)\mathbf{F}^H\right\}\le P_R \tag{7}$$

which leads to the following constrained optimization problem,

$$\begin{aligned} \min_{\mathbf{F}}\quad & tr\{MSE(\mathbf{F})\}=\sigma_x^2 tr\left\{\left\{\mathbf{I}+\frac{\sigma_x^2}{\sigma_n^2}\mathbf{H}^H\left[\mathbf{I}-\left(\mathbf{I}+\frac{\sigma_n^2}{\sigma_z^2}\mathbf{F}^H\mathbf{G}^H\mathbf{GF}\right)^{-1}\right]\mathbf{H}\right\}^{-1}\right\} \\ s.t.\quad & tr\left\{\mathbf{F}\left(\sigma_x^2\mathbf{HH}^H+\sigma_n^2\mathbf{I}_{LK}\right)\mathbf{F}^H\right\}\le P_R \end{aligned} \tag{8}$$

It is worth mentioning that a mutual information based optimization problem using the underlying system model has been considered in our previous work [9]. As the optimization problem therein is in gereral non-convex and very difficult to solve by the existing methods, the difficulty was then overcome by resorting to the diagonalization of the matrix involved in the mutual information expression. Following a similar idea, in this paper, we would like to tackle the non-convex problem (8) by pursuing a diagonal structure for the matrix for MSE ($\mathbf{F}$).

III. OPTIMAL RELAY DESIGN

We now consider the constrained MSE minimization problem (8) under the assumption of perfect CSI to determine the linear relay precoder $\mathbf{F}$. The objective function of (8) can also be expressed as

$$tr\{MSE(\mathbf{F})\}=\sigma_x^2 tr\left\{\mathbf{\Omega}^{-1}\right\} \tag{9}$$

where

$$\mathbf{\Omega}=\mathbf{I}+\frac{\sigma_x^2}{\sigma_n^2}\mathbf{H}^H\left[\mathbf{I}-\left(\mathbf{I}+\frac{\sigma_n^2}{\sigma_z^2}\mathbf{F}^H\mathbf{G}^H\mathbf{GF}\right)^{-1}\right]\mathbf{H} \tag{10}$$

It is proved in [8, Appendix I] that if $\mathbf{A}$ is an $N\times N$ positive definite matrix, the following inequality holds

$$tr\left\{\mathbf{A}^{-1}\right\}\ge\sum_{i=1}^{N}\frac{1}{a_{i,i}} \tag{11}$$

where $a_{i,i}$ is the i-th diagonal element of $\mathbf{A}$. Furthermore, the equality in (11) is attained if and only if $\mathbf{A}$ is diagonal. Then, we will determine the optimal relay matrix $\mathbf{F}$ such that $\mathbf{\Omega}$ is diagonal.

We first perform the singular value decomposition (SVD) of $\mathbf{H}$ to obtain,

$$\mathbf{H}=\mathbf{U}_H\mathbf{\Lambda}_H\mathbf{V}_H^H=\mathbf{U}_{H,1}\mathbf{\Lambda}_{H,1}\mathbf{V}_H^H \tag{12}$$

where $\mathbf{U}_H$ and $\mathbf{V}_H$ are unitary matrices of dimension $LK\times LK$ and $N\times N$, respectively, and $\mathbf{\Lambda}_H=\begin{Bmatrix}\mathbf{\Lambda}_{H,1}\\ \mathbf{0}\end{Bmatrix}$ is an $LK\times N$ matrix, $\mathbf{\Lambda}_{H,1}$ is $N\times N$ diagonal matrix, whose diagonal elements are the nonzero singular values of $\mathbf{H}$ assumed in decreasing order, $\mathbf{U}_{H,1}$ is comprised of the first N column vectors of $\mathbf{U}_H$ corresponding to the nonzero singular values arranged in decreasing order. On the other hand, the eigenvalue decomposition (EVD) of $\mathbf{G}^H\mathbf{G}$ can be written as,

$$\mathbf{G}^H\mathbf{G}=\mathbf{V}_{G,1}\mathbf{D}_{G,1}\mathbf{V}_{G,1}^H \tag{13}$$

where $\mathbf{D}_{G,1}$ is the N-dimension diagonal matrix ($N=rank\left(\mathbf{G}^H\mathbf{G}\right)$) whose diagonal elements are the nonzero eigenvalues of $\mathbf{G}^H\mathbf{G}$ arranged in decreasing order, and $\mathbf{V}_{G,1}\in\mathbb{C}^{LK\times N}$ is comprised of the eigenvectors corresponding to the nonzero eigenvalues. Using (12) and (13) into the objective function of (8), and letting

$$\mathbf{V}_{G,1}^H\mathbf{F}=\mathbf{\Sigma}_F\mathbf{U}_{H,1}^H \tag{14}$$

where $\mathbf{\Sigma}_F$ is a diagonal matrix, we obtain

$$tr\{MSE(\mathbf{F})\}=\sigma_x^2 tr\left\{\left\{\mathbf{I}+\frac{\sigma_x^2}{\sigma_n^2}\mathbf{D}_{H,1}\left[\mathbf{I}-\left(\mathbf{I}+\frac{\sigma_n^2}{\sigma_z^2}\mathbf{\Sigma}_F^H\mathbf{D}_{G,1}\mathbf{\Sigma}_F\right)^{-1}\right]\right\}^{-1}\right\} \tag{15}$$

where $\mathbf{D}_{H,1}=\mathbf{\Lambda}_H^H\mathbf{\Lambda}_H$ is the diagonal matrix of dimension $N=rank\left(\mathbf{HH}^H\right)$ ($N\le LK$) with its diagonal elements being the nonzero eigenvalues of $\mathbf{HH}^H$ arranged in decreasing order. Further, by partitioning $\mathbf{U}_{H,1}$ and $\mathbf{V}_{G,1}$ as

$$\mathbf{U}_{H,1}^H=\left[\underbrace{\left(\mathbf{U}_{H,1}^H\right)_1}_{N\times L},\ \left(\mathbf{U}_{H,1}^H\right)_2,\ \cdots,\ \left(\mathbf{U}_{H,1}^H\right)_K\right],$$

and

$\mathbf{V}_{G,1}^{H}=\left[\underbrace{\left(\mathbf{V}_{G,1}^{H}\right)_1}_{N\times L},\ \left(\mathbf{V}_{G,1}^{H}\right)_2,\ \cdots,\ \left(\mathbf{V}_{G,1}^{H}\right)_K\right]$, respectively, and applying them to (14), we obtain

$$\mathbf{F}_k=\left(\mathbf{V}_{G,1}^{H}\right)_k^{+}\boldsymbol{\Sigma}_F\left(\mathbf{U}_{H,1}^{H}\right)_k \tag{16}$$

where $(\ )^{+}$ denotes the pseudo inverse of a matrix. In obtaining (16), we have assumed $N\le L$. It is clear from (16) that as long as the diagonal matrix $\boldsymbol{\Sigma}_F$ is determined, the optimal relay block-diagonal matrix $\mathbf{F}$ can be obtained immediately.

We now rewrite the relay power constraint in (8) as a function of $\boldsymbol{\Sigma}_F$. First using the partitioned form of $\mathbf{F}$, we have

$$tr\left\{\mathbf{F}\left(\sigma_x^2\mathbf{H}\mathbf{H}^H+\sigma_n^2\mathbf{I}_{LK}\right)\mathbf{F}^H\right\}=\sum_{k=1}^{K}tr\left\{\mathbf{F}_k\left(\sigma_x^2\mathbf{H}_k\mathbf{H}_k^H+\sigma_n^2\mathbf{I}_L\right)\mathbf{F}_k^H\right\}\le P_R \tag{17}$$

Then, by substituting $\mathbf{F}_k=\left(\mathbf{V}_{G,1}^{H}\right)_k^{+}\boldsymbol{\Sigma}_F\left(\mathbf{U}_{H,1}^{H}\right)_k$ into (17) and noting that $tr\left\{\mathbf{AXBX}^H\right\}=\mathbf{x}^H\left(\mathbf{A}\circ\mathbf{B}^T\right)\mathbf{x}$, where $\circ$ denotes the Hadamard product, $\mathbf{A}$ and $\mathbf{B}$ are square matrices, $\mathbf{X}$ is a diagonal matrix and $\mathbf{x}=\mathbf{X1}$ ($\mathbf{1}=[1,\ \cdots,\ 1]^T$) is a vector composed of the diagonal elements of $\mathbf{X}$, the relay power constraint can be expressed in terms of $\boldsymbol{\Sigma}_F$ as

$$\mathbf{1}^H\boldsymbol{\Sigma}_F^H\boldsymbol{\Psi}\boldsymbol{\Sigma}_F\mathbf{1}\le P_R \tag{18}$$

where

$$\boldsymbol{\Psi}=\sum_{k=1}^{K}\left\{\left[\left(\left(\mathbf{V}_{G,1}^{H}\right)_k^{+}\right)^H\left(\mathbf{V}_{G,1}^{H}\right)_k^{+}\right]\circ\left[\left(\mathbf{U}_{H,1}^{H}\right)_k\left(\sigma_x^2\mathbf{H}_k\mathbf{H}_k^H+\sigma_n^2\mathbf{I}_L\right)\left(\mathbf{U}_{H,1}^{H}\right)_k^H\right]^T\right\} \tag{19}$$

Thus, from (15) and (18), the original optimization problem with respect to the block-diagonal matrix $\mathbf{F}$ is simplified as the following constrained problem with respect to the diagonal matrix $\boldsymbol{\Sigma}_F$,

$$\min_{\boldsymbol{\Sigma}_F}\quad f(\boldsymbol{\Sigma}_F)=tr\left\{\sigma_x^2\left\{\mathbf{I}+\frac{\sigma_x^2}{\sigma_n^2}\mathbf{D}_{H,1}\left[\mathbf{I}-\left(\mathbf{I}+\frac{\sigma_n^2}{\sigma_z^2}\boldsymbol{\Sigma}_F^H\mathbf{D}_{G,1}\boldsymbol{\Sigma}_F\right)^{-1}\right]\right\}^{-1}\right\} \tag{20}$$

$$s.t.\quad \mathbf{1}^H\boldsymbol{\Sigma}_F^H\boldsymbol{\Psi}\boldsymbol{\Sigma}_F\mathbf{1}\le P_R$$

The above problem is equivalent to the original problem (8). Without loss of generality, $\boldsymbol{\Sigma}_F$ is assumed to be real, i.e., $\boldsymbol{\Sigma}_F\in\mathbb{R}^{N\times N}$. Moreover, the above minimization problem with respect to $\boldsymbol{\Sigma}_F$ can be rewritten in a scalar form as,

$$\begin{aligned}\min_{\{\sigma_{F,i}\}}\quad&\sigma_x^2\sum_{i=1}^{N}\frac{\lambda_{G,i}\sigma_{F,i}^2+\sigma_z^2/\sigma_n^2}{\left(1+\lambda_{H,i}\,\sigma_x^2/\sigma_n^2\right)\lambda_{G,i}\sigma_{F,i}^2+\sigma_z^2/\sigma_n^2}\\ s.t.\quad&\sum_{i=1}^{N}\psi_{ii}\sigma_{F,i}^2+\sum_{i=1,i<j}^{N}\left(\psi_{ij}+\psi_{ji}\right)\sigma_{F,i}\sigma_{F,j}\le P_R\end{aligned} \tag{21}$$

where $\sigma_{F,i}\in\mathbb{R}$ is the ith diagonal element of $\boldsymbol{\Sigma}_F$, $\lambda_{H,i}$ and $\lambda_{G,i}$ are the ith eigenvalue of $\mathbf{HH}^H$ and that of $\mathbf{G}^H\mathbf{G}$, respectively, both in decreasing order, and ψ_{ij} is the (i,j)-th entry of $\boldsymbol{\Psi}$. Note that $\lambda_{H,i}$ and $\lambda_{G,i}$ in (21) have been arranged in decreasing order such that the summation with such an order gives the minimum value.

It can be proved that the constrained minimization problem (20) or (21) is convex when $\sigma_{F,i}$ satisfies

$$\sigma_{F,i}^2\ge\frac{\sigma_z^2/\sigma_n^2}{3\left(1+\lambda_{H,i}\,\sigma_x^2/\sigma_n^2\right)\lambda_{G,i}} \tag{22}$$

Namely, if the solution to (20) satisfies (22), then the optimization problem is convex and the solution is globally optimum. From (21), it can be conjectured that for a larger value of P_R it is easier for the solution to satisfy (22) and the problem tends to be convex. Hence, the condition (22) indicates that there might be a lowest possible total relay power P_R such that a feasible region of $\sigma_{F,i}$ exists to give optimal relay precoder. As will be seen from our simulation, we need only an adequately large relay power to ensure a convex solution. It should be mentioned that although the above discussion does not solve the optimization problem (21), yet it can be used to check easily whether a solution is globally optimal once it is obtained.

In this paper, we employ the complementary geometric programming (CGP) [6] to solve the optimization problem (21) which is in general not convex. It is known that geometric programming (GP) belongs to nonlinear optimization and has many useful theoretical and computational properties, whose application to wireless communications has already been found in [6]. As an extension of the GP, CGP is expressed as a ratio between two posynomials. Through some trivial mathematical manipulations, the objective function of (21) can be rewritten as,

$$\frac{f_{01}(\mathbf{x})}{f_{02}(\mathbf{x})}=\sigma_x^2\frac{\sum_{i=1}^{N}\left\{\left(\lambda_{G,i}\sigma_{F,i}^2+\sigma_z^2/\sigma_n^2\right)\prod_{j\neq i}\left[\left(1+\lambda_{H,j}\,\sigma_x^2/\sigma_n^2\right)\lambda_{G,j}\sigma_{F,j}^2+\sigma_z^2/\sigma_n^2\right]\right\}}{\prod_{i=1}^{N}\left[\left(1+\lambda_{H,i}\,\sigma_x^2/\sigma_n^2\right)\lambda_{G,i}\sigma_{F,i}^2+\sigma_z^2/\sigma_n^2\right]} \tag{23}$$

where both $f_{01}(\mathbf{x})$ and $f_{02}(\mathbf{x})$ are posynomials. Although the constraint of (21) may be posynomial or signomial depending on the terms $\sum_{i=1,i<j}^{N}\left(\psi_{ij}+\psi_{ji}\right)\sigma_{F,i}\sigma_{F,j}$, yet it can also be expressed as a ratio of two posynomials, namely,

$$\frac{\displaystyle\sum_{i=1}^{N}\frac{\psi_{ii}}{P_R}\sigma_{F,i}^2+\sum_{real(\psi_{ij})>0}\frac{\psi_{ij}+\psi_{ji}}{P_R}\sigma_{F,i}\sigma_{F,j}}{\displaystyle 1+\sum_{real(\psi_{ij})<0}\left|\frac{\psi_{ij}+\psi_{ji}}{P_R}\right|\sigma_{F,i}\sigma_{F,j}}\le 1 \tag{24}$$

Thus, the constrained problem (21) has been converted to a CGP, which can be solved iteratively by using the so-called condensation method, wherein the CGP problem is solved as a series of GPs and moreover the standard barrier-based interior-point method for convex optimization can be used to solve each GP [6]. Note that the CGP does not guarantee a convergence to a global optimum.

IV. SIMULATION RESULTS

Monte-Carlo simulation study of the two-hop relay networks is conducted to demonstrate the performance of the proposed joint relay and destination design. Without loss of generality, we assume that all the random channel matrices are independent identically distributed (i.i.d.), whose elements are zero-mean complex Gaussian random variables with unit variance. We also assume that the signal symbols are obtained from QPSK modulation with the power per antenna given by

$\sigma_x^2 = P_x/N$ and $P_x = 10dB$, and the SNR per bit per antenna at the source and that at the relay are defined as $SNR1 = P_x/(N\cdot\sigma_n^2)$ and $SNR2 = P_R/(LK\cdot\sigma_z^2)$, respectively, with the noise levels $\sigma_n^2 = \sigma_z^2 = 1$.

As shown in the above analysis in Sec.III, the optimization problem is conditionally convex, depending on the total relay power constraint P_R. For fixed relay and antenna configurations, there exists a relay power threshold so that the optimization problem is convex when P_R is above such a threshold. When the system parameters are fixed, one can obtain the threshold of P_R by using the lower bound of $\sigma_{F,i}^2$ in (22) into the constraint of (21). Therefore, it would be of interest to investigate the requirement of the optimization problem on such a threshold. Fig. 1 shows the cumulative distribution function of the relay power threshold for different numbers of relays, *K*, and different numbers of antennas at each relay, *L*, when *N* is fixed at 2. It is seen that the more relays and more antennas at each relay are used, the lower relay power the system requires to guarantee a convex optimization.

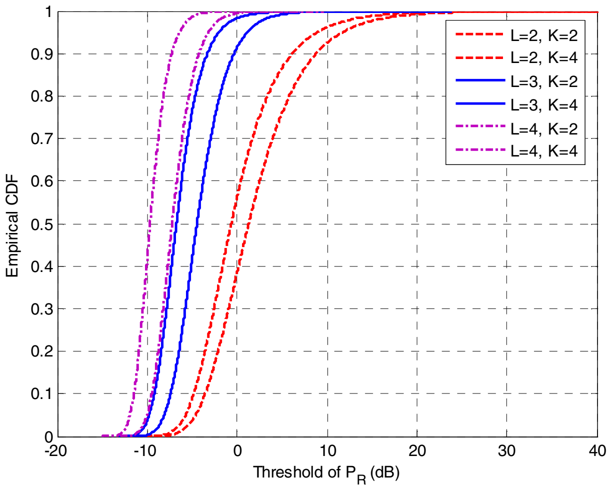

Fig.1. CDF of the relay power threshold over different numbers of relays *K* and relay antennas *L*, P_x =10dB, $\sigma_n^2 = \sigma_z^2 = 1$, *N*=2.

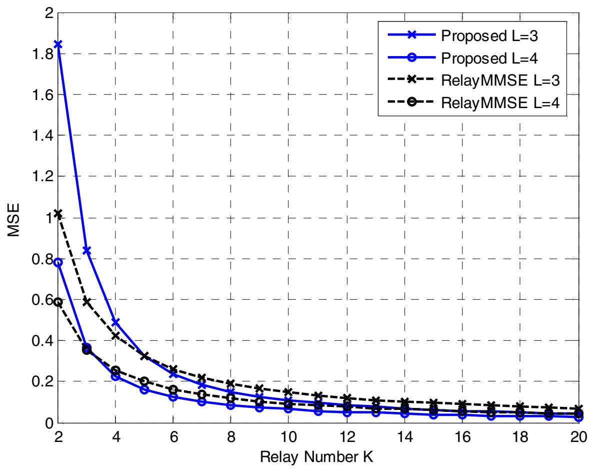

Fig.2. MSE v.s. *K* with *N*=2, P_x =10dB, *SNR*2=5dB, $\sigma_n^2 = \sigma_z^2 = 1$.

Fig.2 compares the MSE of our joint relay-destination processing scheme with that of the MMSE relaying scheme [3] in which Wiener filter is employed at the destination for a fair comparison, where different values of *L* are considered. It is seen from Fig. 2 that, when *K* or *L* is large, the constrained optimization problem is convex. This is why our scheme outperforms the relay-MMSE method for the case of large *K* and *L*. It is also seen that a larger number of relays or relays' antennas gives a lower MSE. Another observation is that the MSEs of all the methods tend to be consistent when *K* is sufficiently large.

V. CONCLUSION

In this paper, an optimization technique in terms of the MSE criterion has been presented for the joint design of relay and destination of MIMO AF relay networks. By assuming the availability of perfect CSI and using Wiener filter in the destination, the relay processing has first been formulated as a constrained optimization problem w.r.t. the block-diagonal relay precoder matrix subject to a total relay transmit power limit. It is then simplified to a problem involving scalar variables only, such that the complementary geometric programming can be used to solve the problem. It has been shown that the proposed relay precoding solution is globally optimal as long as the relay transmit power is adequately large.

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