Performance Analysis and Power Allocation of Multi–hop Multi–branch Relays with Data Storage over Generalized Fading Channels

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Abstract— **Deployment of relays with data storage capability is progressively becoming popular in the literature as it provides significant performance improvement over relays with non–storage capability. In this paper, we consider a multi–branch multi–hop decode–and–forward (DF) relay system. We assume each relay has data storage and the data can be stored temporarily if the next channel is not good enough to transmit through. We derive an asymptotic and approximate outage probability expression for the considered system under generalized fading channel. Based on the derived expression, we formulate an optimization problem which allocates source and relay transmit powers under different system requirements.**

I. Introduction

Multiple relays can be deployed in cooperative communication networks to effectively improve the performance and reliability of wireless communications in the presence of fading [1]–[5]. In conventional multi–branch cooperative systems, relays are placed in parallel between source and destination in which all the relays transmit concurrently in orthogonal channels, leading to a significant loss in spectral efficiency. To alleviate this problem, the best relay is selected for transmission according to the instantaneous signal–to–noise ratio (SNR) which can improve the spectral efficiency at the expense of insignificant performance loss [4]–[9]. In multi–hop scenarios, a broad range of coverage area can be ensured [10], [11]. Therefore, best relay path selection in multi–branch multi–hop cooperative systems can be a potential option to reduce excessive signaling overhead and to improve system performance. However, in the state–of–the–art approach, the best relay path is selected from the end–to–end links, i.e., the bottleneck of the SNRs [12]. Recently, it has been shown that best relay selection based on the bottleneck SNRs may not fully exploit the best channels in each link [13], [14]. To overcome this limitation, Aissa et al propose two new relay selection schemes for relays with buffers [13].

In this paper, we consider a multi–hop multi–branch decode–and–forward (DF) system where the relays have data storage capability. For a definite time instant, a path of a hop that gives the best SNR among all the paths is chosen. We consider independent and identically distributed (i.i.d.) and independent and non–identically distributed (i.n.d.) generalized fading channels at different links. By generalized fading we mean Rayleigh, Rician, Nakagami–q, and Nakagami–m fading channels. We derive a closed form asymptotic outage probability expression for the considered system which is used for performance evaluation at different channel conditions. Moreover, based on the derived expression, we formulate an optimization problem which can be adopted for different power allocation schemes under different system requirements.

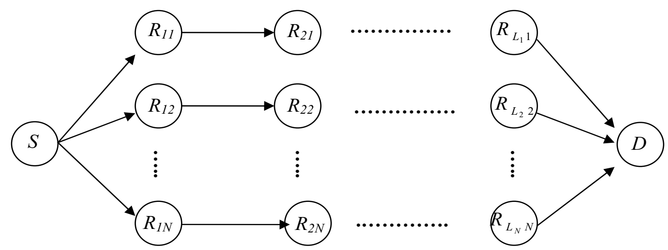

Fig. 1. Block diagram of a multi–hop multi–branch DF system.

The remainder of the paper is organized as follows. In Section II, we describe the system model for multi–hop multi–branch data storage DF best relay–path selection scheme. Section III and IV contain the outage probability and power allocation problem, respectively. Section V analyzes the numerical results and Section VI concludes the paper.

II. System Description

System Model: We consider an N parallel–branch cooperative DF system, as depicted in Fig. 1, where each branch contains $L(= M+1)$, number of hops. M denotes the number of half–duplex relays, R_{im}, $i \in \{1, 2, \cdots, M\}$ in branch, $m \in \{1, 2, \cdots, N\}$. A source terminal, S, sends a message to destination, D via a selected branch of relays equipped with buffers. In particular, all relays have data storage capacity where incoming data can be stored temporarily if the following corresponding channel is not good enough to transmit. We assume the storages have infinite capacity and there is always some data at source and relays to transmit.

We do not consider any direct path from S to D for the sake of simplicity which is a practically feasible model for a multi–hop scenario. As the relays are half–duplex, they can not transmit or receive data simultaneously. The overall transmission is organized in several phases depending on the number of hops. In the first phase, S transmits and only the selected relay (the relay is selected based on the best first hop channel) receives a data packet from the source, decodes it, and stores it to its storage. Then in the second phase, the relay that faces the best second hop channel, forwards its decoded signal (from its storage) to the next relay. In the third phase, the source transmits another data packet to the first hop relay

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TABLE I

COMPLETE PDF AND ASYMPTOTIC PDF PARAMETERS (b, t) OF DIFFERENT FADING CHANNELS. HERE, THE CHANNEL VARIANCES ARE UNITY. A, B, C, AND D DEFINE RAYLEIGH, RICIAN, NAKAGAMI–Q AND NAKAGAMI–M FADING CHANNELS, RESPECTIVELY.

Channel	Complete pdf	b	t
A	$\frac{1}{\bar{\gamma}}\exp\left(-\gamma/\bar{\gamma}\right)$	1	0
B	$\frac{(1+K)}{\bar{\gamma}}\exp\left(-K-\frac{(1+K)\gamma}{\bar{\gamma}}\right)$ $\times I_0\left(2\sqrt{K(1+K)\gamma/\bar{\gamma}}\right)$	$(1+K)\,e^{-K}$	0
C	$\frac{1+q^2}{2q\bar{\gamma}}\exp\left(-\frac{(1+q^2)^2\gamma}{4q^2\bar{\gamma}}\right)$ $\times I_0\left(\frac{(1-q^4)\gamma}{4q^2\bar{\gamma}}\right)$	$\frac{1+q^2}{2q}$	0
D	$\frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma(m)}\exp\left(-m\gamma/\bar{\gamma}\right)$	$\frac{m^m}{\Gamma(m)}$	$m-1$

based on the best first hop channel and simultaneously the relay that faces best third hop channel, forwards its stored decoded signal to the next relay. This process continues and after some delay, the signals are received at D. We assume that the signals from one relay path do not interfere to the relays of other paths. We assume perfect synchronization among all the relays so that the inter–relay interference can be completely mitigated.

Channel Model: The fading gain of each hop, $p \in \{1, 2, \cdots, L\}$ in each branch $m \in \{1, 2, \cdots, N\}$, is denoted by h_{pm}, where we assume independent type of fading in each link. To be specific, $h_{pm} \triangleq a_{pm}e^{j\theta_{pm}}$ is independent complex random variable, where channel amplitude a_{pm} can be Rayleigh, Rician, Nakagami–m, and Nakagami–q distributed with zero or non–zero mean and variance $\Omega_{pm} \triangleq \mathcal{E}\{|h_{pm}|^2\}$. Here, $\mathcal{E}\{\cdot\}$ denotes statistical expectation. θ_{pm} is uniformly distributed in $[-\pi, \pi)$ and it is statistically independent of channel amplitude a_{pm}. Moreover, our model is appropriate for i.n.d. fading in each link, i.e., different fading with equal (or different) variances, or same fading with different variances. n_{im} and $n_{L_m} = n_{M+1}$ denote the additive white Gaussian noise samples at relay $i \in \{1, 2, \cdots, M\}$ of branch m, and D, respectively. The noise sample, n_{im}, has zero mean and variance $\sigma^2_{n_{im}} \triangleq \mathcal{E}\{|n_{im}|^2\}$. Due to the distributed nature of relays, the noise variances can be different at different relays. The instantaneous and average SNR at each p hop of branch m is given by $\gamma_{pm} = (d_0/d_{pm})^\eta P_{(p-1)m}a^2_{pm}/\sigma^2_{n_{pm}}$ and $\bar{\gamma}_{pm} = (d_0/d_{pm})^\eta P_{(p-1)m}\Omega_{pm}/\sigma^2_{n_{pm}}$, respectively. P_{0m} and $P_{(p-1)m}$, for $p \in \{1, 2, \cdots, L\}$ and $m \in \{1, 2, \cdots, N\}$, are the average transmit powers of S and $R_{(p-1)m}$, respectively, and $\Omega_{pm} = \mathcal{E}\{a^2_{pm}\}$. d_{pm} and d_0 denote the distance of p–th hop in the m–th branch and the reference distance, respectively, and η is the propagation exponent. The asymptotic probability density function (pdf) and cumulative distribution function (cdf) of γ_{pm} are given as follow [15]

$$f(\gamma_{pm}) \doteq \frac{b_{pm}\gamma_{pm}^{t_{pm}}}{\bar{\gamma}_{pm}^{t_{pm}+1}} + o(\gamma_{pm}^{t_{pm}})^1 \tag{1}$$

and

$$F(\gamma_{pm}) \doteq \frac{b_{pm}\gamma_{pm}^{t_{pm}+1}}{(t_{pm}+1)\bar{\gamma}_{pm}^{t_{pm}+1}} + o(\gamma_{pm}^{t_{pm}+1}), \tag{2}$$

where b_{pm} and t_{pm} are channel specific parameters that depend on the channel distribution and $o(\cdot)$ is the higher order terms. b_{pm} and t_{pm} for different fading channels can be found in Table I. $f(Y)$ and $F(Y)$ represent the pdf and cdf, respectively, of a random variable Y.

Link Selection: The best link for each hop, p is selected according to

$$W_{p_s} = \underset{m\in\{1,\cdots,N\}}{\arg\max}\ \gamma_{pm}. \tag{3}$$

III. ASYMPTOTIC OUTAGE PROBABILITY ANALYSIS

In this section, we analyze the outage probability of the multi–hop data storage relay system under different fading environments. The outage probability is defined as the probability that the resultant SNR, γ_{eq}, of a system, falls below a certain threshold, $\gamma_{th} \triangleq 2^{2R_{(T)th}} - 1$ with data–rate $R_{(T)th}$, i.e.,

$$P^{\mathrm{ST}}_{\mathrm{OUT}} \triangleq \Pr(\gamma_{eq} \le \gamma_{th}) = F_{\gamma_{eq}}(\gamma_{th}), \tag{4}$$

where $\Pr(X)$ defines the probability of an event X [16]. For our considered system, $\gamma_{eq} = \min\{\gamma_{b_1}, \gamma_{b_2}, \cdots, \gamma_{b_L}\}$, where $\gamma_{b_p} = \max\{\gamma_{p1}, \gamma_{p2}, \cdots, \gamma_{pN}\}$.

Lemma 1: Defining $\gamma_{b_p} \triangleq \max\{\gamma_{p1}, \gamma_{p2}, \cdots, \gamma_{pN}\}$, the asymptotic cdf of γ_{b_p} can be stated as

$$F_{\gamma_{b_p}}(y) \doteq \prod_{m=1}^{N} F_{\gamma_{pm}}(y). \tag{5}$$

Proof: $F_{\gamma_{b_p}}(y) = \Pr(\gamma_{b_p} \le y) = \Pr((\gamma_{p1} \le y), (\gamma_{p2} \le y), \cdots, (\gamma_{pN} \le y))$. This simplifies to

$$F_{\gamma_{b_p}}(y) = F_{\gamma_{p1}}(y)F_{\gamma_{p2}}(y)\cdots F_{\gamma_{pN}}(y). \tag{6}$$

Eq. (6) is valid for any average SNR and this completes the proof. ■

Lemma 2: Defining $\gamma_{eq} \triangleq \min\{\gamma_{b_1}, \gamma_{b_2}, \cdots, \gamma_{b_L}\}$, the asymptotic cdf of γ_{eq} can be stated as

$$F_{\gamma_{eq}}(y) \doteq \sum_{p=1}^{L} F_{\gamma_{b_p}}(y). \tag{7}$$

Proof: $F_{\gamma_{eq}}(y) = \Pr(\gamma_{eq} \le y) = \Pr((\gamma_{b_1} \le y) \cup (\gamma_{b_2} \le y) \cup \cdots \cup (\gamma_{b_L} \le y))$.[2] Thus $F_{\gamma_{eq}}(y)$ can be stated as follows

$$\begin{aligned} F_{\gamma_{eq}}(y) &= F_{\gamma_{b_1}}(y) + F_{\gamma_{b_2}}(y) + \cdots + F_{\gamma_{b_L}}(y) \\ &\quad + o_2(y^2) + o_3(y^3) + \cdots + o_L(y^L). \end{aligned} \tag{8}$$

Here

$$o_s(y^s) = \sum_{\substack{i_1, i_2, \cdots, i_s \\ i_1 \ne i_2 \ne \cdots \ne i_s}}^{\binom{L}{s}} (-1)^{s-1} F_{\gamma_{b_{i_1}}}(y) \cdots F_{\gamma_{b_{i_s}}}(y), \tag{9}$$

for $s \in \{2, 3, \cdots, L\}$. At sufficiently high average SNR, $o_s(y^s)$, $s \in \{2, 3, \cdots, L\}$ provide very small values compared to $F_{\gamma_{b_1}}(y) + F_{\gamma_{b_2}}(y) + \cdots + F_{\gamma_{b_L}}(y)$. Thus $o_s(y^s)$, $s \in \{2, 3, \cdots, L\}$ in (8) can be neglected for high average SNR and hence this concludes the proof. ■

[1] $A \doteq B$ states that A is asymptotically equal to B

[2] $A \cup B$ defines the union operation between A and B.

Using (5) and (7), the outage probability in (4) can be written as follows

$$P_{\mathrm{OUT}}^{\mathrm{ST}} = \sum_{p=1}^{L} \prod_{m=1}^{N} F_{\gamma_{pm}}(\gamma_{th}). \tag{10}$$

Combining (2) and (10) yields the following asymptotic outage probability

$$P_{\mathrm{OUT}}^{\mathrm{ST}} \doteq \sum_{p=1}^{L} \prod_{m=1}^{N} \frac{b_{pm}\gamma_{th}^{t_{pm}+1}}{(t_{pm}+1)\bar{\gamma}_{pm}^{t_{pm}+1}}. \tag{11}$$

Expressing $P_{\mathrm{OUT}}^{\mathrm{ST}}$ as $P_{\mathrm{OUT}}^{\mathrm{ST}} \doteq (G_{S_o}\bar{\gamma}/\gamma)^{-G_{\mathrm{DIV}}}$ where G_{S_o} and G_{DIV} are average–SNR gain and diversity gain respectively. Assuming $\bar{\gamma}_{pm} = \bar{\gamma}$, $b_{pm} = b$, and $t_{pm} = t$ for $p \in \{1, 2, \cdots, L_m\}$ and $m \in \{1, 2, \cdots, N\}$ in (11), gives $G_{\mathrm{DIV}} = N(t+1)$. We observe that G_{DIV} depends on the number of the parallel–branches, but is independent of the number of hops.

Comparison with the End–to–End Relay Selection: In [17] it was shown that the outage probability of a multi–hop multi–branch system with no data storage at the relays is

$$P_{\mathrm{OUT}} \doteq \prod_{m=1}^{N} \sum_{p=1}^{L} \frac{b_{pm}\gamma_{th}^{t_{pm}+1}}{(t_{pm}+1)\bar{\gamma}_{pm}^{t_{pm}+1}}. \tag{12}$$

For i.i.d. fading all over the network, comparing (11) and (12) yields the ratio of the outage probabilities of the storage and non–storage relay systems as follows

$$\frac{P_{\mathrm{OUT}}^{\mathrm{ST}}}{P_{\mathrm{OUT}}} = L^{1-N}. \tag{13}$$

Thus it can be inferred that, for $L \geq 2$ and $N > 1$, using data storage at the relays improves the system performance by increasing the average–SNR gain. This finding was already shown for Rayleigh fading 2 hop multi–branch relay system in [13]. In this work, we extend this result for system with any number of hops and branches under generalized fading channels.

IV. Power Allocation

In this section, we optimize the system performance by allocating the source and relay transmit powers using the derived outage probability expressions in the previous section. We assume that, the average SNR of each hop and channel parameters are known in advance for power allocations. Moreover, several other constraints may be required to know, for example, the total system power and energy usage limitations of each relay and source.

Formulation of Optimization Problem: Based on our derived results in Section III, we formulate an optimum power allocation problem as follows

$$\underset{P_{(p-1)m}\geq 0}{\text{minimize}} \quad \sum_{p=1}^{L} \prod_{m=1}^{N} \frac{b_{pm}\gamma_{th}^{t_{pm}+1}}{(t_{pm}+1)(P_{(p-1)m}\xi_{pm})^{t_{pm}+1}} \tag{14}$$

$$\text{subject to} \quad P_{(p-1)m}\zeta_{pm} \leq E_{(p-1)m,\max}, \forall p, \forall m \tag{15}$$

$$\sum_{p=1}^{L} \sum_{m=1}^{N} P_{(p-1)m}\zeta_{pm} \leq E_T, \tag{16}$$

where, $\xi_{pm} = \bar{\gamma}_{pm}/P_{(p-1)m} = \bar{\xi}\xi_{pm}^{'}$. $E_{(p-1)m,\max}$ and E_T denote the maximum possible transmit energy for pth hop and mth branch and total transmit energy, respectively. ζ_{pm} defines the selection probability of the relay at pth hop and mth branch and $\bar{\xi}$ denotes average channel SNR. The optimization problem described in (14)–(16) defines a large class of optimization problem. For example, when the S and R_{pm} have sources of enough energy, their individual maximum transmit power can be unbounded (with a finite value of E_T for practical system). Sometimes either S or R_{pm} cannot have energy more than a certain limit (for example, due to power limitations in battery operated devices). In such cases, the individual transmit powers (of S and R_{pm}) are bounded by predefined finite values.

Selection Probability of a Link: The selection probability of a link at pth hop and mth branch can be defined as follows

$$\zeta_{pm} = \int_{0}^{\infty} f_{\gamma_{pm}}(\gamma_{pm}) \prod_{n=1, n\neq m}^{N} F_{\gamma_{pn}}(\gamma_{pm}) d\gamma_{pm}. \tag{17}$$

It can be stated that for the convergence of the integral in (17), we need to exploit the complete pdf and cdf of the channel fading. Without loss of generality, we consider Rayleigh fading channels to derive the closed form expression of the selection probability. It is worth mentioning that, ζ_{pm} can be obtained for any fading of interest. Even if the closed–form expression cannot be obtained, (17) involves one integration which can be solved numerically. However, for Rayleigh fading, using $f_{\gamma_{pm}}(\gamma_{pm}) = (1/\bar{\gamma}_{pm})e^{-\gamma_{pm}/\bar{\gamma}_{pm}}$ and $F_{\gamma_{pm}}(\gamma_{pm}) = 1 - e^{-\gamma_{pm}/\bar{\gamma}_{pm}}$ and doing some straightforward manipulation, (17) can be expressed as follows

$$\zeta_{pm} = \sum_{\substack{u_{p1}=0 \\ m\neq 1}}^{1} \cdots \sum_{\substack{u_{pN}=0 \\ m\neq N}}^{1} (-1)^{\sum_{\substack{n=1 \\ n\neq m}}^{N} u_{pn}} \times \frac{1}{1 + P_{(p-1)m}\xi_{pm} \sum_{\substack{n=1 \\ n\neq m}}^{N} \frac{u_{pn}}{P_{(p-1)n}\xi_{pn}}}. \tag{18}$$

Analysis of the Optimization Problem: As $u_{pm} \in \{0, 1\}$, the selection probability defined in (18) is always positive for any p and m. But this expression does not form a posynomial of $P_{(p-1)m}$. However, the objective function in (14) forms a posynomial. Considering all the facts, our optimization problem cannot form a geometric program (GP) and hence the exact solution cannot be obtained in closed–form expression. Therefore, we adopt a monomial approximation approach according to [18, Section 8] to obtain the optimum solution. It is worth noting that, this monomial–approximation approach provides a very efficient solution to the considered optimization problem. Using this approach, ζ_{pm} can form a posynomial and the optimization problem can be solved by solving a series of GP.

We define the denominator of ζ_{pm} as $\nu_{pm}(\boldsymbol{P})$, where $\boldsymbol{P} = [P_{(p-1)1}P_{(p-1)2}\cdots P_{(p-1)N}]$. At first, based on an initial point, $\nu_{pm}(\boldsymbol{P})$ is approximated as a monomial and the optimization problem is solved as GP by any standard software [19]. Then the obtained solution is used as the initial point for the next step. Using this solution, $\nu_{pm}(\boldsymbol{P})$ is again fitted

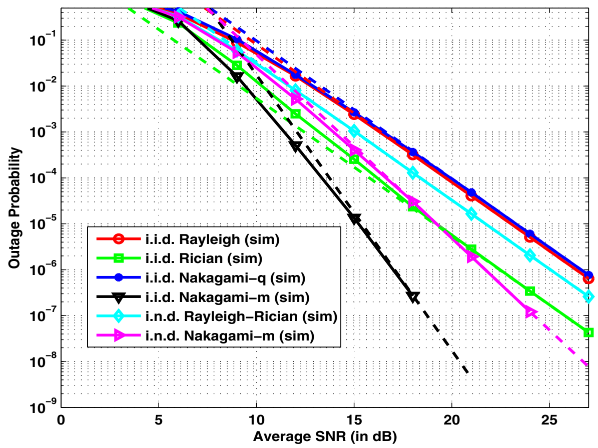

Fig. 2. Outage probability, $P_{\mathrm{OUT}}^{\mathrm{ST}}$ vs. average SNR, $\bar{\gamma}$ of 3–hop 3 branch storage relay system in different fading channel under AWGN. Markers: Simulated $P_{\mathrm{OUT}}^{\mathrm{ST}}$. Dashed lines: Asymptotic $P_{\mathrm{OUT}}^{\mathrm{ST}}$ (theory).

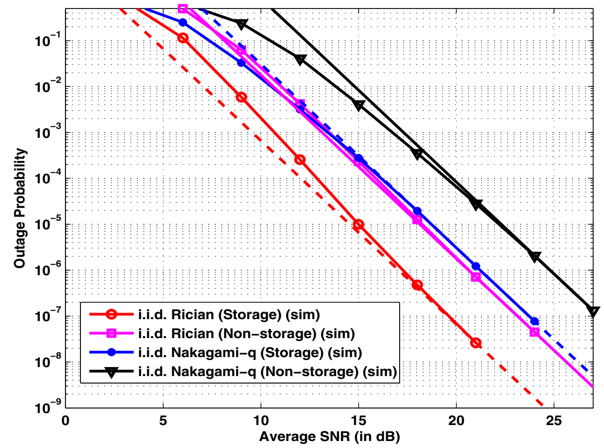

Fig. 3. Outage probability, $P_{\mathrm{OUT}}^{\mathrm{ST}}$ vs. average SNR, $\bar{\gamma}$ of 3–hop 4 branch storage and non–storage relay systems in two different i.i.d. fading channel under AWGN. Markers: Simulated $P_{\mathrm{OUT}}^{\mathrm{ST}}$ and P_{OUT}. Dashed lines: Asymptotic $P_{\mathrm{OUT}}^{\mathrm{ST}}$ (theory). Solid Line: Asymptotic P_{OUT} (theory).

as monomial and the optimization problem is solved. This procedure is continued until the $\boldsymbol{P}$ does not change noticeably between two consecutive iterations.

Monomial Approximation Approach: For $(i+1)$th iteration, the initial point, $\boldsymbol{P}^{(i)} = [P_{(p-1)1}^{(i)} P_{(p-1)2}^{(i)} \cdots P_{(p-1)N}^{(i)}]$ is obtained from the optimization of the previous iteration, i (or is taken as an arbitrary initial point for the first iteration). Then the approximated monomial, $\hat{\nu}_{pm}(\boldsymbol{P})$ can be obtained as follows

$$\hat{\nu}_{pm}(\boldsymbol{P}) = \frac{\nu_{pm}(\boldsymbol{P}^{(i)}) \prod_{n=1}^{N} \left(P_{(p-1)n}\right)^{a_n}}{\prod_{n=1}^{N} \left(P_{(p-1)n}^{(i)}\right)^{a_n}}, \tag{19}$$

where,

$$a_n = \frac{P_{(p-1)n}^{(i)}}{\nu(\boldsymbol{P}^{(i)})} \frac{\partial \nu_{pm}(\boldsymbol{P})}{\partial P_{(p-1)n}} |_{\boldsymbol{P}=\boldsymbol{P}^{(i)}} \tag{20}$$

Combining (19) and (20) gives a posynomial for ζ_{pm}. Thus the optimization problem forms a GP and it can be solved by [19].

V. Numerical Results

In this section, we verify the derived analytical expressions in Sections III and IV with computer simulations. To this end, we set a target rate, $R_{(T)th} = 1$ bit/sec/Hz for necessary calculation of $P_{\mathrm{OUT}}^{\mathrm{ST}}$ and P_{OUT}. For Figs. 2 and 3, we adopt equal distance in each link with $d_0 = 1$ and $\eta = 2$. However, the noise variances and transmit powers are considered as unity. Therefore, the average SNR of each link are equal and denoted as $\bar{\gamma}$.

Performance Analysis under Different Fading Environment: Fig. 2 shows the performance of the outage probability, $P_{\mathrm{OUT}}^{\mathrm{ST}}$ vs. $\bar{\gamma}$ for different i.i.d. and i.n.d. fading channels. 3 branch of relays with 3 hops each (i.e. 2 relays in each link) have been considered in between S and D. $K = 2$, $q = 0.707$, and $m = 2$ have been taken for i.i.d. Rician, Nakagami–q, and Nakagami–m fadings, respectively. It is worth noting that, K in Rician fading channel defines the strength of the direct path in comparison to that of the reflected paths, whereas q in Nakagami–q fading denotes the ratio of real–component and complex–component variances of the corresponding random variable. The fading parameter, m in Nakagami–m represents the severity of fading, with high (low) values denoting small (large) amount of fading. However, for i.n.d. Rayleigh–Rician fading channel, 2 links are subject to Rayleigh fading whereas the rest one faces Rician fading ($K = 2$). In i.n.d. Nakagami–m fading, $m = 2$ has been considered for the first link, whereas $m = 1$ has been adopted for second and third links. We note that all the simulated results are in perfect agreement with the analysis (theory) for sufficiently high average SNR. We observe that i.i.d. Rayleigh, Rician, Nakagami–q, and i.n.d. Rayleigh–Rician fading channels have diversity order, $G_{\mathrm{DIV}} = 3$. But their performance differs because of their average–SNR gain, G_{S_o}. For example, i.i.d. Rician fading shows around 3 dB, 4 dB, and 4.3 dB SNR gains over i.n.d. Rayleigh–Rician, i.i.d. Rayleigh, and i.i.d. Nakagami–q fading channels, respectively. I.i.d. Nakagami–m fading shows the highest diversity order ($G_{\mathrm{DIV}} = 6$) among all the considered cases. In case of i.n.d. Nakagami–m fading, G_{DIV} lies between 3 and 6.

Comparison between Storage and Non–storage Relay Systems: Fig. 3 shows relative comparison between data storage and non–storage relay system for different fading channels. 4 parallel branches of relays have been considered where each branch contains 3 hops between S and D. Two scenarios of channels have been considered here. In the first scenario, all the links are subject to Rician fading with $K = 2$, whereas in the second scenario, the links are Nakagami–q faded with $q = 0.707$. We show outage probability vs. $\bar{\gamma}$ for storage and non–storage relay systems for each of the scenarios. We see that storage relay system outperforms the non–storage relay system by the same amount for both the scenarios. For a particular $\bar{\gamma}$, $P_{\mathrm{OUT}}/P_{\mathrm{OUT}}^{\mathrm{ST}} = 3^{(4-1)} = 27$ is valid for both Rician and Nakagami–q fading channels and for high average SNR. Therefore, we conclude in agreement with the findings in Section III that, the performance improvement of storage

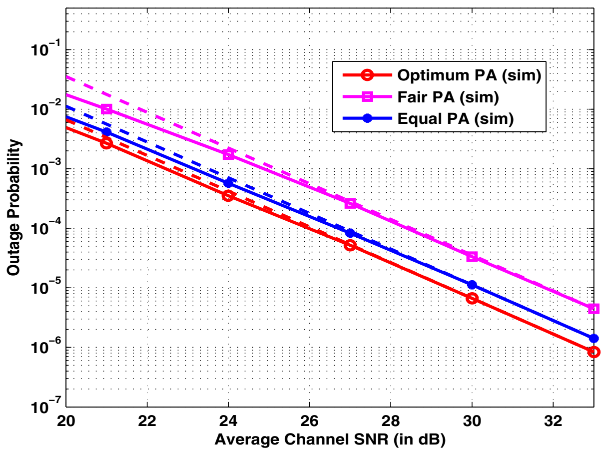

Fig. 4. Outage probability, $P_{\mathrm{OUT}}^{\mathrm{ST}}$ vs. average channel SNR, $\bar{\xi}$ of 2–hop 3 branch storage relay systems for different power allocation schemes in i.n.d. Rayleigh fading channel under AWGN. Markers: Simulated $P_{\mathrm{OUT}}^{\mathrm{ST}}$. Dashed lines: Asymptotic $P_{\mathrm{OUT}}^{\mathrm{ST}}$ (theory).

system over non–storage one is independent of the type of fading.

Power Allocation: Fig. 4 shows the usefulness of the optimization problem described in Section IV through performance analysis (outage probability) of different types of power allocation schemes. We show the outage probability vs. average channel SNR of a 2–hop 3 branch storage relay system with different power allocation schemes. We consider Rayleigh fading for all the links with $\xi_{11} = \bar{\xi}$, $\xi_{21} = \bar{\xi}$, $\xi_{12} = \bar{\xi}/2$, $\xi_{22} = \bar{\xi}/2$, $\xi_{13} = \bar{\xi}/10$, and $\xi_{23} = \bar{\xi}/10$. In particular, the first and the third branch face the best and the worst channels, respectively, for both the hops. Three types of power allocations (PA), optimum PA, fair PA, and equal PA, are considered in this analysis. For all the considered PA schemes, we set $E_T = 1$. However, in optimum PA, we make the individual non–negative power constraints unbounded, i.e., $P_{\max,(p-1)m}\zeta_{pm} \leq \infty$, for $p \in \{1, 2\}$ and $m \in \{1, 2, 3\}$ (we note that, $P_0 = P_{0m}$ for $\forall m$). This allocation is suitable for such system, where the source and the relays have sufficient sources of energy. In fair PA, we set $P_{\max,(p-1)m}\zeta_{pm} \leq E_T/4$, $\forall p$ and $\forall m$. This allocation ensures that the powers are balanced out among the source and the relays while even ensuring higher power for good channels. In equal PA, we do not optimize the powers, rather we set $P_0 = 1/2$ and $P_{11} = P_{12} = P_{13} = 1/2$. We observe in Fig. 4 that, optimum PA scheme achieves around 1 dB and 2.5 dB performance improvements over equal and fair PA schemes, respectively. Therefore, we conclude that fair PA can ensure fairness (in using total energy) among the relays and source at the expense of performance loss.

VI. Conclusion

In this paper, we derive the asymptotic outage probability expression for multi–hop multi–branch DF relays with data storage system under generalized fading channel. By using storage at the relays, the average SNR gain improves over the non–storage relay system whereas, the diversity order remains unaffected. Moreover, this improvement remains same for any type of fading for a definite number of hops and branches. We show that optimizing the outage probability, transmit power of source and relays can be allocated which can provide overall performance improvement over equal power allocation scheme.

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