Impact of Nonlinear Devices in Software Radio Signals

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Abstract - Software radio signals with several channels have high dynamic range, making them very prone to nonlinear distortion effects, namely those inherent to an efficient power amplification.

In this paper we present analytical approach evaluation for evaluating the impact of nonlinear devices in software radio signals. As an application, we will consider the nonlinear amplification of software radio signals where we have a high number of channels with substantially different powers. [1]

I. Introduction

Software radio architectures were proposed in [1] as a flexible and efficient way of implementing versatile multi-standard base stations and/or terminals. They were also proposed for cognitive radio systems [2], [3].

The main idea behind software radio is to employ a wideband ADC (Analog-to-Digital Converter) and all subsequent processing (channel separation, detection, etc.) is implemented digitally, therefore avoiding the need for radio frequency hardware. Moreover, software radio terminals can be programmed for different standards; they can also be programmed for undefined future standards [4], [5]. Multitone software radio architectures can be used in programmable base stations where we have a single terminal for a large number of frequency channels [6].

The implementation of software radio terminals presents many difficulties. The ADC converter is a key component, since it should be able to sample and quantize an wideband signal that can correspond to a large number of users, with different bands and different powers [7], [6]. The power amplification is another challenge, especially when we have a large number of channels, since the software radio signal will have high envelope fluctuations, making them very prone to nonlinear distortion effects.

The analytical evaluation of nonlinear distortion effects in software radio signals is a difficult problem that is not well studied in the literature. As far as we know, the only work on the topic is [6], which considered the nonlinear effects inherent to the quantization procedure.

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In this paper we consider the impact of bandpass memoryless nonlinear devices (the typical model for common power amplifiers used with bandpass signals [8]), contrarily to [6] where the the in-phase and quadrature parts of the software radio signal are submitted two identical memoryless nonlinear devices. As in [6], we take advantage of the Gaussian-like nature of software radio signals with a high number of channels to characterize statistically the transmitted signals. This statistical characterization is then employed to optimize the power amplifier.

This paper is organized as follows: In sec. II we present the software radio architecture considered in this paper. Sec. III presents an analytical statistical characterization of the signal at the output of a memoryless nonlinear device. In sec. IV we present some numerical results and sec. V is concerned with the conclusions of this paper.

II. Power Amplification in Software Radio Architectures

Fig. 1 presents the software radio architecture considered in this paper. We consider the downlink transmission (i.e., the transmission from the base station to the mobile terminals), although our approach could easily be extended to the uplink transmission.

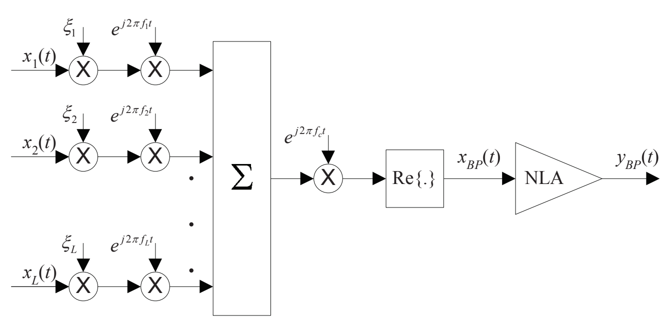

Fig. 1. Software radio architecture.

We have L channels, each one with a bandpass signal centered on the frequency f_l, $l = 1, 2, \ldots, L$. Without loss of generality, it is assumed that the PSDs (Power Spectral Densities) associated to the different channels do not overlap. The complex envelope of the overall signal to be transmitted is

$$x(t) = \sum_{l=1}^{L} \xi_l x_l(t), \tag{1}$$

where $x_l(t)$ is the signal associated to the lth channel and ξ_l is an weighting factor for power control purposes (clearly, the power associated to the lth channel is proportional to $|\xi_l|^2$). Typically, the signal associated to each channel has a PAM format (Pulse Amplitude Modulation), i.e.,

$$x_l(t) = \sum_n a_n^{(l)} r_l(t - nT_l - \tau_l), \tag{2}$$

where $r_t(t)$ is the pulse shape adopted for the lth channel, the complex-valued amplitudes $a_n^{(l)}$ are selected from a given constellation (we can have different constellations for different channels but we assume that each constellation has zero mean), T_l is the symbol separation for the lth channel (the corresponding symbol rate is $1/T_l$) and τ_l is a suitable delay.

When the number of channels is high (say several tens of channels) and there is no channel that has associated a significant fraction of the total power then the complex envelope $x(t)$ is approximately Gaussian. If the delays τ_l are uncorrelated and uniformly distributed in the interval $[0, T_l]$ then $x(t)$ can be regarded as a realization of a stationary Gaussian process that has zero mean and with the variance of the real and imaginary parts σ^2.

If we have uncorrelated signals on the different channels, then the PSD of $x(t)$ is

$$S_x(f) = \sum_{l=1}^{L} \xi_l^2 S_{x_l}(f - f_l) \tag{3}$$

(see fig. 1) where $S_{x_l}(f)$ is the PSD of $x_l(t)$, given by

$$S_{x_l}(f) = \frac{E[|a_n^{(l)}|^2]}{T_l} |R_l(f)|^2, \tag{4}$$

with $R_l(f)$ denoting the Fourier transform of the pulse shape $r_l(t)$.

The corresponding autocorrelation function is

$$\begin{aligned} R_x(\tau) &= E[x(t)x^*(t-\tau)] = \\ &= \sum_{l=1}^{L} \xi_l^2 R_{x_l}(\tau) \exp(j2\pi f_l \tau), \end{aligned} \tag{5}$$

with $R_{x_l}(\tau)$ denoting the autocorrelation function of $x_l(t)$.

The signal to be transmitted is then amplified by a nonlinear amplified that can be modeled as a bandpass memoryless nonlinearity characterized by the AM-to-AM and AM-to-PM characteristics $A(R)$ and $\Theta(R)$, respectively, as shown in fig. 2. This means that the transmitted signals have complex envelope

$$y(t) = A(|x(t)|) \exp(j \arg(x(t)) + j\Theta(|x(t)|)). \tag{6}$$

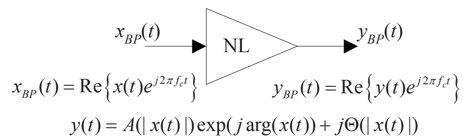

Fig. 2. Bandpass memoryless nonlinear device.

III. Statistical Characterization of the Transmitted Signals

As it was already mentioned, when the number of channels is high $x(t)$ can be approximately regarded a zero-mean complex Gaussian process with PSD given by (4), autocorrelation given by (5) and the variance of both real and imaginary parts of $x(t)$ denoted by

$$\sigma^2 = \frac{1}{2} R_x(0) = \frac{1}{2} \sum_{l=1}^{L} R_{x_l}(0). \tag{7}$$

In the following, we take advantage of the quasi-Gaussian nature of $x(t)$ to obtain the statistical characterization of the transmitted signal $y(t)$. It can be shown that the output can be written as the sum of two uncorrelated components, an useful one, proportional to the input, and a self-interference one [9], i.e.,

$$y(t) = \alpha x(t) + d(t), \tag{8}$$

with

$$\begin{aligned} \alpha &= \frac{E[x^*(t)y(t)]}{E[|x(t)|^2]} = \frac{E[RA(R)\exp(j\Theta(R))]}{E[R^2]} = \\ &= \frac{1}{2\sigma^2} \int_0^{+\infty} RA(R) \exp(j\Theta(R)) \frac{R}{\sigma^2} \exp\left(-\frac{R^2}{2\sigma^2}\right) dR, \end{aligned} \tag{9}$$

where $R \triangleq |x(t)|$. The average power of the useful component is $P_u = |\alpha|^2 \sigma^2$ and the average power of the nonlinear distortion component is $P_d = P_y - P_u$, with

$$\begin{aligned} P_y &= E[|y(t)|^2] = \frac{1}{2} E[A^2(R)] = \\ &= \frac{1}{2} \int_0^{+\infty} A^2(R) \frac{R}{\sigma^2} \exp\left(-\frac{R^2}{2\sigma^2}\right) dR. \end{aligned} \tag{10}$$

It is shown in the Appendix that the autocorrelation of the signal at the output of the nonlinear device can be expressed as a function of the autocorrelation of the signal at its input in the following way:

$$R_y(\tau) = E[y(t)y^*(t-\tau)] = 2 \sum_{\gamma=0}^{+\infty} P_{2\gamma+1} \frac{R_x(\tau)^{\gamma+1} R_x^*(\tau)^{\gamma}}{R_x(0)^{2\gamma+1}}, \tag{11}$$

with $P_{2\gamma+1}$ denoting the total power associated to the Inter-Modulation Product (IMP) of order $2\gamma + 1$.

To obtain the coefficients $P_{2\gamma+1}$ we can employ the approach described in [12]. This means that

$$P_{2\gamma+1} = \frac{|\nu_{2\gamma+1}|^2}{2\gamma!(\gamma+1)!} \tag{12}$$

with

$$\nu_{2\gamma+1} = \frac{2}{\sigma^2}\int_0^{+\infty} RA(R)\exp(j\Theta(R))W_{2\gamma+1}\left(\frac{R}{\sqrt{2\sigma^2}}\right)dR \tag{13}$$

and

$$W_{2\gamma+1}(x) = \frac{\gamma!}{2}\exp(-x^2)xL_\gamma^{(1)}(x^2), \tag{14}$$

where $L_\gamma^{(1)}(x)$ denotes a generalized Laguerre polynomial of order γ [10].

Clearly, $P_1 = |\alpha|^2\sigma^2$ and

$$R_d(\tau) = E[d(t)d^*(t-\tau)] = 2\sum_{\gamma=1}^{+\infty} P_{2\gamma+1}\frac{R_x(\tau)^{\gamma+1}R_x^*(\tau)^\gamma}{R_x(0)^{2\gamma+1}}, \tag{15}$$

The PSD (Power Spectral Density) of $y(t)$ will be

$$\begin{aligned} G_y(f) &= \mathcal{F}\{\mathcal{R}_\dagger(\tau)\} = \\ &= 2\sum_{\gamma=0}^{+\infty}\frac{P_{2\gamma+1}}{(R_x(0))^{2\gamma+1}}\cdot \\ \cdot \underbrace{G_x(-f) * \ldots * G_x(-f)}_{\gamma} &* \underbrace{G_x(f) * \ldots * G_x(f)}_{\gamma+1}. \end{aligned} \tag{16}$$

($\mathcal{F}\{\cdot\}$ denotes 'Fourier transform'). Similarly, the PSD of $d(t)$ will be

$$\begin{aligned} G_d(f) &= \mathcal{F}\{\mathcal{R}_\lceil(\tau)\} = \\ &= 2\sum_{\gamma=1}^{+\infty}\frac{P_{2\gamma+1}}{(R_x(0))^{2\gamma+1}}\cdot \\ \cdot \underbrace{G_x(-f) * \ldots * G_x(-f)}_{\gamma} &* \underbrace{G_x(f) * \ldots * G_x(f)}_{\gamma+1}. \end{aligned} \tag{17}$$

As in [6], this method for statistical characterization of the transmitted blocks is appropriate whenever the power series in (11) and (15) can be reasonably truncated while ensuring an accurate computation. To check if the number IMP considered is enough we compare the total power associated to the nonlinear distortion component $d(t)$ with the total power associated to the first γ_{max} IMP, we can compare

$$P_d = P_y - P_u, \tag{18}$$

where P_y and P_u are obtained as described above, with

$$P_d^{\gamma_{max}} = 2\sum_{\gamma=1}^{\gamma_{max}} P_{2\gamma+1}. \tag{19}$$

Since typical nonlinear amplification characteristics are relatively smooth, we usually just need a few IMP products, say $\gamma_{max} = 5$.

IV. Performance Results

In this section, we present a set of performance results concerning the impact of a nonlinear amplifier in transmitted software radio signal. We have 32 channels with the same bandwidth, each one with a QPSK modulation and square-root raised cosine pulses with roll-off factor 0.2. The different channels have uncorrelated delays and can have the same average power or not. The power amplifier is a SSPA (Solid State Power Amplifier) characterized by the AM-to-AM conversion

$$A(R) = \frac{R}{\sqrt[2p]{(1+(R/s_M)^{2p}}}, \tag{20}$$

with the parameter $p = 1$ and $s_M/\sigma = 2$, and AM-to-PM conversion $\Theta(R) = 0$.

Let us first assume that all channels have the same average power. Fig. 3 shows the PSD of the software radio signal at the input of the amplifier and the PSD of the signal at its output. The PSD of the output signal is obtained analytically, using the approach described in this paper, or by simulation. Clearly, there is a close matching between theoretical and simulated PSD, which means that our analytical method is very accurate. Fig. 4 shows the PSD of the useful and nonlinear distortion terms at the output of the amplifier. We include theoretical and simulated values of the PSD of the nonlinear distortion term. Once again, there is a close matching between theoretical and simulated values. Clearly, the nonlinear amplifier leads to performance degradation due to the nonlinear distortion component, which is more severe for channels in the center of the band. We also have out-of-band radiation due to the nonlinear amplifier.

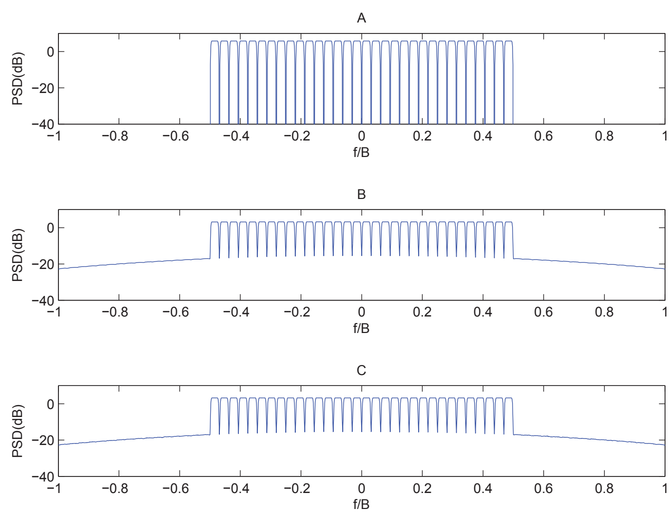

Fig. 3. PSD of the original signal (A) and transmitted signal (theoretical values in (B) and simulated values in (C)).

Let us consider now the case where different channel have different power. The corresponding PSD are presented in figs. 5 and 6 (under the same conditions of figs. 3 and 4). Once again, there is a close matching between theoretical and simulated PSD. An important aspect to highlight is that the PSD of the nonlinear distortion component changes little over the band (in fact, it is not too different from the case where all channels have the same average power). This means that the channels with smaller average power will suffer stronger nonlinear distortion levels, something that we should have in mind when designing software radio architectures where the PSD levels of different channel can be substantially different.

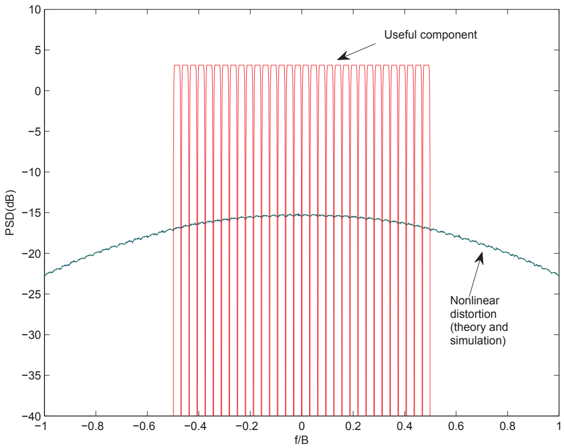

Fig. 4. Theoretical and simulated PSD for the nonlinear distortion component, together with the PSD of the useful component.

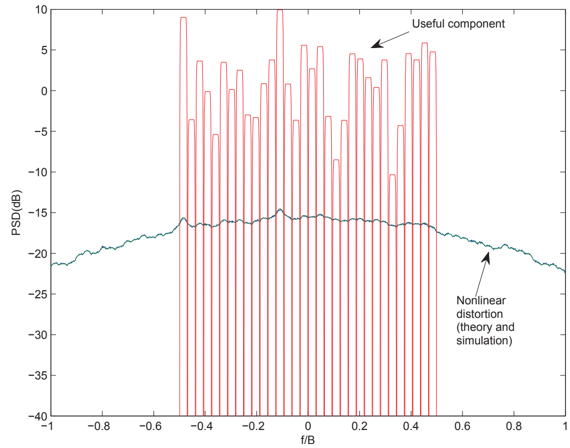

Fig. 6. Theoretical and simulated PSD for the nonlinear distortion component, together with the PSD of the useful component.

From the PSD of the useful and nonlinear distortion components we can easily obtain the signal-to-interference levels for each channel.

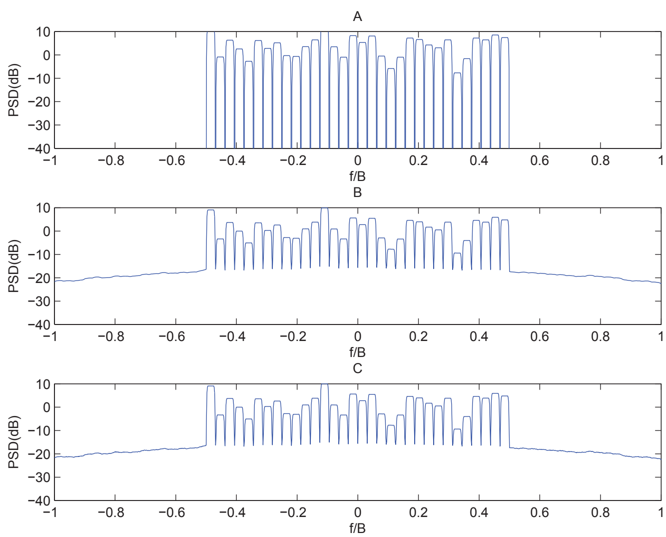

Fig. 5. PSD of the original signal (A) and transmitted signal (theoretical values in (B) and simulated values in (C)).

V. Conclusions

An analytical approach for analyzing the impact of bandpass memoryless nonlinear devices in software radio signals was presented. This approach takes advantage of the Gaussian behavior of software radio signals with a large number of channels. Our results show that the nonlinear distortion effects can be particularly severe when we have channels with substantially different PSD, with the channels with lower PSD levels facing stronger nonlinear distortion effects.

Appendix

In the following we show how we can write the that the autocorrelation of the complex envelope of the sinal at the output of a bandpass memoryless nonlinear device as a function of the autocorrelation of the complex envelope of the sinal at its input, which is assumed Gaussian.

Let us consider a bandpass Gaussian signal with complex envelope $x(t)$ that is submitted to a bandpass memoryless nonlinear device with AM-to-AM can AM-to-PM conversion functions $A(R)$ and $\Theta(R)$, respectively (see fig. 2). The autocorrelation of the complex envelope of the sinal at its output, $y(t)$, is

$$R_y(\tau) = E[y(t)y^*(t-\tau] = \\ = E[A(R_1)A(R_2)\exp(j\Theta(R_1) - j\Theta(R_2))\exp(j\varphi_1 - j\varphi_2)], \quad (21)$$

where $y(t)$ is related with $x(t)$ by (6), i.e., $y(t) = A(R_1)\exp(j\Theta(R_1)\exp(j\varphi_1)$ and $y(t-\tau) = A(R_2)\exp(j\Theta(R_2)\exp(j\varphi_2)$, with $x(t) = R_1\exp(j\varphi_1) = Z_1$ and $x(t-\tau) = R_2\exp(j\varphi_2) = Z_2$. We have

$$R_y(\tau) = \int_0^{+\infty}\int_0^{+\infty}\int_0^{2\pi}\int_0^{2\pi} A(R_1)A(R_2)\cdot \\ \cdot\exp(j\Theta(R_1) - j\Theta(R_2))\cdot \\ \cdot\exp(j\varphi_1 - j\varphi_2)p(R_1,\varphi_1,R_2,\varphi_2)dR_1dR_2d\varphi_1d\varphi_2 \quad (22)$$

with $p(R_1,\varphi_1,R_2,\varphi_2) = p(Z_1,Z_2)$ denoting the joint PDF (Probability Density Function) of Z_1 and Z_2. Since $x(t)$ is Gaussian, Z_1 and Z_2 are two jointly Gaussian complex random variables, and their PDF can be written as

$$p(Z_1,Z_2) = p(R_1,\varphi_1,R_2,\varphi_2) = \frac{1}{\pi^2|R_Z|}\exp\left(-ZR_Z^{-1}Z^H\right) \quad (23)$$

($(\cdot)^H$ denotes the Hermitian of a matrix), where $Z = [Z_1\ Z_2]$ and

$$R_Z = E[Z^TZ^*] = \begin{bmatrix} \rho_0 & \rho_1 \\ \rho_1^* & \rho_0 \end{bmatrix} \quad (24)$$

with $\rho_0 = E[|Z_1|^2] = E[|Z_2|^2] = 2\sigma^2$, $\rho_1 = E[Z_1Z_2^*]$ and σ^2 denotes the variance of the real and imaginary parts of $x(t)$.

After some manipulations we obtain

$$R_y(\tau) = \exp(j\arg(\rho_1)) \cdot$$
$$\cdot \int_0^{+\infty}\int_0^{+\infty}\int_0^{2\pi}\int_0^{2\pi} A(R_1)A(R_2)\exp(j\Theta(R_1) - j\Theta(R_2)) \cdot$$
$$\cdot \exp(j\varphi_1 - j\varphi_2')p(R_1,\varphi_1,R_2,\varphi_2')dR_1 dR_2 d\varphi_1 d\varphi_2',$$

where $\varphi_2' = \varphi_2 - \arg(\rho_1) = \arg(Z_2')$, with Z_2' given by $Z_2' = Z_2\exp(-j\arg(\rho_1))$. Clearly, Z_2' is Gaussian, with $E[Z_1 Z_2'^{*}] = |\rho_1|$ and $E[|Z_2'|^2] = \rho_0$, which means that the joint PDF of Z_1 and Z_2' is

$$p(Z_1, Z_2') = p(R_1,\varphi_1,R_2,\varphi_2') =$$
$$= \frac{1}{\pi^2\rho_0^2(1-\rho^2)} \cdot$$
$$\cdot \exp\left(-\frac{R_1^2 + R_2^2 - 2R_1R_2\rho\cos(\varphi_1 - \varphi_2')}{\rho_0(1-\rho^2)}\right) \tag{25}$$

with $\rho = \frac{|\rho_1|}{\rho_0} = \frac{|R_x(\tau)|}{R_x(0)}$.

By replacing (25) in (25), we obtain

$$R_y(\tau) = \frac{\exp(j\arg(\rho_1))}{\pi^2\rho_0^2(1-\rho^2)}\int_0^{+\infty}\int_0^{+\infty} A(R_1)A(R_2) \cdot$$
$$\cdot \exp(j\Theta(R_1) - j\Theta(R_2))\exp\left(-\frac{R_1^2+R_2^2}{\rho_0(1-\rho^2)}\right) \cdot$$
$$\cdot \left(\int_0^{2\pi}\int_0^{2\pi}\exp\left(\frac{2R_1R_2\rho\cos(\varphi_1-\varphi_2')}{\rho_0(1-\rho^2)}\right)\right.$$
$$\left.\exp(j\varphi_1 - j\varphi_2')d\varphi_1 d\varphi_2'\right) dR_1 dR_2 \tag{26}$$

Since

$$\int_0^{2\pi}\int_0^{2\pi}\exp(x\cos(\varphi_1-\varphi_2))\exp(j(\varphi_1-\varphi_2))d\varphi_1 d\varphi_2 =$$
$$= 2\pi\int_0^{2\pi} 2\pi I_1(x)d\varphi_2 = 4\pi^2 I_1(x),$$

where $I_n(x)$ denotes a modified Bessel function of order n [10], then

$$R_y(\tau) = \frac{\exp(j\arg(\rho_1))}{\pi^2\rho_0^2(1-\rho^2)} \cdot$$
$$\cdot \int_0^{+\infty}\int_0^{+\infty} A(R_1)A(R_2)\exp(j\Theta(R_1) - j\Theta(R_2)) \cdot$$
$$\cdot \exp\left(-\frac{R_1^2+R_2^2}{\rho_0(1-\rho^2)}\right)(4\pi)^2 I_1\left(\frac{2R_1R_2\rho}{\rho_0(1-\rho)^2}\right)dR_1 dR_2. \tag{27}$$

By writing $I_1(x)$ as

$$I_1(x) = \frac{x}{2}\sum_{k=0}^{+\infty}\frac{\left(\frac{x}{2}\right)^{2k}}{k!(k+1)!} \tag{28}$$

(see [10]), we obtain

$$R_y(\tau) = \frac{\rho\exp(j\arg(\rho_1))}{\rho_0^3(1-\rho^2)^2}\sum_{k=0}^{+\infty}\frac{1}{k!(k+1)!}\cdot\left(\frac{\rho}{\rho_0(1-\rho^2)}\right)^{2k} \cdot$$
$$\cdot \left|\int_0^{+\infty} R^{2k+1}A(R)\exp(j\Theta(R))\exp\left(-\frac{R^2}{\rho_0(1-\rho^2)}\right)dR\right|^2 \tag{29}$$

It should be noted that (29) could be regarded as the generalization of [11] for a complex autocorrelation at the nonlinearity input. Clearly, ρ and ρ_1 are constant for a given τ and the output autocorrelation is an odd function of ρ. Therefore, it can be expanded in Taylor series as

$$R_y(\tau) = \sum_{\gamma=0}^{+\infty} 2P_{2\gamma+1}\rho^{2\gamma+1}\exp\left(j\arg(\rho_1)\right) =$$
$$= 2\sum_{\gamma=0}^{+\infty} P_{2\gamma+1}\frac{R_x(\tau)^{\gamma+1}R_x^*(\tau)^{\gamma}}{(R_x(0))^{2\gamma+1}}, \tag{30}$$

where $P_{2\gamma+1}$ denotes the power associated to the IMP (Inter-Modulation Product) of order $2\gamma+1$, which is independent of the shape of the PSD of the sinal at the input.

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