Utilization of Partial Common Information in Distributed Compressive Sensing

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***Abstract*—In a densely distributed sensor network, it is known that it is advantageous to exploit both intra- and inter- signal correlation structures during recovery procedure. Based on this observation, the bound of the required number of measurement in distributed compressive sensing (DCS) is shown to be reduced by exploiting this structure[2]. In this paper, we generalize the model for distributed compressive sensing and show that elaborated signal structure can reduce the required number of measurements further. To this end, we introduce a new concept of partial common information which is shared by some parts of signals, but not by every signal. Numerical results show that with the proposed model, more robust signal recovery can be achieved.**

I. Introduction

Compressive sensing (CS) is an emerging technique that has an advantage in reducing the number of signal samples. With only small samples of signal, the original signal can be recovered if the signal is sparse enough [1][3].

While a research of CS for single-signal and image processing has been studied well [4,5], a framework for distributed multiple signals has been rarely suggested, even though the property of CS can be applied to distributed multiple signals. Some schemes for distributed multi sensors have been proposed by [6], but they have not used potential benefits of multi sensors e.g. inter- signal correlation.

Baron et al, [2], introduced Distributed Compressive Sensing (DCS) that enables new distributed coding algorithms to exploit both intra- and inter- signal correlation structures. In DCS, each sensor compresses their N size signal to M size signal by projecting on randomly generated measurement matrix. In [2], the bound of the number of measurements to recover the original signals is obtained, by exploiting both intra-, and inter- signal correlation information.

However, for a system with multiple sensors, it is unrealistic to assume that there exist inter- signal correlation shared by whole sensors. Since correlation between sensors is likely to depend on spatial distribution, it is more natural that partial inter- signal correlation, shared by only part of the sensors exists. For example, assume that sensors are placed in a concert hall for measuring frequencies in the hall. Some frequencies might be strong enough to be measured by multiple sensors. Clearly, it is more realistic to assume that parts of the sensors measure the frequencies than to assume that all of the sensors measure the frequencies.

In this paper, we introduce a concept of partial common information and obtain more refined theoretical bound for a characterizing example with considering the partial common information, we also utilize it to improve the signal recovery. We provide a numerical result of the characterizing example to verify the efficiency of the proposed signal structure and corresponding algorithms.

II. Compressive Sensing

For many signals sensed in real world, it is possible to represent them as sparse coefficients with the sparse basis. Let $\mathbf{x} \in \mathbf{R}^N$ denote the signal of interest. With an orthonormal particular sparse basis $\mathbf{\Psi}$, we can obtain sparse coefficient vector $\omega \in \mathbf{R}^N$ corresponding to $\mathbf{x}$ by $\omega = \mathbf{\Psi}^T\mathbf{x}$ satisfying $\|\omega\|_0 = K$, where $K \ll N$. Here, $\|\|_0$ indicates the l_0 norm. In this paper, $\mathbf{I}$, the identity matrix, is used as a sparse basis $\mathbf{\Psi}$ for convenience. Arbitrary sparse basis can be easily incorporated into the developed structure without loss of generality.

In CS, the compression is processed by multiplying measurement matrix $\mathbf{\Phi} \in \mathbf{R}^{M\times N}$ with the signal $\mathbf{x}$. It can be written as $\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi\Psi}\omega$, where $\mathbf{y} \in \mathbf{R}^M$. In $\mathbf{y} = \mathbf{\Phi x}$, since the number of equations (M) is smaller than the number of values (N) the system is ill posed. However, because the solution is sparse, the desired signal $\mathbf{x}$ can be recovered by solving below l_0 minimization problem.

$$\hat{\omega} = \arg\min \|\omega\|_0 \quad s.t.\ \mathbf{y} = \mathbf{\Phi\Psi}\hat{\omega} \tag{1}$$

In practical uses of CS, however, l_0 minimization cannot be used practically since it is NP-hard problem. Instead of this, the desired signal is estimated as the solution of l_1 minimization.

$$\hat{\omega} = \arg\min \|\omega\|_1 \quad s.t.\ \mathbf{y} = \mathbf{\Phi\Psi}\hat{\omega} \tag{2}$$

It is proved in [1][3] that when ω is sparse enough and measurement matrix $\mathbf{\Phi}$ satisfies restricted isometric property (RIP), the signal recovery error goes to zero with overwhelming probability. However, the price to pay for l_1 minimization is significantly larger number of measurements is required than when l_0 minimization is used.

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III. JOINT SPARSE SIGNAL MODEL

In [2], the joint sparse signal model is defined. Using the same notation, let $\Lambda := \{1, 2, ..., J\}$ denote the set of indices for J signals in the ensemble, and the ensemble contains the signal $\mathbf{x}_j \in \mathbf{R}^N, \quad j \in \Lambda$. We also use $\mathbf{x}_j(n)$ as the nth sample in the signal j. Each signal j is given a distinct measurement matrix $\mathbf{\Phi}_j \in \mathbf{R}^{M_j \times N}$, composed with i.i.d Gaussian entries. M_j denotes the number of measurements of the signal j. The received signal $\mathbf{y}_j \in \mathbf{R}^{M_j}$ can be written as $\mathbf{y}_j = \mathbf{\Phi}_j \mathbf{x}_j$. Concatenating all signals, we can define $\mathbf{X} = \left[\begin{array}{ccc} \mathbf{x}_1^T & \cdots & \mathbf{x}_J^T \end{array} \right]^T \in \mathbf{R}^{JN}$, $\mathbf{Y} = \left[\begin{array}{ccc} \mathbf{y}_1^T & \cdots & \mathbf{y}_J^T \end{array} \right]^T \in \mathbf{R}^{\sum_j M_j}$ and $\mathbf{\Phi} = diag\left(\begin{array}{ccc} \mathbf{\Phi}_1 & \cdots & \mathbf{\Phi}_J \end{array} \right) \in \mathbf{R}^{\sum_j M_j \times JN}$, where $\mathbf{Y} = \mathbf{\Phi X}$.

In the existing joint sparse signal model, [2], each signal $\mathbf{x}_j$ is decomposed into two parts, which are common information $\mathbf{z}_C$, which is shared by every received signal, and innovation information $\mathbf{z}_j, \quad j \in \Lambda$, which is uniquely measurable by the signal j only, respectively. The signal $\mathbf{x}_j$ can be written accordingly as

$$\mathbf{x}_j = \mathbf{z}_C + \mathbf{z}_j, \quad j \in \Lambda \tag{3}$$

However, there is very significant limitation on applicability of this model on general DCS. Some common information may not be always measured by every signal. If there is a type of information which is shared by multiple signals but not by every signal, then this information may not be properly incorporated into this model.

To overcome this problem, we propose a more refined DCS model to consider a more general DCS environment. To this end, we introduce two kinds of common information, which are full common information and partial common information. Full common information is defined as information shared by every signal in the same way as in [2]. Partial common information is one shared by multiple signals but not by every signal. For ease of explanation, we consider the simple case of existing partial common information when $J = 3$ as in Figure 1. We let $\mathbf{z}_{C_{\{j_1, j_2\}}}$ where $j_1, j_2 \in \Lambda$ and $j_1 \neq j_2$ denote pairwise partial common information that shared by signals $\mathbf{x}_{j_1}$ and $\mathbf{x}_{j_2}$. Since we consider DCS with the three signals in this paper, only pairwise partial common information will be addressed. If there are large number of signals in the system, partial information may be characterized further depending on how many signals share the information. To avoid confusion of notation, we newly define the innovation information as $\mathbf{z}_{i_j}$, which means the information measured only by signal j only. With this notion, we can write the signal $\mathbf{x}_j$ in a following form.

$$\begin{array}{l} \mathbf{x}_1 = \mathbf{z}_C + \mathbf{z}_{C_{\{1,2\}}} + \mathbf{z}_{C_{\{1,3\}}} + \mathbf{z}_{i_1} \\ \mathbf{x}_2 = \mathbf{z}_C + \mathbf{z}_{C_{\{2,1\}}} + \mathbf{z}_{C_{\{2,3\}}} + \mathbf{z}_{i_2} \\ \mathbf{x}_3 = \mathbf{z}_C + \mathbf{z}_{C_{\{3,2\}}} + \mathbf{z}_{C_{\{3,1\}}} + \mathbf{z}_{i_3} \end{array} \tag{4}$$

By relaxing the definition of the innovation information of existing DCS model such that it can include every type of information other than common information, we can represent the general DCS system with the existing DCS model in a consistent way as follows.

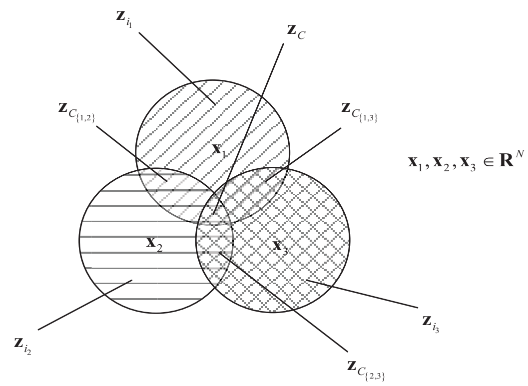

Fig. 1. Venn Diagram Description of the Proposed Signal Model for DCS

$$\begin{array}{l} \mathbf{x}_1 = \mathbf{z}_C + \mathbf{z}_1 \\ \mathbf{z}_1 = \mathbf{z}_{C_{\{1,2\}}} + \mathbf{z}_{C_{\{1,3\}}} + \mathbf{z}_{i_1} \\ \mathbf{x}_2 = \mathbf{z}_C + \mathbf{z}_2 \\ \mathbf{z}_2 = \mathbf{z}_{C_{\{2,1\}}} + \mathbf{z}_{C_{\{2,3\}}} + \mathbf{z}_{i_2} \\ \mathbf{x}_3 = \mathbf{z}_C + \mathbf{z}_3 \\ \mathbf{z}_3 = \mathbf{z}_{C_{\{3,2\}}} + \mathbf{z}_{C_{\{3,1\}}} + \mathbf{z}_{i_3} \end{array} \tag{5}$$

Next, for consistency in expression of the signals with [2], we also adopt the signal model in [2] which uses location matrix P and value vector θ. For further development, we give an example as an arbitrary vector $\mathbf{V} \in \mathbf{R}^N$ satisfying $\|\mathbf{V}\|_0 = K_V$. It can be written as in the following way with this model.

$$\mathbf{V} = P_{\mathbf{V}} \theta_{\mathbf{V}} \tag{6}$$

where $P_{\mathbf{V}} \in \mathbf{R}^{N \times K_V}$ is an identity submatrix, which consists of K_V column vectors chosen from $N \times N$ identity matrix, and a vector $\theta_{\mathbf{V}} \in \mathbf{R}^{K_V}$ contains nonzero elements of signal $\mathbf{V}$. Exploiting (5) and existing DCS model [2], the example signal may be expressed as follows.

$$\begin{array}{l} \mathbf{X} = \left[\begin{array}{ccc} \mathbf{x}_1^T & \cdots & \mathbf{x}_3^T \end{array} \right]^T \in \mathbf{R}^{3N} \\ \mathbf{X} = P\theta, \text{ where} \\ P \in \mathbf{R}^{3N \times \left(K_C(P) + \sum_{j=1}^{3} K_j(P) \right)}, \quad \theta \in \mathbf{R}^{\left(K_C(P) + \sum_{j=1}^{3} K_j(P) \right)} \\ P = \left[\begin{array}{cccc} P_C & P_1 & 0 & 0 \\ P_C & 0 & P_2 & 0 \\ P_C & 0 & 0 & P_3 \end{array} \right], \quad \theta = \left[\begin{array}{c} \theta_C \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{array} \right] \end{array} \tag{7}$$

where $K_C(P)$ and $K_j(P)$ denote the sparsity of $\mathbf{z}_C$ and $\mathbf{z}_j$ respectively. P_C and P_j are location matrices of common information and innovation information, and θ_C and θ_j are value vectors of common and innovation information. Obviously, it can be refinable.

Similarly, we can express the proposed signal model with location matrix and value vector in the following way.

$$
\begin{aligned}
&\mathbf{X} = \begin{bmatrix} \mathbf{x}_1^T & \cdots & \mathbf{x}_3^T \end{bmatrix}^T \in \mathbf{R}^{3N} \\
&\mathbf{X} = P'\theta', \quad \text{where} \\
&P' \in \mathbf{R}^{3N \times \left(K_C(P') + \sum\limits_{j_1,\, j_2,\, j_1 \neq j_2} K_{C_{\{j_1,j_2\}}}(P') + \sum\limits_{j=1}^{3} K_{i_j}(P') \right)}, \\
&\theta' \in \mathbf{R}^{\left(K_C(P') + \sum\limits_{j_1,\, j_2,\, j_1 \neq j_2} K_{C_{\{j_1,j_2\}}}(P') + \sum\limits_{j=1}^{3} K_{i_j}(P') \right)} \\
&P' = \begin{bmatrix} P_C & P_{C_{\{1,2\}}} & P_{C_{\{1,3\}}} & 0 & P_{i_1} & 0 & 0 \\ P_C & P_{C_{\{1,2\}}} & 0 & P_{C_{\{2,3\}}} & 0 & P_{i_2} & 0 \\ P_C & 0 & P_{C_{\{1,3\}}} & P_{C_{\{2,3\}}} & 0 & 0 & P_{i_3} \end{bmatrix}, \\
&\theta' = \begin{bmatrix} \theta_C \\ \theta_{C_{\{1,2\}}} \\ \theta_{C_{\{1,3\}}} \\ \theta_{C_{\{2,3\}}} \\ \theta_{i_1} \\ \theta_{i_2} \\ \theta_{i_3} \end{bmatrix}
\end{aligned} \tag{8}
$$

where $K_C(P')$, $K_{C_{\{j_1,j_2\}}}(P')$ and $K_{i_j}(P')$ denote the sparsity of $\mathbf{z}_C$, $\mathbf{z}_{C_{\{j_1,j_2\}}}$ and $\mathbf{z}_{i_j}$ respectively. In comparison of (7) with (8), it can be easily noted that P and θ in (7) are more refined in (8) as P' and θ' so that it can properly model the signal structure depending on signal correlation. The meaning of the location matrices and the value vectors in (8) can be readily understood since it is similar with the former case.

IV. Theoretical Bound For Three Signal Example

In this section, we articulate the theoretical bound of the number of measurements for distributed compressive sensing by exploiting the proposed generalized DCS model. Before proceeding further, we review the concept of overlap defined in [2].

Definition(quoted from [2]) 1 (Size of overlaps)**.** *The overlap size for the set of signals $\Gamma \subseteq \Lambda$, denoted $K_C(\Gamma, P)$, is the number of indices in which there is overlap between the common and the innovation information supports at all signals $j \notin \Gamma$:*

$$K_C(\Gamma, P) = \left| \left\{ n \in \{1, ..., N\} \;\middle|\; \begin{array}{l} \mathbf{z}_C(n) \neq 0 \text{ and...} \\ \forall j \notin \Gamma, \mathbf{z}_j(n) \neq 0 \end{array} \right\} \right| \tag{9}$$

If both $\mathbf{z}_C(n)$ and $\mathbf{z}_j(n)$ are nonzero, we cannot recover the original signal with this signal alone. It can be recovered, instead, with the help of other signals, which have common information not overlapped by the other information. We also define $K_C(\Lambda, P) = K_C(P)$ and $K_C(\phi, P) = 0$.

Simply, $K_C(\Gamma, P)$ where $\Gamma \subseteq \Lambda$, implies the number of entries which require measurements to recover common information in $\Gamma \subseteq \Lambda$. With above definition, the minimum number of measurements for reconstructing the desired signals can be determined from the following theorem.

Theorem(quoted from [2]) 1. *Assume that a signal ensemble $\mathbf{X}$ is obtained from a common/innovation information JSM (Joint Sparsity Model). Let $M = \{ M_1, \; M_2, \; M_3 \}$ be a measurement tuple, let $\{\mathbf{\Phi}_j\}_{j \in \Lambda}$ be random matrices having M_j rows of i.i.d. Gaussian entries for each $j \in \Lambda$. Suppose there exists a full rank location matrix $P \in P_F(X)$ where $P_F(X)$ is the set of feasible location matrices such that*

$$\sum_{j \in \Gamma} M_j \geq K_{cond}(\Gamma, P) = \left(\sum_{j \in \Gamma} K_j(P) \right) + K_C(\Gamma, P) \tag{10}$$

for all $\Gamma \subseteq \Lambda$. Then with probability one over $\{\mathbf{\Phi}_j\}_{j \in \Gamma}$, there exists a unique solution $\hat{\theta}$ to the system of equations $\mathbf{Y} = \mathbf{\Phi} P \hat{\theta}$; hence, the signal ensemble $\mathbf{X}$ can be uniquely recovered as $\mathbf{X} = P\hat{\theta}$.

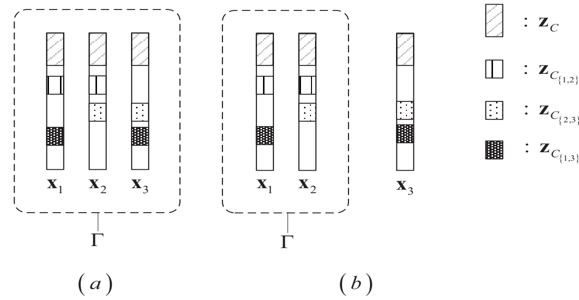

Fig. 2. Two Possible Cases of Subset Γ Including Signals

This theorem is proved in [2] using graph theory. Intuitive explanation of above theorem is as follows : Assume that we want to obtain the minimum number of measurements required in subset $\Gamma \subseteq \Lambda$ for reconstruction of the original signal. If some signals outside of Γ, have common information which is not overlapped by innovation information, we has no need to take measurement in Γ for the common information, because it can be recovered by outside of Γ. However, if all signals outside of Γ has overlapped common information, the measurements must be taken inside of Γ for recovering common information, since there is no way of recovering common information from outside of Γ.

We consider the theoretical bound of the number of measurements for the system consisting of three signals described in Figure 1. Figure 2 shows two possible cases of subset $\Gamma \subseteq \Lambda$ including signals. The case of subet Γ containing only one signals has not been considered since it is equivalent to the case in Figure 2-(b). The innovation information is omitted for convenience. The case in Figure2-(a), denotes that the subset Γ contains all signals. Again, assuming that we want to obtain the minimum number of measurements required in subset $\Gamma \subseteq \Lambda$, we need measurements enough to recover $\mathbf{z}_C$, $\mathbf{z}_{C_{\{1,2\}}}$, $\mathbf{z}_{C_{\{2,3\}}}$ and $\mathbf{z}_{C_{\{1,3\}}}$ that must be recovered inside of Γ. The necessary number of measurements is as follows.

$$
\begin{aligned}
&\sum_{j \in \Gamma} M_j \geq K_C(P) + \sum_{j_1, j_2 \in \Gamma} K_{C_{\{j_1,j_2\}}}(P) \\
&\text{where } \Gamma = \Lambda
\end{aligned} \tag{11}
$$

For the subset Γ in Figure2-(b), we can consider two different cases separately. First, we assume that there are no overlaps. In this case, all we have to recover in Γ is only $\mathbf{z}_{C_{\{1,2\}}}$, since other kinds of information can be recovered outside of Γ. By this principle, required measurements are as follows.

$$\begin{aligned}&\sum_{j\in\Gamma} M_j \geq K_{C_{\{1,2\}}}\\ &\text{where } \Gamma=\{1,2\}\end{aligned} \tag{12}$$

However, if overlaps occur, more measurements are needed in Γ since the overlapped information has to be recovered with help of signals inside of Γ. The necessary number of measurements is as follows.

$$\begin{aligned}&\sum_{j\in\Gamma} M_j \geq K_{C_{\{1,2\}}}+O\\ &\text{where } \Gamma=\{1,2\}\end{aligned} \tag{13}$$

where O denotes additionally required number of measurements because of the overlaps. To obtain the theoretical bound we need to redefine the number of overlaps more specifically, since the proposed model has the different types of common information. That is two types of overlap can happen where each overlap depends on the type of common information.

[Size of overlaps of full common information] The overlap size of full common information overlapped by information for the set of signals $\Gamma \subseteq \Lambda$, denoted $O_C(\Gamma, P)$, is the number of indices for which there is overlap between the full common and other information supports at all signals $j \notin \Gamma$. In three signal example, we have to consider the overlap between full common and innovation, and full common and partial common.

$$\begin{aligned}&O_C(\Gamma,P)=\\ &\left|\left\{ n\in\{1,...,N\} \left| \begin{array}{l} \mathbf{z}_C(n)\neq 0 \;\; and\; \forall j\notin\Gamma,\; \mathbf{z}_{i_j}\neq 0\\ or\;\; \mathbf{z}_C(n)\neq 0\;\; and \;...\\ \forall j_1\notin\Gamma,\; j_2\in\Lambda,\; \mathbf{z}_{C_{\{j_1,j_2\}}}\neq 0\end{array}\right.\right\}\right|\end{aligned} \tag{14}$$

We also need to quantify the overlaps between partial common information and other kinds of information.

Definition 2 (Size of overlaps of partial common information)**.** *Assume that $j_1\in\Gamma$, and $j_2\notin\Gamma$. The overlap size of partial common information shared by signals $\{j_1, j_2\}$ i.e. denoted by $\mathbf{z}_{C_{j_1,j_2}}$, between the other information(including full common, partial common except for $\mathbf{z}_{C_{j_1,j_2}}$ and innovation information) for the set of signals $\Gamma\subseteq\Lambda$, denoted $O_{C_{\{j_1,j_2\}}}(\Gamma,P)$, is the number of indices for which there is overlap between the partial common and the other component supports at a signal $j=j_2$.*

$$\begin{aligned}&O_{C_{\{j_1,j_2\}}}(\Gamma,P)=...\\ &\left|\left\{ n\in\{1,...,N\} \left| \begin{array}{l} \mathbf{z}_{C_{\{j_1,j_2\}}}(n)\neq 0\; and\; ..\; .\\ j=j_2,\; \mathbf{z}_{i_j}(n)\neq 0\\ or\; \mathbf{z}_{C_{\{j_1,j_2\}}}(n)\neq 0\; and\; \mathbf{z}_C(n)\neq 0\\ or\; \mathbf{z}_{C_{\{j_1,j_2\}}}(n)\neq 0\; and\; ...\\ \forall j_3\neq j_1, \forall j_3\neq j_2,\; \mathbf{z}_{C_{\{j_2,j_3\}}}\neq 0\end{array}\right.\right\}\right|\\ &\text{where } j_1\in\Gamma,\; j_2\notin\Gamma\end{aligned} \tag{15}$$

TABLE I
SIMULATION ENVIRONMENT

Simulation Environment	Value
The number of signals	3
The length of individual signal	50
The sparsity of full common information $\mathbf{z}_C$	3
The sparsity of partial information $\mathbf{z}_{C_{\{1,2\}}}$, $\mathbf{z}_{C_{\{2,3\}}}$, $\mathbf{z}_{C_{\{1,3\}}}$	2
The sparsity of innovation component $\mathbf{z}_{i_1}$, $\mathbf{z}_{i_2}$, $\mathbf{z}_{i_3}$	2

With these definitions, we can decide the theoretical bound of measurements for recovering the original signal for distributed compressive sensing in the following way.

Theorem 2. *Assume that there are three signals, which consist of full common, partial common, and innovation information respectively, as described in Figure 1. Suppose there exists a full rank location matrix $P \in P_F(X)$ such that*

$$\begin{aligned}\sum_{j\in\Gamma} M_j \geq O_C(\Gamma,P) + \sum_{j_1\in\Gamma, j_2\notin\Gamma} O_{C_{\{j_1,j_2\}}}(\Gamma,P) +\\ \sum_{j_1\in\Gamma, j_2\in\Gamma} K_{C_{\{j_1,j_2\}}}(P) + \sum_{j\in\Gamma} K_{i_j}(P)\end{aligned} \tag{16}$$

for all $\Gamma\subseteq\Lambda$. Then with probability one, there exists a unique solution $\hat{\theta}$ to the system of equations $\mathbf{Y}=\mathbf{\Phi}P\hat{\theta}$; hence, the signal ensemble $\mathbf{X}$ can be uniquely recovered as $\mathbf{X}=P\hat{\theta}$.

Significant difference between Theorem 1 of the theoretical bound in [2] is that our theorem considers the overlap of partial common component to provide a refined bound on the number of measurements for more general distributed compressive sensing. The proof of the Theorem 2 can be found in the journal version of our paper.

V. NUMERICAL RESULTS

For numerical simulations, we follow a system model described in Figure 1. Each signal is generated by i.i.d. Normal Gaussian i.e. $N(0,1)$. The simulation environment is described in the Table 1. Since each signal has same sparsity, which means symmetric case, the number of measurements of each signal is set to be the same. The measurement matrix is composed of i.i.d Normal Gaussian entries. Our simulation assumes noiseless environment.

For separate recovery, we can represent the given information and target problem as (17).

$$\begin{aligned}&\mathbf{Z}:=\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{x}_3\end{bmatrix}\in\mathbf{R}^{3N} \text{ and}\\ &\tilde{\Phi}:=\begin{bmatrix}\Phi_1 & 0 & 0\\ 0 & \Phi_2 & 0\\ 0 & 0 & \Phi_3\end{bmatrix}\in\mathbf{R}^{3M\times 3N},\\ &\mathbf{Y}=\tilde{\Phi}\mathbf{Z}\\ &\hat{\mathbf{Z}}=\arg\min\|\mathbf{x}_1\|_1+\|\mathbf{x}_2\|_1+\|\mathbf{x}_3\|_1\\ &s.t.\;\; \mathbf{Y}=\tilde{\Phi}\hat{\mathbf{Z}}\end{aligned} \tag{17}$$

In the view point of existing DCS model, the given information is formulated as (18). The weighted l_1 minimization is used to joint recovery.

$$
\begin{aligned}
&\mathbf{Z} := \begin{bmatrix} \mathbf{z}_C \\ \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \end{bmatrix} \in \mathbf{R}^{4N} \quad \text{and} \\
&\tilde{\Phi} := \begin{bmatrix} \Phi_1 & \Phi_1 & 0 & 0 \\ \Phi_2 & 0 & \Phi_2 & 0 \\ \Phi_3 & 0 & 0 & \Phi_3 \end{bmatrix} \in \mathbf{R}^{3M \times 4N}, \\
&\mathbf{Y} = \tilde{\Phi}\mathbf{Z} \\
&\hat{\mathbf{Z}} = \arg\min \gamma_C \|\mathbf{z}_C\|_1 + \gamma_1 \|\mathbf{z}_1\|_1 + \gamma_2 \|\mathbf{z}_2\|_1 + \gamma_3 \|\mathbf{z}_3\|_1 \\
&s.t. \;\; \mathbf{Y} = \tilde{\Phi}\hat{\mathbf{Z}}
\end{aligned} \tag{18}
$$

We use the same γ_C with section 5.1.6 in [2], since the sparsity is the same with [2]. When we consider proposed DCS model, the formulation becomes more complicated.

$$
\begin{aligned}
&\mathbf{Z} := \begin{bmatrix} \mathbf{z}_C \\ \mathbf{z}_{C_{\{1,2\}}} \\ \mathbf{z}_{C_{\{2,3\}}} \\ \mathbf{z}_{C_{\{1,3\}}} \\ \mathbf{z}_{i_1} \\ \mathbf{z}_{i_2} \\ \mathbf{z}_{i_3} \end{bmatrix} \in \; \mathbf{R}^{7N} \text{and} \\
&\tilde{\Phi} := \begin{bmatrix} \Phi_1 & \Phi_1 & 0 & \Phi_1 & \Phi_1 & 0 & 0 \\ \Phi_2 & \Phi_2 & \Phi_2 & 0 & 0 & \Phi_2 & 0 \\ \Phi_3 & 0 & \Phi_3 & \Phi_3 & 0 & 0 & \Phi_3 \end{bmatrix} \in \mathbf{R}^{3M \times 7N}, \\
&\mathbf{Y} = \tilde{\Phi}\mathbf{Z} \\
&\hat{\mathbf{Z}} = \arg\min \gamma_C \|\mathbf{z}_C\|_1 + \gamma_{C_{\{1,2\}}} \left\|\mathbf{z}_{C_{\{1,2\}}}\right\|_1 \\
&\qquad + \gamma_{C_{\{2,3\}}} \left\|\mathbf{z}_{C_{\{2,3\}}}\right\|_1 + \gamma_{C_{\{1,3\}}} \left\|\mathbf{z}_{C_{\{1,3\}}}\right\|_1 \\
&\qquad + \gamma_{i_1} \|\mathbf{z}_{i_1}\|_1 + \gamma_{i_2} \|\mathbf{z}_{i_2}\|_1 + \gamma_{i_3} \|\mathbf{z}_{i_3}\|_1 \\
&s.t. \;\; \mathbf{Y} = \tilde{\Phi}\hat{\mathbf{Z}}
\end{aligned} \tag{19}
$$

The weights used in (19) can be calculated by line search algorithm.

Performance comparison with the existing joint recovery for DCS is presented in Figure 3 and Figure 4. When we use proposed joint recovery, approximately 9 measurements are saved at exact reconstruction, compared to separate recovery while existing joint recovery saves approximately 4 measurements. Only 83.9% of measurements are needed when we use proposed joint recovery rather than existing joint recovery.

VI. Discusstion And Conclusions

In this paper, we elaborate the DCS framework to be applicable to a general DCS system environment, which typically involves partial common information. With proposed model, refined bound on the number of measurements for the DCS is obtained. The simulation results show that the proposed joint recovery taking advantage of the proposed signal structure provides the far better performance than the existing joint signal recovery for the DCS.

The DCS framework is well applied to distributed applications e.g. sensor network because of its properties. In sensor network, there is no communication between sensors therefore compression technique requiring correlation of whole signal structure is hardly used. In DCS, the encoder has no need to know the correlation structure during compression, which makes it matched well with sensor network. Additionally, centralized complexity at joint decoder makes it easy1 to set low hardware limit of sensors. This paper would be a first step of research on practical version DCS. However, this work relies on the assumption of the known signal structures at the joint decoder and no noise perturbation. We are studying more practical application of the DCS in the journal version of our paper which assumes unknown signal structure.

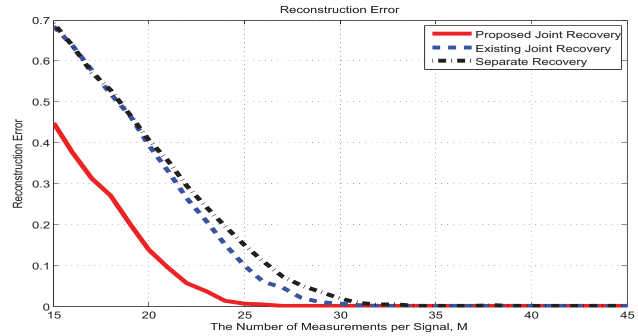

Fig. 3. Comparison of Reconstruction Error between Proposed Joint Recovery, Existing Joint Recovery and Separate Recovery

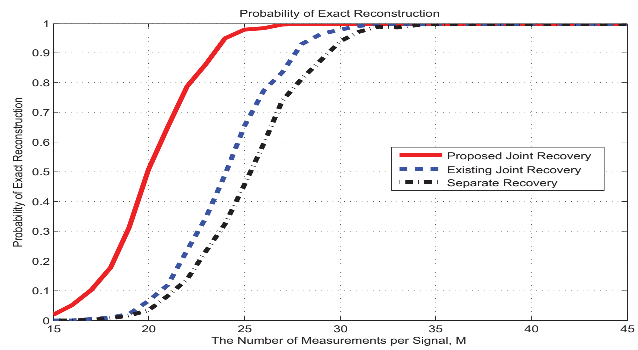

Fig. 4. Comparison of Probability of Exact Reconstruction between Proposed Joint Recovery, Existing Joint Recovery and Separate Recovery

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