Application of Tagged User Analysis to FU-FB Slotted ALOHA Performance over Frequency Selective Fading Channels

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***Abstract*—Slotted ALOHA (S-ALOHA) is a simple random multiple access technique used in many data and cellular networks for random access phase. We use tagged user analysis (TUA), an approximate analysis technique, for evaluation of performance parameters of finite user finite buffer (FU-FB) S-ALOHA system over frequency selective fading channels. We also derive analytical expressions for several performance metrics. TUA does not suffer from the complexity issues normally associated with Markov model, where a state space of complexity in the exponential order of buffer size and the number of users must be solved. On comparison of analytical results with simulations, it is observed that TUA is able to accurately analyze the system for a much lower computational cost.**

***Keywords*-TUA; S-ALOHA; FU-FB; multipath channel; queueing analysis; Rayleigh fading;**

I. Introduction

Random access protocols are widely used in multiple access wireless communication systems such as packet satellite communications, wireless LAN, and cellular mobile systems. The analysis of these systems is of particular importance. Most of the existing studies assume that there is always a packet available for transmission in each station (saturation) [1], [2] or use the assumption of infinite population without buffer for the simplicity of analytical modeling and derivation. The available methods for the analysis of random access methods fall in one of three classes, namely S-G analysis, Markov analysis, and equilibrium point analysis (EPA) [3].

S-G analysis generates an aggregate traffic of S packets/slot. It cannot treat the case of buffered users as it assumes infinite population whereas practical systems are FU-FB systems. Markov chain or state-space method provides a way to perform exact analysis of FU-FB systems [4], [5]. Though it can be applied to all types of multiple access protocols, the analysis becomes too cumbersome when dealing with buffered systems. The number of states increases exponentially with the number of users and queue size. EPA is an approximate analysis technique that has been used to analyze FU-FB systems [6], but the size of its state space becomes prohibitively large quickly with an increase in packet transmission time or buffered capacity. Thus, the existing analytical techniques either employ strong assumptions (S-G analysis) or have high complexity (Markov and EPA analysis methods) that quickly become prohibitive. It is worth mentioning that these existing analytical techniques start from basic principles and do not utilize the existing results of the vast literature on queuing theory.

The current demand of high data rates and mobility means many future wireless systems would be required to operate in frequency selective environments. The complex nature of the frequency selective channels renders the analysis with Markov chain even more computationally intensive. This is because in addition to the size of user population and the buffer, the number of paths in the channel add to the dimension of state space.

TUA is a relatively new approximate analytical approach that was proposed in [7]. Its motivation was to take advantage of the existing results in queueing theory and simplify the analysis of buffered systems by reducing the multi-dimensional state system into a single dimension system. It decouples the channel contention behavior from the user queueing behavior and allows the use of classical queueing theory results to be directly applicable to the analysis of random access protocols. TUA has been successfully applied to the analysis of FU-FB systems for various random access protocols under steady channels that are typical of computer networks and also to flat fading channels for wireless communications [7], [8].

In this paper, we apply TUA method for analyzing a FU-FB system operating over in a frequency selective fading environment and derive analytical expressions for system performance indices. It is shown that for a moderate number of active users, the simulation and analytical results fit closely; proving the accuracy of TUA method. Thus, we establish the viability of TUA in analyzing broadband systems where the multipath fading environment manifests as frequency selective fading.

The paper organization is as follows. The system model is explained in Section II and the TUA technique is elaborated in Section III. The frequency selective multipath fading channel model adopted in the analysis and simulation is described in Section IV. The numerical results of TUA and system simulations are compared in Section V while Section VI gives the concluding remarks.

II. System Model

We consider a homogeneous centralized S-ALOHA system in which a finite population of N users is distributed randomly

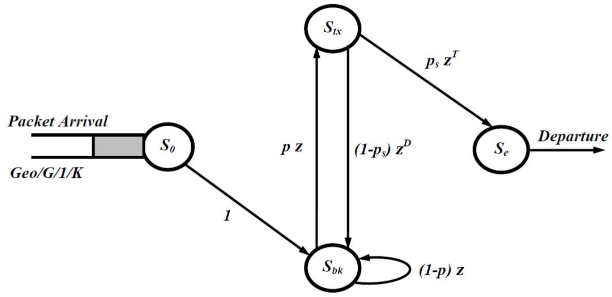


Fig. 1. State flow diagram of a tagged user

on a circle around the base station (equal pathloss). The packet arrival of each user is assumed to be a Bernoulli process with an average arrival rate of λ packets per slot, while channel access is assumed to have a geometric distribution with access probability, p. All packets reach the base station through a frequency selective fading channel. If more than one user transmit packets during a given time slot, the user whose power exceeds the total power of all other contending users by the capture threshold, z_0, is deemed successful [9]. The state flow graph for the tagged user under steady state condition is shown in Figure 1.

The tagged user can be modeled as a Geo/G/1/K queueing system [8]. It has four states, S_0, S_{tx}, S_{bk} and S_e that represent packet ready, transmission start, back off and departure states, respectively. At the start of a slot, the user can be in any one of the above states. Following the late arrival with delayed replacement rule [10], an arriving packet is dropped if the queue of the user is full, otherwise it enters the queue. When a new packet arrives in the queue and the server is busy, it waits until the previous packet already in the server is successfully transmitted. A successful transmission is represented by the state S_e. After a successful departure, which takes T slots, the packet at the head-of-queue enters the virtual server and the user is now in the S_0 state. From S_0 the user moves to the S_{bk} state without any delay. The user can also enter S_{bk} from S_{tx} if the previous transmission attempt was unsuccessful.

III. Tagged User Analysis Model

TUA uses the fact that at equilibrium, all users have statistically equivalent behavior. This allows us to find the system performance from the analysis of any one user. TUA framework decouples the contention analysis from the queueing process analysis.

A. Channel Contention Analysis

The contention analysis deals with the determination of virtual packet service time (VPST) distribution, which is dependent on the number of users and interference from these users, their busy probabilities and channel condition. The expression for probability generating function (PGF) of VPST is derived from the model described by the state transition diagram shown in Figure 1 under the assumption that the system is in steady state.

$$B(z) = \frac{pp_s z^{T+1}}{1-(1-p)z - p(1-p_s)z^{D+1}} \tag{1}$$

where T is the packet transmission time. After packet transmission, a busy user can be in any one of the two modes: CON (contending for the channel) or WAI (waiting for acknowledgement) that takes $D-1$ slots. Let p_c be the probability that the user is in CON mode, then probability p_s that the transmission is successful is given as

$$p_s = \sum_{i=0}^{N-1} \left(\begin{array}{c} N-1 \\ i \end{array} \right) (pp_c)^i (1-pp_c)^{N-1-i} p_{s|i} \tag{2}$$

where, $p_{s|i}$ is the probability that the transmission is a success given there are i interfering users. This probability is dependent on the channel condition and the number of simultaneous transmissions from other users. From (1), the mean packet service time is given by

$$B'(1) = b = (T-D) + \frac{D}{p_s} + \frac{1}{pp_s} \tag{3}$$

On average the user waits for acknowledgment for $\frac{D-1}{p_s}$ slots, hence

$$p_c = \frac{p_b}{b}\left(b - \frac{D-1}{p_s}\right) \tag{4}$$

where p_b is the user busy probability. Using (1) to (4) we can compute $B(z)$ for the given p_c.

B. Queueing Analysis

The queueing analysis is applied to determine the user busy probabilities, given the packet service time distribution and arrival process. These relations are non-linear but sufficient to determine VPST and the user busy probabilities. In order to find p_b from $B(z)$, we use the algorithm given in [10] for Geo/G/1/K system. The PGF for a_k, which is the probability that k packets arrive during a service time, is given as

$$A(z) = \sum_{k=0}^{L-1} a_k z^k = B(1-\lambda+\lambda z) \tag{5}$$

since at most $L-1$ packets arrive in one service time. Denoting π_k, where $k = 0, 1, \ldots, L-1$, as the probability that a leaving packet sees k packets in queue, we have (from [10])

$$\pi_k = \pi_0 a_k + \sum_{j=1}^{k+1} \pi_j a_{k-j+1} \tag{6}$$

$$\pi_0 = \left(\sum_{k=0}^{L-1} \acute{\pi}_k\right)^{-1} \tag{7}$$

where $\acute{\pi}_k = \frac{\pi_k}{\pi_0}$ which can be computed from the recursion

$$\acute{\pi}_{k+1} = \frac{1}{a_0}\left(\acute{\pi}_k - \sum_{j=1}^{k} \acute{\pi}_j a_{k-j+1} - a_k\right) \tag{8}$$

with $\pi'_0 = 1$, and where p_k is the probability that there are k packets present in the system at any slot boundary. For $0 \leq k \leq L-1$, we have

$$p_k = \frac{\pi_k}{\pi_0 + \rho} \quad (9)$$

$$p_L = 1 - \frac{1}{\pi_0 + \rho} \quad (10)$$

where $\rho = \lambda b$ and b is defined in (3). From (1) to (4), we see that that $B(z)$ is a function of p_c or equivalently p_b. The queueing theory results of (5) to (9) affirm the dependence of p_b on $B(z)$. These relationships are independent and hence they can be solved simultaneously. However, due to non-linearity of the equations, numerical methods have to be used.

IV. Frequency Selective Fading Channel and the Success Probability

For a frequency selective channel, the signal due to every active user has M multipath components, each undergoing independent Rayleigh fading. Let P_t be the received power of the tagged user and P_i be the total received power due to i interfering users where $0 \leq i \leq N-1$, then the tagged user transmission is successful only if

$$P_t > z_0 P_i \quad (11)$$

where z_0 is the receiver signal capture ratio. Hence the conditional success probability of the tagged user, $p_{s|i}$, can be written as

$$p_{s|i} = P_r[P_t > z_0 P_i] = 1 - P_r[\frac{P_t}{P_i} < z_0] \quad (12)$$

$$= 1 - P_r[Z_i < z_0] \qquad i = 0, ..., N-1 \quad (13)$$

Using independence of P_t and P_i, and simple transformation of random variables, we get

$$p_{s|i} = 1 - F_{Z_i}(z_0) = \int_{z_0}^{\infty} \int_{0}^{\infty} f_{P_t}(zw) f_{P_i}(w)\ w\ dw\ dz \quad (14)$$

At the base station, the signal component due to the unsuccessful users is termed as the interference while that due to the successful user is termed as the signal of interest (SOI). Under realistic channel conditions, the phases of the interference signals vary sufficiently fast, the result is a *non-coherent addition* of the phasors (sum of powers) at the base station. As the signal from each user is composed of M multipaths, the SOI at the base station can have the following two cases.

A. SOI only one path

The SOI is taken to be the strongest path of the tagged user and the remaining $M-1$ paths of the tagged user along with any signal component due to users other than tagged user act as interference terms. The expression for the conditional success probability of the tagged user, $p_{s|i}$, can be shown to be given as

$$p_{s|i} = \prod_{m=1}^{M} \left(\frac{\overline{P}_M}{\overline{P}_m}\right)^{\alpha_m} \frac{1}{(z_0 \frac{\overline{P}_M}{\overline{P}_1} + 1)^{M(i+1)-1}} \sum_{k=0}^{\infty} \frac{\delta_k}{(z_0 \frac{\overline{P}_M}{\overline{P}_1} + 1)^k} \quad (15)$$

where $\overline{P}_1$ and $\overline{P}_M$ are $\max \overline{P}_m$ and $\min \overline{P}_m$ respectively, for $1 \leq m \leq M$,

$$\alpha_m = \begin{cases} i, & \text{if} \quad m = 1 \\ i+1, & \text{if} \quad m \geq 2 \end{cases} \quad (16)$$

and the coefficient δ_k can be found by recursively solving the following equation

$$\delta_{k+1} = \frac{1}{k+1} \sum_{l=1}^{k+1} \left[\sum_{j=1}^{M} \alpha_j \left(1 - \frac{\overline{P}_M}{\overline{P}_j}\right)^l \right] \delta_{k+1-l} \quad (17)$$

with $\delta_0 = 1$.

B. SOI power sum of M paths

When all paths of the multipath frequency selective channel have distinct average powers, we can combine the M multipaths to form the SOI as a power sum [11]. In this case the $p_{s|i}$ can be shown to be given as

$$p_{s|i} = \prod_{m=1}^{M} \left(\frac{\overline{P}_M}{\overline{P}_m}\right)^i \sum_{j=1}^{M} \frac{\overline{P}_j^{M-1}}{\prod_{k=1, k \neq j}^{M} (\overline{P}_j - \overline{P}_k)} \times \frac{1}{(z_0 \frac{\overline{P}_M}{\overline{P}_j} + 1)^{Mi}} \sum_{k=0}^{\infty} \frac{\delta_k}{(z_0 \frac{\overline{P}_M}{\overline{P}_j} + 1)^k} \quad (18)$$

where all the $\overline{P}_j$ are distinct and the coefficient δ_k are given by

$$\delta_{k+1} = \frac{1}{k+1} \sum_{l=1}^{k+1} \left[\sum_{j=1}^{M} (i) \left(1 - \frac{\overline{P}_M}{\overline{P}_j}\right)^l \right] \delta_{k+1-l} \quad (19)$$

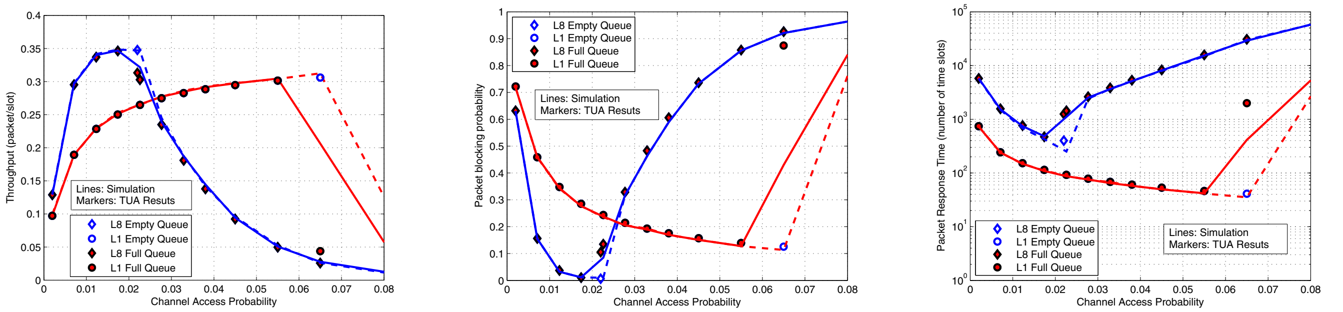
and $\delta_0 = 1$. It should be noted that (15) and (18) reduce to the case of flat fading with non-coherent addition of interference derived in [8], if the paths are reduced to one.

V. Numerical Results and Discussion

We consider a scenario where a homogeneous group of users attempt to communicate with a central base station using S-ALOHA protocol. The channel is assumed to be frequency selective and follows the ITU channel model [12] as given in Table I. The system is simulated for $100,000$ time slots and averaged over 10 runs. The number of users in the system, N, is taken as 100 and the capture ratio, z_0, is set to 4 dB. The packet arrival rate, λ, is set to 0.0035 which corresponds to a system with moderate traffic. The system is simulated for two different queue lengths ($L = 1$ and $L = 8$). Though we have simulated the system for a range of packet arrival rate

TABLE I
ITU Channel Model for Outdoor to Indoor and Pedestrian Test Environment

Taps	Relative delay (ns)	Average power (dB)	Doppler spectrum
1	0	0	Classic
2	110	-9.7	Classic
3	190	-19.2	Classic
4	410	-22.8	Classic



(a) Throughput (b) Packet blocking probability (c) Packet response time

Fig. 2. Analytic and simulation results for an S-ALOHA system with immediate acknowledgement, SOI dominant path only. $N = 100$, $\lambda = 0.0035$

and different queue lengths, those results have been omitted due to the space limitations.

A. Performance Measures

We use the following performance measures to analyze the performance of the S-ALOHA system.

1) Blocking Probability: Blocking probability, p_B, is the probability that an arriving packet finds the queue full and is given as

$$p_B = p_L = 1 - \frac{1}{\pi_0 + \rho} \tag{20}$$

2) System Throughput: The system throughput, Θ, is defined as the average number of packets transmitted per slot.

$$\Theta = N\theta \tag{21}$$

where θ is the throughput of the tagged user which is given as

$$\theta = \lambda(1 - p_B)T \tag{22}$$

where $\lambda(1 - p_B)$ is the average packet acceptance rate.

3) Mean Packet Response time: Packet response time is defined only for those packets that are accepted into the system. It is the total time spent by the packet from its arrival into the queue to its successful departure from the virtual server. The mean packet response time, $E[t_r]$, is given by

$$E[t_r] = \frac{E[I_q]}{\lambda(1 - p_B)} \tag{23}$$

where $E[I_q]$ is the mean queue length, defined as the average number of packets in the queue of a user of maximum queue size L and given by

$$E[I_q] = \sum_{k=0}^{L} k \ p_k \tag{24}$$

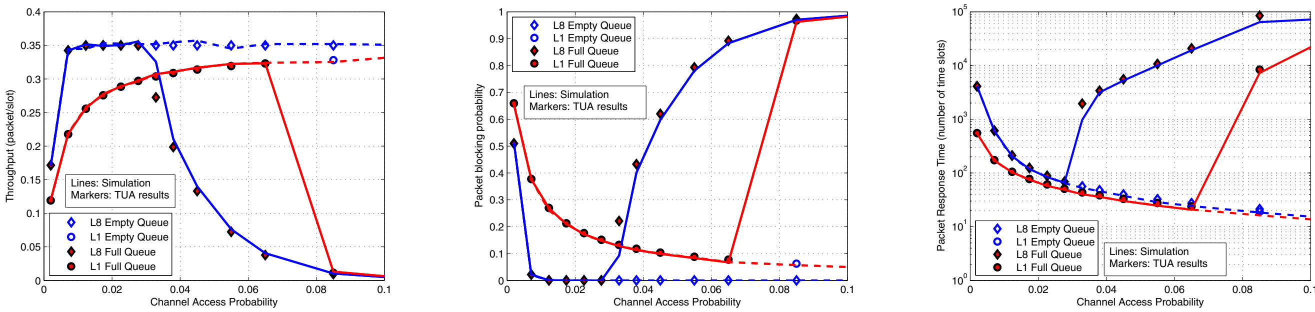
B. SOI is the dominant path

Figure 2(a)-2(c) show a set of curves for the case when SOI comprises of only the strongest path of the dominant user. The figures show the throughput, blocking probability and packet response time for a range of channel access probabilities. Queue length $L = 1$ corresponds to a system with no queue. So if user k has a packet to serve, it will not accept any more packets until the current packet is successfully transmitted and its acknowledgement is received.

From Figure 2(a), we see that for $L = 8$, throughput is reasonably high (more than 0.3355 packets/slots) for p between 0.012 and 0.0226. Outside this range, the throughput decreases dramatically. This behavior is explained as follows: The channel access probability, p, corresponds to the likelihood of a user to attempt transmission. Low p means that most users will be in idle state while high p means most users will attempt to transmit in a given time slot, provided they have a packet in the queue. Although a low p reduces the chances of collision, it will cause the user queue to fill up as shown in Figure 2(b) and result in a large packet response time as shown in Figure 2(c). A high p results in more active users per time slot which reduces the throughput due to interference from other users as well as self interference, which again results in large packet response time. The packet response time shown in Figure 2(c) sets the limits for practical values of p based on acceptable latency in packet transmission. For a system with $L = 8$, the minimum packet response time occurs for $p = 0.0226$.

C. SOI is power sum of M multipaths

Figure 3(a)-3(c) show a set of curves for the case when SOI comprises of the power sum of all the multipath components of the dominant user. The difference in the results of empty and full queue initial conditions has to do with the packet arrival rate and packet response time. If the rate at which the packets leave the queue is less than the rate at which packets arrive at the queue, not only will the queue fill up, but subsequent arrivals will be dropped. This can be observed from Figure 3(b) where the p_B increase rapidly for $p > 0.0277$. A high value of p will allow more users to be active in a given time slot provided they have a packet to transmit. If the queues of all or most of the users are always full, a larger number of users will contend for the channel during every time slot as compared to the case when the user queues are initially empty. Thus, a system with the initial condition of full queue will experience critical failure due to congestion at a lower value of p as compared to a system which start with an initially empty queue. Comparing the results of queue length $L = 1$



(a) Throughput

(b) Packet blocking probability

(c) Packet response time

Fig. 3. Analytic and simulation results for an S-ALOHA system with immediate acknowledgement, SOI sum of multipaths. $N = 100$, $\lambda = 0.0035$

and $L = 8$ in Figure 3(a), we see that increasing the queue length results in better throughput and much lower blocking probability. This is expected as larger queue length means that an arriving packet is less likely to find the queue full and thus less likely to be dropped. Figure 3(a) also shows that for $N = 100$, $\lambda = 0.0035$ and $L = 8$, the maximum throughput is achieved for $p \geq 0.0123$ (empty queue initial condition) and for $0.0123 \leq p \leq 0.0277$ (full queue initial condition).

Figure 3(c) shows that for the case of $L = 8$ and full queue initial condition, the minimum packet response time occurs at $p = 0.0277$ while for the empty queue initial condition, the packet response time follows the same shape as that of full queue up to $p = 0.0277$ and afterwards it keeps on decreasing for larger values of p. The optimum value of p would thus be given by the minimum of the common portion of the two curves, i.e., $p = 0.0277$. We note that using TUA, we can accurately find the optimum operating point of the system.

D. Comparison between SOI dominant path and SOI sum of multipaths

Comparing Figures 2 and 3, we observe the optimum channel access probability is different for the two cases discussed. Further comparison of Figures 2(c) and 3(c) reveals that for the system with $L = 8$, the packet response time of the scheme that performs a power sum of the multipaths of the dominant user to form the SOI is seven times less than the scheme which takes only the strongest path as the SOI (from 481.8 time slots to 66.03 time slots). This is a significant reduction in packet response time and provides an argument for combining the multipaths of the dominant user to form the SOI.

VI. Conclusions

The paper presents an approximate analysis for a homogeneous FU-FB S-ALOHA system using TUA. The system operates in a frequency selective environment with terminals uniformly distributed along a ring. Non-coherent addition (power sum) of interference signals has been assumed. This can easily be extended to the case where interference signals are added coherently. For the SOI, two cases have been discussed and it is found that the system throughput is better for the case when SOI is the sum of multipaths of the desired user. System performance parameters which include throughput, packet blocking probability and packet response time have been analyzed for different queue lengths. Simulation results match very well with those of analyses, which shows the versatility of TUA. Currently, we are working to apply the analysis to heterogeneous systems and systems with other random access protocols operating over frequency selective channels.

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