# Interference Alignment: Improved design via precoding vectors

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***Abstract*—The degree of freedom of the Single Input Single Output (SISO) fading interference channel is asymptotically upperbounded by K/2. This upperbound can be achieved using the Interference Alignment approach (IA), proposed by Cadambe et *al.*. In this work, a new optimized design of the IA scheme is presented. It involves introducing, for each user, a combination matrix so as to maximize the sum rate of the network. The optimal design is obtained via an iterative algorithm proposed in the $K$-user IA network, and a convergence to a local optimum is achieved. Numerical results enable us to evaluate the performance of the new algorithm and to compare it with other designs.**

## I. Introduction

Untill recently, the capacity region of the interference channels was an open problem. Several researches have pursued some special cases among which [1]–[3]. In [3], a new upperbound on the channel capacity have been demonstrated for the two-user $X$ channel. The achievability of this upperbound involves introducing a new approach of interference management, known as Interference Alignment (IA).

The basic idea of the IA is to design the transmit signals such that interfering signals at each receiver overlap while the desired signal remains distinct from interferences. Cadambe and Jafar (CJ) have exploited this approach in order to show that the maximum achieved Degrees of Freedom (DoF) for $K$-user time-varying interference channels, in the $n$-dimensional Euclidean space, is $\frac{K}{2}$ [4]. However, when the channel is quasi-static the capacity region remains unknown. Therefore some researches have discussed the maximum achievable DoF for the quasi-static channels [5], [6].

In [7], the authors have extended the idea of IA from space/time/frequency dimensions to the signal level dimensions. Based on the field of Diophantine approximation in number theory, it has been proven that interference can be aligned in the rational spaces using the properties of the rational and irrational numbers, thus, the full DoF, $\frac{K}{2}$, can be achieved. This demonstration hold when the users are synchronous and the received signal at each receiver node is a synchronized linear combination of the transmitted signals. In practice, such an assumption is not realistic. Therefore, in [8] the asynchronous transmission in the $K$-user fading interference channels with quasi-static coefficients is considered. It is shown that the total DoF of this channel is the same as that of the corresponding synchronous channel.

In the IA schemes described above, the full DoF is achieved for dimensional precoding vectors and signal-to-noise ratio (SNR) close to infinity. The maximum achievable DoF is defined as

$$\lim_{snr\rightarrow\infty} \frac{C(snr)}{\log_2(snr)} = \frac{K}{2} \ , \tag{1}$$

where $K$ is the total user number in the fading interference channel, and $C(snr)$ represents the channel capacity.

In order to optimize the IA beamforming design and to improve the data rate performance, some methods have been proposed such as in [9]–[11]. One of these methods suggests to introduce a combination matrix at each transmitter, and to optimize it so as to maximize the sum rate for high SNR, assuming a Zero-Forcing (ZF) decoding scheme. The optimal solution of the introduced matrix is the one that orthonormalizes the precoding matrix for each user. In this paper we aim to maximize the sum rate of the IA network when a Minimum Mean Square Error (MMSE) decoding scheme is employed in the time-varying fading interference channels. Compared to the ZF decoder, the main advantage of an MMSE decoder is a better error performance while a matched filter receiver can be implemented at lower computational cost. An MMSE shows an equivalent error performance to a ZF for high SNR level.

This paper is organized as follows. In Section II, we describe the system model of the $K$-user fading interference channel. Section III reviews some related work to highlight our contribution. The proposed optimization algorithm is given in Section IV. Numerical results are shown in the $K$-user SISO fading interference channel in Section V. Finally, Section VI concludes the paper.

Notations: boldface upper case letters and boldface lower case letters denote matrices and vectors, respectively. For the transpose, transpose conjugate and conjugate matrix we use $(.)^t$, $(.)^H$ and $(.)^*$, respectively. $|.|$ and tr$(.)$ denote the determinant the trace of a matrix, respectively. The vec$(.)$ operator indicates the vectorization of a matrix, and J$(.)$ denotes the Jacobian of a matrix.

## II. System Model

Consider the $K$-user SISO time-varying block fading interference channel as illustrated in Fig. 1, with $K$ transmit receive pairs. A wireless channel links each receiver to one transmitter, and each transmitter intends to have its signal

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decoded by its destination. The adopted IA scheme in this paper is the one proposed in [12], which shows more efficiency than the CJ scheme for $K > 3$ and higher DoF is achieved for reduced channel extension. A synchronized scheme is adopted with full and perfect channel knowledge at each source and destination [1]. The DoF per user is obtained using the following combinations

$$d^1 = \binom{m^* + M + 1}{M} \quad \text{and} \quad d^3 = \binom{m^* + M}{M}$$

where $m^*$ is a given nonnegative integer, $M$ is a parameter depending on the user number $M = (K-1)(K-2) - 1$, and $d^i$ is the DoF of the $i^{th}$ user. Provided $d^i = d^3$ , $d^1 > d^3$, $i \in \mathcal{K} \backslash \{1, 3\}$, IA can be then satisfied. $\mathcal{K}$ represents the set of the user indices $\{1, 2, ..., K\}$. The precoding vector length, obtained using the channel extensions, is given as $N = d^1 + d^2$. The channel is supposed to be frequency selective, then the Orthogonal Frequency Division Multiplexing (OFDM) transmission technique can be applied (in this case $N$ will denote the subcarrier number). At the $k^{th}$ destination, the channel output is given by

$$\boldsymbol{y}_k = \sum_{j=1}^{K} \boldsymbol{H}_{kj}\, \boldsymbol{V}_j\, \boldsymbol{x}_j + \boldsymbol{z}_k, \ \forall k \in \mathcal{K} \ , \tag{2}$$

where $\boldsymbol{H}_{kj}$ is the $N \times N$ diagonal channel fading matrix between the $j^{th}$ transmitter and the $k^{th}$ receiver. $\boldsymbol{V}_j$ is the $N \times d^j$ precoding matrix of the $j^{th}$ transmitter. The $j^{th}$ transmitted information $\boldsymbol{x}_j$ is defined as a $d^j \times 1$ vector. $\boldsymbol{z}_k$ is the $N \times 1$ circular symmetric complex Gaussian noise vector at the receiver $k$, with independent and identically distributed (i.i.d.) components; i.e. $\boldsymbol{z}_k \sim \mathcal{N}_c(0, \boldsymbol{I}_N)$.

According to [12], the beamforming design criterion of the IA scheme is defined as

$$\boldsymbol{V}_1 = \{ \prod_{k,l \in \mathcal{K}\backslash 1,\ k \neq l,\ (k,l) \neq (2,3)} \left( (\boldsymbol{T}_{23})^{-1} \boldsymbol{T}_{kl} \right)^{n_{kl}} \boldsymbol{w} \ | \sum_{k,l \in \mathcal{K}\backslash 1,\ k \neq l,\ (k,l) \neq (2,3)} n_{kl} \leq m^* + 1 \} \tag{3}$$

and

$$\boldsymbol{V}_3 = \{ (\boldsymbol{T}_{23})^{-1} \prod_{k,l \in \mathcal{K}\backslash 1,\ k \neq l,\ (k,l) \neq (2,3)} \left( (\boldsymbol{T}_{23})^{-1} \boldsymbol{T}_{kl} \right)^{n_{kl}} \boldsymbol{w} \ | \sum_{k,l \in \mathcal{K}\backslash 1,\ k \neq l,\ (k,l) \neq (2,3)} n_{kl} \leq m^* \} \ , \tag{4}$$

and

$$\boldsymbol{V}_j = \boldsymbol{H}_{1j}^{-1} \boldsymbol{H}_{13} \boldsymbol{V}_3 \quad \text{for} j \neq (1, 3), \tag{5}$$

where $\boldsymbol{w}$ is the $N \times 1$ vector determining the precoding subspace of each user. For an optimized data rate performance, $\boldsymbol{w}$ should be judiciously chosen in order to maximize the data rate as described in [9], [10]. $\boldsymbol{T}_{kl}$ is an $N \times N$ diagonal matrix defined as

$$\boldsymbol{T}_{kl} = (\boldsymbol{H}_{k1})^{-1} \boldsymbol{H}_{kl} (\boldsymbol{H}_{1l})^{-1} \boldsymbol{H}_{13}. \tag{6}$$

[1]It is important also to recall that this scheme can be easily extended to the one proposed in the quasi-static interference channels (cf. Section IV in [8]).

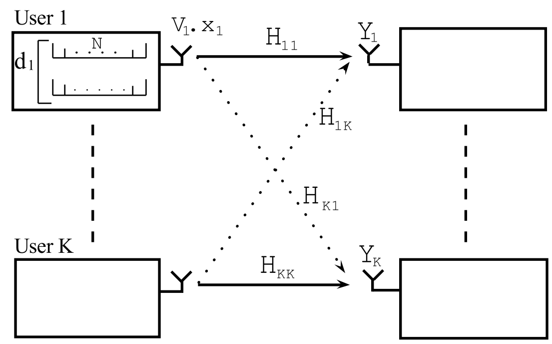


Fig. 1. The $K$-user SISO fading interference channel

In the remaining of the paper, we aim to optimize the precoding vectors of the precoding subspace at each trasmitter node. Therefore, the precoder is modified by introducing a combination matrix at each transmitter. The precoding matrix $\boldsymbol{V}_k$ defined in (2) will be replaced by

$$\boldsymbol{P}_k = \boldsymbol{V}_k \cdot \boldsymbol{C}_k, \tag{7}$$

where $\boldsymbol{C}_k$ is a $d^k \times d^k$ matrix that will be designed so as to maximize the sum rate while keeping the IA scheme satisfied, and thus, ensuring a full asymptotically achievable DoF for the network. Note that the matrices $\boldsymbol{P}_k$ and $\boldsymbol{V}_k$ are still in the same subspace, then the combination matrix.

## III. Review works about the IA beamforming optimized designs

In this section, we review the two algorithms that seek to find the combination matrices $\boldsymbol{C}_k$, for all $k$, that maximize the data rate performance. We also recall the expression of the maximum individual rate for an MMSE receiver.

### A. Optimization of the precoding vectors

The optimization of the precoding vectors within the subspaces, proposed by CJ at each transmitter, was firstly proposed in [11]. This perspective aims to maximize, in the 3-user SISO fading interference channel, the high SNR offset of the sum rate under power limitation constraints (cf. eq. (5) and (8)-(20) in [11]) while preserving the full asymptotically achievable DoF. The optimized precoders is given by

$$\begin{aligned} \tilde{\boldsymbol{V}}_1 &= \frac{\boldsymbol{V}_1}{||\boldsymbol{V}_1||} \\ \tilde{\boldsymbol{V}}_2 &= \boldsymbol{V}_2\, \boldsymbol{F}, \\ \tilde{\boldsymbol{V}}_3 &= \boldsymbol{V}_3\, \boldsymbol{E}. \end{aligned} \tag{8}$$

Considering the optimization of the high SNR offset of the sum rate for a ZF receiver, the maximum solutions of $\boldsymbol{E}$ and $\boldsymbol{F}$ are $d^k \times d^k$ matrices that orthonormalizing the $d^k$ columns of $\boldsymbol{V}_2$ and $\boldsymbol{V}_3$, respectively, while allocating an average power of $\frac{N}{d^k}$ for each stream of the $k^{th}$ transmitter.

Unfortunately, the ZF increases the level noise at the reception leading to a degradation error performance. Therefore MMSE criterion has been employed by Sung et *al.* and the optimization problem has been proposed throughout the SNR region [13]. However, a suboptimal strategy was shown where the $\boldsymbol{C}_k$, at each transmitter, has been chosen as a unitary matrix in order to reduce the computational complexity.

### B. Maximum sum rate expression

The proposed algorithm in this paper is based on the sum rate expression that depends on the MMSE decoding matrix. Therefore, the decoding scheme of each user must be designed carefully so as to maximize the information rate for all users. The expression of the maximum individual information rate for an MMSE receiver is obtained as

$$R_k = \log_2 \frac{|\boldsymbol{I} + p\ \sum_{j=1}^{K} \boldsymbol{H}_{kj}\boldsymbol{P}_j\boldsymbol{P}_j^H\boldsymbol{H}_{kj}^H|}{|\boldsymbol{I} + p\ \sum_{j\neq k}^{K} \boldsymbol{H}_{kj}\boldsymbol{P}_j\boldsymbol{P}_j^H\boldsymbol{H}_{kj}^H|}. \tag{9}$$

Based on this information rate expression, a new improved design for the IA scheme is proposed in the upcoming section.

## IV. Iterative Maximization of the sum rate function

### A. Iterative algorithm

Maximizing the sum rate function requires finding the optimal precoding vectors of each precoder subspace, therefore a new matrix $\boldsymbol{C}_k$ is combined with each precoding matrix $\boldsymbol{V}_k$. A closed-form solution of this combined matrix is nontrivial. Therefore, we seek to find out the solution iteratively using the sum rate maximization criterion. Using the MMSE maximum information rate in (9), the sum rate maximization problem, constrained by the individual power limitation constraint, is formulated as follows

$$\underset{\boldsymbol{C}_k,\ k\in\mathcal{K}}{\text{argmax}} \sum_{k=1}^{K}\ \frac{1}{N} R_k =$$

$$\underset{\boldsymbol{C}_k,\ k\in\mathcal{K}}{\text{argmax}}\ \frac{1}{N}\sum_{k=1}^{K}\log_2\ \frac{|\boldsymbol{I} + p\sum_{j=1}^{K}\bar{\boldsymbol{H}}_{kj}\boldsymbol{C}_j\left(\bar{\boldsymbol{H}}_{kj}\boldsymbol{C}_j\right)^H|}{|\boldsymbol{I} + p\sum_{j\neq k}^{K}\bar{\boldsymbol{H}}_{kj}\boldsymbol{C}_j\left(\bar{\boldsymbol{H}}_{kj}\boldsymbol{C}_j\right)^H|}$$

$$\text{subject to} \qquad \text{tr}(\boldsymbol{V}_k\boldsymbol{C}_k\boldsymbol{C}_k^H\boldsymbol{V}_k^H) = N,\ \ k\in\mathcal{K}. \tag{10}$$

with $\bar{\boldsymbol{H}}_{kj} = \boldsymbol{H}_{kj}\boldsymbol{V}_j$. Due to the variation of the matrices $\boldsymbol{C}_k$ at each iteration, for all $k$, the optimization problem is non concave, which results in a local optimal solution. This kind of constrained problem can be solved by transforming it to an unconstrained problem and then applying a first order optimization method such as the gradient descent method. Other methods can also be used such as Newton method. However, a higher computation cost is required. Let us start by defining the matrix $\boldsymbol{C}_k$ as

$$\boldsymbol{C}_k = \sqrt{\alpha_k}\bar{\boldsymbol{C}}_k, \quad \text{with} \quad \alpha_k = \frac{1}{\text{tr}(\boldsymbol{V}_k\bar{\boldsymbol{C}}_k\bar{\boldsymbol{C}}_k^H\boldsymbol{V}_k^H)}. \tag{11}$$

Substituting (11) into (10) and considering $\bar{\boldsymbol{C}}_k$ as the new variable, the constraint in (10) is satisfied for any $\bar{\boldsymbol{C}}_k$ and the resulting optimization problem gets unconstrained. The optimization problem is now formulated as

$$\underset{\bar{\boldsymbol{C}}_k,\ k\in\mathcal{K}}{\text{argmax}}\ R;\ R \equiv \sum_{k=1}^{K}\log_2\ \frac{|\boldsymbol{I} + p\sum_{j=1}^{K}\alpha_j\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\left(\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\right)^H|}{|\boldsymbol{I} + p\sum_{j\neq k}^{K}\alpha_j\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\left(\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\right)^H|}$$

The expression of the sum rate can be rewritten in the following compact form

$$R \equiv \sum_{k=1}^{K}\log_2|\boldsymbol{X}_k| - \log_2|\boldsymbol{Y}_k| \tag{12}$$

with

$$\begin{aligned}\boldsymbol{X}_k &= \boldsymbol{I} + p\sum_{j=1}^{K}\alpha_j\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\left(\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\right)^H \\ \boldsymbol{Y}_k &= \boldsymbol{I} + p\sum_{j\neq k}^{K}\alpha_j\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\left(\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\right)^H\end{aligned} \tag{13}$$

For such a problem, the gradient descent algorithm which has a simple implementation, could be applied. However, it takes too many iterations to converge. The optimal $\bar{\boldsymbol{C}}_k$ is obtained using the Jacobian of $R(\bar{\boldsymbol{C}}_k)$. This Jacobian can be computed from the differential of the function $R\left(\bar{\boldsymbol{C}}_k\right)$ as described in [14]. As $\bar{\boldsymbol{C}}_k$ is a complex matrix, then $\text{d}R = 2\partial R/\bar{\boldsymbol{C}}_k^*$. The differential of $\log_2|\boldsymbol{X}_k|$ is computed as

$$\text{d}\log_2|\boldsymbol{X}_k| = \text{tr}(\boldsymbol{X}_k^{-1}d\boldsymbol{X}_k)$$

$$\text{d}\boldsymbol{X}_k = p\ \alpha_k\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\text{d}\bar{\boldsymbol{C}}_j^H\bar{\boldsymbol{H}}_{kj}^H + p\ \text{d}\alpha_k\bar{\boldsymbol{H}}_{kj}\bar{\boldsymbol{C}}_j\bar{\boldsymbol{C}}_j^H\bar{\boldsymbol{H}}_{kj}^H, \tag{14}$$

Using the property tr($\boldsymbol{A}d\boldsymbol{B}^H$) =tr($\boldsymbol{A}^t d\boldsymbol{B}^*$) and referring to [14] that describes the first-order differentials and the Jacobian matrix properties, we obtain

$$\begin{aligned}\text{d}\log_2|\boldsymbol{X}_k| = \frac{2p}{\ln 2}\big[\ \ &\alpha_k\text{vec}\left(\bar{\boldsymbol{H}}_{kk}^H\ \boldsymbol{X}_k^{-1}\ \bar{\boldsymbol{H}}_{kk}\ \bar{\boldsymbol{C}}_k\right)^t \\ -\alpha_k^2\text{tr}(\bar{\boldsymbol{C}}_k^H\bar{\boldsymbol{H}}_{kk}^H\boldsymbol{X}_k^{-1}\bar{\boldsymbol{H}}_{kk}\bar{\boldsymbol{C}}_k)\ &\text{vec}(\boldsymbol{V}_k^H\boldsymbol{V}_k\bar{\boldsymbol{C}}_k)^t\ \big]\text{vec}(\text{d}\bar{\boldsymbol{C}}_k^*).\end{aligned} \tag{15}$$

Thus, the derivative of $R$ with respect to $\bar{\boldsymbol{C}}_k$ is obtained as follows

$$\begin{aligned}\text{J}\left(R(\bar{\boldsymbol{C}}_k)\right) &= \frac{2p}{\ln 2}\alpha_k\sum_{i=1}^{K}\bar{\boldsymbol{H}}_{ik}^H\boldsymbol{X}_i^{-1}\bar{\boldsymbol{H}}_{ik}\bar{\boldsymbol{C}}_k \\ &-\frac{2p}{\ln 2}\alpha_k^2\sum_{i=1}^{K}\text{tr}\left[\bar{\boldsymbol{C}}_k^H\bar{\boldsymbol{H}}_{ik}^H\boldsymbol{X}_i^{-1}\bar{\boldsymbol{H}}_{ik}\bar{\boldsymbol{C}}_k\right]\boldsymbol{V}_k^H\boldsymbol{V}_k\bar{\boldsymbol{C}}_k \\ &-\frac{2p}{\ln 2}\alpha_k\sum_{i\neq k}^{K}\bar{\boldsymbol{H}}_{ik}^H\boldsymbol{Y}_i^{-1}\bar{\boldsymbol{H}}_{ik}\bar{\boldsymbol{C}}_k \\ &-\frac{2p}{\ln 2}\alpha_k^2\sum_{i\neq k}^{K}\text{tr}\left[\bar{\boldsymbol{C}}_k^H\bar{\boldsymbol{H}}_{ik}^H\boldsymbol{Y}_i^{-1}\bar{\boldsymbol{H}}_{ik}\bar{\boldsymbol{C}}_k\right]\boldsymbol{V}_k^H\boldsymbol{V}_k\bar{\boldsymbol{C}}_k.\end{aligned} \tag{16}$$

Using this derivative, the matrix $\bar{\boldsymbol{C}}_k$ at the $k^{th}$ transmitter is iteratively computed as described in the following algorithm:

1) Fix the $d^k \times d^k$ matrix $\bar{\boldsymbol{C}}_k^{(0)}$ to the identity matrix for all $k$
2) for k=1:K
   - Calculate J $\left(R(\bar{\boldsymbol{C}}_k)\right)$; the Jacobian of $R$.
   - Update $\bar{\boldsymbol{C}}_k^{(l+1)} = \bar{\boldsymbol{C}}_k^{(l)} + \mu \nabla_{\bar{\boldsymbol{C}}_k^{(l)}} R$.
3) Iterate step 2 until convergence

In this algorithm, the step size $\mu$ is a determining factor to ensure a faster convergence, thus, it must be judiciously selected. In [15], two line search methods are proposed: exact line search and inexact line search methods. Most line searches are inexact in practice, and many methods have been proposed. Herein, the backtracking line search method is used, which is very simple to implement and quite effective. The stop criterion of this iterative algorithm is supposed to be achieved either when

$$\sum_k ||\text{J}\left[R(\bar{\boldsymbol{C}}_k)\right]|| < \epsilon \tag{17}$$

or when a maximum number of iterations is achieved, where $\epsilon$ is defined as a tolerence factor.

The main drawback in the above algorithm is that the convergence to a global maximum is not ensured. However, as long as the gradient descent method is applied, the matrix $\bar{\boldsymbol{C}}_k$ will move into the direction of the Jacobian, which could imply a convergence to a local maximum. The convergence will be illustrated in the next section.

### *B. Extension to the quasi-static scheme system*

In a Gaussian Interference Channels (GIC), the optimized IA scheme above supposes all recived signals as a linear combination of the synchronized transmitted signals. Unfortunately, this seems to be an unrealistic assumption, where propagation delays appear even though synchronization providers exist. Therefore, it is useful to extend the proposed improvements of this paper to the IA scheme proposed in the asynchronous GIC [8].

Referring to the equation (37) in [8], the optimized design proposed above can be applied at once on the IA scheme in the asynchronous GIC. In this scheme, there is no need of any channel knowledge at the transmitters. However, the relative delays between all transmitters and receivers are required. In order to adapt our proposed improvement to the one proposed in the asynchronous GIC, firstly, the channel matrix $\boldsymbol{H}_{kj}$ have to be replaced by the circulant convolution matrix $\boldsymbol{\Gamma}_{kj}$ (equation (13) in [8]), which depends on the values of the relative asynchronous delay. Then the Jacobian of the sum rate function with the new $\boldsymbol{\Gamma}_{kj}$ have to be calculated, and finaly the proposed algorithm can be applied.

## V. Numerical results

In this section, we present the numerical results of the improved design, derived in IV, in the 3-user SISO fading interference channel. The simulation results are based on 1000 channel realizations. Channel coefficients have an i.i.d. circular symmetric complex Gaussian distribution with zero mean and unit variance. In the next figures and interpretations, the following abbreviations are used

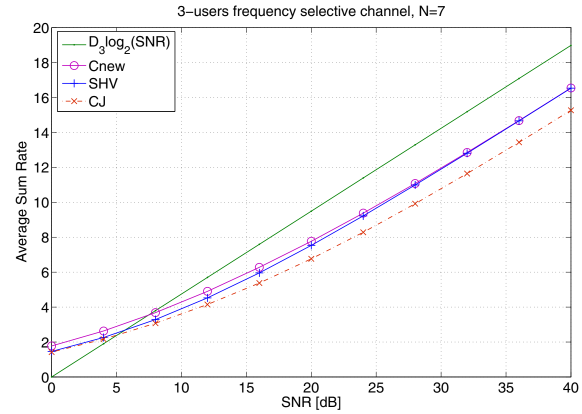


Fig. 2. Average sum rate performance comparison between the different designs for $K = 3$ and $N = 7$ assuming an MMSE receiver

- CJ- The IA scheme proposed by Cadambe and Jafar [4]
- SHV- The IA Improved design proposed by Shen, Host-Madsen and Vidal reviewed in Section III
- Cnew- The IA design with the improvement proposed in this paper
- $D_n \log(snr)$- The maximum DoF for high SNR with $n = m^* + 1$

Depending on $n$, $D_n$ is computed as

$$D_n = \frac{d_1 + d_2}{\sum_{k=1}^{K} d_k} , \tag{18}$$

where $d_k$ is given in (2) (cf. Section II). In all the above designs, we fix $\boldsymbol{w}$ as defined by the equation (29) in [10]. The CJ design is the IA scheme with precoding matrices defined in (4). In the following results, the data rate performance of the following designs: CJ, SHV and Cnew, are compared for different values of $N$. For the Cnew design, the maximum is supposed to be achieved either when

$$||\text{J}\left[R(\bar{\boldsymbol{C}}_k)\right]|| < \epsilon \tag{19}$$

where $\epsilon$ is the tolerance factor for stopping the iterations, or when a maximum number of iterations is achieved. The step size is chosen using the backtracking line search method as in [15] to provide faster convergence.

In Fig. 2, it is clear that the combination with a new matrix while preserving the IA scheme, improves the data rate performance of the network. For $n = 3$, the Cnew design outperforms the CJ design by 0.35-0.47 bits/s/Hz for SNR values comprised between 0dB and 8dB. This gain increases with the SNR. However, compared to the design with SHV combination, which optimizes the IA scheme for high SNR, we observe that the gain of the proposed design towards the SHV design decreases as the SNR increases. For the SNR values between 0dB and 8dB, a gain of about 0.35 bits/s/Hz is shown over the SHV design. However, this gain decreases with SNR to achieve 0.01 bits/s/Hz at 40dB.

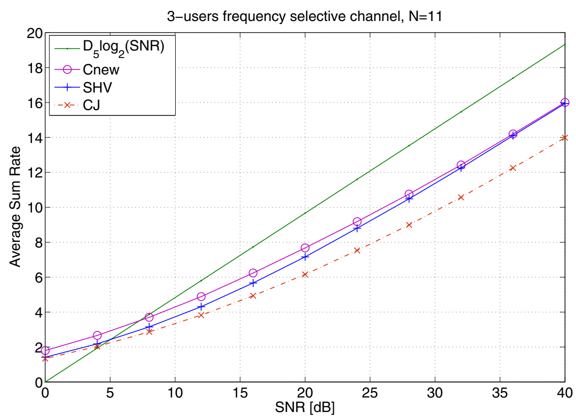


Fig. 3. Average sum rate performance comparison between the different designs for $K = 3$ and $N = 11$ assuming an MMSE receiver

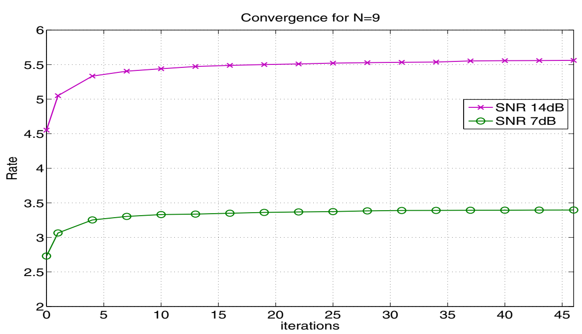


Fig. 4. Average sum rate convergence using the new design for $N = 9$ at different SNR values

Increasing the DoF to $n = 5$, a higher gain between these different designs is shown. The Cnew design outperforms the CJ by a 0.45-0.84 bits/s/Hz between 0dB and 8dB as illustrated in Fig. 3. Increasing the SNR value to 20dB, a gain of about 1.56 bits/s/Hz is obtained. Similar improvements are achieved over the SHV design. The average sum rate gain is 0.38 bits/s/Hz at 0dB and 0.57 bits/s/Hz at 16dB. The sum rate gain compared with the SHV design increases with SNR until the SNR value of 16 dB is achieved, and then decreases with SNR to achieve the same performance at high SNR levels. Moreover, the difference for low and mid SNR regime increases with $N$. These simulation results show that the new design outperforms the SHV design for low to mid SNR values, and performs the same in the high SNR regime.

Finally, the provable convergence of the proposed iterative algorithm is illustrated in Fig. 4. The sum rate of the new design grows up with the number of iterations while an improvement in performance at each additional iteration is observed. Comparing the convergence at 7dB and 14dB, we observe that more iterations are required for higher SNR values while higher gap between the optimal design and the non optimized design is obtained. The number of iterations could be reduced with a better initialization.

## VI. Conclusion

In this paper, a new algorithm is proposed in order to improve the data rate performance of the $K$-user SISO fading synchronous interference channel. This improvement is achieved by optimizing the precoding vectors for each user. It consists in introducing a combination matrix at each transmitter while preserving the IA scheme. The introduced matrices are computed using the gradient descent method. However, the non concavity of the problem makes the solution converge to a local optimal. Simulation results illustrate the performance of the proposed algorithm, which is also compared to other optimized designs. It outperforms existing schemes in the targeted low to medium SNR regime and performs the same in the high SNR regime. It is shown that this scheme can be easily extended to the asynchronous GIC.

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