Robust transmit beamforming for multigroup multicasting

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***Abstract*—This paper addresses a robust downlink beamforming optimization problem for the multigroup multicast scenario, when only imperfect channel state information (CSI) is available at the transmitter. We consider two different optimization criteria: minimizing the total transmit power subject to quality of service (QoS) constraints at each receiver; max-min fair (MMF) signal-to-interference-plus-noise ratio (SINR) subject to total power constraint. With the aid of S-lemma, the infinite non-convex QoS constraints of robust downlink beamforming problem are transformed into finite linear matrix inequalities (LMI). By applying the semidefinite relaxation (SDR) method, the robust downlink beamforming problem can be relaxed and solved efficiently. Simulation results are presented to corroborate our design.**

I. Introduction

Multicast service of a relevant feature within the context of next-generation wireless systems has attracted emerging attention, not only because of its ability to meet mass content distribution requirement but also the enhanced radio efficiency it can offer. In a multicast network, the system performance is constrained by its worst link and when it is extended to multigroup scenario, the problem is further exacerbated due to intergroup interference. In this paper, we deal with the core problem of multigroup multicast downlink beamforming for a multiantenna wireless cellular system while taking the imperfect channel state information (CSI) into account.

The problem of single group multicast beamforming under quality of service (QoS) (minimizing the total transmit power subject to minimum signal-to-interference-plus-noise ratio (SINR) requirements per user) and max-min fair (MMF) (maximizing the overall worst SINR subject to transmit power constraint) criterion was proposed in [1], where it was shown to be a NP-hard problem and high-quality approximate solution can be obtained based on semidefinite relaxation (SDR) method. In [2], Hunger *et al.* suggested a low complexity single group beamforming design by introducing a successive beamforming filter under QoS constraint and then developed an iterative algorithm under MMF constraint. On multigroup case, a nonlinear solution was depicted in [3] by employing dirty paper coding (DPC) and taking a block-triangular channel into account. An extension of [1] to multigroup case was shown in [4][5] and a joint multigroup power control, which was entailed by SDR method, was solved as well. Besides, some techniques of low complexity on multiuser unicast beamforming have been extended to the multigroup multicast case [6][7][8]. While most of the existing designs assume that the transmitter has the perfect CSI, however,

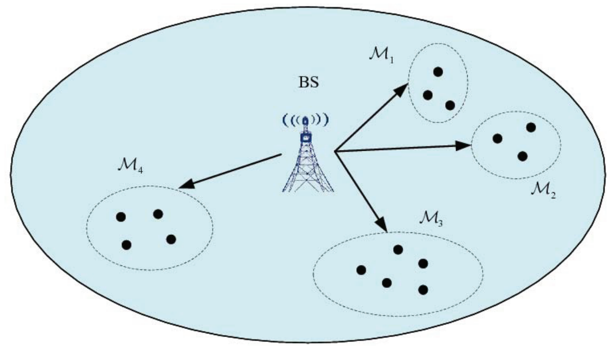

Fig. 1: A multigroup multicast system with one BS and four groups.

channel knowledge may not be perfect in practical systems even with sophisticated training. The conventional beamforming designs are very sensitive to channel estimation errors and may cause performance outage. Hence, it is imperative to design beamformers taking into account the CSI errors.

In this paper, we assume elliptically bounded CSI errors, and propose a robust downlink beamforming design for a multigroup multicast system. The robust beamforming design in multiuser downlink channel has been studied in [9][10], which can be seen as a special case of multigroup multicast where each multicast group only consists of a single receiver. Specifically, we consider two different optimization criteria: 1) minimizing the total transmit power subject to QoS constraints at each receiver; 2) max-min fair SINR subject to total power constraint. In this paper, we show that the imperfect CSI embedded QoS constraints can be converted to a finite number of linear matrix inequalities (LMI) with the aid of S-lemma [12] and the original non-convex problem can be approximated to a convex one by applying SDR method [11], which, thereby, can be efficiently solved. Simulation results demonstrate the satisfying performance of the proposed robust design.

Notations: $\mathbb{C}$, $\mathbb{H}$, $\mathbb{R}$ and $\mathbb{R}_{++}$ stand for the set of complex, Hermitian matrix, real and positive real numbers, respectively. Column vectors and matrices are written in boldface lowercase and uppercase letters. The superscripts $(\cdot)^T, (\cdot)^H$ represent the transpose and Hermitian conjugate transpose respectively. $\mathrm{tr}(\cdot)$ denotes the trace operation of a matrix. $\mathrm{rank}(\cdot)$ denotes the rank of a matrix. $\|\cdot\|$ denotes the Euclidean norm. $\mathbb{E}\{\cdot\}$ represents the statistical expectation.

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II. System Model

Consider a multigroup multicast system (see Fig.1) that consists of a single base station (BS), which is equipped with N antennas, and M single-antenna mobile stations (MS). There are G multicast groups, $\{\mathcal{M}_1, \ldots, \mathcal{M}_G\}$, where $\mathcal{M}_i$ denotes the index set of the users paricipating in the i-th group and $M_i \triangleq |\mathcal{M}_i|$. Assume that each MS listens to a single multicast, i.e., $\mathcal{M}_i \cap \mathcal{M}_{\hat{i}} = \emptyset, i \neq \hat{i}$, and $M = \sum_{i=1}^{G} M_i$.

We denote MS_{ij} as the j-th MS in the i-th group, for all $i \in \mathcal{G} \triangleq \{1, \ldots, G\}$, $j \in \mathcal{M}_i \triangleq \{1, \ldots, M_i\}$. Let $s_i(t) \in \mathbb{C}$ be the information data stream for $\mathcal{M}_i$, $\mathbf{w}_i \in \mathbb{C}^N$ be the associated beamforming vector. Assume that $s_i(t)$ are statistically independent, with zero mean and $\mathbb{E}\{|s_i(t)|^2\} = 1$ for all $i \in \mathcal{G}$. Therefor, the signal transmitted by BS is given by

$$\mathbf{x}(t) = \sum_{i=1}^{G} \mathbf{w}_i s_i(t) \tag{1}$$

The received signal of MS_{ij} can be expressed as

$$\begin{aligned} y_{ij}(t) &= \mathbf{h}_{ij}^H \mathbf{x}(t) + z_{ij}(t) & (2)\\ &= \mathbf{h}_{ij}^H \mathbf{w}_i s_i(t) + \underbrace{\sum_{l \neq i}^{G} \mathbf{h}_{ij}^H \mathbf{w}_l s_l(t)}_{\text{intergroup}} + z_{ij}(t) & (3) \end{aligned}$$

where $\mathbf{h}_{ij} \in \mathbb{C}^N$ is the complex channel vector from BS to MS_{ij}, and $z_{ij}(t) \in \mathbb{C}$ is complex Gaussian random variable with $\mathcal{CN}(0, \sigma_{ij}^2)$. In this paper, we take into account the quantization errors and model the CSI error to be bounded in a hyper spherical region. Then the actual channel can be expressed as

$$\mathbf{h}_{ij} = \hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}, \quad \forall i \in \mathcal{G}, j \in \mathcal{M}_i \tag{4}$$

where $\hat{\mathbf{h}}_{ij} \in \mathbb{C}^N$ is the estimation of CSI at BS, while $\mathbf{e}_{ij} \in \mathbb{C}^N$ is the corresponding error with the inequality condition as

$$\mathbf{e}_{ij}^H \mathbf{C}_{ij} \mathbf{e}_{ij} \leq 1 \tag{5}$$

where $\mathbf{C}_{ij} \in \mathbb{H}^{N \times N}, \mathbf{C}_{ij} \succ 0$ specifies the size and shape of the ellipsoid. When $\mathbf{C}_{ij} = \left(1/\epsilon_{ij}^2\right) \mathbf{I}_{N \times N}$ and $\epsilon_{ij}^2 > 0$, (5) reduces to the popular spherical error model $\|\mathbf{e}_{ij}\|^2 \leq \epsilon_{ij}^2$ [9]. Assume that each MS employs single user detection, the SINR of MS_{ij} is thus given by

$$\mathrm{SINR}_{ij} = \frac{\left|\left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \mathbf{w}_i\right|^2}{\sum_{l \neq i}^{G} \left|\left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \mathbf{w}_l\right|^2 + \sigma_{ij}^2} \tag{6}$$

III. Robust Transmit Beamforming Design

A. Power Minimization with QoS constraints

We firstly attempt to minimize the total transmit power subject to QoS constraints, which provides a guaranteed minimum received SINR to every user, and consequently, obtain the following design problem

$$\min_{\{\mathbf{w}_i\}} \quad \sum_{i=1}^{G} \|\mathbf{w}_i\|^2 \tag{7}$$

$$s.t. \quad \frac{\left|\left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \mathbf{w}_i\right|^2}{\sum_{l \neq i}^{G} \left|\left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \mathbf{w}_l\right|^2 + \sigma_{ij}^2} \geq \gamma_{ij} \tag{8}$$

$$\forall\ \mathbf{e}_{ij}^H \mathbf{C}_{ij} \mathbf{e}_{ij} \leq 1, \quad \forall\ i, l \in \mathcal{G}, j \in \mathcal{M}_i.$$

where γ_{ij} is the minimum SINR required for MS_{ij}. Note that the power minimization problem is NP-hard and cannot be directly solved in an efficient way. To handle this problem, we present a suboptimal method via SDR [11]. By introducting $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H \in \mathbb{C}^{N \times N}$, and $\mathbf{W}_i \succeq 0$, the constraints (8) can be rewritten as

$$\left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \left(\tfrac{1}{\gamma_{ij}} \mathbf{W}_i - \sum_{l \neq i}^{G} \mathbf{W}_l\right) \left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right) \geq \sigma_{ij}^2 \tag{9}$$

$$\forall\ \mathbf{e}_{ij}^H \mathbf{C}_{ij} \mathbf{e}_{ij} \leq 1, \quad \mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H, \quad \forall\ i, l \in \mathcal{G}, j \in \mathcal{M}_i.$$

Then, the problem (7) has its equivalent form

$$\min_{\{\mathbf{W}_i \succeq 0\}} \quad \sum_{i=1}^{G} \mathrm{tr}\left(\mathbf{W}_i\right) \tag{10}$$

$$s.t. \left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \left(\frac{1}{\gamma_{ij}} \mathbf{W}_i - \sum_{l \neq i}^{G} \mathbf{W}_l\right) \left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right) \geq \sigma_{ij}^2 \tag{11}$$

$$\forall\ \mathbf{e}_{ij}^H \mathbf{C}_{ij} \mathbf{e}_{ij} \leq 1, \quad \mathbf{W}_i \succeq 0,$$

$$\mathrm{rank}(\mathbf{W}_i) = 1, \quad \forall\ i, l \in \mathcal{G}, j \in \mathcal{M}_i.$$

By applying SDR method, we drop the rank constraints of (11) and obtain the following problem

$$\min_{\{\mathbf{W}_i \succeq 0\}} \quad \sum_{i=1}^{G} \mathrm{tr}\left(\mathbf{W}_i\right) \tag{12}$$

$$s.t. \left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right)^H \left(\frac{1}{\gamma_{ij}} \mathbf{W}_i - \sum_{l \neq i}^{G} \mathbf{W}_l\right) \left(\hat{\mathbf{h}}_{ij} + \mathbf{e}_{ij}\right) \geq \sigma_{ij}^2 \tag{13}$$

$$\forall\ \mathbf{e}_{ij}^H \mathbf{C}_{ij} \mathbf{e}_{ij} \leq 1, \quad \mathbf{W}_i \succeq 0, \quad \forall\ i, l \in \mathcal{G}, j \in \mathcal{M}_i.$$

Note that (13) contain infinite number of linear inequality constraints and cannot be directly handled. To deal with this problem, we will apply the following S-lemma to reformulate (13) to finite linear inequalitys.

Lemma 1 *(S-lemma[12]) Let $f_i(\boldsymbol{x}) = \boldsymbol{x}^H \boldsymbol{A}_i \boldsymbol{x} + \boldsymbol{x}^H \boldsymbol{b}_i + \boldsymbol{b}_i^H \boldsymbol{x} + c_i$, for $i = 1, 2$, where $\boldsymbol{A}_i \in \mathbb{H}^{N \times N}$, $\boldsymbol{b}_i \in \mathbb{C}^N$ and $c_i \in \mathbb{R}$. Suppose that there exists an $\hat{\boldsymbol{x}} \in \mathbb{C}^N$ satisfying $f_1(\hat{\boldsymbol{x}}) < 0$. Then the two conditions are equivalent:*
(1) $f_1(\boldsymbol{x}) \geq 0$ for all $\boldsymbol{x}$ satisfying $f_2(\boldsymbol{x}) \leq 0$.
(2) There exists a λ such that

$$\lambda \geq 0, \quad \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{b}_1 \\ \boldsymbol{b}_1^H & c_1 \end{bmatrix} + \lambda \begin{bmatrix} \boldsymbol{A}_2 & \boldsymbol{b}_2 \\ \boldsymbol{b}_2^H & c_2 \end{bmatrix} \succeq 0$$

By applying the above S-lemma, we can eliminate the error $\mathbf{e}_{ij}$ and rewrite (13) into an equivalent expression as

$$\Phi_{ij}\left(\{\mathbf{W}_i\},\{\mathbf{W}_l\},\lambda_{ij}\right) \triangleq \begin{bmatrix}\mathbf{I}\\ \hat{\mathbf{h}}_{ij}^H\end{bmatrix}\left(\frac{1}{\gamma_{ij}}\mathbf{W}_i - \sum_{l\neq i}^{G}\mathbf{W}_l\right)\begin{bmatrix}\mathbf{I}\\ \hat{\mathbf{h}}_{ij}^H\end{bmatrix}^H + \begin{bmatrix}\lambda_{ij}\mathbf{C}_{ij} & 0\\ 0 & -\sigma_{ij}^2-\lambda_{ij}\end{bmatrix} \succeq 0 \quad (14)$$

where $\mathbf{I}$ is the $N\times N$ identity matrix and $\lambda_{ij}\geq 0$ is an auxiliary variable.

Then, we have a more compact form of the problem (12)

$$\begin{aligned}\min_{\{\mathbf{W}_i\succeq 0\}} \quad & \sum_{i=1}^{G}\mathrm{tr}\left(\mathbf{W}_i\right) \quad (15)\\ s.t. \quad & \Phi_{ij}\left(\{\mathbf{W}_i\},\{\mathbf{W}_l\},\lambda_{ij}\right)\succeq 0\\ & \mathbf{W}_i\succeq 0, \quad \forall\ i,l\in\mathcal{G}, j\in\mathcal{M}_i.\end{aligned}$$

which belongs to the class of convex semidefinite problems (SDP) that can be efficiently solved by a lot of convex tools using interior point methods, i.e., CVX [13].

Due to the rank relaxation, the optimal solution $\mathbf{W}_i^{opt}$ obtained by solving the SDP of (15) will not be rank one in general. If $\mathbf{W}_i^{opt}$ is rank one, then its principal component will be the optimal solution to the primal problem (7). Otherwise, the obtained solution is a lower bound of the optimal solution. In this case, *randomization* techniques [4][5][11] can be used to produce a rank one solution, whose main idea is to generate a set of random vectors $\{\hat{\mathbf{w}}_{i,k}\}_{k=1}^{K}$ using $\mathbf{W}_i^{opt}$ and choose the best one that yields the best solution to the primal problem (7). Here, K is the number of used randomizations. We calculate the eigen-decomposition of $\mathbf{W}_i^{opt}$ in the form

$$\mathbf{W}_i^{opt} = \mathbf{U}\Sigma\mathbf{U}^H \text{ and } \hat{\mathbf{w}}_{i,k} = \mathbf{U}\Sigma^{1/2}\mathbf{v}_k \quad (16)$$

where $\mathbf{v}_k\sim\mathcal{CN}(0,\mathbf{I})$, so that $\mathbb{E}\{\hat{\mathbf{w}}_{i,k}\hat{\mathbf{w}}_{i,k}^H\} = \mathbf{W}_i^{opt}$. Furthermore, a multigroup mutlicast power control (MMPC) problem, which is a derivative problem by exploiting SDR method [5], could be solved at the same time. Summarizing, an approximate rank-one solution of problem (7) can be generated by Algorithm 1.

B. Max-Min Fair Beamforming

In this subsection, we consider the max-min fair beamforming problem

$$\begin{aligned}\max_{\{\mathbf{w}_i\}} \quad \min_{i\in\mathcal{G}} \quad \min_{j\in\mathcal{M}_i} \quad & \frac{1}{\beta_{ij}}\frac{\left|\left(\hat{\mathbf{h}}_{ij}+\mathbf{e}_{ij}\right)^H\mathbf{w}_i\right|^2}{\sum_{l\neq i}^{G}\left|\left(\hat{\mathbf{h}}_{ij}+\mathbf{e}_{ij}\right)^H\mathbf{w}_l\right|^2+\sigma_{ij}^2} \quad (17)\\ s.t. \quad & \sum_{i=1}^{G}\|\mathbf{w}_i\|^2\leq P\\ & \forall\ \mathbf{e}_{ij}^H\mathbf{C}_{ij}\mathbf{e}_{ij}\leq 1, \quad \forall\ i,l\in\mathcal{G}, j\in\mathcal{M}_i.\end{aligned}$$

where $1/\beta_{ij}\in\mathbb{R}_{++}$ is a predetermined weight factor to account for possibly different grades of service and P is the maximum power of BS. By introducing an auxiliary variable t, the problem (17) can be transformed to

$$\begin{aligned}\max_{\{\mathbf{w}_i\}} \quad & t \quad (18)\\ s.t. \quad & \frac{1}{\beta_{ij}}\frac{\left|\left(\hat{\mathbf{h}}_{ij}+\mathbf{e}_{ij}\right)^H\mathbf{w}_i\right|^2}{\sum_{l\neq i}^{G}\left|\left(\hat{\mathbf{h}}_{ij}+\mathbf{e}_{ij}\right)^H\mathbf{w}_l\right|^2+\sigma_{ij}^2}\geq t \quad (19)\\ & \sum_{i=1}^{G}\|\mathbf{w}_i\|^2\leq P, \quad t\geq 0\\ & \forall\ \mathbf{e}_{ij}^H\mathbf{C}_{ij}\mathbf{e}_{ij}\leq 1, \quad \forall\ i,l\in\mathcal{G}, j\in\mathcal{M}_i.\end{aligned}$$

Algorithm 1 Obtaining Rank-One Solution for Power Minimization Problem (7)

Input: $\hat{\mathbf{h}}_{ij},\sigma_{ij}^2,\gamma_{ij},\mathbf{C}_{ij},K$.
Output: an approximate rank-one solution $\mathbf{w}_i$ of problem (7).
Slove (15) using SDP solver, find the optimal solution $\{\mathbf{W}_i^{opt},\lambda_{ij}^{opt}\}$.
if rank($\mathbf{W}_i^{opt}$) = 1 **then**
  output $\mathbf{w}_i$ with $\mathbf{w}_i\mathbf{w}_i^H = \mathbf{W}_i^{opt}$;
  break ;
else
  for $k = 1,\ldots,K$,
    generate random vectors $\hat{\mathbf{w}}_{i,k}$ using (16) ;
    normalize $\mathbf{u}_{i,k} = \hat{\mathbf{w}}_{i,k}/\|\hat{\mathbf{w}}_{i,k}\|$;
    substitute $\mathbf{w}_{i,k} = \sqrt{p_{i,k}}\mathbf{u}_{i,k}$ into (15). The problem (15), which is now over $p_{i,k}$, is a linear programming(LP) problem ;
    solve (15) with fixed $\hat{\mathbf{w}}_{i,k}$ and find the optimal power control factor $p_{i,k}$;
  end
  pick up $\mathbf{w}_{i,k}$ such that $k = \mathrm{argmin}\{\sum_{i=1}^{G}p_{i,k}, k = 1,\ldots,K\}$.
end if

Contrary to the power minimization problem, problem (18) always admits a feasible solution. The relation between problem (7) and (18) has been discussed in [5], which was shown to be inverse problems. For a given t, the problem (18) can be relaxed and the constraints (19) can be converted into LMI as shown in earlier subsection. Therefore, the solution to problem (18) can be solved by bi-section search over t and checking the feasibility at each step [5]. After obtaining the optimal solution $\mathbf{W}_i^{opt}$, a similar approximate rank-one solution and a solution to MMPC problem can be obtained as well as earlier subsection.

IV. Extension to Multicell Multigroup Scenario

Throughout this paper, we focus on the single-cell multigroup multicast case. However, our robust designs can be easily extended to multicell multigroup scenario. Let there be a total of N_c cells, and the i-th cell is composed of a single base station BS_i and G_i multicast groups. The N_c BSs are assumed

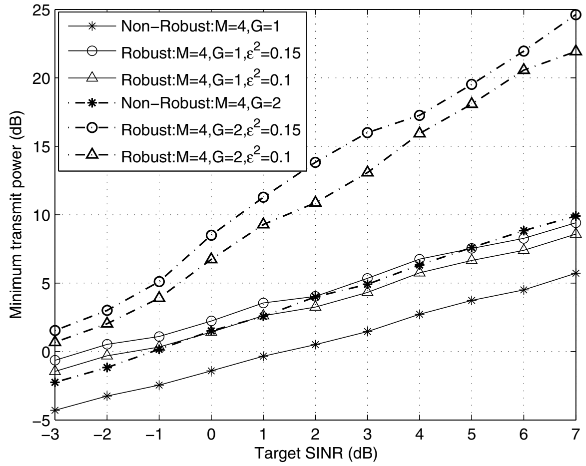

Fig. 2: Minimum transmit power required versus the target SINR γ.

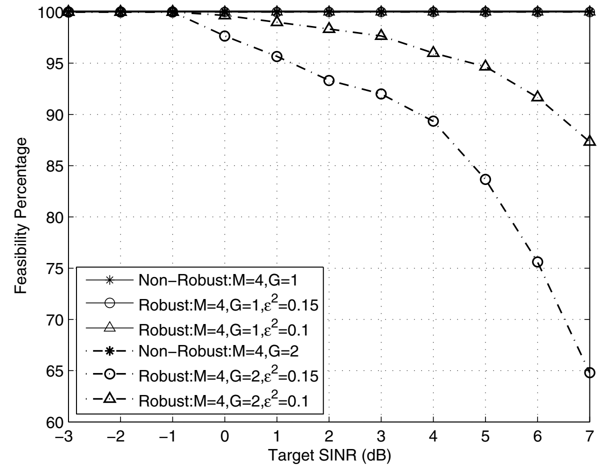

Fig. 3: Feasibility rate (%) versus the target SINR γ.

to operate over a common frequency band and each MS listens to a single multicast. Consequently, the corresponding signal-to-interference-plus-noise ratio (SINR) of the k-th MS in the j-th group in the i-th cell can be given by

$$\mathrm{SINR}_{ijk} = \frac{|(\hat{\mathbf{h}}_{ijk,i} + \mathbf{e}_{ijk,i})^H \mathbf{w}_{ij}|^2}{\sum_{l \neq j}^{G_i} \Gamma_{ijk,il} + \sum_{m \neq i}^{N_c} \sum_{n=1}^{G_m} \Gamma_{ijk,mn} + \sigma_{ijk}^2} \geq \gamma_{ijk} \quad (20)$$

where $\Gamma_{ijk,il} = |(\hat{\mathbf{h}}_{ijk,i} + \mathbf{e}_{ijk,i})^H \mathbf{w}_{il}|^2$ and $\Gamma_{ijk,mn} = |(\hat{\mathbf{h}}_{ijk,m} + \mathbf{e}_{ijk,m})^H \mathbf{w}_{mn}|^2$.

Since the constraint of (20) is similar with its single-cell counterparts of (6), multicell multigroup multicasting can be solved through same methods discussed in previous sections. Note that the interference is expanded from intracell interference to intercell interference, the corresponding computational burden ascends greatly and even the chances of problems being feasible are, however, affected.

V. Simulation Results And Discussions

In this section, we present some simulation results to examine the performance of the proposed robust transmit beamforming design. A multigroup multicast system that consists of a single BS ($N = 4$) and four MSs in all ($M = 4$) is considered here. The MSs are assumed to be distributed uniformly, i.e., $M_i = M/G, \forall i = 1, \ldots, G$. Each MS communicates with the BS over independent rayleigh fading channel, i.e., the elements of each channel vector are circularly symmetric zero mean complex Gaussian random variables of variance 1. In the simulations, we assume the target SINR required for all MSs is equal to γ and the noise variance is set to $\sigma^2 = 1$. For simplicity, each MS has the same priority with $\beta_{ij} = 1$ and the same error radius with $\epsilon_{ij} = \epsilon$ for all $i \in \mathcal{G}, j \in \mathcal{M}_i$. The simulation results presented in this section are obtained by averaging over 300 Monte Carlo simulations and the number of randomizations in approximating rank-one solution is set to $K = 2000$.

In Fig.2, we examine the average transmit power required versus the target SINR γ for $M = 4, G = 1, 2$ and $\epsilon^2 = 0.1, 0.15$. Each point of the curve is obtained by averaging over the *feasible* channel realizations at each target SINR. It can be seen that the robust beamforming design requires higher average transmit power than the non-robust design as a price for worst-case QoS guarantee, and the minimum transmit power required to satisfy QoS requirement increases with the increasing of γ. It is observed that for a certain γ, higher ϵ requires higher transmit power.

Fig.3 shows the feasiblity rate versus the target SINR γ. As seen, for $M = 4, G = 1$, the feasiblity rates are almost 100%. When the number of multicast groups G increases, the robust design problem (15) is getting more difficult to solve due to the higher intergroup interference. Due to the CSI error, the robust design has lower feasibility rates compared to its non-robust counterparts. It is also observed that both higher target SINR and higher ϵ lead to lower feasibility rates.

In Fig.4, we present the maximum achievable SINR versus the total power constraint. For a given total power constraint, it can be seen that the maximum achievable SINR decreases with the increasing of ϵ. When $G = 1$, the maximum achievable SINR increases approximately linearly with the total power due to no intergroup interference. When $G > 1$, the maximum achievable SINR curve saturates and has an upper bound.

VI. Conclusion

In this paper, we design robust transmit beamformers for the multigroup multicast scenario with imperfect CSI at the transmitter. Two design criteria, i.e., to minimize the transmit power subject to QoS constraints at each receiver and to maximize the minimum received SINR subject to total power constraint, were considered and numerically achieved. We show that the imperfect CSI embedded QoS constraints can be converted to a finite number of LMI with the aid of S-lemma and the original non-convex problem can be approximated to a convex one by applying SDR method, which, thereby, can

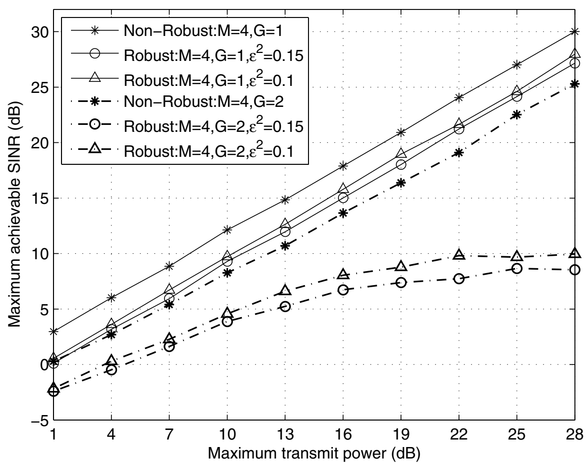

Fig. 4: Maximum achievable SINR versus the total power constraint.

be efficiently solved. Further, simulation results demonstrate the satisfying performance of the proposed robust design.

Acknowledgment

This work is supported in part by Natural Science Foundation of Education Department of Anhui, China (No.KJ2010A333) and National High Tech. Development Program of China (No.2010ZX03003-002 and No.2011ZX03004-002-01).

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