# Incremental Redundancy for LDPC Codes of 2nd Generation DVB Systems


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***Abstract*— In 2nd generation Digital Video Broadcasting (DVB) systems, low-density parity-check codes (LDPCCs) are applied. In this paper, we propose an extension of the standardized LDPCCs to allow for additional incremental redundancy (IR). Several use-cases of IR in broadcasting systems are outlined. We derive an algorithm to extend a given LDPCC with optimized degree sequences. The extended LDPCC yields both original codeword and additional IR. We suggest to transmit IR part at later instances as the standard LDPC codewords. If the receiver fails to decode the standard LDPCC, it can use in addition the IR part. In all other cases, the receiver can fall into sleep mode during transmission of IR, allowing for power saving. Examples are given to indicate both coding gain and power saving capabilities of IR.**



***Index Terms*—incremental redundancy, LDPC, EXIT, DVB**


## I. Introduction and Motivation

Powerful low-density parity-check codes (LDPCCs) [1] have been chosen by Digital Video Broadcasting (DVB) for 2nd generation systems to allow for reliable broadcasting. For broadcasting over satellite and cable (via DVB-S2 or -C2, respectively), the channel is usually quite predictable, and the broadcaster might select a sufficiently small code rate to guarantee a certain quality of service. However, for terrestrial broadcasting via DVB-T2 [2], the receiver could be stationary, portable or even highly mobile, thus facing fading and shadowing effects. In such cases, the receiver might not be able to decode all codewords successfully, hence degrading the viewing experience for the user. In this paper, we investigate the application of incremental redundancy (IR) for broadcasting, a concept well known from bidirectional communications such as High-Speed Downlink Packet Access (HSDPA) or Long Term Evolution (LTE). In bidirectional communications, a receiver that failed successful decoding requests IR over a return channel. For a broadcast channel, several applications of IR are possible. IR on demand via a unicasting network such as LTE allows a hybrid form of broadcast and broadband and would be feasible for non-real time broadcasting. Using only the broadcasting network, IR could be embedded into the same multiplex, e.g., if scheduled at later time instances as the original codewords. If decoding of these codewords fails (or if the receiver detects a priori that decoding is impossible due to a certain decision criterion), the IR part could be evaluated as well.

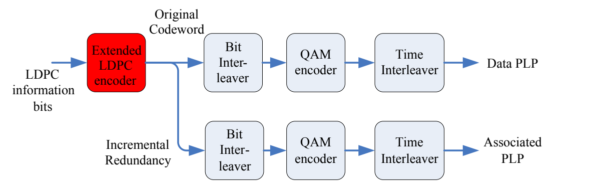



Fig. 1. Generation of original codeword and IR part in the DVB-T2 context

Fig. 1 outlines the generation of IR for the example of a DVB-T2 system. The information bits at the input of the LDPC encoder are encoded, bit interleaved, mapped to quadrature amplitude modulation (QAM) symbols and time interleaved on QAM symbol basis. The upper branch in this figure corresponds to the standard-conform physical layer pipe (PLP). An extended LDPCC would in addition generate IR, that is treated as an independent, but associated PLP (thus, different QAM and interleaver settings could be chosen). Another application of IR in broadcasting would be its usage for scalable video coding (SVC), where the original LDPCC is used to encode both base (low-resolution) and enhancement (high-resolution) layer, while the former is further protected by additional IR. In good channel conditions, the receiver ignores IR and decodes both layers using the same decoder. For severe channels, the enhancement layer will be ignored, and the decoder switches to the extended LDPCC, which allows more robust detection of the base layer, yielding at least a low-resolution video.

IR schemes for LDPCCs have been investigated in [3]–[5], where a mother code was designed first, followed by the definition of puncturing patterns. In our case, we start from the opposite direction, where the punctured LDPCC is defined by the DVB standards and an IR extension of the original parity-check matrix would yield the mother code. Puncturing this IR extension should thus result in the original LDPCC again. We apply the extrinsic information transfer (EXIT) chart [6] to optimize degree sequences of the IR extension.

This paper is organized as follows: In Section II, we describe the extensions of DVB's LDPCCs, which are optimized in Section III. We present simulation results in Section IV and conclude with Section V.

## II. Extension of LDPC Code

### A. General description of code extension

Let us start by describing the original LDPCC in terms of its degree spectra with a similar notation as defined in [7]. The fraction of all variable nodes (VNs) of degree $i$ is denoted as $a_{0,i}$. The index 0 is used for the original LDPCC. In the same way we denote the fraction of all check nodes (CNs) of degree $j$ as $c_{0,j}$, with $j \in C_0$, where $C_0$ is the set of all indices occurring in the CN degree spectrum. Let us further denote the number of information and parity bits as $K_0$ and $M_0$, respectively, such that the total number of VNs is $N_0 = K_0 + M_0$. Thus, the absolute number of VNs having degree $i$ (i.e. being involved in $i$ parity-checks) is $a_{0,i}N_0$.

Now consider the extended LDPCC as shown in Fig. 2. Note that the upper left part of the extended parity-check matrix coincides with the original LDPC matrix. In addition to the original $M_0$ parity bits, $M_{\mathrm{IR}}$ parities are appended, such that the overall number of parities becomes $M_1 = M_0 + M_{\mathrm{IR}}$ and the overall codeword length increases in the same way to $N_1 = N_0 + M_{\mathrm{IR}}$. The index 1 is used for the extended LDPCC. The IR extension reduces the original code rate of $R_0 = K_0/N_0$ to $R_1 = K_0/N_1$. Fig. 2 also depicts two properties that all LDPCCs from DVB possess: a quasi-cyclic structure and the accumulated parities. The latter is indicated by a staircase structure (twin diagonal line) in the right part of the parity-check matrix, similar as in extended irregular repeat accumulate (eIRA) codes [8], while the former is used to impose more structure into the code by specifying the positions of 1s per column only once every 360 columns. The remaining 359 columns of each subgroup are obtained by cyclically down-shifting (modulo $M_0$) the previous column by a factor $Q_0 = M_0/360$. The IR extension should maintain these two properties. In order not to alter the original LDPC matrix, cyclically down-shifting of the additionally introduced 1s is only applied inside the lower $M_{\mathrm{IR}} \times K_0$ submatrix by a factor $Q_1 = M_{\mathrm{IR}}/360$. It is worth mentioning that each LDPCC from

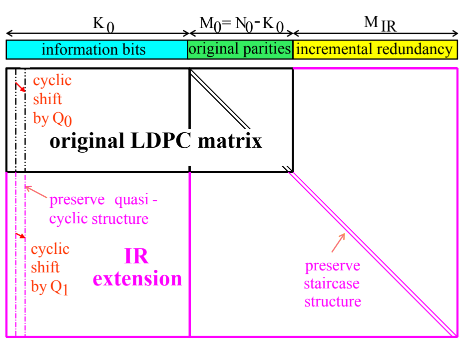



Fig. 2. Extended LDPCC with original matrix and IR extension

DVB has the same VN degree structure: the last parity bit has degree 1, all previous $M_0 - 1$ parities have degree 2 (due to the staircase structure), while the $K_0$ information bits fall into two groups, one with degree $i_1$ (usually 3) and one with $i_2 \geq i_1$.

We restrict the new parity-checks to have one fixed degree $d_{c,\mathrm{IR}}$ and allow for additional VN degrees up to $d_{v,\max}$. As additional 1s are added in the lower left submatrix, $d_{v,\max} \geq i_2$, i.e. the VN degrees are increased. For the extended LDPCC, we now consider the first $M_0$ parities, where the fraction of those CNs having degree $j$ is obtained by simple rescaling

$$c_{1,j} = c_{0,j} \cdot M_0/M_1. \tag{1}$$

Thus, the average CN degree of the extended code is

$$\overline{d_{c,1}} = d_{c,\mathrm{IR}} \cdot \frac{M_{\mathrm{IR}}}{M_1} + \sum_{j \in C_0} j \cdot c_{1,j}. \tag{2}$$

In the same manner, we can describe the average VN degree of the extended code by

$$\overline{d_{v,1}} = \sum_{i=1}^{d_{v,\max}} i \cdot a_{1,i} = \overline{d_{c,1}} \cdot (1 - R_1) = \overline{d_{c,1}} \cdot \left(1 - \frac{K_0}{N_1}\right), \tag{3}$$

where the right hand side is due to the fact that the code rate is related to the average degrees [1], [7]. Hence, for given $M_{\mathrm{IR}}$ and $d_{c,\mathrm{IR}}$, the value $\overline{d_{v,1}}$ is determined by (2), (3).

### B. EXIT functions of extended LDPCC

The optimization of the IR extension is based on EXIT chart analysis for the AWGN channel. Thus, we now describe the EXIT functions for the extended LDPCC of both VN decoder (VND) and CN decoder (CND) based on [7].

Let us denote the EXIT function of a VND, which consists only of degree $i$ VNs, as $I_{E,\mathrm{VND}}(I_A, i, \gamma)$, where $\gamma$ is the signal-to-noise ratio (SNR). Its closed-form for the AWGN channel is given in [7, equ. (4)], where $\gamma$ here corresponds to $R \cdot \frac{E_b}{N_0}$. For the irregular LDPCC with several values for $i$, we can apply the mixing property of the EXIT chart. Hence, the overall EXIT function of the VND is given as the weighted sum [7, equ. (14)] by

$$I_{E,\mathrm{VND}}(I_A, \gamma) = \sum_{i=1}^{d_{v,\max}} a_{1,i} \frac{i}{\overline{d_{v,1}}} \cdot I_{E,\mathrm{VND}}(I_A, i, \gamma), \tag{4}$$

where the weighting factors $a_{1,i} i/\overline{d_{v,1}}$ describe the fractions of *edges* belonging to VNs of degree $i$ (correspond to $b_i$ in [7]). The factors $a_{1,i}$ will be treated as variables in the next section, when we optimize the IR extension. However, the values $a_{1,i}$ for $i = 1, 2$ are fixed due to the imposed staircase structure: $a_{1,1} = 1/N_1$ and $a_{1,2} = (M_1 - 1)/N_1$. Thus, we can split (4) into

$$I_{E,\mathrm{VND}}(I_A, \gamma) = \sum_{i=3}^{d_{v,\max}} a_{1,i} \frac{i}{\overline{d_{v,1}}} \cdot I_{E,\mathrm{VND}}(I_A, i, \gamma) + g, \tag{5}$$

where the function $g$ includes degrees 1 and 2,

$$g = \frac{1}{N_1 \overline{d_{v,1}}} I_{E,\mathrm{VND}}(I_A, 1, \gamma) + \frac{2(M_1 - 1)}{N_1 \overline{d_{v,1}}} I_{E,\mathrm{VND}}(I_A, 2, \gamma). \tag{6}$$

The CND's EXIT function $I_{E,\mathrm{CND}}(I_A, j)$ for one degree $j$ can also be derived in closed-form by [7, equ. (9)]. In a similar manner, we can compute the overall CND's EXIT function as

$$\begin{aligned} I_{E,\mathrm{CND}}(I_A) \quad = \quad & \frac{M_{\mathrm{IR}}}{M_1} \frac{d_{c,\mathrm{IR}}}{\overline{d_{c,1}}} \cdot I_{E,\mathrm{CND}}(I_A, d_{c,\mathrm{IR}}) \\ & + \sum_{j \in C_0} c_{1,j} \frac{j}{\overline{d_{c,1}}} \cdot I_{E,\mathrm{CND}}(I_A, j), \end{aligned} \tag{7}$$

where the weighting factors are used again to describe the edge perspective. If both functions (4), (7) should be plotted into the same EXIT chart, we need to invert the CND curve

$$h = I_{E,\mathrm{CND}}^{-1}(I_A). \tag{8}$$

## III. Optimization of IR Extension

### A. Formulation of design target

From [7] it is known that a capacity-approaching LDPCC should have matching EXIT functions, i.e., code optimization reduces to a curve fitting problem. Similar as in [9], we will optimize the VN degree spectra by minimizing the squared error $J$ between $I_{E,\mathrm{VND}}(I_A,\gamma)$ and $h$,

$$J = \int_0^1 \left(I_{E,\mathrm{VND}}(I_A,\gamma) - h\right)^2 dI_A. \tag{9}$$

Let us define a (row) vector

$$\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_P) = \left(a_{1,3}, a_{1,4},\ldots,a_{1,d_{v,\max}}\right), \tag{10}$$

which includes the remaining $P = d_{v,\max} - 2$ variables $\alpha_n = a_{1,n+2}$ for optimization that fully describe the VN degree spectrum for the IR extension. Inserting (5) into (9), we can state the optimization problem as

$$\min_{\boldsymbol{\alpha}} J = \min_{\boldsymbol{\alpha}} \ \boldsymbol{\alpha} Q \boldsymbol{\alpha}^{\mathrm{T}} - 2\boldsymbol{\alpha}\mathbf{v}^{\mathrm{T}} + w, \tag{11}$$

where $(.)^{\mathrm{T}}$ denotes transposition. The symmetric matrix $Q$ consists of the components

$$Q_{i,j} = \frac{(i+2)\cdot(j+2)}{\overline{d_{v,1}}^2} \cdot \int_0^1 I_{E,\mathrm{VND}}(I_A, i+2,\gamma)\cdot I_{E,\mathrm{VND}}(I_A, j+2,\gamma)\, dI_A, \tag{12}$$

the $i$-th entry of the row vector $\mathbf{v}$ is

$$v_i = \frac{i+2}{\overline{d_{v,1}}} \int_0^1 f\cdot I_{E,\mathrm{VND}}(I_A, i+2,\gamma)\, dI_A, \tag{13}$$

with $f = h - g$. The positive constant $w$ is given as

$$w = \int_0^1 f^2\, dI_A. \tag{14}$$

In the minimization of $J$, $w$ can be neglected. For a given original LDPCC and a fixed set of parameters $\gamma, M_{\mathrm{IR}}, d_{c,\mathrm{IR}}, d_{v,\max}$, the equations (12), (13) can be computed numerically with arbitrary precision.

### B. Formulation of constraints

The vector $\boldsymbol{\alpha}$ that should be optimized in (11) is subject to the following conditions:

$$\sum_{n=1}^{P} \alpha_n = 1 - a_{1,1} - a_{1,2} = 1 - \frac{1}{N_1} - \frac{M_1 - 1}{N_1} = \Theta \tag{15}$$

$$\sum_{n=1}^{P} (n+2)\cdot\alpha_n = \overline{d_{c,1}}\cdot(1-R_1) - a_{1,1} - 2\cdot a_{1,2} \tag{16}$$

$$\alpha_{i_1-2} \le a_{0,i_1}\cdot\frac{N_0}{N_1}, \text{ all other } \alpha_n \le \Theta, n \ne i_1 - 2 \tag{17}$$

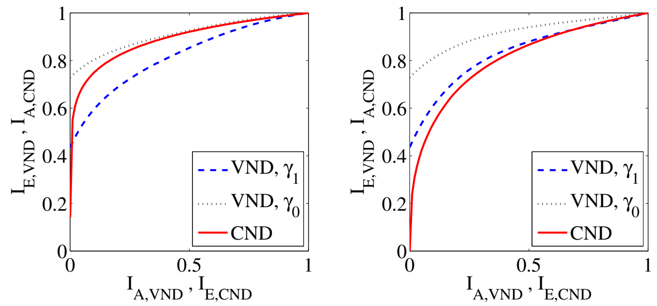
$$\alpha_n \ge 0 \ \text{ and } \sum_{n=i_2-2}^{P} \alpha_n \ge a_{0,i_2}\cdot\frac{N_0}{N_1}. \tag{18}$$

The first condition (15) comes from the fact that all fractions $a_{1,i}$ must add up tp 1 ($\Theta$ is an abbreviation to be used in (17)), the second condition is a reformulation of (3). The lower limit in (17) for $n \ne i_1 - 2$ is a consequence of (15). For $\alpha_{i_1-2}$, we need to consider that additional 1s are only added to the first $K_0$ VNs (originally with degrees $i_1$ or $i_2$). Adding new 1s only to those VNs of degree $i_2$ will yield $\alpha_{i_1-2} = a_{1,i_1} = a_{0,i_1}\cdot N_0/N_1$ (rescaling similar as in (1)), and any additional 1 on VNs of degree $i_1$ will decrease the VN fraction $\alpha_{i_1-2} = a_{1,i_1}$. If the number of these additional 1s on VNs of degree $i_1$ exceeds $i_2 - i_1$, there will be a larger (rescaled) fraction of VNs having degree $i_2$ or larger, which yields the lower limit on the right hand side of (18), while the left hand side is simply due to the fact that the $\alpha_n$ correspond to ratios.

The optimization problem (11) subject to (15)-(18) can be solved by constrained quadratic programming methods [10]. We have applied the sequential quadratic programming method *fmincon* from MATLAB to solve the optimization. It should be noted that the outlined approach only yields optimized degree sequences, which in turn optimize an LDPCC w.r.t. the decoding threshold (waterfall region). The construction of a particular LDPCC based on the optimized VN degree sequence should take several other factors into consideration, such as elimination of small girths and avoidance of stopping sets. This task however is not so crucial, as the error floor of the extended LDPCC can only be lower than that of the original code (as the minimum distance of an extended code can not be smaller than that of the original code), and all DVB codes have been carefully selected to fulfill quasi-error free conditions. Note further that $\alpha_n \cdot N_1$ has to be a multiple of 360 to preserve the quasi-cyclic structure. Hence, in some cases hand-crafted rounding of the fractions $\alpha_n$ has to be performed after degree optimization.

### C. Optimization example

Let the original LDPCC be the 16k code of rate $R_0 = 2/3$ from DVB-T2 [2], i.e., $N_0 = 16200, K_0 = 10800, M_0 = 5400$. It can be observed that all except one CNs have degree 10, while the first parity bit is only of degree 9 (no previous accumulated parities in the staircase). Thus, the fractions for the CNs are $c_{0,9} = 1/M_0$ and $c_{0,10} = 5399/M_0$. The VNs belonging to the $K_0$ information bits have degree $i_2 = 13$ at the first 3 subgroups and degree $i_1 = 3$ at the remaining 27 subgroups, each subgroup consisting of 360 bits. Thus, the VNs have the following ratios: $a_{0,1} = 1/N_0, a_{0,2} = (M_0 - 1)/N_0, a_{0,3} = 27\cdot 360/N_0$, and $a_{0,13} = 3\cdot 360/N_0$. Fig. 3(a) depicts the EXIT chart for this code at SNR $\gamma_0 = 0.1\,\mathrm{dB}$, which corresponds to the decoding threshold of this LDPCC for binary phase shift keying (BPSK) signalling over the AWGN channel, and at a lower SNR $\gamma_1 = -3.7\,\mathrm{dB}$. Note that there is no open tunnel between the VND and CND curve at this low SNR.



(a) original 16k LDPCC from DVB-T2 of rate $R_0 = 2/3$

(b) extended 32k LDPCC of rate $R_1 = 1/3$

Fig. 3. Analytical EXIT chart analysis of original (a) and extended (b) LDPCC at SNRs $\gamma_0 = 0.1\,$dB and $\gamma_1 = -3.7\,$dB over AWGN

We now optimize the IR extension based on this original LDPCC for the target SNR $\gamma_1$. We choose $M_{\mathrm{IR}} = N_0 = 16200$, thus doubling the codeword length and halving the code rate to $R_1 = R_0/2 = 1/3$. Note that we should expect at least 3 dB SNR gain for the extended LDPCC, as this is the gain from simple repetition of the original codeword. In our case we thus aim for a *coding gain* of $\gamma_0 - \gamma_1 - 3 = 0.8\,$dB due to IR. We found that $d_{c,\mathrm{IR}} = 4$ yields the smallest error $J$ after optimization (11) for this example. Allowing for a maximum VN degree of $d_{v,\max} = 20$, the optimal solution is given by $\alpha_4 = 0.2, \alpha_5 = 0.1, \alpha_{13} = 0.033$ and $\alpha_n = 0$ for all other $n$. Thus, $\alpha_4 \cdot N_1/360 = 18$ subgroups should include VNs of degree $4+2 = 6$, $\alpha_5 \cdot N_1/360 = 9$ subgroups should include VNs of degree $5+2 = 7$, and the originally 3 subgroups of degree 13 should remain unchanged (here, no rounding had to be performed). VN degrees 6 and 7 are obtained by adding 3 and 4 additional edges to VNs of the original code of degree 3. Fig. 3(b) shows that the VND's EXIT curve is slightly above the CND curve even for $\gamma_1$, indicating that convergence is possible at the target SNR.

## IV. Simulation Results

We continue the example from subsection III-C. Based on the optimized VN degree spectrum, we have designed the extended LDPCC, which yields the original codeword plus additional IR parities. We examine bit and frame error rate (BER / FER) for both original and extended LDPCC over the AWGN as well as a fading channel. LDPCC decoding applied sum-product algorithm and a maximum of 50 iterations.

### A. Performance over AWGN channel

Fig. 4 shows BER / FER performance over SNR of both codes if BPSK is used over the AWGN channel. The original DVB-T2 code has its decoding threshold at about SNR $\gamma_0 = 0.1\,$dB, whereas the extended LDPCC converges already at $\gamma_1 = -3.7\,$dB, as was predicted by the EXIT chart analysis in Fig. 3. Note that the extended code not only benefits from its lower code rate, but also from its longer codeword length $N_1 = N_0 + M_{\mathrm{IR}} = 32400$. Using the DVB notation, we could thus denote this code as a 32k LDPCC.

It is further worth noting that in DVB-S2 (satellite), there exists a 16k LDPCC of rate 1/3, which has its decoding threshold already at $\gamma = -4.4\,$dB (not shown in Fig. 4), which is 0.7 dB earlier than the optimized IR presented here. However, this code was optimized from scratch and does not extend an original code of higher code rate. Thus, this code would not allow for decoding just parts of the received codewords and does not benefit from the IR advantages such as power saving.

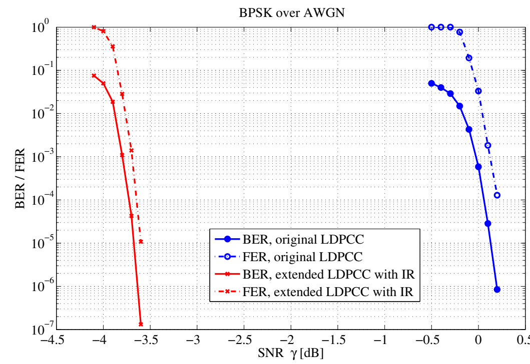



Fig. 4. BER / FER performance over SNR of original DVB-T2 code (16k, $R_0 = 2/3$) and extended code (32k, $R_1 = 1/3$), AWGN channel

### B. Performance over TU6 channel

Let us now consider as a more realistic simulation environment the complete DVB-T2 chain, where we have used the following parameters (for more details on parameter description, the reader is referred to [2]): quadrature phase shift keying (QPSK) is used for the extended LDPCC as described above ($R_1 = 1/3$). The original codewords (which are part of the extended LDPCC) and the additional IR parities are scheduled into two independent PLPs of Type 1 (i.e. without subslicing), each with a time interleaver depth of 7 FECFRAMES (one FECFRAME equals $N_0 = 16200$ bits). The PLP containing IR is scheduled with one T2-Frame delay compared with the PLP carrying the original codewords, where the T2-Frame had a duration of about 200 ms. The T2 system uses 8k FFT in an 8 MHz channel (subcarrier spacing 1.116 kHz), relative guard interval 1/4 and pilot pattern PP1. The channel is Typical Urban 6 (TU6) [11] at 100 Hz maximum Doppler frequency (corresponding to 180 km/h at 600 MHz carrier frequency). Perfect channel estimation is assumed. The BER after the LDPC decoder is depicted in Fig. 5. If the LDPC decoder decodes only the original codewords, a BER of $2 \cdot 10^{-4}$ is obtained at 10 dB. If further the IR parities are collected and the extended LDPCC is decoded, the same BER is achieved already at 3 dB. This large SNR gain of 7 dB is due to higher time diversity, as the IR part is transmitted at later time instances. Note that if the SNR is sufficiently large (here e.g. well above 10 dB), the PLP carrying IR can be ignored at the receiver side. By falling into sleep mode during this time, IR offers power saving capabilities, as will be discussed in more detail in the next subsection.

### C. Power saving capabilities of IR

We now consider the number of events in which the original LDPCC failed decoding over the time-varying TU6 channel.

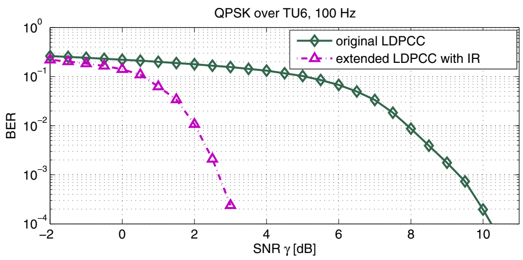



Fig. 5. BER performance over SNR of original DVB-T2 code (16k, $R_0 = 2/3$) and extended code (32k, $R_1 = 1/3$), TU6 channel

The original and extended LDPCC are still of rates $R_0 = 2/3$ and $R_1 = 1/3$, respectively, but of reduced sizes $N_0 = 4320$ and $N_1 = 2N_0$, respectively. These codes have been discussed during the standardization of DVB-NGH (next generation handheld). In Fig. 6, the original LDPCC is investigated over TU6 channel at 1 Hz maximum Doppler frequency and 10 dB SNR. The time-scale is normalized to the duration of a FECFRAME (of size $N_0$). The solid curve reflects the average channel power during one FECFRAME. Each black dot at ordinate position 1 corresponds to successful decoding of one codeword, each red star at position 2 indicates failed decoding. Further, the number of iterations is shown for each FECFRAME (in a scaled version). During discrete times about 130-270, the channel has average power larger than 1. As the SNR is sufficiently large, all FECFRAMES could be decoded during this interval after only 1 iteration. However, there are several deep fades, e.g. around time index 600, where decoding failed after 50 iterations.

In Fig. 7, a similar analysis is shown for the extended LDPCC, which is transmitted instead of the original code (i.e., we did not introduce one T2-Frame delay between original codeword and IR extension). It can be seen that there are several intervals, in which decoding is successful, even though the original code would have failed. For all other times, the required number of iterations is on average in the same range as for the original code. The original LDPCC had a FER of 30% in this example, while the FER for the extended code is 7%. Thus, during $(100-30)\% = 70\%$ of the time, the IR part could be ignored by the receiver, therby offering power saving capability. Several options are possible to detect a priori, if the IR part is required or not, e.g. analyzing the average channel power as was done here and setting a decision threshold.

## V. Conclusion

We presented an algorithm to optimize degree sequences for LDPCC extension. Based on LDPCCs of 2nd generation DVB systems, we examined incremental redundancy as a means to offer more robustness, while allowing for power saving capabilities. An example was presented for the TU6 channel, where halving the code rate due to IR allowed for 7dB SNR gain. Currently, IR is included in the upcoming DVB-NGH standard, where a 4k LDPCC of rate 1/2 is to be used for protection of signalling information. IR in this case allows for additional parities, which are transmitted in a different frame, exploiting both coding gain and time diversity.

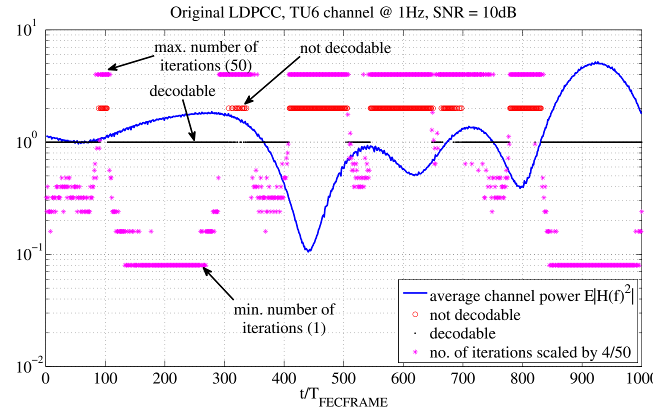



Fig. 6. Decoding events of original code and required number of iterations

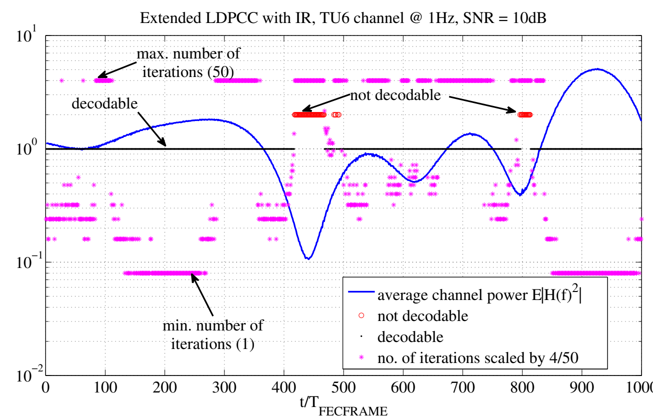



Fig. 7. Decoding events of extended code and required number of iterations

## References


[1] R. G. Gallagher, *Low Density Parity-Check Codes*. Cambridge: MIT Press, 1963, ISBN 0-262-57177-3.

[2] *Digital Video Broadcasting (DVB); Frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (DVB-T2)*, ETSI EN 302 755 Std., Oct. 2008.

[3] N. Varnica, E. Soljanin, and P. Whiting, "LDPC code ensembles for incremental redundancy hybrid ARQ," in *ISIT*, sept. 2005, pp. 995–999.

[4] S. Sesia, G. Caire, and G. Vivier, "Incremental redundancy hybrid ARQ schemes based on low-density parity-check codes," *IEEE Trans. Commun.*, vol. 52, no. 8, pp. 1311–1321, Aug. 2004.

[5] J. Kim, W. Hur, A. Ramamoorthy, and S. McLaughlin, "Design of rate-compatible irregular LDPC codes for incremental redundancy hybrid ARQ systems," in *ISIT*, July 2006, pp. 1139–1143.

[6] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1727–1737, Oct. 2001.

[7] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 670–678, Apr. 2004.

[8] M. Yang, W. Ryan, and Y. Li, "Design of efficiently encodable moderate-length high-rate irregular LDPC codes," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 564–571, Apr. 2004.

[9] N. S. Muhammad, "Short inner block codes for serial concatenated codes and bit-interleaved coded modulation," in *IEEE Vehicular Technology Conference (VTC)*, vol. 70, Sept. 2009.

[10] E. Çela, *The Quadratic Assignment Problem: Theory And Algorithms*. Kluwer Academic Publishers, 1998.

[11] M. Failli, "COST 207 digital land mobile radio communications," *Commission of the European Communities*, 1988.