Optimal Power Allocation in a Spectrum Sharing System with Partial CSI

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***Abstract*—This paper studies an optimal power allocation strategy in spectrum-sharing cognitive radio systems. In reality, it is difficult for the secondary users (SUs) to obtain perfect channel state information (CSI) related to the primary users (PUs) due to the lack of cooperation between the SUs and the PUs. We assume that only instantaneous CSI of the secondary link and statistical CSI of the other links is available to the SUs. This assumption not only reveals the realistic challenges, but also brings mathematical challenges in solving the optimization problem in closed form, since the performance evaluated with statistical CSI involves expectation operations. We reformulate the objective function in closed form and derive the optimal power allocation solution. The feasibility of the solution is discussed. Furthermore, we provide several interesting insights into the achievable performance based on the properties of the solution.**

I. Introduction

In cognitive radio (CR) systems, secondary users (SUs) improve their transmission performance by dynamically accessing the spectrum of primary users (PUs). The essential requirement for the SUs is to guarantee that the performance degradation caused to the PUs is properly controlled.

Among several models of dynamic spectrum access, spectrum-sharing, also known as the underlay model [1], has raised a lot of research interest due to its potential to achieve high spectral efficiency [2]. In this model, the SUs simultaneously transmit with the PUs provided that the interference received at the PUs is kept below some predefined threshold. In single-antenna CR networks, power allocation is an effective way to control the interference. The optimal power control strategy over additive white Gaussian noise (AWGN) channels was studied in [3]. The work [4] discussed the optimal power control method under an interference power constraint and analyzed the ergodic capacity over different fading channel models. When both a peak and an average interference power constraints are imposed, the authors in [5] designed optimal power allocation schemes to maximize the average transmission rate or minimize the outage probability of the CR system. Furthermore, the work in [6] additionally considered a transmit power constraint imposed on the secondary transmitter and derived optimal power control methods to achieve the ergodic capacity, the delay-limited capacity, and the outage capacity, respectively. Another interesting result was discovered in [7], in which a power allocation scheme with a capacity-loss constraint outperformed that with an interference power constraint. We note that all the above work required instantaneous channel state information (CSI).

In reality, the SUs can hardly estimate instantaneous CSI of the primary links due to limited cooperation with the PUs. However, statistical CSI can be obtained by exploiting some side information, e.g., location information [8]. The authors in [9] studied power control methods aiming at achieving the minimum required mean rate of the SU subject to either a transmit power constraint or an outage constraint. They assume that both the secondary transmitter (ST) and the secondary receiver (SR) have instantaneous CSI of the ST-SR link and the link from the ST to the primary receiver (PR), but they only have statistical CSI for the link from primary transmitter (PT) to the PR and the PT-SR link. However, it is challenging for the ST or the SR to obtain instantaneous CSI of the ST-PR link, since it requires the PR to estimate the CSI and feed it back to the ST. In our recent work [10], we studied the ergodic capacity in a spectrum sharing system, in which the ST and the SR can obtain instantaneous CSI of the ST-SR link and the PT-SR link and statistical CSI of the ST-PR link. Nevertheless, under such circumstances, it also requires to some extent, a cooperation between the SUs and the PUs, since the SR needs to synchronize with the primary transmission and estimate the CSI using the primary pilot symbols.

In this paper, we investigate the optimal power control in a spectrum sharing system with realistic CSI assumptions. We aim at maximizing the average secondary transmission rate over Rayleigh channels subject to both a peak transmit power constraint and an average interference power constraint. Instantaneous CSI of the ST-SR link is perfectly known at the ST and the SR, while only statistical CSI of the PT-SR and the ST-PR link is available. This assumption not only reveals the realistic challenges, but also brings mathematical challenges in solving the optimization problem. Specifically, we derive a closed-form expression of the objective function. By applying the Karush-Kuhn-Tucker (KKT) conditions to the optimization problem, we address the optimal power allocation strategy, which includes solving a nonlinear equation. We check the feasibility of the solution and propose a simple numerical method to solve it. Furthermore, We analyze some properties of the solution and provide some insights into the achievable performance, which are validated by simulation results.

II. System Model

We consider the CR system in Fig. 1 where a secondary link with a single-antenna at the ST and the SR coexists with a primary link with a single antenna at the PT and the PR. The transmit symbols of the PT and the ST at the i-th time instant are given by the zero-mean circularly symmetric Gaussian complex random variables $x_1[i]$ and $x_2[i]$ with $\mathbb{E}\{|x_1[i]|^2\} = 1$ and $\mathbb{E}\{|x_2[i]|^2\} = 1$, respectively. The instantaneous CSI

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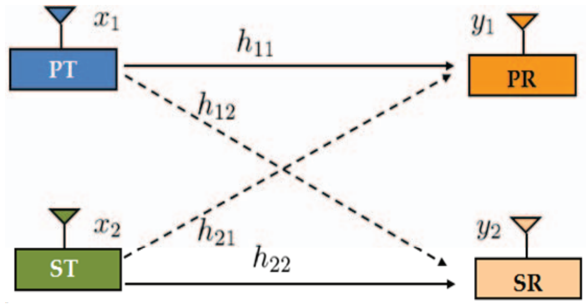

Fig. 1. Spectrum sharing system

of the PT-PR link and the ST-SR link is given by $h_{11}[i]$ and $h_{22}[i]$, respectively, while the instantaneous CSI of the interfering PT-SR link and the ST-PR link is $h_{12}[i]$ and $h_{21}[i]$, respectively. All the channels are assumed to be stationary, ergodic, mutually independent, and also independent of the noises. The noise of the secondary link n_s follows a zero-mean circularly symmetric complex Gaussian distribution with variance σ_s^2. Consequently, the received signal at the SR is

$$y_2[i] = P_2[i]h_{22}[i]x_2[i] + P_1 h_{12}[i]x_1[i] + n_s[i] \tag{1}$$

where we assume the power P_1 is fixed at the PT and the power $P_2[i]$ is optimized at the ST according to the channel variation at the i-th time instant. Assuming Rayleigh flat fading channels for all links in the network, the channel power gain $g_{ij}[i] = |h_{ij}[i]|^2$ follows an exponential distribution with mean $1/l_{ij}$, $i, j = 1, 2$. Due to limited cooperation between the PUs and the SUs, the ST and the SR can acquire $g_{22}[i]$ perfectly, but only know the statistical CSI parameters l_{12} and l_{21}. The time index i is omitted from now on for simplicity.

Two power constraints are imposed on the secondary system. Firstly, we consider the average interference power constraint P_I to protect the PUs. Secondly, the transmit power of the ST is limited to P_S. We assume single-user detection at the SR. The optimization problem aiming at maximizing the achievable rate of the secondary link subject to both aforementioned power constraints is formulated as

$$\max_{P_2} \ \tilde{R} = \mathbb{E}_{g_{12}}\left\{\ln\left(1 + \frac{P_2 g_{22}}{P_1 g_{12} + \sigma_s^2}\right)\right\} \tag{2}$$

$$\text{s.t.} \ \ P_2 \le P_S, \tag{3}$$

$$P_2 \ge 0, \tag{4}$$

$$\mathbb{E}_{g_{22}, g_{21}}\left\{P_2 g_{21}\right\} \le P_I \tag{5}$$

where $\mathbb{E}_x\{y(x)\}$ indicates the expectation of $y(x)$ over x. The unit of $\tilde{R}$ is nats/s/Hz.

Since the instantaneous CSI g_{21} is unknown at the ST/SR, the transmit power P_2 is independent of g_{21}. Using this property, (5) is converted into

$$\mathbb{E}_{g_{22}}\left\{P_2\right\} \le \frac{P_I}{\mathbb{E}_{g_{21}}\left\{g_{21}\right\}} = P_I l_{21}.$$

To evaluate the achievable rate over different realizations of g_{22}, we define the metric R in bits/s/Hz as

$$R = (\log_2 e)\mathbb{E}_{g_{22}}\left\{\tilde{R}\right\}. \tag{6}$$

III. Optimal Power Allocation

The objective function of (2) is to maximize a concave function of P_2 and all the constraints are affine functions of P_2. Thus, the optimization problem (2) is a convex optimization problem and can be optimally solved in theory. However, solving it still involves several challenges, e.g., it may be hard to obtain an explicit analytical solution. The reason is that since both the objective function and the constraints include expectation expressions, directly applying the KKT optimality conditions to (2) would result in involved integral expressions.

In this section, we study the optimal power allocation strategy by solving (2). Specifically, we first give a closed form expression of the objective function. Second, we solve the optimization problem by applying the KKT optimality conditions. The resulting solution requires solving a nonlinear equation. We then investigate the property of this equation and propose simple numerical methods to solve it. Finally, we analyze some properties of the derived solution and provide some interesting insights into the relation between the achievable performance and the power constraints.

A. Reformulation of the Objective Function

According to the assumptions in Section II, we assume that the instantaneous power gain of the ST-SR link g_{22} is available, while only statistical CSI of the ST-PR and the PT-SR links is known at the ST and the SR. Consequently, the optimal power P_2 is a function of g_{22} and the CSI parameters l_{12} and l_{21}. Motivated by this fact, we aim at evaluating the objective function to get an explicit expression with respect to (w.r.t.) the known CSI and the optimization variable P_2.

Considering that g_{12} follows an exponential distribution with parameter l_{12}, we calculate the expectation of the objective function $\tilde{R}$

$$\begin{aligned}
\tilde{R} &= \mathbb{E}_{g_{12}}\left\{\ln\left(1 + \frac{P_2 g_{22}}{\sigma_s^2 + P_1 g_{12}}\right)\right\} \\
&= \int_0^\infty \ln\left(1 + \frac{P_2 g_{22}}{\sigma_s^2 + P_1 x}\right) l_{12} e^{-l_{12}x} dx \\
&= \ln\left(1 + \frac{P_2 g_{22}}{\sigma_s^2}\right) \\
&\quad - \frac{P_2 g_{22}}{P_1}\int_0^\infty \frac{1}{x^2 + \frac{2\sigma_s^2 + P_2 g_{22}}{P_1}x + \frac{\sigma_s^4 + P_2 g_{22}\sigma_s^2}{P_1^2}} e^{-l_{12}x} dx \\
&\overset{(a)}{=} \ln\left(1 + \frac{P_2 g_{22}}{\sigma_s^2}\right) - \int_0^\infty \left(\frac{1}{x + x_0} - \frac{1}{x + x_1}\right) e^{-l_{12}x} dx \\
&\overset{(b)}{=} \ln\left(1 + \frac{P_2 g_{22}}{\sigma_s^2}\right) - e^{l_{12}x_0} E_1(l_{12}x_0) + e^{l_{12}x_1} E_1(l_{12}x_1)
\end{aligned} \tag{7}$$

where in step (a) we use

$$x_1 = \frac{\sigma_s^2 + P_2 g_{22}}{P_1}, \quad x_0 = \frac{\sigma_s^2}{P_1}$$

and in step (b) we use the definition of the exponential integral $E_1(z) = \int_z^\infty \frac{e^{-t}}{t} dt$.

B. Optimal Power Allocation Strategy

Since the optimization problem is convex, solving the Lagrangian dual problem yields the optimal solution [11]. We write the Lagrangian as

$$L = \tilde{R} - \lambda_0(P_2 - P_S) + \lambda_1 P_2 - \lambda_2(\mathbb{E}_{g_{22}}\{P_2\} - P_I l_{21})$$

with $\tilde{R}$ represented by (7). The Lagrangian multipliers λ_i, $i = 0, 1, 2$, are nonnegative and correspond to the three power constraints (3), (4), and (5), respectively.

The solution satisfies the KKT optimality conditions:

$$\left.\frac{\partial L}{\partial P_2}\right|_{P_2 = P_2^*} = be^{a+bP_2^*}E_1(a + bP_2^*) - \lambda_0^* + \lambda_1^* - \lambda_2^* = 0 \quad (8)$$

$$\lambda_0^*(P_2^* - P_S) = 0 \quad (9)$$

$$\lambda_1^* P_2^* = 0 \quad (10)$$

$$\lambda_2^*\left(\mathbb{E}_{g_{22}}\{P_2^*\} - P_I l_{21}\right) = 0 \quad (11)$$

$$\lambda_0^*,\ \lambda_1^*,\ \lambda_2^* \geq 0 \quad (12)$$

$$(3), (4) \text{ and } (5)$$

where in (8)

$$a = \frac{l_{12}\sigma_s^2}{P_1}, \quad b = \frac{l_{12}g_{22}}{P_1} \quad (13)$$

and $(\cdot)^*$ denotes the optimal value. We give three cases for P_2^* according to different values of g_{22}.

1) $P_2^* = P_S$. According to (10), $\lambda_1^* = 0$. Inserting $\lambda_1^* = 0$ and $P_2^* = P_S$ into (8) and using $\lambda_0^* \geq 0$, we have

$$be^{a+bP_S}E_1(a + bP_S) \geq \lambda_2^*. \quad (14)$$

2) $P_2^* = 0$. According to (9), $\lambda_0^* = 0$. Integrating it into (8) and using $\lambda_1^* \geq 0$, we have

$$be^a E_1(a) \leq \lambda_2^*. \quad (15)$$

3) $0 < P_2^* < P_S$. According to (9) and (10), we have $\lambda_0^* = \lambda_1^* = 0$. Inserting these values into (8), the optimal P_2^* is the solution P^* of

$$be^{a+bP^*}E_1(a + bP^*) = \lambda_2^*. \quad (16)$$

There are two main problems in the above-derived strategy. First, for the third case $0 < P_2^* < P_S$, P_2^* needs to be solved through the nonlinear equation (16), which requires a efficient numerical method. Second, it is difficult to interpret the meaning of the conditions (14), (15), and (16) w.r.t. the variation of the instantaneous CSI g_{22}. We focus on solving these two problems in the following.

- We seek for an efficient and feasible method to solve the nonlinear equation (16). In Appendix A, we prove that $h(x) = e^{a+bx}E_1(a+bx)$ monotonically decreases with x, $\forall\, x > 0$. Furthermore, $h(0) - \lambda_2^*/b$ and $h(P_S) - \lambda_2^*/b$ have opposite signs[1] according to (15) and (14). Therefore, there exists a feasible and unique solution to solve (16). The nonlinear equation (16) can be solved either by the bisection method or the more efficient Newton method using the results in (23). The convergence of the Newton method also depends on the selection of the initial point.
- We investigate the relation between P_2^* and the variation of g_{22} by revising the conditions (14), (15), and (16).
  The first condition (14) is a nonlinear function of g_{22}. In Appendix B, we show that $be^{a+bP_S}E_1(a + bP_S)$ is a monotonically increasing function of g_{22}, in which b is a function of g_{22} as shown in (13). Therefore, in order to fulfill (14), it is required that $g_{22} \geq v_1$, in which v_1 is a function of λ_2^* and can be evaluated numerically.
  Considering the second condition (15), we rewrite it as $g_{22} \leq v_0$ with $v_0 = \lambda_2^* P_1/\left(l_{12}e^a E_1(a)\right)$. The optimal strategy is to switch off the ST, when the power gain between the ST and the SR is smaller than v_0.
  Concerning the third condition, it corresponds to the case when $v_0 < g_{22} < v_1$, and the optimal P_2 can be obtained by efficiently solving (16).

To summarize, the optimal power allocation strategy is

$$P_2^* = \begin{cases} 0, & g_{22} \leq v_0 \\ P^*, & v_0 < g_{22} < v_1 \\ P_S, & g_{22} \geq v_1 \end{cases}. \quad (17)$$

As analyzed before, both thresholds v_0 and v_1 are monotonically increasing functions of λ_2^* for $\lambda_2^* > 0$. The average power is defined as

$$Q_{\text{avg}} = \mathbb{E}_{g_{22}}\{P_2^*\} = \int_{v_0}^{v_1} P^*(x) f_{g_{22}}(x)dx + P_S\int_{v_1}^{\infty} f_{g_{22}}(x)dx \quad (18)$$

where $f_{g_{22}}(x)$ is the probability density function (pdf) of g_{22}. Calculating the derivative of Q_{avg} over P_S and λ_2^* yields

$$\frac{dQ_{\text{avg}}}{dP_S} \overset{(c)}{=} \int_{v_1}^{\infty} f_{g_{22}}(x)dx \geq 0 \quad (19)$$

$$\frac{dQ_{\text{avg}}}{d\lambda_2^*} \overset{(d)}{=} \int_{v_0}^{v_1} \frac{dP^*(x)}{d\lambda_2^*} f_{g_{22}}(x)dx \overset{(e)}{\leq} 0 \quad (20)$$

where both steps (c) and (d) use the property that the optimal power P_2^* at v_1 is equal to P_S. The inequality (e) uses the property derived in Appendix A that the optimal solution P^* decreases as λ_2^* increases.

The selection of λ_2^* is essential to obtain the optimal power allocation in (17). From the KKT condition in (11), we conclude that $\lambda_2^* = 0$ if $Q_{\text{avg}} < P_I l_{21}$, i.e., the interference constraint is fulfilled with inequality. For the case in which the interference constraint is fulfilled with equality, i.e., $Q_{\text{avg}} = P_I l_{21}$, λ_2^* should be chosen to satisfy this equality.

C. Remarks

In order to obtain more insight into the performance of the derived optimal power solution, we study the change of the resulting achievable secondary rate $\tilde{R}$ w.r.t. the peak transmit power constraint P_S or the interference constraint P_I.

First, we consider the relation between $\tilde{R}$ and the average interference constraint P_I for a given value of P_S. We analytically explain that if we increase P_I from zero to infinity, the achievable rate $\tilde{R}$ increases at the beginning until P_I reaches some threshold $\widehat{P_I} = P_S/l_{21}$, $\tilde{R}$ saturates.

[1]Here the special cases with inequality in (14) and (15) are excluded, since under such circumstances, P_2^* is chosen to be at boundary, i.e., 0 or P_S.

- Increasing P_I from zero to $\widehat{P_I}$, the interference power constraint is always stricter than the peak transmit power constraint, thus the constraint $Q_{\text{avg}} \leq P_I l_{21}$ is fulfilled with equality. From (20), we note that increasing P_I yields a decrease in λ_2^*. We also prove that the optimal $\tilde{R}$ is a monotonically decreasing function of λ_2^*, which indicates an increase of $\tilde{R}$ in this P_I region. The detailed proof is omitted here for brevity.
- If P_I keeps increasing and fulfills $P_I > \widehat{P_I}$, we note that even though the SU always transmits with the maximum power P_S, the average interference (5) is strictly smaller than the constraint. According to (11), λ_2^* is zero. Taking this into the condition (14), since $be^{a+bP_S}E_1(a+bP_S)$ is positive for all a, b, $P_S > 0$, the condition (14) is always fulfilled and the optimal transmit power is a constant and equal to P_S. This indicates that the achievable rate $\tilde{R}$ is a constant in this case.

Second, we consider the relation between the achievable rate $\tilde{R}$ and the transmit power constraint P_S provided that the interference constraint P_I is fixed. We also give an analytical explanation for the observations if we increase P_S from zero to infinity, the achievable rate $\tilde{R}$ increases at the beginning until P_S reaches some threshold $\widehat{P_S}$, $\tilde{R}$ saturates.

- Increasing P_S from zero but fulfilling $P_S < P_I l_{21}$, the interference power is strictly smaller than the constraint even if the ST transmit with the peak power, i.e., the optimal power is always equal to P_S. Consequently, it is easy to show that $\tilde{R}$ increases monotonically with P_S.
- We consider P_S to increase beyond $P_I l_{21}$ but to be smaller than some limit $\widehat{P_S}$. The average interference power constraint is fulfilled with equality, i.e., $Q_{\text{avg}} = P_I l_{21}$ is fixed. Combining (19) and (20), we conclude that an increase of P_S causes an increase of λ_2^*.
- We prove that keeping P_S increasing beyond some boundary point, the achievable rate $\tilde{R}$ remains constant. First, applying (24) to the left side of (14), we have $be^{a+bP_S}E_1(a+bP_S) \leq b/(a+bP_S) < 1/P_S$ which means that given a certain λ_2^*, increasing P_S beyond some boundary point $\widehat{P_S}$, (14) is never fulfilled, i.e., $v_1 = \infty$. As a result, the derivative in (19) is equal to zero, which indicates that increasing P_S further does not change the value of Q_{avg}.
  We prove that in this case, λ_2^* remains constant by contradiction. If λ_2^* changes in this case, then according to (20), Q_{avg} also changes w.r.t. λ_2^*. Revising that the variation of Q_{avg} only depends on two variables, P_S and λ_2^*, and combining the fact that increasing P_S beyond $\widehat{P_S}$ does not change Q_{avg}, we conclude that Q_{avg} changes only due to the change of λ_2^*, which contradicts our condition that Q_{avg} is fixed due to the given fixed P_I.
  Considering λ_2^* is fixed and $v_1 = \infty$, we prove that the achievable rate $\tilde{R}$ also remains constant. The detailed proof is omitted here.

IV. Simulation Results

We consider the system given in Fig. 1. The channel power gain of each fading link g_{ij}, $i, j = 1, 2$ follows an exponential distribution with $l_{11} = 1$, $l_{21} = 5$ and $l_{22} = 1$. The value l_{12} is specified in each figure. The noise variance of the SU is $\sigma_s^2 = 0.08$. The transmit power of the PT is $P_1 = 0\,\text{dB}$.

The performance in two scenarios is compared. The reference scenario [10] assumes that both the instantaneous CSI g_{22} and g_{12} are perfectly known at the ST and the SR, which is given by the dashed lines. The other scenario is the one introduced in this paper which assums that only instantaneous g_{22} is known. The results are shown by the solid lines.

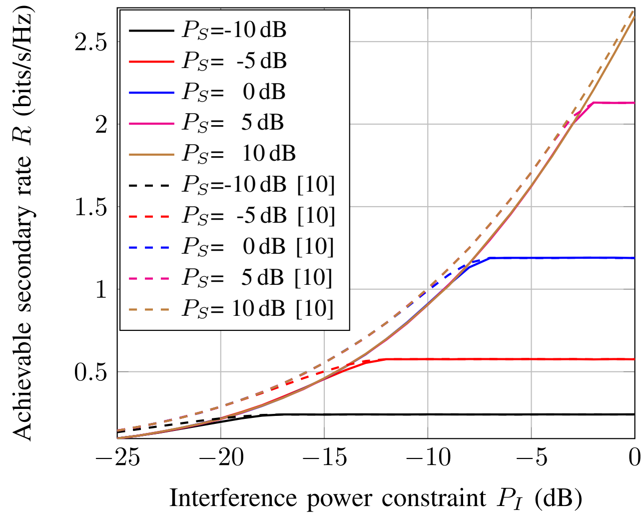

Fig. 2. Achievable rate R versus P_I, $l_{12} = 1$.

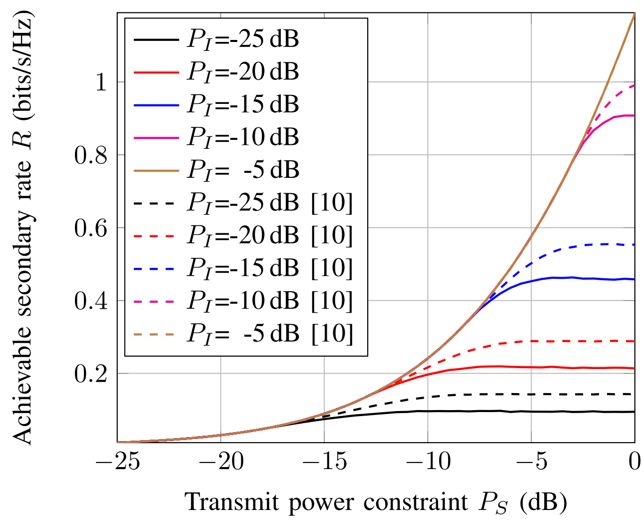

Fig. 3. Achievable rate R versus P_S, $l_{12} = 1$.

Fig. 2 depicts the achievable secondary rate R versus the interference power constraint P_I under different transmit power constraints P_S with $l_{12} = 1$. First, compared with the results in [10], we observe the performance gap regarding R due to the absence of the CSI g_{12}. Second, we observe that the rate saturates with an increasing P_I provided that P_S is fixed, which matches the explanation in III-C. Given certain P_S, the boundary P_I is given by $\widehat{P_I} = P_S/l_{21}$.

Fig. 3 plots the achievable rate R versus the peak transmit power constraint P_S under different interference power

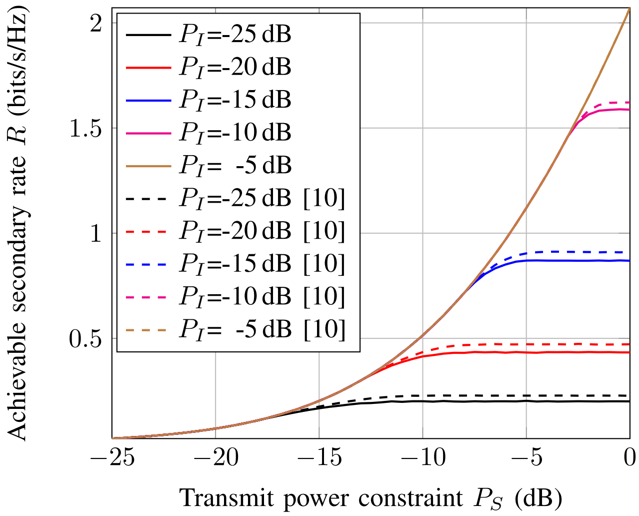

Fig. 4. Achievable rate R versus P_S, $l_{12} = 5$.

constraints P_I with $l_{12} = 1$. First, it is shown that with an increasing P_S, R increases up to some boundary, beyond which it remains constant. This phenomenon is explained in III-C. Second, we observe that compared with the results in [10], the performance gap is large. Since in the simulation setup, the PT-SR link gain is as strong as the ST-SR link, as seen from $l_{22} = l_{12} = 1$. Knowing the instantaneous CSI of the PT-SR link brings large gain by exploiting its dynamic variation. In Fig. 4, we set $l_{12} = 5$, which means that the PT-SR link is weaker. Such the performance gap is reduced.

V. Conclusion

This paper investigates optimal power allocation in a spectrum sharing system where one secondary link coexists with one primary link. Due to the difficulty to cooperate with the PUs, we assume that instantaneous CSI of the ST-SR link and statistical CSI of the PT-SR and ST-PR links is known at the ST and the SR. We reformulate the optimization objective in closed form and then solve the optimization problem by applying the KKT optimality conditions. Based on the resulting analytical expression, some interesting insights are discussed and verified by simulation.

Appendix A

We first rewrite the considered function as

$$e^{a+bx}E_1(a+bx) = f\left(g\left(x\right)\right) \tag{21}$$

where a and b are given in (13) and

$$f(y) = e^y E_1(y), \quad g(x) = a + bx. \tag{22}$$

Applying the chain rule in calculating the derivative of (21) over x, we obtain

$$\frac{df\left(g\left(x\right)\right)}{dx} = \frac{df\left(g\left(x\right)\right)}{dg\left(x\right)}\frac{dg\left(x\right)}{dx} = b\left(e^{g(x)}E_1(g\left(x\right)) - \frac{1}{g\left(x\right)}\right). \tag{23}$$

Using [12, Eq. 5.1.19]

$$\frac{1}{y+1} < e^y E_1(y) \le \frac{1}{y}, \ \ \forall y > 0 \tag{24}$$

and $g(x) > 0$, $\forall x \ge 0$, (23) is smaller than zero. We conclude that $f\left(g\left(x\right)\right) = e^{a+bx}E_1(a+bx)$ is a monotonically decreasing function in x, $\forall x > 0$.

Appendix B

Consider the function $f_1(b) = be^{a+bP_S}E_1(a+bP_S)$, where a and b are given in (13). Applying the chain rule in calculating the derivative of $f_1(bg_22))$ over g_{22}, we obtain

$$\frac{df_1(b(g_{22}))}{dg_{22}} = \frac{df_1(b)}{db}\frac{db}{dg_{22}} = \frac{df_1(b)}{db}\frac{l_{12}}{P_1} \tag{25}$$

where the first term is evaluated by

$$\begin{aligned}
&\frac{df_1(b)}{db} = e^{a+bP_S}E_1(a+bP_S) \\
&\quad + bP_S\left(e^{a+bP_S}E_1(a+bP_S) - \frac{1}{a+bP_S}\right) \\
&\overset{(f)}{=} e^y E_1(y) + bP_S\left(e^y E_1(y) - \frac{1}{y}\right) \\
&= e^y E_1(y) + yP_S\left(e^y E_1(y) - \frac{1}{y}\right) - a\left(e^y E_1(y) - \frac{1}{y}\right) \\
&= (y+1)\left(e^y E_1(y) - \frac{1}{y+1}\right) - a\left(e^y E_1(y) - \frac{1}{y}\right)
\end{aligned} \tag{26}$$

where in step (f) we use $y = a + bP_S > 0$. Applying the property (24), we recognize that (26) is positive. We conclude that $f_1(b)$ is a monotonically increasing function of g_{22}.

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