Joint Channel Information Estimation and Data Detection for OFDM-Based Systems Under Unknown Interference

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Abstract—**In this paper we consider an orthogonal frequency-division multiplexing (OFDM)-based system under unknown narrow-band interference (NBI). We propose an iterative receiver to jointly estimate the channel information, which consists of channel coefficients and noise-plus-interference variances of each sub-carrier, and detect the transmitted signals. The simulation results show that our proposed receiver provides an extremely close bit-error-rate (BER) to that of the case where perfect channel information is available at the receiver. Besides, Cramér-Rao lower bound (CRLB) of interested parameters are also derived. The mean-square-error (MSE) of the estimated parameters given by our proposed algorithm reaches the CRLB.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) modulation scheme [1] has been adopted in some standards to cater for diverse communication services with high speed requirement nowadays [2, 3]. With the adoption of spectrum sharing systems [4], the operation of OFDM-based systems under narrow-band interference (NBI) has attracted much attentions. As shown in [5, 6], NBI affects the performance of the system greatly. Therefore, some solutions have been proposed to alleviate the impact of NBI on the system. One solution to the problem is to use erasure decoders, e.g. [7], which ignore interfered signals in decoding stage. This kind of decoders does not require the interference power but the indices of interfered sub-carriers need to be obtained by null pilot blocks. Another way to deal with the interference is to modify the conventional decoder to take into account the presence of interference, e.g. [8]. In [9], authors propose a channel coefficients and interference power estimation algorithm based on training signals in NBI-affected OFDM systems. Please note that the mentioned methods do not exploit the information given by the data transmission stage to boost the quality of parameter estimation which is needed in decoding process.

In this paper we consider an OFDM-based system under unknown NBI. We treat the interference as an inherent parameter of the system's channel information, try to estimate it and make use of the estimated parameter to decode the transmitted signals. Furthermore, in this paper, the information given by the data transmission period is also made use to boost the quality of estimated channel information. Therefore, the proposed receiver will work in an iterative manner. The better channel information helps to obtain better information of transmitted data. In return, this better information of transmitted data is used in the channel information estimator to provide better estimation of channel information. The channel information estimator is an application of the Expectation-Conditional Maximization (ECM) algorithm [10]. The ECM algorithm is a variation of the general Expectation-Maximization (EM) algorithm [11] when large parameters are dealt with. To evaluate the quality of estimated information, Cramér-Rao lower bound (CRLB) of the interested parameters are also derived. Simulation results show that the bit-error-rate (BER) of the proposed iterative receivers is close to the perfect channel information case. Besides, the mean-square-error (MSE) of estimated parameters approach the CRLB.

The rest of the paper is organized as follows. Section II describes the system model. In Section III, the proposed iterative receiver is presented in details. Then, the CRLB derivations are given in Section IV. Section V provides some simulation results to illustrate the performance of the iterative receiver in terms of BER and MSE criterions. Finally, Section VI concludes the paper.

Notation: Upper case bold letters denote matrices and the small bold letters denote row/column vectors. Transpose and Hermitian transpose of a vector/matrix are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively. The identity matrix of size n is denoted by $\boldsymbol{I}_n$. $\mathbf{0}_{m\times n}$ denotes a zero matrix of size $m \times n$. Trace of a matrix $\boldsymbol{A}$ is denoted by $\mathrm{trace}(\boldsymbol{A})$. $[\boldsymbol{A}]_{m,n}$ is the (m,n)th element of $\boldsymbol{A}$. $\boldsymbol{W}$ is the discrete Fourier transform (DFT) matrix of size N with $[\boldsymbol{W}]_{m,n} = \frac{1}{\sqrt{N}} \exp\{-j\frac{2\pi nm}{N}\}$. $\boldsymbol{F}$ is the first L columns of matrix $\sqrt{N}\boldsymbol{W}$. $\Re(\cdot)$, $\Im(\cdot)$ and $(\cdot)^*$ are the real part, imaginary part and complex conjugate of a complex number, respectively. $\mathbb{E}\{\cdot\}$ is the expectation operation and $f(\boldsymbol{a}|\boldsymbol{b})$ is the probability density function of $\boldsymbol{a}$ given $\boldsymbol{b}$.

II. System Model and Problem Statement

A. System Model

We consider an OFDM-based system in which an information sequence $\boldsymbol{b}$ is encoded by a convolutional encoder and the output is passed though an interleaver. The output of the interleaver is fed to the binary phase shift keying (BPSK) modulator with energy per symbol of 1. The number of sub-

978-1-4673-0990-5/12/$31.00

carriers in use is N. We assume that the channel between the transmitter and the receiver is modeled as a frequency-selective fading channel with a channel response vector $\boldsymbol{h}$ of length L, $\boldsymbol{h} = [h_0\ h_1\ \cdots\ h_{L-1}]^T$. The channel response $\boldsymbol{h}$ is assumed to be static over the transmission of B OFDM symbols. At the receiver, the signals transmitted from the transmitter are not only disturbed by the background noise but also NBI.

Let $\boldsymbol{x}(n) = [x_0(n)\ \cdots x_c(n)\ \cdots\ x_{N-1}(n)]^T$ be the nth OFDM block transmitted from the transmitter in frequency domain. The received signal vector at the receiver before DFT operation can be written as

$$\boldsymbol{y}(n) = \boldsymbol{H}\boldsymbol{W}^H\boldsymbol{x}(n) + \boldsymbol{v}(n),\ n = 1, 2, \cdots, B, \tag{1}$$

where $\boldsymbol{H}$ is a circulant matrix whose first column is $[\boldsymbol{h}^T\ \ \boldsymbol{0}_{1\times(N-L)}]^T$. The *total* noise vector $\boldsymbol{v}(n) = [v_0(n)\ \cdots\ v_c(n)\ \cdots\ v_{N-1}(n)]^T$ consists of the background noise and the interference from other transmitters. Due to the presence of the interference, the noise components of the interfered carriers have different variances compared with those of the non-interfered carriers. We assume that indices of N sub-carriers are divided into two sets: $\mathcal{J}_0$ contains indices of non-interfered sub-carriers and $\mathcal{J}_1$ consists of those of interfered sub-carriers. Let σ_c be the variance of the noise component of the cth sub-carrier. Following [8, 9], we have the following assumption

$$\sigma_c = \begin{cases} N_0 & \text{if } c \in \mathcal{J}_0 \\ N_0 + N_1 & \text{if } c \in \mathcal{J}_1 \end{cases}, \tag{2}$$

where N_0 is the variance of the background noise and N_1 is the variance of the interference. Here, we *do not* have any information on $\mathcal{J}_0$ and $\mathcal{J}_1$. Furthermore, we assume that N_0 and N_1 *are not* available at the receiver.

The received signal vector in time domain $\boldsymbol{y}(n)$ in (1) is then DFT-transformed as follows

$$\begin{aligned}\boldsymbol{r}(n) &= \boldsymbol{W}\boldsymbol{y}(n)\\ &= \boldsymbol{W}\boldsymbol{H}\boldsymbol{W}^H\boldsymbol{x}(n) + \boldsymbol{W}\boldsymbol{v}(n),\ n = 1, 2, \ldots, B.\end{aligned} \tag{3}$$

Note that the circulant matrix $\boldsymbol{H}$ can be decomposed as [12]

$$\boldsymbol{H} = \boldsymbol{W}^H\boldsymbol{\Lambda}\boldsymbol{W}, \tag{4}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}\{H(0), \cdots, H(c), \cdots, H(N-1)\}$ and

$$H(c) = \sum_{l=0}^{L-1} h(l)e^{-j\frac{2\pi lc}{N}},\ c = 0, 1, \cdots, N-1. \tag{5}$$

Therefore, (3) can be written as

$$\boldsymbol{r}(n) = \boldsymbol{\Lambda}\boldsymbol{x}(n) + \boldsymbol{w}(n),\ n = 1, 2, \ldots, B, \tag{6}$$

where $\boldsymbol{w}(n) = \boldsymbol{W}\boldsymbol{v}(n)$. Note that the probabilistic characteristics of $\boldsymbol{w}(n)$ are the same as those of $\boldsymbol{v}(n)$.
An equivalent form of (6) is

$$\boldsymbol{r}(n) = \boldsymbol{X}(n)\boldsymbol{F}\boldsymbol{h} + \boldsymbol{w}(n),\ n = 1, 2, \ldots, B, \tag{7}$$

where $\boldsymbol{X}(n) = \mathrm{diag}\{\boldsymbol{x}(n)\}$.

B. Problem Statement

Our objective is to decode the information sequence $\boldsymbol{b}$ transmitted from the transmitter. To do so we need to have the values of channel response $\boldsymbol{h}$ and effective noise variance vector $\boldsymbol{\sigma} \triangleq [\sigma_0\ \cdots\ \sigma_c\ \cdots\ \sigma_{N-1}]^T$. If we use the brute force search for the three above parameters, the complexity will be prohibitively high. In the next section, we will present an iterative algorithm to estimate the channel information, i.e., the two vectors $\boldsymbol{h}$ and $\boldsymbol{\sigma}$, as well as to decode the information bits. The algorithm needs to have the initial information of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$ which are provided by pilot symbols.

III. Proposed Iterative Receiver

The proposed receiver is given in Fig. 1 which comprises of two main parts. The first part is the ECM-based channel information estimator whose task is to provide better estimates of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$ based on the set of received signal vector $\{\boldsymbol{r}(n)\}_{n=1}^B$, and the soft information of transmitted symbols given by the channel decoder. The second part of the receiver is the data detection part which makes use of the channel information given by the ECM-based estimator to refine the soft information of transmitted symbols. The details of the proposed receiver are given below.

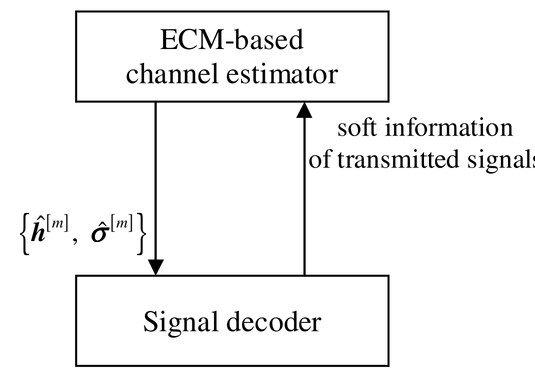

Fig. 1. Block diagram of the proposed iterative receiver

A. Data Detection

Suppose that after m iterations of the ECM-based estimator we have the estimates $\hat{\boldsymbol{h}}^{[m]}$ and $\hat{\boldsymbol{\sigma}}^{[m]} = [\hat{\sigma}_0^{[m]}\ \cdots\ \hat{\sigma}_c^{[m]}\ \cdots\ \hat{\sigma}_{N-1}^{[m]}]^T$. These two estimates, along with $\{\boldsymbol{r}(n)\}_{n=1}^B$, are then used to find the log-likelihood ratio (LLR) $\hat{\gamma}_c^{[m]}(n)$ of the transmitted symbol $x_c(n)$, $c = 0, 1, \cdots, N-1$, and $n = 1, 2, \ldots, B$. $\hat{\gamma}_c^{[m]}(n)$ is calculated as follows

$$\begin{aligned}\hat{\gamma}_c^{[m]}(n) &= \log\frac{\Pr\big(x_c(n) = +1|\boldsymbol{r}(n), \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big)}{\Pr\big(x_c(n) = -1|\boldsymbol{r}(n), \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big)}\\ &= \log\frac{\frac{1}{\pi^2(\hat{\sigma}_c^{[m]})^2}e^{\frac{1}{(\hat{\sigma}_c^{[m]})^2}|r_c(n)-\hat{H}_c^{[m]}|^2}}{\frac{1}{\pi^2(\hat{\sigma}_c^{[m]})^2}e^{\frac{1}{(\hat{\sigma}_c^{[m]})^2}|r_c(n)+\hat{H}_c^{[m]}|^2}}\\ &= \frac{4}{(\hat{\sigma}_c^{[m]})^2}\Re\big(r_c^*(n)\hat{H}_c^{[m]}\big),\end{aligned} \tag{8}$$

where $\hat{H}_c^{[m]}$ is the cth element of the product $\boldsymbol{F}\hat{\boldsymbol{h}}^{[m]}$.

Those LLR values are then fed to the decoder [13]. The decoder exploits the code structure to refine the LLRs. We assume that the outputs of the decoder are the updated LLR $\hat{\theta}_c^{[m]}(n)$ of $x_c(n)$ for $c = 0, 1, \cdots, N-1$ and $n = 1, 2, \ldots, B$.

B. ECM-based Channel Information Estimation

The ECM-based channel information estimation algorithm is to make use of the refined LLRs at the output of the decoder to have better values of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$. We define the set $\left(\{\boldsymbol{r}(n)\}_{n=1}^B, \{\boldsymbol{x}(n)\}_{n=1}^B\right)$ as the complete data space for the set of parameters $(\boldsymbol{h}, \boldsymbol{\sigma})$. The estimation algorithm has the following two steps.

1) E-step: The E-step is to determine the following quantity

$$Q\big(\boldsymbol{h}, \boldsymbol{\sigma} \big| \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big) = \mathbb{E}\big\{\log f\big(\{\boldsymbol{r}(n)\}_{n=1}^B, \{\boldsymbol{x}(n)\}_{n=1}^B \big| \boldsymbol{h}, \boldsymbol{\sigma}\big) \big| \{\boldsymbol{r}(n)\}_{n=1}^B, \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big\}, \quad (9)$$

The probability density function $f\big(\{\boldsymbol{r}(n)\}_{n=1}^B, \{\boldsymbol{x}(n)\}_{n=1}^B \big| \boldsymbol{h}, \boldsymbol{\sigma}\big)$ can be determined by

$$f\big(\{\boldsymbol{r}(n)\}_{n=1}^B, \{\boldsymbol{x}(n)\}_{n=1}^B \big| \boldsymbol{h}, \boldsymbol{\sigma}\big) = f\big(\{\boldsymbol{r}(n)\}_{n=1}^B \big| \{\boldsymbol{x}(n)\}_{n=1}^B, \boldsymbol{h}, \boldsymbol{\sigma}\big) f\big(\{\boldsymbol{x}(n)\}_{n=1}^B \big| \boldsymbol{h}, \boldsymbol{\sigma}\big), \quad (10)$$

where

$$f\big(\{\boldsymbol{r}(n)\}_{n=1}^B \big| \{\boldsymbol{x}(n)\}_{n=1}^B, \boldsymbol{h}, \boldsymbol{\sigma}\big) = \frac{1}{\pi^N \prod_{c=0}^{N-1} \sigma_c} \times \exp\Big(-\sum_{n=1}^B \big(\boldsymbol{r}(n) - \boldsymbol{X}(n)\boldsymbol{F}\boldsymbol{h}\big)^H \boldsymbol{\Phi}^{-1} \big(\boldsymbol{r}(n) - \boldsymbol{X}(n)\boldsymbol{F}\boldsymbol{h}\big)\Big), \quad (11)$$

and $\boldsymbol{\Phi} \triangleq \mathrm{diag}(\boldsymbol{\sigma})$. In (10), $f\big(\{\boldsymbol{x}(n)\}_{n=1}^B \big| \boldsymbol{h}, \boldsymbol{\sigma}\big)$ is not related to the parameters $\boldsymbol{h}$ and $\boldsymbol{\sigma}$; therefore, it can be omitted when we calculate (9). Eq. (11) is substituted into (9) to obtain (12) on the top of next page.

To determined (12), the following quantities are needed.

- We define

  $$\bar{\boldsymbol{X}}^{[m]}(n) = \mathbb{E}\big\{\boldsymbol{X}(n) \big| \{\boldsymbol{r}(n)\}_{n=1}^B, \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big\} = \mathrm{diag}(\bar{x}_0^{[m]}(n), \cdots, \bar{x}_c^{[m]}(n), \cdots, \bar{x}_{N-1}^{[m]}(n)), \quad (13)$$

  where $\bar{x}_c(n)$ is determined based on the LLR information from the channel decoder as follows

  $$\bar{x}_c(n) = \tanh\left(\frac{\hat{\theta}_c^{[m]}(n)}{2}\right). \quad (14)$$

- We observe that

  $$\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n) = \boldsymbol{X}^H(n)\boldsymbol{X}(n)\boldsymbol{\Phi}^{-1} = \boldsymbol{\Phi}^{-1}. \quad (15)$$

  Therefore, we obtain

  $$\sum_{n=1}^B \mathbb{E}\big\{\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n) \big| \{\boldsymbol{r}(n)\}_{n=1}^B, \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big\} = B\boldsymbol{\Phi}^{-1}. \quad (16)$$

Based on (14) and (16), (12) becomes

$$Q\big(\boldsymbol{h}, \boldsymbol{\sigma} \big| \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big) = \sum_{c=0}^{N-1} \sigma_c - \sum_{n=1}^{N} \boldsymbol{r}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n) + \sum_{n=1}^B \boldsymbol{r}^H(n)\boldsymbol{\Phi}^{-1}\bar{\boldsymbol{X}}^{[m]}(n)\boldsymbol{F}\boldsymbol{h} - B\boldsymbol{h}^H\boldsymbol{F}^H\boldsymbol{\Phi}^{-1}\boldsymbol{F}\boldsymbol{h} + \sum_{n=1}^B \boldsymbol{h}^H\boldsymbol{F}^H(\bar{\boldsymbol{X}}^{[m]}(n))^H\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n). \quad (17)$$

2) CM-step: This step is to calculate the updated values $\boldsymbol{h}$ and $\boldsymbol{\sigma}$, i.e., to find $\hat{\boldsymbol{h}}^{[m+1]}$ and $\hat{\boldsymbol{\sigma}}^{[m+1]}$. Those values are determined by the following two steps.

a) Step 1: In this smaller step, $\hat{\boldsymbol{\sigma}}^{[m+1]}$ is found by the following equation

$$\hat{\boldsymbol{\sigma}}^{[m+1]} = \arg\max_{\boldsymbol{\sigma}} \; Q\big(\hat{\boldsymbol{h}}^{[m]}, \boldsymbol{\sigma} \big| \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big). \quad (18)$$

We have

$$Q\big(\hat{\boldsymbol{h}}^{[m]}, \boldsymbol{\sigma} \big| \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big) = \sum_{c=0}^{N-1} \sigma_c - \sum_{n=1}^B \sum_{c=0}^{N-1} \frac{|r_c(n)|^2}{\sigma_c} + 2\sum_{n=1}^B \sum_{c=0}^{N-1} \frac{\mathrm{Re}\big(r_c^*(n)\bar{x}_c^{[m]}(n)\hat{H}_c^{[m]}\big)}{\sigma_c} - B\sum_{c=0}^{N-1} \frac{|\hat{H}_c^{[m]}|}{\sigma_c}. \quad (19)$$

Based on (19) the value $\hat{\sigma}_c^{[m+1]}$, $c = 0, 1, \cdots, N-1$, is determined by

$$\hat{\sigma}_c^{[m+1]} = \frac{1}{B}\sum_{n=1}^B \Big(|r_c(n)|^2 - 2\mathrm{Re}\big(r_c^*\bar{x}_c^{[m]}(n)\hat{H}_c^{[m]}\big) + |\hat{H}_c^{[m]}|^2\Big). \quad (20)$$

b) Step 2: In this smaller step we determine $\hat{\boldsymbol{h}}^{[m+1]}$ by the following equation

$$\hat{\boldsymbol{h}}^{[m+1]} = \arg\max_{\boldsymbol{h}} \; Q\big(\boldsymbol{h}, \hat{\boldsymbol{\sigma}}^{[m+1]} \big| \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big) \quad (21)$$

in which

$$Q\big(\boldsymbol{h}, \hat{\boldsymbol{\sigma}}^{[m+1]} \big| \hat{\boldsymbol{h}}^{[m]}, \hat{\boldsymbol{\sigma}}^{[m]}\big) = \sum_{c=0}^{N-1} \hat{\sigma}_c^{[m+1]} - \sum_{n=1}^{N} \boldsymbol{r}^H(n)(\hat{\boldsymbol{\Phi}}^{[m+1]})^{-1}\boldsymbol{r}(n) + \sum_{n=1}^B \boldsymbol{r}^H(n)(\hat{\boldsymbol{\Phi}}^{[m+1]})^{-1}\bar{\boldsymbol{X}}^{[m]}(n)\boldsymbol{F}\boldsymbol{h} + \sum_{n=1}^B \boldsymbol{h}^H\boldsymbol{F}^H(\bar{\boldsymbol{X}}^{[m]}(n))^H(\hat{\boldsymbol{\Phi}}^{[m+1]})^{-1}\boldsymbol{r}(n) - B\boldsymbol{h}^H\boldsymbol{F}^H(\hat{\boldsymbol{\Phi}}^{[m+1]})^{-1}\boldsymbol{F}\boldsymbol{h}, \quad (22)$$

and $\hat{\boldsymbol{\Phi}}^{[m+1]} = \mathrm{diag}\big(\hat{\sigma}_0^{[m+1]}, \cdots, \hat{\sigma}_c^{[m+1]}, \cdots, \hat{\sigma}_{N-1}^{[m+1]}\big)$. We differentiate (22) with respect to $\boldsymbol{h}$ [14] and equate the result to $\mathbf{0}$ to have

$$\hat{\boldsymbol{h}}^{[m+1]} = \frac{1}{B}\big(\boldsymbol{F}(\hat{\boldsymbol{\Phi}}^{[m+1]})^{-1}\boldsymbol{F}\big)^{-1} \times \sum_{n=1}^B \big(\bar{\boldsymbol{X}}^{[m]}(n)\big)^H(\hat{\boldsymbol{\Phi}}^{[m+1]})^{-1}\boldsymbol{r}(n). \quad (23)$$

$$Q(\boldsymbol{h},\boldsymbol{\sigma}|\hat{\boldsymbol{h}}^{[m]}\,\hat{\boldsymbol{\sigma}}^{[m]}) = \sum_{c=0}^{N-1}\sigma_c - \sum_{n=1}^{B}\boldsymbol{r}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n) + \sum_{n=1}^{B}\boldsymbol{r}^H(n)\boldsymbol{\Phi}^{-1}\mathbb{E}\{\boldsymbol{X}(n)|\{\boldsymbol{r}(n)\}_{n=1}^{B},\hat{\boldsymbol{h}}^{[m]},\hat{\boldsymbol{\sigma}}^{[m]}\}\boldsymbol{F}\boldsymbol{h}$$
$$+\sum_{n=1}^{B}\boldsymbol{h}^H\boldsymbol{F}^H(\mathbb{E}\{\boldsymbol{X}^H(n)|\{\boldsymbol{r}(n)\}_{n=1}^{B},\hat{\boldsymbol{h}}^{[m]},\hat{\boldsymbol{\sigma}}^{[m]}\})^H\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n)$$
$$-\boldsymbol{h}^H\boldsymbol{F}^H\Big(\sum_{n=1}^{B}\mathbb{E}\{\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n)|\{\boldsymbol{r}(n)\}_{n=1}^{B},\hat{\boldsymbol{h}}^{[m]},\hat{\boldsymbol{\sigma}}^{[m]}\}\Big)\boldsymbol{F}\boldsymbol{h} \qquad (12)$$

IV. Cramér-Rao Lower Bound (CRLB) Derivations

Let $\boldsymbol{h}_R$ and $\boldsymbol{h}_I$ be the real and imaginary parts of $\boldsymbol{h}$, respectively. To gather all information of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$, we define $\boldsymbol{\theta} \triangleq [\boldsymbol{h}_R^T\ \boldsymbol{h}_T^T\ \boldsymbol{\sigma}^T]^T$. The Fischer information matrix of $\boldsymbol{\theta}$ is determined by

$$\boldsymbol{I}_{\boldsymbol{\theta}} = \mathbb{E}\Big\{\frac{\partial \log f\big(\{\boldsymbol{r}(n)\}_{n=1}^{B}|\boldsymbol{\theta}\big)}{\partial\boldsymbol{\theta}} \Big(\frac{\partial \log f\big(\{\boldsymbol{r}(n)\}_{n=1}^{B}|\boldsymbol{\theta}\big)}{\partial\boldsymbol{\theta}}\Big)^T\Big\}, \qquad (24)$$

where $f\big(\{\boldsymbol{r}(n)\}_{n=1}^{B}|\boldsymbol{\theta}\big)$ is given in (25)
After long derivations we have

$$\boldsymbol{I}_{\boldsymbol{\theta}} = \begin{bmatrix}\boldsymbol{I}_{\boldsymbol{h}} & \boldsymbol{0}_{L\times N}\\ \boldsymbol{0}_{N\times L} & \boldsymbol{I}_{\boldsymbol{\sigma}}\end{bmatrix}, \qquad (26)$$

where $\boldsymbol{I}_{\boldsymbol{h}}$ is given in (27) and

$$[\boldsymbol{I}_{\boldsymbol{\sigma}}]_{c_1,c_2} = \begin{cases}\frac{B}{\sigma_{c_1}^2} & \text{if } c_1=c_2\\ 0 & \text{otherwise}\end{cases}. \qquad (28)$$

Therefore, we have the CRLBs of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$ as follows

$$\mathrm{CRB}(\boldsymbol{h}) = \mathrm{trace}(\boldsymbol{I}_{\boldsymbol{h}}^{-1}), \qquad (29)$$

$$\mathrm{CRB}(\boldsymbol{\sigma}) = \mathrm{trace}(\boldsymbol{I}_{\boldsymbol{\sigma}}^{-1}) = \frac{\sum_{c=0}^{N-1}\sigma_c^2}{B}. \qquad (30)$$

V. Simulation Results

In this section we present some simulation results for a system which has the following parameters. The number of sub-carriers is $N = 128$. The channel impulse response has length $L = 7$ and uniform power delay profile is used. we assume that the channel impulse response is static over $B = 20$ OFDM symbols. The transmitter uses a recursive convolutional code generator with half rate, memory of 2 and generator $[5\ 7]$. The first OFDM symbol carries pilot signals.

In our proposed receiver we need to have initial values of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$, i.e. $\hat{\boldsymbol{h}}^{[0]}$ and $\hat{\sigma}_c^{[0]}$, $c = 0,1,\cdots,N-1$, to start our iterative algorithm. The cth element of the FFT-transformed of $\hat{\boldsymbol{h}}^{[0]}$ can be found by

$$\hat{H}^{[0]}(c) = \frac{r_c(0)}{x_c(0)}, \qquad (31)$$

where $x_c(0)$ is the pilot signal on the cth subcarrier. If we define $\hat{\boldsymbol{\mathcal{H}}}^{[0]} = [\hat{H}^{[0]}(0)\ \cdots\ \hat{H}^{[0]}(c)\ \cdots \hat{H}^{[0]}(N-1)]^T$ then the initial value $\hat{\boldsymbol{h}}^{[0]}$ is determined by

$$\hat{\boldsymbol{h}}^{[0]} = \boldsymbol{F}^{\dagger}\hat{\boldsymbol{\mathcal{H}}}^{[0]}, \qquad (32)$$

where $\boldsymbol{F}^{\dagger}$ is the MoorePenrose pseudoinverse of $\boldsymbol{F}$. The initial value $\hat{\sigma}_c^{[0]}$ is calculated by $\hat{\sigma}_c^{(0)} = \frac{1}{B}\sum_{n=1}^{B}|r_c(n)|^2$. The signal-to-noise ratio is defined as $\mathrm{SNR} = 1/N_0$ and the signal-to-interference ratio (SIR) is defined as $\mathrm{SIR} = 1/N_1$.

We first consider the case that 16 consecutive sub-carriers of the system are under interference. Fig. 2 illustrates the BER of the system in case $\mathrm{SIR} = 0$dB. The "Initial" curve is the BER if $\hat{\boldsymbol{h}}^{(0)}$ and $\hat{\boldsymbol{\sigma}}^{(0)}$ are used at the decoder. We observe a huge improvement with only 1 iteration of our proposed algorithm. The second or third iteration only has a small improvement. It is observed from Fig. 2 that the performance of our proposed algorithm with 3 iterations is very close to the performance of the decoder using perfect channel information, i.e., perfect $\boldsymbol{h}$ and $\boldsymbol{\sigma}$.

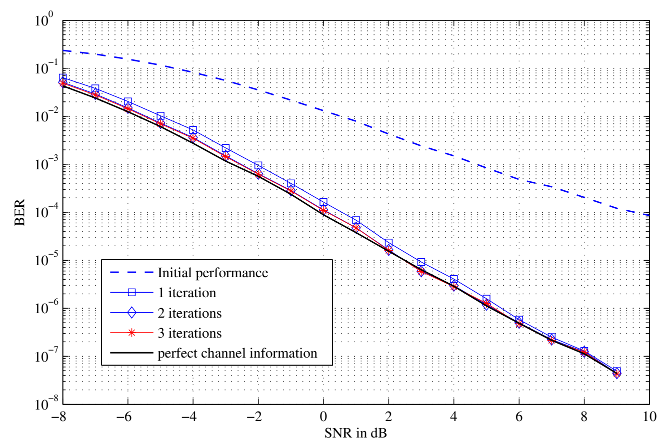

Fig. 2. BER performance of a system with $N = 128$ carriers, 16 carriers under interference and $\mathrm{SIR} = 0$dB

Obtaining the excellent BER performance above is due to ability of the ECM-based estimator in providing good quality of channel information, i.e., $\boldsymbol{h}$ and $\boldsymbol{\sigma}$. Fig. 3 provides the mean-square-error (MSE) performance of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$. Compared with the performance given by pilot symbols, our algorithm greatly improve the qualities of estimated parameters.

We then consider the system at $\mathrm{SIR} = 0$dB and $\mathrm{SNR} = 4$dB but the number of sub-carrier(s) under interference varies. Fig. 4 illustrates the BER as the function of number of subcarrier(s). We observe that our algorithm can reach very near to the performance of perfect channel information case

$$f\left(\{\boldsymbol{r}(n)\}_{n=1}^{B}|\boldsymbol{\theta}\right) = \frac{1}{(\pi^N \prod_{c=0}^{N-1} \sigma_c)^B} \exp\Big\{-\sum_{n=1}^{B} \big(\boldsymbol{r}(n) - \boldsymbol{X}(n)\boldsymbol{F}\boldsymbol{h}\big)^H \boldsymbol{\Phi}^{-1} \big(\boldsymbol{r}(n) - \boldsymbol{X}(n)\boldsymbol{F}\boldsymbol{h}\big)\Big\} \tag{25}$$

$$\begin{aligned}\boldsymbol{I_h} &= \begin{bmatrix} 2\Re\big(\boldsymbol{F}^H\big(\sum_{n=1}^{B}\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n)\big)\boldsymbol{F}\big) & -2\Im\big(\boldsymbol{F}^H\big(\sum_{n=1}^{B}\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n)\big)\boldsymbol{F}\big) \\ 2\Im\big(\boldsymbol{F}^H\big(\sum_{n=1}^{B}\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n)\big)\boldsymbol{F}\big) & 2\Re\big(\boldsymbol{F}^H\big(\sum_{n=1}^{B}\boldsymbol{X}^H(n)\boldsymbol{\Phi}^{-1}\boldsymbol{X}(n)\big)\boldsymbol{F}\big) \end{bmatrix} \\ &= 2B \begin{bmatrix} \Re\big(\boldsymbol{F}^H\boldsymbol{\Phi}^{-1}\boldsymbol{F}\big) & -\Im\big(\boldsymbol{F}^H\boldsymbol{\Phi}^{-1}\boldsymbol{F}\big) \\ \Im\big(\boldsymbol{F}^H\boldsymbol{\Phi}^{-1}\boldsymbol{F}\big) & \Re\big(\boldsymbol{F}^H\boldsymbol{\Phi}^{-1}\boldsymbol{F}\big) \end{bmatrix}. \end{aligned} \tag{27}$$

with just a few iterations.

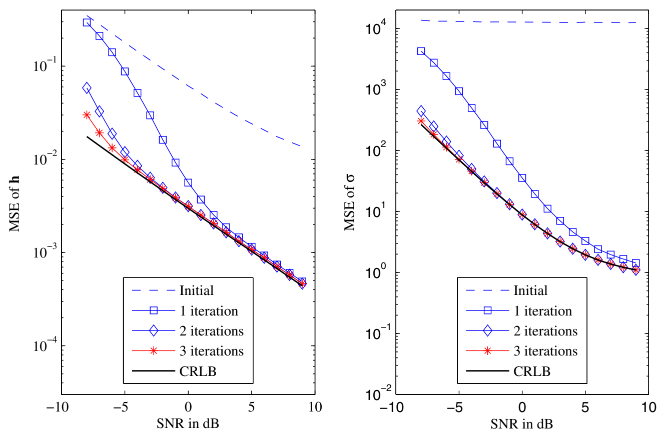

Fig. 3. MSE performance of $\boldsymbol{h}$ and $\boldsymbol{\sigma}$ in a system with $N = 128$ carriers, 16 carriers under interference and $\mathrm{SIR} = 0\mathrm{dB}$

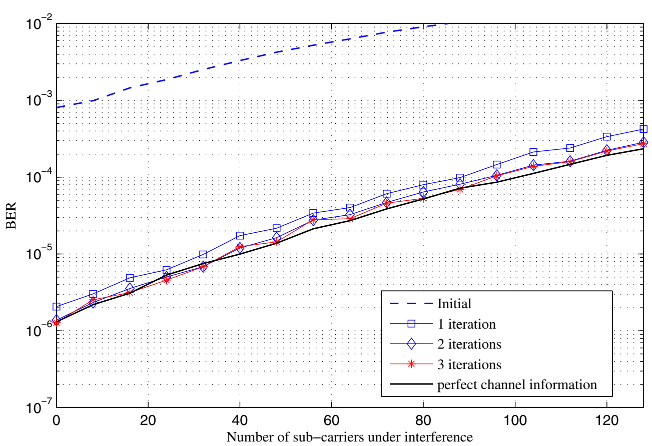

Fig. 4. BER performance as a function of subcarrier(s) under interference in a system with $N = 128$ carriers, $\mathrm{SIR} = 0\mathrm{dB}$ and $\mathrm{SNR} = 4\mathrm{dB}$

VI. Conclusions

In this paper we considered an OFDM-based system under unknown NBI. We proposed an iterative receiver to jointly estimate the necessary channel information and detect the transmitted signals. The iterative receiver consists of the channel information estimator and data decoder. The estimator, an application of the ECM algorithm, provides better estimates of channel information to boost the results of the detection process. In turn, better detected signals are used to improve the quality of estimated parameters. The simulation results have shown that our proposed iterative receiver has excellent performance in terms of BER and MSE.

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