Approaching the Maximum Likelihood Performance with Nonlinearly Distorted OFDM Signals

João Guerreiro[(1)], Rui Dinis[(1,2)] and Paulo Montezuma[(1,3)]
[(1)] DEE, FCT Universidade Nova de Lisboa, Monte de Caparica, Portugal
[(2)] IT, Instituto de Telecomunicações, Lisboa, Portugal.
[(3)] UNINOVA, Monte de Caparica, Portugal

Abstract - The high envelope fluctuations of OFDM signals (Orthogonal Frequency Division Multiplexing) make them very prone to nonlinear distortion effects. In typical OFDM implementations the nonlinear distortion component is regarded as a noise-like term that leads to performance degradation. To achieve optimum performance we should employ a ML (Maximum Likelihood) receiver where we take into account all the information associated to the transmitted signals that is in the nonlinearly-distorted signal. Although the performance of an ML is substantially better than traditional receivers, its complexity is prohibitively high.

In this paper we consider nonlinearly distorted OFDM schemes and we present sub-optimum receivers that try to approach the ML performance. It is shown that, contrarily to what was expectable, the ML performance with nonlinear transmitters can be better than with ideal, linear transmitters. Our suboptimal receivers allow remarkable performance improvements, being able to reduce significantly the gap between the ML performance and the performance of typical OFDM receivers.[1]

Keywords: OFDM signals, nonlinear distortion effects, maximum likelihood receiver

I. Introduction

It is widely recognized that OFDM schemes (Orthogonal Frequency Division Multiplexing) [1] are excellent candidates for broadband wireless communications. This is mainly due to their good performance over severely time-dispersive channels without the need for complex receiver implementations [2]. However, OFDM signals have an important drawback: their high envelope fluctuations, make them susceptible to nonlinear distortion effects. For this reason, several methods have been proposed to reduce the envelope fluctuations of OFDM signals, from especially designed codes [3] to the use of multiple signal representations [4], [5]. However, the simpler and most promising techniques to reduce the envelope fluctuations of OFDM signals involve the use of nonlinear clipping operations [6], [7], [8], [9], [10]. However, this means that we will have nonlinear distortion effects on the transmitted OFDM signals. In fact, since OFDM signals with high number of subcarriers have a Gaussian-like nature, the nonlinearly-distorted signal can be decomposed as the sum of two uncorrelated components: an useful component, proportional to the original OFDM signal, and a distortion component [11]. Since conventional receiver implementations treat the nonlinear distortion component as an additional noise-like term, we can have significant performance degradation. To improve the performance we can use iterative receivers where we estimate and cancel nonlinear distortion effects [12], [13]. However, the nonlinear distortion component has information concerning the transmitted signal that can be used to improve the performance [14].

The optimum receiver is an ML (Maximum Likelihood) receiver where we compare the received signal with all possible transmitted signals and to select the data estimates associated to the transmitted signal that has smaller Euclidean distance relatively to the received signal. This means that the optimum receiver should take into account not just the useful component but also the information inherent to the nonlinear distortion component. The major problem associated to an ML receiver is its complexity.

In this paper we consider strong nonlinear effects in OFDM and we study the performance of ML receivers. It is shown that, contrarily to what one could expect, the optimum performance of nonlinearly distorted OFDM can be better than with an ideal, linear transmitter. We also present and evaluate several suboptimal ML-based receivers that try to approach the ML performance.

This paper is organized as follows: an overview of the analytical characterization of nonlinearly distorted OFDM signals is made in Sec. II and Sec. III concerns the ML (Maximum Likelihood) behavior for OFDM signals with nonlinear distortion effects. Sec. IV presents several suboptimal ML-based receiver and V has some performance results for those receivers. Finally, Sec. VI concludes this paper.

II. Nonlinear Effects in OFDM Signals

In this paper we consider the use of clipping and filtering techniques to reduce the envelope fluctuations of OFDM signals [10]. However, our results could be easily extended to other nonlinear characteristics. The transmitter structure is described in fig. 1.

[1]This work was partially supported by the FCT (Fundao para a Cincia e a Tecnologia) under projects PEst-OE/EEI/LA0008/2011, ADCOD PTDC/EEA - TEL/099973/2008 and MPSat PTDC/EEA-TEL/099074/2008.

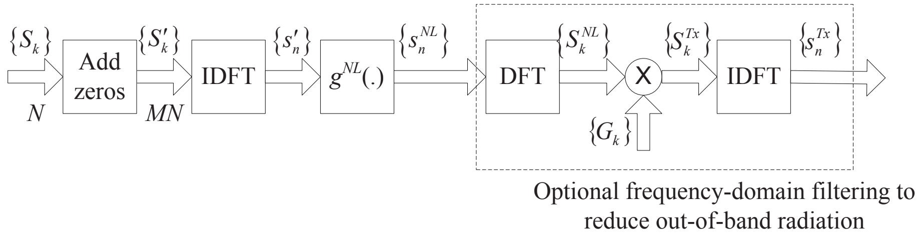

Fig. 1. Transmitter structure.

The frequency-domain block to be transmitted is $\{S_k; k = 0, 1, \ldots, N-1\}$, where S_k is the data symbol associated to the kth subcarrier. An augmented block $\{S'_k; k = 0, 1, \ldots, MN-1\}$ is formed by adding $(M-1)N/2$ idle subcarriers and we compute its IDFT, leading to the block of time-domain samples $\{s'_n; n = 0, 1, \ldots, MN-1\}$. This can be regarded as an oversampled version of the original OFDM block with the oversampling factor M (typically an oversampling factor $M = 4$ is enough to avoid aliasing effects in the nonlinearly-distorted signal). The oversampled time-domain samples are submitted to a nonlinear device leading to the samples

$$s_n^{NL} = g^{NL}(|s'_n|)\exp(j\arg(s'_n)). \quad (1)$$

In this paper we consider an ideal envelope clipping with normalized clipping level s_M/σ, which means that

$$g^{NL}(R) = \begin{cases} R, & R \leq s_M \\ s_M, & R > s_M \end{cases}, \quad (2)$$

with $R = |s'_n|$.

The frequency-domain block associated to the nonlinearly distorted signal is $\{S_k^{NL}; k = 0, 1, \ldots, MN-1\}$ = DFT $\{s_n^{NL}; n = 0, 1, \ldots, MN-1\}$ and we can have a subsequent frequency-domain filtering operation to reduce the out-of-band radiation levels inherent to the nonlinear operation can also be included (the dashed block in fig. 1). In this case, the frequency-domain samples S_k^{NL} are replaced by the samples $S_k^{Tx} = G_k S_k^{NL}$, with the filtering coefficients G_k usually selected to be $G_k = 1$ for the N in-band subcarriers and $G_k = 0$ for the remaining $N(M-1)$ out-of-band subcarriers.

If the number of active subcarriers N is high then the OFDM signal has a Gaussian-like nature. In this case, it can be shown that

$$s_n^{NL} = \alpha s'_n + d_n\ , \quad (3)$$

where the distortion component d_n has zero mean,

$$E[s'_n d^*_{n'}] = 0 \quad (4)$$

and

$$\alpha = \frac{E[s_n^{NL} s_n^{'*}]}{E[|s'_n|^2]} = \frac{E[R g_{NL}(R)]}{E[R^2]}, \quad (5)$$

with $R = |s'_n|$.

and the autocorrelation of the distortion component can be computed as described in [10]. Consequently, the frequency-domain block to be transmitted $\{S_k^{NL}; k = 0, 1, \ldots, MN-1\}$ can also be decomposed into useful and nonlinear self-interference components, i.e.,

$$S_k^{NL} = \alpha S'_k + D_k. \quad (6)$$

Similarly, the frequency-domain block to be transmitted when we have a subsequent frequency-domain filtering operation is

$$S_k^{Tx} = \alpha S'_k + G_k D_k, \quad (7)$$

where $\{D_k; k = 0, 1, \ldots, ; MN-1\}$ is the DFT of $\{d_n; n = 0, 1, \ldots, MN-1\}$. The statistical characterization of D_k is made in [10], and it can be shown that it is approximately Gaussian with zero mean.

III. Performance of ML Receivers

In conventional OFDM receiver implementations this nonlinear distortion component is regarded as an additional noise component that leads to performance degradation. However, it should be noted that the distortion component $\{D_k; k = 0, 1, \ldots, MN-1\}$ has information on the transmitted block that could be used to improve the performance. To take advantage of the potential information inherent to the nonlinear distortion component we need to consider an ideal ML (Maximum Likelihood) receiver. For the sake of simplicity, we consider an ideal AWGN (Additive White Gaussian Noise) channel (the extension to other cases is straightforward). The received frequency-domain block will be

$$Y_k = S_k^{Tx} + N_k = G_k \alpha S'_k + G_k D_k + N_k, \quad (8)$$

where N_k denotes the channel noise component.

Without loss of generality we focus our analysis on the frequency-domain samples. We define the the average bit energy as[2]

$$E_b \triangleq \frac{1}{2N} \sum_{k=0}^{MN-1} E[|S_k^{Tx}|^2] =$$

$$\frac{1}{2N} \sum_{k=0}^{MN-1} G_k^2 \left(|\alpha E[|S_k^{'}|^2] + E[|D_k|^2] \right) =$$

$$= \frac{1}{2N} \sum_{k \in \Psi_G} \left(|\alpha E[|S_k^{'}|^2] + E[|D_k|^2] \right), \quad (9)$$

[2]Naturally, the average bit energy is related to the time domain samples, but since $\sum_{k=0}^{N-1} |X_k|^2 = N^2 \sum_{n=0}^{N-1} |x_n|^2$ when $\{X_k; k = 0, 1, \ldots, N-1\}$ denotes the DFT of the block $\{x_n; n = 0, 1, \ldots, N-1\}$, the SNR and other related parameters are identical in the time domain and frequency domain domains.

with Ψ_G denoting the set of subcarriers where $G_k = 1$ (i.e., the in-band subcarriers, when we employ the frequency-domain filtering operation after the clipping operation. It should be noted that $E_b = 1$ for conventional OFDM signals with a linear transmitter and normalized QPSK constellations with $S_k = \pm 1 \pm j$; for clipped OFDM signals with normalized clipping level s_M/σ we have

$$E_b = 1 - \exp\left(-\frac{s_M^2}{2\sigma^2}\right) < 1 \tag{10}$$

in general and $E_b << 1$ for very small clipping levels.

To understand how one can take advantage of the nonlinear distortion to improve the performance let us consider an OFDM signal with $N = 64$ subcarriers and a QPSK constellation. The normalized clipping level is $s_M/\sigma = 0.5$, which corresponds to a nonlinear device that introduces severe distortion. Fig. 2 shows the absolute value of the transmitted signal $\{S_k^{NL(1)} = \alpha S_k^{'(1)} + D_k^{(1)}; k = 0, 1, ..., MN-1\}$ associated to a given sequence $\{S_k^{(1)}; k = 0, 1, ..., N-1\}$ and the signal $\{S_k^{NL(2)} = \alpha S_k^{'(2)} + D_k^{(2)}; k = 0, 1, ..., MN-1\}$ associated to the sequence $\{S_k^{(2)}; k = 0, 1, ..., N-1\}$ which is identical to the sequence $\{S_k^{(1)}; k = 0, 1, ..., N-1\}$, in all bits except one bit at the subcarrier with index 0, as well as the difference between the two signals $S_k^{NL(1)} - S_k^{NL(2)} = \alpha S_k^{'(1)} + D_k^{(1)} - \alpha S_k^{'(2)} - D_k^{(2)}$. Clearly, the signals differ in almost all frequencies (in-band and out-of-band), not only the frequency where we modified the bit. In fact, the Euclidean distance between these two sequences (also defined in the frequency domain) is

$$D^2 = \sum_k |S_k^{NL(1)} - S_k^{NL(2)}|^2 =$$
$$= \sum_{k \in \Psi_G} |\alpha S_k^{'(1)} + D_k^{(1)} - \alpha S_k^{'(2)} - D_k^{(2)}|^2 \approx 7.7 E_b > 4E_b, \tag{11}$$

Since these values are typical for that clipping level, we can expect an asymptotic gain of about 2.7dB relatively to the linear case (when the out-of-band radiation is completely filtered, as in [10], the asymptotic gain becomes around 2.2dB, which is only slightly lower). Since these sequences differ in many subcarriers there is an intrinsic diversity effect that could be employed to improve the performance in fading channels.

To take advantage of this we should employ an ideal ML receiver that selects $\{\hat{S}'_k; k = 0, 1, \ldots, MN-1\}$ (and, inherently, $\{\hat{S}_k; k = 0, 1, \ldots, N-1\}$) that minimizes

$$J = \sum_{k \in \Psi_G} |Y_k - S_k^{NL}|^2 = \sum_{k \in \Psi_G} |Y_k - \alpha S'_k - D_k|^2, \tag{12}$$

The asymptotic performance of an ideal ML receiver is conditioned by the relation between the average bit energy, E_b, and the minimum Euclidean distance between two data sequences,

$$D_{\min}^2 \triangleq \min_{\{S_k^{(i)}\} \neq \{S_k^{(i')}\}} D_{i,i'}^2 =$$
$$= \sum_k |\alpha S_k^{'(1)} + D_k^{(1)} - \alpha S_k^{'(2)} - D_k^{(2)}|^2, \tag{13}$$

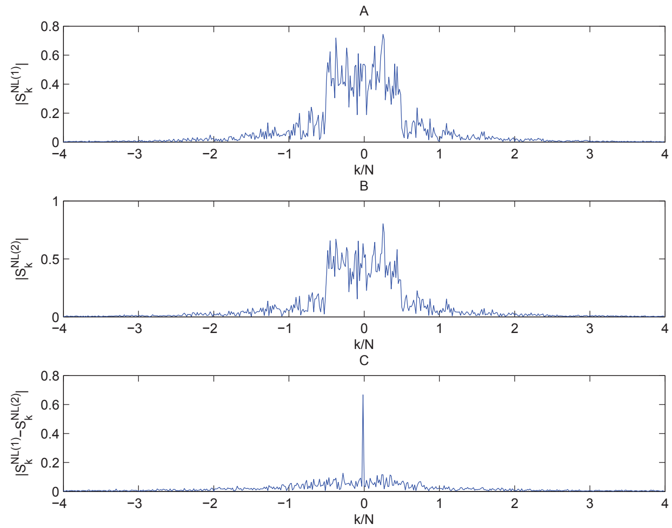

Fig. 2. Absolute value of the transmitted frequency-domain signal for two OFDM signals differing in a single bit that are submitted to an ideal envelope clipping with normalized clipping level $s_M/\sigma = 0.5$ ((A) and (B)) and the absolute value of the corresponding difference (C).

where $\{S_k^{'(i)}; k = 0, 1, ..., MN-1\}$ is the augmented block associated to $\{S_k^{(i)}; k = 0, 1, ..., N-1\}$ and $\{D_k^{(i)}; k = 0, 1, ..., MN-1\}$ the corresponding nonlinear distortion ($i = 1$ or 2). Naturally, for a linear transmission we have

$$D_{\min}^2 = 4E_b. \tag{14}$$

Therefore, one can define the asymptotic gain (or degradation)

$$G = \frac{D_{\min}^2}{4E_b}. \tag{15}$$

Although there are fluctuations on the gain G, it is almost always higher than 1 (the value for a linear transmitter), especially when we have strong nonlinear distortion effects. This means that the performance can be better that the one with a linear transmitter. The reason for this is that the nonlinear distortion component should be considered as an additional information component, instead of an undesirable noise-like component. To confirm this we obtained the approximate BER performance of nonlinear OFDM schemes with an ideal ML receiver. Since the Euclidean distance between nonlinearly distorted signals associated to data sequences differing in more than one bit is much higher than for sequences that differ in only one bit, the BER of an ML receiver was approximated by

$$BER \approx E_{\{S_k^{(1)}\}}\left[\frac{1}{2N} \sum_{\{S_k^{(2)}\} \in \Phi_1(\{S_k^{(1)}\})} Q\left(\sqrt{\frac{D_{1,2}^2/2}{N_0}}\right)\right], \tag{16}$$

where $\Phi_1(\{S_k^{(1)}\})$ denotes the set of sequences that differ from $\{S_k^{(1)}\}$ in only 1 bit (clearly, $\#\Phi_1(\{S_k^{(i)}\}) = 2N$, since there are $2N$ bits in N QPSK symbols). Naturally, for a conventional linear OFDM transmitter we have $D_{1,2}^2 = 4E_b$, leading to the well-known BER expression

$$BER \approx Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad (17)$$

Fig. 3 shows the approximate ML BER performance for clipped OFDM schemes, with or without the subsequent frequency-domain filtering procedure (the ideal performance of conventional linear OFDM was included for the sake of comparisons). From this figure, it is clear that the ideal ML performance of nonlinear OFDM is better than the performance of conventional linear OFDM schemes, with gains between 1 and 2dB, that are higher for lower clipping levels (i.e., stronger nonlinear distortion effects). As expected, the filtering operation (employed to eliminate out-of-band radiation levels) leads to elimination of some nonlinear distortion terms, reducing the Euclidean distance between error events and the achievable BER performance gain when we have nonlinear OFDM with an ML receiver.

It should be pointed out that our analysis considers the transmission of ”typical” sequences and the conclusions can be substantially different for some rare sequences such as sequences where S_k is constant. However, since these sequences are very rare (and, in fact, they can be avoided through the use of suitable scrambling procedures), its effect on the overall performance can be neglected.

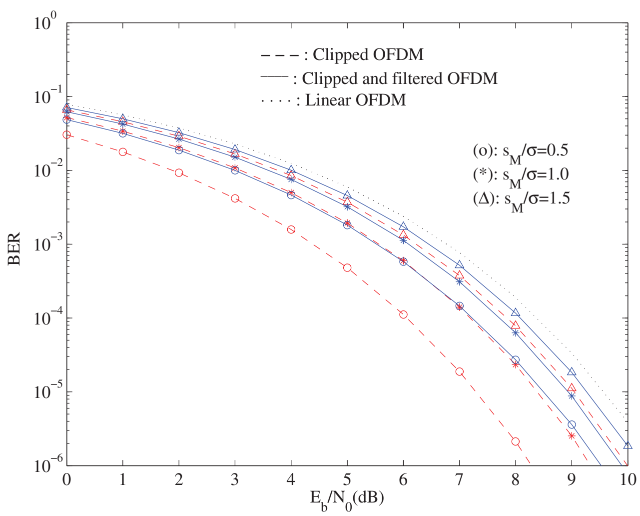

Fig. 3. BER performance of ML receivers.

IV. Suboptimal ML-Based Receivers

Clearly, it is not possible to compare the received signal with all possible transmitted signals, even for a moderate number of subcarriers N and small constellations. For instance, if we consider $N = 64$ and a QPSK constellation need to test $4^N = 2^{128}$ possible combinations. For this reason, in this section we present several suboptimal methods to approach the ML receiver performance. For all these methods we start with the estimated signal associated to a conventional OFDM receiver (which will be denoted *hard decision sequence* in the remaining of the paper) and preform variations of their bits, obtain the corresponding nonlinear signal to be transmitted (i.e., the corresponding signal is submitted to the clipping operation and subsequent frequency-domain filtering that was employed in the receiver), compute the corresponding Euclidean distance relatively to the received sequence and choose the sequence among the original hard decision sequence and all variations of it that were tested that has smaller Euclidean distance to the received signal. The motivation for our techniques is that usually the optimal ML sequence differs in a small number of bits of the *hard decision sequence*. Therefore, we could test only a small fraction of all possible sequences and still have the optimal ML sequence among them. If the ML sequence is among the sequences that we tested than we obtain the optimal performance.

Method I

Starting with the *hard decision sequence* we select the L bits with smaller reliability (i.e., the bits where the corresponding signal is closer to the decision threshold) and perform all 2^L possible variations among those bits.

Method II

In this method we switch each one of the $2N$ bits of the *hard decision sequence* to obtain $2N$ variations of it.

Method III

In this method we start with the *hard decision sequence* and switch the first bit. If the Euclidean distance relatively to the received sequence improves the bit remains changed, if not we return to the original bit. Next we proceed to the second bit and do the same. This procedure is repeated until we reach the last bit. After this we end up with a sequence that has Euclidean distance relatively to the received sequence that is smaller (or at least equal) to the distance from the *hard decision sequence* to the received signal. Since some of the bits might be changed with this procedure we can restart changing the first bit and repeat the procedure K times.

V. Performance Results

In this section we present the BER performance for the suboptimal ML-based methods described above. The OFDM signal has $N = 64$ useful subcarriers with QPSK constellation and an oversampling factor $M = 4$. The QPSK symbols are selected from the data signal under a Gray mapping rule. The nonlinear device corresponds to an ideal envelope clipping with normalized clipping level $s_M/\sigma = 1$. We considered both the case where there is no subsequent frequency domain filtering and the case where the subsequent frequency-domain filter removes completely the out-of-band radiation created by the nonlinear device, i.e., $G_k = 0$ for all out-of-band subcarriers and $G_k = 1$ for the in-band subcarriers. We assumed an ideal AWGN channel and perfect synchronization at the receiver.

Fig. 4 shows the BER for Method I with different values of L. From this figure it is clear that we can improve significantly the performance relatively to conventional OFDM schemes, especially for larger values of L (i.e., when we modify a larger number of bits). The frequency-domain filtering after the clipping operation has only a minor effect on the performance of our receiver. However, for the relatively small values of L that lead to practical implementations the performance is always worse than the performance with a linear transmitter.

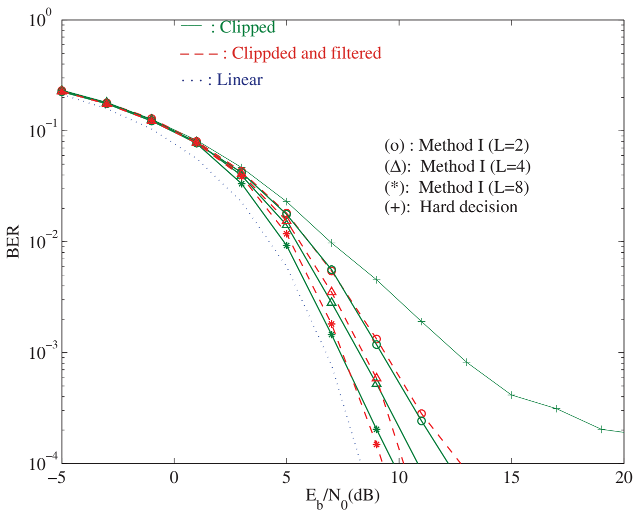

Fig. 4. BER performance for method I.

Fig. 5 shows the performance with methods II and III. Clearly, method II is not recommendable, but method III yields excellent performance, especially if we have 2 iterations (the gains with more than 2 iterations are negligible). Moreover, the BER values can be better than the ones with linear transmitters, even when we employ the filtering operation to eliminate completely the out-of-band radiation generated by the clipping.

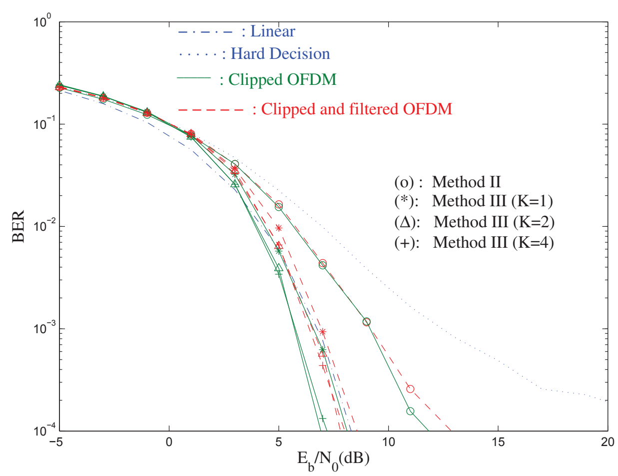

Fig. 5. BER performance for methods II and III.

VI. Conclusions

In this paper we considered nonlinearly distorted OFDM schemes and we present sub-optimum receivers that try to approach the ML performance. It was shown that, contrarily to what was expectable, the ML performance with nonlinear transmitters can be better than with ideal, linear transmitters. Our suboptimal receivers allowed remarkable performance improvements, being able to reduce significantly the gap between the ML performance and the performance of typical OFDM receivers.

References

[1] L.Cimini Jr., “Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing”, *IEEE Trans. on Comm.*, Vol. 33, No. 7, pp. 665–675, Jul. 1985.

[2] R.Nee and R.Prasad, *OFDM for Wireless Multimedia Communications*, Artech House, 2000.

[3] A.Jones and T.Wilkinson, ”Combined Coding for Error Control and Increased Robustness to System Nonlinearities in OFDM”, *IEEE VTC’96*, Atlanta, May 1996.

[4] S.Müller, R.Bräuml, R.Fischer and J.Huber, ”OFDM with Reduced Peak-to-Average Power Ratio by Multiple Signal Representation”, *Annales of Telecommunications*, Vol. 52, Feb. 1997.

[5] L.Cimini Jr. and N.Sollenberger, ”Peak-to-Average Power Reduction of an OFDM Signal Using Partial Transmit Sequences”, *IEEE Comm. Letters*, Nov. 1999. Jul. 1985.

[6] R.O’Neill and L.Lopes, ”Envelope Variations and Spectral Splatter in Clipped Multicarrier Signals”, *IEEE PIMRC’95*, Sep. 1995.

[7] X.Li and L.J.Cimini, Jr., ”Effects of Clipping and Filtering on the Performance of OFDM”, *IEEE Comm. Let.*, Vol. 2, No. 5, pp. 131–133, May 1998.

[8] H.Ochiai and H.Imai, ”Performance of Deliberate Clipping with Adaptive Symbol Selection for Strictly Band-Limited OFDM Systems”, *IEEE JSAC*, Vol. 18, No. 11, Nov. 2000.

[9] J.Armstrong, ”New OFDM Peak-to-Average Power Reduction Scheme”, IEEE VTC’2001(Spring), Rhodes, Greece, May 2001.

[10] R.Dinis and A.Gusmão, “A Class of Nonlinear Signal Processing Schemes for Bandwidth-Efficient OFDM Transmission with Low Envelope Fluctuation”, *IEEE Trans. on Comm.*, Vol. 52, No. 11, pp. 2009–2018, Nov. 2004.

[11] H.Rowe, “Memoryless Nonlinearities with Gaussian Input: Elementary Results”, *Bell System Tech. Journal*, Vol. 61, Sep. 1982.

[12] J.Tellado, L.Hoo and J.Cioffi, ”Maximum Likelihood Detection of Nonlinearly Distorted Multicarrier Symbols by Iterative Decoding”, *IEEE Trans. on Comm.*, Vol. 51, Feb. 2003.

[13] A. Gusmão, R. Dinis and P. Torres, “Low-PMEPR OFDM Transmission with an Iterative Receiver Technique for Cancelation of Nonlinear Distortion”, *IEEE VTC’05(Fall)*, Vol. 4, pp. 2367–2371, Sep. 2005.

[14] P.Carvalho, R.Dinis and M.Luzio, “Analytical Characterization of Nonlinearly Distorted TC-OQAM Signals”, *IEEE ICSPCS*, Omaha, USA, Sep. 2009.