Analytical Study of Multi-Antenna Relaying Systems in the Presence of Co-Channel Interference

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***Abstract*—The outage probability of relay networks with multiple antennas at the relay is analyzed. The relay uses maximal ratio combining to combine the received signals, while it uses maximal ratio transmission for the transmission. It is assumed that the reception at the relay and the destination are corrupted by multiple co-channel interfering signals. Closed-form outage probability expressions are derived when the relay is operated in the amplify-and-forward mode with fixed gain. Both cases of equal power and unequal power interferers are considered.**

I. Introduction

Relay assisted wireless communications has become a topic of great interest among wireless communications researchers during the past decade. It has been shown that relaying can be advantageous in achieving diversity and extending the coverage of a network. Several relaying protocols have been introduced and their performances have been studied extensively in the literature. A large proportion of the performance results available for relaying protocols has assumed that the only additive interference corrupting the received signal is additive white Gaussian noise (AWGN) at the relays and the destination. However, this will not be true for many practical wireless communication systems. The practice of frequency reuse to enhance the spectrum efficiency of wireless systems brings the harmful effect of co-channel interference to the system. Therefore, when one investigates the performance of a relaying protocol, it is crucial to include the effects of co-channel interference on the system. This is essential for conservative system design.

The performance of dual-hop relaying systems with interference has been investigated in several studies [1]–[7]. The outage performance of a dual-hop relaying system with reception at the destination limited by interference was investigated in [1]. Reference [2] studied the performance of a dual-hop relaying system with co-channel interference present only at the relay. The outage probability of a fixed gain dual-hop relaying system under the influence of interference at the relay and the destination is studied in [7]. The approximate performance of a dual-hop relaying system, where additive noise and multiple co-channel interference signals are present at both the relay and the destination, was considered in [3]. An extension of the results in [3] to the multihop relaying case is found in [8]. In all these previous works, it has been assumed that the nodes are equipped with a single antenna element.

A recent paper [9] proposed a relay network configuration where the infrastructure-based relay is equipped with multiple antennas, while the source and the destination are mobile single-antenna nodes. The performance of this system was investigated under the assumption that the only additive interference at the relay and the destination is AWGN.

In this paper, we investigate the performance of the system model introduced in [9], when the relay and the destination are affected by multiple co-channel interfering signals that are present in the network. Closed-form outage probability results are derived for different configurations of the multi-antenna relay network with interference. The results presented in this paper are valid for the case where the interfering signals have unequal powers and non-identical fading statistics. To the best of the authors' knowledge, there are no performance results available for relay networks with multi-antenna relays operating under the effects of co-channel interference.

The remainder of this paper is organized as follows. In Section II, we present the system model considered in this paper. Closed-form outage probability results for different scenarios are given in Section III. Numerical examples and simulation results are presented in Section IV, while Section V concludes this paper. The detailed derivations of the outage probability expressions are given in appendices.

II. System Model

The following notations will be used throughout this paper. Vectors are denoted using lower case bold letters and the $N \times N$ identity matrix is denoted $\mathbf{I}_N$. The Hermitian transpose of a vector or a matrix is denoted $(\cdot)^*$. The symbol $\mathbb{E}[\cdot]$ denotes mathematical expectation while $||\cdot||$ is used to denote the vector norm operation. The probability of an event A is denoted $\Pr\{A\}$. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X is denoted $f_X(x)$ and $F_X(x)$, respectively. A $N \times N$ diagonal matrix whose diagonal elements are $X_1, \cdots, X_N$ is denoted $\mathrm{diag}(X_1, \cdots, X_N)$.

The general system model considered in this paper is given in Fig. 1. We assume a wireless network where a source node (S) communicates with the destination node (D), with the assistance of an amplify-and-forward (AF) relay node (R). Similarly to the model used in [9], the relay node is equipped with multiple antennas while the source and the destination are single antenna nodes. It is assumed that the source transmission and the relay transmission occur in two different time slots. During the first time slot, the source transmits data to the relay. Then the relay processes the received signal and transmits the processed version to the destination. Furthermore, we assume that the nodes operate in the half-duplex mode. The reception at the relay and at the

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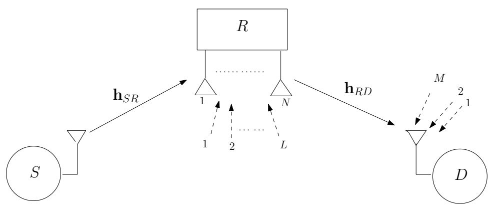

Fig. 1. Multi-antenna relay system with co-channel interference.

destination are corrupted by multiple interference signals and AWGN.

Similarly as in [9], the signals received at the relay are combined using maximal ratio combining (MRC) prior to the amplification. It is well known that MRC is not the optimal combiner in the presence of interfering signals. However, many practical receivers often employ MRC due to its satisfactory performance with lower complexity. The received signal at the relay after MRC can be written as

$$y_{SR} = \mathbf{w}_{SR}\left(\mathbf{h}_{SR}\sqrt{E_s}x + \sum_{l=1}^{L}\mathbf{h}_l\sqrt{E_{Rl}}I_{Rl} + \mathbf{n}_R\right) \quad (1)$$

where $\mathbf{w}_{SR} = \mathbf{h}_{SR}^*/\left|\left|\mathbf{h}_{SR}\right|\right|$ is the MRC weight vector, $\mathbf{h}_{SR}$ is the $N \times 1$ complex Gaussian channel gain vector between the source and the relay, x is the desired data symbol with energy E_s, $\mathbf{h}_l$ is the $N \times 1$ complex Gaussian channel gain vector between the l^{th} interferer and the relay, I_{Rl} is the signal transmitted by the l^{th} interferer with energy E_{Rl}, L is the number of interfering signals at the relay and $\mathbf{n}_R$ is a $N \times 1$ AWGN vector at the relay with $\mathbb{E}\left[\mathbf{n}_R\mathbf{n}_R^*\right] = \sigma_R^2\mathbf{I}_N$.

The relay amplifies the signal after MRC with amplification factor G and transmits the amplified version to the destination using maximal ratio transmission (MRT). The received signal at the destination can be formulated as

$$y_{RD} = \mathbf{w}_{RD}\mathbf{h}_{RD}G\sqrt{\frac{E_s}{N}}y_{SR} + \sum_{k=1}^{M}g_k\sqrt{E_{Dk}}I_{Dk} + n_D \quad (2)$$

where $\mathbf{w}_{RD} = \mathbf{h}_{RD}^*/\left|\left|\mathbf{h}_{RD}\right|\right|$ is the MRT weight vector, $\mathbf{h}_{RD}$ is the $N \times 1$ channel gain vector between the relay and the destination, g_k is the channel gain coefficient between the k^{th} interferer and the destination with mean-square value Φ_k, I_{Dk} is the signal transmitted by the k^{th} interferer with energy E_{Dk}, M is the number of interfering signals at the destination and n_D is AWGN at the relay with variance σ_D^2.

III. Outage Probability Analysis

In this section, we derive exact closed-form outage probability results for the system model introduced in Sec. II. Similarly as in [9], we limit our analysis only to the fixed gain relaying case. The analysis can be extended to the variable gain case using the approximations adopted in [3]. However, due to length restrictions, we avoid the analysis of variable gain relaying in this paper. In fixed gain relaying, the relay scales the signal based on the average values of the received signal power and the amplification gain is computed as [7]

$$G = \sqrt{\frac{1}{E_s\Omega_{SR} + \sum_{l=1}^{L}E_{Rl}\Omega_l + \sigma_r^2}} \quad (3)$$

where $\Omega_{SR} = \mathbb{E}\left[\left|\left|\mathbf{h}_{SR}\right|\right|^2\right]$, and Ω_l is the mean-square value of each component in vector $\mathbf{h}_l$.

Case 1: Noisy Relay with Interference Limited Destination

We first consider the case where the interference is present only at the destination, while the only additive interference present at the relay is AWGN. We assume that σ_d^2 can be neglected compared to $\sum_{k=1}^{M}E_{Dk}|g_k|^2$. For this case, G is computed as

$$G = \sqrt{\frac{1}{E_s\Omega_{SR} + \sigma_r^2}}. \quad (4)$$

Then the signal-to-interference ratio (SIR) at the destination can be found as

$$\gamma = \frac{\frac{E_s\left|\left|\mathbf{h}_{RD}\right|\right|^2}{N}\frac{E_s\left|\left|\mathbf{h}_{SR}\right|\right|^2}{\sigma_r^2}}{\frac{E_s\left|\left|\mathbf{h}_{RD}\right|\right|^2}{N} + \frac{1}{G^2\sigma_r^2}\sum_{k=1}^{M}E_{Dk}|g_k|^2}. \quad (5)$$

We observe that the SIR is in the form of

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_2 + C\gamma_D} \quad (6)$$

where $\gamma_1 = \frac{E_s\left|\left|\mathbf{h}_{SR}\right|\right|^2}{\sigma_r^2}$, $\gamma_2 = \frac{E_s\left|\left|\mathbf{h}_{RD}\right|\right|^2}{N}$, $\gamma_D = \sum_{k=1}^{M}E_{Dk}|g_k|^2$ and $C = \frac{E_s\Omega_{SR}}{\sigma_r^2} + 1$.

We consider that an outage event occurs when the SIR at the destination falls below a given threshold SIR value, γ_{th}, and the outage probability is given by

$$P_{\text{out}} = \Pr(\gamma < \gamma_{\text{th}}) = \Pr\left(\frac{\gamma_1\gamma_2}{\gamma_2 + C\gamma_D} < \gamma_{\text{th}}\right). \quad (7)$$

For the case of identical power interferers with identically distributed fading statistics, the outage probability can be found in closed-form as

$$P_{\text{out}} = 1 - \exp\left(-\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)\sum_{n=0}^{N-1}\sum_{j=0}^{n}\binom{n}{j}\frac{\gamma_{\text{th}}^{a_1}C^{a_2}}{n!\Gamma(N)\Gamma(M)\bar{\gamma}_1^{a_1}\bar{\gamma}_2^{a_2}\bar{\gamma}_D^{a_3}} \Gamma\left(M+N\right)\Gamma\left(M+j\right)\exp\left(\frac{C\gamma_{\text{th}}\bar{\gamma}_D}{2\bar{\gamma}_2\bar{\gamma}_1}\right)W_{\frac{-\mu}{2},\frac{\nu}{2}}\left(\frac{C\gamma_{\text{th}}\bar{\gamma}_D}{\bar{\gamma}_2\bar{\gamma}_1}\right) \quad (8)$$

where $a_1 = \frac{2n+N-j-1}{2}$, $a_2 = \frac{j+N-1}{2}$, $a_3 = \frac{-N-j+1}{2}$, $\mu = 2M - 1 + N + j$, $\nu = N - j$, $\bar{\gamma}_1 = \frac{E_s}{\sigma_r^2}\mathbb{E}[|h_{\text{SR}i}|^2], \forall i \in \{1, \cdots, N\}$ is the average received SNR from the source to each antenna at the relay, $\bar{\gamma}_2 = \frac{E_s}{N}\mathbb{E}[|h_{\text{RD}i}|^2], \forall i \in \{1, \cdots, N\}$, is the average signal power received from each antenna at the relay to the destination, $\bar{\gamma}_D = \mathbb{E}[E_{Dk}|g_k|^2]$, $\Gamma(\cdot)$ is the Gamma function defined in [10, eq. 8.310.1], and $W_{\alpha,\beta}(\cdot)$ is the Whittaker function defined in [10, 9.221.1]. See Appendix A for the derivation of (8).

The extension of outage probability results to the case when the interferers have unequal powers and non-identically distributed fading statistics can be done using [7, eqs. (8) and (9)] and following a similar procedure as for the case of equal power and identically distributed fading. The outage probability can be computed using (9) given at the top of the next page, where $\mathcal{A} = \text{diag}\left(E_{D1}\Phi_1, \cdots, E_{DM}\Phi_M\right)$,

$$P_{\text{out}} = 1 - \exp\left(-\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right) \sum_{n=0}^{N-1} \sum_{k=0}^{n} \sum_{i=1}^{\rho(\mathcal{A})} \sum_{j=1}^{\tau_i(\mathcal{A})} \binom{n}{k} \frac{\gamma_{\text{th}}^{x_1} C^{x_2}}{n!\Gamma(N) \bar{\gamma}_1^{x_1} \bar{\gamma}_2^{x_2}} \mathcal{X}_{i,j}(\mathcal{A}) \frac{P_{D,[i]}^{x_3}}{(j-1)!} \Gamma(N+j)\Gamma(k+j) \exp\left(\frac{C\gamma_{\text{th}} P_{D,[i]}}{2\bar{\gamma}_2\bar{\gamma}_1}\right) W_{\frac{-\mu_1}{2}, \frac{\nu_1}{2}}\left(\frac{C\gamma_{\text{th}} P_{D,[i]}}{\bar{\gamma}_2\bar{\gamma}_1}\right) \tag{9}$$

$\rho(\mathcal{A})$ is the number of distinct diagonal elements of $\mathcal{A}$, $P_{D,[1]} > P_{D,[2]} > \cdots > P_{D,[\rho(\mathcal{A})]}$ are the distinct diagonal elements in decreasing order, $\tau_i(\mathcal{A})$ is the multiplicity of $P_{D,[i]}$, $\mathcal{X}_{i,j}(\mathcal{A})$ is the (i,j)th characteristic coefficient of $\mathcal{A}$ [11], $x_1 = n + \frac{N-k-1}{2}$, $x_2 = \frac{N+k-1}{2}$, $x_3 = -j + \frac{\mu_1}{2}$, $\mu_1 = 2j + N + k - 1$ and $\nu_1 = N - k$.

Case 2: Interference Limited Relay and Noisy Destination

Next, we consider the case where the co-channel interference is present only at the relay while the destination is corrupted by AWGN. Also we assume that σ_r^2 is negligible compared to $\sum_{l=1}^{L} E_{Rl}\Omega_l$. The amplification gain G is computed as

$$G = \sqrt{\frac{1}{E_s\Omega_{SR} + \sum_{l=1}^{L} E_{Rl}\Omega_l}}. \tag{10}$$

Now, the SINR at the destination can be found as

$$\gamma = \frac{\frac{E_s||\mathbf{h}_{RD}||^2}{N\sigma_d^2} E_s\,||\mathbf{h}_{SR}||^2}{\frac{E_s||\mathbf{h}_{RD}||^2}{\sigma_d^2 N} \sum_{l=1}^{L} E_{Rl} Z_l + \frac{1}{G^2}} \tag{11}$$

where Z_l has similar statistics as the RV z_i defined in [12, Section III].

It can be observed that the signal-to-interference-plus-noise ration (SINR) is in the form

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_2\gamma_R + C_2} \tag{12}$$

where $\gamma_1 = E_s\,||\mathbf{h}_{SR}||^2$, $\gamma_2 = \frac{E_s||\mathbf{h}_{RD}||^2}{N\sigma_d^2}$, $\gamma_R = \sum_{l=1}^{L} E_{Rl} Z_l$ and $C_2 = E_s\Omega_{SR} + \sum_{l=1}^{L} E_{Rl}\Omega_l$.

For the case of equal power interferers with identically distributed fading statistics, the outage probability can be found in closed-form as[1]

$$P_{\text{out}} = 1 - 2\sum_{n=0}^{N-1}\sum_{i=0}^{n} \binom{n}{i} \frac{\gamma_{\text{th}}^{k_1} C_2^{k_2} \bar{\gamma}_2^{k_3} (i+L-1)!}{n!\bar{\gamma}_R^L \Gamma(L)\Gamma(N)\bar{\gamma}_1^{k_1}} \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_R}\right)^{-L-i} \mathcal{K}_{N-(n-i)}\left(2\sqrt{\frac{C_2\gamma_{\text{th}}}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \tag{13}$$

where $k_1 = \frac{2n+N-(n-j)}{2}$, $k_2 = \frac{(n-j)+N}{2}$, $k_3 = \frac{N-(n-j)}{2}$, $\bar{\gamma}_R = \mathbb{E}[E_{Rl} Z_l]$ and $\mathcal{K}_n(\cdot)$ is the n^{th} order modified Bessel function of the second kind [10, eq. (8.407)].

In order to derive the outage probability expression for unequal power interferers with non-identical fading statistics, we can use [7, eq. (8)] as the PDF of γ_R and follow similar methodology to that used for the identically distributed case. The closed-form outage probability can be given as

$$P_{\text{out}} = 1 - \left[\sum_{n=0}^{N-1}\sum_{k=0}^{n}\sum_{i=1}^{\rho(\mathcal{B})}\sum_{j=1}^{\tau_i(\mathcal{B})} \binom{n}{k} \frac{\mathcal{X}_{i,j}(\mathcal{B})(j-1+k)!P_{R,[i]}^{-j}}{(j-1)!} \frac{2\gamma_{\text{th}}^{y_1} C_2^{y_2}}{n!\Gamma(N)\bar{\gamma}_1^{y_1}\bar{\gamma}_2^{y_2}} \mathcal{K}_{N-(n-k)}\left(2\sqrt{\frac{C_2\gamma_{\text{th}}}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1} + \frac{1}{P_{R,[i]}}\right)^{-j-k}\right] \tag{14}$$

where $\mathcal{B} = \text{diag}\left(E_{R1}\Omega_1, \cdots, E_{RL}\Omega_L\right)$, and $\rho(\mathcal{B})$, $P_{R,[i]}$ and $\mathcal{X}_{i,j}(\mathcal{B})$ are defined similarly to the matrix $\mathcal{A}$ in Case 1, $y_1 = n + \frac{N-(n-k)}{2}$, and $y_2 = \frac{N+n-k}{2}$.

1) Outage Probability With Source-Destination Link: In the presence of a direct link between the source and the destination, the destination can combine the received signals it receives from the source and the relay according to the well known selection diversity combining scheme. In such systems, the resultant SINR or SNR at the destination can be found as

$$\gamma_{\text{eq}} = \max(\gamma, \gamma_{SD}) \tag{15}$$

where γ_{SD} is the SNR of the source-destination link. The outage probability of the system is then given by

$$P_{\text{out}} = \Pr(\gamma_{\text{eq}} < \gamma_{\text{th}}) = F_{\gamma_{\text{eq}}}(\gamma_{\text{th}}) = F_\gamma(\gamma_{\text{th}})F_{\gamma_{SD}}(\gamma_{\text{th}}). \tag{16}$$

For a Rayleigh faded $S-D$ link, the SNR γ_{SD} is exponentially distributed, and the CDF of γ_{SD} is given by

$$F_{\gamma_{SD}}(x) = 1 - \exp\left(-\frac{x}{\bar{\gamma}_{SD}}\right) \tag{17}$$

where $\bar{\gamma}_{SD}$ is the average SNR of the $S-D$ link. The CDF $F_\gamma(x)$ is used to evaluate the outage probability in (13). For example, the outage probability with equal power interferers and identically distributed fading, in the presence of the direct $S-D$ link can be obtained in closed-form as

$$P_{\text{out}} = \left[1 - 2\sum_{n=0}^{N-1}\sum_{i=0}^{n}\binom{n}{i} \frac{\gamma_{\text{th}}^{k_1} C_2^{k_2}\bar{\gamma}_2^{k_3}(i+L-1)!}{n!\bar{\gamma}_R^L\Gamma(L)\Gamma(N)\bar{\gamma}_1^{k_1}} \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_R}\right)^{-L-i} \mathcal{K}_{N-(n-i)}\left(2\sqrt{\frac{C_2\gamma_{\text{th}}}{\bar{\gamma}_1\bar{\gamma}_2}}\right)\right] \times \left[1 - \exp\left(-\frac{\gamma_{\text{th}}}{\bar{\gamma}_{SD}}\right)\right]. \tag{18}$$

Case 3: Interference Limited Relay and Destination

In this section, we present the general case where the relay and the destination both suffers from the effects of co-channel interference. In this scenario, the relay amplification gain is

[1]The proof is available from the authors.

$$P_{\text{out}} = 1 - \sum_{n=0}^{N-1}\sum_{k=0}^{n}\sum_{i=1}^{\rho(\mathcal{B})}\sum_{j=1}^{\tau_i(\mathcal{B})}\binom{n}{k}\frac{\mathcal{X}_{i,j}(\mathcal{B})(j-1+k)!P_{R,[i]}^{-j}}{(j-1)!}\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}+\frac{1}{P_{R,[i]}}\right)^{-j-k}\sum_{v=1}^{\rho(\mathcal{A})}\sum_{w=1}^{\tau_v(\mathcal{A})}\frac{\gamma_{\text{th}}^{l_1}C_3^{l_2}}{n!\Gamma(N)\bar{\gamma}_1^{l_1}\bar{\gamma}_2^{l_2}}\mathcal{X}_{v,w}(\mathcal{A})\frac{P_{D,[v]}^{l_2}}{(w-1)!}$$
$$\Gamma(N+w)\Gamma(k+w)\exp\left(\frac{C_3\gamma_{\text{th}}P_{D,[v]}}{2\bar{\gamma}_2\bar{\gamma}_1}\right)W_{\frac{-\mu_3}{2},\frac{\nu_3}{2}}\left(\frac{C_3\gamma_{\text{th}}P_{D,[v]}}{\bar{\gamma}_2\bar{\gamma}_1}\right) \quad (21)$$

same as the gain given in (10). The SIR at the destination is given by

$$\gamma = \frac{\frac{E_s||\mathbf{h}_{RD}||^2}{N}E_s\,||\mathbf{h}_{SR}||^2}{\frac{E_s||\mathbf{h}_{RD}||^2}{N}\sum_{l=1}^{L}E_{Rl}Z_l+\frac{1}{G^2}\sum_{k=1}^{M}E_{Dk}|g_k|^2}. \quad (19)$$

It can be observed that the SIR is in the form

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_2\gamma_R + C_3\gamma_D} \quad (20)$$

where $\gamma_1 = E_s\,||\mathbf{h}_{SR}||^2$, $\gamma_2 = \frac{E_s||\mathbf{h}_{RD}||^2}{N}$, $\gamma_R = \sum_{l=1}^{L}E_{Rl}Z_l$, $\gamma_D = \sum_{k=1}^{M}E_{Dk}|g_k|^2$ and $C_3 = E_s\Omega_{SR} + \sum_{l=1}^{L}E_{Rl}\Omega_l$.

For the case of equal power interferers with identically distributed fading statistics, the outage probability of the system can be found in closed-form as[2]

$$P_{\text{out}} = 1 - \sum_{n=0}^{N-1}\sum_{k=0}^{n}\binom{n}{k}\frac{\gamma_{\text{th}}^{v_1}C_3^{v_2}(k+L-1)!}{n!\bar{\gamma}_1^{v_1}\bar{\gamma}_2^{v_2}\bar{\gamma}_R^{L}\bar{\gamma}_D^{v_3}\Gamma(L)\Gamma(N)\Gamma(M)}$$
$$\left(\frac{1}{\bar{\gamma}_R}+\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{-k-L}\Gamma(M+N)\Gamma(M+(n-k))$$
$$\exp\left(\frac{C_3\gamma_{\text{th}}\bar{\gamma}_D}{2\bar{\gamma}_2\bar{\gamma}_1}\right)W_{\frac{-\mu_2}{2},\frac{\nu_2}{2}}\left(\frac{C_3\gamma_{\text{th}}\bar{\gamma}_D}{\bar{\gamma}_2\bar{\gamma}_1}\right) \quad (21)$$

where $v_1 = \frac{n+N+k-1}{2}$, $v_2 = \frac{(n-k)+N-1}{2}$, $v_3 = \frac{1-N-(n-k)}{2}$, $\mu_2 = 2M-1+N+(n-k)$ and $\nu_2 = N-(n-k)$.

This result can be extended to the case when the interferers have unequal power and non-identical fading statistics by replacing the PDFs of γ_R and γ_D by [7, eq. (8)]. The outage probability for this can be found in closed-form as (21), given at the top of this page, where $l_1 = \frac{N+n+k-1}{2}$, $l_2 = \frac{N+n-k-1}{2}$, $\mu_3 = n-k+N+2w-1$ and $\nu_3 = N-(n-k)$.

IV. Numerical Results

In this section, we present some numerical results obtained by evaluating the expressions derived in Sec. III. For brevity, in numerical results, we assume equal power interferers with identically distributed fading statistics. We compare the numerical results with results obtained through computer simulations. In the figures, the lines (solid, dashed and dotted) denote the results obtained with theoretical expressions while the markers on the lines denote the corresponding simulation results. One can observe the excellent agreement between the theoretical results and the simulations.

Fig. 2 shows the outage probability of the system when the interference signals are present only at the relay. The $\text{SIR}_R = \frac{\bar{\gamma}_1}{\bar{\gamma}_R}$ at the relay is 15 dB. The threshold SINR at the destination is normalized by the average SNR of the $R-D$ link. The performance degradation with the number of interference signals can be clearly identified and quantified. We observe that the diversity order is not affected by the number of interference signals. It is recalled that the desired signal replicas are combined coherently whereas the interfering signal replicas are combined noncoherently.

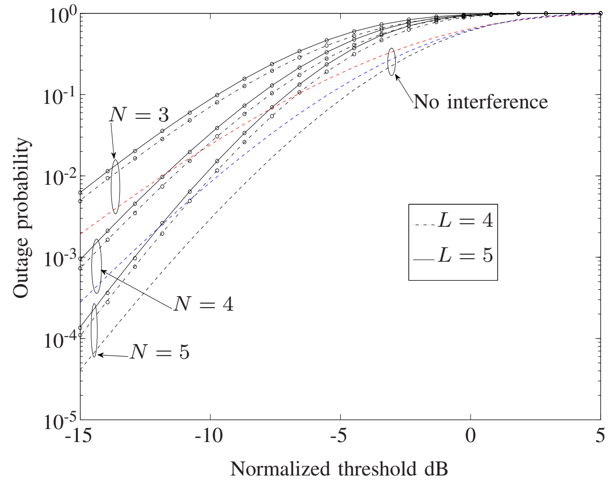

Fig. 2. The outage probability of the system when interferers are present only at the relay.

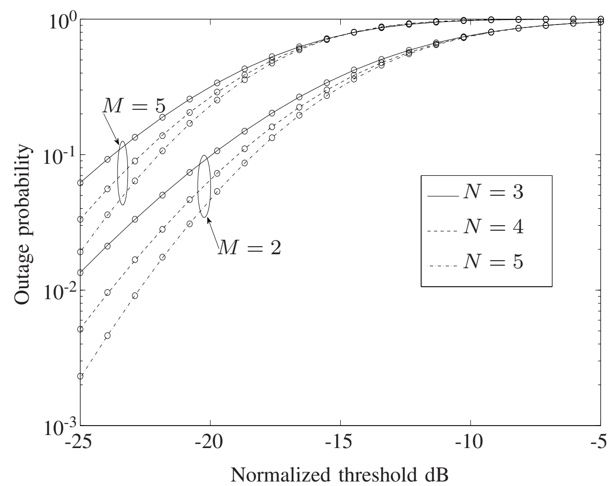

Fig. 3. The outage probability of the system when interferers are present only at the destination.

Fig. 3 shows the effect of the number of antennas and the number of interference signals on the outage probability of the system when the co-channel interference signals are present only at the destination. The SIR $= \frac{\bar{\gamma}_2}{\bar{\gamma}_D}$ at the destination is assumed to be 15 dB. The performance loss due to interference can be quantified. It can be observed that the interference at

[2]The proof is available from the authors.

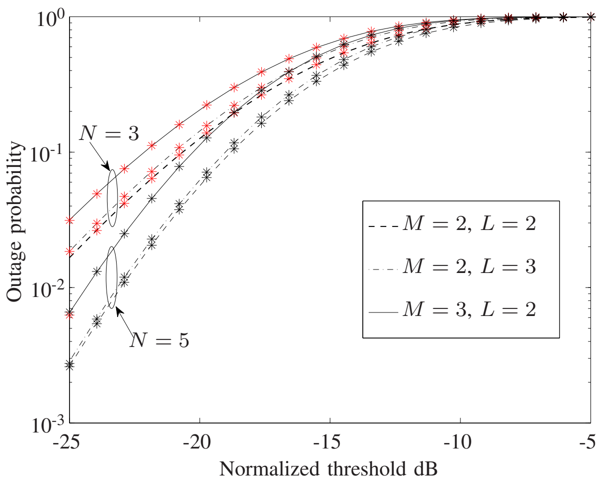

Fig. 4. The outage probability of the system when interferers are present at the relay and the destination.

the destination does not affect the diversity order, similarly to the case when the interference is present only at the relay.

Fig. 4 presents the outage probability of the system when co-channel interference signals are present at both the relay and the destination. The SIRs $\mathrm{SIR}_R = \frac{\bar{\gamma}_1}{\bar{\gamma}_R}$ and $\mathrm{SIR}_D = \frac{\bar{\gamma}_2}{\bar{\gamma}_D}$ are assumed to be 15 dB. Furthermore, we assume the power and the mean-square values of fading gains of the interferers at the relay and the destination are identical. From the results in the figure, it can be observed that the outage probability is more sensitive to the number of interferers at the destination than to the number of interferers at the relay.

V. Conclusion

The outage probability of a recently proposed relay network model was analyzed, assuming that the network is operating in the presence of multiple co-channel interference signals. Closed-form expressions for the outage probability were derived for different scenarios that may prevail in the network. The closed-form expressions can be easily computed with common mathematical software packages. Computer simulations were used to verify the accuracy of the theoretical analysis.

Appendix A

In this appendix, we present the derivation of (8). The outage probability is computed as

$$\begin{aligned} P_{\text{out}} &= \Pr(\gamma < \gamma_{\text{th}}) = \Pr\left(\gamma_1 < \frac{\gamma_{\text{th}}(\gamma_2 + C\gamma_D)}{\gamma_2}\right) \\ &= \int_0^\infty \int_0^\infty F_{\gamma_1}\left(\frac{\gamma_{\text{th}}(\gamma_2 + C\gamma_D)}{\gamma_2}\right) f_{\gamma_2}(\gamma_2) f_{\gamma_D}(\gamma_D) \\ &\qquad d\gamma_2 d\gamma_D. \end{aligned} \tag{23}$$

The RV γ_1 is chi-square distributed with $2N$ degrees of freedom, and the CDF $F_{\gamma_1}(x)$ is given by

$$F_{\gamma_1}(x) = 1 - \exp\left(-\frac{x}{\bar{\gamma}_1}\right) \sum_{n=0}^{N-1} \frac{x^n}{n! \bar{\gamma}_1^n}. \tag{24}$$

The RVs γ_2 and γ_D also follow chi-square distributions with $2N$ and $2M$ degrees of freedom, respectively. Their PDFs can be given as

$$f_{\gamma_2}(x) = \frac{\exp\left(-\frac{x}{\bar{\gamma}_2}\right) x^{N-1}}{\Gamma(N) \bar{\gamma}_2^N} \tag{25}$$

$$f_{\gamma_D}(x) = \frac{\exp\left(-\frac{x}{\bar{\gamma}_D}\right) x^{M-1}}{\Gamma(M) \bar{\gamma}_D^M}. \tag{26}$$

Substituting (24) in (23), and using the binomial expansion, we obtain

$$\begin{aligned} P_{\text{out}} &= 1 - \exp\left(-\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right) \int_0^\infty \int_0^\infty \sum_{n=0}^{N-1} \frac{\gamma_{\text{th}}^n}{n! \bar{\gamma}_1^n} \sum_{j=0}^{n} \binom{n}{j} C^k \\ &\left(\frac{\gamma_D}{\gamma_2}\right)^k \exp\left(-\frac{C\gamma_{\text{th}}\gamma_D}{\bar{\gamma}_1 \gamma_2}\right) f_{\gamma_2}(\gamma_2) f_{\gamma_D}(\gamma_D) d\gamma_2 d\gamma_D. \end{aligned} \tag{27}$$

The double integral (27) can be solved in closed-form to obtain (8) by first using [10, eq. (3.471.9)] and then applying the result [10, eq. (6.631.3)].

When the interferers have unequal powers and non-identical fading statistics, the PDF of γ_D has the form given in [7, eq. (8)]. Following a similar methodology used to solve (27), we can find the outage probability in closed-form for the case of unequal power interferers with non-identical fading.

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