

# FG-based Cooperative Group Localization for Next-generation Communication Networks

Xuefei Zhang, Qimei Cui, Yulong Shi, Xiaofeng Tao

Key Laboratory of Universal Wireless Communications, Ministry of Education

Wireless Technology Innovation Institute, Beijing University of Posts and Telecommunications  
Beijing, China

Email: zhangxuefeikate@gmail.com

**Abstract**—In restricted communication environment, multiple-target localization is of important practical significance. So the cooperative group localization (CGL) was firstly put forward, which has been verified the effectiveness on localization performance gain and simultaneous multiple-target localization in ill conditions. However, it exists two inherent difficulties: the strict demand for CGL topology and the high complexity. By the rational use of information to relax restrictions on topology, and by dividing the complex problem into some simple local ones, the factor graph (FG) together with the sum-product algorithm is a perfect candidate for the problems above. This paper firstly proposes a novel FG-based CGL algorithm. Numerical results indicate that compared with the existing CGL algorithm, the proposed algorithm not only performs better in relaxing CGL topology requirement, but also enjoys high localization accuracy under low complexity.

**Index Terms**—Cooperative group localization, factor graph, next-generation communication networks.

## I. INTRODUCTION

Recently, the rapid growth and the wide appliance of smart terminals suggest a sharp rising in communication services as well as enhanced wireless connectivity [1], largely depending on mobile localization. From the earlier car navigation to recent diverse anytime and anywhere views for location map, more and more people rely on the instant localization, which may make the communication services more user-friendly and provide more value. So, accurate and rapid localization has been a crucial problem for mobile communication networks.

Most of current academic achievements in localization field focus on the enhancement localization techniques in the well condition. The well condition means that mobile terminal (MT) to be localized has the sufficient signal resources. For example, if one MT can be accurately located by the measurement parameters of time of arrival (TOA) or time difference of arrival (TDOA) or angle of arrival (AOA) or received signal strength (RSS), the MT connects well with at least three base stations (BSs) and line of sight (LOS) propagation paths exist between BSs and MT [2]. Nevertheless, the scenarios with sufficient signal resources do not always happen in real circumstances. Sometimes, the parameter measurements may be corrupted by non line of sight (NLOS) error, or MT can not simultaneously connect with three BSs [2, 3]. These ill conditions with insufficient signal resources can make the accurate localization unrealistic.

In order to solve the above ill-conditioned localization problem, the cooperative group localization (CGL) is proposed in [4, 5]. The existing research indicates that the CGL scheme not only has higher localization accuracy over the traditional methods in ill conditions, but also can effectively enhance the robustness and precision of localization. However, it also exists two inherent difficulties: 1) the strict demand for CGL topology and 2) the high complexity. According to the rigid graph theory [6], the CGL topology must be global rigidity so as to satisfy the unique solvability. Furthermore, CGL has to face the problem of high complexity, especially with the increasing number of cooperative MTs.

In [7], factor graph (FG) theory is proposed, which can simplify a complex problem into multiple simple sub-problems. The optimal or the near-optimal solution can be obtained by solving the sub-problems respectively. According to its feasibility and simplicity, it has been introduced into cellular or sensor network based on measurement parameters TOA, TDOA, AOA and RSS [8-12] and reaches the good tradeoff between the localization accuracy and complexity. In [13], Henk Wymeersch, et al. explain the feasibility of FG theory applied into the cooperative localization.

In this paper, we firstly combine FG theory with CGL, and propose a novel FG-based CGL algorithm to effectively solve the above problems of strict demand for CGL topology and high complexity. The corresponding theoretical analyses are described. Numerical simulation results verify that compared to the existing CGL method, the proposed algorithm can relax CGL topology requirement and provide the high localization accuracy under low complexity.

The paper is organized as follows. In section II, CGL model is introduced. In section III, the principles of factor graph and sum-product algorithm are presented. Section IV proposes a FG-based CGL algorithm. Section V evaluates the performance and the complexity of the proposed algorithm by numerical simulation. And compare it with the existing CGL algorithm. Finally, the conclusion is included in Section VI.

## II. MATHEMATICAL MODEL OF CGL

For the description convenience, take the scenario shown in Fig.1 for example, which presents the principle of CGL in the following sections. In CGL, the conception of the terminal group (TG) is firstly put forward. TG refers to a group of MTs

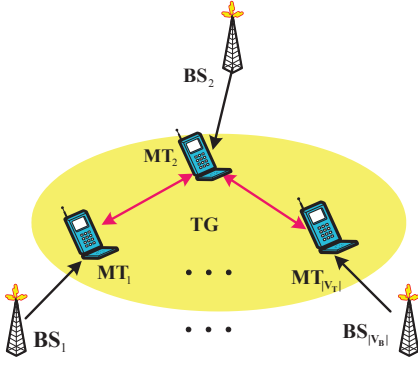


Fig. 1. CGL topology diagram

supporting peer-to-peer communication mode. To represent the most general situation, all MTs are supposed to be the localization-unknown nodes and the relative distances between some certain MT pairs are known. As shown in Fig. 1, each MT in TG connects with less than three BSs, but the FG-based CGL algorithm proposed below is able to accurately localize all of the MTs in the TG.

The CGL graph  $G=(V, E)$  can be divided into three sub-graphs: the first one is BS sub-graph  $G_B=(V_B, E_B)$ , where  $V_B$  represents the BSs and  $E_B$  represents the existing edges between any two vertices of  $V_B$ ; the second one is TG sub-graph  $G_T=(V_T, E_T)$ , where  $V_T$  includes all the MTs in TG and  $E_T$  represents the distances determined by direct communication among the MTs; the third is cooperative sub-graph  $G_C=(V_C, E_C)$ , where  $V_C$  includes the BSs or the cooperative MTs in TG,  $E_C$  represents the edges existing between  $G_B$  and  $G_T$ . The operator  $|\bullet|$  of the sets denotes the number of the elements in a set.

Given the CGL graph, the aim of CGL is to simultaneously solve the positions of all the MTs in TG. However, not all topologies of CGL have unique solvability. The unique solvability means that the positions of MTs in a TG can be located without ambiguity. It can be investigated by the rigid graph theory [6].

### III. FACTOR GRAPH AND SUM-PRODUCT ALGORITHM

#### A. Factor Graph

A factor graph [7] is a bipartite graph that illustrates how a complicated global function with many variables factors is simplified into the product of several simple local functions. Each local function is a function with few variables. A factor graph has the variable node for each variable, the function node for each local function, and the edge connecting variable node to factor node if and only if the variable is an argument of the function. For example,  $f(x_1, x_2, x_3, x_4, x_5)$  is a real-valued function with five variables, which is divided into the product of five local functions  $f_A, f_B, f_C, f_D$  and  $f_E$ , as is shown in Fig.2.

$$f(x_1, x_2, x_3, x_4, x_5) = f_A(x_1)f_B(x_2)f_C(x_1, x_2, x_3)f_D(x_3, x_4)f_E(x_3, x_5) \quad (1)$$

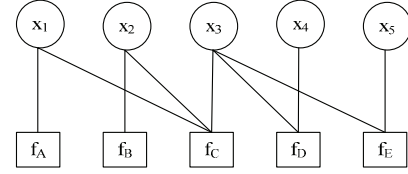


Fig. 2. An example of Factor Graph

#### B. Sum-Product Algorithm

Sum-Product Algorithm [7] is one of the important algorithms in Graphical Model, which provides the convenient solution to marginal probability. The algorithm works by the passing messages along the edges between the nodes. There are two types of messages:

(1) A message from a variable node  $x$  to a function node  $f$  is the product of the messages from all other neighboring function nodes except  $f$  to  $x$ :

$$SI(x, f) = \prod_{h \in n(x) \setminus \{f\}} SI(h, x) \quad (2)$$

where  $SI(a, b)$  represents the message from node  $a$  to node  $b$ ,  $n(x)$  is the set of neighboring (function) nodes to  $x$ .

(2) A message from a function node  $f$  to a variable node  $x$  is the product of the messages from all other nodes, summarized over all variables except  $x$ :

$$SI(f, x) = \sum_{\sim \{x\}} \left( f(X) \prod_{y \in n(f) \setminus \{x\}} SI(y, f) \right) \quad (3)$$

where  $n(f)$  is the set of neighboring (variable) nodes to  $f$ .

With two rules above, information is passed around among neighboring nodes. Each variable node modifies its own value according to the received message. After a few times, the information converges. The variable can be estimated according to the product of overall message from the corresponding node:

$$SI(x) = \prod_{h \in n(x)} SI(h, x) \quad (4)$$

### IV. FG-BASED CGL ALGORITHM

In the following analysis, the positions of several MTs simultaneously are estimated based on TOA measurements. In order to reduce the complexity of the 2-D problem, the problem is divided into two 1-D problems [8], respectively representing two groups of all nodes, x-coordinate group and y-coordinate group, as illustrated in Fig.3. The messages are processed and passed around between variable nodes and function nodes.

#### A. The Functions of All Nodes

(1) Variable  $x_i$  and  $y_i$  (in the  $k^{th}$  iteration)

The message from variable node  $x_i^k$  to function node  $A_{i,j}^k$  or  $M_{i,t}^k$  is a Gaussian pdf of  $x_i^k$  and can be expressed as

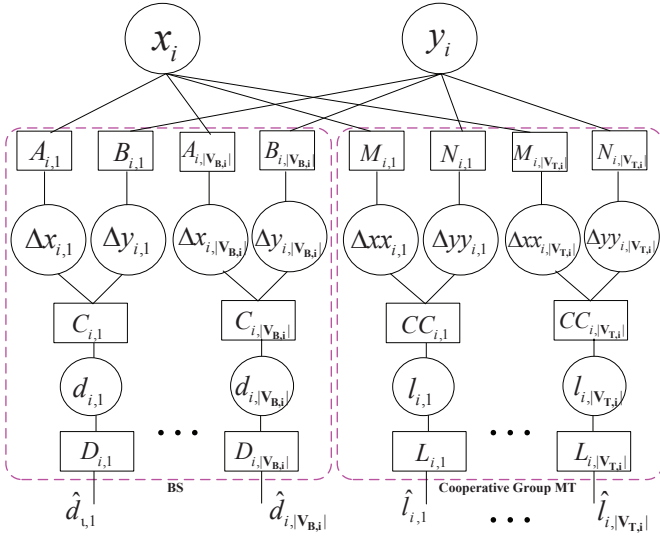


Fig. 3. FG in CGL (Take one of the MT as an example)

$$SI(x_i^k, A_{i,j}^k) = \prod_{u=1, u \neq j}^{|V_{B,i}|} SI(A_{i,u}^k, x_i^k) \prod_{v=1}^{|V_{T,i}|} SI(M_{i,v}^k, x_i^k) \quad (5)$$

$$SI(x_i^k, M_{i,t}^k) = \prod_{u=1}^{|V_{B,i}|} SI(A_{i,u}^k, x_i^k) \prod_{v=1, v \neq t}^{|V_{T,i}|} SI(M_{i,v}^k, x_i^k) \quad (6)$$

where  $i=1, 2, \dots, |V_T|$ ,  $j=1, 2, \dots, |V_{B,i}|$ ,  $t = 1, 2, \dots, |V_{T,i}|$ ,  $t \neq i$ .  $|V_T|$  is the number of whole MTs to be located.  $|V_{B,i}|$  and  $|V_{T,i}|$  represent the number of BSs and other cooperative MTs connecting to the  $i^{th}$  MT, respectively.  $SI(A_{i,u}^k, x_i^k)$  and  $SI(M_{i,v}^k, x_i^k)$  are Gaussian pdf of  $x_i^k$ . Note that the product of any  $U$  Gaussian pdf is also Gaussian and can be derived as [7]

$$\prod_{u=1}^U N(x, m_u, \sigma_u^2) \propto N(x, m_\Lambda, \sigma_\Lambda^2) \quad (7)$$

where  $N(x, m, \sigma^2) \propto \exp[-\frac{(x-m)^2}{2\sigma^2}]$ ,  $\sigma_\Lambda^2 = 1/(\sum_{u=1}^U 1/\sigma_u^2)$  and

$m_\Lambda = \sigma_\Lambda^2 \sum_{u=1}^U (m_u/\sigma_u^2)$ . Similarly, variable group  $y_i$  can be calculated.

(2) Variable node  $\Delta x_{i,j}$ ,  $\Delta y_{i,j}$ ,  $\Delta xx_{i,t}$ ,  $\Delta yy_{i,t}$ ,  $d_{i,j}$  and  $l_{i,t}$

Variable node  $\Delta x_{i,j}$  can transmit the receiving message directly,

$$SI(\Delta x_{i,j}^k, A_{i,j}^k) = SI(C_{i,j}^k, \Delta x_{i,j}^k) \quad (8)$$

$$SI(\Delta x_{i,j}^k, C_{i,j}^k) = SI(A_{i,j}^k, \Delta x_{i,j}^k) \quad (9)$$

$$SI(\Delta xx_{i,t}^k, M_{i,t}^k) = SI(CC_{i,t}^k, \Delta xx_{i,t}^k) \quad (10)$$

$$SI(\Delta xx_{i,t}^k, CC_{i,t}^k) = SI(M_{i,t}^k, \Delta xx_{i,t}^k) \quad (11)$$

Similarly, variable node  $\Delta y_{i,j}$ ,  $\Delta xx_{i,t}$ ,  $\Delta yy_{i,t}$ ,  $d_{i,j}$  and  $l_{i,t}$  all play the role as a message transfer between the corresponding two function nodes.

(3) Function node  $A_{i,j}$ ,  $B_{i,j}$ ,  $M_{i,t}$  and  $N_{i,t}$

The function of these nodes is to convert the relative position information into the absolute position information, and vice versa.

$$\begin{cases} \Delta x_{i,j} = X_{i,j} - x_i \\ \Delta y_{i,j} = Y_{i,j} - y_i \end{cases} \quad (12)$$

where  $X_{i,j}$  and  $Y_{i,j}$  represent the position of the  $j^{th}$  BS connecting to the  $i^{th}$  MT.

$$\begin{cases} \Delta xx_{i,t} = x_t - x_i \\ \Delta yy_{i,t} = y_t - y_i \end{cases} \quad t = 1, 2, \dots, |V_{T,i}| \quad (13)$$

$$SI(A_{i,j}^k, \Delta x_{i,j}^k) = N(\Delta x_{i,j}^k, X_{i,j} - x_i^k, \sigma_{x_i^k}^2) \quad (14)$$

$$SI(A_{i,j}^k, x_i^k) = N(x_i^k, X_{i,j} - \Delta x_{i,j}^k, \sigma_{\Delta x_{i,j}^k}^2) \quad (15)$$

where  $\sigma_{x_i^k}^2$  and  $\sigma_{\Delta x_{i,j}^k}^2$  are the variance of the Gaussian soft-information from  $x_i^k$  and  $\Delta x_{i,j}^k$ . Similarly, the message from  $B_{i,j}$ ,  $M_{i,t}$  and  $N_{i,t}$  can be calculated.

(4) Function node  $C_{i,j}$  and  $CC_{i,t}$

The function of these nodes is to merge the separate information from x-coordinate group and the y-coordinate group and then compare it with real measurement data.

$$(\Delta x_{i,j})^2 + (\Delta y_{i,j})^2 = d_{i,j}^2 \quad (16)$$

$$(\Delta xx_{i,t})^2 + (\Delta yy_{i,t})^2 = l_{i,t}^2 \quad (17)$$

$\Delta y_{i,j}$  updates itself according to the messages from  $\Delta x_{i,j}$  and  $d_{i,j}$ ,  $\Delta x_{i,j}$  updates itself according to the messages from  $\Delta y_{i,j}$  and  $d_{i,j}$ .

$$SI(C_{i,j}^k, \Delta y_{i,j}^{k+1}) = N(\Delta y_{i,j}^{k+1}, \pm \sqrt{\frac{(\hat{d}_{i,j}^k)^2 - (\Delta x_{i,j}^k)^2}{(\Delta x_{i,j}^k)^2 (\sigma_{\Delta x_{i,j}^k}^2) + (\hat{d}_{i,j}^k)^2 (\sigma_{d_{i,j}^k}^2)}}, \frac{(\hat{d}_{i,j}^k)^2 - (\Delta x_{i,j}^k)^2}{(\hat{d}_{i,j}^k)^2 - (\Delta x_{i,j}^k)^2}) \quad (18)$$

$$SI(C_{i,j}^k, \Delta x_{i,j}^{k+1}) = N(\Delta x_{i,j}^{k+1}, \pm \sqrt{\frac{(\hat{d}_{i,j}^k)^2 - (\Delta y_{i,j}^k)^2}{(\Delta y_{i,j}^k)^2 (\sigma_{\Delta y_{i,j}^k}^2) + (\hat{d}_{i,j}^k)^2 (\sigma_{d_{i,j}^k}^2)}}, \frac{(\hat{d}_{i,j}^k)^2 - (\Delta y_{i,j}^k)^2}{(\hat{d}_{i,j}^k)^2 - (\Delta y_{i,j}^k)^2}) \quad (19)$$

where  $\hat{d}_{i,j}$  and  $\hat{l}_{i,t}$  are the measurement information between BSs and MTs, and between MTs. Similarly,  $SI(CC_{i,t}^k, \Delta xx_{i,t}^{k+1})$ ,  $SI(CC_{i,t}^k, \Delta yy_{i,t}^{k+1})$  can be calculated.

(5) Function node  $D_{i,j}$  and  $L_{i,t}$

The measurement information  $\hat{d}_{i,j}$  and  $\hat{l}_{i,t}$  enter FG through the function node  $D_{i,j}$  and  $L_{i,t}$ ,

$$SI(D_{i,j}^k, d_{i,j}^k) = N(d_{i,j}, \hat{d}_{i,j}, \sigma_{d_{i,j}}^2) \quad (20)$$

$$SI(L_{i,t}^k, l_{i,t}^k) = N(l_{i,t}, \hat{l}_{i,t}, \sigma_{l_{i,t}}^2) \quad (21)$$

After  $K$  iterations, information in variable nodes  $x_i$  and  $y_i$  converges.  $x_i$  and  $y_i$  can be estimated.

$$SI(x_i^k) = \prod_{u=1}^{|V_{B,i}|} SI(A_{i,u}^k, x_i^k) \prod_{v=1}^{|V_{T,i}|} SI(M_{i,v}^k, x_i^k) \quad (22)$$

$$SI(y_i^k) = \prod_{u=1}^{|\mathbf{V}_{B,i}|} SI(B_{i,u}^k, y_i^k) \prod_{v=1}^{|\mathbf{V}_{T,i}|} SI(N_{i,v}^k, y_i^k) \quad (23)$$

### B. FG-based CGL Algorithm

According to the above analysis, the procedures of FG-based CGL algorithm are described as follows:

**Step1:** Initialization: all variable nodes need deterministic initial values, except  $d_{i,j}$  and  $l_{i,t}$ . Initial values can be made at random or by some specific algorithms, not detailed here. Set  $k = 0$ .

**Step2:** Entrance of measurement information in the  $k^{th}$  iteration: the measurement information  $\hat{d}_{i,j}^k$  and  $\hat{l}_{i,t}^k$  from TOA measurement enter FG through function nodes  $D_{i,j}^k$  and  $L_{i,t}^k$ , according to (20) and (21).

**Step3:** Uplink calculation in the  $k^{th}$  iteration: Function node  $C_{i,j}$  updates  $\Delta y_{i,j}$  and  $\Delta x_{i,j}$  according to (18), (19). Similarly, function node  $CC_{i,t}$  updates  $\Delta x_{i,t}$  and  $\Delta y_{i,t}$ . Function nodes  $A_{i,j}$ ,  $B_{i,j}$ ,  $M_{i,t}$  and  $N_{i,t}$  update the messages for  $x_i$  and  $y_i$  according to (15) and other similar formulas.

**Step4:** Downlink calculation in the  $k^{th}$  iteration: Variable nodes  $x_i$  transmits message to  $A_{i,j}$  and  $M_{i,t}$  according to (5), (6). Similarly,  $y_i$  transmits message to  $B_{i,j}$  and  $N_{i,t}$ . Function node  $A_{i,j}$  updates  $\Delta x_{i,j}$  according to (14). Similarly, function nodes  $B_{i,j}$ ,  $M_{i,t}$  and  $N_{i,t}$  update  $\Delta y_{i,j}$ ,  $\Delta x_{i,t}$  and  $\Delta y_{i,t}$ .

**Step5:** In the  $k^{th}$  iteration, variable node  $x_i$  and  $y_i$  are updated and estimate the position of the  $i^{th}$  MT according to (22), (23).

**Step6:** Targeting for the  $m^{th}$  MT who has not been updated the position in the  $k^{th}$  iteration, we make  $i = m$ , return to **Step2**. Else, when all of the cooperative MTs in a TG have completed the position updation in the  $k^{th}$  iteration, we make  $k \leftarrow k + 1$ .

**Step7:** If  $k < K$ , return to **Step2**. Else,  $x_i = x_i^k$  and  $y_i = y_i^k$ , the algorithm stops.

## V. SIMULATION RESULTS

Simulation evaluation compares the proposed algorithm with the CGL-based Taylor-series algorithms [5] in the same scenarios. The topologies are based on Fig.4 (a) and (b). In (a), the topology is global rigidity, which has the unique solvability. In (b), the topology is rigidity, which has no unique solvability and converges in the finite positions. Anyone topology scenario is constructed by the random birth of MTs in the efficient range of BSs. Cycle Number of per topology is 10000. The edges existing between BSs and MTs or between MTs are LOS pathes. The distance measurements from TOA between MTs and BSs is  $d = d_0 + n$ , the distance measurements between MTs is  $l = l_0 + q$ , where  $d_0$  denotes the real distance between BS and MT,  $l_0$  denotes the real distance between MTs,  $n$  and  $q$  denote the corresponding measurement error and follow the Gaussian distribution with zero mean value. The iteration time  $K$  of the proposed algorithm is 5. The localization accuracy is analyzed by means of root mean square error (RMSE) between the real MTs' position and the estimated position. The basic parameter configuration is listed in Table I.

TABLE I  
PARAMETERS CONFIGURATION

The number of BSs ( $ \mathbf{V}_B $ )	3
The number of MTs in a TG ( $ \mathbf{V}_T $ )	3
Cell radius (m)	3000
TG radius (m)	750

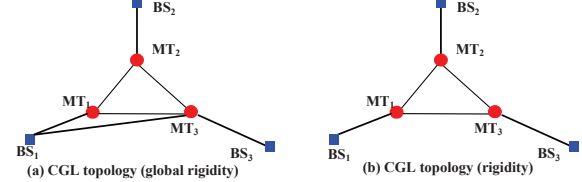


Fig. 4. CGL topology

### A. Performance analysis

As is shown in Fig.5, compare with the CGL-based Taylor-series algorithm, the proposed algorithm has higher localization accuracy in the same scenario. From Fig.5 and Fig.6, they indicate that the standard deviation between MT and BS  $\sigma_n$  has stronger effects on the localization accuracy than the standard deviation between MTs  $\sigma_q$  in the proposed algorithm. The reason is that BS can provide the information of higher accuracy, compared to the cooperative MT. Furthermore, they also showed that the proposed algorithm performs better in global rigidity topology than in rigidity topology. It is obvious that the global rigidity topology can provide more localization information.

Remarkably, the proposed algorithm in rigidity has the similar localization accuracy in global rigidity, as is shown in Fig.6. It shows the strong robustness of the proposed FG-based algorithm for different CGL topologies. The localization accuracy depends on how to utilize the available information. Generally, the variances can represent the practical environment effectively. The proposed algorithm makes full use of the variances in nearly each node of FG, which is considered as a great help for performance upgrade in rigidity, even with less information than global rigidity. Furthermore, the power of the proposed algorithm lies in its ability to distributively process the problem and to provide the optimum or near-optimum solutions, which makes it a good approach to solving distributed problem. On the other hand, CGL is a distributed problem, i.e., measurement parameters are only locally related. Therefore, the excellent combination between FG and CGL can relax the CGL topology requirement under ensuring the localization accuracy to some extent.

### B. Complexity analysis

This section analyzes the computation complexity of the proposed algorithms and Taylor-series algorithm, respectively.

In FG-based CGL algorithm, it is shown that the amount of messages passed between nodes is limited. And the iteration times is pre-defined, commonly  $K = 5$ . Because after  $K = 5$  the performance has no significant improvement with the growth of  $K$ . The performance of the proposed algorithm approaches high accuracy after 5 iterations, which shows that

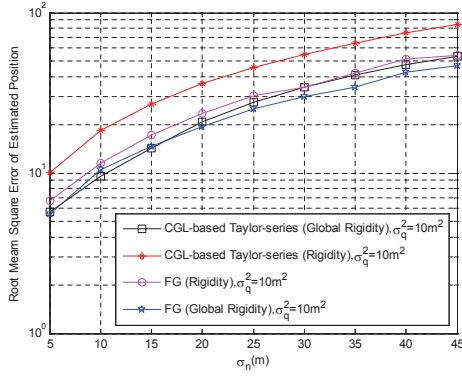


Fig. 5. RMSE comparison between FG-based and Taylor-series in CGL

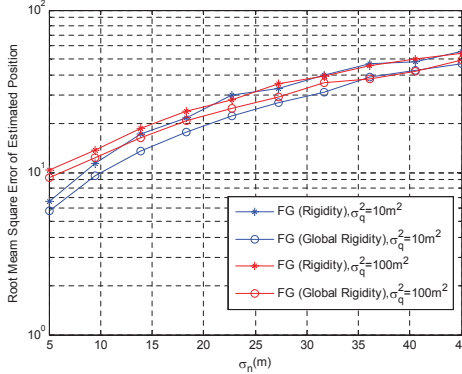


Fig. 6. RMSE of FG-based CGL algorithm in different CGL topologies

the convergence speed of the proposed algorithm is reasonably high. On the other hand, the convergence probability of the proposed algorithm is shown in Fig.7. The proposed algorithm has a very high convergence probability. Even when the standard derivation is big or the topology is not unique solvability, the convergence rate of the proposed algorithm is still acceptable. Those all lead to the low computational complexity and transmission loads while ensuring the localization accuracy.

In CGL-based Taylor-series localization algorithm [5], the most complex step is the solution to pseudo-inverse of large scale matrix. In each iteration, the computation complexity is  $O(N^3)$ , where  $N$  is the number of edges in CGL topology. Especially when the TG includes more cooperative MTs, the complexity of CGL-based Taylor-series localization algorithm increases sharply. Moreover, the iteration number is not definite, because it is determined by the tolerable localization error and the distance measurement error.

From the analysis above and simulation results, it can be concluded that the proposed algorithm can relax the CGL topology requirement and ensure the localization accuracy with low complexity to some extent.

## VI. CONCLUSION

This paper proposes a novel FG-based CGL algorithm in ill conditions. The corresponding analysis and simulations show that the proposed algorithm can relax the demand for CGL topology and provide high localization accuracy under low complexity, compared with the existing CGL algorithm. Next, the scenario of mobile MTs and the analysis of Cramer Rao

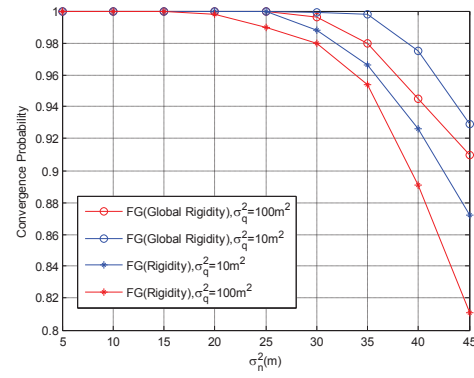


Fig. 7. Convergence probability of the proposed algorithm

Lower Bound (CRLB) will be considered in the future work.

## ACKNOWLEDGEMENT

This work is supported by the National Nature Science Foundation of China Project (Grant. 61001119), Fund for Creative Research Groups of China Project (Grant. 61121001) and Scientific and technological project of Guangdong Province Department of Transportation (Grant. 201102090).

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