Modified Tomlinson Harashima Precoding Using Square Root for Multi-User MIMO Systems

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***Abstract*—Tomlinson Harashima Precoding (THP) for Multi-User Multiple Input Multiple Output (MU-MIMO) system has been proposed as an effective way to achieve near capacity to the theoretical limit in MU-MIMO channels. In order to achieve better performance, other THP methods such as minimum mean square error (MMSE) THP and lattice reduction aided (LRA) THP are proposed. Those methods outperform the conventional THP, however, the computational complexity also increases. In this paper, we propose a modified THP by new approach using square root with less increase of computational complexity. Computer simulation shows that proposed method can achieve better trade-off between bit error rate and computational complexity than MMSE THP and LRA THP.**

I. INTRODUCTION

In recent years, wireless communication systems are installed in many kinds of devices such as mobile phones, notebook computers, and portable game devices. As these devices get widespread, the number of wireless communication users has increased. As a result, faster transmission speeds and better transmission performance are required. To meet these requirements, Multi-User Multiple Input Multiple Output (MU-MIMO) systems are likely to be adopted for downlink transmission in IEEE 802.11ac, 3G LTE, and 4G IMT-Advanced. MU-MIMO system can be seen as a MIMO system [1], but in the downlink its receivers are decentralized.

In MU-MIMO systems, spatial division multiple access (SDMA) technique which transmits data simultaneously to multiple users in the same frequency band , plays an important role in improving the transmission performance. One of SDMA methods called Channel Inversion (CI) [2] cancels inter-user-interference (IUI) by precoding transmitted symbols by the inverse of channel matrix. CI has low complexity, but the channel capacity is much worse than the theoretical limit in MU-MIMO channels. Meanwhile, non-linear precoding method, which achieves near capacity to the theoretical limit in MU-MIMO channels, is proposed. Vector Perturbation (VP) [3] is one of the non-linear precoding method. This method introduces a perturbation vector to minimize the precoded signal power. However, VP is not practical because obtaining an optimum solution requires very high complexity. On the other hand, Tomlinson Harashima Precoding (THP) [4],[5], which is also non-linear precoding, is proposed for MU-MIMO systems. In THP, the transmitter compensates for IUI in advance by using QR decomposition, and modulo operations are performed on transmitted signals in order to constrain the transmitted power for the compensation. In [6], V-BLAST THP which uses sorted QR decomposition is proposed to improve the transmission performance than QR based THP. THP has much smaller complexity than VP, and achieves better performance than linear precoding methods [7], therefore, there have been many proposals regarding THP [8],[9]. For example, in order to improve transmission performance, minimum mean square error (MMSE) THP [8] and lattice reduction aided (LRA) THP [9] are proposed as an improved THP algorithm. Their bit error rate (BER) performances achieve more improvement than conventional THP method, however computational complexity also increases. In this paper, we propose a modified THP scheme by new approach using square root with less increase of computational complexity. We also report the result of comparisons with existing THP methods in terms of transmission performance and the complexity required.

The rest of this paper is organized as follows. We describe the MU-MIMO system model in Section II. Conventional THP methods are described in Section III. The proposed THP method is explained in Section IV. Our computer simulation results and discussions are presented in Section V, and our conclusions are provided in Section VI.

II. MULTI-USER MIMO SYSTEM MODEL

In this section, we describe the MU-MIMO system model used in this paper. The composition of the MU-MIMO system assumes that the number of space streams used for data transmission is the same as the total number of user's receiver antennas. For the MU-MIMO system model, we assume a transmission link from a transmitter with M antennas to the receivers to K users. The number of k-th user's receiver antennas is L_k $(k = 1 \cdots K)$, and the number of total receiver antennas is $L = \sum_{k=1}^{K} L_k$. Furthermore, there is a constraint of $M \geq L$. In order to cancel IUI, we need to precode transmitted symbols, and transmission processes are explained as follows.

First, we precode a $L \times 1$ transmitted symbol vector $\mathbf{u}$, which is generated from bit sequence, by a $M \times L$ weight matrix $\mathbf{W}$. The $M \times 1$ precoded signal $\mathbf{s}$ is given by

$$\begin{aligned}\mathbf{s} &= \mathbf{W}\mathbf{u} \\ &= \begin{bmatrix} W_{1,1} & \cdots & W_{1,L} \\ \vdots & \ddots & \vdots \\ W_{M,1} & \cdots & W_{M,L} \end{bmatrix} \begin{bmatrix} u_1 & u_2 & \cdots & u_L \end{bmatrix}^T \end{aligned} \tag{1}$$

where m-th row l-th column elements of $\mathbf{W}$ is represented as $W_{m,l}$, and l-th element of $\mathbf{u}$ is described as u_l. A function

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$(\cdot)^T$ denotes the transpose. Second, we normalize the signal power of $\mathbf{s}$. If we define a power constraint on transmitted signals as $E\left[\|\mathbf{x}\|^2\right]=1$ where $E\left[\cdot\right]$ represents the expectation, a normalized transmitted signal vector can be represented by

$$\mathbf{x}=\frac{\mathbf{s}}{\sqrt{\gamma}}=\frac{1}{\sqrt{\gamma}}\begin{bmatrix}s_1 & s_2 & \cdots & s_M\end{bmatrix}^T \quad \left(\gamma=E\left[\|\mathbf{s}\|^2\right]\right) \tag{2}$$

where the m-th element of $\mathbf{s}$ is represented as s_m. Then, the $M\times 1$ signal $\mathbf{x}$ is transmitted from an access point to users. When there is no IUI, $\mathbf{y}_k$, which is the $L_k\times 1$ signal received by user k, can be described as

$$\begin{aligned}\mathbf{y}_k &= \mathbf{H}_k\mathbf{x}+\mathbf{n}_k\\ &= \begin{bmatrix}H_{1,1}^{(k)} & \cdots & H_{1,M}^{(k)}\\ \vdots & \ddots & \vdots\\ H_{L_k,1}^{(k)} & \cdots & H_{L_k,M}^{(k)}\end{bmatrix}\begin{bmatrix}x_1\\ \vdots\\ x_M\end{bmatrix}+\begin{bmatrix}n_1^{(k)}\\ \vdots\\ n_{L_k}^{(k)}\end{bmatrix}\end{aligned} \tag{3}$$

$$\mathbf{H}=\begin{bmatrix}\mathbf{H}_1^T & \mathbf{H}_2^T & \cdots & \mathbf{H}_k^T & \cdots & \mathbf{H}_K^T\end{bmatrix}^T \tag{4}$$

$$\mathbf{n}=\begin{bmatrix}\mathbf{n}_1^T & \mathbf{n}_2^T & \cdots & \mathbf{n}_k^T & \cdots & \mathbf{n}_K^T\end{bmatrix}^T \tag{5}$$

where l_k-th row m-th column elements of $\mathbf{H}_k$ is represented as $H_{l_k,m}^{(k)}$, m-th element of $\mathbf{x}$ is described as x_m, and l_k-th element of $\mathbf{n}_k$ is denoted as $n_{l_k}^{(k)}$. $\mathbf{H}_k$ in (3) is the $L_k\times M$ channel matrix from the access point to user k, and it is a partial matrix of $\mathbf{H}$ which represents an $L\times M$ channel matrix from the transmitter to individual users within a network with K users. We assume that each elements of channel $\mathbf{H}$ follows a complex Gaussian distribution with mean 0 and variance 1 (Rayleigh distribution). We also assume that the time variation of channel $\mathbf{H}$ is sufficiently long compared to the packet length and that the MU-MIMO system is not affected by the time variation of channel $\mathbf{H}$. $\mathbf{n}_k$ in (3) is a $L_k\times 1$ partial vector of $\mathbf{n}$, and $\mathbf{n}$ is a $L\times 1$ normalized complex Gaussian noise vector added to transmitted signals and its power is defined as $E[\mathbf{n}^H\mathbf{n}]=\sigma^2$.

The $L\times 1$ received signal vector $\mathbf{y}$ which is transmitted through the MU-MIMO network as a whole is given by

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{n} \tag{6}$$

If we use the channel inversion (CI) algorithm [2], the weight matrix $\mathbf{W}_{CI}$ is equal to $\mathbf{H}^\dagger$, where $(\cdot)^\dagger$ denotes the pseudo-inverse given by (7).

$$\mathbf{H}^\dagger=\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1} \tag{7}$$

where a function $(\cdot)^H$ denotes the Hermitian transpose. Therefore, equation (6) can be rewritten as

$$\mathbf{y}=\frac{\mathbf{1}}{\sqrt{\gamma_{CI}}}\mathbf{u}+\mathbf{n} \quad \left(\gamma_{CI}=E\left[\|\mathbf{s}\|^2\right]\approx\|\mathbf{H}^\dagger\|^2\right) \tag{8}$$

In (8), $\|\mathbf{Z}\|$ is defined by $\sqrt{\mathrm{trace}\left(\mathbf{Z}\mathbf{Z}^H\right)/I}$, where $\mathbf{Z}$ is any $I\times J$ matrix, and trace$(\cdot)$ is a function which gives sum of diagonal elements of any matrix. If a normalization gain is big, noise enhancement becomes significant.

III. Conventional Tomlinson Harashima Precoding Methods

In this section, we provide an explanation about two types of conventional THP methods. The conventional THP is based on zero forcing criteria to cancel IUI, and modulo operation is needed at the receiver side. We will provide an overview of the conventional THP method in reference [6] (Type 1), then, explain the method of reference [7] (Type 2).

A. THP : Type 1

First, we apply a sorted QR decomposition (SQRD) algorithm [10] to a Hermitian matrix of channel, as follows

$$\mathbf{H}^H\mathbf{P}=\mathbf{Q}\mathbf{R} \tag{9}$$

where $\mathbf{R}$ is a $L\times L$ upper triangular matrix, $\mathbf{Q}$ is a $M\times L$ unitary matrix, and $\mathbf{P}$ is a $L\times L$ permutation matrix which sorts columns of $\mathbf{H}^H$ so that $R_{i,i}\leq R_{j,j}$ for $i<j<L$ is approximated where ith row jth column elements of $\mathbf{R}$ is represented as $R_{i,j}$ [10]. If we define $\mathbf{D}$ is the $L\times L$ diagonal matrix which has the same diagonal elements of $\mathbf{R}$, equation (1) is rewritten as

$$\mathbf{s}=\mathbf{H}^\dagger\mathbf{u}=\mathbf{Q}\mathbf{D}^{-1}\left(\mathbf{R}^H\mathbf{D}^{-1}\right)^{-1}\mathbf{P}^H\mathbf{u} \tag{10}$$

The operation of $\left(\mathbf{R}^H\mathbf{D}^{-1}\right)^{-1}\mathbf{P}^H\mathbf{u}$ in (10) represents the compensation of IUI. By this operation, a transmit power becomes large, so the modulo operation represented by (11) is performed on this part in order to restrain the transmit power. Here, we assume $f_\tau\left(\left(\mathbf{R}^H\mathbf{D}^{-1}\right)^{-1}\mathbf{P}^H\mathbf{u}\right)=\mathbf{v}$, where the $\mathbf{v}$ is a $L\times 1$ vector. The function $f_\tau(\cdot)$ is a modulo operation which is given by

$$f_\tau(z)=\left(z-\left\lfloor\frac{z+\tau/2}{\tau}\right\rfloor\tau\right) \tag{11}$$

where the function $\lfloor\cdot\rfloor$ is the floor function, and τ is a positive real number which represents a symbol mapping width. The modulo operation lets I-ch and Q-ch of signals go into $[-\tau/2,\tau/2]$. The modulo operation can be regarded as a perturbation operation by $\tau\mathbf{l}$, where $\mathbf{l}$ is a $L\times 1$ complex integer vector. Then, a precoded signal and a normalized transmitted signal using THP (Type I) is expressed as

$$\mathbf{s}=\mathbf{Q}\mathbf{D}^{-1}\mathbf{v}=\mathbf{H}^\dagger\left(\mathbf{u}+\tau\mathbf{l}\right) \tag{12}$$

$$\mathbf{x}=\frac{\mathbf{s}}{\sqrt{\gamma_{THP1}}} \quad \left(\gamma_{THP1}=E\left[\|\mathbf{s}\|^2\right]\approx\|\mathbf{D}^{-1}\|^2\right) \tag{13}$$

Now, received signals and estimated signals are given by

$$\mathbf{y}=\frac{1}{\sqrt{\gamma_{THP1}}}\left(\mathbf{u}+\tau\mathbf{l}\right)+\mathbf{n} \tag{14}$$

$$\hat{\mathbf{y}}=\sqrt{\gamma_{THP1}}\mathbf{y}=\mathbf{u}+\tau\mathbf{l}+\sqrt{\gamma_{THP1}}\mathbf{n} \tag{15}$$

In (15), we define the average noise enhancement of whole system as follows

$$G_{t1}=\sqrt{\gamma_{THP1}}\approx\|\mathbf{D}^{-1}\| \tag{16}$$

In order to obtain the demodulated symbol vector, modulo operations are also performed on the estimated signal.

B. THP : Type 2

In THP Type 2 [7], the signal vector $\mathbf{s}$ is given by,

$$\mathbf{s} = \mathbf{Q}\left(\mathbf{D}^{-1}\mathbf{R}^H\right)^{-1}\mathbf{P}^H\mathbf{u} \tag{17}$$

If we assume $f_\tau\left(\left(\mathbf{D}^{-1}\mathbf{R}^H\right)^{-1}\mathbf{P}^H\mathbf{u}\right) = \mathbf{v}$, a precoded signal and a normalized transmitted signal are given by

$$\mathbf{s} = \mathbf{Q}\mathbf{v} = \mathbf{Q}\left(\mathbf{R}^H\right)^{-1}\mathbf{D}\mathbf{P}^H(\mathbf{u}+\tau\mathbf{l}) \tag{18}$$

$$\mathbf{x} = \frac{\mathbf{s}}{\sqrt{\gamma_{THP2}}} \quad \left(\gamma_{THP2} = E\left[\|\mathbf{s}\|^2\right] \approx 1\right) \tag{19}$$

Since the equation (18) does not follow the information of $\mathbf{H}^\dagger$ completely, the diagonal elements of matrix $\mathbf{R}$ remain in received signals. The received signals are given by

$$\mathbf{y} = \frac{1}{\sqrt{\gamma_{THP2}}}\mathbf{P}\mathbf{D}\mathbf{P}^H(\mathbf{u}+\tau\mathbf{l}) + \mathbf{n} \tag{20}$$

In (20), $\mathbf{PDP}^H$ becomes a $L\times L$ diagonal matrix. Under the assumption that receivers can estimate CSI perfectly, we multiply the received signal $\mathbf{y}$ by inverse of $\mathbf{PDP}^H$ and normalization gain as follows,

$$\begin{aligned}\hat{\mathbf{y}} &= \sqrt{\gamma_{THP2}}\left(\mathbf{PDP}^H\right)^{-1}\mathbf{y} \\ &= \mathbf{u}+\tau\mathbf{l}+\sqrt{\gamma_{THP2}}\left(\mathbf{PDP}^H\right)^{-1}\mathbf{n}\end{aligned} \tag{21}$$

In (21), we define the average noise enhancement factor of whole system as follows

$$G_{t2} = \sqrt{\gamma_{THP2}}\left\|\left(\mathbf{PDP}^H\right)^{-1}\right\| \approx \|\mathbf{D}^{-1}\| \tag{22}$$

In (21), the SNR of each stream depends on the diagonal elements of $\mathbf{R}$.

IV. Proposed Tomlinson Harashima Precoding Method

From THP Type 1 and Type 2, a general expression of precoded signal for THP can be described as (23). Fig. 1 shows the equivalent time discrete system model used for THP, where $\mathbf{F}$ is a $L\times L$ feedback matrix defined by $\mathbf{F} = \mathbf{B}\mathbf{R}^H\mathbf{A} - \mathbf{I}$.

$$\mathbf{s} = \mathbf{Q}\mathbf{A}f_\tau\left(\left(\mathbf{B}\mathbf{R}^H\mathbf{A}\right)^{-1}\mathbf{P}^H\mathbf{u}\right) \tag{23}$$

where $\mathbf{A}$ and $\mathbf{B}$ are $L\times L$ diagonal matrices, and there is a constraint of $\mathbf{AB} = \mathbf{D}^{-1}$. We assume $A_{l,l}$ and $B_{l,l}$ are the l-th row l-th column elements of $\mathbf{A}$ and $\mathbf{B}$ respectively. Next, a general expression of average noise enhancement factor is given by

$$\begin{aligned}G &= \|\mathbf{A}\|\|\mathbf{B}\| = \sqrt{\frac{\mathrm{trace}\left(\mathbf{AA}^H\right)}{L}}\sqrt{\frac{\mathrm{trace}\left(\mathbf{BB}^H\right)}{L}} \\ &= \frac{1}{L}\sqrt{\sum_{l=1}^{L}A_{l,l}^2\sum_{l=1}^{L}B_{l,l}^2}\end{aligned} \tag{24}$$

Our purpose is to decrease the noise enhancement factor G. The values of $\mathbf{A}$ and $\mathbf{B}$ of conventional THP methods can be described as $\mathbf{A} = \mathbf{D}^{-1}$ and $\mathbf{B} = \mathbf{I}$ in Type 1, and $\mathbf{A} = \mathbf{I}$ and $\mathbf{B} = \mathbf{D}^{-1}$ in Type 2. If we define the average noise enhancement factor of conventional THP as G_c, G_c is expressed as

$$G_c = G_{t1} = G_{t2} = \sqrt{\frac{1}{L}\sum_{l=1}^{L}\frac{1}{D_{l,l}^2}} \tag{25}$$

In order to obtain the minimum value of G, we use the theorem of Cauchy-Schwarz inequality in (24) as follows

$$G = \frac{1}{L}\sqrt{\sum_{l=1}^{L}A_{l,l}^2\sum_{l=1}^{L}B_{l,l}^2} \geq \frac{1}{L}\sum_{l=1}^{L}A_{l,l}B_{l,l} \tag{26}$$

According to the Cauchy-Schwarz inequality, G in (26) becomes minimum when $B_{1,1}/A_{1,1} = B_{2,2}/A_{2,2} = \cdots = B_{L,L}/A_{L,L}$, that is $\mathbf{A} = \mathbf{B} = \sqrt{\mathbf{D}^{-1}}$. We define that $\sqrt{\mathbf{Z}}$ is a matrix which has square root of each element of $\mathbf{Z}$, where $\mathbf{Z}$ is any matrix. Therefore we assume that $\mathbf{A} = \sqrt{\mathbf{D}^{-1}}$ and $\mathbf{B} = \sqrt{\mathbf{D}^{-1}}$. Then the noise enhancement factor of proposed method G_p is given by

$$G_p = \frac{1}{L}\sum_{l=1}^{L}\frac{1}{D_{l,l}} \tag{27}$$

Then, we can obtain next relationship.

$$G_c = \sqrt{\frac{1}{L}\sum_{l=1}^{L}\frac{1}{D_{l,l}^2}} \geq \frac{1}{L}\sum_{l=1}^{L}\frac{1}{D_{l,l}} = G_p \tag{28}$$

where the equality is attained if and only if all diagonal elements of $\mathbf{D}$ have same values, and (28) can be proved by mathematical induction. Therefore, from (28), we can obtain the relationship of $G_c \geq G_p$. Hence, the precoded signal of proposed method is given by (29).

$$\mathbf{s} = \mathbf{Q}\sqrt{\mathbf{D}^{-1}}f_\tau\left(\left(\sqrt{\mathbf{D}^{-1}}\mathbf{R}^H\sqrt{\mathbf{D}^{-1}}\right)^{-1}\mathbf{P}^H\mathbf{u}\right) \tag{29}$$

If we assume $\mathbf{v} = f_\tau\left(\left(\sqrt{\mathbf{D}^{-1}}\mathbf{R}^H\sqrt{\mathbf{D}^{-1}}\right)^{-1}\mathbf{P}^H\mathbf{u}\right)$, a precoded signal and a normalized transmitted signal are given by

$$\mathbf{s} = \mathbf{Q}\sqrt{\mathbf{D}^{-1}}\mathbf{v} = \mathbf{Q}\left(\mathbf{R}^H\right)^{-1}\sqrt{\mathbf{D}}\mathbf{P}^H(\mathbf{u}+\tau\mathbf{l}) \tag{30}$$

$$\mathbf{x} = \frac{\mathbf{s}}{\sqrt{\gamma_{THPP}}} \quad \left(\gamma_{THPP} = E\left[\|\mathbf{s}\|^2\right] \approx \|\sqrt{\mathbf{D}^{-1}}\|^2\right) \tag{31}$$

Then, the received signal is described as

$$\mathbf{y} = \frac{1}{\sqrt{\gamma_{THPP}}}\mathbf{P}\sqrt{\mathbf{D}}\mathbf{P}^H(\mathbf{u}+\tau\mathbf{l}) + \mathbf{n} \tag{32}$$

In (32), $\mathbf{P}\sqrt{\mathbf{D}}\mathbf{P}^H$ also becomes a diagonal matrix. Next, we multiply the received signal $\mathbf{y}$ by inverse of $\mathbf{P}\sqrt{\mathbf{D}}\mathbf{P}^H$ and normalization gain as before.

$$\begin{aligned}\hat{\mathbf{y}} &= \sqrt{\gamma_{THPP}}\left(\mathbf{P}\sqrt{\mathbf{D}}\mathbf{P}^H\right)^{-1}\mathbf{y} \\ &= \mathbf{u}+\tau\mathbf{l}+\sqrt{\gamma_{THPP}}\left(\mathbf{P}\sqrt{\mathbf{D}}\mathbf{P}^H\right)^{-1}\mathbf{n}\end{aligned} \tag{33}$$

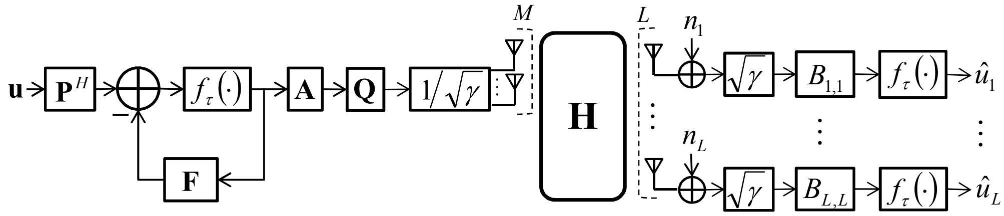

Fig. 1. General expression of downlink transmission model for THP

TABLE I
COMPARISON OF NORMALIZATION FACTOR

	A	**B**	G
Type 1	$\mathbf{D}^{-1}$	$\mathbf{I}$	$\left\|\mathbf{D}^{-1}\right\|$
Type 2	$\mathbf{I}$	$\mathbf{D}^{-1}$	$\left\|\mathbf{D}^{-1}\right\|$
Proposed	$\sqrt{\mathbf{D}^{-1}}$	$\sqrt{\mathbf{D}^{-1}}$	$\left\|\sqrt{\mathbf{D}^{-1}}\right\|^2$

In (33), we can obtain the noise enhancement factor of proposed method as follows

$$G_p = \sqrt{\gamma_{THPP}} \left\| \left(\mathbf{P}\sqrt{\mathbf{D}}\mathbf{P}^H \right)^{-1} \right\| \approx \left\| \sqrt{\mathbf{D}^{-1}} \right\|^2 \qquad (34)$$

Table I shows the comparison of noise enhancement factor between conventional THP and Proposed THP.

V. EVALUATION OF TRANSMISSION PERFORMANCE

In this section, we compare the proposed THP method and existing THP methods in terms of transmission performance and computational complexity. For comparison, we use CI [2], MMSE THP method [8] and LRA THP [9] together with conventional THP Type 1 and Type 2, which were described in Section III. We also consider the case when we use QR decomposition (QRD), that uses an identity matrix $\mathbf{I}$ as a permutation matrix $\mathbf{P}$, as a decomposition algorithm for THP.

A. Evaluation of Noise Enhancement

Firstly, we will verify the magnitude of noise enhancement factor of proposed method and conventional THP Type 1 and Type 2. Figs. 2 and 3 give the complementary cumulative distribution function (CCDF) of the noise enhancement factor of QRD-THP and SQRD-THP respectively. In Fig. 2, the noise enhancement factor of proposed method is smaller than that of conventional method by approximately 2 dB at CCDF=10^{-2}. Meanwhile, in Fig. 3, the noise enhancement factor of proposed method is smaller by 2 dB at CCDF=10^{-2}. Hence, we can confirm that there is a relationship of $G_c \geq G_p$ at the same probability.

From these results, the noise enhancement factor gets small by using proposed method.

B. Evaluation of Bit Error Rate Performance

Table II shows the parameters of simulation. Fig. 4 shows the BER-SNR property in the case of QRD. The BER performance of the proposed THP is improved compared to CI, THP Type 1, Type 2 by 9 dB, 4 dB and 5 dB at BER = 10^{-4} respectively. Note that proposed THP is also improved

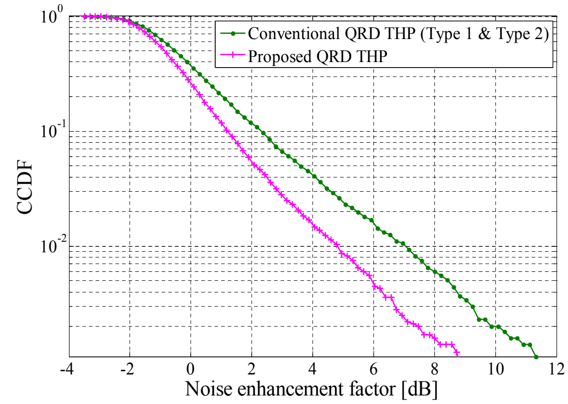

Fig. 2. CCDF of average noise enhancement factor of QRD-THP

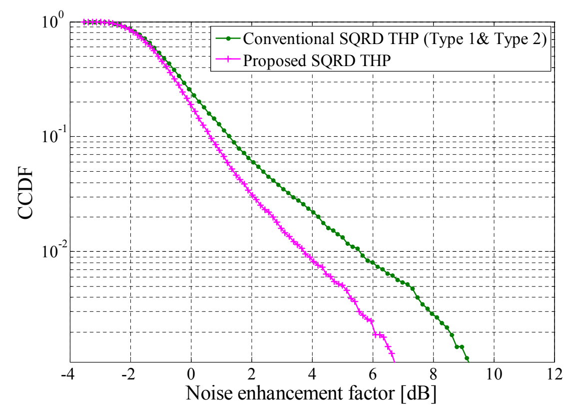

Fig. 3. CCDF of average noise enhancement factor of SQRD-THP

compared to MMSE THP of Type 1 and Type 2 by 1 dB and 3 dB respectively. Meanwhile, LRA THP has better performance than proposed THP by 5 dB.

The BER-SNR property in SQRD case is shown in Fig. 5. The BER performance of proposed THP method is improved compared to CI, THP Type 1, Type 2 and MMSE THP Type 1, Type 2 by 12 dB, 4 dB, 3 dB, 2 dB and 1 dB at BER = 10^{-4} respectively. On the other hand, the BER performance of proposed THP is degraded than that of LRA THP by 2 dB, which is smaller than the degradation in QRD case.

C. Comparison of the Computational Complexity

In order to compare the computational complexity between the proposed THP method and existing THP methods, we assess the number of additional complex multiplications compared to conventional THP method. LRA THP is based on

TABLE II
MU-MIMO SIMULATION PARAMETER

Number of Tx antennas	4
Number of Rx antennas	2 x 2 users
Modulation	64QAM
Coding	Convolutional encoding/Viterbi decording (coding rate : 3/4)
Precoding	CI, THP (Type 1 & Type 2) MMSE THP (Type 1 & Type 2) LRA THP, Proposed THP (QRD & SQRD for THP)
Number of symbols	100
SNR	10-40dB
Channel	Rayleigh
Channel estimation	Ideal
Iteration	10,000

TABLE III
ADDITIONAL COMPUTATIONAL COMPLEXITY COMPARED TO CONVENTIONAL METHOD

Proposed	MMSE [8]	LRA [9]
$O(M)$	$O(M^3)$	$O(M^4 logM)$

THP Type 1, therefore we set THP Type 1 as a reference for comparison.

Table III shows the comparison of additional computational complexity. The proposed THP method requires an additional multiplication of a diagonal matrix, and its additional computational complexity becomes $O(M)$. Meanwhile, MMSE THP and LRA THP require more computational complexity. In the case of MMSE, we need to calculate a pseudo inverse of channel matrix including a regularization parameter of SNR. Therefore its additional complexity could be $O(M^3)$. In the case of LRA THP, Lenstra-Lenstra-Lovasz (LLL) algorithm is employed, and its theoretic average complexity is $O(M^4 logM)$ which is derived in [11].

From these results, the proposed THP method could be achieved with smaller additional computational complexity than MMSE THP and LRA THP.

VI. CONCLUSION

In this paper, we proposed a novel THP method using square root. The proposed method was compared with CI and existing THP in terms of transmission performance and complexity. The simulation result showed that the proposed THP outperformed conventional THP and MMSE THP. Furthermore, the proposed THP achieves nearly the same level of transmission performance of LRA THP with less increase of computational complexity. These results lead us to conclude that the proposed method, which has better trade off between transmission performance and computational complexity than existing THP methods, is a sufficiently effective method.

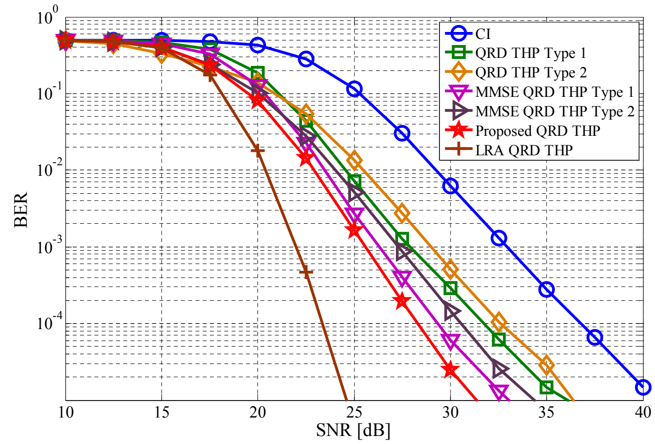

Fig. 4. BER-SNR for QRD-THP

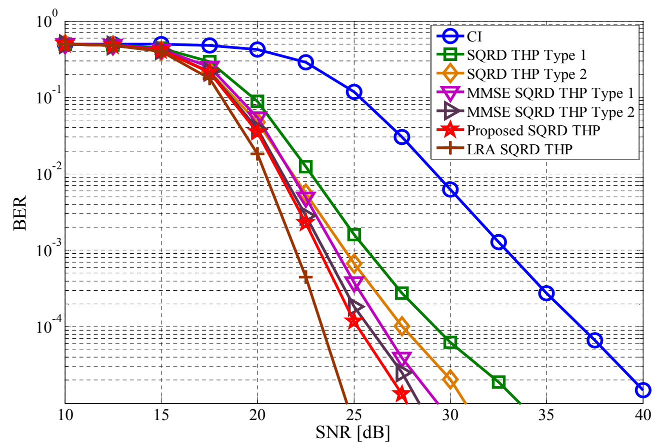

Fig. 5. BER-SNR for SQRD-THP

REFERENCES

[1] G.J. Foschini, and M.J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, 1998.

[2] C.B. Peel, B.M. Hochwald, and A.L. Swindlehurst, "A Vector- Perturbation Technique for Near-Capacity Multiantenna Multiuser Communication - Part I: Channel Inversion and Regularization," *IEEE Trans. Commun.*, vol. 53, Issue. 1, pp. 195-202, Jan. 2005.

[3] C.B. Peel, B.M. Hochwald, and A.L. Swindlehurst, "A Vector- Perturbation Technique for Near-Capacity Multiantenna Multiuser Communication -Part II: Perturbation," *IEEE Trans. Commun.*, vol. 53, Issue. 3, pp. 537-544, Mar. 2005.

[4] M. Tomlinson, "New automatic equalizer employing modulo arithmetic," *Electronics Letters*, vol. 7, no. 5/6, pp. 138-139, Mar. 1971.

[5] H. Harashima and H. Miyakawa, "Matched-transmission technique for channels with intersymbol interference," *IEEE Trans. Commun.*, vol. 20, no. 4, pp. 774-780, Aug. 1972.

[6] R. Habendorf, and G. Fettweis, "On Ordering Optimization for MIMO Systems with Decentralized Receivers," *IEEE Vehicular Technology Conference, 2006. VTC2006-Spring*. vol. 4, pp. 1844-1848, May. 2006.

[7] C.B. Peel, Q.H. Spencer, A.L. Swindlehurst, M. Haardt, and B.M. Hochwald, "Linear and Dirty-Paper Techniques for the Multi-User MIMO Downlink," in *Space-Time Processing for MIMO Communications*, ed., A. Gershman, and N. Sidiropoulos, pp. 209-244, John Wiley & Sons, Ltd, New York, 2005.

[8] J. Liu, and W.A. Krzymien "Improved Tomlinson-Harashima precoding for the downlink of multiple antenna multi-user systems," *IEEE Wireless Communications and Networking Conference, 2005*, vol. 1, pp. 466-472, May. 2005.

[9] C. Stierstorfer, and R.F.H. Fischer "Lattice-Reduction-Aided Tomlinson-Harashima Precoding for Point-to-Multipoint Transmission," *AEU - International Journal of Electronics and Communications*, vol. 60, pp. 328-330, April. 2006.

[10] D. Wübben, R. Böhnke, V. Kühn, and K. Kammeyer, "MMSE Extension of V-BLAST based on Sorted QR Decomposition," *IEEE Vehicular Technology Conference, VTC2003-Fall*. vol. 1, pp. 508-512, Oct. 2003.

[11] A.K. Lenstra, H.W. Lenstra, Jr., and L. Lovász, "Factoring polynomials with rational coefficients." *Math. Ann.*, vol. 261, pp. 515-534, 1982.