On Reliable Multicast with Network coding-ARQ for Relay Cooperation Cells

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***Abstract*—As a substantial means for improving throughtput, network coding has recently attracted much attention to apply in wireless multicast services. To guarantee the reliable multicast, packet retransmission schemes are employed. With one source, network coding has been well studied for the reliable multicast. But for the multicast cell with two sources, the presented schemes are not good enough to approach an impressive effect with the multiple receivers, demands and link situations. In this paper, for the 2-1-(D_1,D_2) multicast cell, which has two sources, one relay and two groups users with the receivers numbers of D_1 and D_2, we propose three schemes to reduce the number of retransmission, which combine the lost packets and retransmit them with network coding, and obtains the least number of retransmissions. Some theoretical results are derived on the bandwidth efficiency of the traditional automatic repeat-request scheme (ARQ), network coding with ARQ scheme (XOR-ARQ) and the improved network coding with ARQ scheme (iXOR-ARQ). Compared with ARQ and XOR-ARQ, iXOR-ARQ is more advantageous on the bandwidth efficiency and the coding gain, which has been confirmed by both simulations and theoretical analysis.**

Keywords-network coding; two sources; multicast; bandwidth efficiency

I. INTRODUCTION

Network coding is a powerful technique introduced at the turn of the millennium, which can allow intermediate nodes not only to forward, but also to combine and process the incoming information flows [1]. The ability, to recode the data at the intermediate nodes, results in a substantial bandwidth improvement over that of the traditional store and forward network [2]-[6]. Notably, network coding techniques have been applied to increase bandwidth efficiency in wireless ad hoc network [2]-[4]. In this paper, we focus on the bandwidth efficiency for multicast cells with two sources network coding.

Multicast is a way to distribute information from one source to multiple intended destinations. It has been extensively investigated in [2]-[9], and widely applicable in many scenarios. On the other hand, the channels, between the source and destinations, may not be good enough all the time to guarantee reliable transmissions, thus appropriate multicast schemes are required.

Multicast with network coding has been discussed in [2]-[6] for its validity. At the same time, network coding schemes can also be used to enhance the reliability, bandwidth efficiency and the capacity of multicast scheme. In [2]-[4], the reliability of multicast schemes was well studied. Network coding schemes were proposed in [2], [3] to reduce the number of multicast transmissions. In [4], the reliability gain of network coding was quantified for reliable multicasting and went deep into a tree-based multicast. Multiuser ARQ was extended to multicasting to pursue a throughput optimal, low en-/decoding complexity enabling and low overhead algorithm in [5]. In [6], a new recovery scheme joint with the application of network coding was proposed to enhance the reliability and the capacity of multicast communication.

For the cooperative system with one source, one relay and numerous destinations [7], the analysis show that cooperation substantially increased the maximum stable throughput rate and higher throughput gain could be obtained by using random linear network coding at the relay. An optimal network coding scheme was presented in [8] to maximize throughput subject to delay and queue length constrains. Numerical results demonstrated that network coding could bring significant gains in terms of throughput.

Previous works mainly pay attention to cooperative multicast system of one source. Basic multicast cell with two sources was studied in [10]. J. Li deduced the optimal statistical channels state information based power allocation and gave comparison among schemes.

In this paper, bandwidth efficiency of different network coding schemes for a general multicast cell will be discussed. The theoretical analysis and simulations will demonstrate the advantages of the proposed scheme. The rest of this paper is organized as follows. In Section II, we illustrate the two sources system model 2-1-(D_1,D_2) and its parameters. In Section III, we will describe different retransmission-based multicast schemes with and without network coding. We then analyze their performance in terms of bandwidth efficiency and coding gain in Section IV. Section V illustrates the simulation results that verify our theoretical predictions. Section VI concludes the paper.

II. SYSTEM MODEL

In Fig.1, a basic multicast cell is presented with 2 sources 1relay and 2 sets of destinations $d_1 = \{d_{11}, d_{12},..., d_{1D1}\}$, $d_2 = \{d_{21}, d_{22},\ldots, d_{2D2}\}$, which is 2-1-($D_1$,$D_2$) model. Suppose that both S_1 and S_2 transmit their messages to the two destination sets d_1 and d_2 simultaneously. However, d_1 (or d_2) is out of the transmission range of S_2 (or S_1), so they use the shared relay R to get the messages of S_2 (or S_1).

To describe our proposed schemes, we make the following assumptions for all the multicast schemes.

1. Each set of d_1, d_2 has D_i receivers, $i=\{1,2\}$ and $D_i \geqslant 1$.
2. Messages are sent in packets, and each packet is sent in a time slot of fixed duration.
3. The sender has access to the information on packet loss of all the receivers at any time slot through the use of positive and negative acknowledgments (ACK/NAKs) and instantaneous ACK/NAKs are assumed for simplicity.
4. A packet loss at each receiver follows a Bernoulli trial with parameter p. As shown in Fig.1, the packet loss from S_1 to R and from S_2 to R is p_1, packet loss from R to d_1 and d_2 is p_2, and packet loss from S_1 to d_1 and from S_2 to d_2 is p_3, $p_1 \leqslant p_2 \leqslant p_3$.

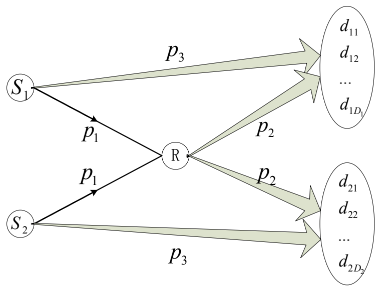

Figure 1. 2-1-(D_1,D_2) wireless multicast system

Assume d_1 and d_2 are to receive M packets of S_1 and M packets of S_2, then the process can be divided into to two phases – Sending Phase and Retransmitting Phase. Let F_1 denotes the packets of S_1, and F_2 denotes that of S_2. Then we have $F_1=\{F_{11}, F_{12}, \ldots, F_{1M}\}$, $F_2=\{F_{21}, F_{22}, \ldots, F_{2M}\}$.

In Sending Phase, both S_1 and S_2 sending one packet to d_1 and d_2 will take 3 transmissions,

1. $S_1 \rightarrow \{R, d_1\}$ with F_{1i},
2. $S_2 \rightarrow \{R, d_2\}$with F_{2i},
3. $R \rightarrow \{d_1, d_2\}$ with $f(F_{1i}, F_{2i})$, if R has received the above two packets correctly.

where $f(\cdot)$ denotes the network coding protocol. Here we use XOR network coding protocol. In this way, the three steps repeat as i increases from 1 to M, and the Sending Phase ends till all $2M$ packets are sent out.

In Retransmitting Phase, the proposed schemes will be used to reduce the transmissions.

III. MULTICAST SCHEMES

A. typical ARQ Scheme

In this scheme, the sender maintains a list of lost packets and their corresponding receivers for which their packets are lost. The sender retransmits each lost packets then. The receiver immediately sends a NAK if a packet is not received in the current time slot nor in any previous time slot. Thus the list of lost packets is immediately refreshed, and the retransmission will be carried on.

The ARQ Scheme consists of 3steps;

1. S_1 retransmits the lost packets of $\{R, d_1\}$ of F_1 using the above ARQ scheme, to make sure that each of $\{R,d_1\}$ receives F_1 correctly.
2. S_2 retransmits the lost packets of $\{R, d_2\}$ of F_2 using the above ARQ scheme, to make sure that each of $\{R,d_2\}$ receives F_2 correctly.
3. R retransmits the lost packets of $\{d_1,d_2\}$ of $XOR(F_1,F_2)$ using the above ARQ scheme, to make sure that each of $\{d_1, d_2\}$ receives $XOR(F_1, F_2)$ correctly.

B. XOR-ARQ Scheme

During the Retransmitting Phase, the sender forms a new packet by XORing a maximum set of the lost packets from different receivers before retransmitting the combined packets to all the receivers. Each receiver could have no more than one lost packet in this set. If the combined packet is lost in the retransmission, it will be immediately retransmitted until all the receivers who need it receive the packet correctly. Then the sender carries on sending the combined packets of other sets of packets, until there are no lost packets on the list. The receiver could recover its lost packet by XORing the received combined packet with the appropriate correct packets.

The XOR-ARQ Scheme also consists of 3steps which is similar to that of the ARQ Scheme by replacing the *ARQ Scheme* by *XOR-ARQ Scheme.*

C. Improved XOR-ARQ Scheme (iXOR-ARQ)

Obviously, the XOR-ARQ Scheme is more optimal than the ARQ Scheme, but when the loss probability from the source to the relay and the destinations differs largely, the advantage of XOR-ARQ Scheme turns dim. With the main idea of sending the most information through one transmission, we propose the Improved XOR-ARQ Scheme.

R	$XOR(F_1,F_2)$				
d_1	x	2	3	x	5
d_2	1	x	3	x	5

Figure 2. Packets combining for XOR-ARQ at R

R	F_1					F_2				
d_1	1	2	x	4	5	x	x	x	4	5
d_2	x	2	x	4	5	1	2	3	x	5

Figure 3. Packets combining for iXOR-ARQ at R

For simplification, define *XORCOD* as *form a new packet by XORing a maximum set of the lost packets from different receivers*. Then the iXOR-ARQ Scheme consists of 3steps;

1. S_1 retransmits the lost packets of $\{R, d_1\}$ of F_1 using *XORCOD*ing, to make sure R receives F_1 correctly, until there are no more lost packet on the state list of R.
2. S_2 retransmits the lost packets of $\{R, d_2\}$ of F_2 using *XORCOD*ing, to make sure R receives F_1 correctly.

3. Join the list of F_1 and F_2 of d_1 at R, join the list of F_1 and F_2 of d_2 at R. Then R retransmits the lost packets of $\{d_1, d_2\}$ of F_1F_2 using *XORCOD*ing for the joint lost packets list, to make sure that each of $\{d_1, d_2\}$ receives F_1, F_2 correctly.

The difference between the latter two schemes is shown for the situation of D_1=D_2=1 in Fig.2 and 3.

IV. TRANSMISSION BANDWIDTH ANALYSIS

To compare the three proposed schemes, we define the transmission bandwidth as the average number of transmissions that are required to successfully transmit a packet to all the destinations. Let η denote the transmission bandwidth. Let D_1, D_2 denote the numbers of receivers. Let M denote the number of packets each source has to transmit, and let n denote the number of transmissions when each receiver receives all transmitted packets successfully. Thus

$$\eta=\frac{E[n]}{2M}\ , \tag{1}$$

where $E[n]$ denotes expectation of n.

A. Specified occasion with D_1=D_2=1

Proposition 1: The transmission bandwidths of ARQ scheme, XOR-ARQ scheme and iXOR-ARQ scheme with D_1=D_2=1 are

$$\eta_{ARQ}=\eta_B(p_1,p_3)+\frac{1}{2}\eta_B(p_2,p_2) \tag{2}$$

$$\begin{aligned}\eta_{XOR-ARQ}&=\eta_C(p_1,p_3)+\frac{1}{2}(1-p_1)^2\\&+\frac{1}{2}\eta_B(p_2,p_2)\left[1-(1-p_1)^2(1-p_2)\right]\end{aligned} \tag{3}$$

$$\begin{aligned}\eta_{iXOR-ARQ}&=1+\frac{p_1}{1-p_1}+\frac{1}{2}(1-p_1)^2\\&+\frac{1}{2}\begin{bmatrix}1+p_3-p_1(1-p_3)\\-(1-p_1)^2(1-p_2)\end{bmatrix}\eta_B(p_2,p_2)\end{aligned} \tag{4}$$

where

$$\eta_B(p_1,p_2)=\frac{1}{1-p_1}+\frac{1}{1-p_2}-\frac{1}{1-p_1p_2} \tag{5}$$

$$\eta_C(p_1,p_2)=1+\frac{p_1}{1-p_1}+\frac{p_2}{1-p_2}-\frac{p_1}{1-p_1p_2} \tag{6}$$

Proof: From [3], for the case of one source and two destinations with loss probability p_1 and p_2, we get that the transmission bandwidth of ARQ is as (5), and the transmission bandwidth of XORARQ is as (6).

1) For ARQ Scheme

For D_1=D_2=1, ARQ is used for $S_1\to\{R, d_1\}$, $S_2\to\{R, d_2\}$and $R\to\{d_1, d_2\}$.Therefore, the total number of transmissions that are required to successfully deliver all $2M$ packets to two receivers is

$$n=2M\eta_B(p_1,p_3)+M\eta_B(p_2,p_2) \tag{7}$$

So the average number of transmissions per successful packet is

$$\begin{aligned}\eta_{ARQ}&=\frac{E[n]}{2M}=\frac{2M\eta_B(p_1,p_3)+M\eta_B(p_2,p_2)}{2M}\\&=\eta_B(p_1,p_3)+\frac{1}{2}\eta_B(p_2,p_2)\end{aligned} \tag{8}$$

2) For XOR-ARQ Scheme

Let random variables X_1, X_2, X_3 donate the numbers of transmission attempts for the successful transmission for $S_1\to\{R, d_1\}$, $S_2\to\{R, d_2\}$and $R\to\{d_1, d_2\}$. Then we have

$$E[n]=E[X_1]+E[X_2]+E[X_3], \tag{9}$$

and from (6), we have

$$E[X_1]=E[X_2]=M\eta_C(p_1,p_3) \tag{10}$$

For $R\to\{d_1, d_2\}$, R sends $M(1-p_1)^2$ packets in the Sending Phase and retransmit $M[1-(1-p_1)^2(1-p_2)]$ packets in the Retransmitting Phase, therefore

$$E[X_3]=(1-p_1)^2+\eta_B(p_2,p_2)\left[1-(1-p_1)^2(1-p_2)\right] \tag{11}$$

Therefore we get the derivation of (3).

3) For iXOR-ARQ Scheme

In the Sending Phase, we have the number of transmissions to be n_1, then that in the Retransmitting Phase to be n_2,

$$n_1=2M+M(1-p_1)^2 \tag{12}$$

$$\begin{aligned}n_2&=2Mp_1\frac{1}{1-p_1}\\&+[Mp_3-Mp_1(1-p_3)]\eta_B(p_2,p_2)\\&+[M-M(1-p_1)^2(1-p_2)]\eta_B(p_2,p_2)\end{aligned} \tag{13}$$

For n_1, the derivation is simple because S_1, S_2 send M packets independently and R send $M(1-p_1)^2$ packets. When it comes to n_2, things turn more complex. In (13), the first item denotes the retransmissions needed for $S_1\to\{R, d_1\}$ and $S_2\to\{R, d_2\}$; the other two items denote the retransmissions needed for $R\to\{d_1, d_2\}$; take d_1 for instance, the polynomial in the square brackets of the second item denotes the residual lost packets after the retransmission for $S_1\to\{R, d_1\}$, and that of the third item denotes the lost packets after the sending procedure of $R\rightarrow\{d_1, d_2\}$; d_2 has the loss packets as d_1 because of the same loss probability on the corresponding links; thus the loss packets of d_1 and d_2 could be combined and retransmitted.

And

$$n = n_1 + n_2 \tag{14}$$

Thus according to (1), (4) is derived.

B. General occasion with D_1, D_2

Proposition 2: The transmission bandwidths of ARQ scheme, XOR-ARQ scheme and iXOR-ARQ scheme with general D_1 and D_2 are

$$\eta_{ARQ} = 0.5\left(\eta_{BS}(D_1)+\eta_{BS}(D_2)+\eta_{BR}\right) \tag{15}$$

$$\eta_{XOR-ARQ} = \tfrac{1}{2}\begin{Bmatrix}\eta_C(D_1)+\eta_C(D_2)+(1-p_1)^2\\ +\left[1-(1-p_1)^2(1-p_2)\right]\eta_{BR}\end{Bmatrix} \tag{16}$$

$$\begin{aligned}\eta_{iXOR-ARQ} &= 1+\frac{p_1}{1-p_1}+\frac{1}{2}(1-p_1)^2\\ &+\frac{1}{2}\begin{bmatrix}1+p_3-p_1(1-p_3)\\ -(1-p_1)^2(1-p_2)\end{bmatrix}\eta_{BR}\end{aligned} \tag{17}$$

$$\eta_{BS}(D) = \sum_{k=1}^{D}\frac{(-1)^{k-1}C_D^k}{1-p_3^k}+\sum_{k=0}^{D}\frac{(-1)^k C_D^k}{1-p_1p_3^k} \tag{18}$$

$$\eta_{BR} = \sum_{k=1}^{D_1+D_2}\frac{(-1)^{k-1}C_{D_1+D_2}^k}{1-p_2^k} \tag{19}$$

$$\eta_C(D) = 1+p_1\eta_{BS}(D)+(p_3-p_1)\sum_{k=1}^{D}\frac{(-1)^{k-1}C_D^k}{1-p_3^k} \tag{20}$$

Equation (18) denotes the average transmissions for a successful packet for $S_1\rightarrow\{R, d_1\}$ and $S_2\rightarrow\{R, d_2\}$ using ARQ, (20) denotes that using XORARQ; equation (19) denotes the average transmissions for a successful packet for $R\rightarrow\{d_1, d_2\}$.

Proof: Using the analytical method in [3], take $S_1\rightarrow\{R, d_1\}$ for instance, the number of transmissions that are needed to successfully deliver a packet to all receivers is the random variable $Y=\max\{X, X_1, X_2, \ldots, X_{D1}\}$, where X is the random variable denoting the number of attempts to successfully deliver a packet to R, and X_i is that to d_{1i}. According the geometric distribution, we have that $P[Y\le k]=P[max\{X, X_1, X_2, \ldots, X_{D1}\}\le k]$. Therefore,

$$P[Y \le k] = (1-p_1)\prod_{i=1}^{D_1}(1-p_3{}^k) \tag{21}$$

$$P[Y = k] = (1-p_1{}^k)\prod_{i=1}^{D_1}(1-p_3{}^k)-(1-p_1{}^{k-1})\prod_{i=1}^{D_1}(1-p_3{}^{k-1}) \tag{22}$$

Therefore the average number of transmissions per successful packet is

$$\begin{aligned}\eta_{BS}(D_1) &= \sum_{k=1}^{\infty}kP[Y=k]\\ &= \sum_{k=1}^{D_1}\frac{(-1)^{k-1}C_{D_1}^k}{1-p_3^k}+\sum_{k=0}^{D_1}\frac{(-1)^k C_D^k}{1-p_1p_3^k}\end{aligned} \tag{23}$$

Thus (18) is derived, so similar derivation is taken for (19).

Then it comes to (20), we still take $S_1\rightarrow\{R,d_1\}$ for instance. The numbers of lost packets at R, $d_{11}, d_{12}, \ldots, d_{1D1}$ are Mp_1, Mp_3, $Mp_3, \ldots$, Mp_3 and $Mp_1 \leqslant Mp_3$. For the retransmitting with XOR-ARQ, we can count the number of combinations and transmit the combined packets in different rounds. In round 1, we have Mp_1 combined packets, and in round 2, we have $M(p_3-p_1)$combined packets. So the average number of transmissions needed for the successful delivery of M packets is

$$n = M + Mp_1\theta_1 + M(p_3-p_1)\theta_2 \tag{24}$$

θ_i denotes the average transmissions needed to successfully deliver a combined packet in round i. the derivation of θ_i is same as the above analysis for (18). Thus (20) is derived with (1), (18) and (24).

With the derivations, the transmission bandwidths of the three schemes are derived in the same way as the specified occasion of $D_1=D_2=1$.

C. Network coding gain

Choose the transmission bandwidth of ARQ to be the basic one, the coding gain of the other schemes will be

$$G_{XOR-ARQ} = \frac{\eta_{ARQ}}{\eta_{XOR-ARQ}} \tag{25}$$

$$G_{iXOR-ARQ} = \frac{\eta_{ARQ}}{\eta_{iXOR-ARQ}} \tag{26}$$

V. SIMULATION RESULTS AND DISCUSSION

In this section, we test the performance of our proposed multicast schemes and verify the analytical derivations for the transmission bandwidth. For simplification, we use packet loss rates to characterize the wireless channel instead of Raleigh fading parameters.

Fig. 4 shows the different performance of the three schemes for the simplest situation $D_1=D_2=1$. We use M=1000 packets to minimize the effect of buffer size. As shown, the transmission bandwidth of ARQ Scheme is the largest and the XOR-ARQ performs better. The iXOR-ARQ Scheme has the smallest transmission number per successful packet, whose advantage

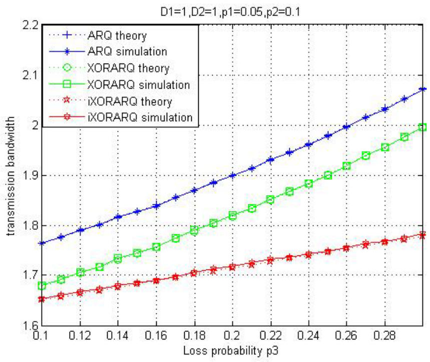

Figure 4. Transmission bandwidth versus packet loss probability for D_1=D_2=1

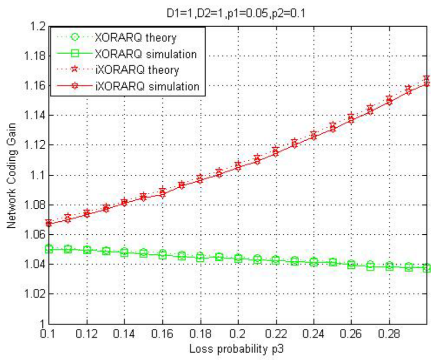

Figure 5. Network coding gain versus packet loss probability for D_1=D_2=1

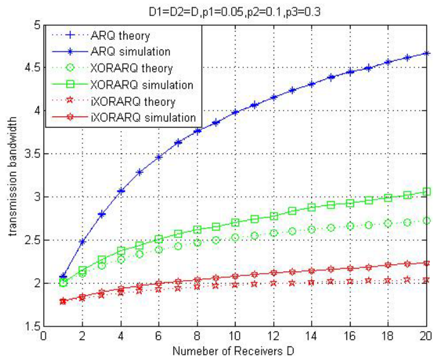

Figure 6. Transmission bandwidth versus the number of receivers

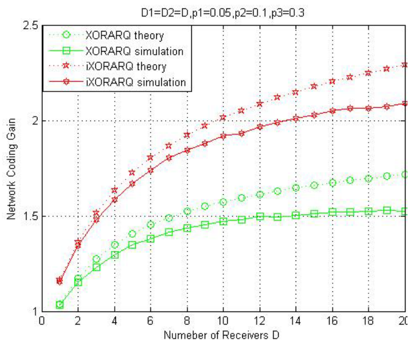

Figure 7. Network coding gain versus the number of receivers

rises up as p_3 grows. Furthermore, the simulation and theoretical curves match almost exactly for all the schemes, verifying the results of our derivations.

Fig.5 shows the coding gain of the XOR-ARQ scheme and the iXOR-ARQ Scheme over ARQ Scheme as functions of the packet loss probability of p3. The coding gain of XOR-ARQ Scheme goes down as p3 grows while that of iXOR-ARQ Schemes increases.

What's more, if p1$\leqslant$p2$\leqslant$p3 is not required as in the realistic situation, the iXOR-ARQ Scheme will degenerate into XOR-ARQ Scheme for there will not be reminded packets to be combined at R.

Fig.6 shows the different performance of the proposed schemes versus various numbers of receivers. p_1, p_2, p_3 are set, and D_1 is equal to D_2 which varies from 1 to 20. It is obvious that the iXOR-ARQ Scheme obtains the best performance for the transmission bandwidth increases tardy as the number of receivers grows. Fig.7 shows the network coding gain versus the number of receivers. The two figures convey the same information that the more the number of receivers grows, the more obvious the advantage of iXOR-ARQ Scheme over the ARQ Scheme and XOR-ARQ Scheme comes out.

Finally, we conclude from all the figures that the iXOR-ARQ Scheme performs much better than both ARQ Scheme and XOR-ARQ Scheme on the variable loss probability and receivers number.

VI. CONCLUSION

In this paper, we designed and proposed a network coding scheme to improve the bandwidth efficiency in wireless multicast cell. Our scheme changes the retransmitting node and combines different packets from different receivers to make the receivers recover their lost packets using the least number of transmissions. Both the simulations and theoretical analysis indicate the advantages of the proposed scheme.

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