Throughput and Delay Analysis of a QoS Differentiated *p*-persistent CSMA Protocol with Multirate

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Abstract—In this paper, a comprehensive and new closed form analytical approach to calculate the saturated throughput and delay of a differentiated *p*-persistent CSMA protocol with a multirate capability is presented for the first time. The *p*-persistent analysis can be used to model accurately 802.11e EDCA for WLAN applications. The analysis allows 802.11e to be characterised in the case of multirate operation without undue numerical complexity unlike state-of-the-art analytical approaches.

Keywords-WLAN, *p*-CSMA, service differentiation, multirate, throughput anomaly

I. INTROCUCTION

The throughput performance of high-rate 802.11 stations is significantly degraded by low-rate stations even though the former instantaneously transmit with a high link data rate. This phenomena is often called the "*performance anomaly*" of IEEE 802.11 whereby an unexpected drop in the throughput performance of the stations with high bit-rates is observed. Multirate modeling of DCF has received considerable attention recently after the problem was discovered experimentally in [1]. An analytical investigation of the *performance anomaly* using a Markov chain model to calculate the saturated throughput of 802.11 was detailed in [2, 3]. Due to the complexity of a Markov chain model their work was limited to 802.11 DCF with two groups and the service differentiation case (i.e. 802.11e) was not considered. In [2] an adjustment of initial backoff window and/or frame size was introduced to alleviate the *performance anomaly* while in [3] an adaptive scheme to adjust the packet size according to the data rate was proposed to improve the throughput *performance anomaly*. QoS support over IEEE 802.11e in a multirate network was studied by simulation only in [4] using NS-2. To alleviate the *anomaly*, the authors in [4] proposed a hybrid method based on a Transmit Opportunity (TXOP) feature combined with a contention window to maintain the quality of the highest priority applications and improve the overall system throughput. Although their study considered the 802.11e case, there was no mathematical formulation of the problem. In [5] the saturated throughput of the *p*- persistent CSMA protocol with multiple traffic types was analysed for the case of a single rate. The noramlized delay for this study was presented in [6].

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In this paper, the model in [5] and [6] is significantly extended to provide a closed form analytical solution, which accurately predicts the throughput, and delay performance of 802.11e with multirate capability. The new analysis is significantly less complex than the Markov analysis used in [2, 4]. Validation of our new model is provided through simulation results and in the basic homogenous condition, our model is also validated by comparison to the model in [2].

II. SYSTEM MODEL

The *p*-persistent CSMA system contains an Access Point (AP) giving coverage to M stations, where $M = M_1+M_2+...+M_{d\max}$, and M_d represents the number of stations with type d traffic and $d_{\max}$ is the total number of traffic types in the system. Each station has only one traffic type and all stations with the same traffic type are statistically identical. Stations with different traffic types transmit at different bit rates and all stations are saturated. Within the busy period there is a regenerative cycle of busy subperiods (B), that consist of a delay time (R) and a transmission time (T) as shown in Figure 1. Each traffic type is given the *p*-persistent CSMA parameter p_d and each station will start its transmission with probability p_d and defer with probability (1- p_d). A transmission will be successful if only one station commences a transmission at the start of the timeslot which corresponds to the vulnerable period.

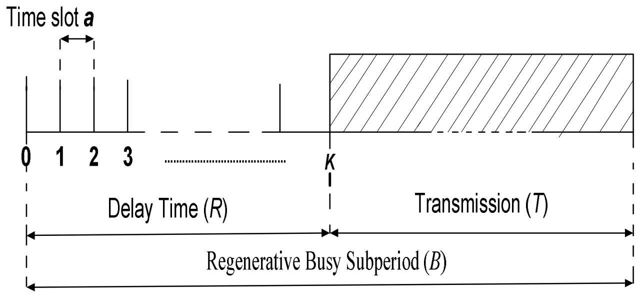

Fig. 1 : Channel state cycle for saturated case

III. THROUGHPUT ANALYSIS

In [5] the throughput s_1 of p-persistent CSMA for traffic type-1 was calculated using (1).

$$s_1 = \frac{E[U_1]}{E[B]} \tag{1}$$

Where $E[U_1]$ is the expected time of useful transmission of traffic type-1 .The probability that only a type-1 commences transmission at timeslot boundary k and that no transmissions have started before this for k =0,1,2,...,∞ is given by (2).

$$P(1tx_1) = M_1 p_1(1-p_1)^k[(1-p_1)^{k+1}]^{M_1-1} \tag{2}$$

Here, the $p_1(1-p_1)^k$ term accounts for the station that transmits only at the kth time slot boundary, the $[(1-p_1)^{k+1}]^{M_1-1}$ term accounts for all other stations abstaining from transmitting beyond at least the kth timeslot boundary and the term , M_1, represents the number of possible combinations in which this situation can happen (i.e, different M_1 stations could be the sending station).

The probability that no transmission, defined by $P(0tx_d)$ has occurred before time slot boundary k+1 for each other traffic type-d is calculated from (3).

$$P(0tx_d) = [(1-p_d)^{k+1}]^{M_d} \tag{3}$$

The probability of successful transmission ($P(s_1)$) of traffic type-1 is calculated as shown in (4) then an expression for $E[U_1]$ is given by (5).

$$P(s_1) = \left(P(1tx_d)\prod_{d\neq 1} P(0tx_d)\right) \tag{4}$$

$$E[U_1] = b_1 \times T_1 \times P(s_1) \tag{5}$$

The value, b_1, represents the percentage of a successful transmission that contains useful data for traffic type-1. The expected busy period, $E[B]$, is the sum of the expected deferral period $E[R]$and expected transmission period $E[T]$ given by (6).

$$E[B] = E[R] + E[T] \tag{6}$$

$E[R]$ can be calculated in the same way as in [5] as shown in (7).

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$$E[R] = a\sum_{k=1}^{\infty}\left(\prod_{d=1}^{d_{max}}(1-p_d)^{M_d k}\right) \tag{7}$$

The expected length of the transmission time can be calculated by (8).

$$E[T] = E[T_s] + E[T_c] \tag{8}$$

where $E[T_s]$ and $E[T_c]$ are the expected successful and collision times , respectively. Up to this point the model considers only the single rate case where $E[T]$ equals 1 as originally stated in [6]. However, in order to modify the model to include multirate capabililty, $E[T]$ should be calculated differently as follows: firstly the successful transmission time for each traffic type, $E(T_{s\_d})$, should be calculated, for example, the expected successful transmission time of traffic type 1, $E(T_{s\_1})$, is given by (9) .

$$E(T_{s\_1}) = \sum_{k=0}^{\infty}\left(P(1tx_d)\prod_{d\neq 1}^{d_{max}} P(0tx_d)\right)\times T_1 \tag{9}$$

Then, the total expected successful transmisson time $E(T_s)$ is given by (10)

$$E[T_s] = \sum_{d=1}^{d_{max}} E[T_{s\_d}] \tag{10}$$

$E(T_c)$ is due to two types of collisions: internal collisions where a collision between two or more packets from stations within the same traffic type occur; and external collisions where a collision between two or more packets from stations of different traffic types occur. In order to calculate $E(T_c)$, the probability (P_d) of more than one station (i.e. $n_d > 2$) transmitting at the same time must be calculated first as shown in (11). The probability of an internal collision of traffic type-1 (P_{i1}) is given by (12).

$$P_d = \binom{M_d}{n_d}(p_d(1-p_d)^k)^{n_d}[(1-p_d)^{k+1}]^{M_d-n_d} \tag{11}$$

$$P_{i1} = \binom{M_1}{n_1}(p_1(1-p_1)^k)^{n_1}[(1-p_1)^{k+1}]^{M_1-n_1} \times \prod_{d\neq 1}^{d_{max}}[(1-p_d)^{k+1}]^{M_d} \tag{12}$$

The probability of an external collision (P_{e1}) for traffic type-1 is given by (13).

$$P_{e1} = \sum_{n_1=1}^{M_1}\sum_{n_2=0}^{M_2}\cdots\sum_{n_{d_{max}}=x}^{M_{d_{max}}}\left(\prod_{d=1}^{d_{max}} P_d\right) \tag{13}$$

Where x = 0 if any other traffic type has a packet to send and x = 1 if all other traffic types have no packets to send. When calculating the external colision of, for example, traffic type-2 the collsion between traffic type-1 and type-2 should be excluded as it is already included in the case of traffic type-1 and so on. The total collision time of traffic type-1, $E[T_{c\_1}]$, can be calculated using (12) and (13) as shown in (14).

$$E[T_{c\_1}] = \sum_{k=0}^{\infty}\sum_{n_1=1}^{M_1}\sum_{n_2=0}^{M_2}\cdots\sum_{n_{d_{max}}=x}^{M_{d_{max}}}\left(\prod_{d=1}^{d_{max}} P_d\right)\times T_{\max} + \binom{M_1}{n_1}(p_1(1-p_1)^k)^{n_1}[(1-p_1)^{k+1}]^{M_1-1} \times \prod_{d\neq 1}^{d_{max}}[(1-p_d)^{k+1}]^{M_d}\times T_1 \quad (14)$$

Where T_{max} is the maximum collision time of a transmitted packet in the network. The total collision time $E[T_c]$ is given by (15).

$$E[T_c] = \sum_{d=1}^{d_{max}}\left[E[T_{c\_d}]\right] \quad (15)$$

The total transmission time $E[T]$ of the QoS differentiated *p*-persistent CSMA protocol with multirate capability is given by (16) .

$$E[T] = \sum_{d=0}^{d_{max}} E[T_{s\_d}] + \sum_{d=1}^{d_{max}}\left[E[T_{c\_d}]\right] \quad (16)$$

Then, the throughput of traffic type -1 is given by (17).

$$S_1 = \frac{b_d \times \mathrm{T}_1 \times \sum_{k=0}^{\infty}\left(P(1tx_d)\prod_{d\neq 1}^{d_{max}} P(0tx_d)\right)}{a\sum_{k=1}^{\infty}\left(\prod_{d=1}^{d_{max}}(1-p_d)^{M_d k}\right) + E[T]} \quad (17)$$

Finally, the total system throughput is given by (18).

$$S = \sum_{d=1}^{d_{max}} S_d \quad (18)$$

IV. DELAY ANALYSIS

The delay analysis does not include a retry limit and is defined as the time from when a packet becomes head of the line at a station until it is successfully transmitted. In *p*-persistent CSMA the delay begins at the start of R following the station's previous successful transmission and ends once the station's next successful transmission has completed. The analytical approach for the delay of *p*-persistent CSMA with multirate will be provided. This model matches that in [6] except that here the expected transmission time ($E[T]$) is calculated differently as in equation (16) to include the multirate. The delay is calculated for a station with traffic type *d*= 1, however this analysis can be applied to stations of any of the traffic types in the system by simple substitution. In order to calculate the average delay, the probability that a particular type-1 station is successful, $P_{s,1}$, during each transmission period must be calculated first. This is shown in (19).

$$P_{s,1} = \frac{P(s_1)}{M_1} \quad (19)$$

The number of busy subperiods N_1 before that particular type-1 station successfully transmits, forms a geometric distribution of mean $\overline{N_1}$ as shown in (20).

$$P(N_1 = X) = P_{s,1}\left(1-P_{s,1}\right)^{X-1}$$

$$\overline{N_1} = \frac{1}{P_{s,1}} \quad (20)$$

The overall delay for a type-1 packet, $\overline{D_1}$ can then be calculated as shown in (21)

$$\overline{D_1} = \overline{N_1} \times E[B] = \frac{E[R]+E[T]}{P_{s,1}} \quad (21)$$

V. RESULTS

Initially a similar DCF network topology was considered to that in [4] with the topology composed of two stations in Group-1 (G1) transmitting at 11 Mbit/s while varying the number of stations from 2 to 14 in Group-2 (G2) transmitting at 1Mbit/s. The parameters of IEEE 802.11b/n used are the same as in the standards and shown in Table 1. In order to do a fair comparison, the same topology also in [4] was used for the *p*-CSMA protocol. Access Category 1 (AC1) corresponds to G1 while Access Category 2 (AC2) corresponds to G2. The same experiment was repeated for 802.11n with data rates 26 and 6.5 Mb/s for DCF and p-persistent CSMA with a multirate capability. In order to carry out these experiments, a MATLAB simulator was developed to model 802.11 DCF with multirate capability. The QoS differentiated *p*-persistent CSMA with multirate was also simulated using MATLAB. A *p* value of 0.03 was assigned to both access categories which corresponds to a homogeneous case the of *p*-persistent CSMA protocol with multiple traffic types. The *p*-values in our model are implemented in an 802.11 DCF simulator through a fixed size contention window (CW) where $CW_{\min}$ = $CW_{\max}$. The relationship between p and CW is $p = 2/(CW+2)$ as calculated in [7]. The results for *p*-persistent CSMA were obtained both analytically and by simulations while the results for DCF with multirate capability were obtained by simulations only. Figure 2a shows graphs of throughput versus number of stations for the 802.11b DCF and *p*-CSMA. The results show that the throughput of both G1 and G2 are less than 1Mbit/s even though stations in G1 transmit at 11 Mbit/s. The same throughput trend was obtained for AC1 and AC2. Figure 2b shows the results for 802.11n and similar trends are obtained as for the results in Figure 2a. the results illustrate the exact behaviour of the performance anomaly. The results show very close agreement between 802.11 DCF and the homogeneous case of *p*-persistent CSMA.

Table 1: system simulation parameters of 802.11b/n

Parameters	802.11b	802.11n
Packet size	2324 byte	1500 byte
ACK	192 µs	32 µs
SIFS	10 µs	16 µs
DIFS	50 µs	34 µs
Slot	20 µs	9 µs

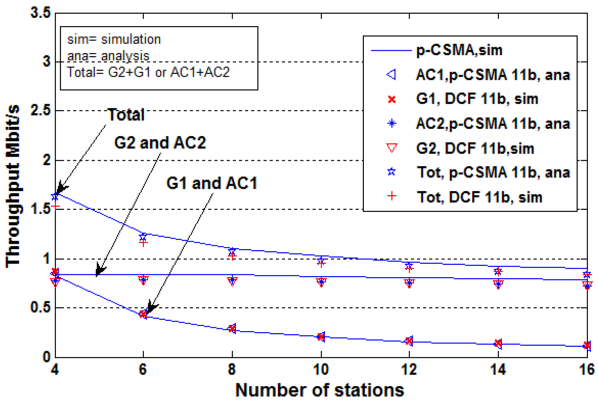

Fig. 2a Saturated throughput per group and traffic type of 802.11b stations

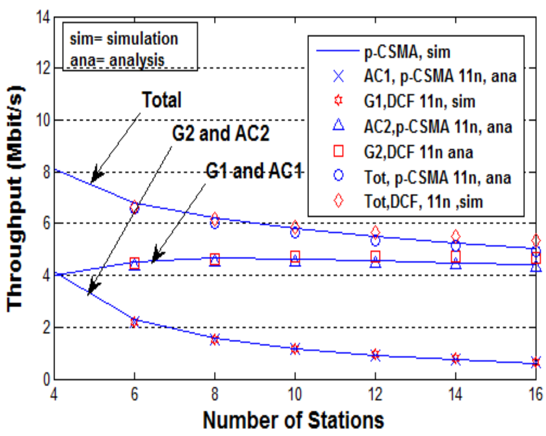

Fig.2b: Saturated throughput per group and traffic type for 802.11n stations

Figure 3 shows for the first time, the throughput performance per station of a *p*-persistent CSMA protocol with QoS differentiation for 4 traffic types (5 stations per traffic type) when transmitting at different date rates. A p_1 value equal to 0.03 was used while p_2, p_3 and p_4 were calculated in order to achieve throughput ratios $S_1=2S_2$, $S_2=2S_3$ and $S_3=2S_4$ according to Hui's ratio [8]. In order to show this new capability of our model (i.e. multirate), two scenarios were investigated . In the first scenario, all station of all traffic types transmit at channel rate (i.e. physical layer date rate) 58.5 Mbit/s. Each Station of traffic type-1 achieves an average throughput of 3.23 Mbit/s. In the second scenario, stations of traffic type-2 to type-4 drop their channel rates to 39, 26, 6.5 Mbit/s, respectively, due to adverse channel conditions. In this scenario, Each station of traffic type-1 can only achieve an average throughput of 2.07 Mbit/s, even though its channel rate is 58.5 Mb/s which correspond to a 36.4% drop in the throughput due to the performance anomaly effect. Figure 3 demonstrates an excellent agreement between the numerical and simulations results which validates the accuracy of our model. Figure 4 shows the delay performance per traffic type of the two scenarios. The results show that in the first scenario the average delay of traffic type 1 is increased from 3.6 ms to 5.7 ms.

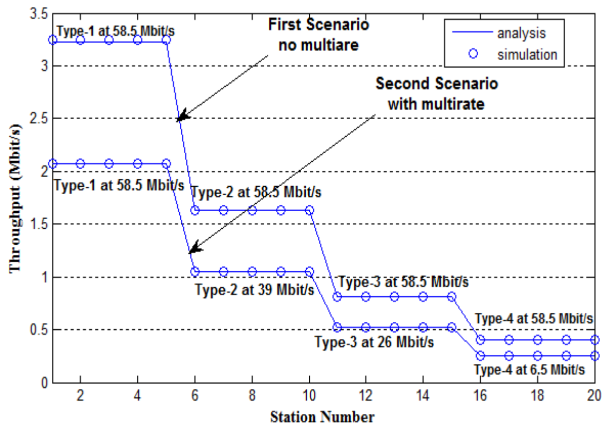

Fig.3 :Throughput of stations transmitting at different data rates for 802.11n with different priorities

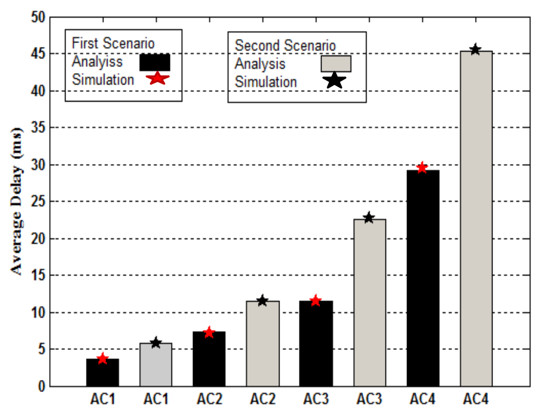

Fig.4 :Delay of stations transmitting at different data rates for 802.11n with different priorities

VI. CONCLUSIONS

In this paper a new throughput and delay analysis for a QoS differentiated *p*-persistent CSMA protocol with multirate capability was developed by modifying the model in [5]. While the previous studies based on Markovian analysis could only access fewer configurations, limited to a single traffic type. This new analysis is an effective way of modeling 802.11 DCF and EDCF in the case of the *performance anomaly*. The results show that due to *performance anomaly*, the throughput performance of stations with a high priority is degraded by 34 % even though they transmit at 58 Mbit/s. The size of these changes demonstrate the need to model the multirate scenario in order to obtain an accurate characterization of the network performance. Results have been produced to validate the analysis by comparing it to the single traffic type model in [2] and through simulations. Modeling the performance of *p*-persistent CSMA in the case of non-saturated and heterogeneous systems with multirate capability is a subject of on-going research by the authors.

REFERNCES

[1] M. Heusse*, et al.*, "Performance anomaly of 802.11b," in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications. IEEE Societies*, 2003, pp. 836-843 vol.2.

[2] Y. Duck-Yong*, et al.*, "Performance enhancement of multirate IEEE 802.11 WLANs with geographically scattered stations," *Mobile Computing, IEEE Transactions on,* vol. 5, pp. 906-919, 2006.

[3] M. Ergen and P. Varaiya, "Formulation of Distributed Coordination Function of IEEE 802.11 for Asynchronous Networks: Mixed Data Rate and Packet Size," *Vehicular Technology, IEEE Transactions on,* vol. 57, pp. 436-447, 2008.

[4] L. Hao-Ming*, et al.*, "QoS Support over IEEE 802.11e in Multirate Networks," in *Wireless Pervasive Computing, 2009. ISWPC 2009. 4th International Symposium on*, 2009, pp. 1-5.

[5] R. MacKenzie and T. O'Farrell, "Throughput Analysis of a p-Persistent CSMA Protocol with QoS Differentiation for Multiple Traffic Types," in *Communications, 2008. ICC '08. IEEE International Conference on*, 2008, pp. 3220-3224.

[6] R. MacKenzie and T. O'Farrell, "Achieving Service Differentiation in 802.11e WLANs using p-Persistent CSMA Throughput/Delay Analysis," presented at the London Communications symposium, London, 2009.

[7] G. Bianchi*, et al.*, "Performance evaluation and enhancement of the CSMA/CA MAC protocol for 802.11 wireless LANs," in *Personal, Indoor and Mobile Radio Communications, 1996. PIMRC'96., Seventh IEEE International Symposium on*, 1996, pp. 392-396 vol.2.

[8] J. H. a. M. Devetsikiotis, "Designing improved MAC packet schedulers for 802.11e WLAN " presented at the IEEE Gloecom, 2003.