Near-Optimal Spectrum Allocation for Cognitive Radio Networks

Tsung-Cheng Wu*, Yaqing Mao†, Yi-Sheng Su‡
Department of Communication Engineering, I-Shou University, Kaohsiung City, Taiwan*
Department of Information and Communication Engineering, Beijing Jiaotong University, Beijing, PRC†
Department of Computer Science and Information Engineering, Chang Jung Christian University, Tainan, Taiwan‡
tcwu@isu.edu.tw*, 08251014@bjtu.edu.cn†, yssu@mail.cjcu.edu.tw‡

***Abstract*—This paper investigates network-wide spectrum allocation based on the cross-entropy (CE) method in cognitive radio network (CRN). A few spatial spectrum allocation techniques in CRN have been proposed in the literature. However, the optimum spectrum allocation requires an exhaustive search over all combinations of available channels and constraints on secondary users, whose complexity increases exponentially with the number of users and channels. Simulation results show that the proposed modified CE-based scheme is an efficient method to greatly reduce the complexity while still reaching optimal network utility. Most of the solutions obtained by the modified CE algorithm are coincided with the optimal values.**

I. Introduction

Cognitive radio (CR) that opportunistically exploit unused spectrum of primary users has been proposed as a solution to utilize the spectrum more efficiently [1]. Cognitive radio is the key technology that enables dynamic spectrum access (DSA) networks to access without interfering with licensed primary users. Secondary users who have no spectrum licenses can sense the empty licensed channel, select the best available channel according to constraints, and change channel as the primary users re-occupy the channel.

Dynamic spectrum sharing among secondary users can be classified into temporal and spatial spectrum allocation. The temporal spectrum allocation is exploited in time-varying access models. The spatial spectrum allocation is usually employed for static environment. Spatial spectrum allocation heavily depends on the relative positions between primary and secondary users. Given a users' topology, multiple available channels assigned to secondary users in network-wide CRN turn out to be a complicated problem.

The optimum spectrum allocation in network-wide CRN belongs to a NP-hard problem and a non-convex optimization. The computational complexity will increase exponentially in proportional to number of users and channels. In the literature, a few methods have been proposed to reduce the complexity. Auction-based [2] and game theory [3][4] have been proposed to spatial spectrum allocation. In addition, network-wide spatial spectrum allocation in CRN can be seen as an combinatorial optimization problem. In [5][6], spectrum allocation has been modeled as the equivalent graph coloring problem. Evolutionary algorithms such as genetic algorithm and particle swarm optimization have been proposed for cognitive radio spectrum allocation [7] to solve the combinatorial optimization problem. Chemical reaction optimization is developed in spectrum allocation and show to outperform the other evolutionary algorithms [8].

In this paper, we propose cross-entropy (CE) methods to search for the optimal spectrum allocation. The optimal spectrum allocation requires an exhaustive search over all combinations whose complexity increases exponentially with the number of users and channels. The CE method was first proposed by Rubinstein[9] for solving rare event probability estimation and was soon successfully applied to solve combinatorial optimization problems. Simulation results show that the proposed scheme greatly reduces the computational complexity while still achieving near optimum solution.

The rest of the paper is organized as follows. We begin in Section II by describing the problem. Optimization problem is formulated in Section III. Cross-entropy algorithm is introduced in Section IV. Section V gives simulation results and Section VI the conclusions.

II. Problem Formulation

We define the spatial spectrum allocation model according to [6] which includes channel availability matrix, channel reward matrix, interference constraint matrix, and conflict free channel assignment matrix. Given a topology for spatial spectrum allocation, three matrices, $\mathbf{L}, \mathbf{B}$, and $\mathbf{C}$, are determined.

The channel availability matrix is indicated by $\mathbf{L} = \{l_{n,m},\}_{N\times M}$, where $l_{n,m} \in \{0,1\}$. If $l_{n,m} = 1$, the m-th channel is available for the n-th secondary user. If $l_{n,m} = 0$, the m-th channel is unavailable for the n-th secondary user. The reward obtained by the n-th user while occupies the m-th channel is represented as channel reward matrix $\mathbf{B} = \{b_{n,m}\}_{N\times M}$, where $b_{n,m} \in \{0,1\}$. If $b_{n,m} = 1$, it represents the bandwidth/throughput when the n-th secondary occupies the m-th channel. Interference constraints imposed on secondary users are expressed by interference constraint matrix $\mathbf{C} = \{c_{n,k,m}\}_{N\times N\times M}$ where $c_{n,k,m} \in \{0,1\}$. If $c_{n,k,m} = 1$, the n-th user and k-th user interfere with each other if they occupy the m-th channel simultaneously. If $c_{n,k,m} = 0$, otherwise.

Assume that there are P primary users, N secondary user, and total M spectrum channels. Central spectrum allocation and interweave spectrum access among secondary users are considered. Secondary users conduct spectrum sensing and co-operatively exchange channel information with primary users and other secondary users. Therefore, the spatial allocation scheme has perfect knowledge of available channels for each secondary user.

All primary users choose their desired channel randomly and determine the transmitted powers . The transmit power of the primary user determines the interference range for which secondary user located within the range cannot utilize the particular spectrum chosen by the primary user. We assume that the primary users have uniform protection ranges of d_p. In addition, secondary users can control their transmit power and also interference range. We assume that the interference range of secondary users are bounded between d_{min} and d_{max} given by

$$d_{min} \leq d(n) \leq d_{max}, \;\; 1 \leq n \leq N,$$

where $d(n)$ denotes the interference range of the n-th user.

The spectrum allocated to secondary users is presented by assignment channel $\mathbf{A}$. It describes the assignment of available channels to the secondary user. $\mathbf{A} = \{a_{n,m}\}_{N\times M}$, where $a_{n,m} \in \{0, 1\}$. If we assign the n-th user to use the m-th channel, then $a_{n,m} = 1$. If otherwise exists, then $a_{n,m} = 0$.

III. Combinatorial Optimization

The network utility of network-wide spectrum allocation includes maximum-sum-reward (MSR), maximum-min-reward (MMR), and maximum-proportional fair (MPF) and are all expressed in cost function format as shown:

1) Sum-Reward (SR)

$$C(\mathbf{A}) = \sum_{n=1}^{N}\sum_{m=1}^{M} a_{n,m} b_{n,m}$$

2) Min-Reward (MR)

$$C(\mathbf{A}) = \min_{1\leq n\leq N} \sum_{m=1}^{M} a_{n,m} b_{n,m}$$

3) Proportional-Fair (PF)

$$C(\mathbf{A}) = \left(\prod_{n=1}^{N} (a_{n,m} b_{n,m} + 10^{-6})\right)^{1/N}$$

If a secondary user does not be assigned any channels, the PF becomes 0 without the additional 10^{-6}.

The spectrum allocation is now considered to find the matrix $\mathbf{A}$. Moreover, to find the matrix $\mathbf{A}$ is equivalent to a combinatorial optimum problem described by,

$$\mathbf{A}^{\star} = \arg\max_{\mathbf{A}\in\Gamma} C(\mathbf{A}), \tag{1}$$

where $\mathbf{A}^{\star}$ presents the optimal conflict free channel assignment matrix and Γ denotes the set of conflict free channel assignment for a given $\mathbf{L}$ and constraints $\mathbf{C}$. The constraint matrix $\mathbf{C}$ can be presented as

$$a_{n,m} + a_{k,m} \leq 1, \text{ for any } c_{n,k,m} = 1, \tag{2}$$

where $1 \leq n, k \leq N$, and $1 \leq m \leq M$. That is, the k-th user and the n-th user do not use the same m-th channel if $c_{n,k,m} = 1$.

The optimal solution of cost function SR is called max-sum-reward (MSR), that of MR is called maximum min-reward (MMR), and that of PF is called maximum proportional-fair (MPF). To reduce the number of searching variables, we convert those available entries in the assignment matrix $\mathbf{A}$ into a vector format and delete those unused entries. In this way, the matrix solution $\mathbf{A}$ becomes a vector $\mathbf{v}$and the mapping is indicated in Fig. 1.

$$\mathbf{A} = \begin{bmatrix} 0 & a_{1,2} & 0 \\ a_{2,1} & a_{2,2} & 0 \\ 0 & 0 & a_{3,3} \end{bmatrix} \;\; \rightarrow \;\; \mathbf{v} = [a_{1,2} \; a_{2,1} \; a_{2,2} \; a_{3,3}]$$

Fig. 1. The mapping from matrix $\mathbf{A}$ to vector $\mathbf{v}$.

IV. Cross-Entropy Algorithm

The CE method is applied for the evaluation of rare event probabilities and also for combinatorial optimization problems [9]. It cast the deterministic optimum into associated stochastic optimum by minimizing the cross entropy of the distribution between importance sampling and crude Monte-Carlo simulation.

The CE method can be divided into two-phase iteration: 1) generating a set of random samples according to a probability distribution and 2) then updating the parameters of the probability distribution to produce better samples in the next iteration. It updates iteratively and then converges to optimal value. For a more thorough discussion of the CE method, the reader is referred to [9].

To solve (1) by the CE method, each element of vector $\mathbf{v}$ is modeled as an independent Bernoulli random variable with the probability mass function $P_r\{v_i = 1\} = p_i$, $P_r\{v_i = 0\} = 1 - p_i$, $1 \leq i \leq K$. The probability distribution of $\mathbf{v}$ is given by

$$f(\mathbf{v}, \mathbf{p}) = \prod_{i=1}^{K} p_i^{v_i}(1 - p_i)^{1-v_i},$$

where K is the length of vector $\mathbf{v}$, and $\mathbf{p} = [p_1, \ldots, p_K]$.

A. CE-Based Spectrum Allocation

1) Initialize the probability vector $\mathbf{p}^t = (p_1^t, \ldots, p_K^t)$ with $t = 0$. i.e., $p_j^0 = \frac{1}{2}, j = 1\ldots, K$. Set the iteration counter $t = 1$.
2) Randomly generate Q samples of vectors $\mathbf{v}_1^t, \ldots, \mathbf{v}_Q^t$ according to $f(\mathbf{v}, \mathbf{p}^{t-1})$. Each vector sample $\mathbf{v}_q^t$,

$q = 1, \ldots, Q$, represents K-dimensional binary vector, whose elements are denoted by $v^t_{q,j}$, for $j = 1, \ldots, K$.

3) Compute cost function $C(\mathbf{v}^t_q)$ for $q = 1, \ldots, Q$ and order them from the smallest to the largest such that $C(\mathbf{v}^t_{(1)}) \leq \cdots \leq C(\mathbf{v}^t_{(Q)})$.
   Set $\gamma^t = C(\mathbf{v}^t_{(\lceil \rho Q \rceil)})$, where $\rho \in (0, 1)$ denotes the fraction of the "elite" samples and $\lceil \cdot \rceil$ is the ceiling function.
4) Evaluate $\mathbf{p}^t$ with (3)

$$p^t_j = \frac{\sum_{q=1}^{Q} I_{\{v^t_{q,j}=1\}} I_{\{C(\mathbf{v}^t_q) \leq \gamma^t\}}}{\sum_{q=1}^{Q} I_{\{C(\mathbf{v}^t_q) \leq \gamma^t\}}}, \; j = 1, \ldots, K, \quad (3)$$

   where $\mathbf{v}^t_q = (v^t_{q,1}, \ldots, v^t_{q,K})$, and $I_{\{\cdot\}}$ denotes the indicator of an event $\{\cdot\}$.
5) Update $\mathbf{p}^t$ smoothly via

$$\mathbf{p}^t = \lambda \mathbf{p}^t + (1 - \lambda) \mathbf{p}^{t-1}$$

   to prevent fast convergence to a local optimum, where $\lambda \in (0, 1)$ is called a smoothing parameter.
6) The iteration ends if the predefined number T of iterations is reached. Otherwise, increase t by 1 and return to Step 2.

B. Repairing Consideration

In general, the CE algorithm is applied to search for unconstrained optimization solutions. In this paper, however, the assignment matrix $\mathbf{A}$ should satisfy constraints according to the interference matrix $\mathbf{C}$. The CE algorithm cannot apply to this constrained optimization problem if we do not make any modification. To tackle the problem, we consider to conduct a two stages of repair procedure in CE algorithm. Firstly, we check the random binary sequence generated in step 2 of the CE algorithm whether satisfy the interference constraint (2). Secondly, if any two secondary users are allocated to the same channel and the corresponding entry of the interference constraint matrix identical to 1, we will randomly choose any one of the two users to delete its allocation. That is, if $a_{n,m} = 1$, $a_{k,m} = 1$, and $c_{n,k,m} = 1$, the repairing procedure assign one of $a_{n,m}$ and $a_{k,m}$ to be zero.

C. Modified CE-based Spectrum Allocation

Computer simulation demonstrates that solutions of the CE method reach local optimal values for most of topologies. the It is because that one of allocated channels is randomly deleted when interference appears within allocated channels of two secondary users. That is, the repairing procedure randomizes the solution of the pure CE algorithm. Therefore, we propose a modified CE method that employ CE algorithm many times and choose the solution of the maximal cost function. The idea of proposed method comes from the mythology that CE algorithm generates samples in random and repairing procedure remove randomly the allocated channels with interference. The optimal solution may appear if we implement several CE method in parallel and select the largest one.

V. Simulation Results

We follow the pseudo-code developed in [6] for modeling topologies. All our simulation are based on the following common settings. Primary and secondary users are randomly placed in a 10×10 area. An example of such random placement is shown in Fig. 2.

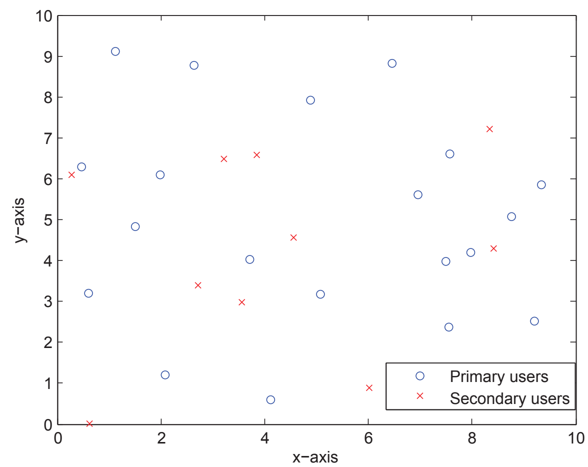

Fig. 2. A 10×10 topology with 20 primary users and 10 secondary users.

We assume that primary users have uniform interference range, i.e. $d_p = 2$. In addition, $d_{min} = 1, d_{max} = 4$. We assume that $P = 10$, $N = 5$, $M = 5$. Parameters of CE algorithm are set to be $Q = 30$, $\lambda = 0.3$, $\rho = 0.1$, $T = 50$, K equal to the length of vectors. The proposed modified CE algorithm run pure CE algorithm 50 times repeatedly and choose the largest one.

All simulation are conducted in MATLAB software and run in the computer with Intel Core 2 Q6800 2.4GHz and 2G DDR2 RAM. The running of the modified CE algorithm time takes 70 to 250 sec with $M = 10$, $N = M = 5$, and the number of repeated pure CE algorithm $= 50$. However, we are working on finding the best parameters for CE algorithm to reduce the run time in the future work.

We generate 20 topologies in random for demonstration. The network utility MSR, MMR, and MPF of channel allocation among secondary users for optimum, pure CE algorithm, and modified CE algorithm versus sample topologies are shown in Fig. 3– Fig. 5, respectively. In our demonstrated results, the network utility performance of the proposed modified CE algorithm are as well as that of optimal allocation but with greatly reduced complexity. Compared to [7], our schemes consider bounded interference range among secondary users and discover solution of the constrained optimization problem. Compared to [8], we introduce a CE algorithm and propose a modified scheme to spectrum allocation. The proposed modified CE method results in near-optimum utility functions.

In Table I, the relative difference are summarized for different network utilities averaged over the the same 20 topologies used in Fig. 3–Fig.5. The relative difference is defined as

TABLE I
COMPARISON TO OPTIMUM VALUES

	Relative difference (%)		
	MSR	MMR	MPF
CE algorithm	1.23	58	6.5
modified CE algorithm	0.93	3.29	0.63

$(C(\mathbf{A}^*)-C(\mathbf{A}))/C(\mathbf{A}^*)\times 100\%$, where $C(\mathbf{A}^*)$ describes the cost function of the optimal assignment. It is observed that the pure CE algorithm has nonzero relative difference, especially a 58% appeared in MMR. On the contrary, the modified CE algorithm greatly reduces the relative difference compared to that of pure CE algorithm. Particularly, the nonzero relative difference of the modified CE algorithm, listed in Table I, come from the 11th topology whose solution is different from the optimal value as shown in Fig. 3–Fig.5.

VI. CONCLUSION

Network-wide spatial spectrum allocation in CRN can be seen as an combinatorial optimization problem. The optimum solution requires an exhaustive search whose complexity increases exponentially. In this paper, a CE-based spectrum allocation scheme for CRN is proposed. The proposed modified CE algorithm satisfies the interference constraints and greatly improve the network utility. Computer simulation results show that MSR, MMR, and MPF of the proposed modified CE algorithm are near to optimal values. In general, most solutions obtained by the proposed modified CE algorithm the optimum spectrum allocation requires an exhaustive search over all combinations of available channels and constraints on secondary users, whose complexity increases exponentially with the number of users and channels. are coincided with the optimal values.

REFERENCES

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.,* vol. 23, no. 2, pp. 201–220, 2005.

[2] J. Huang, R. Berry, and M. L. Honig, "Auction-based spectrum sharing," *ACM Mobile Networks and Applications (MONET),* vol. 11, no. 3, pp. 405–418, 2006.

[3] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," in *Proc. IEEE DySPAN* 2005, pp. 269–278, 2005.

[4] M. Halldorsson, J. Halpern, L. Li, and V. Mirrokni, "On spectrum sharing games," in *Proc. 23rd Annual ACM Symp. Principles Distributed Computing*, 2004, pp. 107-114.

[5] W. Wang and X. Liu, "List-coloring based channel allocation for open-spectrum wireless networks," in *Proc. IEEE Veh. Technol. Conf.*, Sept. 2005, pp. 690–694.

[6] C. Peng, H. Zheng, and B. Y. Zhao, "Utilization and fairness in spectrum assignment for opportunistic spectrum access," *ACM Mobile Networks and Applications (MONET)*, vol. 11, no. 4, pp. 555–576, 2006.

[7] Z. Zhao, Z. Peng, S. Zheng, and J. Shang, "Cognitive radio spectrum allocation using evolutionary algorithms," *IEEE Trans. Wireless Commun.,* vol. 8, no. 9, pp. 4421 –4425, Sept. 2009.

[8] A. Y. S. Lam and V. O. K. Li, "Chemical reaction optimization for cognitive radio spectrum allocation," in *Proc. IEEE Global Telecommun. Conf. (Globecom)*, Dec. 2010.

[9] R. Y. Rubinstein and D. P. Kroese, *The Cross-Entropy Method*, Berlin, Germany: Springer, 2004.

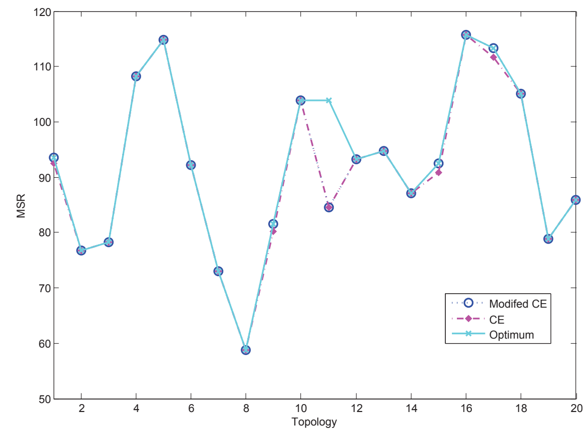

Fig. 3. MSR for the optimum solutions, modified CE algorithm, and CE algorithm illustrated in 20 topologies. $P = 10,\ N = 5,\ M = 5$.

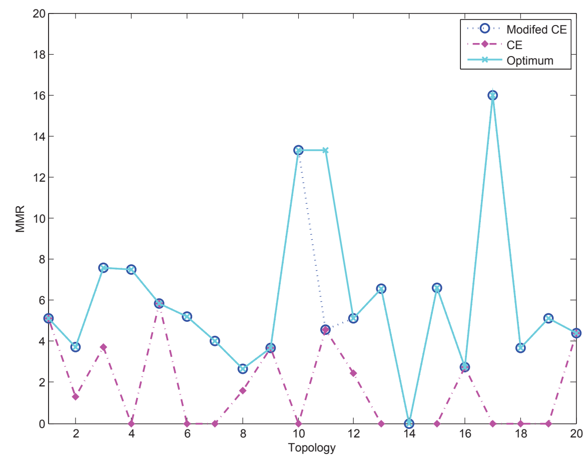

Fig. 4. MMR for the optimum solutions, modified CE algorithm, and CE algorithm illustrated in 20 topologies. $P = 10,\ N = 5,\ M = 5$.

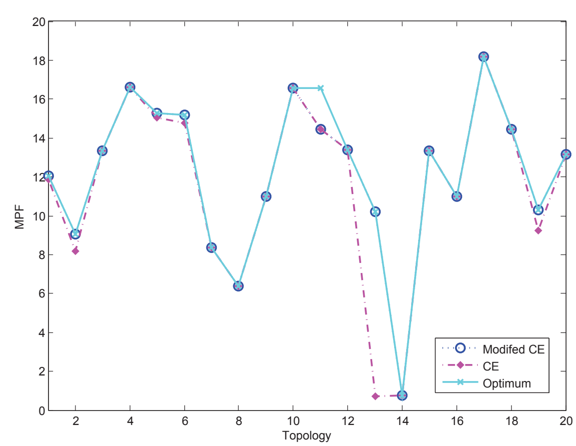

Fig. 5. MPF for the optimum solutions, modified CE algorithm, and CE algorithm illustrated in 20 topologies. $P = 10,\ N = 5,\ M = 5$.