Towards Improved QoS in 802.16e Mobile WiMAX

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***Abstract*— The potential benefits of deploying Matrix C in mobile WiMAX applications are investigated for improving quality of service (QoS). Also, it is shown that Matrix C in the IEEE 802.16e standard is not a Golden code, as previously thought, but a threaded algebraic space-time (TAST) code.**

***Index Terms*—Coding, Golden code, Matrix C, QoS, space-time coding, TAST code, WiMAX.**

I. Introduction

The concept and design of 802.16e WiMAX emphasized that users at any range should experience solid quality of service (QoS). Towards this end, the 802.16e version of WiMAX incorporated options for transmitter antenna diversity and multiple input multiple output (MIMO) antenna technology in the standard. The MIMO antenna technology is based on space-time coding schemes that significantly improve the gain of transmission-reception, thereby increasing throughput as well as improving the integrity of the received data. Meanwhile, operational experience has been that the service providers are desirous of better performance than what is achieved by current chip sets marketed by the major vendors. In this regard, note that the 802.16e standard provides for three optional space-time coding schemes, identified as Matrix A, Matrix B, and Matrix C [1]. It is important to note that currently only the space-time coding schemes Matrix A and Matrix B are available on commercial chip sets.

In this paper, we examine the potential benefits of deploying Matrix C in mobile WiMAX applications. This investigation is germane and timely because Matrix C is already in the existing 802.16e standard as amended in February 2006, so means to achieve improved QoS in 802.16e WiMAX that falls inside existing regulatory and standardization constraints is available.

The major obstacle that has prevented adoption of Matrix C is the complexity of the decoding of Matrix C, and more specifically the power requirements of standard decoding schemes for space-time codes. This fact together with the need for enhanced QoS in 802.16e WiMAX has motivated research into reduced complexity decoding of Matrix C [2]- [6]. At the same time this work has been done, some misunderstanding of the mathematical structure of Matrix C code seems to have arisen.

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We begin the paper by first clarifying the structure of Matrix C in Section II. Recent developments in the search for reduced complexity decoding of Matrix C are surveyed and summarized in Section III.

II. Structure of Matrix C

Mobile WiMAX systems are based on the scalable OFDMA mode of IEEE 802.16e-2005 specifications and use a subset of the different options. In the IEEE 802.16e-2005 standard [1], there is specified a space-time code for two transmit antennas with full rate and full diversity, which is called Matrix C. There is some misunderstanding about the structure of the Matrix C space-time code, and this misunderstanding is propagating in the literature. A correct understanding of the structure and class of the Matrix C space-time code is important since it is incorporated in the WiMAX standard. Reference [3] states that Matrix C is a Golden code, while references [4]- [7] proclaim that Matrix C is a variant of the Golden code. In this section, we prove that Matrix C is not a Golden code, but a threaded algebraic space-time (TAST) code.

Matrix C in the WiMAX standard is given as [1]

$$C = \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} s_1 + jr \cdot s_4 & r \cdot s_2 + s_3 \\ s_2 - r \cdot s_3 & jr \cdot s_1 + s_4 \end{pmatrix}$$

where $r = \frac{-1+\sqrt{5}}{2}$, $j = \sqrt{-1}$, and s_1, s_2, s_3, s_4 are 2^b-ary quadrature amplitude modulation (QAM) signals with inphase and quadrature components equal to $\pm 1, \pm 3, ...$ and having b bits per symbol. This space-time code was originally proposed to the standards committee in [8].

The Golden code was presented and mathematically defined in [9]. The Golden code is defined as [9]

$$G = \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha(x_1 + x_2\theta) & \alpha(x_3 + x_4\theta) \\ j\bar{\alpha}(x_3 + x_4\bar{\theta}) & \bar{\alpha}(x_1 + x_2\bar{\theta}) \end{pmatrix}$$

where $\theta = \frac{1+\sqrt{5}}{2}$, $\bar{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1 + j\bar{\theta}$, $\bar{\alpha} = 1 + j\theta$, and x_1, x_2, x_3, x_4 are 2^b-ary QAM signals with inphase and quadrature components equal to $\pm 1, \pm 3, ...$ and having b bits per symbol. The Golden code is a full rate and full diversity code based on cyclic division algebra.

The TAST codes were presented and mathematically defined in [10]- [11]. The main advantage of the TAST code design methodology is that it induces a partitioning of the space-time code into multiple independent codes. The information vector is first partitioned into a set of disjoint component vectors. Each one of the component vectors is then encoded independently

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using a constituent encoder. The composite channel encoder consists of constituent encoders operating on independent information streams, and there is a corresponding partitioning of the composite codeword into a set of constituent codewords. A component space-time formatter assigns the constituent codeword to a thread and sets all off-thread elements to zero.

We prove that Matrix C is not a Golden code by contradiction. Suppose that Matrix C can be derived from the Golden code. Then, there exists s_1, s_2, s_3, s_4 and x_1, x_2, x_3, x_4 such that

$$\frac{1}{\sqrt{1+r^2}} \begin{pmatrix} s_1 + jr \cdot s_4 & r \cdot s_2 + s_3 \\ s_2 - r \cdot s_3 & jr \cdot s_1 + s_4 \end{pmatrix} = k \cdot \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha(x_1 + x_2\theta) & \alpha(x_3 + x_4\theta) \\ j\bar{\alpha}(x_3 + x_4\bar{\theta}) & \bar{\alpha}(x_1 + x_2\bar{\theta}) \end{pmatrix}$$

where k is a scaling factor. We need to consider every case of any permutation of s_1, s_2, s_3, s_4 being the same as x_1, x_2, x_3, x_4. First, we consider the case of $x_1 = x_2 = x_3 = x_4$ and $s_1 = s_2 = s_3 = s_4$. Denoting $x_1, x_2, x_3, x_4, s_1, s_2, s_3, s_4$ by x,

$$\frac{1}{\sqrt{1+r^2}} \begin{pmatrix} x + jr \cdot x & r \cdot x + x \\ x - r \cdot x & jr \cdot x + x \end{pmatrix} = k \cdot \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha(x + x\theta) & \alpha(x + x\theta) \\ j\bar{\alpha}(x + x\bar{\theta}) & \bar{\alpha}(x + x\bar{\theta}) \end{pmatrix}.$$

Then,

$$\frac{x}{\sqrt{1+r^2}} \begin{pmatrix} 1 + jr & r + 1 \\ 1 - r & jr + 1 \end{pmatrix} = k \cdot \frac{x}{\sqrt{5}} \begin{pmatrix} \alpha(1 + \theta) & \alpha(1 + \theta) \\ j\bar{\alpha}(1 + \bar{\theta}) & \bar{\alpha}(1 + \bar{\theta}) \end{pmatrix}$$

which cannot be satisfied because the matrix on the right side has the same elements in the first row and the matrix on the left side has different elements in the first row. Since we have a contradiction, Matrix C cannot be derived as a case of the Golden code by scaling or by permutation of QAM symbols. Thus, Matrix C is not a Golden code.

For a 2×2 TAST code, two threads are needed with one diagonal thread and the other anti-diagonal thread [10]. Each thread is generated using a full diversity rotation matrix. The diagonal entries and anti-diagonal entries of Matrix C can be generated as

$$\frac{1}{\sqrt{1+r^2}} \begin{pmatrix} 1 & jr \\ jr & 1 \end{pmatrix} \begin{pmatrix} s_1 \\ s_4 \end{pmatrix}$$

and

$$\frac{1}{\sqrt{1+r^2}} \begin{pmatrix} r & 1 \\ 1 & -r \end{pmatrix} \begin{pmatrix} s_2 \\ s_3 \end{pmatrix}$$

respectively. Note that both matrices $\begin{pmatrix} 1 & jr \\ jr & 1 \end{pmatrix}$ and $\begin{pmatrix} r & 1 \\ 1 & -r \end{pmatrix}$ have orthogonal columns to achieve full diversity [12]. Since Matrix C has the TAST structure with two threads, Matrix C is a TAST code.

Finally, we note that Document IEEE C802.16e-04/434r2 [8] clearly identifies that Matrix C is not a Golden code [8, p. 1], and that in comparison the two space-time codes have the same bit error rate performance [8, p. 3], but Matrix C requires fewer multiplications for encoding than does the Golden code.

III. Performance Improvement Using Matrix C

The Golden code has been known to have the best performance (the greatest coding gain) among the full diversity space-time codes in public for rate 2 transmission with two transmit antennas [9]. In Fig. 1, it is verified by computer simulations that the Golden code and Matrix C have the same normalized product distance. In Fig. 2, the frame error rates are shown to be the same for the Golden code and Matrix C. One advantage of Matrix C is that each symbol is multiplied by either a real number or a complex number with only imaginary part while it is multiplied by complex numbers consisting of both nonzero real and imaginary parts in the Golden code. Thus, the number of multiplications required for encoding is smaller for Matrix C than for the Golden code, as previously noted in [8].

In Fig. 3, we show the bit error probability (BEP) vs. signal-to-noise ratio (SNR), E_s/N_0, for Matrix A with 16-QAM signaling, and Matrix B and Matrix C with quadrature phase shift keying (QPSK) signaling for $M_\mathrm{r} = 2$ in Rayleigh fading channels. The average symbol energy and single-sided noise spectral density are denoted by E_s and N_0, respectively. For Matrix A, Matrix B, and Matrix C, maximum likelihood (ML) decoding is used. Matrix A with 16-QAM, Matrix B with QPSK, and Matrix C with QPSK have the same data rate, giving a fair and meaningful comparison. Note that the transmitted energy for Matrix A and Matrix C is $4E_s$ for two time slots. Also, the transmitted energy for Matrix B is $2E_s$ for one time slot. Observe that the curves for Matrix A and Matrix C decay with the same slope since they have the same diversity order. Observe from Fig. 3 that Matrix C provides 3.5 dB gain over Matrix A transmission. To achieve this same 3.5 dB improvement, transmitter power would have to be increased by 124%. This observation strongly motivates using Matrix C instead of Matrix A and Matrix B in mobile WiMAX.

IV. Reduced Complexity Decoding Schemes

In this section, recent developments in the search for reduced complexity decoding of Matrix C are surveyed. In [2], an exhaustive search and zero-forcing (ES-ZF) decoder and an exhaustive search and nulling canceling (ES-NC) decoder for Matrix C decoding in uncoded systems are proposed. The reduced complexity decoders, ES-ZF decoder and ES-NC decoder, can be thought of as a hybrid decoder that uses the exhaustive search and a simple decoding algorithms, ZF or NC algorithms. The computational complexity of the ES-ZF decoder for Matrix C decoding is shown to be the same as the complexity of the ZF decoder for Matrix B decoding with twice the number of receive antennas times the complexity of the ML decoder for Matrix B decoding with twice the number of receive antennas. Similarly, the computational complexity of the ES-NC decoder for Matrix C decoding is shown to be the same as the complexity of the NC decoder for Matrix B decoding with twice the number of receive antennas times the complexity of the ML decoder for Matrix B decoding with twice the number of receive antennas. Note that the ES-ZF decoder and the ES-NC decoder for Matrix C decoding in uncoded systems can be implemented using existing hardware adopted for Matrix B decoding. Thus, compared with the ML decoder for Matrix C decoding, the ES-ZF decoder and the ES-NC decoder can reduce not only computational complexity but also chip size for hardware implementation. For coded systems, double pruned trees using the zero-forcing (DPT-ZF) algorithm and double pruned trees using the nulling canceling (DPT-NC) algorithm are proposed in [2]. A pruned tree in the DPT-ZF decoder and the DPT-NC decoder has only one-half of the number of leaf nodes as the tree for the Max-Log decoding of Matrix C, since ZF and NC algorithms reduce the number of leaf nodes by implementing hard decision of two symbols among four symbols in Matrix C.

V. Conclusion

There is a need for improved QoS in 802.16e mobile WiMAX. There already exists a means for improved QoS in 802.16e, namely, the Matrix C space-time code specification option. We have corrected a misunderstanding and proved that Matrix C is not a Golden code, but a TAST code. We have examined and quantified the potential benefits of deploying Matrix C in mobile WiMAX applications. Deployment of Matrix C will contribute to the enhancement of QoS of WiMAX systems. The benefit is equivalent to a 124% increase transmitter power.

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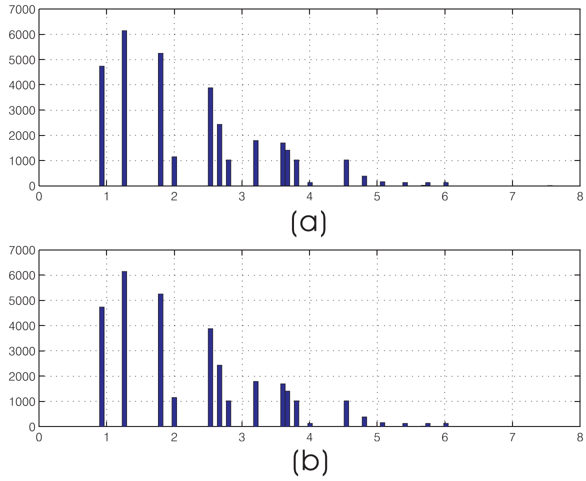

Fig. 1. (a) Histogram of the normalized product distance of the Golden code used with QPSK signal constellation (b) Histogram of the normalized product distance of Matrix C used with QPSK signal constellation.

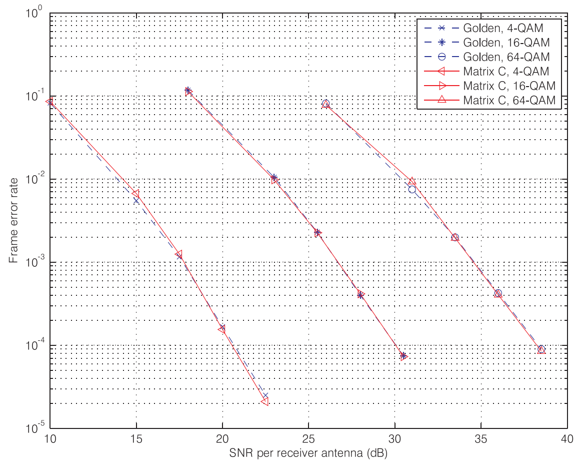

Fig. 2. The frame error rate as a function of SNR for Matrix A with 16-QAM signaling, and Matrix B and Matrix C with QPSK signaling for $M_{\rm r} = 2$.

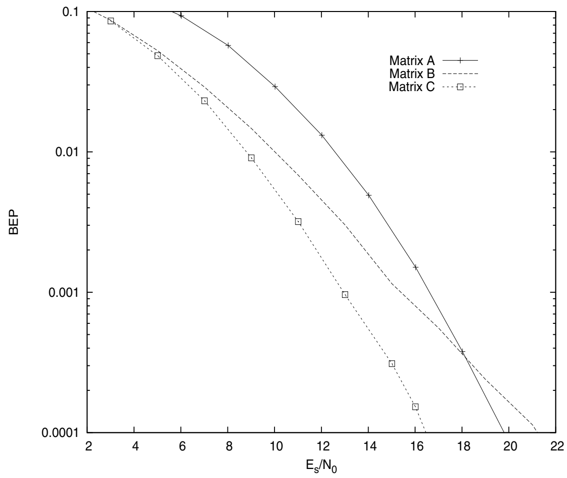

Fig. 3. The BEP as a function of E_s/N_0 for Matrix A with 16-QAM signaling, and Matrix B and Matrix C with QPSK signaling for $M_{\rm r} = 2$.

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