A Differential Codebook Using 8-PSK Alphabets for Slowly Fading Channels

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***Abstract*—In this paper, a novel differential precoding scheme which employs a differential codebook to update the precoding matrices, is proposed for LTE systems. Due to the temporal correlation of the adjacent channel matrices, the consecutive precoding matrices are likely to be similar. This approach quantizes only the differential information of the channel instead of the whole channel subspace, which virtually increases the codebook size to realize more accurate quantization of the channel. Moreover, the proposed precoding utilizes only 8-PSK constellations for codebook design so that it is perfectly compatible with LTE codebooks. The capacity of the proposed codebook performs almost $1.5dB$ better than those of any other non-differential codebooks with the same amount of feedback information.**

***Index Terms*—MIMO, precoding, differential codebook, equal gain transmission.**

I. Introduction

Transmit beamforming for multiple-input multiple-output (MIMO), which is also known as precoding, have been widely adopted in wireless communication standards [1]. It uses quantized channel state information (CSI) at the transmitter to offer a good tradeoff between precoding gain and the required amount of feedback information [2] [3] [4]. In [2] and [3], a codebook is designed by the use of the Grassmannian subspace packing algorithm where codebook matrices are generated to maximize the minimum chordal distances. In [4], a codebook is designed by the use of the Lloyd algorithm to search the right singular value decomposition (SVD) of the channel.

Some codebooks with constant modulus property have also been proposed and it can be called as quantized equal gain transmission (QEGT) [5]. The QEGT codebook is designed from the Grassmannian subspace packing algorithm, preserving equal gain property. In [6] and [7], the codebooks are designed with scalar quantization and vector quantization methods, respectively. QEGT schemes always show suboptimum system performances compared with the quantized maximum ratio transmission (QMRT) schemes. But the constant modulus property of QEGT stands out especially in designing low-cost transmit power amplifiers as well as high-end transmitters of radar systems. That is why the standardization forum seriously considers the constant modulus property in the discussion of codebook design [8] [9]. Moreover, LTE release-8 and release-9 systems already adopted a simple codebook using only 8-PSK constellations as an alphabet of codebook elements to obtain the reduced codebook index search complexity and the constant modulus property [8].

Those previous approaches have commonly assumed the flat fading channel model where the temporal correlation of the wireless channel is neglected. Over temporally correlated channels, some well-defined differential codebook precoding techniques were proposed in [10] and [11]. In [10], the consecutive MIMO channels are assumed to change very slowly so that the differential precoders become quasi-diagonal matrices. In [11], the differential rotation feedback technique was proposed and analyzed by introducing spherical cap radius optimization and applying Procrustes or Gram-Schmidt orthogonalization techniques. LTE release 10 has recently discussed a dual-codebook structure, which has a similarity with the differential or adaptive codebook [12]. In IEEE 802.16m, a transformation-based differential codebook was thoroughly discussed and adopted for the standardization draft [13] [14].

Although the previous differential or conventional codebooks show good performances in capacity analyses, they do not comply with LTE codebook design criteria such as complexity reduction, nested property and constant modulus property by renouncing 8-PSK constellation constraint. In this paper, the proposed differential codebook follows the LTE codebook design criteria. The elements of the proposed codebook consist of the 8-PSK constellations in order to reduce user equipment (UE) complexity and to avoid unnecessary increase in peak-to-average power ratio (PAPR). When high rank codebooks are designed, the nested property is observed.

The remainder of this paper is organized as follows. In Section II, our system model and problem statement are presented. In Section III, the differential precoding scheme is explained and a new differential codebook methodology using 8-PSK constellations is illustrated. Simulation results and related discussions are given in Section IV, and the conclusions of this paper are shown in Section V.

The following notations are used in this paper. $\mathcal{U}(k,l)$ denotes the set of $k \times l$ matrices with orthonormal columns, $\mathbf{I}_k$ denotes the $k \times k$ identity matrix, a bold capital letter $\mathbf{A}$ denotes the matrix, a bold lower case letter $\mathbf{a}$ denotes the vector, $\mathbf{A}^T$ denotes the transposition of matrix $\mathbf{A}$, and $\mathbf{A}^*$ denotes the conjugation transposition of matrix $\mathbf{A}$, $\mathbb{C}^k$ denotes the k-dimensional complex space, and $\mathbf{A} \in \mathbb{C}^{k \times l}$ denotes

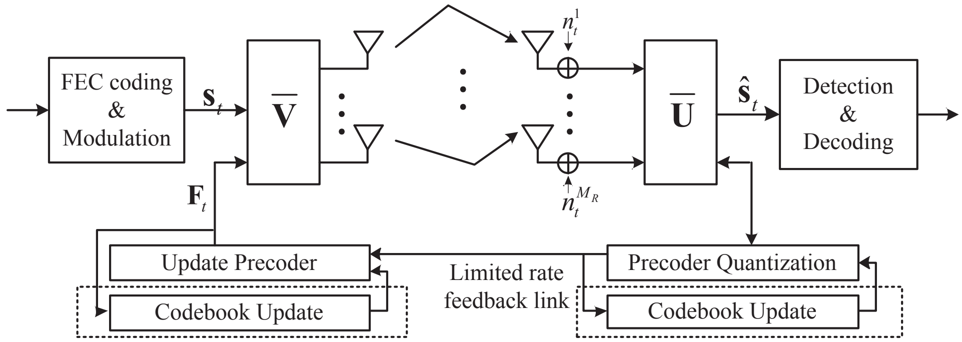

Fig. 1. Block diagram of a limited feedback precoding MIMO system.

complex matrix $\mathbf{A}$ has k rows and l columns. $\mathbf{AB}$ denotes the matrix multiplication of $\mathbf{A}$ and $\mathbf{B}$, while $\mathbf{A} \circ \mathbf{B}$ denotes the Hadamard matrix multiplication of $\mathbf{A}$ and $\mathbf{B}$.

II. System Model

A limited feedback MIMO system with M_t transmit and M_r receive antennas is considered. It is assumed that M_s spatial streams are transmitted simultaneously. Both the transmitter and the receiver have a common codebook with N matrices, $\mathcal{F}_\tau = \{\mathbf{F}_\tau^1 \cdots \mathbf{F}_\tau^N\}$ where τ is the time index and $\mathbf{F}_\tau^n \in \mathbb{C}^{M_t \times M_s}$, $n = 1, \cdots, N$ are precoding matrices. Note that $\mathbb{C}^{a \times b}$ denotes a set of $a \times b$ complex matrices. According to the instantaneous channel estimates, the receiver selects its preferable codebook matrix index, n_{opt}, then the respective codebook index is sent back to the transmitter. The transmitter selects the precoding matrix, $\mathbf{F}_\tau^{n_{opt}}$ among the current codebook, $\mathcal{F}_\tau$. The received signal vector $\mathbf{y}_\tau$ at the M_r receive antennas is described by

$$\mathbf{y}_\tau = \sqrt{\frac{\rho}{M}} \mathbf{H}_\tau \mathbf{F}_\tau \mathbf{s}_\tau + \mathbf{n}_\tau, \tag{1}$$

where $\mathbf{H}_\tau \in \mathbb{C}^{M_r \times M_t}$ denotes the uncorrelated Rayleigh flat fading channel matrix with i.i.d. entries distributed according to $\mathcal{CN}(0,1)$, τ is a discrete time instant, $\mathbf{n}_\tau \in \mathbb{C}^{M_r \times 1}$ denotes the noise vector whose entries are i.i.d. and distributed according to $\mathcal{CN}(0,1)$, and ρ represents the signal-to-noise power ratio (SNR). We assume that $M_r \geq M_s$.

In a channel with temporal correlation, the channel variation can be modeled as a first-order Gauss-Markov process,

$$\mathbf{H}_\tau = \epsilon \mathbf{H}_{\tau-1} + \sqrt{1-\epsilon^2} \mathbf{G}_\tau, \tag{2}$$

where $\mathbf{G}_\tau \in \mathbb{C}^{M_r \times M_t}$ denotes the innovation process having i.i.d. entries distributed according to $\mathcal{CN}(0,1)$. We assume that the noise process $\mathbf{n}_\tau$ is independent of $\mathbf{G}_\tau$ and $\mathbf{H}_0$, and the initial state $\mathbf{H}_0$ is independent of $\mathbf{G}_{\tau+1}$ for all τ. The time correlation coefficient ϵ $(0 \leq \epsilon \leq 1)$ represents the correlation between elements $\mathbf{H}_\tau(i,j)$ and $\mathbf{H}_{\tau+1}(i,j)$. We assume that all the elements of $\mathbf{H}_\tau$ have the same time correlation coefficient ϵ. Multi-user and spatial correlation are not considered in this system model.

III. Differential Precoding

A. Conventional Differential Codebook Operation

In differential quantized feedback systems, only a part of channel space is quantized in order to reduce the quantization distortion. And the quantized subspace depends on the previous precoder. Thereby the differential codebook virtually increases the codebook size to realize finer quantization with the same feedback overhead. The transmitter and receiver are sharing a common codebook update strategy to guarantee that they are holding the same codebook in the general differential codebook-employed system. In [10] or [15], the codebook update process can be represented by

$$\mathbf{F}_{\tau,i} = \mathbf{F'}_i \mathbf{F}_{\tau-1}, \tag{3}$$

where $\mathbf{F'}_i (1 \leq i \leq N)$ denotes the codeword in differential codebook $\mathcal{F}'$ which is made up of random square matrices $\mathbf{F}' \in \mathcal{U}(M_t, M_t)$, the matrices close to the identity matrix (called quasi-diagonal matrix). These matrices refine the orthogonality of the previous precoder and the current precoder. The codebook is optimized to maximize the average mutual information. By using the analysis in [11], the adaptive quasi-diagonal codebook can be found more systematically by the differential codebook adaptation to the temporal correlation variation.

In [16] and [17], the differential codebook generates perturbation points in a spherical cap and projects the perturbed matrices onto unitary space $\mathcal{U}^{M_t \times V}$. The radius of the spherical cap r_m is successively determined by integrating the effects of the channel directional variation and the accumulated quantization error [15]. The codebook update process of perturbation and projection-based adaptive schemes can be expressed as the following

$$\mathbf{F}_{\tau,i} = proj\left(r_m \mathbf{I}_{M_t} + \sqrt{1-r_m} \mathbf{F'}_i\right) \mathbf{F}_{\tau-1}, \tag{4}$$

where $proj(\cdot)$ denotes the projection function. Either Procrustes orthonormalization or Gram-Schmidt column orthonormalization can be used as the projection function [18]. Note that the spherical cap on Grassmann manifold lets the codebook to be per-stream power constraint and total power constraint. The main focus in our paper is to develop a differential codebook with per-antenna equal gain constraint.

B. Proposed Differential Codebook

Since the channel is assumed to change very slowly, the consecutive precoding matrices are almost similar. That is described in the previous differential codebook papers in the expression of quasi-diagonal matrix. Quasi-diagonal matrix is likely to be an identity matrix, which means that the precoding matrix does not change much. In this paper, we design a multi-stream differential codebook having only 1, $e^{j\pi/4}$, and $e^{-j\pi/4}$ not all the 8-PSK constellations for the elements of precoding matrices. Therefore, the proposed codebook can concentrate on the limited part of channel matrix space so that we can design a codebook of higher resolution. But in the process of differential encoding and decoding, the precoders have all 8-PSK constellations by equation (3) or (4). As long as we consider 8-PSK constellations as elements of differential codebooks, the design criterion of [19] is utilized in this paper. For example, when M_t is four and M_s is one, the prime precoding vector in the proposed differential codebook is $\mathbf{F}_1 = [1111]^T$ where the consecutive precoding vector does not change and $\mathbf{F}_1$ is vector form of $\mathbf{F}_1$. When the codebook size, N is fixed to be 16, the remaining 15 precoding vectors having the nearer chordal distance to the prime precoding vector, $\mathbf{F}_1$, can be found by exhaustive search. The proposed rank one differential codebook is shown in Table I. When rank is more than and equal to two, the differential codebook can be found in the same way. When $\mathbf{F}_1 = [1111; 1111]^T$, the differential codebook for rank 2 can be searched as in Table II. At the first feedback of precoding matrix or vector, any kind of the conventional codebook e.g. LTE codebook should be used. Then the selected precoding matrix of the first feedback is multiplied by the proposed differential codebook, which produces the new codebook for the next time instant. The codebook for the time instant, τ can be written as

$$\mathcal{F}_\tau = \mathcal{F}_{diff} \circ \mathbf{F}_{\tau-1}^{n_{opt}}. \tag{5}$$

In the proposed codebook, the matrix or vector multiplication is element-by-element array multiplication so that the differential codebook changes like a series of combination locks [19].

TABLE I
PROPOSED DIFFERENTIAL CODEBOOK FOR RANK 1.

Index	Codeword
1	$[1\ 1\ 1\ 1]^T$
2	$[1\ e^{j\pi/4}\ 1\ 1]^T$
3	$[1\ 1\ e^{j\pi/4}\ 1]^T$
4	$[1\ 1\ 1\ e^{j\pi/4}]^T$
5	$[1\ e^{-j\pi/4}\ 1\ 1]^T$
6	$[1\ 1\ e^{-j\pi/4}\ 1]^T$
7	$[1\ 1\ 1\ e^{-j\pi/4}]^T$
8	$[1\ e^{j\pi/4}\ 1\ e^{-j\pi/4}]^T$
9	$[1\ e^{j\pi/4}\ e^{j\pi/4}\ 1]^T$
10	$[1\ 1\ e^{j\pi/4}\ e^{j\pi/4}]^T$
11	$[1\ e^{j\pi/4}\ 1\ e^{j\pi/4}]^T$
12	$[1\ e^{j\pi/4}\ e^{j\pi/4}\ e^{j\pi/4}]^T$
13	$[1\ e^{-j\pi/4}\ e^{-j\pi/4}\ 1]^T$
14	$[1\ 1\ e^{-j\pi/4}\ e^{-j\pi/4}]^T$
15	$[1\ e^{-j\pi/4}\ 1\ e^{-j\pi/4}]^T$
16	$[1\ e^{-j\pi/4}\ e^{-j\pi/4}\ e^{-j\pi/4}]^T$

TABLE II
PROPOSED DIFFERENTIAL CODEBOOK FOR RANK 2.

Index	Codeword
1	$[1\ 1\ 1\ 1]^T$
2	$[e^{-j\pi/4}\ 1\ 1\ e^{-j\pi/4}]^T$
3	$[e^{-j\pi/4}\ 1\ 1\ 1]^T$
4	$[e^{-j\pi/4}\ 1\ e^{j\pi/4}\ e^{-j\pi/4}]^T$
5	$[e^{-j\pi/4}\ 1\ 1\ e^{j\pi/4}]^T]^T$
6	$[1\ 1\ e^{-j\pi/4}\ 1]^T$
7	$[e^{j\pi/4}\ e^{j\pi/4}\ e^{j\pi/4}\ 1]^T$
8	$[e^{-j\pi/4}\ e^{-j\pi/4}\ e^{-j\pi/4}\ e^{-j\pi/4}]^T$
9	$[e^{j\pi/4}\ 1\ e^{j\pi/4}\ e^{-j\pi/4}]^T$
10	$[e^{-j\pi/4}\ 1\ e^{j\pi/4}\ 1]^T$
11	$[1\ e^{-j\pi/4}\ 1\ 1]^T$
12	$[e^{j\pi/4}\ e^{j\pi/4}\ e^{j\pi/4}\ e^{j\pi/4}]^T$
13	$[1\ 1\ e^{j\pi/4}\ e^{-j\pi/4}]^T$
14	$[1\ 1\ e^{j\pi/4}\ 1]^T$
15	$[e^{j\pi/4}\ e^{j\pi/4}\ e^{j\pi/4}\ e^{j\pi/4}]^T$
16	$[1\ e^{-j\pi/4}\ e^{-j\pi/4}\ 1]^T$

IV. Simulation Results

Monte Carlo simulations were performed to illustrate the capacity performances of the proposed precoding scheme. Also the performances are compared with other existing precoding schemes in Fig. 1. The simulations were run with over 1.5 million iterations per SNR points. The variation of the channel with temporal correlation is modeled as a first-order Gauss-Markov process, and the time correlation coefficient ϵ for the mobile speed of 3km/h and carrier frequency of 2.6GHz have been announced to be 0.999 approximately. The feedback channel is assumed to be error free.

Fig. 2 shows the ergodic capacity of the proposed codebook, LTE codebook (release 8), selection diversity technique (SDT), and the ideal maximum ratio transmission (MRT) employing SVD technique. In this simualtion, $M_t = 4$, $M_r = 1$, and $M_s = 1$ is assumed. For 3km/h ($\epsilon = 0.999$), the proposed scheme outperforms the conventional codebook schemes. The SNR gain is almost $1.5dB$.

The achievable throughput performance of the proposed codebook along with those of [10] and [11], and LTE release-8 codebook, are shown in Fig. 3. Those curves are denoted with the legends of TJ_KIM, T_ABE, and LTE, respectively. When $(M_t,\ M_r,\ M_s) = (4,\ 1,\ 1)$, the proposed codebook shows $0.9dB$ better performance than LTE CB and $0.3dB$ worse than the differential rotation codebook or the quasi-diagonal codebook. When $(M_t,\ M_r,\ M_s) = (4,\ 4,\ 2)$, the proposed codebook shows $0.53dB$ better performance than LTE codebook and $0.15dB$ worse than the differential rotation codebook or the quasi-diagonal codebook. Moreover, as ϵ varies from 0.999 $(1\ km/h)$ to 0.872 $(10\ km/h)$, we can obtain the similar simulation results.

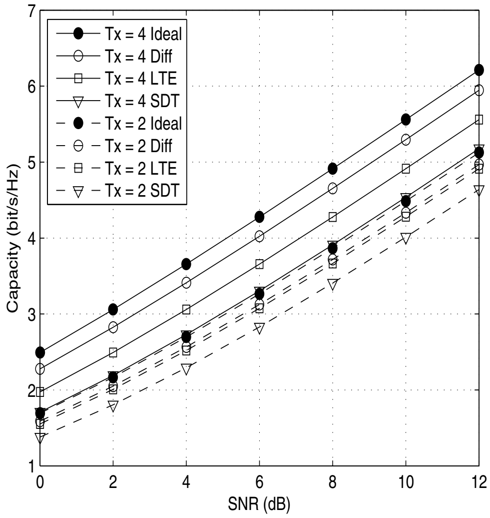

Fig. 2. Capacity performances of the various transmission schemes when $(M_t,\ M_r,\ M_s) = (4,\ 1,\ 1)$ with the user speed of 3km/h ($\epsilon = 0.999$).

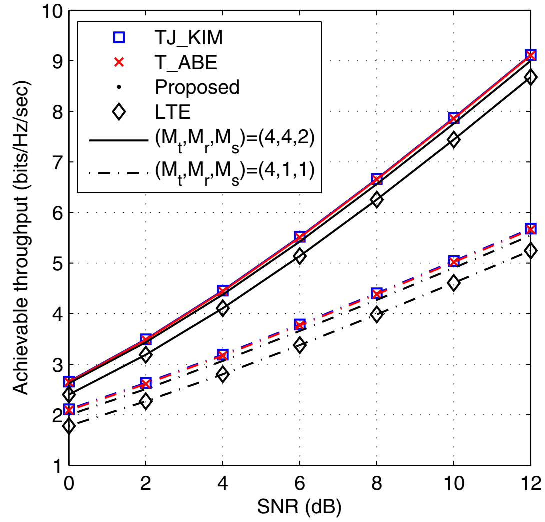

Fig. 3. Capacity performances of the various differential codebook schemes when $(M_t,\ M_r,\ M_s) = (4,\ 1,\ 1)$ and $(4,\ 4,\ 2)$ with the user speed of 3km/h ($\epsilon = 0.999$).

V. Conclusions

In this paper, a simple differential feedback MIMO system for slowly varying channels was proposed to preserve the per-antenna equal gain constraint property. We investigated the channel tracking ability, achievable throughput and BER performances when the number of transmit streams are one and two. The simulation results show that the proposed schemes outperform conventional LTE codebooks while preserving per-antenna equal power constraint. The proposed codebook can be quickly designed and reduce the transceiver complexity greatly. From the simulation results, the proposed codebook showed significant throuput gain, preserving perfect compatibility with LTE and LTE-Advanced systems.

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