Error Compensated MMSE-based Multi-User Precoding for Coordinated Multi-Point Transmission

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***Abstract*—Coordinated multi-point (CoMP) transmission is a general method for mitigating the inter-cell interference in a cellular network through the use of multi-user precoding across neighboring basestations. In this paper, we investigate the system performance of MMSE-based multi-user precoding in a cellular network with CoMP transmission. An iterative algorithm that exploits the duality between the downlink broadcast channel and the uplink multi-access channel is used to compute the multi-user precoding matrices. The algorithm uses the knowledge of the channel error covariance during the precoder computation to alleviate the impact of channel uncertainty at the network. We simulate the system performance of various MMSE-based precoders under the CoMP setting using a realistic MIMO channel model and compare them with those of the zero-forcing precoders. We show that the error-compensated MMSE precoder is more robust towards channel uncertainty and can significantly outperform zero-forcing precoding at high loads.**

I. Introduction

Inter-cell interference has become a major source of signal disturbance in cellular communication networks, limiting not only the service quality of the cell-edge users but also the overall system throughput. An interesting technique for mitigating inter-cell interference that has received much attention recently is the coordinated multi-point (CoMP) transmission, cf. [1][2]. A conceptually simple way of implementing CoMP transmission in the downlink is to connect multiple basestations (BSs) from several adjacent sites with a joint processing unit to form a CoMP cluster. Transmissions to multiple user equipments (UEs) within each CoMP cluster can then be coordinated to reduce or even avoid mutual interference among UEs through multi-user precoding across all antennas within the cluster. Such coordination requires certain channel state information (CSI) to be made available at the BSs either through measurement or feedback on the reverse link.

One well-known technique for multi-user precoding is zero-forcing (ZF) precoding, cf. [2][3], which allows simultaneous transmission to multiple users over the same frequency band without creating any mutual interference within the CoMP cluster. This is done by sending signal to each user in a "direction" orthogonal to the channels of other users. ZF precoding has low computational complexity and performs well when the network has low load. At high load, however, the transmitter may run out of orthogonal dimensions, forcing BSs to transmit in compromised directions. This can in turn lead to low signal-to-noise-plus-interference-ratio and thus limit the overall system throughput.

Linear multi-user precoding based on the minimum mean-squared error (MMSE) criteria [4][5][6][7] is a natural alternative to the ZF precoding. An MMSE precoder weights down the importance of those users with weak channel responses and can thus "open up" the available signal dimensions at high load. However, unlike their receiver counterpart, MMSE precoders for users with multiple receive antennas do not in general have closed-form expressions.

In this paper, we investigate the system performance of MMSE-based multi-user precoding in a cellular network with CoMP transmission. An iterative algorithm that exploits the duality between the uplink multi-access channel and the downlink broadcast channel is used to compute the precoding weights for all antennas within each CoMP cluster. Due to the inevitable channel estimation error incurred in practical systems, the CSI available at the transmitter can only approximately represent the true channel response. To compensate for the uncertainty, we model the channel errors as zero-mean random variables with covariance assumed to be known at the transmitter. We modify the precoder computation algorithm accordingly to take into account the channel error covariance. We evaluate the performance of MMSE-based precoding schemes with or without error-compensation and compare it with that of ZF precoding schemes in a cellular network with CoMP transmission through elaborate system simulations. We show that the error-compensated MMSE precoder is more robust towards channel uncertainty and can significantly outperform zero-forcing precoding at high loads.

II. System Model

Consider a downlink CoMP transmission scenario where n_b BSs are used to transmit to K UEs simultaneously over the same spectrum. For each UE $k \in \{1, 2, \cdots, K\}$, let $\tilde{\mathbf{H}}_k$ be the $n_{r,k}$-by-$n_t n_b$ channel matrix from all the cooperating BSs in a CoMP cluster to UE k, where $n_{r,k}$ denotes the number of receive antennas at UE k, n_b denotes the number of BSs in the CoMP cluster, and n_t denotes the number of transmit antenna in each BS. Also let $\mathbf{P}_k$ denote the $n_t n_b$-by-$n_{s,k}$ precoding matrix of UE k, where $n_{s,k}$ denotes the number of data streams transmitted to UE k, and let $\mathbf{G}_k$ denote the $n_{s,k}$-by-$n_{r,k}$ receiver matrix used by UE k to combine the signals from its receive antennas into $n_{s,k}$ signal streams for subsequent demodulation. For notational simplicity, we let $\mathbf{P} = [\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_K]$ and $\mathbf{G} = \mathrm{diag}\{\mathbf{G}_1, \mathbf{G}_2, \cdots, \mathbf{G}_K\}$.

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Let $\mathbf{s}_k$ denote a $n_{s,k}$-by-1 symbol vector transmitted from the BSs to UE k, normalized such that $E\mathbf{s}_k\mathbf{s}_k^H = \mathbf{I}$. The signal received at UE k is given by

$$\mathbf{r}_k = \tilde{\mathbf{H}}_k\mathbf{P}\mathbf{s} + \tilde{\mathbf{n}}_k,$$

based on which an estimate $\hat{\mathbf{s}}_k$ of $\mathbf{s}_k$ is constructed, where $\mathbf{s} = \left[\mathbf{s}_1^H, \mathbf{s}_2^H, \cdots, \mathbf{s}_K^H\right]^H$, and $\tilde{\mathbf{n}}_k$ denotes the noise-plus-other-CoMP-cluster-interference observed by UE k with covariance matrix $\mathbf{R}_k = E\,\tilde{\mathbf{n}}_k\tilde{\mathbf{n}}_k^H$. Also let $\mathbf{H}_k \equiv \mathbf{R}_k^{-1/2}\tilde{\mathbf{H}}_k$ denote the whitened channel response matrix for UE k, where $\mathbf{R}_k^{-1/2}$ denotes a square root of $\mathbf{R}_k^{-1}$ used to whiten $\tilde{\mathbf{n}}_k$.

To study the impact of channel error on MSE, we model the whitened channel $\mathbf{H}_k$ of the kth user as $\mathbf{H}_k = \hat{\mathbf{H}}_k + \mathbf{E}_k$ for $k = 1, 2, \cdots, K$, where $\hat{\mathbf{H}}_k$ denotes an estimate of $\mathbf{H}_k$ that is available to the transmitter, and $\mathbf{E}_k$ denotes the error of the estimate, which is assumed to be statistically independent of the noise vector $\tilde{\mathbf{n}}$ at the receivers and the channel errors of other users $\{\mathbf{E}_j\}_{j\neq k}$. The channel error matrix $\mathbf{E}_k$ is assumed to have zero mean and cross-covariance matrices $\{\mathbf{R}_{\mathbf{e}_{k,i,j}} \equiv E[\mathbf{e}_{k,i}\,\mathbf{e}_{k,j}^H]\}$ that are known to the transmitter, where $\mathbf{e}_{k,i}$ denotes the ith column of $\mathbf{E}_k$. For notational simplicity, let $\mathbf{E} \equiv \left[\mathbf{E}_1^H, \mathbf{E}_2^H, \cdots, \mathbf{E}_K^H\right]^H$, $\hat{\mathbf{H}} \equiv \left[\hat{\mathbf{H}}_1^H, \hat{\mathbf{H}}_2^H, \cdots, \hat{\mathbf{H}}_K^H\right]^H$ and $\mathbf{R}_{\mathbf{E}} \equiv E\left[\mathbf{E}\mathbf{E}^H\right] = \mathrm{diag}\left(\left\{\sum_{i=1}^{n_t n_b}\mathbf{R}_{\mathbf{e}_{k,i,i}}\right\}_{k=0}^K\right)$. In practice, the error covariance matrices $\{\mathbf{R}_{\mathbf{e}_{k,i,j}}\}$ can often be derived from correlation properties of the reference signal and/or parameters of the quantizer (e.g. its resolution) used to digitize the channel estimate for feedback.

The quality of the communication link between the cooperating BSs and UE k can be measured by the MSE between $\mathbf{s}_k$ and its linear estimate $\hat{\mathbf{s}}_k = \mathbf{G}_k\mathbf{R}_k^{-1/2}\mathbf{r}_k$ constructed at UE k given by $\mathrm{mse}_k^{\mathrm{DL}}(\mathbf{P}, \mathbf{G}_k) \equiv E\left\|\mathbf{s}_k - \hat{\mathbf{s}}_k\right\|^2$, where the expectation is taken over the symbol vector $\mathbf{s}_k$ and the channel error matrix $\mathbf{E}$, and where $\mathbf{G}_k$ denotes the receiver combining matrix employed by UE k.

We seek to find the best set of precoding matrices $\{\mathbf{P}_k\}_{k=1}^K$ that, when combining with the corresponding best set of receiver matrices $\mathbf{G}$, yields the minimum sum MSE under a total power constraint. More precisely, we seek to compute

$$\mathbf{P}_{\mathrm{MMSE}} \equiv \mathop{\arg\min}_{\mathbf{P}:\,\mathrm{tr}\{\mathbf{P}\mathbf{P}^H\}\leq P_{\max}} \min_{\mathbf{G}} \sum_{k=1}^K \mathrm{mse}_k^{\mathrm{DL}}(\mathbf{P}, \mathbf{G}_k) \tag{1}$$

where $P_{\max}$ denotes the maximum total power to be transmitted from all antennas. The optimization problem in (1) in general does not have a closed-form solution. Hence, numerical (e.g. gradient-based) algorithms or algorithms of iterative nature are needed to compute the MMSE precoders.

III. Optimality Conditions

Consider the Lagrangian function for the minimization problem in (1),

$$L(\mathbf{P}, \mathbf{G}) \equiv \sum_{k=1}^K \mathrm{mse}_k^{\mathrm{DL}}(\mathbf{P}, \mathbf{G}_k) + \mu\left[\mathrm{tr}\left(\mathbf{P}\mathbf{P}^H\right) - P_{\max}\right],$$

where $\mu \geq 0$ is the Lagrange multiplier. By taking derivatives of $L(\mathbf{P}, \mathbf{G})$ with respect to $\mathbf{P}_k^*$ and $\mathbf{G}_k^*$, it follows that the Karush-Kuhn-Tucker (KKT) necessary conditions for the optimal $\{\mathbf{P}, \mathbf{G}\}$ are given by

$$\mathbf{G}_k\left(E\left[\mathbf{H}_k\mathbf{P}\mathbf{P}^H\mathbf{H}_k^H\right] + \mathbf{I}\right) = \mathbf{P}_k^H\hat{\mathbf{H}}_k^H, \tag{2}$$

for $k = 1, 2, \cdots, K$, and

$$\left(E\left[\mathbf{H}^H\mathbf{G}^H\,\mathbf{G}\mathbf{H}\right] + \mu\,\mathbf{I}\right)\mathbf{P} \;=\; \hat{\mathbf{H}}^H\mathbf{G}^H, \tag{3}$$

along with the complementary-slackness condition for the power constraint given by

$$\mu\left(\mathrm{tr}\left\{\mathbf{P}\mathbf{P}^H\right\} - P_{\max}\right) \;=\; 0, \tag{4}$$

where $\mathbf{H} = [\mathbf{H}_1^H, \mathbf{H}_2^H, \cdots, \mathbf{H}_K^H]^H$. Note that (2) specifies the optimal receiving matrices in terms of the optimal precoding matrices as

$$\mathbf{G}_k = \mathbf{P}_k^H\hat{\mathbf{H}}_k^H\left[\hat{\mathbf{H}}_k\mathbf{P}\mathbf{P}^H\hat{\mathbf{H}}_k^H + \sum_{i,j}\left[\mathbf{P}\mathbf{P}^H\right]_{i,j}\mathbf{R}_{\mathbf{e}_{k,i,j}} + \mathbf{I}\right]^{-1} \tag{5}$$

where $[\mathbf{A}]_{i,j}$ denotes the element of matrix $\mathbf{A}$ at the ith row and the jth column, while (3) specifies the optimal precoding matrices in terms of the optimal receiving matrices and the Lagrange multiplier. By multiplying (2) by $\mathbf{G}_k^H$ from the right for each k, multiplying (3) the left side by $\mathbf{P}^H$, and summing the traces of all resulting equations, we obtain

$$\mu \;=\; \frac{\sum_{k=1}^K \mathrm{tr}\left\{\mathbf{G}_k\mathbf{G}_k^H\right\}}{P_{\max}} = \frac{\mathrm{tr}\left\{\mathbf{G}^H\mathbf{G}\right\}}{P_{\max}}. \tag{6}$$

From (2), it follows that unless $\{\hat{\mathbf{H}}_k\}$ are zero for all users, μ is positive, which in turn implies, from (4), that the power constraint must be satisfied with an equality.

A suboptimal precoder, which performs well in practice, can be obtained from (3) and (6) by setting $\mathbf{G}_k = \beta^{-1}\,\mathbf{I}_{n_{r,k}}$ for all k,

$$\mathbf{P}^{\mathrm{TxWF}} = \beta\hat{\mathbf{H}}^H\left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \mathbf{R}_{\mathbf{E}} + \frac{\sum_{k=1}^K n_{r,k}}{P_{\max}}\mathbf{I}\right)^{-1} \tag{7}$$

where β is chosen to satisfy the power constraint. We refer to this precoder as the transmit Wiener filter (TxWF), as is known [5] for the special case of $n_{r,k} = 1$ for all k and $\mathbf{R}_{\mathbf{E}} = \mathbf{0}$. When the noise level is small or when the available power is large, this precoder approximately equals to the ZF precoder.

IV. Uplink Dual Problem

To motivate the iterative algorithm we used for computing the MMSE precoder, we describe in this section a virtual dual uplink multi-access problem, where K UEs are transmitting to n_b BSs simultaneously on the same spectrum. For this uplink dual, the channel response matrix for UE k is given by $\mathbf{H}_k^H$. Let $\mathbf{P}_k^{\mathrm{UL}}$ denote a $n_{r,k}$-by-$n_{s,k}$ precoding matrix of UE k, where $n_{s,k}$ denotes the number of data streams transmitted from UE k, and let $\mathbf{G}_k^{\mathrm{UL}}$ denote the $n_{s,k}$-by-$n_t n_b$ receiver combining matrix for UE k. Similar to downlink, for notational simplicity, we let $\mathbf{G}^{\mathrm{UL}} = [\mathbf{G}_1^{\mathrm{UL},H}, \mathbf{G}_2^{\mathrm{UL},H}, \cdots, \mathbf{G}_K^{\mathrm{UL},H}]^H$ and $\mathbf{P}^{\mathrm{UL}} = \mathrm{diag}\{\mathbf{P}_1^{\mathrm{UL}}, \mathbf{P}_2^{\mathrm{UL}}, \cdots, \mathbf{P}_K^{\mathrm{UL}}\}$. For clarity, we will attach superscript DL on variables in the downlink primal problem whenever necessary to distinguish them from those in the uplnk dual problem.

Let $\mathbf{s}_k^{\mathrm{UL}}$ denote a $n_{s,k}$-by-1 symbol vector transmitted from UE k to the BSs normalized such that $E\mathbf{s}_k^{\mathrm{UL}}\mathbf{s}_k^{\mathrm{UL},H} = \mathbf{I}$. The signal received by all BSs is given by

$$\mathbf{r}^{\mathrm{UL}} = \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{P}_k^{\mathrm{UL}} \mathbf{s}_k^{\mathrm{UL}} + \mathbf{n}^{\mathrm{UL}}, \tag{8}$$

based on which an estimate $\hat{\mathbf{s}}_k^{\mathrm{UL}} = \mathbf{G}_k^{\mathrm{UL}}\mathbf{r}^{\mathrm{UL}}$ of $\mathbf{s}_k^{\mathrm{UL}}$ is constructed, where $\mathbf{s}^{\mathrm{UL}} = [\mathbf{s}_1^{\mathrm{UL},H}, \mathbf{s}_2^{\mathrm{UL},H}, \cdots, \mathbf{s}_K^{\mathrm{UL},H}]^H$ and $\mathbf{n}^{\mathrm{UL}}$ denotes the noise vector observed by the UEs with covariance matrix $E\,\mathbf{n}^{\mathrm{UL}}\mathbf{n}^{\mathrm{UL},H} = \mathbf{I}$.

The MSE between $\mathbf{s}_k^{\mathrm{UL}}$ and its estimate $\hat{\mathbf{s}}_k^{\mathrm{UL}}$ is given by

$$\mathrm{mse}_k^{\mathrm{UL}}(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}_k^{\mathrm{UL}}) \equiv E\left\|\mathbf{s}_k^{\mathrm{UL}} - \hat{\mathbf{s}}_k^{\mathrm{UL}}\right\|^2 \tag{9}$$

From (9) it is clear that the optimal receiver matrix of each user for a given set of precoding matrices $\mathbf{P}^{\mathrm{UL}}$ is the well-known linear MMSE receiver matrix that treats signals transmitted from other users as interference given by

$$\mathbf{G}_k^{\mathrm{UL}} = \mathbf{P}_k^{\mathrm{UL},H}\hat{\mathbf{H}}_k \begin{pmatrix} \hat{\mathbf{H}}^H\mathbf{P}^{\mathrm{UL}}\mathbf{P}^{\mathrm{UL},H}\hat{\mathbf{H}} \\ +E\left[\mathbf{E}^H\mathbf{P}^{\mathrm{UL}}\mathbf{P}^{\mathrm{UL},H}\mathbf{E}\right] + \mathbf{I} \end{pmatrix}^{-1}, \tag{10}$$

where the (i,j)th element of the matrix $E\left[\mathbf{E}^H\mathbf{P}^{\mathrm{UL}}\mathbf{P}^{\mathrm{UL},H}\mathbf{E}\right]$ is given by $\mathrm{tr}\left\{\sum_k \mathbf{P}_k^{\mathrm{UL}}\mathbf{P}_k^{\mathrm{UL},H}\mathbf{R}_{\mathbf{e}_{k,j,i}}\right\}$.

A. Transformations

As demonstrated in [4], for each set of precoding matrices and receiver matrices $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$ in the downlink primal problem, there exist a corresponding set of precoding matrices and receiver matrices $\left(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}^{\mathrm{UL}}\right)$ such that

$$\mathrm{mse}_k^{\mathrm{UL}}(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}_k^{\mathrm{UL}}) = \mathrm{mse}_k^{\mathrm{DL}}(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}_k^{\mathrm{DL}}). \tag{11}$$

for all k. Moreover, the corresponding precoding matrices $\mathbf{P}^{\mathrm{DL}}$ in the downlink (or $\mathbf{P}^{\mathrm{UL}}$ in the uplink) are related to the corresponding receiver matrices $\mathbf{G}^{\mathrm{UL}}$ (or, respectively, $\mathbf{G}^{\mathrm{DL}}$ in the downlink) that yield the same mean-squared error through a set of simple per-user scalar transformation factors, i.e.

$$\mathbf{P}_k^{\mathrm{DL}} = \alpha_k \mathbf{G}_k^{\mathrm{UL},H} \quad \text{and} \quad \mathbf{P}_k^{\mathrm{UL}} = \alpha_k \mathbf{G}_k^{\mathrm{DL},H}. \tag{12}$$

Specifically, for a given precoder-receiver pair $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$ in the downlink, the per-user transformation factors $\{\bar{\alpha}_k\}_{k=1}^K$ from downlink to uplink can be obtained by eliminating the variables $\mathbf{P}^{\mathrm{UL}}$ and $\mathbf{G}_k^{\mathrm{UL}}$ in (11) using (12) and replacing $\{\alpha_k\}_{k=1}^K$ with $\{\bar{\alpha}_k\}_{k=1}^K$. This yields a set of linear equations of $\{\bar{\alpha}_k\}_{k=1}^K$ in terms of $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$ given by

$$\left[\bar{\alpha}_1^2, \bar{\alpha}_2^2, \cdots, \bar{\alpha}_K^2\right]\ \bar{\mathbf{T}} \ = \ \left[\bar{\gamma}_1, \bar{\gamma}_2, \cdots, \bar{\gamma}_K\right] \tag{13}$$

where $\bar{\gamma}_i \equiv \mathrm{tr}\{\mathbf{P}_i^{\mathrm{DL}}\mathbf{P}_i^{\mathrm{DL},H}\}$ and

$$\left[\bar{\mathbf{T}}\right]_{j,i} = \begin{cases} \mathrm{tr}\left\{\mathbf{G}_i^{\mathrm{DL}}\begin{bmatrix} \hat{\mathbf{H}}_i\mathbf{P}_{-i}^{\mathrm{DL}}\mathbf{P}_{-i}^{\mathrm{DL},H}\hat{\mathbf{H}}_i^H \\ +\boldsymbol{\Phi}_i\left(\mathbf{P}_{-i}^{\mathrm{DL}}\mathbf{P}_{-i}^{\mathrm{DL},H}\right) + \mathbf{I}\end{bmatrix}\mathbf{G}_i^{\mathrm{DL},H}\right\} & \text{for } \ j = i \\ -\mathrm{tr}\left\{\mathbf{G}_j^{\mathrm{DL}}\begin{bmatrix}\hat{\mathbf{H}}_j\mathbf{P}_i^{\mathrm{DL}}\mathbf{P}_i^{\mathrm{DL},H}\hat{\mathbf{H}}_j^H \\ +\boldsymbol{\Phi}_j\left(\mathbf{P}_i^{\mathrm{DL}}\mathbf{P}_i^{\mathrm{DL},H}\right)\end{bmatrix}\mathbf{G}_j^{\mathrm{DL},H}\right\} & \text{for } \ j \neq i \end{cases}$$

where $\mathbf{P}_{-i}^{\mathrm{DL}}$ denotes the matrix $\mathbf{P}^{\mathrm{DL}}$ with $\mathbf{P}_i^{\mathrm{DL}}$ taken out and $\boldsymbol{\Phi}_j(\mathbf{A}) \equiv \sum_{p,q}[\mathbf{A}]_{p,q}\mathbf{R}_{\mathbf{e}_{j,p,q}}$ for any matrix $\mathbf{A}$. Since $\bar{\mathbf{T}}$ is a strictly diagonally dominant real-valued matrix with positive diagonal entries and negative off-diagonal entries, it is invertible, and all the entries of its inverse $\bar{\mathbf{T}}^{-1}$ are non-negative, as pointed out in [4] for a similar matrix. Therefore, the solution vector $\left[\bar{\alpha}_1^2, \bar{\alpha}_2^2, \cdots, \bar{\alpha}_K^2\right]$ of (13) must have positive entries. After taking square roots, the resulting scaling factors $\{\bar{\alpha}_k\}_{k=1}^K$ can then be used to transform $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$ to the corresponding $\left(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}^{\mathrm{UL}}\right)$ while preserving the mean-squared error of each user. Moreover, summing up the columns of (13) and combining with (12), one can see that [4],

$$\begin{aligned}\mathrm{tr}\left\{\mathbf{P}^{\mathrm{DL}}\mathbf{P}^{\mathrm{DL},H}\right\} &= \sum_{k=1}^{K}\bar{\alpha}_k^2\,\mathrm{tr}\left\{\mathbf{G}_k^{\mathrm{DL}}\mathbf{G}_k^{\mathrm{DL},H}\right\} \\ &= \mathrm{tr}\left\{\mathbf{P}^{\mathrm{UL}}\mathbf{P}^{\mathrm{UL},H}\right\}.\end{aligned}$$

In other words, the transformation also preserves the total transmit power.

Similarly, for a given precoder-receiver pair $\left(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}^{\mathrm{UL}}\right)$ in the dual uplink, the corresponding per-user transformation factors $\{\tilde{\alpha}_k\}_{k=1}^K$ from dual uplink to downlink can be obtained by eliminating the variables $\mathbf{P}^{\mathrm{DL}}$ and $\mathbf{G}_k^{\mathrm{DL}}$ in (11) using (12) and by replacing $\{\alpha_k\}_{k=1}^K$ with $\{\tilde{\alpha}_k\}_{k=1}^K$. This yields a set of linear equations of $\{\tilde{\alpha}_k\}_{k=1}^K$ in terms of $\left(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}^{\mathrm{UL}}\right)$ given by

$$\left[\tilde{\alpha}_1^2, \tilde{\alpha}_2^2, \cdots, \tilde{\alpha}_K^2\right]\ \tilde{\mathbf{T}} \ = \ \left[\tilde{\gamma}_1, \tilde{\gamma}_2, \cdots, \tilde{\gamma}_K\right] \tag{14}$$

where $\tilde{\gamma}_i \equiv \mathrm{tr}\{\mathbf{P}_i^{\mathrm{UL}}\mathbf{P}_i^{\mathrm{UL},H}\}$,

$$\left[\tilde{\mathbf{T}}\right]_{j,i} = \begin{cases} \mathrm{tr}\left\{\mathbf{G}_i^{\mathrm{UL}}\begin{bmatrix}\sum_{l\neq i}\hat{\mathbf{H}}_l^H\mathbf{P}_l^{\mathrm{UL}}\mathbf{P}_l^{\mathrm{UL},H}\hat{\mathbf{H}}_l \\ +\sum_{l\neq i}\Psi_l\left(\mathbf{P}_l^{\mathrm{UL}}\mathbf{P}_l^{\mathrm{UL},H}\right) \\ +\mathbf{I}\end{bmatrix}\mathbf{G}_i^{\mathrm{UL},H}\right\} & \text{for } \ j = i \\ -\mathrm{tr}\left\{\mathbf{G}_j^{\mathrm{UL}}\begin{bmatrix}\hat{\mathbf{H}}_i^H\mathbf{P}_i^{\mathrm{UL}}\mathbf{P}_i^{\mathrm{UL},H}\hat{\mathbf{H}}_i \\ +\Psi_i\left(\mathbf{P}_i^{\mathrm{UL}}\mathbf{P}_i^{\mathrm{UL},H}\right)\end{bmatrix}\mathbf{G}_j^{\mathrm{UL},H}\right\} & \text{for } \ j \neq i, \end{cases}$$

and $[\Psi_i(\mathbf{A})]_{p,q} \equiv \mathrm{tr}\left\{\mathbf{A}\mathbf{R}_{\mathbf{e}_{i,p,q}}\right\}$ for any matrix $\mathbf{A}$, whose solution can be used to obtain the transformation scaling factors $\{\tilde{\alpha}_k\}_{k=1}^K$, which in turn can be used to transform $\left(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}^{\mathrm{UL}}\right)$ to the corresponding $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$ while preserving the mean-squared error of each user as well as the total transmit power.

V. Precoder Computation Algorithm

Here we describe the algorithm we adopted for computing the MMSE precoding matrices through iterative transformation between a virtual uplink dual problem and the original downlink (primal) problem. An attractive property of this algorithm is that each step in the algorithm has a closed form expression.

The algorithm begins with an initialization of the downlink precoders $\mathbf{P}^{\mathrm{DL}}$ that are normalized to fulfill the total power constraint $\mathrm{tr}\left\{\mathbf{P}^{\mathrm{DL}}\mathbf{P}^{\mathrm{DL},H}\right\} = P_{\max}$. For example, $\mathbf{P}^{\mathrm{DL}}$ can be initialized by a ZF precoder given by

$$\mathbf{P}^{\mathrm{DL}} = \left[\mathbf{P}_1^{\mathrm{DL}}, \mathbf{P}_2^{\mathrm{DL}}, \cdots, \mathbf{P}_K^{\mathrm{DL}}\right] = \hat{\mathbf{H}}^H\left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H\right)^{-1}\Lambda$$

where Λ denotes a diagonal matrix of power levels allocated to each data stream to maximize the throughput of each individual user under the total power constraint, which may be computed using standard water-filling power-allocation algorithms. With the precoding matrices initialized, the algorithm computes the optimal receiver matrices $\mathbf{G}^{\mathrm{DL}}$ using (5). With $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$, the per-user transformation scaling factors $\{\bar{\alpha}_k\}_{k=1}^{K}$ is then computed by solving (13). These factors are then used to transform $\mathbf{G}^{\mathrm{DL}}$ into the precoding matrices $\mathbf{P}^{\mathrm{UL}}$ for the dual uplink as described in (12). With $\mathbf{P}^{\mathrm{UL}}$, the optimal receiver matrices $\mathbf{G}^{\mathrm{UL}}$ for the dual uplink are then computed using (10). With $\left(\mathbf{P}^{\mathrm{UL}}, \mathbf{G}^{\mathrm{UL}}\right)$, the per-user transformation scaling factors $\{\tilde{\alpha}_k\}_{k=1}^{K}$ is then computed by solving (14). These factors are then used to transform $\mathbf{G}^{\mathrm{UL}}$ back into the precoding matrices $\mathbf{P}^{\mathrm{DL}}$ for downlink as in (12). This process can be repeated multiple times until a certain stopping criteria, such as that the maximum number of iteration is reached or that the percentage reduction in the sum-MSE falls below certain predefined threshold, is satisfied. Note that the algorithm is guaranteed to converge in the sense that the sum-MSE of the resulting pair of precoding and receiver matrices $\left(\mathbf{P}^{\mathrm{DL}}, \mathbf{G}^{\mathrm{DL}}\right)$ monotonically decreases after each iteration. The algorithm applies to any number of receive antennas for each user.

Note that an alternative way of computing $\mathbf{P}^{\mathrm{DL}}$and $\mathbf{G}^{\mathrm{DL}}$ is to directly iterate among the KKT necessary conditions for minimizing MSE, namely (2), (3) and (6). However, the resulting algorithm does not guarantee to monotonically reduce the MSE after each iteration and, consequently, often leads to an inferior choice of $\mathbf{P}^{\mathrm{DL}}$, since computing the optimal $\mathbf{P}^{\mathrm{DL}}$ for a given $\mathbf{G}^{\mathrm{DL}}$ based on (3) requires the knowledge of the Lagrange multiplier μ, which depends on the optimal choice of $\mathbf{G}^{\mathrm{DL}}$ instead of the given $\mathbf{G}^{\mathrm{DL}}$.

VI. Numerical Results

In this section, we present system simulation results of an OFDM cellular communication system employing CoMP transmission in the downlink using the linear multi-user precoding techniques described above. A hexagonal cell layout, as depicted in Fig. 1, is adopted. Each CoMP cluster is formed by grouping 9 neighboring cells (or sectors) served by 3 sites with sectorized antennas. Each site is connected to a CoMP-cluster-specific centralized joint processor, which may be physically located in one of the sites within the CoMP cluster. Each cell is assumed to serve one scheduled UE in each time frame and at each frequency subcarrier. Based on the acquired information about the channel state from each UE, the centralized processor computes the precoding matrices for all UEs and sends the combined precoded signals for all UEs to each basestation within the CoMP cluster. All UEs are assumed to be moving at a speed of 3 km per hour. All cells of each CoMP cluster are assumed to be synchronized in time, and the cyclic prefix is assumed to be sufficiently long for the propagation delays within each CoMP cluster. As shown in Fig. 1, a total of 7 CoMP clusters (or 63 cells) are simulated with radio signals wrapped around from end to end. Each UE is assumed to have 2 receive antennas. It is assumed here that the network exploits the channel reciprocity between uplink and downlink in the TDD mode of communications and acquires the downlink channel response by estimating the uplink channel over an uplink reference signal. A delay of 2 ms is also assumed for the channel response estimated at the transmitter. Other system simulation parameters are summarized in Table I.

Fig. 2 compares the system performance of multi-user MMSE precoding and ZF precoding in terms of system throughput (sum data rates of all UEs) versus data throughput of the 5-percentile UEs, which is indicative of the data throughput of UEs located near cell edges. Each basestation is assumed to have 2 transmit antennas for each cell. Three different precoding schemes are shown in Fig. 2, namely ZF, basic MMSE (with no error compensation), and error-compensated MMSE (MMSE-EC) precoding . The solid curves represent results with ideal channel knowledge at the transmitter,

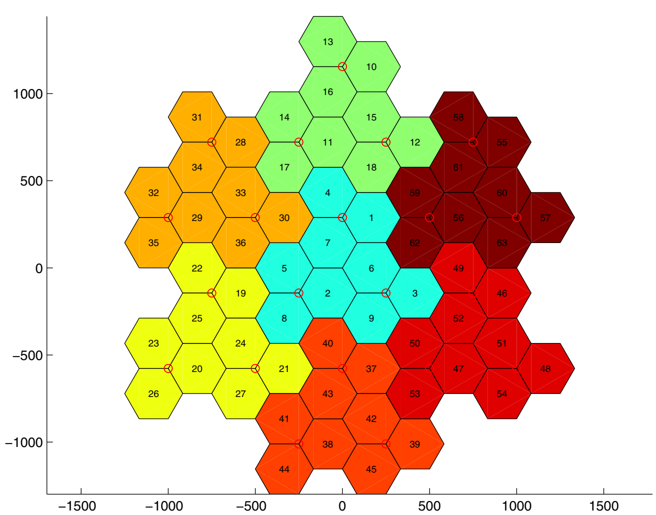

Fig. 1. Cell Layout with Sectorized antennas with 9 Cells Per CoMP cluster (red dots denote site locations; numerical values on each axis are in meters)

TABLE I
SYSTEM SIMULATION PARAMETERS

Modulation	QPSK, 16QAM, 64QAM
Coding	Practical Turbo Codes
Link Adaptation	Ideal (i.e. based on perfect channel quality measurements)
Channel Model	3GPP SCM [8]
Scattering Environment	Suburban-macro [8]
Base Antennas	120-degree antenna (with no down tilt)
Bandwidth	5 [MHz]
Frequency Reuse	1/1
Inter-site distance	500 [m]
UE receiver	Ideal MMSE-SIC
Data Traffic Model	Full buffer
Scheduling	Round Robin
Tx Power per antenna	5 [placeWatts]
Antennas	Base: 4 or 2 ; UT: 2
Sectors per CoMP cluster	Centralized: 9 sectors
Number of CoMP clusters	7
Precoding Schemes	ZF = Zero Forcing MMSE = Minimum Mean-Squared Error TxWF = Transmit Wiener Filter

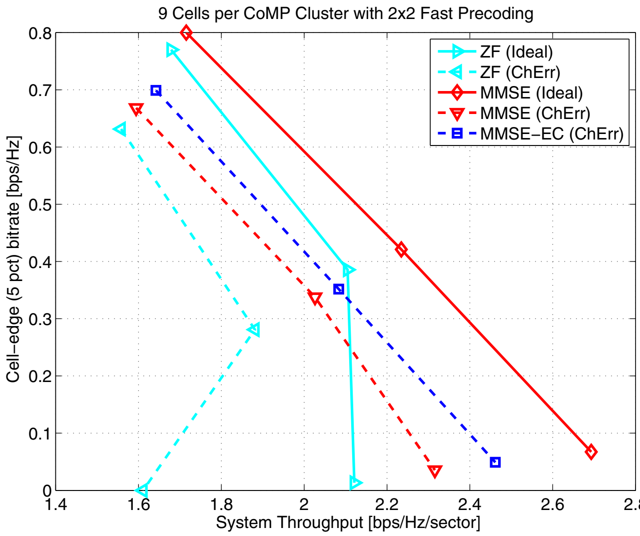

Fig. 2. System performance with coherent multi-user precoding for a CoMP system with 2 transmit antennas per BS and 2 receive antennas per UE

while the dashed curves represent results with uplink channel estimation error. As shown in Fig. 2, with ideal channel knowledge, the MMSE precoder performs substantially better than the ZF precoder. The performance of ZF precoders deteriorates significantly as the network load increases, since it is considerably less likely that the transmitter find compatible directions with good signal-to-noise-rate as the total number of UE receive antennas approaches the total number of transmit antenna per CoMP cluster. In addition, the performance loss of MMSE precoding caused by channel uncertainty at the transmitter is also noticeably less than that of ZF precoding. In the presence of channel error, the error-compensated MMSE precoder provides additional performance gain over the basic MMSE precoder.

Fig. 3 shows a similar system performance plot with 4 transmit antennas per cell instead of 2 transmit antennas. The performance of the TxWF precoders and that of its variant

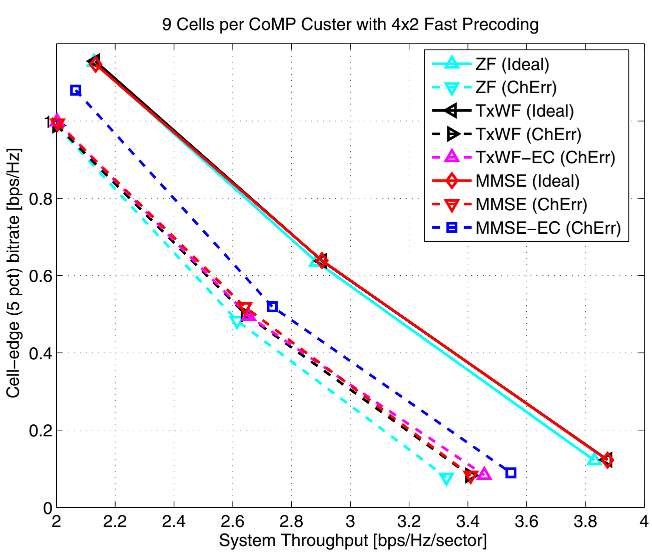

Fig. 3. System performance with coherent multi-user precoding for a CoMP system with 4 transmit antennas per BS and 2 receive antennas per UE

with error compensation (TxWF-EC) are also included in this plot. As shown in the figure, due to the availability of abundant transmit signal dimensions in this case, the performance of the ZF precoder improves substantially compared to the case with 2 transmit antenna per basestation. Without perfect channel knowledge, the performances of all precoders are about the same. However, with channel error, all MMSE-based precoders yield noticeably less performance degradation than the ZF precoder. The TxWF precoder performs remarkably well despite its simplicity. However, the lack of joint optimization with the receiver matrices limits the gain achievable by the TxWF-EC precoder through error compensation. On the other hand, the MMSE-EC precoder is noticeably more robust to channel uncertainty than the other precoders.

VII. Conclusion

We studied the performance of MMSE-based multi-user precoding techniques in a cellular network with CoMP transmission. A cluster-wise centralized architecture for CoMP where basestations are divided into non-overlapping groups was considered. We found that taking into account the extent of channel uncertainty in the precoder computation at the network can make the MMSE-based precoders more robust toward channel errors, which leads to a significantly better system performance in the presence of channel errors than the zero-forcing precoders, especially at high network loads. We showed that there exists simplified, low-complexity versions of the MMSE precoders that can achieve performance close to that of the true MMSE precoder under the CoMP setting.

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