Joint Source-Relay Precoder and Decoder Designs for Amplify-and-Forward MIMO Relay System with Imperfect Channel State Information

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***Abstract*—This paper addresses joint source-relay precoder and decoder designs for a single data flow transmission in amplify-and-forward (AF) multiple-input-multiple-output (MIMO) relay networks with imperfect channel state information (CSI). First, the precoder is obtained by improving the lower bound of the received SNR under power constraints at source and relay. Then, we derive the decoder to maximize the average received signal-to-noise ratio (SNR). Numerical results show a great improvement on the received SNR by applying our proposed schemes. Besides, we also make a discussion about the impact which brought by the channel correlation coefficients based on our derived received SNR expression and the numerical results.**

I. Introduction

Wireless relay networks have attracted significant interest due to their capability of enhancing the system capacity and providing the spatial diversity by the employment of relay nodes [1]. Conventional relay schemes assume single antenna at all of the nodes [2]. However, it has been pointed out that the application of multiple antennas, i.e., multiple-input-multiple-output (MIMO), is a promising scheme for AF-based relay networks in [3], [4]. In AF MIMO relay networks, the system performance can be further improved by applying precoder at source and relay nodes with full channel state information (CSI) or partial CSI.

There are lots of works about the precoder design for AF MIMO relay networks. In [2], the authors jointly optimized the relay precoder and decoder with full or partial channel side information to maximize the received signal-to-noise ratio (SNR). However, our system model is different from [2], and the partial CSI assumption in our paper is more practical. In [4], the authors designed a precoder at relay node to optimize the capacity between the source and the destination in MIMO relay networks with full CSI. In [5], the authors proposed a relay precoder to maximize the upper bound of the capacity in AF MIMO relay networks with imperfect channel state information. An optimal precoder was derived for AF multiple-antenna relay systems to maximize the output SNR and analyses the average symbol error probability in high region over Rayleigh fading channels in [6]. In [7], the authors developed a joint source-relay precoder and decoder to maximize the output SNR with full CSI.

In this study, we consider relay precoder and decoder designs to optimize the received SNR for a single data flow in AF MIMO relay networks. Most of the previous works about precoder designs focused on simple optimization problems by considering precoding at relay node only or joint source-relay precoding with full CSI [4], [2], [7]. However, the precoder and decoder designs in AF MIMO relay networks with imperfect CSI has not been explored deeply yet. In this work, we consider a typical time-division-duplex (TDD) relay network and derive the received SNR expression with imperfect CSI. Based on this expression, we develop the precoder at source and relay with power limits. Then, the optimal decoder to maximize the received SNR is obtained. Moreover, we also make a discussion about the impact brought by the channel correlation coefficients. From the simulation results, we can see significant improvement in terms of the received SNR performance compared with average power allocation precoding scheme even if the CSI is not perfect. Furthermore, the impact of the channel correlation coefficients will be also presented.

This paper is organized as follows. The system model is described in Section II. In Section III, we derive the precoder and decoder to improve the received SNR with imperfect CSI. Simulation results are presented in Section IV. Finally, conclusions are drawn in Section V.

Notations: Vectors and matrices are boldface small and capital letters, respectively; the transpose, Hermitian, inverse of matrix of $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $\mathbf{A}^H$ and $\mathbf{A}^{-1}$, respectively. $E\{\cdot\}$ indicates expectation and $tr\{\cdot\}$ represents the trace function. $\mathbf{I}_N$ and $\mathbf{0}_N$ stand for identitiy and zero matrices of size N, respectively.

II. System Model

Consider a three-node MIMO relay network, where the numbers of antennas at the source, relay and destination are assumed to be M, N, L, respectively, as depicted in Fig. 1. The single data flow is first precoded by $\mathbf{f}_s$ at source node, and then the signal will be transmitted to the relay and destination in the first phase. In the second phase, the signal from source is first precoded at relay node. Then, it will be sent to the destination using AF relay strategy. Destination will combine the signal from source and relay, and then demodulate

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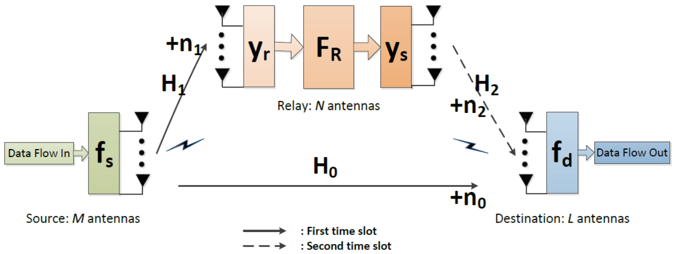

Fig. 1. AF-based MIMO relay system with multiple antennas installed at each node

through the decoder $\mathbf{f}_d$. Flat fading MIMO relay channels are assumed and channel fading remains unchanged for at least two transmission phases.

Let $\mathbf{H}_0$ be the $L \times M$ random channel matrix of the source-destination link, $\mathbf{H}_1$ be the $N \times M$ random channel matrix of the source-relay link, and $\mathbf{H}_2$ be the $L \times N$ random channel matrix of the relay-destination link, all consisting of i.i.d. complex Gaussian random variables denoted as $CN(0, \sigma_0^2)$, $CN(0, \sigma_1^2)$, $CN(0, \sigma_2^2)$, respectively.

As noted above, there are lots of works which studied the precoder design with full CSI. However, in practical environments, receiver and transmitter cannot achieve perfect knowledge of each link. Due to the estimation errors, quantization errors and feedback delay errors, channel information obtained at receiver and transmitter (feedback from the receiver) is usually not identical to the exact channel which the signals pass through.

In general, we can model the discrepancies [5], [8] as complex Gaussian random variables given by

$$\mathbf{H_i} = \rho_i \mathbf{\hat{H}_i} + \mathbf{\Delta_i} \tag{1}$$

where $\rho_i \, (i = 0, 1, 2)$ is the correlation coefficients between the entries of the real channel $\mathbf{H}_i$ and the estimated channel $\hat{\mathbf{H}}_i$. $\mathbf{\Delta}_i$ is zero mean complex Gaussian random matrix uncorrelated with $\mathbf{H}_i$ and has covariance matrix $m\sigma_{\mathbf{\Delta}_i}^2 \mathbf{I} = m\sigma_i^2(1 - \rho_i^2)\mathbf{I}$ (m=M, N, L for i=0, 1, 2, respectively).

We also assume that the CSI error statistics of the channel $\mathbf{H}_i$ is known to the transmitter and receiver. This assumption is reasonable if we take a long time observation during each link in our system.

In the first phase, the signals received at relay and destination are given by

$$\begin{aligned} \mathbf{y_r} &= \sqrt{E_s}\mathbf{H_1 f_s}x + \mathbf{n_1} \\ &= \rho_1\sqrt{E_s}\mathbf{\hat{H}_1 f_s}x + \sqrt{E_s}\mathbf{\Delta_1 f_s}x + \mathbf{n_1} \end{aligned} , \tag{2}$$

$$\begin{aligned} \mathbf{y_1} &= \sqrt{E_s}\mathbf{H_0 f_s}x + \mathbf{n_0} \\ &= \rho_0\sqrt{E_s}\mathbf{\hat{H}_0 f_s}x + \sqrt{E_s}\mathbf{\Delta_0 f_s}x + \mathbf{n_0} \end{aligned} . \tag{3}$$

In the second phase, the signal received at destination is expressed as

$$\begin{aligned} \mathbf{y_2} &= \mathbf{H_2 F_R y_r} + \mathbf{n_2} \\ &= \sqrt{E_s}\mathbf{H_2 F_R H_1 f_s}x + \mathbf{H_2 F_R n_1} + \mathbf{n_2} \\ &= \rho_1\rho_2\sqrt{E_s}\mathbf{\hat{H}_2 F_R \hat{H}_1 f_s}x + \rho_2\sqrt{E_s}\mathbf{\hat{H}_2 F_R \Delta_1 f_s}x \\ &\quad + \rho_1\sqrt{E_s}\mathbf{\Delta_2 F_R \hat{H}_1 f_s}x + \sqrt{E_s}\mathbf{\Delta_2 F_R \Delta_1 f_s}x \\ &\quad + \rho_2\mathbf{\hat{H}_2 F_R n_1} + \mathbf{\Delta_2 F_R n_1} + \mathbf{n_2} \end{aligned} , \tag{4}$$

where x is the transmitted signal satisfying $E\{|x|^2\} = 1$ with power E_s. $\mathbf{n}_i$ (i=0, 1, 2) is a zero mean AWGN at the receiver with its covariance matrix $E\{\mathbf{n}_i{\mathbf{n}_i}^H\} = \sigma_{\mathbf{n}_i}^2\mathbf{I}$ as shown in Fig. 1.(The dimension of matrix $\mathbf{I}$ depends on the location of the receiver.)

We can combine the received vectors at destination into a single vector denoted as

$$\begin{aligned} \mathbf{y} &= [\mathbf{y_1}^T, \mathbf{y_2}^T]^T \\ &= \begin{bmatrix} \sqrt{E_s}\mathbf{H_0 f_s}x \\ \sqrt{E_s}\mathbf{H_2 F_R H_1 f_s}x \end{bmatrix} + \begin{bmatrix} \mathbf{n_0} \\ \mathbf{H_2 F_R n_1} + \mathbf{n_2} \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \rho_0\sqrt{E_s}\mathbf{\hat{H}_0 f_s} \\ \rho_1\rho_2\sqrt{E_s}\mathbf{\hat{H}_2 F_R \hat{H}_1 f_s} \end{bmatrix}}_{\mathbf{\hat{h}}} x + \underbrace{\begin{bmatrix} \sqrt{E_s}\mathbf{\Delta_0 f_s}x + \mathbf{n_0} \\ \mathbf{n_a} \end{bmatrix}}_{\mathbf{n}} \end{aligned} \tag{5}$$

where $\hat{\mathbf{h}}$ and $\mathbf{n}$ are the equivalent estimated channel and noise vector at destination, and we define

$$\begin{aligned} \mathbf{n_a} &= \rho_2\sqrt{E_s}\mathbf{\hat{H}_2 F_R \Delta_1 f_s}x + \rho_1\sqrt{E_s}\mathbf{\Delta_2 F_R \hat{H}_1 f_s}x \\ &\quad + \sqrt{E_s}\mathbf{\Delta_2 F_R \Delta_1 f_s}x + \rho_2\mathbf{\hat{H}_2 F_R n_1} + \mathbf{\Delta_2 F_R n_1} + \mathbf{n_2} \end{aligned} .$$

After receiving the combined signal vector, destination can demodulate the signal by decoder $\mathbf{f}_d$

$$\hat{x} = \left({\mathbf{f}_d}^H\mathbf{\hat{h}}x + {\mathbf{f}_d}^H\mathbf{n}\right) \Big/ {\mathbf{f}_d}^H\mathbf{\hat{h}}. \tag{6}$$

From (5), the equivalent average received SNR can be calculated as

$$\mathrm{SNR} = \frac{E_s \cdot E\left({\mathbf{f}_d}^H\mathbf{\hat{h}}xx^H\mathbf{\hat{h}}^H\mathbf{f}_d\right)}{E\left({\mathbf{f}_d}^H\mathbf{nn}^H\mathbf{f}_d\right)} = \frac{E_s \cdot E\left({\mathbf{f}_d}^H\mathbf{\hat{h}\hat{h}}^H\mathbf{f}_d\right)}{E\left({\mathbf{f}_d}^H\mathbf{nn}^H\mathbf{f}_d\right)}. \tag{7}$$

Under the power constraints at source and relay, our problem can be formulated as

$$\begin{aligned} &\max_{\mathbf{f}_s, \mathbf{F_R}, \mathbf{f}_d} \frac{E_s \cdot E\{{\mathbf{f}_d}^H\mathbf{\hat{h}\hat{h}}^H\mathbf{f}_d\}}{E\{{\mathbf{f}_d}^H\mathbf{nn}^H\mathbf{f}_d\}} \\ &s.t. \\ &tr\{E(E_s\mathbf{f}_s xx^H{\mathbf{f}_s}^H)\} \le p_s \\ &tr\{E(E_s\mathbf{F_R H}_1\mathbf{f}_s xx^H{\mathbf{f}_s}^H{\mathbf{H}_1}^H{\mathbf{F_R}}^H + \mathbf{F_R n}_1{\mathbf{n}_1}^H{\mathbf{F_R}}^H)\} \le p_r \end{aligned} \tag{8}$$

where p_s, p_r are the maximum transmitter power at source and relay, respectively.

III. Precoder and Decoder Designs

Compared with precoder design under full CSI, the imperfect CSI such as channel estimation errors and channel correlation coefficients make our design work more complicated. For joint source-relay precoder and decoder design, we always resort to the iterative methods to find the optimal solution

$$
\mathbf{\Phi} = E(\hat{\mathbf{h}}\hat{\mathbf{h}}^H) = \begin{bmatrix} {\rho_0}^2 E_s \hat{\mathbf{H}}_{\mathbf{0}} \mathbf{f_s} {\mathbf{f_s}}^H \hat{\mathbf{H}}_{\mathbf{0}}^H & \rho_0\rho_1\rho_2 E_s \hat{\mathbf{H}}_{\mathbf{0}} \mathbf{f_s} {\mathbf{f_s}}^H \hat{\mathbf{H}}_{\mathbf{1}}^H {\mathbf{F_R}}^H \hat{\mathbf{H}}_{\mathbf{2}}^H \\ \rho_0\rho_1\rho_2 E_s \hat{\mathbf{H}}_{\mathbf{2}} \mathbf{F_R} \hat{\mathbf{H}}_{\mathbf{1}} \mathbf{f_s} {\mathbf{f_s}}^H \hat{\mathbf{H}}_{\mathbf{0}} & {\rho_1}^2{\rho_2}^2 E_s \hat{\mathbf{H}}_{\mathbf{2}} \mathbf{F_R} \hat{\mathbf{H}}_{\mathbf{1}} \mathbf{f_s} {\mathbf{f_s}}^H \hat{\mathbf{H}}_{\mathbf{1}}^H {\mathbf{F_R}}^H \hat{\mathbf{H}}_{\mathbf{2}}^H \end{bmatrix} \tag{10}
$$

$$
(\mathbf{\Psi}^{H/2})^{-1}\mathbf{\Phi}(\mathbf{\Psi}^{1/2})^{-1} = tr\left(\frac{{\rho_0}^2 E_s tr\left(\hat{\mathbf{H}}_0 \mathbf{f}_s {\mathbf{f}_s}^H \hat{\mathbf{H}}_0^H\right)}{E_s \sigma_{\mathbf{\Delta}_0}^2 tr\left(\mathbf{f}_s {\mathbf{f}_s}^H\right) + \sigma_{\mathbf{n}_0}^2} + tr\left({\mathbf{f}_s}^H \hat{\mathbf{H}}_1^H {\mathbf{F_R}}^H \hat{\mathbf{H}}_2^H (\alpha \hat{\mathbf{H}}_2 \mathbf{F_R} {\mathbf{F_R}}^H \hat{\mathbf{H}}_2^H + \beta\mathbf{I})^{-1} \hat{\mathbf{H}}_2 \mathbf{F_R} \hat{\mathbf{H}}_1 \mathbf{f}_s \right)\right) \tag{13}
$$

$$
\begin{aligned}
&\max_{\mathbf{F_R},\mathbf{f}_s} \frac{E_s \cdot tr\left({\mathbf{f}_s}^H ({\rho_0}^2 \hat{\mathbf{H}}_{\mathbf{0}}^H \hat{\mathbf{H}}_{\mathbf{0}} + {\rho_1}^2{\rho_2}^2 \hat{\mathbf{H}}_{\mathbf{1}}^H {\mathbf{F_R}}^H \hat{\mathbf{H}}_{\mathbf{2}}^H \hat{\mathbf{H}}_{\mathbf{2}} \mathbf{F_R} \hat{\mathbf{H}}_{\mathbf{1}}) \mathbf{f}_s\right)}{L\left(p_s\sigma_{\mathbf{\Delta}_0}^2 + \sigma_{\mathbf{n}_0}^2 + p_s{\rho_1}^2\sigma_{\mathbf{\Delta}_2}^2 tr(\mathbf{F_R}\hat{\mathbf{H}}_1\hat{\mathbf{H}}_1^H{\mathbf{F_R}}^H) + p_s\sigma_{\mathbf{\Delta}_1}^2\sigma_{\mathbf{\Delta}_2}^2 tr(\mathbf{F_R}{\mathbf{F_R}}^H) + \sigma_{\mathbf{n}_1}^2\sigma_{\mathbf{\Delta}_2}^2 tr(\mathbf{F_R}{\mathbf{F_R}}^H) + \sigma_{\mathbf{n}_1}^2\right) + \left(p_s{\rho_2}^2\sigma_{\mathbf{\Delta}_1}^2 + {\rho_2}^2\sigma_{\mathbf{n}_1}^2\right) tr\left(\hat{\mathbf{H}}_2\mathbf{F_R}{\mathbf{F_R}}^H\hat{\mathbf{H}}_2^H\right)} \\
&s.t. \\
&tr\{E(E_s\mathbf{f}_s x x^H {\mathbf{f}_s}^H)\} \le p_s \\
&tr\{E(E_s\mathbf{F_R}\mathbf{H_1}\mathbf{f}_s x x^H {\mathbf{f}_s}^H {\mathbf{H_1}}^H {\mathbf{F_R}}^H + \mathbf{F_R}\mathbf{n}_1{\mathbf{n}_1}^H{\mathbf{F_R}}^H)\} \le p_r
\end{aligned} \tag{14}
$$

to our problem. However, sometimes there is even no close-form solution with imperfect CSI in MIMO relay systems. If possible, the optimal solution will be also very complicated and hard to analyse. To solve the problem, we explore a design method for source-relay precoder and decoder to achieve high received SNR with low complexity in this section.

A. Precoder Design

We first rewrite (7) as

$$
SNR = \frac{E_s \cdot {\mathbf{f}_d}^H \mathbf{\Phi} \mathbf{f}_d}{{\mathbf{f}_d}^H \mathbf{\Psi} \mathbf{f}_d} \tag{9}
$$

where

$$
\begin{aligned}
\mathbf{\Psi} &= E(\mathbf{n}\mathbf{n}^H) \\
&= \begin{bmatrix} (E_s\sigma_{\mathbf{\Delta}_0}^2 tr(\mathbf{f_s}{\mathbf{f_s}}^H) + \sigma_{\mathbf{n}_0}^2)\mathbf{I} & \mathbf{0}_L \\ \mathbf{0}_L & \alpha\hat{\mathbf{H}}_{\mathbf{2}}\mathbf{F_R}{\mathbf{F_R}}^H\hat{\mathbf{H}}_{\mathbf{2}}^H + \beta\mathbf{I} \end{bmatrix} .
\end{aligned} \tag{11}
$$

The correlation matrix (10) of $\hat{\mathbf{h}}$ is shown on the top of this page.

In (11), we define

$$
\begin{aligned}
&\alpha = E_s{\rho_2}^2\sigma_{\mathbf{\Delta_1}}^2 tr(\mathbf{f_s}{\mathbf{f_s}}^H) + {\rho_2}^2\sigma_{\mathbf{n_1}}^2, \\
&\beta = E_s{\rho_1}^2\sigma_{\mathbf{\Delta_2}}^2 tr(\mathbf{F_R}\hat{\mathbf{H}}_{\mathbf{1}}\mathbf{f_s}{\mathbf{f_s}}^H\hat{\mathbf{H}}_{\mathbf{1}}^H{\mathbf{F_R}}^H) + \sigma_{\mathbf{n_2}}^2 \\
&+E_s\sigma_{\mathbf{\Delta_1}}^2\sigma_{\mathbf{\Delta_2}}^2 tr(\mathbf{f_s}{\mathbf{f_s}}^H) tr(\mathbf{F_R}{\mathbf{F_R}}^H) + \sigma_{\mathbf{n_1}}^2\sigma_{\mathbf{\Delta_2}}^2 tr(\mathbf{F_R}{\mathbf{F_R}}^H)
\end{aligned} .
$$

Before deriving precoder $\mathbf{F_R}$ and $\mathbf{f}_s$, we first include the following proposition [9].

Proposition 1: Assume $\mathbf{U}$ is Hermitian and $\mathbf{V}$ is positive definite Hermitian. For any row vector $\mathbf{x}$,

$$
\frac{tr(\mathbf{U})}{tr(\mathbf{V})} \le \frac{\mathbf{x}\mathbf{U}\mathbf{x}^H}{\mathbf{x}\mathbf{V}\mathbf{x}^H} \le \lambda_{\max} \tag{12}
$$

where $\lambda_{\max}$ is the largest eigenvalue of $(\mathbf{V}^{H/2})^{-1}\mathbf{U}(\mathbf{V}^{1/2})^{-1}$. The second equality holds if $\mathbf{x} = c\mathbf{u}_{\lambda_{\max}} \cdot (\mathbf{V}^{H/2})^{-1}$, where c can be any non-zero constant and $\mathbf{u}_{\lambda_{\max}}$ is the eigenvector of $(\mathbf{V}^{H/2})^{-1}\mathbf{U}(\mathbf{V}^{1/2})^{-1}$ corresponding to $\lambda_{\max}$.

Based on *proposition 1*, the optimal precoder and decoder need to maximize the largest eigenvalue of $(\mathbf{\Psi}^{H/2})^{-1}\mathbf{\Phi}(\mathbf{\Psi}^{1/2})^{-1}$ with power constraints. After some manipulations, $(\mathbf{\Psi}^{H/2})^{-1}\mathbf{\Phi}(\mathbf{\Psi}^{1/2})^{-1}$ can be written as equation (13).

Observing (13), we can easily prove that the largest eigenvalue of $(\mathbf{\Psi}^{H/2})^{-1}\mathbf{\Phi}(\mathbf{\Psi}^{1/2})^{-1}$ has an upper bound under the power limits at source and relay. However, solving this constrained problem to find an explicit solution is very difficult. So, we choose to optimize the lower bound of received SNR.

Based on the first inequality of (12) and trace inequality $tr(\mathbf{AB}) \le tr(\mathbf{A})tr(\mathbf{B})$ [10], we replace $tr\left(\mathbf{f_s}{\mathbf{f_s}}^H\right)$ with p_s/E_s, and find the lower bound. Therefore, our optimization problem can be given as (14). Let define $\mathbf{W} = {\rho_0}^2\hat{\mathbf{H}}_{\mathbf{0}}^H\hat{\mathbf{H}}_{\mathbf{0}} + {\rho_1}^2{\rho_2}^2\hat{\mathbf{H}}_{\mathbf{1}}^H{\mathbf{F_R}}^H\hat{\mathbf{H}}_{\mathbf{2}}^H\hat{\mathbf{H}}_{\mathbf{2}}\mathbf{F_R}\hat{\mathbf{H}}_{\mathbf{1}}$

Note that $\mathbf{W}$ is a positive semi-definite Hermitian matrix and can be decomposed as $\mathbf{W} = \mathbf{U_W}\mathbf{\Sigma_W}{\mathbf{U_W}}^H$ where $\mathbf{\Sigma_W}$ is the eigenvalue matrix of $\mathbf{W}$ with decreasing order and $\mathbf{U_W}$ is its corresponding eigenvector matrix.

Observing (14), we find the $\mathbf{f}_s$ only appears at numerator in (14). So, the optimal $\mathbf{f}_s$ is the eigenvector corresponding to the maximum eigenvalue $\lambda_{\mathbf{W}}$ of $\mathbf{W}$. By applying $\mathbf{f}_s$, the transmitter can focus its transmission power on the best sub-channel to the destination. We assume $\mathbf{u_W}$ is the eigenvector corresponding to the maximum eigenvalue, and $\mathbf{f}_s = \eta\mathbf{u_W}$, where η is a coefficient to satisfy the power limit at source node. That is

$$
\begin{aligned}
&tr\{E(E_s\mathbf{f}_s x x^H {\mathbf{f}_s}^H)\} = tr\left(E_s \cdot E\left(\mathbf{f}_s x x^H {\mathbf{f}_s}^H\right)\right) \\
&= \eta^2 E_s tr\left(\mathbf{u_W}{\mathbf{u_W}}^H\right) = p_s \rightarrow \eta = \sqrt{\tfrac{p_s}{E_s}}
\end{aligned} . \tag{15}
$$

Then, we will find a way to design $\mathbf{F_R}$. However, a direct solution to (14) would be very difficult due to its complicated expression with $\mathbf{F_R}$. So, we choose to maximize the lower bound of (14) by applying Weyl's inequality [10]. Substituting $\mathbf{f}_s$ into (14), our problem becomes (16), where λ_1 and λ_2 are the minimum eigenvalue of $\hat{\mathbf{H}}_{\mathbf{0}}^H\hat{\mathbf{H}}_{\mathbf{0}}$ and the maximum eigenvalue of $\hat{\mathbf{H}}_{\mathbf{1}}^H{\mathbf{F_R}}^H\hat{\mathbf{H}}_{\mathbf{2}}^H\hat{\mathbf{H}}_{\mathbf{2}}\mathbf{F_R}\hat{\mathbf{H}}_{\mathbf{1}}$, respectively.

In order to avoid iterative operation between $\mathbf{f}_s$ and $\mathbf{F_R}$ which means low complexity, we define $\mathbf{F_R}$ to diagonalize the matrix $\hat{\mathbf{H}}_{\mathbf{1}}^H{\mathbf{F_R}}^H\hat{\mathbf{H}}_{\mathbf{2}}^H\hat{\mathbf{H}}_{\mathbf{2}}\mathbf{F_R}\hat{\mathbf{H}}_{\mathbf{1}}$. That is,

$$
\mathbf{F_R} = \mathbf{U}_2\mathbf{\Sigma}_R{\mathbf{U}_1}^H \tag{17}
$$

$$\max_{\mathbf{F_R}} \frac{p_s\cdot(\lambda_1+\lambda_2)}{L\left(p_s\sigma^2_{\mathbf{\Delta}_0}+\sigma^2_{\mathbf{n}_0}+p_s{\rho_1}^2\sigma^2_{\mathbf{\Delta}_2}tr(\mathbf{F_R}\hat{\mathbf{H}}_1\hat{\mathbf{H}}_1^H{\mathbf{F_R}}^H)+p_s\sigma^2_{\mathbf{\Delta}_1}\sigma^2_{\mathbf{\Delta}_2}tr(\mathbf{F_R}{\mathbf{F_R}}^H)+\sigma^2_{\mathbf{n}_1}\sigma^2_{\mathbf{\Delta}_2}tr(\mathbf{F_R}{\mathbf{F_R}}^H)+\sigma^2_{\mathbf{n}_1}\right)+\left(p_s{\rho_2}^2\sigma^2_{\mathbf{\Delta}_1}+{\rho_2}^2\sigma^2_{\mathbf{n}_1}\right)tr\left(\hat{\mathbf{H}}_2\mathbf{F_R}{\mathbf{F_R}}^H\hat{\mathbf{H}}_2^H\right)}$$
$$s.t.$$
$$tr\{E(p_s\mathbf{F_R}\mathbf{H_1}{\mathbf{H_1}}^H{\mathbf{F_R}}^H+\mathbf{F_R}\mathbf{n}_1{\mathbf{n}_1}^H{\mathbf{F_R}}^H)\}\leq p_r \tag{16}$$

$$SNR_{LB}=\frac{p_s{\rho_0}^2\lambda_1+p_s{\rho_1}^2{\rho_2}^2\omega_1\psi_1{\xi_1}^2}{L(p_s\sigma^2_{\mathbf{\Delta}_0}+\sigma^2_{\mathbf{n}_0}+p_s{\rho_1}^2\sigma^2_{\mathbf{\Delta}_2}\sum{\xi_i}^2\omega_i+p_s\sigma^2_{\mathbf{\Delta}_1}\sigma^2_{\mathbf{\Delta}_2}\sum{\xi_i}^2+\sigma^2_{\mathbf{n}_1}\sigma^2_{\mathbf{n}_2}\sum{\xi_i}^2+\sigma^2_{\mathbf{n}_2})+(p_s{\rho_2}^2\sigma^2_{\mathbf{\Delta}_1}+{\rho_2}^2\sigma^2_{\mathbf{n}_1})\sum\psi_i{\xi_i}^2} \tag{19}$$

$$\frac{Lp_s{\rho_1}^2{\rho_2}^2\omega_1\psi_1(p_s\sigma^2_{\mathbf{\Delta}_0}+\sigma^2_{\mathbf{n}_0}+\sigma^2_{\mathbf{n}_2})-p_s{\rho_0}^2\lambda_1(Lp_s{\rho_1}^2\sigma^2_{\mathbf{\Delta}_2}\omega_1+Lp_s\sigma^2_{\mathbf{\Delta}_1}\sigma^2_{\mathbf{\Delta}_2}+L\sigma^2_{\mathbf{n}_1}\sigma^2_{\mathbf{\Delta}_2}+p_s{\rho_2}^2\sigma^2_{\mathbf{\Delta}_1}\psi_1+{\rho_2}^2\sigma^2_{\mathbf{n}_1}\psi_1)}{\left(L(p_s\sigma^2_{\mathbf{\Delta}_0}+\sigma^2_{\mathbf{n}_0}+p_s{\rho_1}^2\sigma^2_{\mathbf{\Delta}_2}\sum{\xi_i}^2\omega_i+p_s\sigma^2_{\mathbf{\Delta}_1}\sigma^2_{\mathbf{\Delta}_2}\sum{\xi_i}^2+\sigma^2_{\mathbf{n}_1}\sigma^2_{\mathbf{n}_2}\sum{\xi_i}^2+\sigma^2_{\mathbf{n}_2})+(p_s{\rho_2}^2\sigma^2_{\mathbf{\Delta}_1}+{\rho_2}^2\sigma^2_{\mathbf{n}_1})\sum\psi_i{\xi_i}^2\right)^2} \tag{20}$$

---

where $\mathbf{U}_2$, $\mathbf{U}_1$ are the eigenvector matrix corresponding to the eigenvalue of $\hat{\mathbf{H}}_2^H\hat{\mathbf{H}}_2$ and $\hat{\mathbf{H}}_1\hat{\mathbf{H}}_1^H$ in decreasing order, respectively; $\mathbf{\Sigma}_R$ is a diagonal matrix with decreasing order entries ξ_i to be determined. Furthermore, we assume Hermitian matrix $\hat{\mathbf{H}}_1\hat{\mathbf{H}}_1^H$ and $\hat{\mathbf{H}}_2^H\hat{\mathbf{H}}_2$ can be decomposed as

$$\hat{\mathbf{H}}_1\hat{\mathbf{H}}_1^H=\mathbf{U}_1\mathbf{\Lambda}{\mathbf{U}_1}^H,\ \ \hat{\mathbf{H}}_2^H\hat{\mathbf{H}}_2=\mathbf{U}_2\mathbf{\Gamma}{\mathbf{U}_2}^H \tag{18}$$

where $\mathbf{\Lambda}$ and $\mathbf{\Gamma}$ are diagonal matrix with entries ω_i and ψ_i in decreasing order.

Substituting (17) and (18) into (16), we arrive at (19), shown on the top of this page. Observing (19), we can easily find that any power allocated to $\xi_i (i\geq 2)$ would degrade the received SNR. However, whether put power into ξ_1 depends on the specific parameters in (19). Taking derivative of (19) with respect to ${\xi_1}^2$, (20) is obtained. From (20), we get the conclusion: if (20) ≥ 0, we shall put all of the power at relay into ξ_1. On the other hands, the relay should keep silence and do not participate in the cooperative communication.

Suppose the relay node takes participate in the cooperative communication, ξ_1 should satisfy the power limits at relay node. Substituting (17) and (18) into the power constraint in (16), and after some simple manipulations, the constraints can be given as

$${\xi_1}^2(p_s{\rho_1}^2\omega_1+p_s\sigma^2_{\mathbf{\Delta}_1}+\sigma^2_{\mathbf{n}_1})\leq p_r. \tag{21}$$

Hence, the maximum ξ_1 is

$$\xi_1=\sqrt{p_r/(p_s{\rho_1}^2\omega_1+p_s\sigma^2_{\mathbf{\Delta}_1}+\sigma^2_{\mathbf{n}_1})}. \tag{22}$$

B. Decoder Design

Since we have got the precoder $\mathbf{f}_s$ and $\mathbf{F_R}$, the correlation matrix $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are available now.

Note that $\mathbf{\Phi}$ is a Hermitian matrix and $\mathbf{\Psi}$ is positive definite Hermitian matrix which satisy Proposition 1. Under these conditions, we obtain the optimal decoder

$$\mathbf{f}_d=\left(\mathbf{u}_{\lambda_{\max}}\cdot\left(\mathbf{\Psi}^{H/2}\right)^{-1}\right)^H. \tag{23}$$

$\mathbf{u}_{\lambda_{\max}}$ is the row eigenvector of $(\mathbf{\Psi}^{H/2})^{-1}\mathbf{\Phi}(\mathbf{\Psi}^{1/2})^{-1}$ corresponding to the largest eigenvalue $\lambda_{\max}$.

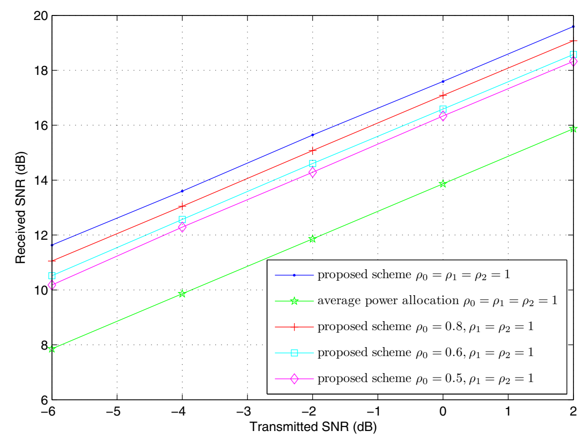

Fig. 2. Received SNR for Proposed precoding scheme with Various ρ_0 and Average Power Allocation Precoding Scheme

$\mathbf{\Psi}^{1/2}$ can be calculated as

$$\mathbf{\Psi}^{1/2}=\begin{bmatrix}\sqrt{E_s\sigma^2_{\mathbf{\Delta}_0}tr(\mathbf{f_s}{\mathbf{f_s}}^H)+\sigma^2_{\mathbf{n}_0}}\mathbf{I} & \mathbf{0}_L\\ \mathbf{0}_L & \mathbf{U}_T(\alpha\mathbf{\Sigma}_T+\beta\mathbf{I})^{1/2}{\mathbf{U}_T}^H\end{bmatrix} \tag{24}$$

where $\mathbf{\Sigma}_T$ is the eigenvalue matrix of $\hat{\mathbf{H}}_2\mathbf{F_R}{\mathbf{F_R}}^H\hat{\mathbf{H}}_2^H$ with decreasing order and $\mathbf{U}_T$ is its corresponding eigenvector matrix.

IV. Simulation Results

In this section, the received SNR performance is evaluated by simulations in relay system with imperfect CSI. Besides, we also present the simulation results which uses average power allocation precoding scheme under the same power limits as proposed schemes for comparison. We assume BPSK modulation in our system with unit signal power. The power limits at source and relay are $p_s=5E_s$ and $p_r=10E_s$, respectively. All the channels are assumed to be i.i.d. complex Gaussian random variable with zero mean and unit variance. The transmitted SNR is $SNR_T=10\log_{10}(E_s/N_0)$ where N_0 is the power of AWGN which is same at each channel.

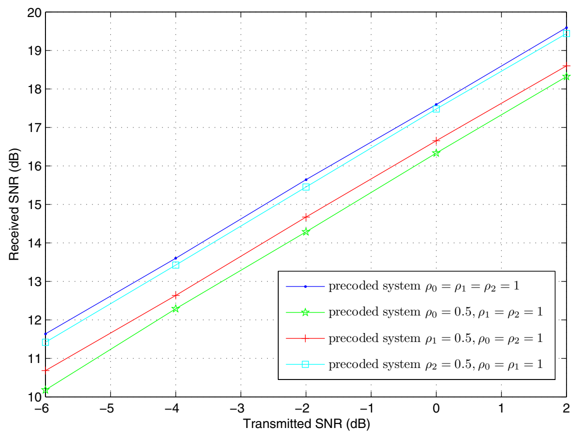

Fig. 3. Received SNR for the Same ρ in Different Link

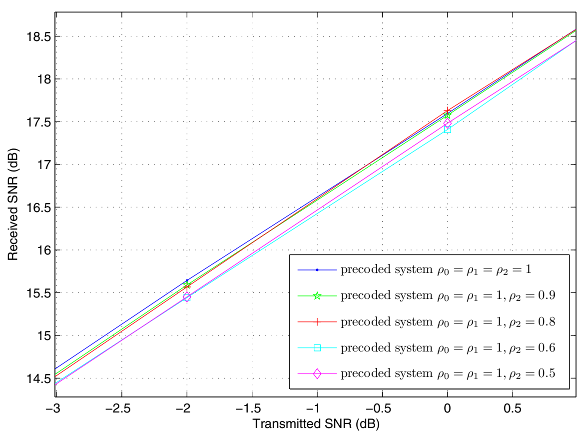

Fig. 4. Received SNR for Various ρ_2

We consider a setup with M=N=L=4. Totally 10^5 Monte-Carlo runs are adopted for average.

The received SNR comparison is given in Fig. 2, 3, 4. In Fig. 2, we can see the significant improvement by applying our proposed schemes. Compared with the average power allocation precoding systems, we can achieve nearly $4dB$ improvements under the same power limits and correlation coefficients. Besides, the impact on received SNR brought by channel correlation coefficients ρ_0 is presented. From (22), we theoretically know that the received SNR increases with the increase in ρ_0. Apparently, Fig. 2 validates our derived results.

Since imperfect CSI degrades our received SNR level, we need to seek which link is the main cause. In Fig. 3, under the same condition, we can see the impact of imperfect CSI on direct link degrades received SNR the most while the relay-destination link is the least. This is not a special case. We get the same results when the correlation coefficient is set to be 0.9, 0.8, 0.6. Thus, the conclusion is, the accuracy of direct link is the most important, followed by the source-relay link and relay-destination link.

Observing (22), we find an interesting conclusion which goes against our conventional understanding. Larger correlation coefficients ρ_1 and ρ_2 do not imply higher received SNR. In other words, better CSI knowledge may also degrade received SNR. In Fig. 4, when the transmitted SNR is -$2dB$, the received SNR corresponding to $\rho_2 = 0.5$ is $15.45dB$ while the received SNR corresponding to $\rho_2 = 0.6$ is $15.44dB$ only. The same occasion happened when the transmitted SNR is $0dB$ between $\rho_2 = 0.5$ and $\rho_2 = 0.8$. So, the degradation impact on our system depends on the specific parameters in our received expression and is not linear correlation with channel correlation coefficients.

V. Conclusion

In this paper, we presented a source-relay precoder and decoder design method in imperfect CSI environment to optimize the received SNR. The precoder to improve the lower bound of received SNR was first derived, expressed with respect to the discrepancies. Then, the decoder under imperfect CSI was obtained. Numerical results show that, the received SNR performance has a significant improvement by applying our proposed schemes. Furthermore, the impact on received SNR brought by the channel correlation coefficient was been discussed based on the numerical results.

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