Joint Hierarchical Modulation and Network Coding for Two Way Relay Networks

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***Abstract*- The performance of a joint Hierarchical Modulation (HM) and Network Coding (NC) scheme for two way relay networks is introduced and evaluated in this paper. The proposed scheme uses selective relaying based on a signal-to-noise (SNR) threshold at the relay. In particular, a two way cooperative network with two sources and one relay is considered. Two different protection classes are modulated by a hierarchical 4/16-Quadrature Amplitude Modulation (QAM) constellation at the source. Based on the instantaneous received SNR at the relay, the relay decides to retransmit both classes by using a hierarchical 4/16-QAM constellation, the more-protection class by using a Quadrature Phase Shift Keying (QPSK) constellation, or remains silent. These thresholds at the relay give rise to multiple transmission scenarios in a two way cooperative network. Simulation results verify our analysis and efficacy of the scheme.**

I. INTRODUCTION

Within the last decade, cooperative communications has gained a lot of attention in the research community. This is due to the fact that the broadcast nature of wireless networks which was earlier considered a drawback can now be used to provide spatial diversity to increase throughput and network resilience. However, the main challenge of cooperative communications is that it requires more bandwidth compared to traditional communication systems. There are different types of forwarding strategies based on the type of processing at the relay, namely Decode and Forward (DF) [1]. In DF, the relay node first decodes the data and then re-encodes it before transmitting it to the destination. This basic forwarding strategy requires advanced techniques which can be used at the intermediate relay nodes to improve spectrum utilization.

Network Coding (NC) is an efficient technique for two way cooperative communications [2]. Here, nodes in a network are capable of combining packets for transmission. The main gain of using NC is throughput improvement, robustness and efficient spectrum utilization. Another gradually emerging technique for efficient spectrum utilization is Hierarchical Modulation (HM) or Superposition Coding [3,4]. This technique allows transmission of multiple data streams simultaneously with different priorities. This gives rise to Unequal Error protection (UEP), where important data is given higher protection as compared to less important data. Under poor channel conditions, the receiver can recover the High Priority (*HP*) classes and with better channel conditions it is possible to recover the Low Priority (*LP*) classes as well.

In this paper, we have proposed our scheme inspired by the results in [5]. In [5], authors have proposed an adaptive transmission protocol with selective relaying at the relay based on two thresholds. According to this protocol, in phase 1, the source transmits the signal with HM. At relay the instantaneous received SNR is compared against the two thresholds to decide 1) to retransmit both classes, or , 2) to retransmit *HP* class only, or 3) to remain silent. Park [6] proposed hierarchically modulated network coding which allows using HM at the source and the relay applies NC on the received data before broadcasting it. In the above schemes authors have shown the bit error rate (BER) performance, however, most of the literature [7,8] in this area uses rate and capacity as performance metrics. Chun-Hung [7] jointly used NC and superposition coding and concluded that the maximum coding gain is achieved when the transmission rate of each node is one third of the sum rate. Modula and Forward [8] scheme apply superposition coding on two lattice coded messages. Sum rate plots show that this scheme performs better compared to traditional schemes under some conditions.

This paper proposes joint HM and NC based relaying scheme. It further evaluates the performance of the scheme in terms of the BER in Rayleigh fading channel. This schemes aims to make use of the two way transmission to achieve better performance. The main features of this scheme are: 1) ability to use HM by all sources and relay, 2) relay's ability to use threshold for detection and NC when forwarding, 3) respective destination's ability to use Selection Combining (SC). The major contribution of this paper thorough investigation on the BER of the proposed scheme by varying the power distribution between *HP* and *LP* bits and threshold at the relay.

II. SYSTEM MODEL

Consider the scenario shown in Fig. 1, consisting of two sources and one relay. The two sources and the relay are capable of using HM consisting of two classes of data i.e. *HP* bits and *LP* bits. There are three phases in this scheme. In phase 1, Source 1 (S_1) transmits a hierarchically modulated data to the relay (R) and directly to Source 2 (S_2). In phase 2, S_2 transmits to R and directly to S_1. In the last phase R forwards the data (after possibly applying NC) to both destinations based on the SNR threshold conditions. The final forwarding strategy is based on the threshold conditions of the data received from the two sources and is depicted in Fig. 2. At the

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respective destination, SC is applied to choose the signal with better SNR.

III. Performance analysis

As mentioned above, all the nodes are capable of using HM 4/16-QAM and use two levels of protection. The relay uses thresholds to determine the form of the next transmission. At the relay R thresholds are defined for the two classes of data from sources S_1 and S_2. As the relay receives bits from both sources it can additionally combine the two bits (by network coding) from different sources into a single bit.

In phase 1, S_1 transmits hierarchically modulated data A symbols composed of high priority symbols A_{HP} and low priority symbols A_{LP}. We define α as the power distribution between HP and LP.

$$A = \sqrt{\alpha} A_{HP} + \sqrt{1-\alpha} A_{LP} \tag{1}$$

Symbols received via the direct link are represented by

$$y_{d1} = h_{S_1 S_2} A + n_{S_1 S_2} \tag{2}$$

where y_{d1} is the received signal at the destination through direct transmission, $h_{S_1 S_2}$ is the channel coefficient, A is defined in (1) and $n_{S_1 S_2}$ is additive white Gaussian noise (AWGN) with zero mean and variance σ^2. Symbols received via the relay link are represented as

$$y_{r1} = h_{S_1 R} A + n_{S_1 R} \tag{3}$$

where y_{r1} is the signal received at the relay from source. Similarly in phase 2, S_2 transmits data B composed of high priority symbols B_{HP} and low priority symbols B_{LP} not shown here due to space constraint. Similarly the symbols of S_2 received via the relay link are denoted by y_{r2}. In phase 3, the relay decides on the forwarding based on the decoded symbols as per the transmission scenarios summarized in Fig. 2.

The BER expressions for the cases described in the proposed scheme are shown below. We have only shown the expressions for cases 1, 2, 3 5 and 9, as the rest of the cases are similar to one of these cases due to a two way relay network.

A. Case 1: Silent Relay (use direct paths only)

In this case, R is not used and we just use the direct path between S_1 to S_2 and vice versa. The BER at S_2 for HP bits resulting from the direct link is [9]. At S_2, the BER of HP of S_1 is:

$$\begin{aligned} P\left\{\varepsilon_{S_1 S_2}^{HP_{S_1}} \,|\lambda\right\} &= \frac{1}{2}\left(\frac{1}{2} erfc\,\frac{d_1^{'}}{\sqrt{N_0}} + \frac{1}{2} erfc\,\frac{d_1^{'} + 2d_2}{\sqrt{N_0}}\right) \\ &= \frac{1}{4}\begin{pmatrix} I(1,0;\lambda;M,\overline{\gamma}_{S_1 S_2}) \\ +I(1,2;\lambda;M,\overline{\gamma}_{S_1 S_2}) \end{pmatrix} \end{aligned} \tag{4}$$

where $P\left\{\varepsilon_{S_1 S_2}^{HP_{S_1}} \,|\lambda\right\}$ is the probability of error for HP bits of S_1 via link S_1 and S_2. M is the constellation size i.e. 16 for 16 QAM and 4 for QPSK, $\overline{\gamma}_{ij}$ is the average SNR (for link between i and j which can be S_1, S_2 or R), d_1, d_2 and $d_1^{'}$ are the distances between the constellation points of 4/16-QAM HM as shown in Fig. 3, λ is the constellation priority parameter and is the ratio of d_2 to $d_1^{'}$, a and b are coefficients of $d_1^{'}$ and d_2, *erfc* is complementary error function and N_0 is the noise. Where we use the following simplification from [6] under Rayleigh fading to evaluate the expressions in (4) and (5):

$$I(a,b;\lambda,M,\overline{\gamma}_{ij}) = erfc\left(1 - \sqrt{\frac{G(a,b;\lambda,M)\overline{\gamma}_{ij})}{1+G(a,b;\lambda,M)\overline{\gamma}_{ij})}}\right), \text{ where}$$

$$G(a,b;\lambda,M) = \frac{(a+b\lambda)^2}{\left(2\left[1+\lambda\left[\sqrt{\frac{M}{2}}-1\right]\right]^2 + \left(\frac{2}{3}\right)\left[\frac{M}{4}-1\right]\lambda^2\right)}$$

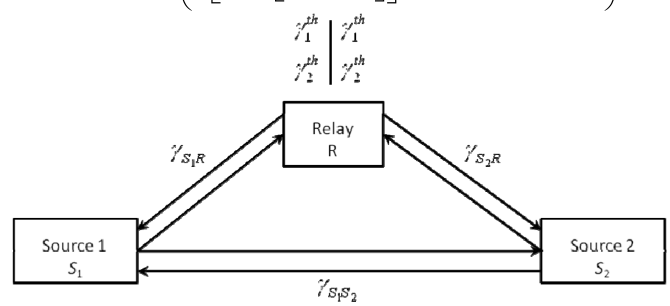

Figure 1. System model .

S_2 \ S_1	$< \gamma_1^{th}$	γ_1^{th} – γ_2^{th}	$> \gamma_2^{th}$
$< \gamma_1^{th}$	**Case 1:** Silent Relay	**Case 2:** Forward high priority bits of S_1	**Case 3:** Forward high and low priority bits of S_1
γ_1^{th} – γ_2^{th}	**Case 4:** Forward high priority bits of S_2	**Case 5:** Neglect low priority bits and network code high priority bits of S_1 and S_2	**Case 6:** Neglect low priority bits of S_1 and network code high priority bits of S_1 and S_2
$> \gamma_2^{th}$	**Case 7:** Forward high and low priority bits of S_2	**Case 8:** Neglect low priority bits of S_2 and network code high priority bits of S_1 and S_2	**Case 9:** Full Network Coding

Figure 2. Forwarding strategies.

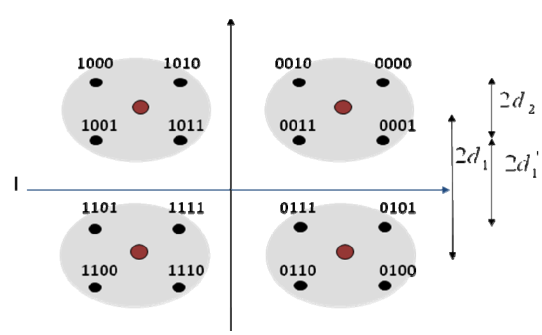

Figure 3. Generalized hierarchical 4/16-QAM.

Similarly the expression for the *LP* bits is [9]:

$$P\left\{\varepsilon_{S_1S_2}^{LP_{S_1}}\left|\lambda\right.\right\}=\frac{1}{2}\left(\begin{array}{l}erfc\,\dfrac{d_2}{\sqrt{N_0}}\\ +\dfrac{1}{2}erfc\,\dfrac{2d_1^{'}+d_2}{\sqrt{N_0}}\\ -\dfrac{1}{2}erfc\,\dfrac{2d_1^{'}+3d_2}{\sqrt{N_0}}\end{array}\right)=\frac{1}{4}\left\{\begin{array}{l}2I\left(0,1;\lambda,M,\overline{\gamma}_{S_1S_2}\right)\\ +I\left(2,1;\lambda,M,\overline{\gamma}_{S_1S_2}\right)\\ -I\left(2,3;\lambda,M,\overline{\gamma}_{S_1S_2}\right)\end{array}\right. \tag{5}$$

where $P\left\{\varepsilon_{S_1S_2}^{LP_{S_1}}\left|\lambda\right.\right\}$ is the probability of error for *LP* bits of S_1 via link S_1 and S_2. The rest of the cases require detection at relay based on the threshold condition [5] for which we need to define the outage probability for each case based on the threshold, followed by the BER at the relay.

$$P_1(S_1)=P(\gamma_{S_1R}\le\gamma_1^{th})=1-e^{-\gamma_1^{th}/\overline{\gamma}_{S_1R}} \tag{6}$$

$$P_2(S_1)=P(\gamma_1^{th}<\gamma_{S_1R}<\gamma_2^{th})=e^{-\gamma_1^{th}/\overline{\gamma}_{S_1R}}-e^{-\gamma_2^{th}/\overline{\gamma}_{S_1R}} \tag{7}$$

$$P_3(S_1)=P(\gamma_{S_1R}>\gamma_2^{th})=e^{-\gamma_2^{th}/\overline{\gamma}_{S_1R}} \tag{8}$$

where $\gamma_1^{th}=\lambda_1^{th}E_s/N_0$ and $\gamma_2^{th}=\lambda_1^{th}E_s/N_0$ are the respective thresholds for *HP* and *LP* symbols of S_1. γ_{S_1R}, γ_{S_2R} and $\gamma_{S_1S_2}$ are the SNR for different links. $P_1(S_1)$ is the probability that the received SNR is below γ_1^{th}, $P_2(S_1)$ is the probability that *HP* symbols of S_1 are between γ_1^{th} and γ_2^{th} and $P_3(S_1)$ is the probability that both *HP* and *LP* symbols are above γ_2^{th}. Equations (6 – 8) represents the Cumulative density function (CDF). Next we calculate the probability density function (PDF) which are truncated exponential, and do the compensation as follows:

$$p_1(\gamma_{S_1R})=\frac{dP_1(S_1)}{d\gamma_{S_1R}}=\frac{1}{1-e^{-\gamma_1^{th}/\overline{\gamma}_{S_1R}}}\frac{e^{-\gamma_{S_1R}/\overline{\gamma}_{S_1R}}}{\overline{\gamma}_{S_1R}} \tag{9}$$

$$p_2(\gamma_{S_1R})=\frac{dP_2(S_1)}{d\gamma_{S_1R}}=\frac{1}{e^{-\gamma_1^{th}/\overline{\gamma}_{S_1R}}-e^{-\gamma_2^{th}/\overline{\gamma}_{S_1R}}}\frac{e^{-\gamma_{S_1R}/\overline{\gamma}_{S_1R}}}{\overline{\gamma}_{S_1R}} \tag{10}$$

$$p_3(\gamma_{S_1R})=\frac{dP_3(S_1)}{d\gamma_{S_1R}}=\frac{1}{e^{-\gamma_2^{th}/\overline{\gamma}_{S_1R}}}\frac{e^{-\gamma_{S_1R}/\overline{\gamma}_{S_1R}}}{\overline{\gamma}_{S_1R}} \tag{11}$$

where $p_1(.)$, $p_2(.)$ and $p_3(.)$ represent three threshold conditions. Next, we calculate the average BER of the *HP* at relay for the case when the threshold of forwarding the *HP* is met. We use the simplification from [5,10] which is as follows:

$$I(a,\overline{\gamma}_{ij},x)=e^{-x/\overline{\gamma}_{ij}}Q(\sqrt{ax})-\left(\sqrt{\frac{a}{\frac{2}{\overline{\gamma}_{ij}}+a}}Q\left(\sqrt{x\left(\frac{2}{\overline{\gamma}_{ij}}+a\right)}\right)\right)$$

where $\overline{\gamma}_{ij}$ is the average SNR. We also know that we can represent the complementary error function as a Q-function and vice versa i.e. $Q(x)=1/2*erfc\left(x/\sqrt{2}\right)$. Therefore,

$$\frac{1}{2}erfc\,\frac{d_1^{'}}{\sqrt{N_0}}=Q\left(\frac{d_1^{'}\sqrt{2\gamma_{ij}}}{\sqrt{E_s}}\right)=Q\left(\frac{(d_1-d_2)\sqrt{2\gamma_{ij}}}{\sqrt{d_1^2+d_2^2}}\right)$$

where E_s is the normalized symbol energy. Next we show the average BER of the *HP* at relay for the threshold condition $\gamma_1^{th}<\gamma_{S_1R}<\gamma_2^{th}$. Therefore, the expression for BER for *HP* bits at the relay $P_2(\varepsilon_R,HP_{S_1})$ is given by:

$$\begin{aligned}P_2(\varepsilon_R,HP_{S_1})&=\frac{1}{e^{-\gamma_1^{th}/\overline{\gamma}_{S_1R}}-e^{-\gamma_2^{th}/\overline{\gamma}_{S_1R}}}\frac{1}{2\overline{\gamma}_{S_1R}}\times\int_{\gamma_1^{th}}^{\gamma_2^{th}}\left\{\begin{array}{l}Q\left(\dfrac{(d_1-d_2)\sqrt{2\gamma_{S_1R}}}{\sqrt{d_1^2+d_2^2}}\right)\\ +Q\left(\dfrac{(d_1+d_2)\sqrt{2\gamma_{S_1R}}}{\sqrt{d_1^2+d_2^2}}\right)\end{array}\right\}e^{-\gamma_{S_1R}/\overline{\gamma}_{S_1R}}d\gamma_{S_1R}\\ &=\frac{1}{2}\frac{1}{e^{-\gamma_1^{th}/\overline{\gamma}_{S_1R}}-e^{-\gamma_2^{th}/\overline{\gamma}_{S_1R}}}\left\{\begin{array}{l}I\left(\dfrac{2(d_1-d_2)^2}{d_1^2+d_2^2},\overline{\gamma}_{S_1R},\gamma_1^{th}\right)-\\ I\left(\dfrac{2(d_1-d_2)^2}{d_1^2+d_2^2},\overline{\gamma}_{S_1R},\gamma_2^{th}\right)+\\ I\left(\dfrac{2(d_1+d_2)^2}{d_1^2+d_2^2},\overline{\gamma}_{S_1R},\gamma_1^{th}\right)-\\ I\left(\dfrac{2(d_1+d_2)^2}{d_1^2+d_2^2},\overline{\gamma}_{S_1R},\gamma_2^{th}\right)\end{array}\right\}\end{aligned} \tag{12}$$

We then derive the BER for *HP* bits $P_3(\varepsilon_R,HP_{S_1})$ and *LP* bits $P_3(\varepsilon_R,LP_{S_1})$ at relay meeting the threshold condition $\gamma_{S_1R}>\gamma_2^{th}$.

$$P_3(\varepsilon_R, HP_{S_1}) = \frac{1}{2}\frac{1}{e^{-\gamma_2^{th}/\bar{\gamma}_{S_1R}}}\left\{\begin{array}{l} I\left(\frac{2(d_1-d_2)^2}{d_1^2+d_2^2}, \bar{\gamma}_{S_1R}, \gamma_2^{th}\right) \\ -I\left(\frac{2(d_1+d_2)^2}{d_1^2+d_2^2}, \bar{\gamma}_{S_1R}, \gamma_2^{th}\right) \end{array}\right\} \quad (13)$$

$$P_3(\varepsilon_R, LP_{S_1}) = \frac{1}{2}\frac{1}{e^{-\gamma_2^{th}/\bar{\gamma}_{S_1R}}}\left\{\begin{array}{l} 2I\left(\frac{2d_2^2}{d_1^2+d_2^2}, \bar{\gamma}_{S_1R}, \gamma_2^{th}\right) + \\ I\left(\frac{(2d_1-d_2)^2}{d_1^2+d_2^2}, \bar{\gamma}_{S_1R}, \gamma_2^{th}\right) \\ -I\left(\frac{(2d_1+d_2)^2}{d_1^2+d_2^2}, \bar{\gamma}_{S_1R}, \gamma_2^{th}\right) \end{array}\right\} \quad (14)$$

B. *Case 2: Forward HP bits of S_1*

In this case, we apply full power to *HP* symbols received from S_1 at the relay i.e. we forward QPSK modulated data containing only *HP* symbols and the BER expression is shown in (15) below. At S_2, we use SC to compare the SNR of the direct and relay links and to decide which signal to choose.

$$P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} = \frac{1}{2}\left\{I(1,0;0,4,\bar{\gamma}_{RS_2})\right\} \quad (15)$$

where $M = 4$ i.e. the case when full power is applied to one set of symbols, thus the resulting constellation is a simple QPSK. For this the combined error probability of the two relay links i.e. S_1-R and R-S_2 is given by following expression [11]

$$P_{error} = P_{error1} + P_{error2} - 2P_{error1}P_{error2} \quad (16)$$

where error probability of first relay link S_1-R is $P_{error1} = P_2(\varepsilon_R, HP_{S_1})$ and the error probability of second relay link is $P_{error2} = P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\}$. Therefore, substituting the respective values in above equation we get:

$$\begin{aligned} P_{error} &= P_2(\varepsilon_R, HP_{S_1}) + P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} \\ &\quad -2P_2(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} \end{aligned} \quad (17)$$

For SC, $y_{recvd} = \max(\gamma_{RS_2}, \gamma_{S_1S_2})$ i.e.

$$\begin{aligned} P\left\{\varepsilon_{SC}^{HP_{S_1}} \,|\lambda\right\} &= P_{e1}^{HP_{S_1}} \left|\gamma_{RS_2}\ selected + P_{e2}^{HP_{S_1}}\right|\gamma_{S_1S_2}\ selected \\ &= P_{e1}^{HP_{S_1}} P(\gamma_{RS_2} > \gamma_{S_1S_2}) + P_{e2}^{HP_{S_1}} P(\gamma_{S_1S_2} > \gamma_{RS_2}) \end{aligned} \quad (18)$$

When the relay link has higher SNR, the *HP* bits for P_{e1} is: $P_{e1}^{HP_{S_1}} = P_{error}$ and when SNR of direct link is higher *HP* bits for P_{e2} is $P\left\{\varepsilon_{S_1S_2}^{HP_{S_1}} \,|\lambda\right\}$ Finally, the BER for *HP* bits when using SC is given as:

$$\begin{aligned} P\left\{\varepsilon_{SC}^{HP_{S_1}} \,|\lambda\right\} &= P_{e1}^{HP_{S_1}} P(\gamma_{RS_2} < \gamma_{S_1S_2}) + \\ &\quad P(\gamma_{S_1S_2} < \gamma_{RS_2})\frac{1}{4}\left\{\begin{array}{l} I(1,0;\lambda, M, \bar{\gamma}_{S_1S_2}) \\ +I(1,2;\lambda, M, \bar{\gamma}_{S_1S_2}) \end{array}\right\} \\ &= \frac{\bar{\gamma}_{RS_2}}{\bar{\gamma}_{S_1S_2}+\bar{\gamma}_{RS_2}}\left[\begin{array}{l} P_1(\varepsilon_R, HP_{S_1}) + P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} \\ -2P_1(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} \end{array}\right] \\ &\quad +\frac{1}{4}\left\{\begin{array}{l} I(1,0;\lambda, M, \bar{\gamma}_{S_1S_2}) \\ +I(1,2;\lambda, M, \bar{\gamma}_{S_1S_2}) \end{array}\right\}\frac{\bar{\gamma}_{S_1S_2}}{\bar{\gamma}_{S_1S_2}+\bar{\gamma}_{RS_2}} \end{aligned} \quad (19)$$

The final expression for *LP* bits is based on the symbols received from direct link and is same as (5). Case 4 is similar to the above case except that we forward the *HP* bits of S_2.

C. *Case 3: Forward HP and LP bits of S_1*

In this case, we use HM at the relay i.e. we forward both *HP* and *LP* symbols, and we use SC at S_2. The BER of *HP* bits for the direct link is same as (4) and BER for HP bits of the relay link is as follows:

$$P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} = \frac{1}{4}\left\{I(1,0;\lambda, M, \bar{\gamma}_{RS_2}) + I(1,2;\lambda, M, \bar{\gamma}_{RS_2})\right\} \quad (20)$$

The overall error probability of the two relay links based on (16) is given by the following expression:

$$\begin{aligned} P_{e1}^{HP_{S_1}} &= P_{error} = P_3(\varepsilon_R, HP_{S_1}) + P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} \\ &\quad -2P_3(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}} \,|\lambda\right\} \end{aligned} \quad (21)$$

Based on (18), the equation for SC is given by

$$\begin{aligned} P\left\{\varepsilon_{SC}^{HP_{S_1}} \,|\lambda\right\} &= P_{e1}^{HP_{S_1}} \frac{\bar{\gamma}_{RS_2}}{\bar{\gamma}_{S_1S_2}+\bar{\gamma}_{RS_2}} + \\ &\quad \frac{1}{4}\left\{\begin{array}{l} I(1,0;\lambda, M, \bar{\gamma}_{S_1S_2}) \\ +I(1,2;\lambda, M, \bar{\gamma}_{S_1S_2}) \end{array}\right\}\frac{\bar{\gamma}_{S_1S_2}}{\bar{\gamma}_{S_1S_2}+\bar{\gamma}_{RS_2}} \end{aligned} \quad (22)$$

The *LP* bits are also obtained in a similar way

$$P\left\{\varepsilon_{RS_2}^{LP_{S_1}} \,|\lambda\right\} = \frac{1}{4}\left\{\begin{array}{l} 2I(0,1;\lambda, M, \bar{\gamma}_{RS_2}) \\ +I(2,1;\lambda, M, \bar{\gamma}_{RS_2}) - I(2,3;\lambda, M, \bar{\gamma}_{RS_2}) \end{array}\right\} \quad (23)$$

$$P\left\{\varepsilon_{S_1S_2}^{LP_{S_1}} \,|\lambda\right\} = \frac{1}{4}\left\{\begin{array}{l} 2I(0,1;\lambda, M, \bar{\gamma}_{S_1S_2}) + I(2,1;\lambda, M, \bar{\gamma}_{S_1S_2}) \\ -I(2,3;\lambda, M, \bar{\gamma}_{S_1S_2}) \end{array}\right\} \quad (24)$$

$$\begin{aligned} P_{e1}^{LP_{S_1}} &= P_{error} = P_3(\varepsilon_R, LP_{S_1}) + P\left\{\varepsilon_{RS_2}^{LP_{S_1}} \,|\lambda\right\} \\ &\quad -2P_3(\varepsilon_R, LP_{S_1})P\left\{\varepsilon_{RS_2}^{LP_{S_1}} \,|\lambda\right\} \end{aligned} \quad (25)$$

$$\begin{aligned} P\left\{\varepsilon_{SC}^{LP_{S_1}} \,|\lambda\right\} &= P_{e1}^{LP_{S_1}} \frac{\bar{\gamma}_{RS_2}}{\bar{\gamma}_{S_1S_2}+\bar{\gamma}_{RS_2}} + \\ &\quad \frac{1}{4}\left\{\begin{array}{l} 2I(0,1;\lambda, M, \bar{\gamma}_{S_1S_2}) + \\ I(2,1;\lambda, M, \bar{\gamma}_{S_1S_2}) - \\ I(2,3;\lambda, M, \bar{\gamma}_{S_1S_2}) \end{array}\right\}\frac{\bar{\gamma}_{S_1S_2}}{\bar{\gamma}_{S_1S_2}+\bar{\gamma}_{RS_2}} \end{aligned} \quad (26)$$

Case 7 is similar to the above case except that we forward the *HP* and *LP* bits of S_2.

D. Case 5: Neglect LP of S_1 and S_2 and NC HP bits of S_1 and S_2

In case 5, we use NC on the *HP* bits of the two received sequences at the relay. This network coded symbols are broadcasted to both S_1 and S_2 where they are used to retrieve bits of the other source. We use SC to decide on the BER of *HP* bits, if SNR of relay link is higher. The BER for the relay link is the same as (15) i.e. whole power is applied to *HP* symbols only. The BER expression for *LP* from direct link is the same as (5). BER expressions for cases 5, 6, 8 and 9 are derived conditioned on the two relay links. The BER for the source – relay – destination with NC link is based on the following equation:

$$\begin{aligned} P_{error} &= P_{error1} + P_{error2}(1-P_{error3}) - 2P_{error1}P_{error2}(1-P_{error3}) \\ &= P_{error1} + P_{error2} - P_{error2}P_{error3} - 2P_{error1}P_{error2} \\ &+2P_{error1}P_{error2}P_{error3} \end{aligned} \quad (27)$$

Where we catered for the fact that an error in phase 1 transmission affects detection at the receiver due to network coding. In (27), (1-P_{error3}) represents the condition for the network coded symbols. P_{error3} is the relay link from S_2 to R given by

$$P\left\{\varepsilon_{RS_2}^{HP_{S_2}}\left|\lambda\right.\right\} = \frac{1}{2}\left\{I(1,0;0,4,\overline{\gamma}_{RS_2})\right\} \quad (28)$$

$$\begin{aligned} P_{e1}^{HP_{S_1}} &= P_{error} = P_2(\varepsilon_R, HP_{S_1}) + P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\} \\ &-P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\}P_2(\varepsilon_R, HP_{S_2}) \\ &-2P_2(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\} \\ &+2P_2(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\}P_2(\varepsilon_R, HP_{S_2}) \end{aligned} \quad (29)$$

Finally, the BER for *HP* bits when SC is used is based on (18) and is given as:

$$\begin{aligned} P\left\{\varepsilon_{SC}^{HP_{S_1}}\left|\lambda\right.\right\} &= P_{e1}^{HP_{S_1}} \frac{\overline{\gamma}_{RS_2}}{\overline{\gamma}_{S_1S_2} + \overline{\gamma}_{RS_2}} + \\ &\frac{1}{4}\left\{\begin{array}{l} I(1,0;\lambda,M,\overline{\gamma}_{S_1S_2}) \\ +I(1,2;\lambda,M,\overline{\gamma}_{S_1S_2}) \end{array}\right\} \frac{\overline{\gamma}_{S_1S_2}}{\overline{\gamma}_{S_1S_2} + \overline{\gamma}_{RS_2}} \end{aligned} \quad (30)$$

The BER for *LP* bits is derived from direct path and is the same as (5). Cases 6 and 8 are similar to the above case i.e. we neglect *LP* bits of S_1 and S_2 and forward network coded *HP* bits of S_1 and S_2. Similarly *LP* bits are derived from direct path.

E. Case 9: Full Network Coding

In case 9, we use NC on the *HP* and *LP* symbols of the two received sequences at the relay. These network coded symbols are then broadcasted to both S_1 and S_2 where they are used to retrieve bits of the other source. We also use SC to decide whether to choose the direct or the relay link. *HP* bits from relay link are given in (21) and *HP* bits from direct link is given in (5). The BER of *HP* bits from R to S_1:

$$P\left\{\varepsilon_{RS_1}^{HP_{S_2}}\left|\lambda\right.\right\} = \frac{1}{4}\left\{I(1,0;\lambda,M,\overline{\gamma}_{RS_1}) + I(1,2;\lambda,M,\overline{\gamma}_{RS_1})\right\} \quad (31)$$

The BER for source to destination via relay link is based on (27) and is given below:

$$\begin{aligned} P_{e1}^{HP_{S_1}} &= P_{error} = P_3(\varepsilon_R, HP_{S_1}) + P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\} \\ &-P_3(\varepsilon_R, HP_{S_2})P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\} \\ &-2P_3(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\} \\ &+2P_3(\varepsilon_R, HP_{S_1})P_3(\varepsilon_R, HP_{S_1})P\left\{\varepsilon_{RS_2}^{HP_{S_1}}\left|\lambda\right.\right\} \end{aligned} \quad (32)$$

Finally, the BER for *HP* bits with SC is:

$$\begin{aligned} P\left\{\varepsilon_{SC}^{HP_{S_1}}\left|\lambda\right.\right\} &= P_{e1}^{HP_{S_1}} \frac{\overline{\gamma}_{RS_2}}{\overline{\gamma}_{S_1S_2} + \overline{\gamma}_{RS_2}} \\ &+\frac{1}{4}\left\{\begin{array}{l} I(1,0;\lambda,M,\overline{\gamma}_{S_1S_2}) \\ +I(1,2;\lambda,M,\overline{\gamma}_{S_1S_2}) \end{array}\right\} \frac{\overline{\gamma}_{S_1S_2}}{\overline{\gamma}_{S_1S_2} + \overline{\gamma}_{RS_2}} \end{aligned} \quad (33)$$

Similarly the BER for *LP* bits when SC is derived based on expressions (5), (24) and expression for *LP* bits from R to S_1:

$$P\left\{\varepsilon_{RS_1}^{LP_{S_2}}\left|\lambda\right.\right\} = \frac{1}{4}\left\{\begin{array}{l} 2I(0,1;\lambda,M,\overline{\gamma}_{RS_1}) + I(2,1;\lambda,M,\overline{\gamma}_{RS_1}) \\ -I(2,3;\lambda,M,\overline{\gamma}_{RS_1}) \end{array}\right\} \quad (34)$$

$$\begin{aligned} P_{e1}^{LP_{S_1}} &= P_{error} = P_3(\varepsilon_R, LP_{S_1}) + P\left\{\varepsilon_{RS_2}^{LP_{S_1}}\left|\lambda\right.\right\} \\ &-P_3(\varepsilon_R, LP_{S_2})P\left\{\varepsilon_{RS_2}^{LP_{S_1}}\left|\lambda\right.\right\} \\ &-2P_3(\varepsilon_R, LP_{S_1})P\left\{\varepsilon_{RS_2}^{LP_{S_1}}\left|\lambda\right.\right\} \\ &+2P_3(\varepsilon_R, LP_{S_1})P_3(\varepsilon_R, LP_{S_2})P\left\{\varepsilon_{RS_2}^{LP_{S_1}}\left|\lambda\right.\right\} \end{aligned} \quad (35)$$

$$\begin{aligned} P\left\{\varepsilon_{SC}^{LP_{S_1}}\left|\lambda\right.\right\} &= P_{e1}^{LP_{S_1}} \frac{\overline{\gamma}_{RS_2}}{\overline{\gamma}_{S_1S_2} + \overline{\gamma}_{RS_2}} \\ &+\frac{1}{4}\left\{\begin{array}{l} 2I(0,1;\lambda,M,\overline{\gamma}_{S_1S_2}) + \\ I(2,1;\lambda,M,\overline{\gamma}_{S_1S_2}) - \\ I(2,3;\lambda,M,\overline{\gamma}_{S_1S_2}) \end{array}\right\} \frac{\overline{\gamma}_{S_1S_2}}{\overline{\gamma}_{S_1S_2} + \overline{\gamma}_{RS_2}} \end{aligned} \quad (36)$$

To calculate the overall BER we sum the product of individual BER of all cases along with their respective probabilities of occurrence. The final BER expression is calculated as:

$$\begin{aligned} BER(XP) &= BER_{case1}^{XP}P_1(S_1)P_1(S_2) + BER_{case2}^{XP}P_2(S_1)P_1(S_2) \\ &+BER_{case3}^{XP}P_3(S_1)P_1(S_2) + BER_{case4}^{XP}P_1(S_1)P_2(S_2) \\ &+BER_{case5}^{XP}P_2(S_1)P_2(S_2) + BER_{case6}^{XP}P_3(S_1)P_2(S_2) \\ &+BER_{case7}^{XP}P_1(S_1)P_3(S_2) + BER_{case8}^{XP}P_2(S_1)P_3(S_2) \\ &+BER_{case9}^{XP}P_3(S_1)P_3(S_2). \end{aligned} \quad (37)$$

where *XP* can be either *HP* or *LP* bits. $P_x(S_x)$ represents the probability of occurrence of each case based on the two sources. Similarly, all expressions can be derived for BER at S_1.

IV. Numerical Results

Fig. 4. shows the average BER of *HP* and *LP* bits for a system with α = 0.8. It is clear from the figure the perfect match

between the derived results and the simulation. In Fig. 5. the impact of changing α for same threshold values is shown. Increasing α means more protection is given for *HP* bits and this is reflected in the curves as compared to Fig. 4. There is a gain in the *HP* bits and performance of *LP* is worse as compared to previous figure. In Fig. 6., thresholds are increased for both *HP* and *LP* resulting in slight improvement in *HP* and *LP*.

V. CONCLUSIONS

In this paper, we have evaluated the performance of a scheme for two way relay networks which jointly uses HM and NC in a Rayleigh fading environment. We derived the BER for the above scheme consisting of two sources and a relay for two priority classes i.e. *HP* and *LP* modulated by hierarchical 4/16-QAM. This scheme is adaptive and opportunistic thus, it makes use of HM and NC for increasing the spectral efficiency. Our results show good match between simulation and theory. In addition, NC also results in reduced number of transmissions. In future, we plan to find the optimal threshold to maximize the BER and also extend the scheme to a multiple relay scenario.

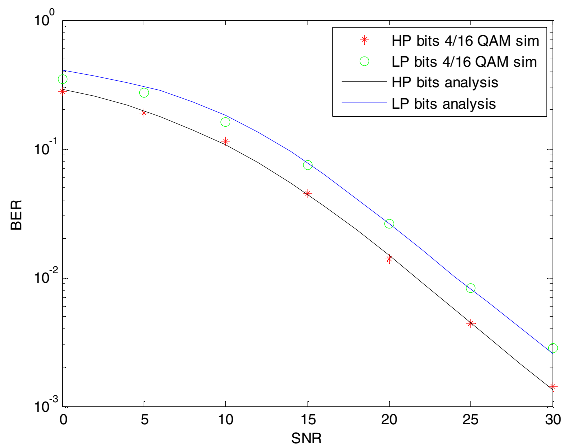

Figure 4. BERs of *HP* and *LP* for proposed scheme (α = 0.8, $\lambda_1^{th} = 0.03$, $\lambda_2^{th} = 0.07$, $\lambda = 1$, $d_1' = d_2$).

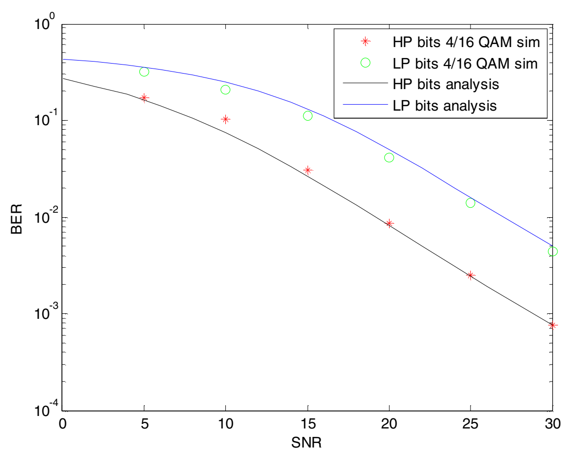

Figure 5. BERs of *HP* and *LP* for proposed scheme (α = 0.9, $\lambda_1^{th}= 0.03$, $\lambda_2^{th}= 0.07$, $\lambda = 0.5$, $2d_1' = d_2$).

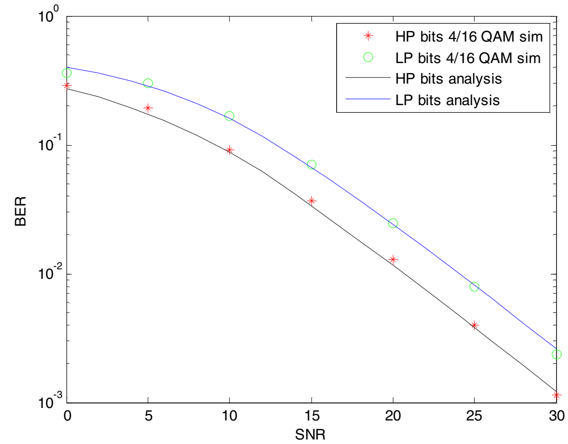

Figure 6. BERs of *HP* and *LP* for proposed scheme (α = 0.8, $\lambda_1^{th} = 0.3$, $\lambda_2^{th} = 0.7$, $\lambda = 1$, $d_1' = d_2$) .

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