Sequential Compressive Sensing in Wireless Sensor Networks

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***Abstract*—Compressive sensing (CS) is a new signal acquisition framework, which allows for a signal recovery from far fewer samples than what is required by traditional sampling methods. In this paper we propose new strategies for adaptively adjusting the number of CS samples in wireless sensor networks (WSNs). Additionally, in the signal reconstruction procedure we apply homotopy algorithm to update the reconstructed signals. The reduction of CS samples and the homotopy update reduce the computational complexity and save processing time and energy for both the fusion centre and wireless sensors. The proposed techniques are investigated numerically in various WSN scenarios.**

I. Introduction

In recent years a new emerging data acquisition framework, known as compressive sensing (CS) [4], [7], has attracted considerable attention among researchers. The main assert of CS is that most natural signals can be perfectly reconstructed after being sampled at a rate much lower than the Nyquist. The Nyquist rate can be very high in some applications such as image processing, implying a great number of samples are required. CS proves to be a much more efficient sampling method since it enables signals to be recovered from far fewer samples than that obtained by sampling at Nyquist rate. Therefore this data acquisition theory has great potential for a wide number of applications such as radar [8], medical imaging processing [11], analog-to-information conversion [13] and wireless sensor networks (WSN) [9]. In this paper we explore the application of CS in WSNs.

A WSN usually consists of a large number of small independent sensors capable of transmitting and receiving information to and from a fusion centre (FC). One big challenge is to make the data collection process as efficient as possible in terms of data transmission and power consumption at the sensor nodes. In [2] authors propose a data gathering algorithm based on multi-hop routing to reduce the communication cost and prolong the WSN lifetime. This algorithm involves all of the sensor components and the communication complexity is still daunting. In [5] a CS algorithm based signal acquisition approach is proposed in order to reduce energy consumption of a WSN. This approach shows significant reduction in the number of samples but the benefits are substantial only for long time based signals. The time delay may limit its applications in practice.

In this paper we design a sequential compressive sensing system in WSNs and utilize random sampling to sequentially detect measurements from randomly chosen sensors. The main goal is to obtain the full sensor field signal by transmitting only a small number of sensor measurements. Sequential CS theory guarantees the least number of measurements is obtained and homotopy updating algorithm [1] is applied to greatly reduce the computational complexity. We also propose appropriate stopping rules to ensure the measurements are sufficient for successful reconstructions.

The rest of the paper is organized as follows. Section II gives a general overview of the theory of compressive sensing and homotopy algorithms. Section III describes the complete sequential compressive sensing system in wireless sensor networks. Then simulations in Rayleigh fading channel are conducted and results are displayed to show the performance of this designed system in section IV. Conclusions and future work are discussed in the last section.

II. Theory overview

A. Compressive Sensing

The CS theory relies on the fact that most real life signals are sparse or compressible in the sense that they have concise representation in a suitable basis $\mathbf{\Psi}$. Consider an N-dimensional sparse signal $\mathbf{x} \in \mathbb{R}^N$; it can be represented in a certain orthonormal $N \times N$ basis matrix $\mathbf{\Psi}$:

$$\mathbf{x} = \sum_{i=1}^{N} s_i \psi_i \qquad \text{or} \qquad \mathbf{x} = \mathbf{\Psi}\mathbf{s} \tag{1}$$

where $\mathbf{s}$ denotes the $N \times 1$ column vector of weighting coefficients of signal $\mathbf{x}$. We say that signal $\mathbf{x}$ is K-sparse in basis $\mathbf{\Psi}$ if $\mathbf{s}$ has only K $(K \ll N)$ non-zero elements. We consider signals produced in WSNs which can be modeled as 2 dimensional (2-D) field signals. Thus the discrete cosine transform (DCT) basis [10] is applied in this paper, which is commonly used to compress 2-D images.

CS obtains M linear measurements by using a $M \times N$ measurement matrix $\mathbf{\Phi}$. Given a signal vector $\mathbf{x} \in \mathbb{R}^N$, which

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is K-sparse in basis $\mathbf{\Psi}$, a measurements vector $\mathbf{y} \in \mathbb{R}^M$ can be achieved from the linear process

$$\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi \Psi s} = \mathbf{A s} \tag{2}$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$. It is demonstrated that unique and exact recovery exists if matrix $\mathbf{A}$ satisfies the *restricted isometry property* (RIP) [3]: $(1-\epsilon)\|\mathbf{v}\|_2 \leq \|\mathbf{A v}\|_2 \leq (1+\epsilon)\|\mathbf{v}\|_2$ for every vector $\mathbf{v}$ that has at most $2K$ nonzero entries and $\epsilon > 0$.

Generally, in CS $\mathbf{\Phi}$ is simply chosen as a random matrix whose entries are independent and identically distributed (i.i.d) variables. However, in WSNs applying CS using a random measurement matrix involves all sensors and consumes extra power since the measurements need encoding. In this paper a more attractive sampling method is utilized: we apply random sampling of the sensor data. For a 1 dimensional (1-D) field signal vector, the measurement matrix contains all zeros except for M ones. There is one "1" in each row, such that the support does not overlap:

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ & & \vdots & & & & \ddots & \vdots \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{M \times N}$$

By this method, measurements of randomly chosen sensors are directly collected by the fusion centre.

Reconstruction can be achieved by solving an l-1 minimisation problem [4], [7], which is also known as basis pursuit (BP).

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}'} \|\mathbf{s}'\|_1, \text{ such that } \mathbf{A s}' = \mathbf{y}. \tag{3}$$

In practice signal measurements are usually accompanied by interference and/or noise. Then the measurement process can be modeled by adding an additional noise parameter $\mathbf{y} = \mathbf{A s} + \mathbf{n}$ and the new problem to solve reads:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}'} \left\{ \frac{1}{2}\|\mathbf{A s}' - \mathbf{y}\|_2^2 + \lambda \|\mathbf{s}'\|_1 \right\} \tag{4}$$

where the parameter λ is typically set to a value slightly larger than the noise level. This is known as basis pursuit de-noising (BPDN) (or Lasso).

B. Homotopy algorithm

Homotopy [1] is a very efficient method to speed up the solution of minimisation (4) by solving a sequence of simple intermediate problems. This sequential process is controlled by the homotopy parameter, which varies slowly to ensure that optimality conditions are maintained.

In (4) λ can be the homotopy parameter. The solution $\hat{\mathbf{s}}$ at any given λ must satisfy the following constraint

$$\|\mathbf{A}^{\mathrm{T}}(\mathbf{A}\hat{\mathbf{s}} - \mathbf{y})\|_\infty \leq \lambda \tag{5}$$

This condition can be expanded as

$$\mathbf{A}_\Gamma^{\mathrm{T}}(\mathbf{A}\hat{\mathbf{s}} - \mathbf{y}) = -\lambda \mathbf{z};\ \|\mathbf{A}_{\Gamma^c}^{\mathrm{T}}(\mathbf{A}\hat{\mathbf{s}} - \mathbf{y})\|_\infty < \lambda \tag{6}$$

where Γ denotes the support of $\hat{\mathbf{s}}$[1], $\mathbf{A}_\Gamma$ is the $M \times \Gamma$ matrix obtained from the columns of $\mathbf{A}$ corresponding to Γ, and $\mathbf{z}$ is the vector of signs of $\hat{\mathbf{s}}$ in Γ. Thus $\hat{\mathbf{s}}$ can be calculated directly from the support Γ and sign vector $\mathbf{z}$: $\hat{\mathbf{s}}$ equals $\left(\mathbf{A}_\Gamma^{\mathrm{T}}\mathbf{A}_\Gamma\right)^{-1}\left(\mathbf{A}_\Gamma^{\mathrm{T}}\mathbf{y} - \lambda\mathbf{z}\right)$ on Γ and 0 elsewhere. If we change the homotopy parameter λ, the solution would move along a path with direction $\left(\mathbf{A}_\Gamma^{\mathrm{T}}\mathbf{A}_\Gamma\right)^{-1}\mathbf{z}$ until either an element of $\hat{\mathbf{s}}$ is reduced to zero, or another constraint in (5) is active. In these cases, called *critical points*, the support of $\hat{\mathbf{s}}$ would either be discarded or a new element is added correspondingly. Additionally, the value of λ is also calculated at any critical point. Therefore, BPDN can be solved by starting with a very large λ and reducing it to the desired value.

We also expect to update the solution using homotopy when a new measurement $w = \mathbf{b s}$ ($\mathbf{b}$ is a new row in $\mathbf{A}$) is added to the system if the original signal is not recovered in the previous iteration. A homotopy parameter ϵ is introduced in order to continuously approach the solution. Thus, the BPDN problem can be rewritten as:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}'} \left\{ \frac{1}{2}(\|\mathbf{A s}' - \mathbf{y}\|_2^2 + \epsilon\left|\mathbf{b s}' - w\right|^2) + \lambda\|\mathbf{s}'\|_1 \right\} \tag{7}$$

Clearly if ϵ varies from 0 to 1, this equation goes from the solution of the old problem to the new problem where a new measurement is added. Assuming homotopy parameter ϵ changes by a small step size θ between two critical points, we can gradually approach the new solution with a series of continuous steps

$$\epsilon_{k+1} = \epsilon_k + \theta, \qquad \mathbf{s}'_{k+1} = \mathbf{s}'_k + \theta\partial\mathbf{s}'$$

where $\partial\mathbf{s}'$ provides the direction of variation of the solution $\mathbf{s}'$. In [12] the method of determining the step size θ and direction $\partial\mathbf{s}'$ is explained in detail.

The homotopy algorithm is shown to be extremely fast in updating the solutions when new measurements are sequentially added to the system. In WSNs, the FC could save significant time as well as computational power when applying homotopy algorithm to update the solutions of the reconstruction problem.

III. System design

In this section the sequential compressive sensing system in a WSN is designed. A new stopping rule is also proposed. We consider that the WSN detects a 2-D field image signal. To exemplify the framework in which we apply the CS algorithm in WSNs, a simple model for an image field signal is considered: a 2-D image having a K-sparse DCT coefficients matrix whose nonzero entries are in top left region. Assuming a 30×30 DCT coefficient matrix has 100 nonzero elements in the top left square region (low frequency region), then the vectorized DCT coefficients vector $\mathbf{s}$ has dimension of $N = 900$ and sparsity $K = 100$. We produce a 900×900 1-D DCT basis $\mathbf{\Psi}$ and the vectorized image data vector $\mathbf{x}$ then

[1] The support Γ of a vector is defined as the set of indexes where the entries are nonzeros i.e., $\text{supp}(\mathbf{s}) = \{i : s_i \neq 0\}$.

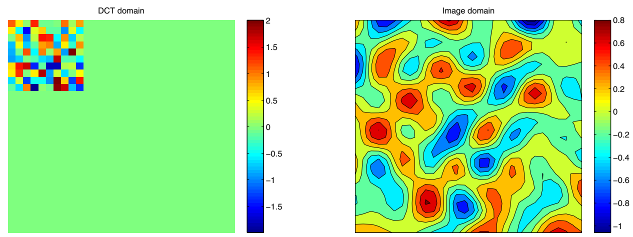

Figure 1. The sensor field image and the corresponding DCT coefficients

reads: $\mathbf{x} = \mathbf{\Psi s}$. The corresponding 2-D image and coefficients matrix is illustrated in Figure 1.

From the CS theory we know that if we obtain M measurements, which is somewhat larger than K, the original image can be reconstructed using a reconstruction algorithm. However in practice prior knowledge about the sparsity of the image field is typically unavailable, making it difficult to determine how many sensors are needed to deliver measurements. Too many sensor communications would cause a waste of energy. Therefore the sequential compressive sensing theory [6] is employed in order to ensure the least number of measurements is obtained to recover the original signal.

A. Sequential compressive sensing system for WSNs

We aim to obtain the original image from the least number of measurements without prior knowledge about the sparsity. Thus, in the designed system sensor data are sequentially detected by the FC using random sampling and this process stops once there are enough measurements for recovery. We start from 10 measurements in this WSN model and we use the BPDN homotopy solution to initially recover the image data. Then sensor data are sequentially added to the system. Every time a new measurement is received, the FC performs the homotopy update rather than reconstructing the original signal from scratch. When a certain stopping rule, which will be introduced later, is satisfied, the FC stops requesting new measurements and the original image is expected to be successfully recovered.

Figure 2 illustrates the whole process described above. The system stops when $M = 331$, which is the least number of measurements required. It is worth remarking that this minimum number of measurements obtained by our system is not constant. Since the sensors are randomly chosen, two or more neighbouring sensors may sometimes be chosen and they may provide very similar measurements. In that case these sensors measurements would be redundant, and the resulting number of measurements would be larger. If all chosen sensors are widely distributed, the total sensor measurements would be much smaller. Later we will give more discussion about this least number obtained in the system.

B. The stopping rule

As discussed above, the performance of the system greatly relies on the appropriate stopping rule. In this section we propose a new stopping rule which is shown to provide immediate stop of the system once the original signal is approached.

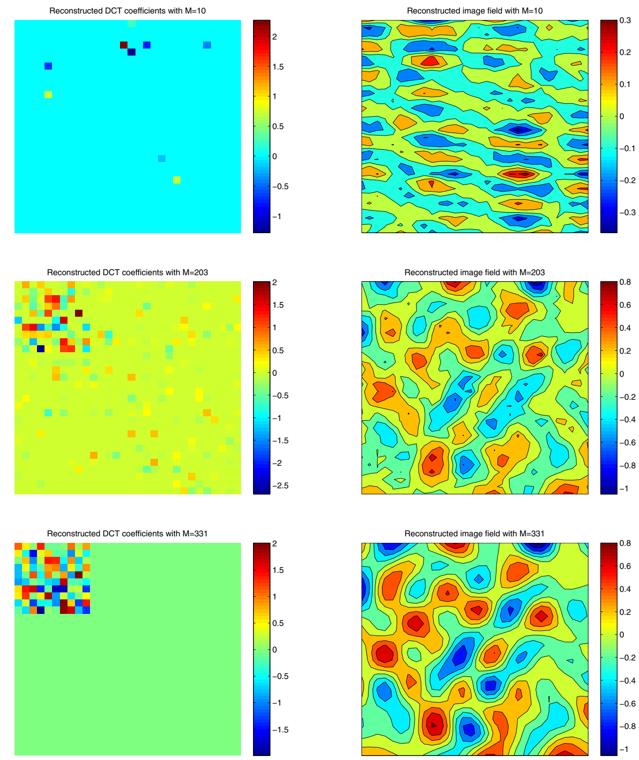

Figure 2. Sequential compressive sensing process

If the reconstructed DCT coefficients vector $\hat{\mathbf{s}}$ is exactly the same to the the original $\mathbf{s}$, then $\mathbf{y}$ would have a linear relationship with $\hat{\mathbf{s}}$: $\mathbf{y} = \mathbf{A}\hat{\mathbf{s}}$. We can simply restrict the matrix $\mathbf{A}$ to the columns corresponding to the support of $\hat{\mathbf{s}}$. Then the set of nonzero coefficients $\hat{\mathbf{s}}_{\mathbf{\Gamma}}$ will satisfy the following relationship with the observations $\mathbf{y}$:

$$\hat{\mathbf{s}}_{\Gamma} = \mathbf{A}_{\Gamma}^{\dagger}\mathbf{y} = \left(\mathbf{A}_{\Gamma}^{\mathrm{T}}\mathbf{A}_{\Gamma}\right)^{-1}\mathbf{A}_{\Gamma}^{\mathrm{T}}\mathbf{y} \tag{8}$$

where $\mathbf{A}_{\mathbf{\Gamma}}^{\dagger}$ is the pseudoinverse of $\mathbf{A}_{\mathbf{\Gamma}}$, which denotes the restricted matrix. This equation simply examines the linear relationship between $\hat{\mathbf{s}}_{\mathbf{\Gamma}}$ and observations $\mathbf{y}$. Note that $\mathbf{A}_{\mathbf{\Gamma}}$ is an $M \times |\Gamma|$ matrix, where M denotes the number of measurements and $|\Gamma|$ denotes the sparsity of the coefficients vector $\mathbf{s}$. The CS theory indicates that original signal can be uniquely recovered when the number of measurements is larger than the sparsity. Therefore if M is larger than $|\Gamma|$, the above equation will only be true when $\hat{\mathbf{s}}$ is exactly the same as the original coefficients vector $\mathbf{s}$. The above equation can be further modified as

$$\mathbf{A}_{\Gamma}\hat{\mathbf{s}}_{\Gamma} = \mathbf{A}_{\Gamma}\left(\mathbf{A}_{\Gamma}^{\mathrm{T}}\mathbf{A}_{\Gamma}\right)^{-1}\mathbf{A}_{\Gamma}^{\mathrm{T}}\mathbf{y} \tag{9}$$

$$\hat{\mathbf{y}} = \mathbf{A}_{\Gamma}\left(\mathbf{A}_{\Gamma}^{\mathrm{T}}\mathbf{A}_{\Gamma}\right)^{-1}\mathbf{A}_{\Gamma}^{\mathrm{T}}\mathbf{y} = \mathbf{P}\mathbf{y} \tag{10}$$

Combing with the restriction of M, the condition $\mathbf{y} = \hat{\mathbf{y}}$ can be our stopping rule, which can be written as

$$If: \quad \begin{cases} (\mathbf{I}-\mathbf{P})\,\mathbf{y}=0 \\ M>|\Gamma| \end{cases}$$

$$Then, \qquad \textbf{stop}$$

However it is notable that this stopping rule cannot be applied in the noisy case, when measurements are accompanied by random noise and other inaccuracies. The reason is clear: $\hat{\mathbf{y}}$ is derived from reconstructed $\hat{\mathbf{s}}$ and contains no noise information, while $\mathbf{y}$ denotes the real measurements vector and would deviate greatly in the measurement process with the presence of noise.

In order to explore the performance of the stopping rule, we produce a signal that contains little information and its coefficient matrix has only ones and zeros. This kind of signal presents directly sparsity information. Then it is expected that once the sparsity of the coefficients vector is achieved the signal would be also reconstructed and the stopping rule would stop the system at this time.

Figure 3 illustrates the typical mean square errors (MSEs) between the reconstructed image data vector $\hat{\mathbf{x}}$ and the original data vector $\mathbf{x}$ during the whole sequential detection process. The dashed line shows a sharp decrease for the MSE at the stopping point, indicating good performance for the stopping rule. The full line gives a typical MSE for the original model where the DCT coefficients are random variables. In general during the updating process the MSE are decreasing all the way and the system stops when the MSE has a quite small value, which indicates an appropriate stopping point given by the stopping rule.

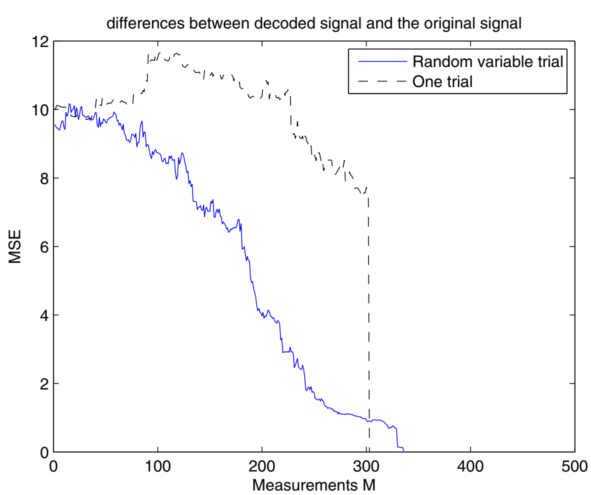

Figure 3. MSEs between recovered signal and the original signals

By now all discussions are under the assumption of no noise present in the system. Our proposed new stopping rule works well in noiseless case. In noisy case, a simple modified stopping rule based on the comparison between contiguous recovered signals [6] is applied: the system stops immediately when the following equation is true.

$$(\| \mathbf{x}^{\mathrm{M-T}} - \mathbf{x}^{\mathrm{M-(T-1)}} \|_2 < \text{threshold})\,\&$$

$$(\| \mathbf{x}^{\mathrm{M-(T-1)}} - \mathbf{x}^{\mathrm{M-(T-2)}} \|_2 < \text{threshold})\,\&$$

$$\ldots\,\&\,(\| \mathbf{x}^{\mathrm{M-1}} - \mathbf{x}^{\mathrm{M}} \|_2 < \text{threshold}) \tag{11}$$

where $\mathbf{x}^{\mathrm{M}}$ denotes recovered signal from M measurements. This rule compares the last T reconstructed signals and would require more memory in practice. The recovered signal data is supposed to be extremely close to the original with a large T and small threshold, which may result in a few more measurements compared to other rules.

IV. Numerical results

In practical application scenarios sensors in WSNs cover large areas. Communications between FC and sensors can be assumed to follow typical propagation models. It is usually the case that no light-of-sight is available between the FC and sensors, and signals tend to experience multipath effect. It is reasonable to assume that the channel model is Rayleigh.

A. Flat fading channel

In this case we assume that the signals generated by the sensors are narrowband. Signal from different sensors also experience independent fading. This can be simply modeled by a diagonal matrix $\mathbf{H}$ whose entries in the diagonal are i.i.d Gaussian random variables.

At the FC, the channel matrix $\mathbf{H}$ is embedded in the measurements signal: $\mathbf{y}=\mathbf{H}\mathbf{\Phi}\mathbf{x}=\mathbf{H}\mathbf{A}\mathbf{s}$, such that reconstruction of the original image data requires the knowledge of channel $\mathbf{H}$. We assume the FC can estimate the channel response $\mathbf{H}$ by using e.g., dedicated training sequences. In the recovery process, we employ the channel response matrix into the reconstruction algorithm. Considering that $\mathbf{y}=\mathbf{H}\mathbf{A}\mathbf{s}=\mathbf{B}\mathbf{s}$, it is clear that $\mathbf{B}$ maintains the incoherency property between different rows, which is a sufficient condition for unique reconstruction. This implies that the sparse coefficients vector $\mathbf{s}$ can be recovered from $\mathbf{y}$ and $\mathbf{B}$ by using the reconstruction algorithm.

B. Frequency selective fading channel

If the sensor signals have a larger bandwidth than the channel coherent bandwidth, they experience frequency selective fading. We model the channel response $\mathbf{H}$ containing channel memories which follow exponential decline. Thus the measurements are obtained from $\mathbf{y}=\mathbf{H}\mathbf{\Phi}\mathbf{x}=\mathbf{H}\mathbf{A}\mathbf{s}$. Similar to flat fading case, we introduce channel matrix $\mathbf{H}$ into the reconstruction algorithm.

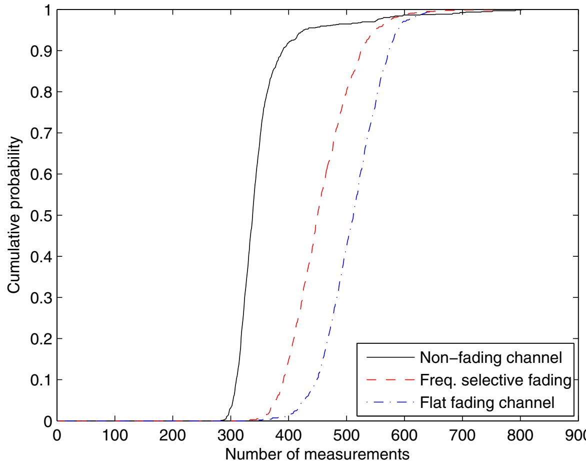

Figure 4. The CDF performance comparison among three channels

It is mentioned previously that in every simulation the number of required measurements is likely to be slightly different. This minimum number of measurements relies on a random sampling matrix, which determines which sensors are chosen. A group of more distributed sensors would generally enable the signal reconstruction with smaller number of measurements. Therefore we produce the statistic results for each practical case by performing a large number of trials. Figure 4 displays the cumulative probability distribution function (CDF) of the least measurement numbers required for reconstruction for 1000 trials in three different channels: non-fading noiseless channel, flat fading channel and frequency selective fading channel. Note that in both the Rayleigh fading channels the received signals are also accompanied by additive white Gaussian noise (AWGN).

For the noiseless case it can be seen that in most trials the system stops when the number of measurements is between 300 and 400. It shows that 90% of all trails require fewer measurements than 390 and the mean value of the smallest number of required measurements is 338. In flat fading channel the required smallest measurement number tend to be much larger as most of trials give numbers around 500. It is clear that the mean value of the least number of required measurements is about 512. Therefore we can conclude that in flat fading channel signal reconstructions would be more difficult and more measurements are required compared to the noiseless case. The results in frequency selective fading channel show a compromise between the noiseless case and the flat fading case. Almost all tests result in a number of measurement between 350 and 550 and the mean value is 450, which is smaller than the number in flat fading case and larger than that for non-fading noiseless case. In frequency selective channel the performance of sequential compressive sensing system is better than that in flat fading channel, implying that it is easier for the algorithms to recover the original signal in frequency selective fading channel.

V. Conclusion

In this paper we exploited the application of CS theory in WSNs. Our main goal is to acquire full signals in WSNs from the least number of sensor measurements to save resources and energy. A complete design of a sequential CS system has been proposed. Our proposed system utilized random sampling to significantly reduce the involved sensors in a WSN. We also applied the homotopy updating algorithms in the reconstruction process during the sequential detection and this was shown to be able to significantly improve the efficiency and save power and energy for FC. Additionally, we have also proposed a new stopping rule for the sequential compressive sensing process and the rule showed good performance in noiseless channel case. The results of our simulation have demonstrated that FC can successfully recover the whole sensor field signal from a small number of sensor measurements. Therefore, this system can greatly improve the resources, time, and energy efficiency for WSNs. Future work will focus on improving the stopping rules.

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