Distributed Auction for Self-Optimization in Wireless Cooperative Networks

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***Abstract*—This paper addresses the relay assignment problem in a wireless cooperative network. We propose a distributed algorithm which does not require global information, hence is more practical in a real network environment. It is based on the distributed auction structure that can achieve the optimal network capacity as centralized ones but with much lower complexity and higher scalability. We give formal proofs for optimality and convergence of this algorithm, and then we analyze its time complexity and further improve it by reducing the iterations in cost of little capacity degradation. Extensive simulations also show huge computation reduction on proposed algorithms than centralized ones with comparable capacity performance.**

I. Introduction

Wireless communications suffer a lot from channel fading which severely decreases the capacity of wireless channels. As an effective technique to overcome this negative effect, spatial diversity, also known as antenna diversity, has shown great success to improve the quality and reliability of a wireless link in modern systems such as multi-input multi-output (MIMO) networks [1]. Equipping multiple antennas at one single wireless node, however, may be impractical in many application scenarios, especially for those small-size battery-powered devices. For this reason, cooperative communications [2], [3] have been proposed to achieve the spatial diversity among distributed nodes. Its basic idea is to exploit the broadcast nature of wireless transmissions and the spatial relaying capability of idle nodes. Depending on the processing capability of relay nodes, two commonly used cooperative transmission modes are amplify-and-forward (AF) and decode-and-forward (DF), both of which can achieve the spatial diversity by reproducing the signal received from the source. Nevertheless, in a network scenario where it consists of many transmission pairs and relays, a problem naturally raised is how to assign the relay nodes to help all these pairs achieving certain network design objectives. Two centralized algorithms for this problem have been proposed in a static network scenario with the aims of maximizing the sum capacity [4] and the minimal link capacity [5], respectively. Unfortunately, wireless networks usually show great dynamics on channel state and network topology in most practical scenarios due to the time-varying wireless channels, node mobility or even burst transmissions. Although both algorithms in [4] and [5] achieve the polynomial-time complexity, such centralized decision making architecture, i.e. global information collection mechanism, is still time-consuming and unscalable, for example in an ad-hoc network. This motivated us to design a distributed algorithm for the relay assignment problem.

There are few studies on the distributed algorithm design for this problem in the literature. In [6], Shastry and Adve proposed a pricing-based system to stimulate the cooperation via payment to the relay nodes. The goal in their algorithm is to ensure both the access point and the relay nodes benefit from cooperation. In [7], Wang et al. employed a Stackelberg game to jointly consider the benefits of source nodes and relay nodes. This work focused on the power control problem which could result in one source node selects multiple relays. In the most related work [8], Huang et al. proposed two auction mechanisms, which are essentially repeated games. In each auction mechanism, each user iteratively updates its bid to maximize its own utility function with the knowledge of others' previous bids. With a common characteristic, all the related studies were analyzed based on game theory and assumed that the network nodes belong to different owners. They tried to prevent negative effects on performance from selfish behaviors by encouraging cooperation. However, we consider another case in this paper that there is only one owner who administrates the whole network, and thus, the nodes can sacrifice for the benefit of the whole network. Specifically, our main contributions in this paper is to propose a distributed algorithm that are flexible and low-complexity to optimally maximize the network capacity, which were achieved using a centralized algorithm in [4]. In addition, our algorithm do not enforce bids sharing among source nodes, which greatly reduces the iterative auction overhead comparing to [8].

The rest of the paper is organized as follows. Section II introduces the system model and revisits the relay assignment problem in [4]. Section III-A demonstrates the details of the distributed auction algorithm. We show some properties such as convergence and optimality in Section III-B, and in Section III-C, we analyze its time complexity and then give a close-optimal solution with much less iterations. Simulations results are presented in Section IV and Section V concludes this work.

II. System Model

Consider a dynamic wireless network, where all the transmissions are in a frame-by-frame fashion and synchronized

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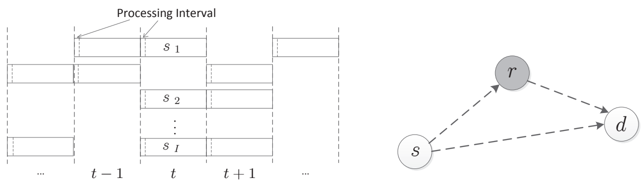


(a) sychronous transmissions (b) single relay model

Fig. 1. Cooperative communications

at each frame, which is simply demonstrated in Fig. 1a. At frame t, some of the nodes may play the roles of source or destination, while, the rest of idle nodes can be the potential relays to help source nodes so long as it can benefit the total network capacity. Denote $\mathcal{S} = \{s_1, \ldots, s_I\}$ and $\mathcal{D} = \{d_1, \ldots, d_I\}$ as I source nodes and I destination nodes at frame t, respectively. The rest of J idle nodes, which are the potential relays, are denoted as $\mathcal{R} = \{r_1, \ldots, r_J\}$. For simplicity, orthogonal channels are assumed so that all the simultaneous transmissions are interference-free. Therefore, with a proper length of the frames, the frame-synchronous dynamic network can be taken as a sequence of the static network, where the channel states are constant and the roles of source, destination and relay are determined. However, even one frame is considered, is more complicated than the static network as in [4] in the following aspects:

1) There is no centralized controller or mechanism that makes a centralized management possible, so the practicable algorithms have to work in a distributed manner.
2) Only local information is available at each node. For example, a source node is only aware of the channel gains from it to the destination node and nearby relays.
3) Limited time for the algorithms to converge, since all the processing should be done within a much shorter period than frame length as shown in Fig. 1a.

Clearly, as shown in next section, our algorithms can directly be applied to the static network case, and we thereby focus our analysis within a frame in the following sections since the relay assignment is periodically updated at a frame basis.

In what follows, we choose a single relay example to explain the channel model of cooperative communications, which is shown in Fig. 1b. The reason why we only consider one relay node for each source-destination pair is mainly based on the conclusions from [4] and [9]. The work in [4] shows that each relay node is assigned to at most one source node for achieving the capacity of the whole network, i.e., for the best relay assignment, no relay sharing among source nodes within a frame. Moreover, in case there are multiple relays available for a source node, the authors in [9] found that it is sufficient for the source node to choose the best one to achieve its link capacity. Therefore, it is also sufficient to consider only one relay for cooperative channel model. In this example, s is the source node that transmits information, d is the destination node that receives information and r is the relay node that may relays the information from the source node to improve the channel capacity between the source and the destination.

Without loss of generality, we only consider large-scale fading for the channel propagation, assuming that small-scale fading is averaged out. Suppose maximum transmit power for each node is P_{max}, when a node x transmits to another node y with power P_t, the signal-to-noise ratio (SNR) at node y, denoted as SNR_{xy}, is

$$SNR_{xy} = \frac{P_t}{N_o \cdot \|x, y\|^{\alpha}}, \tag{1}$$

where N_o is the background noise, $\|x, y\|$ is the distance between node x and y, and α is the path loss factor.

In case of *cooperative transmission*, we focus our discussions on the AF mode, other modes can be analyzed in a similar way. Whatever cooperative mode it adopt, it basically proceeds transmissions sd and rd in two steps within a frame. In this paper, we consider equal time allocation for the two transmission steps that is called time slots below. Source node s transmits data to destination node d in the first slot. Due to the broadcast nature, relay r can simultaneously overhear this signal. In the second slot, s stops transmission, and r amplified the previously received signal to d. The total achievable capacity from s to d is [9]

$$C_{sr} = \frac{W}{2} \log_2(1 + SNR_{sd} + \frac{SNR_{sr} \cdot SNR_{rd}}{SNR_{sr} + SNR_{rd} + 1}). \tag{2}$$

In case of *direct transmission*, i.e. no desirable relay to use, s transmits data to d throughout all two time slots. The achievable capacity is

$$C_{s\phi} = W \log_2(1 + SNR_{sd}). \tag{3}$$

Note that there is no interference between communication links, and both (2) and (3) are simply increasing functions of P_t, so the sources and relays always adopt the maximum transmit power P_{max} to achieve maximum capacity.

From a network point of view, although the scenario considered is more complicated than that in [4], we still aimed to pursuit the same network objective that is to find a relay assignment such that the total capacity is maximized. Using i and j to simply denote source node $s_i \in \mathcal{S}$ and relay node $r_j \in \mathcal{R}$, and denoting λ as any feasible relay assignment. As $j = \lambda(i)$ stands for relay j is assigned to source i, we have the following definition:

Definition 1. Distributed Relay Assignment Problem (DRAP): Given a set $(\mathcal{S}, \mathcal{D})$ of source-destination pairs and a set $\mathcal{R}$ of relay nodes, each node only has its local information and no central controller, the DRAP seeks for a source-relay matching λ^* such that the total capacity $C^{\lambda} = \sum_{i \in \mathcal{S}} C_{i\lambda(i)}$ is maximized among all the possible relay assignment λ.

III. Distributed Auction Algorithm

In this section, we first introduce the distributed auction relay assignment (DARA) algorithm which can lead to the optimal network capacity. Then we give formal proofs for the convergence and optimality. At the end of this section, we analyze the time complexity and improve the algorithm with less iterations.

A. Distributed Auctions

To the best of our knowledge, the centralized version of Definition 1 as in [4] could be transformed to the Maximum Weighted Bipartite Matching (MWBM) problem, which can be further solved by many efficient algorithms. Among these, one interesting algorithm is auction algorithm [10], where it disassembles the problem into many parallel auction processes. Inspired by this, we extended the auction algorithm with the specific structure of DRAP to a fully distributed cooperative network environment.

We interpret each relay assignment as an auction process. Let relay j have a price $p_j \geq 0$, for any source i occupied it has to pay. Define $\beta_{ij} = C_{ij} - C_{i\phi}$, the benefit (capacity increment) of assigning relay j to source i. Denote σ_i as the available relays for source i to have a positive capacity increment. Note that $\lambda(i) \in \sigma_i \cup \phi$, where empty set ϕ means direct transmission. Since the net profit of source node i with the help of a relay j is $\beta_{ij} - p_j$, source node i would like to choose the relay j^* that provides it with a maximum net profit by

$$j^* = \arg\max_{j \in \sigma_i}\{\beta_{ij} - p_j\}. \tag{4}$$

Correspondingly, its net profit is

$$v_i = \beta_{ij^*} - p_{j^*}, \tag{5}$$

then, the source i bids for j^* by

$$b_{ij^*} = p_{j^*} + v_i - w_i, \tag{6}$$

where w_i is defined as the second largest net profit of source i by

$$w_i = \max_{j \neq j^*, j \in \sigma_i}\{\beta_{ij} - p_j\}. \tag{7}$$

In case there is no secondary largest profit (including the case that the profits are the same), we define $w_i = 0$. When receiving new prices from the relays, the source nodes proceed a new iteration to update their bids. The whole biding process at source nodes is summarized in Algorithm 1.

Algorithm 1 Biding Process at source node i

Require: $\beta_{ij} \leftarrow (C_{ij} - C_{i\phi}), p_j \leftarrow 0, \forall j \in \sigma_i, j^* \leftarrow \phi$;
while received new price p_j **do**
  if $\beta_{ij} - p_j \leq 0$ **then**
    $\sigma_i = \sigma_i - \{j\}$: Delete unprofitable relays;
  end if
  if $v_i > 0$: Profitable relay exists **then**
    $j^* \leftarrow \arg\max_{j \in \sigma_i}\{\beta_{ij} - p_j\}$: Choose the best profitable relay;
    $w_i \leftarrow \max_{j \neq j^*, j \in \sigma_i}\{\beta_{ij} - p_j\}$;
    $b_{ij^*} \leftarrow (p_{j^*} + v_i - w_i)$: Bidding for j^*;
  else
    $\lambda(i) \leftarrow \phi$;
  end if
  $\lambda(i) \leftarrow j^*$;
end while
return $\lambda(i)$

While, at the relay side, a relay j simply chooses the highest bid from all the bidders as the new price, by

$$p_j = \max_{i \in \delta_j}\{b_{ij}\}, \tag{8}$$

where δ_j denote the set of all source nodes that are bidding for j. Relay j then broadcasts the new price to its neighbor source nodes immediately.

B. Convergence and Optimality

In last subsection, we gave the details of DARA. However, an important property of a distributed iterative algorithm should be investigated is the convergence. In the following, we analyze the convergence of DARA, and have a further discussion on its optimality.

Proposition 1 (Convergence)**.** *DARA terminates in a finite number of iterations if at least one assignment exists.*

Proof: Based on (6) and (8), we see that the prices are monotonically increased through the bidding. However, substituting (5) into (6), we have

$$b_{ij^*} = \beta_{ij^*} - w_i. \tag{9}$$

As the minimal value of w_i is 0, the bid p_j is upper bounded by the benefit of relay j to source node i, which means no source node raise such high price that they cannot afford. Thus, the prices also has an upper bound that is the maximum benefit of all sources bidding for it. In addition, the only possibility of not being able to converge, is cycles in the iterations. However, it is eliminated by the increment of bids $v_i - w_i$ that is strictly positive. The number of iterations is finite because the choice is countable and finally w_i becomes 0. Actually, in game theory, the convergence point is a Nash equilibrium, where all source nodes try to maximize their net profits. ■

As the convergence property is established, we further investigate the optimality of the converged point. Before that, we have the following lemma:

Lemma 1. *Given the final prices, The sum price of assigned relays in assignment λ^* is not less than that in any other assignment λ, i.e. $\sum_{i \in \mathcal{S}} p_{\lambda^*(i)} \geq \sum_{i \in \mathcal{S}} p_{\lambda(i)}$.*

Proof: First, the final $p_j, \forall j$ are auxiliary variables in DARA, and fixed when DARA converged. Second, any relay should have a zero price provided there is no source node biding for it. Meanwhile, based on (6) and (8), we can conclude that a relay with nonzero price must be competed by at least one source node and finally got assigned. Therefore, all nonzero prices are included in assignment λ^* as $\sum_{i \in \mathcal{S}} p_{\lambda^*(i)} \geq \sum_{i \in \mathcal{S}} p_{\lambda(i)}$, since other combination of these prices in any other assignment λ may not include all nonzero prices. ■

Proposition 2 (Optimality)**.** *DARA terminates at a relay assignment λ^* such that the system capacity is maximized.*

Proof: For assignment λ^*, we have

$$\sum_{i \in \mathcal{S}} \beta_{i\lambda^*(i)} = \sum_{i \in \mathcal{S}} p_{\lambda^*(i)} + \sum_{i \in \mathcal{S}} (\beta_{i\lambda^*(i)} - p_{\lambda^*(i)}), \tag{10}$$

where the second item on the right hand side is the sum of the net profit of all source nodes. Since the assignment λ^* achieves the maximum individual net profit for each source node, we have $\sum_{i\in\mathcal{S}}(\beta_{i\lambda^*(i)}-p_{\lambda^*(i)}) \geq \sum_{i\in\mathcal{S}}(\beta_{i\lambda(i)}-p_{\lambda(i)})$, combines with Lemma 1, we obtain

$$\sum_{i\in\mathcal{S}}\beta_{i\lambda^*(i)} \geq \sum_{j\in\mathcal{S}}p_{\lambda(i)} + \sum_{i\in\mathcal{S}}(\beta_{i\lambda(i)} - p_{\lambda(i)}) = \sum_{i\in\mathcal{S}}\beta_{i\lambda(i)}, \tag{11}$$

which means the relay assignment λ^* obtains the maximum of the sum of benefit. While, to achieve maximum network capacity, we have

$$\max_{\lambda}\sum_{i\in\mathcal{S}}C_{i\lambda(i)} = \max\sum_{i\in\mathcal{S}}\beta_{i\lambda(i)} + \sum_{i\in\mathcal{S}}C_{i\phi} \tag{12}$$

where maximizing total network capacity is equivalent to maximizing the sum of benefit of all source nodes, therefore, the maximum sum capacity is achieved by relay assignment λ^*. ■

C. Time complexity

Another desirable property of our algorithm is the reduced time complexity comparing to the centralized algorithm. In [4], the authors proved that the time complexity for their algorithm is $\mathcal{O}(I^2J)$, which scales with the number of nodes in the network, and specially grows fast as the number of source nodes increases. However, the time complexity of our algorithm is essentially scalable to any size of networks. The reason is that the complexity of DARA is mainly determined by the number of biding iterations with very lightweight computation at each node. Thus, as we show in the next paragraph, our further work is to reduce the number of iterations.

Due to the fast changing of wireless environment, sometimes we need a much faster converged algorithm that has much less iterations. Even DARA largely reduces the time complexity than the centralized one, we can further improve it but with little capacity performance decrease. Recall that the source nodes iteratively adjust the bids to control its net profit to the maximum in the procedure of DARA, which requires many round-trip signaling when the competition for a relay is intense. In this section, we propose a Fast Distributed Auction (FDA) algorithm that have the relay decide which source node to assign to. The procedure is briefly listed as follows and illustrated in Fig. 2.

1) **Source**, bidding for the relay with maximum benefit.
2) **Relay**, choosing the highest bid, and acknowledge all the rejected bidders.
3) **Source**, if got a reject ACK, then it bids the next relay with maximum benefit.
4) **Relay**, acknowledge the chosen source when converged.

Comparing with DARA, FDA does not need every source nodes carefully adjust their bids. The relay nodes, thereby, just greedily decide the source nodes with maximum benefit, which can largely reduce the iterations.

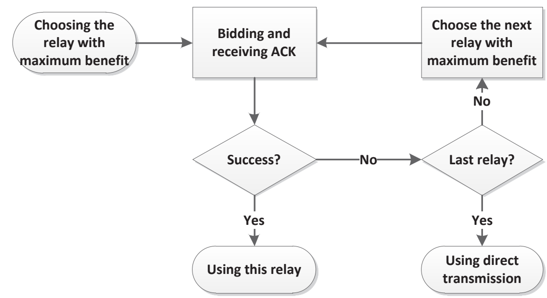


Fig. 2. Diagram of FDA at source nodes

IV. Numerical Results

In this section, we evaluate the performance of our algorithm through extensive simulations. The evaluation focuses on the network capacity and time complexity comparing to the OPRA proposed in [4].

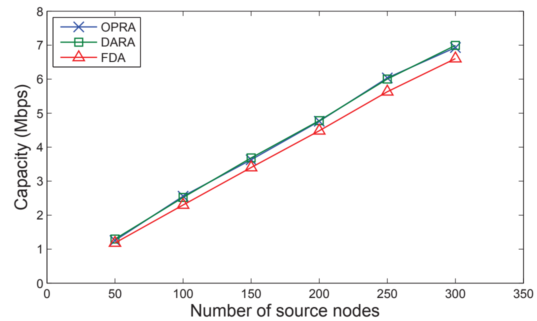
A. Simulation Settings

We follow a similar simulation parameter settings as in [4]. Consider a wireless network, where all the nodes are randomly distributed in a 1000 m $\times$ 1000 m square. For simplicity, we assume the bandwidth W is 20 kHz for each channel. All the nodes transmit with the same power $P = 1$ Watt. We set the path loss exponent $\alpha = 4$ and the background white noise $N_o = 10^{-10}$ Watt. In the simulation, we have two parameters, source nodes number I and relay nodes number J. We varied either I or J from 50 to 300 with increment of 50 while fixed the other as 100. For each setting, we randomly generated 1000 instances and collected the statistic results. All tests were performed on the same PC with 2.67 GHz CPU clock speed.

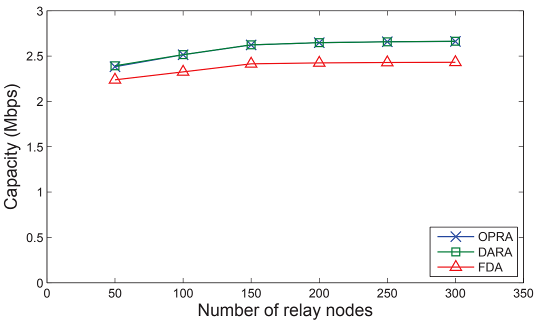
B. Results Analysis

Network capacity varying with the amount of nodes is shown in Fig. 3. Specifically, Fig. 3a shows the network capacity almost linearly increases with the number of source nodes with fixed number of relays. Note that the increase of network capacity gets slightly slower at the region of large number of source nodes since all the relays have been fully utilized. In addition, FDA shows close performance as the optimal algorithms in terms of network capacity. Fig. 3b shows the network capacity is not so affected by the number variation of relay nodes. We can see small capacity increase at the region of small number of relays when the network is not saturated with relays.

Time complexity is a huge advantage of our proposed algorithms. If we do not consider any signaling cost as in most previous related studies, results on time complexity are shown in Fig. 4a. No matter How many nodes in this network, both distributed algorithms greatly outperform the centralized ones, which shows the beauty of distributed algorithms. However, for a complete understanding and comparison, we also take the signaling delay into consideration. Therefore, in practical

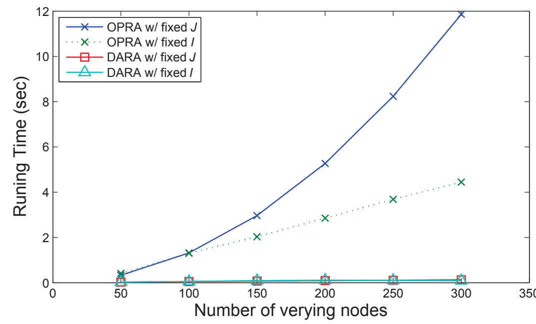


(a) Varing the number of source nodes

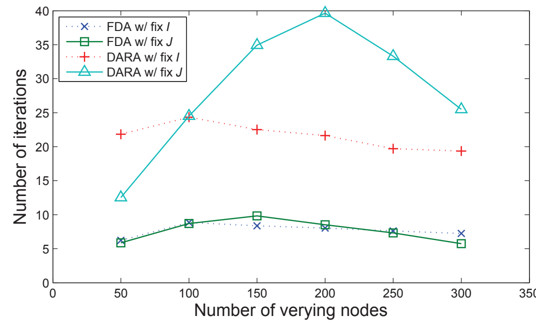


(b) Varing the number of relay nodes

Fig. 3. Network Capacity



(a) Processing time



(b) Number of Iterations

Fig. 4. Time complexity

scenarios, the total running time of distributed algorithms largely depend on the number of biding iterations. Fig. 4b compares the averaged worst-case iterations for DARA and FDA. Solid lines show the case that varying the number of source nodes while fix the number of relays, and dotted lines show the opposite case. First, even with the conservative estimates of round-trip delay, 40 iterations is trivial to the total delay. Second, FDA have much less iterations than DARA in both cases, which are desirable for some critical scenarios. Last and interestingly, all curves goes up then goes down suggesting that the competition is most intense at some point, where the number of source nodes is comparable with that of effective relay nodes. We also believe that for lager-scale networks, distributed algorithms show great advantage than any other centralized ones.

V. Conclusions

In this paper, we proposed two distributed relay assignment algorithms to achieve network capacity and low time complexity. DARA can obtain the optimal capacity but with several biding iterations, which may not be practical for fast changing dynamic network. However, we further improved its speed just with a slight capacity performance drop. It is obvious that the proposed algorithms can replace the centralized one in static network as well by once execution. For the dynamic network, our algorithms can run smoothly if all the transmissions are synchronized at each frame. However, our effort will concentrate on an asynchronous scenario for next step.

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