An Efficient Spectrum-Sensing Method based on Analog-to-Information Converter

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***Abstract* – Spectrum sensing for detecting channel occupancy is one of the key issues in cognitive radio networks. This paper investigates a power spectrum sensing method that uses an analog-to-information converter (AIC). In the literature, power spectrum reconstruction must be conducted before performing spectrum sensing. In this work, a new approach for AIC-based power spectrum sensing is developed, which is capable of directly estimating the average power in each channel from the samples produced by AIC. In addition, we demonstrate that the proposed method requires less computational operations because the reconstruction of power spectrum is not required. Furthermore, the simulation results illustrate that the proposed approach has improved performance in not only the MSE of average power estimation, but also in the detection probability.**

***Index Terms* – Power spectrum sensing, analog-to-information converter (AIC), cognitive radio (CR), sub-Nyquist rate sampling.**

I. INTRODUCTION

Utilization of the radio spectrum by licensed wireless systems (e.g., television broadcasting and aeronautical telemetry) is low. Therefore, the United States Federal Communications Commission approved a report and order on the cognitive use of TV white space [1], which indicates unused TV channels. Specifically, the licensed bands may be utilized by the secondary users when the licensed bands are unused. In cognitive radio (CR) networks [2, 3], the secondary users must possess channel occupancy information to avoid interference with primary users. This information can be obtained via power spectrum sensing.

Several studies have investigated spectrum sensing for CR networks [4–6]. A CR network is generally operated in a very wide frequency bandwidth. Thus, the conventional narrow band spectrum-sensing schemes using a tunable narrow band bandpass filter (BPF) [5] require considerable processing time. Alternatively, multiple BPFs are employed in [6] to reduce the processing time, which increases the RF components. Many researchers have therefore investigated wideband spectrum sensing.

The Nyquist sampling theorem indicates that a baseband signal with bandwidth B Hz can be perfectly recovered from its samples if the sampling rate is equivalent to or higher than $2B$ Hz. Therefore, when B is very large, wideband spectrum sensing requires an analog-to-digital converter (ADC) operating at a very high sampling rate. Unfortunately, this causes high power consumption for the ADC [7].

Performing spectrum sensing via an ADC operating at the sub-Nyquist rate has elicited considerable attention. In [8], compressive sensing was introduced to wideband spectrum sensing by utilizing the sparsity of the edge spectrum. However, the ADC must still operate at the Nyquist rate. Accordingly, based on [8], a spectrum-sensing method using an analog-to-information converter (AIC) is proposed in [9]; the ADC is able to operate at the sub-Nyquist rate. Several studies [10-12] were further developed based on [8]. In these approaches, the reconstruction of the power spectrum is necessary. However, the reconstruction procedure is computationally complex.

In this work, an efficient method for power spectrum sensing using AIC is developed. The information of the average power over each channel is sufficient for spectrum sensing. Thus, the problem of spectrum sensing using AIC is reformulated. Based on the reformulated problem, a new approach is derived to directly estimate the average power in each channel. Notably, power spectrum reconstruction is not required in our method. The proposed method is demonstrated to be computationally simpler than that proposed in [9]. Moreover, the proposed approach has improved performance in not only the MSE of average power estimation, but also in the detection probability.

The remainder of this paper is organized as follows. Section II describes the signal model of wideband spectrum sensing. Section III comprises three parts. Compressive spectrum sensing using AIC is discussed in Section III-A. In Section III-B, an approach is proposed that directly estimates the average power in each channel by using the output samples from AIC. The corresponding computational complexities are detailed in Section III-C. Finally, Section IV investigates the performance via simulation results and Section V concludes this paper.

II. CONVENTIONAL WIDEBAND SPECTRUM SENSING

In the present work, $s(t)$ denotes the wideband signal with bandwidth B Hz. In addition, U non-overlapping channels exist among the frequency range of interest. The average signal power over the

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uth channel is given by $p(u)$, where $u \in \{0,1,...,U-1\}$.

Considering the noise effect, the received analog signal $x(t)$ may be represented by

$$x(t) = s(t) + w(t). \quad (1)$$

According to the Shannon theorem, $s(t)$ can be perfectly recovered from its samples $x[n]$ if the sampling rate is higher than or equivalent to the Nyquist rate $2B$ Hz. Thus, conventionally, $x(t)$ is sampled at the Nyquist rate and then stacked into an $N\times 1$ vector $\mathbf{x}$:

$$\mathbf{x} = \{x[0], x[1], ..., x[N-1]\}^T. \quad (2)$$

The samples $x[n]$ may be represented as

$$x[n] = s[n] + w[n], \quad (3)$$

where $w(t)$ stands for the additive white Gaussian noise (AWGN) with a zero mean and variance σ^2, $\mathrm{E}[w(t)] = 0$ and $\mathrm{var}\left[|w(t)|^2\right] = \sigma^2$.

An $2N\times 1$ vector $\mathbf{r}_x$ is used to denote the auto-correlation function vector of $\mathbf{x}$:

$$\mathbf{r}_x = \left[0, r_x(-N-1), ..., r_x(0), ..., r_x(N-1)\right]^T, \quad (4)$$

where $r_x(m)$ is defined as $r_x(m) \equiv \mathrm{E}\left[x(l)x(l-m)^*\right]$. Subsequently, the corresponding power spectrum vector $\mathbf{S}_x$ can be obtained by taking the discrete Fourier transform (DFT) on the auto-correlation function vector $\mathbf{r}_x$:

$$\mathbf{S}_x = \mathbf{F}\mathbf{r}_x, \quad (5)$$

where $\mathbf{F}$ is the DFT matrix of $2N \times 2N$.

Many various power spectrum-sensing methods are proposed by exploiting $\mathbf{S}_x$ [4-6]. Unfortunately, when B is very large, sampling the signal $x(t)$ at the Nyquist rate results in high power consumption for the ADC [7]. Therefore, performing spectrum sensing via an ADC operating at the sub-Nyquist rate has elicited considerable attention [8-12].

III. Spectrum Sensing Based on Analog-to-Information Converter

This section investigates the spectrum sensing using an analog-to-information converter (AIC), where the sampling rate of ADC is generally much smaller than the Nyquist rate. The details about AIC design have been studied in [13, 14].

An AIC can be theoretically considered as an ADC operating at the Nyquist rate, followed by a matrix multiplication [9]. In defining the $M\times N$ matrix, $\mathbf{\Phi}$ is taken as the compressive sampling matrix. The output sequence of AIC is given by an $M\times 1$ vector $\mathbf{y}$.

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x}. \quad (6)$$

In general, M is much smaller than N. In addition, M/N is defined as the compression rate.

Subsequently, the auto-correlation function of $\mathbf{y}$ as a $2N\times 1$ vector is expressed as $\mathbf{r}_y$:

$$\mathbf{r}_y = \left[0, r_y(-M-1), ..., r_y(0), ..., r_y(M-1)\right]^T, \quad (7)$$

where $r_y(m)$ is defined as $r_y(m) \equiv \mathrm{E}\left[y(l)y(l-m)^*\right]$. According to [9], $\mathbf{r}_x$ and $\mathbf{r}_y$ can be related by

$$\mathbf{r}_y = \tilde{\mathbf{\Phi}}\mathbf{r}_x, \quad (8)$$

where $\tilde{\mathbf{\Phi}}$ is a $2M \times 2N$ matrix, as defined in Eq. (16) of [9].

Equation (5) can be written as

$$\mathbf{r}_x = \mathbf{F}^H\mathbf{S}_x. \quad (9)$$

Then, (9) is substituted into (8) to obtain

$$\mathbf{r}_y = \tilde{\mathbf{\Phi}}\mathbf{F}^H\mathbf{S}_x. \quad (10)$$

In inspecting (10), the power spectrum vector $\mathbf{S}_x$ is related to the autocorrelation function vector $\mathbf{r}_y$, which can be obtained from the output sequence $\mathbf{y}$ of the AIC. Thus, the AIC can be utilized in power spectrum sensing. However, since $\tilde{\mathbf{\Phi}}\mathbf{F}^H$ is a singular matrix ($M < N$), (10) cannot be solved easily.

A. Compressive spectrum sensing

In [8], the technique of compressive sensing was exploited to reconstruct the power spectrum vector $\mathbf{S}_x$ through $\mathbf{r}_y$. Furthermore, based on [8], a number of compressive spectrum sensing methods have been developed [9-12]. Subsequently, the AIC-based compressive spectrum sensing (AIC-CSS) presented in [8] is studied.

The derivative of the power spectrum results in the edge spectrum. In [8], the power spectrum vector $\mathbf{S}_x$ is related to the edge spectrum vector $\mathbf{Z}_x$ by introducing a $2N \times 2N$ matrix $\mathbf{\Gamma}$:

$$\mathbf{Z}_x = \mathbf{\Gamma}\mathbf{S}_x. \quad (11)$$

where $\mathbf{\Gamma}$ is defined as

$$\mathbf{\Gamma} \equiv \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & & & \vdots \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & & -1 & 1 & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \quad (12)$$

After substituting (11) into (10), we obtain that

$$\mathbf{r}_y = \tilde{\mathbf{\Phi}}(\mathbf{\Gamma}\mathbf{F})^{-1}\mathbf{Z}_x = \tilde{\mathbf{\Phi}}\mathbf{\Psi}\mathbf{Z}_x, \quad (13)$$

where $\mathbf{\Psi}$ is defined as $\mathbf{\Psi} \equiv (\mathbf{\Gamma}\mathbf{F})^{-1}$. By using the sparse property of the edge spectrum $\mathbf{Z}_x$, an estimate $\hat{\mathbf{Z}}_x$ of the edge spectrum can be obtained by resolving the compressive sensing problem [15]:

$$\hat{\mathbf{Z}}_x = \arg\min_{\mathbf{Z}_x} \|\mathbf{Z}_x\|_1 \quad \text{s.t.} \quad \mathbf{r}_y = \tilde{\mathbf{\Phi}}\mathbf{\Psi}\mathbf{Z}_x. \quad (14)$$

The optimization problem given in (14) is commonly resolved using the orthogonal matching pursuit (OMP) algorithm [16, 17].

Subsequently, an estimate $\hat{\mathbf{S}}_x$ of the power spectrum can be obtained by the cumulative summation over $\hat{\mathbf{Z}}_x$:

$$\hat{S}_x(k) = \sum_{q=1}^{k} \hat{Z}_x(q) . \quad (15)$$

Finally, the estimated average power in each channel can be calculated using $\hat{\mathbf{S}}_x$:

$$\hat{\alpha}(u) = \frac{1}{K} \sum_{k=(p-1)K}^{K-1} \hat{S}_x(k) . \quad (16)$$

The uth channel is determined as available if the estimated average power over the corresponding interval is less than a predefined threshold.

B. Proposed spectrum sensing

In performing AIC-CSS, power spectrum reconstruction is necessary. This section presents the reformulated AIC-based spectrum sensing problem. Subsequently, we propose a novel spectrum sensing scheme that is capable of directly estimating the average power over each channel.

A $U \times 1$ vector $\boldsymbol{\alpha}$ is defined with the uth element $\alpha(u) = \frac{1}{K} \sum_{k=(u-1)K}^{K-1} S_x(k)$, where K is the number of samples in each channel. Specifically, $\alpha(u)$ stands for the average power over the uth channel, and the expectation of $\alpha(u)$ is given by

$$\mathrm{E}\left[\alpha(u)\right] = \mathrm{E}\left[\rho(u)\right] + \sigma^2 . \quad (17)$$

Then, the power spectrum $S_x(k)$ may be represented as

$$S_x(k) = \alpha\left(\left\lfloor \frac{k}{K} \right\rfloor\right) + n(k) , \quad (18)$$

where $\lfloor \bullet \rfloor$ denotes taking the integer part, and $n(k)$ stands for the fluctuation of the signal spectrum $S_x(k)$. The influence of AWGN is also involved in $n(k)$. Moreover, (18) can be rewritten into vector form, i.e.,

$$\mathbf{S}_x = \boldsymbol{\Lambda}\boldsymbol{\alpha} + \mathbf{n} , \quad (19)$$

where $\mathbf{n}$ is a $2N \times 1$ vector and $\boldsymbol{\Lambda}$ is a $2N \times U$ matrix defined as

$$\boldsymbol{\Lambda} = \frac{1}{K} \begin{bmatrix} \mathbf{1}_K & & & \\ & \mathbf{1}_K & & \\ & & \ddots & \\ & & & \mathbf{1}_K \end{bmatrix} , \quad (20)$$

where $\mathbf{1}_K$ is a $K \times 1$ vector of ones.

By substituting (19) into (10), the autocorrelation function vector $\mathbf{r}_y$ can be rewritten as

$$\begin{aligned} \mathbf{r}_y &= \tilde{\boldsymbol{\Phi}} \mathbf{r}_x \\ &= \tilde{\boldsymbol{\Phi}} \mathbf{F}^{-1} \mathbf{S}_x \\ &= \tilde{\boldsymbol{\Phi}} \mathbf{F}^{-1} \left(\boldsymbol{\Lambda}\boldsymbol{\alpha} + \boldsymbol{\varepsilon}\right) \\ &= \tilde{\boldsymbol{\Phi}} \tilde{\boldsymbol{\Psi}} \boldsymbol{\alpha} + \tilde{\boldsymbol{\Phi}} \mathbf{F}^{-1} \mathbf{n} \end{aligned} \quad (21)$$

where $\tilde{\boldsymbol{\Psi}} \equiv \mathbf{F}^{-1}\boldsymbol{\Lambda}$ is a matrix of $2N \times U$. For simplicity, a $2M \times U$ matrix $\boldsymbol{\Theta}$ is defined as $\boldsymbol{\Theta} \equiv \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{\Psi}}$. When $\boldsymbol{\Theta}^H \boldsymbol{\Theta}$ has a full rank, which can be achieved when $2M \geq U$, the least square (LS) estimate of $\boldsymbol{\alpha}$ is given by

$$\begin{aligned} \tilde{\boldsymbol{\alpha}} &= \boldsymbol{\Theta}^{\dagger} \mathbf{r}_y \\ &= \boldsymbol{\alpha} + \boldsymbol{\Theta}^{\dagger} \tilde{\boldsymbol{\Phi}} \mathbf{F}^{-1} \boldsymbol{\varepsilon} \end{aligned} , \quad (22)$$

where $\boldsymbol{\Theta}^{\dagger}$ is the pseudo inverse of $\boldsymbol{\Theta}$, i.e., $\boldsymbol{\Theta}^{\dagger} \equiv \left(\boldsymbol{\Theta}^H \boldsymbol{\Theta}\right)^{-1} \boldsymbol{\Theta}^H$.

In (22), the term $\boldsymbol{\Theta}^{\dagger} \tilde{\boldsymbol{\Phi}} \mathbf{F}^{-1} \boldsymbol{\varepsilon}$ can be viewed as the effective noise with zero mean in power spectrum sensing. Thus, the value of $\tilde{\alpha}(u)$ indicates the average power in the uth channel and can be used for power spectrum sensing. Accordingly, the uth channel is available if $\tilde{\alpha}(u)$ is less than a predefined threshold. Specifically, the decision rule is given by

$$\tilde{\alpha}(u) \underset{H_1}{\overset{H_0}{\lessgtr}} T , \quad (23)$$

where the hypothesis H_0 represents the absence of licensed signal and the hypothesis H_1 represents the presence of licensed signal, respectively. In addition, the parameter T stands for the decision threshold, which is determined according to system requirements.

As a brief summary, the average power in each channel can be estimated directly from $\mathbf{r}_y$, as shown in (22). The procedure of spectrum reconstruction is not necessary in the proposed method. For the sake of simplicity, the proposed spectrum sensing scheme is termed as AIC-based direct spectrum sensing (AIC-DSS).

C. Comparison of computational complexity

The computational complexity of AIC-CSS is approximately equivalent to that of the OMP. According to [17], the OMP requires $O\left(r^3 + r^2 M + rMN\right)$ operations, where r denotes the iteration number. On the other hand, AIC-DSS involves a pseudo-inverse operation and requires $O\left(MU^2\right)$ computational flops [18].

Subsequently, the computational operations of AIC-CSS and AIC-DSS are compared. N is set as 400, and the compression rate M/N varies from 0.08 to 0.3. Figure 1 shows the corresponding computational complexities, where $U = 20$ and $r = 12$. The numerical results reveal that the proposed AIC-DSS requires less computational operations compared with AIC-CSS.

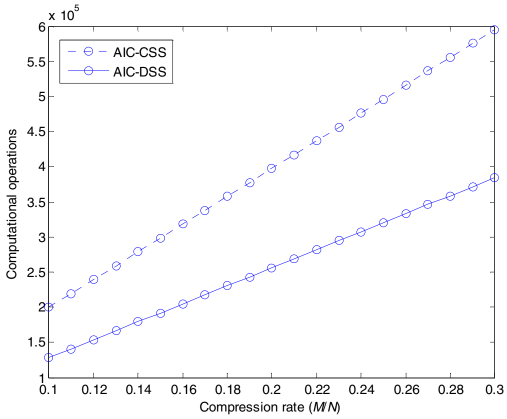

Fig. 1 The comparison for computational complexity

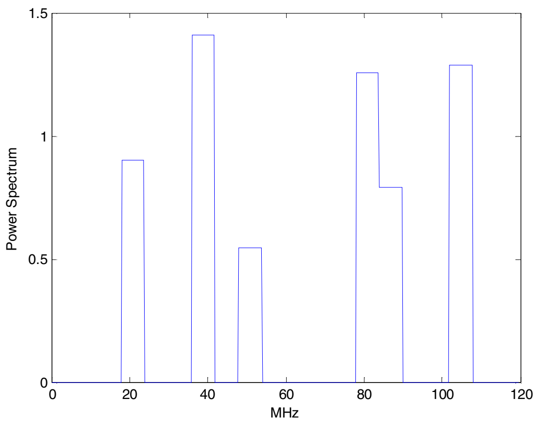

Fig. 2 An illustration example for power spectrum, where the $\{4,7,9,14,15,18\}$th channels are occupied

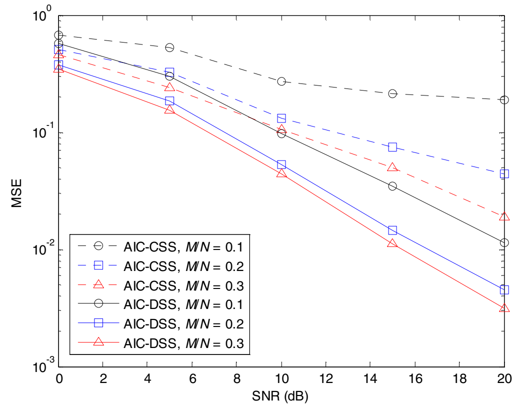

Fig. 3 The comparison for MSEs of average power estimation versus SNR

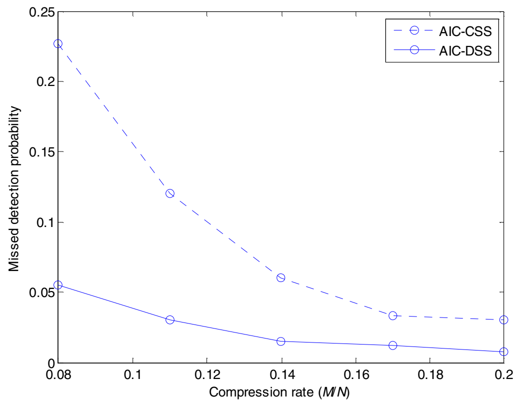

Fig. 4 The miss detection probability of power spectrum sensing versus compression rate (SNR=10dB)

IV. Simulation Results and Discussions

In this section, the performance of AIC-DSS is compared with that of AIC-CSS by conducting simulation results. The signal model is described. A wide frequency band of 120 MHz containing P=20 non-overlapping channels with equal bandwidth of 6 MHz is considered. In addition, six channels are randomly selected to be occupied by licensed signal. The average power of a licensed signal is assumed to be uniformly distributed between 0.5 and 1.5, i.e., $\rho(u)\sim\mu[0.5,1.5]$. Figure 2 shows an example of the corresponding power spectrum, where the indices of occupied channels are $\{4,7,9,14,15,18\}$.

In performing spectrum sensing, N is assumed to be 400. Moreover, the compression sampling matrix $\mathbf{\Phi}$ of $M\times N$ is randomly generated and then fixed in each Monte Carlo run. The elements of $\mathbf{\Phi}$ are independent complex Gaussian distributed random variables with mean zero and variance $\frac{1}{M}$.

For the AIC-DSS, the mean square error (MSE) of the average power estimation is defined as

$$MSE\left[\tilde{\alpha}(u)\right]=\frac{\mathrm{E}\left[\left|\rho(u)-\tilde{\alpha}(u)\right|^2\right]}{\left|\rho(u)\right|^2}. \tag{24}$$

On the other hand, for AIC-CSS, the MSE of the average power estimation has a similar definition:

$$MSE\left[\hat{\alpha}(u)\right]=\frac{\mathrm{E}\left[\left|\rho(u)-\hat{\alpha}(u)\right|^2\right]}{\left|\rho(u)\right|^2}. \tag{25}$$

In Fig. 3, the simulation result shows the MSE of the average power estimation as a function of SNR for various compression rate M/N. The simulation indicates that the proposed AIC-DSS outperforms the AIC-CSS in terms of the MSE of average power estimation.

Subsequently, the performances of missed detection probability in power spectrum sensing are examined. The decision rule is shown in (23). The missed detection probability is defined as the probability where the decision is H_0 but H_1 is true. On the other hand, the false alarm probability is defined as that where the decision is H_1 but H_0 is true. In addition, the threshold T is designed to guarantee to achieve the requirement on false alarm probability P_f. Figure 4 shows the missed detection probability as a function of compression rate M/N, where $P_f = 0.01$, and SNR is 15 dB. It is worthy to note that the proposed AIC-DSS has improved missed detection probability compared with AIC-CSS.

V. CONCLUSIONS

This paper studies the AIC-based power spectrum sensing scheme. The power spectrum sensing problem is reformulated. In addition, this work proposes an efficient spectrum sensing method using AIC, which is capable of estimating the average power in each channel directly. Unlike the AIC-CSS, the proposed power spectrum sensing scheme does not require to perform power spectrum reconstruction. Consequently, the corresponding computational complexity is significantly reduced. Moreover, the simulation results demonstrate that the proposed spectrum sensing scheme performed better than the AIC-CSS.

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