Precoding with Known Transmit Coupling and Spatial Covariance Matrices

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***Abstract*—A closed-form precoding scheme using the knowledge of transmit coupling and spatial covariance matrices is proposed, which ensures that the achievable rate increases with the number of transmit antennas when the size of the transmit antenna array is fixed and the radiated power, but not the amplifier output power, is constrained. Eigenvalue-proportional power allocation is employed in this precoding scheme. Simulation shows that the performance of eigenvalue-proportional power allocation is close to that of covariance matrix based waterfilling power allocation. Energy efficiency of different schemes is evaluated and compared.**

I. Introduction

At a transceiver equipped with multiple antennas, increasing antennas in a fixed-size antenna array reduces antenna spacing and consequently introduces mutual coupling between antennas [1]. Mutual coupling degrades system performance such as signal-to-inference-plus-noise ratio (SINR) [2], direction finding [3], [4], achievable rate [5], [6], *etc.*.

For transmitting, if the effect of mutual coupling is not taken into account in transmitter design, increasing antennas in a fixed-size antenna array can cause rate degradation instead of growth. If, considering limiting interference and electromagnetic radiation, the radiated power but not the power amplifier (PA) output power is the mandatory constraint on a wireless system, coupling compensation at the transmitter can reduce the rate loss caused by transmitter-end mutual coupling [6]. Fig. 1 illustrates the solutions in [6] for MIMO systems with full channel state information (CSIT) and with no CSIT.

In MIMO systems, other than full channel state information, channel statistics such as spatial covariance matrix is an alternative kind of channel knowledge possibly known at the transmitter. In contrast to CSI feedback, covariance feedback is of much less feedback overhead and is more accurate for fast-varying channels. In [7]–[9], spatial covariance matrix based precoding with eigenmode match and waterfilling power allocation is analyzed and compared with equal power allocation and single-stream beamforming. Apparently, covariance matrix based precoding is applicable only for spatially correlated channels and can achieve relatively high gain for highly correlated channels; whereas, in a rich scattering environment, high spatial correlation means closely-spaced antennas, where the mutual coupling issue arises.

In this paper, we consider spatial covariance matrix based precoding for coupled transmit antennas. The coupling compensation method in [6] is integrated into the proposed linear precoding scheme. Considering processing complexity, we suggest to employ eigenvalue-proportional power allocation instead of waterfilling power allocation. Under the radiated power constraint, achievable rates and energy efficiency are evaluated.

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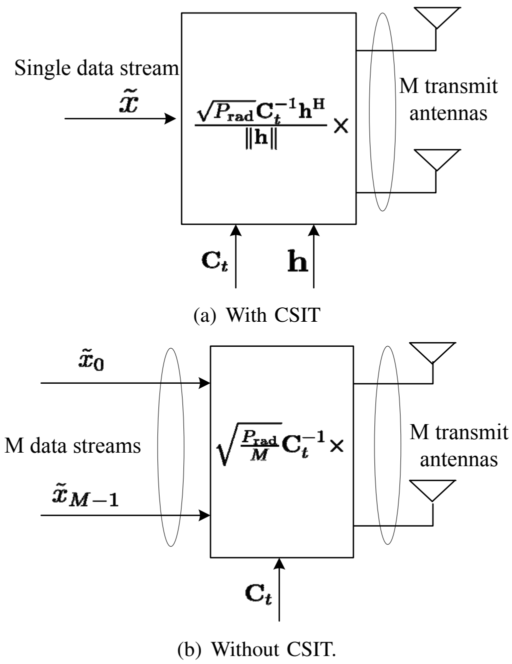

Fig. 1. Transmitter-end coupling compensation schemes with full CSIT and with no CSIT in [6].

II. System Model

Consider a transmitter with M transmit antennas and a receiver with N antennas. The flat wireless channel can be represented by the $N \times M$ matrix $\mathbf{H}$. The $N \times 1$ received vector $\mathbf{y}$ per channel use is

$$\mathbf{y} = \mathbf{C}_r\mathbf{H}\mathbf{C}_t\mathbf{F}\mathbf{x} + \mathbf{z} \tag{1}$$

where the $L \times 1$ vector $\mathbf{x}$ represents L data streams, $\mathbf{F}$ is the $M \times L$ precoding matrix, $\mathbf{C}_t$ is the $M \times M$ transmit coupling matrix, $\mathbf{C}_r$ is the $N \times N$ receive coupling matrix, and $\mathbf{z}$ is the $N \times 1$ additive noise vector at the receiver, whose entries are i.i.d. $\mathcal{CN}(0, \sigma_z^2)$.

Let $\mathbf{R}_t$ denote the transmit spatial covariance matrix. Suppose the receive transmit covariance matrix is an identity matrix, *i.e.*, no spatial correlation at the receiver. We have

$$\mathbf{H} = \mathbf{H}_w\mathbf{R}_t^{H/2} \tag{2}$$

where the entries in $\mathbf{H}_w$ are uncorrelated [10]. Under the assumption that scattering in the environment is sufficiently rich, $\mathbf{R}_t$ is a positive-definite Hermitian matrix.

In realistic wireless networks, considering co-channel interference and electromagnetic radiation, the allowable radiated power can be an important constraint on radio transmission. Assume the value of the radiated power constraint is P_{rad} per channel use. Under the radiated power constraint, given (1), the precoder $\mathbf{F}$ needs to satisfy

$$\|\mathbf{C}_t\mathbf{F}\mathbf{x}\|_{\mathrm{F}}^2 \le P_{\mathrm{rad}} \tag{3}$$

where $\|\cdot\|_{\mathrm{F}}^2$ denotes the Frobenius norm. The power amplifier (PA) output power is

$$P_{\mathrm{A}} = \|\mathbf{F}\mathbf{x}\|_{\mathrm{F}}^2. \tag{4}$$

From (3) and (4), we see that, if mutual coupling and antenna impedance are not taken into account in setting up the system model, *i.e.*, $\mathbf{C}_t$ is assumed to be an identity matrix, the radiated power constraint P_{rad} is also the constraint on PA output power P_{A}, *i.e.*, $P_{\mathrm{A}} \le P_{\mathrm{rad}}$. In the practical case, considering mutual coupling and antenna impedances, $\mathbf{C}_t$ may be a non-diagonal matrix, which makes considerable difference between the radiated power and the PA output power. In this case, if only the radiated power constraint needs to be satisfied (3), the PA output power can be greater than the radiated power constraint, *i.e*, $P_{\mathrm{A}} > P_{\mathrm{rad}}$.

III. Precoding Scheme

By eigendecomposition,

$$\mathbf{R}_t = \mathbf{U}_t\mathbf{\Lambda}_t\mathbf{U}_t^{\mathrm{H}} \tag{5}$$

where $\mathbf{U}_t$ is a unitary matrix whose columns are the basis eigenvectors of $\mathbf{R}_t$, and $\mathbf{\Lambda}_t$ is a diagonal matrix whose diagonal elements are the corresponding eigenvalues.

Suppose transmit coupling and spatial covariance matrices are known at the transmitter. We propose to integrate coupling compensation, covariance matrix based eigenmode match [7], [8]and eigenvalue-proportional power allocation, and design the precoding matrix as

$$\mathbf{F}_{\mathrm{PCC}} = \sqrt{\frac{P_{\mathrm{rad}}}{\mathrm{Tr}(\mathbf{R}_t)}}\mathbf{C}_t^{-1}\mathbf{R}_t^{1/2} \tag{6}$$

where $\mathbf{R}_t^{1/2}$ is defined as

$$\mathbf{R}_t^{1/2} = \mathbf{U}_t\sqrt{\mathbf{\Lambda}_t}, \tag{7}$$

for transmitting M data streams simultaneously under the radiated power constraint P_{rad}. The M symbols per channel use from the M data streams are supposed to be i.i.d. $\mathcal{CN}(0,1)$,

$$E(\mathbf{x}\mathbf{x}^{\mathrm{H}}) = \mathbf{I}. \tag{8}$$

Fig. 2 illustrates the precoding scheme.

Under the radiated power constraint P_{rad}, the achievable rate is

$$C_{\mathrm{PCC}} = \log_2\det\left(\mathbf{I}_N + \frac{P_{\mathrm{rad}}}{\sigma_z^2\mathrm{Tr}(\mathbf{R}_t)}\mathbf{C}_r\mathbf{H}\mathbf{R}_t\mathbf{H}^H\mathbf{C}_r^H\right), \tag{9}$$

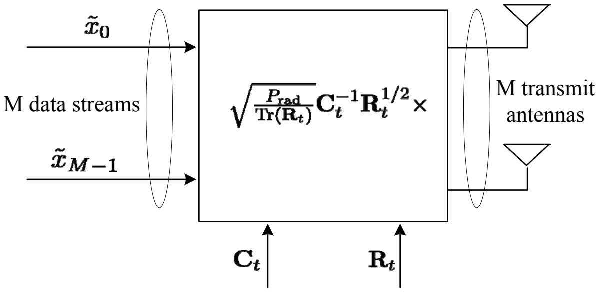

Fig. 2. Precoding M independent data streams with known transmit coupling and spatial covariance matrices

and the PA output power is

$$P_{\mathrm{A,PCC}} = \frac{\|\mathbf{C}_t^{-1}\mathbf{R}_t^{1/2}\|_{\mathrm{F}}^2}{\mathrm{Tr}(\mathbf{R_t})}P_{\mathrm{rad}}. \tag{10}$$

Let η denote the ratio of achievable rate to PA output power, representing energy efficiency. The energy efficiency in this precoding scheme is

$$\eta_{\mathrm{PCC}} = \frac{C_{\mathrm{PCC}}}{P_{\mathrm{A,PCC}}}. \tag{11}$$

In the following numerical results, we define the unit of energy efficiency as bits/channel use/mWatt.

IV. Comparison

In this section, different transmission schemes and antenna configurations are compared via simulation with respect to achievable rate and energy efficiency.

A. Simulation assumptions

The model of receive coupling matrix is

$$\mathbf{C}_r = Z_L^r(\mathbf{Z}^r + Z_L^r\mathbf{I})^{-1}, \tag{12}$$

where $\mathbf{Z}^r$ is the array impedance matrix at the receiver, and Z_L^r is the load impedance [2]. And, the model of transmit coupling matrix is

$$\mathbf{C}_t = Z_L^t(\mathbf{Z}^t + Z_L^t\mathbf{I})^{-1}, \tag{13}$$

where $\mathbf{Z}^t$ is the impedance matrix at the transmitter, and Z_L^t is the effective load impedance [6]. Note that the diagonal entries in an array impedance matrix are self impedances, and the non-diagonal entries are mutual impedances. In our simulations, we assume load impedances are self conjugate, *i.e.*,

$$Z_L^r = Z_{nn}^r, \quad Z_L^t = Z_{nn}^t, \tag{14}$$

where Z_{nn}^r and Z_{nn}^t represents self impedances at the receiver and at the transmitter, respectively. Suppose uniform linear arrays composed of identical dipole antennas with length 0.5λ and radius 0.025λ are employed, all receivers have two far-spaced antennas, and antenna configurations at the transmitter are different. Receive and transmit coupling matrices are calculated in terms of the formulas for calculating self and mutual impedances given in [1]. Clearly, for the arrays with far-spaced antennas, the corresponding coupling matrix can be approximated by a diagonal matrix whose diagonal elements are determined by antenna impedances.

The spatial correlation model given in [10], based on the "one-ring" model by Jakes [12], is used to simulate spatial covariance matrices at the transmitter. That is, assuming the angle spread $\Delta = 1$, the (i, j)-th entry of $\mathbf{R}_t$ is calculated as

$$R_{t,ij} = J_0(2\pi d_{ij}/\lambda) \tag{15}$$

where λ is the wavelength, $J_0(\cdot)$ is the zeroth-order Bessel function, and d_{ij} is the distance between the i-th and the j-th transmit antennas. With far-spaced antennas at the receiver, the receive covariance matrix is supposed to be an identity matrix, $\mathbf{R}_r = \mathbf{I}_N$.

The $2 \times M$ channel matrix $\mathbf{H}$ is simulated with the model

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{\mathrm{H}/2} \tag{16}$$

where $\mathbf{H}_w$ are the $2 \times M$ channel matrix for uncorrelated channels, whose entries are i.i.d. $\mathcal{CN}(0, 1)$.

B. Power allocation: proportional vs. waterfilling

Proportional power allocation is employed in the proposed precoding scheme (see (6) and (7)), that is, the radiated power for each data stream is proportional to the corresponding eigenvalue.

With known transmit covariance matrix, the optimal power allocation scheme should be waterfilling based on the eigenvalues of the covariance matrix [7]. With waterfilling, the precoder is

$$\mathbf{F}_{\mathrm{W}} = \sqrt{\frac{P_{\mathrm{rad}}}{\mathrm{Tr}(\mathbf{R}_t)}} \mathbf{C}_t^{-1} \mathbf{U}_t \sqrt{\mathbf{\Lambda}_w}. \tag{17}$$

where $\mathbf{\Lambda}_w$ is figured out according to the principle of waterfilling applied to the eigenvalues of the covariance matrix.

For comparison, the achievable rate with beamforming is also evaluated. The precoder for beamforming is supposed to be

$$\mathbf{F}_{\mathrm{BF}} = P_{\mathrm{rad}} \mathbf{C}_t^{-1} \mathbf{u}_1 \tag{18}$$

with $\mathbf{u}_1$ the dominant eigenvector of $\mathbf{R}_t$, *i.e.*, the first column of $\mathbf{U}_t$, and $E(|x|^2) = 1$.

From Fig. 3, we see that with respect to achievable rate, the performance of proportional power allocation is near optimal, and beamforming cannot compete with the multi-stream precoding schemes. Therefore, considering computational complexity, proportional power allocation is suggested.

C. Compact array vs. sparse array

Another possible precoding scheme for compact transmit arrays, alternative to $\mathbf{F}_{\mathrm{PCC}}$, is treating $\mathbf{C}_t \mathbf{R}_t \mathbf{C}_t^H$ as the effective transmit covariance matrix. The precoder based on this design principle is supposed to be

$$\mathbf{F}_{\mathrm{EC}} = \frac{P_{\mathrm{rad}}}{\|\mathbf{C}_t \mathbf{C}_t^H \mathbf{R}_t^{1/2}\|_{\mathrm{F}}^2} \mathbf{C}_t^H \mathbf{R}_t^{1/2}. \tag{19}$$

If only the coupling matrix $\mathbf{C}_t$ is known at the transmitter and coupling compensation in [6] is employed, the precoder is

$$\mathbf{F}_{\mathrm{CC}} = \frac{P_{\mathrm{rad}}}{M} \mathbf{C}_t^{-1}. \tag{20}$$

In the following, achievable rate and energy efficiency are evaluated by simulation. The plots are shown in Fig. 4-6.

From Fig. 4, we see that, under the radiated power constraint, when transmit coupling and spatial covariance matrix are known at the transmitter, employing $\mathbf{F}_{\mathrm{PCC}}$ makes the achievable rate increase with the number of antennas in a fixed-length array, whereas the system with sparse transmitting array cannot benefit from increasing antennas. In contrast to $\mathbf{F}_{\mathrm{PCC}}$, $\mathbf{F}_{\mathrm{EC}}$ and $\mathbf{F}_{\mathrm{CC}}$ cannot make compact transmit antenna arrays outperform sparse trasnmit antenna arrays with respect to achievable rate.

Fig. 5 illustrates the reason why compact transmit arrays with $\mathbf{F}_{\mathrm{PCC}}$ can outperform sparse transmitting arrays with respect to achievable rate: for achieving higher rate under the same radiated power constraint, a transmitter employing $\mathbf{F}_{\mathrm{PCC}}$ with

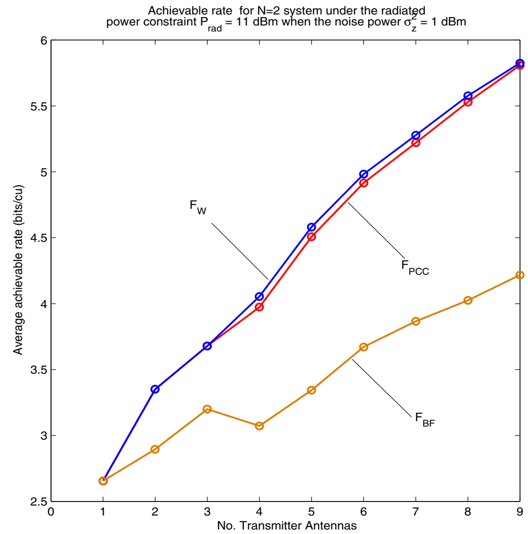

Fig. 3. Comparison of achievable rate for different precoding schemes with known transmit coupling and spatial covariance matrices. The abscissa is the number of dipole antennas equi-spaced in a λ-length linear transmit array. Shown are curves for multiple-stream precoding schemes with waterfilling/proportional power allocation strategies, and beamforming precoding.

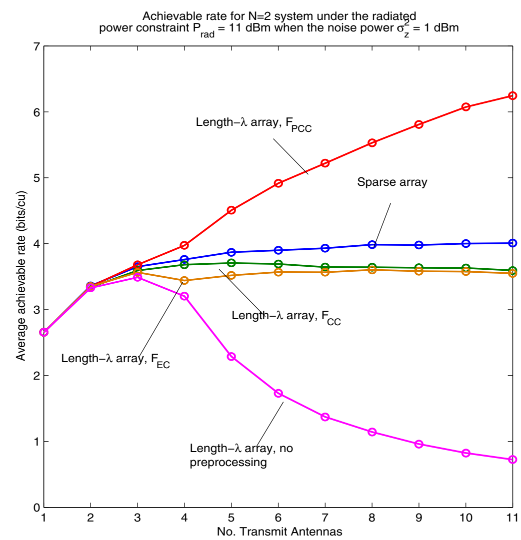

Fig. 4. Comparison of achievable rate for different transmission schemes and antenna configurations. The abscissa is the number of dipole antennas equi-spaced in a linear transmitting array. Sparse array refers to a linear array without length limitation, whose element antennas are far spaced. Length-λ array refers to a linear array of the fixed length λ.

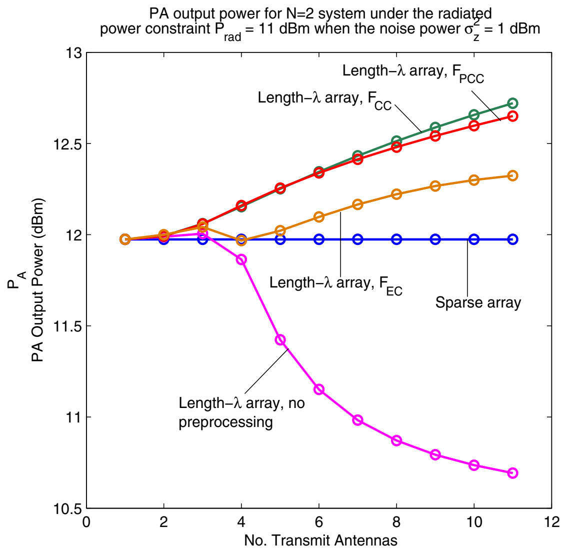

Fig. 5. Comparison of PA output power for different transmission schemes and antenna configurations. PA output power indicates energy consumption at the transmitter.

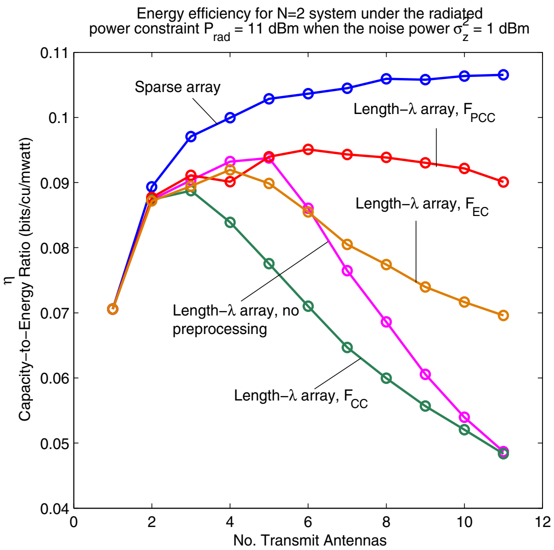

Fig. 6. Comparison of energy efficiency for different transmission schemes and antenna configurations. The ratio of data rate to PA output power is used to represent transmitter energy efficiency.

a compact transmitting array requires more PA output power than a transmitter with a sparse transmitting array.

The plots of energy efficiency are shown in Fig. 6. It is seen that with the same number of transmit antennas, sparse arrays can achieve higher energy efficiency than compact arrays. It is because no mutual coupling exists in sparse arrays and thus no energy is dissipated due to mutual impedances. For compact transmit arrays, employing $\mathbf{F}_{\mathrm{PCC}}$ is much more energy efficient than other transmission schemes. With $\mathbf{F}_{\mathrm{PCC}}$, increasing the antenna number in a a fixed-length transmitting array can increase or at least does not reduce much energy efficiency, while the achievable rate is significantly improved.

Although deploying sparse arrays is more energy efficient, deploying compact transmitting arrays with $\mathbf{F}_{\mathrm{PCC}}$ is meaningful in the sense that it can achieve higher data rate under certain radiated power constraint and is more practical considering physical space is often limited.

V. Conclusion

In this paper, for compact transmit arrays, a precoding scheme using the knowlege of transmit coupling and spatial covariance matrices has been proposed. Under the radiated power constraint, it ensures that, for a system equipped with a fixed-size transmit array, the achievable rate increases with the number of antennas. Deploying compact transmit arrays with the proposed precoding scheme can achieve higher data rate than deploying sparse transmitting arrays, though the energy efficiency is lower because of the existence of mutual impedances.

Eigenvalue-based proportional power allocation is used in the proposed precoding scheme. It has been shown that this simple power allocation scheme achieves the near-optimal performance.

References

[1] C. A. Balanis, *Antenna Theory: Analysis & Design*. New York: Wiley, 1997.

[2] I. Gupta and A. Kienski, "Effect of mutual coupling on the performance of adaptive arrays," *IEEE Trans. Antennas and Propagat.*, vol. 31, pp. 785–791, 1983.

[3] H. Steyskal and J. S. Herd, "Mutual coupling compensation in small array antennas," *IEEE Trans. Antennas and Propagat.*, vol. 38, pp. 1971–1975, 1990.

[4] B. Friedlander and A. J. Weiss, "Direction finding in the presence of mutual coupling," *IEEE Trans. Antennas and Propagat.*, vol. 39, pp. 273–284, 1991.

[5] R. Janaswamy, "Effect of element mutual coupling on the capacity of fixed length linear arrays," *IEEE Antennas and Wireless Propagat. Lett.*, vol. 1, pp. 157–160, 2002.

[6] J. Chen and K. Wu, "Mutual coupling cancelation for compact transmit antenna arrays," in *Proc. IEEE Wireless Commmunications and Networking Conf. (WCNC)*, Cancun, Mar. 2011.

[7] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Trans. Inf. Theory*, vol. 47, pp. 2632–2639, 2001.

[8] S. A. Jafar, S. Vishwanath, and A. Goldsmith, "Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback," in *Proc. IEEE Int. Conf. on Communication (ICC)*, Salt Lake City, U.S., May 2001.

[9] S. H. Simon and A. L. Moustakas, "Optimizing MIMO antenna systems with channel covariance feedback," *IEEE J. Sel. Areas Commun.*, vol. 21, pp. 406–417, 2003.

[10] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Commun.*, vol. 48, pp. 502–513, 2000.

[11] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.