An Optimal Multiuser Beamforming Scheme based on the Worst SNR in Cellular Systems

Da Wang*, Lin Bai†, Chen Chen*, Wenyang Guan‡, Ye Jin*, and Jinho Choi‡
*School of Electronics Engineering and Computer Science, Peking University, Beijing, China
†School of Electronic and Information Engineering, Beihang University, Beijing, China
‡College of Engineering, Swansea University, Swansea, United Kingdom
Email: *{wangda, c.chen, jinye}@pku.edu.cn, †l.bai@buaa.edu.cn, ‡{440501, j.choi}@swansea.ac.uk

***Abstract*—In the future cellular communication systems, multiple relay nodes can be used for distributed virtual multiple-input multiple-output (MIMO) beamforming to improve the performance of downlink transmissions. In this paper, we study a distributed relay scheme in cellular systems which consists of a base station (BS), multiple amplify-and-forward (AF) relay nodes, and multiple users. We propose an optimal distributed relay beamforming scheme that maximizes the worst-case received signal-to-noise ratio (SNR) under two different types of power constraints, which are the total relay transmit power constraint and individual relay transmit power constraints. We show that the distributed relay beamforming solution for multiusers can be obtained optimally by solving second-order cone programming (SOCP) problems. Numerical results validate our theoretical analysis and demonstrate an effective gain of the proposed scheme over other conventional schemes.**

I. Introduction

Since multiple-input multiple-output (MIMO) systems can increase the spectral efficiency for given transmit power and bandwidth in cellular systems [1], [2], various MIMO-based approaches for reliable signal transmissions have been investigated. These MIMO-based approaches can be extended for cell-edge users who are not close to any base stations (BSs). For example, in order to obtain a better spectral efficiency and improve the throughput for cell-edge users, multiple collaborative relay nodes can be used to form a distributed virtual multi-antenna system in next-generation cellular systems [3], [4].

Among various relaying schemes for the cooperative diversity, the amplify-and-forward (AF) scheme is the most attractive one due to its low implementation complexity. In the AF scheme, relay nodes can forward scaled (or amplified) signals from the BS to user equipments for downlink transmissions [5]. The AF relays can work in a way similar to a MIMO system to produce a virtual beam towards multiple destinations with the aid of channel state information (CSI) [6]. Song *et al.* [7], [8] proposed an analog network coding scheme with differential modulation using AF protocol for bidirectional relay networks. In [9], a relay power allocation algorithm was developed to minimize the total relay transmission power with individual relay power constraints for non-coherent and coherent AF relay networks. Gharavol *et al.* [10] studied a MIMO relay channel with imperfect CSI to form linear beam towards receivers obtained in point to point relay systems. Zheng *et al.* [11] optimized the relay weights jointly to maximize the received signal-to-noise ratio (SNR) for single user with both individual and total power constraints at the relays with the aid of perfect CSI. For the case of imperfect CSI at a transmitter, Zheng *et al.* [12] proved that the optimal collaborative relay beamforming weights can be obtained under a condition on the quality of the estimated CSI. Choi [13] investigated the distributed beamforming scheme with the AF protocol when a consensus algorithm was employed and different MMSE criteria were proposed in [14]. While existing approaches focused on optimal or sub-optimal beamforming for the single user or bidirectional relay networks, multiple relay nodes can be used to support multiple users for downlink transmissions. In this paper, we propose a beamforming scheme that can be used for multiple users in cellular system.

In this paper, we consider the problems of optimizing the distributed relay beamforming weights for maximizing SNR to form a virtual beam towards multiusers in the cellular system when perfect CSI is available. Our aim is to maximize the worst receive SNR under two different types of power constraints, which are the total relay transmit power constraint and individual relay transmit power constraints. We show that the optimization problem of the beamforming algorithm can be formulated as a second-order cone programming (SOCP) problem which can be efficiently solved using interior point methods in polynomial time [15].

The rest of the paper is organized as follows. Section II presents the system model of the distributed relay beamforming. In Section III, we propose the optimal distributed beamforming algorithm to maximize the worst-case SNR for multiple users with the total relay transmit power constraint and individual relay transmit power constraints. Simulations results are provided and discussed in Section IV. We conclude the paper with remarks in Section V.

II. System Model and Problem Formulation

As shown in Fig. 1, we consider a distributed relay system which consists of a BS, M relay nodes and K users. The relays work collaboratively to relay signals from the BS to K users

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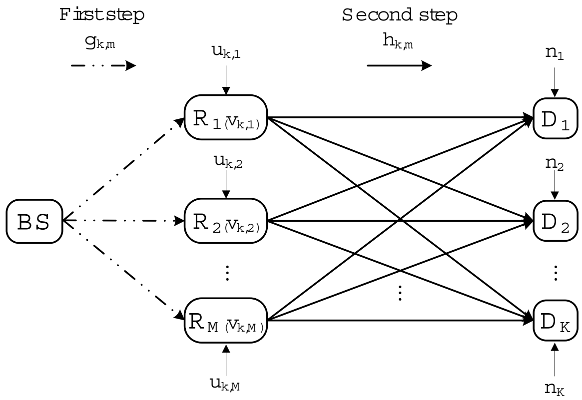

Fig. 1. The distributed relay system model

using the AF protocol. We assume that due to the poor qualities of the channels between the BS and users, there is no direct link between them. It is assumed that the coherence bandwidth of an orthogonal channel is larger than the bandwidth of the signal in our distributed relay system. Therefore, each channel can be seen as a flat fading channel. We assume that user k can receive its signal over a dedicated orthogonal channel through relays. The complex channel coefficients from the BS to the mth relay and from the mth relay to user k are denoted by $g_{k,m} \in \mathbb{C}$ and $h_{k,m} \in \mathbb{C}$, respectively. Let $\mathbf{g}_k = [g_{k,1}, g_{k,2}, \ldots, g_{k,M}]^T$ and $\mathbf{h}_k = [h_{k,1}, h_{k,2}, \ldots, h_{k,M}]^T$. For convenience, the propagation loss is contained in $g_{k,m}$ and $h_{k,m}$ to reflect the relative locations from the BS to relays and from relays to multiple users.

The AF relay protocol can be divided into two steps. In the first step, the user's signal s_k is sent from the BS to the relays. The signal $r_{k,m}$ received at the relay node m is given by

$$r_{k,m} = g_{k,m}s_k + u_{k,m}, \;\; k = 1, 2, \ldots, K, \tag{1}$$

where s_k is the signal transmitted to user k and $u_{k,m} \sim \mathcal{CN}(0, N_{k,m})$ is the independent Gaussian white noise at relay node m with zero mean and variance of $N_{k,m}$. It is assumed that the s_k's are independent with $\mathbb{E}[s_k] = 0$ and $\mathbb{E}[|s_k|^2] = P_k$.

During the second step, relay m sends the amplified signal to user k through sub-channel k (we assume that the index of sub-channels is assigned to that of users throughout this paper). The amplified signal is given by

$$x_{k,m} = v_{k,m}r_{k,m},$$

where $v_{k,m}$ is the complex beamforming weight (both transmit power and signal phase) at relay m for user k. Since the channels between some relay nodes and user k have poor qualities, the weights of such relay nodes could be zero to make them inactive.

The total transmission power of the relay node m becomes

$$\begin{aligned} P_m(v_{1,m}, ..., v_{K,m}) &= \sum_{k=1}^{K} \mathbb{E}[|x_{k,m}|^2] \\ &= \sum_{k=1}^{K} |v_{k,m}|^2 \; \mathbb{E}[|r_{k,m}|^2] \\ &= \sum_{k=1}^{K} |v_{k,m}|^2 \left(|g_{k,m}|^2 P_k + N_{k,m}\right). \end{aligned} \tag{2}$$

User k receives a superposition of the signals transmitted by active relay nodes as follows

$$\begin{aligned} y_k &= \sum_{m=1}^{M} h_{k,m}v_{k,m}r_{k,m} + n_k \\ &= \sum_{m=1}^{M} h_{k,m}g_{k,m}v_{k,m}s_k + w_k, \end{aligned} \tag{3}$$

where $n_k \sim \mathcal{CN}(0, N_k)$ is the independent Gaussian white noise at user k, and

$$w_k = \sum_{m=1}^{M} h_{k,m}v_{k,m}u_{k,m} + n_k.$$

Thus, $w_k \sim \mathcal{CN}(0, W_k)$, where $W_k = \sum_{m=1}^{M} |h_{k,m}|^2 |v_{k,m}|^2 N_{k,m} + N_k$. The receive SNR of user k, denoted by γ_k, is given by

$$\begin{aligned} \gamma_k &= \frac{P_k|\sum_{m=1}^{M} h_{k,m}g_{k,m}v_{k,m}|^2}{W_k} \\ &= \frac{P_k|\sum_{m=1}^{M} h_{k,m}g_{k,m}v_{k,m}|^2}{\sum_{m=1}^{M} |h_{k,m}|^2 |v_{k,m}|^2 N_{k,m} + N_k}. \end{aligned} \tag{4}$$

For coherent combining, the phase of weight $v_{k,m}$ at the relay node m should be the conjugate of the phase of composite channel $h_{k,m}g_{k,m}$, which can guarantee that the received signals at user k are added constructively. Thus, we only need to find the optimal power $|v_{k,m}|$. We define the equivalent (real-valued) channel as $\bar{h}_{k,m} \triangleq |h_{k,m}|$ and $\bar{g}_{k,m} \triangleq |g_{k,m}|$ and the transmit power at relay node as $\bar{v}_{k,m} \triangleq |v_{k,m}|$. Then, the receive SNR (4) can be rewritten as

$$\begin{aligned} \gamma_k &= \frac{P_k(\sum_{m=1}^{M} \bar{h}_{k,m}\bar{g}_{k,m}\bar{v}_{k,m})^2}{\sum_{m=1}^{M} (\bar{h}_{k,m})^2 (\bar{v}_{k,m})^2 N_{k,m} + N_k} \\ &= \frac{P_k(\mathbf{a}_k^T \mathbf{x}_k)^2}{\mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k}, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathbf{x}_k &= [\bar{v}_{k,1}, \ldots, \bar{v}_{k,M}]^T, \\ \mathbf{a}_k &= [\bar{h}_{k,1}\bar{g}_{k,1}, \ldots, \bar{h}_{k,M}\bar{g}_{k,M}]^T, \\ \mathbf{B}_k &= \mathbf{diag}\left((\bar{h}_{k,1})^2 N_{k,1}, \ldots, (\bar{h}_{k,M})^2 N_{k,M}\right). \end{aligned}$$

The superscript T stands for the transpose of a vector or matrix and the matrix $\mathbf{diag}\left((\bar{h}_{k,1})^2 N_{k,1}, \ldots, (\bar{h}_{k,M})^2 N_{k,M}\right)$ is a diagonal matrix with elements $(\bar{h}_{k,1})^2 N_{k,1}, \ldots, (\bar{h}_{k,M})^2 N_{k,M}$.

We can also rewrite the total transmission power of the mth relay node as

$$P_m = \sum_{k=1}^{K} (\bar{v}_{k,m})^2 \left((\bar{g}_{k,m})^2 P_k + N_{k,m}\right). \tag{6}$$

III. Max-Min SNR with Relay Power Constraints

Since the transmission power of relays is limited, we need to formulate optimization problems for the power allocation at the relays. In this section, we propose an optimal scheme to maximize the worst receive SNR for multiple users under two different types of relay power constraints. We first investigate the case where the total relay power is constrained, and then study the scenario where the individual relay transmit power is constrained.

A. Total Relay Power Constraint

In this subsection, we aim to maximize the worst receive SNR for multiusers under a total relay power constraint P_{total}. In this case, we can formulate the problem as

$$\begin{array}{ll} & \max_{\mathbf{x}_k} \min_k \gamma_k \\ \text{subject to} & \sum_{m=1}^{M} P_m \leq P_{\text{total}},\ m = 1, 2, \ldots, M. \end{array} \tag{7}$$

Note that the optimization problem (7) is quasi-convex. It is closely related to the following problem for some given Γ:

$$\begin{array}{ll} \min\limits_{\mathbf{x}_k} & \sum_{m=1}^{M} P_m \\ \text{subject to} & \begin{cases} \sum_{m=1}^{M} P_m \leq P_{\text{total}},\ m = 1, 2, \ldots, M, \\ \gamma_k \geq \Gamma,\ k = 1, 2, \ldots, K. \end{cases} \end{array} \tag{8}$$

Denote the optimal solution of (7) by γ_{opt}. If $\Gamma = \gamma_{\text{opt}}$ in (8), then the optimal $\mathbf{x}_k$ of (8) is also optimal for (7). Thus we can use a bisection search method to repeatedly search for the optimal Γ [16]. In the following, we focus on solving the problem in (8) for a given Γ.

With the receive SNR in (5) and the mth relay transmit power in (6), we can express Problem (8) as

$$\begin{array}{ll} \min\limits_{\mathbf{x}_k} & \sum_{k=1}^{K} \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} & \begin{cases} \sum_{k=1}^{K} \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \leq P_{\text{total}}, \\ \frac{P_k}{\Gamma} (\mathbf{a}_k^T \mathbf{x}_k)^2 \geq \mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k, \\ k = 1, 2, \ldots, K, \end{cases} \end{array} \tag{9}$$

where

$$\mathbf{C}_k = \textbf{diag}\left(((\bar{g}_{k,1})^2 P_k + N_{k,1}), \ldots, ((\bar{g}_{k,M})^2 P_k + N_{k,M}) \right).$$

As can be seen, Problem (9) can be decomposed into K independent problems and one total relay power constraint. The kth problem is expressed as

$$\begin{array}{ll} \min\limits_{\mathbf{x}_k} & \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} & \frac{P_k}{\Gamma} (\mathbf{a}_k^T \mathbf{x}_k)^2 \geq \mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k, \end{array} \tag{10}$$

and the total relay power constraint is

$$\sum_{k=1}^{K} \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \leq P_{\text{total}}. \tag{11}$$

For the given Γ, we can obtain K solutions $\mathbf{x}_k$, $k = 1, 2, \ldots, K$ from K problems in (10). Then we calculate the total relay power by $\sum_{m=1}^{M} P_m = \sum_{k=1}^{K} \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k$. If the total relay power is less than or equal to the total relay power constraint P_{total}, the solutions are feasible for Problem (9).

Now we turn to solve Problem (10). We denote by $\mathbf{L}_{ck}$ and $\mathbf{L}_{bk}$ as the Cholesky factors of $\mathbf{C}_k$ and $\mathbf{B}_k$, respectively. Thus, we have $\mathbf{L}_{ck}^T \mathbf{L}_{ck} = \mathbf{C}_k$ and $\mathbf{L}_{bk}^T \mathbf{L}_{bk} = \mathbf{B}_k$. Problem (10) can be rewritten as

$$\begin{array}{ll} \min\limits_{\mathbf{x}_k} & ||\mathbf{L}_{ck} \mathbf{x}_k||^2 \\ \text{subject to} & \frac{P_k}{\Gamma} (\mathbf{a}_k^T \mathbf{x}_k)^2 \geq ||\mathbf{L}_{bk} \mathbf{x}_k||^2 + N_k. \end{array} \tag{12}$$

Note that the inequality constraint in Problem (12) becomes

$$\frac{P_k}{\Gamma} (\mathbf{a}_k^T \mathbf{x}_k)^2 \geq \left|\left| \begin{pmatrix} \mathbf{L}_{bk} \mathbf{x}_k \\ \sqrt{N_k} \end{pmatrix} \right|\right|^2. \tag{13}$$

Since $\mathbf{a}_k^T \mathbf{x}_k$ is real and nonnegative, we can take the square root of the equation (13). Moreover, the objective function in Problem (12) is equivalent to the problem of minimizing $||\mathbf{L}_{ck} \mathbf{x}_k||$. We can convert the problem (12) into the following form:

$$\begin{array}{ll} \min\limits_{\mathbf{x}_k} & ||\mathbf{L}_{ck} \mathbf{x}_k|| \\ \text{subject to} & \sqrt{\frac{P_k}{\Gamma}} (\mathbf{a}_k^T \mathbf{x}_k) \geq \left|\left| \begin{pmatrix} \mathbf{L}_{bk} \mathbf{x}_k \\ \sqrt{N_k} \end{pmatrix} \right|\right|. \end{array} \tag{14}$$

The constraint becomes a second-order cone (SOC) constraint, which is convex. Thus, the optimization problem in (14) becomes an SOCP problem [16]. It can be optimally solved using interior-point methods, whose computational complexities are polynomial [15]. As shown in [17], the SOCP methods need at most $O\left(\sqrt{\alpha}(\alpha^2 \sum_i \beta_i)\right)$ arithmetic operations, where α is the number of optimization variables and β_i is the dimension of the ith constraint. In our SOCP problem of (14), the number of optimization variables is $\alpha = M$ and the dimension of the only one constraint is $\beta_1 = M + 2$. Therefore, our SOCP problem of (14) needs at most $O(M^{3.5})$, where O means the upper bound on the computational complexity order. We will present the algorithm for distributed relay beamforming with the total relay power constraint for multiusers in Subsection III-C.

B. Individual Relay Power Constraints

Given the individual relay power constraints $P_m^*, m = 1, 2, \ldots, M$, we can maximize the worst receive SNR for multiusers. Note that the individual relay power constraints are important when the battery lifetime of the relay nodes is limited. We can formulate the problem with the individual relay power constraints as

$$\begin{array}{ll} & \max_{\mathbf{x}_k} \min_k \gamma_k \\ \text{subject to} & P_m \leq P_m^*,\ m = 1, 2, \ldots, M. \end{array} \tag{15}$$

Since the optimization problem in (15) is quasi-convex, it can be solved through solving the following problem for some given Γ

$$\begin{array}{ll} \min\limits_{\mathbf{x}_k} & \sum_{m=1}^{M} P_m \\ \text{subject to} & \begin{cases} P_m \leq P_m^*,\ m = 1, 2, \ldots, M, \\ \gamma_k \geq \Gamma,\ k = 1, 2, \ldots, K. \end{cases} \end{array} \tag{16}$$

We can obtain the optimal solution of (15) by solving (16) repeatedly using the bisection search method to search for the

Γ [16]. Therefore, we solve the problem in (16) for a given Γ in the following.

With (5) and (6), we can express Problem (16) as

$$\begin{aligned} \min_{\mathbf{x}_k} \quad & \textstyle\sum_{k=1}^{K} \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} \quad & \begin{cases} \sum_{k=1}^{K} (\bar{v}_{k,m})^2 \left((\bar{g}_{k,m})^2 P_k + N_{k,m} \right) \le P_m^*, \\ \frac{P_k}{\Gamma} (\mathbf{a}_k^T \mathbf{x}_k)^2 \ge \mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k, \\ m = 1, 2, \ldots, M, k = 1, 2, \ldots, K, \end{cases} \end{aligned} \tag{17}$$

where

$$\mathbf{C}_k = \mathbf{diag} \left(((\bar{g}_{k,1})^2 P_k + N_{k,1}), \ \ldots, \ ((\bar{g}_{k,M})^2 P_k + N_{k,M}) \right).$$

Problem (17) can be decomposed into K independent problems and M individual relay power constraints. The kth problem is expressed as

$$\begin{aligned} \min_{\mathbf{x}_k} \quad & \mathbf{x}_k^T \mathbf{C}_k \mathbf{x}_k \\ \text{subject to} \quad & \tfrac{P_k}{\Gamma} (\mathbf{a}_k^T \mathbf{x}_k)^2 \ge \mathbf{x}_k^T \mathbf{B}_k \mathbf{x}_k + N_k, \end{aligned} \tag{18}$$

and the M individual relay power constraints are

$$\sum_{k=1}^{K} (\bar{v}_{k,m})^2 \left((\bar{g}_{k,m})^2 P_k + N_{k,m} \right) \le P_m^*, \ m = 1, \ldots, M. \tag{19}$$

For the given Γ, we can obtain K solutions $\mathbf{x}_k$ from K problems in (18), then we calculate each of the relay power for the mth relay by $P_m = \sum_{k=1}^{K} (\bar{v}_{k,m})^2 \left((\bar{g}_{k,m})^2 P_k + N_{k,m} \right)$. If each of the relay power of the M relay nodes is less than or equal to the individual relay power constraints $P_m^*, m = 1, 2, \ldots, M$, the solutions are feasible for Problem (17).

As can be seen, the problem in (18) is the same as Problem (10). We can also formulate the optimization problem (18) as the SOCP problem (14). Thus, the computational complexity is also at most $O(M^{3.5})$. We can obtain the power allocation vector $\mathbf{x}_k$ by solving the SOCP problem in (14) with the individual relay power constraints (19).

C. Optimal Algorithm

Based on the above results, we can propose an efficient algorithm (see Algorithm 1) to solve the primal problem in (7) or (15) for distributed relay beamforming in multiuser systems. Due to the quasi-convexity of (7) and (15), they can be optimally solved though the bisection search method by repeatedly solving (8) and (16), respectively. Denote the pair of upper and lower bounds on the achievable worst-case SNR by γ_{UB} and γ_{LB}, respectively. In practice, γ_{UB} can be obtained by removing the relay power constraints, while γ_{LB} can be simply set to 0.

The algorithm searches for the achievable worst SNR to find the optimal relay power allocation with the total relay power constraint or individual relay power constraints. The bisection search method requires $\lceil \log_2((\gamma_{UB} - \gamma_{LB})/\varepsilon) \rceil$ iterations [16]. Note that our primal problem in (7) or (15) consists of K subproblems in (14). The computational complexity of Algorithm 1 is $O\left(KM^{3.5} \lceil \log_2((\gamma_{UB} - \gamma_{LB})/\varepsilon) \rceil\right)$, where $\lceil z \rceil$ rounds an element of z to the nearest integers. With Algorithm 1, the relays can form virtual beams towards K users and it can be guaranteed to achieve the maximization of the worst receive SNR of multiple users.

Algorithm 1 Optimal Algorithm

Initialize $\bar{h}_{k,m}, \bar{g}_{k,m}, k = 1, 2, \ldots, K, m = 1, 2, \ldots, M$, γ_{LB}, γ_{UB} and P_{total} or P_m^*
For some given $\varepsilon > 0$
while $\gamma_{UB} - \gamma_{LB} > \varepsilon$ **do**
  for all $k = 1, 2, \ldots, K$ **do**
    Using interior-point methods to solve the SOCP problem (14) with $\Gamma = (\gamma_{LB} + \gamma_{UB})/2$
    if the problem (14) is feasible **then**
      Get the solution $\mathbf{x}_{k\text{opt}}$
    end if
  end for
  if the total power constraint (11) is satisfied or M individual power constraints (19) are satisfied **then**
    $\gamma_{LB} = \Gamma$
  else
    $\gamma_{UB} = \Gamma$
  end if
end while
Get the maximization of the worst receive SNR $\gamma_{\text{opt}} = \Gamma$
Obtain the power of weight $v_{k,m}$ from $\mathbf{x}_{k\text{opt}}$
Adjust the phase of weight $v_{k,m}$ to the conjugate of the phase of the composite channel coefficients $h_{k,m} g_{k,m}$
return the complex beamforming weight $v_{k,m}, \forall k, m$ and γ_{opt}

IV. Simulation Results

Simulation results are presented to see the performance of the proposed beamforming scheme. In the simulations, we assume that the relays have the same individual power constraint, i.e., $P_m^* = P_{\text{total}}/M$. The BS transmission SNR, denoted by $P_k/N_0, k = 1, 2, \ldots, K$, is the same for all users. For convenience, we set the transmission SNR to be 10 dB. The channels are assumed to be Rayleigh flat fading channels, i.e., $\mathbf{g}_k, \mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{I})$ in particular. In addition, we assume that the noise at all the nodes, including the relays and the users, has the same power spectral density, i.e., $N_{k,m} = N_k = N_0$. The phase for the mth AF relay node is adjusted to be the conjugate of the phase of the composite channel coefficients $h_{k,m} g_{k,m}$. Then, we can use the proposed algorithm to determine the relay power allocation with total relay power constraint and individual relay power constraints. We use YALMIP [18] to numerically solve the SOCP problems. Results for the following schemes are presented: a) the proposed optimal scheme to maximize the worst SNR for multiusers in Section III; b) a conventional scheme, as described below.

We present a conventional scheme to compare with our optimal beamforming scheme. In the conventional scheme, the relay nodes transmit fixed power and each relay weight's phase is adjusted to the conjugate of the phase of the composite channel coefficients $h_{k,m} g_{k,m}$. We assume that the fixed relay power is the same for K users. Thus, according to (6), the transmit power of relay node m towards the kth user can be

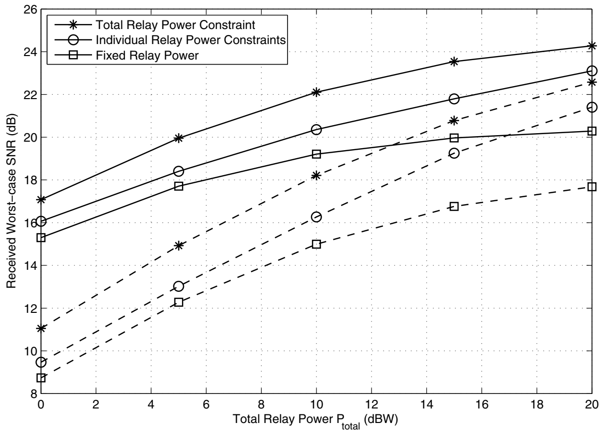

Fig. 2. Expected worst SNR against the total relay power for 20 relays with one user (solid line), and five users (dashed line)

written as

$$|v_{k,m}| = \sqrt{\frac{P_{\text{total}}}{MK\left((\bar{g}_{k,m})^2 P_k + N_{k,m}\right)}}. \tag{20}$$

Fig. 2 demonstrates the receive worst SNR result against the total relay power for our proposed algorithm with 20 relays and different numbers of users. The curves with solid line represent that the system has 20 relays and 1 user, while the curves with dashed line represent the scenario that has 20 relays and 5 users in the system. The single user system (solid line) can be seen as a special case of our proposed scheme for multiuser systems. As can be seen, with the same number of relays, the single user system can obtain the better receive SNR than the multiuser system. The results also confirm that the proposed optimal algorithm with the total power constraint and the individual power constraints achieves $2 \sim 4$ (dB) and $1 \sim 3$ (dB) higher SNR than the conventional scheme with fixed relay power, respectively.

V. Conclusion

This paper has proposed a distributed AF relay scheme to form virtual beams towards multiusers in cellular systems for effective downlink transmissions. In order to derive relay gains, we maximized the worst receive SNR with two different constraints: *i)* the total relay transmit power constraint and *ii)* individual relay transmit power constraints. To find optimal solutions, we used SOCP that can be efficiently solved using interior point methods in polynomial time. Simulation results demonstrated that our optimal scheme had an effective gain compared to the conventional scheme.

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