Bit Error Rate Analysis in WiMAX Communication at Vehicular Speeds Using Nakagami-*m* Fading Model

Biswojit Bose, Iftekhar Ahmad and Daryoush Habibi
Centre for Communications Engineering Research, Edith Cowan University, Australia.

***Abstract*—High speed wireless communication technologies such as Worldwide Interoperability for Microwave Access (WiMAX) have revolutionized the way of our day-to-day communication and opened opportunities for many innovative applications. The 802.6m version of WiMAX offers data rates up to 1 Gbps for fixed communications and supports mobility up to 350 km/h. While WiMAX technology's capacity to deliver high data rates in a fixed environment is beyond any doubt, the standard is not fully optimized yet for mobile communication at high vehicular speeds. At high vehicular speeds, rapid changes in surrounding environments, cause severe fading at the receiver, resulting a drastic fall in throughput and unless any proactive measure is taken to combat this problem, throughput becomes insufficient to support many applications, particularly those with multimedia contents. Bit Error Rate (BER) estimation is an integral part of any proactive measure and recent studies suggest that Nakagami-m model performs better for modeling channel fading in wireless communications at high vehicular speeds. No work has been reported in literature that estimates BER at high vehicular speeds in WiMAX communication using Nakagami-*m* model. In this paper, we develop and present an analytical model to estimate BER in WiMAX at vehicular speeds using Nakagami-*m* fading model. The proposed model is adaptive and can be used with resource management schemes designed for fixed, nomadic, and mobile WiMAX communications.**

***Index Terms*—WiMAX, vehicular speeds, bit error rate, Nakagami-m.**

I. Introduction

WiMAX is a popular next generation wireless technology that currently serves more than 620 million people in approximately 147 countries. WiMAX is popular for its capacity to deliver high throughput at a fixed communication environment. In a mobile communication environment at high vehicular speeds, this throughput decreases sharply, often providing a connection only service with no guaranteed data rate. The 802.16m version of WiMAX standard acknowledges this problem and indicates that the 802.16m is fully optimized for stationary and pedestrian speeds (0-10km/h) [1]. At speeds between 10-120km/h, WiMAX users experience a gradual degradation of service and at speeds above 120 km/h, only a connection can be maintained without any assurance on data rate.

WiMAX standard incorporates Orthogonal Frequency Division Multiplexing (OFDM) and Orthogonal Frequency Division Multiple Access (OFDMA) [2] for achieving better spectral efficiency and data rates. The OFDM is a robust technique that overcomes the frequency selectivity problem of the channel and provides higher throughput. In OFDM systems, total bandwidth is divided into multiple sub-carriers using Fast Fourier Transformation (FFT) operation where the sub-carriers are orthogonal to each other. Sub-carriers are divided into data, pilot, DC and guard sub-carrier. The data, pilot and guard sub-carriers are used for transmitting data, pilot symbols and guard information for limiting interference, respectively. The OFDMA technique is based on OFDM, which provides multiuser access to a channel by dividing the sub-carriers into subsets of sub-carriers. For supporting mobility, mobile WiMAX extends its physical layers and incorporates the scalable OFDMA (S-OFDMA) technique. By adopting scalable PHY architecture, it can support a wide range of bandwidths. The scalability is implemented by varying the FFT from 128 and multiple of 128. Table-1 summarizes the primitives used for mobile WiMAX PHY. At high speeds, doppler shift causes inter-carrier interference (ICI) and due to small sub-carrier spacing, ICI possibilities on larger FFT is higher than lower FFT. WiMAX forum therefore recommends smaller FFT and simpler modulation at high vehicular speeds.

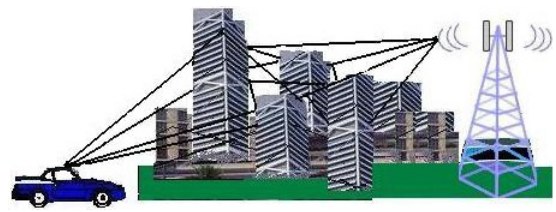

Figure 1. Multipath fading problem in wireless communication at vehicular speeds.

In a high mobility scenario, relative motion between transmitters and receivers results in rapid time variation and high doppler shift. Accumulating dynamically changed multipath effects and noise, a significant fluctuation in received signal strength is observed in the channel. Fading like this is often modelled in literature with Rayleigh fading model. Rayleigh fading model has not been challenged until very recently when researchers started to focus on the throughput problem at vehicular speeds. Rayleigh model works on the assumption that the resultant fading arises from a large number of uncorrelated partial waves with identically distributed amplitudes and uniformly $[0, 2\pi]$ distributed random phases. This assumption is highly optimistic in a mobile communication environment at high vehicular speeds and the more realistic assumption

978-1-4673-1881-5/12/$31.00 ©2012 IEEE

Table I
WIMAX FORUM SPECIFICATION PRIMITIVES FOR MOBILE WIMAX PHY.

Parameters	Value .	Value
Number of Sub-carriers	512	1024
Bandwidth (BW)	3.5, 5	7, 8.75, 10
Sampling Factor (n)	BW is multiple of 1.75 is 8/7, multiple of 1.25, 1.50, 2, and 2.75 is 28/25 not otherwise specified then n=8/7	
Cyclic Prefix time ratio (G)	1/8	
Used sub-carriers	433	865
Pilot Sub-carriers	48	96
Guard sub-carriers, Left	40	80
Guard sub-carriers, Right	39	79
DC sub-carrier	1	1
Data sub-carriers	384	768

is to have many partial waves with amplitudes that follow distributions that are not identical, yet partially correlated [3], [4]. In an environment like this, signal fluctuations are better modelled by Nakagami-*m* distribution [4], [3] and as a result, estimated BER is more accurate in Nakagami-*m* model than in Rayleigh model. Moreover, Nakagami-m model can be made adaptive to suit fixed, pedestrian and high speeds mobility environments by changing the fading parameter m, which is used to reflect the fading severity. The parameter value $m < 1$ is considered as Nakagami/sub-Rayleigh fading and the fading process is considered as a product of complex Gaussian process and a square root beta process. Rayleigh distribution ($m = 1$) and Rician distribution ($m > 1$) are considered as a special case of Nakagami distribution [4], [5]. In this paper, we develop and present an analytical model for BER estimation in WiMAX communication systems using Nakagami-*m* fading model.

II. PREVIOUS WORKS

Wireless communication at vehicular speeds in general has attracted increasing attentions in recent years mainly because of its relevance to intelligent transportation system. Onsy $et\ al.$ [6] conducted a simulation study in the IEEE 802.16e at high vehicular speeds for channels with Additive White Gausian Noise (AWGN). Dong $et\ al.$ conducted a comparative study of channel estimation for mobile WiMAX at high mobility [7] and proposed a piecewise linear interpolation based channel estimator. Ahmad $et\ al.$ investigated the BER performance in WiMAX at high vehicular speeds using Rayleigh fading and proposed a mathematical scheme to compute the optimum packet size based on the estimated BER [8]. Ahmad $et\ al.$ conducted another study [9] to develop an adaptive error correction scheme based on estimated BER in WiMAX communications. Boudali $et\ al.$ evaluated the BER in a convolutional coded OFDM system for the return link in a mobile environment. [10]. Aguado $et\ al.$ performed a simulation study on mobile WiMAX deploying CCTV on a public transport [11].

Subotic $et\ al.$ [12] analysed the BER for an equalized OFDM system using Nakagami, m< 1 distribution using pilot assisted linear channel estimation and channel equalization. A BER analysis for frequency selective slow fading channel has been conducted by Zhang $et\ al.$[13] using Nakagami–m distribution . Performance of maximal-ratio diversity systems in a correlated Nakagami fading environment is analysed by Aalo [14] for urban and vehicular communications. A research was conducted by He $et\ al.$ [5] to estimate the BER on QAM and MPSK in Nakagami fading channel with space time transmit diversity. Guo $et\ al.$ [15] conducted a research to analyse the BER for MIMO-OFDM system over Nakagami-m fading channels using Rate Compatible Punctured Convolution (RCPC) and Space Time Block Code (STBC) at transmitters and maximum ratio combining (MRC) at receivers. Statistical characteristics of ICI and BER performance of OFDM system has been analysed by Vivek $et\ al.$ [16] over a correlated Nakagami fading channel using MRC at receiver. BER analysis for OFDM-BPSK and QPSK over a flat fading channel using Nakagami-m distribution has been conducted by Neetu $et\ al.$ [17]. There is no research reported in the literature that presents BER estimation in WiMAX communication at vehicular speeds using Nakagami-m model. An analytical model is required for estimating BER at various vehicular speeds for efficient resource management, which provides the motivation for this research.

Table II
DERIVED PARAMETERS FOR IEEE 802.16M.

Parameters	Equations
Sampling frequency (F_s)	$F_s = floor(n.BW/8000).8000$
Sub-carrier spacing (∇f)	F_s/N_{fft}
Useful symbol time(T_b)	$1/\nabla F$
Cyclic Prefix (CP) time (T_g)	$G.T_b$
OFDMA symbol time $ofdmT_s$	$T_b + T_g$
Sampling time (T_s)	Sampling time T_b/N_{fft}

III. CHANNEL MODEL FOR SUB-RAYLEIGH FADING

When mobile nodes move at high vehicular speeds, doppler shift and dynamic multipath effects cause complex fading and reduce throughput. Following [4], [12], this complex fading process $x(t)$ can be modelled as a product of two independent processes $y(t)$ and $z(t)$ where $y(t)$ is the zero mean complex gaussian random process and $z(t)$ is exponentially correlated random process. The correlation function $\Re_x(\tau)$, of the process $x(t)$ is a product of correlation process of $y(t)$ and $z(t)$, which can be given as

$$R_x(\tau) = |y(t)y(t+\tau)z(t)z(t+\tau)| = R_y(\tau)R_z(\tau) \quad (1)$$

The marginal distribution $p_x(x)$ of $x(t)$ does not depend on the correlation of these component. Therefore, the marginal distribution $p_z(x)$ of the modulation component $z(t)$ affects on the desired distribution. If γ is the symbol energy to noise ratio at the receiver, then the pdf of Nakagami process is given by

$$pdf(\gamma) = \frac{2m^m\gamma^{2m-1}}{\Gamma(m)\Omega^m} exp\left(-\frac{m}{\Omega}\gamma^2\right), \quad \gamma \geq 0, 0.5 \leq m < 1 \quad (2)$$

where $\Gamma(.)$ is the gamma function and

$$\Omega = E[\gamma^2] \tag{3}$$

$E(.)$ is the mathematical expectation and the fading parameter m is defined as

$$m = \frac{\Omega^2}{E[(\gamma^2 - \Omega^2)2]} \tag{4}$$

$z^2(t)$ has a standard beta distribution with a pdf given as

$$pdf(z^2(\gamma)) = \frac{z^{m-1}(1-z)^m}{\beta(m, 1-m)}, \quad 0 \leq z \geq 1 \tag{5}$$

where $\beta(x, y)$ is the beta function.

Considering the Markov exponentially correlated process [12], $z(t)$ is given by

$$R_z(\tau) = exp(-\lambda\tau/2) \tag{6}$$

where λ is a positive constant.

$$R_y(\tau) = R_x(\tau).exp(\lambda\tau/2) \tag{7}$$

If $R_x(\tau)$ is approximated by a correlation function with a rational spectrum, it can be rewritten as

$$R_x(\tau) = \sum_{k=1}^{\propto} c_k^2 exp(-\tau_k|\tau|) \tag{8}$$

where c_k is the complex number. By setting $\tau = 0.05min(\tau_k)$, $R_y(\tau)$ defines a proper correlation function of a Gaussian process.

IV. Proposed Analytical Model for Bit Error Rate Estimation

WiMAX OFDM system consists of N orthogonal sub-carriers with a frequency spacing ∇f . The cyclic guard is used to combat inter-symbol interference (ISI). Each of the sub-carrier is modulated independently with complex modulation symbol. During an OFDMA symbol transmission, the transmitted signal strength to the antenna [18] is given by

$$S(t) = Re\left\{ e^{j2\pi f_c t} \sum_{k=-(N_{used}-1)/2}^{(N_{used}-1)/2} c_k.e^{2\pi k \nabla f(t-T_g)} \right\} \tag{9}$$

where f_c is the center frequency, t is time elapsed since the beginning of the OFDMA symbol, which also specify a point in a QAM constellation, T_g is the guard time.

In a high mobility scenario, fading is caused by dynamic multipath effects and doppler shift, and the average received symbol energy to noise ratio ($\overline{\gamma_s}$) of this channel is given by [8]

$$\overline{\gamma_s} = \frac{1}{1 - \frac{1}{N^2}\left[N + 2\sum_{i=1}^{N-1}(N-i)J_0(2\pi f_d T_s i)\right] + \frac{NT_s}{\frac{E_s}{N_0}}} \tag{10}$$

where N is the number of used sub-carrier, j_0 is the Bessel function, T_s is the duration of each *M*-ary QAM symbol transmitted on a sub-carrier, NT_s is the useful OFDM symbol time, E_s is the average symbol energy, N_0 is the energy of the noise and f_d is the doppler shift of the channel, which is defined as

$$f_d = f_c(\frac{v}{c}) \tag{11}$$

where f_c is carrier frequency, v is the speed of the mobile station and c is the speed of the light.

The average bit energy to noise ratio ($\bar{\gamma_b}$) of a multipath fading is given by

$$\overline{\gamma}_b = \frac{\overline{\gamma_s}}{log_2 M} \tag{12}$$

where M is the number of symbol for *M*-ary QAM modulation scheme (for QPSK $M = 4$ and for 16 QAM $M = 16$). From equation (10) (12), bit energy to noise ratio can be written as

$$\overline{\gamma_b}(v) = \frac{\frac{1}{log_2 M}}{1 - \frac{1}{N^2}\left[N + 2\sum_{i=1}^{N-1}(N-i)J_0(2\pi f_c(\frac{v}{c})T_s i)\right] + \frac{NT_s}{log_2 M}\cdot\frac{1}{\frac{E_b}{N_0}}} \tag{13}$$

where E_b is the average energy per bit.

For a WiMAX OFDMA air interface, the main source of bit error is inter-carrier interference instead of interference between OFDMA users. This is because interference between OFDMA users is averaged over many users and therefore it remains almost constant over time and absorbed in the Gaussian thermal noise. Additive white Gaussian noise (AWGN) is used successfully to approximate the OFDMA inter-carrier interference. Taking AWGN into consideration, the bit error probability for a generalized maximum likelihood (ML) receiver is given by

$$P_b(v) = Q\left(\sqrt{2\gamma_b(v)}\right) \tag{14}$$

where bit error probability P_b and symbol error probability P_M of the OFDM are related to each other in the form of

$$P_b \approx \frac{P_M}{log_2 M} \tag{15}$$

For $M = 2$ or $M = 4$, the average bit error probability is as

$$P_b(v) = \int_0^{\infty} P_b(x) p_{y_b}(x) dx \tag{16}$$

$$P_b(v) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_b(v)}{1 + \bar{\gamma}_b(v)}}\right] \tag{17}$$

Bit error probability for a higher order of M also be obtain from

$$P_b(v) = \frac{\int_0^{\infty} p_M(x) P_{\gamma_M(x)dx}}{log_2 M} \tag{18}$$

Bit error rate, however, becomes excessively high for higher order of M (>4) and therefore, is not recommended by WiMAX forum for use in practical applications at vehicular communications. It is also evident that bit error probability remains almost the same for PSK and QAM for M up to 4, a reason why, in this work we develop an analytical model for QPSK modulation scheme only.

In Nakagami fading, all the sub-carrier frequencies are multiplied by complex channel gains whose square root beta modulating component (c) arises from the same parameter distribution and this modulating component is constant over the OFDM symbol [12]. The independent and identically distributed (i.i.d) Gaussian process is conditioned on c and the average bit energy to noise ratio for Nakagami *m*<1 fading becomes

$$\tilde{\bar{\gamma}}_b = c\bar{\gamma}_b \tag{19}$$

and the bit error probability of a Nakagami-m fading channel is given by:

$$P_b(E) = \int_0^1 P(E|c).P_c(c) = \int_0^1 P(E|z).Pc^2(z)dz \tag{20}$$

where $P(E|c)$ is a Rayleigh pdf conditioned on c and $z = c^2$ is distributed according to equation (5).

Solving equation (20), bit error probability for QPSK modulation in WiMAX using Nakagami-m fading becomes:

$$P_b(v) = \int\limits_0^1 \frac{1}{2}\left[1 - \sqrt{\frac{c\gamma_b(v)}{1 + c\gamma_b(v)}}\right] \frac{z^{(m-1)}(1-z)^{-m}}{\beta(m, 1-m)} dz \tag{21}$$

In the above mentioned equation, the Nakagami parameter m indicates the fading severity which can be computed using equation (2) from observed data. The m value can be calibrated at various vehicular speeds using data collected from a hardware set-up (a work in progress that will provide information about speed groups that cause severe, mild or marginal fading), This calibrated value of m can be used in equation (21) to reflect various fading in different environments such as Rician ($m > 1$ for LOS fixed communication), Rayleigh fading ($m = 1$ for low speed communication) and Nakagami fading ($m < 1$). Once the m value is calibrated, the proposed analytical model can be used with a resource management scheme (e.g., to compute how much bandwidth is required for a node that moves at a mean speed of 80 km/h) to be used in WiMAX communication at vehicular speeds.

V. Result Analysis

A simulation study was conducted for a 2.6 GHz system with a 5 MHz bandwidth (BW) and QPSK modulation. Primitives and derived parameters were taken from table-I and table-II, respectively. We used MATLAB signal processing toolbox for BER analysis at various vehicular speeds from 0 to 200 km/h with different combinations of SNR and Nakagami parameter m. Figure 2 shows BER at various vehicular speeds from 0 to 50 km/h against different SNR values for *m* = 0.5. It is evident that the BER increases with increasing vehicular speeds and the system experiences severe fading at high vehicular speeds (Figure 3). It was observed that at low vehicular speeds, SNR has a big impact on overall BER, but as the speed increases, SNR has insignificant impact on BER (i.e., the system becomes ICI limited), a reason why special attention is required for resource management at vehicular speeds for facilitating applications requiring high data rates. In Nakagami-*m* model, the fading severity of the channel is depicted by the parameter m . Figure 4 shows BER of the channel for various values of m at a speed ranging from 0 to 200 km/h. Figure 5: shows BER of the system, which decreases with increasing Nakagami parameter m. When the value of m is 0.1, BER is relatively high due to the severe fading and when the value of m is 0.9, BER is relatively low. It is shown in Figure 6 that Rayleigh fading fails to model severe

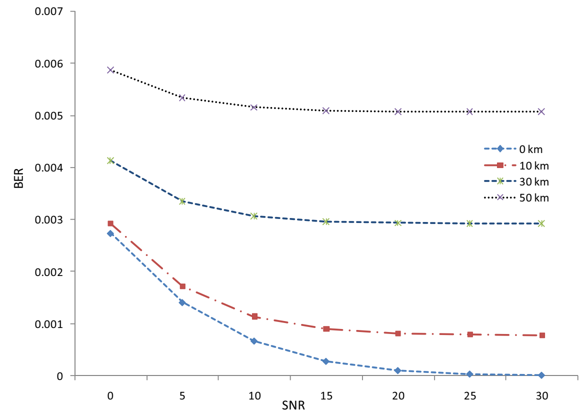

Figure 2. BER at vehicular speeds for Nakagami-*m* (*m*=0.5) fading channel.

fading at high vehicular speeds and the BER increases only marginally even at very high vehicular speeds. One biggest advantage of the proposed analytical model (equation (21) is that the same model works perfectly fine when the mobile node changes its speed resulting various scales of fading severity. For example, when the mobile node is static and has good line of sight communication, the fading model can be modeled using Rician fading and this can be achieved by setting m value greater than 1 in Equ. (21). When the node moves at low to medium speed, Rayleigh fading can be modeled by setting the m parameter value as 1. A more severe fading can be reflected by setting the m value less than 1 in Equ. (21). As such, the proposed model is a perfect fit for BER estimation in WiMAX communication at vehicular communications. Our future works will investigate the changes of Nakagami parameter m in response to the speed of a moving node.

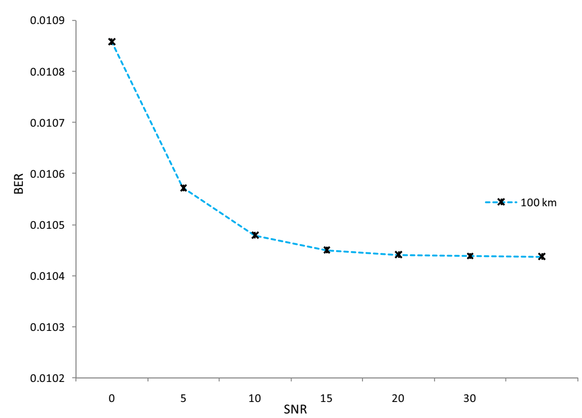

Figure 3. BER at 100 km/h for various SNR values.

VI. Conclusions

In this paper, an analytical model has been proposed that estimates BER in WiMAX communication at vehicular speeds using Nakagami-m fading model. Due to its high estimation accuracy in wireless communications, Nakagami-m model has attracted increasing attentions from researchers in recent years and challenged the popularity of other models such as

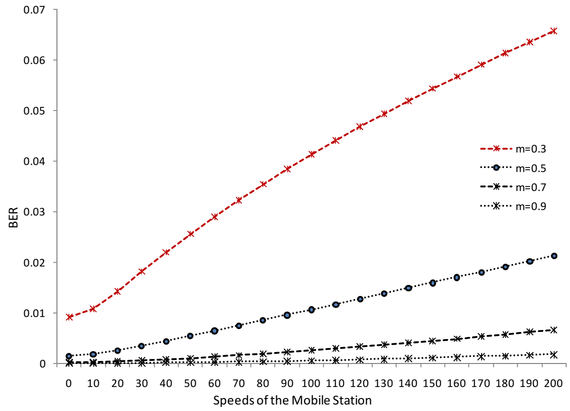

Figure 4. BER at various speeds for various values of m.

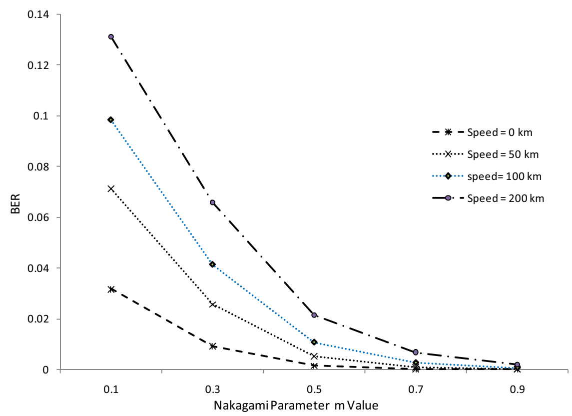

Figure 5. BER for different values of Nakagami parameter m.

Rayleigh model, which warrants an in-depth investigation of Nakagami-m model in 4G technologies. WiMAX, a popular 4G standard, has been fully optimized for fixed communication environments and is capable of delivering high data rates. At vehicular speeds, however, spectral efficiency of WiMAX becomes low mainly due to multipath fading problem and further research is needed for designing resource management schemes at vehicular speeds so that multimedia applications can be supported at high vehicular speeds. A key requirement

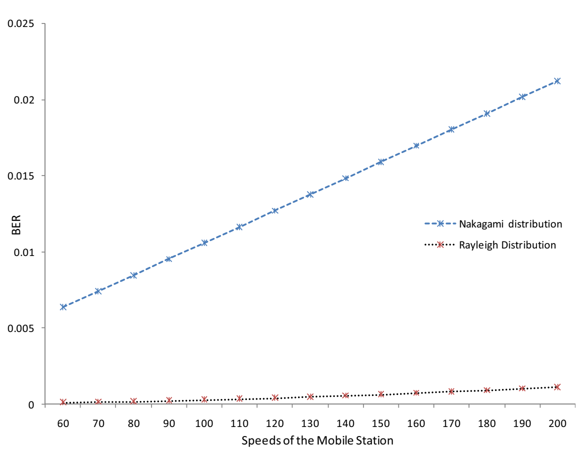

Figure 6. Comparative BER at vehicular speeds between Nakagami-m and Rayleigh model.

for such a resource management scheme is to have an analytical model that can estimate BER at high vehicular speeds so that proactive actions can be taken and proper planning can be done. The proposed analytical model in this paper is adaptive to reflect fading severity at various speeds and is a perfect fit for WiMAX communication. The proposed model can also be used with long term evolution (LTE) down link channel which uses similar OFDMA technique.

References

[1] IEEE, "Draft ieee 802.16m requirments, ieee802.16m broadband wireless access working group, www.ieee802.org/16/tgm/docs/80216m-070-024r.pdf,, last accessed 23 sep 2011."

[2] I. A. Habibi and D., "Improving quality of service in wimax communication at vehicular speeds: A new call admision control solution (in press doi:10.1002/wcm.111),," *Wireless communication and mobile computing,2011*.

[3] P. Lombardo, G. Fedele, and M. Rao, "Mrc performance for binary signals in nakagami fading with general branch correlation," *Communications, IEEE Transactions on*, vol. 47, no. 1, pp. 44 –52, jan 1999.

[4] K.-W. Yip and T.-S. Ng, "A simulation model for nakagami-m fading channels, m lt;1," *Communications, IEEE Transactions on*, vol. 48, no. 2, pp. 214 –221, feb 2000.

[5] R. He and J. Xie, "Ber performance of mqam and mpsk in nakagami fading channel with sttd," in *Personal, Indoor and Mobile Radio Communications, 2003. PIMRC 2003. 14th IEEE Proceedings on*, vol. 2, sept. 2003, pp. 1327 – 1330 vol.2.

[6] O. Alim, H. Abdallah, and A. Elaskary, "Simulation of wimax systems," in *Communications Workshop, 2008. LCW 2008. IEEE Lebanon*, may 2008, pp. 11 –16.

[7] D. H. Lee, S. C. Kim, D. C. Park, and Y. il Kim, "A comparative study of channel estimation for mobile wimax system in high mobility," in *Advanced Communication Technology, 2008. ICACT 2008. 10th International Conference on*, vol. 1, feb. 2008, pp. 781 –785.

[8] I. Ahmad and D. Habibi, "A novel mobile wimax solution for higher throughput," in *Networks, 2008. ICON 2008. 16th IEEE International Conference on*, dec. 2008, pp. 1 –5.

[9] ——, "A proactive forward error control scheme for mobile wimax communication," in *Communication Systems, 2008. ICCS 2008. 11th IEEE Singapore International Conference on*, nov. 2008, pp. 1647 – 1649.

[10] B. Ouarzazi, M. Berbineau, I. Dayoub, and A. Menhaj-Rivenq, "Channel estimation of ofdm system for high data rate communications on mobile environments," in *Intelligent Transport Systems Telecommunications,(ITST),2009 9th International Conference on*, 2009, pp. 425–429.

[11] M. Aguado, E. Jacob, J. Matias, C. Conde, and M. Berbineau, "Deploying cctv as an ethernet service over the wimax mobile network in the public transport scenario," in *Communications Workshops, 2009. ICC Workshops 2009. IEEE International Conference on*, 2009, pp. 1–5.

[12] S. P. Vanja Subotic, "Ber analysis of equalized ofdm systems in nakagami, m < 1 fading," *Wireless Personal Communications*, vol. Volume 40, Number 3, 281-290,, no. Number 3, pp. 281–290, 2006.

[13] D. Zheng, J. Cheng, and N. C. Beaulieu, "Accurate error-rate performance analysis of ofdm on frequency-selective nakagami-m fading channels," *Communications, IEEE Transactions on*, vol. 54, no. 2, pp. 319–328, 2006.

[14] V. Aalo, "Performance of maximal-ratio diversity systems in a correlated nakagami-fading environment," *Communications, IEEE Transactions on*, vol. 43, no. 8, pp. 2360 –2369, aug 1995.

[15] G. Rui and L. Ji-lin, "Ber performance analysis of rcpc encoded mimo-ofdm in nakagami-m channels," in *Information Acquisition, 2006 IEEE International Conference on*, 2006, pp. 1416–1420.

[16] V. K. Dwivedi and G. Singh, "Improved ber analysis of ofdm communication system on correlated nakagami-m fading channel," in *Recent Advances in Microwave Theory and Applications, 2008. MICROWAVE 2008. International Conference on*, 2008, pp. 536–539.

[17] N. Sood, A. K. Sharma, and M. Uddin, "Ber performance of ofdm-bpsk and-qpsk over nakagami-m fading channels," in *Advance Computing Conference (IACC), 2010 IEEE 2nd International*, 2010, pp. 88–90.

[18] "Ieee standard for local and metropolitan area networks part 16: Air interface for broadband wireless access systems amendment 3: Advanced air interface," *IEEE Std 802.16m-2011(Amendment to IEEE Std 802.16-2009)*, pp. 1 –1112, 5 2011.