Enhanced DFT-based Channel Estimation for LTE Uplink

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***Abstract*—Discrete Fourier transform (DFT) based channel estimation (CE) has been widely studied as a practical CE scheme over the OFDM based wireless systems. The conventional DFT-based channel estimation utilizes a transform domain cut-off filter to suppress the noise in the time domain. However, this method can suffer significant performance loss due to the channel impulse response (CIR) energy leakage, especially when the available pilot sub-carriers are confined to a small portion of the system bandwidth. In this paper, we propose an enhanced DFT-based channel estimation technique for the long term evolution (LTE) based cellular uplink. A sinc-null based noise power estimation method in conjunction with a dynamic noise removal technique is proposed to suppress the noise in the time domain and achieve better performance while keeping the complexity in check. Simulation results show that the proposed scheme not only achieves better mean square error (MSE) and block error rate (BLER) performance but also exhibits robustness to timing offsets compared to existing DFT based CE schemes.**

I. Introduction

In orthogonal frequency-division multiplexing (OFDM) based wireless systems, minimum mean squared error (MMSE) and least squares (LS) criteria have been widely employed for frequency domain channel estimation [1], [2]. MMSE based CE has a better performance but at the cost of high complexity which is incurred due to matrix inversion. In addition, its requires an a-priori knowledge of second order channel statistics and the operating signal to noise ratio (SNR). On the other hand, the LS based CE has low complexity but it suffers performance degradation especially at low SNR due to its negligence of noise and channel statistics. Recently, other channel estimation methods based on approximated MMSE, discrete cosine transform (DCT), singular value decomposition (SVD) [3]–[5] have been proposed to achieve a better complexity and performance tradeoff. Unfortunately, the complexity is still much higher compared to the LS based estimation. In this work, our focus is on designing a channel estimator suitable for practical implementation in an LTE uplink and we thus propose enhanced schemes building upon on the low complexity LS based scheme.

The DFT-based channel estimation is also referred to as a transform domain (TD) CE technique which denoises the LS estimates in the transform domain (time domain) [6]–[8]. In [8], a transform domain cut-off filter with its passband length set as the cyclic prefix (CP) length is applied in the time domain to retain the useful time domain samples and set the samples outside "energy concentration" region to be zero. This is based on the fact that in OFDM systems the symbol length is much longer than the maximum channel delay tap. In [6], [7], the noise within the "energy concentration" region is further suppressed by removing the insignificant channel coefficients whose amplitudes are smaller than a threshold, which in turn is determined by the estimated average noise power. Therefore a properly designed threshold is decisive for the noise suppression and final channel estimation performance. In [2], [6], the noise power is estimated by averaging transform domain samples (deemed to have insignificant channel coefficients) that are located in a "noise-only" region.

There are two drawbacks of the existing DFT-based CE schemes. The first one is the performance loss due to a hard cut-off window which essentially ignores the CIR energy leaked into the "noise-only" region. This is especially problematic for small window sizes and indeed results in a severe mean square error (MSE) error floor [9]. The second one is due to the inaccurate noise power estimation leading to removal of useful CIR samples within the "energy concentration" region. This results in further MSE performance loss. [10] proposes an improved scheme to estimate the in-band noise variance and uses it for an approximated MMSE based CE. However, this method entails a higher complexity and its performance is also susceptible to timing offsets.

In this paper, we propose an enhanced DFT-based CE to overcome above mentioned drawbacks while keeping the advantage of low complexity. Our major contribution is a novel method for noise power estimation by averaging over the time domain samples in the vicinity of the nulls of a sinc function, which carry the least contribution from the useful CIR. Then, a dynamic noise removal windowing technique is devised based on the estimated noise power. In the LTE uplink, demodulation reference signals (DMRS) occupy contiguous resource blocks (RBs) which are exactly those allocated to data and control channels for each user. Note that the RB allocation for a given user equipment (UE) is generally only a small portion of the overall uplink bandwidth[1]. Small RB allocation to a UE introduces severe CIR energy leakage and leads to an MSE error floor [9] for the conventional DFT-based CE method. In contrast, our proposed method is particularly efficient in the small RB allocation regime and not only provides an accurate

[1]In [11], it has been observed that, the average numbers of RBs allocated to each UE per transmit time interval (TTI) is 4.1 RBs for a 20MHz LTE system with 30 active UEs and 2.6 RBs for a system with 50 active UEs respectively. This indicates that a small RB allocation is quite common in practical LTE uplink.

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noise power estimation but also exhibits robust performance in the presence of timing offsets.

The remainder of this paper is organized as follows. Section II introduces the LTE uplink channel model and the conventional DFT-based CE techniques. In Section III, the enhanced DFT-based CE is elaborated with a novel noise power estimation method and a dynamic noise removal filter. A robust scheme resisting the degradation caused by timing offsets is also presented. Section IV compares the MSE performance between the proposed scheme and the other existing DFT-based CE schemes. Finally, the concluding remarks are made in Section V.

II. System model and conventional channel estimation for LTE uplink

A. System model

In LTE uplink, DMRS is transmitted as pilot signal to perform channel estimation for coherent demodulation of uplink data and/or control signaling. The DMRS is sent at the four-th and ten-th OFDM symbols in each TTI (consisting of two slots with seven OFDM symbols in each slot for a normal CP) and it occupies the same RB resources as those allocated for data transmission of each UE. The UEs are orthogonally separated in frequency domain due to single carrier frequency domain multiple access (SC-FDMA) so the channel estimation can be performed independently for each UE. Unlike the data signal, the reference signal will not pass through the DFT spread block.

Supposing that the CP length N_c is longer than CIR length L, the received signal at eNodeB (after CP removal) in a symbol interval can be expressed as

$$y(n) = (h \otimes x)(n) + w(n), n = 0, 1, ..., N-1. \qquad (1)$$

where n indicates time domain sample index and $\otimes$ denotes the circular convolution operator. $\{h(\ell)\}_{\ell=0}^{L-1}$ denotes the CIR and $w(n)$ the independently and identically distributed (i.i.d) additive white Gaussian noise (AWGN) in the time domain with zero mean and variance σ_w^2.

Next, the frequency domain received signal of DMRS sequence at subcarrier k is given by

$$Y(k) = H(k)C(k) + W(k), \qquad (2)$$

where $C(k)$ is the k-th sample taken from a Zadoff-Chu sequence [12] with unit power and perfect auto-correlation property. $H(k)$ is the channel frequency response (CFR) at the k-th tone. $W(k)$ is additive noise in frequency domain.

A low complexity channel estimation based on LS criteria can be obtained for each DMRS sub-carrier [1]

$$\hat{H}_{LS}(k) = \frac{Y(k)}{C(k)} = H(k) + \frac{W(k)}{C(k)}. \qquad (3)$$

The LS CE results in an unacceptable MSE especially at low SNR region. DFT-based CE has been widely studied [6]–[8], [13] to combat the noise degradation in LS CE.

B. DFT-based CE with transform domain cut-off filter

The conventional DFT-based channel estimation [6], [7], [13] exploits the feature of OFDM systems having symbol length much longer than the length of CIR. For each UE, the LS estimates are first extended to a size-N block by padding zeros at the unallocated tones, i.e.

$$\hat{H}_{ext}(k) = \begin{cases} \hat{H}_{LS}(k), & k \in \mathcal{S}, \\ 0, & k \notin \mathcal{S}, 0 \le k \le N-1 \end{cases} \qquad (4)$$

where $\mathcal{S}$ denotes the contiguous chunk of sub-carriers allocated to a UE. The extended block is then transformed via a size-N Inverse DFT (IDFT) to obtain transform domain or time domain estimates $\hat{h}_{LS}(n)$.

$$\hat{h}_{LS}(n) = \frac{1}{N}\sum_{k=0}^{N-1}\hat{H}_{ext}(k)e^{j2\pi\frac{nk}{N}}, \quad 0 \le n \le N-1 \qquad (5)$$

A de-noising filter is then applied in time domain to reduce noise. The transform domain cut-off filter $w_{CF}(n)$ can be designed by simply keeping the transform domain samples at the "energy concentration" region as useful CIR samples and setting the samples at the "noise-only" region to be zeros [8], i.e.,

$$w_{CF}(n) = \begin{cases} 1, & 0 \le n \le f_c - 1, N - f_c \le n \le N-1 \\ 0, & \text{otherwise} \end{cases} \qquad (6)$$

where f_c is the "cut-off" point of the transform domain filter. f_c is commonly chosen as channel length L or CP length N_c if there is no knowledge about channel length L [2], [8]. Note that $N - 2f_c$ samples have been removed by this hard cut-off filtering, which might also contain useful CIR information smearing into the "noise-only" region. The transform domain estimates after noise-removing are then given by

$$\hat{h}_{nr}(n) = w_{CF}(n)\hat{h}_{LS}(n), \quad 0 \le n \le N-1 \qquad (7)$$

Lastly, the time domain filtered samples are transformed via a DFT block to get the final channel estimates back in frequency domain.

$$\hat{H}_{DFT}(k) = \sum_{n=0}^{N-1}\hat{h}_{nr}(n)e^{-j2\pi\frac{nk}{N}}, \quad 0 \le k \le N-1 \qquad (8)$$

One additional improvement is to further suppress the noise effect in the transform domain by comparing time domain estimates' powers with a threshold determined by the estimated noise power [7]. Thus the noise removal filter can be further updated as (9). In (9), $\hat{\sigma}_n^2$ denotes the estimated noise power and α a scaling factor that can be adjusted as a noise margin.

Assuming all the samples outside the energy concentration region contain noise only, [6] and [7] apply a noise power estimator by averaging the samples located in a "noise-only" region, as is given by

$$\hat{\sigma}_n^2 = \frac{1}{N-2N_c}\sum_{n=N_c}^{N-N_c-1}|\hat{h}_{LS}(n)|^2. \qquad (10)$$

Thus the final channel estimates in frequency domain with

$$w_{CFNR}(n) = \begin{cases} 1, & |\hat{h}_{LS}(n)|^2 \geq \alpha\hat{\sigma}_n^2, \quad 0 \leq n \leq N_c - 1, \quad N - N_c \leq n \leq N - 1 \\ 0, & |\hat{h}_{LS}(n)|^2 < \alpha\hat{\sigma}_n^2, \quad 0 \leq n \leq N_c - 1, \quad N - N_c \leq n \leq N - 1 \\ 0, & N_c \leq n < N - N_c \end{cases} \tag{9}$$

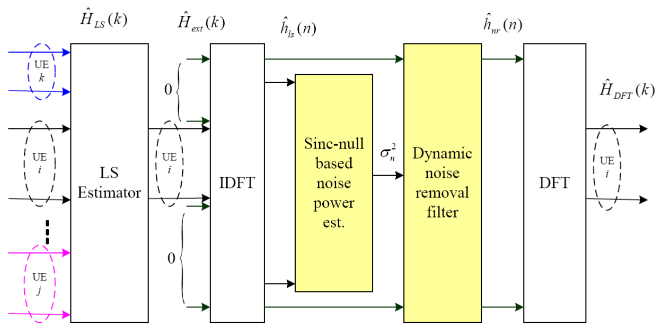

Fig. 1. Proposed enhanced DFT-based channel estimation.

transform domain cut-off filter and in-band noise removal can be obtained as

$$\hat{H}_{CFNR}(k) = \sum_{n=0}^{N-1} w_{CFNR}(n)\hat{h}_{LS}(n)e^{-j2\pi\frac{nk}{N}}, \quad k \in \mathcal{S}. \tag{11}$$

Note that the above DFT-based CE method does not require any information about channel. And DFT/IDFT units are available blocks in the system. Thus it has very low implementation complexity.

III. Enhanced DFT-based channel estimation

The conventional method resists noise interference by applying a cut-off filter and an in-band noise suppression. This method is effective when all sub-carriers (after interpolation) in an OFDM symbol are assigned for channel estimation since there is no need for zero-padding extension and the channel taps with detectable energy usually fall into the CP region. However, for SC-FDMA in LTE uplink with large number of users, the reference signals are transmitted in localized chunks consisting of a relatively small number of RBs. This method suffers significant performance loss due to CIR energy leakage.

A. Enhanced noise power estimation and noise removal windowing

Fig. 1 illustrates a block diagram of DFT-based CE with proposed enhancements (noise power estimation and noise removal windowing) in the transform domain. Note that each UE will perform the DFT-based CE independently. Without loss of generality, we assume the first $M = 12 * n_{RB}$ tones are allocated to a current UE of interest, where n_{RB} is the number of allocated RBs with 12 sub-carriers per RB. Note that the zero-padding in (4) imposes a rectangular windowing W_f in frequency domain, i.e.,

$$W_f = \begin{cases} 1, & 0 \leq k \leq M-1, \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

Thus the resulting transform domain channel estimate $\hat{h}_{LS}(n)$ is equal to the convolutional output of the CIR $h(n)$ and the time domain response of W_f plus a colored noise term, i.e.,

$$\hat{h}_{LS}(n) = (h \otimes g_W)(n) + \varepsilon(n), \quad 0 \leq n \leq N-1, \tag{13}$$

where g_W is the time domain response of W_f and $\varepsilon(n)$ the residual colored noise.

For any UE with a given RB assignment, g_W is a known sinc function having the nulls occurring every Δn samples where the sinc null set is given by

$$\Omega_0(i) = i * \Delta n, \qquad i = 1, 2, ..., \lfloor \frac{N}{\Delta n} \rfloor, \tag{14}$$

where $\Delta n \triangleq \lfloor \frac{N}{12 * n_{RB}} \rfloor$ and $\lfloor . \rfloor$ is a floor function.

For small RB allocations, Δn is relatively large so that the null points of convolutional output time domain samples are approximately those of the known sinc function itself. Using this fact, we can improve the noise power estimation by averaging the samples in the vicinity of the sinc nulls. Thus the noise power estimation can be improved as

$$\hat{\sigma}_{n1}^2 = \frac{\sum_{n=0}^{N-1} w_{noise}(n)|\hat{h}_{LS}(n)|^2}{\sum_{n=0}^{N-1} w_{noise}(n)}, 0 \leq n < N, \tag{15}$$

where w_{noise} is given by

$$w_{noise}(n) = \begin{cases} 1, & \Omega_0(i) - \lfloor \frac{\Delta n}{\beta} \rfloor \leq n \leq \Omega_0(i) + \lfloor \frac{\Delta n}{\beta} \rfloor, \\ & \qquad i = 1, 2, ..., \lfloor \frac{N}{\Delta n} \rfloor \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

β is a factor determining the number of samples collected near each sinc null point for noise estimation. In our simulations, $\beta = 8$ is chosen with a best performance and complexity tradeoff (Increasing β doesnot result in noticeable performance gain).

With the estimated noise power, we now eliminate the noise in transform domain by applying a dynamic noise removal window (instead of a hard boundary cut-off filter) based on

$$w_{NR1}(n) = \begin{cases} 1, & |\hat{h}_{LS}(n)|^2 \geq \alpha\hat{\sigma}_{n1}^2 \\ 0, & |\hat{h}_{LS}(n)|^2 < \alpha\hat{\sigma}_{n1}^2 \end{cases}, \quad 0 \leq n \leq N-1 \tag{17}$$

Similarly, α is a scaling factor that can be adjusted as a noise margin.

After suppressing the insignificant channel coefficients, the noise-removed channel coefficients are converted into frequency domain channel estimates given by

$$\hat{H}_{DFT1}(k) = \sum_{n=0}^{N-1} w_{NR1}(n)\hat{h}_{LS}(n)e^{-j2\pi\frac{nk}{N}}, \quad k \in \mathcal{S}. \tag{18}$$

B. Enhanced channel estimation resisting timing offset

In above sinc-null based noise power estimation method, we assume perfect timing synchronization in LTE uplink. Timing offset in time domain introduces a phase ramp effect over the tones or equivalently selectivity in frequency domain [14]. Assuming there is a $\tilde{\theta}$-sample offset, the LS estimates over the allocated RBs are given by

$$\hat{H}_{LS}(k) = e^{\frac{-j2\pi k\tilde{\theta}}{N}}H(k) + \frac{W(k)}{C(k)}. \tag{19}$$

This phase ramp can be absorbed in the RB allocation window function so that its corresponding time domain signal is a shifted sinc function. Further, for small RB allocation, we can use the null points of this shifted sinc function for noise power estimation. However, since the shift is not known we instead use a moving window technique to determine a good set of null points for noise power estimation.

Let P denote the total number of moving windows being accumulated where $P = \lfloor \frac{N-2*N_c}{\Delta n} \rfloor$ and θ a sample shift between $-\frac{\Delta n}{2}$ and $\frac{\Delta n}{2}$, i.e. $-\lfloor \frac{\Delta n}{2} \rfloor \leq \theta \leq \lfloor \frac{\Delta n}{2} \rfloor$. The accumulated energy from all moving windows at a θ-sample offset can be expressed as

$$\hat{\sigma}_w^2(\theta) = \frac{\sum_{n=0}^{N-1} w_{noise}(n,\theta)|\hat{h}_{LS}(n)|^2}{\sum_{n=0}^{N-1} w_{noise}(n,\theta)}, 0 \leq n < N, \tag{20}$$

where $w_{noise}(n,\theta)$ indicates a shifted window given by

$$w_{noise}(n,\theta) = \begin{cases} 1, \Omega_0(i) - \lfloor \frac{\Delta n}{\beta} \rfloor + \theta \leq n \leq \Omega_0(i) + \lfloor \frac{\Delta n}{\beta} \rfloor + \theta, \\ \qquad\qquad\qquad\qquad i = 1, 2, ..., P \\ 0, \text{otherwise.} \end{cases} \tag{21}$$

where $\Omega_0(i)$ denotes the i-th null point of the assocated sinc function g_W given by (14). Note that all windows share the same timing offset. Thus the detected offset can be found as

$$\theta^* = \arg \min_{-\lfloor \frac{\Delta n}{2} \rfloor \leq \theta \leq \lfloor \frac{\Delta n}{2} \rfloor} \hat{\sigma}_w^2(\theta). \tag{22}$$

Now the updated noise power estimated with robustness to timing offset can be found as

$$\hat{\sigma}_{n2}^2 = \hat{\sigma}_w^2(\theta^*) \tag{23}$$

Finally, the channel estimates for current DMRS signal can be obtained by performing the dynamic noise removal in (17) albeit using the updated noise power (23) followed by an IDFT operation as given in (18). In practical LTE systems, the channel estimates from the two DMRS signals in each TTI will be combined (such as using equal gain combining) to obtain the final channel state information for coherent demodulation and link adaptation.

Parameter	**Assumption**
Bandwidth	10.0 MHz
Total number of RBs	50
N	1024
N_c CP length	128
Number of antennas at eNode-B	2
Number of antennas of UE	1
Modulation	QPSK,16QAM,
Coding rate	3/4
Channel encoder	$(13, 15)_8$
α noise power offset	4
β sample collecting factor	8
N_{TTI} for MSE evaluation	1000
Channel model	SCM-C

TABLE I
SIMULATION PARAMETERS

IV. PERFORMANCE ANALYSIS AND SIMULATION

We now evaluate the performance of proposed scheme and other DFT-based CE methods. The general simulation parameters are listed in Table I. The MSE performance is evaluated from $N_{TTI} = 1000$ channel realizations based on

$$MSE = \frac{1}{N_{TTI}} \frac{1}{|\mathcal{S}|} \sum_{t=1}^{N_{TTI}} \sum_{k\in\mathcal{S}} \left| \hat{H}(k,t) - H(k,t) \right|^2 \tag{24}$$

where $\hat{H}(k,t)$ is the estimated frequency domain channel based on existing CE and proposed CE methods. $H(k,t)$ is the actual CFR. t is TTI index and k is the tone index.

Fig. 2 illustrates the MSE performance for 1 RB allocation in a system without timing offset. It can be seen that the raw LS estimation has severe MSE at low SNR. And the existing DFT-based CE with transform domain cut-off filter (CF) suffers significant MSE loss (with a error floor). In addition, the in-hand noise removal (CFNR) based on existing noise estimation method gives only sight gain in low SNR region. However the proposed scheme achieves much better MSE performance throughout the SNR region.

In Fig. 3, the MSE performance are compared for the LTE uplink in the presence of timing offset with $\theta = 20$ and $\theta = 60$ samples for two-RB allocations. The proposed scheme exhibits robust performance at different level of timing offset while keeping the best MSE performance. In addition, the proposed scheme has no performance loss for large RB allocation compared to existing DFT-based CE, which is not presented due to limitation of space.

In Fig. 4, the link level block error rate (BLER) performance is evaluated based on different channel estimation schemes for $n_{RB} = 1$ without timing offset. The simulation parameters are detailed in Table I. As illustrated in the figure, the proposed scheme shows a better BLER gain compared to LS and existing DFT-based channel estimation attributed to its best MSE performance.

V. CONCLUSIONS

In this paper, we have proposed an enhanced DFT-based channel estimation scheme for LTE uplink. The proposed

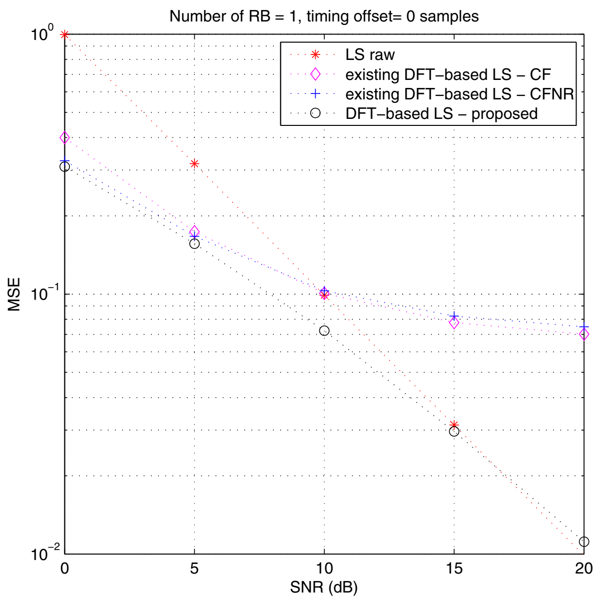

Fig. 2. MSE for 1 RB allocation with perfect synchronization and SCM-C.

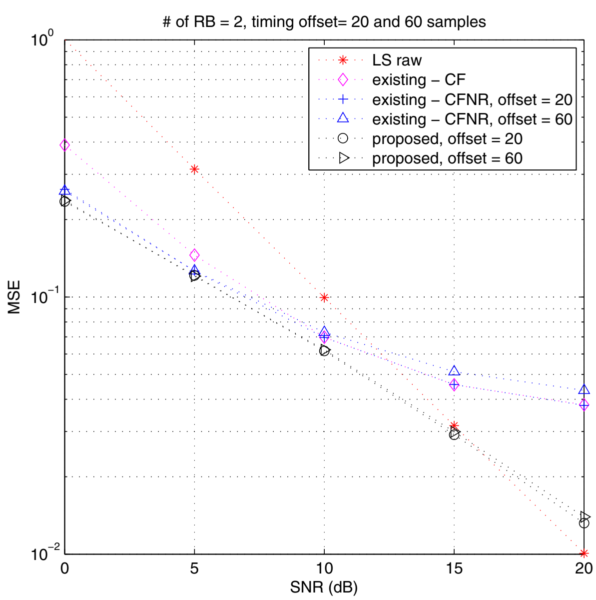

Fig. 3. MSE for 2 RB allocation with timing offset and SCM-C.

scheme is based on a a dynamic noise removal window in conjunction with a robust noise power estimation. It has same complexity with the existing DFT-based CE method with in-band noise removal but exhibits superior MSE and BLER performance at no additional cost. Thus it is quite suitable for practical implementation in LTE or LTE-A systems.

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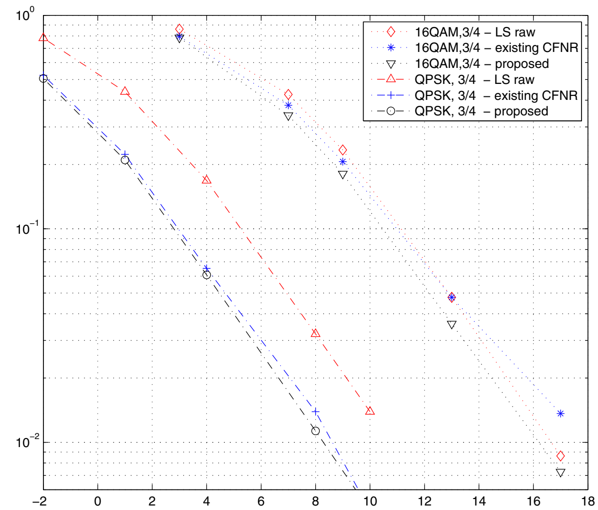

Fig. 4. BLER performance, 16 subcarriers allocation with $n_{RB} \sim 1.3$ RBs and timing offset $\theta = 0$.

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