Decoding of Distributed Alamouti STBC in DF Based Cooperative System

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***Abstract*—In this paper, we derive a maximum-likelihood (ML) decoder for the demodulate-and-forward (DF) based cooperative communication system using Alamouti space-time block code (STBC) in a distributed manner. We also propose a sub-optimal low complexity piece-wise linear (PL) decoder of the distributed Alamouti code in a DF cooperative system in which one relay out of two relays is in outage. The proposed PL decoder does not lead to any significant performance degradation and performs very close to the proposed ML decoder. Moreover, the proposed ML decoder of the DF cooperative system significantly outperforms an amplify-and-forward (AF) based cooperative system when both systems use the same data rate and distributed Alamouti STBC.**

I. Introduction

Cooperative communication provides diversity and throughput advantages of multi-antenna systems by sharing the resources among the single antenna distributed nodes [1], [2]. The data from the source reaches the destination via one or more cooperative agents called the *relays*. The relays can apply different protocols like amplify-and-forward (AF) [3] and demodulate-and-forward (DF) [4]–[6] in a *symbol-wise* manner to the data of the source. In AF protocol, the relay amplifies the data from the source and forwards it to the destination whereas, the *uncoded*[1] data from the source is decoded and forwarded (in uncoded form) to the destination in the DF protocol. Because of the possibility of erroneous relaying, the DF protocol looses the diversity in its pure form [5]. The DF protocol, combined with coding techniques [7], [8] is known as decode-and-forward protocol [9], where the source and relays transmit the *coded* data and employ forward error correction (FEC) coding. It is shown in [4], [6] that the performance of the DF based cooperative system with a source, a destination, and a single relay can be enhanced with the use of a maximum-likelihood (ML) decoder in the destination receiver. An ML decoder of a multi-antenna based DF cooperative system using binary phase-shift keying (BPSK) data is considered in [10], which is an extension of [4].

To achieve diversity gains, the space-time block codes (STBCs) can also be utilized in a *distributed* manner [11]–[13] by the multi-relay cooperative systems, where each row of the STBC is transmitted by a different relay using non-orthogonal transmission. However, erroneous transmissions from the relays in DF protocol spoil the well designed structure of STBC. A distributed STBC transmission with relay selection strategy is considered in [9] for uncoded DF cooperative system. It is shown in [13] that a DF based distributed STBC with relay selection performs much worse than the AF protocol based distributed STBC utilizing uncoded transmissions.

In this paper, our main contributions are: 1) We derive an ML decoder for the *distributed* Alamouti STBC [14] in a DF based cooperative system utilizing an arbitrary M-point constellation. 2) A sub-optimal piece-wise linear (PL) decoder is also proposed for the DF cooperative system with one out of two relays in outage. 3) It is shown by simulations that the proposed ML decoder of distributed Alamouti STBC in the DF protocol based system significantly outperforms the distributed Alamouti code with AF protocol.

II. System Model

Let us consider a cooperative system containing a single pair of source (S) and destination (D) node with two relays R_1 and R_2. Each node is equipped with single antenna and works in half-duplex mode. It is assumed that the S-D link is very poor, hence, direct transmission between the source and the destination is not possible. The source takes help of two relays for transmitting its *uncoded* data to the destination. Transmission from the source to the destination can now be decoupled into two orthogonal phases. In Phase I, the source transmits uncoded data sequentially to both relays. The relays symbol-wise demodulate the data of the source and transmit Alamouti STBC in a distributed manner to the destination in Phase II. In this phase, the source remains silent. The destination decodes the data by utilizing an ML decoder. It is assumed that the channel gains of all links involved in cooperation follow a Rayleigh block fading model [15] and remain constant for a block of at least two consecutive time intervals.

Let $s_1, s_2 \in \mathcal{A}$, where $\mathcal{A}$ is a complex-valued M-point constellation, be the two *uncoded* symbols transmitted by the source in a block[2]. The transmission of $\boldsymbol{s} = [s_1, s_2]$

[1] Here from uncoded we mean that without forward error correction (FEC) coding.

[2] Since each block is independent of other because of the block fading model, we skip the notation for a block in the article.

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to the destination is performed in two decoupled phases. In Phase I, the received data $\boldsymbol{y}_m = [y_{m,1}, y_{m,2}]$ at the m-th relay, $m \in \{1, 2\}$ can be written as

$$\boldsymbol{y}_m = h_m \boldsymbol{s} + \boldsymbol{e}_m, \tag{1}$$

where $y_{m,n}$, $n = 1, 2$, denotes the signal received in the m-th relay in the n-th time interval in a block, $h_m \sim \mathcal{CN}(0, \sigma_m^2)$ is the channel gain of the S-R$_m$ link, and $\boldsymbol{e}_m = [e_{m,1}, e_{m,2}]$ denotes the complex-valued additive white Gaussian noise (AWGN) vector with each element having zero mean and N_0 variance.

The relays demodulate the symbols transmitted by the source by using an ML demodulator and transmit the Alamouti STBC in a distributed manner in two consecutive time intervals in Phase II. It is further assumed that the transmissions from the relays are perfectly synchronized. Let $\hat{x}_{n,m}$ is the estimated symbol by the m-th relay in the n-th time interval, then we can write the data received at the destination in the two consecutive time-intervals as

$$\begin{aligned} y_1 &= f_1 \hat{x}_{1,1} + f_2 \hat{x}_{2,2} + z_1, \\ y_2 &= -f_1 \hat{x}_{2,1}^* + f_2 \hat{x}_{1,2}^* + z_2, \end{aligned} \tag{2}$$

where $f_m \sim \mathcal{CN}(0, \Omega_m^2)$, $m = 1, 2$ is the channel gain of the R$_m$-D link, and $\boldsymbol{z}_d = [z_1, z_2]$ is a 1×2 row vector containing zero mean AWGN noise elements with N_1 variance. It is assumed that f_m remains constant over the transmission period of one distributed Alamouti block. We can rewrite (2) as follows:

$$\boldsymbol{y}_d = [f_1, f_2] \begin{bmatrix} \hat{x}_{1,1} & -\hat{x}_{2,1}^* \\ \hat{x}_{2,2} & \hat{x}_{1,2}^* \end{bmatrix} + \boldsymbol{z}_d, \tag{3}$$

where $\boldsymbol{y}_d = [y_1, y_2]$ is a 1×2 received data vector. From (3), we can observe that the distributed Alamouti STBC in the DF cooperative system will be

$$\boldsymbol{S} = \begin{bmatrix} \hat{x}_{1,1} & -\hat{x}_{2,1}^* \\ \hat{x}_{2,2} & \hat{x}_{1,2}^* \end{bmatrix}. \tag{4}$$

It can be seen from (4), that when $\hat{x}_{n,i} \neq \hat{x}_{n,j}$, where $n, i, j \in \{1, 2\}$, and $i \neq j$, then $\boldsymbol{S}\boldsymbol{S}^H$ is not necessarily proportional to the identity matrix. Therefore, $\boldsymbol{S}$ is *not* necessarily an orthogonal STBC when the relays commit error in demodulation of the symbols transmitted by the source.

III. ML Decoder of Distributed Alamouti STBC

We assume that the relays utilize an ML decoder for demodulating the symbols. The ML demodulator of arbitrary constellations over co-located MIMO links is a well established concept [16]. Next, we will derive the ML decoder of the distributed Alamouti code in the destination.

A. ML Decoder in the Destination Node

An ML decoder of the symbols s_1, s_2 can be obtained by maximizing the conditional probability density function (p.d.f.) of the received data vector $\boldsymbol{y}_d$, given in (3), in the destination. It is equivalent to maximizing a likelihood ratio for decoding of the data [17]. By using the analysis given in [17, Section 2.3], it can be shown that the destination needs to find the following log likelihood ratio (LLR) to decide between the two possible source's transmissions $\boldsymbol{x}_p = [x_{p_1}, x_{p_2}]$ and $\boldsymbol{x}_q = [x_{q_1}, x_{q_2}]$, where $x_{p_1}, x_{p_2}, x_{q_1}, x_{q_2} \in \mathcal{A}$, and $p_1, p_2, q_1, q_2 = 1, 2, ..., M$

$$\Lambda^d_{p_1,p_2,q_1,q_2} = \ln \left(\frac{p_{\boldsymbol{y}_d | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,1}, \hat{x}_{1,2}, \hat{x}_{2,1}, \hat{x}_{2,2}\} \in \mathcal{A}^4}}{p_{\boldsymbol{y}_d | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_q, \{\hat{x}_{1,1}, \hat{x}_{1,2}, \hat{x}_{2,1}, \hat{x}_{2,2}\} \in \mathcal{A}^4}} \right), \tag{5}$$

where $p_{\boldsymbol{y}_d | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,1}, \hat{x}_{1,2}, \hat{x}_{2,1}, \hat{x}_{2,2}\} \in \mathcal{A}^4}$ is the conditional joint p.d.f. of the received data vector $\boldsymbol{y}_d$ given that channels of the S-R$_1$, S-R$_2$, R$_1$-D, and R$_2$-D links, the symbols transmitted by the source s_1, s_2, and the symbols transmitted by the relay $\hat{x}_{1,1}, \hat{x}_{1,2}, \hat{x}_{2,1}, \hat{x}_{2,2}$ are perfectly known at the destination. Let $\boldsymbol{p} = [p_1, p_2]$ and $\boldsymbol{q} = [q_1, q_2]$, then in (5), p_i and q_i, $i = 1, 2$ should be chosen such that $\boldsymbol{p} \neq \boldsymbol{q}$ i.e., the vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are different in at least one component. The LLR of (5) is used to decide about $\boldsymbol{x}_p$ or $\boldsymbol{x}_q$ as follows:

$$\Lambda^d_{\boldsymbol{p},\boldsymbol{q}} \underset{\boldsymbol{x}_q}{\overset{\boldsymbol{x}_p}{\gtrless}} 0. \tag{6}$$

Since the AWGN noises z_1 and z_2 are independent of each other, y_1 and y_2 are independent of each other when h_m, f_m, s_n, and $\hat{x}_{n,m}$ are known for all m, n. Therefore,

$$\begin{aligned} &p_{\boldsymbol{y}_d | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,1}, \hat{x}_{1,2}, \hat{x}_{2,1}, \hat{x}_{2,2}\} \in \mathcal{A}^4} \\ &\quad = p_{y_1 | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,1}, \hat{x}_{2,2}\} \in \mathcal{A}^2} \\ &\quad \times p_{y_2 | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,2}, \hat{x}_{2,1}\} \in \mathcal{A}^2}. \end{aligned} \tag{7}$$

Depending on the channel quality of the S-R$_1$ and S-R$_2$ links, there exist the following four possibilities: 1) R$_1$ demodulates the data erroneously, and R$_2$ takes a correct decision. 2) R$_2$ demodulates the data erroneously, and R$_1$ takes a correct decision. 3) R$_1$ and R$_2$ both take wrong decisions. 4) R$_1$ and R$_2$ both demodulate the data correctly.

Let ϵ_1 and ϵ_2 be the uncoded instantaneous probability of errors of decoding an M point constellation in R$_1$ and R$_2$, respectively, then considering the four cases discussed above we can write

$$\begin{aligned} &p_{y_1 | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,1}, \hat{x}_{2,2}\} \in \mathcal{A}^2} \\ &\quad = \epsilon_1 (1 - \epsilon_2)\, p_{y_1 | f_1, f_2, \hat{x}_{1,1} \neq x_{p_1}, \hat{x}_{2,2} = x_{p_2}} \\ &\quad + (1 - \epsilon_1)\, \epsilon_2 p_{y_1 | f_1, f_2, \hat{x}_{1,1} = x_{p_1}, \hat{x}_{2,2} \neq x_{p_2}} \\ &\quad + \epsilon_1 \epsilon_2 p_{y_1 | f_1, f_2, \hat{x}_{1,1} \neq x_{p_1}, \hat{x}_{2,2} \neq x_{p_2}} \\ &\quad + (1 - \epsilon_1)(1 - \epsilon_2)\, p_{y_1 | f_1, f_2, \hat{x}_{1,1} = x_{p_1}, \hat{x}_{2,2} = x_{p_2}}. \end{aligned} \tag{8}$$

Similarly,

$$\begin{aligned} &p_{y_2 | f_1, f_2, h_1, h_2, \boldsymbol{s}=\boldsymbol{x}_p, \{\hat{x}_{1,2}, \hat{x}_{2,1}\} \in \mathcal{A}^2} \\ &\quad = \epsilon_1 (1 - \epsilon_2)\, p_{y_2 | f_1, f_2, \hat{x}_{1,2} = x_{p_1}, \hat{x}_{2,1} \neq x_{p_2}} \\ &\quad + (1 - \epsilon_1)\, \epsilon_2 p_{y_2 | f_1, f_2, \hat{x}_{1,2} \neq x_{p_1}, \hat{x}_{2,1} = x_{p_2}} \\ &\quad + \epsilon_1 \epsilon_2 p_{y_2 | f_1, f_2, \hat{x}_{1,2} \neq x_{p_1}, \hat{x}_{2,1} \neq x_{p_2}} \\ &\quad + (1 - \epsilon_1)(1 - \epsilon_2)\, p_{y_2 | f_1, f_2, \hat{x}_{1,2} = x_{p_1}, \hat{x}_{2,1} = x_{p_2}}. \end{aligned} \tag{9}$$

Since z_1 and $z_2 \sim \mathcal{CN}(0, N_1)$, we have

$$\begin{aligned} p_{y_1 | f_1, f_2, \hat{x}_{1,1} = x_{p_1}, \hat{x}_{2,2} = x_{p_2}} &= \frac{1}{\pi N_1} e^{-\frac{1}{N_1} \left| y_1 - f_1 x_{p_1} - f_2 x_{p_2} \right|^2}, \\ p_{y_2 | f_1, f_2, \hat{x}_{1,2} = x_{p_1}, \hat{x}_{2,1} = x_{p_2}} &= \frac{1}{\pi N_1} e^{-\frac{1}{N_1} \left| y_2 + f_1 x_{p_2}^* - f_2 x_{p_1}^* \right|^2}. \end{aligned} \tag{10}$$

From [18, Section III], it can be deduced that p.d.f.s $p_{y_1|f_1,f_2,\hat{x}_{1,1}\neq x_{p_1},\hat{x}_{2,2}=x_{p_2}}$, $p_{y_1|f_1,f_2,\hat{x}_{1,1}=x_{p_1},\hat{x}_{2,2}\neq x_{p_2}}$, and $p_{y_1|f_1,f_2,\hat{x}_{1,1}\neq x_{p_1},\hat{x}_{2,2}\neq x_{p_2}}$ denote the p.d.f.s of Gaussian mixture random variables. With these observations, it follows that:

$$
\begin{aligned}
p_{y_1|f_1,f_2,\hat{x}_{1,1}\neq x_{p_1},\hat{x}_{2,2}=x_{p_2}} &= k_1 \sum_{\substack{l=1,\\ l\neq p_1}}^{M} e^{-\frac{1}{N_1}\left|y_1-f_1x_l-f_2x_{p_2}\right|^2},\\
p_{y_1|f_1,f_2,\hat{x}_{1,1}=x_{p_1},\hat{x}_{2,2}\neq x_{p_2}} &= k_1 \sum_{\substack{l=1,\\ l\neq p_2}}^{M} e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_l\right|^2},\\
p_{y_1|f_1,f_2,\hat{x}_{1,1}\neq x_{p_1},\hat{x}_{2,2}\neq x_{p_2}} &= k_2 \sum_{\substack{l=1,\\ l\neq p_1}}^{M}\sum_{\substack{k=1,\\ k\neq p_2}}^{M} e^{-\frac{1}{N_1}|y_1-f_1x_l-f_2x_k|^2}.
\end{aligned}
\tag{11}
$$

where $k_1 = 1/[\pi N_1(M-1)]$, and $k_2 = k_1/(M-1)$. Similarly, conditional p.d.f.s of y_2 can be obtained. The ML decoder for distributed Alamouti STBC can be obtained from (5) -(11) and with help of conditional p.d.f.s of y_2 as follows:

$$
\Lambda^d_{\boldsymbol{p},\boldsymbol{q}} = \ln\left(\frac{a_1+a_2+a_3+a_4}{b_1+b_2+b_3+b_4}\right) + \ln\left(\frac{c_1+c_2+c_3+c_4}{d_1+d_2+d_3+d_4}\right), \tag{12}
$$

where

$$
\begin{aligned}
a_1 &= \epsilon_1(1-\epsilon_2)(M-1)\sum_{\substack{l=1,\\ l\neq p_1}}^{M} e^{-\frac{1}{N_1}\left|y_1-f_1x_l-f_2x_{p_2}\right|^2},\\
a_2 &= (1-\epsilon_1)\epsilon_2(M-1)\sum_{\substack{l=1,\\ l\neq p_2}}^{M} e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_l\right|^2},\\
a_3 &= \epsilon_1\epsilon_2\sum_{\substack{l=1,\\ l\neq p_1}}^{M}\sum_{\substack{k=1,\\ k\neq p_2}}^{M} e^{-\frac{1}{N_1}|y_1-f_1x_l-f_2x_k|^2},\\
a_4 &= (1-\epsilon_1)(1-\epsilon_2)(M-1)^2 e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{p_2}\right|^2},
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
c_1 &= \epsilon_1(1-\epsilon_2)(M-1)\sum_{\substack{l=1,\\ l\neq p_1}}^{M} e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2x^*_l\right|^2},\\
c_2 &= (1-\epsilon_1)\epsilon_2(M-1)\sum_{\substack{l=1,\\ l\neq p_2}}^{M} e^{-\frac{1}{N_1}\left|y_2+f_1x^*_l-f_2x^*_{p_1}\right|^2},\\
c_3 &= \epsilon_1\epsilon_2\sum_{\substack{l=1,\\ l\neq p_1}}^{M}\sum_{\substack{k=1,\\ k\neq p_2}}^{M} e^{-\frac{1}{N_1}|y_2+f_1x^*_k-f_2x^*_l|^2},\\
c_4 &= (1-\epsilon_1)(1-\epsilon_2)(M-1)^2 e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2x^*_{p_1}\right|^2},
\end{aligned}
\tag{14}
$$

and b_i's, d_i's, $i = 1, 2, 3, 4$ can be obtained by substituting $\{q_1, q_2\}$ in place of $\{p_1, p_2\}$ in (13) and (14), respectively.

Remark 1: *It can be seen from* (12) *that the ML decoding of the data transmitted by the source is performed jointly. Since the distributed Alamouti STBC $\boldsymbol{S}$, in DF based cooperative system is not necessarily an orthogonal design under the estimation errors in relays, decoupled decoding of the symbols is not possible.*

Further, it can be noticed from (12) that for decoding the data transmitted by the source, the destination requires the knowledge of the uncoded instantaneous error probabilities ϵ_1 and ϵ_2. In practice, the values of h_1 and h_2 can be forwarded by the relays to the destination and the destination can calculate the values of ϵ_1 and ϵ_2.

For BPSK constellation ($M = 2$), the ML decoder of (12) gets simplified into

$$
\Lambda^d_{\boldsymbol{p},\boldsymbol{q}} = \ln\left(\frac{A_p}{A_q}\right) + \ln\left(\frac{B_p}{B_q}\right), \tag{15}
$$

where

$$
\begin{aligned}
A_p &= \epsilon_1(1-\epsilon_2)e^{-\frac{1}{N_1}\left|y_1-f_1\bar{x}_{p_1}-f_2x_{p_2}\right|^2}\\
&\quad + (1-\epsilon_1)\epsilon_2 e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2\bar{x}_{p_2}\right|^2}\\
&\quad + \epsilon_1\epsilon_2 e^{-\frac{1}{N_1}\left|y_1-f_1\bar{x}_{p_1}-f_2\bar{x}_{p_2}\right|^2}\\
&\quad + (1-\epsilon_1)(1-\epsilon_2)e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{p_2}\right|^2},\\
B_p &= \epsilon_2(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2\bar{x}^*_{p_1}\right|^2}\\
&\quad + (1-\epsilon_2)\epsilon_1 e^{-\frac{1}{N_1}\left|y_2+f_1\bar{x}^*_{p_2}-f_2x^*_{p_1}\right|^2}\\
&\quad + \epsilon_1\epsilon_2 e^{-\frac{1}{N_1}\left|y_2+f_1\bar{x}^*_{p_2}-f_2\bar{x}^*_{p_1}\right|^2}\\
&\quad + (1-\epsilon_1)(1-\epsilon_2)e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2x^*_{p_1}\right|^2},
\end{aligned}
\tag{16}
$$

and A_q, B_q can be found by replacing the variables $\{p_1, p_2\}$ with $\{q_1, q_2\}$ in the expressions of A_p, B_p, respectively. Also, in (16), $\bar{x}_k = -x_k$, for $k \in \{p_1, p_2, q_1, q_2\}$.

Remark 2: *If the channel between the source and the relays is very good i.e., $\epsilon_1, \epsilon_2 \to 0^+$, then it can be shown after some algebra that the decoder of* (12) *reduces into the ML decoder of the Alamouti code in co-located antennas system [14, Eq. (13)] which provides decoupled decoding of both transmitted symbols.*

IV. ML Decoder in Destination with a Single Relay in Outage

Let us assume that only one relay is in outage. This scenario exists when one relay is very close to the source such that the channel between the source and one of the relays is very good. Whereas another relay is relatively far from the source, and, hence, experience its source-relay channel in outage. If R_1 is the relay in outage then $\epsilon_2 = 0$, and from (12), we get the following ML decoder for an M-ary constellation:

$$
\begin{aligned}
\Lambda^d_{\boldsymbol{p},\boldsymbol{q}} &= \ln\left(\frac{\frac{\epsilon_1}{(M-1)}\sum_{\substack{l=1,\\ l\neq p_1}}^{M} e^{-\frac{1}{N_1}\left|y_1-f_1x_l-f_2x_{p_2}\right|^2} + (1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{p_2}\right|^2}}{\frac{\epsilon_1}{(M-1)}\sum_{\substack{l=1,\\ l\neq q_1}}^{M} e^{-\frac{1}{N_1}\left|y_1-f_1x_l-f_2x_{q_2}\right|^2} + (1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_1-f_1x_{q_1}-f_2x_{q_2}\right|^2}}\right)\\
&\quad + \ln\left(\frac{\frac{\epsilon_1}{(M-1)}\sum_{\substack{l=1,\\ l\neq p_2}}^{M} e^{-\frac{1}{N_1}\left|y_2+f_1x^*_l-f_2x^*_{p_1}\right|^2} + (1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2x^*_{p_1}\right|^2}}{\frac{\epsilon_1}{(M-1)}\sum_{\substack{l=1,\\ l\neq q_2}}^{M} e^{-\frac{1}{N_1}\left|y_2+f_1x^*_l-f_2x^*_{q_1}\right|^2} + (1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{q_2}-f_2x^*_{q_1}\right|^2}}\right).
\end{aligned}
\tag{17}
$$

If $\boldsymbol{p}\neq\boldsymbol{q}$ such that $x_{p_1}\neq x_{q_1}$, then we can rewrite the first term in the right hand side (R.H.S.) of LLR decoder of (17) as

$$\Lambda^{d(1)}_{\boldsymbol{p},\boldsymbol{q}}=\ln\left(\frac{\alpha_{p,q}+\frac{\epsilon_1}{(M-1)}e^{-\frac{1}{N_1}\left|y_1-f_1x_{q_1}-f_2x_{p_2}\right|^2}+(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{p_2}\right|^2}}{\alpha_{p,q}+\frac{\epsilon_1}{(M-1)}e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{q_2}\right|^2}+(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_1-f_1x_{q_1}-f_2x_{q_2}\right|^2}}\right), \quad (18)$$

where $\alpha_{p,q}=\frac{\epsilon_1}{(M-1)}\sum_{\substack{l=1\\ l\neq p_1,q_1}}^{M}e^{-\frac{1}{N_1}\left|y_1-f_1x_l-f_2x_{p_2}\right|^2}$. It is difficult to simplify (18) further. However, let us neglect the term $\alpha_{p,q}$ in the numerator and denominator in (18) to obtain a suboptimal decoder. It will be shown by using the QPSK signaling scheme in Fig. 2 that neglecting these terms does not degrade the performance of the decoder significantly at all signal-to-noise ratio (SNR) values. Therefore, we get the following approximate LLR from (18):

$$\Lambda^{d(1)}_{\boldsymbol{p},\boldsymbol{q}}\approx\ln\left(\frac{\frac{\epsilon_1}{(M-1)}e^{-\frac{1}{N_1}\left|y_1-f_1x_{q_1}-f_2x_{p_2}\right|^2}+(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{p_2}\right|^2}}{\frac{\epsilon_1}{(M-1)}e^{-\frac{1}{N_1}\left|y_1-f_1x_{p_1}-f_2x_{q_2}\right|^2}+(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_1-f_1x_{q_1}-f_2x_{q_2}\right|^2}}\right). \quad (19)$$

If $\boldsymbol{p}\neq\boldsymbol{q}$ such that $x_{p_1}\neq x_{q_1}$ and $x_{p_2}=x_{q_2}$, then (19) can be approximated as

$$\Lambda^{d(1)}_{\boldsymbol{p},\boldsymbol{q}}\approx\phi(t_1)=\ln\left(\frac{\epsilon_1+(M-1)(1-\epsilon_1)e^{t_1}}{\epsilon_1e^{t_1}+(M-1)(1-\epsilon_1)}\right), \quad (20)$$

where, for M-PSK constellation

$$t_1=\frac{2}{N_1}\text{Re}\left\{(y_1-f_2x_{p_2})f_1^*\left(x_{p_1}-x_{q_1}\right)^*\right\}. \quad (21)$$

It can be seen from (20) that when $\epsilon_1=0$, $\phi(t_1)=t_1$, and for very large and very small values of t_1, $\phi(t_1)$ is clipped to $T_1=\pm\ln\left[(M-1)(1-\epsilon_1)/\epsilon_1\right]$. Therefore, we can approximate $\phi(t_1)$ by a PL function as follows:

$$\phi(t_1)\approx\phi_{\text{PL}}(t_1)\triangleq\begin{cases}-T_1, & \text{if} \quad t_1<-T_1,\\ t_1, & \text{if} \quad -T_1\leq t_1\leq T_1,\\ T_1, & \text{if} \quad t_1>T_1.\end{cases} \quad (22)$$

Following a similar procedure as stated above, we can get an approximation for the second term on the R.H.S. of (17) for $x_{p_2}\neq x_{q_2}$ as follows:

$$\Lambda^{d(2)}_{\boldsymbol{p},\boldsymbol{q}}\approx\ln\left(\frac{\frac{\epsilon_1}{(M-1)}e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{q_2}-f_2x^*_{p_1}\right|^2}+(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2x^*_{p_1}\right|^2}}{\frac{\epsilon_1}{(M-1)}e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{p_2}-f_2x^*_{q_1}\right|^2}+(1-\epsilon_1)e^{-\frac{1}{N_1}\left|y_2+f_1x^*_{q_2}-f_2x^*_{q_1}\right|^2}}\right). \quad (23)$$

We can further rewrite (23) for $x_{p_2}\neq x_{q_2}$ and $x_{p_1}=x_{q_1}$ as

$$\Lambda^{d(2)}_{\boldsymbol{p},\boldsymbol{q}}\approx\phi(t_2)=\ln\left(\frac{\epsilon_1+(M-1)(1-\epsilon_1)e^{t_2}}{\epsilon_1e^{t_2}+(M-1)(1-\epsilon_1)}\right), \quad (24)$$

where, for M-PSK constellation

$$t_2=\frac{2}{N_1}\text{Re}\left\{(y_2^*-f_2^*x_{p_1})f_1\left(x_{q_2}-x_{p_2}\right)^*\right\}. \quad (25)$$

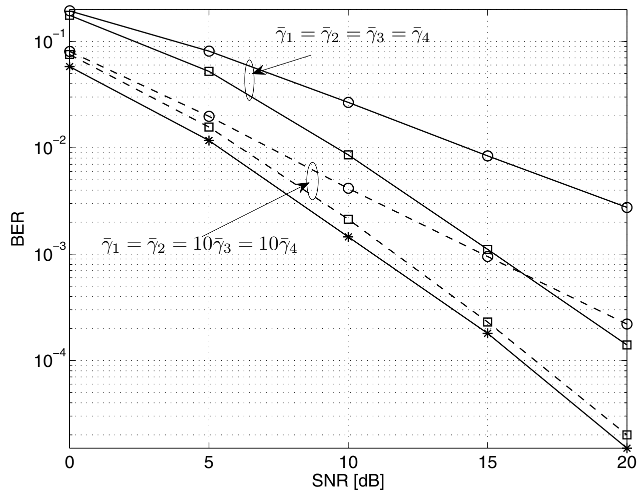

Fig. 1. Performance of the proposed ML decoder □, existing decoder ○ [14, Eq. (13)], distributed Alamouti code with no decoding error in the relays $*$ under BPSK modulation.

Hence, we can also approximate $\phi(t_2)$ by a PL function similar to (22) with t_1 replacing t_2. Therefore, we get a low complexity and approximate LLR decoder as follows:

$$\Lambda^{d}_{\boldsymbol{p},\boldsymbol{q}}\approx\begin{cases}\phi_{\text{PL}}(t_1)+\Lambda^{d(1)}_{\boldsymbol{p},\boldsymbol{q}}, & \text{if} \quad x_{p_1}\neq x_{q_1} \text{ and } x_{p_2}=x_{q_2},\\ \Lambda^{d(1)}_{\boldsymbol{p},\boldsymbol{q}}+\phi_{\text{PL}}(t_2), & \text{if} \quad x_{p_1}=x_{q_1} \text{ and } x_{p_2}\neq x_{q_2},\\ \Lambda^{d(1)}_{\boldsymbol{p},\boldsymbol{q}}+\Lambda^{d(2)}_{\boldsymbol{p},\boldsymbol{q}}, & \text{if} \quad x_{p_1}\neq x_{q_1} \text{ and } x_{p_2}\neq x_{q_2}.\end{cases} \quad (26)$$

V. Simulation Results

We consider Rayleigh fading channels for simulations. Let $\bar{\gamma}_1,\bar{\gamma}_2,\bar{\gamma}_3$, and $\bar{\gamma}_4$ be the average SNRs of S-R$_1$, S-R$_2$, R$_1$-D, and R$_2$-D links, respectively. We have simulated the performance of the proposed ML decoder and an existing decoder [14, Eq. (13)] of distributed Alamouti code with BPSK constellation in Fig. 1 under the following two scenarios: 1) $\bar{\gamma}_1=\bar{\gamma}_2=\bar{\gamma}_3=\bar{\gamma}_4$. 2) $\bar{\gamma}_1=\bar{\gamma}_2=10\bar{\gamma}_3=10\bar{\gamma}_4$. The existing decoder of distributed Alamouti STBC is an ML decoder which considers the relays to be error free. It can be seen from Fig. 1 that the proposed ML decoder significantly outperforms the existing decoder of the distributed Alamouti STBC at all SNRs for BPSK constellation and provides better diversity. For example, a gain of approximately 5 dB is obtained at SER=10^{-2} by the proposed ML decoder as compared to the existing ML decoder [14, Eq. (13)] when the SNRs of all links are same. The performance of the distributed Alamouti STBC with *no* decoding error in the relays is also plotted in Fig. 1. It can be seen from Fig. 1 that if we increase the SNR of the source-relay links, the proposed ML decoder performs close to the distributed Alamouti STBC with error free relays at all SNRs considered in the figure. If the relays are error free, then the cooperative system reduces into the 2×1 co-located multiple-input single-output (MISO) system. Since Alamouti STBC in a 2×1 MISO system achieves second order diversity, the proposed ML decoder achieves full diversity of *two* under both SNR-scenarios for BPSK constellation as can be seen from Fig. 1.

In Fig. 2, we have shown the performance of the proposed ML and PL decoders for the uncoded cooperative communication system with single relay in outage. The simulations

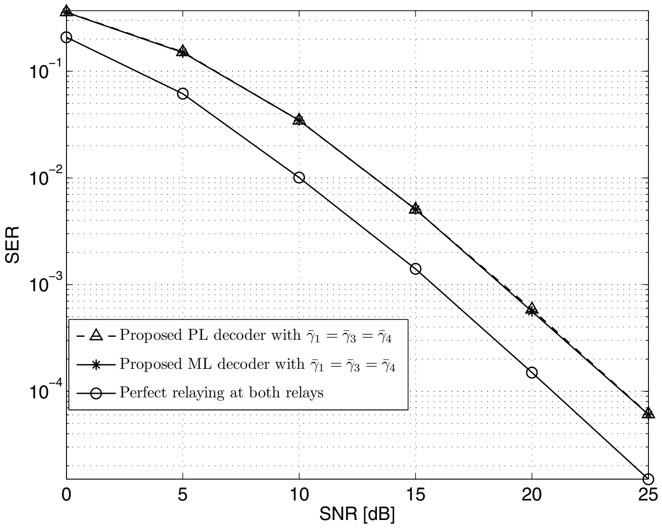

Fig. 2. Performance of the proposed ML and PL decoders with QPSK constellation.

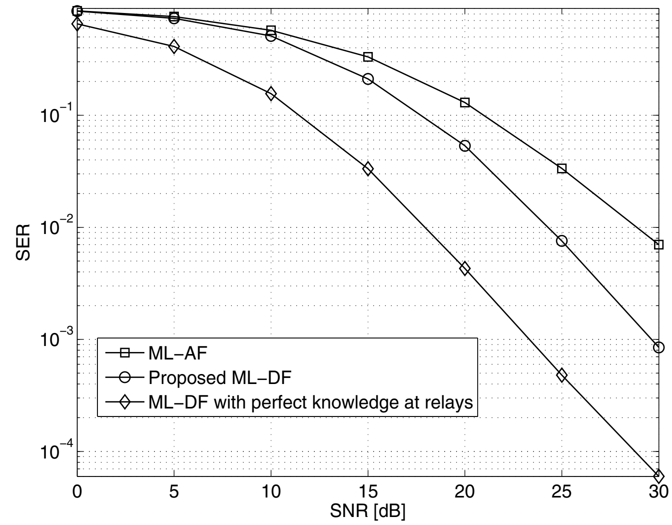

Fig. 3. Comparison of the DF and AF based distributed Alamouti code with 16-QAM constellation.

are performed with the uncoded symbols based distributed Alamouti STBC and QPSK constellation. It is assumed that $\bar{\gamma}_2 \to \infty$, i.e., R_2 has perfect knowledge of the data transmitted by the source and $\bar{\gamma}_1 = \bar{\gamma}_3 = \bar{\gamma}_4$. It can be seen from Fig. 2 that the proposed PL decoder works similar to the proposed ML decoder at all SNRs considered in the figure. Further, it can also be deduced from Fig. 2 that the proposed ML and PL decoders achieve full diversity of *two* under the considered scenario.

We have compared the performance of the proposed ML decoder (12) of the DF system with same-rate AF based cooperative system [13] for 16-QAM constellation in Fig. 3 by assuming that both cooperative systems utilize distributed Alamouti STBC. Further, we assume that all links have the same SNR and the destination has perfect knowledge of the channel gains of the S-R_m and R_m-D channels. It can be seen from Fig. 3 that the DF based distributed Alamouti STBC with the proposed ML decoder significantly outperforms the AF based distributed Alamouti STBC for SNR larger than 5 dB. In Fig. 3, we have also shown the performance of the proposed ML decoder when the relays are error free. It can be noticed from Fig. 3 that the proposed ML decoder enables the distributed Alamouti code based DF system to achieve full diversity with erroneous relays contrary to the AF relaying.

VI. Conclusions

We have derived an ML decoder for DF based distributed Alamouti STBC in a cooperative system utilizing complex-valued M-ary constellations. Moreover, the proposed ML decoder enables the DF based distributed Alamouti STBC to significantly outperform the AF based uncoded distributed Alamouti STBC. We have also derived a low complexity decoder of DF based cooperative system with one out of two relays in outage.

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