# Direction of arrival estimation for MIMO systems employing constellation-based precoding

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***Abstract*—A method is proposed to derive direction of arrival (DOA) information in a multiple-input single-output (MISO) or single-mode multiple-input multiple-output (MIMO) communications system that uses constellation-based limited feedback precoding. The DOA is estimated based on the distribution of precoders selected over multiple feedback intervals. The results show that coarse DOA estimates can be made with little additional system resource allocation.**

## I. Introduction

Direction of arrival (DOA) or positioning information is a requirement in many wireless communications applications, including search and rescue, tracking and enhanced 911 services. The ability to share DOA information among users in wireless networked applications is vital for situational awareness, location service and network protection applications. Sharing this positioning information, however, requires additional use of spectrum resources that are already overtaxed in wireless communications applications.

Limited feedback precoding was introduced in [1] as a means to approximate eigenbeamforming in multiple-input multiple-output (MIMO) systems while significantly reducing the required overhead. In this method, constellations of precoding matrices known to both the transmitter and receiver are used to direct the transmitted signal through a subspace that is well matched to the channel. This method allows rapid adaptation to varying channel conditions and enables multimode operation, wherein multidimensional codebooks of constellations can be used to vary the transmission rate as a mobile unit traverses the environment. When a single-dimensional subspace is used for transmission, e.g. in line-of-sight (LOS) or multiple-input single-output (MISO) circumstances, the precoders selected can yield information about the DOA of a signal at the multiple antenna receiver.

While DOA and positioning have been studied extensively for radar sytems employing MIMO, little attention has been given to MIMO positioning techniques in wireless communications applications [2–4]. In this paper, it will be shown that when using a limited feedback precoding system under LOS conditions, the observed distribution of selected precoders can provide a coarse DOA estimate. Furthermore, this method can be leveraged by neighbouring networked users who are able to overhear the feedback messages detailing precoder selection for increased location service information. DOA estimation is accomplished by matching the distribution of selected codewords to a set of reference precoder index distributions generated using an idealised Ricean model under a range of K-factors. The DOA is selected to correspond to that of the reference distribution minimising the Bhattacharyya distance between the distributions. Herein, this method of estimating DOA is tested using time-varying channels generated with the WINNER2 rural LOS and non-line-of-sight (NLOS) models. While the results presented here are promising, many aspects of propagation are difficult to reproduce accurately with models; testing with a prototype system will more accurately demonstrate the efficacy of the proposed algorithm.

### A. Precoder constellations

Constellations of precoding matrices are designed with their specific application in mind. Some constellations, such as those generated using Grassmannian subspace packing techniques, are designed to spread the precoders as far apart as possible on the Grassmann manifold. Optimisation of the constellation spacing can yield maximal diversity for precoder transmissions [1, 5], or an improvement in error performance for Grassmannian-based symbol modulation [6]. Because these precoders are designed to maximally span complex space, the beampattern of each of the precoders is not designed to correspond to a specific azimuth in relation to the array. Fig. 1(a) shows the beampatterns for a constellation designed using the method outlined by Hochwald in [7] when used with a linear array at half-wavelength spacing. The first precoder in the constellation is highly directional (directed perpendicular to the linear array) as it was arbitrarily chosen in initialisation. The other precoders are not tuned to specific directions and thus their beampatterns do not reach unity for any direction.

The long term evolution (LTE) constellations were selected to meet specific design criteria. The single symbol stream (layer) codebook in LTE was designed partly for peak-to-average power ratio (PAPR) reduction [8], with the transmit power evenly distributed across all transmit antennas for all precoders. In this constellation, the first 8 precoders are directional, with main lobes pointed roughly at 0 (and 180), 41, 60, 75, 90, 105 and 139 degrees in relation to the axis of the linear array. The remaining 8 precoders are not tuned to specific directions. Fig. 1(b) shows beampatterns for three LTE single layer codebook precoders; two of these are directional precoders.

When the channel has a significant LOS signal component,

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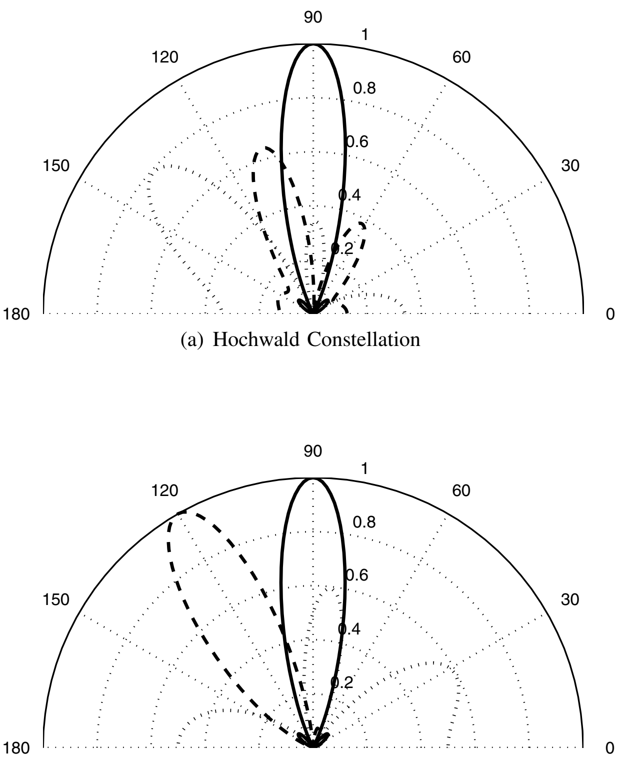


(a) Hochwald Constellation

(b) LTE Constellation

Fig. 1. Beampatterns for three of the precodes for a Hochwald constellation and the LTE single layer constellation.

the match between the channel and the precoders selected has a direct relationship with the beampattern of the precoders in the constellation. For example, for a signal arriving perpendicular to the array, the receiver would likely select the highly directional precoders for both of the examples shown in Fig. 1. However, due to signal components arriving through reflection, shadowing and scattering, there is a non-zero probability that the overall channel can be perturbed in such a way that any of the other precoders would be selected instead of the precoder best corresponding to the LOS signal component. The relative probability of each of the precoders being selected is a function of the constellation as well as the relative powers of the LOS and other signal components.

### *B. System Model*

For ease of explanation, the following is described in terms of a MISO system. However, the method proposed herein is equally applicable to the single dimensional application in MIMO systems where the precoders are matched to the right singular vector corresponding to the largest singular value of the MIMO channel matrix, $\mathbf{H}$.

Consider a MISO system with $N_T$ transmit antennas and a single receive antenna that uses a codebook of $L$ complex, $N_T \times 1$ precoding vectors $\mathbf{f}_l, l = 1, 2, \ldots, L$, in order to match the spatial signature of transmission to the perfectly estimated $1 \times N_T$ channel vector, $\mathbf{h}$. The received signal due to precoder $\mathbf{f}$ is written

$$y = \mathbf{hf}s + n, \tag{1}$$

where $s$ is the transmitted symbol and $n$ is receiver noise.

In a MISO system, the precoding vector maximising the inner product of the precoder with the channel, i.e.,

$$\mathbf{f} = \arg\max_{\mathbf{f}_l} |\mathbf{hf}_l|, \tag{2}$$

is selected in order to maximise the receiver signal-to-noise ratio (SNR).

Over a series of $M$ feedback intervals, let $Q(l)$ represent the number of instances that the $l$th precoder was selected. The distribution of precoder indices, $q(l)$, during $M$ feedback intervals is thus given by

$$q(l) = \frac{Q(l)}{M}. \tag{3}$$

In the following, the angle between the fixed antenna array and the mobile unit, denoted $\theta$, is estimated by finding the minimum distance between $q(l)$ and a series of reference distributions that are generated for known $\theta$ values using a Ricean model with a variety of power factors.

### *C. Ricean model*

For a scenario where a single antenna mobile communicates with a multiple-antenna base station, with the received signal characterised by a LOS component, represented by $\mathbf{h}_{LOS}$, and a diffuse component, represented by $\mathbf{h}_{NLOS}$, the ratio of the powers of these components is given by the Ricean factor, $K$, and the complete channel vector can be written

$$\mathbf{h} = \sqrt{\frac{K}{K+1}}\mathbf{h}_{LOS} + \sqrt{\frac{1}{K+1}}\mathbf{h}_{NLOS}, \tag{4}$$

where $\mathbf{h}_{NLOS}$ is assumed to be zero-mean i.i.d. complex Gaussian [9]. If the mobile's angle with respect to the base station array is $\theta$, and the phase offset is $\phi$, $\mathbf{h}_{LOS}$ is

$$\mathbf{h}_{LOS} = \left[\begin{array}{cccc} e^{j(\alpha_0+\phi)} & e^{j(\alpha_1+\phi)} & \ldots & e^{j(\alpha_{N_T-1}+\phi)} \end{array}\right], \tag{5}$$

where, for a linear array with half-wavelength spacing, $\alpha_\beta = \beta\pi\cos\theta$. For a circular array with half-wavelength spacing, $\alpha_\beta = \frac{\pi\cos\theta - \beta\frac{2\pi}{N_T}}{2\sin\frac{\pi}{N_T}}$.

## II. Proposed algorithm

In this section the method used to determine an estimate of $\theta$ based on the distribution of selected precoder indices, $q(l)$, is described.

In a Ricean propagation environment with factor $K$ and LOS component DOA $\theta$, the probability that the $l$th precoder will be will be selected, $\tilde{p}_{\theta,K}(l)$, is

$$\tilde{p}_{\theta,K}(l) = P(|\mathbf{hf}_l| > |\mathbf{hf}_n|, \forall n \neq l|\theta, K). \tag{6}$$

Closed-form expressions for $\tilde{p}_{\theta,K}(l)$ are left for future work. For the work reported here, numerical approximations of reference precoder distributions, $p_{\theta,K}(l)$, were generated

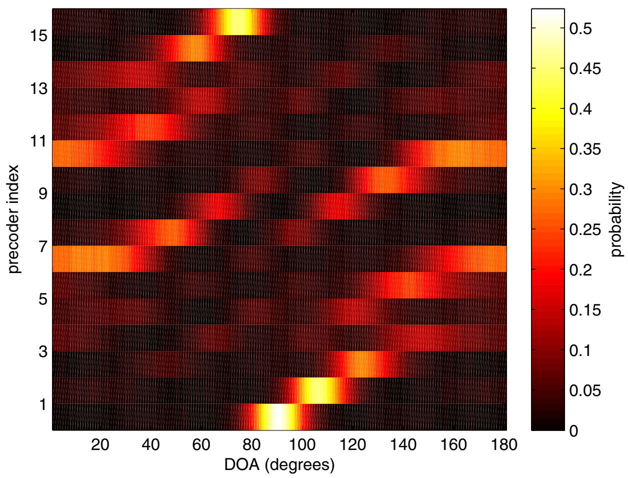


Fig. 2. Probability distribution of the expected precoder indices that would be selected as a function of the direction of arrival for a 16-precoder constellation when used for a linear, half-wavelength spaced, array for $\frac{K}{K+1} = 0.7$.

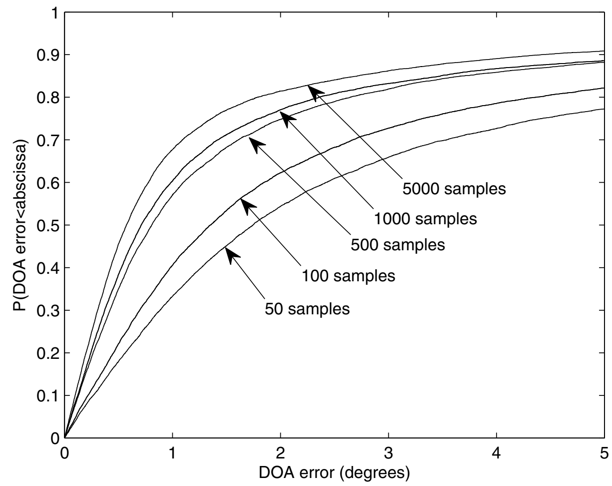


Fig. 3. DOA error cdfs as a function of the number of independent channel vectors for a specific $\theta$ used to generate $q(l)$.

based on 10000 $\mathbf{h}$ realisations using (2), (4) and (5) for $\theta \in \{1^o, 2^o, \ldots, 360^o\}$ and $\frac{K}{K+1} \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. A set of reference precoder distributions was generated for each precoder constellation when used with a linear or circular array, as required.

Fig. 2 shows reference precoder index distributions, assuming $\frac{K}{K+1} = 0.7$, using a four-element linear array with a 16-precoder constellation generated using the method outlined in [7]. As the figure represents $p_{\theta,K}(l)$ when used with a linear array, it is only plotted for 180 single-degree increments; the reference distributions corresponding to $\theta$ ranging from 180 to 360 degrees is a mirror image of this plot.

Fig. 2 shows the distribution of the expected precoder indices that would be selected as a function of the direction of arrival. For example, for signals arriving from $50^o$ with 70 percent LOS power, it would be expected that precoders 7, 11 and 14 would be selected most frequently, with precoders 12 and 13 being selected less frequently, and the other precoders selected infrequently. It is this weaving pattern of selection probabilities shown in the figure which ultimately gives an estimate of $\theta$. When a given constellation and array configuration results in reference precoder distributions that are distinct for all $\theta$ values of interest in a specific application, better estimates will result than in cases where similar reference precoder distributions occur for multiple $\theta$ values, which results in ambiguous estimates.

The Bhattacharyya distance measure is used to quantify the similarity between between two probability distributions $p(l)$ and $q(l)$. For discrete probability distributions, the Bhattacharyya distance measure is given by [10]

$$D_B(p, q) = -ln \left( \sum_{l \in L} \sqrt{p(l)q(l)} \right). \tag{7}$$

In the work reported herein, the direction estimate, $\hat{\theta}$, is set equal to the angle of the reference precoder distribution, $p_{\theta,K}$ which has the minimum Bhattacharyya distance measure to the distribution of precoder indices, $q(l)$, observed over $M$ precoder intervals; i.e.,

$$\hat{\theta} = \arg\min_{\theta,K} D_B(p_{\theta,K}(l), q(l)). \tag{8}$$

### A. Proof of concept

As a proof of concept test, the capability of the algorithm described above to estimate a known $\theta$ was tested with $\theta \sim U(0^o, 180^o)$. In order to avoid cases of extremely low LOS component power, $\frac{K}{K+1} \sim U(0.2, 1.0)$. Reference precoder index distributions, $p_{\theta,K}(l)$, were generated for a 4-element linear array and a constellation generated as in [7]. Fig. 3 shows cdf plots which have been averaged over 10000 independent $\theta$ realisations, with $q(l)$ distributions each generated as in (3) based on the precoder indices selected using $M \in \{50, 100, 500, 1000, 5000\}$ i.i.d. Gaussian $\mathbf{h}_{NLOS}$ realisations for each $\mathbf{h}_{LOS}$.

Fig. 3 shows that with single-degree resolution in the reference precoder index distributions, estimates are made with less than one degree of median DOA error when at least 500 samples are used to generate $q(l)$ values. Even with as few as 50 samples, a median DOA error of less than two degrees is achieved. Although Fig. 3 shows promising results, the channel model used both to generate the reference precoder index distributions and to verify performance assumes independent samples. As (4) does not give a representation of mobility, in the following section the performance of the algorithm is evaluated using the WINNER2 channel mobility model.

## III. Results

In this section, the ability of the algorithm described in Sec. II to estimate mobile direction is demonstrated with time-varying channel data generated using the IST-WINNER2 model [11],

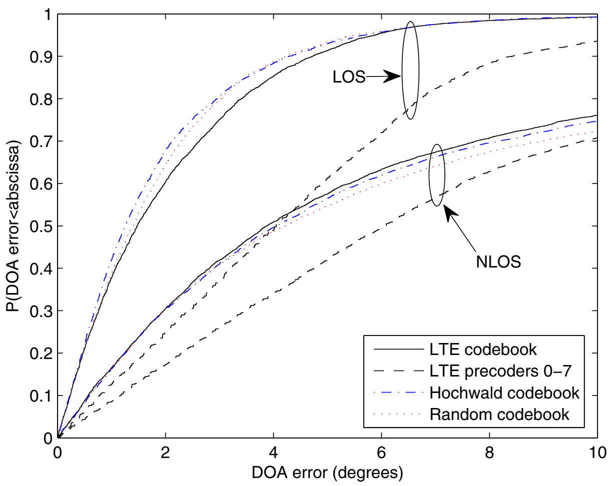


Fig. 4. DOA error cdfs when generating $q(l)$ using 1000 samples over 4 m of mobile motion for the WINNER2 rural LOS and NLOS scenarios.

using both the rural LOS and NLOS scenarios at 2 GHz. Based on the structure of the model, the MISO channels were generated for a fixed antenna array base station communicating with a single-antenna mobile unit. For all of the results reported herein, the distance between the base station and the mobile station was fixed at 500 m. A variety of codebooks and antenna array geometries were used to illustrate different aspects of using the algorithm in a practical system. In all cases, $p_{\theta,K}(l)$, were generated using 10000 independent realisations of (4) for each angle of arrival with one-degree resolution at each value of $\frac{K}{K+1}$ given above. It is assumed that the choice of precoder at every instance is based on perfect estimation of the channel vector, $\mathbf{h}$. In practice, channel state information is never perfect. However, the quality of channel estimates must be good enough to allow acceptable bit error performance. This level of channel estimation error would have little effect on the distribution of selected precoder indices, $q(l)$, since small constellations are used in this work.

For the following results, unless otherwise stated, cdfs are based on $q(l)$ distribution estimates calculated with 1000 feedback samples evenly distributed across 25 wavelengths, i.e., roughly 4 m.

### A. Linear sectorised base station

The first array configuration considered is that of a 4-element, half-wavelength spaced, linear array that forms a $120^o$ sector at a base station. This implementation avoids array processing on signals arriving along the axis of the array, where beamformers have low resolution. Fig. 4 shows mobile angle estimate error cdfs for the WINNER2 rural scenario using the algorithm described in Sec. II with three different codebooks. The codebooks used are: a 16-precoder Hochwald codebook designed as in [7]; a randomly generated codebook consisting of complex unit vectors; and the single-layer LTE codebook [8].

Fig. 4 shows that good direction estimates can be obtained with the algorithm presented herein, with median DOA estimation errors of less than $2^o$, regardless of the constellation used. Although the LTE codebook was designed with several directional precoders, the Hochwald constellation yields slightly better LOS error estimates. Since the Hochwald constellation is designed to be spatially diverse, the resulting reference precoder index distributions are more distinct than those of the LTE constellation, resulting in better DOA estimates. Fig. 4 also shows that, although no LOS signal component is present, usable mobile angle estimates can be obtained using the WINNER2 rural NLOS model, with median DOA estimation error of roughly $4^o$ demonstrated for the three 16-precoder constellations tested.

A possible alternate method for mobile angle estimation using the LTE constellation would be to assign an angle estimate, $\hat{\theta}$, based on the most frequently selected of the directional precoders, i.e., precoder 0-7. Cdfs using this method are plotted in Fig. 4 for comparison purposes. Since only eight directional precoders are present in the LTE single layer codebook, and only seven of these are directed within the $120^o$ sector, only very coarse angle estimates can be made using this alternate method. Using this method results in median mobile angle estimation error of $4^o$ and $6^o$ for the LOS and NLOS WINNER2 models, respectively.

In Fig. 3, error cdfs are shown as a function of the number of independent samples used to generate $q(l)$. With the WINNER2 mobility model, a better idea of the distance a mobile unit must move in order to obtain good mobile angle estimates can be obtained. Fig. 5 shows angle estimate error cdfs for $q(l)$ generated with 50 and 500 feedback samples evenly distributed across 50, 25, 12.5, 6.25 and 3.125 wavelengths of motion, using the Hochwald constellation for the rural LOS scenario. Increasing the sampling density has virtually no benefit above two samples per wavelength of motion. It is clear that as the number of wavelengths of mobile motion is increased, better DOA estimates are obtained, provided that the spacing between the base station and mobile is large enough to make the DOA change small over the number of wavelengths sampled. Good DOA estimates can be obtained even with very little motion; e.g., 3.25 wavelengths corresponds to about 0.5 m at 2 GHz.

### B. Circular array at base station

Fig. 6 shows error cdfs when the proposed algorithm is used with a circular array for the WINNER2 rural LOS model. In order to understand the impact of array size and constellation size on error performance, Hochwald constellations of 4, 16 and 64 precoders were generated for both 4 and 8 element arrays, and reference distributions were generated for their use with half-wavelength spaced circular arrays.

Fig. 6 shows that reasonable mobile angle estimation can be obtained using a circular array with 16 and 64 precoder constellations. This problem is slightly different than that of the single sector base station as the mobile is not limited to being within a specific sector. The quality of the mobile angle estimates depend greatly on the array and constellation sizes.

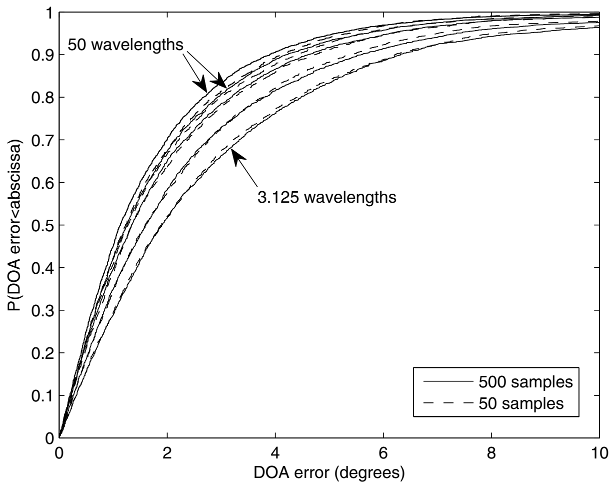


Fig. 5. DOA error cdfs when generating $q(l)$ using 50 and 500 samples over 3.125-50 wavelengths of mobile motion for the WINNER2 rural LOS scenario.

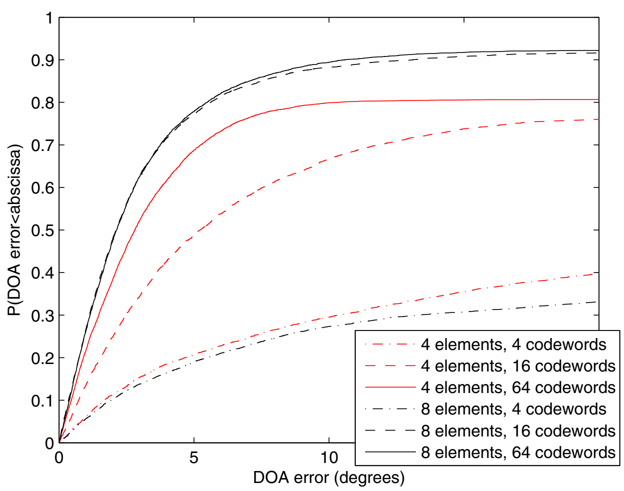


Fig. 6. DOA error cdfs when generating $q(l)$ using 1000 samples over 4 m of mobile motion for the WINNER2 rural LOS with 4- and 8-element circular arrays with $L = 4$, 16 and 64 precoder constellations.

In Sec. II, it was mentioned that when a given constellation and array configuration result in similar reference precoder index distributions for multiple $\theta$ values, ambiguous mobile direction estimates result. With only 4 precoders in a constellation using both the 4 and 8 element circular arrays, poor mobile angle estimates are due to a general lack of distinction across the reference distributions; signals arriving from a wide variety of directions can have similar precoder index distributions. With 4 elements and 16 or 64 precoders, performance is improved. However, the 64 element constellation plot in Fig. 6 clearly shows the cdf levelling off at 0.8. This plateau in the cdf is the direct result of the ambiguity in direction found for signals arriving perpendicular to the edges of the square array; these signals cannot be distinguished from signals arriving from the opposite side of the array. A reduced plateau effect is also seen with the 8-element circular array, with directional ambiguity resulting in large DOA error estimates for just under 10% of the cases.

It should be noted that, for the work reported herein, no effort was made to test for constellations yielding reference distributions providing good spatial distinction. It is possible that two constellations yielding matching bit error rate performance, for the purposes of limited feedback MIMO precoding, could yield appreciably different reference precoder index distributions, resulting in substantially different mobile angle estimate error performance. The goal of this work was to show that, regardless of the constellation used in a limited feedback system, useful mobile angle estimates can be obtained from the distribution of selected precoder indices. It may be possible to design constellations yielding improved DOA estimates while maintaining the main MIMO functionality; this is a topic for future work.

## IV. Conclusion

The distribution of the precoder indices selected in a limited feedback MIMO system can yield valuable direction of arrival information, regardless of whether the precoders are directional. For the base station, the mobile, or other users within the network, mobile angle estimates can be obtained using network information which is already being passed through the system, thus requiring little additional resource allocation.

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