Joint Transmit/Receive MMSE-FDE for MIMO Analog Network Coding in Single-Carrier Bi-Directional Relay Communications

Hiroyuki MIYAZAKI† Masayuki NAKADA† Tatsunori OBARA† and Fumiyuki ADACHI‡
Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
†{miyazaki, nakada, obara}@mobile.ecei.tohoku.ac.jp, ‡adachi@ecei.tohoku.ac.jp

Abstract— **In this paper, we propose a joint transmit/receive frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion for multi-input multi-output (MIMO) analog network coding (ANC) in single-carrier (SC) bi-directional relay communications. In the proposed scheme, the relay station (RS) equipped with multiple antennas carries out antenna diversity and one-tap transmit FDE, and the base station (BS) and mobile terminal (MT) receivers, both equipped with single antenna, carry out one-tap receive FDE only. The FDE weights at RS, BS and MT are jointly optimized so as to minimize the end-to-end mean square error (MSE). We evaluate, by computer simulations, the bit error rate (BER) performance when using the proposed scheme, and discuss how the transmit FDE at RS operates and affects the BER performance.**

Keywords-component; Analog network coding, frequency-domain equalization, single-carrier transmission

I. INTRODUCTION

In broadband data transmissions, the received signal suffers from the propagation path loss, the shadowing loss and the frequency selective fading [1]. Cooperative relay is a promising technique to overcome the propagation path loss and the shadowing loss [2]. However, cooperative relay requires four time slots for bi-directional communications while the conventional direct communications achieve them for two time slots. Therefore, the achievable maximum throughput decreases to half of the direct transmission.

An application of network coding is an effective way to improve the throughput performance for bi-directional relay communications. Network coding schemes are classified into two types: digital network coding [3] and analog network coding (ANC) [4-6]. Recently, ANC has been attracting much attention. ANC uses two time slots for bi-directional relay communications and hence, it can achieve the same maximum throughput as the direct communications. To further improve the throughput performance, ANC using multi-antenna relay was studied in [7,8]. The spatial diversity gain can be obtained by employing transmit diversity at relay station (RS). We call this scheme as MIMO-ANC.

In broadband single-carrier (SC) MIMO-ANC, the transmission performance degrades due to the inter-symbol interference (ISI) caused in a frequency-selective fading channel. The joint transmit/receive frequency domain equalization (FDE) based on minimum mean square error (MMSE) criterion [9] can be applied to MIMO-ANC.

In this paper, we propose a joint transmit/receive MMSE-FDE for MIMO-ANC in SC bi-directional relay communications. In the proposed scheme, RS equipped with multiple antennas carries out antenna diversity and one-tap transmit FDE, and the base station (BS) and mobile terminal (MT) receivers, both equipped with single antenna, carry out one-tap receive FDE only. The FDE weights at RS, BS and MT are jointly optimized so as to minimize the end-to-end mean square error (MSE). By using the joint transmit/receive MMSE-FDE, the spatial and frequency diversity gains can be obtained.

However, in ANC relay, the received noise powers are different between BS and MT due to the noise which is amplified and broadcast by RS. Therefore, the transmit FDE weight which can simultaneously minimize the MSEs for both uplink and downlink does not exist. We derive the transmit FDE weights optimized for uplink and downlink separately. The receive FDE weights at BS and MT are derived by considering the concatenation of the transmit FDE and the propagation channel as an equivalent channel. We evaluate, by computer simulations, the BER performance, and discuss how the transmit FDE weight operates and affects the BER performance.

The remainder of this paper is organized as follows. The bi-directional relay using MIMO-ANC is introduced in Sect. II. Section III explains the proposed joint transmit/receive FDE. Section IV discusses the computer simulation results, and Sect. V offers conclusion.

II. BI-DIRECTIONAL RELAY USING MIMO ANC

A. System model

Figure 1 illustrates the system model of bi-directional relay using MIMO-ANC considering in this paper. BS, RS and MT are linearly located on the line. We assume that RS has J antennas, and BS and MT have single antenna, respectively. In this paper, we do not consider the propagation path loss and the shadowing loss for simplicity. Ideal channel estimation at RS, BS and MT is assumed.

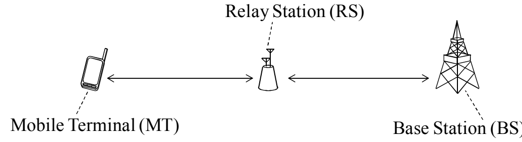
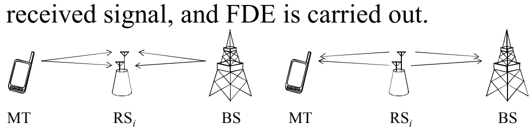


Figure 1. System model.

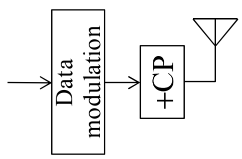
B. Behavior of MIMO-ANC

Figure 2 and 3 show the behavior and signal processing of MIMO-ANC. MIMO-ANC requires two time slots for bi-directional relay communications. In the first time slot, BS and MT simultaneously transmit their signals to RS. Then, RS applies FDE to the received signal. In the second time slot, RS amplifies the equalized received signal and broadcasts it to BS and MT. At BS and MT receivers, self-interference is removed from the received signal, and FDE is carried out.

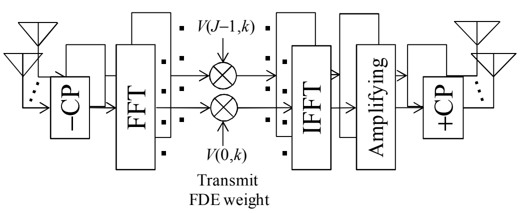


(a) 1st time slot (b) 2nd time slot

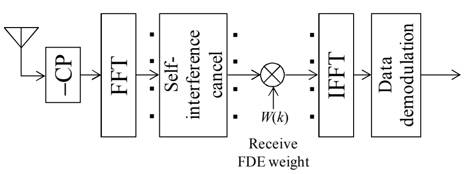
Figure 2. Behavior of MIMO ANC.



(a) Transmit structure of BS and MT.



(b) Structure of RS.



(c) Receiver structure of BS and MT.

Figure 3. Signal processing at BS, RS, and MT.

C. Signal Representation

In this paper, symbol-spaced discrete time signal representation is used.

(a) First time slot

The data symbol blocks of N_c symbols at BS and MT are denoted as $\{x_B(t) : t=0,\ldots,N_c-1\}$ and $\{x_M(t) : t=0,\ldots,N_c-1\}$, respectively. After insertion of N_g sample cyclic prefix (CP) into the beginning of each block, BS and MT simultaneously transmit their symbol blocks to RS. At RS, after CP removal, the received signal is transformed into the frequency-domain signal by N_c-point fast Fourier transform (FFT). The frequency-domain received signal, $\{Y_R(j,k) : k=0,\ldots,N_c-1\}$, at jth RS antenna can be expressed as

$$\begin{aligned}Y_R(j,k) &= \sqrt{2P_B}H_{B-R}(j,k)X_B(k)\\ &\quad+\sqrt{2P_M}H_{M-R}(j,k)X_M(k)+N_R(j,k)\end{aligned}\quad. \tag{1}$$

In Eq. (1), P_B and P_M are the transmit power of BS and MT respectively. $H_{B-R}(j,k)$ and $H_{M-R}(j,k)$ denote the channel transfer functions between BS and jth RS antenna and between MT and jth RS antenna, respectively. $N_R(j,k)$ is the independent zero-mean complex-valued additive white Gaussian noise (AWGN) having variance $2N_0/T_s$ with N_0 and T_s being the single-sided power spectrum density of the AWGN and the symbol duration. $X_B(k)$ and $X_M(k)$ are the transmit signal components at BS and MT, respectively. They are given as

$$\begin{cases}X_B(k)=\dfrac{1}{\sqrt{N_c}}\displaystyle\sum_{t=0}^{N_c-1}x_B(t)\exp(-j2\pi kt/N_c)\\ X_M(k)=\dfrac{1}{\sqrt{N_c}}\displaystyle\sum_{t=0}^{N_c-1}x_M(t)\exp(-j2\pi kt/N_c)\end{cases}. \tag{2}$$

RS applies FDE to the frequency-domain received signal $\{Y_R(j,k) : k=0,\ldots,N_c-1\}$. The frequency-domain signal, $\{\hat{Y}_R(j,k) : k=0,\ldots,N_c-1\}$, after FDE at jth RS antenna can be expressed as

$$\hat{Y}_R(j,k)=Y_R(j,k)V(j,k)\,, \tag{3}$$

where, $V(j,k)$ denotes the transmit FDE weight at jth RS antenna. The transmit FDE weight has a constraint in order to keep the average transmit power of RS constant as

$$\frac{1}{N_c}\sum_{k=0}^{N_c-1}\sum_{j=0}^{J-1}|V(j,k)|^2=1\,. \tag{4}$$

The frequency-domain signal after FDE is transformed back to the time-domain signal by N_c-point inverse FFT (IFFT) and amplified. The transmit signal $\{x_R(j,t): t=0,\ldots,N_c-1\}$ of jth RS antenna is given as

$$x_R(j,t)=\frac{\beta(j)}{\sqrt{N_c}}\sum_{t=0}^{N_c-1}\hat{Y}_R(j,k)\exp(j2\pi kt/N_c)\,, \tag{5}$$

where, $\beta(j)$ is the amplifying factor at jth RS antenna. The amplifying factor $\beta(j)$ is set so as to keep the average transmit power of RS constant as

$$\beta(j)=\sqrt{\frac{P_R}{\dfrac{P_M}{N_c}\displaystyle\sum_{k=0}^{N_c-1}|H_{M-R}(j,k)|^2+\dfrac{P_B}{N_c}\displaystyle\sum_{k=0}^{N_c-1}|H_{B-R}(j,k)|^2+N}}\,, \tag{6}$$

where, P_R is the transmit power of RS, and $N=N_0/T_s$ is the noise power. In this paper, we consider the total transmit power constraint. The total transmit power P_T for bi-directional relay communications is defined as

$$P_M+P_B+P_R=P_T\,. \tag{7}$$

(b) Second time slot

After insertion of N_g sample CP into the beginning of transmit signal block, RS broadcasts it to BS and MT. After CP removal, the received signals at BS and MT are transformed into the frequency-domain signals by N_c-point FFT. The frequency-domain received signals, $\{Y_B(k) : k=0,\ldots,N_c-1\}$ and $\{Y_M(k) : k=0,\ldots,N_c-1\}$, at BS and MT can be respectively expressed as

$$\begin{cases} Y_B(k)=\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)\hat{Y}_R(j,k)+N_B(k) \\ Y_M(k)=\sum_{j=0}^{J-1}\beta(j)H_{M-R}(j,k)\hat{Y}_R(j,k)+N_M(k) \end{cases}, \tag{8}$$

where, $N_B(k)$ and $N_M(k)$ are the zero mean AWGNs at BS and MT having variance $2N_0/T_s$. From Eq. (1) and (3), Eq. (8) can be rewritten as

$$\begin{cases} Y_B(k)=\sqrt{2P_M}\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)H_{M-R,}(k)V(j,k)X_M(k) \\ \quad +\sqrt{2P_B}\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)H_{B-R}(j,k)V(j,k)X_B(k) \\ \quad +\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)V(j,k)N_R(j,k)+N_B(k) \\ Y_M(k)=\sqrt{2P_B}\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)H_{M-R,}(k)V(j,k)X_B(k) \\ \quad +\sqrt{2P_M}\sum_{j=0}^{J-1}\beta(j)H_{M-R}(j,k)H_{M-R,}(k)V(j,k)X_M(k) \\ \quad +\sum_{j=0}^{J-1}\beta(j)H_{M-R}(j,k)V(j,k)N_R(j,k)+N_M(k) \end{cases}. \tag{9}$$

The first and the second terms are the desired signal and the self-interference, respectively. The third and the forth terms are the noises added at RS and each receiver, respectively. The self-interference is removed from the received signal as

$$\begin{cases} \widetilde{Y}_B(k)=Y_B(k) \\ \quad -\sqrt{2P_B}\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)H_{B-R}(j,k)V(j,k)X_B(k) \\ \widetilde{Y}_M(k)=Y_M(k) \\ \quad -\sqrt{2P_M}\sum_{j=0}^{J-1}\beta(j)H_{M-R}(j,k)H_{M-R}(j,k)V(j,k)X_M(k) \end{cases}. \tag{10}$$

After self-interference removal, the receive FDE is carried out. The received signals after the receive FDE at BS and MT, $\{\hat{Y}_B(k) : k=0,\ldots,N_c-1\}$ and $\{\hat{Y}_M(k) : k=0,\ldots,N_c-1\}$ can be respectively expressed as

$$\begin{cases} \hat{Y}_B(k)=\widetilde{Y}_B(k)W_B(k) \\ \hat{Y}_M(k)=\widetilde{Y}_M(k)W_M(k) \end{cases}, \tag{11}$$

where, $W_B(k)$ and $W_M(k)$ are the receive FDE weights at BS and MT receivers, respectively. The received signals after FDE are transformed back to the time-domain signal by N_c-point IFFT, and the data demodulation is carried out.

III. Derivation of MMSE-FDE weights

In this section, we derive the FDE weights at RS, BS and MT which minimize the end-to-end MSEs of uplink and downlink. The MSEs, e_{up} and e_{down}, of uplink and downlink are defined as

$$\begin{cases} e_{up}=\sum_{k=0}^{N_c-1}E\left[\left|X_M(k)-\hat{Y}_B(k)/\sqrt{2P_M}\right|^2\right] \\ e_{down}=\sum_{k=0}^{N_c-1}E\left[\left|X_B(k)-\hat{Y}_M(k)/\sqrt{2P_B}\right|^2\right] \end{cases}. \tag{12}$$

From Eq. (9), (10) and (11), Eq. (12) can be rewritten as

$$\begin{cases} e_{up}=\sum_{k=0}^{N_c-1}\left[\begin{array}{l}\{\widetilde{H}(k)W_B(k)-1\}\{\widetilde{H}(k)W_B(k)-1\}^* \\ +\left\{\sum_{j=0}^{J-1}|\beta(j)H_{B-R}(j,k)V(j,k)|^2+1\right\}|W_B(k)|^2\left(\frac{P_M}{N}\right)^{-1}\end{array}\right] \\ e_{down}=\sum_{k=0}^{N_c-1}\left[\begin{array}{l}\{\widetilde{H}(k)W_M(k)-1\}\{\widetilde{H}(k)W_M(k)-1\}^* \\ +\left\{\sum_{j=0}^{J-1}|\beta(j)H_{M-R}(j,k)V(j,k)|^2+1\right\}|W_M(k)|^2\left(\frac{P_B}{N}\right)^{-1}\end{array}\right] \end{cases}, \tag{13}$$

where,

$$\widetilde{H}(k)=\sum_{j=0}^{J-1}\beta(j)H_{B-R}(j,k)H_{M-R}(j,k)V(j,k). \tag{14}$$

At first, the receive FDE weights at BS and MT are derived by considering the concatenation of the given transmit FDE and the propagation channel gain as an equivalent channel, and then, we want to derive the optimal transmit FDE weight at RS which minimizes the MSEs of uplink and downlink. However, in ANC relay, the noise which is amplified by RS is broadcast to BS and MT. Then, the received noise powers at BS and MT are different because the channel gains between BS and RS and between MT and RS are different. Therefore, the transmit FDE weight at RS which simultaneously minimizes both e_{up} and e_{down} does not exist. In this paper, we derive the transmit FDE weights at RS optimized for uplink and downlink separately.

At first, we derive the receive FDE weights. From $\partial e_{up}/\partial W_B(k)=0$ and $\partial e_{down}/\partial W_M(k)=0$, the optimal receive FDE weights at BS and MT are obtained as

$$\begin{cases} W_B(k)=\dfrac{\widetilde{H}^*(k)}{\left|\widetilde{H}(k)\right|^2+\left\{\sum_{j=0}^{J-1}|\beta(j)H_{B-R}(j,k)V(j,k)|^2+1\right\}\left(\frac{P_M}{N}\right)^{-1}} \\ W_M(k)=\dfrac{\widetilde{H}^*(k)}{\left|\widetilde{H}(k)\right|^2+\left\{\sum_{j=0}^{J-1}|\beta(j)H_{M-R}(j,k)V(j,k)|^2+1\right\}\left(\frac{P_B}{N}\right)^{-1}} \end{cases}. \tag{15}$$

Substituting Eq. (15) to Eq. (13), we can obtain

$$\begin{cases} e_{up}=\sum_{k=0}^{N_c-1}\dfrac{\left(\sum_{j=0}^{J-1}|\beta(j)H_{B-R}(j,k)V(j,k)|^2+1\right)\left(\frac{P_M}{N}\right)^{-1}}{\left|\widetilde{H}(k)\right|^2+\left(\sum_{j=0}^{J-1}|\beta(j)H_{B-R}(j,k)V(j,k)|^2+1\right)\left(\frac{P_M}{N}\right)^{-1}} \\ e_{down}=\sum_{k=0}^{N_c-1}\dfrac{\left(\sum_{j=0}^{J-1}|\beta(j)H_{M-R}(j,k)V(j,k)|^2+1\right)\left(\frac{P_B}{N}\right)^{-1}}{\left|\widetilde{H}(k)\right|^2+\left(\sum_{j=0}^{J-1}|\beta(j)H_{M-R}(j,k)V(j,k)|^2+1\right)\left(\frac{P_B}{N}\right)^{-1}} \end{cases}. \tag{16}$$

There are two types of the transmit FDE weight at RS; one is the transmit FDE weight which minimizes e_{up} and the other is that which minimizes e_{down}.

(a) The transmit FDE weight which minimizes e_{up}

The optimization problem for the transmit FDE weight can be expressed as

$$\begin{aligned}&\text{minimize} \quad e_{up}\\ &s.t. \quad \sum_{k=0}^{N_c-1}\sum_{j=0}^{J-1}|V(j,k)|^2 - N_c = 0, \\ &\qquad |V(j,k)|^2 \geq 0, \qquad j=0,..,J-1,\ k=0,..,N_c-1\end{aligned} \tag{17}$$

The optimal transmit weight for Eq. (17) can be derived by using the Cauchy-Schwarz inequality and the Lagrange multiplier method [10] under the transmit power constraint. The optimal transmit weight which minimizes the MSE of uplink is given as

$$V(j,k) = \frac{\beta(j)H^*_{M-R}(j,k)H^*_{B-R}(j,k)}{\sqrt{A(k)}} \times \max\left[\left\{\frac{\sqrt{\frac{A(k)}{\lambda_B}\left(\frac{P_M}{n}\right)^{-1}} - \left(\frac{P_M}{n}\right)^{-1}}{A(k)+\left(\frac{B_B(k)}{A(k)}\right)\left(\frac{P_M}{n}\right)^{-1}}\right\}^{\frac{1}{2}},\ 0\right], \tag{18}$$

where,

$$\begin{cases} A(k) = \sum_{j=0}^{J-1}|\beta(j)H_{B-R}(j,k)H_{M-R}(j,k)|^2 \\ B_B(k) = \sum_{j=0}^{J-1}|\beta(j)H_{B-R}(j,k)H_{M-R}(j,k)|^2|\beta(j)H_{B-R}(j,k)|^2 \end{cases}, \tag{19}$$

and λ_B is chosen so as to satisfy the constraint given as Eq. (4).

(b) The transmit FDE weight which minimizes e_{down}

The optimization problem for the transmit FDE weight can be expressed as

$$\begin{aligned}&\text{minimize} \quad e_{down}\\ &\text{s.t.} \quad \sum_{k=0}^{N_c-1}\sum_{j=0}^{J-1}|V(j,k)|^2 - N_c = 0, \\ &\qquad |V(j,k)|^2 \geq 0, \qquad j=0,..,J-1,\ k=0,..,N_c-1\end{aligned} \tag{20}$$

Similarly to (a), the optimal FDE weight which minimizes the MSE of downlink is derived as

$$V(j,k) = \frac{\beta(j)H^*_{M-R}(j,k)H^*_{B-R}(j,k)}{\sqrt{A(k)}} \times \max\left[\left\{\frac{\sqrt{\frac{A(k)}{\lambda_M}\left(\frac{P_B}{N}\right)^{-1}} - \left(\frac{P_B}{N}\right)^{-1}}{A(k)+\left(\frac{B_M(k)}{A(k)}\right)\left(\frac{P_B}{N}\right)^{-1}}\right\}^{\frac{1}{2}},\ 0\right], \tag{21}$$

where,

$$B_M(k) = \sum_{j=0}^{J-1}|\beta(j)H_{M-R}(j,k)H_{B-R}(j,k)|^2|\beta(j)H_{M-R}(j,k)|^2 , \tag{22}$$

and λ_M is chosen so as to satisfy the constraint given as Eq. (4).

IV. COMPUTER SIMULATION

The simulation condition is shown in Table 1. We consider QPSK data modulation. FFT block size N_c and CP length N_g are set to N_c=128, and N_g=16. We assume a frequency-selective block Rayleigh fading having symbol-spaced L=16-path uniform power delay profile. The total transmit power allocation to RS, BS and MT is set as $P_R=P_T/2$ and $P_B=P_M=P_T/4$.

TABLE I. COMPUTER SIMULATION CONDITION

Transmitter /recceiver	Modulation	QPSK
	FFT size	N_c=128 samples
	CP length	N_g=16 samples
	Channel estimation	Ideal
	Power allocation	$P_R=P_T/2$, $P_B=P_M=P_T/4$
Channel	Fading type	L=16 frequency-selective block Rayleigh
	Power delay profile	Uniform power delay profile
	Time delay	$\tau_l=l$ (l=0~L−1)

A. Behavior of transmit FDE weight

Figure 4 shows the magnitudes of the transmit FDE weight at RS, that of the channel gain between MT and the jth RS antenna, and that of the channel gain between the jth RS antenna and BS. $H_{B-R}(j.k)H_{M-R}(j,k)$ is the equivalent channel gain between MT and BS when using the jth RS antenna. The total transmit power-to-noise power ratio P_T/N is set to P_T/N=15dB. The two antennas (J=2) are used at RS.

It can be seen from Figs. 5(a) and 5(b) that the most of transmit power is allocated to the antenna whose equivalent channel gain is high to maximize the antenna diversity gain. Moreover, when the channel gain is high, the transmit FDE acts as the zero-forcing equalization to try to remove the residual inter-symbol interference (ISI).

Figures 5(b) and 5(c) show that the transmit FDE weight is almost identical for the up/downlinks. In ANC relaying transmission, the noise enhancement is produced. The channel gain between MT and RS and that between RS and BS are in general different, resulting in the different noise enhancement in the up/downlinks. However, only a slight difference is seen between transmit FDE weights for the up/downlinks because of high SNR obtained by the antenna diversity.

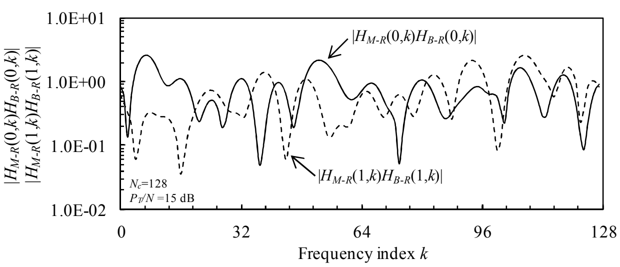
B. BER Performance

Figure 5 plots the average BER performance when using the proposed scheme as a function of the transmit P_T/N. For comparison, the performance when using receive FDE only is also plotted. It can be seen from Fig. 5 that the proposed scheme can improve the BER performance compared to when using receive FDE only. The joint transmit/receive FDE can suppress the residual ISI and can reduce by about 2 dB the required transmit power for achieving BER=10^{-5} compared to when using receiver FDE only. Furthermore, increasing the number J of RS antennas improves the BER performance. Also seen from Fig. 5 is that when using transmit FDE, the almost identical performance is obtained when using transmit FDE weights for the up and downlink.

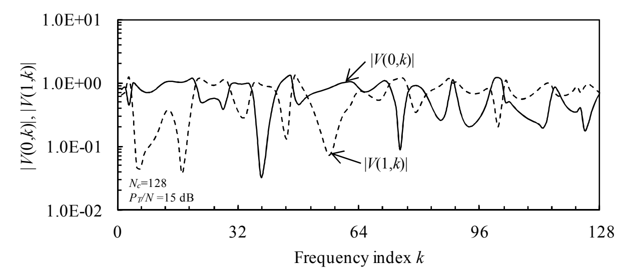
V. CONCLUSION

In this paper, we proposed the joint transmit/receive MMSE-FDE for MIMO-ANC in SC bi-directional relay communications. The jointly optimized FDE weights at RS, BS and MT were derived so as to minimize the end-to-end MSE. It was shown by the computer simulations that the proposed scheme improves the BER performance compared to when using the receive FDE only and that when using transmit FDE, almost identical performance is obtained for the up/downlinks.

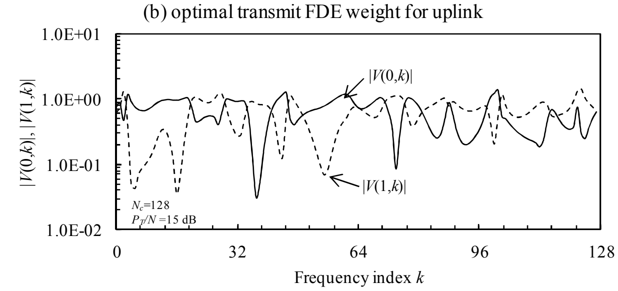
In this paper, BS was assumed to have single antenna. The joint transmit/receive MMSE-FDE for the case multiple antennas are used at both BS and MT is left as our future work.



(a) Equivalent channel between BS and MT

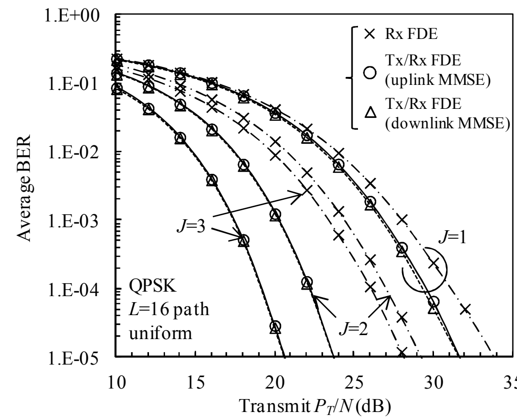


(b) optimal transmit FDE weight for uplink

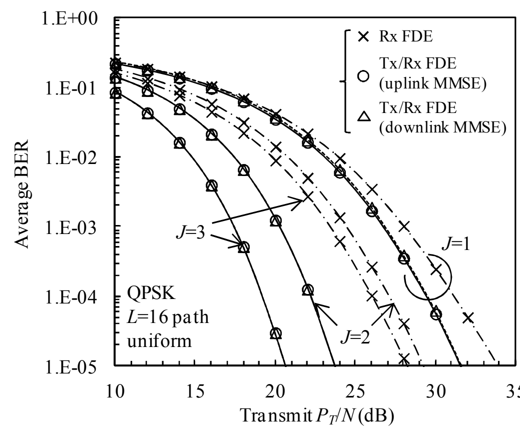


(c) optimal transmit FDE weight for downlink

Figure 4. Transmit FDE weights.



(a) uplink



(b) downlink

Figure 5. Average BER performance.

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