

Factor-Graph-Based Iterative Receiver Design in the Presence of Strong Phase Noise

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Abstract—In this paper, we propose an improved iterative receiver scheme for low density parity check (LDPC) codes transmitted over unknown channels affected by a strong phase noise. For achieving joint channel parameter estimation, data detection and decoding, the proposed algorithm utilizes the sum product algorithm (SPA) implemented on the factor graph (FG) that represents the joint *a posteriori* probability of information symbols and channel parameters given the channel output. Through the iterative use of the soft information on coded symbols from channel decoder, the proposed algorithm employs forward-backward recursions for message passing on the graph. Numerical results for binary LDPC codes show that, the proposed algorithm can be able to cope with a strong phase noise under unknown channel information and achieve nearly the same performance as the optimal coherent receiver under DVB-S2 compliant ESA phase noise model, and only a slightly decrease under strong Wiener model.

Keywords—factor graph; sum-product algorithm (SPA); phase noise; LDPC codes; iterative receiver; Tikhonov parameterization.

I. INTRODUCTION

In recent years, low density parity check (LDPC) codes, typically employed in the 2nd generation Digital Video Broadcasting standards (DVB-S2), have been shown to be capable for achieving near-capacity performance in a very low signal-to-noise ratio (SNR) regime on memoryless channels [1]. However, the optimal coherent receiver in practical satellite communication systems is intractable as the receivers are always affected by phase disturbances, such as a time-varying phase noise due to the quality of the local oscillators. Therefore, the efficient algorithm for phase tracking has become necessary for coherent receiver design in practice.

The iterative detection and decoding algorithm has been proven to be a good solution for the problem of the channel with unknown phase in the last few years [2]–[5]. Recently, propagating messages on a suitable factor graph has been introduced as a systematic tool for deriving efficient low-complexity iterative algorithms [6]. Based on the application of sum-product algorithm (SPA) on the factor graphs, the researchers have proposed some new algorithms to resolve various transmission problems [7]–[10]. A unified approach for iterative receiver design using factor graphs was first proposed in [7], in which the *canonical distributions* are

used for handling continuous random variables. Particularly, Colavolpe *et al.* [10] presented a factor-graph representation of coded system over the random-walk phase channel and proposed various receivers based on some reasonable approximations for the underlying factor graph. Among various receivers, the receiver based on Tikhonov parameterization achieves near-optimal performance with very low complexity. However, the algorithm based on Tikhonov parameterization is deeply dependent on the channel SNR information, which was assumed exactly known at receiver in [10] impractically. In practice, SNR estimation is an essential problem in satellite communication systems [11]. The powerful channel coding like LDPC codes employed enables the systems to operate in extremely low SNR values, which makes the received SNR more difficult to be derived [12]. In [13], a data-aided (DA) gain and SNR estimator was proposed to extend the algorithm in [10] based on the known preamble only in the DVB-S2 scenario. Nevertheless, DA estimators rely on the insertion of pilot symbols in the data frame and lead to power and spectrum efficiency loss, which is aggravated at low SNR for the unacceptable long preambles required to meet certain estimation accuracy. On the other hand, the DA estimator in [13] is only appropriate for DVB-S2 compliant ESA phase noise model which is much minor compared to the strong Wiener phase noise under normal circumstances.

In this paper, we consider a more practical scenario with the channel information unknown at receiver in the presence of strong phase noise. Based on a stretching factor graph and sum-product algorithm (SPA), we extend the Tikhonov parameterization algorithm to obtain a joint iterative detection and decoding algorithm for channel parameter estimation and phase noise tracking. Numerical results by computer simulations show that the proposed algorithm can estimate the channel efficiently and obtain a very close performance compared to the receiver with the channel information exactly known under both Wiener and ESA phase noise models.

The rest of paper is organized as follows. In Section II, the system model is described. Message passing algorithm working on the corresponding factor graph is described in Section III. In Section IV, the joint iterative algorithm is proposed and numerical results are presented and discussed in Section V. Finally, we conclude this paper in Section VI.

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II. SYSTEM MODEL

We consider a typical passband coded transmission system, where the coded symbols are transmitted over an additive white Gaussian noise (AWGN) channel affected by strong phase noise. Assuming the ideal frequency synchronization and perfect symbol timing, the equivalent baseband complex channel model at the receiver is given by

$$r_k = c_k e^{j\theta_k} + n_k, \quad k = 0, \dots, K-1 \quad (1)$$

where r_k is the sample of the matched filter output at time index k . By encoding a information bits sequence $\mathbf{b} \triangleq \{b_k\}$, the sequence $\mathbf{c} \triangleq \{c_k\}$ is a codeword of channel code C modulated over a M -PSK constellation \mathcal{A} . n_k is an independent and identically distributed (i.i.d.), complex white Gaussian noise sample with the variance σ^2 per dimension. Then the received signal-to-noise ratio (SNR) can be defined as $\gamma = 1/2\sigma^2$. θ_k is carrier phase noise introduced by channel at the k th symbol which is unknown to both transmitter and receiver. Phase noise process can be commonly modeled as a random walk (Wiener) process as

$$\theta_k = (\theta_{k-1} + \Delta_k) \bmod 2\pi \quad (2)$$

where $\{\Delta_k\}$ are real i.i.d. Gaussian random variables with zero mean and variance σ_Δ^2 . Assuming θ_0 is uniformly distributed within $[0, 2\pi)$, we can get

$$\begin{aligned} p(\theta_k | \theta_{k-1}, \dots, \theta_0) &= p(\theta_k | \theta_{k-1}) = p_\Delta(\theta_k - \theta_{k-1}) \\ &\triangleq \sum_{l=-\infty}^{\infty} g(0, \sigma_\Delta^2; \theta_k - \theta_{k-1} - l2\pi) \end{aligned} \quad (3)$$

where $g(\eta, \sigma^2; x)$ is a Gaussian distribution in x with mean η and variance σ^2 .

III. MESSAGE PASSING ALGORITHM BASED ON FACTOR GRAPH

Now we will use the factor graph framework and message passing algorithm to develop a low-complexity iterative receiver which performs joint channel parameter estimation, data detection and decoding in the presence of strong phase noise. The optimal decision rule with respect to bit error rate follows the maximum *a posteriori* probability (MAP) criterion, given by

$$\hat{b}_i = \arg \max_{b_i} P(b_i | \mathbf{r}) \quad (4)$$

where $P(b_i | \mathbf{r})$ denotes the *a posteriori* probability mass function (pmf) of the i th information bit b_i given the received symbol vector $\mathbf{r} = \{r_k\}_{k=0}^{K-1}$. This can be obtained by marginalizing the joint posteriori probability distribution function $P(\mathbf{b}, \boldsymbol{\theta}, \sigma | \mathbf{r})$ of the information bits vector \mathbf{b} , the phase noise vector $\boldsymbol{\theta}$ and unknown channel noise standard

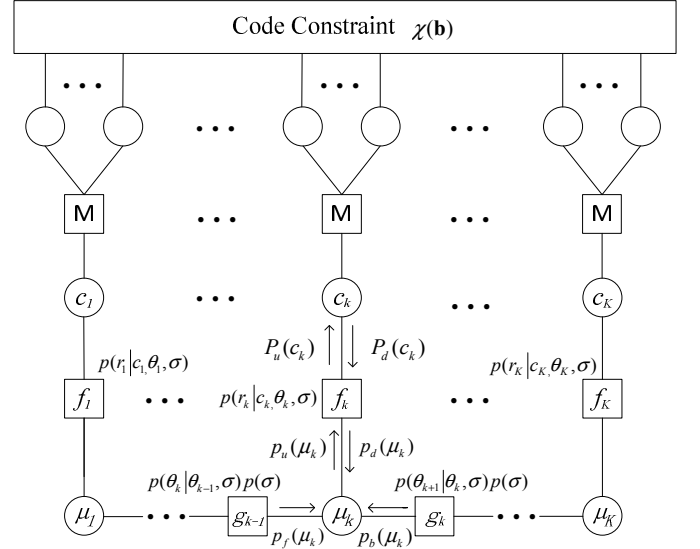


Fig. 1. Factor graph of the joint posteriori distribution function in (5).

deviation σ . We assume the channel keeps constant over a codeword and changes independently from one codeword to another. Then the joint posteriori probability distribution function can be factorized as

$$\begin{aligned} P(\mathbf{b}, \boldsymbol{\theta}, \sigma | \mathbf{r}) &\propto p(\mathbf{r} | \mathbf{b}, \boldsymbol{\theta}, \sigma) p(\mathbf{b}) p(\boldsymbol{\theta} | \sigma) p(\sigma) \\ &\propto \chi[\mathbf{c} = \mu_c(\mathbf{b})] p(\mathbf{r} | \mathbf{c}, \boldsymbol{\theta}, \sigma) p(\boldsymbol{\theta} | \sigma) p(\sigma) \\ &\propto \chi[\mathbf{c} = \mu_c(\mathbf{b})] p(\sigma) p(\theta_0) \prod_{k=1}^{K-1} p(\theta_k | \theta_{k-1}, \sigma) \prod_{k=0}^{K-1} p(r_k | c_k, \theta_k, \sigma) \end{aligned} \quad (5)$$

where $\chi[\mathbf{c} = \mu_c(\mathbf{b})]$ denotes the code constraint function and the receiver constraint function $p(r_k | c_k, \theta_k, \sigma)$ is given by

$$p(r_k | c_k, \theta_k, \sigma) \propto \exp \left\{ -\frac{1}{2\sigma^2} |r_k - c_k e^{j\theta_k}|^2 \right\} \quad (6)$$

The corresponding factor graph of $P(\mathbf{b}, \boldsymbol{\theta}, \sigma | \mathbf{r})$ is shown in Fig. 1. For achieving the MAP decoding more exactly, we remove the cycles of length four in the FG by stretching [6] the variable node σ to every variable node θ_k with a slightly increase in complexity. The new variable node $\mu_k \triangleq (\theta_k, \sigma)$ represents for parameters that need to be estimated in receiver.

Applying sum-product algorithm (SPA) on the factor graph depicted in Fig. 1, we can obtain a suboptimal but low-complexity iterative message passing algorithm [6]. Since exchange of messages in the upper part of FG corresponds to the standard belief propagation based on the structure of channel code, we will only concentrate on the message propagation process in the lower part of the graph.

Some of the messages are shown in the graph. Message from variable node c_k to function node f_k is denoted by $P_d(c_k)$, and $P_u(c_k)$ for the message on the opposite direction, which represent the *a posteriori* probabilities of the coded symbols received from and sent to the channel decoder, respectively. Similarly, the messages from function node f_k to variable node μ_k and the opposite are denoted as $p_d(\mu_k)$ and $p_u(\mu_k)$ respectively. According to SPA, we have

$$p_d(\mu_k) \propto \sum_{x \in \mathcal{X}} P_d(c_k = x) p(r_k | c_k = x, \mu_k) \quad (7)$$

For message evolution on variable node μ_k , a forward-backward recursive schedule is adopted, where the messages $p_f(\mu_k)$ and $p_b(\mu_k)$ represent the *a posteriori* probability density functions (pdfs) of μ_k , given the past and the future received signals respectively, i.e.,

$$p_f(\mu_k) \triangleq p(\theta_k, \sigma | \mathbf{r}_0^{k-1}) = p(\sigma | \mathbf{r}_0^{k-1}) p(\theta_k | \sigma, \mathbf{r}_0^{k-1}) \quad (8)$$

$$p_b(\mu_k) \triangleq p(\theta_k, \sigma | \mathbf{r}_{k+1}^K) = p(\sigma | \mathbf{r}_{k+1}^K) p(\theta_k | \sigma, \mathbf{r}_{k+1}^K) \quad (9)$$

Then a recursive computation for these messages can be obtained as

$$p_f(\mu_k) \propto \int_0^\infty \int_0^{2\pi} p_d(\mu_{k-1}) p_f(\mu_{k-1}) p(\theta_k | \theta_{k-1}) p(\sigma) d\theta_{k-1} d\sigma \quad (10)$$

$$p_b(\mu_k) \propto \int_0^\infty \int_0^{2\pi} p_d(\mu_{k+1}) p_b(\mu_{k+1}) p(\theta_{k+1} | \theta_k) p(\sigma) d\theta_{k+1} d\sigma \quad (11)$$

both of which have the uniformly distributed initial condition. Then the message $p_u(\mu_k)$ is obtained by combining the forward and the backward messages as

$$p_u(\mu_k) \propto p_f(\mu_k) p_b(\mu_k) \quad (12)$$

Finally, the message $P_u(c_k)$ can be calculated as

$$P_u(c_k) \propto \int_0^\infty \int_0^{2\pi} p_u(\mu_k) p(r_k | c_k, \theta_k, \sigma) d\theta_k d\sigma \quad (13)$$

After message propagation in the lower part of the factor graph, the message $P_u(c_k)$ is sent to the upper part of the graph, where the message $P_d(c_k)$ is updated at each iteration based on the standard belief propagation algorithm of the channel code. Then the *a posteriori* probability $P(b_i | \mathbf{r})$ for each information bit can be updated for decision.

IV. PROPOSED ITERATIVE RECEIVER DESIGN SCHEME

The complete message passing process on the factor graph is shown in Section III. However, the channel parameters are always continuous random variables and thus the messages

propagating on adjacent edges are probability density functions (pdfs), such as the messages $p_f(\mu_k)$ and $p_b(\mu_k)$. The SPA for continuous random variables involves the integration and computation of continuous pdfs, which is extremely difficult for practical implementation. Thus, some approximation is needed. We employ the parameterized *canonical distributions* method suggested in [7] for the outgoing messages of the continuous variables. First, we assume the channel parameter σ as a discrete random variable taking on L values within $[0, \sigma_0]$, denoted by $\{\sigma^{(l)}\}_{l=1}^L$. Thus, pdfs $p(\sigma | \mathbf{r}_0^{k-1})$ and $p(\sigma | \mathbf{r}_{k+1}^K)$ in (8) and (9) become probability mass functions (pmfs), given by

$$P(\sigma = \sigma^{(l)} | \mathbf{r}_0^{k-1}) \triangleq \lambda_{f,k}^{(l)}, \quad l=1, \dots, L \quad (14)$$

$$P(\sigma = \sigma^{(l)} | \mathbf{r}_{k+1}^K) \triangleq \lambda_{b,k}^{(l)}, \quad l=1, \dots, L \quad (15)$$

Second, the pdfs $p(\theta_k | \sigma, \mathbf{r}_0^{k-1})$ and $p(\theta_k | \sigma, \mathbf{r}_{k+1}^K)$ can be approximated as Tikhonov distributions as in [10], denoted by

$$p(\theta_k | \sigma = \sigma^{(l)}, \mathbf{r}_0^{k-1}) \propto t(a_{f,k}^{(l)}; \theta_k) \quad (16)$$

$$p(\theta_k | \sigma = \sigma^{(l)}, \mathbf{r}_{k+1}^K) \propto t(a_{b,k}^{(l)}; \theta_k) \quad (17)$$

where $t(\xi; x)$ represents a Tikhonov distribution in x with complex parameter ξ as

$$t(\xi; x) = \frac{1}{2\pi I_0(|\xi|)} \exp\{\text{Re}[\xi e^{-jx}]\} \quad (18)$$

$I_0(\cdot)$ denotes the zeroth-order modified Bessel function of the first kind. Now, instead of continuous pdfs, we just need to propagate the parameters $\lambda_{f,k}^{(l)}$, $\lambda_{b,k}^{(l)}$, $a_{f,k}^{(l)}$ and $a_{b,k}^{(l)}$ for updating messages in SPA calculation. Furthermore, another continuous message $p_d(\mu_k)$ is still need to be simplified in (7). We approximate this message by the nearest Gaussian pdf in the sense of divergence [14], given by

$$p_d(\theta_k, \sigma^{(l)}) \approx g(\rho_k e^{j\theta_k}, 2\sigma^{(l)^2} + \eta_k - |\rho_k|^2; r_k) \propto t(\omega_k^{(l)}; \theta_k) \quad (19)$$

where

$$\omega_k^{(l)} \triangleq \frac{2r_k \rho_k^*}{2\sigma^{(l)^2} + \eta_k - |\rho_k|^2} \quad (20)$$

ρ_k and η_k are the *a posteriori* mean and the *a posteriori* mean square value of transmitted symbol c_k , i.e.,

$$\rho_k \triangleq \sum_{c \in \mathcal{A}} c P_d(c_k = c) \quad (21)$$

$$\eta_k \triangleq \sum_{c \in \mathcal{A}} |c|^2 P_d(c_k = c) \quad (22)$$

Therefore, by substituting the above messages into the forward-backward schedule in (10) and (11), we can finally be able to update the messages $p_f(\mu_k)$ and $p_b(\mu_k)$ recursively. Taking the forward recursion into account first and neglecting the irrelative terms, we can get

$$p_f(\theta_{k+1}, \sigma^{(l)}) = \int_0^{2\pi} p_f(\theta_k, \sigma^{(l)}) p_d(\theta_k, \sigma^{(l)}) p(\theta_{k+1} | \theta_k, \sigma^{(l)}) d\theta_k \quad (23)$$

Substituting (8), (14), (16) and (19) into (23), after some complicated algebra calculation neglected here, we can obtain

$$\lambda_{f,k+1}^{(l)} t(a_{f,k+1}^{(l)}; \theta_{k+1}) \propto \lambda_{f,k}^{(l)} \frac{I_0(|a_{f,k}^{(l)} + \omega_k^{(l)}|)}{I_0(|a_{f,k}^{(l)}|) I_0(|\omega_k^{(l)}|)} \cdot t\left(\frac{a_{f,k}^{(l)} + \omega_k^{(l)}}{1 + \sigma_\Delta^2 |a_{f,k}^{(l)} + \omega_k^{(l)}|}; \theta_{k+1}\right) \quad (24)$$

Using the approximation of $I_0(x) \approx e^x$ for simplification, we can finally get the recursive calculation for the coefficients as

$$\begin{aligned} a_{f,k+1}^{(l)} &= (a_{f,k}^{(l)} + \omega_k^{(l)}) / (1 + \sigma_\Delta^2 |a_{f,k}^{(l)} + \omega_k^{(l)}|) \\ \lambda_{f,k+1}^{(l)} &= \lambda_{f,k}^{(l)} \exp\{|a_{f,k}^{(l)} + \omega_k^{(l)}| - |a_{f,k}^{(l)}| - |\omega_k^{(l)}|\} \end{aligned} \quad (25)$$

Then the backward coefficients recursion can be obtained as the same way, given by

$$\begin{aligned} a_{b,k-1}^{(l)} &= (a_{b,k}^{(l)} + \omega_k^{(l)}) / (1 + \sigma_\Delta^2 |a_{b,k}^{(l)} + \omega_k^{(l)}|) \\ \lambda_{b,k-1}^{(l)} &= \lambda_{b,k}^{(l)} \exp\{|a_{b,k}^{(l)} + \omega_k^{(l)}| - |a_{b,k}^{(l)}| - |\omega_k^{(l)}|\} \end{aligned} \quad (26)$$

Both coefficients need to be initialized as

$$a_{f,0}^{(l)} = a_{b,K-1}^{(l)} = 0; \quad \lambda_{f,0}^{(l)} = \lambda_{b,K-1}^{(l)} = 1/L, \quad \forall l = 1, \dots, L$$

It is implied that the complex parameters $\{a_{f,k}\}$ and $\{a_{b,k}\}$ implicitly represent the channel phase estimation at time index k from the past and the future received symbols respectively. Finally, we can calculate the message $P_u(c_k)$ which is sent to the decoder for updating $P_d(c_k)$, given by

$$\begin{aligned} P_u(c_k) &\propto \sum_l \int_0^{2\pi} p_f(\theta_k, \sigma^{(l)}) p_b(\theta_k, \sigma^{(l)}) p(r_k | c_k, \theta_k, \sigma^{(l)}) d\theta_k \\ &\propto \sum_l \exp\left(-\frac{1}{2\sigma^{(l)^2}} |c_k|^2\right) \lambda_{f,k}^{(l)} \lambda_{f,k}^{(l)} \\ &\quad \cdot \int_0^{2\pi} t(a_{f,k}^{(l)}; \theta_k) t(a_{b,k}^{(l)}; \theta_k) \exp\left\{-\frac{|r_k - c_k e^{j\theta_k}|^2}{2\sigma^{(l)^2}}\right\} d\theta_k \end{aligned} \quad (27)$$

After some algebra calculation, we can obtain

$$P_u(c_k) \propto \sum_l e^{-\frac{|c_k|^2}{2\sigma^{(l)^2}}} \cdot \lambda_{f,k}^{(l)} \lambda_{f,k}^{(l)} \cdot \frac{I_0(|a_{f,k}^{(l)} + a_{b,k}^{(l)} + r_k c_k^* / \sigma^{(l)^2}|)}{I_0(|a_{f,k}^{(l)}|) I_0(|a_{b,k}^{(l)}|)} \quad (28)$$

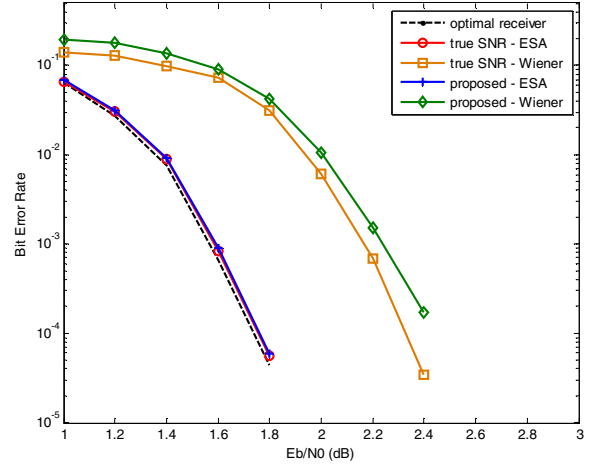


Fig. 2. BER Performance under ESA and Wiener phase noise model.

Finally, we get a practical message propagation algorithm for combating the phase noise and unknown channel parameter, where the computational complexity of the algorithm is linearly proportional to the parameters K and L .

V. NUMERICAL RESULTS

In this Section, the numerical results of the proposed iterative receiver are shown by Monte-Carlo simulation. The channel code considered here is a (3,6)-regular LDPC code with codeword length of 4000, then coded symbols are modulated by QPSK constellation. In each frame, 1 pilot has been inserted in every 20 transmitted symbols to make the iterative algorithm bootstrap [10]. A maximum of 50 iterations of the SPA on the overall graph is allowed. 11 quantization levels are employed at the first iteration for channel parameter σ equally spaced in the interval that corresponds to the received E_b / N_0 distributed in $[1, 3]$ dB. After that, we can only keep three quantization levels which take the maximum values for complexity consideration.

The time-varying phase noise model we consider here includes two types: one is a strong Wiener phase noise model with $\sigma_\Delta = 6^\circ$ described in Section II, the other is DVB-S2 compliant ESA phase noise model described in [15] and [16], where the phase noise $\{\theta_k\}$ is the sum of the outputs of two IIR filters driven by the same white Gaussian noise with unit variance. The corresponding parameter σ_Δ is selected as 0.3° for iterative receiver, which is optimized by simulation.

The bit error rate (BER) performance is shown in Fig. 2. The “optimal receiver” is also shown as a lower bound which corresponds to the performance of an ideal coherent receiver that perfectly knows the phase noise and the channel parameter information with the same code and modulation format. Moreover, an increase of the required SNR due to the insertion of the pilot symbols has also been considered and introduced artificially in this ideal case for fair comparison.

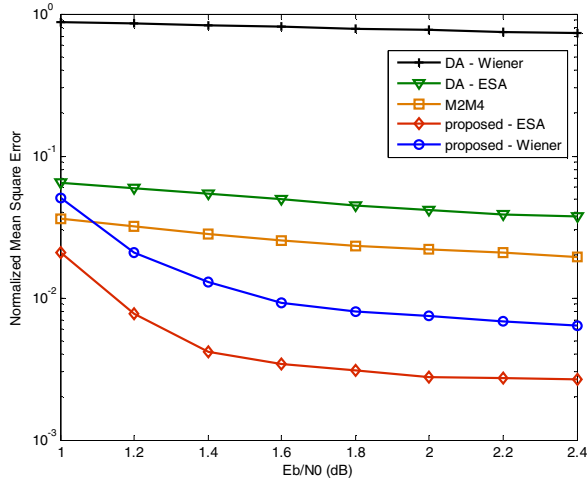


Fig. 3. Normalized MSE of SNR estimation under both phase noise models.

The performance of the iterative algorithm under the SNR exactly known condition (“true SNR” case) is also shown in Fig.2, which performs very closely to the “optimal receiver” case under ESA phase noise model, and only a performance degradation of about 0.6 dB at $\text{BER}=10^{-4}$ under a more severe Wiener phase noise. However, we can see in Section IV that the phase noise estimation process is vitally dependent on the channel SNR information. When the channel SNR is unknown, the performance of the iterative estimation and detection algorithm for the phase noise will deteriorate badly and the information bits cannot be decoded correctly, which is verified through simulation but not shown here due to space constraints. Finally, we can see in Fig.2 that the proposed algorithm by considering channel parameter estimation joint with phase noise tracking greatly coincide with the “true SNR” cases under both phase noise model, only a slight performance degradation about 0.1 dB at $\text{BER}=10^{-4}$ under strong Wiener model.

Fig.3 shows the normalized mean square error (NMSE) performance for SNR estimation of the proposed algorithm under both phase noise models. The performance of an explicit data-aided (DA) estimator and a M2M4 estimator are also shown for comparison. As the M2M4 algorithm is based on the moment estimation, almost the same performance can be obtained under both models. It can be seen that the DA estimator performs very badly when phase noise becomes stronger. Although the M2M4 estimator is hardly affected by phase noise, as we know [12], it cannot get the optimal performance even in the high SNR regime due to blind estimation. Compared with these traditional estimators, we can see the proposed algorithm which performs an implicit and iterative estimation outperforms them under both ESA and Wiener phase noise models. This accuracy of the estimation has ensured the advantage of the algorithm in the BER performance shown in Fig.2.

VI. CONCLUSION

In this paper, the iterative decoding over channels with strong phase noise has been considered. We have proposed an improved iterative receiver scheme based on a message passing algorithm for joint channel parameter estimation, phase noise tracking and decoding under a practical scenario. Instead of explicit estimator, an implicit SNR estimation is embedded into the message propagation process which performs the forward-backward recursions for channel phase estimation. Simulation results demonstrate that the proposed receiver can effectively resist the strong phase noise when the channel parameter is unknown, with performance very close to the ideal coherent receiver.

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