Low-Complexity Channel Estimation for CoMP Multi-user Systems

Xin Wang, Xiaohui Li and Yongqiang Hei
State Key Laboratory of Integrated Services Networks
Xidian University
Xi'an, 710071 Shaanxi, China
Email: shuying4626@163.com, {xhli, yqhei}@mail.xidian.edu.cn

***Abstract*—This paper presents a novel reduced-complexity channel estimator for multi-user Coordinated Multi-point (CoMP) systems. The proposed scheme is based on Space Alternating Generalized-EM (SAGE) algorithm combining with the Least Square (LS) criterion. Aiming at the problem that the conventional joint multi-user LS estimator will introduce the complex inversion of large size matrix when using non-orthogonal pilots, the proposed estimator avoids the inverse operation by transforming the joint multi-user estimation into a series of single-user estimating process in an iterative way. Simulation results verify that the proposed algorithm can reduce the complexity while achieving the optimal performance of joint LS channel estimation for the CoMP systems.**

Keywords- coordinated multipoint transmission(CoMP); multi-user; Space Alternating Generalized-EM (SAGE); channel estimation

I. INTRODUCTION

Coordinated Multi-point transmission/reception(CoMP), which is adopted as one of the key techniques for the 3GPP Long Term Evolution-Advanced(LTE-A), is an effective way to mitigate inter-cell interference and boost system spectrum efficiency[1]. For downlink CoMP, Joint Processing (JP) and Coordinated Scheduling/Beamforming (CS/CB) are applied. For uplink CoMP, performance gain is obtained by joint reception of the transmitted signal at multiple reception points.

The performance of CoMP system is limited by the accuracy of channel estimation, especially in multi-user scenarios because either precoding or scheduling depends on CSI feedback obtained by channel estimation technique. In CoMP multi-user uplink, all the channels between the Base Stations (BSs) and User Equipment (UEs) scattered in multiple coordinated cells need to be estimated. According to the LTE specification[2], the BSs can estimate the channel condition by Reference Signal (RS), which is referred to the pilots. To avoid the co-channel interference (CCI), UEs send RSs which are orthogonal among different sectors or sites and it can be achieved by employing the Zadoff-Chu sequences[2]. However, the number of sequences is limited and the orthogonality between RSs of different UEs could not be completely satisfied. In this case, CCI occurs. In [3], it is shown that if the training sequences for the UEs in different cells are not orthogonal, CCI severely degrades the performance of the whole system.

To mitigate CCI in multi-cell environment, many methods have been researched. In [4], popular channel estimation schemes and their optimal training sequences are presented based on the interference statistics. The criterion of pilot design is further investigated in [5], which focuses on choosing an appropriate DFT phase shift in frequency domain. The schemes mentioned above are designed for multi-cell non-cooperative systems, and not suitable for the CoMP systems. This is because they ignore the distinguishing feature of the CoMP technique, that is, the pilot information of all the served UEs can be shared in the coordinated BSs. In this case, a joint multi-user channel estimation method is put forward in [6]. The Base Station can estimate the channels between BS and UEs in one time by using the conventional Least Square (LS) or the Minimum Mean Square Error (MMSE) algorithms. This method is easy to implement when the pilots of the served users are orthogonal, however, when they are non-orthogonal and the number of the served users or the multipath is large, the complexity is unaffordable.

In this paper, a Space Alternating Generalized-EM (SAGE) based channel estimation algorithm is proposed for multi-user CoMP systems to reduce the computational complexity. At first, we divide the received pilots into a series of subsets, and each subset consists of the pilots of one served user. Then, we alternately update the relevant parameters of the selected subset in a iterative way in combination with LS criterion. Simulation results show that the proposed scheme performs the same as the conventional joint multi-user estimators while its complexity is much lower than the conventional one.

The remainder of the paper is organized as follows. In Section II, the system model of CoMP multi-user uplink is described. Section III gives the existing joint multi-user channel estimation algorithms and develops the proposed SAGE-based estimator. Simulation results are given in Section IV to verify the theoretical analysis. Summary follows in Section V.

Notations: The capital and lowercase letters in the boldface denote matrices and vectors, respectively. $(\cdot)^T$, $(\cdot)^H$, $\mathrm{diag}(\cdot)$ stand for the transpose, the conjugate transpose, the diagnal matrix with $(\cdot)$ on its diagnal. $\mathbf{E}(\cdot)$, $\mathrm{tr}(\cdot)$, $\mathrm{mod}(\cdot)$ denote the expectation, trace, and modulus operator, respectively. $\mathbf{I}$ is the identity matrix.

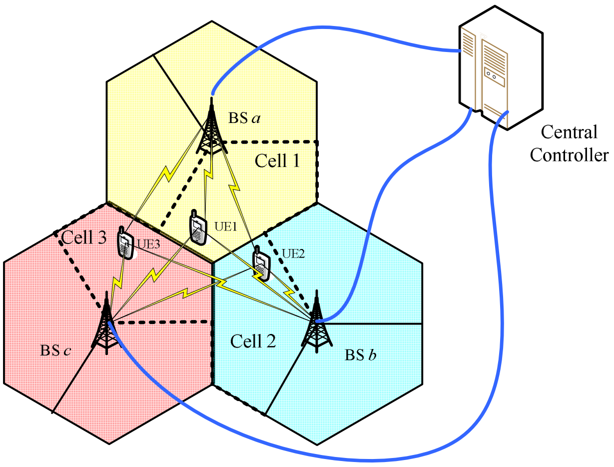

Fig.1 Application scenario

II. SYSTEM MODEL

Considering a CoMP multi-user uplink system with B cooperating cells, each cell is comprised of one BS with $N_r \geq 1$ antennas and M single-antenna cell-edge UEs. The application scenario is shown in Fig. 1, where the CoMP system is modeled as three BSs jointly serving three UEs at the same time. The scheduling and precoding are controlled by a central controller which is connected to each BS by the fiber. In the CoMP systems, data and control information are shared in all the coordinated BSs. In this case, the scheduler at the central controller chooses three UEs, located in different cells, to transmit pilot signal in the same time-frequency resources. Assuming that the pilots of the served UEs are non-orthogonal, CCI occurs.

The diagram of a M-user CoMP channel estimation system is illustrated in Fig. 2. At the transmitter, user i ($i = 1, 2 \ldots M$) generate its pilot signal $\mathbf{s}_i \in \mathbb{C}^N$ in frequency domain, which occupying N subcarriers, then, after the N point Inverse Fast Fourier Transform (IFFT) processing and the introduction of cyclic prefix (CP), the pilot signal is transmitted through its own channel with L multipath .

Without loss of generality, we select the BS b as the receiver. After the removal of CP, the base station can get the estimated CIR vector $\hat{\mathbf{h}}_{b,n_r} = \left[\hat{\mathbf{h}}^{\mathrm{T}}_{b,n_r,1}, \hat{\mathbf{h}}^{\mathrm{T}}_{b,n_r,2} \ldots \hat{\mathbf{h}}^{\mathrm{T}}_{b,n_r,M}\right] \in \mathbb{C}^{LM}$ from M served UEs to its receive antenna n_r by channel estimation technique. FFT mapping is performed in order to get the channel frequency respond.

The CIR from user i to antenna n_r of BS b can be modeled as

$$\mathbf{h}_{b,n_r,i} = \sum_{l=1}^{L} h^l_{b,n_r,i}\,\delta(\tau - \tau_l) \tag{1}$$

where $h^l_{b,n_r,i}$ is the channel coefficient of the l^{th} resolvable path, $h^l_{b,n_r,i} = \alpha_{b,i} g^l_{b,n_r,i}$, $\alpha_{b,i}$ and $g^l_{b,n_r,i}$ represent the large scale and the small scale channel fading coefficient respectively. $g^l_{b,n_r,i}$ is subject to Gaussian distribution with zero mean and variance $\sigma^2_{g_l}$, τ_l is the quantized delay of the l^{th} path.

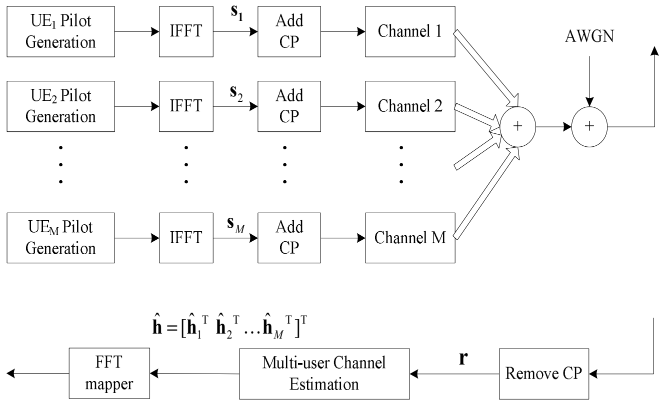

Fig.2 Diagram of a M-user CoMP channel estimation system

The received signal is a superposition of pilot signals transmitted from M different served users. At the receive antenna n_r of coordinated BS b, the time domain received signal is given by [8]

$$\mathbf{r}_{b,n_r} = \mathbf{X}_1\mathbf{h}_{b,n_r,1} + \mathbf{X}_2\mathbf{h}_{b,n_r,2} + \ldots + \mathbf{X}_M\mathbf{h}_{b,n_r,M} + \mathbf{n} \tag{2}$$

where $\mathbf{X}_i = [\mathbf{s}_i^{\Delta cs_1}, \mathbf{s}_i^{\Delta cs_2}, \ldots \mathbf{s}_i^{\Delta cs_L}] \in \mathbb{C}^{N \times L}$ is the equivalent time domain pilot signal of user i , $\mathbf{s}_i^{\Delta cs_1}$ is the right cyclic shift vector of $\mathbf{s}_i$ by ΔCS_i times, in which ΔCS_i is determined by the channel path delay, $\mathbf{h}_{b,n_r,i} = [h^1_{b,n_r,i}, h^2_{b,n_r,i} \ldots h^L_{b,n_r,i}]^T \in \mathbb{C}^L$, $\mathbf{n} \in \mathbb{C}^N$ is the zero-mean complex Gaussian noise vector with $\mathbf{E}\left(\mathbf{n}\mathbf{n}^H\right) = \sigma_n^2 \mathbf{I}_N$.

Eq. (2) can also be expressed in a more compact form

$$\mathbf{r}_{b,n_r} = \mathbf{X}\mathbf{h}_{b,n_r} + \mathbf{n} \tag{3}$$

where $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \ldots \mathbf{X}_M] \in \mathbb{C}^{N \times LM}$ is the reconstructed pilot matrix consist of the sending pilot of served users, $\mathbf{h}_{b,n_r} = \left[\mathbf{h}^T_{b,n_r,1}, \mathbf{h}^T_{b,n_r,2} \ldots \mathbf{h}^T_{b,n_r,M}\right]^T \in \mathbb{C}^{LM}$ is the real CIR between served users and n_r^{th} receive antenna of BS b .

III. MULTI-USER CHANNEL ESTIMATION

In this section, the conventional joint multi-user channel estimation algorithms for CoMP system is outlined first. Then, to reduce the realizing complexities, we derive a SAGE based channel estimation method.

A. Conventional joint multi-user channel estimation

Define the pilot correlation matrix as $\mathbf{Q}_{\mathbf{X}}$, the joint LS estimation of CIR vector, which is from M served users to receive antenna n_r of BS b , is given by

$$\hat{\mathbf{h}}^{LS}_{b,n_r} = (\mathbf{X}^{\mathrm{H}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{H}}\mathbf{r}_{b,n_r} = \mathbf{Q}_{\mathbf{X}}^{-1}\mathbf{X}^{\mathrm{H}}\mathbf{r}_{b,n_r} \tag{4}$$

$$\hat{\mathbf{h}}^{MMSE}_{b,n_r} = \mathbf{R}_{b,n_r}[\mathbf{R}_{b,n_r} + \sigma_n^2 \mathbf{Q}_{\mathbf{X}}^{-1}]^{-1}\mathbf{Q}_{\mathbf{X}}^{-1}\mathbf{X}^{\mathrm{H}}\mathbf{r}_{b,n_r} \tag{5}$$

where $\mathbf{R}_{b,n_r} = \mathbf{E}\left(\mathbf{h}_{b,n_r}\mathbf{h}_{b,n_r}^H\right)$ is the channel covariance matrix. $\mathbf{Q}_\mathbf{X} = \mathbf{X}^H\mathbf{X} \in \mathbb{C}^{LM\times LM}$, in the case the pilot of served users are non-orthogonal, $\mathbf{Q}_\mathbf{X}$ is not a diagonal matrix.

Clearly, the inversion of $LM \times LM$ non-diagonal pilot correlation matrix and the multiplication of two $N \times LM$ matrices will lead to huge computational complexities, especially when the number of served users and channel resolvable path are large.

B. Low-complexity channel estimation for COMP systems

To avoid the huge computational complexities resulting from the inversion and multiplication in Eq.(4) and Eq.(5), a SAGE-based low-complexity channel estimator for CoMP multi-user systems is derived. The algorithm transforms the estimation process of multi-user channels into the estimation of a series of independent single user channels. Unlike the conventional SAGE algorithm for multiple input system [7] [8], we view the received pilot $\mathbf{r}_{b,n_r}$ as the "incomplete data" and define the "complete data" $\mathbf{Y}_{b,n_r,i}$ as the pilot signal received from the i^{th} served user, which is given by

$$\mathbf{Y}_{b,n_r,i} = \mathbf{X}_i\mathbf{h}_{b,n_r,i} + \mathbf{n}_i \tag{6}$$

where $\sum_{i=1}^{M}\mathbf{Y}_{b,n_r,i} = \mathbf{r}_{b,n_r}$, $\sum_{i=1}^{M}\mathbf{n}_i = \mathbf{n}$, $\mathbf{n}_i \in \mathbb{C}^N$ is the zero-mean complex Gaussian noise vector with $\mathbf{E}\left(\mathbf{n}_i\mathbf{n}_i^H\right) = \sigma_{\mathbf{n}_i}^2\mathbf{I}_N$, $\sigma_{\mathbf{n}_i}^2$ is determined by

$$\sigma_{\mathbf{n}_i}^2 = \frac{P_{\mathbf{X}_i}\left|\alpha_{b,i}\right|^2}{\sum_{j=1}^{M}P_{\mathbf{X}_j}\left|\alpha_{b,j}\right|^2} \tag{7}$$

Here $P_{\mathbf{X}_i}$ represents the transmission power of user i. $\left|\alpha_{b,i}\right|^2$ is the large scale channel fading energy of user i.

In this way, we divide the received pilot into a series of subsets, and each subset consists of one served user. At each iteration, the relevant parameters of the selected subset are updated, and the parameters of the unselected subsets remain the same as the last iteration. Based on this, we propose the following three estimators.

The SAGE-LS estimation for the antenna n_r of BS b in CoMP multi-user system can be shown to be:

- Initialization: For: $1 \le i \le M$

$$\hat{\mathbf{Y}}_{b,n_r,i}^{(0)} = \mathbf{X}_i\hat{\mathbf{h}}_{b,n_r,i}^{(0)} \tag{8}$$

- At the k^{th} iteration,($k = 1,2\ldots$):

For $i = 1 + \mathrm{mod}(k-1, M)$, compute:

$$\hat{\mathbf{r}}_{b,n_r,i}^{(k)} = \mathbf{r}_{b,n_r} - \sum_{j=1,j\neq i}^{M}\hat{\mathbf{Y}}_{b,n_r,i}^{(k-1)} \tag{9}$$

$$\hat{\mathbf{h}}_{b,n_r,i}^{(k)} = \left(\mathbf{X}_i^H\mathbf{X}_i\right)^{-1}\mathbf{X}_i^H\hat{\mathbf{r}}_{b,n_r,i}^{(k)} = \frac{1}{P_{\mathbf{X}_i}}\mathbf{X}_i^H\hat{\mathbf{r}}_{b,n_r,i}^{(k)} \tag{10}$$

$$\hat{\mathbf{Y}}_{b,n_r,i}^{(k)} = \mathbf{X}_i\hat{\mathbf{h}}_{b,n_r,i}^{(k)} \tag{11}$$

For $1 \le j \le M$ and $j \ne i$:

$$\hat{\mathbf{h}}_{b,n_r,j}^{(k)} = \hat{\mathbf{h}}_{b,n_r,j}^{(k-1)}, \hat{\mathbf{Y}}_{b,n_r,j}^{(k)} = \hat{\mathbf{Y}}_{b,n_r,j}^{(k-1)} \tag{12}$$

The selection of the initial value of $\hat{\mathbf{h}}_{b,n_r,i}$ is closely related to the convergence speed of the algorithm. Assuming that all the pilot transmitted from other than the i^{th} user to be zeros, $\hat{\mathbf{h}}_{b,n_r,i}^{(0)}$ can be set as

$$\hat{\mathbf{h}}_{b,n_r,i}^{(0)} = \left(\mathbf{X}_i^H\mathbf{X}_i\right)^{-1}\mathbf{X}_i^H\mathbf{r}_{b,n_r} = \frac{1}{P_{\mathbf{X}_i}}\mathbf{X}_i^H\mathbf{r}_{b,n_r} \tag{13}$$

Obviously, compared with Eq.(4) of the joint LS algorithm, Eq.(10) avoids the complex inversion based on the autocorrelation property of the Zadoff-Chu sequences. Suppose the estimated CIR achieves its stable point after N_{iter} iterations, the complexity comparison of joint LS and SAGE-LS algorithm is given in table Ⅰ.

TABLE I. COMPLEXITY COMPARISION

Algorithm	*complex multiplications*	*complex devisions*
Joint LS	$N_r\left[(LM)^3 + 2(LM)^2N + LMN\right]$	$N_r(LM)^2$
SAGE-LS	$N_rLMN + N_{iter}N_rL(2N+1)$	0

The channel estimation of BS a and BS c can be obtained by the same method.

IV. SIMULATION RESULTS

In this section, we simulate the channel estimation error in the form of Normalized Mean Square Error(NMSE) to compare different channel estimators mentioned in this paper.

Consider the CoMP multi-user uplink system with three BSs cooperatively serving three single-antenna UEs located in three cells. Each BS is configured with two receive elements. The system bandwidth is assumed to be 10MHz, and FFT size equals to 1024. We apply the 3GPP Spatial Channel Model (SCM) to generate the local and cross links between BSs and UEs. Each link consists of six resolvable paths, and we assume that the delay of the local and cross channel taps is known perfectly at the BS. Also, the large scale channel fading energy can be known in advance. Channel condition is obtained by non-orthogonal Sounding Reference Signal (SRS) transmitted from different UEs, which occupy $N = 480$ subcarriers. The receive SNR is denoted by $SNR_{b,i} = P_{\mathbf{X}_i}\left|\alpha_{b,i}\right|^2 / \sigma_n^2$.

A. NMSEs of different channel estimation algorithms

Normalized Mean Square Error (NMSE), which is defined as $NMSE_{b,n_r,i} = MSE_{b,n_r,i} / \left|\alpha_{b,i}\right|^2$, is implemented as a metric to characterize the estimation performance of small scale fading channels.

Fig. 3 compares the NMSE performance of SAGE-LS and joint LS estimation algorithms. It is not hard to see that the NMSE of SAGE-LS estimation converge to its optimum after a few iterations, the same as joint LS algorithm. Besides, the number of iterations for convergence is reduced with the receive SNR decreasing. As is shown, the convergence is reached after 3, 6 and 9 times of iteration when SNRs are set to be -2, 8 and 18dB.

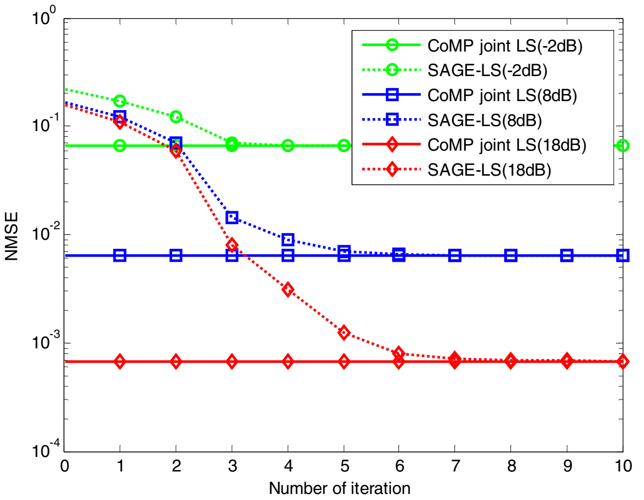

Fig.3. NMSE of SAGE-LS channel estimation for different numbers of iteration. The receive SNR is defined as -2dB, 8dB and 18dB respectively.

B. *Complexity of different channel estimation algorithms*

The complexities of different channel estimators are approximately measured by the numbers of complex multiplications and divisions. Assume that the inversion of a P order matrix requires P^3 multiplication as well as P^2 divisions, the multiplication of a matrix of size $S\times T$ and a matrix of size $T\times U$ needs $S\times T\times U$ complex multiplication.

Table II gives a complexity calculation example of different channel estimation algorithms. According to the system parameters and the simulation result mentioned above, the iteration numbers N_{iter} are set to be 3, 6 and 9, which is in accordance with the receive SNR to guarantee convergence. Take $N_{iter}=3$ for example, it can be seen that the joint LS estimation needs 651024 complex multiplications as well as 648 divisions, meanwhile, the numbers reduced to 51867 multiplications and 0 divisions for the SAGE-LS algorithm, which is approximately one tenth of the original joint LS method.

TABLE II. EXAMPLE OF COMPLEXITY COMPARISON

Algorithm	*complex multiplications*			*complex divisions*
	$N_{iter}=3$	$N_{iter}=6$	$N_{iter}=9$	$N_{iter}=3,6,9$
Joint LS	***651024***	***651024***	***651024***	***648***
SAGE-LS	***51876***	***86472***	***121068***	***0***

V. CONCLUSIONS

In this paper, we have derived a novel SAGE-based LS channel estimator for CoMP multi-user systems. The estimator transforms the estimation process of multi-user channels into the estimation of a series of independent single user channels with iterations, which can avoid the complex inversion of large size matrix. Simulation results validate that the proposed algorithm obtain huge complexity reduction, while achieving the optimal performance of joint LS channel estimation.

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