

Analysis of Multiband Joint Detection Framework for Waveform-based Sensing in Cognitive Radios

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Abstract—An optimal joint multiband-sensing framework is studied to maximize the throughput in cognitive radio networks. Convexity range for this non-convex framework is calculated under certain channel conditions keeping either sensing time or threshold constant. Also joint convexity range for both sensing time and threshold is determined. Waveform based detection, which is accurate and better than other detection schemes in terms of reliability and convergence time, is applied.

Index Terms – *Waveform detection, convex optimization, cognitive radio, spectrum sensing*

I. INTRODUCTION

Cognitive radio (CR) represents a class of terminals that can adjust their transmission parameters according to different environment. It can provide solution to the spectral congestion problem by giving the notion of opportunistic spectrum usage. The presence or absence of licensed users must be detected accurately by the CR user in order to mitigate the interference to licensed communications.

In wideband environment, sensing a large number of primary channels is a difficult task. To alleviate this issue, the bandwidth of interest is divided into sub bands and then analyzed sequentially [1], [2] or in parallel [3]-[5].

Spectrum sensing in cognitive radio networks can be categorized into local sensing and cooperative sensing. Cooperative sensing can be implemented in a centralized or a distributed mode. The detection accuracy has been studied to increase the sensing efficiency subjected to the interference avoidance constraint [7].

The various detection techniques are being applied in local sensing like energy detection, matched filter and cyclostationary feature detection etc [8], [9].

In this work, we have exploited waveform based signal detection. Waveform based detection is one of the most accurate spectrum sensing techniques and is slightly more complex than energy detection. It outperforms other detection schemes in terms of reliability and convergence time [10]. A multiband detection framework [6], [12] is investigated that jointly detects a Binary Phase Shift Keying signal (BPSK) over non-overlapping sub bands of Gaussian channels. Convexity range for maximizing the aggregate opportunistic

throughput of CR user under the constraint of tolerable interference to licensed primary user (PU) is found.

II. SYSTEM DESCRIPTION

Consider a CR system model comprising of licensed primary users, CR users and a fusion center. When the unlicensed users are sensing the Gaussian channel, hypothesis H_1 denotes the licensed user is active whereas hypothesis H_0 denotes the licensed user is inactive.

A. Individual sub band sensing

The wideband spectrum to be sensed is distributed into K non-overlapping sub bands. L primary users share the spectrum, where each PU occupies subset $R_L \in R$ of the K sub bands. Here R is the signal sample space. At any instant of time, some of the K sub bands might be unused and could be exploited by the CR users. On detection of an idle channel, CR user could tune its transmission parameters to avail this spectrum opportunity. However, the CR user has to monitor the behavior of LU continuously and vacate the channel immediately if PU reappears.

B. Waveform based Detection:

Waveform detection is performed for each sub band during the sensing time slot τ . The received signal in time domain has the following form [10]

$$y(n) = \Re e[y(n)s^*(n)]$$

where “*” denotes the conjugate, $y(n)$ is received signal and $s(n)$ is the transmitted primary signal which is assumed to be an identical and independent random process (iid) with zero mean and variance σ_s^2 . Each medium access control frame has length T having one sensing timeslot τ and one data transmission slot $T - \tau$. For a given sub band, the number of samples to be sensed is given as $N = \tau f_s$, where f_s is the sampling frequency and τ is sensing time.

The decision statistic T_k for each of the K sub bands can be written as

$$T_k(y) = \Re e \left[\sum_{n=1}^N [y(n)s^*(n)] \right]$$

$$H_{0,k} : T_k(y) = \Re e \left[\sum_{n=1}^N [w(n)s^*(n)] \right]$$

$$H_{1,k} : T_k(y) = \sum_{n=1}^N |s(n)|^2 + \Re e \left[\sum_{n=1}^N [w(n)s^*(n)] \right]$$

where $w(n)$ is additive white Gaussian noise with zero mean and variance σ_w^2 .

Lemma 1: For large N , if received signal is BPSK, noise is real-valued and both are iid Gaussian variables, PDF of $T(y)$ under H_0 can be given as Gaussian distribution with zero mean $\mu_0 = 0$ and variance $\sigma_0^2 = \frac{1}{N}(\sigma_w^2\sigma_s^2)$

Proof: Detailed proof can be found in appendix A. ■

The probability of false alarm P_f , probability of detection P_d , probability of missed detection P_m and threshold vector \mathcal{E}_K , can be written as [3]

$$\mathcal{E} = [\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_K]^T$$

$$P_f(\mathcal{E}, \tau) = [P_f^{(1)}(\mathcal{E}_1, \tau), \dots, P_f^{(K)}(\mathcal{E}_K, \tau)]^T$$

$$P_m(\mathcal{E}, \tau) = [P_m^{(1)}(\mathcal{E}_1, \tau), \dots, P_m^{(K)}(\mathcal{E}_K, \tau)]^T$$

$$P_d(\mathcal{E}, \tau) = [P_d^{(1)}(\mathcal{E}_1, \tau), \dots, P_d^{(K)}(\mathcal{E}_K, \tau)]^T$$

The test statistics T_k is compared with threshold \mathcal{E}_K . The received SNR γ of the K_{th} sub band can be given as

$$\gamma_k = \frac{E(|S_k|^2)}{\sigma_w^2} = \frac{\sigma_s^2}{\sigma_w^2}$$

where $E(\cdot)$ denotes expectation.

Definition 1: Probability of false alarm for the k th sub band is defined as

$$P_f^{(k)}(\mathcal{E}_k, \tau) = P_r(T_k > \mathcal{E}_k | H_{0,k}) = Q\left(\frac{\mathcal{E}_k - \mu_{0,k}}{\sqrt{\sigma_{0,k}^2}}\right)$$

If both signal and noise are BPSK variables, P_f for any k sub band may be given as:

$$P_f^{(k)}(\mathcal{E}_k, \tau) = Q\left(\frac{\mathcal{E}_k}{\sigma_w^2} \sqrt{\frac{\tau f_s}{\gamma}}\right)$$

where $Q(x)$ is the complementary distribution function of the

standard Gaussian. i.e. $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt$

Lemma 2: For a large N , if received signal is BPSK and noise is real-valued signal and both are iid Gaussian variables, the PDF of $T(y)$ under hypothesis H_1 can be approximated by a Gaussian distribution with mean $\mu_1 = \sigma_s^2$ and variance $\sigma_1^2 = 1/N(\sigma_s^4\gamma)$

Proof: Detailed proof can be found in appendix A. ■

Definition 2: Probability of detection for the k th sub band is defined as

$$P_d^{(k)}(\mathcal{E}_k, \tau) = P_r(T_k > \mathcal{E}_k | H_{1,k}) = Q\left(\frac{\mathcal{E}_k - \mu_{1,k}}{\sqrt{\sigma_{1,k}^2}}\right)$$

If the received signal is BPSK and noise is real-valued, P_d for any k sub band may be given as:

$$P_d^{(k)}(\mathcal{E}_k, \tau) = Q\left(\left(\frac{\mathcal{E}_k}{\sigma_w^2} - \gamma_k\right) \sqrt{\frac{\tau f_s}{\gamma_k}}\right)$$

III. MULTIBAND JOINT DETECTION

Let the opportunistic throughput of the CR user at the k th sub band be e_k and vector $e = [e_1, e_2, \dots, e_K]^T$. The available throughput can be defined as

$$Y(\mathcal{E}, \tau) = \left(\frac{T-\tau}{T}\right) e^T (1 - P_f(\mathcal{E}, \tau))$$

where $1 - P_f$ is probability of detection of vacant sub bands by the CR user, $\left(\frac{T-\tau}{T}\right)$ is time for opportunistic transmission and

1 denotes the all-ones vector. Here \mathcal{E} and τ are jointly obtained to achieve the highest opportunistic throughput of the CR user under the constraint of minimum interference to PU network [14]. Increasing τ reduces the data transmission time ($T-\tau$) and increases probability of detection of vacant sub band $1 - P_f$. On the contrary, maximizing $Y(\mathcal{E}, \tau)$ for given τ , increases the probability of missed detection P_m and interference with PU.

Let ς_k be the cost of interfering with PU in the k th sub band and $\boldsymbol{\varsigma} = [\varsigma_1, \varsigma_2, \dots, \varsigma_K]^T$ be the relative priorities of the sub bands from the licensed network perspective [12]. The cumulative interference to j th PU may be defined as

$$I_j(\mathcal{E}, \tau) = \sum_{j \in R_j} \varsigma_j P_m^j(\mathcal{E}_j, \tau)$$

where $j=1, 2, \dots, J$. For a single-user network, all sub bands would be used by one primary user and $J=1$.

Mathematically the optimization problem is stated as

$$\max_{(\boldsymbol{\varepsilon}, \tau)} Y(\boldsymbol{\varepsilon}, \tau) \quad (P1)$$

$$s.t. \quad I_j(\boldsymbol{\varepsilon}, \tau) \leq \xi_j \quad (C1)$$

$$P_m(\boldsymbol{\varepsilon}, \tau) \leq \nu \quad (C2)$$

$$P_f(\boldsymbol{\varepsilon}, \tau) \leq \delta \quad (C3)$$

where ξ_j denotes the maximum cumulative interference tolerated by the j th primary user given the probability of missed detection does not exceed the limit $\nu = [v_1, v_2, \dots, v_j]^T$. Each sub band should achieve a minimum opportunistic spectral utilization given by $[1 - \delta_1, 1 - \delta_2, \dots, 1 - \delta_k]^T$ [3].

A. Convexity Analysis

Problem (P1) is reformulated to simplify analysis [5] as

$$\min_{(\boldsymbol{\varepsilon}, \tau)} Y_{loss}(\boldsymbol{\varepsilon}, \tau) \quad (P2)$$

$$Y_{loss}(\boldsymbol{\varepsilon}, \tau) = e^T \left[P_f(\boldsymbol{\varepsilon}, \tau) \left(1 - \frac{\tau}{T} \right) + \frac{\tau}{T} \right]$$

where Y_{loss} is the opportunistic throughput loss.

IV. SENSING PARAMETERS OPTIMIZATION

The above problem can be considered in the convex optimization category under some practical conditions.

1) Convexity Range: Joint Multiband Sensing Duration Configuration

(P2) can be considered as a convex optimization problem under following conditions

Lemma 3: The function $P_f^{(k)}(\boldsymbol{\varepsilon}_k, \tau)$ is convex in $\boldsymbol{\varepsilon}_k$ and τ if

$$P_f^{(k)}(\boldsymbol{\varepsilon}_k, \tau) \leq Q\left(\frac{1}{\sqrt{3}}\right)$$

Proof: The Hessian of the function can be calculated as

$$c_k \times \begin{bmatrix} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right) \frac{1}{\sigma_w^2} \frac{\tau f_s}{\gamma_k} & \frac{1}{2} \left(\frac{\tau f_s}{\gamma_k} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right)^2 - 1 \right) \\ \frac{1}{2} \left(\frac{\tau f_s}{\gamma_k} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right)^2 - 1 \right) & \left(\frac{1}{4\sigma_w^2} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right) \left(\frac{f_s}{\gamma_k} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right)^2 + \frac{1}{\tau} \right) \right) \end{bmatrix}$$

$$\text{where } c_k = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{\sigma_w^2} \sqrt{\frac{f_s}{\gamma_k \tau}} \right) \exp \left(- \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right)^2 \frac{\tau f_s}{2\gamma_k} \right)$$

$$\text{The determinant is } c_k^2 \times \left(\frac{3}{4} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right)^2 \frac{\tau f_s}{2\gamma_k} \right) - \frac{1}{4}$$

$$\text{which is non negative if } \left(\left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} \right) \sqrt{\frac{\tau f_s}{\gamma_k}} \right) \geq \frac{1}{\sqrt{3}}$$

Consequently the matrix is positive semi-definite implying that $P_f^{(k)}(\boldsymbol{\varepsilon}_k, \tau)$ is convex under the stated condition. ■

Lemma 4: For BPSK case the function $P_m^{(k)}(\boldsymbol{\varepsilon}_k, \tau)$ is convex in $\boldsymbol{\varepsilon}_k$ and τ if $P_m^{(k)}(\boldsymbol{\varepsilon}_k, \tau) \leq Q\left(\frac{1}{\sqrt{3}}\right)$

Proof: The Hessian of the function can be calculated as

$$d_k \times \begin{bmatrix} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right) \left(\frac{1}{\sigma_w^2} \frac{\tau f_s}{\gamma_k} \right) & \left(\frac{\tau f_s}{2\gamma_k} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right)^2 - \frac{1}{2} \right) \\ \left(\frac{\tau f_s}{2\gamma_k} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right)^2 - \frac{1}{2} \right) & \left(\frac{f_s}{4\gamma_k \sigma_w^2} \frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} (-\gamma_k)^3 + \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right) \frac{1}{4\tau \sigma_w^2} \right) \end{bmatrix}$$

$$\text{where } d_k = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{\sigma_w^2} \sqrt{\frac{f_s}{\gamma_k}} \right) \exp \left(- \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right)^2 \frac{\tau f_s}{2\gamma_k} \right)$$

The determinant of the equation is

$$Det(.) = d_k^2 \times \left(\frac{3}{4} \left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right)^2 \frac{\tau f_s}{\gamma_k} \right) - \frac{1}{4}$$

$$\text{which is non negative if } \left(\left(\frac{\boldsymbol{\varepsilon}_k}{\sigma_w^2} - \gamma_k \right) \sqrt{\frac{\tau f_s}{\gamma_k}} \right) \geq \frac{1}{\sqrt{3}}$$

Consequently, the matrix is positive semi-definite implying that $P_m^{(k)}(\boldsymbol{\varepsilon}_k, \tau)$ is convex under the stated condition. ■

Lemma 5: The function $Y_{loss}(\boldsymbol{\varepsilon}, \tau)$ is convex in $\boldsymbol{\varepsilon}_k$ and τ if

$$P_f^{(k)}(\boldsymbol{\varepsilon}_k, \tau) \leq Q\left(\frac{1}{\sqrt{3}}\right) \text{ and } \tau/T \leq 0.5$$

Proof: The Hessian of the function can be calculated as

$$\begin{bmatrix} \left(1 - \frac{\tau}{T} \right) \frac{\partial^2 P_f^{(k)}}{\partial \boldsymbol{\varepsilon}_k^2} & \left(1 - \frac{\tau}{T} \right) \frac{\partial P_f^{(k)}}{\partial \boldsymbol{\varepsilon}_k \partial \tau} - \frac{1}{T} \frac{\partial P_f^{(k)}}{\partial \boldsymbol{\varepsilon}_k} \\ \left(1 - \frac{\tau}{T} \right) \frac{\partial P_f^{(k)}}{\partial \boldsymbol{\varepsilon}_k \partial \tau} - \frac{1}{T} \frac{\partial P_f^{(k)}}{\partial \boldsymbol{\varepsilon}_k} & \left(1 - \frac{\tau}{T} \right) \frac{\partial^2 P_f^{(k)}}{\partial \tau^2} - \frac{2}{T} \frac{\partial P_f^{(k)}}{\partial \tau} \end{bmatrix}$$

It is positive definite for $P_f^{(k)}(\epsilon_k, \tau) \leq Q\left(\frac{1}{\sqrt{3}}\right)$ and $\tau/T \leq 0.5$ ■

Under Lemmas 3, 4 and 5, P2 is a convex optimization problem where both the objective and constraint functions are convex which can be easily solved with numerical algorithms such as interior point method [13].

2) Convexity Range : Constant τ

It is a crucial task to sense the channel periodically by the CR user as the spectrum must be immediately vacated when the primary user reappears. This can form the basis of a two stage framework. In the first step, the optimal threshold vector ϵ is determined based on the assumption of constant sensing slot duration. Later τ is updated based on the information obtained from the previous stage [6]. The multiband joint detection framework makes decisions over multiple frequency bands jointly which is essential for implementing efficient CR networks [12]. (P2) can accordingly be simplified into an equivalent form as

$$\min_{(\epsilon)} Y_{loss}(\epsilon) \quad (P3)$$

$$s.t. \quad I_j(\epsilon) \leq \xi_j \quad (C4)$$

$$\epsilon_{k,\min} \leq \epsilon_k \leq \epsilon_{k,\max} \quad (C5)$$

where $\epsilon_{k,\min}$ and $\epsilon_{k,\max}$ can be determined owing to the monotonically non-increasing nature of the Q-function as

$$\epsilon_{k,\min} = \sigma_w^2 \left(\sqrt{\frac{\gamma_k}{\tau f_s}} Q^{-1}(\delta_k) \right)$$

$$\text{and} \quad \epsilon_{k,\max} = \sigma_w^2 \left(\sqrt{\frac{\gamma_k}{\tau f_s}} Q^{-1}(1 - v_k) + \gamma_k \right)$$

Although the constraint (C5) is linear, the problem (P3) can be transformed into a convex optimization problem under the following conditions.

$$0 < v_k \leq 1/2 \quad \text{and} \quad 0 < \delta_k \leq 1/2$$

Lemma 6: The function $P_f^{(k)}(\epsilon_k)$ is convex in ϵ_k if $P_f^{(k)}(\epsilon_k) \leq 1/2$ and the function $P_m^{(k)}(\epsilon_k)$ is convex in ϵ_k if $P_m^{(k)}(\epsilon_k) \leq 1/2$

Proof: Taking the second derivative of $P_f^{(k)}(\epsilon_k)$ gives

$$\frac{d^2 P_f^{(k)}}{d\epsilon_k^2} = \frac{(\tau f_s)^{3/2}}{\sqrt{2\pi}\sigma_w^4 \gamma_k^{3/2}} \left(\frac{\epsilon_k}{\sigma_w^2} \right) \exp \left(- \left(\frac{\epsilon_k}{\sigma_w^2} \right)^2 \frac{\tau f_s}{2\gamma_k} \right)$$

For the second derivative to be larger than or equal to zero ϵ_k should be greater or equal to zero. Hence the maximum value of $P_f^{(k)}$ would be 0.5 and the range $P_f^{(k)} \leq 1/2$ also holds.

Similarly by taking the second derivative of $P_d^{(k)}(\epsilon_k)$ gives

$$\frac{d^2 P_d^{(k)}}{d\epsilon_k^2} = \frac{(\tau f_s)^{3/2}}{\sqrt{2\pi}\sigma_w^4 \gamma_k^{3/2}} \left(\frac{\epsilon_k}{\sigma_w^2} - \gamma_k \right) \exp \left(- \left(\frac{\epsilon_k}{\sigma_w^2} - \gamma_k \right)^2 \frac{\tau f_s}{2\gamma_k} \right)$$

The second derivative is concave for $\epsilon_k \leq \gamma_k \sigma_w^2$. Hence $P_m=1-P_d$ is convex under the given condition. ■

Thus the problem (P3) becomes a convex problem. The solutions would thus provide the joint optimal thresholds for K narrowband detectors.

3) Convexity Range : Uniform Threshold

When a uniform threshold $\epsilon_1 = \epsilon_2 = \dots = \epsilon_k = \epsilon$ is chosen for all the sub bands and the throughput loss is to be minimized with the channel sensing time as the decision variable, (P2) is simplified into an equivalent form as follows

$$\min_{(\tau)} Y_{loss}(\tau) \quad (P4)$$

$$s.t. \quad I_j(\tau) \leq \xi_j \quad (C6)$$

$$\tau_{\min} \leq \tau \leq \tau_{\max} \quad (C7)$$

$$\text{where} \quad \tau_{\min} = \left(Q^{-1}(\delta) \sqrt{\gamma_k} / \sqrt{f_s} \left(\frac{\epsilon}{\sigma_w^2} \right) \right)^2$$

$$\text{and} \quad \tau_{\max} = \left(Q^{-1}(1-v) / \sqrt{\frac{f_s}{\gamma_k}} \left(\frac{\epsilon}{\sigma_w^2} - \gamma_k \right) \right)^2$$

Bounds on the problem (P4) can be determined by introducing the conditions $0 < v_k \leq 1/2$ and $0 < \delta_k \leq 1/2$ classifying it into the convex optimization category.

Lemma 7: The function $P_f^{(k)}(\tau)$ is convex in τ if $P_f^{(k)}(\tau) \leq 1/2$ and the function $P_m^{(k)}(\tau)$ is convex in ϵ_k if $P_m^{(k)}(\tau) \leq 1/2$

Proof: Taking the second derivative of $P_f^{(k)}(\tau)$ gives

$$\frac{d^2 P_f^{(k)}}{d\tau^2} = \frac{1}{4\sqrt{2\pi}} \left(\frac{\epsilon}{\sigma_w^2} \right) \sqrt{\frac{f_s}{\gamma_k}} \exp \left(- \left(\frac{\epsilon}{\sigma_w^2} \right)^2 \frac{\tau f_s}{2\gamma_k} \right) \left(\frac{f_s}{\gamma_k} \left(\frac{\epsilon}{\sigma_w^2} \right)^2 + \frac{1}{\tau} \right)$$

Since $P_f^{(k)} \leq 1/2$ holds, we get $\epsilon_{\max} \geq \sigma_w^2 \sqrt{\frac{\gamma_k}{\tau f_s}}$. The second derivative is greater than or equal to zero, implying that $P_f^{(k)}$ is concave in τ . Following the same procedure for $P_d^{(k)}$, it is

found to be concave whereas $\varepsilon_{\min} \leq \left(\gamma_k - \sqrt{\frac{\gamma_k}{f_s \tau_k}} \right) \sigma_w^2$. Hence

$P_m = 1 - P_d$ is a convex function. ■

(P4) also takes the form of convex optimization problem and can be solved with simpler algorithms. The solutions would thus provide the optimal sensing slot duration for the K narrowband detectors.

APPENDIX A

Proof of Lemma 1:

Mean:

$$\begin{aligned} \mu_0 &= \frac{1}{N} E \left[\Re \{ w(n) s^*(n) \} \right] \\ &= \frac{1}{N} E \left[\Re \{ w_r(n) + j w_i(n) \} [s_r(n) - j s_i(n)] \right] \\ &= \frac{1}{N} E [w_r(n) s_r(n)] + E [w_i(n) s_i(n)] = 0 \end{aligned}$$

since $s(n)$ and $w(n)$ are iid and zero mean random variables.

Variance:

$$\begin{aligned} \sigma_0^2 &= \frac{1}{N} E \left| \Re \{ w(n) s^*(n) \} - E \left[\Re \{ w(n) s^*(n) \} \right] \right|^2 \\ &= \frac{1}{N} E \left[w_r^2(n) s_r^2(n) + w_i^2(n) s_i^2(n) \right. \\ &\quad \left. + 2 E [w_r(n) w_i(n) s_r(n) s_i(n)] \right] \\ &= \frac{1}{N} \sigma_w^2 \sigma_s^2 \end{aligned}$$

since $s(n)$ is BPSK and $w(n)$ is real valued.

Proof of Lemma 2:

Mean:

$$\mu_1 = [E |s(n)|^2] + E \left[\Re \{ w(n) s^*(n) \} \right] = \sigma_s^2$$

since $s(n)$ and $w(n)$ are iid and zero mean random variables.

Variance:

$$\begin{aligned} \sigma_1^2 &= \frac{1}{N} E \left[|s(n)|^2 + \Re \{ w(n) s^*(n) \} - E [|s(n)|^2 + \Re \{ w(n) s^*(n) \}] \right|^2 \\ &= \frac{1}{N} E \left[|s(n)|^4 + w^2(n) s^2(n) + 2 E [|s(n)|^2 w(n) s(n)] - \sigma_s^4 \right] \\ &= \frac{1}{N} E \left[|s(n)|^4 + \sigma_s^2 \sigma_w^2 - \sigma_s^4 \right] \\ &= \frac{1}{N} [\sigma_s^4 + \sigma_s^2 \sigma_w^2 - \sigma_s^4] = \frac{1}{N} \sigma_w^2 \sigma_s^2 \end{aligned}$$

since for BPSK, $E[|s(n)|^4] = \sigma_s^4$

CONCLUSION

In this paper waveform based spectrum sensing technique is exploited which is one of the most accurate and less complex. Unlike energy detection, it gives good performance even at low SNR and performs similarly as energy detection at high SNR. A joint spectrum sensing technique that detects the licensed user signal, which is assumed to be BPSK in this paper, over multiple frequency bands, is explored. The spectrum sensing problem is formulated as a set of optimization problems and convexity range of these problems is determined under certain constraint, maximizing the opportunistic throughput of a cognitive radio system. Due to space/page limitation, numerical results are deferred for full/journal paper.

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