A New Noise Variance Based Layered Pruning ML-DFE Algorithm

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***Abstract* - A new noise variance based reduced maximum likelihood decision feedback equalization (ML-DFE) algorithm has been developed. This algorithm reduces the calculation complexity by exploring the intrinsic statistical properties layer by layer. Through setting layered thresholds, part of the nodes in the searching process will be cut by comparing with the thresholds. Simulation results show that the complexity drops lots while the performance drops small.**

Keywords* - *decision feedback equalization (DFE), maximum likelihood decision feedback equalization (ML-DFE), maximum likelihood (ML) algorithm, multi-input multi-output (MIMO) systems

I. INTRODUCTION

The multiple-input multiple-output (MIMO) system has emerged in recent years as one of the most significant technical breakthroughs in modern communications since it has the potential to increasing the spectrum efficiency and transmission capacity [1][2]. Minimum mean-square error (MMSE) or zero-forcing (ZF) detection algorithm can be applied in MIMO detection. But the linear detectors suffer significant performance loss in fading channel. The scheme of maximum likelihood (ML) detection offers the best decoding performance [3]. However, the computational complexity increases exponentially with the number of transmit antennas and modulation order. Sphere decoding is a suboptimal decoding method that has been proposed to reduce complexity by reducing the size of the search space for determining the optimum transmitted symbol vector [4]. Although it can reduce the average complexity, in some certain case the complexity of sphere decoding is still too high since a large number of candidates need to be searched. Thus, there is still a need for reduced complexity MIMO detection and many works have been done [5][6]. The maximum likelihood decision feedback equalization (ML-DFE) has been proposed [7] to solve the problem. But, the complexity of the ML-DFE is still high for some utilizing [8]. So we introduce a new noise variance based layered pruning ML-DFE algorithm in this article.

This paper is organized as follows. The model of MIMO is introduced in section II. The ML-DFE algorithm is explained in section III. In section IV, the proposed algorithm is presented. The simulation results will be shown in section V. The conclusion will be drawn in section VI.

II. MIMO SYSTEMS

The MIMO system can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_{Nt}]^T$ and $\mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_{Nr}]^T$ are the $Nt \times 1$ and $Nr \times 1$ transmit and receive vector, and $\mathbf{n} = [n_1 \quad n_2 \quad \cdots \quad n_{Nr}]^T$ is a $Nr \times 1$ independent and identical distributed (i.i.d.) complex zero-mean Gaussian noise vector with the variance σ^2 per dimension. $\mathbf{H}$ is an $Nr \times Nt$ matrix, where h_{ij} is the complex channel gain and satisfies

This work is supported by the National Science Foundation of China (Grant No. NSFC #61071083). Corresponding author: Jianjun Wu; phone: 86-010-62752848; e-mail: just@pku.edu.cn

$E\left[\left\|h_{ij}^2\right\|\right]=1$. In this article we only talk about the case of *Nt=Nr=N/2*, but all the analysis and results can be applied to the case when $Nt \neq Nr$.

The ML algorithm is to calculate

$$\hat{\mathbf{x}}_{ML} = \arg\min_{\mathbf{x}\in\aleph} \|\mathbf{y} - \mathbf{Hx}\|^2 \quad (2)$$

III. ML-DFE ALGORITHM

A. Tree search and DFE algorithm

The ML-DFE algorithm can be executed in the complex system and all the variables are complex. In this article, we use the real variables instead of the complex variables for simplicity. Actually the proposed algorithm can work in the complex system.

The real form of equation (1) is

$$\begin{pmatrix}\Re(\mathbf{y})\\ \Im(\mathbf{y})\end{pmatrix} = \begin{pmatrix}\Re(\mathbf{H}) & -\Im(\mathbf{H})\\ \Im(\mathbf{H}) & \Re(\mathbf{H})\end{pmatrix}\begin{pmatrix}\Re(\mathbf{x})\\ \Im(\mathbf{x})\end{pmatrix} + \begin{pmatrix}\Re(\mathbf{n})\\ \Im(\mathbf{n})\end{pmatrix} \quad (3)$$

where $\Re(\bullet)$ and $\Im(\bullet)$ represent the real or the image part of $(\bullet)$.

Then we change the channel matrix into the upper triangular matrix as

$$\boldsymbol{\rho} = \mathbf{Rs} + \boldsymbol{\eta} \quad (4)$$

where $\begin{pmatrix}\Re(\mathbf{H}) & -\Im(\mathbf{H})\\ \Im(\mathbf{H}) & \Re(\mathbf{H})\end{pmatrix} = \mathbf{QR}$, $\mathbf{s} = \begin{pmatrix}\Re(\mathbf{x})\\ \Im(\mathbf{x})\end{pmatrix}$, $\boldsymbol{\rho} = \mathbf{Q}^H\begin{pmatrix}\Re(\mathbf{y})\\ \Im(\mathbf{y})\end{pmatrix}$ and $\boldsymbol{\eta} = \mathbf{Q}^H\begin{pmatrix}\Re(\mathbf{n})\\ \Im(\mathbf{n})\end{pmatrix}$. The ML algorithm can be expressed as

$$\hat{\mathbf{s}} = \arg\min_{\Omega} \|\boldsymbol{\rho} - \mathbf{Rs}\|^2 \quad (5)$$

where Ω is the real counterpart of $\aleph$, $\hat{\mathbf{s}} = \begin{pmatrix}\hat{s}_1 & \hat{s}_2 & \cdots & \hat{s}_N\end{pmatrix}^T$ is the estimated value of transmitted symbol.

(4) can also be written as

$$\begin{pmatrix}\rho_1\\ \rho_2\\ \vdots\\ \rho_N\end{pmatrix} = \begin{pmatrix}h_{11} & h_{12} & \cdots & h_{1N}\\ 0 & h_{22} & \cdots & h_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & h_{NN}\end{pmatrix}\begin{pmatrix}s_1\\ s_2\\ \vdots\\ s_N\end{pmatrix} + \begin{pmatrix}\eta_1\\ \eta_2\\ \vdots\\ \eta_N\end{pmatrix} \quad (6)$$

From (6), assuming the previous decisions are correct, the DFE can be described as follows

$$\begin{cases}\hat{s}_N = round\left(\dfrac{\rho_N}{h_{NN}}\right)\\ \cdots\\ \hat{s}_i = round\left(\dfrac{1}{h_{ii}}\left(\rho_i - \left(\displaystyle\sum_{k=i+1}^{N} h_{ik}\bullet\hat{s}_k\right)\right)\right)\\ \cdots\\ \hat{s}_1 = round\left(\dfrac{1}{h_{11}}\left(\rho_1 - \left(\displaystyle\sum_{k=2}^{N} h_{1k}\bullet\hat{s}_k\right)\right)\right)\end{cases} \quad (7)$$

So the solution of (5) can be searched layer by layer from the N^{th} to the 1^{st}. This is the DFE algorithm, like Fig. 1.

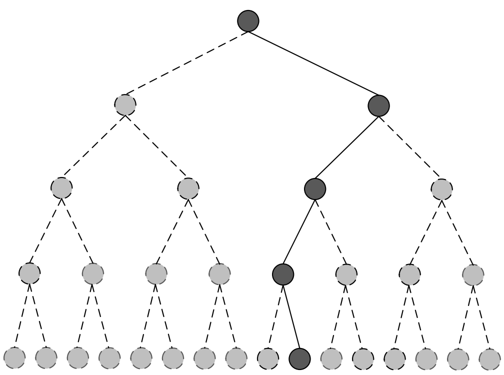

Fig. 1. DFE algorithm.

B. ML algorithm

ML algorithm calculates all nodes in the tree, then select the node which satisfies equation (5) from all the possible nodes. It provides the best performance among all the detection algorithms.

C. Euclidean distance and the partial Euclidean distance

The Euclidean distance (ED) between transmit and receive symbol can be expressed as

$$D = \left(\sum_{j=1}^{N}\left(\rho_j - \sum_{k=j}^{N} r_{jk}\bullet\hat{s}_k\right)^2\right)^{1/2} \quad (8)$$

Define the partial Euclidean distance (PED) as

$$D_i = \left(\sum_{j=i}^{N}\left(\rho_j - \sum_{k=j}^{N} r_{jk}\bullet\hat{s}_k\right)^2\right)^{1/2} \quad (9)$$

Obviously, the value of PED is part of that of ED.

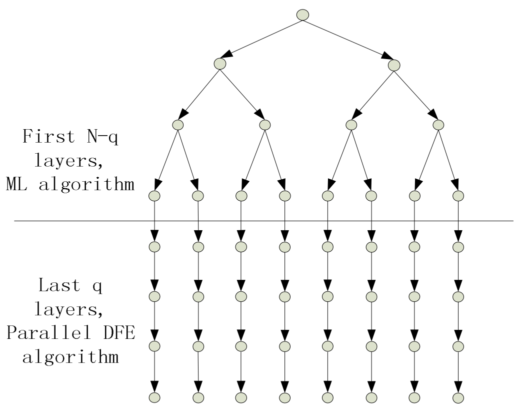

Fig. 2. ML-DFE algorithm.

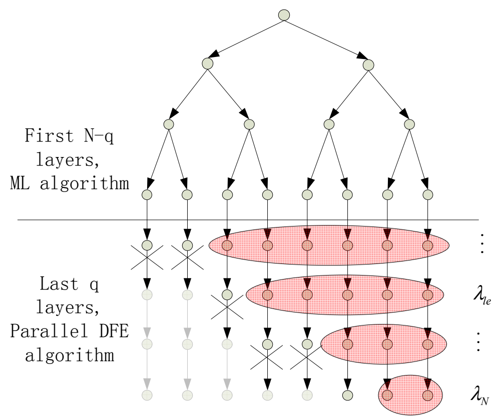

Fig. 3. NVLP-MD algorithm.

D. ML-DFE algorithm

The ML-DFE algorithm is to combine the ML and DFE algorithm. During the first *N-q* layers calculation, all the possible nodes should be concerned. In the remained layers, several DFE routes will be searched parallelly, like Fig. 2.

IV. The New Noise Variance Based Layered Pruning ML-DFE Algorithm

A. Principle

We propose a noise variance based layered pruning ML-DFE (NVLP-MD) algorithm here. If there are errors occurred in a transmit symbol, it will have larger ED and PED. So the nodes having very large ED or PED can be cut in the process of tree search. We know that the ED of the ML solution can be described by the noise variance. So the pruning of the nodes can be based on the noise variance. Before the search of the ML-DFE, we set a vector of pruning radius $\boldsymbol{\lambda} = (\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_N)$. For each layer, there is a pruning radius for it. Then cut nodes whose PED are larger than the corresponding pruning radius, like Fig. 3.

$$\lambda_i = \sqrt{\beta(N-i+1)\sigma^2} \tag{10}$$

where *i* is the number of layer. In Fig. 3, the nodes in the big light-colored circles will be kept and others will be cut.

B. The algorithm

The proposed algorithm is given as follows:

1) Set β according to the toleration of performance drop and complexity, and then get pruning radius $\boldsymbol{\lambda} = (\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_N)$. Set layer number of ML algorithm *q*.
2) QR decompose the channel matrix, then left multiply **Q** to **y** to get $\boldsymbol{\rho}$.
3) Set *le*=*N*
4) Extend the surviving nodes to all their children nodes as the ML algorithm.
5) *le*=*le*-1
6) if *le*>*q*, then go to step 4)
7) Several parallel DFE algorithms are worked to get new layer's nodes and their PEDs.
8) Compare the PED of the kept nodes after step 7). If the PED of a certain node is larger than λ_{le}, this node will be pruned. At the same time at least one node must be kept.
9) *le*=*le*-1
10) If *le*>1, go to step 7).
11) If *le*=1, select the node that has the smallest ED as the solution of the algorithm.

V. Simulation Results

The simulation result of the proposed algorithm performance is given through Monte Carlo. The simulations are implemented in a 4×4 antennas, 16QAM modulation system. The breadth of the ML-DFE algorithm and the NVLP-MD algorithm is 16, or say, *N-q*=2.

Fig. 4 shows the performance of the proposed algorithm, the DFE and the ML-DFE algorithm when *q*=2. From this figure, it can be found that the performance drop of the proposed algorithm is different according to β. When BER=10^{-2}, if $\beta = 0.1$, 1, 2 or 4, the performance drops are

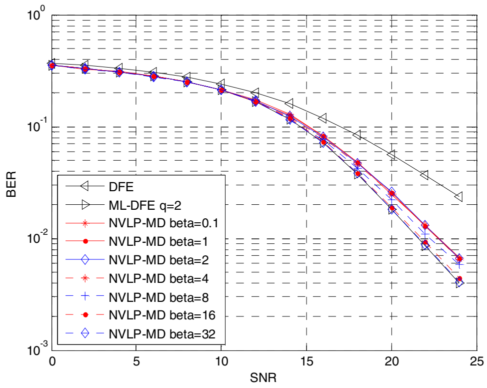

Fig. 4. Performance of different algorithms.

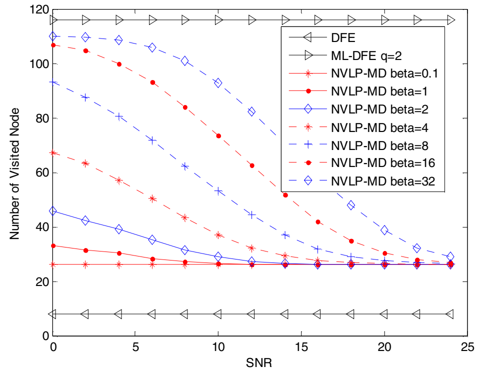

Fig. 5. Complexity of different algorithms.

about 2.2dB. The performance drops about 1.0dB, 0.2dB, 0.1dB or less than 0.1dB, when $\beta = 8$, 16 or 32.

Fig. 5 shows the complexity of the proposed algorithm, the DFE and the ML-DFE algorithm when *q*=2. The complexity drops lot especially in high SNR. When SNR=24, the numbers of trimmed nodes are about 84, 76.4% searching nodes are cut during the tree search. When $\beta = 32$, the number of trimmed nodes are about 81, 73.8% searching nodes are cut during the tree search.

VI. Conclusion

In this article, we proposed a simplified ML-DFE algorithm for MIMO detection. Some of the searching nodes are trimmed during the tree search process according to their PED and the preset pruning radiuses. The pruning radiuses are set based on the noise variance and the number of layer. Simulation results show that the proposed algorithm reduces the complexity of the ML-DFE algorithm with the performance drops small.

References

[1] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, “An overview of MIMO communications - a key to gigabit wireless,” *Proceedings of the IEEE*, vol. 92, no. 2, pp. 198-218, 2004.

[2] Lingyang Song, Youhui Li, and Bingli Jiao, “Differential Modulation for Bi-directional Relaying with Network Coding,” *IEEE Transactions on Signal Processing*, Vol. 58, No. 7, pp. 3933-3938, July 2010.

[3] Xu Zhu and R. D. Murch, “Performance analysis of maximum likelihood detection in a MIMO antenna system,” *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 187-191, Feb. 2002.

[4] O. Damen, A. Chkeif, and J.-C. Belfiore, “Lattice code decoder for space-time codes,” *Communications Letters, IEEE*, vol. 4, no. 5, pp. 161-163, 2000.

[5] X. Mao, S. Ren, L. Lu, and H. Xiang, “A Reduced Complexity K-Best SD Algorithm Based on Chi-Square Distribution for MIMO Detection,” in *2011 IEEE Vehicular Technology Conference (VTC Fall)*, San Francisco, United States, 2011, pp. 1-4.

[6] X. Mao, S. Ren, and H. Xiang, “Adjustable Reduced Metric-First Tree Search,” in *2011 7th International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM)*, 2011, pp. 1-4.

[7] Won-Joon Choi, R. Negi, and J. M. Cioffi, “Combined ML and DFE decoding for the V-BLAST system,” in *Communications, 2000. ICC 2000. 2000 IEEE International Conference on*, 2000, vol. 3, pp.1243-1248.

[8] X. Su, K. Yi, B. Tian, and Y. Sun, “An enhanced detection algorithm for V-BLAST system,” *Journal of Electronics (China)*, vol. 23, no. 5, pp. 773-776, Sep. 2006.