Interference-Aware Random Beam Selection for Spectrum Sharing Systems

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***Abstract*—Spectrum sharing systems have been introduced to alleviate the problem of spectrum scarcity by allowing secondary unlicensed networks to share the spectrum with primary licensed networks under acceptable interference levels to the primary users. In this paper, we develop interference-aware random beam selection schemes that provide enhanced throughput for the secondary link under the condition that the interference observed at the primary link is within a predetermined acceptable value. For a secondary transmitter equipped with multiple antennas, our schemes select a random beam, among a set of power-optimized orthogonal random beams, that maximizes the capacity of the secondary link while satisfying the interference constraint at the primary receiver for different levels of feedback information describing the interference level at the primary receiver. For the proposed schemes, we develop a statistical analysis for the signal-to-noise and interference ratio (SINR) statistics as well as the capacity of the secondary link. Finally, we present numerical results that study the effect of system parameters including number of beams and the maximum transmission power on the capacity of the secondary link attained using the proposed schemes.**

***Index Terms*—Beamforming, spectrum sharing, cognitive radio, multi-antenna systems.**

I. Introduction

Reliable high-speed data communication systems that support multimedia applications for both indoor and outdoor mobile users is a fundamental requirement for next generation wireless networks, which requires a dense deployment of physically coexisting different network architectures. Due to the limited spectrum available, recently, a novel interference-aware spectrum sharing concept is introduced where networks that suffer from congested spectrum; called secondary networks are allowed to share the spectrum with other networks with available licensed spectrum; called primary networks under the condition that limited interference is applied at the primary network [1], [2]. Multiple-antenna techniques associated with power optimization can be utilized as a power-efficient technique for improving the data rate of the secondary link while satisfying the interference constraint by allowing the secondary user to adapt its transmitting antenna, power and rate according to the channel state information of the secondary channel between the secondary transmitter and the secondary receiver, and the interference channel between the secondary transmitter and the primary receiver [3], [4], [5].

Beamforming has been utilized in multiple-antenna multiple-user systems as a well-known technique for providing high signal-to-interference-noise (SINR) to an intended user while minimizing the interference at non-intended users. In spectrum sharing systems, assuming different level of channel state information available at the secondary transmitter, various beamforming techniques have been developed that find optimal beamformers while maintaining the interference to the primary networks within an acceptable level [6], [7]. However, in order to achieve the desired performance for these schemes, a certain level of information of the channel coefficients must be available at the secondary transmitter, which can be impractical in terms of the amount of feedback bandwidth needed. To alleviate this problem, opportunistic random beamforming techniques have been proposed in wireless networks as an efficient means for providing capacity gains by generating random orthogonal beams for which the receivers can find the SINR attained by each beam and send to the transmitter the index of the beam that maximizes the SINR [8], [9].

In this paper, we develop a multiple-antenna access scheme for the secondary network based on opportunistically selecting a random beam among a set of orthogonal beams that satisfies the interference threshold at the primary link and maximizes the capacity of the secondary link. In particular, we consider a secondary link composed of a secondary transmitter equipped with multiple antennas and a single antenna receiver sharing the spectrum with a primary link composed of a single antenna transmitter and receiver. We develop random beam selection techniques that optimize the performance of the secondary link in terms of its throughput while maintaining the interference level at the primary receiver within an acceptable value. In particular, the secondary transmitter first sends an initial set of random orthogonal beams to the primary receiver at which the interference level for each beam is computed. Based on the level of feedback information conveyed by the primary receiver to the secondary transmitter, a new optimized set

This publication was made possible by NPRP grant numbers NPRP 08-152-2-043 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

of orthogonal random beams is formed which is sent to the secondary receiver, at which the best beam that maximizes the secondary SINR is selected. We develop mathematical analysis for the capacity values of the secondary link achieved by the proposed techniques. We then show the effect of system parameters such as the number of beams and transmission power on the capacity of the secondary link using numerical analysis.

The remainder of the paper is organized as follows. In the next section, we present the system model. In Sec. III, we present the interference-aware random beam selection schemes for different levels of interference feedback information. Then, in Sec. IV, we present mathematical analysis for the SINR statistics of the secondary link as well as its capacity. In Sec. V, we present numerical results for the proposed techniques as well as their discussion.

II. System Model

We consider the system model shown in Fig. 1 whereby the secondary transmitter is equipped with L antennas and the secondary receiver has a single antenna, while the primary link is composed of a single antenna transmitter and a single antenna receiver. The primary transmitter is assumed to send with a constant power P_p. We assume discrete-time slowly Rayleigh fading channels where the channel gains between the secondary transmitter of the lth antenna and the secondary receiver and the primary receiver, denoted by $h_{ss,l}$ and $h_{sp,l}$, respectively, are modeled as complex Gaussian random variables (CGRVs) with zero mean and unit variances. We also assume the channel gain between the primary transmitter and the secondary receiver, denoted by h_{ps}, is modeled as a CGRV with zero mean and unit variance. We further assume that all channels involved in communications are statistically independent and remain constant for the duration of a frame of size much larger than L. We finally note that, although, we limit our system model to a single user case, the proposed schemes in this paper can be readily generalized to multiple secondary users. During each symbol period, the transmitted signal at the secondary transmitter is given by $\mathbf{x}_{s,m} = \mathbf{u}_m s$, where s is the information symbol and $\mathbf{u}_m$ is an $L \times 1$ random beam vector selected from a set of orthogonal vectors $\mathbf{U} = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_M\}$, where M is the number of the random beams. The selection process will be explained later. The transmitted signal has a maximum peak power given by $\mathrm{E}[\mathbf{x}_{s,m}\mathbf{x}_{s,m}^H] = 1$, where $\mathrm{E}[\cdot]$ is the expectation operator and $(\cdot)^H$ is the Hermitian transpose.

As such, by letting $\mathbf{h}_{ss} = [h_{ss,1}, h_{ss,2}, \cdots, h_{ss,L}]$, the received signal at the secondary receiver is given by

$$y_{s,m} = \sqrt{P_s}\,\mathbf{h}_{ss}^T\,\mathbf{x}_{s,m} + \sqrt{P_p}\,h_{ps}\,x_p + n \qquad (1)$$

where P_s is the peak secondary transmission power, x_p represents the primary transmitted signal, P_p is the primary transmitted power, and n is a CGRV with zero mean and unit variance that represents the thermal noise at the secondary receiver. Similarly, by letting $\mathbf{h}_{sp} = [h_{sp,1} h_{sp,2} \cdots h_{sp,L}]$, the received signal at the primary receiver is given by $y_{p,m} = \mathbf{h}_{sp}^T\mathbf{x}_{s,m}$ where its peak power $I_{p,m} = |y_{p,m}|^2$ should be less than an acceptable threshold denoted by Q. Finally, we assume that there exists an error-free delay-free feedback channels between the both the primary and secondary receivers and the secondary transmitter used for conveying the information required by the secondary transmitter.

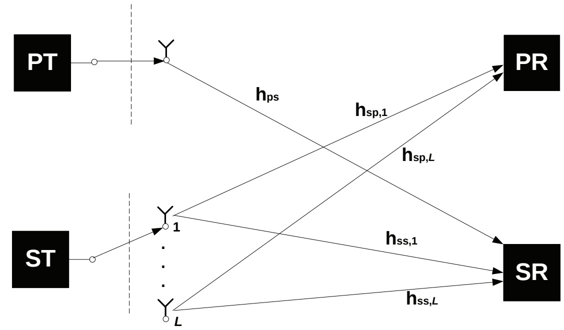

Figure 1. System model for spectrum-sharing cognitive networks with secondary transmit antenna selection.

III. Interference-Aware Random Beam Selection

In this paper, we consider the problem of finding the optimal beam $\mathbf{u}_m$ that satisfies the peak interference constraint at the primary receiver and optimizes the performance of the secondary link in terms of capacity. In particular, we develop an interference-aware random beam selection algorithm which can be explained in the following steps.

- **Step 1:** The secondary transmitter forms an initial set of random orthogonal beams $\mathbf{U}$ where the power of each beam is set to the maximum power value P_s.
- **Step 2:** These beams are then transmitted to the primary receiver where we assume that the primary receiver can estimate fully the value of the received signal for each beam $y_{p,m} = \mathbf{h}_{sp}^T\mathbf{x}_{s,m}$.
- **Step 3:** Based on the level of information describing the received signal $y_{p,m}$, which is feedbacked by the primary receiver to the secondary transmitter, the secondary transmitter formulates a new set of orthogonal beams that satisfy the interference constraint taking into consideration the performance of the secondary link. The new set of orthogonal beam is obtained by multiplying with an orthogonal matrix $\mathbf{R}$ to preserve the orthogonality of the initial beams. In this paper, we consider three different schemes for finding $\mathbf{R}$ based on the level of feedback information describing $y_{p,m}$; 1) scheme A: the primary receiver feedbacks the magnitude value of $\mathbf{h}_{sp}^T\mathbf{x}_{s,m}$ or equivalently $I_{p,m}$, 2) Scheme B: the primary receiver feedbacks the magnitude and phase of $y_{p,m}$, and 3) Scheme C: the primary receiver feedbacks only one bit indicating whether the interference value $I_{p,m}$ is above or less than Q.
- **Step 4:** Finally, the new set of beams are transmitted to the secondary receiver to determine the one that achieves the best signal to noise and interference ratio (SINR).

The index of this beam is feedbacked to the secondary transmitter to utilize for data transmission.

In the following, we explain the formation of the orthogonal beams described at **Step 3** for the three different schemes and find the associated SINR observed at the secondary receiver.

A. Scheme A

In this scheme, we assume that the primary receiver feedbacks the interference level $I_{p,m} = |y_{p.m}|^2$ for all the M beams that can be augmented in the following vector $\mathbf{I}_p = [I_{p,1} I_{p,2}, \dots I_{p,M}]$ for $M < L$. Based on this feedback information, the secondary transmitter adjusts the power of each beam to ensure that the interference is less than Q, and hence the orthogonal matrix $\mathbf{R}_A$ is a diagonal matrix that has the following form $\mathbf{R}_A = \mathrm{diag}\left(\alpha_1,\ \alpha_2,\ \dots,\ \alpha_M\right)$, where α_m is formed to adjust the power for the mth beam power to satisfy the maximum interference level while maintaining the maximum power to P_s, or equivalently $\alpha_m = \min(1, \frac{Q}{P_s g_{sp,m}})$ where $g_{sp,m} = |\mathbf{h_{sp}}^T \mathbf{x}_{s,m}|^2$. After applying **Step 4**, the maximum SINR attained at the secondary receiver is given by

$$\mathrm{SINR}_A = \max_{1<m<M} \frac{P_s \alpha_m g_{ss,m}}{P_p g_{ps} + 1}. \tag{2}$$

where $g_{ss,m} = |\mathbf{h_{ss}}^T \mathbf{x}_{s,m}|^2$ and $g_{ps} = |h_{ps}|^2$ are exponential random variables with unit mean.

B. Scheme B

For this scheme, we assume that the primary receiver feedbacks the exact value of $y_{p,m}$ (magnitude and phase) to the secondary transmitter that can be augmented in the following vector $\mathbf{y}_p = [y_{p,1} y_{p,2}, \dots y_{p,M}]$ for $M < L$. This information can be exploited by the secondary transmitter to rotate the initial orthogonal beams (transmitted with maximum power P_s) in new directions that achieve interference levels less than Q and thus maximizing the diversity order attained at the secondary receiver. In particular, our objective is to find an orthogonal matrix $\mathbf{R}_B$ such that number of the orthogonal beams, which transmits with maximum power and creates interference less than or equal to Q, is maximized. This objective should increase the probability that the SINR achieved at the secondary receiver is maximized given no prior information of the secondary channel state information is available at the secondary transmitter. In achieving this objective, we assume that $\mathbf{R}_B$ can be written in the following form,

$$\mathbf{R}_B = \mathbf{R}_{B,D} \mathbf{R}_{B,U} \tag{3}$$

where $\mathbf{R}_{B,D}$ and $\mathbf{R}_{B,U}$ denote a diagonal matrix and a unitary matrix, respectively. Based on this form, we design the matrix $\mathbf{R}_B$ in two steps. In the first step, we apply a unitary transformation, which preserves the power as well as the orthogonality of the initial set of beams U using the unitary matrix $\mathbf{R}_{B,U}$ such that the number of beams that satisfy the interference constraint is maximized or equivalently $\mathbf{R}_{B,U} \mathbf{y}_p^T = \mathbf{z}_p$, where the number of the elements of $\mathbf{z}_p$ whose magnitude is less than or equal Q is maximized. In the second step, we apply the diagonal matrix $\mathbf{R}_{B,D}$, which have the same form as $\mathbf{R}_A$, that adjusts the power for the beams to satisfy the interference constraint. In the next, we prove the following theorem

Theorem 1. *Assuming that the exact value of $y_{p,m}$ is available at the secondary transmitter, we can construct $\mathbf{R}_{B,U}$ to achieve at least $M-1$ beams that transmit with maximum power P_s for all values of Q.*

Proof: In proving the theorem, we first note the following

- Any unitary transformation must preserve the length of the vectors. Therefore, for Eq. (3), the length of the vectors $\mathbf{y}$ and $\mathbf{z}$ should be equal ($||\mathbf{y}_p||^2 = ||\mathbf{z}_p||^2$).
- If the magnitudes of the whole elements of $\mathbf{z}_p$ are less than Q, then its length $||\mathbf{z}_p||^2$ must be less than $M\ Q$.

Based on these notes, we consider two cases where for the first case $||\mathbf{y}_p||^2 < M\ Q$ and the second case $||\mathbf{y}_p||^2 > M\ Q$. For the first case, we can find $\mathbf{R}_{B,U}$ such that the all the elements of the vector $\mathbf{z}_p$ have magnitudes less than Q while maintaining $||\mathbf{y}_p||^2 = ||\mathbf{z}_p||^2$. This can be achieved by setting $\mathbf{z}_p$ as $\mathbf{z}_p = \frac{||\mathbf{y}_p||}{\sqrt{M}} \quad \mathbf{1}_{M\times 1}$ whose length is equal to $\mathbf{y}_p$ by formation as well as the magnitude of each element of $\mathbf{z}_p$ is less than Q since we assume that $||\mathbf{y}_p||^2 < M\ Q$ and finally $\mathbf{1}$ is the unit vector. In this case, the diagonal matrix $\mathbf{R}_{B,D}$ will be the identity matrix since all the beams will satisfy the interference constraint with maximum power P_s.

For the second case where $||\mathbf{y}_p||^2 > M\ Q$, it is not feasible to find $\mathbf{R}_{B,U}$ that sets all the magnitudes of the elements of $\mathbf{z}_p$ to be less than Q. However, we can design the $\mathbf{R}_{B,U}$ such that first $M-1$ beams satisfy the interference constraint with maximum power P_s while setting the interference created by the Mth beam to a value that maintains the relation $||\mathbf{y}_p||^2 = ||\mathbf{z}_p||^2$. In particular, we design $\mathbf{R}_{B,U}$ such that the vector $\mathbf{z}_p$ is given by $\mathbf{z}_p = \sqrt{Q}\left[\mathbf{1}_{(M-1)\times 1} \quad \sqrt{|\frac{||\mathbf{y}_p||^2}{Q} - M + 1}\right]$ whose length is equal to that of $\mathbf{y}_p$ by formation. It is clear that the Mth element will create interference level greater than Q since $||\mathbf{y}_p||^2 > M\ Q$. The unitary matrix $\mathbf{R}_{B,U}$, for given vectors $\mathbf{y}_p$ and $\mathbf{z}_p$, can be computed using the elementary unitary transformation which is given by [10]

$$\mathbf{R}_{B,U} = \mathbf{I}_M - \frac{2}{1+j\eta} \frac{\mathbf{w}\mathbf{w}^T}{\mathbf{w}^T\mathbf{w}} \tag{4}$$

where $\mathbf{I}_M$ is the $M \times M$identity matrix, $\mathbf{w} = \mathbf{y}_p - \mathbf{z}_p$, $\eta = \frac{\mathrm{imag}(\mathbf{y}_p^T \mathbf{z}_p)}{\mathbf{y}_p^T \mathbf{z}_p - \mathrm{real}(\mathbf{y}_p^T \mathbf{z}_p)}$ and $j = \sqrt{-1}$. In this case, the diagonal matrix $\mathbf{R}_{B,D}$ will have its first $M-1$ elements set to one while the Mth diagonal element will be set to adjust the power of the Mth beam to satisfy the interference constraint.

We note that the above analysis can be achieved for any value of Q. Therefore, we can state that for each channel frame, we can construct at least $M-1$ beams that transmit with maximum power P_s for all values of Q. ■

Based on these results, the performance of scheme B can be very well approximated, especially for large M, by an interference-free system whereby the secondary transmitter selects the best beam that maximizes the SINR out of $M-1$ beams with transmit power P_s

$$\mathrm{SINR}_B \approx \max_{1<m<M-1} \frac{P_s g_{ss,m}}{P_p g_{ps}+1}. \qquad (5)$$

As it will be evident later by simulations, this scheme outperforms substantially the performance of scheme A especially for high transmitted power values.

C. Scheme C

In this scheme, the primary receiver just feedbacks a one bit that describes whether the interference level for each beam is above or below Q. This can be viewed as an on/off beam selection where the orthogonal matrix is a diagonal matrix which can be written as $\mathbf{R}_C = \mathrm{diag}\left(c_1,\ c_2,\ \ldots,\ c_M\right)$, where $c_k = \mathrm{sgn}(Q - I_{p,k})$ and $\mathrm{sgn}(x) = 1$ for $x < 0$ and 0 otherwise. In this case, the SINR at the secondary receiver depends on the number of beams selected that satisfy the interference constraint per channel period. In particular the probability of having k beams selected at each period is given by $\pi_k = \left(\begin{array}{c} M \\ k \end{array}\right)[Pr(c_k=1)]^k[1-Pr(c_k=1)]^{M-k}$, where $Pr(c_k=1) = 1-\mathrm{e}^{-Q/P_s}$ and e^x is the exponential function. Hence, the SINR of scheme C can be computed as

$$\mathrm{SINR}_{C_k} = \max_{1<m<k} \frac{P_s g_{ss,m}}{P_p g_{ps}+1}. \qquad (6)$$

This performance of this scheme should serve as a lower bound to the performance of scheme A.

IV. Capacity Analysis for the Secondary Link

In this section, we find the SINR statistics for the three schemes which can be then utilized to compute numerically the capacity of each scheme using the formula $C = \int_0^\infty \frac{1}{1+z}\left(1-F_\gamma(z)\right)dz$ [9].

1) Scheme A: Let γ_A denotes the instantaneous SINR for scheme A. Hence Eq. (2) can be rewritten as $\gamma_A = \dfrac{P_s}{P_p g_{ps}+1}\zeta_A$, where $\zeta_A = \max_{1<m<M}\zeta_{A,m}$ and $\zeta_{A,m} = \alpha_m g_{ss,m}$. The cumulative distribution function (CDF) of γ_A can be obtained as

$$F_{\gamma_A}(z) \;=\; \int\limits_0^\infty F_{\zeta_A}\left[\frac{z}{P_s}\left(P_p x+1\right)\right]\mathrm{e}^{-x}dx. \qquad (7)$$

The CDF of ζ_A can be readily calculated as the product of the CDFs of $\zeta_{A,m}$. The CDF of $\zeta_{A,m}$ can be obtained as $F_{\zeta_{A,m}}(y) = 1-\mathrm{e}^{-y} + \dfrac{y}{y+\frac{Q}{P_s}}\mathrm{e}^{-\left(y+\frac{Q}{P_s}\right)}$ [11]. Hence, the CDF of ζ_A can be obtained as $F_{\zeta_A}(y) = F_{\zeta_{A,m}}(y)^M$. Inserting $F_{\zeta_A}(y)$ into (7) yields

$$\begin{aligned} F_{\gamma_A}(z) \;&=\; \frac{P_s \mathrm{e}^{\frac{1}{P_p}}}{zP_p}\int\limits_{\frac{z}{P_s}}^\infty \left[1+\mathrm{e}^{-u}\left[\frac{u\mathrm{e}^{-\frac{Q}{P_s}}}{u+\frac{Q}{P_s}}-1\right]\right]^M \\ &\quad \times \mathrm{e}^{-\frac{P_s}{P_p z}u}du. \end{aligned} \qquad (8)$$

Using the binomial theorem [12] and doing some mathematical manipulation, (8) can be represented as

$$\begin{aligned} F_{\gamma_A}(z) \;&=\; \frac{P_s\mathrm{e}^{\frac{1}{P_p}}}{zP_p}\sum_{n=0}^{M}\sum_{l=0}^{n}\binom{M}{n}\binom{n}{l}(-1)^{n+l} \\ &\quad \times \mathrm{e}^{-l\frac{Q}{P_s}}\mathrm{e}^{\left(n+\frac{P_s}{P_p z}\right)\frac{Q}{P_s}} I, \end{aligned} \qquad (9)$$

where $I = \int_{\frac{z+Q}{P_s}}^\infty \left(1-\frac{Q}{P_s v}\right)^l \mathrm{e}^{-\left(n+\frac{P_s}{P_p z}\right)v}dv$. Likewise, using the binomial theorem to expand $\left(1-\frac{Q}{P_s v}\right)^l$ and using [13, Eq. (3.381.3)], I can be obtained as

$$I = \begin{cases} \dfrac{\mathrm{e}^{-\left(\frac{n}{P_s}+\frac{1}{P_p z}\right)(z+Q)}}{n+\frac{P_s}{P_p z}} + \sum_{s=1}^{l}\binom{l}{s}(-1)^s\left(\frac{Q}{P_s}\right)^s & l \neq 0 \\ \times\left(n+\frac{P_s}{P_p z}\right)^{s-1}\Gamma\left(1-s, (\frac{n}{P_s}+\frac{1}{P_p z})(z+Q)\right) & \\ \dfrac{P_p z}{P_s+nP_p z}\mathrm{e}^{-(\frac{nz}{P_s}+\frac{1}{P_p})} & l=0 \end{cases} \qquad (10)$$

Substituting (10) in (9), the CDF of γ_A can be obtained as

$$\begin{aligned} F_{\gamma_A}(z) \;&=\; 1+\sum_{r=1}^{M}\binom{M}{r}(-1)^r\frac{P_s}{P_s+rP_p z}\mathrm{e}^{-\frac{rz}{P_s}} \\ &\quad + \frac{P_s\mathrm{e}^{\frac{1}{P_p}}}{zP_p}\sum_{n=1}^{M}\sum_{l=1}^{n}\binom{M}{n}\binom{n}{l}(-1)^{n+l}\mathrm{e}^{-l\frac{Q}{P_s}} \\ &\quad \times\left[\frac{\mathrm{e}^{-\left(\frac{n}{P_s}+\frac{1}{P_p z}\right)z}}{n+\frac{P_s}{P_p z}} + \mathrm{e}^{(n+\frac{P_s}{P_p z})\frac{Q}{P_s}}\sum_{s=1}^{l}\binom{l}{s}\right. \\ &\quad \times(-1)^s\left(\frac{Q}{P_s}\right)^s\left(n+\frac{P_s}{P_p z}\right)^{s-1} \\ &\quad \left.\times\Gamma\left(1-s,\left(\frac{n}{P_s}+\frac{1}{P_p z}\right)(z+Q)\right)\right]. \end{aligned} \qquad (11)$$

2) Scheme B: Let γ_B denotes the instantaneous SINR for scheme B which can be represented as $\gamma_B = \dfrac{P_s}{P_p g_{ps}+1}\zeta_B$, where $\zeta_B = \max_{1<m<M-1} g_{ss,m}$. Following similar analysis to the one used in scheme A, we can obtain the CDF of γ_B in closed form as $F_{\gamma_B}(z) = \sum_{n=0}^{M-1}\binom{M-1}{n}(-1)^n\dfrac{P_s}{P_s+nP_p z}\mathrm{e}^{-\frac{nz}{P_s}}$.

3) Scheme C: Let γ_C denotes the instantaneous SINR for scheme C and γ_{C_k} denotes the instantaneous SINR for k selected beams. The CDF of γ_{C_k} can be readily obtained using Eq. (6) as $F_{\gamma_{C_k}}(z) = \sum_{n=0}^{k}\binom{k}{n}(-1)^n\dfrac{P_s}{P_s+nP_p z}\mathrm{e}^{-\frac{nz}{P_s}}$. Since the probability that k beams are selected out of M beams is π_k, then the average capacity can be written in terms of the capacities of the k-selected beams as $C = \sum_{k=1}^{M}\pi_k C_k$.

V. Numerical Results

In this section, we present some selected numerical results that determine the behaviour of the capacity of the secondary

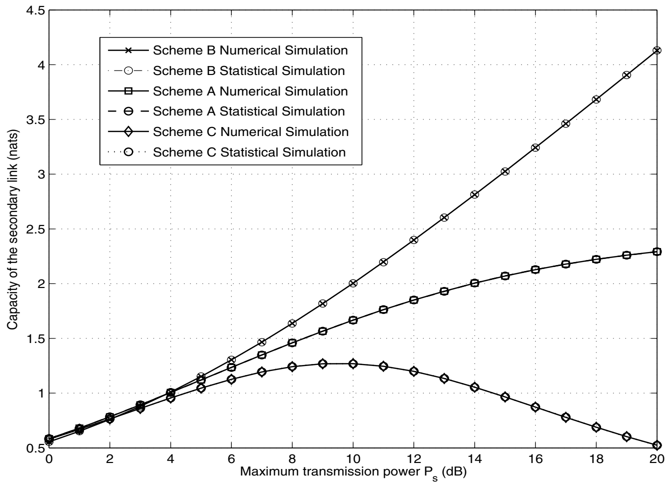

Figure 2. Capacity of the secondary link (in nats) as a function of the maximum transmission power P_s (in dB) for $L = 10$, primary transmission power $P_p = 5$ dB, peak interference threshold $Q = 5$ dB, and $M = 6$.

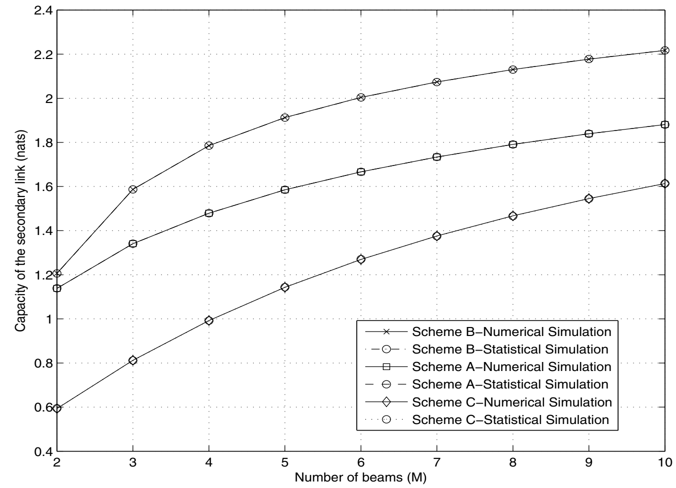

Figure 3. Capacity of the secondary link (in nats) as a function of the number of beams M for for $L = 10$, primary transmission power $P_p = 5$ dB, peak interference threshold $Q = 5$ dB, and $P_s = 10$ dB.

links for the proposed schemes. In the following, we assume that the number of antennas $L = 10$, the primary transmission power $P_p = 5$ dB, and the peak interference threshold $Q = 5$ dB. Fig. 2 depicts the capacity of the secondary link as a function of the transmit power P_s at $M = 6$ orthogonal beams. As evident from figure 2, scheme B achieves the best performance and the capacity almost increases linearly with P_s. This is attributed that in this scheme for each power value, we can find $M - 1$ beams with maximum power P_s which results in that the capacity increases as P_s increases. For scheme A, the figure reveals that the capacity saturates as P_s increases since increasing P_s leads to a higher interference level and hence the power for the beams needs to be further reduced to satisfy the interference constraint. Finally for scheme C, the figure shows that there exists an optimal value for the transmission power P_s at which the capacity is maximized. This is due to the fact that as the power increases, the secondary link increases, however, the probability of finding a beam that satisfies the interference constraint decreases until it reaches zero for infinite P_s and hence zero capacity.

Fig. 3 depicts the capacity of the secondary link as the number of orthogonal beams increases from $M = 2$ to $M = 10$ for the three schemes for $P_s = 10$ dB. As the figure reveals, the capacity of scheme B that utilizes both magnitude and phase of the interference value achieves the best performance while scheme C achieves the lowest performance. It is evident from figures 2 and 3 that the numerical results are inline with the statistical simulations.

VI. Conclusion

In this paper, we presented random beam selection schemes for spectrum sharing systems with different levels of feedback information describing the interference level observed at the primary receiver. For each scheme, we presented a closed-form expression for the received SINR at the secondary receiver. Finally, we presented numerical results for the average capacity of the secondary link versus the peak secondary transmit power and versus the number of transmitted beams for each scheme.

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