Optimization for Outage Probability Constrained Robust Downlink Collaborative Beamforming

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***Abstract*—In this paper, we design an outage probability constrained robust collaborative beamforming approach for the distributed multi-relay network in the downlink, where the channel state information (CSI) available is imperfect. We aim to minimize the total transmit power of the relay nodes whilst keeping the outage probability at the destination node below the predefined threshold. Assuming that the CSI mismatches follow Gaussian distribution, the equivalent counterpart for the outage probability constraint on the required signal-to-noise-ratio (SNR) is given explicitly. We show that though the original optimization problem is non-convex thus very intractable, it could be *optimally* solved by using the well-known interior-point method together with an efficient one-dimension search. Simulation results reveal that our proposed approaches can guarantee the quality-of-service (QoS) in term of outage probability in statistical sense while the non-robust scheme fails to do so.**

***Index Terms*—Robust collaborative beamforming, convex, one-dimension search.**

I. Introduction

Distributed relay beamforming technique, where spatially separated nodes are allowed to work collaboratively to form a virtual multiple-input and multiple-output (MIMO) array for enlarged network coverage and improved spectral efficiency has attracted much attention recently [1]–[4]. It is well known that, the performance of such network can be substantially threatened by the inevitable mismatches in the CSI that arise from many issues, such as low training SNR, quantization, limited feedback and feedback delay in practical environment. Therefore, the CSI mismatches must be taken into consideration by seeking techniques robust to the channel uncertainties. The robust design criteria can be roughly categorized into two main types: 1) the worst-case analysis based design [3], [4] and 2) the probability constrained optimization technique [5], [6]. Although the worst-case based design can maintain guaranteed robustness for the bounded CSI uncertainties, however, it may be overly conservative since the worst-case seldomly occurs in practice and thus leads to unnecessary performance loss. Moreover, the hypothesis of bounded CSI is somewhat impractical with regard to the training process for the channel estimation. Motivated by the above facts, it is stimulating and logical to exploit the probability model for the channel mismatches. Previously, in [5], an outage probability specified beamforming strategy was proposed to resist Gaussian uncertainties in channel covariance matrices of the downlink. Then, the probabilistic robustness against Gaussian channel uncertainties for a multiuser single-input multiple-output (SIMO) antenna system in the uplink was considered in [6].

In this work, we consider the robust downlink collaborative relay beamforming approach for the multi-relay network with imperfect knowledge of the CSI. By explicitly modeling the CSI errors as Gaussian, the equivalent condition for the QoS constraint measured by outage probability is derived. In [7], the same problem was investigated and resorted to an iteration based suboptimal solution. In this paper, we show that, though the original optimization problem is non-convex and quite intractable, it can be *optimally* solved by the well-known interior-point method together with an efficient one-dimension search technique. In order to validate our proposed approach, computer simulations are carried out against an conventional non-robust method. Simulation results demonstrate that, the QoS in term of outage probability can be guaranteed by the proposed approaches while it is violated by the non-robust scheme.

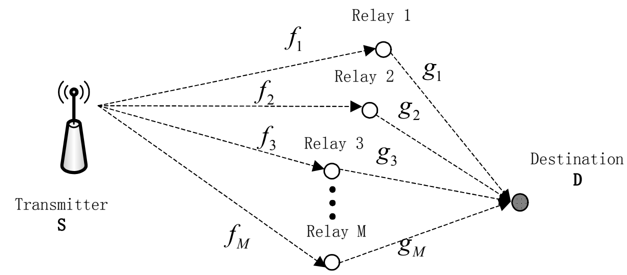

Fig. 1. System Model.

Notations: Vectors are denoted by boldface lowercase letters while matrices are represented by boldface uppercase letters. $[\cdot]^T$, $[\cdot]^*$ and $[\cdot]^\dagger$ denote transpose, conjugate and Hermitian operations, respectively. $[\mathbf{A}]_{ij}$ is the (i,j)th element of the matrix $\mathbf{A}$. $\|\cdot\|$ represents the Frobenius norm, and $\mathbb{E}[\cdot]$, $\mathtt{tr}\{\cdot\}$ denotes the expectation and trace operators, respectively. $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A}-\mathbf{B}$ is positive semi-definite. $\odot$ denotes the (element-wise) Hadamard product, $\mathbf{0}$ and $\mathbf{I}$ represents matrix with all 0 elements and unit matrix of appropriate dimension, respectively.

II. System Model and Problem Formulation

A. System Model

Consider a downlink relay network consisting of a base station S, a destination D and M synchronized relay nodes, namely $\{\mathrm{R}_m\}_{m=1}^{M}$, as depicted in Fig.1. It is assumed that there is no direct link between the terminal node S and D due to transmission attenuation and all the channels undergo identical independent distributed (i.i.d) Rayleigh flat fading. The communication process takes place in two stages with the amplify-and-forward (AF) protocol. Denote the links from S to $\{\mathrm{R}_m\}_{m=1}^{M}$ and $\{\mathrm{R}_m\}_{m=1}^{M}$ to D as $f_m \in \mathbb{C}$ and $g_m \in \mathbb{C}$, respectively. In the first stage, the received signal at R_m is given by

$$r_m = \sqrt{P_{\mathrm{S}}} f_m \mathrm{s} + n_{R,m} \tag{1}$$

where the information symbol s satisfies $\mathrm{E}[|\mathrm{s}|^2] = 1$ and P_{S} is the transmit power of S. $n_{R,m}$ is a zero mean circular complex Gaussian noise with variance σ_R^2. In the second stage, the received signal r_m is weighted by the complex beamforming coefficient w_m before forwarded to D. Hence, the total transmit power of the relay nodes is given by

$$P_{\mathrm{R}} = \sum_{m=1}^{M} |w_m|^2 \mathbb{E}\left[|r_m|^2\right] = \mathbf{w}^\dagger \mathbf{D} \mathbf{w} \tag{2}$$

where $\mathbf{w} \triangleq [w_1, w_2, \cdots, w_M]^\dagger$, $[\mathbf{D}]_{m,m} = P_{\mathrm{S}}|f_m|^2 + \sigma_R^2$. The received signal at D is

$$d = \sum\nolimits_{m=1}^{M} \left[g_m w_m \left(\sqrt{P_{\mathrm{S}}} f_m \mathrm{s} + n_{R,m}\right)\right] + n_D \tag{3}$$

where n_D is a zero mean circular complex Gaussian noise with variance σ_D^2. For simplicity, we reformulate (3) in vector form as

$$d = \underbrace{\sqrt{P_{\mathrm{S}}} \mathbf{g}^\dagger \mathbf{W} \mathbf{f} \mathrm{s}}_{\text{signal component}} + \underbrace{\mathbf{g}^\dagger \mathbf{W} \mathbf{n}_R + n_D}_{\text{noise component}} \tag{4}$$

where $\mathbf{f} \triangleq [f_1, f_2, \cdots, f_M]^T$, $\mathbf{g} \triangleq [g_1, g_2, \cdots, g_K]^\dagger$, $\mathbf{n}_R \triangleq [n_{R,1}, n_{R,2}, \cdots, n_{R,M}]^T$, and $\mathbf{W} \triangleq \mathtt{diag}(w_1, w_2, \cdots, w_M)$. Therefore, the achieved instantaneous SNR at D is expressed as

$$\Gamma = \frac{\mathbf{w}^\dagger \mathbf{G} \mathbf{w}}{\mathbf{w}^\dagger \mathbf{S} \mathbf{w} + \sigma_D^2} \tag{5}$$

in which

$$\begin{cases} \mathbf{G} = P_{\mathrm{S}} (\mathbf{g}^* \odot \mathbf{f}) (\mathbf{g}^* \odot \mathbf{f})^\dagger \\ \mathbf{S} = \sigma_R^2 \mathtt{diag}\left([\mathbf{g}^* \mathbf{g}^T]_{11}, [\mathbf{g}^* \mathbf{g}^T]_{22}, \ldots, [\mathbf{g}^* \mathbf{g}^T]_{MM}\right) \end{cases} \tag{6}$$

B. Problem Formulation

We assume that the collaborative beamforming procedure takes place in the base station S, then the channel link $\mathbf{f}$ can be estimated directly from training by S, e.g. in time division duplex (TDD) systems, the downlink CSI available for BS can be directly estimated at the BS by exploiting the downlink-uplink reciprocity in the successive time slots. However, the channel link $\mathbf{g}$ must be estimated by either the relay node $\{\mathrm{R}_m\}_{m=1}^M$ or the terminal node D, which tend to have low training power budget, then fed back to S, and therefore is erroneous [3]. To model the CSI error in $\mathbf{g}$, we denote

$$\mathbf{g} \triangleq \tilde{\mathbf{g}} + \Delta\tilde{\mathbf{g}} \tag{7}$$

where $\tilde{\mathbf{g}}$ is the nominal CSI known at S and $\Delta\tilde{\mathbf{g}} \sim \mathcal{CN}\left(0, \sigma_g^2 \mathbf{I}\right)$ is the estimation error assumed to be Gaussian and independent of $\tilde{\mathbf{g}}$ [8]. Consider an outage probability specified constraint on the QoS, where the SNR at the destination D is required to be above the predefined threshold γ, with certain preselected probability ε, else it is impossible for node D to decode correctly and outage occurs. We aim to minimize the total transmit power of the relay nodes, which is a crucial performance measure for relay network limited by battery life. Mathematically, that is

$$\min_{\mathbf{w}} \quad \mathbf{w}^\dagger \mathbf{D} \mathbf{w} \quad \text{s.t.} \;\; \mathcal{P}(\Gamma \geq \gamma) \geq \varepsilon \tag{8}$$

where $\mathcal{P}$ is the probability operation. The above probability constraint is equivalent to

$$\mathcal{P}\left(x \geq \gamma \sigma_D^2\right) \geq \varepsilon \tag{9}$$

in which $x \triangleq \mathbf{w}^\dagger (\mathbf{G} - \gamma \mathbf{S}) \mathbf{w}$. The cumulative-density-function (CDF) of x turns out to be too difficult to utilize in our optimization problem. To obtain a mathematically tractable formulation, we note that x is the sum of M independent random variables[1]. Adopting the central limit theorem (CLT), if M is sufficient large, the error of x is anticipated to be Gaussian characterized by the mean $\mu_x = \mathrm{E}[x]$ and variance $\sigma_x^2 = \mathrm{var}[x]$. Meanwhile, for practical use of the received signal, it is reasonable to have $\varepsilon > 0.5$, i.e. $\varepsilon \in [0.5, 1]$. Now, the QoS constraint can be approximated by

$$\begin{aligned} &\mathcal{P}\left(x \geq \gamma \sigma_D^2\right) \geq \varepsilon \Leftrightarrow Q\left(\frac{\gamma \sigma_D^2 - \mu_x}{\sigma_x}\right) \geq \varepsilon \\ &\Leftrightarrow \mu_x - \sqrt{2}\mathrm{erf}^{-1}(2\varepsilon - 1)\, \sigma_x \geq \gamma \sigma_D^2 \end{aligned} \tag{10}$$

where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-u^2/2} du$ is the Q-function and erf^{-1} is the inverse of $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$. Hence, the optimization problem in (8) becomes

$$\min_{\mathbf{w}} \quad \mathbf{w}^\dagger \mathbf{D} \mathbf{w} \quad \text{s.t.} \;\; \mu_x - \eta_x \sigma_x \geq \gamma \sigma_D^2 \tag{11}$$

where $\eta_x \triangleq \sqrt{2}\mathrm{erf}^{-1}(2\varepsilon - 1)$ is positive for $\varepsilon \in [0.5, 1]$.

C. Gaussian Approximation

To show the Gaussian approximation explicitly and for ease of exposition, we reformulate x as

$$x = \mathbf{g}^\dagger \mathbf{Q} \mathbf{g} \tag{12}$$

where $\mathbf{Q} \triangleq \mathbf{T} \odot \left(\mathbf{w}^* \mathbf{w}^T\right)$, with $\mathbf{T} \triangleq \left(P_{\mathrm{S}} \mathbf{f}\mathbf{f}^\dagger - \gamma \sigma_R^2 \mathbf{I}\right)$. Inserting $\mathbf{g} = \tilde{\mathbf{g}} + \Delta\tilde{\mathbf{g}}$ into x, we have

$$x = \mathtt{tr}\left\{\mathbf{Q}\left(\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger + \tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger + \Delta\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger + \Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right)\right\} \tag{13}$$

Hence, the expectation of x against the channel error $\Delta\tilde{\mathbf{g}}$ is

$$\mathbb{E}[x] = \mathtt{tr}\left\{\mathbf{Q}\left(\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger + \mathbb{E}\left[\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right] + \mathbb{E}\left[\Delta\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\right] + \mathbb{E}\left[\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right]\right)\right\} \tag{14}$$

As the CSI errors are zero-mean Gaussian, we have $\mathbb{E}\left[\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right] = 0$, $\mathbb{E}\left[\Delta\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\right] = 0$, and $\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger$ follows a wish-hart distribution, i.e. $\mathcal{CW}\left(\sigma_g^2 \mathbf{I}, 1\right)$, where $\mathbb{E}\left[\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right] = \sigma_g^2 \mathbf{I}$, and $\mathbb{E}\left[\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger \mathbf{Q} \Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right] = \sigma_g^4 \mathtt{tr}\{\mathbf{Q}\}\mathbf{I} + \sigma_g^4 \mathbf{Q}$ [9]. Therefore

$$\mathbb{E}(x) = \mathtt{tr}\left\{\mathbf{Q}\left(\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger + \sigma_g^2 \mathbf{I}\right)\right\} = \tilde{\mathbf{g}}^\dagger \mathbf{Q} \tilde{\mathbf{g}} + \sigma_g^2 \mathtt{tr}\{\mathbf{Q}\} \tag{15}$$

For notational simplicity, denote $\hat{x} \triangleq x - \tilde{\mathbf{g}}^\dagger \mathbf{Q} \tilde{\mathbf{g}}$, then $\mathbb{E}[\hat{x}] = \sigma_g^2 \mathtt{tr}\{\mathbf{Q}\}$ and $\sigma_{\hat{x}}^2 = \mathbb{E}\left[\hat{x}^2\right] - (\mathbb{E}[\hat{x}])^2$. It is worth noting that $\hat{x}$ and x has the same variance regarding on $\Delta\tilde{\mathbf{g}}$. Then the second moment of $\hat{x}$ is

[1] It is assumed that the elements of channel vector $\mathbf{g}$ (i.e. $g_m, \forall m$) are estimated individually, where M is the total relay number.

$$
\begin{aligned}
\mathbb{E}\left[\hat{x}^2\right] &= \mathbb{E}\left[(\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} + \tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} + \Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}})\right. \\
&\qquad\qquad \left.\times\ (\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} + \tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} + \Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}})^\dagger\right] \\
&= \mathbb{E}\left[\begin{array}{l}
\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} + \Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} \\
+\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} + \tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} \\
+\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} + \tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} \\
+\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} + \Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} \\
+\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}
\end{array}\right] \\
&= \mathbb{E}\Big[\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}} + \tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} \\
&\qquad\qquad\qquad + \Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Big] \\
&= \mathtt{tr}\left\{\left(\mathbf{Q}\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\mathbf{Q}\right)\mathbb{E}\left[\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right]\right\} \\
&\qquad + \mathtt{tr}\left\{\left(\mathbf{Q}\tilde{\mathbf{g}}\tilde{\mathbf{g}}^\dagger\mathbf{Q}\right)\mathbb{E}\left[\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right]\right\} \\
&\qquad\qquad + \mathtt{tr}\left\{\mathbf{Q}\mathbb{E}\left[\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\mathbf{Q}\Delta\tilde{\mathbf{g}}\Delta\tilde{\mathbf{g}}^\dagger\right]\right\} \\
&= 2\sigma_g^2\left(\tilde{\mathbf{g}}^\dagger\mathbf{Q}\mathbf{Q}\tilde{\mathbf{g}}\right) + \sigma_g^4\mathtt{tr}\left\{\mathbf{Q}\mathbf{Q}\right\} + \sigma_g^4\left(\mathtt{tr}\left\{\mathbf{Q}\right\}\right)^2
\end{aligned}
\tag{16}
$$

Now, $\mathrm{var}[x]$ can be expressed as

$$
\sigma_x^2 = \mathbb{E}\left[\hat{x}^2\right] - \left(\mathbb{E}\left[\hat{x}\right]\right)^2 = 2\sigma_g^2\left(\tilde{\mathbf{g}}^\dagger\mathbf{Q}\mathbf{Q}\tilde{\mathbf{g}}\right) + \sigma_g^4\mathtt{tr}\left\{\mathbf{Q}\mathbf{Q}\right\} \tag{17}
$$

After some algebraic manipulations, the mean and variance of x can be expressed by

$$
\begin{cases}
\mu_x = \tilde{\mathbf{g}}^\dagger\mathbf{Q}\tilde{\mathbf{g}} + \sigma_g^2\mathtt{tr}\left\{\mathbf{Q}\right\} = \mathbf{w}^\dagger\mathbf{B}\mathbf{w} \\
\sigma_x^2 = 2\sigma_g^2\left\|\tilde{\mathbf{g}}^\dagger\left[\mathbf{T}\odot\left(\mathbf{w}^*\mathbf{w}^T\right)\right]\right\|^2 + \sigma_g^4\left\|\mathbf{T}\odot\left(\mathbf{w}^*\mathbf{w}^T\right)\right\|^2
\end{cases}
\tag{18}
$$

where $\mathbf{B} \triangleq \tilde{\mathbf{G}} - \gamma\tilde{\mathbf{S}} + \mathbf{V}$, in which

$$
\begin{cases}
\tilde{\mathbf{G}} = P_{\mathrm{S}}\left(\tilde{\mathbf{g}}^*\odot\mathbf{f}\right)\left(\tilde{\mathbf{g}}^*\odot\mathbf{f}\right)^\dagger \\
\tilde{\mathbf{S}} = \sigma_R^2\mathtt{diag}\left(\left[\tilde{\mathbf{g}}^*\tilde{\mathbf{g}}^T\right]_{11}, \left[\tilde{\mathbf{g}}^*\tilde{\mathbf{g}}^T\right]_{22}, \ldots, \left[\tilde{\mathbf{g}}^*\tilde{\mathbf{g}}^T\right]_{MM}\right) \\
\mathbf{V} = \sigma_g^2\left[P_{\mathrm{S}}\mathtt{diag}\left(\left[\mathbf{f}\mathbf{f}^\dagger\right]_{11}, \left[\mathbf{f}\mathbf{f}^\dagger\right]_{22}, \ldots, \left[\mathbf{f}\mathbf{f}^\dagger\right]_{MM}\right) - \gamma\sigma_R^2\mathbf{I}\right]
\end{cases}
\tag{19}
$$

III. The Proposed Approach

A. The Sub-Optimal Solution

With the above Gaussian approximation to x in (18), the optimization problem of (11) can be reformulated by

$$
\begin{aligned}
\min_{\mathbf{w}}\quad & \mathbf{w}^\dagger\mathbf{D}\mathbf{w} \\
\text{s.t.}\quad & \mathbf{w}^\dagger\mathbf{B}\mathbf{w} - \eta_x\sqrt{\psi(\mathbf{w})} \geq \gamma\sigma_D^2
\end{aligned}
\tag{20}
$$

where $\psi(\mathbf{w}) = 2\sigma_g^2\left\|\tilde{\mathbf{g}}^\dagger\left[\mathbf{T}\odot\left(\mathbf{w}^*\mathbf{w}^T\right)\right]\right\|^2 + \sigma_g^4\left\|\mathbf{T}\odot\left(\mathbf{w}^*\mathbf{w}^T\right)\right\|^2$. By introducing a new variable $\mathbf{X} = \mathbf{w}\mathbf{w}^\dagger$, the optimization problem in term of $\mathbf{X}$ is equivalent to

$$
\begin{aligned}
\min_{\mathbf{X}}\quad & \mathtt{tr}\{\mathbf{D}\mathbf{X}\} \\
\text{s.t.}\quad & \mathtt{tr}\{\mathbf{B}\mathbf{X}\} - \eta_x\sqrt{\Psi(\mathbf{X})} \geq \gamma\sigma_D^2, \\
& \mathbf{X}\succeq\mathbf{0}, \mathrm{rank}\left(\mathbf{X}\right) = 1
\end{aligned}
\tag{21}
$$

where $\Psi(\mathbf{X}) = 2\sigma_g^2\left\|\tilde{\mathbf{g}}^\dagger\left(\mathbf{T}\odot\mathbf{X}^*\right)\right\|^2 + \sigma_g^4\left\|\mathbf{T}\odot\mathbf{X}^*\right\|^2$. It can be seen that, the optimization problem (21) is non-convex and turns out to be very intractable [10]. We find that the non-convex term $\sqrt{\Psi(\mathbf{X})}$ is upper-bounded as

$$
\sqrt{\Psi(\mathbf{X})} \leq \sqrt{2}\sigma_g\left\|\tilde{\mathbf{g}}^\dagger\left(\mathbf{T}\odot\mathbf{X}^*\right)\right\| + \sigma_g^2\left\|\mathbf{T}\odot\mathbf{X}^*\right\| \tag{22}
$$

Hence, the QoS constraint can be safely replaced by the upper-bound. Consequently, the conservative optimization problem is

$$
\begin{aligned}
\min_{\mathbf{X}}\quad & \mathtt{tr}\{\mathbf{D}\mathbf{X}\} \\
\text{s.t.}\quad & \mathtt{tr}\left(\mathbf{B}\mathbf{X}\right) - \eta_x\Phi(\mathbf{X}) \geq \gamma\sigma_D^2, \\
& \mathbf{X}\succeq\mathbf{0}, \mathrm{rank}\left(\mathbf{X}\right) = 1
\end{aligned}
\tag{23}
$$

where $\Phi(\mathbf{X}) = \sqrt{2}\sigma_g\left\|\tilde{\mathbf{g}}^\dagger\left(\mathbf{T}\odot\mathbf{X}^*\right)\right\| + \sigma_g^2\left\|\mathbf{T}\odot\mathbf{X}^*\right\|$. If the non-convex rank-1 constraint is dropped, the problem becomes convex and could be efficiently solved by the well-known interior-point method, e.g. CVX [11]. It is worth noting that, the original outage probability constraint is replaced by the conservative upper-bound (22), which will lead to a smaller feasible region and therefore this design is suboptimal. For that reason, the minimal transmit power of the suboptimal solution will be larger than transmit power of the potential optimal solution, which can be observed from the following simulation results.

B. The Near-Optimal Solution

By adding an upper-bound constraint to the variance of x, i.e. $\sqrt{\psi(\mathbf{w})} \leq t$, the optimization problem in (20) becomes

$$
\begin{aligned}
\min_{\mathbf{w}}\quad & \mathbf{w}^\dagger\mathbf{D}\mathbf{w} \\
\text{s.t.}\quad & \mathbf{w}^\dagger\mathbf{B}\mathbf{w} - \eta_x t \geq \gamma\sigma_D^2, \psi(\mathbf{w}) \leq t^2
\end{aligned}
\tag{24}
$$

Similarly, introducing $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^\dagger$, problem (24) is equivalent to

$$
\begin{aligned}
\min_{\mathbf{X}}\quad & \mathtt{tr}\{\mathbf{D}\mathbf{X}\} \\
\text{s.t.}\quad & \mathtt{tr}\{\mathbf{B}\mathbf{X}\} - \eta_x t \geq \gamma\sigma_D^2, \Psi(\mathbf{X}) \leq t^2, \\
& \mathbf{X}\succeq\mathbf{0}, \mathrm{rank}\left(\mathbf{X}\right) = 1
\end{aligned}
\tag{25}
$$

If the non-convex rank-1 constraint is dropped, for certain t, the optimization problem has affine objective, together with SDP and SOC constraint, hence it is convex and can be optimally solved by interior-point method, e.g. CVX. If the final solution $\mathbf{X}$ to the rank-relaxed problem of (23) and (25), happens to be rank-1, then $\mathbf{w}$ could be exactly recovered by eigenvalue decomposition. If not, we could pick the "best" rank-1 solution from all the rank-1 iterations, see [12] and references therein. It is worth mentioning that, the scenario where each relay node has there own power constraint can also be easily incorporated by adding constraint to $\mathbf{X}$ due to convexity. The global optimal solution can be obtained with a efficient one-dimension search on t, such as DIRECT [13]. The above one-dimension search needs the range of t be bounded by an closed-interval and we will tackle this in subsection C.

C. The Feasible Region of the One-dimension Search

The exact bound for the one-dimension search is generally intractable since it involves the transmit power of the relay nodes, which is difficult to know in advance. Since $\psi(\mathbf{w})$ is the sum of two matrix norm, then obviously $t \geq 0$. As a result, our main focus is on obtaining the upper-bound for t. The one-dimension search is anticipated to be more efficient if the upper-bound is tighter.

If problem (20) is feasible and the possible optimal solution for $\mathbf{w}$ is denoted by $\mathbf{w}_{\text{opt}}$, the following constraint must be fulfilled at the optimal solution

$$
\mathbf{w}_{\text{opt}}^\dagger\mathbf{B}\mathbf{w}_{\text{opt}} - \eta_x t_{\text{opt}} \geq \gamma\sigma_D^2 \tag{26}
$$

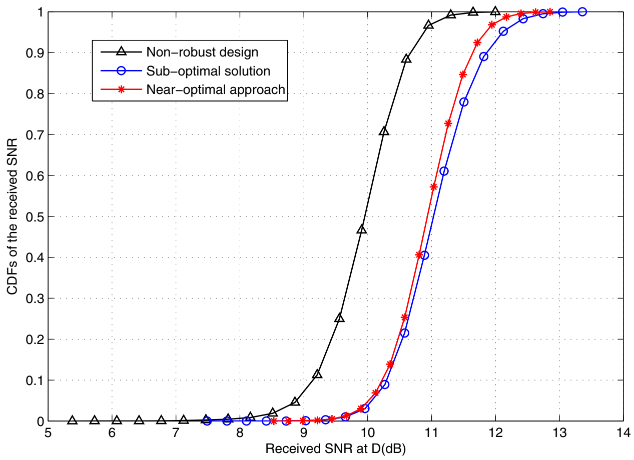

Fig. 2. CDFs of the achieved SNR with target SNR of 10dB, outage probability constraint $\varepsilon = 0.95$ and channel uncertainty variance $\sigma_g^2 = 0.1$.

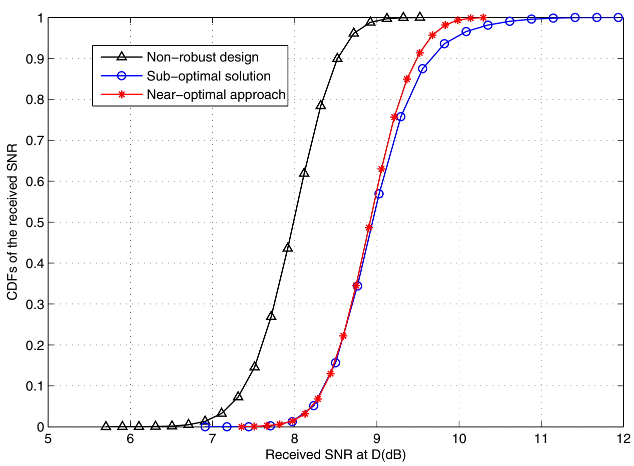

Fig. 3. CDFs of the achieved SNR with target SNR of 8dB, outage probability constraint $\varepsilon = 0.98$ and channel uncertainty variance $\sigma_g^2 = 0.05$.

where t_{opt} is the possible optimal value of t by the one-dimension search. Denoting the minimal transmit power of the suboptimal solution to (23) as P_{sub}. Hence

$$\mathbf{w}_{\text{opt}}^{\dagger}\mathbf{D}\mathbf{w}_{\text{opt}} \leq P_{\text{sub}} \tag{27}$$

Based on (26) and (27), we have

$$t_{\text{opt}} \leq \frac{\mathbf{w}_{\text{opt}}^{\dagger}\mathbf{B}\mathbf{w}_{\text{opt}} - \gamma\sigma_D^2}{\eta_x} \leq \frac{P_{\text{sub}}\lambda_{\max}\left(\mathbf{D}^{-1}\mathbf{B}\right) - \gamma\sigma_D^2}{\eta_x} \tag{28}$$

Then the search region is given by $t \in [0, \tau]$. where

$$\tau = \frac{P_{\text{sub}}\lambda_{\max}\left(\mathbf{D}^{-1}\mathbf{B}\right) - \gamma\sigma_D^2}{\eta_x}. \tag{29}$$

IV. Simulation Results and Discussion

In this section, computer simulations are carried out to verify the performance of our proposed approaches compared with a conventional non-robust scheme described in [1]. Without loss of generality, we consider a network with $M = 10$ relays and all the links are subject to identical and independent zero mean circular complex Gaussian distribution, i.e. $\mathbf{f}, \mathbf{g} \sim \mathcal{CN}\left(0, \mathbf{I}\right)$. The CSI mismatches are generated according to $\Delta\tilde{\mathbf{g}} \sim \mathcal{CN}\left(0, \sigma_g^2\mathbf{I}\right)$. Throughout the simulation process, the source power is defined as $P_{\text{S}} = 10$dB and all the nodes in the network have the same noise power, i.e. $\sigma_R^2 = \sigma_D^2 = 1$.

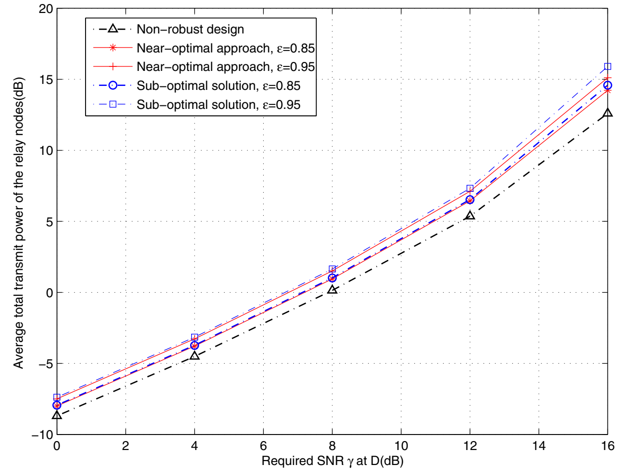

Fig. 4. Average total transmit power of the relay nodes versus target SNR(dB) with service probability requirements $\varepsilon = 0.85, 0.95$, and channel uncertainty variance $\sigma_g^2 = 0.1$.

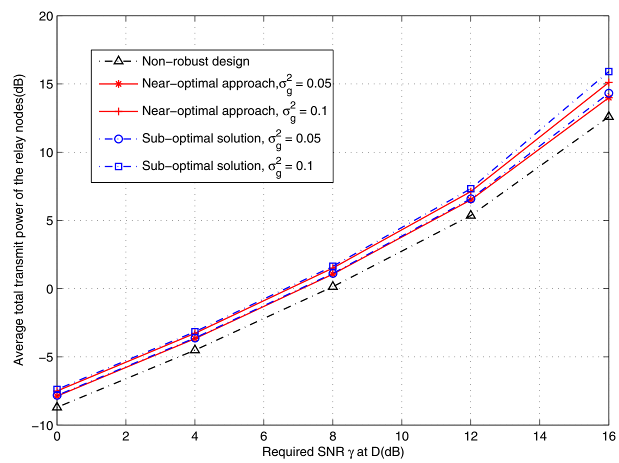

Fig. 5. Average total transmit power of the relay nodes versus target SNR(dB) with service probability requirements $\varepsilon = 0.95$, and channel uncertainty variance $\sigma_g^2 = 0.05, 0.1$.

We first study the achieved outage probability resulting from CSI mismatches of all the schemes. For that goal, the CDFs of the received SNR at D targeting at 10dB with the QoS requirements determined by $\varepsilon = 0.95$ and $\sigma_g^2 = 0.1$, are plotted in Fig. 2. It can be observed that, the constraint on outage probability can be exactly met by the proposed approaches, while the destination D declares outage with about 50% of the channel realizations for the non-robust method, since it is impossible to decode correctly when the received SNR falls below the preselected threshold. Fig. 3 provides another illustration for the resulting outage performance with different QoS constraints. Both figures show that, the distribution range of the received SNR of the suboptimal solution is slightly larger than the near-optimal approach, which therefore, will lead to energy efficiency reduction.

To investigate the power consumption of the proposed approaches, we plot the average total transmit power of the relay nodes versus the required SNR threshold γ, with different outage probability constraints measured by ε, in Fig. 4. It can be seen that the transmit power rises accordingly with the increase of the required SNR γ for all the schemes and better QoS measured by lower outage probability can be achieved at the cost of more transmit power consumption. Fig. 5 gives another depiction of the energy efficiency for the proposed approaches with different channel uncertainty variances σ_g^2. We can find that, more transmit power is needed to compensate larger channel uncertainty variance. Both figures reveal that, the non-robust design has the minimal transmit power since no robustness is guaranteed and both the proposed approaches have mild energy efficiency reduction.

V. Conclusion

In this paper, we propose a robust downlink collaborative relay beamforming approach with the imperfect knowledge of CSI. The equivalent condition for outage probability specified constraint is derived. Subsequently, we show that the optimization problem can be optimally solved by using the interior-point method together with an efficient one-dimension search technique. Simulation results demonstrate that the QoS measured by outage probability can be exactly maintained in statistical sense while it is violated by the non-robust technique.

Acknowledgment

This work was supported in part by National Natural Science Foundation of China (60872024); Cultivation Fund of the Key Scientific and Technical Innovation Project (708059); Open Fund of State Key Laboratory of Integrated Services Networks (ISN12-10); Independent Innovation Foundation of Shandong University (2010JC007), Natural Science Foundation of Shandong (ZR2011FM027); open research fund of National Mobile Communications Research LaboratorySoutheast University(2010D10); open research fund of National Digital Multimedia Key Laboratory(2011-1-1557); China Postdoctoral Science Foundation funded project(2011M501092); Special Fund for Postdoctoral Innovative Projects of Shandong Province(201103003); Shandong Technical Innovative Project (201110201007).

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