Uplink interference protection as a non-cooperative game over OFDMA networks

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***Abstract*—We propose in this paper a distributed mechanism for intercell interference (ICI) reduction with full frequency reuse. Our algorithm is based on a non-cooperative game, where users try to maximize a utility function which is proportional to the Shannon capacity. An intelligent pricing scheme, based on the allocated user rate is proposed. This pricing method allows the system to adjust the mobile's transmit power, in order to prioritize users which are close to the cell edge or with low signal-to-interference plus noise ratios (SINR) levels. An optimal centralized algorithm is also developed to provide a performance bound. Our simulations show that the proposed algorithm achieves considerable gains in energy efficiency and fairness using a reduced set of channel path gains for the optimization process.**

***Index Terms*—Game theory; energy efficiency; interference protection; non-cooperative games.**

I. Introduction

The main design goals behind the fourth generation (4G) of wireless systems are higher user bit rates, low delays and increased spectral efficiency [1]. How to accommodate all these requirements has become an important research issue in wireless networks. A smart resource allocation method based on a joint consideration of power control and rate adaptation is one effective method to improve network performance, since the available radio resources such as bandwidth and power are very limited [2]. Previous work has been done for multiple-cell multiuser scenario perspective, with the purpose of improving the efficiency of Orthogonal Frequency Division Multiple Access (OFDMA) systems under different objectives and constraints [2]–[4]. In such systems, intercell interference (ICI) is a major issue, as the cell edge performance is particularly susceptible to ICI [3]. Current research has been focused on maximizing the system performance due to capacity or energy efficiency without considering a fair distribution of the system resources. In most of the recent proposals, users which are close to the base station (BS) are prioritized which can be unfair for other users which are close to the cell edge. Furthermore, for practical implementations distributed resource allocation schemes are preferred since they are simple and require less information exchange [5].

Game theory provides a natural framework for developing pricing mechanisms of direct relevance to the resource allocation problem in wireless networks. We can say that in such networks the users behave non-cooperatively, since each user attempts to maximize its own utility function in response to the actions of other users. This approach naturally leads to distributed control as we will see further on.

In this paper, we present a distributed non-cooperative game (NCG) theory mechanism which provides interference protection for interference-prone users. Our proposal enhances fairness and energy efficiency without inter-cell exchange information. The rest of the paper is structured as follows. In Section II, we present our problem scenario and the proposed interference mitigation technique based on a game theory framework. In Section III, we demonstrate that our NCG theory framework fulfills the necessary conditions for the existence and uniqueness of the equilibrium. A summary of the state of art schemes and our simulation scenario are described in Section IV. Simulation results are presented in Section V. Finally, Section VI offers concluding remarks.

II. Interference Scenario and System Model

In Fig. 1 the interference scenario is presented. There are three mobile stations (MSs) transmitting on the uplink (UL) simultaneously on the same resource block (RB), which are served by three different base stations (BSs). The vulnerable MS_{v1} is served by the base station BS_v and the interfering MS_{i1} and MS_{j1} are served by BS_i and BS_j respectively. The uplink interference that is caused by MS_{i1} and MS_{j1} to the vulnerable MS_{v1} will decrease the received SINR of the MS_{v1} at its serving BS. Hence, in order to protect vulnerable users, which are close to the cell edge or with low SINR levels, we present in this paper a game theory framework for uplink interference protection.

A. Joint Power Control and Rate Allocation Utility Function Design

We describe here the model proposed in this paper for a multiple cell OFDMA system with up to M users per RB. Let $G = [M, \{P_k\}, \{U_k(\cdot)\}]$ denote the NCG where $k = \{1, 2, \ldots, M\}$ is the set of mobiles transmitting on the same RB; P_k is the strategy set and U_k is the utility function of user k. Each user selects the power level p_k, such that $p_k \in P_k$. The choices in this strategy space are limited to the maximum transmitted power of the mobile,

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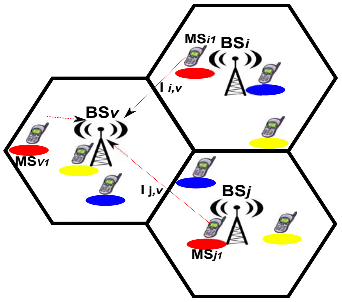


Fig. 1. UL interference scenario

$P_k \in \{P_{\min}, P_{\max}\},\ \ k = 1, \ldots, M$. This is a convex set with minimum and maximum power constraints. By definition $P_{\max} \geq P_{\min}$. We denote as the outcome of the game $\mathbf{p} = (p_1, \ldots, p_M) \in P$, which is the vector composed by the selected power levels of the M users. The resulting utility levels of the k-th user is $U_k(\mathbf{p})$. We define the utility function of the k-th user, as the difference between the throughput of user k and its pricing function. The user throughput function is chosen as a logarithmic function of the k-th user's SINRs, which we denote by γ_k. Furthermore, the pricing function defines the "price" μ_k that is paid by the k-th user by using power p_k to transmit. This means that the pricing factor reduces linearly the utility by a factor $\mu_k p_k$, for some $\mu_k \geq 0$. Accordingly, the utility function of the k-th user is defined as:

$$U_k(p_k, p_{-k}) = \log(1 + \gamma_k) - \mu_k p_k, \tag{1}$$

where $p_k \geq 0$ and p_{-k} denote the vector of power levels of all the users except the k-th one, and γ_k is the SINR of user k given by

$$\gamma_k = \frac{h_{kl} p_k}{\sum\limits_{j \neq k} h_{jl} p_j + \eta}. \tag{2}$$

The parameter $h_{kl} > 0$ is the channel path gain of the k-th user to the base station l, and $\eta > 0$ is the power of the thermal noise. We consider the same channel model used in [3], which is stated as follows

$$h_{kl} = \mid H_{kl} \mid^2 10^{-\frac{-L(d) + X_\sigma}{10}}, \tag{3}$$

where h_{kl} is the gain between the transmitter k and the receiver l separated by d m. H_{kl} describes the channel transfer function between the transmitter k and receiver l, X_σ is the log-normal shadowing value (dB) with standard deviation σ and L(d) is the distance-dependent path loss (dB) which is calculated as follows:

$$L(d) = 15.3 + 37.6 \log_{10}(d). \tag{4}$$

Note that (1) shows the interdependence between users. The utility of user k depends of the transmitted power of the k-th user and the power strategy that the remaining users have chosen. We conceive of the power control scheme as a utility maximization problem, where users play an NCG in order to get the highest payoff based on their utility function. This premise can be formulated as

$$\max U_k(p_{k,} p_{-k})\ p_k \ \in \ P_k\ k = 1, \ldots, M. \tag{5}$$

B. Pricing Based on Adaptative Rate Allocation

In a non-cooperative wireless network, pricing is an important factor, which creates an incentive for the users to adjust their power levels in order to achieve the common goals of the network. This pricing factor makes the game more "Pareto efficient" than one with no pricing [2]. In the system we propose, we are studying the price per unit of power that the k-th user consumes. The power scaling μ_k is determined by the base station in order to relate the power allocation with the k-th user's spectral efficiency. As has been stated in [1], it is possible to obtain larger SINR improvements if ICI is reduced at the cell edge than for the cell center, this is because at the cell center spectral efficiency increases logarithmically with SINR, while it increases linearly at the cell edge. Based on this premise, in this work we define three frequency bands: *high priority*, *mid priority* and *low priority*, as shown in Fig. 2. Hence for an RB that has assigned a high priority status in one cell, the same RB is assigned *mid* and *low priority* status in the neighboring cells, as no RB can have the same priority between neighboring cells. This technique is useful to control the power that is consumed by each terminal. As is shown in Fig. 2, in a fair allocation scheme, *high priority* RBs should be allocated to mobiles with bad SINR conditions or low spectral efficiency. This will allow mobiles with low spectral efficiency to transmit up to P_{max}. Terminals with *low priority* and *mid priority* RBs are restricted to transmit with less power in order to reduce ICI. *Low priority* RBs will be allocated to cell center users which are able to achieve a high spectral efficiency due to more favorable propagation conditions. The algorithm utilizes the average rate over the previous *x* time slots (where *x* is defined as an algorithm parameter). This information is normally available at the BS.

$$\kappa_{kl} : \{\bar{T}_{1,l}, \bar{T}_{2,l}, \ldots, \bar{T}_{N,l}\}, \tag{6}$$

where κ_{kl} denotes the set of average users throughput in the cell, $\bar{T}_{kl}$, denotes the average throughput of MS$_k$ in cell l, and N denotes the number of mobiles in the cell. Thus, the next step is to sort the $\bar{T}_{kl}$, in ascending order, with the purpose of identifying mobiles with lower spectral efficiency

$$\kappa_k^* = f(\kappa_k) = \{\bar{T}_{(1),l}, \bar{T}_{(2),l}, \ldots, \bar{T}_{(N),l}\}, \tag{7}$$
$$\text{s.t } \bar{T}_{(1),l} \leq \bar{T}_{(2),l} \leq \ldots \leq \bar{T}_{(N),l},$$

where κ_k^* is the ordered set of throughput measurements per cell. The function $f(\cdot)$ that defines this ordering can now be applied to the cell of interest. Furthermore, the set of high priority users ($S_{hp,l}$) under the cell l can be obtained in the following way. For simplicity, define $S_{T,l} = N$, hence $S_{hp,l}$

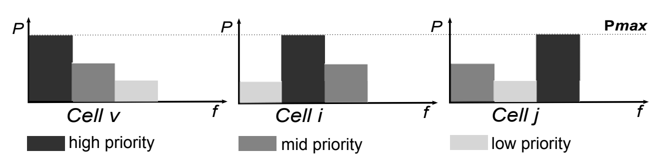


Fig. 2. Allocation of priority bands in neighboring cells a, b, and c (after [3]).

can be found by applying:

$$S_{hp,l} = \left\{ s \in S_{T,l} : f(s) \leq \left[\frac{N}{n}\right] \right\} \; S_{hp,l} \in S_{T,l}, \tag{8}$$

where n denotes the number of priority bands in the system (n = 3, for our system). In (8) the *high priority* RBs are allocated to the $\left[\frac{N}{3}\right]$ users with the lowest spectral efficiency. The *low priority* RBs are allocated to the mobiles with the highest spectral efficiency, thus the $\left[\frac{N}{3}\right]$ users with the highest spectral efficiency. Finally the *mid priority* RBs are allocated to the remaining $\left[\frac{N}{3}\right]$ mobiles. Due to operational requirements in the proposed system, the price of the k-th user μ_k is fixed by the base station based on the average user's throughput in the cell κ_k^*. The price of the k-th user in the cell can be defined by:

$$\mu_k = y(\kappa_k^*) = \left\{ \frac{\bar{T}_{(1),l}}{c_k}, \frac{\bar{T}_{(2),l}}{c_k}, \ldots, \frac{\bar{T}_{(N),l}}{c_k} \right\} \tag{9}$$

$$\text{s.t } \frac{\bar{T}_{(1),l}}{c_k} \leq \frac{\bar{T}_{(2),l}}{c_k} \leq \ldots \leq \frac{\bar{T}_{(N),l}}{c_k},$$

where $c_k = \bar{T}_{(1),l} \times \; P_{max}$. From (9), if a mobile is allocated to a *high priority* RB, the price factor imposed on the design of the utility function should be lower than for a mobile which is allocated to a *mid* or *low priority* RB. In this way, our allocation scheme fulfills the conditions imposed by the priority bands, shown in Fig. 2.

III. Existence and uniqueness of the equilibrium

In a non-cooperative game (NCG), the users will try to maximize their utility functions, and after multiple iterations, the game will converge to an *equilibrium* point (if it exists). At equilibrium, all the users should be satisfied with the utilities that they obtain from the NCG, if so the *equilibrium* point is called *Nash Equilibrium* [4,6].

Definition 1: A power vector $\mathbf{p} = (p_1 \ldots p_M)$ is a *Nash equilibrium* of the NCG, $G = [M, \{P_k\}, \{U_k(\cdot)\}]$ if, for every $k \in M$, $U_k(p_k, p_{-k}) \geq U_k(p'_k, p_{-k})$ for all $p_k \in P_k$.

Theorem 1: A *Nash equilibrium* exists in game $G = [M, \{P_k\}, \{U_k(\cdot)\}]$ if, for all $k = 1, \ldots, M$:

1) P_k is a nonempty, convex and compact subset of some Euclidean space $\Re^n$

2) $U_k(\mathbf{p})$ is continuous in p and quasi-concave [7], in p_k.

For the first part of *Theorem 1*, we have already stated in Section II, that P_k is a compact convex set. For the second part using (1) and (2), it is easy to show that U_k is twice differentiable over p_k and the second derivative is always negative for any value of p_k. Therefore, the second order conditions for concavity are fulfilled [7]. Hence, the inner solution if it exists, is the unique point maximizing the cost function and it is defined by (10). The boundary solution $p_k = 0$, is the other possible maximization point for the optimization problem. If the user utility function $U_k(p_k, p_{-k})$ reaches a value less than zero, the optimal solution will be the boundary point

$$p_k^{\cdot} = \frac{1}{\mu_k} - \frac{\sum\limits_{j \neq k} h_{jl} p_j + \eta}{h_{kl}}. \tag{10}$$

Theorem 2: The NCG has a unique equilibrium.

For notational convenience, let us denote a user specific parameter a_k for the k-th user and the user interference at the base station l as $I(p_{-k})$

$$a_k = \frac{h_{kl}}{\mu_k} - \eta, \tag{11}$$

$$I(p_{-k}) = \sum_{j \neq k} h_{jl} p_j. \tag{12}$$

Hence, the optimal response $b_k(p_{-k})$ of the k-th user is:

$$b_k(p_{-k}) = \begin{cases} \frac{1}{h_{kl}} \left[a_k - I(p_{-k}) \right], & a_k \geq I(p_{-k}) \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

The solution for the maximization problem will be the best response of user k to the other users strategies, which are denoted by p_{-k}.

Definition 2: The best response $b_k(p_{-k})$ of the player k to the profile strategies p_{-k} is the strategy p_k such that

$$b_k(p_{-k}) = \{ p_k \in P_k : \text{argmax } U_k(p_k, p_{-k}) \}. \tag{14}$$

A *Nash equilibrium* for the NCG can be stated as the power vector $\mathbf{p}$ which fulfills: $p_k \in b_k(p_{-k})$. When conditions of *Theorem 1* are satisfied, the correspondence $b(\cdot)$ is nonempty, convex-valued, and upper semicontinous for all k. Thus, there exists a fixed point $\mathbf{p}$ such that $p_k \in b_k(p_{-k})$ for all $k \in M$ [6]. This fixed point is by definition the *Nash equilibrium.* From (13), the set of linear fixed point equations which converges to the equilibrium solution, if it exists, can be written in matrix form (15):

$$\begin{bmatrix} 1 & \frac{h_{21}}{h_{11}} & \frac{h_{31}}{h_{11}} & \ldots \frac{h_{M1}}{h_{11}} \\ \frac{h_{12}}{h_{22}} & 1 & \frac{h_{32}}{h_{22}} & \ldots \frac{h_{M2}}{h_{22}} \\ \vdots & \vdots & \vdots & \ddots \vdots \\ \frac{h_{1M}}{h_{MM}} & \frac{h_{2M}}{h_{MM}} & \frac{h_{3M}}{h_{MM}} & \ldots 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} = \begin{bmatrix} \frac{a_1}{h_{11}} \\ \frac{a_2}{h_{22}} \\ \vdots \\ \frac{a_M}{h_{MM}} \end{bmatrix} \tag{15}$$

$$\Leftrightarrow \mathbf{Ap=c.}$$

For *Theorem 2*, we need to show that the $M \times M$ matrix **A** is non-singular. This means that the system has a unique solution given by **p=** $\mathbf{A}^{-1}\mathbf{c}$. Therefore if $\mathbf{A}^{-1}$ exists, it is equivalent to prove that **Ax**$= 0 \rightarrow \mathbf{x} = 0$ [8]. Furthermore, define the vector **x**$= (x_1 x_2 \ldots x_M)^T \neq 0$ such that **Ax**$= 0$. This equation can be written as: $\sum h_{jl} x_j$=0. Summing up this set of equations for all the M users, we have:

$$\sum_{l=1}^{M} \sum_{j=1}^{M} h_{jl} x_j = 0. \tag{16}$$

In (16), the channels gains as we have stated before are greater than 0. The only value that can be zero is the vector **x**. If $\mathbf{x} = 0$, we are at the boundary solution of our problem, which means that no mobile will be active. For any other case $\mathbf{A}^{-1}$ exists, provided that the rows of **A** are linearly independent. Hence, the matrix **A** is non-singular and the system has a unique solution given by **p**.

IV. State of Art Schemes and Simulation Scenario

In order to compare the performance of our algorithm, we present two state of art schemes in Subsections A and B, below. We then present resource allocation schemes that improve the system performance by achieving a fair allocation for the distribution of limited system resources.

A. Centralized SINR Balancing

On one hand, we need to find an optimal solution as a performance bound, since distributed algorithms cannot always achieve global optima [5]. The most straightforward idea is to let the system centrally decide how to allocate the system resources. As an example, [5] introduces the concept of SINR balancing, which allows a fair distribution of the system's resources. Since all the users in the same resource block target similar throughput and SINR levels, it proposes a hypothetical global power control algorithm (PCA) scheme that minimizes the interference probability. This algorithm requires full knowledge of base-mobile and interference path gains.

B. Uplink Intercell Interference Coordination (ICIC)

On the other hand, low complexity distributed algorithms are preferred since they can be implemented without the need for new network entities in current wireless systems. Furthermore, they require only a small set of channel path gains for the optimization process. As an example, [3] proposes a distributed uplink ICIC technique. It introduces the use of frequency bands in order to prioritize users with low SINR levels, as is shown in Fig. 2. Therefore, MSs allocated in the *mid* and *low priority* bands are generally required to adjust their transmit power in order to allow the *high priority* mobiles to achieve a common target SINR. *High priority* users always transmit at maximum power. MSs in the *mid priority* band scale their power first, based on the interference level caused to the *high priority* users. MSs in the *low priority* band must then reduce their power, taking into account the minimum interference metric of both the *high* and *mid priority* users in the neighboring cells for the same RB. The main drawback of this method is when *high priority* users are affected by high interference levels, since this situation make *mid* and *low priority* users transmit under the common target SINR. Furthermore, optimization is performed independently by each mobile, so only local optima are achieved.

C. Non-Cooperative Game (NCG) Theory Algorithm

The users will try to maximize the utility function (1) playing an NCG until the equilibrium is achieved. Furthermore, the mobiles will update their powers at time instants given by $t = \{t_1, t_2, t_3, \ldots\}$, as is shown in the following algorithm:

Algorithm 1: Consider the NCG defined above. Generate the sequence of power as follows:

1) Set the initial power vector $\mathbf{p}(0) = \mathbf{p}$, where $\mathbf{p}$ is a vector where all the mobiles transmit at $P_{\max}$. Set $j = 1$.

2) For all j such that $t_j \in t$ and for all terminals given $\mathbf{p}(t_{j-1})$, compute, $\hat{p_k}(t_j) = b_k(p_{-k})$ $k = 1, 2, \ldots M$.

TABLE I
Channel Path Gains

Method	Propagation paths
Uplink ICIC	$RB_{num} \times (M + hp_u \times (M - hp_u)$ $+ (mp_u \times lp_u))$
Centralized SINR Balancing	$RB_{num} \times M^2$
Game Theory	$RB_{num} \times M$

TABLE II
Simulation Parameters

Parameter	Value
MSs per macro-cell, N	6
Intersite-site distance	200m
Number of available RBs, RB_{num}	6
Number of cells, M	7
RB bandwidth, B_{RB}	180kHz
Subcarriers per RB, k_{sc}	12
Symbol rate per subcarrier, ϱ_s	15ksps
Subframe duration	1ms
Thermal noise, η	-174dBm/Hz
Total MS transmit power	23dBm
Shadowing, Std. Dev., σ	4dB

3) Repeat the same procedure for the next time slot.

Algorithm 1 should converge to the *Nash equilibrium* as described in Section III. In our simulation, the users achieve the equilibrium after four iterations.

D. Simulation Scenario

For this simulation, in order to obtain performance measurements, our method is compared with a *baseline* algorithm, where all the mobiles transmit at maximum power, and the two algorithms presented above. For comparing the performance of our algorithm, we consider the simulation parameters shown in Table II. The simulation is comprised of a single-tier, tessellated hexagonal cell distribution. However, statistics are only taken from the center cell. Each cell is served by a single omnidirectional BS with the users uniformly distributed in the cell, each RB is assigned to a single MS per cell. The priority bands are allocated in a way to make the priorities mutually orthogonal, as it was explained in Section II. For our own proposal, RB allocation and the price charged to each user for transmission are calculated based on the techniques introduced in Section II. Finally, each simulation is run over 20 time slots to provide the scheduler with the necessary and sufficient information to perform the resource allocation.

For comparison, the channel path gains that are needed for the optimization process are presented for the two state of art algorithms and our proposed method in Table I. Where RB_{num} is the number of available RBs in the system, lp_u, mp_u and hp_u stands *low*, *mid and high priority* users respectively, and M is the number of mobiles that share the same RB in different cells. For this work, we consider that each RB is assigned to a single MS per cell.

E. Performance Statistics

After the transmit power adjustments in each cell, the performance statistics can be gathered, which are composed by the following three metrics: the UL throughput, energy efficiency and Jain fairness [3], as in the following equations:

$$C_k(\gamma_k) = n_u^{RB} k_{sc} \varrho_s \varepsilon_s(\gamma_k) \ \text{[bits]} \tag{17}$$

$$\beta_k = \frac{C_k}{p_k} \left[\frac{\text{bits}}{\text{J}}\right] \tag{18}$$

$$\Gamma = \frac{(\sum_{i=1}^{n} C_k)^2}{n \sum_{i=1}^{n} C_k^2} \tag{19}$$

Where n_u^{RB} is the number of RBs assigned to MS_u and ε_s the symbol efficiency, previously given in [3].

V. Results

From the simulations, the CDFs are generated based on the performance of the algorithms presented in the last section. In Fig. 3 the overall user's performance in terms of throughput is displayed. As we can see from Fig. 3, our approach provides gains for *high* and *mid priority* users. On one hand, while sacrificing high-end throughput gains of approximately 13% are achieved at the 30^{th} percentile compared to the baseline method and the ICIC approach. Furthermore, the centralized algorithm outperforms our method with gains of 12%. On the other hand, the losses for *low priority* users are perceived at the 85^{th} percentile, since the centralized, the ICIC and the NCG method have losses of 57%, 10% and 20% respectively, compared to the baseline method. This is due to the displacement of throughput from the cell center to the cell edge, making the system fairer. Therefore, in Fig. 4(a) we observe that our algorithm performs 6% and 4% better than the baseline and the ICIC approach regarding fairness. However, the centralized method outperforms our NCG algorithm with gains of 15%, since the centralized method forces the users to target similar data rates. Regarding energy efficiency, it can be seen in Fig. 4(b) that we have gains of 54% and 40% at the 40^{th} percentile compared with the baseline and the ICIC approach. However, the centralized approach outperforms our algorithm with further gains of 57%. In the same situation for the 90^{th} percentile we have gains of 64% and 48%, and the centralized approach has gains of 55% compared to our method. Hence, we can infer that our algorithm is more energy efficient compared with the baseline and the ICIC method. These gains compared to the ICIC method and the baseline are related to the intelligent pricing mechanism that allows the mobile users close to the cell edge to transmit at full power only if it is really necessary, due to the interference conditions at its serving BS. In most cases, this pricing mechanism together with the NCG approach forces the users to transmit in a more energy efficient manner enhancing the energy savings and increasing fairness. Regarding the complexity of the schemes, the number of channel path gains that have to be estimated are: 42 for the NCG method, 132 for the ICIC method, and 294 for the centralized approach. We are using the 14% of channel path gains that the centralized method requires for its optimization process.

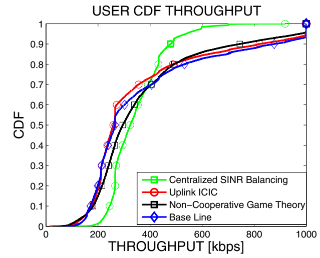
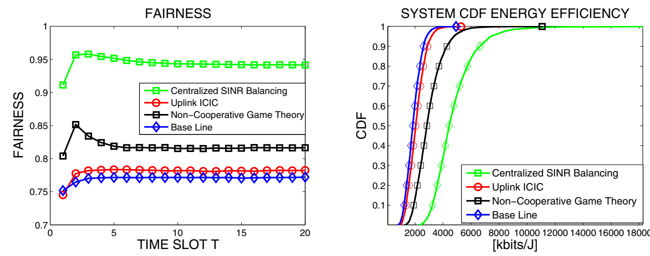


Fig. 3. User Average CDF Throughput



(a) Fairness (b) System CDF Energy Efficiency

Fig. 4. Performance comparison for Fairness and System Energy Efficiency

VI. Conclusions

The mechanism we presented here is a viable and distributed alternative for centralized ICI methods in cellular networks. Based on an NCG, it dynamically adjusts the power values of the users between a defined set of RBs. Our method allows the system to achieve substantial gains in energy efficiency and fairness with a low system complexity, since it does not require intercell communication to function properly. Given the promising results presented in this paper, future research will investigate the performance of our method in different scenarios. A further prospect can be the use of improved utility functions for the NCG.

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