Joint Utility Maximization in Two-tier Networks by Distributed Pareto-Optimal Power Control

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Abstract—**This paper addresses the critical problem of interference management in two-tier networks, where the newly-deployed femtocell users (FUEs) operate in the licensed spectrum owned by the existing macrocell. A Pareto-optimal power-control algorithm is devised that jointly maximizes the utilities of both macrocell and femtocell networks while robustly guaranteeing the macrocell's quality-of-service (QoS) requirements. After effectively enforcing the minimum signal-to-interference-plus-noise ratios (SINRs) prescribed by the macrocell users (MUEs) with the use of a penalty function, the Pareto-optimal boundary of the strongly-coupling SINR feasible region is characterized. Based upon the specific network utility functions and also the target SINRs of the MUEs, a unique operating SINR point is determined, and transmit power adapted to achieve such a design objective. We prove that the developed algorithm converges to the global optimum, and more importantly, it can be distributively implemented at individual links. Effective mechanisms are also available to flexibly designate the access priority between macrocells and femtocells, as well as to fairly share the system resources among different users. The merits of the proposed approach are verified by numerical examples.**

I. Introduction

Femtocells have recently emerged as a promising technology to increase wireless network capacity and extend cellular coverage. Since femtocells operate in the licensed spectrum owned by wireless operators and share this spectrum with macrocell networks, limiting the cross-tier interference from femtocell users (FUEs) at a macrocell BS becomes an indispensable condition. One of the key research issues is to develop interference management schemes such that the quality-of-service (QoS) requirements of all macrocell users (MUEs) are maintained, while the residual network capacity is optimally exploited by FUEs [1]. Given that only limited signaling information can be exchanged over backhaul wired networks, it is desirable to accomplish such an optimization by distributed algorithms.

Taking a game-theoretical approach, the studies in [2], [3] devise different power-control schemes for femtocell networks. In most instances, although the underlying games settle at the Nash equilibria (NE), such socially optimal operating points do not guarantee to be Pareto-efficient. With a novel pricing scheme, the work of [4] shows that the outcome of the non-cooperative power control game in single-cell systems is a unique and Pareto-efficient NE. In multicell settings where the transmit powers of all users need to be jointly optimized across different cells, intercell interference, however, cannot simply be treated as noise, making the solution of [4] inapplicable.

The authors of [5] propose a distributed power-control algorithm that enables users to eventually achieve their fixed target signal-to-interference-plus-noise ratios (SINRs). For feasible SINRs, the developed algorithm converges to a Pareto-optimal solution. In the context of two-tier networks, a fixed SINR assignment, however, is certainly not suitable as femtocells typically serve data users. SINRs should instead be adjusted to the extent that the system capacity can still support. A high SINR is translated into better throughput and reliability while a low SINR implying lower data rates. For data-service multicell systems, reference [6] performs a joint optimization of SINR assignment and power control to come up with distributed Pareto-optimal solutions. Yet, the results here apply to homogeneous networks where there exists no differentiated classes of users with distinct access priority and design specifications.

In a heterogeneous network deployment, the choices of target SINRs assigned to the lower-tier users are much more limited, and strict QoS guarantees need to be enforced to meet the demands of more prioritized users. Our earlier work [7] considers a mixed macro/femto network with higher-tier MUEs and lower-tier FUEs. Unlike [6], the devised solution accounts for the complicated coupling and strong interdependency among users in multi-tier settings. A Pareto-optimal power-control scheme has been developed, where FUEs have their utility maximized and all MUEs have their predefined SINR targets exactly met.

On the contrary, this paper proposes a power-control algorithm to deal with a more general problem formulation in which the utilities of *both* macrocell and femtocell networks are *jointly* maximized. In this practical scenario, the Pareto-optimal SINR frontier is no longer confined within the space spanned only by the number of FUEs, but rather the whole dimension of all MUEs and FUEs. Locating the particular SINR point on such a boundary to maximize the joint utilities is not trivial at all, since one must also satisfy the QoS requirements of the MUEs. This implies that the approach used in [7] is no longer applied. To effectively deal with this issue, we employ the penalty function method and utilize the load-spillage parametrization of [6] to provide an approximate solution. Through the proper tuning of penalty and control parameters, a *globally optimal* solution is eventually realized that can be implemented in a *distributed* manner. The developed algorithm can designate the access priority to MUEs and FUEs by adjusting the weight given of each type of users. The utility function here can also be regulated to fairly allocate radio resources to individual users.

II. System Model and Problem Formulation

Consider an uplink scenario in a two-tier wireless network, in which M MUEs establish links to its servicing macrocell base station (BS) while K FUEs also communicate with their respective femtocell BSs. Assume that the association of a certain FUE with its own femtocell BS is fixed during the runtime of the underlying power control. Without the loss of generality, denote the set of MUEs and FUEs by $\mathcal{L}_m := \{1, \cdots, M\}$ and $\mathcal{L}_f := \{M+1, \cdots, M+K\}$, respectively. The set of all users is then simply $\mathcal{L} = \mathcal{L}_m \cup \mathcal{L}_f$ whose cardinality is $|\mathcal{L}| = M+K$. An

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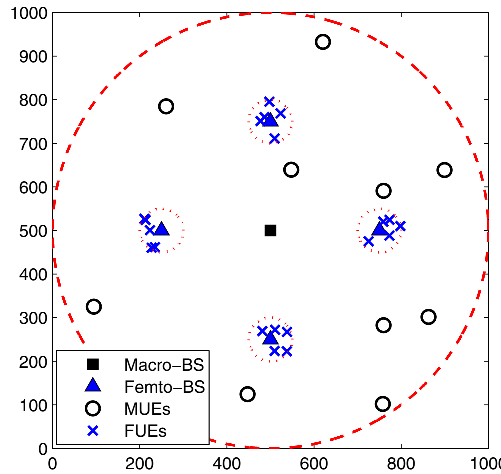

Fig. 1. Network topology and user placement in a two-tier wireless network.

example of the system under investigation is illustrated in Fig. 1. We assume that data-transmission time scale is far shorter than that of the underlying optimization process, allowing for any short-term statistical variations to be averaged out.

Denote by σ_i the receiving BS for user $i \in \mathcal{L}$ (either an MUE or a FUE). For brevity, the path between user i and its servicing BS σ_i shall be referred to as link i. Also, let $\bar{h}_{k,j}$ be the absolute channel gain from user j to BS k and define its corresponding normalization as $h_{k,j} := \bar{h}_{k,j}/\bar{h}_{\sigma_j,j}$. To represent the normalized channel gain from user j to the servicing BS of user i, we define the $(M+K)\times(M+K)$ channel matrix $\mathbf{H} = [H_{i,j}]$ as [6]

$$H_{i,j} \quad := \quad \begin{cases} 0, & \text{if } i = j, \\ 1, & \text{if } \sigma_i = \sigma_j,\, i \neq j, \\ h_{\sigma_i,j}, & \text{if } \sigma_i \neq \sigma_j. \end{cases} \tag{1}$$

Suppose that user j transmits to its serving BS σ_j, and let $p^{(j)}$ be the received power at σ_j by such a transmission. Since $\bar{h}_{\sigma_j,j}$ is the channel gain from j to σ_j, it is clear that j must have transmitted at a power level of $p^{(j)}/\bar{h}_{\sigma_j,j}$. At any BS k, this signal appears with a power of $\bar{h}_{k,j}\big(p^{(j)}/\bar{h}_{\sigma_j,j}\big) = h_{k,j}p^{(j)}$. Denote by $\mathcal{S}_i$ the set of users whose transmit powers appear as interference to link i. If $j \in \mathcal{S}_i$, this transmission interferes link i with power $h_{\sigma_i,j}p^{(j)} = H_{i,j}p^{(j)}$. The total interference at BS σ_i serving user $i \in \mathcal{L}$ on link i is $q^{(i)} := \sum_{j\in\mathcal{S}_i} H_{i,j}p^{(j)} + \varphi^{(i)} = \sum_{j=1}^{M+K} H_{i,j}p^{(j)} + \varphi^{(i)}$, where $\varphi^{(i)}$ is the noise power at the receiving end of link i. Throughout this paper, we also make the reasonable assumption that $\boldsymbol{\varphi} = [\varphi^{(1)}, \cdots, \varphi^{(M+K)}]^T \neq \mathbf{0}$.

In matrix form, one has $\mathbf{q} = \mathbf{H}\mathbf{p} + \boldsymbol{\varphi}$. Denote by $\gamma^{(i)} := p^{(i)}/q^{(i)}$ the signal-to-interference-plus-noise ratio (SINR) at link $i \in \mathcal{L}$. Then, $\mathbf{p} = \mathbf{D}(\boldsymbol{\gamma})\mathbf{q}$ where $\mathbf{D}(\boldsymbol{\gamma}) = \mathrm{diag}\big(\gamma^{(1)}, \cdots, \gamma^{(M+K)}\big)$. Some simple manipulations yield $\mathbf{q} = \mathbf{H}\mathbf{D}(\boldsymbol{\gamma})\mathbf{q} + \boldsymbol{\varphi}$, and $\mathbf{p} = \mathbf{D}(\boldsymbol{\gamma})\mathbf{H}\mathbf{p} + \mathbf{D}(\boldsymbol{\gamma})\boldsymbol{\varphi}$. Since we do not consider totally isolated groups of links, it is assumed that $\mathbf{H}\mathbf{D}(\boldsymbol{\gamma})$ and $\mathbf{D}(\boldsymbol{\gamma})\mathbf{H}$ are primitive[1].

This paper aims to devise a jointly optimal power allocation $\mathbf{p}$ and SINR assignment $\boldsymbol{\gamma}$ for the two types of users (i.e., MUEs and FUEs) with different service priorities and design objectives. On one hand, the prioritized MUEs with higher access rights demand their ongoing operations be, at least, unaffected regardless of any femtocell deployment. Minimum SINR targets $\gamma^{\min}$ prescribed by the MUEs must therefore be supported in the first place, i.e., $\gamma_i \geq \gamma^{\min(i)}$, $\forall i \in \mathcal{L}_m$. On the other hand, for the existence of a feasible $\mathbf{p} \succ \mathbf{0}$, [9] requires that $\rho\big(\mathbf{H}\mathbf{D}(\boldsymbol{\gamma})\big) < 1$, where $\rho(\mathbf{X})$ denotes the spectral radius of matrix $\mathbf{X}$, i.e., the maximum modulus eigenvalue of $\mathbf{X}$. Furthermore, it is assumed that both the MUEs and the FUEs attempt to maximize their own utilities, defined by the following α-fair function [10]:

$$U_i(\gamma^{(i)}) \quad := \quad \begin{cases} \log(\gamma^{(i)}), & \text{if } \alpha = 1, \\ (1-\alpha)^{-1}(\gamma^{(i)})^{1-\alpha}, & \text{if } \alpha \neq 1. \end{cases} \tag{2}$$

By regulating α, the available resource can be fairly allocated to individual users. In particular, $\alpha = 1$ corresponds to proportional fairness whereas $\alpha \to \infty$ max-min fairness.

It is worth mentioning that in the limit $\rho\big(\mathbf{H}\mathbf{D}(\boldsymbol{\gamma})\big) = 1$, an infinite amount of transmit power is needed to support the SINR target $\boldsymbol{\gamma}$ [9]. As this is impossible to realize in practice, we insist that $\rho\big(\mathbf{H}\mathbf{D}(\boldsymbol{\gamma})\big) \leq \bar{\rho}$ where $0 \leq \bar{\rho} < 1$, for the existence of a feasible solution with $0 \leq p_i < \infty$, $\forall i \in \mathcal{L}$. Given $\bar{\rho} \in [0, 1)$, we consider the following design problem:

$$\begin{aligned} \max_{\boldsymbol{\gamma}\in\mathbb{R}_+^{M+K},\ \mathbf{p}\in\mathbb{R}_+^{M+K}} \quad & w_m \sum_{i\in\mathcal{L}_m} U_i(\gamma^{(i)}) + w_f \sum_{i\in\mathcal{L}_f} U_i(\gamma^{(i)}) \\ \text{s.t.} \quad & \rho\big(\mathbf{H}\mathbf{D}(\boldsymbol{\gamma})\big) \leq \bar{\rho}, \\ & \gamma^{(i)} \geq \gamma^{(i)\min},\ \forall i \in \mathcal{L}_m. \end{aligned} \tag{3}$$

The formulation in (3) includes the design problem in [7] as a special case. Here, $w_m \geq 0$ and $w_f \geq 0$ are the weights designated to macrocell and femtocell network, respectively.

Clearly, problem (3) is not necessarily convex since $\big\{\boldsymbol{\gamma} \in \mathbb{R}_+^{M+K} \mid \rho\big(\mathbf{H}\mathbf{D}(\boldsymbol{\gamma})\big) \leq \bar{\rho}\big\}$ is not a convex set. However, if we let $\boldsymbol{\Gamma} := \log\boldsymbol{\gamma}$ then its equivalent set $\big\{\boldsymbol{\Gamma} \in \mathbb{R}^{M+K} \mid \rho\big(\mathbf{H}\mathbf{D}(e^{\boldsymbol{\Gamma}})\big) \leq \bar{\rho}\big\}$ is actually convex [11]. Through such a change of variable and upon denoting $\Gamma^{(i)\min} := \log(\gamma^{(i)\min})$, the following problem, obviously equivalent to (3), is considered instead.

$$\begin{aligned} \max_{\boldsymbol{\Gamma}\in\mathbb{R}^{M+K},\ \mathbf{p}\in\mathbb{R}_+^{M+K}} \quad & w_m \sum_{i\in\mathcal{L}_m} U_i(\Gamma^{(i)}) + w_f \sum_{i\in\mathcal{L}_f} U_i(\Gamma^{(i)}) \\ \text{s.t.} \quad & \rho\big(\mathbf{H}\mathbf{D}(e^{\boldsymbol{\Gamma}})\big) \leq \bar{\rho}, \\ & \Gamma^{(i)} \geq \Gamma^{(i)\min},\ \forall i \in \mathcal{L}_m. \end{aligned} \tag{4}$$

The utility function now becomes

$$U_i(\Gamma^{(i)}) \quad := \quad \begin{cases} \Gamma^{(i)}, & \text{if } \alpha = 1, \\ (1-\alpha)^{-1}e^{(1-\alpha)\Gamma^{(i)}}, & \text{if } \alpha \neq 1, \end{cases} \tag{5}$$

which is increasing, twice-differentiable and concave with respect to $\Gamma^{(i)}$. Problem (4) is indeed a convex optimization program. However, due to the complicated coupling in the feasible region, centralized algorithms, if available, are typically needed to resolve this kind of problem. Given the nature of two-tier networks where central coordination and processing are usually inaccessible, we aim at developing optimal solutions that can be distributively implemented by individual users.

III. Distributed Power Control for Joint Utility Maximization with Macrocell QoS Protection

A. Approximate Solution via Penalty Method

Proposition 1: The Pareto-optimal SINR[2] for problem (4) lies on the boundary $\partial\mathcal{G}_{\bar{\rho}} = \big\{\boldsymbol{\Gamma} \in \mathbb{R}^{M+K} \mid \rho\big(\mathbf{H}\mathbf{D}(e^{\boldsymbol{\Gamma}})\big) =$

[1] A non-negative matrix is called *primitive* if it is irreducible and has only one eigenvalue of maximum modulus [8, Def. 8.5.0].

[2] A feasible SINR is called *Pareto-optimal* if it is impossible to increase the SINR of any one link without simultaneously reducing the SINR of some others.

$\bar{\rho}$ and $\Gamma^{(i)} \geq \Gamma^{(i)\min}, \forall i \in \mathcal{L}_m\}$.

Proof: The proof can be adapted from that for [6, Th.1], with special care taken in dealing with the minimum SINR requirements $\Gamma^{(i)} \geq \Gamma^{(i)\min}, \forall i \in \mathcal{L}_m$. ∎

Prop. 1 indicates that the search space for Pareto-optimal SINR assignments of (4) is $\mathbb{R}^{M+K}$, confined within $\partial\mathcal{G}_{\bar{\rho}}$ specified by $\bar{\rho}$ and $\Gamma^{(i)\min}, \forall i \in \mathcal{L}_m$. However, locating the particular SINR point on such a boundary that maximizes the objective in (4) is not trivial at all. In this situation, the key difficulty lies in how to satisfy $\Gamma^{(i)} \geq \Gamma^{(i)\min}, \forall i \in \mathcal{L}_m$. Developed for homogeneous networks, the solution in [6] is not directly applied here. Toward this end, we propose to represent the QoS constraints by:

$$I_{-}(\Gamma^{(i)}) \;:=\; \begin{cases} 0, & \text{if } \Gamma^{(i)} \geq \Gamma^{(i)\min}, \\ \infty, & \text{otherwise}, \end{cases} \tag{6}$$

for all $i \in \mathcal{L}_m$. Accordingly, problem (4) is now:

$$\begin{aligned} \max_{\mathbf{\Gamma} \in \mathbb{R}^{M+K},\ \mathbf{p} \in \mathbb{R}_{+}^{M+K}} \quad & w_m \sum_{i \in \mathcal{L}_m} U_i(\Gamma^{(i)}) + w_f \sum_{i \in \mathcal{L}_f} U_i(\Gamma^{(i)}) \\ & - \sum_{i \in \mathcal{L}_m} I_{-}(\Gamma^{(i)}) \qquad (7) \\ \text{s.t.} \quad & \rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}})\big) \leq \bar{\rho}. \end{aligned}$$

However, it is noted that the objective function in (7) is not differentiable. Therefore, we approximate $I_{-}(\Gamma^{(i)})$ by

$$\hat{I}_{-}(\Gamma^{(i)}) := \begin{cases} -(1/a)\log\big(\Gamma^{(i)} - \Gamma^{(i)\min}\big), & \text{if } \Gamma^{(i)} \geq \Gamma^{(i)\min}, \\ \infty, & \text{otherwise}, \end{cases}$$

where $a > 0$ is a parameter to control the accuracy of the approximation. As a increases, the approximation becomes more accurate. Since $\hat{I}_{-}(\Gamma^{(i)})$ is convex, non-increasing and differentiable, the concavity of the objective function in (7) is preserved. Let $\Phi(\mathbf{\Gamma}) := -\sum_{i \in \mathcal{L}_m} \log(\Gamma^{(i)} - \Gamma^{(i)\min})$, whose domain is $\{\mathbf{\Gamma} \in \mathbb{R}^{M+K} \mid \Gamma^{(i)} > \Gamma^{(i)\min}, \forall i \in \mathcal{L}_m\}$. Let us consider the following problem, which approximates (7):

$$\begin{aligned} \max_{\mathbf{\Gamma} \in \mathbb{R}^{M+K},\ \mathbf{p} \in \mathbb{R}_{+}^{M+K}} \quad & a\big[w_m \sum_{i \in \mathcal{L}_m} U_i(\Gamma^{(i)}) + w_f \sum_{i \in \mathcal{L}_f} U_i(\Gamma^{(i)})\big] \\ & -\Phi(\mathbf{\Gamma}) \qquad (8) \\ \text{s.t.} \quad & \rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}})\big) \leq \bar{\rho}. \end{aligned}$$

With the proposed penalty function, the macrocell SINR constraints are effectively eliminated from the constraint set, leaving the Pareto-optimal SINR surface simply $\big\{\mathbf{\Gamma} \in \mathbb{R}^{M+K} \mid \rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}})\big) = \bar{\rho}\big\}$. To distributively realize all points on this surface, we make use of the load-spillage framework [6] and parameterize $\mathbf{\Gamma}$ through a new variable $\mathbf{s} \succ \mathbf{0}$ such that $\mathbf{s}^T\mathbf{HD}(e^{\mathbf{\Gamma}}) = \bar{\rho}\,\mathbf{s}^T$. Let $\mathbf{v} := \mathbf{H}^T\mathbf{s}$, then the resulting SINR

$$\Gamma^{(i)} \;=\; \log\big(\bar{\rho}\, s^{(i)}/v^{(i)}\big), \ \forall i \in \mathcal{L} \tag{9}$$

is indeed Pareto-optimal. It is also clear that $\mathbf{v}$ is the left eigenvector of $\mathbf{D}(e^{\mathbf{\Gamma}})\mathbf{H}$ with associated eigenvalue $\bar{\rho}$ because $\mathbf{v}^T\mathbf{D}(e^{\mathbf{\Gamma}})\mathbf{H} = \bar{\rho}\mathbf{v}^T$. By fixing $\bar{\rho} \in [0,1)$ and upon applying the aforementioned parametrization of $\mathbf{\Gamma}$ in terms of $\mathbf{s}$, (7) can be solved via an equivalent problem in the variable $\mathbf{s}$. The resolution of this new problem can be accomplished by updating $\mathbf{s}$ as

$$s^{(i)}[t+1] = s^{(i)}[t] + \delta\,\Delta s^{(i)}[t+1], \quad \forall i \in \mathcal{L} \tag{10}$$

$$\Delta s^{(i)}[t+1] = \begin{cases} \frac{w_m U_i'(\Gamma^{(i)})}{\bar{\rho} q^{(i)}} + \frac{1}{a\bar{\rho}q^{(i)}(\Gamma_i - \Gamma_i^{\min})} - s^{(i)}[t], \\ \hfill \forall i \in \mathcal{L}_m \\ w_f U_i'(\Gamma^{(i)})/\big(\bar{\rho}q^{(i)}\big) - s^{(i)}[t],\ \forall i \in \mathcal{L}_f. \end{cases} \tag{11}$$

Algorithm 1 Proposed JUM-QoS Algorithm

Require: $\mathbf{s}[1] \succ \mathbf{0}$ satisfying (17)-(18); $\gamma^{(i)\min} > 0, \forall i \in \mathcal{L}_m$; $\bar{\rho} \in [0,1); a > 0; k > 1; \delta > 0; 0 < b < 1; \epsilon > 0$; $\mathbf{p}[0] \succ \mathbf{0}$.

while $M/a \geq \epsilon$ **do**
- Set $t_s := 1$.
- **repeat**
  - User $i \in \mathcal{L}$ computes $v^{(i)}[t_s]$ by (16) and SINR target $\Gamma^{(i)}[t_s] = \log\big(\bar{\rho}s^{(i)}[t_s]/v^{(i)}[t_s]\big)$.
  - Set $t_p := 0$. Using the Foschini-Miljanic's algorithm [5], user $i \in \mathcal{L}$ measures the actual SINR $\hat{\gamma}^{(i)}[t_s]$ and update power $p^{(i)}[t_p+1] := p^{(i)}[t_p]e^{\Gamma^{(i)}}/\hat{\gamma}^{(i)}[t_s]$.
  - User $i \in \mathcal{L}$ measures interference $q^{(i)}[t_s]$ and finds $\Delta s^{(i)}[t_s]$ according to (11).
  - Scale $\delta := b\,\delta$ until the resulting SINR target is strictly greater than $\gamma^{(i)\min},\ \forall i \in \mathcal{L}_m$.
  - Update $s^{(i)}[t_s+1] := s^{(i)}[t_s] + \delta\Delta s^{(i)}[t_s]$; $t_s := t_s + 1$.
- **until** $\mathbf{s}[t_s]$ converges to $\mathbf{s}^*$
- Set $\mathbf{s}[1] := \mathbf{s}^*$ and update $a := k\,a$.

end while

Using similar arguments as [6], it can be shown that $\nabla f^T\Delta\mathbf{s} = (\partial f/\partial\mathbf{\Gamma})^T(\partial\mathbf{\Gamma}/\partial\mathbf{s})\Delta\mathbf{s} > 0, \forall\mathbf{s} \succ \mathbf{0}$ where $f(\mathbf{s})$ denotes the objective function of (8). This means that (11) actually represents an ascent search direction of $f(\mathbf{s})$.

In the penalty method, it is crucial to ensure that the condition $\Gamma^{(i)} > \Gamma^{(i)\min},\ \forall i \in \mathcal{L}_m$ be always met after every update step. Otherwise, the resulting $\mathbf{\Gamma}$ would lie outside the domain of $\Phi(\mathbf{\Gamma})$, making the objective function of (8) unbounded below. To this end, as long as $\Gamma^{(i)} \leq \Gamma^{(i)\min}$ for any $i \in \mathcal{L}_m$, we propose to scale the step size in (10) as $\delta := b\delta$ where $0 < b < 1$.

Proposition 2: As $\bar{\rho} \to 1$, the update of $\mathbf{s}$ in (10)-(11) allows the global optimum of the approximate problem (8) to be found.

Proof: The Lagrangian of (8) is defined as $\mathcal{L}(\mathbf{\Gamma},\mu) := a\Big[w_m\sum_{i\in\mathcal{L}_m} U_i(\Gamma^{(i)}) + w_f\sum_{i\in\mathcal{L}_f} U_i(\Gamma^{(i)})\Big] - \Phi(\mathbf{\Gamma}) - \mu\big[\rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}})\big) - \bar{\rho}\big]$, where multiplier $\mu \geq 0$. From the Karush-Kuhn-Tucker (KKT) condition [12], $\nabla\mathcal{L}(\mathbf{\Gamma},\mu) = 0$, i.e.,

$$a\,w_m U_i'(\Gamma^{(i)}) + \frac{1}{\Gamma^{(i)} - \Gamma^{(i)\min}} = \mu\, v^{(i)}\tilde{q}^{(i)}e^{\Gamma^{(i)}}, i \in \mathcal{L}_m \tag{12}$$

$$a\,w_f U_i'(\Gamma^{(i)}) = \mu\, v^{(i)}\tilde{q}^{(i)}e^{\Gamma^{(i)}}, i \in \mathcal{L}_f \tag{13}$$

where $\mathbf{v} = [v^{(i)}]$ is the left eigenvector of $\mathbf{D}(e^{\mathbf{\Gamma}})\mathbf{H}$ and $\tilde{\mathbf{q}} = [\tilde{q}^{(i)}]$ the right eigenvector of $\mathbf{HD}(e^{\mathbf{\Gamma}})$ (normalized such that $\mathbf{s}^T\tilde{\mathbf{q}} = 1$), both associated with eigenvalue $\bar{\rho}$.

On the other hand, $\Delta\mathbf{s}^* = 0$ at convergence point $\mathbf{s}^*$, i.e.,

$$a\,w_m U_i'(\Gamma^{(i)*}) + \frac{1}{\Gamma^{(i)*} - \Gamma^{(i)\min}} = a\,\bar{\rho}\,q^{(i)*}s^{(i)*}, i \in \mathcal{L}_m \tag{14}$$

$$a\,w_f U_i'(\Gamma^{(i)*}) = a\,\bar{\rho}\,q^{(i)*}s^{(i)*}, i \in \mathcal{L}_f. \tag{15}$$

Upon noting that $\bar{\rho}s^{(i)*} = e^{\Gamma^{(i)*}}v^{(i)*}$ and that $q^{(i)*} \to \tilde{q}^{(i)}/[(1-\bar{\rho})\mathbf{s}^{*T}\boldsymbol{\varphi}]$ as $\bar{\rho} \to 1$, these are exactly (12)-(13) for $\mathbf{\Gamma} = \mathbf{\Gamma}^*$ and $\mu = a/[(1-\bar{\rho})\mathbf{s}^{*T}\boldsymbol{\varphi}] > 0$. Since (8) is a convex problem, any point satisfying the KKT condition is indeed its global optimum.

With $\mathbf{s}^*$ known and upon recalling that $\mathbf{v}^* = \mathbf{H}^T\mathbf{s}^*$, the optimal SINR $\mathbf{\Gamma}^*$ are determined according to (9). By the Foschini-Miljanic's algorithm [5], the globally optimal power allocation $\mathbf{p}^*$ that achieves these SINR targets can be found. ∎

B. Distributed Algorithm for Globally Optimal Solution

We present in Alg. 1 the JUM-QoS scheme to jointly maximize the total utility of both the MUEs and the FUEs in problem (4). Recall that resolving (8) only provides an approximate solution to problem (7), and in turn (4). Once problem (8) has been solved, control parameter a needs to be accordingly regulated to make the approximation more accurate. Specifically, there are two levels of execution in this algorithm: the outer loop is to update a whereas the inner loop to find optimal solution to the approximate problem (8). The resulting $\mathbf{s}$ of the current inner loop will be used in the next iteration of the outer loop. If a is chosen to be too large, there would likely be many more inner iterations. In contrast, if a is too small, the algorithm would require extra outer iterations to converge.

The proposed solution can be distributively implemented at each individual link with limited information exchange being required, either via a broadcast channel or over the available backhaul networks. By assuming that channel gains between the downlink and the uplink are identical, and upon noticing that

$$v^{(i)} = \sum_{j\in\mathcal{L}} H_{j,i}s^{(j)} = \sum_{j\neq i,\, \sigma_j=\sigma_i} s^{(j)} + \sum_{l\neq\sigma_i} h_{l,i} \sum_{j,\, \sigma_j=l} s^{(j)}, \quad (16)$$

the value of $v^{(i)}$ in Step 4 can be computed and managed by user $i \in \mathcal{L}$. Specifically, we require each BS l to broadcast the quantity $\sum_{j,\,\sigma_j=l} s^{(j)}$ at a constant power. This permits user i to also measure the all channel gains $h_{l,i} = h_{i,l}$ required for the calculation of $\sum_{l\neq\sigma_i} h_{l,i} \sum_{j,\,\sigma_j=l} s^{(j)}$. In Step 7, to check the feasibility of the resulting target SINR $\bar{\Gamma}_i = \log\big(\bar{\rho}\,\bar{s}^{(i)}/\bar{v}^{(i)}\big)$ associated with the search direction $\Delta s^{(i)}$, each user $i \in \mathcal{L}$ computes $\bar{s}^{(i)} := s^{(i)}[t_s] + \delta\Delta s^{(i)}[t_s]$, and subsequently $\bar{v}^{(i)}$ as a function of $\bar{\mathbf{s}}$ [similar to (16)]. With channel gains $h_{l,i} = h_{i,l}$ already known, the computation of $\bar{v}^{(i)}$ only requires $\bar{s}^{(i)}$ to be exchanged. Also note that we need to initialize the algorithm with a strictly feasible solution to ensure $\Gamma^{(i)} > \Gamma^{(i)\min}, \forall i \in \mathcal{L}_m$. Since $\Gamma^{(i)} = \log\big(\bar{\rho}\, s^{(i)}/v^{(i)}\big)$ and $v^{(i)} = \sum_{i\in\mathcal{L}} H_{j,i}s^{(j)}$, this involves solving the following linear inequalities:

$$\bar{\rho}s^{(i)} - \sum_{j\in\mathcal{L}\setminus\{i\}} e^{\Gamma^{(i)\min}} H_{j,i}s^{(j)} > 0, \ \forall i \in \mathcal{L}_m, \quad (17)$$

$$s^{(i)} > 0, \ \forall i \in \mathcal{L}_f. \quad (18)$$

Theorem 1: The proposed JUM-QoS algorithm converges to the global optimum of the original problem (4).

Proof: Let $\mathbf{\Gamma}_a^*$ be the optimal solution of (8). It follows that $\Gamma_a^{(i)*} > \Gamma^{(i)\min}, \forall i \in \mathcal{L}_m$ (strictly feasible), and $\rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}_a^*})\big) = \bar{\rho}$ (operating on the Pareto-optimal boundary of SINR). With $\mu \in \mathbb{R}$, the stationarity condition implies

$$a\, w_m \sum_{i\in\mathcal{L}_m} U_i^{'}(\Gamma_a^{(i)*}) + 1/[\Gamma_a^{(i)*} - \Gamma^{(i)\min}] - \mu\tilde{q}^{(i)}(\Gamma_a^{(i)*})v^{(i)}(\Gamma_a^{(i)*})e^{\Gamma_a^{(i)*}} = 0, \quad (19)$$

for all $i \in \mathcal{L}_m$, and

$$a\, w_f \sum_{i\in\mathcal{L}_f} U_i^{'}(\Gamma_a^{(i)*}) - \mu\tilde{q}^{(i)}(\Gamma_a^{(i)*})v^{(i)}(\Gamma_a^{(i)*})e^{\Gamma_a^{(i)*}} = 0, \quad (20)$$

for all $i \in \mathcal{L}_f$.

Now let $\lambda_i^* := 1/[a(\Gamma_a^{(i)*} - \Gamma^{(i)\min})] > 0, \forall i \in \mathcal{L}_m$ and $\mu^* = \mu/a$, then $w_m \sum_{i\in\mathcal{L}_m} U_i^{'}(\Gamma_a^{(i)*}) + \lambda_i^* - \mu^*\tilde{q}^{(i)}(\Gamma_a^{(i)*})v^{(i)}(\Gamma_a^{(i)*})e^{\Gamma_a^{(i)*}} = 0, \ \forall i \in \mathcal{L}_m$, and also $w_f \sum_{i\in\mathcal{L}_f} U_i^{'}(\Gamma_a^{(i)*}) - \mu^*\tilde{q}^{(i)}(\Gamma_a^{(i)*})v^{(i)}(\Gamma_a^{(i)*})e^{\Gamma_a^{(i)*}} = 0, \ \forall i \in \mathcal{L}_f$. This means that $\mathbf{\Gamma}_a^*$ maximizes the Lagrangian $\mathcal{L}(\mathbf{\Gamma}, \boldsymbol{\lambda}, \xi) := w_m \sum_{i\in\mathcal{L}_m} U_i(\Gamma^{(i)}) + w_f \sum_{i\in\mathcal{L}_f} U_i(\Gamma^{(i)}) + \sum_{i\in\mathcal{L}_m} \lambda_i(\Gamma^{(i)} - \Gamma^{(i)\min}) - \xi\big[\rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}})\big) - \bar{\rho}\big]$ of (4) for $\lambda_i = \lambda_i^* > 0$ and $\xi = \mu^*$. From this, it is apparent that $(\boldsymbol{\lambda}^*, \mu^*)$ are a dual feasible pair of (4). The dual function $g(\boldsymbol{\lambda}^*, \mu^*)$ is thus finite and

$$\begin{aligned} g(\boldsymbol{\lambda}^*, \mu^*) &= w_m \sum_{i\in\mathcal{L}_m} U_i(\Gamma_a^{(i)*}) + w_f \sum_{i\in\mathcal{L}_f} U_i(\Gamma_a^{(i)*}) \\ &+ \sum_{i\in\mathcal{L}_m} \lambda_i^*\big(\Gamma_a^{(i)*} - \Gamma^{(i)\min}\big) - \mu^*\big[\rho\big(\mathbf{HD}(e^{\mathbf{\Gamma}_a^*})\big) - \bar{\rho}\big] \\ &= w_m \sum_{i\in\mathcal{L}_m} U_i(\Gamma_a^{(i)*}) + w_f \sum_{i\in\mathcal{L}_f} U_i(\Gamma_a^{(i)*}) + M/a. \end{aligned} \quad (21)$$

Denote the optimal value of (4) as $U^* := \min_{\boldsymbol{\lambda}^*\succ\mathbf{0},\mu^*} g(\boldsymbol{\lambda}^*, \mu^*)$. From (21), $U^* - M/a \leq w_m \sum_{i\in\mathcal{L}_m} U_i(\Gamma_a^{(i)*}) + w_f \sum_{i\in\mathcal{L}_f} U_i(\Gamma_a^{(i)*}) \leq U^*$. As $a \to \infty$, $\mathbf{\Gamma}_a^*$ approaches the globally optimal solution of (4). ∎

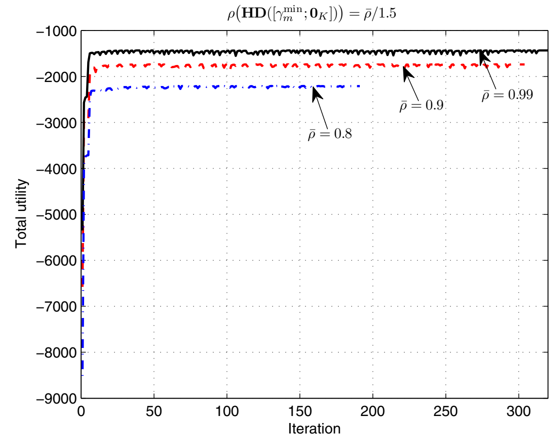

Fig. 2. Convergence of the proposed JUM-QoS algorithm.

IV. Illustrative Examples

The network setting in our numerical examples is shown in Fig. 1, where MUEs and FUEs are randomly deployed inside circles of radii of 1000m and 50m, respectively. Assume there are $M = 10$ MUEs, whereas $K = 20$ FUEs are divided equally among 4 femtocells (i.e., 5 FUEs per femtocell). Uplink is considered in all simulations. We calculate the channel gains as $h^0_{\sigma_i,j} = d^{-\beta}_{\sigma_i,j}$, where $d_{\sigma_i,j}$ is the corresponding distance and $\beta = 3$ the pathloss exponent. Noise power is $\varphi^{(i)} = 10^{-6}$, $\forall i \in \mathcal{L}$. The values of SINR target, set equal for all the MUEs, i.e., $\gamma_i^{\min} = \gamma^{\min}, \forall i \in \mathcal{L}_m$, are chosen such that $\rho\big(\mathbf{HD}([\boldsymbol{\gamma}_m^{\min}; \mathbf{0}_K])\big) \leq \bar{\rho} < 1$. Unless stated otherwise, $\alpha = 3$ in (5). Error tolerance values for the convergence of the JUM-QoS scheme and the Foschini-Miljanic's algorithm are $\epsilon = 10^{-4}$ and $\epsilon_p = 10^{-10}$, respectively. Also, $a = 5, k = 3, \delta = 1, b = 0.8$.

Fig. 2 demonstrates the convergence of the proposed algorithm with $w_m = w_f = 0.5$. It is clear that the algorithm almost attains the optimal point after only $10 - 20$ iterations, and eventually converges in some $200 - 300$ iterations. Notice that the total utility of both MUEs and FUEs increases as $\bar{\rho}$ tends to 1. This is because the feasible SINR region becomes larger as $\bar{\rho}$ grows, essentially implying that more system capacity is available.

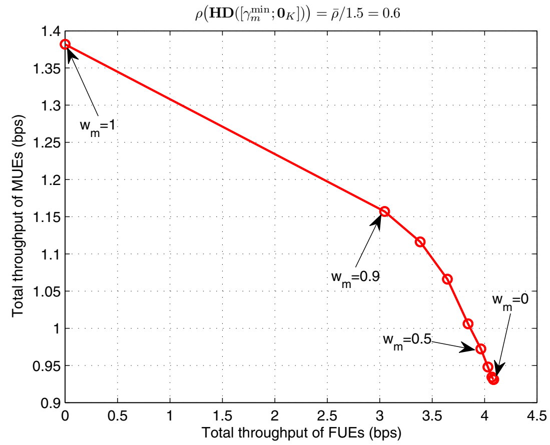

Fig. 3. Throughput tradeoff between macrocell and femtocell networks.

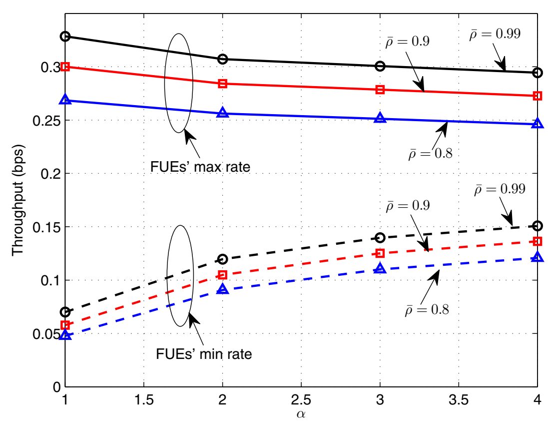

Fig. 4. Fairness achieved by the use of different utility functions.

TABLE I
PERFORMANCE OF JUM-QOS ALGORITHM (FOR $\bar{\rho} = 0.9$)

$\gamma^{\min}$	0.0667	0.0714	0.0769	0.0833	0.0909
$\max\{\boldsymbol{\gamma}_{\text{MUE}}\}$	0.0719	0.0723	0.0774	0.0856	0.1089
$\min\{\boldsymbol{\gamma}_{\text{MUE}}\}$	0.0667	0.0714	0.0769	0.0833	0.0909
Macrocell throughput	0.9755	0.9996	1.0712	1.1623	1.2966
$\max\{\boldsymbol{\gamma}_{\text{FUE}}\}$	0.2152	0.1908	0.2022	0.1781	0.9967
$\min\{\boldsymbol{\gamma}_{\text{FUE}}\}$	0.0548	0.0514	0.0420	0.0297	0.0075
Femtocell throughput	4.0450	3.8121	3.5434	2.9568	2.5774

With $\bar{\rho} = 0.9$, Table I shows that all MUEs see their final SINRs being above the prescribed threshold $\gamma^{\min}$. It is noteworthy that while approaching very closely to the Pareto-optimal boundary, the values of these SINR targets still lie inside the interior of the feasible SINR region. Therefore, although appearing to be equal to $\gamma^{\min}$, the minimum of γ_m in Table I is indeed strictly larger that $\gamma^{\min}$ by a small value, as the direct result of Step 7 of the JUM-QoS algorithm. Recall that this step ensures that $\Phi(\mathbf{\Gamma})$, and hence the objective function of (8), is bounded below. Also clear from Table I is that the capacity remained for the femtocell network becomes more limited as the prioritized MUEs demand for higher SINRs.

To flexibly share the radio resources among the MUEs and the FUEs, the proposed algorithm can designate the importance toward either the macrocell or the femtocell network by varying w_m and w_f. Fig. 3 displays the achieved throughput of both networks for $w_m = 0 : 0.1 : 1$ and $w_f = 1 - w_m$. By allowing femtocell deployment and accepting a small loss in the MUEs' throughput, a large gain can be realized for the FUEs. Remarkably, by changing from $w_m = 1$ to $w_m = 0$, i.e., the MUEs only require to have their minimum QoS requirements maintained rather than their utility solely optimized, the throughput improvement in the femtocell network is above ten-fold the amount of rate loss in the macrocell. This can be explained by noting that the FUEs are in close proximity to their corresponding BSs and thus are able to achieve potentially much higher data rates compared with the MUEs. Notice also that the macrocell sum rate is not zero even with $w_m = 0$ because their minimum SINR targets are always maintained.

With $w_m = 0$ and $w_f = 1$, Fig. 4 shows the minimum and maximum throughput achieved by the femtocells. Clearly, for larger values of α, the FUE whose data rate is the highest (likely due to its advantageous link conditions) experiences a decline in its throughput, whereas the FUE with the lowest throughput has its data-rate gradually enhanced. Such an observation again confirms that by regulating α in the utility function, the available resources can be more fairly shared among the FUEs.

V. CONCLUSION

Given that a central coordination is usually inaccessible in two-tier networks, this paper devises a Pareto-optimal power control scheme that can be distributively implemented at individual links. The proposed algorithm jointly maximizes the utilities of both macrocell and femtocell networks while protecting the minimum prescribed SINRs of the MUEs at all times. Effective mechanisms are provided to flexibly designate the access priority and to fairly share the radio resources among different users. The developed solution is shown to converge to the global optimum, and its potentials are illustrated by numerical results.

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