Outage Performance Of OFDM Ad-hoc Routing With and Without Subcarrier Grouping in Multihop Network

A. Gouissem, M.O. Hasna R. Hamila
Electrical Engineering Department
Qatar University

H. Besbes, F. Abdelkefi
Research Lab. COSIM
Sup'Com, University of Carthage

***Abstract*—In this contribution, we investigate Decode and Forward (DF) OFDM Ad-hoc Routing strategy with bottleneck maximization. We derive the exact outage probability and diversity order for the different scenarios: with and without subcarrier grouping and with and without joint selection, where joint selection refers to the selection of the last two hops together. When joint selection is performed, numerical results show that a power gain can be obtained when using subcarrier grouping technique.**

***Index*— Cooperative OFDM, Outage probaility, joint selection, subcarrier grouping, diversity order, power gain.**

I. Introduction

With the increasing demand on high speed wireless applications, cooperative communication has emerged as a powerful technique to exploit the broadcast nature of the wireless medium and the numerous nodes in the network, to achieve Multiple Input Multiple Output (MIMO) diversity with single antenna terminals[1], [2], [3]. Hence, the authors in [4] presented a selection scheme entitled "Selective OFDMA" (per subcarrier basis selection) which give full diversity order.

Full diversity is also achieved with Ad-hoc routing strategy studied in [5] for flat fading channel. In [6] the authors extended this strategy to a broadband channel and carried out a performance analysis based on some valid approximations for high SNR.

In this contribution, this strategy is studied with and without subcarrier grouping and with and without joint selection. The end-to-end outage performance of Selective DF (Decode and Forward) OFDM with Ad-hoc routing based on the bottleneck maximization is evaluated with exact expressions for the different scenarios and compared to some approximated results given in [6]. The diversity order is analytically derived for the different cases and the power gain is evaluated in the case of subcarrier grouping technique with and without joint selection.

In a decode-and-forward multi hop network with L relays cooperating with each other per hop, the maximum diversity gain is L-fold regardless of the number of hops, we prove that this diversity is achieved with this strategy when joint selection is performed in the last two hops and all the selection gain will be lost when no joint selection is performed. Moreover, we show that when dividing the subcarriers into n_G groups routed independently from each others a power gain of ${n_G}^{L-1}$ is achieved.

The paper is organized as follows. In section II, the system model and the selection scheme are presented. In Section III, we perform the outage analysis for the system. Section IV presents diversity order and power gain analysis. Simulation results are given in section V. Finally, section VI summarizes and concludes the paper.

II. System Model

A wireless communication scenario is considered where the source terminal S transmits information to the destination D with the assistance of an equidistant clustered M-hop network with L relays at each hop. Transmission is based on Orthogonal Frequency-Division Multiplexing (OFDM) with N subcarriers. To avoid inter block interference, a cyclic prefix of length L_{CP} larger than the maximum channels dispersion is added between adjacent information blocks. We assume that each subchannel is independently and identically distributed (i.i.d.) and the channel remains constant during an OFDM block interval but varies from block to block (slow fading).

In frequency domain, the complex baseband equivalent received signal for one signaling interval, over the nth subcarrier in hop "i" and selected relay l, has the following expression:

$$r_{l,n,i} = H_{l,n,i}.A_{l,n,i}.\exp(j\theta_{l,n,i}) + n_{l,n,i}, \tag{1}$$

where, $H_{l,n,i} = h_{l,n,i}\exp(j\phi_{l,n,i})$ is the random complex channel that has a Rayleigh-distributed amplitude $h_{i,k}$ and a uniformly distributed phase angle $\phi_{l,n,i}$.

$A_{l,n,i}.\exp(j\theta_{l,n,i})$ is the complex base band equivalent transmitted signal in frequency domain from the hop $i-1$ with amplitude $A_{l,n,i}$ and phase angle $\theta_{l,n,i}$ and average power E_s. $n_{l,n,i}$ is a zero-mean complex Gaussian random variable representing the Additive White Gaussian Noise (AWGN) with variance $\frac{N_0}{2}$, where N_0 is the noise power spectral density.

The received SNR for one signaling interval over the nth subcarrier in hop i and for the lth link can be written as follows:

$$\gamma_{l,n,i} = \frac{E_s}{N_0}.|H_{l,n,i}|^2. \tag{2}$$

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Because $h_{l,n,i}$ follows Rayleigh distribution, then $\gamma_{l,n,i}$ is an exponential random variable with mean $\gamma_c \triangleq \frac{E_s}{N_0}$.

For each subcarrier, the best path (denoted by l^*) is chosen referring to the subcarrier with the worst SNR i.e

$$l^* = \underset{1\leq l\leq L}{\operatorname{argmax}}\big(\min_{1\leq n\leq N}(\gamma_{l,n,i})\big). \quad (3)$$

If no joint selection is performed, there will be no selection in the last hop, otherwise the selection in the hop $M-1$ will not be based only on $\gamma_{l,n,M-1}$ but on $\min(\gamma_{l,n,M-1}, \gamma_{l,n,M})$.

If subcarriers grouping technique is used, the selection method described for selective OFDM is performed independently for each joint subcarriers group.

III. Outage analysis

For subchannel n in hop i, we assume that the demodulation is correct when the SNR of this subchannel is greater than some SNR threshold ϵ, which can be determined as in [7].

The performance of the system will be evaluated in this section in terms of Outage Probability (denoted P_{out}), where an outage occurs if the demodulation of at least one symbol is incorrect i.e.the received SNR of any one of the OFDM symbol subcarriers (for at least one hop) is below the threshold ϵ,

$$P_{out} = 1 - \prod_{i=1}^{M}(1 - P_{out}^i), \quad (4)$$

where P_{out}^i denotes the outage probability for hop i.

In this section, we denote by $f_X(\gamma)$ and $F_X(\gamma)$ the probability density function and the cumulative density function of x, respectively.

A. Selective OFDM Without Joint Selection

If we assume that no joint selection is taken into account, the outage probability of the first $M-1$ hops is expressed as follows:

$$\begin{aligned} P_{out}^i &= P(\gamma_{l^*,1,i} \leq \epsilon \ Or\ \gamma_{l^*,2,i} \leq \epsilon\ \ldots\ Or\ \gamma_{l^*,N,i} \leq \epsilon) \\ &= P(\gamma_{l^*,n^*_{min,i},i} \leq \epsilon) \\ &= P_{out}^{l^*,n^*_{min,i},i} \\ &= \int_{-\infty}^{\epsilon} f_{\Gamma_{l^*,n^*_{min,i},i}}(\gamma).d\gamma, \end{aligned} \quad (5)$$

where, $P_{out}^{l^*,n^*_{min,i},i}$ is the outage probability in hop $i = 1,\ldots,M-1$ and of the selected path l^* the subcarrier corresponding to the lowest γ (denoted by the $n^*_{min,i}$th subcarrier). i.e;

$$n^*_{min,i} = \underset{1\leq n\leq N}{\operatorname{argmin}}(\gamma_{l^*,n,i}). \quad (6)$$

Let

$$\gamma_{l,n_{min,i},i} = \min_{1\leq n\leq N}(\gamma_{l,n,i}). \quad (7)$$

This is equivalent to consider:

$$\gamma_{l^*,n^*_{min,i},i} = \max_{1\leq l\leq L}(\gamma_{l,n_{min,i},i}). \quad (8)$$

As $\{\gamma_{l,n,i}, n = 1,\ldots,N\}$ are assumed to be i.i.d. [8]

$$f_{\Gamma_{l,n_{min,i},i}}(\gamma) = \frac{N}{\gamma_c}.\exp(-\frac{N.\gamma}{\gamma_c}). \quad (9)$$

then

$$\begin{aligned} F_{\Gamma_{l,n_{min,i},i}}(\gamma) &= \int_{-\infty}^{\gamma} f_{\Gamma_{l,n_{min,i},i}}(\gamma).d\gamma \\ &= 1 - \exp(-\frac{N.\gamma}{\gamma_c}). \end{aligned} \quad (10)$$

Since $\{\gamma_{l,n,i}, n = 1,\ldots,N\ , l = 1,\ldots,L\}$ are assumed to be i.i.d. then $\{\gamma_{l,n_{min,i},i}, l = 1,\ldots,L\}$ are also i.i.d. and the density function of $\gamma_{l^*,n^*_{min,i},i}$ can be written as follows [8]:

$$\begin{aligned} f_{\Gamma_{l^*,n^*_{min,i},i}}(\gamma) &= L(F_{\Gamma_{l,n_{min,i},i}}(\gamma))^{L-1} f_{\Gamma_{l,n_{min,i},i}}(\gamma) \\ &= \frac{L.N}{\gamma_c}.exp(-\frac{N.\gamma}{\gamma_c}).(1 - exp(-\frac{N.\gamma}{\gamma_c}))^{L-1}. \end{aligned} \quad (11)$$

The outage probability of each hop $i \in \{1,\ldots M-1\}$, can be easily deduced:

$$\begin{aligned} P_{out}^i &= P(\gamma_{n^*_{min,i},i,l^*} \leq \epsilon) \\ &= \int_{-\infty}^{\epsilon} f_{\Gamma_{n^*_{min,i},i,l^*}}(\gamma).d\gamma \\ &= (1 - exp(-\frac{N.\epsilon}{\gamma_c}))^L. \end{aligned} \quad (12)$$

Since the dependency of P_{out}^i on the index i is dropped, we denote by P_a the outage probability for the first $M-1$ hops:

$$\begin{aligned} P_a &\triangleq P_{out}^i\ for\ i = 1,\ldots,M-1 \\ &= (1 - exp(-\frac{N.\epsilon}{\gamma_c}))^L. \end{aligned} \quad (13)$$

Concerning the last hop, the outage probability is denoted by P_b and can be expressed as follows:

$$\begin{aligned} P_b &\triangleq P_{out}^M = P_a|_{L=1} (\text{i.e.no selection in the last hop}) \\ &= 1 - exp(-\frac{N.\epsilon}{\gamma_c}). \end{aligned} \quad (14)$$

Substituting (12) and (14) in (4), we obtain the outage probability when no joint selection is taken into account (denoted by $P_{out,njs}$):

$$\begin{aligned} P_{out,njs} &= 1 - (1 - P_a)^{M-1}.(1 - P_b) \\ &= 1 - (1 - (1 - exp(-\frac{N.\epsilon}{\gamma_c}))^L)^{M-1}.exp(-\frac{N.\epsilon}{\gamma_c}). \end{aligned} \quad (15)$$

B. Selective OFDM with joint selection

For the first $M-2$ hops, there is no difference between this model and the last one, thus:

$$P_{out}^{i} = P_a \ for \ i = 1, \ldots, M-2$$
$$= (1 - exp(-\frac{N.\epsilon}{\gamma_c}))^L. \quad (16)$$

The outage probability for the last two hops together (denoted by P_c) is:

$$P_c = P(\gamma_{l^*,n^*_{min,M-1},M-1} \leq \epsilon \ or \ \gamma_{l^*,n^*_{min,M},M} \leq \epsilon)$$
$$= P(min(\gamma_{l^*,n^*_{min,M-1},M-1}, \gamma_{l^*,n^*_{min,M},M}) \leq \epsilon)$$
$$= P(\gamma^* \leq \epsilon), \quad (17)$$

where $\gamma^* = min(\gamma_{l^*,n^*_{min,M-1},M-1} \quad , \quad \gamma_{l^*,n^*_{min,M},M})$.

Let

$$\gamma^*_{l,n} \triangleq \ min(\gamma_{l,n,M-1} \quad \gamma_{l,n,M}), \quad (18)$$
$$\gamma^*_l \triangleq \min_{1\leq n\leq N}(\gamma^*_{l,n}). \quad (19)$$

Then, we have:

$$\gamma^* = \max_{1\leq l\leq L}(\gamma^*_l). \quad (20)$$

Since $\{\gamma_{l,n,i}, n = 1, \ldots, N \ , i = 1, \ldots, M \ , l = 1, \ldots, L\}$ are i.i.d. then, $\left\{\gamma^*_{l,n}, n = 1, \ldots, N \ , l = 1, \ldots, L\right\}$ and $\{\gamma^*_l, n = 1, \ldots, N\}$ are also i.i.d. then the density function of $\gamma^*_{l,n}$ is given by [8] as follows:

$$f_{\Gamma^*_{l,n}}(\gamma) = 2.f_\Gamma(\gamma).(1 - F_\Gamma(\gamma))$$
$$= \frac{2}{\gamma_c}.exp(-\frac{2.\gamma}{\gamma_c}). \quad (21)$$

Then, its cumulative density function will be equal to:

$$F_{\Gamma^*_{l,n}(\gamma)} = \int_{-\infty}^{\gamma} f_{\Gamma^*_{l,n}}(\gamma).d\gamma$$
$$= 1 - exp(-\frac{2.\gamma}{\gamma_c}). \quad (22)$$

By using (21) and (22), we obtain the density function of γ^*_l (23) and its cumulative density function (24).

$$f_{\Gamma^*_l}(\gamma) = N.f_{\Gamma_{1,min}}(\gamma).(1 - F_{\Gamma_{1,min}}(\gamma))^{N-1}$$
$$= 4.N.exp(-\frac{2.N.\gamma}{\gamma_c}), \quad (23)$$

It follows that

$$F_{\Gamma^*_l(\gamma)} = \int_{-\infty}^{\gamma} f_{\Gamma^*_l}(\gamma).d\gamma$$
$$= 1 - exp(-\frac{2.N.\gamma}{\gamma_c}). \quad (24)$$

After the manipulations of (23) and (24), we obtain the density function of Γ^*.

$$f_{\Gamma^*}(\gamma) = L.f_{\Gamma^*_l}(\gamma)\,(F_{\Gamma^*_l}(\gamma))^{L-1}$$
$$= \frac{2.L.N}{\gamma_c}.exp(-\frac{2.N\gamma}{\gamma_c}).(1 - \exp(-\frac{2.N.\gamma}{\gamma_c}))^{L-1}. \quad (25)$$

Then, combining (25) and (17), we obtain the outage probability for the last two hops together:

$$P_c = \int_{-\infty}^{\epsilon} f_{\Gamma^*}(\gamma).d\gamma = (1 - exp(-\frac{2.N.\epsilon}{\gamma_c}))^L. \quad (26)$$

Substituting (16) and (26) in (4), we get the end-to-end outage probability when joint selection is performed (denoted by $P_{out,js}$):

$$P_{out,js} = 1 - (1 - (1 - \exp(-\frac{N.\epsilon}{\gamma_c}))^L)^{M-2}$$
$$* (1 - (1 - \exp(-\frac{2N\epsilon}{\gamma_c}))^L). \quad (27)$$

C. Subcarrier Grouping

In this section, we propose to divide the N subcarriers into n_G groups of n_{spg} subcarriers ($n_{spg} = \frac{N}{n_G}$), each group will be routed independently from the others to choose not only one best relay for all the subcarriers but a one for each group. Then, the outage probability of such system (denoted by $P_{out,sg}$) can be expressed as follows:

$$P_{out,sg} = P(\text{any one of the subcarriers is in outage})$$
$$= P(\text{any one of the groups is in outage})$$
$$= 1 - \prod_{i=1}^{n_G}(1 - P^i_{out,sg}), \quad (28)$$

where, $P^i_{out,sg}$ denotes the outage probability of the ith group. Since the subcarriers are i.i.d. and the selection for each group is independent from the others, then the end-to-end outage probability can be written as:

$$P_{out,sg} = 1 - (1 - P_{out \ |N=n_{spg}})^{n_G}, \quad (29)$$

where, $P_{out \ |N=n_{spg}}$ is the outage probability of the selective OFDM system without subcarrier grouping with n_G subcarriers.

1) With joint selection: When joint selection is performed in the last two hops, combining (29) and (27), we obtain the end-to-end outage probability (denoted in this case by $P_{out,sg,js}$)

$$P_{out,sg,js} = 1 - (1 - P_{out,js \ /N=n_{spg}})^{n_G}$$
$$= 1 - (1 - (1 - (1 - exp(-\frac{n_{spg}\,\epsilon}{\gamma_c}))^L)^{M-2}$$
$$(1 - (1 - exp(-\frac{2.n_{spg}\,\epsilon}{\gamma_c}))^L))^{n_G}. \quad (30)$$

2) Without joint selection: If no joint selection is taken place, combining (29) and (15), we obtain the end-to-end outage probability (denoted in this case by $P_{out,sg,njs}$)

$$P_{out,sg,njs} = 1 - (1 - P_{out,njs \ /N=n_{spg}})^{n_G}$$
$$= 1 - (1 - (1 - (1 - \exp(-\frac{n_{spg}\,\epsilon}{\gamma_c}))^L)^{M-1}$$
$$\exp(-\frac{n_{spg}\,\epsilon}{\gamma_c}))^{n_G}. \quad (31)$$

IV. Diversity oder and power gain

The performance of the minimum SNR criteria in terms of diversity order and power gain for the different scenarios is evaluated in this section. The diversity order is defined as

$$D = -\lim_{\gamma_c \to +\infty} \frac{Ln(P_{out})}{Ln(\gamma_c)}. \quad (32)$$

Doing approximations to (13),(14) and (26) when $\gamma_c \to +\infty$, we obtain these expressions:

$$P_a = (\frac{N\epsilon}{\gamma_c})^L + o(\frac{\epsilon}{\gamma_c}), \quad (33)$$

$$P_b = (\frac{N\epsilon}{\gamma_c}) + o(\frac{\epsilon}{\gamma_c}), \quad (34)$$

$$P_c = (\frac{2N\epsilon}{\gamma_c})^L + o(\frac{\epsilon}{\gamma_c}). \quad (35)$$

Then, substituting (33) and (34) in (15), we obtain an approximation of the outage probability when $\gamma_c \to +\infty$ for a selective OFDM system without joint selection:

$$P_{out,njs} = (\frac{N\epsilon}{\gamma_c}) + o(\frac{\epsilon}{\gamma_c}). \quad (36)$$

From (36), we conclude that the diversity order for such system (denoted by D_{njs}) is:

$$D_{njs} = -\lim_{\gamma_c \to +\infty} \frac{Ln(P_{out,njs})}{Ln(\gamma_c)} = 1. \quad (37)$$

When no joint selection is taken place, not only we do not have any diversity order but the outage probability remains also constant for different values of M and L. However, With joint selection the outage probability will be given by:

$$P_{out,js} = (M - 2 + 2^L)(\frac{N\epsilon}{\gamma_c})^L + o(\frac{\epsilon}{\gamma_c}). \quad (38)$$

(We obtained the same expression as in [6] (without correcting code)).From (38), we conclude that the diversity order when joint selection is taken place in the last two hops (denoted by D_{js}) is:

$$D_{js} = -\lim_{\gamma_c \to +\infty} \frac{Ln(P_{out,js})}{Ln(\gamma_c)} = L. \quad (39)$$

Thus, we conclude that with minimum SNR criteria, we obtain a full diversity order when joint selection is performed and we lose all this diversity if there is no joint selection. In [4], it is is proven that when using combined SNR there is no diversity gain. In this work, we proved that when using minimum SNR criteria a full diversity order is achieved.

When subcarrier grouping is used without joint selection, nothing changes neither in terms of diversity order nor in terms of power gain. In effect, the outage probability have the same approximated expression as (36) when only one best relay is chosen for all the subcarriers.

$$P_{out,sg,njs} = (\frac{N\epsilon}{\gamma_c}) + o(\frac{\epsilon}{\gamma_c}). \quad (40)$$

But, when Subcarrier grouping is used with joint selection, the expression of the outage probability will be:

$$P_{out,sg,js} = (\frac{M - 2 + 2^L}{{n_G}^{L-1}})(\frac{N\epsilon}{\gamma_c})^L + o(\frac{\epsilon}{\gamma_c}). \quad (41)$$

Consequently, the corresponding diversity order will be:

$$D_{js} = -\lim_{\gamma_c \to +\infty} \frac{Ln(P_{out,sg,js})}{Ln(\gamma_c)} = L. \quad (42)$$

Thus, we conclude that subcarrier grouping technique do not improve diversity order but gives a power gain of ${n_G}^{L-1}$

Table (I) recapitulates the approximated expressions of outage probability for the different scenarios (with and without joint selection, with and without subcarrier grouping).

TABLE I
APPROXIMATED OUTAGE PROBABILITY FOR HIGH SNR

	without subcarrier grouping	with subcarrier grouping
J.S.[1]	$(M-2+2^L)(\frac{N\epsilon}{\gamma_c})^L + o(\frac{\epsilon}{\gamma_c})$	$(\frac{M-2+2^L}{{n_G}^{L-1}})(\frac{N\epsilon}{\gamma_c})^L + o(\frac{\epsilon}{\gamma_c})$
N.J.S.[2]	$(\frac{N\epsilon}{\gamma_c}) + o(\frac{\epsilon}{\gamma_c})$	$(\frac{N\epsilon}{\gamma_c}) + o(\frac{\epsilon}{\gamma_c})$

[1]:J.S. refers to Joint selection.

[2]:N.J.S. refers to No Joint selection.

Table (II) recapitulates the diversity order for the different scenarios:

TABLE II
DIVERSITY ORDER

	without subcarrier grouping	with subcarrier grouping
J.S.[1]	L	L
N.J.S.[1]	1	1

V. Simulation results

This section presents the simulation results which validates the analytics for the different studied scenarios. The number of subcarriers used for the simulation is 16. To determine the SNR threshold ϵ, we proceed similarly as in [7]:

To determine ϵ, a fixed throughput target r is chosen, then, ϵ is calculated as the minimum needed SNR to satisfy this throughput constraint:

$$r = log_2(1+\epsilon) \;\; i.e. \;\; \epsilon = 2^r - 1 \quad (43)$$

where r is the spectral efficiency. We assume that $r = 3bits/s/Hz$ so we obtain $\epsilon = 7$.

Fig.1 presents the theoretical and simulation results for outage probability when Joint selection takes place into the last two hops with different values of L and M=2. The theoretical expression is given by (27) and fits perfectly the simulation results. It can be seen that full diversity order is achieved.

When using subcarrier grouping with joint selection, it is proven in section IV that full diversity order is achieved with power gain of n_G^{L-1}. In Fig. 2, 2 relays are used, so for $n_G = 2$

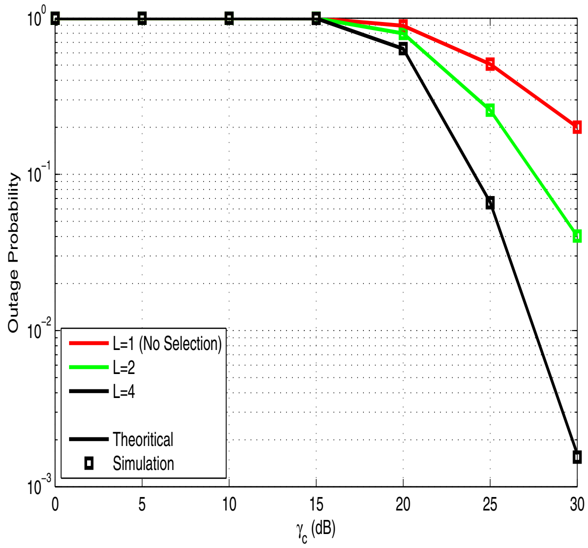

Fig. 1. Outage Probability for selective OFDM with joint selection for $\epsilon = 7, \quad M = 2, \quad and \quad L = 1, 2, 4$

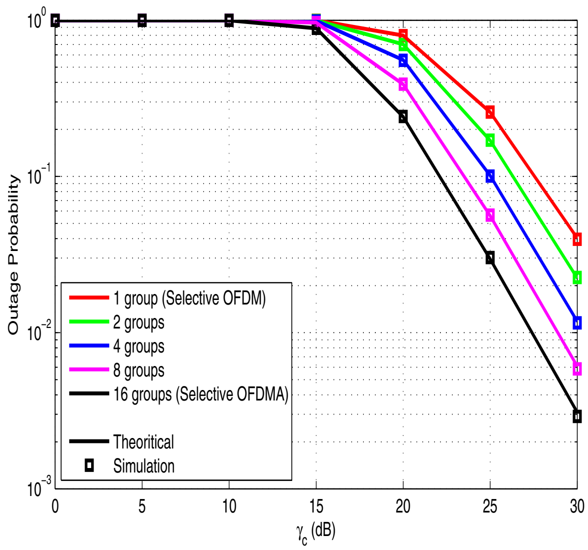

Fig. 2. Outage Probability for selective OFDM with joint selection and subcarrier grouping for $\epsilon = 7, \quad M = 2, \quad L = 2, \quad and \quad n_G = 1, 2, 4, 8, 16$

- for example - the power gain should be $10.log_{10}(2) = 3dB$ which is proved by the simulation.

As it is proven in section IV, for high SNR, outage probability remains constant for different values of L and M when no joint selection is taken into account. Thus, there is neither diversity gain nor power gain for $L \geq 2$. This can be clearly seen in Fig. 3. The lonely gain that can be seen is in low SNR and it decreases as long as M increases.

VI. Conclusion

In this contribution, outage performance of Ad-hoc Routing strategie for broadband channel with bottleneck maximization

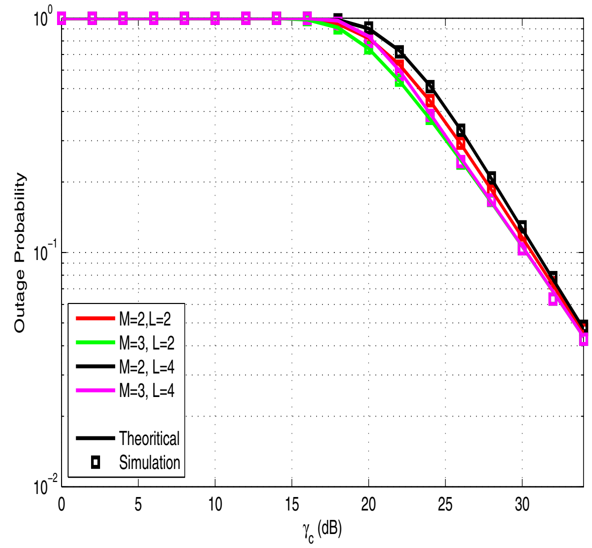

Fig. 3. Outage Probability for selective OFDM without joint selection for $\epsilon = 7, \quad M = 2, 4, \quad and \quad L = 2, 4$

was analyzed and then validated by simulations. Based on these analysis, we showed that dividing the subcarriers into n_G groups (subcarrier grouping) will conserve the same diversity gain but bring n_G^{L-1} fold power gain. If no joint selection is performed in the last two hops, it is proved that the system already lost its selection gain (Outage probability independent from $L \, \forall L \geq 2$). Thus, there is no need for subcarrier grouping when no joint selection is used.

Future work will take into account correlation between the subchannels which will be more realistic and will improve the performance of subcarrier grouping.

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