Optimal Progressive Precoder Design for ARQ Packet Retransmissions in Nonregenerative MIMO Relay Systems

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***Abstract*—This paper investigates the precoder design for packet retransmissions in nonregenerative multi-input multi-output (MIMO) relay systems. Assuming the channel state information (CSI) only available at the relay and the destination, the precoder design performed at the relay is considered. To fully utilize the time diversity provided by automatic repeat request (ARQ), progressive precoders are designed with the objective of maximizing the mutual information delivered by multiple transmissions of the same packet. We derive the form of the optimal ARQ precoder by restating the optimization problem as a matrix diagonalization problem, then the power allocation is formulated as a convex optimization problem where the karush-kuhn-tucker conditions are used to obtain the optimal solution. Numerical results show that the proposed precoders can improve the system performance significantly.**

I. Introduction

The use of multiple-input multiple-output (MIMO) technique in a wireless relay network has gained special attentions in recent years due to its ability to extend propagation distance and improve spectral efficiency [1-2]. In practice, relays can be either regenerative or nonregenerative [3]. Generally, nonregenerative relaying is more simple and with smaller processing delay since the relays just amplify and forward (AF) the received signals [4-5]. In a nonregenerative MIMO relay system, the studies have shown that performing linear precoder at the relays leads to improved system performance compared to conventional AF relaying. The precoder design performed at the relay to maximize the channel capacity of a MIMO relay network is studied in [4-5]. Other relaying strategies regarding multiple relays are presented in [6] while the precoder is designed to minimize the mean squared error (MSE) of the estimated symbols in [7].

Despite the high capacity and diversity promised by MIMO relay links, practical systems would inevitably encounter channel distortions, noise, or fading, leading to potential packet failures. The automatic repeat request (ARQ) technique is an effective means for packet error correction through incrementally retransmitting the failed packet. However, unlike the linear precoder design for the single transmission without ARQ [4-7], the precoder design for ARQ retransmissions must take into account previous receptions and cannot go back to change precoders for previous transmsssions. In related works, the ARQ precoder design is investigated deeply in point-to-point MIMO systems [8-11]. The optimal progressive ARQ precoder is derived with the objective of maximizing the retransmission capacity in [8], and the progressive precoder is studied to minimize the MSE in [9]. Moreover, in [10] and [11-12] the multiuser progressive ARQ precoder and the codebook-based ARQ precoder are proposed.

However, to the best of our knowledge, there are no studies investigating the ARQ precoder in nonregenerative MIMO relay systems, where the corresponding problem becomes more complex since two hop channel information needs to be considered jointly. Motivated by such fact, the paper investigates the ARQ precoder design for nonregenerative MIMO relay systems. The optimization objective is to maximize the retransmission capacity. We restate the optimization problem as a matrix diagonalization problem, based on which the form of the optimal ARQ precoder is derived. Then the power allocation is formulated as a convex optimization problem where the karush-kuhn-tucker (K.K.T) conditions are employed to obtain the optimal solution. Compared to the conventional precoder design, our proposed scheme shows significant gains.

The paper is organized as follows: Section II describes the system model for nonregenerative MIMO relay ARQ systems. Section III presents the optimal precoder for the single transmission without ARQ. The optimal progressive ARQ precoder is proposed in Section IV. The numerical results are presented in Section V. The conclusion is in Section VI.

Notationwise, we use bold upper case symbols to denote matrices and bold lower case symbols to denote column vectors. $\{\cdot\}^H$ denotes matrix conjugate transpose. $E(\cdot)$ denotes statistical expectation. $(x)_+$ returns $\max(x,0)$. $Tr(\mathbf{A})$ and $|\mathbf{A}|$ represent the trace and the determinant of a square matrix $\mathbf{A}$.

II. System Model

We consider a three-node precoded AF relay system over flat fading channels as depicted in Figure 1, in which the

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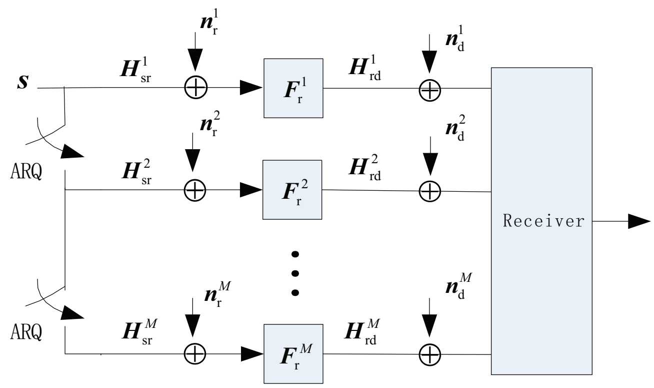

Fig. 1. System model of AF MIMO Relay ARQ systems

source, relay, and destination are all equipped with N antennas and N streams are sent from the source. The maximum number of retransmission times is M. Using the typical two-phase transmission scheme [4], for the m_{th} transmission we can express the received signal at the destination as

$$\begin{aligned}\mathbf{y}_{\mathrm{d}}^{m}&=\mathbf{H}_{\mathrm{rd}}^{m}\mathbf{F}_{\mathrm{r}}^{m}\left(\mathbf{H}_{\mathrm{sr}}^{m}\mathbf{s}+\mathbf{n}_{\mathrm{r}}^{m}\right)+\mathbf{n}_{\mathrm{d}}^{m}\\&=\mathbf{H}_{\mathrm{rd}}^{m}\mathbf{F}_{\mathrm{r}}^{m}\mathbf{H}_{\mathrm{sr}}^{m}\mathbf{s}+\mathbf{H}_{\mathrm{rd}}^{m}\mathbf{F}_{\mathrm{r}}^{m}\mathbf{n}_{\mathrm{r}}^{m}+\mathbf{n}_{\mathrm{d}}^{m}\end{aligned}\tag{1}$$

where s is the transmitted signal vector with zero mean and $\mathbf{R_s}=E(\mathbf{s}\mathbf{s}^H)=\frac{P_s}{N}\mathbf{I}$, and the P_s is the power constraint in the source. The channel state information (CSI) is assumed only available at the relay and the destination such as in [4-7], so the power is averaged by antennas in the source. $\mathbf{H}_{\mathrm{sr}}^{m}$ and $\mathbf{H}_{\mathrm{rd}}^{m}$ are the channels of the first hop and the second hop during the m_{th} transmission. The precoder employed at the relay is denoted as $\mathbf{F}_{\mathrm{r}}^{m}$. The white noise in the relay and the destination is denoted as $\mathbf{n}_{\mathrm{r}}^{m}$ and $\mathbf{n}_{\mathrm{d}}^{m}$ with zero mean and covariance matrix $\mathbf{R}_{\mathbf{n}_{\mathrm{r}}^{m}}=E\left(\mathbf{n}_{\mathrm{r}}^{m}(\mathbf{n}_{\mathrm{r}}^{m})^H\right)=\sigma^2\mathbf{I},\mathbf{R}_{\mathbf{n}_{\mathrm{d}}^{m}}=E\left(\mathbf{n}_{\mathrm{d}}^{m}(\mathbf{n}_{\mathrm{d}}^{m})^H\right)=\sigma^2\mathbf{I}$.

The average power used by the relay is upper bounded by P_r. Since the transmitted signal from the relay is $\mathbf{F}_{\mathrm{r}}^{m}\mathbf{H}_{\mathrm{sr}}^{m}\mathbf{s}+\mathbf{F}_{\mathrm{r}}^{m}\mathbf{n}_{\mathrm{r}}^{m}$, the power constraint on the relay leads to the following constraint on $\mathbf{F}_{\mathrm{r}}^{m}$:

$$tr\left\{\mathbf{F}_{\mathrm{r}}^{m}\left(\mathbf{I}+\rho_1\mathbf{H}_{\mathrm{sr}}^{m}(\mathbf{H}_{\mathrm{sr}}^{m})^H\right)(\mathbf{F}_{\mathrm{r}}^{m})^H\right\}\le\rho_2 N\tag{2}$$

where $\rho_1=\frac{P_s}{\sigma^2 N}$ and $\rho_2=\frac{P_r}{\sigma^2 N}$ denote the signal-to-noise ratio (SNR) of the first hop and the second hop.

After combining the previous receptions, the signal during the m_{th} retransmission is

$$\mathbf{y}_D^m=\underbrace{\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^{1}\mathbf{F}_{\mathrm{r}}^{1}\mathbf{H}_{\mathrm{sr}}^{1}\\\mathbf{H}_{\mathrm{rd}}^{2}\mathbf{F}_{\mathrm{r}}^{2}\mathbf{H}_{\mathrm{sr}}^{2}\\\vdots\\\mathbf{H}_{\mathrm{rd}}^{m}\mathbf{F}_{\mathrm{r}}^{m}\mathbf{H}_{\mathrm{sr}}^{m}\end{bmatrix}}_{\mathbf{H}}\mathbf{s}+\underbrace{\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^{1}\mathbf{F}_{\mathrm{r}}^{1}\mathbf{n}_{\mathrm{r}}^{1}+\mathbf{n}_{\mathrm{d}}^{1}\\\mathbf{H}_{\mathrm{rd}}^{2}\mathbf{F}_{\mathrm{r}}^{2}\mathbf{n}_{\mathrm{r}}^{2}+\mathbf{n}_{\mathrm{d}}^{2}\\\vdots\\\mathbf{H}_{\mathrm{rd}}^{m}\mathbf{F}_{\mathrm{r}}^{m}\mathbf{n}_{\mathrm{r}}^{m}+\mathbf{n}_{\mathrm{d}}^{m}\end{bmatrix}}_{\mathbf{n}}\tag{3}$$

For a given retransmission, the receiver can only utilize the current retransmission and the previously unsuccessful (re)transmissions of the same data packet to decode. Moreover, future retransmissions may not be needed. Thus, the relay must progressively find the best linear precoder based on the knowledge it has of its previous precoders. In other words, the m_{th} transmission precoder $\mathbf{F}_{\mathrm{r}}^{m}$ is optimized based on the current channel $\{\mathbf{H}_{\mathrm{sr}}^{m},\mathbf{H}_{\mathrm{rd}}^{m}\}$ and the channels $\{\mathbf{H}_{\mathrm{sr}}^{i},\mathbf{H}_{\mathrm{rd}}^{i}\}$ and the precoders $\mathbf{F}_{\mathrm{r}}^{i}, i=1,2,...m-1$ in previous transmissions.

A. Problem Formulation

The objective of this paper is to maximize the mutual information between the source and the destination. The mutual information of the m_{th} transmission is [13]

$$\begin{aligned}\mathrm{C}&=\tfrac{1}{2}\log\left|\mathbf{I}+\mathbf{H}\mathbf{R_s}\mathbf{H}^H\mathbf{R_n}^{-1}\right|\\&=\tfrac{1}{2}\log\left|\mathbf{I}+\tfrac{P_s}{N}\mathbf{H}^H\mathbf{R_n}^{-1}\mathbf{H}\right|\end{aligned}\tag{4}$$

where we applies the property that $|\mathbf{I}+\mathbf{AB}|=|\mathbf{I}+\mathbf{BA}|$ if $\mathbf{AB}$ is complex conjugate symmetric. The factor 1/2 in (4) comes from the fact that the signal vector is actually transmitted in two time instances, so the efficiency drops by one half. In the following discussion, we will ignore the factor.

Thus, the problem of progressive ARQ precoder design can be formulated as

$$\begin{aligned}&\max_{\mathbf{F}_r^m}\log\left|\mathbf{I}+\tfrac{P_s}{N}\mathbf{H}^H\mathbf{R_n}^{-1}\mathbf{H}\right|\\&s.t.\\&tr\left\{\mathbf{F}_{\mathrm{r}}^{m}\left(I+\rho_1\mathbf{H}_{\mathrm{sr}}^{m}(\mathbf{H}_{\mathrm{sr}}^{m})^H\right)(\mathbf{F}_{\mathrm{r}}^{m})^H\right\}\le\rho_2 N\end{aligned}\tag{5}$$

III. Optimal Precoder for Single Transmission

In this section, we briefly summarize the optimal precoder for AF MIMO relay systems without ARQ.

Let the eigenvalue decompositions of $\mathbf{H}_{\mathrm{sr}}^{1}\left(\mathbf{H}_{\mathrm{sr}}^{1}\right)^H$ and $\left(\mathbf{H}_{\mathrm{rd}}^{1}\right)^H\mathbf{H}_{\mathrm{rd}}^{1}$ be

$$\begin{aligned}\mathbf{H}_{\mathrm{sr}}^{1}\left(\mathbf{H}_{\mathrm{sr}}^{1}\right)^H&=\mathbf{U}_{\mathrm{sr}}^{1}\mathbf{\Lambda}_{\mathrm{sr}}^{1}\left(\mathbf{U}_{\mathrm{sr}}^{1}\right)^H\\\left(\mathbf{H}_{\mathrm{rd}}^{1}\right)^H\mathbf{H}_{\mathrm{rd}}^{1}&=\mathbf{V}_{\mathrm{rd}}^{1}\mathbf{\Lambda}_{\mathrm{rd}}^{1}\left(\mathbf{V}_{\mathrm{rd}}^{1}\right)^H\end{aligned}\tag{6}$$

where $\mathbf{U}_{\mathrm{sr}}^{1}$ and $\mathbf{V}_{\mathrm{rd}}^{1}$ are unitary matrices, $\mathbf{\Lambda}_{\mathrm{sr}}^{1}=diag\left(\alpha_1,\alpha_2...\alpha_N\right)$ with $\alpha_n\ge 0$, and $\mathbf{\Lambda}_{\mathrm{rd}}^{1}=diag\left(\beta_1,\beta_2...\beta_N\right)$ with $\beta_n\ge 0$, and all eigenvalues are arranged in the descending order.

In [4-5], the authors derive the optimal linear precoder to maximize the channel capacity for the single transmission as follows.

$$\mathbf{F}_{\mathrm{r}}^{1}=\mathbf{V}_{\mathrm{rd}}^{1}\mathbf{G}\left(\mathbf{U}_{\mathrm{sr}}^{1}\right)^H\tag{7}$$

where the optimal power allocation matrix $\mathbf{G}=diag\left(g_1,g_2,...g_N\right)$ is derived as

$$|g_n|^2=\frac{1}{2\beta_n(1+\rho_1\alpha_n)}\left[\sqrt{\rho_1^2\alpha_n^2+4\rho_1\alpha_n\beta_n\mu^*}-\rho_1\alpha_n-2\right]_+\tag{8}$$

where μ^* must be chosen such that the power allocation meets the power constraint $\sum_{n=1}^{N}\left(1+\rho_1\alpha_n\right)|g_n|^2=\rho_2 N$.

IV. Optimal Progressive Precoder for Retransmissions

Progressive precoder design is a unique feature in ARQ systems as subsequent retransmission is only needed when previous transmissions failed to deliver satisfactory packet reception. As previous transmissions cannot be altered once a new

$$
\begin{aligned}
&\left|\mathbf{I}+\tfrac{P_s}{N}\mathbf{H}^{\mathrm{H}}\mathbf{R_n}^{-1}\mathbf{H}\right| \\
&=\left|\mathbf{I}+\rho_1\begin{bmatrix}\left(\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{H}_{\mathrm{sr}}^1\right)^H & \left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)^H\end{bmatrix}\begin{bmatrix}\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\left(\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\right)^H\right)^{-1} & \mathbf{0}\\ \mathbf{0} & \left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{H}_{\mathrm{sr}}^1\\ \mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\end{bmatrix}\right| \\
&=\left|\mathbf{I}+\rho_1\left(\left(\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{H}_{\mathrm{sr}}^1\right)^H\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\left(\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{H}_{\mathrm{sr}}^1+\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)^H\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)\right| \\
&=\left|\mathbf{I}+\rho_1\left(\mathbf{V}_{\mathrm{sr}}^1\mathbf{\Lambda}_{\mathrm{sr}}^1\mathbf{\Lambda}_{\mathrm{rd}}^1\mathbf{G}^2\left(\mathbf{I}+\mathbf{\Lambda}_{\mathrm{rd}}^1\mathbf{G}^2\right)^{-1}\left(\mathbf{V}_{\mathrm{sr}}^1\right)^H+\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)^H\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)\right| \\
&=\left|\mathbf{V}_{\mathrm{sr}}^1\left(\mathbf{I}+\rho_1\mathbf{\Lambda}_{\mathrm{sr}}^1\mathbf{\Lambda}_{\mathrm{rd}}^1\mathbf{G}^2\left(\mathbf{I}+\mathbf{\Lambda}_{\mathrm{rd}}^1\mathbf{G}^2\right)^{-1}\right)\left(\mathbf{V}_{\mathrm{sr}}^1\right)^H+\rho_1\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)^H\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right| \\
&=\left|\mathbf{V}_{\mathrm{sr}}^1\mathbf{G}_{pre}\left(\mathbf{V}_{\mathrm{sr}}^1\right)^H+\rho_1\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)^H\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right|
\end{aligned}
\tag{12}
$$

ARQ request is sent and future retransmissions may not be needed, the relay can only process the current (re)transmission for optimum effects.

Without loss of generality, we start with the design of optimal precoder for the second transmission. Then we extend the design to multiple retransmissions.

A. Optimal Progressive Precoder for the Second Transmission

After combining the first reception, the signal during the second transmission is

$$
\mathbf{y}_D^2=\underbrace{\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{H}_{\mathrm{sr}}^1\\ \mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\end{bmatrix}}_{\mathbf{H}}\mathbf{s}+\underbrace{\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{n}_{\mathrm{r}}^1+\mathbf{n}_{\mathrm{d}}^1\\ \mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{n}_{\mathrm{r}}^2+\mathbf{n}_{\mathrm{d}}^2\end{bmatrix}}_{\mathbf{n}}
\tag{9}
$$

The covariance matrix of the noise vector received at the destination is

$$
\begin{aligned}
\mathbf{R_n}&=\mathrm{E}\left(\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{n}_{\mathrm{r}}^1+\mathbf{n}_{\mathrm{d}}^1\\ \mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{n}_{\mathrm{r}}^2+\mathbf{n}_{\mathrm{d}}^2\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\mathbf{n}_{\mathrm{r}}^1+\mathbf{n}_{\mathrm{d}}^1\\ \mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{n}_{\mathrm{r}}^2+\mathbf{n}_{\mathrm{d}}^2\end{bmatrix}^H\right)\\
&=\sigma^2 diag\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\left(\mathbf{H}_{\mathrm{rd}}^1\mathbf{F}_{\mathrm{r}}^1\right)^H,\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)
\end{aligned}
\tag{10}
$$

Let $\mathbf{R}_i=\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^i\mathbf{F}_{\mathrm{r}}^i\left(\mathbf{H}_{\mathrm{rd}}^i\mathbf{F}_{\mathrm{r}}^i\right)^H, i=1,2$, then we can get

$$
\mathbf{R}_n^{-1}=\tfrac{1}{\sigma^2}diag\left((\mathbf{R}_1)^{-1},(\mathbf{R}_2)^{-1}\right)
\tag{11}
$$

Then, we can obtain the equation (12) at the top of the page, where $\mathbf{G}_{pre}=\left(\mathbf{I}+\rho_1\mathbf{\Lambda}_{\mathrm{sr}}^1\mathbf{\Lambda}_{\mathrm{rd}}^1\mathbf{G}^2\left(\mathbf{I}+\mathbf{\Lambda}_{\mathrm{rd}}^1\mathbf{G}^2\right)^{-1}\right)$ is a diagonal matrix with positive entries, and it denotes the information of the previous transmissions. Then we have the following lemma.

Lemma: If we let $\mathbf{A}=\mathbf{V}_{sr}^1\mathbf{G}_{pre}\left(\mathbf{V}_{sr}^1\right)^H$, the optimization problem in (5) can be restated as follows.

$$
\begin{aligned}
&\max_{\mathbf{F}_{\mathrm{r}}^2}\left(\log|\mathbf{A}|+\log|\mathbf{M}|\right)\\
&where \quad \mathbf{M}=\left(\mathbf{I}+\rho_1\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\mathbf{A}^{-1/2}\right)^H\right.\\
&\left.\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\mathbf{A}^{-1/2}\right)\\
&s.t \quad \mathbf{M} \text{ is diagonal and}\\
&tr\left\{\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{I}+\rho_1\mathbf{H}_{\mathrm{sr}}^2\left(\mathbf{H}_{\mathrm{sr}}^2\right)^H\right)\left(\mathbf{F}_{\mathrm{r}}^2\right)^H\right\}\le\rho_2 N
\end{aligned}
\tag{13}
$$

Proof: If $\mathbf{A}\in\mathbb{C}^{N\times N}$ and $\mathbf{B}\in\mathbb{C}^{N\times N}$ are two positive definite matrices, then [14]

$$
|\mathbf{A}+\mathbf{B}|=|\mathbf{A}|\left|\mathbf{I}+\mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right|
\tag{14}
$$

If we let $\mathbf{A}=\mathbf{V}_{sr}^1\mathbf{G}_{pre}\left(\mathbf{V}_{sr}^1\right)^H$, then $\mathbf{A}^{-1/2}=\left(\mathbf{A}^{-1/2}\right)^H=\mathbf{V}_{sr}^1\left(\mathbf{G}_{pre}\right)^{-1/2}\left(\mathbf{V}_{sr}^1\right)^H$. From (12) and (14) we can derive the following equation (due to limitation of space, the proof of the positive definite property is neglected).

$$
\begin{aligned}
&\log\left|\mathbf{I}+\tfrac{P_s}{N}\mathbf{H}^{\mathrm{H}}\mathbf{R_n}^{-1}\mathbf{H}\right|\\
&=\log|\mathbf{A}|+\log\left|\mathbf{I}+\rho_1\left(\mathbf{A}^{-1/2}\right)^H\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\right)^H\right.\\
&\left.\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\mathbf{A}^{-1/2}\right|
\end{aligned}
\tag{15}
$$

where the first term $\log|\mathbf{A}|$ is constant since it doesn't depend on $\mathbf{F}_{\mathrm{r}}^2$. Now our objective is to maximize the second term.

In [14], we have the following Hadamard inequality: let $\mathbf{M}\in\mathbb{C}^{N\times N}$ be a positive definite matrix, and $\mathbf{M}(i,j)$ be its entry of the i_{th} row and the j_{th} column, then

$$
|\mathbf{M}|\le\prod_{i=1}^{N}\mathbf{M}(i,i)
\tag{16}
$$

where the equality holds when $\mathbf{M}$ is a diagonal matrix.

Based on the aforementioned Hadamard inequality, let

$$
\begin{aligned}
&\mathbf{M}=\mathbf{I}+\rho_1\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\mathbf{A}^{-1/2}\right)^H\\
&\left(\mathbf{I}+\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\left(\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\right)^H\right)^{-1}\mathbf{H}_{\mathrm{rd}}^2\mathbf{F}_{\mathrm{r}}^2\mathbf{H}_{\mathrm{sr}}^2\mathbf{A}^{-1/2}
\end{aligned}
$$

, it turns out that when $\mathbf{M}$ is diagonalized, the cost function in (15) is then maximized. So the optimization problem in (5) can be restated as (13). *ProofEnd*

■

The diagonalization requirement in (13) motivates us to consider the following singular value decomposition

$$
\begin{aligned}
&\mathbf{H}_{\mathrm{sr}}^2\mathbf{A}^{-1/2}=\mathbf{U}_{\mathrm{sr}}^{2'}\left(\mathbf{\Lambda}_{\mathrm{sr}}^{2'}\right)^{1/2}\left(\mathbf{V}_{\mathrm{sr}}^{2'}\right)^H\\
&\left(\mathbf{H}_{\mathrm{rd}}^2\right)^H\mathbf{H}_{\mathrm{rd}}^2=\mathbf{V}_{\mathrm{rd}}^2\mathbf{\Lambda}_{\mathrm{rd}}^2\left(\mathbf{V}_{\mathrm{rd}}^2\right)^H
\end{aligned}
\tag{17}
$$

where $\mathbf{U}_{\mathrm{sr}}^{2'}$ and $\mathbf{V}_{\mathrm{rd}}^2$ are unitary matrices, $\mathbf{\Lambda}_{\mathrm{sr}}^{2'}=diag\left(a_1,a_2,...a_N\right)$ with $a_n\ge 0$, and

$\mathbf{\Lambda}_{\rm rd}^2 = diag\left(b_1, b_2, ...b_N\right)$ with $b_n \geq 0$, and all eigenvalues are arranged in the descending order. To have a full diagonalization of $\mathbf{M}$, it turns out that the optimum $\mathbf{F}_{\rm r}^2$ have the following structure.

$$\mathbf{F}_{\rm r}^2 = \mathbf{V}_{\rm rd}^2 \widetilde{\mathbf{G}} \left(\mathbf{U}_{\rm sr}^{2'}\right)^H \tag{18}$$

where $\widetilde{\mathbf{G}} = diag\left(\widetilde{g}_1, \widetilde{g}_2, ...\widetilde{g}_N\right)$ is the power allocation matrix.

Then the second term of the optimization objective in (13) becomes

$$\log|\mathbf{M}| = \log\left|\mathbf{I} + \rho_1 \mathbf{\Lambda}_{\rm sr}^{2'} \mathbf{\Lambda}_{\rm rd}^2 \left(\widetilde{\mathbf{G}}\right)^2 \left(\mathbf{I} + \mathbf{\Lambda}_{\rm rd}^2 \left(\widetilde{\mathbf{G}}\right)^2\right)^{-1}\right| \tag{19}$$

Now we turn to deal with the power constraint. From the equation (17) we can obtain

$$\begin{aligned}
&\mathbf{H}_{\rm sr}^2 \left(\mathbf{H}_{\rm sr}^2\right)^H \\
&= \mathbf{U}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{\frac{1}{2}} \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A}^{\frac{1}{2}} \left(\mathbf{U}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{\frac{1}{2}} \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A}^{\frac{1}{2}}\right)^H \\
&= \mathbf{U}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{1/2} \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A} \mathbf{V}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{1/2} \left(\mathbf{U}_{\rm sr}^{2'}\right)^H
\end{aligned} \tag{20}$$

Substituting (18) and (20) into the power constraint, we can get the following equation

$$\begin{aligned}
&tr\left\{\mathbf{F}_{\rm r}^2 \left(\mathbf{I} + \rho_1 \mathbf{H}_{\rm sr}^2 \left(\mathbf{H}_{\rm sr}^2\right)^H\right) \left(\mathbf{F}_{\rm r}^2\right)^H\right\} \\
&= tr\left\{\mathbf{V}_{\rm rd}^2 \widetilde{\mathbf{G}} \left(\mathbf{U}_{\rm sr}^{2'}\right)^H \left(\mathbf{I} + \rho_1 \mathbf{U}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{1/2} \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A} \right.\right. \\
&\left.\left. \mathbf{V}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{1/2} \left(\mathbf{U}_{\rm sr}^{2'}\right)^H\right) \left(\mathbf{V}_{\rm rd}^2 \widetilde{\mathbf{G}} \left(\mathbf{U}_{\rm sr}^{2'}\right)^H\right)^H\right\} \\
&\overset{(a)}{=} tr\left\{\widetilde{\mathbf{G}} \left(\mathbf{I} + \rho_1 \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{1/2} \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A} \mathbf{V}_{\rm sr}^{2'} \left(\mathbf{\Lambda}_{\rm sr}^{2'}\right)^{1/2}\right) \widetilde{\mathbf{G}}^H\right\} \\
&\overset{(b)}{=} tr\left\{\left(\mathbf{I} + \rho_1 \mathbf{\Lambda}_{\rm sr}^{2'} \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A} \mathbf{V}_{\rm sr}^{2'}\right) \widetilde{\mathbf{G}}^2\right\} \\
&= tr\left\{\left(\mathbf{I} + \rho_1 \mathbf{\Lambda}_{\rm sr}^{2'} \mathbf{Q}\right) \widetilde{\mathbf{G}}^2\right\}
\end{aligned} \tag{21}$$

where $\mathbf{Q} = \left(\mathbf{V}_{\rm sr}^{2'}\right)^H \mathbf{A} \mathbf{V}_{\rm sr}^{2'}$. In the step (a) we applies the property $tr\left(\mathbf{BDB}^{-1}\right) = tr\left(\mathbf{D}\right)$ while the property $tr\left(\mathbf{BD}\right) = tr\left(\mathbf{DB}\right)$ is used in the step (b).

Then, to compute the elements of diagonal matrix $\widetilde{\mathbf{G}}$ we need to solve the following scalar problem

$$\begin{aligned}
&\text{Max} \sum_{n=1}^{\text{N}} \log_2\left(1 + \rho_1 \frac{a_n b_n |\widetilde{g}_n|^2}{1 + b_n |\widetilde{g}_n|^2}\right) \\
&s.t \\
&|\widetilde{g}_n|^2 \geq 0 \qquad \text{n} = 1, 2, ..., N \\
&\sum_{n=1}^{N} \left(\rho_1 a_n \mathbf{Q}(n,n) + 1\right) |\widetilde{g}_n|^2 = \rho_2 N
\end{aligned} \tag{22}$$

The problem in (22) is a standard concave optimization problem (the objective function and the inequality constraint function are concave, while the equality constraint function is affine with respect to $|\widetilde{g}_n|^2$), which can be solved by means of the K.K.T conditions [15] to obtain the optimum value for $|\widetilde{g}_n|^2, n = 1, 2, ..., N$. After some tedious calculations, the optimum solution for $|\widetilde{g}_n|^2$ is given by

$$|\widetilde{g}_n|^2 = \frac{1}{2b_n(1+\rho_1 a_n)}\left[\sqrt{\rho_1^2 a_n^2 + 4\rho_1 a_n b_n \upsilon^*} - \rho_1 a_n - 2\right]_+ \tag{23}$$

where υ^* must be chosen such that the power allocation meets the power constraint $\sum_{n=1}^{N} \left(\rho_1 a_n \mathbf{Q}(n,n) + 1\right) |\widetilde{g}_n|^2 = \rho_2 N$, which can be solved by using a numerical root-finding algorithm, such as Bisection etc.

B. Optimal ARQ Precoders for Multiple Retransmissions

Based on the above analysis, we can obtain the optimal progressive precoder for the second transmission. Moreover, the method can be directly applied to optimal sequential precoder design for multiple ARQ transmissions. Note that, by induction, we can assume that at the m_{th} ARQ transmission, all the previous $m-1$ precoders have been sequentially determined. In order to determine the optimal precoder $\mathbf{F}_{\rm r}^m$, let $\mathbf{A} = \mathbf{I} + \rho_1 \sum_{k=1}^{m-1} \left(\mathbf{H}_{\rm rd}^k \mathbf{F}_{\rm r}^k \mathbf{H}_{\rm sr}^k\right)^H \left(\mathbf{I} + \mathbf{H}_{\rm rd}^k \mathbf{F}_{\rm r}^k \left(\mathbf{H}_{\rm rd}^k \mathbf{F}_{\rm r}^k\right)^H\right)^{-1} \mathbf{H}_{\rm rd}^k \mathbf{F}_{\rm r}^k \mathbf{H}_{\rm sr}^k$, $\mathbf{M} = \mathbf{I} + \rho_1 \left(\mathbf{H}_{\rm rd}^m \mathbf{F}_{\rm r}^m \mathbf{H}_{\rm sr}^m \mathbf{A}^{-1/2}\right)^H \left(\mathbf{I} + \mathbf{H}_{\rm rd}^m \mathbf{F}_{\rm r}^m \left(\mathbf{H}_{\rm rd}^m \mathbf{F}_{\rm r}^m\right)^H\right)^{-1} \mathbf{H}_{\rm rd}^m \mathbf{F}_{\rm r}^m \mathbf{H}_{\rm sr}^m \mathbf{A}^{-1/2}$, then the similar optimization problem can be formulated as (13) which also gives the similar solution as (18) and (23) with a minor change of indices.

V. Simulations

In this section, we compare the optimal ARQ precoder shown in the previous section, with the conventional individual precoder design scheme in terms of the ergodic capacity. In the individual precoder design scheme, the precoders of retransmissions are designed individually as Section III.

In the simulation, the channels of the first hop and the second hop are both modeled as the rayleigh channels which have independent and identically distributed (i.i.d.) circularly

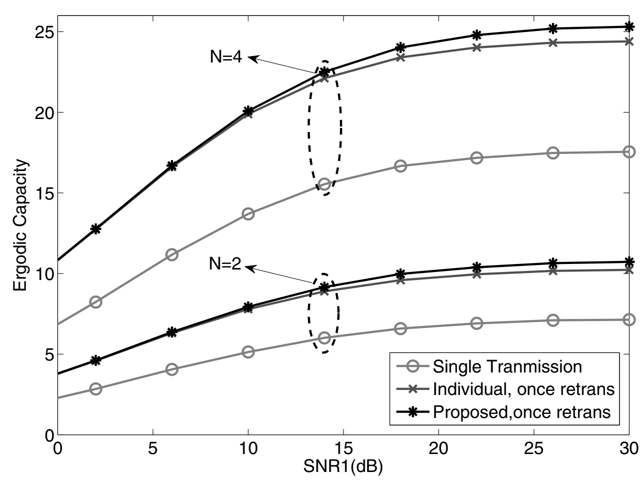

Fig. 2. Ergodic capacity as a function of ρ_1. $\rho_2 = 10dB$

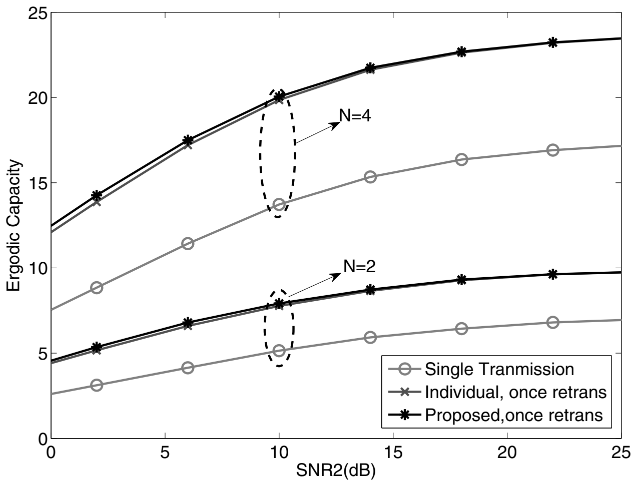

Fig. 3. Ergodic capacity as a function of ρ_2. $\rho_1 = 10dB$

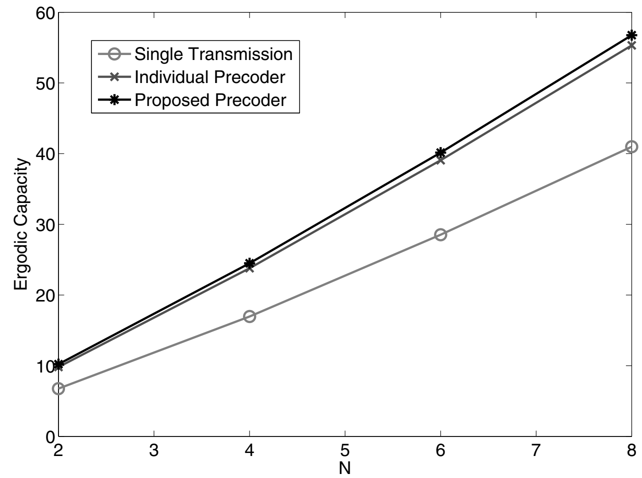

Fig. 4. Ergodic capacity as a function of N. $\rho_1 = 20dB$. $\rho_2 = 10dB$

symmetric complex Gaussian entries with zero mean and unit variance. In particular, the channels between different ARQ retransmissions are assumed to be independent. Moreover, once retransmission is considered.

Figure 2 shows the ergodic capacity of the MIMO relay ARQ system as a function of ρ_1 when $\rho_2 = 10dB$. Figure 3 shows the ergodic capacity as a function of ρ_2 when $\rho_1 = 10dB$. It can be noticed that our proposed scheme is superior to the conventional individual precoder design scheme.

Figure 4 presents the ergodic capacity as a function of the antennas number N when $\rho_1 = 20dB$ and $\rho_2 = 10dB$. It can be observed that the performance gain increases with the increase of the number of transceiver antennas.

VI. Conclusion

In this paper, we address the optimal precoder design problem for flat fading MIMO relay ARQ systems. The objective of precoder design is to maximize the channel capacity provided by each ARQ transmission. Based on the assumption that the relay and the destination have the perfect CSI, we derive the optimal linear sequential precoder at the relay. Numerical results show that the proposed scheme outperforms the conventional precoder design scheme.

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