Impact of Outdated Feedback on the Performance of M-QAM Adaptive Modulation in User Selection Diversity Systems with OSTBC over MIMO Rayleigh Fading Channels

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***Abstract*—This paper presents an evaluation of the impact of channel time variations on performance of multiuser selection diversity systems employing orthogonal space-time block coding (OSTBC) over MIMO Rayleigh fading channels. A rate-adaptive M-QAM modulation along with the maximum signal-to-noise-ratio (SNR) based user selection diversity is considered. Analytical expressions are derived for the average spectral efficiency, average bit error rate (BER), and bit error outage (BEO) of the system. Using the results obtained from the analytical expressions as well as from Monte-Carlo simulations, the impact of channel time variations on the system performance is evaluated.**

I. Introduction

Multiuser selection diversity has been used to exploit the multiuser diversity of multiuser multiple-input multiple-output (MIMO) systems [1]. On the other hand, adaptive transmission techniques are well-known efficient techniques for improving the system performance. Their goal is to increase the spectral efficiency up to the Shannon capacity limit, in which the modulation parameters, transmission rate and/or transmit power are chosen according to the channel state information (CSI) satisfying some given Quality of Service (QoS) requirements, usually a target bit error rate (BER) [2]. Most of the work on user selection schemes that have been presented for the single-input single-output (SISO) systems [3] and for MIMO systems, assuming a perfect CSI and delayless feedback [4]. Furthermore, the performance of adaptive modulation and user selection is highly dependent on the accuracy of the available CSI. Due to several factors such as channel time variations, channel estimation errors, feedback delay and/or scheduling delay, the CSI used to select the best user and modulation parameters may differ from its suitable values at the time of transmission, which may yield a considerable degradation of the system performance. A particular problem is that in a time-varying channel, the delay between the modulation parameter selection and the transmission may result in a violation of the QoS requirements.

In the literature there have been so far a few investigations on the impact of outdated and/or erroneous CSI on the performance of single user and multiuser SISO systems [5]–[7] and for single user OSTBC systems [8]. Recently, in [9] a performance analysis of a normalized-SNR based user selection scheme and adaptive modulation in OSTBC-MIMO systems under outdated feedback CSI has been presented over non-identically distributed channels. In distinction from [9], in this paper, we present a performance analysis to evaluate the impact of outdated CSI and thereby the feedback delay on multiuser OSTBC-MIMO with rate-adaptive modulation over i.i.d channels and employing a maximum SNR-based user scheduling scheme, yielding much simpler mathematical expressions for each performance criteria, compared to those presented in [9]. We derive analytical expressions for the average spectral efficiency, bit error outage (BEO) and average BER. Finally, we compare the numerical results obtained for different cases.

The paper is organized as follows. Section II presents the system model and underlying assumptions. An analysis for average spectral efficiency analysis, average BER and bit error outage is given in Section III. Numerical results are presented in Section IV, and finally, Section V concludes the paper.

II. System Model

We consider a downlink multiuser OSTBC-MIMO transmission system, employing $n_{\rm T}$ transmit antennas at the base station (BS) and K user-stations, each with $n_{\rm R}$ receive antennas. We assume a Rayleigh fading MIMO channel between the u-th user and transmit antennas at the BS which can be expressed by matrix $\mathbf{H}_u$ of size $n_{\rm R} \times n_{\rm T}$, with elements $h_u^{(j,i)}$ corresponding to the complex channel gain between the i-th transmit and j-th receive antennas for the u-th user. The PDF and CDF of the received SNR at time t for each user, assumed to be i.i.d. and denoted $\gamma(t) = \widehat{\gamma}$, in OSTBC can be respectively expressed as [4]

$$f_{\widehat{\gamma}}(\widehat{\gamma}) = \frac{1}{(\lambda-1)!}\left(\frac{\mu}{\overline{\gamma}}\right)^{\lambda} \widehat{\gamma}^{\,\lambda-1} \exp\left(-\frac{\mu\,\widehat{\gamma}}{\overline{\gamma}}\right) \tag{1}$$

$$F_{\widehat{\gamma}}(\widehat{\gamma}) = 1 - \exp\left(-\frac{\mu\,\widehat{\gamma}}{\overline{\gamma}}\right)\sum_{l=0}^{\lambda-1}\frac{1}{l!}\left(\frac{\mu\,\widehat{\gamma}}{\overline{\gamma}}\right)^{l} \tag{2}$$

where $\lambda = n_{\rm T} n_{\rm R}$, $\mu = n_{\rm T} R_c$, R_c is the coding rate of the OSTBC, $\widehat{\gamma}^{\rm OSTBC} = (\overline{\gamma}/(n_{\rm T} R_c))\ \|\mathbf{H}_u\|_F^2$, $u = 1, 2, \cdots, K$, and $\overline{\gamma}$ is the average receive SNR per antenna, and the term OSTBC is omitted from $\gamma_u^{\rm OSTBC}$, for the sake of brevity. In addition, $\|\mathbf{H}_u\|_F^2 = \sum_{j=1}^{n_{\rm R}}\sum_{i=1}^{n_{\rm T}}\left|h_u^{(j,i)}\right|^2$ and it is assumed that $h_u^{(j,i)}$ values are i.i.d. with respect to j, i and u. Furthermore, the CDF expression in (2) can be written as $F_{\widehat{\gamma}}(\widehat{\gamma}) = \widetilde{\Gamma}\left(\lambda, \frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)$ where $\widetilde{\Gamma}(.,.)$ is the incomplete Gamma function given by $\widetilde{\Gamma}(\alpha, x) = \frac{1}{\Gamma(\alpha)}\int_0^x t^{\alpha-1}e^{-t}dt$.

Assuming a reliable channel state information (CSI) obtained from the K receivers, the multiuser scheduler can select the best user (user b) according to the CSI of the users

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in time-slot t such that $b = \arg\max_{u \in \mathcal{U}} \{\gamma_u(t)\}$. Then the discrete rate-adaptive modulator chooses the highest discrete modulation mode for that best user while ensuring it satisfies a given target BER noted BER_t.

In this case, using theory of order statistics, the CDF of the best user (with SNR $\widehat{\gamma}_b$) selected from K users can be expressed as

$$F_{\widehat{\gamma}_b}(\widehat{\gamma}) = [F_\gamma(\widehat{\gamma})]^K = \left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)\right]^K. \tag{3}$$

Therefore, the PDF $f_{\widehat{\gamma}_b}(\widehat{\gamma})$ can be obtained by taking the derivative of the CDF $F_{\widehat{\gamma}_b}(\widehat{\gamma})$ in (3) with respect to $\widehat{\gamma}$, yielding:

$$\begin{aligned} f_{\widehat{\gamma}_b}(\widehat{\gamma}) &= K\, f_{\widehat{\gamma}}(\widehat{\gamma}) \left[F_{\widehat{\gamma}}\left(\frac{\widehat{\gamma}}{\overline{\gamma}}\right)\right]^{K-1} \\ &= \frac{K\widehat{\gamma}^{\lambda-1}}{(\lambda-1)!}\left(\frac{\mu}{\overline{\gamma}}\right)^{\lambda} \exp\left(\frac{-\mu\widehat{\gamma}}{\overline{\gamma}}\right)\left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)\right]^{K-1}. \end{aligned} \tag{4}$$

In the following, we first review a rate-adaptive modulation scheme, and then we perform a mathematical analysis and obtain formulations for evaluation the impacts of channel time variations and feedback delay on the average spectral efficiency, average BER and bit error outage of the system.

III. Rate-Adaptive Modulation

Using the channel state information (CSI), the base station in the OSTBC-MIMO system adapts the transmission rate and selects the best modulation mode which satisfies a given target BER, BER_t. We consider a discrete-rate adaptive modulation where the numbers of bits/symbol (β_n) are integer values. For discrete-rate adaptive modulation, we consider $N+1$ transmission modes where the integer numbers of bits/symbol for discrete-rate adaptive modulation are denoted by $\beta_n, \quad n \in \{0, 1, \cdots, N\}$. The $N+1$ modes include a no-transmission mode ($\beta_0 = 0$) and N-transmission modes employing M-QAM modulation with constellation sizes $M_n = 2^{\beta_n}$, $n \in \{1, 2, \cdots, N\}$. Adaptive modulation selection is performed by dividing the entire SNR region into $N+1$ fading regions defined by SNR thresholds $0 < \alpha_1 < \cdots < \alpha_N < \alpha_{N+1} = \infty$. For an instantaneous SNR γ in the fading region $\alpha_n \leq \gamma < \alpha_{n+1}$, β_n bits/symbol will be allocated for the M-QAM modulation while for the region $\gamma < \alpha_1$, no data is sent (i.e., $\beta_0 = 0$). The threshold values α_n's act as switching levels and can approximately be computed for a certain BER_t as

$$\alpha_n = \begin{cases} \frac{-(2^n-1)}{1.5}\ln(5\,\mathrm{BER}_t); & n = 1, \cdots, N \\ +\infty & n = N+1. \end{cases} \tag{5}$$

From a practical point of view, the CSI required for selecting the best user and the best constellation size (i.e., best modulation mode), may be outdated at transmission time because of feedback delay. According to Jakes model for channel time variations and the definition given in [10], the power correlation coefficient ρ, $(0 \leq \rho \leq 1)$ of the users' SNR at the transmission time $t+\tau$ denoted $\gamma(t+\tau) = \gamma_b$, and the users' SNR at time t (outdated SNR) denoted $\gamma(t) = \widehat{\gamma}$ can be expressed by $\rho = J_0^2(2\pi f_D \tau)$, where $J_0(.)$ denotes the zero-order Bessel function of the first kind, and where f_D is the maximum Doppler frequency shift. Furthermore, $f_{\gamma|\widehat{\gamma}}(\gamma|\widehat{\gamma})$, the conditional PDF of the SNR of users at transmit time $t+\tau$ given the measured SNR at the destination at time t, is as follows [11]:

$$\begin{aligned} f_{\gamma|\widehat{\gamma}}(\gamma|\widehat{\gamma}) = \frac{\mu}{(1-\rho)\,\overline{\gamma}}\left(\frac{\gamma}{\rho\widehat{\gamma}}\right)^{\mu-1} \exp\left(-\frac{\mu(\rho\,\widehat{\gamma}+\gamma)}{(1-\rho)\,\overline{\gamma}}\right) \\ \times I_{\lambda-1}\left(\frac{2\mu\sqrt{\rho\,\widehat{\gamma}\,\gamma}}{(1-\rho)\,\overline{\gamma}}\right) \end{aligned} \tag{6}$$

where $I_{\lambda-1}(.)$ is the modified Bessel function of the first kind of order $\lambda-1$. Note that the conditional PDF of the current channel SNR for a particular user (the best user) is determined based on the outdated channel SNR for that particular user [7], therefore $f_{\gamma_b|\widehat{\gamma}_b}(\gamma|\widehat{\gamma}) = f_{\gamma|\widehat{\gamma}}(\gamma|\widehat{\gamma})$.

In the following, we obtain analytical expressions for three metrics: the average spectral efficiency, average bit error rate and bit error outage of the system under study. The expressions for the average spectral efficiency (SE) and outage probability are closed-form whereas the expression for the average BER involves an integral which can be computed numerically.

A. Average Spectral Efficiency

The achievable average spectral efficiency denoted $\langle SE \rangle$, expressed in terms of bits/sec/Hz can be defined as [2]

$$\langle SE \rangle = \sum_{n=1}^{N} \beta_n \left[F_{\widehat{\gamma}_b}(\alpha_{n+1}) - F_{\widehat{\gamma}_b}(\alpha_n)\right]. \tag{7}$$

In the highest SNR region $\alpha_N \leq \widehat{\gamma}_b < \alpha_{N+1} = \infty$, we set $F_{\widehat{\gamma}_b}(\alpha_{N+1}) = 1$. Substituting (3) into (7), and taking the OSTBC code rate (R_c) into account, we obtain

$$\langle SE \rangle = \sum_{n=1}^{N} R_c \beta_n \left\{\left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\alpha_{n+1}}{\overline{\gamma}}\right)\right]^K - \left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\alpha_n}{\overline{\gamma}}\right)\right]^K\right\}. \tag{8}$$

Observe that the system spectral efficiency given by (8) is independent of ρ and thereby is independent of feedback delay τ, since the best user and the constellation size of the modulation and the SNR switching levels have been chosen according to the CSI information at the receiver side at time t without considering the time delay at transmission time $t+\tau$.

B. Average Bit Error Rate

The average BER for discrete-rate adaptive modulation denoted $\langle \mathrm{BER} \rangle$ can be expressed as the sum of the BERs of the mode-specific modulations divided by the average spectral efficiency $\langle SE \rangle$, and can be formulated as follows [2]:

$$\langle \mathrm{BER} \rangle = \frac{1}{\langle SE \rangle} \sum_{n=1}^{N} \beta_n\, \overline{\mathrm{BER}}_n \tag{9}$$

where $\langle SE \rangle$ is defined earlier and $\overline{\mathrm{BER}}_n$ is the average BER in the n-th SNR region ($\alpha_n \leq \widehat{\gamma}_b < \alpha_{n+1}$), in which $\widehat{M}_n$ modulation is selected at time t for the transmission which takes place at time $t+\tau$, and it can be obtained from

$$\overline{\mathrm{BER}}_n = \int_{\alpha_n}^{\alpha_{n+1}} \mathrm{BER}_n(\widehat{\gamma})\, f_{\widehat{\gamma}_b}(\widehat{\gamma})\, d\widehat{\gamma} \tag{10}$$

where

$$\mathrm{BER}_n(\widehat{\gamma}) = \int_0^{+\infty} \mathrm{BER}_n(\gamma, \widehat{M}_n) f_{\widehat{\gamma}}(\gamma|\widehat{\gamma}) d\gamma. \tag{11}$$

$\mathrm{BER}_n(\gamma, \widehat{M}_n)$, the BER for a SNR γ given that the constellation was selected based on the SNR $\widehat{\gamma}$ is as follows:

$$\begin{aligned}\mathrm{BER}_n(\gamma, \widehat{M}_n) &= 0.2 \, \exp\left(-\frac{1.5\,\gamma}{\widehat{M}_n - 1}\right) \\ &= 0.2 \exp\left(-g_n \gamma\right)\end{aligned} \tag{12}$$

Substituting the expressions of $\mathrm{BER}(\gamma|\widehat{\gamma})$, and of $f_{\widehat{\gamma}}(\gamma|\widehat{\gamma})$, and using $\int_0^\infty x^n e^{-\alpha x} I_{\lambda-1}\left(2\sqrt{\beta x}\right) dx = \frac{(n+\lambda-0.5)!}{(\lambda-1)!} \beta^{(\lambda-1)/2} \alpha^{-(n+(\lambda+1)/2)} \exp\left(\frac{\beta}{\alpha}\right)$, given in (6.614.3) and (9.220.2) in [12], we obtain, after some manipulations

$$\mathrm{BER}_n(\widehat{\gamma}) = \frac{0.2}{\left[1 + g_n(1-\rho)\overline{\gamma}/\mu\right]^\lambda} \exp\left(\frac{-g_n \rho\,\widehat{\gamma}}{1 + g_n(1-\rho)\overline{\gamma}/\mu}\right). \tag{13}$$

Substituting (4) and (13) into (10) and using (22), the expression for $\overline{\mathrm{BER}}_n$ can be written in terms of finite series expansions by the following expression, which is proved in the appendix:

$$\begin{aligned}\overline{\mathrm{BER}}_n = {} & \frac{0.2\,K\,A_{n,\rho}}{(\lambda-1)!} \sum_{i=0}^{K-1} \sum_{t=0}^{i(\lambda-1)} \binom{K-1}{i} (-1)^i\, a_{i,t} \\ & \times \left(\frac{\mu}{\overline{\gamma}}\right)^{\lambda+t} \left[\exp\left(-\left[B_{n,\rho} + \frac{(i+1)\mu}{\overline{\gamma}}\right] \alpha_{_n}\right)\right. \\ & \times \sum_{j=0}^{\lambda+t-1} \frac{(\lambda+t-1)!\,\alpha_{_n}^j}{j!\left[B_{n,\rho} + \frac{(i+1)\mu}{\overline{\gamma}}\right]^{\lambda+t-j}} \\ & - \exp\left(-\left[B_{n,\rho} + \frac{(i+1)\mu}{\overline{\gamma}}\right] \alpha_{_{n+1}}\right) \\ & \left. \times \sum_{j=0}^{\lambda+t-1} \frac{(\lambda+t-1)!\,\alpha_{_{n+1}}^j}{j!\left[B_{n,\rho} + \frac{(i+1)\mu}{\overline{\gamma}}\right]^{\lambda+t-j}}\right],\end{aligned} \tag{14}$$

where $A_{n,\rho} = [1 + g_n(1-\rho)\overline{\gamma}/\mu]^{-\lambda}$, $B_{n,\rho} = \frac{g_n \rho}{1+g_n(1-\rho)\overline{\gamma}/\mu}$, and $a_{i,t}$ is the coefficient of $\left(\frac{\mu}{\overline{\gamma}}\,\gamma\right)^t$ in the expansion $\left[\sum_{l=0}^{\lambda} \frac{1}{l!}\left(\frac{\mu}{\overline{\gamma}}\,\gamma\right)^l\right]^i$, defined by [1]

$$a_{i,t} = \begin{cases} 1 & \text{for} \quad t = 0 \\ i & \text{for} \quad t = 1 \\ \frac{1}{t}\sum_{n=1}^{\min(t,\,m\lambda-1)} \frac{n(i+1)-t}{n!}\, a_{i,t-n} & \text{for } 2 \le t < i(\lambda-1) \\ [(\lambda-1)!]^{-i} & \text{for} \quad t = i(\lambda-1). \end{cases} \tag{15}$$

Finally, substituting (14) into (9) yields the overall average BER expression as follows

$$\langle \mathrm{BER} \rangle = \frac{(1/R_c)\sum_{n=1}^{N} \beta_{_n}\, \overline{\mathrm{BER}}_n}{\sum_{n=1}^{N} \beta_{_n}\left(\left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\alpha_{_{n+1}}}{\overline{\gamma}}\right)\right]^K - \left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\alpha_{_n}}{\overline{\gamma}}\right)\right]^K\right)}, \tag{16}$$

where $\overline{\mathrm{BER}}_n$ is given in (14).

C. Bit Error Outage

The bit error outage (BEO) probability is the probability that the instantaneous BER at the transmission time violates the target BER value BER_t [13]–[15] and it is an important indicator of the QoS that can be delivered to a user in a time-varying system with feedback delay. Let BEO_n denote the probability that $\mathrm{BER}_n(\gamma_b)$ for the n-th mode modulation in the n-th SNR region ($\alpha_n \le \widehat{\gamma}_b < \alpha_{n+1}$) goes beyond BER_t. BEO_n can be formulated as

$$\begin{aligned}\mathrm{BEO}_n &= \mathrm{Prob}(\mathrm{BER}_n(\gamma_b) > BER_t | \alpha_n \le \widehat{\gamma}_b < \alpha_{n+1}) \\ &= \mathrm{Prob}(\gamma_b < \xi_n | \alpha_n \le \widehat{\gamma}_b < \alpha_{n+1})\end{aligned} \tag{17}$$

where ξ_n is the SNR in which $\mathrm{BER}_n(\xi_n) = BER_t$ and therefore, can be expressed by $\xi_n = (-1/g_n)\ln(5BER_t)$.

Using the total probability theorem, the total bit error outage occurred over all $N+1$ SNR regions, which includes a no-transmission mode (causing an outage event with a probability given by $P_0 = F_{\widehat{\gamma}_b}(\alpha_1)$) and N M-QAM modulation modes can be expressed by

$$\begin{aligned}\mathrm{BEO} &= P_0 + \sum_{n=1}^{N} \mathrm{BEO}_n\, P_n \\ &= P_0 + \sum_{n=1}^{N} \mathrm{Prob}(\gamma_b < \xi_n | \alpha_n \le \widehat{\gamma}_b < \alpha_{n+1}) \\ &\qquad \times \mathrm{Prob}(\alpha_n \le \widehat{\gamma}_b < \alpha_{n+1}) \\ &= P_0 + \sum_{n=1}^{N} \mathrm{Prob}(\gamma_b < \xi_n,\ \alpha_n \le \widehat{\gamma}_b < \alpha_{n+1}) \\ &= P_0 + \sum_{n=1}^{N} \int_{\alpha_n}^{\alpha_{n+1}} \int_0^{\xi_n} f_{\gamma_b, \widehat{\gamma}_b}(\gamma, \widehat{\gamma})\, d\gamma d\widehat{\gamma} \\ &= P_0 + \sum_{n=1}^{N} \int_{\alpha_n}^{\alpha_{n+1}} \int_0^{\xi_n} f_{\gamma_b|\widehat{\gamma}_b}(\gamma|\widehat{\gamma})\, f_{\widehat{\gamma}_b}(\widehat{\gamma})\, d\gamma d\widehat{\gamma}\end{aligned} \tag{18}$$

Using (6) it can be shown that

$$\int_0^{\xi_n} f_{\gamma_b|\widehat{\gamma}_b}(\gamma|\widehat{\gamma}) d\gamma \simeq 1 - Q_\lambda\left(\sqrt{\frac{2\mu\,\rho\,\widehat{\gamma}}{\overline{\gamma}(1-\rho)}}, \sqrt{\frac{2\mu\,\xi_n}{\overline{\gamma}(1-\rho)}}\right) \tag{19}$$

where $Q_\lambda\,(.,.)$ is the λth order marcum-Q-function, therefore, expression (18) can be written as

$$\begin{aligned}\mathrm{BEO} = {} & P_0 + \sum_{n=1}^{N} \int_{\alpha_n}^{\alpha_{n+1}} f_{\widehat{\gamma}_b}(\widehat{\gamma})\, d\widehat{\gamma} \\ & - \sum_{n=1}^{N} \int_{\alpha_n}^{\alpha_{n+1}} Q_\lambda\left(\sqrt{\frac{2\mu\,\rho\,\widehat{\gamma}}{\overline{\gamma}(1-\rho)}}, \sqrt{\frac{2\mu\,\xi_n}{\overline{\gamma}(1-\rho)}}\right) f_{\widehat{\gamma}_b}(\widehat{\gamma})\, d\widehat{\gamma} \\ = {} & 1 - \sum_{n=1}^{N} \int_{\alpha_n}^{\alpha_{n+1}} Q_\lambda\left(\sqrt{\frac{2\mu\,\rho\,\widehat{\gamma}}{\overline{\gamma}(1-\rho)}}, \sqrt{\frac{2\mu\,\xi_n}{\overline{\gamma}(1-\rho)}}\right) f_{\widehat{\gamma}_b}(\widehat{\gamma})\, d\widehat{\gamma} \\ = {} & 1 - \frac{K}{(\lambda-1)!}\left(\frac{\mu}{\overline{\gamma}}\right)^\lambda \sum_{n=1}^{N} \int_{\alpha_n}^{\alpha_{n+1}} \widehat{\gamma}^{\,\lambda-1} \exp\left(-\frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right) \\ & \times \left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)\right]^{K-1} Q_\lambda\left(\sqrt{\frac{2\mu\,\rho\,\widehat{\gamma}}{\overline{\gamma}(1-\rho)}}, \sqrt{\frac{2\mu\,\xi_n}{\overline{\gamma}(1-\rho)}}\right) d\widehat{\gamma}\end{aligned} \tag{20}$$

where the integral can be computed numerically.

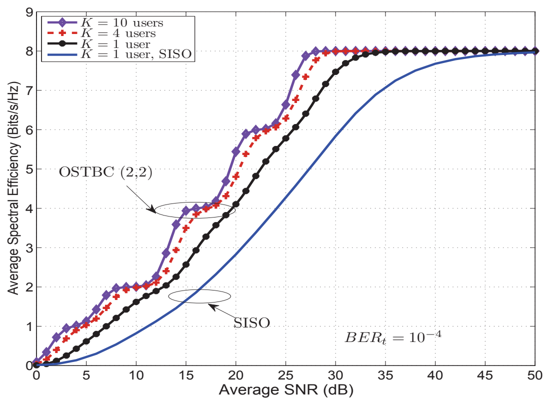

Fig. 1. Average spectral efficiency versus average SNR $\overline{\gamma} = E_s/N_0$.

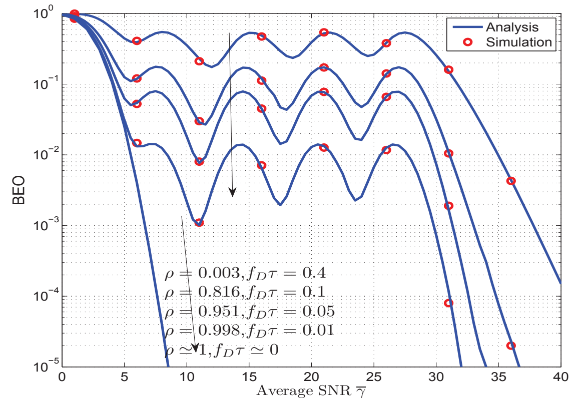

Fig. 2. Bit error outage versus average SNR $\overline{\gamma} = E_s/N_0$ for different values of $f_D\,\tau$, $K = 4$ users.

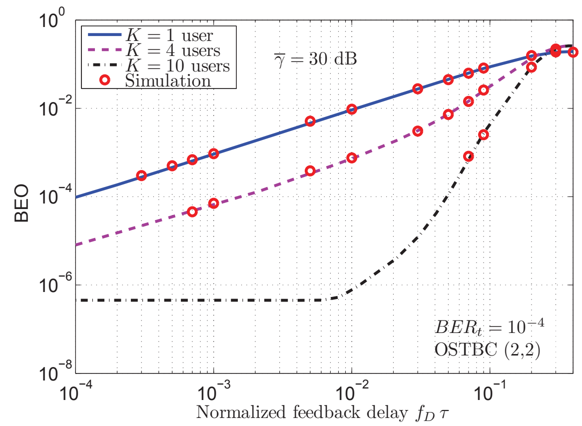

Fig. 3. Bit error outage versus Normalized feedback delay $f_D\,\tau$.

IV. Simulation and Numerical Results

In this section, we present the numerical results for discrete-rate adaptive modulation scheme. We consider the following square M-QAM modulation modes: $M_n \in \{2, 4, 16, 64, 256\}$, which corresponds to the transmission of $\beta_n \in \{1, 2, 4, 6, 8\}$ bits/symbol, $n = 1, 2, \cdots, 5$. Therefore, the modulation schemes are BPSK, and square M-QAM, ($M = 4$, 16, 64, and 256). Thus, for example, for a target BER of 10^{-4}, using (5), we obtain 5 SNR switching levels as $\alpha_n \in \{7.05, 11.82, 18.81, 25.04, 31.11\}$ dB.

The results shown in Fig.s 1 to 6 are obtained from the various analytical formulas derived in the paper, where $\overline{\gamma} = E_s/N_0$. However, in some of those Figures, Monte carlo simulation results are also provided to verify the analysis.

Fig. 1 shows the average spectral efficiency of the system, obtained from both analytical formulae and Monte carlo simulation results. It can be observed that increasing the number of users from $K = 1$ to $K = 10$, increases the average spectral efficiency. This is due to the fact that increasing the number of relays increases the cooperative diversity. As has been observed from expression (8) and from Fig. 1 the spectral efficiency of the system is independent of the coefficient ρ and hence are independent of the feedback delay τ. This is due to the fact that user selection and the constellation size of the modulation and SNR switching levels have been chosen according to the CSI knowledge at the receiver side at time t without considering the time delay at transmission time $t + \tau$. As can be seen in Fig.s 2 to 6, the insensitivity of the spectral efficiency to the feedback delay is obtained at the cost of violating the target BER when the feedback delay is relatively high. The small fluctuations in the spectral efficiency curves in Fig. 1 are due to the use of discrete-rate adaptive modulation. This fluctuating behavior occurs when the number of users and/or antennas are large. Fig.s 2 and 3 show the impact of feedback delay on the bit error outage versus the average SNR and normalized feedback delay $f_D\,\tau$, respectively, respecting a target BER of $\text{BER}_t = 10^{-4}$. The results are obtained from Monte carlo simulation as well as from the analytical formula derived earlier. We consider the delayless feedback ($\rho = 1$) case, and outdated CSI cases. It can be seen that as the feedback delay increases the bit error outage deteriorates. In fact, for the delayless feedback ($\rho = 1$) bit error outage is equal to the probability of no transmission mode which is determined by P_0, defined earlier. However, as the feedback delay increases the instantaneous BER of N transmission modes are deteriorated and may go beyond the target BER, hence causing an outage event. In the worst case, when $\rho \simeq 0$, all the modulation modes violate the target BER, i.e., $\text{BER}_n(\gamma) > BER_t \quad (n = 1, 2, \cdots, N)$ and in each SNR reagin $\alpha_n \leq \widehat{\gamma} < \alpha_{n+1}$ there exists a bit error outage.

Fig.s 4-6 show the impact of channel time variations on the average BER of the system, assuming a target BER of $\text{BER}_t = 10^{-4}$. It is shown that increasing the value of normalized feedback delay $f_D\,\tau$ may cause a significant degradation in the average BER of the system in comparison to that of delayless feedback, i.e., $f_D\,\tau = 0$. It can be seen that the average BER may become larger than $\text{BER}_t = 10^{-4}$, thus violating the target BER requirement. However, as can be seen in Fig. 4 for $\overline{\gamma} = 30$ dB, a normalized feedback delay value up to $f_D\,\tau = 0.06$, the target BER of 10^{-4} can be guaranteed. In addition, in Fig. 5 it can be observed that MIMO OSTBC system is more robust to the channel time variations (the target BER is more satisfied) compared with the SISO system. This suggests that in the systems with more channel variations in time MIMO systems are better options.

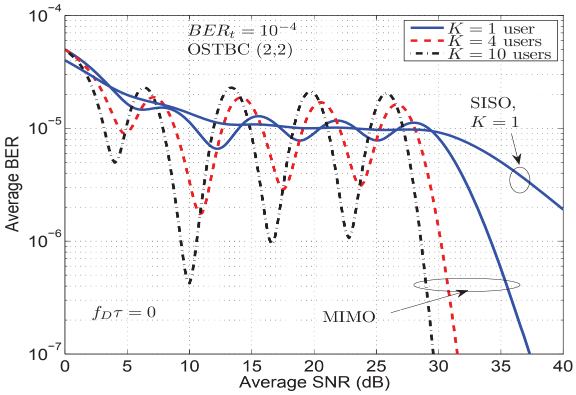

Fig. 4. Average BER versus Average SNR for delayless feedback $f_D\,\tau = 0$.

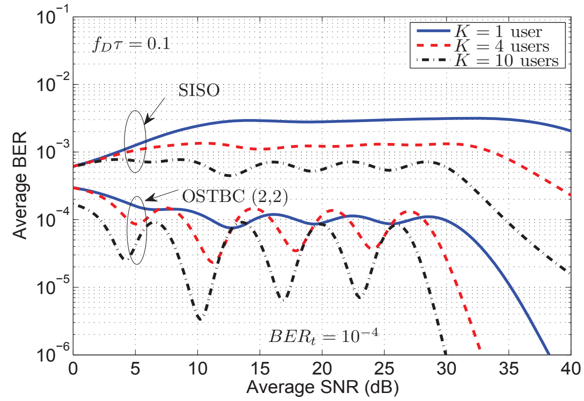

Fig. 5. Average BER versus Average SNR for the normalized feedback delay $f_D\,\tau = 0.1$.

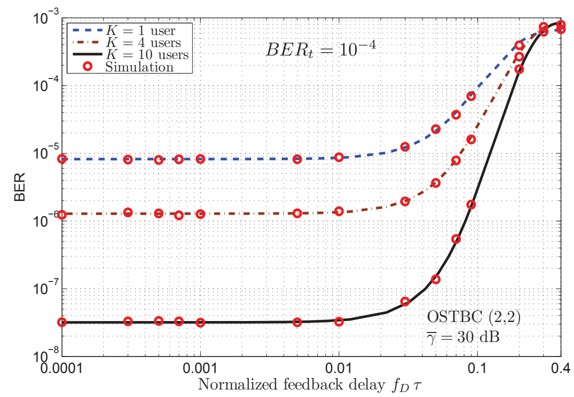

Fig. 6. Average BER versus Normalized feedback delay $f_D\,\tau$.

V. Conclusion

A performance analysis is presented evaluating the impact of channel time variation for the multiuser selection diversity systems with rate-adaptive modulation employing MIMO OSTBC over Rayleigh fading channels. Closed-form and analytical expressions are derived for the average spectral efficiency, average BER, and bit error outage of the system. Using numerical evaluation of the closed-form and analytical expressions, as well as using Monte-Carlo simulation results the impact of channel time variations of the system performance have been evaluated and different cases are compared and discussed.

Appendix

The expression $\left[\widetilde{\Gamma}\,(.,.)\right]^{K-1}$ in (4) can be written in terms of finite series expansions

$$\left[\widetilde{\Gamma}\left(\lambda, \frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)\right]^{K-1} = \left[1 - \exp\left(\frac{-\mu\widehat{\gamma}}{\overline{\gamma}}\right)\sum_{l=0}^{\lambda}\frac{1}{l!}\left(\frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)^{l}\right]^{K-1}$$
$$= \sum_{i=0}^{K-1}\sum_{t=0}^{i(\lambda-1)}\binom{K-1}{i}(-1)^{i}\,a_{i,t}\,\exp\left(\frac{-i\mu\widehat{\gamma}}{\overline{\gamma}}\right)\left(\frac{\mu\widehat{\gamma}}{\overline{\gamma}}\right)^{t} \tag{21}$$

where $a_{i,t}$ is defined in (15). Therefore, substituting (21) into the PDF expression (4), yields

$$f_{\widehat{\gamma}_b}(\widehat{\gamma}) = \frac{K}{(\lambda-1)!}\sum_{i=0}^{K-1}\sum_{t=0}^{i(\lambda-1)}\binom{K-1}{i}(-1)^{i}\,a_{i,t} \times\left(\frac{\mu}{\overline{\gamma}}\right)^{\lambda+t}\widehat{\gamma}^{\lambda+t-1}\exp\left(-\frac{(i+1)\mu\widehat{\gamma}}{\overline{\gamma}}\right). \tag{22}$$

Substituting (22) into (10), after some manipulations and solving the integral, we obtain (14) which concludes the proof.

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