Beamforming Optimization for Generalized MIMO Y Channels with Both Multiplexing and Diversity

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***Abstract*—We consider a K-user multiple-input multiple-output (MIMO) relay channel, where each user sends independent messages to the other $K-1$ users via a common relay in two time slots. All users and the relay are equipped with multiple antennas. In contrast to existing work, we consider systems with both multiplexing and diversity, where each user message contains multiple data streams and there are extra degrees of freedom to optimize the transmit beamforming matrices. We propose a novel iterative beamforming optimization algorithm based on orthogonal projection optimization with the signal subspace alignment. An optimal power allocation is also considered to maximize the system sum rate. The sum rate performance of the proposed scheme in various channel configurations is verified by simulations, which shows that the proposed scheme produces significant improvement over existing one.**

I. Introduction

Relaying technique has reemerged in recent years as a powerful approach to improve the reliability and throughput of wireless networks. Recently, a novel three-user relay channel model, referred to as the 'MIMO Y channel', has been proposed in [1]. In this new model, each of the three users, equipped with multiple antennas, intends to convey independent messages to the other two users via a common multi-antenna relay node. The exchange of a total of six messages all happens in two time slots, multiple access (MA) and broadcast (BC). In [2] the authors generalized the model to any number of K users ($K \geq 3$).

The principles of interference alignment [3] and physical layer network coding [4] were applied in the MIMO Y channel, and the authors showed that given enough antennas at the relay, the multiplexing gain of a MIMO Y channel is $3n_S$ where n_S is the number of antennas at each user. Although the authors pointed out that signal subspace alignment and interference nulling beamforming should be used in MA and BC stages, respectively, they did not address the optimal design of the beamforming vectors in MIMO Y channels. Furthermore, information rate was the main focus of [1], [2], where the authors did not address receiver signal processing and assumed ideal reception at both relay and users. So far, the only work in the open literature that addressed the beamforming design of MIMO Y channels appeared in [5]. The authors proposed a random beamforming algorithm, where the beamforming vectors are randomly generated within the intersection subspace of the corresponding pair of channels. However, due to its random nature, for any given channel realization, the beamforming design could be far from optimal. Also, [5] only considered single data stream per user message, which limits the multiplexing gain if the number of source antennas increases.

In this paper, we consider a generalized K-user MIMO-Y channel with both multiplexing and diversity. Multiple data streams per user message are allowed and we propose a deterministic beamforming design aimed at maximizing the effective signal to noise ratios (SNRs) for any given channel realization. After beamforming is designed, an optimal power allocation scheme is proposed to maximize the sum rate. Monte Carlo simulation results are provided and compared with the random beamforming results from [5]. The proposed scheme shows a significant improvement over the existing one.

Notations: We use lowercase normal letters to represent scalars and lower/upper-case boldface letters to represent vectors/matrices. $(\cdot)^T$ and $(\cdot)^H$ denote vector/matrix transpose and Hermitian transpose, respectively. $(\cdot)^\dagger$ denotes the Moore-Penrose pseudoinverse. n_i represents the i-th element of a vector $\mathbf{n}$; $[\mathbf{A}]_{i,j}$ represents the element at the i-th row and the j-th column of $\mathbf{A}$; $\mathbf{a}_i$ represents the i-th column of $\mathbf{A}$. $\mathcal{E}[\cdot]$ stands for expectation. $|\mathbf{x}|$ means 2-norm of vector $\mathbf{x}$. ran$(\mathbf{H})$ denotes the range (column space) of matrix $\mathbf{H}$. $\langle \mathbf{x} \rangle$ and $\langle \mathbf{A} \rangle$ denote normalization operation on vector $\mathbf{x}$ and the columns of $\mathbf{A}$, respectively, i.e. $\langle \mathbf{x} \rangle = \frac{\mathbf{x}}{|\mathbf{x}|}$, $\langle \mathbf{A} \rangle = [\langle \mathbf{a}_1 \rangle, \cdots, \langle \mathbf{a}_n \rangle]$. We define an equality $\mathbf{A} \rightleftharpoons \mathbf{B}$ that means $\langle \mathbf{A} \rangle = \langle \mathbf{B} \rangle$.

II. System Model

We consider a generalized MIMO Y channel model [2], [5], where K source nodes/users, $U_1, \cdots, U_K$ ($K \geq 3$), each equipped with n_S antennas, communicate with each other via the help of a relay station (RS), which has n_R antennas. Each user has $K-1$ independent messages to be sent to the other $K-1$ users and expects to receive $K-1$ independent messages from the other users. Assuming an uncorrelated flat fading channel, the channel coefficients between user i and the relay are collected in an $n_R \times n_S$ matrix $\mathbf{H}_i$ ($i = 1, \cdots, K$). It is also assumed that full channel state information (CSI) is known to all nodes. In Fig. 1 we exemplify the system model with $K = 3$. The exchange of $K(K-1)$ messages happens in two time slots: the MA stage and the BC stage. In the MA stage, all K users transmit their messages to the relay using properly designed beamforming. The relay receives these messages and, after certain processing (receive

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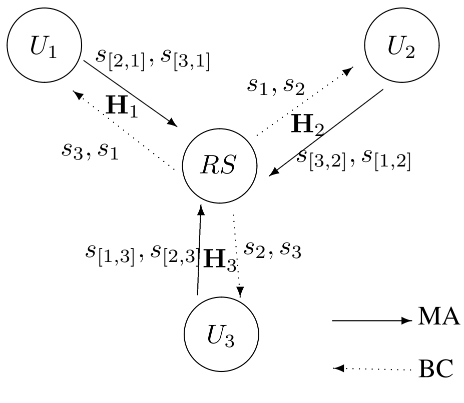

Fig. 1. System model for $K = 3$

combining, network coding, etc.), broadcasts them to the users in the BC stage. In this paper, we focus on the MA stage, as it is illustrated in [5] that the same transmit beamforming and receive combining vectors from the MA stage can be readily used as the receive combining and transmit beamforming vectors, respectively, in the BC stage.

A. Signal subspace alignment

In contrast to existing work [2], [5], where only single data stream per message is assumed, we consider the generalized scenario where each message contains m data streams[1]. We denote the modulated signal carrying the message that user i sends to user j as an $m \times 1$ vector $\mathbf{s}_{[j,i]}$ and assume normalized constellation and independent data streams such that $\mathcal{E}[\mathbf{s}_{[j,i]}\mathbf{s}_{[j,i]}^H] = \mathbf{I}$. User i sends all $K-1$ vector signals $\{\mathbf{s}_{[j,i]}\}_{j\neq i}$ simultaneously from its n_S antennas by transmitting a signal vector $\mathbf{x}_i$ as follows.

$$\mathbf{x}_i = \sum_{j=1, j\neq i}^{K} \mathbf{V}_{[j,i]}\mathbf{\Phi}_{[j,i]}\mathbf{s}_{[j,i]}, \tag{1}$$

where $\mathbf{\Phi}_{[j,i]}$ is an $m \times m$ diagonal matrix with power scaling factors $\{\sqrt{\kappa_{[j,i]l}}\}_{l=1}^m$ as its diagonal elements, i.e. $\mathbf{\Phi}_{[j,i]} = \text{diag}[\sqrt{\kappa_{[j,i]1}}, \cdots, \sqrt{\kappa_{[j,i]m}}]$; and $\mathbf{V}_{[j,i]}$ is an $n_S \times m$ transmit beamforming matrix, associated with $\mathbf{s}_{[j,i]}$. Without loss of generality, we assume that the columns of $\mathbf{V}_{[j,i]}$ are normalized, i.e. $|\mathbf{v}_{[j,i]l}| = 1, l = 1, \cdots, m$. Since a total of $(K-1)m$ data streams are transmitted by user i, Eq. (1) implies a requirement on the number of source antennas $n_S \geq (K-1)m$.

The received signal by the relay at the MA stage is

$$\mathbf{r} = \sum_{i=1}^{K} \mathbf{H}_i\mathbf{x}_i + \boldsymbol{\nu}_R, \tag{2}$$

where $\boldsymbol{\nu}_R$ is the additive white Gaussian noise (AWGN) vector with variance σ_ν^2. Without loss of generality, we normalize all noise variances to be unity, $\sigma_\nu^2 = 1$.

[1]In fact, the number of data streams m can be different for different users. We use equal m only for notation simplification. Extension of the algorithm proposed hereafter to different m values is straightforward. It is the same case for the number of source antennas n_S.

Substituting (1) into (2), we get

$$\mathbf{r} = \sum_{i=1}^{K} \sum_{j=1, j\neq i}^{K} \mathbf{H}_i\mathbf{V}_{[j,i]}\mathbf{\Phi}_{[j,i]}\mathbf{s}_{[j,i]} + \boldsymbol{\nu}_R. \tag{3}$$

We pair the reciprocal messages $\mathbf{s}_{[j,i]}$ and $\mathbf{s}_{[i,j]}$ ($\forall i < j$). Then (3) contains a total of $n_K \triangleq K(K-1)/2$ message pairs. We use k to index those pairs ($k = 1, \cdots, n_K$), and define a one-to-one index mapping function $k = \pi(i,j), i < j$. Based on this pairing and index mapping, Eq. (3) can be rewritten as

$$\mathbf{r} = \sum_{k=1}^{n_K} \left(\mathbf{H}_i\mathbf{V}_{[j,i]}\mathbf{\Phi}_{[j,i]}\mathbf{s}_{[j,i]} + \mathbf{H}_j\mathbf{V}_{[i,j]}\mathbf{\Phi}_{[i,j]}\mathbf{s}_{[i,j]}\right) + \boldsymbol{\nu}_R. \tag{4}$$

The basic idea of signal subspace alignment is to design the beamforming matrices such that the two vector signal components within each pair are aligned, i.e. $\text{ran}(\mathbf{H}_i\mathbf{V}_{[j,i]}) = \text{ran}(\mathbf{H}_j\mathbf{V}_{[i,j]}), \forall i < j$.

Lemma 1: As long as $\min(2n_S - n_R, n_R) \geq m$, the signal subspace alignment is achievable.

Proof: The intersection subspace $\text{ran}(\mathbf{H}_i) \cap \text{ran}(\mathbf{H}_j)$ has a dimension of $n_I \triangleq \min(2n_S - n_R, n_R)$ [1]. Assume an orthonormal basis of $\text{ran}(\mathbf{H}_i) \cap \text{ran}(\mathbf{H}_j)$ is $\{\mathbf{b}_{k1}, \cdots, \mathbf{b}_{kn_I}\}$. We define $\mathbf{B}_k \triangleq [\mathbf{b}_{k1}, \cdots, \mathbf{b}_{kn_I}]$. Then any unit-norm vector in the intersection can be expressed as $\mathbf{B}_k\mathbf{c}_{kl}$, where $\mathbf{c}_{kl}$ is an $n_I \times 1$ unit-norm vector. If $n_I \geq m$, we can always find m independent vectors, $\mathbf{c}_{k1}, \cdots, \mathbf{c}_{km}$, such that $\text{ran}(\mathbf{B}_k\mathbf{C}_k) \subseteq \text{ran}(\mathbf{H}_i) \cap \text{ran}(\mathbf{H}_j)$, where $\mathbf{C}_k \triangleq [\mathbf{c}_{k1}, \cdots, \mathbf{c}_{km}]$. Now we set

$$\mathbf{V}_{[j,i]} = \langle \mathbf{H}_i^\dagger \mathbf{B}_k\mathbf{C}_k \rangle, \ \mathbf{V}_{[i,j]} = \langle \mathbf{H}_j^\dagger \mathbf{B}_k\mathbf{C}_k \rangle. \tag{5}$$

Using (5) we obtain

$$\mathbf{H}_i\mathbf{V}_{[j,i]} \rightleftharpoons \mathbf{H}_i\mathbf{H}_i^\dagger\mathbf{B}_k\mathbf{C}_k = \mathbf{B}_k\mathbf{C}_k, \tag{6}$$

where we use the fact that $\mathbf{H}_i\mathbf{H}_i^\dagger\mathbf{B}_k = \mathbf{B}_k$, since $\mathbf{H}_i\mathbf{H}_i^\dagger$ is an orthogonal projector onto $\text{ran}(\mathbf{H}_i)$ and $\mathbf{B}_k$ contains the basis of $\text{ran}(\mathbf{H}_i) \cap \text{ran}(\mathbf{H}_j)$. Similarly, we can derive

$$\mathbf{H}_j\mathbf{V}_{[i,j]} \rightleftharpoons \mathbf{B}_k\mathbf{C}_k. \tag{7}$$

Therefore, the signal subspace alignment is achieved. ■

According to (6) and (7), with a proper power allocation, we can write the aligned matrices as

$$\mathbf{H}_i\mathbf{V}_{[j,i]}\mathbf{\Phi}_{[j,i]} = \mathbf{H}_j\mathbf{V}_{[i,j]}\mathbf{\Phi}_{[i,j]} \triangleq \mathbf{U}_k, \forall k = \pi(i,j). \tag{8}$$

Using (8), Eq. (4) becomes

$$\begin{aligned}\mathbf{r} &= \sum_{k=1}^{n_K} \mathbf{U}_k(\mathbf{s}_{[j,i]} + \mathbf{s}_{[i,j]}) + \boldsymbol{\nu}_R \\ &= \sum_{k=1}^{n_K} \mathbf{U}_k\mathbf{s}_k + \boldsymbol{\nu}_R = \mathbf{U}\mathbf{s} + \boldsymbol{\nu}_R,\end{aligned} \tag{9}$$

where $\mathbf{s}_k \triangleq \mathbf{s}_{[j,i]} + \mathbf{s}_{[i,j]}$ is the physical layer network coded symbol vector [4], $\mathbf{U} \triangleq [\mathbf{U}_1, \cdots, \mathbf{U}_{n_K}]$ and $\mathbf{s} \triangleq [\mathbf{s}_1^T, \cdots, \mathbf{s}_{n_K}^T]^T$. Eq. (9) shows an equivalent MIMO channel with $n_K m$ transmit antennas and n_R receive antennas, where the channel input is $\mathbf{s}$ and channel matrix is $\mathbf{U}$. The MIMO principle tells that as long as $n_R \geq n_K m$, the relay can decode $\mathbf{s}$.

In summary, in the generalized MIMO Y channel, there are three requirements on the numbers of source and relay antennas: 1) $n_S \geq (K-1)m$; 2) $n_I \geq m$; 3) $n_R \geq n_K m$. The first requirement is for the sources to transmit all data streams; the second one is for signal subspace alignment; and the third one is for the relay to decode all composite messages.

B. Systems with diversity

If $n_I = m$, the intersection subspace is only big enough to accommodate the message pair with m composite data streams. In this case, $\mathbf{C}_k$ is a full rank square matrix and

$$\text{ran}(\mathbf{B}_k\mathbf{C}_k) = \text{ran}(\mathbf{B}_k) = \text{ran}(\mathbf{H}_i) \cap \text{ran}(\mathbf{H}_j), \quad (10)$$

which means that the signal subspace for message pair k is uniquely determined by $\mathbf{H}_i$ and $\mathbf{H}_j$. There is no degree of freedom in the signal subspace optimization. This kind of systems are multiplexing only without diversity, whose error performance is usually very poor [5].

In contrast, if $n_I > m$, the intersection has more dimensions than needed to accommodate the m data streams in a message pair. So there are extra degrees of freedom in the signal subspace optimization. In other words, $\mathbf{C}_k$ is a free variable whose choice determines the m-dimensional subspace within the n_I-dimensional intersection subspace of each message pair. We will see in Section IV that proper signal subspace optimization plays an important role in ensuring a good performance. However, the job of optimizing $\{\mathbf{C}_k\}$ is not easy due to the coupling both between message pairs and within each pair.

In the following, we propose a deterministic design of the beamforming vectors/matrices as well as power allocation scheme for multiple data stream scenarios, where we try to maximize the effective SNRs for each channel realization.

III. Beamforming Optimization and Power Allocation

A. Beamforming optimization

To decode the network coded symbol vector $\mathbf{s}_k$ in (9), the relay applies an $n_R \times m$ combining matrix $\mathbf{W}_k$ to $\mathbf{r}$.

$$\mathbf{W}_k^H\mathbf{r} = \mathbf{W}_k^H\mathbf{U}_k\mathbf{s}_k + \sum_{k' \neq k} \mathbf{W}_k^H\mathbf{U}_{k'}\mathbf{s}_{k'} + \mathbf{W}_k^H\boldsymbol{\nu}_R, \quad (11)$$

where $\mathbf{W}_k$ is designed according to the following two zero-forcing (ZF) criteria.

$$\mathbf{W}_k^H\mathbf{U}_{k'} = \mathbf{0}, \forall k' \neq k \quad (12)$$

$$\mathbf{W}_k^H\mathbf{U}_k \rightleftharpoons \mathbf{I}. \quad (13)$$

Without loss of generality, we assume that the columns of $\mathbf{W}_k$ are normalized, i.e. $|\mathbf{w}_{kl}| = 1, l = 1, \cdots, m$. Thus,

$$\mathcal{E}\left[(\mathbf{W}_k^H\boldsymbol{\nu}_R)(\mathbf{W}_k^H\boldsymbol{\nu}_R)^H\right] = \mathbf{W}_k^H\mathbf{W}_k \quad (14)$$

has its diagonal elements to be all ones.

Eq. (11) contains m data streams. For the moment, for message pair k, we only consider the l-th data stream s_{kl} while assuming the other data streams do not exist. i.e. $s_{kl'} = 0, \forall l' \neq l$. Note that we still allow for multiple data streams for the other message pairs $k' \neq k$.

Under this assumption, Eq. (11) reduces to

$$\mathbf{w}_{kl}^H\mathbf{r} = \mathbf{w}_{kl}^H\mathbf{u}_{kl}s_{kl} + \sum_{k'=1,k'\neq k}^{n_K} \mathbf{w}_{kl}^H\mathbf{U}_{k'}\mathbf{s}_{k'} + \mathbf{w}_{kl}^H\boldsymbol{\nu}_R, (15)$$

where $\mathbf{w}_{kl}$ should be designed to maximize the effective channel gain $\lambda_{kl} \triangleq |\mathbf{w}_{kl}^H\mathbf{u}_{kl}|^2$ while satisfying the ZF criterion $\mathbf{w}_{kl}^H\mathbf{U}_{k'} = \mathbf{0}, \forall k' \neq k$. Given $\{\mathbf{U}_k\}$, we can derive that the optimal combining vector is [6]

$$\mathbf{w}_{kl} = \langle \mathbf{M}_k\mathbf{u}_{kl} \rangle, \quad (16)$$

and the maximum effective channel gain is

$$\lambda_{kl} = \mathbf{u}_{kl}^H\mathbf{M}_k\mathbf{u}_{kl}, \quad (17)$$

where

$$\mathbf{M}_k = \mathbf{I} - \mathbf{G}_k(\mathbf{G}_k^H\mathbf{G}_k)^{-1}\mathbf{G}_k^H \quad (18)$$

$$\mathbf{G}_k = \left[\langle \mathbf{U}_1 \rangle \cdots \langle \mathbf{U}_{k-1} \rangle \, \langle \mathbf{U}_{k+1} \rangle \cdots \langle \mathbf{U}_{n_K} \rangle\right]. \quad (19)$$

With the optimal $\mathbf{w}_{kl}$, Eq. (15) becomes

$$\mathbf{w}_{kl}^H\mathbf{r} = \sqrt{\lambda_{kl}}s_{kl} + \mathbf{w}_{kl}^H\boldsymbol{\nu}_R, \quad (20)$$

and the effective SNR for data stream l of message pair k is

$$\gamma_{kl} = 2\lambda_{kl}/\sigma_\nu^2, \; k = 1, \cdots, n_K, \quad (21)$$

where the factor 2 comes from the power of the network coded symbol s_{kl}. According to (8)

$$\mathbf{u}_{kl} = \mathbf{H}_i\mathbf{v}_{[j,i]l}\sqrt{\kappa_{[j,i]l}} = \mathbf{H}_j\mathbf{v}_{[i,j]l}\sqrt{\kappa_{[i,j]l}}. \quad (22)$$

Thus, Eq. (17) becomes

$$\begin{aligned} \lambda_{kl} &= \kappa_{[j,i]l}\mathbf{v}_{[j,i]l}^H\mathbf{H}_i\mathbf{M}_k\mathbf{H}_i\mathbf{v}_{[j,i]l} \\ &= \kappa_{[i,j]l}\mathbf{v}_{[i,j]l}^H\mathbf{H}_j\mathbf{M}_k\mathbf{H}_j\mathbf{v}_{[i,j]l}. \end{aligned} \quad (23)$$

We define the net effective channel gains

$$\lambda_{[j,i]l} \triangleq \mathbf{v}_{[j,i]l}^H\mathbf{H}_i^H\mathbf{M}_k\mathbf{H}_i\mathbf{v}_{[j,i]l} \quad (24)$$

$$\lambda_{[i,j]l} \triangleq \mathbf{v}_{[i,j]l}^H\mathbf{H}_j^H\mathbf{M}_k\mathbf{H}_j\mathbf{v}_{[i,j]l}, \quad (25)$$

which represent the effective channel gains without power scaling. On the one hand, we want to maximize the net gains (24) and (25). On the other hand, we do not want to see the two net gains differing too much. Otherwise, the power will be largely wasted in compensating the gain difference as in (23) rather than improving the system performance. So we formulate the following subproblem for message pair $k = \pi(i,j)$.

$$\underset{\mathbf{v}_{[j,i]l}, \mathbf{v}_{[i,j]l}}{\text{maximize}} \left(\lambda_{[j,i]l}^{-1} + \lambda_{[i,j]l}^{-1}\right)^{-1} \quad (26)$$

$$\text{subject to } \mathbf{u}_{kl} \rightleftharpoons \mathbf{H}_i\mathbf{v}_{[j,i]l} \rightleftharpoons \mathbf{H}_j\mathbf{v}_{[i,j]l} \quad (27)$$

$$|\mathbf{v}_{[j,i]l}| = |\mathbf{v}_{[i,j]l}| = 1. \quad (28)$$

Note that the objective function (26) is the harmonic mean of the two net channel gains. This is to achieve balanced net gains. According to (5)

$$\mathbf{v}_{[j,i]l} = \langle \mathbf{H}_i^\dagger \mathbf{B}_k \mathbf{c}_{kl} \rangle \tag{29}$$

$$\mathbf{v}_{[i,j]l} = \langle \mathbf{H}_j^\dagger \mathbf{B}_k \mathbf{c}_{kl} \rangle^2. \tag{30}$$

Using (29) and (30) in (24) and (25), respectively, we get

$$\begin{aligned}\lambda_{[j,i]l} &= \frac{\mathbf{c}_{kl}^H (\mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{B}_k)^H \mathbf{M}_k (\mathbf{H}_i \mathbf{H}_i^\dagger \mathbf{B}_k) \mathbf{c}_{kl}}{|\mathbf{H}_i^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2} \\ &= \frac{\mathbf{c}_{kl}^H \mathbf{B}_k^H \mathbf{M}_k \mathbf{B}_k \mathbf{c}_{kl}}{|\mathbf{H}_i^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2} \end{aligned} \tag{31}$$

$$\lambda_{[i,j]l} = \frac{\mathbf{c}_{kl}^H \mathbf{B}_k^H \mathbf{M}_k \mathbf{B}_k \mathbf{c}_{kl}}{|\mathbf{H}_j^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2}. \tag{32}$$

Thus, the objective function (26) can be written as

$$\begin{aligned}\left(\lambda_{[j,i]l}^{-1} + \lambda_{[i,j]l}^{-1}\right)^{-1} &= \left(\frac{|\mathbf{H}_i^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2 + |\mathbf{H}_j^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2}{\mathbf{c}_{kl}^H \mathbf{B}_k^H \mathbf{M}_k \mathbf{B}_k \mathbf{c}_{kl}}\right)^{-1} \\ &= \frac{\mathbf{c}_{kl}^H \mathbf{E}_k \mathbf{c}_{kl}}{\mathbf{c}_{kl}^H \mathbf{F}_k \mathbf{c}_{kl}}, \end{aligned} \tag{33}$$

where we define

$$\mathbf{E}_k \triangleq \mathbf{B}_k^H \mathbf{M}_k \mathbf{B}_k \tag{34}$$

$$\mathbf{F}_k \triangleq \mathbf{B}_k^H (\mathbf{H}_i^{\dagger H} \mathbf{H}_i^\dagger + \mathbf{H}_j^{\dagger H} \mathbf{H}_j^\dagger) \mathbf{B}_k. \tag{35}$$

The optimization (26)-(28) reduces to

$$\underset{\mathbf{c}_{kl}}{\text{maximize}} \ \frac{\mathbf{c}_{kl}^H \mathbf{E}_k \mathbf{c}_{kl}}{\mathbf{c}_{kl}^H \mathbf{F}_k \mathbf{c}_{kl}} \tag{36}$$

$$\text{subject to } |\mathbf{c}_{kl}| = 1. \tag{37}$$

This is a generalized eigenvalue problem. The optimal $\mathbf{c}_{kl}$ should be the generalized eigenvector corresponding to the largest generalized eigenvalue of $(\mathbf{E}_k, \mathbf{F}_k)$. Once $\mathbf{c}_{kl}$ is obtained, we can calculate $\mathbf{v}_{[j,i]l}$ and $\mathbf{v}_{[i,j]l}$ according to (29)-(30). Note that because $\mathbf{E}_k$ depends on $\mathbf{M}_k$, which is a function of all other users' beamforming matrices, the proposed algorithm should be performed iteratively from user to user.

Now we are ready to consider multiple data streams for message pair k. We denote the first m generalized eigenvectors corresponding to the largest m generalized eigenvalues of $(\mathbf{E}_k, \mathbf{F}_k)$ as $\mathbf{c}_{k1}, \cdots, \mathbf{c}_{km}$, then we set

$$\mathbf{C}_k = [\mathbf{c}_{k1}, \cdots, \mathbf{c}_{km}], \tag{38}$$

and set the relay combining matrix as

$$\mathbf{W}_k = \langle \mathbf{M}_k \mathbf{B}_k \mathbf{C}_k \rangle. \tag{39}$$

We show that such chosen $\mathbf{C}_k$ and $\mathbf{W}_k$ meet the ZF criteria (12) and (13).

Firstly, we consider two different message pairs $k \neq k'$. According to (6)-(8)

$$\mathbf{U}_k \rightleftharpoons \mathbf{B}_k \mathbf{C}_k \rightleftharpoons \mathbf{H}_i \mathbf{V}_{[j,i]} \rightleftharpoons \mathbf{H}_j \mathbf{V}_{[i,j]}, \forall k = \pi(i,j). \tag{40}$$

[2]If $n_S > n_R$, the choice of $\mathbf{v}_{[j,i]l}$ and $\mathbf{v}_{[i,j]l}$ is not unique, but pseudoinverse results in minimum transmit power.

Using (39) and (40), we have

$$\mathbf{W}_k^H \mathbf{U}_{k'} \rightleftharpoons \mathbf{C}_k^H \mathbf{B}_k^H \mathbf{M}_k \mathbf{U}_{k'} \rightleftharpoons \mathbf{U}_k^H \mathbf{M}_k \mathbf{U}_{k'}. \tag{41}$$

According to the definition of $\mathbf{M}_k$ in (18)-(19), we know

$$\mathbf{M}_k \mathbf{U}_{k'} = \mathbf{0}, \forall k \neq k'. \tag{42}$$

Therefore,

$$\mathbf{W}_k^H \mathbf{U}_{k'} = \mathbf{0}, \forall k \neq k', \tag{43}$$

which meets the condition (12).

Secondly, within a message pair k, using (39) and (40)

$$\mathbf{W}_k^H \mathbf{U}_k \rightleftharpoons \mathbf{C}_k^H \mathbf{B}_k^H \mathbf{M}_k \mathbf{B}_k \mathbf{C}_k = \mathbf{C}_k^H \mathbf{E}_k \mathbf{C}_k. \tag{44}$$

Since $\mathbf{E}_k$ and $\mathbf{F}_k$ are Hermitian and $\mathbf{F}_k$ is positive definite and $\mathbf{C}_k$ is the collection of the first m generalized eigenvectors according to (38), both $\mathbf{C}_k^H \mathbf{E}_k \mathbf{C}_k$ and $\mathbf{C}_k^H \mathbf{F}_k \mathbf{C}_k$ are diagonal matrices. Therefore, $\mathbf{W}_k^H \mathbf{U}_k$ is diagonal, which meets condition (13).

Now we determine the net gains for each data stream. According to (40)

$$\mathbf{W}_k^H \mathbf{U}_k \rightleftharpoons \mathbf{W}_k^H \mathbf{H}_i \mathbf{V}_{[j,i]} \rightleftharpoons \mathbf{W}_k^H \mathbf{H}_j \mathbf{V}_{[i,j]}, \tag{45}$$

which are all diagonal matrices. So using (5) we can derive the net effective channel gains for the l-th data stream of message pair k as

$$\lambda_{[j,i]l} = \left[\mathbf{W}_k^H \mathbf{H}_i \mathbf{V}_{[j,i]}\right]_{l,l} = \frac{\mathbf{c}_{kl}^H \mathbf{E}_k \mathbf{c}_{kl}}{|\mathbf{H}_i^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2} \tag{46}$$

$$\lambda_{[i,j]l} = \left[\mathbf{W}_k^H \mathbf{H}_j \mathbf{V}_{[i,j]}\right]_{l,l} = \frac{\mathbf{c}_{kl}^H \mathbf{E}_k \mathbf{c}_{kl}}{|\mathbf{H}_j^\dagger \mathbf{B}_k \mathbf{c}_{kl}|^2}. \tag{47}$$

It is interesting to note that the net gains (46) and (47) have exactly the same form as (31) and (32) as if there is only one data stream per message pair. This is attributed to the particular selection of $\mathbf{C}_k$ according to (38).

B. Power allocation

Once beamforming is determined, we can optimize the power scaling factors $\{\mathbf{\Phi}_{[j,i]}\}$ such that: 1) the difference of the two net gains (46) and (47) within each pair is compensated; 2) certain system performance is optimized, which depends on specific system design. As an example, we consider the maximization of sum rate of all data streams under per-source power constraint. The sum rate of a single-antenna two-way relay channel given balanced channel gain γ is $2\log_2(1+\gamma)$ [4]. Thus, we can formulate the power allocation problem as

$$\underset{\{\kappa_{[j,i]l}\}_{i\neq j}}{\text{maximize}} \sum_{k=1}^{n_K} \sum_{l=1}^{m} 2\log_2(1+\lambda_{kl}) \tag{48}$$

$$\text{subject to } \lambda_{kl} = \kappa_{[j,i]l}\lambda_{[j,i]l} = \kappa_{[i,j]l}\lambda_{[i,j]l}, \quad \forall i<j, \ l=1,\cdots,m \tag{49}$$

$$\sum_{j=1, j\neq i}^{K} \sum_{l=1}^{m} \kappa_{[j,i]l} \leq P_i, \ i=1,\cdots,K, \tag{50}$$

where P_i is the transmit power limit for user i. Using (49), we substitute $\kappa_{[j,i]l}$ for $\kappa_{[i,j]l}$ for all $i < j$. Then, we arrive at

$$\underset{\{\kappa_{[j,i]l}\}_{i<j}}{\text{maximize}} \sum_{k=1}^{n_K}\sum_{l=1}^{m} 2\log_2(1+\kappa_{[j,i]l}\lambda_{[j,i]l}) \quad (51)$$

$$\text{subject to} \sum_{j=1}^{i-1}\sum_{l=1}^{m}\frac{\lambda_{[i,j]l}}{\lambda_{[j,i]l}}\kappa_{[i,j]l} + \sum_{j=i+1}^{K}\sum_{l=1}^{m}\kappa_{[j,i]l} \le P_i$$
$$i = 1, \cdots, K. \quad (52)$$

It is easy to verify that this is a convex optimization problem, which can be efficiently solved by the interior point method.

IV. Numerical results

In this section, we present simulation results for the proposed scheme in various system configurations and make comparison with the random beamforming results from [5]. A system configuration is represented by a quadruplet (K, m, n_R, n_S). We assume the power constraint on each source is the same, i.e. $P_i = P_S/K$, where P_S is the maximum total transmit power from all sources. The SNR in the figures is defined as P_S/σ_ν^2. We choose the bit-wise XOR [5] as the network coding protocol for the purpose of fair comparison.

Firstly, we present in Fig. 2 the sum rate results for the $(3, 2, 6, 5)$ channel configuration with up to eight iterations. It should be noted that we initialize the iterative beamforming algorithm by a random matrix generated in the intersection subspace, thus the curve of iteration 0 in Fig. 2 is in fact the random beamforming algorithm proposed in [5]. We can

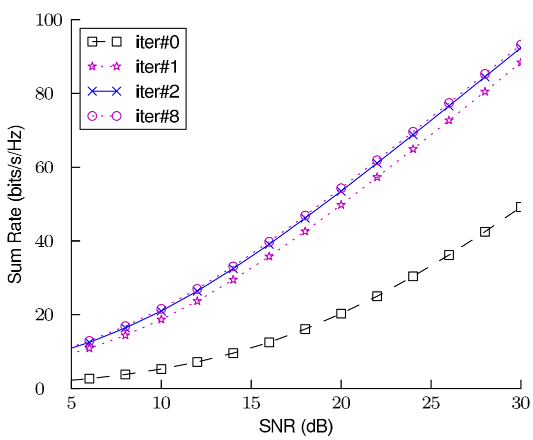

Fig. 2. Sum rate: $(K, m, n_R, n_S) = (3, 2, 6, 5)$

clearly see that the proposed iterative algorithm improves the performance significantly. Even with only one iteration, the SNR is improved by about 10 dB at sum rate of 40 bits/s/Hz. The iteration gain reduces quickly after two iterations. It is easy to verify in Fig. 2 that a multiplexing gain of $n_K m = 12$ is achieved by the proposed scheme.

In Fig. 3, we consider four channel configurations with $K = 3$ or 4, $m = 2$ and various antenna combinations. Note that all of the simulated channel configurations have more source antennas than the minimum required. For instance, for the (3,2,6,5) channel, the minimum source antenna number is $(m + n_R)/2 = 4$ and the actual antenna number is $n_S = 5$. Thus, there are extra degrees of freedom, with which beamforming optimization can be done to maximize system performance. For comparison, the results for the random beamforming algorithm [5] are also included in Fig. 3. It is obvious that the proposed scheme outperforms the random beamforming one significantly.

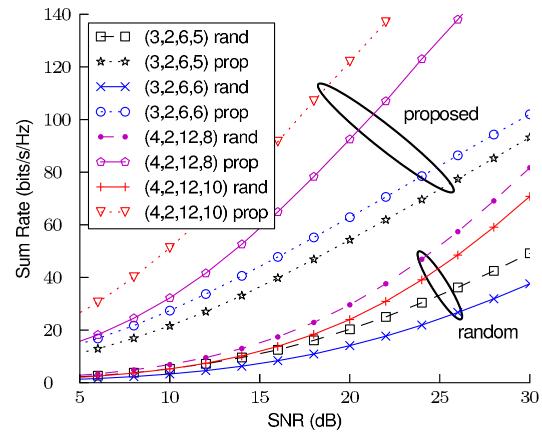

Fig. 3. Sum rate: $K = 3$ or 4, $m = 2$

V. Conclusion

We have proposed a novel iterative beamforming optimization algorithm and a power allocation scheme for the MA stage of a K-user MIMO Y channel. Multiple data streams per user message are considered and extra degrees of freedom are available so that beamforming matrices can be optimized to improve the system performance. The beamforming optimization algorithm maximizes the net effective channel gains while maintaining a gain balance among user messages. Simulation results have been presented for various channel configurations. Compared to the existing random beamforming algorithm [5], our proposed scheme produces superior performance gains in all channel configurations.

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