# A Minimum Bit Error-Rate Detector for Amplify and Forward Relaying Systems

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***Abstract*—In this paper, a new detector is being proposed for amplify-and-forward (AF) relaying system when communicating with the assistance of $L$ number of relays. The major goal of this detector is to improve the bit error rate (BER) performance of the system. The complexity of the system is further reduced by implementing this detector adaptively. The proposed detector is free from channel estimation. Our results demonstrate that the proposed detector is capable of achieving a gain of more than 1-dB at a BER of $10^{-5}$ as compared to the conventional minimum mean square error detector when communicating over Rayleigh fading channel.**

## I. Introduction

With the advancement in telecommunications, quality of service (QoS) has become a very important criteria for wireless systems [1–3]. The wireless systems usually suffer from fading which effects this desired QoS [1]. An easy approach to combat fading is to transmit multiple copies of the desired signal, which can be achieved with the assistance of any diversity technique such as spatial, time, frequency or code [1, 2]. However, due to the small size of mobile equipments, it is difficult to deploy multiple antennas at the mobile terminals [1]. Cooperative communications assist in forming a distributed network where these communication devices are able to share their transmit antennas, enabling these devices to achieve spatial diversity at the mobile terminals [3, 4]. This spatial diversity can also be combined with other forms of diversities to further improve the QoS of the desired system [3].

One of the key parameters to determine QoS in a wireless system is the bit error rate (BER) performance. It is a well known fact that maximum likelihood (ML) detection is the optimal scheme in terms of BER performance [5]. However, due to the high complexity of ML detectors, sub-optimal linear equalizers are required to be considered [5–7]. In sub-optimal linear detectors, minimum mean square error (MMSE) detection is preferred over decorrelating and the matched filter, due to its improved BER performance [6]. Furthermore, the complexity of the MMSE assisted detector can be easily reduced with the help of adaptive techniques [6]. The MMSE detector minimizes the mean square error (MSE) between the transmitted signal and the detected signal. However, in communication systems it is the BER performance which needs to be minimized rather then the MSE. In [8, 9] it has been shown that minimizing the MSE does not necessarily minimize the BER performance of the system. Therefore, a BER detector is required for a cooperative communication environment. This BER detector can be easily implemented adaptively, like the MMSE detectors, so as to reduce its computational complexity.

Low-complexity cooperative diversity protocols have been developed and analyzed for cooperative communications in different operating conditions and environments. According to [3], the family of fixed relaying arrangements which have the lowest complexity as compared to all the other families consists of decode-and-forward (DF) and amplify-and-forward (AF) protocols. Furthermore, in [10], it has been proved that the low complexity AF protocols have the ability to achieve similar BER performance as the more complicated DF protocol. Therefore, in our contribution only AF protocol is considered.

In this paper, a BER detector for a cooperative communication system implementing AF protocol is derived and implemented when communicating over a Rayleigh fading channels. Furthermore, in this contribution an adaptive BER detection scheme, which is free from channel estimation, is proposed. Our simulation results show that with the assistance of training sequence of reasonable length, this adaptive detector can provide a gain of more than 1-dB as compared to the conventional MMSE detector especially at a BER of $10^{-5}$.

This paper is organized as follows. In Sec. II, a detailed explanation of the system model and the basic assumptions are elaborated. Sec. III investigates the conventional relay-assisted MMSE and BER detectors for a cooperative communication environment. Furthermore, in Sec. IV the adaptive detector is presented. The attainable BER performance of the system is presented in Sec. V. Finally, the paper is concluded in Sec. VI .

## II. Uplink Communication Model

The block diagram for the considered cooperative communication system is shown in Fig. 1. In this system, the source, $S$, transmits the information to the destination, $D$, with the assistance of a direct link and $L$ relays, $R_1, R_2, \cdots R_L$. For all transmissions, the channels are assumed to be mutually independent and follow the Rayleigh fading distribution model with a normalized doppler frequecy of $1e-05$. Two phases are required for the complete transmission of the data. In Phase-I, the source $S$ transmits data to the destination and the relays, while in phase-II, the data is transmitted to the destination

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through the relays. It is assumed that all the channels are orthogonal to each other using either frequency or time [11]. Therefore, each full transmission consists of $(L+1)$ slots, where the first slot is for the direct link and each relay is assigned one of the remaining $L$ slots. The interference between the relays and the direct link is minimized due to orthogonality. Moreover, perfect synchronization is assumed between the source, relays and destination. Let us first consider the transmission of the desired signal.

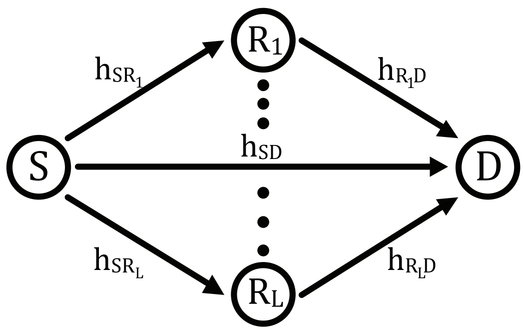


Figure 1. Schematic block diagram of a cooperative communication system assisted with direct link transmission and $L$-relays.

### *A. Phase-I: Transmission from Source*

We assume for simplicity that the source employs binary phase-shift keying (BPSK) baseband modulation. The source transmits $b=\{\pm 1\}$ with equal probability. In the first time-slot, the transmitted data bit is forwarded to the destination, $D$, and all the relays, $R_1, R_2, \cdots R_L$.

*1) Transmission Through Direct link:* The received signal at the destination can be represented as

$$y_0 = h_{SD} b + n_D, \tag{1}$$

where $n_D$ is the additive white Gaussian noise (AWGN) with mean zero and variance $\sigma_0^2 = \sigma_D^2/2$ per dimension. The channel coefficient, $h_{SD}$, is modeled as a complex Gaussian random variable with variance $\sigma_2^2/2$ per dimension. The channel is assumed to follow the Rayleigh fading distribution. Thus, the probability density function (PDF) for the direct link is given by [2]

$$p_{h_{D_l}}(z) = \frac{z}{\sigma_2^2} \exp\left(-\frac{z^2}{2\sigma_2^2}\right), \quad z \geq 0. \tag{2}$$

Furthermore, the received signal $y_0$ can be separated into the noise-free part, denoted as $x_0$, and the noise part, denoted as $n_0$, i.e.,

$$y_0 = x_0 + n_0. \tag{3}$$

*2) Transmission To Relays:* As each relay receives the broadcast from the source, the received signals at the relays can be represented as

$$y_{R_l} = h_{SR_l} b + n_{R_l}, \quad l = 1, 2, \cdots, L. \tag{4}$$

where $n_{R_l}$ is the AWGN with mean zero and variance $\sigma_{R_l}^2/2$ per dimension, and where $h_{SR_l}$ $(l = 1, 2, \cdots, L)$ denote the complex fading coefficients where the amplitudes are modeled according to Rayleigh model with the PDF as shown in (2). After completion of phase-I, let us proceed to phase-II of transmission.

### *B. Phase-II: Transmission from Relays to Destination*

In this phase, after receiving signal $y_{R_l}$, the respective relay, i.e., the $l$th terminal, normalizes this signal by a factor of $\sqrt{E[|y_{R_l}|^2]}$, where $E(.)$ denotes the expectation operator. After this normalization, the resulting signal is transmitted to the destination. This operation is performed at all relays. As such, the signals received at the destination can be expressed as follows

$$y_l = h_{R_l D} \frac{y_{R_l}}{\sqrt{E[|y_{R_l}|^2]}} + n_D, \quad l = 1, 2, \cdots, L. \tag{5}$$

where $h_{R_l D}$ denotes the complex fading coefficient of the link between relay $l$ and destination $D$, with distribution as shown in (2). Substituting (4) in (5), we obtain

$$\begin{aligned} y_l &= \frac{h_{R_l D} h_{SR_l}}{\sqrt{E[|y_{R_l}|^2]}} b + \frac{h_{R_l D} n_{R_l}}{\sqrt{E[|y_{R_l}|^2]}} + n_D \\ &= x_l + n_l, \quad l = 1, 2, \cdots, L. \end{aligned} \tag{6}$$

where

$$x_l = \frac{h_{R_l D} h_{SR_l}}{\sqrt{E[|y_{R_l}|^2]}} b, \tag{7}$$

and

$$n_l = \frac{h_{R_l D} n_{R_l}}{\sqrt{E[|y_{R_l}|^2]}} + n_D. \tag{8}$$

In the above, $x_l$ and $n_l$ represent the noise free term and noise part of $y_l$, respectively. If the channel knowledge is available, $n_l$ can be approximated as a Gaussian noise with zero mean and variance per dimension given by

$$\sigma_l^2 = \frac{|h_{R_l D}|^2 \sigma_{R_l}^2}{\sqrt{E[|y_{R_l}|^2]}} + \sigma_D^2, \quad l = 1, 2, \cdots, L. \tag{9}$$

### *C. Receiver Structure*

In order to detect the desired signal $b$, we need to collect $(L+1)$ copies of the desired signal arriving at the receiver through the direct link and the $L$ relays. Therefore, the received signal at the destination can finally be represented as

$$\boldsymbol{y} = \boldsymbol{h} b + \boldsymbol{n}, \tag{10}$$

where $\boldsymbol{h}$ is defined as

$$\boldsymbol{h} = \left[ h_{SD}, \frac{h_{R_1 D} h_{SR_1}}{\sqrt{E[|y_{R_1}|^2]}}, \cdots, \frac{h_{R_L D} h_{SR_L}}{\sqrt{E[|y_{R_L}|^2]}} \right]^T, \tag{11}$$

and $\boldsymbol{n}$ can be represented as

$$\begin{aligned} \boldsymbol{n} &= \left[ n_D, \frac{h_{R_1 D} n_{R_1}}{\sqrt{E[|y_{R_1}|^2]}} + n_D \cdots, \frac{h_{R_L D} n_{R_L}}{\sqrt{E[|y_{R_L}|^2]}} + n_D \right]^T \\ &= [n_0, n_1, \cdots, n_L]^T, \end{aligned} \tag{12}$$

where $n_0$ is the noise part of the received signal through source directly and is represented in (3) and $n_1, \cdots, n_L$ are the received signal through relays $1, \cdots, L$, respectively and is represented in (8). Also $\boldsymbol{n}$ is an AWGN with mean zero and variance $\boldsymbol{\Lambda}$. The variance $\boldsymbol{\Lambda}$ can be expressed as

$$\boldsymbol{\Lambda} = \text{diag}[\sigma_0^2, \sigma_1^2, \cdots, \sigma_L^2]. \tag{13}$$

$\boldsymbol{\Lambda}$ is a diagonal matrix of size $(L+1)$. We now proceed towards detection of our desired signal $b$.

## III. Detection for Cooperative Communication Systems

Linear detectors are preferred over the optimal and other sub-optimal detectors due to their low complexity [5]. Furthermore, in systems like sensor networks, ultra-wide band, etc., where the battery life is very important, low complexity detectors play a significant role. As such, in our contribution, low-complexity linear detectors are only considered. The receiver consists of a linear filter characterized by

$$\begin{aligned} z &= \boldsymbol{w}^H \boldsymbol{y} \\ &= \boldsymbol{w}^H \boldsymbol{h} b + \boldsymbol{w}^H \boldsymbol{n}, \end{aligned} \tag{14}$$

where

$$\boldsymbol{w} = [w_0, w_1, \cdots, w_L]^T, \tag{15}$$

and $w_l$ is the $l$-th tap complex valued filter coefficient. As our transmitted signal $b$ is BPSK-modulated, only the real part is of interest, which is given by

$$z_R = \Re(z) = \Re(\boldsymbol{w}^H \boldsymbol{h} b) + \Re(\boldsymbol{w}^H \boldsymbol{n}). \tag{16}$$

The estimate of the desired bit $b$ is expressed as

$$\hat{b} = sgn(z_R), \tag{17}$$

where $\Re(z)$ is the real value of $z$ and $sgn(z)$ is the sign function. The PDF of $z_R$ can be represented as

$$p_{z_R}(z_R) = \frac{1}{\sqrt{2\pi \boldsymbol{w}^H \Lambda \boldsymbol{w}}} \exp\left(-\frac{\left(z_R - \Re\left(\boldsymbol{w}^H \boldsymbol{h} b\right)\right)^2}{2\boldsymbol{w}^H \boldsymbol{\Lambda} \boldsymbol{w}}\right). \tag{18}$$

### A. Relay-assisted MMSE Detector

We now consider the basic MMSE detector. The classical Wiener filter design will be based on minimizing the MSE criterion given as [12]

$$\begin{aligned} J(w) &= E[|b - \hat{b}|^2] \\ &= E[bb^*] - \boldsymbol{w}^H E[\boldsymbol{y} b] - E[\boldsymbol{y}^H]\boldsymbol{w} \\ &+ \boldsymbol{w}^H E[\boldsymbol{y}\boldsymbol{y}^H]\boldsymbol{w}. \end{aligned} \tag{19}$$

The optimal weights can be calculated by derivation of (19) with respect to $\boldsymbol{w}$ and setting the result to zero. The optimal weights in a relay assisted MMSE detector can be easily determined as [12]

$$\boldsymbol{w} = \boldsymbol{R}^{-1} \boldsymbol{\rho}, \tag{20}$$

where $\boldsymbol{\rho}$ is the cross-correlation between $\boldsymbol{y}$ and $b$, and $\boldsymbol{R}$ is the auto-correlation of $\boldsymbol{y}$:

$$\boldsymbol{\rho} = E[\boldsymbol{y} b] = \boldsymbol{h} \tag{21}$$

$$\boldsymbol{R} = E[\boldsymbol{y}\boldsymbol{y}^H] = E[\boldsymbol{h}\boldsymbol{h}^H] + 2\boldsymbol{\Lambda}. \tag{22}$$

In wireless communication systems, the performance criteria is measured in terms of BER not with respect to MSE. MMSE detector minimizes the MSE, which may not translate into the lowest BER performance of the system [8]. Accordingly, a detector that minimizes the BER performance of the system needs to be designed for a cooperative communication environment.

### B. Relay Assisted BER Detector

In order to establish the minimum BER detector for equally likely $b = \{\pm 1\}$, we need to minimize the BER. This is done as follows. First, we have

$$\begin{aligned} P_E(\boldsymbol{w}) &= \int_{-\infty}^{0} p(z_R | b = +1) dz_R \\ &= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi \boldsymbol{w}^H \Lambda \boldsymbol{w}}} \exp\left(-\frac{\left(z_R - \Re\left(\boldsymbol{w}^H \boldsymbol{h}\right)\right)^2}{2\boldsymbol{w}^H \Lambda \boldsymbol{w}}\right) dz_R \\ &= Q\left(\frac{\Re\left(\boldsymbol{w}^H \boldsymbol{h}\right)}{\sqrt{\boldsymbol{w}^H \boldsymbol{\Lambda} \boldsymbol{w}}}\right), \end{aligned} \tag{23}$$

where $Q(\cdot)$ is the standard Gaussian Q-function:

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} \exp\left(-\frac{t^2}{2}\right) dt. \tag{24}$$

The gradient of $P_E(\boldsymbol{w})$ can be represented as

$$\begin{aligned} \nabla P_E(\boldsymbol{w}) &= \frac{1}{\sqrt{2\pi}} \frac{\Re\left(\boldsymbol{w}^H \boldsymbol{h}\right) \boldsymbol{\Lambda} \boldsymbol{w} - \Re(\boldsymbol{h}) \left(\boldsymbol{w}^H \boldsymbol{\Lambda} \boldsymbol{w}\right)}{(\boldsymbol{w}^H \boldsymbol{\Lambda} \boldsymbol{w})^{\frac{3}{2}}} \\ &\times \exp\left(-\frac{\left(\Re\left(\boldsymbol{w}^H \boldsymbol{h}\right)\right)^2}{2\boldsymbol{w}^H \boldsymbol{\Lambda} \boldsymbol{w}}\right). \end{aligned} \tag{25}$$

Now the weights of the BER detector could be found with the help of steepest-descent algorithms, which can be represented as

$$\boldsymbol{w}(i+1) = \boldsymbol{w}(i) - \mu \nabla P_E(\boldsymbol{w}), \tag{26}$$

where $\mu$ is the step-size and $i$ represents the number of iteration.

Finally, the BER algorithm can be summarized as follows.

1) *Parameters*: $\mu > 0$ a suitable step-size and a terminating constant $\gamma$;
2) *Initialization*: $\boldsymbol{w}(0)$;
3) *Calculation*: $\nabla P_E(w)$ with help of (21) and update $\boldsymbol{w}$ with help of (23).
4) *Threshold*: Compare $|\nabla P_E(w)| < \gamma$; if condition is met stop, otherwise go to step 3.
5) *Weights*: $\boldsymbol{w}(i+1)$ are the optimal weights.

It can be observed from (20), that in order to obtain the weights of a relay-assisted MMSE detector we need to invert the correlation matrix $\boldsymbol{R}$. Furthermore, we need to know the exact channel from source to destination, source to relays and then relays to destination, which may be difficult to acquire. Furthermore, it can also be observed from (23) and (25) that the channel knowledge is required for the BER detector. However, by implementing the MMSE and BER adaptively, the above mentioned problems can be overcomed.

## IV. Adaptive Implementation.

In adaptive algorithms, the optimum weight vector $\boldsymbol{w}$ is determined by iterative computing when converging to the optimal MMSE or BER solutions [12]. Initially, the filter weights are trained with the assistance of training sequences. After converging to the optimal MMSE or BER solution, the detector is switched to decision-directed mode. In this mode, the detected bit $\hat{b}$ is fed back to the filter to update the weight vector $\boldsymbol{w}$ according to the given criteria. Therefore, when employing adaptive detectors, only complete knowledge about the training sequence is required.

### A. Adaptive MMSE Detector

MMSE detectors can be easily implemented with the aid of least mean square (LMS) or normalised mean square algorithms (NLMS) [12]. As the main emphasis is towards lower complexity, the LMS approach is preferred. The LMS adaptive detector can be summarized as follows.

- *Parameters*:
  $\mu =$ a suitable step-size, $0 < \mu < \dfrac{2}{E[||\boldsymbol{y}||^2]}$.
- *Initialization*:
  $\boldsymbol{w}(0)$; $\boldsymbol{w}(0) = \mathbf{0}$, when without *prior* knowledge.
- *Weight vector update*:
  For $i = 0, 1, 2, \ldots$, compute
  estimation error: $e(i) = b(i) - \boldsymbol{w}^H(i)\boldsymbol{y}(i)$, and
  weight vector: $\boldsymbol{w}(i+1) = \boldsymbol{w}(i) + \mu\boldsymbol{y}(i)e^*(i)$.

### B. Adaptive MBER Detector

For adaptive detection of the BER algorithm, we need to estimate the PDF of $z_R$. Kernel density estimation is known to produce reliable estimates with the help of training bits [9]. Given a training length $TL$, the estimate of the PDF can be computed as

$$\begin{aligned}\hat{p}_{z_R}(z_R) &= \frac{1}{TL}\sum_{i=1}^{TL}\frac{1}{\sqrt{2\pi}\beta} \\ &\times \exp\left(-\frac{\left(z_R - \Re\left(\boldsymbol{w}^H(i)\boldsymbol{y}(i)\right)\right)^2}{2\beta^2}\right), \quad (27)\end{aligned}$$

where $\beta$ is the kernel width. Now the approximated $\hat{P}_E(w)$ will be

$$\begin{aligned}\hat{P}_E(\boldsymbol{w}) &= \int_{-\infty}^{0}\hat{p}(z_R, \boldsymbol{w})dz_R \\ &= \frac{1}{TL}\sum_{i=1}^{TL}Q\left(\frac{\Re\left(\boldsymbol{w}^H(i)\boldsymbol{y}(i)\right)}{\beta}\right) \quad (28)\end{aligned}$$

and the $\nabla\hat{P}_E(\boldsymbol{w})$ can be calculated as [8]

$$\begin{aligned}\nabla\hat{P}_E(\boldsymbol{w}) &= \frac{-1}{2\sqrt{2\pi}TL\beta}\sum_{i=1}^{TL}b(i) \\ &\times \exp\left(-\frac{\left(\Re\left(\boldsymbol{w}^H(i)\boldsymbol{y}(i)\right)\right)^2}{2\beta^2}\right)\boldsymbol{y}(i) \quad (29)\end{aligned}$$

Therefore, for the $(i+1)$-th bit, the adaptive BER detector will be

$$\begin{aligned}\boldsymbol{w}(i+1) &= \boldsymbol{w}(i) + \mu\frac{b(i)}{2\sqrt{2\pi}\beta} \\ &\times \exp\left(-\frac{\left(\Re\left(\boldsymbol{w}^H(i)\boldsymbol{y}(i)\right)\right)^2}{2\beta^2}\right)\boldsymbol{y}(i) \quad (30)\end{aligned}$$

Next, we provide simulation results sustaining the above analysis.

## V. Simulation Results and Discussion

In this section BER performance of the proposed cooperative communication system with $L$ relays is investigated. In our simulations, the channel gains were assumed to obey the Rayleigh distribution. Furthermore, the normalized Doppler frequency of the channel was fixed to $1e-05$. Here the direct link between the source and the destination is ignored because of the large path loss assumed between these nodes.

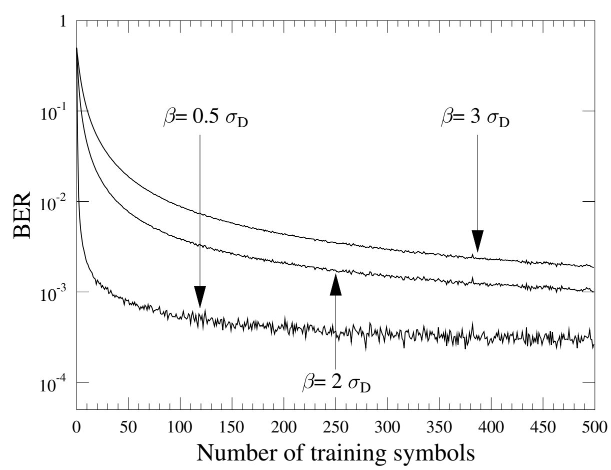


Figure 2. Learning curves of the cooperative communication system when communicating with respect to different kernel width values. The other parameters for the simulations were $E_b/N_0 = 14$dB, $\mu = 0.25$ and $L = 3$.

Fig. 2 illustrates the impact of the kernel width on the learning performance of the cooperative communication system carried out with the assistance of $L = 3$ relays. In our simulations, the BER performance was obtained by averaging over $100,000$ independent realizations of the channel. Based on the results, it can be observed that for a given length of training, an appropriate kernel width $\beta$ needs to be adjusted to achieve minimal BER. From the figure, it can be observed that $\beta = 0.5\sigma_D$ converges much faster and also achieves a lower

BER as compared to the other values of $\beta$. Furthermore, it can be observed that less than 200 training symbols are required for the algorithm to converge. Therefore, in the sequel, kernel width density is fixed to $\beta = 0.5\sigma_D$ and training length is set to 200.

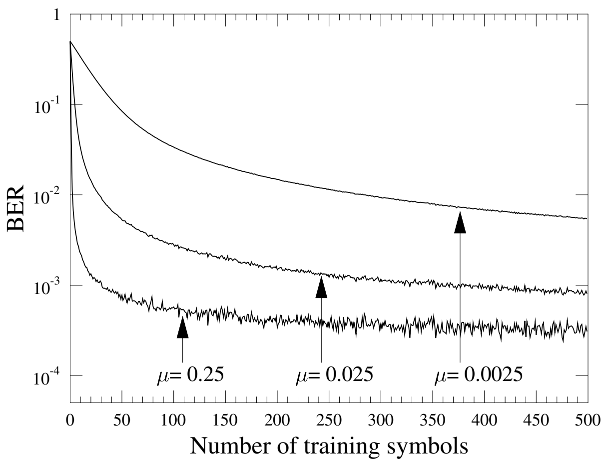


Figure 3. Learning curves of the cooperative communication system when communicating with respect to different step-sizes. The other parameters for the simulations were $E_b/N_0 = 14$dB, $\beta = 0.5\sigma_D$ and $L = 3$.

Fig. 3 shows the learning curves of the BER algorithm with different step-sizes for the AF relaying system at $E_b/N_0 = 14$dB, when the communication is carried over a Rayleigh fading channel. The ensemble average was taken over $100,000$ independent realizations of the channel. It can be observed from Fig. 3 that the step-size $\mu$ determines the BER performance and convergence of the algorithm. From the figure, it can also be observed that $\mu = 0.25$ yields much faster convergence and also achieves a better BER performance. Therefore, the step-size is fixed to $\mu = 0.25$ in the sequel.

Fig.4 shows the BER performance of the cooperative communication system, as a function of the SNR per bit, when communicating over Rayleigh fading channel. The training length was fixed to 200 while the frame length was fixed to 1000 bits. It can be observed from the Fig. 4 that the BER performance of the system improves as the number of relays are increased, due to the increased diversity. Furthermore, it can be observed that a gain of more than 1dB can be obtained by employing the relay assisted BER detector as compared to the conventional relay assisted MMSE detector at higher SNR values.

## VI. Conclusions

In this paper, we have proposed a new detector for AF relaying systems. This detector has the capability of improving the BER performance when communicating over Rayleigh fading channels. Furthermore, this detector is independent from channel estimation. From our simulations, it was attested that a gain of more than 1dB is possible at a BER of $10^{-5}$ as compared to the conventional MMSE detection scheme at higher SNRs.

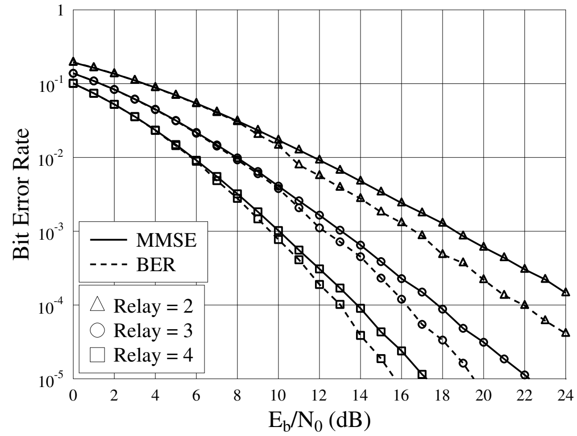


Figure 4. BER performance of a cooperative communication system when communicating with the assistance of $L$ relays. Relays experiences independent Rayleigh fading. The frame length was fixed to 1000 bits, where the first 200 bits were used for training. The other parameters for the simulation was $\mu = 0.25$ and $\beta = 0.5\sigma_D$.


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