Effects of Feedback Delay on the Performance of Multiple Relay Network over Nakagami-m Fading Channels

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***Abstract*—This paper studies the effect of feedback delay on the performance of amplify-and-forward (AF) relay selection over Nakagami-m fading environment. Spatial diversity can be improved by employing multiple relays with selection, however, this diversity gain cannot be fully realized when the feedback delay is present in the decision metric. Hence, we try to quantify this detrimental effect by deriving the exact closed form solution to outage probability and ergodic capacity for channel state information (CSI)-assisted relay. Further, we derive simple asymptotic results for outage to provide insight of the system performance and diversity. Monte Carlo simulation is used to verify the analytical work.**

I. Introduction

Multiple relay selection network has been a topic of significant importance in the recent research work. The selection of maximum gain relay for the transmission increases the system performance and, importantly, the spatial diversity. There are several ways of selecting a relay for transmission. One method is referred to as the opportunistic relay selection [1], [2] in which the relay with maximum instantaneous end-to-end signal to noise ratio (SNR) relay is considered. Synchronization is very important fact in this case. Another is the partial relay selection method, which can be carried out in two ways; by selecting either the maximum instantaneous SNR first-hop relay [3], [4] or the second hop relay [1], [5], [6]. The second-hop SNR based method is referred to as reactive opportunistic relay [1] or selection cooperation (SC) [5], [6]. All these studies have assumed perfect channel state information (CSI) in selecting the maximum SNR relay.

Although the use of perfect CSI to select the relay increases the spatial diversity gain, when the available CSI is outdated due to feedback delay, the expected gain can not be realized. The effect on the performance of feedback delay on different relay selection schemes are investigated in [7]–[9]. Authors in [8] have investigated the partial relay selection according first-hop SNR and best relay selection is considered in [9]. The above mentioned analysis has been in the independent Rayleigh fading environment.

A. Contributions

The use of perfect CSI for relay selection increases the diversity gain and the performance significantly, however, imperfect CSI due to feedback delay works to reverse these gains. Hence, it is important to quantify this detrimental performance loss. In this paper, we investigate the performance of AF relay selection network over Nakagami-m fading environment. Selection cooperation is used with outdated CSI to find the best relay. CSI-assisted relaying will be considered with an arbitrary m parameter for first and second hop. We derive the exact closed form solution to outage probability and ergodic capacity which have not been presented in previous literature. Moreover, the performance in high SNR is analyzed and a simple asymptotic expression for outage is derived. Further, our outage probability expression can be easily used to derive average SER and moments of SNR. All the analytical works are verified with the Monte Carlo simulations. A comprehensive analysis and more results are available in [10].

II. System Models

A. Relay selection network

Consider a source ($\mathtt{S}$) which communicates with the destination ($\mathtt{D}$) via a selected amplify-and-forward (AF) relay ($\mathtt{R_m}$). Selection cooperation [1], [5], [6] is used to obtain the best relay among M relays. Downlink half-duplex communication is considered. The direct link between source to destination is assumed to be not available. As presented in numerous literature, communication happens in two time slots; during the first time slot $\mathtt{S}$ transmits the signal to $\mathtt{R_m}$ and the received signal is given as

$$y_{\mathtt{R_m}} = \sqrt{P_s} h_{\mathtt{SR_m}} x + n_{\mathtt{R_m}} \tag{1}$$

where $h_{\mathtt{SR_m}}$ denotes the $\mathtt{S} - \mathtt{R_m}$ channel coefficient and $n_{\mathtt{R_m}}$ denotes the additive white Gaussian noise (AWGN) component with N_r variance. Then $\mathtt{R_m}$ multiplies the received signal by gain G and retransmits during the second time slot to $\mathtt{D}$ and the received signal at $\mathtt{D}$ is given as

$$\begin{aligned} y_{\mathtt{R_mD}} &= \sqrt{P_r} h_{\mathtt{R_mD}} G y_{\mathtt{R_m}} + n_{\mathtt{D}} \\ &= \sqrt{P_r} h_{\mathtt{R_mD}} G (\sqrt{P_s} h_{\mathtt{SR_m}} x + n_{\mathtt{R_m}}) + n_{\mathtt{D}} \end{aligned} \tag{2}$$

where $h_{\mathtt{R_mD}}$ is the channel coefficient of $\mathtt{R_m} - \mathtt{D}$ link and $n_{\mathtt{D}}$ is the AWGN component at $\mathtt{D}$ with variance N_u. After some mathematical manipulations, the end-to-end SNR without time delay can be obtained from

$$\gamma_{\mathtt{E_m}} = \frac{P_s |h_{\mathtt{SR_m}}|^2 P_r |h_{\mathtt{R_mD}}|^2 G^2}{G^2 P_r |h_{\mathtt{R_mD}}|^2 N_r + N_u} \tag{3}$$

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End-to-end SNR can be rewritten as

$$\gamma_{\mathtt{E_m}} = \frac{\gamma_s \gamma_m}{\gamma_m + \frac{1}{G^2 N_r}} \tag{4}$$

where $\gamma_s = \bar{\gamma}_s |h_{\mathtt{SR_m}}|^2$ and $\bar{\gamma}_s = \frac{P_s}{N_r}$.

Maximum instantaneous second hop SNR gain relay ($\mathtt{R_m}$) is selected from M relays for the transmission [5]. $\mathtt{R_m}$ is selected using outdated CSI as [1], [5]

$$\gamma_m = \bar{\gamma}_u |h_{\mathtt{R_mD}}|^2 = \max_{[1<k<M]} \bar{\gamma}_u |h_{\mathtt{R_kD}}|^2 \tag{5}$$

where $\bar{\gamma}_u = \frac{P_r}{N_u}$ and let $\gamma_k = \bar{\gamma}_u |h_{\mathtt{R_kD}}|^2$. Now we can write the received time delayed end-to-end SNR at $\mathtt{D}$ as

$$\tilde{\gamma}_{\mathtt{E_m}} = \frac{\gamma_s \tilde{\gamma}_m}{\tilde{\gamma}_m + \frac{1}{G^2 N_r}} \tag{6}$$

where $\tilde{\gamma}_m$ is the τ_d time delayed version of γ_m. Let $\tilde{\gamma}_k$ be the τ_d time delayed version of γ_k. We model the relation between γ_k and $\tilde{\gamma}_k$ as in [9]

$$\tilde{\gamma}_k = \sqrt{\rho}\gamma_k + \sqrt{1-\rho}\omega_k \tag{7}$$

where $|\omega_k|^2$ is Gamma distributed random variable with same variance as γ_k. ρ is the correlation coefficient.

B. Fading channel model

First hop SNR, γ_s takes the form of Gamma distribution with parameter m_1 and probability density function (pdf) of γ_s is given as [11]

$$f_{\gamma_s}(z) = \frac{m_1^{m_1} z^{m_1-1}}{\bar{\gamma}_s^{m_1}\Gamma(m_1)} e^{-\frac{m_1 z}{\bar{\gamma}_s}} \tag{8}$$

where $\Gamma(\cdot)$ is the Gamma function defined in [12, Eq. 8.310.1]. Similarly the pdf of γ_k can be written as in (8) for Nakagami parameter m_2 and cumulative density function (cdf) of γ_k for integer m_2 is given by [11]

$$F_{\gamma_k}(z) = 1 - e^{-\frac{m_2 z}{\bar{\gamma}_k}} \sum_{i=1}^{m_2-1} \frac{1}{i!} \left(\frac{m_2 z}{\bar{\gamma}_k}\right)^i \tag{9}$$

By assuming same average SNR at all relays [3]–[9], the pdf of γ_m can be derived as

$$f_{\gamma_m}(z) = M \left[F_{\gamma_k}(z)\right]^{M-1} f_{\gamma_k}(z) \tag{10}$$

Pdf of of $\tilde{\gamma}_m$ can be derived using [8], [9]

$$f_{\tilde{\gamma}_m} = \int_0^\infty f_{\tilde{\gamma}_m|\gamma_m}(y|z) f_{\gamma_m}(z)dz \tag{11}$$

where

$$f_{\tilde{\gamma}_m|\gamma_m}(y|z) = \frac{f_{\tilde{\gamma}_k,\gamma_k}(y,z)}{f_{\gamma_k}(z)} \tag{12}$$

Joint pdf of γ_r and $\tilde{\gamma}_r$ for correlation factor is ρ is given as [11]

$$f_{\tilde{\gamma}_k,\gamma_k}(x,y) = \left(\frac{m_2}{\bar{\gamma}_u}\right)^{m_2+1} \frac{\left(\frac{xy}{\rho}\right)^{\frac{m_2-1}{2}}}{(1-\rho)\Gamma(m_2)} e^{-\left(\frac{x+y}{1-\rho}\right)\frac{m_2}{\bar{\gamma}_u}} \times I_{m_2-1}\left(\frac{2m_2\sqrt{\rho xy}}{\bar{\gamma}_u(1-\rho)}\right) \tag{13}$$

where $I_n(\cdot)$ denotes the n^{th} order modified Bessel function of first kind.

III. Performance analysis

First we derive the pdf of $\tilde{\gamma}_m$ as

$$f_{\tilde{\gamma}_m}(x) = M \sum_{n_1=0}^{M-1} (-1)^{n_1} \binom{M-1}{n_1} \frac{\Psi}{\Gamma(m_2)} \frac{\beta!}{1-\rho} \sum_{p=0}^{\beta} \times \binom{\beta+m_2-1}{\beta-p} \frac{\lambda^{2p+m_2-1}}{\alpha^{\beta+m+p} p! \rho^{\frac{m_2-1}{2}}} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2} \times x^{p+m_2-1} e^{-\frac{\varsigma m_2 x}{\bar{\gamma}_u}} \tag{14}$$

where $\lambda = \frac{\sqrt{\rho}}{1-\rho}$, $\alpha = \frac{\rho}{1-\rho} + n_1 + 1$, $\varsigma = \frac{1}{1-\rho} - \frac{\lambda^2}{\alpha}$,

$$\Psi = \sum_{n_2=0}^{n_1} \sum_{n_3=0}^{n_2} \cdots \sum_{n_{m_2}=0}^{n_{m_2}-1} \prod_{i=0, n_{m_2+1}=0}^{m_2-1} \left(\frac{1}{i!}\right)^{n_{i+1}-n_{i+2}} \binom{n_{i+1}}{n_{i+2}} \tag{15}$$

and $\beta = \sum_{j=0}^{m_2-1} j(n_{j+1} - n_{j+2})$ where $n_{m_2+1} = 0$. The parameters defined above will appear in the equations to follow.

Proof: see Appendix A

A. CSI-assisted relay

As in numerous previous literature, we select the gain G at the relay as [13], then end-to-end SNR is given by [13]

$$\tilde{\gamma}_{\mathtt{E1_m}} = \frac{\gamma_s \tilde{\gamma}_m}{\gamma_s + \tilde{\gamma}_m + 1} \tag{16}$$

1) Outage Probability: Outage probability can effectively be used to quantify the performance, which is defined as the probability that the end-to-end SNR drops below a predefined threshold Λ and it can be given as

$$P_o = Pr\left(\gamma_{\mathtt{E_m}} < \Lambda\right) \tag{17}$$

Theorem 1 (Outage Probability of CSI-assisted relay): Outage probability of $\gamma_{\mathtt{E1_m}}$ for CSI-assisted relay can be derived as

$$F_{\gamma_{\mathtt{E1_m}}}(\Lambda) = 1 - \frac{2M}{1-\rho} \sum_{n_1=0}^{M-1} (-1)^{n_1} \binom{M-1}{n_1} \frac{\Psi\beta!}{\Gamma(m_2)} \sum_{p=0}^{\beta} \times \binom{\beta+m_2-1}{\beta-p} \frac{\lambda^{2p+m_2-1}}{\alpha^{\beta+m_2+p} p! \rho^{\frac{m_2-1}{2}}} \sum_{i=0}^{m_1-1} \frac{1}{i!} \times \sum_{q=0}^{i} \binom{i}{q} c^{i-q} \sum_{s=0}^{q+p+m_2-1} \binom{q+p+m_2-1}{s} \times \left(\frac{1}{\varsigma}\right)^{\frac{s-i+1}{2}} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{\frac{2p+2m_2-s+i-1}{2}} \left(\frac{m_1}{\bar{\gamma}_s}\right)^{\frac{s+i+1}{2}} \times \Lambda^{\frac{2(q+p+m_2)-s-1+i}{2}} (\Lambda+c)^{\frac{s-i+1}{2}} e^{-\frac{m_1\Lambda}{\bar{\gamma}_s} - \frac{\varsigma m_2 \Lambda}{\bar{\gamma}_u}} \times K_{s-i+1}\left(2\sqrt{\frac{m_1 \varsigma m_2 \Lambda(\Lambda+c)}{\bar{\gamma}_s \bar{\gamma}_u}}\right) \tag{18}$$

where $K_v(\cdot)$ is the v^{th} order modified Bessel function of second kind.

Proof: See Appendix B

2) Ergodic Capacity: Ergodic capacity can be mathematically represented as

$$C_{erg} = \frac{1}{2} E_{\gamma_{\mathtt{E1_m}}} \left[\log_2(1+\gamma_{\mathtt{E1_m}})\right] \tag{19}$$

Theorem 2 (Ergodic capacity of CSI-assisted relay): Exact closed form expression of ergodic capacity for CSI-assisted relay can be derived as

$$C_{erg} = \frac{1}{2\ln 2}\left[\zeta_1 + \zeta_2 - \zeta_3\right] \tag{20}$$

where

$$\zeta_1 = \sum_{u=0}^{m_1-1} \frac{(m_1-1)!(-1)^{m_1-1-u}}{\Gamma(m_1)(m_1-1-u)} \left(\frac{m_1}{\bar{\gamma}_s}\right)^{m_1-u-1} \Bigg[e^{\frac{m_1}{\bar{\gamma}_s}} \tag{21}$$

$$\times \mathtt{E}_1\left(\frac{m_1}{\bar{\gamma}_s}\right) + \sum_{k=1}^{m_1-1-u} (-1)^k (k-1)! \left(\frac{\bar{\gamma}_s}{m_1}\right)^k \Bigg]$$

$$\zeta_2 = M \sum_{n_1=0}^{M-1} \binom{M-1}{n_1} \frac{(-1)^{n_1}\Psi\beta!}{\Gamma(m_2)(1-\rho)} \sum_{p=0}^{\beta} \binom{\beta+m_2-1}{\beta-p} \tag{22}$$

$$\times \frac{\lambda^{2p+m_2-1}}{\alpha^{\beta+m+p} p! \rho^{\frac{m_2-1}{2}}} \sum_{v=0}^{p+m_2-1} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2-v-1}$$
$$\times \frac{(p+m_2-1)!(-1)^{p+m_2-1-v}}{(p+m_2-1-v)!\varsigma^{v+1}} \Bigg[e^{\frac{\varsigma m_2}{\bar{\gamma}_u}} \mathtt{E}_1\left(\frac{\varsigma m_2}{\bar{\gamma}_u}\right)$$
$$+ \sum_{k=1}^{p+m_2-1-v} (-1)^k (k-1)! \left(\frac{\bar{\gamma}_u}{\varsigma m_2}\right)^k \Bigg]$$

and

$$\zeta_3 = \begin{cases} \zeta_{31}, & \frac{m_1}{\bar{\gamma}_s} = \frac{\varsigma m_2}{\bar{\gamma}_u} \\ \zeta_{32}, & \frac{m_1}{\bar{\gamma}_s} \neq \frac{\varsigma m_2}{\bar{\gamma}_u} \end{cases} \tag{23}$$

where ζ_{31} and ζ_{32} are given in (24) and (25) respectively. $\mathtt{E}_1(\cdot)$ is exponential integral. *Proof:* see Appendix C

IV. Asymptotic analysis

Outage probability of $\gamma_{\mathtt{E1_m}}$ for CSI-assisted relay in high SNR can be derived as

$$F_{\gamma_{\mathtt{E1_m}}}(z) = \begin{cases} \frac{\psi_1 z^{m_1}}{m_1} + o(z^{m_1+1}), & m_1 < m_2 \\ \frac{\psi_2 z^{m_2}}{m_2} + o(z^{m_2+1}), & m_1 > m_2 \\ \frac{\psi_3 z^{m}}{m} + o(z^{m+1}), & m_1 = m_2 = m \end{cases} \tag{26}$$

where

$$\psi_1 = \frac{m_1^{m_1}}{\Gamma(m_1)}, \tag{27}$$

$$\psi_2 = \sum_{n_1=0}^{M-1} \frac{\binom{M-1}{n_1} M (-1)^{n_1} \Psi \beta! \lambda^{m_2-1} m_2^{m_2}}{\Gamma(m_2)(1-\rho)\alpha^{\beta+m_2}\rho^{\frac{m_2-1}{2}}\kappa^{m_2}} \binom{\beta+m_2-1}{\beta} \tag{28}$$

and

$$\psi_3 = \psi_1 + \psi_2 \tag{29}$$

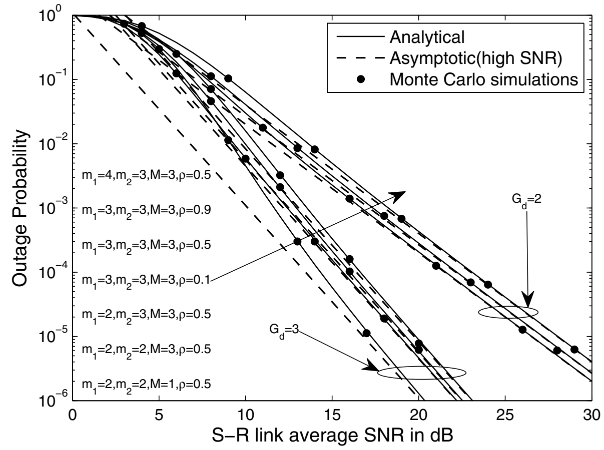

Fig. 1. Outage probability of CSI-assisted relay, $\Lambda = 1$dB.

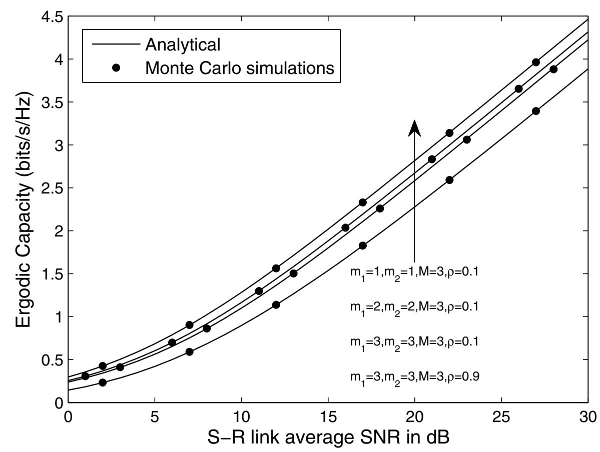

Fig. 2. Ergodic Capacity for CSI-assisted relay.

where $\bar{\gamma}_u = \kappa\bar{\gamma}_s$. It is observed from high SNR expression that the diversity order (G_d) of the system is $min[m_1, m_2]$. *Proof:* We expand the $e^{-\frac{m_1 x}{\bar{\gamma}_s}}$ using Maclaurin series and rewrite the pdf of γ_s in (8). Then we simplify to observe that $x^t, t < m_1 - 1$ terms sum to zero, hence, by limiting to higher order terms we can rewrite the pdf of γ_s. By integrating and making the variable change as $z = \frac{x}{\bar{\gamma}_s}$, cdf of γ_s can be obtained. Similarly, we obtain the high SNR cdf for γ_u. Then we use derived cdf of γ_s and γ_u to express outage probability of $\gamma_{\mathtt{E1_m}}$ in high SNR as in (26). The detailed proof is given in [10].

V. Numerical Analysis

In this section we give numerical validations for our derived analytical results. Monte Carlo simulation is presented to verify our work. Without loss of generality we assume $\bar{\gamma}_s = \bar{\gamma}_u$ ($\kappa = 1$) in all the figures.

Fig. 1 shows the outage probability variation with different m_1, m_2, M and ρ. It is noticed that with the increase of ρ (decrease of feedback delay), the outage probabilities improve. This can be clearly noticed in the three curves of

$$\zeta_{31} = \sum_{n_1=0}^{M-1} \binom{M-1}{n_1} \frac{(-1)^{n_1}\Psi\beta!}{\Gamma(m_2)(1-\rho)} \sum_{p=0}^{\beta} \binom{\beta+m_2-1}{\beta-p} \frac{M\lambda^{2p+m_2-1}}{\alpha^{\beta+m+p}p!\rho^{\frac{m_2-1}{2}}} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2} \left(\frac{m_1}{\bar{\gamma}_s}\right)^{m_1} \sum_{q=0}^{m_1-1} \binom{m_1-1}{q} \sum_{u=0}^{p+m_2+m_1-1} \tag{24}$$
$$\times \frac{(p+m_2+m_1-1)!(-1)^{p+m_2+m_1+q-1-u}}{(p+m_2+m_1-1-u)!(q+p+m_2)\Gamma(m_1)} \left(\frac{\bar{\gamma}_s}{m_1}\right)^{u+1} \left[e^{\frac{m_1}{\bar{\gamma}_s}} \mathtt{E}_1\left(\frac{m_1}{\bar{\gamma}_s}\right) + \sum_{k=1}^{p+m_2+m_1-1-u} (-1)^k (k-1)! \left(\frac{\bar{\gamma}_s}{m_1}\right)^k\right]$$

$$\zeta_{32} = M \sum_{n_1=0}^{M-1} \binom{M-1}{n_1} \frac{(-1)^{n_1}\Psi\beta!}{\Gamma(m_2)(1-\rho)} \sum_{p=0}^{\beta} \binom{\beta+m_2-1}{\beta-p} \frac{\lambda^{2p+m_2-1}}{\alpha^{\beta+m+p}p!\rho^{\frac{m_2-1}{2}}} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2} \left(\frac{m_1}{\bar{\gamma}_s}\right)^{m_1} \sum_{q=0}^{m_1-1} \binom{m_1-1}{q} \tag{25}$$
$$\times \frac{(p+m_2+q-1)!}{\Gamma(m_1)\left(\frac{\varsigma m_2}{\bar{\gamma}_u} - \frac{m_1}{\bar{\gamma}_s}\right)^{p+q+m_2}} \left[\sum_{u=0}^{m_1-q-1} \frac{(m_1-q-1)!(-1)^{m_1-1-u}}{(m_1-q-1-u)!} \left(\frac{\bar{\gamma}_s}{m_1}\right)^{u+1} \left(e^{\frac{m_1}{\bar{\gamma}_s}} \mathtt{E}_1\left(\frac{m_1}{\bar{\gamma}_s}\right) + \sum_{k=1}^{m_1-1-q-u} (-1)^k\right.\right.$$
$$\left.\times (k-1)! \left(\frac{\bar{\gamma}_s}{m_1}\right)^k\right) + \sum_{k=0}^{p+m_2+q-1} \frac{\left(\frac{\varsigma m_2}{\bar{\gamma}_u} - \frac{m_1}{\bar{\gamma}_s}\right)^k}{k!} \sum_{v=0}^{m_1-q-1+k} \frac{(m_1-q+k-1)!(-1)^{m_1-1+k-v}}{(m_1-1-q+k-v)!} \left(\frac{\bar{\gamma}_u}{\varsigma m_2}\right)^{v+1}$$
$$\left.\times \left(e^{\frac{\varsigma m_2}{\bar{\gamma}_u}} \mathtt{E}_1\left(\frac{\varsigma m_2}{\bar{\gamma}_u}\right) + \sum_{r=1}^{m_1-1-q+k-v} (-1)^r (r-1)! \left(\frac{\bar{\gamma}_u}{\varsigma m_2}\right)^r\right)\right]$$

$\rho = 0.1, 0.5, 0.9$. Due to feedback delay in selecting the relay, full diversity order of $min(m_1, Mm_2)$ cannot be fully realized and this fact is observed from three right most curves that there is no diversity gain although M increases from 1 to 3. Further, this point is verified from the high SNR curves. Monte Carlo simulations exactly coincide with the analytical ones and which verify the accuracy of our analytical results.

Ergodic capacity variations with average SNR are plotted in Fig. 2. Capacity variation is highlighted for parameters m_1, m_2 and ρ. Here we fixed the $M = 3$ since the variations of the curves for different Ms are marginal. When we look at the bottom three curves we see that increases in m_1, m_2 improve the capacity, however, there is a large improvement when m_1, m_2 increase from 1 to 2 than from 2 to 3. Rayleigh fading case ($m_1 = m_2 = 1$) is also plotted. As expected from previous observations, here also we see an improvement in capacity with the increase of ρ and it is highlighted in two top most curves.

VI. Conclusion

We have investigated the performance of CSI assisted AF relay selection network over Nakagami-m fading environment in the presence of feedback delay. We have derived the exact closed form expressions to outage probability and ergodic capacity. Our results can be used to quantify the feedback delay effect of selecting the relay according to selection cooperation scheme with the loss of spatial diversity as a result. Moreover, we have presented simple asymptotic analysis and it shows that the desired diversity order of $min(m_1, Mm_2)$ cannot be achieved. Finally Monte Carlo simulations verify the accuracy of our analytical work.

Appendix A

We substitute the pdf and cdf of γ_k in (10) and using the multinomial theorem and then carrying out some mathematical manipulation we obtain the $f_{\gamma_m}(z)$ as

$$f_{\gamma_m}(z) = \sum_{n_1}^{M-1} \binom{M-1}{n_1} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{m_2+\beta} \frac{M\Psi(-1)^{n_1} z^{\beta+m_2-1}}{\Gamma(m_2) e^{\frac{(n_1+1)m_2 z}{\bar{\gamma}_u}}} \tag{30}$$

where Ψ and β are defined in (14) and (15). Now we use the pdf of γ_k and (13) in (12) to obtain the conditioned pdf $f_{\tilde{\gamma}_m|\gamma_m}(x|y)$. Then using the conditioned pdf and (30) in (11) and proceeding we obtain

$$f_{\tilde{\gamma}_m}(x) = \sum_{n_1}^{M-1} \frac{\binom{M-1}{n_1} M(-1)^{n_1}\Psi}{\Gamma(m_2)(1-\rho)} \left(\frac{m_2}{\bar{\gamma}_u}\right)^{m_2+\beta+1} \left(\frac{x}{\rho}\right)^{\frac{m_2-1}{2}} \tag{31}$$
$$\times e^{-\frac{x m_2}{\bar{\gamma}_u(1-\rho)}} \int_0^\infty y^{\frac{2\beta+m_2-1}{2}} e^{-\frac{\alpha m_2 y}{\bar{\gamma}_u}} I_{m_2-1}\left(\frac{2\lambda m_2\sqrt{xy}}{\bar{\gamma}_u}\right)$$

Now we use [12, Eq.8.406.3] to replace $I_{m_2-1}(\cdot)$ by Bessel function of first kind $J_{m_2-1}(\cdot)$ and then use [12, Eq.6.643.2] to find Δ. Then using [12, Eq. 8.970.1] to expand the Laguerre polynomial and with some mathematical simplification, we obtain the desired pdf of $\tilde{\gamma}_m$ as in (14).

Appendix B
Theorem 1

We write the outage probability of $\gamma_{\mathtt{E_m}}$ as in [14]

$$F_{\gamma_{\mathtt{E_m}}}(\Lambda) = 1 - \int_0^\infty \tilde{F}_{\gamma_s}\left(\frac{\Lambda(x+\Lambda+1)}{x}\right) f_{\tilde{\gamma}_m}(x+\Lambda)dx \tag{32}$$

where $\tilde{F}(\cdot) = 1 - F(\cdot)$. We substitute cdf of γ_s and (14) in (32). With some mathematical simplification we obtain

$$\tilde{F}_{\gamma_{\mathtt{E1_m}}}(\Lambda) = \sum_{n_1=0}^{M-1} \binom{M-1}{n_1} \frac{(-1)^{n_1} M\Psi\beta!}{\Gamma(m_2)(1-\rho)} \sum_{p=0}^{\beta} \binom{\beta+m_2-1}{\beta-p} \tag{33}$$

$$\times \frac{\lambda^{2p+m_2-1}\left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2}}{\alpha^{\beta+m+p}p!\rho^{\frac{m_2-1}{2}}} \sum_{i=0}^{m_1-1} \frac{e^{-\frac{m_1\Lambda}{\bar{\gamma}}-\frac{\varsigma m_2\Lambda}{\bar{\gamma}_u}}\left(\frac{m_1\Lambda}{\bar{\gamma}_s}\right)^i}{i!}\Delta_1$$

where

$$\Delta_1 = \int_0^\infty \frac{(x+\Lambda+c)^i(x+\Lambda)^{p+m_2-1}}{x^i} e^{-\frac{m_1\Lambda(\Lambda+c)}{\bar{\gamma}_s x}-\frac{m_2\varsigma x}{\bar{\gamma}_u}}dx \tag{34}$$

By using the Binomial theorem twice with some steps we get

$$\Delta_1 = \sum_{q=0}^{i}\binom{i}{q}c^{i-q}\sum_{s=0}^{q+p+m_2-1}\binom{q+p+m_2-1}{s} \tag{35}$$

$$\times \Lambda^{q+p+m_2-1-s}\int_0^\infty x^{s-i}e^{-\frac{m_1\Lambda(\Lambda+c)}{\bar{\gamma}_s x}-\frac{m_2\varsigma x}{\bar{\gamma}_u}}dx$$

Now use of [12, Eq. 3.471.9] with simplification we find the outage probability of $\gamma_{\mathtt{E1_m}}$ as in (18).

Appendix C
Theorem 2

Ergodic capacity expression mentioned in (19) can be rewritten as

$$C_{erg} = \frac{1}{2}E_{\tilde{\gamma}_{\mathtt{E1_m}}}\left[\log_2\left(1+\frac{\gamma_s\tilde{\gamma}_m}{\gamma_s+\tilde{\gamma}_m+1}\right)\right] \tag{36}$$

After some mathematical manipulation we obtain C_{erg} as in [15]

$$C_{erg} = \frac{1}{2\ln 2}\Big[E_{\gamma_s}\left[\ln(1+\gamma_s)\right] + E_{\tilde{\gamma}_m}\left[\ln(1+\tilde{\gamma}_m)\right] \tag{37}$$

$$- E_{\gamma_s+\tilde{\gamma}_m}\left[\ln(1+\gamma_s+\tilde{\gamma}_m)\right]\Big]$$

$\zeta_1 = E_{\tilde{\gamma}_{\mathtt{E1_m}}}\left[\ln(1+\gamma_s)\right]$ and $\zeta_2 = E_{\tilde{\gamma}_{\mathtt{E1_m}}}\left[\ln(1+\tilde{\gamma}_m)\right]$ can be derived taking the expectation with respect to the pdf in (8) and (14) respectively with the help [12, Eq. 4.337.5]. Pdf of $\gamma_s + \tilde{\gamma}_m$ can be derived by taking the convolution and using [12, Eq. 3.351.1] as

$$f_{\gamma_s+\tilde{\gamma}_m}(x) = \begin{cases} \delta_1(x), & \frac{m_1}{\bar{\gamma}_s} = \frac{\varsigma m_2}{\bar{\gamma}_u} \\ \delta_2(x), & \frac{m_1}{\bar{\gamma}_s} \neq \frac{\varsigma m_2}{\bar{\gamma}_u} \end{cases} \tag{38}$$

where

$$\delta_1(x) = M\sum_{n_1=0}^{M-1}\binom{M-1}{n_1}\frac{(-1)^{n_1}\Psi\beta!}{\Gamma(m_2)(1-\rho)}\sum_{p=0}^{\beta}\binom{\beta+m_2-1}{\beta-p} \tag{39}$$

$$\times \frac{\lambda^{2p+m_2-1}}{\alpha^{\beta+m+p}p!\rho^{\frac{m_2-1}{2}}}\left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2}\left(\frac{m_1}{\bar{\gamma}_s}\right)^{m_1}$$

$$\times \sum_{q=0}^{m_1-1}\binom{m_1-1}{q}\frac{(-1)^q}{q+p+m_2}x^{p+m_1+m_2-1}e^{-\frac{m_1 x}{\bar{\gamma}_s}}$$

$$\delta_2(x) = M\sum_{n_1=0}^{M-1}\binom{M-1}{n_1}\frac{(-1)^{n_1}\Psi\beta!}{\Gamma(m_2)(1-\rho)}\sum_{p=0}^{\beta}\binom{\beta+m_2-1}{\beta-p} \tag{40}$$

$$\times \frac{\lambda^{2p+m_2-1}}{\alpha^{\beta+m+p}p!\rho^{\frac{m_2-1}{2}}}\left(\frac{m_2}{\bar{\gamma}_u}\right)^{p+m_2}\left(\frac{m_1}{\bar{\gamma}_s}\right)^{m_1}\sum_{q=0}^{m_1-1}\frac{(-1)^q}{\Gamma(m_1)}$$

$$\times \binom{m_1-1}{q}\frac{(p+m_2+q-1)!}{\left(\frac{\varsigma m_2}{\bar{\gamma}_u}-\frac{m_1}{\bar{\gamma}_s}\right)^{p+q+m_2}}\Bigg[x^{m_1-1-q}e^{-\frac{m_1 x}{\bar{\gamma}_s}}$$

$$- e^{-\frac{\varsigma m_2 x}{\bar{\gamma}_u}}\sum_{k=0}^{p+m_2+q-1}\frac{1}{k!}\left(\frac{\varsigma m_2}{\bar{\gamma}_u}-\frac{m_1}{\bar{\gamma}_s}\right)^k x^{m_1-1-q+k}\Bigg]$$

$\zeta_3 = E_{\gamma_s+\tilde{\gamma}_m}\left[\ln(1+\gamma_s+\tilde{\gamma}_m)\right]$ can be derived by taking the expectation with respect to (38) with the help of [12, Eq. 4.337.5]. By using the derived ζ_1, ζ_2 and ζ_3 we can obtain the exact closed form solution for ergodic capacity as in (20).

References

[1] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless. Commun.,* vol. 6, no. 9, pp. 3450-3460, Sep. 2007.

[2] B. Barua, H. Q. Ngo and H. Shin, "On the SEP of Cooperative Diversity with Opportunistic Relaying," *IEEE Commun. Lett.*, vol. 12, no. 10, pp. 727-729, Oct. 2008.

[3] I. Krikidis, J. Thompson, S. McLaughlin and N. Goertz, "Amplify-and-Forward with Partial Relay Selection," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 557-559, Apr. 2008.

[4] D. B. da Costa and S. Aïssa, "End-to-end performance of dual-hop semi-blind relaying systems with partial relay selection," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4306-4315, Aug. 2009.

[5] ——, "Performance Analysis of Relay Selection Techniques With Clustered Fixed-Gain Relays," *IEEE Signal Proc. Lett.*, vol. 17, no. 2, pp. 201-204, Feb. 2010.

[6] ——, "Amplify-and-Forward Relaying in Channel-Noise-Assisted Cooperative Networks with Relay Selection," *IEEE Commun. Lett.*, vol. 14, no. 7, pp. 608-610, Jul. 2010.

[7] M. Torabi and D. Haccoun,"Capacity Analysis of Opportunistic Relaying in Cooperative Systems with Outdated Channel Information," *IEEE Commun. Lett.*, vol. 14, no. 2, pp. 1137-1139, Dec. 2010.

[8] H. A. Suraweera, M. Soysa, C. Tellambura and H. K. Garg, "Performance Analysis of Partial Relay Selection With Feedback Delay," *IEEE Signal Proc. Lett.*, vol. 17, no. 6, pp.531-534, Jun. 2010.

[9] D. S. Michalopoulos, H. A. Suraweera, G. K. Karagiannidis and R. Schober, "Amplify-and-Forward Relay Selection with Outdated Channel State Information," *in IEEE Globecom 2010*, Miami, Florida, Dec. 2010, pp. 1-6.

[10] N. S. Ferdinand, N. Rajatheva and M. Latva-aho, "Effects of the feedback delay in partial relay selection over Nakagami-m fading channels," *IEEE Trans. Veh. Tech.*, vol. 61, no. 4, pp. 1620-1634, May 2012.

[11] M. K. Simon and M.-S. Alouini, *Digital Communications over Fading Channels: A Unified Approach to Performance Analysis*, 1st ed., New York: John Wiley and Sons, 2000.

[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed., New York: Academic, 2007.

[13] M. O. Hasna and M.S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans.Wireless Commun.,* vol. 2, no. 6, pp. 1126-1131, Nov. 2003.

[14] D. Senaratne and C. Tellambura, "Unified exact performance analysis of two-hop amplify-and-forward relaying in Nakagami fading," *IEEE Trans. on Veh. Tech.*, vol. 59, pp. 1529-1534, Mar. 2010.

[15] L. Fan, X. Lei, and W. Li, "Exact Closed-Form Expression for Ergodic Capacity of Amplify-and-Forward Relaying in Channel-Noise-Assisted Cooperative Networks with Relay Selection" *IEEE Commun. Lett.*, vol. 15, no. 3, pp. 332-333, Mar. 2011.