Clique-based Capacity Analysis of Wireless Ad-hoc Networks with Cooperative Relaying in Multi-flow Scenario

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***Abstract*—Cooperative relaying techniques have been shown to provide spatial diversity in fading wireless environment. As a result, they increase link reliability, provide higher capacity, reduce transmit power, and extend transmission range as opposed to non-cooperative transmission (direct transmission). Although the use of cooperative relaying has been proven to achieve those gains in the absence of interference (single flow), it is not clear how much gain we can expect from using cooperative relaying in multi-flow scenario specifically the total network capacity. The network capacity in multi-hop multi-flow settings is severely affected by interference between links and, this effect increases when the cooperative relaying is imposed. In this paper, we use a clique based capacity analysis to investigate the performance gain (loss) on network capacity for wireless ad hoc networks by using cooperative relaying. It is observed that the throughput drops significantly when cooperative links are imposed in the network.**

Keywords: Cooperative relaying, Cliques, throughput, interference.

I. Introduction

Cooperative relaying has emerged as a promising technique to combat multi-path fading in wireless communications and also showed performance improvement in wireless relay networks. The advantage of this type of cooperation is that each node needs only a single antenna, and a virtual antenna array is formed through multiple nodes in the network. The basic idea of cooperative relaying is that between the transmitter and the receiver nodes, there can be another node (or multiple nodes), which can be used to provide spatial diversity by forming a virtual multi-antenna system. An interesting feature of the cooperative relaying approach is that the additional relaying nodes carrying the same copy of the packet can be used to boost the received signal quality and mitigate channel fading, thus improving the reliability of the network.

Design of efficient routing schemes for cooperative wireless network is more challenging compared to the traditional non-cooperative case. It has been an active research area in recent years with different objectives such as to minimize the end-to-end energy consumption [1], [2], prolong the network life time [3], minimize the end-to-end outage probability [4] and achievable bit error rate [5] etc. In our previous works, we showed that using cooperative relaying at the physical layer we can save energy [2] by solving an optimization problem that jointly allocates power, relay and routing, and extends the transmission range by a factor of $\omega(\geq 1)$ compared to direct transmission [6] using the same transmit power. It has been revealed that the above mentioned objectives can be achieved by using of cooperative transmission due to the achievable diversity gain.

Most of these works including ours, however, did not consider the impact of using cooperation when there are multiple flows in the network and they studied the benefits of cooperative transmission for only one flow (i.e., no interference) as opposed to direct transmission. As we will show in the next sections, using another node as a cooperative relay changes the interference distribution in the network which will affect the aggregated capacity of the network in the sense that links that are able to simultaneously transmit in non-cooperative network may not be able to do so in the case when cooperative transmission is used. In an ad-hoc network, when a node is transmitting to one of its neighbors, other neighbors need to be silent due to the interference in the medium. Adding cooperative relay for transmission blocks other nodes located in the relay's transmission range. This effect could seriously limit the usefulness of cooperation in terms of total network capacity.

Our contribution of this paper is that we investigate the performance gain (loss) on the network capacity for wireless ad hoc networks by using cooperative relaying. Also an analytical model for interference calculation using conflict graph is presented. We use cliques to derive the capacity constraints within each clique and hence derive the upper bound on the maximum capacity of the network.

The rest of this paper is organized as follows. Section II gives the motivation of this work. Section III describes the system architecture including network and channel model, general interference model, and some concepts of graph theory. Section IV presents the problem formulation. Section V presents the numerical results. Finally, we conclude in Section VI.

II. MOTIVATION

In this section, we give an example to show that adding cooperative relaying to direct links could block new links, thus affect the total throughput of multi-source multi-destination communication in wireless networks.

Consider a network as shown in Fig. 1(a) and suppose there are two flows in the network. The source, destination and intermediate nodes of flow 1 and flow 2 are $\{S1 \rightarrow N1 \rightarrow N2 \rightarrow D1\}$ and $\{S2 \rightarrow M1 \rightarrow M2 \rightarrow D2\}$ respectively. We assume that the interference range equals to the transmission range and all the links use traditional transmission. Interference range is shown as circle around each node. We can see that there is no conflict between the links scheduled for flow 1 and flow 2. Thus, the network achieves the maximum possible network capacity.

Now consider that node $K1$ is involved as a cooperative relay for the link between nodes $M1$ and $M2$ as shown in Fig. 1(b). As a result, the two flows conflict with each other in node $K1$. Thus, the two links $S1 \rightarrow N1$ and $M1 \rightarrow M2$ will take turn and the per-flow-throughput for each link is decreased by 50% compared with traditional transmission scenario discussed above. Thus, the network may not achieve the maximum network capacity.

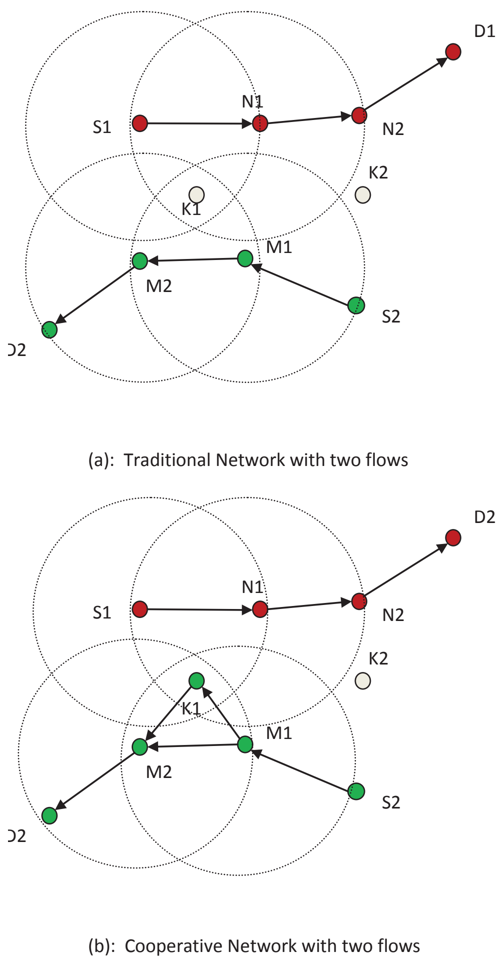

Fig. 1. An example to illustrate the conflict between flows in case of (a) direct transmission, and (b) cooperative transmission.

The above example illustrates that cooperative strategy is beneficial in the case of single flow scenario; when used in the network with multiple flows, it may cause collision due to interference caused by the relay and, thus reduce the overall system throughput. Moreover, increasing the number of cooperative links in the network results more links being blocked and the total throughput will degrade. Thus, we need to investigate the capacity of the cooperative networks with multi-flows and how we can benefit efficiently by using cooperation in such situations.

III. SYSTEM ARCHITECTURE

In this section, we discuss the network and channel model, interference analysis, and the conflict graph.

A. Network and Channel Model

We consider a multi-hop wireless ad hoc network consisting of N nodes, where each node is equipped with single omni-directional antenna. These N nodes are assumed to be uniformly distributed in a square area. The decode-and-forward (DF) cooperation scheme is employed. We model the multi-hop wireless network as a directed graph $G = (V, E)$ where V is the set of vertices (nodes) and E is the set of edges (links). Without loss of generality, we assume all nodes transmit with the same power level, employ similar modulation and coding scheme.

The channel between any two nodes i and j in the network is modeled using a combination of small-scale fading and path loss [7]. The small-scale fading is quasi-static Rayleigh fading in nature, where the channel remains the same for several transmission blocks, i.e., inter-node channels change very slowly (i.e., the channel coherence time is much longer than the block transmission duration). The transmitted signal also suffers from propagation path loss that causes the signal to attenuate with distance. The signal received at the receiving node j from transmitting node i is modeled as,

$$y_{i,j} = \sqrt{p.d_{i,j}^{-\alpha}}.h_{i,j}.x_i + n_{i,j}; \quad i,j \in \{1...N\}, \ i \neq j, \quad (1)$$

where p is the transmitting power (for simplicity analysis, we assume that all nodes transmit at the same power level), $d_{i,j}$ is the distance between nodes (i and j), α is the path loss exponent ranging between 2-4, $h_{i,j}$ captures the channel fading gain, x_i is the transmitted signal with average unit power, and $n_{i,j}$ is the AWGN with zero mean and variance N_0.

B. Generic Interference Model

In a wireless ad hoc network, not all links can transmit simultaneously due to interference and resource contention. To characterize the radio transmission in the presence of interference, two different interference models have been used in the literature physical and protocol model.

1) Physical interference model: In this model, a message can be transmitted successfully between two nodes if the received signal to interference plus noise ratio ($SINR$) exceeds a given threshold β, that is

$$SINR = \frac{p_{r,i}}{N_0 + \sum_{j \neq i} p_{r,j}} \geq \beta, \quad (2)$$

where β is the minimum $SINR$ required for a successful message reception. N_0 is the ambient noise power, $p_{r,i}$ is the power received from the source, and $p_{r,j}, (j \neq i)$ is the power received from a set of transmitters that are transmitting simultaneously with the source that is considered as interference. Although the physical interference model imposes realistic condition for successful reception and it is widely considered as a reference model for physical layer, it is not appropriate for constructing the conflict graphs and the application of this model is limited in some scenarios specifically in multi-hop wireless networks due to its higher complexity. Thus, in this work, we adopt the protocol interference model.

2) Protocol interference model: In this model, any two nodes can communicate and their transmission is successful if and only if the receiving node is located within the transmission range of the intended transmitting node and is outside the interference range of any other node that is actively transmitting on the same band. In protocol interference model, the transmission from node i to node j is successful if both of the following conditions are satisfied for every other node k that is simultaneously transmitting or receiving.

$$d_{i,j} < r_C \text{ and } d_{k,j} > r_I, \tag{3}$$

where r_C and r_I are a radio transmission and interference range sensitivity region respectively. We assume each node knows its position (e.g. using GPS) and disseminates its position information to other stations in the local neighborhood. Each station then geometrically computes which stations are within an interference radius; we call such stations interfering neighbors.

Assume that each node transmits with fixed power and has the same interference sensitive range r_I. The interference sensitive region is the combination of two end-nodes plus cooperative relay for cooperative links as shown in Fig. 2. In this figure, the direct and cooperative link's interference sensitive range combination are modeled using a unit disc graph [8] and is approximated by a disk with a radius of $r_{direct} = d_{direct}/2$ and $r_{coop} = d_{coop}/2$ centered at the median between S-D and S-R-D respectively, where d_{direct} and d_{coop} are given by.

$$d_{direct} = r_I + r_C = (\lambda + 1) r_C \tag{4}$$

$$d_{coop} = \begin{cases} r_I + r_C = (\lambda+1) r_C & : r_1^2 + r_2^2 \le r_C^2 \\ r_I + \frac{1}{2} r_C \left(\sqrt{1 + \frac{(r_1^2 + r_2^2 - r_C^2)^2}{4 r_1^2 r_C^2 (r_c^2 + r_1^2 - r_2^2)^2}} + 1 \right) & : \text{Otherwise}, \end{cases} \tag{5}$$

where $\lambda = \frac{r_I}{r_C}$ is refereed to as the interference sensitive range and; r_1, r_2 are the distance between source-relay and relay-destination respectively.

It is clear from (5) that a cooperative link has a much larger interference-sensitive region than direct link which leads to degradation of the total network throughput.

C. Conflict Graph and Cliques

Graph theory has been used as a modeling technique to analyze several properties of the wireless network including interference relationships between all of the links in a network.

1) Connectivity Graph: A given network can be modeled as a connectivity graph $G = (V, E)$ where V is the set of vertices (nodes) and E is the set of edges (directed links). A link $l_{i,j} \in E$ exists if the range between nodes i and j is less than transmission range, i.e., there is a directed link from vertex i to vertex j if $d_{i,j} < r_C$. The graph $G(V, E)$ is said to be connected if there is a path connecting any two vertices

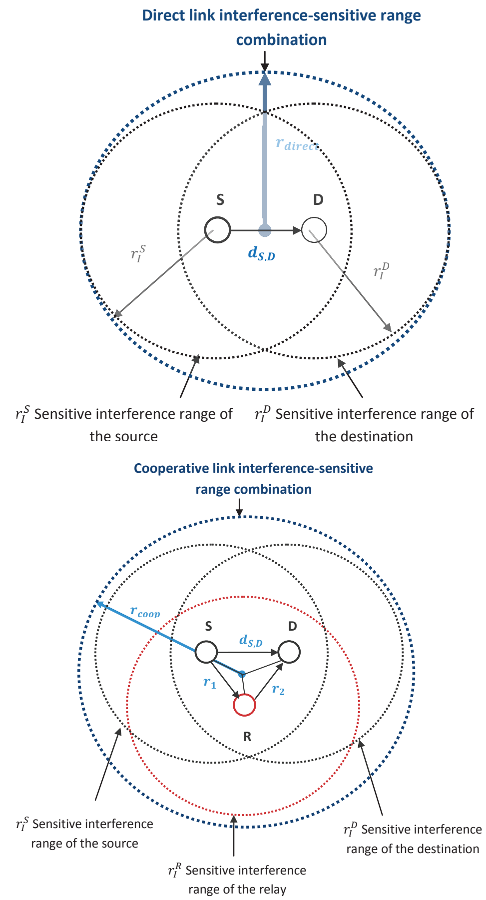

Fig. 2. An example to illustrate the sensitive interference range for (a) direct transmission, and (b) cooperative transmission

(nodes) in V. The cardinality of the graph is the number of vertices V, and the degree of vertices is the number of edges attached to that vertex.

When cooperative transmission is employed in the network, some direct links will emerge into a virtual link (cooperative link), therefore, a virtual link based connectivity graph $G = (V, E')$ is constructed based on the original connectivity graph $G = (V, E)$, where, $E' = E \cup E_c$, and E_c is the set of new edges (virtual links) introduced by using cooperation.

2) Conflict Graph: Conflict graph (CG) has been used to describe the contention relationship between links and to obtain the link capacity constraints imposed by the interference between links, which is derived from the connectivity graph. The vertices of the conflict graph $CG(V^C, E^C)$ represent links in the connectivity graph $G(V, E)$ and there is an edge between any two vertices (links in the connectivity graph) if they interfere with each other.

Generally, any two links in the network graph that have a node in common are connected in the conflict graph, but the definition of the link in cooperative transmission has changed, which may include one or more relays helping the source to forward its data and should have an edge in a generic conflict graph.

Fig. 3 illustrates an example of the conflict graph formation in a simple network with four links. As we can see in

Fig. 3(a) where no cooperation is used, link 1 (l_1) could simultaneously transmit with link 4 (l_4). However, they may not be active simultaneously when cooperative transmission is imposed with link 1 (l_1) as shown in Fig. 3 (b). Therefore, we generalized the traditional CG to reflect the new contention relation between direct and cooperative links (new virtual links), therefore, new edges should be added to the graph accordingly.

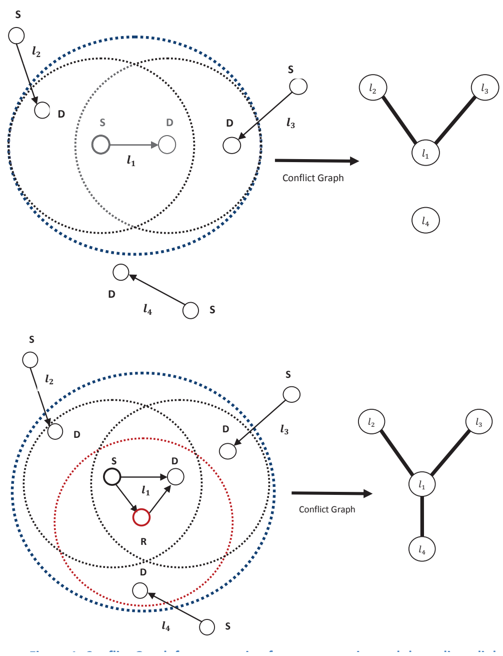

Fig. 3. An example to illustrate the conflict graphs for two scenarios (a) with four direct transmission links (b) with one cooperative and three direct transmission links.

3) Cliques: A clique in an undirected graph $G = (V, E)$ is a subset of the vertex set $H \in V$, such that for every two vertices in H, there exists an edge connecting the two (i.e., a clique is a complete subgraph where all vertices are adjacent). We use the concept of cliques on the conflict graph to capture the interference relation among all links when cooperative relaying is considered. In the interference graph, a clique represents the links which can not be active at the same time. Therefore, corresponding to each clique H in the interference graph, we get a constraint.

$$\sum_{(i,j)\in H} \mu_{ij} \leq 1 \tag{6}$$

where, $\mu_{i,j}$ is a fraction of time link (i, j) is active.

IV. Problem Formulation and Solution

We use cliques to obtain the conflict regions and then derive new constraints by identifying the various maximal cliques in the conflict graph. The clique constraints derived are necessary but not always sufficient for link scheduling [9] and hence, the solution of the optimization problem is the upper bound for the network capacity.

Given a CG, we use a linear programming to compute the optimal network capacity in the cooperative networks. In particular, we formulate the problem as a multi-commodity flow (MCF) optimization problem, augmented by the additional interference constraints derived from the the cooperative CG due to imposing cooperative relaying in some links in the network. The solution of the MCF formulation not only presents the maximum achievable capacity for wireless cooperative networks, but also jointly indicates the optimal scheduling of link transmissions, and the optimal routing.

Define that each commodity is associated with a source destination pairs (S, D). A common formulation is to maximize the total throughput (or capacity) over all source destination pairs, however such objective function may lead to starvation of some commodity flows[1]. Given $G(V, E')$ and a set of M commodities each with source destination pair S^m, D^m, our MCF formulation can be expressed as a mixed-integer linear programming problem as follows:

$$\max \sum_{m\in M} f^m \tag{7}$$

subject to:

$$\sum_{(i,j)\in E'} x_{i,j}^m - \sum_{(j,i)\in E'} x_{j,i}^m = \begin{cases} f^m, & i = S^m \\ -f^m, & i = D^m \\ 0, & \text{Otherwise} \end{cases} \tag{8}$$

$$\sum_{(i,j)\in q} \sum_{m\in M} x_{i,j}^m \leq C \quad \forall q, \tag{9}$$

$$\sum_{m\in M} x_{i,j}^m . \phi_{i,j}^m > 0 \quad \forall i,j \in E_c, \tag{10}$$

$$\phi_{i,j}^m \in \{0, 1\} \quad \forall i,j \in E_c, \tag{11}$$

$$x_{i,j}^m \geq 0, \quad \forall (i,j), m \tag{12}$$

where $x_{i,j}^m$ is the amount of flow from the m^{th} commodity over link $(i, j \in E')$ normalized with respect to the capacity of the channel. The term f^m denotes the normalized flow coming out from source S^m, and q is the maximal clique belongs to the set of maximal cliques Q.

In the above formulation, the objective in (7) is to maximize the total throughput of all commodities. The first constraint in (8) represents the flow conservation constraints at each node for each commodity. The constraint in (9) represents the clique's capacity constraint, the sum of all flows on all the link belonging to each maximal clique is bounded by the channel capacity C. Constraints (10) and (11) are used to ensure that at least one cooperative link is scheduled in the network, where $\phi_{i,j}^m$ is a set of binary value integer variables taking values 0 or 1. The constraint in (12) ensures that the flow over each link is a positive quantity.

[1] In this work we are interested on the total capacity, considering the fairness issue presented in some literature that seeks to maximize the total throughput of the network and at least some amount of throughput can be ensured for each commodity.

V. Performance Evaluation and Numerical Results

In this section, we present some numerical results based on our formulation and solution. We use MATLAB to implement our network topology and ILOG CPLEX optimizer [10] to compute the maximum flow by solving our multi-commodity flow problem for multi-hop wireless networks with and without cooperative transmission.

To show the effectiveness of using cooperative relaying in multi flow scenarios, we consider a static multi-hop wireless network with variant number of nodes between 10 and 25 nodes randomly deployed in 5000m x 5000m area. A set of five flows is randomly chosen. The link capacity is normalized to 1. In each topology, three nodes are selected to form a cooperative link[2] in the network.

Figure 4 shows the differences of the average total number of edges in the conflict graph (conflicts between links) between direct and cooperative conflict graph for different network densities. As expected, the network has slightly larger number of conflicts between links when cooperative transmission is used due to the interference caused by the relays and increases exponentially with the network size.

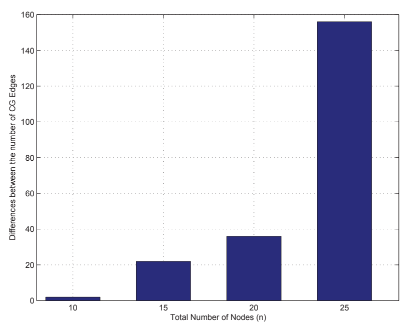

Fig. 4. Differences of the number of conflict graph edges between cooperative and non-cooperative network with different node densities.

Figure 5 shows the upper bound on the maximum flow in the network with respect to the number of nodes with one cooperative zone in the network. Here, the number of flows is set to be 5. With the number of nodes increasing, the total capacity of the network, when cooperative transmission is used between some nodes slightly decreased as compared to the traditional transmission. That is because, with cooperative transmission links the interference range is increased and hence more links are contending for same wireless medium.

These results are expected since the benefits of cooperative transmission have been studied in non-interference scenario. To avoid the degradation of the total network capacity which is much more likely to occur, more cost on channel allocation should be paid which will be our focus for the future work.

VI. Conclusion and Future work

In this paper, we investigated the performance of multi-hop wireless ad hoc networks with cooperative transmission.

[2]Select a group of three nodes so as to create a cooperative link between any pair by using the third node as a relay.

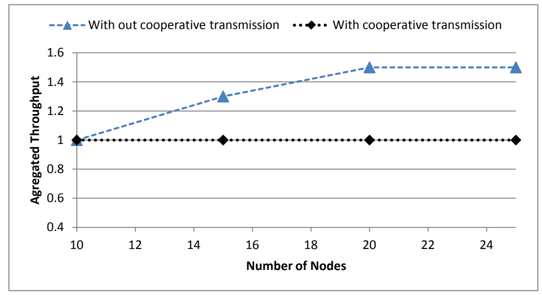

Fig. 5. Maximum flow of the network with and without cooperative transmission.

We addressed of the feasibility of a given set of flows on an arbitrary ad-hoc cooperative network, by formulating the problem as a multi-commodity flow problem and derived the upper bound on the capacity of such networks. We used the conflict graph and its cliques to derive the interference constraints. We then used these cliques to write constraints that provide sufficient conditions for feasibility within a constant bound of the optimal.

Our solution results show that the capacity of cooperative multi-hop wireless networks can be significantly decreased even with only one cooperative zone. Our results are realistic since the sensitive interference range of cooperative links is greater than direct links which means that some new links will be blocked.

To mitigate the impact of interference among concurrent transmissions in multi-hop settings and thus enhance the performance of cooperative networks, we plan to formulate the problem with multiple channels that could reduce the wireless interference giving rise to diversity gain and thus greatly improve the overall network capacity.

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