

# Elastic Game Based Radio Resource Management

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**Abstract**—With the abundance of diverse air interfaces in the same operating area, a mobile user is able to connect concurrently to different wireless access networks in order to meet more easily its target performance. In this paper, we consider the downlink of a multi-class hybrid network with two Radio Access Technologies (RAT): WiMAX [1] and WiFi [2]. We devise a distributed Radio Resource Management (RRM) scheme for elastic traffic that coexists with streaming traffic. The proposed scheduling policy is original in the sense that elastic users have a counterintuitive behaviour: they will try to occupy the least amount possible of bandwidth owing to their delay tolerance to accommodate QoS stringent streaming users. A non-cooperative submodular game is used to load balance the traffic of elastic users between the two available RATs aiming at minimizing their bandwidth consumption. We characterize the Nash Equilibriums (NE) of the resource management game and study the efficiency of a distributed algorithm based on best response dynamics to achieve those equilibriums. The game is played upon every new arrival and an admission control scheme is used to limit the number of ongoing connections so that admitted elastic flows are sustained with a guaranteed minimal rate.

## I. INTRODUCTION

The migration of wireless networking towards the 4G era is distinguished by the proliferation of radio access technologies. In this composite radio environment, mobile users will be able to simultaneously utilize services through diverse RATs. The load balancing policy among the available RATs is crucial and must be designed astutely to avoid resource wastage.

In this paper, we devise a scheduling policy for elastic traffic in a realistic environment where it potentially coexists with streaming traffic in a multi-class hybrid network. A WiMAX-WiFi network is considered because of the predominance of these two technologies. The proposed RRM scheme is transversal as a distributed approach is adopted but with a "system-like" objective. In fact, the load balancing made by every elastic user aims at minimizing its perceived capacity as the performance target is global welfare. The idea is to profit from elastic traffic delay tolerance, to ensure that streaming users benefit from large enough bandwidth so that their packets are forwarded with very low latency. As elastic users must be guaranteed a minimal rate to enjoy good performances, the RRM policy is defined as a constrained minimization problem. We resort to non-cooperative game theory to attain that goal and obtain distributed elastic traffic allocation among the different RATs. We show that, under some legitimate conditions, the proposed game have the S-modular type structure introduced by Topkis in [3]. The practical importance of this type of non-cooperative games is twofold: the existence of NEs; and the fact that these equilibriums can be attained using greedy best-response type algorithms.

RRM as a non-cooperative game theory is an emerging area of research and few related work is available in the literature. In [4], power consumption is devised as a submodular game. In [5], the approach is based on a 2 players finite game that is played by access networks (which is rather a centralised approach). In [6], the selection between WiFi et UMTS is formulated as an interesting symmetric non-cooperative game that reduces to a threshold policy. The work in [7] lies on the maximization of some marginal cost pricing where the network notifies users of some reward to enhance the system performance. All existent work is different from our proposed policy where a fully distributed algorithm for elastic traffic is devised in the scope of accomodating QoS stringent streaming traffic while ensuring elastic traffic gets a vital minimal rate.

The rest of the paper is organized as follows. Traffic and capacity models are described in sections II and III. The proposed traffic allocation is formulated as a non-cooperative game in section IV where the admission control scheme is sketched. Submodularity of the game is proven in V. In section VI, we outline the set of optimal strategies and how to attain those equilibriums through a best response algorithm. Section VII gives numerical results and shows how our optimal policy outperforms a common sense policy. We conclude in VIII.

## II. TRAFFIC MODEL

Flows are basically produced by two types of applications: streaming and elastic. For the two types of flows to coexist without degrading mutually their performance, priority must be given to streaming flows while a minimum rate must be guaranteed for elastic flows. In [8], it is proven that giving the largest share of bandwidth to streaming packets ensures maximal responsiveness for the underlying streaming applications. Elastic flows share the remaining bandwidth unused by streaming traffic. To emulate this approach, we propose a RRM scheme where each elastic flow load balances its traffic among WiMAX and WiFi access networks, aiming at minimizing its rate provided that it stays greater than a threshold  $C_{min}$ .

To guarantee elastic traffic that minimal vital rate, it is necessary to adopt an admission control scheme in line of that proposed in [9]. Indeed, in overload, the performance of elastic flows in an uncontrolled system would rapidly deteriorate. Streaming flows would suffer much less in view of the preferential treatment they are given. Thus, whenever the rate of an elastic flow goes below  $C_{min}$ , all new flows are blocked. New flows are admitted back in the system whenever the individual rate of existing elastic flows grows beyond the minimal threshold.

### III. CAPACITY MODEL

The WiFi and WiMAX base stations are co-localized in every cell. The index  $x$  is used throughout the paper to designate a given RAT  $x$  where  $x = WM$  is used for WiMAX and  $x = WF$  for WiFi. We denote by  $n$  the total number of admitted elastic users in the system.

The aim of the article is to appropriately allocate the traffic of each elastic user between the available RATs. For that, we denote by  $\theta_k$  the instantaneous fraction of time user  $k$  is assigned to WiMAX and hence  $1 - \theta_k$  of time is spent in WiFi. In both WiFi and WiMAX standards, the set of achievable instantaneous peak rates is not continuous. Indeed, coding constraints result in a discrete set of achievable rates (given in [1] and [2]) denoted by  $C_{k,x}$ ,  $k \in [1, \dots, M_x]$  where  $M_x$  is the maximum number of achievable rates in RAT  $x$ . Thus, the instantaneous rate  $\mathfrak{R}_{k,x}$  of user  $k$  in RAT  $x$  will be given by one of the achievable peak rates.

#### A. Capacity in WiMAX

In WiMAX, the wireless resource is time-shared between active users. We consider a Fair Time sharing model where all users are given the same chance to access resources. The mean capacity of elastic user  $k$  assigned to RAT  $WM$  is:

$$\begin{aligned} C_k^{WM} &= \mathbb{E} \left[ \frac{\mathfrak{R}_{k,WM}}{\sum_{i=1}^n \mathbf{1}_{\{\text{user } i \text{ admitted in RAT } WM\}}} \right] \\ &= C_{k,WM} \cdot \mathbb{E} \left[ \frac{1}{1 + \sum_{i \neq k} \mathbf{1}_{\{\text{user } i \text{ admitted in RAT } WM\}}} \right] \end{aligned}$$

Using the Jensen inequality, a lower bound on the mean capacity is given by:

$$R_k^{WM} = \frac{C_{k,WM}}{1 + \sum_{i=1, i \neq k}^n \theta_i} \quad (1)$$

#### B. Capacity in WiFi

In this paper, the uplink traffic is neglected which leads to a fair access scheme on the downlink channel. However, when a low rate user captures the channel, it will use it for a long time which penalizes high rate users and reduces the fair access strategy to a case of fair rate sharing (assuming a constant MAC frame size and neglecting the 802.11 waiting times (DIFS, SIFS, ...) in comparison with transmission times). Thus, the mean capacity of each elastic user is:

$$C_k^{WF} = \mathbb{E} \left[ 1 / \sum_{i=1}^n \frac{\mathbf{1}_{\{\text{user } i \text{ admitted in RAT } WF\}}}{\mathfrak{R}_{i,WF}} \right]$$

where  $n$  is the total number of admitted users in the system. We notice that the index  $k$  can be omitted as all users have the same rate. Using the Jensen inequality, a lower bound on the mean capacity is given by:

$$R_k^{WF} = 1 / \left( \frac{1}{C_{k,WF}} + \sum_{i=1, i \neq k}^n \frac{1 - \theta_i}{C_{i,WF}} \right) \quad (2)$$

### IV. TRAFFIC ALLOCATION AS A NON-COOPERATIVE GAME

The objective of the present traffic allocation is to set the percentage of traffic that every elastic user should convey through each RAT so that it occupies the least possible amount of resources.

#### A. Cost Function

To analyze user cost for the above RAT assignment, the cost function of an elastic user  $k$  is defined as its mean rate in this hybrid environment and given by the following:

$$U_k(\theta) = R_k^{WM} \cdot \theta_k + R_k^{WF} \cdot (1 - \theta_k) \quad (3)$$

where  $R_k^{WM}$  and  $R_k^{WF}$  are given respectively by (2) and (1).

We formulate a non-cooperative game among elastic users where the objective of each user  $k$  is to minimize its individual utility function  $U_k(\theta)$ .

#### B. The Non-cooperative Game

Game theory models the interactions between players competing for a common resource. Therefore, it is a well adapted tool for radio resource management modelling.

For each state of the system, defined by the number of elastic users  $n$ , we define a multi-player non-cooperative game  $G$  between those  $n$  users present in a double covered area. In this model, there is a sequence of one-stage games, each corresponding to a given state of the system, defined by the number of elastic users. Whenever a new elastic mobile is admitted in the system, the game is played again with an additional player. We assume that players have complete information on each other. Elastic users are assumed to make their decisions without knowing the decisions of each other.

The formulation of this non-cooperative game  $G = \langle N, S, U_k \rangle$  can be described as follows:

- A finite set of players:  $N = (1, \dots, n)$ .
- The space of pure strategies  $S$  formed by the Cartesian product of each set of pure strategies  $S = S_1 \times S_2 \times \dots \times S_n$ . An action of a player is the choice of traffic amount placed on each RAT (or equivalently the probability of choosing a given RAT). Hence  $S_k = [0, 1]$ .
- A set of utility functions  $\{U_1, U_2, \dots, U_n\}$  that quantify the players' preferences over the possible outcomes of the game. Outcomes are determined by the particular action  $\theta_k$  chosen by player  $k$  and the particular actions chosen by all other players  $\theta_{-k}$ .
- The game is as follows for each elastic user  $k$ :  
Minimize  $U_k(\theta)$  subject to  $U_k(\theta) \geq C_{min}$

1) *The Admission Control Scheme:* On arrival of elastic user  $k$ , if the rate it perceives in both RATs is lower than the minimal threshold  $C_{min}$ , it means that the mean rate  $U_k(\theta)$  will be lower than  $C_{min}$  for any strategy  $\theta_k$ . Such a user cannot be admitted in the system. Hence, we apply the following the admission control to ensure that all active elastic users earn at least the minimal rate as preconized in II:

- User  $k$  is accepted if  $R_k^{WM} \geq C_{min}$  or  $R_k^{WF} \geq C_{min}$ ,  
If not, block user  $k$ . (4)

2) *The Constraint Policy*: Each elastic user  $k$  aims at restricting its policy to a subset of policies within  $S_k$  that depends on  $\theta_{-k}$ . We denote this subclass by  $S_k(\theta_{-k})$  and term it constraint policy. Using equation (3), the constraining condition  $U_k(\theta) \geq C_{min}$  rewrites into:

$$\theta_k \cdot (R_k^{WM} - R_k^{WF}) \geq (C_{min} - R_k^{WF})$$

We distinguish the following cases:

- 1) If  $(R_k^{WM} - R_k^{WF}) > 0$ , we have two possibilities:
  - a) If  $R_k^{WF} \geq C_{min}$  then  $S_k(\theta_{-k}) = \{\theta_k : 0 \leq \theta_k \leq 1\}$
  - b) If  $R_k^{WF} < C_{min}$  and further  $R_k^{WM} \geq C_{min}$  then  $S_k(\theta_{-k}) = \{\theta_k : I_k^1(\theta_{-k}) \leq \theta_{k,x} \leq 1\}$  where  $I_k^1(\theta_{-k}) = \frac{C_{min} - R_k^{WF}}{R_k^{WM} - R_k^{WF}}$
- 2) If  $(R_k^{WM} - R_k^{WF}) < 0$  and further  $R_k^{WF} \geq C_{min}$  then  $S_k(\theta_{-k}) = \{\theta_k : 0 \leq \theta_k \leq \min[I_k^2(\theta_{-k}), 1]\}$  where  $I_k^2(\theta_{-k}) = \frac{R_k^{WF} - C_{min}}{R_k^{WF} - R_k^{WM}}$ .
- 3) If  $(R_k^{WM} - R_k^{WF}) = 0$ , any strategy is feasible as long as the mean capacity in any RAT is greater than  $C_{min}$  meaning that  $S_k(\theta_{-k}) = \{\theta_k : 0 \leq \theta_k \leq 1\}$ .

Other possibilities lead to empty strategy sets which is avoided by applying the proposed admission rule in IV-B1.

## V. SUBMODULAR GAME

Under some realistic conditions, the present game is submodular (see [3], [10]). Such games have always have a Nash Equilibrium and it can be attained using a greedy best response type algorithm (called algorithm I in both references).

The present game is submodular if it verifies the following properties for all  $k$ :

- 1) The strategy space  $S_k$  is a compact sublattice<sup>1</sup> of  $\mathbb{R}^m$  (we consider the case where  $m = 1$ ).
- 2) The utility function  $U_k(\theta_k)$  is submodular which means that if it is twice differentiable, it verifies  $\frac{\partial^2 U_k}{\partial \theta_k \partial \theta_j} \leq 0$ , for all  $j \in N - \{k\}$  and  $\forall \theta_k \in [0, 1]$ .
- 3) *Continuity*: The constraint policy  $S_k(\theta_{-k})$  is lower semi continuous in  $\theta_{-k}$ . We say that the point to set mapping that maps elements  $\theta_{-k}$  to  $S_k(\theta_{-k})$  is lower semi continuous if for every  $\theta_{-k}^j \rightarrow \theta_{-k}^*$  and  $\theta_{-k}^* \in S_k(\theta_{-k}^*)$ , there exist  $\{\theta_k^j\}$  s.t.  $\theta_k^j \in S_k(\theta_{-k}^j)$  for each  $k$ , and  $\theta_k^j \rightarrow \theta_k^*$ .
- 4) *Monotonicity*: The constraint policy has the descending property that states the following:

$$\theta'_{-k} \leq \theta_{-k} \implies S_k(\theta_{-k}) \prec S_k(\theta'_{-k})^2 \quad (5)$$

Contrary to the first two properties that are obvious, establishing the last property is tedious.

### A. Submodularity of the Utility Function

The utility function of every elastic user  $k$  is submodular as the following result is non-positive for all  $\theta_k \in S$  and  $k \neq j$ :

$$\frac{\partial U_k(\theta)}{\partial \theta_k \partial \theta_j} = -\frac{C_{k,x}}{(1 + \sum_{i=1, i \neq k}^n \theta_i)^2} - \frac{1/C_{j,\bar{x}}}{(1 + \sum_{i=1, i \neq k}^n \frac{1-\theta_i}{C_{i,\bar{x}}})^2}$$

<sup>1</sup>A is a sublattice of  $\mathbb{R}^m$  if  $a \in A$  and  $a' \in A$  imply  $a \wedge a' \in A$  and  $a \vee a' \in A$

<sup>2</sup>Let  $A$  and  $B$  be sublattices. We say that  $A \prec B$  if for any  $a \in A$  and  $b \in B$ ,  $a \wedge b \in A$  and  $a \vee b \in B$ .

### B. The Descending Property

We first examine the case where constraint policies are of the same type (i.e. both  $S_k(\theta_{-k})$  and  $S_k(\theta'_{-k})$  belong to the same classification in IV-B2).

The descending property is straightforward when constraint policies are either of type 1a or 3. When constraint policies are either of type 1b or 2, the functions  $I_k^1(\theta_{-k})$  and  $I_k^2(\theta_{-k})$  must be non-increasing and continuous for both the monotonicity and continuity properties to be satisfied (see [10]).

**Proposition** The functions  $I_k^1(\theta_{-k})$  and  $I_k^2(\theta_{-k})$  are decreasing for:

$$R_k^{WM} \geq C_{min} \quad (6)$$

**Proof** If  $I_k^1(\theta_{-k})$  (resp.  $I_k^2(\theta_{-k})$ ) is non-decreasing then  $\frac{\partial I_k^1(\theta_{-k})}{\partial \theta_j} \geq 0$  (resp.  $\frac{\partial I_k^2(\theta_{-k})}{\partial \theta_j} \geq 0$ ),  $\forall j$ . As in 1b and 2,  $(C_{min} - R_k^{WF})$  and  $(R_k^{WM} - R_k^{WF})$  have the same sign, the positivity of the partial derivatives of either  $I_k^1(\theta_{-k})$  or  $I_k^2(\theta_{-k})$  amount to the following:

$$\frac{\frac{\partial(C_{min} - R_k^{WF})}{\partial \theta_j}}{(C_{min} - R_k^{WF})} \geq \frac{\frac{\partial(R_k^{WM} - R_k^{WF})}{\partial \theta_j}}{(R_k^{WM} - R_k^{WF})} \implies R_k^{WM} \leq C_{min}$$

The latter result is equivalent to:

$$R_k^{WM} \geq C_{min} \implies \frac{I_k^1(\theta_{-k})}{\partial \theta_j} \leq 0 \text{ (resp. } \frac{I_k^2(\theta_{-k})}{\partial \theta_j} \leq 0), \forall j.$$

■

For constraint policies that belong to different classifications in IV-B2, we examine the types that they can belong to concurrently. Having  $\theta'_{-k} \leq \theta_{-k}$  leads to:

$$R_k^{WM}(\theta'_{-k}) \geq R_k^{WM}(\theta_{-k}) \quad (7)$$

$$R_k^{WF}(\theta'_{-k}) \leq R_k^{WF}(\theta_{-k}) \quad (8)$$

The results in (7) and (8) give the following:

$$R_k^{WM}(\theta_{-k}) - R_k^{WF}(\theta_{-k}) \leq R_k^{WM}(\theta'_{-k}) - R_k^{WF}(\theta'_{-k})$$

Again, we distinguish two cases:

- (1) In the first case we have  $R_k^{WM}(\theta_{-k}) - R_k^{WF}(\theta_{-k}) \geq 0$  which gives  $R_k^{WM}(\theta'_{-k}) - R_k^{WF}(\theta'_{-k}) \geq 0$ 
  - (i) If additionally  $(C_{min} - R_k^{WF}(\theta_{-k})) \leq 0 \implies S_k(\theta_{-k})$  is of type 1a. In view of inequality (8),  $S_k(\theta'_{-k})$  is either of type 1a or 1b. In either case, the descending property is preserved.
  - (ii) Otherwise, if  $(C_{min} - R_k^{WF}(\theta_{-k})) \geq 0 \implies S_k(\theta_{-k})$  is of type 1b. In view of inequality (8),  $S_k(\theta'_{-k})$  is also of type 1b which preserves the descending property under condition (6). The latter condition is fulfilled by enforcing the admission rule in (4).
- (2) In the second case, we have  $(R_k^{WM}(\theta_{-k}) - R_k^{WF}(\theta_{-k})) \leq 0$  which leads us to another distinction depending on the sign of  $(R_k^{WM}(\theta'_{-k}) - R_k^{WF}(\theta'_{-k}))$ :
  - (i) If  $(R_k^{WM}(\theta'_{-k}) - R_k^{WF}(\theta'_{-k})) \leq 0$ , the constraint policies belong to the second classification in 2.

We can further make two distinctions:

- (a) If  $(C_{min} - R_k^{WF}(\theta_{-k})) > 0 \Rightarrow (C_{min} - R_k^{WF}(\theta'_{-k})) > 0$  and hence the descending property can not be verified.
- (b) If  $(C_{min} - R_k^{WF}(\theta_{-k})) \leq 0$ , we have the following two possibilities:
  - (1) One possibility corresponding to  $(C_{min} - R_k^{WF}(\theta'_{-k})) \leq 0$  where both constraint policies are of the same type 2. The descending property is satisfied under condition (6).
  - (2) The other possibility corresponds to the reverse inequality  $(C_{min} - R_k^{WF}(\theta'_{-k})) \geq 0$  where the descending property cannot be met.

For all cases in (2)(i), the admission rule in (4) is enforced. It will override the cases (2)(i)(a) and (2)(i)(b)(2) where the descending property cannot be met. As for (2)(i)(b)(1), it will fulfill condition (6).

- (ii) If  $R_k^{WM}(\theta'_{-k}) - R_k^{WF}(\theta'_{-k}) \geq 0$ ,  $S_k(\theta_{-k})$  is of type 2 and  $S_k(\theta'_{-k})$  is of type 1 which again preserves the descending property.

## VI. ATTAINING THE NASH EQUILIBRIUM

### A. The Admission Control Algorithm

Surprisingly, we deduce that obtaining the constraining policy and verifying the descending property boils down to applying the aforementioned vital admission control policy explained in IV-B1 through the admission rule in (4). This rule is necessary to guarantee elastic users a minimal rate  $C_{min}$  and render the present game submodular.

### B. Optimal Strategies

Every elastic user  $k$  makes the following comparisons successively to find its optimal strategy depending on the mean capacities obtained in each RAT:

- 1) If  $R_k^{WM} > R_k^{WF}$  and  $R_k^{WF} \geq C_{min}$  then the optimal strategy is  $\theta_k^* = 0$ .
- 2) If  $R_k^{WM} > R_k^{WF}$  and  $R_k^{WF} < C_{min}$  then the optimal strategy is  $\theta_k^* = I_k^1(\theta_{-k})$ .
- 3) If  $R_k^{WM} < R_k^{WF}$  and  $R_k^{WF} \geq C_{min}$  then the optimal strategy is  $\theta_k^* = \min[I_k^2(\theta_{-k}), 1]$ .
- 4) If  $R_k^{WM} = R_k^{WF}$ , we enforce only two possible strategies  $\theta_k^* = \{0; 1\}$ , chosen randomly where user  $k$  is present solely in one RAT to avoid costly load balancing.

**Proof** We re-write the cost function as follows:

$$U_k(\theta) = (R_k^{WM} - R_k^{WF}) \cdot \theta_k + R_k^{WF}$$

It is easy to notice that when  $(R_k^{WM} - R_k^{WF}) > 0$ ,  $U_k(\theta_k)$  is minimized for the lowest possible value of  $\theta_k$ , that is 0 and  $I_k^1(\theta_{-k})$  in resp. 1a and 1b. Equivalently, when  $(R_k^{WM} - R_k^{WF}) < 0$ ,  $U_k(\theta)$  is minimized for the largest possible value of  $\theta_k$ , that is  $I_k^2(\theta_{-k})$  in 2. ■

### C. The Updating Algorithm

With a feasible starting point, e.g.  $\theta_k(0) \in S_k$  for  $k = 1, \dots, n$ , the following best-response algorithm iterates until convergence, that is  $\theta_k(t) = \theta_k(t-1)$  for all  $k \in N$ .

**Input:**  $\theta_k(0)$  for  $k = 1, \dots, n$  and  $t = 1$ .

- 1) At time  $t$ , for  $k = 1, \dots, n$ , do the following:

- **Compute**  $R_k^x(\theta_1(t), \dots, \theta_{k-1}(t), \theta_{k+1}(t-1), \dots, \theta_n(t-1))$  for both RATs.
- **Make** the set of comparisons proposed in (VI-B) and set  $\theta_k(t)$  accordingly.

- 2) **Update**  $k = k+1$ . If  $(\theta_k(t) - \theta_k(t-1)) \leq \epsilon \forall k$ , **Output:**  $\theta_k(t) \forall k$ . Else **Do**  $t = t+1$  and **Go to** step 1.

Simulations carried out in the next section show the fast convergence of the proposed algorithm.

## VII. NUMERICAL RESULTS

We consider a cell with a co-localised WiMAX Base Station and a WiFi hotspot. Elastic users arrive in sequence in the system, get one of the achievable peak rates in both RATs and choose a random strategy  $\theta_k$ . Although servicing streaming flows is out of the scope of this paper, for every elastic arrival, we have a streaming arrival that gets in terms of consumed bandwidth twice as much as an elastic user. Performance of streaming users is irrelevant to our work and hence not displayed due to lack of space.

The minimal guaranteed  $C_{min}$  is set to 1.5Mbit/s. The value of this threshold limits the maximum number of elastic users admitted in the system to 22 for our optimal approach.

In order to show the improvements achieved by our approach, we compare it with a common sense approach deemed *trivial* where every arriving elastic user  $k$  simply chooses a random strategy  $\theta_k$  that fixes once for all the amount of traffic sent by the latter through each RAT. We define the average Global Rate as the sum of mobile users individual rates, given by the following:  $\sum_{k=1}^n U_k(\theta)$ , where  $n$  denotes the number of active users. It is denoted by  $U_{opt}$  and  $U_{triv}$  for resp. the optimal and trivial approach.

In Figure 1, for both approaches, the average Global Rate (in Mbit/s) is depicted resp. as a function of the number of elastic users present in the system and as a function of the number of blocked users. The values presented are averaged over a set of 20 simulations. We can clearly see the economy in resources brought on by our proposed approach: at least 20Mbit/s are saved in the optimal approach in comparison with the trivial one as soon as there are 10 elastic users in the system. As for blocking, we see that the trivial approach starts loosing incoming users whenever there are at least 15 active users admitted whereas our approach blocks in average only one user when they are already 21 users in the system.

We illustrate in Figure 2, the evolution of the optimal strategy for the first 5 admitted users in a scenario where 22 elastic users arrive in sequence in the system. At each new arrival, the game is played again and optimal strategies reached are depicted. As WiFi offers lower mean rates than WiMAX, elastic users choose to convey exclusively their traffic through

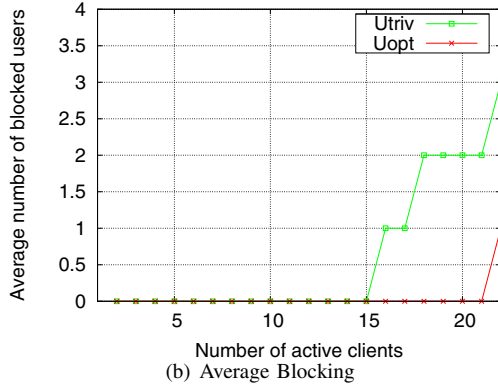
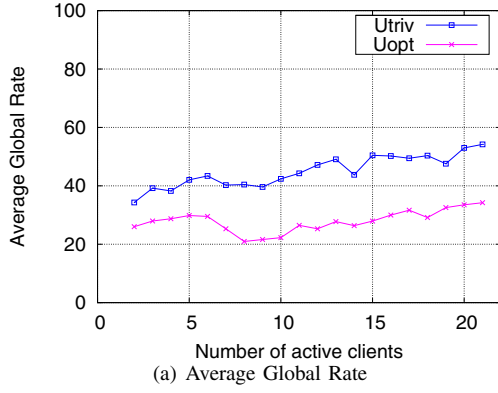


Figure 1. Optimal vs. Trivial Approach

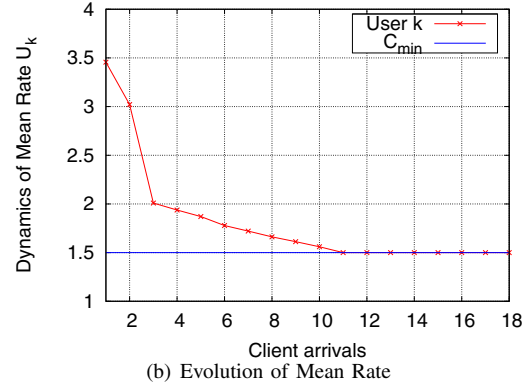
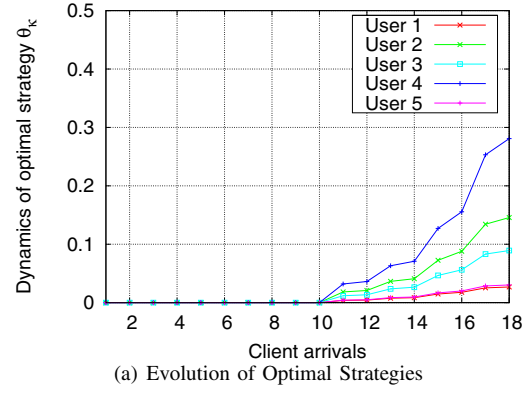


Figure 2. Updates for 5 users initially admitted in the system

the WiFi hotspot. However, as the number of users increases (above 10), WiFi constitutes a bottleneck that can no longer offer  $C_{min}$  and elastic traffic starts migrating to WiMAX by load balancing their traffic through both RATs. To further highlight this behaviour, we reported the mean rate  $U_k(\theta_k)$  variations of a randomly chosen elastic user. We notice that below 10 users in the system, the pinpointed user profits from a rate greater than the minimal threshold. At 10 users, it gets no more than  $C_{min}$  which coincides with the moment it starts using the WiMAX RAT.

## VIII. CONCLUSION

In this paper, radio resources are astutely allocated in a hybrid WiMAX/WiFi network. We devise a game based distributed scheduling scheme that profits from the delay tolerance of elastic users in order to grant them the least possible amount of bandwidth provided that it is greater than a minimal vital threshold. The proposed RRM policy has two goals: on the one hand, it economizes resources; and on the other hand, it accommodates stringent streaming traffic. Owing to the proposed submodular game characteristics, a best response algorithm is used to learn Nash equilibriums. Numerical simulations that compare our approach and a common sense approach allocating resources randomly were conducted. They showed the relevance of our approach through the amount of resources that was saved up.

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