Amplify-and-Forward MIMO Y Channel: Power Allocation Based Signal Space Alignment

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***Abstract*—In this paper, we propose a power allocation based signal space alignment for analog network coding (SSA-ANC) scheme for the MIMO Y channel with amplify-and-forward (AF) strategy. The specific SSA-ANC scheme to achieve the maximum degrees is discussed first, with an emphasize on the design of the precoding vectors. Then, we propose a suboptimal algorithm called OMAC to optimize the power allocation at three users aiming at maximizing the sum rate of the MIMO Y system with zero-forcing (ZF) receiver. It is shown that the proposed power allocation algorithm is a convex optimization problem and therefore a global optimal solution can be obtained. Numerical results show that the OMAC algorithm can achieve improvement in sum rate performance compared to the non-optimization case.**

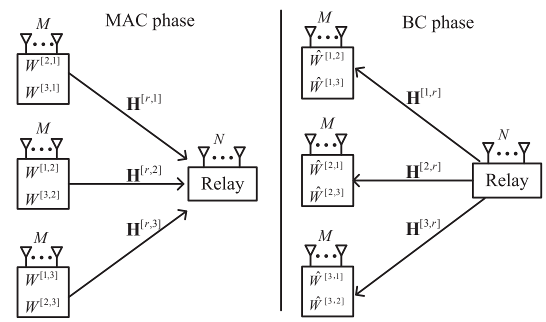

Fig. 1. MIMO Y channel

I. Introduction

Network coding has become a promising transmission technology to improve spectral efficiency and system throughput [1]. Although it was originally introduced in the context of a wired network, the idea of network coding can also be applied to wireless networks due to the broadcast nature of radio transmissions. Furthermore, network coding can be considered as an effective solution to the interference problem, since it transforms the traditional interference avoidance or mitigation into a new perspective of interference exploitation.

The two-way relay channel (TWRC) is one of the most important scenarios for the applications of network coding. The simplest TWRC is a wireless network where two source nodes exchange information via the help of a relay node. By using either physical-layer network coding (PNC) [2] or analog network coding (ANC) [3] in the TWRC, only two time slots are required for two source nodes to exchange one round of information. Specifically, during the first time slot, two source nodes transmit messages to the relay node simultaneously. During the second time slot, the relay processes the received signals and broadcasts them to two source nodes. At last, two source nodes can decode their own desired signals without interference by exploiting side information which they transmit during the first time slot. However, it is well known that three time slots are needed in digital network coding [4] and four time slots are needed if network coding is not used.

As an extension of TWRC, the multi-way relay channel was proposed in [5], where multiple users exchange information via a single relay. Another setup called MIMO Y channel was proposed in [6] and its extension in [7], which consists of three users and a single relay and each node is equipped with multiple antennas. In this system, each user intends to deliver two independent messages to the other two users via the intermediate relay while receiving two independent messages from the other two users. A novel signaling called signal space alignment for network coding (SSA-NC) was proposed in [6], [7], and it is demonstrated in [7] that the number of degrees of freedom for the MIMO Y channel is $3M$ if $N \geq \lceil \frac{3M}{2} \rceil$ with M antennas at each user node and N antennas at the relay node. Here, the relay performs decode-and-forward (DF) strategy by decoding and encoding the network coded messages using the PNC modulation-demodulation mapping principle [2]. In [8], the feasibility conditions of SSA-NC for K-user MIMO Y channel is investigated and achievable degrees of freedom is studied.

Considering that amplify-and-forward (AF) is another popular strategy with simple relay operations, we wonder that whether the AF MIMO-Y channel can provide the same degrees of freedom as the DF MIMO Y channel does. So in this paper, we consider the AF MIMO Y channel where three users transmit their independent messages to the relay simultaneously during the multiple access (MAC) phase, then the relay just amplifies the received signals and broadcasts them to three users during the broadcasting (BC) phase. Accordingly, a power allocation based signal space alignment for analog network coding (SSA-ANC) scheme to is proposed. Specifically, the precoding vectors for both the MAC phase and the BC phase are discussed in detail to achieve the maximum multiplexing gain. Furthermore, a low-complexity algorithm for the optimization of power allocation at three user nodes under the assumption of zero-forcing (ZF) receiver is proposed to maximize the sum rate performance. It is shown that the proposed algorithm is convex and therefore a global optimal solution can be obtained.

The organization of this paper is as follows. System model is described in Section II and the SSA-ANC scheme is proposed in Section III. A power optimization algorithm is proposed in Section IV and numerical results are provided in Section V. At last, the paper is concluded in Section VI.

Notations: We use $tr(\bullet)$, $(\bullet)^T$, $(\bullet)^H$ and $\| \bullet \|$ for trace of a matrix, transpose, conjugation transpose, Euclidian norm, respectively. For an arbitrary $m \times n$ matrix $\mathbf{A}$, we express its singular value decomposition (SVD) as $\mathbf{A} = \mathbf{U\Sigma V}$, where $\mathbf{\Sigma}$ is an $m \times n$ matrix with only diagonal entries nonzero. $\mathbf{U}$ and $\mathbf{V}$ are $m \times m$ and $n \times n$ unitary matrices, respectively.

II. System Model

The system model of the MIMO Y channel, where each user is equipped with M antennas and the relay equipped with N antennas, is illustrated in Fig. 1. Without loss of generality, we assume that M is even and $N = 3M/2$. We also assume that all nodes are half-duplex and that perfect global channel state information (CSI) is available.

During the MAC phase, three users transmit signals to the relay simultaneously, and the relay receives

$$\mathbf{y}_r = \sum_{i=1}^{3} \mathbf{H}^{[r,i]} \mathbf{x}_i + \mathbf{n}_r \tag{1}$$

where $\mathbf{x}_i$ is the $M \times 1$ transmit vector at user i with transmit power constraint $\mathbb{E}[tr(\mathbf{x}_i \mathbf{x}_i^H)] \le P_i, i \in \{1, 2, 3\}$. $\mathbf{y}_r$ and $\mathbf{n}_r$ are the $N \times 1$ received signal vector and noise vector at the relay, respectively. The elements of $\mathbf{n}_r$ are independent and identically distributed (i.i.d) zero mean complex Gaussian variables with variance σ_r^2. $\mathbf{H}^{[r,i]}$ is the $N \times M$ channel matrix from user i to the relay. Each entry of $\mathbf{H}^{[r,i]}$ is assumed to be an i.i.d zero mean complex Gaussian random variable with unit variance.

During the BC phase, the relay generates new signals and broadcasts them to all users. The received signal vector at the ith user is described as

$$\mathbf{y}_i = \mathbf{H}^{[i,r]} \mathbf{x}_r + \mathbf{n}_i, \quad \forall i \in \{1, 2, 3\} \tag{2}$$

where $\mathbf{x}_r$ is the $N \times 1$ transmit vector at the relay with power constraint $\mathbb{E}[tr(\mathbf{x}_r \mathbf{x}_r^H)] \le P_r$. $\mathbf{y}_i$ and $\mathbf{n}_i$ are the $M \times 1$ received signal vector and noise vector at the ith user, respectively. Each element of $\mathbf{n}_i$ is i.i.d zero mean complex Gaussian variable with variance σ_i^2. $\mathbf{H}^{[i,r]}$ is the $M \times N$ channel matrix from the relay to user i and its elements have the same distribution as that of $\mathbf{H}^{[r,i]}$.

In [7], the signal $\mathbf{x}_r$ is obtained by following decode-and-forward strategy at the relay. In this paper, we discuss the amplify-and-forward scheme. A power allocation based SSA-ANC scheme will be discussed with an emphasize on the design of the signal $\mathbf{x}_r$ and the optimization of the power allocation. It will be shown that the maximum multiplexing gain, i.e. $3M$, can also be achievable.

III. Power Allocation Based SSA-ANC Scheme

If both power allocation and precoding are employed at each user, the signal that user i transmits to the relay during the MAC phase can be written as

$$\mathbf{x}_i = \sum_{j=1, j \ne i}^{3} \mathbf{V}^{[j,i]} \mathbf{P}_{\pi(j,i)} \mathbf{\Sigma}^{[j,i]} \mathbf{s}^{[j,i]} = \sum_{j=1, j \ne i}^{3} \mathbf{F}^{[j,i]} \mathbf{\Sigma}^{[j,i]} \mathbf{s}^{[j,i]} \tag{3}$$

where $\mathbf{s}^{[j,i]}$ is the $\frac{M}{2} \times 1$ encoded signal vector satisfying $\mathbb{E}[\mathbf{s}^{[j,i]} \mathbf{s}^{[j,i]H}] = \mathbf{I}_{\frac{M}{2}}$, $\mathbf{F}^{[j,i]} = \mathbf{V}^{[j,i]} \mathbf{P}_{\pi(j,i)}$ represents the precoding matrix and $\mathbf{\Sigma}^{[j,i]}$ is a $\frac{M}{2} \times \frac{M}{2}$ diagonal matrix representing the power allocation of each data stream in $\mathbf{s}^{[j,i]}$. The subscript $\pi(i, j)$ represents an index function for the combination among sets $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ comprised of two unordered user indices $\{i, j\}$ and the function outputs are $\pi(1, 2) = 1, \pi(1, 3) = 2, \pi(2, 3) = 3$.

Remark 1: According to (3), the power constraint at user i can be expressed as

$$\begin{aligned} & tr\{ \sum_{j=1, j \ne i}^{3} \mathbf{F}^{[j,i]} \mathbf{\Sigma}^{[j,i]} \mathbf{\Sigma}^{[j,i]H} \mathbf{F}^{[j,i]H} \} \\ = & \sum_{j=1, j \ne i}^{3} \sum_{k=1}^{M/2} (\omega_k^{[j,i]})^2 \|\mathbf{f}_k^{[j,i]}\|^2 = P_i \end{aligned} \tag{4}$$

where $\mathbf{f}_k^{[j,i]}$ is the kth column of matrix $\mathbf{F}^{[j,i]}$ and $\omega_k^{[j,i]}$ is the kth diagonal entry of matrix $\mathbf{\Sigma}^{[j,i]}$ for all $k \in \{1, 2, \cdots, \frac{M}{2}\}$.

Remark 2: From (3), we can see that the total data streams transmitted to the relay is $3M$, while the antenna number N at the relay is limited to $3M/2$. Therefore, it is impossible for the relay to decode all the data streams without interference due to the limitation of degrees of freedom. One solution available is signal space alignment. According to [7], the $M \times \frac{M}{2}$ precoding matrix $\mathbf{V}^{[j,i]}$ in (3) should satisfy that $\mathbf{H}^{[r,i]} \mathbf{V}^{[j,i]} = \mathbf{H}^{[r,j]} \mathbf{V}^{[i,j]}$.

Remark 3: Since $\pi(i, j) = \pi(j, i)$, we must have $\mathbf{H}^{[r,i]} \mathbf{V}^{[j,i]} \mathbf{P}_{\pi(j,i)} = \mathbf{H}^{[r,j]} \mathbf{V}^{[i,j]} \mathbf{P}_{\pi(i,j)}$ given that $\mathbf{H}^{[r,i]} \mathbf{V}^{[j,i]} = \mathbf{H}^{[r,j]} \mathbf{V}^{[i,j]}$ holds. In other words, the existence of precoding matrix $\mathbf{P}_{\pi(i,j)}$ is harmless in the sense of signal space alignment.

Let $\mathbf{U}_{\pi(i,j)} = \mathbf{H}^{[r,i]} \mathbf{V}^{[j,i]} = \mathbf{H}^{[r,j]} \mathbf{V}^{[i,j]}$, the received signal at the relay can be represented as

$$\mathbf{y}_r = \sum_{i<j, i,j \in \{1,2,3\}} \mathbf{U}_{\pi(i,j)} \mathbf{P}_{\pi(i,j)} \mathbf{s}^{[r,\pi(i,j)]} + \mathbf{n}_r \tag{5}$$

where $\mathbf{s}^{[r,\pi(i,j)]} = \mathbf{\Sigma}^{[j,i]} \mathbf{s}^{[j,i]} + \mathbf{\Sigma}^{[i,j]} \mathbf{s}^{[i,j]}$ represents the corresponding aligned signal.

Noting that the right hand side of (5) is composed of three aligned signals $\mathbf{s}^{[r,i]}, i \in \{1, 2, 3\}$, the relay should conduct a signal processing to separate these signals first. To do so, we choose a $\frac{M}{2} \times N$ separated matrix $\mathbf{H}_i$ for $\mathbf{s}^{[r,i]}$ satisfying the following conditions:

$$
\begin{aligned}
span(\mathbf{U}_2, \mathbf{U}_3) &\subset null(\mathbf{H}_1) \\
span(\mathbf{U}_1, \mathbf{U}_3) &\subset null(\mathbf{H}_2) \\
span(\mathbf{U}_1, \mathbf{U}_2) &\subset null(\mathbf{H}_3)
\end{aligned} \tag{6}
$$

Thus, the three aligned signals can be separated as $\mathbf{y}^{[r,i]} = \mathbf{H}_i\mathbf{y}_r = \mathbf{H}_i\mathbf{U}_i\mathbf{P}_i\mathbf{s}^{[r,i]} + \mathbf{H}_i\mathbf{n}_r, i \in \{1,2,3\}$.

Under the AF assumption, the relay will broadcast $\mathbf{y}^{[r,i]}$ to all users after some precoding and power allocation operations. Let $\tilde{\mathbf{y}}^{[r,i]} = \mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{W}_i\mathbf{y}^{[r,i]}, i \in \{1,2,3\}$ be the broadcasting signals, where $\mathbf{\Sigma}^{[i,r]}$ is a $\frac{M}{2} \times \frac{M}{2}$ diagonal matrix indicating the power allocation of the corresponding separated aligned signal, both $\mathbf{V}^{[i,r]}$ and $\mathbf{W}_i$ are precoding matrices. The transmit signal at the relay can be written as

$$
\begin{aligned}
\mathbf{x}_r &= \sum_{i=1}^{3}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{W}_i\mathbf{H}_i\mathbf{U}_i\mathbf{P}_i\mathbf{s}^{[r,i]} + \sum_{i=1}^{3}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{W}_i\mathbf{H}_i\mathbf{n}_r \\
&= \mathbf{x}_r^s + \mathbf{x}_r^n
\end{aligned} \tag{7}
$$

where $\mathbf{x}_r^s = \sum_{i=1}^{3}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{W}_i\mathbf{H}_i\mathbf{U}_i\mathbf{P}_i\mathbf{s}^{[r,i]}$ and $\mathbf{x}_r^n = \sum_{i=1}^{3}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{W}_i\mathbf{H}_i\mathbf{n}_r$ denote the useful and noise signal contained in the relay transmit signal, respectively.

Remark 4: Since the number of antennas at each user is $M < N$, so the precoding matrix for each $\frac{M}{2}$-dimensional aligned signal at the relay should be designed so that no more than M-dimensional signal is received for each user. Hence, $\mathbf{V}^{[i,r]}$ is designed to cancel the selective interfering signals and its columns can be chosen according to the conditions (29) in [7]. The other precoding matrix $\mathbf{W}_i$ is introduced only to simplify the power constraint expression of $\mathbf{x}_r$, which will be discussed later. And it can also be designed to have no influence on the cancellation of selective interfering signals.

Consider the useful signal $\mathbf{x}_r^s$ in (7), the SVD of matrix $\mathbf{H}_i\mathbf{U}_i$ can be represented as

$$
\mathbf{H}_i\mathbf{U}_i = \mathbf{A}_i\mathbf{\Sigma}_i\mathbf{B}_i^H \quad \forall i \in \{1,2,3\} \tag{8}
$$

where $\mathbf{A}_i$ and $\mathbf{B}_i$ are $\frac{M}{2} \times \frac{M}{2}$ unitary matrices and $\mathbf{\Sigma}_i$ is a $\frac{M}{2} \times \frac{M}{2}$ diagonal matrix. We choose precoding matrices $\mathbf{P}_i$ and $\mathbf{W}_i$ respectively as

$$
\mathbf{P}_i = \mathbf{B}_i, \quad \mathbf{W}_i = \mathbf{A}_i^H \tag{9}
$$

Thus, (7) can be rewritten as

$$
\begin{aligned}
\mathbf{x}_r &= \sum_{i=1}^{3}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{\Sigma}_i\mathbf{s}^{[r,i]} + \sum_{i=1}^{3}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{A}_i^H\mathbf{H}_i\mathbf{n}_r \\
&= \mathbf{x}_r^s + \mathbf{x}_r^n
\end{aligned} \tag{10}
$$

where $\mathbf{x}_r^s$ and $\mathbf{x}_r^n$ have the similar meanings as those in (7).

Remark 5: Comparing (7) with (10), we can see that the influence of $\mathbf{B}_i^H$ and $\mathbf{A}_i$ on the power constraint of $\mathbf{x}_r^s$ is balanced out by precoding matrices $\mathbf{P}_i$ and $\mathbf{W}_i$, respectively. The main advantage of such process is that the design of the power allocation can be simplified. However, it should be pointed out that the result might not be optimal. Therefore, how to design precoding matrices $\mathbf{P}_i$ and $\mathbf{W}_i$ to optimize the performance of the MIMO Y system with SSA-ANC transmission scheme is a very interesting topic and remain to be studied further.

Assuming that the columns of $\mathbf{V}^{[i,r]}$ are normalized and orthogonal with each other for each $i \in \{1,2,3\}$, the power of the useful signal $\mathbf{x}_r^s$ is

$$
P_{r_s} = \sum_{i=1}^{3}\sum_{j=1,j\neq i}^{3}\sum_{k=1}^{M/2}(\alpha_k^{[\pi(i,j),r]}\beta_{[k,\pi(i,j)]}\omega_k^{[i,j]})^2 \tag{11}
$$

where $\beta_{[k,i]}$ and $\alpha_k^{[i,r]}$ represent the kth diagonal entry of matrices $\mathbf{\Sigma}_i$ and $\mathbf{\Sigma}^{[i,r]}$, respectively.

Therefore, the power at the relay should satisfy

$$
E[tr(\mathbf{x}_r\mathbf{x}_r^H)] = P_{r_s} + P_{n_r} = P_r \tag{12}
$$

where P_{n_r} represents the power of the noise signal $\mathbf{x}_r^n$.

Taking user 1 as an example, the received signal under the previous assumption can be written as

$$
\begin{aligned}
\mathbf{y}_1 &= \mathbf{H}^{[1,r]}\sum_{i=1}^{2}\tilde{\mathbf{y}}^{[r,i]} + \mathbf{n}_1 \\
&= \mathbf{Q}^{[1,r]}\begin{bmatrix} \mathbf{s}^{[r,1]} \\ \mathbf{s}^{[r,2]} \end{bmatrix} + \mathbf{n}_1'
\end{aligned} \tag{13}
$$

where $\mathbf{n}_1' = \mathbf{H}^{[1,r]}\sum_{i=1}^{2}\mathbf{V}^{[i,r]}\mathbf{\Sigma}^{[i,r]}\mathbf{A}_i^H\mathbf{H}_i\mathbf{n}_r + \mathbf{n}_1$ and $\mathbf{Q}^{[1,r]} = \begin{bmatrix} \mathbf{H}^{[1,r]}\mathbf{V}^{[1,r]}\mathbf{\Sigma}^{[1,r]}\mathbf{\Sigma}_i & \mathbf{H}^{[1,r]}\mathbf{V}^{[2,r]}\mathbf{\Sigma}^{[2,r]}\mathbf{\Sigma}_i \end{bmatrix}$ denotes the effective channel through which signal vector $\begin{bmatrix} \mathbf{s}^{[r,1]T} & \mathbf{s}^{[r,2]T} \end{bmatrix}^T$ has passed. At user 1, the self-interference $\mathbf{s}^{[2,1]}$ and $\mathbf{s}^{[3,1]}$ can be subtracted from the received signal and therefore the desired signals $\mathbf{s}^{[1,2]}$ and $\mathbf{s}^{[1,3]}$ can be decoded.

Similarly, user 2 and user 3 can also decode their own desired signals, which means that $3M$ degrees of freedom can be achieved by using the SSA-ANC scheme for the MIMO Y channel with AF strategy.

From the power constraint at user i in (4) and the power expression of the useful signal contained in the relay transmit signal in (11), it is obvious that only the entries of the power allocation matrices $\mathbf{\Sigma}^{[j,i]}$ and $\mathbf{\Sigma}^{[i,r]}$ remain to be determined. In the following section, we will propose an OMAC algorithm (i.e., optimization in the MAC phase) to optimize the power allocation at three users aiming to maximize the sum rate.

IV. Optimization of Power Allocation

A. Sum Rate of the SSA-ANC based MIMO Y Channel

In this subsection, we derive the achievable sum rate of the system under the assumption that zero forcing receiver is employed at each user. Taking user 1 as an example, the received signal is

$$
\begin{aligned}
\mathbf{y}_1 = {} & \mathbf{H}_{eff}^{[1,r]}\begin{bmatrix} \mathbf{\Sigma}^{[1,r]}\mathbf{\Sigma}_1\mathbf{s}^{[r,1]} \\ \mathbf{\Sigma}^{[2,r]}\mathbf{\Sigma}_2\mathbf{s}^{[r,2]} \end{bmatrix} \\
& + \mathbf{H}_{eff}^{[1,r]}\begin{bmatrix} \mathbf{\Sigma}^{[1,r]}\mathbf{A}_1^H\mathbf{H}_1\mathbf{n}_r \\ \mathbf{\Sigma}^{[2,r]}\mathbf{A}_2^H\mathbf{H}_2\mathbf{n}_r \end{bmatrix} + \mathbf{n}_1
\end{aligned} \tag{14}
$$

where $\mathbf{H}_{eff}^{[1,r]} = \left[\begin{array}{cc} \mathbf{H}^{[1,r]}\mathbf{V}^{[1,r]} & \mathbf{H}^{[1,r]}\mathbf{V}^{[2,r]} \end{array} \right]$ denotes the effective channel from the relay to user 1. Subtracting the self-interference from $\mathbf{y}_1$ yields

$$\begin{aligned} \hat{\mathbf{y}}_1 &= \mathbf{H}_{eff}^{[1,r]} \left[\begin{array}{c} \mathbf{\Sigma}^{[1,r]}\mathbf{\Sigma}_1\mathbf{\Sigma}^{[1,2]}\mathbf{s}^{[1,2]} \\ \mathbf{\Sigma}^{[2,r]}\mathbf{\Sigma}_2\mathbf{\Sigma}^{[1,3]}\mathbf{s}^{[1,3]} \end{array} \right] \\ &\quad + \mathbf{H}_{eff}^{[1,r]} \left[\begin{array}{c} \mathbf{\Sigma}^{[1,r]}\mathbf{A}_1^H\mathbf{H}_1\mathbf{n}_r \\ \mathbf{\Sigma}^{[2,r]}\mathbf{A}_2^H\mathbf{H}_2\mathbf{n}_r \end{array} \right] + \mathbf{n}_1 \end{aligned} \tag{15}$$

Using zero forcing receiver $\mathbf{H}_{zf}^1 = (\mathbf{H}_{eff}^{[1,r]H})^{-1}$ at user 1, we can obtain

$$\begin{aligned} \mathbf{H}_{zf}^1\hat{\mathbf{y}}_1 &= \left[\begin{array}{c} \mathbf{\Sigma}^{[1,r]}\mathbf{\Sigma}_1\mathbf{\Sigma}^{[1,2]}\mathbf{s}^{[1,2]} \\ \mathbf{\Sigma}^{[2,r]}\mathbf{\Sigma}_2\mathbf{\Sigma}^{[1,3]}\mathbf{s}^{[1,3]} \end{array} \right] \\ &\quad + \left[\begin{array}{c} \mathbf{\Sigma}^{[1,r]}\mathbf{A}_1^H\mathbf{H}_1\mathbf{n}_r \\ \mathbf{\Sigma}^{[2,r]}\mathbf{A}_2^H\mathbf{H}_2\mathbf{n}_r \end{array} \right] + \left[\begin{array}{c} \mathbf{h}_{12} \\ \mathbf{h}_{13} \end{array} \right] \mathbf{n}_1 \end{aligned} \tag{16}$$

where $\mathbf{h}_{12}$ and $\mathbf{h}_{13}$ are $\frac{M}{2} \times M$ matrices with $\mathbf{H}_{zf}^1 = \left[\begin{array}{c} \mathbf{h}_{12} \\ \mathbf{h}_{13} \end{array} \right]$.

At the output of the matched filter, the signal-to-noise ratio (SNR) of each data stream can be computed as

$$SNR(j,i,k) = \frac{(\alpha_k^{[\pi(j,i),r]})^2\beta_{[k,\pi(j,i)]}^2(\omega_k^{[j,i]})^2}{\sigma_r^2(\alpha_k^{[\pi(j,i),r]})^2 + \sigma_j^2(\mathbf{h}_{ji}\mathbf{h}_{ji}^H)_{k,k}} \tag{17}$$

where $SNR(j,i,k), k \in \{1,2,\cdots,\frac{M}{2}\}$ represents the receiving SNR of the kth data stream in $\mathbf{s}^{[j,i]}, j = 1, i \in \{2,3\}$ and $A_{k,k}$ denotes the (k,k)th entry of matrix $\mathbf{A}$. we also assume that the rows of $\mathbf{H}_i$ are normalized and orthogonal with each other.

Similarly, the SNR of each data stream at the other two users can also be computed according to (17). For each channel realization, the achievable data rate of each data stream is $\frac{1}{2}log_2(1+\text{SNR}(j,i,k))$. The factor $\frac{1}{2}$ is due to the half duplex assumption. As a result, the achievable sum rate of the SSA-ANC based MIMO Y channel can be written as

$$R = \sum_{j=1,j\neq i}^{3}\sum_{i=1}^{3}\sum_{k=1}^{M/2}\frac{1}{2}\mathbb{E}[log_2(1+SNR(j,i,k))] \tag{18}$$

B. OMAC Algorithm

It can be seen from (17) that both $\mathbf{\Sigma}^{[i,r]} = diag(\alpha_k^{[i,r]})$ and $\mathbf{\Sigma}^{[j,i]} = diag(\omega_k^{[j,i]})$ affect the value of $SNR(j,i,k)$, which means that all the entries in $\mathbf{\Sigma}^{[i,r]}$ and $\mathbf{\Sigma}^{[i,j]}$ should be designed to maximize the sum rate given in (18). However, it is very difficult to optimize $\mathbf{\Sigma}^{[i,r]}$ and $\mathbf{\Sigma}^{[i,j]}$ simultaneously. Therefore, we first fix the entries of $\mathbf{\Sigma}^{[i,r]}$ at the relay under the assumption that equal power allocation is employed for each data stream at each user. Then, the obtained value of $\mathbf{\Sigma}^{[i,r]}$ is fixed as the initialization, based on which the power allocation matrix $\mathbf{\Sigma}^{[j,i]}$ is optimized.

Initialization of power allocation matrices at the relay: According to the discussion above, we first assume that equal power allocation is performed for each data stream at each user. Thus, from (4), we have

$$(\omega_k^{[j,i]})^2\|\mathbf{f}_k^{[j,i]}\|^2 = P_i/M \quad i = 1,2,3. \tag{19}$$

Therefore, the value of $\omega_k^{[j,i]}$ can be determined by

$$\hat{\omega}_k^{[j,i]} = \sqrt{P_i/(M\|\mathbf{f}_k^{[j,i]}\|^2)} \tag{20}$$

Furthermore, let

$$\theta_{k,\pi(i,j)}^2 = (\alpha_k^{[\pi(i,j),r]}\beta_{[k,\pi(i,j)]})^2[(\hat{\omega}_k^{[i,j]})^2 + (\hat{\omega}_k^{[j,i]})^2]$$

denote the power of the kth data stream of the aligned signal from user i and user j in $\mathbf{x}_r^s$. We assume that each data stream of the aligned signals has equal power, i..e $\theta_{k,\pi(i,j)}^2 = \lambda^2$ for all $j,i \in \{1,2,3\}, j \neq i$. Substituting this condition into (11) and (12), we can determine the value of λ and obtain the value of $\alpha_k^{[\pi(i,j),r]}$ as

$$\hat{\alpha}_k^{[\pi(i,j),r]} = \frac{\lambda}{\beta_{[k,\pi(i,j)]}\sqrt{(\hat{\omega}_k^{[i,j]})^2 + (\hat{\omega}_k^{[j,i]})^2}} \tag{21}$$

OMAC algorithm: Fixing the values of entries in the relay power allocation matrices $\mathbf{\Sigma}^{[i,r]}$ as $\alpha_k^{[i,r]} = \hat{\alpha}_k^{[i,r]}$, the optimization of power allocation at three users can be conducted via the following theorem.

Theorem 1: The sum rate in (18) for the AF-based MIMO Y channel with ZF receiver is a concave function with respect to $p_k^{[j,i]}, k \in \{1,2,\cdots,M/2\}, i,j \in \{1,2,3\}, i \neq j$. It can be expressed as the following optimization problem

$$\underset{p_k^{[j,i]}}{\text{Max}}\, R = \sum_{j=1,j\neq i}^{3}\sum_{i=1}^{3}\sum_{k=1}^{M/2}\frac{1}{2}\mathbb{E}[log_2(1+SNR(j,i,k))] \tag{22}$$

subject to

$$\sum_{j=1,j\neq i}^{3}\sum_{k=1}^{M/2}p_k^{[j,i]} = P_i \qquad i = 1,2,3 \tag{23}$$

$$\begin{aligned} &\sum_{k=1}^{M/2}[\eta_{1,k}^2(\frac{p_k^{[1,2]}}{\gamma_k^{[1,2]}} + \frac{p_k^{[2,1]}}{\gamma_k^{[2,1]}}) + \eta_{2,k}^2(\frac{p_k^{[1,3]}}{\gamma_k^{[1,3]}} + \frac{p_k^{[3,1]}}{\gamma_k^{[3,1]}}) \\ &\quad + \eta_{3,k}^2(\frac{p_k^{[2,3]}}{\gamma_k^{[2,3]}} + \frac{p_k^{[3,2]}}{\gamma_k^{[3,2]}})] = P_r - P_{n_r} \end{aligned} \tag{24}$$

where $\gamma_k^{[j,i]} = \|\mathbf{f}_k^{[j,i]}\|^2$ and $\eta_{i,k} = \hat{\alpha}_k^{[i,r]}\beta_{[i,k]}$. $p_k^{[j,i]} = (\omega_k^{[j,i]})^2\|\mathbf{f}_k^{[j,i]}\|^2$ represents the power of the kth data stream in encoded signal vector $\mathbf{s}^{[j,i]}$.

Two constraint functions in (23) and (24) can be easily obtained by substituting these substitutions into (4) and (11). It is easily to prove that the sum rate objective function in (22) is concave since the second derivative of the objective function respect to each $p_k^{[j,i]}$ is less than zero. Furthermore, two constraint functions are also convex because they are linear functions respect to $p_k^{[j,i]}$. Therefore, the optimization problem can be effectively solved through Khun-Tucker conditions.

By using the Khun-Tucher conditions, we can obtain the global optimal solution of the optimization problem as

$$p_k^{[j,i]} = \left[\frac{1}{(\tau_i + \frac{\eta_{\pi(j,i),k}^2}{\gamma_k^{[j,i]}}\tau_r)ln2} - \frac{\delta_{[j,i,k]}^2}{\eta_{\pi(j,i),k}^2}\right]^+ \tag{25}$$

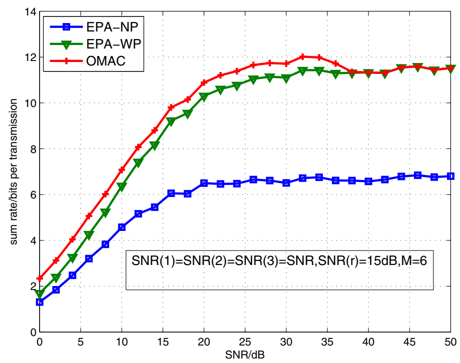

Fig. 2. Comparison for Various Scheme, SNR(r)=15dB, [M,N]=[6,9]

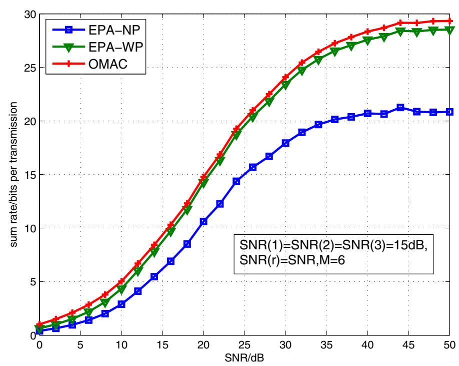

Fig. 3. Comparison for Various Scheme,SNR(i)=15dB(i=1,2,3), [M,N]=[6,9]

where $[x]^+ = max\{0, x\}$ and ln represents natural logarithm. $\delta^2_{[j,i,k]} = \gamma_k^{[j,i]} D_k^{[j,i]}$ and $D_k^{[j,i]}$ represents the denominator of the receiving SNR in (17) for the kth data stream of $\mathbf{s}^{[j,i]}$. $\tau_i, i \in \{1, 2, 3\}$ and τ_r are the lagrange multipliers of the constraints at user i and the relay, respectively.

Solutions in (25) combining with the power constraint of the optimization problem can be solved for $\tau_i, i \in \{1, 2, 3\}$ and τ_r. Thus, the values of the power allocated to each data stream and entries of the power allocation matrix $\mathbf{\Sigma}^{[j,i]}$ can be determined according to $\omega_k^{[j,i]} = \sqrt{p_k^{[j,i]}/(M\|\mathbf{f}_k^{[j,i]}\|^2)}$.

V. Numerical Results

In this section, we provide numerical results to illustrate the ergodic sum rate performance of the proposed OMAC algorithm. In all the simulation results, it is assumed that the noise variance at each receive antenna is fixed as σ^2, while the transmitting power is variable. Let $SNR(i) = P_i/\sigma^2$, $i = 1, 2, 3$ and r denotes the SNR of the links between user i and the relay, and the SNR of the links between the relay and each user, respectively. We uses EPA-NP as the abbreviation of equal power allocation at each node without additional precoding (we call $\mathbf{P}_i$ and $\mathbf{W}_i$ as additional precoding matrices) and EPA-WP as that of equal power allocation at each node with additional precoding.

Assuming the $SNR(r)$ is fixed at 15dB and the $SNR(i)$ for $i = 1, 2, 3$ are changed symmetrically, Fig. 2 and Fig. 3 compare the sum rate performance of various schemes with different antenna configurations. It is obvious that the proposed OMAC algorithm achieves better sum rate performance than the other schemes with the increase of the number of antennas for lower and moderate $SNR(i), i = 1, 2, 3$. Surprisingly, although both EPA-NP and EPA-WP employ equal power allocation at each node, Fig. 2 and Fig. 3 show that the EPA-WP achieves significant improvement in the sum rate performance than the EPA-NP scheme, which implies that the existence of additional precoding matrices is very necessary for improving the sum rate performance of the system.

VI. Conclusion

We propose an SSA-ANC scheme for the MIMO Y channel with AF strategy to achieve the maximum degrees of freedom. An OMAC algorithm is also proposed to optimize the power allocation at three user nodes with the aim of maximizing the sum rate performance. Numerical results show that the proposed algorithm can achieve improvement in sum rate performance compared with the non-optimization case.

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References

[1] R. Ahlswede, N. Cai, "Network information flow," *IEEE Trans. Inform. Theory*, vol. 46,no. 4, pp. 1204-1216, Jul. 2000.

[2] S. Zhang, S.-C. Liew, and P. P. Lam, "Physical layer network coding," [Online]. Available: arXiv:0704.2475v1

[3] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," Comput. Sci. Artif. Intell. Lab. Tech.Rep., Cambridge, MA, MIT-CSAIL-TR-2007-012, 2007.

[4] Y. Wu, P. A. Chou, and S.-Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," in *Proc. 39th Annual Conf. inf. Sciences and Systems (CISS)*, Mar. 2005.

[5] D. Gunduz, A. Yener, A. Goldsmith, and H. V. Poor, "The multi-way relay channel," in *proc. IEEE Int. Symp. Inf. Theory*, 2009, pp. 339-343.

[6] N. Lee, J. Lim, "A Novel Signaling for Communication on MIMOY Channel: Signal Space Alignment for Network Coding," in *proc. IEEE Int. Symp. Inf. Theory*, 2009, pp. 2892-2896.

[7] N. Lee, J. Lim, and J. Chun, "Degrees of Freedom of the MIMO Y Channel: Signal Space Alignment for Network coding," *IEEE Trans. Inform. Theory*, vol. 56, no. 7, pp. 3332-3342, Jul. 2010.

[8] K. Lee, N. Lee, and I. Lee, "Feasibility Conditions of Signal Space Alignment for Network Coding on K-user MIMO Y channels," in *proc. IEEE ICC*, 2011.