Spectrum Sensing for Networked System Using 1-bit Compressed Sensing with Partial Random Circulant Measurement Matrices

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Abstract—**Recently developed compressed sensing theory enables signal acquisition and reconstruction from incomplete information with high probability provided that the signal is sparsely represented in some basis. This paper applies compressed sensing for spectrum sensing in a networked system. To tackle the calculation and communication cost problems, this paper also applies structured compressed sensing and 1-bit compressed sensing. Measurement using the partial random circulant matrices can reduce the calculation cost at the sacrifice of a slightly increased number of measurements by utilizing the fact that a circulant matrix is decomposed by multiplications of structured matrices. This paper investigates the tradeoff between calculation cost and compression performance. 1-bit compressed sensing extracts only sign data (1-bit quantization) from measured data, and reconstructs the original signal from the extracted sign data. Therefore, 1-bit compressed sensing can save communication costs associated with spectrum sensing in a networked system. This paper evaluates the efficiency of 1-bit compressed sensing. In addition, this paper also proposes a block reconstruction algorithm for 1-bit compressed sensing that uses the block sparsity of the signals. Empirical study shows that partial random circulant matrices work as efficient as completely random measurement matrices for spectrum sensing and that 1-bit compressed sensing can be used for spectrum sensing with greatly reduced communication costs.**

I. Introduction

Compressed sensing is a new framework that uses signal sparsity to reduce the amount of data that needs to be measured [1, 2]. The measurement of an M-dimensional vector onto an N-dimensional ($N<<M$) vector loses some information in general and the inverse problem (N equations with M variables) has an infinite number of solutions. However, when the signal only has K nonzero coefficients ($K<N<<M$) in some convenient basis and the measurement matrix is incoherent with the basis, or equivalently satisfies the restricted isometry property (RIP [24, 25]), the inverse problem has, with high probability, a unique and exact solution. Inspired by above seminal work of Donoho [1] and Candes [2], a plethora of related research has taken place from theory to applications including astronomy, magnetic resonance imaging, and digital imaging by exploiting the natural sparsity of their underlying data [3-5].

Compressed sensing has also been actively researched in wireless communication literature [6-12]. It is used for channel estimation by exploiting the sparse nature of the multipath channel profile in ultra-wideband channel [6] and underwater acoustics channel [7]. [8] used compressed sensing theory for occupied channel detection in wideband cognitive radio by utilizing the sparse frequency usage. [9] proposed the usage of the Gabor time-frequency decomposition to sense the GSM band signal by applying compressed sensing. [10] proposed the spectrum sensing method for cognitive radio using matrix completion and joint sparsity in a collaborative manner. [11] proposed the parallel spectrum sensing for cognitive radios using parallel compressed sensing blocks.

With respect to the advantages of compressed sensing over the conventional sample-then-compress transform, the following properties are commonly argued: First, the number of measurements depends not only on the size of the representation basis (M) but also on the amount of information (K). In conventional transform coding, all M transforms are performed regardless of the size of K, then the K most significant coefficients are isolated. However, $N(=O(K\log(M/K)))$ measurements are sufficient in compressed sensing, which is highly efficient given a large M and a small K. Secondly, the processing load of the measurement is reduced at the sacrifice of the increased processing load of the reconstruction, which is beneficial in tiny device deployed system such as a sensor network. Thirdly, signal-independent universal measurement is possible thanks to the random measurement nature, which eliminates the necessity of signal-specific processing in measurement. It is beneficial in dealing with unknown signals such as blind spectrum sensing.

In practice, however, some empirical observations yield contrary results against the above stated arguments in terms of calculation and communication costs. Regarding calculation costs, compressed sensing needs $O(NM)$ matrix-vector multiplication while FFT based transform coding needs $O(M\log M)$ FFT processing and thresholding costs. In most practical cases, the transform coding is faster than compressed sensing despite the necessity of thresholding costs. Regarding the communication cost in a networked system, the necessary bandwidth for transferring a measured data is increased q times when the quantization depth is q. For example, when N/M is 0.3 and quantization depth is 10 bits, $3M$ bits should be

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transferred to carry K-sparse information in an M-dimensional vector. This is undesirable in a bandwidth-insufficient system.

To tackle these calculation and communication cost problems, this paper applies structured compressed sensing [12-14] and 1-bit compressed sensing [15, 16]. Recent research has proved that $O(K^{1.5}\log^{1.5}M)$ measurements of partial random circulant matrices satisfies RIP in many reconstruction algorithms [14]. Utilizing the fact that a circulant matrix is decomposed by multiplications of structured matrices, measurement using a partial random circulant matrix can be done with less calculation cost at the sacrifice of a slightly increased number of measurements. The tradeoff between calculation cost and compression performance (the number of measurements) is investigated in Section Ⅳ.

1-bit compressed sensing extracts only sign data (1-bit quantization) from measured data, and reconstructs the original signal from the extracted sign data [15, 16]. Although it seems impossible, [16] has proven that 1-bit compressed sensing can work in many scenarios using the new concept of *binary ε-stable embedding* (BεSE) which works similar to RIP. Details will be discussed in Section Ⅲ. Additionally, we also propose a block reconstruction algorithm for 1-bit compressed sensing by slightly modifying the algorithm proposed in [16].

To explore the validity of 1-bit compressed sensing using a structured matrix for spectrum sensing in a networked system, experiments and simulations have been conducted using a unified wireless platform, called the Flexible Wireless System, which has been previously proposed and implemented by us [17, 18].

The remainder of this paper is organized as follow: Section Ⅱ describes the basic theory of compressed sensing. Section Ⅲ explains the concept of structured and 1-bit compressed sensing. Section Ⅳ describes the system model and provides results of empirical study, and Section Ⅴ concludes this paper.

II. COMPRESSED SENSING

A. Measurement and Reconstruction

Let an $M\times 1$ signal vector $\mathbf{X}$ be sparsely represented with an $M\times M$ basis matrix $\mathbf{\Psi}$ and an $M\times 1$ sparse vector $\mathbf{s}$ as $\mathbf{X}=\mathbf{\Psi s}$. It follows that an $N\times 1$ measured vector $\mathbf{Y}$ is obtained as $\mathbf{Y}=\mathbf{\Phi X}=\mathbf{\Phi\Psi s}=\mathbf{\Theta s}$, where $\mathbf{\Theta}$ equals to $\mathbf{\Phi\Psi}$. Typically, the inverse problem $\mathbf{s}=\mathbf{\Theta}^{-1}\mathbf{Y}$ is ill-posed and has no unique solution. However, it has been proven that this inverse problem is cast to the following l1-norm minimization problem if $\mathbf{s}$ is sparse [1-2].

$$\min\|\tilde{\mathbf{s}}\|_1 \text{ subject to } \mathbf{Y}=\mathbf{\Theta}\tilde{\mathbf{s}} \text{ and } \tilde{\mathbf{s}}\in\mathbf{R^M}, \quad (1)$$

where, $\|\tilde{\mathbf{s}}\|_1=\sum_i|\tilde{s}_i|$ and $\mathbf{R^M}$ is the set of $M\times 1$ vector. If the RIP is satisfied, the solution, $\tilde{\mathbf{s}}$, is unique and identical to $\mathbf{s}$ with overwhelming probability.

Numerous algorithms have been proposed for solving this l1-norm minimization problem [19]. Most algorithms are based on basis pursuit [20] or the iterative greedy algorithm [21]. Some algorithms modify the above optimization problem to consider the effect of noise [22] or to reduce processing time [23]. This paper uses the fast reconstruction algorithm in [23], the gradient projection for sparse reconstruction (GPSR), because our main concern is calculation cost reduction.

B. Restricted Isometry Property and δ-stable embedding

The RIP provides the sufficient condition of measurement matrix $\mathbf{\Phi}$ and basis matrix $\mathbf{\Psi}$ for solving the l1-norm minimization problem [24-25].

When the compressed sensing matrix $\mathbf{\Theta}$ $(=\mathbf{\Phi\Psi})$ satisfies the following inequality for all S-sparse vectors, matrix $\mathbf{\Phi}$ is said to obey the RIP of order S.

$$(1-\delta)\|\mathbf{s}\|_2^2\le\|\mathbf{\Theta s}\|_2^2\le(1+\delta)\|\mathbf{s}\|_2^2, \quad (2)$$

where, $\|\mathbf{s}\|_2=\sum_i s_i^2$ and $\delta(0\le\delta<1)$ is the smallest constant that satisfies Eq. (2).

The RIP is further generalized to δ-stable embedding as Eq. (3) in a geometric perspective [26].

$$(1-\delta)\|\mathbf{X}_1-\mathbf{X}_2\|_2^2\le\|\mathbf{\Phi X}_1-\mathbf{\Phi X}_2\|_2^2\le(1+\delta)\|\mathbf{X}_1-\mathbf{X}_2\|_2^2, \quad (3)$$

where, $\mathbf{X}_1$ and $\mathbf{X}_2$ belong to the signal vector set $\mathbf{R^M}$. δ-stable embedding guarantees not only the preservation of information but also the preservation of the Euclidean distances between two vectors in $\mathbf{R^M}$.

Amongst numerous research regarding RIP and δ-stable embedding, the benchmark result yields that $O(K\log(M/K))$ measurements satisfy the RIP for random Gaussian measurement matrices [24].

C. Implementaion

Implementation of compressed sensing can be done in an analog or a discrete manner.

In the case of analog compressed sensing in which the measurement is conducted by analog circuits and the low-speed analog-to-digital converter (ADC), the measurement matrix is usually derived into the form of the partial Toeplitz matrix. For example, the measurement matrix of Xampling [27, 28], which directly yields the sub-Nyquist rate measurement signal without high-speed ADC by mixing the input signal with the over-Nyquist rate sign alternating mixer and feeding mixed signal into low pass filter, is represented in the form of the partial Toeplitz matrix. Since Toeplitz matrices can be implemented by linear convolution, the hardware implementation by the partial Toeplitz measurement matrices is more feasible than completely random measurement matrices.

In case of the discrete compressed sensing in which the measurement is conducted by the matrix-vector multiplication using stored measurement matrix in the memory and discrete samples obtained by Nyquist rate ADC, the calculation cost issue of digital processing arises rather than the implementation issue. Partial random circulant measurement matrices may be attractive for discrete compressed sensing because their computation cost can be reduced by FFT and IFFT operations.

Although analog compressed sensing eliminates the necessity of the high-speed ADC, new hardware front-end is needed and its development is still premature. The hardware modification or replacement is difficult in many scenarios, e.g., the spectral density mapping using commercial devices such as the universal software radio peripheral (USRP) [29]. Or, the speed limitation of ADC is not a bottle neck for the system configuration in many scenarios such as wide band spectrum sensing in a manner by sweeping truncated narrow band spectrum sensing.

Therefore, this paper considers discrete compressed sensing and focuses on possibilities of calculation cost reduction by using partial random circulant measurement matrices.

III. Sturctred Compressed Sensing and 1-Bit Compressed Sensing

A. Structured Compressed Sensing

Difficulties in hardware implementation and expensive calculation costs of completely random measurement matrices motivated the use of structured matrices as the measurement matrices. Two such promising candidates are the Toeplitz matrix (T) and the circulant matrix (C). Measurement by Toeplitz matrices and circulant matrices are realized by linear and circular convolutions, respectively.

In terms of causality of the physical signal, Toeplitz matrices are suitable for practical hardware implementations [13, 27-28]. For circulant matrices, however, due to the noncausality caused by the circular convolution, additional effort is necessary. As discussed in the previous section, this paper exclusively considers discrete compressed sensing and applies circulant matrices to save calculation costs because: 1) The noncausality problem is relaxed in discrete compressed sensing, and 2) Matrix operations of circulant matrices are simpler than those of Toeplitz matrices.

Measurement matrices in this paper are generated by the partial random circulant matrix as $\mathbf{\Phi} = \mathbf{RC} = \mathbf{RUDU}^*$. $\mathbf{R}$ is an $N \times M$ matrix ($N<M$) consists of partial rows of the $M \times M$ identity matrix. $\mathbf{C}$ is an $M \times M$ random circulant matrix where the first row consists of $M \times 1$ random numbers and every left-to-right descending diagonal is constant. $\mathbf{U}$ and $\mathbf{U}^*$ are the FFT and IFFT matrices of which size are $M \times M$. $\mathbf{D}$ is an $M \times M$ diagonal matrix with diagonal elements generated by the DFT of the first row of $\mathbf{C}$. Therefore, calculation costs of partial random circulant measurement matrices are $O(2M\log M+M)$ including FFT, IFFT, and diagonal matrix multiplication operations. This is much lower than that of completely random measurement matrix, $O(NM)$.

Due to the fewer degrees of freedom, an increased number of measurements is necessary when partial random circulant matrices are used. It has been proven that if $\mathbf{R}$ is the randomly selected rows of the identity matrix and $\mathbf{C}$ is generated by Rademacher variables, $O(K^{1.5}\log^{1.5}M)$ measurements satisfy RIP in many reconstruction algorithms [14]. Although this is larger than the case of the benchmark result given by $O(K\log(M/K))$, our empirical observations for spectrum sensing showed that partial random circulant matrices work as efficiently as completely random matrices.

B. 1-bit Compressed Sensing

Increased communication costs from quantization, particularly in a networked system, motivate us to use 1-bit compressed sensing. Boufounous [15] first proposed 1-bit compressed sensing, which only extracts sign data from measured data, $\mathbf{Y} = \mathrm{sign}(\mathbf{\Theta s})$, and then reconstructs the original signal from the extracted sign data. Although amplitude information is lost during the measurement stage, [15] showed the possibility of the reconstruction. The reconstruction method proposed in [15] solved the sparse signal inversion problem by enforcing that the solution ($\tilde{\mathbf{s}}$) lies on the unit sphere to artificially resolve the amplitude ambiguity as bellows.

$$\min\|\tilde{\mathbf{s}}\|_1 \text{ subject to } \mathbf{Y} = \mathrm{sign}(\mathbf{\Theta}\tilde{\mathbf{s}}), \|\tilde{\mathbf{s}}\|_2 = 1, \text{and } \tilde{\mathbf{s}} \in \mathbf{R}^{\mathbf{M}}. \quad (4)$$

From the geometric perspective, the surface of the unit sphere and rows of $\mathbf{\Theta}$ represent the collections of all $M \times 1$ signal vectors bereft of the amplitude information and intersection planes of the unit sphere, respectively. Note that plenty of intersection planes with different directions and their sign information can specify the specific area of the surface of the unit sphere. If $M \times 1$ signal vectors are K-sparse, their location on the surface of the unit sphere can be exactly specified provided that the numbers of intersection planes are numerous enough.

Jacques [16] provides the theoretical analysis on the best achievable performance of 1-bit compressed sensing by introducing the BεSE, defined as:

$$d_S(\mathbf{X_1},\mathbf{X_2}) - \varepsilon \le d_H\big(sign(\mathbf{\Phi X_1}) - sign(\mathbf{\Phi X_2})\big) \le d_S(\mathbf{X_1},\mathbf{X_2}) + \varepsilon, (5)$$

where $d_S (0 \le d_S \le 1)$ is the normalized angle between two $M \times 1$ vectors and $d_H (0 \le d_H \le 1)$ is the normalized hamming distance between two $N \times 1$ sign vectors, respectively. $\varepsilon (0 \le \varepsilon < 1)$ is the smallest constant that satisfies Eq. (5).

Similarities of BεSE in Eq. (5) and δ-stable embedding in Eq. (3) provide brilliant geometric intuition to comprehend how reconstruction of 1-bit compressed sensing is guaranteed. The Euclidean distances between two $M \times 1$ vectors and between two $N \times 1$ measured vectors correspond to the normalized angle between two $M \times 1$ amplitude-bereft vectors and the hamming distance between two $N \times 1$ sign vectors, respectively. Therefore, BεSE guarantees the preservation of the angle between two amplitude-bereft vectors in $\mathbf{R}^{\mathbf{M}}$ just as the δ-stable embedding in the conventional compressed sensing. [16] showed that $O(K\log(M))$ measurement satisfies the BεSE when completely random Gaussian measurement matrices are used. Theoretical guarantee of the BεSE with structured random measurement matrices has not yet been investigated in the literature.

IV. Empirical Study

A. System Model

Experiments and simulations were performed using 310 MHz band periodic FSK signals transmitted by commercial RFID tags over The AWGN channel using our previously implemented prototype [17, 18]. Total bandwidth, data rate of RFID, the signal transmission period, the signal duration, and the signal to noise ratio were 10 MHz, 19.2 Kbit/s, 1 sec, 10

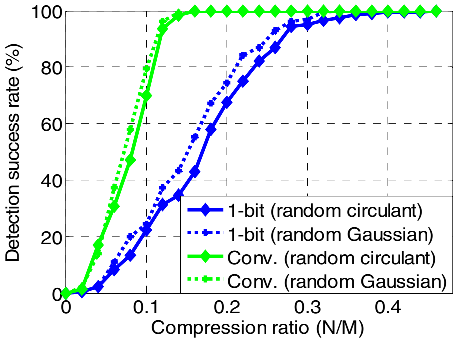

Fig.1 Pd of the conventional and 1-bit CS.

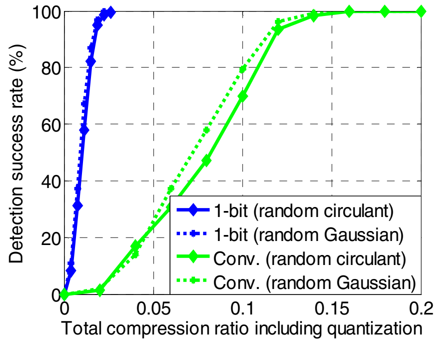

Fig.2 Pd of the conventional and 1-bit CS in terms of total compression ratio (q=16).

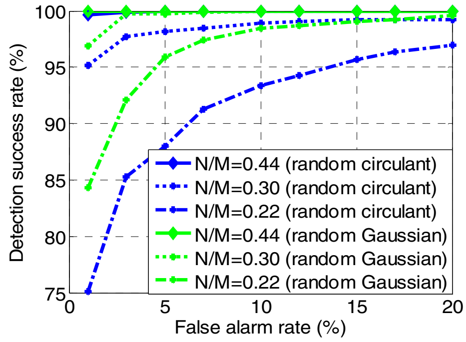

Fig.3 Receiver operating curve of 1-bit CS.

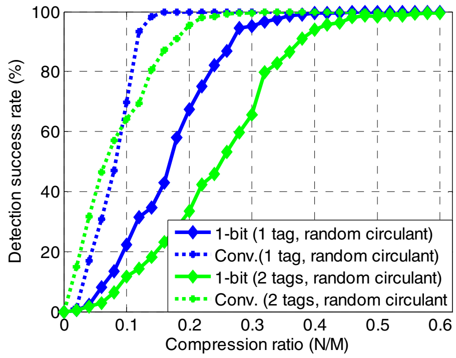

Fig.4 Pd of the conventional and 1-bit CS with different sparsity.

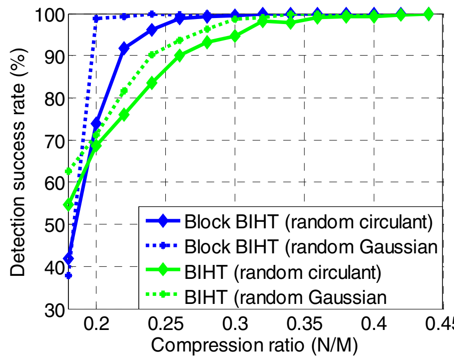

Fig.5 Pd of 1-bit CS (Block BIHT vs BIHT).

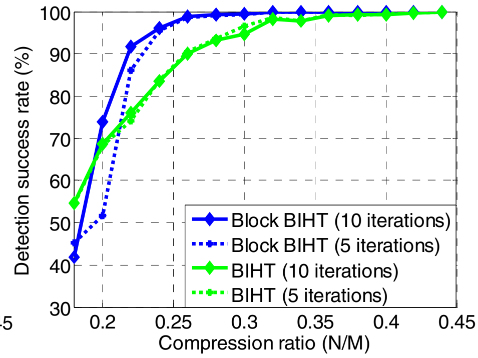

Fig.6 Pd of 1-bit CS with constrained iteration numbers.

msec, and 27 dB, respectively. Due to the frequency separation of FSK signals and spectral leakage, the channel bandwidth of each signal occupies up to 300 KHz, which yields 0.03 of the frequency domain sparsity.

Received signals were sampled at 20 Msps with 16 quantization depth, measured, and then transferred to the central server through a wired network. Transferred signals were reconstructed at the central server. For the convenience of iterative verifications, measurements and reconstructions of the power spectral density were conducted using the stored signal at the receiver with various parameter settings.

GPSR and binary iterative hard thresholding (BIHT) [16] are used as reconstruction algorithms for the conventional and 1-bit compressed sensing, respectively. We also proposed a block BIHT algorithm for the block-wise sparse signal reconstruction by slightly modifying the BIHT. Given an initial estimate $\mathbf{s}^0 = \mathbf{0}$, the l^{th} iteration of the BIHT computes $\boldsymbol{\alpha}^{l+1} = \mathbf{s}^l + (\tau/2)\boldsymbol{\Theta}^T(\mathbf{Y} - \text{sign}(\boldsymbol{\Theta}\mathbf{s}^l))$ and $\mathbf{s}^{l+1} = \eta_K(\boldsymbol{\alpha}^{l+1})$, where τ is the scalar that controls the step-size of the gradient decent and $\eta_K(\mathbf{v})$ is the thresholder that eliminates all but *K*-significant entries of $\mathbf{v}$. The block BIHT algorithm modifies the thresholder in order to eliminate all but *J*-significant blocks of $\mathbf{v}$. Since most practical signals are sparse in block, the block BIHT algorithm can improve the reconstruction performance.

B. Results

This section provides results of empirical study for evaluation of the performance of partial circulant measurement matrices, 1-bit compressed sensing, and block BIHT algorithm with various parameter settings. All curves of the detection success rate (P_d) are obtained within 0.01 of the detection false alarm rate (P_f) except curves in Fig. 3. Completely random measurement matrices are generated by Gaussian random matrices.

Figure 1 shows two comparisons: 1) completely random measurement matrices vs. partial random circulant measurement matrices and 2) conventional vs. 1-bit compressed sensing in terms of P_d. In the first comparison, P_d of the partial random circulant matrices yield almost similar results to those of the completely random measurement matrices. Since the RIP is the sufficient condition, these empirical observations showed more favorable results than theoretical analysis. With respect to the second comparison, *N/M* for P_d over 0.99 are 0.16 and 0.38 in the conventional and 1-bit compressed sensing, respectively, when the partial random measurement matrices were used. 1-bit compressed sensing needs a larger number of measurements because it uses only sign information.

Figure 2 shows the same results of Fig. 1 in terms of the total compression rate including quantization effects. Note that curves of 1-bit compressed sensing are shifted to the left and a compression rate of 0.024 yields P_d over 0.99. This proves that 1-bit compressed sensing can save communication costs for spectrum sensing in a networked system.

Figure 3 shows the receiver operating curve (ROC) comparison between the completely random measurement matrices and the partial random circulant matrices in 1-bit compressed sensing. Although the ROC of the partial random circulant matrices shows poorer performance than the completely random measurement matrices at low *N/M*, both show good performance at high N/M. This also indicates that partial random circulant matrices work as efficient as the completely random measurement matrices provided that a reasonably sufficient number of measurements is obtained.

Figure 4 shows the P_d of the conventional and 1-bit compressed sensing with different signal sparsity. As expected, the necessary number of measurements is increased as signal sparsity. N/M for P_d over 0.99 for overlapped two tags' signal in the conventional and 1-bit compressed sensing are 0.26 and 0.56, respectively, when the partial random measurement matrices are used.

Figure 5 shows performance comparisons between block BIHT and BIHT in terms of P_d . P_d of block BIHT with completely random measurement matrices and partial random circulant measurement matrices are over 0.99 when N/M is 0.22 and 0.28, respectively. On the other hand, those of BIHT are 0.32 and 0.38, respectively. This shows that block BIHT can save more an communication costs than BIHT with fewer measurements.

Figure 6 shows performance comparisons between block BIHT and BIHT with constrained iteration numbers for reconstruction. Iteration numbers are limited to 5 and 10 in the reconstruction process. It can be confirmed that block BIHT outperforms BIHT with a limited number of iterations, which indicates that calculation costs for the reconstruction at the server can be also saved by using a block BIHT algorithm.

V. CONCLUSION

This paper applied the recently developed compressed sensing for the blind spectrum sensing in a networked system. To tackle the calculation and communication cost problems caused by completely random measurement matrices and quantization, this paper also applied structured compressed sensing and 1-bit compressed sensing. Calculation cost of partial random circulant measurement matrices is reduced to $O(2MlogM+M)$ using the property of circulant matrices. Despite the reduced degrees of freedom, empirical evidence showed that partial random circulant matrices work as efficiently as completely random measurement matrices for spectrum sensing. It has also been proved that communication costs for spectrum sensing in a networked system can be saved by the usage of 1-bit compressed sensing. Furthermore, empirical study also proved that the performance of 1-bit compressed sensing can be improved when block sparsity of the signals is used in reconstruction process in terms of both calculation and communication costs. Results obtained in this paper are beneficial not only for our system but also for other networked spectrum sensing scenarios such as spectrum sensing for cognitive radio or distributed spectral density mapping.

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