

Grouped Interference Alignment in Inter-Vehicle Communications

Takayuki Shimizu*, Akihisa Yokoyama[†] and Hisato Iwai*

*Graduate School of Engineering, Doshisha University

1-3 Tatara Miyakodani, Kyotanabe, Kyoto, 610-0394 Japan

Email: {etj1101@mail4, iwai@mail}.doshisha.ac.jp

[†]TOYOTA InfoTechnology Center, U.S.A., Inc.

465 Bernardo Avenue, Mountain View, CA, 94043 USA

Email: ayokoyama@us.toyota-itc.com

Abstract—This paper investigates the application of interference alignment (IA) to inter-vehicle communications for further performance improvement. In conventional IA in the spatial domain, the number of antennas of each terminal limits the number of terminals, and therefore the sum rate performance is upper-bounded. This paper shows that when IA is applied to inter-vehicle communications at an intersection, it is possible to exceed the sum rate of the conventional IA by taking into account the effect of path loss. To this end, we consider multiple-input multiple-output interference channels with two IA groups, where all interference signals from the same group are aligned at each receiver by virtue of IA, but interference signals from the other group are not. Simulation results show that two IA groups can geographically co-exist by utilizing high propagation loss at a corner of the intersection to mitigate inter-group interference, and thus the achievable sum rate can be significantly improved, compared with that of single-group IA.

I. INTRODUCTION

Intelligent transportation systems (ITS) are the integrated application of information and communication technologies to improve traffic flow efficiency and traffic safety. One of the key technologies for ITS is inter-vehicle communications [1], [2]. It enables information exchange among vehicles such as the information of vehicle position, speed, and moving direction. In the U.S. and Europe, 5.9 GHz band is allocated for ITS applications, whereas in Japan, 700 MHz band and 5.8 GHz band are allocated. The recently released IEEE 802.11p [3] defines the physical and medium access control layers tailored to inter-vehicle communications, which is a slightly modified version of the IEEE 802.11a.

It is expected that in the future all vehicles on a road will be able to communicate with each other. However, it causes heavy congestion in the network. For such congested wireless networks, the throughput of inter-vehicle communication systems based on the IEEE 802.11p is significantly degraded due to severe interference caused by concurrent transmissions. Therefore, more sophisticated techniques to deal with interference must be investigated. One of the promising solutions in the physical layer is interference alignment (IA) [4], [5].

IA is a transmission technique for K -user interference channels, where K pairs of transmitters and receivers communicate simultaneously. This technique enables interference-free communications without data sharing among transmitters. The key

idea of IA is to overlap interference signals at each receiver to minimize the dimensions occupied by the interference signals, while keeping the interference-free signaling dimensions. In [4], it was shown that when all transmitters and receivers have a single antenna each, IA achieves the maximum degrees-of-freedom $K/2$ (i.e., interference-free signaling dimensions) with infinite symbol extensions in time or frequency selective fading channels. In [4], IA in K -user multiple-input multiple-output (MIMO) interference channels was also considered. For K -user MIMO interference channels, iterative algorithms to find precoding matrices achieving IA without symbol extensions was proposed in [5].

In conventional IA in the spatial domain, the number of antennas of each terminal limits the number of terminals, and therefore the sum rate performance is upper-bounded. This paper shows that when IA is applied to inter-vehicle communications at an intersection, it is possible to exceed the multiplexing gain of the conventional IA by taking into account the effect of path loss. For this purpose, we consider MIMO interference channels with two IA groups around an intersection, where all interference signals from the same group are aligned at each receiver, but interference signals from the other group are not. Through numerical simulations, we show that the achievable sum rate can be significantly improved by utilizing high propagation loss at a corner of the intersection to mitigate inter-group interference. The proposed transmission scheme is useful when many vehicles aim at establishing unicast communications.

Notations: We use boldface capital letters to represent matrices and boldface lower-case letters to represent vectors. Let $\mathbf{A} \in \mathbb{C}^{m \times n}$ be a complex-valued $m \times n$ matrix, where \mathbb{C} denotes the complex field. The notations \mathbf{A}^T , \mathbf{A}^H , and $\|\mathbf{A}\|$ represent the transpose, the Hermitian transpose, and the Frobenius norm of \mathbf{A} , respectively. The $m \times m$ identity matrix is denoted by \mathbf{I}_m . The expectation operator is denoted by $\mathbb{E}[\cdot]$.

II. INTERFERENCE ALIGNMENT FOR K -USER MIMO INTERFERENCE CHANNELS

In this section, we review the concept of IA [4]. Let us consider the K -user MIMO interference channel shown in Fig. 1. In this system, there are K pairs of transmitters and

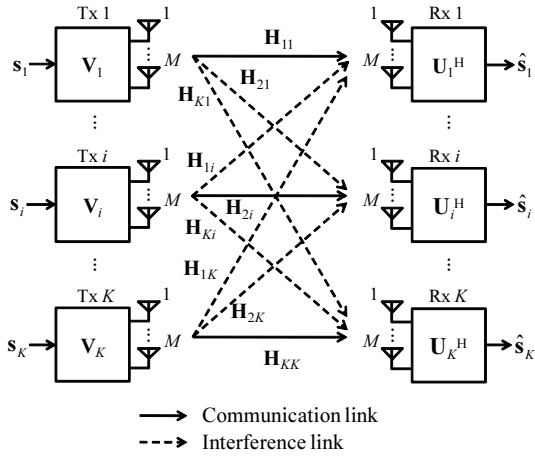


Fig. 1. K -User MIMO Interference Channel.

receivers, where all the transmitters transmit signals simultaneously, and the transmitter i wants to send d data streams to its corresponding receiver (the receiver i). Each transmitter and receiver has M antennas. Assuming frequency-flat fading channels, the complex MIMO fading channel matrix from the transmitter j to the receiver i is represented by $\mathbf{H}_{ij} \in \mathbb{C}^{M \times M}$, where each channel coefficient is drawn independently from a certain continuous distribution. The received signal at the receiver i is given by

$$\mathbf{y}_i = \mathbf{H}_{ii} \mathbf{V}_i \mathbf{s}_i + \sum_{j=1, j \neq i}^K \mathbf{H}_{ij} \mathbf{V}_j \mathbf{s}_j + \mathbf{n}_i, \quad (1)$$

where $\mathbf{s}_i \in \mathbb{C}^{d \times 1}$ is the data vector of the transmitter i , $\mathbf{V}_i \in \mathbb{C}^{M \times d}$ is the precoding matrix of the transmitter i , and $\mathbf{n}_i \in \mathbb{C}^{M \times 1}$ is the complex circularly symmetric zero-mean additive white Gaussian noise (AWGN) vector at the receiver i . Here, we assume that $\mathbb{E}[|\mathbf{V}_i \mathbf{s}_i|^2] = P$ and $\mathbb{E}[\mathbf{n}_i \mathbf{n}_i^H] = \sigma^2 \mathbf{I}_M$, where P is the transmission power and σ^2 is the noise power. Due to simultaneous transmission, each receiver experiences interference from $K-1$ transmitters as well as a desired signal.

The key idea of IA is to design the precoding matrices in such a way that all interference signals are aligned into a signal subspace at each receiver in order to minimize the dimensions of the signal subspace occupied by the interference signals, while maintaining the interference-free subspace at each receiver. To this end, the precoding and filtering matrices are carefully designed to satisfy the following conditions:

$$\begin{aligned} \mathbf{U}_i^H \mathbf{H}_{ij} \mathbf{V}_j &= \mathbf{0}, \forall j \neq i, \\ \text{rank}[\mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i] &= d, \end{aligned} \quad (2) \quad (3)$$

where \mathbf{U}_i^H is the filtering matrix at the receiver i .

The received signal at the receiver i after interference suppression is given by

$$\begin{aligned} \mathbf{r}_i &= \mathbf{U}_i^H \left(\mathbf{H}_{ii} \mathbf{V}_i \mathbf{s}_i + \sum_{j=1, j \neq i}^K \mathbf{H}_{ij} \mathbf{V}_j \mathbf{s}_j + \mathbf{n}_i \right) \\ &= \mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i \mathbf{s}_i + \mathbf{U}_i^H \mathbf{n}_i, \end{aligned} \quad (4)$$

where we use the fact that all the interference signals at the receiver i is completely eliminated by the filtering matrix according to Eq. (2). The condition of Eq. (3) is satisfied almost surely if the channels are drawn independently from continuous distributions. Thus, the K -user MIMO interference channel is converted into K parallel MIMO channels with the effective channel $\mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i$ seen by the receiver i .

In this setting, the achievable sum rate for Gaussian signals [5] is given by

$$R_\Sigma = \sum_{i=1}^K \log_2 \det \left(\mathbf{I}_d + \frac{1}{\sigma^2} \tilde{\mathbf{H}}_{ii} \mathbf{Q}_i \tilde{\mathbf{H}}_{ii}^H \right), \quad (5)$$

where $\tilde{\mathbf{H}}_{ii} = \mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i$ and $\mathbf{Q}_i = \mathbb{E}[\mathbf{s}_i \mathbf{s}_i^H]$.

The closed-form expressions for the precoding and filtering matrices are only available for certain cases [4]. For general cases, however, it has been an open problem to give the closed-form expressions. As an alternative solution, iterative algorithms to derive the precoding and filtering matrices were proposed in [5]. In these iterative algorithms, the precoding and filtering matrices are alternately optimized in an iterative manner to minimize an interference leakage or maximize a signal-to-interference-plus-noise power ratio (SINR).

In [6], [7], the feasibility condition of the K -user MIMO interference channel was analyzed and shown that IA is feasible if and only if the following inequality is satisfied:

$$2M - (K+1)d \geq 0. \quad (6)$$

Therefore, for a given number of antennas M and a given number of data streams d , the maximum number of terminals K is limited according to $K \leq \frac{2M}{d} - 1$. For example, the maximum number of terminals for $d = 1$ is $K = 2M - 1$. Therefore, conventional IA is not feasible for the number of terminals greater than $K > \frac{2M}{d} - 1$, and thus all interference signals cannot be eliminated. Nevertheless, the condition of Eq. (6) is derived for perfect IA (i.e., all interference can be aligned and eliminated at every receiver) and does not take into account the effect of path loss. When small residual interference is allowed and the effect of path loss is taken into account, there is no need to align all interference at every receiver. For instance, if the path loss for residual interference is sufficiently high at every receiver, compared with that of a desired signal, there is obviously no point in aligning the residual interference. In the following sections, we show that such a situation arises in inter-vehicle communications. As a simplified model, we consider the MIMO interference channel with two IA groups, where all interference from the same group is aligned, but interference from the other group is not.

III. INTERFERENCE ALIGNMENT WITH MULTIPLE INTERFERENCE ALIGNMENT GROUPS

In this section, we consider the scenario where there are two IA groups. The system model is depicted in Fig. 2. In this system, there are two IA groups (the group A and group B), where the IA group g consists of K_g ($g \in \{A, B\}$) pairs of transmitters and receivers equipped with M antennas,

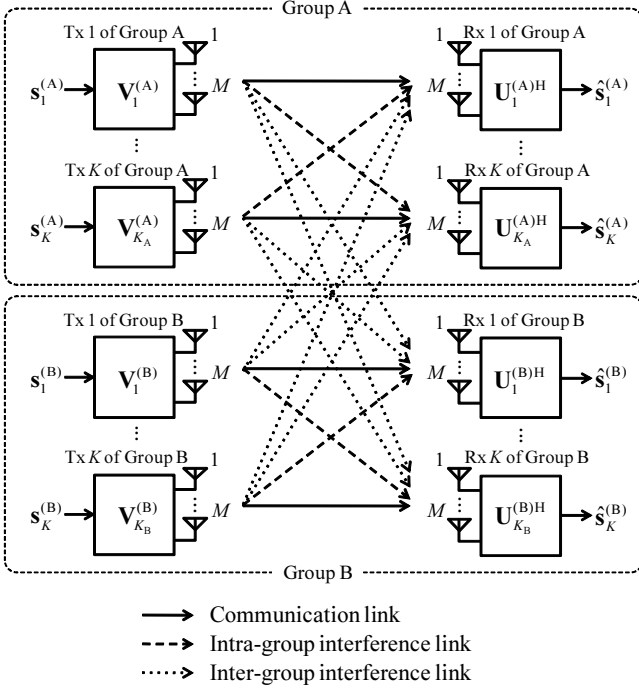


Fig. 2. $(K_A + K_B)$ -User MIMO Interference Channel.

all the transmitters of the group A and B transmit signals simultaneously, and the transmitter i of the group g wants to send d_g data streams to its corresponding receiver (the receiver i) of the same group.

Let $\mathbf{s}_i^{(A)} \in \mathbb{C}^{d_A \times 1}$, $\mathbf{s}_i^{(B)} \in \mathbb{C}^{d_B \times 1}$ denote the transmitting data vector from the transmitter i of the group A and that from the transmitter i of the group B, respectively. The transmitting data vectors $\mathbf{s}_i^{(A)}$ and $\mathbf{s}_i^{(B)}$ are precoded by $\mathbf{V}_i^{(A)} \in \mathbb{C}^{M \times d_A}$ and $\mathbf{V}_i^{(B)} \in \mathbb{C}^{M \times d_B}$, respectively. Here, we assume $\mathbb{E}[\|\mathbf{V}_i^{(A)} \mathbf{s}_i^{(A)}\|^2] = \mathbb{E}[\|\mathbf{V}_i^{(B)} \mathbf{s}_i^{(B)}\|^2] = P$.

The received signal at the receiver i of the group $g \in \{A, B\}$ after interference suppression is given by

$$\begin{aligned} \mathbf{r}_i^{(g)} &= \sqrt{\gamma_{ii}^{(g)}} \mathbf{U}_i^{(g)H} \mathbf{H}_{ii}^{(g)} \mathbf{V}_i^{(g)} \mathbf{s}_i^{(g)} \\ &+ \sum_{j=1, j \neq i}^K \sqrt{\gamma_{ij}^{(g)}} \mathbf{U}_i^{(g)H} \mathbf{H}_{ij}^{(g)} \mathbf{V}_j^{(g)} \mathbf{s}_j^{(g)} \\ &+ \sum_{j=1}^K \sqrt{\gamma_{ij}^{(g\bar{g})}} \mathbf{U}_i^{(g)H} \mathbf{H}_{ij}^{(g\bar{g})} \mathbf{V}_j^{(\bar{g})} \mathbf{s}_j^{(\bar{g})} + \mathbf{n}_i^{(g)} \\ &= \sqrt{\gamma_{ii}^{(g)}} \mathbf{U}_i^{(g)H} \mathbf{H}_{ii}^{(g)} \mathbf{V}_i^{(g)} \mathbf{s}_i^{(g)} \\ &+ \sum_{j=1}^K \sqrt{\gamma_{ij}^{(g\bar{g})}} \mathbf{U}_i^{(g)H} \mathbf{H}_{ij}^{(g\bar{g})} \mathbf{V}_j^{(\bar{g})} \mathbf{s}_j^{(\bar{g})} + \mathbf{n}_i^{(g)}, \quad (7) \end{aligned}$$

where \bar{g} denotes the complementary group of the group g (i.e., $\bar{g} = \{A, B\} \setminus \{g\}$), $\mathbf{U}_i^{(g)H} \in \mathbb{C}^{d_g \times M}$ is the filtering matrix at the receiver i of the group g , $\gamma_{ij}^{(g)}$ and $\gamma_{ij}^{(g\bar{g})}$ are the path gain from the transmitter j of the group g to the receiver i of the group g and that from the transmitter j of

the group \bar{g} to the receiver i of the group g , respectively, $\mathbf{H}_{ij}^{(g)}, \mathbf{H}_{ij}^{(g\bar{g})} \in \mathbb{C}^{M \times M}$ are the MIMO fading channel matrix from the transmitter j of the group g to the receiver i of the group g and that from the transmitter j of the group \bar{g} to the receiver i of the group g , respectively, and $\mathbf{n}_i^{(g)} \in \mathbb{C}^{M \times 1}$ is the complex circularly symmetric zero-mean AWGN noise vector with $\mathbb{E}[\mathbf{n}_i^{(g)} \mathbf{n}_i^{(g)H}] = \sigma^2 \mathbf{I}_M$ at the receiver i of the group g .

In this setting, the total achievable sum rate (i.e., the sum of the achievable sum rates of the groups A and B) for Gaussian signals [8] is given by

$$R_\Sigma = R_\Sigma^{(A)} + R_\Sigma^{(B)} \quad (8)$$

with

$$R_\Sigma^{(g)} = \sum_{i=1}^K \log_2 \det \left\{ \mathbf{I}_d + \gamma_{ii}^{(g)} \tilde{\mathbf{H}}_{ii}^{(g)} \mathbf{Q}_i^{(g)} \tilde{\mathbf{H}}_{ii}^{(g)H} \left(\sigma^2 \mathbf{I}_d + \sum_{j=1}^K \gamma_{ij}^{(g\bar{g})} \tilde{\mathbf{H}}_{ij}^{(g\bar{g})} \mathbf{Q}_j^{(\bar{g})} \tilde{\mathbf{H}}_{ij}^{(g\bar{g})H} \right)^{-1} \right\}, \quad (9)$$

where $\tilde{\mathbf{H}}_{ii}^{(g)} = \mathbf{U}_i^{(g)H} \mathbf{H}_{ii}^{(g)} \mathbf{V}_i^{(g)}$, $\tilde{\mathbf{H}}_{ij}^{(g\bar{g})} = \mathbf{U}_i^{(g)H} \mathbf{H}_{ij}^{(g\bar{g})} \mathbf{V}_j^{(\bar{g})}$, and $\mathbf{Q}_i^{(g)} = \mathbb{E}[\mathbf{s}_i^{(g)} \mathbf{s}_i^{(g)H}]$.

In Eqs. (7) and (9), we assume that the interference signals from the same group (the intra-group interference) are completely eliminated. However, the interference signals from the different group (the inter-group interference) still remain because they are not coordinated with each other for the receivers of the group g . Nevertheless, if the path gains $\gamma_{i1}^{(g\bar{g})}, \dots, \gamma_{iK}^{(g\bar{g})}$ are much lower than the path gain $\gamma_{ii}^{(g)}$ (i.e., $\gamma_{i1}^{(g\bar{g})}, \dots, \gamma_{iK}^{(g\bar{g})} \ll \gamma_{ii}^{(g)}$), the interference signals from the other group is considered to be negligible. This suggests that both groups can co-exist with negligible performance degradation if the pass gains for the inter-group interference satisfy the above condition. Such a scenario arises when channels in the same group are line-of-sight (LOS) and channels between different groups are non-line-of-sight (NLOS). One of the examples of such a scenario is inter-vehicle communications around an intersection within street canyons, which is detailed in the following section. In the following section, we evaluate the achievable sum rate in this system to show the positive impact of path loss for IA with multiple groups.

IV. SIMULATION RESULTS

A. Simulation Setup

The simulation environment is depicted in Fig. 3. There are two IA groups around the intersection between the roads A and B, where transmitters and receivers of the same group communicate across the intersection. We assume that the transmitters and receivers of the group A and group B are located on the road A and road B, respectively, and the transmitters of the group A and B transmit signals simultaneously. Each road consists of six lanes, and transmitters and receivers are placed on each lane at the equal interval of 10 m. We define $x_{\min}/2$ as the minimum distance from the center of the intersection to transmitters or receivers of each group. Each group is formed

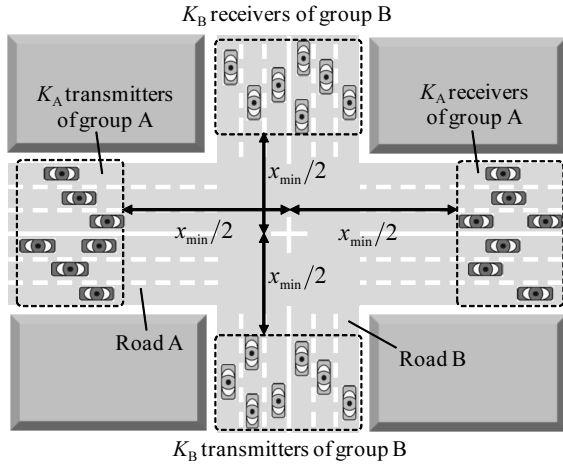


Fig. 3. Simulation environment.

by the K_g ($g \in \{A, B\}$) nearest transmitters and receivers from the center of the intersection but at least $x_{\min}/2$ [m] away from that point, and K_g pairs of transmitters and receivers are randomly determined.

Simulation parameters are as follows. The carrier-frequency is set to 5.9 GHz, which corresponds to the carrier-frequency for inter-vehicle communications in the U.S. and Europe. The transmission power P of each terminal is set to 10 dBm and the noise power σ^2 is set to -100 dBm. The number of antennas of each terminal M is assumed to be $M = 4$. For the sake of simplicity, we assume that $\mathbf{H}_{ij}^{(g)}$ and $\mathbf{H}_{ij}^{(g\bar{g})}$ are independent and identically distributed (i.i.d.) quasi-static frequency-flat MIMO Rayleigh fading channels. The path loss models used in the simulations are detailed in the following section. To obtain transmit and filtering matrices, we employ the iterative IA algorithm based on the minimum interference leakage criterion in [5]. For the sake of simplicity, we assume that IA in the same group is perfectly done, and thus no intra-group interference occurs. It is also assumed that the transmission power is equally allocated to each data stream.

B. Propagation Model

In this simulation, we adopt propagation loss models recommended by the ITU-R P.1411-5 [9]. The ITU-R P.1411-5 provides path loss models for outdoor LOS and NLOS situations within street canyons. In the following subsections, we briefly summarize both path loss models.

1) Line-of-Sight (LOS) Situations Within Street Canyons:

For the UHF frequency range and LOS situations within street canyons, path loss can be characterized by two slopes and a single breakpoint. Let λ be the wavelength and h_t and h_r be the transmitter and receiver antenna heights, respectively. For the distance between the transmitter and the receiver denoted

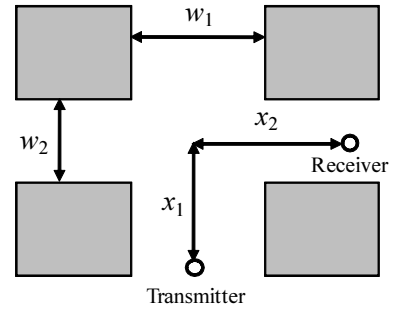


Fig. 4. Non-line-of-sight (NLOS) situations within street canyons.

by x , a median value of path loss is given by

$$L_{\text{LOS}} = L_{\text{bp}} + 6 + \begin{cases} 20 \log_{10} \left(\frac{x}{R_{\text{bp}}} \right), & \text{for } x \leq R_{\text{bp}} \\ 40 \log_{10} \left(\frac{x}{R_{\text{bp}}} \right), & \text{for } x > R_{\text{bp}} \end{cases}, \quad (10)$$

where R_{bp} is the break point distance and is approximately given by

$$R_{\text{bp}} \approx \frac{4h_t h_r}{\lambda}, \quad (11)$$

$L_{\text{bp}}+6$ is the propagation loss at the break point and L_{bp} is given by

$$L_{\text{bp}} = \left| 20 \log_{10} \left(\frac{\lambda^2}{8\pi h_t h_r} \right) \right|. \quad (12)$$

In this simulation, we set $h_t = h_r = 1.5$ m.

2) *Non-Line-of-Sight (NLOS) Situations Within Street Canyons:* For a frequency range from 2 to 16 GHz and NLOS situations within street canyons, the path loss model is derived based on measurements. For x_1 , x_2 , w_1 , and w_2 , as shown in Fig. 4, path loss beyond the corner region ($x_2 > w_1/2 + 1$) is given by

$$L_{\text{NLOS}} = L_{\text{LOS}} + L_c + L_{\text{att}}, \quad (13)$$

with

$$L_c = \begin{cases} \frac{L_{\text{corner}}}{\log_{10}(1 + d_{\text{corner}})} \log_{10} \left(x_2 - \frac{w_1}{2} \right), & \text{for } \frac{w_1}{2} + 1 < x_2 \leq \frac{w_1}{2} + 1 + d_{\text{corner}} \\ L_{\text{corner}}, & \text{for } x_2 > \frac{w_1}{2} + 1 + d_{\text{corner}} \end{cases} \quad (14)$$

and

$$L_{\text{att}} = \begin{cases} 0, & \text{for } x_2 \leq \frac{w_1}{2} + 1 + d_{\text{corner}} \\ 60 \log_{10} \left(\frac{x_1 + x_2}{x_1 + \frac{w_1}{2} + d_{\text{corner}}} \right), & \text{for } x_2 > \frac{w_1}{2} + 1 + d_{\text{corner}} \end{cases}, \quad (15)$$

where L_{LOS} is the path loss in the LOS street for x_1 (> 20 m) given by Eq. (10), L_{corner} is 20 dB for an urban environment and 30 dB for a residential environment, and d_{corner} is 30 m for both environments. In this simulation, we set $L_{\text{corner}} = 20$ dB and $w_1 = w_2 = 30$ m.

C. Performance Evaluations

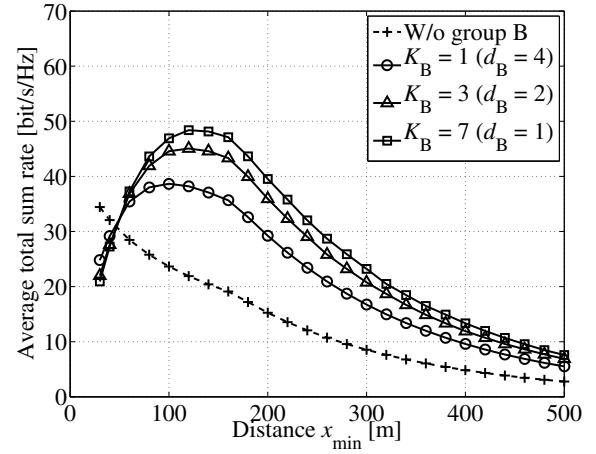
Figure 5 shows the averaged total sum rate R_Σ as a function of the distance x_{\min} for different values of K_A and K_B . For given K_A and K_B , the number of data streams of each transmitter in the group g ($g \in \{A, B\}$) is set to $d_g = \lfloor 2M/(K_g + 1) \rfloor$ to satisfy the condition of Eq. (6). For comparison, we also indicate the averaged sum rate of the group A when the transmitters and receivers of the group B are absent (i.e., single-group transmission), denoted by “W/o group B” in the figure. From this figure, we observe that the total sum rate performance is improved by multiple-group transmission when x_{\min} is more than 100 m, compared with that of single-group transmission. However, for a small x_{\min} , the total sum rate of multiple-group transmission is degraded due to significant inter-group interference, and single-group transmission achieves a higher sum rate. This degradation becomes prominent as the number of terminals K_A increases. This suggests that the mode of transmission must be adaptively switched between single-group and multiple-group transmission according to the locations of groups.

V. CONCLUSION

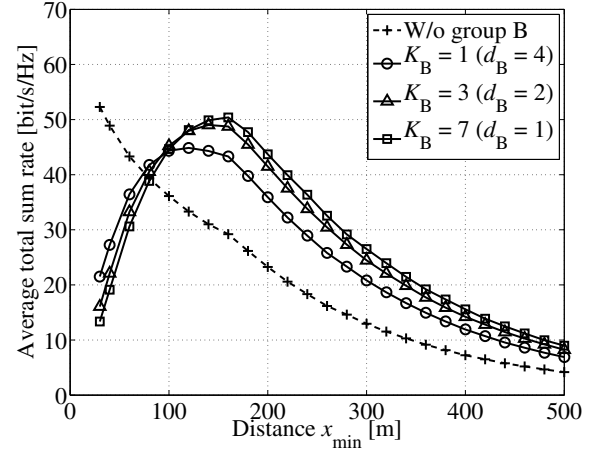
In this paper, we considered MIMO interference channels with multiple IA groups for the application of IA to inter-vehicle communications. Through numerical simulations, we evaluated the achievable sum rate in an inter-vehicle communication system where there are two IA groups around an intersection. The simulation results showed that the sum rate performance is improved by multiple-group transmission and utilizing high propagation loss at a corner of the intersection, compared with that of single-group transmission, when both groups are located a certain distance away from each other.

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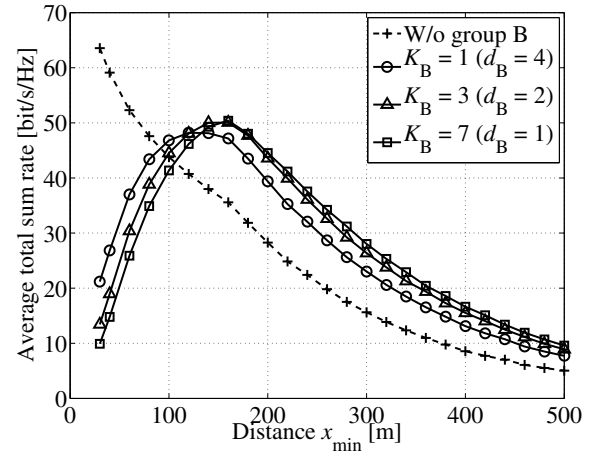
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(a) $K_A = 1$ ($d_A = 4$)



(b) $K_A = 3$ ($d_A = 2$)



(c) $K_A = 7$ ($d_A = 1$)

Fig. 5. Average total sum rate R_Σ as a function of the distance x_{\min} for the number of antennas $M = 4$.

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