Power Allocation in Energy Harvesting Relay Systems

Imtiaz Ahmed[†], Aissa Ikhlef[†], Robert Schober[†], and Ranjan K. Mallik[††]
[†]University of British Columbia, Vancouver, Canada, [††]Indian Institute of Technology, Delhi, India

***Abstract*— Using energy harvesting nodes can be a viable solution for energy limited cooperative communication systems. In this paper, we consider the optimization of an energy harvesting (EH) relay system, where an EH source communicates with the destination via an EH decode–and–forward (DF) relay. Our objective is to maximize the system throughput over a finite number of transmission intervals. To this end, we propose an offline and several online joint source and relay transmit power allocation schemes. For offline power allocation, we formulate a convex optimization problem which can be solved either in closed form or using standard optimization tools. For the online case, we propose a dynamic programming (DP) approach to compute the optimal online transmit power. To alleviate the complexity inherent to DP, we propose several suboptimal online power allocation schemes. Our simulation results show that the developed suboptimal schemes provide a good complexity–performance tradeoff compared to optimal online power allocation.**

I. Introduction

In cooperative communication systems, the source and the cooperating relays expend their own energy in processing and transmitting data. For some applications, connecting source and relays to the power grid is cumbersome or may even not be possible. Precharged batteries can be a viable solution to overcome this problem. In practice, the limited storage capacity of the batteries and high transmit powers of the source and the relays may result in quick drainage of the batteries. As a result, the batteries need to be replaced/recharged periodically which can be sometimes impractical. An alternative solution is the deployment of energy harvesting (EH) nodes. EH nodes harvest energy from their surrounding environment to carry out their functions. Energy can be harvested using solar, thermoelectric, and motion effects, or by other physical phenomena [1]. An EH node that has used up its stored energy can harvest new energy and become again available for transmission. Thus, EH nodes can be regarded as a promising option for deployment as they ensure a long system lifetime without the need for periodic battery replacements.

In EH cooperative systems, the energy can be harvested by the EH source and/or EH relays independently during the course of data transmission at random times and in random amounts. For data transmission (and for other signal processing tasks), EH nodes expend the energy from their storage and only the unused energy remains in the batteries. In particular, at each time slot, the source and the relays are constrained to use at most the available energy from their storage. These constraints lead to the need of designing new transmission strategies for the source and the relays to ensure optimum performance in an EH environment.

Recently, transmission strategies for and performance analyses of EH nodes in wireless communication systems have been provided in [2]–[7]. In [2], an EH source is considered in a single source–destination non–cooperative link and the optimal offline along with the optimal and several sub–optimal online transmission policies for allocating transmit power to the source according to the random variations of the channel and the energy storage conditions are provided. In [3], a similar system model is considered where dynamic programming (DP) is employed to allocate source transmit power for the cases where causal and full channel state information (CSI) is available. Several higher layer issues e.g. transmission time minimization and transmission packet scheduling in EH systems are considered in [5]–[7]. In fact, the deployment of EH sensors in sensor networks has been extensively discussed in the literature [1], [8]. The use of EH relays in cooperative communication has been introduced in [4], where a comprehensive performance analysis is performed for relay selection in a cooperative network employing EH relays. However, to the best of our knowledge, the transmission power management in a cooperative communication system with EH nodes has not been considered before.

In this paper, we consider a simple single link cooperative system where a source communicates with the destination via a decode–and–forward (DF) relay. Both the source and the relay are EH nodes. We propose offline and different online (real–time) power allocation schemes that maximize the end–to–end system throughput over a finite number of transmission intervals. The offline scheme is of interest when the amount of harvested energy and the channel signal–to–noise ratio (SNR) for all transmission intervals are known a–priori. However, in practice, the amount of harvested energy and channel SNR are random in nature and cannot be predicted in advance. Hence, in this case, optimum online power allocation is considered and a DP approach is adopted. To avoid the complexity inherent to DP, we also propose several sub–optimal online algorithms.

II. System Model

Signal Model: We consider an EH relay system, where the source, S, communicates with the destination, D, via a cooperative relay, R. S and R are EH devices and their participation in signal transmission and processing solely depends on the harvested energy. They are equipped with batteries, which have limited storage capacity and store the harvested energy for future use. In particular, the batteries of S and R can store at most $B_{S,max}$ and $B_{R,max}$ units of energy, respectively.

We assume the transmission is organized in equal duration time intervals and each interval is comprised of two time slots of duration T, respectively. In the following, we set $T = 1$. The total transmission time is equal to K intervals. During the first time slot, S transmits and R receives, and during the second time slot, R transmits and D receives. The received

978-1-4673-0990-5/12/$31.00 ©2012 IEEE

packet at R in the first slot of transmission of time interval $k \in \{1, 2, \cdots, K\}$ is given by $y_{R,k} = h_{S,k} x_k + n_{R,k}$, where $h_{S,k}$ is the fading gain of the S–R link, and $n_{R,k}$ denotes the noise sample at R. The transmitted packet x_k contains symbols that are taken from an M–ary alphabet, $\mathcal{A}$, such as M–ary quadrature amplitude modulation (QAM) or M–ary phase shift keying (PSK). Assuming DF relaying at R, the detected packet, $\hat{x}_k$, is transmitted from R during the second time slot of time interval k. Thus, the received packet at D is given by $y_{D,k} = h_{R,k}\hat{x}_k + n_{D,k}$, where $h_{R,k}$ and $n_{D,k}$ denote the fading gain of the R–D link and the noise sample at D, respectively. $h_{S,k}$ and $h_{R,k}$ can follow any fading distribution, e.g., Rayleigh, Rician, Nakagami–q, or Nakagami–m fading. $n_{R,k}$ and $n_{D,k}$ are additive white Gaussian noise (AWGN) samples having zero mean and unit variance.

System Throughput: We assume the channels are quasi–static within each interval and the channel SNRs of the S–R and the R–D links are $\gamma_{S,k}$ and $\gamma_{R,k}$, respectively. In particular, $\gamma_{S,k} = |h_{S,k}|^2$ and $\gamma_{R,k} = |h_{R,k}|^2$. We assume $\gamma_{S,k}$ and $\gamma_{R,k}$ are independent of each other and independent and identically distributed (i.i.d.) over the time intervals. For future reference, we introduce the average SNRs of the S–R and the R–D links as $\bar{\gamma}_{S,k}$ and $\bar{\gamma}_{R,k}$, respectively. When x_k is transmitted from S with transmit power $P_{S,k}$ during the first time slot of transmission interval k, $\xi_{S,k} \triangleq \log_2(1 + \gamma_{S,k} P_{S,k})$ bits of data are transmitted via the S–R link. Similarly, when $\hat{x}_k$ is transmitted from R with transmit power $P_{R,k}$, $\xi_{R,k} \triangleq \log_2(1 + \gamma_{R,k} P_{R,k})$ bits of data are transmitted via the R–D link. We assume R ensures error free detection by employing standard error correction coding and hence $\hat{x}_k = x_k$. Therefore, the end–to–end (S–D) system throughput in interval k is given by $\frac{1}{2}\min\{\xi_{S,k}, \xi_{R,k}\}$ bits/s/Hz where the factor $\frac{1}{2}$ is due to the half-duplex constraint.

Battery Dynamics: The battery energies of S and R in interval k are $B_{S,k}$ and $B_{R,k}$, respectively. During transmission interval k, the transmit powers of S and R are bounded by their battery energies, i.e., $0 \le P_{S,k} \le B_{S,k}$ and $0 \le P_{R,k} \le B_{R,k}$. We assume the energy consumed by the internal circuitry of S and R is negligible compared to the transmit power [3]. While transmitting a packet, the energy harvester at S collects $H_{S,k} \le B_{S,max}$ units of energy from the beginning of the first time slot to the end of the second time slot of interval k. Similarly, the energy harvester at R collects $H_{R,k} \le B_{R,max}$ units of energy from the beginning of the second time slot of the kth interval to the end of the first time slot of the $(k+1)$th interval. Let $H_{S,E} \triangleq E\{H_{S,k}\}$ and $H_{R,E} \triangleq E\{H_{R,k}\}$ denote the average energy harvesting rate of S and R over the time intervals, respectively. Here, $E\{\cdot\}$ denotes the expectation. We assume $H_{S,k}$ and $H_{R,k}$ are independent of each other and i.i.d. over time intervals. Similar to [3], we assume the stored energies at S and R increase and decrease linearly provided the maximum storage capacities, $B_{S,max}$ and $B_{R,max}$ are not exceeded, i.e.,

$$B_{S,k+1} = \min\{(B_{S,k} - P_{S,k} + H_{S,k}), B_{S,max}\}, \ \forall k \quad (1)$$
$$B_{R,k+1} = \min\{(B_{R,k} - P_{R,k} + H_{R,k}), B_{R,max}\}, \ \forall k. \quad (2)$$

Furthermore, $B_{S,1} = H_{S,0} \ge 0$ and $B_{R,1} = H_{R,0} \ge 0$, respectively, denote the available energies at S and R before transmission starts.

III. Power Allocation with Energy Constraints

A. *Offline Optimal Power Allocation*

We consider maximizing the total number of transmitted bits (from S to D) delivered by a deadline of K intervals over a fading channel assuming offline (prior) knowledge of the full CSI and the energy arrivals at S and R in each time interval. The maximization problem is subject to a causality constraint on the harvested energy and the (maximum) storage constraint for the battery at both S and R.

The offline optimization problem for maximizing the throughput of the considered system for K intervals can be formulated as follows

$$\max_{\mathcal{T} \succeq 0} \quad \sum_{k=1}^{K} \min\{\xi_{S,k}, \xi_{R,k}\} \quad (3)$$
$$\text{s.t.} \quad \sum_{k=1}^{l} (P_{S,k} + \lambda_{S,k}) \le \sum_{k=0}^{l-1} H_{S,k}, \ \forall l \quad (4)$$
$$\sum_{k=1}^{l} (P_{R,k} + \lambda_{R,k}) \le \sum_{k=0}^{l-1} H_{R,k}, \ \forall l \quad (5)$$
$$\sum_{k=0}^{m} H_{S,k} - \sum_{k=1}^{m} (P_{S,k} + \lambda_{S,k}) \le B_{S,max}, \ \forall m \quad (6)$$
$$\sum_{k=0}^{m} H_{R,k} - \sum_{k=1}^{m} (P_{R,k} + \lambda_{R,k}) \le B_{R,max}, \ \forall m \quad (7)$$
$$\gamma_{S,k} P_{S,k} = \gamma_{R,k} P_{R,k}, \ \forall k, \quad (8)$$

where $\mathcal{T} \triangleq [P_{S,k}\ P_{R,k}\ \lambda_{S,k}\ \lambda_{R,k}]$. $\forall l$, $\forall m$, and $\forall k$ stand for $l = 1, 2, \cdots, K$, $m = 1, 2, \cdots, K-1$, and $k = 1, 2, \cdots, K$, respectively. The slack variables $\lambda_{S,k}$ and $\lambda_{R,k}$ ensure that constraints (6)–(8) can be met for all realizations of $\gamma_{S,k}$, $\gamma_{R,k}$, $H_{S,k}$, and $H_{R,k}$. In particular, these slack variables represent the energy (possibly) wasted in each transmission interval. Constraints (4) and (5) stem from the causality requirement on the energy harvested at S and R, respectively. Moreover, (6) and (7) ensure the harvested energy does not exceed the limited storage capacity of the batteries at S and R, respectively. Constraint (8) ensures that the amount of information transmitted from S to R is identical to that transmitted from R to D so as to avoid data loss at R.

Using (8) in (3)–(7) the considered offline optimization problem can be rewritten as

$$\max_{\mathcal{T}' \succeq 0} \quad \sum_{k=1}^{K} \xi_{S,k} \quad (9)$$
$$\text{s.t.} \quad \sum_{k=1}^{l} \left(\frac{\gamma_{S,k} P_{S,k}}{\gamma_{R,k}} + \lambda_{R,k} \right) \le \sum_{k=0}^{l-1} H_{R,k}, \ \forall l \quad (10)$$
$$\sum_{k=0}^{m} H_{R,k} - \sum_{k=1}^{m} \left(\frac{\gamma_{S,k} P_{S,k}}{\gamma_{R,k}} + \lambda_{R,k} \right) \le B_{R,max}, \ \forall m. \quad (11)$$

Constraints (4) and (6),

where $\mathcal{T}' \triangleq \mathcal{T} \setminus P_{R,k}$. The problem in (9)–(11) with (4) and (6) forms a convex optimization problem and the optimum solution can be obtained either in closed form by using the Karush-Kuhn-Tucker (KKT) conditions or by using any standard technique that solves convex optimization problems [9], [10]. Let $P^*_{S,k}$ denote the optimum solution of the considered optimization problem. Then, the optimum $P_{R,k}$ can be obtained as

$$P^*_{R,k} = \frac{\gamma_{S,k} P^*_{S,k}}{\gamma_{R,k}}. \quad (12)$$

B. Optimal Online Power Allocation by DP

In practice, only causal information of channels and harvested energies are available for power allocation. Therefore, the offline power allocation scheme is not readily applicable as at a given time interval k the future CSI and the upcoming harvested energy are not known in advance. We propose to employ a stochastic DP approach for optimum online power allocation [3], [11].

Let $\boldsymbol{c}_k \triangleq (\gamma_{S,k}, \gamma_{R,k}, H_{S,k-1}, H_{R,k-1}, B_{S,k}, B_{R,k})$ denote the state for time interval k. Our aim is to maximize the average throughput over K intervals. We assume the initial state $\boldsymbol{c}_1 = (\gamma_{S,1}, \gamma_{R,1}, H_{S,0}, H_{R,0}, B_{S,1}, B_{R,1})$ is always known. We define a policy $p = \{(P_{S,k}(\boldsymbol{c}_k), P_{R,k}(\boldsymbol{c}_k)), \forall \boldsymbol{c}_k, k = 1, 2, \cdots, K\}$, as feasible if the energy harvesting constraints $0 \le P_{S,k}(\boldsymbol{c}_k) \le B_{S,k}$ and $0 \le P_{R,k}(\boldsymbol{c}_k) \le B_{R,k}$ are satisfied for all k. Hence, the objective function to be maximized can be reformulated as [3]

$$R(p) = \sum_{k=1}^{K} E\{\min\{\xi'_{S,k}, \xi'_{R,k}\} | \boldsymbol{c}_1, p\} \quad (13)$$

where we defined $\xi'_{S,k} \triangleq \log_2(1 + \gamma_{S,k} P_{S,k}(\boldsymbol{c}_k))$, $\xi'_{R,k} \triangleq \log_2(1 + \gamma_{R,k} P_{R,k}(\boldsymbol{c}_k))$, and the expectation is with respect to the SNRs of the channels and the harvested energies. In particular, for a given $\boldsymbol{c}_1$, the maximum throughput can be obtained as

$$R^* = \max_{p \in \mathcal{P}} R(p), \quad (14)$$

where $\mathcal{P}$ denotes the space of all feasible policies.

The maximum throughput at time interval k is denoted by $J_k(B_{S,k}, B_{R,k})$. For a given $\boldsymbol{c}_1$, the maximum throughput, $J_1(B_{S,1}, B_{R,1})$, can be recursively obtained from $J_K(B_{S,K}, B_{R,K})$, $J_{K-1}(B_{S,K-1}, B_{R,K-1})$, $\cdots$, $J_2(B_{S,2}, B_{R,2})$ [3]. For the last time interval K, we have

$$J_K(B_{S,K}, B_{R,K}) = \max_{\substack{0 \le P_{S,K} \le B_{S,K} \\ 0 \le P_{R,K} \le B_{R,K} \\ \gamma_{S,K} P_{S,K} = \gamma_{R,K} P_{R,K}}} \frac{1}{2} \min\{\xi_{S,K}, \xi_{R,K}\} \quad (15)$$

and for time interval k, we obtain

$$J_k(B_{S,k}, B_{R,k}) = \max_{\substack{0 \le P_{S,k} \le B_{S,k} \\ 0 \le P_{R,k} \le B_{R,k} \\ \gamma_{S,k} P_{S,k} = \gamma_{R,k} P_{R,k}}} \frac{1}{2} \min\{\xi_{S,k}, \xi_{R,k}\} + \bar{J}_{k+1}(B_{S,k} - P_{S,k}, B_{R,k} - P_{R,k}), \quad (16)$$

where,

$$\bar{J}_{k+1}(B'_{S,k+1}, B'_{R,k+1}) = E_{\tilde{\gamma}_{S,k+1}, \tilde{\gamma}_{R,k+1}, \tilde{H}_{S,k}, \tilde{H}_{R,k}}\{J_{k+1}(\min\{B'_{S,k+1} + \tilde{H}_{S,k}, B_{S,max}\}, \min\{B'_{R,k+1} + \tilde{H}_{R,k}, B_{R,max}\})\}. \quad (17)$$

Here, $B'_{S,k+1} \triangleq B_{S,k} - P_{S,k}$, $B'_{R,k+1} \triangleq B_{R,k} - P_{R,k}$, $\tilde{\gamma}_{S,k+1}$ ($\tilde{\gamma}_{R,k+1}$) represents the SNR of the S–R (R–D) link in the $(k+1)$th interval given the SNR $\gamma_{S,k}$ in the kth interval, and $\tilde{H}_{S,k}$ ($\tilde{H}_{R,k}$) denotes the harvested energy at S (R) in the kth interval given the harvested energy $H_{S,k-1}$ in the $(k-1)$th interval. It can be shown that the cost functions in (15) and (16) are concave in $P_{S,k}$ and $P_{R,k}$. Thus, (15) and (16) are convex optimization problems and can be solved very efficiently [9]. Further simplification of (15) yields

$$J_K(B_{S,K}, B_{R,K}) = \frac{1}{2} \log_2(1 + \gamma_{S,K} \rho_K), \quad (18)$$

where $\rho_K = \min\{B_{S,K}, \gamma_{R,K} B_{R,K} / \gamma_{S,K}\}$. Therefore, $P^*_{S,K} = \min\{B_{S,K}, \frac{\gamma_{R,K} B_{R,K}}{\gamma_{S,K}}\}$, and $P^*_{R,K}$ follows from (12). Similarly, (16) can be simplified to

$$J_k(B_{S,k}, B_{R,k}) = \max_{0 \le T_{S,k} \le \min\{B_{S,k}, \gamma_{R,k} B_{R,k} / \gamma_{S,k}\}} \frac{1}{2} \xi_{S,k} + \bar{J}_{k+1}(B_{S,k} - P_{S,k}, B_{R,k} - \gamma_{S,k} P_{S,k} / \gamma_{R,k}), \quad (19)$$

Using (18) and (19), $P^*_{S,k}$ and $P^*_{R,k}$, $k \in \{1, 2, \cdots, K\}$, can be obtained for different possible values of $\gamma_{S,k}$, $\gamma_{R,k}$, $B_{S,k}$, and $B_{R,k}$. The results are stored in look–up tables. This is done before transmission starts. When transmission starts, for a given realization of $\gamma_{S,k}$, $\gamma_{R,k}$, $B_{S,k}$, and $B_{R,k}$ in time interval k, those values of $P^*_{S,k}$ and $P^*_{R,k}$ are picked that correspond to that realization from the look–up tables.

C. Suboptimal Online Power Allocation

In the proposed DP–based optimal online power allocation algorithm, for a certain transmission time interval k, we consider the average effect of all the succeeding time intervals. Due to the recursive nature of DP, the computational complexity of this approach increases alarmingly with increasing K. For this reason, in the following, we propose three different suboptimal online power allocation schemes, which perform close to the optimal DP approach with reduced complexity.

1) Suboptimal Simplified DP Power Allocation ("DP–I$_2$" and "DP–I$_1$" Schemes): In this scheme, we use the average effect of only 1 (or 2) following time interval(s) to allocate the transmit power in each time interval. In particular, we assume for the current time interval that all the energies have to be spent over the following 1 (or 2) time interval(s). Moreover, in the last time interval, either S or R uses up all of its stored energy. This scheme reduces the computational complexity at the expense of a performance degradation. We refer to the suboptimal DP schemes with 2 and 1 intervals as "DP–I$_2$" and "DP–I$_1$", respectively.

2) Suboptimal Harvesting Rate (HR) Assisted Power Allocation ("HR Assisted" Scheme): In this scheme, we constrain the transmit powers $P_{S,k}$ and $P_{R,k}$ by the average energy harvesting rates $H_{S,E}$ and $H_{R,E}$, respectively. This scheme is referred to as "HR Assisted" power allocation. For a given time interval $k \in \{1, 2, \cdots, K-1\}$ the resulting optimization problem can be stated as

$$\max_{P_{S,k} \geq 0} \quad \xi_{S,k} \tag{20}$$

$$\text{subject to} \quad P_{S,k} \leq B_{S,k}, \tag{21}$$

$$P_{S,k} \leq H_{S,E}, \tag{22}$$

$$P_{S,k} \leq \frac{\gamma_{R,k} B_{R,k}}{\gamma_{S,k}}, \tag{23}$$

$$P_{S,k} \leq \frac{\gamma_{R,k} H_{R,E}}{\gamma_{S,k}}. \tag{24}$$

Therefore, the optimum solution for this optimization problem is given by $P^*_{S,k} = \min\{B_{S,k}, H_{S,E}, \frac{\gamma_{R,k} B_{R,k}}{\gamma_{S,k}}, \frac{\gamma_{R,k} H_{R,E}}{\gamma_{S,k}}\}$, and $P^*_{R,k}$ is obtained from (12). For the Kth time interval, we ensure that either S or R uses up all of its stored energy.

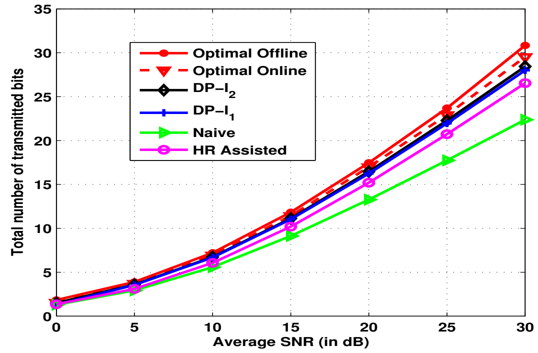
3) Suboptimal Naive Power Allocation ("Naive" Scheme): In this suboptimal "naive" approach, for each time interval, only the stored energies at hand determine the transmit power, i.e., this approach does not take into account the effect of the following time intervals. We note that either S or R uses up all of its available energy in each transmission interval. To be specific, for a particular time interval $k \in \{1, 2, \cdots, K\}$, $P^*_{S,k} = \min\{B_{S,k}, \frac{\gamma_{R,k} B_{R,k}}{\gamma_{S,k}}\}$, and $P^*_{R,k}$ follows directly from (12).

IV. Simulation Results

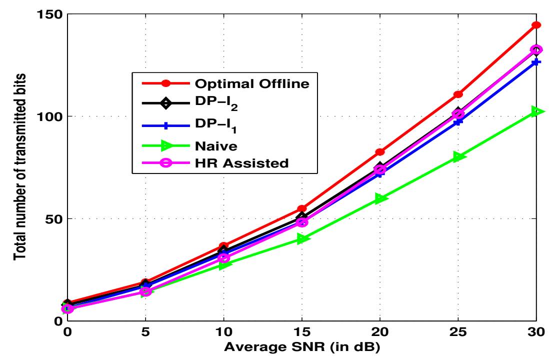
In this section, we evaluate the performance of the proposed offline and online power allocation schemes. The (overall) average harvesting rate is $H_{S,E} = H_{R,E} = H_E$ and we assume $H_{S,k}$ and $H_{R,k}$ independently take a value from the set $\{0, H_E, 2H_E\}$, where all elements of the set are equiprobable. For Figs. 1(a), 1(b), 2(a), and 2(b), we assume $H_E = 0.5$. For all the considered simulations, we assume an i.i.d. Rayleigh fading channel for all time intervals and $B_{max} = 4$. For all simulations, we assume $\bar{\gamma}_{S,k} = \bar{\gamma}_{R,k} = \bar{\gamma}$ and use 10^4 randomly generated realizations of the S–R and the R–D channels to obtain the average throughput.

A. Total number of transmitted bits vs. $\bar{\gamma}$

Figs. 1(a) and 1(b) show the total number of transmitted bits for the proposed power allocation schemes vs. the average channel SNR, $\bar{\gamma}$, for $K = 5$ and $K = 20$, respectively. For all considered schemes, the total throughput increases as $\bar{\gamma}$ increases. The optimal offline scheme, which allocates the transmit power while knowing the complete CSI and the harvested energy, performs better than the online power allocation schemes for all $\bar{\gamma}$ and for both $K = 5$ and $K = 20$. Moreover, as expected, the optimal online scheme outperforms all considered suboptimal online schemes and performs close to the optimal offline scheme. As the complexity of implementation of DP increases rapidly with increasing K, we show the performance of the optimal online scheme only for $K = 5$, i.e., in Fig. 1(a). For $K = 5$, the suboptimal online schemes DP–I$_2$ and DP–I$_1$ perform close to each other for all $\bar{\gamma}$. On the contrary, for $K = 20$, DP–I$_2$ outperforms DP–I$_1$ at sufficiently high $\bar{\gamma}$. The consideration of two succeeding time intervals for power allocation in the current interval has less impact for low K, whereas for high K considering two time intervals leads to performance gains. The HR assisted scheme performs better than the naive scheme for both $K = 5$ and $K = 20$ at sufficiently high values of $\bar{\gamma}$. Moreover, since the transmitted powers are constrained by the average energy harvesting rate, the HR assisted scheme yields better results for large K. Indeed, for $K = 20$, the HR assisted scheme outperforms DP–I$_1$ and achieves a performance similar to that of DP–I$_2$ at high $\bar{\gamma}$.



(a) $K = 5$.

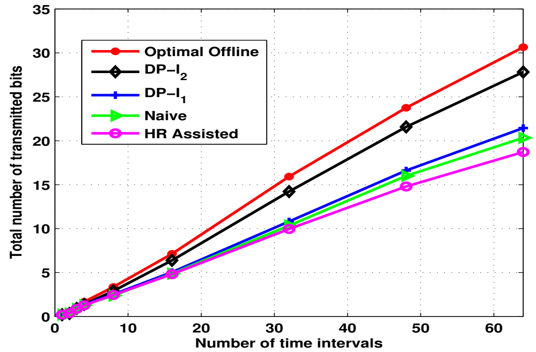


(b) $K = 20$.

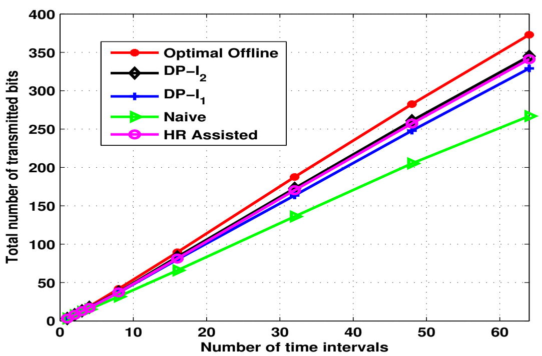
Fig. 1. Total number of transmitted bits vs. average channel SNR $\bar{\gamma}$ for different K.

B. Total number of transmitted bits vs. K

In Figs. 2(a) and 2(b), we show the total number for transmitted bits of the proposed power allocation schemes vs. the number of time intervals K for $\bar{\gamma} = 0$ dB and $\bar{\gamma} = 25$ dB, respectively. We observe that the optimal offline method is the best among all considered power allocation schemes in both figures. Among different suboptimal online schemes, DP–I$_2$ shows significant performance improvement over all other suboptimal schemes for large K and for low $\bar{\gamma}$ (Fig. 2(a)), whereas this improvement is relatively small for high $\bar{\gamma}$ (Fig. 2(b)). Surprisingly, the HR assisted scheme, which provides a similar performance as DP–I$_2$ for $\bar{\gamma} = 25$ dB, shows degraded performance at $\bar{\gamma} = 0$ dB. For low $\bar{\gamma}$, DP–I$_1$ and the naive scheme have similar performance, whereas for high $\bar{\gamma}$, DP–I$_1$ outperforms the naive scheme.



(a) $\bar{\gamma} = 0$ dB.



(b) $\bar{\gamma} = 25$ dB.

Fig. 2. Total number of transmitted bits vs. number of time intervals K for different $\bar{\gamma}$.

C. Total number of transmitted bits vs. H_E

Fig. 3 depicts the total number of transmitted bits for the proposed power allocation schemes vs. the average harvesting rate, H_E, for $\bar{\gamma} = 25$ dB and $K = 30$. We observe that the throughput increases with increasing H_E for all considered power allocation schemes. We note that the slope of the throughput curves is large for small H_E and decreases with increasing H_E. This behavior is mainly due to the fact that the performance of all the schemes is limited by the finite storage capability of the batteries. For large H_E, additional energy cannot be stored in the batteries and therefore the extra amount is wasted. Similar to the previous figures, the optimal offline scheme is the best among all considered power allocation schemes. We also observe that the HR assisted scheme performs better than the other online algorithms at low H_E, whereas at high H_E, DP–I_1 and the HR assisted scheme show similar performance.

V. Conclusions

In this paper, we have considered transmit power allocation schemes for a single link relay system, where the source and the relay transmit signals using the energy harvested from the surrounding environment. The harvesting process is random and the stored energy at the batteries depends on the energy harvesting rate, the previous status of the batteries, and the previous transmit powers. These factor make optimal power allocation in EH systems more challenging than in conventional

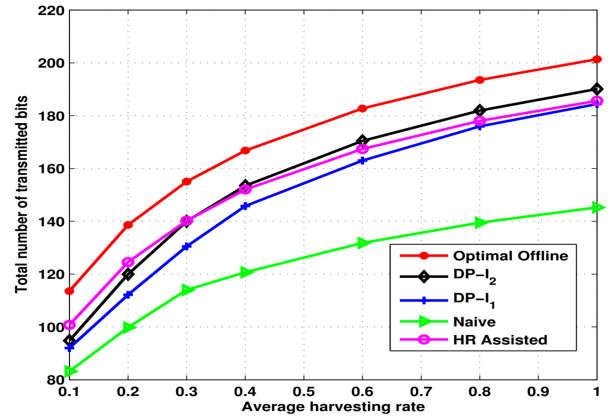


Fig. 3. Total number of transmitted bits vs. H_E for $K = 8$ and $\bar{\gamma} = 25$ dB.

systems. We have proposed an offline and several online power allocation schemes maximizing the system throughput over a finite number of time intervals. The proposed offline scheme performs optimum among all considered power allocation schemes, whereas the DP assisted optimal power allocation scheme performs the best among the online schemes. However, the suboptimal online schemes provide less complexity at the expense of a degraded performance in comparison to the optimal online scheme.

References

[1] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava. Power Management in Energy Harvesting Sensor Networks. *ACM Trans. Embed. Comput. Syst.*, 6:1–35, September 2007.

[2] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener. Transmission with Energy Harvesting Nodes in Fading Wireless Channels: Optimal Policies. *IEEE J. Select. Areas Commun.*, 29:1732–1743, September 2011.

[3] C. K. Ho and R. Zhang. Optimal Energy Allocation for Wireless Communications with Energy Harvesting Constraints. *[Online]: http://arxiv.org/abs/1103.5290.*

[4] B. Medepally and N. B. Mehta. Voluntary Energy Harvesting Relays and Selection in Cooperative Wireless Networks. *IEEE Trans. on Wireless Commun.*, 9:3543–3553, November 2010.

[5] J. Yang and S. Ulukus. Transmission Completion Time Minimization in an Energy Harvesting System. *Proceedings of Information Sciences and Systems (CISS)*, pages 1–6, March 2010.

[6] J. Yang and S. Ulukus. Optimal Packet Scheduling in an Energy Harvesting Communication System. *[Online]: http://arxiv.org/abs/1010.1295.*

[7] J. Yang, O. Ozel, and S. Ulukus. Broadcasting with an Energy Harvesting Rechargeable Transmitter. *IEEE Trans. Inform. Theory (submitted) [Online]: http://www.ece.umd.edu/ ulukus/papers/journal/bc-enerharv.pdf.*

[8] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta. Optimal Energy Management Policies for Energy Harvesting Sensor Nodes. *IEEE Trans. on Wireless Commun.*, 9:1326–1336, April 2010.

[9] S. Boyd and L. Vandenberghe. Convex Optimization. *Cambridge, UK: Cambridge University Press*, 2004.

[10] CVX: Matlab Software for Disciplined Convex Programming. *[Online]: http://www.cvxr.com/cvx/.*

[11] D. P. Bertsekas. Dynamic Programming and Optimal Control Vol. 1. *Belmont, MA: Athens Scientific.*