Performance Analysis of Distributed Beamforming in a Spectrum Sharing System

Liang Yang*◇
*Department of Electrical Engineering
Jinan University
Guangzhou, China

Mohamed-Slim Alouini †
†Electrical Engineering Program
KAUST
Thuwal,Saudi Arabia

Khalid Qaraqe ◇
◇Electrical and Computer Engineering Program
Texas A&M University at Qatar
Doha, Qatar

***Abstract*— In this paper, we consider a distributed beamforming scheme (DBF) in a spectrum sharing system where multiple secondary users share the spectrum with the licensed primary users under an interference temperature constraint. We assume that DBF is applied at the secondary users. We first consider optimal beamforming and compare it with the user selection scheme in terms of the outage probability and bit-error rate performance. Since perfect feedback is difficult to obtain, we then investigate a limited feedback DBF scheme and develop an outage probability analysis for a random vector quantization (RVQ) design algorithm. Numerical results are provided to illustrate our mathematical formalism and verify our analysis.**

I. INTRODUCTION

Recently spectrum sharing systems have received considerable attention. Until now, spectrum sharing system has been widely studied [1]-[3]. In [1][2], the authors assume that the secondary transmitter needs to obtain both the channel state information (CSI) of the channel gains from the secondary transmitter to the primary and secondary receivers, respectively. Their results are based on the assumption that both CSI are perfect. However, in practice, it is difficult to obtain perfect CSI due to delayed feedback or channel estimation error and this results into a degradation of the system performance. In this paper, we will take the channel estimation error into consideration.

For a multiple secondary users network, we can apply a distributed beamforming (DBF) scheme to further improve the system performance. Beamforming with imperfect feedback is more practicable in realistic wireless applications due to the large amount of feedback overheads in the ideal case. Some work related to the limited feedback (LFB) was performed for the vector quantization LFB [4][5]. Recent work extends LFB to the cooperative diversity system. For instance, the authors in [6] investigated the DBF in wireless relay networks with random quantized feedback (RVQ). However, all the papers did not consider a cognitive radio (CR) environment.

Recently, applying beamforming in CR environments has received some interest [7]-[10]. However, to the best of the authors' knowledge, there has been no work on DBF with limited feedback in a spectrum sharing environment. In this paper, we consider this problem and assume that DBF is applied at the multiple secondary users. Analytical results are developed to characterize the performance of the above mentioned system with perfect or quantized feedback. The objective is two folds and the detailed contributions of this paper are summarized in what follows:

1) We consider a DBF network in a spectrum sharing environment with multiple single-antenna secondary users, one primary receiver, and one secondary receiver. We assume that perfect CSI of the links between the secondary transmitters and the secondary receiver is sent back to the secondary transmitters to form the optimal beamformer under peak received power constraint [1]. Our motivation is to analyze the diversity order and present a performance comparison with the user selection (US) scheme discussed in [2].

2) We consider a RVQ-LFB scheme where the beamforming vector is randomly selected from a given codebook through a finite-rate feedback channel [4]. Specifically, based on the method in [5], we derive the density function of the squared inner product between the quantized and the optimal beamforming vectors. With it, we analyze the approximate system performance.

II. SYSTEM MODEL

We consider a cognitive radio network where N secondary users share the same spectrum with a licensed primary user. Similar to the models in [1][2], we also adopt the interference temperature concept which only allows the secondary users whose interference power received at the primary receiver is less than a given threshold Q to utilize the spectrum. Let α_i and β_i denote the channel gains from the secondary transmitter to the primary receiver and secondary receiver, respectively. In [1], the authors have investigated the channel capacity of the secondary systems under an average or peak power constraint at the primary receiver. Here, we also adopt a peak received-power constraint $P_i|\alpha_i|^2 \leq Q$ where P_i is the transmitted power of the i-th secondary transmitter. This constraint implies that the secondary transmitter can obtain the CSI of α_i and β_i. In [1][2], the authors assume that the CSI of α_i is perfect. However, the assumption that the secondary transmitter has perfect CSI of α_i is unrealistic. Denoting the estimated channel gain as $\hat{\alpha}_i$, we model the channel as [11]

$$\alpha_i = \hat{\alpha}_i + e_i \tag{1}$$

where e_i is the error term with variance $1-\rho^2=\sigma_e^2$ and ρ denotes the correlation coefficient between the estimated and the exact channel gain. We assume $\mathcal{E}[|\alpha_i|^2]=1$, which implies

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that $\mathcal{E}[|\hat{\alpha}_i|^2]=\rho^2$ and e_i is independent of $\hat{\alpha}_i$. σ_e^2 reflects the accuracy of estimating CSI.

The entire transmission procedure needs two stages. Firstly, with the CSI of $\hat{\alpha}_i$, the secondary transmitter computes the transmit power and compares it with the interference temperature Q to satisfy the power constraint. If the secondary transmitter can obtain the CSI of β_i, the secondary transmitters then send the weighted signals to the secondary receiver, which built up a distributed beamforming CR network. Like [1], we employ the maximum instantaneous transmit power $Q/|\hat{\alpha}_i|^2$ of the i-th secondary transmitter allowed by the primary user as the transmit power. Therefore, the received signal at the secondary receiver can be modeled as

$$y = \sum_{i=1}^{N} \sqrt{\frac{Q}{|\hat{\alpha}_i|^2}} \beta_i w_i s + n, \tag{2}$$

where w_i is the complex weight number and s is the transmitted signal with unit energy. To keep the power constraint, we require that $\sum_{i=1}^{N} |w_i|^2 = 1$. In (2), n is the Gaussian noise with zero mean and variance 1. Let us define an equivalent channel vector $\mathbf{h} = [\beta_1/|\hat{\alpha}_1|, ..., \beta_N/|\hat{\alpha}_N|]$. The resulting SNR at the secondary receiver is given by

$$\gamma = Q \sum_{i=1}^{N} |\beta_i/\hat{\alpha}_i w_i|^2 = Q|\mathbf{h}\mathbf{w}|^2 = Q\mathbf{w}^\dagger \mathbf{h}\mathbf{h}^\dagger \mathbf{w}, \tag{3}$$

where $\mathbf{w}$ denotes the weight vector with a length of N.

If the channel estimation of β_i at the secondary receiver is error-free and the bandwidth of the feedback links is unlimited, the optimal weight vector is chosen as $\mathbf{w}^*=\mathbf{h}/\|\mathbf{h}\|_2$ in order to maximize the received SNR. Therefore, (3) becomes

$$\gamma = \sum_{i=1}^{N} Q|\beta_i|^2/|\hat{\alpha}_i|^2 = \sum_{i=1}^{N} \gamma_i, \tag{4}$$

where γ_i is the received SNR between the secondary receiver and the i-th secondary transmitter. Since both $|\hat{\alpha}_i|^2$, $|\beta_i|^2$ are independent exponentially distributed random variables, the probability density function (PDF) of γ_i is readily given by

$$f_{\gamma_i}(\gamma)=Q\rho^2/(Q+\rho^2\gamma)^2, \ \gamma \geq 0, \tag{5}$$

The resulting cumulative density function (CDF) is given by

$$F_{\gamma_i}(\gamma)=1-Q(Q+\rho^2\gamma)^{-1}, \ \gamma \geq 0. \tag{6}$$

III. PERFECT FEEDBACK BEAMFORMING VERSUS USER SELECTION

In [2], multiuser diversity gain was investigated in spectrum sharing systems. The used performance metric was the system capacity. We now focus on the outage probability and bit error rate (BER) performance. To get additional insight, we present the performance comparison between the user selection scheme developed in [2] and our proposed optimal DBF scheme.

A. Analysis of User Selection

For a fair comparison, we employ the maximum transmit power $Q/|\hat{\alpha}_i|^2$ for the user selection scheme. The secondary user with the maximum γ_i will be chosen to send signals to the secondary receiver. According to the order statistics, the PDF of $\max_{1\leq i\leq N} \gamma_i$ can be expressed as

$$f_{\gamma_{max}}(\gamma)=\frac{NQ\rho^{2N}}{(Q+\rho^2\gamma)^{N+1}}\gamma^{N-1}, \ \gamma \geq 0. \tag{7}$$

1)Outage Probability Analysis

Using (7), the system outage probability for a given threshold γ_{th} can then be computed as

$$P_{out}^S=\Pr(\gamma_{max}<\gamma_{th})=NQ\rho^{2N}\int_0^{\gamma_{th}} \frac{\gamma^{N-1}}{(Q+\rho^2\gamma)^{N+1}}d\gamma. \tag{8}$$

With the help of identity [12, eq.(3.194.1)] and using the fact ${}_2F_1(a,b;a;z) = (1-z)^{-b}$[13], we obtain

$$P_{out}^S = \left(\frac{\rho^2\gamma_{th}}{Q+\rho^2\gamma_{th}}\right)^N, \tag{9}$$

where ${}_2F_1(a,b;c;z)$ the Gaussian hypergeomtric function [12].

2) BER Analysis

From [14], we know the average BER performance can be computed according to the CDF $F_{\gamma_{max}}(\gamma)$ of γ_{max} if we introduce a standard normal distribution random variable V . Then, the average BER is given by [14]

$$P_b^S(E) = \mathcal{E}_V\left\{F_{\gamma_{max}}\left(\frac{V^2}{2}\right)\right\}. \tag{10}$$

Note that $F_{\gamma_{max}}(\gamma)$ can be directly obtained from (9) by replacing γ_{th} with γ. Hence, we can evaluate the BER performance as follows

$$\begin{aligned} P_b^S(E) &= \int_0^\infty \left(\frac{\rho^2v^2}{2Q+\rho^2v^2}\right)^N \frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv \qquad (11)\\ &\overset{(a)}{=} \frac{\rho^{2N}}{2\sqrt{2\pi}}\int_0^\infty y^{N-0.5}(2Q+\rho^2y)^{-N}e^{-\frac{y}{2}}dy \\ &\overset{(b)}{=} \frac{1}{2\sqrt{\pi}}\left(\frac{Q}{\rho^2}\right)^{1/2}\Gamma(N+0.5)\Psi\left(N+0.5,1.5,\frac{Q}{\rho^2}\right) \\ &\overset{(c)}{=} \frac{1}{2\sqrt{\pi}}\left(\frac{\rho^2}{Q}\right)^N\Gamma(N+0.5){}_2F_0\left(N+0.5,N;;-\frac{\rho^2}{Q}\right), \end{aligned}$$

where $\Psi(\cdot,\cdot,\cdot)$ is the Tricomi confluent hypergeometric function and defined in [12, eq.(9.211.4)] and ${}_pF_q(a_1,...,a_p;b_1,...,b_q;z)$ is the generalized hypergeometric function. In step (a), we use the variable transformation $y=v^2$ and the equality (b) follows from [12, eq.(3.383.5)]. The last identity (c) is used the fact ${}_2F_0(a,b;;-1/x) = x^a\Psi(a,a-b+1,x)$ [15].

To investigate the diversity order of the user selection scheme, we need to find an asymptotic expression. For high Q, we have $(2Q+\rho^2y)^{-N} \approx (2Q)^{-N}$. Then, the BER in (11) can be approximated by

$$P_b^S(E) \approx \frac{\rho^{2N}}{Q^N 2\sqrt{\pi}}\Gamma(N+0.5). \tag{12}$$

The above equation clearly indicates that the diversity order of a CR system with multiuser diversity is N.

B. Analysis of Optimal Beamforming

1)Outage Probability Analysis

Since γ is a sum of identically independent distributed (i.i.d) RVs, the exact outage probability analysis is complicated and the resulting expression may not give much insight. Thus, we rely on an asymptotic analysis. Specifically, like [6], we can use the inequality $\gamma \leq N\gamma_{max}$ to lower bound the outage probability. Therefore, the outage probability for the optimal beamforming scheme in a spectrum sharing communication system can be calculated as

$$P_{out}^{OBF} \approx \Pr[N\gamma_{max} < \gamma_{th}] = \frac{\rho^{2N}\gamma_{th}^N}{(\rho^2\gamma_{th} + NQ)^N}. \quad (13)$$

With (9) and (13), we can now consider the ratio of the outage probability for the user selection and optimal beamforming schemes leading to

$$\frac{P_{out}^S}{P_{out}^{OBF}} = \left(\frac{NQ + \rho^2\gamma_{th}}{Q + \rho^2\gamma_{th}}\right)^N \approx N^N, \ Q \to \infty. \quad (14)$$

We can observe that the optimal beamforming scheme leads to a higher system performance with the increase of the number of secondary users. However, as mentioned in [6], optimal beamforming is not practical in realistic wireless application due to its high amount of feedback. For this reason, we will consider the RVQ limited feedback scheme in section IV.

2) BER Analysis

Using the moment generating function (MGF)-based method [16] and after some mathematical manipulation, the BER of a spectral sharing system with perfect beamforming feedback can be shown to be given by

$$P_b^{OBF}(E) = \frac{1}{\pi}\int_0^{\pi/2}\left(1 - \Delta(\theta)e^{\Delta(\theta)}E_1\left(\Delta(\theta)\right)\right)^N d\theta, (15)$$

where $\Delta(\theta)$=$Qg/(\rho^2\sin^2\theta)$ and $E_1(x)$ is the exponential integral function and g=1 for binary phase-shift-keying (BPSK) and g=0.5 for binary frequency-shift-keying (BFSK).

Eq.(15) can not reveal much information about the diversity order. Also, we can not directly use it to make a comparison with (12). Thus, we again apply the bound $\gamma \leq N\gamma_{max}$ to compute the asymptotic BER. Using the same steps that lead to (12), it can be shown that the approximate BER of perfect beamforming feedback is given by

$$P_b^{OBF}(E) \approx \frac{\rho^{2N}}{N^N Q^N 2\sqrt{\pi}}\Gamma(N + 0.5). \quad (16)$$

From (16), we can see that the optimal DBF scheme also achieves a diversity order N. In addition, looking at (12) and (16) allow us to conclude that

$$P_b^S \approx N^N P_b^{OBF}, \ Q \to \infty. \quad (17)$$

In addition to the asymptotic formula (16), we can use the technique developed in [17] to derive another asymptotic BER expression for (15). From (5) and (6), we observe that $f_{\gamma_i}(0)$≠0 and $F_{\gamma_i}(0)$=0 which satisfies the condition required in [17]. Thus, according to [17], the asymptotic average BER expression is readily given by

$$P_b^{OBF}(E) \approx \frac{\prod_{n=1}^{N+1}(2n-1)}{(N+1)!2^{N+2}}(f_{\gamma_i}(0))^N = \frac{\prod_{n=1}^{N+1}(2n-1)\rho^{2N}}{(N+1)!2^{N+2}Q^N}. \quad (18)$$

Using the fact that $\Gamma(N+1.5)$=$\frac{\sqrt{\pi}}{2^{N+1}}\prod_{n=1}^{N+1}(2n-1)$ [16, Eq.(8.339.2)], we finally get

$$P_b^{OBF}(E) \approx \frac{\rho^{2N}}{(N+1)!Q^N 2\sqrt{\pi}}\Gamma(N + 1.5). \quad (19)$$

IV. ANALYSIS FOR THE RVQ LIMITED FEEDBACK

In the previous section, the analysis is based on the perfect feedback assumption where the bandwidth of the feedback link from the secondary receiver to the secondary transmitter is unlimited. In this section, we assume that only a limited number of bits is sent from the secondary receiver to the secondary transmitter to indicate which beamforming vector is chosen. According to the definition in [4][5], in a RVQ scheme, the beamforming vector which maximizes the received SNR is chosen form a randomly generated codebook $\mathcal{W} = \{\mathbf{w}_1, ..., \mathbf{w}_M\}$ known to both the transmitter and receiver ends. Therefore, the conditional SNR is given by

$$\gamma = \max_{w\in\mathcal{W}} Q|\mathbf{hw}|^2. \quad (20)$$

Notice that only the index number is conveyed to the secondary transmitter through an error-free and no-delay feedback link.

Like [4][5], the SNR in (20) can be rewritten as

$$\gamma = QXU, \quad (21)$$

where $X = \max_{w\in\mathcal{W}} |\mathbf{hw}|^2/\|\mathbf{h}\|_2^2$, $U = \|\mathbf{h}\|_2^2$. Under the i.i.d assumption, X is independent of U.

Lemma 1: If $\mathbf{h} = \sum_{i=1}^N \beta_i/|\hat{\alpha}_i|$ where $\beta_i \sim \mathcal{CN}(0,1)$ and $\hat{\alpha}_i \sim \mathcal{CN}(0,\rho^2)$, then the CDF of X in a spectral sharing system with RVQ limited feedback can be bounded as

$$\left[1 - \frac{1}{N}{}_2F_1\left(N-1, 1; N+1; \frac{1-2x}{1-x}\right)\right]^M < F_X(x) \quad (22)$$
$$< \left[1 - \frac{1}{N}{}_2F_1\left(N-1, 1; N+1; \frac{1-Nx}{1-x}\right)\right]^M, \ x \in (0,1)$$

Proof: See Appendix A.

Based on (22), we can evaluate the outage probability as shown in what follows. From (21), the system outage probability can be expressed as

$$P_{out}^{Li} = \Pr(QXU \leq \gamma_{th}) = \int_0^1 F_U\left(\frac{\gamma_{th}}{Qx}\right)f_X(x)dx. \quad (23)$$

where the superscript *Li* denotes the limited feedback beamforming, $F_U(u)$ is the CDF of U, and $f_X(x)$ is the PDF of X. As stated before, the exact analysis for the statistics of U is difficult. Hence, similar to the analysis for the CDF of X, we can obtain upper and lower bounds as

$$[1 - N(N + \rho^2 u)^{-1}]^N < F_U(u) < [1 - (1 + \rho^2 u)^{-1}]^N. \quad (24)$$

Using integration by parts, (23) can be rewritten as

$$P_{out}^{Li} = F_U\left(\frac{\gamma_{th}}{Q}\right) + \frac{\gamma_{th}}{Q}\int_0^1 \frac{1}{x^2} F_X(x) f_U\left(\frac{\gamma_{th}}{Qx}\right) dx, \quad (25)$$

where $f_U(u)$ is the PDF of U. Note that the first term represents the optimal feedback case and the second term denotes the performance loss due to the quantization of the beamforming vector.

By substituting the density functions of U and X into (25), the approximate outage probability is given by

$$\begin{aligned} &\frac{\Omega^N}{(\Omega+N)^N} + N^2\Omega^N \\ &\times \int_0^1 \frac{\left[1 - \frac{1}{N}{}_2F_1\left(N-1,1;N+1;\frac{1-2x}{1-x}\right)\right]^M}{(\Omega+Nx)^{(N+1)}} dx < P_{out}^{Li} \\ &< \frac{\Omega^N}{(\Omega+1)^N} + N\Omega^N \\ &\times \int_0^1 \frac{\left[1 - \frac{1}{N}{}_2F_1\left(N-1,1;N+1;\frac{1-Nx}{1-x}\right)\right]^M}{(\Omega+x)^{(N+1)}} dx, \quad (26) \end{aligned}$$

where the constant $\Omega = \rho^2\gamma_{th}/Q$. Unfortunately, no closed-form is available for the integrals in (26) and numerical integration is therefore needed.

V. NUMERICAL RESULTS AND CONCLUSIONS

In this section, we present numerical results to illustrate our analysis. Unless otherwise specified, we assume that $\rho^2 = 1$.

The comparison for the outage probability between the user selection and our proposed DBF schemes are shown in Fig.1. The SNR threshold γ_{th} is set to 5 dB and N is set to 3. The analytical results are plotted by using (9) and (13). Since (13) is a lower bound on the exact outage probability, we also present the exact results obtained by simulations. As expected, the proposed DBF scheme has better performance than the user selection scheme. We also observe that the simulation results match their corresponding analytical results perfectly.

Fig.2 shows the BER results for the optimal DBF and user selection scheme when $N = 3$. For the user selection case, the analytical result (11) is in agreement with the simulation. Since the analysis (16) is a lower bound and obtained in term of $N\gamma_{max}$, the simulation curve according to $N\gamma_{max}$ is also plotted. Compared with their corresponding simulation results, it is clearly observed that the approximate results (12),(16) and (19) are tight at high SNR. We can also see that formula (19) is a good approximation for the exact BER performance of the OBF scheme. Furthermore, we can observe that the proposed scheme can achieve better performance.

Fig.3 shows the outage probability comparison between the lower and upper bounds (26) and the exact simulation results at a SNR threshold of 5 dB when $N = 3$ and $M = 4, 8$. The result for the perfect feedback beamforming is also plotted to show the quantization loss. The M vectors are randomly generated and all are uniformly distributed on the complex unit-norm sphere. We observed that these bounds can reflect the scaling law of the system outage probability. In Fig.4, we plot the outage probability curves for different M. As expected, the system performance due to quantization is decreased. However, with the increase of the number of random vectors in the codebook, the quantized feedback scheme approaches the perfect feedback case.

ACKNOWLEDGEMENT

This publication was made possible by NPRP grant #08-152-2-043 and NPRP grant #09-341-2-128 from the Qatar National Research Fund (a member of Qatar Foundation).

APPENDIX A
PROOF OF LEMMA 1

By using the approach developed in [5], we can decompose the equivalent channel $\mathbf{h}$ into two parts in terms of $\mathbf{w}$ and its corresponding orthogonal complements $\mathbf{w}^\perp$. Using the result in [5], we have

$$Z = \frac{|\mathbf{hw}|^2}{\|\mathbf{h}\|_2^2} = \frac{|h_a|^2}{|h_a|^2 + \|\mathbf{h_b}\|_2^2}, \quad (27)$$

where h_a is a scalar and $\mathbf{h_b}$ is a vector with $N-1$ elements. Unlike in [5], here $|h_a|^2$ and $Y = \|\mathbf{h_b}\|_2^2$ are not simple Gamma distributed RVs. From (5), the PDF of $V = |h_a|^2$ can be easily obtained. To obtain the CDF of Z, we have first to find the PDF of Y. In the following, we use the bounds $(Y_L = \max_{1\le i\le N-1}|\beta_i|^2/|\hat{\alpha}_i|^2) < Y < (Y_U = (N-1)\max_{1\le i\le N-1}|\beta_i|^2/|\hat{\alpha}_i|^2)$ to analyze the CDF of Z.

Thus, from (7), we can obtain the PDFs of Y_L and Y_U as

$$f_{Y_L}(y) = (N-1)\rho^{2(N-1)}y^{N-2}(1+\rho^2 y)^{-N}. \quad (28)$$

$$f_{Y_U}(y) = (N-1)^2\rho^{2(N-1)}y^{N-2}(N-1+\rho^2 y)^{-N}. \quad (29)$$

From (27), the CDF of Z is given by

$$F_Z(z) = \int_0^\infty F_V\left(\frac{zy}{1-z}\right) f_Y(y)dy. \quad (30)$$

Substituting (28) and (29) into (30) and after some mathematic manipulation, the CDF of Z can be bounded as

$$\begin{aligned} &1 - \frac{1}{N}{}_2F_1\left(N-1,1;N+1;\frac{1-2z}{1-z}\right) < F_Z(z) \\ &< 1 - \frac{1}{N}{}_2F_1\left(N-1,1;N+1;\frac{1-Nz}{1-z}\right). \quad (31) \end{aligned}$$

To get (31), the integral table [12, Eq.(3.197.1)] and the equality [12, Eq.(9.131.1)] were used. Based on ${}_2F_1(a,b;c;x) \to 0$ when $x \to \infty$ and [12, Eq.(9.122.1)], we can verify that $0 \le F_Z(z) \le 1$ for $z \in (0,1)$. Then, using the order statistics, we finish the proof for Lemma 1.

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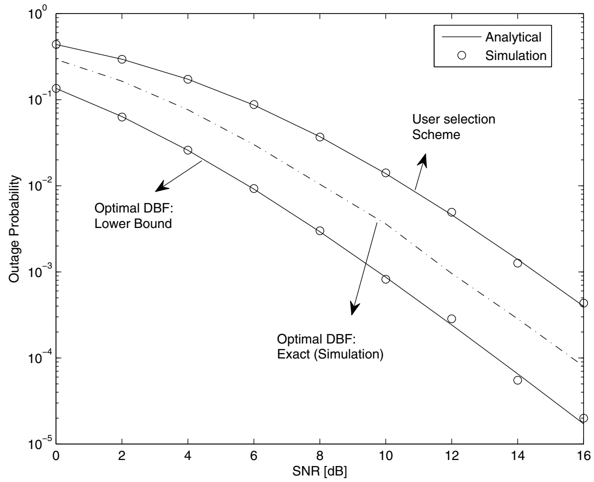

Fig. 1. Outage probability comparison between perfect-feedback DBF and user selection.

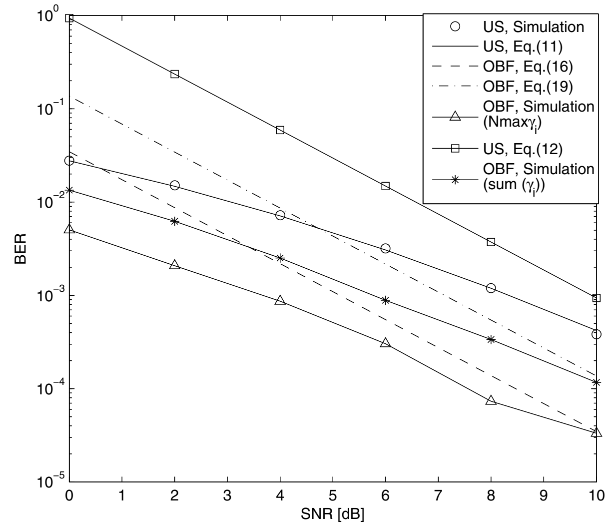

Fig. 2. BER Comparison between perfect-feedback DBF and user selection.

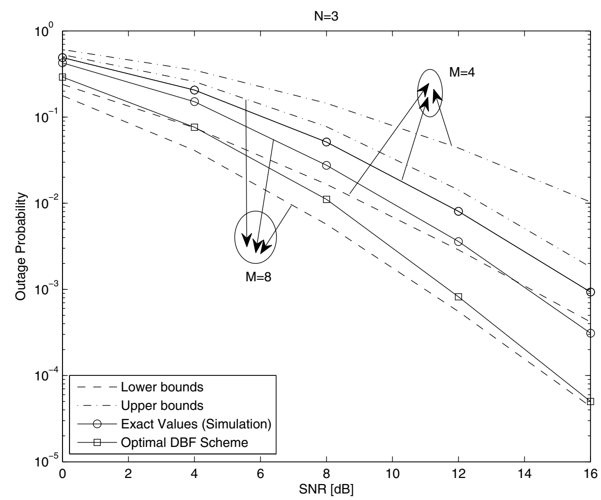

Fig. 3. Outage probability comparison between the perfect-feedback and limited feedback beamforming schemes.

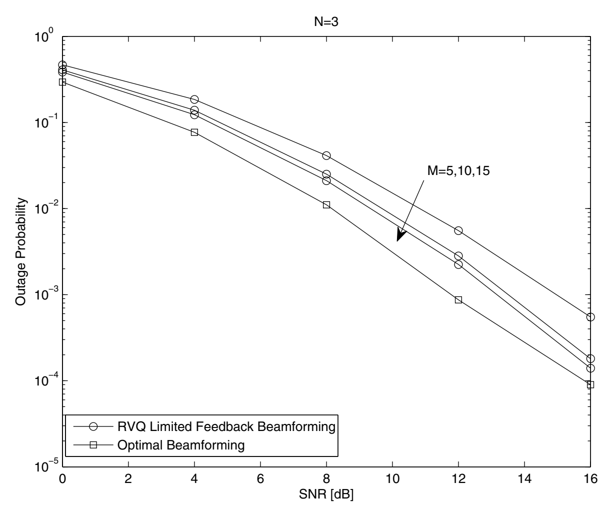

Fig. 4. Outage probability comparison for different value of M.

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