Proportionally Fair User Scheduling for Multiuser MIMO Systems with Unequal Average SNR Users

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Abstract—**Multiple-input multiple-output (MIMO) wireless communications systems have attracted a lot of attentions, because of their potential for high gain in channel capacity. Recently, attention has shifted to multiuser MIMO (MU-MIMO) systems. In the schemes for downlink MU-MIMO system, dirty paper coding (DPC) and block diagonalization (BD) algorithms can provide higher sum rate capacity of MU-MIMO systems; however, exhaustive user scheduling for them may cause prohibitive computation complexity. The greedy scheduling algorithms were proposed for DPC and for BD; and proportionally fair (PF) scheduling for MU-MIMO was also discussed in the previous works. However, the discussions in where mainly based on the system model with equal average signal to noise power ratio (SNR) users. In this paper, we focus on the user scheduling for the downlink of MU-MIMO systems with DPC transmission scheme, where each user is assumed to have only one receive antenna, which will simplify the analysis and is expected to give more insight of PF scheduling. We also assume each individual user has different average transmit SNR in the system. We modify the conventional user scheduling algorithm for DPC to modified Gram-Schmidt orthogonalization (M-GSO) based algorithm and extend it into PF scheduling. We also propose a modified PF metric corresponding to the unequal SNR users system model, that increases the sum rate capacity of MU-MIMO system.**

I. Introduction

Multiple-input multiple-output (MIMO) wireless communications systems have attracted a lot of attentions, because of their potential for high gain in channel capacity. Recently, attention has shifted to multiuser MIMO (MU-MIMO) systems, which serve several users simultaneously in frequency and time domain, that can exploit the benefit of space division multiple access (SDMA) [1]–[4]. However, the number of simultaneous users is limited by the environments as well as the number of antennas equipped in the base station (BS). When the number of active users in the BS is larger than the supportable number of simultaneous users, it is necessary to schedule (or select) a set of users from all the active users to maintain the performance of the MU-MIMO systems.

In the schemes for downlink MU-MIMO, dirty paper coding (DPC) [3] and block diagonalization (BD) [4] algorithms can provide higher sum rate capacity of MU-MIMO systems; however, exhaustive user scheduling for DPC or BD MIMO systems, that maximizes the capacity of the systems, may cause prohibitive computation complexity. The suboptimal user scheduling algorithms (or called greedy scheduling, that the BS selects the users one by one for complexity reducing) were proposed for DPC in [5] and for BD in [6], [7]; in where, [7] also discussed proportionally fair (PF) user scheduling for BD MIMO systems.

Scheduling algorithms based on throughput maximization may result in some users being denied service due to bad channel conditions. PF scheduling provides fairness among users, and still maintains the multiuser diversity gain [8]; and PF scheduling for MU-MIMO was also discussed in [9]. However, beside [7], the discussions in the previous works are based on the system model with equal average signal to noise power ratio (SNR) users. Even [7], the discussion and analysis are also mainly based on equal SNR users system that is not practical.

In this paper, we focus on the user scheduling for the downlink of MU-MIMO systems with DPC transmission scheme, where each user is assumed to have only one receive antenna, which will simplify the analysis and is expected to give more insight of PF scheduling. We also assume each individual user has different average transmit SNR in the system. Gram-Schmidt orthogonalization (GSO) based simplified user scheduling was proposed in [5]; We modify the user scheduling algorithm to modified GSO (M-GSO) based algorithm and extend it into PF scheduling with the user scheduling metric similar to that of [9]; and we also propose a modified PF metric corresponding to the unequal SNR users system model, that increases the sum rate capacity of system.

A. Notation

In this paper the following notations are used. Uppercase boldface type is used for matrices and lowercase boldface is used for vectors. All scalar quantities are not in boldface. The $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the conjugate transpose of a matrix, respectively. The $E[\cdot]$ denotes the expectation or average operator. Finally, the $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a scalar and the norm of a vector, respectively.

II. System Model

We consider a downlink of an MU-MIMO system with N_{t} transmit antennas at the BS and N_{r} single receive antenna users, i.e., the total number of receive antennas is N_{r}. The channel matrix $\mathbf{H}$ becomes an $N_{\mathrm{r}} \times N_{\mathrm{t}}$ matrix, where we assume the entries of $\mathbf{H}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Firstly, we consider the case $N_{\mathrm{r}} \leq N_{\mathrm{t}}$, so it is not necessary to consider user selection.

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Because we employ DPC as the MU-MIMO scheme, we define the QR decomposition (QRD) of the channel matrix as $\mathbf{H} = \mathbf{L}\mathbf{Z}^H$, where $\mathbf{Z}^H$ is an orthonormal set of row vectors for the row basis of $\mathbf{H}$ and $\mathbf{L}$ is an $N_\mathrm{r} \times N_\mathrm{r}$ lower triangular matrix. The BS precodes the signal vector with the precoding matrix is $\mathbf{Z}$ and transmits the precoded signal to the users, then the received signal vector at the users' side becomes

$$\mathbf{r} = \mathbf{HZs} + \mathbf{n} = \mathbf{Ls} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is a vector in $\mathbb{C}^{N_\mathrm{r}}$ and denotes the zero mean additive Gaussian noises.

The average total transmit power is $P_\mathrm{t} = E(\mathbf{s}^H\mathbf{Z}^H\mathbf{Z}\mathbf{s}) = E(\mathbf{s}^H\mathbf{s})$, for $\mathbf{Z}$ is orthonormal. With Tomlinson-Harashima precoding (THP) [10], the BS can pre-cancel the interferences between the data streams before transmission without increasing the average total transmit power, such that the individual receive antenna can receive their own data stream just like there are no interferences. The received signal for the ith receive antenna becomes

$$r_i = l_{i,i}s_i + n_i, \tag{2}$$

where $l_{i,i}$ is the ith diagonal entry of $\mathbf{L}$; and n_i is the ith entry of $\mathbf{n}$ with variance is σ_i^2. In this paper, we define the average transmit SNR of the ith receive antenna as $\rho_i = P_\mathrm{t}/\sigma_i^2$; and because we assume the users have different SNR levels, ρ_i can have different value for different user i, which is including the effect of channel pathloss.

The sum rate capacity of the DPC system then becomes

$$C_\mathrm{DPC} = \sum_{i=1}^{N_\mathrm{r}} \log_2\left(1 + \eta_i\rho_i|l_{i,i}|^2\right), \tag{3}$$

where η_i is the ratio of the average power allocated to the ith data stream for the ith receive antenna to the average total transmit power P_t, and $\sum_{i=1}^{N_\mathrm{r}} \eta_i = 1$ (i.e., the ith diagonal entry of $E(\mathbf{s}\mathbf{s}^H)$ is $\eta_i P_\mathrm{t}$).

When equal power (EP) allocation is employed, the power ratio becomes $\eta_i = 1/N_\mathrm{r}$ for the ith data stream, $\forall i$; and when water filling (WF) allocation is employed, the optimal power ratios can be obtained over the values of $\rho_i|l_{i,i}|^2$, $\forall i$.

When $N_\mathrm{r} > N_\mathrm{t}$, it is necessary to select a subset of users from the N_r users, such that the number of the selected users $n \leq N_\mathrm{t}$, to guarantee no interference transmission. The optimal user scheduling strategy is to select the subset of users that maximizes the sum rate capacity of the MU-MIMO channel at each time slot.

However, it is clear that an exhaustive search over the N_r users is computationally prohibitive if $N_\mathrm{r} \gg n$; so suboptimal user selection algorithm was proposed to reduce the complexity of finding the user set.

III. Simplified User Scheduling for DPC

The QRD of channel matrix can be calculated with GSO for reducing calculation complexity. However, it is reported that the classical GSO is more numerically unstable [11]. We modify the GSO based user scheduling algorithm proposed in [5] as modified GSO (M-GSO) based algorithm, such that the resultant orthonormal set can be used in the precoding process. We summarize the M-GSO based user (antenna) selection algorithm as bellow, where the input is the channel matrix $\mathbf{H}$ and the average transmit SNRs of the users. The ith *row vector* of $\mathbf{H}$ is denoted as $\mathbf{h}_i$ that is the channel vector of the ith user.

Algorithm I: M-GSO based User Scheduling.

1) Initially, let $\mathcal{U} = \{1,\ 2,\ ...,\ N_\mathrm{r}\}$ and $\mathcal{U}_\mathrm{s} = \{\phi\}$.
   a. Calculate the norm of the channel vectors: $\nu_k = \|\mathbf{h}_k\|^2$, $\forall k \in \mathcal{U}$.
   b. Select the first user u_1, such that $u_1 = \arg\max_{k\in\mathcal{U}} \rho_k\nu_k$.
   c. Set $g_1 = \sqrt{\nu_{u_1}}$, $\dot{\mathbf{z}}_1 = \mathbf{h}_{u_1}^H/g_1$ and $\dot{\mathbf{h}}_k^{(1)} = \mathbf{h}_k$ ($\forall k \in \mathcal{U}$).
   d. Let $\mathcal{U} = \mathcal{U} \setminus \{u_1\}$, $\mathcal{U}_\mathrm{s} = \mathcal{U}_\mathrm{s} \cup \{u_1\}$, $n = 1$ and $C_\mathrm{temp} = \log_2(1 + \rho_{u_1}\nu_{u_1})$.
2) For $i = 2$ to N_t
   a. Calculate $\dot{\mathbf{h}}_k^{(i)} = \dot{\mathbf{h}}_k^{(i-1)} - (\dot{\mathbf{h}}_k^{(i-1)}\mathbf{z}_{i-1})\mathbf{z}_{i-1}^H$, $\forall k \in \mathcal{U}$; and the norm of $\dot{\mathbf{h}}_k^{(i)}$: $\nu_k = \|\dot{\mathbf{h}}_k^{(i)}\|^2$, $\forall k \in \mathcal{U}$.
   b. Select the ith user u_i, such that $u_i = \arg\max_{k\in\mathcal{U}} \rho_k\nu_k$.
   c. Set $g_i = \sqrt{\nu_{u_i}}$, $\dot{\mathbf{z}}_i = \dot{\mathbf{h}}_{u_i}^{(i)H}/g_i$ and calculate $C_\mathrm{temp}^{(i)} = \sum_{j=1}^{i} \log_2(1 + \rho_{u_j}\nu_{u_j}/i))$.
   d. If $C_\mathrm{temp}^{(i)} < C_\mathrm{temp}$, terminate the algorithm and the selected user set is $\mathcal{U}_\mathrm{s}$.
   e. Let $\mathcal{U} = \mathcal{U} \setminus \{u_i\}$, $\mathcal{U}_\mathrm{s} = \mathcal{U}_\mathrm{s} \cup \{u_i\}$, $n = i$ and $C_\mathrm{temp} = C_\mathrm{temp}^{(i)}$.
3) Endfor: Return the row basis $\dot{\mathbf{Z}}^H = [\dot{\mathbf{z}}_1\ \dot{\mathbf{z}}_2\ ...\ \dot{\mathbf{z}}_n]^H$ of the channel matrix of the selected users, that is $\dot{\mathbf{H}}(\mathcal{U}_\mathrm{s}) = [\mathbf{h}_{u_1}^T\ \mathbf{h}_{u_2}^T\ ...\ \mathbf{h}_{u_n}^T]^T$. The channel gain of the ith subchannel becomes ν_i for the channel matrix $\dot{\mathbf{H}}(\mathcal{U}_\mathrm{s})$.

In the algorithm above, $\mathcal{U}$ and $\mathcal{U}_\mathrm{s}$ are the sets of the unselected and selected users, respectively. The QRD of $\dot{\mathbf{H}}(\mathcal{U}_\mathrm{s})$ is defined as $\dot{\mathbf{H}}(\mathcal{U}_\mathrm{s}) = \dot{\mathbf{L}}\dot{\mathbf{Z}}^H$ and $\dot{\mathbf{L}}$ is also a lower triangular matrix with the ith diagonal entry is $g_i = \sqrt{\nu_{u_i}}$, such that the channel gain of the subchannel for the ith selected user, u_i, is ν_{u_i}.

The algorithm selects the users one by one, that reduces the complexity of scheduling. In each step (or loop), the BS calculates the values of norm of the unselected users' channel vectors or components of channels lying on the null space of the already selected users' channel. The BS also calculates the temporary sum rate capacity of each step.

In step 2. d, the BS compares the temporary capacity with that of the previous step (or loop); and if the capacity of the current step is smaller than that of the previous one, the BS considers the capacity will no more increase with the newly selected users and terminates the loop, then the number of the selected user is $n < N_\mathrm{t}$.

With Algorithm I, the sum rate capacity of the channel matrix of the selected users becomes

$$C_\mathrm{DPC}\big(\dot{\mathbf{H}}(\mathcal{U}_\mathrm{s})\big) = \sum_{i=1}^{n} \log_2(1 + \eta_i\rho_{u_i}\nu_{u_i}), \tag{4}$$

where the power allocation can be EP (i.e., $\eta_i = 1/n$) or optimized with WF over the SNR gains $\rho_{u_i}\nu_{u_i}$ ($i = 1, \ldots, n$).

The supportable rate of the subchannel for the ith selected user is $\log_2(1 + \eta_i\rho_{u_i}\nu_{u_i})$, that depends on the selected user set including the order of selection of the users. However, for simplifying the analysis latter, we define the supportable rate for user k as below, when $k \in \mathcal{U}_\mathrm{s}$ and user k is the ith selected user, u_i, at time slot t.

$$R_k(t) = R_k(\mathcal{U}_\mathrm{s}) = \log_2\left(1 + \eta_i\rho_{u_i}\nu_{u_i}\right), \quad (5)$$

A. Proportionally Fairness (PF)

PF scheduling provides fairness among users, and still maintains the multiuser diversity gain. We modify the steps 1.b and 2.b in Algorithm I as the metric below for PF scheduling the ith user (In the case of step 1.b, $i = 1$; and power allocation is ignored for simplifying the user scheduling).

$$u_i = \arg\max_{k\in\mathcal{U}} \mu_k(t)\log_2(1 + \rho_k\nu_k), \quad (6)$$

where $\mu_k(t) = 1/\bar{R}_k(t)$ is the priority weight of the kth user at time slot t. $\bar{R}_k(t)$ is the average supportable rate of user k up to time slot $(t-1)$, which is updated as below.

$$\bar{R}_k(t+1) = \begin{cases} \delta\bar{R}_k(t) + (1-\delta)R_k(t), & k \in \mathcal{U}_\mathrm{s} \\ \delta\bar{R}_k(t), & k \notin \mathcal{U}_\mathrm{s} \end{cases}, \quad (7)$$

where $\delta = 1 - (1/T_\mathrm{c})$ is the forgetting factor, that means the algorithm keeps tracking the average throughput of each user in a past window of length T_c.

B. Modified PF Metric (MPF)

When the average of transmit SNRs of the users are unequal, the average supportable rate of each user increases proportionally with the SNR [dB] of the user in the system with PF user selection (The same results are also shown in Fig. 4 in the next section); thus the user selection metric in (6) can be approximated as below.

$$\frac{\log_2(1 + \rho_k\nu_k)}{\bar{R}_k(t)} \approx \frac{\log_2(1 + \rho_k\nu_k)}{\alpha + \beta\left[10\log_{10}(\rho_k)\right]}, \quad (8)$$

where we use the ergodic average of $\bar{R}_k(t)$ instead of the instantaneous value of it; and α and β are the coefficients of the linear function of SNR [dB].

However, α and β change with the number of users, the distribution of the SNRs and users, as well as the user scheduling algorithm. It is difficult to theoretically analyse (8), so we show an example in Fig. 1, where ν_k is 1, 2 or 4, $\alpha = 0.2$ and $\beta = 0.04$. We can see that the value of (8) is not only dependent on the value of ν_k, but also highly dependent on the SNRs of users.

In this paper we propose the user scheduling metric as below instead of (6) in step 2.b in Algorithm I.

$$u_i = \arg\max_{k\in\mathcal{U}} \left[a\log_2(1 + \nu_k) + \mu_k(t)\log_2(1 + \rho_k\nu_k)\right], \quad (9)$$

where we add the part of $\log_2(1 + \nu_k)$ that weighted with the parameter a, and the added parameter increases the effect of

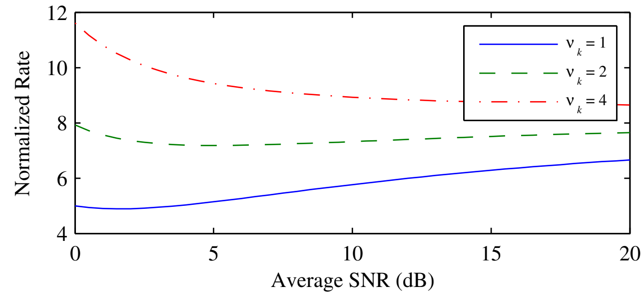

Fig. 1. Approximate normalized sum rate capacity vs. average SNR of users.

TABLE I
SIMULATION PARAMETERS

Parameter	Value or Setting
No. of Tx antennas, N_t	8
No. of users, N_r	10, 15, 20, 30, 40, 60, 80
Channel model	Flat i.i.d Rayleigh Channel
Maximum Doppler frequency, f_d	67 Hz
Frame period, T_f	0.5 ms
Width of sliding window, T_c	100 time slots
SNR range of the users	From 0 dB to 20 dB

ν_k to the user selection. ν_k can be considered as an indicator of *how the channel of the current selecting user matches with those of the already selected users.*

The design of a will become a tradeoff problem between the sum rate capacity and the variance of latency time of each user. We will discuss this problem with the simulation results in the next section. In addition, (9) induces a problem that the lower SNR user becomes more easily to be selected in the earlier step, which reduces the total number of selected users finally, that is also because of step 2.d in Algorithm I. The problem causes some inefficiency of the MU-MIMO systems, so we modify step 2.d in Algorithm I as below that keep the number of the finally selected users more than $N_\mathrm{t} - 1$.

d. When $i = N_\mathrm{t}$ and if $C_\mathrm{temp}^{(i)} < C_\mathrm{temp}$, terminate the algorithm and the selected user set is $\mathcal{U}_\mathrm{s}$.

In the statement above, the comparison of the temporary sum rate capacities is executed only when the number of selected users reaches $N_\mathrm{t} - 1$.

IV. SIMULATION RESULTS

In this section we show the simulation results with the system model introduced in Section II and the system parameters are shown in Table I, where the elements in the channel matrix are i.i.d. to each other and each channel element suffers block Rayleigh fading with $T_\mathrm{f} = 0.5$ ms and $f_\mathrm{d} = 67$ Hz. The SNR of the users is ranged from 0 dB to 20 dB, that linearly increase with the indices of user (i.e., the SNR of the first user and the last user are 0 dB and 20 dB, respectively; and the intervals [dB] of SNRs of the neighbour index users are the same).

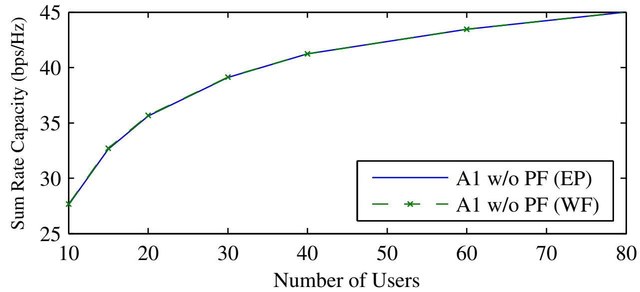

Fig. 2. Average sum rate capacities of systems without PF.

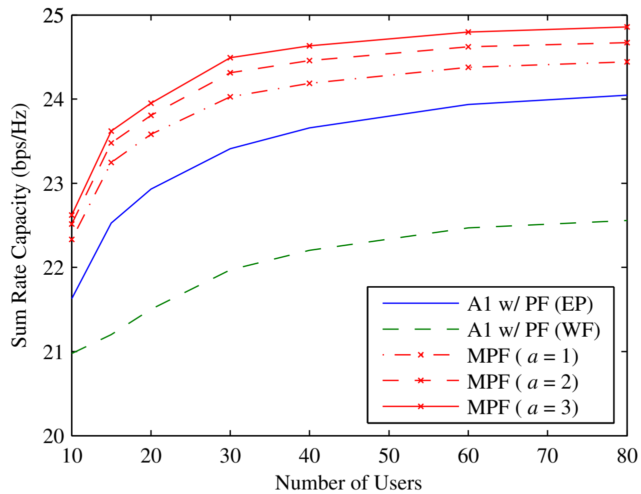

Fig. 3. Average sum rate capacities of systems with PF.

In the figures, "A1 w/o PF" and "A1 w/ PF" denote the results of Algorithm I without PF and with PF, respectively; "EP" or "WF" denotes the results of algorithm with the power allocations are EP or WF; and "MPF" denotes the results of the proposed modified PF user metric and the power allocations of MPF results are all EP (the reason will be shown later). In Figs. 2 and 3, we can see the results of algorithms without PF are much larger than that of algorithms with PF or MPF metric; however, the fairness between the users is poor (shown in Fig. 4).

In Fig. 2, the results of "A1 w/o PF (EP)" and "A1 w/o PF (WF)" are similar to each other. However, in Fig. 3, the result of "A1 w/ PF (EP)" is much larger than that of "A1 w/ PF (WF)", because of that WF cuts the power for the selected lower SNR users, that wastes the chance of increasing the data rate for the lower SNR users and increases the selection rate of the lower SNR users (shown in Fig. 6). As the results, the proposed algorithm only employs EP.

Comparing the results of "MPF" and that of "A1 w/ PF (EP)" in Fig. 3, we can see the results of the proposed metric are larger than that of "A1 w/ PF (EP)"; and the result of the proposed metric increases with the value of a; however, that also increase the variance (deviation) of latency time (shown in Fig. 7).

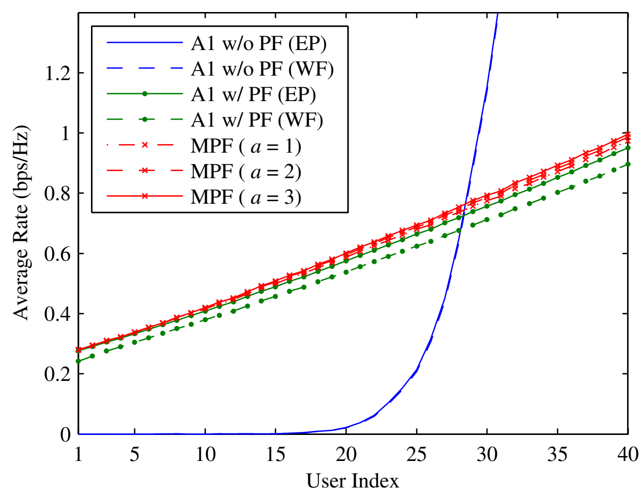

Fig. 4. Average rate of individual user.

In Fig. 4, we show the average rate of individual user when the number of users is 40. In the results of "A1 w/o PF", the average rates of higher SNR user are much larger than that of lower SNR users; the average rates of the users with indices smaller than 20 are 0 or near 0; that is quite unfair between the users. In the results of algorithms with PF or MPF, the average rates are increase almost linearly with the index of user (i.e., the SNR [dB] of user). The result of "A1 w/ PF (EP)" is larger than that of "A1 w/ PF (WF)"; and the results of "MPF" are somewhat larger than that of "A1 w/ PF (EP)", especially when index of user is large.

In Fig. 5, we show the average number of selected users results of the algorithms with different total number of users. The results increase and close to $N_{\mathrm{t}} = 8$ when the total number of users increases. With MPF, when the parameter a increases, the average number of selected user decreases. With the modified step 2.d, the results of "MPF" are close to that of "A1 w/ PF (EP)", that keeps the benefit of SDMA.

In Figs. 6 and 7, we show the results of the average and deviation of the latency time, respectively. The latency time is defined as the time between the current selection and the next selection of each user, and the results are for the system with different algorithms and with total number of users that is 40.

In Fig. 6, we can see the results of average latency of "A1 w/o PF" for higher SNR users is much smaller than that of lower SNR users; that implies the algorithm selects the higher SNR users frequently. The results of "MPF" and "A1 w/ PF (EP)" increase almost linearly with the user index, while the results of "A1 w/ PF (WF)" for lower SNR users is smaller than that of other metrics, that implies it selects the lower SNR users more frequently than the other metrics.

In Fig. 7, we can see the deviation of the latency time results of "A1 w/o PF" for higher SNR users is much smaller than that of lower SNR users; that also implies the algorithm selects the higher SNR users frequently. We also can see the results of "MPF" increase when a increase, that may effect the quality

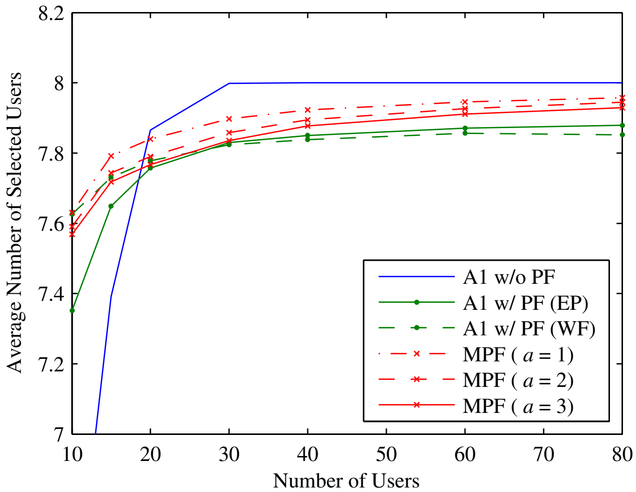

Fig. 5. Average number of selected users.

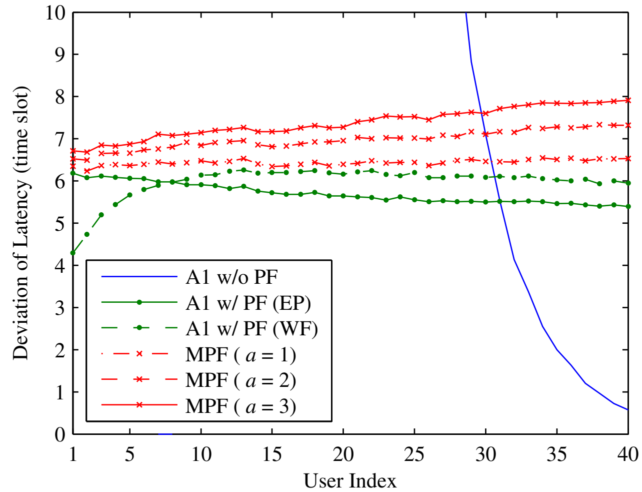

Fig. 7. Deviation of latency time of individual user.

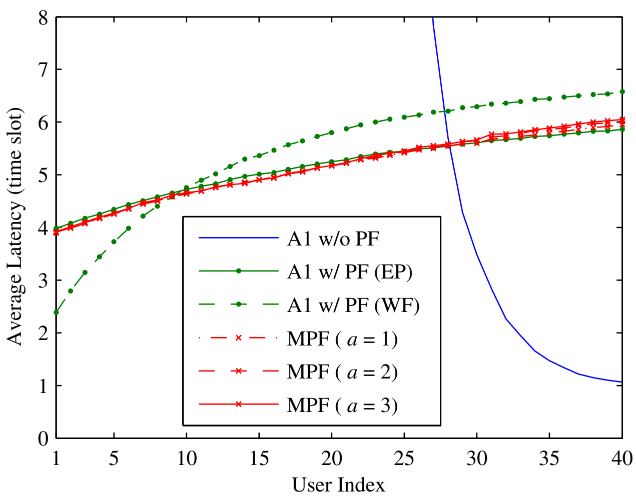

Fig. 6. Average latency time of individual user.

of service (QoS) of the channel for each user. The design of the parameter a may become: choosing a reasonable value of a, that provides an acceptable deviation of the latency time.

V. Conclusion

In this paper, we focused on the user scheduling for the downlink of MU-MIMO systems with DPC transmission scheme, where each user was assumed to have only one receive antenna. We also assumed the individual user has different average transmit SNR in the system. We modified the user scheduling algorithm proposed in [5] to M-GSO based algorithm and extended it into PF scheduling with the user scheduling metric similar to that of [9]; and we also proposed a MPF metric corresponding to the unequal SNR users system model, that increased the sum rate capacity of system.

In simulation results, power allocation with EP was better than that with WF when system had unequal SNR users; sum rate capacity of system increased with the parameter a in MPF metric, that also increased the deviation of latency time of individual user. The design of a becomes a tradeoff problem and still needs more analysis. In the future works, we expect to further optimize or suboptimize the power allocation and the PF metric design for MU-MIMO systems.

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