Antenna Placement Designs for Distributed Antenna Systems with Multiple-Antenna Ports

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***Abstract*—In this paper, we optimize antenna locations for a distributed antenna system (DAS) with distributed antenna (DA) ports equipped with multiple antennas under per-DA port power constraint. Maximum ratio transmission and scaled zero-forcing beamforming are employed for single-user and multi-user DAS, respectively. Instead of maximizing the cell average ergodic sum rate, we focus on a lower bound of the expected signal-to-noise ratio (SNR) for the single-cell scenario and the expected signal-to-leakage ratio (SLR) for the two-cell scenario to determine antenna locations. For the single-cell case, optimization of the SNR criterion generates a closed form solution in comparison to conventional iterative algorithms. Also, a gradient ascent algorithm is proposed to solve the SLR criterion for the two-cell scenario. Simulation results show that DAS with antenna locations determined from the proposed algorithms achieves capacity gains over traditional centralized antenna systems.**

I. Introduction

Over the past decade, distributed antenna system (DAS) has drawn attention as a new structure for wireless communication due to its advantages over conventional centralized antenna systems (CAS). With distributed antenna (DA) ports separated geographically within a cell, DAS provides an increase in system capacity and coverage and a reduction in the access distance, which lead to decreased transmit power and co-channel interference [1] and [2]. Capacity analysis of DAS in a composite fading was derived in [3] and authors in [4] proposed transmission schemes based on sum rate analysis.

Recently, several papers proposed algorithms for determining the location of DA ports which are equipped with a single antenna. In [5] and [6], the squared distance criterion was presented to design the antenna location which maximizes a lower bound of the cell average ergodic capacity for the single-cell DAS. In [7], sub-optimal DA port locations which maximize lower bounds of the expected signal-to-noise ratio (SNR) and the expected signal-to-leakage ratio (SLR) were suggested for single-cell and two-cell DAS. However, aforementioned works are not suitable for DAS which is equipped with multiple antennas. The authors of [8] proposed an iterative algorithm to determine optimal deployment of antenna locations based on stochastic approximation theory for DAS with multiple antenna DA ports. However, this algorithm is feasible only for the single-cell single-user case and requires an iterative method.

In this paper, we extend the works in [7] to more generalized DAS scenarios with DA ports equipped with multiple antennas under per-DA port power constraint. The MRT in [9] and the scaled ZFBF in [10] are applied for single-user and multi-user DAS, respectively. The locations of DA ports are computed by considering the following performance metrics. For the single-cell scenario, a lower bound of the expected SNR is maximized to obtain the locations of DA ports. This optimization problem is convex and generates a closed form solution. Simulation results show that DA port locations determined by the proposed algorithms are almost the same as the optimal locations from conventional algorithms in [8] with much lower complexity. For the two-cell scenario, locations of DA ports are found by maximizing a lower bound of the expected SLR. The SLR criterion is suitable for multi-cell environments since other-cell interference is taken into consideration. In this case, we apply an iterative gradient ascent algorithm as the optimization problem is non-convex.

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II. System Model

We consider a multi-cell downlink DAS. There are N DA ports equipped with T antennas, and K Mobile stations (MS) equipped with a single antenna in each cell where all MSs are assumed to be uniformly distributed within a cell. We assume a cell with the radius of R with circular antenna layout and the distance between centers of adjacent two cells is set to $\sqrt{3}R$. Each cell is separated into N equal area regions and each DA port with multiple antennas is located in each region. The location of the i-th DA port is expressed as

$$P_i = R_i \exp(j\theta_i), \quad \text{for} \quad i = 1, \cdots, N \tag{1}$$

where R_i and θ_i are the magnitude and the phase of the i-th DA port with $(R_{min,i} \leq R_i \leq R_{max,i})$ and $(\theta_{min,i} \leq \theta_i \leq \theta_{max,i})$, respectively.

The channel model for DAS encompasses not only small scale fadings but also large scale fadings including shadowing and pathloss [2]. It is assumed that perfect channel state information and MS locations are known to all DA ports. We concentrate on fully cooperative networks in which the DA ports are connected via ideal backhaul in each cell but with no inter-cell cooperation. First, we define the precoded signal vector as $\mathbf{x}^{(l)} = \sum_{k=1}^{K} \mathbf{f}_k^{(l)} u_k^{(l)}$ at cell l, where $\mathbf{f}_k^{(l)} = [\mathbf{f}_{k,1}^{(l)T} \cdots \mathbf{f}_{k,N}^{(l)T}]^T$ indicates the $NT \times 1$ beamforming vector for the k-th MS in cell l with $\|\mathbf{f}_{k,n}^{(i)}\|^2 \leq P$ and $u_k^{(l)}$ represents the complex-valued data symbol for the k-th MS in cell l with $\mathbb{E}[|u_k^{(l)}|^2] = 1$. Here, P denotes the power available at each DA port.

Then, the $T \times 1$ channel vector from the n-th DA port in cell i to the k-th MS in cell j is defined as

$$\mathbf{g}_{k,n}^{(i,j)} = \sqrt{s_{k,n}^{(i,j)} / \left(d_{k,n}^{(i,j)}\right)^{\alpha}} \cdot \left[h_{k,(n,1)}^{(i,j)} \cdots h_{k,(n,T)}^{(i,j)}\right]^T$$

where $h_{k,(n,t)}^{(i,j)}$ equals an independent and identically distributed zero-mean unit-variance complex Gaussian random variable representing small scale fadings from the t-th antenna of n-th DA port in cell i to the k-th MS in cell j, $s_{k,n}^{(i,j)}$ is a log-normal random variable corresponding to large scale fadings from the n-th DA port in cell i to the k-th MS in cell j, i.e.

$10\log_{10} s_n^{(i,j)}$ is zero-mean Gaussian with variance σ_{sh}^2, $d_{k,n}^{(i,j)}$ stands for the distance between the n-th DA port in cell i to the k-th MS in cell j, and α indicates the path loss exponent.

The received signal for the k-th MS in cell l can be expressed as

$$y_k^{(l)} = \mathbf{g}_k^{(l,l)H}\mathbf{x}^{(l)} + \sum_{l'\neq l}\mathbf{g}_k^{(l',l)H}\mathbf{x}^{(l')} + z_k^{(l)}$$

where $z_k^{(l)}$ is the additive complex Gaussian noise at the k-th MS in cell l with zero mean and variance σ_z^2 and $\mathbf{g}_k^{(i,j)}$ $(k = 1,\cdots,K)$ equals an $NT\times 1$ channel vector from all DA ports in cell i to the k-th MS in cell j. Here, $\mathbf{g}_k^{(i,j)}$ can be decomposed into $\mathbf{g}_k^{(i,j)} = [\mathbf{g}_{k,1}^{(i,j)T}\cdots\mathbf{g}_{k,N}^{(i,j)T}]^T$.

For the single-user DAS, the MRT is applied for each DA port since it is the optimal precoding design in the single-cell single-user DAS under per-DA port power constraint [9]. Then, we obtain the precoding vector of the n-th DA port in cell i as

$$\mathbf{f}_{1,n}^{(i)} = \sqrt{P}\frac{\mathbf{g}_{1,n}^{(i,i)}}{\left\|\mathbf{g}_{1,n}^{(i,i)}\right\|}. \tag{2}$$

Then, the ergodic sum rates for the single-user DAS in single-cell and two-cell cases are given, respectively, as

$$R_{SC}^{SU} = \mathbb{E}_{h,s,p}\left[\log_2\left(1+\frac{P\left(\sum_{n=1}^N\left\|\mathbf{g}_{1,n}^{(1,1)}\right\|\right)^2}{\sigma_z^2}\right)\right] \tag{3}$$

$$R_{TC}^{SU} = \mathbb{E}_{h,s,p}\left[\sum_{l=1}^2\log_2\left(1+\frac{P\left(\sum_{n=1}^N\left\|\mathbf{g}_{l,n}^{(l,l)}\right\|\right)^2}{\sigma_z^2+\left|\mathbf{g}_1^{(\bar{l},l)H}\mathbf{f}_1^{(\bar{l})}\right|^2}\right)\right]$$

where $\mathbb{E}_{h,s,p}$ indicates the expectation with respect to small scale fadings, shadowing and the position of MS and we define $\bar{l} = 1$ if $l = 2$ and $\bar{l} = 2$ if $l = 1$.

For the case of multi-user DAS, there is no closed form solution for the optimal beamforming scheme because of the complexity in solving the optimization problem [11]. Therefore, we apply scaled ZFBF as precoding vectors for the multi-user DAS. The precoding matrix for cell i can be determined as $\mathbf{F}^{(i)} = \sqrt{P}\mathbf{W}^{(i)}/\max_{1\leq n\leq N}\|\mathbf{W}_n^{(i)}\|_F^2$ [10] where $\mathbf{W}^{(i)} = [\mathbf{W}_1^{(i)T}\cdots\mathbf{W}_N^{(i)T}]^T = \mathbf{G}^{(i,i)}(\mathbf{G}^{(i,i)H}\mathbf{G}^{(i,i)})^{-1}$ is the right Pseudo inverse of $\mathbf{G}^{(i,i)H}$ with $\mathbf{G}^{(i,i)} = [\mathbf{g}_1^{(i,i)}\cdots\mathbf{g}_K^{(i,i)}]$, $\mathbf{W}_n^{(i)}$ equals the non-scaled beamforming matrix of the n-th DA port for all MSs in cell i and $\|\cdot\|_F$ defines the Frobenius norm. Then, we can compute the precoding vector with the scaled ZFBF for the k-th MS in cell i as

$$\mathbf{f}_k^{(i)} = \mathbf{F}_{(k)}^{(i)} \tag{4}$$

where $\mathbf{A}_{(k)}$ denotes the k-th column vector of a matrix $\mathbf{A}$.

For multi-user DAS, the cell average ergodic sum rates for single-cell and two-cell cases are expressed as

$$R_{SC}^{MU} = \mathbb{E}_{h,s,p}\left[\sum_{k=1}^K\log_2\left(1+\frac{\left|\mathbf{g}_k^{(1,1)H}\mathbf{f}_k^{(1)}\right|^2}{\sigma_z^2}\right)\right] \tag{5}$$

$$R_{TC}^{MU} = \mathbb{E}_{h,s,p}\left[\sum_{l=1}^2\sum_{k=1}^K\log_2\left(1+\frac{\left|\mathbf{g}_k^{(l,l)H}\mathbf{f}_k^{(l)}\right|^2}{\sigma_z^2+\sum_{k'=1}^K\left|\mathbf{g}_k^{(\bar{l},l)H}\mathbf{f}_{k'}^{(\bar{l})}\right|^2}\right)\right].$$

Note that precoding vectors in both scenarios satisfy per-DA port power constraint P.

III. Antenna Placement

The optimization problem of antenna locations which maximize the cell average ergodic sum rate (3) and (5) are quite complicated to solve in general. Therefore, in this section, we formulate the performance metrics derived by lower bounds of the expected SNR and SLR for the single-cell and two-cell scenarios, respectively as suggested in [7]. Then, we introduce new algorithms to determine antenna locations by maximizing the derived performance metrics.

A. SNR Criterion

In the single-cell DAS scenario, the cell average ergodic sum rate can be approximated by applying Jensen's inequality as

$$R_{SC} \approx \sum_{k=1}^K\log_2\left(1+\mathbb{E}_{h,s,p}\left[\text{SNR}_k^{(1)}\right]\right)$$

where $\text{SNR}_k^{(1)} = \frac{1}{\sigma_z^2}\left|\mathbf{g}_k^{(1,1)H}\mathbf{f}_k^{(1)}\right|^2$. From this relation, instead of maximizing the cell average ergodic capacity, we focus on maximizing the SNR criterion derived by a lower bound of the expected SNR.

1) Single-user Case: For the single-cell single-user case, each DA port uses a beamforming vector with MRT in (2). Then, a lower bound of the expected SNR can be written as

$$\begin{aligned}\mathbb{E}_{h,s,p}\left[\text{SNR}_1^{(1)}\right] &\geq \frac{P}{\sigma_z^2}\mathbb{E}_{h,s,p}\left[\sum_{n=1}^N\left\|\mathbf{g}_{1,n}^{(1,1)}\right\|^2\right]\\ &\geq \frac{PC_{\text{min}}^{(1,1)}}{\sigma_z^2}\mathbb{E}_p\left[\sum_{n=1}^N\left(d_{1,n}^{(1,1)}\right)^{-\alpha}\right]\end{aligned}$$

where the first inequality comes from Cauchy-Schwarz inequality and the fact that $\|\mathbf{f}_{1,n}^{(1)}\|^2 = P$ and the second inequality follows from $C_{\text{min}}^{(1,1)} \triangleq \min[|s_{1,1}^{(1,1)}|\|\mathbf{h}_{1,1}^{(1,1)}\|^2,\cdots,|s_{N,1}^{(1,1)}|\|\mathbf{h}_{N,1}^{(1,1)}\|^2]$.

Assuming that MS 1 is located in the region of DA port i, we focus on $(d_{1,i}^{(1,1)})^{-\alpha}$ for a simple derivation. Finally, applying Jensen's inequality from the fact that $\left(\frac{1}{x}\right)^a$ is convex for $a \geq 0$, a lower bound of the expected SNR for the single-cell single-user scenario can be determined as

$$\mathbb{E}_{h,s,p}\left[\text{SNR}_1^{(1)}\right] \geq \frac{PC_{\text{min}}^{(1,1)}}{\sigma_z^2}\mathbb{E}_p\left[\left(d_{1,i}^{(1,1)}\right)^2\right]^{-\frac{\alpha}{2}}. \tag{6}$$

2) Multi-user Case: For the single-cell multi-user case, the beamforming vector with scaled ZFBF in (4) is applied. Since every MS has the same formulation of the expected SNR, we take the expected SNR of MS 1 into consideration without loss of generality. Then, the expected SNR of MS 1 can be lower bounded as

$$\begin{aligned}\mathbb{E}_{h,s,p}\left[\text{SNR}_1^{(1)}\right] &= \frac{1}{\sigma_z^2}\mathbb{E}_{h,s,p}\left[\sum_{n=1}^N\left|\mathbf{g}_{1,n}^{(1,1)H}\mathbf{f}_{1,n}^{(1)}\right|^2\right]\\ &\geq \frac{D_{\text{min}}^{(1,1)}}{\sigma_z^2}\sum_{n=1}^N\mathbb{E}_p\left[\left(d_{1,n}^{(1,1)}\right)^{-\alpha}\right]\end{aligned}$$

where the equality comes from $\mathbb{E}_{h,s}[\mathbf{g}_{1,n}^{(1,1)}\mathbf{g}_{1,m}^{(1,1)*}] = 0$ for $n \neq m$ and the inequality follows from $D_{\text{min}}^{(1,1)} \triangleq \min[|s_{1,1}^{(1,1)}||\mathbf{h}_{1,1}^{(1,1)}\mathbf{f}_{1,1}^{(1)}|^2,\cdots,|s_{1,N}^{(1,1)}||\mathbf{h}_{1,N}^{(1,1)}\mathbf{f}_{1,N}^{(1)}|^2]$.

Similar to the single-cell single-user scenario, a lower bound of the expected SNR can be derived as

$$\mathbb{E}_{h,s,p}\left[\mathrm{SNR}_1^{(1)}\right] \geq \frac{D_{\min}^{(1,1)}}{\sigma_z^2}\mathbb{E}_{p_1}\left[\left(d_{k,i}^{(1,1)}\right)^2\right]^{-\frac{\alpha}{2}}. \quad (7)$$

Maximizing (6) and (7) is equivalent to minimizing $\mathbb{E}_{p_1}[(d_{k,i}^{(1,1)})^2]$. Finally, for the single-cell DAS, we determine the SNR criterion which minimizes the SNR metric $\Gamma_{\text{SNR}}^{(i)}$ as

$$\Gamma_{\text{SNR}}^{(i)} = \mathbb{E}_{p_1}\left[\left(d_{k,i}^{(1,1)}\right)^2\right]. \quad (8)$$

B. SLR criterion

In the two-cell DAS scenario, the cell average ergodic sum rate can be computed by

$$R_{TC} = \mathbb{E}_{h,s,p_1,p_2}\left[\sum_{l=1}^{2}\sum_{k=1}^{K}\log_2\left(1+\mathrm{SINR}_k^{(l)}\right)\right] \quad (9)$$

where $\mathrm{SINR}_k^{(l)} = \frac{\left|\mathbf{g}_k^{(l,l)H}\mathbf{f}_k^{(l)}\right|^2}{\sigma_z^2+\sum_{k'=1}^{K}\left|\mathbf{g}_k^{(\bar{l},l)H}\mathbf{f}_{k'}^{(\bar{l})}\right|^2}$. Similar to the single-cell scenario, we try to optimize a new metric instead of maximizing the cell average ergodic sum rate. Maximizing the expected signal-to-interference plus noise ratio (SINR) may be suitable to take other-cell interference into consideration. However, the expected SINR is still difficult to deal with because of its coupled nature in two cells causing high computational complexity. Therefore, we focus on the expected SLR as an alternative approach.

With high SINR approximation and interference limited assumptions, (9) can be expressed as a function of SLR as

$$\begin{aligned} R_{TC} &\approx \mathbb{E}_{h,s,p_1,p_2}\left[\sum_{l=1}^{2}\sum_{k=1}^{K}\log_2\left(\mathrm{SLNR}_k^{(l)}\right)\right] \\ &\approx \mathbb{E}_{h,s,p_1,p_2}\left[\sum_{l=1}^{2}\sum_{k=1}^{K}\log_2\left(\mathrm{SLR}_k^{(l)}\right)\right] \end{aligned}$$

where p_1 and p_2 indicate the position of MSs in cell 1 and 2, respectively, and $\mathrm{SLNR}_k^{(l)} = \frac{\left|\mathbf{g}_k^{(l,l)H}\mathbf{f}_k^{(l)}\right|^2}{\sigma_z^2+\sum_{k'=1}^{K}\left|\mathbf{g}_k^{(l,\bar{l})H}\mathbf{f}_{k'}^{(l)}\right|^2}$ and $\mathrm{SLR}_k^{(l)} = \frac{\left|\mathbf{g}_k^{(l,l)H}\mathbf{f}_k^{(l)}\right|^2}{\sum_{k'=1}^{K}\left|\mathbf{g}_k^{(l,\bar{l})H}\mathbf{f}_{k'}^{(l)}\right|^2}$. Here, $\mathrm{SLR}_k^{(l)}$ is defined as the ratio of the signal power of the k-th MS in cell l to the leakage power from all MSs in cell l to the k-th MS in cell $\bar{l}$.

After applying Jensens's inequality, the cell-averaged sum rate can be bounded as

$$R_{TC}^{MU} \leq \sum_{l=1}^{2}\sum_{k=1}^{K}\log_2\left(\mathbb{E}_{h,s,p_1,p_2}\left[\mathrm{SLR}_k^{(l)}\right]\right).$$

1) Single-user Case: For the two-cell single-user case, each DA port uses a beamforming vector with MRT in (2). Then, a lower bound of the expected SLR can be derived as

$$\begin{aligned} \mathbb{E}_{h,s,p_1,p_2}\left[\mathrm{SLR}_1^{(1)}\right] &= \mathbb{E}_{h,s,p_1,p_2}\left[\frac{P\left(\sum_{n=1}^{N}\left\|\mathbf{g}_{1,n}^{(1,1)}\right\|\right)^2}{\left|\mathbf{g}_1^{(1,2)H}\mathbf{f}_1^{(1)}\right|^2}\right] \\ &\geq \frac{1}{N}\mathbb{E}_{h,s,p_1}\left[\left(\sum_{n=1}^{N}\left\|\mathbf{g}_{1,n}^{(1,1)}\right\|\right)^2\right]\mathbb{E}_{h,s,p_2}\left[\frac{1}{\sum_{n=1}^{N}\left\|\mathbf{g}_{1,n}^{(1,2)}\right\|^2}\right] \end{aligned} \quad (10)$$

where the inequality comes from Cauchy Schwarz and the fact that $\|\mathbf{f}_{1,n}^{(1)}\|^2 = P$.

With the result of (6), the first term of (10) can be lower bounded by $C_{\min}^{(1,1)}/\mathbb{E}_{p_1}[(d_{1,n}^{(1,1)})^2]^{\frac{\alpha}{2}}$. Also, the second term of (10) can be expressed as

$$\begin{aligned} \mathbb{E}_{h,s,p_2}\left[\frac{1}{\sum_{n=1}^{N}\left\|\mathbf{g}_{1,n}^{(1,2)}\right\|^2}\right] &\geq \frac{1}{C_{\max}^{(1,2)}}\mathbb{E}_{p_2}\left[\frac{1}{\sum_{n=1}^{N}\left(d_{1,n}^{(1,2)}\right)^{-\alpha}}\right] \\ &\approx \frac{1}{C_{\max}^{(1,2)}N}\mathbb{E}_{p_2}\left[\frac{1}{\left(d_{1,i}^{(1,2)}\right)^{-\alpha}}\right] \end{aligned} \quad (11)$$

where the inequality comes from $C_{\max}^{(1,2)} \triangleq \max[|s_{1,1}^{(1,2)}|\|\mathbf{h}_{1,1}^{(1,2)}\|^2,\cdots,|s_{1,N}^{(1,2)}|\|\mathbf{h}_{1,N}^{(1,2)}\|^2]$ and (11) is approximated by assuming that distances from any DA port in cell 1 to MS 1 in cell 2 are equidistant. Finally, inserting the two terms into (10), a lower bound of the expected SLR for MS in cell 1 for the two-cell single-user scenario can be determined as

$$\mathbb{E}_{h,s,p_1,p_2}\left[\mathrm{SLR}^{(l)}\right] \geq \frac{C_{\min}^{(1,1)}}{C_{\max}^{(1,2)}N^2}\frac{\mathbb{E}_{p_2}\left[\left(d_{1,i}^{(1,2)}\right)^{\alpha}\right]}{\mathbb{E}_{p_1}\left[\left(d_{1,i}^{(1,1)}\right)^2\right]^{\frac{\alpha}{2}}}. \quad (12)$$

2) Multi-user Case: For the two-cell multi-user case, the beamforming vetor with scaled ZFBF (4) is employed similar to the single-cell multi-user case. We focus on the expected SLR of MS 1 in cell 1 without loss of generality. Then, a lower bound of the expected SLR of MS 1 in cell 1 can be written as

$$\begin{aligned} \mathbb{E}_{h,s,p_1,p_2}\left[\mathrm{SLR}_1^{(1)}\right] &\geq \mathbb{E}_{h,s,p_1,p_2}\left[\frac{\left|\mathbf{g}_1^{(1,1)H}\mathbf{f}_1^{(1)}\right|^2}{\sum_{k=1}^{K}\sum_{n=1}^{N}\left\|\mathbf{g}_1^{(1,2)}\right\|^2\sum_{n=1}^{N}\left\|\mathbf{f}_k^{(1)}\right\|^2}\right] \\ &\geq \frac{1}{KNP}\mathbb{E}_{h,s,p_1}\left[\left|\mathbf{g}_1^{(1,1)H}\mathbf{f}_1^{(1)}\right|^2\right]\mathbb{E}_{h,s,p_2}\left[\frac{1}{\sum_{n=1}^{N}\left\|\mathbf{g}_{1,n}^{(1,2)}\right\|^2}\right] \end{aligned} \quad (13)$$

where inequalities comes from the Cauchy-Schwarz inequality and per-DA port power constraint as in the two-cell single-user scenario.

From (7), the first term of (13) can be lower bounded by $D_{\min}^{(1,1)}/\mathbb{E}_{p_1}[(d_{k,i}^{(1,1)})^2]^{\frac{\alpha}{2}}$. Also, the second term of (13) can be expressed as

$$\mathbb{E}_{h,s,p_2}\left[\frac{1}{\sum_{n=1}^{N}\left\|\mathbf{g}_{1,n}^{(1,2)}\right\|^2}\right] \geq \frac{1}{C_{1,\max}^{(1,2)}N}\mathbb{E}_{p_2}\left[\frac{1}{\left(d_{1,i}^{(1,2)}\right)^{-\alpha}}\right]$$

where the inequality comes from $C_{1,\max}^{(1,2)} \triangleq \max[|s_{1,1}^{(1,2)}|\|\mathbf{h}_{1,1}^{(1,2)}\|^2,\cdots,|s_{1,N}^{(1,2)}|\|\mathbf{h}_{1,N}^{(l,2)}\|^2]$ and equidistance assumption. Finally, by combining the two terms of (13), a lower bound of the expected SLR of MS 1 in cell 1 for the two-cell multi-user scenario can be given as

$$\mathbb{E}_{h,s,p_1,p_2}\left[\mathrm{SLR}_1^{(1)}\right] \geq \frac{D_{\min}^{(1,1)}}{C_{1,\max}^{(1,2)}KN^2P}\frac{\mathbb{E}_{p_2}\left[\left(d_{1,i}^{(1,2)}\right)^{\alpha}\right]}{\mathbb{E}_{p_1}\left[\left(d_{1,i}^{(1,1)}\right)^2\right]^{\frac{\alpha}{2}}}. \quad (14)$$

Maximizing lower bounds of the expected SLR (12) and (14) is equivalent to maximizing $\mathbb{E}_{p_2}[(d_{1,i}^{(1,2)})^{\alpha}]/\mathbb{E}_{p_1}[(d_{1,i}^{(1,1)})^2]^{\frac{\alpha}{2}}$. Since this term cannot be computed with general α, we reformulate the cost function in order to make it easy to deal with. After putting the logarithm operation and Jensen's inequality, the cost function γ can be lower bounded as

$$\gamma \geq \frac{\alpha}{2}\left\{\mathbb{E}_{p_2}\left[\ln\left(d_{1,i}^{(1,2)}\right)^2\right] - \ln \mathbb{E}_{p_1}\left[\left(d_{1,i}^{(1,1)}\right)^2\right]\right\}.$$

Then, for the two-cell DAS, we can obtain the SLR criterion which maximizes the SLR metric $\Gamma_{\text{SLR}}^{(i)}$ as

$$\Gamma_{\text{SLR}}^{(i)} = \mathbb{E}_{p_2}\left[\ln\left(d_{1,i}^{(1,2)}\right)^2\right] - \ln \mathbb{E}_{p_1}\left[\left(d_{1,i}^{(1,1)}\right)^2\right]. \tag{15}$$

C. Optimization of DA Port Placement

As shown in Sections III-A and III-B in DAS with multiple-antenna ports, the SNR and SLR criteria are expressed through deriving lower bounds of the expected SNR and SLR for the single-cell and two-cell scenarios, respectively. Therefore, an antenna placement optimization problem is determined by the SNR criterion (8) for the single-cell case and the SLR criterion (15) for the two-cell case with DA ports given in (1).

1) Single-Cell Case: For the single-cell scenario, the location of the i-th DA port can be determined by the following optimization problem:

$$\left\{\hat{R}_i, \hat{\theta}_i\right\} = \arg\min_{\{R_i,\theta_i\}} \Gamma_{\text{SNR}}^{(i)} \quad \text{for} \quad i = 1,\cdots,N \tag{16}$$

$$\text{subject to} \quad R_{min,i} \leq R_i \leq R_{max,i}, \quad \theta_{min,i} \leq \theta_i \leq \theta_{max,i}.$$

By solving (16) using the result in [7], a closed form solution for the location of the i-th DA port is given as

$$\hat{R}_i = \frac{2\left(R_{max,i}^3 - R_{min,i}^3\right)\left(\sin\left(\theta_{max,i} - \hat{\theta}_i\right) + \sin\left(\hat{\theta}_i - \theta_{min,i}\right)\right)}{3\left(R_{max,i}^2 - R_{min,i}^2\right)\left(\theta_{max,i} - \theta_{min,i}\right)} \tag{17}$$

$$\hat{\theta}_i = \tan^{-1}\left(\frac{\cos\theta_{max,i} - \cos\theta_{min,i}}{\sin\theta_{max,i} - \sin\theta_{min,i}}\right). \tag{18}$$

2) Two-Cell Case: For the two-cell scenario, the location of the i-th DA port in cell 1 can be derived by solving the optimization problem:

$$\left\{\hat{R}_i, \hat{\theta}_i\right\} = \arg\max_{\{R_i,\theta_i\}} \Gamma_{\text{SLR}}^{(i)} \quad \text{for} \quad i = 1,\cdots,N \tag{19}$$

$$\text{subject to} \quad R_{min,i} \leq R_i \leq R_{max,i}, \quad \theta_{min,i} \leq \theta_i \leq \theta_{max,i}.$$

Since (19) is not a convex problem, we employ a gradient ascent method to find a local optimal solution.

Applying the results in [7], the two gradients of $\Gamma_{\text{SLR}}^{(i)}$ with respect to R_i and θ_i are computed as

$$\nabla_{R_i}\Gamma_{\text{SLR}}^{(i)} = \begin{cases} \frac{2}{R^2}\left(R_i - \sqrt{3}R\cos\theta_i\right) - \frac{\nabla_{R_i}\Gamma_{\text{SNR}}^{(i)}}{\Gamma_{\text{SNR}}^{(i)}} & \text{for } \rho < R \\ \frac{2}{\rho^2}\left(R_i - \sqrt{3}R\cos\theta_i\right) - \frac{\nabla_{R_i}\Gamma_{\text{SNR}}^{(i)}}{\Gamma_{\text{SNR}}^{(i)}} & \text{for } \rho \geq R \end{cases} \tag{20}$$

$$\nabla_{\theta_i}\Gamma_{\text{SLR}}^{(i)} = \begin{cases} 2\sqrt{3}\frac{R_i}{R}\sin\theta_i - \frac{\nabla_{\theta_i}\Gamma_{\text{SNR}}^{(i)}}{\Gamma_{\text{SNR}}^{(i)}} & \text{for } \rho < R \\ 2\sqrt{3}\frac{RR_i}{\rho^2}\sin\theta_i - \frac{\nabla_{\theta_i}\Gamma_{\text{SNR}}^{(i)}}{\Gamma_{\text{SNR}}^{(i)}} & \text{for } \rho \geq R \end{cases} \tag{21}$$

where $\rho = \sqrt{R_i^2 + 3R^2 - 2\sqrt{3}RR_i\cos\theta_i}$ and $\nabla_{R_i}\Gamma_{\text{SNR}}^{(i)}$ and $\nabla_{\theta_i}\Gamma_{\text{SNR}}^{(i)}$ are the gradients of $\Gamma_{\text{SNR}}^{(i)}$ with respect to R_i and θ_i, respectively.

Now, with (20) and (21), we propose an iterative algorithm which solves (19) as follows:

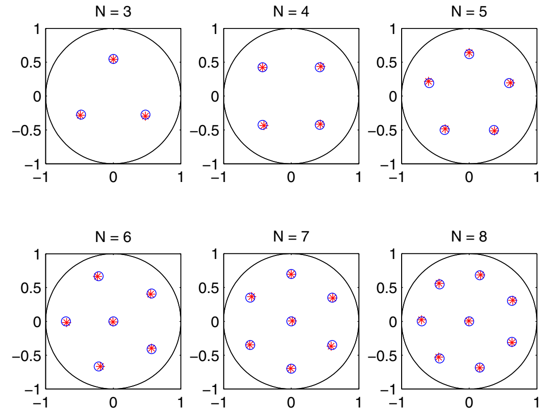

Fig. 1. Locations of DA ports for DAS in single-cell

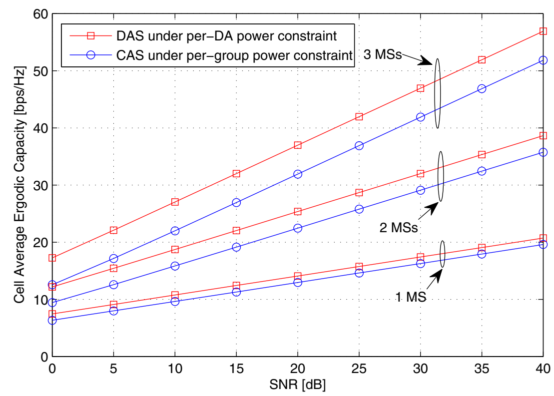

Fig. 2. The cell average ergodic sum rate for the single-cell DAS and CAS with $N = 7$

1) Initialize the position of the i-th DA port in cell 1.
2) Update the position with $R_i \leftarrow R_i + \delta_R \cdot \nabla_{R_i}\Gamma_{\text{SLR}}^{(i)}$ and $\theta_i \leftarrow \theta_i + \delta_\theta \cdot \nabla_{\theta_i}\Gamma_{\text{SLR}}^{(i)}$.
3) Compute $\Gamma_{\text{SLR}}^{(i)}$ with the updated position.
4) Go back to 2) until convergence.

In our algorithm, we choose the step sizes δ_R and δ_θ adopting Armijo's rule which provides provable convergence [12]. After the positions of DA ports are decided in cell 1, the positions of DA ports in cell 2 can be determined by using symmetry between cell 1 and 2.

IV. Numerical Results

In this section, simulation results are presented to evaluate the performance of the proposed SNR and SLR critera. We adopt the MRT and the scaled ZFBF for the single-user and mult-user scenario, respectively, and set $R = 1$ km, $\sigma_{sh} = 4$ dB, and $\alpha = 3.75$ throughout the simulations. In Figure 1, we plot the locations of DA ports for $N = 3,\cdots,8$ for the single-cell case which are computed from the SNR criterion (17) and (18). In this figure, the asterisks represent the optimal locations of DA ports from the algorithms given by [8], and the circles indicate the locations of DA ports from the proposed algorithm. Existence of a center antenna is decided by comparing the SNR criteria [7]. Note that the proposed closed form gives DA port locations almost identical to the scheme in [8] with much lower computational complexity.

For the single-cell DAS and CAS, the cell average ergodic sum rate curves are illustrated as a function of SNR in Figure

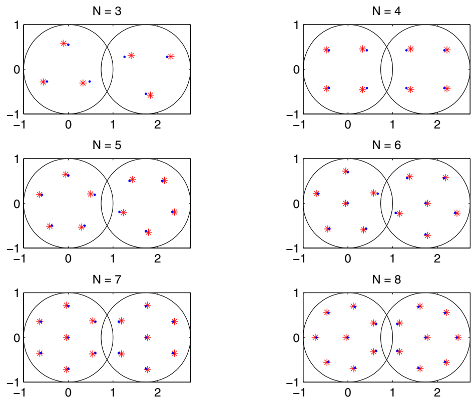

Fig. 3. Locations of DA ports for DAS in two-cell

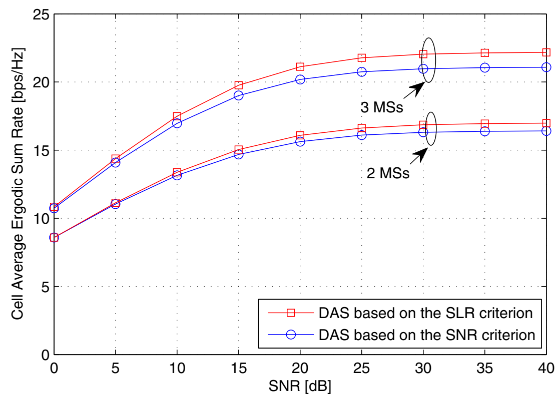

Fig. 4. The cell average ergodic sum rate for two-cell DAS with $N = 3$

2. We adopt antenna locations illustrated in Figure 1 for the DAS and set $T = 2$, $N = 7$ for $K = 1$, 2 and 3. For fair comparison between DAS and CAS, CAS with per-group power constraint is considered where antennas are divided into N groups and which contains T antennas in each group with power constraint $\|\mathbf{f}_{k,n}^{(1)}\|^2 \leq P$ for $n = 1, \cdots, N$. At an SNR of 20 dB, DAS with the proposed antenna locations provides sum rate gains of about 9%, 13% and 16% over the CAS with per-group power constraint for $K = 1$, 2 and 3.

In Figure 3, we plot the locations of DA ports for $N = 3, \cdots, 8$ for the two-cell case determined by the proposed iterative algorithm in Section III-C. The dots indicate the locations of DA ports decided by the SNR criterion while the asterisks stand for the locations of DA ports determined by the proposed SLR criterion. As shown in this figure, port locations with the SLR criterion are shifted against the other cell to reduce inter-cell interference.

For two-cell environment, we compare the performance of DAS with the port locations determined from the SNR criterion with DAS from the SLR criterion. We set $T = 2$ and $N = 3$ for following simulation. In Figure 4, we illustrate the cell average ergodic sum rate curves for the two-cell case with respect to SNR. Simulation shows that DAS with the proposed SLR criterion has performance gains of about 5% compared to DAS with the SNR criterion for $K = 2$, 3 at a SNR of 30 dB. This performance gain results from the fact that the SLR criterion takes the leakage power term into consideration.

Now, we present the performance of the DAS with antenna locations from the proposed SLR criterion in terms of the cell average ergodic sum rate in Figure 5. We set $T = 2$, $N = 7$ for $K = 2$ and 3. At 20 dB SNR, simulation results show that

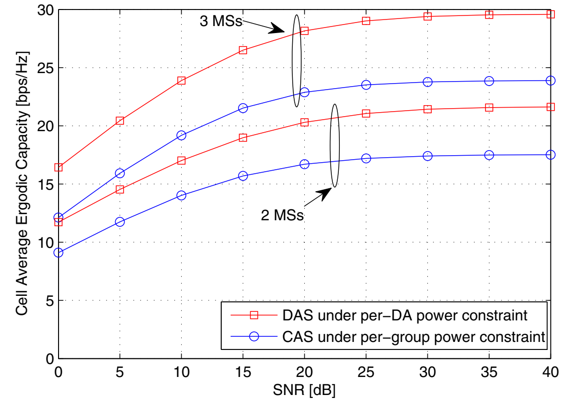

Fig. 5. The cell average ergodic sum rate for the two-cell DAS and CAS with $N = 7$

DAS with antenna locations from the proposed SLR criterion has sum rate gains of about 22% and 23% over the CAS with per-group power constraint for $K = 2$ and 3, respectively.

V. Conclusion

In this paper, we have addressed the problem of antenna placements for the single-cell and two-cell DAS with DA ports equipped with multiple antennas under per-DA port power constraint. We have formulated the optimization problems and proposed algorithms to obtain locations of DA ports. For the single-cell scenario, we derived the SNR criterion and computed a closed form solution. The locations determined from the proposed SNR criterion are quite close to the optimal locations attained by conventional iterative algorithms. For the two-cell scenario, we adopted the SLR criterion and presented an iterative gradient ascent algorithm. Simulation results exhibit that both single-cell and two-cell DAS outperforms CAS in terms of the cell average ergodic sum rates.

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