

Successive Interference Cancellation via Rank-Reduced Maximum Likelihood Detection

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Abstract—This paper proposes a codeword-based iterative detecting and decoding (IDD) algorithm using a rank-reduced maximum likelihood (ML) detector with pre-whitened interference over multiple-input multiple-output (MIMO) channels. This iterative algorithm operates on the principle of successive interference cancellation (SIC) where all the layers per codeword are decoded together at each iteration. The recent wireless standard requires to encode data bits per codeword, not per layer at multiple antennas. Thus, SIC algorithms based on layer-separating detectors could lose joint information between layers in a single codeword. Instead, the proposed algorithm minimizes this loss by using rank-reduced ML detectors over soft feedback. Simulations are performed on a space-time bit-interleaved coded modulation over Rayleigh fading channels, and demonstrate the proposed SIC algorithm is superior to comparable iterative and non-iterative IDD algorithms.

I. INTRODUCTION

There has been great interest in multiple-input multiple output (MIMO) systems ever since it is known that the capacity of wireless channels can significantly increase over the benefit of spatial diversity gain [1]. In order to achieve such a potential gain on the capacity, many space-time schemes for practical standards such as 3GPP Long Term Evolution (LTE) are proposed. For example, [2] reforms channels to be orthogonal and [3] uses the orthogonal structure of space-time block codes to achieve the maximum diversity order. Also, [4] designs an architecture of offering spatial multiplexing over multiple antenna wireless channels called Bell Lab layered space-time (BLAST). In [5], the vertical BLAST (V-BLAST) scheme is introduced to eliminate interference successively. This detection algorithm assumes that each layer is independently coded and iterative layer-by-layer detecting is available.

The space-time detection scheme is combined with channel codes to achieve the channel capacity promised by MIMO technologies. This combined scheme evolved to iteratively detect and decode (IDD) symbols by exchanging soft information between MIMO detectors and channel decoders. Under the IDD principle, various MIMO detectors have been extensively researched to aim the objective that achieves high throughput with less computational burden. In [6], the maximum *a posteriori* (MAP) detector directly computes log-likelihood ratios (LLRs) for space-time bit-interleaved coded modulation (BICM). [7] extends the scheme to be applicable for OFDM systems. Even though the MAP detector achieves the optional performance, it also has a drawback that the computational complexity grows exponentially as using multiple antennas

and high order modulations. Instead, [8] modifies the sphere decoding technique originally proposed in [9] such that a list of computable candidates are provided at the detector. Moreover, [10] thoroughly studied the less computational detector using a linear minimum mean square error (MMSE) with *a posteriori* information. In [11], soft feedback based on *a priori* LLR is employed to eliminate interference in an iterative fashion. As the V-BLAST detector opens the algorithm for successively canceling interference, these detectors have been developed under the assumption that all layers are separable and distinctly coded. Thus it is assumed that the decoded information for a single layer could be used for other undecoded layers.

However, recent LTE standards support codeword-based coded signals rather than layer-based coded signals [12]. This paper proposes the codeword-based successive interference cancellation (SIC) algorithm using a rank-reduced maximum likelihood (ML) detector with pre-whitened interference. In particular, the rank-reduced ML detector can be replaced with efficient techniques to closely approach the ML performance while reducing the complexity with exponential orders. For example, [13] employs a simple slicer block that uses *a priori* information and shows that it suffers only negligible performance loss at high modulation. Under the rank-reduced ML detector, the proposed algorithm minimizes to lose joint information among multiple layers in a single codeword.

This paper is organized as follows: Sec. II introduces the framework of both the transmitter and the receiver, and describes a receiver architecture for SIC. Sec. III depicts the proposed SIC algorithm compared with a conventional MMSE-based SIC algorithm. Sec. IV analyzes the detection complexity for the proposed SIC algorithm. In Sec. V, the performance of the proposed algorithm is evaluated in terms of modulation order, coding rates and the number of iterations. The conclusion is followed in Sec. VI.

II. SYSTEM MODEL

This paper considers a point-to-point MIMO channel where both a transmitter and a receiver are equipped with n_t antennas and n_r antennas, respectively. In a transmitter, the sequence of binary information bits are encoded as following the principle of BICM. The binary bit sequence is lengthened depending on the code rate. The coded bit sequence is permuted by randomly chosen interleaving patterns that belong to each codeword. It is assumed that the permuted sequence is statistically independent with large interleaving patterns. The coded sequences are

A. Existing MMSE-based SIC

Compared with the ML detector, the MMSE detector is a suboptimal method that nullifies the interference so as to separate layers and maximize the signal on the layer being detected. Suppose that the n th layer is currently detected, the corresponding MMSE filtering vector is given by

$$\mathbf{w}_n = (\mathbf{H}\mathbf{Q}_n\mathbf{H}^\dagger + \sigma^2\mathbf{I}_{n_r})^{-1}\mathbf{h}_n \quad (12)$$

where \mathbf{Q}_n , \mathbf{I}_{n_r} and \mathbf{h}_n are the covariance matrix obtained from *a posteriori* LLRs of decoded codewords, the identity matrix of size n_r , and the n th column vector of \mathbf{H} , respectively. The covariance matrix \mathbf{Q}_n consists of sub-diagonal matrices \mathbf{Q}_{cw_i} for codeword i as

$$\mathbf{Q}_n = \begin{bmatrix} \mathbf{Q}_{\text{cw}_0} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{Q}_{\text{cw}_{n_c-1}} \end{bmatrix} \quad (13)$$

where n_c is the number of codewords. As reasoning from Eq. (9), the diagonal element of the covariance matrix \mathbf{Q}_n is derived as

$$\mathbf{Q}_n(k, k) = \begin{cases} 1 & \text{if } k \in \text{detecting codeword} \\ E[|s_k|^2|\mathbf{L}_{k,1:M}^{(\cdot)}] - |\bar{s}_k^{(\cdot)}|^2 & \text{otherwise} \end{cases} \quad (14)$$

where, the second order soft estimate given LLRs can be similarly derived by exchanging s into \bar{s} in Eq. (11). For undetected codewords, the LLRs remain initialized so as to be zero. As *a posteriori* LLR being updated from the channel decoder, the sub-covariance matrices approach to zero. This implies that a series of SIC iterations could lead to an interference-free channel. Suppose that the n th layer of the channel matrix \mathbf{H} is a part of codeword i . The MMSE estimated symbol \tilde{s}_n of s_n is obtained by filtering \mathbf{y}_{cw_i} with the MMSE filtering vector \mathbf{w}_n as

$$\begin{aligned} \tilde{s}_n &= \mathbf{w}_n^\dagger \mathbf{y}_{\text{cw}_i} \\ &= \mathbf{w}_n^\dagger \mathbf{h}_n s_n + \sum_{\substack{i \in \text{cw}_i \\ i \neq n}} \mathbf{w}_n^\dagger \mathbf{h}_i s_i + \sum_{j=1}^{n_c} \sum_{\substack{k \in \text{cw}_j \\ j \neq i}} \mathbf{w}_n^\dagger \mathbf{h}_k (s_k - \bar{s}_k^{(A,2)}) + \mathbf{w}_n^\dagger \mathbf{n} \\ &= \mu_n s_n + \eta_n \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mu_n &= \mathbf{w}_n^\dagger \mathbf{h}_n \\ \eta_n &= \mathbf{w}_n^\dagger \left(\sum_{\substack{i \in \text{cw}_i \\ i \neq n}} \mathbf{h}_i s_i + \sum_{j=1}^{n_c} \sum_{\substack{k \in \text{cw}_j \\ j \neq i}} \mathbf{H}_{\text{cw}_j} (\mathbf{s}_{\text{cw}_j} - \bar{\mathbf{s}}_{\text{cw}_j}^{(A,2)}) + \mathbf{n} \right). \end{aligned} \quad (16)$$

Consequently, Eq. (15) becomes a SISO channel where the residual interference is combined with the noise. In [14], it is revealed that the residual interference plus noise can be well approximated to a Gaussian random variable with mean zero and variance $\tilde{\sigma}_n^2$ that is given by

$$\begin{aligned} \tilde{\sigma}_n^2 &= E[|\eta_n|^2] \\ &= \mathbf{w}_n^\dagger (\mathbf{H}\mathbf{Q}_n\mathbf{H}^\dagger + \sigma^2\mathbf{I}_{n_r} - \mathbf{h}_n\mathbf{h}_n^\dagger) \mathbf{w}_n. \end{aligned} \quad (18)$$

From the definition of *a posteriori* LLR, the LLR of the m th bit in the n th estimated symbol in Eq. (15) is obtained as

$$L_{n,m}^{(A,1)} = \log \frac{P(b_{n,m} = +1|\hat{s}_n)}{P(b_{n,m} = -1|\hat{s}_n)} \quad (19)$$

$$= L_{n,m}^{(a,1)} + L_{n,m}^{(\text{ext},1)} \quad (20)$$

where both *a priori* LLR and the extrinsic LLR are respectively expressed as

$$L_{n,m}^{(a,1)} = \log \frac{P(b_{n,m} = +1)}{P(b_{n,m} = -1)} \quad (21)$$

$$\begin{aligned} L_{n,m}^{(\text{ext},1)} &= \log \frac{\sum_{\mathbf{b} \in \{b_{n,j}=+1\}_{j=1}^M} P(\hat{s}_n|\mathbf{b}) \exp\left(\frac{1}{2}\mathbf{b}_{[m]}^\dagger \mathbf{L}_{n,[m]}^{(a,1)}\right)}{\sum_{\mathbf{b} \in \{b_{n,j}=-1\}_{j=1}^M} P(\hat{s}_n|\mathbf{b}) \exp\left(\frac{1}{2}\mathbf{b}_{[m]}^\dagger \mathbf{L}_{n,[m]}^{(a,1)}\right)} \\ &\approx \max_{\theta: \{b_{n,m}=+1\}} \left(-\frac{1}{\sigma_n^2} |\hat{s}_n - \mu_n \theta|^2 + \frac{1}{2} \mathbf{b}_{[m]}^\dagger \mathbf{L}_{n,[m]}^{(a,1)} \right) \\ &\quad - \max_{\theta: \{b_{n,m}=-1\}} \left(-\frac{1}{\sigma_n^2} |\hat{s}_n - \mu_n \theta|^2 + \frac{1}{2} \mathbf{b}_{[m]}^\dagger \mathbf{L}_{n,[m]}^{(a,1)} \right) \end{aligned} \quad (22)$$

where $\mathbf{b}_{[m]}$ refers to the bit vector excluding the m th bit. Similarly, *a priori* LLR vector, $\mathbf{L}_{n,[m]}^{(a,1)}$, also denotes the set excluding the LLR for the m th bit. Using the max-log approximation to reformulating the equation to Eq. (22), the computational burden to consider all constellation points can be alleviated.

B. Proposed Rank-Reduced ML SIC

The MMSE detector transforms a single MIMO channel into multiple SISO channels. This channel modification becomes a reason that the MMSE detector loses joint information among multiple layers in a single codeword. Instead, this section proposes an alternative SIC algorithm to use a rank-reduced ML detector with pre-whitened interference. The max-log approximation is also employed for this detector to mitigate the complexity. Recently, [13] shows that the impairment caused by the max-log approximation can be minimized by using *a priori* LLR at a slicer block that adjusts the decision boundary between QAM constellation points.

The proposed scheme operates over the whitened residual interference plus noise in Eq. (9). Suppose that codeword i is detected. The whitening matrix for codeword i is given by

$$\mathbf{W}_{\text{cw}_i} = \left(\sum_{j=1, j \neq i}^{n_c} \mathbf{H}_{\text{cw}_j} \mathbf{Q}_{\text{cw}_j} \mathbf{H}_{\text{cw}_j}^\dagger + \sigma^2 \mathbf{I}_{n_r} \right)^{-\frac{1}{2}}. \quad (23)$$

Once this whitening matrix is applied to the receive signal, the pre-whitened symbol $\hat{\mathbf{s}}_{\text{cw}_i}$ of \mathbf{s}_{cw_i} is derived from a vector equation as

$$\begin{aligned} \hat{\mathbf{s}}_{\text{cw}_i} &= \mathbf{W}_{\text{cw}_i}^\dagger \mathbf{y}_{\text{cw}_i} \\ &= \mathbf{W}_{\text{cw}_i}^\dagger \mathbf{H}_{\text{cw}_i} \mathbf{s}_{\text{cw}_i} + \sum_{j=1, j \neq i}^{n_c} \mathbf{W}_{\text{cw}_i}^\dagger \mathbf{H}_{\text{cw}_j} (\mathbf{s}_{\text{cw}_j} - \bar{\mathbf{s}}_{\text{cw}_j}^{(A,2)}) + \mathbf{W}_{\text{cw}_i}^\dagger \mathbf{n}_{\text{cw}_i} \\ &= \mu_{\text{cw}_i} \mathbf{s}_{\text{cw}_i} + \eta_{\text{cw}_i} \end{aligned} \quad (24)$$

where

$$\mu_{cw_i} = \mathbf{W}_{cw_i}^\dagger \mathbf{H}_{cw_i} \quad (25)$$

$$\eta_{cw_i} = \sum_{j=1, j \neq i}^{n_c} \mathbf{W}_{cw_i}^\dagger \left(\mathbf{H}_{cw_j} (\mathbf{s}_{cw_j} - \hat{\mathbf{s}}_{cw_j}^{(A,2)}) + \mathbf{n}_{cw_i} \right). \quad (26)$$

where the residual noise vector η_{cw_i} follows a complex Gaussian distribution with zero mean and identity variance, i.e., $\mathcal{CN}(0, I_{n_r})$. Since the vector equation rank in Eq. (24) is the same as the number of transmit antennas divided by the number of codewords, the required rank for the ML detector can be decreased by the number of codewords.

IV. COMPLEXITY COMPARISON

This section evaluates the complexity of each algorithm by counting the number of ED calculation per symbol period, which is tabulated in Table. I. For the ML detector, the complexity exponentially increases in terms of the number of transmit antennas while the MMSE-based SIC detector only requires linearly increased complexity. Since the MMSE-based SIC detector works on the principle of a layer-separating algorithm, its complexity is independent of the number of codewords. Regarding the proposed SIC detector, the exponential complexity of the ML algorithm is mitigated by the order of the number of codewords. In addition, it can be reduced more by one layer order when using a slicer block from [13]. For example, suppose to take typical parameters for LTE standards that the number of transmit antennas is 4 and the modulation is 16 QAM with 2 codewords. In this scenario, the full-dimensional ML and the MMSE-based SIC detector require $16^4 = 65536$ and $4 \times 16 = 64$ ED calculations, respectively. On the other hand, the proposed SIC detector needs $4 \times 16 = 64$ that is the same as the MMSE-based SIC detector. The complexity can vary according to simulation scenarios. However, this section shows that the proposed SIC detector is away from impractical burden of computational complexity.

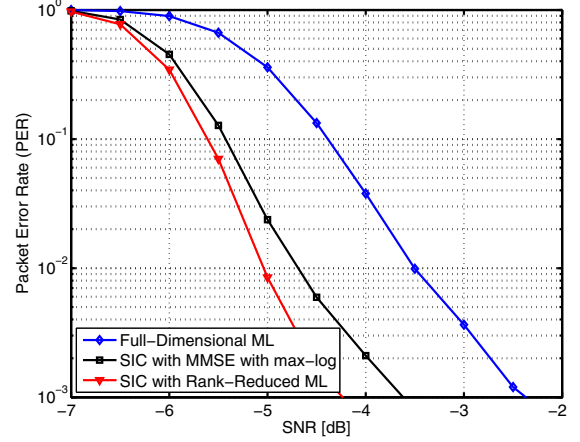
V. SIMULATION RESULTS

This section presents the results of computer simulation, using MATLAB to assess the gain of the proposed SIC algorithm. The number of transmit and receive antennas is assumed to be 4, respectively. The channel distribution follows a Rayleigh fading channel model. The modulation orders used in this section are 4 and 16 QAMs with Gray mapping. A single packet is assumed to have uncoded 400 bits. Using a turbo code with the rate $R_c = 1/3, 1/2$ or $3/4$, the coded packet with $400/R_c$ bits is sent over 10 subcarriers with 2 codewords. A random interleaver is used to permute the bit sequence from the turbo code with generator polynomials (7, 5). This section also assumes that channels are perfectly estimated at the receiver. The algorithm performance is measured with packet error rates (PERs) that are simulated with 10000 packets. For fair comparisons, a non-iterative receiver such as the ML detector was simulated with 8 inner iterations at the decoder, while iterative receivers using SIC algorithms

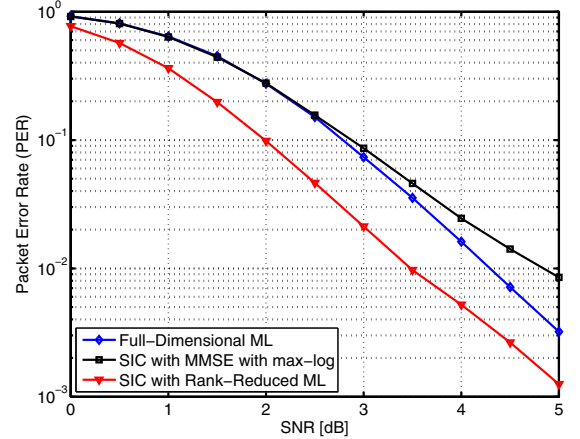
was simulated with 2 outer iterations per codeword and 4 inner iterations at the decoder.

TABLE I
NUMBER OF EUCLIDEAN DISTANCE CALCULATIONS

Detector	Complexity
Full-Dimensional ML	2^{Mn_t}
MMSE-based SIC	$n_t 2^M$
Rank-Reduced ML-SIC	$n_c 2^M \left(\frac{n_t}{n_c} - 1 \right) + 1$



(a) 4 QAM with code rate 0.33



(b) 4 QAM with code rate 0.75

Fig. 2. The PERs are compared in terms of code rates.

Fig. 2 compares the proposed SIC algorithm using rank-reduced ML with other detection algorithms: the first is full-dimensional ML detection as a representative of non-iterative algorithms and the second is MMSE-based detection for iterative algorithms. It is shown that the performance remarkably depends on coding rates. At the low code rate, much redundancy is given from the coding gain so that the detection becomes more accurate. On the other hand, the coding gain is reduced at high code rates and simultaneously the SIC gain is also mitigated compared to low code rate. In particular, Fig. 2(b) shows that the PER of MMSE-based SIC

detection is reversed against the PER of ML detection, while the proposed algorithm is not. Overall, the proposed algorithm is shown to outperform the ML detection up to 3 dB.

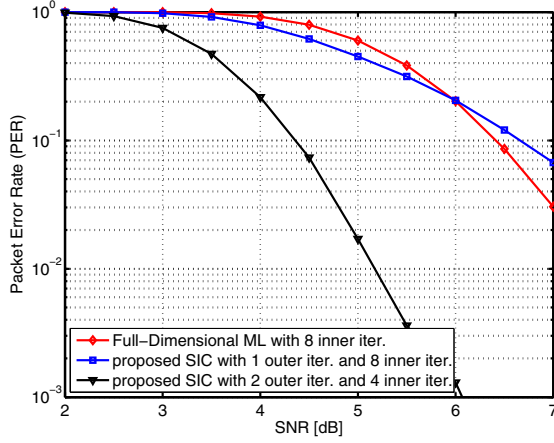


Fig. 3. The PERs are shown on the fixed total number of iteration at 16 QAM with code rate 0.50.

Fig. 3 demonstrates the effect on the number of inner and outer iterations under the fixed total number of iteration. Once data bits are encoded by turbo codes, the PER performance could be enhanced by increasing the number of iteration between two MAP decoders inside the turbo decoder. Alternatively, the PER performance could also be enhanced by increasing the number of iteration between detectors and decoders. However, the complexity of practical hardware devices is upper-limited so that both types of total iteration should be restricted accordingly. In Fig. 3, the advantage of outer iteration for SIC operation is clearly observed. When the number of inner iteration is close to a certain level, it is more beneficial to increase the number of outer iteration.

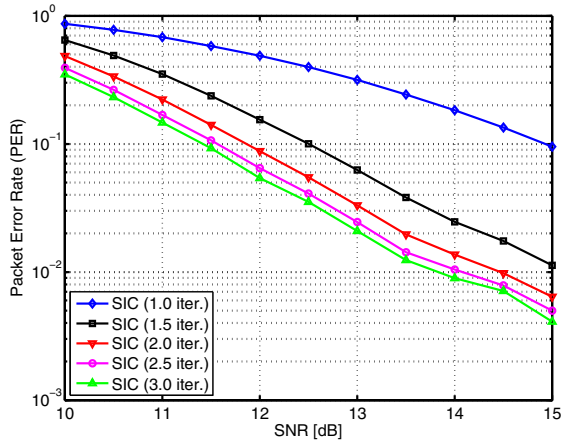


Fig. 4. The PERs of the proposed algorithm at 16 QAM with code rate 0.83 are plotted in terms of the number of iteration.

Fig. 4 presents the performance of the proposed algorithm in terms of the number of outer iteration when the number of inner iteration is fixed. Since this paper is assumed to use 2

codewords, a half iteration denotes that codeword 0 is detected one more than codeword 1. It is observed that the SIC gain is bigger at the first iteration and becomes gradually reduced as the number of iteration increases.

VI. CONCLUSION

This paper investigated the PER gain achievable at the receiver using the SIC-typed algorithm over soft feedback. This paper proposed the rank-reduced ML detector combined with pre-whitening filter to minimize the loss of joint information in a codeword. It was shown that the PER performance was improved a lot by increasing the number of SIC iteration rather than by increasing the number of iteration between two MAP decoders inside a turbo decoder. Also, it was observed that the proposed SIC algorithm outperformed both a MMSE-based SIC algorithm and a full dimensional ML algorithm at both low and high code rates. However, the demonstrated result was based on the perfect knowledge of channel estimation at the receiver. In practice, channel estimation errors could degrade the PER performance. Thus, a further study is required to compensate the PER degradation caused by the imperfect channel knowledge.

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