

A Study on Transmit Beamforming at Source Node in MISO-SISO/MIMO-MIMO AF Relays

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Abstract—A cooperative relay has been studied as a method to improve transmission quality in wireless mesh networks and cellular systems. Multiple antenna elements at a source node enable adaptive beamforming and thus improve total performance of relay transmission. In this paper, we propose simple transmit beamforming at a source node when the source node has two or more antenna elements and knows all channel information in MISO-SISO/MIMO-MIMO amplify-and-forward relays with direct link and evaluate the packet error rate. The simulation results show that the proposed method achieves both low complexity and high performance.

I. INTRODUCTION

Cooperative relaying has been under intense study in recent years as a method for range extension [1], [2]. The relay retransmits the received signal to the destination and is generally categorized into the two types: (i) amplify-and-forward (AF) and (ii) decode-and-forward (DF). The AF relay retransmits the received signal itself with power adjustment whereas the DF relay regenerates the signal for retransmission after decoding. In the paper, we focus on the AF relay which is less complex.

Also, the channel model has two major types with and without the direct link connection between the source and destination nodes. In both cases, the source node equipped with multiple antennas is expected to improve the performance by appropriate transmit beamforming.

The simplest method is selecting a appropriate antenna or set of ones from the given antennas [3], [4]. Adaptive beamforming is more complex and thus expected to yield higher gain. When there is no direct link connection between the source and destination nodes, the optimum beamforming weight is solved from the channel between the source and relay nodes only [5], [6]. However, when the direct link exists, the problem becomes extensively difficult. Thus, an approximation using three-step transmission [7] or iterative solutions [8], [9] have been proposed.

In terms of computational complexity, analytical solutions are more preferable than iterative ones. Therefore, we propose to introduce an approximation for deriving the optimum weight in the paper. The rest of the paper is organized as follows. In Section II, a general system model and some preparations for later formulation are described. In Sections III and IV, beamforming weights in MISO-SISO/MIMO-MIMO

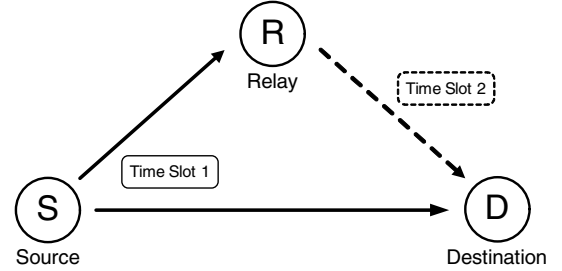


Fig. 1. A 2-step relay.

cases are discussed. The numerical results and conclusions are shown in Sections V and VI, respectively.

II. SYSTEM MODEL

In this paper we consider a general relay model with source (S), relay (R), and destination (D) nodes as shown in Fig. 1. In the first time interval, the source node transmits a signal to the destination and relay nodes. Then, in the second time interval, the relay node transmits the received signal to the destination. At the destination node, the two signals received in both time slots are mixed with a maximal-ratio combining (MRC) weight. This two-step relay causes no interference at the expense of time resource and is called a half duplex relay [10].

A. MISO-SISO Relay Transmission

First, let us consider a MISO-SISO relay as shown in Fig. 2(a). The source node has N_S antenna-elements. The relay and destination nodes have a single antenna. Let us denote the transmit signal as s , the S-D and S-R channels as row vectors $\mathbf{h}_{SD} \in \mathbb{C}^{1 \times N_S}$ and $\mathbf{h}_{SR} \in \mathbb{C}^{1 \times N_S}$, and the R-D channel response as $h_{RD} \in \mathbb{C}$. Then, the received signal at each node is expressed as

$$y_{SD} = \sqrt{P_S} \mathbf{h}_{SD} \mathbf{w}_S s + n_{SD} \quad (1)$$

$$y_{SR} = \sqrt{P_S} \mathbf{h}_{SR} \mathbf{w}_S s + n_{SR} \quad (2)$$

$$y_{RD} = h_{RD} G y_{SR} + n_{RD}, \quad (3)$$

where P_S and $\mathbf{w}_S \in \mathbb{C}^{N_S \times 1}$ are the total transmit power and the transmit weight vector (a column vector) at the source node, respectively, and n_{SD} , n_{SR} , and n_{RD} are the Gaussian

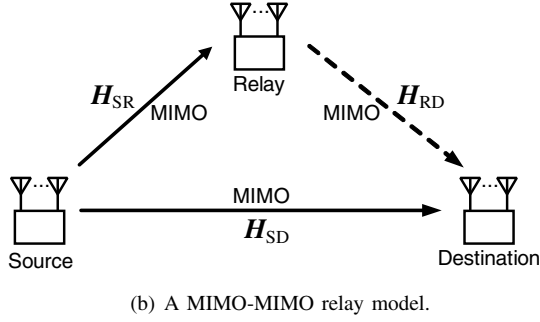
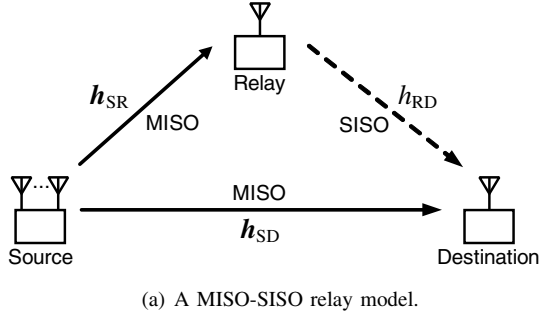


Fig. 2. Relay models.

noises at the receivers. G is a factor to maintain the transmit power of the relay node at a constant level and is given by

$$G = \sqrt{P_R / (P_S |\mathbf{h}_{SR} \mathbf{w}_S|^2 + \sigma^2)}, \quad (4)$$

where P_R is the transmit power at the relay node. For the sake of convenience, we assume all noise powers are equal to σ^2 .

The MRC signal at the destination node is then written as

$$y_D = \frac{\sqrt{P_S} (\mathbf{h}_{SD} \mathbf{w}_S)^*}{\sigma^2} y_{SD} + \frac{\sqrt{P_S} (\mathbf{h}_{SR} \mathbf{w}_S)^* h_{RD}^* G}{|h_{RD}|^2 G^2 \sigma^2 + \sigma^2} y_{RD}. \quad (5)$$

B. MIMO-MIMO Relay Transmission

Next, let us extend to the case with multiple antennas at all nodes where the numbers of transmit and receive antennas at the relay node are N_{Rtx} and N_{Rrx} , respectively, and the number of receive antennas at the destination node is N_D .

Let the channels between each node be $\mathbf{H}_{SD} \in \mathbb{C}^{N_D \times N_S}$, $\mathbf{H}_{SR} \in \mathbb{C}^{N_{Rrx} \times N_S}$, and $\mathbf{H}_{RD} \in \mathbb{C}^{N_D \times N_{Rtx}}$. Then, the received signal at each node is expressed as

$$\mathbf{y}_{SD} = \sqrt{P_S} \mathbf{H}_{SD} \mathbf{w}_S + \mathbf{n}_{SD} \quad (6)$$

$$\mathbf{y}_{SR} = \sqrt{P_S} \mathbf{H}_{SR} \mathbf{w}_S + \mathbf{n}_{SR} \quad (7)$$

$$\mathbf{y}_{RD} = \mathbf{H}_{RD} \mathbf{w}_R G \mathbf{r}_R \mathbf{y}_{SR} + \mathbf{n}_{RD}, \quad (8)$$

where

$$G = \sqrt{P_R / (P_S \|\mathbf{H}_{SR} \mathbf{w}_S\|^2 + \sigma^2)}. \quad (9)$$

$\|\cdot\|$ denotes Euclidean norm, i.e., $\|\mathbf{x}\|^2 = \mathbf{x}^H \mathbf{x}$ for an arbitrary column vector \mathbf{x} . $\mathbf{w}_R \in \mathbb{C}^{N_{Rtx} \times 1}$ and $\mathbf{r}_R \in \mathbb{C}^{N_{Rrx} \times 1}$ are the transmit and receive weight vectors, respectively. \mathbf{w}_R is given

by the eigenvector corresponding to the maximum eigenvalue of $\mathbf{H}_{RD}^H \mathbf{H}_{RD}$. The receive weight is simply expressed as a MRC weight

$$\mathbf{r}_R = (\mathbf{H}_{SR} \mathbf{w}_S)^H / \|\mathbf{H}_{SR} \mathbf{w}_S\|. \quad (10)$$

Finally, the signal at the destination nodes becomes

$$y_D = \frac{\sqrt{P_S} (\mathbf{H}_{SD} \mathbf{w}_S)^H}{\sigma^2} \mathbf{y}_{SD} + \frac{\sqrt{P_S} G \|\mathbf{H}_{SR} \mathbf{w}_S\| (\mathbf{H}_{RD} \mathbf{w}_R)^H}{G^2 \|\mathbf{H}_{RD} \mathbf{w}_R\|^2 \sigma^2 + \sigma^2} \mathbf{y}_{RD}. \quad (11)$$

III. BEAMFORMING AT SOURCE NODE IN MISO-SISO RELAY

Here, we discuss the different transmit weights at the source node in the MISO-SISO relay under an assumption that full channel information is known at the source node.

A. Omni-directional

As the reference without beamforming, we consider the omni-directional weight as

$$\mathbf{w}_S = [1, 0, \dots, 0]^T, \quad (12)$$

where only the first element is used. This weight is commonly defined in both MISO-SISO and MIMO-MIMO cases.

B. Maximizing Sum SNR at S-R and S-D Links

Next, let us consider the weight solved using channels \mathbf{h}_{SD} and \mathbf{h}_{SR} only, which are available at the source node. For the sake of convenience, we employ a criterion maximizing the sum of SNR observed at both destination and relay nodes [11]

$$\begin{aligned} & \frac{P_S |\mathbf{h}_{SD} \mathbf{w}_S|^2}{\sigma^2} + \frac{P_S |\mathbf{h}_{SR} \mathbf{w}_S|^2}{\sigma^2} \\ &= (|\mathbf{h}_{SD} \mathbf{w}_S|^2 + |\mathbf{h}_{SR} \mathbf{w}_S|^2) \frac{P_S}{\sigma^2}. \end{aligned} \quad (13)$$

Then, we can define a cost function with a Lagrange multiplier μ as

$$J_A = |\mathbf{h}_{SD} \mathbf{w}_S|^2 + |\mathbf{h}_{SR} \mathbf{w}_S|^2 - \mu (\mathbf{w}_S^H \mathbf{w}_S - 1). \quad (14)$$

$\partial J_A / \partial \mathbf{w}_S = \mathbf{0}$ yields

$$(\mathbf{h}_{SD}^H \mathbf{h}_{SD} + \mathbf{h}_{SR}^H \mathbf{h}_{SR}) \mathbf{w}_S = \mu \mathbf{w}_S. \quad (15)$$

Thus, the weight is given by the eigenvector corresponding to the maximum eigenvalue of $\mathbf{h}_{SD}^H \mathbf{h}_{SD} + \mathbf{h}_{SR}^H \mathbf{h}_{SR}$.

C. Maximizing SNR After MRC at the Destination Node

The optimum weight is defined as one that maximizes the SNR after MRC

$$\frac{P_S |\mathbf{h}_{SD} \mathbf{w}_S|^2}{\sigma^2} + \frac{P_S |\mathbf{h}_{SR} \mathbf{w}_S|^2 |h_{RD}|^2 G^2}{|h_{RD}|^2 G^2 \sigma^2 + \sigma^2}. \quad (16)$$

Substituting (4) into (16) yields

$$\left(|\mathbf{h}_{SD} \mathbf{w}_S|^2 + \frac{P_R |\mathbf{h}_{SR} \mathbf{w}_S|^2 |h_{RD}|^2}{P_R |h_{RD}|^2 + P_S |\mathbf{h}_{SR} \mathbf{w}_S|^2 + \sigma^2} \right) \frac{P_S}{\sigma^2}. \quad (17)$$

Here, let us define a cost function omitting the last coefficient in (17) as

$$J_B = |\mathbf{h}_{SD}\mathbf{w}_S|^2 + \frac{P_R|\mathbf{h}_{SR}\mathbf{w}_S|^2|h_{RD}|^2}{P_R|h_{RD}|^2 + P_S|\mathbf{h}_{SR}\mathbf{w}_S|^2 + \sigma^2}. \quad (18)$$

The optimum weight can be obtained by the solution maximizing J_B . Since the analytical solution is unavailable, the following two suboptimal methods are used.

1) *Gradient Descent*: In [8], an iterative method with gradient descent algorithm has been proposed. The partial derivative of J_B in (18) with respect to \mathbf{w}_S provides

$$\frac{\partial J_B}{\partial \mathbf{w}_S} = \mathbf{h}_{SD}^H \mathbf{h}_{SD} \mathbf{w}_S + \frac{P_R |h_{RD}|^2 \rho \mathbf{h}_{SR}^H \mathbf{h}_{SR}}{(P_S |\mathbf{h}_{SR}\mathbf{w}_S|^2 + \rho)^2} \mathbf{w}_S, \quad (19)$$

where

$$\rho = P_R |h_{RD}|^2 + \sigma^2. \quad (20)$$

The update rule is then written as

$$\mathbf{w}_S^{(i+1)} = \mathbf{w}_S^{(i)} + \eta \frac{\partial J_B(\mathbf{w}_S^{(i)})}{\partial \mathbf{w}_S}, \quad (21)$$

where i is the iteration index and η is a step size. Note that the constraint $\mathbf{w}_S^H \mathbf{w}_S = 1$ should be applied after every update. The gradient descent can produce the near optimum solution. However, the computational complexity is determined by the number of iterations which depends on the initial value, step size, and channel condition.

2) *Proposal*: Our objective is to solve the problem analytically by introducing an approximation. We rewrite the cost function J_B with the constraint $\mathbf{w}_S^H \mathbf{w}_S = 1$ and a Lagrange multiplier μ as

$$J'_B = J_B - \mu(\mathbf{w}_S^H \mathbf{w}_S - 1). \quad (22)$$

Solving $\partial J'_B / \partial \mathbf{w}_S = \mathbf{0}$ yields

$$\left(\mathbf{h}_{SD}^H \mathbf{h}_{SD} + \frac{P_R |h_{RD}|^2}{\rho + \beta} \mathbf{h}_{SR}^H \mathbf{h}_{SR} \right) \mathbf{w}_S = \mu \mathbf{w}_S, \quad (23)$$

where

$$\beta = \left(\frac{P_S |\mathbf{h}_{SR}\mathbf{w}_S|^2}{\rho} + 2 \right) P_S |\mathbf{h}_{SR}\mathbf{w}_S|^2. \quad (24)$$

If \mathbf{w}_S in β is ignorable, the solution maximizing J'_B can be easily obtained. The scalar variable $|\mathbf{h}_{SR}\mathbf{w}_S|^2$ is non-negative and upper-bounded by $\|\mathbf{h}_{SR}\|^2$, i.e., $0 \leq |\mathbf{h}_{SR}\mathbf{w}_S|^2 \leq \|\mathbf{h}_{SR}\|^2$. Therefore, we apply the following approximation:

$$|\mathbf{h}_{SR}\mathbf{w}_S|^2 \simeq \alpha \|\mathbf{h}_{SR}\|^2, \quad (25)$$

where $0 \leq \alpha \leq 1$. Then, (24) is written as a constant

$$\beta \simeq \left(\frac{P_S \alpha \|\mathbf{h}_{SR}\|^2}{\rho} + 2 \right) P_S \alpha \|\mathbf{h}_{SR}\|^2. \quad (26)$$

Consequently, \mathbf{w}_S is given by the eigenvector corresponding to the maximum eigenvalue of the eigen equation (23). Although α must be determined numerically, the impact on the performance is very low as will be shown in Appendix.

IV. BEAMFORMING AT SOURCE NODE IN MIMO-MIMO RELAY

The discussions above can be easily extended to the MIMO-MIMO relay case.

A. Maximizing Sum SNR at S-R and S-D

The weight maximizing the sum of SNR observed at both destination and relay nodes is given by the eigenvector corresponding to the maximum eigenvalue of the following eigen equation.

$$(\mathbf{H}_{SD}^H \mathbf{H}_{SD} + \mathbf{H}_{SR}^H \mathbf{H}_{SR}) \mathbf{w}_S = \mu \mathbf{w}_S. \quad (27)$$

B. Maximizing SNR after MRC

The SNR at the destination node after MRC is expressed as

$$\frac{P_S \|\mathbf{H}_{SD}\mathbf{w}_S\|^2}{\sigma^2} + \frac{P_S \|\mathbf{H}_{SR}\mathbf{w}_S\|^2 \|\mathbf{H}_{RD}\mathbf{w}_R\|^2 G^2}{\|\mathbf{H}_{RD}\mathbf{w}_R\|^2 G^2 \sigma^2 + \sigma^2}, \quad (28)$$

and thus the cost function can be defined as

$$J_C = \|\mathbf{H}_{SD}\mathbf{w}_S\|^2 + \frac{P_R \|\mathbf{H}_{SR}\mathbf{w}_S\|^2 \|\mathbf{H}_{RD}\mathbf{w}_R\|^2}{P_R \|\mathbf{H}_{RD}\mathbf{w}_R\|^2 + P_S \|\mathbf{H}_{SR}\mathbf{w}_S\|^2 + \sigma^2}. \quad (29)$$

The partial derivative of J_C with respect to \mathbf{w}_S provides

$$\frac{\partial J_C}{\partial \mathbf{w}_S} = \mathbf{H}_{SD}^H \mathbf{H}_{SD} \mathbf{w}_S + \frac{P_R \lambda_{RD\max} \rho \mathbf{H}_{SR}^H \mathbf{H}_{SR} \mathbf{w}_S}{(\rho + P_S \|\mathbf{H}_{SR}\mathbf{w}_S\|^2)^2}, \quad (30)$$

where

$$\lambda_{RD\max} = \|\mathbf{H}_{RD}\mathbf{w}_R\|^2 \quad (31)$$

$$\rho = P_R \lambda_{RD\max} + \sigma^2. \quad (32)$$

Here $\lambda_{RD\max}$ is the maximum eigenvalue of $\mathbf{H}_{RD}^H \mathbf{H}_{RD}$. When using gradient descent, the update rule (21) can be similarly applied with (30).

To extend our proposal to this MIMO-MIMO case, we may replace the cost function J_C by $J'_C = J_C - \mu(\mathbf{w}_S^H \mathbf{w}_S - 1)$. Then, $\partial J'_C / \partial \mathbf{w}_S = \mathbf{0}$ yields

$$\left(\mathbf{H}_{SD}^H \mathbf{H}_{SD} + \frac{P_R \lambda_{RD\max}}{\rho + \beta} \mathbf{H}_{SR}^H \mathbf{H}_{SR} \right) \mathbf{w}_S = \mu \mathbf{w}_S, \quad (33)$$

where

$$\beta = \left(\frac{P_S \|\mathbf{H}_{SR}\mathbf{w}_S\|^2}{\rho} + 2 \right) P_S \|\mathbf{H}_{SR}\mathbf{w}_S\|^2. \quad (34)$$

Now let us use a property $0 \leq \|\mathbf{H}_{SR}\mathbf{w}_S\|^2 \leq \lambda_{SR\max}$ where $\lambda_{SR\max}$ is the maximum eigenvalue of $\mathbf{H}_{SR}^H \mathbf{H}_{SR}$. Thus, we apply the following approximation:

$$\|\mathbf{H}_{SR}\mathbf{w}_S\|^2 \simeq \alpha \lambda_{SR\max}. \quad (35)$$

Then, (34) becomes

$$\beta \simeq \left(\frac{P_S \alpha \lambda_{SR\max}}{\rho} + 2 \right) P_S \alpha \lambda_{SR\max}. \quad (36)$$

Finally, the solution can be obtained as the eigenvector corresponding to the maximum eigenvalue of the eigen equation (33) where β is a constant.

TABLE I
SIMULATION PARAMETERS

Modulation	QPSK
Number of antenna elements at source node	2
Packet length	128 symbols
Number of packets	100,000
Channel coding	Binary convolutional code (constraint length 3, coding rate 1/2)
Channel decoder	soft Viterbi decoder
Channel statistics	block Rayleigh fading
Noise	white Gaussian noise
Transmit power at relay node	$P_R = P_S$
α	0.4 for MISO-SISO case and 0.6 for MIMO-MIMO case (see Appendix)

V. NUMERICAL ANALYSIS

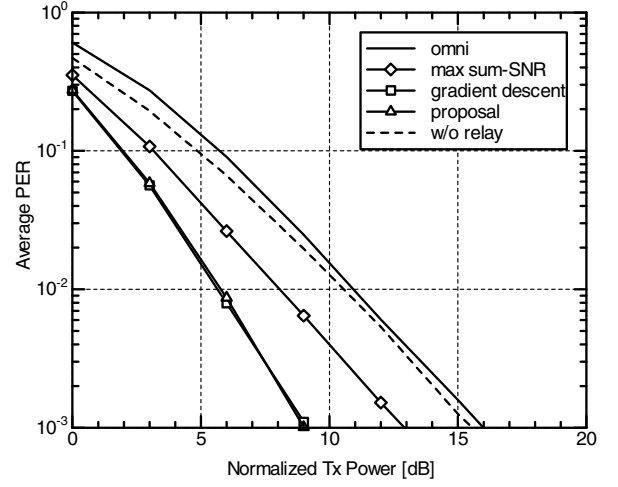
We applied computer simulations for evaluation of the above beamforming methods. The simulation parameters are shown in Table I. The gradient descent algorithm was iterated 10,000 times with step size $\eta = 0.01$ and the initial weight $\mathbf{w}_S^{(0)} = [1, 0]^T$. The number of transmit antennas at the source node was fixed to 2. For the MIMO-MIMO case, the number of antenna elements at each node was set to 2. The packet error rate (PER) performance was used as the performance measure whilst changing the SNR of the S-D link. Here, the SNR of the S-R and R-D links are defined relatively to the one of the S-D link. In the following discussions, we have evaluated two cases: (i) S-R: +3 dB, R-D: +3 dB and (ii) S-R: +10 dB, R-D: +3 dB.

A. MISO-SISO Relay

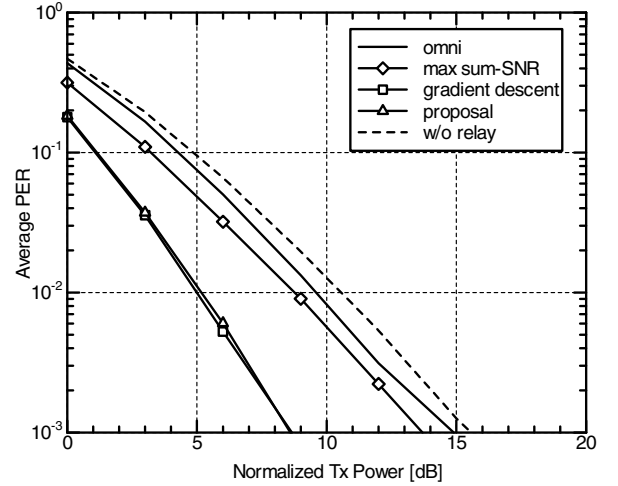
The PER performance is shown in Fig. 3. As the SNR measure at the receiver, we used the normalized transmit power, i.e., the transmit power divided by the one achieving an average E_s/N_0 of 0 dB for the S-D link in the single antenna transmission. Also, we tested the case without relay (w/o relay). In this case, the transmit weight at the source node was simply determined as transmit MRC: $\mathbf{w}_S = \mathbf{h}_{SD}^H / \|\mathbf{h}_{SD}\|$.

From Fig. 3, it is clearly shown that the performance of gradient descent outperforms the other methods. That is, the gradient descent algorithm works properly with enough iterations. Note that the performance of the proposed method is almost the same as the one of gradient descent.

Both omni and w/o relay cases achieve two-branch diversity gain. The former is from the receiver side, and the latter is from the transmitter side. Thus, the observed performances are almost the same. However, the case without relay is more effective when the S-R link quality is low. Although the method maximizing the sum SNR for S-R and S-D (max sum-SNR) also has some gain compared to these methods, the gain was inferior to the one of gradient descent and our proposed method.



(a) S-R: +3dB, R-D: +3dB



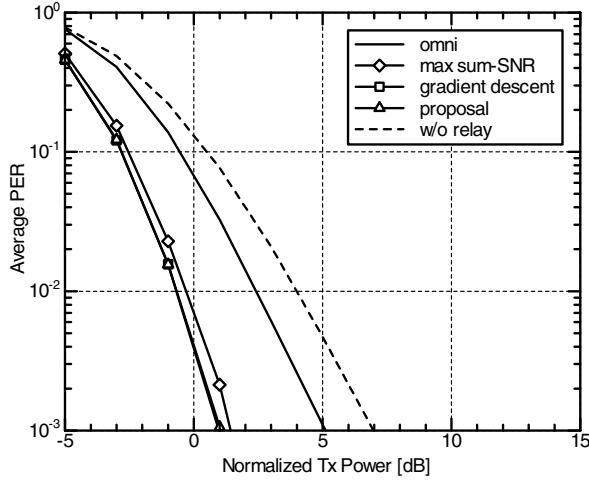
(b) S-R: +10dB, R-D: +3dB

Fig. 3. Average PER performance in a MISO-SISO relay case.

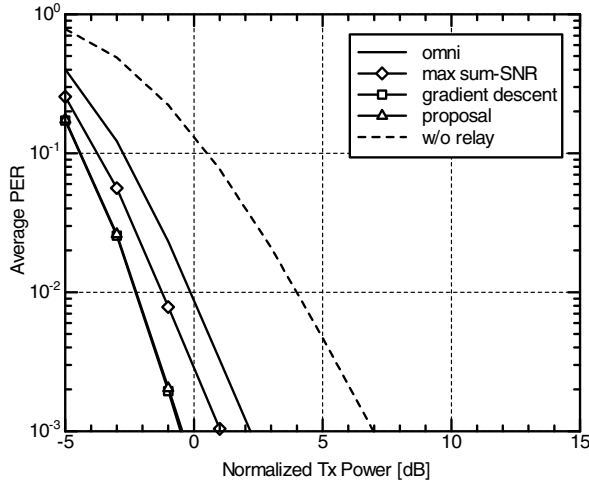
B. MIMO-MIMO Relay

Finally, the PER performance in the MIMO-MIMO relay case is shown in Fig. 4. The transmit weight at the source node for w/o relay was determined as MIMO-MIMO transmit MRC: the eigenvector corresponding to the maximum eigenvalue of $\mathbf{w}_S = \mathbf{H}_{SD}^H \mathbf{H}_{SD}$.

Fig. 4 indicates similar tendency compared to the one in the MISO-SISO case. However, an increase in the number of antenna elements at the relay node highly improves the performances with relay. In addition, the performance of max sum-SNR approaches the one of gradient descent. This means that optimization with limited channel information at the source node potentially provides large gain when a MIMO-MIMO relay is considered. When the R-D link condition is relatively bad, however, the total optimization is required to obtain the proper gain. Note that the proposal has accomplished almost the same performance as the one of the gradient descent.



(a) S-R: +3dB, R-D: +3dB



(b) S-R: +10dB, R-D: +3dB

Fig. 4. Average PER performance in a MIMO-MIMO relay case.

VI. CONCLUSION

In this paper, we discussed the optimum transmit beamforming for cooperative MISO-SISO/MIMO-MIMO relays and proposed the approximation method for a simple solution. It has been shown that our proposal provides almost the same performance as obtained using gradient descent but without any iterations. Considering the low complexity of the proposal, we can conclude that the proposed method is very effective for cooperative MISO-SISO/MIMO-MIMO relays when the source node knows full channel information.

APPENDIX

Fig. 5 shows the PER performance versus α varying with 0.2 step for both MISO-SISO and MIMO-MIMO cases in an SNR condition for S-R and R-D links. It is clearly shown that the impact of α is very small, except for $\alpha = 0$. By changing the SNR conditions, we chose the best values in the

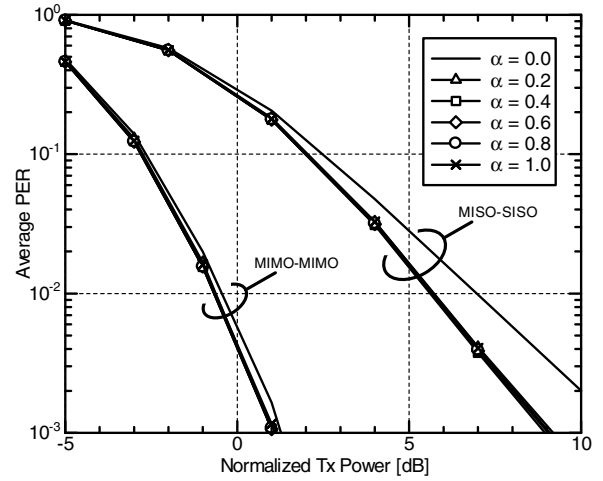


Fig. 5. An impact of α on the PER performance of proposed method where the SNR of S-R and R-D links are 3 dB higher than that of S-D link.

average sense for the MISO-SISO and MIMO-MIMO cases, respectively.

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