Performance Analysis of Spatial Modulation over Correlated Fading Channels

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***Abstract*—We present a general upper bounding framework for the average bit error probability of spatial modulation over correlated Rayleigh and Rician channels. The proposed approach provides a closed form upper bound for correlated Rayleigh fading conditions whereas for correlated Rician channels it leads to the numerical evaluation of a single integral formula. The framework is applicable to a general class of linear modulation alphabets and any number of transmit/receive antennas. Theoretical derivations are validated via simulation results.**

I. Introduction

Spatial modulation (SM), proposed in [1], is an emerging multiple-input multiple-output (MIMO) technology where the information is conveyed simultaneously over both the signal and the antenna space. In the conventional scheme, part of the incoming information bits is mapped onto a transmit antenna index while only the rest is used to select a constellation symbol within the modulation alphabet. Then only one antenna - designated by the antenna bits- is activated to send the selected symbol. This approach has a number of advantages such as the avoidance of inter-channel interference (ICI), reduction in transmitter and receiver complexity and improvement in the error performance, especially at large spectral efficiencies.

Despite its advantages, the unique nature of SM, that is, the transmission of two different forms of information simultaneously, creates a challenge in its analysis and design. For instance, in the error analysis of conventional communication systems, the envelope of channel fading effects appears in the error performance metric. The distribution of this envelope or its square can be obtained for most known fading conditions. Then using the probability density function (PDF) or more commonly the moment generating function (MGF) [2], [3] and the characteristic function (CHF) [4] approaches, the average error rate can be computed for a wide class of digital modulation methods. As pointed out in [5]–[7], the difficulty in SM arises from the fact that the error metric consists of the envelope of not a single fading variable/vector but instead a complex weighted mixture of two complex random vectors formed by the channel fading coefficients (the mixture weights are the constellation symbols). This makes it challenging to obtain a PDF or MGF for the envelope and therefore to derive a closed form expression for the average pairwise error probability (APEP). Notice that in [5], a maximum likelihood (ML) detection rule is presented along with a performance analysis using the well-known union bound method of [8]. A closed form formula is derived only in the case of real constellations and over uncorrelated Rayleigh fading channels. For correlated fading conditions (Rayleigh or Rician) and/or when the constellations are complex, the error upper bound is evaluated using the Monte Carlo simulation methods. Because of its inherent difficulty in the general MIMO setup, the SM performance analysis problem is studied with only one receive antenna in [6] for uncorrelated and in [7] for correlated Rayleigh fading channels. Among these, [6] is particularly important as the authors make a detailed analysis of the error metric and show that by employing a Gaussianity approximation it is possible to derive tight upper bounds for certain digital modulation schemes such as phase-shift keying (PSK) and multilevel quadrature amplitude modulation (QAM). However, the analytical results of [6] and [7] are not extendable to general SM schemes with multiple receive antennas and/or with other modulation schemes such as rectangular QAM. To the best of our knowledge, performance analysis of SM with an arbitrary number of transmit/receive antennas and general family of linear modulation alphabets is still an open problem even for uncorrelated Rayleigh channels.

In this paper, we propose an upper bounding technique for SM that is applicable to an arbitrary number of transmit/receive antennas and general modulation alphabets for uncorrelated/correlated Rayleigh and Rician fading channels. We extend the insight of [5] that is limited to SM with real constellations to the case of complex constellations. Specifically, we show that for a number of fading scenarios, the random variable/vector that appears in the SM error metric is a "proper complex random variable/vector" defined in [9] and can be described with a joint multivariate Gaussian PDF. In the case of Rayleigh fading channels, the framework results in a closed-form solution for the error bound whereas over Rician channels, it reduces to a simple integral that can conveniently be computed using numerical integration tools. Notice that all correlated fading scenarios considered in this paper which are commonly used in SM literature, the framework eliminate the

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need to resort to long Monte Carlo averaging methods for the ABEP upper bound computation.

The contributions of this paper can be summarized as follows:

- The general statement of previous works on SM performance analysis, [5]–[7] is that "unless the modulation symbols are not drawn from real constellations, a distribution of the error metric or its envelope is not easily obtained". We show that for a class of widely used correlated Rayleigh and Rician channel models, the SM error vector indeed has a distribution for both real and complex symbol constellations.
- Using this distribution and employing the methodology of [10] to obtain error performance expressions for diversity receivers, we present a framework to compute the upper bounding approach of SM error performance for correlated/uncorrelated Rayleigh and Rician fading channels.
- To the best of our knowledge, this work is the first to address the problem of deriving an error upper bound for SM over Rician channels (correlated or uncorrelated) and to propose one. In addition for both Rician and Rayleigh fading, contrary to [6], [7], the approach presented here is not limited to single antenna receivers but can be used with the general MIMO communication.
- The proposed approach relies only on the values of the constellation symbols to form the SM error vector and not on the constellation geometry. Notice that a similar insight also exists in [5] but only for real constellations. As a results, as opposed to the approach presented in [6] presenting approximately tight upper bounds only in the case of phase shift keying (PSK) or multi-level quadrature modulation alphabets (QAM), our method is conveniently applicable to a wide family of digital modulation alphabets such as PAM, PSK, rectangular and multilevel QAM...etc.

The rest of this paper is organized as follows. In Section II, the signal and channel models are presented. The framework is presented in Section III followed by the simulation results validating the theoretical derivations in Section IV. This is ended with concluding remarks and discussions in Section V.

II. System Model

We consider the conventional SM system model with N_t transmit and N_r receive antennas employing optimal detection in [5]. We also assume that the number of transmit antennas is an integer power of 2, i.e., $N_t = 2^n$, and the transmitter employs M-ary digital modulation to an m-bit message where $M = 2^m$ and the modulation symbol set is $\boldsymbol{\mathcal{X}} = \left\{ X_1, \ \dots \ X_k, \ \dots \ X_M \right\}$. Notice that the spectral efficiency of the system is $R = n+m = \log_2(N_t M)$. At each transmission instant, each set of $n+m$ of bits is split into groups of n and m bits, and the former is used to select one of the N_t antennas and the latter to be mapped onto one of M possible complex constellation points determined by the particular digital modulation method. The method used in the bit-to-antenna index and bit-to-symbol mappings is immaterial to the central discussion, so throughout the paper we assume uniform mapping. The transmitted signal vector is $\mathbf{x} = \left[\begin{array}{ccccc} x_1, & \dots & x_\ell, & \dots & x_{N_t} \end{array} \right]^T$ where all but one entry is zero because only one antenna is active for transmission. Notice that if the ℓ-th antenna is selected, then all entries other than x_ℓ is zero and $x_\ell \in \boldsymbol{\mathcal{X}}$. In other words, the position of the non-zero element denotes the antenna index and its value indicates the transmitted symbol. Similar to [5], we assume a power constraint of unity, i.e., $E_{\mathbf{x}}\left[\mathbf{x}^\dagger \mathbf{x} = 1\right]$. The received signal model is expressed as

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \boldsymbol{\nu} \tag{1}$$

where $\rho = E_s/N_0$ is the average signal to noise ratio (SNR) observed at each receiver branch, $\mathbf{y}$ and $\boldsymbol{\nu}$ are the $N_r \times 1$ received signal and channel noise vectors, respectively, and $\mathbf{H}$ is the $N_r \times N_t$ dimensional channel matrix. The elements of $\boldsymbol{\nu}$ are modelled as independent identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance, i.e., $\nu_k \sim \mathcal{CN}(0,1)$ for $k = 1, \dots, N_r$.

We assume a slow fading MIMO channel with the sum of an average (or fixed, possibly line-of-sight - LOS) component and a variable (or random) component. Accordingly, $N_r \times N_t$ dimensional channel matrix $\mathbf{H}$ is described as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\tilde{\mathbf{H}} \tag{2}$$

where $\bar{\mathbf{H}}$ and $\tilde{\mathbf{H}}$ are the fixed and variable components, respectively. The square-root terms are the normalization weights with K being the Rician factor. The fixed part $\bar{\mathbf{H}}$ is such that $[\bar{\mathbf{H}}]_{p,q} = \bar{h}_{p,q}$, for $p = 1, \dots, N_r$ and $q = 1, \dots, N_t$. The variable component of the channel matrix, $\tilde{\mathbf{H}}$, consists of -possibly correlated- complex Gaussian variables. Given $[\tilde{\mathbf{H}}]_{p,q} = \tilde{h}_{p,q}$, for all channel coefficient pairs $(\tilde{h}_{p,q}, \tilde{h}_{\hat{p},\hat{q}})$ $(p, \hat{p} = 1, \dots, N_r$ and $q, \hat{q} = 1, \dots, N_t)$ we assume that

$$\begin{aligned} \mathrm{E}\left[\tilde{h}^R_{p,q}\tilde{h}^R_{\hat{p},\hat{q}}\right] &= \mathrm{E}\left[\tilde{h}^I_{p,q}\tilde{h}^I_{\hat{p},\hat{q}}\right], && (3) \\ \mathrm{E}\left[\tilde{h}^R_{p,q}\tilde{h}^I_{\hat{p},\hat{q}}\right] &= \mathrm{E}\left[\tilde{h}^I_{p,q}\tilde{h}^R_{\hat{p},\hat{q}}\right] = 0 && (4) \end{aligned}$$

that is to say, the auto-correlations of the real and imaginary parts are the same and there is no correlation between real and imaginary parts. With this condition, the correlated channel matrix can be described by the well-known Kronecker correlation model in which $\tilde{\mathbf{H}}$ is expressed as

$$\tilde{\mathbf{H}} = \mathbf{\Sigma}_r^{\frac{1}{2}}\breve{\mathbf{H}}\mathbf{\Sigma}_t^{\frac{1}{2}T} \tag{5}$$

where $\boldsymbol{\Sigma}_t$ and $\boldsymbol{\Sigma}_r$ are the real valued and Hermitian symmetric transmit and receive correlation matrices, respectively, with the elements defined as $[\boldsymbol{\Sigma}_t]_{q,\hat{q}} = \sigma^t_{q,\hat{q}}$ for $q, \hat{q} = 1, \dots, N_t$ and $[\boldsymbol{\Sigma}_r]_{p,\hat{p}} = \sigma^r_{p,\hat{p}}$ for $p, \hat{p} = 1, \dots, N_r$. $\breve{\mathbf{H}}$ is the independent Rayleigh fading channel matrix whose elements of $\breve{\mathbf{H}}$ are described as independent identically distributed complex Gaussian random variables, i.e., $[\breve{\mathbf{H}}]_{p,q} = \breve{h}_{p,q} \sim \mathcal{CN}(0,1)$ for $p = 1, \dots, N_r$ and $q = 1, \dots, N_t$. Notice that the Kronecker correlation model separating the transmit and receive correlation effects has been used extensively within the context of

SM literature (e.g. [7], [11], [12]). The correlation matrices can be formed according to a number of models as detailed in the discussion of the simulation parameters and results.

Combining (2) and (5), the general channel model can be rewritten as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\mathbf{\Sigma}_r^{\frac{1}{2}}\breve{\mathbf{H}}\mathbf{\Sigma}_t^{\frac{1}{2}T} \tag{6}$$

which, via the appropriate selection of parameters, makes it possible to characterize uncorrelated/correlated Rayleigh and Rician fading scenarios.

III. Average Error Probability Bound

The optimum antenna and symbol index pair $(u_{\text{ML}}, v_{\text{ML}})$ in the maximum likelihood (ML) sense is expressed as

$$(u_{\text{ML}}, v_{\text{ML}}) = \arg\max_{u,v} f_{\mathbf{Y}}(\mathbf{y} \mid \mathbf{x}, \mathbf{H}) = \arg\min_{u,v} D(\mathbf{y}, \mathbf{h}_u X_v)$$

where $\mathbf{h}_u$ is the u-th column of $\mathbf{H}$ for $u = 1, \ldots, N_t$ and X_v is the v-th element of the modulation alphabet $\boldsymbol{\mathcal{X}}$ for $v = 1, \ldots, M$. $D(\mathbf{y}, \mathbf{h}_u X_v)$ is the -modified- distance metric between $\mathbf{y}$ and $\mathbf{h}_u X_v$ defined as

$$D(\mathbf{y}, \mathbf{h}_u X_v) = \sqrt{\rho} \parallel \mathbf{h}_u X_v \parallel^2 - 2\text{Re}\{\mathbf{y}^\dagger \mathbf{h}_u X_v\}. \tag{7}$$

Then, the average bit error probability (ABEP) with optimum ML detection can be computed using the well-known upper bounding technique in [8] such that:

$$\bar{P}_b \leq \frac{1}{N_t M - 1} \sum_{u=1}^{N_t} \sum_{\hat{u}=1}^{N_t} \sum_{v=1}^{M} \sum_{\hat{v}=1}^{M} \frac{N(u, \hat{u}, v, \hat{v})}{\log_2(N_t M)} \bar{P}_s(u, \hat{u}, v, \hat{v}). \tag{8}$$

where, $N(u, \hat{u}, v, \hat{v})$ is the number of bits in error between the respective channel and symbol pairs, $(\mathbf{h}_u, X_v)$ and $(\mathbf{h}_{\hat{u}}, X_{\hat{v}})$. The term $\log_2(N_t M)$ in expression in (8) represents the total number of antenna and symbol bits and division with this term indicates the summation weight for the corresponding pairwise error probability (PEP). $\bar{P}_s(u, \hat{u}, v, v)$ is the average pairwise symbol error probability (APEP) such that

$$\bar{P}_s(u, \hat{u}, v, \hat{v}) = E_{\mathbf{H}} \left[Q\left(\sqrt{\parallel \mathbf{z} \parallel^2} \right) \right] \tag{9}$$

where the $N_r \times 1$ vector $\mathbf{z}$ is defined as [5]:

$$\mathbf{z} = \sqrt{\frac{\rho}{2}} \left(\mathbf{h}_u X_v - \mathbf{h}_{\hat{u}} X_{\hat{v}} \right) = \bar{\mathbf{z}} + \tilde{\mathbf{z}}. \tag{10}$$

Notice that in [5], it is shown that so long as the symbol constellation $\boldsymbol{\mathcal{X}}$ is real, the real and imaginary parts of the expression in (10) are statistically independent of each other in the case of uncorrelated Rayleigh fading channels ($K = 0$, $\tilde{\mathbf{H}} = \breve{\mathbf{H}}$). Then for any symbol pair $(X_v, X_{\hat{v}})$, the term $\parallel \mathbf{z} \parallel^2$ becomes a Chi-squared random variable with $2N_r$ degrees of freedom and therefore the average of the Q-function in (8) can be computed in closed form. On the other hand, as also indicated in [5], when $(X_v, X_{\hat{v}})$ come from complex constellations the distribution of the envelope of random variable $\tilde{z}_k$ (or z_k in general) cannot be easily obtained, due to statistical dependency of the real and imaginary parts of $\tilde{z}_k$. In this case, the Monte Carlo averaging methods are adopted in both [5] and the subsequent works [6], [7] for both SM error analysis.

Notice that so long as the conditions in (3) and (4) are valid in the correlated channel model in (6) and given its definition in (10), $\mathbf{z}$ satisfies the following properties:

1) Both its real and imaginary parts are Gaussian multivariate vectors as they are real weighted sum of Gaussian vectors.
2) The mean of $\mathbf{z}$ is

$$\mathbf{m}_{\mathbf{z}} = \text{E}[\mathbf{z}] = \sqrt{\frac{\rho K}{2(K+1)}} \left(\bar{\mathbf{h}}_u X_v - \bar{\mathbf{h}}_{\hat{u}} X_{\hat{v}} \right). \tag{11}$$

3) The covariance matrix of $\mathbf{z}$ is expressed as

$$\begin{aligned} \boldsymbol{\Sigma}_{\mathbf{z}} &= \text{E}\left[(\mathbf{z} - \mathbf{m}_{\mathbf{z}})(\mathbf{z} - \mathbf{m}_{\mathbf{z}})^\dagger \right] \qquad (12) \\ &= \frac{\rho}{2(K+1)} \left[\mid X_v \mid^2 + \mid X_{\hat{v}} \mid^2 - 2\text{Re}\{\sigma^t_{u,\hat{u}} X_v X^*_{\hat{v}}\} \right] \boldsymbol{\Sigma}_r. \end{aligned}$$

4) The pseudo-covariance matrix of $\mathbf{z}$ satisfies:

$$\breve{\boldsymbol{\Sigma}}_{\mathbf{z}} = \text{E}\left[(\mathbf{z} - \mathbf{m}_{\mathbf{z}})(\mathbf{z} - \mathbf{m}_{\mathbf{z}})^T \right] = \mathbf{0}.$$

The properties (1) - (4) together imply that the vector $\mathbf{z}$ is a proper complex Gaussian vector as defined in [9] with the conditional joint PDF given as

$$f_{\mathbf{Z}}(\mathbf{z} \mid \mathbf{h}_u, \mathbf{h}_{\hat{u}}, X_v, X_{\hat{v}}) = \frac{e^{-(\mathbf{z}-\mathbf{m}_{\mathbf{z}})^\dagger \boldsymbol{\Sigma}_{\mathbf{z}}^{-1} (\mathbf{z}-\mathbf{m}_{\mathbf{z}})}}{\pi^{N_r} \det(\boldsymbol{\Sigma}_{\mathbf{z}})} \tag{13}$$

where det$(\cdot)$ represents the matrix determinant operation and $\mathbf{m}_{\mathbf{Z}}$ and $\boldsymbol{\Sigma}_{\mathbf{Z}}$ are as given in (11) and (12), respectively. Notice that this is the main difference from the argument of [5]. Proper complex random variables/vectors do not require the real and imaginary components to be statistically independent. So long as the pseudo-covariance matrix vanishes, a proper complex random variable (vector) has a distribution.

Next we use the distribution of $\mathbf{z}$ in (13) and employing the methodology of [10] we derive the conditional APEP expression in (9) as

$$\begin{aligned} \bar{P}_s(u, \hat{u}, v, \hat{v}) &= \int_{\mathbf{z}} Q\left(\sqrt{\parallel \mathbf{z} \parallel^2} \right) f_{\mathbf{Z}}(\mathbf{z} \mid \mathbf{h}_u, \mathbf{h}_{\hat{u}}, X_v, X_{\hat{v}}) d\mathbf{z} \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{\exp\left(-\tilde{\mathbf{m}}_{\mathbf{z}}^\dagger [\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}} + \sin^2\theta \mathbf{I}]^{-1} \tilde{\mathbf{m}}_{\mathbf{z}} \right)}{\det\left(\frac{\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}}}{\sin^2\theta} + \mathbf{I} \right)} d\theta \qquad (14) \end{aligned}$$

where $\tilde{\mathbf{m}}_{\mathbf{z}} = \frac{\mathbf{m}_{\mathbf{z}}}{\sqrt{2}}$ and $\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}} = \frac{\boldsymbol{\Sigma}_{\mathbf{z}}}{2}$. Even though this integral does not simplify any further to yield closed form expressions for cases where $\mathbf{m}_{\mathbf{z}} \neq \mathbf{0}$, it can be conveniently evaluated numerically for any given mean vector and covariance matrix. This makes it possible to compute (14) and thus (8) for any constellation and MIMO configuration without resorting to Monte Carlo averaging methods, providing a significant simplification to the SM error analysis.

Notice that in the case of Rayleigh fading further simplification is possible. Notice that when the LOS component of the channel is absent, i.e., when $K = 0$, $\mathbf{m}_{\mathbf{z}} = \mathbf{0}$ and

$$\boldsymbol{\Sigma}_{\mathbf{z}} = \frac{\rho}{2} \left[\mid X_v \mid^2 + \mid X_{\hat{v}} \mid^2 - 2\text{Re}\{\sigma^t_{u,\hat{v}} X_v X^*_{\hat{v}}\} \right] \boldsymbol{\Sigma}_r.$$

As a result the the conditional APEP in (14) reduces to

$$\bar{P}_s^{\text{Rayleigh}}(u,\hat{u},v,\hat{v}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left[\det\left(\frac{\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}}}{\sin^2\theta}+\mathbf{I}\right)\right]^{-1} d\theta. \tag{15}$$

Suppose λ_k for $k=1,\ldots,N_r$ denote the eigenvalues of the matrix $\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}}$. In the fully correlated Rayleigh fading case ($\mathbf{m_z}=\mathbf{0}$, rank $\left[\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}}\right]=N_r$), all the eigenvalues of $\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}}$ are distinct. Then

$$\begin{aligned}\left[\det\left(\frac{\tilde{\boldsymbol{\Sigma}}_{\mathbf{z}}}{\sin^2\theta}+\mathbf{I}\right)\right]^{-1} &= \prod_{k=1}^{N_r}\left(\frac{\lambda_k}{\sin^2\theta}+1\right)^{-1} d\theta \\ &= \sum_{k=1}^{N_r}\xi_k\left(\frac{\lambda_k}{\sin^2\theta}+1\right)^{-1} d\theta\end{aligned}$$

where $\xi_k = \prod_{j\neq k}\left(1-\frac{\lambda_j}{\lambda_k}\right)^{-1}$ is the k-th residual in the partial fraction expression for $k=1,\ldots,N_r$. With this result and changing the order of integration and summation operations, (14) can be rewritten as

$$\begin{aligned}\bar{P}_s^{\text{Rayleigh}}(u,\hat{u},v,\hat{v}) &= \sum_{k=1}^{N_r}\xi_k\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\lambda_k}{\sin^2\theta}+1\right)^{-1} d\theta \\ &= \sum_{k=1}^{N_r}\xi_k\left(1-\sqrt{\frac{\lambda_k}{1+\lambda_k}}\right). \end{aligned} \tag{16}$$

Notice that the second line of (16) is due to ([3], Appendix B) stating that

$$\begin{aligned} I_n(c) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta+c}\right)^n d\theta \\ &= \left[\frac{c}{1+c}\right]^{\frac{n}{2}}\sum_{k=0}^{n-1}\binom{n-1+k}{k}\left[1-\sqrt{\frac{c}{1+c}}\right]^k. \end{aligned}$$

If the correlation matrix has repeated eigenvalues, the partial fraction expansion in (16) has terms in the form of $\left(\frac{\lambda_k}{\sin^2\theta}+1\right)^{-t}$ where $t>1$ for which the expansion coefficients can be found and ABEP can still be computed. The extreme case is when the receiver correlation does not exist, which corresponds to $t=N_r$. In this case, the APEP can be expressed directly as

$$\begin{aligned}&\bar{P}_s^{\text{Rayleigh}}(u,\hat{u},v,\hat{v}) \\ &= \left[\frac{\bar{\rho}}{1+\bar{\rho}}\right]^{\frac{N_r}{2}}\sum_{k=0}^{N_r-1}\binom{N_r-1+k}{k}\left[1-\sqrt{\frac{\bar{\rho}}{1+\bar{\rho}}}\right]^k\end{aligned}$$

where the value of $\bar{\rho}$ depends on the values of X_v and $X_{\hat{v}}$ and the presence of transmit correlation such that

$$\bar{\rho} = \frac{\rho}{4}\left(\mid X_v\mid^2+\mid X_{\hat{v}}\mid^2 - 2\mathrm{Re}\left[\sigma_{u,\hat{u}}^t X_v X_{\hat{v}}^*\right]\right). \tag{17}$$

In summary, given any two pairs of channel and symbol indices, the APEP, $\bar{P}_s(u,\hat{u},v,\hat{v})$, can be conveniently computed in closed form. Then, as in the Rician case, the computation of the ABEP upper bound in (8) becomes straightforward.

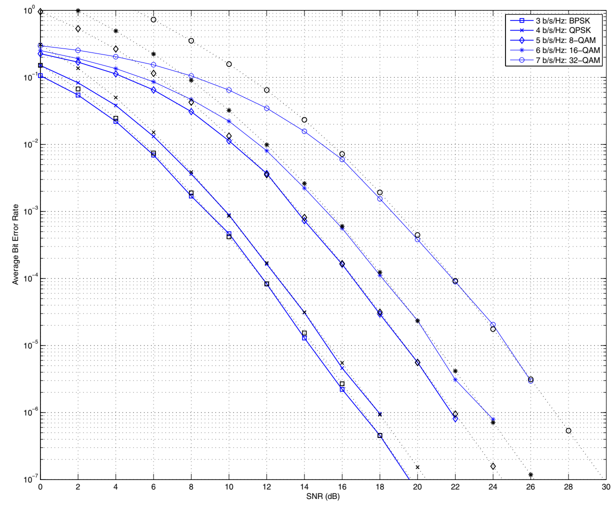

Figure 1: SM error performance with $N_t=4$ and $N_r=4$ for $R=3$–7 b/s/Hz. Uncorrelated Rayleigh fading.

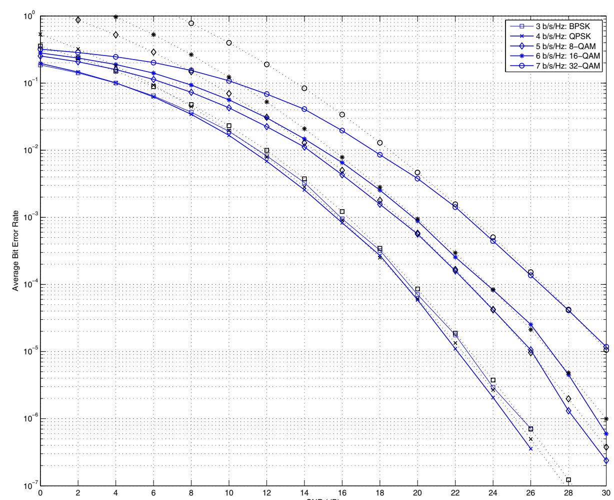

Figure 2: SM error performance with $N_t=4$ and $N_r=4$ for $R=3$–7 b/s/Hz. Correlated Rayleigh fading. Exponential correlation model of [13] with parameters $\gamma_t=\gamma_r=0.8$.

IV. Simulation Results

In this section, simulation results are presented to validate the proposed error performance calculation method for the SM systems under various channel fading conditions. For all simulations, a MIMO system with 4 transmit and 4 receive antennas is considered. We simulate transmission schemes with spectral efficiencies of $R=3$ b/s/Hz to $R=7$ b/s/Hz. In a $N_t=4$ system this corresponds to employing modulation alphabets of 2 up to 32 symbols. For small R, BSPK and QPSK constellations are used while for large R values they are replaced by rectangular QAM constellations with 8, 16

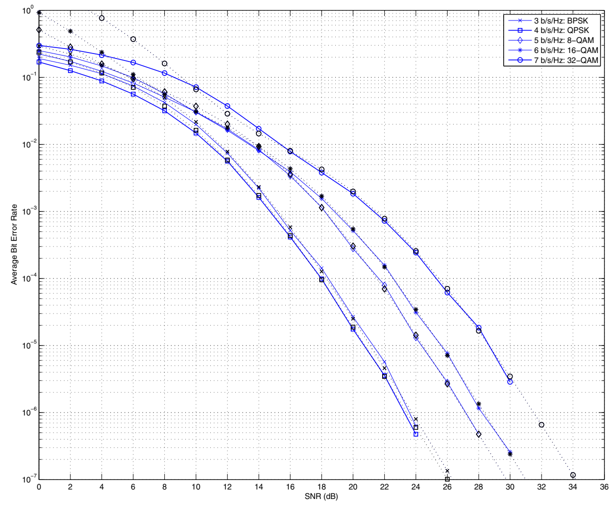

Figure 3: SM error performance with $N_t = 4$ and $N_r = 4$ for $R = 3$–7 b/s/Hz. Uncorrelated Rician fading with K=5.

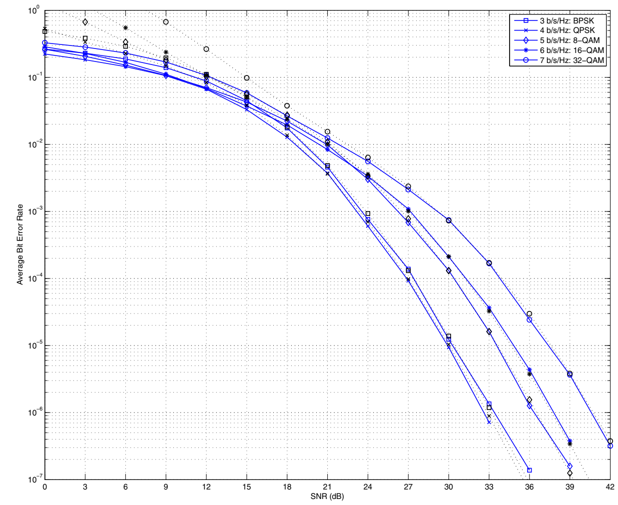

Figure 4: SM error performance with $N_t = 4$ and $N_r = 4$ for $R = 3$–7 b/s/Hz. Correlated Rician fading, K=5. Exponential correlation model of [13] with parameters $\gamma_t = \gamma_r = 0.8$.

and 32 points. Notice that rectangular QAM constellations are chosen in order to show that the approach presented in this paper is not dependent on a particular constellation geometry.

Transmit and receive correlation matrices in the Kronecker model are formed according to the exponential model of [13]. Here, the correlation matrix entries are formed as $[\boldsymbol{\Sigma}]_{p,q} = \sigma_{p,q} = \gamma^{|p-q|}$ where γ is a fixed correlation coefficient between adjacent antennas. In simulations, we consider $\gamma_t = \gamma_r = 0.8$. Similar to other SM works such as [11], the fixed channel matrix $\bar{\mathbf{H}}$ is modeled as an all one matrix in the simulations but any other fixed matrix choice does not change the algorithm. The Rician factor is chosen as $K = 5$.

Theoretical error upper bounds are compared with the ABEP's obtained through simulations in Fig. 1 and in Fig. 2 for uncorrelated and correlated Rayleigh fading channels, respectively. Similarly Fig. 3 and Fig. 4 depicts the theoretical and simulated results for uncorrelated and correlated Rician fading channel conditions, respectively. In all of the results presented, the blue curves corresponding to simulations are plotted with solid lines whereas those from analytical derivations are represented with black dotted lines with identical markers. For reasons of clarity the marker of only one curve (simulation) is included in the legends. All curves are drawn down to at least 1×10^{-6} bit error rate (BER) levels and simulations indicate the upper bounds obtained with analytical framework are in close accordance with the simulated performances under all channel conditions considered.

V. CONCLUSION

The main contribution of this paper is to show that the error analysis of SM relies on a proper complex Gaussian vector and to derive its distribution. Then this distribution is used to compute the ABEP upper bound for SM. This is made possible for correlated Rician and Rayleigh channel models frequently used in literature with any number of transmit/receive antennas and for a wide class of modulation schemes.

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