Joint Relay and Receive Beamforming in Cognitive Relay Networks with Hybrid Relay Strategy

Tao Yi, Li Guo, and Jiaru Lin
Key Laboratory of Universal Wireless Communication
Beijing University of Posts and Telecommunications
Beijing 100876, R.P. China
Email: {kzdg2005, guoli, jrlin}@bupt.edu.cn

***Abstract*—This paper investigates the joint design of relay and receive beamforming vectors in cognitive relay networks with the secondary network (SN) using the same frequency band allocated to the primary network (PN). To guarantee the QoS of the primary user (PU), the interference from source and relays in the SN to PU must be lower than what the PU can tolerate. A hybrid relay strategy is adopted by relays that can use amplify-and-forward (AF) or decode-and-forward (DF) strategies to retransmit signal according to signal to interference pulse noise ratio (SINR). The capacity of the source-destination in the SN is maximized with the transmit power and the interference at PU constraints. The simulation results show that the maximum relay transmit power will affect the gain of both the joint beamforming and the hybrid relay strategy, while the maximum interference power at PU will only affect that of the hybrid relay strategy.**

Key words: Joint design, cognitive relay networks, hybrid relay strategy

I. Introduction

Due to the rapid development of wireless communications, the radio spectrum—it is believed—is becoming crowded. But the measurements by FCC show that a large amount of the allocated spectrum is not being utilized [1]. Cognitive radio, which would allow secondary users/networks to share the allocated spectrum with primary users/networks under some conditions, has been proposed as a way to improve spectrum utilization. Sometimes the PN or the SN is too large to ensure reliable communication for every link. An efficient way to address this issue is to transmit signals, with the help of one or more relays, which can compensate for the effects of signal fading and shadowing [2]–[4].

Many studies have examined the cooperative beamforming in the relay network. In [5], the relay weights were designed to maximize the received signal-to-noise (SNR) at the destination terminal with power constraint at the relays. In [6], the authors considered the problem of joint beamforming and power control in a cooperative and cognitive radio system where two secondary users exchange information through a "cognitive" relay station via two-way relaying. Relay beamforming has been studied to maximize the sum-rate of the two-way relay network in [7].

However, there are few studies which look at joint relay and receive beamforming in the relay network. In [8], the source covariance matrix and the relay amplifying matrix were jointly designed to maximize the source-destination capacity with AF relays. The problem of joint design of transmit and cooperative beamforming vectors to maximize the received SNR in cooperative systems with a multi-antenna source and multiple single-antenna relays was addressed in [9].

In [8] and [9] the system models are all traditional relay networks, not the cognitive relay networks; moreover, only AF strategy is being used by the relays. In this paper, we study the joint relay and receive beamforming in the cognitive relay networks with single-antenna multiple relays and a multi-antenna receiver. The relay can adopt AF or DF according to the SINR in the first slot. An iterative algorithm which fixes one beamforming vector to get the other is proposed in order for the relay and receive beamforming vectors to maximize the capacity or SINR of the SN. In addition, the gains of the joint beamforming and the hybrid relay strategy are focused on by comparing the performance of the SN in different cases.

Throughout this paper, scalars, vectors and matrices are denoted by lower-case letters, boldface lower-case letters and boldface upper-case letters respectively. We define $\mathbb{E}\{a\}$ as the expectation of a. Let $|b|$, b^*, $\boldsymbol{b^T}$, $\boldsymbol{b^*}$, $\boldsymbol{b^H}$, $\|\boldsymbol{b}\|$, and $\boldsymbol{B^H}$ denote the amplitude and conjugate of complex number b, the transpose, conjugate, conjugate transpose and Euclidean norm of vector $\boldsymbol{b}$, the conjugate transpose of matrix $\boldsymbol{B}$. In particular, an identity matrix with an appropriate dimension is denoted by $\boldsymbol{I}$. We represent the trace and rank of matrix $\boldsymbol{C}$ as $trace(\boldsymbol{C})$ and $rank(\boldsymbol{C})$ respectively. Matrix $diag(\boldsymbol{d})$ is a diagonal matrix with the diagonal element given by vector $\boldsymbol{d}$. For matrices, "$\succeq$" is used to indicate the generalized inequality, $\lambda_{max}(\boldsymbol{X}, \boldsymbol{Y})$ and $\nu(\boldsymbol{X}, \boldsymbol{Y})$ denote the maximum generalized eigenvalue and primary eigenvector of matrix $\boldsymbol{X}$ and $\boldsymbol{Y}$.

The remainder of this paper is organized as follows. Section II presents the system model and the signal model of the cognitive relay network. The joint relay and receive beamforming optimal problem and the corresponding iterative algorithm are provided in III. The simulation results are provided in section IV. Section V concludes this paper.

II. System Model and Signal Model

The cognitive relay networks with a PN and a SN is shown in Fig. 1. The PN consists of a primary transmitter (PT) and a primary receiver (PD), while the SN contains a secondary transmitter (ST), N relays SR and a secondary receiver (SD).

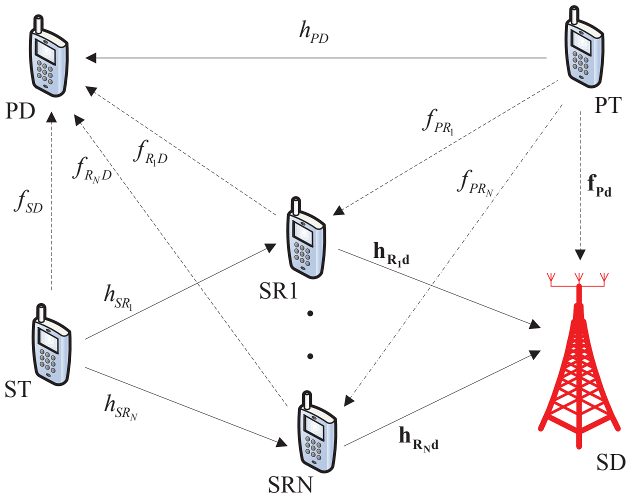

Fig. 1. The system model

SD is equipped with M antennas, while all other nodes only have one antenna. We assume the link between ST and SD is very weak and thus ignored, requiring them to communicate with the help of the relays.

The system operates in half-duplex mode, and the transmit scheme occurs in two slots. In the first slot, PT transmits m_1 to PD with power P_P where $\mathbb{E}\{|m_1|^2\} = 1$, and ST transmits x to SR where $\mathbb{E}\{|x|^2\} = 1$. Then the received signals at PD and the ith ($i = 1, 2, \cdots N$) relay can be written as

$$y_{D1} = \sqrt{P_P} h_{PD} m_1 + \sqrt{P_S} f_{SD} x + n_{D1} \quad (1)$$
$$y_{R_i} = \sqrt{P_S} h_{SR_i} x + \sqrt{P_P} f_{PR_i} m_1 + n_{R_i} \quad (2)$$

where h_{PD}, f_{SD}, h_{SR_i} and f_{PR_i} denote the channel coefficients from PT to PD, from ST to PD, from ST to the ith relay and from PT to the ith relay respectively, $n_{D1} \sim \mathcal{CN}(0, N_D)$ and $n_{R_i} \sim \mathcal{CN}(0, N_R)$ represent the complex Gaussian noise at PD and the ith relay. To simplify the expression, (2) can be rewritten in the matrix form as

$$\boldsymbol{y_R} = \sqrt{P_S} \boldsymbol{h_{SR}} x + \sqrt{P_P} \boldsymbol{f_{PR}} m_1 + \boldsymbol{n_R} \quad (3)$$

where $\boldsymbol{y_R} = [y_{R_1}, y_{R_2}, \cdots, y_{R_N}]^T_{N\times1}$, $\boldsymbol{h_{SR}} = [h_{SR_1}, h_{SR_2}, \cdots, h_{SR_N}]^T_{N\times1}$, $\boldsymbol{f_{PR}} = [f_{PR_1}, f_{PR_2}, \cdots, f_{PR_N}]^T_{N\times1}$, and $\boldsymbol{n_R} = [n_{R_1}, n_{R_2}, \cdots, n_{R_N}]^T_{N\times1}$.

In the second slot, PT transmits m_2 to PD with the same power P_P where $\mathbb{E}\{|m_2|^2\} = 1$, and the relays retransmit the signal to PD with AF or DF strategy according to the SINR in the first slot. When the SINR at the relays in the first slot is higher than the threshold denoted by γ, the DF will be used, otherwise, the AF will be adopted. The threshold has to be set reasonably to make sure the relays can decode the symbol x successfully. If so, the retransmitted signal at the ith relay can be given by

$$x_{R_i} = \begin{cases} w^*_{R_i} \beta_i y_{R_i} & \text{if } \gamma_i < \gamma, \\ w^*_{R_i} x & \text{if } \gamma_i \geq \gamma. \end{cases} \quad (4)$$

Here, $w_{R_i} \in \mathbb{C}$ is the beamforming weight, β_i and γ_i are the scaling factor and the SINR at the ith relay in the first slot can be determined as

$$\beta_i = 1/\sqrt{P_S |h_{SR_i}|^2 + P_P |f_{PR_i}|^2 + N_R} \quad (5)$$
$$\gamma_i = P_S |h_{SR_i}|^2 / (P_P |f_{PR_i}|^2 + N_R) \quad (6)$$

Accordingly, (4) can be written in a matrix form as

$$\boldsymbol{x_R} = \boldsymbol{W D_1 y_R} + \boldsymbol{W D_2 e} x \quad (7)$$

where $\boldsymbol{W} = diag(\boldsymbol{w_R})$ and $\boldsymbol{w_R} = [w^*_{R_1}, w^*_{R_2}, \cdots, w^*_{R_N}]^T_{N\times1}$, $\boldsymbol{e}$ is the unit-vector, $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$ are all diagonal matrices with the diagonal elements $(\boldsymbol{D_1})_{ii} = 0$ if $\gamma_i \geq \gamma$, otherwise, $(\boldsymbol{D_1})_{ii} = \beta_i$, and $(\boldsymbol{D_2})_{ii} = 1$ if $\gamma_i \geq \gamma$, otherwise, $(\boldsymbol{D_2})_{ii} = 0$.

The SD receives the signal with the receive beamforming vector $\boldsymbol{w_d} = [w_{d1}, w_{d2}, \cdots, w_{dM}]^T_{M\times1}$ normalized to unit power $||\boldsymbol{w_d}||^2 = 1$, then the received signals at PD and SD in the second slot can be obtained as

$$y_{D2} = \sqrt{P_P} h_{PD} m_2 + \boldsymbol{f^T_{RD} x_R} + n_{D2} \quad (8)$$
$$y_d = \boldsymbol{w^H_d} (\boldsymbol{H_{Rd} x_R} + \sqrt{P_P} \boldsymbol{f_{Pd}} m_2 + \boldsymbol{n_d}) \quad (9)$$

Here, $\boldsymbol{f_{RD}} = [f_{R_1D}, f_{R_2D}, \cdots, f_{R_ND}]^T_{N\times1}$, $\boldsymbol{H_{Rd}} = [\boldsymbol{h_{R_1d}}, \boldsymbol{h_{R_2d}}, \cdots, \boldsymbol{h_{R_Nd}}]_{M\times N}$ where $\boldsymbol{h_{R_id}} = [h_{R_id_1}, h_{R_id_2}, \cdots, h_{R_id_M}]^T_{M\times1}$, $\boldsymbol{f_{Pd}} = [f_{Pd_1}, f_{Pd_2}, \cdots, f_{Pd_M}]^T_{M\times1}$ are the channel coefficients between SR and PD, SR and SD, PT and SD respectively, $\boldsymbol{n_d}$ is the $M \times 1$ vector of noise at SD with $\mathbb{E}\{\boldsymbol{n_d n^H_d}\} = N_d \boldsymbol{I}$.

After submitting (3), (7) to (8) and (9), we can reorganize y_{D2} and y_d as

$$\begin{aligned} y_{D2} = &\underbrace{\sqrt{P_P} h_{PD} m_2}_{\text{signal}} + \underbrace{\boldsymbol{f^T_{RD} W D_1 n_R} + n_{D2}}_{\text{noise}} \\ &+ \underbrace{\boldsymbol{f^T_{RD} W a} x + \sqrt{P_P} \boldsymbol{f^T_{RD} W D_1 f_{PR}} m_1}_{\text{interference}} \end{aligned} \quad (10)$$

$$\begin{aligned} y_d = &\underbrace{\boldsymbol{w^H_d H_{Rd} W a} x}_{\text{signal}} + \underbrace{\boldsymbol{w^H_d H_{Rd} W D_1 n_R} + \boldsymbol{w^H_d n_d}}_{\text{noise}} \\ &+ \underbrace{\sqrt{P_P} \boldsymbol{w^H_d} (\boldsymbol{H_{Rd} W D_1 f_{PR}} m_1 + \boldsymbol{f_{Pd}} m_2)}_{\text{interference}} \end{aligned} \quad (11)$$

where $\boldsymbol{a} = \sqrt{P_S} \boldsymbol{D_1 h_{SR}} + \boldsymbol{D_2 e}$

Assuming that the information symbols x, m_1, m_2, the noise n_{D1}, n_{D2}, $\boldsymbol{n_R}$, $\boldsymbol{n_d}$ and the channel coefficients are all statistically independent, the SINR at SD can be derived from (11) as

$$\gamma_d = \frac{\boldsymbol{w^H_d H_{Rd} W a a^H W^H H^H_{Rd} w_d}}{\boldsymbol{w^H_d H_{Rd} W D_1 A D^H_1 W^H H^H_{Rd} w_d} + \boldsymbol{w^H_d B w_d}} \quad (12)$$

where $\boldsymbol{A} = P_P \boldsymbol{f_{PR} f^H_{PR}} + N_R \boldsymbol{I}$, $\boldsymbol{B} = P_P \boldsymbol{f_{PR} f^H_{PR}} + N_d \boldsymbol{I}$.

III. Joint Relay and Receive Beamforming

In this section, we consider joint relay and receive beamforming as a way to maximize the capacity of the SN. To do so, the relay transmit power and the interference power at PD have to be constrained. From (4) in last section, it is known that the transmit power of the ith relay is $|w_{R_i}|^2$ no matter which strategy is used; thus, the relay transmit power is $||\boldsymbol{w_R}||^2$. Using (1) and (10), the interference power at PD in the two slots is given by

$$I_1 = P_S|f_{SD}|^2 \tag{13}$$

$$I_2 = \boldsymbol{w_R^H F_{RD}}(\boldsymbol{a}\boldsymbol{a^H} + P_P \boldsymbol{D_1 f_{PR} f_{PR}^H D_1^H})\boldsymbol{F_{RD}^H w_R} \tag{14}$$

where $\boldsymbol{F_{RD}} = diag(\boldsymbol{f_{RD}})$.

Using (13), the transmit power of ST can be configured as

$$P_S = min\{P_{ST}, I_{th}/|f_{SD}|^2\} \tag{15}$$

where P_{ST} is the maximum transit power of ST and I_{th} is the maximum interference power at PD.

The capacity of the SN can be expressed as

$$C = \frac{1}{2}log_2(1+\gamma_d) \tag{16}$$

where the factor $\frac{1}{2}$ in front of log-function is due to the transmit scheme with two slots. The capacity is the monotonic function of SINR at SD denoted by γ_d; therefore, the problem can be formulated as

$$\begin{aligned} \max_{\boldsymbol{w_R},\boldsymbol{w_d}} \quad & \gamma_d \\ s.t. \quad & I_2 \le I_{th} \\ & ||\boldsymbol{w_R}||^2 \le P_{RT} \\ & ||\boldsymbol{w_d}||^2 = 1 \end{aligned} \tag{17}$$

where P_{RT} is the maximum relay transmit power.

For any fixed $\boldsymbol{w_R}$, the optimization problem (17) is a generalized eigenvalue problem from which we can get the closed form solution of $\boldsymbol{w_d}$. Then the optimization problem (17) is converted to an optimization problem that has only one optimization variable $\boldsymbol{w_R}$. Unfortunately, the converted optimization problem is still difficult for us to solve. However, it is observed that the optimization problem (17) becomes a generalized eigenvalue problem when $\boldsymbol{w_R}$ is fixed. It turns to a fraction program when $\boldsymbol{w_d}$ is fixed. Inspired by [10], an iterative algorithm which gets the receive beamforming vector $\boldsymbol{w_d}$ and the relay beamforming vector $\boldsymbol{w_R}$ iteratively is proposed to solve the optimization problem (17).

A. Fixed Relay Beamforming Vector

In this subsection, the relay beamforming vector $\boldsymbol{w_R}$ is fixed to get the receive beamforming vector $\boldsymbol{w_d}$. Given the following definitions,

$$\boldsymbol{A_1} = \boldsymbol{H_{Rd} W a a^H W^H H_{Rd}^H},$$
$$\boldsymbol{B_1} = \boldsymbol{H_{Rd} W D_1 A D_1^H W^H H_{Rd}^H} + \boldsymbol{B},$$

we can rewrite the SINR maximization problem (17) as

$$\begin{aligned} \max_{\boldsymbol{w_d}} \quad & \frac{\boldsymbol{w_d^H A_1 w_d}}{\boldsymbol{w_d^H B_1 w_d}} \\ s.t. \quad & ||\boldsymbol{w_d}||^2 = 1 \end{aligned} \tag{18}$$

The optimization problem (18) is a generalized eigenvalue problem, and the corresponding optimal solution is given by

$$\boldsymbol{w_d^\star} = \xi\nu[\boldsymbol{A_1}, \boldsymbol{B_1}] \tag{19}$$

where ξ is a parameter to make $\boldsymbol{w_d^\star}$ satisfy the power constraint $||\boldsymbol{w_d^\star}||^2 = 1$.

B. Fixed Receive Beamforming Vector

In this subsection, the receive beamforming vector $\boldsymbol{w_d}$ is fixed to get the relay beamforming vector $\boldsymbol{w_R}$.

Note that $\boldsymbol{\alpha^T} diag(\boldsymbol{\beta}) = \boldsymbol{\beta^T} diag(\boldsymbol{\alpha})$ and given the following definitions,

$$a_2 = \boldsymbol{w_d^H B w_d},$$
$$\boldsymbol{b} = \boldsymbol{H_{Rd}^H w_d},$$
$$\boldsymbol{A_2} = diag(\boldsymbol{b^*})\boldsymbol{a a^H} diag(\boldsymbol{b}),$$
$$\boldsymbol{B_2} = diag(\boldsymbol{b^*})\boldsymbol{D_1 A D_1^H} diag(\boldsymbol{b}),$$
$$\boldsymbol{C_2} = \boldsymbol{F_{RD}}(\boldsymbol{a a^H} + P_P \boldsymbol{D_1 f_{PR} f_{PR}^H D_1^H})\boldsymbol{F_{RD}^H},$$

the SINR maximization problem (17) can be rewritten as

$$\begin{aligned} \max_{\boldsymbol{w_R}} \quad & \frac{\boldsymbol{w_R^H A_2 w_R}}{\boldsymbol{w_R^H B_2 w_R} + a_2} \\ s.t. \quad & \boldsymbol{w_R^H C_2 w_R} \le I_{th} \\ & \boldsymbol{w_R^H w_R} \le P_{RT} \end{aligned} \tag{20}$$

We transform the maximum to minimum and rewrite the optimization problem (20) as

$$\begin{aligned} \min_{\boldsymbol{w_R}} \quad & -\frac{\boldsymbol{w_R^H A_2 w_R}}{\boldsymbol{w_R^H B_2 w_R} + a_2} \\ s.t. \quad & \boldsymbol{w_R^H C_2 w_R} \le I_{th} \\ & \boldsymbol{w_R^H w_R} \le P_{RT} \end{aligned} \tag{21}$$

The optimization problem (21) is a fractional problem; inspired by the method in [11] we can use the following algorithm to solve it:

Table 1 Iteration algorithm for the fraction program

1: initialization L_0, $U_0 = 0$, $k = 0$, tolerance $\varepsilon \ge 0$
2: **repeat**
3: $\quad k = k+1$, $t_k = (L_{k-1} + U_{k-1})/2$
4: $\quad$ solve QP-2 problem to get the optimal value η_k,
5: $\quad$ **if** $\eta_k \le 0$ **then**
6: $\quad\quad L_k = L_{k-1}$, $U_k = t_k$
7: $\quad$ **else**
8: $\quad\quad L_k = t_k$, $U_k = U_{k-1}$
9: $\quad$ **end if**
10: **until** $U_k - L_k \le \varepsilon$
11: **output** $w_R^* \in \operatorname*{argmin}_{\boldsymbol{w_R} \in S}\{-\boldsymbol{w_R^H}(\boldsymbol{A_2} + \boldsymbol{U_k B_2})\boldsymbol{w_R}\}$, where S is the feasible set of the optimization problem (17)

The QP-2 problem in step 4 of the algorithm is given by

$$\begin{aligned} \min_{\boldsymbol{w_R}} \quad & -\boldsymbol{w_R^H A_2 w_R} - t_k(\boldsymbol{w_R^H B_2 w_R} + a_2) \\ s.t. \quad & \boldsymbol{w_R^H C_2 w_R} \le I_{th} \\ & \boldsymbol{w_R^H w_R} \le P_{RT} \end{aligned} \tag{22}$$

Using the definition $\boldsymbol{W_R} = \boldsymbol{w_R w_R^H}$, the optimization problem (22) can be rewritten as

$$\begin{aligned} \min_{\boldsymbol{W_R}} \quad & -trace[(\boldsymbol{A_2} + t_k\boldsymbol{B_2})\boldsymbol{W_R}] - t_k a_2 \\ s.t. \quad & trace(\boldsymbol{C_2 W_R}) \le I_{th} \\ & trace(\boldsymbol{W_R}) \le P_{RT} \\ & \boldsymbol{W_R} \succeq 0, rank(\boldsymbol{W_R}) = 1 \end{aligned} \tag{23}$$

The optimization problem (23) is not convex duo to the last rank constraint [12]. We can drop the rank constraint after applying convex relaxation; then the optimization problem becomes a semi-definite problem (SDP) as follows:

$$\begin{aligned} \min_{\boldsymbol{W_R}} \quad & -trace[(\boldsymbol{A_2}+t_k\boldsymbol{B_2})\boldsymbol{W_R}]-t_k a_2 \\ s.t. \quad & trace(\boldsymbol{C_2}\boldsymbol{W_R}) \le I_{th} \\ & trace(\boldsymbol{W_R}) \le P_{RT} \\ & \boldsymbol{W_R} \succeq 0 \end{aligned} \tag{24}$$

The optimal solution of the SDP denoted by $\boldsymbol{W_R^\star}$ can be obtained using effective algorithms [13]. Then, the optimal solution of the optimizaiton problem (20) $\boldsymbol{w_R^\star}$ can be extracted from $\boldsymbol{W_R^\star}$. When $rank(\boldsymbol{W_R^\star})=1$, we can write $\boldsymbol{W_R^\star}=\boldsymbol{w_R^\star}(\boldsymbol{w_R^\star})^{\boldsymbol{H}}$ with nothing to do, while $\boldsymbol{w_R^\star}$ can be extracted from $\boldsymbol{W_R^\star}$ with the randomization method [14] when $rank(\boldsymbol{W_R^\star})$ is higher than one.

The initialization of L_0 in step 1 of the algorithm can be achieved using the following optimization problem:

$$\begin{aligned} \min_{\boldsymbol{w_R}} \quad & -\frac{\boldsymbol{w_R^H A_2 w_R}}{\boldsymbol{w_R^H B_2 w_R}+a_2} \\ s.t. \quad & \boldsymbol{w_R^H w_R} \le P_{RT} \end{aligned} \tag{25}$$

The optimization problem (25) is a generalized problem with the optimal solution and the optimal value as follows:

$$\boldsymbol{w_{R0}} = \mu\nu[\boldsymbol{A_2},\boldsymbol{B_2}+a_2 I/P_{RT}] \tag{26}$$

$$L_0 = -\lambda_{max}[\boldsymbol{A_2},\boldsymbol{B_2}+a_2\boldsymbol{I}/P_{RT}] \tag{27}$$

where the parameter μ is to make $\boldsymbol{w_{R0}}$ satisfy the power constraint $\boldsymbol{w_{R0}^H w_{R0}}=P_{RT}$.

The iteration algorithm transforms the primary optimization problem (20) to the QP-2 problem in (22) which is easily solved. This algorithm finds an output $\boldsymbol{w_R^\star}$ which is an $\varepsilon-$optimal solution of the optimization problem (20) with $\lceil ln(\frac{U_0-L_0}{\varepsilon})/ln(2)\rceil$ iterations, i.e., $a^*-\varepsilon \le u^* \le a^*$, where a^* is the global optimal value of the optimization problem (20) and u^* is the suboptimal value determined by the iteration algorithm.

C. Joint Relay and Receive Beamforming Vector

The iterative algorithm, which is designed to solve the joint relay and receive beamforming problem in (17), is given by the following table:

Table 2 Iteration algorithm for the joint optimization program

1: initialization $\boldsymbol{w_{R0}}$, $k=0$ and tolerance $\tau \ge 0$
2: **repeat**
3: $k=k+1$;
4: fixed $\boldsymbol{w_R}$ at $\boldsymbol{w_{R(k-1)}}$ and solve the optimization problem (18) to get $\boldsymbol{w_{dk}}$ and optimal value U_{dk}
5: fixed $\boldsymbol{w_d}$ at $\boldsymbol{w_{dk}}$ and solve the optimization problem (20) to get $\boldsymbol{w_{Rk}}$ and optimal value U_{Rk}
6: **until** $U_{Rk}-U_{dk} \le \tau$

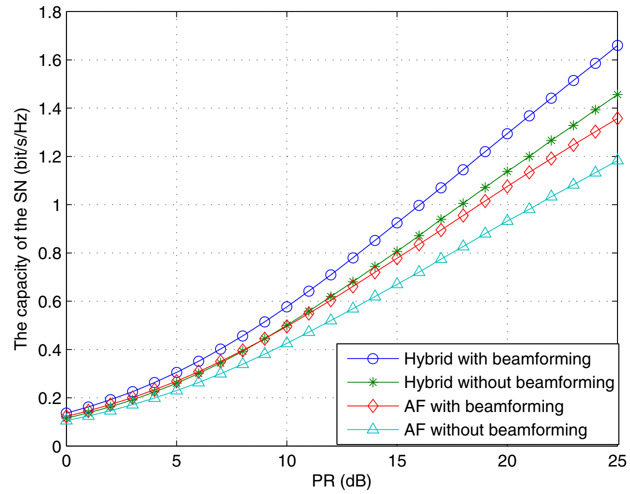

Fig. 2. The capacity of the SN vs the maximum relay transmit power

The algorithm alternates between the maximization of $\gamma_d(\boldsymbol{w_R},\boldsymbol{w_d})$ with respect of $\boldsymbol{w_R}$ for given $\boldsymbol{w_d}$ and maximization of $\gamma(\boldsymbol{w_R},\boldsymbol{w_d})$ with respect of $\boldsymbol{w_d}$ for given $\boldsymbol{w_R}$. Thus, it possesses the property:

$\gamma(\boldsymbol{w_{Rk}},\boldsymbol{w_{dk}}) \ge \gamma(\boldsymbol{w_{R(k-1)}},\boldsymbol{w_{dk}}) \ge \gamma(\boldsymbol{w_{R(k-1)}},\boldsymbol{w_{d(k-1)}})$

From the first inequality it follows that $\boldsymbol{w_{Rk}}$ is the optimal solution of the optimization problem (17) when $\boldsymbol{w_d}$ is fixed at $\boldsymbol{w_{dk}}$. Given the second inequality, it follows that $\boldsymbol{w_{dk}}$ is the optimal solution of the optimization problem (17) when $\boldsymbol{w_R}$ is fixed at $\boldsymbol{w_{R(k-1)}}$. Thus, it is clear that the solution is monotone increasing.

IV. Numerical Results

In this section, numerical results will be provided to verify the performance of the SN denoted by $\mathbb{E}\{\gamma\}$ in different cases. In the simulation, SD is equipped with four antennas, while all the other nodes have only one antenna. The location of PT, PD, ST and SD are $(-1,0)$, $(1,0)$, $(-2,-2)$ and $(2,-2)$ respectively, and the relays are uniformly distributed over the region whose center is at $(0,-2)$ and radius is 1. We set the channel to be independent Raleigh flat-fading channels as $h=d^{-\alpha/2}\chi$, where d is the distance between the transmitter and the receiver, α is the path loss exponent (set to 3.8), and χ is independent zero-mean complex Gaussian random variable with variance 1. The variances of the noise at SR and SD are set as $N_R=N_d=1$.

The capacity of the SN is investigated in four cases. “Hybrid with beamforming” is the system studied in this paper, where the relay and receive beamforming vectors are jointly designed, and the relays use AF or DF strategy according to the SINR. “Without beamforming” is the system with no beamforming at the relays or the destination. In addition, “AF” denotes the system where the relays only use AF strategy.

In Fig. 2, the capacity of the SN versus the maximum relay transmit power P_{RT} is provided for the cognitive relay networks with $P_P=15dB$, $P_{ST}=15dB$, and $I_{th}=-5dB$.

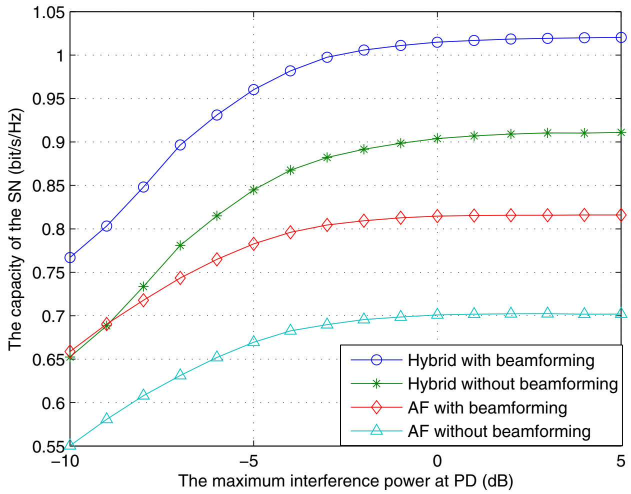

Fig. 3. The capacity of the SN vs the maximum interference power at PD

As we can see from the figure, the gain of the joint beamforming and the hybrid relay strategy is revealed gradually as the maximum relay transmit power increases. In the low P_{RT} region, the gaps between the performances of the four cases are very small, i.e., the joint beamforming and the hybrid relay strategy make a relatively small impact on the performance of SN in the cognitive relay networks, while the gaps become very obvious in the medium and high P_{RT} regions.

Fig. 3 shows the capacity of the SN versus the maximum interference power at PD denoted by I_{th} when the maximum relay transmit power P_{RT} is fixed at $15dB$. It is shown that the gap between "Hybrid with beamfomring" and "Hybrid without beamforming" is unchanged. This means that the advantage of the joint beamforming is irrelevant to the maximum interference power at SD. It is also observed that the capacity of the SN is still when I_{th} exceeds $0dB$. This means that the effect of increasing the maximum interference power at PD will be saturated. This is because bottlenecks are due to the transmit power of SD and the relays, and the interference constraints are always satisfied when I_{th} exceeds $0dB$.

V. Conclusion

In this paper, we have examined joint relay and receive beamforming in the cognitive relay networks with hybrid relays which can use AF or DF strategy according to the SINR. An iterative algorithm which gets the relay beamforming vector and the receive beamforming vector iteratively was developed to solve the capacity or the SINR maximization optimization problem. The simulation results compare the performance of the SN in four cases to illustrate the gain of the joint beamforming and the hybrid relay strategy. It is shown that the gain of the joint beamforming and hybrid relay strategy are related to the maximum relay power, while only the hybrid relay strategy is related to the maximum interference power at the primary user.

Acknowledgment

This work is supported by the National Basic Research Program of China (973 Program) 2009CB320401.

References

[1] Facilitating Opportunities for Flexible, Efficient, and Reliable Spectrum Use Employing Cognitive Radio Technologies, Notice of Proposed Rule Making and Order, FCC 03-322 Federal Communications Commission, 2003.

[2] Shahram Shahbazpanahi and Min Dong, "Achievable rate region and sum-rate maximize for network beamforming for bi-derectional relay networks," *Proc. Acoustics, Speech and Signal Processing (ICASSP)*, pp.2510-2513, Mar. 2010.

[3] R. Pabst, B. H. Walke, D. C. Schultz, D. C. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, pp. 80-89, Sep. 2004.

[4] Liping Luo, Ping Zhang, Guangchi Zhang, and Jiayin Qin, "Outage Performance for Cognitive Relay Networks with Underlay Spectrum Sharing" *IEEE Communications Letters*, vo1. 15, no. 7, pp. 710-712, Jul. 2011.

[5] Gan Zheng, Kai-Kit Wong, Arogyaswami Paulraj, and Bjorn Ottersten, "Collaborative-Relay Beamforming With Perfect CSI:Optimum and Distributed Implementation," *IEEE Signal Processing Letters*, vol. 16, no. 4, pp: 257 - 260, Apr. 2009.

[6] Kommate Jitvanichphaibool, Ying-Chang Liang, and Rui Zhang, "Beamforming and Power Control for Multi-Antenna Cognitive Two-Way Relaying", *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, pp. 1-6, Apr. 2009.

[7] Wenjing Cheng, Mounir Ghogho, Qinfei Huang, Dongtang Ma, and Jibo Wei, "Maximizing the Sum-Rate of Amplify-and-Forward Two-Way Relaying Network", *IEEE Signal Processing Letters*, vol. 18, no. 11, pp: 635 - 638, Nov. 2011.

[8] Z. Fang, Y. Hua, and J. C. Koshy, "Joint source and relay optimization for a non-regenerative MIMO relay," *Proc. IEEE Workshop Sensor Array Multi-Channel Signal Process.*, Waltham, WA, pp. 239-243, Jul. 2006.

[9] X. Chen, S.H. Song and K. B. Letaief, "Transmit and cooperative beamforming in multi-relay systems," *Proc. IEEE International Conference on Communications (ICC)*, Cape Town, South Africa, pp. 1-5, May 2010.

[10] Petre Stoica and Yngve Seln, "Cyclic Minimizers, Majorization Techniques, and the Expectation-Maximization Algorithm: A Refresher," *IEEE Signal Processing Magazine*, vol. 21, no. 1, pp. 112-114, Jan. 2004.

[11] Amir beck, Aharon Ben-Tal and Marc Teboulle, "Finding a global optimal solution for a quadratically constrained Fractional Quadratic Problem with Applications to the Regularized total least squares", SIAM J. Matrix Anal. Appl. 28(2):425-445, 2006.

[12] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2239-2251, Jun. 2006.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[14] Z.-Q. Luo, Wing-kin Ma, Anthony So, Y. Ye, and S. Zhang, "Semidefinite Relaxation of Quadratic Optimization Problems", *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20-34, May 2010.