# Reduced-Complexity Single-Carrier E-SDM for Wideband Transmissions

Cristian Davidescu, Yasutaka Ogawa, Julian Webber, Toshihiko Nishimura and Takeo Ohgane
Graduate School of Information Science and Technology,
Hokkaido University,
Kita 14, Nishi 9, Kita-ku, Sapporo, 060-0814 Japan
Email: [davidescu, ogawa, julian.webber, nishim, ohgane]@m-icl.ist.hokudai.ac.jp

*Abstract*—**In the case of broadband wireless access systems operating over non-line-of-sight (NLOS) links, the multipath can be severe. This leads to frequency-selective fading channel which causes distortions of the received signal. The single-carrier (SC) modulation method combined with frequency-domain equalization (FDE) at the receiver has been proposed as an alternative to the orthogonal frequency division multiplexing (OFDM) system due to its relative small peak-to-average power ratio (PAPR) achieved at a similar complexity. Furthermore, an eigenbeam-space division multiplexing (E-SDM) technique can be applied to the single-carrier system to improve the spectrum efficiency. This scheme makes use of both spatial and frequency diversity while maintaining a low PAPR. The computation load still remains a challenge for the uplink transmission where the power resources are limited, therefore, in this paper we propose some reduced-complexity techniques. The transmission performance of these techniques and the trade-off between complexity and bit error rate (BER) are evaluated in a frequency-selective Rayleigh channel by computer simulations.**

## I. Introduction

A major challenge for high-speed broadband transmissions is the inter-symbol interference (ISI) spanning over tens of symbols. Orthogonal frequency division multiplexing (OFDM), adopted as the downlink transmission scheme in 3GPP long term evolution (LTE), is a promising technique to solve the ISI mitigation problem. One of the drawbacks of this technique that makes it unsuitable for uplink transmissions is its high PAPR.

One alternative solution for ISI mitigation is the single-carrier system with frequency domain equalization (SC-FDE) that has similar performance and efficiency while maintaining the same low signal processing complexity of OFDM as reported in [1], [2] and the references therein. The equalization is performed on a block of data, and the operation is that of simply inversing the channel. Since the SC modulation uses just one carrier, the PAPR for its signals will be smaller than in the case of ODFM modulation. That makes it more suitable for mobile devices that have a limited power supply.

Moreover, multiple-input multiple-output (MIMO) systems using an eigenbeam-space division multiplexing (E-SDM) technique can be employed for further improvement of spectrum efficiency [3], [4]. E-SDM is also called a singular value decomposition (SVD) system [5] or MIMO eigenmode transmission system [6]. It has been well known that the E-SDM technique can be used in OFDM systems [7], [8]. Also, Ozaki et al. have proposed the application of the E-SDM technique to the SC system [9]. In the SC E-SDM system, the optimum transmit weight is determined for each frequency component. Then, we need SVD for every frequency component. The computational load is heavy. At the receiver side, demultiplexing and FDE are done separately.

In this paper, we propose reduction of the computational burden in the SC E-SDM system. Common values for some groups of weights are used at the transmitter side. This decreases the number of SVDs.

This paper is organized as follows. Section II describes the conventional frequency-domain E-SDM for SC transmission and the proposed low complexity methods for computation reduction. Section III describes the resource allocation method. We present the simulation results and draw conclusions in Sections IV and V, respectively.

## II. System Configuration

### A. Conventional Transmitter for SC E-SDM

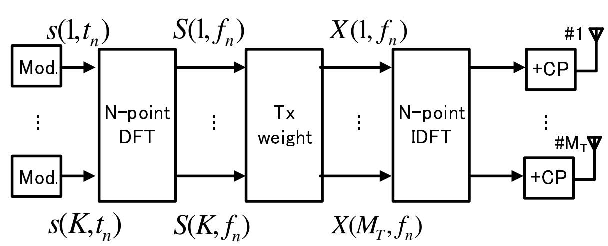


Fig. 1. Transmitter Structure

We assume that a user terminal has $M_T$ antennas as shown in Fig.1. As seen from the figure, the $K$ substreams $\mathrm{s}(k, t_n)$ are multiplexed adaptively using the channel state information (CSI) from the receiver. Here, the time domain signal $\mathrm{s}(k, t_n)$ shows the information from the $k$-th substream at the $t_n$-th sampling time. We also assume that we have $N$ data symbols in a block for each substream.

First, an $N$-point discrete Fourier transformation (DFT) is applied to the modulated signals and then a transmit weight matrix $\mathbf{W}_t$ of $[M_T \times K]$ dimensions will be used in the

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frequency domain to construct the orthogonal channels. The determination of the transmit weight matrix will be stated later. We employ the $\mathbf{S}(f_n)$ notation for the frequency-domain equivalent of signal vector $\mathbf{s}(t_n)$ and define it as $\mathbf{S}(f_n)$ = $[S_0(f_n),\ S_1(f_n),\ ..\ ,\ S_{K-1}(f_n)]^T$ where $f_n$ represents the $n$-th frequency component and $[.]^T$ is the transpose operation. The transmitted signal $\mathbf{X}(f_n)$ can be written as

$$\mathbf{X}(f_n) = \mathbf{W_t}(f_n)\mathbf{S}(f_n). \tag{1}$$

After restoring the signal to the time domain by an $N$-point IDFT operation, a cyclic prefix (CP) that prevents inter-block interference is added.

*B. Conventional Receiver for SC E-SDM*

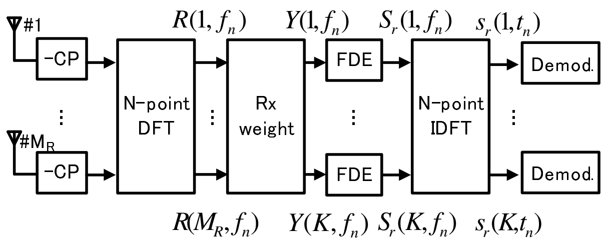


Fig. 2. Receiver Structure

We assume that a receiver has $M_R$ antennas as shown in Fig. 2. We express the channel matrix between the TX and RX antennas as $\mathbf{H}(f_n)$ of $[M_R,\ M_T]$ dimensions. The channel is time-dispersive and consists of $L$ propagation paths.

At the receiver side, first the cyclic prefix is removed. Similarly, an $N$-point DFT operation is used to transform the signal block into the frequency domain. The obtained signal vector is

$$\mathbf{R}(f_n) = \mathbf{H}(f_n)\mathbf{W}_t(f_n)\mathbf{S}(f_n) + \mathbf{N}(f_n), \tag{2}$$

where $\mathbf{N}(f_n)$ represents the $n$-th frequency component of the additive white Gaussian noise (AWGN). Next, a receive weight $\mathbf{W}_r(f_n)$ is applied for orthogonalizing the channel as shown in Fig. 2. The $f_n$ frequency component of each signal block obtained after this procedure can be written as

$$\begin{aligned}\mathbf{Y}(f_n) &= \mathbf{W}_r(f_n)\mathbf{R}(f_n)\\ &= \mathbf{W}_r(f_n)\mathbf{H}(f_n)\mathbf{W}_t(f_n)\mathbf{S}(f_n) + \mathbf{W}_r(f_n)\mathbf{N}(f_n).\end{aligned} \tag{3}$$

In order to determine the TX and RX weights, we apply SVD to each frequency component of the channel $\mathbf{H}(f_n)$. Namely, we have $\mathbf{H}(f_n)$ = $\mathbf{U}(f_n)\mathbf{\Lambda}(f_n)\mathbf{V}^H(f_n)$, where $\mathbf{U}(f_n)$ and $\mathbf{V}(f_n)$ are $M_R \times M_R$ and $M_T \times M_T$ unitary matrices, respectively. Also, $\mathbf{\Lambda}(f_n)$ is an $M_R \times M_T$ matrix with the singular values. We express the $M_T \times K$ and $M_R \times K$ matrices consisting of the first $K$ columns of $\mathbf{V}(f_n)$ and $\mathbf{U}(f_n)$ as $\mathbf{V}'(f_n)$ and $\mathbf{U}'(f_n)$, respectively. These matrices provide the TX and RX weights. That is, we have $\mathbf{W}_T(f_n)$ = $\mathbf{V}'(f_n)$ and $\mathbf{W}_R(f_n)$ = $\mathbf{U}'^H(f_n)$. By replacing these weights in Eq.(3), the signal vector $\mathbf{y}(f_n)$ can be written as

$$\begin{aligned}\mathbf{Y}(f_n) = \mathbf{U}'^H(f_n)\mathbf{U}(f_n)\mathbf{\Lambda}(f_n)\mathbf{V}^H(f_n)\mathbf{V}'(f_n)\mathbf{S}(f_n)\\ + \mathbf{U}'^H(f_n)\mathbf{N}(f_n)\,.\end{aligned} \tag{4}$$

Since $\mathbf{V}(f_n)$ and $\mathbf{U}(f_n)$ are unitary matrixes, $\mathbf{Y}(f_n)$ becomes

$$\mathbf{Y}(f_n) = \mathbf{\Lambda}'(f_n)\mathbf{S}(f_n) + \mathbf{U}'^H(f_n)\mathbf{N}(f_n)\,, \tag{5}$$

where $\mathbf{\Lambda}'(f_n)$ is a $K \times K$ diagonal matrix with the singular values.

As seen from Eq.(5), by using the TX and RX weights obtained from the SVD of the frequency-domain channel matrix, the inter-stream interference can be suppressed. However, inter-symbol interference remains. Then, frequency-domain equalization based on the ZF or MMSE criterion is applied. The ZF weight is defined as

$$\mathbf{W}_{ZF}(f_n) = \mathbf{\Lambda}'^{-1}(f_n) \tag{6}$$

The computational load at the transmitter side for calculating the TX weight is therefore proportional to the size of the signal block. Indeed, we need to compute the SVD operation $N$ times. Next, we use the frequency spectrum properties of the channel and propose some methods for reducing the computational load at the transmitter side.

*C. Proposed methods*

*1) Reduction of SVD operations:* In Figs. 3 and 4 we have an example of the characteristics of a frequency selective channel in the time domain and frequency domain, respectively. Here, we assume that the multipath channel consists of 16 paths in the time domain, and that the data block contains 256 symbols. Then, we have 256 DFT points. As seen from Fig. 4, since the frequency-domain channel response has similar values for neighboring frequency components, we can reduce the number of SVD procedures by using the same weight for the neighboring frequencies. For example, let $\mathbf{W}_t(f_P)$ and $\mathbf{W}_r(f_P)$ be the weights obtained by the SVD procedure. The weights at the neighboring frequencies are given by

$$\begin{aligned}\mathbf{W}_t(f_n) = \mathbf{W}_t(f_P),\ \mathbf{W}_r(f_n) = \mathbf{W}_r(f_P)\\ P - Q + 1 \leqq n \leqq P + Q\end{aligned} \tag{7}$$

where $Q$ is some integer. In this case, we use the conventional receiver structure shown in Fig. 2. By calculating the TX and RX weights in this way, the condition in (5) will not be met which means that the inter-stream interference will not be completely removed before the FDE procedure. This shortcoming is therefore expected to bring some degradation of the bit error rate (BER) with decreased amount of the number of SVD procedures.

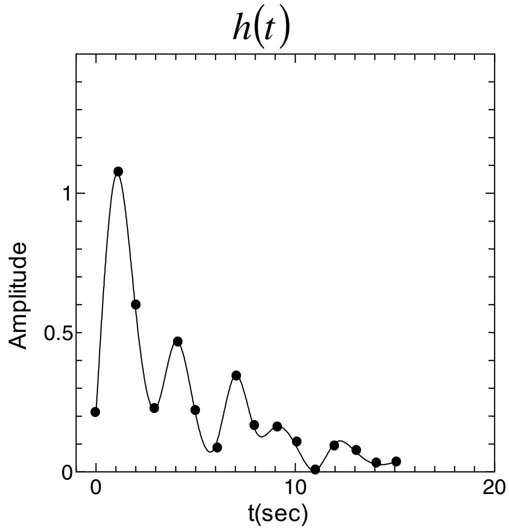


Fig. 3. Example of Channel Response in Time-Domain

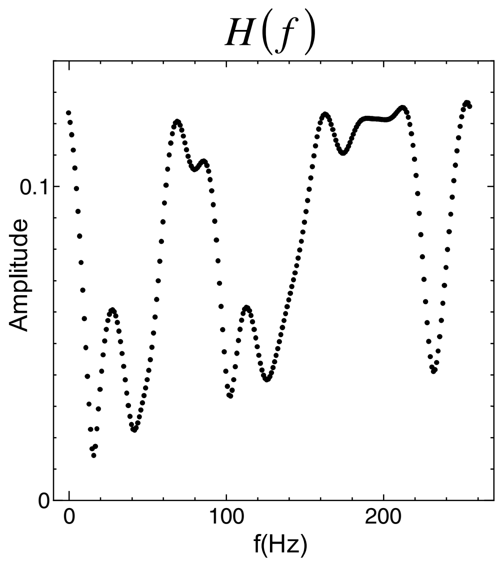


Fig. 4. Example of Channel Response in Frequency-Domain

*2) ZF/MMSE RX weight:* As stated above, the proposed reduced-complexity scheme does not completely remove the inter-stream interference at the receiver, so here we will propose a different approach, based on the zero-forcing (ZF) or minimum mean square error (MMSE) algorithms. Here, the TX weight is defined in the same way with (7). In (2) we replace $\mathbf{H}(f_n)\mathbf{W}_t(f_n)$ component with $\mathbf{H}'(f_n)$, which is interpreted as an effective channel including the TX weight. Then, the RX weight based on the ZF criterion is given by

$$\mathbf{W}_{R_1}(f_n) = (\mathbf{H}'^H(f_n)\mathbf{H}'(f_n))^{-1}\mathbf{H}'^H(f_n), \quad (8)$$

and the one based on the MMSE criterion is given by

$$\mathbf{W}_{R_2}(f_n) = (\mathbf{H}'^H(f_n)\mathbf{H}'(f_n) + \frac{1}{\gamma'}\mathbf{I}_K)^{-1}\mathbf{H}'^H(f_n), \quad (9)$$

where $\gamma'$ is the SNR at the output of each receive antenna and $\mathbf{I}_K$ is a $K \times K$ identity matrix. In this case, the FDE procedure is included in the RX weight, therefore the receiver structure can be represented as Fig. 5.

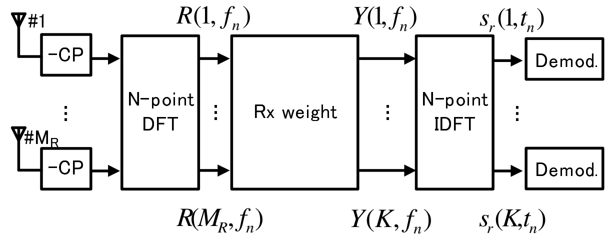


Fig. 5. Proposed Receiver Structure

## III. Power Allocation

As noted above, if the channel information is available at the transmitter side, we can realize the E-SDM transmission. Moreover, since the quality of a substream is given by its SNR, allocating more resources to substreams with a higher SNR, results in an overall increase in transmission throughput.

Here, we determine the average BER probability for all the streams and distribute the resources in order to minimize it [3]. For modulation techniques based on the Gray code criterion, the BER probability is approximately given by

$$P_{eb} = \alpha \text{ erfc}\left(\sqrt{\frac{\gamma}{\beta}}\right), \quad (10)$$

where $\gamma$ is the SNR at the receiver and $\alpha$, $\beta$ are constants defined for each modulation technique as shown in Table I.

TABLE I
CONSTANTS $\alpha$ AND $\beta$ FOR EACH MODULATION

| modulation | $\alpha$ | $\beta$ |
|---|---|---|
| QPSK | 1/2 | 2 |
| 16QAM | 3/8 | 10 |
| 64QAM | 7/24 | 42 |
| 256QAM | 15/64 | 170 |

For simplicity, the Chernoff upper bound, given by (11), is used to calculate the average BER probability for all the streams in the block of data as shown in (12).

$$P_{eb} = \alpha\text{erfc}\left(\sqrt{\frac{\gamma}{\beta}}\right) \leq 2\alpha e^{-\gamma/\beta} \quad (11)$$

$$\overline{P_{eb}} \leq \frac{1}{2M_T}\sum_{k=1}^{K} 2\alpha_k m_k e^{-P_k\gamma_k/\beta_k} \quad (12)$$

Here $m_k$ represents the number of bits sent through the $k$-th substream. We assume that the overall bit number is defined according to (13). $\gamma_k$ is the output signal-to-noise ratio at the output from the $k$-th substream.

$$\sum_{k=1}^{K} m_k = 2M_T \tag{13}$$

The $P_k$ component from (12) represents the power allocated to each substream $k$. We use the restriction $\sum_{k=1}^{K} P_k = 1$ and determine each component $P_k$ by the Lagrange undetermined multipliers method. The power components are calculated as follows:

$$P_k = max\left\{\frac{\beta_k}{\gamma_k}\left(\log\frac{\alpha_k m_k \gamma_k}{\beta_k} - \xi\right), 0\right\} \tag{14}$$

After determining the power components for each substream, we calculate the average BER in (12) for all the possible resource distributions. The adaptive modulation and power allocation is done in order to minimize the overall BER.

## IV. Numerical Analysis

### A. Simulation Parameters

TABLE II
SIMULATION ENVIRONMENT

| | |
|---|---|
| Number of Antennas | $(M_T, M_R) = (4, 4)$ |
| Bit Rate (bps/Hz) | 8 |
| Multiplexing Technique | E-SDM |
| Data Modulation | QPSK, 16QAM, 64QAM, 256QAM |
| Block Length | $N = 128$ symbols |
| Number of Blocks | 10,000 |
| Number of SVD operations | $N_{SVD} = 4$, 8, 16, 32, 64, 128 |
| Noise | AWGN |
| Channel Profile | $L$ = 16 paths (1dB att./symbol) |
| Fading | Quasi-Static Rayleigh Fading |
| FDE | ZF |

We carried out numerical analysises on the above three receiver structures using computer simulations. The simulation parameters are shown in Table II. For the sake of convenience, we assumed that the number of transmit antennas is equal to the number of receive antennas. The 4×4 channels are assumed to be frequency-selective quasi-static Rayleigh fading channels. Each channel has 16 paths with an attenuation factor of 1 dB. The channel estimation at both the transmitter and receiver is assumed to be ideal. We assumed a total data rate of 8 bps/Hz and the block length was 128 symbols. Therefore, at both the TX and RX sides we used 128-point DFT and IDFT processing. The FDE at the conventional receiver structure was based on the ZF algorithm.

### B. Performance Comparison for Proposed Receiver Structures

The BER performance of the proposed methods is shown in Figs. 6–8. The horizontal axis of the graphs represents the normalized Tx power which is the total transmit power normalized by the power yielding $E_s/N_0 = 0\,\text{dB}$ in the case of a single omni-antenna transmission. The vertical axis represents the average BER. The power allocation and adaptive modulation are achieved assuming ZF weight for all the cases. The number of SVD operations performed is represented by $N_{SVD}$.

In Fig. 6 we show the BER characteristics for the case when using the conventional receiver structure. Since the block length is 128, performing $N_{SVD} = 128$ SVD operations, i.e. one for each subcarrier, provides the interference-free BER characteristic. As the number of SVD calculations decreases, a degradation in BER performance can be seen.

The residual inter-stream interference cannot be removed by the conventional RX weight, so it results in a degradation of the BER performance with decreased amount of the number of SVD operations, $N_{SVD}$. A degradation of approximately 14 dB is observed between the ideal case $N_{SVD} = 128$ and the least accurate case $N_{SVD} = 4$ at the BER of $10^{-2}$.

In Fig. 7 we have the BER characteristics for the case when using the ZF RX weight. The TX weight is still calculated at some fixed interval so the computational load can be reduced for the uplink transmission, however the RX weight is defined in order to counter both the residual inter-stream and inter-symbol interference using the zero-forcing algorithm. Therefore, the FDE operation is included in the RX weight. As seen from Fig. 7, the ideal case $N_{SVD} = 128$ is identical with the one in Fig. 6, while all the other cases of $N_{SVD}$ reflect the improvement in interference reduction. The degradation between the ideal case $N_{SVD} = 128$ and the least precise case $N_{SVD} = 4$ in here is reduced to approximately 5 dB at the BER of $10^{-2}$.

Furthermore, in Fig. 8 we have the BER characteristics for the case when using the MMSE criterion at the receiver side. By using the MMSE algorithm instead of the ZF one, we can further reduce the amount of interference and have an approximately 0.5–1dB improvement of BER performance.

In Fig. 9 we show a comparisson between the three proposed methods for $N_{SVD}$ of 4. The dashed line illustrates the ideal case of $N_{SVD}$ = 128.

## V. Conclusion

In this paper, we considered a single-carrier E-SDM system. We proposed three reduced complexity methods for calculating the TX and RX weights and made numerical analysises on the trade-off between computational load and BER performance. The TX weight calculation is done at a user terminal that has less computational power than a basestation. The reduction of the TX weight calculation is important.

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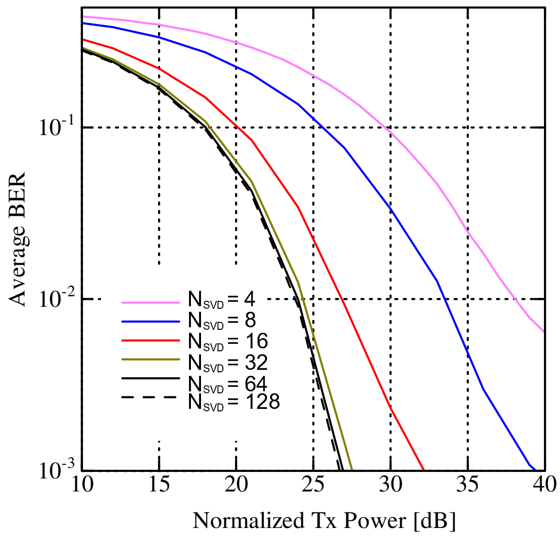


Fig. 6. Average BER (1) : Conventional receiver structure using FDE


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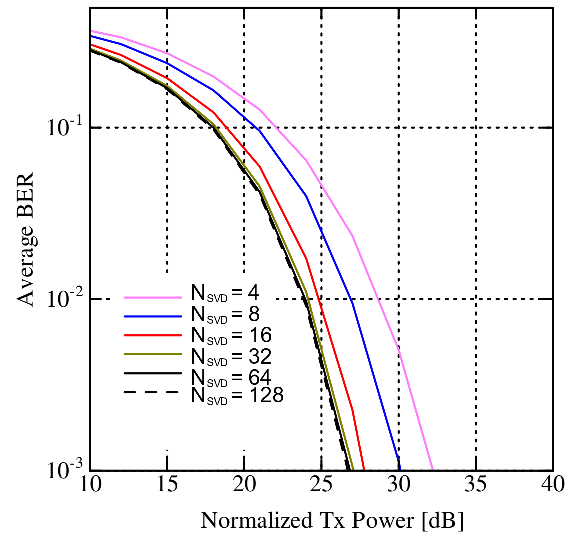


Fig. 7. Average BER (2) : Proposed receiver using ZF weight

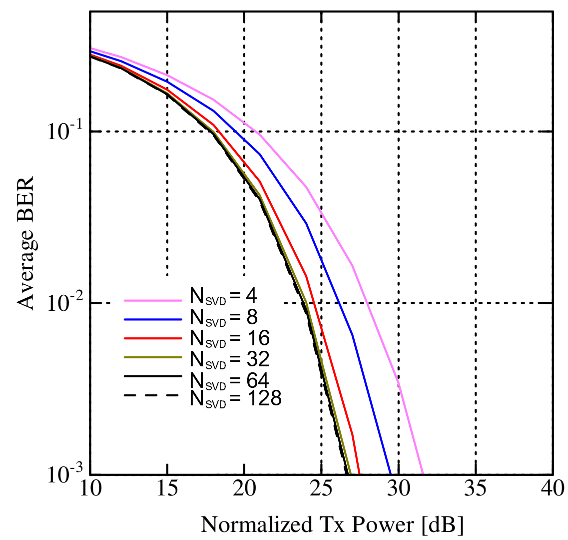


Fig. 8. Average BER (3) : Proposed receiver using MMSE weight

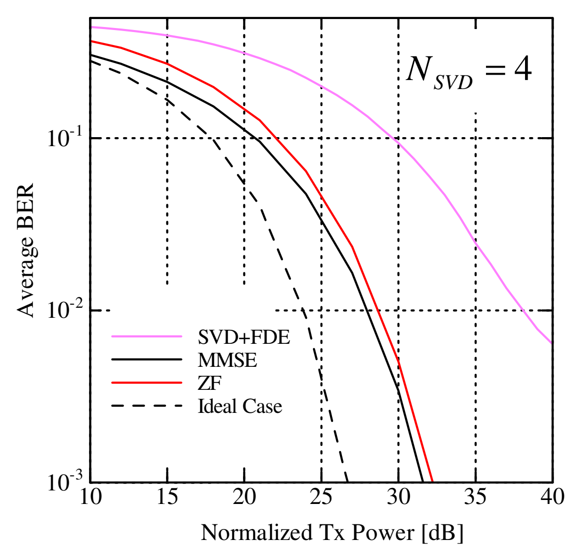


Fig. 9. Average BER (4) : Comparisson among proposed methods