MIMO Two-Way Relaying: A Comparison of Beamforming and Antenna Selection

Nan Yang[†], Phee Lep Yeoh[‡], Maged Elkashlan[§], and Iain B. Collings[†]
[†]Wireless and Networking Technologies Laboratory, CSIRO ICT Centre, Australia
[‡]Department of Electrical and Electronic Engineering, University of Melbourne, Australia
[§]School of Electronic Engineering and Computer Science, Queen Mary, University of London, UK

***Abstract*—We propose and analyze two MIMO protocols with analog network coding (ANC) in two-way amplify-and-forward (AF) relaying where multi-antenna nodes communicate via a single antenna relay. Specifically, we present a new framework for the comparative analysis of beamforming and antenna selection in two-way relaying with non-identical Rayleigh fading between the hops. To facilitate the comparison, we derive new closed-form expressions for the exact and asymptotic sum symbol error rate (SSER). We show that beamforming and antenna selection offer the same diversity order of $\min\{N_{\rm A}, N_{\rm B}\}$, where $N_{\rm A}$ and $N_{\rm B}$ are the number of antennas at the two nodes. We proceed to characterize the fundamental difference between the two protocols in terms of their array gains. A pivotal conclusion is reached that when either of the two nodes is equipped with a single antenna, antenna selection provides identical performance to beamforming at medium and high signal-to-noise ratios without the added hardware and signaling overhead.**

I. Introduction

Two-way relaying [1] is a cost-effective solution to overcome the spectral efficiency loss from the half-duplex constraint in one-way relaying. In one-way relaying, four transmission phases are utilized to support the information exchange between two nodes. By employing two-way relaying with analog network coding (ANC) [2], the information exchange utilizes only two transmission phases, as follows: 1) the two nodes transmit concurrently to an amplify-and-forward (AF) relay in the first phase, and 2) the AF relay broadcasts to the two nodes in the second phase. Two-way relaying with ANC is relatively well studied for single antennas at the nodes and the relay (e.g., [3–5] and the citations therein).

In this paper, we consider multiple-input multiple-output (MIMO) two-way relaying [6] where multiple co-located antennas are employed to boost link reliability. Previously, beamforming was utilized at the nodes and/or the relay to reap the benefits of multiple antennas [7, 8]. Nevertheless, the multiple radio-frequency (RF) chains required for beamforming inevitably brings a cost of complex transceivers and power intensive signal processing module. Motivated by this, antenna selection was applied at the multi-antenna relay to reduce the hardware overhead of MIMO signaling [9, 10]. While the aforementioned contributions stand on their own merits, a thorough and quantitative performance comparison between beamforming and antenna selection has not been explored in the open literature.

In contrast to [7–10], we focus on MIMO two-way relaying with two multiple antenna nodes equipped with $N_{\rm A}$ and $N_{\rm B}$ antennas, respectively, communicating with the help of a single antenna relay. This has practical applications, for example, in next generation cellular networks where a multi-antenna base station communicates with a multi-antenna user via a dedicated relay which is limited to a single antenna due to size and cost constraints.

In this paper, we propose and analyze beamforming and antenna selection in MIMO two-way relaying with ANC. In beamforming, each node transmits using maximal-ratio transmission (MRT) and receives using maximal-ratio combining (MRC). In antenna selection, each node transmits and receives using a single antenna which maximizes the instantaneous signal-to-noise ratio (SNR). We present new closed-form expressions for the cumulative distribution function (CDF) of the end-to-end SNR in Rayleigh fading, based on which the exact sum symbol error rate (SSER) is derived. We next examine the asymptotic SSER by deriving new easy-to-compute first order expansions of the CDF. Our asymptotic SSER expressions highlight that beamforming and antenna selection achieve the same diversity order of $\min\{N_{\rm A}, N_{\rm B}\}$. We further conclude that the fundamental difference between beamforming and antenna selection lies in the array gain. An interesting conclusion is reached that when $N_{\rm A} = 1$ or $N_{\rm B} = 1$, antenna selection achieves the same SSER as beamforming in the medium and high SNR regime.

II. Beamforming and Antenna Selection

Consider MIMO two-way relaying where the exchange of information between node $\rm A$ and node $\rm B$ is facilitated by an intermediate AF relay $\rm R$. Nodes $\rm A$ and $\rm B$ are equipped with $N_{\rm A}$ and $N_{\rm B}$ antennas, respectively, while $\rm R$ is equipped with a single antenna. The information exchange occurs over two transmission phases. In the sequel, we denote the first phase as the uplink and the second phase as the downlink. We further assume channel reciprocity between uplink and downlink.

We outline the proposed MIMO two-way relaying protocols as follows:

1) Beamforming: In the uplink, all antennas at $\rm A$ and $\rm B$ simultaneously transmit $x_{\rm A}$ and $x_{\rm B}$, respectively, to $\rm R$. The $N_{\rm A}$ and $N_{\rm B}$ antennas are weighted according to the channel coefficients between the nodes and the relay such that the instantaneous SNR at $\rm R$ is maximized. The received signal at $\rm R$ is given by

$$y_{\rm R_{BF}} = \sqrt{P_{\rm A}}\mathbf{h}_{\rm A}^{\dagger}\mathbf{w}_{\rm A}x_{\rm A} + \sqrt{P_{\rm B}}\mathbf{h}_{\rm B}^{\dagger}\mathbf{w}_{\rm B}x_{\rm B} + n_{\rm R}, \quad (1)$$

where P_{A} denotes the transmit power at A, P_{B} denotes the transmit power at B, and n_{R} is the additive white Gaussian noise (AWGN) component with mean power σ_{R}^2. We denote $\mathbf{h}_{\mathrm{A}} = [h_{\mathrm{A},1}, h_{\mathrm{A},2}, \cdots, h_{\mathrm{A},N_{\mathrm{A}}}]^T$ as the $N_{\mathrm{A}} \times 1$ channel vector between A and R with independent and identically distributed (i.i.d.) Rayleigh fading entries, and $\mathbf{h}_{\mathrm{B}} = [h_{\mathrm{B},1}, h_{\mathrm{B},2}, \cdots, h_{\mathrm{B},N_{\mathrm{B}}}]^T$ as the $N_{\mathrm{B}} \times 1$ channel vector between B and R with i.i.d. Rayleigh fading entries. According to MRT [11], the $N_{\mathrm{A}} \times 1$ transmit weight vector at A is $\mathbf{w}_{\mathrm{A}} = \mathbf{h}_{\mathrm{A}}/\|\mathbf{h}_{\mathrm{A}}\|_F$ and the $N_{\mathrm{B}} \times 1$ transmit weight vector at B is $\mathbf{w}_{\mathrm{B}} = \mathbf{h}_{\mathrm{B}}/\|\mathbf{h}_{\mathrm{B}}\|_F$. Throughout this paper, $(\cdot)^T$ denotes the transpose, $(\cdot)^\dagger$ denotes the conjugate transpose, and $\|\cdot\|_F$ denotes the Frobenius norm.

In the downlink, R applies a scaling gain G_{BF} to $y_{\mathrm{R_{BF}}}$ and forwards the scaled signal to A and B with transmit power P_{R}. All antennas at A and B combine the received signals as per MRC. The received signals at A and B are given by $z_{\mathrm{A_{BF}}} = \mathbf{w}_{\mathrm{A}}^\dagger \left(G_{\mathrm{BF}}\sqrt{P_{\mathrm{R}}}\mathbf{h}_{\mathrm{A}} y_{\mathrm{R_{BF}}} + \mathbf{n}_{\mathrm{A}}\right)$ and $z_{\mathrm{B_{BF}}} = \mathbf{w}_{\mathrm{B}}^\dagger \left(G_{\mathrm{BF}}\sqrt{P_{\mathrm{R}}}\mathbf{h}_{\mathrm{B}} y_{\mathrm{R_{BF}}} + \mathbf{n}_{\mathrm{B}}\right)$, respectively, where $\mathbf{n}_{\mathrm{A}}$ and $\mathbf{n}_{\mathrm{B}}$ are the $N_{\mathrm{A}} \times 1$ and $N_{\mathrm{B}} \times 1$ AWGN vectors with mean power $\mathbf{I}_{N_{\mathrm{A}}}\sigma_{\mathrm{A}}^2$ and $\mathbf{I}_{N_{\mathrm{B}}}\sigma_{\mathrm{B}}^2$, respectively. At nodes A and B, the self-interference is subtracted from $z_{\mathrm{A_{BF}}}$ and $z_{\mathrm{B_{BF}}}$, resulting in

$$z_{\mathrm{A_{BF}}}^* = G_{\mathrm{BF}}\sqrt{P_{\mathrm{R}}P_{\mathrm{B}}}\mathbf{w}_{\mathrm{A}}^\dagger \mathbf{h}_{\mathrm{A}} \mathbf{h}_{\mathrm{B}}^\dagger \mathbf{w}_{\mathrm{B}} x_{\mathrm{B}} + G_{\mathrm{BF}}\sqrt{P_{\mathrm{R}}}\mathbf{w}_{\mathrm{A}}^\dagger \mathbf{h}_{\mathrm{A}} n_{\mathrm{R}} + \mathbf{w}_{\mathrm{A}}^\dagger \mathbf{n}_{\mathrm{A}} \tag{2}$$

and

$$z_{\mathrm{B_{BF}}}^* = G_{\mathrm{BF}}\sqrt{P_{\mathrm{R}}P_{\mathrm{A}}}\mathbf{w}_{\mathrm{B}}^\dagger \mathbf{h}_{\mathrm{B}} \mathbf{h}_{\mathrm{A}}^\dagger \mathbf{w}_{\mathrm{A}} x_{\mathrm{A}} + G_{\mathrm{BF}}\sqrt{P_{\mathrm{R}}}\mathbf{w}_{\mathrm{B}}^\dagger \mathbf{h}_{\mathrm{B}} n_{\mathrm{R}} + \mathbf{w}_{\mathrm{B}}^\dagger \mathbf{n}_{\mathrm{B}}, \tag{3}$$

respectively. Based on the rules of variable gain AF relaying, the scaling gain G_{BF} is defined as

$$G_{\mathrm{BF}} = \left(P_{\mathrm{A}}\|\mathbf{h}_{\mathrm{A}}\|_F^2 + P_{\mathrm{B}}\|\mathbf{h}_{\mathrm{B}}\|_F^2 + \delta\sigma_{\mathrm{R}}^2\right)^{-\frac{1}{2}}. \tag{4}$$

Setting $\delta = 1$ in (4) corresponds to channel-noise-assisted AF (CNA-AF) where noise variance is available at the relay. Setting $\delta = 0$ corresponds to channel-assisted AF (CA-AF) where noise variance is not available at the relay.

As such, the instantaneous end-to-end SNR at A and B is written as

$$\gamma_{i_{\mathrm{BF}}} = \frac{\alpha_i \gamma_{i\mathrm{R_{BF}}} \gamma_{j\mathrm{R_{BF}}}}{(\alpha_i + 1)\gamma_{i\mathrm{R_{BF}}} + \gamma_{j\mathrm{R_{BF}}} + \delta}, \tag{5}$$

where $(i,j) \in \{(\mathrm{A},\mathrm{B}), (\mathrm{B},\mathrm{A})\}$, $\alpha_{\mathrm{A}} = P_{\mathrm{R}}\sigma_{\mathrm{R}}^2/P_{\mathrm{A}}\sigma_{\mathrm{A}}^2$, and $\alpha_{\mathrm{B}} = P_{\mathrm{R}}\sigma_{\mathrm{R}}^2/P_{\mathrm{B}}\sigma_{\mathrm{B}}^2$. In (5), we denote $\gamma_{\mathrm{AR_{BF}}} = P_{\mathrm{A}}\|\mathbf{h}_{\mathrm{A}}\|_F^2/\sigma_{\mathrm{R}}^2$ and $\gamma_{\mathrm{BR_{BF}}} = P_{\mathrm{B}}\|\mathbf{h}_{\mathrm{B}}\|_F^2/\sigma_{\mathrm{R}}^2$ as the instantaneous SNRs of A-R link and B-R link, respectively.

2) Antenna Selection: In the uplink, a single antenna at A and B simultaneously transmit x_{A} and x_{B}, respectively, to R. The transmit antennas are selected such that the instantaneous SNR at R is maximized. The received signal at R is given by

$$y_{\mathrm{R_{AS}}} = \sqrt{P_{\mathrm{A}}} h_{\mathrm{A}}^{\max} x_{\mathrm{A}} + \sqrt{P_{\mathrm{B}}} h_{\mathrm{B}}^{\max} x_{\mathrm{B}} + n_{\mathrm{R}}, \tag{6}$$

where $|h_k^{\max}| = \max_{1 \le n_k \le N_k} |h_{k,n_k}|$, $k \in \{\mathrm{A},\mathrm{B}\}$ is the Rayleigh fading coefficient between the nodes and the relay.

In the downlink, R applies a scaling gain G_{AS} to $y_{\mathrm{R_{AS}}}$ and forwards the scaled signal to A and B. A single receive antenna at A and B is selected such that the instantaneous SNRs are maximized. Due to the channel reciprocity between the uplink and the downlink, the receive antenna is identical to the transmit antenna. The received signals at A and B are given by $z_{\mathrm{A_{AS}}} = G_{\mathrm{AS}}\sqrt{P_{\mathrm{R}}} h_{\mathrm{A}}^{\max} y_{\mathrm{R_{AS}}} + n_{\mathrm{A}}$ and $z_{\mathrm{B_{AS}}} = G_{\mathrm{AS}}\sqrt{P_{\mathrm{R}}} h_{\mathrm{B}}^{\max} y_{\mathrm{R_{AS}}} + n_{\mathrm{B}}$, respectively, where n_{A} and n_{B} are the AWGN components with mean power of σ_{A}^2 and σ_{B}^2, respectively. Subtracting the self-interference from $z_{\mathrm{A_{AS}}}$ and $z_{\mathrm{B_{AS}}}$ results in

$$z_{\mathrm{A_{AS}}}^* = G_{\mathrm{AS}}\sqrt{P_{\mathrm{R}}P_{\mathrm{B}}} h_{\mathrm{A}}^{\max} h_{\mathrm{B}}^{\max} x_{\mathrm{B}} + G_{\mathrm{AS}}\sqrt{P_{\mathrm{R}}} h_{\mathrm{A}}^{\max} n_{\mathrm{R}} + n_{\mathrm{A}} \tag{7}$$

and

$$z_{\mathrm{B_{AS}}}^* = G_{\mathrm{AS}}\sqrt{P_{\mathrm{R}}P_{\mathrm{A}}} h_{\mathrm{B}}^{\max} h_{\mathrm{A}}^{\max} x_{\mathrm{A}} + G_{\mathrm{AS}}\sqrt{P_{\mathrm{R}}} h_{\mathrm{B}}^{\max} n_{\mathrm{R}} + n_{\mathrm{B}}, \tag{8}$$

respectively. As such, the scaling gain G_{AS} is defined as

$$G_{\mathrm{AS}} = \left(P_{\mathrm{A}}|h_{\mathrm{A}}^{\max}|^2 + P_{\mathrm{B}}|h_{\mathrm{B}}^{\max}|^2 + \delta\sigma_{\mathrm{R}}^2\right)^{-\frac{1}{2}}, \tag{9}$$

where $\delta = 1$ corresponds to CNA-AF and $\delta = 0$ corresponds to CA-AF.

Therefore, the instantaneous end-to-end SNR at A and B is written as

$$\gamma_{i_{\mathrm{AS}}} = \frac{\alpha_i \gamma_{i\mathrm{R_{AS}}} \gamma_{j\mathrm{R_{AS}}}}{(\alpha_i + 1)\gamma_{i\mathrm{R_{AS}}} + \gamma_{j\mathrm{R_{AS}}} + \delta}, \tag{10}$$

where $(i,j) \in \{(\mathrm{A},\mathrm{B}), (\mathrm{B},\mathrm{A})\}$. In (10), we denote $\gamma_{\mathrm{AR_{AS}}} = P_{\mathrm{A}}|h_{\mathrm{A}}^{\max}|^2/\sigma_{\mathrm{R}}^2$ and $\gamma_{\mathrm{BR_{AS}}} = P_{\mathrm{B}}|h_{\mathrm{B}}^{\max}|^2/\sigma_{\mathrm{R}}^2$ as the instantaneous SNRs of A-R link and B-R link, respectively.

III. Exact Sum Symbol Error Rate

In two-way relaying, a common metric for assessing the error performance is the SSER [4], which is defined as the sum of the SER of A and the SER of B. In this section, we derive new closed-form expressions for the exact SSER of beamforming and antenna selection for a wide range of modulations. Based on [12], we express the SSER directly in terms of the CDF of γ_{A_τ}, $F_{\gamma_{\mathrm{A}_\tau}}(\gamma)$, and the CDF of γ_{B_τ}, $F_{\gamma_{\mathrm{B}_\tau}}(\gamma)$, as

$$P_{s_\tau} = \frac{a}{2}\sqrt{\frac{b}{\pi}} \int_0^\infty \gamma^{-\frac{1}{2}} e^{-b\gamma} \left(F_{\gamma_{\mathrm{A}_\tau}}(\gamma) + F_{\gamma_{\mathrm{B}_\tau}}(\gamma)\right) d\gamma, \tag{11}$$

where $\tau \in \{\mathrm{BF}, \mathrm{AS}\}$, and a and b are modulation constants for several Gray-mapped modulations such as quadrature phase-shift keying (QPSK) with $a = 1$ and $b = 0.5$.

A. Beamforming

In this subsection, we derive an exact closed-form expression for the SSER of beamforming, $P_{s_{\mathrm{BF}}}$. To this end, we present a new result for the CDF of $\gamma_{i_{\mathrm{BF}}}$, as given in the following lemma.

Lemma 1: The CDF of the instantaneous end-to-end SNR with beamforming, defined as $\gamma_{i_{\mathrm{BF}}}$ in (5), is derived as

$$F_{\gamma_{i_{\mathrm{BF}}}}(\gamma) = 1 - \frac{2}{\Gamma(N_j)\,\overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}^{N_j}} e^{-\gamma\left(\frac{1}{\alpha_i \overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}}} + \frac{\alpha_i+1}{\alpha_i \overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}}\right)} \times \sum_{n=0}^{N_i-1} \sum_{p=0}^{n} \sum_{q=0}^{N_j-1} \frac{1}{n!}\binom{n}{p}\binom{N_j-1}{q}\left(\frac{\gamma}{\alpha_i \overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}}}\right)^{n+\frac{1-p+q}{2}} \times \left(\frac{\alpha_i+1}{\alpha_i}\gamma+\delta\right)^{\frac{1+p+q}{2}} \left(\frac{\alpha_i+1}{\alpha_i}\gamma\right)^{N_j-q-1} \overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}^{\frac{1-p+q}{2}} \times K_{1-p+q}\left(2\sqrt{\frac{\gamma}{\alpha_i \overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}} \overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}}\left(\frac{\alpha_i+1}{\alpha_i}\gamma+\delta\right)}\right), \quad (12)$$

where $(i,j) \in \{(\mathrm{A},\mathrm{B}),(\mathrm{B},\mathrm{A})\}$ and $K_v(x)$ is the vth-order modified Bessel function of the second kind. In (12), $\overline{\gamma}_{\mathrm{AR}_{\mathrm{BF}}} = \mathbb{E}\left[\|\mathbf{h}_{\mathrm{A}}\|_F^2\right] P_{\mathrm{A}}/N_{\mathrm{A}}\sigma_{\mathrm{R}}^2$ denotes the average SNRs from each antenna at A, and $\overline{\gamma}_{\mathrm{BR}_{\mathrm{BF}}} = \mathbb{E}\left[\|\mathbf{h}_{\mathrm{B}}\|_F^2\right] P_{\mathrm{B}}/N_{\mathrm{B}}\sigma_{\mathrm{R}}^2$ denotes the average SNRs from each antenna at B, where $\mathbb{E}[\cdot]$ is the expectation.

Proof: We express the CDF of $\gamma_{i_{\mathrm{BF}}}$ as

$$F_{\gamma_{i_{\mathrm{BF}}}}(\gamma) = \Pr\left[\frac{\alpha_i \gamma_{i\mathrm{R}_{\mathrm{BF}}} \gamma_{j\mathrm{R}_{\mathrm{BF}}}}{(\alpha_i+1)\gamma_{i\mathrm{R}_{\mathrm{BF}}} + \gamma_{j\mathrm{R}_{\mathrm{BF}}} + \delta} \le \gamma\right] = 1 - \int_{\zeta\gamma}^{\infty} \widetilde{F}_{\gamma_{i\mathrm{R}_{\mathrm{BF}}}}(\beta_1)\, f_{\gamma_{j\mathrm{R}_{\mathrm{BF}}}}(\gamma_{j\mathrm{R}_{\mathrm{BF}}}) d\gamma_{j\mathrm{R}_{\mathrm{BF}}}, \quad (13)$$

where $\beta_1 = \gamma(\gamma_{j\mathrm{R}_{\mathrm{BF}}}+\delta)/(\alpha_i\gamma_{j\mathrm{R}_{\mathrm{BF}}} - (\alpha_i+1)\gamma)$, $\zeta = (\alpha_i+1)/\alpha_i$, $\widetilde{F}_{\gamma_{s\mathrm{R}_{\mathrm{BF}}}}(\gamma) = 1 - F_{\gamma_{s\mathrm{R}_{\mathrm{BF}}}}(\gamma)$, and $(i,j) \in \{(\mathrm{A},\mathrm{B}),(\mathrm{B},\mathrm{A})\}$. In Rayleigh fading, the probability density function (PDF) and the CDF of $\gamma_{k\mathrm{R}_{\mathrm{BF}}}$ are given by $f_{\gamma_{k\mathrm{R}_{\mathrm{BF}}}}(\gamma) = \gamma^{N_k-1} e^{-\frac{\gamma}{\overline{\gamma}_{k\mathrm{R}_{\mathrm{BF}}}}}/\Gamma(N_k)\overline{\gamma}_{k\mathrm{R}_{\mathrm{BF}}}^{N_k}$ and $F_{\gamma_{k\mathrm{R}_{\mathrm{BF}}}}(\gamma) = 1 - e^{-\frac{\gamma}{\overline{\gamma}_{k\mathrm{R}_{\mathrm{BF}}}}} \sum_{n=0}^{N_k-1} \frac{1}{n!}\left(\gamma/\overline{\gamma}_{k\mathrm{R}_{\mathrm{BF}}}\right)^n$, respectively, where $k \in \{\mathrm{A},\mathrm{B}\}$. To solve the integral in (13), we set $\omega = \gamma_{j\mathrm{R}_{\mathrm{BF}}} - \zeta\gamma$ and substitute the PDF and the CDF into (13). We then apply the binomial expansion [13, eq.(1.111)] and utilize [13, eq. (3.471.9)] to solve the resultant integral to obtain the desired closed-form result in (12). ■

Based on $F_{\gamma_{i_{\mathrm{BF}}}}(\gamma)$ in Lemma 1, we proceed to derive the SSER of beamforming. We focus on CA-AF with $\delta = 0$ given its mathematical tractability and the fact that it asymptotes CNA-AF in the low SER regime. The exact SSER is derived by substituting (12) into (11) and using [13, eq. (6.621.3)] to solve the integrals, which results in

$$P_{s_{\mathrm{BF}}} = P_{s,\mathrm{A}_{\mathrm{BF}}} + P_{s,\mathrm{B}_{\mathrm{BF}}}, \quad (14)$$

where $P_{s,\mathrm{A}_{\mathrm{BF}}}$ and $P_{s,\mathrm{B}_{\mathrm{BF}}}$ is derived as (15) at the top of next page. In (15), $\varrho_{1i} = n+p+N_j-q-0.5$, $\varrho_{2i} = n-p+N_j+q+1.5$, $\varrho_{3i} = 4(\alpha_i+1)\overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}}\overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}$, $\varrho_{4i} = (\alpha_i+1)\overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}} + \overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}} + \alpha_i b \overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}}\overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}$, $\varrho_{5i} = n+N_j+1$, $\varrho_{6i} = 1-p+q-N_j$, and $(i,j) \in \{(\mathrm{A},\mathrm{B}),(\mathrm{B},\mathrm{A})\}$. We denote $\Gamma(x)$ as the gamma function and ${}_2F_1(a,b;c;z)$ as the hypergeometric function. Our result in (15) involves finite summations of exponentials, powers, and hypergeometric functions, which are efficiently evaluated using mathematical software packages such as MATLAB.

B. Antenna Selection

In this subsection, we derive an exact closed-form expression for the SSER of antenna selection, $P_{s_{\mathrm{AS}}}$. The CDF of $\gamma_{i_{\mathrm{AS}}}$ is presented in the following lemma.

Lemma 2: The CDF of the instantaneous end-to-end SNR with antenna selection, defined as $\gamma_{i_{\mathrm{AS}}}$ in (10), is derived as

$$F_{\gamma_{i_{\mathrm{AS}}}}(\gamma) = 1 - 2N_j \sum_{n=1}^{N_i} \sum_{q=0}^{N_j-1} \binom{N_i}{n}\binom{N_j-1}{q}(-1)^{n+q-1} \times e^{-\frac{\gamma}{\alpha_i}\left(\frac{n}{\overline{\gamma}_{i\mathrm{R}_{\mathrm{AS}}}} + \frac{(q+1)(\alpha_i+1)}{\overline{\gamma}_{j\mathrm{R}_{\mathrm{AS}}}}\right)} \times \sqrt{\frac{n\gamma}{(q+1)\,\alpha_i \overline{\gamma}_{i\mathrm{R}_{\mathrm{AS}}} \overline{\gamma}_{j\mathrm{R}_{\mathrm{AS}}}}\left(\frac{\alpha_i+1}{\alpha_i}\gamma+\delta\right)} \times K_1\left(2\sqrt{\frac{n(q+1)\gamma}{\alpha_i \overline{\gamma}_{i\mathrm{R}_{\mathrm{AS}}} \overline{\gamma}_{j\mathrm{R}_{\mathrm{AS}}}}\left(\frac{\alpha_i+1}{\alpha_i}\gamma+\delta\right)}\right), \quad (16)$$

where $\overline{\gamma}_{\mathrm{AR}_{\mathrm{AS}}} = \mathbb{E}\left[|h_{\mathrm{A}}^{\max}|^2\right] P_{\mathrm{A}}/\sigma_{\mathrm{R}}^2$ is the average SNR from the selected antenna at A, $\overline{\gamma}_{\mathrm{BR}_{\mathrm{AS}}} = \mathbb{E}\left[|h_{\mathrm{B}}^{\max}|^2\right] P_{\mathrm{B}}/\sigma_{\mathrm{R}}^2$ is the average SNR from the selected antenna at B, and $(i,j) \in \{(\mathrm{A},\mathrm{B}),(\mathrm{B},\mathrm{A})\}$.

Proof: In Rayleigh fading, the PDF of $\gamma_{k\mathrm{R}_{\mathrm{AS}}}$ is given by $f_{\gamma_{k\mathrm{R}_{\mathrm{AS}}}}(\gamma) = (N_k/\overline{\gamma}_{k\mathrm{R}})\, e^{-\frac{\gamma}{\overline{\gamma}_{k\mathrm{R}}}}\left(1 - e^{-\frac{\gamma}{\overline{\gamma}_{k\mathrm{R}}}}\right)^{N_k-1}$ and the CDF of $\gamma_{k\mathrm{R}_{\mathrm{AS}}}$ is given by $F_{\gamma_{k\mathrm{R}_{\mathrm{AS}}}}(\gamma) = \left(1 - e^{-\frac{\gamma}{\overline{\gamma}_{k\mathrm{R}}}}\right)^{N_k}$. Using these statistics, the CDF in (16) is derived following the steps outlined in the proof of Lemma 1. ■

Based on $F_{\gamma_{i_{\mathrm{AS}}}}(\gamma)$ in Lemma 2, we derive the SSER of antenna selection with CA-AF (i.e., $\delta = 0$). Substituting (16) into (11) and evaluating the resultant integrals with [13, eq. (3.324.1)], the exact SSER is

$$P_{s_{\mathrm{AS}}} = P_{s,\mathrm{A}_{\mathrm{AS}}} + P_{s,\mathrm{B}_{\mathrm{AS}}}, \quad (17)$$

where $P_{s,\mathrm{A}_{\mathrm{AS}}}$ and $P_{s,\mathrm{B}_{\mathrm{AS}}}$ is derived as (18) at the top of next page. In (18), $\rho_{1i} = 4n(q+1)(\alpha_i+1)\overline{\gamma}_{i\mathrm{R}_{\mathrm{AS}}}\overline{\gamma}_{j\mathrm{R}_{\mathrm{AS}}}$, $\rho_{2i} = (q+1)(\alpha_i+1)\overline{\gamma}_{i\mathrm{R}_{\mathrm{AS}}} + n\overline{\gamma}_{j\mathrm{R}_{\mathrm{AS}}} + \alpha_i b \overline{\gamma}_{i\mathrm{R}_{\mathrm{AS}}}\overline{\gamma}_{j\mathrm{R}_{\mathrm{AS}}}$, and $(i,j) \in \{(\mathrm{A},\mathrm{B}),(\mathrm{B},\mathrm{A})\}$. Our result in (18) is easy to compute since it contains simple exponentials, powers, and standard hypergeometric functions.

IV. Asymptotic Sum Symbol Error Rate

In this section, we examine the SSER behavior in the high SNR regime, leading to new design insights. Our solutions are valid for both CNA-AF and CA-AF since they are asymptotically the same at high SNRs.

A. Beamforming

We commence our asymptotic SSER analysis by deriving the first order expansion of $F_{\gamma_{i_{\mathrm{BF}}}}(\gamma)$ in (12). To this end, we denote $\kappa_{i_{\mathrm{BF}}} = \overline{\gamma}_{j\mathrm{R}_{\mathrm{BF}}}/\overline{\gamma}_{i\mathrm{R}_{\mathrm{BF}}}$ as the ratio of the average SNRs in (12), and apply [13, eq. (1.211.1)] and [13, eq.(8.446)] to expand the exponential function and the Bessel function, respectively. We derive the first order expansion of $F_{\gamma_{i_{\mathrm{BF}}}}(\gamma)$ by retaining the dominant term in the series expansion as

$$P_{s,i_{\rm BF}}=\frac{a}{2}-\frac{a\sqrt{\alpha_i b}}{\Gamma\left(N_j\right)}\sum_{n=0}^{N_i-1}\sum_{p=0}^{n}\sum_{q=0}^{N_j-1}\frac{1}{n!}\binom{n}{p}\binom{N_j-1}{q}\frac{\overline{\gamma}_{i\rm R_{BF}}^{\varrho_{1i}-n}\overline{\gamma}_{j\rm R_{BF}}^{n+\frac{1}{2}}\Gamma\left(\varrho_{1i}\right)\Gamma\left(\varrho_{2i}\right){}_2F_1\left(\frac{\varrho_{1i}}{2},\frac{\varrho_{1i}+1}{2};\varrho_{5i};1-\frac{\varrho_{3i}}{\varrho_{4i}^2}\right)}{2^{\varrho_{2i}}\varrho_{4i}^{\varrho_{1i}}\left(\alpha_i+1\right)^{\varrho_{6i}}\Gamma\left(\varrho_{5i}\right)}. \tag{15}$$

$$P_{s,i_{\rm AS}}=\frac{a}{2}-\frac{aN_j}{4}\sqrt{\frac{\alpha_i b\overline{\gamma}_{i\rm R_{AS}}\overline{\gamma}_{j\rm R_{AS}}}{2}}\sum_{n=1}^{N_i}\sum_{q=0}^{N_j-1}\binom{N_i}{n}\binom{N_j-1}{q}\frac{(-1)^{n+q-1}\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{5}{2}\right){}_2F_1\left(\frac{1}{4},\frac{3}{4};2;1-\frac{\rho_{1i}}{\rho_{2i}^2}\right)}{(q+1)\sqrt{\rho_{2i}}}. \tag{18}$$

$\overline{\gamma}_{i\rm R_{BF}}\to\infty$. Specifically, we consider the following three cases of N_i relative to N_j.

Case 1: For $N_i<N_j$, the first order expansion of $F_{\gamma_{i_{\rm BF}}}(\gamma)$ is derived as $F^{\infty}_{\gamma_{i_{\rm BF}}}(\gamma)\approx\left(\gamma/\overline{\gamma}_{i\rm R_{BF}}\right)^{N_i}/N_i!\alpha_i^{N_i}$.

Case 2: For $N_i>N_j$, we have $F^{\infty}_{\gamma_{i_{\rm BF}}}(\gamma)\approx\left(\gamma/\overline{\gamma}_{i\rm R_{BF}}\right)^{N_j}(\alpha_i+1)^{N_j}/N_j!\alpha_i^{N_j}\kappa_{i_{\rm BF}}^{N_j}$.

Case 3: For $N_i=N_j$, we have $F^{\infty}_{\gamma_{i_{\rm BF}}}(\gamma)\approx\left(\gamma/\overline{\gamma}_{i\rm R_{BF}}\right)^{N_i}\left(1+\left((\alpha_i+1)^{N_i}/\kappa_{i_{\rm BF}}^{N_i}\right)\right)/N_i!\alpha_i^{N_i}$.

Substituting these first order expansions into (11) and solving the resulting integrals, we derive the asymptotic SSER of beamforming as

$$P^{\infty}_{s_{\rm BF}}=\left(G_{a_{\rm BF}}\overline{\gamma}_{\rm AR_{BF}}\right)^{-G_{d_{\rm BF}}}+o\left(\overline{\gamma}_{\rm AR_{BF}}^{-G_{d_{\rm BF}}}\right), \tag{19}$$

where the diversity order $G_{d_{\rm BF}}$ is

$$G_{d_{\rm BF}}=\min\left\{N_{\rm A},N_{\rm B}\right\} \tag{20}$$

and the array gain $G_{a_{\rm BF}}$ is

$$G_{a_{\rm BF}}=b\left(\frac{a\Gamma\left(G_{d_{\rm BF}}+\frac{1}{2}\right)\Delta_{\rm BF}}{2\sqrt{\pi}}\right)^{-\frac{1}{G_{d_{\rm BF}}}}, \tag{21}$$

with

$$\Delta_{\rm BF}=\begin{cases}\frac{1}{N_{\rm A}!}\left(\frac{1}{\alpha_{\rm A}^{N_{\rm A}}}+\frac{(\alpha_{\rm B}+1)^{N_{\rm A}}}{\alpha_{\rm B}^{N_{\rm A}}}\right), & N_{\rm A}<N_{\rm B}\\ \frac{1}{N_{\rm B}!\kappa_{\rm BF}^{N_{\rm B}}}\left(\frac{(\alpha_{\rm A}+1)^{N_{\rm B}}}{\alpha_{\rm A}^{N_{\rm B}}}+\frac{1}{\alpha_{\rm B}^{N_{\rm B}}}\right), & N_{\rm A}>N_{\rm B}\\ \begin{array}{c}\frac{1}{N_{\rm A}!}\left(\frac{1}{\alpha_{\rm A}^{N_{\rm A}}}+\frac{(\alpha_{\rm B}+1)^{N_{\rm A}}}{\alpha_{\rm B}^{N_{\rm A}}}\right)+\\ \frac{1}{N_{\rm A}!\kappa_{\rm BF}^{N_{\rm A}}}\left(\frac{(\alpha_{\rm A}+1)^{N_{\rm A}}}{\alpha_{\rm A}^{N_{\rm A}}}+\frac{1}{\alpha_{\rm B}^{N_{\rm A}}}\right)\end{array}, & N_{\rm A}=N_{\rm B}\end{cases}. \tag{22}$$

and $\kappa_{\rm BF}=\overline{\gamma}_{\rm BR_{BF}}/\overline{\gamma}_{\rm AR_{BF}}$.

B. Antenna Selection

We now focus on the asymptotic SSER of antenna selection. Similar to beamforming, we denote $\kappa_{i_{\rm AS}}=\overline{\gamma}_{j\rm R_{AS}}/\overline{\gamma}_{i\rm R_{AS}}$ as the ratio of the average SNRs in $F_{\gamma_{i_{\rm AS}}}(\gamma)$ given in (16). By expanding the exponential and the Bessel function, we derive the first order expansion of $F_{\gamma_{i_{\rm AS}}}(\gamma)$ as $\overline{\gamma}_{i\rm R_{AS}}\to\infty$ for the following three cases.

Case 1: For $N_i<N_j$, the first order expansion of $F_{\gamma_{i_{\rm AS}}}(\gamma)$ is derived as $F^{\infty}_{\gamma_{i_{\rm AS}}}(\gamma)\approx\left(\gamma/\overline{\gamma}_{i\rm R_{AS}}\right)^{N_i}/\alpha_i^{N_i}$.

Case 2: For $N_i>N_j$, we have $F^{\infty}_{\gamma_{i_{\rm AS}}}(\gamma)\approx\left(\gamma/\overline{\gamma}_{i\rm R_{AS}}\right)^{N_j}(\alpha_i+1)^{N_j}/\alpha_i^{N_j}\kappa_{i_{\rm AS}}^{N_j}$.

Case 3: For $N_i=N_j$, we have $F^{\infty}_{\gamma_{i_{\rm AS}}}(\gamma)\approx\left(\gamma/\overline{\gamma}_{i\rm R_{AS}}\right)^{N_i}\left(1+\left((\alpha_i+1)^{N_i}/\kappa_{i_{\rm AS}}^{N_i}\right)\right)/\alpha_i^{N_i}$.

Therefore, we substitute these first order expansions into (11) to derive the asymptotic SSER of antenna selection as

$$P^{\infty}_{s_{\rm AS}}=\left(G_{a_{\rm AS}}\overline{\gamma}_{\rm AR_{AS}}\right)^{-G_{d_{\rm AS}}}+o\left(\overline{\gamma}_{\rm AR_{AS}}^{-G_{d_{\rm AS}}}\right), \tag{23}$$

where the diversity order $G_{d_{\rm AS}}$ is given by

$$G_{d_{\rm AS}}=\min\left\{N_{\rm A},N_{\rm B}\right\}, \tag{24}$$

and the array gain $G_{a_{\rm AS}}$ is given by

$$G_{a_{\rm AS}}=b\left(\frac{a\Gamma\left(G_{d_{\rm AS}}+\frac{1}{2}\right)\Delta_{\rm AS}}{2\sqrt{\pi}}\right)^{-\frac{1}{G_{d_{\rm AS}}}}, \tag{25}$$

with

$$\Delta_{\rm AS}=\begin{cases}\frac{1}{\alpha_{\rm A}^{N_{\rm A}}}+\frac{(\alpha_{\rm B}+1)^{N_{\rm A}}}{\alpha_{\rm B}^{N_{\rm A}}}, & N_{\rm A}<N_{\rm B}\\ \frac{1}{\kappa_{\rm AS}^{N_{\rm B}}}\left(\frac{(\alpha_{\rm A}+1)^{N_{\rm B}}}{\alpha_{\rm A}^{N_{\rm B}}}+\frac{1}{\alpha_{\rm B}^{N_{\rm B}}}\right), & N_{\rm A}>N_{\rm B}\\ \begin{array}{c}\left(\frac{1}{\alpha_{\rm A}^{N_{\rm A}}}+\frac{(\alpha_{\rm B}+1)^{N_{\rm A}}}{\alpha_{\rm B}^{N_{\rm A}}}\right)+\\ \frac{1}{\kappa_{\rm AS}^{N_{\rm A}}}\left(\frac{(\alpha_{\rm A}+1)^{N_{\rm A}}}{\alpha_{\rm A}^{N_{\rm A}}}+\frac{1}{\alpha_{\rm B}^{N_{\rm A}}}\right)\end{array}, & N_{\rm A}=N_{\rm B}\end{cases}. \tag{26}$$

and $\kappa_{\rm AS}=\overline{\gamma}_{\rm BR_{AS}}/\overline{\gamma}_{\rm AR_{AS}}$. Comparing (24) and (20), we confirm that antenna selection attains the same diversity order as beamforming. Furthermore, the performance gap between beamforming and antenna selection is determined by the array gains as $\frac{P^{\infty}_{s_{\rm BF}}}{P^{\infty}_{s_{\rm AS}}}=\frac{\Delta_{\rm BF}}{\Delta_{\rm AS}}\left(\frac{\overline{\gamma}_{\rm A_{BF}}}{\overline{\gamma}_{\rm A_{AS}}}\right)^{-\min\{N_{\rm A},N_{\rm B}\}}$.

V. Numerical Results

In this section, we present numerical results to examine the impacts of antenna configuration and transmit power on the SSER. Throughout this section, we consider $P_{\rm A}=P_{\rm B}=P_{\rm R}=P_{\rm T}/3$ and $\sigma_{\rm A}^2=\sigma_{\rm B}^2=\sigma_{\rm R}^2=N_0$ for simplicity. Of course, these considerations do not invalidate the generality of our results. We further normalize $\mathbb{E}\left[|h_{{\rm A},n_{\rm A}}|^2\right]=\mathbb{E}\left[|h_{{\rm B},n_{\rm B}}|^2\right]=1$, $\forall\ n_{\rm A}\in\{1,\cdots,N_{\rm A}\}$, $\forall\ n_{\rm B}\in\{1,\cdots,N_{\rm B}\}$. All our results are based on BPSK. The Monte Carlo simulation points of beamforming and antenna selection are marked with '$\circ$' and '$\bullet$', respectively. In Figs.

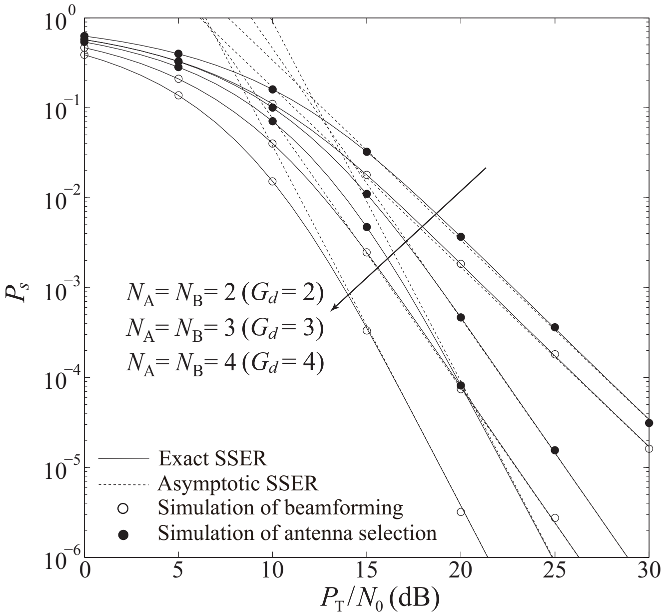

Fig. 1. SSER of beamforming and antenna selection with $N_{\mathrm{A}} = N_{\mathrm{B}}$.

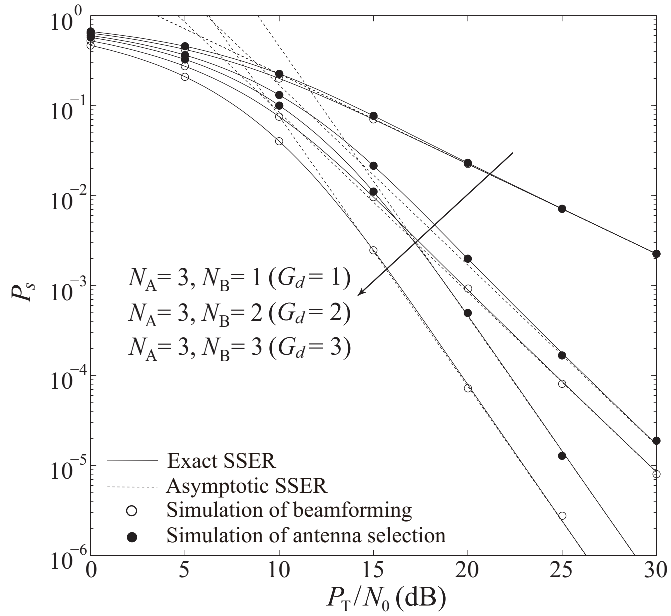

Fig. 2. SSER of beamforming and antenna selection with fixed N_{A}.

1 and 2, there is a precise agreement between the simulation and the exact curves generated from (14) and (17). We further confirm that the asymptotic curves generated from (19) and (23) accurately predict the diversity order and the array gain.

Fig. 1 compares the SSER of beamforming and antenna selection for $N_{\mathrm{A}} = N_{\mathrm{B}}$. We first observe that the SSER is progressively reduced as N_{A} and N_{B} increase. This is due to the fact that the diversity order increases with N_{A} and N_{B}. We also observe that beamforming outperforms antenna selection. Furthermore, we observe that the SNR advantage of beamforming over antenna selection increases with N_{A} and N_{B}.

Fig. 2 compares the SSER of beamforming and antenna selection for fixed N_{A}. We observe that when $N_{\mathrm{B}} = 1$, antenna selection and beamforming offer the same SSER in the medium and high SNR regime. This behavior is not surprising since the end-to-end performance is dominated by B-R link when $N_{\mathrm{A}} > N_{\mathrm{B}}$. When $N_{\mathrm{B}} = 1$, antenna selection achieves the same performance as beamforming in B-R link. Although not shown, we confirm that beamforming and antenna selection also exhibit identical performance when $N_{\mathrm{B}} > N_{\mathrm{A}} = 1$.

VI. Conclusion

We examined MIMO two-way AF relaying in which the communication between two nodes, with N_{A} and N_{B} antennas, is aided by a single antenna relay. For this network, we proposed and compared two protocols, namely beamforming and antenna selection. Based on our new asymptotic results for the SSER, the following points are clarified: 1) An important design parameter is $\min\{N_{\mathrm{A}}, N_{\mathrm{B}}\}$ which has a direct impact on the diversity order and the array gain; 2) Beamforming and antenna selection offer the same diversity order; and 3) The fundamental difference between the two protocols lies in the array gain.

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