Regenerative Multi-way Relaying: Relay Precoding and Ordered MMSE-SIC Receiver

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***Abstract*—Consider a wireless network, in which multiple users exchange data with each other via a multi-antenna relay. The users transmit to the relay simultaneously. Then the relay regenerates the transmitted signal and precodes it before broadcasting to the users. In particular, we propose a minimum mean square error (MMSE) based precoder, which takes the relay detection errors into account. We indicate that this precoder can improve the bit error performance when uplink channel is in a bad condition. Afterwards, we deploy the ordered MMSE successive interference cancellation (MMSE-SIC) receiver, which exploits temporal diversity gains over the multiple downlink slots. Simulation results show that the ordered MMSE-SIC receiver can achieve sufficient diversity gains over traditional MMSE receiver.**

I. INTRODUCTION

Relaying helps wireless networks extend coverage as well as improve energy efficiency [1]. In this paper, we study a set-up in which multiple users communicate with each other via a multi-antenna relay. To clearly describe the procedure of information exchange, we divide it into two cascaded phases, i.e., the multiple access phase and the broadcast phase. In the multiple access phase, all users transmit to the relay simultaneously. Afterwards, the relay detects the received signal, and then precodes and forwards the regenerated signal in the broadcast phase.

Prior work that investigated multiple users exchanging data via relay includes [1–6]. In [1], a single-antenna relay with different forwarding strategies is considered in Gaussian channel. The technique of direct-sequence code-division-multiple-access (DS-CDMA) is employed to facilitate the information exchange in [2]. However, multiple antennas at the relay could help forward the received signal while exploiting spatial degrees of freedom. In [3, 4], the users exchange data with each other via a multi-antenna relay by using a specific permutation matrix. The framework in [3] is more general than that of [4]. However, only amplify-and-forward and zero-forcing precoder are studied in [3].

In the first part of this paper, we derive a minimum mean square error (MMSE) based precoder specifically for regenerative multi-way relaying (MWR), which takes the effect of uplink errors into account. As is indicated by simulation, this precoder can achieve significant gains especially for low SNR regime of the uplink channel.

The specific space-time structure of MWR is firstly considered in [5, 6]. Each transmission comprises of one uplink slot and multiple downlink slots. Due to the special structure of the latter, the temporal degrees of freedom can thus be exploited by single-antenna users via multi-stage receivers.

In the second part of this paper, we focus on the design of multi-stage receivers at users to achieve diversity gains. As each user has only one antenna, spatial diversity can not be achieved. However, within the multiple downlink slots, we apply the ordered MMSE successive interference cancellation (MMSE-SIC) receiver at each user, which is proposed for uplink multiple access channel in [7, 9], to exploit the temporal diversity gains. Particularly, owing to the fact that the interference-plus-noise term can be well approximated by Gaussian distribution, we measure the detection reliability according to signal to interference-plus-noise ratio (SINR) and/or the output of linear MMSE estimate, and then order the SIC. Note that this multi-stage receiver may enlarges delay and complexity when compared with linear receiver in [6], however, it harvests the temporal diversity gains in the broadcast phase.

The remainder of this paper is organized as follows. Section II formulates the multiple access phase and the broadcast phase. In Section III, a MMSE precoder is proposed specifically for the regenerative MWR. Section IV investigates how to measure the reliability in three different ways and presents the corresponding MMSE-SIC receivers. The simulation results on bit error rate (BER) are provided in Section V. Section VI concludes this paper.

II. SYSTEM DESCRIPTION

Consider a multi-way relaying channel as shown in Fig. 1. There are K single-antenna users, which communicate with the aid of a relay with N antennas. We assume no direct link between any two of the users and half-duplex transmission in this slotted system. For each transmission of information exchange, all the users transmit to the relay in the uplink slot. The uplink slot is followed by multiple downlink slots. In each downlink slot, the relay precodes and broadcasts the regenerated symbols to all the users.

A. Multiple Access Phase

The received signal of the uplink slot at the relay is

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{v}, \tag{1}$$

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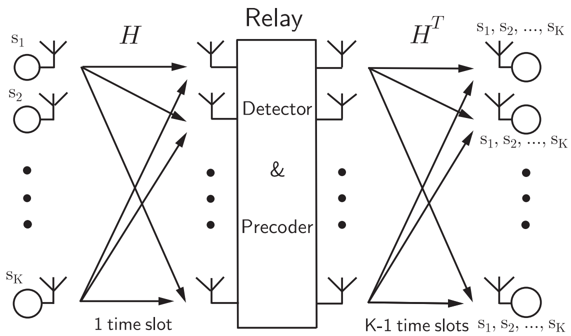

Fig. 1. Paradigm of regenerative multi-way relay system where K single antenna users are exchanging data via a multi-antenna relay with N antennas.

where $\mathbf{s} = [s_1, s_2, ..., s_K]^T$ is the symbol vector of the K users satisfying $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_K] \in \mathbb{C}^{N\times K}$ whose column vector $\mathbf{h}_k = [h_{1k}, h_{2k}, ..., h_{Nk}]^T$ represents channel impulse response (CIR) from the kth user to the relay, $\mathbf{v} \in \mathbb{C}^{N\times 1}$ is a zero-mean Gaussian noise vector at the relay with a covariance matrix $\mathbb{E}\{\mathbf{v}\mathbf{v}^H\} = \sigma_r^2\mathbf{I}_N$, and σ_r^2 is the noise variance at the relay. For simplicity, in this paper, we use the BPSK signalling between the relay and users. Note that more general signalling schemes, such as M-QAM, M-PSK, can be employed in our proposed precoder and receivers [8] in the forthcoming sections. Within one transmission, the channel is assumed to be static. In addition, we require the antenna number of the relay $N \geq K$.

B. Broadcast Phase

When the relay receive the uplink signals $\mathbf{r}$ of the K users, the relay detects $\mathbf{r}$ to regenerates $\mathbf{x}_m \in \mathbb{C}^{K\times 1}$ of downlink slot m, which will be shown in the sequel. We assume the uplink channel and downlink channel are reciprocal, such that the observation of downlink slot m at users is written as

$$\mathbf{y}_m = \mathbf{H}^T\mathbf{P}_m\mathbf{x}_m + \mathbf{w}_m, \quad 1 \leq m \leq K-1, \tag{2}$$

where $\mathbf{P}_m \in \mathbb{C}^{N\times K}$ is the precoding matrix of downlink slot m, $\mathbf{w}_m \in \mathbb{C}^{K\times 1}$ denotes the zero-mean Gaussian noise vector with covariance matrix $\mathbb{E}\{\mathbf{w}_m\mathbf{w}_m^H\} = \sigma^2\mathbf{I}_K$ for $\forall m$, and σ^2 is the noise variance at all users. $\mathbf{D}_m \in \mathbb{R}^{K\times K}$ shown in the following section is a permutation matrix for a specific traffic demand, and all the permutations should satisfy

$$\sum_{m=1}^{K-1}\mathbf{D}_m = \mathbf{J} - \mathbf{I}, \tag{3}$$

where $\mathbf{J}$ is a matrix with all elements to be "1", and $\mathbf{I}$ is an identity matrix [3]. Next, we will delicately design the precoding matrix $\mathbf{P}_m$.

III. MMSE Precoding for Regenerative Relay

In this section, we first illustrate the regeneration at the relay via linear MMSE detector. Afterwards, we propose a MMSE precoder specifically designed for the regenerative relay regarding the errors incurred by relay detection.

Given the observation vector $\mathbf{r}$ and uplink channel $\mathbf{H}$ at the relay, the estimate of the linear MMSE detector is written as

$$\mathbf{z} = \Re\{\mathbf{\Psi}^H\mathbf{r}\}, \tag{4}$$

where $\mathbf{\Psi} = \mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \sigma_r^2\mathbf{I}\right)^{-1} \in \mathbb{C}^{N\times K}$. Due to the simplicity of BPSK signalling, the decision of the received signal is $\hat{\mathbf{s}} = \operatorname{sgn}(\mathbf{z})$ with correlation matrix $\mathbb{E}\{\hat{\mathbf{s}}\hat{\mathbf{s}}^H\} = \mathbf{I}_K$. Note that in the literature there are various multiuser detectors, e.g., maximum likelihood (ML) detector, multi-stage detectors, etc. Here, the reason we adopt the linear MMSE detector lies in that its pair-wised error probability is analytical, which is an essential component when we propose the following MMSE precoder regarding relay detection error.

In particular, let $\mathbf{x}_m = \mathbf{D}_m\hat{\mathbf{s}}$ for downlink slot m. The precoding for downlink channel is designed according to $\mathbf{x}_m$ in downlink slot m. We assume full channel information is available at the relay. In order to minimize the mean square error between the original symbol vector $\mathbf{s}$ and its observation vector $\mathbf{y}_m$ in downlink slot m, we need to consider the error of relay decision.

Proposition 1: The MMSE precoder for regenerative relay in downlink slot m is

$$\mathbf{P}_{m,\text{MMSE}} = \left(\mathbf{H}^*\mathbf{H}^T + \lambda\mathbf{I}\right)^{-1}\mathbf{H}^*\mathbf{D}_m\hat{\mathbf{R}}_s\mathbf{D}_m^T, \tag{5}$$

where $\hat{\mathbf{R}}_s$ and λ are defined and calculated in appendix.

IV. Ordered MMSE-SIC Receiver

Without loss of generality, we next focus on one specific user and here let us say user 1 who obtains only its local CSI, i.e., $\mathbf{h}_1$. This user desires to detect the other $K-1$ symbols from the other users via ordered MMSE-SIC receivers.

To this end, we first establish equivalent channel for user 1 in time domain when using pseudo random precoding at the relay. Subsequently, we propose three MMSE-SIC schemes which have distinct ordering metrics. Note that different from the SIC schemes applied at multi-antenna receiver [7], in this paper, we exploit these SIC receivers over the multiple downlink slots where the non-cooperative users with single antenna harvest the diversity gain in time domain.

A. Equivalent Channel

In downlink slot m, the received signal at user 1 is

$$y_{1,m} = \mathbf{h}_1^T\mathbf{G}_m\mathbf{D}_m\hat{\mathbf{s}} + n_{1,m}, \;\; m \in [1, ..., K-1], \tag{6}$$

where $\mathbf{G}_m$ is the random precoding matrix at the relay in downlink slot m. For brevity, we next drop the subscript of user 1. During $K-1$ downlink time slots, user 1 collects its received signal into vector $\tilde{\mathbf{y}} = [y_{1,1}, y_{1,2}, ..., y_{1,K-1}]^T$ which can be expressed as

$$\tilde{\mathbf{y}} = \frac{1}{\beta}\tilde{\mathbf{H}}\tilde{\mathbf{G}}\tilde{\mathbf{D}}\hat{\mathbf{s}} + \mathbf{n} = \mathbf{H}_{eq}\hat{\mathbf{s}} + \mathbf{n}, \tag{7}$$

where $\tilde{\mathbf{H}} = \operatorname{diag}\left(\mathbf{h}_1^T, \mathbf{h}_1^T, ..., \mathbf{h}_1^T\right) \in \mathbb{C}^{(K-1)\times(K-1)N}$, $\tilde{\mathbf{G}} = \operatorname{diag}\left(\mathbf{G}_1, \mathbf{G}_2, ..., \mathbf{G}_{K-1}\right) \in \mathbb{R}^{(K-1)N\times(K-1)K}$, $\tilde{\mathbf{D}} = \left(\mathbf{D}_1^T, \mathbf{D}_2^T ..., \mathbf{D}_{K-1}^T\right)^T \in \mathbb{R}^{(K-1)K\times K}$, $\mathbf{n} = [n_{1,1}, n_{1,2}, ..., n_{1,K-1}]^T$ with $\mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma^2\mathbf{I}_{K-1}$ and $\operatorname{diag}(\cdot)$ stands for the diagonal operation on matrices or vectors.

In (7), we denote the equivalent channel established between the relay and user 1 during $K-1$ downlink slots as $\mathbf{H}_{eq}$, which can also be expressed as $\mathbf{H}_{eq} = [\mathbf{h}_{eq,1}, \mathbf{h}_{eq,2}, ..., \mathbf{h}_{eq,K}] \in$

$\mathbb{C}^{(K-1)\times K}$. Moreover, the power allocation factor β of each downlink slot can be derived as

$$\beta = \left(\frac{\mathbb{E}\left(\left(\mathbf{G}_m \mathbf{D}_m \hat{\mathbf{s}} \right)^H \mathbf{G}_m \mathbf{D}_m \hat{\mathbf{s}} \right)}{p} \right)^{\frac{1}{2}} = \left(\frac{K}{p} \right)^{\frac{1}{2}}. \quad (8)$$

In (8), we have $\mathbf{D}_m^H \mathbf{D}_m = \mathbf{I}_K$ for $\forall m$ according to the property of the rotation matrix and $\mathbf{G}_m^H \mathbf{G}_m = \mathbf{I}_K$ following the characteristic of pseudo random precoding at the relay which will be explained in the next subsection.

B. Pseudo Random Precoding at Relay

To exploit the MMSE-SIC receivers at users, we apply the pseudo random precoding [10] which facilitates the measurement process on detection reliability. Specifically, for downlink slot m, we randomly generate the matrix $\mathbf{\Phi}_m \in \mathbb{R}^{N\times N}$ with zero mean and unit variance Gaussian entries, then with the aid of QR decomposition, we decompose it as $\mathbf{\Phi}_m = \mathbf{G}_m^{all}\mathbf{S}_m$. As a consequence, $\mathbf{G}_m^{all}$ and $\mathbf{S}_m$ are orthonormal and upper triangle matrix, respectively. In order to randomly precode the vector $\hat{\mathbf{s}}$, the first K column vectors of $\mathbf{G}_m^{all}$ are selected and denoted as $\mathbf{G}_m$.

Notice that the pseudo random precoding matrix can be known at users via two potential ways: 1) the seed for generating random matrix is shared between the relay and users; 2) the random matrix are generated offline priori to the broadcast phase.

C. Ordered MMSE-SIC Receiver

In the sequel, we propose three MMSE-SIC receivers whose interference cancellation order depends on the output of linear MMSE estimate, SINR or both of them, respectively. For user 1, we first derive the first two metrics determining the SIC order via linear MMSE estimate. Subsequently, aiming to measure the third metric in the sense of ML, we adopt Gaussian approximation and explain how to achieve this metric when the relay applies the random precoding.

Before carrying out the SIC, the self-interference from user 1 should be cancelled, such that we have $\bar{\mathbf{y}} = \tilde{\mathbf{y}} - \mathbf{h}_{eq,1}s_1$. With the equivalent channel at hand, the initial MMSE estimate can be derived as

$$\mathbf{u} = \Re\{\mathbf{W}^H \bar{\mathbf{y}}\}, \quad (9)$$

in which $\mathbf{W} = \mathbf{R}_{\bar{y}}^{-1}\bar{\mathbf{H}}_{eq}$ is the linear MMSE weight matrix, $\bar{\mathbf{H}}_{eq} = [\mathbf{0}_{K-1}, \mathbf{h}_{eq,2}, ..., \mathbf{h}_{eq,K}] \in \mathbb{C}^{K-1\times K}$ is obtained by replacing the first column of $\mathbf{H}_{eq}$ with appropriate zero vector and the correlation matrix of $\bar{\mathbf{y}}$ is

$$\mathbf{R}_{\bar{y}} = \bar{\mathbf{H}}_{eq}\bar{\mathbf{H}}_{eq}^H + \sigma^2\mathbf{I}_{K-1}. \quad (10)$$

Next, for the purpose of measuring the detection reliability of each symbol individually, here we assume the reliability between symbols are mutually independent. Specifically, to measure the kth symbol from user k, we note down the kth column of $\mathbf{W}$ as

$$\omega_k = \frac{\mathbf{R}_k^{-1}\mathbf{h}_{eq,k}}{1+\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\mathbf{h}_{eq,k}}, \quad k \in [2, ..., K], \quad (11)$$

where $\mathbf{R}_k = \sum_{l\neq k, l\neq 1} \mathbf{h}_{eq,l}\mathbf{h}_{eq,l}^H + \sigma^2\mathbf{I}_{K-1}$.

1) Magnitude of MMSE Estimate-based Receiver: To be explicit, we express the kth entry of $\mathbf{u}$ as

$$\begin{aligned} u_k = &\frac{\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\mathbf{h}_{eq,k}}{1+\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\mathbf{h}_{eq,k}}\hat{s}_k \\ &+ \underbrace{\Re\left\{ \frac{\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}}{1+\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\mathbf{h}_{eq,k}} \left(\sum_{l\neq k, l\neq 1} \mathbf{h}_{eq,l}\hat{s}_l + \mathbf{n} \right) \right\}}_{G_k}. \end{aligned} \quad (12)$$

Then straightforwardly, in case of BPSK modulation, the magnitude of the linear MMSE estimate's output, i.e., $|u_k|$ can be viewed as the metric indicating the detection reliability. To be more specific, the higher value $|u_k|$ has, the higher detection reliability can be obtained for symbol $\hat{s}_k$.

2) SINR-based Receiver: Alternatively, we may measure the detection reliability by deriving $\text{SINR}_k, k \in [2, ..., K]$. When given (12), we derive the SINR_k as follows

$$\gamma_k = \mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\mathbf{h}_{eq,k} = \frac{\mathbf{h}_{eq,k}^H\mathbf{R}_{\bar{y}}^{-1}\mathbf{h}_{eq,k}}{1-\mathbf{h}_{eq,k}^H\mathbf{R}_{\bar{y}}^{-1}\mathbf{h}_{eq,k}}, \quad (13)$$

which can be used in the SINR-based MMSE-SIC receiver.

3) ML-based Receiver: In what follows, we are going to demonstrate how the metric on detection reliability is achieved in the sense of ML. Fortunately, once the aforementioned pseudo random precoding is used at the relay, the equivalent interference-plus-noise term observed by user 1 can be well approximated by zero mean Gaussian random variable G_k [11] with variance

$$\begin{aligned} \sigma_k^2 = \mathbb{E}\left\{G_k^2\right\} = &\frac{1}{2(1+\gamma_k)^2}\mathbb{E}\Bigg\{\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\times \\ &\left(\sum_{l\neq k, l\neq 1} \mathbf{h}_{eq,l}\hat{s}_l + \mathbf{n} \right)\left(\sum_{l\neq k, l\neq 1} \mathbf{h}_{eq,l}\hat{s}_l + \mathbf{n} \right)^H \mathbf{R}_k^{-1}\mathbf{h}_{eq,k}\Bigg\} \\ = &\frac{1}{2(1+\gamma_k)^2}\left(\mathbf{h}_{eq,k}^H\mathbf{R}_k^{-1}\mathbf{R}_k\mathbf{R}_k^{-1}\mathbf{h}_{eq,k}\right) \\ = &\frac{\gamma_k}{2(1+\gamma_k)^2}. \end{aligned} \quad (14)$$

Hence given $\hat{s}_k$, u_k can be considered as a Gaussian random variable with mean $m_k = \frac{\gamma_k}{1+\gamma_k}\hat{s}_k$ and variance σ_k^2.

In particular, the reliability of $\hat{s}_k, k \in [2, ..., K]$ in the context of maximum a posteriori (MAP) is given as

$$L_k^{map} = \left| \ln\left[\frac{f(\hat{s}_k = +1|\mathbf{u})}{f(\hat{s}_k = -1|\mathbf{u})} \right] \right|, \quad (15)$$

where $f(\hat{s}_k = \pm 1|\mathbf{u})$ denotes the probability density function (PDF) conditioned on $\mathbf{u}$. However based on (15), the exhaustive searching for the reliability may require unaffordable complexity, i.e., $O(2^{K-1})$ when the number of users involved is relatively high.

For the purpose of reducing the searching burden, we further assume the entries in $\mathbf{u}$ are mutually independent random variables, therefore, we may evaluate the detection reliability using the ML criteria in the form of log-likelihood ratio (LLR) as

$$L_k^{ml} = \left| \ln\left[\frac{f(u_k|\hat{s}_k = +1)}{f(u_k|\hat{s}_k = -1)} \right] \right|, \quad (16)$$

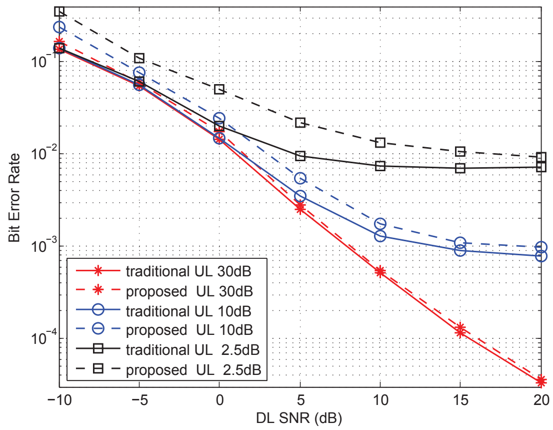

Fig. 2. Comparisons between the proposed MMSE precoder and the traditional precoder at the low, mediate and high uplink SNR regimes.

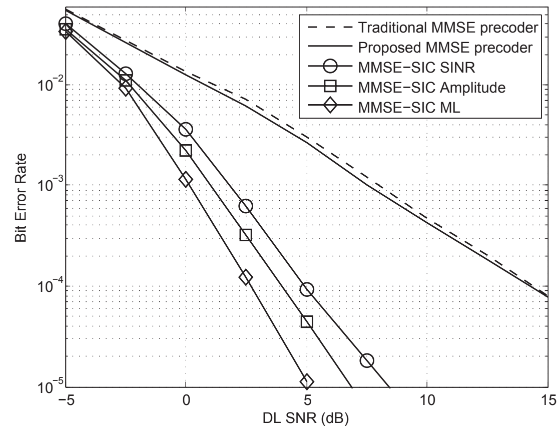

Fig. 3. Performance achieved by various MMSE-SIC detectors and linear MMSE precoders when the uplink SNR is 30dB.

where the conditional PDF is listed below

$$f(u_k|\hat{s}_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left[-\frac{(u_k - m_k)^2}{2\sigma_k^2}\right]. \tag{17}$$

Upon applying the Gaussian approximation and substituting (17) into (16), we consequently obtain the detection reliability of $\hat{s}_k, k \in [2, ..., K]$ as

$$L_k^{ml} = 4\left(1 + \gamma_k\right)|u_k|, \tag{18}$$

in which the metric on detection reliability invokes both the SINR_k and $|u_k|$ to arrange the SIC order.

V. Simulation Results

In this section, we provide the simulation results on average bit error rate (BER) conditioned on fully loaded MWR system. Without other statement, we specify $K = N = 10$ and assume independent quasi-static Rayleigh fading channel of each link.

In Fig. 2, we illustrate the error performance achieved by the proposed MMSE precoder in contrast with that of the traditional MMSE precoder. To highlight the effect of relay decision error, we set the uplink SNR in the low, mediate and high regime, respectively. One may observe that when the decision error at the relay dominates the performance and incurs error floor in bad uplink channel, the proposed MMSE precoder outperforms its counterpart in achieving the error floor faster. As expected, once the relay regenerates the symbols with lower error rate, the performance gap shrinks. In the high uplink SNR regime, the two MMSE precoders converge as the relay regenerates user symbols with even higher confidence. By checking the formulation of the proposed MMSE precoder in (5), one may find that as $\hat{\mathbf{R}}_s$ approaches to $\mathbf{I}_K$ in the high uplink SNR regime, the proposed precoder mathematically converges to the traditional MMSE precoder.

Next, to avoid the error floor incurred by linear relay detection, we allocate high uplink SNR to all the users, i.e., 30dB. It can be seen in Fig. 3 that the ML-based MMSE-SIC receiver is advantageous to the other multi-stage receivers because of the optimal detection order measured in the sense of ML. Meanwhile, the magnitude-based MMSE-SIC receiver outperforms the SINR-based MMSE-SIC receiver slightly. And not surprisingly, the proposed linear MMSE precoder achieves no diversity gain in the fully loaded MWR system. Notice that the proposed MMSE-SIC receivers reap the diversity gain in time domain at the expense of detection complexity and time delay when compared with linear precoder.

Finally, in Fig. 3, we concentrate on the ML-based MMSE-SIC receiver and show the error performance when we classify $K-1$ different levels on detection reliability. And we observe huge performance gap between various BER curves ranging from the most reliable to the most unreliable.

VI. Conclusion

In this paper, we employed the regenerative multi-antenna relay to support the information exchange among multiple users. First, we proposed a MMSE precoder while taking the relay detection error into account. This proposed MMSE precoder is superior to the traditional MMSE precoder in terms of BER, especially in the low uplink SNR regime. Second, we apply pseudo random precoding at the relay to make the interference-plus-noise term approach Gaussian, and then apply three ordered MMSE-SIC receivers at single-antenna users. Simulation results on BER indicated that these MMSE-SIC receivers can reap the temporal diversity gains over multiple downlink slots for each user.

Appendix
Derivation of Proposition 1

We henceforth drop the subscripts of $\mathbf{y}_m$, $\mathbf{P}_m$ and $\mathbf{D}_m$ during the derivation for brevity. However, the precoders vary in different downlink slots. We aim to determine the precoding matrix $\mathbf{P}_{\text{MMSE}}$ in order to minimize the mean square error between the observation vector $\mathbf{y}$ and its original vector $\mathbf{s}$, i.e.

$$\mathbf{P}_{\text{MMSE}} = \arg\min_{P} J\left(\mathbf{P}\right), \tag{19}$$

where the MSE function is defined as

$$J\left(\mathbf{P}\right) = \mathbb{E}\left(||\mathbf{y} - \mathbf{D}\mathbf{s}||^2\right). \tag{20}$$

Subsequently, by substituting (2) into (20), we rewrite the MSE function as

$$\begin{aligned} &J\left(\mathbf{P}\right) \\ &= \text{Tr}\left\{\mathbb{E}[(\mathbf{H}^T\mathbf{P}\mathbf{D}\hat{\mathbf{s}} + \mathbf{w} - \mathbf{D}\mathbf{s})(\mathbf{H}^T\mathbf{P}\mathbf{D}\hat{\mathbf{s}} + \mathbf{w} - \mathbf{D}\mathbf{s})^H]\right\} \\ &= \text{Tr}\Big\{\mathbf{H}^T\mathbf{P}\mathbf{D}\mathbf{R}_s\mathbf{D}^T\mathbf{P}^H\mathbf{H}^* - \mathbf{H}^T\mathbf{P}\mathbf{D}\hat{\mathbf{R}}_s^H\mathbf{D}^T \\ &\qquad -\mathbf{D}\hat{\mathbf{R}}_s\mathbf{D}^T\mathbf{P}^H\mathbf{H}^* + \mathbf{D}\mathbf{R}_s\mathbf{D}^T + \sigma^2\mathbf{I}\Big\}, \end{aligned} \tag{21}$$

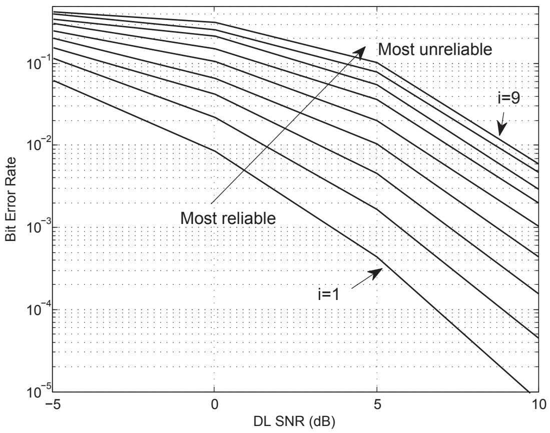

Fig. 4. Layered BER performance achieved at each user with ML-based MMSE-SIC receiver where $i = 1$ and $i = 9$ denote the most reliable and most unreliable bits, respectively.

where $\mathbf{R}_s = \mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbb{E}\{\hat{\mathbf{s}}\hat{\mathbf{s}}^H\} = \mathbf{I}_K$ and $\hat{\mathbf{R}}_s = \mathbb{E}\{\mathbf{s}\hat{\mathbf{s}}^H\}$ denotes the correlation matrix between original and estimated symbol vectors.

Specifically, the kth diagonal element in $\hat{\mathbf{R}}_s$ can be achieved according to the law of total probability as

$$\hat{\mathbf{R}}_{s\ (k,k)} = \sum_{s_k \in \mathbf{\Omega_k}} \sum_{\hat{s}_k \in \mathbf{\Omega_k}} s_k \hat{s}_k^* Pr\left(s_k\right) Pr\left(s_k \rightarrow \hat{s}_k\right), \quad (22)$$

where $\mathbf{\Omega}_k$ is the constellation set used by user k, $Pr\left(s_k\right)$ denotes the probability that symbol s_k is selected and $Pr\left(s_k \rightarrow \hat{s}_k\right)$ denotes the pair-wised transition probability from s_k to $\hat{s}_k$. For instance, if the BPSK signalling is applied, then we have the kth diagonal entry

$$\begin{aligned}\hat{\mathbf{R}}_{s\ (k,k)}^{BPSK} &= \mathbb{E}_{s_k}\{s_k s_k^* | \text{Correct}\} P_c^{(k)} + \mathbb{E}_{s_k}\{-s_k s_k^* | \text{Error}\} P_e^{(k)} \\ &= 1 - 2P_e^{(k)}, \quad (23)\end{aligned}$$

where $P_e^{(k)}$ denotes the error probability of user k at the relay.

We consider the relay subject to a transmit power constraint,

$$\text{Tr}\{\mathbf{P}\mathbf{P}^H\} \leq p. \quad (24)$$

Therefore, regarding the MSE in (20), we have the Lagrangian dual function as

$$\bar{J}\left(\mathbf{P}\right) = J(\mathbf{P}) + \lambda\left(\text{Tr}\left\{\mathbf{P}\mathbf{P}^H\right\} - p\right), \quad (25)$$

where λ is Lagrange multiplier. Differentiating (25) with respect to $\mathbf{P}^*$ yields

$$\frac{\partial \bar{J}(\mathbf{P})}{\partial \mathbf{P}^*} = \mathbf{H}^*\mathbf{H}^T\mathbf{P} - \mathbf{H}^*\mathbf{D}\hat{\mathbf{R}}_s\mathbf{D}^T + \lambda\mathbf{P}. \quad (26)$$

Then let (26) be zero, we derive the optimal precoding matrix $\mathbf{P}_{\text{MMSE}}$ as

$$\mathbf{P}_{\text{MMSE}} = \left(\mathbf{H}^*\mathbf{H}^T + \lambda\mathbf{I}\right)^{-1} \mathbf{H}^*\mathbf{D}\hat{\mathbf{R}}_s\mathbf{D}^T. \quad (27)$$

Consider the power constraint of the relay. Plugging (27) into (24) gives

$$\text{Tr}\left\{\left(\mathbf{H}^*\mathbf{H}^T + \lambda\mathbf{I}\right)^{-1} \mathbf{H}^*\mathbf{D}\hat{\mathbf{R}}_s^2\mathbf{D}^T\mathbf{H}^T \left(\mathbf{H}^*\mathbf{H}^T + \lambda\mathbf{I}\right)^{-1}\right\} \leq p, \quad (28)$$

which involves high computational complexity when calculating λ. To solve the parameter λ without incurring complicated numerical calculation, we here exploit the eigenvalue decomposition, i.e., $\mathbf{H}^*\mathbf{H}^T = \mathbf{Q}\Lambda\mathbf{Q}^H$, where $\Lambda = \text{diag}\{\lambda_1, ..., \lambda_K, 0, ..., 0\} \in \mathbb{R}^{N \times N}$ consists of K eigenvalues of matrix $\mathbf{H}^*\mathbf{H}^T$ with rank K. In addition, let $\tilde{\Lambda} = \text{diag}\{\lambda, ..., \lambda\} \in \mathbb{R}^{N \times N}$. As a consequence, we have

$$\left(\mathbf{H}^*\mathbf{H}^T + \lambda\mathbf{I}_N\right)^{-1} = \mathbf{Q}\left(\Lambda + \tilde{\Lambda}\right)^{-1}\mathbf{Q}^H. \quad (29)$$

Then substituting (29) into (28), we get

$$\begin{aligned}&\text{Tr}\left\{\mathbf{Q}\left(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{Q}^H\mathbf{H}^*\mathbf{D}\hat{\mathbf{R}}_s^2\mathbf{D}^T\mathbf{H}^T\mathbf{Q}\left(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{Q}^H\right\} \\ &= \text{Tr}\left\{\left(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}\right)^{-2}\left(\mathbf{H}^T\mathbf{Q}\right)^H\mathbf{D}\hat{\mathbf{R}}_s^2\mathbf{D}^T\mathbf{H}^T\mathbf{Q}\right\} \leq p. \quad (30)\end{aligned}$$

Recall the eigenvalue decomposition used previously, the downlink channel matrix can thus be decomposed as

$$\mathbf{H}^T = \mathbf{U}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{Q}^H, \quad (31)$$

and the relay power consumption (30) is simplified as

$$\text{Tr}\left\{\mathbf{\Lambda}\left(\mathbf{\Lambda} + \tilde{\mathbf{\Lambda}}\right)^{-2}\mathbf{U}^H\mathbf{D}\hat{\mathbf{R}}_s^2\mathbf{D}^T\mathbf{U}\right\} \leq p. \quad (32)$$

Assume the relay transmits with the maximum power, we can express (32) as

$$\sum_{i=1}^{K} \frac{\lambda_i}{\left(\lambda_i + \lambda\right)^2}\left[\mathbf{U}^H\mathbf{D}\hat{\mathbf{R}}_s^2\mathbf{D}^T\mathbf{U}\right]_{(i,i)} = p, \quad (33)$$

where $[\cdot]_{(i,i)}$ denotes the ith diagonal element of a matrix. Therefore, the Lagrangian multiplier λ can be solved numerically.

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