Bound Analysis of Physical Layer Network Coding in Interference-Limited Two-way Relaying System

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***Abstract*—In this paper, performance of three-time slot (3TS) physical layer network coding (PNC) is investigated in interference-limited two-way relaying system. Effective lower and upper bounds are derived for important performance metrics like outage probability and average bit error rate (BER) with different modulation modes. Asymptotic behavior is also proposed to intuitively exhibit the trend of performance in high signal-to-interference and noise ratio (SINR) regime. We differentiate practical applications with different relay power for the bounds. A particular scenario with positional grid structure of sources, relay and co-channel interferers is established for numerical analysis, and the effects of some critical parameters, such as relay's location, forwarding capability and power allocation factor, are compared and discussed. The theoretic solutions are finally verified by simulation results.**

***Index Terms*—Two-way relaying, physical layer network coding, co-channel interference, outage probability, average BER.**

I. Introduction

Two-way relaying, in which two sources exchange their information with the aid of one relay, was early introduced in [1]. Instead of regarding it as an immediate concatenation of two one-way relaying process, spectrum efficiency is remarkably enhanced by allowing two sources transmitting/receiving with shared channel resource. Zhang *et al.* suggested a physical-layer network coding (PNC) scheme for the two-way structure [2], which needs only two time slots for the two-source communication by making use of the broadcast nature of radio. Critical performance metrics like outage probability, average bit error rate (BER) and sum rate for diverse evolved PNC schemes were derived in explicit or approximate forms for Rayleigh channels in [3], and the authors suggested that a three-time slot (3TS) scheme was valuable since it offered a good compromise between data rate and error probability.

Interference is a widespread problem in actual wireless networks, and performance evaluation of interference-limited relay system have recently gained a lot of attraction. Lee *et al.* considered a multi-cell environment where co-channel interference was generated from adjacent cells, and decode-and-forward (DF) outage probability was analyzed [4]. Zhong *et al.* studied the explicit outage probability of a simplified model in which interference only impaired the destination while relay had only noise [5]. An opposite model of interfered relay was analyzed in [6]. [7] proposed a more general model considering independent interference at both relay and destination; noise was ignored for mathematical simplicity. Recent research [8] extended the relaying structure to an arbitrary N-hop case, utilizing some effective signal-to-interference and noise ratio (SINR) bounds for outage and BER calculation. However, all existing works above focus on one-way relaying system. A multiuser two-way relaying model was considered in [9] with interference among spreading signatures of different users at relay, but the analysis is too limited to be applied to systems other than CDMA; besides, few interference study has been seen on two-way relaying to the best of our knowledge, which may be due to the intricate impairment of interference on end-to-end (E2E) performance in the bi-directional forwarding.

In this paper, we investigate a two-way relaying system with co-channel interference, and evaluate the system performance for 3TS PNC scheme in terms of outage probability and average BER. Since the original E2E SINR expression is mathematically untractable, we turn to derive a couple of effective lower and upper bounds of performance, which are well supported by numerical simulations. Asymptotic form of the bounds are also proposed to intuitively exhibit high SINR behavior. A practical classification according to relay's forwarding power is introduced, *i.e.* a dedicated relay station may have greater power over that of a mobile terminal (source), while a cooperative user terminal would like to contribute only a small fraction of its power to relay for others. We demonstrate that lower bounds of outage and BER matches the former scenario well, and upper bounds for the latter one. A particular scenario with relay, sources and interferers positioned in a grid structure is then established for numerical analysis. Some critical parameters of the system are further investigated, such as relay's location, forwarding power and power allocation factors, along with their impact on the accuracy of different bounds.

II. System Model

Consider a two-way relaying system consisting of two source nodes A, B which need to exchange message via a relay node R using amplify-and-forward (AF) protocol, and several co-channel interferers denoted by the universal interferer set $\mathcal{I} = \{I_1, \cdots, I_N\}$. An interferer may transmit at arbitrary time slot during the relay process, and notation $\mathcal{I}_n \subseteq \mathcal{I}$ is used to indicate the subset of activated interferers in the n-th time slot. All channels are modeled as slow Rayleigh fading, and are denoted by: h_A (channel between A and R), h_B (B and R), $h_{A,B}$ (A and B); $g_{A,j}$ (A and interferer I_j), $g_{B,j}$ (B and I_j), and $g_{R,j}$ (R and I_j). Signal x_A, x_B of source A and B have equal transmission power of P_S, and power of relay's forwarding signal x_R is denoted by $P_R = cP_S$, where c is used to weigh the relay's forwarding power versus the sources. Interference signal $z_{i,j}$ generated by I_j in the i-th time slot has fixed power Q_j. For self-interference cancellation, source A (or B) shall have the exact knowledge of instantaneous channel state information (CSI) h_A (or h_B).

Received signal at R in the 1st time slot is:

$$y_{1,R} = h_A x_A + n_{1,R} + \sum_{i \in \mathcal{I}_1} g_{R,i} z_{1,i} \tag{1}$$

and in the 2nd time slot:

$$y_{2,R} = h_B x_B + n_{2,R} + \sum_{i \in \mathcal{I}_2} g_{R,i} z_{2,i} \tag{2}$$

Using PNC, forwarded signal of R in the 3rd time slot is [3]:

$$x_{3,R} = G\left(a_1 y_{1,R} + a_2 y_{2,R}\right) \tag{3}$$

where $n_{1,R}, n_{2,R}, n_{3,A} \sim \mathcal{CN}(0, \sigma_n^2)$ denote the additive white Gaussian noise (AWGN),

$$G = \sqrt{\frac{P_R}{a_1^2 P_A |h_A|^2 + a_2^2 P_B |h_B|^2}} \tag{4}$$

is the amplitude forwarding gain of R, and a_1, a_2 are the power allocation factors satisfying $a_1^2 + a_2^2 = 1$. The power allocation factors are constants invariant with instantaneous CSI, which makes the system more practical, and simplifies latter analysis as well; they may be optimized based on statistical CSI. After self interference cancellation, A receives

$$\begin{aligned} y'_{3,A} \quad &= \quad G a_2 h_A h_B x_B + n_{3,A} + \sum_{j \in \mathcal{I}_3} g_{A,j} z_{3,j} \\ &\quad + G a_1 h_A \left(n_{1,R} + \sum_{i \in \mathcal{I}_1} g_{R,i} z_{1,i}\right) \\ &\quad + G a_2 h_A \left(n_{2,R} + \sum_{i \in \mathcal{I}_2} g_{R,i} z_{2,i}\right) \end{aligned} \tag{5}$$

Here we only consider the two-hop receiving via relay and disregard the direct link between the sources as [3]. Final SINR expressions of A's received signal is derived by (6), where $\rho_A = P_S|h_A|^2, \rho_B = P_S|h_B|^2$, $\eta_{R,j} = Q_j|g_{R,j}|^2, \eta_{A,j} = Q_j|g_{A,j}|^2$. For B the derivation is just similar, and we only consider the single source (say, A) performance in the rest of this paper if not specified.

A. SINR Bounds of 2-Hop Transmission

The original SINR expression (6) are too complicated for performance analysis, therefore we turn to some appropriate upper and lower bounds as follows:

$$\gamma_{A,\text{ub}} \quad = \quad \min\left(\frac{P_B}{\sigma_n^2 + ||\mathbf{Q}_R||_1}, \frac{c\rho_A}{\sigma_n^2 + ||\mathbf{Q}_A||_1}\right) \tag{7}$$

$$\gamma_{A,\text{lb}} \quad \approx \quad \frac{P_{A,B}}{N_A + ||\mathbf{Q}_{R,A}||_1} \tag{8}$$

where the notations are given by (9) and (10), and approximation $\frac{1}{2}H(x_1, x_2) \approx \min(x_1, x_2)$ is utilized ($H(x_1, x_2) = 2(x_1^{-1} + x_2^{-1})^{-1}$ is the harmonic mean function) in derivation. $||\mathbf{Y}||_1 = \sum_{i=1}^n |Y_i|$ denotes the L_1 norm of vector $\mathbf{Y}$.

Remarks:

1) SINR Upper Bound and Dedicated Relay Scenario: A Large c Case: Dedicated relay station, which are deployed in the service region as component of network infrastructure (*e.g.* in 802.16j [10]), may have much greater capability of processing and power amplifying over the mobile terminals (sources). This application results in a large value of parameter c. Meanwhile, the SINR upper bound (7) is derived by neglecting $a_1^2\rho_A$ in the $\frac{1}{c}(a_1^2\rho_A + a_2^2\rho_B)$ term in the denominator of (6), which may be of little deviation when c is sufficiently large.

2) SINR Lower Bound and User Cooperation Scenario: A Small c Case: User cooperation is a distributed structure exploiting the potential of multiuser diversity. Idle user terminals may forward the signal for other users who are experiencing poor channel condition to the destination, with some elaborate cooperation algorithm which should not harm the experience of the cooperators. As a result, a relay of user terminal kind should not be expected to forward with great power, which corresponds to a small c case. On the other hand, the lower bound of SINR (8) is obtained by substituting ρ_A in the first term of the denominator with $\frac{1}{a_1^2}(a_1^2\rho_A + a_2^2\rho_B)$ (in (6)), which is still tight when c is sufficiently small since the second term of the denominator is dominating.

III. Outage Analysis

Outage event of single source A is defined by the E2E received SINR of A falling below an acceptable threshold γ_{th}, which has the probability

$$\mathcal{P}_{\mathcal{O},A}(\gamma_{\text{th}}) = F_{\gamma_A}(\gamma_{\text{th}}) \tag{11}$$

A. Exact Outage Bounds of 2-Hop Transmission

Before further exhibition to exact outage bounds, following lemmas are proposed to facilitate the expressions, which generalizes the distribution of the elements in the bounds formulas (7) (8).

Lemma 1 (PDF of chi-square distribution): $\mathbf{Y} = [Y_1, Y_2, \cdots, Y_n]$ is an exponentially distributed random vector with all elements independent. The summation $||\mathbf{Y}||_1$ follows a generalized chi-square distribution of which the probability density function (PDF) is (see [7])

$$f_{||\mathbf{Y}||_1}(u) = \sum_{k=1}^{r(\bar{\mathbf{Y}})} \sum_{l=1}^{v_k(\bar{\mathbf{Y}})} \frac{\varphi_{k,l}(\bar{\mathbf{Y}})}{\Gamma(l)\bar{Y}_{[k]}^l} u^{l-1} e^{-\frac{u}{\bar{Y}_{[k]}}} \tag{12}$$

where $\bar{Y}_{[k]}, k = 1, 2, \cdots, r(\bar{\mathbf{Y}})$ is the unique values in the mean vector $\bar{\mathbf{Y}} = [\bar{Y}_1, \bar{Y}_2, \cdots, \bar{Y}_n]$ with repetition number $v_k(\bar{\mathbf{Y}})$, which satisfies $\sum_{k=1}^{r(\bar{\mathbf{Y}})} v_k(\bar{\mathbf{Y}}) = n$. Coefficients $\varphi_{k,l}(\bar{\mathbf{Y}})$ is determined by (13), in which the set for summation $\Omega_{k,l}(\bar{\mathbf{Y}})$ is given by (14).

Lemma 2 (CDF of elements in (7) (8)*):* Suppose X is an exponentially distributed random variable, $\mathbf{Y} = [Y_1, Y_2, \cdots, Y_n]$

$$\gamma_A^{\text{3TS}} = \frac{a_2^2\rho_A\rho_B}{\rho_A\left(\sigma_n^2 + a_1^2\sum_{i\in\mathcal{I}_1}\eta_{R,i} + a_2^2\sum_{i\in\mathcal{I}_2}\eta_{R,i}\right) + \frac{1}{c}(a_1^2\rho_A + a_2^2\rho_B)\left(\sigma_n^2 + \sum_{j\in\mathcal{I}_3}\eta_{A,j}\right)} \tag{6}$$

$$P_B = a_2^2\rho_B; \quad \mathbf{Q}_R = \left[a_1^2[\eta_{R,i\in\mathcal{I}_1\setminus\mathcal{I}_2}], a_2^2[\eta_{R,j\in\mathcal{I}_2\setminus\mathcal{I}_1}], [\eta_{R,k\in\mathcal{I}_1\cap\mathcal{I}_2}]\right]; \quad \mathbf{Q}_A = [\eta_{A,l\in\mathcal{I}_3}] \tag{9}$$

$$P_{A,B} = \min(a_1^2\rho_A, a_2^2\rho_B);\ N_A = \left(1+\frac{a_1^2}{c}\right)\sigma_n^2;\ \mathbf{Q}_{R,A} = \left[a_1^2[\eta_{R,i\in\mathcal{I}_1\setminus\mathcal{I}_2}], a_2^2[\eta_{R,j\in\mathcal{I}_2\setminus\mathcal{I}_1}], [\eta_{R,k\in\mathcal{I}_1\cap\mathcal{I}_2}], \frac{a_1^2}{c}[\eta_{A,l\in\mathcal{I}_3}]\right] \tag{10}$$

$$\varphi_{k,l}(\bar{\mathbf{Y}}) = \left(-\bar{Y}_{[k]}\right)^{-v_k(\bar{\mathbf{Y}})+l}\sum_{\Omega_{k,l}(\bar{\mathbf{Y}})}\prod_{\substack{j=1\\ j\neq k}}^{r(\bar{\mathbf{Y}})}\binom{v_j(\bar{\mathbf{Y}})+q_j-1}{q_j}\ \bar{Y}_{[j]}^{q_j}\left(1-\frac{\bar{Y}_{[j]}}{\bar{Y}_{[k]}}\right)^{-v_j(\bar{\mathbf{Y}})-q_j} \tag{13}$$

$$\Omega_{k,l}(\bar{\mathbf{Y}}) = \left\{\left(q_1,\cdots,q_{r(\bar{\mathbf{Y}})}\right)\in\mathbb{Z}^{r(\bar{\mathbf{Y}})}\,\middle|\, q_k = 0, \sum_{j=1}^{r(\bar{\mathbf{Y}})} q_j = v_k(\bar{\mathbf{Y}}) - l\right\} \tag{14}$$

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is an exponential random vector as stated in Lemma 1, and w a positive constant, then the cumulative distribution function (CDF) of a new random variable $Z = \frac{X}{w+||\mathbf{Y}||_1}$ is

$$\begin{aligned} F_Z(z) &= 1 - \sum_{k=1}^{r(\bar{\mathbf{Y}})}\sum_{l=1}^{v_k(\bar{\mathbf{Y}})}\varphi_{k,l}(\bar{\mathbf{Y}})\left(1+\frac{\bar{Y}_{[k]}}{\bar{X}}z\right)^{-l} e^{-\frac{w}{\bar{X}}z} \\ &\triangleq \Theta(z; w, \bar{X}, \bar{\mathbf{Y}}) \end{aligned} \tag{15}$$

where $\bar{X}$ is the mean value of X, and parameters related to $\bar{\mathbf{Y}}$ are as introduced in Lemma 1.

Theorem 1: The outage probability of 3TS PNC scheme is bounded by the following lower bound:

$$\begin{aligned} \mathcal{P}_{\mathcal{O},A}(\gamma_{\text{th}})|_{\text{lb}} &= 1 - \left(1 - \Theta\left(\gamma_{\text{th}}; \sigma_n^2, \bar{P}_B, \bar{\mathbf{Q}}_R\right)\right) \times \\ &\quad \left(1 - \Theta\left(\gamma_{\text{th}}; \sigma_n^2, c\bar{\rho}_A, \bar{\mathbf{Q}}_A\right)\right) \end{aligned} \tag{16}$$

and the approximate upper bound

$$\mathcal{P}_{\mathcal{O},A}(\gamma_{\text{th}})|_{\text{ub}} \approx \Theta\left(\gamma_{\text{th}}; N_A, \bar{P}_{A,B}, \bar{\mathbf{Q}}_{R,A}\right) \tag{17}$$

The parameters, *e.g.* $\bar{\mathbf{Q}}_R$, $r(\bar{\mathbf{Q}}_R)$, $v_k(\bar{\mathbf{Q}}_R)$, $\varphi_{k,l}(\bar{\mathbf{Q}}_R)$ and $\Omega_{k,l}(\bar{\mathbf{Q}}_R)$ *etc.* are determined as stated in Lemma 1, and

$$\bar{P}_B = a_2^2\bar{\rho}_B, \quad \bar{P}_{A,B} = \frac{1}{2}H(a_1^2\bar{\rho}_A, a_2^2\bar{\rho}_B) \tag{18}$$

are the mean values of P_B and $P_{A,B}$.

Theorem 1 is an immediate consequence by applying Lemma 2 to the SINR bound expressions (7) (8). Note that a upper (lower) bound of SINR corresponds to a lower (upper) bound of outage probability.

B. Asymptotic Behavior

Asymptotic form of outage is useful for evaluating the performance in high SINR regime in a more intuitive and concise way, and follow lemma is to be utilized.

Lemma 3: The asymptotic form of CDF (15) as $\bar{X}\to\infty$ is

$$\Theta(z; w, \bar{X}, \bar{\mathbf{Y}}) = \Theta_1(w, \bar{\mathbf{Y}})\frac{z}{\bar{X}} + o\left(\frac{z}{\bar{X}}\right) \tag{19}$$

where the multiplier

$$\Theta_1(w, \bar{\mathbf{Y}}) = w + \sum_{k=1}^{r(\bar{\mathbf{Y}})}\sum_{l=1}^{v_k(\bar{\mathbf{Y}})}\varphi_{k,l}(\bar{\mathbf{Y}})\bar{Y}_{[k]}l \tag{20}$$

is only related to the strength of interference and noise.

Lemma 3 is the first order Taylor extension of (15) with $\frac{z}{\bar{X}} \to 0$. Applying the lemma to Theorem 1, following asymptotic solutions are derived for the outage bounds.

Corollary 1: The asymptotic form of 2-hop outage bounds are given by

$$\mathcal{P}_{\mathcal{O},A}(\gamma_{\text{th}})|_{\text{lb,asym}} = \frac{\gamma_{\text{th}}}{\bar{P}_B}\Theta_1(\sigma_n^2, \bar{\mathbf{Q}}_R) + \frac{\gamma_{\text{th}}}{c\bar{\rho}_A}\Theta_1(\sigma_n^2, \bar{\mathbf{Q}}_A) \tag{21}$$

$$\mathcal{P}_{\mathcal{O},A}(\gamma_{\text{th}})|_{\text{ub,asym}} = \frac{\gamma_{\text{th}}}{\bar{P}_{A,B}}\Theta_1\left(N_A, \bar{\mathbf{Q}}_{R,A}\right) \tag{22}$$

As $\bar{P}_B$, $\bar{\rho}_A$ and $\bar{P}_{A,B}$ are all proportional to transmission power of the sources P_S, the outage bounds are shown to descend log-linearly with the power in high SINR regime.

IV. BER Analysis

Consider a common form of average BER v.s. SINR expression that covers a variety of modulation techniques (see [6], [11]):

$$\begin{aligned} \mathcal{P}_{b,A} &= \int_0^{+\infty} s\sum_{j=0}^{L-1} Q\left(\sqrt{t_j\gamma}\right) f_{\gamma_A}(\gamma)\,\mathrm{d}\gamma \\ &= \frac{s}{\sqrt{2\pi}}\sum_{j=0}^{L-1}\int_0^{+\infty} F_{\gamma_A}\left(\frac{x^2}{t_j}\right) e^{-\frac{x^2}{2}}\,\mathrm{d}x \end{aligned} \tag{23}$$

in which s, L and $\mathbf{t} = [t_0, t_2\cdots, t_{L-1}]$ are parameters determined by the modulation adopted, *e.g.* for binary phase shift keying (BPSK) $s=1, L=1, t_0=2$, and for M-ary quadrature amplitude modulation (M-QAM) $s = \frac{4}{\log_2 M}\left(1-\frac{1}{\sqrt{M}}\right)$, $L = \frac{\sqrt{M}}{2}$, $t_j = \frac{3(1+2j)^2}{M-1}$.

Theorem 2: Average BER of 3TS PNC has following approximate bounds:

$$\mathcal{P}_{b,A}|_{\text{lb}} \approx \Lambda\left(s, \mathbf{t}; \sigma_n^2, \bar{P}_B, \bar{\mathbf{Q}}_R\right) + \Lambda\left(s, \mathbf{t}; \sigma_n^2, c\bar{\rho}_A, \bar{\mathbf{Q}}_A\right) \quad (24)$$

$$\mathcal{P}_{b,A}|_{\text{ub}} \approx \Lambda\left(s, \mathbf{t}; N_A, \bar{P}_{A,B}, \bar{\mathbf{Q}}_{R,A}\right) \quad (25)$$

where the function notation is

$$\begin{aligned} &\Lambda\left(s, \mathbf{t}; w, \bar{X}, \bar{\mathbf{Y}}\right) \\ = \quad &\frac{s}{2} - \frac{s}{2} \sum_{j=0}^{L-1} \sum_{k=1}^{r(\bar{\mathbf{Y}})} \sum_{l=1}^{v_k(\bar{\mathbf{Y}})} \varphi_{k,l}(\bar{\mathbf{Y}}) \sqrt{\frac{t_j \bar{X}}{2\bar{Y}_{[k]}}} \times \\ &U\left(\frac{1}{2}, \frac{3}{2} - l; \frac{t_j \bar{X} + 2w}{2\bar{Y}_{[k]}}\right) \end{aligned} \quad (26)$$

where $U(a, b; x)$ is the confluent hypergeometric function of the second kind (see [12]).

The two bounds above are immediately obtained by substituting the CDF bounds (*i.e.* outage expressions) (16) and (17) into (23). However, the results are not intuitive due to the complicated U function, and following corollary are proposed to reveal the asymptotic performance.

Corollary 2: The asymptotic BER bounds of 3TS PNC are

$$\mathcal{P}_{b,A}|_{\text{lb,asym}} = \frac{s}{2} \sum_{j=0}^{L-1} \frac{1}{t_j} \left(\frac{\Theta_1(\sigma_n^2, \bar{\mathbf{Q}}_R)}{\bar{P}_B} + \frac{\Theta_1(\sigma_n^2, \bar{\mathbf{Q}}_A)}{c\bar{\rho}_A} \right) \quad (27)$$

$$\mathcal{P}_{b,A}|_{\text{ub,asym}} = \frac{s}{2} \sum_{j=0}^{L-1} \frac{1}{t_j} \frac{\Theta_1(N_A, \bar{\mathbf{Q}}_{A,R})}{\bar{P}_{A,B}} \quad (28)$$

These asymptotic bounds for BER are derived by substituting asymptotic outage expressions (21) and (22) into (23), respectively.

V. Numerical Results and Discussion

For numerical analysis, following scenario is established as illustrated in Fig.1. Distance of A and B is normalized $d_{A,B} = 1$, and R locates on the line connecting them, with $d_{A,R}$ denoting its distance to A. There are $N = 6$ co-channel interferers placed in a grid pattern along with the sources and relay, constituting an interference environment. Interferer subsets of the 1st, 2nd and 3rd time slot are assumed to be generalized randomly with average size of 2, *i.e.* $\mathbf{E}[|\mathcal{I}_i|] = 2, i = 1, 2, 3$. Note that the subset assignments do not impact the rationality of bounds and the performance trend. Pass loss of channel h with distance d is assumed $\mathbf{E}[|h|^2] = (\alpha d)^{-\beta}$, which is also the variance of Rayleigh fading. In following numerical analysis we fix $\alpha = 2, \beta = 3$.

The outage performance is shown in Fig.2 with transmission power varying from 0 to 40dB to verify the effectiveness of analytic bounds. All interferers are assumed equal power of 0dB, and the SINR threshold is set 3dB. Lower and upper bounds of outage are separately displayed for $c = 4$ and $c = 1/4$ cases, representing for dedicated relay station and user cooperation scenarios, respectively. We plot exact bound curves as well as asymptotic lines, and the latter are shown to converge upon the former from medium SINR regime, *i.e.*

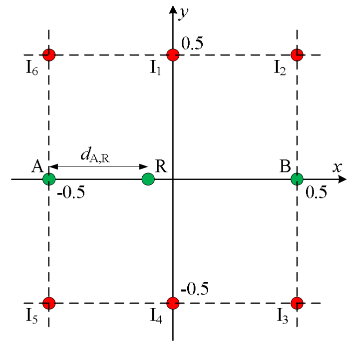

Fig. 1. Simulation scenario: the grid structure.

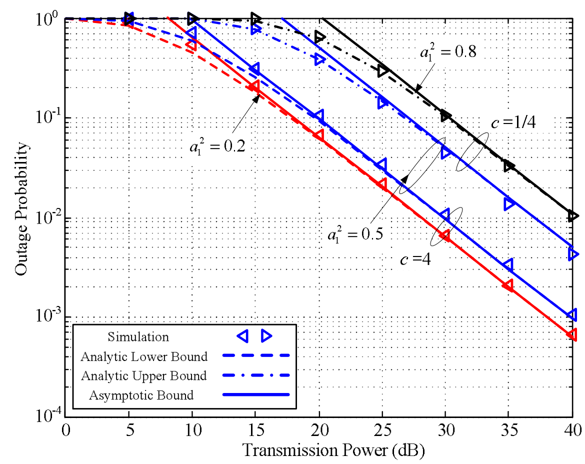

Fig. 2. Outage performance of A versus transmission power. R is located at the midpoint of A and B, $d_{A,R} = 0.5$.

about $P_S = 15$dB for the large c case, and 25dB for the small c case, in which the relay power is relatively lower, resulting in the asymptotic regime starts from a higher power.

A. Impact of Relay Forwarding Power

In Fig.2 basic effect of different relay power capability c has been revealed. It is worthy proposing a more comprehensive exhibition to the impact of wide-range varying c, as in Fig.3. Here we use the BER asymptotic curves as well as simulation dots. The conclusion of different bounds match different c value is strongly validated by behavior of equal power allocation at relay (which is a rational decision for fairness): the asymptotic lower bound becomes rather tight when c is sufficiently large, and the upper bound matches in small c regime. However, when $a_1^2 = 0.2$ (corresponding to a worse case for A's receiving), only the lower bound is plotted and coincides well with actual values, for the SINR upper bound is more reasonable in this case (see the remarks in Sec.II.A). A converse result is given for $a_1^2 = 0.8$. With the relay capability increasing, BER performance get to a bottleneck limited by the transmission power of sources.

B. Impact of Relay Location

In the numerical analysis above, R is always deployed at the very middle of the two sources. Nevertheless, deviation

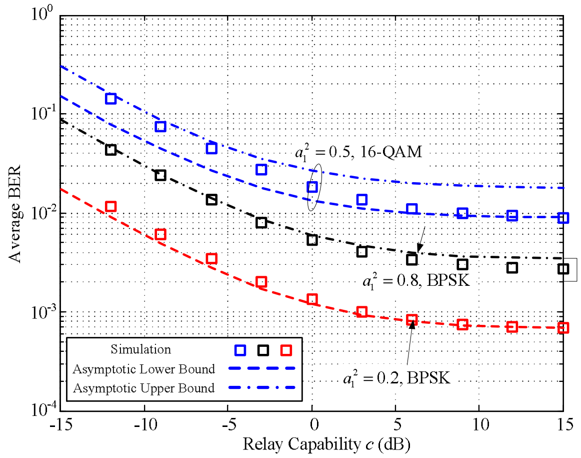

Fig. 3. Average BER of A versus relay power capability c. Transmission power of sources are fixed $P_A = P_B = 30$dB. R is located at the midpoint of A and B, $d_{A,R} = 0.5$.

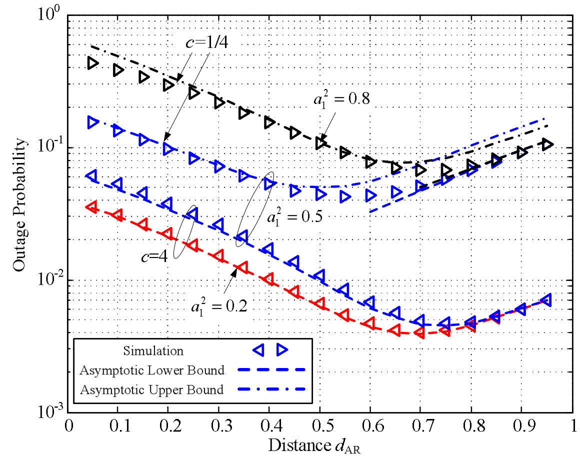

Fig. 4. Outage performance of A vs distance of A and R $d_{A,R}$. SINR threshold is set $\gamma_{\text{th}} = 3$dB.

of R position may arise new problems. A close-to-A location of R leads to a bad channel to B, which finally harms the E2E performance since the 2-hop quality is limited by the worse hop. Fig.4 shows the impact of varying relay location on outage of A. Consider an equal power allocation, when $c = 1/4$, the optimal location of R seems to be rather near the midpoint, mainly because deviation from midpoint may lead to considerable increase of interference strength (see Fig.1); when $c = 4$, the R to A link is statistically superior to that of B to R, thus the optimal distance move towards B to lessen their disparity. Since relay location not only effect two-way relay channels but also interference channels, further quantitative analysis of optimal $d_{A,R}$ is too complicated and beyond our concern. The upper bound for case $c = 1/4$ becomes a little loose when $d_{A,R}$ get large (especially when a_1^2 is greater), because $\bar{\rho}_B$ get to dominate, and the upper bound obtained by adding a term of $a_2^2\rho_B$ in the SINR expression is not that suitable. However, it is observed the lower bound matches well in this case because its derivation is by ignoring $a_1^2\rho_A$ (which is relatively small) in (6).

VI. Conclusions

In this paper, performance of two-way relaying system with co-channel interference is investigated in terms of outage probability and average BER for 3TS PNC scheme. A couple of effective bounds are derived for performance analysis due to the complexity of original SINR expression, as well as asymptotic forms to intuitively exhibit high SINR behavior, which are all validated by numerical simulation results. We further demonstrate that performance lower bounds of outage and BER matches well with the case c is relatively high, and upper bounds for the opposite situation, which is meaningful and corresponding to different relay applications. A particular scenario with multiple interferers located as a grid along with the relay and sources is then established for investigation on impacts of some critical parameters other than transmission power, *i.e.* a large scale of c value, relay location and power allocation factors.

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