Time Delays Estimation from DS-CDMA Multipath Transmissions using Expectation Maximization

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***Abstract*—In this paper, we develop a new implementation of maximum likelihood (ML) time delay estimation from direct-sequence CDMA (DS-CDMA) multipath transmissions. The formulation of the problem obtained from the DS-CDMA post-correlation model (PCM) leads to a non-linear likelihood function which is maximized by the iterative expectation maximization (EM) algorithm. In this approach, we avoid eigen-decomposition of the autocorrelation matrix widely used for multiple parameters estimation. Instead, we transpose the problem of multidimensional maximization to simpler one-dimensional maximizations carried out in parallel, thereby reducing the computational cost considerably. We also extend the application of the proposed single-carrier (SC) algorithm to multicarrier (MC)-CDMA systems by exploiting the frequency gain over subcarriers. Simulations suggest that the proposed EM-based algorithm provides accurate estimates even in the challenging case of closely-spaced delays from one transmitted symbol.**

***Index Terms*—Array processing, post-correlation model (PCM), time delay estimation, maximum likelihood (ML), expectation maximization (EM), DS-CDMA, single/multi-carrier.**

I. Introduction

Multipath reception constitutes one diversity form created by the multiple paths instead of multiple carriers, antennas or time slots. In this context, time delay estimation arises to a full exploitation of these different paths. In particular, for single- or multi-carrier CDMA-type receivers, timing recovery is an essential operation for CDMA receivers. It is involved in channel coding, power control and more so in signal combining, making its impact crucial on capacity and throughput performance [1]. Therefore, low-complexity yet accurate estimators of this parameter have been the subject of intense research.

In previous works, the Root-MUSIC-based estimator [2] was adapted to the post correlation model (PCM) of the despread data as a simple suboptimal estimator. However, in challenging scenarios such as closely-spaced delays or low SNR values, the performance of eigenvector-based methods is considerably poor compared to the maximum likelihood technique. Nonetheless, estimating the delays considering the ML criterion may be a difficult task. In our case, a closed-form solution of the maximum is intractable due to the complexity of the likelihood function and a direct implementation requires a multidimensional grid search whose complexity increases with the number of parameters to estimate. One alternative is to resort to the concept of importance sampling in the context of DS-CDMA [3] . While this method performs close to the Cramér-Rao bound, its complexity may increase significantly with the number of paths.

In this paper, we apply the expectation maximization (EM) algorithm to find the global maximum of the likelihood function and hence the maximum likelihood estimate of the delays. While this method has received much attention in multiple parameters estimation [4], [5], none of the previous works has exploited the EM approach in the context of single-carrier or multi-carrier CDMA systems. Roughly speaking, the proposed EM algorithm is able to decompose the observed signal into different replicas, each one coming from one path, then maximize a cost function that depends on one replica of the signal to find its corresponding delay. Therefore the multidimensional optimization problem is converted into much simpler parallel one-dimensional optimizations problems. In the deterministic case, when the channel coefficients are constant but unknown, the likelihood function depends on the complex amplitudes of the channel coefficients in addition to the time delays. However, under the effects of path loss, Rayleigh fading and shadowing, the channel coefficients become stochastic and, hence, we need to include the channel covariance matrix in the expression of the likelihood function. Therefore some additional steps are added in the proposed estimator to simultaneously evaluate the channel covariance matrix and the time delays of interest.

This paper is organized as follows. In section II, we briefly introduce the DS-CDMA model. In section III, we outline the different steps of the new EM estimator for DS-CDMA systems before extending it to MC-CDMA systems in section IV. In section V, we evaluate the performance of the proposed estimator through some numerical examples. Finally some conclusions are drawn out in section VI.

II. System Model

We consider a CDMA communication system on the uplink direction (portable-to-base station) where the base station is equipped by M receiving antenna branches. Note that we can apply the proposed algorithm on the downlink as well when the terminal is equipped with two or more antenna branches. We also assume that the transmitted signal travels through a multipath environment. The number of paths P is supposed to be known. For a discussion on how to determine P, we refer the reader to [2]. The post-correlation model of the spatio-temporal observation of the n^{th} received symbol can be written as [1], [2]:

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$$\boldsymbol{Z}_n = \boldsymbol{G}_n \boldsymbol{\Upsilon}_n \boldsymbol{D}^T(\boldsymbol{\tau}) s_n + \boldsymbol{N}_n, \tag{1}$$

where s_n carries the unknown transmitted symbol, $\boldsymbol{G}_n$ is the $M \times P$ spatial propagation matrix $\boldsymbol{\Upsilon}_n$ represents the $P \times P$ diagonal matrix of the power partition over the different paths. For notation compactness, the two matrices $\boldsymbol{G}_n$ and $\boldsymbol{\Upsilon}_n$ are gathered into one spatial-response matrix $\boldsymbol{J}_n$ (i.e., $\boldsymbol{J}_n = \boldsymbol{G}_n \boldsymbol{\Upsilon}_n$). $\boldsymbol{\tau} = [\tau_1, \ \tau_2, \ldots, \ \tau_P]$ is the time delays vector to be estimated. Finally, the matrix $\boldsymbol{D}(\boldsymbol{\tau}) = [\mathbf{d}_1, \ \mathbf{d}_2, \ldots, \ \mathbf{d}_P]$ is the time response matrix whose p^{th} column, $\mathbf{d}_p$, is given by:

$$\mathbf{d}_p = [\rho_c(-\tau_p), \ \rho_c(T_c - \tau_p), \ldots, \ \rho_c((L-1)T_c - \tau_p)]^T, \tag{2}$$

where L is the processing gain and $\rho_c(.)$ is the correlation function of the spreading code. Note that the correlation function of a perfect spreading code is reduced to the correlation function of the shaping pulse. This property still holds as a very reliable approximation with practical codes and will be later exploited in the development of the estimator. Including the scalar term s_n in the matrix[1] $\boldsymbol{J}_n$, an easier form of $\boldsymbol{Z}_n$ is given by:

$$\boldsymbol{Z}_n^T = \boldsymbol{D}(\boldsymbol{\tau}) \boldsymbol{J}_n^T + \boldsymbol{N}_n^T. \tag{3}$$

The reason behind the use of the transpose operator in (3) is to interpret the problem as estimating the time delays from M time snapshots observed on L virtual antennas. If we suppose that the delay vector $\boldsymbol{\tau}$ remains constant over N transmitted symbols, a compact representation of (3) over N symbols is given by:

$$\begin{aligned} \boldsymbol{Z} &= [\boldsymbol{Z}_1^T, \ \boldsymbol{Z}_2^T, \ldots, \ \boldsymbol{Z}_N^T] \\ &= \boldsymbol{D}(\boldsymbol{\tau}) \boldsymbol{J}^T + \boldsymbol{N}^T, \end{aligned} \tag{4}$$

with $\boldsymbol{J}^T = [\boldsymbol{J}_1^T, \ \boldsymbol{J}_2^T, \ldots, \boldsymbol{J}_N^T]$ and $\boldsymbol{N}^T = [\boldsymbol{N}_1^T, \ \boldsymbol{N}_2^T, \ldots, \boldsymbol{N}_N^T]$.

Exploiting the analogy between the delays in the time domain and the frequencies in the frequency domain, it is more convenient to convert the problem in the spectral domain, after which high-resolution methods, such as the Root-MUSIC algorithm can be applied to estimate the delays [2]. Therefore, we perform a column-by-column fast Fourier transform (FFT) of $\boldsymbol{Z}^T$ to obtain:

$$\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) \boldsymbol{J}^T + \boldsymbol{\mathcal{N}}, \tag{5}$$

where $\boldsymbol{\mathcal{N}}$ is the resulting noise matrix and the columns of $\boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) = [\boldsymbol{d}(\tau_1), \ \boldsymbol{d}(\tau_2), \ldots, \ \boldsymbol{d}(\tau_P)]$ are given by:

$$\boldsymbol{d}(\tau_p) = [c_0, \ c_1 e^{-\frac{j2\pi\tau_p}{L}}, \ldots, \ c_{L-1} e^{-\frac{j2\pi(L-1)\tau_p}{L}}]^T, \tag{6}$$

and $\{c_l\}_{l=1}^{L-1}$ are the FFT coefficients of the shaping pulse, reliably approximated to be quasi-constant [1], [2].

III. Expectation Maximization Algorithm

Each column of the matrix $\boldsymbol{\mathcal{Z}}$ in (5), $\{\boldsymbol{\mathcal{Z}}_i\}_{i=1}^{MN}$, is distributed according to a complex Gaussian probability density function (pdf) with zero mean and covariance matrix $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}} = \boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) \boldsymbol{R}_{\boldsymbol{J}} \boldsymbol{\mathcal{D}}(\boldsymbol{\tau})^H + \sigma^2 \boldsymbol{I}_L$, where $\boldsymbol{R}_{\boldsymbol{J}}$ is the covariance matrix of the i^{th} column of $\boldsymbol{J}^T$ (easily verified to be identical for $i = 1, \ldots, \ MN$), $\boldsymbol{I}_L$ is the $L \times L$ identity matrix and σ^2 is the noise variance. Under these assumptions, the probability density function (pdf) of $\boldsymbol{\mathcal{Z}}$, parameterized by $\boldsymbol{\tau}$ and $\boldsymbol{R}_{\boldsymbol{J}}$, is given by:

$$\begin{aligned} \overline{p}(\boldsymbol{\mathcal{Z}}; \boldsymbol{\tau}, \boldsymbol{R}_{\boldsymbol{J}}) &= \frac{1}{\pi^{MNL}} \frac{1}{\left(\det(\boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) \boldsymbol{R}_{\boldsymbol{J}} \boldsymbol{\mathcal{D}}(\boldsymbol{\tau})^H + \sigma^2 \boldsymbol{I}_L)\right)^{MN}} \\ &\exp\left\{ -\sum_{i=1}^{MN} \boldsymbol{\mathcal{Z}}_i^H \left(\boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) \boldsymbol{R}_{\boldsymbol{J}} \boldsymbol{\mathcal{D}}(\boldsymbol{\tau})^H + \sigma^2 \boldsymbol{I}_L \right)^{-1} \boldsymbol{\mathcal{Z}}_i \right\}, \end{aligned} \tag{7}$$

where $\det(.)$ refers to the determinant of a given matrix. While the noise variance σ^2 is unknown, it can be easily estimated by exploiting the estimated power carried out in the previous stage of the receiver [2] or by averaging the $L - M$ smallest eigenvalues of $\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}$. The resulting log-likelihood function, with the constant term discarded, is given by:

$$\begin{aligned} L(\boldsymbol{\tau}, \boldsymbol{R}_{\boldsymbol{J}}) &= -\ln\left(\det(\boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) \boldsymbol{R}_{\boldsymbol{J}} \boldsymbol{\mathcal{D}}^H(\boldsymbol{\tau}) + \sigma^2 \boldsymbol{I}_L)\right) - \\ &\frac{1}{MN} \sum_{i=1}^{MN} \boldsymbol{\mathcal{Z}}_i \left(\boldsymbol{\mathcal{D}}(\boldsymbol{\tau}) \boldsymbol{R}_{\boldsymbol{J}} \boldsymbol{\mathcal{D}}^H(\boldsymbol{\tau}) + \sigma^2 \boldsymbol{I}_L \right)^{-1} \boldsymbol{\mathcal{Z}}_i \\ &= -\ln\left(\det(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}})\right) - \mathrm{trace}\left\{ \boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{-1} \widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}} \right\}, \end{aligned} \tag{8}$$

with $\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}$ being the sample covariance matrix defined as:

$$\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}} = \frac{1}{MN} \sum_{i=1}^{MN} \boldsymbol{\mathcal{Z}}_i \boldsymbol{\mathcal{Z}}_i^H. \tag{9}$$

The formulation of the log-likelihood function in (8) depends on both the time delays vector $\boldsymbol{\tau}$ and the covariance matrix $\boldsymbol{R}_{\boldsymbol{J}}$. Hence optimization should be performed simultaneously on $\boldsymbol{\tau}$ and $\boldsymbol{R}_{\boldsymbol{J}}$. A quick look at the log-likelihood function reveals that it is a non-linear function of $\boldsymbol{\tau}$, which makes the estimation task difficult and its resolution in a closed-form solution intractable. Thus we consider an iterative method to maximize this function. The literature on solving optimization problems in an iterative manner is abundant. In the following, we adopt the expectation maximization (EM) algorithm [6] because of its association with the maximum likelihood problem and its ingenious way to decouple the computation task into a parallel processors implementation. More precisely, we decompose the observations $\{\boldsymbol{\mathcal{Z}}_i\}_{i=1}^{MN}$ into P complete data, then we estimate the delay of each complete data separately. Thus we transform the optimization problem in a P-dimensional space into P identical optimization problems in one-dimensional spaces. Therefore, we define the complete data as:

$$\begin{aligned} \boldsymbol{z}^{(p)}(i) &= \boldsymbol{J}^T(i,p) \boldsymbol{d}(\tau_p) + \boldsymbol{n}^{(p)}(i), \\ &p = 1, \ 2, \ldots, \ P, \ i = 1, \ 2, \ldots, \ MN. \end{aligned} \tag{10}$$

The vector $\boldsymbol{z}^{(p)}(i)$ is interpreted as the received signal on the i^{th} spatio-temporal snapshot from the p^{th} path (i.e., $\boldsymbol{\mathcal{Z}}_i = \sum_{p=1}^{P} \boldsymbol{z}^{(p)}(i)$) and $\boldsymbol{n}^{(p)}(i)$ is an arbitrary decomposition of the noise. It follows from (10) that the covariance matrix of $\boldsymbol{z}^{(p)}(i)$ is given by:

$$\boldsymbol{R}_{\boldsymbol{z}^{(p)}} = \varepsilon_p^2 \boldsymbol{d}(\tau_p) \boldsymbol{d}(\tau_p)^H + \frac{\sigma^2}{P} \boldsymbol{I}_L, \tag{11}$$

where $\{\varepsilon_p^2\}_{p=1}^{P}$ are the diagonal elements of $\boldsymbol{R}_{\boldsymbol{J}}$ (i.e., $\varepsilon_p^2 = E\{|\boldsymbol{J}^T(i,p)|^2\}$). The EM algorithm for this problem is defined

[1] For the sake of simplicity, we keep the same notation $\boldsymbol{J}_n$ for $\boldsymbol{J}_n s_n$. Hence, please note that the following formulation holds, unless specified otherwise, for both data-aided (i.e., s_n is a known reference signal) and non-data-aided transmissions.

as follows. In the expectation-step (E-step), we are interested in finding the conditional expectations of the sample covariance matrices $\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}$ of the complete data given by:

$$\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}} = \frac{1}{MN}\sum_{i=1}^{MN} \boldsymbol{z}^{(p)}(i)\boldsymbol{z}^{(p)}(i)^H. \tag{12}$$

At iteration q, the expectation of $\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}$, given $\boldsymbol{R}_{\boldsymbol{J}}^{\{q-1\}}$, $\boldsymbol{\tau}^{\{q-1\}}$ (estimated at iteration $q-1$) and $\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}$, is computed from the classical formulas of the conditional expectation with Gaussian distributed random vectors and results in:

$$\begin{aligned}\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}} &= E\left\{\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}|\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}};\boldsymbol{R}_{\boldsymbol{J}}^{\{q-1\}};\boldsymbol{\tau}^{\{q-1\}}\right\}\\ &= \boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{\{q\}}\left(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}\right)^{-1}\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}\left(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}\right)^{-1}\boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{\{q\}}\\ &+ \boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{\{q\}} - \boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{\{q\}}\left(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}\right)^{-1}\boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{\{q\}},\end{aligned} \tag{13}$$

where the matrix $\boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{\{q\}}$ is computed at each iteration by injecting $\tau_p^{\{q-1\}}$ and $\varepsilon_p^{2\{q-1\}}$ in (11). Now, we only need to estimate $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}$. A quick study of some previous EM algorithms in [4] and [5] reveals that the considered covariance matrix of the received signal is diagonal, which is not the case in our work. Therefore, we add some additional steps to estimate the covariance matrix $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}$. First, due to stationarity, the covariance between $\boldsymbol{\mathcal{Z}}_i(m_1)$ and $\boldsymbol{\mathcal{Z}}_i(m_2)$ depends only on the difference between m_1 and m_2, which gives $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}$ a Toeplitz matrix. Therefore, to estimate it, we adopt the method proposed in [7] also based on the iterative EM, which makes its integration very simple. First, we denote by $\boldsymbol{R}_s$ the $N_s \times N_s$ circulant extended version of $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}$. Precisely, $\boldsymbol{R}_s$ is the covariance matrix of the extended vector $\widetilde{\boldsymbol{\mathcal{Z}}}_i$, where $\widetilde{\boldsymbol{\mathcal{Z}}}_i$ is built by augmenting $\boldsymbol{\mathcal{Z}}_i$ by a $(N_s - L)$-dimensional null vector. The circulant matrix $\boldsymbol{R}_s$ can be factorized as follows:

$$\boldsymbol{R}_s = \boldsymbol{F}^H\boldsymbol{R}_{\boldsymbol{C}}\boldsymbol{F}, \tag{14}$$

where $\boldsymbol{F}$ is the $N_s \times N_s$ matrix of normalized orthogonal discrete Fourier transform (DFT) and $\boldsymbol{R}_{\boldsymbol{C}}$ is the diagonal matrix of the eigenvalues of $\boldsymbol{R}_s$. We also define the rotated vector $\boldsymbol{C}_i = \boldsymbol{F}\widetilde{\boldsymbol{\mathcal{Z}}}_i$ so that the covariance matrix of $\boldsymbol{C}_i$ is $\boldsymbol{R}_{\boldsymbol{C}}$. Therefore, an estimate of the matrix $\boldsymbol{R}_{\boldsymbol{C}}$, denoted $\widehat{\boldsymbol{R}}_{\boldsymbol{C}}$, is obtained from the set of vectors $\{\boldsymbol{C}_i\}_{i=1}^{M}N$ as follows:

$$\widehat{\boldsymbol{R}}_{\boldsymbol{C}} = \frac{1}{MN}\sum_{i=1}^{MN}\boldsymbol{C}_i\boldsymbol{C}_i^H. \tag{15}$$

The expectation of $\widehat{\boldsymbol{R}}_{\boldsymbol{C}}$ conditioned on $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}$ and $\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}$ is obtained in the same way as in (13):

$$\begin{aligned}E(\widehat{\boldsymbol{R}}_{\boldsymbol{C}}|\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}},\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}) = \boldsymbol{R}_{\boldsymbol{\mathcal{Z}C}}&\left(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{-1}\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}\ (\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{-1})^H - \boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{-1}\right)\times\\ &\boldsymbol{R}_{\boldsymbol{\mathcal{Z}C}} + \boldsymbol{R}_{\boldsymbol{C}},\end{aligned} \tag{16}$$

where $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}C}}$ is the cross-covariance between $\boldsymbol{\mathcal{Z}}_i$ and $\boldsymbol{C}_i$ By defining $\widetilde{\boldsymbol{F}} = \boldsymbol{F}[\boldsymbol{I}_L\boldsymbol{0}]$ and $\boldsymbol{0}$ the $(L \times N_s - L)$ null matrix, we can write $\boldsymbol{\mathcal{Z}}_i = \widetilde{\boldsymbol{F}}^H\boldsymbol{C}_i$ and therefore the cross-covariance matrix $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}C}}$ is equal to $\boldsymbol{R}_{\boldsymbol{C}}\widetilde{\boldsymbol{F}}$. Then, using (16) and the fact that $\boldsymbol{R}_{\boldsymbol{C}}$ is diagonal, the estimate of $\boldsymbol{R}_{\boldsymbol{C}}$ at iteration q is given by:

$$\begin{aligned}\boldsymbol{R}_{\boldsymbol{C}}^{\{q\}} = \mathrm{diag}&\left(\boldsymbol{R}_{\boldsymbol{C}}^{\{q-1\}}\widetilde{\boldsymbol{F}}\left((\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q-1\}})^{-1}\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q-1\}})^{-1}\right.\right.\\ &\left.\left.-(\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q-1\}})^{-1}\right)\widetilde{\boldsymbol{F}}^H\boldsymbol{R}_{\boldsymbol{C}}^{\{q-1\}} + \boldsymbol{R}_{\boldsymbol{C}}^{\{q-1\}}\right).\end{aligned} \tag{17}$$

Considering again the transformation $\boldsymbol{\mathcal{Z}}_i = \widetilde{\boldsymbol{F}}^H\boldsymbol{C}_i$, $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}$ is simply obtained as follows:

$$\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}} = \widetilde{\boldsymbol{F}}^H\boldsymbol{R}_{\boldsymbol{C}}^{\{q\}}\widetilde{\boldsymbol{F}}. \tag{18}$$

This concludes the E-step of the algorithm.

In the maximization-step (M-step), we maximize the log-likelihood function of the complete data with respect to the unknown parameters $\{\tau_p\}_{p=1}^P$. This function is obtained by replacing in (8) $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}$ by $\boldsymbol{R}_{\boldsymbol{z}^{(p)}}$ and $\widehat{\boldsymbol{R}}_{\boldsymbol{\mathcal{Z}}}$ by $\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}$. Thus the log-likelihood function of the complete data, $L_p(\tau_p, \boldsymbol{R}_{\boldsymbol{z}^{(p)}})$, to be maximized is given by:

$$L_p(\tau_p, \boldsymbol{R}_{\boldsymbol{z}^{(p)}}) = -\ln\left(\det(\boldsymbol{R}_{\boldsymbol{z}^{(p)}})\right) - \mathrm{trace}\left\{\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{-1}\right\}. \tag{19}$$

The maximization of (19) is solved using the eigen-decomposition of $\boldsymbol{R}_{\boldsymbol{z}^{(p)}}$. The matrix $\varepsilon_p^2\boldsymbol{d}(\tau_p)\boldsymbol{d}(\tau_p)^H$ is of rank one with $L-1$ null eivenvalues and one equal to ε_p^2. Therefore, it is easy to show that:

$$\det(\boldsymbol{R}_{\boldsymbol{z}^{(p)}}) = \left(\varepsilon_p^2 + \frac{\sigma^2}{P}\right)\left(\frac{\sigma^2}{P}\right)^{L-1}, \tag{20}$$

and

$$\boldsymbol{R}_{\boldsymbol{z}^{(p)}}^{-1} = \frac{P}{\sigma^2}\boldsymbol{I}_L - \boldsymbol{d}(\tau_p)\boldsymbol{d}(\tau_p)^H\left(\frac{P}{\sigma^2} - \frac{1}{\frac{\sigma^2}{P} + \varepsilon_p^2}\right). \tag{21}$$

Injecting (20) and (21) in (19), we obtain the following expression of $L_p(\tau_p, \boldsymbol{R}_{\boldsymbol{z}^{(p)}})$:

$$\begin{aligned}L_p(\tau_p, \boldsymbol{R}_{\boldsymbol{z}^{(p)}}) = &-\ln\left(\varepsilon_p^2 + \frac{\sigma^2}{P}\right) - (M-1)\ln\left(\frac{\sigma^2}{P}\right)\\ &-\left(\frac{1}{\varepsilon_p^2 + \frac{\sigma^2}{P}} - \frac{P}{\sigma^2}\right)\boldsymbol{d}(\tau_p)^H\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\boldsymbol{d}(\tau_p)\\ &-\frac{P}{\sigma^2}\,\mathrm{trace}\left(\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\right).\end{aligned} \tag{22}$$

For a given $\tau_p^{\{q\}}$, the value of $\varepsilon_p^{2\{q\}}$ which maximizes $L_p(.)$ is given in closed form:

$$\varepsilon_p^{2\{q\}} = \boldsymbol{d}(\tau_p^{\{q\}})^H\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\boldsymbol{d}(\tau_p^{\{q\}}) - \frac{\sigma^2}{P}. \tag{23}$$

For $\varepsilon_p^2 = \varepsilon_p^{2\{q\}}$, the time delay τ_p at iteration q is obtained by solving the following one-dimensional maximization problem:

$$\begin{aligned}\tau_p^{\{q\}} = \arg\max_{\tau_p} &-\ln\left\{\boldsymbol{d}(\tau_p)^H\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\boldsymbol{d}(\tau_p)\right\}\\ &+\frac{P}{\sigma^2}\boldsymbol{d}(\tau_p)^H\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\boldsymbol{d}(\tau_p),\end{aligned} \tag{24}$$

or, in an equivalent way, by simply maximizing:

$$\tau_p^{\{q\}} = \arg\max_{\tau_p}\boldsymbol{d}(\tau_p)^H\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}\boldsymbol{d}(\tau_p), \tag{25}$$

since the function $f(x) = -\ln(x) + P/\sigma^2 x$ is monotonic.

The EM algorithm can be summarized by the following steps:

1) Compute $\boldsymbol{R}_{\boldsymbol{\mathcal{Z}}}^{\{q\}}$ from (17) and (18).
2) Evaluate $\widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}}$ as defined in (13).

3) Maximize $\boldsymbol{d}(\tau_p)^H \widehat{\boldsymbol{R}}_{\boldsymbol{z}^{(p)}}^{\{q\}} \boldsymbol{d}(\tau_p)$ to find $\tau_p^{\{q\}}$ and evaluate $\varepsilon_p^{2\{q\}}$ from (23) if another iteration is needed.
4) Repeat the two last steps to find the delay of each path.

The main advantage of this algorithm is that it decouples the complex multidimensional optimization problem into P simple one-dimensional optimizations. And since these optimization processes could be carried out in parallel, the complexity of the algorithm is not significantly affected by the increase in number of paths.

IV. Extension to MC-CDMA systems

By combining multicarrier modulation with CDMA, it is possible to transmit several DS-CDMA waveforms in parallel with the spreading operation performed in time. The transmitted data are converted into $N_c = 2K + 1$ parallel sequences[2], $s_{-K,n}, \ldots, s_{0,n}, \ldots, s_{K,n}$. The data $s_{k,n}$ is then modulated, after serial/parallel conversion, at rate $1/T_{MC}$, where $T_{MC} = N_c T$. The resulting parallel input is spread with a spreading code and modulated by the inverse discrete Fourier transform (IDFT).

An extension of the post-correlation model for MC-CDMA of the spatio-temporal observation of frame number n and for the k^{th} subcarrier is given by [8]:

$$\boldsymbol{Z}_{k,n} = s_{k,n} \boldsymbol{J}_{k,n} \boldsymbol{D}_k^T(\boldsymbol{\tau}) + \boldsymbol{N}_{k,n}, \tag{26}$$

where $s_{k,n}$ is the signal component on the k^{th} subcarrier and $\boldsymbol{J}_{k,n}$ is the spatial response matrix. We can assume that the propagation time delays are the same for all subcarriers [8]. The columns of the time response matrix $\boldsymbol{D}_k(\boldsymbol{\tau}) = [\mathbf{d}_{k,1}, \ \mathbf{d}_{k,2}, \ldots, \ \mathbf{d}_{k,P}]$ are hence defined as:

$$\begin{aligned}\mathbf{d}_{k,p} = e^{-j2\pi k\lambda\frac{\tau_p}{T_{MC}}}[\rho_c(-\tau_p), \ \rho_c(T_c/k_s - \tau_p)e^{j2\pi k\frac{\lambda}{Lk_s}}, \ldots, \\ \rho_c((Lk_s - 1)T_c/k_s - \tau_p)e^{j2\pi k\frac{\lambda(Lk_s-1)}{Lk_s}}],\end{aligned} \tag{27}$$

where λ is a parameter used to determine the frequency f_k of the k^{th} subcarrier ($f_k = \lambda k/T_{MC}$) and k_s is the oversampling ratio [8]. Unlike the single-carrier case, we cannot implement directly the algorithm over the column-wise FFT of $\boldsymbol{Z}_{k,n}$. Alternatively, we introduce an intermediate transformation of $\boldsymbol{Z}_{k,n}$, denoted as $\boldsymbol{Z}_{k,n}^c$, given by:

$$\begin{aligned}\boldsymbol{Z}_{k,n}^c &= \boldsymbol{Z}_{k,n} * (\boldsymbol{a} \ \mathbf{1}_M^T) \\ &= s_{k,n} \boldsymbol{J}_{k,n} \boldsymbol{D}_k^{cT}(\boldsymbol{\tau}) + \boldsymbol{N}_{k,n}^c,\end{aligned} \tag{28}$$

where the M-dimensional vector $\mathbf{1}_M = [1, ..., 1]^T$ and the p^{th} column of $D_k^c(\boldsymbol{\tau})$ is:

$$\mathbf{d}_{k,p}^c = \mathbf{d}_{k,p} * \boldsymbol{a}, \tag{29}$$

where the operator $*$ denotes the element by element product and $\boldsymbol{a} = [1, \ e^{-j2\pi\frac{\lambda}{Lk_s}}, \ldots, \ e^{-j2\pi\frac{\lambda(Lk_s-1)}{Lk_s}}]^T$. Hence we eliminate in $\boldsymbol{D}_k^c(\boldsymbol{\tau})$ the dependence row-wise of the phase slope of each column vector on k in the spectral domain [see (32)].

Over N transmitted symbols per carrier, we concatenate all the resulting observation into one matrix $\boldsymbol{Z}_k^c$ as:

$$\begin{aligned}\boldsymbol{Z}_k^c &= [\boldsymbol{Z}_{k,1}^{cT}, \ \boldsymbol{Z}_{k,2}^{cT}, \ldots, \ \boldsymbol{Z}_{k,N}^{cT}] \\ &= \boldsymbol{D}_k^c(\boldsymbol{\tau}) \boldsymbol{J}_k^T + \boldsymbol{N}_k^T,\end{aligned} \tag{30}$$

[2] An even number of carriers could be considered as well

where $\boldsymbol{J}_k^T = [\boldsymbol{J}_{k,1}^T, \ldots, \ \boldsymbol{J}_{k,N}^T]$. Adopting the same approach as above, we perform a column-by-column FFT of $\boldsymbol{Z}_k^c$ to obtain:

$$\boldsymbol{\mathcal{Z}}_k^c = \boldsymbol{\mathcal{D}}_k^c(\boldsymbol{\tau}) \boldsymbol{J}_k^T + \boldsymbol{\mathcal{N}}_k^c, \tag{31}$$

in which the columns of $\boldsymbol{\mathcal{D}}_k^c(\boldsymbol{\tau}) = [\boldsymbol{d}_k^c(\tau_1), \ldots, \ \boldsymbol{d}_k^c(\tau_P)]$ are given by:

$$\boldsymbol{d}_k^c(\tau_p) = e^{-j2\pi\frac{\lambda\tau_p}{T_{MC}}}[1, \ e^{-j2\pi\frac{\tau_p}{Lk_s}}, \ldots, \ e^{-j2\pi\frac{\tau_p}{Lk_s}(Lk_s-1)}]. \tag{32}$$

At this stage, we apply the EM algorithm on each subcarrier. Since we can assume that the time delays are the same for all subcarriers, we can exploit the frequency gain [8] to improve the accuracy of the time delay vector estimator by averaging the vector estimates $\boldsymbol{\tau}_k$ obtained over different subcarriers:

$$\widehat{\boldsymbol{\tau}} = \frac{1}{N_c} \sum_{k=-K}^{K} \widehat{\boldsymbol{\tau}}_k. \tag{33}$$

V. Simulation Results

In all scenarios, we consider a multipath propagation environment with 3 propagations paths and small delays separation, which, of course, is the factor that limits estimation performance. The delay vector $\boldsymbol{\tau}$ is equal to $[0.12T, \ 0.15T, \ 0.18T]^T$. Unless specified otherwise, the estimation is performed from a single shot to evaluate the performance in the most critical case of no latency. The processing gain is set to $L = 64$ and the received power is assumed to be equally distributed over the three paths on average. We consider for comparisons the IS-based ML algorithm [3], Root-MUSIC [2] and the Cramér-Rao lower bound derived in [3].

Please note that the EM algorithm is iterative in nature. Therefore, initialization is a critical issue. Hence, the initial values are selected from a random variable centered at the real time delays with variance of $0.05T^2$.

In the first simulation, we fix the number of antenna branches to $M = 4$. In Fig. 1, we compared the proposed EM algorithm to Root-MUSIC and the IS-based algorithm in terms of the mean square error (MSE) performance versus the signal-to-noise ratio (SNR). Clearly, the EM algorithm outperforms Root-MUSIC over all the SNR range. Moreover, the EM ML and the IS-based algorithm have almost the same performance, which proves that the EM ML converges to the exact maximum of the likelihood function. Therefore, it is suggested in practice to use the EM ML approach since it offers less computational complexity compared to the IS-based algorithm. We also note that at high SNR values above 7 dB, the performance of the EM algorithm is close to the CRLB. This is hardly surprising since the developed method is an implementation of the ML criterion.

Usually, the estimation performance is closely linked to the number of samples. In our problem, the number of samples is fixed by the array antenna size and the number of received samples. Therefore, it is of interest to study the effect of these parameters on the two estimators. We fix the SNR value at 10 dB. In Fig. 2, we plot the MSE versus M, considering a single received sample ($N = 1$). For a small number of antennas, Root-MUSIC exhibits poor performance. In fact, Root-MUSIC

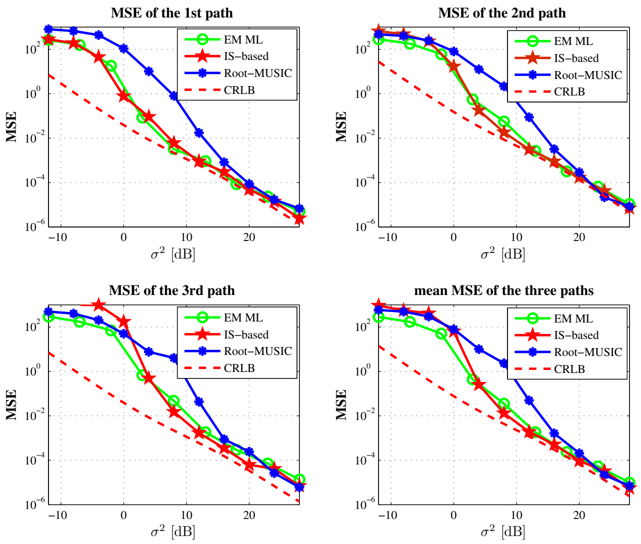

Fig. 1. MSE of the EM ML, IS-based and Root-MUSIC algorithms vs. noise variance, σ^2, for closely-separated delays, $M = 4$, $N = 1$.

is based on an estimate of the covariance matrix of $\boldsymbol{J}^T$ from the columns of $\boldsymbol{\mathcal{Z}}$. And an accurate estimate of this covariance matrix needs a large array size. The performance of the EM algorithm depends also on M but it shows more robustness against smalls values of this parameter. In Fig. 3, we plot, the MSE versus N for $M = 4$. As expected, the performance of the two estimators improves as N increases, with a noticeable advantage for the EM ML.

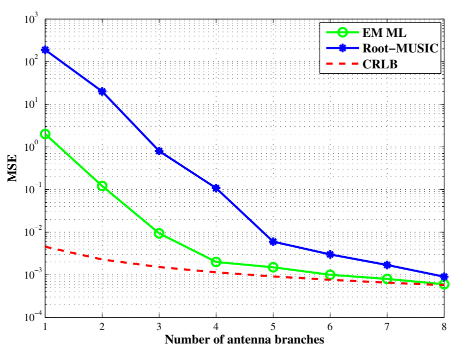

Fig. 2. MSE vs. M for EM ML and Root-MUSIC algorithm at SNR= 10 dB, $N = 1$.

To assess the robustness of the EM algorithm to initialization, we plot in Fig. 4 the MSE versus the variance of initial delays values at 5 dB SNR. The proposed EM ML approach performs well over a large range of initialization values.

VI. Conclusion

In this paper, we developed a computationally modest implementation of the ML estimator for time delays estimation from DS-CDMA multipath transmissions. This implementation, which can be applied to both SC and MC air interfaces, is based on the EM method which has the advantage of decoupling the multidimensional maximization of the actual likelihood function into parallel one-dimensional maximizations, resulting in a significant reduction of the computational cost. Given the iterative nature of the proposed approach,

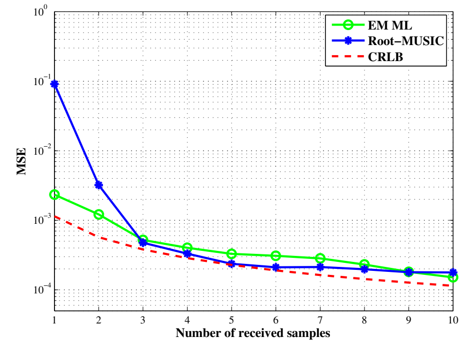

Fig. 3. MSE vs. N for EM ML and Root-MUSIC algorithm at SNR= 10 dB, $M = 4$.

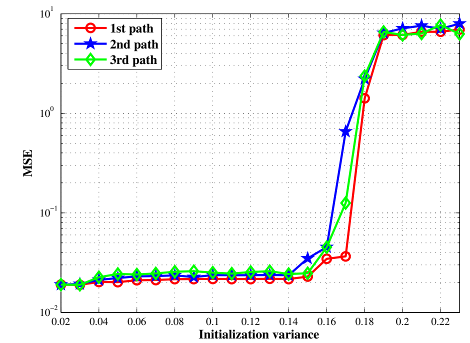

Fig. 4. MSE vs. initial variance estimate at SNR = 5 dB.

initialization is a critical issue. Yet we demonstrate through simulations that ML convergence is guaranteed over a wide range of initialization values.

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