Detection of Stochastic Noise in Vehicular Applications

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Abstract—**This paper considers observer design and controllability issues on vehicular systems with stochastic noises. The noised disturbances are modeled as independent Brownian motions for affections such as radiation, heat and material fatigue. Under this framework, this paper proposes the design and controllability conditions on a Proportional-Integral observer in terms of four crucial theorems. The results can be applied to state and disturbance estimations in related applications.**

Keywords-Proportional-Integral observer; Brownian motions; stochastic noise model; state estimation, controllability.

I. INTRODUCTION

Modern vehicles utilize many vehicular electronics to enhance the driving safety, navigation, stability, and recreations. Wireless technology is one of these convenient solutions under this need. However the wireless transmission has the radiation problems. Radiation raises extra noise to sensors for feedback control [1]. The operation heat and material fatigue also may increase noise during the operation [2]-[4]. Definitely, driving of modern vehicles needs the assists of electronics. Functions such as traction control, electronic stability control, and navigation all depends on the measuring sensor to construct a feedback mechanism. Therefore if the malfunction happens in these sensors, serious failure may occur with huge costs. Although most of these noises can be filtered, if one can detect it, the driving safety can be further enhanced.

Conventional Proportional-Integral (PI) control have been adopted in many industrial applications, in which the proportional (P) part provides a constant gain control and the integral (I) gain provides the stiffness to eliminate the DC drop [5]-[9]. The PI-type compensator is utilized in the observer structure since it provides better robustness against model inaccuracies and step disturbances than the conventional Luenberger observer [7]-[9]. It can estimate both the system states and external disturbances. The observer design for linear systems are well-known [10]-[11] techniques. However, the observer design issues on a linear system with stochastic noised disturbances are still a challenge. Especially, for the applications such as systems biology [12] and/or vehicular electronics [2], the disturbance may appeal in a probability model that deteriorate the performance of state estimation in the observer mechanism. In order to investigate these phenomena, this study considers applying the Brownian motions to sketch the noised disturbing model. The Brownian motions are widely used in physics, astronomy, applications of the random walks, and so on [13]-[15]. In this paper, one is devoted to present a new design of PI observer under the stochastic noise injections. Due to the affections of stochastic noises [16], the performance under standard observer design procedure [17]-[18] may not satisfy the specifications of feedback requirements. This engineering challenge attracts our interests of reformulating the PI observer design. In this paper, issues of the controllability and observer gains relevant to PI observer will be presented.

This paper aims to propose an aspect of applying the PI observer on the vehicular applications. The presented approach allows the user to evaluate the controllability on a relevant vehicular system. This paper is structured as follows. Section 2 describes the PI observer with stochastic noise model. Details of the controllability issues are presented in Section 3. Section 4 gives observer gains and a numerical example for the presented system. Finally, Section 5 offers some concluding remarks.

II. PI OBSERVER WITH STOCHASTIC NOISE MODEL

As shown in Fig. 1, consider the state space realization of linear systems with stochastic noise disturbance as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \sigma^{(1)} W^{(1)}(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the internal state, $y(t) \in R^m$ is the output, $u(t) \in R^r$ is a random input, $A \in R^{n\times n}$, $\sigma^{(1)} \in R^{n\times n}$, $B \in R^{n\times r}$, and $C \in R^{m\times n}$ are constant matrices as well as an n-dimensional Brownian motion $\left\{W^{(1)}(t)\right\}_{t=0}^{\infty}$.

According to [5], a PI observer with stochastic noise model as illustrated in Fig. 2 can be obtained as

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$$
\begin{aligned}
\dot{\hat{x}}(t) &= (A-LC)\hat{x}(t)+Fz(t)+Bu(t) \\
&\quad +Ly(t)+\sigma^{(2)}W^{(2)}(t), \\
\dot{z}(t) &= -KC\hat{x}(t)+Ky(t), \\
\hat{W}^{(1)}(t) &= z(t).
\end{aligned} \tag{2}
$$

where $K \in R^{p\times m}$ and $L \in R^{n\times m}$ are observer gains, $\sigma^{(2)} \in R^{n\times n}$, $F \in R^{n\times p}$, and $\left\{W^{(2)}(t)\right\}_{t=0}^{\infty}$ is another n-dimensional Brownian motion independent of $\left\{W^{(1)}(t)\right\}_{t=0}^{\infty}$.

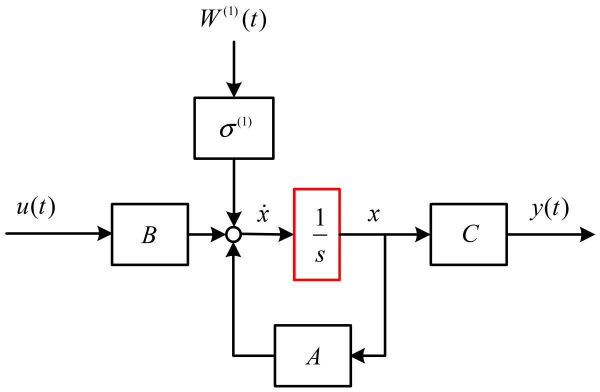

Figure 1. A linear system with stochastic noise disturbance.

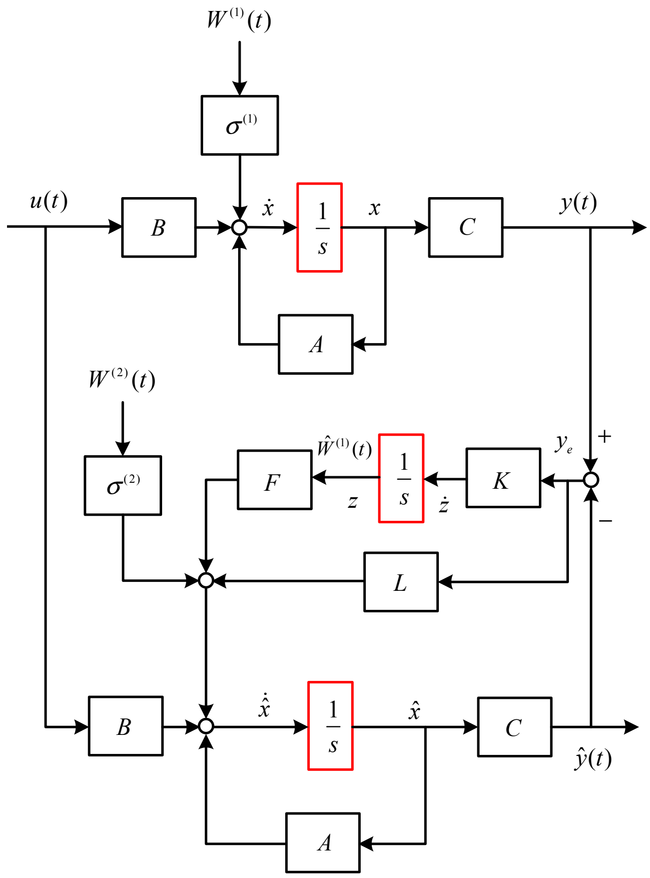

Figure 2. A PI observer with stochastic noise model.

Under the stochastic models defined by (1) and (2), primary objectives of this paper are as follows.

1) To compute

$$
V(t)=E\left\{\left[e(t)-m(t)\right]\left[e(t)-m(t)\right]^{T}\right\} \tag{3}
$$

and

$$
m(t)=E\{e(t)\} \tag{4}
$$

where $e(t)=\begin{bmatrix}\hat{x}(t)-x(t)\\ z(t)\end{bmatrix}$ *and* $E\{\cdot\}$ *denotes the expectation.*

2) To obtain the sufficient condition that the stochastic model of (1) is controllable, where (1) is said to be controllable if and only if, for any initial state x_0 *and any state* x_1*, there exists an adapted input* $u(t)$ *that transfers* x_0 *to* x_1 *in a finite time.*

3) To obtain the sufficient condition that the stochastic model (2) is solvable, where (2) is said to be solvable if and only if, there exist $L \in R^{n\times m}$ *and* $K \in R^{p\times m}$ *such that* $\lim_{t\to\infty} E\{\hat{x}(t)-x(t)\}=0$ *and* $\lim_{t\to\infty} E\{z(t)\}=0$.

In the following sections, the crucial theorems for the aforementioned issues are formulated by theoretical discussion.

III. CONTROLLABILITY

In order to apply the presented observer for sate feedback control, the controllability conditions is need. In the following, from the theoretical derivation, one can obtain the main results.

Theorem 1: *The functions of* $m(t)$ *and* $V(t)$ *given by (3) and (4) are*

$$
m(t)=e^{Gt}m(0), \tag{5}
$$

and

$$
V(t)=e^{Gt}\left\{V(0)+\int_0^t e^{-Gs}\Sigma\Sigma^{T}e^{-G^{T}s}ds\right\}e^{G^{T}t}, \tag{6}
$$

where $G=\begin{bmatrix}A-LC & F\\ -KC & 0\end{bmatrix}$ *and* $\Sigma=\begin{bmatrix}-\sigma^{(1)} & \sigma^{(2)}\\ 0 & 0\end{bmatrix}$.

Proof: By means of solving the two differential equations $\dot{m}(t)=Gm(t)$ and $\dot{V}(t)=GV(t)+V(t)G^{T}+\Sigma\Sigma^{T}$, one can obtain the desired results.

■

Theorem 2: *The stochastic model defined by (1) is controllable if and only if,* $e^{At}BB^{T}e^{A^{T}t}$ *is nonsingular for any fixed* $t>0$.

Proof: Let $G(t)=e^{At}BB^{T}e^{A^{T}t}$ and $H(t)=\int_0^t G(s)ds$. Given a finite time $t_1>0$, one has

$$
\begin{aligned}
x(t_1) &= e^{At_1}x_0+\int_0^{t_1}e^{A(t_1-s)}Bu(s)ds \\
&\quad +\int_0^{t_1}e^{A(t_1-s)}\sigma^{(1)}dW^{(1)}(s),
\end{aligned} \tag{7}
$$

for any initial state x_0. Herein, for any final state x_1, take

$$u(s) = -B^T e^{A^T(t_1-s)} H^{-1}(t_1)\left\{e^{At_1}x_0 - x_1\right\} - B^T e^{-A^T s} G^{-1}(s)\sigma^{(1)} dW^{(1)}(s), \quad (8)$$

which implies $x(t_1) = x_1$.

■

From Theorem 2, clearly, the sufficient and necessary condition of the controllability with noises is become stronger than that without noises [10]. Moreover, if input $u(t)$ is built without any stochastic component, it is not difficult to see that the specified stochastic model is not controllable.

IV. Observer Gains and Numerical Example

In the following, the design of observer gains will be revealed.

Theorem 3: *Assume that Λ is a nonsingular diagonal matrix in which the diagonal entries are complex numbers with negative real parts, if there exist $P \in R^{n\times(n+p)}$ and $Q \in R^{m\times(n+p)}$ such that $P^T A + Q^T C = \Lambda P^T$ and $\begin{bmatrix} P \\ F^T P\left(\Lambda^{-1}\right)^T \end{bmatrix}$ is nonsingular, then the stochastic model defined by (2) is solvable, in the meanwhile, $L \in R^{n\times m}$ and $K \in R^{p\times m}$ are given by*

$$\begin{bmatrix} L \\ K \end{bmatrix} = -\begin{bmatrix} P^T & \Lambda^{-1}P^T F \end{bmatrix}^{-1} Q^T. \quad (9)$$

Proof: The observe gains $L \in R^{n\times m}$ and $K \in R^{p\times m}$ establish

$$Q^T = -P^T L + \left(\Lambda^{-1}P^T F\right)K. \quad (10)$$

Substituting (10) into $P^T A + Q^T C = \Lambda P^T$, one can obtain $P^T A - \left(P^T L + \Lambda^{-1}P^T FK\right)C = \Lambda P^T$ which leads

$$P^T(A - LC) - \Lambda^{-1}P^T FKC = \Lambda P^T. \quad (11)$$

Hence, one obtains $\begin{bmatrix} P^T & \Lambda^{-1}P^T F \end{bmatrix} G = \Lambda \begin{bmatrix} P^T & \Lambda^{-1}P^T F \end{bmatrix}$. Since the diagonal entries of Λ are complex numbers with negative real parts, one gets $\lim_{t\to\infty} e^{Gt} = 0$. In consequence,

$$\lim_{t\to\infty}\begin{bmatrix} E\left\{\hat{x}(t) - x(t)\right\} \\ E\left\{z(t)\right\} \end{bmatrix} = \lim_{t\to\infty} m(t) = \lim_{t\to\infty}\left\{e^{Gt}m(0)\right\} = 0. \quad (12)$$

This completes that the stochastic model (1) is controllable.

■

Theorem 4: *Assume that the stochastic model defined by (2) is solvable. If $\sigma^{(1)}$ and $\sigma^{(2)}$ have full rank n, respectively, then $x(t) = \hat{x}(t) + S(t)$, where $S(t)$ is given by*

$$\begin{bmatrix} S(t) \\ z(t) \end{bmatrix} = e^{Gt}\begin{bmatrix} \hat{x}(0) - x(0) \\ z(0) \end{bmatrix} + \int_0^t e^{G(t-s)}\sigma^{(2)} dW^{(2)}(s) - \int_0^t e^{G(t-s)}\sigma^{(1)} dW^{(1)}(s). \quad (13)$$

Proof: Since $e(t)$ satisfies the following stochastic differential equation:

$$\dot{e}(t) = Ge(t) + \Sigma W(t), \quad e(0) = \begin{bmatrix} \hat{x}(0) - x(0) \\ z(0) \end{bmatrix}, \quad (14)$$

where $W(t) = \begin{bmatrix} W^{(1)}(t) & W^{(2)}(t) \end{bmatrix}^T$, one can obtain

$$e(t) = e^{Gt}\begin{bmatrix} \hat{x}(0) - x(0) \\ z(0) \end{bmatrix} + \int_0^t e^{G(t-s)}\sigma^{(2)} dW^{(2)}(s) - \int_0^t e^{G(t-s)}\sigma^{(1)} dW^{(1)}(s), \quad (15)$$

which gives the desired result.

■

Herein, Theorem 4 offers a new aspect of state estimation. In addition, it is clear that the quantities of $m(t)$ and $V(t)$ play roles in the stochastic model due to the estimation error between $x(t)$ and $\hat{x}(t)$ after taking the expectation.

Remark 1: Since $\int_0^t e^{G(t-s)}\sigma^{(2)} dW^{(2)}(s)$ and $\int_0^t e^{G(t-s)}\sigma^{(1)} dW^{(1)}(s)$ are the types of n-dimensional Ornstein-Uhlenbeck processes, in terms of Theorem 4, $x(t)$ can be estimated by $\hat{x}(t)$ and n-dimensional Ornstein-Uhlenbeck processes facilitates this estimation.

In the following, one utilizes a numerical example for further verification. Let a linear system $G(s)$ with $\sigma^{(1)}$ as

$$G(s) \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] = \left[\begin{array}{cccc|c} -3 & -2 & -3 & -1 & 0.8 \\ 2 & 0 & 2 & 5 & 0.3 \\ 3 & 1 & 0 & -1 & 0.6 \\ 1 & 0 & 1 & 0 & 1 \\ \hline 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & 5 & 0 \end{array}\right], \quad (16)$$

$$\sigma^{(1)} = \begin{bmatrix} 1 & 0.3 & 1 & 0.7 \end{bmatrix}^T.$$

Let $\sigma^{(1)} = F$ and $\sigma^{(2)} = \begin{bmatrix} 0.0015 & 0.023 & 0.0375 & 0.056 \end{bmatrix}^T$. Note that from (16) it is not easy to find that $G(s)$ is observable. Since $G(s)$ is observable, one then can assign the eigenvalues of the PI observer according to the Λ matrix as

$$\Lambda = \begin{bmatrix} -10 & 0 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 & 0 \\ 0 & 0 & -25 & 0 & 0 \\ 0 & 0 & 0 & -30 & 0 \\ 0 & 0 & 0 & 0 & -50 \end{bmatrix}. \quad (17)$$

Consequently, from Theorem 3, one can obtain

$$P = \begin{bmatrix} -0.0023 & -0.936 & 0.3348 & 6.494 & -2.3994 \\ -0.0071 & -1.364 & -0.3335 & -8.214 & -12.3104 \\ -0.0149 & -2.758 & -1.5363 & -36.644 & -38.9971 \\ -0.0063 & -1.252 & -2.0898 & -48.978 & -40.9426 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.0813 & 28.166 & 0 & 0 & 295.3265 \\ 0 & 0 & 10.5422 & 296.052 & 354.3918 \end{bmatrix},$$

$$L = \begin{bmatrix} -417.6595 & -95.8677 \\ 968.6099 & -129.5644 \\ -247.5040 & 171.2406 \\ 9.7121 & -97.8105 \end{bmatrix},$$

$$K = [930.3245 \quad 334.8257].$$

In the following simulation, one assumes that $W^{(1)}(t)$ and $W^{(2)}(t)$ are independent Brownian motions. Figure 3 illustrates the disturbance estimation performance between $W^{(1)}(t)$ and $\hat{W}^{(1)}(t)$. As can be seen in this evaluation, although the disturbance input of the linear system is unpredictable, the proposed system still has the ability to detect the trend of this perturbed source. Note that due to the input is constructed under random process; the simulation results will be different in each testing. The reliability can be verified by the mean value of adequate tests.

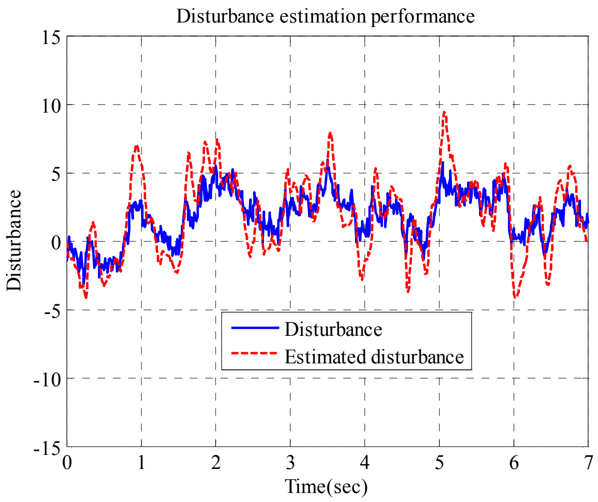

Figure 3. Simulation results for disturbance estimation.

To sum up, the results presented in this paper can be applied to state and disturbance estimations in relevant vehicular systems.

V. Conclusions

This paper has proposed a Proportional-Integral observer design under the stochastic noise injections. The noises are modeled as independent Brownian motions for applications such as such as systems biology and/or vehicular applications. The related design such as controllability, stochastic model, and observer gains were derived and given, respectively. In the presented system, one has showed that the $m(t)$ and $V(t)$ play important roles in the stochastic process. In addition, crucial theorems to probe the state estimation performance have been formulated by theoretical discussion. The results can be applied to relevant applications for improving the performance of feedback controls.

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