Joint Optimization of Transmit Power and Codebook Size for Multiuser MISO Systems

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***Abstract*— Transmit power and feedback bandwidth are two limited and interrelated resources which are crucial to system performance in wireless channel with feedback, so it is necessary to maximize their utilization efficiencies in the joint sense. In this paper, we investigate the inherent relationship between transmit power and codebook size in multiuser limited feedback MISO system by making use of Grassmann line packing theory, as an effort to provide an insight on how to jointly distribute the two resources to fulfill the diverse requirements. Then, the impact of feedback delay on the tradeoff relation is characterized in detail, and we find that even with relatively small delay, there is considerable performance loss with respect to the ideal case. Thereby, much more transmit power or feedback bandwidth should be consumed to achieve the same performance target. Finally, the theoretical claims are validated by numerical results.**

I. Introduction

In order to satisfy the demand for users to have advanced wireless services, considerable works focus on MIMO techniques which can effectively improve spectral and power efficiency during the past decades [1]. Recently, multiuser MIMO (MU-MIMO) communication attracts considerable attentions owing to its numerous advantages compared with traditional single user MIMO (SU-MIMO) counterpart, e.g. multiuser diversity and distributed spatial multiplexing [2].

In MU-MIMO systems, channel state information (CSI) at transmitter plays an extremely important role on user scheduling and performance enhancement [3]. However, for FDD systems, due to the rate constraint of backward link, the transmitter can only obtain partial CSI by feedback from the users. As shown in previous work, in order to exploit the gain of MU-MIMO system, the transmitter should know both channel quality information (such as SNR or channel gain) and channel direction information (CDI). As a scalar, the amount of feedback of channel quality information can be neglected. In general, CDI is commonly conveyed via a quantization codebook [3]. Whereas, due to multiple users, the amount of feedback is large even with a codebook of small size. Thereby, it is imperative to choose a codebook of optimal size while guaranteeing the performance requirement. In [4], the author revealed the relationship between the sum rate and feedback bandwidth with zero-forcing beamforming at the transmitter, which provided a theoretical tool to select the proper codebook size. On the other hand, power allocation is also helpful to exploit the gain of MU-MIMO systems. In [5], the authors proposed an algorithm that could find the unique optimum power allocation policies of all users to maximize the sum rate. Intuitively, we can sacrifice power for the CSI feedback amount in feedback bandwidth limited systems. Similarly, transmit power can be saved by enlarging codebook size in power limited systems. Because transmit power and feedback bandwidth are both scarce resources, we expect to achieve the performance objective with the most efficient combination of them according to the requirement of system. Motivated by this, it is necessary to character the intrinsic relation between the two resources.

This work was supported by NUAA Research Funding (No. NP2011010), National Key Basic Research Program of China (Nos. 2009CB320405 and 2012CB316104), National Natural Science Foundation Program of China (No.61172077).

A potential drawback of limited feedback systems is that there is always more or less feedback delay due to channel dynamics. The mismatch of CSI caused by feedback delay consequentially results in gain penalty [6]. To be specific, with the same performance objective, more power or larger codebook size is required when the other resource is given. To quantitatively analyze the impact of feedback delay, we first qualify it with a parameter called correlation coefficient between the channel realization when transmitting data and the one when selecting codeword. Thereby, the intrinsic relation can be obtained under the condition with arbitrary feedback delay.

II. System Model

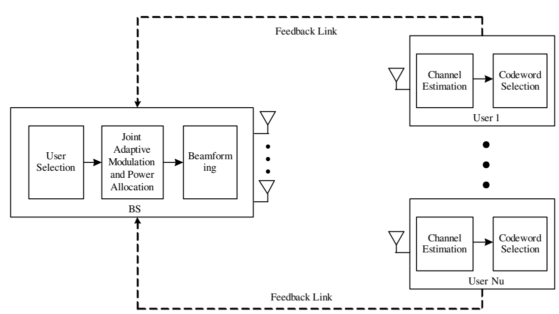

Fig. 1. An overview of the considered system model.

Consider a multiuser downlink including a base station (BS) equipped with N_t antennas and N_u users each with one antenna as seen in Fig.1. Assume the channels from the BS

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to all users are independent and identically distributed (i.i.d) and all channels are quasi-stationary and ergodic. Generally speaking, the channels keep constant during a transmission block and vary block by block according to Jakes channel model. The Grassmann codebook $\mathcal{W}$ of size B bits (containing 2^B codewords) is designed in advanced and stored at both transmit and receive sides.

At the beginning of a transmission block, each user selects the optimal codeword that maximizes its effective channel gain and computes the corresponding nominal SNR γ with average power. Then, the index of the optimal codeword and γ is conveyed to the BS. Based on the feedback information from all users, the BS chooses the user k with the largest γ as the unique communication object and applies joint adaptive modulation and power allocation to its data. Before transmission, the modulated data x_k is multiplexed by the optimal beam $\mathbf{w}_i$. Hence, the received signal y_k can be written as

$$y_k = \sqrt{\bar{P}\varphi(\gamma_k)}\mathbf{h}_k\mathbf{w}_i x_k + n_k \tag{1}$$

where $\mathbf{h}_k$ is the channel vector with i.i.d zero mean and unit variance complex Gaussian entries, n_k is zero mean and unit variance complex Gaussian noise, $\bar{P}$ is average transmit power, $\varphi(\gamma_k)$ is the power allocation strategy with the following constraint:

$$\int_0^\infty \varphi(\gamma_k) f(\gamma_k) d\gamma_k = 1 \tag{2}$$

where $\gamma_k = \bar{P}|\mathbf{h}_k\mathbf{w}_i|^2$ is the instantaneous received SNR with average power, and $f(\gamma_k)$ is the pdf of γ_k. The detail of power allocation will be discussed in section III.

From (2), it is known that $f(\gamma_k)$ plays an important role on establishing power allocation strategy. Define $\Upsilon_k = |\mathbf{h}_k\mathbf{w}_i|^2$ as the effective channel gain of user k. Since $\bar{P}$ is a constant, it is easy to derive $f(\gamma_k)$ from the pdf of Υ_k according to probability theory. For convenience, we first consider the case of single user and omit the subscript k. Thus, the effective channel gain Υ when given $\mathbf{w}_i$ can be expressed as $\Upsilon = |\mathbf{h}\mathbf{w}_i|^2 = \eta\theta_i$ where $\eta = \|\mathbf{h}\|^2$ is amplitude component, $\theta_i = |\vartheta\mathbf{w}_i|^2$ is the phase component, and $\vartheta = \mathbf{h}/\|\mathbf{h}\|$. Since $\mathbf{h}$ is an N_t dimensional vector with i.i.d zero mean and unit variance complex Gaussian entries, η complies with $\chi^2(2N_t)$ distribution, so its pdf can be written as

$$g_\eta(x) = \frac{x^{N_t-1}}{\Gamma(N_t)}\exp(-x) \qquad x \geq 0 \tag{3}$$

where $\Gamma(\cdot)$ denotes the Gamma function. As mentioned above, $\theta_i = \frac{|\mathbf{h}\mathbf{w}_i|^2}{\|\mathbf{h}\|^2} = \frac{|\mathbf{h}\mathbf{w}_i|^2}{|\mathbf{h}\mathbf{w}_i|^2+\|\mathbf{h}\mathbf{w}_i^\perp\|^2} = \frac{q}{q+p}$, where $\mathbf{w}_i^\perp$ is the orthogonal components of $\mathbf{w}_i$. Because q and p have $\chi^2(2)$ and $\chi^2(2N_t-2)$ distributions (denoted by $g_p(\cdot)$) respectively, the pdf of θ_i can be computed as

$$\begin{aligned} g_{\theta_i}(y) &= \int_0^\infty g_{\theta_i|p}(y|p)g_p(p)dp \\ &= \int_0^\infty \frac{p}{(1-y)^2}\exp(-\frac{y}{1-y}p)\frac{p^{N_t-2}\exp(-p)}{\Gamma(N_t-1)}dp \\ &= (N_t-1)(1-y)^{N_t-2} \end{aligned} \tag{4}$$

where $0 \leq y \leq 1$.

For Grassmann codebook of size 2^B, it is equivalent to a set $\mathcal{W}$ as a collection of lines, which are chosen to maximize the codeword minimum distance defined as

$$\delta = \sqrt{1-|\mathbf{w}_l^*\mathbf{w}_m|^2} \qquad 1 \leq l < m \leq 2^B \tag{5}$$

Given N_t and B, the maximum minimum distance $\delta_{\max} = 2 * 2^{-\frac{B}{2(N_t-1)}}$ according to Grassmann manifold theory [7]. Hence, all codewords partition the whole channel space into 2^B subspaces $\mathcal{R}_j, j = 1, \cdots, 2^B$. If instantaneous channel realization belongs to subspace $\mathcal{R}_i$, or the distance between channel realization and codeword $\mathbf{w}_i$ is smaller than $\delta_{\max}/2$, $\mathbf{w}_i$ is selected as the beam for the current transmission block. Thus, the probability that $\mathbf{w}_i$ is the optimal beam can be computed as

$$\begin{aligned} P_r(\mathbf{w}_i) &= P(\sqrt{1-\theta_i} < \delta_{\max}/2) \\ &= \int_{1-\kappa}^1 g_{\theta_i}(y)dy \\ &= 2^{-B} \end{aligned} \tag{6}$$

where $\kappa = \left(\frac{\delta_{\max}}{2}\right)^2$. Clearly, all codewords have the same probability to be selected. Hence, the pdf of the phase component can be written as

$$g_\theta(y) = 2^B(N_t-1)(1-y)^{N_t-2} \quad 1-\kappa \leq y < 1 \tag{7}$$

Then, we can calculate the pdf of Υ as

$$\begin{aligned} g(\Upsilon) &= \int_{1-\kappa}^1 \frac{1}{y} g_\eta(\Upsilon/y) g_\theta(y) dy \\ &= \frac{2^B}{\Gamma(N_t-1)}\int_{1-\kappa}^1 \Upsilon^{N_t-1}(1-y)^{N_t-2}\exp\left(-\frac{\Upsilon}{y}\right)dy \\ &= \frac{2^B}{\Gamma(N_t-1)}\int_{1-\kappa}^1 \sum_{t=0}^{N_t-2} C_{N_t-2}^t(-1)^t\Upsilon^{N_t-1} \\ &\times \int_{1-\kappa}^1 y^{t-N_t}\exp\left(-\frac{\Upsilon}{y}\right)dy \\ &= \frac{2^B}{\Gamma(N_t-1)}\sum_{t=0}^{N_t-2} C_{N_t-2}^t(-1)^t\Upsilon^t \sum_{m=0}^{N_t-2-t}\frac{(N_t-2-t)!}{m!} \\ &\times\left(\exp\left(-\Upsilon\right)\Upsilon^m - \exp\left(-\frac{\Upsilon}{1-\kappa}\right)\left(\frac{\Upsilon}{1-\kappa}\right)^m\right) \end{aligned} \tag{8}$$

Given the number of transmit antennas N_t and codebook size B, we could calculate $g(\Upsilon)$ according to (8). In this paper, we consider the case of $N_t = 3$. Under this condition, the pdf of effective channel gain for single user can be expressed as

$$g(\Upsilon) = 2^B\left(\exp(-\Upsilon) - (1+\frac{\kappa}{1-\kappa}\Upsilon)\exp(-\frac{\Upsilon}{1-\kappa})\right) \tag{9}$$

In the above analysis, we consider the case of $N_u = 1$. We now turn our attention to deriving a closed-form expression for $g(\Upsilon, N_u, B)$ with arbitrary N_u and B. Under multiuser condition, the user with largest γ or Υ is selected for each transmission block, so the cdf of effective channel gain can be reckoned as

$$\begin{aligned} G(\Upsilon, N_u, B) &= G(z_1 < \Upsilon, \cdots, z_{N_u} < \Upsilon) \\ &= G^{N_u}(\Upsilon) \end{aligned} \quad (10)$$

where (10) follows from the fact that the channels of the N_u users are i.i.d. Clearly, $g(\Upsilon, N_u, B) = G^{'}(\Upsilon, N_u, B) = N_u g(\Upsilon) G^{N_u-1}(\Upsilon)$. Specially, for $N_t = 3$, the pdf of effective channel gain can be expressed as

$$\begin{aligned} & g\left(\Upsilon, N_u, B\right) = N_u 2^{N_u B} \\ & \times \left(\exp(-\Upsilon) - (1 + \frac{\kappa}{1-\kappa}\Upsilon) \exp(-\frac{\Upsilon}{1-\kappa}) \right) \\ & \times (\kappa^2 - \exp(-\Upsilon) + (1 - \kappa^2 + \kappa\Upsilon) \exp(-\frac{\Upsilon}{1-\kappa}))^{N_u-1} \end{aligned} \quad (11)$$

III. Joint Optimization of Transmit Power and Codebook Size

In this section, we study the achievable maximum spectral efficiency based on a joint adaptive modulation and power allocation strategy and the long-term statistical information of effective channel gain for a limited feedback MU-MISO system from the viewpoint of information theory. Thereby, we are able to derive the spectral efficiency in terms of transmit power and codebook size, which forms the basis of resource joint optimization.

In this paper, we consider the scenario where the transmitter employs adaptive MQAM modulation. As revealed in [8], the BER for MQAM modulation can be approximately bounded by

$$\mathrm{BER} \leq 0.2 \exp\left(-\frac{1.5\varphi(\gamma)\gamma}{M-1}\right) \quad (12)$$

where M is the modulation rank. Rearrange (12), we can obtain the instantaneous data rate when given a BER requirement as

$$\begin{aligned} R(\gamma) &= \log_2(M) \\ &= \log_2(1 + A\varphi(\gamma)\gamma) \end{aligned} \quad (13)$$

where $A = -\frac{1.5}{\ln(5\mathrm{BER})}$. Thus, the joint strategy is equivalent to the solution of the following optimization problem

$$\begin{aligned} \max \quad & \log_2(1 + A\varphi(\gamma)\gamma) \\ \text{s.t.} \quad & \int_0^\infty \varphi(\gamma) f(\gamma) d\gamma = 1 \end{aligned} \quad (14)$$

where $f(\gamma) = \frac{1}{\bar{P}} g(\gamma/\bar{P}, N_u, B)$, which is obtained directly as discussed above. By Lagrange multiplexer method, the solution of (14) can be expressed as

$$\varphi(\gamma) = \begin{cases} \frac{1}{A\gamma_0} - \frac{1}{A\gamma}, & \text{if } \gamma \geq \gamma_0 \\ 0, & \text{if } \gamma < \gamma_0 \end{cases} \quad (15)$$

where γ_0 is the cutoff SNR, which can be obtained by solving the following equation

$$\int_{\gamma_0}^\infty \left(\frac{1}{A\gamma_0} - \frac{1}{A\gamma}\right) f(\gamma) d\gamma = 1 \quad (16)$$

As we know, for adaptive modulation, the key is to determine the switching thresholds in the form of SNR. Generally speaking, we should partition the whole SNR range into several regions and distribute each region an optimal modulation format to maximize the spectral efficiency. However, (15) does not give any insight to determine the boundaries of SNR regions. Substituting (15) into (13), we can obtain the relationship between receive SNR and modulation format as

$$\gamma = M\gamma_0 \quad (17)$$

As the modulation rank, M only can be some discrete values. In this paper, we let $M = 2^n, n = 1, \cdots, N-1$. In other words, $N-1$ modulation formats is usable in total. From (17), it is known that when the nth modulation format is adopted, the received SNR must be greater than or equals to $2^n\gamma_0$ so that the average BER is able to satisfy the given requirement. Hence, we could let $\Omega_0 = 0, \Omega_n = 2^n\gamma^\star, n = 1, \cdots, N-1$ and $\Omega_N = \infty$ be the switching thresholds, when $\Omega_n \leq \gamma < \Omega_{n+1}$, the nth modulation format is optimal in the sense of spectral efficiency under the BER constraint. In specialty, when $\Omega_0 \leq \gamma < \Omega_1$, no data is transmitted.

Given the SNR thresholds for adaptive modulation, for the transmission block whose SNR is between two thresholds, its partial power are wasted if we allocate power according to (15). For example, if $\Omega_0 \leq \gamma < \Omega_1$, no power needs to be loaded because the BS does not transmit any data. In fact, we could make the instantaneous SNR of all transmission blocks equal to their left boundaries of the SNR regions where they belong to, so that the BER requirement is satisfied with the minimum power. By fixing the BER in (13), we obtain

$$\varphi(\gamma) = \begin{cases} \frac{2^n-1}{A\gamma}, & \text{if } n = 1, \cdots, N-1 \\ 0, & \text{if } n = 0 \end{cases} \quad (18)$$

After getting the power allocation strategy, we should recompute the cutoff SNR $\gamma^\star$ to determine the switching thresholds. By substituting (18) into (16), it is obtained that

$$\sum_{n=1}^{N-1} \int_{2^n\gamma_\star}^{2^{n+1}\gamma_\star} \frac{2^n-1}{A\gamma} f(\gamma) d\gamma = 1 \quad (19)$$

Although it is difficult to get a closed-form expression of $\gamma_\star$, it is likely to obtain the approximate value by numerical method as long as the searching step is sufficiently small.

Based on the pdf of SNR and the joint adaptive modulation and power allocation strategy, the spectral efficiency can be expressed as

$$\begin{aligned} \varrho &= \sum_{n=1}^{N-1} n \int_{\Omega_n}^{\Omega_{n+1}} f(\gamma) d\gamma \\ &= N - 1 - \sum_{n=1}^{N-1} F(\Omega_n) \end{aligned} \quad (20)$$

where $F(\Omega_n) = \int_0^{\Omega_n} f(\gamma) d\gamma$, which is a function of B and $\bar{P}$. For $N_t = 3$, $F(\Omega_n) = 2^{N_u B}(\delta_{\max}^2/4 - \exp(-\frac{\Omega_n}{\bar{P}})(1 - \delta_{\max}^2/4 + \delta_{\max}\frac{\Omega_n}{2\bar{P}})\exp(-\frac{\Omega_n}{\bar{P}(1-\delta_{\max}/2)}))^{N_u-1}$. Hence, Given a requirement of spectral efficiency, we could perform joint optimization of B and $\bar{P}$ according to the characteristics of system. As we know, the codebook size B can only be some discrete values and the transmit power is continuous. For feedback bandwidth limited systems, the smallest transmit power can be computed by giving the codebook size and the requirement of spectral efficiency. For power limited systems, it is relatively complicated. With the given power, we first obtain the smallest codebook size while satisfying the requirement of spectral efficiency, and then we calculate the smallest power with the above codebook size.

IV. The Impact of Feedback Delay

In the earlier analysis, we consider the case under ideal condition. However, there is always more or less feedback delay in practical systems. Due to the existence of feedback delay, the pdf of effective channel gain is inconsistent with (8). As a result, the power and feedback bandwidth efficiencies are no longer optimal if the above joint adaptive modulation and power allocation strategy is adopted directly. In other words, we need to find a new adaptive strategy to maximize the spectral efficiency and establish the corresponding joint optimization strategy in the presence of feedback delay.

Following [6], the relationship between the real and predicted channel realizations can be expressed as

$$\mathbf{h}_r = \rho\mathbf{h}_p + \sqrt{1-\rho^2}\mathbf{n}_p \tag{21}$$

where $\mathbf{h}_r$ and $\mathbf{h}_p$ are the real and the predicted channel realizations, respectively. $\mathbf{n}_p$ is the prediction noise with i.i.d zero mean and unit variance complex Gaussian entries. ρ is the correlated coefficient between $\mathbf{h}_r$ and $\mathbf{h}_p$, which is a function of f_d, T and prediction method. The small doppler frequency shift or feedback delay denotes high correlation. If $\rho = 1$, then $\mathbf{h}_r$ equals to $\mathbf{h}_p$, the impact of feedback delay can be canceled completely. If $\rho = 0$, the users have no any knowledge about $\mathbf{h}_r$, so that the feedback information is unreliable fully. In the scenario with feedback delay, the received signal can be written as

$$\begin{aligned} y &= \sqrt{\bar{P}\varphi(\xi)}\mathbf{h}_r\mathbf{w}_i x + n \\ &= \sqrt{\bar{P}\varphi(\xi)}\rho\mathbf{h}_p\mathbf{w}_i x + \sqrt{\bar{P}\varphi(\xi)}\sqrt{1-\rho^2}\mathbf{n}_p\mathbf{w}_i x + n \end{aligned} \tag{22}$$

where, ξ is the received SNR, which can be expressed as

$$\xi = \frac{\bar{P}\varphi(\xi)\rho^2|\mathbf{h}_p\mathbf{w}_i|^2}{\bar{P}\varphi(\xi)(1-\rho^2)+1} \tag{23}$$

Different from the case under ideal condition, the SNR in (23) is not a monotone increasing function of power due to the existence of prediction noise. As a result, it seems to be difficult to give a closed-form of the joint adaptive modulation and power allocation strategy. Examining (23), if we make $\varphi(\xi) = 1$, namely constant power allocation, the SNR varies only with the effective channel gain. Hence, the pdf of SNR can be obtained easily from the pdf of effective channel gain as

$$p(\xi) = \frac{1}{\beta}g(\frac{\xi}{\beta}, N_u, B) \tag{24}$$

where $\beta = \frac{\bar{P}\varphi(\xi)\rho^2}{\bar{P}\varphi(\xi)(1-\rho^2)+1}$. Having gotten the pdf, we could derive the adaptive modulation for this scenario.

Let $\Pi_n, n = 0, 1, \cdots, N$ be the $N+1$ SNR thresholds for adaptive modulation, where $\Pi_0 = 0$ and $\Pi_N = \infty$. Once $\Pi_n \leq \xi < \Pi_{n+1}$, MQAM modulation format is adopted where $M = 2^n$. Noticeably, if $\Pi_0 \leq \xi < \Pi_1$, no data is transmitted. When the nth modulation format is used, the corresponding average BER can be casted as

$$\overline{\mathrm{BER}}_n = \frac{1}{\tau_n}\int_{\Pi_n}^{\Pi_{n+1}} \mathrm{BER}(2^n, \xi)p(\xi)d\xi \tag{25}$$

where $\mathrm{BER}(2^n, \xi)$ is the BER expression as (12) with $M = 2^n$ and received SNR ξ. $\tau_n = \int_{\Pi_n}^{\Pi_{n+1}} p(\xi)d\xi$ is the probability that the nth modulation format is chosen when given $\bar{P}, N_t, B, N_u, \rho, N$. Hence, the average BER over all modulation formats can be expressed as

$$\overline{\mathrm{BER}} = \frac{\sum\limits_{n=1}^{N-1} n\tau_n\overline{\mathrm{BER}}_n}{\sum\limits_{n=1}^{N-1} n\tau_n} \tag{26}$$

Let $\overline{\mathrm{BER}}$ equal to the objective BER, then the residual $N-1$ SNR thresholds can be obtained by numerical method as follows: given $\Pi_N = \infty$, search the minimum Π_{N-1} so that $\overline{\mathrm{BER}}_{N-1}$ equals to the objective BER; with the same method, $\Pi_{N-2}, \cdots, \Pi_1$ can be obtained in turn.

Similarly, we could derive the spectral efficiency in the form of B, $\bar{P}$ and ρ as

$$\varsigma = N - 1 - \sum_{n=1}^{N-1} P(\Pi_n) \tag{27}$$

where $P(\Pi_n) = \int_0^{\Pi_n} p(\xi)d\xi$. For $N_t = 3$, it is obtained from (10) and (24) that $P(\Pi_n) = 2^{N_u B}(\delta_{\max}^2 - \exp(-\frac{\Pi_n}{\beta})(1-\delta_{\max}^2 + \delta_{\max}\frac{\Pi_n}{\beta})\exp(-\frac{\Pi_n}{\beta(1-\delta_{\max})}))^{N_u-1}$. Hence, the joint optimization with feedback delay can be performed according to (27) as long as the correlated coefficient ρ is known.

V. Simulation Results

To examine the effectiveness of the proposed strategy, we present several numerical results in different scenarios. For all scenarios, we set $N_t = 3$, $N_u = 5$ and $N = 7$ for convenience. However, the proposed strategy is applicable to the case with arbitrary N_t, N_u and N. The BER requirement is fixed as 10^{-3}. In addition, SNR is denoted as the ratio of average transmit power to the variance of noise.

Fig.2 addresses the joint optimization of $\bar{P}$ and B without feedback delay. Clearly, given an objective of spectral efficiency, we could achieve it with different combinations of $\bar{P}$ and B. For example, at the spectral efficiency of 2.5, the joint strategy with $B = 6$ has a about 2 dB gain than that with $B = 1$. Hence, we could sacrifice feedback amount for power. Otherwise, we could use more power to reduce feedback. Noticeably, for large N_u, the amount of feedback is vary large even with $B = 1$. Under this condition, we could set

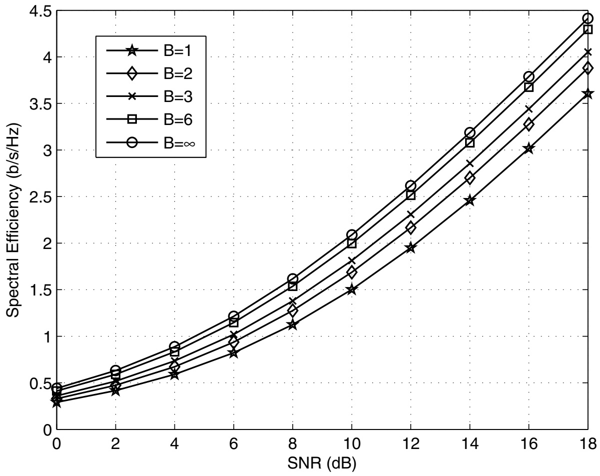

Fig. 2. Transmit power-codebook size tradeoff without feedback delay.

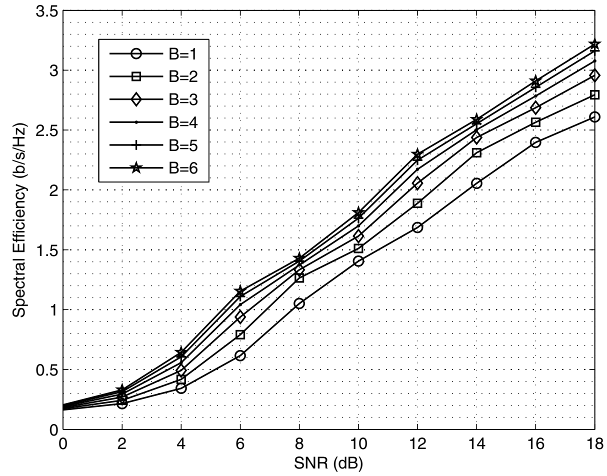

Fig. 4. Transmit power-codebook size tradeoff with feedback delay.

a SNR threshold so that only the users with larger SNR than this value would convey their information. Hence, we could achieve the good performance with the acceptable amount of feedback.

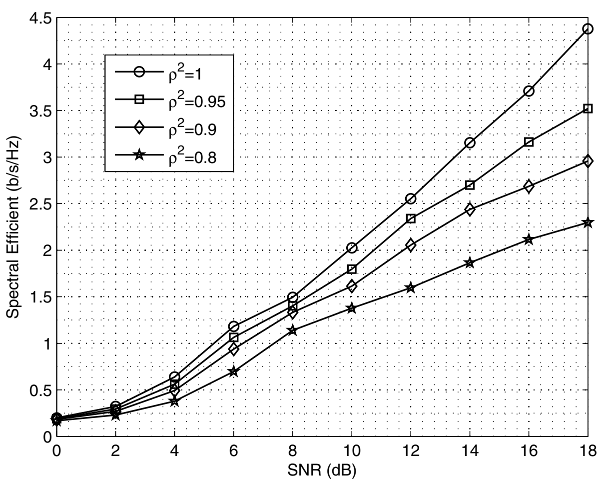

Fig. 3. Performance loss due to feedback delay.

Next, we turn our attentions to the impact of feedback delay on the joint optimization. Here, we denote ρ as the degree of feedback delay and set $B = 3$. As seen in Fig.3, even small delay ($\rho^2 = 0.95$) would lead to an obvious performance loss because of the extra interference and low power efficiency. With the increase of ρ, the performance gap becomes larger with respect to the ideal case. Thereby, we should use more power to reduce the same codebook size compared with the ideal case. As an example, for $\rho^2 = 0.9$, more 4 dB transmit power should be utilized to reduce the codebook size from 6 bits to 1 bit at the spectral efficiency of 2.5 as shown in Fig.4, which is larger one time than that without delay. In addition, it is found that with the increase of transmit power, the received SNR is interference-limited, which can be expressed as $\xi = \frac{\rho^2}{1-\rho^2}|\mathbf{h}_p\mathbf{w}_i|^2$, so that the spectral efficiency is also limited. In other words, the spectral efficiency would not be improved even with more transmit power when given the codebook size.

VI. Conclusion

In this paper, we analyze the relationship among average data rate, the number of transmit antennas, codebook size, the number of users, average transmit power and the BER requirement by Grassmann line packing theory and information theory, which provides a guideline on the joint optimization of transmit power and codebook size according to the characteristics of the considered system. In addition, the impact of feedback delay on the joint optimization is also investigated in some detail and it is found that there is an obvious performance loss compared with the ideal case due to the existence of interference term caused by feedback delay.

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