Distributed Resource Allocation Scheme for Multicell OFDMA Networks Based on Combinatorial Auction

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***Abstract*—Users' competition in a multicell OFDMA network has been modeled as auction game. A combinatorial auction, which takes into account the interference from adjacent cells is presented. Users' valuation for a set of subcarriers is sent to the base station, which acts as an auctioneer and makes the decisions on the allocation. Auction objective is to minimize the interference, while power of users is limited. Our proposed valuation function will enforce truth-telling of the users. Due to the complexity of original problem, we apply a heuristic approach, which orders the bids based on the linear programming approximation of combinatorial auction, and then makes local improvements in the order of bids. Our iterative approach along with the proposed load control scheme provides fair distribution of resources to the users, regardless of their position in the cell. Simulation results demonstrate the performance of our proposed method.**

I. Introduction

The Orthogonal Frequency Division Multiple Access (OFDMA), is a promising solution for next generation wireless communication systems. OFDM offers robustness to channel distortion and the ability to exploit channel diversity to increase the spectral efficiency through a dynamic resource allocation. In recent years single cell resource allocation for OFDMA networks has been a subject of many studies [1] [2].

In a multicell OFDMA system, a user not only competes for resources with the users in its own cell, but also generates interference for adjacent cells. Advanced and intelligent radio resource management (RRM) schemes which consider the dynamic co-channel interference has to be designed. Moreover, RRM must take advantage of the opportunities and flexibilities of OFDMA system, and address the critical notion of fairness. Taking these challenges into consideration, introduces additional complexity into the problem of resource allocation. Meanwhile, the conventional centralized approaches do not scale well to the multicell networks where there is a need for distributed algorithm. Interference avoidance techniques have been proposed for OFDMA systems to reduce the interference. The authors in [3] showed that dynamic packet assignment (DPA) based on interference avoidance can improve system throughput. A performance analysis of DPA in cellular OFDMA systems was presented in [4]. [5] compared the performance between the interference average technique and the interference avoidance technique. However, since these approaches are based on frequency allocation, they require greater computational complexity [2]. Furthermore, reuse partitioning which is based on geographical channel allocation improves the system throughput in multicell environments [6].

Auction is a decentralized market mechanism for allocating resources, which makes it a good candidate to address the competitive behavior of nodes for available resources. Some previous works discuss the single unit auctions or multi-unit auctions with identical values. But in OFDM systems, the quality of every subcarrier varies for different users. Thus, the resource allocation problem should be formulated as a multi-unit auction with non-identical values, which is usually referred as combinatorial auction. In combinatorial auctions, users are able to express their preferences and bid for bundle of subcarriers. The main challenges in combinatorial auctions come from the computational intractability of the problem along with the incentive compatibility requirement where users reveal their true valuation for the available resources. We address both problems in this paper.

[7] proposed an auction algorithm for time slot allocation in wireless networks. In proposed algorithm, users submit a bid for a time slot to the Base Station (BS) which acts as an auctioneer, then BS allocates the slot to the user that made the highest bid. The proposed approach in [7] is appropriate for single unit auction, however, in multicarrier environment such as OFDMA systems, we need a multi unit auction scenario. An auction approach to resource allocation in multicell OFDMA is studied in [1]. In a single-unit auction, both single cell and multicell scenarios are considered. A subcarrier allocation algorithm based on auction game is presented in [8]. A margin adaptive problem has been modeled, and the results are provided for a single-cell OFDMA network. Then, based on the result of the single-cell scenario, an extension to multi-cell network is proposed. Compare to the proposed work, our algorithm will force the truthful behavior of the user in their bidding strategy, and take advantage of load control to provide

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fair distribution of resources, particularly for cell-edge users.

In this paper, we present a distributed and near-optimum resource allocation scheme in multicell OFDMA network. The objective of the proposed approach is to minimize the interference. We focus on subcarrier and power allocation schemes that are possibly amendable for a distributed implementation. Our proposed valuation function provides a truth-telling behavior of users. Moreover, it reduces the computational complexity and increases scalability in terms of the number of cells, users and subcarriers. Also, based on the users' valuation for a bundle of subcarriers, we perform a load control scheme which mitigates the interference experienced by cell-edge users. We apply a heuristic approach, which orders the bids based on the linear programming approximation of combinatorial auction, and then makes local improvements in the order of bids. Our algorithm is designed to be implemented in a distributive manner with low complexity.

The organization of this paper is as follows. In section II, system model and proposed auction solution are described. Then performance evaluation is provided through simulation results in section III. Finally section IV concludes the paper.

II. System Model

Consider a multi-cell OFDMA system with M base stations that cover a geographic area. In each cell, there are a BS and K users competing for the available resources. Here resources mean the subcarriers to be allocated through auction process. The overall network uses a total bandwidth B divided into N OFDM subcarriers. We assume complete reuse of the available frequency resources. Each user k is assigned a subset of subcarriers $\mathcal{N}_k$ to send data over the wireless media. Note that subcarriers are not shared by different users. We suppose the subcarrier separation is narrower than the coherent bandwidth. Moreover, in each cell the base station knows the channel state information (CSI) and the amount of interference relative to all users within the cell which is then made known to the users via a dedicated feedback channel. On the basis of this information, users send their valuation to the base station as the offering bid. Let $h_{k,i,j}$ be the radio channel gain between BS j and user k on subcarrier i. Each user k has a minimum data rate requirement, denoted as $r^k_{\min}$.

In a cellular OFDMA system, the computation of the SINR achieved at subcarrier i in the receiver of user k served by BS j, is obtained as follows

$$\mathrm{SINR}_{k,i,j} = \frac{p_{k,i,j} \left|h_{k,j}\left(i\right)\right|^2}{I_j\left(i\right) + BN_0} \tag{1}$$

where $I(j)_i$ is the total interference in subcarrier i, cell j which is

$$I_j\left(i\right) = \sum_{l=1, l\neq j}^{M} p_{U(l,i)}\left(i\right) \left|h_{U(l,i),j}\left(i\right)\right|^2 \tag{2}$$

$p_{k,i,j}$ is the transmit power of user k on subcarrier i, of cell j, $U(l,i)$ are the users transmitting on subcarrier i in cell l, and $h_{U(l,i),j}(i)$ is the channel gain of users in $U(l,i)$ to base station j. The bit rate of user k on allocated subcarrier i of cell j would be

$$r_{k,i,j} = \log_2(1 + \mathrm{SINR}_{k,i,j}) \tag{3}$$

In the rest of the paper, without loss of generality, we drop the index j as the cell number. The auction consists of K users and N subcarriers. q_k denotes the price that user k is paying for a bundle of subcarriers. Let P_k denote the power constraint of user k. $\mathcal{N}_k$ is the set of subcarriers which user k requests. The objective of allocation is maximizing the total utility of all users. U_k is the utility of user k defined as allocated rate $U_k = \sum_{i=1}^{N} r_{k,i}$.

As combinatorial auction suggests, users are allowed to bid for a bundle of subcarriers. From game theoretic perspective, the auction problem may formulated as $\Gamma = (\mathcal{K}, \{S_k\}, \{u_k\})$. The players participating in the game are the users of the network, so the set of players are the set of users $\mathcal{K}$. Each user k reports his preferences to the base station by expressing a subset of subcarriers $\mathcal{S}_k \subseteq \mathcal{N}$, and his valuation $v_k \in \mathbb{R}^+$ on the subcarrier set. So, the strategy of user k would be the pair $\langle \mathcal{S}_k, v_k \rangle$. Note that the whole strategy space of user k is $\mathcal{I}_k = \mathcal{C}_{\mathcal{N}_k}(\mathcal{N}) \times \mathbb{R}^+$, which $\mathcal{C}_{\mathcal{N}_k}(\mathcal{N})$ is the combination of choosing N_k subcarriers from set of subcarriers $\mathcal{N}$. The payoff of user k is defined as $u_k = [v_k]^+ - q_k$, which

$$[v_k]^+ = \begin{cases} v_k & \text{if user } k \text{ granted the bundle} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and q_k is the price user k pays for a bundle. Later we define the bundle price according to the users' demand for that subcarrier.

In the following, we propose a computationally efficient heuristic approach along with a load control scheme which provides a fair distribution of resources to the users regardless of their location in the cell. Moreover, the valuation function enforces users to bid according to their true demand. First, the number of subcarriers a user may apply for is determined according to the rate requirement ratio of the users, particularly

$$r^k_{\min} : r^d_{\min} = \beta_k : \beta_v \qquad \forall k, d \in \mathcal{K} \tag{5}$$

where β is the normalized proportionality constants, where $\sum_{k=1}^{K} \beta_k = 1$. The number of subcarriers N_k is determined to satisfy

$$N_1 : N_2 : ... : N_K = \beta_1 : \beta_2 : ... : \beta_K \tag{6}$$

This step is based on the reasonable assumption also made in [7] that the proportion of subcarriers assigned to each user is approximately the same as their eventual rates after power allocation. The step is accomplished by $N_k = \lfloor \beta_k N \rfloor$, where $\lfloor . \rfloor$ denotes the floor function. The subcarrier set $\mathcal{N}_k$ which user k requests is the N_k best available subcarriers, as each user acts rational. Since each user has a power budget P_k, and the number of subcarriers allocated to the user k is determined to be N_k, the power level at each subcarrier for this user is then P_k/N_k. We define the valuation for the bundle of subcarriers as

$$v_k = \sum_{i \in \mathcal{N}_k} \exp\left(- \frac{(\mathrm{SINR}_{k,i} - \mathrm{SINR}_{\text{target}})^2}{(\mathrm{SINR}_{\text{target}})^2} \right) \tag{7}$$

where $\text{SINR}_{\text{target}}$ is the required SINR to achieve the minimum rate requirement. The bell-shape valuation function reaches its maximum point where the real SINR and the required SINR meets. When the subcarrier SINR cannot meet the user requirement, valuation will decrease from maximum point. Moreover, when SINR exceeds the user requirement, the user valuation for that subcarrier will decrease. Note that, this property motivates the users to bid for the subcarriers which meet their requirements. Note that, the property of utility function motivates users to bid their true valuation for the subcarriers. Moreover, the definition of users' valuation for subcarriers results in an allocation of subcarriers which reduces the interference from adjacent cells.

Our applied heuristic approach is as follows. In the approximated linear program algorithm, a variant of primal-dual approximation algorithm is used, and a scaled price for each subcarrier is maintained. Finally, a fractional allocation for every user and a price for every subcarrier are assigned. Then, in the hill climbing algorithm, the bids are sorted by the descending order of a defined ratio followed by a greedy allocation of bids which provide an initial allocation. After initial allocation is attained, a stepwise random updates are made through local improvement algorithm. The detailed of the procedure is as follows. In Algorithm 1, we will maintaining a scaled price θ_i for each subcarrier. The main concept is the fractional allocation to a bid from user k at prices $\theta_1 ... \theta_N$ to be

$$A_k\left(\theta_1 ... \theta_N\right) = \exp\left(-\sum_{i \in \mathcal{N}_k} \frac{\theta_i}{v_k}\right) \tag{8}$$

So if the offered price i.e. valuation, is small compared to the subcarrier prices, the allocation is nearly zero, while if it is large, the allocation is nearly 1. We also keep a demand for subcarrier i at prices $\theta_1 ... \theta_N$ which is the sum of allocation for it, particularly

$$D_i\left(\theta_1 ... \theta_N\right) = \sum_{k | i \in \mathcal{N}_k} A_k\left(\theta_1 ... \theta_N\right) \tag{9}$$

In another word, in each iteration the algorithm chooses the subcarrier i with highest demand at the current price, and then increases the price by a small quantity, then it updates the allocation and demand vectors. Note that, the users' valuation in the first step is according to the fractional allocation vector of A_k. The linear programming approximation provides a fractional allocation $0 \leq A_k \leq 1$ for every bid from users. It also creates a scaled price for each subcarrier which depends on the demands and users' valuation for subcarriers. The basic idea is that the algorithm tries to do the allocation to the users with larger A_k, and for those with $A_k = 0$, do the allocation to the users with larger $v_k \big/ \sum_{i \in \mathcal{N}_k} \theta_k$. Note that, the inputs to the hill climbing step, are the set of users' bid for subset of subcarriers, (N_k, v_k), as well as the outputs of algorithm 1 including scaled prices and scaled allocations. In summary, the greedy algorithm orders the bids according to the ranking. It then goes over the users' bid according to the order, trying to satisfy each user that does not request a subcarrier which has been previously allocated.

To achieve better performance in terms of optimality, we then try improving the results using local improvements. The main concept is to reorder the bids and run the greedy algorithm. The local improvement is done by moving a bid to the first place and then re-running the greedy algorithm on the modified order. We keep the new ordering if the solution of the greedy algorithm has improved, then new local improvement is performed, until no more local improvements are possible. It is shown in the simulation that only a small number of improvements are usually required.

Algorithm 1 Linear programming approximation

Initialization

$\forall i \ \theta_i \leftarrow 0$, $\forall k \ \alpha_k \leftarrow 0$, $\forall k \ A_k \leftarrow 1$

while $\frac{\sum_i \theta_i}{\alpha} \leq (1+\varepsilon) \frac{\sum_k A_k v_k}{D_t}$ **do**

1. $D_i \leftarrow \sum_{k | i \in \mathcal{N}_k} A_k \ \ \forall i$
2. find t such that $D_t = \max_i D_i$
3. $\theta_t \leftarrow \theta_t + \rho_t$ where $\rho_t = \varepsilon \frac{\sum_{k \left| t \in \mathcal{N}_k \right.} A_k}{\sum_{k \left| t \in \mathcal{N}_k \right.} \frac{A_k}{v_k}}$
4. $\forall k$ such that $t \in \mathcal{N}_k$: $\alpha_k \leftarrow \alpha_k + \frac{\rho_t}{v_k}$
5. $\alpha \leftarrow \min_k \alpha_k$

end while

The output of the linear programming approximation algorithm is the scaled prices of $\overline{\theta_i}$ and the scaled allocation of $\overline{A_k}$. Note that, the payment q_k which user k is paying for a bundle is thus $q_k = \sum_{i \in S_k} \bar{\theta}_i$. It must be highlighted that the price of each subcarrier is determined by the system demand for that subcarrier. This specific pricing approach resolve the problem of multiple requests for a specific resource. In the proposed approach, by keeping a demand vector for subcarriers, true price would be determined. In each iteration, subcarriers' prices are updated according to the users' demand. Note that if the offered price is small compared to the item price $v_k \ll \sum_{i \in \mathcal{N}_k} \theta_i$ the allocation is almost zero, while if the item price is large $v_k \gg \sum_{i \in \mathcal{N}_k} \theta_i$ the allocation is nearly one. Moreover, the described algorithm is distributed to both users and BS. Each cell updates the resource allocation and calculates the SINR according to (1), and autonomously allocates its resources. The implementation of a distributed resource allocation leads to an iterative allocation procedure: sequentially, each cell assigns its radio resources with the aim of minimizing the interference. Every time a cell changes its allocation it also modifies the way it interferes with the neighboring cells; therefore, the allocation procedure requires a certain number of iterations before reaching a steady state. To prevent the case in which cell-edge users starve [9], we propose a load control algorithm which controls the load of an entire cell. The load control algorithm reduces the total number of subcarriers that the RRM can allocate in a cell. In this way, it mitigates the total level of interference that a cell generates in the whole system. At the end of each scheduling period, the users' valuation is compared to a threshold value δ. Note that, low valuation is a sign of high level of interference experienced by the user. Then, the worst quality subcarrier is chosen to be removed from the allocation list. When the condition in load control is not satisfied, all cells in the system will use all the bandwidth, otherwise they use only a subset of the available subcarriers. We give priority in the next scheduling interval to the user whose subcarrier has been removed. In this way, the system provide more long-term fairness by compensating that user in the next interval.

Algorithm 2 Hill Climbing Algorithm

1. **Initialization**
for $A_k \neq 0$ **do**
  $\Omega_{init} \leftarrow \text{argsort}_k A_k$
end for
for $A_k = 0$ **do**
  $\Omega_{init} \leftarrow \text{argsort}_k v_k / \sum_{i \in \mathcal{N}_k} \theta_k$
end for
$\Omega \leftarrow \Omega_{init}$
2. **Locally improve** Ω **until no improvement is possible**.
for $k = 1..K$ **do**
  $\hat{\Omega} \leftarrow \Omega$, element j of Ω moved to the first to make $\hat{\Omega}$.
end for
if $\Pi_\Omega > \Pi_{\hat{\Omega}}$ **then**
  Return $\hat{\Omega}$
end if
3. **Do the Greedy algorithm over** Ω .
$W \leftarrow \emptyset$, $Q \leftarrow \emptyset$
for $k = 1...K$ **do**
  if $\mathcal{N}_k \cap Q = \emptyset$ **then**
    $W \leftarrow W \cup \{k\}$
    $Q \leftarrow Q \cup \mathcal{N}_k$
  end if
end for
Return W with value $\Pi = \sum_{k \in W} v_k$.
4. **Load Control**.
if $v_k < \delta$ **then**
  $n^* \leftarrow \text{argmin}_n \text{SINR}_{k,i}$ for $i \in \mathcal{N}_k$
  Remove n^* from $\mathcal{N}_k$
  In next scheduling period $v_k = v_k + \mu$
end if

III. Performance Evaluation

In this section, we examine the performance of our proposed resource allocation scheme over a multicell OFDMA system and compare it with other approaches, namely Hungarian method, adaptive allocation with fractional frequency reuse (AAFFR) scheme. We consider an OFDMA system consisting of three adjacent cells with radius of $R = 500$m where the base stations are situated in the middle of the cell and have omnidirectional antennas. There are ten users randomly located in each cell, transmitting in a bandwidth of 5MHz.

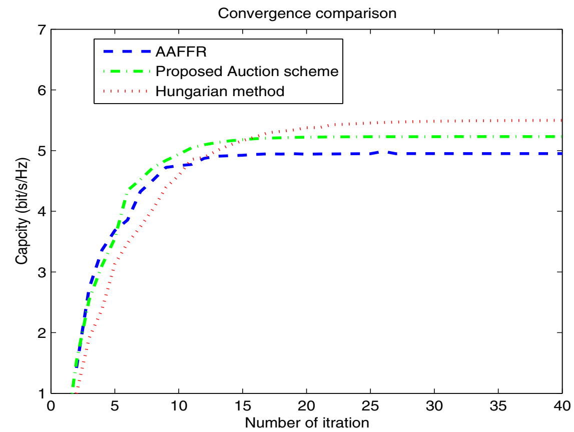

Fig. 1. Convergence comparison of different methods.

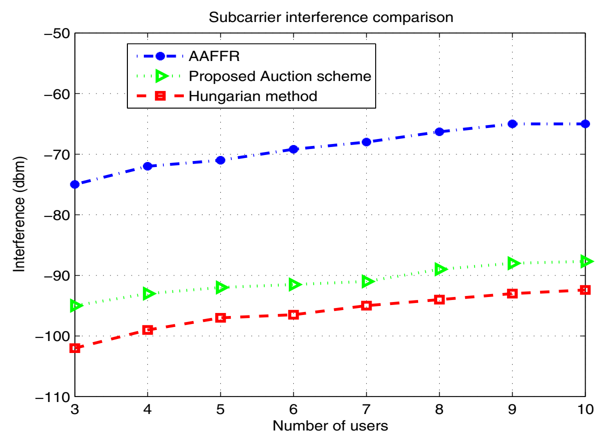

Fig. 2. Average interference per subcarrier for different methods.

The six-path Rayleigh model is taken into consideration to simulate the frequency selective fading channels, which has an exponential power profile. We consider a channel attenuation due to the distance between the BS and the user with the path loss exponent of $\alpha = 4$. The number of subcarriers in the system is 32.

The first numerical result evaluates the convergence performance of the proposed algorithm. Fig. 1 shows the convergence rate of the proposed method compared to the AAFFR and Hungarian approaches. As expected, the Hungarian scheme takes more iteration for convergence, while AAFFR settle faster. The distributed algorithm based on auction needs 14 iterations to converge, while Hungarian need 31 iterations. Moreover, Hungarian algorithm shows better performance as it reaches higher capacity, a result rather expected since it is optimal solution. The performance of our proposed algorithm is slightly inferior compared to that of Hungarian algorithm.

Fig. 2 represents the interference per subcarrier as the number of users increase from 3 users per cell to 10 users per cell. Our approach generates less interference, as the utility of users aims at reducing the interference. As the number of users

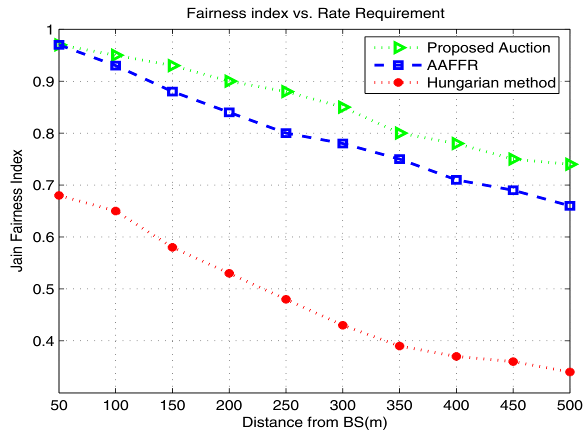

Fig. 3. Jain fair index comparison.

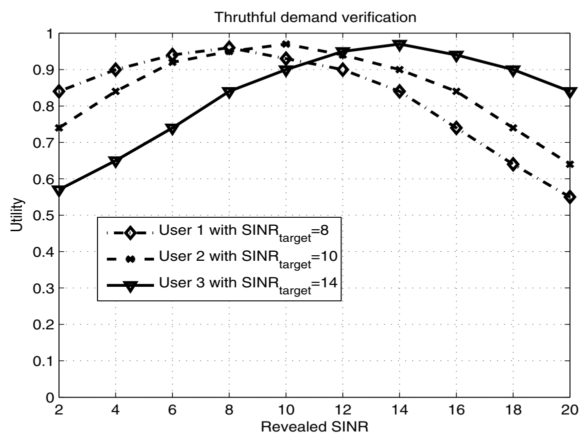

Fig. 4. Truthful demand verification.

increases, the interference changes in small values. Superiority of the Hungarian algorithm, which generates less interference is obvious. Better performance of central approach is true as the number of users increases.

It is important to evaluate the effects of the algorithm on fairness among users. A commonly used measure of fairness is Jain fair index.

$$\text{Jain Index} = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2} \tag{10}$$

The Jain fairness index varies from 0 and 1. A rate allocation is perfectly fair if $\text{Jain Index} = 1$. Higher Jain index indicates higher fairness among the users. Fig. 3 depicts the comparison of our distributed auction with AAFFR and Hungarian methods. Due to the limited radio channel resource, as the rate requirements of users increases the fairness index decreases. Since users are allowed to express their willingness for available resources, auction allocates the resources in a fair approach.

Fig. 4 shows the utility of the users when the revealed SINR changes. Three users with different target SINR i.e., $\text{SINR}_1 = 8$, $\text{SINR}_2 = 10$, $\text{SINR}_3 = 14$, are considered. As the revealed SINR in the biding function increases, the utility of users improves until it reaches its maximum value at the target SINR (i.e. truthful demand), beyond which the utility degrades. Hence, the proposed algorithm enforces truth telling and each user must report its target demand to maximize its utility.

IV. Conclusion

A combinatorial auction is used to model the users' competition in a multicell OFDMA network. Auction objective is to minimize the interference, while power of users is limited. Our proposed valuation function provides a truth-telling behavior of users. Moreover, based on the users' valuation for a bundle of subcarriers, we perform a load control scheme which mitigates the interference experienced by cell-edge users. Due to the complexity of solving the original problem, we apply a heuristic approach to find near optimum allocation. In the approximated linear program phase, a variant of primal-dual approximation algorithm is used. A scaled price for each subcarrier is maintained. Finally, a fractional allocation for every bid and a subcarrier price for every subcarrier are assigned. Then, the bids are sorted by the descending order of a defined ratio, then greedy allocation of bids are made for attaining initial allocation. After initial allocation is attained, a stepwise random updates are made. Our algorithm is designed to be implemented in a distributive manner with low complexity. Simulation results demonstrate that the results offer near-optimal performance.

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