Communications Receivers Employing Wavelet-Domain Zero-Forcing Equalization of Multipath Fading Channels

Canute Vaz and David G. Daut
Department of Electrical and Computer Engineering
Rutgers, The State University of New Jersey
Piscataway, NJ, USA
email: kenvaz@ieee.org, daut@ece.rutgers.edu

***Abstract*— In this paper communications receivers that implement Zero-Forcing (ZF) equalization using the Discrete Wavelet Transform (DWT) are examined. In such receivers, noisy faded received signals are first decomposed using the DWT into detail and approximation coefficients. Instead of the received signals being directly equalized in the discrete time-domain the DWT coefficients of the signals are equalized using ZF equalization. In this work the performance of such receivers under various transmission channel conditions is evaluated. Both slow- and fast- fading multipath channels are considered. In addition, the effect of using different wavelets on receiver performance is also examined. Overall system performance is evaluated using Monte Carlo simulations employing 16-QAM modulation. For all cases considered, the transmitted signals have also been corrupted by additive White Gaussian Noise (WGN). It is found that performance of receivers using wavelet-domain ZF equalization is similar to the performance of receivers using discrete time-domain ZF equalization.**

***Keywords*— Wavelet transforms, communications receiver, zero-forcing equalization, channel equalization**

I. INTRODUCTION

Several studies conducted recently have shown that certain communication strategies exhibit surprisingly powerful properties in the wavelet-domain [1-6].

The first strategy is that of Automatic Modulation Recognition (AMR). Newly devised wavelet-domain AMR algorithms [1-3] have been able to blindly recognize digitally modulated signals under harsh Signal-to-Noise Ratio (SNR) conditions. For example, 256-QAM signals can been correctly classified with 100% accuracy at -5 dB SNR [3].

It has also been discovered that after such a modulation recognition step the signal can be demodulated in the wavelet-domain itself and baseband data symbols can be recovered. Bit Error Rates (BERs) from this second wavelet-domain strategy of interest have been found to be similar to traditional direct matched filter detection of signals [3].

The third interesting strategy is of de-noising received digitally modulated signals in the wavelet-domain before matched filter demodulation. It has been found in the case of PSK signals that BERs can be reduced by three orders of magnitude [4].

When combined these three wavelet-domain strategies have the potential to enable a new class of radio receivers, namely agile radio receivers in which signals are processed in the wavelet-domain [5].

A. Scope of this Work

When considering any radio receiver channel equalization is an essential component. This would certainly be the case in a receiver operating in the wavelet-domain as well.

The first objective of this work is, therefore, to establish a general filter bank-based approach that will allow for traditional channel equalization algorithms to be used in the wavelet-domain. To demonstrate that this approach works, the specific method of zero-forcing equalization is used in a 16-QAM receiver and operates on the DWT coefficients of received signals (i.e., operates in the wavelet-domain) instead of directly on the received signal itself.

The second objective is to determine whether the approach of zero-forcing channel equalization in the wavelet-domain works well in the context of communications channels. For this purpose three multipath channel models, each with slow and fast fading conditions, as well as AWGN are used in Monte Carlo simulations. The AWGN results in SNRs from 0 dB to 30 dB.

To lend a further measure of rigor, two wavelets, the Haar and symlet5, are used in the simulations. The results of the combinations of all of these variables are presented in this paper.

It must be mentioned that the concept of wavelet-based channel equalization has been the subject of a few studies in the past, in which various approaches have been devised with success [7-11]. The approach taken in this work, however, is to implement equalization within a filter bank framework, which will be ultimately beneficial in the design of a wavelet-domain agile radio receiver.

B. *Organization of the Paper*

In Section II the technique for DWT-based convolution called the Forward Merge Approach is first explained [5]. It is then used to develop the general framework for wavelet-domain channel equalization. The communications system incorporating such an equalizer is also presented.

Section III contains details of the setup and parameters of the Monte Carlo simulations, descriptions of the three channel models. The simulation results are also presented in Section III by way of Symbol Error Rate (SER) curves.

Conclusions about the performance of receivers using wavelet-domain channel equalization are presented in Section IV.

II. Theoretical Background

A. *Generalized Technique for Wavelet-Domain Equalization*

Consider a system in which a message signal, $x(n)$, is transmitted through a channel with an impulse response $y(n)$. Assuming this transmission process is modeled by a convolution operation, the resulting received signal, $r(n)$, is obtained from the expression

$$x(n)*y(n)=r(n) \tag{1}$$

or, equivalently, in the z-domain as

$$X(z)Y(z)=R(z). \tag{2}$$

It is desired, however, to process the received signal prior to demodulation using a suitable equalizer filter, $B(z)$. The equalized output, $\hat{x}(n)$, can now be described in the z-domain as

$$X(z)Y(z)B(z)=\hat{X}(z). \tag{3}$$

The relationship described in (3) can be used to introduce the DWT into the equalization process. Using the DWT a signal can be decomposed into its multiresolution detail and approximation coefficients. By using the Inverse Discrete Wavelet Transform (IDWT) these coefficients can be used to reconstruct the original signal. Such decomposition (analysis) followed by reconstruction of a signal (synthesis) means that the output of the equalizer $B(z)$, i.e., $\hat{X}(z)$, can be input to a DWT-IDWT filter bank. The output of the filter bank is simply a perfectly reconstructed copy of $\hat{X}(z)$ [12, 13]. It could be said that there is no net effect of introducing a DWT filter bank and then an IDWT filter bank right after it. However, there is a distinct, albeit subtle, advantage to doing so. This procedure allows the equalizer filter, $B(z)$, to be merged into the DWT portion of the DWT-IDWT filter bank, as shown in Fig. 1. This technique is called the Forward Merge Approach for convolution using the DWT [11].

In Fig. 1, the merged equalizer filter, $B(z)$, is now positioned in two locations: following both the high-pass and low-pass DWT analysis filters, $G(z)$ and $H(z)$, respectively. The filters $B(z)$ are highlighted in Fig. 1.

Now consider the case where a pilot signal, $u(n)$, is transmitted prior to the message signal, $x(n)$, through the same channel having an impulse response, $y(n)$. A replica of the uncorrupted transmitted pilot signal, $u(n)$, is available *a priori* at the receiver, and it is used, along with the noisy faded received pilot signal, $v(n)$, to determine the filter tap coefficients of the desired equalizer. In this paper the ZF equalization algorithm is used, whose action is well known to be that of a z-domain inversion of the transfer function of the channel being equalized. In particular, this can be expressed as

$$B(z)=Y^{-1}(z). \tag{4}$$

The coefficients of the ZF equalizer that are computed based on the transmitted and received pilot signals, $u(n)$ and $v(n)$, respectively, are denoted by $\{b_n\}$. These coefficients are used in filters to operate on the detail and approximation coefficients of the received message signal, $r(n)$, as shown in Fig. 2.

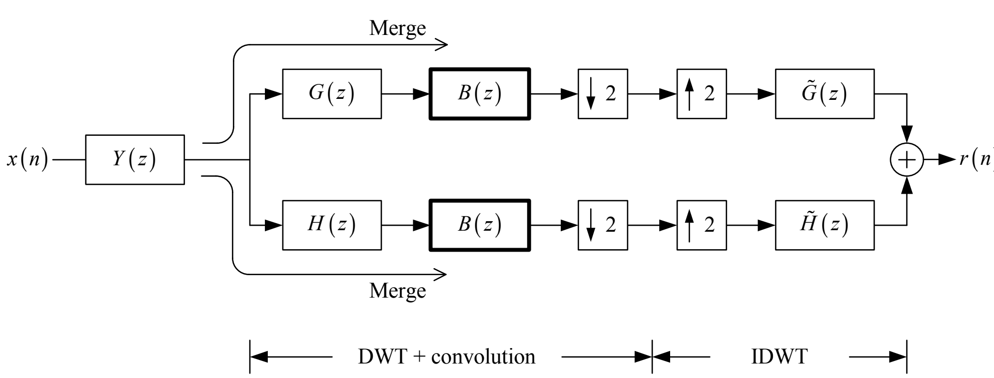

Fig. 1. Merging the equalizer filter with a DWT filter bank.

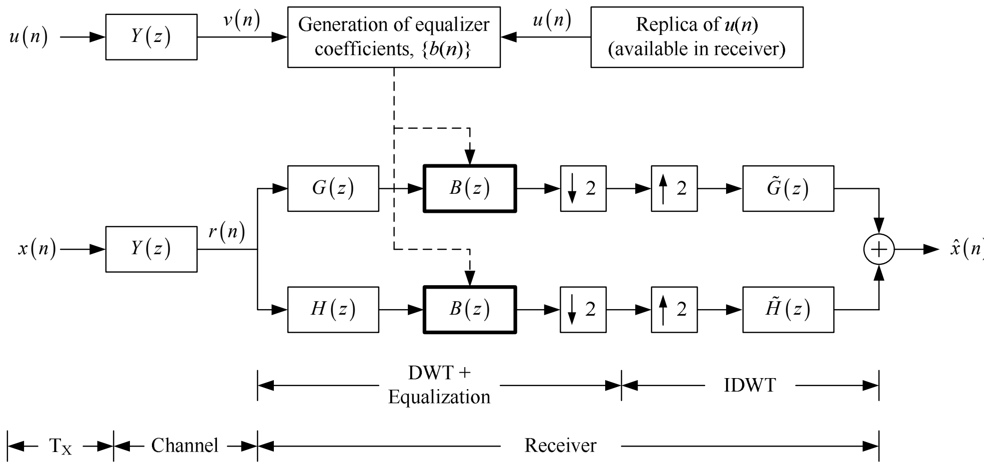

Fig. 2. Framework for equalizing DWT coefficients of received signals.

B. Communications Receiver Model

The communications system model used in this paper, and implemented in MATLAB, is that of a 16-QAM system. This modulation scheme has been chosen for illustrative purposes. The block diagram of the system is shown in Fig. 3. A baseband symbol sequence, $\{I_n\}$, is used to modulate a carrier signal as defined by the 16-QAM scheme [1] and is transmitted over a communications channel having a known impulse response.

The channel introduces fading distortions in the signal, and WGN is added to the faded signal. Upon acquisition by a receiver, the noisy faded message signal is decomposed in the wavelet-domain with the aid of a DWT filter bank. The DWT coefficients of the signal are equalized using identical ZF equalizer filters, and these equalized coefficients are then reconstructed into a time-domain signal with the aid of an IDWT filter bank resulting in a desired equalized signal.

The equalized signal is then input to a demodulator, which is composed of matched filters and samplers. A decision device follows the demodulator in order to produce an estimate the transmitted message symbol sequence.

III. Simulations

A. Simulation Parameters

Each simulation experiment consists of a pilot signal and a message signal being convolved separately with the same channel impulse response to produce a faded pilot signal and a faded message signal. White Gaussian noise is added to each faded signal. Using a copy of the transmitted pilot signal, stored locally at the receiver, and the noisy faded received pilot signal the coefficients of the corresponding ZF equalizer are computed. The noisy faded message signal is then decomposed using the DWT to the first level of DWT resolution. The resulting level 1 detail and approximation coefficients are filtered by the ZF equalizer. These equalized DWT coefficients are then input to an IDWT reconstruction filter bank, the output of which is the equalized message signal in the discrete time-domain. This signal is demodulated and the baseband symbol sequence is estimated.

The entire procedure described above, i.e., of pilot and

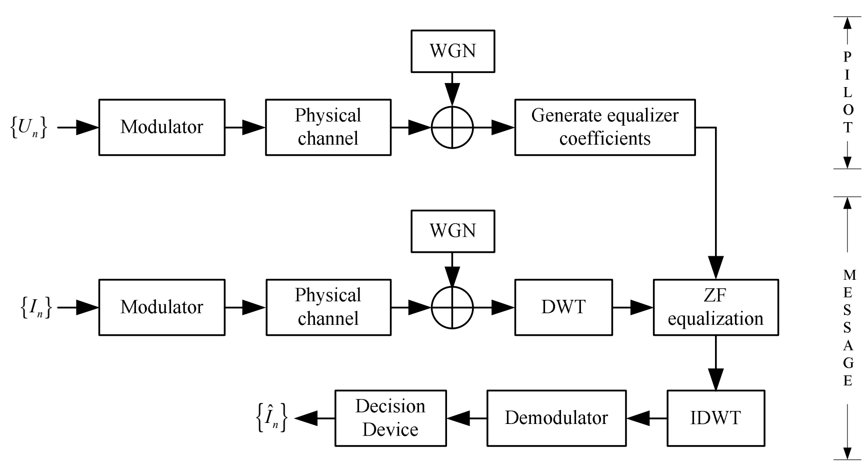

Fig. 3. Communications system model including wavelet-domain ZF equalization.

message signal transmission, corruption of signals by channels and AWGN, generation of equalizer filter coefficients, and channel equalization, is repeated in every Monte Carlo simulation trial. In all simulations, the first parameter that is varied is the power level of the AWGN. For every value of E_b/N_0 1,001 signal transmissions, receptions, equalizations, and demodulations are performed with new symbol, channel, noise, and filter sequences generated for each run.

Each message signal transmission consists of 10 data symbols. There are 10 symbols in each pilot signal as well. The symbol errors that occur are recorded, from which the Symbol Error Rate (SER) is determined so as to assess the performance of the receiver under specific channel conditions and choice of wavelet.

The simulations are first performed for three channel PDP models and two fading conditions while using the Haar wavelet. Next, the same set of simulations are repeated but this time using the symlet5 wavelet. These two particular wavelets have been chosen as representatives of two important wavelet families: the Daubechies and symlet families, respectively. They have also been chosen because the Haar wavelet corresponds to DWT and IDWT filters having two taps, while the symlet5 wavelet corresponds to filters having 10 taps. These wavelets have been selected so that any effects DWT and IDWT filter tap lengths might have on the equalization of noisy faded 16-QAM signals can be determined.

B. Channel Models

For the three channel PDPs presented herein, the Root-Mean-Squared (RMS) delay spread of the channel, τ, is defined as

$$\tau = dT_s \tag{5}$$

where d is the normalized delay time of the channel and T_s is the baseband data symbol period.

The first is a Gaussian channel PDP that is defined as [14]

$$\phi_c(t) = \frac{1}{\sqrt{2\pi\tau}} e^{\frac{-(t/\tau)^2}{2}}. \tag{6}$$

The second is an exponential PDP is defined as [14]

$$\phi_c = \begin{cases} \frac{1}{\tau} e^{-t/\tau}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

The final PDP represents a "hilly area" transmission link that is a double-exponential model described by

$$\phi_c(t) = \begin{cases} e^{-10t/\tau}, & 0 < t \leq 0.5\tau \\ 0.5e^{-5(0.5\tau - t)/\tau}, & 0.5 < t \leq \tau \end{cases}. \tag{8}$$

The fading conditions for all channels are controlled by varying the normalized delay time, d. More precisely, for slow fading the value of d is 2.5, while for fast fading the value of d is 0.2.

C. Results

In Fig. 4 the Symbol Error Rates (SERs) for the case of slow-fading channels and the Haar wavelet are illustrated. The receiver performance SER curves when fast-fading channels and the Haar wavelet were considered are shown in Fig. 5.

Note that in Figs. 4-7 there are two SER performance curves for each channel. The solid lines are obtained from wavelet-domain ZF equalization whereas the dashed lines are from discrete time-domain ZF equalization, i.e., from the control experiments.

In Figs. 6 and 7, the results of the Monte Carlo simulations when the symlet5 wavelet is used are shown. Fig. 6 contains the SER curves of the receiver when all three channel PDPs are slow-fading. In Fig. 7 the SER performance curves are shown for the cases when fast-fading channels, along with the symlet5 wavelet, are used.

From Figs. 4-7 it is seen that for every channel used, and for both slow- and fast-fading, and for both the Haar and symlet5

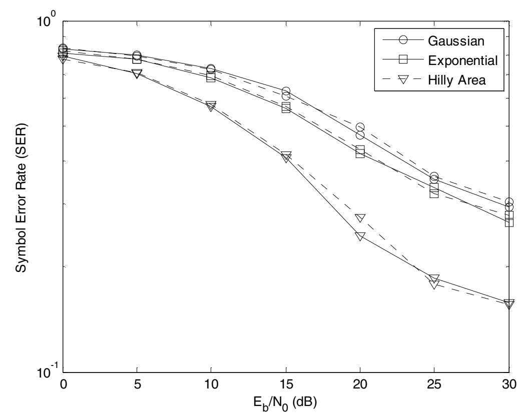

Fig. 4. SER curves for all three slow-fading channels, for the Haar wavelet case. Solid lines depict wavelet-domain ZF equalization while dashed lines depict the control experiments, receivers using time-domain ZF equalization.

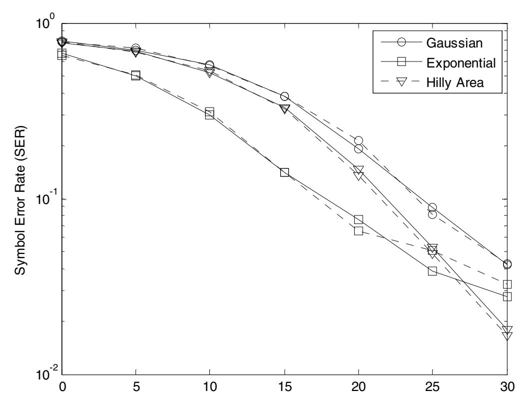

Fig. 5. SER curves for all three fast-fading channels, for the Haar wavelet case. Solid lines depict wavelet-domain ZF equalization while dashed lines depict the control experiments, receivers using time-domain ZF equalization.

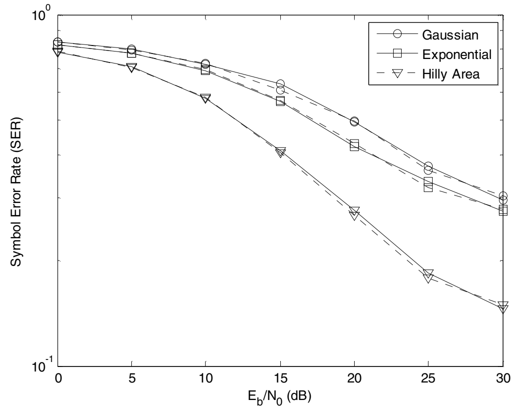

Fig. 6. SER curves for all three slow-fading channels, for the symlet5 wavelet case. Solid lines depict wavelet-domain ZF equalization while dashed lines depict the control experiments, receivers using time-domain ZF equalization.

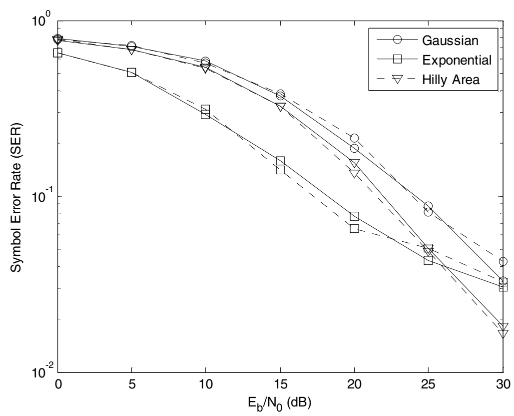

Fig. 7. SER curves for all three fast-fading channels, for the symlet5 wavelet case. Solid lines depict wavelet-domain ZF equalization while dashed lines depict the control experiments, receivers using time-domain ZF equalization.

wavelets, receivers that use wavelet-domain ZF equalization perform similarly to receivers that employ discrete time-domain equalization.

When comparing fading conditions it is generally seen that symbol error rates are greater for the fast-fading channels than for slow-fading channels. This is seen to occur when either discrete time- or wavelet-domain ZF equalization is used in the receivers, and for all values of E_b/N_0 .

IV. Conclusions

The intent of this study was to determine whether receivers that use a wavelet-domain ZF equalization sub-system have similar performance to identical receivers that use discrete time-domain ZF equalization.

It has been seen from the results of these simulations, i.e., the SER performance curves illustrated in Figs. 4-7, that receivers in which wavelet-domain ZF equalization was used performed in a manner similar to receivers that used discrete time-domain ZF equalization. The observation is consistent across all cases of three, slow- and fast-fading, multipath channel models that were used along with AWGN.

Overall receiver performance evaluation using Monte Carlo simulations was obtained systematically, in which three important parameters, e.g., channel model, wavelet and E_b/N_0 , were varied one-at-a-time.

It was found that the both the Haar and symlet5 wavelets provided similar ZF equalization performance. The Haar wavelet corresponds to DWT and IDWT filters with two taps whereas the symlet5 wavelet corresponds to filters with 10 taps. This observation is important because it implies that long wavelet filter lengths do not contribute to degradation in the performance of wavelet-domain channel equalization.

It has been established in this paper that wavelet-domain ZF equalization provides accurate results for the time-invariant channel cases simulated. Ongoing work is focused on developing this fundamental approach further to realize a generalized adaptive equalization scheme in the wavelet-domain.

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