Low Complexity Beamforming Methods for MIMO-OFDM Systems

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Abstract—**Multiple-input multiple-output (MIMO) systems can significantly improve the network capacity and the quality of service (QoS). We propose a novel beamforming method for MIMO orthogonal frequency division multiplexing (OFDM) systems in this work. The presented method is based on the Rayleigh Quotient (RQ) of the Fourier transform of the channel matrix. Two RQ analysis algorithms, simple steepest descent and Luo's algorithms, are used to obtain the new beamforming method. It is demonstrated that the proposed method can considerably reduce the complexity and needs only (2*K*+1)*L* complex multiplications and 2*KL*-1 complex additions for a single run, where *K* and *L* are the number of antennas at the transmitter and the receiver, respectively. We evaluate the performance of the presented method in AWGN and Rayleigh fading channels. Simulation results show that the new method reduces the bit error rate (BER) significantly while having low computational complexity. It is also shown that the presented method has a very fast convergence rate.**

Keywords-MIMO; OFDM; beamforming; complexity.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is an efficient technique in reducing severe multipath fading effects and providing high speed digital data transmission [1]. Due to its advantages and flexibilities, OFDM is widely used in different communication networks [2], [3]. On the other hand, beamforming by adaptive antenna arrays can significantly increase the network capacity and improve the quality of service (QoS) in wireless systems [4], [5].

Consequently, improving the performance of OFDM systems by implementing antenna arrays and beamforming has been an interesting research area in recent years. Several beamforming methods for OFDM systems have been presented so far [6]-[13].

Although these methods lead to the satisfactory results, their application in real time systems is restricted by their computational complexity. Heavy computational load not only makes these beamforming methods time consuming, but also increases the power consumption which is a degrading parameter especially in mobile users.

Extensive research has been done in order to alleviate this problem and to propose beamforming algorithms which can be applied to real-time OFDM systems. A beamforming method based on eigenvectors analysis of the channel matrix is proposed in [8]. This method considers the antenna arrays at both base and mobile stations, but its required number of complex multiplications for a single run is $(K^2+L^2)(K+L)$, where K and L are the number of antennas at the base and mobile stations, respectively. A minimum mean squared error (MMSE) based algorithm is presented in [9]. Although the presented method has a lower complexity than [8], it computational load is still of order 3 of the number of antennas. In [10], Sandel et al reduced the complexity of the smooth beamforming method presented in [11]. Though, using wiener filter for channel smoothing and QR algorithm for singular value decomposition (SVD), the algorithm complexity is still of order 3. In [12], a switched-beam beamformer technique is described. The switched-beam technique forms multiple fixed beams simultaneously covering the serving area with each beam directing a particular direction and selects the optimum beam. Although this method has a lower complexity than adaptive beamforming methods, its accuracy and performance is degraded compared to the adaptive methods. A recursive least square (RLS) beamforming method is presented in [13]. This method offers the complexity of order 2 of the number of antennas but it only considers the antenna array at one side of the link i.e. either at the transmitter or at the receiver.

In this paper, a new adaptive beamforming method for multiple-input multiple-output (MIMO) OFDM communication systems is proposed. Rayleigh quotient (RQ) objective function is the basis for developing the presented method. Variations of the RQ-based algorithms have been investigated by researchers [14]-[17]. Two of these algorithms, simple steepest descent [14] and Luo algorithm [15], are used to obtain updating equation for array weight vectors in this work. It is shown that the proposed method needs only $(2K+1)L$ complex multiplications and $2KL$-1 complex additions for a single run, where K and L are the number of antennas in the transmitter and receiver arrays, respectively. We evaluate the proposed method performance in terms of the bit error rate (BER) in additive white Gaussian noise (AWGN) and Rayleigh fading channels. It is illustrated that the proposed method not only has a considerably low computational load but also provides a desirable performance in reducing the BER. We compare the performance of the proposed method with that of the method presented in [8] and it is shown that they provide similar BER reduction while the proposed method has a considerably lower computational load. It is demonstrated that the proposed method has a fast convergence rate too.

The remainder of this paper is organized as follows. In section II, the OFDM system with antenna array is described. In section III, the beamforming method is proposed based on the system description in section II. Simulation results are presented in section IV and finally, section V concludes the paper.

Throughout this paper, bold lower letters denote vectors and bold upper letters denote matrices. $(.)^*$, $(.)^T$, and $(.)^H$ denote complex conjugate, transpose and Hermitian transpose, respectively. $\|.\|$ is used to denote the Euclidean norm.

II. SYSTEM MODEL

Figure 1 shows the configuration of the OFDM system with the proposed beamformer. It is assumed that there are K and L antennas at the transmitter and receiver sides, respectively. In this paper, downlink transmission is investigated, i.e., base and mobile stations are considered as transmitter and receiver, respectively. Similar procedure can be applied for uplink transmission.

Initially, a serial sequence of input data is converted to the parallel form. Then, in each parallel path the input data is mapped into one of the 2^{d_c} transmitted symbols, where, d_c is the number of bits assigned to the subcarrier *c*. Different number of bits could be assigned to the different subcarriers based on the quality of the related subchannels. In general, subcarriers with higher quality can carry more data in order to provide more network capacity [18].

After generating the transmitted symbols, these symbols are multiplied by the weight vector at the transmitter side. It should be noted that all subcarriers will pass through the antenna array and since each subcarrier experiences its own channel, different weight vector is calculated for each subcarrier. The transmitter side weight vector related to the subcarrier c is shown by $\mathbf{w}_t^c = [w_{t,1}^c, w_{t,2}^c, ... w_{t,K}^c]^T$.

After applying the weight vector, the resulted signal will be modulated on the subcarriers by using inverse fast Fourier transform (IFFT). A separate IFFT module is used for each antenna. Finally, guard interval is inserted in order to reduce the Inter-symbol Interference (ISI), parallel to serial conversion is performed, and the signal is transmitted via antenna array.

At the receiver side, receiver antenna array collects the data. Serial to parallel conversion is performed and guard interval is removed. Then, the received signal is demodulated (i.e., is transformed to the frequency domain) by applying fast Fourier transform (FFT). After that, the received signal from each subcarrier is multiplied by the related weight vector at the receiver side ($\mathbf{w}_r^c = [w_{r,1}^c, w_{r,2}^c, ... w_{r,L}^c]^T$). The weighted signals from L antennas will be summed up to form the scalar output for each subcarrier. For a subcarrier *c*, signal is then de-mapped from one of the 2^{d_c} symbols into binary sequence. Finally, all these binary sequences are converted from parallel to serial in order to generate the output data sequence.

We model the wireless channel as a multipath fading model represented by its channel impulse response. The impulse response of the channel between *k*th transmitter and *l*th receiver antennas can be written as [8]

$$h_{k,l}(t) = \sum_{i=0}^{M-1} \alpha_{k,l}^i \delta(t - \tau_{k,l}^i) \ , \tag{1}$$

where M is the number of paths, $\alpha_{k,l}^i$ and $\tau_{k,l}^i$ are the complex gain and time delay of the *i*th path between *k*th transmitter and *l*th receiver antennas, respectively, and $\delta(\cdot)$ is delta function.

III. PROPOSED METHOD

Considering (1), the discrete-time channel matrix at the *n*th sampling time (i.e., $t = nT$, where T is the sampling period) would be

$$\mathbf{H}_t(n) = \begin{bmatrix} h_{1,1}(n) & h_{1,2}(n) & ... & h_{1,L}(n) \\ h_{2,1}(n) & h_{2,2}(n) & ... & h_{2,2}(n) \\ \vdots & \vdots & & \vdots \\ h_{K,1}(n) & h_{K,2}(n) & ... & h_{K,L}(n) \end{bmatrix}. \tag{2}$$

Suppose the OFDM system has N_c subcarriers. If we take the N_c-point Fourier transform of each of the channel matrix ($\mathbf{H}_t(n)$) elements, we obtain N_c $K \times L$ matrices each of which corresponds to a subcarrier. We denote by $\mathbf{H}^c$ the matrix corresponding to the *c*th subcarrier. In the proposed method, we assume that channel matrix $\mathbf{H}_t$ is known in order to calculate the weight vector. Some OFDM channel estimation methods can be used to obtain the channel matrix [19, 20].

The transmitted symbols form the matrix $\mathbf{s} = [s_0, s_1, ..., s_{N_c-1}]$, where s_c is the symbol transmitted by the *c*th subcarrier. Consequently, the received signal $\hat{s}_c$ at subcarrier *c* could be written as

$$\hat{s}_c = (\mathbf{w}_r^c)^H (\mathbf{H}^c)^T \mathbf{w}_t^c s_c + (\mathbf{w}_r^c)^H \mathbf{n}_c , \tag{3}$$

where $\mathbf{n}_c$ is a L dimensional vector of complex additive white Gaussian noise (AWGN) with variance σ_n^2. To obtain the maximum SINR using maximal ratio combining (MRC) [8], we set the weight vector at the receiver side as

$$\mathbf{w}_r^c = (\mathbf{H}^c)^T \mathbf{w}_t^c . \tag{4}$$

Therefore, considering that $\left((\mathbf{H}^c)^T\right)^H = (\mathbf{H}^c)^*$, (3) could be written as

$$\hat{s}_c = (\mathbf{w}_t^c)^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}_t^c s_c + (\mathbf{w}_t^c)^H (\mathbf{H}^c)^* \mathbf{n}_c . \tag{5}$$

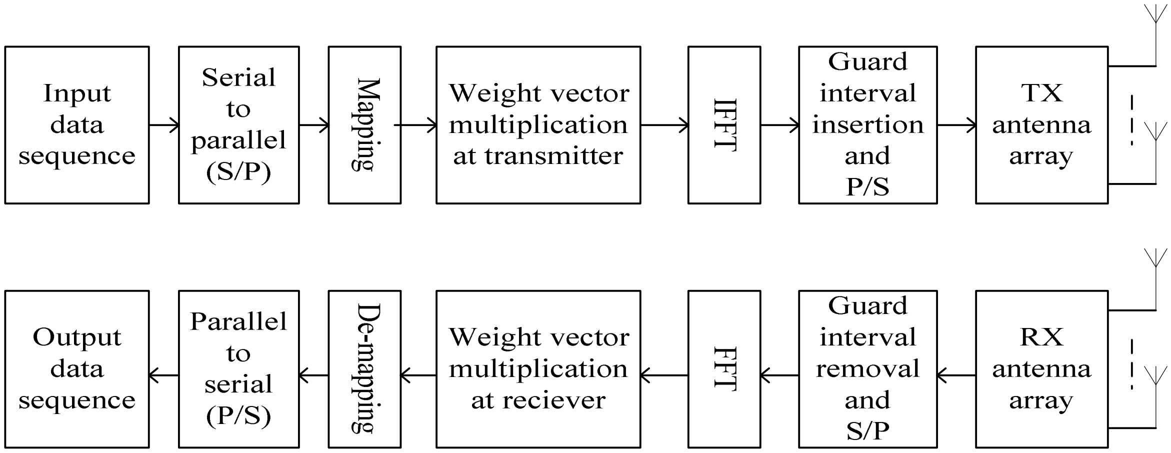

Fig. 1. OFDM/beamformer system configuration

Assuming the fix values for the power of signal and noise, the optimum weight vector for maximizing the signal to noise ratio (SNR) at the receiver array output will be obtained as

$$\left(\mathbf{w}_t^c\right)_{opt} = \arg\max_{\mathbf{w}_t^c} \frac{(\mathbf{w}_t^c)^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}_t^c}{(\mathbf{w}_t^c)^H \mathbf{w}_t^c}. \quad (6)$$

This means that the optimum weight vector would be vector wish maximize the RQ of the covariance of the Fourier transform of the channel matrix.

Regarding the aforementioned explanations, in this section, we propose a new beamforming method based on two RQ-based algorithms, simple steepest descent [16] and Luo [17]. The RQ objective function for the Fourier transform of the channel matrix of the cth subcarrier ($\mathbf{H}^c$) can be written as

$$J(\mathbf{w}) = -\left(\frac{\mathbf{w}^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}}{\mathbf{w}^H \mathbf{w}}\right). \quad (7)$$

The gradient of (7) with respect to $\mathbf{w}$, which is used to obtain the weight vector updating equation in both methods, would be

$$\nabla J(\mathbf{w}) = -\frac{1}{\mathbf{w}^H \mathbf{w}}\left((\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w} - \mathbf{w}\frac{\mathbf{w}^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}}{\mathbf{w}^H \mathbf{w}}\right). \quad (8)$$

A. Steepest Gradient Method

Considering the simple steepest gradient method [16], the equation for updating the weight vector regarding the objective function $J(\mathbf{w})$ is

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu \nabla J(\mathbf{w}), \quad (9)$$

where $\mathbf{w}(k)$ denotes the weight vector at the kth sampling time, μ is a real number which is determined for the convergence of the adaptive procedure, and $\nabla J(\mathbf{w})$ is the gradient vector of $J(\mathbf{w})$ with respect to $\mathbf{w}$.

Using (8), we can rewrite (9) as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\frac{1}{\mathbf{w}^H \mathbf{w}}\left((\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w} - \mathbf{w}\frac{\mathbf{w}^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}}{\mathbf{w}^H \mathbf{w}}\right). \quad (10)$$

If we impose the criterion $\mathbf{w}^H \mathbf{w} = 1$ (for normalizing the array gain) to (10), we would have

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu((\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w} - \mathbf{w}\mathbf{w}^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}). \quad (11)$$

Now, if we define the L dimensional vector $\mathbf{a}$ as

$$\mathbf{a} = (\mathbf{H}^c)^T \mathbf{w}, \quad (12)$$

which is in fact the combiner vector at the receiver, the final form of the updating equation for the weight vector would be

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu((\mathbf{H}^c)^* \mathbf{a} - \mathbf{w}\|\mathbf{a}\|^2). \quad (13)$$

B. Luo Method

In this section, Luo algorithm for RQ analysis [17] is used to obtain the updating equation. According to Luo algorithm, the weight vector's updating equation can be written as

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\left(\mathbf{w}^H \mathbf{w}\right)^2 \nabla J(\mathbf{w}), \quad (14)$$

Replacing $\nabla J(\mathbf{w})$ from (8) into in (14), we obtain

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\, \mathbf{w}^H \mathbf{w}\left((\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w} - \mathbf{w}\frac{\mathbf{w}^H (\mathbf{H}^c)^* (\mathbf{H}^c)^T \mathbf{w}}{\mathbf{w}^H \mathbf{w}}\right), \quad (15)$$

Again, applying $\mathbf{w}^H \mathbf{w} = 1$ and substituting $\mathbf{a}$ from (12) into (15) we obtain exactly the same equation as (13) for updating equation of the weight vector.

The number of complex multiplications and complex additions required for a single run of the proposed method is shown in Table I. We observe from Table I that each run of the proposed method needs only $(2K+1)L$ complex multiplications and $2KL$-1 complex additions, where K and L are the number of antennas in the transmitter and receiver arrays, respectively. It can be seen that although in the proposed method antenna arrays are used at both transmitter and receiver sides, the computational load is still of order two of the number of antennas. In case of using single antenna at the mobile station (receiver side), the number of complex multiplications and complex additions will be $2K$+1 and $2K$-1, respectively which is of order one. The number of required complex multiplications for a single run of the proposed method is compared to that of method in [8] in Fig. 2. Noting that a complex multiplication requires four real multiplications and two real additions and that a complex addition only requires two real additions, number of complex multiplications is the determining factor for the computational load of an algorithm.

TABLE I

NUMBER OF COMPLEX OPERATIONS NEEDED FOR A SINGLE RUN OF THR PROPOSED METHOD

	Number of complex multiplications	**Number of complex additions**
$\mathbf{a} = (\mathbf{H}^c)^T \mathbf{w}$	KL	$(K\text{-}1)L$
$\boldsymbol{\alpha} = (\mathbf{H}^c)^* \mathbf{a}$	KL	$K(L\text{-}1)$
$\beta = \|\mathbf{a}\|^2$	L	$L\text{-}1$
$(1-\mu\beta)\mathbf{w}(k) + \mu\boldsymbol{\alpha}$	0	K
Total	$(2K+1)L$	$2KL\text{-}1$

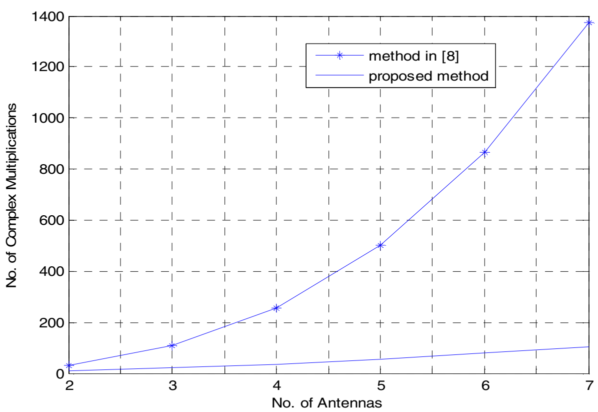

Fig. 2. Number of complex multiplications required for a single run of beamforming methods, when the same number of antennas is used at the transmitter and the receiver.

IV. Simulation Results

In this section, we evaluate the performance of the proposed method via software simulations. For this purpose, the effectiveness of the proposed method in reducing the BER is investigated. The simulated OFDM system has 32 subcarriers and for simplicity, binary phase shift keying (BPSK) modulation is used for all subcarriers. It is assumed that each subcarrier experiences a flat Rayleigh fading with maximum Doppler frequency of 80Hz and sampling time of 10µs. We also assume that the guard interval is larger than the maximum delay spread of the channel. Channel noise is considered as additive white Gaussian. In all simulations, we consider only one iteration for the convergence of the proposed method.

Fig. 3 shows the BER vs. SNR, for three different cases: single antenna in transmitter and receiver (i.e., no beamforming), two antennas at the transmitter and single antenna at receiver, and finally two antennas at the both sides.

It can be seen from Fig. 3 that using two antennas only at the transmitter side can decrease the BER. But, this reduction is more considerable if diversity is provided at both sides of the link. When no beamforming is used, the BER varies from 0.2 when SNR is -4 dB to almost 0.004 when SNR is 16dB. While using two antennas at the transmitter reduces the BER from almost 0.1 for SNR of -4 dB to slightly less than 10^{-4} for SNR of 16 dB. It can be observed from Fig. 3 that in the presented range of SNR, using two antennas at transmitter could improve the SNR by approximately 6 dB in average.

More significant improvement can be obtained while the second antenna is also used at the receiver side. In this case, the BER goes from 0.02 when SNR is -4 dB to less than 10^{-5} for SNR of 12 dB. It can be seen from Fig. 3 that for the BER of 10^{-3}, the SNR can be further improved with a gain of approximately 5 dB when two antennas are utilized at both transmitter and receiver sides.

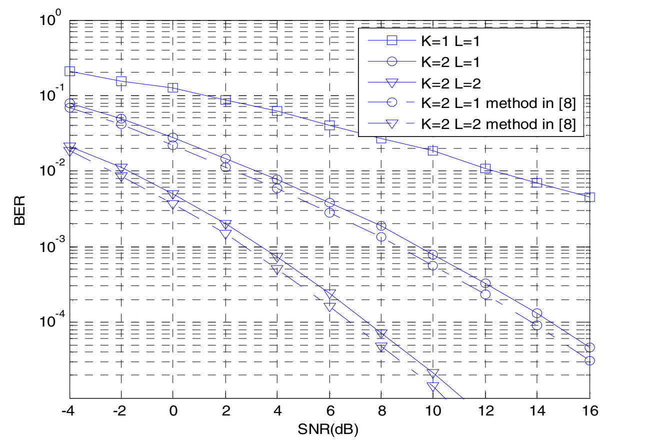

Fig. 3. BER vs. SNR in s a Rayleigh fading channel with Doppler frequency of 80Hz.

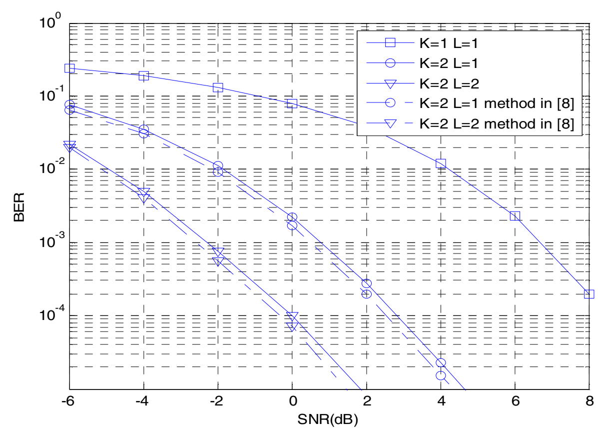

Fig. 4. BER vs. SNR in an AWGN channel

To evaluate the performance of the proposed method in reducing the noise effect, we assume that the channel fading fluctuation is negligible for a long period of time. That is, the channel is simplified as non-fading AWGN channel. The BER vs. the SNR in such channel is illustrated in Fig. 4, for the three previously mentioned cases. It can be seen from Fig. 4 that when two antennas are used only at the transmitter, the SNR improvement with respect to when no beamforming is used is approximately 6dB. For the case of using two antennas in both sides, this improvement is almost 9dB. It can be seen from fig. 3 and fig.4 that although the proposed method has considerably lower computational complexity than the method in [8], it offers the close performance.

Now, we investigate the convergence rate of the obtained method. It is shown in [17] that the convergence of the iterative RQ algorithm is exponentially related to the maximum eigenvalue ($\lambda_{\max}$) of that matrix. To investigate the convergence rate of the proposed method, we generate 50,000 random samples of $(\mathbf{H}^c)^*(\mathbf{H}^c)^T$ and plot the probability distribution function (pdf) of the maximum eigenvalues. In this case, 2 antennas are considered at both transmitter and receiver sides which results in a 2×2 channel matrix. Fig. 5 illustrated the resulted pdf. Then we examine the number of iterations required for the proposed method to converge to different eigenvalues in lower, middle, and upper parts of the pdf. Table II shows the $\lambda_{\max}$ computed in each iteration vs. the exact $\lambda_{\max}$.

It can be seen from Table II that in the second iteration the normalized error is less than 0.2 and in the third iteration this error would be lower than 0.05. It should be noted that Table II is drown for the first sampling time in which the initial value is considered as $\mathbf{w}_0 = (1/\sqrt{K})\mathbf{1}_K$ where $\mathbf{1}_K$ is a $K\times1$ vector that all of its elements are equal to one. In following sampling times, the weight vector in the previous sampling time is considered as the initial value. In this case, considering the limited change of the channel between two consecutive sampling times, a single iteration will be enough for the convergence of the proposed method. In all simulations of this work, a single iteration is used to compute the weight vector

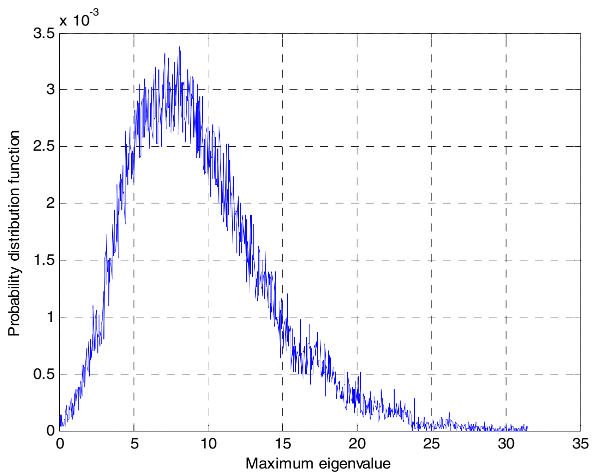

Fig. 5 Probability density function (pdf) of the maximum eigenvalues of $(\mathbf{H}^c)^*(\mathbf{H}^c)^T$ with 2 antennas at the transmitter and the receiver

TABLE II.
CONVERGENCE OF THE PROPOSED METHOD

1st iteration	2nd iteration	3rd iteration	Exact $\lambda_{\max}$
0.3412	0.6143	0.7416	0.7622
8.6426	8.8634	8.8908	8.8916
18.4894	19.1764	19.1786	19.2242

V. CONCLUSIONS

A new low complexity beamforming method for MIMO-OFDM systems has been presented in this paper. The proposed method works based Rayleigh quotient (RQ) of the Fourier transform of the channel matrix. It was shown that the proposed method needs only $(2L+1)K$ complex multiplications and $2KL$-1 complex additions for a single run, where K and L are the number of antennas in the transmitter and receiver arrays, respectively. This means the proposed method has a considerably lower computational load than other existing algorithms. Besides its simplicity, the proposed method can effectively reduce BER in Rayleigh fading and AWGN channels. It has been also shown that the proposed method will converge to the desired weight vector within two iterations. However, considering the actual fluctuation of wireless channels, a single iteration is enough for convergence as we used it in our simulations.

ACKNOWLEDGMENT

This work is supported by AFOSR under grant FA9550-09-0630.

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