# Ordered Precoder Designs for MIMO Interference Channels Based on Interference Alignment

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***Abstract*—Recent research shows that $K$ user time-varying interference channel is capable of achieving $K/2$ degrees of freedom based on interference alignment (IA). In multiple-input-multiple-output (MIMO) interference channels, appropriate transmit precoder design plays a pivotal role in efficiently suppressing the co-channel interference. In this paper, we propose two heuristic ordered precoder design algorithms for MIMO interference channels based on global search and chordal distance, respectively. Simulation results are provided to demonstrate that the proposed ordered precoder designs are capable of significantly improving the system sum capacity.**

***Index Terms*—interference alignment, interference channel, precoder, MIMO**

## I. Introduction

It has long been an accepted wisdom that only one degree of freedom is available for a network supporting $K$ transmitters and $K$ receivers in time-varying wireless interference channel, although $K$ non-interfering spatial signaling dimensions could potentially be created through joint signal processing among the $K$ transmitters and $K$ receivers [1]. The recent emergence of interference alignment (IA) has demonstrated that the achievable degrees of freedom of wireless interference networks can be significantly higher than what has been believed before [1–3]. The essence of IA is to restrict all interference at every receiver to approximately half of the received signal space through appropriate precoder design, leaving the other half interference-free for the desired signal [1–3]. To elaborate further, $K/2$ degrees of freedom can be obtained for the $K$-user time-varying interference channel [1]. Consequently, every user is capable of achieving reliable communication at rates approaching half of the capacity that he/she could achieve in an interference-free scenario. Nevertheless, one of the major concerns in IA schemes is the requirement of global channel knowledge. While a node may acquire channel state information for its own channels, it becomes more challenging to obtain the channel knowledge between other pairs of nodes. Therefore, a distributed precoder design is considered in [2] with iterative algorithms based on channel reciprocity. Although theoretical bound on degree of freedom for interference channel system has been well addressed in [1–3], precoder design can be further optimized from the perspective of improving the system sum capacity. In [4], a two-stage precoder optimization is proposed to improve the system sum capacity of the conventional IA scheme, while simultaneously maintaining the optimality of the degree of freedom. Multiple-input-multiple-output (MIMO) technology has attracted extensive attention due to its capability of offering significant increase in data throughput and improved link reliability without incurring extra bandwidth or transmission power [6]. It has been recognized as one of the most significant technical breakthroughs in modern communications. MIMO technology has been adopted in wide-ranging wireless communication standards such as IEEE 802.11n, the 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE) and Worldwide Interoperability for Microwave Access (WiMAX) [7]. In this paper, we propose ordered precoder designs for MIMO interference channels based on IA. Specifically, we consider two types of ordered precoder design methods for MIMO interference channels based on global search and chordal distance, respectively. As demonstrated by the simulation results, our proposed precoder designs are capable of significantly improving the system sum capacity.

The rest of the paper is organized as follows: Section II provides an introduction to the system model of the MIMO interference channel based on IA. In Section III, we discuss the conventional precoder design for MIMO interference channels. Section IV provides the proposed ordered precoder designs based on global search and chordal distance. Section V presents the simulation results. Finally, concluding remarks are given in Section VI.

*Notation:* $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote conjugate, transpose and Hermitian transpose operations, respectively. $\boldsymbol{I}_M$ denotes an $(M \times M)$ identity matrix.

## II. System Model

We consider a $K$ user interference channel model as depicted in Fig. 1, where the $k$th transmitter is equipped with $M$ antennas, while the $k$th receiver is equipped with $N$ antennas. Specifically, the $k$th receiver is the desired receiver of the $k$th transmitter. Without loss of generality, as in [4], we illustrate the precoder method by considering the scenario of $K = 3$ in our forthcoming discourse.

Let us denote the channel spanning from the $i$th transmitter to the $j$th receiver as an $(N \times M)$ matrix $\boldsymbol{H}_{j,i}$. Then the received

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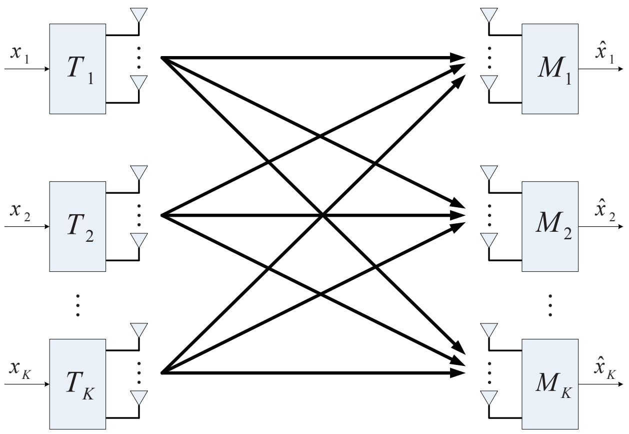


Fig. 1. System schematic diagram of a multiuser transmission downlink.

signal vector at the $i$th receiver can be expressed as

$$\boldsymbol{y}_i = \boldsymbol{H}_{i,i}\boldsymbol{T}_i\boldsymbol{x}_i + \sum_{j=1, j\neq i}^{K} \boldsymbol{H}_{j,i}\boldsymbol{T}_j\boldsymbol{x}_j + \boldsymbol{n}_i \tag{1}$$

where $\boldsymbol{T}_i$ represents an $(M \times \frac{N}{2})$ precoder associated with the $i$th transmitter, which satisfies trace$(\boldsymbol{T}_i\boldsymbol{T}_i^H) \leq P_i$ for $i = 1, \cdots, K$, with $P_i$ denoting the total transmitter power of the $i$th transmitter. In (1), $\boldsymbol{x}_i$ is an $(\frac{N}{2} \times 1)$ data vector in association with the $i$th transmitter, which obeys $E[\boldsymbol{x}_i\boldsymbol{x}_i^H] = \boldsymbol{I}_{\frac{N}{2}}$, while the $(N \times 1)$ vector $\boldsymbol{n}_i$ denotes the zero-mean circularly symmetric additive white Gaussian noise at the $i$th receiver.

At the receiver side, upon applying an $(\frac{N}{2} \times N)$ decoding matrix $\boldsymbol{M}_i$ at the $i$th receiver, the corresponding output vector at user $i$ can be expressed as

$$\hat{\boldsymbol{x}}_i = \boldsymbol{M}_i\boldsymbol{H}_{i,i}\boldsymbol{T}_i\boldsymbol{x}_i + \boldsymbol{M}_i \sum_{j=1, j\neq i}^{K} \boldsymbol{H}_{j,i}\boldsymbol{T}_j\boldsymbol{x}_j + \boldsymbol{M}_i\boldsymbol{n}_i \tag{2}$$

In order to cancel the inter-user interference [4], zero-forcing (ZF) algorithm can be employed at each receiver such that

$$\boldsymbol{M}_i \sum_{j=1, j\neq i}^{K} \boldsymbol{H}_{j,i}\boldsymbol{T}_j\boldsymbol{x}_j = 0 \tag{3}$$

Note that the ZF operation is capable of fully cancelling the interuser interference, but makes the noise in (2) correlated as well. Let us denote the noise covariance matrix at the $i$th receiver as $\boldsymbol{Z}_i$, then we can obtain

$$\boldsymbol{Z}_i = \mathrm{E}\left[(\boldsymbol{M}_i\boldsymbol{n}_i)(\boldsymbol{M}_i\boldsymbol{n}_i)^H\right] = \sigma_i^2\boldsymbol{M}_i\boldsymbol{M}_i^H \tag{4}$$

where $\sigma_i^2$ represents the noise variance at the $i$th receiver. The noise in (2) can be made uncorrelated through noise whitening process. Specifically, Cholesky factorization [5] can be applied to $\boldsymbol{Z}_i$ as

$$\boldsymbol{Z}_i = \boldsymbol{W}_i\boldsymbol{W}_i^H \tag{5}$$

Then the signal vector with uncorrelated noise can be expressed as

$$\tilde{\boldsymbol{x}}_i = \boldsymbol{W}_i^{-1}\boldsymbol{M}_i\boldsymbol{H}_{i,i}\boldsymbol{T}_i\boldsymbol{x}_i + \tilde{\boldsymbol{n}}_i \tag{6}$$

where $\tilde{\boldsymbol{n}}_i = \boldsymbol{W}_i^{-1}\boldsymbol{M}_i\boldsymbol{n}_i$ with the covariance matrix being the identity matrix of $\boldsymbol{I}_{\frac{N}{2}}$. Consequently, the capacity of the $i$th user can be written as

$$C_i = \log\det\left(\boldsymbol{I}_{\frac{N}{2}} + (\boldsymbol{W}_i^{-1}\boldsymbol{M}_i\boldsymbol{H}_{i,i}\boldsymbol{T}_i)(\boldsymbol{W}_i^{-1}\boldsymbol{M}_i\boldsymbol{H}_{i,i}\boldsymbol{T}_i)^H\right) \tag{7}$$

The system sum capacity $C$ is therefore obtained as

$$C = \sum_{i=1}^{K} C_i \tag{8}$$

Without loss of generality, we assume in our forthcoming discourse that each of the transmitters and receivers is equipped with the same number of antennas, i.e., $M = N$, where $M$ is assumed to be an even number.

## III. Conventional IA Precoder

In order to decode the desired signals at the corresponding receivers more efficiently, the following constraints must be satisfied [1]

$$\begin{aligned} \mathtt{span}(\boldsymbol{H}_{1,2}\boldsymbol{T}_2) &= \mathtt{span}(\boldsymbol{H}_{1,3}\boldsymbol{T}_3) && (9)\\ \mathtt{span}(\boldsymbol{H}_{2,1}\boldsymbol{T}_1) &= \mathtt{span}(\boldsymbol{H}_{2,3}\boldsymbol{T}_3) && (10)\\ \mathtt{span}(\boldsymbol{H}_{3,1}\boldsymbol{T}_1) &= \mathtt{span}(\boldsymbol{H}_{3,2}\boldsymbol{T}_2) && (11) \end{aligned}$$

where $\mathtt{span}(\boldsymbol{X})$ denotes the vector space spanned by the column vectors of $\boldsymbol{X}$. If the above-mentioned constraints are satisfied, the inter-user interference can be well suppressed at each receiver. In the following analysis, we assume that zero-forcing (ZF) algorithm is employed at each receiver in order to cancel the inter-user interference.

The precoder design for symmetric system can be carried out with the aid of a special solution satisfying the interference alignment constraints. Specifically, for the sake of finding a special solution, the IA constraints can be further restricted as

$$\begin{aligned} \mathtt{span}(\boldsymbol{H}_{1,2}\boldsymbol{T}_2) &= \mathtt{span}(\boldsymbol{H}_{1,3}\boldsymbol{T}_3) && (12)\\ \boldsymbol{H}_{2,1}\boldsymbol{T}_1 &= \boldsymbol{H}_{2,3}\boldsymbol{T}_3 && (13)\\ \boldsymbol{H}_{3,1}\boldsymbol{T}_1 &= \boldsymbol{H}_{3,2}\boldsymbol{T}_2 && (14) \end{aligned}$$

The above expressions can be equivalently written as

$$\begin{aligned} \mathtt{span}(\boldsymbol{T}_1) &= \mathtt{span}(\boldsymbol{B}\boldsymbol{T}_1) && (15)\\ \boldsymbol{T}_2 &= (\boldsymbol{H}_{3,2})^{-1}\boldsymbol{H}_{3,1}\boldsymbol{T}_1 && (16)\\ \boldsymbol{T}_3 &= (\boldsymbol{H}_{2,3})^{-1}\boldsymbol{H}_{2,1}\boldsymbol{T}_1 && (17) \end{aligned}$$

where we have $\boldsymbol{B} = \boldsymbol{H}_{3,1}^{-1}\boldsymbol{H}_{3,2}\boldsymbol{H}_{1,2}^{-1}\boldsymbol{H}_{1,3}\boldsymbol{H}_{2,3}^{-1}\boldsymbol{H}_{2,1}$. Then $\boldsymbol{T}_1$ can be designed as

$$\boldsymbol{T}_1 = [\boldsymbol{t}_1, \boldsymbol{t}_2, \cdots, \boldsymbol{t}_{2/N}] \tag{18}$$

where $\boldsymbol{t}_i, i = 1, \cdots, N/2$ are eigenvectors of $\boldsymbol{B}$.

Upon obtaining $\boldsymbol{T}_i (i = 1, \cdots, K)$ subject to the signal space constraints as mentioned above, we can optimize the precoder matrix tailored to individual rate. Specifically, denote the QR decomposition [9] of $\boldsymbol{T}_i (i = 1, \cdots, K)$ as

$$\boldsymbol{T}_i = \boldsymbol{Q}_i \boldsymbol{R}_i \tag{19}$$

where $\boldsymbol{Q}_i$ is an $(M \times \frac{N}{2})$ semi-unitary matrix and $\boldsymbol{R}_i$ is a full-rank upper tri-angular matrix with a dimension of $\frac{N}{2} \times \frac{N}{2}$. Consequently, the precoder based on the conventional IA methodology can be optimized as

$$\boldsymbol{T}_i = \boldsymbol{Q}_i \boldsymbol{G}_i \tag{20}$$

where $\boldsymbol{G}_i$ is a diagonal matrix subject to transmit power constraint trace$(\boldsymbol{G}_i^H \boldsymbol{G}) \leq P_i (i = 1, \cdots, K)$, where $P_i$ denotes the total transmitted power of the $i$th transmitter.

From what has been discussed above, we can see that the precoders associated with user $i$ $(i \neq 1)$ are coupled with their counterpart for user 1. Therefore, it is crucial to design the first precoder, which may significantly impact the overall system performance. In our forthcoming discussions, two heuristic ordered precoder designs are proposed in order to enhance the system performance in terms of sum capacity.

## IV. Ordered Precoder Design

In this section, we propose two ordered precoder designs with the aid of global search and chordal distance based on the IA algorithm in [1], which can be referred to as G-IA and C-IA, respectively. Furthermore, the G-IA includes the best ordering scheme that is capable of achieving largest performance gain and the worst ordering scheme that may result in largest performance degradation, compared to the baseline performance of the conventional IA scheme in terms of the system sum capacity. These two schemes are referred to as G-B-IA and G-W-IA, respectively.

### A. Global Search Based Ordering

The algorithm for finding the appropriate first node based on global search is described as

1) Initialization: Sum capacity $C^{(i)} = 0, i = 0, \boldsymbol{T}_i = 0$.
2) For $i < 3$, follow the below procedure:
   a) $i = i + 1$;
   b) Calculate $\boldsymbol{B}_i = \boldsymbol{H}_{k,i}^{-1} \boldsymbol{H}_{k,j} \boldsymbol{H}_{i,j}^{-1} \boldsymbol{H}_{i,k} \boldsymbol{H}_{j,k}^{-1} \boldsymbol{H}_{j,i}, k \neq j \neq i, \quad i, j, k \in \{1, 2, 3\}$; Obtain $\boldsymbol{T}_i =$ eigenvectors$(\boldsymbol{B}_i)$;
   c) Obtain $\boldsymbol{T}_j$ and $\boldsymbol{T}_k$ based on (15).
   d) Calculate the sum capacity $C^{(i)}$ based on (8).
3) a) G-B-IA: find the optimal first candidate which maximizing the system sum capacity

$$C_B = \max_{\{i=1,2,3\}} \{C^{(i)}\} \tag{21}$$

   b) G-W-IA: find the worst first candidate which minimizing the system sum capacity

$$C_W = \min_{\{i=1,2,3\}} \{C^{(i)}\} \tag{22}$$

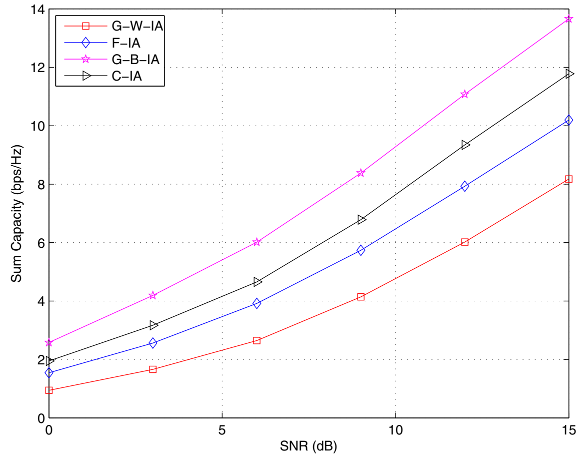


Fig. 2. Achievable sum capacity of IA schemes with different precoder designs. In the simulation we assume $K = 3$ pairs of transmit and receive nodes. Furthermore, the number of antennas per node is assumed to be $M = N = 2$.

### B. Chordal Distance Based Ordering

The chordal distance between an $(m \times n_1)$ matrix $\boldsymbol{A}$ and an $(m \times n_2)$ matrix $\boldsymbol{B}$ is defined as [4, 11]

$$\begin{aligned} d(\boldsymbol{A}, \boldsymbol{B}) &= \frac{1}{\sqrt{2}} \parallel \mathbb{O}(\boldsymbol{A})\mathbb{O}(\boldsymbol{A})^H - \mathbb{O}(\boldsymbol{B})\mathbb{O}(\boldsymbol{B})^H \parallel_F \\ &= \sqrt{\frac{n_1 + n_2}{2} - \parallel \mathbb{O}(\boldsymbol{A})\mathbb{O}(\boldsymbol{B})^H \parallel_F^2} \end{aligned} \tag{23}$$

where $\mathbb{O}(\boldsymbol{A})$ denotes an $(m \times n_1)$ matrix that consists of the orthonormal basis vectors that the column space of $A$, so is similar with $\mathbb{O}(\boldsymbol{B})$. The ordering algorithm based on chordal distance ban be summarized as

1) Initialization: Sum capacity $C^{(i)} = 0$,i=0,$\boldsymbol{T}_i = 0$.
2) For $i < 3$, follow the below procedure:
   a) $i = i + 1$;
   b) Calculate $\boldsymbol{B}_i = \boldsymbol{H}_{k,i}^{-1} \boldsymbol{H}_{k,j} \boldsymbol{H}_{i,j}^{-1} \boldsymbol{H}_{i,k} \boldsymbol{H}_{j,k}^{-1} \boldsymbol{H}_{j,i}, k \neq j \neq i, \quad i, j, k \in \{1, 2, 3\}$; Obtain $\boldsymbol{T}_i =$ eigenvectors$(\boldsymbol{B}_i)$;
   c) Calculate the chordal distance $D_i$ as

$$\begin{aligned} D_i = \; & d(\boldsymbol{H}_{i,i}\boldsymbol{T}_i, \boldsymbol{H}_{i,j}(\boldsymbol{H}_{k,j})^{-1}\boldsymbol{H}_{k,i}\boldsymbol{T}_i) \\ & + d(\boldsymbol{H}_{j,j}(\boldsymbol{H}_{k,j})^{-1}\boldsymbol{H}_{k,i}\boldsymbol{T}_i, \boldsymbol{H}_{j,i}\boldsymbol{T}_i) \\ & + d(\boldsymbol{H}_{k,k}(\boldsymbol{H}_{j,k})^{-1}\boldsymbol{H}_{j,i}\boldsymbol{T}_i, \boldsymbol{H}_{k,i}\boldsymbol{T}_i) \end{aligned} \tag{24}$$

3) C-IA: find the optimal first candidate with the maximal chordal distance $\max_{\{i=1,2,3\}} \{D_i\}$. Then the corresponding $\boldsymbol{T}_i$ for maximizing the chordal distance is used to calculate the system sum capacity.

## V. Simulation Results

In this section, we provide a range of simulation results to demonstrate the achievable sum capacity of IA schemes with different precoder designs. For the sake of simplicity, we assume three pairs of transmit and receive nodes in our

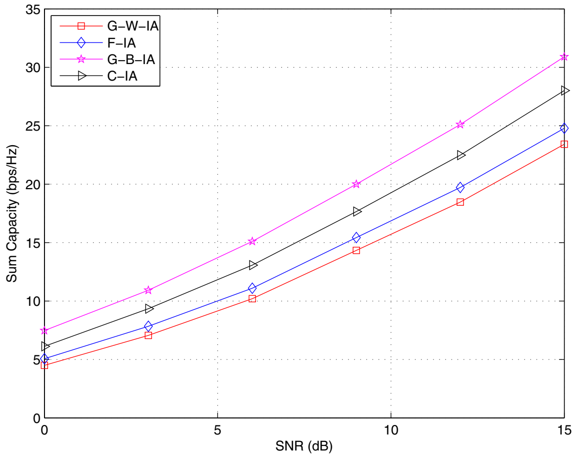


Fig. 3. Achievable sum capacity of IA schemes with different precoder designs. In the simulation we assume $K = 3$ pairs of transmit and receive nodes. Furthermore, the number of antennas per node is assumed to be $M = N = 4$.

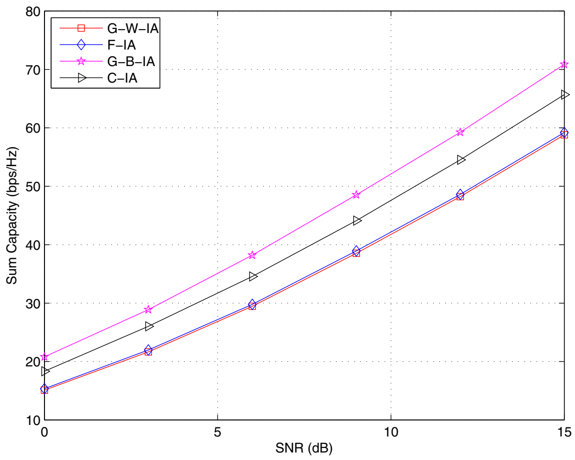


Fig. 4. Achievable sum capacity of IA schemes with different precoder designs. In the simulation we assume $K = 3$ pairs of transmit and receive nodes. Furthermore, the number of antennas per node is assumed to be $M = N = 8$.

simulations. Specifically, the $i$th receiver is the desired receiver of the $i$th transmitter, while data from the $j$th ($j \neq i$) transmitter is the interference for the $i$th receiver. Furthermore, in our simulations we assume independent flat Rayleigh fading for all the links between the transmitters and receivers. Each transmit node is assumed to have the same transmit power $P_0$ and adopt the equal power allocation for rank-2 transmissions. Fig. 2 shows the achievable sum capacity of IA schemes with different precoder designs, when the number of antennas per node is assumed to be $M = N = 2$. Specifically, F-IA corresponds to the conventional IA precoder scheme [1], where the first candidate is fixed in the simulations. G-B-IA and G-W-IA denote the IA precoder schemes based on the best and worst ordering global search, which are capable of maximizing or minimizing the system sum capacity, respectively. Note that it may not be reasonable to design the precoder that results in minimal sum capacity. Nonetheless, the performance of G-W-IA is provided for comparison. Specifically, the performance margin between G-B-IA and G-W-IA schemes indicates the necessity for appropriately ordered precoder design in order to achieve improved system performance. From Fig. 2, it can be observed that C-IA is also capable of achieving improved sum capacity, compared to the baseline performance of the conventional IA scheme. Furthermore, G-B-IA is capable of achieving the best performance at the cost of increasing computational complexity. In Figs. 3 and 4, we provide the achievable sum capacity of IA schemes with different precoder designs, when the number of antennas per node is assume to be $M = N = 4$ and $M = N = 8$, respectively. It can be seen that C-IA and G-B-IA schemes are always capable of outperforming the conventional IA scheme in terms of the system sum capacity. Finally, the performance loss of G-W-IA scheme with respect to F-IA scheme decreases as the number of antennas per node increases, which once again implies the necessity the appropriately ordered precoder design.

## VI. Conclusions

In this paper, we show the necessity of ordered precoder design for IA schemes. We propose two heuristic ordered precoder design algorithms based on global search and chordal distance in order to improve the system sum capacity. Simulation results are provided to demonstrate that appropriately ordered precoder designs are capable of significantly improving the performance of IA systems in terms of sum capacity.

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