# Lateral Disturbance Compensation Using Motor Driven Power Steering

Kyuwon Kim, Jaewoong Choi, Kyongsu Yi School of Mechanical and Aerospace Engineering, Seoul National University 599 Gwanangno, Gwanak-Gu, Seoul, 151-742, Korea ghostkitten@snu.ac.kr, loar00@snu.ac.kr, kyi@snu.ac.kr

Abstract— This paper deals with lateral disturbance compensation algorithm for an application to a Motor Driven Power Steering (MDPS) based driving assistant system. The lateral disturbance such as wind force and load from bank angle reduces the driver refinement and increases the possibility of an accident. In order to reduce the maneuvering effort of the driver in the disturbing situation, the lateral disturbance compensation algorithm has been proposed. The characteristics of the compensation system including a human driver model and the steering system have been mathematically analyzed. The control strategy using the motor overlay torque as a control input which improves the human steering behavior under the lateral disturbance has been proposed. A numerical simulation of the proposed algorithm has been conducted by full vehicle model and lateral driver model which represents steering behavior of human driver. The human torque and lateral deviation under the lateral disturbance are confirmed to be reduced by the simulation results.

Keywords-component; Crosswind, Bank angle, Disturbance compensation, Motor Driven Power Steering (MDPS), Human torque, Overlay torque

# I. INTRODUCTION

In recent years, a lateral disturbance on vehicle stability causes a big concern and has met with the greatest attention by researchers[1]. The main factors of the lateral disturbance in the actual driving are considered as crosswind which is usually blown to the road on the bridge or near the sea shores and road bank angle. Strong crosswind blowing to the lateral direction of the vehicle induced accidents of the vehicle being bodily blown over, or the vehicle being blown off course. The stability of the vehicle and the course deviation under strong crosswind has been analyzed in the previous research [2].

In the previous research, the lateral control system with steering torque under lateral disturbance has been conducted with the steering – vehicle dynamics model [3]. The proposed lateral disturbance compensation strategy includes the overall driver, steering and vehicle system. In order to analyze the overall driver, steering system and vehicle system with Motor Driven Power Steering (MDPS) overlay torque control input under the lateral disturbance, each system dynamics is unified as state space equation. The following section includes the basic vehicle dynamic model under the lateral disturbance and the lateral disturbance characteristics. The driver characteristics

which intends to keep the lane with the preview distance is introduced in the next section, and the overall steering system including the motor assist torque characteristic and the tire self-aligning torque characteristics will be described. The vehicle lateral position error dynamics which is modified from the basic vehicle dynamics will be covered in the next section, and finally, the overall state space equation of the driver-steering-vehicle dynamics has been developed.

The control target of the proposed compensation algorithm is to get rid of the effect of the lateral disturbance in the human steering behavior which is the same as the normal driving situation without the lateral. The linear analysis of the proposed system ignoring the nonlinear characteristics in the mild driving situation of straight driving or small curvature curved road driving under the lateral disturbance has been conducted to choose the appropriate overlay torque control strategy. The estimated lateral disturbance is used to improve the human steering behavior in the feedforward control input. The performance of the proposed control strategy has been confirmed by applying the developed driver model and the full vehicle model.

#### II. VEHICLE DYNAMIC MODEL

### A. Vehicle Body dynamics

The simplified vehicle model has been developed from the full vehicle model in the previous researches[4]. The proposed vehicle model is 3 DOF planar motion model under the lateral disturbance with the angle of incidence  $\theta$  represented as following equations.

$$\begin{split} m(\dot{v}_{x} - \gamma v_{y}) &= F_{r,x} + F_{f,x} \cos \delta_{f} - F_{f,y} \sin \delta_{f} - F_{w} \cos \theta \\ m(\dot{v}_{y} + \gamma v_{x}) &= F_{r,y} + F_{f,y} \cos \delta_{f} - F_{f,x} \sin \delta_{f} - F_{w} \sin \theta \\ I_{z}\dot{\gamma} &= I_{f} F_{f,y} \cos \delta_{f} - I_{r} F_{r,y} - I_{f} F_{f,x} \sin \delta_{f} \\ &+ \frac{d}{2} (\Delta F_{r,x} + \Delta F_{f,x} \cos \delta_{f}) - \sum_{i=1}^{4} T_{align,i} + M_{w} \end{split} \tag{1}$$

# B. Lateral Disturbance Dynamics

In the driving under crosswind, the flow around a vehicle becomes asymmetric. By the results from the experience, it is known that only yaw moment and the lateral force of these components are dominant to a vehicle's behavior [1]. The crosswind coefficients on i-th direction,  $C_{f,i}$ ,  $C_{n,i}$  are determined by the reference shape of the vehicle [5]. General mathematical expressions of the force and moment are represented as follows:

$$F_{CW,i} = \frac{1}{2} \rho C_{f,i} v_r^2 A$$

$$M_{CW,i} = C_{n,i} \frac{\rho}{2} l A v_r^2$$
(2)

The crosswind coefficients on i-th direction,  $C_{f,i}$ ,  $C_{n,i}$  are determined by the reference shape of the vehicle. The main disturbances from crosswind causing the dangerous situation can be considered as yaw moment and the lateral force. The lateral disturbance comes from the road bank angle is represented as Eqn(4). The yaw dynamics of the vehicle are not disturbed by the bank angle.

$$F_{BANK} = mg \frac{\Delta z}{2t_f} \tag{4}$$

Since a lateral disturbance exerting on a vehicle is hard to measure in an actual driving, several methods for estimating the lateral disturbance are proposed [7,8].

# C. Steering Dynamics

A mathematical steering dynamics around the king-pin represented in Fig(1) is described as following equation.

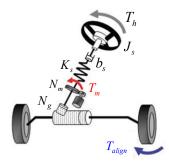


Figure 1. Modeling of MDPS System

$$J_s \ddot{\mathcal{S}}_f = -b_s \dot{\mathcal{S}}_f - K_s \mathcal{S}_f + N_g N_m T_M + N_g T_h - T_{align}$$
 (5)

Where,  $\delta_f$  is the front steering angle,  $N_g$  is the steering gear ratio,  $N_m$  is the motor gear ratio and  $T_h$  is the driver's torque. The driver's torque is applied on the steering wheel in order to follow the desired path.

The desired steering angle for tracking the desired path from the perception of the environment within the preview distance is described as follows:

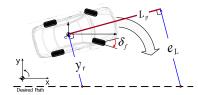


Figure 2. Lane Keeping Control with Preview Distance

$$\delta_{des} = -k_1 \cdot y_r - k_2 \cdot e_L$$

$$= -k_1 \cdot y_r - k_2 \left( y_r + L_p \cdot e_w \right)$$
(6)

The driver's torque considering the human neuromuscular system delay is determined to follow the desired path with the difference of the desired steering angle and the current steering angle considering the time from the perception of the environment to the moment of applying steering torque as shown in Eqn(7). A representation of the steering torque originating from the steering wheel angle for the steer-by-wire system has been studied in the previous research [9].

$$T_h = \frac{k_T}{1 + \tau_{re} s} \left( \delta_f - \delta_{des} \right) \tag{7}$$

 $T_{align}$  represents the tire self-aligning torque which is generated by the discrepancy between the point of the lateral force acting on the tire and the center of the tire. The tire self-aligning torque is varying linearly within the certain slip angle, which is around 4deg in the passenger car. A mathematical representation considering the nonlinear characteristics of the tire self-aligning torque is:

$$T_{align,i} = 2C_{i,y}\xi_i\alpha_i + \Delta T_{align,i}$$
 (8)

The assist torque characteristic of MDPS is determined by the velocity of the vehicle, and the steering torque of the driver. In order to prevent the driver from feeling uncomfortable when he or she maneuvers a vehicle, the direction of the motor assist torque has the same direction with the driver's torque. These characteristics are shown in the following figure.

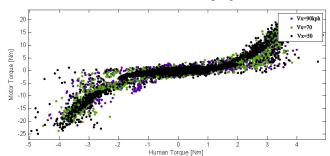


Figure 3. Motor Assist Torque According to Human Torque

The motor assist torque shows linear characteristic within the certain region. The proportional rate of the motor assist torque is shown to be small as the speed of the vehicle increases. The overall motor torque can be represented as following equations.

$$T_{assist} = k_a \cdot T_h + \Delta T_{assist}$$

$$where, k_a = f(v_x)$$

$$T_m = T_{assist} + T_{overlav}$$
(9)

# D. Vehicle Position Error Dynamics

The following figure shows the driver-vehicle system based on the preview distance of the driver. The driver intends follow the desired path which comes from the driver's perception of the driving environment.

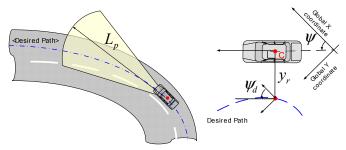


Figure 4. Schematic Diagram of Human Driver-Vehicle System

The lateral position error is defined as the lateral distance between the centerline of the desired path and the center of gravity of the vehicle. The first derivative of the lateral position error and yaw angle error are represented as following equation.[11]

$$\Delta y_{y} \simeq v_{y} \cdot \Delta t + v_{x} \cdot \Delta t \cdot e_{\psi} \qquad \dot{y}_{r} = v_{y} + v_{x} \cdot e_{\psi}$$

$$\psi_{d} \simeq v_{x} \cdot \Delta t \cdot \rho_{des} \qquad \dot{\psi}_{d} = v_{x} \cdot \rho_{des}$$
(10)

A mathematical nonlinear tire model has been studied widely so far, the various tire formula were developed to represent the behavior of the actual tire [4]. The proposed tire models represent the characteristics of the tire forces with the lateral position error and yaw angle error as follows:

$$F_{f,y} = C_f \alpha_f + \Delta f_{f,y}$$

$$= C_f \left( \delta_f - \frac{\dot{y}_r + l_f \cdot \dot{e}_{\psi}}{v_x} + e_{\psi} - \frac{l_f}{v_x} \cdot \dot{\psi}_d \right) + \Delta f_{f,y}$$
(11)

In order to minimize the state variable of a planar model in Eqn(1), a 2DOF bicycle model is designed based on Eqn(14) with following assumption; Constant longitudinal velocity and the same slip angle between the left / right wheel.

From Eqn(5)~(11), the state space representation is formed with following states:  $x = \begin{bmatrix} y_r & \dot{y}_r & e_\psi & \dot{e}_\psi & \delta_f & \dot{\delta}_f & T_h \end{bmatrix}^T$ 

$$\frac{d}{dt} \begin{bmatrix} y_r \\ \dot{y}_r \\ e_y \\ \dot{\delta}_f \\ T_h \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & A_{24} & A_{25} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{65} & A_{67} \\ A_{71} & 0 & A_{73} & 0 & A_{75} & 0 & A_{77} \end{bmatrix} \begin{bmatrix} y_r \\ \dot{y}_r \\ e_y \\ \delta_f \\ T_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} T_{overlay} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_w \\ M_w \end{bmatrix} + \begin{bmatrix} 0 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rho_{dev} + f_{nl}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

 $f_{nl}$  in Eqn(12) contains the nonlinear terms in the tire force, the tire self-aligning torque and the assist torque. The nonlinearities of the equation are assumed to be neglected in the case of the small steering behavior and vehicle side slip angle, which is shown in a straight driving or a curved road driving with the small curvature.

# III. MOTOR OVERLAY TORQUE CONTROL STRATEGY FOR LATERAL DISTURBANCE COMPENSATION

When a vehicle is exposed to the lateral disturbance, a driver tries to steer the vehicle to the desired path with high frequency comparing with the normal driving situation. In the previous research, the linear controller of steering-vehicle system for the lateral control under the lateral disturbance has been proposed with feedback input based on LQ control theory and feedforward control input considering the lateral disturbance and the driver's torque has been assumed to be zero [3]. In this part, the motor overlay torque control strategy has been proposed on the basis of the human steering characteristics including steering torque and the vehicle dynamics by the analysis of the developed mathematical system model.

### A. Overlay Torque Input with Human Torque Feedback

In order to reduce the magnitude of the human torque for keeping the desired path and the frequency of the steering behavior, a feedback overlay torque of the human torque has been considered. The human torque which can be measured from the torque sensor of the MDPS module is represented as Eqn(7) by using the state variables in Eqn(12).

$$v = T_h = C \cdot x$$

$$= \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \cdot \underbrace{\begin{bmatrix} y_r & \dot{y}_r & e_{\psi} & \dot{e}_{\psi} & \delta_f & \dot{\delta}_f & T_h \end{bmatrix}}^{T}$$
(13)

The steering torque can be measured from the torque sensor of the MDPS module, and the feedback overlay torque of the human torque can be represented as follows:

$$u_{fb} = -K_{fb} \cdot T_b \tag{14}$$

Neglecting the nonlinear terms and road curvature shown in Eqn(12) in the situation of straight driving with small steering, the overall driver-steering-vehicle system can be represented with feedback input as following equation.

$$\dot{x} = A \cdot x + B \cdot u_{fb} + F \cdot W 
= (A - B \cdot K)x + F \cdot W$$
(15)

In order to choose the appropriate feedback gain, the variation of the dominant pole location of the transfer function in Eqn(15) according to the variation of the feedback gain has been plotted in Fig.5.

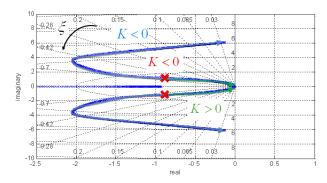


Figure 5. Dominant Pole Location with Feedback Gain

### B. Feedforward Overlay Torque Input

The angle difference between the direction of the vehicle and the direction of the tire alignment becomes large comparing with the normal driving situation by the lateral disturbance. Because of the angle difference, the reactive torque from the tire self-aligning torque is generated and the driver imposes the steering torque to cancel out the reactive torque. In order to assist the human torque by the reactive torque, the feedforward overlay torque has been proposed.

The various methods to measure the lateral disturbance by the lateral wind have been proposed so far [6], however the real-time measuring the lateral disturbance in driving are very complicated. Various estimation methods of the lateral disturbance was proposed as an alternative, which are direct force and moment estimation method [7,8]. By the use of the estimated force and moment, the tire self-aligning torque has been calculated.

Eqn(12) can be rewritten with the reduced state space equation form with the steering angle input as shown in Eqn(16).

$$x_{r} = \begin{bmatrix} y_{r} & \dot{y}_{r} & e_{\psi} & \dot{e}_{\psi} \end{bmatrix}^{T}, \quad W = \begin{bmatrix} F_{w} & M_{w} \end{bmatrix}^{T}$$

$$\frac{d}{dt} \begin{bmatrix} y_{r} \\ \dot{y}_{r} \\ e_{\psi} \\ \dot{e}_{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & A_{22} & A_{23} & A_{24} \\ 0 & 0 & 0 & 1 \\ 0 & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} y_{r} \\ \dot{y}_{r} \\ e_{\psi} \\ \dot{e}_{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_{25} \\ 0 \\ A_{45} \end{bmatrix} \delta_{f} + \begin{bmatrix} 0 & 0 \\ F_{21} & 0 \\ 0 & 0 \\ 0 & F_{42} \end{bmatrix} \begin{bmatrix} F_{w} \\ M_{w} \end{bmatrix}$$

$$(16)$$

The steering angle input in Eqn(16) is the summation of the feedback angle for tracking the desired path in Eqn(6) and the feedforward steering angle for reducing the effect of the lateral disturbance as follows:

$$\begin{split} \delta_{f} &= -k_{1} \cdot y_{r} - k_{2} \cdot e_{L} = -k_{1} \cdot x_{1} - k_{2} \cdot \left(x_{1} + L_{p} \cdot x_{3}\right) + \delta_{ff} \\ &= -K_{d} \cdot x + \delta_{ff}, \text{ where, } K_{d} = \begin{bmatrix} k_{1} + k_{2} & 0 & k_{2} \cdot L_{p} & 0 \end{bmatrix} = \begin{bmatrix} k_{1}^{'} & k_{2}^{'} \end{bmatrix} \end{split}$$

$$(17)$$

Substituting Eqn(17) into Eqn(16), the system can be rewritten as following form:

$$\dot{x}_r = (A_r - B_r \cdot K_d) \cdot x_r + B_r \cdot \delta_{ff} + F_r \cdot W \tag{18}$$

In order to appear the steady state value of the lateral position error, which is the first state of the reduced states, the

steady state value of the reduced states is represented as following equation by the final value theorem.

$$x_{ss} = \lim_{t \to \infty} x(t) = \lim_{s \to \infty} s \cdot X(s)$$

$$= -\left(A - B \cdot K\right)^{-1} \cdot \left\{B \cdot \left(\delta_{ff}\right) + F \cdot \begin{bmatrix} \hat{F}_{w} \\ \hat{M}_{w} \end{bmatrix}\right\} = 0$$
(19)

The feedforward steering angle which makes the steady lateral position error go to zero can be calculated as shown in Eqn(26). Each parameter of the equation including the cornering stiffness, and the distance from the center of gravity of the vehicle to the front / rear axle is the nominal value of the normal driving condition.

$$\delta_{ff} = \frac{-\hat{F}_{w}c_{1} - \hat{M}_{w}c_{2}}{2\bar{C}_{f}c_{1} + 2\bar{C}_{f}\bar{I}_{f}c_{2}}, \quad where, c_{1} = \bar{C}_{r}\bar{I}_{r} - \bar{C}_{f}\bar{I}_{f} + \bar{C}_{f}k_{2}'\bar{I}_{f}}$$

$$c_{2} = \bar{C}_{f} + \bar{C}_{r} - \bar{C}_{f}k_{2}'$$
(20)

The feedforward steering angle calculated in Eqn(20) shows high frequency behavior due to the estimated lateral disturbance. In order to prevent the high frequency maneuvering from the estimated lateral disturbance, the random-walk Kalman filtering method has been proposed as shown in Eqn(21) [12].

$$\begin{split} & \delta_{ff}(k+1\big|k) = \delta_{ff}(k\big|k) + w(k) \qquad w(k) \sim N(0,Q_{\delta_{ff}}) \\ & z(k) = \delta_{ff}(k\big|k) + v(k) \qquad v(k) \sim N(0,R_{\delta_{ff}}) \end{split} \tag{21}$$

The tire self-aligning torque generated by the lateral disturbance can be represented as Eqn(28). The tire self-aligning torque of the feedforward steering angle is determined by the slip angle of the feedfoward steering angle. The slip angle in the equation can be calculated using the feedforward steering angle in Eqn(20) and estimated lateral velocity which has been proposed by various methods. The feedforward overlay torque is represented by the nominal parameters as following equation:

$$T_{overlay,ff} = \sum_{i=1}^{2} \hat{T}_{align,i} \left( \delta_{ff} \right) = 2 \overline{C}_{i,y} \overline{\xi}_{i} \hat{\alpha}_{i} \left( \delta_{ff} \right)$$
 (22)

Consequently, the proposed overlay torque input can be represented as a summation of the steering torque feedback input which reduces the magnitude and the frequency of the human torque and the feedforward steering torque for compensating the tire self-aligning torque generated by the lateral disturbance as follows:

$$u = T_{overlav} = -K \cdot T_h + T_{overlav \ ff} \tag{23}$$

### IV. SIMULATION RESULTS

A simulation has been conducted with the full vehicle model with the developed driver model[10]. Fig.6(a) and Fig.6(b) show the crosswind force and moment with 12~15m/s wind velocity along the vehicle path applied to the vehicle which drives at the speed of 80kph. Fig.7(a) shows the human torque, and Fig.7(b) and Fig.7(c) show the course deviations which represent the lateral distance error and yaw angle error respectively. The feedback gain has been chosen through the result of the pole location analysis considered in the previous

section. The magnitude of the course deviation is confirmed to be reduced significantly through the following results. From the simulation results, the performance of the proposed algorithm can be expected in the actual test.

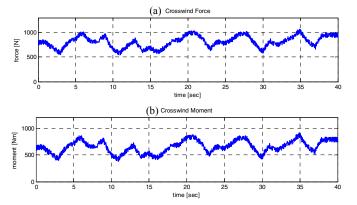


Figure 6. Lateral Force and Yaw Moment due to Lateral Disturbance

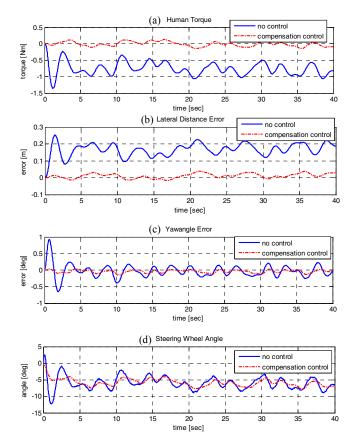


Figure 7. Simulation Results

### V. CONCLUSION

In this paper the lateral disturbance compensation algorithm for application to an MDPS steering assistant system has been evaluated. In order to analyze the overall driver-steering-vehicle behavior, each model has been developed. The steering dynamics with MDPS and the driver model considering the torque delay from the perception and muscular system have

considered. The vehicle lateral error dynamics are derived from the basic vehicle dynamics, and finally the unified state space equation including the above dynamics has been proposed to represent the overall driving system. In order to choose the appropriate control gain of the motor overlay torque which is the control input of the unified system, the linear system analysis by pole location has been conducted. The feedforward overlay torque has been proposed to compensate the tire aligning torque which is generated by the lateral disturbance. The estimated lateral force and yaw moment from the lateral disturbance are used to calculate the feedforward overlay torque. The proposed control strategy is the summation of the human torque feedback and the feedforward overlay torque, and the performance of the compensation control has been evaluated by the full vehicle simulation model with the developed driver The proposed algorithm was shown to have desirable driver and vehicle behaviors which are similar to the normal driving situation even under the lateral disturbance.

# ACKNOWLEDGEMENT

This work was supported by the BK21 program, SNU-IAMD, the Korea Research Foundation Grant funded by the Korean Government (MEST) (KRF-2009-200-D00003), and the National Research Foundation of Korea Grant Funded by the Korean Government (2011-0001277)

### REFERENCES

- [1] C.J.Baker, Measures to control vehicle movement at exposed sites during windy periods, Journal of Wind Engineering and Industrial Aerodynamics, Vol. 25, 1987, p. 151-161
- [2] R.G.Gawthorpe, Wind effects on ground transportation, Journal of Wind Engineering and Industrial Aerodynamics, Vol.52, 1994, p. 73-92
- [3] T. Ohta, T. Mimuro and J. Lee, Robust Lateral Control System with Steering Torque Assist, AVEC'02, Hiroshima, Japan, 2002, p. 225-230
- [4] R.Rajamani, Vechicle Dynamics and Control, 2006
- [5] A.D.Quinn, C.J.Baker, N.G.Wright, Wind and vehicle induced forces on flat plates-Part 1: Wind induced force, Journal of Wind Engineering and Industrial Aerodynamics, Vol. 89, 2001, p. 817-829
- [6] Abe, Masato, Vehicle Handling Dynamics: Theory and Application, 2009
- [7] S.Glaser, S.Mammar, D.Dakhlallah, Lateral wind force and torque estimation for a driving assistance, International Federation of Accountants, Proceedings of the 17th World Congress, 2008, p. 5688-5693
- [8] K.Kyuwon, et el, Estimation of Lateral Force due to Lateral Disturbance for Application to an MDPS-Based Driving Assistant System, SAE Int.J.Mater.Manuf. Vol. 4, No. 1, 2011, p. 1014-1024
- [9] Ryouhei Hayama, et al, Resistance torque control for steer-by-wire system to improve human-machine interface, Vehicle System Dynamics, Vol. 48, No. 9, 2010, p. 1065-1075
- [10] T.Lee, et el, Integration of longitudinal and lateral human driver models for Evaluation of the Vehicle Active Safety Systems, SAE 2010 World Congress, 2010
- [11] H.Peng, T.Masayoshi, Vehicle Lateral Control for Highway Automation, American Control Conference, 1990. Proceedings of the 1990, Vol.5, p.788-794
- [12] J.Lee and K.Yi, Development of a combined steering torque overlay and differential braking strategy for unintended lane departure avoidance, Intelligent Transportation Systems (ITSC), 2011 14th International IEEE Conference, p. 1223 - 1230