A Reliable Broadcast Transmission Approach Based on Random Linear Network Coding

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***Abstract*—Recently, XOR-based network coding has been demonstrated as an effective approach to improve the network performance for wireless broadcast. In this way, the source XOR multiple lost packets from different receivers into one packet and retransmits it, then multiple receivers are able to recover their lost packets, which can effectively reduce the number of retransmissions and utilize wireless bandwidth. However, to find the optimal XOR coding set that minimizes the total number of transmissions has been proved as NP-hard and of high complexity. In this paper, we proposed a new wireless broadcast retransmission scheme based on linear random network coding, in which the source code combines all lost packets to a single one by linear network coding for retransmission, and receivers are able to decode the original packets by method of Gaussian elimination when they receive enough coded-packets. Simulation results are given to demonstrate superior performance of our algorithm over previously proposed works.**

***Index Terms*—wireless broadcasting, retransmission, linear network coding, network coding**

I. Introduction

Broadcast is an important mechanism in wireless networks for transmitting identical data to multiple receivers. It can be applied in content delivery services like IPTV, Video on Demand (VoD), device configuration, etc. Reliable broadcast requires that each receiver must receive the correct data sent by the source. ARQ is the most common method for guaranteeing reliable communications, however, it is not so efficient in broadcast transmissions. Using this method, the source need to rebroadcast the lost packet if at least one receiver does not receive the correct packet, and a large number of control frames like ACK/NAK are needed, which will both result in poor throughput performance. Network coding, which was firstly proposed by Ahlswede [1], has been demonstrated as a promising approach to improve throughput in wireless networks. The core idea of network coding is to allow intermediate nodes to combine packets before forwarding, instead of traditional store-and-forward method. It has been proved that for many problems such as multicast and broadcast, using appropriate encoding schemes can achieve the network capacity. D.Nguyen [2] proposed XOR network coding as a retransmission scheme in wireless broadcasting. The source maintains a table recording if corresponding packet is correctly received by each receiver, and we will name this table as feedback-matrix later. During the retransmission phase, the source forms a new packet by XORing a set of lost packets from different receivers for retransmitting. Upon receiving a XOR-ed packet, multiple receivers can recover their lost packets through one retransmission, which can effectively reduce the number of retransmissions and utilize wireless bandwidth.

However, D.Nguyen paid little attention on how to find the optimal XOR coding set for any given feedback-matrix T using XOR network coding. X. Xiao et al. [3] proposed a packet-combination algorithm, NCWBR, as a solution for this problem. However, NCWBR suffers from drawbacks from two aspects (which will be explained later in detail in Section II), making it impractical for general use.

In this paper, we proposed a new wireless broadcast retransmission scheme based on random linear network coding(LNCR). Our approach can effectively save the number of retransmissions, also the overhead caused by feedback signals. Besides, this algorithm has a low complexity and no restrict on the distribution of packets loss.

The rest of this paper is organized as follows. We first discuss the research motivation and a few of related works in Section II. Section III gives the detail of LNCR. The simulation and discussion are given in Section IV, and Section V concludes this paper.

II. Motivation

Recently, XOR-based network coding was proposed by Nguyen et al. [2] to reliable broadcast in wireless networks. This work can be described as follows: the source maintains a $K \times M$ table T in which each entry $T(i,j)$ $(1 \leq i \leq K, 1 \leq j \leq M)$ is used to indicate whether the receiver R_i has received packet P_j or not, i.e., $T(i,j)$=1 if P_j is correctly received by R_i and $T(i,j)$=0 otherwise. We name this table as feedback-matrix T. During the retransmission phase, the source forms a new packet by XORing a set of the lost packets according to T and retransmit it. Then multiple receivers can recover their lost packets through one retransmission. The source generates new packets until no packet is found left in T, and then starts a new transmission of next M packets.

Fig. 1(a) illustrates an example of feedback-matrix with 4 receivers after 4 packets are broadcast. For example, P_1, P_2, P_3 is successfully received at R_1 while P_4 is not received. In traditional scheme, the source need to retransmit P_1, P_2, P_3 and P_4 separately until each packet are successfully received by all receivers. However, using XOR-based network coding scheme, the source only need XOR P_1, P_2, P_3 and P_4 together to get packet $P = P_1 \oplus P_2 \oplus P_3 \oplus P_4$ and broadcast P. Once P is received at all receivers, each receiver can recover their

978-1-4673-0990-5/12/$31.00

	P_1	P_2	P_3	P_4
R_1	1	1	1	0
R_2	1	1	0	1
R_3	1	0	1	1
R_4	0	1	1	1

(a) matrix 1

	P_1	P_2	P_3	P_4
R_1	0	0	1	0
R_2	1	1	0	0
R_3	0	0	0	1
R_4	1	0	0	0

(b) matrix 2

Fig. 1. Two Examples of Feedback Matrix

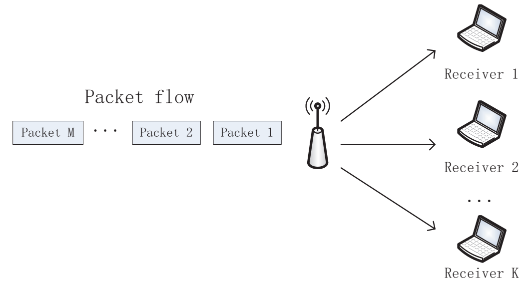


Fig. 2. System Model

lost packets. Take R_1 for example, through $P \oplus P_1 \oplus P_2 \oplus P_3$, P_4 is recovered. Similarly, R_2, R_3, R_4 can recover their lost packets respectively.

However, D.Nguyen paid little attention on how to combine the lost packets for any given feedback-matrix T. For example, how to perform combination for feedback-matrix T shown in Fig. 1(b) is a difficulty. X. Xiao et al. [3] proposed a method named NCWBR to solve this problem. It searches the first entry equal to 0 in each row from T and combines corresponding packets together by XOR-ing. Upon receiving a XOR-ed packet, a receiver will attempt to recover its lost packet, if successfully, the source will update corresponding entry in feedback-matrix T as 1, otherwise still 0. After updating the whole matrix according to all receivers' feedback signals, the source starts a new combining and retransmitting phase until all entries equal 1 in T. Take matrix T in Fig. 1(b) for example, NCWBR will firstly XOR P_1, P_2 and P_3 together, from which only R_2 can recover its lost packet P_3. And the source will update $T(2,3)$ as 1 according to feedback signal by R_2. However, other receivers cannot decode any lost packets, so corresponding entries such as $T(1,1)$, $T(1,2)$ are still 0. Then the source starts a new combining and retransmitting phase.

NCWBR suffers from drawbacks from two aspects. On one hand, finding the optimal way to combine lost packets has been proved as a complex NP-complete problem [4]. Besides, the source need to update feedback-matrix after each packet is broadcasted, which will cost a large number of control frames, bringing about high overhead. On the other hand, the algorithm is very much affected by the distribution of packets loss, which results in a unstable performance. For Fig. 1(a), the source just need to retransmit 1 packet, which is $P = P_1 \oplus P_2 \oplus P_3 \oplus P_4$. However, for Fig. 1(b), the source would transmit $P_1 \oplus P_2 \oplus P_3$, $P_1 \oplus P_2 \oplus P_4$, $P_1 \oplus P_2$, $P_1 \oplus P_3$, $P_1 \oplus P_2 \oplus P_4$, $P_1 \oplus P_2$, $P_1 \oplus P_4$, P_2, P_3 sequentially, resulting a even larger number of retransmissions than traditional ARQ scheme, who will retransmit P_1, P_2, P_3 and P_4 separately.

To address this challenge, we propose in this paper a new retransmission scheme based on random linear network coding to achieve higher transmission efficiency in wireless networks.

III. LNCR

A. System Model

Firstly, we make the following model for wireless broadcasting transmission, as shown in Fig. 2.

1. The broadcast system consists of 1 source node and K ($K > 2$) receivers. A group of packets needs to be broadcasted to K receivers. It is assumed that transmissions occur in time slots and only one packet can be delivered per slot, and the source broadcasts packets in a fixed time slot Δt.

2. All receivers use Acknowledgements/Negative Acknowledgements (ACK/NAKs) to feed back, and the source maintains a $K \times M$ feedback-matrix T in which each entry $T(i,j)$ $(1 \le i \le M, 1 \le j \le K)$ is used to indicate whether the receiver R_i has received packet P_j.

3. For simplicity, we assume all ACK/NAKs are instantaneous and never lost.

4. Packets lost ratio at R_i follows the Bernoulli distribution with parameter $p_i (i = 1, 2, ..., K)$.

B. Linear Random Network Coding

Consider a set of m packets $X_1, X_2, \ldots, X_m$ with the same length need to be retransmitted. The source encodes all lost packets by random linear codes, generating a new coded packet:

$$Y_i = \sum_{j=1}^{m} g_{ij} X_j \tag{1}$$

where coding coefficients $g_{ij} (1 \le j \le m)$ are random elements from a selected finite field F_q. After m coding packets are received, each receiver can recover the m original packets by solving linear equations as:

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix} = \begin{pmatrix} g_{11} & \cdots & g_{1m} \\ \vdots & \ddots & \vdots \\ g_{m1} & \cdots & g_{mm} \end{pmatrix}^{-1} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} \tag{2}$$

where m coefficients vectors: $\overrightarrow{g_i} = (g_{i1}, g_{i2}, \ldots, g_{im})$ $(1 \le i \le m)$ compose a matrix, which is named as coefficients matrix and denoted by G later. An original packet can be considered with unit vector as its coefficient vector.

According to Ho and Li's linear coding theory [6] [7], the solvability of the above equation for a certain receiver is: the rank of coefficients vector matrix G must equal to the number of unknown original packets, which means G should be of full rank or m sets of coefficients should be independent. According to [8], when F_q is sufficiently large, e.g. $F_q = 2^8$, the probability of decoding is over %99.6, and decoding failure caused by dependent coefficients vectors can be ignored.

Assume that in a broadcast system consists of 1 source and K ($K > 2$) receivers, node R_i has the highest packet lost ratio and lost D original packets of M. It has been proved that in retransmission phase, we rebroadcast D linear network coding packets and all receivers will recover their lost packets [6]. Actually, under such circumstances, R_i needs the maximum number of coding packets, which is D, to make its vector matrix achieve full rank and decode its lost packets. So by the time D coding-packets are retransmitted, other receivers $R_j (1 \leq j \leq K, j \neq i)$ have already achieved full rank because they need less than D packets. That is to say, all receivers will meet solvability of Equation. (2).

C. LNCR DETAIL

The transmission in LNCR scheme incorporates regular broadcast phase and retransmission phase. Step details are given below:

1. The source broadcasts M packets to K receivers, and each packet is sent in a time slot of fixed duration. The source maintains a feedback-matrix T according to all receivers' ACK/NAK feedbacks.

2. After M packets have been broadcasted, the source enters the retransmission phase at time $M\Delta t$. All lost packets compose a set $D = \{X_1, X_2, \ldots, X_m\}$, and $N_{\max}$ sets of coefficients vectors $\overrightarrow{g_i} = (g_{i1}, g_{i2}, \ldots, g_{im})$ $(1 \leq i \leq N_{\max})$ (randomly selected from finite field F_q) are used to encode all lost packets in D, generating $N_{\max}$ coding-packets. $N_{\max}$ is the largest number of lost packets among all receivers, which can be determined by:

$$N_{\max} = \max_{i \in \{1,2,\ldots,K\}} \{\sum_{j=1}^{K} T(i,j)\} \tag{3}$$

3. After $N_{\max}$ coding-packets are retransmitted, each receiver evaluates the rank of its coding-vector matrix G, which is denoted by r_i. If r_i is not equal to M, meaning G does not achieve full rank for receiver R_i, then R_i will notify the source how many coding-packets it still needs to let G achieve full rank. The number of packets is denoted by N_i, and can be determined by:

$$N_i = \begin{cases} M - r_i, r_i \leq M \\ 0, r_i \geq M \end{cases} \qquad i = 1, 2, \ldots, K \tag{4}$$

For example, if R_i received all $N_{\max}$ coding-packets during the previous transmission, N_i would be 0, however, if 2 coding-packets is lost at R_i, then $N_i = 2$.

In order to let each receiver feedback N_i to the source, we add a new type of control frame, which we name as INFO frame. Its format is almost the same as ACK, with the only difference that a new field named *Count* is added to carry N_i. N_i would not be greater than 255 in general use, so a byte is enough for it.

4. The source updates $N_{\max}$ according to each receiver's feedback, and generate $N_{\max}$ coding-packets in the new retransmission phase:

$$N_{\max} = \max_{i \in \{1,2,\ldots,K\}} \{\sum_{j=1}^{K} T(i,j)\} \tag{5}$$

5. Repeat step 3 and 4, until the rank of vector matrix equals to M at all receivers ($N_{\max}$=0). Receivers can recover the original packets by Gaussian elimination.

As we can see, LNCR and NCWBR defers from the following aspects:

1. LNCR has a low complexity in combining lost packets. Instead of updating feedback-matrix and determining which packets to XOR in NCWBR, LNCR just combines all lost packets for retransmitting, and the number of coding-packets depends on $N_{\max}$.

2. LNCR is not effected by the distribution of packets loss, which defers from NCWBR. Take Fig. 1(b) for example, as we have analyzed, the source need to retransmit 9 packets using NCWBR, even more than traditional ARQ scheme. However, using LNCR, only 3 coding packets: $Y_1 = g_{11}P_1 + g_{12}P_2 + g_{13}P_3 + g_{14}P_4$, $Y_2 = g_{21}P_1 + g_{22}P_2 + g_{23}P_3 + g_{24}P_4$, $Y_3 = g_{31}P_1 + g_{32}P_2 + g_{33}P_3 + g_{34}P_4$ will make receivers recover their lost packets.

3. LNCR reduces the overhead caused by feedback signals during retransmission phase. For NCWBR, receivers should feedback ACK/NACK after each XOR-ed packet is transmitted, so that the source can update matrix T. When there is a large number of users in broadcasting group, overhead caused by large-number and frequent feedback signals would be expensive. Using LNCR, all receivers only give feedback after $N_{\max}$ coding-packets are transmitted. With not so frequent feedback, the overhead can be reduced on a certain scale.

4. LNCR has a larger decoding time-delay. NCWBR has a much simpler decoding operation than LNCR. For LNCR, receivers would not decode until a certain quantity of packets are received, bringing about a larger decoding time-delay. LNCR applies for services not sensitive to time-delay.

D. Mathematics Analysis

We define the average number of transmissions per packet as the analysis parameter. Let L_1, L_2 and L_3 denote the parameter using traditional retransmission approach, NCWBR, LNCR respectively.

Reference [2] gives the average number of transmissions using traditional retransmission approach:

$$L_1 = \sum_{i_1, i_2 \ldots i_K} \frac{(-1)^{\sum_{j=1}^{K-1} i_j}}{1 - \prod_{j=1}^{K} P_j^{i_j}} \qquad i_1, i_2, \ldots i_K \in \{0, 1\} \tag{6}$$

where K denotes the number of receiver nodes. The average number of transmissions using NCWBR is given in reference

[3] :

$$L_2 = \frac{1}{1 - \max\{P_1, P_2, \ldots, P_K\}} \tag{7}$$

***Theorem 1*:** The average number of transmissions per packet of LNCR with K receiver nodes is:

$$L_3 = \frac{1}{1 - \max\{P_1, P_2, \ldots, P_K\}} \tag{8}$$

***Proof*:** We assume the node j with the highest lost rate, and then we have $P_j = \max\{P_1, P_2, \ldots, P_K\}$. Using LNCR, the total number of rebroadcasts is determined by the receiver who has the highest lost rate, which is node j here, and only when the rank of coding vector matrix G at node j achieves full rank, the transmission will end. That is to say, at least M packets should be successfully received by node j (including original packets and coding-packets). This implies that the transmission scene of LNCR is equivalent to the status where the source code will transmit M packets to node j. Then, we have the total number of transmissions:

$$L_{total} = \frac{M}{1 - P_j} \tag{9}$$

therefore get the average number of transmissions per packet as Equation. (8).

IV. PERFORMANCE EVALUATION

In this section, we investigate average number of transmissions per packet of our algorithm, NCWBR and traditional ARQ scheme under different sizes of the packet amount in one broadcast, different packet loss rate and different numbers of receivers respectively.

Fig. 3 illustrates the average transmission number of different schemes versus the lost rate p (assume all the nodes have the same lost rate) with 4 receivers and 20 packets to broadcast. As Figure 1 indicates, when the lost rate is relatively small, the average number of transmissions is relatively closely among the 3 approaches. As the lost rate increases, our algorithm significantly outperforms ARQ and NCWBR schemes. This is because LNCR combines packets to retransmit instead of rebroadcasting them one by one under ARQ scheme. And compared with NCWBR, the number of coding-packets for LNCR is determined by the receiver who has the highest lost rate, not influenced by the distribution of packets loss, which makes the performance of NCWBR retransmission unstable.

Fig. 4 illustrates the average transmission number of different schemes versus different number of receivers (p_i=0.2, M=20). When there are only two broadcast receivers, our algorithm provides a little improvement to the other two schemes. As K increases, the gap of the transmission bandwidth between the other two schemes and our algorithm becomes much larger. This is because as receiver nodes increase, the distribution of packets loss becomes more complex for NCWBR, and the probability that a certain receiver can't recover lost packets from the XOR-packets becomes larger, causing the average number of transmission growing. However, for our algorithm, the number of transmissions is

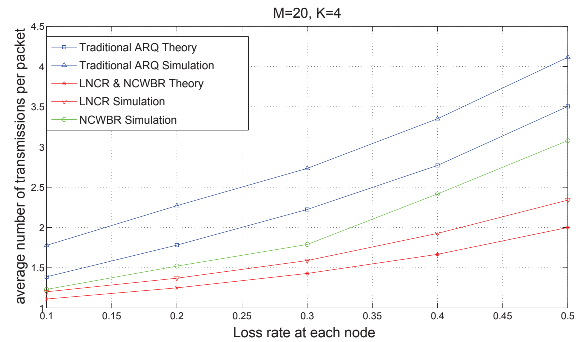


Fig. 3. Transmission bandwidth versus packets lost rate

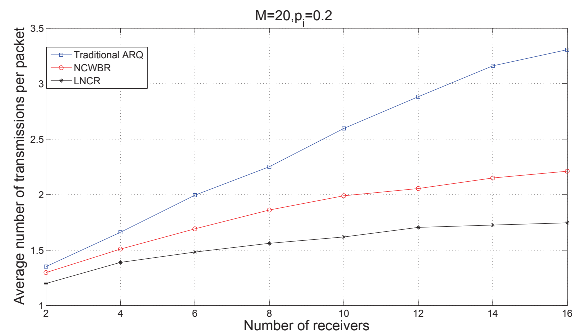


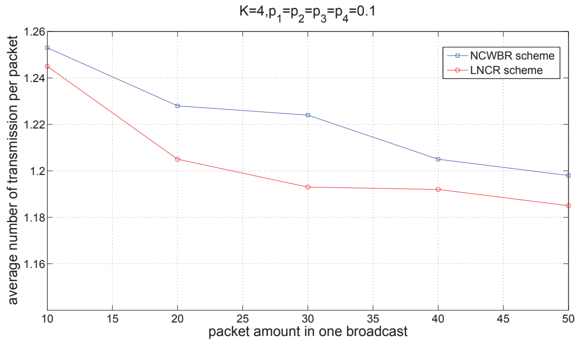
Fig. 4. Transmission bandwidth versus different number of receivers

mostly determined by the largest number of lost packets, which will not increase significantly under our simulation with the constant packet loss.

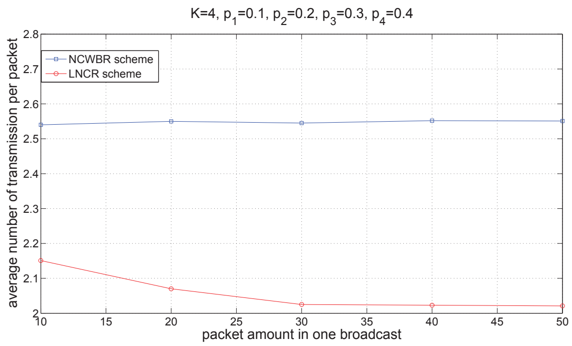
Fig. 5 illustrates the average transmission number of different schemes versus packets account in one broadcast, with 4 receivers. We can see that in general the average number of transmission of both schemes decrease as the packet amount increases, and our algorithm is more efficient than NCWBR. In low packet loss rate scenarios(Fig. 5(a)), the improvement of LNCR over NCWBR is not obvious, while in high packet loss scenarios(Fig. 5(b)), LNCR outperforms NCWBR. This is because when the packet loss rate are very small, there are only a few of lost packets need to be retransmitted, Therefore, the chances for combining packets through XOR are also very limited. However, as the packet loss rate increases, the distribution of packets loss becomes more non-uniform, which brings lower probability for receivers to recover their lost packets from XOR-packets and larger average transmission number as a consequence.

V. CONCLUSION

In this paper, we proposes a novel retransmission approach in wireless broadcasting based on random linear network coding (LNCR), which effectively reduces the average number of transmissions. The approach's performance is evaluated through extensive simulations under different conditions. Simulation results show that compared with existing approaches using XOR-based network coding, as NCWBR, our approach can achieve higher transmission efficiency, especially in the



(a) Low packets loss environment



(b) High packets loss environment

Fig. 5. Transmission bandwidth versus packets account in one broadcast

environment with larger number of broadcast receivers. We will focus on how to reduce packet-delay during decoding in the future.

ACKNOWLEDGEMENT

The authors are supported by National Natural Science Foundation of China (No. 60972076, No. 61072052) and by the National Major Specialized Project of Science and Technology (No. 2010ZX03003-004-03).

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