Computational Complexity and Energy Consumption Analysis of Dynamic Resource Scheduling Algorithms for LTE

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Abstract: This paper presents a detailed analysis of computational complexity, RF and operational energy consumption of a number of radio resource management strategies. Those scheduling algorithms are evaluated within a LTE downlink simulator and the results show that the adaptive scheduling algorithm is able to achieve a significant energy saving – up to average operational energy reduction gain of 71.75% and 47.86% over a conventional nonenergy aware resource allocation scheme in the pessimistic and optimistic case, respectively. In addition, the computational complexity of the algorithm is estimated, and the trade-off between RF and DSP processing for power efficiency is evaluated.

Index Terms—Green Radio, Energy Efficiency, Radio Resource Management

I. INTRODUCTION

Currently, around 80% of the whole world population are mobile users. The energy consumption of mobile phone networks is contributing to the global climate change as the worldwide telecommunications industry is currently responsible for 183 million tones or 0.7% of the total carbon dioxide emissions [1]. In addition, the growing energy costs are becoming significant OPEX (Operational Expense). The core 5 Green Radio programme of mobile VCE (MVCE) [2][3] aims to develop more power efficient wireless networks in order to reduce CO2 emissions and operating expenditure without compromising the Quality of Service (QoS) of the end user.

Long term evolution (LTE) is the latest standard in the mobile network technology tree that previously realised the GSM/EDGE and UMTS/HSPA network technologies that currently dominate over 85% of the mobile phone global market. It is introduced as Release 8 in the 3rd Generation Partnership Project (3GPP) [4]. LTE employs advanced physical layer (PHY) techniques, including orthogonal frequency division multiplexing (OFDM) for downlink transmission and multiple-input and multiple-output (MIMO) techniques [5]. Space-time block coding (STBC) is able to achieve full transmit diversity and enable reliable communication. Thus in LTE, an Alamouti [6] based Space-Frequency Block Coding (SFBC) technique is adopted and will be considered in this paper. Spatial multiplexing (SM) MIMO technique is also considered and its gain is accomplished by simultaneously sending different data streams over the same radio resource, which can dramatically increase throughput and bandwidth efficiency.

Radio resource management (RRM) strategies can improve the utilisation of limited radio spectrum resources and radio network infrastructure. Multi-user diversity gains in both the time and frequency domain can be exploited by adopting an appropriate joint time/frequency scheduling

strategy. Here, an initial user pre-selection stage is implemented in the time domain, imposing fairness constraints to all users. In the frequency domain, dynamic multi-user scheduling approaches are then applied to the subset of eligible users to further improve the system performance. Whilst the conventional target of those scheduling and allocation algorithms is to improve system throughput, this can be translated to energy reduction to achieve a specific rate target. A scheduling approach, which is designed for systems under light load conditions, is also considered here. A low order modulation scheme generally requires less transmit power but requires more bandwidth to maintain a specific link quality requirement. Therefore, if the system is not operating at full load, an opportunity exists to allocate the spare spectrum to users and allow them to switch from highly spectrally efficient modes to modes with lower spectral efficiency to achieve energy savings while meeting the rate target. It is referred as "trading bandwidth for energy efficiency", or "bandwidth expansion" strategy.

This paper will present a detailed performance analysis of the scheduling algorithms in terms of both RF and operational energy consumption based on a LTE macrocell base station model for both pessimistic and optimistic cases. The results show how the operational and RF transmit power varies with traffic load and the important role of the RRM scheme plays in achieving energy savings at the base station. For different scheduling strategies, they not only experience different degrees of performance trade-offs in terms of rate, energy and fairness, but also differences in computational complexity can be widely diverse. The question then arises as to whether the energy requirement for scheduling algorithm processing is significant compared to its achievable energy savings. The computational complexity in terms of MIPS (millions instructions per second) costs for different algorithms are calculated and their DSP power consumption are estimated based on a current DSP platform for LTE. The trade-off between RF and DSP processing for power efficiency is evaluated. The energy, rate and fairness trade-offs between non-rate guaranteed and rate-guaranteed scheduling approaches are also evaluated.

II. BASE STATION POWER CONSUMPTION MODEL

The efficiency of the base station is governed by the power consumption of various components, including the core radio devices which are radio transceivers, power amplifiers and transmit antennas. On the basis that the base station is made of several components, some of which will have a power consumption dependent only on the RF power, some on the traffic load, some jointly on both and some on

neither, the following model is proposed for the pessimistic bound [3][7]:

$$P_{\text{supply}} = 2.85 P_{RFMax} + 557 \tag{1}$$

and for the optimistic bound [3][7]:

$$P_{\text{sup}\,plv} = 0.0121LP_{RFMax} + 0.115P_{RFMax} + 2.419L + 37 \tag{2}$$

where L represents the traffic load, and the peak load L = 100 is equated to the maximum throughput of a SISO system.

Table 1 Macrocell power consumption figures

Description	Optimistic case	Pessimistic case		
Radiated power	18dBi antenna gain	18dBi antenna gain		
(per sector)				
Antenna	90% efficiency	70% efficiency		
Switch/Duplexer	0.2dB switch loss	0.2dB switch loss		
Feeder	1dB feeder loss	3dB feeder loss		
Power Amplifier	85% PA efficiency	40% PA efficiency		

III. SYSTEM AND CHANNEL MODEL

The channel model used in the simulations is the Spatial Channel Model Extension [8] (SCME) Urban Macro scenario, specified by 3GPP [9]. A low spatially correlated channel is assumed for all the users where 10λ spacing at the BS is employed.

Table 2: Simulation Parameters for LTE OFDMA Downlink

Transmission Bandwidth	20 MHz			
Time Slot/Sub-frame duration	0.5ms/1ms			
Sub-carrier spacing	15kHz			
Sampling frequency	30.72MHz (8x3.84MHz)			
FFT size	2048			
Number of occupied sub-carriers	1201			
Physical resource block (PRB) size	180 KHz (12 sub-carriers)			
Total number of PRBs C	100			
Symbols per time slot/sub-frame	7 / 14			
Packet Size	54 Bytes			
Channel coding	Turbo coding			
Interleaver	Random			
BS Tx Power	46dBm (40W)			
Propagation Model	SCM Urban Macro			
Path Loss Model	Cost-Hata [9]			
Cell Radius	750m			
Noise Power	-104dBm			
User Equipment Noise Figure	6dB			
Total number of users K_{total}	25			

Table 3: Modulation and Coding Schemes

Mode	Modula-	Coding		Spectral Efficiency		
	tion	Rate	(SISO, SIMO,	η (SISO, SIMO,		
			SFBC/SM)Mbps	SFBC/SM) bps/Hz		
1	QPSK	1/3	11.2/22.4	0.56 / 1.12		
2	QPSK	1/2	16.8/33.6	0.84 / 1.68		
3	QPSK	3/4	25.2/50.4	1.26 / 2.52		
4	16 QAM	1/3	22.4/44.8	1.12 / 2.24		
5	16 QAM	1/2	33.6/67.2	1.68 / 3.36		
6	16 QAM	3/4	50.4/100.8	2.52 / 5.04		
7	64 QAM	3/5	60.48/120.96	3.024 / 6.048		
8	64 QAM	3/4	75.6/151.2	3.78 / 7.56		
9	64 QAM	6/7	86.4/172.8	4.32 / 8.64		

Considering a multi-user scenario, the analysis in this paper is performed on the downlink of a 3GPP LTE Orthogonal Frequency Division Multiple Access (OFDMA) system. In LTE, each time frame consists of 10 sub-frames which last 1ms each, and half of a sub-frame is called a time

slot. The total system bandwidth is divided into 100 virtual resource blocks (VRBs), which is the smallest unit of allocation in LTE [10]. Each VRB is comprised of a pair of physical resource blocks (PRBs). A PRB spans 12 subcarriers in the frequency domain and 7 symbols in time domain (if the shorter cyclic prefix is used). The key parameters of the considered LTE OFDMA downlink system are given in Table 2. Perfect channel estimation is also assumed. A representative sample of the modulation and coding schemes (MCSs) adopted by LTE are used, indicated in Table 3. The effective data rate of a MCS is $R = \text{(number of data subcarriers per symbol} \times \text{coding rate} \times \text{number of coded bits per symbol} \times \text{Number of OFDM symbols per time slot)/duration of time slot.}$

Figure 1 shows the spectral efficiency performance of the overall system employing those schemes in the SCME channel. The achievable average throughput is given by, Throughput = R(1-PER) where R and PER are the bit rate and the residual packet error rate for a specific mode respectively and this in turn is used to calculate the spectral efficiency. The overall throughput envelope is obtained by assuming ideal adaptive modulation and coding (AMC) based on the (throughput) optimum switching point.

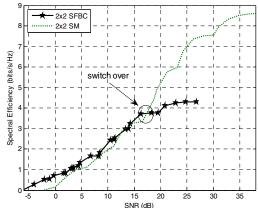


Figure 1: Switching point for SFBC and SM schemes

III. Computational Complexity Analysis

A. Assumptions

The energy consumption requirement of the considered algorithm for DSP processing is estimated using data available for the TMS320C6474 [16], a DSP platform recently developed by Texas Instruments. The chip integrates three of TI's TMS320C64x+ cores running at 1 GHz each, delivering 3 GHz of raw DSP performance that consumes 1/3 less power at 2/3 less DSP cost over discrete processing solutions. The processor can support 24,000 million instructions per second (MIPS) and consumes an average power of 6W to give a performance of 4MIPS/mW, which equates to 4nJ/instruction. The computational load of the scheduling algorithms is measured by MIPS cost and the number of instructions is calculated based on the following assumptions [17]:

- 1. All operations are confined to either multiplications/divisions or additions/substractions.
- 2. Memory operations run in parallel behind computing.

B. Rate Guranteed Scheduling Strategy

While a low order modulation scheme is less spectrally efficient, it generally requires less transmit power to maintain a specific link quality requirement. Therefore, if the system is

not operating at full load, an opportunity exists to allocate the spare spectrum to users and allow them to switch from highly spectrally efficient modes to modes with lower spectral efficiency to achieve energy savings while maintaining the rate target. An adaptive multi-user resource and power allocation (MRPA) algorithm is illustrated in Figure 2. The objective is defined as minimising the total transmit power requirement P_r subject to a given user's rate requirement:

$$P_{T} = \min \sum_{k=1}^{K} P_{k}(i, \mathbf{C}_{k}) = \min \sum_{k=1}^{K} \frac{C_{k} f_{k}(D(i)_{k}, PER_{k})}{\alpha_{k}^{2}(\mathbf{C}_{k})}$$
(3)

subject to:

$$R_k = C_k M_k \text{ for all } k \tag{4}$$

The information of the supported amount of bits D(i) and minimum received SNR requirements for the 9 MCSs can be obtained from Table 3. For MCS index i, the required received power with unity channel gain for achieving D(i) bits per PRB per time slot while meeting the PER target is $f_k(M_k, PER_k) \in \{f_k(D(i), PER_k), ..., f_k(D(I), PER_k)\}$. M_k is the number of transmission bits per PRB. There are C PRBs available for transmission, $\mathbf{C} = \{1, ..., c, ..., C\}$, and each user is assigned a number of PRBs denoted as C_k . The resource allocation here jointly considers fair resource allocation and link adaptation. In MRPA, un-allocated PRBs are assigned to the users which can achieve the most transmit energy reduction through trading bandwidth. It lowers the received SNR requirement by using a lower modulation and coding scheme through assigning more PRBs.

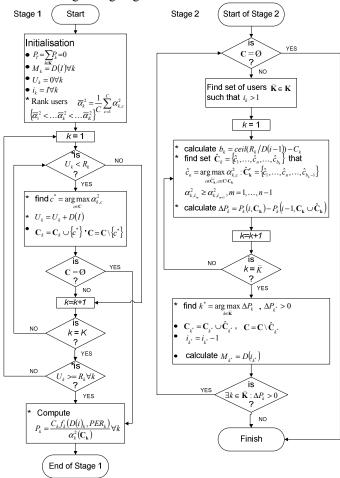


Figure 2. Flow Chart of Adaptive Multi-user Resource and Power Allocation Scheme

To calculate the computational complexity, in stage 1, to initialise the process, the cost of ranking K users is $K \log K$. The scheduler then searches for the best PRB for each user and the process continues until all the user's rate requirements are met by adopting the highest MCS level. The maximum cost occurs if the allocation continues for all users until $C_1(\leq C)$ PRBs are assigned, i.e. $\sum_{c \log c}^{c}$. In addition,

calculating the new achievable rate after a PRB is allocated to a user costs C_1 and comparing the current rate with the target rate at the beginning and end of the loop costs another $2C_1$. At the end of the first stage, the cost for computing the transmit power is 3K. Therefore, the maximum total cost for running stage 1 of the MRPA is $K \log K + \sum_{i=1}^{C_1} c \log_i c^{i} + 2C_1 + 3K$.

Assuming C_1 PRBs are allocated at stage 1, and that each time a lower MCS level is adopted, b PRBs are needed for a user to maintain the target rate. To estimate the maximum total cost, the scheduler is assumed to continually examine the conditions of all the users until all PRBs are assigned. Searching for a set of PRBs $\hat{\mathbf{C}}_k$ with the highest channel gains for all users cost $K \sum_{k=1}^{(C-C_1)/b} (bn) \log(bn)$. Calculating the change in

power ΔP_k requires 2 multiplications and 1 subtraction every time and therefore the total cost is $3K(C-C_1)/b$. Finding the user that is able to achieve the most energy saving through bandwidth expansion costs $(C-C_1)K\log K/b$. As a result, the overall cost for stage 2 is $K\sum_{i=1}^{(C-C_1)/b}(bn)\log(bn) + \frac{K(C-C_1)}{b}(3+\log K)$.

C. Joint Time and Frequency Scheduling Strategy

Considering a joint time and frequency scheduling approach (JTDFD), the first layer of scheduling is implemented in the time domain (TD) and the second layer in the frequency domain (FD) [13]. The TD scheduler attempts to identify users with relatively good channels whilst maintaining an overall fairness for all users.

Time Domain Scheduler

In the time domain, a well known proportional fair (PF) scheduler is adopted to assign approximately the same number of resources to all users (averaged over a period of time) and to try to allocate resource in any given scheduling interval to a user whose channel condition is near its peak. At any time slot, proportional fairness can be achieved by transmitting to user k^* having the highest priority based on:

$$k^* = \underset{k}{\operatorname{arg\,max}} P_k(t) = \underset{k}{\operatorname{arg\,max}} R_k(t) / T_k(t)$$
 (5)

where $R_k(t)$ represents the current requested transmission rate chosen from the set of available MCS. $T_k(t)$ represents the user's average throughput over a window in the past:

$$T_{k,c}(t) = \begin{cases} (1 - (1/t_c))T_k(t-1) + (1/t_c)R_k(t) & k = k^* \\ (1 - (1/t_c))T_k(t-1) & k \neq k^* \end{cases}$$
(6)

where t_c is the window size of the average throughput. For larger t_c , the scheduler maximises throughput but has a lower latency tolerance to some applications. t_c is set to 500 to compromise between throughput and delay constraints here. In the context of OFDMA, the subset of users with the highest priorities, the number of which is termed 'multiplexing users' and denoted by k, is selected for subsequent FD scheduling.

For each PRB, computing the decision metric and selecting the user with the highest value of the decision metric costs $K_{total} + K_{total} \log K_{total}$. Consider 60% of the users are selected for the next stage, $K = 0.6K_{total}$, to update the average throughput for the next scheduling interval, it costs $2(K_{total} - K)$ for the non-scheduled users and 4K for the scheduled users. Therefore, the total cost of allocating all the PRBs within one OFDM symbol is $K_{total}(3 + \log K_{total}) + 2K$.

Frequency Domain Scheduler

A number of scheduling algorithms are integrated to an OFDMA system, allocating resources in the frequency domain to the subset of the selected users. The algorithms considered are:

1) Round Robin (RR)

The selected users are serviced in a round-robin fashion across different PRBs. The cost of implementing a round robin scheduler is considered to be negligible.

2) Greedy Algorithm (GA)

It has low complexity but is unfair as it simply selects the best user to reduce the total energy consumption.

3) Proportional Fair Algorithm (PFA) Scheme 1 and 2

The PFA, which is taken as a standard reference scheduler and which is able to trade off throughput versus short term fairness by controlling a window size parameter. Two variants are considered here [11] returning different degrees of tradeoffs in terms of complexity and fairness.

PF I: The average throughput metric $T_k(t)$ is updated for each new time interval (after all PRBs are allocated).

PF II: The average throughput metric $T_k(t)$ is updated after the allocation of each PRB This enhances fairness at the cost of greater complexity.

4) Equal Gain Dynamic Allocation (EG-DA)

The equal gain dynamic allocation (EGDA) algorithm [12] attempts to allocate PRBs so as to improve the perceived channel gain in a user without minimising the perceived channel gain in other users. If users experience similar SNR levels, this is a fair approach and ensures different users achieve similar PER and BER performances.

5) Fair Cluster Algorithm (FCA)

The algorithm allocates a fair number of PRBs to all selected users, and achieve both short term and long term fairness [14].

6) Relative Strength Scheduling Algorithm (RSSA)

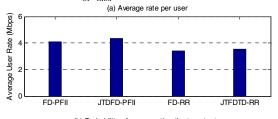
The relative strength scheduling algorithm (RSSA) proposed in [15], gives enhanced scheduling priority of weak users on their strong PRBs, resulting in a more equally distributed resource allocation process across an OFDM symbol via variation of tuning parameter.

Following the methods introduced in section III A and B, the computational complexity of the scheduling strategies discussed in this section are calculated and listed in Table 4.

V. RF, Operational and DSP Processing Energy Requirement of Different Resource Scheduling Algorithms

Let $E_{\it DSP-RRM}$ and $E_{\it DSP-FFT}$ denote the required DSP processing energy of different radio resource management strategies and IFFT function, respectively. Let $E_{\it BE-BRM}$ be the

base station (BS) RF energy requirement of different scheduling algorithms and $E_{\mathit{OP-RRM}}$ be the related base station operational energy requirement. Assuming that the energy is measured over one transmission time interval (TTI), for a maximum transmit power of 20W at the base station, the energy requirements and DSP usage in percentage are listed in Table 4 for those joint time and frequency domain schedulers and MRPA. For a half-loaded system in the considered scenario, the required RF transmit power of different scheduling algorithms $E_{\mathit{RF-RRM}}$ are presented in Table 4. The pessimistic bound and optimistic bound of the operational energy $E_{\mathit{OP-RRM}}$ are also listed in Table 4.



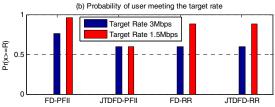


Figure 3: Rate Performance of Joint-Time and Frequency Domain (JTDFD) PFII, RR Scheduling and Frequency Domain (FD) only PFII, RR Scheduling (a) Average rate per user (b) Probability of individual user's rate meeting the target

The estimates show that the required DSP energy consumption of implementing different scheduling algorithms is negligible compared to the RF energy saving that it achieves. In addition, although the number of instructions required to perform a particular scheduling algorithm can be a lot more than for some others, the differences in the estimated processing energy requirement among different scheduling algorithms are insignificant. That is due to the fact that most recently developed DSP platform are able to support very computationally intensive algorithms at low energy cost. Although the complexity of some algorithms depend on the assumptions of certain parameters, all of the scheduling algorithms consume around or much less than 7% of the total resources on the chip in the considered scenarios. In particular, comparing the two PFA variants, variant 2 is just over 50% more complex for the considered parameters and the difference between the two variants in DSP processing energy consumption is only around 0.01mJ per sub-frame. On the other hand, PFA variant 2 significantly outperforms variant 1 in terms of throughput and RF energy requirement. Therefore, overall it is shown that PFA variant 2 is preferred.

For the rate-guaranteed scheduling approach, MRPA, when the user target rate is reduced from 3 Mbps to 1.5 Mbps, more spectrum resources becomes available and assigning more PRBs to especially the weak users allows the lower level of MCS to be adopted to save RF energy significantly. However, the computational complexity analysis shows greater processing time is required by the searching process. As a result, the DSP processing energy is significantly increased as the requested data rate is reduced. Figure 3 shows that the average user rate achieved by PFII and RR both improve as a joint-time and frequency domain scheduling approach is adopted compared to a frequency

domain only scheduling approach. For similar RF transmit power requirements, although both PFII and RR achieve greater overall system rate performance than MRPA, 24% and 40% of their users fail to meet the 3Mbps rate target respectively. For a target rate or 1.5Mbps, 4% and 12% of users are not able to meet the target if they are scheduled by a PFII and RR approach respectively. Therefore, if the system is at low or medium load, for rate-guaranteed applications, MRPA is preferred whereas PFII is preferred at high load or to improve overall system rate at slightly poorer fairness performance than MRPA.

V. CONCLUSIONS

This paper provides a detailed estimate of the computational complexity of different algorithms. Based on a recent DSP platform developed for LTE systems, the DSP usage and processing energy requirement for different scheduling algorithms are estimated and the trade off between operational energy consumption, DSP processing energy, overall system throughput and fairness in resource allocation is evaluated. All of the scheduling algorithms consume around or much less than 7% of the total resources on the chip in the considered scenarios. This percentage estimate is expected to be reduced further as much faster and more energy efficient DSP platforms for LTE have been released lately. By comparing the non-rate guaranteed and rateguaranteed scheduling approaches, results show that if a system is at low or medium load, for rate-guaranteed applications, MRPA is preferred whereas PFII is preferred at high load or to improve overall system rate at slightly poorer fairness performance than MRPA.

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Table 4: RF, Operational and DSP Energy Consumption of Non-Rate Guaranteed Schedulers

Scheme	Computational Complexity	Instructions per sub-frame	$E_{\it RF-RRM}$ / sub-frame	Pessimistic $E_{\mathit{OP-RRM}}$	Optimistic $E_{\mathit{OP-RRM}}$	DSP Usage	$E_{\it DSP-RRM}$ / sub-frame
IFFT	$(N_{fi}/2)\log_2 N_{fi}$	157696				0.660%	0.0394mJ
TD-PF	$K_{total}(3 + \log K_{total}) + 2K$	2597				0.011%	0.0007mJ
(TD-PF)+GA	$CK \log K$	59466	18.8mJ	563.34 mJ	279.58 mJ	0.248%	0.0149mJ
(TD-PF)+PF 1	$CK(1+\log K)+2C+5K$	84316	111.1mJ	594.45 mJ	282.92 mJ	0.351%	0.0211mJ
(TD-PF)+PF 2	$CK(3 + \log K) + 2C$	125270	311.87mJ	662.11 mJ	290.19 mJ	0.522%	0.0313mJ
(TD-PF)+ EGDA	$\sum_{k=1}^{K} (C-k) \log(C-k) + \sum_{c=1}^{C-K} c \log c + C$	293430	261.7mJ	645.20 mJ	288.37 mJ	0.122%	0.0734mJ
(TD-PF)+FCA	$KC(\log C + 4) + KN_c - C + K$	183380	278.1mJ	650.73 mJ	288.96 mJ	0.764%	0.0458mJ
(TD-PF)+RSSA	$KC(\log K + \gamma + 3) + K$	1172700	272.3mJ	648.78 mJ	288.75 mJ	4.886%	0.2932mJ
(TD-PF)+RR		2597	338.17mJ	670.98 mJ	291.14 mJ	0.011%	0.0007mJ
MRPA (Required rate per person: 3 / 1.5 Mbps)	$K \log K + \sum_{c=1}^{C_1} c \log c + 2C_1 + 3K + K \sum_{n=1}^{(C-C_1)/b} (bn) \log(bn) + \frac{K(C-C_1)}{b} (3 + \log K)$	522930 / 1700700	666.87mJ/ 577.21 mJ	40.22mJ / 38.25mJ	326.27mJ / 59.95mJ	2.179% / 7.086%	0.1307mJ / 0.4252mJ