On Subspace Noise Estimation for OFDM

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***Abstract*— The performance of the subspace-based noise power estimation algorithm for an OFDM receiver is investigated. It is shown that due to the correlation of the channel in the frequency domain, the dimension of the autocorrelation matrix does not need to be larger than the number of signal paths. Therefore, due to the reduction of the dimension of the signal space, the required complexity and computation for subspace decomposition is reduced. It is shown that the algorithm performs well in various speed and channel conditions.**

***Index Terms*— Noise power, noise estimation, subspace, OFDM, pilot subcarrier, receiver, blind estimation, LTE.**

I. Introduction

There are several measurements performed in a wireless receiver. The measurements are generally used to characterize the operating environment and the channel. Upon the characterization, the receiver optimizes its algorithms to adapt and match to the operating condition. Due to the limited battery power and the physical size of a handset, an efficient design and implementation of signal processing algorithms is required. Noise power estimation is among the measurements performed in a User Equipment (UE) or handset. Noise power measurement is generally used as an input to higher level measurements such as Signal to Interference and Noise Ratio (SINR) and Channel Quality Indicator (CQI) whereas such measurements are further used for various important functions such as Adaptive Modulation and Coding (AMC), Radio Link Monitoring (RLM) and handover [1].

In this paper, noise estimation for a Long Term Evolution (LTE) handset is investigated. In LTE, the downlink reference signals are based on OFDM where pilots are defined in the frequency domain to assist downlink channel estimation. Figure 1 shows the mapping of the Common Reference Signals (CRS). There are 4 sets of orthogonal pilots, shown as T_1-T_4, each assigned to an antenna port. For example, for the first antenna, there are two staggered groups of reference signals located at the first and fifth symbols of every slot. The spacing of the reference signals in the frequency domain is set to $6\Delta f$, where Δf is the subcarrier spacing [2].

Papers [3]-[7] investigate different methods for noise estimation in an OFDM-based receiver. The proposed techniques in [4]-[5] rely on channel estimation that may not be always acceptable. The scheme in [5] discusses the noise estimation for an OFDM-MIMO 2×2 system using specific training signals. It is shown that the proposed algorithm does not perform well in time-varying frequency selective channels as it relies heavily on a quasi-static time invariant channel with small delay spread assumptions. In [6], a technique based on time domain autocorrelation property of a symmetric preamble signal is presented. In [7], a subspace-based noise estimation approach is discussed for a SISO-OFDM system. The main restriction of the technique is that the dimension of the overall observation space has to be larger than the order of multipath. Such requirement may impose a significant load of the computations in the receiver.

In this paper, using subspace analysis, it is shown that the stringent requirement in [7] may not be always necessary, and accurate noise estimation can be performed without increasing the processing complexity. The performance of the proposed approach is examined under different channel conditions.

In section II, the general signal model and basic subspace noise estimation are discussed. In section III, the signal dimension requirement for noise estimation is discussed. Simulation results and final remarks are presented in section IV and V, respectively.

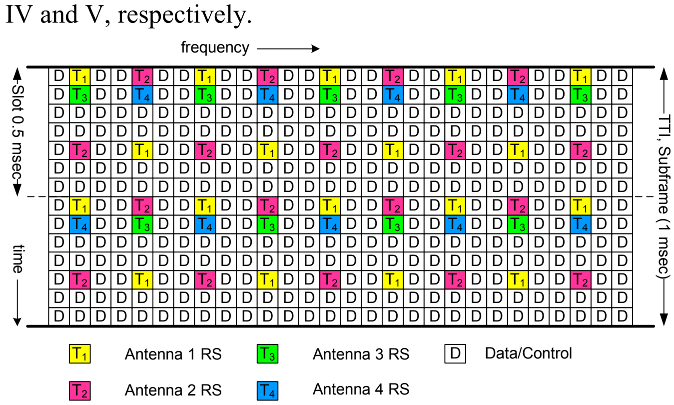

Figure 1: Reference signal structure in downlink LTE

II. Signal Model and Problem Formulation

An OFDM-based MIMO system with a dimension of $N_T \times N_R$, and an FFT size of N is assumed. For the analysis, a frequency selective fading channel with P resolvable path is considered. The time domain channel model can be defined as,

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$$h^{ji}(t,\tau)=\sum_{p=1}^{P} h_p^{ji}\delta\left(t-\tau_p^{ji}\right) \quad (1)$$

where h_p^{ji} and τ_p^{ji} are complex gain and delay of p_{th} channel path, respectively. For the analysis, a low Doppler channel is assumed, so that h_p^{ji} and τ_p^{ji} do not vary over the duration of the estimation period. Let Y_k^i presents the received pilot subcarrier at the antenna *i*, after removing the cyclic prefix and performing DFT on the received OFDM signal,

$$Y_k^i=\sum_{j=1}^{N_T} XH_k^{ji}+Z_k^i \quad (2)$$

where X , H_k^{ji} and Z_k^i are the transmitted pilot symbol from the j_{th} antenna, complex channel gain and an additive noise term for the k_{th} pilot subcarrier, respectively. Noise is assumed complex Additive White Gaussian Noise (AWGN) with a power of σ_N^2 .

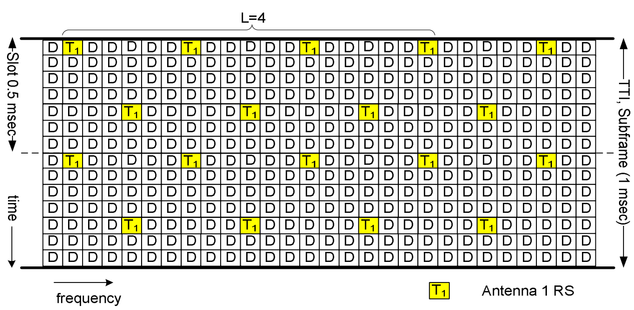

Figure 2: Pilot grouping (L=4) at the receiver

For MIMO channel estimation, all the pilots are used to fully characterize the spatial channel. However, since the noise power does not have any relation with the dimension or the spatial characteristics of the channel, one can simply use only one set of the transmitted orthogonal pilots, such as T_1, for the noise estimation. Hence, the Equation (2) can be further simplified to

$$Y_k^i=XH_k^i+Z_k^i \quad (3)$$

and the equivalent channel response per pilot subcarrier can be shown as

$$H_k^i=\sum_{p=1}^{P} h_p^i e^{-j2k\pi\Delta f\tau_p} . \quad (4)$$

As demonstrated in Figure 2, the frequency band can be divided into *M* subbands, where in each subband, there exist *L* pilot subcarriers. Then, a vector $\mathbf{y}^i$ with a length of *L* can be defined as

$$\mathbf{y}^i=\begin{bmatrix}Y_k^i & Y_{k+1}^i & \cdots & Y_{k+L-1}^i\end{bmatrix}^T . \quad (5)$$

The autocorrelation matrix of the vector $\mathbf{y}^i$ can be estimated as

$$\mathbf{R}=E\left\{\mathbf{y}^i\mathbf{y}^{iH}\right\}=\mathbf{R}_S+\mathbf{R}_N \quad (6)$$

where

$$\mathbf{R}_S=|X|^2E\left\{\mathbf{h}^i\mathbf{h}^{iH}\right\} \quad (7)$$

$$\mathbf{R}_N=\sigma_N^2\mathbf{I} \quad (8)$$

$$\mathbf{h}^i=\begin{bmatrix}H_k^i & H_{k+1}^i & \cdots & H_{k+L-1}^i\end{bmatrix}^T \quad (9)$$

If *L* is selected large enough, due to the orthogonality of the signal and noise, subspace decomposition of the autocorrelation matrix $\mathbf{R}$ yields two orthogonal subspaces of signal and noise. An estimate of the noise power can be obtained by finding the smallest eigenvalue of $\mathbf{R}$,

$$\sigma_N^2=\lambda_{Smallest} . \quad (10)$$

In a practical receiver, the described scheme can be independently performed on each set of the transmitted pilots T_i, and then use the average the results as the estimate of the noise power.

In general, the main drawback of the subspace noise estimation seems to be its requirement to have a large *L*, as the complexity of eigenvalue computations required for the subspace analysis grows exponentially with increasing the *L* [9]. Therefore, it is important to investigate the impact of selection of *L* for practical receiver application.

III. Revisiting the Dimension Requirement

To reduce the complexity of the overall algorithm, it is essential to avoid using a large value for *L*, however *L* has to be larger than *P* to have a clear separation of signal and noise subspaces. In the following, it is shown that due to the correlation of the channel this requirement may not be necessary for many channels. Hence, the complexity of the estimation algorithm can be reduced significantly.

The autocorrelation matrix of the pilot signals $\mathbf{R}_S$ can be expressed as

$$\mathbf{R}_S=|X|^2E\left\{\begin{bmatrix}\left|H_k^i\right|^2 & H_k^iH_{k+1}^{i\,*} & \cdots & H_k^iH_{k+L-1}^{i\,*}\\ H_{k+1}^iH_k^{i\,*} & \left|H_{k+1}^i\right|^2 & & \\ H_{k+2}^iH_k^{i\,*} & & \ddots & \vdots\\ \vdots & & \ddots & \\ H_{k+L-1}^iH_k^{i\,*} & & \cdots & \left|H_{k+L-1}^i\right|^2\end{bmatrix}\right\} \quad (11)$$

where by using (4)

$$E\left\{H_k^iH_{k+l}^{i\,*}\right\}=\sum_{p=1}^{P}\left|h_p^i\right|^2e^{-j2l\pi\Delta f\tau_p} \quad (12)$$

Since,

$$E\left\{H_k^iH_{k+l}^{i\,*}\right\}=E\left\{H_{k'+l}^iH_{k'}^{i\,*}\right\},\qquad k=k' \quad (13)$$

$$E\left\{\left|H_k^i\right|^2\right\}=E\left\{\left|H_j^i\right|^2\right\}=1,\qquad \forall\, k,j \quad (14)$$

then $\mathbf{R}_S$ is a positive semi-definite matrix, with the form

$$\mathbf{R}_S = |X|^2 \begin{bmatrix} 1 & r_1 & r_2 & \cdots & r_{L-1} \\ r_1^* & 1 & r_1 & \ddots & \vdots \\ r_2^* & r_1^* & \ddots & \ddots & r_2 \\ \vdots & \ddots & \ddots & 1 & r_1 \\ r_{L-1}^* & \cdots & r_2^* & r_1^* & 1 \end{bmatrix}_{L\times L} \quad (15)$$

Furthermore, the above structure has a nested property that allows partitioning of $\mathbf{R}_S$ as

$$\mathbf{R}_S = |X|^2 \left[\begin{array}{c|c} 1 & \mathbf{r}_{1,L-1}{}^T \\ \hline \mathbf{r}_{1,L-1} & \mathbf{Q}_1 \end{array}\right] \quad (16)$$

where

$$\mathbf{r}_{1,L-1} = [r_1 \quad r_2 \quad \cdots \quad r_{L-1}]^T \quad (17)$$

$$\mathbf{Q}_1 = |X|^2 \begin{bmatrix} 1 & r_1 & \cdots & r_{L-2} \\ r_1^* & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_1 \\ r_{L-2}^* & \cdots & r_1^* & 1 \end{bmatrix}_{L-1\times L-1} \quad (18)$$

Similarly, matrix $\mathbf{Q}_1$ can now be partitioned down as

$$\mathbf{Q}_1 = |X|^2 \left[\begin{array}{c|c} 1 & \mathbf{r}_{1,L-2}{}^T \\ \hline \mathbf{r}_{1,L-2} & \mathbf{Q}_2 \end{array}\right] \quad (19)$$

where

$$\mathbf{r}_{1,L-2} = [r_1 \quad r_2 \quad \cdots \quad r_{L-2}]^T \quad (20)$$

$$\mathbf{Q}_2 = |X|^2 \begin{bmatrix} 1 & r_1 & \cdots & r_{L-3} \\ r_1^* & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_1 \\ r_{L-3}^* & \cdots & r_1^* & 1 \end{bmatrix}_{L-2\times L-2} \quad (21)$$

The partitioning process can be repeated $L-2$ times, so that at each step, the dimensions of the remaining matrix are reduced by one. At the end, including $\mathbf{R}_S$, there will be $L-1$ matrices $\{\mathbf{R}_S, \mathbf{Q}_1, \mathbf{Q}_2, \cdots, \mathbf{Q}_{L-2}\}$ where the last matrix will have a dimension of 2×2. If $\eta_{(\mathrm{Q}_1)_1} < \eta_{(\mathrm{Q}_1)_2} < \cdots < \eta_{(\mathrm{Q}_1)_{L-1}}$ are the $L-1$ eigenvalues of $\mathbf{Q}_1$, the following relations hold between the eigenvalues of $\mathbf{R}_S$ and $\mathbf{Q}_1$ [8]

$$\lambda_1 < \eta_{(\mathrm{Q}_1)_1} < \lambda_2 < \eta_{(\mathrm{Q}_1)_2} < \lambda_3 < \cdots < \eta_{(\mathrm{Q}_1)_{L-1}} < \lambda_L \,. \quad (22)$$

Since $\mathbf{R}_S$ is a nested positive semi-definite matrix, this relation can be extended to $\mathbf{Q}_1$ and $\mathbf{Q}_2$ and so forth. Therefore, it can be concluded that

$$\lambda_1 < \eta_{(\mathrm{Q}_1)_1} < \eta_{(\mathrm{Q}_2)_1} < \cdots < \eta_{(\mathrm{Q}_{L-2})_1} \,. \quad (23)$$

In order to use the smallest eigenvalue of $\mathbf{R}$ as an estimate of the noise power,

$$\lambda_1 << \sigma_N^2 \quad (24)$$

then by considering Equation (23), the condition expressed in Equation (24) is met if

$$\eta_{(\mathrm{Q}_{L-2})_1} << \sigma_N^2 \quad (25)$$

Since $\mathbf{Q}_{L-2}$ is a 2×2 matrix, its eigenvalues can be found easily by solving

$$\begin{vmatrix} 1-\eta_{(\mathrm{Q}_{L-2})_{1,2}} & r_1 \\ r_1^* & 1-\eta_{(\mathrm{Q}_{L-2})_{1,2}} \end{vmatrix} = 0 \quad (26)$$

then the smallest eigenvalue of $\mathbf{Q}_{L-2}$ will be

$$\eta_{(\mathrm{Q}_{L-2})_2} = 1 - |r_1| \quad (27)$$

Hence the condition expressed in Equation (24) can be re-stated as

$$|X|^2 (1 - |r_1|) << \sigma_N^2 \quad (28)$$

and consequently

$$\frac{|X|^2}{\sigma_N^2} << \frac{1}{1-|r_1|} \quad (29)$$

Therefore, given a Signal to Noise Ratio (SNR), it may be possible that the required dimension of the signal space L, becomes independent of the number of signal paths P. Such relaxation could be realized according to the amount of the correlation between the adjacent frequency domain pilots, and also the operating SNR. From (29), it can be concluded that the requirement on L becomes more relaxed at low SNR region.

	P1	*P2*	*P3*	*P4*	*P5*	*P6*
Delay (us)	0	0.2	0.6	1.6	2.4	5
Relative Power (dB)	-3	0	-2	-6	-8	-10

Table 1 – TU-6 channel profile

IV. Simulations

The impact of L, the length of the observation vector, on the performance of the subspace noise estimation method is studied under different UE speeds. The simulated system is based on a 10 MHz LTE system. Since there are many pilot subcarriers per training symbol, the algorithm is implemented by dividing the frequency band into several subbands, each containing L subcarriers. The initial noise estimates are based on the measurements performed on each subband, and then the final estimate is obtained by averaging the initial estimates. Due to the high number of subbands, an accurate estimation of the noise power is obtained only by a few training symbols.

For the simulation, a TU-6 multipath channel model, with a delay profile, as shown in Table 1, is used. As such, it is expected that an accurate noise estimation is achieved only with an L of at least $L \geq 7$. For the simulation, a noise power of 10^{-9} W is assumed. Figures 3 and 4 show the performance of the noise estimation algorithm assuming different dimensions for the autocorrelation matrix. Due to the correlation of the received pilots in the frequency domain, the subspace noise estimation algorithm performs well with autocorrelation matrix dimensions of $L = 5, 6$. It can be seen that $L = 4$ could still provide relatively accurate estimation for a low speed UE in low/medium SNR range.

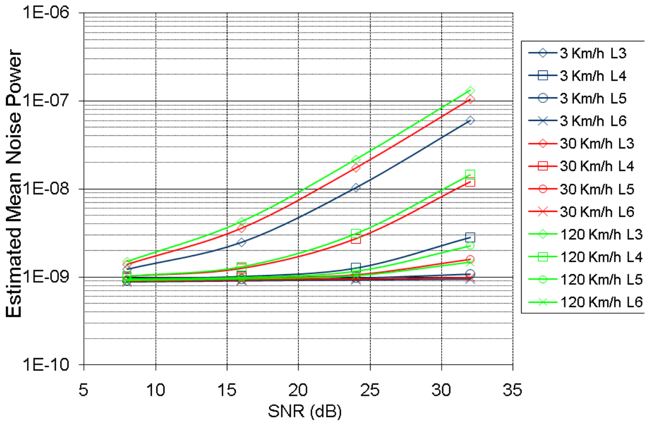

Figure 3- Estimated mean noise power

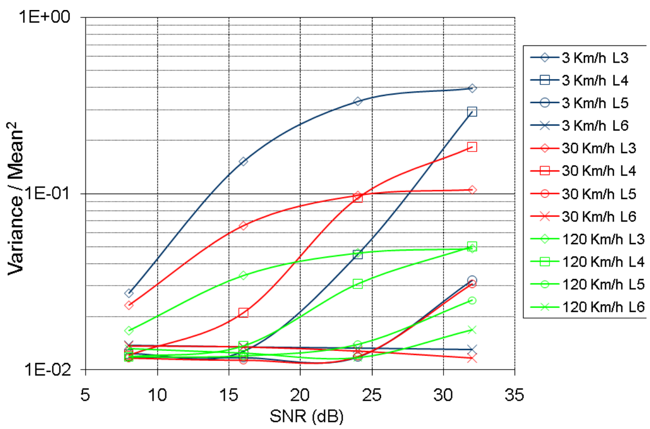

Figure 4- Normalized variance of the noise power measurement

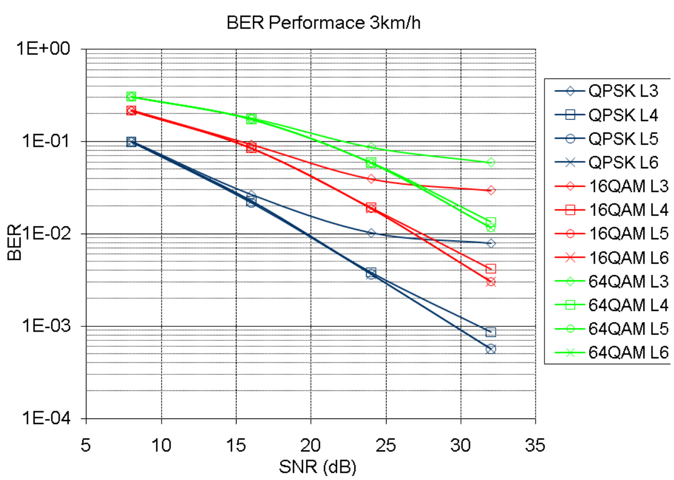

Figure 5: Pilot grouping (L=4) at the receiver

To better examine the impact of the performance of the noise estimation algorithm with different L lengths, the raw Bit Error Rate of a 2×2 open loop MIMO assuming an MMSE receiver is studied. An MMSE receiver relies on the estimated noise power required for MIMO detection. The UE speed is fixed at 3 Km/h. Similar to the earlier results; observation lengths of $L=5,6$ cause no degradation of the performance. A choice of $L=4$, results in a minor degradation in the system performance. Clearly, the higher modulation schemes exhibit more sensitivity to the inaccuracy of the noise estimation.

The computational complexity of different eigenvalue methods are generally estimated as $O(L^3)$ [9]. Therefore, a reduction of dimension from 7 to 4 or even 5, results in a significant saving in the required computations. Furthermore, we may also optimize the power consumption of the noise estimation block by making adjustments to L, according to the used modulation, speed and the operating SNR point.

V. Conclusion

A subspace-based algorithm for noise power estimation is investigated for use in a 3GPP LTE OFDM-MIMO receiver. Using the downlink orthogonal pilot subcarriers, noise power at each receive antenna is estimated. It is shown that in such scenario, due to the correlation between the received pilot subcarriers in the frequency domain, the dimension of the autocorrelation matrix does not need to be larger than the number of signal paths. Simulation results indicate that for a TU6 channel, a signal dimension of 5 provides a satisfactory dimension for noise estimation using the subspace analysis. As a result of reduction of the dimension of the observation space, the required complexity and computation for subspace decomposition is significantly reduced. It is shown that the algorithm with the reduced dimension autocorrelation matrix performs well in various speed and channel conditions.

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