End-to-End Performance of Satellite Mobile Communications with Multi-Beam Interference

Fei Yang*, *Student Member IEEE*, Meiyu Huang*, Sihai Zhang*† and Wuyang Zhou*, *Member IEEE*
*Wireless Information Network Laboratory,
University of Science and Technology of China, Hefei, Anhui, China, 230027
Email: {genyang, myhuang}@mail.ustc.edu.cn, {shzhang, wyzhou}@ustc.edu.cn
†Key Laboratory of Wireless Sensor Network & Communication,
Shanghai Institute of Microsystem and Information Technology,
Chinese Academy of Sciences, 865 Changning Road, Shanghai, China, 200050

***Abstract*—In this paper, we study the communication between terrestrial source and destination terminals via a geosynchronous (GEO) multi-beam satellite in satellite mobile communication (SMC) system. The end-to-end transmission is modeled by a relaying process, in which inter-beam interference plays a key role due to frequency reuse among adjacent beam cells and side-lobe leaking of practical beam antenna. Through theoretic analysis we obtain the closed-form expressions of end-to-end outage probability and system achievable throughput as the major performance metrics. Both theoretic and simulation results, which coincide well with each other under diverse beam frequency planning, suggest that reuse factor of 1/3 is an appropriate compromise between system throughput and reliability in the GEO SMC system.**

***Index Terms*—Satellite mobile communication, inter-beam interference, Rician fading, outage probability, achievable throughput.**

I. Introduction

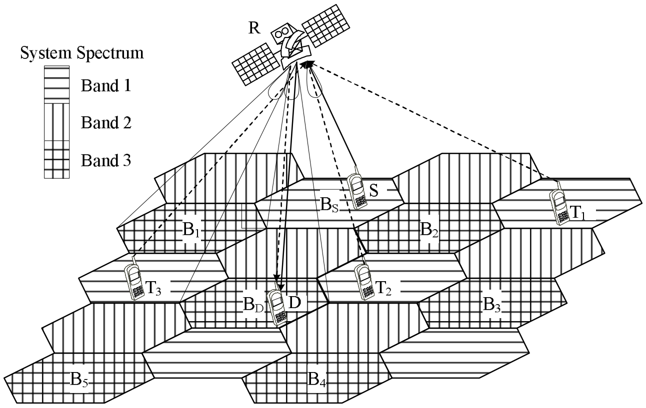
Satellite mobile communication (SMC) has gained scientific and engineering attention for decades, since it provides vast coverage with relatively low cost and complexity of infrastructure, compared with terrestrial cellular networks [1]. Modern SMC systems have adopted multi-beam structure, *e.g.* Thuraya, ACeS and Inmarsat 4 (all the listed systems consist of geosynchronous [GEO] satellites), and the frequency can be reused by multiple isolated spot beams to enhance system capacity, compared to old-fashioned single beam coverage. However, there exists a contradictory issue of reuse level and inter-beam interference: reuse among sufficiently isolated beams keeps low interference but offers limited capacity enhancement, while dense reuse causes serious interference. This inter-beam interference problem is similar to the inter-cell interference in terrestrial cellular networks like 3GPP LTE, and is even more severe with dense reuse, because all beams are generated by the satellite array antenna and the signal strength out of desired beam coverage is only suppressed by radiation pattern of beam antenna [2], in contrast to terrestrial cellular networks where cells are served by geographically separated base stations and interference is more effectively suppressed by propagation distance (path loss).

C. Loo *etc.* had reviewed the terrestrial-satellite channel modeling issue and proposed fundamental stochastic models including Rayleigh, Rician and log-normal for different scenarios [3] (Rician fading corresponds to the case a line-of-sight (LOS) component between satellite and terminal exists together with multi-path fading, which characteristic the open areas such as farm land or open fields). A general discussion on interference can be found in [2], in which the authors considered several kinds of satellite systems with different orbits, and modeled the large scale attenuation due to radiation pattern of spot beam; Rician fading effect was only briefly mentioned and approximately analyzed for the case of high signal-to-interference ratio (SIR). More complicated Rice-lognormal fading was studied in [4] for an inter-satellite interference case, but no closed-form solution was obtained due to its complexity. Interference at satellite in a satellite/terrestrial hybrid network, which originated from other terminals or base station using the same frequency, was considered in [5] with only free-space path loss but no fading effect. System capacity was investigated from the information theory point of view in [6] with Wyner's interference model.

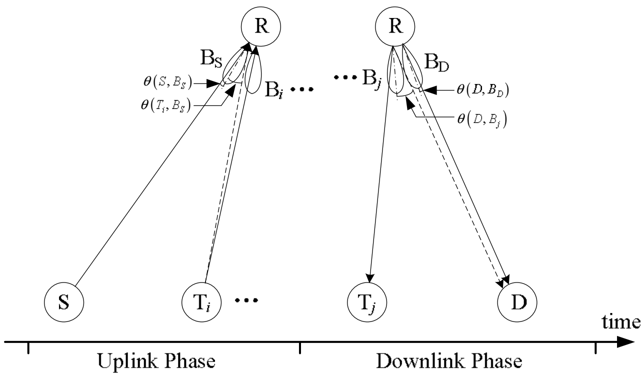
In this paper, we consider the end-to-end communication process between two terrestrial mobile terminals with the aid of a GEO satellite. With the presence of inter-beam interference, an interference-limited relaying model is established and analyzed to obtain the closed-form solutions for the significant performance metrics such as end-to-end outage probability and system achievable throughput. Distinguished from terrestrial cellular networks and multi-satellite systems, the downlink interference generated by adjacent co-frequency beams are not independent with the channel from serving beam antenna to destination terminal. Theoretic solutions turn out to match well with simulation results and validates our analysis. It also suggests that frequency reuse factor of 1/3 is an appropriate compromise between system throughput and transmission reliability.

II. Satellite Relaying Model

Consider two terrestrial mobile terminals S and D, S has message destined for D with the aid of satellite R. Denote $\mathsf{B_S}$, $\mathsf{B_D}$ as the serving beams of S and D. There are M terminals



(a) SMC scenario.



(b) Relaying model for the end-to-end communication in SMC system.

Fig. 1. Multi-beam SMC system model. Frequency reuse factor of 1/3 is taken for example in sub-figure (a). Solid lines denote the desired channels for transmission, while dashed lines the interference channels. The interference-limited relaying model is given by sub-figure (b).

$\mathsf{T}_1, \mathsf{T}_2, \cdots, \mathsf{T}_M$ that reuse the same channel as S, and M beams $\mathsf{B}_1, \mathsf{B}_2, \cdots, \mathsf{B}_M$ share the same band as B_D. The scenario is illustrated in Fig.1(a). Path loss between terminal X and beam Y is denoted by $L_{\mathsf{X},\mathsf{Y}}$, and fading coefficient of channel X-R is $h_{\mathsf{X},\mathsf{R}}$. Noise power is denoted by N_R, N_D at satellite and destination, and transmit signal strength is P_X at terminal X for uplink and P_Y at beam Y for downlink.

The three nodes $\mathsf{S}, \mathsf{R}, \mathsf{D}$ form a typical relay system. In the uplink phase when S transmits to R, there are also co-channel interferers $\mathsf{T}_i, i \in \{1, 2, \cdots, M\}$ that disturb the S-R link. If angle between the axis of beam B_S and the direction of R-T_i is not sufficiently large, the uplink interference is not well suppressed and the impairment is severe. In the downlink phase, R forwards the signal to D through beam B_D, while interference leaking from the side lobe of beams $\mathsf{B}_j, j \in \{1, 2, \cdots, N\}$ also reaches D.

End-to-end signal-to-interference and noise ratio (SINR, denoted by alphabet γ) for a relay system can be expressed by following formula [7]:

$$\gamma_{\text{e2e}} = \min(\gamma_{\mathsf{S},\mathsf{R}}, \gamma_{\mathsf{R},\mathsf{D}}) \tag{1}$$

where

$$\gamma_{\mathsf{S},\mathsf{R}} = \frac{P_\mathsf{S} L_{\mathsf{S},\mathsf{B}_\mathsf{S}} |h_{\mathsf{S},\mathsf{R}}|^2}{N_\mathsf{R} + \sum_{i=1}^{M} P_{\mathsf{T}_i} L_{\mathsf{T}_i,\mathsf{B}_S} |h_{\mathsf{T}_i,\mathsf{R}}|^2} \tag{2}$$

$$\gamma_{\mathsf{R},\mathsf{D}} = \frac{P_{\mathsf{B}_\mathsf{D}} L_{\mathsf{D},\mathsf{B}_\mathsf{D}} |h_{\mathsf{D},\mathsf{R}}|^2}{N_\mathsf{D} + \sum_{j=1}^{M} P_{\mathsf{B}_j} L_{\mathsf{D},\mathsf{B}_j} |h_{\mathsf{D},\mathsf{R}}|^2} \tag{3}$$

are the instantaneous SINRs of uplink and downlink, respectively. The expression (1) has been proved an exact solution for decode-and-forward (DF) relaying which asks for satellite's on-board processing capability, and also a well approximation for amplify-and-forward (AF) with a bent-pipe satellite.

A. Large Scale Path Loss

Free-space path loss between terrestrial terminal X and satellite beam Y is

$$L_{\mathsf{X},\mathsf{Y}} = G_\mathsf{X} G_\mathsf{Y} \left(\frac{\lambda}{4\pi \cdot d(\mathsf{X}, \mathsf{R})} \right)^2 \tag{4}$$

where $G_\mathsf{X}, G_\mathsf{Y}$ are the transmit/receive antenna gain, λ is the carrier wave length and $d(\mathsf{X}, \mathsf{R})$ is the distance between the terminal and satellite. A simple user terminal may equip an omnidirectional antenna and $G_\mathsf{X} = 0$dB; on the other hand, a key character of satellite antenna is its radiation pattern [9], of which antenna gain is determined by the angle θ between directions of beam Y's axis and $\overrightarrow{\mathsf{RX}}$. Thus $G_\mathsf{Y} = G(\theta(\mathsf{X}, \mathsf{Y}))$.

B. Small Scale Rician Fading

Rician fading is assumed for all channels throughout this paper. Note that channels between a particular terminal X and all the beam antennas undergo identical fading due to the fact that satellite size is rather tiny compare to the terrestrial-satellite distance, which is distinguished from terrestrial cellular networks [6]. Channel coefficient $h_{\mathsf{X},\mathsf{R}}$ has been normalized, *i.e.* $\nu_{\mathsf{X},\mathsf{R}}^2 + 2\sigma_{\mathsf{X},\mathsf{R}}^2 = 1$, where $\nu_{\mathsf{X},\mathsf{R}}, \sigma_{\mathsf{X},\mathsf{R}}$ correspond to the LOS and scatter strength, respectively, and $\kappa_{\mathsf{X},\mathsf{R}} = \nu_{\mathsf{X},\mathsf{R}}^2 / 2\sigma_{\mathsf{X},\mathsf{R}}^2$ is the Rice factor. Thus the cumulative distribution function (CDF) of channel gain is ([11], 2-1-124):

$$F_{|h_{\mathsf{X},\mathsf{R}}|^2}(x) = 1 - Q_1 \left(\sqrt{2\kappa_{\mathsf{X},\mathsf{R}}}, \sqrt{2(1+\kappa_{\mathsf{X},\mathsf{R}})x} \right) \tag{5}$$

where $Q_1(\cdot, \cdot)$ is the Marcum Q-function.

III. Performance Analysis

System outage is declared when current supportable end-to-end rate falls below the transmit rate, thus the outage probability is given by

$$\begin{aligned} \mathcal{P}_{\text{out}}(r) &= \Pr\left[\log_2\left(1 + \gamma_{\text{e2e}}\right) < r\right] \\ &= 1 - \left[1 - F_{\gamma_{\mathsf{S},\mathsf{R}}}(\gamma_{\text{th}})\right]\left[1 - F_{\gamma_{\mathsf{R},\mathsf{D}}}(\gamma_{\text{th}})\right] \end{aligned} \tag{6}$$

where r (in bits/symbol or bits/Sec/Hz) represents the transmit rate of source S, and $\gamma_{\text{th}} = 2^r - 1$ corresponds to the receive SINR threshold for successful decoding. System achievable throughput with rate r is defined as the successfully delivered proportion of data stream:

$$T(c, r) = cW \cdot \frac{r}{2}\left[1 - \mathcal{P}_{\text{out}}(r)\right] \tag{7}$$

where W is system bandwidth, c is the frequency reuse factor; r is divided by 2 since a relaying process needs two temporal phases.

A. Uplink Phase

Receive SINR in the uplink phase is given by (2). The first step is to analyze the co-channel interference from terminals served by other beams that reuse the same frequency. The distribution of the sum of Rician interference power is too complicated to get an exact closed-form solution. Since channel coefficient of Rician fading is actually complex Gaussian random variable with non-zero mean value, an approximate approach proposed in [10] can be utilized here and concluded as following lemma.

Lemma 1: For an n-length complex Gaussian vector $\mathbf{x}$ with mean vector μ_x and covariance matrix $\mathbf{R}_x$, *i.e.* $\mathbf{x} \sim \mathcal{CN}(\mu_x, \mathbf{R}_x)$, $\mathbf{x}^\dagger \mathbf{x}$ can be approximated by a chi-square random variable $\alpha_x \chi^2(\ell_x)$ with degrees of freedom ℓ_x and scaling factor α_x, where the parameters are determined by:

$$\ell_x = \frac{2\left[\sum_{k=1}^{n} \lambda_{x,k}\left(1+|\tilde{\mu}_{x,k}|^2\right)\right]^2}{\sum_{k=1}^{n} \lambda_{x,k}^2\left(1+2|\tilde{\mu}_{x,k}|^2\right)} \tag{8}$$

$$\alpha_x = \frac{\left[\sum_{k=1}^{n} \lambda_{x,k}^2\left(1+2|\tilde{\mu}_{x,k}|^2\right)\right]^2}{2\sum_{k=1}^{n} \lambda_{x,k}\left(1+|\tilde{\mu}_{x,k}|^2\right)} \tag{9}$$

where

$$\Lambda_x = \mathrm{diag}\{\lambda_{x,1}, \cdots, \lambda_{x,n}\} = \mathbf{U}_x^\dagger \mathbf{R}_x \mathbf{U}_x$$

is the diagonal matrix of the eigenvalues of $\mathbf{R}_x$, $\mathbf{U}_x$ consists of corresponding eigenvectors in the columns, and

$$\tilde{\mu}_x = [\tilde{\mu}_{x,1}, \cdots, \tilde{\mu}_{x,n}]^T = \Lambda_x^{-1/2}\mathbf{U}_x^\dagger \mu_x$$

is the normalized mean vector.

Theorem 1: Signify interference vector

$$\xi = \left[\sqrt{\frac{P_{T_1} L_{T_1,B_S}}{N_R}} h_{T_1,R}\ , \cdots, \sqrt{\frac{P_{T_M} L_{T_M,B_S}}{N_R}} h_{T_M,R}\right]^T \tag{10}$$

in which the elements represent the interference from other beams. The approximate CDF expression of $\gamma_{\mathsf{S,R}}$ is derived as (11), in which $\kappa_1 = \kappa_{\mathsf{S,R}}$ and $\tilde{\gamma}_{\mathsf{R,th}} = N_{\mathsf{R}}\gamma_{\mathsf{th}}/(P_{\mathsf{S}} G_{\mathsf{S,B_S}})$ are signified for simplicity, ℓ_ξ, α_ξ are calculated according to Lemma 1 for ξ.

$$\begin{aligned} F_{\gamma_{S,R}}(\gamma_{\mathsf{th}}) \approx 1 - &\frac{\exp\left(-\kappa_1 - (1+\kappa_1)\tilde{\gamma}_{\mathsf{R,th}}\right)}{\Gamma(\ell_\xi/2)(2\alpha_\xi)^{\ell_\xi/2}} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \\ &\times \frac{\kappa_1^{m+n}[(1+\kappa_1)\tilde{\gamma}_{\mathsf{R,th}}]^n}{n!(m+n)!} \sum_{p=0}^{n} \binom{n}{p} \Gamma\left(p+\frac{\ell_\xi}{2}\right) \\ &\times \left[(2\alpha_\xi)^{-1} + (1+\kappa_1)\tilde{\gamma}_{\mathsf{R,th}}\right]^{-(p+\ell_\xi/2)} \end{aligned} \tag{11}$$

Proof: Please refer to Appendix A. ■

B. Downlink Phase

As demonstrated in Sec. II B, the downlink SINR behavior of SMC system is distinct from terrestrial cellular network, as the fading states of signal link and interference links are identical. Thus the CDF of downlink SINR is given by a new formula in following theorem.

Theorem 2: The CDF of downlink received SINR is given by

$$F_{\gamma_{\mathsf{R,D}}}(\gamma_{\mathsf{th}}) = \begin{cases} 1 - Q_1\left(\sqrt{2\kappa_2}, \sqrt{\frac{2(1+\kappa_2)\gamma_{\mathsf{th}}}{a-b\gamma_{\mathsf{th}}}}\right) & \text{, if } \gamma_{\mathsf{th}} < \frac{a}{b} \\ 1 & \text{, otherwise} \end{cases} \tag{12}$$

where $\kappa_2 = \kappa_{D,R}$, with notations

$$a = \frac{P_{\mathsf{B_S}} L_{\mathsf{D,B_S}}}{N_{\mathsf{D}}} \tag{13}$$

$$b = \frac{\sum_{j=1}^{M} P_{\mathsf{B}_j} L_{\mathsf{D,B}_j}}{N_{\mathsf{D}}} \tag{14}$$

Proof: Please refer to Appendix B. ■

IV. Numeric Results and Discussion

Monte Carlo simulations are carried out to manifest the system performance under interference and fading, as well as to validate our theoretic analysis. Parameters utilized in simulation are given in Table I (some can be found in [5]) unless otherwise specified. Beam antenna radiation pattern is based on that of single feed circular antenna ([9], Fig.1). The simulation scenario is given by Fig.1(a), and all terminals' (*i.e.* $\mathsf{S}, \mathsf{D}, \mathsf{T}_1, \cdots, \mathsf{T}_M$) location are randomly generated for 100 times in all hexagon beam cells and the performance metrics are averaged and given as follows.

For an intuitive insight, Table II gives the average interference to noise ratio (I/N) with different terminal position (beam center or edge) and frequency reuse ($c = 1, 1/3, 1/7$). Uplink interference strength is equal for arbitrary source terminal location. Downlink I/N value is relatively higher because transmit power of satellite is configured greater than terminal power, while noise at terminal is lower than that in

TABLE I
Parameters of SMC system.

parameter	value
beam radius	200km
terminal transmit power $P_{\mathsf{S}}, P_{\mathsf{D}}, P_{\mathsf{T}_i}$ (typical)	26dBm
satellite transmit power $P_{\mathsf{B_S}}, P_{\mathsf{B}_j}$ (typical)	30dBm
beam antenna gain $G(0)$ (peak value)	54.4dBi
terminal antenna gain	0dBi
carrier frequency of uplink	1995MHz
carrier frequency of downlink	2185MHz
system bandwidth of uplink/downlink W	30MHz
single user bandwidth	180kHz
noise temperature at satellite	27.97dB-K
noise temperature at terminal	290K
Rice factor $\kappa_1, \kappa_2, \kappa_{T_i,R}$	4.8794

TABLE II
I/N OF DIFFERENT CASES.

I/N (dB)	Uplink	Downlink	
		Center	Edge
c=1	9.3	14.9	16.7
c=1/3	-2.4	3.3	6.1
c=1/7	-3.3	3.3	3.3

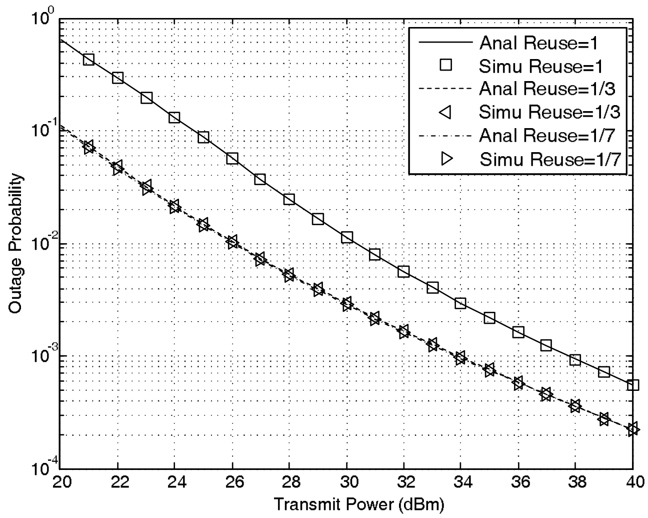


Fig. 2. Outage probability vs. transmit power. Target rate is $r = 0.5$bps/Hz.

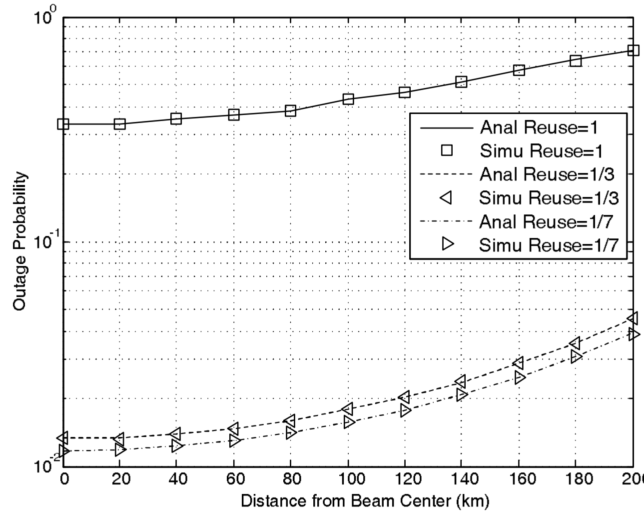


Fig. 3. Outage probability vs. distance from the center of beam cell.

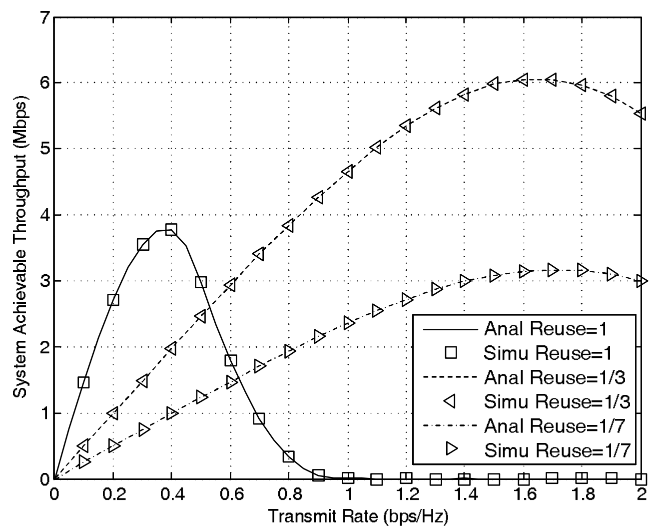


Fig. 4. System achievable throughput vs. transmit rate.

the space. $c = 1$ corresponds to the most severe interference; with other reuse factors, interference becomes not that drastic, which indicates that it is not suitable to neglect the noise in multi-beam satellite system. In contrast, the average receive signal-to-noise (SNR) of uplink is 0dB at the beam center, -3dB at the edge; in downlink it is 15.5dB at the center and 12.5dB at the edge.

In Fig.2 the outage behavior versus signal transmit power is shown, where we suppose $P_{\mathsf{S}} = P_{\mathsf{B}_{\mathsf{D}}}$, and all interference strength ($P_{\mathsf{T}_i}, i = 1, \cdots, M$, $P_{\mathsf{B}_j}, j = 1, \cdots, N$) is fixed 20dBm. Radiation pattern of beam antenna determines that inter-beam interference is strong in adjacent cells, but is attenuated rapidly in farther ones. As a result, reuse factors of 1/3 and 1/7 have very close outage performance, which are more than 4dB better than that with full frequency reuse.

In practice SINR of one communication process cannot be raised infinitely from the network point of view, otherwise the other simultaneous transmissions are all interfered and blocked. In general cases all power values of terminals or beams are equal, and the outage behavior is investigated for diverse terminal location in Fig.3 with parameters configured as Table I and $r = 0.1$bps/Hz. The result suggests that outage probability with full frequency reuse may be the most unacceptable even for error-tolerant real time application (the outage probability exceeds 30% even at the beam center).

Fig.4 illustrates the performance of system achievable throughput, and the variable here is transmit rate of source signal, or equivalently number of bits one symbol can bear, determined by coding rate and modulation scheme adopted. Throughput first increases with the ascent of transmit rate in relatively low r regime, where the rate is supportable in most cases; when r gets sufficiently high, the performance deteriorates because outage event occurs frequently, leading to lower percentage of effective transmission. Thus there exists a rate adaption issue of optimizing r, which is however beyond our discussion in this paper. Although reuse factor 1/3 doesn't make full use of the system spectrum in every single beam, it achieves the highest maximal throughput at about 1.6 bps/Hz, as it efficiently suppresses the inter-beam interference compare with full reuse while saves a half resource in contrast to reuse factor 1/7.

V. CONCLUSIONS

We model the end-to-end communication in multi-beam SMC system by interference-limited relay channel with Rician fading in this paper, and study performance of outage as well as throughput in both theoretic and simulation ways. Impacts of some significant variables such as transmit power and rate, terminal location, as well as frequency reuse schemes, are derived and exhibited, providing meaningful guidance for system evaluation and design. Reuse factor 1/3 is recommended by numeric results based on some practical satellite parameters, which gets to a satisfactory compromise between system throughput and transmission reliability. However, the study on orthogonal frequency planning is just an initial work, in which reuse factor is the only key parameter to be designed. Our future research subject is to innovate upon interference coordination methods to further enhance SMC capacity.

ACKNOWLEDGEMENT

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APPENDIX A
PROOF OF THEOREM 1

According to Lemma 1, the total strength of uplink interference has an approximate probability density function (PDF) as follows:

$$f_{\xi^\dagger\xi}(y) = \frac{y^{\ell_\xi/2-1}}{\Gamma(\ell_\xi/2)(2\alpha_\xi)^{\ell_\xi/2}} \exp\left(-\frac{y}{2\alpha_\xi}\right) \tag{15}$$

Denote

$$\eta = \sqrt{\frac{P_{\mathsf{S}} L_{\mathsf{S},\mathsf{B}_\mathsf{S}}}{N_{\mathsf{R}}}} h_{\mathsf{S},\mathsf{R}} \tag{16}$$

the SINR of the uplink can be rewritten $\gamma_{S,R} = |\eta|^2/(1+\xi^\dagger\xi)$ and its CDF is calculated as follows

$$\begin{aligned} F_{\gamma_{\mathsf{S},\mathsf{R}}}(\gamma_{\text{th}}) &= \int_0^{+\infty} f_{\xi^\dagger\xi}(y) F_{|\eta|^2}(\tilde{\gamma}_{\mathsf{R},\text{th}}(1+y))\,\mathrm{d}y \\ &= 1 - \int_0^{+\infty} \frac{y^{\ell_\xi/2-1}}{\Gamma(\ell_\xi/2)(2\alpha_\xi)^{\ell_\xi/2}} \exp\left(-\frac{y}{2\alpha_\xi}\right) \\ &\quad \times Q_1\left(\sqrt{2\kappa_1}, \sqrt{2(1+\kappa_1)\tilde{\gamma}_{\mathsf{R},\text{th}}(1+y)}\right) \mathrm{d}y \end{aligned} \tag{17}$$

Use the series expansion of Marcum Q-function ([11], Eq.(2-1-123))

$$\begin{aligned} Q_1(x,y) &= \exp\left(-\frac{x^2+y^2}{2}\right) \sum_{m=0}^{\infty} \left(\frac{x}{y}\right)^m I_m(xy) \\ &= \exp\left(-\frac{x^2+y^2}{2}\right) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{x^{2(m+n)} y^{2n}}{2^{m+2n} n!(m+n)!} \end{aligned}$$

where $I_m(\cdot)$is the modified Bessel function of the first kind with order m, (17) can be further expanded as

$$\begin{aligned} F_{\gamma_{\mathsf{S},\mathsf{R}}}(\gamma_{\text{th}}) &= 1 - \frac{\exp(-\kappa_1 - (1+\kappa_1)\tilde{\gamma}_{\mathsf{R},\text{th}})}{\Gamma(\ell_\xi/2)(2\alpha_\xi)^{\ell_\xi/2}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \\ &\quad \times \frac{\kappa_1^{m+n} [(1+\kappa_1)\tilde{\gamma}_{\mathsf{R},\text{th}}]^n}{n!(m+n)!} \int_0^{+\infty} (1+y)^n y^{\frac{\ell_\xi}{2}-1} \\ &\quad \times \exp\left[-\left(\frac{1}{2\alpha_\xi} + (1+\kappa_1)\tilde{\gamma}_{\mathsf{R},\text{th}}\right) y\right] \mathrm{d}y \end{aligned} \tag{18}$$

take the binomial expansion of $(1+y)^n$ and the integration form of gamma function, final expression of (11) is obtain.

APPENDIX B
PROOF OF THEOREM 2

The probability of $\gamma_{\mathsf{R},\mathsf{D}} < \gamma_{\text{th}}$ is rewritten as

$$\begin{aligned} F_{\gamma_{\mathsf{R},\mathsf{D}}}(\gamma_{\text{th}}) &= \Pr\left(\frac{a|h_{\mathsf{D},\mathsf{R}}|^2}{1+b|h_{\mathsf{D},\mathsf{R}}|^2} < \gamma_{\text{th}}\right) \\ &= \Pr\left((a - b\gamma_{\text{th}})|h_{\mathsf{D},\mathsf{R}}|^2 < \gamma_{\text{th}}\right) \end{aligned} \tag{19}$$

1) if $a - b\gamma_{\text{th}} \leqslant 0$, *i.e.* $\gamma_{\text{th}} \geqslant \frac{a}{b}$, the probability above degenerates to

$$\Pr\left(|h_{\mathsf{D},\mathsf{R}}|^2 > \frac{\gamma_{\text{th}}}{a - b\gamma_{\text{th}}}\right) = \Pr(|h_{\mathsf{D},\mathsf{R}}|^2 \geqslant 0) = 1 \tag{20}$$

2) else, $a - b\gamma_{\text{th}} > 0$, *i.e.* $\gamma_{\text{th}} < \frac{a}{b}$, the probability is

$$\Pr\left(|h_{\mathsf{D},\mathsf{R}}|^2 < \frac{\gamma_{\text{th}}}{a - b\gamma_{\text{th}}}\right) = F_{|h_{\mathsf{D},\mathsf{R}}|^2}\left(\frac{\gamma_{\text{th}}}{a - b\gamma_{\text{th}}}\right) \tag{21}$$

The two results above leads to Theorem 2.

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