# Low-Complexity Spectral Precoding for Rectangularly Pulsed OFDM

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Abstract—In the context of cognitive radio, this paper proposes a simple spectral precoding for orthogonal frequency-division multiplexing (OFDM) to suppress its out-of-band radiation for a better adjacent-channel coexistence. Additionally, the proposed scheme is also aimed to improve the in-band subcarrier notching to efficiently utilize the fragmented spectrum. Through the proper system modeling and optimal selection for the precoder's weights, it is demonstrated to achieve a much better spectral compactness than the conventional OFDM, meanwhile possess only a trivial cost for computational complexity. Compared with other OFDM variant schemes, it is thereby concluded that the proposed scheme can be one of the promising candidates for the physical layer of future's cognitive radio devices.

## I. Introduction

In recent years, the mobile Internet has been driving a rapid growth in the demand for the radio spectral resource. Meanwhile, a large portion of spectrum among the licensed bands is underutilized. By sensing and opportunistically accessing spectral holes in temporal, frequency, and spatial domains, the concept of cognitive radio (CR) [1], [2] and dynamic spectrum access (DSA) [3] has been strongly proposed and supported by both the academia and industry.

Orthogonal frequency-division multiplexing (OFDM) [4], as one prevailing multicarrier technique, has been extensively applied to wired/wireless broadband transmission systems. However, despite its robustness in multi-path fading channels and an efficient implementation using discrete Fourier transform (DFT), the rectangularly-pulsed OFDM signal suffers from severe spectral power leakage that could potentially lead to high adjacent-channel interference to other systems. In addition, to avoid the co-channel collision in the overlay CR systems, a modified multicarrier scheme referred to as noncontiguous OFDM (NC-OFDM) has been proposed in [5]. A group of discontinuous spectrum fragments can be aggregated for an unified broadband transmission by turning off the OFDM subcarriers, which are overlapping with other narrowband licensed users (LU). Besides this concept of NC-OFDM where perfect subcarrier-nulling is preferred, CR users are also under rather strict regulations on the out-of-band (OOB) power radiation. Therefore, to enhance for better coexistence, many research efforts (such as [6]-[8]) have being made to improve the spectral compactness for the conventional OFDM.

Recently, the data-independent approaches called *spectral precoding* [9] are proposed. The state-of-the-art precoding

scheme, called *N-Continuous OFDM* [10], [11] can achieve very high sidelobe suppression by making the signal and its derivatives continuous in the border of consecutive OFDM symbols. However, the matrix operations required in both transmitter and receiver will also bring a considerable computation burden to implementation.

In this paper, a simple but effective spectral precoding is proposed to suppress the sidelobe emission for both OFDM and NC-OFDM signals. Through weighting each data symbol by M predefined precoding coefficients and mapping these coded symbols to M adjacent subcarriers prior to DFT modulation, the proposed scheme can achieve a suppression gain of 15dB and 30dB with  $M{=}2$  and 3, respectively. Compared with the existing precoding schemes, the computational complexity in terms of complex multiplications has been dramatically reduced from  $O(N^2)$  to O(N), where N denotes the length of DFT. In practice, it will be a promising transmission scheme for the complexity-constrainted systems operating in television white space (TVWS), such as long-term evolution (LTE) Femto base stations (BS), IEEE 802.11ah super-WiFi access points (AP) and IEEE-802.15.4m-based ZigBee nodes.

This paper is organized as follows. The system model of spectral precoding for OFDM and NC-OFDM is introduced in Section II. The proposed scheme including the precoder and the decoder are described in Section III. In Section IV, the computational complexity is analyzed and compared with other schemes. The estimation of power spectral density (PSD) for different schemes are presented in Section V, and some concluding remarks are given in Section VI.

# II. SYSTEM MODEL

The continuous-time baseband transmitted signal of OFDM can be written as

$$s(t) = \sum_{k=-\infty}^{+\infty} \sum_{n=0}^{N-1} x_{n,k} p_{n,k}(t), \tag{1}$$

where  $x_{n,k}$  denotes the data symbol transmitted on nth subcarrier during kth symbol interval, and N is the length of DFT. Derived from the time-frequency translation of the prototype filter p(t), the synthesis filters  $p_{n,k}(t)$  in (1) can be given by

$$p_{n,k}(t) = p(t - kT)e^{j2\pi n f_0(t - kT)},$$
(2)

where  $j=\sqrt{-1}$ , the inter-subcarrier frequency spacing  $f_0$ , and the symbol duration T. To simplify the equalization in time-dispersive channels, a cyclic prefix (CP) with duration of  $T_{cp}$  is added before each effective OFDM symbol with duration of  $T_0$ , thereby  $T=T_{cp}+T_0$ . The prototype filter p(t) in the OFDM system is a rectangular pulse, i.e.

$$p(t) = \begin{cases} \frac{1}{T}, & -T/2 \leqslant t < T/2 \\ 0, & otherwise \end{cases}$$
 (3)

The Fourier transform of a single unmodulated subcarrier described in (2) can thus be derived as

$$S_{n,k}(f) = \int_{-\infty}^{+\infty} p_{n,k}(t)e^{-j2\pi ft}dt$$

$$= \operatorname{sinc}(T(f - nf_0))e^{-j2\pi kfT},$$
(4)

where  $sinc(x)=sin(\pi x)/\pi x$ . Similarly, the Fourier transform of OFDM signal in (1) can be expressed as

$$S(f) = \sum_{k=-\infty}^{+\infty} \sum_{n=0}^{N-1} x_{n,k} S_{n,k}(f).$$
 (5)

In terms of [12], the power spectra of a band-limited signals such as the OFDM signal can be estimated by observing a truncated version during the time  $[-\kappa T, \kappa T]$  with a large number of  $\kappa$ . Assuming the Fourier transform of the truncated signal is denoted by  $S_{\kappa T}(f)$  and the symbols carried on different subcarriers are independent, estimated PSD can hence be formulated as:

$$P(f) = \lim_{\kappa \to \infty} \frac{\mathbb{E}\left[\left|S_{\kappa T}(f)\right|^{2}\right]}{\kappa T}$$

$$= \frac{1}{\kappa T} \sum_{k=-\infty}^{\kappa} \sum_{n=0}^{N-1} |x_{n,k}|^{2} |S_{n,k}(f)|^{2}, \tag{6}$$

where  $\mathbb{E}[\cdot]$  denotes the ensemble average. Suppose data symbols  $d_{p,k},\ p{=}0,...,P{-}1$  for kth OFDM symbol can be collected in a data vector

$$\mathbf{d}_{k} = [d_{0,k}, \dots, d_{P-1,k}]^{T}, \tag{7}$$

where superscript  $^{T}$  denotes the transpose.

According to [9], by determining a precoding matrix denoted by G with  $N \times P$  optimized complex-valued coefficients  $g_{n,p}$ , the spectral sidelobe of OFDM signals is possible to be deeply suppressed. In this paper, the spectral precoding by the matrix G is formulated as:

$$\mathbf{x}_k = \mathbf{G}\mathbf{d}_k,\tag{8}$$

where  $\mathbf{x}_k = [x_{0,k}, x_{1,k}, ..., x_{N-1,k}]^T$  denotes an  $N \times 1$  vector carrying coded data symbols. After the precoding process,  $x_{n,k}$  is mapped to the corresponding nth subcarrier during kth OFDM symbol, and followed by the DFT operation and insertion of CP to form the baseband OFDM signal with the desired spectral shape.

In particular, if the matrix G is set as an N-dimensional identity matrix  $I_N$ , the whole precoding is equivalent to the

conventional unprecoded OFDM scheme. Further, suppose consecutive subcarriers indexed from  $(\alpha+1)$  to  $(\alpha+\beta)$  are overlapped with other narrow-band licensed users, the following diagonal matrix  $\mathbf{G}_{nc}$  is equivalent to the effect of subcarrier-nullifying in NC-OFDM signals, i.e.

$$\mathbf{G}_{nc} = \begin{bmatrix} \mathbf{I}_{\alpha} \\ \mathbf{0}_{\beta} \\ \mathbf{I}_{N-\alpha-\beta} \end{bmatrix}, \tag{9}$$

where  $I_x$  denotes x-dimensional identity matrix, and  $\mathbf{0}_x$  represents  $x \times x$  zero matrix.

## III. THE PROPOSED SCHEME

In this paper, a simple spectral precoding is proposed to shape the signal spectrum by make sidelobe self-cancellation among a group of adjacent subcarriers. All effective OFDM subcarriers are divided into P non-overlapping groups, each of which consists of M adjacent subcarriers. By weighting a data symbol with M precoding coefficients and mapping these coded symbols to M adjacent subcarriers prior to the DFT modulation, the proposed scheme can achieve considerable suppression with a much lower computational complexity. Note that each group is used to carry coded symbols weighted from a single data symbol instead of modulating M different symbols in the conventional OFDM system.

A normalized weight vector is defined as

$$\mathbf{g}_{M} = [w_{1}, w_{2}, ..., w_{M}]^{T}, \tag{10}$$

where the norm  $\|\mathbf{g}_M\|_2 = \sqrt{|w_1|^2 + ... + |w_M|^2} = 1$ . Each data symbol  $d_{p,k}$  in (7) is weighted by coefficients in  $\mathbf{g}_M$  into M coded symbols, i.e.

$$\mathbf{x}_{k}^{p} = \mathbf{g}_{M} d_{p,k} = [w_{1} d_{p,k}, w_{2} d_{p,k} ..., w_{M} d_{p,k}]^{T},$$
(11)

which mapped upon pth subcarrier group. The symbol correlation within the same group of adjacent subcarriers are introduced by the weighting, thus P(f) in (12) should be rewritten as:

$$P(f, \mathbf{g}_{M}) = \lim_{\kappa \to \infty} \frac{\mathbb{E}\left[|S_{\kappa T}(f)|^{2}\right]}{\kappa T}$$

$$= \frac{1}{\kappa T} \sum_{k=-\kappa}^{\kappa} \sum_{p=0}^{P-1} \left| \sum_{m=1}^{M} w_{m} d_{p,k} S_{(pM+m-1),k}(f) \right|^{2}$$

$$(12)$$

To measure the performance of sidelobe suppression, we define a parameter  $D(\mathbf{g}_M)$  as the difference between the peak and the valley of PSD in decibel:

$$D(\mathbf{g}_{M}) = \max [10 \lg(P(f, \mathbf{g}_{M}))] - \min [10 \lg(P(f, \mathbf{g}_{M}))]$$
(13)

The peak of PSD corresponds to the transmitted power, while the valley indicates the out-of-band spectral notch, thereby  $D(\mathbf{g}_M)$  can directly reflect the suppression ability. Hence, the optimal weight vector  $\hat{\mathbf{g}}_M$  can be determined by the following optimization problem, i.e.

$$\hat{\mathbf{g}}_{M} = \arg\max_{\mathbf{g}_{M}} D(\mathbf{g}_{M}) \tag{14}$$

The numerical search for the optimal value was done by computer. In the first cycle, the phase of  $\theta$  is set as  $\theta \in [0, 2\pi]$  and the amplitude of r=1, where the weights in (10) can be written as  $w_m=re^{j\theta}$ . The same calculation are also dong by reset r, such as r=1.5 or r=0.5. It can be observed that the optimal weight vector in case of M=2 is

$$\hat{\mathbf{g}}_2 = \frac{1}{\sqrt{2}} [1.0, -0.9816 - 0.1908j]^T.$$
 (15)

Meanwhile, the optimal vector for M=3 can be given by

$$\hat{\mathbf{g}}_{3} = \frac{1}{\sqrt{6}} \left[ -0.8290 - 0.5592i, \\ 1.3894 + 1.4386i, -0.5299 - 0.8480i \right]^{T}.$$
(16)

## A. Precoder

In essence, the spectral precoding in (8) introduces the correlation among all data symbols, and the precoding granularity is regarded as N. Each element in  $\mathbf{x}_k$  contains ingredient from all data symbols in  $\mathbf{d}_k$ , i.e.,

$$x_{n,k} = \sum_{p=0}^{P-1} g_{n,p} d_{p,k}.$$
 (17)

In (11), only M coded symbols within each subcarrier group are correlated. The precoding granularity is reduced to M, which is far less than the length of DFT  $(M \ll N)$ . The precoding matrix can be simplified to a *block* diagonal matrix consisting of the vector  $\mathbf{g}_M$  as the following (18), instead of a full matrix  $\mathbf{G}$  with  $N \times P$  nonzero weights.

$$\mathbf{G}_{M} = \begin{bmatrix} \mathbf{g}_{M} & & \\ & \ddots & \\ & & \mathbf{g}_{M} \end{bmatrix}_{N \times P} \tag{18}$$

In case of M=2, the concrete precoding matrix is

$$\mathbf{G}_{2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \hat{w}_{2} \\ \ddots \\ 1 \\ \hat{w}_{2} \end{bmatrix}_{N \times \frac{N}{2}}$$
 (19)

where the optimal weights of  $\hat{w}_2 = -0.9816 - 0.1908j$  in terms of (15). To provide stronger sidelobe suppression, the number of subcarriers in each group can be increased to M = 3 or more. Similarly, we can use the normalized weight vector  $\mathbf{g}_3$  in (16) to construct a precoding matrix  $\mathbf{G}_3 = diag[\mathbf{g}_3, ..., \mathbf{g}_3]$ .

Moreover, the proposed scheme is well-suited for the fragmented spectrum bands in NC-OFDM systems. Its precoding matrix can be easily obtained by cascading the matrix in (9) and (18) as follows.

$$\mathbf{G}_{nc,M} = \mathbf{G}_{nc}\mathbf{G}_{M} \tag{20}$$

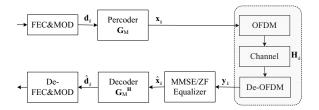


Fig. 1. Schematic diagram of the transmitter and receiver in the proposed spectral precoding scheme.

#### B. Decoder

As one linear approach, the decoding matrix corresponding to  $\mathbf{G}_M$  can be defined by

$$\mathbf{G}_{M}^{H} = \begin{bmatrix} \mathbf{g}_{M}^{H} & & \\ & \ddots & \\ & \mathbf{g}_{M}^{H} \end{bmatrix}_{P \times N}$$
 (21)

where  $\mathbf{g}_{M}^{H} = [w_{1}^{*}, w_{2}^{*}, ..., w_{M}^{*}]$ , and superscript  $^{H}$  and  $^{*}$  denote the Hermitian and the conjugate transpose, respectively. It can be easily derived that:

$$\mathbf{G}_{M}^{H}\mathbf{G}_{M} = \begin{bmatrix} \mathbf{g}_{M}^{H}\mathbf{g}_{M} & & \\ & \ddots & \\ & & \mathbf{g}_{M}^{H}\mathbf{g}_{M} \end{bmatrix}_{P \times P} = \mathbf{I}_{P}$$
 (22)

where  $\mathbf{g}_{M}^{H}\mathbf{g}_{M}=1$  due to the normalization of weight vectors.

The schematic diagram of the transceiver is illustrated in Fig.1. At the transmitter side, the data bits are first forward-error-correction (FEC) coded and modulated (MOD) into symbol blocks denoted by  $\mathbf{d}_k$ . Through the precoder, the coded symbols  $\mathbf{x}_k = \mathbf{G}_M \mathbf{d}_k$  are acquired. At the receiver side, the received signals are denoted by  $\mathbf{y}_k = [y_{0,k},...,y_{N-1,k}]$ . Thus, the system model can be expressed by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \tag{23}$$

where  $\mathbf{H}_k$  is the channel response matrix during the kth OFDM symbol, and  $\mathbf{z}_k = [z_{0,k},...,z_{N-1,k}]^T$  is the vector of addition white Gaussian noise (AWGN). The coded symbols can be estimated via zero-forcing (ZF) detection as:

$$\hat{\mathbf{x}}_k = (\mathbf{H}_k^H \mathbf{H}_k)^{-1} \mathbf{H}_k^H \mathbf{y}_k \tag{24}$$

Then, the data symbols can be restored by  $\hat{\mathbf{d}}_k = \mathbf{G}_M^H \hat{\mathbf{x}}_k$ . In summary, by combining (22), (23), (24), we can derived that

$$\hat{\mathbf{d}}_{k} = \mathbf{G}_{M}^{H} \hat{\mathbf{x}}_{k}$$

$$= \mathbf{G}_{M}^{H} \left[ (\mathbf{H}_{k}^{H} \mathbf{H}_{k})^{-1} \mathbf{H}_{k}^{H} \mathbf{y}_{k} \right]$$

$$= \mathbf{G}_{M}^{H} \left[ (\mathbf{H}_{k}^{H} \mathbf{H}_{k})^{-1} \mathbf{H}_{k}^{H} (\mathbf{H}_{k} \mathbf{G}_{M} \mathbf{d}_{k} + \mathbf{z}_{k}) \right]$$

$$= \mathbf{d}_{k} + \mathbf{z}_{k}^{\prime} \tag{25}$$

where  $\mathbf{z}_k' = \mathbf{G}_M^H (\mathbf{H}_k^H \mathbf{H}_k)^{-1} \mathbf{H}_k^H \mathbf{z}_k$  is the received noise vector after the spectral decoding. Consequently, it is verified that the data symbols can be properly restored according to the proposed decoding matrix  $\mathbf{G}_M^H$ .

TABLE I
COMPUTATIONAL COMPLEXITY IN TERMS OF THE COMPLEX
MULTIPLICATION

Methods	Complexity
OFDM	$\frac{N}{2}\log_2 N$
The proposed scheme	$\begin{array}{c} \frac{N}{2}\log_2 N \\ \frac{N}{2}\log_2 N + N \\ \frac{N}{2}\log_2 N + 2N^2 \end{array}$
N-Continuous Memory [10]	$\frac{N}{2}\log_2 N + 2N^2$
N-Continuous Memoryless [11]	$\frac{\frac{N}{2}\log_2 N + N^2}{2}$

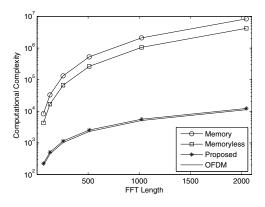


Fig. 2. Computational complexity as a function of the FFT length N, in terms of the number of complex multiplication per OFDM symbol duration.

## IV. COMPLEXITY COMPARISON

In this section, the computational complexity of different spectral precoding schemes have been comparatively investigated. In terms of the number of required complex multiplication (CM), the complexity expressions for the generation of each OFDM symbol at the transmitter's side has been taken into account. The results are summarized in Table I.

The conventional OFDM scheme requires  $\frac{N}{2}\log_2 N$  CM in case that N is a power of 2 and the fast Fourier transform (FFT) is used. The precoding in [10], called *memory N-Continuous OFDM*, requires a feedback operation in the transmitter. The data symbols are processed in terms of

$$\bar{\mathbf{x}}_k = \mathbf{G}_0 \mathbf{x}_k + \mathbf{G}_1 \bar{\mathbf{x}}_{k-1},\tag{26}$$

where  $\mathbf{G}_i, i \in \{0,1\}$  is  $N \times N$  precoding matrix,  $\mathbf{x}_k$  denotes the symbol vector in kth OFDM symbol prior to the precoding, and  $\bar{\mathbf{x}}_{k-i}$  denotes the coded symbol vectors. Without taking into account the FFT module, it it easy to get that the number of CM required per OFDM symbol is  $2N^2$ . Concerning the precoding formulated by  $\bar{\mathbf{x}}_k = \mathbf{G}\mathbf{x}_k$  in [11], called memoryless N-Continuous OFDM, the number of required CM is  $\frac{N}{2}\log_2 N + N^2$  for generating an OFDM symbol. In the proposed scheme, N CM is required for weighting all subcarriers per OFDM symbol since  $\mathbf{G}_M$  is a block diagonal matrix with large number of 0. Thus, the total complexity amounts to  $\frac{N}{2}\log_2 N + N$ , as given in Table I.

The complexity expressions in Table I versus the length of DFT have been plotted in Fig.2. It can be seen that the number of CM required by the existing precoding follows  $O(N^2)$ , resulting in a considerably complexity exceeding that of conventional OFDM by nearly up to 3 orders of

TABLE II SIMULATION PARAMETERS

Parameters	Values
Sampling frequency	$30.72 \mathrm{MHz}$
FFT length	2048
CP length	144
Bandwidth	10MHz
Number of subcarriers	600
Subcarrier spacing	15KHz
Modulation	16 QAM
Channel coding	Convolutional codes, rate=1/2
	generator=[1167 1545]
Channel model	3GPP Vehicular Channel A

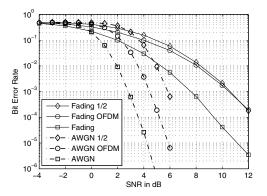


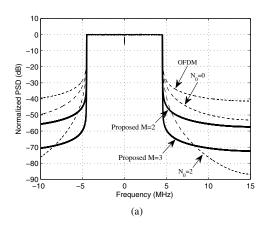
Fig. 4. BER performance comparison between the conventional OFDM and the proposed scheme with half or full transmitted power, in AWGN and fading channels

magnitude. On the basis of  $\frac{N}{2}\log_2 N$  CM in the conventional OFDM signal, the proposed spectral precoding lead to an extra complexity of N per OFDM symbol, which is negligible. As illustrated by Fig.2, its complexity is almost overlapped with that of the conventional OFDM. In summary, the proposed scheme possess only a trivial cost for complexity, which is dramatically decreased from  $O(N^2)$  to O(N) with respect to the existing schemes.

# V. NUMERICAL RESULTS

In this section, the suppression performance of different precoding schemes for OFDM signals are evaluated via computer simulations. To obtain a performance of practical relevance, the simulation parameters illustrated in Table II are compliant with the specification of LTE standards. The power spectra have been estimated by the classical Welch [13] method.

Fig.3a presents the normalized PSD for different precoding schemes and the original OFDM signal. Note that the existing schemes in [10] and [11] achieved the identical performance, thereby only the memoryless N-Continuous precoding with the derivative order of  $N_0$ =0 and  $N_0$ =2 have been plotted for simplicity. The adjacent-channel leakage-power ratio (ACLR) defined by federal communication commission (FCC) and  $3^{rd}$  generation partnership project (3GPP) are  $-72.8 \mathrm{dB}$  and  $-45 \mathrm{dB}$ , respectively. However, it is observed from this figure that at the center of the adjacent channel, i.e.  $10 \mathrm{MHz}$  on the



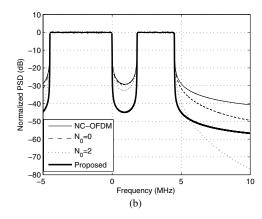


Fig. 3. (a) Normalized power spectra for different spectral precoding schemes and the conventional OFDM. (b) Normalized power spectra for different spectral precoding schemes and NC-OFDM in fragmented spectrum bands

horizontal axis, the original OFDM signal with a total guard band of  $1 \rm MHz$  can achieve only  $-40 \rm dB$  power attenuation. The existing scheme can achieve an ACLR of  $-50 \rm dB$  and  $-76 \rm dB$  in case of  $N_0{=}0$  and  $N_0{=}2$ , respectively, but the price of complexity is considerably higher as mentioned before. The proposed scheme with the optimal weight vectors in (15) and (16) indicated by  $M{=}2$  and  $M{=}3$  in Fig.3a, can achieve  $-55 \rm dB$  and  $-70 \rm dB$ , respectively. In summary, it is concluded that the proposed scheme achieved a considerable sidelobe suppression as strong as the existing schemes, while the complexity has been dramatically decreased.

The performance of precoding schemes in the fragmented bands have been simulated. Suppose that 120 subcarriers are occupied by the licensed user, the transmitted signal of the rental user should form a deep spectral notch. As illustrated in Fig.3b, the NC-OFDM signal can achieve only  $-30\mathrm{dB}$  power attenuation, which may introduce considerable interferences. Applying the existing schemes with the derivative order of  $N_0$ =0 and  $N_0$ =2, the decrease of the power leakage are marginal. On the contrary, it can be seen that the proposed scheme achieve  $-45\mathrm{dB}$  attenuation in the center of spectral notch, i.e. achieved a gain of 15dB.

Furthermore, we have evaluated bit-error rate (BER) performance of the proposed scheme, as given in Fig.4. Compared with that of OFDM, the total transmit power of the proposed scheme will be halved since the weighting of  $\mathbf{g}_2 d_{n,k}$  split the energy of a data symbol for a pair of subcarrier. Thereby, both the performance with the same transmit power as the OFDM and the halved power (indicated by  $\frac{1}{2}$  in the figure legend) have been plotted. In AWGN channel, as shown in Fig.4, the proposed scheme suffers from only 1.5dB signal-to-noise ratio (SNR) loss even if the total transmit power are decreased 3dB. In fading channel, about 3dB gain can be achieved with the same power constraint due to the diversity gain derived from the coherent combining of the adjacent subcarriers.

# VI. CONCLUSION

In this paper, a simple but effective precoding scheme is proposed to enhance the spectral compactness for both OFDM and NC-OFDM signals. By deploying a precoder, a group of adjacent subcarriers are conveying correlated data symbols with different optimized weights. Compared to the existing schemes, considerable sidelobe suppression is achieved by the proposed scheme, while the computational complexity in terms of complex multiplications is reduced from  $O(N^2)$  to O(N). Therefore, it is concluded that our proposed scheme can be very useful to low-cost TVWS devices such as cognitive Femto BS, Wi-Fi AP and ZigBee nodes.

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