New Decoding Algorithms for Matrix C in the 802.16e WiMAX Standard

Young Gil Kim, *Member, IEEE* and Norman C. Beaulieu, *Fellow, IEEE*

***Abstract*—We examine the decoding of Matrix C in the 802.16e WiMAX standard. An exhaustive search and zero-forcing (ES-ZF) decoder and an exhaustive search and nulling canceling (ES-NC) decoder are proposed for uncoded systems. The computational complexity of Matrix C decoding using the ES-ZF decoder is shown to be the same as the complexity of the ZF decoder for Matrix B decoding with twice the number of receive antennas times the complexity of the maximum likelihood (ML) decoder for Matrix B decoding with twice the number of receive antennas. Matrix C can be implemented in a 2×2 multiple-input multiple-output (MIMO) system using the ES-NC decoder with reduced complexity compared to ML decoding with no performance loss. For coded systems, double pruned trees using a zero-forcing (DPT-ZF) algorithm or nulling canceling (DPT-NC) algorithm are proposed. The DPT-NC decoder can be implemented in a 2×2 MIMO coded system with reduced complexity compared to the Max-Log decoding with no performance loss.**

***Index Terms*—Matrix C, space-time codes, WiMAX standard.**

I. Introduction

Mobile WiMAX systems are based on the scalable OFDMA mode of IEEE 802.16e-2005 specifications and use a subset of the different options. Multiple-input multiple-output (MIMO) techniques based on using multiple antennas at both transmitter and receiver can provide spatial diversity, multiplexing gain, and interference suppression, as well as make various trade-offs among them. These techniques have been incorporated in all of the recently developed wireless communications system specifications including the IEEE 802.11n standard for local area networks and the IEEE 802.16e-2005 standard for mobile broadband wireless access systems [1]. Regarding MIMO options, the WiMAX Forum has specified two mandatory profiles for use on the downlink. Denoted in the WiMAX standard as Matrix A, one of them is based on the space-time code (STC) proposed by Alamouti for transmit diversity [2]. This code provides perfect second-order diversity when used with a single receive antenna and fourth-order diversity when used with two receive antennas. However, it is only half-rate, because it only transmits two symbols using two time slots and two transmit antennas. Denoted Matrix B in the WiMAX standard, the other profile is spatial multiplexing (SM), which uses two transmit antennas to transmit two independent data streams. This scheme is full-rate, but it does not benefit from any diversity gain at the transmitter, and at best, it provides second-order diversity with two receive antennas.

Reports have indicated that the desired quality of service cannot always be achieved using Matrix A or Matrix B. For future evolutions of the WiMAX standard, it is highly desirable to include a new code combining the respective advantages of the Alamouti code and the SM, while avoiding their drawbacks. Such a code actually already exists in the IEEE 802.16e-2005 specifications, where it is referred to as Matrix C [3]. The Matrix C code is known to be one of the best STCs of size 2x2. However, this code has a high decoding complexity which grows with the fourth power of the modulation order for maximum likelihood (ML) decoding. The decoding complexity has likely hindered the adoption of Matrix C, despite its superior performance over Matrix A and Matrix B.

In this paper, we propose an exhaustive search and zero-forcing (ES-ZF) decoder and an exhaustive search and nulling canceling (ES-NC) decoder for Matrix C decoding in uncoded systems. The computational complexity of the ES-ZF decoder for Matrix C decoding is shown to be the same as the complexity of the ZF decoder for Matrix B decoding with twice the number of receive antennas multiplied by the complexity of the ML decoder for Matrix B decoding with twice the number of receive antennas. Similarly, the computational complexity of the ES-NC decoder for Matrix C decoding is shown to be the same as the complexity of the NC decoder for Matrix B decoding with twice the number of receive antennas multiplied by the complexity of the ML decoder for Matrix B decoding with twice the number of receive antennas. The ES-ZF decoder and the ES-NC decoder for Matrix C decoding in uncoded systems can be implemented using existing hardware adopted for Matrix B decoding. Thus, compared with the ML decoder for Matrix C decoding, the ES-ZF decoder and the ES-NC decoder can reduce not only computational complexity but also chip size for hardware implementation. Using computer simulation, the ES-ZF decoder and the ES-NC decoder are shown to give up 4.2 dB and 2.1 dB power gain, respectively, to ML decoding for a single receive antenna at a target value of bit error probability (BEP) $= 10^{-3}$ for uncoded systems. When two receive antennas are used, the ES-ZF decoder and the ES-NC decoder give up 1.4 dB and 0 dB, respectively, to ML decoding for BEP $= 10^{-3}$ for uncoded systems. For coded systems, double pruned trees using the zero-forcing (DPT-ZF) algorithm and double pruned trees using the nulling canceling (DPT-NC) algorithm are proposed. A pruned tree in the DPT-ZF decoder and the DPT-NC decoder has only one-half of the number of leaf nodes as the tree for the Max-Log decoding of Matrix C, since ZF and NC

Y. G. Kim is with the Department of Electrical and Computer Engineering, University of Seoul, Seoul, 130-743, Korea. (e-mail: ygkim@ieee.org). N. C. Beaulieu is with the iCORE Wireless Communications Laboratory, University of Alberta, Edmonton, AB, T6G 2V4, Canada. (e-mail: beaulieu@ece.ualberta.ca).

978-1-4673-1881-5/12/$31.00 ©2012 IEEE

algorithms reduce the number of leaf nodes by hard decision of two symbols among four symbols in Matrix C. For a coded system using a $1/2$ convolutional code, the DPT-ZF decoder and the DPT-NC decoder are shown to give up 1.7 dB and 0.8 dB power gain, respectively, to the Max-Log decoding for a single receive antenna and target value of bit error probability (BEP) $= 10^{-3}$. When two receive antennas are used, the DPT-ZF decoder gives up 0.1 dB to the Max-Log decoder for BEP $= 10^{-3}$.

This paper has the following organization. In Section II, the system model is described. In Section III, the ES-ZF decoder and the ES-NC decoder for Matrix C decoding in uncoded systems are investigated. In Section IV, the DPT-ZF decoder and the DPT-NC decoder for Matrix C decoding of coded systems are examined. Numerical examples are presented in Section V. Conclusions are drawn in Section VI.

II. System Model

We consider a MIMO system with two transmit antennas and M_{r} receive antennas. Matrix C in the WiMAX standard is given by [3]

$$C = \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} s_1 + jr \cdot s_4 & r \cdot s_2 + s_3 \\ s_2 - r \cdot s_3 & jr \cdot s_1 + s_4 \end{pmatrix} \qquad (1)$$

where s_1, s_2, s_3, s_4 are M-ary quadrature amplitude modulation (QAM) signals with average symbol energy E_s, $r = \frac{-1+\sqrt{5}}{2}$, and $j = \sqrt{-1}$. We will compare the complexity of Matrix C decoding with that of Matrix B decoding. Matrix B is given by

$$B = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}. \qquad (2)$$

We denote the average bit energy by E_b; it is equal to $E_s/\log_2 M$. The received signal is given by

$$\mathbf{R} = \mathbf{H}C + \mathbf{n} \qquad (3)$$

where $\mathbf{H}$ is the $M_{\mathrm{r}} \times 2$ channel gain matrix whose entries h_{lm} represent the gain from the mth transmit antenna to the lth receive antenna. The gains h_{lm} are independent and identically distributed (iid) circularly symmetric complex Gaussian random variables of unit variance. The vector $\mathbf{n}$ is a $M_{\mathrm{r}} \times 2$ additive white Gaussian noise (AWGN) vector with variance $N_0/2$ per dimension. The symbol $||\cdot||$ is used for vector two-norm.

III. Matrix C Decoding in Uncoded Systems

A. One Receive Antenna

In this subsection, the ES-ZF decoder and the ES-NC decoder with one receive antenna for uncoded systems are described. The received signal for the ES-ZF decoder can be written as

$$R_1 = \frac{1}{\sqrt{1+r^2}} \cdot (h_1 s_1 + h_1 jr s_4 + h_2 s_2 - h_2 r s_3) + n_1 \qquad (4a)$$

$$R_2 = \frac{1}{\sqrt{1+r^2}} \cdot (h_1 r s_2 + h_1 s_3 + h_2 jr s_1 + h_2 s_4) + n_2 \qquad (4b)$$

where R_1 and R_2 are the received signals in the first and the second time slot, respectively, and h_1 and h_2 are the channel gain from the first and the second transmit antennas, respectively. We fix the values of s_1 and s_4 to any symbol in the signal constellation, so one has

$$\begin{aligned} R_1 - \frac{1}{\sqrt{1+r^2}} (h_1 s_1 + h_1 jr s_4) &= \\ \frac{1}{\sqrt{1+r^2}} (h_2 s_2 - h_2 r s_3) + n_1 & \\ R_2 - \frac{1}{\sqrt{1+r^2}} (h_2 jr s_1 + h_2 s_4) &= \\ \frac{1}{\sqrt{1+r^2}} (h_1 r s_2 + h_1 s_3) + n_2. & \end{aligned}$$

Defining $\beta_1 = R_1 - \frac{1}{\sqrt{1+r^2}} (h_1 s_1 + h_1 jr s_4)$ and $\beta_2 = R_2 - \frac{1}{\sqrt{1+r^2}} (h_2 jr s_1 + h_2 s_4)$,

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} h_2 & -h_2 r \\ h_1 r & h_1 \end{pmatrix} \begin{pmatrix} s_2 \\ s_3 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}. \qquad (5)$$

Given β_1 and β_2, eq. (5) can be solved using ZF decoding. Multiplying by the inverse of the matrix of $\begin{pmatrix} h_2 & -h_2 r \\ h_1 r & h_1 \end{pmatrix}$ on both sides of (5) gives

$$\begin{aligned} \frac{1}{h_1 h_2 \sqrt{1+r^2}} \begin{pmatrix} h_1 & h_2 r \\ -h_1 r & h_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} &= \\ \begin{pmatrix} s_2 \\ s_3 \end{pmatrix} + \frac{1}{h_1 h_2 \sqrt{1+r^2}} \begin{pmatrix} h_1 & h_2 r \\ -h_1 r & h_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}. & \end{aligned} \qquad (6)$$

Symbols s_2 and s_3 can be detected using (6). We repeat ZF decoding for every s_1 and s_4 in the signal constellation. Using exhaustive search, we find s_1 and s_4 to minimize

$$\begin{aligned} &||R_1 - \frac{1}{\sqrt{1+r^2}} (h_1 s_1 + h_1 jr s_4 + h_2 \hat{s}_2 - h_2 r \hat{s}_3)||^2 + \\ &||R_2 - \frac{1}{\sqrt{1+r^2}} (h_1 r \hat{s}_2 + h_1 \hat{s}_3 + h_2 jr s_1 + h_2 s_4)||^2 \end{aligned} \qquad (7)$$

where $\hat{s}_2$ and $\hat{s}_3$ are symbols detected using the ZF decoder given s_1 and s_4. Note that (5) can be viewed as a problem of detecting Matrix B with two receive antennas. Eq. (7) can then be viewed as the distance metric of Matrix B decoding with two receive antennas. Thus, the complexity of the ES-ZF decoder for Matrix C decoding with one receive antenna, $K_{\mathrm{ES-ZF}}(M_{\mathrm{r}} = 1, C)$, is the same as the complexity of the ZF decoder for Matrix B decoding with two receive antennas, $K_{\mathrm{ZF}}(M_{\mathrm{r}} = 2, B)$, multiplied by the complexity of the ML decoder for Matrix B decoding with two receive antennas, $K_{\mathrm{ML}}(M_{\mathrm{r}} = 2, B)$. That is, one has

$$K_{\mathrm{ES-ZF}}(M_{\mathrm{r}} = 1, C) = K_{\mathrm{ZF}}(M_{\mathrm{r}} = 2, B) \cdot K_{\mathrm{ML}}(M_{\mathrm{r}} = 2, B). \qquad (8)$$

For the ES-NC decoder, we use NC decoding for detecting s_2 and s_3 in (5). If $||h_1|| > ||h_2||$, s_3 is detected first. Then, putting the detected $\hat{s}_3$ into (5), we can detect s_2 adopting NC decoding [5]. Then, we repeat NC decoding for every s_1 and s_4 in the signal constellation. Symbols s_1 and s_4 are detected by exhaustive search. For the ES-NC decoder, a similar complexity equality is valid as holds for the ES-ZF decoder. The complexity of the ES-NC decoder for Matrix C decoding with one receive antenna, $K_{\mathrm{NCD}}(M_{\mathrm{r}}=1,C)$, is the complexity of the NC decoder for Matrix B decoding with two receive antennas, $K_{\mathrm{NC}}(M_{\mathrm{r}}=2,B)$, multiplied by the complexity of the ML decoder for Matrix B decoding with two receive antennas, $K_{\mathrm{ML}}(M_{\mathrm{r}}=2,B)$. That is,

$$K_{\mathrm{ES-NC}}(M_{\mathrm{r}}=1,C)=K_{\mathrm{NC}}(M_{\mathrm{r}}=2,B)\cdot K_{\mathrm{ML}}(M_{\mathrm{r}}=2,B). \tag{9}$$

B. Two Receive Antennas

Now, the ES-ZF decoder and the ES-NC decoder with two receive antennas for uncoded systems are described. First, the ES-ZF decoder is described. The received signal with two receive antennas is written as

$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} = \frac{1}{\sqrt{1+r^2}} \times \begin{pmatrix} h_{11}(s_1+jrs_4)+h_{12}(s_2-rs_3) & h_{11}(rs_2+s_3)+h_{12}(jrs_1+s_4) \\ h_{21}(s_1+jrs_4)+h_{22}(s_2-rs_3) & h_{21}(rs_2+s_3)+h_{22}(jrs_1+s_4) \end{pmatrix} + \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \tag{10}$$

where R_{lm} and n_{lm} are the received signal and the noise signal, respectively, over the lth receive antenna and the mth time slot. Then, fix the values of s_1 and s_4 to any symbol in the signal constellation, and one has

$$\begin{pmatrix} R_{11}-\frac{h_{11}(s_1+jrs_4)}{\sqrt{1+r^2}} & R_{12}-\frac{h_{12}(jrs_1+s_4)}{\sqrt{1+r^2}} \\ R_{21}-\frac{h_{21}(s_1+jrs_4)}{\sqrt{1+r^2}} & R_{22}-\frac{h_{22}(jrs_1+s_4)}{\sqrt{1+r^2}} \end{pmatrix} = \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} h_{12}(s_2-rs_3) & h_{11}(rs_2+s_3) \\ h_{22}(s_2-rs_3) & h_{21}(rs_2+s_3) \end{pmatrix} + \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}. \tag{11}$$

Defining $\beta_{11}=R_{11}-\frac{h_{11}(s_1+jrs_4)}{\sqrt{1+r^2}}$, $\beta_{12}=R_{12}-\frac{h_{12}(jrs_1+s_4)}{\sqrt{1+r^2}}$, $\beta_{21}=R_{21}-\frac{h_{21}(s_1+jrs_4)}{\sqrt{1+r^2}}$, and $\beta_{22}=R_{22}-\frac{h_{22}(jrs_1+s_4)}{\sqrt{1+r^2}}$, (11) are written as

$$\begin{pmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{21} \\ \beta_{22} \end{pmatrix} = \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} h_{12} & -h_{12}r \\ h_{11}r & h_{11} \\ h_{22} & -h_{22}r \\ h_{21}r & h_{21} \end{pmatrix} \begin{pmatrix} s_2 \\ s_3 \end{pmatrix} + \begin{pmatrix} n_{11} \\ n_{12} \\ n_{21} \\ n_{22} \end{pmatrix}. \tag{12}$$

Eq. (12) can be solved given $\beta_{11},\beta_{12},\beta_{21}$, and β_{22} using ZF decoding. We repeat detecting s_2 and s_3 by ZF decoding for all symbols s_1 and s_4 in the signal constellation. Using exhaustive search, we detect s_1 and s_4. Note that (12) can be viewed as describing the problem of detecting Matrix B with four receive antennas. Thus, the complexity of the ES-ZF decoder for Matrix C decoding with two receive antennas, $K_{\mathrm{ES-ZF}}(M_{\mathrm{r}}=2,C)$, is the complexity of the ZF decoder for Matrix B decoding with four receive antennas, $K_{\mathrm{ZF}}(M_{\mathrm{r}}=4,B)$, multiplied by the complexity of the ML decoder for Matrix B decoding with four receive antennas, $K_{\mathrm{ML}}(M_{\mathrm{r}}=4,B)$. That is,

$$K_{\mathrm{ES-ZF}}(M_{\mathrm{r}}=2,C)=K_{\mathrm{ZF}}(M_{\mathrm{r}}=4,B)\cdot K_{\mathrm{ML}}(M_{\mathrm{r}}=4,B). \tag{13}$$

For the ES-NC decoder, we adopt NC decoding for detecting s_2 and s_3 in (12). For the ES-NC decoder, a similar complexity equality is valid as holds for the ES-ZF decoder. The complexity of the ES-NC decoder for Matrix C decoding with two receive antenna, $K_{\mathrm{ES-NC}}(M_{\mathrm{r}}=2,C)$, is the complexity of the NC decoder for Matrix B decoding with four receive antennas, $K_{\mathrm{NC}}(M_{\mathrm{r}}=4,B)$, multiplied by the complexity of the ML decoder for Matrix B decoding with four receive antennas, $K_{\mathrm{ML}}(M_{\mathrm{r}}=4,B)$, viz

$$K_{\mathrm{ES-NC}}(M_{\mathrm{r}}=2,C)=K_{\mathrm{NC}}(M_{\mathrm{r}}=4,B)\cdot K_{\mathrm{ML}}(M_{\mathrm{r}}=4,B). \tag{14}$$

We show that detecting s_2 and s_3 in Matrix C with M_{r} receive antennas given s_1 and s_4 is equivalent to detecting Matrix B with $2M_{\mathrm{r}}$ receive antennas in the Appendix. Thus, the complexity of the ES-ZF decoder for Matrix C decoding with M_{r} receive antennas is the complexity of the ZF decoder for Matrix B decoding with $2M_{\mathrm{r}}$ receive antennas, $K_{\mathrm{ES-ZF}}(2M_{\mathrm{r}},B)$, multiplied by the complexity of the ML decoder for Matrix B decoding with $2M_{\mathrm{r}}$ receive antennas, $K_{\mathrm{ML}}(2M_{\mathrm{r}},B)$. That is,

$$K_{\mathrm{ES-ZF}}(M_{\mathrm{r}},C)=K_{\mathrm{ZF}}(2M_{\mathrm{r}},B)\cdot K_{\mathrm{ML}}(2M_{\mathrm{r}},B). \tag{15}$$

Similarly,

$$K_{\mathrm{ES-NC}}(M_{\mathrm{r}},C)=K_{\mathrm{NC}}(2M_{\mathrm{r}},B)\cdot K_{\mathrm{ML}}(2M_{\mathrm{r}},B). \tag{16}$$

Since the computational complexity of the ZF decoder and the NC decoder does not depend on the modulation order M, the complexity of the ES-ZF decoder and the ES-NC decoder is proportional to M^2, which originates from the exhaustive search operation. However, the complexity of the ML decoder for Matrix C decoding is proportional to M^4. A discussion on the complexity of the ZF decoder and the NC decoder is found in [6, p. 8]. Another advantage of the ES-ZF decoder and the ES-NC decoder for Matrix C decoding is that they can reuse hardware already implemented for Matrix B decoding. For example, if NC decoder hardware for Matrix B decoding is implemented on a chip, then the additional hardware needed for the ES-NC decoder for Matrix C decoding is only the ML decoder for searching M^2 candidates since the NC decoder hardware for Matrix B decoding can be reused. In Table I, the computational complexities of the ML decoder, the ES-ZF decoder, and the ES-NC decoder are shown. The Matrix C decoding for uncoded systems can be thought as a tree-searching problem. For the ML decoder, we have a tree with M^4 leaf nodes as in Fig. 1 (a). For the ES-ZF and ES-NC decoders, we have a pruned tree with only M^2 leaf nodes as in Fig. 1(b). This fact clarifies why the computational complexity can be reduced by adopting the ES-ZF decoder or the ES-NC decoder.

IV. Matrix C Decoding in Coded Systems

In this section, we describe the DPT-ZF decoder and the DPT-NC decoder for coded systems. For coded systems, an error correction code is used as an outer code and Matrix C can be considered as an inner code. Soft-input for the outer error correction decoder can approximated as [8]

$$L(x_{j,b}) = \min_{\{C^0_{j,b}|C^0_{j,b}\in\chi^0_{j,b}\}} ||\mathbf{R} - \mathbf{H}C^0_{j,b}||^2 - \min_{\{C^1_{j,b}|C^1_{j,b}\in\chi^1_{j,b}\}} ||\mathbf{R} - \mathbf{H}C^1_{j,b}||^2 \quad (17)$$

where $x_{j,b}$ is the b^{th} bit in the j^{th} symbol in Matrix C, $j = 1,2,3,4$, $b = 1,2,...,\log_2 M$, $\chi^0_{j,b}$ is a set of Matrix C's that have zero at the b^{th} bit in the j^{th} symbol and vice versa. The signal-to-noise ratio (SNR) loss of the max-log approximation in (17) is around 0.25 dB over a large range of SNRs [8]. We will call this max-log approximation, Max-Log decoding. The Max-Log decoding for coded systems requires computation of (17) for $j = 1,2,3,4$, $b = 1,2,...,\log_2 M$. For each j and b, the number of required comparisons is $2(M^4/2-1)$ since the number of possible Matrix C's that have zero at the b^{th} bit in the j^{th} symbol is $M^4/2$. Note that Matrix C can be represented as a tree that has M^4 leaf nodes since Matrix C consists of four M-ary symbols. Using the ZF or NC algorithm, the tree of Matrix C can be pruned to a tree that has only M^2 leaf nodes since s_2 and s_3 can be fixed to certain values as in Section 3. For this pruned tree as in Fig. 1(b), $L(x_{j,b})$ in (17) for $j = 1,4$ exists for $b = 1,2,...,\log_2 M$. However, $L(x_{j,b})$ in (17) for $j = 2,3$ could not exist for some $b = 1,2,...,\log_2 M$. Thus, we make another pruned tree so that $L(x_{j,b})$ in (17) for $j = 2,3$ exists for all $b = 1,2,...,\log_2 M$. Using these double pruned trees, $L(x_{j,b})$s in (17) are calculated. For each j and b, the number of required comparisons is $2(M^2/2-1)$, which is far fewer than $2(M^4/2-1)$ of the Max-Log decoding. In Table II, the computational complexities of the Max-Log decoder, the DPT-ZF decoder, and the DPT-NC decoder are shown. We can see that the number of comparisons of the DPT-ZF decoder and the DPT-NC decoder is reduced from the order of M^4 to the order of M^2 by pruning the tree in Fig. 1(a).

V. Numerical results

In this section, we present numerical results for the BEP performance. The results are obtained by computer simulation. The FFT size is chosen as 1024 which corresponds to a system bandwidth of 10 MHz in the mobile WiMAX OFDMA PHY specifications [3], [7]. We have used downlink Partial Usage of SubChannel (PUSC) permutation for a 10 MHz bandwidth as in [7]. Jake's channel model is used in a Pedestrian B environment with a speed of 3 km/h in [7] in the simulations.

In Fig. 2, we show the BEP vs. E_b/N_0 for $M_{\text{r}} = 1,2$ and quadrature phase shift keying (QPSK) signaling. Observe that when there is one receive antenna, $M_{\text{r}} = 1$, the ES-ZF decoder gives up 4.2 dB in power gain at BEP $= 10^{-3}$ to the ML decoder in return for its reduced complexity compared to the ML decoder. However, the ES-NC decoder gives up only 2.1 dB in power gain at BEP $= 10^{-3}$ to the ML decoder when one receive antenna is used, while offering needed reduced complexity. When two receive antennas are used, the ES-ZF decoder gives up only 1.4 dB to the ML decoder for BEP $= 10^{-3}$. Significantly, the ES-NC decoder provides virtually the same performance as that of the ML decoder, at reduced complexity. This is an important result as it implies that Matrix C can be implemented in 2×2 MIMO systems, greatly improving performance over Matrix A and Matrix B, with reduced decoding complexity using the ES-NC decoder with essentially no performance loss relative to optimal ML decoding.

Fig. 3 shows the BEP vs. E_b/N_0 for $M_{\text{r}} = 1,2$ and QPSK signaling. A $1/2$ convolutional code with constraint length 7 that has the generator polynomials 171 and 133 in octal format is used for coded system simulations employing soft-input Viterbi decoding. For one receive antenna, the DPT-ZF decoder gives up 1.7 dB in power gain at BEP $= 10^{-3}$ to the Max-Log decoder in return for its reduced complexity compared to the Max-Log decoder. However, the DPT-NC decoder gives up only 0.8 dB in power gain at BEP $= 10^{-3}$ to the Max-Log decoder when one receive antenna is used, while offering needed reduced complexity. When two receive antennas are used, the DPT-ZF decoder gives up only 0.1 dB to the ML decoder for BEP $= 10^{-3}$. Significantly, the DPT-NC decoder provides virtually the same performance as that of the Max-Log decoder, at reduced complexity.

VI. Conclusion

The 802.16e 2005 WiMAX Standard specifies a Matrix C space-time code that offers the potential of significantly improved QoS, however, Matrix C has not yet been adopted owing to the large power requirements for its decoding. In this paper, we have proposed the ES-ZF decoder and the ES-NC decoder for decoding Matrix C in the WiMAX standard for uncoded systems. It is shown that Matrix C can be implemented in a 2×2 MIMO system without the cost of ML decoding. The ES-NC decoder used with Matrix C performs virtually as well as ML decoding with the advantage of reduced complexity and power consumption. Also, the DPT-ZF decoder and the DPT-NC decoder have been proposed for Matrix C decoding in coded systems. The computational complexity is reduced by pruning the tree by the ZF or NC algorithm as in an uncoded system.

References

[1] S. Sezginer and H. Sari, "Full-rate full-diversity 2×2 space-time codes for reduced decoding complexity," *IEEE Commun. Lett.*, vol. 11, No. 12, pp. 1-3, Dec. 2007.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

[3] IEEE 802.16e-2005: IEEE Standard for Local and Metropolitan Area Networks - Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems, Feb. 2006.

[4] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The golden code: a 2×2 full rate space-time code with nonvanishing determinants," *IEEE Trans. Inf. Theory*, vol. 51, pp. 1432-1436, April 2005.

[5] G. B. Giannakis, Z. Liu, X. Ma, and S. Zhou, *Space-Time Coding for Broadband Wireless Communications.* Wiley, 2007.

[6] T. Kailath, H. Vikalo, and B. Hassibi, "MIMO Receive Algorithms," in *Space-Time Wireless Systems: From Array Processing to MIMO Communications*, Cambridge University Press, 2005.

[7] R. Kobeissi, S. Sezginer, and F. Buda, "Downlink performance analysis of full-rate STCs in 2×2 MIMO WiMAX systems," in *Proc. IEEE Veh. Tech. Conf.*, pp. 1-5, April 2009.

[8] C. Studer, A. Burg, and H. Bolcskei, "Soft-output sphere decoding: algorithms and VLSI implementation," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 290-300, Feb. 2008.

TABLE I

COMPUTATIONAL COMPLEXITY OF ML DECODING, ES-ZF DECODING, AND ES-NC DECODING FOR UNCODED SYSTEMS

	ML	ES-ZF	ES-NC
Number of comparisons	M^4	$5M^2$	$5M^2$
Number of multiplications	$18M^4M_{\mathrm{r}}$	$34M^2M_{\mathrm{r}}$	$40M^2M_{\mathrm{r}}$
Number of additions	$11M^4M_{\mathrm{r}}$	$15M^2M_{\mathrm{r}}$	$18M^2M_{\mathrm{r}}$

APPENDIX

In this Appendix, we show that detecting s_2 and s_3 in Matrix C with M_{r} receive antennas given s_1 and s_4 is equivalent to detecting Matrix B with $2M_{\mathrm{r}}$ receive antennas.

For a MIMO system with two transmit antennas and M_{r} receive antennas, the received signal for Matrix C transmission in (3) can be written as

$$\begin{pmatrix} \mathbf{R}_1 & \mathbf{R}_2 \end{pmatrix} = \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{pmatrix} \begin{pmatrix} s_1 + jr\cdot s_4 & r\cdot s_2 + s_3 \\ s_2 - r\cdot s_3 & jr\cdot s_1 + s_4 \end{pmatrix} + \begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix} \quad (18)$$

where $\mathbf{R}_1$ and $\mathbf{R}_2$ are the received signals in the first and the second time slot, respectively, $\mathbf{h}_1$ and $\mathbf{h}_2$ are the channel gain for the first and the second transmit antennas, respectively, and $\mathbf{n}_1$ and $\mathbf{n}_2$ are AWGN in the first and the second time slot, respectively. The vectors $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{h}_1$, $\mathbf{h}_2$, $\mathbf{n}_1$, and $\mathbf{n}_2$ have size $M_{\mathrm{r}} \times 1$. From (18),

$$\begin{aligned} &\begin{pmatrix} \mathbf{R}_1 & \mathbf{R}_2 \end{pmatrix} = \\ &\frac{1}{\sqrt{1+r^2}} \left(\mathbf{h}_1(s_1 + jr\cdot s_4) + \mathbf{h}_2(s_2 - r\cdot s_3)\right. \\ &\left.\mathbf{h}_1(r\cdot s_2 + s_3) + \mathbf{h}_2(jr\cdot s_1 + s_4)\right) + \begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix} \\ &= \frac{1}{\sqrt{1+r^2}} \left\{ \begin{pmatrix} \mathbf{h}_1(s_1 + jr\cdot s_4) & \mathbf{h}_2(jr\cdot s_1 + s_4) \end{pmatrix} \right. \\ &\left. + \begin{pmatrix} \mathbf{h}_2(s_2 - r\cdot s_3) & \mathbf{h}_1(r\cdot s_2 + s_3) \end{pmatrix} \right\} \\ &+ \begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix}. \end{aligned}$$

Defining $\boldsymbol{\beta}_1 = \mathbf{R}_1 - \frac{1}{\sqrt{1+r^2}}\mathbf{h}_1(s_1 + jrs_4)$ and $\boldsymbol{\beta}_2 = \mathbf{R}_2 - \frac{1}{\sqrt{1+r^2}}\mathbf{h}_2(jrs_1 + s_4)$,

$$\begin{aligned} \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{pmatrix} &= \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} \mathbf{h}_2(s_2 - r\cdot s_3) \\ \mathbf{h}_1(r\cdot s_2 + s_3) \end{pmatrix} + \begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{pmatrix} \\ &= \frac{1}{\sqrt{1+r^2}} \begin{pmatrix} \mathbf{h}_2 & -r\mathbf{h}_2 \\ r\mathbf{h}_1 & \mathbf{h}_1 \end{pmatrix} \begin{pmatrix} s_2 \\ s_3 \end{pmatrix} + \begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{pmatrix}. \quad (19) \end{aligned}$$

Eq. (19) can be viewed as describing the problem of detecting Matrix B with $2M_{\mathrm{r}}$ receive antennas.

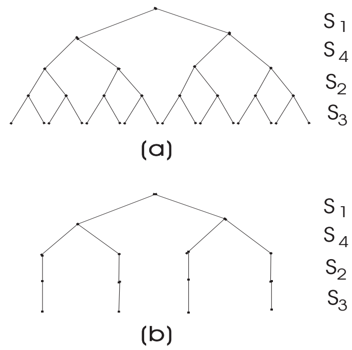

Fig. 1. (a) A tree for the ML decoding in uncoded systems and the Max-Log decoding in coded systems (b) A pruned tree for the ES-ZF and the ES-NC decoders in uncoded systems and the DPT-ZF and the DPT-NC decoders in coded systems. Note that we assume that the modulation order, M, is two in this figure for simplicity. However, in WiMAX systems, M should be one of 4, 16, 64 [3].

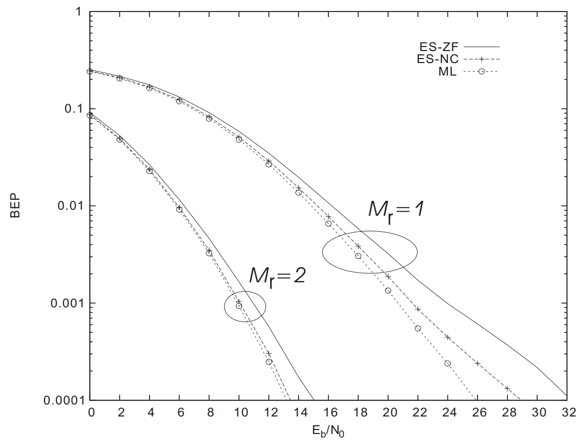

Fig. 2. The BEP as a function of E_b/N_0 for uncoded systems with $M_{\mathrm{r}} = 1, 2$ and QPSK signaling.

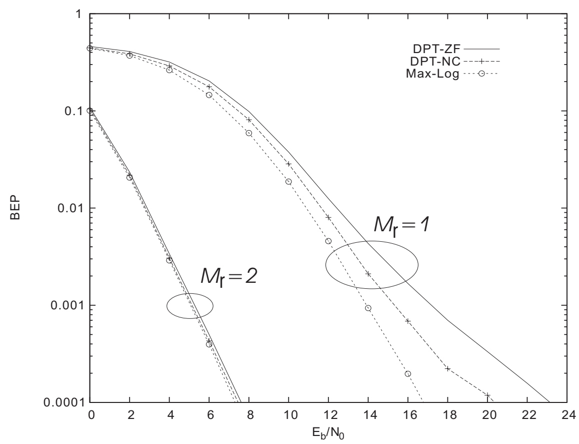

Fig. 3. The BEP as a function of E_b/N_0 for coded systems with $1/2$ convolutional code, $M_{\mathrm{r}} = 1, 2$, and QPSK signaling.

TABLE II

COMPUTATIONAL COMPLEXITY OF MAX-LOG DECODING, DPT-ZF DECODING, AND DPT-NC DECODING FOR CODED SYSTEMS

	Max-Log	DPT-ZF	DPT-NC
Number of comparisons	M^4	$9M^2$	$9M^2$
Number of multiplications	$18M^4M_{\mathrm{r}}$	$68M^2M_{\mathrm{r}}$	$80M^2M_{\mathrm{r}}$
Number of additions	$11M^4M_{\mathrm{r}}$	$30M^2M_{\mathrm{r}}$	$36M^2M_{\mathrm{r}}$