# Sequential Cooperative Spectrum Sensing Technique for Cognitive Radio System in Correlated Channel

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Abstract—Cognitive radio is proposed to opportunistically access the spectrum while the licensed user is idle. As a result, spectrum sensing procedure to observe primary user's existence is vital to cognitive radio. In this paper, we investigate the energy detection based sequential cooperative spectrum sensing technique in time varying channel. By utilizing past local observations from previous sensing slots, cognitive radio nodes can aggregate the current and previously received energy values to improve the detection performance. We explored the moving-average technique using equal and exponential weighting. Simulation results show that this technique provides better detection performance compared to conventional energy detection based cooperative technique. Equal weighting provides the best performance when primary user's activity is not varying. If the primary users often change states, the proposed exponential weighting approach provides better detection performance.

# I. INTRODUCTION

Nowadays, wireless network is regulated by the fixed spectrum assignment policy. Its usage is limited to a certain part of the spectrum, while the rest of the spectrum remains under-utilized [1]. With the high demand of mobile service, a new scheme that efficiently utilizes the spectrum is needed. By opportunistically accessing the licensed spectrum without interfering the licensed user, Cognitive Radio (CR) can improve the efficiency of spectrum usage [2]. It determines the existence of the licensed users (or primary user) by sensing the wireless spectrum. If the spectrum is vacant, it will start communicating to its receiver. On the other hand, if the spectrum is occupied or the licensed users retransmit again, the CR users should stop their transmission. Hence, spectrum sensing is a vital procedure for CR.

Spectrum sensing is based on the CR user's observation. When the CR user experiences a deep fade or shadowing from the licensed transmitter. The hidden terminal problem will occur which leads to a missed detection, and causes interference to the licensed users.

In order to tackle this problem, cooperative detection has been proposed to gather sensing information from multiple CR users [3]. Each CR user senses the spectrum and forwards its local observation to the fusion centre. Based on the multiple observations, the fusion centre will then decide whether the licensed user exists or not. As users are spatially separated, cooperative detection can minimize the uncertainty in single user detection.

Sequential spectrum sensing in CR has been introduced in [4] to make use of past observations. A local decision is made when the CR user is confident on its observation, based on the log-likelihood ratio (LLR). Otherwise, it will not make any local decision but simply adds the new LLR value into the previous one. However, computing the LLR involves more computation.

In this work, we investigate energy detection based sequential spectrum sensing technique in time varying channels. Weighted sum of the past observations are utilized to improve the detection performance. To avoid hidden terminal problem, cooperative sensing is also considered.

# II. SYSTEM MODEL

In this paper, energy detection based sequential cooperative sensing is used to exploit temporal correlation of the channel. Primary user's activity is also considered. These models are first discussed in this section.

The sequential sensing procedure is shown in Fig. 1. The sensing period occurs periodically at the beginning of each packet, followed by the data transmission part. The number of observation samples taken in the sensing period is defined by the time-bandwidth product.

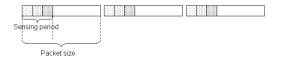


Fig. 1. sequential sensing procedure

To detect the primary user's existence, we adopt the energy detection technique [5]. It is a simple and effective approach whereby the received signal energy is compared to a detection threshold to determine the existence of the primary users. The received energy value at the cognitive radio user can be expressed as

$$O = \begin{cases} \sum_{i=1}^{m} |n_i|^2 & H_0\\ \sum_{i=1}^{m} |h_i s_i + n_i|^2 & H_1 \end{cases}$$
 (1)

where m is the time-bandwidth product,  $h_i$ ,  $s_i$  and  $n_i$  is channel gain, primary user's signal and noise in  $i^{th}$  time slot

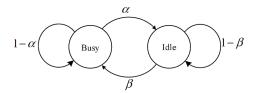


Fig. 2. Two-state Markov chain on primary user's activity model

and  $H_0$ ,  $H_1$  represents the case when primary user is idle and active respectively.

Conventional cooperative spectrum sensing works such that the local decision on the primary user's existence is forwarded to the fusion center from every CR users. Then, the fusion center makes the final decision according to the rules, such as OR-rule, AND-rule or the voting-rule [6].

Due to the time varying nature of wireless channels, there is a correlation between the previous and current observations of the received energy value. This correlation is affected by the Doppler frequency. When the Doppler frequency is low, channel gain changes slowly and the difference between observations is small. Hence, the channel correlation is high. Conversely, high Doppler frequency leads to high channel variation, leading to low channel correlation.

The primary user activity is modelled according to a twostate Markov chain [7] as shown in Fig. 2. The parameter  $\alpha$ and  $\beta$  represents the probability of primary user changing its state from active  $(H_1)$  to idle  $(H_0)$  and vice versa, while the parameter  $1-\alpha$  and  $1-\beta$  represents the probability of primary user remains in active  $(H_1)$  and idle  $(H_0)$  state respectively.

# III. PROPOSED SEQUENTIAL COOPERATIVE SENSING SCHEME

In this section, we propose a novel cooperative spectrum sensing technique that uses the energy observation from previous slots. In order to exploit the channel correlation, the current and previous observations are weighted and aggregated before comparing to the threshold locally at the CR node. The local decision is then forwarded to the fusion center. If any CR node detects the primary user, the fusion center will inform all other nodes about the decision. In other words, this approach make use of soft combining for local decision and the OR-rule for global decision at the fusion center.

We employed the moving average model, where the output comes from the summation of the weighted input value. The mathematical representation of this moving average model is  $T[i] = \sum\limits_{k=0}^{N-1} W[k] \cdot O[i-k]$ , where  $W, \ N, \ O[i], \ T[i]$  represents weighting vector, number of input taken, input and output of the moving average filter at the  $i^{th}$  time slot. [8]

The sensing procedure is as follows.

1) First, the energy observations are weighted and aggregated such that  $T = \mathbf{WO} = \sum_{i=1}^{N} w_i O_i$ , where T is the summation of weighted observations.  $\mathbf{O}$  is

the local observation for each sensing slot denoted as  $\mathbf{O} = \begin{bmatrix} O_1 & O_2 & \cdots & O_N \end{bmatrix}^T$ , where  $O_N$  is the current observation from the energy detection, and  $O_{N-i}$  is the observation from the  $i^{th}$  sensing slot before.  $\mathbf{W}$  is the weighting vector  $\mathbf{W} = \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}$ , where  $w_N$  is the weighting factor for the current observation  $O_N$  and  $w_{N-i}$  is that for  $O_{N-i}$  with  $(w_1 + w_2 + \ldots + w_N) = 1$ .

- 2) Then, the decision D is made by comparing T to the threshold  $\lambda$ .  $D = \begin{cases} H_0 \text{ ,if } T < \lambda \\ H_1 \text{ ,otherwise} \end{cases}$
- 3) Finally, the local decision *D* is sent to the fusion centre to make a final decision on primary user's existence using the OR-rule.

# IV. WEIGHT COMPUTATION

A key component of the sequential cooperative sensing technique is the weight vector. Ideally, this vector should be optimized by minimizing the false alarm and miss detection probability [6]. However, this optimization is complicated as the probability of false alarm  $(P_f)$  and detection  $(P_d)$  becomes the complementary cumulative distribution function (CCDF) for weighted sum of N-random variables as shown in Eq. (2) and Eq. (3). Hence, the closed-form and optimal solution is difficult to obtain. [9]

$$\begin{array}{lcl} P_f & = & P(T > \lambda | H_0) \\ P_f & = & P(w_1 O_1 + w_2 O_2 + \ldots + w_N O_N > \lambda | H_0) \end{array} \eqno(2)$$

and similarly for  $P_d$ 

$$P_d = P(w_1O_1 + w_2O_2 + \dots + w_NO_N > \lambda | H_1)$$
 (3)

In this section, we explain our sub-optimal weight calculation approach. An example of the probability density function (pdf) of  $H_0$  and  $H_1$  is shown in Fig. 3. The optimal detection approach is one that can jointly minimize the false alarm probability  $p(H_1|H_0)$  and missed detection  $p(H_0|H_1)$ . Therefore, the properties of the pdf of  $H_0$  and  $H_1$  are important in computing the weight vector. We first define the following proposition.

Proposition 1: For a normalized weight vector  $(w_1 + w_2 + ... + w_N = 1)$ , the mean of the weight sum energy equals to the mean of the local observation (i.e. E[T] = E[O])

*Proof:* 

$$\begin{split} E\left[T\right] = & E\left[\sum_{i=1}^{N} w_{i} O_{i}\right] \\ = & w_{1} \cdot E\left[O_{1}\right] + w_{2} \cdot E\left[O_{2}\right] + \ldots + w_{N} \cdot E\left[O_{N}\right] \\ \text{As } E\left[O\right] = & E\left[O_{1}\right] = E\left[O_{2}\right] = \ldots = E\left[O_{N}\right], \\ E\left[T\right] = & \left(w_{1} + w_{2} + \ldots + w_{N}\right) \cdot E\left[O\right] = E\left[O\right] \end{split}$$

The importance of this proposition is that the mean of the weighted sum energy will not be affected by the choice of W. In other words, the separation between the mean of  $H_0$ 

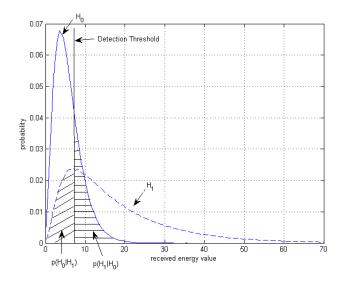


Fig. 3. Probability density function of  $H_0$  and  $H_1$ 

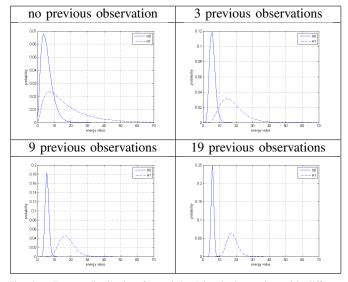


Fig. 4. Energy distribution for weighted local observation with different number of previous observations taken

and  $H_1$  will not be affected by varying **W**. Hence, to separate  $H_0$  and  $H_1$  distribution, the variance for both cases plays an important role. The effect of variance on the distribution of  $H_0$  and  $H_1$  is illustrated in Fig. 4. When more observations are taken, the variance for both  $H_0$  and  $H_1$  will be reduced. Hence, the separation between the distribution of  $H_0$  and  $H_1$  is more evident and this leads to better detection performance. We also show mathematically that when equal weighting is applied, the variance decreases proportionally to the number of samples taken in Appendix A.

# A. Static Primary User Activity

As discussed above, the optimal weighting is one that can minimise the variances of T in  $H_0$  and  $H_1$ . However, its complexity involves minimizing both distributions at the same time. Hence, we propose a sub-optimal approach to obtain

the weighting vector by minimising the variance  $H_0$  or  $H_1$  separately. When the primary user's activity is static (i.e. the primary user is either always on or always off), the following can be defined.

Proposition 2: In quasi-static fading channel, the optimal weight vector that minimises the variance of  $H_0$  and  $H_1$  is the equal weighting.

Proof: See Appendix B.

Corollary 1: In time-varying channel, the optimal weight vector that minimises the variance of  $H_0$  is the equal weighting.

**Proof:** As the primary user does not exist in  $H_0$ , the channel will not affect the received energy observation. Hence, the proof follows that of Proposition 2.

Now the remaining problem is the optimization of weight vector for  $H_1$  in time varying channel. This involves the computation of the covariances between observations. A closed form solution could not be obtained and thus we resolved to computing it using numerical methods.

# B. Varying Primary User Activity

When primary user's activity is varying, the received energy value at the cognitive radio could change suddenly between observations. Hence, a practical approach is to rely more on the newer observations for detection. Thus, we propose the use of exponential weighting, whereby each past observation is weighted less by a factor of  $\frac{1}{e}$ , when N observations are taken, the weight vector  $\mathbf{W} = \frac{1}{K} \begin{bmatrix} e^1 & e^2 & \dots & e^N \end{bmatrix}$ , where

$$K = \sum_{i=1}^{N} e^{i}$$
 is the normalisation factor.

# V. SIMULATION RESULTS

In this section, the proposed schemes are evaluated using computer simulation. Three spatially separated users are considered for cooperative sensing. Three previous observations are taken with the time-bandwidth product=3 for each observation. Each packet has length of 100 symbols and the sampling frequency is  $100\ kHz$ . The primary user's SNR is 3dB and the channel is modelled as a time-varying Rayleigh fading channel with Doppler frequency of  $50\ Hz$ . First, the simulation result with static primary user activity is investigated. Then, the primary user's activity is modelled and the detection performance of the proposed model with equal and exponential weighting vector is evaluated.

#### A. Simulation Results for static primary user activity

As shown in the last section that to optimise the  $H_0$  case, the equal weighting  $\mathbf{W} = [\begin{array}{ccc} 0.25 & 0.25 & 0.25 \\ \end{array}]$  is optimal when three previous observations are taken. However for the  $H_1$  case, we found by numerical methods that the set of optimal weighting vector which minimizes the variance of  $H_1$  is  $\mathbf{W} = [\begin{array}{ccc} 0.334 & 0.166 & 0.166 & 0.334 \\ \end{array}]$ .

The detection performance for the proposed sequential cooperative sensing scheme in 50~Hz correlated channel with the optimized weight vectors is shown in Fig. 5. The proposed scheme provides significantly better detection performance

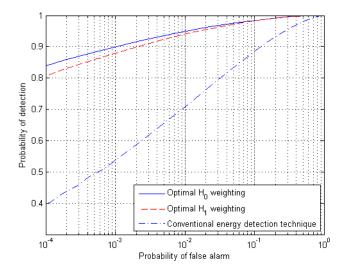


Fig. 5. Detection performance for proposed scheme in  $50\ Hz$  correlated channel case

than the conventional cooperative OR-rule energy detection scheme. This is due to the use of past observations that are available locally. The equal weighting is shown to outperform the weighting that minimizes the  $H_1$  variance. Moreover, the advantage of using equal weighting is that it is SNR and channel correlation independent, while the optimization by search algorithm for minimum variance in  $H_1$  case is dependent upon the SNR and channel correlation.

# B. Simulation Results with varying primary user activity

In this section, the primary user's activity is considered and the detection performance for the proposed scheme is evaluated, for both equal and exponential weighting.

1) Equal weighting: We first investigate the performance of our proposed scheme using equal weighting when primary user's activity is considered. The detection performance in 50 Hz correlated channel case is shown in Fig. 6. The primary user activity  $\alpha$  and  $\beta$  are both set equal to 0.001 and 0.1 for the case that primary user seldom and regularly changes its state respectively.

Simulation result shows that the primary user activity affects the detection performance. When  $\alpha$  and  $\beta$  equal to 0.1, the detection performance is worsened and it is even worse than the conventional energy detection technique which does not take any previous observation. This is because when the primary user activity varies a lot, equal weighting relied too heavily on past observation that is already outdated. Hence, it is better to simply use the current observation. However, when primary user movement low in the  $\alpha$  and  $\beta$  equal to 0.001 case, the proposed scheme with equal weighting is better than the conventional technique in high false alarm region, but still worse than the conventional technique in low false alarm region.

2) Exponential weighting: The detection performance with the proposed exponential weighting vector is shown in Fig. 7.

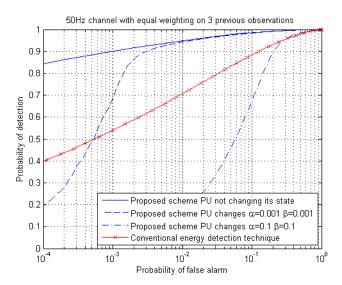


Fig. 6. Detection performance on equal weighting with primary user activity in  $50\ Hz$  correlated channel

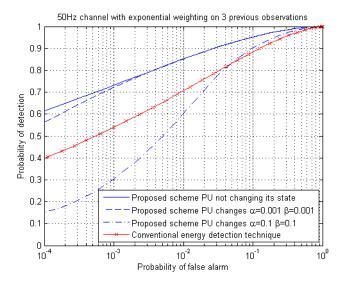


Fig. 7. Detection performance on exponential weighting with primary user activity in  $50\ Hz$  correlated channel

The simulation parameters are the same with the one above with equal weighting.

The result shows that the proposed exponential weighting provides an improvement on detection performance compared to Fig. 6 on the same primary user activity. When  $\alpha$  and  $\beta$  equal to 0.1, the proposed exponential weighting improves the detection performance in the high false alarm area above the conventional technique. Moreover, it helps improves the detection performance in the low false alarm area, also for the  $\alpha$  and  $\beta$  equal to 0.001 case.

By applying exponential weighting, the more recent observations get higher weighting than the older observation, while the equal weighting one gives the same weight for every observation. When primary user changes its activity between  $H_0$  and  $H_1$ , previous observations in further time slot can be outdated and affect the sum T, which then also affect sensing result and the detection performance.

# VI. CONCLUSIONS

In this paper, a novel sequential cooperative spectrum sensing technique is proposed. By taking energy observations in previous sensing slots, we exploit the time varying nature of the channel. First, when primary user's activity is not varying, we show that the equal weighting is the optimal weighting vector for aggregating the observations. However, when primary user's activity is considered, the primary user's activity affects the detection performance. To improve the detection performance, we propose the exponential weighting, which gives higher weighting to the current observation and reduces the reliance on the previous ones. Simulation results show that the performance of the proposed schemes improve the detection performance.

#### APPENDIX

Appendix A

First, the variance of T in correlated channel  $H_1$  case is derived

$$Var\left(T\right) = Var\left(\sum_{i=1}^{N} w_i O_i\right)$$

From the property for variance of the weighted sum of several random variables [10], it leads to that

$$Var(T) = \sum_{i=1}^{N} w_i^2 \cdot Var(O_i)$$

$$+2 \sum_{i,j:i < j} w_i w_j \cdot Cov(O_i, O_j) \qquad (4)$$

For equal weighting, the weighting vector is  $\mathbf{W} = \begin{bmatrix} \frac{1}{N} & \frac{1}{N} & \dots & \frac{1}{N} \end{bmatrix}$ , and thus

$$Var\left(T\right) = \sum_{i=1}^{N} \left(\frac{1}{N}\right)^{2} \cdot Var\left(O_{i}\right)$$

$$+2 \sum_{i,j;i < j} \left(\frac{1}{N}\right)^{2} \cdot Cov\left(O_{i}, O_{j}\right)$$
(5)

In quasi-static fading channel, observation in each frame is independent to its previous and next observation. Hence  $Cov\left(O_i,O_j\right)=0$  for all i and j, then

$$Var\left(T\right) = \left(\frac{1}{N}\right)^{2} \cdot \sum_{i=1}^{N} Var\left(O_{i}\right)$$

For each  $O_i$ , its variance is the same such that  $Var(O_1) = Var(O_2) = ... = Var(O)$  which leads to

$$Var\left(T\right) = \frac{Var\left(O\right)}{N} \tag{6}$$

This concludes that when equal weighting is applied in  $H_0$  and  $H_1$  in quasi-static channel, variance of T equals to variance of O divided by the number of total observations taken.

Appendix B

To prove that equal weighting is the optimal weighting in case when primary user is idle  $(H_0)$  and when primary user is active  $(H_1)$  in quasi-static channel.

In quasi-static fading case, there is no correlation between each observation, hence

$$Var(T) = \sum_{i=1}^{N} w_i^2 \cdot Var(O_i).$$

The optimization criteria to minimize variance of T is

Minimize 
$$\sum_{i=1}^{N} w_i^2 \cdot Var\left(O\right)$$
Minimize 
$$\sum_{i=1}^{N} w_i^2$$
s.t.  $(w_1 + w_2 + ... + w_N) = 1$ 

By using standard Lagrange multiplier, the solution is

$$w_A = w_B = \dots = w_N = \frac{1}{N}.$$
 (7)

Hence, in quasi-static fading channel, equal weighting is optimal as it gives the lowest variance.

#### REFERENCES

- I. Akyildiz, W. Lee, M. Vuran and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks*, vol. 50, no. 13, pp. 2127-2159, 2006.
- [2] J. Mitola III, "Software radios: Survey, critical evaluation and future directions," Aerospace and Electronic Systems Magazine, IEEE, vol. 8, no. 4, pp. 25-36, 1993.
- [3] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio networks", New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on , pp.137-143, 8-11 Nov. 2005
- [4] S. Mitsuya, K. Kobayashi, T. Yamazato and M. Katayama, "Improvement of sequential-test-based cooperative spectrum sensing systems in band limited control channels," *Communications and Information Technologies* (ISCIT), 2010 International Symposium on , vol., no., pp.968-973, 26-29 Oct. 2010
- [5] H. Urkowitz, "Energy detection of unknown deterministic signals," Proceedings of the IEEE, vol. 55, no. 4, pp.523-531, 1967.
- [6] Z. Wei, R. Mallik and K. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," Wireless Communications, IEEE Transactions on , vol.8, no.12, pp.5761-5766, December 2009
- [7] M.G. Khoshkholgh, K. Navaie and H. Yanikomeroglu, "On the impact of the primary network activity on the achievable capacity of spectrum sharing over fading channels," Wireless Communications, IEEE Transactions on, vol.8, no.4, pp.2100-2111, April 2009
- [8] J. Proakis and D. Manolakis, *Digital signal processing: principles, algorithms, and applications.* Prentice Hall, 1996.
- [9] F.F. Digham, M.-S. Alouini and M.K. Simon, "On the energy detection of unknown signals over fading channels," *Communications*, 2003. ICC '03. IEEE International Conference on , vol.5, no., pp. 3575- 3579 vol.5, 11-15 May 2003
- [10] K. Subrahmaniam and K. Kocherlakota, A primer in probability. CRC, 1990