Bargaining Solutions for Multicast Subgroup Formation in LTE

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***Abstract*—This paper focuses on the design of radio resource management policies for efficient delivery of multicast services in Long Term Evolution (LTE) systems. In order to find alternative solutions to the Conventional Multicast Scheme (CMS), an effective framework is proposed that splits multicast destinations into different subgroups, depending on the User Equipments (UEs) channel quality, and applies a subgroup based Adaptive Modulation and Coding (AMC) scheme. This approach allows a more efficient spectrum exploitation. Game theoretic notions are used for the multicast subgroups formation and the resource allocation problems. The effectiveness of the proposed framework is evaluated and four different bargaining solutions are compared in terms of data rate, fairness, and subgroup configurations.**

***Index Terms*—LTE, Multicast, Game Theory, Scheduling**

I. Introduction

Recently, the growing demand for group-oriented services (i.e., multicast and broadcast) has led to the definition of new standards and multimedia applications for mobile terminals. Long Term Evolution (LTE) [1] is expected as the most promising wireless system able to support such services in the next years. Being characterized by a simple but innovative network infrastructure, it increases the performance of 3G technology by introducing several benefits: *(i)* low latency, *(ii)* high spectrum efficiency, *(iii)* high data rate.

In order to meet such expectations, Orthogonal Frequency Division Multiple Access (OFDMA) plays an important role in LTE systems owing to its great flexibility in spectrum management (e.g., high robustness against fading phenomena, high scalability). Spectrum is divided into several sub-channels, named Resource Blocks (RBs), each one formed by 12 consecutive and equally spaced sub-carries. RBs are allocated to a given User Equipment (UE) by radio resource management unit according to the channel conditions experienced by the mobile terminal. In details, UE estimates the Signal to Interference and Noise Ratio (SINR) and forwards the Channel Quality Indicator (CQI) feedback to the eNodeB (the LTE base station) which indicates the maximum *modulation and coding scheme* (MCS) supported by the UE.

Despite the many advantages introduced by OFDMA, still important and challenging research issues derive from the different fairness requirements and diverse channel quality experienced by multicast users belonging to the same Multicast Group (MG). In Conventional Multicast Scheme (CMS) [2] data rate is usually restricted by the data rate of the user with the worst channel condition. As a consequence, though CMS might be considered as "fair" since all the UEs are served at the same data rate, it achieves a poor performance in terms of network throughput and satisfaction experienced by the users with good channel conditions.

The aforementioned issues motivated the recent research activities on multicast radio resource allocation policies. An interesting approach to overcome limitations of CMS algorithm is based on multicast *subgrouping*: the idea is to split multicast destinations into different subgroups depending on the UE channel quality. Open issue for subgrouping techniques is the evaluation of the appropriate *(i)* number of subgroups to enable with their respective MCSs and *(ii)* number of RBs to assign to each subgroup. Interesting studies that investigate subgrouping resource allocation are reported in [3] and [4] for High-Speed Downlink Packet Access (HSDPA) and OFDMA-based systems, respectively. Such approaches overcome the limitation of the CMS policy through the implementation of a Maximum Throughput (MT) algorithm which maximizes the network throughput but neglects other important performance parameters such as fairness.

In the present paper a novel approach is introduced, based on game theoretic solutions which intrinsically consider the differences between involved UEs. The focus will be on the class of problems belonging to the Non-Transferable Utility (NTU) game theory, known as *bargaining problems*. Four different bargaining solutions, each one promoting different objectives, will be compared in terms of throughput and fairness performance. Cooperative game theory and bargaining have found their natural application in several wireless communication fields and in particular also in scheduling and bandwidth allocation problems. In particular, already in 1991 in [5] the Nash Bargaining Solution was applied to scheduling problems in Jackson networks. Bargaining solutions have been also applied to OFDMA networks in [6] and to radio aware scheduling in WiMAX networks in [7]. An application to LTE OFDMA scheduling for unicast communications has been introduced in [8]. Nonetheless, to the best of author's knowledge, no contribution is available from the literature addressing a game theoretic approach for subgroups definition and resource allocation in LTE networks.

The remaining of the paper is structured as follows. In the next section, the reference system model is introduced in more details. Section III introduces the considered bargaining solutions, while results of a performance evaluation are given in section IV. Conclusive remarks are given in the last section.

II. System model

Let us consider a general case where the MG is composed by N UEs. Let c be the CQI value associated to a generic user, where c can vary from 1 to 15 in the LTE system. For a given CQI value c, the attainable data rate depends on the number of assigned RBs, which can vary from 1 to R, where R depends on the system bandwidth configuration [1]. Table I lists the CQI values defined in LTE systems with the related minimum and maximum data rate.

TABLE I
CQI - Data Rates for 10 MHz Channel Bandwidth

CQI value	MCS	Minimum Rate [kbps]	Maximum Rate [kbps]
1	QPSK/0.076	25.59	1279.32
2	QPSK/0.120	39.38	1968.96
3	QPSK/0.190	63.34	3166.80
4	QPSK/0.300	101.07	5053.44
5	QPSK/0.440	147.34	7366.80
6	QPSK/0.590	197.53	9876.72
7	16-QAM/0.370	248.07	12403.44
8	16-QAM/0.480	321.57	16078.44
9	16-QAM/0.600	404.26	20212.92
10	64-QAM/0.450	458.72	22936.20
11	64-QAM/0.550	558.15	27907.32
12	64-QAM/0.650	655.59	32779.32
13	64-QAM/0.750	759.93	37996.56
14	64-QAM/0.850	859.35	42967.68
15	64-QAM/0.930	933.19	46659.48

The proposed resource allocation starts from the *CQI collection phase*, where the eNodeB collects the CQI feedbacks from each UE belonging to the MG. At the end of this phase, the required information to perform the resource allocation is available and the *subgroups creation phase* can begin. This is the key phase where the resource allocation policy splits the multicast destinations into subgroups, by accounting for the collected CQI feedbacks; then it selects the appropriate number of RBs and the transmission parameters for each enabled subgroup. The assumption at the basis of the subgrouping policy is that all the UEs experiencing the same CQI value will be associated to the same subgroup. Of course, a subgroup could serve UEs with different CQI values. With this assumption, the possible number of subgroups to activate varies from 1 to 15. Finally, once subgroups are created, the multicast service is provided during the *radio resource allocation phase*.

A. Subgrouping and Resource Allocation Algorithm

Let $\mathbf{V} = \{v_1, v_2, \ldots, v_{15}\}$ be the vector containing the number of UEs per CQI level. Being G a generic bargaining solution to be adopted, the proposed framework foresees the following steps:

1) Consider the $\tilde{\mathcal{S}}$ set representing all the possible configurations of subgroups to be activated. Let $\mathcal{S} \subseteq \tilde{\mathcal{S}}$ be the subset of all potential configurations belonging to $\tilde{\mathcal{S}}$ which assure, according to the $\mathbf{V}$ vector, that: *(i)* all UEs of the MG are served; *(ii)* each UE supports the CQI it is associated to; *(iii)* only subgroups with a CQI reported by at least one of the UEs are considered.
2) For each configuration $S_i \in \mathcal{S}$, the generic UE with a CQI value equal to c is assigned to the subgroup j associated to the closest CQI supported by the user. Let $V_j^{S_i}$ be the number of UEs served by the subgroup j in the configuration S_i;
3) For every configuration $S_i \in \mathcal{S}$, define the best resource allocation solution given by the selected game theoretic bargaining solution G (details on the game theoretic bargaining solutions are given in section III);
4) Select the configuration $S_t \in \mathcal{S}$ that maximizes a performance index P. S_t determines the CQIs to be activated and the resources allocated to each multicast group according to the bargaining solution G.

In the present paper the chosen index P is the *Aggregate Utility (AU)*, that is the total utility of the system for a given configuration S_i. Any of the activated CQIs will be considered as a player of the game, thus, the utility (or payoff) assigned to each player is computed as the total data rate obtained at the specific CQI. The total utility for a generic player j of the generic configuration S_i is strictly related to the number of associated UEs and the number of assigned RBs:

$$u_j^{S_i} = V_j^{S_i} \cdot RB_j^{S_i} \cdot b_j^{S_i} \tag{1}$$

where $b_j^{S_i}$ is the data rate achieved with 1 RB assigned to the CQI related to the subgroup j (the *Minimum Rate* column in Table I), and $RB_j^{S_i}$ are the RBs assigned to the *j*-th subgroup ($1 \leq RB_j^{S_i} \leq R$) by the game theoretic solution G. With this utility definition, the Aggregate Utility for the configuration S_i is defined as: $AU^{S_i} = \sum_{j=1}^{J^{S_i}} u_j^{S_i}$, where J^{S_i} is the total number of enabled subgroups.

III. Bargaining Solutions

In this section the game theoretic notions for the reference bargaining problem are introduced. In a cooperative bargaining game, players bargain with each other and if an agreement is reached, then players act accordingly, otherwise they act in a non-cooperative way. The reached agreement must be binding and players are not allowed to deviate from what they agreed on. In the reference problem, the centralized network coordination guarantees that the solution is adopted without further incentive for cooperation needed by the users.

A bargaining problem is characterized as follows. Let $\mathbf{K} = \{1, 2, ..., K\}$ be the set of players and $\mathbf{F} \subset \mathbb{R}^K$ be a closed and convex set, also called *feasible set*, that is the set of utilities the players potentially can reach following any bargaining process. Let $\mathbf{u} = (u_1, u_2, ..., u_K) \in \mathbf{F}$ be a generic possible game theoretic utility distribution solution also called *agreement point* and let $\mathbf{d} = (d_1, d_2, ..., d_K) \in \mathbb{R}^K$ be the *disagreement point* representing the utility the players will obtain when they don't reach an agreement; where the utility for any player at any agreement point $\mathbf{u}$ is at least as much as the utility achieved at the disagreement point $\mathbf{d}$. For the reference problem, each element of $\mathbf{u}$ is defined as in equation (1), thus the feasible set $\mathbf{F}$ collects the possible utilities assigned to each activated CQI determined by the RB assignment to each group. To best model the *disagreement point*, following the choice that all users in the network should be served, the disagreement point $\mathbf{d}$ assigns the minimum of one RB to the single player at the given CQI. The pair $(\mathbf{F}, \mathbf{d})$ is called a K-person bargaining problem and a *bargaining solution* is a map that assigns a solution to the bargaining problem. Many solutions for bargaining problems exist, but the four most important solutions are considered in this paper; each of them defined through axiomatic definitions.

A. Nash Bargaining Solution

Definition 1: **Nash Bargaining Solution (NBS)**. $\mathbf{u}$ is said to be a **NBS** in $\mathbf{F}$ for $\mathbf{d}$, i.e. $\mathbf{u} = \phi(\mathbf{F}, \mathbf{d})$, if the following axioms are satisfied:

1) *Individual Rationality*: $u_i \geq d_i$, $\forall i$.
2) *Feasibility*: $\mathbf{u} \in \mathbf{F}$.
3) *Pareto Optimality*: if there exists $\hat{\mathbf{u}} \in \mathbf{F}$, such that $\hat{u}_i \geq u_i, \forall i$ then $\hat{u}_i = u_i, \forall i$.
4) *Symmetry*: if $\mathbf{F}$ is symmetric with respect to i and j, $d_i = d_j$, and $\hat{\mathbf{u}} = \phi(\mathbf{F}, \mathbf{d})$, then $\hat{u}_i = \hat{u}_j$.
5) *Invariance to affine transformations*: for any linear scale transformation ψ, $\psi(\phi(\mathbf{F}, \mathbf{d})) = \phi(\psi(\mathbf{F}), \psi(\mathbf{d}))$.
6) *Independence of irrelevant alternatives*: if $\hat{\mathbf{u}} \in \mathbf{F'} \subset \mathbf{F}$, $\hat{\mathbf{u}} = \phi(\mathbf{F}, \mathbf{d})$, then $\hat{\mathbf{u}} = \phi(\mathbf{F'}, \mathbf{d})$.

Axioms 1-3 represent the binding conditions for any agreement point given by a bargaining solution, while axioms 4-6 are the so called fairness axioms. In particular, the symmetry axiom states that if the feasible set is completely symmetric for all users and they have the same disagreement point then the NBS is the same for all users. The invariance to affine transformations states that the NBS is scale-invariant. Finally, the independence axiom states that eliminating those solutions that would not be chosen from the feasible set does not affect the NBS.

Theorem 2: **Existence and Uniqueness of NBS**. There is a unique solution function $\phi(\mathbf{F}, \mathbf{d})$ that satisfies all six axioms in Definition 1, namely the maximizer of the Nash Product:

$$\phi(\mathbf{F}, \mathbf{d}) = arg \max_{\mathbf{u} \in \mathbf{F}, u_i \geq d_i \forall i} \prod_{i=1}^{K} (u_i - d_i) \tag{2}$$

The intuitive idea is that after a minimal requirement is satisfied for all players, the remaining resources are allocated according to the conditions of each player. An interesting property of the NBS is that it satisfies the strong individual rationality, which means that all players have an utility larger than the disagreement point. As demonstrated in the literature, the *proportional fairness* in scheduling problems (see e.g. [6] and [8]) is a special case of the NBS. Further discussion about the convexity of $\mathbf{F}$ for the specific problem and the mapping of an integer number of RBs to the bargaining solution is not presented here due to space constraints.

B. Kalai-Smorodinsky Solution

Axiom 6 in Definition 1 has suffered some criticism [9] because it is not taking into account how much other players have given up. For this reason, another interesting fairness criterion was introduced by modifying this axiom, namely the Kalai-Smorodinsky, whereby utilities are proportional to their maximum possible values. In fact, this solution, assigns the maximal point of the feasible set on the segment connecting the disagreement point to the so-called *utopia point*. This ideal point, in which each player would get his maximum possible benefit, is defined as $h_i(\mathbf{F}, \mathbf{d}) = \max\{u_i | \mathbf{u} \in \mathbf{F}\}, \forall i \in \mathbf{K}$. For the reference problem, the *utopia point* is considered as the point where all available RBs are assigned to the given CQI value. From the axiomatic point of view, the independence of irrelevant alternatives in the NBS is substituted by the so-called *Individual monotonicity* axiom:

Definition 3: **Kalai-Smorodinsky Solution (KSS)**. $\mathbf{u}$ is said to be a **KSS** in $\mathbf{F}$ for $\mathbf{d}$, i.e. $\mathbf{u} = \phi(\mathbf{F}, \mathbf{d})$, if the following axioms are satisfied:

1) *Individual Rationality*.
2) *Feasibility*.
3) *Pareto Optimality*.
4) *Symmetry*.
5) *Invariance to affine transformations*.
6) *Individual monotonicity*: if $\mathbf{G} \subset \mathbf{F}$ and $\mathbf{h}(\mathbf{F}, \mathbf{d}) = \mathbf{h}(\mathbf{G}, \mathbf{d})$ then $\phi(\mathbf{F}, \mathbf{d}) \geq \phi(\mathbf{G}, \mathbf{d})$.

$$\phi(\mathbf{F}, \mathbf{d}) = \max_{\mathbf{u} \in \mathbf{F}, u_i \geq d_i \forall i} \left\{ |\mathbf{u} - \mathbf{d}|, \left(\frac{u_i - d_i}{h_i - d_i} = \frac{u_j - d_j}{h_j - d_j} \right), \forall i, j \in \mathbf{K} \right\} \tag{3}$$

The *individual monotonicity* property implies that if in a configuration $S_i \in \mathcal{S}$ the worst performing player improves the CQI level, its utility would be increased without any reduction for the other players. An alternative formulation for the KSS can be given in the form of a weighted max-min fairness solution, which focuses on improving the payoff of the weakest players, weighted according to the best-case payoff:

$$KSS = \max_{\mathbf{u} \in \mathbf{F}} \min_{j \in \mathbf{K}} \left(\frac{u_j - d_j}{h_j - d_j} \right) \tag{4}$$

C. Egalitarian Solution

The third bargaining solution considered in this work is the Egalitarian solution assigning the point in the feasible set where all players achieve maximal equal increase in utility with respect to the disagreement point.

Definition 4: **Egalitarian Solution (ES)**. $\mathbf{u}$ is said to be a **ES** in $\mathbf{F}$ for $\mathbf{d}$, i.e. $\mathbf{u} = \phi(\mathbf{F}, \mathbf{d})$, if the following axioms are satisfied:

1) *Individual Rationality*.
2) *Feasibility*.
3) *Pareto Optimality*.
4) *Symmetry*.
5) *Strong monotonicity*: if $\mathbf{G} \subset \mathbf{F}$ then $\phi(\mathbf{F}, \mathbf{d}) \geq \phi(\mathbf{G}, \mathbf{d})$.

$$\phi(\mathbf{F}, \mathbf{d}) = \max_{\mathbf{u} \in \mathbf{F}, u_i \geq d_i \forall i} \{ |\mathbf{u} - \mathbf{d}|, (u_i - d_i = u_j - d_j), \forall i, j \in \mathbf{K} \} \tag{5}$$

The *strong monotonicity* property implies that if in a configuration $S_i \in \mathcal{S}$ the worst performing player improves the

CQI level, all the players would improve their utility. Also for the ES, an alternative formulation can be given in the form of a strict max-min fairness:

$$ES = \max_{\mathbf{u} \in \mathbf{F}} \min_{j \in \mathbf{K}} (u_j - d_j) \tag{6}$$

D. Utilitarian Solution

The last bargaining solution considered in this work is the Utilitarian solution.

Definition 5: **Utilitarian Solution (US)**. $\mathbf{u}$ is said to be a **US** in $\mathbf{F}$ for $\mathbf{d}$, i.e. $\mathbf{u} = \phi(\mathbf{F}, \mathbf{d})$, if the following axioms are satisfied:

1) *Individual Rationality*.
2) *Feasibility*.
3) *Pareto Optimality*.
4) *Symmetry*.
5) *Linearity*: $u(F_i + F_j) = u(F_i) + u(F_j)$.

The Linearity axiom states that the players are indifferent between solving problems separately or in a single problem. The Utilitarian solution maximizes the sum of the utilities and is, therefore, also called as *Maximum throughput scheduler* in scheduling problems (e.g. in [8]):

$$\phi(\mathbf{F}, \mathbf{d}) = arg \max_{\mathbf{u} \in \mathbf{F}, u_i \geq d_i \forall i} \sum_{i=1}^{K} u_i \tag{7}$$

IV. Performance Analysis

The performance evaluation is based on the guidelines for the multi-cell system model defined in [10]. According to the LTE assumptions, 50 RBs are available on a bandwidth equal to 10 MHz. Channel conditions for each UE are evaluated in terms of Signal to Interference and Noise Ratio (SINR) experienced over each sub-carrier where path loss, shadow and fast fading affect the signal reception. The Exponential Effective SIR Mapping (EESM) is adopted in order to map the channel state into the instantaneous effective SINR, and, finally, this is mapped onto the CQI level ensuring a block error rate (BLER) smaller than 10% [11]. Outputs are achieved by averaging a sufficient number of simulation results to obtain a 95% confidence interval. More details on simulation assumptions are listed in Table II.

A numerical evaluation has been performed in Matlab, aiming at understanding how the proposed bargaining solutions answer to the needs of the investigated scenario. Main focus of the evaluation is on the data rate obtained by the single subgroups and the fairness in the proposed solutions. In particular, the following Group Fairness Index (GFI) is defined as term of comparison among the different solutions:

$$GFI^{S_i} = \frac{(\sum_{j=1}^{J^{S_i}} u_j^{S_i})^2}{J^{S_i} \sum_{j=1}^{J^{S_i}} (u_j^{S_i})^2} \tag{8}$$

where J_{S_i} is the number of activated subgroups in configuration S_i.

TABLE II
Main Simulation Assumption

Parameters	Value
Distance attenuation	128.1+37.6*log(d), d [km]
Shadow fading	Log-normal, $\sigma = 8$ [dB]
Fast Fading	ITU-R PedB (extended for OFDM)
Cell layout	Hexagonal, 18 Interfering cells
Inter site Distance	1732, Macro Case 3
CQI scheme	Full Bandwidth
eNodeB transmit power	20 W, 13 dB
Maximum antenna gain	11.5 dB
Thermal Noise	-100 dBm
Multicast group size	100

A. A case of study: on-campus distributed users

A sample study case is considered, with $N = 100$, representing a typical on-campus scenario where students are distributed over a concentrated area and forming an LTE multicast group; a sample UE distribution is plotted in Fig. 1.

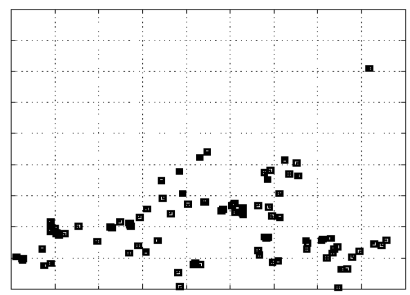
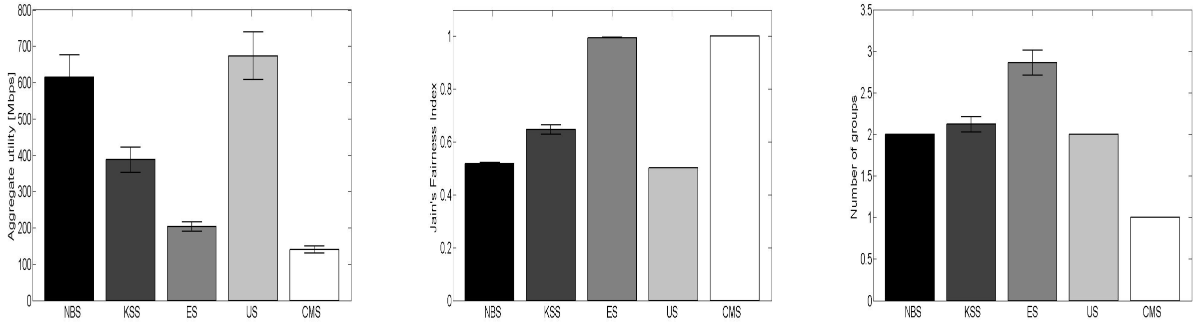


Fig. 1. Snapshot of users distribution in the network.

The first evaluation is in terms of overall system utility. In Fig. 2 (a) the four different bargaining solutions and the conventional CMS scheme are compared. The overall system utility reflects the total aggregate data rate that the system can handle according to the subgroup definition and the resource allocation proposed by each solution. As it can be noticed, all the bargaining solutions proposed with the novel framework outperform the CMS solution. As expected the US reaches the highest value, 674 Mbps in the average, followed by the NBS with 615 Mbps in the average. The ES has the lowest performance among the proposed bargaining solutions, 203 Mbps on average, and the KSS shows an average aggregate data rate of 387 Mbps. The CMS instead, reaches an average 140 Mbps aggregate data rate.

When focusing on the fairness aspect, the behavior is somewhat dual to the data rate results. As it clearly appears in Fig. 2 (b), the US has a very low value for the GFI (i.e. 0.5 on average), the NBS has almost the same behavior, while the ES reaches a value very close to 1, as for the CMS solution. Also this result is expected, since the US is maximizing the throughput, while the ES guarantees equal increase in the utility distribution for the players w.r.t. the disagreement point (see section III-C for the details). The KSS falls again in between the other solutions, showing an average value for the GFI of about 0.65. Interesting is also to see how the solutions



(a) Aggregate utility for each solution. (b) Group fairness index for each solution. (c) Number of subgroups formed in the system.

Fig. 2. Performance figures.

differ in number of activated multicast subgroups. As it can be noticed in Fig. 2 (c), the ES has the highest average number of activated subgroups, while both the US and the NBS actually always choose only two subgroups to be formed.

Next a sample study case is shown, in order to compare the behavior of the four bargaining solutions in terms of number of RBs assigned per activated CQI, number of UE per group and data rate per activated CQI in Fig. 3. It can be noticed that the RBs assigned by the NBS and the US are very close. The same behavior is observed for all the tested cases and is independent from the number of activated groups and the number of UE per group. Interesting it is also to see how the ES is the only solution that activates more than two groups. For the US we observe a group with low CQI (CQI=1 in this case) with only one RB assigned (notice that a requirement for the proposal is that none of the UE is excluded from the system), and one with high CQI (CQI=11 in this case) to which all the remaining 49 RBs are assigned. Also this choice is expected as this maximizes the aggregate utility for the system, while the choice of which CQIs to activate depends on the specific distribution of users.

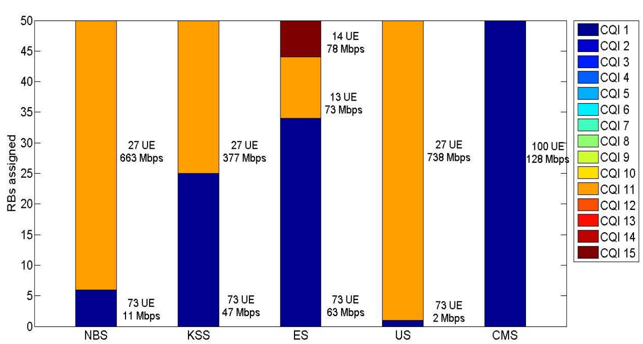


Fig. 3. RB assignment per activated multicast subgroup in the system.

As a final analysis it is worth commenting on the aggregate utility for the activated CQIs according to the solutions proposed. This gives a clearer idea of how the disparities in the per group utility are the highest for the US and the lowest for the ES; please refer to the values reported in Fig. 3.

V. Conclusions

In this paper an effective framework is proposed for an efficient delivery of multicast services in LTE network based on subgroups formation. The use of cooperative bargaining solutions has been investigated and four game theoretic solutions have been considered. Besides showing how the proposed framework outperforms the Conventional Multicast Scheme in terms of system utility, a detailed analysis on the subgroups resource allocation and performance has been evaluated. It is worth noting that data rate and fairness are conflicting parameters and none of the solutions outperforms the other with respect to both parameters simultaneously. From the network point of view, maximizing the throughput could represent the main objective, thus US is the solution to be adopted. From the UE point of view instead, fairness in the utility distribution is also of high concern, making the ES preferable. The NBS and KSS give somehow trade-off solutions in matching both the network and the user interests.

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