# Spatial Multiplexing with Opportunistic Multiuser Scheduling in Ad Hoc Networks

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*Abstract*—**In this paper, we investigate the performance of multiuser scheduling with open-loop spatial multiplexing in ad hoc networks. Closed-form expressions for outage probability and spatial throughput are derived for spatial multiplexing transmission with zero forcing (ZF) receiver and multiuser scheduling. Monte Carlo simulation results reveal that these expressions accurately demonstrate the performance of multiuser scheduling with open-loop spatial multiplexing. Based on these expressions, transmission capacity performance is analyzed and the effects of system parameters over network performance metrics are studied. Our finding indicates that spatial multiplexing with opportunistic multiuser scheduling can provide boosted network performance for ad hoc network by exploiting multiuser diversity gain, whilst requiring very low feedback requirements.**

## I. Introduction

Multiple-input multiple-output (MIMO) schemes are promising technologies which can offer significant performance gains by providing high data rates and reliable transmission links in wireless communication systems [1]. In multiuser environments, these performance gains can be further enhanced when multiuser scheduling is employed. In the context of point-to-point systems operating in absence of interference, research has been done to characterize the benefits of opportunistic multiuser scheduling for the broadcast channel scenarios [2].

In ad hoc networks, the notion of transmission capacity has been introduced in [3] as a performance metric, defined as the maximum allowable spatial density of successful transmissions multiplied by the corresponding data rate under an outage constraint. Based on this notion, various prior work has been done on the evaluation of different MIMO technologies in ad hoc network. In [4], different spatial diversity approaches are considered, including single-antenna transmission with maximum ratio combining receivers (MRC) and orthogonal space-time block codes (OSTBC). In [5], spatial multiplexing schemes with MRC receivers and ZF receivers are considered and new closed-form results for spatial multiplexing in ad hoc network are derived. Based on the results of [5], [6] studies spatial multiplexing with transmit antenna selection schemes in ad hoc network and characterizes the benefits of transmit antenna selection exploiting limited-feedback. Multiuser MIMO transmission is analyzed in [7], in which ZF beamforming with limited feedback scheme is considered in broadcast channel.

All the above work addressed multi-antenna transmission with either multiuser scheduling in point-to-point scenario or non-scheduling in ad hoc networks. However, few work has been focused on the performance of multiuser scheduling with MIMO spatial multiplexing in ad hoc network.

In this paper, we aim to investigate the performance of multiuser scheduling with spatial multiplexing in ad hoc network, and to characterize the benefits which it can achieve over single user scenarios. In particular, we assume a non-cooperative network scenario, each transmitter communicating with a certain number of corresponding receivers while all other links in the network are treated as interference. We adopt the randomized network model, assuming that the transmit nodes are randomly distributed on an infinite 2-D plane according to a homogeneous Poisson point process (PPP) [8]. This approach has the advantage of corresponding to realistic network scenarios. Moreover, randomized network model is simple enough for the evaluation of network performance measures.

On the basis of randomized network model, we consider spatial multiplexing scheme in multiuser environment, in which receive nodes are located at a certain distance away from the transmitter. We consider spatial multiplexing scheme in conjunction with low complexity linear zero forcing (ZF) receivers. Receivers report incomplete channel state information (CSI), namely signal to interference-plus-noise ratio (SINR), to the transmitter. With limited feedback in this practical scenario, precoding is not available at the transmitter side, which can be seen as open-loop spatial multiplexing. In the scheduling algorithm, the transmitter sends data streams to the receiver with the optimum corresponding SINR.

The key finding of this paper is that opportunistic multiuser scheduling can provide boosted network performance by exploiting multiuser diversity gain, whilst requiring very low feedback. We establish these results by deriving closed-form expressions for the outage probability and spatial throughput. Based on these expressions, we study the transmission capacity of the network and investigate the effects of system parameters over these network performance metrics.

The remainder of this paper is organized as follows. The network structure and system model are described in Section II. The performance of spatial multiplexing with multiuser scheduling is investigated in Section III. Simulation results are given and analyzed in Section IV. Finally, our work is concluded in Section V.

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## II. Network and System Model

### A. Network Model

We consider an ad hoc network comprising transmit nodes placed according to a homogeneous PPP denoted as $\Pi_\lambda$ of intensity $\lambda$ in $\mathbb{R}^2$. In this model, each transmitter communicates with its corresponding receivers, forming a serving cluster. Since assuming the same pathloss will not change the multiuser diversity order [9], we assume that all receive nodes are located at $r$ distance away from the corresponding transmit node for simplicity. Each transmitter chooses a receiver from its serving cluster and transmits with probability $p$ according to a slotted ALOHA medium access protocol. In such case, the effective intensity of active transmitter is $p\lambda$.

Our object is to investigate network performances such as outage probability, spatial throughput and transmission capacity. To obtain these metrics, it is sufficient to focus on a typical transmitter-receivers cluster due to the stationarity of the PPP [10]. In addition, the statistics of the received signal are the same for each receiver in the cluster. Thus, it is reasonable to place the typical transmit node, denoted as $T_0$, at the origin and corresponding receive node at distance $r$ away from $T_0$. The receive node associated with $T_0$ are denoted as $R_0^{(l)}$, where $l$ is the receiver index in the cluster. The transmitted signal is attenuated by $r^{-\alpha}$ where $\alpha > 2$ is the pathloss exponent. $\mathbf{H}_i^{(l)}$ models the Rayleigh fading channel matrix between $T_i$ and $R_0^{(l)}$ $(i \geq 0)$, where for $i > 0$, $T_i$ represents interference from other clusters.

### B. Spatial Multiplexing with ZF Receivers

At the physical layer, we consider spatial multiplexing scheme where each transmitter is equipped with $M$ antennas to send $M$ independent data streams. Each transmitter is assumed to use the same transmission power $P$ equally divided among antennas. Each receiver is equipped with $N\,(\geq M)$ antennas, so we have the channel matrices $\mathbf{H}_i^{(l)} \overset{d}{\sim} \mathcal{CN}_{N\times N}\left(\mathbf{0}_{N\times N}, \mathbf{I}_{N\times N}\right)$ $(i \geq 0)$, which are independent and identically distributed (i.i.d.). The received signal for the $k$th stream at the typical receiver $R_0^{(l)}$ is given by

$$\mathbf{y}_{0,k}^{(l)} = \sqrt{\frac{1}{r^\alpha}}\mathbf{h}_{0,k}^{(l)}x_{0,k} + \sqrt{\frac{1}{r^\alpha}}\sum_{j=1,j\neq k}^{M}\mathbf{h}_{0,j}^{(l)}x_{0,j} + \sum_{i\in\Pi_\lambda/T_0}\sqrt{\frac{1}{|X_i|^\alpha}}\sum_{j=1}^{M}\mathbf{h}_{i,j}^{(l)}x_{i,j} + \mathbf{n}_0^{(l)}, \quad (1)$$

where $\mathbf{h}_{i,j}^{(l)} \in \mathbb{C}^{N\times 1}$ is the $j$th column of $\mathbf{H}_i^{(l)}$ representing the vector channel from the $j$th transmitting antenna of the $i$th transmitter to $R_0^{(l)}$. $x_{0,k}$ is the desired signal, while $x_{0,j}$ is the inter-stream interference satisfying $\mathbb{E}\left[|x_{0,k}|^2\right] = \mathbb{E}\left[|x_{0,j}|^2\right] = \frac{P}{M}$. $x_{i,j}$ is the inter-cluster interference signal sent from the $j$th transmitting antenna of the $i$th transmitter $(i > 0)$, also satisfying $\mathbb{E}\left[|x_{i,j}|^2\right] = \frac{P}{M}$. $|X_i|$ is the distance between transmitter $T_i$ and receiver $R_0^{(l)}$, and $\mathbf{n}_0^{(l)} \sim \mathcal{CN}_{N\times 1}\left(\mathbf{0}_{N\times 1}, N_0\mathbf{I}_N\right)$ is the complex additive white Gaussian noise vector.

We assume that each receiver has the CSI to its corresponding transmitter, but does not know the CSI to the other transmitters. Moreover, with limited feedback of SINR, precoding is not available at the transmitter side. With these assumption, we adopt a linear ZF receiver to eliminate the interference between streams, for which the estimated signal is given by $\hat{x}_{0,k} = \mathbf{g}_k^\dagger\mathbf{y}_{0,k}^{(l)}$ where receiving filter $\mathbf{g}_k^\dagger$ is the $k$th row of $(\mathbf{H}_0^\dagger\mathbf{H}_0)^{-1}\mathbf{H}_0^\dagger$. Note that as $\mathbf{H}_i^{(l)}$ for different $l$ are all i.i.d., we omit the superscript $(l)$ of $\mathbf{H}_i^{(l)}$ for simplicity. Thus, the received signal for the $k$th stream at receiver $R_0^{(l)}$ is given by

$$\hat{x}_{0,k} = \sqrt{\tfrac{1}{r^\alpha}}x_{0,k} + \sum_{i\in\Pi_\lambda/T_0}\sqrt{\tfrac{1}{|X_i|^\alpha}}\mathbf{g}_k^\dagger\mathbf{H}_i\mathbf{x}_i + \mathbf{g}_k^\dagger\mathbf{n}_0^{(l)}. \quad (2)$$

The SINR for the ZF receiver is given as

$$\gamma_{0,k}^{(l)} = \frac{\frac{\rho}{Mr^\alpha\left[\left(\mathbf{H}_0^\dagger\mathbf{H}_0\right)^{-1}\right]_k}}{\frac{\rho\left[\left(\mathbf{H}_0^\dagger\mathbf{H}_0\right)^{-1}\mathbf{H}_0^\dagger\left(\sum_{i\in\Pi_\lambda/T_0}\frac{1}{|X_i|^\alpha}\mathbf{H}_i\mathbf{H}_i^\dagger\right)\mathbf{H}_0\left(\mathbf{H}_0^\dagger\mathbf{H}_0\right)^{-1}\right]_k}{M\left[\left(\mathbf{H}_0^\dagger\mathbf{H}_0\right)^{-1}\right]_k}+1}, \quad (3)$$

where $\rho = \frac{P}{N_0}$ is the signal to noise ratio (SNR), and $[\cdot]_k$ denotes the $(k,k)$th element in the matrix. Note that the distributions of the numerator and denominator are concluded in [5] that

$$\frac{\rho}{Mr^\alpha\left[(\mathbf{H}_0^\dagger\mathbf{H}_0)^{-1}\right]_k} \sim \text{Gamma}\left(m, \theta\right) \quad (4)$$

and

$$\frac{\rho}{M}\frac{\left[(\mathbf{H}_0^\dagger\mathbf{H}_0)^{-1}\mathbf{H}_0^\dagger(\mathbf{H}_i\mathbf{H}_i^\dagger)\mathbf{H}_0(\mathbf{H}_0^\dagger\mathbf{H}_0)^{-1}\right]_k}{\left[(\mathbf{H}_0^\dagger\mathbf{H}_0)^{-1}\right]_k} \sim \text{Gamma}\left(n, \Omega\right), \quad (5)$$

where $m = N - M + 1$, $\theta = \frac{\rho}{Mr^\alpha}$, $n = M$ and $\Omega = \frac{\rho}{M}$.

### C. Multiuser Scheduling

We assume a multiuser environment, where $L$ receivers together with the corresponding transmitter constitute a serving cluster. In the transmission procedure, limited feedback is provided through a dedicated error and delay free feedback channel from these $L$ receivers to the corresponding transmitter. For each receiver, SINR is measured from each transmit antenna separately and ordered. The lowest SINR will be quantized and fed back as CSI to the transmitter. At transmitter side, feedbacks from each receiver within the same cluster are gathered and will be used in the scheduling procedure to decide a receiver to serve.

In the scheduling procedure, a simple opportunistic scheduling algorithm is adopted, where transmitter will select the receiver with the largest reported SINR and send data streams to this receiver. It should be noted that as receivers are located at the same distance away from the corresponding transmitter, the reported SINRs of different receivers are mainly affected by the fast fading of the channel. The large scale factor contributes little to the variation of SINR, which will guarantee the fairness between receivers.

## III. Performance Analysis

In this section, we analyze the performance of spatial multiplexing with multiuser scheduling and derive closed-form expression for outage probability. Based on the expression of outage probability, we analyze the spatial throughput and transmission capacity performance of the network.

### A. Outage Probability

The outage probability $\mathrm{F}(\beta)$ for the concerned cluster is defined as the probability that the SINR of any data stream falls below a certain threshold $\beta$, and is given by

$$\begin{aligned} \mathrm{F}(\beta) &= \Pr\left(\bigcap_{l=1}^{L}\left\{\bigcup_{k=1}^{M}\left\{\gamma_{0,k}^{(l)} \le \beta\right\}\right\}\right) \\ &= \Pr\left(\bigcap_{l=1}^{L}\left\{\gamma_{0,k,\min}^{(l)} \le \beta\right\}\right) \\ &= \Pr\left(\gamma_{0,k,\min}^{(l^*)} \le \beta\right), \end{aligned} \tag{6}$$

where $\gamma_{0,k,\min}^{(l)}$ is the minimum SINR out of the $M$ streams for receiver $R_0^{(l)}$, and $\gamma_{0,k,\min}^{(l^*)}$ is the SINR for the selected receiver $R_0^{(l^*)}$ with the largest $\gamma_{0,k,\min}^{(l)}$ in the cluster. The following lemma presents the closed-form expression for the outage probability of ZF receiver with multiuser scheduling.

*Lemma 1:* The outage probability of spatial multiplexing with ZF receivers and multiuser scheduling in ad hoc network is given by (7) on the top of next page, where $\Gamma(\cdot)$ is the Gamma function, $\eta(M,\lambda,\alpha,\beta) = -\frac{\pi p\lambda\Gamma\left(M+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\beta^{\frac{2}{\alpha}}r^2}{\Gamma(M)}$, $b(i,a)$ is the coefficient of $x^i$ in the expansion of $\left(\sum_{k=0}^{m-1}\frac{x^k}{k!}\right)^{M(a+1)-1}$. Also, $s(\cdot,\cdot)$ and $S(\cdot,\cdot)$ are the Stirling number of the first kind and second kind respectively given in [11].

*Proof:* To derive the outage probability, the first step is to obtain the distribution of $\gamma_{0,k,\min}^{(l)}$, which is the minimum SINR out of the $M$ streams for receiver $R_0^{(l)}$. Then, the distribution of $\gamma_{0,k,\min}^{(l^*)}$ is required to be calculated, which is the largest one among $L$ reported $\gamma_{0,k,\min}^{(l)}$. However, this derivation is difficult due to the complexity of the expression of $\gamma_{0,k}^{(l)}$ and the correlation between different data streams. To solve these two problems, we first approximate the user selection procedure by channel gains $W_{0,k,\min}^{(l^*)}$ instead of SINR. Specifically, $W_{0,k,\min}^{(l^*)}$ is the maximum of $L$ reported channel gains $W_{0,k,\min}^{(l)}$, where $W_{0,k,\min}^{(l)}$ is the minimum of channel gains $W_{0,k}^{(l)} = 1/\left[(\mathbf{H}_0^{\dagger}\mathbf{H}_0)^{-1}\right]_k$ out of $M$ data streams. Then, we assume that the channel gains for each data stream are mutually independent. With this assumption, the complementary cumulative distribution function (CCDF) of $\mathrm{F}(\beta)$ can be rewritten in the form as

$$\mathrm{F}(\beta) = \Pr\left(\frac{W}{\sum_{i\in\Pi_\lambda/T_0}|X_i|^{-\alpha}Y_i+1} \le \beta\right), \tag{8}$$

where

$$W = \frac{\rho W_{0,k,\min}^{(l^*)}}{Mr^\alpha} \tag{9}$$

and $Y_i \sim \mathrm{Gamma}(n,\Omega)$. The distribution of $W$ can be obtained with the result of order statistics from [12] and given as

$$\begin{aligned} f_W(w) &= LMw^{m-1}\frac{e^{-\frac{w}{\theta}}}{\theta^m\Gamma(m)}\left[\sum_{i=0}^{m-1}\frac{\left(\frac{w}{\theta}\right)^i}{i!}e^{-\frac{w}{\theta}}\right]^{M-1} \\ &\times\left\{1-\left[\sum_{i=0}^{l-1}\frac{\left(\frac{w}{\theta}\right)^i}{i!}e^{-\frac{w}{\theta}}\right]^M\right\}^{L-1}. \end{aligned} \tag{10}$$

By exploiting the results of [5] and [6], the outage probability expression (7) can be obtained. ■

It will be shown in Fig. 1 that the approximation of user selection and the assumption of stream independence is accurate.

### B. Spatial Throughput

The spatial throughput is a practical metric for the network which is defined as the total number of successfully transmitted symbols in a time slot in an area. The general expression of the spatial throughput is given as

$$\mathrm{T} = MR\lambda p(1-\mathrm{F}(\beta)), \tag{11}$$

where $R$ denotes the throughput of a stream in a cluster and is related with the SINR threshold as $R = \log_2(\beta+1)$. Substituting (7) into (11) and letting $\beta = 2^R - 1$, the spatial throughput for spatial multiplexing with ZF receivers and multiuser scheduling can be reached. This can be easily achieved through simple algebraic manipulation and the expression is omitted due to space limitation.

### C. Transmission Capacity

Generally, the spatial throughput is obtained at the expense of high outage levels, which is undesirable for many applications. This shortage of spatial throughput has motivated the introduction of the notion of transmission capacity [8]. It is defined as the maximum throughput subject to an outage constraint $\epsilon$ and is given by

$$c(\epsilon) = MR\lambda(\epsilon)(1-\epsilon), \tag{12}$$

where $\lambda(\epsilon)$ is the optimal contention density guaranteeing an outage probability of $\epsilon$ for the network and can be defined as the inverse of $\epsilon = \mathrm{F}(\beta;p\lambda)$ taken with respect to $p\lambda$. Note that the maximization is performed over all cluster, which brings in the same $R$ for each cluster.

Although the expressions of transmission capacity for general spatial multiplexing cases are obtained in [5], we find

$$
\begin{aligned}
\mathrm{F}(\beta) &= 1-\left(\frac{\beta}{\theta}\right)^{m-1}\frac{LM}{\Gamma(m)}\sum_{a=0}^{L-1}\binom{L-1}{a}e^{-(a+1)M\frac{\beta}{\theta}}\sum_{i=0}^{(m-1)[M(a+1)-1]}b(i,a)\sum_{q=0}^{m+i-1}\binom{m+i-1}{q}\left(\frac{\beta}{\theta}\right)^{i-q} \\
&\quad\times(-1)^{i-q}\sum_{h=0}^{q}\left(\frac{2}{\alpha}\right)^{h}s(q+1,h+1)\sum_{v=0}^{h}\eta^{v}\left[M(a+1)\right]^{\frac{2v}{\alpha}-q-1}S(h,v)\,e^{\eta[M(a+1)]^{\frac{2}{\alpha}}}
\end{aligned}
\tag{7}
$$

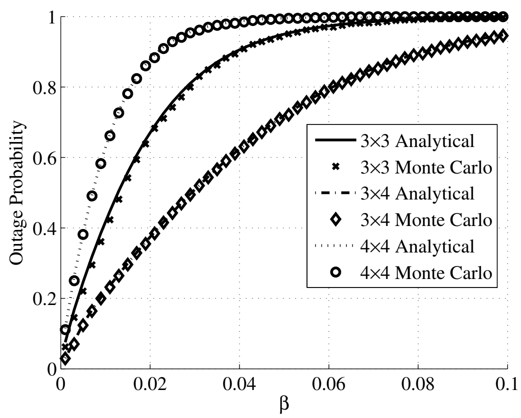


Fig. 1. Outage probability vs. SINR threshold for multiuser scheduling with ZF receivers, and with $L = 10, \alpha = 3, r = 2$ m, $\rho = 10$ dB, $\lambda = 0.25$ and $p = 1$.

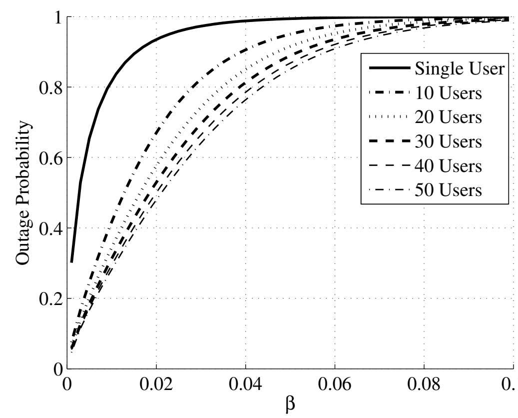


Fig. 2. Outage probability vs. SINR threshold for ZF receivers with different serving user numbers, and with $M = N = 3, \alpha = 3, r = 2$ m, $\rho = 10$ dB, $\lambda = 0.25$ and $p = 1$.

that in multiuser scheduling environment, the exact analysis of the transmission capacity turns out to be intractable due to the complexity involved with inverting the outage probability expression. Thus, we use numerical calculation method to obtain the transmission capacity performance under different network setups.

## IV. Simulation Results

In this section, we evaluate the performance of ZF receivers with multiuser scheduling in ad hoc networks. At first, we demonstrate the accuracy of the approximation of outage probability. Then, through simulation results, we compare the performance of multiuser scheduling system with non-scheduling system and show the impacts of different network parameters on the outage probability, spatial throughput and transmission capacity.

In Fig. 1, we plot the outage probability vs. SINR threshold curves based on our outage probability expression and Monte Carlo simulation labeled ‘Analytical’ and ‘Monte Carlo’ respectively. It can be observed in the figure that the ‘Analytical’ curves closely match the ‘Monte Carlo’ curve, which confirms the accuracy of the approximation in the calculation of outage probability. The impact of antenna configuration is also revealed in the figure that the outage probability falls as the equipped antenna number increases. In particular, when the amount of receive antennas equals the amount of transmit antennas ($N = M$), outage probability falls slowly with the increase of antenna number as all the $N$ degrees of freedom are used to cancel the interference between different streams in a cluster. When the receive node is equipped with more antennas than the transmit node ($N > M$), additional $N - M$ degrees of freedom are provided and can be utilized to strengthen the power of the signal, which will significantly reduce the outage probability.

Fig. 2 illustrates the multiuser scheduling gain through the outage probability vs. SINR threshold curves based on our outage probability expression. In the single user environment, the accuracy of the expression was confirmed in [5], and is the same as (7) when $L = 1$. In Fig. 2, we investigate the multiuser scheduling gain simply by adjusting the user number $L$. As demonstrated in the figure, the outage probability falls rapidly with the increase of user number. However, the gain is not linear to the user number $L$ and turns out to be converging with $L$ increasing in large regime. This is due to the fact that with $L$ in large regime, the fading conditions of the selected receivers are nearly the same and the performance is limited by the interference from other transmitters.

In Fig. 3, the spatial throughput performances are compared under different network parameters. It is shown in the figure that the peak value exists on the curve of spatial throughput vs. data rate. With the increase of data rate $R$, spatial throughput firstly increase to reach the peak point at $R^*$. After $R^*$, the spatial throughput falls. Moreover, a significant gain can be observed in spatial throughput performance with multiuser scheduling and with the increase of user number, $R^*$ grows larger. It’s also revealed in the figure that the $R^*$ point appears at a high $R$ level, bringing in a high outage probability, which

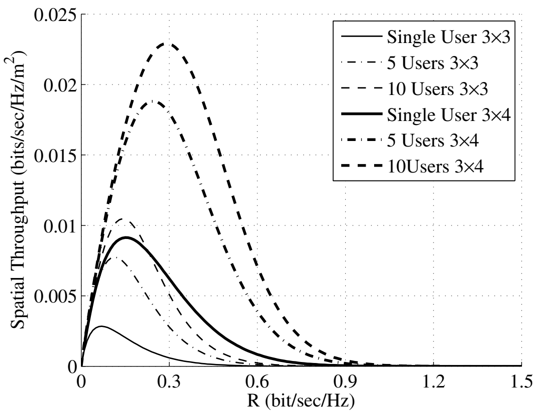


Fig. 3. Spatial Throughput vs. data rate for ZF receivers with different serving user numbers and antenna configuration, and with $\alpha = 3, r = 2$ m, $\rho = 10$ dB, $\lambda = 0.05$ and $p = 1$.

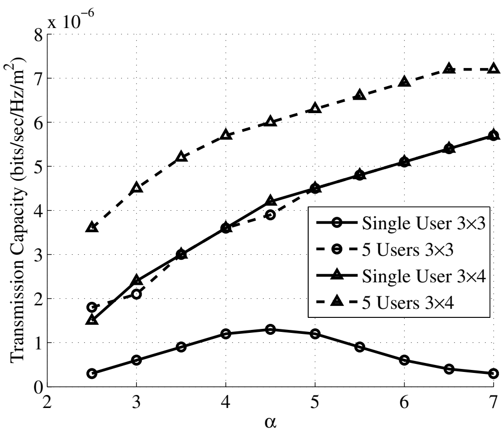


Fig. 4. Transmission Capacity vs. path loss exponent for ZF receivers with different serving user numbers and antenna configuration, and with $\beta = 2$ dB, $r = 3$ m and $p = 1$.

is unexpected in several quality guaranteed scenarios.

In Fig. 4, with throughput $R$ normalized, the transmission capacity performance curves are plotted and the impacts of path loss exponent, user number and antenna configuration are compared. On the one hand, we can see in the figure that additional receive antennas can also provide significant gains in transmission capacity by enhancing the power of the signal. On the other hand, peak value also exists on the curve in single user environment when $N = M$. This is due to the lack of multiuser diversity and receive diversity which can perform against the fading of the signal. In the low path loss exponent regime, the influence of the signal fading is insignificant and the transmission capacity is limited by the interference, so increasing the path loss exponent will decrease interference and in turn increase the transmission capacity. In the high path loss exponent regime, both the signal fading and interference fading are affected, where the signal power decreases quicker than the interference power. So with no multiuser diversity and receive diversity, the transmission capacity will decrease.

## V. Conclusion

In this paper, we investigated the performance of multiuser scheduling with ZF receivers in random access ad hoc networks. We have derived new expressions for the outage probability and the spatial throughput. Based on these expressions, we further studied the impact of different network parameters on the transmission capacity with numerical calculation method. Our results quantified the benefit of multiuser scheduling in ad hoc networks and revealed the relationship between path loss exponent and transmission capacity under different network parameters. Possible directions for future work may be focused on limited feedback for transmitter side precoding with multiuser scheduling to exploit multiuser diversity and transmit diversity.

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## References

[1] D. Tse and P. Viswanath, *Fundamentals of Wirelss Communications*, 1st ed. New York: Cambridge, 2005.

[2] R. H. Y. Louie, M. R. McKay, and I. B. Collings, "Maximum sum-rate of mimo multiuser scheduling with linear receivers," *IEEE Trans. Commun.*, vol. 57, pp. 3500–3510, Oct. 2009.

[3] S. P. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4091–4102, Dec. 2005.

[4] A. M. Hunter, J. G. Andrews, and S. Weber, "Transmission capacity of ad hoc networks with spatial diversity," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 5058–5071, Dec. 2008.

[5] R. H. Y. Louie, M. R. McKay, and I. B. Collings, "Open-loop spatial multiplexing and diversity communications in ad hoc networks," *IEEE Trans. Inf. Theory*, vol. 57, pp. 317–344, Jan. 2011.

[6] Y. Wu, R. H. Y. Louie, M. R. McKay, and I. B. Collings, "Benefits of transmit antenna selection in ad hoc networks," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM'2010)*, Miami, FL, USA, Dec. 2010, pp. 1–5.

[7] M. Kountouris and J. G. Andrews, "Multiuser zero-forcing beamforming with limited feedback in wireless ad hoc networks," in *Proc. IEEE International Conference on Communications (ICC'2011)*, Kyoto, Japan, Jun. 2011, pp. 1–5.

[8] S. Weber, J. G. Andrews, and N. Jindal, "An overview of the transmission capacity of wireless networks," *IEEE Trans. Commun.*, vol. 58, pp. 3593–3604, Dec. 2010.

[9] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[10] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*, 2nd ed. England: John Wiley and Sons, 1995.

[11] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover Publications, 1970.

[12] H. David and H. Nagaraja, *Order Statistics*, 3rd ed. New Jersey: John Wiley and Sons, 2003.