Improved Detection of ACK/NACK Messages in the LTE Uplink Control Channel

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***Abstract*—In this paper, we present an improved detector for ACK/NACK message detection in the LTE uplink control channel with imperfect channel state information at the receiver. The detector is based on the generalized likelihood-ratio test (GLRT) paradigm. We derive detection metrics for the cases when the noise variances at the receiver are known and unknown. Noise here may comprise both thermal noise and interference. Simulation results show remarkable performance gains of the GLRT-based detector with unknown noise variances compared to the training-based maximum-likelihood detector with unknown noise variances when the noise variances in two slots are different. Furthermore, the performance of the GLRT-based detector with unknown noise variances is nearly the same as that of the training-based maximum-likelihood detector with known noise variances.**

I. Introduction

In order to guarantee that the packets can be delivered over the radio link with the required level of reliability, LTE employs a combination of automatic repeat request (ARQ) and forward error correction mechanisms, also known as Hybrid-ARQ (HARQ), for both uplink and downlink transmission. In the downlink, if the decoding of a data block is successful the user equipment (UE) will send a positive ACK message through the uplink control channel to the evolved NodeB (eNB), and the data will be handed over to other medium access control (MAC) module. If decoding fails the UE will send a negative ACK message through the uplink control channel to the eNB for retransmission. Since the ultimate reliability of the packet delivery in the downlink cannot exceed the reliability of the ACK/NACK messages in the uplink control channel, the ACK/NACK message detector for the uplink control channel is an important factor that influences the overall downlink performance, especially for cell-edge users who experience large path losses and high inter-cell interference. Improving the error performance of the ACK/NACK message detector in the LTE uplink control channel is thus important. To this end, several papers, for example [1] [2] and [3], have studied methods to enhance the performance of ACK/NACK detector in the LTE uplink control channel. All these studies assume that the channel state information and noise variance at the receiver are known or estimated. Noise here may comprise both thermal noise and intra-cell/inter-cell interference.

In this paper, we investigate the problem of ACK/NACK message detection in the LTE physical uplink control channel (PUCCH) when the channel state information and the noise variances at the receiver are unknown. The motivation for this is that the LTE uplink is typically characterized by high interference variability. Interference variations from one subframe to the next in the order of 15-20 dB make it a hard task to estimate accurately the uplink noise variance [4]. This is especially true for the LTE PUCCH where slot-level frequency hopping is used for providing frequency diversity. As a result, the noise variance in the LTE PUCCH can be regarded as unknown by the receiver. Furthermore, it is also not clear what a *priori* distribution for the noise variance one can assume. With these observations, a direct optimal detection method which tries to marginalize the conditional likelihood over the channel distribution (conditioned on the known transmitted symbol) becomes numerically intricate. Hence a simpler, suboptimal solution is desirable.

Contribution and relation to previous work: Signal detection with unknown channel state information and unknown noise variance has been studied before, for example in [5] for a multiple-antenna diversity system under flat fading channel model. So far, no work has been done on signal detection for the frequency-selective LTE PUCCH where the noise variances at the receiver are unknown. In this paper, we formulate a basic framework for ACK/NACK message detection in the LTE PUCCH with imperfect channel state information and unknown noise variances at the receiver. Due to the slot-level frequency hopping, we assume that the noise variances are different from one slot to the next in one subframe. We present a GLRT-based detection metric that implicitly estimates both the channel state information and the noise variances. For comparison, we have also derived detection metrics for the conventional training-based maximum-likelihood detector and GLRT-based detector with known noise variances. Simulation results show remarkable performance gains of the GLRT-based detectors compared to the conventional training-based maximum-likelihood detector when the noise variances in two slots are different.

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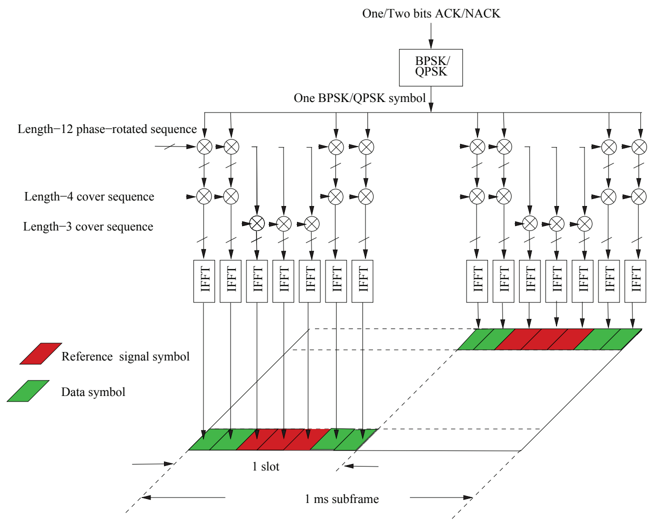

Fig. 1. Block diagram of PUCCH format 1 for normal CP (reproduced freely from [6]).

II. System Model

In this paper, we focus on the ACK/NACK message detection in the PUCCH with transmission format 1a/1b where one or two (in case of two MIMO codewords downlink transmission) ACK/NACK bits are transmitted in the two slots of a subframe. Fig. 1 shows the time-frequency structure of PUCCH format 1a/1b with normal cyclic prefix (CP). As we can see from this figure, the PUCCH consists of two resource blocks located at the edges of the total available cell bandwidth. Two resource blocks of PUCCH have the same structure with each resource block consisting of $N = 12$ sub-carriers in the frequency domain and $L = 7$ continuous OFDM symbols (1 slot) in the time domain. In order to provide frequency diversity to the ACK/NACK message, frequency hopping is used on a slot-by-slot basis. In each slot, the hybrid-ARQ acknowledgement bits are modulated into a BPSK or QPSK symbol s. This symbol s will be transmitted simultaneously in 4 OFDM symbols (with indices 1, 2, 6, and 7) over 12 sub-carriers. The remaining 3 OFDM symbols will be used for the transmission of the reference signal (RS) symbol s_p. To efficiently exploit the resource blocks for the ACK/NACK message transmission, multiple terminals can share the same resource block. This is done in two steps as shown in Fig. 1. Firstly, the ACK/NACK signals and RSs are multiplied by a cell-specific length-12 cyclic shift sequence. Secondly, the ACK/NACK signals and RSs are further code spread by length-4 and length-3 orthogonal cover sequences. Details about these sequences can be found in [6].

We assume that the channel gains are constant in one time slot, but change from one time slot to the next time slot. To simplify the notation, we also assume that the OFDM symbols which carry the ACK/NACK information are contiguous, i.e., there is no reference signal symbol between them. At the receiver, after the fast Fourier transform (FFT) operation and undoing the effect of cyclic shift sequence and orthogonal cover sequences, the received RSs and ACK/NACK signals in slot l over 12 sub-carriers are given by

$$\mathbf{Y}_l = [\mathbf{Y}_l^p \ \mathbf{Y}_l^s] = \mathbf{h}_l[\mathbf{s}_p^T \ \mathbf{s}^T] + \mathbf{E}_l, \quad l = 1, 2 \tag{1}$$

where

- $\mathbf{Y}_l$ is an $N \times L$ matrix of received signals.
- $\mathbf{Y}_l^p$ and $\mathbf{Y}_l^s$ are an $N \times 3$ matrix of received RS and an $N \times 4$ matrix of received ACK/NACK, respectively.
- $\mathbf{h}_l$ is an $N \times 1$ vector of frequency domain channel gains.
- $\mathbf{s}_p \triangleq [s_p \, s_p \, s_p]^T$ and $\mathbf{s} \triangleq [s \, s \, s \, s]^T$ are vectors that contain the RS and modulated ACK/NACK symbols, respectively.
- $\mathbf{E}_l$ is an $N \times L$ matrix of additive noise with i.i.d. $\mathcal{CN}(0, \sigma_l^2)$ elements.

Assuming that $\mathbf{h}_l$ and $\sigma_l^2, l = 1, 2$ are perfectly known to the receiver, the optimal detector for s is obtained by maximizing the conditional probability $p_{coh}(\mathbf{Y}_1^s, \mathbf{Y}_2^s | \mathbf{h}_1, \mathbf{h}_2, \sigma_1^2, \sigma_2^2, s)$. It can be shown that:

$$\begin{aligned} &p_{coh}(\mathbf{Y}_1^s, \mathbf{Y}_2^s | \mathbf{h}_1, \mathbf{h}_2, \sigma_1^2, \sigma_2^2, s) \\ &= \frac{1}{(\pi^2\sigma_1^2\sigma_2^2)^{4N}} \exp\left\{ -\frac{\|\mathbf{Y}_1^s - \mathbf{h}_1\mathbf{s}^T\|^2}{\sigma_1^2} - \frac{\|\mathbf{Y}_2^s - \mathbf{h}_2\mathbf{s}^T\|^2}{\sigma_2^2} \right\} \end{aligned} \tag{2}$$

Hence the maximum likelihood (ML) detector for this system will have the following form

$$\hat{s}_{coh} = \arg\max_s \, p_{coh}(\mathbf{Y}_1^s, \mathbf{Y}_2^s | \mathbf{h}_1, \mathbf{h}_2, \sigma_1^2, \sigma_2^2, s). \tag{3}$$

After omitting the constant factor and irrelevant terms in the metric calculation, the detector can be rewritten as

$$\hat{s}_{coh} = \arg\max_s \, \mathrm{tr}\left\{ \frac{\mathrm{Re}\left(\mathbf{Y}_1^s\mathbf{s}^*\mathbf{h}_1^\dagger\right)}{\sigma_1^2} + \frac{\mathrm{Re}\left(\mathbf{Y}_2^s\mathbf{s}^*\mathbf{h}_2^\dagger\right)}{\sigma_2^2} \right\}. \tag{4}$$

III. Detectors for Unknown $\mathbf{h}_l$, Known σ_l^2

In practice, $\mathbf{h}_l, l = 1, 2$ and $\sigma_l^2, l = 1, 2$ will be unknown or only partially known to the receiver. In this section we present three detectors for unknown $\mathbf{h}_l$ but known σ_l^2.

A. Optimal noncoherent detector

In this subsection, we consider the problem of detecting the transmitted symbol s without any prior estimate of the channel gains $\mathbf{h}_l, l = 1, 2$. We assume that $\sigma_l^2, l = 1, 2$ and the distribution $p(\mathbf{h}_l), l = 1, 2$ are perfectly known to the receiver. The optimal noncoherent detection problem can be written as

$$\begin{aligned} \hat{s}_{non} &= \arg\max_s \, p_{non}(\mathbf{Y}_1, \mathbf{Y}_2 | \sigma_1^2, \sigma_2^2, s) \\ &= \arg\max_s \, \mathrm{E}_{\mathbf{h}_1, \mathbf{h}_2}\left[p(\mathbf{Y}_1, \mathbf{Y}_2 | \mathbf{h}_1, \mathbf{h}_2, \sigma_1^2, \sigma_2^2, s) \right] \end{aligned} \tag{5}$$

where

$$\begin{aligned} &p(\mathbf{Y}_1, \mathbf{Y}_2 | \mathbf{h}_1, \mathbf{h}_2, \sigma_1^2, \sigma_2^2, s) \\ &= \frac{1}{(\pi^2\sigma_1^2\sigma_2^2)^{7N}} \exp\left\{ -\frac{\|\mathbf{Y}_1 - \mathbf{h}_1\bar{\mathbf{s}}^T\|^2}{\sigma_1^2} - \frac{\|\mathbf{Y}_2 - \mathbf{h}_2\bar{\mathbf{s}}^T\|^2}{\sigma_2^2} \right\} \end{aligned} \tag{6}$$

$$
\begin{aligned}
\mathrm{E}_{\mathbf{h}_1,\mathbf{h}_2}\left[p(\mathbf{Y}_1,\mathbf{Y}_2|\mathbf{h}_1,\mathbf{h}_2,\sigma_1^2,\sigma_2^2,s)\right] &= \frac{1}{(\pi^2\sigma_1^2\sigma_2^2)^{7N}}\mathrm{E}_{\mathbf{h}_1}\left[\exp\left\{-\frac{\|\mathbf{Y}_1-\mathbf{h}_1\bar{\mathbf{s}}^T\|^2}{\sigma_1^2}\right\}\right]\mathrm{E}_{\mathbf{h}_2}\left[\exp\left\{-\frac{\|\mathbf{Y}_2-\mathbf{h}_2\bar{\mathbf{s}}^T\|^2}{\sigma_2^2}\right\}\right] \\
&= \frac{1}{(\pi^2\sigma_1^2\sigma_2^2)^{7N}\det(\mathbf{R}_{hh}^2A_1A_2)}\exp\left\{-\frac{\mathrm{tr}\left[\mathbf{Y}_1\mathbf{Y}_1^\dagger\right]}{\sigma_1^2}+\frac{\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger\mathbf{A}_1^{-1}\mathbf{Y}_1\bar{\mathbf{s}}^*}{\sigma_1^4}-\frac{\mathrm{tr}\left[\mathbf{Y}_2\mathbf{Y}_2^\dagger\right]}{\sigma_2^2}+\frac{\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger\mathbf{A}_2^{-1}\mathbf{Y}_2\bar{\mathbf{s}}^*}{\sigma_2^4}\right\}.
\end{aligned}
\tag{7}
$$

and $\bar{\mathbf{s}} \triangleq [\mathbf{s}_p^T\,\mathbf{s}^T]^T$ is an $L\times 1$ vector of transmitted reference signals and ACK/NACK signals. Consider the system model in (1), and suppose that $\mathbf{h}_l \sim \mathcal{CN}(\mathbf{0},\mathbf{R}_{hh})$, where $\mathbf{R}_{hh}$ is the covariance matrix of the channel gains on all $N=12$ subcarriers, i.e.,

$$
\mathbf{R}_{hh} = \mathrm{E}[\mathbf{h}_1\mathbf{h}_1^\dagger] = \mathrm{E}[\mathbf{h}_2\mathbf{h}_2^\dagger]. \tag{8}
$$

We further define $\mathbf{A}_l \triangleq \frac{\|\bar{\mathbf{s}}\|^2}{\sigma_l^2}\mathbf{I}+\mathbf{R}_{hh}^{-1}, l=1,2$ and assume that $\mathbf{h}_1$ and $\mathbf{h}_2$ are independent. With these assumptions, we can obtain $\mathrm{E}_{\mathbf{h}_1,\mathbf{h}_2}\left[p(\mathbf{Y}_1,\mathbf{Y}_2|\mathbf{h}_1,\mathbf{h}_2,\sigma_1^2,\sigma_2^2,s)\right]$ as shown in (7) on the top of the page. Insertion of (7) in (5) and dropping constant terms gives the optimal noncoherent detector

$$
\begin{aligned}
\hat{s}_{non} &= \arg\max_s \frac{1}{\det(\mathbf{I}+\frac{\|\bar{\mathbf{s}}\|^2}{\sigma_1^2}\mathbf{R}_{hh})\det(\mathbf{I}+\frac{\|\bar{\mathbf{s}}\|^2}{\sigma_2^2}\mathbf{R}_{hh})} \\
&\quad\cdot\exp\left\{\frac{\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger\mathbf{A}_1^{-1}\mathbf{Y}_1\bar{\mathbf{s}}^*}{\sigma_1^4}+\frac{\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger\mathbf{A}_2^{-1}\mathbf{Y}_2\bar{\mathbf{s}}^*}{\sigma_2^4}\right\}.
\end{aligned}
\tag{9}
$$

B. Mismatched maximum-likelihood (MML) detector

A commonly used receiver in practice is to first estimate the channel gains $\mathbf{h}_l$ from the RS symbols, and then insert the estimate into the coherent metric (4) as if the channel were perfectly known. There are several possible mismatched receivers depending on the type of channel estimation algorithm used. If least-squares estimation is used, the estimated channel gain can be expressed as

$$
\begin{aligned}
\hat{\mathbf{h}}_l &= \arg\min_{\mathbf{h}_l}\|\mathbf{Y}_l^p-\mathbf{h}_l\mathbf{s}_p^T\|^2 = \mathbf{Y}_l^p\mathbf{s}_p^*(\mathbf{s}_p^T\mathbf{s}_p^*)^{-1} \\
&= \frac{\mathbf{Y}_l^p\mathbf{s}_p^*}{\|\mathbf{s}_p\|^2}
\end{aligned}
\tag{10}
$$

Then, using (4), we get the following MML detector:

$$
\hat{s}_{MML} = \arg\max_s \mathrm{tr}\left\{\frac{\mathrm{Re}\left(\mathbf{Y}_1^s\mathbf{s}^*\hat{\mathbf{h}}_1^\dagger\right)}{\sigma_1^2}+\frac{\mathrm{Re}\left(\mathbf{Y}_2^s\mathbf{s}^*\hat{\mathbf{h}}_2^\dagger\right)}{\sigma_2^2}\right\}. \tag{11}
$$

C. GLRT detector

Due to the properties of the LTE PUCCH as discussed in Section I, it is not possible to know the exact probability density function (PDF) $p(\mathbf{h}_l), l=1,2$. This limits the usefulness of the optimal noncoherent detector. Furthermore, the detector (9) is optimal only when $\mathbf{h}_l$ and $\mathbf{E}_l$ have the assumed PDF which is not necessarily true in practice. Regarding the MML detector, its performance will significantly degrade when the UEs are moving at high speed or when the noise variances at the receiver is large. Therefore, it is important to design a new detector which is robust to the channel uncertainty.

Here we derive a detector based on the GLRT. GLRT has been used in many composite hypothesis testing problems where one or more parameters in the likelihood functions are unknown. The basic idea of GLRT is to first obtain the maximum likelihood estimate (MLE) of the unknown parameters by maximizing the likelihood functions, and then forms the detection metric by substituting the unknown parameters in the likelihood functions with the MLE.

For the system considered here, the maximization of the likelihood function in (6) with respect to $\mathbf{h}_1$ amounts to requiring that

$$
\frac{\partial p(\mathbf{Y}_1,\mathbf{Y}_2|\mathbf{h}_1,\mathbf{h}_2,\sigma_1^2,\sigma_2^2,s)}{\partial\mathbf{h}_1} = 0 \tag{12}
$$

which yields the equation

$$
\mathbf{h}_1\bar{\mathbf{s}}^T\bar{\mathbf{s}}^* - \mathbf{Y}_1\bar{\mathbf{s}}^* = 0. \tag{13}
$$

Hence the MLE of $\mathbf{h}_1$ that maximizes the likelihood function (6) can be obtained as

$$
\mathbf{h}_1 = \mathbf{Y}_1\bar{\mathbf{s}}^*(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}. \tag{14}
$$

Similarly, we can get the MLE of $\mathbf{h}_2$ as

$$
\mathbf{h}_2 = \mathbf{Y}_2\bar{\mathbf{s}}^*(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}. \tag{15}
$$

Insertion of (14) and (15) into (6) yields

$$
\begin{aligned}
&p(\mathbf{Y}_1,\mathbf{Y}_2|\sigma_1^2,\sigma_2^2,s) \\
&= \frac{1}{(\pi^2\sigma_1^2\sigma_2^2)^{7N}}\exp\left\{-\frac{\mathrm{tr}\left[\mathbf{Y}_1\mathbf{Y}_1^\dagger-\mathbf{Y}_1\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]}{\sigma_1^2}\right\} \\
&\quad\cdot\exp\left\{-\frac{\mathrm{tr}\left[\mathbf{Y}_2\mathbf{Y}_2^\dagger-\mathbf{Y}_2\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]}{\sigma_2^2}\right\}.
\end{aligned}
\tag{16}
$$

Hence the GLRT detector will have the following form

$$
\hat{s}_{GLRT} = \arg\max_s p(\mathbf{Y}_1,\mathbf{Y}_2|\sigma_1^2,\sigma_2^2,s). \tag{17}
$$

Omitting the constant factor and irrelevant terms in the metric calculation, the detector can finally be rewritten as

$$
\begin{aligned}
&\hat{s}_{GLRT} = \\
&\arg\max_s \mathrm{tr}\left\{\frac{\mathbf{Y}_1\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}}{\sigma_1^2}+\frac{\mathbf{Y}_2\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}}{\sigma_2^2}\right\}.
\end{aligned}
\tag{18}
$$

After some mathematical operations and omitting the constant factor, we can show that the GLRT-based metric in (18) is the same as the training-based metric in (11). In other words, the MML detector and the GLRT detector are equivalent under the assumption of known σ_l^2. This can also be clearly seen from the simulation results presented in Section V.

IV. Detectors for Unknown $\mathbf{h}_l$, Unknown σ_l^2

We now proceed to derive detectors for unknown $\mathbf{h}_l$ and σ_l^2. In principle, all three detectors described in Section III can be extended to this case. However, to marginalize the conditional density of noncoherent detector in (7) over a prior $p(\sigma_l^2)$ results in computationally very intractable problem. Hence we only present MML and GLRT detectors in this section.

A. *Mismatched maximum-likelihood (MML) detector*

MML detector will first estimate σ_l^2 form the RS symbols. Then the estimated σ_l^2 will be used for ACK/NACK message detection. With the estimated channel gain $\hat{\mathbf{h}}_l$ in (10), the maximum-likelihood (ML) estimate of the noise variance can be expressed as

$$\hat{\sigma}_l^2 \quad = \quad \frac{1}{3N}\|\mathbf{Y}_l^p - \hat{\mathbf{h}}_l\mathbf{s}_p^T\|^2. \tag{19}$$

Then, using (11), we get the following MML detector:

$$\hat{s}_{MML} = \arg\max_s \text{tr}\left\{\frac{\text{Re}\left(\mathbf{Y}_1^s\mathbf{s}^*\hat{\mathbf{h}}_1^\dagger\right)}{\hat{\sigma}_1^2} + \frac{\text{Re}\left(\mathbf{Y}_2^s\mathbf{s}^*\hat{\mathbf{h}}_2^\dagger\right)}{\hat{\sigma}_2^2}\right\}. \tag{20}$$

B. *GLRT detector*

We next consider the GLRT detector where the explicit estimation of σ_l^2 is not needed. With the GLRT paradigm, we will form the detection metric by substituting the unknown parameters σ_l^2 in the likelihood functions (18) with the MLE of σ_l^2. Using likelihood functions in (16), the MLE of σ_l^2 that maximizes $p(\mathbf{Y}_1, \mathbf{Y}_2|\sigma_1^2, \sigma_2^2, s)$ can be obtained from

$$\frac{\partial p(\mathbf{Y}_1, \mathbf{Y}_2|\sigma_1^2, \sigma_2^2, s)}{\partial \sigma_1^2} = 0 \tag{21}$$

which yields equation

$$\frac{\text{tr}\left[\mathbf{Y}_1\mathbf{Y}_1^\dagger - \mathbf{Y}_1\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]}{\sigma_1^4} - \frac{7N}{\sigma_1^2} = 0. \tag{22}$$

Hence the MLE of σ_1^2 that maximizes $p(\mathbf{Y}_1, \mathbf{Y}_2|\sigma_1^2, \sigma_2^2, s)$ can be obtained as

$$\sigma_1^2 = \frac{\text{tr}\left[\mathbf{Y}_1\mathbf{Y}_1^\dagger - \mathbf{Y}_1\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]}{7N}. \tag{23}$$

Similarly, we can get the MLE of σ_2^2 that maximizes $p(\mathbf{Y}_1, \mathbf{Y}_2|\sigma_1^2, \sigma_2^2, s)$ as

$$\sigma_2^2 = \frac{\text{tr}\left[\mathbf{Y}_2\mathbf{Y}_2^\dagger - \mathbf{Y}_2\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]}{7N}. \tag{24}$$

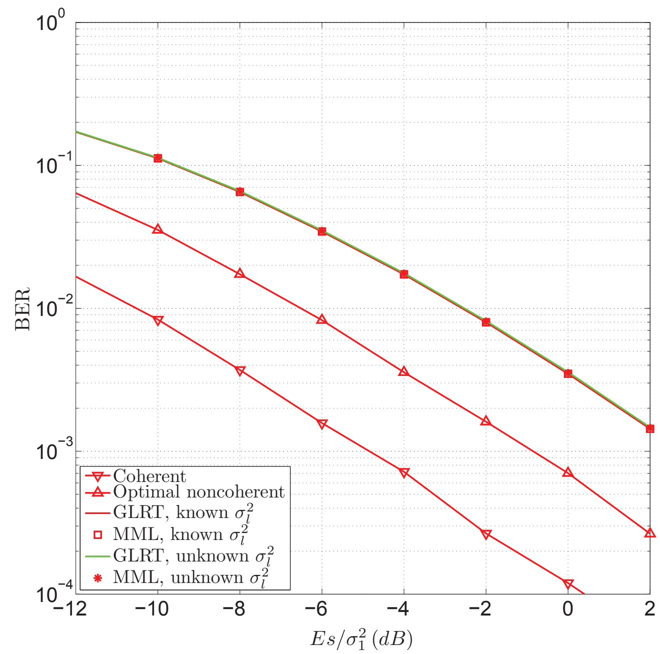

Fig. 2. Performance comparison of all detectors for the case $\sigma_1^2 = \sigma_2^2$.

Insertion of (23) and (24) into (16) yields

$$\begin{aligned} p(\mathbf{Y}_1, \mathbf{Y}_2|s) = &\frac{(7N)^{14N}}{(\pi^2\text{tr}\left[\mathbf{Y}_1\mathbf{Y}_1^\dagger - \mathbf{Y}_1\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right])^{7N}} \\ &\cdot \frac{1}{(\text{tr}\left[\mathbf{Y}_2\mathbf{Y}_2^\dagger - \mathbf{Y}_2\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right])^{7N}}\exp\left(-14N\right). \end{aligned} \tag{25}$$

After taking the logarithm of the likelihood function (25) and dropping irrelevant constants, we get the following GLRT detector:

$$\begin{aligned} \hat{s}_{GLRT} = \arg\min_s\ &\log\left\{\text{tr}\left[\mathbf{Y}_1\mathbf{Y}_1^\dagger - \mathbf{Y}_1\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_1^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]\right\} \\ &+ \log\left\{\text{tr}\left[\mathbf{Y}_2\mathbf{Y}_2^\dagger - \mathbf{Y}_2\bar{\mathbf{s}}^*\bar{\mathbf{s}}^T\mathbf{Y}_2^\dagger(\bar{\mathbf{s}}^T\bar{\mathbf{s}}^*)^{-1}\right]\right\}. \end{aligned} \tag{26}$$

V. Numerical Results

In this section, we evaluate and compare the performances of the proposed detectors. We consider a LTE uplink system with 300 subcarriers and a carrier spacing of 15 KHz. We assume that the ACK/NACK message is transmitted in format 1a. In our simulation, we adopt a Rayleigh fading channel that follows the 3GPP Extended Vehicular A power delay profile with maximum Doppler frequency $f_{\max} = 20$ Hz. The channel coefficients are assumed to be constant over the duration of one slot interval, but vary from slot to slot. The temporal autocorrelation of the complex channel gain between slots is described by the autocorrelation functions $R(\tau) = J_0(2\pi f_{\max}\tau)$, where $J_0(x)$ is the zeroth order Bessel function of the first kind.

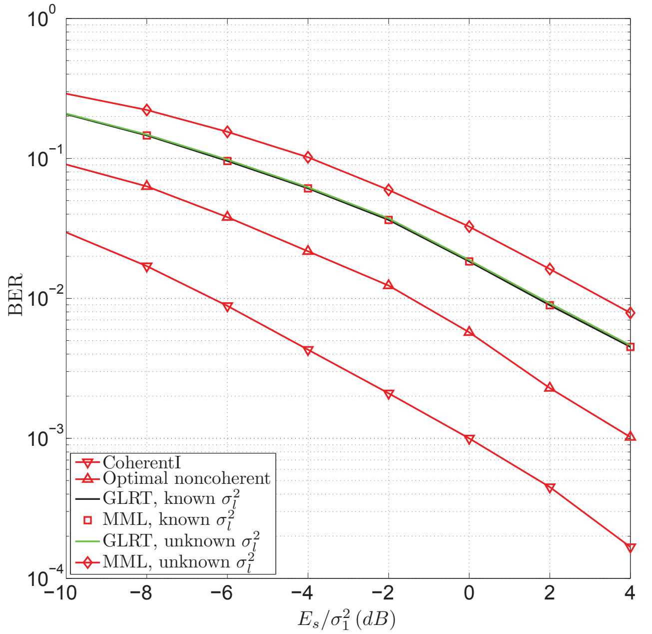

Fig. 3. Performance comparison of all detectors for the case $\sigma_2^2/\sigma_1^2 = 10$ dB.

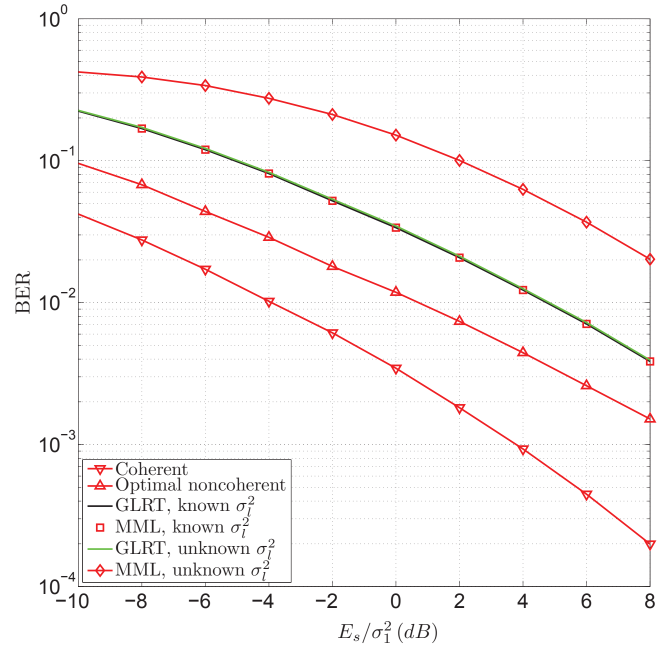

Fig. 4. Performance comparison of all detectors for the case $\sigma_2^2/\sigma_1^2 = 20$ dB.

First we show the bit-error rate (BER) performance when we set $\sigma_1^2 = \sigma_2^2$. Fig. 2 shows the performance comparison of the detector with the MML metric (11), the GLRT metric (18), the MML metric (20), and the GLRT metric (26). The BER performances of the detectors with coherent metric (4) and optimal noncoherent metric (9) are also included for reference. We can see that all the MML and GLRT detectors have nearly the same performance. The simulation results also verify the fact that the MML detector and the GLRT detector are equivalent under the assumption of known σ_l^2.

Next, we look at the BER performance when $\sigma_1^2 \neq \sigma_2^2$. The simulation results of the BER are shown in Fig. 3 and Fig. 4 for $\sigma_2^2/\sigma_1^2 = 10$ dB and $\sigma_2^2/\sigma_1^2 = 20$ dB, respectively. It can be observed that for the case $\sigma_2^2/\sigma_1^2 = 10$ dB the GLRT detector yields an improvement in SNR of about 1.5 dB over the MML detector at BER of 10^{-2} if the noise variances are unknown to the receiver. The performance improvement increases to 6 dB if $\sigma_2^2/\sigma_1^2 = 20$ dB. This demonstrates that the GLRT detector with unknown noise variances provides a significant gain compared to its training-based counterpart when $\sigma_1^2 \neq \sigma_2^2$. It can also be observed that the MML detector and the GLRT detector have the same performance if the noise variances are known to the receiver. Furthermore, it can be found that the GLRT detector with unknown noise variances has nearly the same BER performance as the GLRT detector with known noise variances. In other words, the GLRT detector with unknown noise variance can perform as well as a training-based detector with known noise variances.

VI. Conclusions

We have developed GLRT detectors for ACK/NACK message detection in the LTE uplink control channel. Simulation results show that the GLRT detector with unknown noise variances offers a significant gain over the MML detector with unknown noise variance when $\sigma_1^2 \neq \sigma_2^2$. For all the other cases, GLRT detector has nearly the same BER performance as MML detector. This shows that GLRT detector is a robust detector for ACK/NACK message in the LTE uplink control channel.

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