

# In-Network Local Distributed Estimation for Power-constrained Wireless Sensor Networks

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**Abstract**—In this paper, we consider the problem of power-efficient distributed estimation of a localized event in the large-scale Wireless Sensor Networks (WSNs). In order to increase the power efficiency in these networks, we develop a joint optimization problem that involves both selecting a subset of active sensors and the routing structure so that the quality of estimation at a given querying node is the best possible subject to a total imposed communication cost. We first formulate our problem as an optimization problem and show that it is NP-Hard. Then, we propose a local distributed optimization algorithm that is based on an *Estimate-and-Forward* (EF) strategy, which allows to perform sequentially this joint optimization in an efficient way. We also provide a lower bound for our optimization problem and show that our local distributed optimization algorithm provides a performance that is close to this bound. Although there is no guarantee that the gap between this lower bound and the optimal solution of the main problem is always small, our numerical experiments support that this gap is actually very small in many cases. An important result from our work is that because of the interplay between the communication cost over the links and the gains in estimation accuracy obtained by choosing certain sensors, the traditional Shortest Path Tree (SPT) routing structure, widely used in practice, is no longer optimal, that is, our routing structures provide a better trade-off between the overall power efficiency and the final estimation accuracy obtained at the querying node. Our experimental results show that our algorithms yield a significant energy saving.

## I. INTRODUCTION

In this paper, we focus on the deterministic parameter produced by a localized source target, and consider the scenario where the network performs distributed estimation of this parameter. In a distributed estimation setting of the limited power resources in WSNs, because there is a need to choose a subset of active sensors whose data can be used for estimation and a routing tree to route the information to a querying (sink) node, which performs the final estimation. This motivates us to perform joint optimization on the sensor selection and routing so that the quality of estimation has to be maximized for a given power budget.

There has been already a substantial amount of research in sensor selection and data transmission for WSNs with an application view to estimation. The problem of choosing a subset of sensor measurements from a set of possible sensor measurements has been analyzed in [1], [2], where they transmit data directly to the sink, therefore, no routing. In

[1], they propose a centralized solution based on performing a relaxation of an integer optimization problem using an efficient interior point method. A distributed version of this interior point method is introduced in [2].

In [3], [4], a tradeoff between the number of active sensors and the energy used (in data transmission to the sink) by each active sensor is presented to minimize the Mean Square Error (MSE) estimation [5]. A problem of bandwidth constrained distributed estimation of a deterministic parameter has been proposed in [6], [7], where each sensor compresses its observation into a few bits and transmit directly to the fusion center for parameter estimation. However, to the best of our knowledge none of these works consider jointly the sensor selection and routing problem.

In this paper, we show that the SPT based only on communication cost (SPT-CC) is not the optimal routing structure. Fig. 1 illustrates a simple example where the SPT-CC is not the optimal routing structure when an EF [8] is used, that is, when each sensor performs a parameter estimation and fuses all other measurements it receives from its child sensors together with its own measurement and then sends only one flow of data to its parent sensor in the chosen routing structure, instead of simply forwarding the data, which would generate more flows. As shown in Fig. 1, choosing in this particular case all other sensors, the same gains in estimation are obtained however, taking into account that  $f_c(\cdot)$  is increasing with distance, the total communication cost corresponding to SPT-CC is larger, assuming that we generate observations at each sensor.

The main contributions of this paper are: (a) we define and analyze the problem of joint sensor selection and routing for distributed estimation, in terms of both algorithm design and complexity; (b) we show that this problem is NP-Hard; (c) we provide a local distributed optimization algorithm using an EF strategy; (d) we also provide a lower bound for the optimal solution of our optimization problem and show experimentally

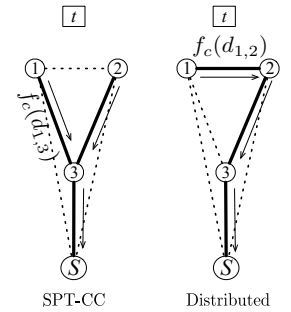


Fig. 1. SPT based on communication cost is not optimal in this case for the distributed estimation scenario, when an EF approach is used, since  $f_c(d_{1,3}) > f_c(d_{1,2})$  where  $f_c(\cdot)$  is a function of communication cost between two sensors. Here  $S$  and  $t$  denote the sink node and target respectively. Discontinuous lines show all potential connectivities.

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that our local distributed optimization algorithm provides a performance that is close to this bound. The rest of the paper is structured as follows: In section II, we present our problem formulation and analyze its NP-hardness. Section III describes our joint sensor selection and routing algorithms and their complexity analysis. In section IV, we present our simulation results, showing the performance of our algorithms. Finally, conclusions in section V.

## II. PROBLEM FORMULATION

We consider a WSN consisting of  $N$  sensors and each sensor has a unique identity  $k \in \{1, 2, \dots, N\}$ . We assume also that all sensors are using the same transmission power  $P_0$  and equipped with omnidirectional antennas. Then, there is a certain distance range  $d_{th}$  such that if the distance  $d_{i,j}$  between two sensors  $i$  and  $j$  is less than or equal to  $d_{th}$ , then, there is a communication link between them. This results in an undirected network connectivity symmetric graph  $G$ , as illustrated in Fig. 2. We denote the  $(N+1)$ -th sensor as the sink node, where the final estimation is to be obtained. We denote the communication cost model for a link between sensor  $i$  and  $j$  as  $f_c(d_{i,j}) \propto d_{i,j}^\alpha$ , where  $\alpha$  is a path-loss exponent and without loss of generality, it is assumed to be an increasing function of the distance  $d_{i,j}$  between them. This simple communication cost model has been experimentally supported in [10], as an approximately valid model for analyzing routing structures.

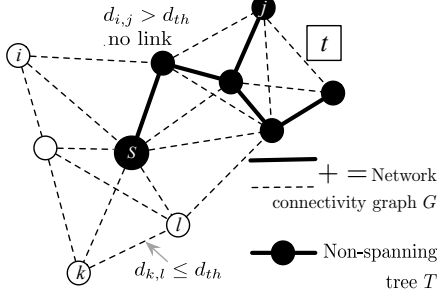


Fig. 2. Symmetric network connectivity graph  $G$ , where  $S$  is the sink node.

We consider the problem of estimating a deterministic parameter produced by a spatially localized source. There is a sink node, whose aim is to get the best possible estimation under some power constraints, as usually in WSNs. We consider stationary sensors with one-to-one communication links assuming the existence of some MAC protocol that resolves the network collisions. This problem can be naturally formulated as an optimization problem [9] as follows:

$$\begin{aligned} & \underset{T \subset G}{\text{minimize}} && \text{Distortion} \\ & \text{subject to} && \text{Comm\_cost} \leq P_{max} \end{aligned}$$

where  $P_{max}$  is the maximum power budget for the WSN in terms of communication, our objective is the average estimation error (*Distortion*) obtained at the sink node,  $T \subset G$  is in general a non-spanning tree (as represented in Fig. 2 by the continuous links) and *Comm\_cost* is the total communication cost associated to the non-spanning tree  $T$ . Thus, our optimization problem involves optimizing jointly both: selecting a subset of sensors and selecting a routing structure.

The cost of delivering a measurement to the sink node grows usually as the number of sensors grows, although it depends strongly on the chosen routing structure. On the other hand, the distortion associated to the estimation reduces usually as the number of chosen sensors grows (notice that it depends on the SNRs). Thus, there is a trade-off between distortion and communication cost, which motivates us to perform pareto optimization over these two metrics.

We use a linear function in our signal model because of its simplicity, and because it leads to practical estimation approaches that give closed form estimators. As we see later, even with a linear model our optimization problem is already NP-hard, that is, a linear model still maintaining essentially the complexity of our problem. In fact, assuming a linear model, the optimal estimator, as well as its performance, can be readily obtained. Let us consider the model:

$$y_k = h_k x + z_k \quad k = 1, 2, \dots, N \quad (1)$$

where  $y_k \in \mathbb{R}$  is a scalar observation of sensor  $k$ ,  $x \in \mathbb{R}$  is an unknown deterministic parameter to be estimated, whose observation is distorted by a scalar  $h_k \in \mathbb{R}$  and corrupted by additive Gaussian noise  $z_k$ , which is taken to be independent and identically distributed (i.i.d.) with pdf  $\mathcal{N}(0, \sigma^2)$ , where  $\sigma^2$  is assumed to be known. We assume that the scalar  $h_k$  follows the usual path loss model, that is,  $h_k \propto 1/d_{k,t}^\beta$  is a decreasing function of the distance  $d_{k,t}$  from the sensor  $k$  to the source target  $t$  of the parameter that we are interested to estimate, where  $\beta$  has been experimentally estimated in [11].

### A. Parameter Estimation

The well known Best Linear Unbiased Estimator (BLUE) is the optimal estimator for linear problem (1) giving the smallest possible MSE, thus, it coincides in this case with the Minimum Mean Square Error (MMSE) estimator. The optimal estimator of  $x$  is readily given by:

$$\hat{x} = \frac{\sum_{k=1}^N h_k y_k}{\sum_{k=1}^N h_k^2} \quad (2)$$

and the associated MSE is given by:

$$\text{MSE}_{\hat{x}} = \left( \frac{\sum_{k=1}^N h_k^2}{\sigma^2} \right)^{-1} \quad (3)$$

### B. Joint Sensor Selection and Routing Optimization Problem

Let us assume a variable  $b_k \in \{0, 1\}$ ,  $k = 1, 2, \dots, N$ , denoting the status of each sensor, namely,  $b_k = 1$  denotes that the  $k$ -th sensor is active (i.e. chosen) and  $b_k = 0$  denotes that the  $k$ -th sensor is inactive (i.e. not chosen). We want to minimize the overall distortion (MSE) subject to a communication cost, and where we can choose both a subset of sensors and a routing structure. This can be formulated analytically as follows:

$$\begin{aligned} & \underset{\{b_k, b_{k_p}, \Delta\}}{\text{minimize}} && \text{MSE}_{\hat{x}} = \left( \frac{\sum_{k=1}^N b_k h_k^2}{\sigma^2} \right)^{-1} \\ & \text{subject to} && \sum_{k=1}^N b_k c_k = P_{max} - \Delta \\ & && b_k \leq b_{k_p}, \text{ where } k_p = \text{parent of } k \\ & && \Delta \geq 0; b_k \in \{0, 1\} \end{aligned} \quad (4)$$

where  $\{b_k, b_{k_p}, \Delta\}$  are chosen so that the resulting routing structure  $T$  is a subtree of the  $G$  and  $\Delta$  is the power gap, that is, maximum power allowed  $P_{max}$  minus the total power incurred. Here, the second constraint ensures that no sensor is selected if its parent on the tree is not selected, and therefore, it ensures that the selected sensors form a valid routing subtree  $T \subset G$  rooted at the sink node.

### C. NP-hardness

**Lemma 1.** *The joint optimization problem of sensor selection and routing structure is NP-Hard.*

**Proof.** The proof is based on performing a polynomial time reduction [12] from the Undirected Hamiltonian Path (UHP) to our problem, that is, mapping every instance from the UHP problem to our problem<sup>2</sup>.

## III. JOINT SENSOR SELECTION AND ROUTING ALGORITHMS

### A. Fixed-Tree Relaxation-based Algorithm

Considering the idea of an EF, a relaxed version of problem (4) is given by:

$$\begin{aligned} & \underset{\{b_k^r, b_{k_p}^r, \Delta\}}{\text{minimize}} && MSE_{\hat{x}} = \left( \frac{\sum_{k=1}^N b_k^r h_k^2}{\sigma^2} \right)^{-1} \\ & \text{subject to} && \sum_{k=1}^N b_k^r c_k = P_{max} - \Delta \\ & && b_k^r \leq b_{k_p}^r; \Delta \geq 0 \\ & && 0 \leq b_k^r \leq 1 \end{aligned} \quad (5)$$

where  $b_k^r$  is the relaxed version of the variable  $b_k$ . Let us denote  $\{b_k^{r*}\}_{k=1}^N$  a solution of this relaxed problem.

Notice that problem (5) is a convex problem, but it is not equivalent to the original problem (4) since  $\{b_k^{r*}\}_{k=1}^N$  will not be binary in general. We use the solution of problem (5) to perform a suboptimal subset selection  $\mathcal{S}$  by sorting the optimal values  $\{b_k^{r*}\}_{k=1}^N$  in descending order and selecting the subset of  $K$  largest  $b_k^{r*}$ s satisfying the power constraint as long as  $\Delta \geq 0$ . Then, denoting  $\{b_k^r\}_{k=1}^N$  a binary values such that  $\hat{b}_k^r = 1$  if  $k \in \mathcal{S}$  and  $\hat{b}_k^r = 0$  if  $k \notin \mathcal{S}$ , we have that:

$$L_{fxd} = \left( \frac{\sum_{k=1}^N \hat{b}_k^r h_k^2}{\sigma^2} \right)^{-1} \geq p^* \quad (6)$$

When making the sorting, because of the constraint  $b_k^r \leq b_{k_p}^r$ , it is a tree and because of the relaxation the routing solution will be a subset of SPT-CC.

In our network setting, sink node is not allowed to take the measurement of the event, that is,  $h_{N+1} = 0$ , then the lower bound is obtain by the following:

$$\begin{aligned} & \underset{\{b_k^r, b_{k_p}^r, \Delta\}}{\text{minimize}} && MSE_{\hat{x}} = \left( \frac{\sum_{k=1}^{N+1} b_k^r h_k^2}{\sigma^2} \right)^{-1} \\ & \text{subject to} && \sum_{k=1}^{N+1} b_k^r f_c(d_{k,k_p}) = P_{max} - \Delta \\ & && b_{N+1}^r = 1; \Delta \geq 0 \\ & && 0 \leq b_k^r \leq 1 \end{aligned} \quad (7)$$

<sup>2</sup>We do not include the full proof due to the lack of space.

We use the solution  $\{b_k^{r*}\}_{k=1}^{N+1}$  of the problem (7) to obtain an optimal routing structure that has to be routed at the sink node. We use the minimum spanning tree of the directed graph  $G' = (V, E')$ , where it is assumed that the edge between sensor  $i$  and  $j$  has a cost  $c_{ij} = b_i^{r*} f_c(d_{i,j})$ . We obtain the direction of the measurement flow by calculating the costs  $c_{ij} = b_i^{r*} f_c(d_{i,j})$  and  $c_{ji} = b_j^{r*} f_c(d_{i,j})$ . Later, we update the routing structure based on the condition that if  $c_{ij} < c_{ji}$  then sensor  $j = i_p$  will be the parent of sensor  $i$  otherwise,  $i = j_p$  will be the parent of sensor  $j$ . The constraint  $b_{N+1}^r = 1$  in (7) enforces this tree to be routed at the sink node. Finally, we store the new set  $\{k, k_p\}$  based on the condition at each sensor pair, then calculate the total communication cost of this routing structure and the lower bound  $L$  that is given by the objective in (7), where  $h_{N+1} = 0$ , that is:

$$L = \left( \frac{\sum_{k=1}^{N+1} b_k^{r*} h_k^2}{\sigma^2} \right)^{-1} \leq p^* \quad (8)$$

By observing the gap  $\delta_{fxd} = L_{fxd} - L$ , we can have an assessment on how good this suboptimal approximation is.

We can verify our lower bound  $L$  given in (8) using Newton's method by solving our relaxed problem (5) approximately using the *log barrier method* ([9], Sec. 11.2) and using the new set  $\{k, k_p\}$ , that is, problem (5) can be also posed as:

$$\begin{aligned} & \underset{\{b_k^r, b_{k_p}^r, \Delta\}}{\text{minimize}} && \phi(b_k^r) = \left( \frac{\sum_{k=1}^N b_k^r h_k^2}{\sigma^2} \right)^{-1} \\ & && - \frac{1}{\nu} \sum_{k=1}^N \left( \log(b_k^r) + \log(1 - b_k^r) + \log(b_{k_p}^r - b_k^r) + \log \Delta \right) \\ & \text{subject to} && \sum_{k=1}^N b_k^r c_k = P_{max} - \Delta, 0 \leq b_k^r \leq 1 \end{aligned} \quad (9)$$

where  $\nu > 0$  is a parameter that sets the accuracy of the approximation. The log barrier function  $\phi$  is convex and smooth, therefore, problem (9) can be efficiently solved by the Newton method. Let  $p^*$  denote the solution of problem (9), which depends on  $\nu$ . A standard result in interior-point methods (this can be seen in [9], Sec. 11.2.2) is that  $p^*$  is no more than  $\frac{2N}{\nu}$  suboptimal for the problem (5), that is:

$$\left( \frac{\sum_{k=1}^N b_k^{r*} h_k^2}{\sigma^2} \right)^{-1} \leq \left( \frac{\sum_{k=1}^N b_k^{r*} h_k^2}{\sigma^2} \right)^{-1} + \frac{2N}{\nu} = L + \frac{2N}{\nu}$$

which implies that:

$$L' = \left( \frac{\sum_{k=1}^N b_k^{r*} h_k^2}{\sigma^2} \right)^{-1} - \frac{2N}{\nu} \leq p^* \quad (10)$$

and as  $\nu \rightarrow \infty$ , we get the lower bound  $L$ . We can use this bound to choose  $\nu$  so that increase in gap contributed by term  $2N/\nu$  is small.

### B. Local Distributed Optimization Algorithm

Finding a possible subset  $\mathcal{S}$  of  $K$  selected sensors and the associated routing structure  $T$  from the solution of the fixed-tree relaxation based problem (5), can be improved by a local distributed optimization method, as explained next. First, we

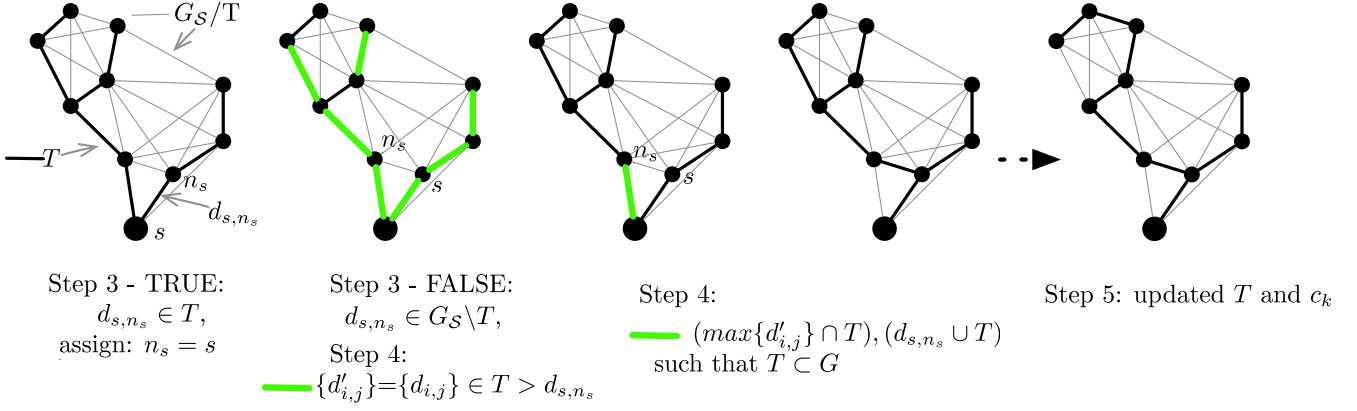


Fig. 3. First part of the local distributed optimization algorithm: the number of fixed sensors in  $\mathcal{S}$  and updating  $T$ , Step 1 to Step 5 from **Algorithm 1**.

optimize the total communication cost  $c_k$  of the subset  $\mathcal{S}$  by the following edge-swap method [13]:

We start by keeping fixed the number of sensors  $K$  in subset  $\mathcal{S}$  and updating  $T$  so that it results in an important power saving, thus allowing us to activate additional sensors while maintaining the same total given power budget, which will give rise to an estimation gain. To achieve this, let us define  $G_S$  as the subgraph resulting from restricting the graph  $G$  to the subset of  $K$  sensors in  $\mathcal{S}$ , next we need to perform the swaps among the edges in  $T$  and the edges in  $G_S \setminus T$  in such a way that the new resulting tree after the update (swap) in the edges remains non-spanning tree of a graph  $G$ . The procedural steps are given in the **Algorithm 1**: from Step 1 to Step 5 and then are illustrated in Figure 3.

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**Algorithm 1** Local Distributed Optimization Algorithm

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**Require:**  $\mathcal{S}$ ,  $T$ ,  $c_k$

Initialization:

starting node  $s = N + 1$ ;  $j_p =$  parent of  $j$ ;  $k=1$

$n_s =$  nearest sensor to  $s$ ;  $K =$  number of sensors in  $\mathcal{S}$

**Step 1:** find  $n_s$  to  $s$

**Step 2:** if  $k=K$  Stop and go to Step 6; otherwise continue

**Step 3:** if  $d_{s,n_s} \in T$  go to Step 5; otherwise continue

**step 4:** find  $\{d'_{i,j}\} = \{d_{i,j}\} \in T > d_{s,n_s}$

calculate  $\max\{d'_{i,j}\} \cap T, d_{s,n_s} \cup T \ni T \subset G$

update:  $T, c_k$ ; assign:  $n_s = s$ ;  $k = k + 1$  go to Step 1

**Step 5:** assign  $n_s = s$ ;  $k = k + 1$  go to Step 1

**Step 6:** if  $c_k > P_{max}$  Stop and go to Step 8; otherwise continue

assign:  $\mathcal{A}_{i,j}$ : sensor  $i$  has  $j$  neighbors ( $j \in \mathcal{N}(i)$ )

calculate:  $\{i, j\} = \arg \min_{\{i \in \mathcal{S}, j \in \mathcal{A}_{i,j} \setminus \mathcal{S}\}} (h_j^{-2} + \gamma_j f_c(d_{i,j}))$

update:  $T = T \cup (i, j)$ ;  $\mathcal{S} = \mathcal{S} \cup \{j\}$ ;  $c_k = c_k + f_c(d_{i,j})$

**Step 7:** if  $c_k > P_{max}$  go to Step 8; otherwise continue

update  $\gamma_j$ :

**if**  $h_j > h_{j_{pre}}$  **then** Decrease  $\gamma_{j_{next}}$

**else** Increase  $\gamma_{j_{next}}$

**end if**; go to Step 6

**Step 8:** calculate:  $L_{loc}, \delta_{loc}$

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In the following, in order to activate additional sensors, we perform a local distributed estimation for the remaining power

budget, which we gain using an edge-swap method, taking into account the interplay between two metrics: distortion and communication cost, taking  $\mathcal{S}$  as the starting subset. This is done by the steps given in **Algorithm 1**: from Step 6 to Step 8. Notice that our local distributed optimization problem involves minimizing the distortion under a bound in the total given communication cost, while involves minimizing a weighted sum of both metrics. Considering this, the basis of our local distributed algorithm will be that at a given (already activated) sensor  $i$ , we will perform the following local minimization:

$$\min_{j \in \mathcal{N}(i)} (h_j^{-2} + \gamma_j f_c(d_{i,j})) \quad (11)$$

where  $j \in \mathcal{N}(i)$ , that is, the index  $j$  goes through all the neighbors of an active sensor  $i$ , and  $\gamma_j$  is positive space-variant factor ( $\gamma_j > 0$ ) that trades-off approximated distortion and communication cost when considering a sensor  $j$  that is neighbor of sensor  $i$ .

The whole process starts from the sensor (among  $K$  sensors) that first detects the phenomenon on an average with the largest SNR, equivalently, the largest  $h_j$ . In the next phase for each sensor in the  $\mathcal{S}$ , we check the objective (11) for every neighbor. Finally, we activate the best neighbor that minimize the objective function among all 1-hop neighbors, and where we assume that the communication between sensors can be done by message passing. This local objective function is a combination of the estimation gain of each candidate corresponding inactive sensor and the communication cost incurred when selecting that sensor.

For a given sensor these two metrics (estimation gain and communication cost) are weighted by a factor  $\gamma_j$  to perform the suitable trade-off locally at each sensor. We start with the value of  $\gamma_j = 1$  and then, we update  $\gamma_j$  that determines the relative importance of the estimation gain and the communication cost in the objective function of the algorithm. We use the following update scheme: if the sensor estimation gain  $h_j$  of the currently selected sensor  $j$  is greater than the gain  $h_{j_{pre}}$  in the preceding step (associated to the sensor  $j_{pre}$  previously chosen), the value of next  $\gamma_{j_{next}}$  is lowered as we are heading to the location of the event and more importance is given to the estimation gain metric. Otherwise, the value of next  $\gamma_{j_{next}}$  is increased to force heading towards the correct direction.



### C. Computational Complexity Analysis

Next, we provide a complexity analysis for the algorithms we described previously.

1) *Fixed-Tree Relaxation-based Algorithm*: The main computational complexity in the fixed-tree relaxation-based algorithm comprises the following parts: SPT-CC from Bellman-Ford algorithm that runs in  $O(MN)$  (where  $M$  is the number of edges), solving the relaxed optimization problem requires  $O(N^3)$  operations (using interior-point methods [9]), since this problem is convex (objective is to be minimized with convex constraints) at the sink node. These methods typically require a few tens of iterations and each iteration can be carried out with a complexity of  $O(N^3)$  operations.

2) *Local Distributed Optimization Algorithm*: Our local distributed optimization performs the following operations: (a) start from the solution of the fixed-tree relaxation-based algorithm that takes  $O(N^3)$  operations; (b) subtree  $T$  of  $K$  sensors can be updated in polynomial time that takes  $O(K^2)$  operations, improving the subset of  $K$  sensors among  $N$  sensors by finding their best 1-hop neighbors takes  $O(KN \log N)$  operations. Then it takes  $O(N^3) + O(KN \log N)$  operations.

## IV. SIMULATION RESULTS

In this section, we show the performance comparison of the proposed algorithms through numerical simulations. We have tested algorithms using 100 different network topologies. For the sake of consistency and ease in comparing the results, we consider only a fixed target location. Each topology contains  $N = 200$  sensors placed randomly in a square region. The communication cost between sensors  $i$  and  $j$  is given by  $f_c(d_{i,j}) \propto d_{i,j}^\alpha$  with  $\alpha = 4$ . Moreover, we take the measurement coefficient at sensor  $i$  to be inversely proportional to the distance from  $i$  to the target, i.e  $h_i \propto 1/d_{i,t}^\beta$  with  $\beta = 1$ .

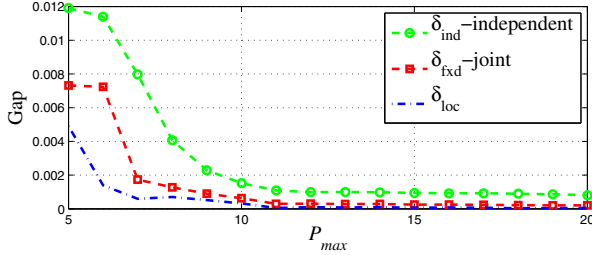


Fig. 4. Performance gaps  $\delta_{ind}$ ,  $\delta_{fxd}$ , and  $\delta_{loc}$ .

In order to show the performance of any algorithm obtained when selecting a subset of sensors and a certain routing structure, we evaluate the gaps:  $\delta_{ind} = L_{ind} - L$ ;  $\delta_{fxd} = L_{fxd} - L$ ; and  $\delta_{loc} = L_{loc} - L$  as shown in Fig. 4. These plots show that our local distributed optimization algorithm performs better than the fixed-tree relaxation-based for each power budget. It also shows that our algorithm is close to the lower bound when the power budget is large enough. As shown by our results above, the routing tree based on the SPT-CC routing structure is not optimal in general. In many situations, we see a notable gap between the maximum power allowed and the

<sup>3</sup>  $L_{ind}$  is obtained while optimizing the distortion in (5) independently from communication cost, later select a subset of sensors and associated routing structure from SPT-CC while satisfying the given power budget.

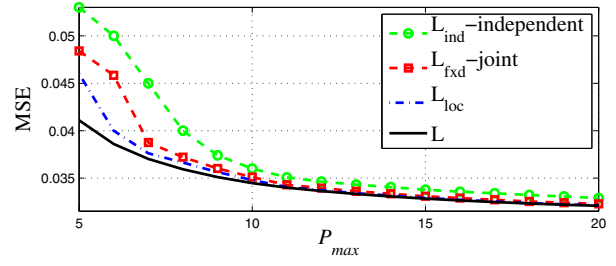


Fig. 5. A performance bound comparison of the proposed algorithms.

total incurred power when using fixed-tree relaxation-based algorithm. Fig. 5 shows the performance in terms of MSE for different maximum power budgets  $P_{max}$  with the sink located at the center. Our local distributed optimization algorithm outperforms the fixed-tree relaxation-based by a 30-70 % as compare to the lower bound  $L$ , depending on the available power budget.

## V. CONCLUSION

This paper considers the estimation problem in a scenario where the WSN covers a large geographical area, and is monitoring a localized event. In the recent literature, some solutions have been proposed, but most of them try only to reduce the problem to only the cardinality of the selected sensor set, ignoring the optimization of the network topology (routing). However, network topology is indeed an important factor in the problem since in general, the cost of receiving one measurement that is far from the fusion center consumes more energy than one that is closer to it.

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