Outage Probability of Joint Relay Selection and Power Allocation for Two-Way Relay Networks over Rayleigh Fading Channels

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***Abstract*—This paper investigates the outage probability of joint relay selection and power allocation for two-way relay networks over Rayleigh Fading channels. Firstly, a tight lower bound of outage probability with equal power allocation is derived. Furthermore, a joint relay selection and optimum power allocation scheme is proposed, which minimizes the outage probability, then, a lower bound of outage probability is also obtained. Based-on the closed-form outage probabilities, we obtain a relay selection strategy, which requires only a single integer parameter to be broadcasted to all relays. The analysis is verified by using Monte-Carlo simulations.**

***Index Terms*—two-way relay, outage probability, power allocation, relay selection.**

I. Introduction

In conventional cooperative communication networks, when all terminals are operated in half-duplex fashion, the transmission of one information symbol from the source to the destination terminal occupies two channel uses [1]. This leads to a loss in spectral efficiency due to the pre-log factor one-half in corresponding to sum-rate expressions. To improve the spectral efficiency , two way relaying was proposed in [1], [2], where two source nodes simultaneously send their information to the relay node in the multiple-access (MA) phase and the relay node broadcasts the received signal to the two source nodes in the broadcast (BC) phase. In [1], Rankov and Wittneben have introduced the two-way relaying protocols, known as two-way amplify-and-forward (AF) relaying and two-way decode-and-forward (DF) relaying.

Outage probability is one of the most important performance measures for coded systems. Li $etal.$ and Louie $etal.$ studied independently outage performance of the physical network coding (PNC) protocol in [3], [4]. It has been shown that opportunistic amplify-and-forward (AF) relaying is outage optimal among single relay selection methods and significantly outperforms an AF strategy based on equal-power multiple relay transmissions with local channel knowledge [5], [6]. Furthermore, optimum power allocation of the PNC protocol is very crucial and useful because it can enhance the system performance such as the outage probability. In [7], an optimum power allocation scheme is proposed, which can minimize the outage probability. In [8], an optimal joint relay selection and power allocation scheme is presented based on the maximization of the smaller of the received signal-to-noise ratios (SNRs) of the two transceivers under a total transmit power budget. To the best of our knowledge, however, there has been no work which investigated performance for joint relay and optimum power allocation on the basis of the outage probability. This has motivated our work.

In this paper, we first analyze the performance of relay selection with equal power allocation and derive a tight lower bound of outage probability in closed-form, which is very close to the exact outage probability obtained by simulation, irrespective of the values of channel variances. Furthermore, we propose a joint relay selection and optimum power allocation scheme, whose lower bound outage probability goes close to the exact outage probability obtained by simulation with the increasing E. Based-on outage probability analysis, we obtain a relay selection strategy, which requires only a single integer parameter (i.e, the index of the $optimal$ relay) to be broadcasted, while in [9], in order to allow each relay to calculate its own beamforming coefficient, the two transceivers are required to broadcast three analog parameters to all relays.

The remainder of this paper is organized as follows. In Section II, we present the system model. In Section III, we analyze the outage probabilities in two cases (i.e,relay selection with equal power allocation and optimum power allocation), specifically, we derive a lower bound of the outage probability in closed-form, which is close to the exact outage probability obtained by simulation. Furthermore, we investigate how to select the optimal relay. Simulation results are presented in Section IV and conclusions are drawn in Section V.

II. System Model

We consider a bidirectional network, where two terminals S_1 and S_2 want to exchange information with the help of K relay nodes. We assume no direct communication between the two transceivers because of the poor quality of the channels between them. Transceivers and the relays, with single antenna units, are capable of either transmitting or receiving using a half-duplex single-antenna communication scheme. Assuming a Rayleigh-fading scenario, let h_i represent the channel between S_1 and R_i, and f_i represent the channel between R_i and S_2, where all the channels are reciprocal. Furthermore, we assume that h_i and f_i are complex Gaussian random variables with zero mean and variances Ω_{h_i} and Ω_{f_i}, respectively. The

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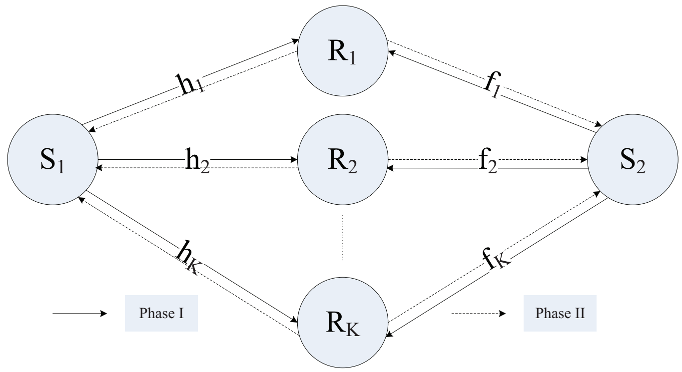

Fig. 1. System model for the PNC protocol.

additive white Gaussian noise (AWGN) associated with every channel is assumed to be a complex Gaussian random variable with zero mean and unit variance. As in many previous publications on the PNC protocol, we assume that the two sources as well as the relay know h_i and f_i.

The PNC protocol is illustrated in Fig.1. In phase 1, S_1 and S_2 transmit their signals simultaneously to the relay nodes. In phase 2, a *best* relay is chosen as the forwarding node, Let x_1 and x_2 be the information symbols transmitted by S_1 and S_2, respectively. They are normalized as E$\{|x_1|^2\}$ = E$\{|x_2|^2\}$,where E$\{.\}$ stands for the expectation and $|.|$ represents the absolute value of a complex number. The received signal at i is given by

$$x_i = \sqrt{E_1} h_i x_1 + \sqrt{E_2} f_i x_2 + n_i \tag{1}$$

where x_i is the baseband (complex) signal received at the ith relay, E_1 and E_2 are the transmit powers of S_1 and S_2, respectively, and n_i is the complex noise at the ith relay. In the second step, the ith relay multiplies its received signal by a complex weight ρ_i and transmits the so-obtained signal. The signals y_{i1} and y_{i2} received at S_1 and S_2 can be represented respectively, as

$$y_{i1} = \sqrt{E_1} \rho_i h_i^2 x_1 + \sqrt{E_2} \rho_i h_i f_i x_2 + \rho_i h_i n_i + n_1 \tag{2}$$

$$y_{i2} = \sqrt{E_2} \rho_i h_i f_i x_1 + \sqrt{E_1} \rho_i f_i^2 x_2 + \rho_i f_i n_i + n_2 \tag{3}$$

where n_k is the noise at the kth transceiver, for k = 1,2. All noise are assumed to be i.i.d.Gaussian with zero-mean and unit variance. The first term in (2), known as self-interference, can be subtracted from y_{i1}. Similarly, the second term in (3) can be subtracted from y_{i2}. The residual signals and after self-interference cancellation are defined as

$$\widetilde{y}_{i1} = y_{i1} - \sqrt{E_1} \rho_i h_i^2 x_1 = \sqrt{E_2} \rho_i h_i f_i x_2 + \rho_i h_i n_i + n_1 \tag{4}$$

$$\widetilde{y}_{i2} = y_{i2} - \sqrt{E_2} \rho_i f_i^2 x_2 = \sqrt{E_1} \rho_i h_i f_i x_1 + \rho_i f_i n_i + n_2 \tag{5}$$

The residual signals $\widetilde{y}_{i1}$ and $\widetilde{y}_{i2}$ can be used to decode the information symbols x_2 and x_1 at S_1 and S_2, respectively.

III. OUTAGE PROBABILITY ANALYSIS

A. Relay Selection With Equal Power Allocation

Assuming the total transmission power of the bidirectional network is constrained to be 3E, while S_1 and S_2 both transmit their symbols with power E(E_1 = E_2 = E), thus, in order to ensure that the transmission power at ith relay is always E, the amplifying coefficient ρ_i can be chosen as

$$\rho_i = \sqrt{\frac{E}{\mathrm{E}\,|h_i|^2 + \mathrm{E}\,|f_i|^2 + 1}} \tag{6}$$

Note that the relay node R does not need to estimate the individual channel gains , because the denominator of (6) is the received symbol power. This choice of ρ_i, however, makes the analysis intractable. In this paper, therefore, it is approximated as follows:

$$\rho_i = \sqrt{\frac{1}{|h_i|^2 + |f_i|^2}} \tag{7}$$

which has been a widely adopted approximation in many other works. This approximation was originally proposed for high SNR range [10], [11]. It was also demonstrated that the average bit error rate derived based on this approximation was actually very close to the exact value even when the SNR was as low as 0 dB [10]. Through (4) and (5), the instantaneous SNR at S_1 is given by

$$\gamma_{i1} = \frac{\mathrm{E}\,|h_i f_i|^2}{2\,|h_i|^2 + |f_i|^2} \tag{8}$$

In a similar way to (3), the instantaneous SNR at S_2 is given by

$$\gamma_{i2} = \frac{\mathrm{E}\,|h_i f_i|^2}{2\,|f_i|^2 + |h_i|^2} \tag{9}$$

Note that γ_{i1} of (8) and γ_{i2} of (9) obtained based on (7) are actually upper-bounds of the exact SNRs based on exact ρ_i of (6).

Now, we can continue to derive a lower bound of the outage probability of the PNC protocol. From (8) and (9), the mutual information at S_1 and S_2 for the PNC protocol is given by [1]

$$I_{i1} = \frac{1}{2} log_2(1 + \gamma_{i1}), \qquad I_{i2} = \frac{1}{2} log_2(1 + \gamma_{i2}) \tag{10}$$

Note that the pre-log factor 1/2 is used because the information exchange between the two sources takes two time slots [1]. Since the two sources in this network are equivalent terminals, it is fair to set the target rate of each source as R/2, where R denotes the target rate of the whole bidirectional network. As in [12], We define the outage probability of the PNC protocol as the probability that either I_1 or I_2 is smaller than the target rate R/2,i.e.

$$P_{outage}(R_{ith}) = Pr(I_{i1} < \frac{R}{2} \quad or \quad I_{i2} < \frac{R}{2}) \qquad (11)$$

Since the two mutual information I_1 and I_2 are highly correlated, it is very difficult to directly solve the outage probability of (11) even after we adopt the approximation for ρ_i of (7). In order to make the analysis feasible, we take a bounding approach. Specifically, we adopt the following well-known inequality, where the harmonic mean of two positive numbers can be upper-bounded by the minimum of those two numbers as follows [10]:

$$\frac{xy}{x+y} < min(x, y). \qquad (12)$$

Using this method, we upper-bound the instantaneous SNRs and derive a lower bound of the outage probability in the following theorem as was shown in [7].

$Theorem 1$: The outage probability of the single relay selection can be lower-bounded as follows:

$$\begin{aligned} P_{outage}(R_{ith}) &> \widetilde{P}_{outage}(R_{ith}) \\ &= 1 - exp(-\frac{2(\Omega_{h_i} + \Omega_{f_i})(2^R - 1)}{E\Omega_{h_i}\Omega_{f_i}}) \end{aligned} \qquad (13)$$

Note that derived outage probability of (13) in Theorem 1 is indeed a strict lower bound of the exact outage probability, because $xy/(x+y+1) < xy/(x+y)$.

Then, the outage probability of the best relay selection strategy is

$$\begin{aligned} P_{opp-outage}(R_{ith}) &> \widetilde{P}_{opp-outage}(R_{ith}) \\ &= \prod_{i=1}^{K}(1 - exp(-\frac{2(\Omega_{h_i} + \Omega_{f_i})(2^R - 1)}{E\Omega_{h_i}\Omega_{f_i}})) \end{aligned} \qquad (14)$$

From the numerical results above, we will notice that, irrespective of the channel variances, the lower bound $\widetilde{P}_{outage}(R_{ith})$ is very close to $P_{outage}(R_{ith})$ in the whole SNR range. Due to this accuracy, we use this lower bound to find the *best* relay. Now our problem can be represented as

$$\min_{i \in \{1,2,\cdots,K\}} \qquad \widetilde{P}_{outage}(R_{ith}) \qquad (15)$$

The optimization problem in (14) is equivalent to the following one

$$\min_{i \in \{1,2,\cdots,K\}} \qquad (\frac{\Omega_{h_i} + \Omega_{f_i}}{\Omega_{h_i}\Omega_{f_i}}) \qquad (16)$$

The optimal RS can be conducted as follows. Both transceivers, who know all channels, can select the *best* relay by calculating $\frac{\Omega_{h_i}+\Omega_{f_i}}{\Omega_{h_i}\Omega_{f_i}}$, for $i \in \{1, 2, \cdots, K\}$, as in (16) and pick the relay which results in the minimum $\frac{\Omega_{h_i}+\Omega_{f_i}}{\Omega_{h_i}\Omega_{f_i}}$. Then, one transceiver broadcasts the index of the best relay to all relays over a control channel. Those relays which do not *hear* their own indices, will not participate in relaying. The optimally selected relay, upon hearing its index, will use its local channel state information to calculate the minimum $\widetilde{P}_{outage}(R_{ith})$, as given in (13). The amount of overhead bits required for broadcasting the index of the optimally selected relay is $log_2 K$ [8].

B. Joint Relay Selection And Optimum Power Allocation

In this section, we propose a joint relay selection and optimum power allocation scheme, which can simultaneously select the best relay and obtain the minimization of the outage probability of the PNC protocol. In the A section, we derived a tight lower bound of the outage probability of the PNC protocol when every terminal had the same transmissions power E. Since every terminal knows the values of h_i and f_i , it is more desirable to allocate the transmission power according to channel conditions in order to maximize system performance. The network is still assumed to have a total transmit power constraint $E_T^{max} = 3E$. Denoting the transmit power at the ith relay as E_r, consequently, the instantaneous SNRs at $S1$ and S_2 should be rewritten as

$$\begin{aligned} \gamma_{i1} &= \frac{\mathrm{E_r E_1}|h_i f_i|^2}{(\mathrm{E_r+E_1})|h_i|^2+\mathrm{E_2}|f_i|^2}, \\ \gamma_{i2} &= \frac{\mathrm{E_r E_2}|h_i f_i|^2}{(\mathrm{E_r+E_1})|f_i|^2+\mathrm{E_1}|h_i|^2} \end{aligned} \qquad (17)$$

The main problem can thus be represented as

$$\min_{E_1,E_2,E_r,i} \quad (\widetilde{P}_{outage}(R_{ith})) \quad subject \quad to \quad E_{T,i} \le 3E \quad (18)$$

This is equivalent to first optimizing over E_1, E_2, E_r, which is the optimal power allocation problem, then optimizing over i, which is the optimal relay selection problem. In the following, we consider the two separately.

The power allocation problem, represented as

$$\min_{E_1,E_2,E_r} \quad (\widetilde{P}_{outage}(R_{ith})) \quad subject \quad to \quad E_{T,i} \le 3E \quad (19)$$

It follows from (11) that the optimization problem in (17) is equivalent to the following one

$$\max_{E_1,E_2,E_r} \quad min(\gamma_{i1}, \gamma_{i2}) \quad subject \quad to \quad E_{T,i} \le 3E \quad (20)$$

The minimax problem in (18) is solved in the following theorem as was shown in [7].

Theorem 2: When $E_1+E_2+E_r = 3E$, the optimum power allocation that minimizes the outage probability $\widetilde{P}_{outage}(R_{ith})$ of the PNC protocol is given by

$$\begin{aligned} E_r &= \tfrac{3}{2}E, \\ E_1 &= \tfrac{3|f_i|}{2(|h_i|+|f_i|)}E, \\ E_2 &= \tfrac{3|h_i|}{2(|h_i|+|f_i|)}E \end{aligned} \qquad (21)$$

We notice that the optimum power allocation scheme in Theorem 2 allocates more power to the week traffic flow in

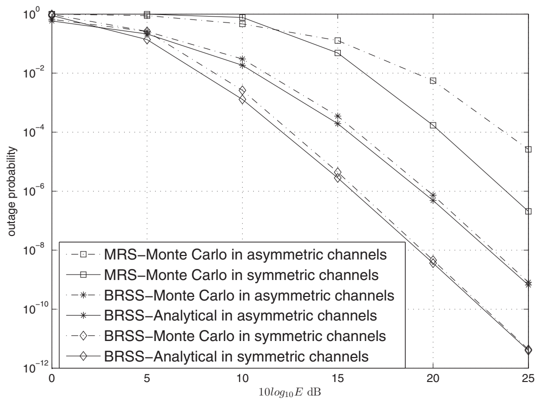

Fig. 2. Comparison of outage probabilities with equal power allocation.

order to minimize the outage probability. With the optimal power allocation solution found, we can get the SNRs as follows:

$$\gamma_{i1} = \quad \gamma_{i2} = \frac{3E}{2}(\frac{h_i f_i}{h_i + f_i})^2 \tag{22}$$

Furthermore, the outage probability can be obtained as follows:

$$\begin{aligned} P_{outage}(R_{ith}) &> \widetilde{P}_{outage}(R_{ith}) \\ &= 1 - exp(-\frac{2(\Omega_{h_i} + \Omega_{f_i})(2^R - 1)}{3E\Omega_{h_i}\Omega_{f_i}}) \end{aligned} \tag{23}$$

Proof:See Appendix A

Now, we can get the outage probability of the best relay selection strategy with optimum power allocation as followed

$$\begin{aligned} P_{opp-outage}(R_{ith}) &> \widetilde{P}_{opp-outage}(R_{ith}) \\ &= \prod_{i=1}^{K}(1 - exp(-\frac{2(\Omega_{h_i} + \Omega_{f_i})(2^R - 1)}{3E\Omega_{h_i}\Omega_{f_i}})) \end{aligned} \tag{24}$$

Then our problem in (17) reduces to the following RS problems:

$$\min_{i \in \{1,2,\cdots,K\}} \quad \widetilde{P}_{outage}(R_{ith}) \tag{25}$$

The optimal RS can be conducted as follows. Both transceivers, who know all channels, can select the *best* relay by calculating $\widetilde{P}_{outage}(R_{ith})$, for $i \in \{1, 2, \cdots, K\}$, as in (16) and pick the relay which results in the minimum $\widetilde{P}_{outage}(R_{ith})$. Then, one transceiver broadcasts the index of the best relay to all relays over a control channel and the selected relay calculate the minimum outage probability as in section A.

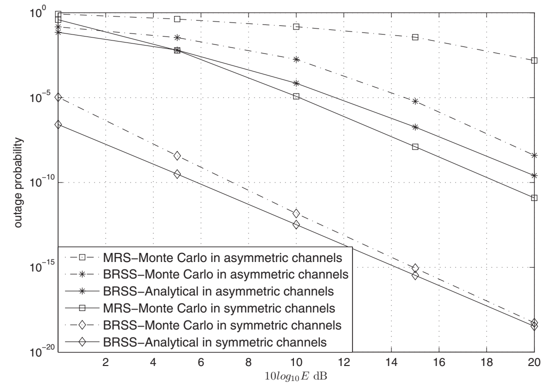

Fig. 3. Comparison of outage probabilities with optimum power allocation.

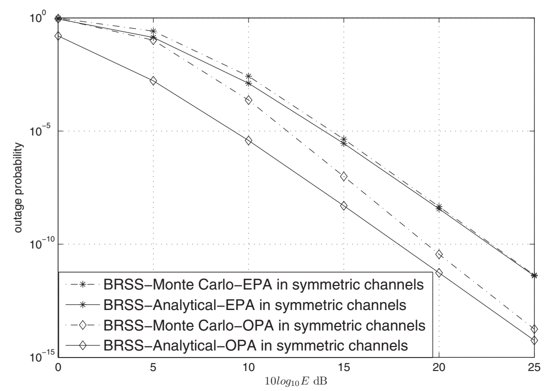

Fig. 4. Comparison of outage probabilities with both equal and optimum power allocation.

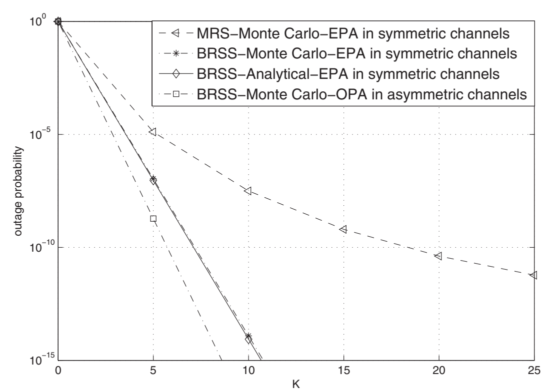

Fig. 5. Comparison of outage probabilities with increasing relay numbers.

IV. NUMERICAL RESULTS

In this section, simulation is provided to demonstrate the validity and usefulness of our analytical expressions. We have set $K = 6$, $R = 1bps/Hz$, $\Omega_{h_i} = \Omega_{f_i} = 1, i \in \{1, 2, \cdots, K\}$ for the symmetric channels, and $\Omega_{h_i} = \Omega_{f_i} = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$ for the asymmetric channels.

In Fig.2 and Fig.3, we compare the performance of (i) best relay selection strategy(BRSS) by simulation, (ii) best relay selection strategy by analysis, (iii) AF relaying with equal-power multiple-relay strategy(MRS) in scenarios of equal power allocation(EPA) and optimum power allocation(OPA), respectively. From Fig.2 and Fig.3, it is evident that a significant gain of best relay selection strategy compared to multiple-relay transmission is achieved. What's more, we can find that the gap between the lower bound and the actual performance becomes smaller and smaller with the increasing E. All above stated is true both for the symmetric and asymmetric scenarios.

In Fig 4, we compare the performance of both best relay selection with equal power allocation and best relay selection with optimum power allocation in both simulation and analysis, we can find that for both equal and optimum power allocation, the tight lower bound of outage probability is very close to the exact outage probability, and the gap between the lower bound and the actual performance becomes smaller and smaller with the increasing E. Furthermore, it is obviously that the outage probability of optimum power allocation is much smaller than that of equal power allocation.

In Fig.5, the outage probabilities as a function of K with $10log_{10}E = 20$ in symmetric channels is shown. Particularly, the outage probabilities decrease according to the number of relays K. From Fig.5, we can find that the outage probability of optimum power allocation is smaller than that of equal power allocation, which is smaller than that of multiple-relay with equal-power. What's more, the outage probability of equal power allocation by analysis is very close to that by simulation.

V. CONCLUSIONS

In this paper, we study the outage probability of joint RS and power allocation scheme for bidirectional relay networks consisting of two transceivers and multiple relay nodes. We have first derived a tight lower bound of the outage probability assuming equal power allocation in closed-form. Then, we developed the best relay selection strategy. Finally, we have proposed a joint relay selection and optimum power allocation scheme, which could simultaneously select the best relay and complete the optimum power allocation. Our analysis has been verified by simulation.

APPENDIX A

Let $X = |h_i|^2$ and $Y = |f_i|^2$. Thus, X and Y are exponential random variables with means Ω_{h_i} and Ω_{f_i}, respectively. By using the equality(22), the outage probability can be lower-bounded as follows:

$$\begin{aligned} P_{outage}(R_{ith}) &= Pr(I_{i1} < \frac{R}{2}) \\ &= 1 - Pr(I_{i1} > \frac{R}{2}) \\ &= 1 - Pr(\gamma_i > 2^R - 1) \end{aligned} \qquad (26)$$

By using the inequality(12), the probability can be evaluated in the following way

$$\begin{aligned} Pr(\gamma_i > 2^R - 1) &< Pr(2^R - 1 < \frac{3}{2}E\{min(h_i, f_i)\}^2) \\ &= Pr(\frac{3E}{2}min(X,Y) > 2^R - 1) \\ &= Pr(\frac{3E}{2}X > 2^R - 1, X > Y) \\ &+ Pr(\frac{3E}{2}Y > 2^R - 1, Y > X) \end{aligned} \qquad (27)$$

Among the above inequality, the first probability can be evaluated

$$\begin{aligned} &Pr(\frac{3E}{2}X > 2^R - 1, X > Y) \\ &= \frac{\Omega_{h_i}}{\Omega_{h_i} + \Omega_{f_i}} exp(-\frac{2(\Omega_{h_i} + \Omega_{f_i})(2^R - 1)}{3E\Omega_{h_i}\Omega_{f_i}}) \end{aligned} \qquad (28)$$

Similarly, the second probability can be evaluated:

$$\begin{aligned} &Pr(\frac{3E}{2}X > 2^R - 1, X > Y) \\ &= \frac{\Omega_{f_i}}{\Omega_{h_i} + \Omega_{f_i}} exp(-\frac{2(\Omega_{h_i} + \Omega_{f_i})(2^R - 1)}{3E\Omega_{h_i}\Omega_{f_i}}) \end{aligned} \qquad (29)$$

By substituting (28) and (29) into (26), we obtain the lower bound $\widetilde{P}_{outage}(R_{ith})$ of the joint relay selection and optimum power allocation strategy.

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