Viterbi Demodulation in Two-Level FH-CDMA Wireless Systems Based on Reed-Solomon Codes

Lun-Cheng Tsao, Hou-Shou Chen, and Guu-Chang Yang
Department of Electrical Engineering
National Chung Hsing University
Taichung 402, Taiwan, R.O.C.
email: houshou@dragon.nchu.edu.tw
email: gcyang@dragon.nchu.edu.tw

Cheng-Yuan Chang
Department of Electrical Engineering
National United University
Miaoli 360, Taiwan, R.O.C.
email: chengyuan@nuu.edu.tw

Wing C. Kwong
Department of Engineering
Hofstra University
Hempstead, NY 11549, U.S.A.
email: wing.c.kwong@hofstra.edu

***Abstract*—Coded modulation with Reed-Solomon (RS) codes has recently been proposed to use atop of frequency-hopping code-division multiple access (FH-CDMA) in order to increase data transfer rate. In this paper, a new method on implementing the modulation and demodulation processes of the RS codes in RS/FH-CDMA is proposed. With the use of our specially "pruned" trellis and the Viterbi algorithm, both processes are simplified and so are their computational complexities. The performance and computational complexities of this RS/FH-CDMA system are analyzed and verified with computer simulation.**

I. Introduction

The application of frequency-hopping code-division multiple access (FH-CDMA) to wireless communications is receiving attention lately [1], [2]. Allowing many simultaneous users to share the same channel bandwidth, FH-CDMA is based on assigning a distinct FH pattern (with a low cross-correlation function) to each user in order to minimize multiple-access interference (MAI). Further, coded modulation has been proposed to add atop FH-CDMA to increase data transfer rate by transmitting symbols (of multiple data bits) [3], [4]. In [3], we studied the use of Reed-Solomon (RS) codes [5]–[7] as modulation codes, representing the transmitting symbols. The RS/FH-CDMA scheme was compared to Goodman's M-ary frequency-shift-keying (MFSK/FH-CDMA) scheme. Our study showed that MFSK/ and RS/FH-CDMA provided a tradeoff between performance and data rate. If data rate is more important, RS/FH-CDMA is more preferable.

One bottleneck in RS/FH-CDMA is on the speed of demodulating the RS codes. At a receiver, an arriving RS/FH-CDMA signal is first dehopped by the receiver's FH pattern to form a decoding matrix. The matrix elements are compared with those of all RS codes for demodulation; the desired RS code (i.e., symbol) is the one with the minimum distance from this matrix [3], [4]. The comparison process is time-consuming and grows with the symbol size. In this paper, we propose a "pruned-trellis" method with the Viterbi algorithm [8], [9] to reduce the modulation and demodulation complexities and, thus, improve the processing speed in RS/FH-CDMA.

In Section II, we review how to design the trellis of the RS codes [7]. Representing k_b data bits per symbol, there must exist 2^{k_b} symbols and so is the number of the RS codes. However, the RS codes constructed in Section II do not usually have such an exact cardinality of 2^{k_b}. In Section III, we formulate a pruning method to remove the RS codes in the trellis so that the remaining 2^{k_b} RS codes result in less demodulation errors. The demodulation process with this pruned trellis and the Viterbi algorithm is studied. In Section IV, the performances of this RS/FH-CDMA system is analyzed under the effects of MAI, Rayleigh fading, and additive white Gaussian noise (AWGN) and verified with computer simulation. Finally, the computation complexity with and without the Viterbi algorithm are compared in Section V.

II. Construction of Reed-Solomon Codes

The RS codes are constructed in Galois field GF(p^s), in which p is a prime number and s is a positive integer [5]–[7]. For illustration purpose, we here consider $s = 1$. Thus, these RS codes, denoted RS$(n, k, n-k+1)$, have length $n = p-1$, cardinality p^k, maximum cross-correlation function of $k-1$, and minimum Hamming distance of $n-k+1$, where $k \geq 1$ is the code dimension. If erasure-insertion correction in [6] is employed, these RS codes can correct up to $n-k$ symbol errors; otherwise, only at most $(n-k)/2$ symbol errors can be corrected, as assumed in this paper.

The code construction begins with a generator polynomial $g(x) = (x-\alpha)(x-\alpha^2)\cdots(x-\alpha^{n-k}) = g_0 + g_1 x + \cdots + g_{n-k}x^{n-k}$, where α is a primitive element in GF(p) and $g_j \in$ GF(p) is the coefficient of x^j for all $j \in [0, n-k]$. Let the input message polynomial be $m(x) = m_0 + m_1 x + \cdots + m_{k-1}x^{k-1}$, where $m_j \in$ GF(p) is the coefficient of x_j for all $j \in [0, k-1]$. The collections of the code coefficients, $(c_0, c_1, \ldots, c_{n-1})$, in $c(x) = m(x)g(x) = c_0 + c_1 x + \cdots + c_{n-1}x^{n-1}$, result in the RS$(n, k, n-k+1)$ codes. Equivalently, the jth code coefficient of x_j in $c(x)$ is given by

$$c_j = m_0 g_j + m_1 g_{j-1} + \cdots + m_j g_0 \qquad (1)$$

where $g_j = 0$ for all $j > n-k$ and $m_j = 0$ for all $j \geq k$.

These RS codes can be generated graphically by means of a trellis with nodes and branches [7]. Each branch has a value determined by one code coefficient; each node is labeled with the input-message coefficients involved in the computation of the code coefficient. To effectively construct the trellis and

978-1-4673-0990-5/12/$31.00 ©2012 IEEE

TABLE I
RS$(4,2,3)$ CODES FROM TRAVERSING THE PATHS IN FIG. 1.

m_0 \ m_1	0	1	2	3	4
0	0000	0341	0132	0423	0214
1	3410	3201	3042	3333	3124
2	1320	1111	1402	1243	1034
3	4230	4021	4312	4103	4444
4	2140	2431	2222	2013	2304

prune the trellis (later in Section III), we consider $n \geq 2k$. Also, we focus on $k = 2$ (i.e., maximum cross-correlation function of one) in order to minimize demodulation errors.

The trellis starts from an initial state 0 of one node. This node branches out to p nodes (in state 1) and the value of each branch is computed by $c_0 = g_0 m_0$ (code coefficient of x^0) for nodes labeled with $m_0 = \{0, 1, \ldots, p-1\}$, respectively. Each of these p nodes branches out to another p nodes and each branch value is computed by $c_1 = g_0 m_1 + m_0 g_1$ (code coefficient of x^1) for nodes labeled with $(m_0, m_1) = \{(0,0), (0,1), \ldots, (p-1, p-1)\}$, correspondingly, giving a total of p^2 nodes in state 2. From state 2 to state $n-2$, only one branch leaves from each of the p^2 nodes in each state. Each branch at state $j \in [2, n-2]$ has its value of $c_j = m_0 g_j + m_1 g_{j-1}$ for nodes labeled with $(m_0, m_1) = \{(0,0), (0,1), \ldots, (p-1, p-1)\}$, correspondingly, giving the same number of p^2 nodes in each state. From state $n-2$ to state $n-1$, the trellis contracts and converges to p nodes labeled with $m_1 = \{0, 1, \ldots, p-1\}$ in state $n-1$. Only one branch leaves from each of these p nodes with its value of $c_{n-1} = g_{n-2} m_1$. These branches all converge to a single node in the final state n. Thus, there are totally p^2 paths spanning from state 0 to state n and each path consists of n branches. By traversing these n branches in each path, each of the p^2 RS$(n, 2, n-1)$ codes is obtained, where $n \geq 2k$ and $n = p-1$.

For example, using $\alpha = 2$ in GF(5), the RS$(4,2,3)$ codes have $g(x) = 3 + 4x + x^2$ and $c(x) = 3m_0 + (4m_0 + 3m_1)x + (m_0 + 4m_1)x^2 + m_1 x^3$, where the code coefficients are given on the branches of the trellis in Fig. 1. Traversing the twenty-five paths from left to right and recoding the values on the four branches of each path, the RS$(4,2,3)$ codes with cross-correlation functions of at most one are shown in Table I.

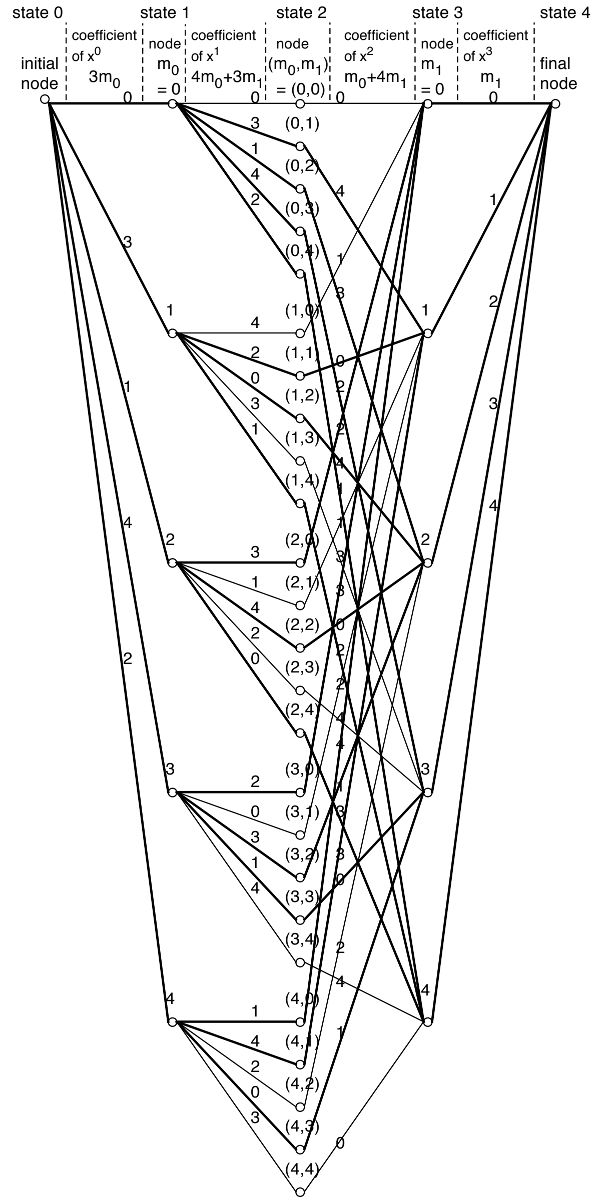

Fig. 1. The trellis of the RS$(4,2,3)$ codes. (The branches drawn with light lines correspond to the removed codes in the pruned trellis in Section III.)

III. RS/FH-CDMA SCHEME WITH PRUNED TRELLIS

In our two-level RS/FH-CDMA scheme [3], [4], we can generally choose any family of $(M_m \times L_m, w_m, \lambda_{c,m})$ modulation codes and any family of $(M_h \times L_h, w_h, \lambda_{c,h})$ FH patterns as long as $w_h \geq L_m$, where M_m and M_h denote the numbers of carrier frequencies, L_m and L_h denote the numbers of time slots (i.e., code length), w_m and w_h are the numbers of elements (i.e., code weight), and $\lambda_{c,m}$ and $\lambda_{c,h}$ denote the maximum cross-correlation functions of the modulation codes and FH patterns, respectively.

At the front of a transmitter is the modulation stage, in which every $k_b = \lfloor \log_2 \phi_m \rfloor$ serial data bits are first mapped into a symbol and each symbol is represented by a modulation code, where ϕ_m is the cardinality of the modulation codes in use and $\lfloor \cdot \rfloor$ is the floor function. Afterward, the modulation code combines with the FH pattern (address signature) of the destined receiver, determining the carrier frequencies of the final FH-CDMA signal [3], [4]. While an element of a modulation code defines the carrier frequency used in a frequency band in a given time slot, an element of the FH pattern determines which frequency band to use in that time slot. In other words, the available transmission bandwidth is divided into M_h frequency bands with M_m carrier frequencies in each band, giving a total of $M_m M_h$ carrier frequencies.

At a receiver, the arrival RS/FH-CDMA signals are considered as a freqeuncy-time matrix, which contains the desired user's signal and the effects of MAI, fading, and AWGN (i.e., hits, deletions, and false alarms). Each receiver uses its own

FH pattern (address signature) to decode this matrix. Afterward, the decoded matrix, which contains the modulation-code (i.e., symbol) information, is compared with the elements of all modulation codes. The desired code (i.e., symbol) is the one with the minimum distance from the matrix. Finally, the recovered symbol is demapped to generate the data bits.

Assume that the RS$(n, 2, n-1)$ codes, constructed by the trellis in Section II, are used as the $(p \times p-1, p-1, 1)$ modulation codes to represent the symbols of every $k_b = \lfloor \log_2 p^2 \rfloor$ serial data bits in our RS/FH-CDMA scheme, where $n = p-1$. To represent the 2^{k_b} symbols, we only need 2^{k_b} RS codes. In other words, there exist $p^2 - 2^{k_b}$ unused RS codes and, equivalently, paths to be pruned from the trellis.

In our pruning method, we first identify the message vectors (m_0, m_1) of the RS$(n, 2, n-1)$ codes that have the same code coefficients (e.g., 0000, 1111, etc in Table I). There are totally p of them, where $n = p - 1$. If more RS codes need to be removed, we continue on the codes with $m_0 + 1 \pmod p$, where m_0 comes from the (m_0, m_1) vectors of the p same-coefficient codes, giving p additional RS codes for removal. We apply $m_0 + 2 \pmod p$ and so on for further code removal, if needed. The pruned trellis is obtained by removing the nodes labeled with these (m_0, m_1) values. By removing the codes with the same coefficients first, this method lowers the amount of interference in the remaining RS codes.

For example, the nine RS$(4, 2, 3)$ codes to be removed from Table I are 0000, 1111, 2222, 3333, 4444, 3410, 4021, 0132, and 1243, corresponding to $(m_0, m_1) = (0, 0)$, $(2, 1)$, $(4, 2)$, $(1, 3)$, $(3, 4)$, $(1, 0)$, $(3, 1)$, $(0, 2)$ and $(2, 3)$, respectively. The pruned trellis is obtained by removing the nodes labeled with these (m_0, m_1)-values in state 2 and their associated (those drawn in light lines) branches in Fig. 1. The pruned trellis has sixteen paths (i.e., RS codes), supporting sixteen symbols or, equivalently, four bits per symbol.

With this pruned trellis, demodulation of the RS codes can be efficiently performed by the Viterbi algorithm [8], [9]. Since the Viterbi algorithm is well known, only a brief description is presented here in the context of finding the maximum-likelihood path in the pruned trellis, for sake of completeness. Assume that the demodulator's RS code is expressed as $c = (c_0, c_1, \ldots, c_{n-1})$, and the received RS code is expressed as $r = (r_0, r_1, \ldots, r_{n-1})$. The logarithm of the likelihood ratio of the received RS code (given the demodulator's RS code) is defined as a path metric $M(r|c) = \log p(r|c) = \sum_{i=0}^{n-1} \log p(r_i|c_i)$, where $p(r_i|c_i)$ is a channel transition probability and $M(r_i|c_i)$ is called the branch metric.

To find the path (i.e., RS code) in a trellis with the largest path metric, the Viterbi algorithm defines two parameters while traversing the trellis: store the paths and the path metrics in each state of the trellis, and apply the following steps:

1) Begin at state $i = 1$ of the trellis. Compute the branch metric for each path entering a node, and store the path and its metric.
2) Increase i by one. Compute the path metric for each path entering a node by adding the branch metric entering the node to the metric of the connecting path from the previous state. In each state, compare the metrics of all paths entering every node, select the path with the largest metric, store it along with its metric, and eliminate all other paths.
3) If $i < n$, repeat step 2; otherwise, stop.

The algorithm terminates at the final state $i = n$ and the final path is the maximum-likelihood path. By traversing the branches of this path, the RS code with the minimum distance from the demodulator's RS (signature) code is recovered.

IV. Performance Analysis

In this section, we analyze the performance of our RS/FH-CDMA scheme under the effects of MAI, Rayleigh fading, and AWGN. A receiver, based on the presence or absence of energy, makes a decision by selecting the row containing the greatest number of entries in the decoded matrix. Coded modulation creates additional interference and the maximum cross-correlation function of the RS codes needs to be considered, in addition to the usual MAI created by the FH patterns. Assume that $(M_h \times L_h, w, 1)$ FH patterns are used. We divide the transmission band into M_h frequency bands and each frequency band is subdivided into $M_m = p$ frequencies for the RS modulation. The probability that there are v entries in an undesired row is given by [1], [3]

$$P(v) = \binom{w}{v} \sum_{i=0}^{v} (-1)^i \binom{v}{i} \left(1 - q + \frac{vq}{w} - \frac{iq}{w}\right)^{K-1} \tag{2}$$

where $q = w^2/(pM_hL_h)$ is the probability that a frequency of the desired user is being hit by an interferer and K is the number of simultaneous users.

In a Rayleigh fading channel, the probability that the decoded matrix contains v entries in an undesired row is

$$\begin{aligned} &P_s(v) \\ &= \sum_{j=0}^{v} \sum_{r=0}^{\min[v-j, w-v]} P(v-j) \binom{v-j}{r} p_d^r (1-p_d)^{v-j-r} \\ &\quad \times \binom{w-v+j}{r+j} p_f^{r+j} (1-p_f)^{w-v-r} \\ &\quad + \sum_{j=1}^{w-v} \sum_{r=j}^{\min[v+j, w-v]} P(v+j) \binom{v+j}{r} p_d^r (1-p_d)^{v+j-r} \\ &\quad \times \binom{w-v-j}{r-j} p_f^{r-j} (1-p_f)^{w-v-r} \end{aligned} \tag{3}$$

where $p_d = 1 - \exp\{-\beta^2/[2 + 2(\overline{E_b}/N_o)(k_b/w)]\}$ is the deletion probability that a receiver misses a transmitted frequency, $p_f = \exp(-\beta^2/2)$ is the false-alarm probability that a tone is detected in the receiver when none has actually been transmitted, β denotes the threshold level normalized to the root-mean-squared signal level, and $\overline{E_b}/N_o$ is the energy-per-bit to noise-power spectral density ratio [2].

Since the maximum number of hits between any two RS$(n, 2, n-1)$ codes is one (i.e., $\lambda_{c,m} = 1$). We have to consider the probability of increasing the number of entries in an undesired row of the decoded matrix from v to $v+1$ due to

$\lambda_{c,m} = 1$. The probability of having v entries in an undesired row of the decoded matrix with up to one of these entries coming from the demodulation process is then modified as

$$P'_s(v) = xP_s(v-1) + \begin{cases} (1-y)P_s(v) & \text{for } v < w \\ P_s(v) & \text{for } v = w \end{cases} \tag{4}$$

where $P_s(z) = 0$ if $z < 0$. If the extra RS codes are removed randomly in the trellis, the probability of adding one entry due to $\lambda_{c,m} = 1$ is given by [4, eq. (12)]

$$x = y = \frac{p-1}{p^2}. \tag{5}$$

If the extra RS codes are removed by the pruning method in Section III for lowering demodulation errors, we have

$$x = \frac{\sum_{i=0}^{2^{k_b}-1} \sum_{j=0, i\neq j}^{2^{k_b}-1} h_1(c_i^{(v)}, c_j^{(v)}) \times \frac{1}{\binom{v}{1}}}{2^{k_b} \times 2^{k_b}} \tag{6}$$

$$y = \frac{\sum_{i=0}^{2^{k_b}-1} \sum_{j=0, i\neq j}^{2^{k_b}-1} h_1(c_i^{(v+1)}, c_j^{(v+1)}) \times \frac{1}{\binom{v+1}{1}}}{2^{k_b} \times 2^{k_b}} \tag{7}$$

where $x = 0$ if $v = 0$ and $y = 0$ if $v = w$. Eqs. (6) and (7) average the total number of interfering RS codes, c_j's, contributing one-hits to the cross-correlation functions with the desired code c_i. The function $h_1(c_i^{(v)}, c_j^{(v)})$ counts the total number of one-hits between c_i and c_j. If such a one-hit occurs, the number of entries in the undesired row will be increased from $v-1$ to v and we set $h_1(c_i^{(v)}, c_j^{(v)}) = 1$; otherwise, $h_1(c_i^{(v)}, c_j^{(v)}) = 0$. The term $1/\binom{v}{1}$ represents the probability of such an increase. The two terms in the denominator represent the probabilities of choosing c_i and c_j, out of the 2^{k_b} codes. Similarly, we set $h_1(c_i^{(v+1)}, c_j^{(v+1)}) = 1$ when c_i is hit by c_j and this increases the number of entries from v to $v+1$. The term $1/\binom{v+1}{1}$ represents the probability of such an increase.

For example, when $p = n+1 = 7$ and $k_b = \{5, 4, 3\}$, we find from (6) that $x = 0$ if $v = 0$, and $x = \{116/(32 \times 32), 22/(16 \times 16), 2/(8 \times 8)\}$, respectively, if $v \in [1, 6]$, and also from (7) that $y = \{116/(32 \times 32), 22/(16 \times 16), 2/(8 \times 8)\}$, respectively, if $v \in [0, 5]$, and $y = 0$ if $v = 6$. When $p = n+1 = 11$ and $k_b = \{6, 5, 4\}$, we find from (6) that $x = 0$ if $v = 0$, and $x = \{310/(64 \times 64), 62/(32 \times 32), 10/(16 \times 16)\}$, respectively, if $v \in [1, 11]$, and also from (7) that $y = \{310/(64 \times 64), 62/(32 \times 32), 10/(16 \times 16)\}$, respectively, if $v \in [0, 10]$, and $y = 0$ if $v = 11$.

Since there exist up to $2^{k_b} - 1$ incorrect rows, the probability that exactly t undesired rows contain n entries is given by

$$P(n,t) = \binom{2^{k_b}-1}{t} [P'_s(n)]^t \left[\sum_{m=0}^{n-1} P'_s(m)\right]^{2^{k_b}-1-t}. \tag{8}$$

The probability of having an entry in the desired row is $1 - p_d$. Therefore, the probability that there exist n entries in the desired row is given by [1], [3]

$$P_c(n) = \binom{w}{n} (1-p_d)^n \, p_d^{w-n}. \tag{9}$$

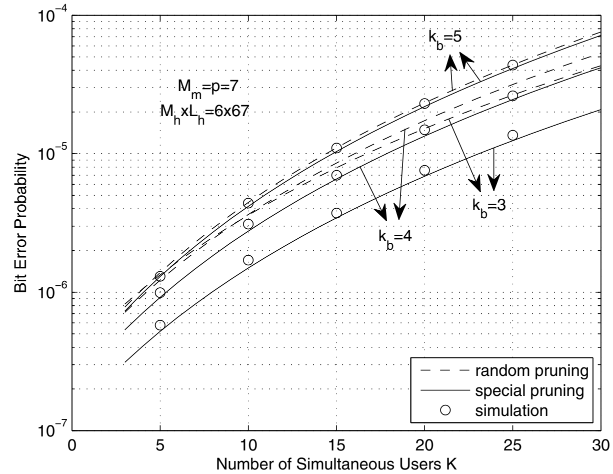

Fig. 2. Bit error probability of the RS/FH-CDMA scheme in a Rayleigh fading channel with the $\text{RS}(6, 2, 5)$ codes and $k_b = \{3, 4, 5\}$.

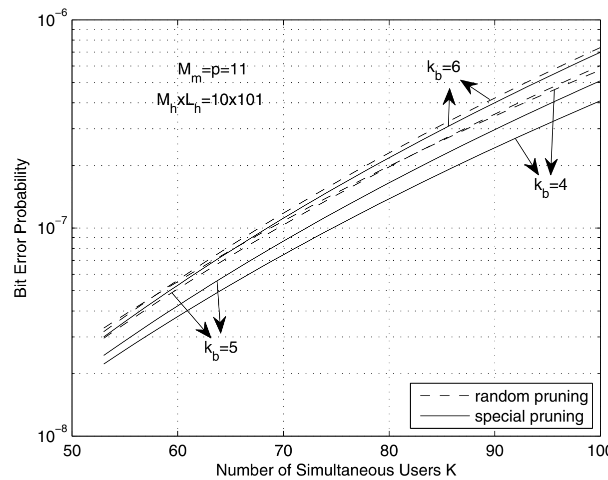

Fig. 3. Bit error probability of the RS/FH-CDMA scheme with the $\text{RS}(10, 2, 9)$ codes and $k_b = \{4, 5, 6\}$.

Finally, the bit error probability is given by

$$P_e = \frac{2^{k_b}}{2(2^{k_b}-1)} \left[1 - \sum_{n=1}^{w} P_c(n) \sum_{t=0}^{2^{k_b}-1} \frac{1}{t+1} P(n,t)\right]. \tag{10}$$

Fig. 2 plots the bit error probability, from (10), of the RS/FH-CDMA scheme with the $\text{RS}(6, 2, 5)$ codes of cardinality $p^2 = 49$ as the modulation codes in a Rayleigh fading channel with $\beta = 3$ and $\overline{E_b}/N_o = 25$ dB, for $k_b = \{3, 4, 5\}$. One-hit $M_h \times L_h = 6 \times 67$ FH patterns of weight $w_h = 6$ are used, corresponding to the use of $7 \times 6 = 42$ carrier frequencies. With the same FH patterns, Fig. 3 plots the bit error probability of the RS/FH-CDMA scheme with the $\text{RS}(10, 2, 9)$ codes of cardinality $p^2 = 121$ for $k_b = \{4, 5, 6\}$, corresponding to the use of $11 \times 10 = 110$ carrier frequencies. The curves labeled with "random pruning" are based on (5), in which the extra RS codes are removed randomly from the trellis. The curves labeled with "special pruning" are based on (6), in which the special pruning method in Section III is used for lowering demodulation errors. In general, the performance of the RS/FH-CDMA scheme with special pruning is better than that with random pruning. It is because the remaining RS codes are ensured to have lower hit probabilities. As more codes are removed from the trellis by special pruning, the performance gets better because of further reduction in the hit probabilities among the remaining codes. The accuracy of the theoretical error-probability calculations based on the special pruning method is validated by the computer-simulation results, which closely match with the theoretical ones from (6). In the computer simulation, each user maps every k_b random data bits to one of the 2^{k_b} RS$(6, 2, 5)$ codes, and the mapped modulation code is combined with the FH pattern of the user's destination to forming the final RS/FH-CDMA signal. The error probability seen at a receiver is calculated by correlating all arrival RS/FH-CDMA signals (for a given K) with its own FH pattern over a sufficient number of data bits. The total number of data bits involved in the simulation needs to be about 100 times of the reciprocal of the targeted error probability for providing sufficient simulation iterations.

V. Complexity Analysis

Trellis complexity is generally measured in terms of the node and branch complexities (i.e., the numbers of nodes and branches). These two complexities determine the storage and computation requirements of a trellis-based decoding algorithm. We here compute the complexity of using the Viterbi algorithm to decode the pruned trellis of the RS codes.

Let N be the node complexity and B be the branch complexity. There are $n + 1$ states of nodes and branches in the trellis of the RS$(n, 2, n - 1)$ codes, where $n = p - 1$ and $n \geq 2k$. The Viterbi algorithm [9] requires simple arithmetic operations: "addition" and "comparison" [10]. The total number of additions required by the algorithm is B and the total number of comparisons is $B - N + 1$, giving a total of $\Phi = 2B - N + 1$ arithmetic operations [10].

For example, the total complexity of traversing the (heavy-lined) branches of the pruned trellis of the RS$(4, 2, 3)$ codes in Fig. 1 with the Viterbi algorithm is given by $\Phi = 57$ (i.e., $B = 5+16+16+5 = 42$ and $N = 1+5+16+5+1 = 28$). Without the Viterbi algorithm, the decoded matrix is compared with the sixteen RS$(4, 2, 3)$ codes one by one, as done in [3], and the total demodulation complexities can be found as $\Phi = 79$ (i.e., $B = 16+16+16+16 = 64$ and $N = 1+16+16+16+1 = 50$.

Table II compares the complexities of demodulating the RS$(n, 2, n - 1)$ pruned trellis with the Viterbi algorithm for $n = p - 1 = \{4, 6, 10, 12\}$. [Note: "Viterbi complexity" denotes the total number of arithmetic operations ($\Phi = 2B - N + 1$) required by the Viterbi algorithm; "non-Viterbi complexity" denotes the total arithmetic operations required by the one-by-one comparison in [3]; and "complexity reduction" denotes the ratio of the "(non-Viterbi complexity − Viterbi complexity)" to the "non-Viterbi complexity."] When n or p increases, both complexities grow larger because there are more states to compute. However, the "Viterbi complexity" is alway better and the "complexity reduction" slows down as n or p increases.

TABLE II
Complexity comparison of demodulating the RS$(n, 2, n - 1)$ codes of $n = p - 1 = \{4, 6, 10, 12\}$.

	RS(4,2,3)	RS(6,2,5)	RS(10,2,9)	RS(12,2,11)
Viterbi complexity	57	173	597	1433
Non-Viterbi complexity	79	223	703	1663
Complexity reduction	0.28 fold	0.23 fold	0.16 fold	0.14 fold

VI. Conclusions

To represent k_b data bits per symbol, we formulated a systematical method to remove the RS codes in the trellis so that the remaining 2^{k_b} RS codes result in lower demodulation errors in our two-level RS/FH-CDMA scheme. The use of the pruned trellis and Viterbi algorithm was detailed and simplified the modulation and demodulation processes. Our analyses showed that the pruned trellis and Viterbi algorithm provided better performance and substantial savings in the computation complexity than the non-trellis method in [3].

Acknowledgment

This work was supported in part by the National Science Council of Republic of China under Grant NSC 98-2221-E-005-035-MY3 and Grant NSC 100-2221-E-239-020.

References

[1] G.-C. Yang and W.C. Kwong, "Frequency-hopping codes for multimedia services in mobile telecommunications," *IEEE Trans. Veh. Technol.*, vol. 48, no. 6, pp. 1906–1915, Nov. 1999.

[2] G.-C. Yang and W.C. Kwong, *Prime Codes with Applications to CDMA Optical and Wireless Networks*, Norwood, MA: Artech House, 2002.

[3] M.-F. Lin, G.-C. Yang, C.-Y. Chang, Y.-S. Lin, and W.C. Kwong, "Frequency-hopping CDMA with Reed-Solomon code sequences in wireless communications" *IEEE Trans. Commun.*, vol. 55, no. 11, pp. 2051–2055, Nov. 2007.

[4] S.-M. Wu, G.-C. Yang, C.-Y. Chang, and W.C. Kwong, "A two-level FH-CDMA scheme for wireless communication systems under fading channel," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 38–44, Jan. 2011.

[5] G.-C. Yang and J.-Y. Jaw, "Performance analysis and sequence designs of synchronous code-division multiple-access systems with multimedia services," *IEE Proc.–Commun.*, vol. 141, no. 12, pp. 371–378, Dec. 1994.

[6] L. Hanzo, L-L. Yang, E-L. Kuan and K. Yen, *Single- and Multi-Carrier DS-CDMA: Multi-User Detection, Space-Time Spreading, Synchronization and Standard*, West Sussex, England: John Wiley & Sons, 2003.

[7] S. Lin and D.J. Costello, *Error Control Coding: Fundamentals and Applications*, 2nd ed., Upper Saddle River, NJ: Pearson Education, 2004.

[8] A.J. Viterbi, "Error bounds for convolutional codes and asymptotically optimum decoding algorithm," *IEEE Trans. Inform. Theory*, vol. 13, no. 2, pp. 260–269, Apr. 1967.

[9] G.D. Forney, Jr., "The Viterbi algorithm," *Proc. IEEE*, vol. 61, no. 3, pp. 268–278, Mar. 1973.

[10] R.J. McEliece, "On the BCJR trellis for linear block codes," *IEEE Trans. Inform. Theory*, vol. 42, no. 4 , pp. 1072–1092, July 1996.