High Power Amplifier Linearization Using Zernike Polynomials in a LTE Transmission

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***Abstract*—Digital predistortion (DPD) is a highly cost-effective technique to linearize high power amplifiers (HPAs) in wireless applications. The polynomial-type behavioral models are widely used both for HPAs and predistorters, but they often suffer from the numerical instability when high order nonlinearities are included. In this paper, we propose a new polynomial predistorter model considering the use of a orthogonal polynomial model based on a Zernike polynomial basis which is very robust and stable when classical identification techniques are applied.**

Computer simulations in a LTE uplink transmission confirm the predistorter computed with the Zernike polynomial model provides improvements in adjacent channel leakage ratio (ACLR) with stability and moderate complexity in the DPD coefficient estimation.

***Index Terms*—Digital predistortion, Zernike polynomials, linearization, LTE, power amplifier.**

I. Introduction

The Long Term Evolution (LTE) standard for mobile communication systems and its upgraded version LTE-Advanced, developed by the 3rd Generation Partnership Project (3GPP) [1], are being widely accepted in communication business, since they increase the data rate and the speed of mobile communications. LTE provides adaptive modulation schemes (QPSK, 16-QAM or 64-QAM) in order to achieve the optimum rate for each user. The use of denser modulation schemes increases the number of effective bits per symbol and therefore the overall bit rate. However, since the constellation of these modulation schemes is more complex, they are more sensitive to interferences and require more restrictive power and linearity requirements. In a LTE uplink transmission, the radio-frequency (RF) power amplifiers (PA) are the most critical components for the battery lifetime of mobile handsets, which typically dominate the power consumption. Particularly, at LTE systems the efficiency of handset PA is a very important issue with high linearity.

In order to maintain linearity while improve power efficiency, several linearization techniques have been investigated in modern wireless communications. Among all PA linearization methods, digital predistortion (DPD) [2], [3] becomes one of the most cost-effective due to its high precision and relative simplicity, although it requires the use of signal processing techniques. A digital baseband predistorter creates a nonlinearity that is complementary to the nonlinear characteristic of the power amplifier. Ideally, the cascade of the predistorter and the amplifier becomes a linear system which amplifies the original input without generating new harmonics. In this situation, the PA can be used with the DPD up to its saturation point while maintaining good linearity and increasing the efficiency.

Models used for the DPD coefficients extraction are usually based on mathematical methods which arise in the description of nonlinear systems. Most of them were developed using the Volterra series approach [4], which provides a very powerful way to model PAs and describe both nonlinearities and memory effects. Unfortunately, the classical Volterra model is too complex and it is usually difficult to apply it in a real system. Therefore, several simplified models have been developed to characterize PAs with enough accuracy under certain conditions. Polynomial models provide a significant complexity reduction while keeping a reasonable accuracy [5]–[7]. One of the main drawbacks of these models is that they usually do not provide unconditioned stability. Thus, when the PA works in a more nonlinear state the predistorter operation is not guaranteed.

In this paper, we present a study of a new DPD model based on Zernike polynomials. These polynomials are orthogonal in the unit disk and thus, every complex input signal in that domain (like LTE waveforms) can be perfectly matched with them. In addition, their orthogonal features guarantees numerical stability.

The paper is outlined as follows. Section II presents the proposed digital predistortion model. In Section III we validate the proposed DPD with the aid of different computer simulations carried out with a high nonlinear amplifier. Finally, conclusions are summarized in Section IV.

II. DPD Models

A. Polynomial Model

Polynomial models are classical mathematical representations used in wideband applications for both power amplifier modeling and digital predistortion design [6]. We can formulate the polynomial model (PM) as

$$y(n) = \sum_{q=1}^{M} \sum_{p=1}^{N} a_{pq}\; x(n-d_q)\,|x(n-d_q)|^{p-1}\,, \qquad (1)$$

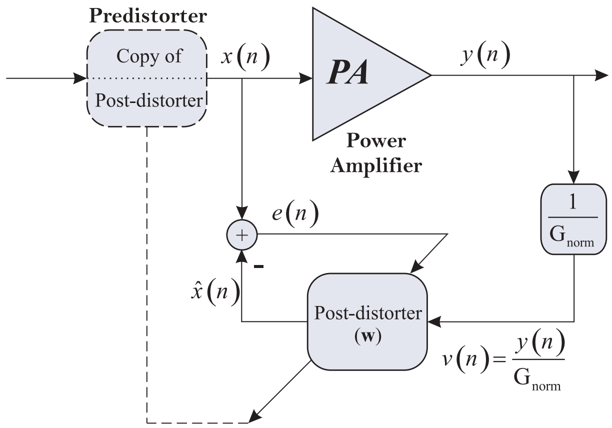

Fig. 1. DPD Schematic.

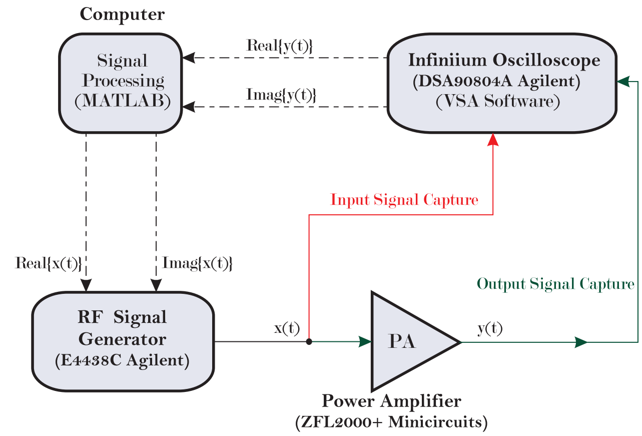

Fig. 2. Experimental Set Up.

where N is the nonlinear order, M is the total number of delays, d_q is the delay value for qth delay tap, a_{pq} are the model coefficients, $x(n)$ is the input signal, and $y(n)$ is the PA output signal. We can express (1) as

$$\mathbf{y} = \mathbf{X}\mathbf{a}, \tag{2}$$

being $\mathbf{a}$ the vector representation of the model coefficients, $\mathbf{y}$ the vector representation of the PA output $y(n)$, and $\mathbf{X}$ the matrix expression of the PA input signal,

$$\mathbf{X} = \begin{bmatrix} \mathbf{X_1} & \cdots & \mathbf{X_p} & \cdots & \mathbf{X_N} \end{bmatrix}, \tag{3}$$

where

$$\mathbf{X_p} = \begin{bmatrix} \mathbf{x}_{p,1}(n) & \cdots & \mathbf{x}_{p,M}(n) \\ \mathbf{x}_{p,1}(n) & \cdots & \mathbf{x}_{p,M}(n+1) \\ \vdots & \ddots & \vdots \\ \mathbf{x}_{p,1}(n) & \cdots & \mathbf{x}_{p,M}(n+L-1) \end{bmatrix} \tag{4}$$

and

$$\mathbf{x}_{p,q}(n) = x(n-d_q)\,|x(n-d_q)|^{p-1}, \tag{5}$$

being L the input signal length.

Fig. 1 shows the indirect learning structure used for predistorter identification. Using this scheme DPD coefficients are calculated in a first training stage in the feedback path whose input is the normalized PA output sequence $v(n)$, calculated as

$$v(n) = y(n)/\mathsf{G}_{\mathsf{norm}}, \tag{6}$$

where $\mathsf{G}_{\mathsf{norm}}$ is the complex gain of the PA. The Polynomial Predistorter (P-DPD) solution that estimates the PA inverse characteristic is obtained by means of the least squares minimization, whose solution is the pseudo-inverse of the normalized PA output $v(n)$

$$\mathbf{w}^{\mathrm{PM}} = (\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{x}. \tag{7}$$

If matrix $\mathbf{V}$ is poorly conditioned, the inversion of the matrix $\mathbf{V}^H\mathbf{V}$ in (7) can experience a numerical instability problem and the solution may be unsatisfactory. This is quite usual when considering high nonlinearity order in polynomial models and it is prone to instability and divergence. In our context, large PA output power values will favor the bad conditioning of the matrix. Under this circumstances, the predistorter operation is not guaranteed and it is very usual the output power increases dramatically.

B. Zernike Polynomial Model

To alleviate the numerical instability problem associate to the polynomials models for the DPD estimation, we propose the use of orthogonal polynomials [8]. This is a change of basis from the algebra point of view, which can be expressed as

$$\psi_{qp}(x(n)) = \sum_{l=1}^{p} b_{lp}\; x(n-d_q)\,|x(n-d_q)|^{l-1}, \tag{8}$$

being b_{lp} the linear coefficients of these polynomial functions. Thus, eq. (1) can be expressed as

$$y(n) = \sum_{q=1}^{M}\sum_{p=1}^{N} a_{pq}\; \psi_{qp}(x(n)). \tag{9}$$

Substituting (8) in (9) we obtain the PM in a general orthogonal basis representation

$$y(n) = \sum_{q=1}^{M}\sum_{p=1}^{N} a_{pq}\; \sum_{l=1}^{p} b_{lp}\; x(n-d_q)\,|x(n-d_q)|^{l-1}. \tag{10}$$

In [9], [10] authors have developed basis with classical orthogonal polynomials which focus on real plane as Legendre or Hermite. Thus, they are sensible only to the magnitude of the sequence. However, for a suitable model performance, phase information should be included. Under this approach, we propose the use of a set of orthogonal functions called Zernike Polynomials. This basis is commonly used in image processing and it considers both magnitude and phase signal information. According to [11], [12] for an input signal $x(n)$, Zernike Polynomials have the form

$$Z_{pq}(|x(n)|, \theta[x(n)]) = R_{pq}(|\overline{x}(n)|)\; e^{jq\theta[x(n)]}, \tag{11}$$

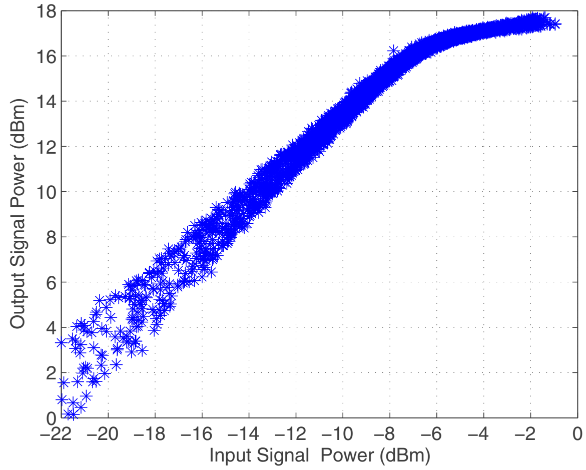

Fig. 3. AMAM conversion of the tested power amplifier.

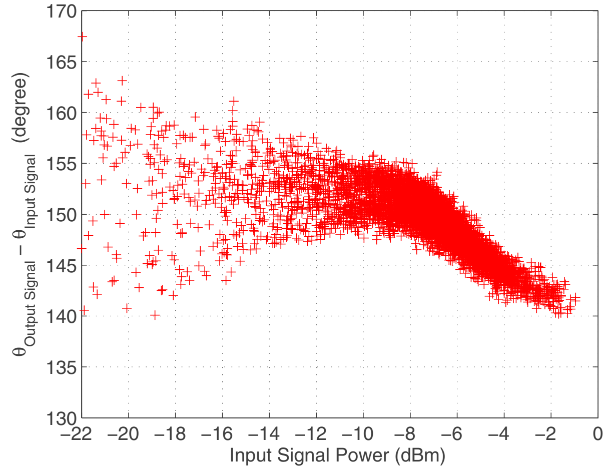

Fig. 4. AMPM conversion of the tested power amplifier.

where $\overline{x}$ denotes the normalized expression of signal x, p is a non-negative integer, q denotes the positive and negative integers subject to constraints that $p - |q|$ is even, and $|q| \leq p$ and $R_{pq}(|\overline{x}(n)|)$ are the Zernike radial polynomials defined in [11] and [12].

With equations (10) and (11) we obtain the mathematical expression of the Zernike Polynomial Model (ZPM)

$$y(n) = \sum_{q=1}^{M} \sum_{p=1}^{N} \sum_{l=1}^{p} \beta_{qpl} \; Z_{pl}(|\overline{x}(n-d_q)|, \theta[x(n-d_q)]), \quad (12)$$

where β_{qpl} are the model coefficients, called Zernike Moments [13], [15]. In the same way than in (2), if we denote $\mathbf{\Theta}$ as the vector expression of the Zernike Moments, $\mathbf{Z}_{[\mathbf{X}]}$ the matrix representation of the PA input signal by means of the Zernike Polynomials $Z_{pq}(|\overline{x}(n)|, \theta[x(n)])$, and $\mathbf{y}$ the vector representation of the PA output $y(n)$, we can express (12) as

$$\mathbf{y} = \mathbf{Z}_{[\mathbf{X}]}\mathbf{\Theta}. \quad (13)$$

The Zernike Predistorter (Z-DPD) is computed by the least squares minimization, as the Polinomial Predistorter. Following the same procedure than in (7) (Fig. 1), the Z-DPD coefficients will be obtained as

$$\mathbf{w}^{\mathbf{ZPM}} = (\mathbf{Z}_{[\mathbf{V}]}^{H}\mathbf{Z}_{[\mathbf{V}]})^{-1}\mathbf{Z}_{[\mathbf{V}]}^{H}\mathbf{x}, \quad (14)$$

where $\mathbf{Z}_{[\mathbf{V}]}$ is the Zernike Polynomial matrix representation of the normalized PA output signal $v(n)$.

III. Simulations and Results

A. Experimental Set-Up

The experimental set-up used in this work (Fig. 2) consists of a PC (computer running MATLAB) used in the signal processing, a Signal Generator (Agilent E4438C), and an Oscilloscope (Agilent Infiniium DSA90804A) with the Vector Signal Analyzer application (VSA). The Device Under Test (DUT) used in this research is the Minicircuits Model ZFL-2000+ amplifier [16] (Figs. 3 and 4), with a P_{1dB} = 16 dBm, and a typical gain of 22 dB at a test frequency of 1.8 GHz. The PA is operating with 3 dB of output back-off (OBO).

The linearization strategy is based on capturing the calibration sequences at the PA input and output in order to extract the predistortion parameters for both P-DPD and Z-DPD. Then, we estimate the predistorters coefficients with these PA input and output signals. Once the predistortion functions are computed we test the whole system, that is, the DPD block with the power amplifier. We apply the different DPD on a LTE transmission and perform linearity measurements in terms of DPD convergence and adjacent channel leakage ratio (ACLR) enhancements. In this research, measurements were performed transmitting a 64-QAM LTE uplink signal with a 5-MHz bandwidth, and a PAPR of 9.345 dB.

B. Simulation Process

The numerical problems associated with estimating the predistorter coefficients will affect the performance of the predistorter. We will examine the problem from the view point of the spectral regrowth suppression and the normalized mean squared error obtained in the DPD estimation process.

In this work, memory effects are neglected by setting $M = 0$ in (1) and (12) in order to simplify the PA characterization and the DPD design processes. However, to obtain the full predistorter performances and improve their accuracy it would be necessary to consider memory effects. The parameter $\mathsf{G}_{\mathsf{norm}}$ of (6) is chosen as the quotient between the maximum envelope values of the PA input and output values. This solution notably reduces the PA gain regarding the nonlinearized PA, but helps the predistortion algorithm to converge and simplifies the implementation because the original input and the predistorted signal can be normalized by the same scaling factor, which facilitates power control. This parameter is not so critical in the Zernike model, due to the signal normalization [12]. In this work, simulation parameters are carefully chosen in order to compare both DPDs in the same operating conditions.

TABLE I
COMPUTER SIMULATION RESULTS OBTAINED WITH ZERNIKE AND POLYNOMIAL PREDISTORTERS.

DPD coefficients (N)	**DPD model**	**NMSE** (dB)	**Power Channel** (dBm)	**ACLR** (dBc)
0	no-DPD	–	13.50	27.51
9	Z–DPD	−20.51	13.30	27.54
11	Z–DPD	−25.02	13.24	27.55
13	Z–DPD	−25.03	13.23	32.12
15	Z–DPD	−25.04	13.23	32.19
17	Z–DPD	−25.04	13.20	34.71
19	Z–DPD	−28.75	12.27	34.73
21	Z–DPD	−28.77	12.26	34.80
23	Z–DPD	−28.80	12.23	34.85
25	Z–DPD	−28.82	12.22	34.88
27	Z–DPD	−28.91	12.18	37.82
29	Z–DPD	−29.70	12.19	37.92
31	Z–DPD	−29.72	12.19	37.93
33	Z–DPD	−29.74	12.18	38.02
9	P–DPD	−29.45	11.70	30.16
11	P–DPD	−29.62	11.66	35.95
13	P–DPD	−29.70	11.65	37.23
15	P–DPD	−29.70	11.65	37.44
17	P–DPD	−29.71	11.65	37.38
19	P–DPD	−29.71	11.65	37.39
21	P–DPD	−29.71	11.65	37.39
23	P–DPD	−29.71	11.65	37.39
25	P–DPD	−29.71	11.65	37.39
27	P–DPD	−29.71	11.65	37.39
29	P–DPD	−29.71	11.65	37.39
31	P–DPD	−29.71	11.65	37.39
33	P–DPD	−29.71	11.65	37.39

C. Obtained Results

In Table I both Polynomial (P-DPD) and Zernike (Z-DPD) predistorters are compared using different nonlinear-order values (N). The Normalized Power Spectral Densities (NPSD) of some of the PA output LTE signal described in Table I are depicted as examples in Fig. 5 (with Z-DPD) and Fig. 6 (with P-DPD).

We can observe that the P-DPDs outcome Zernike predistorters in terms of normalized mean square error (NMSE), specially for low nonlinearity orders. The P-DPD enhances the NMSE obtained with a Z-DPD until N = 17, not obtaining

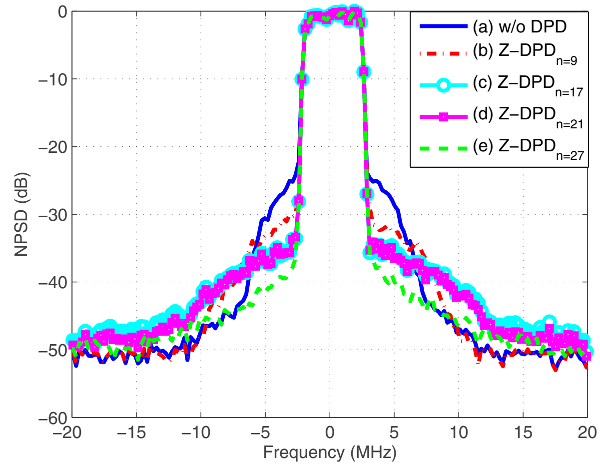

Fig. 5. Normalized power spectral density of the 5-MHz LTE signal at the PA output using a Zernike Predistorter (Z-DPD): (a) without DPD. (b) Z-DPD, N = 9. (c) Z-DPD, N = 17. (d) Z-DPD, N = 21. (e) Z-DPD, N = 27.

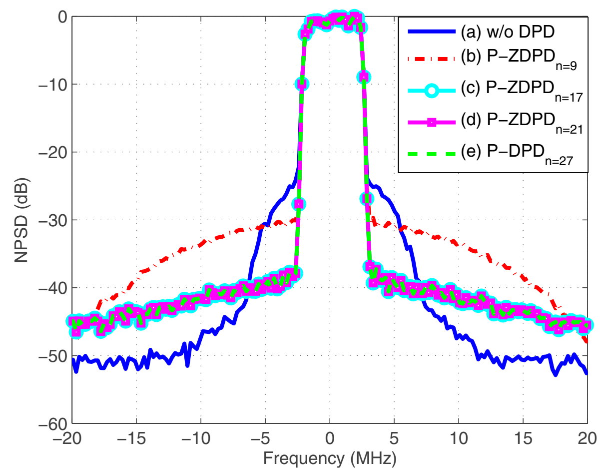

Fig. 6. Normalized power spectral density of the 5-MHz LTE signal at the PA output using a Polynomial Predistorter (P-DPD): (a) without DPD. (b) P-DPD, N = 9. (c) P-DPD, N = 17. (d) P-DPD, N = 21. (e) P-DPD, N = 27.

better results when increasing N over this value.

With regard to the adjacent channel leakage ratio (ACLR), Z-DPD achieves better results than the P-DPD when the parameter N becomes high ($N \geq 27$), reaching an ACLR up to 38 dBc. That means an improvement in the out-of-band nonlinear distortion of around 10.5 dB with respect to the non-DPD situation. P-DPD accomplishes a slightly minor improvement in ACLR (9.9 dB) with 1.85 dB of in-band power losses, while the Z-DPD power losses are lower than 1.4 dB.

P-DPD reaches better nonlinear distortion reduction than Z-DPD for low N values , but with a minor power level in the interest bandwidth (the PA output power decreases 1.5 dB compared with Z-DPD when N = 9), which reduces the P-DPD efficiency. When increasing the nonlinear order (N) over

17 the enhancements accomplished by P-DPD in ACLR keep constant (around 37.39 dBc), as it occurs with the NMSE. This is due to the polynomial model can not exactly match the power amplifier behavior and thus, the estimated predistorter is not able to compensate additional nonlinear distortion.

As we have commented, the DPD includes a power loss level that is inherent to the linearization process. This power loss is related to the AMAM conversion of the PA, the $\mathsf{G}_{\mathsf{norm}}$ value in the case of a P-DPD, and the input signal normalization in the Z-DPD approach. The PA output power using a P-DPD is around 13.24 dBm. If we increase $\mathsf{G}_{\mathsf{norm}}$ in order to maximize the PA output power, the algorithm does not converge because of the numerical instability. This fact does not occur with the Z-DPD, which allows obtaining a major PA output signal level with the same nonlinearity order (N) than P-DPD but without compromising stability. The Zernike DPD achieves a PA output power 0.3 dB lower than without DPD for low N values ($N = 9 - 15$). As this parameter N increases, the PA output signal power decreases up to 1.4 dB.

In Figs. 5 and 6 it can be observed that Z-DPD outperforms P-DPD in the second adjacent channel non-linear distortion reduction, specially for high N values, while the Polynomial DPD generates new nonlinear distortion in this band.

IV. Conclusion

In this paper, we have proposed the use of a mathematical model based on orthogonal polynomials for a digital predistortion design. The orthogonal basis is built using the Zernike polynomials. These polynomials focus on the complex plane, considering information from both magnitude and phase. We have analyzed the performance of the proposed method compared with the classical polynomial model. Several predistorters have been generated using both DPD algorithms by modifying their nonlinear order N. Several comparisons in terms of the ACLR values, the PA output power levels and the normalized mean square errors have been carried out.

The simulation results show that the proposed predistorter (Z-DPD) outperforms the spectral regrowth achieved by P-DPD when the nonlinear order (N) is high. Zernike predistorter also allows numerical stability. The in-band power losses related to Zernike predistorters are lower than 1.5 dB for all tested cases. The Z-DPD with $N = 33$ enhances the ACLR value in about 10.5 dB with respect to the non-DPD situation.

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