Hadamard Transform Based Codebook Design for Uniform Circular Arrays in Mobile Radio Communications

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Abstract—Multiple-input multiple-output (MIMO) has been adopted by long term evolution (LTE) and its updated version LTE-Advanced (LTE-A) to increase spectrum efficiency. However, the benefits of multi-user (MU) MIMO highly rely on the accurate channel knowledge at transmitter. So proper codebook design is a key problem for frequency division multiplexing (FDD) downlink systems. In this paper, an efficient codebook design based on Hadamard transform is proposed for uniform circular arrays. Then a two-stage feedback scheme is further proposed to reduce the feedback overhead. The average cell spectrum efficiency and cell-edge user spectrum efficiency of the proposed scheme are evaluated through system level simulation and compared with LTE Rel-10 feedback. The results show that the proposed codebook significantly improves feedback efficiency and system performance.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) spatial multiplexing is recognized to be capable of dramatically increasing spectrum efficiency of wireless communication [1] and has been adopted by the 3rd Generation Partnership Project (3GPP) long term evolution (LTE) and its updated version LTE-Advanced (LTE-A) as one of the key techniques.

It is well-known that multi-user (MU) MIMO performance is very sensitive to the channel estimation accuracy at transmitter. In frequency division multiplexing (FDD) systems, receiver generally sends several bits to indicate channel state information including channel direction information (CDI) and channel quality, respectively denoted as precoding matrix indicator (PMI) and channel quality indicator (CQI) in LTE and LTE-A specifications. When the probability density function of channel is known a priori, an optimal codebook can be obtained by the Lloyd algorithm [2] to quantize the possible channel directions. However, when channel statistics vary, it is impractical to use Lloyd algorithm due to latency and complexity. Among various antenna array configurations, linear antenna array is most commonly employed in current cellular and personal communication systems. However, such type of antenna array has a limited angular view. In order to obtain operation for larger angular views, uniform circular arrays (UCA) can be a good choice [3]. In literatures, many practical codebook schemes [4][5][6] have been proposed mainly for linear antenna arrays, but the practical feedback scheme for UCA especially on multiple streams has not been

extensively studied yet.

In this paper, a codebook based on Hadamard transform is proposed to effectively match the channel characteristics of UCA with omni antennas. The proposed UCA codebook has constant modulus, unitary and nested properties, which are preferred for practical usage due to simplicity. In order to reduce the total feedback overhead, a two-stage feedback for the proposed codebook is further designed, which consists of two separate matrices W1 and W2 as in LTE Rel-10 feedback[7]. In the proposed two-stage feedback framework, the matrix W1 is a cluster of beams targeting wideband channel property, and the other matrix W2 performs beam selection for subband channel property. Based on W1 and W2, Hadamard transform is performed to construct the corresponding CDI estimation. System-level simulation results demonstrate that implementing the proposed practical feedback scheme achieves much better system performance than the existing feedback solution in LTE Rel-10 specification.

The remainder of this paper is organized as follows. Section III introduces downlink system model. Section III proposes an UCA codebook first, then derives a two-stage feedback framework for the proposed codebook. System level simulation results are presented in Section IV. Finally, Section V provides some concluding remarks.

Notation: $(\cdot)^T$ is transpose, and $n \mod N$ denotes the modulus function. $\operatorname{diag}(a_1, a_2, \cdots, a_N)$ represents a diagonal matrix with a_1, a_2, \cdots, a_N being the main diagonal elements. $\mathbf{A}(:,k)$ and $\mathbf{A}(:,1:k)$ respectively denote the k-th column of matrix \mathbf{A} and the matrix constructed by the first k columns of matrix \mathbf{A} .

II. DOWNLINK SYSTEM MODEL

Consider a downlink system employing MIMO and orthogonal frequency division multiplexing (OFDM) technologies. Each base station (BS) is equipped with a uniform circular array with N_t omni antennas. For example, an 8-element uniform circular array is illustrated in Figure 1. Let (α, β) denotes the angle of departure (AOD), where α and β respectively represent the azimuth angle and elevation angle. The azimuth angle of the n-th antenna is denoted as θ_n . Since antennas are

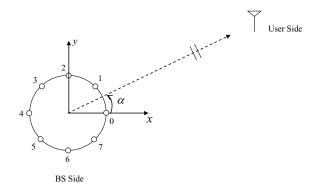


Fig. 1. An 8-element Uniform Circular Array

allocated uniformly on a circular, θ_n can be written as

$$\theta_n = \frac{2\pi n}{N_t}, \quad n = 0, 1, \dots N_t - 1$$
 (1)

The array manifold vector **v** corresponding to each incoming plane wave for a uniform circular array can be written as [8]

$$\mathbf{v} = \left[e^{-j\frac{2\pi R}{\lambda}} \sin\beta \cos(\alpha - \theta_0), e^{-j\frac{2\pi R}{\lambda}} \sin\beta \cos(\alpha - \theta_1), \cdots, e^{-j\frac{2\pi R}{\lambda}} \sin\beta \cos(\alpha - \theta_{N_{t-1}}) \right]^T$$
(2)

where $\alpha \in [0, 2\pi)$, $\beta \in [0, \pi)$, R denotes the radius of UCA and λ is the wavelength of the center frequency of interest. For simplicity, only azimuth angles are considered in the propagation geometry, i.e. $\beta = \pi/2$.

At BS, the transmitted signal for subcarrier k can be expressed by

$$\mathbf{x}(k) = \mathbf{F}(k)\mathbf{s}(k) \tag{3}$$

where $\mathbf{s}(k) = [s_1(k), s_2(k), \cdots, s_r(k)]^T$ is a $r \times 1$ vector containing the encoded MIMO complex data symbols at subcarrier k, r is the rank of transmission, and $\mathbf{F}(k)$ is a $N_t \times r$ complex precoding matrix.

Codebook based precoding is employed for LTE-A FDD system. Each user selects the preferred codeword in the predefined codebook to represent the CDI from the served BS, and sends the codeword index to the BS. There are some criterias to select codeword, such as maximum capacity and chordal distance. Then BS retrieves the downlink CDI according to the codeword index fed back from user, and performs proportional fairness scheduling by greedy search based on maximum weighted sum capacity. If single user (SU) mode is scheduled, the precoding matrix $\mathbf{F}(k)$ is set as the codeword fed back by the corresponding user. Otherwise, if MU mode is scheduled, $\mathbf{F}(k)$ is usually calculated by zero-forcing (ZF) method based on the CDI estimation from each scheduled user.

III. CODEBOOK DESIGN

How to design the codebook effectively quantizing channel direction is a very important question for FDD downlink

systems. In LTE Rel-10 specification, the 8Tx PMI feedback is composed of two matrices selected from two separate codebooks. One matrix targets wideband channel property, and the other matrix targets subband channel property. Actually, the LTE Rel-10 codebooks are mainly designed for linear antenna arrays. Therefore, in the cases of UCA, the system performance cannot be guaranteed due to the inaccuracy of the LTE Rel-10 codebooks.

In this section, through analysis, the codebook suitable for UCA is proposed first. Then a two-stage PMI feedback is further designed to reduce the total feedback overhead.

A. Proposed UCA Codebook

Unlike the form of discrete Fourier transform exhibited in the LTE Rel-10 codebook for linear antenna arrays, UCA has a quite different array manifold vector shown in equation (2). Hence, to well match the channel statistics of UCA in high spatial correlation, the rank-1 UCA codebook should be designed in the form of equation (2). Firstly, use several bits with number $\log_2(N)$ to uniformly quantize the azimuth angle α as

$$\alpha = 2\pi n/N, \quad n = 0, 1, \dots, N - 1$$
 (4)

Then by substituting equation (4) into the expression of the UCA array manifold vector in equation (2), the rank-1 codebook $C^{(1)}$ with size N can be easily derived as

$$C^{(1)} = \{\mathbf{b}_0, \mathbf{b}_1, \cdots, \mathbf{b}_{N-1}\}$$
 (5)

where the n-th codeword is exactly the n-th beam written as

$$\mathbf{b}_{n} = \left[e^{-j\frac{2\pi R}{\lambda}} \cos\left(\frac{2\pi n}{N} - \theta_{0}\right), e^{-j\frac{2\pi R}{\lambda}} \cos\left(\frac{2\pi n}{N} - \theta_{1}\right), \cdots, e^{-j\frac{2\pi R}{\lambda}} \cos\left(\frac{2\pi n}{N} - \theta_{N_{t}-1}\right) \right]^{T}, \quad n = 0, 1, \cdots, N - 1$$
(6)

In industry, a codebook structure with constant modulus and unitary properties is usually preferred. Since Hadamard matrix is composed of several orthogonal columns with element being 1 or -1, it is used herein to derive the higher rank codeword based on the rank-1 codeword in equation (6). Firstly, define \mathbf{U}_n as a $N_t \times N_t$ matrix of which the k-th column is constructed by

$$\mathbf{U}_{n}(:,k) = \operatorname{diag}(\mathbf{D}(:,k)) * \mathbf{b}_{n}, \quad k = 1, 2, \cdots, N_{t}$$
 (7)

where \mathbf{D} is the $N_t \times N_t$ Hadamard matrix with all elements in the first column being 1. The k-th column of \mathbf{D} is used to construct the k-th column of \mathbf{U}_n . Taking 8Tx for example, i.e. $N_t = 8$, the Hadamard matrix \mathbf{D} is

By use of the Hadamard transform on the rank-1 codeword as in equation (7), the rank-r $(r=1,2,\cdots,N_t)$ codebook $C^{(r)}$ is proposed as follows

$$C^{(r)} = \left\{ \frac{1}{\sqrt{r}} \mathbf{U}_n (:, 1:r), \quad n = 0, 1, \dots, N-1 \right\}$$
 (9)

Since the elements in the first column of the Hadamard matrix ${\bf D}$ are set to be 1, the above codebook expression with r=1 is equivalent to the rank-1 codebook in equation (5). Obviously, the proposed codebook has constant modulus, unitary and nested properties.

B. Two-stage Feedback for the Proposed UCA Codebook

To reduce the feedback overhead of the UCA codebook proposed above, a two-stage feedback structure is further considered in the following. As in LTE Rel-10, the two-stage codebook consists of two separate matrices W1 and W2. Matrix W1 targets wideband channel property, and the other matrix W2 is designed for subband channel property.

W1 codeword is proposed to be constructed by M adjacent beams. Given the predefined beams according to equation (6), the k-th W1 codeword $\mathbf{W}_{1,k}$ is proposed as follows.

$$\mathbf{W}_{1,k} = \left[\mathbf{b}_{\frac{Mk}{2} \mod N}, \mathbf{b}_{\left(\frac{Mk}{2} + 1\right) \mod N}, \cdots, \mathbf{b}_{\left(\frac{Mk}{2} + M - 1\right) \mod N} \right]^{T},$$

$$k = 0, 1, \cdots, \frac{2(N - M)}{M} + 1 \quad (10)$$

Then W1 codebook is

$$C_1^{(1)} = C_1^{(2)} = \dots = C_1^{(N_t)}$$

$$= \left\{ \mathbf{W}_{1,0}, \mathbf{W}_{1,1}, \dots, \mathbf{W}_{1,\frac{2(N-M)}{M}+1} \right\}$$
(11)

where $C_1^{(r)}$ denotes the rank-r W1 codebook. Codebooks $C_1^{(1)}, C_1^{(2)}, \cdots, C_1^{(N_t)}$ are set as the same with the size of $\log_2\left(2(N-M)/M+2\right)$ bits.

1) Rank-1 W2 codebook: For rank-1 W2 codebook construction, W2 selects one beam in the W1 codeword shown in equation (10) to represent the subband channel direction. So the l-th rank-1 W2 codeword $\mathbf{W}_{2,l}^{(1)}$ is written as

$$\mathbf{W}_{2,l}^{(1)} = \frac{1}{\sqrt{N_t}} \left[\tilde{\mathbf{e}}_l \right], \quad l = 0, 1, \cdots, M - 1$$
 (12)

where $\tilde{\mathbf{e}}_l$ is an $M \times 1$ elementary vector with all zeros except for the (l+1)-th element being 1. The codewords above compose the rank-1 W2 codebook $C_2^{(1)}$ as

$$C_2^{(1)} = \left\{ \mathbf{W}_{2,0}^{(1)}, \mathbf{W}_{2,1}^{(1)}, \cdots, \mathbf{W}_{2,M-1}^{(1)} \right\}$$
 (13)

So the codebook size of $C_2^{(1)}$ is $\log_2 M$ bits.

Based on the W1 codeword $\mathbf{W}_{1,k}$ and the W2 codeword $\mathbf{W}_{2,l}^{(1)}$, the final rank-1 codeword $\mathbf{W}_{k,l}^{(1)}$ is the multiplication of them, i.e.

$$\mathbf{W}_{k,l}^{(1)} = \mathbf{W}_{1,k} \mathbf{W}_{2,l}^{(1)}, \quad k = 0, 1, \cdots, \frac{2(N-M)}{M} + 1;$$

$$l = 0, 1, \cdots, M-1$$
(14)

2) Higher rank W2 codebook: For rank r, W2 codeword $\mathbf{W}_{2,l}^{(r)}$ is constructed by r identical elementary vectors as

$$\mathbf{W}_{2,l}^{(r)} = \frac{1}{\sqrt{rN_t}} \left[\underbrace{\tilde{\mathbf{e}}_l \quad \tilde{\mathbf{e}}_l, \quad \dots, \quad \tilde{\mathbf{e}}_l}_{r} \right]$$
(15)

where $l=0,1,\cdots,M-1$. Then the rank-r $(r=1,2,\cdots,N_t)$ W2 codebook $C_2^{(r)}$ is

$$C_2^{(r)} = \left\{ \mathbf{W}_{2,0}^{(r)}, \mathbf{W}_{2,1}^{(r)}, \cdots, \mathbf{W}_{2,M-1}^{(r)} \right\}$$
 (16)

Based on the W1 codeword $\mathbf{W}_{1,k}$ and the W2 codeword $\mathbf{W}_{2,l}^{(r)}$, Hadamard matrix is also employed for higher rank as proposed in Section III-A to derive the final codeword $\mathbf{W}_{k,l}^{(r)}$. Define

$$\mathbf{T}_{k\,l}^{(r)} = \mathbf{W}_{1,k} \mathbf{W}_{2\,l}^{(r)} \tag{17}$$

The *i*-th column of the final codeword $\mathbf{W}_{k,l}^{(r)}$ is derived by

$$\mathbf{W}_{k,l}^{(r)}(:,i) = \operatorname{diag}(\mathbf{D}(:,i)) * \mathbf{T}_{k,l}^{(r)}(:,i),$$

$$i = 1, 2, \dots, r; \quad k = 0, 1, \dots, \frac{2(N-M)}{M} + 1;$$

$$l = 0, 1, \dots, M-1; \quad r = 1, 2, \dots, N_t$$
(18)

Since the elements in the first column of the Hadamard matrix \mathbf{D} are set to be 1, the above final codeword expression for r=1 is equivalent to the final rank-1 codeword in equation (14). Obviously, $\mathbf{W}_{k,l}^{(r)}$ shows constant modulus, unitary and nested properties.

Assume that the whole band is divided into B subbands. Based on the N beams in equation (6), the proposed UCA codebook $\mathcal{C}^{(r)}$ in equation (9) totally needs $B\log_2 N$ bits for channel direction (i.e. PMI) feedback on all subbands. However, in the derived two-stage PMI feedback, at first user selects one codeword in codebook $\mathcal{C}_1^{(r)}$ for the whole band CDI. Then on each subband, user selects one codeword in codebook $\mathcal{C}_2^{(r)}$ to match the subband CDI. Hence, the total overhead of the two-stage feedback is $(\log_2{(2(N-M)/M+2)}+B\log_2{M})$ bits.

IV. SIMULATIONS

This section evaluates the system level performance respectively using the proposed UCA codebook and the two-stage feedback in a downlink FDD system over 19 hexagon-shaped cells. Each BS is equipped with a UCA composed of eight omni antennas, i.e. $N_t=8$. Setting N=64 and M=8, the 6-bit UCA codebook in equation (9) is considered in the simulations, and the two-stage UCA codebook in equations (11) and (16) includes a 4-bit W1 codebook and a 3-bit W2 codebook. The main simulation parameters are summarized in Table I. The overall downlink bandwidth of 10 MHz is divided into 10 subbands, i.e. B=10. Both single user and multi-user scheduling schemes are evaluated here. For SU mode, rank adaption is supported with up to two streams per user. The post-detection signal to interference plus noise ratio (SINR) of each user is calculated and quantized as the SU

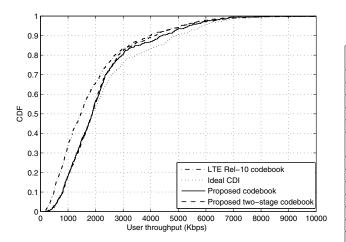


Fig. 2. CDF of User Throughput in SU-MIMO for 8Tx UCA.

CQI. User reports the rank index (RI), the SU PMI and CQI for SU scheduling. In MU mode, one stream per user is a common understanding in LTE Rel-10 due to the sufficient users. So rank-1 PMI feedback is considered here for MU scheduling. As to CQI, since each user has no knowledge of the interference from the other users to be co-scheduled by BS, a lower-bound MU CQI [9] is used here to support MU scheduling. At BS side, the reported CQI is further adjusted according to the number of streams per user in SU mode or the co-scheduled user number in MU mode by multiplying a scaling factor, and equal transmit power is allocated to each data stream. In the following, SU-MIMO performance is evaluated first. Then dynamic SU mode and MU mode switching is further considered, denoted as SU-/MU-MIMO.

Table II lists the PMI feedback overhead of the proposed UCA codebook, the proposed two-stage UCA codebook and the LTE Rel-10 codebook. For subband PMI feedback with total 10 subbands, compared with the LTE Rel-10 codebook, the proposed UCA codebook needs additional 16 bits, but by further employing the proposed two-stage feedback scheme, the feedback overhead can be reduced by 10 bits.

Figure 2 shows the cumulative distribution function (CDF) of user throughput for SU-MIMO, which illustrates that the proposed UCA codebook obviously improves the system performance compared with the LTE Rel-10 codebook. Furthermore, Table III gives the performance comparison in the form of spectrum efficiency (SE) among the proposed UCA codebook, the proposed two-stage UCA codebook, the LTE Rel-10 codebook and the ideal CDI. It can be seen that the proposed UCA codebook outperforms the LTE Rel-10 codebook with 17% and 71% gains on average cell SE and cell-edge user SE respectively, and only has 7% and 2% loss on average cell SE and cell-edge user SE respectively compared with ideal CDI feedback. And the proposed two-stage PMI feedback achieves similar performance as the proposed UCA codebook with only 2% loss on average cell SE.

Furthermore, the performance improvement on MU-MIMO

TABLE I SIMULATION PARAMETERS FOR SYSTEM LEVEL SIMULATION

Deployment scenario	3GPP urban macro,		
	fifteen-degree angle spread		
Cell number	19 cells		
Wrap-around model	Yes		
Site-to-site distance	150 m		
Carrier frequency	2.5 GHz		
Bandwidth	10 MHz		
Pathloss model	$36.7\log_{10}(d) + 22.7 + 26\log_{10}(f)$		
Tumoss moder	(d in m, f in GHz)		
BS antenna gain	5 dBi		
Transmit power	24 dBm		
Shadowing standard deviation	10 dB		
Shadow correlation	0.5 between cells		
Penetration loss	20 dB		
Noise figure at receiver	7 dB		
User dropping	Uniformly dropped, ten users per cell,		
	associated to maximum SINR cell		
Antenna configuration	- BS: 8Tx UCA (omni antennas)		
	- User: 2Tx uniform linear array		
	- Antenna spacing: 0.5λ		
Duplex method	FDD		
Network synchronization	Synchronized		
Traffic model	Full buffer		
Maximum number of			
co-scheduled users	Four		
Scheduler	Proportional fair and frequency		
Scheduler	selective scheduling, scheduling		
	granularity of one subframe (1 ms)		
Link adaptation	Non-ideal CQI quantized according		
Link adaptation	to LTE Rel-10		
Channel estimation	Non-ideal channel estimation		
Feedback assumptions	- SU mode: SU PMI, SU CQI		
	- MU mode:		
	rank-1 PMI, lower bound MU-CQI		
	- Feedback period:		
	5ms, per subband, 6ms delay		
	- Feedback error: bit error rate of 10^{-3}		
Downlink precoding	- SU mode: PMI based precoding		
	- MU mode: ZF scheme		
Codebook	- Ideal CDI: non-quantized CDI		
	- LTE Rel-10 codebook for 8Tx		
	- Proposed codebook: 6 bits		
	- Proposed two-stage codebook:		
	4-bit W1, 3-bit W2		
MIMO schemes	- SU-MIMO:		
	Rank adaptation, up to rank 2		
	- SU-/MU-MIMO:		
	Up to four users with 1 stream per user		
	for MU mode, up to two streams		
	per user for SU mode, dynamic		
Hybrid Automotic	switching between SU and MU		
Hybrid Automatic	C		
Repeat Request (HARQ)	Synchronous HARQ		
	Chase combining		
	Up to four retransmissions		
Control channel overhead	- SU-MIMO: 0.3158		
	- SU-/MU-MIMO: 0.3063		
Test parameter	3 trials, 300 subframes per trial		

TABLE II PMI FEEDBACK OVERHEAD FOR SU-MIMO (N=64, M=8)

Codebook	Feedback Overhead
LTE Rel-10 codebook	4 + 4B = 44 bits
Proposed codebook	6B = 60 bits
Proposed two-stage codebook	4 + 3B = 34 bits

TABLE III SU-MIMO PERFORMANCE FOR 8TX UCA

Codebook	Cell Average SE	Cell-edge User SE	
	(bps/Hz)	(bps/Hz/user)	
LTE Rel-10 codebook	1.88 (100%)	0.035 (100%)	
Ideal CDI	2.37 (126%)	0.061 (174%)	
Proposed codebook	2.20 (117%)	0.060 (171%)	
Proposed two-stage codebook	2.15 (114%)	0.061 (174%)	

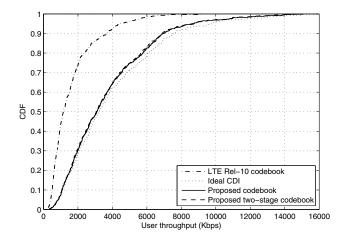


Fig. 3. CDF of User Throughput in SU-/MU-MIMO for 8Tx UCA.

from the proposed UCA codebook is evaluated in Table IV-V and Figure 3. From the ratio of the co-scheduled user number listed in Table IV, it can be seen that compared with the LTE Rel-10 codebook, the proposed codebook enables much more users to be scheduled simultaneously due to the more accurate CDI quantization. Table V shows that the proposed UCA codebook has a prominent MU-MIMO gain of 126% and 105% on average cell SE and cell-edge user SE respectively compared with the LTE Rel-10 codebook, and only has 7% and 1% loss over the ideal CDI feedback on average cell SE and cell-edge user SE respectively. The performance gain on SU-MU-MIMO is much larger than SU-MIMO case, since MU-

TABLE IV RATIO OF CO-SCHEDULED USER NUMBER IN SU-/MU-MIMO FOR 8TX UCA

Codebook	1 User	2 Users	3 Users	4 Users
LTE Rel-10 codebook	65%	14%	15%	6%
Ideal CDI	7%	2%	16%	75%
Proposed codebook	10%	2%	17%	71%
Proposed two-stage codebook	11%	2%	18%	69%

TABLE V SU-/MU-MIMO PERFORMANCE FOR 8TX UCA

Codebook	Cell Average SE	Cell-edge User SE	
	(bps/Hz)	(bps/Hz/user)	
LTE Rel-10 codebook	1.64 (100%)	0.041 (100%)	
Ideal CDI	3.98 (243%)	0.085 (207%)	
Proposed codebook	3.71 (226%)	0.084 (205%)	
Proposed two-stage codebook	3.66 (223%)	0.084 (205%)	

MIMO performance is more sensitive on channel quantization accuracy. By use of the proposed two-stage PMI feedback, similar performance as the UCA codebook can be achieved for SU-/MU-MIMO operation with only 1% loss on average cell-edge.

Through the above analysis on system evaluation, it is demonstrated that the proposed UCA codebook significantly outperforms the existing LTE Rel-10 codebook especially for MU-MIMO mode in UCA configurations, and the further proposed two-stage codebook greatly reduces the feedback overhead without obvious performance degradation.

V. CONCLUSIONS

In this paper, a UCA codebook based on Hadamard transform is proposed to effectively match the channel characteristics of UCA with omni antennas. Furthermore, a two-stage feedback for the proposed codebook is designed to reduce the feedback overhead. System level simulations are performed to evaluate the proposed codebook, and the simulation results demonstrate that the proposed codebook brings significant performance gain on MU-MIMO over the LTE Rel-10 codebook for UCA configurations in FDD systems, and the proposed two-stage feedback greatly reduces the PMI feedback overhead without obvious performance degradation.

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REFERENCES

- [1] A. Paulraj, R. Nabar and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge: Cambridge University Press, 2003.
- [2] A. Gersho and R. M. Gray, Vector Quantization and Signal Compression. Norwell, MA: Kluwer Academics, 1992.
- [3] X. Liu and M. E. Bialkowski, Effective Degree of Freedom and Channel Capacity of a MIMO System Employing Circular and Linear Array Antennas in Proc. IEEE WiCom 2009, Beijing, China, Sep. 2009.
- [4] L. Wu, J. Chen, H. Yang, etc., Codebook Design for LTE-A Downlink System, in Proc. IEEE VTC-2011 Fall, San Francisco, USA, Sep. 2010.
- [5] L. Wu, J. Chen and H. Yang, Codebook Design for Cross-polarized Linear Antenna Array in LTE-A Downlink System, in Proc. IEEE VTC-2011 Fall, San Francisco, USA, Sep. 2010.
- [6] J. Chen, D. Li, H. Yang, etc., On Implementing Spatial-Correlation-Information Aided Feedback in Practical MIMO Systems, in Proc. IEEE VTC-2011 Fall, San Francisco, USA, Sep. 2010.
- [7] 3GPP TS 36.211 v10.3.0, 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 10), Sep. 2011, pp. 61-63.
- [8] J. Tsai, R. M. Buehrer, and B. D. Woerner, BER Performance of a Uniform Circular Array Versus a Uniform Linear Array in a Mobile Radio Environment, IEEE Trans. Wireless Commun., Vol. 3, No. 3, May 2004, pp. 695-700.
- [9] M. Trivellato, F. Boccardi, F. Tosato, User Selection Schemes for MIMO Broadcast Channels with Limited Feedback, in Proc. IEEE VTC2007-Spring, Dublin, Ireland, Apr. 2007, pp. 2089-2093.