A Compressed Analog Feedback Strategy for Spatially Correlated Massive MIMO Systems

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***Abstract*— In multiuser MIMO systems, the amount of the required feedback increases with the number of transmit antennas to improve the sum rate performance. This feedback overhead is critical when it comes to massive multiple antenna wireless systems. Therefore, we propose a compressed analog feedback scheme. The proposed compressed analog feedback strategy exploits natural correlation characteristics among large-scale multiple antennas in order to reduce feedback overhead. By using the sparse approximation and low dimensional random projection at each mobile station (MS), and convex relaxation based feedback decompression at a base station (BS), the proposed scheme outperforms the uncompressed analog feedback which wastes long time duration and energy, in the consideration of the feedback delay, requiring lower feedback overhead from each MS.**

***Keywords*— Analog Feedback, convex relaxation, spatial correlation, multiuser MIMO systems, massive MIMO.**

I. Introduction

In multiuser MIMO broadcast channels, transmit beamforming tremendously increases the sum rate by adding multiple antennas at a base station (BS). The BS equipped with multiple antennas can support the downlink sum rate proportional to the number of transmit antennas over a single antenna BS. Typically, in order to realize this benefit, transmit beamforming requires accurate feedback of channel state information (CSI) from all users to the BS. This CSI at the BS is obtained either implicitly by channel reciprocity or explicitly by feedback. Channel reciprocity, however, does not hold in frequency-division duplexed (FDD) systems. Moreover, with time-division duplexing (TDD), tight calibration of RF devices and overhead are required from the dynamic use of a train of forward and reverse transmission [1], [2]. Since reciprocity may not hold in FDD systems, methods to transfer CSI back to the BS are important for realizing the expected sum rate performance.

Massive MIMO systems with hundreds of low-power multiple antennas offer a plethora of advantages over conventional MIMO systems. In particular, this promotes higher data rates, increased link reliability, and potential power savings since the transmitted energy can be more sharply focused on desired directions. In addition, random impairments can be averaged out. Thus, the research of massive systems has attracted considerable attention in recent years due to their potential to enhance conventional MIMO systems. Massive MIMO scheme with the use of simple transmit beamforming has been recently proposed, achieving unprecedented system spectral efficiency [3]. Despite these major advantages, there are several feedback issues. Indeed, CSI feedback for massive MIMO systems can potentially incur excessive overhead due to a large number of BS antennas. In practice, the increase in the number of antennas with spatial constraints reduces inter-antennas spacing and introduces correlation among antennas [4]. However, conventional feedback schemes seem incapable of exploiting the inherent correlation structures, which leads to overutilization of scarce communication resources. This motivates designing highly efficient CSI feedback strategy.

Many works have investigated the limited digital feedback for multiuser MIMO systems. It has shown that the feedback load per user must be scaled with both the number of transmit antennas as well as the system SNR in order to achieve near-perfect CSI performance and the full multiplexing gain [5]. Also, the optimal vector quantization codebook has been designed based on Grassmannian line packing [6]. The complexity of the quantized feedback, however, increases with the codebook size relevant to the number of transmit antennas, which is challenging since large Grassmanian codebooks are hard to design and encode. Thus, these reasons are, in this paper, why we consider analog feedback for massive MIMO systems.

This paper focuses on massive multiple antenna systems over spatially correlated channels. It addresses its compressed analog CSI feedback scheme. For applying compressed sensing approaches to this problem, sparse approximation is used to obtain a spare structure, and for compression we project large dimensional CSI onto low dimensional random sequences. This compression ratio can be adjusted according to a required feedback delay constraint. Due to the resiliency of random projections, the effect of compression ratio on CSI distortion is gradual.

The remainder of this paper is organized as follows. In the section Ⅱ, multiuser MIMO system with the compressed analog feedback is described. Section Ⅲ provides simple pre-coding method for massive MIMO channels. In section Ⅳ, the

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conventional analog feedback is described. In section Ⅴ, the compressed analog feedback strategy is proposed including sparse approximation and random projection for reducing feedback overhead, and decoding procedure, followed by numerical results and conclusions in section Ⅵ and Ⅶ, respectively.

Notation: A bold face letter denotes a vector or a matrix; $\mathbf{a}^T\left(\mathbf{A}^T\right)$ is the transpose of a vector (a matrix); $\mathbf{a}^*\left(\mathbf{A}^*\right)$ is the conjugate of a vector (a matrix); $\mathbf{a}^H\left(\mathbf{A}^H\right)$ is the conjugate transpose of a vector (a matrix); $\mathbf{A}^{-1}$ denotes the inverse of a square matrix; The $N\times N$ identity matrix is denoted by $\mathbf{I}_N$,

II. SYSTEM MODEL

The compressed analog feedback based multiuser MIMO system illustrates in Fig. 1 which consists of the forward and the reverse link (or CSI feedback link) to be described in the following subsections. We assume the case of the homogeneous network where a BS and K mobile stations (MSs) are equipped with N multiple antennas and a single antenna, respectively.

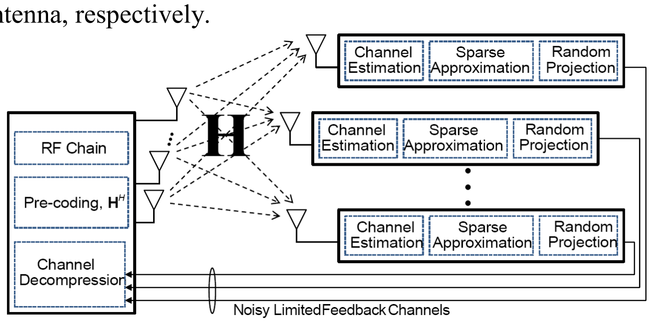

Fig. 1. Multiuser massive systems with the compressed analog feedback

A. Correlated MIMO Channels Model

For simplicity, a uniform linear array at the BS with identical antenna elements and randomly distributed MSs are assumed. The correlated model is studied on the previous work [7]. The spatial correlation matrix can be presented by

$$\mathbf{R}=\mathbf{R}_{\mathrm{MS}}\otimes\mathbf{R}_{\mathrm{BS}}, \tag{1}$$

where $\otimes$ denotes Kronecker product, the $\mathbf{R}_{\mathrm{BS}}$ and $\mathbf{R}_{\mathrm{MS}}$ are the BS antennas and MSs antennas correlation matrices. Each i-th row and j-th column entry of these matrices is given by,

$$\mathbf{R}_{\mathrm{BS}}^{(i,j)}=J_0\left(2\pi\Delta|i-j|\frac{d_{\mathrm{BS}}}{\lambda}\right), \tag{2}$$

where J_0 is zero-order Bessel function, Δ is the angle spread. The spacing between antennas at a BS is d_{BS}. The λ is the wavelength of a narrowband signal with carrier frequency. In this paper, we assume that the correlation matrix $\mathbf{R}_{\mathrm{MS}}$ is identity matrix $\mathbf{I}_K$ since MSs equipped by a single antenna are distributed randomly and separated by more hundreds of wavelengths.

B. Forward Link Signal Model

In the forward link, the BS transmits and MSs receive. The forward link channels are mathematically described as

$$\begin{aligned} y_k &= \underbrace{\sqrt{\frac{P}{K}}\mathbf{z}_k^T\mathbf{R}_{\mathrm{BS}}^{1/2}\mathbf{w}_k u_k}_{\text{desired signal}}+\underbrace{\sqrt{\frac{P}{K}}\sum_{j\neq k}\mathbf{z}_k^T\mathbf{R}_{\mathrm{BS}}^{1/2}\mathbf{w}_j u_j+n_k}_{\text{interfering signal and noise}} \\ &= \sqrt{\frac{P}{K}}\mathbf{h}_k^T\mathbf{w}_k u_k+\sqrt{\frac{P}{K}}\sum_{j\neq k}\mathbf{h}_k^T\mathbf{w}_j u_j+n_k, \end{aligned} \tag{3}$$

where $\mathbf{z}_k\in\mathbb{C}^{N\times1}$ is the i.i.d. channel gain vector with zero mean and unit variance complex Gaussian entries, the vector $\mathbf{w}_k\in\mathbb{C}^{N\times1}$ is the pre-coding vector and u_k is the message-bearing signal for the k-th MS with the power constraint $\mathbb{E}|u_k|^2=1$, n_k is the additive complex Gaussian noise with zero mean and unit variance. We denote the concatenation of the channels by $\mathbf{H}=[\mathbf{h}_1,\ldots,\mathbf{h}_K]\in\mathbb{C}^{N\times K}$ whose k-th column is equal to the channel of the k-th MS where $\mathbf{h}_k^T=\mathbf{z}_k^T\mathbf{R}_{\mathrm{BS}}^{1/2}$. We assume that the channel state varies over each frame according to a first-order Gauss-Markov process as follows

$$\mathbf{h}_k[n]=\alpha\mathbf{h}_k[n-1]+\sqrt{1-\alpha^2}\mathbf{b}[n] \tag{4}$$

where $\mathbf{b}[n]\in\mathbb{C}^{M\times1}$ has i.i.d. Gaussian entries with zero mean and unit variance. The evolution variable obeys Jakes' model according to $\alpha=J_0(2\pi f_D T_S)$ where T_S denotes symbol duration, $f_D=\frac{vf_c}{c}$ is the maximum Doppler frequency with relative velocity v, carrier frequency f_c, and $c=3\times10^8\,m/s$, but is with independent fading between each frame. In addition, we consider equal power allocation due to that water-filling gain is negligible at high SNR.

C. Reverse (Feedback) Link Signal Model

The K MSs transmit information to the BS independently. The reciprocity assumption on the forward and reverse link channels cannot be realized and generally have highly uncorrelated relations since they are sufficiently separated in frequency. The received signal at the BS from k-th MS is

$$\begin{aligned} \mathbf{y}_k &= \underbrace{\sqrt{P_{\mathrm{F}}}\mathbf{R}_{\mathrm{BS}}^{1/2}\mathbf{v}_k x_k}_{\text{desired signal}}+\underbrace{\sqrt{P_{\mathrm{F}}}\sum_{j\neq k}\mathbf{R}_{\mathrm{BS}}^{1/2}\mathbf{v}_j x_j+\mathbf{n}_k}_{\text{interfering signal and noise}} \\ &= \sqrt{P_{\mathrm{F}}}\mathbf{g}_k x_k+\sqrt{P_{\mathrm{F}}}\sum_{j\neq k}\mathbf{g}_j x_j+\mathbf{n}_k, \end{aligned} \tag{5}$$

where P_{F} is the transmit power for transmitting pilot sequences or feedback symbols at each MS, the i.i.d. channel gain vector $\mathbf{v}_k\in\mathbb{C}^{N\times1}$ is with zero mean and unit variance complex Gaussian entries, x_k is transmitted symbol with power constraint $\mathbb{E}|x_k|^2=1$, and $\mathbf{n}_k$ is a complex vector of i.i.d. white Gaussian noise with zero mean and unit variance. The reverse channel matrix, $\mathbf{G}=[\mathbf{g}_1,\ldots,\mathbf{g}_K]\in\mathbb{C}^{N\times K}$ where $\mathbf{g}_k=\mathbf{R}_{\mathrm{BS}}^{1/2}\mathbf{v}_k$, have the block fading model where $\mathbf{G}$ is constant over each frame, and is with independent fading between each frame.

III. Simple Precoding for Massive MIMO Systems

The BS transmits a vector of message-bearing symbols multiplied by pre-coding matrix. The used pre-coding matrix is designed as complex conjugate of the estimated forward link channel matrix $\mathbf{H}=\mathbf{Z}\mathbf{R}_{\mathrm{BS}}^{1/2}$. Thus, pre-coding matrix $\mathbf{W}$ is the normalized $\mathbf{H}^{*}$ of each column. So, the forward link signals in (3) can be rewritten as

$$y_k=\sqrt{\frac{P}{\beta K}}\mathbf{h}_k^T\mathbf{h}_k^{*}u_k+\sqrt{\frac{P}{\beta K}}\sum\nolimits_{j\neq k}\mathbf{h}_k^T\mathbf{h}_j^{*}u_j+n_k. \tag{6}$$

where β is the normalizing factor.

The first term on the right hand side is the desired signal, while the second term is the interference and noise signals. Thus, the received SINR is given by

$$\mathrm{SINR}_k=\frac{\frac{P}{\beta K}\left|\mathbf{h}_k^T\mathbf{h}_k^{*}\right|^2}{1+\frac{P}{\beta K}\sum\nolimits_{j=1,j\neq k}^{K}\left|\mathbf{h}_k^T\mathbf{h}_j^{*}\right|^2}, \tag{7}$$

and the achievable sum-rate is given by

$$R_{\mathrm{sum}}=\mathbb{E}\sum_{k=1}^{K}\log_2\left(1+\mathrm{SINR}_k\right). \tag{8}$$

As N grows infinitely, the forward channel gain vectors for different MSs become asymptotically orthogonal. Hence, interference introduced by different MSs vanishes as follows

$$\lim_{N\to\infty}\frac{\mathbf{h}_k^T\mathbf{h}_j^{*}}{\left\|\mathbf{h}_k\right\|_2\left\|\mathbf{h}_j\right\|_2}=\delta_{kj}. \tag{9}$$

IV. Conventional (Uncompressed) Analog Feedback

We consider a model including the effect of reverse link training and channel feedback. There are two main phase over T period: 1) the MSs transmit pilot symbols for the BS to learn their reverse link channels $\mathbf{G}=[\mathbf{g}_1,\ldots,\mathbf{g}_K]$ during T_{P}, and 2) the forward CSI is fed back to the BS during T_{F}. We assume that instantaneous knowledge of forward link channels $(\mathbf{H})$ are perfectly estimated using maximum likelihood estimator (MLE) by each MS while the BS uses a minimum mean square error (MMSE) estimator for the reverse link channels [8].

A. Reverse Link Training

The K MSs transmit pilot symbols over a period $T_{\mathrm{P}}\geq K$ in order to learn reverse link channels. The MSs collectively transmit a $K\times T_{\mathrm{P}}$ unitary matrix of pilots $\mathbf{P}$, where $\mathbf{P}\mathbf{P}^H=\mathbf{I}_K$.

We can write the $N\times T_{\mathrm{P}}$ received training matrix at the BS as

$$\mathbf{Y}_{\mathrm{P}}=\sqrt{T_{\mathrm{P}}P_{\mathrm{F}}}\mathbf{G}\mathbf{P}+\mathbf{N}_{\mathrm{P}}, \tag{10}$$

where $\mathbf{G}$ is the $N\times K$ combined reverse channel matrix and $\mathbf{N}_{\mathrm{P}}$ is the $N\times T_{\mathrm{P}}$ matrix with i.i.d. complex zero mean and unit variance Gaussian entries. The BS performs the MMSE estimate of reverse channel matrix based on received training information as follows

$$\hat{\mathbf{G}}=\left(\frac{\sqrt{T_{\mathrm{P}}P_{\mathrm{F}}}}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)\mathbf{Y}_{\mathrm{P}}\mathbf{P}^H. \tag{11}$$

The statistical characteristics of estimate results $\hat{\mathbf{G}}$ and its error components $\tilde{\mathbf{G}}=\mathbf{G}-\hat{\mathbf{G}}$ are $\hat{\mathbf{G}}\sim\mathcal{N}_{\mathcal{C}}\left(0,\frac{T_{\mathrm{P}}P_{\mathrm{F}}}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)$ and $\tilde{\mathbf{G}}\sim\mathcal{N}_{\mathcal{C}}\left(0,\frac{1}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)$, respectively.

B. Analog Forward Link CSI Feedback

Subsequent to training the reverse link channels, all MSs directly transmit their un-quantized and un-coded estimates of $\mathbf{h}_k$ over a period T_{F}. In order for transmitting feedback information simultaneously and orthogonally from MSs, each MSs post multiplies its feedback vector $\mathbf{h}_k^T$ with a $N\times T_{\mathrm{F}}$ unitary matrix $\mathbf{O}$, where $\mathbf{O}_k\mathbf{O}_j^H=\mathbf{I}_N\delta_{k,j}$. The transmitted $1\times T_{\mathrm{F}}$ feedback vector $\mathbf{x}_k$ over a period $T_{\mathrm{F}}\geq K\cdot N$ from k-th MS can be written as

$$\mathbf{x}_k=\mathbf{h}_k^T\mathbf{O}_k. \tag{12}$$

The BS collectively receive a $N\times T_{\mathrm{F}}$ matrix as given by

$$\mathbf{Y}_{\mathrm{F}}=\sqrt{\frac{T_{\mathrm{F}}P_{\mathrm{F}}}{N}}\sum_{k=1}^{K}\mathbf{g}_k\mathbf{h}_k^T\mathbf{O}_k+\mathbf{N}_{\mathrm{F}}. \tag{13}$$

where $\mathbf{N}_{\mathrm{F}}$ is a $N\times T_{\mathrm{F}}$ noise matrix with complex zero mean and unit variance Gaussian entries. In order to estimate each forward link channel $\mathbf{h}_k$, the BS extracts the CSI of k-th MS by post multiplying $N\times T_{\mathrm{F}}$ received matrix $\mathbf{Y}_{\mathrm{F}}$ by $\mathbf{O}_k^H$ as given by

$$\mathbf{Y}_{\mathrm{F}}\mathbf{O}_k^H=\sqrt{\frac{T_{\mathrm{F}}P_{\mathrm{F}}}{N}}\sum_{k=1}^{K}\mathbf{g}_k\mathbf{h}_k^T+\mathbf{N}\mathbf{O}_k^H. \tag{14}$$

And at the BS, a least squares (LS) estimate of $\hat{\mathbf{h}}_k$ given by

$$\hat{\mathbf{h}}_k^T=\sqrt{\frac{N}{T_{\mathrm{F}}P_{\mathrm{F}}}}\frac{\hat{\mathbf{g}}_k^H}{\left\|\hat{\mathbf{g}}_k\right\|_2^2}\mathbf{Y}_{\mathrm{F}}\mathbf{O}_k^H. \tag{15}$$

The error vector $\tilde{\mathbf{h}}_k^T$ in the estimate of true channel vector $\mathbf{h}_k^T$ can be written by

$$\tilde{\mathbf{h}}_k^T=\frac{\hat{\mathbf{g}}_k^H}{\left\|\hat{\mathbf{g}}_k\right\|_2^2}\left(\sqrt{\frac{N}{T_{\mathrm{F}}P_{\mathrm{F}}}}\mathbf{N}_{\mathrm{F}}-\tilde{\mathbf{g}}_k\mathbf{h}_k^T\right). \tag{16}$$

And its average power of the error vector can be given by

$$\mathbb{E}\left\|\tilde{\mathbf{h}}_k^T\right\|_2^2=\left(\frac{N}{N-1}\right)\left(\frac{N}{T_{\mathrm{F}}P_{\mathrm{F}}}+\frac{1}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)\left(\frac{T_{\mathrm{P}}P_{\mathrm{F}}}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)^{-1}. \tag{17}$$

V. Proposed Compressed Analog Feedback

In this section, the compressed analog feedback strategy is proposed which exploits spatial correlation among a large number of BS antennas. This strategy includes three steps: 1) the sparse approximation and 2) random projection for compression at each MS, and 3) the channel decompression via convex relaxation at the BS.

A. Sparse Approximation

The forward link channel CSI at each MS is first approximated through two steps: 1) sparse representation in terms of a basis of $N\times 1$ vectors φ_i, and 2) the suppression of smaller values than fixed threshold. Then, for compression into low dimension, each channel vector approximated is projected onto M random sequences, $M \ll N$. The sparse approximation is needed since error bound of decompression results rely on sparsity of a vector to be projected.

Using the orthonormal $N\times N$ matrix $\mathbf{\Psi}=[\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_N]$, the channel vector can be expressed as

$$\mathbf{h}_k=\sum\nolimits_{i=1}^{N} a_i\boldsymbol{\varphi}_i, \tag{18}$$

where a_i is a inner product between a channel vector and a basis of $\mathbf{\Psi}$, $\langle \mathbf{h}_k,\boldsymbol{\varphi}_i\rangle$. The representation vector $\mathbf{a}_k=[a_1,\ldots,a_N]^T$ of a channel vector is compressible since representation vector has many small coefficients and just a few large coefficients. Thus, the compressible vector $\mathbf{a}_k$ can be well approximated by S -sparse representation $\breve{\mathbf{a}}_k$ through the following approximation step. We consider three approximation methods:

Approximation 1(AND):

$$\breve{\mathbf{a}}_k=[\Re(\mathbf{a}_k)+j\cdot\Im(\mathbf{a}_k)]\mathbb{I}[|\Re(\mathbf{a}_k)|>\Gamma\cap|\Im(\mathbf{a}_k)|>\Gamma]$$

Approximation 2(OR):

$$\breve{\mathbf{a}}_k=\Re(\mathbf{a}_k)\mathbb{I}[|\Re(\mathbf{a}_k)|>\Gamma]+j\cdot\Im(\mathbf{a}_k)\mathbb{I}[|\Im(\mathbf{a}_k)|>\Gamma],$$

Approximation 3(Average):

$$\breve{\mathbf{a}}_k=\mathbf{a}_k\times\mathbb{I}\left[\frac{|\Re(\mathbf{a}_k)|+|\Im(\mathbf{a}_k)|}{2}>\Gamma\right],$$

where $\mathbb{I}[\cdot]$ is the indicator function, the $\Re(\cdot)$ and $\Im(\cdot)$ extract a real and imaginary component from a complex value respectively.

B. Random Projection

The compression is performed by projecting each *S*-sparse channel vector onto random sequences $\phi_1,\ldots,\phi_M$, $\langle\breve{\mathbf{a}}_k,\phi_i\rangle$. Its procedure is mathematically described as

$$\mathbf{m}_k=\mathbf{\Phi}\breve{\mathbf{a}}_k, \tag{19}$$

where the $\mathbf{\Phi}$ is the $M\times N$ random matrix whose entries are i.i.d. random variables from complex zero mean and $1/M$ variance Gaussian, the $\mathbf{m}_k$ is $M\times 1$ compressed channel vector.

If $M\geq O(S\log(N/S))$ is satisfied, the Gaussian random matrix has interesting property with high probability as follows

$$(1-\delta_S)\|\breve{\mathbf{a}}_k^T\|_2^2\leq\|\breve{\mathbf{a}}_k^T\mathbf{\Phi}^T\|_2^2\leq(1+\delta_S)\|\breve{\mathbf{a}}_k^T\|_2^2, \tag{20}$$

where a $\delta_S>0$ is some constant [9]. When δ_S is small, this means that the salient information in any sparse vector is not damaged by the compression, i.e., reduction of dimension. This condition is referred to as the restricted isometry property (RIP) and the matrix satisfying this condition is called stable measurement matrix. With this property, the sparse channel vector $\breve{\mathbf{a}}_k\in\mathbb{C}^{N\times 1}$ can be stably recovered from only $M\geq O(S\log(N/S))$ random Gaussian measurement vector $\mathbf{m}_k\in\mathbb{C}^{M\times 1}$.

Each MS transmits the measurement vector $\mathbf{m}_k$ after post multiplying by a $N\times\overline{T}_{\mathrm{F}}$ unitary matrix $\overline{\mathbf{O}}_k$ as follows

$$\mathbf{x}_k=\breve{\mathbf{a}}_k^T\mathbf{\Phi}\overline{\mathbf{O}}_k, \tag{21}$$

where the $\overline{T}_{\mathrm{F}}\geq K\cdot M$ time is a required period to transmit the compressed feedback signal, $\overline{T}_{\mathrm{F}}\ll T_{\mathrm{F}}$. In proposed strategy, the compressed analog feedback exploiting RIP property of Gaussian random matrix results in using far less energy and low latency during transmitting channel information.

The collectively received signal is

$$\overline{\mathbf{Y}}_{\mathrm{F}}=\sqrt{\frac{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}{M}}\sum_{k=1}^{K}\mathbf{g}_k\mathbf{m}_k^T\,\overline{\mathbf{O}}_k+\overline{\mathbf{N}}_{\mathrm{F}}, \tag{22}$$

where $\overline{\mathbf{N}}_{\mathrm{F}}$ is $N\times\overline{T}_{\mathrm{F}}$ additive white Gaussian noise matrix.

At the BS, the unitary matrix $\overline{\mathbf{O}}_k^H$ is post multiplied in order to extract the measurement vector for k -th MS as follows

$$\overline{\mathbf{Y}}_{\mathrm{F}}\overline{\mathbf{O}}_k^H=\sqrt{\frac{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}{M}}\mathbf{g}_k\mathbf{m}_k^T+\overline{\mathbf{N}}_{\mathrm{F}}\overline{\mathbf{O}}_k^H. \tag{23}$$

Then, the BS computes the LS estimate of $\hat{\mathbf{m}}_k^T$ as given by

$$\begin{aligned}\hat{\mathbf{m}}_k^T&=\sqrt{\frac{M}{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}}\frac{\hat{\mathbf{g}}_k^H}{\|\hat{\mathbf{g}}_k\|_2^2}\left(\sqrt{\frac{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}{M}}\mathbf{g}_k\mathbf{m}_k^T+\overline{\mathbf{N}}_{\mathrm{F}}\right)\\&=\sqrt{\frac{M}{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}}\frac{\hat{\mathbf{g}}_k^H}{\|\hat{\mathbf{g}}_k\|_2^2}\left(\sqrt{\frac{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}{M}}(\hat{\mathbf{g}}_k-\tilde{\mathbf{g}}_k)\mathbf{m}_k^T+\overline{\mathbf{N}}_{\mathrm{F}}\right)\\&=\mathbf{m}_k^T+\frac{\hat{\mathbf{g}}_k^H}{\|\hat{\mathbf{g}}_k\|_2^2}\left(\sqrt{\frac{M}{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}}\overline{\mathbf{N}}_{\mathrm{F}}-\tilde{\mathbf{g}}_k\mathbf{m}_k^T\right)=\underbrace{\mathbf{m}_k^T}_{\text{true}}+\underbrace{\tilde{\mathbf{m}}_k^T}_{\text{error}}\end{aligned} \tag{24}$$

In the compressed analog feedback, the average power of error vector is given by

$$\mathbb{E}\|\tilde{\mathbf{m}}_k^T\|_2^2=\left(\frac{\overline{T}_{\mathrm{F}}}{T_{\mathrm{F}}}\right)\left(\frac{N}{N-1}\right)\left(\frac{M}{\overline{T}_{\mathrm{F}}P_{\mathrm{F}}}+\frac{1}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)\left(\frac{T_{\mathrm{P}}P_{\mathrm{F}}}{1+T_{\mathrm{P}}P_{\mathrm{F}}}\right)^{-1}, \tag{25}$$

where $\overline{T}_{\mathrm{F}}/T_{\mathrm{F}}$ stands for compression ratio.

C. CSI Decompression via Convex Optimization

In this section, we present the channel decompression method. The decompressed $\hat{\mathbf{a}}_k$ is obtained by solving noise-aware convex relaxation problem as follows

$$\hat{\mathbf{a}}_k=\arg\min_{\breve{\mathbf{a}}_k\in\mathbb{C}^{N\times 1}}\|\breve{\mathbf{a}}_k\|_1\ \text{subject to}\ \|\mathbf{m}_k-\mathbf{\Phi}\breve{\mathbf{a}}_k\|_2\leq\varepsilon, \tag{26}$$

where $\mathcal{L}_1$ norm is $\|\breve{\mathbf{a}}_k\|_1=\sum_i|\breve{a}_i|$, and the $\mathcal{L}_2$ norm constraint is ε chosen high enough to bound an error vector with high probability. If $M\geq O(S\log(N/S))$ is satisfied, the decompression error bound is [10]-[12]

$$\|\hat{\mathbf{a}}_k-\breve{\mathbf{a}}_k\|_2^2\leq c(N\cdot S\cdot\eta)\log N, \tag{27}$$

where the constant c is small depending on the RIP coefficient δ_S, and $\eta = \mathbb{E}\|\tilde{\mathbf{m}}_k^T\|_2^2$.

Finally, the forward link channel can recovered by inverse orthonormal transform, $\hat{\mathbf{h}}_k = \mathbf{\Psi}^H \hat{\mathbf{a}}_k$, successfully. As a result, the total error for each channel vector obeys the following upper bound,

$$\varepsilon_{\text{total}}^2 \leq \underbrace{\|\mathbf{a}_k - \breve{\mathbf{a}}_k\|_2^2}_{\text{approximation error}} + \underbrace{c(N \cdot S \cdot \eta)\log N}_{\text{noise and compression error}}. \tag{28}$$

Equation (28) shows that there is trade-off between the sparse approximation step and the compression procedure with respect to the total error. The small threshold is used for the lower approximation error, which leads to the large S and an increase in compression error.

VI. SIMULATION RESULTS

In this section, we present simulation results. We consider the BS equipped with $N = 500$ massive multiple antennas, and $K = 5$ MSs. The angle spread is set to $\Delta = 0.1$, and the distances among massive multiple antennas are assumed to be less than $\lambda/2$, which introduces the effect of fading correlations. The correlation coefficient is defined as [13]

$$\rho = \max_{i \neq j} \frac{r_{\text{BS}}(i,j)}{\sqrt{r_{\text{BS}}(j,j) r_{\text{BS}}(i,i)}}, \tag{29}$$

where $r_{\text{BS}}(i,j)$ is the i-th row and j-th column entry of correlation matrix $\mathbf{R}_{\text{BS}}$. We consider highly correlated structure, $\rho = 0.5, 0.7$. In addition, for a temporally correlated channel model, a first-order Gauss-Markov process in (6.4) is used. We assume that a closed-loop operation is with 30km/h velocity, feedback interval of $T_{\text{F}} = 5$ms, and center frequency of 2.5GHz. Then, we calculate the average sum rate of analog feedback based massive multiple antenna system as

$$R_{sum} = \mathbb{E}\sum_{k=1}^{K} \log_2\left(1 + \frac{P}{K}|\mathbf{h}_k \mathbf{w}_k|^2 (1 + I_k)^{-1}\right), \tag{30}$$

where $I_k = \frac{P}{K}\sum_{j=1, j\neq k}^{K} |\mathbf{h}_k \mathbf{w}_j|^2$.

Each pre-coding vector is designed as the normalized vector by operating conjugate transpose of the estimated forward link channel vector, $\mathbf{w}_k = \hat{\mathbf{h}}_k^* / \|\hat{\mathbf{h}}_k\|_2$ as presented in [3]. For sparse representation, the Fourier domain is used.

Figure 2 shows the average sum rate of the proposed scheme with different approximation methods when the feedback link SNR is 25dB, the length of feedback channel vector is M = 300 and correlation coefficient is $\rho = 0.7$. The approximation 3 (average) method shows the best performance at threshold $\Gamma = 0.5$. Thus, we use the approximation 3 method for the following simulations.

Figure 3 shows the average sum rate of the proposed scheme with different values of threshold Γ when feedback link SNR is 30dB. The best performance can be achieved with different threshold values depending on the length of the compressed feedback vector M. Based on simulation result of Fig. 3, each optimal threshold value is selected for Fig. 4.

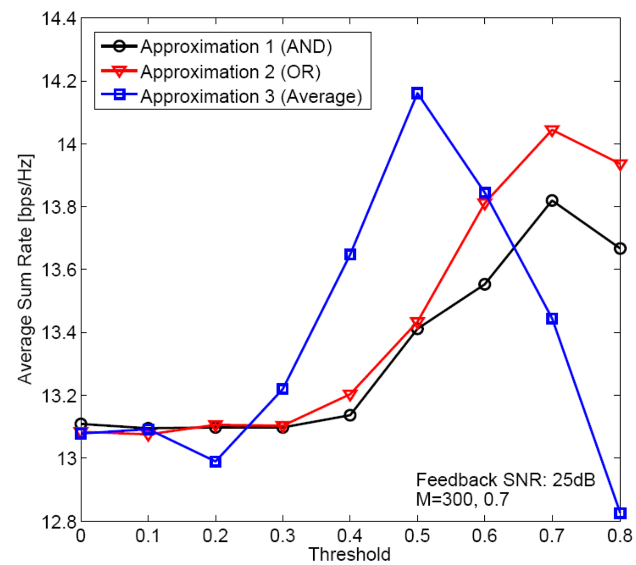

Fig. 2. Average sum rate comparison between the compressed analog feedback with different approximation methods when the forward link SNR=-10dB and M=300 with no feedback delay.

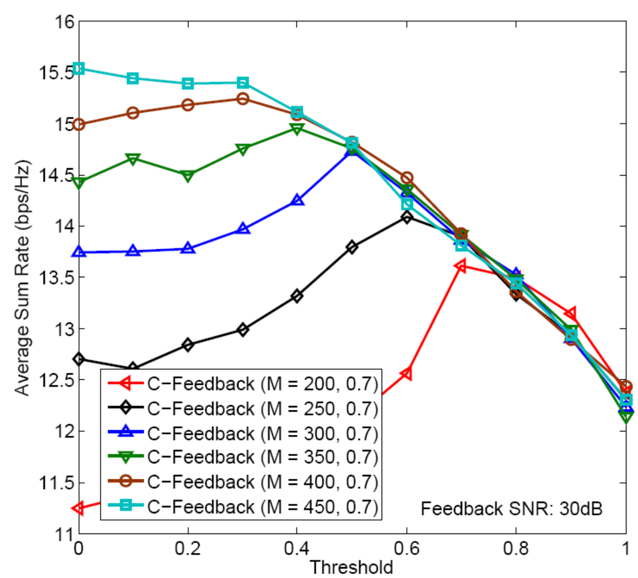

Fig. 3. Average sum rate comparison between the compressed analog feedback with the different threshold when the forward link SNR=-10dB.

For comparison in Fig. 4, we also show the average sum rate of uncompressed analog feedback based transmit beamforming. It is shown that the proposed compressed analog feedback based scheme achieves the comparable performance of the uncompressed feedback based scheme although that does not utilize all resources of both energy and time. In particular, the proposed strategy with 0.8 compression ratio (M=400) achieves the similar sum rate performance to that of uncompressed feedback. This is because the CSI recovery can be attempted using any number of received random measurements and any loss of the smaller number of random measurements leads to the graceful degradation in the CSI recovery quality. On the other hand, more feedback measurements allow the progressively better CSI recovery leading to the higher sum rate performance. Thus, our proposed strategy can adjust the compression ratio according to required feedback delay and energy constraints while being gradual in terms of the sum rate performance.

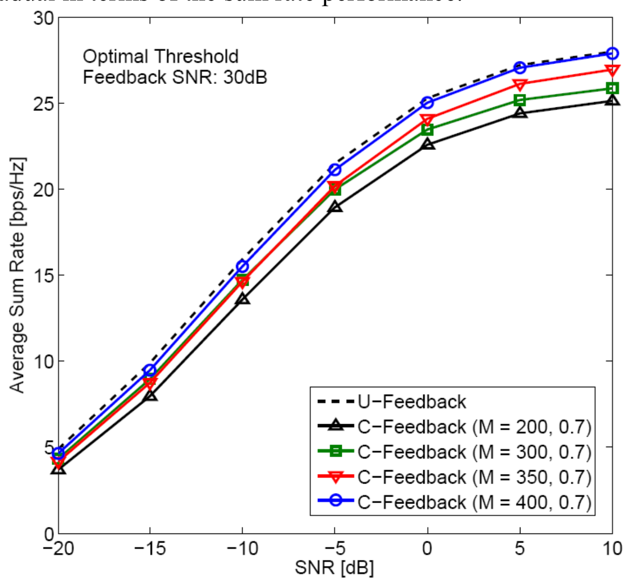

Fig. 4. Average sum rate comparison between the uncompressed and compressed analog feedback with the different compression ratio in no feedback delay.

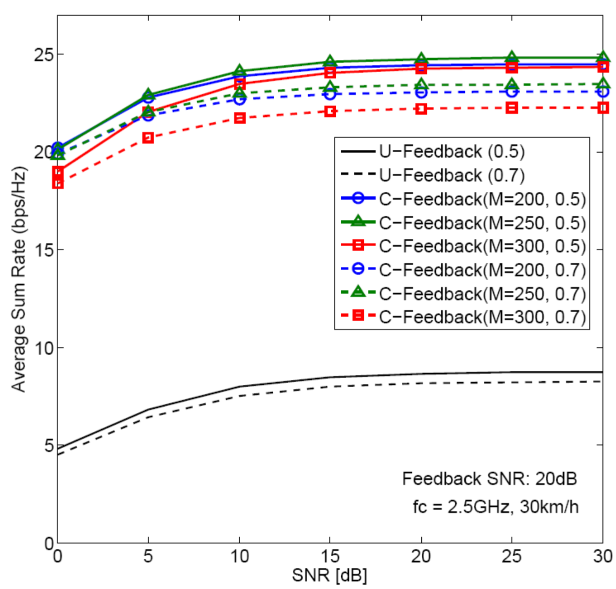

Fig. 5. Average sum rate comparison between the uncompressed and compressed analog feedback with the different compression ratio in the consideration of the feedback delay.

In Fig. 5, it is shown that the proposed compressed analog feedback based scheme outperforms the uncompressed feedback although the proposed scheme does not utilize all resources of both energy and time. This is because the performance degradation mainly results from the outdated channel information. In addition, the sum rate performance with 0.5 compression ratio (M=250) outperforms that of 0.6 compression ratio (M=300). This means there is the trade-off between the degradation induced by the compression loss and the outdated channel information.

VII. CONCLUSION

In this paper, we proposed the compressed analog feedback strategy for spatially correlated massive multiple antenna systems. This is based on spare approximation, random projection and convex relaxation based decompression procedures. Simulation results show the proposed feedback strategy achieves sum rate performance comparable to the case of uncompressed analog feedback strategy, using less energy and latency than those dictated by the uncompressed analog feedback scheme by exploiting the correlation structure of massive multiple antennas, and outperforms that in the consideration of feedback delay.

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