Efficient Feedback Design for Interference Alignment in MIMO Interference Channel

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***Abstract*—Interference alignment (IA) is a joint-transmission technique that achieves the capacity of the interference channel for high signal-to-noise ratios (SNRs). However, most prior works on IA are based on the impractical assumption that perfect and global channel-state information(CSI) is available at all transmitters, resulting in overwhelming feedback overhead. To substantially suppress the feedback overhead, this paper proposes an efficient design of the feedback framework for IA in the K-user multiple-input multiple-output (MIMO) interference channel. The proposed feedback topology supports sequential CSI exchange (feedback and feedforward) between transmitters and receivers and reduces the feedback overhead from a cubic function of K to a linear one, compared to conventional feedback approaches. Given the proposed feedback topology, we consider the limited feedback channel from the receivers to corresponding interferers and analyze the effect of quantization error which generates the residual interference. Also, an efficient feedback-bit allocation algorithm that minimizes the upper-bound of sum residual interference is proposed.**

I. Introduction

In a wireless network, interference alignment (IA) techniques achieve the maximum multiplexing gain or *degrees of freedom* (DoF) of the K-user single-antenna interference channel, namely $K/2$, by asymptotic signal-space extension over time, frequency or space [1]. Given symbol extension, the bounds on the achievable DoF for multiple-input multiple-output (MIMO) interference channel were derived in [2] and the optimal IA solutions were obtained in closed-form for some specific settings. Recently, IA research has been focusing on quantifying the achievable DoF and designing matching IA solutions for a single channel realization over finite spatial dimensions, called the *MIMO constant channel* [3], [4]. In particular, the feasibility conditions of IA were derived in [3] and iterative IA algorithms for achieving such conditions were proposed in [4], which exploit the channel reciprocity to achieve distributive implementation. In addition, the IA principle has been extended to design multi-cell precoding for cellular networks [5], [6].

Despite being a promising technique for interference mitigation, IA is far from being practical. A key challenge in implementing IA algorithms is that each transmitter requires perfect and global channel state information at transmitters (CSIT) of all interference channel, incurring potentially unacceptable CSI feedback overhead. Therefore, efficient CSIT acquisition remains the key challenge for implementing IA techniques, which is the main theme of this paper. In addition, the required CSIT for IA usually has to rely on finite-rate CSI feedback, called *limited feedback* from receivers to their interferers, resulting in imperfect CSIT. In this paper, we consider the K-user constant MIMO interference channel where each transmitter/receiver employs $M = K-1$ antennas and supports a single data stream [7]. The contributions of this paper are summarized as follows.

1) Efficient arrangements of CSI feedback links for reducing the sum feedback overhead of IA are proposed, where the IA beamformers are sequentially computed based on the exchange of pre-determined beamformers (feedback and feedforward) between transmitters and receivers.
2) We consider the impact of limited feedback on the performance of the proposed feedback topology, using random vector quantization (RVQ) in [8]. The expected cross-link interference power at each receiver is upper-bounded by sum of exponential functions of the number of feedback bits. Also, we propose a dynamic feedback-bit allocation algorithm based on the water-filling principle, which minimizes sum residual interference.

II. System Model

We consider a K-user MIMO interference channel where each node employes M antennas and each link supports a single data stream. For tractability, M and K are constrained by $K = M + 1$ so that the IA beamformers can be computed in closed-form as shown in [7].[1] Then, the received signal at the receiver k is given by

$$\mathbf{y}^{[k]} = \mathbf{H}^{[k\,k]}\mathbf{v}^{[k]}s_k + \sum_{j\neq k}\mathbf{H}^{[k\,j]}\mathbf{v}^{[j]}s_j + \mathbf{n}_k \tag{1}$$

where s_k denotes a data symbol sent by the transmitter k with $\mathcal{CN}(0, P)$, the $M \times 1$ vector $\mathbf{v}^{[k]}$ is the corresponding beamformer and $\mathbf{n}_k$ is additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma^2\mathbf{I}_M$. The fading channel between the transmitter j and the receiver k is represented by $M \times M$ matrix $\mathbf{H}^{[kj]}$, which comprises independent and identically distributed (i.i.d) circularly symmetric complex Gaussian random variables with $\mathcal{CN}(0, 1)$. In this section, we

[1] In $M \times N$ MIMO interference channels, the number of users supported by IA is constrained by $K \leq M+N-1$. However, the closed-form solutions have been unknown in general antenna configurations [3], [4].

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introduce the closed-form IA beamformers that provide total K DoF in the K-user interference channel. Also, the overhead of CSI feedback for IA is defined and measured under the conventional approach.

A. Design of IA Beamformer

Consider the set of interference $\left\{\mathbf{H}^{[kj]}\mathbf{v}^{[j]}|\forall j, j\neq k\right\}$ that spans $K-1$ dimensional subspace at receiver k. To achieve one dimensional interference-free link at the k-th transmitter-receiver pair, we align the interference from transmitter $k+1$ and $k+2$ into the same subspace at receiver k so that $K-1$ interferers lie in the $M-1$ dimensional subspace. Then, a simple zero-forcing (ZF) receive filter can be designed for nullifying all interference at receiver k. Using those IA principles, we obtain the following IA conditions that support a single data transmission at each link :

$$\begin{aligned}
\mathsf{span}\left(\mathbf{H}^{[12]}\mathbf{v}^{[2]}\right) &= \mathsf{span}\left(\mathbf{H}^{[13]}\mathbf{v}^{[3]}\right)\\
\mathsf{span}\left(\mathbf{H}^{[23]}\mathbf{v}^{[3]}\right) &= \mathsf{span}\left(\mathbf{H}^{[24]}\mathbf{v}^{[4]}\right)\\
&\vdots\\
\mathsf{span}\left(\mathbf{H}^{[K-1K]}\mathbf{v}^{[K]}\right) &= \mathsf{span}\left(\mathbf{H}^{[K-11]}\mathbf{v}^{[1]}\right)\\
\mathsf{span}\left(\mathbf{H}^{[K1]}\mathbf{v}^{[1]}\right) &= \mathsf{span}\left(\mathbf{H}^{[K2]}\mathbf{v}^{[2]}\right)
\end{aligned} \tag{2}$$

where $\mathsf{span}(\mathbf{A})$ denotes the vector space spanned by the columns of $\mathbf{A}$.

Note that each of IA conditions in (2) can be modified as

$$\begin{aligned}
&\mathsf{span}\left(\mathbf{H}^{[k-1k]}\mathbf{v}^{[k]}\right) = \mathsf{span}\left(\mathbf{H}^{[k-1k+1]}\mathbf{v}^{[k+1]}\right)\\
&\Rightarrow \mathsf{span}(\mathbf{v}^{[k+1]}) = \mathsf{span}\left(\left(\mathbf{H}^{[k-1k+1]}\right)^{-1}\mathbf{H}^{[k-1k]}\mathbf{v}^{[k]}\right)
\end{aligned} \tag{3}$$

and $\left\{\mathsf{span}\left(\mathbf{v}^{[k]}\right)\right\}_{k=1}^{K}$ are concatenated with each other. From (2) and (3), IA beamformers are computed by

$$\begin{aligned}
\mathbf{v}^{[1]} &= \text{any eigenvector of } \mathbf{T}_K\\
\mathbf{v}^{[2]} &= \left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}\mathbf{v}^{[1]}\\
\mathbf{v}^{[3]} &= \left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[12]}\mathbf{v}^{[2]}\\
&\vdots\\
\mathbf{v}^{[K]} &= \left(\mathbf{H}^{[K-2K]}\right)^{-1}\mathbf{H}^{[K-2K-1]}\mathbf{v}^{[K-1]}
\end{aligned} \tag{4}$$

and then normalized to have unit norm, where $\mathbf{T}_K = \left(\mathbf{H}^{[K-11]}\right)^{-1}\mathbf{H}^{[K-1K]}\cdots\left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[12]}\left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}$.

B. Feedback Structure

Although the IA requires perfect and global CSI at each transmitter, the design of feedback topology is not explicitly addressed in [1]-[3]. Existing works in [9],[10] commonly assume that each receiver feeds back the estimated CSI to all transmitters, corresponding to *full-feedback topology*. We consider the full-feedback topology as the conventional feedback method for IA and measure its overhead as

$$N = \sum_{m,k\in\{1,2,\cdots,K\}}\left(N_{\mathsf{TR}}^{[mk]} + N_{\mathsf{RT}}^{[mk]}\right) \tag{5}$$

where $N_{\mathsf{TR}}^{[mk]}$ denotes the number of complex CSI coefficients sent from receiver k to transmitter m and $N_{\mathsf{RT}}^{[mk]}$ from transmitter k to receiver m. In the full-feedback topology, the receiver k feeds back all interfering channels $\left\{\mathbf{H}^{[kj]}|\forall j, j\neq k\right\}$ that consist of $(K-1)M^2$ nonzero coefficients to K transmitters. Since each of K receivers feeds back the same amount of CSI coefficients to all transmitters, total CSI overhead in the K-user interference channel comprises

$$N_{\mathsf{FF}} = K^2(K-1)M^2 \tag{6}$$

where the overhead N_{FF} increases approximately as $O(K^3M^2)$ whereas the network throughput grows linearly with K. Therefore, the CSI overhead becomes potentially a limiting factor of the network throughput.

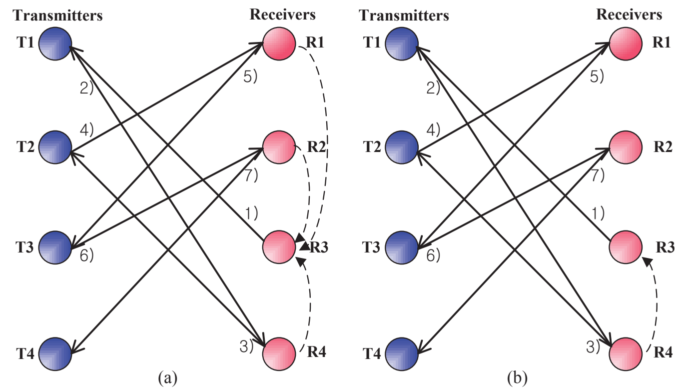

Fig. 1. CSI-exchange topologies for achieving IA with K=4 and M=3.

III. CSI-Exchange Topologies

In this section, we propose the CSI feedback topologies, namely the *CSI-exchange* and *modified CSI-exchange* built upon the IA solution in (4). In the proposed topologies, IA beamformers are computed based on the exchange of pre-determined beamformers between subsets of transmitters and receivers and then CSI overhead is *linearly* scaled with K rather than *cubically* increased in the full-feedback approach. To design the proposed IA feedback topologies, we make the following assumptions: (i) CSI can be exchanged in both directions between a transmitter and receiver through feedforward/feedback channels. (ii) To share CSI between receivers, each receiver is directly connected with others. This connectivity is feasible for the receivers who are linked with local area networks such as Wi-Fi and wireless backhaul networks [11].

A. CSI-Exchange Topology

For simplicity, we consider 4-user interference channel with $M=3$. From (4), the IA beamformers $\mathbf{v}^{[1]}$,$\mathbf{v}^{[2]}$, $\mathbf{v}^{[3]}$ and $\mathbf{v}^{[4]}$ are represented by

$$\begin{aligned}
\mathbf{v}^{[1]} &= \text{any eigenvector of } \mathbf{T}_4\\
\mathbf{v}^{[2]} &= \left(\mathbf{H}^{[42]}\right)^{-1}\mathbf{H}^{[41]}\mathbf{v}^{[1]}\\
\mathbf{v}^{[3]} &= \left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[12]}\mathbf{v}^{[2]}\\
\mathbf{v}^{[4]} &= \left(\mathbf{H}^{[24]}\right)^{-1}\mathbf{H}^{[23]}\mathbf{v}^{[3]}
\end{aligned} \tag{7}$$

where $\mathbf{T}_4 = \left(\mathbf{H}^{[31]}\right)^{-1}\mathbf{H}^{[34]}\left(\mathbf{H}^{[24]}\right)^{-1}\mathbf{H}^{[23]}\left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[12]}\left(\mathbf{H}^{[42]}\right)^{-1}\mathbf{H}^{[41]}$. After computing $\mathbf{v}^{[1]}$, the IA beamformer

Algorithm 1: CSI-exchange topology

1. Computation of $\mathbf{v}^{[1]}$

Receiver k, $\forall k$, forwards the corresponding interference channel $\left(\mathbf{H}^{[k\ \bar{k}+1]}\right)^{-1}\mathbf{H}^{[k\bar{k}]}$ to receiver $K-1$, where $k \neq K-1$ and $\bar{k} = \mathsf{mod}(k+1, K)$.

Then, receiver $K-1$ determines $\mathbf{v}^{[1]}$ as any eigenvector of $\left(\mathbf{H}^{[K-11]}\right)^{-1}\mathbf{H}^{[K-1K]}\cdots\left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[12]}\left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}$ and feeds it back to transmitter 1.

2. Exchange of beamformers $\mathbf{v}^{[1]}, ..., \mathbf{v}^{[K]}$

for k=1:$K-1$ **do**

Transmitter k forwards $\mathbf{v}^{[k]}$ to receiver $\hat{k}$.

Receiver $\hat{k}$ calculates $\mathbf{v}^{[k+1]}$ using (4) and feeds back $\mathbf{v}^{[k+1]}$ to transmitter $k+1$.

$\mathbf{v}^{[k+1]}$ in (7) is sequentially determined by the product of predetermined $\mathbf{v}^{[k]}$ and the estimated interfering channel matrices at receiver $\hat{k}$, where $\hat{k} = \mathsf{mod}(k+(K-2), K)+1$.[2] These properties motivate the design of sequential CSI-exchange topology in Algorithm 1 which only exchanges beamforming vectors between transmitters and receivers after initializing $\mathbf{v}^{[1]}$ at receiver $K-1$.[3] Fig. 1 (a) illustrates CSI-exchange topology for 4 user interference channel and its procedure is $R_3 \overset{\mathbf{v}^{[1]}}{\rightarrow} T_1 \overset{\mathbf{v}^{[1]}}{\rightarrow} R_4 \overset{\mathbf{v}^{[2]}}{\rightarrow} T_2 \overset{\mathbf{v}^{[2]}}{\rightarrow} R_1 \overset{\mathbf{v}^{[3]}}{\rightarrow} T_3 \overset{\mathbf{v}^{[3]}}{\rightarrow} R_2 \overset{\mathbf{v}^{[4]}}{\rightarrow} T_4$, where T_m and R_n represent transmitter m and receiver n, respectively. In Algorithm 1, all receivers transmit CSI of the product channel matrices to receiver $K-1$ which comprises $(K-1)M^2$ nonzero complex-valued coefficients. After computing the beamformer $\mathbf{v}^{[1]}$, each beamformer is determined by iterative exchange of $M \times 1$ complex valued beamformers between transmitters and interfered receivers. Then, total CSI overhead in CSI-exchange topology becomes

$$N_{\mathsf{EX}} = (K-1)M^2 + (2K-1)M. \tag{8}$$

From (8), the proposed topology provides much less sum overhead for achieving K DoF, namely on the order of KM^2.

B. Modified CSI-Exchange Topology ($K \geq 4$)

In the CSI-exchange topology, $\mathbf{v}^{[1]}$ is solved by the eigenvalue problem that incorporates the channel matrices of all interfering links which requires a significant overhead for the case of many links or antennas. To reduce CSI overhead for the computation of $\mathbf{v}^{[1]}$, we propose to align two interferers from transmitter 1 and 2 on the same subspace at receiver $K-1$ and K as following conditions:

$$\begin{aligned} \mathsf{span}(\mathbf{H}^{[K-11]}\mathbf{v}^{[1]}) &= \mathsf{span}(\mathbf{H}^{[K-12]}\mathbf{v}^{[2]}) \\ \mathsf{span}(\mathbf{H}^{[K\,1]}\mathbf{v}^{[1]}) &= \mathsf{span}(\mathbf{H}^{[K\,2]}\mathbf{v}^{[2]}) \end{aligned} \tag{9}$$

[2] $\mathsf{mod}(n,k)$ represents the modulo operation.

[3] We assume that all interfering channels are invariant during the exchange process in Algorithm 1.

Substituting (9) with last two conditions in (2), IA beamformers $\mathbf{v}^{[1]}, \mathbf{v}^{[2]}, ..., \mathbf{v}^{[K]}$ are modified as

$$\begin{aligned} \mathbf{v}^{[1]} &= \text{any eigenvector of } \bar{\mathbf{T}}_K \\ \mathbf{v}^{[2]} &= \left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}\mathbf{v}^{[1]} \\ \mathbf{v}^{[3]} &= \left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[12]}\mathbf{v}^{[2]} \\ &\vdots \\ \mathbf{v}^{[K]} &= \left(\mathbf{H}^{[K-2K]}\right)^{-1}\mathbf{H}^{[K-2\,K-1]}\mathbf{v}^{[K-1]} \end{aligned} \tag{10}$$

where $\bar{\mathbf{T}}_K = \left(\mathbf{H}^{[K-1\,1]}\right)^{-1}\mathbf{H}^{[K-12]}\left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}$. Based on (10), the modified CSI-exchange topology is designed by replacing step 1 in Algorithm 1 with following statement.

1. Computation of $\mathbf{v}^{[1]}$: Receiver K forwards the matrix $\left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}$ to receiver $K-1$. Then, receiver $K-1$ computes $\mathbf{v}^{[1]}$ as any eigenvector of $\left(\mathbf{H}^{[K-11]}\right)^{-1}\mathbf{H}^{[K-12]}\left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}$ and feeds it back to the transmitter 1.

Fig. 1 (b) illustrates the procedure of modified CSI-exchange topology for 4 interference channels that computes $\mathbf{v}^{[1]}$ using received CSI of $\left(\mathbf{H}^{[K2]}\right)^{-1}\mathbf{H}^{[K1]}$. Since the feedback channel for computing $\mathbf{v}^{[1]}$ consists of M^2 complex-valued coefficients in modified CSI-exchange, the corresponding sum feedback overhead is given as follows.

$$N_{\mathsf{MEX}} = M^2 + (2K-1)M. \tag{11}$$

Comparing (11) with (8), both CSI-exchange topologies have the same overhead for the exchange of beamformers. However, the product channel matrices for $\mathbf{v}^{[1]}$ in modified CSI-exchange topology requires constant M^2 overhead in any K user case while that of CSI-exchange topology is linearly scaled with K. In Fig. 2, we compare the CSI overhead of two proposed feedback topologies in (8) and (11) with the full-feedback topology in given number of user, $K = M+1$. The proposed topologies represent a dramatic improvement in the reduction of feedback, while CSI-exchange shows slightly larger overhead than that of modified CSI-exchange.

IV. Analysis on Residual Interference

Proposed feedback topologies are designed under the assumption of perfect CSI exchange. However, due to the limitation of feedback bandwidth, CSI is quantized at receiver and sent back to the corresponding transmitter in practical implementation. In this section, we use RVQ for CSI quantization and analyze the upper-bound of residual interference as the throughput loss in limited feedback channel [12].

Consider the codebook $\mathcal{W}$ known to both transmitters and receivers. Given B_k bits feedback channel, $\mathcal{W}$ consists of 2^{B_k} independently selected random vectors from the isotropic distribution on the M dimensional complex unit sphere, where $\mathcal{W} = \{\hat{\mathbf{v}}_1, ..., \hat{\mathbf{v}}_{2^{B_k}}\}$. The quantized beamformer $\hat{\mathbf{v}}^{[k]}$ is then selected by the minimal chordal distance metric [13]:

$$\hat{\mathbf{v}}^{[k]} = \underset{\hat{\mathbf{v}}_i \in \mathcal{W}}{\arg\min}\, d^2\left(\mathbf{v}^{[k]}, \hat{\mathbf{v}}_i\right) \tag{12}$$

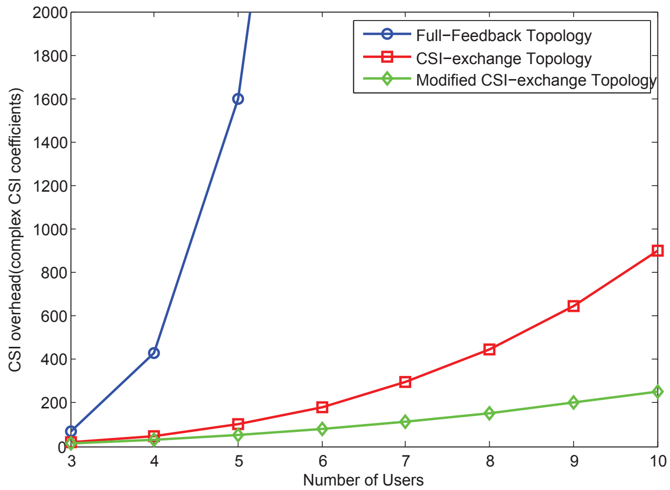

Fig. 2. Comparison of feedback overhead in the proposed topologies.

where $d\left(\mathbf{v}^{[k]}, \hat{\mathbf{v}}_i\right) = \sin\theta_k = \sqrt{1 - \left|\mathbf{v}^{[k]\dagger}\hat{\mathbf{v}}_i\right|^2}$ and θ denotes the principle angle between $\mathbf{v}^{[k]}$ and $\hat{\mathbf{v}}_i$. From [14], [15], we model the quantized beamformer $\hat{\mathbf{v}}^{[k]}$ as

$$\hat{\mathbf{v}}^{[k]} = (\cos\theta_k)\mathbf{v}^{[k]} + (\sin\theta_k)\Delta\mathbf{v}^{[k]}, \tag{13}$$

where $\Delta\mathbf{v}^{[k]}$ represents the quantization error of $\mathbf{v}^{[k]}$ and $E[\sin^2\theta_k] \leq 2^{-\frac{B}{M-1}}$.

A. Residual Interference Relative to Quantization Error

Let denote $\hat{\mathbf{v}}^{[k]}$ and $\hat{\mathbf{r}}^{[k]}$ as the k-th IA beamformer and receive filter calculated in the presence of CSI quantization errors. From the difference between the sum throughput by perfect CSIT-based IA and limited feedback-based IA, the average throughput loss is upper-bounded by [12]

$$\Delta R_{sum} \leq K \cdot \log_2\left(1 + \frac{1}{K} \cdot E\left[\sum_{k=1}^{K} \hat{I}^{[k]}\right]\right) \tag{14}$$

where $\hat{I}^{[k]} = \sum_{k\neq j} P\left|\hat{\mathbf{r}}^{[k]\dagger}\mathbf{H}^{[kj]}\hat{\mathbf{v}}^{[j]}\right|^2$.

In this subsection, we analyze the residual interference $\hat{I}^{[k]}$ in modified CSI-exchange topology that consists of two types of CSI exchange links: (i) Exchange of the interference channel between receiver $K-1$ and K and (ii) Sequential exchange of quantized beamformer between transmitters and receivers through feedforward/feedback links. Under the assumption of perfect CSI exchange in (i), receiver $K-1$ computes $\mathbf{v}^{[1]}$ and feeds back $\mathbf{v}^{[1]}$ to transmitter 1. Subsequently, the following $\mathbf{v}^{[k]}$, $k = 2, \ldots, K$, is sequentially determined by Algorithm 1 and fed back to the corresponding transmitter. However, in limited feedback channel, the feedback/feedforward information is the quantized beamformer $\hat{\mathbf{v}}^{[k]}$, which is modeled as

$$\hat{\mathbf{v}}^{[k]} = \left(\sqrt{1-\sigma_k^2}\right)\mathbf{v}^{[k]} + \sigma_k\Delta\mathbf{v}^{[k]} \tag{15}$$

where $\sigma_k^2 = 2^{-\frac{B_k}{M-1}}$ and $E\left[\left\|\Delta\mathbf{v}^{[k]}\right\|^2\right] = 1$. From (15), we derive the upper-bound of residual interference averaged over all random codebooks $\mathcal{W}$ in Proposition 1.

Proposition 1. *In modified CSI-feedback topology, the upper-bound of expected residual interference at each receiver is represented as*

$$\begin{aligned} &E\left(\hat{I}^{[k]}\right) \leq P\sigma_{k+2}^2\lambda_{\max}^{[kk+2]} \quad k{=}1,\ldots,K-2 \\ &E\left(\hat{I}^{[K-1]}\right) \leq P\left(\sigma_1^2\lambda_{\max}^{[K-11]} + \sigma_2^2\lambda_{\max}^{[K-12]}\right) \\ &E\left(\hat{I}^{[K]}\right) \leq P\left(\sigma_2^2\lambda_{\max}^{[K2]} + \sigma_1^2\lambda_{\max}^{[K1]}\right) \end{aligned} \tag{16}$$

where $\lambda_{\max}^{[ij]}$ is a maximum eigenvalue of $\mathbf{H}^{[ij]}(\mathbf{H}^{[ij]})^H$.

Proof: Note that the residual interference $\hat{I}^{[1]}$ at receiver 1. Since the proposed IA strategy aligns the interference from 2 and 3 in the same subspace at receiver 1, we design the ZF receiver $\hat{\mathbf{r}}^{[1]}$ on the nullspace of $\left[\mathbf{H}^{[12]}\hat{\mathbf{v}}^{[2]}; \mathbf{H}^{[14]}\hat{\mathbf{v}}^{[4]}; \ldots; \mathbf{H}^{[1K]}; \hat{\mathbf{v}}^{[K]}\right]$. Then, the upper-bound of $\hat{I}^{[1]}$ is derived by

$$\begin{aligned} \hat{I}^{[1]} &= P \cdot \left|\sum_{m=2}^{K} \hat{\mathbf{r}}^{[1]\dagger}\mathbf{H}^{[1m]}\hat{\mathbf{v}}^{[m]}\right|^2 \\ &= P \cdot \left|\hat{\mathbf{r}}^{[1]\dagger}\mathbf{H}^{[13]}\hat{\mathbf{v}}^{[3]}\right|^2 \\ &\overset{(a)}{=} P \cdot \left|\alpha \cdot \hat{\mathbf{r}}^{[1]\dagger}\mathbf{H}^{[12]}\hat{\mathbf{v}}^{[2]} + \sigma_3\hat{\mathbf{r}}^{[1]\dagger}\mathbf{H}^{[13]}\Delta\mathbf{v}^{[3]}\right|^2 \\ &= P \cdot \sigma_3^2 \cdot \left|\hat{\mathbf{r}}^{[1]\dagger}\mathbf{H}^{[13]}\Delta\mathbf{v}^{[3]}\right|^2 \\ &\leq P \cdot \sigma_3^2 \cdot \left\|{\mathbf{H}^{[13]}}^H\hat{\mathbf{r}}^{[1]}\right\|^2\left\|\Delta\mathbf{v}^{[3]}\right\|^2 \end{aligned} \tag{17}$$

where (a) follows from $\mathbf{v}^{[3]} \propto {\mathbf{H}^{[13]}}^{-1}\mathbf{H}^{[12]}\hat{\mathbf{v}}^{[2]}$ with a constant value α. The last inequality is obtained by Cauchy-Schwarz inequality, $\left|\mathbf{a}^\dagger\mathbf{b}\right| \leq \|\mathbf{a}\|\|\mathbf{b}\|$, where $\mathbf{a}, \mathbf{b} \in \mathcal{C}^{M\times 1}$. Averaging (17) over $\mathcal{W}$, $E\left(\hat{I}^{[1]}\right)$ is upper-bounded by

$$E\left(\hat{I}^{[1]}\right) \leq P\sigma_3^2\lambda_{\max}^{[13]}. \tag{18}$$

In a same manner, the upper-bound of $E\left(\hat{I}^{[k]}\right)$, $k = 2, \ldots, K$, can also be derived as (16). ■

In Proposition 1, receiver $K-1$ and K are affected by the quantization errors of $\Delta\mathbf{v}^{[1]}$ and $\Delta\mathbf{v}^{[2]}$ since $\mathbf{v}^{[1]}$ and $\mathbf{v}^{[2]}$ are designed based on the IA condition in (9). However, other receiver k sequentially designs $\hat{\mathbf{v}}^{[k+2]}$ based on the pre-determined $\hat{\mathbf{v}}^{[k+1]}$ so that the interference at receiver k is only affected by the quantization error of $\hat{\mathbf{v}}^{[k+2]}$, respectively.

B. Feedback-Bit Allocation Strategy

The throughput loss is characterized by the sum of residual interference in (14) which is detrimental to the system performance in high SNR regime. Therefore, we propose the dynamic feedback-bit allocation strategy that adaptively distributes the number of feedback-bits to each pair of link for minimizing sum residual interference under the constraints of total B_T feedback-bits. To provide an optimal feedback allocation scheme, we formulate the following optimization problem :

$$\min_{\mathbf{B}} \sum_{k=1}^{K} E\left(\hat{I}^{[k]}\right) \tag{19}$$

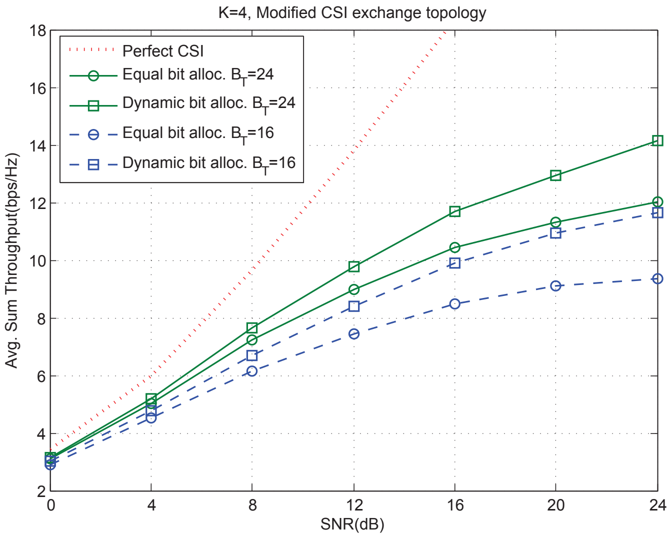

Fig. 3. Average sum throughput of modified CSI-exchange topology in B_T=16 and 24 bits feedback channels.

$$\text{s.t.} \ \sum\nolimits_{k=1}^{K} B_k \le B_T$$

where $\mathbf{B} = \{B_1, ..., B_K\}$ are the non-negative integers. Using the results of Proposition 1, (19) can be transformed to convex optimization problem :

$$\min_{\mathbf{B}} \sum\nolimits_{k=1}^{K} a_k 2^{-\frac{B_k}{M-1}} \tag{20}$$

$$a_k = \begin{cases} a_1 = P\left(\lambda_{max}^{[K1]} + \lambda_{max}^{[K-11]}\right) \\ a_2 = P\left(\lambda_{max}^{[K-12]} + \lambda_{max}^{[K2]}\right) \\ a_k = P\lambda_{max}^{[k-2k]} \ \ k = 3, \ldots, K \end{cases} \tag{21}$$

In order to solve the constrained optimization problem in (20), we formulate the Lagrangian and take derivative with respect to B_k. Then, we have

$$L = \sum_{k \in \mathrm{U}} a_k 2^{-\frac{B_k}{M-1}} + \nu \left(\sum_{k \in \mathrm{U}} B_k - B_T \right) \tag{22}$$

and

$$\frac{\partial L}{\partial B_k} = -2^{-\frac{B_k}{M-1}} \ln 2 \frac{a_k}{M-1} + \nu = 0, \tag{23}$$

where ν is the Lagrange multiplier and U is the set of feedback links $\mathrm{U} = \{1, ..., K\}$. From (23), we obtain B_k as

$$B_k = (M-1) \cdot \log_2 \left(\frac{\mu a_k}{M-1} \right) \tag{24}$$

under the following constraint

$$\sum_{k \in \mathrm{U}} (M-1) \cdot \log_2 \left(\frac{\mu a_k}{M-1} \right) = B_T, \tag{25}$$

where $\mu = \frac{\ln 2}{\upsilon}$. Combining (24) and (25) with $B_k \ge 0$, the number of optimal feedback-bit B_k^* that minimizes the sum residual interference is obtained as

$$B_k^* = \frac{1}{|\mathrm{U}|} \left(\gamma - (M-1) \cdot |\mathrm{U}| \cdot \log_2 \left(\frac{M-1}{a_k} \right) \right)^+ \tag{26}$$

where $|\mathrm{U}|$ denotes the cardinality of U and $\gamma = B_T + \sum_{k \in \mathrm{U}} (M-1) \cdot \log_2 \left(\frac{M-1}{a_k} \right)$. The solution of (26) is found by the waterfilling algorithm in [16]. Since the final decision of feedback-bits should become integer, each of optimal feedback-bits $\{B_k^* : k \in \mathrm{U}\}$ is truncated to $\lfloor B_k^* \rfloor$, where $\lfloor x \rfloor$ is the largest integer not greater than x. Fig. 3 shows the average sum throughput of 4-user 3×3 MIMO IA channel in limited feedback channel, which is constructed by modified CSI-exchange topology with the dynamic feedback-bits algorithm in (26). Comparing the curves for equal feedback-bit allocation in $B_T = 16$ and 24, the proposed feedback-bits allocation scheme shows superior performance than equal feedback-bits allocation, especially in high SNR regime.

V. Conclusion

The efficient feedback topologies for IA have been proposed in K-user MIMO interference channel, which provide the dramatic reduction of feedback overhead compared with conventional feedback framework. In the context of limited feedback channel, we analyze the residual interference that affects the significant throughput loss in high SNR and provide the dynamic feedback-bits allocation scheme which effectively regulates sum of residual interference in high SNR.

References

[1] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom for the K user interference channel," *IEEE Trans. on Inform. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[2] T. Gou and S. A. Jafar, "Degrees of freedom of the K user MxN MIMO interference channel," *IEEE Trans. on Inform. Theory*, vol. 56, no. 12, pp. 6040-6057, Dec. 2010.

[3] C. M. Yetis, T. Gou, S. A. Jafar and A. H. Kayran, "Feasibility conditions for Interference Alignment," *in Proc., of IEEE GLOBECOM*, Nov. 2009.

[4] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," *in Proc., IEEE GLOBECOM*, Dec. 2008.

[5] C. Suh and D. Tse, "Interference alignment for cellular networks," *in Proc., Allerton Conf. on Comm., Cont., and Comp.*, 2008.

[6] C. Suh, M. Ho and D. Tse, "Downlink Interference Alignment," *IEEE Trans. on Commun.*, vol. 59, no. 9, pp. 2616-2626, Sept. 2011.

[7] R. Tresch, M. Guillaud and E. Riegler, "On the achievability of interference alignment in the K-User constant MIMO interference channel," *in Proc. of IEEE Workshop on Statistical Signal Process.*, Sep. 2009.

[8] C. K. Au-Yeung and D. J. Love, "On the Performance of Random Vector Quantization Limited Feedback Beamforming in a MISO System," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 458-462, Feb. 2007.

[9] J. Thukral and H. Bolcskei, "Interference alignment with limited feedback," *in Proc., IEEE Int. Symp. Inform. Theory*, June. 2009.

[10] R. T. Krishnamachari and M. K. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," arXiv:0911.5509, Nov. 2009.

[11] W. Shin, N. Lee, J. Lim, C. Shin and K. Jang, "Interference Alignment Through User Cooperation for Two-cell MIMO Interfering Broadcast Channels," *in Proc. of IEEE GLOBECOM*, Dec. 2010.

[12] O. E. Ayach and R. W. Heath, Jr., "Interference Alignment with Analog Channel State Feedback ," arXiv:1010.2787, Oct. 2010.

[13] D. J. Love, R. W. Heath, Jr. and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. on Inform. Theory*, vol. 49, no. 10, pp. 2735-2747, Oct. 2003.

[14] T. Yoo, N. Jindal and A. Goldsmith, "Multi-antenna downlink channels with Limited feedback and user selection," *IEEE Journal on Sel. Areas in Commun.*, vol. 25, no. 7, pp. 1478-1491, Sep. 2007.

[15] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. on Inform. Theory*, vol. 52, no. 11, pp. 5045–5060, Nov. 2006.

[16] A. Paulraj, R. Nabar, and D. Gore, *Introduction to space-time wireless communications*, Cambridge University Press, Cambridge, 2003.