Adaptive Bit Allocation in Rateless Coded MISO Downlink System with Limited Feedback

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***Abstract*— Rateless coding is a new type of feed-forward incremental redundancy channel coding technique which can be incorporated with MIMO technology to exploit both the diversity and coding gain with possibly reduced channel feedback. In this paper, the benefits of limited feedback beamforming and rateless coding are investigated jointly in a multiple-input single-output (MISO) downlink system. Based on weight enumerator analysis and with the goal of improving the effective channel gain, we propose an adaptive bit allocation scheme by taking advantage of the inherent relationship between the feedback codebook size and the number of transmitted coded bits. Given the feedback codebook size, we derive the required minimum number of coded bits for a reliable data recovery. In addition, for the service with time delay constraint, the required feedback codebook size is also determined. Finally, numerical results are presented to validate our theoretical analysis.**

I. Introduction

Transmit beamforming in multiple-input multiple-output (MIMO) systems is an efficient technique to achieve full spatial diversity to enhance link reliability, at the expense of requiring channel state information (CSI) at the transmitter [1]-[3]. Since a priori CSI is not always available at the transmitter in practice, beamforming with limited feedback is often brought out [4]-[5]. In such cases, the receiver usually uses its channel estimation to select the optimal beam from a quantization codebook and conveys the index of the chosen beam over a feedback channel with limited rate, and the transmitter employs something like precoding based on the feedback information so as to adapt predetermined space-time signal to the instantaneous CSI to achieve better system performance [6]-[8].

Now question arises: how to maintain the reliable transmission in MIMO systems with reduced feedback or even non-feedback while keeping its adaptivity to the channel variation? This question can be partly answered by proper space-time code and precode design as done massively in literatures, or by incorporating some adaptive channel coding and modulation schemes into the MIMO signaling, etc. However, the requests for retransmission in case of transmission failures, in addition to the quantized CSI itself, as hidden in the above approaches, still need considerable and well-designed feedback specific for the coding or precoding schemes employed.

Rateless codes [9], as a special type of feed-forward incremental redundancy channel codes, provide us a new insight on tackling the above challenging problem. A rateless coder generates potentially infinite coded bits from the information data and transmits them to the receiver in a continuous manner. The receiver is able to recover the information data as long as it collects a sufficient number of correctly received coded bits. Even a certain coded bit is not received correctly, there is no need to send any feedback information for initiating a data retransmission. In addition, the number of transmitted coded bits is not fixed but determined adaptively according to the channel variations. This property of rate-adaption turns out to be very effective in MIMO communications with limited feedback beamforming. Intuitively, in a system whose feedback bandwidth is strictly restricted, we can transmit more coded bits for the sake of reducing feedback amount. On the contrary, in a system with a strict transmit delay constraint, such as audio and video transmission system, the transmitted coded bits can be saved by enlarging the feedback codebook size to convey more accurate CSI.

Inspired by the above observations, in this paper, we investigate the inherent relationship between the feedback codebook size and the number of rateless coded bits in a MISO downlink. We propose an adaptive bit allocation scheme according to different requirements of Quality of Service (QoS) guarantee. Based on the relationship between the two important factors, the impacts of other system parameters, such as transmit delay and transmit signal to noise ratio (SNR) are also discussed.

The remainder of this paper is organized as follows. In Section II, we give a brief introduction of our considered MISO downlink and the corresponding transmission protocol employing rateless codes. By applying weight enumerator analysis and the property of effective channel gain, we derive the intrinsic relationship between the Granssmann codebook size and the number of transmitted coded bits in Section III. We present several numerical results in Section IV and conclude our paper in Section V.

II. System Model

We consider the data transmission in a MISO downlink, including a base station (BS) equipped with N_t antennas and a

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single-antenna user. The information data of the user is divided into several blocks, each of which consists of k bits. The data transmission is conducted block by block. More specifically, the BS encodes k information bits into a potentially infinite number of coded bits through rateless coding, and then transmits one coded bit through N_t antennas to the user in each transmission time slot whose duration is T. Once the user collects sufficient coded bits to successfully recover the current block by means of Maximum Likelihood (ML) decoding, an acknowledgement (ACK) message is conveyed to inform the BS to move to the transmission of next block.

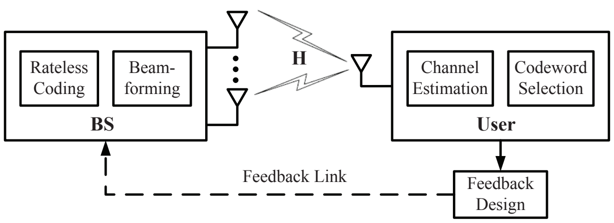

Fig. 1. System model

The system model is shown in Fig.1. Based on the assumption that all the channels from the BS to the user are independent and identically distributed (i.i.d) and quasi-stationary, we denote the channel coefficients by an $1 \times N_t$ vector $\mathbf{h} = (h_1, h_2, ..., h_{N_t})$, in which the element h_i $(i = 1, 2, ..., N_t)$ is a complex Gaussian variable with mean 0 and variance 1, and keeps constant in each transmission time slot. For the convenience of notation, we denote $\widetilde{\mathbf{h}} = \mathbf{h}/||\mathbf{h}||$ as the unit-norm direction of $\mathbf{h}$. A pre-designed Grassmann codebook $\mathcal{W}$ is stored at both the BS and the user, whose size is M bits. Each codeword $\mathbf{w}_j \in \mathcal{W}$ $(j = 1, 2, ..., 2^M)$ is a unit-norm $N_t \times 1$ vector. At the beginning of each transmission time slot, the user employs its channel estimation to select an codeword with the smallest distance to $\widetilde{\mathbf{h}}^H$ as the optimal beam. The distance is defined as [6]:

$$\delta = \sqrt{1 - |\tilde{\mathbf{h}}\mathbf{w}_j|^2} \;\; (j = 1, 2, .., 2^M) \tag{1}$$

Therefore, the beam selection criterion can be written as:

$$\mathbf{w} = \arg\min_{\mathbf{w}_j \in \mathcal{W}} \sqrt{1 - |\tilde{\mathbf{h}}\mathbf{w}_j|^2} = \arg\max_{\mathbf{w}_j \in \mathcal{W}} |\tilde{\mathbf{h}}\mathbf{w}_j|^2 \tag{2}$$

Then, the index of this optimal beam $\mathbf{w}$ is fed back to the BS. The BS multiplies the modulated coded symbol x with this optimal beam, and transmits it to the user. The received signal y by the user thus can be expressed as:

$$y = \sqrt{P}\mathbf{h}\mathbf{w}x + n \tag{3}$$

where P is the transmit power and n is the Gaussian noise with mean 0 and variance σ_n^2.

III. Performance Analysis

In this section, we propose our adaptive bit allocation scheme. First, we employ a weight enumerator analysis to derive the condition for a reliable decoding in our MISO downlink system. Then, based on the concept and property of *effective channel gain*, we study the relationship between the Grassmann codebook size and the average number of coded bits of our adopted rateless codes. Meanwhile, the impacts of transmit delay and transmit SNR are also analyzed.

A. The condition for a reliable decoding

We employ a Raptor code [10] as the rateless coding manner in our scheme. Concretely, an $(n; k)$ regular binary LDPC code is firstly adopted as the outer part, through which k information bits are encoded into n precode bits. Then, an LT code [11] is employed as the inner part to generate a potentially infinite sequence of coded bits. Finally, the coded bits are modulated through binary phase shift keying (BPSK) scheme, and transmitted by the multiple antennas. According to the properties of rateless codes, as long as the user collects an adequate number of coded bits, the information data could be correctly recovered. Thus the number of coded bits represents the effectiveness of our rateless coding manner, and how to achieve a reliable decoding becomes the key point of our bit allocation problem.

[12] introduced the weight enumerator analysis of LDPC codes and Raptor codes. Applying the analogical methods, we could investigate the weight enumerator function in our MISO downlink system in order to establish the condition for a reliable decoding. As we know, the Raptor codeword is generated from the precode LDPC codeword through an LT encoding procedure, namely, each coded bit of the Raptor codeword is obtained by operating exclusive-or of the randomly chosen precode bits from the LDPC codeword. Obviously, if and only if the number of 1's in the chosen pre-code bits is odd, the value of coded bit equals "1", otherwise, the value of coded bit equals "0". Therefore, when the weight of LDPC codeword is u (the "weight" means the number of 1's in the codeword), the probability of a coded bit in the Raptor codeword to be "1" can be computed as [13]:

$$p(u) = \frac{1}{2} - \frac{1}{2}\sum_d \Omega_d (1 - 2u/n)^d \tag{4}$$

where $\Omega(d)$ is the degree distribution of the employed LT code.

We denote the LDPC code ensemble as $[C](n)$, and the average number of codewords with weight u over such ensemble as $\overline{A}_u$. In addition, when the length of Raptor codeword achieves N, we denote the Raptor code ensemble as $[R](N)$, and the average number of codewords with weight v over such ensemble as $\overline{B}_v$. Obviously, given $\overline{A}_u$, $\overline{B}_v$ can be computed as:

$$\overline{B}_v = \sum_{u=1}^{n} \bar{A}_u \cdot \left(\begin{array}{c} N \\ v \end{array} \right) p(u)^v (1 - p(u))^{N-v} \tag{5}$$

We define $\Phi = |\mathbf{h}\mathbf{w}|^2$ as the ***effective channel gain***. Since BPSK modulation is adopted, the expression of the received signal given in (3) is confined in the field of real numbers. Hence we obtain: $\Phi = |\mathbf{h}\mathbf{w}|^2 = (\mathbf{h}\mathbf{w})^2$. We denote the input-output transition probabilities of the channel as $W(y|x = +1)$

and $W(y|x=-1)$, so the *Bhattacharyya noise parameter* γ can be written as:

$$\begin{aligned}\gamma &= \int_{-\infty}^{+\infty} \sqrt{W(y|x=+1)W(y|x=-1)}dy \\ &= \frac{1}{\sqrt{2\pi\sigma_n^2}} \int_{-\infty}^{+\infty} e^{-\frac{y^2+P(\mathbf{hw})^2}{2\sigma_n^2}} dy \\ &= e^{-\frac{P(\mathbf{hw})^2}{2\sigma_n^2}} \\ &= e^{-\frac{P\Phi}{2\sigma_n^2}} \end{aligned} \tag{6}$$

Based on (5) and (6), the average word error rate (WER) of the data transmission with ML decoding in our MISO system can be restricted by the well known *union-bound*:

$$\begin{aligned}\overline{P}_e &\leq \sum_{v=1}^{N} \bar{B}_v \gamma^v \\ &= \sum_{v=1}^{N}\sum_{u=1}^{n} \bar{A}_u \cdot \begin{pmatrix} N \\ v \end{pmatrix} p(u)^v (1-p(u))^{N-v} e^{-\frac{P\Phi v}{2\sigma_n^2}} \\ &= \sum_{u=1}^{n} \bar{A}_u \cdot [1-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)]^N \end{aligned} \tag{7}$$

According to the theorem proposed in [12], it is obtained that:

$$\sum_{u=1}^{u_0} \bar{A}_u \cdot [1-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)]^N \leq \sum_{u=1}^{u_0} \bar{A}_u = O(n^{-\frac{1}{2}}) \tag{8}$$

where u_0 $(1 \leq u_0 \leq n)$ is a sufficiently small integer. In addition, on account of the fact that $1-x \leq e^{-x}$, we have:

$$\begin{aligned} &\sum_{u=u_0+1}^{n} \bar{A}_u \cdot [1-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)]^N \\ \leq &\sum_{u=u_0+1}^{n} \bar{A}_u \cdot e^{-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)N} \\ \leq &\sum_{u=u_0+1}^{n} e^{c_0 u-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)N} \end{aligned} \tag{9}$$

where $c_0 = \max\limits_{u_0<u\leq n} \frac{\log \bar{A}_u}{u}$ is called the *noise threshold*, which can be computed numerically by the analytical method introduced in [14].

Substituting (8) and (9) into (7), it is obtained that:

$$\overline{P}_e \leq O(n^{-1/2}) + \sum_{u=u_0+1}^{n} e^{c_0 u-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)N} \tag{10}$$

When the code length n of the pre-coded is large, $O(n^{-\frac{1}{2}})$ can be omitted. Suppose the average WER is constrained by: $\overline{P}_e \leq \zeta$, it is obtained that:

$$\sum_{u=u_0+1}^{n} e^{c_0 u-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)N} \leq \zeta \tag{11}$$

The condition depicted in (11) can be relaxed as:

$$e^{c_0 u-(1-e^{-\frac{P\Phi}{2\sigma_n^2}})p(u)N} \leq \zeta/n \quad \text{for } \forall u, u_0 < u \leq n$$

Therefore, we have:

$$(1-e^{-\frac{P\Phi}{2\sigma_n^2}})N \geq C_1 \tag{12}$$

where $C_1 = \max\limits_{1\leq u\leq n} \{\frac{c_0 u - \ln(\zeta/n)}{p(u)}\}$. Inequation (12) shows the condition for a reliable decoding in our MISO downlink system. Examining this inequation, C_1 is a constant determined by the adopted Raptor code, and the number of coded bits N is closely related to the effective channel gain Φ. Thereby, it is necessary to analyze the property of Φ in depth to derive our bit allocation scheme.

B. Adaptive bit allocation scheme

[15] analyzed the statistic property of effective channel gain. Given the number of transmit antennas N_t and the Grassmann codebook size M, the probability density function (pdf) of Φ is given by:

$$\begin{aligned} f(\phi) &= \frac{2^M}{\Gamma(N_t-1)} \sum_{t=0}^{N_t-2} \begin{pmatrix} N_t-2 \\ t \end{pmatrix} (-1)^t \phi^t \\ &\times \sum_{m=0}^{N_t-2-t} \frac{(N_t-2-t)!}{m!} \left(e^{-\phi}\phi^m - e^{-\frac{\phi}{1-\kappa}} (\frac{\phi}{1-\kappa})^m \right) \end{aligned} \tag{13}$$

where $\kappa = 2^{-\frac{M}{N_t-1}}$, and $\Gamma(.)$ denotes the Gamma function. In this paper, we consider the condition of $N_t = 3$. In this case, $\kappa = 2^{-\frac{M}{2}}$, and the pdf can be simplified as:

$$f(\phi) = 2^M \left(e^{-\phi} - e^{-\frac{\phi}{1-\kappa}} (1 + \frac{\kappa\phi}{1-\kappa}) \right) \tag{14}$$

Based on (14), we could compute the expectation of $(1-e^{-\frac{P\Phi}{2\sigma_n^2}})$. By denoting $C_2 = \frac{P}{2\sigma_n^2}$, we have:

$$\begin{aligned} E[1-e^{-\frac{P\Phi}{2\sigma_n^2}}] &= 1 - \int_0^{+\infty} e^{-C_2\phi} f(\phi) d\phi \\ &= 1 - \frac{1}{(1+C_2)(1+(1-2^{-\frac{M}{2}})C_2)^2} \end{aligned} \tag{15}$$

Therefore, according to (12), the decoding condition with regard to the average number of coded bits $\overline{N}$ can be expressed as:

$$\left(1 - \frac{1}{(1+C_2)(1+(1-2^{-\frac{M}{2}})C_2)^2} \right) \cdot \overline{N} \geq C_1 \tag{16}$$

Examining (16), C_1 and C_2 are constants determined by the adopted Raptor code and the transmit SNR, respectively. We focus on the relationship between M and $\overline{N}$, and present our adaptive bit allocation scheme as follows.

(i) Given the Grassmann codebook size M, we can adaptively achieve the requirement of $\overline{N}$:

$$\begin{aligned} \overline{N} &\geq g_1(M) \\ &= \frac{C_1}{1-1/(\,(1+C_2)(1+(1-2^{-\frac{M}{2}})C_2)^2\,)} \end{aligned} \tag{17}$$

$g_1(M)$ is a monotonic decreasing function with respect to M. In particular, when $M \to +\infty$,

$$g_1(+\infty) = C_1 \frac{(1+C_2)^3}{(1+C_2)^3 - 1}$$

when $M \to 0$,

$$g_1(0) = C_1(1 + \frac{1}{C_2})$$

This is consistent with physical significance. The larger M is, the more accurate information is fed back, thus the less $\overline{N}$ is required. One one hand, according to the property of rateless codes, the user has to collect sufficient coded bits for a successful data recovery. So $\overline{N}$ can not be decreasing unboundedly. When $M \to +\infty$, the BS obtains completely accurate CSI, and $g_1(+\infty)$ is the corresponding least number of coded bits required for the data recovery. On the other hand, when $M \to 0$, no feedback information is conveyed. In this case, the BS knows nothing about the CSI, so that a much larger number of coded bits $g_1(0)$ is required for the data recovery.

(ii) For the service with delay constraint, we can adaptively achieve the requirement of M:

Following the previous analogous work, we model the arrival of information data at the BS as a Poisson process with an average arrival rate λ block/s. Since the user can decode one block of information data after collects $\overline{N}$ coded bits in average, the average service rate can be expressed as $u = \frac{1}{\overline{N}T}$ block/s. Similar to [16], we employ an M/D/1 model to characterize the arrival-service process. In this context, by denoting $\beta = \frac{\lambda}{\mu} = \lambda \overline{N} T$, the average waiting delay for the user can be written as:

$$D = \beta + \frac{\beta^2}{2(1-\beta)}$$

In our system, the delay constraint is set to be $D \leq D_0$. By solving this inequation while considering the fact that the arrival-service process is steady only when $\beta < 1$ ($\lambda < \mu$), we have:

$$\overline{N} \leq g(D_0) = \frac{(D_0 + 1 - \sqrt{D_0^2 + 1})}{\lambda T} \tag{18}$$

Thereby, $g(D_0)$ is the furthest tolerable number of collected coded bits for the user to guarantee a steady arrival-service process under the transmit delay constraint D_0.

It is observed that, given λ and T, a very small D_0 may result in $g(D_0) < g_1(+\infty)$. In this case, even completely accurate CSI is conveyed to the BS, $\overline{N}$ can not achieve the least number of coded bits required for a successful data recovery, so the user can not satisfy such a small delay constraint D_0. On the contrary, if D_0 is sufficiently large, we have $g(D_0) > g_1(0)$. Under this condition, more than $g_1(0)$ coded bits are allowed to be transmitted, so the user can recover the data even without conveying feedback information to the BS ($M = 0$). If $g_1(+\infty) \leq g(D_0) \leq g_1(0)$, based on (16), the number of feedback bits M has to satisfy the following inequation:

$$\begin{aligned} M &\geq g_2(D_0) \\ &= \left\lceil -2\log_2(1 - \frac{1}{C_2}(\sqrt{\frac{1}{(1 - \frac{C_1}{g(D_0)})(1+C_2)}} - 1)) \right\rceil \end{aligned} \tag{19}$$

where $\lceil . \rceil$ denotes the ceiling function. $g_2(D_0)$ is a monotonic decreasing function with respect to D_0. When the delay constraint D_0 is looser, namely, it is allowed to use more coded bits for the data transmission, the number of required feedback bits decreases.

Remark: The above two cases are both confined with limited transmit SNR. Here we discuss the special case when the transmit SNR goes to infinity. Under this condition, $C_2 = \frac{P}{2\sigma_n^2} \to +\infty$, the inequation given in (16) can be reckoned as: $\overline{N} \geq C_1$. Therefore, it is found that the information data could be successfully recovered as long as the average number of coded bits achieves a constant C_1, without regard for the number of feedback bits M. The reason is evident. In an extremely strong communication environment, the feedback information for CSI is unnecessary. Once the user collects a sufficient number of coded bits, the information data can be recovered.

IV. Simulation Results

In this section, we present several simulation results to examine the validation of our adaptive bit allocation scheme. For all scenarios, we set $N_t = 3$, $T = 0.01s$, and the average WER constraint $\zeta = 0.01$. The transmit power P is normalized to be 1, so the transmit SNR can be computed by: SNR$= 10\lg\frac{1}{\sigma_n^2}$. The parameters of the adopted Raptor code are set as follows: for the LDPC part, the code rate $R_{LDPC} = 0.7$, $n = 100000$; for the LT part, we choose the widely used degree distribution given in [10]:

$$\begin{aligned} \Omega(x) = &\ 0.006495x + 0.495044x^2 + 0.16801x^3 + 0.0679x^4 \\ &+ 0.089209x^5 + 0.041731x^8 + 0.050162x^9 \\ &+ 0.038837x^{19} + 0.015537x^{20} + 0.016298x^{66} \\ &+ 0.010777x^{67} \end{aligned}$$

Table I addresses the requirement of the Grassmann codebook size M according to the transmit delay constraint D_0 with different transmit SNRs, when the average arrival rate $\lambda = 0.0002$(block/s). It is seen that, given SNR, M decreases as D_0 increases. The reason lies in that a larger D_0 allows the user to collect more coded bits for the data recovery. Consequently, a feedback codebook with a smaller size is required in the system.

Fig.2 demonstrates the system throughput η versus the Grassmann codebook size M. The throughput is defined as the ratio of the number of recovered information bits to the corresponding number of transmission time slots. Clearly, given the transmit SNR, η increases as M increases. The reason is that

TABLE I

THE REQUIREMENT OF M ACCORDING TO DELAY CONSTRAINT D_0

	D_0	3.5	4.0	4.5	5.0	5.5	6.0
SNR $= 2dB$	M	11	7	5	5	4	4
SNR $= 2.5dB$	M	5	4	4	3	3	2
SNR $= 3dB$	M	4	3	3	3	2	2

when the number of feedback bits increases, more accurate information of the channel realization is conveyed to the BS, so that less coded bits are required to be transmitted to the user for a successful data recovery. It is noticed that, when SNR is large, such as 8 dB, η varies slightly as we further increase the value of M. It is because that when the communication environment is sufficiently strong, the feedback information counts a little in such a case, which confirms our claims in section III.

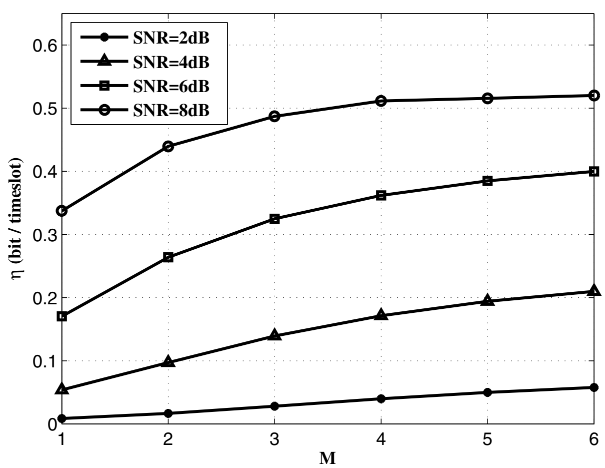

Fig. 2. Throughput versus M

V. CONCLUSION

In this paper, we have revealed the inherent relationship between the feedback codebook size and the number of transmitted coded bits in a MISO downlink system employing rateless codes. On one hand, multi-antenna beamforming is applied to improve transmission efficiency. On the other hand, rateless codes are utilized with the purpose of reducing the feedback amount during the data transmission. Based on weight enumerator analysis and the property of effective channel gain, we have proposed an adaptive bit allocation scheme which can achieve different requirements according to the constraints of the system. Simulation results have verified the validation of our proposed scheme.

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