Gaussian Pulse Shape Optimization of BFDM in Time-Frequency Dispersive Channels

Daisuke Goto
Department of Electrical Engineering
and Computer Science
Nagoya University
Nagoya, 464-8603,JAPAN
Email: dgoto@katayama.nuee.nagoya-u.ac.jp

Takaya Yamazato
Department of Electrical Engineering
and Computer Science
Nagoya University
Nagoya, 464-8603, JAPAN
Email: yamazato@ieee.org

Bayarpurev Mongol
National University of Mongolia
Ulan Bator, MONGOL
Email:bayarpurev@num.edu.mn

***Abstract*—The objective of this research is to reduce intersymbol interference (ISI) and intercarrier interference (ICI) in a highly mobile environment by optimizing the performance of Biorthogonal Frequency Division Multiplexing (BFDM). In time-frequency dispersive channels, reducing the signal interference depends on the shaping of the signal pulse. BFDM was invented to reduce the interference through a flexible design of the signal pulse shaping. However, the shape of the Gaussian pulses used in BFDM is not always optimized in every situation and as a result the interference is not always minimized. This research seeks to optimize the shape of the Gaussian pulse. In particular, we optimize the Gaussian pulse in BFDM by maximizing the ratio of the ambiguity function of the transmitter (Gaussian) and its receiver pulses. Numerical results obtained show that the proposed method is effective for the optimization of BFDM. Furthermore, we show that the optimized BFDM outperforms the conventional OFDM in time-frequency dispersive channels.**

I. Introduction

In wireless communications, orthogonal frequency division multiplexing (OFDM) is a well-known modulation scheme with high data rate transmissions. However, as far as time-frequency dispersive channels in highly mobile environments are concerned, OFDM suffers from both intersymbol interference (ISI) by multipath delays and intercarrier interference (ICI) by Doppler effects. OFDM can overcome ISI by using a cyclic prefix (CP) [1], but OFDM cannot overcome ICI.

To overcome both ISI and ICI, *pulse-shaping* OFDM and *biorthogonal frequency division multiplexing* (BFDM) have been proposed [2], [3], [4]. Pulse-shaping OFDM and BFDM employ time-frequency well-localized pulses to reduce ISI/ICI. In particular, BFDM can employ the Gaussian transmitter pulse, which has the best time-frequency localization[5].And [5] shows the two pulse shaping methods with the biorthogonal condition. A particularly noteworthy feature of BFDM is that it requires no guard interval.

In general, the shape of a Gaussian pulse varies according to its predetermined shaping parameter. If the parameter is small, the shape of the Gaussian pulse spreads in the time domain, which may cause it to suffer from ISI. However, we can design BFDM so that it can suppress ICI because the spectrum of the pulses becomes narrow. Therefore,it is important to adjust the shaping parameter of the transmitter Gaussian pulse to minimize both ISI and ICI.

In this paper, we discuss the optimization Gaussian pulse shape of BFDM in time-frequency dispersive channels and propose an optimization procedure. Because of the biorthogonality condition of both the transmitter Gaussian pulse and the receiver pulse, we use the ambiguity function for the Gaussian pulse shape optimization in the time-frequency domains. Our main objective is to suppress ISI/ICI by setting the appropriate shaping parameter of the transmitter Gaussian pulse of BFDM.

A straightforward way to suppress ISI/ICI is to use the cost function defined as the reciprocal of the signal to interference ratio (SIR) with respect to the transmitter pulse and the receiver pulse (see [4]). However, it requires perfect knowledge of the channel and its scattering function.

In this paper, we relax the requirement by assuming that we only know the maximum multipath spread and the maximum Doppler frequency of the channel. The scattering function can be estimated by the underspread property[6]. However, the scattering function vary greatly depending on the channel condition. Therefore, we assume that the maximum multipath spread and the maximum Doppler frequency can be known statistically and the scattering function cannot be known. We propose using a ratio of the ambiguity function of the transmitter pulse and the receiver pulse. Using the ratio, we try to maximize the SIR by varying the shaping parameter of the transmitter Gaussian pulse of BFDM. As a result, we can design the transmitter Gaussian pulse that matches well with the given time-frequency dispersive channels, and the SIR of BFDM can be maximized.

The paper is organized as follows. In section II, we present the system model and the design procedure of the receiver pulse. Section III shows the time-frequency dispersive channel model. In section IV, we present an ISI/ICI analysis in time-frequency dispersive channels and the proposed Gaussian pulse optimization in BFDM. Section IV provides numerical results to indicate the effectiveness of the pulse optimization.

II. System Model

A. Transmitter

Fig.1 shows a system model of BFDM with K subcarriers, symbol period T and subcarrier frequency spacing F. The baseband BFDM signal can be expressed as

$$s(t) = \sum_{l=-\infty}^{\infty} \sum_{k=0}^{K-1} a_{l,k} g_{l,k}(t) = \sum_{l=-\infty}^{\infty} \sum_{k=0}^{K-1} a_{l,k} g(t-lT) e^{j2\pi kF(t-lT)} \quad (1)$$

where, $a_{l,k}$ is data symbol at symbol time $l \in \mathbb{Z}$ and subcarrier $k \in \{0, ..., K-1\}$l, where $\mathbb{Z}$ denotes the integers. We assumes that data symbol $a_{l,k}$ is i.i.d. with zero mean and has mean power E_s. The time-frequency (TF) shifted version of the transmitter pulse $g(t)$ is given by

$$g_{l,k}(t) = g(t-lT)e^{j2\pi kF(t-lT)}, \quad (2)$$

and we normalize $g(t)$ as

$$\|g\|^2_{\mathbf{L}^2(\mathbb{R})} = \int_{-\infty}^{\infty} |g(t)|^2 dt = 1. \quad (3)$$

For the transmitter pulse, we employ a Gaussian pulse:

$$g(t) = B\exp(-\pi\alpha t^2), \quad (4)$$

where B is a normalization variable. If α is small, the shape of the Gaussian pulse spreads in the time domain. Simultaneously, the spectrum of the subcarrier becomes narrow in the frequency domain. So we can design BFDM that can suppress ICI but suffer from ISI. In contrast, if α is large, Gaussian pulse becomes narrow in the time domain and its spectrum spread in the frequency domain. So BFDM can decrease ISI but increase ICI. Therefore, it is important to set appropriate α that can suppress both ISI and ICI.

B. Receiver

At the receiver, the retrieved data symbol $\hat{a}_{l,k}$ is estimated by computing the inner products of the received signal $r(t)$ with a TF shifted version $\gamma_{l,k}(t) = \gamma(t-lT)e^{j2\pi kF(t-lT)}$ of the receiver pulse $\gamma(t)$:

$$\hat{a}_{l,k} = \langle r, \gamma_{l,k} \rangle = \int_t r(t)\gamma^{\star}_{l,k}(t)dt \quad (5)$$

For the receiver pulse $\gamma(t)$, we employ the pulse that satisfies the biorthogonality condition based on Weyl-Heisenberg frames theory [7]. The biorthogonality condition is given as follows

$$\langle g_{l,k}(t), \gamma(t) \rangle = \delta_l \delta_k \quad (6)$$

For OFDM, (6) reduces to the orthogonality condition: $\langle g_{l,k}(t), g(t) \rangle = \delta_l \delta_k$. In other words, BFDM becomes OFDM system by setting $\gamma(t) = g(t)$.

C. Design of Receiver Pulse

We design the receiver pulse, satisfying biorthogonality condition, by using the Zibulski-Zeevi method [7], [8] based on the Weyl-Heisenberg frames (WHF) [9].

Let us start with the transmitter pulse expressed in discrete form as

$$g_{l,k}[n] = g[n-lN]e^{j2\frac{\pi}{K}k(n-lN)} \quad (7)$$

Here, N is the symbol period and K is the number of subcarriers. Let us consider *discrete Zak transform* (DZT) for the function $\tilde{g}_{l,k}[n] = g[n-lK]e^{j2\frac{\pi}{N}k(n-lK)}$. Note that the order of $\tilde{g}_{l,k}[n]$ is in opposite of $g_{l,k}[n]$. If we consider the pulse having length of KM, then DZT of $\tilde{g}_{l,k}[n]$ can be expressed as

$$Z_{\tilde{g}}[n,m] = \frac{1}{\sqrt{M}} \sum_{l=0}^{M-1} g[n+lK]e^{-j2\pi(m/M)l}$$
$$0 \le n \le K-1, \quad 0 \le m \le M-1. \quad (8)$$

Let $\boldsymbol{S}$ be the frame operator defined as $(\boldsymbol{S}x)[n] = \sum_{l=0}^{M-1}\sum_{k=0}^{K-1}\langle x, \tilde{g}_{l,k}\rangle \tilde{g}_{l,k}[n]$ and $P = N/K$, i.e., the pulse length is MN/P. The eigen values of $\boldsymbol{S}$ can be written in terms of the DZT of $\tilde{g}[n]$ as

$$\lambda_{\tilde{g}}[n,m] = K\sum_{p=0}^{P-1}\left| Z_{\tilde{g}}[n, m - p\frac{MK}{N}]\right|^2 \quad (9)$$

The DZT of $\gamma[n]$ can expressed as

$$Z_{\gamma}[n,m] = \frac{Z_{\tilde{g}}[n,m]}{\lambda_{\tilde{g}}[n,m]}. \quad (10)$$

We obtain $\gamma[n]$ by inverting DZT as

$$\gamma[n] = \frac{1}{\sqrt{M}}\sum_{m=0}^{M-1} Z_{\gamma}[n,m]. \quad (11)$$

III. TIME-FREQUENCY DISPERSIVE CHANNEL

A. Channel Model

In time-frequency dispersive channel, the receive signal is given by

$$r(t) = \int_{-\infty}^{\infty} c(\tau;t)s(t-\tau)d\tau + n(t) \quad (12)$$

Here, $c(\tau;t)$ denotes the time-varying impulse response and $n(t)$ is the additive white Gaussian noise. The channel is assumed to satisfy the *wide-sense stationary uncorrelated scattering* (WSSUS) property and characterized by its time-varying impulse response $c(\tau;t)$ and the transfer function $C(f;t)$ [10]. In this paper, the channel is assumed to be *under-spread*, i.e., their scattering function is effectively supported within a rectangular region $[0, \tau_{\max}] \times [-\nu_{\max}, \nu_{\max}]$ of area $2\tau_{\max}\nu_{\max} \ll 1$.Here, we define the maximum delay $\tau_{\max}$ and the maximum Doppler frequency $\nu_{\max}$,respectively. We define the delay Doppler spread function $h(\tau, \nu)$ [11] by

$$h(\tau,\nu) = \int_{-\infty}^{\infty} c(\tau;t)e^{-j2\pi t\nu}dt. \quad (13)$$

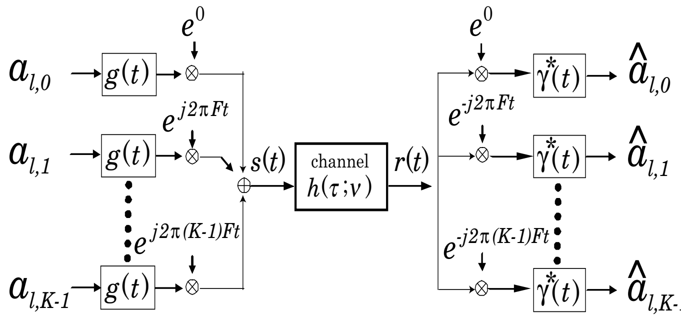

Fig. 1. System model of BFDM system.

Let $S(\tau, \nu)$ be the time-frequency dispersion function, then we obtain the following WSSUS assumption

$$\begin{aligned} E\{h(\tau, \nu)\} &= 0, \\ E\{h(\tau, \nu)h^\star(\tau', \nu')\} &= S(\tau, \nu)\delta(\tau - \tau')\delta(\nu - \nu'). \end{aligned} \quad (14)$$

The example of the scattering function by the channel model of Cost207 [10] is given as follows

$$S(\tau, \nu) = \exp(-\tau \times 10^6) \cdot \frac{1}{\pi\nu_{\max}\sqrt{1 - (\nu/\nu_{\max})^2}}. \quad (15)$$

The scattering function is based on the typical delay profile for suburban and urban areas.

B. Retrieved Data Symbol and Equivalent Channel Parameter

After passing through the time-frequency dispersive channel, the transmit signal loses its biorthogonality, causing ISI/ICI in the receive signal. Therefore, the retrieved data symbol in (6) may be rewritten as:

$$\hat{a}_{l,k} = \sum_{l',k'} a_{l'k'} h_{l,k}^{l',k'} + n' \quad (16)$$

where, n' is a noise component and $h_{l,k}^{l',k'}$ is an equivalent channel parameter and defined as [12]

$$h_{l,k}^{l',k'} = \int\int h(\tau, \nu) g_{l',k'}(t) \gamma_{l,k}(t) d\tau d\nu. \quad (17)$$

The example of $h_{l,k}^{l',k'}$ is shown in Fig.2. In the case of $l \neq l'$ and $k \neq k'$, the equivalent channel parameters represents ISI/ICI caused by the channel. In other words, $h_{l,k}^{l',k'}$ represents the interference coming from $a_{l',k'}$ to $a_{l,k}$.

IV. OPTIMIZATION OF GAUSSIAN PULSE

A. Ambiguity Function

Both the transmitter pulse $g(t)$ and the receiver pulse $\gamma(t)$ must satisfy the biorthogonality condition (6). Since we employ Gaussian pulse and optimize its pulse shape by setting appropriate α, the shape of the corresponding receiver pulse changes according to α. Therefore, we need to consider both transmitter pulse and receiver pulse simultaneously. We use the well-known ambiguity function to design the pulses; $g(t)$ and $\gamma(t)$. The ambiguity function is defined as [13]

$$A_{\gamma,g}(\tau, \nu) = \int_{-\infty}^{\infty} \gamma(t) g^\star(t - \tau) e^{-j2\pi\nu t} dt. \quad (18)$$

If $g(t)$ and $\gamma(t)$ satisfy the biorthogonal condition given by (6), the ambiguity function $A_{\gamma,g}(\tau, \nu)$ is reduced to the relation on the given time-frequency lattice:

$$A_{\gamma,g}(kT, lF) = \delta_k \delta_l. \quad (19)$$

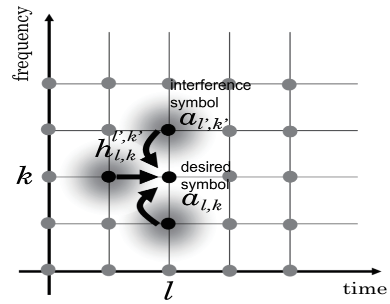

Fig. 2. Channel parameter $h_{l,k}^{l',k'}$ represents interference coming from $a_{l',k'}$ to $a_{l,k}$

B. Representation of Signal to Interference Ratio (SIR) by Ambiguity Function

Using the ambiguity function, the equivalent channel parameter given by (17) may be rewritten as

$$\begin{aligned} h_{l,k}^{l',k'} = \int\int & h(\tau, \nu) A^\star_{\gamma,g}((l' - l)T + \tau, (k' - k)F + \nu)) \\ & e^{-j2\pi k' F\tau} e^{j2\pi lT\nu} e^{j2\pi lFT(k-k')} d\tau d\nu. \end{aligned} \quad (20)$$

The relation between the data symbol $a_{l,k}$ and the retrieved data symbol $\hat{a}_{l,k}$ is given by

$$\hat{a}_{l,k} = a_{l,k} h_{l,k}^{l,k} + \sum_{(l',k') \neq (l,k)} a_{l',k'} h_{l,k}^{l',k'}. \quad (21)$$

The first term of (21) represents the desired data symbol and the second term corresponds the interference coming from the neighboring data symbols. Let σ_S^2 be the average power of the desired data symbol and σ_I^2 be the average power of ISI/ICI. Then the signal to interference ratio (SIR) is defined as

$$SIR = \frac{\sigma_S^2}{\sigma_I^2}. \quad (22)$$

where,

$$\begin{aligned} \sigma_S^2 &= E\{a_{l,k} h_{l,k}^{l,k}\}, && (23) \\ \sigma_I^2 &= E\{|\hat{a}_{l,k} - a_{l,k} h_{l,k}^{l,k}|^2\}. && (24) \end{aligned}$$

Without loss of generality, we set (l, k)=(0,0). Using the ambiguity function, σ_S^2 and σ_I^2 can be respectively described as

$$\sigma_S^2 = E_s \int\int S(\tau, \nu) |A_{\gamma,g}(\tau, \nu)|^2 d\tau d\nu \quad (25)$$

$$\sigma_I^2 = E_s \sum_{(l',k') \neq (0,0)} \int\int S(\tau, \nu) |A_{\gamma,g}(l'T + \tau, k'F + \nu)|^2 d\tau d\nu. \quad (26)$$

Because $S(\tau, \nu)$ is supported within a rectangular region $[0, \tau_{\max}] \times [-\nu_{\max}, \nu_{\max}]$, σ_S^2 takes value in the same region as $S(\tau, \nu)$. Similarly, the σ_I^2 takes value in the region $[l'T, l'T + \tau_{\max}] \times [k'F - \nu_{\max}, k'F + \nu_{\max}]$, except for the case $(l', k') = (0, 0)$.

A straightforward way to maximize SIR (=σ_S^2/σ_I^2) is to minimize the cost function defined as the reciprocal of the SIR with respect to $g(t)$ and $\gamma(t)$. Such maximization using *Joint Nonlinear Optimization* have been proposed in [4]. However, it

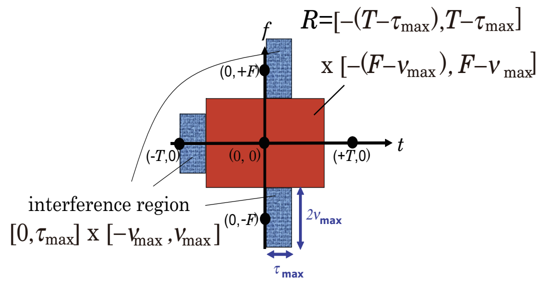

Fig. 3. No interference region $\mathcal{R}$ of desired symbol

requires a perfect knowledge of the channel and its scattering function.

In this paper, we relax the requirement by assuming that we only know the $\tau_{\max}, \nu_{\max}$, the region of the $S(\tau, \nu)$.

C. Optimization of Gaussian pulse

Let us assume that most interference is coming from the adjacent symbols. Fig.3 shows the interference coming from the adjacent symbols $(-1, \pm 1)$ to a desired symbol (0,0). Let us define the region $\mathcal{R}$ ($[-(T-\tau_{\max}), T-\tau_{\max}] \times [-(F-\nu_{\max}), F-\nu_{\max}]$) that represents the no interference area from the adjacent symbols. From the underspread assumption of the channel, the region $\mathcal{R}$ exists because T and F satisfy $T > \tau_{\max}, F > \nu_{\max}$. Instead of the direct maximization of SIR that requires $S(\tau, \nu)$, we consider to concentrate the energy of the ambiguity function within the region $\mathcal{R}$. Let us define the ratio of the ambiguity function power as follows

$$\eta = \frac{\int_{-(T-\tau_{\max})}^{T-\tau_{\max}} \int_{-(F-\tau_{\max})}^{F-\nu_{\max}} |A_{\gamma,g}(\tau, \nu)| d\tau d\nu}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A_{\gamma,g}(\tau, \nu)| d\tau d\nu} \tag{27}$$

If η becomes high, $|A_{\gamma,g}(\tau, \nu)|$ is concentrated in the region $\mathcal{R}$. From (25), this energy concentration contributes to increase of σ_S^2. Furthermore, from (26), the ISI/ICI power becomes low since $|A_{\gamma,g}(\tau, \nu)|$ becomes low in the outside of the region $\mathcal{R}$. As a result, SIR of (22) becomes high.

The optimization of the transmitter Gaussian pulse can be represented as

$$(g(t), \gamma(t))_{\text{opt}} = \arg \max_{\alpha} \eta. \tag{28}$$

V. Numerical Results

We consider the BFDM having a total bandwidth W = 1MHz. Digital mapping is QPSK and no guard interval is used. We consider the transmitted signal with $T = 128\mu$s, $K = 64$, $F = 15.625$kHz, and its pulse length $6T$.

For the time-frequency dispersive channels, we assume WSSUS channels characterized by the scattering function (15). We also assume $\tau_{\max} = 6\mu$s, $\nu_{\max} = 50$Hz$\sim$ 500Hz.

We first show examples of the transmitter Gaussian pulse and its corresponding receiver pulse. Fig.4 shows the transmitter Gaussian pulse with the shaping parameter $\alpha = 5.0 \times 10^{-5}$ and the pulse length $6T$ (left) and its receiver pulse (right). Fig.5 shows their ambiguity function. From the figure, we confirm that the transmitter Gaussian pulse may suppress more ICI than ISI. Similarly, Fig.6 shows pulse pairs with $\alpha = 2.0 \times 10^{-4}$ and the pulse length $6T$. Fig.7 depicts their ambiguity function. For this case, we see that the transmitter Gaussian pulse may suppress more ISI than ICI.

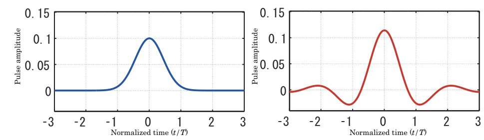

Fig. 4. Gaussian pulse with $\alpha = 5.0 \times 10^{-5}$ (left) and its receiver pulse (right).

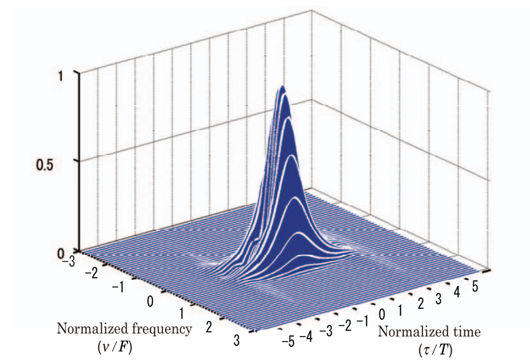

Fig. 5. Ambiguity function ($\alpha = 5.0 \times 10^{-5}$), $\eta = 0.7024$ ($\tau_{\max} = 6\mu$s, $\nu_{\max} = 500$Hz)

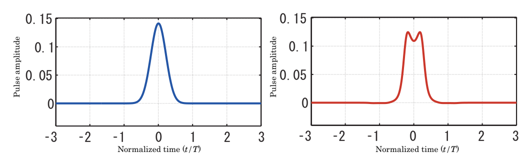

Fig. 6. Gaussian pulse with $\alpha = 2.0 \times 10^{-4}$ (left) and its receiver pulse (right)).

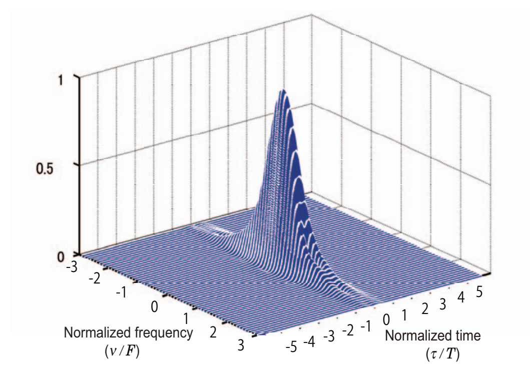

Fig. 7. Ambiguity function ($\alpha = 2.0 \times 10^{-4}$), $\eta = 0.8404$ ($\tau_{\max} = 6\mu$s, $\nu_{\max} = 500$Hz)

If we assume that the maximum delay $\tau_{\max} = 6\mu$s and the maximum Doppler frequency $\nu_{\max} = 500$Hz, we obtain $\eta = 0.7024$ and $\eta = 0.8404$ for the shaping parameters $\alpha = 5.0 \times 10^{-5}$ and $\alpha = 2.0 \times 10^{-4}$, respectively. Because η is higher for $\alpha = 2.0 \times 10^{-4}$ than the other, we observe that the Gaussian pulse with $\alpha = 2.0 \times 10^{-4}$ matches well to the channel.

Fig.8 shows η varying the shaping parameter α from 7.0×10^{-5} to 3.0×10^{-4} for $(\tau_{\max}, \nu_{\max}) = (6\mu$s,50Hz) and (6$\mu$s,500Hz). We confirm that η depends on α and there exists

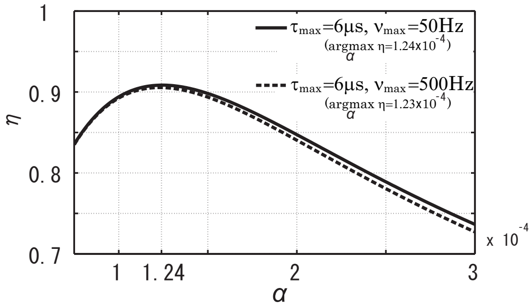

Fig. 8. The ratio η as a function of α ($\tau_{\max}$ = 40μs, $\nu_{\max}$ = 500Hz).

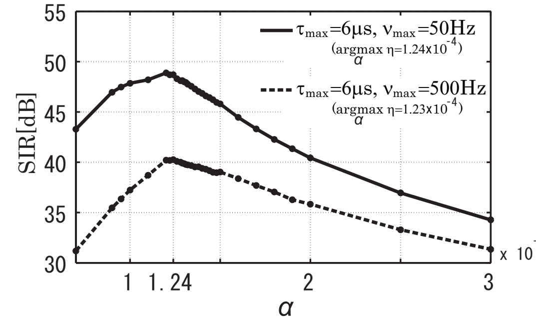

Fig. 9. SIR as a function of σ.

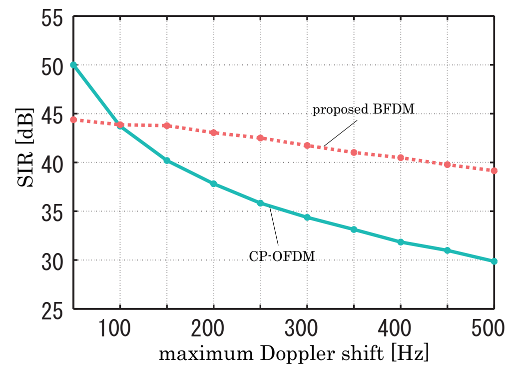

Fig. 10. SIR for BFDM optimized its Gaussian pulse compared with CP-OFDM

α which maximizes η. From Fig.8, the maximum η is obtained with $\alpha = 1.24 \times 10^{-4}$ and $\alpha = 1.23 \times 10^{-4}$ for $\nu_{\max}$ = 50Hz and $\nu_{\max}$ = 500Hz, respectively.

Fig.9 shows the SIR as a function of α. We see that the SIR for the two cases is maximized at $\alpha = 1.20 \times 10^{-4}$ which is almost equal to the value of α which maximizes η. Thus, we can see that the optimization of the transmitter Gaussian pulse of BFDM can be established by selecting a value of α that maximizes η.

Finally, we compare the SIR performance of BFDM and CP-OFDM. Fig.10 shows the SIR as a function of $\nu_{\max}$. For the time-frequency dispersive channels, we consider the same WSSUS channel as described previously but change the $\nu_{\max}$. In detail, we set $\tau_{\max}$ = 6μs and $\nu_{\max}$ varies from 50 to 500Hz. We consider the CP-OFDM designed to achieve ISI-free transmissions by setting the appropriate CP length. We set the symbol period T = 70μs, K = 64, F = 15.625kHz and CP length T_{CP} = 6μs. The system parameters of BFDM are the same as previously described, but in this case we change α in accordance with $\nu_{\max}$ to maximize its SIR.

In $\nu_{\max}$=50-100Hz, the SIR of CP-OFDM is higher than our proposed BFDM. However, in 150-500Hz, our proposed BFDM outperforms CP-OFDM. Furthermore, this superiority is stronger as $\nu_{\max}$ becomes higher. From this result, we confirm that our proposed BFDM is a more efficient system than CP-OFDM in highly mobile environments.

VI. Conclusion

In this paper, we discussed the optimization of the transmitter Gaussian pulse shape of BFDM for time-frequency dispersive channels aiming at the minimization of ISI/ICI. The optimization of the Gaussian pulse shape is performed by maximizing the ratio of the ambiguity function of the transmitter and receiver pulses. This method needs the maximum delay time and the maximum Doppler frequency in the assumed channels. We show that the Gaussian pulse is optimized when the proposed ratio η is maximized.

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