# Performance Analysis and Parameter Optimization of Random Access Backoff Algorithm in LTE

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*Abstract*—**This paper studies the performance of random access backoff algorithm in Long Term Evolution (LTE) system, taking the physical loss into account. Analytical results are developed for system performance metrics including throughout, drop probability and medium access delay. All these analytical results are verified with simulation. Performance of the backoff algorithm under different parameter settings is compared in a typical scenario of LTE. The performance results provide useful insights on optimal backoff parameter settings. For example, under a physical loss probability of 1%, the optimal backoff window size should be set to 1 and the attempt limit set to 2 or 3.**

***Keywords-LTE/LTE-A; random access; backoff; performance analysis; medium access delay; parameter optimization.***

## I. INTRODUCTION

Universal Terrestrial Radio Access Network Long Term Evolution (UTRAN LTE) and its enhancement, LTE Advanced (LTE-A), are advanced radio access standards proposed by the 3rd Generation Partnership Project (3GPP) to provide a smooth migration towards fourth generation (4G) cellular networks. Due to the flat IP-based architecture and continuous IP connectivity demands, LTE/LTE-A has stringent requirements in regards to the performance of fast random access schemes. Specifically, distributed random access schemes for packet networks require algorithms to resolve transmission collisions that occur as multiple uncoordinated users contend for common resources.

A widely used collision resolution protocol is the binary exponential backoff (BEB) algorithm, which is analyzed in [1]. Kwak *et al.* [1] provide analytical results pertaining to the throughput and medium access delay. In [2], Jing *et al.* use theoretical analysis to gauge the performance of another type of backoff algorithm in the context of IEEE 802.11 ad-hoc networks. The authors use a 2-dimensional Markov chain in order to make the analysis of media access delay more concise and clear. In [3], Zhang and Liu analyze the multi-channel slotted Aloha algorithm by making use of recursive expressions in order to determine the probability of packets being successfully transmitted over parallel channels. Combining the ideas proposed in [1] and [3], Seo and Leung in [4] analyze multi-channel slotted Aloha algorithms in the context of LTE and IEEE 802.16 based systems. Further, [4] presents an effective method in which the standard deviation of medium access delay can be analyzed. However, the 2-dimensional Markov chain model used in [4] is not consistent with the LTE standard described in [5]. Also, [4] assumes at least one packet is transmitted in every slot. This assumption becomes inappropriate under low traffic conditions typically supported in LTE [6].

This paper offers two main contributions. First, the paper provides new and accurate analytical results for the backoff algorithm in LTE/LTE-A. The analyzed backoff algorithm is consistent with the LTE standard, and we take into account physical loss and accurately deduce the equation of success probability over parallel channels, which all differ from the work performed in [4]. Furthermore, we achieve new results on the medium access delay and its standard deviation for successful packet transmissions. Second, after verification by simulation results, the proposed analytical solution is used to compare the performance of the backoff algorithm under different parameter settings in a typical scenario of LTE. With this, we can provide some useful insights for the implementation of random access procedure in LTE.

The rest of this paper is organized as follows: Section II introduces the random access channel and backoff algorithm in LTE. Performance of the backoff algorithm in the presence of physical loss is analyzed in Section III. In Section IV, the results of the analysis are verified using simulation, and useful insights on optimal parameter settings are presented. Finally concluding remarks are given in Section V.

## II. RANDOM ACCESS BACKOFF ALGORITHM IN LTE

The random access channel is one of the three uplink channels in LTE. It is used as the initial uplink multiple access to common resources for uncoordinated users, and is also used to restore the synchronization required to perform handover. In order to solve the problem of collision, the performance of random access backoff algorithm has a key influence on the overall system performance. This section describes its system model in LTE through the following aspects: random access channel and backoff procedure.

### A. Random access channel in LTE

Random access blocks can be configured in up to 16 different layouts, as mentioned in [6], assuming a periodic pattern with a slot period of 10 milliseconds (ms) or 20 ms. Depending on the traffic load, one or more random access blocks may be allocated per slot period. Each block occupies 1.08 MHz in the frequency domain and 64 approximately orthogonal preambles are available in each frequency-time block, as shown in Figure 1. Receivers detect these preambles simultaneously at every random access block and these preambles are equivalent to parallel channels. Specified in [7], the minimum probability of correct preamble detection should be equal to or exceed 99%.

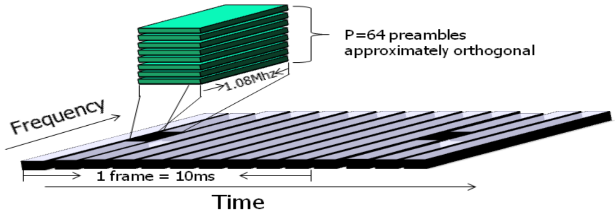



Figure 1. Typical allocation of random access channel

### *B. Backoff Algorithm in Random Access Procedure*

There are contention based and non-contention based random access procedures in LTE [8] and this paper focuses on the former. In order to resolve packet collisions, a fixed window multi-channel slotted backoff algorithm with an attempt limit is adopted in LTE. As shown in Figure 2, the transmission attempt backoff window size $W$, and attempt limit $M$ are broadcasted by each eNodeB. The number of transmission attempts for a packet, denoted by $m$, is increased by one in the slot of every transmission attempt. Before every retransmission, the backoff counter $j$ is selected uniformly in the range [0, $W$-1] at the end of a collision slot and decremented by one at the end of the next slot. If $j=0$, this user transmits its packet again at the start of the next slot.

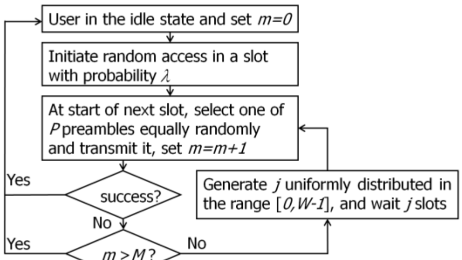



Figure 2. Backoff algorithm in random access procedure

## III. ANALYSIS OF BACKOFF ALGORITHM

Under the assumptions generally used in the literature and taking physical loss into account, this section analyzes the probability of successful transmission over parallel channels via a 2-dimensional Markov chain, and provides a solution for system throughput and drop probability under steady state conditions. After that, we analyze the delay and its standard deviation through total probability and Bayes theorems.

### *A. Assumptions and Notations*

Suppose the system has $P$ parallel preambles in every slot and $N$ users. Each user in the idle state initializes a random access procedure in a slot with probability $\lambda$. In addition to these, the algorithm is analyzed based on the following assumptions, as those made in [1][2][4]:

*1)* Time is divided into equal size slots, where the slot length is equal to the cycle time of random access channel.

*2)* An unsaturated traffic condition is considered. This turns to the saturation condition if the probabilty $\lambda$ becomes equal to 1.

*3)* During each transmission procedure under a fixed value of traffic load, a packet collides with a constant and independent probability with respect to its number of (re)transmissions incurred.

*4)* The system detects collision at the end of the slot that a packet is transmitted in. Analysis can be extended to collision detection delay of $L$ slots with a more complicated formula.

In order to take physical loss into account, the paper further assumes:

*5)* Physical loss of transmission due to, for example, preamble detection failure (but not due to collisions) is constant and independent, regardless of transmission attempts and number of packets simultaneously transmitted. This assumption is feasible because of approximate orthogonality among preambles with very low SNR requirement [7].

Before proceeding, we summarize in Table 1 the notations which will be used in the subsequent sections.

TABLE I. NOTATIONS OF VARIABLES AND PARAMETERS

| Notation | Definition |
|---|---|
| $P$ | Number of orthogonal preambles per slot |
| $N$ | Number of users |
| $\lambda$ | Probability to initiate Random Access for every user in the idle state in every slot |
| $M$ | Attempt limit of (re)transmissions |
| $W$ | Backoff window size |
| $p_f$ | Failure probability for every (re)transmission, including the effects of physical loss and collision. |
| $p_s$ | Success probability for every (re)transmission, $p_s = 1 - p_f$ |
| $p_c$ | Collision probability for every (re)transmission |
| $p_t$ | (Re)transmission probability for every user in every slot |
| $p_{phyloss}$ | The probability of physical layer failure at each attempt *(not due to collisions)*, including preamble detection failure and the failure probability of following steps of random access procedure. |

### *B. Success Probability of Transmissions in Parallel Channels*

In order to analyze the probability of successful transmission, we first follow the ideas of [3][4] in iteratively computing $\gamma(k \mid n,P)$, which is the probability of $k$ successful transmissions among $n$ simultaneous transmissions over $P$ parallel preamble channels. The binomial probability distribution function is denoted by $B(j,n,p) = \binom{n}{j} p^j (1-p)^{n-j}$.

We observe one of $P$ parallel channels and get:

$$\gamma(k \mid n,P) = \sum_{j=0}^{n-k+1} B(j,n,1/P)\gamma(k - I(j) \mid n-j, P-1) \qquad (1)$$

where the indicator function $I(j)$ equals to 1 at $j=1$ and 0, otherwise. From (1), suppose we compute the probability of $k$ successful packet transmission(s) among $n$ packets transmitted over $P$ parallel channels and we take one of $P$ parallel channels to observe. The following conditions can be considered:

*C1) If no packet (j=0) is transmitted on this observed channel, then k successful transmissions among n packets should be transmitted over the other P-1 channels, with probability* $\gamma(k \mid n,P-1)$.

*C2) If only one packet (j=1) is transmitted on the observed channel, it is successful. As a result, the other k-1 successful transmissions among n-1 packets should be transmitted over the other P-1 channels, with probability* $\gamma(k-1|n-1,P-1)$.

*C3) If j>1 packets are transmitted on the observed code channel, then a collision occurs and these j packets are unsuccessful, so k successful transmissions among the other n-j packets should be transmitted over the other P-1 channels with probability* $\gamma(k|n-j,P-1)$.

Equation (1) can be iteratively computed with the following initial conditions:

$$\gamma(0|0,1)=1;\gamma(1|0,1)=0;\gamma(0|1,1)=0;\gamma(1|1,P)=1 \text{ for } P>0;$$
$$\gamma(0|n,1)=1 \text{ for } n>1;\gamma(k|n,1)=0 \text{ for } k>0,n>1;$$
$$\gamma(k|n,P)=0 \text{ for } k<0;\gamma(k|n,P)=0 \text{ for } \min(n,P)<k;$$

We denote the transmission (or retransmission) probability of each user in any slot as $p_t$, take the physical loss into account and the success probability for every transmission attempt can be written as

$$p_s=(1-p_{phyloss})\sum_{n=1}^{M}\left(\sum_{k=1}^{n}k\cdot\gamma(k|n,P)\right)B(n,N,p_t)\Big/(Np_t) \quad (2)$$

### *C. Probability of Transmission, Throughput and Drop Probability*

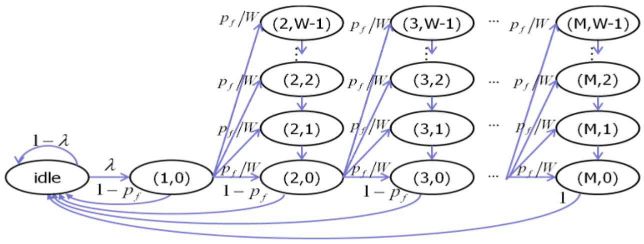



Figure 3. State transition of backoff algorithm of each user

We model the system as a 2-dimensional Markov chain where the observation points are chosen at the end of slots. As shown in Figure 3, the states of each user are expressed by 2-tuples ($m$, $t$), where $m$ is the counter of transmission attempts which increases by one when it begins to backoff and prepares for the current transmission, and $t$ is the counter of backoff slots which decreases by one after the duration of a slot.

Under the assumption that the system is in steady-state, the state probability is computed as follows:

$$\pi_{idle}=(1-p_f)\left[1-p_f+\lambda\left(1+\frac{W-1}{2}p_f-\frac{W+1}{2}p_f^M\right)\right]^{-1} \quad (3)$$

$$\pi_{1,0}=\lambda\pi_{idle} \quad (4)$$

$$\pi_{i,j}=\frac{W-j}{W}p_f^{i-1}\lambda\pi_{idle}, i=2,3,\cdots M, j=0,1,\cdots W-1 \quad (5)$$

The transmission (including retransmission) probability in any slot can be expressed as $p_t=\sum_{i=1}^{M}\pi_{i,0}$ and, substituting (3), (4) and (5) into it, we get

$$p_t=\frac{1-p_f^M}{1-p_f}\lambda\pi_{idle}=\frac{\lambda(1-p_f^M)}{1-p_f+\lambda\left(1+\frac{W-1}{2}p_f-\frac{W+1}{2}p_f^M\right)} \quad (6)$$

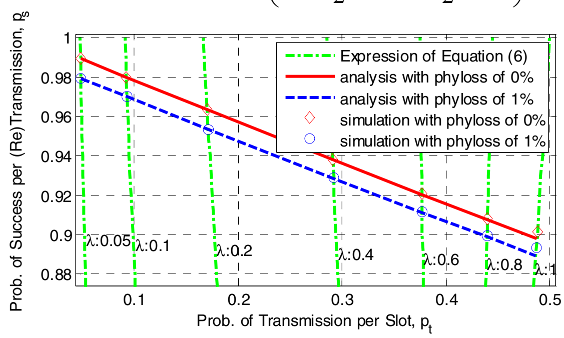



Figure 4. Prob. of transmission attempts per slot & success per (re)transmission w/o and w/ physical loss (#ofusers = 8, #ofpreambles = 32).

Further combining with $p_s$=*1- $p_f$*, (2) and (6) can be solved for $p_t$ and $p_s$. Shown in Figure 4, at each specified value of the parameter $\lambda$, a unique intersection between the two equations gives the values of $p_s$ and $p_t$ by an iterative algorithm.

The system throughput $\tau$, defined as the average number of successful packet transmissions per slot, is then obtained as

$$\tau=p_tp_sN \quad (7)$$

Then the dropped number of transmissions per slot is calculated through $p_{dropperslot}=\pi_{M,0}p_f=p_f^M\lambda\pi_{idle}$ and the drop probability for every transmission procedure is achieved by:

$$p_{drop}=\frac{p_{dropperslot}}{\pi_{1,0}}=p_f^M \quad (8)$$

### *D. Medium Access Delay and its Standard Deviation*

In this section, we first determine the first and second central moments of all transmission delays based on the random access backoff model in LTE, and then further analyze these moments separately for successful and dropped transmissions. The packet medium access delay $D_{i,j}$ is defined as the time for a transmission process starting from the backoff counter value $j$ at the $i$-th backoff stage to reach the idle state and the average delay $\overline{D_{i,j}}=\overline{D_{i,j-1}}+1=\overline{D_{i,0}}+j$ which includes the delay of all packet transmissions. Then we obtain

$$\sum_{j=0}^{W-1}\overline{D_{i,j}}=W\left(\overline{D_{i,0}}+\frac{W-1}{2}\right) \quad (9)$$

Using the definition of expectation we get

$$\overline{D_{i,0}}=(1-p_f)\cdot 1+p_f\left(1+\frac{1}{W}\sum_{j=0}^{W-1}\overline{D_{i+1,j}}\right) \text{ for } i=1,2,\cdots,M-1 \quad (10)$$

Applying the boundary condition of $\overline{D_{M,0}}=1$ and inserting (9) into (10) gives

$$\bar{d}=\overline{D_{1,0}}=\frac{1+\frac{W-1}{2}p_f-\frac{W+1}{2}p_f^M}{1-p_f} \quad (11)$$

and the average delay for successful packet transmissions is

$$\overline{d_{success}} = \frac{\overline{d} - p_{drop}\overline{d_{drop}}}{1 - p_{drop}} \tag{12}$$

where $\overline{d_{drop}} = (M-1)\frac{W+1}{2} + 1$ computed by (11) under the condition that the packet is dropped, that is, $p_f = 1$.

Further we use the boundary condition of $\overline{D_{M,0}{}^2} = 1$ and compute the second moment of delay $\overline{D_{i,0}{}^2}$ from $i = M-1$ to 1:

$$\begin{aligned}\overline{D_{i,0}{}^2} &= (1-p_f)\cdot 1^2 + p_f \frac{1}{W}\sum_{j=0}^{W-1}\overline{(1+D_{i+1,j})^2} \\ &= 1 + p_f\left[\overline{D_{i+1,0}^2} + (W+1)\overline{D_{i+1,0}} + \frac{2W^2+3W-5}{6}\right]\end{aligned} \tag{13}$$

The second moment of delay for all packet transmissions is obtained through $\overline{d^2} = \overline{D_{1,0}^2}$.

In order to observe the second moment of delay for successful packet transmissions, we first compute the second moment of delay for dropped packets

$$\overline{d_{drop}^2} = \overline{D_{1,0|drop}^2} = (M-1)(W+1)\left[\frac{(W+1)(M-2)}{4} + \frac{2W+7}{6}\right] + 1 \tag{14}$$

where $\overline{d_{drop}^2}$ is obtained from (13) under the condition of $p_f = 1$. Then the second moment of delay for successful packets is achieved by the following variation of the law of total probability:

$$\overline{d_{success}^2} = \frac{\overline{d^2} - p_{drop}\overline{d_{drop}^2}}{1 - p_{drop}} \tag{15}$$

Finally, the standard deviation of access delay $std(d) = \sqrt{\overline{d^2} - \left(\overline{d}\right)^2}$ for all packet transmissions and $std(d_{success}) = \sqrt{\overline{d_{success}^2} - \left(\overline{d_{success}}\right)^2}$ for successful transmissions is computed.

## IV. NUMERICAL RESULTS AND PARAMETER OPTIMIZATION

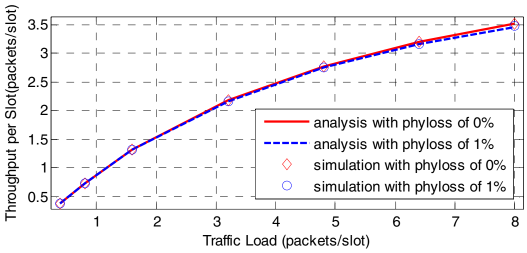
In this section we compare the results between analysis and simulation with or without a physical loss probability of 1% (the minimal requirement in [7], as mentioned in Sector II-A) to validate the analytical solution. Next, we use it to optimize the parameter settings of the backoff algorithm in LTE.

In the comparison, we suppose the system has 8 users and 32 preambles with the backoff parameter setting of window size $W$=4 and attempt limit $M$=4. Through changing the parameter value of the probability $\lambda$, the results at different traffic load are achieved, depicted in Figures 4 and 5. Here the system traffic load in packets/slot is determined approximately as a product of $N$ and $\lambda$. Analytical results, expressed by curves, are computed through solving the equations in Section III; and simulation results, shown by markers, are obtained by executing 14 iterations of Monte Carlo simulation with 400,000 packet transmissions after the warm up period, whose procedure is described in Section II. The figures show that the analytical results are very well consistent with the simulation results which verify the accuracy of the analytical solution. There is one point to be noted: the results of the simulation are slightly better than the analytical results under high traffic load conditions. The dependence between user states under high traffic load accounts for this deviation. This phenomenon can be observed clearly in the system with 2 parallel channels and 2 users with $\lambda = 1$ where each user in the idle state in turn generates packets without any collisions. Fortunately this phenomenon will not impact the following analytical results because RACH in LTE is designed to be used under light traffic load (as mentioned in [6], 0.64packets/slot typically) where $\lambda \ll 1$.

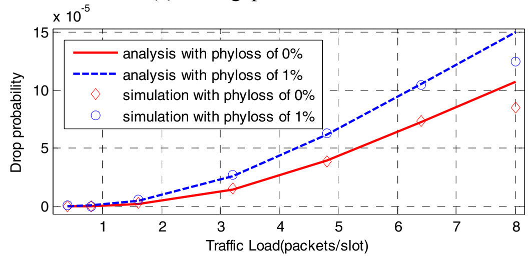
When $\lambda$=1, a user in the idle state in a slot initiates a packet transmission with probability 1 and transmits this packet in the next slot. This means the user takes 2 slots to transmit one packet and then the transmission probability $p_t$ is less than 0.5 under the condition that physical loss probability is less than 50%. There are $N$=8 users and the average number of packets transmitted in every slot is less than 4, a very light load with respect to 32 parallel channels used: random variable $n \geq 1$ is the number of packets transmitted in one instance of the system and we select one of 32 preamble channels where at least one packet is already located. The probability other packets are also located in this preamble channel, i.e., the collision probability, equals $\underset{n\geq 1}{E}\left[\binom{n-1}{1}\frac{1}{32} - \binom{n-1}{2}\left(\frac{1}{32}\right)^2 + \cdots\right] \approx \underset{n\geq 1}{E}\left[\binom{n-1}{1}\frac{1}{32}\right] \approx \frac{Np_t}{32}$ where $\underset{n\geq 1}{E}\left[f(n)\right]$ means the expectation of $f(n)$ with respect to random variable $n$ under the condition $n \geq 1$. Therefore the collision probability increases approximately linearly with $p_t$, shown in Figure 4. In addition to the collision probability, the probability of successful transmission $p_s$ is further reduced by 1% approximately ( $p_s = (1-p_{phyloss})(1-p_c) \approx 1-(p_{phyloss}+p_c)$ ) when additional physical loss probability of 1% is included.

In Figure 5(a), the throughput increases with traffic load, because most of the packets are transmitted successfully at the first time under small collision probability (less than 12% shown in Figure 4). But the collision probability increases with traffic load also, causing the gradient of throughput to decrease. Combining the formulation (8) and the approximate linearity between the total failure probability $p_f$ and traffic load, results in the drop probability increasing in the $M^{th}$ order with traffic load, shown in Figure 5(b). Meanwhile, increasing collision probability results in more packets being retransmitted once, twice, or more, therefore packet transmission delay increases and varies more widely, leading to Figures 5(c) and (d).

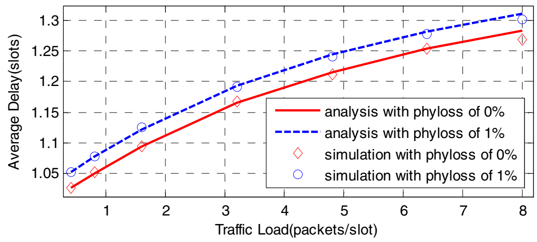
As indicated in Equation (7), the physical loss probability of 1% makes the throughput loss about 1% which is negligible compared to the throughput increase with traffic load in Figure 5(a). But this physical loss influences the failure probability substantially ( $p_f = 1 - p_s \approx p_{phyloss} + p_c$ ), causing even more packets being retransmitted or dropped with large variation in the packet transmission delay, this brings a substantial impact on drop probability and the delay statistics in Figures 5(b)-(d).



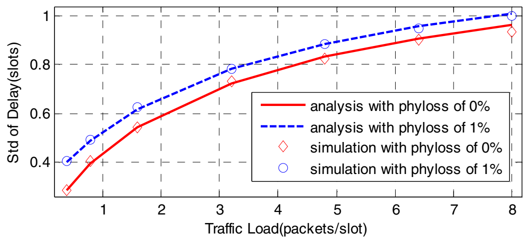

(a) Throughput vs. traffic load




(b) Drop probability vs. traffic load




(c) Medium access delay vs. traffic load




(d) STD of delay vs. traffic load

Figure 5. Comparison between analysis and simulation without and with physical loss probability of 1% (#ofusers = 8, #ofpreambles = 32).

Next we use the verified analytical solution to optimize the backoff parameter settings. We take the worst case into account and suppose the traffic load is 2.56 packets per slot (four times the traffic load expected in LTE [6]) over 64 parallel channels with a physical loss probability of 1%. There are 128 users and the probability of random access by each user in idle state is 0.02. This results in a collision probability of 4.8%. In Figure 6, with an increase in backoff window size, the drop probability and throughput are basically the same but the average delay and its standard deviation increase because of an increase in average delay for each backoff. Therefore the backoff window size $W$ should be set to 1. The reason is that the 64 parallel channels are achieving sufficient probability of successful transmission under the expected traffic load. When the attempt limit is greater than 3, the improvement in performance is very limited. And if the attempt limit is set to 1, the drop probability is not acceptable, given that it is the same as the collision probability.

Without physical loss, or using half of the 64 parallel channels as the resources of the contention based random access channel, the results are still the same as those above. In short, under the typical condition that the collision probability is less than 5% and, under steady state operating condition, the optimal backoff window size should be set to 1 and the attempt limit set to 2 or 3.

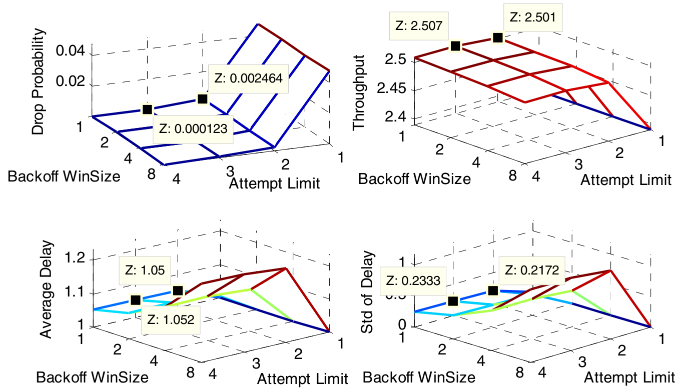



Figure 6. Performances vs. backoff parameters with 1% physical loss probability

## V. CONCLUSIONS

In this paper, we comprehensively analyze the performance of the random access backoff algorithm in LTE in steady state, taking physical loss into account, and further provide its performance under different backoff parameter settings in a typical scenario. This improved analytical scheme provides an effective means to evaluate the performance of multi-channel fixed-window backoff algorithm quickly and the results provide useful insights on LTE backoff parameter settings.

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