# Cramér-Rao Lower Bounds for Hybrid Distance Estimation Schemes

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***Abstract*—This paper presents Cramér-Rao Lower Bounds (CRLBs) for hybrid distance estimation algorithms, i.e., for received signal strength (RSS) and time of arrival (TOA) as well as for RSS and time difference of arrival (TDOA) measurements. The CRLBs are first evaluated with a simple channel model that consists of additive white Gaussian noise (AWGN), path loss (PL), and shadow fading (SF) in contrast to previous works. Several different CRLBs are dereived as the SF can be treated as nuisance parameter. Further, we apply the CRLBs to the WINNER II channel model and to current communication systems. Results show that the hybrid schemes always perform equal or better than RSS, TOA or TDOA only schemes, that the fourth type CRLB is the tightest bound, and that the distance when RSS performs better than TOA or TDOA increases with decreasing bandwidth.**

***Index Terms*—CRLB, TOA, TDOA, RSS, RSSD, GSM, UMTS, LTE, WiMAX, path loss, shadow fading, WINNER II channel**

## I. Introduction

Since the advent of smartphones, location based services (LBS) have become a major source of revenue for cellular network operators, cell-phone vendors, and smartphone application developers. Currently, smartphones employ several technologies that can be used to determine their positions for LBS. The most prominent technology is a Global Positioning Satellite (GPS) navigation receiver [1], which can be assisted through the cellular networks. However, GPS provides reliable position estimates only in good environmental conditions, where mobile stations (MSs) can receive line-of-sight signals from at least four satellites. Especially in urban canyons and indoor environments, positioning with GPS is unreliable.

As a complement to GPS, cellular networks can provide position estimates [1] to MSs, e.g., by measuring TOAs or TDOAs of several base stations (BSs) at the MS. While, received signal strength (RSS) based positioning is quite popular in wireless local area networks (WLANs), it has not been widely used with cellular networks. Recently the 3rd Generation Partnership Project (3GPP) has standardized RSS based positioning in its Long Term Evolution (LTE) [2].

In this paper, we consider hybrid RSS-TOA, RSS difference-TDOA (RSSD-TDOA), RSSD-TOA, or RSS-TDOA based positioning for cellular networks only. The first step in locating the MS is to estimate the distance between the MS and several BSs through distance dependent measurements such as RSS, TOA, RSSD, and TDOA [1]. In the second step, the MS position is estimated from these measurements together with the known positions of the BSs. In this paper, we focus on the distance estimation only as the position accuracy can be easily estimated for a given scenario and geometry of BSs and MS through position CRLBs [3].

The first contribution of this paper is the derivation of CRLBs for the distance estimation with these hybrid methods. Thus, we can assess the variance of the estimation errors for unbiased hybrid RSS-TOA, RSSD-TDOA, RSSD-TOA, and RSS-TDOA estimators. We first evaluate these CRLBs with a simple channel model that consists of AWGN, PL, and SF. In contrast to [3], [4], we do not only consider the PL and SF for the RSS, but we also take into account the AWGN in the derivation of the CRLBs. Moreover, we take into account the distance dependence, i.e., the PL, in the derivation of the CRLBs contrary to [4] for the TOA or TDOA. According to [5], there exist several different ways to compute varyingly tight CRLBs as the SF can be treated as nuisance parameter. We discuss this in detail in this paper. Further, we apply the CRLBs to the WINNER II channel model [6] and to current communication systems such as Global System for Mobile Communications (GSM), Universal Mobile Telecommunications System (UMTS), 3GPP-LTE, and Worldwide Interoperability for Microwave Access (WiMAX).

## II. CRLBs for Distance Estimation

We define the distance $d$ between the MS position $\boldsymbol{x}$ and the BS position $\boldsymbol{x}_{\mathrm{BS}}$ as

$$d = \|\boldsymbol{x}_{\mathrm{BS}} - \boldsymbol{x}\|_2 = \sqrt{(x_{\mathrm{BS}} - x)^2 + (y_{\mathrm{BS}} - y)^2} \qquad (1)$$

for a two-dimensional coordinate system. At distance $d$ we assume that the average received power $P$ is [7] $P = P_0 \cdot \left(\frac{d}{d_0}\right)^{-\varepsilon}$, where $P_0$ is the average received power at the reference distance $d_0$ and $\varepsilon$ denotes the environment dependent PL exponent. Given the transmit signal $s(t)$ and the speed of light $c_0$, the received signal sample $y[n]$ at distance $d$ is then

$$y[n] = a\left(\frac{d}{d_0}\right)^{-\frac{\varepsilon}{2}} s\left(nT - \frac{d}{c_0}\right) + z[n] \qquad (2)$$

for $n = 1, \ldots, N$. Here, $a$, $T$, and $z[n]$ denote the long-term SF coefficient of the wireless channel, the sampling interval, and the complex AWGN with zero mean and variance $\sigma^2$. Note that we consider only long-term and mid-term fading, i.e., the PL and SF [7], and assume that $a$ is constant for the duration of $N$ consecutive samples.

For the above signal model, we define the observation vector $\mathbf{y} = \left[y[1] \ldots y[N]\right]^{\mathrm{T}}$ and write the probability density function (pdf) $p(\mathbf{y}|d,a)$ as

$$p(\mathbf{y}|d,a) = \frac{1}{(\pi\sigma^2)^N} e^{-\frac{\sum_{n=1}^{N}\left|\left(\frac{d}{d_0}\right)^{-\frac{\varepsilon}{2}} \cdot a \cdot s\left(nT - \frac{d}{c_0}\right) - y[n]\right|^2}{\sigma^2}}. \quad (3)$$

The pdf of the SF factor $a$ is

$$p(a) = \frac{1}{\sqrt{2\pi}\zeta a} e^{-\frac{\left(\ln a + \zeta^2\right)^2}{2\zeta^2}}, \quad (4)$$

with $a > 0$ and $\zeta^2$ being the variance of the log-normal distributed random variable $a$ with mean $-\zeta^2$. In this case, the mean and variance of $a$ are $e^{-\zeta^2/2}$ and $1 - e^{-\zeta^2}$. Thus, the second moment of $a$ is $\mathrm{E}\{a^2\} = 1$.

The fading coefficient $a$ in (2) is a nuisance parameter as it does not depend on the distance $d$. According to [5], four different CRLBs exist for the nuisance parameter $a$. The first CRLB B1 is calculated w.r.t. $p(\mathbf{y}|d)$, which is obtained by marginalizing the joint pdf $p(\mathbf{y}, a|d)$ over a. The second CRLB B2 is obtained by inverting the Fisher information matrix (FIM) for estimating $d$ and $a$ jointly. The third CRLB B3 is obtained by computing the FIM w.r.t. $p(\mathbf{y}|d,a)$ and then inverting the expectation over the FIM w.r.t. $a$. Finally, the fourth CRLB B4 is obtained by first inverting the FIM of the third CRLB and then calculating the expectation w.r.t. $a$. In the sequel, we discuss these four CRLBs in more detail.

First, we consider the CRLB B1 based on $p(\mathbf{y}|d)$, i.e.,

$$p(\mathbf{y}|d) = \int_0^\infty p(\mathbf{y}, a|d)\, da = \int_0^\infty p(\mathbf{y}|d,a)p(a)\, da. \quad (5)$$

To the best of our knowledge, the integral in (5) cannot be solved analytically when inserting (3) and (4). Thus, we use (5) directly to calculate the FIM

$$\mathbf{J}(d,d) = -\mathrm{E}_{\mathbf{y}|d}\left\{\frac{\partial^2}{\partial d^2} \ln p(\mathbf{y}|d)\right\} = \mathrm{E}_{\mathbf{y}|d}\left\{\left(\frac{\partial}{\partial d} \ln p(\mathbf{y}|d)\right)^2\right\}, \quad (6)$$

which cannot be solved analytically similar to (5). Note, the second equality holds due to the regularity condition [9].

To obtain the other three bounds, we first derive the FIM based on the conditional pdf $p(\mathbf{y}|d,a)$. With the signal model in (2), we can determine the FIM for estimating the distance $d$ given the fading coefficient $a$

$$\begin{aligned}\mathbf{J}(d,d|a) &= \mathrm{E}_{\mathbf{y}|d,a}\left\{-\frac{\partial^2}{\partial d^2} \ln p(\mathbf{y}|d,a)\right\} \\ &= \frac{2a^2}{\sigma^2}\left(\frac{d}{d_0}\right)^{-\varepsilon}\left(\sum_{n=1}^{N}\left|\frac{\partial s\left(nT - \frac{d}{c_0}\right)}{\partial d}\right|^2 + \frac{\varepsilon^2}{4d^2}\sum_{n=1}^{N}\left|s\left(nT - \frac{d}{c_0}\right)\right|^2\right) \\ &= \frac{2a^2}{\sigma^2 T}\left(\frac{d}{d_0}\right)^{-\varepsilon}\left(\frac{4\pi^2 F_\mathrm{s}^2}{c_0^2}\sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor} k^2 |S[k]|^2 + \frac{\varepsilon^2}{4d^2}\sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor}|S[k]|^2\right),\end{aligned} \quad (7)$$

where we use $s(t) = \frac{1}{\sqrt{N}} \sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor} S[k] e^{j2\pi k F_\mathrm{s} t}$ and $S[k] = S(F_\mathrm{s} k)$. Analyzing the second or third line of (7), we see that $\mathbf{J}(d,d|a)$ can be partitioned into two parts

$$\mathbf{J}(d,d|a) = \mathbf{J}_\mathrm{TOA}(d,d|a) + \mathbf{J}_\mathrm{RSS}(d,d|a), \quad (8)$$

with

$$\mathbf{J}_\mathrm{TOA}(d,d|a) = \frac{2a^2}{\sigma^2 T}\left(\frac{d}{d_0}\right)^{-\varepsilon}\frac{4\pi^2 F_\mathrm{s}^2}{c_0^2}\sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor} k^2 |S[k]|^2 \quad (9)$$

and

$$\mathbf{J}_\mathrm{RSS}(d,d|a) = \frac{2a^2}{\sigma^2 T}\left(\frac{d}{d_0}\right)^{-\varepsilon}\frac{\varepsilon^2}{4d^2}\sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor} |S[k]|^2. \quad (10)$$

Equation (9) contains the FIM of the TOA measurement while (10) contains the FIM of the RSS measurement. Due to the summation in (8), we can either estimate the distance $d$ from the same data $\mathbf{y}$ by estimating the TOA and RSS jointly or independently without any performance loss. Further, it is straightforward to derive the CRLBs for individual TOA or RSS from the partition in (8). Hence, we consider in the following only the joint bounds.

In general, the channel for wideband signals will be frequency selective, e.g., for the $18\,\mathrm{MHz}$ pilot signal in LTE. Hence, we modify the FIM in (7) to

$$\begin{aligned}\mathbf{J}(d,d|a) = \frac{2a^2}{\sigma^2 T}\left(\frac{d}{d_0}\right)^{-\varepsilon}\Bigg(&\frac{4\pi^2 F_\mathrm{s}^2}{c_0^2}\sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor} k^2 |H[k]S[k]|^2 \\ &+ \frac{\varepsilon^2}{4d^2}\sum_{k=\lfloor -\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor}|H[k]S[k]|^2\Bigg),\end{aligned} \quad (11)$$

taking into account the channel transfer functions $H[k]$, $k = 0, \ldots, N-1$. This results in a tighter bound for frequency selective multipath channels. Note that (11) assumes that the channel transfer function $H(F_\mathrm{s} k)$ is perfectly known to the receiver and needs not to be estimated. This is clearly a simplification as any practical receiver can obtain $H[k]$ only with a channel estimation algorithm.

If the number samples $N$ approach $\infty$, we can replace for the sampled spectrum in (7) with

$$\begin{aligned}\mathbf{J}(d,d|a) = \frac{2a^2}{\sigma^2 T}\left(\frac{d}{d_0}\right)^{-\varepsilon}\Bigg(&\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{4\pi^2}{c_0^2} f^2 |S(f)|^2\, df \\ &+ \frac{\varepsilon^2}{4d^2}\int_{-\frac{1}{2T}}^{\frac{1}{2T}} |S(f)|^2\, df\Bigg)\end{aligned} \quad (12)$$

for a continuous spectrum. Employing a rectangular spectrum $S(f)$, i.e., $S(f) = \frac{1}{B}$ for $f \in [-B/2, B/2]$ and $S(f) = 0$

otherwise, (12) becomes

$$\mathbf{J}(d,d|a)=\frac{2a^2}{\sigma^2 T}\left(\frac{d}{d_0}\right)^{-\varepsilon}\left(\frac{\pi^2 B}{3c_0^2}+\frac{\varepsilon^2}{4Bd^2}\right). \tag{13}$$

To obtain the CRLB B3, we now compute the expectation over $a$ for the FIM $\mathbf{J}(d,d|a)$ and then take the inverse. For instance, the CRLB B3 for the rectangular spectrum is

$$\mathrm{var}\{d\}\geq(\mathrm{E}_a\{\mathbf{J}(d,d|a)\})^{-1}=\frac{\sigma^2 T}{2\left(\frac{\pi^2 B}{3c_0^2}+\frac{\varepsilon^2}{4Bd^2}\right)}\left(\frac{d}{d_0}\right)^{\varepsilon}. \tag{14}$$

Note that $\mathrm{E}\{a^2\}$ equals 1 for the pdf $p(a)$ in (4).

The CRLB B4 can be computed by taking the expectation over $a$ for the inverse of the FIM $\mathbf{J}(d,d|a)$. In the case of the rectangular spectrum, the CRLB B4 is

$$\mathrm{var}\{d\}\geq\mathrm{E}_a\left\{(\mathbf{J}(d,d|a))^{-1}\right\}=\frac{\sigma^2 T}{2\left(\frac{\pi^2 B}{3c_0^2}+\frac{\varepsilon^2}{4Bd^2}\right)}\left(\frac{d}{d_0}\right)^{\varepsilon}e^{4\zeta^2}, \tag{15}$$

where we used $\mathrm{E}\{a^{-2}\}=e^{4\zeta^2}$.

For the CRLB B2, we need to derive the FIM that estimates $d$ and $a$ jointly, i.e.,

$$\mathbf{J}\left(\begin{bmatrix}d\\a\end{bmatrix}\right)=\begin{bmatrix}\mathbf{J}(d,d) & \mathbf{J}(d,a)\\ \mathbf{J}(a,d) & \mathbf{J}(a,a)\end{bmatrix}, \tag{16}$$

where $\mathbf{J}(d,d)=\mathrm{E}_a\{\mathbf{J}(d,d|a)\}$ (cf. inverse of (14)). The off-diagonal element $\mathbf{J}(d,a)$ is given by

$$\begin{aligned}\mathbf{J}(d,a)&=\mathrm{E}_a\left\{-\mathrm{E}_{\mathbf{y}|d,a}\left\{\frac{\partial^2}{\partial d\partial a}\ln p(\mathbf{y}|d,a)\right\}\right\}-\underbrace{\mathrm{E}_a\left\{\frac{\partial^2}{\partial d\partial a}\ln p(a)\right\}}_{=0}\\&=\mathrm{E}_a\left\{-\mathrm{E}_{\mathbf{y}|d,a}\left\{\frac{\partial^2}{\partial d\partial a}\ln p(\mathbf{y}|d,a)\right\}\right\}\\&=-\frac{\varepsilon}{\sigma^2 d}\left(\frac{d}{d_0}\right)^{-\varepsilon}e^{-\frac{\zeta^2}{2}}\left(\sum_{n=1}^{N}\left|s\left(nT-\frac{d}{c_0}\right)\right|^2\right)\\&=-\frac{\varepsilon}{\sigma^2 d}\left(\frac{d}{d_0}\right)^{-\varepsilon}e^{-\frac{\zeta^2}{2}}\sum_{k=\lfloor-\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor}|S[k]|^2.\end{aligned} \tag{17}$$

Due to the interchangeable order of the partial derivatives in (17), we have $\mathbf{J}(a,d)=\mathbf{J}(d,a)$. Finally, the diagonal element $\mathbf{J}(a,a)$ is given by

$$\begin{aligned}\mathbf{J}(a,a)&=\mathrm{E}_a\left\{-\mathrm{E}_{\mathbf{y}|d,a}\left\{\frac{\partial^2}{\partial a^2}\ln p(\mathbf{y}|d,a)\right\}\right\}-\mathrm{E}_a\left\{\frac{\partial^2}{\partial a^2}\ln p(a)\right\}\\&=\frac{2}{\sigma^2}\left(\frac{d}{d_0}\right)^{-\varepsilon}\left(\sum_{n=1}^{N}\left|s\left(nT-\frac{d}{c_0}\right)\right|^2\right)+\left(1+\frac{1}{\zeta^2}\right)e^{4\zeta^2}\\&=\frac{2}{\sigma^2}\left(\frac{d}{d_0}\right)^{-\varepsilon}\left(\sum_{k=\lfloor-\frac{N-1}{2}\rfloor}^{\lfloor\frac{N-1}{2}\rfloor}|S[k]|^2\right)+\left(1+\frac{1}{\zeta^2}\right)e^{4\zeta^2}.\end{aligned} \tag{18}$$

With the above equations, we can now calculate the CRLB B2 as

$$\mathrm{var}\{d\}\geq\left(\mathbf{J}^{-1}\left(\begin{bmatrix}d\\a\end{bmatrix}\right)\right)_{d,d}=\left(\mathbf{J}(d,d)-\mathbf{J}(d,a)\,\mathbf{J}^{-1}(a,a)\,\mathbf{J}(a,d)\right)^{-1}. \tag{19}$$

For the equality in (19), we used the formula for the inverse of a block partitioned matrix [5], i.e., the Schur Complement. Concerning the ordering of the different CRLBs, we know [5]

$$\mathrm{var}\{d\}_{\mathrm{B1}}\geq\mathrm{var}\{d\}_{\mathrm{B2}}\geq\mathrm{var}\{d\}_{\mathrm{B3}}\text{ and }\mathrm{var}\{d\}_{\mathrm{B4}}\geq\mathrm{var}\{d\}_{\mathrm{B3}}. \tag{20}$$

No further ordering between the CRLBs B4 and B1 or B2 are possible in general. Our numerical analysis in Section III shows that for a wide range of parameters the CRLB B4 is larger than the CRLB B2. As we have no analytic solution for the CRLB B1, we did not investigate this CRLB further.

When directly using the TOA $d/c_0$ for positioning, the MS needs to be synchronized to the BSs. Otherwise, the clock bias $b_c$ of the MS needs to be estimated to calculate its position. As shown in [8], this is equivalent to using TDOAs for position estimation, where the clock bias $b_c$ is eliminated by subtracting the TOA $d_1/c_0$ of $\mathrm{BS}_1$ from the TOA $d_2/c_0$ of $\mathrm{BS}_2$. An equivalent argument holds for RSS versus RSSD measurements.

Therefore, we define the distance difference $\Delta d$ w.r.t. the first BS as

$$\Delta d=\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}d_1\\d_2\end{bmatrix}. \tag{21}$$

With the vector parameter CRLB for transformations [9], we obtain the CRLB on $\Delta d$ as

$$\begin{aligned}\mathrm{var}\{\Delta d\}&\geq\begin{bmatrix}-1 & 1\end{bmatrix}\mathbf{J}^{-1}(\mathbf{d},\mathbf{d})\begin{bmatrix}-1\\1\end{bmatrix}\\&=\mathbf{J}^{-1}(d_1,d_1)+\mathbf{J}^{-1}(d_2,d_2).\end{aligned} \tag{22}$$

Here, we assume that there exist no dependencies between the measurements $d_1$ and $d_2$. For $\mathbf{J}^{-1}(d_1,d_1)$ and $\mathbf{J}^{-1}(d_2,d_2)$, we can use the distance estimation bounds, e.g., (15). Thus, we can obtain CRLBs for RSSD, TDOA, and hybrid RSSD-TDOA measurements.

In addition to the hybrid RSS-TOA measurements and the hybrid RSSD-TDOA measurements, we can also combine RSSD measurements with TOA measurements or RSS measurements with TDOA measurements and calculate the corresponding CRLBs assuming that the measurements are independent [4]. To simplify the following analysis, we assume a one-dimensional scenario, in which the MS is located on the line segment between $\mathrm{BS}_1$ and $\mathrm{BS}_2$ and the distance $d_{12}$ between the two BSs is known. Thus, we have $d_1+d_2=d_{12}$. For instance, if the MS has measured a TDOA and independently an RSS from $\mathrm{BS}_1$, the resulting CRLB is

$$\mathrm{var}\{d_1,\Delta d\}=\frac{1}{\frac{1}{\mathrm{var}\{d_1\}}+\frac{1}{\mathrm{var}\{\Delta d\}}}. \tag{23}$$

# III. Results

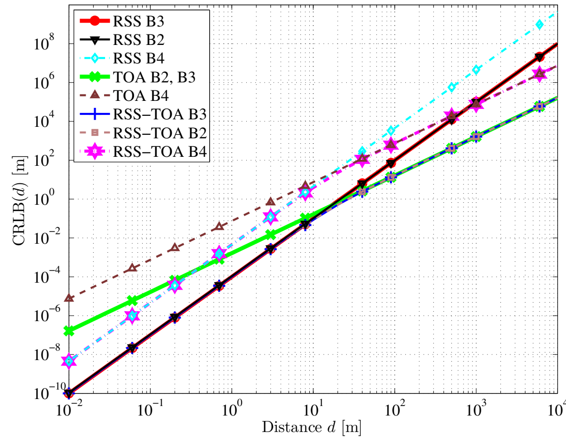
In this section we first present results for the AWGN channel model with PL and SF and subsequently for the more realistic WINNER II typical urban macro cell channel model C2 [6].

Fig. 1 compares CRLBs for the signal model in (2) with noise spectral density $N_0 = \sigma^2 T = -204\,\mathrm{dB-Hz}$, PL exponent $\varepsilon = 4$, SF standard deviation $\zeta = 6\,\mathrm{dB}$, and a rectangular spectrum with a bandwidth of $20\,\mathrm{MHz}$. Note that the chosen values for the PL exponent and SF variance are typical for the urban macro cell environment [6]. The curves in Fig. 1(a) labeled with 'RSS B3', 'TOA B2, B3', and 'RSS-TOA B3' correspond to the CRLB B3 in (14), whereas the curves labeled with 'B2' correspond to the CRLB B2 in (19) and the curves labeled with 'B4' to the CRLB B4 in (15). Note that it can be shown analytically that the TOA CRLBs B2 and B3 are the same. The hybrid RSS-TOA CRLBs always perform equal or better than the corresponding RSS or TOA CRLBs. The B3 CRLBs and the B2 CRLBs coincide whereas the B4 CRLBs results in the largest variance. Thus, the CRLB B4 in (15) is the tightest CRLB for the signal model in (2).

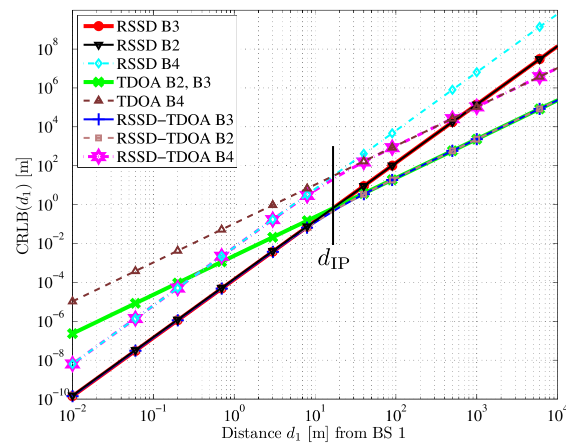
In Fig. 1(b), we plotted the CRLBs for RSSD, TDOA, and hybrid RSSD-TDOA at a distance $d$ in the middle of the two BSs, i.e., $d = d_{12}/2$. Thus, $d_{12}$ increases proportional to $d$. This scenario is the best performance case for distance difference measurements as one can easily infer from, e.g., (22) and (7). From (22), we know that $\mathrm{var}\{\Delta d\} = 2\mathrm{var}\{d_1\}$. Thus, the accuracy of RSSD, TDOA, or hybrid RSSD-TDOA is by a factor of $\sqrt{2}$ worse than that of RSS, TOA, or hybrid RSS-TOA. Otherwise, the behavior of the distance difference CRLBs in Fig. 1(b) is the same as the one of the distance CRLBs in Fig. 1(a). Additionally, Fig. 1(c) shows hybrid RSSD-TOA and RSS-TDOA bounds. These hybrid bounds are closely limited by their corresponding individual RSSD or RSS CRLBs for distances below $10\,\mathrm{m}$ and by their corresponding individual TDOA or TOA CRLBs for distances above $30\,\mathrm{m}$. For instance, consider the curves labeled 'RSSD+TOA, B4', 'TOA B4', and 'RSS B4'.

Fig. 2 investigates how the intersection point distance $d_{\mathrm{IP}}$ (cf. Fig. 1(b)) for different signal strength and timing based CRLBs varies versus the signal bandwidth. Increasing the bandwidth by a factor of 10 decreases $d_{\mathrm{IP}}$ approximately by a factor of 10. No significant difference in $d_{\mathrm{IP}}$ can be seen for different CRLBs.

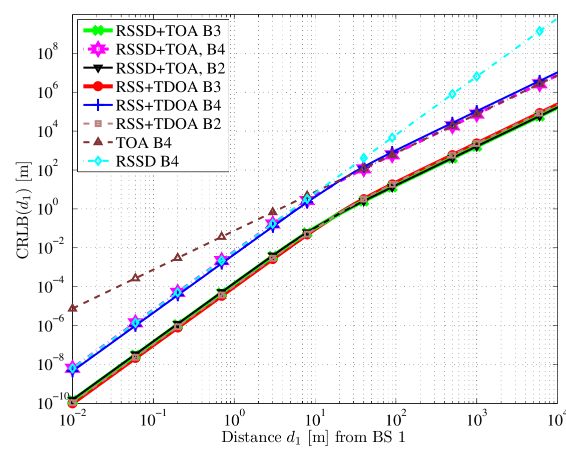
In the remainder, we analyze the distance estimation accuracy of GSM, UMTS, LTE, and WiMAX for the WINNER II C2 channel model [6] using the CRLB B4 in Fig. 3. The WINNER II C2 channel model generates the predictions for PL, SF, and multipath. Therefore, we combine (15) with (11) to account for multipath in the CRLBs. For the RSSD and TDOA measurements, the inter-BS distance is set to $d_{12} = 750\,\mathrm{m}$. Comparing the different systems, RSS or RSSD measurements perform significantly better than TOA or TDOA measurements only for narrow band GSM for the investigated distances. For UMTS, RSS performs better than TOA for distances below $70\,\mathrm{m}$. Comparing the overall accuracies, the wide band WiMAX system ($20\,\mathrm{MHz}$ bandwidth) achieves the lowest CRLB followed by the wide band LTE system ($18\,\mathrm{MHz}$




(a) RSS, TOA, and hybrid RSS-TOA




(b) RSSD, TDOA, and hybrid RSSD-TDOA




(c) Hybrid RSSD-TOA and RSS-TDOA

Fig. 1. Distance estimation CRLBs for AWGN channel model with path loss and shadow fading and for rectangular spectrum with $20\,\mathrm{MHz}$ bandwidth.

bandwidth) for TOA or TDOA measurements. Whereas the RSS or TOA CRLBs in Fig. 3(a) exhibit a strongly distance dependent accuracy (over 4 orders of magnitude), the RSSD or TDOA CRLBs in Fig. 3(b) are only slightly distance dependent at the expense of less accuracy. The hybrid RSSD-TOA and RSS-TDOA CRLBs in Fig. 3(c) again reveal a strong

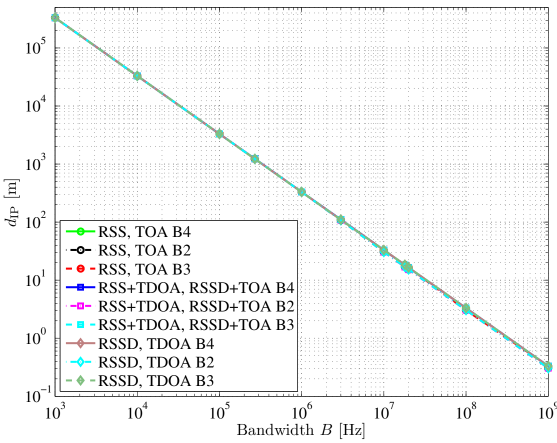
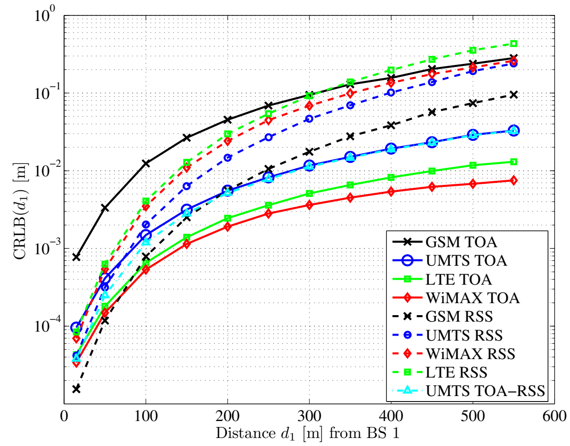



Fig. 2. Intersection point distance $d_{\mathrm{IP}}$ for signal strength and timing based CRLBs versus signal bandwidth $B$: AWGN channel model with path loss and shadow fading, rectangular spectrum.

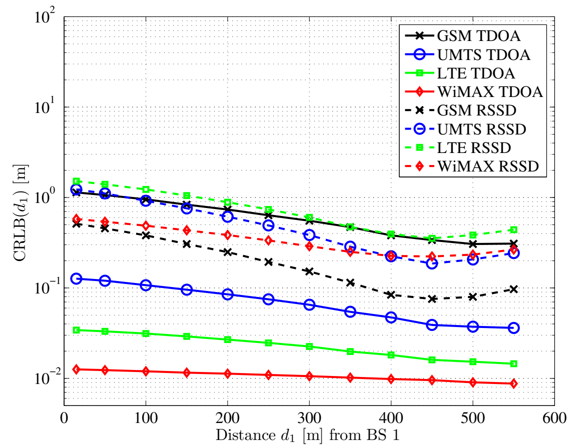
distance dependence when the MS is close to BS 1. Except for UMTS in Fig. 3(a), we omitted the hybrid CRLBs for clarity of presentation as they closely follow the better performing TOA or TDOA and RSS or RSSD bounds.

## IV. Conclusions

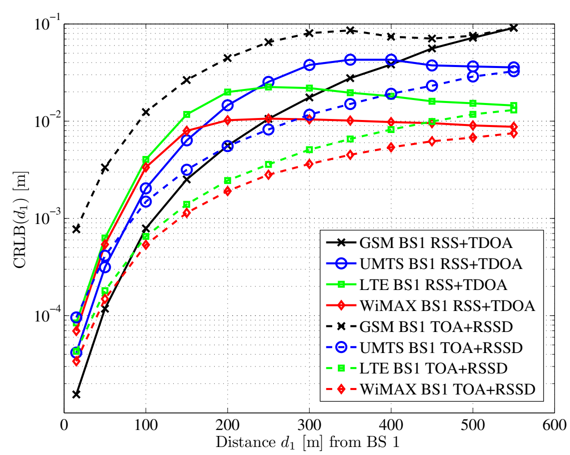
In this paper, we presented CRLBs for hybrid distance estimation algorithms, specifically, RSS-TOA, RSSD-TDOA, RSS-TDOA, and RSSD-TOA. The derived CRLBs take into account PL and SF as well as multipath propagation. As expected, results show that the hybrid schemes always perform equal or better than RSS, RSSD, TOA, or TDOA only schemes, that the forth type CRLB is the tightest bound, and that the distance when RSS or RSSD performs better than TOA or TDOA increases with decreasing bandwidth. When comparing GSM, UMTS, LTE, and WiMAX for the WINNER II channel model, we conclude that the timing based CRLBs perform significantly better for the wide band systems LTE and WiMAX and only for the narrow band GSM system the RSS based CRLBs perform significantly better. Further, RSSDs or TDOAs exhibit a nearly distance independent CRLB compared to RSSs or TOAs at the expense of degraded accuracy when the MS is close to a BS.




(a) RSS, TOA, and hybrid RSS-TOA




(b) RSSD, TDOA, and hybrid RSSD-TDOA




(c) Hybrid RSSD-TOA and RSS-TDOA

Fig. 3. Distance estimation CRLBs for GSM, UMTS, LTE, and WiMAX and WINNER II C2 channel model.

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