Improved and Opportunistic Interference Alignment Schemes for Multi-Cell Interference Channels

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***Abstract*—We study precoder optimization gains and multiuser diversity gains with interference alignment in a two-cell wireless network. In this paper, we propose algorithms to improve the achievable sum rate by optimizing alignment directions. The proposed iterative algorithm can provide a substantial gain in the achievable rate while it requires only a few iterations. In addition, we investigate interference alignment to exploit multiuser diversity gains. A new criterion of user selection scheduling is proposed. This user selection can be done independently in different cells. Therefore, both the searching space and the information exchanged between base stations are significantly reduced compared to joint user scheduling over two cells.**

I. Introduction

Interference is believed as the most significant bottleneck in the performance of next-generation wireless communication systems. A recent breakthrough in managing interference is a new technique called interference alignment [1, 2]. The key idea behind the interference alignment is to consolidate multiple interference into smaller subspace so as to reserve the remaining dimensions for desired signals. Since many insights on interference alignment emerged out from the degrees of freedom (DoF) perspective, the precoder designs for interference alignment have mainly focused on maximizing the achievable DoF of the wireless network. In fact, from the DoF perspective, what matters is only the space spanned by the precoding vectors used at the transmitters. While different signal-space bases can result in the identical DoF performance, the achievable rate can differ according to the the bases. Therefore, we shall optimize the precoders for interference alignment to achieve further gains in the data rate.

Several earlier works have optimized the precoders based on closed-form interference alignment schemes [3–5] and iterative distributed interference alignment schemes [6]. In [3], precoder optimizations based on the closed-form solutions in [2] have been proposed for K-user single-input single-output (SISO) interference channels and 3-user multi-input multi-output (MIMO) interference channels. For general MIMO interference channels, deriving a closed-form solution of zero-leakage interference alignment is cumbersome. In [6, 7], distributed interference alignment algorithms have been proposed to achieve interference alignment in an iterative manner. Based on these algorithms, a precoder optimization algorithm to maximize the sum rate is proposed in [10] for MIMO interference networks.

In this paper, we consider a cellular system consisting of two cells. Closed-form interference alignment schemes have been

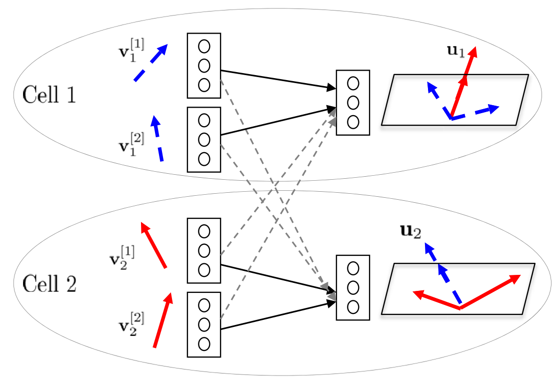

Fig. 1. Uplink interference alignment in two-cell networks.

discussed for this network to achieve interference-free DoF [8, 9]. Let us first review the interference alignment schemes. For uplink communications, the interference alignment scheme for two users in each cell with every node equipped with three antennas is depicted in Fig. 1. Base station (BS) 1 can *arbitrarily* choose an alignment direction $\mathbf{u}_1$. Once this alignment direction is specified, user 1 and 2 in cell 2 design their beamforming vectors $\mathbf{v}_2^{[1]}$ and $\mathbf{v}_2^{[2]}$, respectively such that after going through the channel, they align along the same direction $\mathbf{u}_1$. At the same time, Base station 2 can *arbitrarily* choose an alignment direction $\mathbf{u}_2$, which then determines the beamforming vectors $\mathbf{v}_1^{[1]}$ and $\mathbf{v}_1^{[2]}$ for users in cell 1. After interference alignment, in the three dimensional signal space at each BS, interference occupies one dimension while two desired signals occupy the other two dimensions, and thus the BS can decode its desired signal by zero-forcing interference. For the downlink, the alignment scheme is similar due to a reciprocity of linear interference alignment schemes based on zero-forcing [9]. Specifically, in the downlink, the role of the transmitters and receivers of the uplink switches. We can simply use the receive filter in the uplink as the transmit filter in the downlink and the transmit filter in the uplink as receive filter in the downlink. For example, the transmit filters $\mathbf{v}_1^{[1]}$ and $\mathbf{v}_1^{[2]}$ in cell 1 shown in Fig.1 are used as receive filters in the downlink. As a result, from the BS 2's point of view, two users in the other cell are along the same dimension $\mathbf{u}_2$ in its three dimensional transmit signal space. Therefore, it can restrict its transmit signal in the two dimensional subspace orthogonal to $\mathbf{u}_2$ without causing interference to users in cell 1. As a result, each user can achieve one DoF.

As we can see, for both uplink and downlink interference alignment schemes, the alignment directions can be chosen *arbitrarily*. While different alignment directions achieve the same DoF, they lead to different achievable rates. Motivated by this observation, we investigate the best alignment directions to maximize sum rate achieved for two cells. Our contribution is an iterative algorithm that optimizes the alignment directions. With only a very few number of iterations, e.g. 2 iterations, we can obtain a substantial gain in the sum rate over randomly chosen alignment directions.

In the second part of this paper, we study interference alignment from an opportunistic communication perspective. If there are multiple users in each cell, an arbitrarily given alignment direction is good for some users with high probability. Therefore, even if we do not optimize the alignment directions, we can still achieve a high rate by exploiting the multiuser diversity gains. However, how to efficiently select the users based on interference alignment is another problem. To address this issue, we propose a new criterion for user selection scheduling. The proposed method can considerably reduce the searching space and the overhead communications between base stations. It is demonstrated through computer simulations that our proposed schemes offer a significant performance gain in multi-cell networks.

Notations: We denote vectors and matrices by bold fonts in lower cases and upper cases, respectively. A scalar is denoted using italic font in lower case. The notations $|\mathbf{X}|$, $\mathbf{X}^{-1}$, $\mathbf{X}^{\dagger}$, and $\mathbf{X}^{-\dagger}$ represent the determinant, the inverse, the conjugate transpose and the conjugate transpose inverse of a matrix $\mathbf{X}$, respectively. The complex field is denoted by $\mathbb{C}$, the expectation operator is written by $\mathbf{E}(\cdot)$, $\mathbf{I}$ is an identity matrix, and $\mathrm{abs}(\cdot)$ denotes the absolute value. A multivariate complex-valued Gaussian distribution of mean $\mathbf{m}$ and variance $\mathbf{V}$ is denoted by $\mathcal{CN}(\mathbf{m}, \mathbf{V})$.

II. Interference Alignment

A. System Model

Consider a cellular system consisting of two cells. There are a total of K users in each cell. All users and base stations are equipped with M antennas. We mainly consider the uplink in this paper. Note that the downlink can be solved similarly due to a duality of interference alignment between uplink and downlink [9].

For simplicity, we consider $K = M - 1$ users in each cell and each user sends one data stream with precoding to the corresponding BS. The received signal at the BS in the j^{th} cell is written as

$$\mathbf{y}_j = \sum_{i=1}^{M-1} \sum_{k=1}^{2} \mathbf{H}_{jk}^{[i]} \mathbf{v}_k^{[i]} x_k^{[i]} + \mathbf{n}_j, \quad j \in \{1, 2\},$$

where $\mathbf{H}_{jk}^{[i]} \in \mathbb{C}^{M \times M}$ is the channel matrix from user i in cell k to cell j, $\mathbf{v}_k^{[i]}$ is a unit-norm beamforming vector for user i in cell k, $x_k^{[i]}$ is the transmitting stream from user i in cell k, and $\mathbf{n}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is the additive white Gaussian noise (AWGN) at BS j. The transmitter should also satisfy the average power constraint, i.e., $\mathbf{E}(\|\mathbf{v}_k^{[i]} x_k^{[i]}\|^2) \leq P$ with P being the maximum transmission power.

B. Uplink Interference Alignment

We will design the precoding vectors such that all interference vectors are aligned along the same direction at the undesired BS [9]. Let us denote the direction at BS j by a unit-norm vector $\mathbf{u}_j$. At BS 1, all the interference vectors from users in cell 2 should be received along $\mathbf{u}_1$, i.e.,

$$\frac{\mathbf{H}_{12}^{[i]} \mathbf{v}_2^{[i]}}{\|\mathbf{H}_{12}^{[i]} \mathbf{v}_2^{[i]}\|} = \mathbf{u}_1 \quad \Longrightarrow \quad \mathbf{v}_2^{[i]} = \|\mathbf{H}_{12}^{[i]} \mathbf{v}_2^{[i]}\| \mathbf{H}_{12}^{[i]-1} \mathbf{u}_1. \quad (1)$$

Since $\|\mathbf{v}_2^{[i]}\| = 1$, normalizing $\mathbf{v}_2^{[i]}$ yields

$$\mathbf{v}_2^{[i]} = \frac{\mathbf{H}_{12}^{[i]-1} \mathbf{u}_1}{\|\mathbf{H}_{12}^{[i]-1} \mathbf{u}_1\|}, \quad \forall i \in \{1, \ldots, M-1\}. \quad (2)$$

In an analogous way, the precoding vectors for users in cell 1 are expressed as

$$\mathbf{v}_1^{[i]} = \frac{\mathbf{H}_{21}^{[i]^{-1}} \mathbf{u}_2}{\|\mathbf{H}_{21}^{[i]^{-1}} \mathbf{u}_2\|}, \quad \forall i \in \{1, \ldots, M-1\}. \quad (3)$$

Accordingly, once we specify the alignment direction at BS 1 (and BS 2), i.e., $\mathbf{u}_1$ (and $\mathbf{u}_2$), all beamforming vectors of users in cell 2 (and cell 1) can be determined.

C. Achievable Rate

With the precoding vectors designed, we can calculate the achievable sum rate for both cells. Assuming Gaussian signaling, i.e., $x_k^{[i]} \sim \mathcal{CN}(0, P)$, the achievable sum rate in cell 1 is

$$R_1 = \log \left| \mathbf{I} + \sum_{i=1}^{M-1} P \left(\mathbf{H}_{11}^{[i]} \mathbf{v}_1^{[i]} \mathbf{v}_1^{[i]\dagger} \mathbf{H}_{11}^{[i]\dagger} + \mathbf{H}_{12}^{[i]} \mathbf{v}_2^{[i]} \mathbf{v}_2^{[i]\dagger} \mathbf{H}_{12}^{[i]\dagger} \right) \right| - \log \left| \mathbf{I} + \sum_{i=1}^{M-1} P \mathbf{H}_{12}^{[i]} \mathbf{v}_2^{[i]} \mathbf{v}_2^{[i]\dagger} \mathbf{H}_{12}^{[i]\dagger} \right|. \quad (4)$$

Plugging (3) and (2) into the above expression, we obtain

$$R_1 = \log \left| \mathbf{I} + \sum_{i=1}^{M-1} P \big(a^{[i]2} \mathbf{H}_{11}^{[i]} \mathbf{H}_{21}^{[i]-1} \mathbf{u}_2 \mathbf{u}_2^{\dagger} \mathbf{H}_{21}^{[i]-\dagger} \mathbf{H}_{11}^{[i]\dagger} + b^{[i]2} \mathbf{u}_1 \mathbf{u}_1^{\dagger} \big) \right| - \log \left(1 + \sum_{i=1}^{M-1} b^{[i]2} P \right), \quad (5)$$

where

$$a^{[i]} = \frac{1}{\|\mathbf{H}_{21}^{[i]-1} \mathbf{u}_2\|}, \qquad b^{[i]} = \frac{1}{\|\mathbf{H}_{12}^{[i]-1} \mathbf{u}_1\|}. \quad (6)$$

Similarly, we can compute the sum rate for cell 2 as

$$R_2 = \log \left| \mathbf{I} + \sum_{i=1}^{M-1} P \big(b^{[i]2} \mathbf{H}_{22}^{[i]} \mathbf{H}_{12}^{[i]-1} \mathbf{u}_1 \mathbf{u}_1^{\dagger} \mathbf{H}_{12}^{[i]-\dagger} \mathbf{H}_{22}^{[i]\dagger} + a^{[i]2} \mathbf{u}_2 \mathbf{u}_2^{\dagger} \big) \right| - \log \left(1 + \sum_{i=1}^{M-1} a^{[i]2} P \right) \quad (7)$$

Our goal is to maximize the sum rate over two cells with respect to the two alignment directions, more specifically,

$$\begin{aligned} \max_{\mathbf{u}_1,\mathbf{u}_2} \quad & R = R_1 + R_2, \\ \text{s.t.} \quad & \|\mathbf{u}_1\| = \|\mathbf{u}_2\| = 1. \end{aligned} \tag{8}$$

III. Optimizing Alignment Directions

Since the optimization problem in (8) is non-convex, it is hard to obtain the optimal alignment directions in both analytical and numerical manners. One numerical approach to find a local optima is the gradient method as follows.

A. Gradient Method

The gradient of the rate in terms of the alignment vectors is written by

$$\begin{aligned} \nabla_{\mathbf{u}_1} R_1 = & P\left(\sum_{i=1}^{M-1} b^{[i]2}\right)\mathbf{Q}_1^{-1}\mathbf{u}_1 + \sum_{i=1}^{M-1} b^{[i]4}\mathbf{H}_{12}^{[i]-\dagger}\mathbf{H}_{12}^{[i]-1}\mathbf{u}_1 \\ & P\left(\frac{1}{1+P\sum_{i=1}^{M-1} b^{[i]2}} - \mathbf{u}_1^\dagger\mathbf{Q}_1^{-1}\mathbf{u}_1\right), \end{aligned} \tag{9}$$

$$\begin{aligned} \nabla_{\mathbf{u}_1} R_2 = & \sum_{i=1}^{M-1} Pb^{[i]2}\left(-b^{[i]2}\mathbf{H}_{12}^{[i]-\dagger}\mathbf{H}_{12}^{[i]-1}\mathbf{u}_1\mathbf{u}_1^\dagger + 1\right) \\ & \mathbf{H}_{12}^{[i]-\dagger}\mathbf{H}_{22}^{[i]\dagger}\mathbf{Q}_2^{-1}\mathbf{H}_{22}^{[i]}\mathbf{H}_{12}^{[i]-1}\mathbf{u}_1, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathbf{Q}_1 = \mathbf{I} + & \sum_{i=1}^{M-1} P\big(a^{[i]2}\mathbf{H}_{11}^{[i]}\mathbf{H}_{21}^{[i]-1}\mathbf{u}_2\mathbf{u}_2^\dagger\mathbf{H}_{21}^{[i]-\dagger}\mathbf{H}_{11}^{[i]\dagger} \\ & + b^{[i]2}\mathbf{u}_1\mathbf{u}_1^\dagger\big), \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbf{Q}_2 = \mathbf{I} + & \sum_{i=1}^{M-1} P\big(b^{[i]2}\mathbf{H}_{22}^{[i]}\mathbf{H}_{12}^{[i]-1}\mathbf{u}_1\mathbf{u}_1^\dagger\mathbf{H}_{12}^{[i]-\dagger}\mathbf{H}_{22}^{[i]\dagger} \\ & + a^{[i]2}\mathbf{u}_2\mathbf{u}_2^\dagger\big). \end{aligned} \tag{12}$$

Hence, we have

$$\nabla_{\mathbf{u}_1} R = \nabla_{\mathbf{u}_1} R_1 + \nabla_{\mathbf{u}_1} R_2. \tag{13}$$

The gradient for $\nabla_{\mathbf{u}_2} R$ can be obtained by changing $b^{[i]}$ to $a^{[i]}$ and the indices 1 to 2 and 2 to 1. With the gradient vectors, we can use a gradient-based optimization method:

1: Initialize random vectors $\mathbf{u}_1$ and $\mathbf{u}_2$
2: **for** $t = 1$ to T **do**
3: Calculate the gradient $\nabla_{\mathbf{u}_1} R$ and $\nabla_{\mathbf{u}_2} R$
4: Update $\mathbf{u}_1 \leftarrow \mathbf{u}_1 + \delta\nabla_{\mathbf{u}_1} R$ and $\mathbf{u}_2 \leftarrow \mathbf{u}_2 + \delta\nabla_{\mathbf{u}_2} R$
5: Normalize $\mathbf{u}_1$ and $\mathbf{u}_2$
6: **end for**

Here, δ is the step size and T is the maximum number of iterations. Although the gradient algorithm can find a local optimum, the convergence speed of the gradient method is extremely slow and the sum-rate performance is highly dependent on the initial vectors. In the following, we propose an iterative algorithm to optimize the alignment directions with only a few iterations.

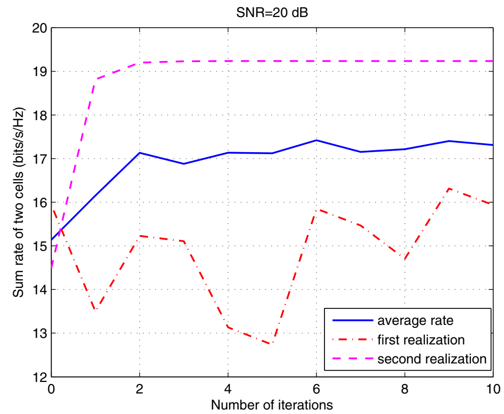

Fig. 2. The sum rate as a function of number of iterations in iterative orthogonalization algorithm ($M = 3$).

B. Iteratively Orthogonalizing Interference

In high signal-to-noise ratio (SNR) regimes, we shall minimize the interference by setting the interference vector orthogonal to the desired signals. Motivated by this intuition, we can align the interference orthogonal to the space spanned by the desired signal at both receivers, i.e.,

$$\mathbf{u}_1 = \text{null}\Big(\Big[\mathbf{H}_{11}^{[1]}\mathbf{v}_1^{[1]} \quad \cdots \quad \mathbf{H}_{11}^{[\text{M}-1]}\mathbf{v}_1^{[\text{M}-1]}\Big]^\dagger\Big), \tag{14}$$

$$\mathbf{u}_2 = \text{null}\Big(\Big[\mathbf{H}_{22}^{[1]}\mathbf{v}_2^{[1]} \quad \cdots \quad \mathbf{H}_{22}^{[\text{M}-1]}\mathbf{v}_2^{[\text{M}-1]}\Big]^\dagger\Big), \tag{15}$$

where $\text{null}(\mathbf{A})$ denotes a null space of a matrix $\mathbf{A}$. Note that $\mathbf{v}_1^{[i]}$ and $\mathbf{v}_2^{[i]}$ are a function of $\mathbf{u}_2$ and $\mathbf{u}_1$ as in (3) and (2), respectively. Since it is not straightforward to obtain a closed-form solution to fulfill the above two conditions at the same time, we propose the following iterative algorithm to solve this problem as below:

1: Initialize an alignment direction $\mathbf{u}_1$ at BS 1
2: **for** $t = 1$ to T **do**
3: Given $\mathbf{u}_1$, calculate the precoding vectors in cell 2, i.e., $\mathbf{v}_2^{[i]}$, according to (2). Set $\mathbf{u}_2$ according to (15).
4: Given $\mathbf{u}_2$, calculate the precoding vectors in cell 1, i.e., $\mathbf{v}_1^{[i]}$, according to (3). Set $\mathbf{u}_1$ according to (14).
5: **end for**

The convergence of this algorithm depends on a specific channel realization. For most channel realizations, regardless of the initialization, the algorithm converges with few iterations to the same sum rate. This is shown by the dashed line in Fig. 2. For some channel realizations, the algorithm does not converge as shown by the dashdot line in Fig. 2. Nevertheless, on average the algorithm will converge with only a few iterations. In Fig. 2, we plot the average sum rate for 100 channel realizations as a function of the number of iterations. As we can see, the average sum rate converges only after 2 iterations, although there are some fluctuations. In addition, we observed through additional simulations that the convergence of the average sum rate does not depend on the initialization.

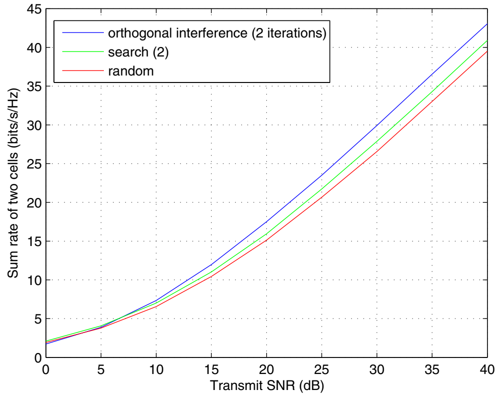

Fig. 3. Sum rate with iterative orthogonalizing interference ($M = 3$).

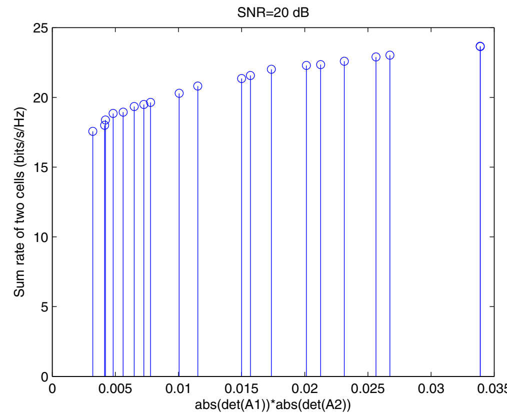

Fig. 4. Sum rate as a function of determinant ($M = 3$).

IV. Simulation Results

In this section, we evaluate the proposed algorithm for the case when $M = 3$, i.e., two users in each cell and every node is equipped with 3 antennas. We assume all entries in the channel matrix and the noise at each antenna are *i.i.d* complex Gaussian with zero mean and unit variance. In addition, all transmitters use the same power, and the path loss from all users to one base station is the same since we are mainly interested in cell-edge scenarios. Through Monte-Carlo simulations with 1000 independent channel realizations, we evaluate the expected sum rate of two cells.

We compare the average sum rates achieved by three schemes: The first scheme is the orthogonalizing interference algorithm with $T = 2$ iterations. For each iteration, we calculate the achieved sum rate, and we choose the larger one among these two. This ensures the selection of the highest sum rate solution in the event that we have a channel realization that exhibits the non convergence behavior depicted in Fig. 2. The second scheme is to randomly generate two alignment direction pairs. For each pair, calculate the sum rate and select the larger one. The last scheme is to randomly generate two independent alignment directions. As shown in Fig. 3, the orthogonalizing interference algorithm gives a substantial gain over the other two schemes at high SNR regimes; approximately 3 bits per second can be increased at an SNR of 40 dB from the conventional random approach. It should be noted that we observed in further simulations that the gradient method has no visible advantage over the random scheme for such a few iterations.

V. Multiuser Diversity Gain

In previous sections, we have investigated the gain provided by optimizing the alignment directions. If there are many users in the cell, an additional *multiuser diversity gain* can be exploited. For this case, even if we randomly generate the alignment directions at the base stations, due to multiple users, with high probability, any directions are good for some users in the sense of sum rate. One way to determine which users to be served is to do exhaustive searching among all user combinations based on the sum rate given by (8). However, if the total number of users is large, such an exhaustive search becomes impractical due to the large searching space for some practical applications. In addition, the searching should be done jointly by two base stations. This is because the sum rate in one cell depends on the users served in the other cell. For example, from (5), the sum rate in the first cell depends on $b^{[i]}$ which is a function of the channels of the users in the other cell. In this section, we derive a new user selection criterion to deal with those two problems.

A. User Selection Scheduling

Let us first intuitively understand what alignment directions perform well. For simplicity, consider the case of $M = 3$. As shown in Fig. 1, at each receiver, there are three vectors, one unit-norm interference vector and two desired vectors. The achievable rate is dependent on two parameters; the angels among these vectors and the amplitudes of two desired signal vectors. To minimize interference, all three vectors should be as orthogonal to each other whenever possible. On the other hand, to obtain a high desired signal power, the norm of the desired signal should be large. One parameter to capture these two factors is the volume of the parallelepiped formed by the two desired signal vectors and the interference vector. If these vectors are orthogonal and the desired signal's norm is large, then the volume should be large. Therefore, good alignment directions should give a large volume at each receiver. Moreover, the volume of the parallelepiped equals to the absolute value of the determinant of the matrix formed by those three vectors. Thus, one possible criterion is the product of the absolute values of the determinants at two base stations. In Fig. 4, we plot the sum rate as a function of the products of two determinants for 20 randomly generated alignment direction pairs for a specific channel realization at an SNR of 20 dB. One can see that the sum rate increases as the product of the determinants increases. Note that it is not the case when SNR is low.

B. Determinant Criterion

Next, we justify this intuition through an approximation of the sum rate expression given in (5) and (7). At high SNRs,

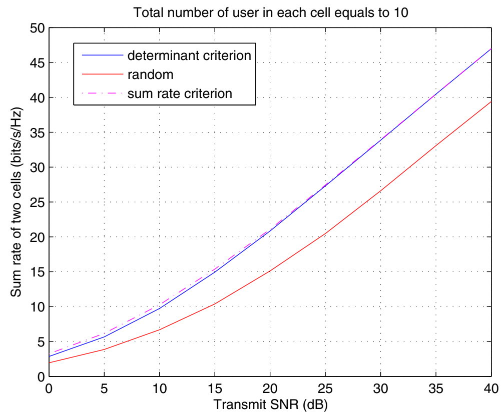

Fig. 5. Determinant Criterion for user selection ($M = 3$).

we can ignore the identity matrix in (5). Factoring out P and ignoring the amplitude of the interference vector, (5) becomes

$$\begin{aligned} R_1 &\approx \log\left|\sum_{i=1}^{M-1}(a^{[i]})^2\mathbf{H}_{11}^{[i]}\mathbf{H}_{21}^{[i]-1}\mathbf{u}_2\mathbf{u}_2^\dagger\mathbf{H}_{21}^{[i]-\dagger}\mathbf{H}_{11}^{[i]\dagger}+\mathbf{u}_1\mathbf{u}_1^\dagger\right| \\ &\quad+(M-1)\log P \\ &= 2\log\left(\mathrm{abs}(|\mathbf{A}_1|)\right)+(M-1)\log P, \end{aligned} \tag{16}$$

where

$$\mathbf{A}_1 = \left[\mathbf{u}_1 \;\; a^{[1]}\mathbf{H}_{11}^{[1]}\mathbf{H}_{21}^{[1]-1}\mathbf{u}_2 \cdots a^{[M-1]}\mathbf{H}_{11}^{[M-1]}\mathbf{H}_{21}^{[M-1]-1}\mathbf{u}_2\right]. \tag{17}$$

Likewise, we have

$$R_2 \approx 2\log\left(\mathrm{abs}(|\mathbf{A}_2|)\right)+(M-1)\log P, \tag{18}$$

where

$$\mathbf{A}_2 = \left[\mathbf{u}_2 \;\; b^{[1]}\mathbf{H}_{22}^{[1]}\mathbf{H}_{12}^{[1]-1}\mathbf{u}_1 \cdots b^{[M-1]}\mathbf{H}_{22}^{[M-1]}\mathbf{H}_{12}^{[M-1]^{-1}}\mathbf{u}_1\right]. \tag{19}$$

From (16) and (18), we can approximate R as

$$R \approx 2\log\left(\mathrm{abs}(|\mathbf{A}_1|)\mathrm{abs}(|\mathbf{A}_2|)\right)+2(M-1)\log P. \tag{20}$$

In consequence, maximizing R corresponds to maximizing the product of determinants, $\mathrm{abs}(|\mathbf{A}_1|)\mathrm{abs}(|\mathbf{A}_2|)$, in high SNR regimes. We use this new metric for user selection scheduling.

It is important to note that the determinant at one base station does not depend on the channels of users in the other cell because we ignore the length of the interference vector, while the sum rate of one cell depends on those users. For example, from (17), the determinant at cell 1 only depends on the channels of users in that cell. This observation directly leads to the conclusion that the user selection can be done *separately* by the base stations. In cell 1, two users can be selected to maximize the determinant of $\mathbf{A}_1$ given by (17). This can be carried out similarly and separately in cell 2. Such a separation considerably reduces the number of searches compared to that required if we select users based on the sum rate expression in (8). For example, if two out of ten users in each cell should be selected, only 90 searches (45 per cell) are required using the proposed determinant criterion. Whereas, $45^2 = 2025$ searches are required if we jointly search for the users in the two cells that achieve highest sum rate.

C. Performance Evaluation

In Fig. 5, we plot the sum rate of two cells in which there are 10 users with $M = 3$ antennas for each cell. Two users are selected out of 10 users in each cell based on the sum rate criterion or the determinant criterion, given a randomly generated alignment direction pair. As we can see from this figure, two criteria perform almost the same in the low SNR regimes and the same in the high SNR regimes. In addition, we plot the performance of randomly chosen two users in each cell, like a round robin scheduling. As shown in the figure, multiuser diversity provides a significant gain over no multiuser-diversity case; approximately 7 bits per second can be improved for SNRs higher than 25 dB. The simulation result shows that although the determinant criterion is based on the approximation in high SNR regimes (specifically, ignoring the length of the interference vectors and doing search separately), it is still a very good criterion over the whole SNR regimes, for user selection scheduling.

VI. Conclusion

We proposed an iterative algorithm to optimize the alignment directions for interference alignment in the uplink multi-cell networks such that the sum rate is maximized. In addition, we provided a new user-selection strategy to exploit multiuser gains with interference alignment. Through performance analysis, it was confirmed that our proposed algorithm and strategy for multi-cell interference alignment perform well. Although we only focused on the uplink interference alignment, the proposed methods can be applied to the downlink interference alignment due to a duality between the downlink and uplink.

References

[1] S. Jafar and S. Shamai, "Degrees of freedom region for the MIMO X Channel," *IEEE Trans. Information Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.

[2] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom for the K user interference channel," *IEEE Trans. Information Theory*, vol. 54, no. 8, Aug. 2008.

[3] H. Sung, S. H. Park, K. J. Lee, and I. Lee, "Linear precoder designs for K-user interference channels," *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, Jan. 2010.

[4] D. Kim and M. Torlak, "Interference alignment via improved subspace conditioning", *IEEE Globecom 2010*, Miami, FL, Dec. 2010.

[5] M. Shen, A. Host-Madsen, and J. Vidal, "An improved interference alignment scheme for frequency selective channels," *IEEE Int. Symposium on Information Theory*, Toronto, ON, Jul. 2008.

[6] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," *IEEE Globecom*, Nov. 2008.

[7] S. W. Peters and R. W. Heath, "Cooperative algorithms for MIMO interference channels," http://arxiv.org/abs/1002.0424.

[8] C. Suh, D. Tse, and M. Ho, "Downlink interfence alignment," *IEEE Globecom*, Dec. 2010.

[9] C. Suh and D. Tse, "Interfence alignment for cellular networks," *Allerton Conference on Communication, Control and Computing*, 2008.

[10] I. Santamaria, O. Gonzalez, R. Heath, and S. W. Peters. "Maximum sum-rate interference alignment algorithms for MIMO channels," *IEEE Globecom*, Miami, FL, Dec. 2010.