Dynamic Decode and Forward With Network Coding

Wei-Cheng Liu and Yu-Neng Chen
Department of Communications Engineering, National Chung Cheng University, Chia-Yi, Taiwan
Email: comwcliu@ccu.edu.tw, travis_151719@hotmail.com

***Abstract*—This paper is divided into two parts. In the first part, we analyze outage probability of dynamic decode and forward (DDF) with network coding (NC) and compare outage probability with using NC and without using NC. In the second part, we use MATLAB programming language to simulate our system model. We simulate outage probability and compare the simulation result with numerical value of outage probability in the first part to prove the correctness of the analytical outage probability. Our results show that network coding can improve the outage probability of considered system model.**

***Index Terms*—Cooperative communications, dynamic decode and forward, network coding, outage probability.**

I. Introduction

Recently, cooperative communications [1] is interesting in the field of wireless communications. Relays can help the source nodes send information to the destination nodes in a cooperative communications system. Cooperative communications is similar to a virtual multiple-input multiple-output (MIMO) system. Clearly, cooperative communications systems can improve diversity gain which is named cooperative diversity and is similar to the space diversity provided by MIMO systems. Cooperative communications can also help ad hoc networks extend coverage and improve capacity [2]. In the multiuser environment, users can use cooperative communications to share the resource. Cooperative communications has also been proved that it requires less power at source nodes.

As we introduced before, a cooperative communications system can be considered as a virtual MIMO system. Therefore, cooperative communications systems have diversity-multiplexing tradeoff (DMT) [3] characteristics as MIMO systems do. In addition, how to improve the diversity and multiplexing gain is an important issue. Integrating the network coding with the cooperative communications, or named the cooperative network coding (CNC) [4] is generally used at relay nodes. It has the potential for improving the multiplexing gain. When we use XOR at a relay node, it can mix two different bit streams. Clearly, CNC uses less time to transmit data. That means CNC has improvement on multiplexing gain. The outage probability [5] is a general measure of the performance of cooperative communications protocols. Through the analysis of outage probability, we can observe that using "eXclusive OR (XOR)" onto relay node could improve the outage probability compared with the classical cooperative protocols.

Decode and forward (DF) [6] is a generally used protocol in cooperative communications. However, it has a shortcoming in which it must listen to all symbols in a codeword. For this reason, it does not have good performance. In [7], Azarian and Gamal proposed a new protocol called dynamic decode and forward (DDF) to solve this problem.

Because DDF has good performance on cooperative communications, we use this protocol with network coding (NC) and find out the outage probability of our system model. In the end, we use the MATLAB programming language to simulate our system model then compare the simulation outage probability with the analytical values.

This paper is organized as follows. We give the introduction in Section I. In Section II, we give a brief introduction to DDF. In Section III, we describe our system model. In Section IV, we calculate the mutual information of the system model. In Section V, we calculate the outage probability of the system model. We also find the outage probability when our system does not use NC. We show the numerical results in Section VI and make the conclusions in Section VII.

II. A Brief Introduction to DDF

When using DDF protocol, a codeword is assumed has l consecutive symbols. The data rate is R (bits/sec/Hz). Relay node will listen to source node until the mutual information is greater than R and then decode and re-encode the received signal and transmit it to destination. l' is defined as how many symbols the relay will listen,

$$l' = \min\left\{l, \left\lceil \frac{lR}{\log(1+|h|^2\rho)} \right\rceil\right\}, \tag{1}$$

where h is the source-relay channel gain, ρ is the signal-to-noise ratio (SNR) from source node to relay node. The DDF protocol has good performance because it achieves full diversity (i.e., when multiplexing gain is zero, the diversity gain is $d(0) = 2$). When the multiplexing gain $r \in [0, 0.5]$, DDF has the optimal DMT. The diversity-multiplexing tradeoff with one relay node of DDF is given by

$$d(r) = \begin{cases} 2(1-r), & \text{if } 0 \le r \le 1/2, \\ (1-r)/r, & \text{if } 1/2 \le r \le 1. \end{cases} \tag{2}$$

We compare the DMT of DDF with other protocols, including DF, amplify and forward (AF) and 2×1 multiple-input single-output (MISO) in Fig. 1. The DMT of 2×1 MISO is the upper bound of the DMT of all the cooperative communications protocol with one relay node. Fig. 1 shows to us the DMT of DDF is very close to the DMT of 2×1 MISO.

III. System Model

Fig. 2 shows our system model of DDF with NC. The system has one relay node. Terminal A has one transmission antenna and one receive antenna and uses frequency 1 (f_1) to transmit symbols $\{x_{k,a}\}_{k=1}^{l}$ to terminal B and relay node C in phase 1, where a is an information bit stream which is known by terminal A. After encoding and modulation, we turn a into $\{x_{k,a}\}_{k=1}^{l}$. Relay node C only listens to l_1' symbols, where

$$l_1' = \min\left\{l, \left\lceil \frac{lR}{\log(1+|h_{\mathrm{A,C}}|^2\rho)} \right\rceil\right\} \tag{3}$$

and $h_{X,Y}$ is the channel gain between nodes X and Y, where $X, Y \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ and $X \neq Y$. Terminal B has one receive antenna and one transmission antenna and uses frequency 2 (f_2) to transmit symbols $\{x_{k,b}\}_{k=1}^{l}$ to terminal A and relay node C in phase 2, where b is an information bit stream which is known by terminal B. After encoding and modulation, we turn b into $\{x_{k,b}\}_{k=1}^{l}$. Relay node C only listens to l_2' symbols, where

$$l_2' = \min\left\{l, \left\lceil \frac{lR}{\log(1+|h_{\mathrm{B,C}}|^2\rho)} \right\rceil\right\}. \tag{4}$$

We assume that information bit streams a and b are independent. Relay node C has one receive antenna and one transmission antenna. In phase 3, relay node C uses XOR operator to combine two information bit streams a and b. The symbols correspond to $a \oplus b$ can be defined as $\{x_{k,a\oplus b}\}_{k=1}^{l}$. Relay node C uses frequency 3 (f_3) to broadcast symbols $\{x_{k,a\oplus b}\}_{k=1}^{l}$ to terminals A and B. Terminals A and B listen to l_1' and l_2' symbols, respectively. Terminal A can obtain message b via the operation $(a \oplus b) \oplus a = b$ and terminal B can obtain message a via the operation $(a \oplus b) \oplus b = a$. All terminals in the system model are half-duplex which cannot transmit and receive symbol at the same time. All channels are independent and identically distributed (i.i.d.) complex normal random variables (RVs) with zero mean and unit variance. Noise at three terminals node are (i.i.d.) complex normal RVs with zero mean and variance N_0.

A. Signal Model of Phase 1

In phase 1, terminal A transmits symbols $\{x_{k,a}\}_{k=1}^{l}$ to terminal B and relay node C. We denote $y_{(1)\mathrm{C},k}$ as the received signal at relay node C and $y_{(1)\mathrm{B},k}$ as the received signal at terminal B.

$$y_{(1)\mathrm{C},k} = h_{\mathrm{A,C}} x_{k,a} + n_{(1)\mathrm{C},k}, 1 \leq k \leq l_1'. \tag{5}$$

$$y_{(1)\mathrm{B},k} = h_{\mathrm{A,B}} x_{k,a} + n_{(1)\mathrm{B},k}, 1 \leq k \leq l. \tag{6}$$

$n_{(i)X,k}$ is the noise at node X in phase i, where $i \in \{1, 2, 3\}$.

B. Signal Model of Phase 2

In phase 2, terminal B transmits symbols $\{x_{k,b}\}_{k=1}^{l}$ to relay node C and terminal A. We denote $y_{(2)\mathrm{C},k}$ as the received signal at relay node C and $y_{(2)\mathrm{B},k}$ as received signal at terminal B.

$$y_{(2)\mathrm{C},k} = h_{\mathrm{B,C}} x_{k,b} + n_{(2)\mathrm{C},k}, 1 \leq k \leq l_2'. \tag{7}$$

$$y_{(2)\mathrm{B},k} = h_{\mathrm{B,A}} x_{k,b} + n_{(2)\mathrm{A},k}, 1 \leq k \leq l. \tag{8}$$

C. Signal Model of Phase 3

In phase 3, relay node C transmits symbols $\{x_{k,a\oplus b}\}_{k=1}^{l}$ to terminal A and terminal B. We denote $y_{(3)\mathrm{A},k}$ as the received signal at terminal A and $y_{(3)\mathrm{B},k}$ as the received signal at terminal B.

$$y_{(3)\mathrm{A},k} = h_{\mathrm{C,A}} x_{k,a\oplus b} + n_{(3)\mathrm{A},k}, 1 \leq k \leq l_1'. \tag{9}$$

$$y_{(3)\mathrm{B},k} = h_{\mathrm{C,B}} x_{k,a\oplus b} + n_{(3)\mathrm{B},k}, 1 \leq k \leq l_2'. \tag{10}$$

IV. Mutual Information of System Model

A. Definition of Mutual Information

The mutual information between two discrete random variables X and Y is denoted by $I(X;Y)$ or $I(Y;X)$. Mutual information is a useful concept to measure the amount of information shared between input and output of noisy channels.

$$I(X;Y) = H(X) - H(X|Y). \tag{11}$$

$$I(Y;X) = H(Y) - H(Y|X). \tag{12}$$

$H(X)$ is the measurement of the amount of uncertainty associated with the value of discrete random variable X. $H(X|Y)$ is the conditional uncertainty of X given random variable Y.

$I(X;Y)$ and $I(Y;X)$ can also be defined as

$$I(X;Y) = I(Y;X) = \sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p(x,y) \log\left(\frac{p(x,y)}{p_X(x)p_Y(y)}\right), \tag{13}$$

where $\mathcal{X}$ and $\mathcal{Y}$ are the alphabets of X and Y, respectively. $p(x,y)$ is the joint probability density function (PDF) of X and Y, and $p_X(x)$ and $p_Y(y)$ are the marginal PDFs of X and Y, respectively.

B. Mutual Information of System Model

In this section, we calculate the maximum mutual information from terminal A to terminal B as (14). Because we transmit two independent information bit streams a and b in three phases and one codeword has l symbol intervals, there is a factor $\frac{2l}{3}$.

$$I_{\mathrm{A,B}} = \frac{2l}{3} \cdot \log(1 + |h_{\mathrm{A,B}}|^2 \mathrm{SNR}) \tag{14}$$

and maximum mutual information from terminal A to relay node C as (15). Because we transmit two independent information bit streams a and b in three phases and one codeword has l_1' symbol intervals, there is a factor $\frac{2l_1'}{3}$.

$$I_{\mathrm{A,C}} = \frac{2l_1'}{3} \cdot \log(1 + |h_{\mathrm{A,C}}|^2 \mathrm{SNR}). \tag{15}$$

The maximum mutual information from terminal B to relay node C is

$$I_{\text{B,C}} = \frac{2l_2'}{3} \cdot \log(1 + |h_{\text{B,C}}|^2 \text{SNR}). \tag{16}$$

V. Outage Probability of System Model

A. Definition of Signal-to-Noise Ratio

Signal-to-noise ratio (SNR) is the ratio between signal power and noise power. It is defined in (17), where $\text{E}[Z]$ is the expected value of random variable Z and $c \in \{a, b, a \oplus b\}$.

$$\text{SNR} = \frac{\text{E}[|x_{k,c}|^2]}{N_0}. \tag{17}$$

B. Slow Fading Channel

The condition of slow fading channel is that the coherence time is greater than the symbol duration. This is also called the *quasi-static scenario* [8]. The maximum rate of slow fading channel is $\log(1 + |h|^2\text{SNR})$ bits/sec/Hz. The transmitter encodes data at a data rate R bits/sec/Hz. The outage probability can be defined as $\text{P}[\log(1 + |h|^2\text{SNR}) < R]$. If the decoding error cannot be arbitrarily small, regardless of which code is used by the transmitter, we can name this event as outage. Therefore, the optimal transmitter can assume that the channel gain is large enough to support the data rate R when it encodes the data. For Rayleigh fading (i.e., h is a complex normal RV with zero mean and unit variance), $|h|^2$ can be seen as an exponential RV with unit mean. We calculate the outage probability as follows

$$\begin{aligned}\text{P}[\log(1 + |h|^2\text{SNR}) < R] &= \text{P}\left[|h|^2 < \frac{2^R - 1}{\text{SNR}}\right] \\ &= \int_0^{\frac{2^R-1}{\text{SNR}}} e^{-x} dx \\ &= 1 - \exp\left(-\frac{2^R - 1}{\text{SNR}}\right). \end{aligned} \tag{18}$$

C. Outage Probability of System Model

The outage probability of system model is analyzed in the follow theorem:

Theorem 1: The outage probability of system model is

$$P_{\text{out}}^{\text{NC}} = \int_0^\infty \int_0^\infty p_1(p_2 + p_3 - p_2 p_3) e^{-u-v} du dv, \tag{19}$$

where

$$p_1 = 1 - \exp\left(-\frac{2^{\frac{3R}{2}} - 1}{\text{SNR}}\right), \tag{20}$$

$$p_2 = 1 - \exp\left(-\frac{2^{\frac{3lR}{2l_1'}} - 1}{\text{SNR}}\right), \tag{21}$$

and

$$p_3 = 1 - \exp\left(-\frac{2^{\frac{3lR}{2l_2'}} - 1}{\text{SNR}}\right). \tag{22}$$

Proof: We denote $E_{X,Y}$ as the outage event of the link between nodes X and Y. Clearly, $E_{X,Y} = E_{Y,X}$. Events $E_{\text{A,B}}, E_{\text{A,C}}$ and $E_{\text{B,C}}$ are independent. Moreover,

$$\begin{aligned}\text{P}[E_{\text{A,B}}] &= \text{P}\left[\frac{2l}{3}\log(1 + |h_{\text{A,B}}|^2\text{SNR}) < lR\right] \\ &= \text{P}\left[|h_{\text{A,B}}|^2 < \frac{2^{\frac{3R}{2}} - 1}{\text{SNR}}\right] \\ &= 1 - \exp\left(-\frac{2^{\frac{3R}{2}} - 1}{\text{SNR}}\right) = p_1. \end{aligned} \tag{23}$$

Moreover,

$$\begin{aligned}\text{P}[E_{\text{A,C}}] &= \text{P}\left[\frac{2l_1'}{3}\log(1 + |h_{\text{A,C}}|^2\text{SNR}) < lR\right] \\ &= \text{P}\left[|h_{\text{A,C}}|^2 < \frac{2^{\frac{3lR}{2l_1'}} - 1}{\text{SNR}}\right] \\ &= 1 - \exp\left(-\frac{2^{\frac{3lR}{2l_1'}} - 1}{\text{SNR}}\right) = p_2. \end{aligned} \tag{24}$$

Similarly,

$$\text{P}[E_{\text{B,C}}] = 1 - \exp\left(-\frac{2^{\frac{3lR}{2l_2'}} - 1}{\text{SNR}}\right) = p_3. \tag{25}$$

The outage event of node A is

$$E_{\text{A}} = E_{\text{B,A}} \cap E_{\text{C,A}}. \tag{26}$$

The equation means outage event of node A will occur when events $E_{\text{B,A}}$ and $E_{\text{C,A}}$ both occur. Similarly, the outage event of node B can be written as

$$E_{\text{B}} = E_{\text{A,B}} \cap E_{\text{C,B}}. \tag{27}$$

The outage event of the overall system can be written as

$$E_{\text{out}}^{\text{NC}} = E_{\text{A}} \cup E_{\text{B}} = E_{\text{A,B}} \cap (E_{\text{A,C}} \cup E_{\text{B,C}}). \tag{28}$$

Let $U = |h_{\text{A,C}}|^2$ and $V = |h_{\text{B,C}}|^2$. The outage probability of system model is

$$\begin{aligned}P_{\text{out}}^{\text{NC}} &= \text{E}_{U,V}[\text{P}[E_{\text{A,B}} \cap (E_{\text{A,C}} \cup E_{\text{B,C}})]] \\ &= \text{E}_{U,V}[\text{P}[E_{\text{A,B}}](\text{P}[E_{\text{A,C}}] + \text{P}[E_{\text{B,C}}] - \text{P}[E_{\text{A,C}} \cap E_{\text{B,C}}])] \\ &= \text{E}_{U,V}[p_1(p_2 + p_3 - p_2 p_3)] \\ &= \int_0^\infty \int_0^\infty p_1(p_2 + p_3 - p_2 p_3) e^{-u-v} du dv \end{aligned} \tag{29}$$

because U and V are i.i.d. exponential RVs with unit mean. ■

D. Outage Probability Without Using NC

In this subsection, we calculate the outage probability of system model without NC. The outage probability of system model without NC is analyzed in the following theorem:

Theorem 2: The outage probability of system model without network coding is

$$P_{\text{out}}^{\text{Non-NC}} = 1 - \exp\left(-\frac{2^{\frac{3R}{2}} - 1}{\text{SNR}}\right). \tag{30}$$

Proof: We assume relay node C can only transmit the information bit stream a to terminal A and terminal B in phase 3. The outage event of node A is

$$E_{\text{A}} = E_{\text{A,B}}. \tag{31}$$

The outage event of node B is

$$E_{\text{B}} = E_{\text{A,B}} \cap E_{\text{C,B}}. \tag{32}$$

The outage event of system model without network coding is

$$E_{\text{out}}^{\text{Non-NC}} = E_{\text{A}} \cup E_{\text{B}} = E_{\text{A,B}} \cup (E_{\text{A,B}} \cap E_{\text{C,B}}) = E_{\text{A,B}}. \tag{33}$$

The outage probability of the overall system model without network coding is

$$\text{P}[E_{\text{out}}^{\text{Non-NC}}] = P[E_{\text{A,B}}] = 1 - \exp\left(-\frac{2^{\frac{3R}{2}} - 1}{\text{SNR}}\right). \tag{34}$$

■

VI. Numerical Results

In this section, we set $l = 5$. That is, a codeword has 5 consecutive symbols.

A. Comparison of the Outage Probabilities With and Without Using NC

In this subsection, we compare the outage probabilities with and without using NC for data rates $R = 1, 2, 3$ bits/sec/Hz. Fig. 3 shows the results, which are calculated using (19) and (30). In this figure, the outage probability without using NC is greater than that of using NC. This proves that using NC can improve the performance of the considered system model. Moreover, when data rate is getting larger, outage probability is also larger.

B. Comparison of Simulation Outage Probabilities With and Without Using NC

In this subsection, we compare simulation outage probabilities with and without NC and show the results in Fig. 4. The results display that outage probability of the system with NC is smaller than that of without NC in a large SNR (dB) region. This outcome shows us that the outage probability of system with NC is lower than that of without NC.

C. Compare Simulation With Analytical Outage Probability

In this subsection, we compare simulation with analytical outage probability. Fig. 5 shows the result when data rate is 3 bits/sec/Hz. We find that the simulation values and analytical values of outage probability are very close. Fig. 5 also shows us the correctness of our calculations of outage probabilities with and without NC in Section V.

VII. Conclusions

In this paper, we analyze the outage probability of DDF with NC and compare the cases of using NC with not using NC. We prove that using network coding can improve the performance of system model which we design. We find that the outage probability of not using NC is larger than using NC. Simulation values of outage probability with and without NC also prove our correctness of calculations of outage probabilities with and without NC.

References

[1] K. J. R. Liu, A. K. Sadek, W. Su, and A. Kwasinski, *Cooperative communications and networking*. Cambridge: Cambridge University Press, 2009.

[2] S. J. Kim, P. Mitran, and V. Tarokh, "Performance bounds for bi-directional coded cooperation protocols," in *27th Int. Conf. Distrib. Comput. Syst. Workshops (ICDCSW'07)*, Toronto, Canada, Jun. 22–29, 2007, p. 83.

[3] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[4] L.-C. Wang, W.-C. Liu, and S.-H. Wu, "Diversity-multiplexing tradeoff analysis of a cooperative network coding system," in *IEEE Sarnoff Symp.*, Mar. 30–Apr. 1, 2009, pp. 1–5.

[5] S. Karmakar and M. Varanasi, "Diversity-multiplexing tradeoff of the dynamic decode and forward protocol on a MIMO half-duplex relay channel," in *IEEE Inter. Symp. Inf. Theory*, Jun. 28–Jul. 3, 2009, pp. 1443–1447.

[6] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[7] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity–multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.

[8] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. Cambridge, U.K.: Cambridge University Press, 2005.

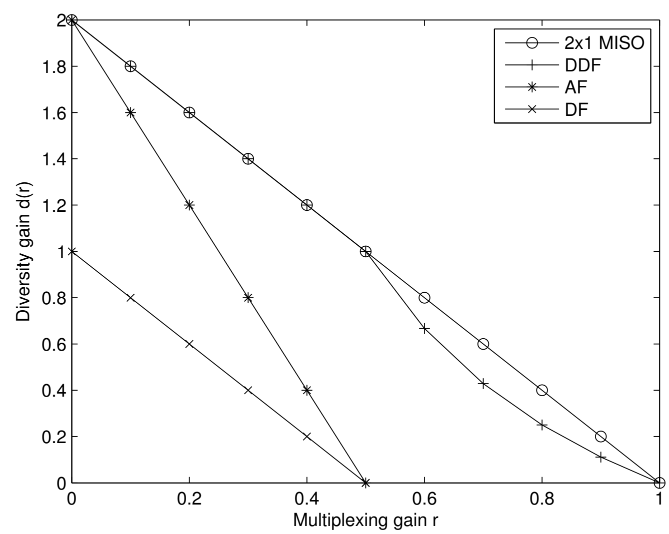

Fig. 1. Comparison of diversity-multiplexing tradeoff of DDF with 2×1 MISO, AF, and DF.

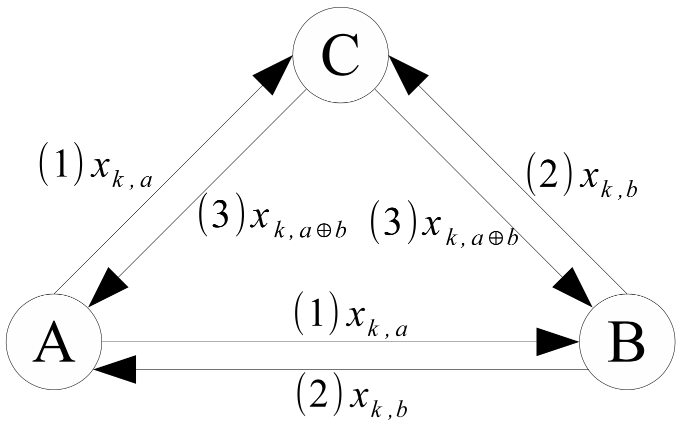

Fig. 2. System model of DDF with network coding.

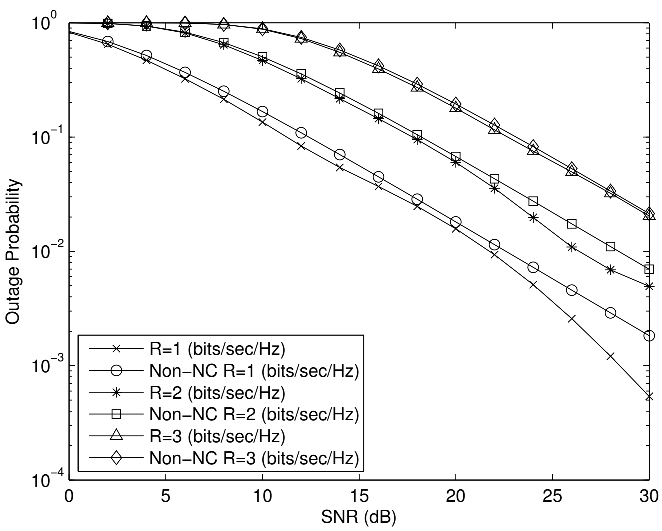

Fig. 3. Comparison of the outage probabilities with and without using NC.

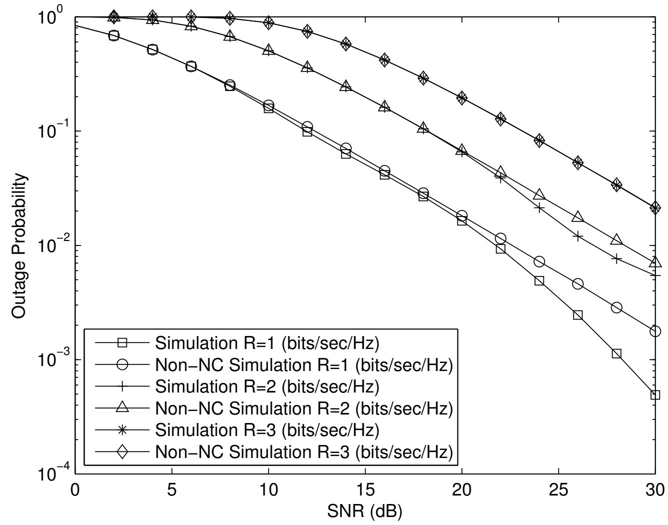

Fig. 4. Simulation of outage probabilities with and without network coding.

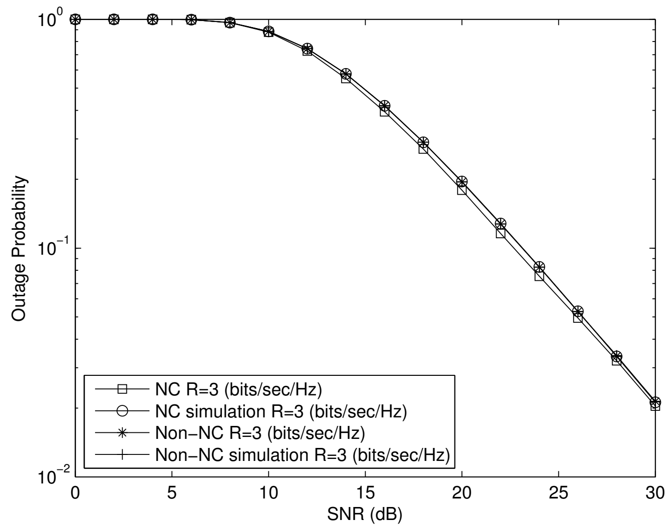

Fig. 5. Compare simulation values with analytical values.