Subcarrier Power Allocation in OFDM with Low Precision ADC at Receiver

Tapan Shah
School of Technology and Computer Science
Tata Institute of Fundamental Research
Mumbai-400005, India
Email: tapanshah@tifr.res.in

Onkar Dabeer
School of Technology and Computer Science
Tata Institute of Fundamental Research
Mumbai-400005, India
Email: onkar@tcs.tifr.res.in

***Abstract*—Orthogonal frequency division multiplexing (OFDM) has been incorporated in standards/draft standards such as IEEE 80.15.3c, IEEE 802.11ad for building multi-Gigabit systems operating in a few GHz of bandwidth. The digital implementation of the receivers for such a system is challenging because high precision (6+ bits/sample) analog-to-digital conversion (ADC) at such high speeds is power hungry and expensive. In this paper, we show that by suitable subcarrier power allocation we can get good performance even with low precision ADC (1-4 bits/sample without oversampling). We derive an analytical expression for the uncoded SER of an M-QAM OFDM system with finite precision ADC. By accounting for automatic gain control (AGC), we show that equal received subcarrier power (ERSP) leads to less quantization noise power than equal transmit subcarrier power (ETSP). Furthermore, for high SNR, ERSP has a lower symbol error rate (SER) than ETSP. But for lower SNR, ETSP is better, and hence we also use convex combinations of ETSP and ERSP power allocations. We illustrate the accuracy of our analytical results with simulations for the Saleh-Valenzuela channel model with log-normal fading. Our results show that for 16-QAM, at SER of 0.01, with a 3-bit ADC and a combination of ERSP and ETSP, we can come within 1 dB of ETSP of the full precision case (while ETSP with 3-bit ADC has a SER floor above 0.04).**

I. INTRODUCTION

Several standards groups such as IEEE 802.11.ad [1], 802.15.3c [2] are furnishing designs for multi-Gbps wireless communication systems. Since modern receivers implement most of their functionality in the digital domain, the analog-to-digital converter (ADC) is an important component of the receiver architecture. However, since high precision ADCs for giga-samples-per-second rates are power hungry and expensive [3], for multi-Gbps systems exploiting few GHz of bandwidth, the ADC becomes a bottleneck. A similar power consumption issue also applies to the digital-to-analog converter (DAC) at the transmitter. In this paper, we consider a transmitter with substantial power resources, such as an access point, transmitting at multi-Gbps rates to a handheld device, which has more stringent power requirements. Hence, we focus on the ADC power consumption at the receiver and do not address the power consumption of the DAC at the transmitter.

This work was supported by a grant from the Ministry of Communications and Information Technology, Govt. of India.

To reduce the power consumption of the ADC, we would like to use a lower precision (1-4 bits/sample without oversampling). But in a conventional receiver this leads to severe loss of performance (see Fig. 1). To improve performance, we study the role of subcarrier power allocation in alleviating this performance degradation. We show that with appropriate power allocation, even with low precision sampling, we can get performance within 1 dB (at an SER of 1%) of a traditional OFDM system with full precision sampling. In the remainder of this section, we briefly outline prior work and our contributions.

A. Prior Work

The ADC bottleneck for multi-Gbps communication systems is well acknowledged and several authors have addressed the implementation of specific receiver functions using low precision ADCs. These works suggest several ways to deal with the ADC nonlinearity and we review them below.

In an OFDM system, due to the severe nonlinearity of a low precision ADC, a traditional OFDM receiver cannot cancel inter-carrier interference and we get an error floor (see Fig. 1). For ultrawideband signals with narrowband interference, a modified receiver front end is suggested in [4], which gives acceptable performance even with 1-bit precision ADC. Motivated by this work, we studied analog interference cancellation for OFDM receivers in [5]. But such an approach requires significant hardware changes to existing OFDM system design.

In [6], it was shown that with a dither signal based on feedback, near full precision performance can be obtained for channel estimation using just 3-bits of precision. The basic idea of dithering has a long history (see for example [7], [8] and references therein) and it is natural to ask if it can be exploited for OFDM demodulation. We have not had much success with this approach.

Instead, in this paper, we explore a third approach based on subcarrier power allocation, which has yielded us good results. Typically in an OFDM system, equal transmit subcarrier power (ETSP) is used. However, due to the wide channel gain variations across the subcarriers, usage of low precision ADC only captures information from a few strong subcarriers. We ask: can an appropriate unequal power allocation alleviate the problem? Such power allocation is motivated by results in

978-1-4673-1881-5/12/$31.00 ©2012 IEEE

[9], [10], [11], which suggest that for full precision sampling, the symbol error rate (SER) is minimized for large SNR by equal received subcarrier power (ERSP). Our results, analysis as well as simulations, show that a combination of ETSP and ERSP indeed allows us to approach near full precision performance with low precision samples. Such power allocation requires channel state information at the transmitter, which is reasonable in the low-mobility applications targeted by IEEE 802.11ad/IEEE 802.15.3c.

B. Main Results

Since the problem of finding SER minimizing power allocation under finite precision ADC is unwieldy, to gain insight, we focus on the case of ETSP, ERSP, and their combinations. We find analytical expressions for uncoded SER of an M-QAM OFDM system with ETSP and ERSP with finite precision ADC at the receiver (Proposition, Parts 1 and 2 , Section III).

We prove that the variance of the quantization noise in the frequency domain is smaller under ERSP (Proposition, Part 3, Section III). Using this, we prove that for high SNR, the uncoded SER of a M-QAM OFDM system with ERSP is smaller than that with ETSP with a finite precision ADC at receiver (Proposition, Part 4, Section III). However, ETSP is better than ERSP for lower SNR values. The use of combinations of ETSP and ERSP power allocations allows us to achieve a better tradeoff between low and high SNR performance.

We also state simulation results to illustrate the improvement in performance of OFDM with ERSP with low precision ADC at receiver and compare them with our analytical results (Section IV). We use the Saleh-Valenzuela model [12] for the channel as suggested in IEEE 802.15.3c [2]. We find that by a suitable combination of ETSP and ERSP power allocations, even with a 3-bit ADC and a traditional OFDM receiver, at an SER of 0.01, we are within 1 dB of ETSP with full precision. The combination saves 3 dB over ERSP with 3-bit ADC and for this case ETSP with 3-bit precision has an error floor above 0.04.

The conclusion is given in Section V and the proofs are given in the Appendix.

II. System Description

A. ISI Channel model

We consider an ISI channel in which the resolved multipath components are grouped into L_c clusters, each having L_i rays. The time domain channel impulse response is given by

$$h(t) = \sum_{c=0}^{L_c-1} \sum_{i=0}^{L_i-1} h_{c,i}\delta\left(t - T_c - \tau_{c,i}\right),$$

where $h_{c,i}$ is the tap weight of the i-th ray of the c-th cluster, T_c is the delay of c-th cluster and $\tau_{c,i}$ is the delay of the i-th cluster relative to T_c. While our results are valid for any statistical channel model, we use the Saleh-Valenzuela (S-V) model [12] since this is preferred in IEEE 802.15.3c [13]. According to this model,

$$f\left(T_c|T_{c-1}\right) = \Lambda \exp\left[-\Lambda\left(T_c - T_{c-1}\right)\right], \quad c > 0$$
$$f\left(\tau_{c,i}|\tau_{c,(i-1)}\right) = \lambda \exp\left[-\lambda\left(\tau_{c,i} - \tau_{c,(i-1)}\right)\right], \quad i > 0$$

where Λ and λ are the cluster and ray arrival rate, respectively, and $f(\cdot)$ is the probability density function. Also, the mean square power of the tap weights are

$$\mathrm{E}[|h_{c,i}|^2] = \mathrm{E}[|h_{0,0}|^2]\exp\left(-\frac{T_c}{\Gamma}\right)\exp\left(-\frac{\tau_{c,i}}{\gamma}\right),$$

where Γ and γ are the cluster and ray decay rates, respectively. Since we are in the wideband regime, the distribution of the channel taps is modelled by a lognormal distribution [13].

The receiver implements a front end filter of sufficient bandwidth and then samples the received analog signal uniformly. The impulse response of the resultant discrete channel is represented by the vector $\mathbf{h} = [h_0 \dots h_{L-1}]$.

B. Traditional OFDM System with Equal Transmit Subcarrier Power (ETSP)

Let N be the number of subcarriers in an OFDM symbol. Let $\mathbf{u} = [u_0 \dots u_{N-1}]$ be the input symbol block. The symbol u_k is mapped to k-th subcarrier. As is common, we assume that $\{u_k\}_{k=0}^{N-1}$ are i.i.d. and chosen from the same constellation. We assume that $\forall k$, $\mathrm{E}[|u_k|^2] = 1$. This input block is transformed to the time domain using a N-point IDFT, $\mathbf{x} = \mathbf{F}^\dagger\mathbf{u}$, where $\mathbf{F}$ is the N point IDFT matrix and $(\cdot)^\dagger$ denotes the Hermitian of a matrix. A cyclic prefix of length L is appended and $\mathbf{x}$ is transmitted over a channel described in II-A. At the receiver, if the received signal is sampled with full precision, then the received vector $\mathbf{r}$ (after removing the cyclic prefix) can be expressed as

$$r_n = \sum_{l=0}^{L-1} h_l x_{n-l} + v_n, \quad L \leq n \leq N + L - 1.$$

where the additive Gaussian noise $\{v_n\}_{n=0}^{N+L-1}$ is i.i.d. $\mathcal{CN}\left(0, \sigma^2\right)$. In vector notation, we write $\mathbf{r} = \mathbf{Cx} + \mathbf{v}$, where $\mathbf{C}$ is circulant. The receiver then applies the DFT:

$$\mathbf{y} = \mathbf{Fr} = \mathbf{FCx} + \mathbf{Fv}.$$

Using the following facts:

- $\mathbf{D} := \mathbf{FCF}^\dagger$ is a $N \times N$ diagonal matrix with the diagonal entries $\{d_k\}_{k=0}^{N-1}$

$$d_k = \sum_{l=0}^{L-1} h_l \exp\left(\frac{-2j\pi lk}{N}\right), \tag{1}$$

- $\mathbf{w} := \mathbf{Fv}$ are i.i.d. $\mathcal{CN}(0, \sigma^2)$,

we can write $\mathbf{y} = \mathbf{Du} + \mathbf{w}$. We assume that $\{h_n\}$'s and hence $\{d_k\}$'s are known. This can be achieved, for example, using the technique in [6].

C. OFDM receiver with finite precision ADC

If at the receiver, the received signal is sampled and quantized with a finite precision ADC, the quantized received vector $\mathbf{r}^q$ is expressed as

$$r_n^q = A\left(\sum_{l=0}^{L-1} h_l x_{n-l} + v_n\right), \quad L \le n \le N+L-1, \quad (2)$$

where $A(\cdot)$ denotes the nonlinear analog-to-digital conversion [1]. In vector notation, we write $\mathbf{r}^q = A(\mathbf{Cx}+\mathbf{v})$, where $A(\cdot)$ is applied elementwise. The ADC $A(\cdot)$ is defined by following two parameters.

$Resolution\ \nu$: If the resolution is ν bits, then the real and imaginary parts are each quantized to 2^ν levels.

$Range$: We assume that $A(\cdot)$ has a constant range of $(-1,+1)$. If the sampled signal exceeds this range, then it is clipped. In practice, an AGC block, with gain G_n at time instant n, is used prior to the quantization to ensure that clipping occurs with low probability.

Modeling Quantization Noise: If N is large enough, it is reasonable to assume that $\{x_n\}_{n=0}^{N+L-1}$ are samples of a zero mean Gaussian process [14], and thus r_n, which is a linear combination of $\{x_n\}$'s and w_n, is a Gaussian random variable with zero mean and variance $\sigma_r^2 = \mathrm{E}[|r_n|^2]$. As the input to the quantizer is Gaussian, we can use the model proposed in [15] to model the quantization noise. However, for cases where clipping ratio ($\mu = \frac{1}{G_n\sqrt{E[|r_n|^2]}}$) $> \mu_{opt}$[2], from [15] we know that the PQN model (Chapter 4 of [16]) is a good approximation. Using the PQN model, we can write (2) as

$$r_n^q = \sum_{l=0}^{L-1} h_l x_{n-l} + v_n + q_n, \quad L \le n \le N+L-1, \quad (3)$$

where $\mathrm{E}[q_n]=0$, $\mathrm{E}[|q_n|^2] = \frac{1}{G_n^2}\frac{2^{-2\nu}}{6}$ and q_n is independent of r_n. In vector notation, we can write $\mathbf{r}^q = \mathbf{Cx}+\mathbf{v}+\mathbf{q}$. The quantized samples $\{r_n^q\}_{n=L}^{N+L-1}$ are transformed to the frequency domain using a N-point DFT:

$$\mathbf{y}^q = \mathbf{F}\mathbf{r}^q = \mathbf{Du}+\mathbf{w}+\mathbf{p}, \quad (4)$$

where $\mathbf{p} := \mathbf{Fq}$ and are modeled as zero mean, complex, independent Gaussian random variables with variance[3]

$$\begin{aligned}\sigma_q^2 := \mathrm{E}[|p_k|^2] &= \frac{1}{N}\sum_{n=0}^{N-1}\mathrm{E}[|q_n|^2] \\ &= \frac{1}{N}\left(\sum_{n=0}^{N-1}\frac{1}{G_n^2}\right)\frac{2^{-2\nu}}{6}, \quad (5)\end{aligned}$$

where (5) follows by substituting the value of $\mathrm{E}[|q_n|^2]$.

[1]For a complex signal $s = s_R + js_I$, $A(s) \equiv A(s_R) + jA(s_I)$.

[2]For large SNRs, μ_{opt} depends on resolution ν and varies between 6-11 dB.

[3]Here we assume p_k and p_l are uncorrelated $\forall k,l$. Although this is not strictly true and we can find the exact expression without making this assumption, we prefer it for its simplicity. Further, it still leads to fairly accurate analytical prediction.

Choice of AGC Gain: For our analysis and simulations, we take

$$G_n^2 = \frac{\alpha}{\mathrm{E}[|r_n|^2]}, \quad (6)$$

where α is such that $\mu \approx \mu_{opt}$ ([15]).

D. OFDM System With Equal Receiver Subcarrier Power (ERSP)

In order to ensure that each subcarrier has equal received power, the input symbols $\{u_k\}$'s is transformed as $\tilde{u}_k = u_k\sqrt{\left(\frac{N|d_k|^{-2}}{\sum_{i=0}^{N-1}|d_i|^{-2}}\right)}$. In vector notation, we can write

$$\tilde{\mathbf{u}} = \sqrt{\frac{N}{\sum_{m=0}^{N-1}|d_m|^{-2}}}\left(\mathbf{DD}^\dagger\right)^{-1/2}\mathbf{u}. \quad (7)$$

The normalization is made so that the total average input power remains the same. Thus, we get system equations of OFDM with ERSP from those of OFDM with ETSP by replacing $\mathbf{u}$ with $\tilde{\mathbf{u}}$.

III. Main Results

In this section, we analyze the uncoded symbol error rate of the OFDM system under finite precision ADC and compare the cases of ETSP and ERSP. Our main result is the following.

Proposition: Under the system modeling assumptions stated in Section II, the following are true.

1) The average SER for M-QAM in an uncoded OFDM system with equal transmit subcarrier power (ETSP) is given by[4]

$$S|_{ETSP} = 4\mathrm{E}[P] + \mathrm{E}[O(P^2)], \quad (8)$$

where

$$\begin{aligned}P = &\left(1-\frac{1}{\sqrt{M}}\right) \\ &\times \frac{1}{N}\sum_{k=0}^{N-1} Q\left(\sqrt{\left(\frac{3}{M-1}\right)\frac{|d_k|^2}{\sigma^2+\sigma_{q,ETSP}^2}}\right).\end{aligned} \quad (9)$$

2) The average SER for M-QAM in an uncoded OFDM system with ERSP (described in II-D) is given by

$$S|_{ERSP} = 4\mathrm{E}[P'] + \mathrm{E}[O(P'^2)], \quad (10)$$

where

$$P' = \left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\left(\frac{3}{M-1}\right)\frac{|d_{ERSP}|^2}{\sigma^2+\sigma_{q,ERSP}^2}}\right) \quad (11)$$

and $|d_{ERSP}|^2 = \frac{N}{\sum_{m=0}^{N-1}|d_k|^{-2}}$.

3) The variance of the quantization noise σ_q^2 in frequency domain, defined in (5), is smaller with ERSP i.e.

$$\sigma_{q,ETSP}^2 \ge \sigma_{q,ERSP}^2. \quad (12)$$

[4]The expectation is over the subcarrier channel gains $\{d_k\}_{k=0}^{N-1}$.

4) Suppose all the arguments of $Q(\sqrt{\cdot})$ in (9) and (11) are > 3 (i.e. the subcarrier SNRs are sufficiently high), then

$$P' < P. \tag{13}$$

The proof is given in the Appendix.

We find that the analytical expressions in part 1) and 2) of the above result are quite accurate even if we ignore the higher order term. For example, in Fig.1, we see a close match between the simulations and the analytical approximation ignoring the higher order terms.

Comparing ETSP and ERSP: Part 3) states that the quantization noise in the frequency domain has a lower variance under ERSP. Under the additional assumption of sufficiently high SNR, part 4) states that ERSP leads to a smaller SER than ETSP. However, in Section IV we see that for lower SNR, ETSP is better. Hence in Section IV, we consider a combination of ETSP and ERSP power allocation such that the transmitted power on subcarrier k is given by

$$\beta \mathrm{E}[|u_k|^2] + (1-\beta)\mathrm{E}[|\tilde{u}_k|^2], \tag{14}$$

where $0 \leq \beta \leq 1$. We expect that for a suitable $0 < \beta < 1$, we can get good performance in the intermediate SNR regime.

IV. Simulation Results

A. Setup

The values of the OFDM symbol and channel model (defined in II-A) parameters used in simulations are specified in [2] and [13], respectively. For the results reported here, the following values were used for parameters defined in Section II.

- $N = 512$, Length of cyclic prefix$= 64$
- $\Lambda = 0.037\ s^{-1}$, $\lambda = 0.641\ s^{-1}$
- $\Gamma = 21.1\ s$, $\gamma = 7.69\ s$
- Cluster lognormal standard deviation, $\sigma_c = 3.01$
- Ray lognormal standard deviation, $\sigma_r = 7.69$
- $L_c = 7\Gamma$, $L_i = 5\gamma$

We use a uniform quantizer for our simulations.

B. Simulation Results and Discussion

In Fig. 2, we compare ETSP (full precision and 3-bit ADC), ERSP (full precision and 3-bit ADC), and a convex combination of ETSP and ERSP with $\beta = 0.15$ in (14) (full precision and 3-bit ADC). As expected, the power combination case with 3-bit ADC is worse than the corresponding full precision case. However, at an uncoded SER of 0.01, it operates within 1 dB of ETSP with full precision sampling. In comparison, ETSP with 3-bit ADC has an error floor above 0.04 and ERSP with 3-bit ADC is about 3 dB worse. For 4-QAM and 64-QAM we observe similar results using 2 and 4 bit precision ADCs respectively. We note that at smaller SNRs, the gap between full precision ETSP and combination of ETSP and ERSP with $\beta = 0.15$ and 3-bit ADC is larger than 1 dB. But this can be reduced with a SNR dependent β. For example, with $\beta = 0.65$ we can reduce the gap to 1 dB at SNR 10 dB. Due to space constraints, we do not discuss the choice of β more, but in a future journal manuscript we plan to add more details.

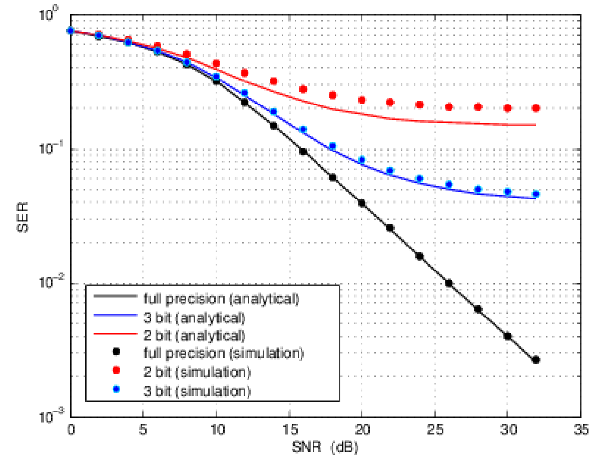

Fig. 1. Comparison of performance of 16-QAM OFDM system with ETSP with different ADC precision at the receiver.

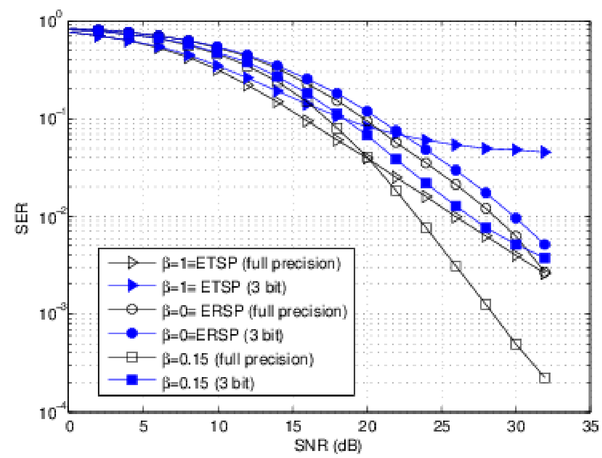

Fig. 2. Comparison of performance of receivers for 16-QAM OFDM with ERSP, ETSP and a combination of them.

V. Conclusion

The main message of this paper is that subcarrier power control can significantly reduce the error floor for an OFDM system with finite precision ADC. In particular, using a convex combination of ETSP and ERSP power allocations, we can come within 1 dB of ETSP with full precision sampling using only 3-bit precision ADC at an SER of 1%. We also present accurate analytical expression for the SER under finite precision sampling. In particular, these analytical expressions are useful to show that ERSP is better than ETSP at high SNRs.

Appendix
Proof of Proposition

Let ρ_k be the received SNR of subcarrier k of an OFDM system. Using the exact expression for symbol error rate (SER) for M-QAM in [17], we get the SER for subcarrier

k conditioned on the channel gain d_k:

$$S^k|_{d_k} = 4P_k + 4P_k^2, \tag{15}$$

where

$$P_k = \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\left(\frac{3}{M-1}\right)\rho_k}\right).$$

From (4), we can write

$$\rho_k\big|_{ETSP} = \frac{|d_k|^2 \mathrm{E}[|u_k|^2]}{\sigma^2 + \sigma^2_{q,ETSP}} = \frac{|d_k|^2}{\sigma^2 + \sigma^2_{q,ETSP}}. \tag{16}$$

Similarly, using (7), we can write

$$\begin{aligned}\rho_k\big|_{ERSP} &= \frac{|d_k|^2 \mathrm{E}[|\tilde{u}_k|^2]}{\sigma^2 + \sigma^2_{q,ERSP}} = \frac{N|d_k|^2|d_k|^{-2}\mathrm{E}[|u_k|^2]}{\left(\sum_{m=0}^{N-1}|d_k|^{-2}\right)} \\ &\times \frac{1}{\left(\sigma^2 + \sigma^2_{q,ERSP}\right)} = \frac{N}{\sum_{m=0}^{N-1}|d_k|^{-2}} \frac{1}{\left(\sigma^2 + \sigma^2_{q,ERSP}\right)}.\end{aligned} \tag{17}$$

In both (16) and (17), we use the assumption that $\mathrm{E}[|u_k|^2] = 1$. For a given channel, the SER of an OFDM system conditioned on $\{d_k\}_{k=0}^{N-1}$ is

$$S|_{\{d_k\}} = \frac{1}{N}\sum_{k=0}^{N-1} S^k|_{d_k}. \tag{18}$$

Proof of part 1): Substituting values of $S^k|d_k$ and $\rho_k\big|_{ETSP}$ from (15) and (16), respectively, into (18), observing that $\frac{1}{N}\sum_{l=0}^{N-1} P_k^2 \leq N\left(\frac{1}{N}\sum_{l=0}^{N-1} P_k\right)^2$ and taking expectation over $\{d_k\}_{k=0}^{N-1}$, we get (8).

Proof of part 2): Substituting values of $S^k|d_k$ and $\rho_k\big|_{ERSP}$ from (15) and (17), respectively, into (18) and taking expectation over $\{d_k\}_{k=0}^{N-1}$, we get (10).

Proof of part 3): For OFDM with ETSP, we have $\mathbf{Fr} = \mathbf{Du} + \mathbf{w}$ or $\mathbf{r} = \mathbf{F}^\dagger\mathbf{Du} + \mathbf{F}^\dagger\mathbf{w}$. Using the assumption that $\mathrm{E}[\mathbf{uu}^\dagger] = \mathbf{I}$ and $\mathrm{E}[\mathbf{F}^\dagger\mathbf{ww}^\dagger\mathbf{F}] = \sigma^2\mathbf{I}$, where $\mathbf{I}$ is $N \times N$ identity matrix, we can write $\mathrm{E}[\mathbf{rr}^\dagger]\big|_{ETSP} = \mathbf{F}^\dagger\mathbf{DD}^\dagger\mathbf{F} + \sigma^2\mathbf{I}$. Therefore,

$$\mathrm{E}[|r_n|^2]\big|_{ETSP} = \sigma^2 + \frac{1}{N}\sum_{k=0}^{N-1}|d_k|^2. \tag{19}$$

From (5), we have $\sigma^2_{q,ETSP} = \frac{1}{N}\left(\sum_{n=0}^{N-1}\frac{1}{G_n^2\big|_{ETSP}}\right)\frac{2^{-2\nu}}{6}$. Using the relation between G_n and $\mathrm{E}[|r_n|^2]$ in (6), we can write

$$\begin{aligned}\sigma^2_{q,ETSP} &= \frac{1}{N}\left(\sum_{n=0}^{N-1}\mathrm{E}[|r_n|^2]\big|_{ETSP}\right)\frac{2^{-2\nu}}{6\alpha} \\ &= \left(\frac{1}{N}\sum_{k=0}^{N-1}|d_k|^2 + \sigma^2\right)\frac{2^{-2\nu}}{6\alpha}.\end{aligned} \tag{20}$$

where the last equality follows from (19). For OFDM with ERSP, we have $\mathbf{r} = \mathbf{F}^\dagger\left(\mathbf{D}\tilde{\mathbf{u}} + \mathbf{w}\right)$. Therefore, $\mathrm{E}[\mathbf{rr}^\dagger] = \mathbf{F}^\dagger\mathbf{D}\mathrm{E}[\tilde{\mathbf{u}}\tilde{\mathbf{u}}^\dagger]\mathbf{D}^\dagger\mathbf{F} + \sigma^2\mathbf{I}$. Using the relation between $\mathbf{u}$ and $\tilde{\mathbf{u}}$ in (7) and using arguments similar to the ones used above, we can write

$$\sigma^2_{q,ERSP} = \left(\frac{N}{\sum_{k=0}^{N-1}|d_k|^{-2}} + \sigma^2\right)\frac{2^{-2\nu}}{6\alpha}. \tag{21}$$

From (20) and (21), we get

$$\sigma^2_{q,ERSP} \leq \sigma^2_{q,ETSP}$$

using the AM-HM inequality.

Proof of part 4): If $\forall k$, $\frac{3\log M}{M-1}\rho_k\big|_{ETSP} > 3$, then P is convex in $\{\frac{1}{|d_k|^2}\}_{k=0}^{N-1}$ [18]. An application of Jensen's inequality [19] for the sum $\frac{1}{N}\sum_{k=1}^{N-1}\frac{1}{|d_k|^2}$ and using (12) yields the desired result.

References

[1] "IEEE 802.11 Task Group ad," http://ieee802.org/11/Reports/tgupdate.htm.

[2] *IEEE Std 802.15.3c-2009 (Amendment to IEEE Std 802.15.3-2003)*, pp. c1 –187, Oct. 2009.

[3] R. Walden, "Analog-to-digital converter survey and analysis," *Selected Areas in Communications, IEEE Journal on*, vol. 17, no. 4, pp. 539 –550, Apr. 1999.

[4] O. Dabeer and U. Madhow, "Detection and interference suppression for ultra-wideband signaling with analog processing and one bit A/D," in *Signals, Systems and Computers, Thirty-Seventh Asilomar Conference on*, vol. 2, Nov. 2003, pp. 1766 – 1770.

[5] T. Shah, "Receiver design for multigigabit system," Master's thesis, School of Technology and Computer Science, TIFR, 2010.

[6] O. Dabeer and U. Madhow, "Channel estimation with low-precision analog-to-digital conversion," in *Communications (ICC), 2010 IEEE International Conference on*, May 2010, pp. 1–6.

[7] O. Dabeer and E. Masry, "Multivariate signal parameter estimation under dependent noise from 1-bit dithered quantized data," *Information Theory, IEEE Transactions on*, vol. 54, no. 4, pp. 1637 –1654, April 2008.

[8] O. Dabeer and A. Karnik, "Signal parameter estimation using 1-bit dithered quantization," *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 5389–5405, 2006.

[9] L. Goldfeld, V. Lyandres, and D. Wulich, "Minimum BER power loading for OFDM in fading channel," *Communications, IEEE Transactions on*, vol. 50, no. 11, pp. 1729 – 1733, Nov 2002.

[10] D. Love and J. Heath, R.W., "OFDM power loading using limited feedback," *Vehicular Technology, IEEE Transactions on*, vol. 54, no. 5, pp. 1773 – 1780, Sept. 2005.

[11] J. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come," *Communications Magazine, IEEE*, vol. 28, no. 5, pp. 5 –14, May 1990.

[12] A. Saleh and R. Valenzuela, "A statistical model for indoor multipath propagation," *Selected Areas in Communications, IEEE Journal on*, vol. 5, no. 2, pp. 128 – 137, Feb. 1987.

[13] "Channel model sub-committee final report," https://mentor.ieee.org/802.15/file/07/15-07-0584-01-003c-tg3c-, channel-modeling-sub-committee-final-report.doc.

[14] T. Araujo and R. Dinis, "Performance evaluation of quantization effects on multicarrier modulated signals," *Vehicular Technology, IEEE Transactions on*, vol. 56, no. 5, pp. 2922 –2930, Sept. 2007.

[15] D. Dardari, "Joint clip and quantization effects characterization in OFDM receivers," *Circuits and Systems I: Regular Papers, IEEE Transactions on*, vol. 53, no. 8, pp. 1741 –1748, Aug. 2006.

[16] B. Widrow and I. Kollár, *Quantization Noise: Roundoff Error in Digital Computation, Signal Processing, Control, and Communications*. Cambridge, UK: Cambridge University Press, 2008.

[17] J. Proakis, *Digital Communications*, 4th ed. McGraw-Hill Science/Engineering/Math, Aug. 2000.

[18] N. Wang and S. D. Blostein, "Power loading for CP-OFDM over frequency-selective fading channels," in *Proc. IEEE Global Telecommun. Conf*, 2003, pp. 2305–2309.

[19] T. Cover and J. Thomas, *Elements of information theory*. New York: Wiley, 1991.