A Complete Framework for Spectrum Sensing based on Spectrum Change Points Detection for Wideband Signals

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***Abstract*—This paper [1] presents a novel technique in spectrum sensing based on a new characterization of primary users signals in wideband communications. First, we have to remind that in cognitive radio networks, the very first task to be operated by a cognitive radio is sensing and identification of spectrum holes in the wireless environment. This paper summarizes the advances in the algebraic approach. Initial results have been already disseminated in few other conferences. This paper aims at finalizing and presenting the last results and the complete framework of the proposed technique based on algebraic spectrum discontinuities detection. The signal spectrum over a wide frequency band is decomposed into elementary building blocks of subbands that are well characterized by local irregularities in frequency. As a powerful mathematical tool for analyzing singularities and edges, the algebraic framework is employed to detect and estimate the local spectral irregular structure, which carries important information on the frequency locations and power spectral densities of the sensed subbands. In this context, a wideband spectrum sensing techniques was developed based on an analog decision function to multi-scale wavelet product. The proposed sensing techniques provide an effective sensing framework to identify and locate spectrum holes in the signal spectrum.**

***Index Terms*—spectrum sensing, cognitive radio, spectrum discontinuities, algebraic detection, wideband signals, change point detection, algebraic approach vs. wavelet approach.**

I. Introduction

Trying to face the shortage of radio resources and its misuse, telecommunication regulators and standardization organisms recommended sharing this valuable resource between the different actors in the wireless environment. The Federal Communications Commission (FCC), for instance, defined a new policy of priorities in the wireless systems, giving some privileges to some users, called Primary Users (PU) and less to others, called Secondary Users (SU), who will use the spectrum in an opportunistic way with minimum interference to PU systems.

Cognitive Radio (CR) as introduced by Mitola [1], is one of those possible devices that could be deployed as SU equipments and systems in Wireless networks. As originally defined, a CR is a self aware and "intelligent" device that can adapt itself to the Wireless environment changes. Such a device is able to detect the changes in Wireless network to which it is connected and adapt its radio parameters to the new opportunities that are detected. This constant track of the environment change is called the "spectrum sensing" function of a cognitive radio device.

Thus, spectrum sensing in CR aims in finding the holes in the PU transmission which are the best opportunities to be used by the SU. Many statistical approaches already exist. The proposed technique was inspired from algebraic Spike detection in EEGs (electroencephalograms) [5] and the recent work developed by Giannakis based on wavelet sensing [2]. Originally, the algebraic detection technique was introduced [5] [6] [7] to detect signals transients in EEGs, called spikes. Given Giannakis work on wavelet approach, and its limitations in complexity and implementation, we suggest in this context of wideband channels sensing, a detector using an algebraic approach to detect and estimate the local spectral irregular structure, which carries important information on the frequency locations and power spectral densities of the subbands.

This document summarizes the work we've been conducting in spectrum sensing for cognitive radio networks. A complete description of the reported work can be found in [11]–[15]

The rest of the paper is organized as following: in II, we introduce the state of the art and the motivations behind our proposed approach. In III, we state the problem as a detection problem with the formalism related to both sensing and detection theories. In section V, we give the results and the simulation framework in which the developed technique was simulated. And finally, in VI, we summarize about the presented work and conclude about its contributions.

II. Related works

Many statistical approaches for spectrum sensing have been developed. The most performing one is the cyclostationary features detection technique [3] [4]. The main advantage

[1]The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement SACRA n°249060 and CROWN project

978-1-4673-0990-5/12/$31.00 ©2012 IEEE

of the cyclostationarity detection is that it can distinguish between noise signal and PU transmitted data. Indeed, noise has no spectral correlation whereas the modulated signals are usually cyclostationary with non null spectral correlation due to the embedded redundancy in the transmitted signal. The cyclostationary features detector is thus able to distinguish between noise and primary users (PU).

The reference sensing method is the energy detector [3], as it is the easiest to implement. Although the Energy Detector can be implemented without any need of apriori knowledge of the PU signal, some difficulties still remain for implementation. First of all, the only PU signal that can be detected is the one having an energy above the threshold. So, the threshold selection in itself can be problematic as the threshold highly depends on the changing noise level and the interference level. Another challenging issue is that the energy detection approach cannot distinguish the primary user from the other secondary users sharing the same channel. Cyclostationary detection is more robust to noise uncertainty than an energy detector. Furthermore, it can work with lower SNR than energy detectors.

Some other techniques, exploiting a wavelet approach to efficient spectrum sensing of wideband channels were also developed [2]. The signal spectrum over a wide frequency band is decomposed into elementary building blocks of subbands that are well characterized by local irregularities in frequency. As a powerful mathematical tool for analyzing singularities and edges, the wavelet transform is employed to detect and estimate the local spectral irregular structure, which carries important information on the frequency locations and power spectral densities of the subbands. Along this line, a couple of wideband spectrum sensing techniques are developed based on the local maxima of the wavelet transform modulus and the multi-scale wavelet products.

III. System Model

In this section we investigate the system model considered through this report. In this system, the received signal at time n, denoted by y_n, can be modeled as:

$$y_n = A_n s_n + e_n \tag{1}$$

where A_n being the transmission channel gain, s_n is the transmit signal sent from primary user and e_n is an additive corrupting noise.

The goal of spectrum sensing is to decide between two conventional hypotheses modeling the spectrum occupancy:

$$y_n = \begin{cases} e_n & \mathrm{H}_0 \\ A_n s_n + e_n & \mathrm{H}_1 \end{cases} \tag{2}$$

The sensed sub-band is assumed to be a white area if it contains only a noise component, as defined in H_0; while, once there exist primary user signals drowned in noise in a specific band, as defined in H_1, we infer that the band is occupied. The key parameters of all spectrum sensing algorithms are the false alarm probability P_F and the detection probability P_D. P_F is the probability that the sensed sub-band is classified as a PU data while actually it contains noise, thus P_F should be kept as small as possible.
P_D is the probability of classifying the sensed sub-band as a PU data when it is truly present, thus sensing algorithm tend to maximize P_D. To design the optimal detector on Neyman-Pearson criterion, we aim on maximizing the overall P_D under a given overall P_F.
According to those definitions, the probability of false alarm is given by:

$$P_F = P(\mathrm{H}_1 \mid \mathrm{H}_0) = P(\text{ PU is detected} \mid \mathrm{H}_0) \tag{3}$$

that is the probability of the spectrum detector having detected a signal given the hypothesis H_0, and P_D the probability of detection is expressed as:

$$\begin{aligned} P_D &= 1 - P_M = 1 - P(\mathrm{H}_0 \mid \mathrm{H}_1) \\ &= 1 - P(\text{ PU is not detected} \mid \mathrm{H}_1) \end{aligned} \tag{4}$$

which represents the probability of the detector having detected a signal under hypothesis H_1, where P_M indicates the probability of missed detection.

In order to infer on the nature of the received signal, we use a decision threshold which is determined using the required probability of false alarm P_F given by (3). The threshold Th for a given false alarm probability is determined by solving the equation:

$$P_F = P(y_n \text{ is present} \mid \mathrm{H}_0) = 1 - F_{H_0}(Th) \tag{5}$$

where F_{H_0} denote the cumulative distribution function (CDF) under H_0.

IV. An algebraic approach to spectrum sensing

A. Problem formulation

First let's suppose that the frequency range available in the wireless network is *B* Hz; so *B* could be expressed as $B = [f_0, f_N]$. Saying that this wireless network is cognitive, means that it supports heterogeneous wireless devices that may adopt different wireless technologies for transmissions over different bands in the frequency range. A CR at a particular place and time needs to sense the wireless environment in order to identify spectrum holes for opportunistic use. Suppose that the radio signal received by the CR occupies *N* spectrum bands, whose frequency locations and PSD levels are to be detected and identified. These spectrum bands lie within $[f_0, f_N]$ consecutively, with their frequency boundaries located at $f_0 < f_1 < ... < f_N$. The *n*-th band is thus defined by:$B_n\ :\ f \in B_n : f_{n-1} f < f_n,\ n = 1, 2, ..., N$. The PSD structure of a wideband signal is illustrated in Fig. IV-A. The following basic assumptions are adopted:

1) The frequency boundaries f_0 and $f_N = f_0 + B$ are known to the CR. Even though the actual received signal may occupy a larger band, this CR regards $[f_0, f_N]$ as the wide band of interest and seeks white spaces only within this spectrum range.
2) The number of bands N and the locations $f_1, ..., f_{N-1}$ are unknown to the CR. They remain unchanged within

a time burst, but may vary from burst to burst in the presence of slow fading.

3) The PSD within each band B_n is smooth and almost flat, but exhibits discontinuities from its neighboring bands B_{n-1} and B_{n+1}. As such, irregularities in PSD appear at and only at the edges of the N bands.
4) The corrupting noise is additive white and zero mean.

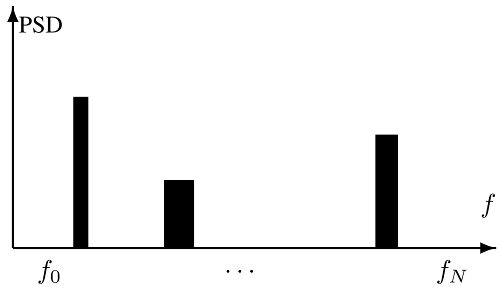

Fig. 1. N frequency bands with piecewise smooth PSD.

B. Detector derivation

In this Section some noncommutative ring theory notions are used [9].We start by giving an overview of the mathematical background leading to the algebraic detection technique. The input signal is the amplitude spectrum of the received noisy signal. We assume that its mathematical representation is a piecewise regular signal:

$$Y(f) = \sum_{i=1}^{K} \chi_i[f_{i-1}, f_i](f) p_i(f - f_{i-1}) + n(f) \tag{6}$$

where: $\chi_i[f_{i-1}, f_i]$: the characteristic function of the interval $[f_{i-1}, f_i]$, $(p_i)_{i\in[1,K]}$: an N^{th} order polynomials series, $(f_i)_{i\in[1,K]}$: the discontinuity points resulting from multiplying each $p_i$by a χ_i and $n(f)$:the additive corrupting noise.

Now, let $X(f)$ the clean version of the received signal given by:

$$X(f) = \Sigma_{i=1}^{K} \chi_i[f_{i-1}, f_i](f) p_i(f - f_{i-1}) \tag{7}$$

And let b, the frequency band, given such as in each interval $I_b = [f_{i-1}, f_i] = [\nu, \nu + b]$, $\nu \geq 0$ maximally one change point occurs in the interval I_b.

Now denoting $X_\nu(f) = X(f+\nu)$,$f \in [0, b]$ for the restriction of the signal in the interval I_b and redefine the change point which characterizes the distribution discontinuity relatively to I_b say f_ν given by:

$$\begin{cases} f_\nu = 0 \text{ if } X_\nu \text{ is continuous} \\ 0 < f_\nu \leq b \text{ otherwise} \end{cases}$$

Now, in order to emphasis the spectrum discontinuity behavior, we decide to use the N^{th} derivative of $X_\nu(f)$, which in the sense of Distributions Theory is given by:

$$\frac{d^N}{df^N} X_\nu(f) = [X_\nu(f)]^{(N)} + \sum_{k=1}^{N} \mu_{N-k} \delta(f - f_\nu)^{(k-1)} \tag{8}$$

where: μ_k is the jump of the k^{th} order derivative at the unique assumed change point:f_ν

$$\mu_k = X_\nu^{(k)}(f_\nu^+) - X_\nu^{(k)}(f_\nu^-)$$

with : $\begin{cases} \mu_k = 0\rfloor_{k=1..N} \text{ if there is no change point.} \\ \mu_k \neq 0\rfloor_{k=1..N} \text{ if the change point is in } I_b. \end{cases}$

$[X_\nu(f)]^{(N)}$ is the regular derivative part of the N^{th} derivative of the signal.

The spectrum sensing problem is now casted as a change point f_ν detection problem. Several estimators can be derived from the equation 8. For example any derivative order N can be taken and depending on this order the equation is solved in the operational domain and back to frequency domain the estimator is deduced.

In a matter of reducing the complexity of the frequency direct resolution, the equation 8 is transposed to the operational domain, using the Laplace transform:

$$\begin{aligned} L(X_\nu(f)^{(N)}) &= s^N \widehat{X_\nu}(s) - \sum_{m=0}^{N-1} s^{N-m-1} \frac{d^m}{df^m} X_\nu(f)\rfloor_{f=0} \\ &= e^{-sf_\nu}(\mu_{N-1} + s\mu_{N-2} + .. + s^{N-1}\mu_0) \end{aligned} \tag{10}$$

Given the fact that the initial conditions, expressed in Eq. 8, and the jumps of the derivatives of $X_\nu(f)$ are unknown parameters to the problem, in a first time we are going to annihilate the jump values μ_0,μ_1,...,μ_{N-1} then the initial conditions. After some calculations steps, we finally obtain:

$$\sum_{k=0}^{N-1} \binom{N}{k} . f_\nu^{N-k} . (s^N \widehat{X_\nu}(s))^{(N+k)} = 0 \tag{11}$$

In the actual context, the noisy observation of the amplitude spectrum $Y(f)$ is taken instead of $X_\nu(f)$. As taking derivative in the operational domain is equivalent to high-pass filtering in frequency domain, which may help amplifying the noise effect. It is suggested to divide the whole equation 11 by s^l which in the frequency domain will be equivalent to an integration if $l > 2N$, we thus obtain:

$$\sum_{k=0}^{N-1} \binom{N}{k} . f_\nu^{N-k} . \frac{(s^N \widehat{X_\nu}(s))^{(N+k)}}{s^l} = 0 \tag{12}$$

Since there is no unknown variables anymore, the equation 12 is now transformed back to the frequency domain, we obtain the polynomial to be solved on each sensed sub-band:

$$\sum_{k=0}^{N-1} \binom{N}{k} . f_\nu^{N-k} . L^{-1}[\frac{(s^N \widehat{X_\nu}(s))^{(N+k)}}{s^l}] = 0 \tag{13}$$

And denoting:

$$\varphi_{k+1} = L^{-1}[\frac{(s^N \widehat{X_\nu}(s))^{(N+k)}}{s^l}] = \int_0^{+\infty} h_{k+1}(f).X(\nu - f).df \tag{14}$$

where: $h_{k+1}(f) = \begin{cases} \frac{(f^l(b-f)^{N+k})^{(k)}}{(l-1)!}, 0 < f < b \\ 0, otherwise \end{cases}$

To summarize, we have shown that on each interval $[0, b]$, for the noise-free observation the change points are located at frequencies solving:

$$\sum_{k=0}^{N} \binom{N}{k} . f_{\nu}^{N-k} . \varphi_{k+1} = 0 \tag{15}$$

In [10], it was shown that edge detection and estimation is analyzed based on forming multiscale point-wise products of smoothed gradient estimators. This approach is intended to enhance multiscale peaks due to edges, while suppressing noise. Adopting this technique to our spectrum sensing problem and restricting to dyadic scales, we construct the multiscale product of $N+1$ filters (corresponding to Continuous Wavelet Transform in [10]), given by:

$$Df = \| \prod_{k=0}^{N} \varphi_{k+1}(f_{\nu}) \| \tag{16}$$

C. Algorithm Discrete Implementation

The proposed algorithm in its discrete implementation is a filter bank composed of N filters mounted in a parallel way. The impulse response of each filter is:

$$h_{k+1,n} = \begin{cases} \frac{(n^l(b-n)^{N+k})^{(k)}}{(l-1)!}, 0 < n < b \\ 0, otherwise \end{cases} \tag{17}$$

where $k \in [0..N-1]$ and l is chosen such as $l > 2 \times N$. The proposed expression of $h_{k+1,n}\rfloor_{k\in[0..N-1]}$ was determined by modeling the spectrum by a piecewise regular signal in frequency domain and casting the problem of spectrum sensing as a change point detection in the primary user transmission. Finally, in each detected interval $[n_{\nu_i}, n_{\nu_{i+1}}]$, we compute the following equation:

$$\varphi_{k+1} = \sum_{m=n_{\nu_i}}^{n_{\nu_{i+1}}} W_m h_{k+1,m} X_m \tag{18}$$

where W_m are the weights for numeric integration defined by:

$$\begin{array}{ll} W_0 = W_M = 0.5 & \\ W_m = 1 & \text{otherwise} \end{array}$$

$$Df = \| \prod_{k=0}^{N} \varphi_{k+1}(n_{\nu}) \| \tag{19}$$

For instance, we consider a frequency band in the range of $[50, 250]$MHz, in order to compare the compressive sensing using the algebraic method and the wavelet approach introduced in [10]. The signal is fully described in [10]. During the observed burst of transmissions in the network, there 6 bands, with frequency boundaries at ${n_{\nu}}_{n=0}^{6} = [50, 120, 170, 200, 220, 224, 250]$MHz.
Comparing with the wavelet approach, in the algebraic detection technique change points are detected only in one shot, while in the wavelets approach, many detections have to be conducted and fused to make a final decision.
Figure 2 shows the algebraic detection performance on this signal.

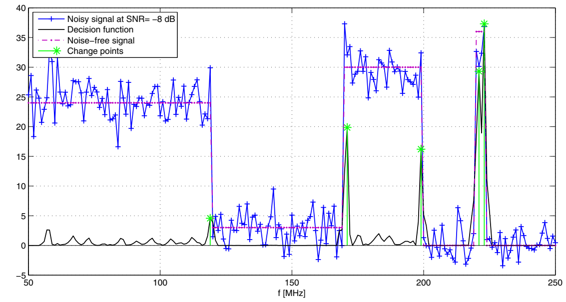

Fig. 2. Edge detection using the algebraic technique. The signal in red is the original signal, the one in blue is the noisy observation with SNR=-8dB. The black signal is the computed decision function and the green stars are the detected change points.

V. Simulations and Results

In this section, we use the ED as a reference technique, since it is the most common method for spectrum sensing because of its non-coherency and low complexity. The energy detector measures the received energy during a finite time interval and compares it to a predetermined threshold. That is, the test statistic of the energy detector is:

$$\sum_{n=1}^{M} \| y_n \|^2 \tag{20}$$

where M is the number of samples of the received signal x_n.
Traditional ED can be simply implemented as a spectrum analyzer. A threshold used for primary user detection is highly sensitive to unknown or changing noise levels. Even if the threshold would be set adaptively, presence of any in-band interference would confuse the energy detector.
For simulation results, the choice of the DVB-T primary user system is justified by the fact that most of the primary user systems utilize the OFDM modulation format [8]. The considered model is an Additive White Gaussian Noise (AWGN) channel. The simulation scenarios are generated by using different combinations of parameters given in Table I.

Bandwidth	8MHz
Mode	2K
Guard interval	1/4
Frequency-flat	Single path
Sensing time	1.25ms
Location variability	10dB

TABLE I
THE TRANSMITTED DVB-T PRIMARY USER SIGNAL PARAMETERS

Fig. 3 reports the comparison in terms of Probability of Detection Vs. SNR between the Energy Detector (ED) and the three first Algebraic Detectors:(AD_1) (AD_2) and (AD_3), for P_F=0.05 and SNR ranging in -40 to 0 dBs.
The threshold level for each detector is computed with function of the probability of false alarm P_F with respect to Equation 5.
This figure clearly shows that the proposed sensing algorithm is quite robust to noise. These curves show also that

the detection rate goes higher as the polynomial order gets higher.

This result is to be expected as the higher the polynomial order is, the more accurate the approximation a polynomial is. Nevertheless, it is to be noticed that this gain in precision is implies a higher complexity in the algorithms implementation.

In Figure 4, we plot the ROC curve at an SNR=-15dB. We clearly see that for the proposed technique, the higher the order, the more performing the detector gets.

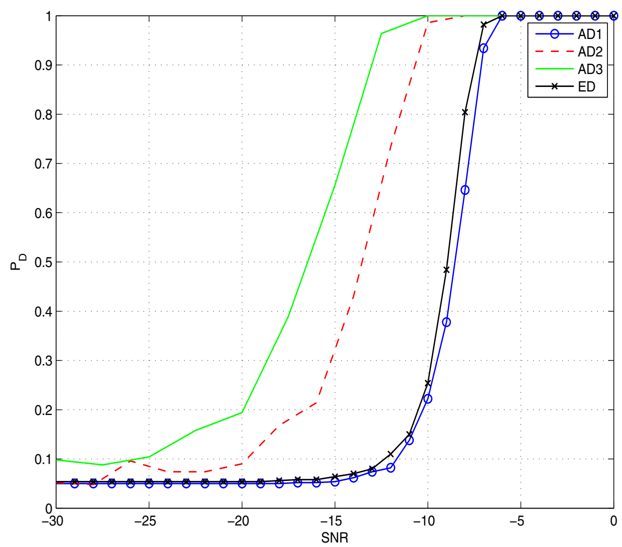

Fig. 3. Probability of detection vs. SNR for the simulated detectors with $P_F = 0.05$.

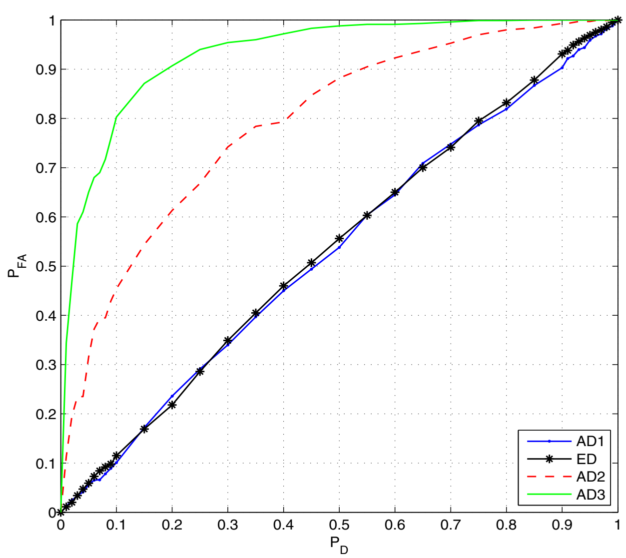

Fig. 4. ROC curves at SNR=-15dB

VI. Summary

In this work, we presented a new standpoint for spectrum sensing emerging in detection theory, deriving from differential algebra, noncommutative ring theory, and operational calculus. The proposed algebraic based algorithm for spectrum sensing by change point detections in order to emphasizes "spike-like" parts of the given noisy amplitude spectrum. Simulations results showed that the proposed approach is very efficient to detect the occupied sub-bands in the the primary user transmissions. We have shown how very simple sensing algorithm with good robustness to noise can be devised within the framework of such unusual mathematical chapters in signal processing. A probabilistic interpretation, in the sense of ROC curve, probability of detection, probability of false alarm ... is shown to be attached to the presented approach. It has allowed us to give a first step towards a more complete analysis of the proposed sensing algorithm.

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