Enhanced Index Assignment for Beamforming with Limited-rate Imperfect Feedback

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***Abstract*—The quantized beamforming systems always need the channel state information, which must be quantized into a finite set of vectors (named codebook), and feedback only sends the index representing the desired vector. Thereby, the impact of feedback errors, caused by feedback overhead and delay, is minimized. In this regard, index assignment methods, exhaustive-search, and group-based schemes, have been presented to minimize performance degradation without the need for additional feedback bits. In this paper, we proposed an enhanced group-based index assignment method, which used the optimal codebook design with chordal distance, having the adaptive properties in the application of the existing index assignment methods. When the number of transmit antennas is 4 and LTE codebook is used, Monte-Carlo simulation results show that the proposed method has a power advantage of 0.5∼1 dB over the methods without index assignment to achieve the same bit error rate, and it achieves 0.1∼0.2 dB better performance compared with the existing index assignment methods with the same simulation environment.**

I. Introduction

In multiple-input multiple-output (MIMO) wireless systems, the transmit-beamforming (TxBF) scheme has been used to exploit diversity and array gains [1]. Typically, the TxBF scheme requires the perfect feedback of channel state information (CSI) from the receiver to the transmitter. In time division duplex systems with suitable time, the perfect CSI may be available. In frequency division duplex systems, however, the perferct CSI must be conveyed through a feedback channel. This is impractical, though, due to the number of channel coefficients that need to be quantized and sent back to the transmitter over limited bandwidth control channels [2]. And thus a small number of feedback bits are sent via a feedback path for the transmitter to recreate the TxBF vector. These systems are known as limited feedback systems (see [3] and the reference therein). In these limited feedback systems, the transmit precoder is chosen from a finite set of precoding matrices, named codebook, and known to both the transmitter and the receiver. The receiver chooses the optimal codeword index from the codebook as s function of the current CSI, and the chosen codeword index is sent back from the receiver to the transmitter. Most studies on the these limited feedback systems with codebook design have assumed perfect feedback channels [1],[4]-[6], but feedback errors are always presenting practice, and they lead to applying the incorrect index to the transmitter. To solve this problem, feedback error protection methods were investigated such as the finite-state Markov channel based detection exploiting the Markov nature of the channel and the Bayesian based detection exploiting knowledge of the average signal-to-noise ratio (SNR) [7],[8], adaptive diversity detection [9], and index assignment methods to maximize the array gain or the average received SNR [10],[11]. In these feedback error protection methods, index assignment (IA) method is more useful than other feedback error protection methods, because it does not require additional feedback bits and it is only assigned to optimize indices. In [11], two IA methods are presented: one is an exhaustive-search based algorithm and the other is the group-based index assignment (GIA) scheme. The GIA method can be useful in such cases where codebook size N is large. This GIA method starts with a small-size good codebook of size N_p $(N_p > N)$ with optimized index-vector mapping, named the parent codebook. The parent codebook must be selected from already made codebooks in [12],[13], and there has restrictions about codebook size, the number of transmitted antennas. In this paper, we consider the optimal codebook selection method with chordal distance, and we propose an enhanced GIA method. Monte-Carlo simulations in the long term evolution (LTE) system are presented to show that the proposed method outperforms the conventional one and it has more adaptive properties.

The rest of the paper is organized as follows. Section II reviews the feedback system model and convention IA methods. In Section III, we propose an enhanced GIA method. In Section IV, the simulation results are demonstrated and the related discussions are given. Finally, the concluding remarks are given in Section V.

This paper uses the following notation. A bold capital letter $\mathbf{A}$ denotes the matrix, $a_{i,j}$ denotes the i-th and j-th elements of $\mathbf{A}$, a bold lower case letter $\mathbf{a}$ denotes the vector, $\mathbf{A}^T$ denotes the transposition of matrix $\mathbf{A}$, $\mathbf{A}^H$ denotes the conjugation transposition of matrix $\mathbf{A}$, and $\mathbf{A} \in \mathbb{C}^{m\times n}$ denotes that complex matrix $\mathbf{A}$ has m rows and n columns.

II. System Overview

A. Feedback Systems

A simple feedback MIMO system model using TxBF is illustrated in Fig. 1, where there are N_T transmit antennas and N_R receive antennas. We assume that the transfer function between the transmitter and receiver can be modelled as a

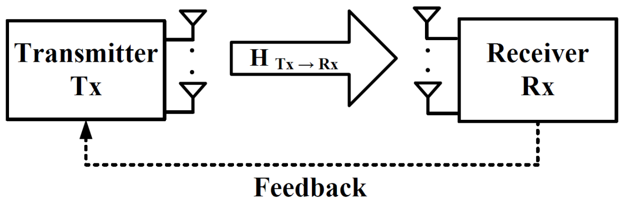

Fig. 1. Block diagram of a limited feedback precoding MIMO system.

memoryless linear channel, which is constant over several codeword transmissions before independently taking on a new value. The received signal $\mathbf{y} \in \mathbb{C}^{N_R \times 1}$ can be written as

$$\mathbf{y} = \mathbf{HWs} + \mathbf{n}, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{R \times 1}$ is the transmitted symbol, $R \leq \min(N_T, N_R)$ is the number of bit streams, $\mathbf{W} \in \mathbb{C}^{N_T \times R}$ is the normalized TxBF matrix, $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix with independent and identically distributed (i.i.d.) as $\mathcal{CN}(0, 1)$, and $\mathbf{n} \in \mathbb{C}^{N_R \times 1}$ is the noise vector with i.i.d. entries distributed according to $\mathcal{CN}(0, N_0)$. In the TxBF system, the key problem is how to design $\mathbf{W}$ to maximize the system performance. For this reason, $\mathbf{W}$ should be chosen to maximize the receive signal-to-noise ratio (SNR) in order to minimize the average probability of error and maximize the capacity. In general, it can be determined by applying the singular vector decomposition (SVD) [14]. However, accurate quantization and feedback of this normalized TxBF matrix can require a large number of feedback bits, especially combined with the orthogonal frequency division multiplexing (OFDM) system employing numerous antennas, subcarriers, and a rapidly varying channel. Quantized TxBF techniques provide a solution for this problem by quantizing the optimal precoder at the receiver. Specifically, the precoder is constrained to be one of N matrices, which as a group is called a codebook. If the codebook of N matrices is known to both the transmitter and the receiver, then $L = log_2 N$ bits of feedback are required for indicating the index of the appropriate TxBF matrix [15]. The codebook $\mathcal{F}$ can be written as

$$\mathcal{F} = \{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N\}. \tag{2}$$

For each channel realization, the receiver chooses one beamforming matrix from the codebook to maximize the receive SNR or the capacity. The i-th $(1 \leq i \leq N)$ optimum codeword with index i_{opt} can be selected using (3), and sends the corresponding index to the transmitter [1]

$$i_{opt} = \underset{H \in \Omega, i \in \{1,2,\ldots,N\}}{\arg\max} \|\mathbf{HW}_i\|_F, \tag{3}$$

where $\|\cdot\|_F$ denotes the Frobenius-norm operation.

B. Exhaustive-search Based IA Method

The codebook IA method is introduced in this section. The codebook within 3 bits feedback can be calculated using exhaustive-search. The illustration of index variation when 1

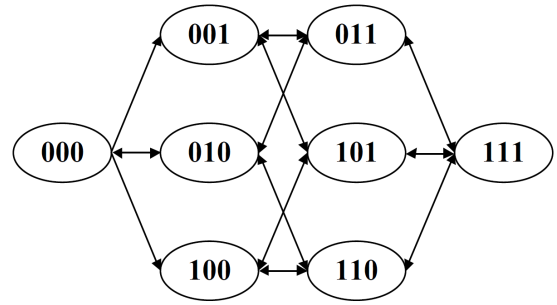

Fig. 2. Example of 1bit error flow chart (3bits feedback).

bit error occurs in 3bit feedback is shown in Fig. 2. Under normal operation conditions, the bit error rate (BER) of the feedback channel would not be too high. Hence, we are assuming that maximally 1 bit error occurs in feedback. Based on the relationship shown in Fig. 2, the codebook index assignment procedure can be applied as follows

1) Generate all possible combinations of the N codewords in the codebook with a different sort. The number of the possible combinations is $N!$.
2) Calculate the cost function C^k of the total Hamming distance between a pair of different codewords of each k-th, $k = 1, 2, \ldots, N!$ combination. This is shown as follows [11]

$$C^k = \sum_{i=0}^{N!-2} \sum_{j>i}^{N!-1} I\left(d_{I_i^k I_j^k}\right) \left|\left(\mathbf{W}_i^k\right)^H \left(\mathbf{W}_j^k\right)\right|^2, \tag{4}$$

where $\mathbf{W}_i^k$ and $\mathbf{W}_j^k$ denote codewords of the corresponding i-th, j-th index in the k-th combination codebook. I_i^k and I_i^k denote the binary format of i-th, j-th codeword index in the k-th combination codebook. $d_{I_i^k I_j^k}$ is the Hamming distance between index I_i^k and index I_j^k, $I\left(d_{I_i^k I_j^k}\right)$ = 1 if $d_{I_i^k I_j^k}$ =1 and $I\left(d_{I_i^k I_j^k}\right)$ = 0 otherwise.
3) The optimal codebook can be found by searching for the largest cost function.

Maximization of the cost function in (4) needs the entire N! possible permutations. If N is small, then it can be solved via brute-force search. However, if N is large, e.g., $N > 16$, then a brute-force search is prohibitive since $16! > 10^{13}$. Hence, the group-based IA method, which is suboptimal, has been proposed.

C. Group-based IA Method

In [11], the group-based IA (GIA) method starts with a smaller size good codebook of size N_p $(N_p > N)$ with optimized index vector mapping, named the parent codebook expressed as $C_p = \left[\mathbf{c}_0, \mathbf{c}_1, \ldots, \mathbf{c}_{N_p - 1}\right]$. Any larger codebook with size N, named the child codebook, is partitioned into N_p non-overlapping groups. The partitioning procedure is given as

1) Initialize i = 0.
2) With $j = [i]_{N_p}$, where $[\cdot]_{N_p}$ denote modulo-N_p operation, select a beamforming vector from the child

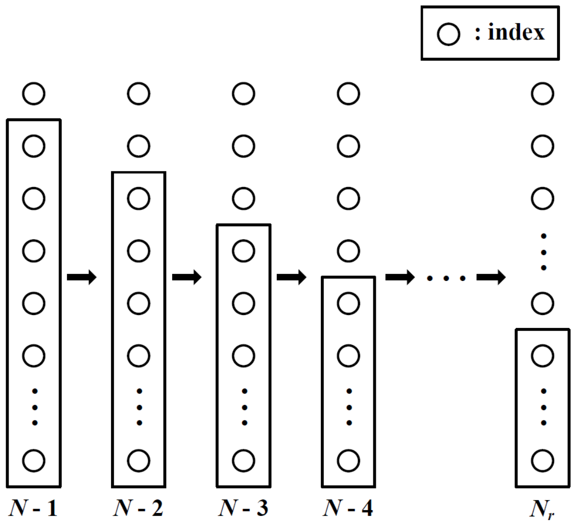

Fig. 3. Generation method for the reference codebook.

codebook that has the largest magnitude of inner product with vector $\mathbf{c}_j$ and add it in the j-th group. A new child codebook with a reduced-size is then formed by removing this vector from the current child codebook.

3) Let $i = i + 1$. If $i < N$, repeat step 2); otherwise, the partition process ends.

We assume that $N = 2^B$ and $N_p = 2^{B'}$, where B and B' are positive integers and $B' < B$. After the partition is completed, there are N_p groups, each of which has $2^{B-B'}$ beamforming vectors. For the j-th group, $0 \le j \le N_p - 1$, we copy the index of $\mathbf{c}_j$ as the B' most significant bits of the index of each beamforming vector within this group. For the remaining $B - B'$ unassigned index bits, we perform random index vector mapping for simplicity. Thus, the resulting index for each beamforming vector in the child codebook has the B' most significant bits to indicate which group it belongs to, and $B - B'$ least significant index bits for index mapping within the group.

Previous research has primarily focused on worldwide interoperability for microwave access (WiMAX), LTE and the Grassmannian line packing (GLP) codebook [1],[12],[13] as the parent codebook. There are restrictions of codebook size, the number of transmitted antennas. Especially, standard documents have recently specified that the number of transmitted antennas is only 2, 4 and 8, while codebook size is only 8, 16 and 32. If some codebooks that are not specified standard documents are needed, then we should use GLP codebook.

III. Proposed Enhanced Group-based IA Method

The main proposition of the enhanced group-based IA (EGIA) method is that the parent codebook can be selected from the child codebook. It is possible to set the parent codebook size as desired. We use a reference codebook instead of the parent codebook in the proposed method. N_r denoted the size of reference codebook. As shown in Fig. 3, the

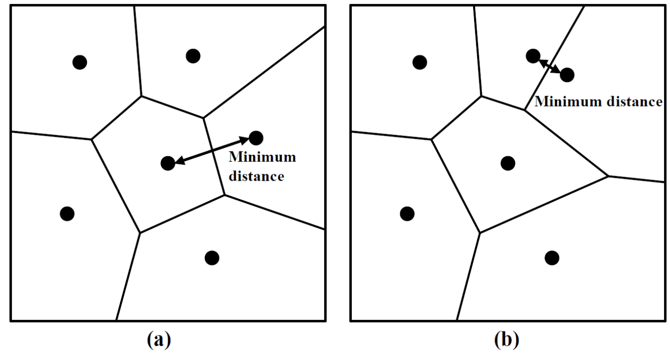

Fig. 4. Comparison of the Voronoi diagram in 2-dimensional Grassmannian manifold by the minimum chordal distance between specific codewords $N = 6$.

reference codebook is formed by removing the indices of non-optimal codewords. The removing progress goes on until the reference codebook size is N_r which is what we need.

The processes of the reference codebook generation can be applied as follows

1) We generate all the possible candidate reference codebooks by deleting one index from the child codebook, and thus each codebook is comprised of $N - 1$ indices; we get N candidate reference codebooks.
2) In order to select the optimal codebook among the candidate reference codebooks, we use the chordal distance which is closely related to codebook performances over Grassmannian subspace packing research [14],[16]. The chordal distance is defined as

$$d(\mathbf{W}_k, \mathbf{W}_l) = \sqrt{1 - \left|\mathbf{W}_k^H, \mathbf{W}_l\right|^2}, \tag{5}$$

where k, l $(k \neq l, 1 \le k, l \le N)$ are integers. The optimal codebook is designed to maximize the minimum chordal distance δ_{min} in Grassmannian subspace packing. This is written as

$$\delta_{min} = \min_{k \neq l, 1 \le k, l \le N} d(\mathbf{W}_k, \mathbf{W}_l). \tag{6}$$

Comparison of the Voronoi diagram [17][18] in 2-dimensional Grassmannian manifold by minimum chordal distance between codewords in specific codebook, of which the size is 6, as is shown in Fig. 4. In order to successfully achieve good performance, we have to maximize the minimum chordal distance, and all codewords are then evenly distributed over subspace in Grassmannian manifold, as is shown in Fig. 4 (a). However, if we choose the shorter one than that in Fig. 4 (a), then all codewords are not evenly distributed over the subspace in Grassmannian manifold, as is shown in Fig. 4 (b). Therefore, we found the optimal reference codebook with maximized minimum chordal distance between each pair of codewords among the all candidate reference codebooks, which is shown as follows

$$p_{opt} = \underset{p\in\{1,2,\ldots,N\}}{\arg\max}\ \delta^{p}_{min}, \tag{7}$$

where δ^{p}_{min} denoted the minimum chordal distance for p-th $(1 \le p \le N)$ candidate reference codebook; p_{opt}-th candidate reference codebook is selected as the best one.

3) Let N_c denote the selected reference codebook size. If N_c and the desired N_r are not equal, then we change the child codebook to the new child codebook which is selected in the previous step. Further, we repeat steps 1) and 2) until the size of reference codebook is as desired N_r, which is possible to use the algorithm introduced in section 2.2, as is shown in Fig. 3.

The proposed EGIA method has the adaptive properties in the application of the existing GIA method. In other words, it is possible to set the reference codebook size as desired. Thereby, it has the advantage of restrictions in terms of codebook size and the number of transmitted antennas.

IV. Simulations

The Monte-Carlo simulation is employed to evaluate the performance of the proposed EGIA method and conventional GIA method. The performances are evaluated by using the LTE link level simulator, and it has 4 transmit antennas and 2 receive antennas, and applies 3GPP LTE 4 transmitted antennas rank 1 codebook from the Table 6.3.4.2.3-2 in [13] with 4 bits feedback. We adopt that the reference codebook size is 8. The bit error rate (BER) is estimated using at least 1 million iterations per the energy per bit to noise power spectral density ratio (EbNo) point. The number of data stream is 1. And quadrature phase shift keying (QPSK) modulation/demodulation is assumed for LTE system whose FFT/IFFT size is 512, sampling rate is 7.68 MHz and cyclic prefix mode is normal. Meanwhile, the channel model is assumed to be ITU-R Pedestrian-A model having 3 multi-paths [19]. Channel is a memoryless fading channel with independent and identically distributed entries according to $\mathcal{CN}(0,1)$, and additive white Gaussian noise is assumed with elements that are i.i.d. entries according to $\mathcal{CN}(0,N_0)$. It is also assumed that the channel estimation and symbol synchronization are ideal, and that there is no spatial correlation amongst the antennas. Details about the simulation parameters are shown in Table I.

In Fig. 5, we show the BER performances with or without index assignment when feedback BER is 10^{-2} and 10^{-3}, respectively. At the feedback BER of 10^{-2}, the proposed EGIA method (curve label 'LTE CB using proposed EGIA') achieves a gain of about 0.5∼1 dB, which is better performance than that of the feedback system without IA method (curve label 'LTE CB without IA') at BER values below 10^{-3}. At the feedback BER of 10^{-3}, the proposed EGIA method achieves a gain of about 0.5 dB, which is better performance than that of the feedback system without IA method at BER of 10^{-4}.

In Fig. 6, we show a comparison between the proposed EGIA and conventional GIA method when the feedback BER is 1 (always 1 bit generation) and 10^{-2}, respectively. We adopt the 3 bits codebook $V(4,1,3)$ from the Table 298o in [12] as the parent codebook. At feedback BER of 1 and 10^{-2}, the proposed EGIA method (curve label 'proposed EGIA') achieves a gain of about 0.1∼0.2 dB, which is better performance than that of the conventional GIA method (curve label 'conventional GIA') at BER values below 10^{-2}.

TABLE I
SIMULATION PARAMETERS

Parameters	Values
Modulation/Demodulation	QPSK
Carrier frequency [GHz]	2.6
Bandwidth [MHz]	5
Sampling rate [MHz]	7.68
Subcarrier spacing [kHz]	15
IDFT/DFT size	512
Number of occupied subcarriers	300
Cyclic prefix mode	Normal
Number of bir streams	1
Number of transmitted antennas	4
Number of received antennas	2
Feedback bits	4
Channel model	Rayleigh fading
Multiu-path model	ITU-R (Pedestrian A)
Number of multi-paths	3
Relative delay [ns]	[0 110 190]
Average power [dB]	[0 -9.7 -19.2]
Codebooks a. 4 bits 3GPP LTE 4 transmitted antennas, Rank 1 codebook b. 3 bits WiMAX codebook $V(4,1,3)$	

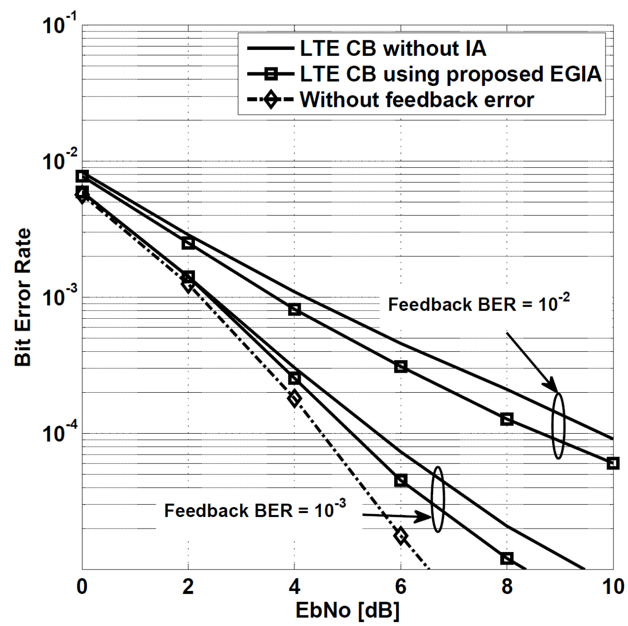

Fig. 5. Comparison of BER performance with or without the proposed EGIA method.

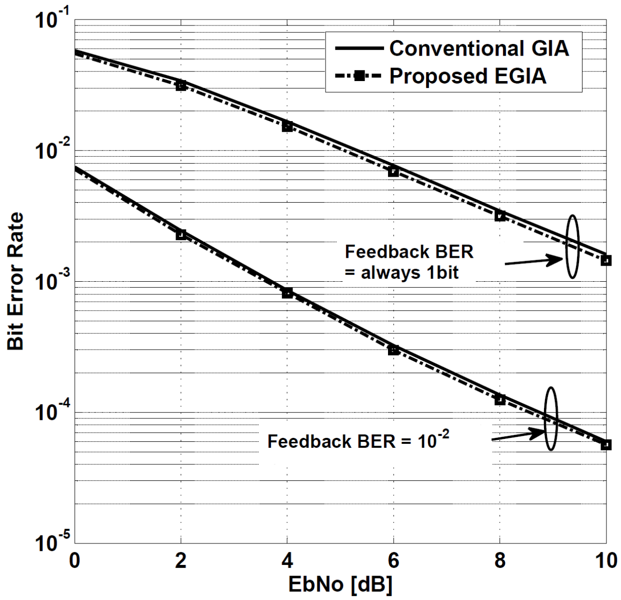

Fig. 6. Comparison of BER performance between the conventional and the proposed GIA methods.

V. Conclusions

In this paper, we proposed an enhanced group-based index assignment method for MIMO feedback system with noisy feedback channels. The proposed method used the optimal codebook design with chordal distance, having the adaptive properties in the application of the existing index assignment methods. Based on the simulation results, the advantage of the proposed method is that it has a power advantage to obtain the same bit error rate better than that of methods without index assignment; also it achieves better performance compared with the conventional group-based index assignment methods.

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