Optimal Frequency Offsets with Doppler Spreads In Mobile OFDM System

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***Abstract*—In highly mobile OFDM systems, the carrier frequency offsets (CFO) with Doppler spreads for downlink detection can be considerably large, which degrades the frequency alignment for uplink transmission, particularly in employing directional antennas for inter-carrier interference (ICI) reduction. In prior works, the directional antenna was investigated with appropriate frequency alignment in receiver's local oscillator to efficiently reduce ICI in fast time varying OFDM systems. To resolve the optimal frequency offset problem with Doppler spreads, this paper develops a simple scheme to capture instant Doppler power spectrum density (PSD) through moving directional antennas with arbitrary gain patterns. Thus, the optimal aligning frequency is derived as the center of gravity of the Doppler PSD. Simulations show our approach acquires the highest carrier to interference (C/I) ratio and the lowest bit error rates (BER) compared with other approaches.**

Keywords- frequency alignment; carrier frequency offset; CFO; directional antenna; ICI reduction; mobile OFDM System

I. INTRODUCTION

In mobile orthogonal frequency division multiplexing (OFDM) systems, a mobile terminal (MT) needs to align frequency with its local oscillator to detect the downlink data streams and serve as synchronization references in the uplink transmission. Conventional frequency tracking schemes exploit the discrete Fourier transform (DFT) with maximum likelihood principle [1-2] to compute estimates of the carrier frequency offsets (CFO), which represent the collective effects of the oscillator instabilities and Doppler shifts in time varying channels [3]. However, with asymmetrical multipath profiles and different antenna pattern between the receivers of MT and base station (BS), aligning the local oscillators with the conventional CFO estimates may have repercussions on the performance of uplink transmission [3-4], particularly in fast fading channels with significant Doppler spreads. To preserve the frequency tracking for uplink transmission in the MTs for highly mobile OFDM communications, it is essential to extract the frequency offsets to the Doppler effects along with the total CFO estimates.

In this paper, we consider the train-to-ground links in trackside infrastructure based OFDM systems for broadband communication in high-speed rail (HSR) systems. The HSR has been recently constructed in many countries to facilitate public transportation, where highly mobile OFDM communication is of significant research importance. In such fast time varying channels, the ICI induced with Doppler effects is one of the factors to degrade system performance.

In the literatures of ICI mitigation in time-varying channels, employing directional antennas for the MT reception presents advantages of high gains on ICI reduction [4-8] with no computing overhead because the directional antennas act as spatial filters and restrain frequency spreads of the receiving signals to certain ranges. In general, the Doppler spectra of the signals reaching the receivers are biased to neutral frequency. Therefore, the demodulating frequency for data detection is aligned with the spectra. The investigation indicates that tracking frequency shift increases 20-40 dB for carrier to interference (C/I) ratios in power as directional antennas are applied for ICI reduction [5]. Furthermore, solving the optimal alignments on local oscillator requires complex analytical techniques [4].

This paper investigates the optimal frequency offsets for the lowest ICI with the Doppler power spectrum density (PSD). The Doppler PSD can be derived either by the experimental methods (e.g., site survey in advance) or by analytical predictions with the "sliding beam" scheme described in the next section. With given Doppler PSD, the optimal frequency offsets that result in the highest C/I ratio are derived as the center of gravity of the Doppler PSD. A system simulation is conducted to confirm the optimal frequency offset gains with 1~3 dB in C/I ratios and the lowest BER as compared to other aligning approaches.

The rest of this paper is organized as followings. In section II, the "sliding beam" schemes to derive Doppler PSD for arbitrary antenna gain patterns and steering directions are presented. In section III, the ICI power is formulated with Doppler PSD. In section IV, the optimal frequency offsets are resolved with the Doppler PSDs. The simulations are demonstrated in section V, and we conclude this paper in section VI.

II. DOPPLER PSD WITH DIRECTIONAL ANTENNAS

In the scenario of the HSR infrastructure-based OFDM system, the downlink signals are transmitted from BS and

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scattered in the wireless channels before reaching the antenna. The signals suffer from severe frequency spread, called Doppler effects, due to the high speed between transmitters and receivers. The maximum Doppler frequency is expressed as

$$f_d = u/\lambda \tag{1}$$

where u is the relative speed from receiver to transmitter, and λ is the wavelength of the carrier.

Due to the effects of multipath in wireless propagation, the amount of frequency offset for each propagating path depends on the angle of arrival (AOA), denoted as θ. The frequency offset can be expressed as

$$f = f_d \cos\theta. \tag{2}$$

In the scenario, the von Mises distribution model is adopted to approximate directional scattering environments as high speed trains operate mostly in suburban and rural areas. Thus, the probability distribution of the AOA is modeled as [9]

$$p(\theta) = \frac{e^{k\cos(\theta-\alpha)}}{2\pi I_0(k)}, \theta \in (-\pi, \pi], \tag{3}$$

where $I_0(k)$ is the zero order modified Bessel function of the first kind; k determines the beamwidth of the directional scattering, and α is the angle between the average scattering direction and the mobile direction.

The Doppler PSD defines the statistical power distribution along with the axis of frequency shifts with the Doppler effects. With the models disciplined in (1)~(3), the Doppler PSD for directional scattering channels is given [9] as

$$S_\alpha(f) = \begin{cases} \frac{1}{K+1} \frac{e^{k\cos\alpha\cdot\frac{f}{f_d}} \cosh\left(k\sin\alpha\sqrt{1-\left(\frac{f}{f_d}\right)^2}\right)}{\pi I_0(k)\sqrt{1-\left(\frac{f}{f_d}\right)^2}} \\ \quad + \frac{K}{K+1}\delta(f - f_d\sin\theta_0), |f| \le f_d \\ 0, \text{otherwise} \end{cases} \tag{4}$$

where K is known as the K-factor that measures the power ratio of LOS to NLOS components, θ_0 denotes the angle that the LOS component with mobile direction.

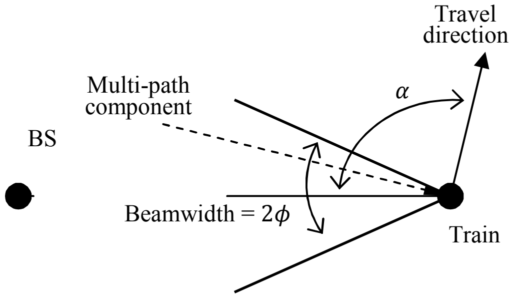

Figure 1. Geometry of train communication.

Figure 1 shows the geometry among BS, the train with its direction, and the antenna beamwidth, denoted as 2ϕ, as those in [10]. With the profiles of the track-side propagations, the average scattering direction is assumed to point to the BS direction, i.e., $\alpha = \theta_0$ in (4). To simplify the discussion, the directional antennas with capability to track the LOS directions [11] are considered in the rest of this paper. To realize frequencies of signal components reaching to the directional antennas at moving trains, we observe with the coordinate system at the train as reference, instead of the ground coordinate. As shown in Figure 2, the relative location of BS moves backward as the observer sits on trains. Therefore, we have the signal components arrive at the front of trains with a frequency shift of the maximum Doppler frequency f_d given in (1). As directional antennas act as spatial filters of receiving signals, the components with the Doppler frequencies $f \in [f_d\cos(\alpha-\phi), f_d\cos\alpha]$ and $f \in [f_d\cos\alpha, f_d\cos(\alpha+\phi)]$ can reach the receivers.

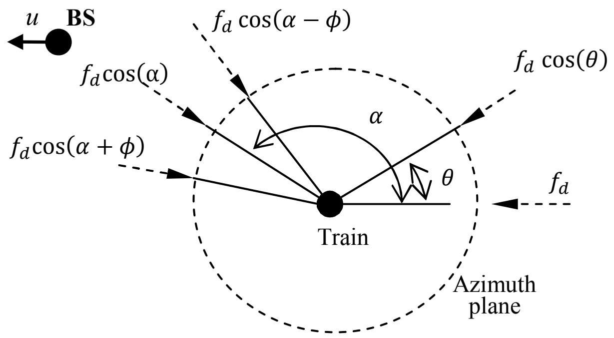

Figure 2. The Doppler frequency with respect to AOA.

Inspired by Figure 2, the scheme named as sliding beam is developed to track the receiving Doppler PSD upon the steer direction of a directional antenna, as illustrated in Figure 3, with following procedures:

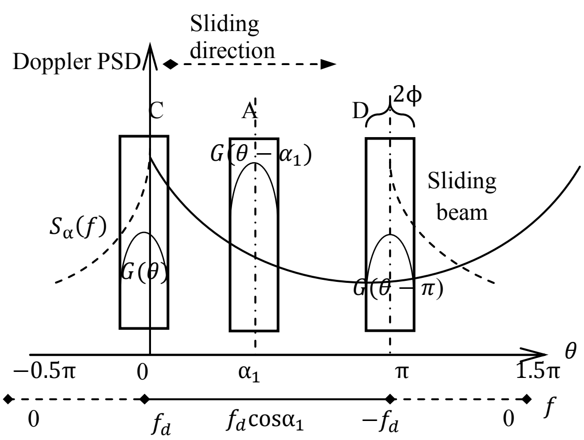

Figure 3. Sliding beam for directional antennas

First, insert (2) into (4) to derive the Doppler PSD $S_\alpha(\theta)$ with $S_\alpha(f)$. Secondly, plot the snapshot $S_\alpha(\theta)$ with α for $\theta \in [0, \pi]$. Map the variable f of Doppler frequency to the AOA variable θ by (2). On the properties of *cosine* function, the Doppler PSD $S_\alpha(\theta)$ for $\theta \in [-0.5\pi, 0]$ and $[\pi, 1.5\pi]$ mirror to those for $\theta \in [0, \pi]$ as shown with dashed lines in

Figure 3. Thirdly, insert a window of 2ϕ in width as the beam of the directional antenna. By aligning the antenna central to the axis $\theta = \alpha$, the antenna gain pattern $G(\theta)$ is shifted to $G(\theta - \alpha)$. Last, slide the antenna beam with trains directions from the position C to A until D, where represent the positions for connecting BS from far site (i.e., $\alpha = 0$), approaching BS (i.e., $\alpha = \alpha_1$) and disconnecting BS (i.e., $\alpha = \pi$), respectively.

Encapsulating those depicted in Figure 3, the power spectrum for the components through the directional antenna are comprehended as the products of Doppler PSD $S_\alpha(\theta)$ and the antenna gain function $G(\theta - \alpha)$in $\theta \in [\alpha - \phi, \alpha + \phi]$, as derived in (5) for $0 \le \alpha \le 0.5\pi$. Those for $0.5\pi < \alpha < \pi$ are inferred symmetrically.

$$\check{S}_\alpha(f) = \begin{cases} S_\alpha(f)\left(G\left(\cos^{-1}\frac{f}{f_d} - \alpha\right) + G\left(\cos^{-1}\frac{f}{f_d} + \alpha\right)\right), \\ \qquad \alpha \le \phi \text{ and } \cos(\phi - \alpha) \le \frac{f}{f_d} \le 1 \\ S_\alpha(f)G\left(\cos^{-1}\frac{f}{f_d} - \alpha\right), \\ \qquad \cos(\phi + \alpha) \le \frac{f}{f_d} < \cos(\phi - \alpha) \\ 0, \text{otherwise} \end{cases} \tag{5}$$

III. ICI POWER FORMULATION

A. Baseband operation in OFDM system

This section concisely formulates the ICI power in OFDM system with given Doppler PSD. The OFDM system use DFT to demodulate received samples into OFDM symbols on the subcarriers. Employing rectangular windows, the frequency response of each subcarrier is expressed as "sinc" function with zero crossings at multiples of $\Delta f = 1/T$, where T denotes symbol duration. The property is expressed with Fourier transform in [12]

$$\mathcal{F}\{\cos(2\pi f_c) \cdot \mathrm{rect}\,(t/T)\} = \mathrm{sinc}\big(T(f - f_c)\big) \tag{6}$$

where $\mathcal{F}\{\cdot\}$ denotes the Fourier transform operation, f_c denotes the carrier frequency, and $\mathrm{rect}(x) = 1$ for $x \in (-0.5, 0.5)$, and $\mathrm{rect}(x) = 0$ elsewhere. Considering an ideal OFDM system with perfect time synchronization and no noise in the wireless channels, the spectrum transformed from the received signals over a complete period of symbol time is the combination of the sinc waveforms, expressed as

$$\textstyle\sum_{m=0}^{N-1} d_m \mathrm{sinc}\big(T(f - f_0 - m\Delta f)\big) \tag{7}$$

where d_m denotes the data mapped to *m*-th subcarrier in the observed period; *N* is the number of subcarriers in the OFDM system; f_0 is the frequency of subcarrier 0; and Δf denotes the subcarrier frequency spacing in the OFDM system.

B. ICI induced with constant frequency offsets

Assuming a wireless channel induces with a constant frequency offset which is normalized by Δf and denoted as ξ. Then, the spectrum for symbol duration in (7) is inserted with a frequency shift of amount $\xi\Delta f$ and expressed as

$$\textstyle\sum_{m=0}^{N-1} d_m \mathrm{sinc}\big(T(f - f_0 - m\Delta f - \xi\Delta f)\big) \tag{8}$$

Clearly, inserting $f = f_0 + m\Delta f$ into (8), the magnitude of ICI on *m*-th adjacent subcarrier is derived as

$$i_m(\xi) = C_0\,\mathrm{sinc}(m - \xi) \tag{9}$$

where C_0 denotes the constant aggregating extra parameters and the d_m in (7)~(8).

The total ICI power on *p*-th subcarrier with a single normalized frequency offset ξ is derived as summation of ICI power from all of the adjacent subcarriers.

$$I_p(\xi) = E\{\textstyle\sum_{m\neq p}^{N-1} i_m(\xi)^2\} = {C_0}^2 \sum_{m\neq p}^{N-1} \mathrm{sinc}^2(m - p - \xi) \tag{10}$$

where $E\{\cdot\}$ denotes mathematical expectation.

C. ICI power induced with given Doppler PSD

The Doppler PSD describes the power distribution for a signal-frequency signal scattered in the wireless channel with Doppler effects. A Doppler PSD can be theoretically the summation of infinite components with individual frequency offset f of magnitude $\check{S}_\alpha(f)$ in (5). Therefore, the ICI power on *p*-th subcarrier echoing to Doppler PSD is expressed as

$$\begin{aligned} ICI_{\alpha,p} &= \int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi) I_p(\xi) d\xi \\ &= {C_0}^2 \textstyle\sum_{m\neq p}^{N-1} \int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi)\mathrm{sinc}^2(m - p - \xi) d\xi \end{aligned} \tag{11}$$

where $\check{S}_\alpha(\xi)$ is derived after inserting $f = \xi\Delta f$ into (5) and $\xi_d \stackrel{\text{def}}{=} f_d/\Delta f$ is the maximum normalized Doppler frequency.

IV. THE OPTIMAL FREQUENCY OFFSETS

The Doppler effects in mobile wireless channel contribute certain spans of frequency offsets described with Doppler PSD. The frequency shift in demodulation that results in the minimal total ICI power in the OFDM system is generally close to the median frequency for the profiles of the Doppler PSD.

Consider a receiver employs its local oscillator with a constant normalized frequency offset, denoted as ξ_s, the ICI power on *p*-th subcarrier in (11) becomes

$$\begin{aligned} ICI_{\alpha,p}^{\xi_s} &= \int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi) I_p(\xi - \xi_s) d\xi \\ &= {C_0}^2 \textstyle\sum_{m\neq p}^{N-1} \int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi)\mathrm{sinc}^2(m - p + \xi_s - \xi) d\xi \end{aligned} \tag{12}$$

It could be difficult to solve the optimal frequency offset ξ_s with the minimal power ICI derived by (12). Alternatively, it is practical to consider the optimal frequency shift ξ_s as that with maximal sampled power on subcarriers. Insert $m = p$ to (12), the average sampled power on subcarriers with normalized frequency offset ξ_s is derived as

$$P(\xi_s) = C_0{}^2 \int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi)\, \mathrm{sinc}^2(\xi_s - \xi) d\xi \tag{13}$$

Set the *derivative* of (13) to zero to determine ξ_s for the maximal $P(\xi_s)$, the condition for the optimal frequency offset ξ_s is described as (see Appendix)

$$\int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi)(\xi_s - \xi) d\xi = 0 \tag{14}$$

Solving (14), the optimal frequency offset is derived as

$$\xi_s = \frac{\int_{-\xi_d}^{\xi_d} \xi \check{S}_\alpha(\xi) d\xi}{\int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi) d\xi} \tag{15}$$

Therefore, the optimal frequency offset ξ_s dynamically aligns to the center of gravity of the instant receiving Doppler PSD $\check{S}_\alpha(\xi)$.

V. NUMERICAL RESULTS AND SYSTEM SIMULATIONS

In this section, we apply the fading wireless channels with the OFDM parameters specified in 802.16 standards for system simulation. The carrier frequency is chosen as 2.4 GHz with bandwidth 5MHz divided into 512 subcarriers.

The train speed in HSR is set at 350 km/hr to incur the normalized Doppler frequency of 0.08. The train travel directions (α in Figure 1) vary from 0 to 0.5π for the journey from the far site to passing a BS. The gain patterns of directional antennas in (5) are set as $G(\theta) = \mathrm{A}$ for $-\phi \le \theta \le \phi$, where A is an arbitrary constant. In the von Mises distribution model, the parameter of directional scattering beamwidth k in (3) and K factor in (4) are set to zeros for the worst case while directional antennas are employed for ICI reduction.

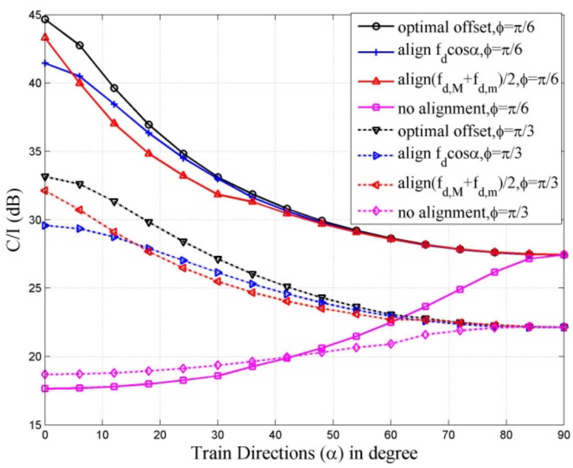

Figure 4. The C/I ratios v.s. train directions for various frequency aligning policies and beamwidths.

Figure 4 compares the C/I ratios of the central subcarrier for various policies of frequency alignments and beamwidths as directional antennas are employed for ICI reduction. The results indicate the proposed frequency offset is superior to other approaches: 1~2 dB against aligning central frequency to mean Doppler frequency [4] and 2~3 dB against aligning central frequency to the maximum Doppler frequency.

To simulate the performance with various frequency aligning policies with directional antennas, the simplified version of DVB-T channel model of static impulse response in [13-14] is adopted. The least square (LS) estimators based on 1/8 comb-type pilot arrangement perform the channel estimation for the mobile OFDM system with 16QAM modulation. The beams of directional antenna are modeled with restricted Jakes Doppler spectrum [15]. The travel direction α is set to zero because it has $\alpha \approx 0$ in the most part of the journey while the train approaches BS.

To perform frequency shifts with local oscillator alignment, the phases proportional to the frequency shifts are imposed on the time-domain samples according to the property of DFT operation:

$$e^{\left(\frac{-2\pi j n \xi}{N}\right)} x[n] \overset{FFT}{\longleftrightarrow} X(k + \xi) \tag{16}$$

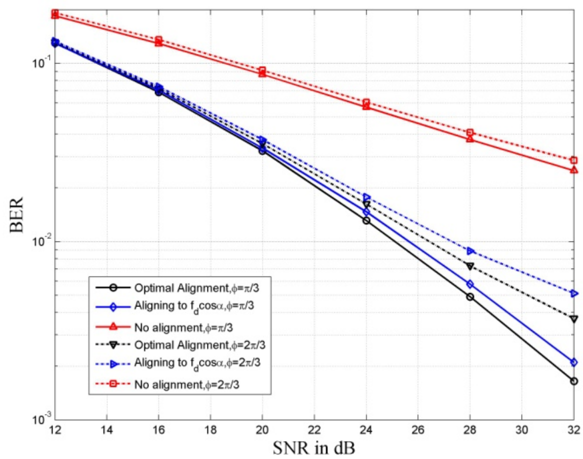

Figure 5. Performance in BER for various frequency alignment and beamwidths v.s. SNR.

Plotting the simulation results in Figure 5, the frequency alignment for Doppler effects achieves considerably lower BERs with higher C/I ratios than those without alignments. Therefore, the frequency alignment is vital for ICI reduction with directional antennas. Furthermore, the frequency shifts using (15) result in the lowest bit error rates for all cases. Thus, the correctness of the optimal frequency offsets developed in this paper is verified.

VI. CONCLUSION

This paper investigates the optimal frequency offsets with Doppler effects for the compensation of CFOs for downlink detection. The acquisition of the optimal frequency offsets is explicitly formulated as the center of gravity of the instant receiving Doppler PSD which can be evaluated with the sliding beam scheme presented in this paper or other methods. The correctness for the proposed approach is examined as the highest C/I ratios and the lowest BER are observed through system simulations.

APPENDIX

The Taylor series expansion of $\mathrm{sinc}(x)$ around $x = 0$ is given as:

$$\mathrm{sinc}(x) = 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \cdots. \tag{17}$$

Therefore, we have the first three terms for the Taylor series expansion of $\mathrm{sinc}^2(x)$ as

$$\mathrm{sinc}^2(x) = 1 - \frac{1}{3}x^2 + \frac{2}{45}x^4 + \cdots. \tag{18}$$

In practical mobile communication systems, the maximum normalized Doppler frequency ξ_d is supposed to be lower than 12%, and the reasonable frequency offset is chosen between ξ_d and $-\xi_d$, i.e.,

$$|\xi_s - \xi_d| \leq 0.12. \tag{19}$$

Therefore, the term of $\mathrm{sinc}^2(\xi_s - \xi)$ in (13) can be closely approached with the first two terms in (18) with deviation less than 0.001%. Thus we have

$$P(\xi_s) \approx {C_0}^2 \int_{-\xi_d}^{\xi_d} \check{S}_\alpha(\xi)\left(1 - \frac{1}{3}(\xi_s - \xi)^2\right) d\xi. \tag{20}$$

The *derivative* of (20) is set to zero to determine ξ_s for the maximal $P(\xi_s)$. Thus, results in (14) can be obtained.

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