Transmission of discrete constellations under strong interference

Dimitris Toumpakaris*, Jungwon Lee† and Rizwan Ghaffar‡
*Wireless Telecommunications Laboratory, Department of Electrical and Computer Engineering
University of Patras, Rio, Greece 265 00, Email: dtouba@upatras.gr
†Samsung Electronics, Mobile Solutions Lab, 4921 Directors Pl., San Diego, CA 92121, Email: jungwon@stanfordalumni.org
‡Department of Electrical and Computer Engineering,
University of Waterloo, Waterloo, ON, N2L 3G1, Canada, Email: rghaffar@uwaterloo.ca

***Abstract*—Transmission with discrete constellations is considered over the two-user Gaussian interference channel. The boundaries between the very strong, strong and weak interference regime are examined. Achievable regions are then found for the two-user Gaussian interference channel under strong interference for rates that can be attained using receivers of small complexity.**

I. Introduction

In addition to noise and fading, modern wireless systems also face the challenge of interference. Orthogonalization of users in frequency, time, code and/or space or treating interference as noise are widely used approaches of reasonable complexity; in some cases, they are close to optimal. One example is treating interference as noise when small [1]. However, avoiding or ignoring interference may not always be the best approach.

The need for efficient use of the system resources has been one of the reasons for the renewed interest in the Interference Channel (IC), in general, and in the Gaussian IC, in particular. When interference is *very strong*, it does not impact transmission [2] if each receiver can decode the signals of all users. The capacity region of the Gaussian IC under *strong* interference is also known [3]. Although the achievable sum rate is smaller compared to very strong interference, it is larger than when using orthogonalization (FDMA/TDMA/CDMA). The capacity region of the Gaussian IC for *weak* interference is still not known except for the sum capacity for very weak interference, achieved by treating interference as noise [1].

The information theoretic results are based on limiting arguments and long codebooks. In practice, to simplify the implementation, uncoded or coded messages are mapped to discrete constellations [4]. Although such transmit schemes are suboptimal, dealing with interference explicitly can often lead to improved rates compared to viewing it as noise.

This paper examines such cases where discrete constellations are used by the transmitter and interference is taken into account for detection at the receiver. A difference with related work [5], [6] is that specific modulation schemes are considered instead of deriving expressions for the constellation-constrained mutual information. The contributions of the paper are summarized below

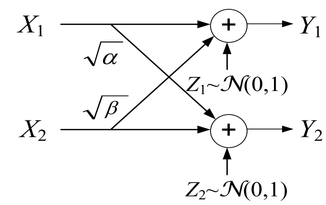

Fig. 1. Two-user Gaussian Interference Channel in standard form.

- It is first shown that, for given transmit power, the cross-channel gains that correspond to the case of very strong interference are smaller when discrete constellations are used compared to capacity-achieving codebooks.
- Then transmission schemes are proposed for the Gaussian IC under strong interference, and the corresponding achievable rate regions are derived.
- Finally, the changes that are needed at the receivers in order to support the proposed schemes are discussed for both strong and very strong interference.

A practical scenario to which the strong interference model applies is in heterogeneous networks where users have some restricted association, i.e. femto users may remain attached to their femto base station in spite of receiving relatively stronger signals from a macro base station.

Full Channel State Information is assumed at both the transmitter and the receiver. It is also assumed that the transmissions of both users are synchronized. To simplify the discussion, baseband transmission and Pulse-Amplitude Modulation (PAM) is considered.

The remainder of the paper is organized as follows. The model of the Gaussian IC is reviewed in Section II. The Gaussian IC with very strong interference is considered in Section III, followed by strong interference in Section IV. Conclusions are drawn in Section V.

II. The Gaussian Interference Channel

The *standard-form* real 2-user Gaussian Interference Channel (G-IC), shown in Fig. 1, is described by the equations

$$\begin{aligned} Y_1 &= X_1 + \sqrt{\beta} X_2 + Z_1, \text{ and} \\ Y_2 &= \sqrt{\alpha} X_1 + X_2 + Z_2, \end{aligned} \tag{1}$$

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where $\mathbb{E}\left[Z_k^2\right] = 1$. The transmit symbols X_1 and X_2 are subject to an average power constraint: $\mathbb{E}\left[X_k^2\right] \leq P_k$, $k = 1, 2$. Any G-IC can be converted to an equivalent standard-form G-IC [7]. When $\alpha \geq 1 + P_2$ and $\beta \geq 1 + P_1$, interference does not affect the transmission [7]: Because $\alpha \geq 1 + P_2$, receiver 2 can first decode X_1 treating X_2 as noise, subtract $\sqrt{\alpha}X_1$ from Y_2 and decode X_2. The same argument can be used for receiver 1. This is the capacity region of the G-IC, since the capacity of the Gaussian channel along each link cannot be exceeded. This regime is called very strong interference and can be generalized to non-Gaussian ICs [3].

When $1 \leq \alpha < 1 + P_2$ and $1 \leq \beta < 1 + P_1$, the G-IC is in the strong interference regime. In this case both users are decodable at both receivers. The capacity region is formed by the intersection of two Multiple Access Channels (MACs), one from transmitters 1 and 2 to receiver 1, and the other from transmitters 1 and 2 to receiver 2 [3]. Again, the result can be generalized to non-Gaussian ICs [8].

III. Transmission of Discrete Constellations in the Very Strong Interference Regime

It is now assumed that discrete constellations are used and symbol-by-symbol detection is employed at each receiver. Moreover, a target maximum probability of error P_e is set. For this scenario, the very strong interference case corresponds to the regime where interference does not affect the rates that each of the users would be able to achieve over a Gaussian channel in the absence of interference.[1]

Hence, the maximum achievable rate of user k is equal to

$$R_k = \log_2(M_k) = \left\lfloor \frac{1}{2}\log_2\left(1 + \frac{P_k}{\Gamma(R_k, P_e)}\right)\right\rfloor, \quad (2)$$

where $M_k = 2^{R_k}$ is the constellation size and Γ is the so-called SNR gap that quantifies the additional power that is required by the suboptimal coding scheme compared to optimal, capacity-achieving Gaussian codebooks [9], [10].[2] For uncoded PAM, $\Gamma(R_k, P_e) = \Gamma(M_k, P_e) = \left[Q^{-1}\left(\frac{M_k P_e}{2(M_k-1)}\right)\right]^2/3$, where $Q(\cdot)$ is the Q function. When coding is also used, the value of Γ needs to be scaled by dividing by the code gain γ, and R_k can take non-integer values. In (2), the fact that $\mathbb{E}\left[Z_k^2\right] = 1$ was used.

If X_1 belongs to an M_1-PAM signal, the addition of $\sqrt{\beta}X_2$ at receiver 1 shifts $X_1 + Z_1$ by a value that belongs to the constellation of user 2 scaled by $\sqrt{\beta}$. Therefore, $X_1 + \sqrt{\beta}X_2$ belongs to one of M_2 cosets, determined by the value of X_2.

If the target P_e is very small, considering only the most likely error events leads to a good approximation. There are two types of most likely events. Events that lead to an erroneous decision inside a coset (and, therefore, affect only the signal of the direct user) and those that cause a wrong coset decision (and affect the probability of error of both users). In order to simplify the expressions, an upper bound is considered, which is close to the exact value for small values of P_e. The upper bound is equal to the probability of error when estimating the value of the aggregate signal, $X_1 + \sqrt{\beta}X_2$ (i.e., the probability of error for the joint estimation of X_1 and X_2). Hence, $P_e \leq 2\left(1 - \frac{1}{M_1M_2}\right)Q\left(\frac{d_{\min,1}}{2}\right)$ [9]; thus, $d_{\min,1} > 2Q^{-1}\left(\frac{M_1M_2P_e}{2(M_1M_2-1)}\right)$ is the minimum distance between the values of the aggregate signal $X_1 + \sqrt{\beta}X_2$. The same result can be derived for receiver 2, *i.e.,* $d_{\min,2} > 2Q^{-1}\left(\frac{M_1M_2P_e}{2(M_1M_2-1)}\right)$.

Therefore, P_1 should be such that the constellation points X_1 at receiver 1 be at least $d_{\min} = d_{\min,1}$ away and such that the constellation points $\sqrt{\alpha}X_1$ at receiver 2 be at least $M_2 d_{\min,2} = M_2 d_{\min}$ away in order not to affect the target P_e. Hence, in order to be in the very strong interference regime,

$$\sqrt{\alpha}d_{\min} \geq M_2 d_{\min} \Rightarrow \alpha \geq M_2^2. \quad (3)$$

Similarly, $\beta \geq M_1^2$.

From (2), $R_k \leq \left\lfloor \frac{1}{2}\log\left(1 + P_k/\Gamma(M_k, P_e)\right)\right\rfloor \Rightarrow M_k^2 \lessapprox 1 + P_k/\Gamma(M_k, P_e)$. Thus,

$$\alpha \gtrapprox 1 + P_2/\Gamma(M_2, P_e) \text{ and } \beta \gtrapprox 1 + P_1/\Gamma(M_1, P_e). \quad (4)$$

Recall that the corresponding conditions when optimal, capacity-achieving codebooks are used are [2]

$$\alpha \geq 1 + P_2 \text{ and } \beta \geq 1 + P_1. \quad (5)$$

Therefore, when a discrete constellation is used instead of an optimal codebook, the boundary between the very strong and the strong interference region moves closer to 1. This is because discrete interference is better than worst-case Gaussian interference [11]. Hence, smaller cross-channel gains suffice in order to be able to fully cancel interference.

In the very strong interference regime, the power of the interferer is larger than the power of the signal of interest. To derive (4), neighboring constellation points of the interferer were forced to be further apart than the outermost constellation points of the direct signal. As will be seen in Section IV, this does not always hold in the strong interference case.

As in the capacity-achieving case, knowledge of the constellation and the power of both users is required at each receiver. Then either joint detection or successive interference cancellation can be used. In fact, decoding of the interferer is not necessary; each receiver k can be simplified by wrapping the received signal in the interval $(-M_k d_{\min}/2, +M_k d_{\min}/2)$.

An example of achievable rate regions is given in Fig. 2. $\alpha = 112 > P_2/\Gamma(2, 10^{-7})$ and $\beta = 1120 > P_1/\Gamma(2, 10^{-7})$. Because (4) hold, for uncoded PAM interference can be fully canceled. On the other hand, (5) are not satisfied, so for optimal codebooks the sum rate is smaller than the sum of the capacities of the direct links. As expected, orthogonalization performs worse. The reason why the orthogonalization curve is not continuous is because of the floor operation in (2); an improved rate region can be obtained using time sharing. Finally,

[1]Note that an "operational" definition of very strong interference is used in this paper. This is different than the information theoretic definition $I(X_k; Y_l) \geq I(X_k; Y_k|X_l)$, $k \neq l$ and for any distributions for X_1 and X_2, which reduces to $\alpha \geq 1 + P_2$ and $\beta \geq 1 + P_1$ for the Gaussian IC.

[2]Strictly speaking, a non-integer rate can be achieved, in general, using time-sharing between R_k and $R_k + 1$. Time-sharing is not used in this paper to avoid over-complicating the presentation.

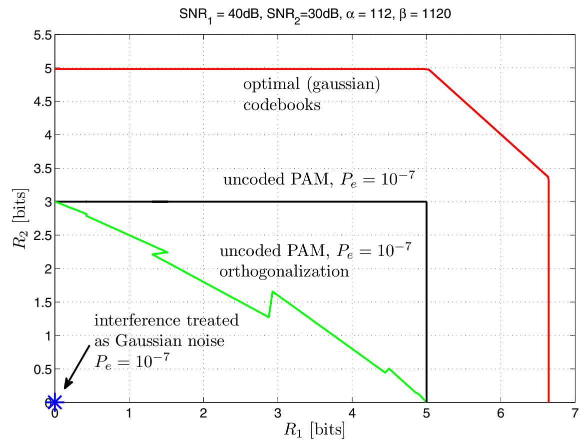

Fig. 2. Comparison of achievable rates. $\alpha = 112$, $\beta = 1120$, $P_1 = 40$dB, $P_2 = 30$dB. The IC is in the very strong interference regime when uncoded PAM is used and $P_e = 10^{-7}$, and in the strong interference regime when optimal, capacity-achieving codebooks are employed.

clearly, designing the receiver assuming that the interference is Gaussian makes it impossible to achieve meaningful rates.

The results can be extended to coded PAM by substituting Γ by Γ/γ, where γ is the coding gain.

IV. The strong interference regime

A. Introduction and achievable regions for the Gaussian MAC

Strong interference occurs when both α and β are between 1 and the values in (4).[3] It can be argued, similarly to the case of optimal codebooks [3] that, when discrete constellations are used, the optimal rate region of the G-IC with strong interference is given by the intersection of the optimal rate regions of two MACs. Let the rate pair (R_1, R_2) belong to an achievable region. Because R_1 is achievable, receiver 1 can decode X_1, subtract it from Y_1 and obtain $\sqrt{\beta}X_2 + Z_1$. Because R_2 is achievable, receiver 2 can decode X_2 in the presence of $\sqrt{\alpha}X_1 + Z_2$. However, $\sqrt{\beta}X_2 + Z_1$ at receiver 1 is only subject to noise Z_1 of equal power as Z_2 and X_2 is amplified by $\sqrt{\beta} \geq 1$ compared to receiver 2. Hence, receiver 1 can also decode X_2. The same argument can be used for receiver 2. Therefore, rate points inside an achievable rate region should be decodable by both receivers.

However, finding the MAC rate regions for discrete constellations appears to be more complicated than the Gaussian codebook case, because of the inter-dependence of the minimum distances of the constellations. In this section, a region that leads to simple receiver implementations is formed. As will be seen, the transmission scheme depends on whether $\beta P_2 \geq P_1$ or $\beta P_2 < P_1$. In some cases, the scheme performs better than orthogonalization. However, the derived region may not necessarily be the largest achievable.

First, two lemmas are introduced for the 2-user Gaussian MAC. The lemmas are proven in [12].

[3] Here, the same definition of strong interference is employed as for optimal, capacity-achieving inputs, i.e., $I(X_k; Y_l|X_l) \geq I(X_k; Y_k|Y_l)$, $l \neq k$ (which results in $\alpha > 1$ and $\beta > 1$ for the Gaussian IC).

Lemma 1 [12]: The achievable rate region of the 2-user Gaussian MAC with $\mathbb{E}\left[Z^2\right] = 1$ when uncoded PAM is employed is upper bounded by

$$\begin{aligned} R_k &\leq \frac{1}{2}\log_2\left(1 + \frac{P_k}{\Gamma(R_k, P_e)}\right), \; k = 1, 2, \\ R_1 + R_2 &\leq \frac{1}{2}\log_2\left(1 + \frac{P_1 + P_2}{\Gamma(R_1 + R_2, P_e)}\right). \end{aligned} \tag{6}$$

Lemma 2 [12]: $(R_{1,in}, 0)$ and $(R_{\text{sum},in} - R_{2,in}, R_{2,in})$ are achievable rate pairs for the 2-user Gaussian MAC with $P_1 \geq P_2$ and $\mathbb{E}\left[Z^2\right] = 1$ when uncoded PAM and superposition of the signals is employed, where

$$\begin{aligned} R_{k,in} &\triangleq \left\lfloor \frac{1}{2}\log_2\left(1 + \frac{P_k}{\Gamma(R_k, P_e)}\right)\right\rfloor, \; k = 1, 2 \text{ and} \\ R_{\text{sum},in} &\triangleq \left\lfloor \frac{1}{2}\log_2\left(1 + \frac{P_1 + P_2}{\Gamma(R_1 + R_2, P_e)}\right)\right\rfloor. \end{aligned} \tag{7}$$

The lemmas are discussed in detail in [12] and is also shown that, in general, the rate pairs of Lemma 2 can be improved by a fraction of a bit. The rate pairs of Lemma 2 are used to facilitate the discussion on the G-IC that follows. The inner and outer bounds of the achievable rate region are depicted in Fig. 3 by the solid and the dashed line, respectively. The improved rate region not discussed in this paper is also shown.

Lemma 1 is straightforward: the rate of each user cannot exceed the rate of the single-user Gaussian channel, and the sum rate cannot exceed the rate achieved when both users can fully cooperate. For Lemma 2, as is shown in [12], the rate pair $(R_{\text{sum},in} - R_{2,in}, R_{2,in})$ is achieved by the weaker user 2 with $P_2 \leq P_1$, forming a constellation of smaller minimum distance compared to the stronger user 1. Other rate pairs on the boundary of the rate region can be achieved using smart time-sharing between $(R_{1,in}, 0)$ and $(R_{\text{sum},1} - R_{2,in}, R_{2,in})$. This is based on the observation that, at $(R_{1,in}, 0)$, user 2 is not transmitting, and therefore conserves energy that can be spent during transmission at $(R_{\text{sum},in} - R_{2,in}, R_{2,in})$.

Returning to the Gaussian IC, note that, for very strong interference, if P_1 and $P_2 \geq \Gamma$ (so that, in the absence of interference, $M_k \geq 2$), $\beta P_2 > P_1$, and $\alpha P_1 > P_2$. Hence, the power of the interfering user always exceeds the power of the user of the direct link. However, for strong interference, this may not always be true at receiver 1. Because $\alpha \geq 1$, $\alpha P_1 > P_2$. Therefore, assuming, without loss of generality, that $P_1 \geq P_2$, two cases need to be considered, namely $\beta P_2 < P_1$ (the power of user 1 is the largest on both receivers), and $\beta P_2 \geq P_1$ (the power of user 2 at receiver 1 is larger than the power of user 1).

B. An achievable rate region when $\beta P_2 < P_1$

When $\beta P_2 < P_1$, the weak user of both MACs is user 2. Transmitter 2 uses $P_2 = \Gamma(M_2, P_e)(M_2^2 - 1)$ to create a signal with sufficient minimum distance to guarantee the target P_e at receiver 2. At receiver 1, P_2 is amplified by β. In order to achieve the inner bound of the sum rate of Lemma 2, transmitter 1 (with $P_1 \geq P_2$) should create a

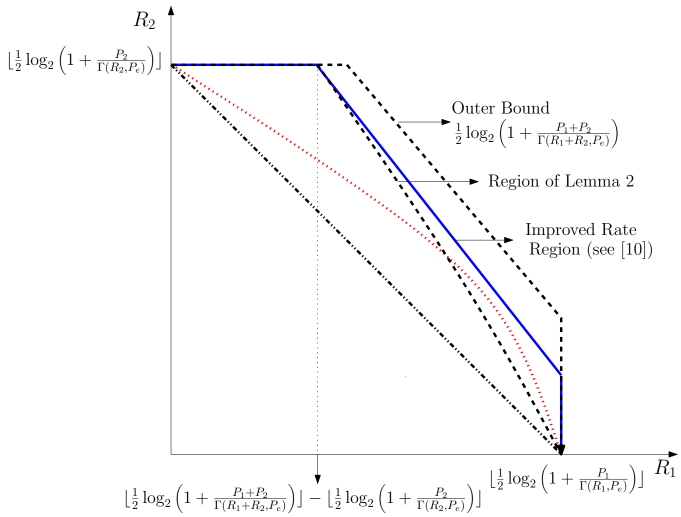

Fig. 3. Rate region of Lemma 2 (dashed curve) and improved rate region (continuous blue line) of 2 user Gaussian MAC channel with PAM alphabets. TDMA with power control, i.e. smart TDMA (red dotted curve) and naive TDMA, i.e. without power control (dashed-dotted line) are also shown.

signal with minimum distance that is at least M_2 times the minimum distance of $\sqrt{\beta}X_2$. This also guarantees decodability at receiver 2 because the minimum distance of $\sqrt{\alpha}X_1$ is larger than the minimum distance of X_1 at receiver 1.

It should be mentioned here that the rate of user 1 can be improved by observing that the distance between the symbols of user 1 at receiver 1 can be reduced from $M_2\sqrt{\beta}d_{\min}$ to $\left[(M_2-1)\sqrt{\beta}+1\right]d_{\min}$, because the distance between the edge points of the cosets indexed by X_1 (each containing M_2 points) can be moved closer without increasing the target P_e. In the following this improvement that is more pronounced for large values of β is ignored to simplify the expressions.

However, at receiver 1, the minimum distance of $\sqrt{\beta}X_2$ is larger than what is required for decoding with probability of error not exceeding P_e and cannot be reduced, because this would reduce the minimum distance of X_2. This, in turn, would render the decoding of X_2 at receiver 2 impossible. This rate penalty is equivalent to requiring a gap of $\Gamma\times\beta$ at receiver 1. Hence, using Lemma 2, the following achievable rate pairs can be obtained.

Theorem 1: For the 2-user Gaussian IC with strong interference, $P_1 \geq P_2$ and $\beta P_2 < P_1$, when uncoded PAM and superposition of the signals is employed, rate pairs $(R_1, 0)$ and $(R_{\text{sum}} - R_2, R_2)$ are achievable, where

$$R_k = \left\lfloor \frac{1}{2}\log_2\left(1+\frac{P_k}{\Gamma(R_k,P_e)}\right)\right\rfloor,\ k=1,2 \text{ and}$$
$$R_{\text{sum}} = \left\lfloor \frac{1}{2}\log_2\left(1+\frac{P_1+\beta P_2}{\beta\Gamma(R_1+R_2,P_e)}\right)\right\rfloor. \quad (8)$$

An example for $\beta P_2 < P_1$ is given in Fig. 4. $\alpha = 2$, $\beta = 3$, $P_1 = 44.77$dB and $P_2 = 30$dB. For example, the rate point $(R_1, R_2) = (3, 2)$, which is not achievable by FDMA/TDMA, can be achieved as follows: Transmitter 2

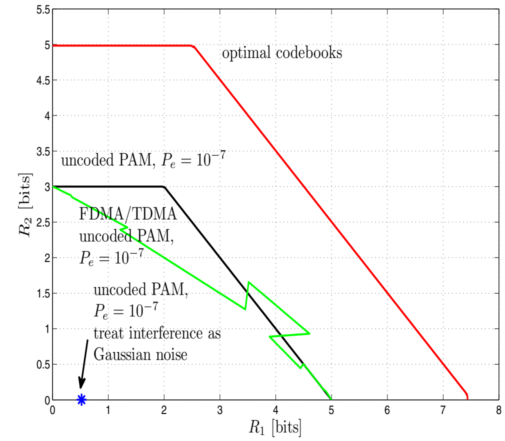

Fig. 4. Comparison of achievable rates for strong interference and $\beta P_2 < P_1$. $\alpha = 2$, $\beta = 3$, $P_1 = 44.77$dB, $P_2 = 30$dB. $P_e = 10^{-7}$.

uses $P_2 = 15\Gamma(R_2 = 2, P_e) = 21.43$dB to form a 4-PAM signal. Transmitter 1 forms an 8-PAM signal with 4 times the minimum distance of $\sqrt{\beta}X_2$. Hence, $P_1 = 16{\cdot}3{\cdot}\Gamma(2,P_e){\cdot}63 = 44.48$dB. Clearly, the minimum distance of $\sqrt{\alpha}X_1$ exceeds by far M_2 times the minimum distance of X_2.

Receiver 2 can either decode X_1 and X_2 jointly to determine X_2; or can first detect the coset (determined by $\sqrt{\alpha}X_1$) and then find the value of X_2; or wrap the received signal in the interval $[-\sqrt{\alpha\beta}M_2d_{\min}/2, +\sqrt{\alpha\beta}M_2d_{\min}/2]$ and then decode X_2. At receiver 1, unless joint decoding is used, what needs to be found is the coset where the received signal belongs, as it corresponds uniquely to the value of X_1.

C. An achievable rate region when $\beta P_2 \geq P_1$

This case appears to be the most difficult to handle. Now, the weak transmitter for each MAC is the one corresponding to the direct link. From Lemma 2, one approach is for each transmitter to create a constellation of minimum distance. Then the interfering signal should be such that the minimum distance of its constellation be at least M_k times larger than the minimum distance of the constellation of the direct user. Let $d_{\min,i}$ be the minimum distances of the constellations of the X_k. Then the following conditions need to be satisfied

$$\sqrt{\alpha}d_{\min,1} \geq M_2 d_{\min,2}, \text{ and } \sqrt{\beta}d_{\min,2} \geq M_1 d_{\min,1}. \quad (9)$$

Combining the two inequalities, $d_1 \geq \frac{M_1M_2}{\sqrt{\alpha\beta}}d_1$ and $d_2 \geq \frac{M_1M_2}{\sqrt{\alpha\beta}}d_2$. Therefore, $M_1M_2 \leq \sqrt{\alpha\beta}$, and, regardless of the available power, $R_1 + R_2 \leq \frac{1}{2}\log(\alpha\beta)$.

The constellations can be constructed as follows: Let R_1 and R_2 be inside the regions of both MACs and $M_1M_2 \leq \sqrt{\alpha\beta}$. If $M_1/\sqrt{\beta} > M_2/\sqrt{\alpha}$, set $d_1 = d_{\min} = \sqrt{12\Gamma(\infty, P_e)}$. Note that, because $\beta \geq 1$, $M_1\sqrt{\beta} \geq 1$. Then pick d_2 so that $\sqrt{\beta}d_2 = M_1 d_{\min} \Rightarrow d_2 = \frac{M_1}{\sqrt{\beta}}d_{\min}$ in order for both X_1 and X_2 to be decodable at receiver 1. Thus, $\sqrt{\alpha}d_{\min} = \frac{\sqrt{\alpha\beta}}{M_1}d_2 \geq M_2d_2$. Therefore, X_1 and X_2 can also be decoded at receiver 2. The power employed by user 1 is equal to $P_{1,\text{required}} =$

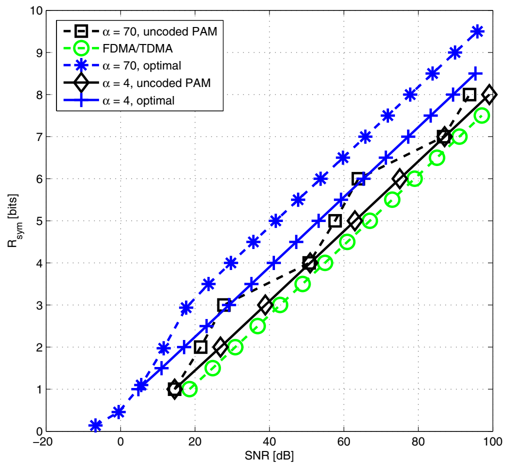

Fig. 5. Comparison of achievable symmetric rate for symmetric Gaussian IC with strong interference. $P_e = 10^{-7}$.

$\Gamma(M_1, P_e)(M_1^2 - 1)$, whereas the power employed by user 2 is $P_{2,\text{required}} = \frac{M_1^2}{\beta}\Gamma(M_2, P_e)(M_2^2 - 1)$.

When $M_1/\sqrt{\beta} < M_2/\sqrt{\alpha}$, construction of the constellation starts by setting $d_2 = d_{\min}$.

Clearly, $M_1 M_2 \leq \sqrt{\alpha\beta}$ is a very restrictive constraint for values of α and β close to 1, especially for large P_k that are much larger than $P_{k,\text{required}}$. In that case, only a portion of the P_k is used to attain a sum rate equal to $\frac{1}{2}\log(\alpha)+\frac{1}{2}\log(\beta)$, and this approach is inferior to orthogonalization. Forming the constellations along the lines of Section IV-B is not an option, either, because the power of the direct signal is smaller than the interference. Hence, it is not possible to construct points in the constellation of the direct signal with power that is larger than the outermost points of the constellation of the interferer (assuming that full power is used by the transmitters).

A way to overcome the limitation of $\frac{1}{2}\log(\alpha\beta)$ is to create constellation "layers" above the first $M_1 M_2$ points. For example, if one more "layer" is created, it can be shown that $\tilde{M}_1\tilde{M}_2 \leq \sqrt{\alpha\beta}$, where $\tilde{M}_k$ are the sizes of the constellations of the new layer. However, there now is a "penalty" $M_1 M_2$ in the minimum distances. Hence, all powers are scaled by $M_1^2 M_2^2$. For example, in the second iteration, if $\tilde{M}_1/\sqrt{\beta} > \tilde{M}_2/\sqrt{\alpha}$, $\tilde{P}_{1,\text{required}} = M_1^2 M_2^2 \Gamma(\tilde{M}_1, P_e)(\tilde{M}_1^2 - 1)$ and $\tilde{P}_{2,\text{required}} = M_1^2 M_2^2 \frac{\tilde{M}_1^2}{\beta}\Gamma(\tilde{M}_2, P_e)(\tilde{M}_2^2 - 1)$ needs to be added to $P_{1,\text{required}}$ and $P_{2,\text{required}}$, respectively. The overall rate for user k is $R_k + \tilde{R}_k$.

An example is shown in Fig. 5 where the symmetric rate, R_{sym}, is plotted for a symmetric G-IC ($P_1 = P_2$ and $\alpha = \beta$). Two cases are considered, $\alpha = 70$ and $\alpha = 4$. As can be seen, use of properly constructed constellations can lead to improved symmetric rates compared to orthogonalization, which cannot take advantage of the cross-channel gains. For $\alpha = 4$, $M = 2$ and for each additional bit, a new constellation "layer" is superimposed on the previous ones. The loss in performance with respect to the capacity-achieving case is constant. For $\alpha = 70$ the channel is in the very strong interference regime for $R_{\text{sym}} < 3.065$ bits and enters in the strong interference area for larger rates (and powers). In each layer, $M = 8$. As can be seen, use of the power is more efficient when the maximum possible size of M can be used in each layer.

The receivers either need to decode jointly X_1 and X_2 from the overall constellation, or apply successive interference cancellation, starting from the outer "layers".

Therefore, in the case of strong interference, there exist several scenarios where simultaneous transmission of discrete constellations by both transmitters can lead to improved performance compared to orthogonalization. Moreover, the required receivers are not prohibitively complex.

Similar to very strong interference, the case of coded PAM can be handled by appropriately modifying the value of Γ.

V. Conclusion

In this paper, transmission of discrete constellations over the Gaussian IC was considered. In the case of very strong interference it was shown that it is possible to decouple the channel to two Gaussian channels, similar to when optimal codebooks are used. Moreover, in some scenarios, the achievable rates over the Gaussian IC with strong interference can be improved by joint design of the constellations of the users, simultaneous transmission and detectors of reasonable complexity.

Such schemes may be applicable to future systems where the need to exploit the available resources efficiently may justify the increased complexity that is required to estimate the channels and decode the signals of interfering users.

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