Multi-Radio Multi-Channel Allocation in Competitive Wireless Ad hoc Networks

Wen Jingrong, Wu Muqing, Tang Xiong
Laboratory of Network System Architecture and Convergence,
Beijing University of Posts and Telecommunications, Beijing, China
Email: jrwen@bupt.edu.cn

Abstract—**Multi-radio multi-channel allocation has been studied in different networks, like Wireless Mesh Networks, Cellular Networks. In this paper, we study the multi-radio multi-channel allocation in competitive wireless Ad hoc networks from the game-theoretic point of view. Nash Equilibrium (NE) and Pareto-optimal are used to study the selfish devices and global optimality. The decreasing utility function is analyzed and a Pareto-optimal channel allocation algorithm based on NE is proposed to achieve the system optimization. A charging scheme is also considered to represent the cost of transmission. Finally, we show that, our proposal can achieve higher utility than NE with both charging and no-charging schemes, especially when there are more players for the no-charging scheme.**

Keywords-channel allocation; game theory; Pareto-optimal; Nash equilibrium

I. INTRODUCTION

With the fast development of emerging wireless technologies, multiple non-overlapping channels can be provided with lower hardware costs [1]. The current IEEE 802.11 standard [2] specifies several orthogonal channels by frequency division multiple access (e.g., 3 such channels in IEEE 802.11b/g and 12 in 802.11a). With multi-radio multi-channel, nodes can transmit and receive simultaneously, which will increase the network capacity greatly by reducing co-channel interference, especially in Wireless Mesh Networks (WMNs) [3] [4].

However, due to the limited number of radios and channels, the interference can not be completely eliminated. An efficient channel allocation is needed to mitigate the performance degradation. Integer Linear Programming (ILP) was used to develop channel allocation schemes in order to achieve optimal performance under a set of necessary conditions as constraints in [3]. Reference [4] also formulated the joint channel assignment, routing and interference-free link scheduling using Linear Programming (LP), considering the number of channels and available radios. Another approach for channel assignment is graph-coloring, which aims to minimize over-all network interference and preserve the whole network connectivity[5][6].

Recently, game theory has been considered to solve the multi-channel allocation problem in competitive wireless networks [7-14]. Felegyhazi et al. first introduced the strategic game model to study the non-cooperative behavior of channel allocation in competitive networks [7]. Each node-pair equips with multiple radio transmitters and multiple available channels. All they are rational and the objective is to maximize their utilities in the network. The necessary and sufficient conditions of Nash equilibrium (NE) were studied, and some load-balancing Nash equilibrium channel allocation algorithms were presented, in spite of the non-cooperative behavior of these players. However, a single collision domain is assumed, that means each pair of nodes in the system will contend and share their wireless channel, interfere with each other. Based on [7], Shila et al. also proposed a Nash Equilibrium solution to achieve load balance in a selfish and topology-blind environment [8]. Lin Gao et al. presented a game-theoretic analysis of multi-radio channel allocation strategies in multi-hop wireless networks [9-11]. The inefficiency of NE was studied in multi-hop networks due to the poor performance of achieved date rate of the multi-hop links. Static cooperative game was introduced, and min-max coalition-proof Nash equilibrium channel allocation scheme was proposed to achieve the maximal date rate of all links, especially for the bottleneck link. While, they also considered all senders and relaying nodes were in a single collision domain, which is unsuitable for the long path. Chen et al. went one step further to study the problem of non-cooperative multi-radio channel assignment in multiple collision domains [12] [13]. They designed incentive-compatible protocols to guarantee complete fair channel assignment and to avoid Pareto suboptimality in multiple collision domains. [14] also presented one centralized algorithm using perfect information and one distributed algorithm using imperfect information in multiple collision domains to balance and maximize all player's total rate.

In the light of non-maximum of system-wide performance in Nash Equilibrium, Wu et al. proposed a strongly dominant strategy equilibrium (SDSE) to achieve the global optimality of system-wide throughput by introducing a payment formula [15]. The repeated game was also considered to get a completely fair channel assignment. What's more, the adaptive-width channel allocation was introduced as a strategic game in non-cooperative multi-radio wireless networks [16]. A charging scheme was also used to guarantee the network to converge to a dominant strategy equilibrium (DSE), which means that the system-wide throughput is maximized. Flows of the network were also considered as players of the game in [17], in order to maximize fairness across non-cooperative multiple collision domains in multi-radio multi-channel wireless mesh networks.

However, all these works assume that the total available rate function $R(k_c)$ is independent of k_c, the number of radios using channel c. Under this assumption, any NE channel allocation will be the Pareto-optimal [7]. Whereas, in fact,

$R(k_c)$ should be a decreasing function due to the packet collision increasing with more radios. This issue leads some theorems to be reconsidered. To the best of our knowledge, our paper is the first to address the decreasing utility function in the game-theoretic analysis of multi-channel allocation.

The rest of this paper is organized as follows. Section II introduces the basic system model along with the game-theoretic description of channel assignment. In section III, we analyze the decreasing of rate functions, and its influence on NE channel allocation. A globally optimal channel allocation algorithm using imperfect information is proposed in section IV, and the result is shown. Finally, we conclude in section V.

II. GAME MODEL

In this section, some basic definitions and game model in competitive wireless Ad hoc networks are introduced. As shown in Fig.1, we assume that all transmitters reside in a single collision domain, which means that they can contend and interfere with each other. What's more, they can hear the transmissions of each other if they are using the same channel. We denote the set of players by P, and the number of players by N. It should be noted that we use the terms "transmitter", "player", "communicating node pair" interchangeably in this paper.

$$P = \{p_1, p_2, ..., p_N\} \tag{1}$$

We assume that the available frequency band is divided into $|C|$ orthogonal channels with the same band-width and channel characteristics, which is denoted as $C = \{c_1, c_2, ..., c_{|C|}\}$. We also assume that each node pair is equipped with k radios, each operating on a particular channel. Each node pair wants to maximize its total transmission rate in the competitive wireless environment. So we define the achieved bit rate as the utility function of the system. The utility of player p_i is written as:

$$U_{p_i}(S) = \sum_{c \in C} R_{p_i,c} \tag{2}$$

where S is the strategy of all players, and $R_{p_i,c}$ indicates the transmission rate of play p_i in channel c.

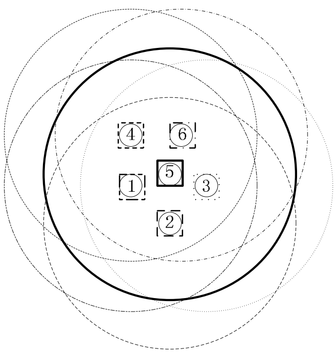

Fig.1. The single collision domain

In the channel allocation game, the strategy of each player is to define the number of radios on each of the channels. We denote the strategy of player p_i as its channel allocation vector:

$$s_{p_i} = \{k_{p_i,1}, k_{p_i,2}, ..., k_{p_i,|C|}\} \tag{3}$$

where $k_{p_i,c}$ indicates the number of radios of player p_i using on channel c. To suppress the co-channel interference, we assume that different radios of the same player cannot employ the same channel, i.e.:

$$k_{p_i,c} = \begin{cases} 1 & \text{if } p_i \text{ is using channel } c \\ 0 & \text{if } p_i \text{ is not using channel } c \end{cases} \tag{4}$$

We denote the number of radios using a particular channel by k_c, i.e., $k_c = \sum_{p_i \in P} k_{p_i,c}$. Similarly, k_{p_i} defines the total number of radios used by player p_i, i.e., $k_{p_i} = \sum_{c \in C} k_{p_i,c}$. The strategies of all players are represented as the matrix S, where the row i of the matrix represents the strategy of player p_i.

$$S = \left(s_{p_1}, s_{p_2}, ..., s_{p_N}\right)^T \tag{5}$$

Furthermore, we denote the strategy matrix except for the strategy of player p_i as S_{-p_i}, i.e.:

$$S = s_{p_i} \cup S_{-p_i} \tag{6}$$

From [7], three important theorems are summarized regarding the NE:

Theorem 1. If $N \cdot k \le |C|$, any channel allocation in which $k_c \le 1$, $\forall c \in C$ is a Pareto-optimal NE; else, S^* is a NE, if and only if both of the following conditions hold: (1) $\delta_{b,c} \le 1$ for $\forall b, c \in C$, where $\delta_{b,c} = |k_b - k_c|$; (2) $k_{p_i,c} \le 1$ for $\forall c \in C$.

Theorem 2. In a NE, each player should use all of its radios to communicate in order to maximize its data rate, i.e., $k_{p_i} = k$.

Theorem 3. Assume that $N \cdot k \ge |C|$, then any NE channel allocation S^* is Pareto-optimal, if the rate function is independent of k_c on any channel c.

It should be noted that NE is usually inefficient from the system point of view, i.e., some of the players might change their strategies mutually to increase each others utility. The concept of Pareto-optimality is introduced to characterize the efficiency of NE. But theorem 3 cannot be guaranteed if the rate function is a decreasing function. We will show the details in section III.

III. UTILITY ISSUES

In this section, the utility of the selfish multi-radio multi-channel allocation game with the decreasing rate function is studied, which is our main contribution.

As shown in (2), the utility of player p_i can be represented as the sum of data rate in all channel of p_i. $R_{p_i,c}$ is the rate of p_i on channel c. Similar to [7], we assume that the total rate $R_c(k_c)$ on channel c is shared equally among the radio transmitters using this channel. So the utility function for player p_i can be rewritten as:

$$U_{p_i}(S)=\sum_{c\in C}R_{p_i,c}=\sum_{c\in C}\frac{k_{p_i,c}}{k_c}R_c(k_c) \tag{7}$$

The previous works consider that $R_c(k_c)$ is independent of k_c by using a TDMA protocol or the CSMA/CA protocol with optimal values [7]. However, it's not the reality. More overhead and collisions make $R_c(k_c)$ become a decreasing function for $k_c>1$. Referring to the results reported by Bianchi in [18], the collision probability ρ is a function of station transmission probability and station number. By combining equation (7) and (9) in [18], ρ can be solved using numerical techniques. Moreover, it's only decided by the number of radio transmitters. Fig.2 shows the collision probability changing with the station number in one single collision domain, got by equation (7) and (9) in [18]. Furthermore, using polynomial fitting method, we can get the fitting curve as shown in Fig.2. It can be seen that the goodness of fit is very well. So we can denote the collision probability function by $\rho=f(n)$, i.e., $\rho=f(k_c)$ in multi-radio multi-channel allocation game.

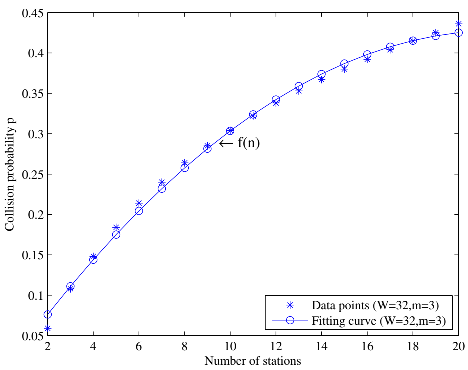

Fig.2. Collision probability function

What's more, we can denote the data rate on channel c by $R_c(k_c)=R_c\cdot(1-f(k_c))$, which is a decreasing function with k_c. The utility of player p_i can be rewritten as:

$$U_{p_i}(S)=\sum_{c\in C}R_{p_i,c}=\sum_{c\in C}\frac{k_{p_i,c}}{k_c}R_c(k_c)=\sum_{c\in C}\frac{k_{p_i,c}}{k_c}\cdot R_c\cdot(1-f(k_c)) \tag{8}$$

So, theorem 3 in section II does not hold, since the players might remove some of their radios to decrease the total number of radios mutually, in order to increase each others' utility. For example in Fig.3, each player distributes all his radios over the channels. It's a NE, i.e., none of players can unilaterally change its strategy to increase its utility, but not Pareto-optimal. The utility of player p_1 is:

$$\begin{aligned}U_{p_1}(S)&=\frac{1}{2}\cdot R_c\cdot(1-f(2))+\frac{1}{2}\cdot R_c\cdot(1-f(2))\\&+\frac{1}{2}\cdot R_c\cdot(1-f(2))=\frac{3}{2}\cdot R_c\cdot(1-f(2))\end{aligned} \tag{9}$$

And the utility of player p_3 is:

$$\begin{aligned}U_{p_3}(S)&=\frac{1}{2}\cdot R_c\cdot(1-f(2))+\frac{1}{2}\cdot R_c\cdot(1-f(2))\\&+\frac{1}{2}\cdot R_c\cdot(1-f(2))=\frac{3}{2}\cdot R_c\cdot(1-f(2))\end{aligned} \tag{10}$$

However, when player p_1 remove one of his radio from channel c_3, and player p_3 remove one of his radio from channel c_1. The utility of player p_1 is:

$$U_{p_1}(S')=R_c\cdot(1-f(1))+\frac{1}{2}\cdot R_c\cdot(1-f(2)) \tag{11}$$

And the utility of player p_3 is:

$$U_{p_3}(S')=R_c\cdot(1-f(1))+\frac{1}{2}\cdot R_c\cdot(1-f(2)) \tag{12}$$

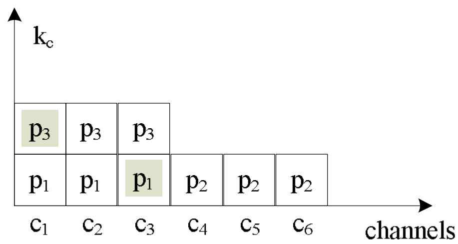

Fig.3. A NE channel allocation, not Pareto-optimal. Here $|C|=6$, $N=3$, $k=3$

Because $f(k_c)$ is an increasing function, we have $U_{p_1}(S')>U_{p_1}(S)$ and $U_{p_3}(S')>U_{p_3}(S)$. So, this NE is inefficient and not Pareto-optimal.

Theorem 4. A NE defined by theorem 1 is not Pareto-optimal if both of the two following conditions hold:

(1) $\exists c_i,c_j,c_k,\ldots\in C_{eq}\in C$ for $k_{c_i}=k_{c_j}=k_{c_k}=\ldots>1$ and $\sum_{c\in C_{eq}}k_{p_n,c}=\sum_{c\in C_{eq}}k_{p_m,c}=\ldots>1$, $\forall p_n,p_m\in P$;

(2) $g(k_c-1)>2\cdot g(k_c)$, where $g(k_c)=\dfrac{1-f(k_c)}{k_c}$.

Proof: Assume that S^* is a NE, and S' is a different strategy in which players remove their radios from some channels. Without loss of generality, we assume that there are two players in cooperation and satisfy the condition (1). Player p_i remove one radio from channel c, and player p_j remove one radio from channel b. Similar analysis can be used for more

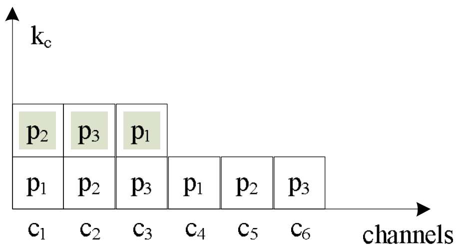

Fig.4. A NE with three players in cooperation

players (Fig.4 shows three players in cooperation). The benefit of change, i.e., the difference in the utility of player p_i is calculated as follow:

$$
\begin{aligned}
\Delta &= U_{p_i}(S^*) - U_{p_i}(S') \\
&= \frac{k_{p_i,b}}{k_b}\cdot(1-f(k_b)) + \frac{k_{p_i,c}}{k_c}\cdot(1-f(k_c)) - \frac{k_{p_i,b}}{k_b-1}(1-f(k_b-1)) \\
&= \frac{(1-f(k_b))}{k_b} + \frac{(1-f(k_c))}{k_c} - \frac{(1-f(k_b-1))}{k_b-1} \\
&= g(k_b) + g(k_c) - g(k_b-1) \\
&= 2\cdot g(k_c) - g(k_c-1)
\end{aligned} \tag{13}
$$

Let us notice the condition (1) and the condition of NE, so $k_{p_i,b} = k_{p_i,c} = 1$, and $k_b = k_c$. If $\Delta < 0$, i.e., $U_{p_i}(S^*) < U_{p_i}(S')$, which means this NE channel allocation is not Pareto-optimal, we can get $g(k_c-1) > 2\cdot g(k_c)$, where $g(k_c) = \frac{1-f(k_c)}{k_c}$. The theorem is proved. □

All mentioned above don't consider the cost to transmit data. Next, we will study how a charging scheme influences players' strategies. We denote the *outcome utility* of player p_i by O_{p_i}:

$$
O_{p_i}(S) = \alpha\cdot\sum_{c\in C}\frac{k_{p_i,c}}{k_c}\cdot R_c\cdot(1-f(k_c)) - co_{p_i}\cdot k_c \tag{14}
$$

where α is a positive constant coefficient to convert the rate utility into the outcome utility. co_{p_i} is a constant, and indicates the cost of p_i for one radio to transmit data.

Similar to the proof of theorem 4, we can change the condition (2) to: $\alpha\cdot R\cdot g(k_c-1) > 2\alpha\cdot R\cdot g(k_c) + co_{p_i}$, which is a more relaxed condition.

IV. A PARETO-OPTIMAL NE SOLUTION

Based on the theorems given above, we propose a practical distributed channel assignment algorithm with imperfect information that enables the selfish players to converge to the Pareto-optimal NE. There are two stages in this solution. The first stage is similar to the algorithm 3 in [7]. Players employ the backoff mechanism to avoid changing their radios simultaneously, and only know the total number of radios on those channels on which they operate a radio. So the collision probability ρ can be counted on these channels. By using the inverse function of $\rho = f(k_c)$, k_c can be got and the utility is calculated. In the second stage, the theorem 4 is used to achieve the Pareto-optimal ground on the NE channel assignment obtained from the first stage. We provide the description of our algorithm in Fig.5.

Algorithm: Pareto-optimal NE channel allocation algorithm

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Stage 1: NE channel allocation with imperfect information (refer to [7]).
Random channel allocation
for i = 1 to N do
   if backoff counter is 0 then
      move the radios to achieve NE
      reset the backoff counter
   else
      decrease the backoff counter value
   end if
end for
Stage 2: Convert to Pareto-optimal
for j = 1 to |C| do
   if the condition (1) in theorem 4 is satisfied
      if ρ = f(k_c) and k_c satisfy the condition (2)
         each player removes one radio from sequential channels,
         i.e., p_n → c_i, p_m → c_k, where n < m and i < k.
      end if
   end if
end for
```

Fig.5. Pareto-optimal NE channel allocation algorithm

We implemented this algorithm in MATLAB, and compared it with the algorithm 3 in [7]. In each simulation, we assume there exists 6 orthogonal channel and the data rate is 54Mbps for each channel, i.e., $|C| = 6$, $R_c = 54$. What's more, no charging scheme and a charging scheme were both considered.

We represent the utilities of each player of Fig.4 on each channel in three dimensions in Fig.6. Player 1, 2, 3 stand for the NE channel allocation using algorithm 3 in [7], and player 1', 2', 3' stand for the Pareto-optimal channel allocation using our proposal. By removing one radio from one channel mutually, players can increase their utilities on other channel greatly, and their total utilities also increase. The main reason is that it can reduce the collision probability on that channel.

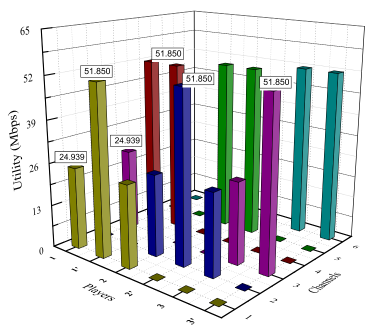

Fig.6. Utilities on the respective channels achieved by each player of Fig.4

In Figs.7 and 8, we compare the total performance of our algorithm with the algorithm 3 in [7]. Both no charging scheme and charging scheme are studied by using different parameter configuration.

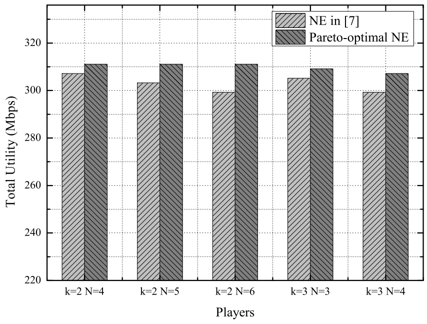

Fig.7. Total utility of the proposed solution as well as the existing solution in [7] without a charging scheme

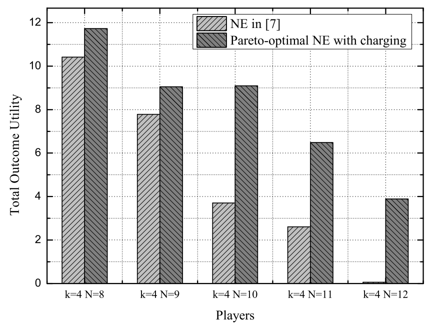

Fig.8. Total outcome utility with a charging scheme for $\alpha = 0.1$ and $co_{p_i} = 0.5$

In Fig.7, without considering the charging scheme, our algorithm can achieve higher data rate based on the fitting function $f(k_c)$ in section II, only when $k_c < 3$, $\forall c \in C$, since the condition (2) is not satisfied for other instances. However, when the charging scheme is considered, our algorithm can achieve higher utility with larger k_c, especially when there are more players, as shown in Fig.8. It's because that more players on one channel means more collision. So removing one radio not only increase the utilities of the two players mutually, but also other players.

V. CONCLUSION

In this paper, we have considered the problem of multi-radio multi-channel allocation from the game-theoretic point of view. Nash equilibrium is used to achieve the efficient channel allocation, but not system optimization. We first analyze the relationship between collision probability and the number of players. More players mean more collisions, which will reduce the data rate. Then, we propose a Pareto-optimal channel allocation algorithm based on NE to achieve the system effectiveness. A charging scheme is also considered to represent the cost of transmission. Finally, simulation results show that our proposal outperforms NE channel allocation in both charging and no-charging schemes.

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