Multi-user Analog Network Coding with Spread Spectrum

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***Abstract*—In this paper, we propose a physical layer network coding technique combined with asynchronous code division multiple access (CDMA). We consider a scenario where there are multiple pair of nodes with a single relay, and each pair of nodes exchange information asynchronously. With the assumption of asynchronous transmission, there is a trade-off between the number of nodes and the inter-user interference because the spreading codes are not orthogonal to each other. The BER and the throughput of the system are analyzed, and simulations are performed to verify the results. It is shown that physical layer (analog) network coding improves throughput compared to the conventional routing protocol with large E_b/N_0, low number of users, and large spreading factor.**

I. Introduction

Network coding was originally introduced in [1] aimed at maximum achievable rate for wireline multicast communication. Thanks to the broadcast nature of wireless links, it is later applied to wireless networks for larger throughput [2], [3]. The idea is illustrated in a bi-directional relay (Fig. 1). When information a is transmitted from a set of node X to another set of node Y, we use the notation of $X \xrightarrow{a} Y$. By conventional (CONV) routing, four time slots are needed for packet exchange between A and B ($A \xrightarrow{a} R, R \xrightarrow{a} B, B \xrightarrow{b} R, R \xrightarrow{b} A$) where a (or b) refers to digital packet of A (or B). By digital network coding, only three time slots are needed ($A \xrightarrow{a} R, B \xrightarrow{b} R, R \xrightarrow{c} \{A, B\}$) where $c = a \oplus b$, and $\oplus$ denotes a XOR operation. A retrieves b by $c \oplus a$, and B retrieves a by $c \oplus b$. Analog network coding which focuses on physical layer [4], [5] reduces the time further into two slots ($\{A, B\} \xrightarrow{S_a, S_b} R, R \xrightarrow{S_c} \{A, B\}$) where $S_c = S_a + S_b$, and S_a (or S_b) is the analog (physical layer) signal of a (or b). A gets S_b (hence b) by $S_c - S_a$ and B gets S_a (hence a) by $S_c - S_b$. The network coding scenario requires fewer time slots for data exchange, and results in higher throughput. It is based on the assumption of a simple topology and ideal conditions of synchronous transmission and noiseless channel.

This paper addresses a more practical case. The three-node topology is extended to the scenario of multiple nodes with a single relay where each pair of users exchanges packets through the relay node (Fig. 2). The ideal assumption of synchronous transmission is replaced by asynchronous transmission. To improve throughput, analog network coding is used for packet exchange. To deal with multiple access, code division multiple access (CDMA) is used. In addition to BER and capacity analysis for the proposed system, the throughput gain of analog network coding is also analyzed.

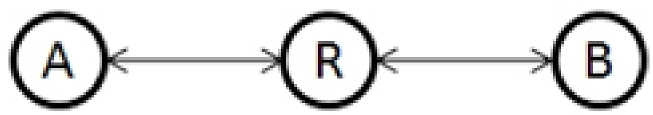

Fig. 1. Three node topology for bi-directional relaying.

There are several existing literatures on network coding for multiple nodes with a single relay. In [6], each user intends to send its packet to all the other users through the single relay. The throughput gain by using digital or analog network coding in [6] comes from the assumptions of overhearing, scheduling, and error-free reception. In [7], a network coding method with spread spectrum which is similar to the system proposed in this paper was introduced. However, a different system model in terms of topology, scenario, network coding scheme is used in [7], and it focuses on how different system parameters influence reliability instead of throughput.

This paper explores a novel combination of analog network coding and CDMA. The proposed system is not the same as multiple independent analog network coding systems, but they are dependent with each other because of the inter-user interference caused by asynchronous transmission. Thus, we focus on the relationship between the interference and the number of users in this paper. We consider both binary and complex spreading, and provides results on the throughput improvement by using analog network coding with CDMA. The paper is organized as follows. The review of spread spectrum techniques is given in Section II, and the proposed system model is discussed in Section III. The analysis of BER of the proposed system is given in Section IV, and the analysis of the throughput gain of analog network coding is presented in Section V. Concluding remarks are given in Section VII.

II. Spread Spectrum

There are two kinds of CDMA systems: frequency-hopping CDMA and direct-sequence CDMA. The paper only considers the latter [8], where each user in the system is assigned with one distinct spreading sequence to spread its signal. A receiver obtains superposed signals from multiple users as well as noise, and detects the desired user's information by

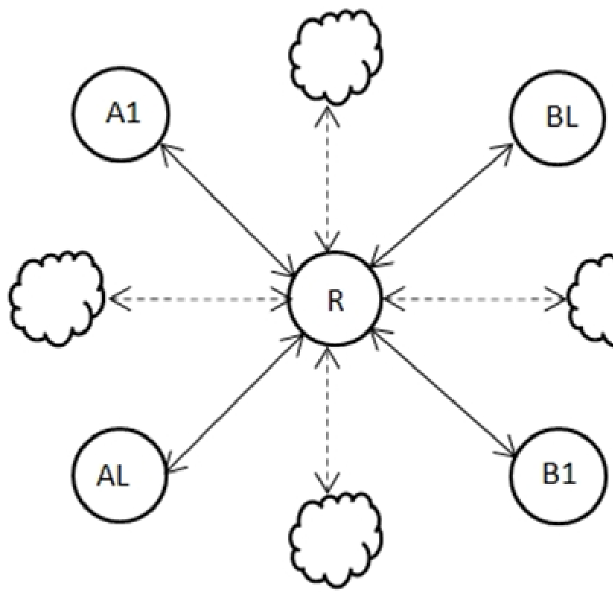

Fig. 2. The system topology.

applying the same spreading sequence of the user so that other interferences and noise can be cancelled out effectively due to the low correlation between spreading sequences.

Two kinds of spreading sequences (codes) are considered in this paper. One is binary code and the other is complex code. A binary spreading code [9], [10] (e.g., M-sequence, Gold code, and Walsh code) uses real-valued chips (± 1) while a complex spreading code (e.g., WCDMA scrambling code [11]) uses complex-valued chips ($\pm 1 \pm j$). The spreading factor (SF) denoted by N_s is defined by the number of chips in a bit. N_s is usually equal to a power of two especially for binary spreading [9], so the spreading factor index (M_s) is defined as $M_s = \lceil \log_2(N_s) \rceil$.

III. System Model

Fig. 2 is the proposed system topology where the ith pair of users A_i and B_i ($i \in \{1, 2, ..., L\}$) exchanges information with each other through R. The remaining section describes three stages for the users to finish one round of packet exchange and the assumptions for each stage.

A. Stage One - Spread Spectrum

The two set of users (A_i's and B_i's) are denoted by $U_{k=2i-1}$ and $U_{k=2i}$ ($k \in \{1, 2, ..., K = 2L\}$), respectively. The bit-level waveform for U_k is given by

$$B_k(t) = \sum_{q=-\infty}^{+\infty} b_{k,q} p_T(t - qT) \tag{1}$$

where $b_{k,q} \in \{\pm 1\}$ refers to the qth data bit of U_k. The rectangular pulse $p_T(t)$ is 1 for $t \in [0, T]$ and 0 elsewhere, and T is the data bit duration.

The chip-level waveform for U_k is given by

$$C_k(t) = \sum_{p=-\infty}^{+\infty} c_{k,p} \psi(t - pT_c) \tag{2}$$

where $c_{k,p}$ refers to the pth sequence chip of U_k, and its polar coordinate representation is $c_{k,p} = \alpha e^{j\beta_{k,p}}$. For binary spreading, $c_{k,p} \in \{\pm 1\}$, so chip magnitude $\alpha = 1$ and chip phase $\beta_{k,p} \in \{0, \pi\}$. For complex spreading, $c_{k,p} \in \{\pm 1 \pm j\}$, so $\alpha = \sqrt{2}$ and $\beta_{k,p} \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$. $\psi(t)$ is the chip waveform and $\int_0^{T_c} \psi^2(t)dt = T_c$ where T_c is the chip duration and $T_c = T/N_s$. The transmitted signal with complex spreading for U_k is expressed as

$$S_k(t) = B_k(t) C_k(t) A_k e^{j(w_c t + \theta_k)} \tag{3}$$

where w_c is the carrier frequency, θ_k is the carrier phase and A_k is the amplitude of the transmitted signal for U_k. When binary spreading is used, the actual transmitted signal is $\sqrt{2} Re\{S_k(t)\}$ instead of $S_k(t)$. In this paper, we assume negligible delay for this stage, no channel coding, the same spreading sequence for each pair of users (different sequences for different pairs), identical packet size and transmission power among all users, and fixed system bandwidth or chip period T_c.

B. Stage Two - Packet Exchange by Analog Network Coding

The exchange process takes two time slots. In the first time slot, all users transmit their signals to R. In the second time slot, R simply broadcasts its received signal $R(t)$ to all users as in

$$R(t) = \sum_{k=1}^{K} S_k(t - \tau_k) + n_R(t) \tag{4}$$

where τ_k is the time delay for U_k, and it is due to asynchronous transmission. The time delay is assumed to be in $[0, T)$, and it is an integer multiple of the chip period ($\tau_k \in \{0, T_c, 2T_c, ..., (N_s - 1)T_c\}$).

The received noise $n_R(t)$ of R comes from the assumption of AWGN channel, and is denoted by $n(t)$ in general. For complex spreading, it has form of $n(t) = \{X(t) + jY(t)\} e^{jw_c t}$ [12]. Binary spreading uses $\sqrt{2} Re\{n(t)\}$ instead of $n(t)$. $X(t)$ and $Y(t)$ are baseband Gaussian noise component, each of which has spectral density $N_0/4$, and thus distribution $N(0, N_0 B_s/4)$ where B_s is the bandwidth of baseband signal $B_k(t) C_k(t)$. Additional assumptions of this stage include no carrier frequency or phase shift and no channel fading since it does not appear to change the trend of the performance that is discussed later.

C. Stage Three - Reception

Due to the symmetry of the system model, U_2 (B_1) receives the information from U_1 (A_1). U_2 first receives $R(t)$ with the noise $n_2(t)$, and subtracts its own signal copy. It then converts the signal into baseband and uses the same spreading sequence of U_1 to obtain the information from U_1. The receiver output Z_2 for complex spreading is given by

$$Z_2 = \int_{\tau_1}^{T+\tau_1} \{R(t) + n_2(t) - S_2(t-\tau_2)\} e^{-jw_c t} C_1^*(t) dt. \quad (5)$$

The assumptions of this stage include negligible delay, zero time delay from R to U_2, perfect channel and delay estimation of U_1 and U_2 for despreading (τ_1) and subtraction (τ_2), and instantaneous and error-free feedback for retransmission in the next round in case of erroneous packet reception.

IV. Bit Error Rate Performance

Based on the signal model in Section III, analytical expressions and simulation results of both complex and binary spreading are provided here in terms of BER, which is one of the key performance metrics of the proposed system.

A. Analysis

1) Decision Statistics: Z_2 can be separated further into $Z_2 = D + I + N$ where the desired signal D is

$$D = A_1 b_{1,0} \alpha^2 T \quad (6)$$

where α is the amplitude of $c_{k,p}$, and T is the bit period. The multiple access interference I from other users' signals is given by

$$I = \sum_{k=3}^{K} A_k e^{jr_k} \int_0^T B_k(t-\tau_k) C_k(t-\tau_k) C_1^*(t) dt \quad (7)$$

where $r_k = \theta_k - w_c \tau_k$. The noise part N for complex spreading is

$$N = \int_0^T [n_R(t) + n_2(t)] e^{-jw_c t} C_1^*(t) dt. \quad (8)$$

Note that $\sqrt{2}N$ instead of N is used for noise when binary spreading is used.

2) Multiple Access Interference Effect: Several assumptions are used for approximation of I. One is that the spreading sequence chips are randomly generated so that $B_k(t)$ is ignored, and τ_k can be replaced by $\tau_k - \left\lfloor \frac{\tau_k}{T_c} \right\rfloor$ which is essentially 0 due to the chip-level synchronization. We also assume $r_k = 0$. The approximated $Re(I)$ thus is

$$Re(I) = \sum_{k=3}^{K} A_k \alpha^2 T_c \left\{ \sum_{p=0}^{N_s-1} \cos(\beta_{k,p} - \beta_{1,p}) \right\}. \quad (9)$$

According to the chip phase distribution, $var\{Re(I)\}$ for complex spreading is given by

$$var\{Re(I)\} = \frac{1}{2} \alpha^4 {T_c}^2 N_s \sum_{k=3}^{K} A_k^2. \quad (10)$$

Note that $var\{Re(I)\}$ for binary spreading is twice that of complex spreading due to the different chip phase distribution.

3) Noise Effect: Based on the noise (N) expression and the distribution of β, $var\{Re(N)\}$ for complex spreading can be calculated as

$$var\{Re(N)\} = \frac{1}{2} \alpha^2 N_0 N_s (B_s T_c). \quad (11)$$

Note that $var\{Re(N)\}$ for binary spreading is twice that of complex spreading.

4) BER Expressions: Let us denote E_b/N_0 by ρ. By Gaussian approximation for Z_2, complex spreading can be shown to have BER expression of

$$P_e = Q\left(\frac{1}{\sqrt{\frac{K-2}{2N_s} + \frac{1}{\rho}}}\right) \quad (12)$$

where P_e is the BER, and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-x^2/2} dx$. Binary spreading has

$$P_e = Q\left(\frac{1}{\sqrt{\frac{K-2}{N_s} + \frac{2}{\rho}}}\right), \quad (13)$$

which is more degraded by interference and noise.

V. ANC Throughput Gain

To assess the effect of $E_b/N_0, L$, and N_s on throughput by using analog network coding with spread spectrum, the analog network coding (ANC) scheme and the conventional (CONV) scheme are compared. Note that both the ANC and the CONV schemes are assumed to be used with CDMA to support many users (multiuser communications).

A. Throughput (goodput)

Assuming that retransmission is being used, the throughput is defined as how many correct packets per unit time a user receives, which is given by

$$R_t = \frac{P_c}{T_e} = \frac{(1-P_e)^{N_p}}{T_e} \quad (14)$$

where R_t is the throughput, T_e is the time to exchange data between the two nodes, and P_c is the correct packet rate (the probability to receive one correct packet). Without channel coding, $P_c = (1-P_e)^{N_p}$ where N_p is the packet size in terms of bits. Note that the definition is similar to the goodput definition in the existing literatures.

The analog network coding (ANC) scheme is the proposed method that combines analog network coding with CDMA, and it was described in Section III. Thus $T_e = 2N_p N_s T_c$. Only complex spreading is considered for analysis because of the better performance and continuous N_s value that does not have to be a power of two. The conventional (CONV) scheme which is also combined with CDMA needs four slots for packet exchange. In the first slot, all A_is transmit signals to R, and R broadcasts the received signal to all B_is in the second slot. Similarly, B_is transmit signals to A_is through R in the third and the fourth slots. Consequently $T_e = 4N_p N_s T_c$. Besides, $P_e = Q(\frac{1}{\sqrt{\frac{L-1}{2N_s} + \frac{1}{\rho}}})$ since each receiver faces $L-1$ interferers.

By using twice many spreading codes in lieu of analog network coding, we can also use two slots for packet exchange. In the first slot, all A_is and B_is spread their packets by their own spreading codes, and transmit to R. In the second slot, R broadcasts the received signal to all A_is and B_is, and each receiver despreads desired information without subtraction. As for throughput comparison, only the four-time slot CONV scheme combined with CDMA is considered here.

VI. Simulation

For BER simulations, we consider the proposed analog network coding system with CDMA, which is fully loaded. The number of pairs (L) is the maximum number of available spreading sequences, which is shown in Table I. Gold code does not exist for $M_s = 2, 4, 8$ according to [9]. Binary spreading analysis uses "Shifted M" parameters and complex spreading analysis uses "Scrambling" parameters. For complex spreading, we use scrambling codes. For binary spreading, we use the other 3 sequences. In simulations, we assume asynchronous transmission, $E_b/N_0 = 9$ dB, and a fully loaded system. The simulation and the theoretical results of the analog network coding system with CDMA are shown in Fig. 3. A larger N_s degrades the BER due to more interference. Among the tested sequences, it is observed that complex spreading performs the best. It is also observed that the analysis and the simulations match well. It should be noted that the overall BER is poor, which indicates the impracticality of serving too many users with the topology of multiple nodes with one single relay in practical situations.

Fig. 4, Fig. 5, and Fig. 6 show the throughput comparison between the ANC and the CONV schemes using the analytical expression of (14). Both ANC and CONV throughputs depend on E_b/N_0, L, and N_s. When L, and N_s are fixed, Fig. 4 shows the E_b/N_0 effect on the throughput. At low E_b/N_0 regime, the throughput of ANC is inferior to that of CONV since the ANC scheme is more susceptible to noise, and thus more time slots are needed for retransmission. At high E_b/N_0 regime, the ANC scheme can achieve throughput gain over the CONV scheme by factor of two roughly, which is similar to the analog network coding gain under ideal conditions with a three-node topology. A similar relationship of capacity (b/s/Hz) versus SNR (dB) can be found in [5]. When E_b/N_0 and N_s are fixed, the ANC scheme has higher throughput than the CONV scheme at small L due to less interference. This effect is shown in Fig. 5.

Fig. 6 shows the N_s effect on the throughput when E_b/N_0 and L are fixed. As N_s increases, both ANC and CONV

Type	Shifted M	Gold	Walsh	Scrambling
Chip	± 1	± 1	± 1	$\pm 1 \pm j$
M_s	2 to 9	3,5,6,7,9	2 to 9	2 to 9
N_s	$2^{M_s} - 1$	$2^{M_s} - 1$	2^{M_s}	2^{M_s}
L	$2^{M_s} - 1$	$2^{M_s} + 1$	2^{M_s}	2^{M_s}

TABLE I
Parameters for BER Simulation for a near full-load System.

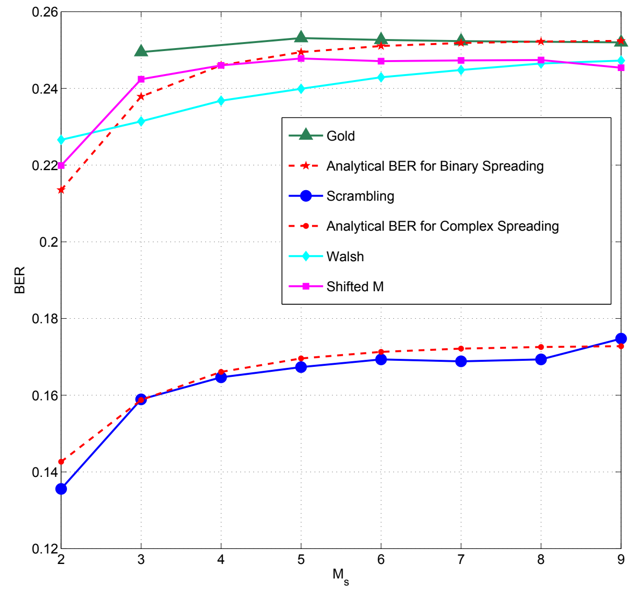

Fig. 3. BER versus M_s with near-full load with $E_b/N_0 = 9$ dB.

schemes have peak throughput. According to the throughput expressions, P_c increases with N_s and retransmission decreases, which is due to spreading gain. Meanwhile, the bit rate (or packet rate) decreases as N_s increases since the bandwidth (the chip period) is fixed. At low N_s, the spreading gain dominates, and the throughput increases. At high N_s, the spreading gain reaches saturation, and the reduced bit rate decreases the throughput.

Based on the expression of P_c and R_t, the ANC scheme and the CONV scheme have the same value when $N_s^{\text{ANC}} = 2N_s^{\text{CONV}}$ where N_s^{ANC} and N_s^{CONV} are the spreading factors for the ANC and the CONV schemes, respectively. The equality may not hold in practice since factors such as overhead are not considered. Based on the theory, the ANC scheme is preferred for a large value of N_s. The ANC throughput expression can be further approximated as

$$R_t^{ANC} \approx \frac{\frac{L}{N_s}\left[1 - Q\left(\frac{1}{\sqrt{\frac{L}{N_s} + \frac{1}{\rho}}}\right)\right]^{N_p}}{2N_pL} \tag{15}$$

where R_t^{ANC} is the throughput for the ANC scheme. Given a certain E_b/N_0 between 0 to 30 dB and a fixed L, numerical calculation shows that the value of L/N_s leading to peak ANC throughput is at around 0.1. Thus for a fixed L, the N_s value that maximizes the ANC throughput is actually proportional to L (about $10L$). The corresponding ANC throughput gain over CONV is about 1.5.

A potential application to use the N_s effects is as follows. For a system with fixed L and E_b/N_0, the N_s values to maximize the throughput of the ANC scheme and the CONV scheme can be found. For a given spreading factor (N_s), a selection algorithm between the ANC or CONV schemes may improve the throughput.

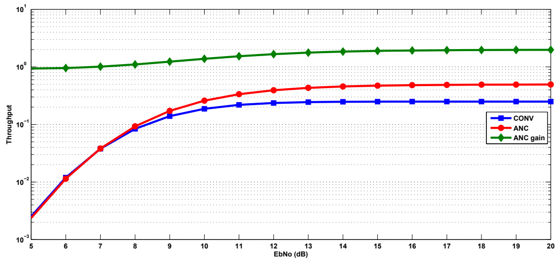

Fig. 4. Comparison of ANC and CONV throughput with respect to E_b/N_0 with $N_s = 64$ and $L = 5$.

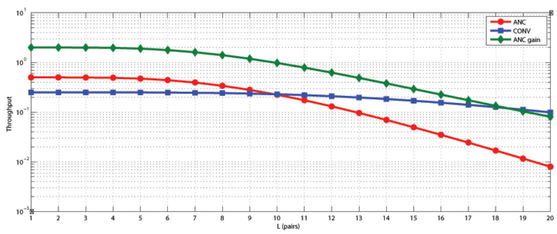

Fig. 5. Comparison of ANC and CONV throughput with respect to L with $E_b/N_0 = 15$ and $N_s = 64$.

VII. Conclusion

In this paper, we proposed an analog network coding with code division multiple access for multi-node wireless networks with a single relay. Due to the assumption of asynchronous transmission, it was observed that there is more inter-user interference in the proposed system when a larger number of users is serviced. From the simulations and the analysis, it is observed that the proposed system improves the performance over the conventional system when we have large signal-to-noise ratio, large spreading factor, and low number of users. The results will be helpful in terms of selecting system parameters such as the spreading factor and the number of users when an analog network coding system with multiple access is designed. Based on the results, a selection algorithm between the proposed scheme and the conventional scheme can also be used for a given spreading factor.

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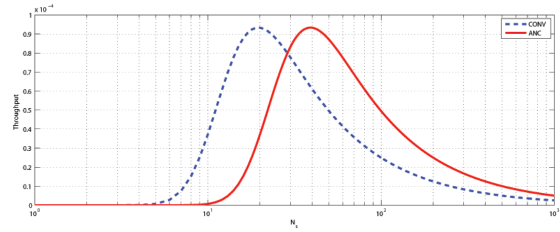

Fig. 6. Comparison of ANC and CONV throughput with respect to N_s with $L = 5$ and $E_b/N_0 = 15$.

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