Joint Spectrum Sensing and Power Allocation Algorithm for Spectrum Efficiency Optimization in Ultra Wideband Cognitive Radio Networks

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***Abstract*—Ultra Wideband (UWB) system uses extremely low signal power to coexist with other primary users (PUs), which limits the use of the UWB's wide spectrum band. Cognitive Radio (CR) enables UWB systems to efficiently use the overlapped spectrum without causing interference to other wireless systems. In this paper, we focuses on the low-complexity joint optimization algorithm design with respect to transmit power allocation and spectrum sensing time (SST) for maximizing the spectrum efficiency of the Orthogonal Frequency Division Multiplexing based CR-UWB system. The SST optimization algorithm minimizes the spectrum sensing time in order to maximize the time length of applying the power allocation algorithm for data transmission. The proposed group power allocation algorithm adaptively assigns the transmit power to the subcarrier groups according to the effective signal-to-noise ratio (SNR) of each subcarrier group based on greedy algorithm. The proposed joint optimization algorithm can maximize the CR-UWB systems spectrum efficiency at a extremely low primary user SNR regime with low complexity.**

I. Introduction

The 3.1-10.6 GHz Ultra Wideband (UWB) operating spectrum overlaps with narrowband systems, such as WiMAX, UMTS and 802.11a/n [1]. To protect the incumbent wireless systems from being interfered by UWB systems, the emission Power Spectral Density (PSD) of a UWB system is strictly constrained by the Federal Communications Commission (FCC) regulations ($\leq$ -41.3 dBm/MHz) [2]. With such a limitation, the UWB systems cannot provide the required Quality of Service (QoS) if the aggregate interference from the Primary Users (PUs) is high [3]. Furthermore, a UWB system can cause intolerable interference to PUs if the transmit (Tx) power of the UWB system rises within the overlapped spectrum. The spectrum efficiency is low because the overlapped spectrum is far from being fully utilized by the PUs [4].

Cognitive Radio (CR) technology [5] enables an Orthogonal Frequency Division Multiplexing (OFDM) based UWB system to efficiently use the overlapped spectrum by operating within the spectrum according to the CR-UWB system's spectrum sensing results. According to the Multiband OFDM (MB-OFDM) UWB system's protocol, the time length for a CR-UWB system's data transmission is limited [6]. Thus, the Spectrum Sensing Time (SST) determines the effective data transmission period in the overlapped spectrum. In the data transmission period, the power allocation algorithm determines the CR-UWB's spectrum efficiency. Thus, the power allocation scheme is coupled with the spectrum sensing time scheduling. To use the spectrum as efficient as possible, joint algorithm design that considers the power allocation and sensing time simultaneously is needed.

For spectrum efficiency maximization, the joint problem is generally nonconvex for nonlinearity of the formulated objective and constraint functions. Thus, the power allocation and sensing time are optimized sequentially to obtain an optimal solution in polynomial time. For capacity-based maximization, the optimal power can be derived as a function of a given sensing time by using convex maximization methods (the joint problem can be transformed into a convex problem with respect to the CR system's transmit power), such as water-filling method [7], subgradient method [8], ellipsoid method [9] and Newton's method [10]. Then, one-dimensional exhaustive search or bisection search method is commonly used to obtain the optimal sensing time since it is NP-hard to derive an analytical form. Using convex maximization method to solve the power allocation problem requires relaxation of constraints, which will cause the maximization algorithm cannot be implemented in practical CR-UWB systems. For example, water-filling method assumes the number of bits allocated on a frequency band is non-integer. Furthermore, the convex maximization algorithm often converges slowly near to the optimum and needs a large number of iterations to reach the desired accuracy [7]. For sensing time maximization, the complexity of the exhaustive search can be high, especially in multiuser CR networks, since the subsets of users is exponentially increasing with the number of users. To design a low-complexity algorithm for more practical spectrum efficiency maximization, the joint problem can be formed as a knapsack problem with respect to the power allocation [11]–[13]. In [11], Zhang and Leung applied the greedy algorithm by allocating a bit to the subcarrier which has the maximum efficiency value in each iteration until one of the constraints is violated. Since there are multiple PUs

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near the signal cognitive OFDM system, there are multiple interference related efficiency values in each subcarrier. Hence, in a subcarrier, the minimum interference efficiency value is chosen to be compared with other subcarriers' minimum interference efficiency values. Choosing the minimum interference efficiency value is to guarantee the PU with the minimum interference margin will not be interfered. The complexity of the maximization algorithm is proportional to the number of source bits, the number of subcarriers and the number of the PUs. In [12], Koufos et al. formulated a multiple choice knapsack problem with respect to the sensing power and power allocation maximization. The authors used a greedy-based maximization algorithm to achieve the optimal tradeoff between the expected throughput over the multiple spectrum bands and the total power spent for sensing. In this paper, we formulate the joint problem into a multi-dimensional knapsack problem with respect to power allocation and develop a suboptimal greedy algorithm that significantly reduces the complexity of maximizing the CR-UWB system's spectrum efficiency. For sensing time maximization, we derive a quasi-analytical solution for the optimal sensing time, which enables the joint algorithm to quickly compute the value of the optimal sensing time.

The rest of the paper is organized as follows. Section II discusses the spectrum sensing model and the transmit power limitation of the CR-UWB system. Next, the spectrum efficiency maximization problem is formulated in Section III. The joint algorithm with respect to group power allocation and quasi-analytical sensing time maximization algorithm is discussed in Section IV. Then, simulation results are presented in Section V to compare the spectrum efficiency enhancement contributed by the use of the proposed joint algorithm. Finally, conclusion is given in Section VI.

II. System Model

We assume that the overlay spectrum sharing mechanism is used in the CR-UWB system, since the FCC's power limitation ($\leq$ -41.3 dBm/MHz) on underlay CR-UWB signals may result in a significantly constrained Quality of Service (QoS) [14]. The CR-UWB's spectrum efficiency is defined as the ratio of the usable information transmitted (in bps) to the spectrum resource (bandwidth in MHz) used for the information transmitting, and is expressed as

$$\eta_{eff} = \frac{\mathbf{B_{cog}}}{T_s W}, \tag{1}$$

where $\mathbf{B_{cog}}$ represents the number of bits allocated on the CR-UWB subcarriers that are used for effective data transmission, W is the bandwidth used by the transmitted OFDM symbol, and T_s denotes the OFDM symbol period.

A. Sensing Model

The spectrum opportunity for a CR-UWB system, i.e., the probability that an overlapped spectrum will contain less than energy threshold power at any instant of time, is determined by the probability that a PU is operating within the overlapped spectrum. Since the Poisson distribution is widely used to model the spectrum occupancy in CR networks, the probability that a PU is activated follows the Poisson process [15].

In MB-OFDM CR-UWB receiver, incoming UWB signals are demodulated by a Fast Fourier Transform (FFT) engine, which facilitates the use of Discrete Fourier Transform (DFT) based energy detection and feature detection for spectrum sensing. Compared with feature detection, energy detection requires much lower computational complexity and less information of PU (the complexity of the feature detection is $Nlog_2N$ times of energy detection [16]). Thus, we assume the CR-UWB system uses energy detection method. The proposed algorithms can be extended when feature detection is applied.

For energy detection, the SST that is required for a set of target probability of detection P_d and probability of false alarm P_f is determined by [17]

$$\tau_s = \frac{2}{\gamma_p^2 f_s}(Q^{-1}(\tilde{P}_f) - Q^{-1}(\tilde{P}_d))^2, \tag{2}$$

where γ_p is the received Signal-to-Noise Ratio (SNR) of PUs' signal at the CR-UWB receiver, and f_s is the CR-UWB's sampling frequency. Furthermore, $Q^{-1}(\cdot)$ denotes the inverse of the Q-function. Thus, $Q^{-1}(P_d)$ and $Q^{-1}(P_f)$ are expressed as

$$Q^{-1}(P_d) = \frac{\epsilon(N)/\sigma_u^2 - N - \gamma_p}{\sqrt{2(2\gamma_p + N)}}, \tag{3}$$

$$Q^{-1}(P_f) = \frac{\epsilon(N)/\sigma_u^2 - N}{\sqrt{2N}}, \tag{4}$$

where $\epsilon(N)$ is the detection threshold with signal samples $N = \tau_s f_s$ at the UWB receiver, σ_u^2 is the power of the additive white Gaussian noise.

In a CR-UWB system, the length of SST determines the time ratio $\alpha = (T_{txop} - \tau_s)/T_{txop}$ for the system to apply the spectrum management function for useful data transmission, where T_{txop} is a pre-defined transmission period in the MB-OFDM UWB MAC layer protocol, called transmission opportunity (TXOP). In ECMA-368 [6]. We assume that the CR-UWB system starts sensing the channel prior to the start of a TXOP.

The effective number of bits that can be allocated on the CR-UWB system is given by

$$\mathbf{B_{cog}} = \mathbf{B}\alpha(1 - P_f)(1 - P(\mathcal{H}_1)), \tag{5}$$

where $\mathbf{B}$ denotes the total number of bits loaded in the UWB subcarriers when all the subcarriers are available, and $\mathcal{H}_1$ represents the hypothesis that a PU is activated. To maximize a CR-UWB system's spectrum efficiency, an optimal SST value is needed to maximize α while meet the target value of P_d and P_f.

B. Transmit Power

In UWB systems, transmit power is allocated on a per MHz basis. The total transmit power can be determined by integrate the average PSD over the UWB bandwidth while the maximum PSD does not exceed the regulatory limits. The maximum allowable transmit power P_{tx} for transmitting an OFDM symbol in a sub-band is expressed as [18]

$$P_{tx} = -41.3 \text{ dBm/MHz} + 10\log_{10}(N_{su} \cdot B_{sc})(dBm), \quad (6)$$

where $B_{sc} = 4.125$ MHz denotes the bandwidth of each OFDM subcarrier, and N_{su} is the number of the used UWB subcarriers in the sub-band. Note that the computed P_{tx} is in dBm.

III. Optimization Problem Formulation

In this paper, we formulate joint problem into a multi-dimensional knapsack problem, as

$$\text{arg max}_{P_i,\alpha} \; \eta_{eff} = \frac{1}{T_s W} \sum_{i=1}^{I} \sum_{j=1}^{J} b_{ij} x_{ij} \quad (7)$$

subject to,

$$P_e \leq \tilde{P}_e, \quad (8)$$

$$P_i \leq P_{mask}, \quad (9)$$

$$\tilde{P}_d \leq P_d \leq 1, \; 0 \leq P_f \leq \tilde{P}_f \quad (10)$$

where P_i is the power allocated to the i-th subcarrier by the user, $b_{ij} = 1$ represents the profit, i.e., the value, of allocating the j-th bit to the user's i-th subcarrier, and x_{ij} indicates whether the CR-UWB's j-th bit would be allocated on its i-th subcarrier. In (8), P_e is the CR-UWB's uncoded average BER, and $\tilde{P}_e$ denotes the average BER threshold. The P_{mask} represents the maximum allowable transmit power on each UWB subcarrier. Furthermore, $\tilde{P}_f$ is the target probability of a false alarm, and $\tilde{P}_d$ is the target probability of detection.

For M-ary QAM, by assuming the channel state information is perfectly known at the UWB receiver and the transmitted symbols are independent and identically distributed (i.i.d.) with the symbol energy, P_e for each CR-UWB subcarrier is expressed as [19]

$$P_b \approx \frac{2(\sqrt{M}-1)}{\sqrt{M}\log_2 M}\left(1 - \sqrt{\frac{3\bar{\gamma}_b \log_2 M}{2(M-1) + 3\bar{\gamma}_b \log_2 M}}\right), \quad (11)$$

where γ_b represents the average received SNR per bit and is approximated by [20]

$$\gamma_b = \frac{P_i |H_i|^2}{2\sigma_u^2 \log_2 M}, \quad (12)$$

where H_i represents the CR-UWB's subcarrier frequency response [21].

Thus, the minimum required power for a certain BER threshold to assign $\log_2 M$ bits on a CR-UWB's subcarrier can be given by

$$P_i(m) = \frac{2\sigma_u^2 (M-1)(1 - \frac{P_e \sqrt{M}\log_2 M}{2(\sqrt{M}-1)})^2}{3 H_i log_2(M)[1 - (1 - \frac{P_e \sqrt{M}\log_2 M}{2(\sqrt{M}-1)})^2]}, \quad (13)$$

where $m = \log_2 M, \; M = 2, 4, 8....$ Then, the cost of assigning one more bit to a CR-UWB's subcarrier can be derived by

$$\Delta P_i = P_i(m) - P_i(m-1), \quad (14)$$

where $P_i(0) = 0$, which means no power will be allocated to the subcarrier if there is no bit assigned to the subcarrier.

IV. Joint Optimization Method

A. Group Power Allocation Algorithm

For power allocation, a greedy algorithm based method can be applied to assign bits to the subcarrier with the lowest cost [22]. The algorithm complexity in [22] is proportional to $\mathcal{O}(\beta \cdot B_{total} N_{used} \log_2 N_{used})$, where N_{used} is the number of the used subcarriers, and β is a ratio. Since N_{used} contributes to the complexity of the spectrum efficiency maximization algorithm, a new group power allocation algorithm is proposed based on the previous algorithm proposed in [22] to lower the computational complexity.

The group power allocation algorithm consists of three steps, they are:

1) Grouping a number of adjacent subcarriers into subcarrier groups, next
2) Allocating power on subcarrier groups by the algorithm proposed in [22], then
3) Allocating bits on the subcarriers in each subcarrier group by equal power allocation.

The coherence bandwidth for each UWB CM are: 53.6 MHz, 28.9 MHz, 20.6 MHz and 12.4 MHz for CM1, CM2, CM3 and CM4, respectively [23]. Hence, the adjacent UWB subcarriers are grouped into blocks whose total bandwidth is smaller than the coherence bandwidth of the UWB channel. By evaluating the channel gain of a certain subcarrier block, the proposed algorithm can modulate the same amount of bits to each subcarrier in the block using M-ary QAM modulation.

The maximum number of subcarriers in a subcarrier block for each UWB channel model is $N_{block} = \left\lfloor \frac{BW_c}{B_{sc}} \right\rfloor$, where BW_c is the coherence bandwidth in a UWB channel model. Thus, the subcarrier grouping process is performed $N_g = \left\lceil \frac{N_{used}}{N_{block}} \right\rceil$, where N_g is the number of subcarrier groups after the grouping process. Thus, the last subcarrier block in an OFDM symbol contains $N_{block} = (N_{used} \mod N_{block})$ subcarriers, where mod represents the modulo operation [15].

The equivalent single channel SNR of each subcarrier group equals to the geometric mean of the SNRs on each of the subcarriers in the group. Hence,

$$SNR_{G_i} = \left(\prod_{j=1}^{N_{block}} SNR_i(j) \right)^{\frac{1}{N_{block_i}}}, \quad (15)$$

where SNR_{G_i} is the equivalent single channel SNR of the i-th subcarrier group, and $SNR_i(j)$ represents the channel SNR of the j-th subcarrier in the i-th subcarrier group. The value of $SNR_i(j)$ is computed by

$$SNR_i(j) = \frac{\varepsilon \cdot |H_i(j)|^2}{\sigma^2} = \frac{|H_i(j)|^2}{Bsc_i(j)\sigma^2}, \quad (16)$$

where $\varepsilon = 1$ denotes a unit power allocation on each subcarrier, $H_i(j)$ is the j-th subcarrier channel gain in the i-th subcarrier group, σ^2 represents the noise PSD of the AWGN channel.

Then, the cost of assigning a number of bits to the subcarrier group can be derived as (13) and (14), and the optimal power allocation algorithm proposed in [22] can be applied. Compared with the power allocation algorithm in [22], the order-of-growth of the proposed spectrum management algorithm for the joint algorithm is reduced to $\mathcal{O}(\beta \cdot B_{total} N_g \log_2 N_g)$. Since the complexity of the two algorithms both take linearithmic time, the reduction of the term N in $N \cdot \log_2 N$ will significantly lower the complexity of the algorithm when the total number of the allocated bits B_{total} is the same in the two algorithms.

B. Sensing Time Optimization Algorithm

Discussions in Section II indicate that an optimal tradeoff can be made between the probability of false alarm and the spectrum efficiency. Thus, by manipulating (3) and (4), P_f can be expressed as a function of P_d and τ_s, as

$$P_f = Q\left(\frac{Q^{-1}(P_d)\sqrt{2(2\gamma_p + N)} + \gamma_p}{\sqrt{2N}}\right), \tag{17}$$

Hence, (5) is re-written as

$$\begin{aligned}\mathbf{B_{cog}} = \mathbf{B}\frac{T_{txop} - \tau_s}{T_{txop}} \cdot \\ \left[1 - Q\left(\frac{Q^{-1}(P_d)\sqrt{2(2\gamma_p + N)} + \gamma_p}{\sqrt{2N}}\right)\right] \cdot \\ (1 - P(\mathcal{H}_1))\end{aligned} \tag{18}$$

The value of $\mathbf{B_{cog}}$ is a function of τ_s and P_d.

For a target $\tilde{P}_d$, the optimal τ_s is computed by finding the root for

$$f_{ratio}(\tau_s) = 0, \tag{19}$$

where $f_{ratio}(x) = F'_{ratio}(x) = \frac{\partial \mathbf{B_{cog}}/\{\mathbf{B}[1-P(\mathcal{H}_1)]\}}{\partial \tau_s}$. The differential of $F_{ratio}(x)$ is expressed as

$$\begin{aligned}F'_{ratio}(\tau_s) = -\frac{1}{T_{txop}} - \\ \left[Q'(\tau_s) - \frac{1}{T}(Q(f(\tau_s)) + Q'(f(\tau_s)))\right]\end{aligned} \tag{20}$$

where $f(\tau_s)$ is a function of τ_s and is given by

$$f(\tau_s) = \frac{Q^{-1}(P_d)\sqrt{2(2\gamma_p + \tau_s f_s)} + \gamma_p}{\sqrt{2\tau_s f_s}}. \tag{21}$$

Furthermore, the differential of $f(\tau_s)$ is computed as

$$\begin{aligned}f'(\tau_s) = \frac{Q^{-1}(P_d) f_s}{2\sqrt{(2\gamma_p + \tau_s f_s)\tau_s f_s}} - \\ \frac{\sqrt{2} f_s (Q^{-1}(P_d)\sqrt{(4\gamma_p + 2\tau_s f_s)} + \gamma_p)}{4(\tau_s f_s)^{3/2}}\end{aligned} \tag{22}$$

However, to find the optimal spectrum sensing time τ_s by solving the equation shown above is complex [15]. Hence, numerical method is used to find a value of τ_s that is approximate to the optimum.

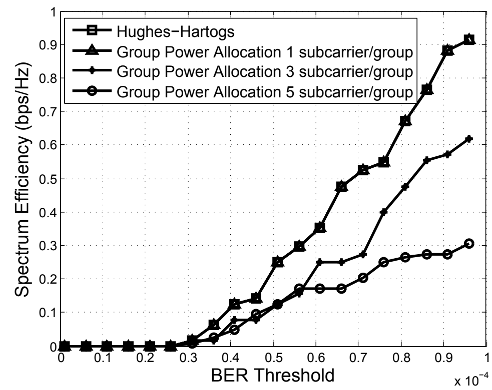

Fig. 1. Spectrum Efficiency of Group Power Allocation in CM1

V. Numerical Results

The UWB CM1 (Line-of-Sight) and CM3 (NLOS) are used to simulate the wireless channel environment. We assume that the PUs are WiMAX systems, the parameter settings for the PUs can be referred to [22]. As shown in Fig. 1, the spectrum efficiency performance of the proposed algorithm is analyzed in CM1 and compared with the Hughes-Hartogs (HH$_{uwb}$) algorithm. The spectrum efficiency performance degradation is higher when more subcarriers are included in one subcarrier group. For example, the spectrum efficiency reached by group power allocation is 50% lower than that of the HH$_{uwb}$ algorithm when 3 subcarriers are included in each subcarrier group as the BER threshold approaches 10^{-4}. However, the algorithm complexity is over 3 times lower in group power allocation algorithm than that in subcarrier-by-subcarrier the HH$_{uwb}$ algorithm.

Figure 2 and Figure 3 compare the spectrum efficiency achieved without using the SST optimization algorithm and the spectrum efficiency obtained when the SST optimization algorithm is applied. Observations in Figure 2 and Figure 3 show that by using the SST optimization algorithm in low γ_p regime (i.e., < -12 dB), the spectrum efficiency is significantly increased. For example, at $\gamma_p = -19$ dB, the spectrum efficiency of the CR-UWB system is 0.49 bps/Hz which is twice of spectrum efficiency that is achieved by the CR-UWB system without using the SST optimization algorithm. With the increase of the γ_p, the difference between the two lines decreases exponentially. At high γ_p regime (i.e., > -10 dB), the spectrum efficiencies of the two CR-UWB systems are very close because the large γ_p value becomes the dominant part of (17), the target P_d is reached at a very small τ_s.

Figure 2 and Figure 3 indicate that the SST optimization algorithm is more suitable for the situation where the received γ_p is low than the situation where the γ_p is high.

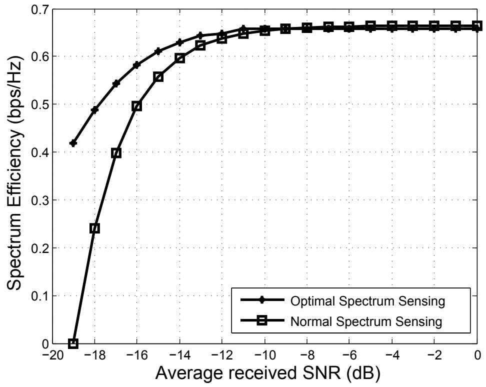

Fig. 2. The maximum spectrum efficiency as a function of received SNR γ_p in CM1.

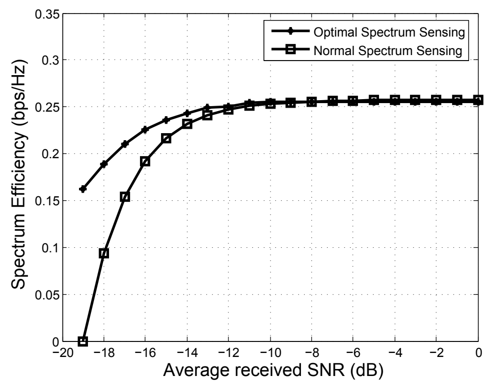

Fig. 3. The maximum spectrum efficiency as a function of received SNR γ_p in CM3.

VI. CONCLUSION

We proposed a joint power allocation and SST optimization algorithm for CR-UWB's spectrum efficiency maximization. For the SST algorithm, an optimal tradeoff was made between SST and probability of detection/false alarm. Thus, the effective time of the CR-UWB's using power allocation algorithm is maximized, which facilitates the spectrum efficiency maximization process. The proposed group power allocation scheme is based on greedy algorithm, and adaptively allocates the transmit power to the subcarrier groups according to the effective SNR of each subcarrier group whose bandwidth is less than the coherence bandwidth of the UWB channel. The CR-UWB system's spectrum efficiency is significantly enhanced with low complexity by jointly using the SST and group power allocation algorithms, especially when the received PUs' SNR is low.

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