

# Simplified Sequential Linear Assignment Algorithm for Energy Efficient Resource Allocation

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**Abstract**—We consider an energy efficient resource allocation method for orthogonal frequency division multiple access (OFDMA) systems consisting of  $M$  users and  $N$  subchannels. Transmit power assignment follows the optimal strategy, i.e., a water-filling strategy, for the given subchannels. For the subchannel allocation, we introduce a recently proposed algorithm called a sequential linear assignment algorithm (SLAA). The SLAA determines how many subchannels are allocated to each user and which subchannels are allocated to each user by solving the outer and inner problems, respectively, in an iterative manner; as a result, it requires  $\mathcal{O}(MN^2(N-M)^2)$  complexity. To reduce the computational complexity of SLAA, we propose a simplified sequential linear assignment algorithm (SSLAA). Based on the observation that a user requiring the higher power would take the more subchannels, SSLAA uses each user's power consumption as a metric to determine who will take an additional subchannel in each iteration. Consequently, contrast to SLAA using network power consumption as a metric, the SSLAA incurs significant reduction of computational complexity, by  $\mathcal{O}(N^2(N-M)^2)$ . Computer simulations in compared with other existing heuristic algorithms verify that the proposed SSLAA results in high energy efficiency and low computational complexity.

**Index Terms**—OFDMA, energy efficiency, resource allocation, linear assignment algorithm.

## I. INTRODUCTION

Due to the NP-hard complexity on subchannel (subcarrier or subband) allocation and power assignment (or rate control) for orthogonal frequency-division multiple access (OFDMA) systems [1], [2], recently, various heuristic resource allocation algorithms minimizing power consumption with polynomial complexity have been studied [3]–[7]. Different criteria have been employed in different algorithms to minimize the network power. A bandwidth assignment based on signal-to-noise-ratio (BABS) amplitude-craving-greedy (ACG), a BABS rate-craving-greedy (RCG), and a successive user integration (SUSI) algorithms have been proposed in [3], [4]. In [3], subchannels are greedily assigned to a user who yields a maximum *rate* and a maximum effective *channel gain* in RCG or ACG. In [4], however, SUSI algorithm assigns subchannels based on the original cost, namely, the *network power*. Once assignment is completed, iteratively subchannels are swapped if further sum power reduction can be achieved [4]. Recently, the authors of this paper have modeled the subchannel allocation problem as a maximum-network-flow problem [6],

where they propose a network-flow-based algorithm (NFBA) to maximize the sum of power saving amount. To further improve energy efficiency, they have introduced a sequential linear assignment algorithm (SLAA), in which an *inner* problem is solved to allocate subchannels under a constraint on the number of subchannels  $s_m$  allocated to user  $m$ , and the *outer* problem is sequentially solved to decide  $s_m$  [7]. Though the SLAA can achieve near optimal performance, it requires high computational complexity  $\mathcal{O}(MN^2(N-M)^2)$  if  $N > M$ .

In this paper, we revisit the SLAA and propose its simplified method called a simplified SLAA (SSLAA). In SSLAA, an additional subchannel is assigned based on the power consumption of each user. As a consequence, the proposed SSLAA requires the complexity of only  $\mathcal{O}(N^2(N-M)^2)$ . Furthermore, we provide a simple example of the LAA-based methods, i.e., SLAA and SSLAA. The resource allocation procedure is illustrated with a cost matrix, which helps readers understand the LAA-based methods better. Some existing heuristic algorithms including RCG and SUSI are compared with the proposed SSLAA in cellular environment. From the numerical results, it is verified that the proposed SSLAA can achieve the closet energy efficiency to the SLAA (with a gap around 5%) with the significant reduction of computational complexity by order  $M$ .

## II. SYSTEM MODEL AND OPTIMIZATION FORMULATION

We consider a downlink consisting of one base station (transmitter) and  $M$  users (receivers) with  $N$  subchannels, where  $N > M$ . One subchannel consists of a few consecutive subcarriers with bandwidth  $B$ . In the paper, it is assumed that channel gain within one subchannel is static, i.e., slow fading channel is assumed, and that the channel coding can be applied for multiple subchannels that is a resource block. Let  $\mathcal{M}$ ,  $\mathcal{N}$ , and  $\mathcal{N}_m$  be the sets of all users, all subchannels, and the assigned subchannels for the  $m$ th user, respectively. Denote the assigned power for the  $n$ th subchannel of user  $m$  by  $P_{m,n}$ . The transmitter is assumed to have perfect information of the instantaneous signal-to-noise ratio (SNR),  $\gamma_{m,n} \triangleq \frac{|h_{m,n}|^2}{\sigma_n^2}$ , where  $\sigma_n^2$  is the noise power and  $h_{m,n}$  is the  $n$ th subchannel between transmitter and the  $m$ th receiver. Here, without loss of generality, a noise level can be assumed as one. The network

power minimization problem is then formulated as

$$\{\mathcal{N}_m^*, P_{m,n}^*\} = \arg \underset{\{\mathcal{N}_m, P_{m,n}\}}{\text{minimize}} P^{\text{net}} \quad (1a)$$

$$\text{s.t. } \sum_{n \in \mathcal{N}_m} \log_2(1 + P_{m,n} \gamma_{m,n}) \geq R_m, \forall m \quad (1b)$$

$$\mathcal{N}_{m_1} \cap \mathcal{N}_{m_2} = \emptyset, \forall m_1 \neq m_2 \in \mathcal{M}, \quad (1c)$$

where  $P^{\text{net}} = \sum_{m \in \mathcal{M}} P_{m, \mathcal{N}_m}$  is the network power and  $P_{m, \mathcal{N}_m}$  is the power consumed by user  $m$  with the multiple subchannels in  $\mathcal{N}_m$ , i.e.,  $P_{m, \mathcal{N}_m} = \sum_{n \in \mathcal{N}_m} P_{m,n}$ . Note that subchannel bandwidth  $B$  can be included in the target rate and it disappears in (1b). The optimization variables are the transmit power  $P_{m,n}$  and subchannel allocation  $\mathcal{N}_m$ , for all user  $m \in \mathcal{M}$  and subchannel  $n \in \mathcal{N}$ . The first constraint (1b) is for each user  $m$  to achieve the given target rate  $R_m$  and the second constraint (1c) is to ensure an orthogonal subchannel allocation. The minimum power with a given subchannel set  $\mathcal{N}_m$  and under the rate constraint (1b) can be solved by using a water-filling power allocation [4], so that we can rewrite (1) as a subchannel allocation problem as

$$\{\mathcal{N}_m^*\} = \arg \underset{\{\mathcal{N}_m \subseteq \mathcal{N}\}}{\text{minimize}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}_m} P_{m,n}^{\text{WF}}(\mathcal{N}_m) \quad (2a)$$

$$\text{s.t. } P_{m,n}^{\text{WF}}(\mathcal{N}_m) = \max \left( 0, \lambda_{\mathcal{N}_m} - \frac{1}{\gamma_{m,n}} \right), \forall n \in \mathcal{N}_m, \quad (2b)$$

$$\sum_{n \in \mathcal{N}_m} \log_2(1 + \gamma_{m,n} P_{m,n}^{\text{WF}}(\mathcal{N}_m)) = R_m, \quad (2c)$$

and (1c),

where  $\lambda_{\mathcal{N}_m}$  is the water-level satisfying (2c). This allocation problem (2) is NP-hard [1], [2]. To reduce the high computational complexity of solving (2), a heuristic algorithm that solves a two-stage problem sequentially has been proposed in [7]. Before we propose a new algorithm, we introduce the two-stage problem, which is equivalent to the original problem in (2), and its approximation.

### III. REFORMULATION OF ORIGINAL PROBLEM AS TWO-STAGE PROBLEM

The original problem (2) can be equivalently reformulated to the two-stage problem including an inner problem and an outer problem [7]. The inner problem determines the optimal allocation given  $\{s_m\}$  and the outer problem determines the optimal  $\{s_m\}$ . Therefore, without any loss of optimality, we can get the optimal allocation of (2) by solving the following inner problem  $P^{\text{in}}$  in (3) for given assignment vector  $\mathbf{s} \triangleq [s_1 \cdots s_M]^T$  and by exploring all possible  $\mathbf{s}$  via the outer problem  $P^{\text{out}}$  in (8).

#### A. Inner Problem

For given assignment vector  $\mathbf{s}$ , we find the optimal subchannel assignment  $\{\mathcal{N}_m\}$  based on the network power con-

sumption as follows:

$$P^{\text{in}}: \{\mathcal{N}_m^o\} = \arg \underset{\mathcal{N}_m \subseteq \mathcal{N}}{\text{minimize}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}_m} P_{m,n}^{\text{WF}}(\mathcal{N}_m) \quad (3a)$$

$$\text{s.t. } |\mathcal{N}_m| = s_m, \forall m \in \mathcal{M}, \quad (3b)$$

$$(1c), (2b), \text{ and } (2c), \forall m \in \mathcal{M}.$$

Note that the additional constraint (3b) is present compared to the original optimization problem in (2).

Since the power  $P_{m,n}^{\text{WF}}$  in (3) is coupled for different  $n$ , i.e.,  $P_{m,n}^{\text{WF}}$  depends on which other subchannels  $n' \neq n$  are allocated to user  $m$ , in [7], we approximate (3) to

$$\{\tilde{\mathcal{N}}_m\} = \arg \underset{\mathcal{N}_m \subseteq \mathcal{N}}{\text{minimize}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}_m} \log_2 \bar{P}_{m,n} \quad (4)$$

$$\text{s.t. } (1c) \text{ and } (3b), \forall m \in \mathcal{M},$$

by using the decoupled (additive) power cost

$$\bar{P}_{m,n} = (2^{R_m} - 1) \gamma_{m,n}^{-1}, \forall m \in \mathcal{M}. \quad (5)$$

The new cost in (5) is the minimum power required to achieve the rate  $R_m$  if only the  $n$ th subchannel with SNR  $\gamma_{m,n}$  is allocated to the  $m$ th user; therefore, it is independent of the other subchannels which are assigned to user  $m$ . For the detailed derivation of the approximations for (4), refer to the proofs in [7].

To apply a simple LAA to (4), a modification of the LAA is required, so as to enable multiple assignment from the  $m$ th user to  $s_m$  subchannels when  $s_m > 1$ . Note that there is no multiple assignment in the original LAA. To allow  $s_m$  multiple assignment, we consider  $(s_m - 1)$  virtual users in the assignment problem. The subchannels assigned to the virtual users are actually assigned to the original user. To incorporate virtual users into (4), define a user set including virtual users as  $\mathcal{V} = \{1, \dots, v, \dots, V\}$ , where  $V = \sum_{m \in \mathcal{M}} s_m \leq N$  is a number of virtual users; a virtual user index is  $v \in \mathcal{V}$ ; and a set of subchannels allocated to the  $v$ th virtual user as  $\mathcal{V}_v$ . Then, (4) can be equivalently modified as follows:

$$\{\mathcal{V}_v\} = \arg \underset{\mathcal{V}_v \subseteq \mathcal{N}}{\text{minimize}} \sum_{v \in \mathcal{V}} \sum_{n \in \mathcal{N}_v} \log_2 \bar{P}_{\text{Map}(v, \mathbf{s}), n} \quad (6a)$$

$$\text{s.t. } \mathcal{V}_{v_1} \cap \mathcal{V}_{v_2} = \emptyset, \forall v_1 \neq v_2, \forall v \in \mathcal{V} \quad (6b)$$

$$|\mathcal{V}_v| = 1, \forall v \in \mathcal{V}, \quad (6c)$$

where  $\text{Map}(v, \mathbf{s})$  is a mapping function from  $v$  to the real user index  $m \in \mathcal{M}$  when  $\mathbf{s}$  is given. The mapping rule is as follows. We map  $v$  to  $m$ , i.e.,  $\text{Map}(v, \mathbf{s}) = m$ , if  $\sum_{m'=0}^{m-1} s_{m'} < v \leq \sum_{m'=0}^m s_{m'}$  and  $s_0 \triangleq 0$ . The orthogonal constraint in (1c) is retained as (6b). Comparing (3b) and (6c), note that number of subchannels allocated to each virtual user is one, i.e.,  $|\mathcal{V}_v| = 1$ . Therefore, (6) can be solved by any LAAs since there is no extrinsic multiple assignment in its assignment from  $V$  users to  $V$  resources.

After finding  $\{\mathcal{V}_v\}$  in (6), we can easily and directly find  $\{\tilde{\mathcal{N}}_m\}$  by following the reverse mapping. The subchannels allocated to virtual user  $v$  such that  $\text{Map}(v, \mathbf{s}) = m$  are actually allocated the  $m$ th user. Consequently, we get  $\tilde{\mathcal{N}}_m$  as

$$\tilde{\mathcal{N}}_m \leftarrow \cup_v \mathcal{V}_v, \text{ s.t. } \text{Map}(v, \mathbf{s}) = m, \forall v. \quad (7)$$

## B. Outer Problem

To obtain the optimal solution of (2), we should perform an exhaustive search over all  $\mathbf{s} \in \mathcal{S}$  by choosing  $\{\mathcal{N}_m^*\}$  which requires the lowest network power  $P^{\text{net}}$  as follows:

$$\mathbf{p}^{\text{out}}: \{\mathcal{N}_m^*\} = \arg \min_{\mathbf{s} \in \mathcal{S}} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}_m^o} P_{m,n}^{\text{WF}}(\mathcal{N}_m^o), \quad (8)$$

where  $\mathcal{S}$  is a vector set including all possible assignment vector  $\mathbf{s}$  and  $\mathcal{N}_m^o$  is obtained by solving the inner problem  $\mathbf{p}^{\text{in}}$  in (3) for given  $\mathbf{s}$ . Although the optimality of the minimization problem is achieved in (8), the exhaustive search over  $\mathbf{s} \in \mathcal{S}$  is computationally intensive. Hence, though the approximated inner problem in (6) can reduce the overall computational complexity, still the overall problem is intractable due to the outer problem requiring the exhaustive search.

To efficiently reduce the computational complexity of the outer problem, SLAA has been proposed in [7]. We set an initial allocation vector as  $\mathbf{s} = [1 \cdots 1]^T$  and determine the virtual user who yields the lowest network power with one additional subchannel, iteratively. As the first step of the first iteration ( $t = 1$ ), we increase only one  $s_c$  to  $s_c + 1$ , while  $s_m$  is unchanged where  $m \neq c$ , and with the updated  $\mathbf{s}$  we find the assignment  $\{\tilde{\mathcal{N}}_{c,m}^t\}$  for all  $m \in \mathcal{M}$ . Then, in the second step of the first iteration, using  $\{\tilde{\mathcal{N}}_{c,m}^t\}$  into (2b) and (2c), we compute the network power consumption as

$$P_c^t = \sum_{m \in \mathcal{M}} \sum_{n \in \tilde{\mathcal{N}}_{c,m}^t} P_{m,n}^{\text{WF}}(\tilde{\mathcal{N}}_{c,m}^t). \quad (9)$$

Comparing all possible  $P_c^t$ 's for all  $c \in \mathcal{M}$ , we determine the virtual user  $c^*$  at iteration  $t$  as

$$c^* = \arg \min_{c \in \mathcal{M}} P_c^t, \quad (10)$$

and update  $s_{c^*}$  as  $s_{c^*} + 1$ . Since the iteration is performed until all subchannels are assigned, i.e.,  $\sum s_m = N$ ,  $(N - M)$  iterations are required to complete the assignment. Furthermore, linear assignment problem (6) should be solved  $M$  times in each iteration to decide  $c^*$ , resulting in high computational complexity.

## IV. SIMPLIFIED SEQUENTIAL LINEAR ASSIGNMENT ALGORITHM (SSLAA)

We propose a simple selection method of virtual user to further reduce the complexity of the outer problem. Instead of the network power comparison, we compare the required power of *each user* defined as

$$Q_c^t \triangleq P_{c,\tilde{\mathcal{N}}_c^t} = \sum_{n \in \tilde{\mathcal{N}}_c^t} P_{c,n}^{\text{WF}}(\tilde{\mathcal{N}}_c^t). \quad (11)$$

In other words, we assign an additional subchannel to a user who requires the highest power consumption at the previous iteration. This is reasonable because the user will require more subchannels to reduce network power with higher priority than the others. Accordingly, the new selection method based on the individual power consumption is as follows:

$$c' = \arg \max_{c \in \mathcal{M}} Q_c^t. \quad (12)$$

This modified algorithm is thus termed a simplified SLAA (SSLAA). In SSLAA, the linear assignment problem (6) is solved only once in each iteration. As a consequence, computational complexity can be reduced significantly. In the subsequent subsections, for ease of understanding, we show a simple example of SLAA and SSLAA with the graphical representation.

### A. Examples

We consider an example to illustrate the LAA-based algorithms in Fig. 1, by using a  $V$ -by- $N$  cost matrix. The  $(m, n)$ th element of the cost matrix is a log-based single resource power  $\bar{P}_{m,n}$  in (5), i.e.,  $\log_2(\bar{P}_{m,n})$ . The illustration shows the procedure to arrive the final assignment  $\{\tilde{\mathcal{N}}_m\}$  when  $(M, N) = (2, 4)$ . The 2-by-4 initial cost matrix is shown in Fig. 1(a), where the initial assignment vector  $\mathbf{s} = [1 \ 1]^T$ .

**SLAA:** In the first iteration ( $t = 1$ ), we consider two cases:  $c = 1$  and  $c = 2$ . When  $c = 1$ , the first user is considered as a virtual user and the corresponding assignment vector  $\mathbf{s} = [2 \ 1]^T$ . When  $c = 2$ , the second user is added as a virtual user and  $\mathbf{s} = [1 \ 2]^T$ . For each case, we apply LAA and find assignment (marked as '★') as shown in Fig. 1(b) and (c), respectively. For example, in Fig. 1(b), the first, the third, and the second subchannels are assigned to the first, the second, and the third virtual users, respectively, i.e.,  $\mathcal{V}_1\{1\}$ ,  $\mathcal{V}_2\{3\}$ , and  $\mathcal{V}_3\{2\}$ . From the mapping rule in (7), we get the final assignment  $\tilde{\mathcal{N}}_{1,1}^1 = \{1, 3\}$  and  $\tilde{\mathcal{N}}_{1,2}^1 = \{2\}$ . Here, the third subchannel assigned to the second virtual user can be interpreted as it is actually assigned to the first user. Note that the fourth and the second subchannels are assigned to nobody in Fig. 1(b) and (c), respectively. Now, we compute the required network power  $P_1^1$  for  $c = 1$  and  $P_2^1$  for  $c = 2$  from (9), and from (10) we determine the virtual user who yields the lower network power. In this example, we assume that  $P_1^1 > P_2^1$ ; thus, the second user is chosen as the virtual user, i.e.,  $c^* = 2$ . According to  $c^*$ , the assignment vector is updated as  $\mathbf{s} = [1 \ 2]^T$ . In the second iteration ( $t = 2$ ), we repeat the same procedure as the first iteration. The assignments are found for two cases as shown in fig. 1(d) and (e). Contrary to the assignments in the second iteration, all subchannels from  $n = 1$  to  $n = 4$  are allocated. Hence, we stop the iteration and complete the assignment. For the completion, we compare the final network power  $P_1^2$  and  $P_2^2$ , and determine the final assignment. In this example, we assume that  $P_1^2 < P_2^2$  and get the final assignment corresponding to  $P_1^2$  as  $\tilde{\mathcal{N}}_1 = \{1, 3\}$  and  $\tilde{\mathcal{N}}_2 = \{2, 4\}$ . A pseudo-code of SLAA for general  $M$  and  $N$  is shown in Algorithm 1.

**SSLAA:** In the first iteration ( $t = 1$ ), we apply an LAA to the initial cost matrix, and get the assignment results as shown in Fig. 1(f). The first and the third subchannels are assigned to the first and the second users, respectively, i.e.,  $\tilde{\mathcal{N}}_1^1 = \{1\}$  and  $\tilde{\mathcal{N}}_2^1 = \{3\}$ . From the assignment results, we can compute the required power  $Q_c^t$  in (11) for each user  $c$  with the assigned subchannel at  $t = 1$ . Based on  $\{Q_c^t\}$ , from (12) we determine a virtual user who requires more power and who is supposed to take more subchannels. In this example,

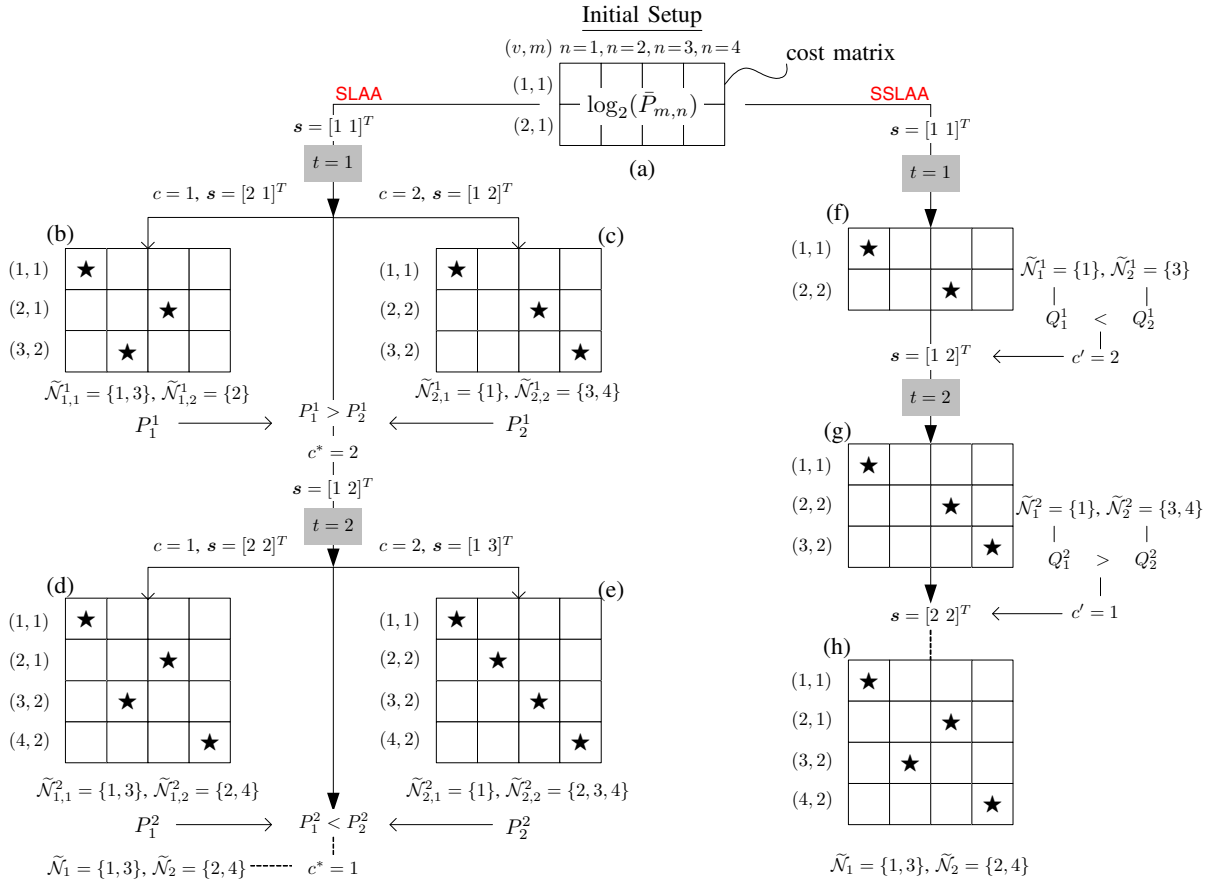


Fig. 1. Graphical illustration of SLAA and SSLAA with  $M = 2$  and  $N = 4$ . ‘★’ represents the assigned resource after assignment from a linear assignment algorithm (LAA). In this example, SLAA and SSLAA yield the identical final assignment  $\tilde{\mathcal{N}}_1$  and  $\tilde{\mathcal{N}}_2$ .

**Algorithm 1: Sequential LAA (SLAA) [7],  $N > M$**

```

1 Set up:  $\mathcal{M} = \mathcal{V} = \{1, \dots, M\}$ ,  $\mathcal{N} = \{1, \dots, N\}$ ,
  and  $s = [1 \dots 1]^T$ , i.e.,  $s_m = 1, \forall m \in \mathcal{M}$ .
  Compute  $\log_2(\bar{P}_{m,n}), \forall m, n$ , from (5)
2 for  $t = 1, \dots, N - M$  do
3   for  $c \leftarrow 1$  to  $M$  do
4     Set  $s_c^t \leftarrow s_c^{t-1} + 1$ .
5     Find  $\{\tilde{\mathcal{N}}_{c,m}^t\}$  with  $s$  and  $\mathcal{V}$ : (6), (7).
6     Compute network power  $P_c^t$ : (9).
7     Reset  $s_c^t \leftarrow s_c^{t-1} - 1$ .
8   end
9   Find a virtual user  $c^*$ : (10).
10  Update  $s_{c^*} \leftarrow s_{c^*} + 1$  and  $\mathcal{V} \leftarrow \mathcal{V} \cup \{M + t + 1\}$ .
11 end
12  $\tilde{\mathcal{N}}_m \leftarrow \tilde{\mathcal{N}}_{c^*}^{N-M}$  for all  $m \in \mathcal{M}$ .

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under the assumption that  $Q_1^1 < Q_2^1$ , we select the second user as the virtual user, i.e.,  $c' = 2$ . According to  $c'$ , we update the assignment vector as  $s = [1 \ 2]^T$ . In the second iteration ( $t = 2$ ), we update the cost matrix and again apply LAA to get the assignment as shown in Fig. 1(g). Following the mapping

**Algorithm 2: Simplified SLAA (SSLAA),  $N > M$**

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1 Set up: same as SLAA.
2 for  $t = 1, \dots, N - M$  do
3   Find  $\{\tilde{\mathcal{N}}_m^t\}$  with  $s$  and  $\mathcal{V}$ : (6), (7).
4   Compute  $Q_c^t$  for all  $c \in \mathcal{M}$ : (11).
5   Find a virtual user  $c'$ : (12).
6   Update  $s_{c'} \leftarrow s_{c'} + 1$  and  $\mathcal{V} \leftarrow \mathcal{V} \cup \{M + t + 1\}$ .
7 end
8 Find  $\{\tilde{\mathcal{N}}_m\}$  with  $s$  and  $\mathcal{V}$ : (6), (7).

```

rule in (7), we get the assignment  $\tilde{\mathcal{N}}_1^2 = \{1\}$  and  $\tilde{\mathcal{N}}_2^2 = \{3, 4\}$ . Since the first user takes only one subchannel, we assume that the first user requires more power consumption than the second user who takes two subchannels. In other words, we assume that  $Q_1^2 > Q_2^2$ , select the first user as a virtual user, i.e.,  $c' = 1$ , and update  $s$  as  $s = [2 \ 2]^T$ . Since  $\sum s_m = N$ , we know that all subchannels can be assigned with one additional execution of LAA. Updating the cost matrix according to  $s$  and applying LAA, we find the final assignments  $\tilde{\mathcal{N}}_1 = \{1, 3\}$  and  $\tilde{\mathcal{N}}_2 = \{2, 4\}$  as shown in Fig. 1(h). A pseudo-code of SSLAA for general  $M$  and  $N$  is shown in Algorithm 2

TABLE I  
SIMULATION PARAMETERS

Cell shape	hexagon
Radius of cell	1 Km
Number of users	10
Number of subchannels	20
Target rates	$R_1 = \dots = R_5 = 5$ $R_6 = \dots = R_9 = 10$ $R_{10} = 20$
UE distribution	Uniform
UE noise STD	$\sigma_U^2 = -131.5$ dBm
Path loss model	Log-normal
Path loss exponent	3.76
Shadowing STD	8.9 dB

### B. Complexity Comparison

For the computational complexity, solving (6) requires  $\mathcal{O}((M+t)(M+N+t)N)$  if we apply an optimal LAA [8] and it is a bottleneck. Since this step is repeated  $M$  times in each iteration and there are  $(N-M)$  iterations, the overall complexity of SLAA is  $\mathcal{O}(M \sum_{t=1}^{N-M} ((M+t)(M+N+t)N)) = \mathcal{O}(MN^2(N-M)^2)$ . On the other hand, as the proposed SSLAA performs LAA only once in each iteration, its overall complexity is reduced to  $\mathcal{O}(N^2(N-M)^2)$ . Corresponding performance loss in terms of power consumption is observed in the next subsection.

### C. Performance Comparison

In this subsection, we evaluate average power consumption of the proposed SSLAA and the various existing algorithms including RCG and SUSI, and SLAA. A cumulative distribution function (CDF) of the network power is shown. To obtain the results, the users' locations are realized 1,000 times uniformly in a cell modeled as a hexagonal array with 1 Km radius. For a fixed user location, 10 independent fading channels are generated, and one averaged network power is obtained for CDF. Since inter-cell interferences can be assumed as an AWGN, the relative performance of the compared methods in the single cell model is sustained in a multi-cell model. Large-scale path loss with shadowing is generated from a log-normal model in [9] with a 3.76 path loss exponent and a 8.9 dB shadowing standard deviation (STD). The noise STD is set by  $-131.5$  dBm at the users. We consider 10 users and 20 subchannels, i.e.,  $M = 10$  and  $N = 20$  with the target rates  $[R_1, \dots, R_{10}] = [5, 5, 5, 5, 5, 10, 10, 10, 10, 20]$ . Hungarian algorithm [8] is employed as the LAA in both SLAA and SSLAA.

The results in Fig. 2 confirms that the proposed SSLAA achieves the closest performance to the SLAA and its performance loss is marginal.

The average network power is computed by averaging 1,000 samples for CDF and depicted in the legend of Fig. 2. The average network power increment of SSLAA is only around 5% compared to SLAA. Comparing the existing algorithms RCG and SUSI, the proposed SSLAA outperforms them with the reduction of average power consumption around 16%.

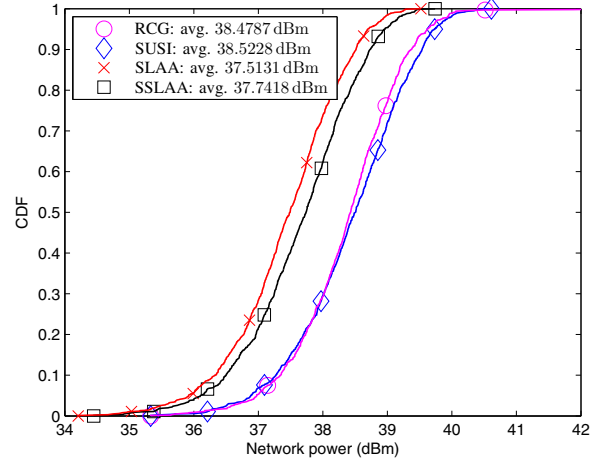


Fig. 2. CDF of average power consumption in a cellular environment summarized in Table I.

## V. CONCLUSION

A low complexity linear assignment algorithm is proposed for resource allocation of OFDMA systems when the number of subchannel  $N$  is larger than the number of user  $M$ . The proposed algorithm, called a simplified sequential linear assignment algorithm (SSLAA), can reduce the computational complexity of a sequential linear assignment algorithm (SLAA) by the order of  $M$  with performance reduction of around 5%. Numerical result in cellular environment verifies that the proposed SSLAA outperforms other existing algorithms which are in the same class (polynomial) of computational complexity.

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