A Novel Subspace Decomposition-Based Detection Scheme with Soft Interference Cancellation for OFDMA Uplink

Yung-Ping Tu, Wen-Hsien Fang, and Yie-Tarng Chen
Department of Electronic Engineering
National Taiwan University of Science and Technology, Taipei, Taiwan, R.O.C.
Email: {D9002204,whf,ytcehn}@mail.ntust.edu.tw

***Abstract*—This paper presents a novel subspace decomposition-based detection scheme with the assistance of soft interference cancellation in the uplink of interleaved orthogonal frequency division multiple access (OFDMA) systems. By utilizing the inherent data structure, the interference is first separated with the desired symbol and then further decomposed into the one caused by the residues of decision errors and the other one by the undetected symbols in the successive interference cancellation (SIC) process. Such an interference decomposition along with the soft processing scheme aim to render more thorough interference cancellation. Moreover, for practical implementations, a low-complexity version, which only copes with the principal components of inter-carrier interference (ICI), is also addressed. Conducted simulations show that the developed receiver and its low-complexity implementation can provide satisfactory performance compared with pervious works and are resilient to the presence of carrier-frequency offsets (CFOs). The low-complexity implementation, in particular, requires substantially lower computational overhead with only slight performance loss.**

I. Introduction

OFDMA systems have received a considerable amount of interest in emerging wireless communications [1]. However, the CFOs induced by oscillator mismatch and/or Doppler shift destroy the orthogonality among subcarriers and incur ICI and multi-user interference (MUI) in the uplink transmission, which seriously impact the system performance.

One effective approach to mitigate ICI and MUI is the multiuser detectors (MUDs). For example, Hsu *et al.* [2] proposed a zero-forcing (ZF)-based CFO compensation scheme. However, it, as the traditional ZF detection, suffered the flaw of noise enhancement. Although the minimum mean-square error (MMSE) detectors [3] can effectively mitigate the interference, their complexity is not light. Since it has been observed in [4]-[7] that the interference for each symbol is mainly concentrated in its neighboring subcarriers, so the remaining subcarrier components can be neglected without substantial performance degradation. In light of this, Cao *et al.* [5] proposed a low-complexity MMSE receiver based on a banded ICI matrix approximation. Also, Ahmed *et al.* [7] considered a sophisticated preprocessing scheme to squeeze the interference into nearby subcarriers and then devised a low-complexity MMSE equalizer with iterative soft interference cancellation (SIC-MMSE). Huang *et al.* [8] proposed a CFO compensation scheme and a PIC iterative approach, referred to as the CLJL-PIC. It is, however, only applicable to small CFOs, as it needs a more precise estimate of CFOs in the initial stage to remove ICI. Arslan *et al.* [9] first used a decorrelator to remove the ICI within a user cluster and then employed SIC to mitigate MUI. It requires manageable complexity, but the decorrelator can not provide precise ICI compensation in the first stage, which in turn impacts the overall performance.

In this paper we propose a novel subspace decomposition-based detection scheme with soft information-assisted interference cancellation in the uplink of interleaved OFDMA systems. The new receiver possesses two salient features: First, in contrast to previous works which treat all types of interferences as a whole, the interference, based on the inherent data structure, is further decomposed into one caused by the residues of decision errors and the other one by the undetected symbols via subspace decomposition. Second, in light of the fact that the soft information approach [10]-[12] can in general result in a more precise symbol estimate, every detector is followed by a soft decision. Such an interference decomposition along with the soft processing scheme aim to render more thorough interference cancellation. Moreover, for practical implementations, a low-complexity version, which only deals with the principal ICI components, is also addressed. Conducted simulations show that the developed detection sheme not only provides satisfactory performance compared with pervious works, but it is also resilient to the presence of CFOs. The low-complexity implementation, in particular, requires substantially lower computational overhead with only slight performance loss.

II. System Model

Consider an OFDMA system with U users simultaneously uplink their symbols over N subcarriers to the base station (BS). Assume that in each OFDM symbol block, the subset of N subcarriers, Ψ_u, is assigned to to user u and $\bigcup_{u=1}^{U} \Psi_u = \{0, \cdots, N-1\}$ and $\Psi_i \bigcap \Psi_j = \phi$ for $i \neq j$ and $i, j = 1, \cdots, U$. For each user, a data frame of N symbols passes through the mechanism of an inverse fast Fourier transform (IFFT) and the result can be expressed as $d_u(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_u(k) e^{j\frac{2\pi}{N}kn}$, $n = 0, \cdots, N-1$, $u = 1, \cdots, U$, where $s_u(k)$ denotes the symbol transmitted from user u by using the k^{th} subcarrier. To prevent the inter-symbol interference (ISI) effect, the cyclic prefix (CP) is also added at the starting position of each data frame. Furthermore, after a parallel-to-serial converter, each frame of the users' symbols is then transmitted serially by using the BPSK modulation. Assume that the transmitted symbols from each user propagate through L different paths and induce a CFO of $f_u = \frac{\omega_u}{2\pi}$ due to the Doppler frequency shift, where ω_u is the angular CFO for user u. Therefore, the BS's received baseband signal at time instant n can be expressed as

$$r(n) = \sum_{u=1}^{U} \left\{ e^{j\omega_u n} \sum_{l=0}^{L-1} h_u(l) d_u(\langle n-l \rangle_N) \right\} + g(n), \qquad (1)$$

where $\langle \cdot \rangle_N$ denotes modulo-N operator; $h_u(l)$ is the complex channel gain of the l^{th} path between user u and the BS; and $g(n)$ denotes a complex white Gaussian noise with zero mean and variance σ^2. After a serial-to-parallel converter and the removal of CP, the BS's received baseband signal vector can be shown as

$$\mathbf{r} = \sum_{u=1}^{U} \mathbf{\Gamma}(\omega_u) \mathbf{F}^H \mathcal{D}(\mathbf{h}_u) \mathbf{s}_u + \mathbf{g},$$

where $\mathbf{r} = [r(0), \; \cdots, \; r(N-1)]^T$, the superscript $(\cdot)^{H(T)}$ denotes the Hermitian (transposition) operation; $\mathbf{\Gamma}(\omega_u) = \mathcal{D}(1 \; e^{j\omega_u} \cdots e^{j\omega_u(N-1)})$ with $\mathcal{D}(\mathbf{v})$ denoting the diagonal matrix whose diagonal elements are $\mathbf{v}$; $\mathbf{b}_u = [b_u(0) \cdots \; b_u(N-1)]^T$, $\mathbf{s}_u = [s_u(0) \cdots \; s_u(N-1)]^T$, $\mathbf{g} = [g(0) \cdots \; g(N-1)]^T$; and $\mathbf{h}_u$ is the frequency-domain channel-impulse-response (CIR) corresponding to user u.

Thereby, the frequency-domain received signal, $\mathbf{y} = \mathbf{Fr}$, obtained by applying the Fourier transform $\mathbf{F}$ to the time-domain signal $\mathbf{r}$, can be expressed as

$$\mathbf{y} = \sum_{u=1}^{U} \mathbf{A}_u \mathcal{D}(\mathbf{h}_u) \mathbf{s}_u + \mathbf{n}, \qquad (2)$$

where $\mathbf{A}_u = \mathbf{F}\mathbf{\Gamma}(\omega_u)\mathbf{F}^H$ is the ICI-induced matrix of user u, in which $[\mathbf{A}_u]_{mn} = \frac{1}{N} \frac{1-e^{j2\pi(n-m+f_u)}}{1-e^{j2\pi(n-m+f_u)/N}}$; and $\mathbf{n} = \mathbf{Fg}$. Note that since each of the N subcarriers is only assigned to one user and $s_u(i)$ is zero if the i^{th} subcarrier does belong to user u, (2) hence can be re-written as

$$\mathbf{y} \;\; = \;\; \mathbf{Hs} + \mathbf{n}, \qquad (3)$$

where $\mathbf{s} = [\mathcal{C}(0), \; \mathcal{C}(1), \cdots, \mathcal{C}(N-1)]^T$, in which $\mathcal{C}(i)$ is the operation to select a nonzero symbol from $\{s_1(i) \cdots s_U(i)\}$; and $\mathbf{H} = \left[\bar{\mathbf{h}}_0, \; \cdots, \; \bar{\mathbf{h}}_{N-1}\right]$ is the corresponding interleaved effective ICI-induced channel matrix.

III. PROPOSED DETECTION SCHEME AND LOW-COMPLEXITY IMPLEMENTATION

A. Subspace Decomposition-Based Detection Scheme

The proposed receiver consists of N symbol detectors, each for the transmitted symbol on every subcarrier, and the soft SIC, as shown in Fig. 1(a). Without loss of generality, the detection is assumed to proceed from the first to the N^{th} symbol. Hence, based on (3) and SIC, the input to the j^{th} detector, devised for the symbol on the j^{th} subcarrier (the j^{th} symbol), can be expressed as

$$\begin{aligned} \mathbf{y}_j \;\; &= \;\; \mathbf{y} - \mathbf{H}(1:j-1)\tilde{\mathbf{s}}_{j-1} \\ &= \;\; \mathbf{H}_{E,j}\tilde{\mathbf{e}}_{j-1} + \bar{\mathbf{h}}_j s_j + \mathbf{H}_{I,j}\mathbf{s}_{I,j+1} + \mathbf{n}, \end{aligned} \qquad (4)$$

where $\mathbf{H}_{E,j} = \mathbf{H}(1:j-1)$, $\mathbf{H}_{I,j} = \mathbf{H}(j+1:N)$, $\tilde{\mathbf{s}}_{j-1} = [\tilde{s}_1 \; \cdots \; \tilde{s}_{j-1}]^T$, $\mathbf{s}_{I,j+1} = [s_{j+1} \; \cdots \; s_N]^T$, and $\tilde{\mathbf{e}}_{j-1} = \mathbf{s}_{j-1} - \tilde{\mathbf{s}}_{j-1}$ denotes the residues of the decision errors occurring in the detection of the previous $(j-1)$ symbols, in which $\mathbf{s}_{j-1} = [s_1 \; \cdots \; s_{j-1}]^T$, $\tilde{s}_m$ denotes the soft-decision output of the m^{th} detector, and $\mathbf{H}(i:j)$ denotes the submatrix of $\mathbf{H}$ consisting of its i^{th} to the j^{th} column.

Note from (4) that there are two types of interferences in $\mathbf{y}_j$: one caused by the residues of decision errors and the other one by the undetected symbols. To follow, we consider a detector which consists of two branches, and followed by a soft demapper, as shown in Fig. 1 (b). The upper branch of the detector is devised to keep the desired symbol intact and the lower branch is comprised of two paths, which are designed to extract either one of these two interferences to cancel those in the input $\mathbf{y}_j$ through an appropriate combination.

In light of the above discussion, the weight vector of the upper branch of the detector, $\mathbf{w}_{S,j}$, is chosen to satisfy

$$\mathbf{w}_{S,j}^H \bar{\mathbf{h}}_j = 1. \qquad (5)$$

Solving for $\mathbf{w}_{S,j}$ in (5) results in

$$\mathbf{w}_{S,j} = (\bar{\mathbf{h}}_j^H \bar{\mathbf{h}}_j)^{-1} \bar{\mathbf{h}}_j. \qquad (6)$$

In the lower branch, in order to extract the interference we first remove the desired symbol component s_j in (4). This can be achieved by passing $\mathbf{y}_j$ through an $N \times (N-1)$ matrix $\mathbf{B}_{S,j}$, referred to as the signal blocking matrix, given by

$$\mathbf{B}_{S,j} = \mathsf{Null}(\bar{\mathbf{h}}_j^H), \qquad (7)$$

where $\mathsf{Null}(\mathbf{A})$ denotes a matrix constructed by the orthonormal basis of the nullspace of $\mathbf{A}$. As such, $\mathbf{y}_j$, when passing through $\mathbf{B}_{S,j}$, is given by

$$\mathbf{z}_j = \mathbf{B}_{S,j}^H \mathbf{y}_j = \mathbf{B}_{S,j}^H \mathbf{H}_{E,j} \tilde{\mathbf{e}}_{j-1} + \mathbf{B}_{S,j}^H \mathbf{H}_{I,j} \mathbf{s}_{I,j+1} + \mathbf{B}_{S,j}^H \mathbf{n},$$

where we have used the fact that $\mathbf{B}_{S,j}^H \bar{\mathbf{h}}_j s_j = \mathbf{0}$. It is noteworthy that the interference caused by the decision residues and the one by the undetected symbols lie in the column space of $\mathbf{B}_{S,j}^H \mathbf{H}_{E,j}$ and of $\mathbf{B}_{S,j}^H \mathbf{H}_{I,j}$, respectively.

Next, we make use of the data structure in (4) to further decompose the interference in $\mathbf{z}_j$ into these two types of interferences. To achieve this, we split the output of $\mathbf{B}_{S,j}$, $\mathbf{z}_j$, into two paths, each of which is devised to extract either one of these two interferences, as shown in Fig. 1 (b). More specifically, the upper path contains an $(N-1) \times (j-1)$ matrix, $\mathbf{B}_{I,j}$, referred to as the undetected-symbol blocking matrix, and followed by a $(j-1) \times 1$ weight vector $\mathbf{w}_{E,j}$; whereas, the lower path contains an $(N-1) \times (N-j)$ matrix, $\mathbf{B}_{E,j}$, referred to as the decision-residue blocking matrix, and followed by an $(N-j) \times 1$ weight vector $\mathbf{w}_{I,j}$.

To extract the interference caused by the decision errors, $\mathbf{B}_{I,j}$, based on (8), can be chosen as $\mathsf{Null}\big((\mathbf{B}_{S,j}^H\mathbf{H}_{I,j})^H\big)$. Thereby, $\mathbf{z}_j$, after passing $\mathbf{B}_{I,j}$, becomes

$$\mathbf{z}_{E,j} = \mathbf{B}_{I,j}^H\mathbf{z}_j = \bar{\mathbf{B}}_{I,j}^H\mathbf{H}_{E,j}\tilde{\mathbf{e}}_{j-1} + \bar{\mathbf{B}}_{I,j}^H\mathbf{n}, \tag{8}$$

where $\bar{\mathbf{B}}_{I,j} = \mathbf{B}_{S,j}\mathbf{B}_{I,j}$ and we have used the fact that $\bar{\mathbf{B}}_{I,j}^H\mathbf{H}_{I,j}\mathbf{s}_{I,j+1} = \mathbf{0}$. Eq. (8) implies that only the interference caused by the decision residues remains. To annihilate this interference, the weight vector $\mathbf{w}_{E,j}$ is determined based on the MMSE criterion given by $E\Big\{\|\mathbf{w}_{S,j}^H\mathbf{y}_j - \mathbf{w}_{E,j}^H\mathbf{z}_{E,j}\|^2\Big\}$. Solving for $\tilde{\mathbf{w}}_{E,j}$ yields

$$\mathbf{w}_{E,j}^o = \big[\bar{\mathbf{B}}_{I,j}^H\mathbf{R}_j\bar{\mathbf{B}}_{I,j}\big]^{-1}\bar{\mathbf{B}}_{I,j}^H\mathbf{R}_j\mathbf{w}_{S,j}, \tag{9}$$

where $\mathbf{R}_j = E\{\mathbf{y}_j\mathbf{y}_j^H\}$ and we have used the fact that $\mathbf{z}_{E,j} = \bar{\mathbf{B}}_{I,j}^H\mathbf{y}_j$ by (8).

Likewise, to extract the interference caused by the undetected symbols in the lower path, $\mathbf{B}_{E,j}$, based on (8), can be chosen as $\mathbf{B}_{E,j} = \mathsf{Null}\big((\mathbf{B}_{S,j}^H\mathbf{H}_{E,j})^H\big)$. As such, $\mathbf{z}_j$, after passing $\mathbf{B}_{E,j}$, becomes

$$\mathbf{z}_{I,j} = \mathbf{B}_{E,j}^H\mathbf{z}_j = \bar{\mathbf{B}}_{E,j}^H\mathbf{H}_{I,j}\mathbf{s}_{I,j+1} + \bar{\mathbf{B}}_{E,j}^H\mathbf{n}, \tag{10}$$

where $\bar{\mathbf{B}}_{E,j} = \mathbf{B}_{S,j}\mathbf{B}_{E,j}$ and we have used the fact that $\bar{\mathbf{B}}_{E,j}^H\mathbf{H}_{E,j}\tilde{\mathbf{e}}_{j-1} = \mathbf{0}$. Again, (10) implies that only the interference caused by the undetected symbol remains. Similarly, to remove this interference, we consider the filter $\mathbf{w}_{I,j}$, determined based on the MMSE criterion given by $E\Big\{\|d_j - \mathbf{w}_{I,j}^H\mathbf{z}_{I,j}\|^2\Big\}$, where $d_j = (\mathbf{w}_{S,j} - \bar{\mathbf{B}}_{I,j}\mathbf{w}_{E,j}^o)^H\mathbf{y}_j$, and can obtain

$$\mathbf{w}_{I,j}^o = \big[\bar{\mathbf{B}}_{E,j}^H\mathbf{R}_j\bar{\mathbf{B}}_{E,j})\big]^{-1}\bar{\mathbf{B}}_{E,j}^H\mathbf{R}_j(\mathbf{w}_{S,j} - \bar{\mathbf{B}}_{I,j}\mathbf{w}_{E,j}^o). \tag{11}$$

Therefore, after the removal of these two detrimental interferences, the output of the filter for the j^{th} symbol, $\bar{s}_j$, can be expressed as

$$\bar{s}_j = \mathbf{w}_j^H\mathbf{y}_j, \tag{12}$$

where $\mathbf{w}_j = \mathbf{w}_{S,j} - \bar{\mathbf{B}}_{I,j}\mathbf{w}_{E,j}^o - \bar{\mathbf{B}}_{E,j}\mathbf{w}_{I,j}^o$ denotes the overall weights of the detector for the j^{th} symbol. Since the transmitted symbols and the noise are uncorrelated, $\mathbf{R}_j$ can be expressed as

$$\mathbf{R}_j = \mathbf{H}\begin{bmatrix} \mathbf{R}_{\tilde{e}_{j-1}} & \mathbf{0}_{(j-1)\times 1} & \mathbf{0}_{(j-1)\times(N-j)} \\ \mathbf{0}_{1\times(j-1)} & 1 & \mathbf{0}_{1\times(N-j)} \\ \mathbf{0}_{(N-j)\times(j-1)} & \mathbf{0}_{(N-j)\times 1} & \mathbf{I}_{N-j} \end{bmatrix}\mathbf{H}^H + \sigma^2\mathbf{I}_N, \tag{13}$$

where $\mathbf{R}_{\tilde{e}_{j-1}} \approx \mathcal{D}(E\{|e_1|^2\,|\tilde{s}_1\},\ \cdots,\ E\{|e_{j-1}|^2\,|\tilde{s}_{j-1}\})$ [4].

Based on the statistical property of the transmitted symbols and the Gaussian noise assumption of the channels, it can be readily shown that $\bar{s}_j$ is approximately Gaussian so $\bar{s}_j = \bar{m}_j s_j + n_j$ [10], [11], where $\bar{m}_j$ is the equivalent strength and n_j is the noise with variance $\bar{\sigma}_j^2$. After some manipulations, we can obtain

$$\bar{m}_j = \mathbf{w}_{S,j}^H\bar{\mathbf{h}}_j = 1, \tag{14}$$

where $\delta(\cdot)$ is the kronecker delta. Also, by using the fact that the interference has been roughly annihilated in the filtering process, $\bar{\sigma}_j^2$ can be approximated by

$$\bar{\sigma}_j^2 \approx E\Big\{(\mathbf{w}_j^H\mathbf{n})(\mathbf{w}_j^H\mathbf{n})^H\Big\} = \sigma^2\|\mathbf{w}_j\|^2, \tag{15}$$

where we have used the fact that the contaminated noise is white with variance σ^2.

The soft-decision output, after the filter and the MAP criterion, is then given by [10], [11]

$$\tilde{\lambda}(s_j) = \log\frac{p(\bar{s}_j|s_j = +1)}{p(\bar{s}_j|s_j = -1)} = \frac{2\bar{s}_j\bar{m}_j}{\bar{\sigma}_j^2}. \tag{16}$$

We can then construct a soft estimate of the j^{th} symbol as $\tilde{s}_j = tanh\Big(\frac{1}{2}\tilde{\lambda}(s_j)\Big)$ [12]. Finally, taking the sigmoid function of $\tilde{s}_j$ yields the estimate of the j^{th} symbol.

The overall procedures can be summarized as follows:

Step 1: (Initialization) Start with $j = 1$ and $\mathbf{R}_{\tilde{e}_{j-1}} = \mathbf{0}$.

Step 2: Determine $\mathbf{w}_{S,j}$ and $\mathbf{w}_{E,j}^o$ by (6) and (9), respectively, and remove the interference caused by the decision residues. Thereafter, determine $\mathbf{w}_{I,j}^o$ by (11) and cancel the interference due to the undetected symbols to obtain $\bar{s}_j$ by (12).

Step 3: Compute the mean and variance of $\bar{s}_j$ by (14) and (15), respectively, and then obtain its soft estimate $\tilde{s}_j$ via (16).

Step 4: Update $\mathbf{R}_{\tilde{e}_j}$ from $\mathbf{R}_{\tilde{e}_{j-1}}$ by computing $E\{|e_j|^2\,|\tilde{s}_j\}$. Perform signal regeneration and cancellation of the soft estimate $\tilde{s}_j$ by $\mathbf{y}_{j+1} = \mathbf{y}_j - \bar{\mathbf{h}}_j\tilde{s}_j$.

Step 5: Repeat Steps 2 to 4 from the first to the N^{th} symbols until all symbols are detected.

B. Low-Complexity Implementation

Note that the detection scheme addressed above requires to take the inverse of the transformed covariance matrices for every transmitted symbol and thus calls for lots of computations. To reduce the complexity overhead, in light of the fact that the power of the ICI entries is mainly concentrated in the neighboring subcarriers of the desired symbol [4]-[7], we can utilize these principal components only and ignore the less significantly ones, entailing a banded-like structure of the ICI-induced matrix. Since the sizes of these data vectors become smaller, the computations required to determine the corresponding filters, which dictate the computational load, can thus be alleviated.

To be specific, the detection for the j^{th} symbol now will only cope with its most significant q neighboring ICI entries in $\mathbf{H}$, $\mathcal{H}_j$, which is given by

$$\mathcal{H}_j := \mathbf{H}(j-q:j+q, j-q:j+q), \tag{17}$$

where $\mathbf{H}(m:n, i:j)$ denotes the submatrix of $\mathbf{H}$ constructed by the intersection of its m^{th} to the n^{th} rows and the i^{th} to the j^{th} columns.

Similar as the discussion in the previous subsection, the input to the j^{th} detector, $\check{\mathbf{y}}_j$, is given by

$$\begin{aligned} \check{\mathbf{y}}_j &:= \mathbf{y}_j(j-q:j+q) \\ &= \mathbf{y}(j-q:j+q) - \mathcal{H}_j(1:q)\check{\mathbf{s}}_{j-1} \\ &= \mathcal{H}_{E,j}\check{\mathbf{e}}_{j-1} + \check{\mathbf{h}}_j s_j + \mathcal{H}_{I,j}\check{\mathbf{s}}_{I,j+1} + \check{\mathbf{n}}, \end{aligned} \tag{18}$$

where $\breve{\mathbf{s}}_{j-1} = \tilde{\mathbf{s}}_{j-1}(j-q:j-1)$, $\mathcal{H}_{E,j} = \mathbf{H}(j-q:j+q, j-q:j-1)$, $\mathcal{H}_{I,j} = \mathbf{H}(j-q:j+q, j+1:j+q)$, $\breve{\mathbf{h}}_j = \bar{\mathbf{h}}_j(j-q:j+q)$, $\breve{\mathbf{e}}_{j-1} = \tilde{\mathbf{e}}_j(j-q:j-1)$, $\breve{\mathbf{s}}_{I,j+1} = \mathbf{s}_{I,j}(j+1:j+q)$ and $\breve{\mathbf{n}} = \mathbf{n}(j-q:j+q)$ denote the principal data/noise vectors and the related principal channel matrices/vectors in (4).

The receiver is now still comprised of N detectors, each of which consists of two branches, along with a soft SIC. The discussion and derivations are the same as the above, except that now the data dealt with are only the principal ICI component vectors. Due to space limitation, these related derivations are omitted here.

C. Computational Complexity

In this subsection we evaluate the proposed detection scheme and its low-complexity version in terms of the computational complexity based on the number of complex multiplications and additions (CMAs) required for the detection of one OFDMA block as compared with previous works.

The complexity of the proposed detection scheme is dictated by the determination of the optimum weight vectors $\mathbf{w}^o_{E,j}$ and $\mathbf{w}^o_{I,j}$ in (9) and (11), which require $[N(j-1)^2 + N(N-1)(j-1) + N(j-1) + (j-1)^3 + (j-1)^2]$ and $[N(N-j)^2 + N(N-1)(N-j) + N(N-j) + (N-j)^3 + (N-j)^2]$ CMAs, respectively. Therefore, the computations for the j^{th} symbol are $\left[(N-j)^2(2N-j+1) + (j-1)^2(N+j) + N^3 + 8N^2 + N\right]$ CMAs, so the overall computational load is $\sum_{j=1}^{N}\left[(N-j)^2(2N-j+1) + (j-1)^2(N+j) + N^3 + 8N^2 + N\right]$ CMAs. For its low-complexity implementation, the $(N-j)\times(N-j)$ and $(j-1)\times(j-1)$ matrices in (9) and (11), respectively, are now both reduced to $q \times q$, the inverse of which calls for q^3 CMAs. Also, the computational load for the determination of the filters in the lower branch is now both reduced to $\left[q^3 + 2q^2(2q+1) + 2q(2q+1)^2 + q(2q+1)\right]$. Consequently, the overall computations become $2\big\{(q+1)^3 + q^3 + 2q(q+1)^2 + q(q+1) + \sum_{j=2}^{q}\left[(q^3 + (j-1)^3) + 3(q+j-1)(q+j)^2 + (q^2 + (j-1)^2 + 2q)(q+j)\right]\big\} + (N-2q)\left[q^3 + 10q(2q+1)^2 + 2q(q+1)\right]$ CMAs.

For easy reference, Table 1 shows the analytic expressions of the numbers of CMAs required by the existing detection algorithms. As an illustration, based on these analytic expressions, the complexity of these detectors for $N = 64$ and 128, and some choices of q's is also provided in Table 2. We can find that the computational overhead required by the proposed detection scheme is lower than that of SIC-MMSE, but higher than the other works due to the SIC scheme. However, the computations called for by the low-complexity version is drastically reduced, since the sizes of the matrices needed to be inverted are greatly reduced.

IV. Simulations and Discussions

Consider the uplink of an OFDMA system, where the number of subcarriers for each OFDM symbol block, N, is 128. Assume that the channels are frequency selective Rayleigh fading, and there are $U = 4$ active users simultaneously communicating with the BS in the uplink systems with the number of resolvable paths for each user, L, being 5.

Six detection algorithms: the conventional MMSE detector [3], the CFO-compensated ZF detector [2], SIC-MMSE [7], CLJL-PIC [8], the decorrelator SIC [9], and the proposed subspace decomposition-based detection scheme are conducted for comparison in terms of the BER performance.

First, we compare the BER performance versus signal-to-noise ratio (SNR) with the (normalized) CFOs being uniformly distributed in 0.5CS, as shown in Fig. 2, from which we can note that CLJL-PIC yields the worst performance, since MUI is not effectively annihilated due to the unprecise initial symbol estimates in the first stage. The CFO-compensated ZF detector shows similar inferior performance, as ICI is not properly dealt with by the CFO compensation scheme in the first stage, which in turn impacts the overall performance. The decorrelator SIC outperforms the previous two detection algorithms, but the decorrelators conducted in the first stage can not effectively compensate for the offset of ICI. Also, we can find that the conventional MMSE detector provides slightly better performance, but it exhibits an error floor as SNR increases because ICI becomes more serious now and substantially impacts its performance. On the other hand, SIC-MMSE and the proposed detection scheme provide similar performance and both outperform the other detectors with the incorporation of SIC.

Next, we compare the BER performance versus SNR for low-complexity implementations of SIC-MMSE and of the proposed scheme with $q = 2,\ 4$ for CFOs being distributed in 0.5CS, as shown in Fig. 3, from which we can observe that the performance of both low-complexity implementations only degrades slightly compared to the counterparts in Fig. 2. This justifies the fact that we can only cope with the nearby dominant subcarrier components and neglect the others. Also, we can find that the proposed scheme outperforms SIC-MMSE due to its enhanced interference cancellation capability.

V. Conclusions

In this paper, we have developed a subspace decomposition-based detection scheme for the interleaved OFDMA uplink systems. The new scheme combines the refined interference decomposition with the soft processing to render more thorough interference cancellation. Conducted simulations verify the efficacy of the new receiver and its low-complexity implementation which only copes with the principal ICI subcarrier components.

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TABLE I

COMPARISON OF COMPUTATIONAL COMPLEXITY, WHERE N DENOTES THE NUMBER OF SUBCARRIERS IN OFDMA, U DENOTES THE NUMBER OF USERS, q DENOTES THE NUMBER OF PRINCIPAL ICI COMPONENTS EMPLOYED, AND I DENOTES THE NUMBER OF ITERATIONS REQUIRED.

Detectors	Complex multiplications/additions
MMSE [3]	$2N^3+3N^2+N$
CFO-compensated ZF[2]	$N^3+2N^2+\sum_{k=1}^{I}2^k(N+N^2+N\log_2\frac{N^2}{U})+\left[(2^k-1)+2^k+(2^k-1)^2\right]$
SIC-MMSE[7]	$IN\left[2N^3+2N^2+4N\right]$
SIC-MMSE (Low) [7]	$IN\left[(2q+1)^3+(4q+1)^2(2q+1)+(4q+1)(2q+1)+2(2q+1)^2+(2q+1)\right]$
CLJL-PIC[8]	$(3UN^2+UN^2\log_2 N)+IU\left[(U-1)(N^2+N\log_2 N)+N\right]$
Decorrelator SIC[9]	$UN\left[2(\frac{N}{U})^3+3(\frac{N}{U})^2\right]$
Proposed	$\sum_{j=0}^{N-1}\left[(N-j)^2(2N-j+1)+(j-1)^2(N+j)+N^3+8N^2+N\right]$
Proposed (Low)	$2\{(q+1)^3+q^3+2q(q+1)^2+q(q+1)+\sum_{j=2}^{q}\left[q^3+(j-1)^3+3(q+j-1)(q+j)^2+(q^2+(j-1)^2+2q)(q+j)\right]\}+(N-2q)\left[(q^3+10q(2q+1)^2+2q(q+1)\right]$

TABLE II

COMPARISON OF COMPUTATIONAL COMPLEXITY, WHERE $N=64$ AND $N=128$, I=3 AND 6 ARE USED FOR [2], [7] AND [8], RESPECTIVELY.

Detectors	$N=64$	$N=128$
MMSE [3]	536,576	4,243,584
CFO-compensated ZF[2]	394,964	2,612,052
SIC-MMSE [7]	204,570,624	3.2468e+009
SIC-MMSE(Low q=2)[7]	120,960	241,920
SIC-MMSE(Low q=4)[7]	701,568	1,403,136
CLJL-PIC [8]	471552	1,902,592
Decorrelator SIC[9]	2,293,760	35,127,296
Proposed	38,118,080	595,670,400
Proposed (Low q=2)	31,684	64,492
Proposed (Low q=4)	191,946	405,962

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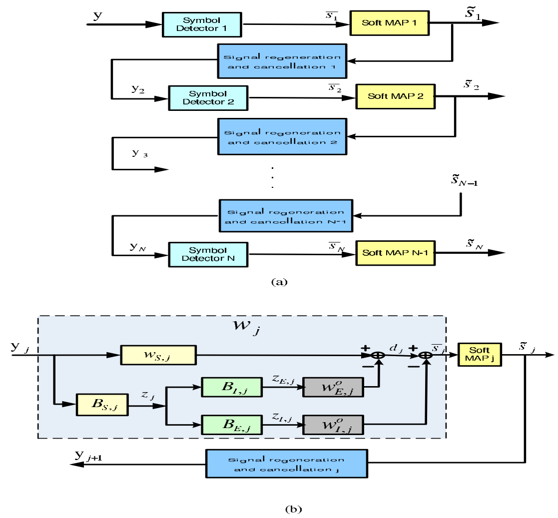

Fig. 1. (a) Structure of the proposed subspace decomposition-based soft receiver, (b) The j^{th} symbol detector

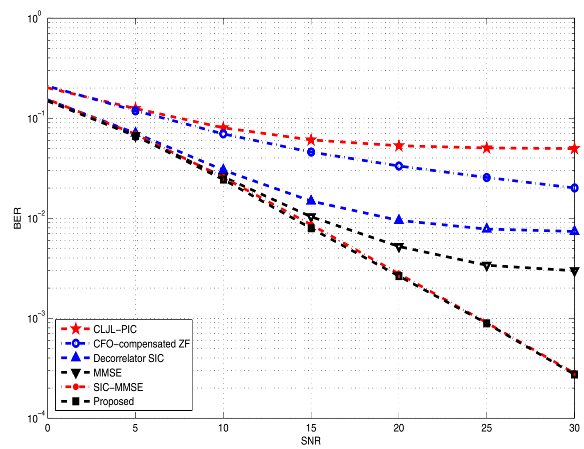

Fig. 2. Comparison of the BER versus SNR for CFO=0.5CS.

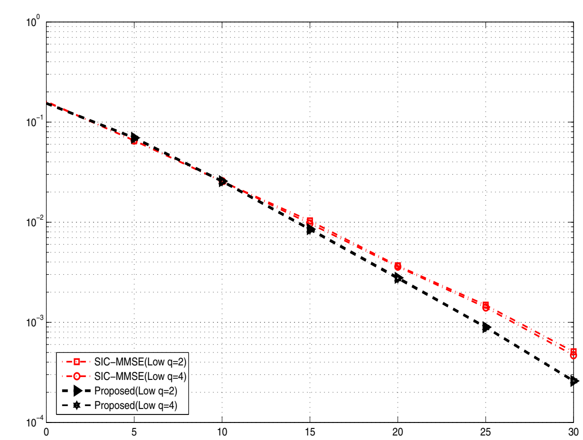

Fig. 3. Comparison of the BER versus SNR for the low-complexity implementations of SIC-MMSE and the proposed detection scheme for CFO=0.5CS and $q=2,\ 4$.