Minimizing Sum Power in Two-Way Amplify-and-Forward Relay Channel Based on Instantaneous Channel State Information

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***Abstract*—This paper considers a two-way relay network in which two source nodes exchange messages during two time slots with the help of a half-duplex amplify-and-forward relay node. Closed form expressions of source and relay node powers are obtained by solving an optimization problem which aims to minimize system total power by constraining source nodes' received signal-to-noise ratios (SNR). Furthermore, the variation of optimum powers for different channel variance scenarios is investigated. For comparison, the distribution in the total power consumption by allocating optimum powers versus equal powers is shown by numerical results. Furthermore, the outage performances of optimum power allocation (OPA) and equal power allocation (EPA) schemes are simulated. Numerical and simulation results show that the OPA scheme outperforms the EPA in all scenarios.**

Keywords-amplify-and-forward; two-way relaying; total power minimization

I. Introduction

Relay networks have recently been the focus of many studies. Although one-way relaying has been extensively studied, it is spectrally inefficient due to the fact that extra channel uses are required for the relay assisted communication of source nodes. On the other hand, two-way relaying proposed by Shannon [1] first, allows two source nodes to simultaneously transmit their signals to the relay by using network coding techniques. Then the relay broadcasts the received signal after some processing such as amplify-and-forward (AF), decode-and-forward, compress-and-forward or estimate-and-forward. Source nodes can recover the desired signal sent by the other one after subtracting its own signal. So, the signal exchange between two source nodes is completed in two time slots, satisfying spectral efficiency.

For practical consideration, AF is desirable due to its non-regenerative processing. Non-regenerative relaying is lower in complexity, processing delay and processing power when compared to regenerative ones. Attracted by the benefits of power allocation, the performance of one-way AF relaying has been extensively studied [2], [3]. For two-way AF relaying, power allocation is considered in [4]-[8]. In single relay networks, [4] provides power allocation maximizing the average sum rate. Minimizing the outage probability is considered in [5]. In [6], optimal power allocation to maximize the minimum data rate is proposed. In [7], sum power minimization satisfying traffic requirements defined by outage probabilities is considered. In [8], total transmit power minimization is discussed in a single relay two-way scheme where beamforming is applied.

In this paper, we consider an AF two-way relaying scenario consisting two source nodes and communicating through a relay node, where all nodes are equipped with single antenna. We aim to minimize the total power consumption by constraining the received signal to noise ratios (SNR) at source nodes. Exact closed form expressions for optimum source and relay powers are derived by solving the optimization problem of total power consumption minimization. Simulation results show that the proposed optimum power allocation (OPA) outperforms that of equal power allocation (EPA) scheme considerably. Variations in the total amount of power values for fixed SNR s and outage probability distributions for fixed power are investigated.

The rest of this paper is organized as follows. In Section II, we describe the system model. In Section III, we formulate the total power optimization problem and provide a derivation of optimum source and relay powers. In Section IV, we show simulations for different channel scenarios. Finally, we state our conclusions in Section V.

II. System Model

As shown in Fig. 1, we consider a wireless two-way network where two source nodes, A and B , communicate with each other through a single relay node R . We assume no direct communication between two source nodes because of poor quality of the channel between them. A , B and R are capable of either transmitting or receiving using a half-duplex single-antenna communication scheme. The data symbols x_A and x_B are transmitted by source nodes A and B with powers P_A and P_B , respectively. We assume $E\left\{\left|x_A\right|^2\right\}=E\left\{\left|x_B\right|^2\right\}=1$, where $E\{.\}$ stands for the expectation and $|.|$ represents the absolute value of a complex number.

The channels between source A and R, and source B and R are denoted by h_A and h_B, respectively. $h_A \sim CN\left(0, \sigma_{h_A}^2\right)$ and $h_B \sim CN\left(0, \sigma_{h_B}^2\right)$ undergo independent and identically distributed (i.i.d.) flat Rayleigh fading and satisfy channel reciprocity.

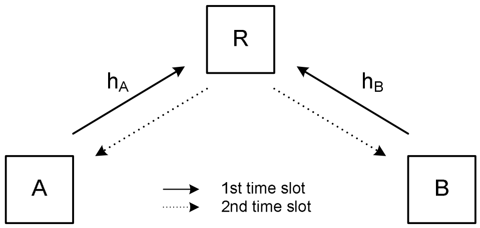

Figure 1. System model for two-way relaying.

We consider total transmission over two time slots. During the first time slot, both sources simultaneously transmit their data to the relay. The signal received at the relay can be represented as

$$y_R = \sqrt{P_A} h_A x_A + \sqrt{P_B} h_B x_B + n_R \tag{1}$$

where n_R is the additive white noise at the relay distributed as $n_R \sim N(0, \sigma_R^2)$. In the second time slot, the relay broadcasts the signal amplified by the factor γ given by

$$\gamma = \sqrt{\frac{P_R}{P_A |h_A|^2 + P_B |h_B|^2 + \sigma_R^2}} \approx \sqrt{\frac{P_R}{P_A |h_A|^2 + P_B |h_B|^2}} \tag{2}$$

where P_R is the transmit power of R. At high signal-to-noise ratio (*SNR*), the noise becomes negligible, and this amplification factor only relies on the instantaneous channel state information (CSI).

The received signals in the second time slot at source A and B are given, respectively, by

$$\begin{aligned} y_A &= \gamma y_R h_A + n_A \\ &= \gamma \sqrt{P_A} h_A^2 x_A + \gamma \sqrt{P_B} h_A h_B x_B + \gamma h_A n_R + n_A \end{aligned} \tag{3}$$

and

$$\begin{aligned} y_B &= \gamma y_R h_B + n_B \\ &= \gamma \sqrt{P_A} h_A h_B x_A + \gamma \sqrt{P_B} h_B^2 x_B + \gamma h_B n_R + n_B \end{aligned} \tag{4}$$

where n_A and n_B are additive white noise at A and B distributed as $n_A \sim CN(0, \sigma_A^2)$ and $n_B \sim CN(0, \sigma_B^2)$, respectively. The first term in (3) and the second term in (4) are known as self-interference and can be subtracted from y_A and y_B, respectively. After self-interference cancellation, *SNR* at source A is

$$SNR_A = \frac{\gamma^2 P_B |h_A|^2 |h_B|^2}{\gamma^2 |h_A|^2 \sigma_R^2 + \sigma_A^2}. \tag{5}$$

Similarly, the *SNR* at source B is

$$SNR_B = \frac{\gamma^2 P_A |h_A|^2 |h_B|^2}{\gamma^2 |h_B|^2 \sigma_R^2 + \sigma_B^2}. \tag{6}$$

Substituting (2) into (5) and (6) and assuming $\sigma_A^2 = \sigma_B^2 = \sigma_R^2 = \sigma^2$, the *SNR*s at A and B are given respectively by

$$SNR_A = \frac{|h_A|^2 |h_B|^2 P_R P_B}{\sigma^2 \left(|h_A|^2 P_R + |h_A|^2 P_A + |h_B|^2 P_B\right)} \tag{7}$$

and

$$SNR_B = \frac{|h_A|^2 |h_B|^2 P_R P_A}{\sigma^2 \left(|h_B|^2 P_R + |h_A|^2 P_A + |h_B|^2 P_B\right)}. \tag{8}$$

III. Power Optimization

In this section, we are interested in optimizing powers allocated to both source nodes and the relay to satisfy the minimization of total power consumption while maintaining a lower bound for the SNRs at sources A and B in the two-way system. The optimization problem can be formulated as

$$\begin{aligned} &\min_{P_A, P_B, P_R} \; P_A + P_B + P_R \\ &\text{subject to} \quad SNR_A \geq \psi_A \\ &\qquad\qquad\quad\; SNR_B \geq \psi_B \\ &\qquad\qquad\quad\; 0 \leq P_A,\, 0 \leq P_B,\, 0 \leq P_R \end{aligned} \tag{9}$$

where SNR_A is lower bounded by ψ_A and similarly, SNR_B by ψ_B. The geometric mean of $|h_A|$ and $|h_B|$ is denoted as $|h|$, where $|h_A||h_B| = |h|^2$ is satisfied. Defining $|h_A| = |h| / \sqrt{k}$ and $|h_B| = \sqrt{k}\,|h|$, we write the lower bounds on SNR_A and SNR_B as in (10) and (11), respectively.

$$SNR_A = \frac{k|h|^2 P_B P_R}{\sigma^2\left(P_A + k^2 P_B + P_R\right)} \ge \psi_A \tag{10}$$

$$SNR_B = \frac{k|h|^2 P_A P_R}{\sigma^2\left(P_A + k^2 P_B + k^2 P_R\right)} \ge \psi_B \tag{11}$$

Let

$$A_1 \triangleq \psi_A \sigma^2 / |h|^2 \quad \text{and} \quad A_2 \triangleq \psi_B \sigma^2 / |h|^2 \tag{12}$$

for notational simplicity. After some algebra on (10) and (11), the first and second inequalities in (9) become

$$A_1(P_A + k^2 P_B + P_R) - kP_B P_R \le 0 \tag{13}$$

and

$$A_2(P_A + k^2 P_B + k^2 P_R) - kP_A P_R \le 0 \tag{14}$$

respectively. Through (13) and (14), the optimization problem in (9) is reformulated by

$$\begin{aligned} &\min_{P_A, P_B, P_R} \; P_A + P_B + P_R \\ &\textit{subject to} \;\; A_1(P_A + k^2 P_B + P_R) - kP_B P_R \le 0 \\ &\qquad\qquad\quad A_2(P_A + k^2 P_B + k^2 P_R) - kP_A P_R \le 0 \\ &\qquad\qquad\quad 0 \le P_A,\; 0 \le P_B,\; 0 \le P_R. \end{aligned} \tag{15}$$

The Lagrangian function for (15) is then given by

$$\begin{aligned} &L(P_A, P_B, P_R, \mu_1, \mu_2, \mu_{11}, \mu_{22}, \mu_{33}) \\ &= (P_A + P_B + P_R) \\ &\quad + \mu_1(A_1(P_A + k^2 P_B + P_R) - kP_B P_R) \\ &\quad + \mu_2(A_2(P_A + k^2 P_B + k^2 P_R) - kP_A P_R) \\ &\quad - \mu_{11} P_A - \mu_{22} P_B - \mu_{33} P_R \end{aligned} \tag{16}$$

where μ_1, μ_2, μ_{11}, μ_{22} and μ_{33} are Karush-Kuhn-Tucker (KKT) multipliers. Applying Karush-Kuhn-Tucker (KKT) conditions for Lagrangian optimality, we can obtain the following equalities.

$$\begin{aligned} &\partial L / \partial P_A = 0 \\ &\partial L / \partial P_B = 0 \\ &\partial L / \partial P_R = 0 \\ &\mu_1(A_1(P_A + k^2 P_B + P_R) - kP_B P_R) = 0 \\ &\mu_2(A_2(P_A + k^2 P_B + k^2 P_R) - kP_A P_R) = 0 \\ &\mu_{11} P_A = 0, \quad \mu_{22} P_B = 0, \quad \mu_{33} P_R = 0 \\ &\mu_1 \ge 0 \;\; \mu_2 \ge 0 \;\; \mu_{11} \ge 0 \;\; \mu_{22} \ge 0 \;\; \mu_{33} \ge 0 \end{aligned} \tag{17}$$

Solving the nonlinear equation sets given in (17), we can obtain the optimal solutions given by (18) through (21) including the total power P_T.

$$P_A = A_2\left(1 + k\right) \tag{18}$$

$$P_B = A_1\left(1 + k^{-1}\right) \tag{19}$$

$$P_R = A_1\left(1 + k\right) + A_2\left(1 + k^{-1}\right) \tag{20}$$

$$P_T = \left(2 + k + k^{-1}\right)\left(A_1 + A_2\right) \tag{21}$$

On the other hand, the equal power scheme should be understood as minimum powers satisfying $P_A = P_B = P_R$, and (10), (11), at least one with equality.

As can be seen in Fig. 2, the equal power allocation scheme is strongly affected from the asymmetry of channels and much more power saving is provided by the optimum power allocation scheme.

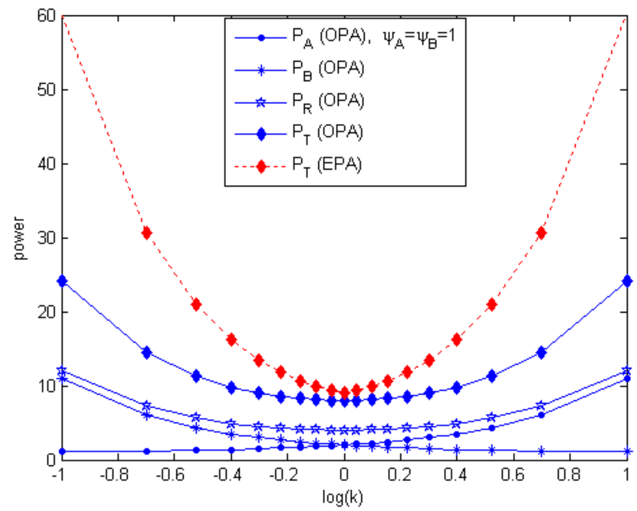

Figure 2. OPA scheme when $\psi_A = \psi_B = 1$ and $\sigma^2 / |h|^2 = 1$.

In fact, two leading inequality constraints in (15) are satisfied with equality, namely, $SNR_A = \psi_A$ and $SNR_B = \psi_B$. Substituting $|h_A||h_B| = |h|^2$ and $k = |h_B| / |h_A|$ into (18) – (21), we obtain the power expressions based on the instantaneous channel states, given by (23) through (26).

$$P_A = \frac{\sigma^2 SNR_B \left(|h_A| + |h_B|\right)}{|h_A|^2 |h_B|} \tag{23}$$

$$P_B = \frac{\sigma^2 SNR_A \left(|h_A| + |h_B|\right)}{|h_A| |h_B|^2} \tag{24}$$

$$P_R = \frac{\sigma^2 \left(|h_A| + |h_B|\right)\left(|h_A| SNR_B + |h_B| SNR_A\right)}{|h_A|^2 |h_B|^2} \tag{25}$$

$$P_T = \frac{\sigma^2 \left(|h_A| + |h_B|\right)^2 \left(SNR_A + SNR_B\right)}{|h_A|^2 |h_B|^2} \tag{26}$$

For the special case where we choose $SNR_A = SNR_B$, the proposed power allocation given by (27) through (29) coincides with the results obtained in [9]. Both schemes aim to balance the two traffic flows between source nodes by allocating more power to the weaker flow. As a result, for this special case, we obtain an optimum power allocation scheme that minimizes total power by constraining the received SNRs at source nodes, or that maximizes the minimum of the received SNRs by constraining the total power P_T.

$$P_A = \frac{|h_B|}{2\left(|h_A| + |h_B|\right)} P_T \tag{27}$$

$$P_B = \frac{|h_A|}{2\left(|h_A| + |h_B|\right)} P_T \tag{28}$$

$$P_R = P_T / 2 \tag{29}$$

IV. Numerical Results

In this section, we present some simulation and numerical results for comparisons of the proposed power allocation and the equal power allocation schemes. The noise power σ^2 is assumed to be unity. We consider three scenarios in which the variances of the channels h_A and h_B have different characteristics: i) $\sigma_{h_A}^2 = \sigma_{h_B}^2$, both variances increasing; ii) $\sigma_{h_A}^2 + \sigma_{h_B}^2 = 10$, the ratio of the variances varying; iii) $\sigma_{h_A}^2 = d^{-\alpha}, \sigma_{h_B}^2 = (1-d)^{-\alpha}$ where d denotes the distance between A and R and assuming A, B and R are located in a straight line. The distance between A and B is fixed to unity. Here, the pathloss factor α is set to 4 to model radio propagation in urban areas [10].

In Fig. 3, we plot the optimum power variation of source and relay nodes versus $\sigma_{h_A}^2$ for scenario (i). Since the variances of the channels change in the same direction and all nodes need less power to transmit in the stronger channel, the total power decreases as the variances of the channels increase. It is also clear that OPA is superior to the EPA in terms of total power.

Fig. 4 illustrates the optimum power variation of the source and relay nodes versus $\sigma_{h_A}^2$ for scenario (ii). In scenario (ii), as $\sigma_{h_A}^2$ increases, $\sigma_{h_B}^2$ decreases to fix the sum of these two variances to 10. Thus, the links between A - R and B - R are unbalanced which lead to the fact that the weaker link will dominate the performance. The gap between the powers of A and B widens as the balance between the channels gets more far apart and narrows as the links get balanced. The advantage over EPA scheme is more significant in the case of strongly unbalanced links.

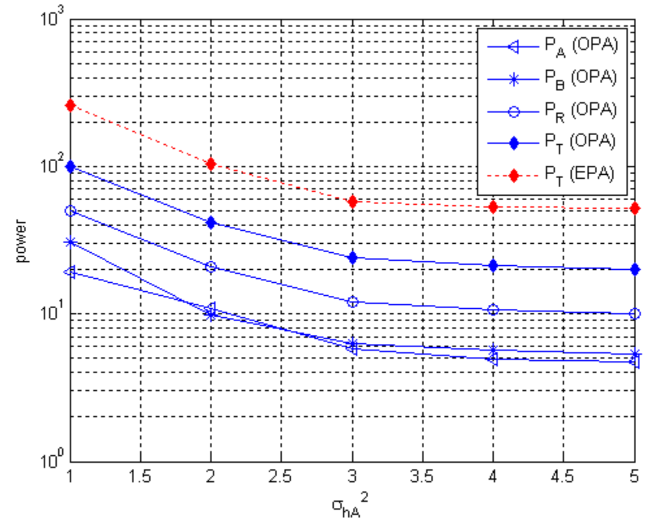

Figure 3. EPA and OPA schemes versus $\sigma_{h_A}^2$ for scenario (i).

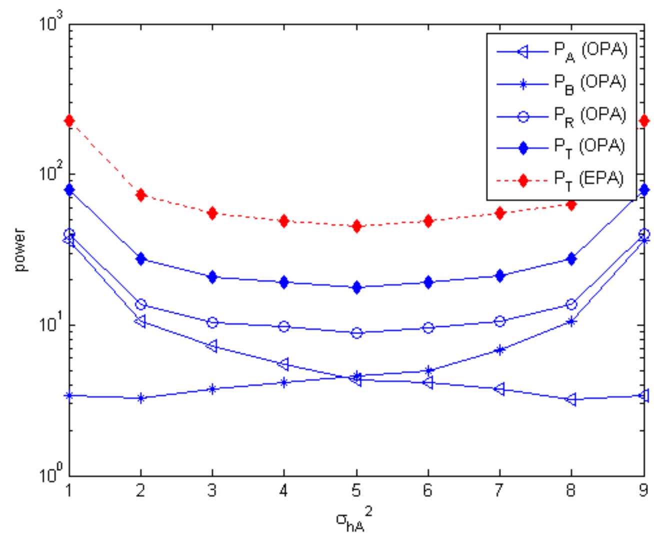

Figure 4. EPA and OPA schemes versus $\sigma_{h_A}^2$ for scenario (ii).

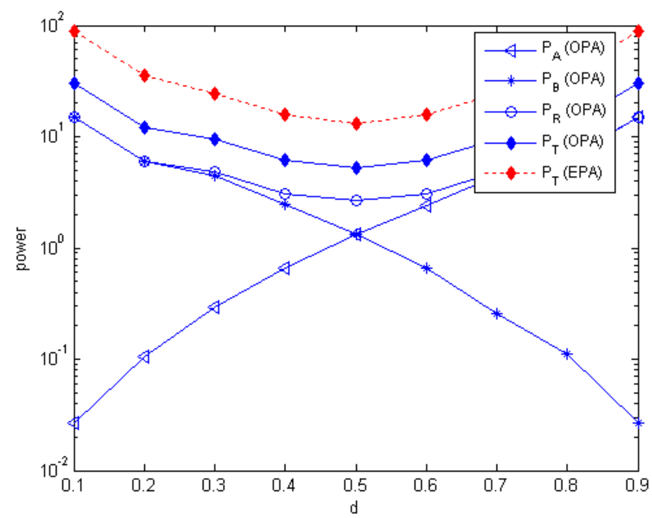

Figure 5. EPA and OPA schemes versus d for scenario (iii).

In Fig. 5, we demonstrate the optimum power variation of the source and relay nodes versus d for scenario (iii). When the relay gets closer to one of the source nodes, the gap between the powers of the source nodes widens since the distance between the source nodes is fixed to unity and the source node closer to the relay transmits with much less power. It is shown that the proposed OPA scheme performs better than the scheme of EPA across the whole range of distances. The advantage is more significant when the relay is located close to either source node.

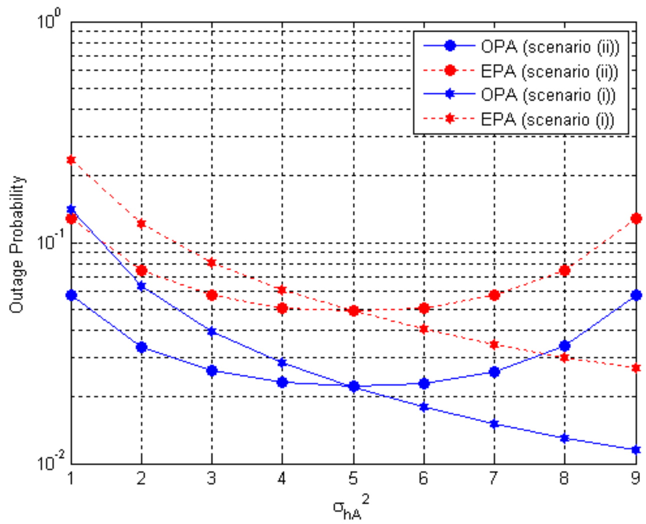

Figure 6. Outage probabilities versus $\sigma_{h_A}^2$ for EPA and OPA schemes in scenario (i) and (ii).

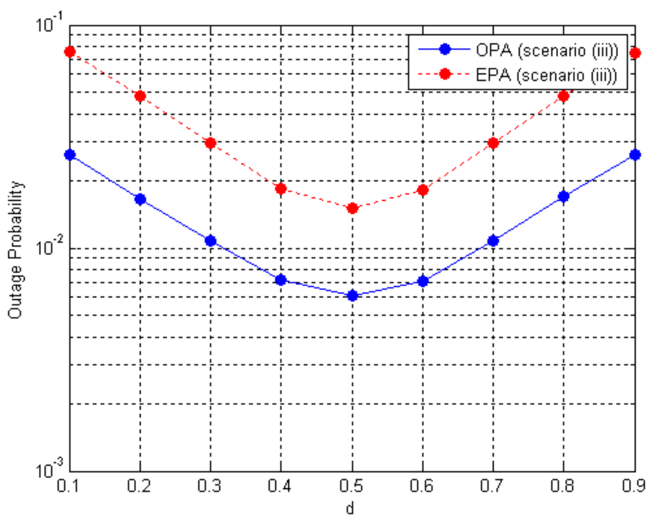

Figure 7. Outage probabilities versus d for EPA and OPA schemes in scenario (iii).

In a new setup, we fix the powers and allow SNRs to vary and we observe the outage probabilities where the outage is defined as SNR dropping below a threshold (minimum of the SNRs in the EPA scheme). In Fig. 6, we plot the outage probability as a function of $\sigma_{h_A}^2$ for scenario (i) and (ii). As expected, OPA performs better than the EPA for both scenarios irrespective of the variances of the channels. Moreover, outage performance of scenario (ii) is symmetric with recpect to the middle value of $\sigma_{h_A}^2$ range as can be foreseen. In scenario (i), channel variances change in the same direction so the outage performance improves as the channel variances increase. In any case, OPA outperforms EPA for both scenarios. Fig. 7 compares the EPA and OPA schemes for scenario (iii). As expected, OPA is lot better and can achieve more significant performance gains in terms of outage probability when the channels are strongly asymmetric. Fig.7 also shows that the relay is best positioned at the middle point of two source nodes.

V. Conclusions

In this paper, we address the problem of finding the optimal relay and source power allocation which minimizes the total power when the received SNRs at the sources are constrained. For this purpose, an optimization problem is solved and closed form expressions for the optimum powers are derived. We compared by simulations the power distribution of OPA and EPA schemes for different channel scenarios. OPA scheme outperforms EPA scheme in terms of total amount of power. We also realized the same scenarios for fixed power. Simulation results show that the OPA outperforms EPA scheme in terms of outage probability for all channel variance scenarios.

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