A Novel OFDM Power Based Estimation for Dynamic Channel Tracking in Downlink LTE

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***Abstract*—In this paper we compare several channel estimation algorithms in the case of LTE (Long Term Evolution) System when applied in a high mobility environment. In particular, we propose a novel algorithm, based on the observation of OFDM block energy, to perform a semi-blind tracking of the channel between pilot blocks. This algorithm is compared to more traditional approaches based on 1D estimation and interpolation. In addition, an Expectation Maximization (EM) algorithm is used to improve the overall channel estimation.**

I. Introduction

3GPP Long-Term Evolution is the beyond 3G system in 3GPP (3rd Generation Partnership Project), which represents the next step toward 4G. LTE's goal is to provide a higher data rate, a lower latency and an increased capacity of wireless data networks. One of LTE's requirements is the ability to maintain the connection with a high speed moving mobile terminal (up to 350 km/h). LTE is considered to be a good candidate for vehicular applications. The concept of ”LTE connected cars” [1] is already being tested. LTE is also proposed as a 4G alternative to the ”Global System for Mobile communication Railway” (GSM-R) currently used for Railway mobile communications [2].

LTE uses orthogonal Frequency Division Multiple Access (OFDMA) as an access technology in Downlink.In wireless systems, channels are usually highly frequency selective. It is well known that frequency selectivity can cause inter-symbol interference (ISI), which may result in a severe capacity lost. Orthogonal Frequency Division Multiplexing (OFDM) provides a low-complexity solution to handle the ISI issue. OFDM is a digital multi-carrier modulation method. It can be seen as decomposing the spectrum of the frequency selective channel into narrow-band parallel sub-channels called sub-carriers. These sub-carriers experience almost flat fading, which simplifies the channel equalization process.

In OFDM systems, the whole knowledge of the channel is required for coherent detection and decoding. In order to perform channel estimation, pilot signals are sent over a few OFDM symbols of the LTE Downlink frame. These pilots are distributed over a limited number of sub-carriers. Channel estimation on pilot OFDM symbols can be obtained using standard OFDM channel estimation techniques. However, the estimation process is not obvious for OFDM symbols that do not carry pilots. This is especially the case for rapidly time varying channels impacting high speed mobile terminals, e.g., high speed trains.

Several techniques have been proposed for the channel estimation in LTE OFDM system. In [3], the least square (LS) estimator was used to obtain the channel impulse response at the pilot symbol, the authors propose to estimate a down-sampled channel impulse response to reduce the ill-conditioned LS matrix effect typical to LTE systems. Indeed, many LTE sub-carriers do not carry any data, leading to ill-conditioned LS matrix. When the channel statistics are known, a linear minimum mean square (LMMSE) error can be used [4], [5]. The LMMSE achieves better performance than the LS algorithm and it does not suffer from ill-conditioned matrices. However, it presents a higher complexity and it requires the knowledge of the second order channel statistics. Several interpolation techniques are proposed in [6] for the case of one-dimensional estimations (1D). A 2D MMSE estimation can also be performed [7]. This method uses the statistical channel information in the frequency domain and in the time domain in order to perform the 2D estimation. In [8] several 2D interpolation techniques that are less complex than 2D MMSE are presented.

In this paper, we propose a two-stage technique for estimating the LTE channel. The first stage consists of implementing a 1D time domain estimation of the channel over the pilot OFDM symbols, and the second stage performs the channel tracking between pilot OFDM symbols using interpolation. The interpolation is either a linear interpolation if no further information on the channel is available, or an MMSE-based interpolation if the channel statistics are known. Furthermore, we propose a novel channel tracking technique, based on the observation of the power of the received OFDM symbols. The power variation between two received OFDM symbols is mainly due to time variations of the channel. The channel estimates on the pilot OFDM symbols and the high correlation between the different sub-carriers at a given time allow the channel tracking between pilot OFDM symbols. Finally, in order to improve the estimation, an iterative estimator based on Expectation Maximization (EM) [9] is implemented. This estimator uses the soft outputs, i.e., the output a posteriori probabilities (APPs), of the decoder to refine the estimation results and to improve the bit error rate (BER).

The current paper is organized as follows. Section II de-

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scribes the general system model in the LTE downlink. Section III presents the two-stage algorithm with different interpolation techniques. In section IV, the power-based channel tracking method is introduced. Section V provides an overview of the EM algorithm to be used. Simulation results of the various estimation algorithms are given in Section VI. Finally, Section VII draws some conclusions.

II. System Model

A. Coded OFDM system

For the system model, we consider a Bit Interleaved Coded Modulation (BICM) OFDM system as shown in Fig.1. At the transmitter side, channel coding is implemented on the data bits S. The encoded bits C are scrambled using an interleaver, before being mapped onto a QAM modulation symbol. Pilot symbols are inserted within the generated QAM symbols. The resulting signal X represents the frequency-domain signal to be sent over the sub-carriers. It is the input of the OFDM modulation block.The null sub-carriers are inserted by placing zeros in X before the IFFT block. Thus, X has a length equal to the total number of sub-carriers N.

The channel is considered to be a stationary time varying multipath channel, having L distinct taps. The channel impulse response is represented as $h_m = [h_m(0)...h_m(l)...h_m(L-1)]$, where m denotes the index of the OFDM symbol in the time domain, and $h_m(l)$ the l-th channel coefficient, corresponding to a l-sample delay. The channel taps are considered statistically independent for different delay taps whereas channel taps having the same delay are correlated with each others in time. Let R be the autocorrelation of the channel h, we have:

$$R(|m-n|) = E\left[h_m(l)h_n(l)^*\right] \quad (1)$$

We consider a normalized square channel impulse response. Thus, for $m = n$ we have $R(0) = \frac{1}{L}$ for all delays l and $E\left[h_m(l')h_n(l)^*\right] = 0$ for $l \neq l'$.

At the receiver, and after the fast Fourier transform (FFT) operation, the received signal may be written as follows:

$$Y_m = \text{Diag}(H_m)X_m + W_m \quad (2)$$

where Y_m is a vector of length N, $\text{Diag}(H_m)$ is a diagonal matrix having H_m as it is diagonal, where H_m is the FFT of the channel impulse response h_m: $H_m = Q_t h_m$, where Q_t is FFT matrix truncated to the length of h_m. The receiver is supposed to be equipped with an iterative estimator that can benefit from the soft output symbols of the decoder in order to improve the final BER of the system.

B. Frame Structure in LTE Downlink

The LTE frame is partitioned into sub-frames of 1 ms duration. Each of these sub-frames consists of 14 OFDM symbols. Pilots are sent on 4 OFDM symbols within a single sub-frame. When pilot symbols are inserted, they are evenly distributed over all the available sub-carriers: One out of 6 sub-carriers carry pilots. The first carrier carrying a pilot is shifted by 3 sub-carriers between two consecutive pilot OFDM symbols. Sub-carrier spacing is 15 kHz, the total number of sub-carriers N is 512, among which $K = 300$ sub-carriers are used for transmission (both for pilots and data symbols). Fig. 2 shows LTE sub-frame for a single transmission Antenna.

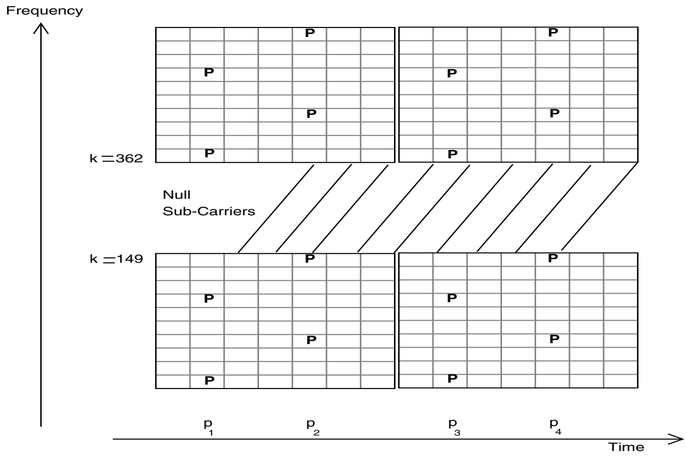

Fig. 2. A typical LTE sub-frame in 2D (Frequency/Time)

III. Two-Stage Estimation Algorithm

In this section, we present a 2-stage estimation algorithm. The first stage consists of a 1D estimation of the channels over the pilot OFDM symbols using the pilot sub-carriers. The second stage estimates the channel change between pilot symbols using interpolation over the pilot channel estimates already obtained on stage 1.

A. Stage one Estimation: Least Square/MMSE

We first use a least square (LS) estimation of the channel at the pilot sub-carriers frequency. The estimation consists of simply dividing the received symbols at the pilot sub-carriers with the known pilot symbols. Let $p_u \in \mathcal{S}_p$ where $\mathcal{S}_p = \{p_1, p_2, p_3, p_4\}$ denotes the set of the time indices corresponding to OFDM symbols where pilots are sent. The LS estimation of the channel at the pilot sub-carrier k_p is given by:

$$\hat{H}_{p_u}(k_p) = H_{p_u}(k_p) + W_{p_u}(k_p)/X_{p_u}(k_p), \quad (3)$$

In order to estimate the channel between the pilot sub-carriers we use the FFT interpolation. This interpolation consists of estimating the channel impulse response in the time-domain and then computing the channel estimates over all sub-carriers using FFT. The time-domain estimation of the channel can be performed using the minimum mean square error (MMSE) estimator:

$$\hat{h}_{p_u} = \left[Q_{ts}^H Q_{ts} + \frac{\sigma_w^2}{R(0)} I\right] Q_{ts}^H \hat{H}_{p_u}. \quad (4)$$

Matrix Q_{ts} is the punctured FFT matrix that corresponds only to the pilot sub-carriers. The frequency-domain estimate is

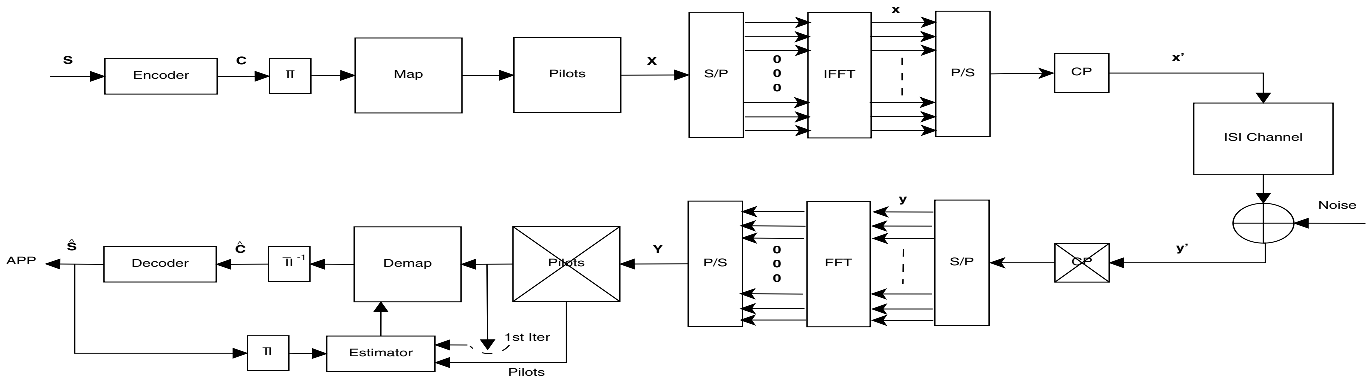

Fig. 1. Block Diagram for a Coded OFDM system with Iterative receiver [9]

simply obtained by:

$$\tilde{H}_{p_u} = Q\hat{h}_{p_u}. \tag{5}$$

B. Stage two Estimation: Interpolation

For OFDM symbols other than pilot OFDM symbols (n $n \notin \mathcal{S}_p$), the channel impulse response h_n is estimated by interpolating between the estimated channels $\hat{h}_{p_u}$. This interpolation can be done in different ways, depending if the channel autocorrelation vector R is known at the receiver or not.

1) Linear Interpolation: Linear interpolation (LI) is the straightforward way to obtain the channel when the channel statistics are not known. In this paper, we propose to use the following interpolator:

$$\hat{h}_{n,LI}(l) = \frac{\sum_{u=1}^{4} \hat{h}_{p_u}(l)/|p_u - n|}{\sum_{u=1}^{4} 1/|p_u - n|}, \tag{6}$$

for all $n \notin \mathcal{S}_p$. With the interpolator in (6), the weight of $\hat{h}_{p_u}(l)$ is inversely proportional to the distance between the time index of the channel to be estimated n and the index of the pilot symbol p_u. The choice of the weights in (6) results from the decreasing correlation of channels corresponding to OFDM symbols when the time separating them is increasing.

2) MMSE Interpolation: When the channel autocorrelation is known, we may use a more efficient interpolation based on a linear MMSE. Linear MMSE interpolation (LMMSEI) minimizes the mean square error between the outcome of the linear interpolation and the actual channel tap. The channel tap of delay l at time index n is estimated using:

$$\hat{h}_{n,LM}(l) = a_n^H \hat{h}_P(l) \tag{7}$$

where $\hat{h}_P(l)$ is a vector representing the channel taps estimates of delay l at the various pilot symbols: $\hat{h}_P(l) = [\hat{h}_{p_1}(l)\ \hat{h}_{p_2}(l)\ \hat{h}_{p_3}(l)\ \hat{h}_{p_4}(l)]^T$. a_n is a vector that satisfies the LMMSE criterion and minimizes the expression :

$$J = E\Big[\big(h_n(l) - a_n^H \hat{h}_P(l)\big)\big(h_n(l) - a_n^H \hat{h}_P(l)\big)^*\Big]$$

Thus,

$$a_n^H = C_{h_n \hat{h}_P}^T C_{\hat{h}_P}^{-1}. \tag{8}$$

$C_{h_n \hat{h}_P}$ is the vector given by:

$$C_{h_n \hat{h}_P} = E\left[h_n(l)\hat{h}_P(l)^*\right] = [R(|n-p_1|)...R(|n-p_i|)...]^T$$

$C_{\hat{h}_P}$ is a covariance matrix given by:

$$C_{\hat{h}_P} = E\left[\hat{h}_P(l)\hat{h}_P^H(l)\right].$$

$C_{\hat{h}_P}^{-1}$ is independent of n and l and can be computed and stored in advance. $C_{h_n \hat{h}_P}$ is only dependent of the time index n. Hence, for a symbol n, the same vector a_n is used for interpolation over all tap delays. a_n is calculated only for one subframe as the pilot structure does not change from one subframe to another.

IV. Channel Tracking using received power observation

The changes of the channel impulse response in the time domain will affect the power of the received OFDM symbols. In the following we use the received power over individual sub-carriers, in order to obtain an initial estimate of the channel gain in the frequency domain. Then, the channel tracking can be made by combining the information obtained from the current channel gain and from the channel estimates over pilot symbols.

Let H_n be the channel to be estimated, and $\hat{H}_{p_u}$ the channel estimate on pilot symbol p_u. The power of the received signal $P_{Y_n}(k)$ on sub-carrier k is given by:

$$P_{Y_n}(k) \quad = \quad |H_n(k)|^2 P_X(k) + \sigma_W^2(k), \tag{9}$$

where $P_X(k)$ represents the power of the transmitted signal $X_n(k)$. The instantaneous power of the received signal is given as:

$$|Y_n(k)|^2 = |H_n(k)|^2|X_n(k)|^2 + |W(k)|^2 + \\ 2|H_n(k)||X_n(k)||W(k)|\cos(\theta_{H_n,k} + \theta_{X_n,k} - \theta_{W,k}) \tag{10}$$

$\theta_{H_n,k}$, $\theta_{X_n,k}$, $\theta_{W,k}$ represent the phases of $H_n(k)$, $X_n(k)$, and $W(k)$, respectively.

To estimate the channel gain at time index n and sub-carrier k, we estimate the received signal power by the instantaneous power $\hat{P}_{Y,n}(k) = |Y_n(k)|^2$. Then we use the following estimator:

$$|\hat{H}_n(k)|^2 = \left(|Y_n(k)|^2 - \sigma_W^2(k)\right)/P_X(k), \tag{11}$$

The proposed channel gain estimator (11) is an unbiased estimator. Assuming that $H_n(k)$ follows a centered Gaussian distribution with a normalized variance, one can express the mean square error (MSE) of the channel gain estimator as:

$$\text{MSE}\left(|\hat{H}_n|^2\right)(k) = 2\left(\frac{E\left[|X|^4\right]}{P_X^2} - 1\right) + \frac{1}{\text{SNR}^2(k)} + \frac{2}{\text{SNR}(k)}, \tag{12}$$

where SNR represents the signal to noise ratio given by SNR $= \frac{P_X}{\sigma_W^2}$. From (12) we can see that there are 2 factors affecting the channel's gain estimation, one factor is related to the channel noise and it is represented by the terms involving the SNR, the other factor is the type of modulation used and it is represented by the term $F_m = 2\left(\frac{E\left[|X|^4\right]}{P_X^2} - 1\right)$. This indicates that the estimator will perform the best when a single-level modulation is used (e.g., QPSK or 8-PSK) as $F_m = 0$ in this case. However, when a multi-level modulation is used (e.g., 16-QAM or 64-QAM) the channel gain estimator performance is degraded as F_m increases.

The next step in the channel tracking is to establish a relationship between the channel h_n to be estimated, and the previously estimated channel h_{p_u} at pilot symbol p_u. We model the channel variation as $h_n = h_{p_u} + e_n$, where e_n is a variation vector. In the frequency domain, the relation between the sub-carriers is given as $H_n(k) = H_{p_u}(k) + E_n(k)$, where $E_n = Q_t e_n$.

Now we can develop the relationship between the channel gain $|H_n(k)|^2$ and the channel H_{p_u} estimated at time index p_u:

$$\begin{aligned} |H_n(k)|^2 &= |H_{p_u}(k)|^2 + |E_n(k)|^2 \\ &+ 2H_{p_u,i}(k)\left(Q_{t,i}(k)e_{n,r} + Q_{t,r}(k)e_{n,i}\right) \\ &+ 2H_{p_u,r}(k)\left(Q_{t,r}(k)e_{n,r} - Q_{t,i}(k)e_{n,i}\right) \end{aligned} \tag{13}$$

where, $H_{p_u,i} = \Im(H_{p_u})$, $H_{p_u,r} = \Re(H_{p_u})$, $e_{n,i} = \Im(e_n)$, $e_{n,r} = \Re(e_n)$, $Q_{t,i} = \Im(Q_t)$, $Q_{t,r} = \Re(Q_t)$, and $Q_t(k)$ represents the row k of the truncated FFT matrix Q_t.

All the terms in equation (13) are already estimated except for the terms related to e_n.

One way to do the channel tracking is to compute the vector e_n that minimizes $\sum_{k=0}^{K}(|H_n|^2(k) - |\hat{H}_n|^2(k))^2$. However, this optimization problem is non-convex and it is complex to solve. Assuming that the channel variation $|E_n|^2$ is small and that it can be neglected in equation (13), we can formulate the following linear system:

$$\begin{aligned} O &= 2\left[\text{Diag}(H_{p_u,i})Q_{t,i} + \text{Diag}(H_{p_u,r})Q_{t,r}\right]e_{n,r} \\ &+ \underbrace{2\left[\text{Diag}(H_{p_u,i})Q_{t,r} - \text{Diag}(H_{p_u,r})Q_{t,i}\right]}_{M_{H,Q}} e_{n,i} \end{aligned} \tag{14}$$

where the observation $O = |H_n|^2 - |H_{p_u}|^2$. In order to keep tracking of the channel, $e_{n,r}$ and $e_{n,i}$ are estimated using an MMSE estimator:

$$\hat{v}_{e_n} = \left(\hat{M}_{H,Q}^T \hat{M}_{H,Q} + \frac{\sigma_O^2}{\sigma_e^2}\right)\hat{M}_{H,Q}^T \hat{O} \tag{15}$$

where $v_{e_n} = [e_{n,r}\ e_{n,i}]^T$, $\hat{M}_{H,Q}$, and $\hat{O}$ are obtained by replacing H_{p_u} and $|H_n|^2$ by their respective estimates in the system (14). σ_O^2 is approximated by the MSE given in (12) and σ_e^2 depends on the associated speed and on $|n - p_u|$. For a delay of one OFDM symbol, i.e., $|n - p_u| = 1$, and for a terminal moving with a velocity of 150 km/h, σ_e^2 can be calculated using Clarck's channel model to be equal 6.5×10^{-3}. Since the algorithm is supposed to work without any a priori knowledge, we use the value $\sigma_e^2 = 6.5 \times 10^{-3}$ regardless of the moving terminal's speed.

V. Expectation Maximization Algorithm

To further improve the estimation, an Expectation Maximization (EM) algorithm is used. EM provides an iterative estimation of the communication channel. It makes use of the complete data set $\mathcal{K} = (X_m, Y_m)$. In this case the estimation is initialized with one of the previous mentioned estimators, the estimation is improved iteratively using the following EM estimator:

$$h_{n,EM}^{i+1} = (Q_t^H \Omega^i Q_t)^{-1} \widetilde{\text{Diag}(X_m^i)}^H Y_m, \tag{16}$$

where i is the current iteration, $\widetilde{\text{Diag}(X_m^i)}^H$ represents the soft estimates of transmitted symbols, given by

$$\widetilde{\text{Diag}(X_m^i)}^H = \sum_X \text{APP}_i \text{Diag}(X_m)$$

and $\Omega^i = \sum_X \text{APP}_i \text{Diag}(X_m)^T \text{Diag}(X_m)^*$.

The derivation of the EM estimator is out of the scope of this article, for further information please refer to [9].

VI. Comparison and Simulation Results

In this section we simulate the different proposed estimation algorithms with LTE Downlink. As described in Section II, LTE subframe is composed of 14 OFDM symbols, which are coded together with a convolutional encoder. Out of the 14 OFDM symbols 0 to 13, pilots are inserted at OFDM symbols $\mathcal{S}_p = \{1, 4, 8, 11\}$. The channel impulse response is a 6-tap rectangular channel and the time variation over each tap is represented by the autoregressive (AR) model $h_{n+1}(l) = \gamma h_n(l) + \sqrt{1-\gamma^2}\nu(l)$ where h_n and ν are Gaussian variables with the same normalized variance. γ is calculated from Clark's model for a carrier frequency $f_c = 2.6$ GHz and for a delay corresponding to one OFDM symbol (71 μs). The estimation algorithms are tested for two velocities $v = 150$ km/h, and $v = 300$ km/h, where γ is found to be equal to 0.9935 and 0.974, respectively.

Fig. 3 presents the BER curves for the different estimation techniques without EM iterations for $v = 150$ km/h. From SNR

= 15 dB, the performances of different estimators diverge. The linear interpolation (LI) estimator is clearly the least efficient one. The power based channel tracking using the nearest channel pilot estimates (NP for Nearest Power) is slightly better than the LMMSE (LM) for an SNR lower than 24 dB. However, for higher SNR, the NP reaches some sort of saturation and it is outperformed by the LMMSE. At 30 dB it has a BER comparable to that of LI. This saturation is due to the fact that for higher SNR the main error factor in the NP estimator is related to the type of modulation in use (the modulation factor F_m) and not to the channel noise level. The EM iterations are not shown for the sake of clarity, but they are roughly 2 dB better for SNR bigger than 20 dB.

Fig. 4 shows the BER results for v = 300 km/h. The increase in velocity severely degrades the performance of LMMSE and linear interpolations. The power based method performance is degraded but not as severely as the other estimators. For example, at iteration 1 and for SNR = 20 dB, the NP estimator experiences a 5 dB gain compared to the LI estimator and a gain of 2.5 dB compared to the LMMSE estimator. The performance gain of the NP estimator in this case is even clearer after the EM iterations, where for SNR=20 dB, NP shows a gain of 5 dB over the LMMSE interpolator and its performance exceeds by far that of the linear interpolator.

a) Complexity: Even though the NP method has a clear advantage from a BER point of view in the case of high vehicular speed, its drawback is its higher complexity. As it was shown earlier, the interpolations method can calculate and store in advance the interpolation weights, which results in a linear complexityduring real time estimation $\mathcal{O}(L)$. The NP method is dominated by the multiplication of large matrices of the order $(2L \times K)$, and by the inversion of a $(2L \times 2L)$ matrix, making its complexity in the order of $\mathcal{O}(L^2K)$ as K is typically larger than L.

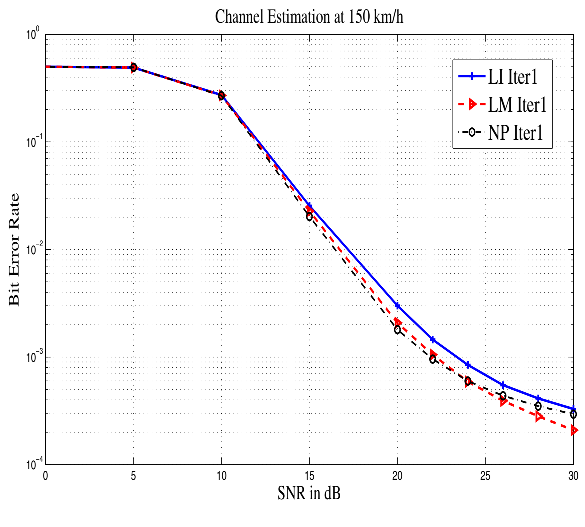

Fig. 3. BER of the 3 estimation techniques obtained for $v = 150$ km/h

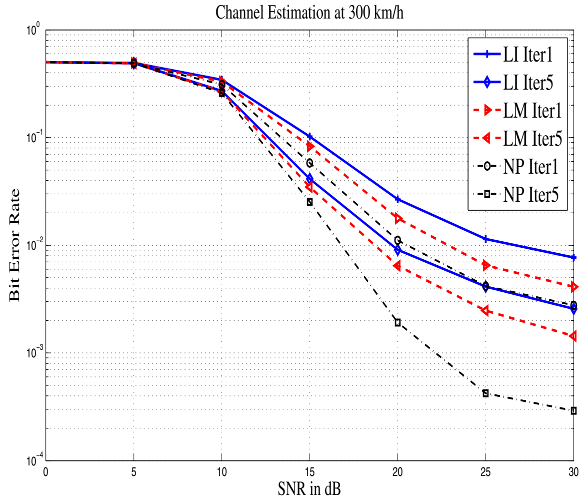

Fig. 4. BER of the 3 estimation techniques obtained for $v = 300$ km/h

VII. Conclusion

In this paper, we have presented three techniques for the channel estimation in LTE. We have proposed a novel method for the channel tracking based on the observation of the instantaneous power of the received OFDM symbols. Even though this method does not assume any previous knowledge on the channel statistics, the simulation showed that it can outperform techniques making use of the total knowledge of the channel autocorrelation like MMSE. An EM iterative receiver was also implemented. This receiver improves the estimation performance drastically for high SNR.

References

[1] J. Mosyagin , "A high-speed railway mobile communication system based on LTE" 2010 12th Intl. Conf. on Transparent Optical Networks (ICTON), Munich, Germany, june 2010.

[2] Gao Tingting and Sun Bin, "A high-speed railway mobile communication system based on LTE" Intl. Conf. On Electronics and Information Engineering (ICEIE), Kyoto, Japan,V1-414 -V1-417, Aug. 2010

[3] A. Ancora, C. Bona, and D.T.M. Slock, "Down-Sampled Impulse Response Least-Squares Channel Estimation for LTE OFDMA" ICASSP 2007 ,III-293 -III-296

[4] C. Shin, J.G. Andrews and E.J Powers "An Efficient Design of Doubly Selective Channel Estimation for OFDM Systems" IEEE Trans. on Wireless Comm. v6 p 3790 -3802 2007

[5] C. Sgraja, J. Lindner "Estimation of rapid time-variant channels for OFDM using Wiener filtering" IEEE Intl. Conf. on Communication v4 p 2390 - 2395 2003

[6] S. Coleri, M. Ergen, A. Puri and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems" IEEE Trans. Broadcasting, pp. 223-229, Sept. 2002.

[7] P. Hoeher, S. Kaiser and P. Robertson, "Two-dimensional pilot-symbol-aided channel estimation by Wiener filtering" IEEE Intl. Conf. Acoustics, Speech, and Signal Processing (ICASSP 97), Munich, Germany, pp. 1845-1848, Apr. 1997.

[8] X. Dong, W. S. Lu and A.C.K. Soong, , "linear Interpolation in Pilot Symbol Assisted Channel Estimation for OFDM" IEEE Trans. on Wireless Communications volume 6 pp 1910 -1920 may 2007

[9] Y. Liu, L. Brunel, J. J. Boutros, "EM-based Channel Estimation for Coded Multi-Carrier Transmissions," IEEE Transactions on Wireless Communications, vol. 10, no. 10, pp. 3185-3195, October 2011.