Distance dependent Call Blocking Probability, and Area Erlang Efficiency of Cellular Networks

Subhendu Batabyal[1], Suvra Sekhar Das[1,2]
[1]G.S.Sanyal School of Telecommunication, Indian Institute of Technology Kharagpur, India.
[2]Dept. of Electronics & Electrical Communication Engineering, Indian Institute of Technology Kharagpur, India

***Abstract*—In this paper the capacity and coverage of a cellular network is analyzed in the perspective of Grade of Service (GoS) constraint. The cellular networks considered, unlike GSM systems, allocate variable radio resource sizes to users even for constant bit-rate (CBR) traffic. The Grade of Service is the call blocking probability. It is shown that the blocking probability (P_b) varies with user distance from the Base Station (BS). It is also shown that the effective cell-radius varies for different services (bit-rates) given a certain GoS. Furthermore the variation of useful area of a cell along with offered load is shown for different services.**

If maximizing energy efficiency is one of the considerations in deploying a cellular network, then it is shown that the optimum transmit power for a cell layout is different when GoS is used as the criterion as compared to minimum throughput or minimum Signal to Interference plus Noise Ratio (SINR) at cell edge.

I. Introduction

The cellular systems such as GSM, cdmaOne etc. allocate fixed radio resource to users. Such networks are designed to carry traditional voice traffic. Modern mobile wireless networks primarily carry data. One of the important improvements introduced in these systems is the capability to allocate multiple radio resources to one user's session (call). Examples of such networks are 3G and 4G (Wimax, 3GPP LTE). In case of 4G networks even voice is carried as VoIP. The capacity of such networks is given in terms of throughput and maximum no. of VoIP users per MHz. The method used to estimate max VoIP capacity is to find the no. of users when present in the system that would allow at least 95% of the users to be satisfied [1] [2]. A satisfied user is one who successfully receives at least 98% of the transmitted packets. Whereas the measure of Erlangs is used to define the capacity of wireline telephone exchanges and earlier cellular systems, where fixed resources are allocated to users making a voice call. The capacity of the systems are defined as the maximum offered traffic intensity in Erlangs for which the the GoS, which is the blocking probability, is less than a target threshold.

The definition of Erlang capacity was developed originally for circuit switched telephone networks. It was assumed that in such networks a resource is allocated to the user for the entire call duration and is not shared with other users even when idle. On the other hand wireless systems are bandwidth (resource) limited due to the broadcast nature of the channel. Furthermore due to the bursty nature of the traffic the protocols are being designed which maximize system capacity by sharing the radio resource using dynamic Radio Resource Allocation (RRA) mechanism. The RRA takes into account the rate of information at the source and the channel condition of the user and then allocates as much resource as required to support the requested bit rate. Thus variable amount of radio resource is allocated to users which may further vary with time. It is not straightforward to derive the Erlang capacity of such systems. We build our work upon the framework in [3]–[6] to develop the Erlang capacity for Orthogonal Frequency Division Multiple Access (OFDMA) wireless cellular networks. These works show cellular capacity from a Grade of Service point of view, and study the cell-average blocking probability. Our objective is to analyze the GoS as a function of user distance from Base Station (BS). We also investigate the percentage ($\%$) area coverage in the perspective of GoS, and call it the useful service area of the cell. This parameter is defined in Section II. The classical definition of cell coverage area [7] is in terms of the received signal strength (and hence the Signal to Interference plus Noise Ratio (SINR)) being above a certain threshold. The variation of the useful service area of the cell as a function of the offered traffic load in Erlangs, the streaming rate and the cell radius is also presented in this work.

Energy efficiency of cellular networks has been studied as [8], [9] the cell spectral efficiency (SE) divided by the total power consumed by the BS. The cell SE is averaged over the cell and unnormalized by the area of the cell. While the above approach may optimize the transmitted power in a cell where only best effort traffic is present, it need not necessarily be optimal in a cell where streaming traffic predominates. Hence we propose to study Energy Efficiency from an Erlang Capacity (and associated GoS) viewpoint.

The rest of the paper is organized as follows. Erlang capacity is introduced and studies of blocking probability variation within a cell and useful service area variation with offered load etc. are presented in Section II. Area Erlang efficiency[1] (AEE) is introduced and Energy Efficiency from Erlang capacity point of view is presented in Section III. Finally the conclusions are presented in section IV.

II. Blocking Probability

The analysis framework and system parameters (Fig. 1) used are as given in [3]. Selected parameters are shown in Table I. Typical data rate for voice is $r = 12.2kbps$ [3], $64kbps$ is the rate for 3G Video Telephony, and $256kbps$ for good

[1]This term is defined in Section III.

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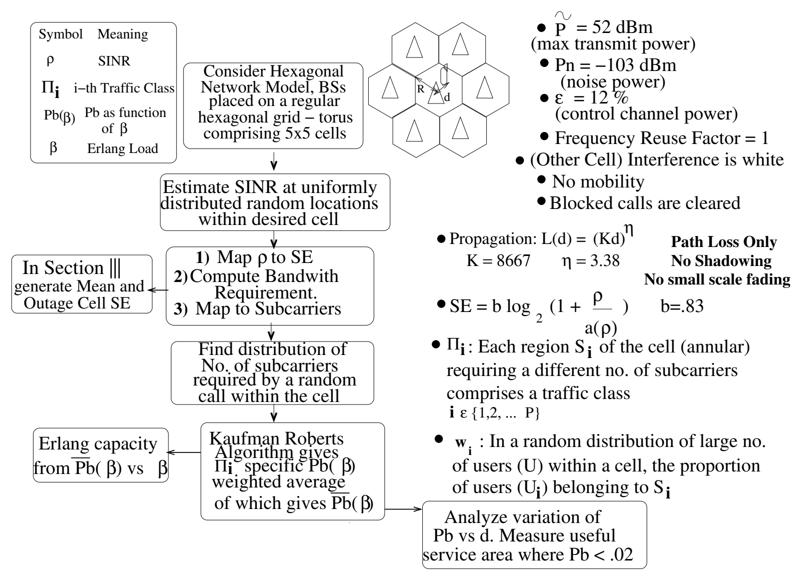

Fig. 1: Flow-chart and key parameters for Erlang Capacity, Useful Service Area of Cell and later Cell Spectral Efficiency Evaluation.

quality streaming video. Fading is accounted for by using a constant margin [10] which is detailed later. The Kaufman Roberts Algorithm (KRA) is used to evaluate the blocking probability in a cellular network employing OFDMA [3]. We use modifications of Shannon formula in order to get closer to real world system (eg. LTE) performance. Typically this sort of modification looks like [11]

$$C = b\log_2(1+\frac{\rho}{(a(\rho)*a_{fading})}), \quad (1)$$

where C is the spectral efficiency, b accounts for system overheads, ρ is the average SNR and $a(\rho)$ accounts for the gap between Shannon limit and performance of real world coding schemes which varies with ρ. Furthermore, to account for fading [10] we bring in another extra factor $a_{fading} = 3dB$. With these modifications, we do not need to use the exaggerated bandwidth requirement of $180kbps$ (instead we use $r = 12.2kbps$) per call [3], thereby enhancing the accuracy of our simulations. We also study the effect of variation in the no. of subcarriers (N_{subc}) (hence in Δ_f - the bandwidth per subcarrier) for a given system bandwidth W_{sys}. For most of the simulation results (unless otherwise specified) $N_{subc} = 128$ corresponding to $\Delta_f = 39.0625KHz$. Some results, such as the maximum voice traffic per cell (Erlangs) and the Area Spectral Efficiency as a function of cell radius are evaluated for both $N_{subc} = 128$, and 512 ($\Delta_f = 9.7656KHz$). The Energy Efficiency study uses $N_{subc} = 512$ for evaluating the Erlang capacity for voice calls, and maps it to an equivalent spectral efficiency as described in section III.

Streaming calls whose inter-arrival times to the network are i.i.d exponential random variables with rate λ (mean $\frac{1}{\lambda}$) are handled by the BSs. Duration of a call is exponentially distributed with mean $\frac{1}{\mu}$. Each call request originates from a point uniformly distributed within the geographical area of the cell (say S_C). Each call must be guaranteed its intrinsic bit-rate (r) for the duration of the call. The measure $\beta = \frac{\lambda}{\mu}$ is

TABLE I: Simulation Parameters

Parameter	Choice
$b, a(1)$	0.83, TU SISO [11]
r	$12.2kbps$ [3], $64, 256kbps$
P_b	$\leq .02$
W_{sys}	$5Mz$
N_{subc}	$128, 512$

called traffic demand density ($Erlang/m^2$). Depending on its distance from the BS, the SINR and hence SE achievable for a MS varies. In our model, since only signal attenuation due to propagation loss is considered, the no. of subcarriers required by a call is constant over a ring surrounding the BS. Hence we can classify the cell into annular regions (S_i, see Fig. 1) within which the blocking probability of a call is constant [3]. Each annular region comprises a traffic class (Π_i) with a specific GoS ($P_b^{\Pi_i}$) for a given load that can be evaluated by KRA. The average blocking probability $\overline{P_b}$ is defined as the weighted average of the ring-wise GoS, i.e. $\overline{P_b} = \sum_{i=1}^{P} w_i P_b^{\Pi_i}$, where $w_i = \frac{U_i}{U}$ is the fraction of users belonging to class Π_i, and P is the no. of distinct traffic classes. The useful cell area from GoS perspective is defined as

$$F_u(P_{b_{max}}) = \Pr(P_{b_{x,y}} < P_{b_{max}} | \overline{P_b} = P_{b_{max}}) \forall x, y \in S_C, \quad (2)$$

where x, y is an arbitrary point within the cell and $P_{b_{max}}$ is typically .02. The RHS of (2) is given by the probability that a randomly selected point in the cell falls within the circle composed of all the rings for which $P_b < P_{b_{max}}$. The Admission Probability is defined as $P_a = 1 - P_b$. The $p\%$ outage P_a is a value $P_a^{o,p}$ such that

$$\Pr(P_{a_{x,y}} < P_a^{o,p}) = \frac{p}{100}. \quad (3)$$

$P_a^{o,p}$ is thus the value of $P_{b_{max}}$ for which F_u is $(100 - p)\%$.

Fig. 2 shows $\overline{P_b}$ as a function of offered load ($\beta = \frac{\lambda}{\mu}$ where λ is call arrival rate, call arrival process being assumed Poisson, and $\frac{1}{\mu}$ is the mean call holding time) for various streaming data rates ($r = 12.2, 64, 256kbps$). The maximum load on the system for which $\overline{P_b} < \overline{P}_{b_{max}}$ (which we denote as β_{max} for $\overline{P}_{b_{max}} = .02$) is defined as the Erlang Capacity which varies with bit-rate and cell radius (R). The variation of β_{max} vs R for various rates is shown in Fig. 3. The Erlang capacity for $12.2kbps$ voice is improved by a maximum of $\approx 50\%$ by increasing N_{subc} from 128 to 512 because of the increased resource allocation granularity, whereas that for streaming video shows a much smaller change – so in Fig. 3 we plot β_{max} for voice (only) with both 128 as well as 512 subcarriers.

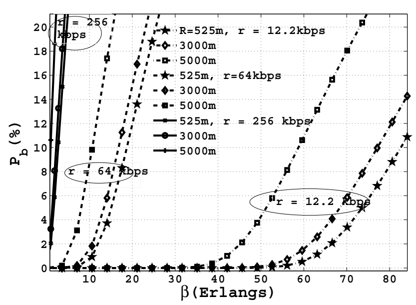

Fig. 2: Blocking probability as a function of offered load in Erlangs for streaming traffic ($r = 12.2, 64$, and $256kbps$).

It is observed that β_{max} stays more or less flat in the interference limited region ($R < 2km$) [3] and falls sharply for $R > 3km$. Also β_{max} decreases sharply with increase in r, having values < 1 for $r = 256kbps$. For supporting these rates Multiple Input Multiple Output (MIMO) and other spectral efficiency enhancement techniques become invaluable. For values of $R < 2km$, $\beta_{max} \approx 66$ with 128 subcarriers and $\beta_{max} \approx 93$ with 512 subcarriers. This is significantly higher than the value ≈ 24 in [3] because of the constant SNR penalty of $\approx 11.7dB^2$ assumed at all average SNRs instead of a penalty that varies with ρ between 4 to $11dB$ as in LTE spectral efficiency [11].

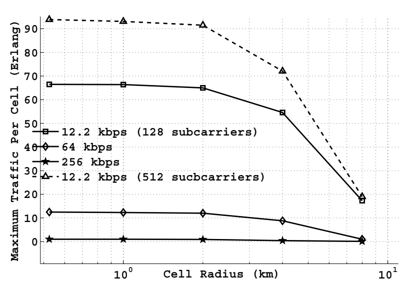

Fig. 3: Maximum traffic load (Erlangs) per Cell vs Cell Radius for various streaming traffic rates ($r = 12.2, 64$, and $256kbps$).

[2] $12.2e3 = W_{sys} \log_2(1 + \rho_{12}); 180e3 = W_{sys} \log_2(1 + \rho_{180})$ SNR penalty = $\rho_{180} - \rho_{12} \approx 11.7dB$

Fig. 4 shows the no. of subcarriers $N_{subcreq}$ needed for various r as a function of d_{BS-MS}, the distance of the MS from the BS within a cell of radius $R = 525m$ (Figs. 5, 6, 7 also assume this value of R). $N_{subcreq}$ varies between 1–4 for $12.2kbps$ because of its low bandwidth requirement. For the higher values of r such as $256kbps$, $N_{subcreq}$ increases steeply with d_{BS-MS} especially for higher values of d_{BS-MS} where the fall in SINR is accelerated by the combined effect of increasing distance from the desired BS and decreasing distance relative to (dominant) interfering BSs. Figure 5 shows P_b (evaluated by KRA, which assigns different P_b to each class of calls with a different $N_{subcreq}$, $\overline{P_b}$ being their weighted average) as a function of d_{BS-MS} for $\beta = \beta_{max}$. To study the impact of higher P_b at and near the cell-edge, two approaches are possible:

1) Study the Jain Fairness measure [12] for the Call Admission Probability metric ($P_a = 1 - P_b$) defined as:

$$F_a(P_a) = \frac{1}{1 + \frac{V(P_a)}{E^2(P_a)}}, \tag{4}$$

where $V(P_a)$ is the variance of P_a, and $E(P_a)$ is its expectation.

2) Compute the $p\%$ outage P_a ($p = 5, 10$) and also F_u.

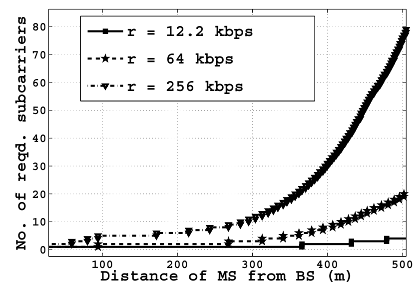

Fig. 4: Subcarrier Requirement ($N_{subcreq}$) for Cell Radius of $525m$ and various Streaming Data Rates as function of distance from BS.

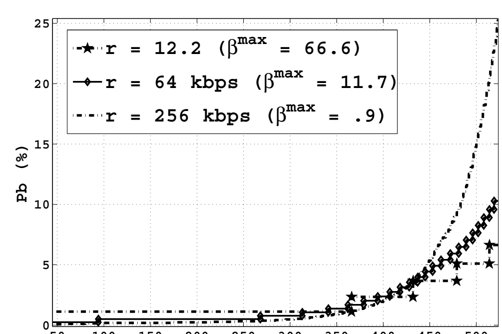

Fig. 5: Blocking Probability (P_b) for Cell Radius of $525m$ ($\overline{P_b} = .02$) and various Streaming Data Rates as function of distance from BS.

Figure 6 shows the CDF of P_a when $\overline{P_b} = .02$. It is seen that for all rates, F_u is less than 80%, i.e. in at least 20% of the possible locations within the cell, $P_a < 98\%$. $F_a(P_a), F_u$ and

$P_a^{o,p}$ for various streaming rates are listed in Table II for $R = 500m$, and $3000m$. It is observed from these tables that though $F_a(P_a)$ decreases with increasing r, it is always fairly close to unity. $P_a^{o,p}$ values, however, are much lower than 98% with the minimum being 91.82% for $p = 5$ ($r = 256kbps, R = 3km$) which corresponds to a P_b of 8.18% and 94% for $p = 10$ ($r = 256kbps, R = 3km$) - which corresponds to a P_b of 6%. $P_a^{o,p}$ decreases with increasing data rate. F_u on the other hand increases with increasing data rate, being the lowest for $12.2kbps$ (.55 for $R = 3km$) and the highest for $256kbps$ (.72 for $R = 3km$). The increase of F_u with increasing rates is because the final rise of P_b vs. d_{BS-MS} (Fig. 5) is steeper for higher rates, hence to maintain the same $\overline{P_b}$, P_b must cross $\overline{P_b}$ later (i.e. at a higher value of d_{BS-MS}). Note that the β_{max} is significantly lower for higher data rates.

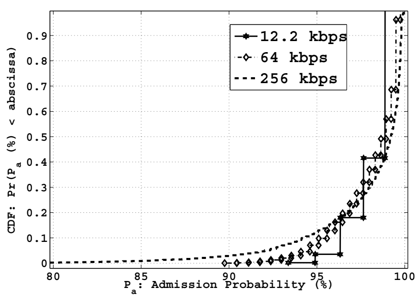

Fig. 6: CDF of Probability of Call Admission (P_a) for Cell Radius of $525m$ ($\overline{P_b} = .02$) and various Streaming Data Rates.

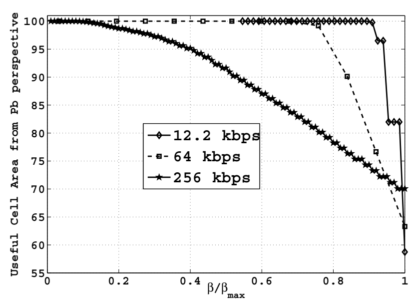

Fig. 7: Useful Cell Area (in P_b sense) for a Cell Radius of $525m$ vs. ratio of β and β_{max} - cell dilation at low load ($\overline{P_b} = .02$ when $\frac{\beta}{\beta_{max}} = 1$).

An interesting question is what happens to F_u when the load in the cell is less than β_{max}. To answer this, Fig 7 contains a plot of F_u vs $\frac{\beta}{\beta_{max}}$ (for $\beta < \beta_{max}$) and it is seen that F_u increases as $\frac{\beta}{\beta_{max}}$ decreases - a phenomenon that can be visualized as the useful cell expanding as the Erlang load decreases, similar to the concept of cell size breathing [13]. This also leads to a possible solution for increasing F_u and $P_a^{o,p}$ which is to use a lower value of $\overline{P}_{b_{max}}$ while analyzing network capacity for dimensioning and accordingly increasing surface density of rolled out BSs so as to reduce β per cell. Overall, Jain Fairness of P_a seems to be inadequate in analyzing P_b variations over the cell.

TABLE II: Call Admission Probability Fairness ($Fa(P_a)$), Useful Cell Area from GoS perspective ($F_u = \Pr(P_b < .02|\overline{P_b} = .02)$), and Outage ($P_b > .02$) Rates for Streaming Calls originating from Random Locations within a Cell (Radius $R_1 = 0.525km, R_2 = 3km$) when $\beta = \beta_{max}$

r $(kbps)$	$Fa(P_a)$ $(0 < Fa() < 1)$		F_u		p% Outage $P_a(\%)$ $(p = 5, 10)$	
	R_1	R_2	R_1	R_2	R_1	R_2
12.2	.9999	.9999	.58	.55	$96.3, 96.3$	$95.26, 96.58$
64	.9997	.9997	.63	.64	$94.6, 95.6$	$94.22, 95.17$
256	.9991	.9990	.7	.72	$92, 94.4$	$91.82, 94.4$

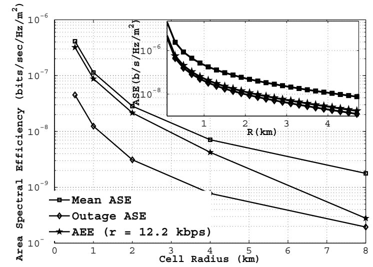

Fig. 8: Mean and Outage Area Spectral Efficiency and Area Erlang ($r = 12.2kbps$) Efficiency as a function of Cell Radius (512 subcarriers, inset - 128 subcarriers).

III. Energy Efficiency

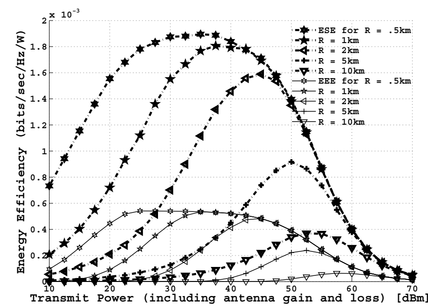

Fig. 9: Energy (Spectral, voice-Erlang) Efficiency as a function of Cell Radius (512 subcarriers).

TABLE III: Transmit Power to achieve Peak Network (Energy) Efficiency - Mean Spectral (ESE) and Erlang (EEE) Efficiency terms

R (km)	P_{tx}^m at ESE_{max} (dBm)	P_{tx}^e at EEE_{max} (dBm)	% EEE loss at P_{tx}^m	Extra Power for ESE_{max} (dB)
0.5	35	25	1.1	10
1	37.5	35	0.92	2.5
2	45	45	0	0
5	50	52.5	7	-2.5
10	52.5	57.5	41.5	-5

The analysis framework and system parameters (see Fig. 1) used for the study of energy efficiency are same as that of [9]. The classical Area Spectral Efficiency (ASE) considers the cell-average of the Shannon spectral efficiency and ignores the impact of QoS constrined transmissions; we extend it by directly incorporating Erlang capacity into the definition [14]. Area Erlang efficiency (Ω) for streaming rate r is defined as:

$$\Omega(r) = \frac{\beta_{max}(r)r}{W_{sys}A_{cell}} \tag{5}$$

where β and β_{max} are as defined before, r is the bit rate of each streaming user ($bits/sec$), and A_{cell} is the cell area $= \frac{3\sqrt{3}}{2}R^2$, R being the cell radius of the hexagonal cell. Area Erlang efficiency (AEE) is thus a (rate-specific) measure of spectral efficiency per unit area inferred from the Erlang capacity. Fig. 8 shows the variation of mean ($\overline{A_e}$) and outage (A_e^o) area spectral efficiency and area Erlang efficiency Ω as a function of cell radius (inset considers AEE with 128 subcarriers while main plot has AEE with 512 subcarriers). The *Outage ASE* in the plot implies a value of SE (A_e^o) such that

$$\Pr(\omega(x,y) < A_e^o) = 0.1; \tag{6}$$

where $\omega(x,y)$ is the LTE spectral efficiency [11] at location defined by coordinates (x,y) in the Argand plane (within the cell of interest). Mean ASE ($\overline{A_e}$) is defined as usual [15] but with modified Shannon formula [11]. Fig. 8 (inset) indicates that for $N_{subc} = 128$, Ω is much closer to the A_e^o than to $\overline{A_e}$. The rate of fall of $\overline{A_e}$, A_e^o and that of Ω are similar. At high values of cell radius, A_e^o and Ω are almost identical. Fig. 8 (main) shows that for $N_{subc} = 512$, Ω is closer to $\overline{A_e}$ for lower cell radii, and to A_e^o for higher cell radii, decreasing at a rate faster than both as R_{cell} increases.

The plot of Energy Efficiency (Fig 9) contains on its y-axis the ratio Energy Spectral Efficiency $(ESE) = \frac{\overline{A_e}A_{cell}}{P_{BS}}$[3] and the transmit power lever (P_{tx}, including antenna gain/loss) in the x-axis. Energy Erlang Efficiency (EEE) is defined similarly, but with Ω in the numerator in place of $\overline{A_e}$. It is observed that for low and high values of R (eg. $= .5km$ and $10km$), the peaks of ESE and EEE occur at different values of P_{tx}. For higher values of R, the peak for ESE requires less transmit power level compared to EEE, whereas for lower values of R, EEE peak requires a lower transmit power level. Thus operating at EEE peak has the advantage of high Erlang efficiency at a lower (by $10dB$) transmit power level for small cell radii ($0.5kms$), resulting in lower capital expenses. Moreover, from Table III we observe that if the Base Station transmit power is set in order to achieve peak ESE (which might be a straightforward conclusion from [9]), then a penalty in terms of Erlang capacity will be paid for $R \geq 5km$ varying from 1% at $R = .5km$ to $\approx 42\%$ at $R = 10km$. Thus the network designer cannot simply operate by traditional energy efficiency if he needs to maximize it from a Erlang capacity perspective.

[3] $P_{BS} = cp + d, c = 21.45, d = 354.44W, p = P_{tx} - G; P_{BS}$ being the total power consumed at the Base Station, $G(= 12dB)$ being the antenna gain - see [9]. A difference from the Energy Efficiency plot in [9] is that we have not multiplied by W_{sys} in the numerator, so ESE has units of bits/sec/Hz/W instead of bits/sec/W.

IV. Conclusion

We have built upon the initial work on Erlang capacity of wireless cellular networks with spectral efficiency models that are closer to real world system performance. We have shown the choice of subchannel bandwidth strongly influences Erlang capacity, and needs to be small enough to permit high resource allocation granularity and hence high Erlang capacity. We have analyzed the distance dependence of call blocking probability and shown that P_b may significantly exceed 2% at the cell-edge even when the cell-average $\overline{P_b}$ is less than this limit. We have extended the classical concept of cell coverage area from GoS viewpoint as useful service area (F_u) of the cell. We have shown that F_u decreases as the load on the cell increases, a concept useful in dimensioning, especially if F_u is significantly less than 100%. We have studied Energy Efficiency from Erlang Capacity perspective. We have shown that use of Area Erlang Efficiency (AEE), which extends the classical concept of Area Spectral Efficiency to account for QoS constrained transmissions, leads to different settings for optimal transmit power of a BS. We have observed that for cells with radii $R > 5km$, the Energy Efficiency from Erlang Capacity perspective may be sacrificed by 7 – 40% if the BS transmit power is set to the optimum from ASE viewpoint.

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