An Iterative Water-filling Based Resource Allocation Scheme in OFDMA Systems for Energy Efficiency Optimization

Zhiyong Feng*, Zhiqing Wei†, Tianping Shuai‡, Qixun Zhang*, Rong Li†
†*Key Laboratory of Universal Wireless Communications, Ministry of Education
Wireless Technology Innovation Institute (WTI)
‡Beijing University of Posts and Telecommunications
Beijing, P.R.China, 100876
email:*{fengzy, zhangqixun}@bupt.edu.cn, †‡{zhiqingwei, tpshuai, lirong0722}@gmail.com

***Abstract*—In this paper, the subcarrier and power allocation problem for energy efficiency maximization is addressed, which is different from traditional throughout maximization. Lagrangian dual decomposition (LDD) is applied in this problem and a multilevel water-filling for power allocation is derived. But the water-filling in this paper is a transcendental equation, which is different from traditional water-filling form. To solve this equation, fixed point iteration is applied. Besides, a sufficient condition for the existence of the fixed point is derived and joint resource allocation algorithms are designed. Finally, numerical results verify our work and the energy efficiency is improved compared with the capacity maximization scheme.**

***Index Terms*—Green Communications; Energy Efficiency; Power Allocation; OFDMA**

I. Introduction

The data rate that wireless communication systems can provide is growing rapidly in the past few decades, and the energy consumption has also increased simultaneously. Statistical data shows that the amount of transmitted data increases approximately by 10% per year for 5 years, which corresponds to an increase of the associated energy consumption by 20% per year [1]. And currently, 3% of the world-wide energy is consumed by the ICT (Information and Communication Technology) infrastructure which causes about 2% of the world-wide CO_2 emissions [2]. Thus, the concept of green communications is proposed to reduce the energy consumption of communication infrastructure and network. To optimize the energy consumption, more attention should be paid on the access segment of the networks, mainly because it consists of the largest number of elements and hence accounts for the most energy consumption [3], and the access segments are mainly network nodes, such as BSs, Node-Bs, etc.

Energy consumption can be reduced by resource allocation [5]. For example, the MA (Margin Adaptive) service in OFDMA, for which the performance margin is maximized while satisfying data rate and BER requirements of the terminals, and can reduce power consumption accordingly. The other is the RA (Rate Adaptive) service, for which the throughput is maximized subject to the constraints on the fixed total transmission power at the base station and bit error rate (BER) required by each terminal. As RA always maximize the capacity, and the performance margin is minimized. So there exists redundant power that is wasted. On the other hand, MA maximize the performance margin, which will reduce the QoS of terminals in the time-varying wireless environment.

Energy efficiency is defined in many versions for different applications, in [5], Energy efficiency of one cellular is defined as the cell's throughput divided by the total power. And [5] models a problem which maximizes the energy efficiency. But [5] didn't solve the model directly, but converted to a RA problem, which is inappropriate, for the energy efficiency maximization is actually a $\frac{\mathrm{RA}}{\mathrm{MA}}$ problem, as it always maximizes network capacity, and simultaneously the power consumption is minimized.

As to resource allocation problem, decomposability structures are widely used for network utility maximization [6]. In multicarrier systems, with dual methods, the LDD (Lagrangian dual decomposition) is developed to maximize network utility subject to resource constraints. And in multicarrier systems, time-sharing condition [7] says that the duality gap is zero even when the optimization problem is not convex. Thus, LDD can be used in multicarrier systems when the time-sharing condition is satisfied. Generally, system with a sufficient large number of subcarriers satisfies the time-sharing condition.

In this research, we address the resource allocation problem in the viewpoint of energy efficiency maximization, and a water-filling form is derived. As the water-filling in this paper is a transcendental equation, the fixed point theory is applied to solve the equations and to prove the existence of fixed point. Finally a sufficient condition for the convergence of the iterations in solving the equations is derived and the algorithms to find the optimal resource allocation scheme are designed. The paper is organized as follow: problem formulation is presented in section II, the sufficient condition for convergence and joint resource allocation algorithms are given in section III, section IV illustrates the numerical simulation results and section V concludes our research.

II. Problem Formulation

In an OFDMA system with K users and N subcarriers. Define the channel gain of user k on subcarrier n as $H_{k,n}$, the total noise power spectral density as $N_{k,n}$, and the allocated power of user k on subcarrier n as $p_{k,n}$. Assume that M-QAM (Multiple Quadrature Amplitude Modulation) is applied with a BER requirement, then the signal to noise ratio (SNR) is $g_{k,n} = \frac{|H_{k,n}|^2}{N_{k,n}\Gamma}$, where $\Gamma = \frac{-\ln(5BER)}{1.5}$ [8] and is a constant SNR gap for QAM modulation. The data rate of user k on subcarrier n is $r_{k,n} = B\log_2(1+p_{k,n}g_{k,n})$, where B is the bandwidth of each subcarrier. An indicator variable $w_{k,n}=1$ if subcarrier n is allocated to user k, and $w_{k,n}=0$ otherwise. The total data rate of user k is

$$R_k = B\sum_{n=1}^{N}\log_2(1+p_{k,n}g_{k,n}) \tag{1}$$

In this paper, energy efficiency is considered rather than merely system capacity. The energy efficiency is defined as follow [5]:

$$b = \frac{\text{transmission rate}}{\text{power consumption}}\ \text{bit/s/W} \tag{2}$$

Thus, the energy efficiency of user k is defined as:

$$E_k = \frac{R_k}{\sum\limits_{n=1}^{N} p_{k,n}} \tag{3}$$

With the decision variables $w_{k,n}$ and $p_{k,n}$, an optimization problem can be formulated as maximizing the sum energy efficiency of each user:

$$\max_{w_{k,n},p_{k,n}} \sum_{k=1}^{K} E_k = \sum_{k=1}^{K} \frac{\sum\limits_{n=1}^{N} w_{k,n}B\log_2(1+p_{k,n}g_{k,n})}{\sum\limits_{n=1}^{N} p_{k,n}} \tag{4}$$

subject to:

$$\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \le P_T \tag{5}$$

$$\sum_{k=1}^{K} w_{k,n} \le 1, \forall n \tag{6}$$

$$\sum_{n=1}^{N} w_{k,n}B\log_2(1+p_{k,n}g_{k,n}) \ge R_{k,\min}, \forall k \tag{7}$$

$$p_{k,n} \ge 0, w_{k,n} \ge 0, \forall k,n \tag{8}$$

Where equation (5) is the constraint of transmit power, the total allocated power can not exceed the limit power of BS transmitter P_T. Equation (6) says that one subcarrier can be allocated to one user at most. And equation (7) is the minimum data rate demand for each user. For simplicity, denote $\mathbf{w} = [w_{k,n}]_{K\times N}$, $\mathbf{p} = [p_{k,n}]_{K\times N}$. We note that although $w_{k,n}$ takes the value of either 0 or 1, it can be relaxed to a real number in $[0,1]$ to make the problem tractable [8].

III. Joint Resource Allocation Algorithm

The most significant difference between the traditional resource allocation problem in OFDMA systems and ours is the objective function. As the energy efficiency is considered, the denominator of objective function contains the decision variables, which brings much more difficulties. Since the resource allocation approaches based on LDD (Lagrangian dual decomposition) are widely used in optimization problems with multiple constraints, we proposed an iterative water-filling (IWF) scheme based on LDD to fulfill resource allocation.

The Lagrangian function for the optimization problem can be formed as follow:

$$\begin{aligned}
&L(\mathbf{w},\mathbf{p},\lambda,\mathbf{u},\pi)\\
&= \sum_{k=1}^{K}\left(\left(\sum_{n=1}^{N} w_{k,n}r_{k,n}\right)\Big/\sum_{n=1}^{N} p_{k,n}\right)\\
&-\lambda\left(\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} - P_T\right) - \sum_{n=1}^{N} u_n\left(\sum_{k=1}^{K} w_{k,n} - 1\right)\\
&+\sum_{k=1}^{K}\pi_k\left(\sum_{n=1}^{N} w_{k,n}r_{k,n} - R_{k,\min}\right)
\end{aligned}$$

Where λ, $\mathbf{u} = [u_n]_{1\times N}$ and $\pi = [\pi_k]_{1\times K}$ are non-negative Lagrangian multipliers for the constraints. And λ and π_k can be interpreted as power price and bidding price respectively. The lagrangian dual form for the primal problem is as follow:

$$\min_{\lambda,\mathbf{u},\pi} D(\lambda,\mathbf{u},\pi) \tag{9}$$

subject to:

$$\lambda \ge 0, \mathbf{u} \succ= 0, \pi \succ= 0 \tag{10}$$

Where the dual function is as follow:

$$\begin{aligned}
D(\lambda,\mathbf{u},\pi) &= \max_{\mathbf{w},\mathbf{p}} L(\mathbf{w},\mathbf{p},\lambda,\mathbf{u},\pi)\\
&= \max_{\mathbf{w},\mathbf{p}} \sum_{k=1}^{K}\left(\begin{array}{c}
\left(\sum\limits_{n=1}^{N} w_{k,n}r_{k,n}\right)\Big/\sum\limits_{n=1}^{N} p_{k,n}\\
-\lambda\sum\limits_{n=1}^{N} p_{k,n} - \sum\limits_{n=1}^{N} u_n w_{k,n}\\
+\pi_k\left(\sum\limits_{n=1}^{N} w_{k,n}r_{k,n} - R_{k,\min}\right)
\end{array}\right)\\
&+\lambda P_T + \sum_{n=1}^{N} u_n
\end{aligned} \tag{11}$$

Using the decomposition method [6], the problem can be separated into two levels, the subproblems (lower level) and the dual problem (higher level). At the lower level, user k deals with its own subproblem, in which the dual problem is decoupled as follow:

$$\begin{aligned}&\max_{\mathbf{w},\mathbf{p}} D_k(\lambda,\mathbf{w},\mathbf{p},\mathbf{u},\pi)\\&=\frac{\sum_{n=1}^{N} w_{k,n} r_{k,n}}{\sum_{n=1}^{N} p_{k,n}}-\lambda\sum_{n=1}^{N} p_{k,n}\\&-\sum_{n=1}^{N} u_n w_{k,n}+\pi_k\left(\sum_{n=1}^{N} w_{k,n} r_{k,n}-R_{k,\min}\right)\end{aligned} \quad (12)$$

subject to:

$$\mathbf{w}\succeq 0,\mathbf{p}\succeq 0 \quad (13)$$

Finally, the dual problem is as follow:

$$\min_{\lambda,\mathbf{u},\pi}\sum_{k=1}^{K} D_k(\lambda,\mathbf{w},\mathbf{p},\mathbf{u},\pi)+\lambda P_T+\sum_{n=1}^{N} u_n \quad (14)$$

subject to:

$$\lambda\geq 0,\mathbf{u}\succeq 0,\pi\succeq 0 \quad (15)$$

The subproblem for user k is solved at first. According to Karush-Kuhn-Tucker (KKT) conditions, the optimal subcarrier allocation $w_{k,n}$ and power allocation $p_{k,n}$ satisfy the following conditions:

$$\begin{aligned}&\mathrm{c1}: \left.\frac{\partial D_k(\lambda,\mathbf{u},\pi)}{\partial w_{k,n}}\right|_{w_{k,n}}=0\\&\mathrm{c2}: \left.\frac{\partial D_k(\lambda,\mathbf{u},\pi)}{\partial p_{k,n}}\right|_{p_{k,n}}=0\\&\mathrm{c3}: \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}\leq P_T\\&\mathrm{c4}: \sum_{k=1}^{K} w_{k,n}\leq 1,\forall n\\&\mathrm{c5}: \sum_{n=1}^{n} r_{k,n}\geq R_{k,\min},\forall k\\&\mathrm{c6}: \lambda\geq 0\\&\mathrm{c7}: \lambda\left(\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}-P_T\right)=0\\&\mathrm{c8}: u_n\left(\sum_{k=1}^{K} w_{k,n}-1\right)=0,\forall n\\&\mathrm{c9}: \pi_k\left(\sum_{n=1}^{N} w_{k,n} r_{k,n}-R_{k,\min}\right)=0,\forall k\end{aligned} \quad (16)$$

By solving c2, an optimal power allocation is calculated by a similar water-filling fashion:

$$p_{k,n}=\left[\frac{B\left(P_k+\pi_k P_k^2\right)}{(\ln 2)\left(R_k+\lambda P_k^2\right)}-\frac{1}{g_{k,n}}\right]^+ \quad (17)$$

Where $P_k=\sum_{n=1}^{N} p_{k,n}$, and $R_k=\sum_{n=1}^{N} r_{k,n}$. Equation (17) is a transcendental equation and can be solved by fixed point iteration. By solving c1, equation (18) is derived:

$$\left.\frac{\partial D_k(\lambda,\mathbf{w},\mathbf{p},\mathbf{u},\pi)}{\partial w_{k,n}}\right|_{w_{k,n}}=\left(\frac{1}{P_k}+\pi_k\right) r_{k,n}-u_n \quad (18)$$

We note that $D_k(\lambda,\mathbf{w},\mathbf{p},\mathbf{u},\pi)$ is a linear function of $w_{k,n}$, $w_{k,n}$ is a variable either 0 or 1. When $\partial D_k/\partial w_{k,n}>0$, D_k increases with $w_{k,n}$ and $w_{k,n}=1$ maximizes the objective function D_k. Similarly, when $\partial D_k/\partial w_{k,n}<0$, $w_{k,n}=0$ maximizes D_k. So the conclusion as equation (19) is got.

$$w_{k,n}=\begin{cases}=0 & (\frac{1}{P_k}+\pi_k) r_{k,n}<u_n\\ \in(0,1) & (\frac{1}{P_k}+\pi_k) r_{k,n}=u_n\\ =1 & (\frac{1}{P_k}+\pi_k) r_{k,n}>u_n\end{cases} \quad (19)$$

Equation (19) implies that subcarrier n tends to be allocated to user with maximum $(\frac{1}{P_k}+\pi_k) r_{k,n}$. Finally, the subcarrier and power allocation scheme is organized in Theorem 1 and 2, which are solutions of the primal problem.

Theorem 1. *To maximize network capacity, subcarrier n should be allocated to user k^* which satisfies*

$$k^*=\arg\max_k\left(\frac{1}{P_k}+\pi_k\right) r_{k,n} \quad (20)$$

Theorem 2. *To maximize network capacity, power allocated to user k^* in subcarrier n is*

$$p_{k^*,n}=\left[\frac{B\left(P_{k^*}+\pi_{k^*} P_{k^*}^2\right)}{(\ln 2)\left(R_{k^*}+\lambda P_{k^*}^2\right)}-\frac{1}{g_{k^*,n}}\right]^+ \quad (21)$$

Conditions c3, c5, c7 and c9 imply that Lagrangian multipliers λ and $\pi_k,\forall k$ converge to 0 when power and rate allocated to each user is not tightly fit the constraints.

As mentioned previously, π_k can be explained as bidding price, when investigating the power and subcarrier allocation results in equation (17) and (20), we note that when user k holds a higher bidding price π_k, it's more likely that subcarrier and power are allocated to user k. Besides, λ can be explained as the price of unit power. the amount of power allocated to user k is related with π_k/λ, which fits the basic economic laws, which is "amount $=\frac{\text{pay money}}{\text{unit price}}$". When $P_k\to\infty$, the result as equation (22) reveals, which is an evidence for the price explanation.

$$\begin{aligned}\lim_{P_k\to\infty} p_{k,n}&=\lim_{P_k\to\infty}\left[\frac{B\left(P_k+\pi_k P_k^2\right)}{(\ln 2)\left(R_k+\lambda P_k^2\right)}-\frac{1}{g_{k,n}}\right]^+\\&=\left[\underbrace{\frac{B}{(\ln 2)}}_{\text{constant}}\cdot\underbrace{\frac{\pi_k}{\lambda}}_{\frac{\text{bidding price}}{\text{power price}}}-\frac{1}{g_{k,n}}\right]^+\end{aligned} \quad (22)$$

As to equation (17), the fixed point theory can help to find the conditions for convergence in the iterations for $p_{k,n}$. But Jacobi matrix of the equations is required, and the spectral radius of which is the basis for the proof of convergence. But Jacobi matrix is in dimensions of $KN\times KN$, and the solution for the spectral radius is impossible. So we simplify the analysis and give a sufficient condition as Theorem 3.

Theorem 3. *For user $k, \forall k$, denote $P_{-k} = \sum_{j=1, j\neq k}^{K} P_j$ and $R_{-k} = \sum_{j=1, j\neq k}^{K} R_j$, then the equations converge if:*

$$\pi_k R_{-k} \leq \lambda P_{-k}, \forall k \tag{23}$$

Proof: For user k and subcarrier n, define a function ϕ as follow:

$$p_{k,n} = \phi(p_{k,n}, p_{-k,n}) = \left[\frac{B\left(P_k + \pi_k P_k^2\right)}{(\ln 2)\left(R_k + \lambda P_k^2\right)} - \frac{1}{g_{k,n}}\right]^+$$

Where $p_{-k,n} = \{p_{j,n} | j \neq k\}$. The partial derivative of ϕ is as follow:

$$\frac{\partial \phi(p_{k,n}, p_{-k,n})}{\partial p_{k,n}} = \frac{B}{\ln 2} \frac{(1+2\pi_k P_k)\left(R_k + \lambda P_k^2\right) - \left(P_k + \pi_k P_k^2\right)\left(\frac{B g_{k,n}}{(\ln 2)\left(1+p_{k,n} g_{k,n}\right)} + 2\lambda P_k\right)}{\left(R_k + \lambda P_k^2\right)^2}$$

We find that $\left.\frac{\partial \phi(p_{k,n}, p_{-k,n})}{\partial p_{k,n}}\right|_{p_{k,n}\to\infty} = 0$, and the partial derivative of ϕ when $p_{k,n} \to 0$ is as follow:

$$\left.\frac{\partial \phi(p_{k,n}, p_{-k,n})}{\partial p_{k,n}}\right|_{p_{k,n}\to 0} = \frac{(1+\pi_k R_{-k})\left(R_{-k} - \frac{B g_{k,n} P_{-k}}{\ln 2}\right) + P_{-k}(\pi_k R_{-k} - \lambda P_{-k})}{\left(R_{-k} + \lambda P_{-k}^2\right)^2} \tag{24}$$

In equation (24), $R_{-k} - \frac{B g_{k,n} P_{-k}}{\ln 2} < 0$. **If** $\pi_k R_{-k} - \lambda P_{-k} \leq 0$, **then** $\left.\frac{\partial \phi(p_{k,n}, p_{-k,n})}{\partial p_{k,n}}\right|_{p_{k,n}\to 0} < 0$. With $\phi(p_{k,n}, p_{-k,n})|_{p_{k,n}\to 0} > 0$ and $\phi(p_{k,n}, p_{-k,n})|_{p_{k,n}\to\infty} = \left[\frac{B}{\ln 2}\frac{\pi_k}{\lambda} - \frac{1}{g_{k,n}}\right]^+$, the fixed point $p_{k,n}^*$ does exists which satisfies the equation $p_{k,n}^* = \phi(p_{k,n}^*)$. Further, as $\left.\frac{\partial \phi(p_{k,n}, p_{-k,n})}{\partial p_{k,n}}\right|_{p_{k,n}\to\infty} = 0$, then $\exists p'_{k,n}$, when $p_{k,n} > p'_{k,n}$, $\left|\frac{\partial \phi(p_{k,n}, p_{-k,n})}{\partial p_{k,n}}\right| < 1$. Thus, when $p_{k,n}^* \in [p'_{k,n}, +\infty]$, $p_{k,n}^*$ can be found through iterations according to Lemma 1. When $p_{k,n}^* \in [0, p'_{k,n}]$, the nearly exact solution of $p_{k,n}^*$ can be found, as we note that the solution space is restricted to a limited interval, which means that the solution is controllable and the exact solution can be found by other approach, such as bisection method. ■

Lemma 1. *The locally convergence of finding a fixed point:*

Denote the fixed point of $\phi(p_{k,n}, p_{-k,n})$ as $p_{k,n}^*$. And $\phi(p_{k,n}, p_{-k,n})$ is continuous in a neighborhood of $p_{k,n}^*$. Then the iterations are locally convergent if:

$$\left|\frac{\partial \phi\left(p_{k,n}, p_{-k,n}\right)}{\partial p_{k,n}}\right|_{p_{k,n}^*} < 1 \tag{25}$$

As to the solution of Lagrangian multipliers λ, u_n and π_k, for the complexity of the dual problem, the closed form is not derived. Generally, subgradient method as equations (26) (27) is used in the iteration process for the optimal λ and π_k to solve the dual problem.

$$\begin{aligned}\lambda(t+1) &= \left[\lambda(t) - \alpha(t)\frac{\partial L}{\partial \lambda}\right]^+ \\ &= \left[\lambda(t) - \alpha(t)\left(P_T - \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}\right)\right]^+\end{aligned} \tag{26}$$

$$\begin{aligned}\pi_k(t+1) &= \left[\pi_k(t) - \beta(t)\frac{\partial L}{\partial \pi_k}\right]^+ \\ &= \left[\pi_k(t) - \beta(t)\left(\sum_{n=1}^{N} w_{k,n} r_{k,n} - R_{k,\min}\right)\right]^+\end{aligned} \tag{27}$$

Where $\alpha(t)$ and $\beta(t)$ are sufficiently small step size and t is the iteration index. The step size can be either fixed or adaptive. To avoid that a subcarrier is occupied by two users simultaneously, a random number between $r_{k^*,n}/P_{k^*} + \pi_k$ and $r_{k',n}/P_{k'} + \pi_k$ is assigned to u_n, with k^* an optimal user and k' a suboptimal user for subcarrier n in equation (20). In each iteration, the dual problem informs Lagrangian multipliers λ, u_n and π_k to the subproblems, and in the reverse direction subproblems reported the resource allocation results to the dual problem. Based on equations (17), (20), (26) and (27), the algorithm is designed as Algorithm 1. To accelerate the convergence speed, the instantaneous data rate R_k and power P_k in equations (17) (20) are substituted with the average rate $\bar{R}_k$ and average power $\bar{P}_k$ respectively, which is calculated as follow, with t the iteration index, and M is the length of moving average window, which is an integer.

$$\begin{aligned}\bar{R}_k(t+1) &= \left(1 - \frac{1}{M}\right) R_k(t+1) + \frac{1}{M}\bar{R}_k(t) \\ \bar{P}_k(t+1) &= \left(1 - \frac{1}{M}\right) P_k(t+1) + \frac{1}{M}\bar{P}_k(t)\end{aligned}$$

Algorithm 1 Subgradient based iterative water-filling (IWF) algorithm for energy efficient subcarrier and power allocation

1: Initialize λ with a larger number and $\pi_k, \forall k$ with a smaller number to satisfy the equation (23). Initialize ε and τ with sufficient small positive number, and iteration index $t = 1$.
2: **repeat**
3:     **repeat**
4:         Allocate power $p_{k,n}, \forall k, n$ by equation (17). Calculate $r_{k,n}, \forall k, n$ and $\bar{R}_k, \forall k$.
5:     **until** $\bar{R}_k, \forall k$ converges
6:     $\forall n$, select the optimal user k^* by equation (20). Assign a random number between $r_{k^*,n}/P_{k^*} + \pi_k$ and $r_{k',n}/P_{k'} + \pi_k$ to u_n.
7:     Update λ and $\pi_k, \forall k$ by equation (26) (27).
8: **until** $|\lambda(t+1) - \lambda(t)| \leq \varepsilon$ and $|\pi_k(t+1) - \pi_k(t)| \leq \tau, \forall k$

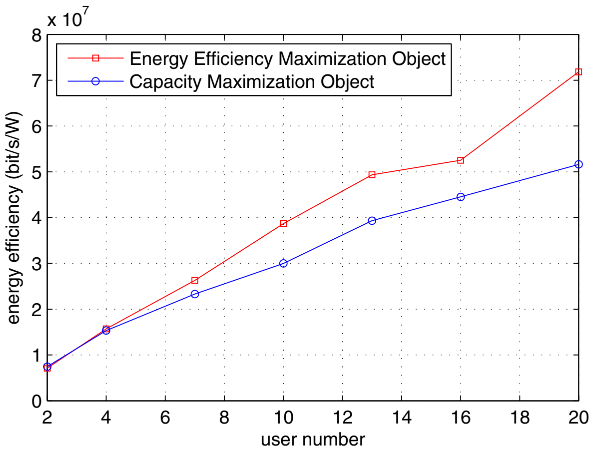

Fig. 1. Energy Efficiency of two objects

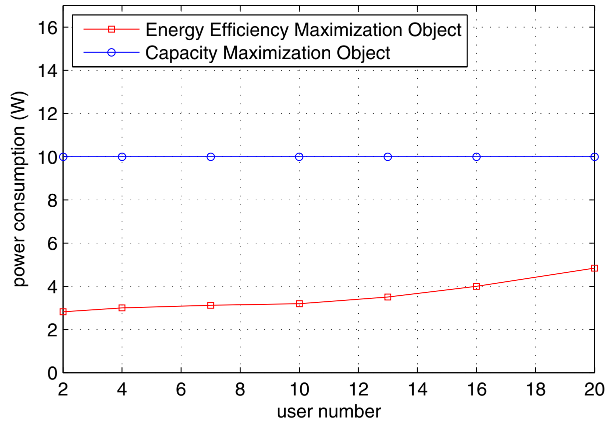

Fig. 2. Power Consumption of two objects

IV. Evaluation and Analysis

A single cellular network with radius of 1km is considered. Users are uniformly generated in the cell. The COST 231 Hata urban propagation model is used for the channel [8]:

$$\begin{cases} 31.5 + 3.5\log(d) & d > 0.035\text{km} \\ 31.5 + 3.5\log(0.035) & d \leq 0.035\text{km} \end{cases}$$

Shadowing is assumed to be lognormally distributed with mean 0 dB and standard deviation 8 dB. Other parameters are as follow: BER is 10^{-4}, user's minimum required rate is 1Mbps, system bandwidth is 20MHz, the number of subcarriers is 100, the total transmit power of BS is 10W and the noise power is -80dB.

Our results are compared with that of capacity maximization object. Fig 1 illustrates the energy efficiency of two objects. It's obvious that our scheme has improved the energy efficiency, and the improvement is more obvious when user number increases, as the improvement gap is enlarged when user number is increased. This can be viewed as an example of large group effects, that the increase of user number can improve the usage efficiency of public resource.

Fig 2 illustrates the power consumption of two objects. As user number increases, the power consumption is increased in our scheme, which is reasonable, for the increase of network load will result in more power consumption. But compared with capacity maximization object, our scheme still saves energy, as the power gap between two objects still exists even when user number is largest in the simulation.

V. Conclusion

The subcarrier and power allocation problem for energy efficiency maximization is addressed in this paper, where Lagrangian dual decomposition (LDD) is applied and a multilevel water-filling for power allocation is derived. As the water-filling here is a transcendental equation, fixed point iteration is applied to solve the equations. Besides, a sufficient condition for the existence of the fixed point is derived. Finally, the joint resource allocation algorithms are designed, and numerical results show that the energy efficiency is reduced compared with the capacity maximization scheme.

Acknowledgment

This work was supported by Program for New Century Excellent Talents in University (NCET-01-0259), the National Natural Science Foundation of China (60832009, 11001030), the National Science and Technology Major Project (2010ZX03003-001-01), Sino-Finland ICT Collaboration Program Project on "Future Wireless Access Technologies" (2010DFB10410).

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