M-PSK Codebook Based Clustered MIMO-OFDM SDMA with Efficient Codebook Search

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***Abstract*—In this paper, we consider a channel quantization problem that selects the best codeword representing a group of consecutive subcarriers (cluster) in multiuser MIMO-OFDM systems. The clustering is required to reduce the amount of feedback information in practical OFDM systems. Based on M-PSK based codebooks, we propose a new codeword search algorithm for the clustered MIMO-OFDM with complexity of $\mathcal{O}(2^{N_T})$ compared to the complexity of $\mathcal{O}(M^{N_T})$ for conventional exhaustive search, where N_T denotes the number of transmit antennas. Through simulations, the proposed clustered codeword selection scheme for the multiuser MIMO-OFDM shows up to 54% performance improvement in throughput compared with the conventional MIMO-OFDM system that performs the channel quantization only on predetermined pilot subcarriers.**

I. Introduction

Recent advances in multiuser downlink communications show that multiuser multi-input/multi-output (MIMO) systems with N_T transmit antennas at a base station (BS) and $N_u \geq N_T$ single antenna users can increase the achievable throughput up to N_T times by applying space-division multiple access (SDMA) technologies. It comes at a cost of perfect channel state information (CSI) at the transmitter.

Feeding back the CSI estimated at user without a form of lossy compression incurs a large amount of feedback overhead that prohibits practical implementation. In practice, particularly in frequency-division duplexed (FDD) systems, a quantized version of the CSI is provided to the transmitter, in which each user quantizes its channel vector into one of codewords in a predefined codebook and feeds back the selected codeword index [1].

In the codebook based feedback, the performance depends on the quantization error that is inversely proportional to the codebook size. Also, in orthogonal frequency division multiplexing (OFDM) systems, the amount of subcarriers that feedback the quantized CSI determines the system performance and the overhead for the channel feedback.

In the multiuser MIMO, the amount of feedback information needs to be scaled with the number of transmit antennas and signal-to-noise ratio (SNR) to achieve the full spatial multiplexing gain while maintaining the performance gap within a certain offset value compared with the performance with the perfect CSI [2]. This requires the codebook to be scalable with respect to the number of transmit antennas and/or the feedback resolution.

In [3], unitary codebook was proposed for multiuser MIMO systems by exploiting Riemannian manifolds. The proposed scheme in [3] requires an exhaustive search to perform channel quantizations and needs new codebook generations for each N_T. The complexity of the exhaustive search grows exponentially, which makes the unitary codebook impractical for multiuser MIMO systems with large number of antennas. For example, the exhaustive search for the codebook in [3] with 24-bit codewords for $N_T = 8$ (i.e., 3 bits per antenna) requires to traverse 16,777,216 (=2^{24}) codewords that is prohibitively huge in practical systems.

In [4], the M-PSK based codebook is employed to quantize channels for a single user MIMO beamforming system. In the N_T antenna case, the codebook is constructed by concatenating the M-PSK alphabet N_T times and codewords in the codebook are formed by normalizing the set of all possible M^{N_T} sequences of length N_T to unit magnitude.

In this paper, we consider a multiuser MIMO-OFDM downlink system with zero-forcing (ZF) precoder employed on each subcarrier at the BS to separate different user streams [2]. This system can be viewed as MIMO-OFDM SDMA transmissions and each subcarrier can be viewed as a multiuser MIMO system with the ZF precoder.

For the multiuser MIMO with the ZF precoder, we propose to use the M-PSK based codebook, and show that the M-PSK based codebook achieves diversity order up to N_T and the diversity order depends on the rank of the concatenated channel with the reported codewords. This M-PSK based codebook has two advantages: 1) it can be easily extended (i.e., scalable) with the increased number of antennas since the codeword construction depends only on the base alphabet, 2) it allows an efficient codebook search with complexity of $\mathcal{O}(N_T \log N_T)$ due to the embedded codeword generation structure.

For the MIMO-OFDM SDMA system, we propose to use the M-PSK codebook to quantize a group of consecutive subcarriers into one codeword to reduce the amount of feedback information, which is our major interest in the paper. To make this quantization practical for large number antenna systems, we propose a new codeword search algorithm with $\mathcal{O}(2^{N_T})$ complexity by exploiting the structure of the M-PSK base codebook. We note that the algorithm with $\mathcal{O}(N_T \log N_T)$ complexity in [5] cannot be used in this case and the complexity of exhaustive search is $\mathcal{O}(M^{N_T})$. The quantization is

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performed to minimizes the aggregated quantization error over the the group of subcarriers based on mean square error (MSE) criteria. The proposed scheme divides the whole OFDM subcarriers into multiple clusters of size L and quantizes L channel vectors into one codeword that minimizes the MSE. By doing so, the amount of feedback information is reduced into L codeword index. We will refer to this scheme as 'clustered' MIMO-OFDM SDMA.

We derive a new upper bound of the quantization error with the M-PSK codebook to provide a mathematical tool for system analysis. Through simulations, in the clustered MIMO-OFDM SDMA, we show that the proposed quantization with the MSE criteria improves the performance significantly compared with the simple clustered system in which channel quantizations are performed on predetermined pilot subcarriers in [1]. When $L = 8$ for the clustered MIMO-OFDM SDMA with $N_T = 4$ and 64 subcarriers, our simulation result exhibits that the proposed scheme improves the achievable throughput up to 54% at the received SNR of 20 dB.

II. System Model

A. Downlink MIMO-OFDM SDMA Transmissions with Perfect CSI

We consider a downlink MIMO-OFDM SDMA system with N_T transmit antennas, N_c subcarriers, and N_u users having a single receive antenna. At the k-th subcarrier of the transmitter, N_s users are selected among N_u users to maximize the sum-rate and a N_s-dimensional vector consisting of N_s user symbols $\mathbf{s}(k) = [s_1(k), s_2(k), \cdots, s_{N_s}(k)]^T$ is transmitted where $(\cdot)^T$ represents the transpose of a matrix and $N_s \leq N_T$ as in [3]. Without loss of generality, we denote the index of the selected N_s users selected as $1, \ldots, N_s$. Prior to the transmission, $\mathbf{s}(k)$ is precoded by $N_T \times N_s$ matrix $\mathbf{W}(k)$ yielding $\mathbf{x}(k) = \mathbf{W}(k)\mathbf{s}(k)$ to suppress the inter-user interference at receivers. Denoting the average transmit power as P such that $E[\|\mathbf{s}(k)\|^2] = P$, the precoder matrix should satisfy the transmit power constraint of $\|\mathbf{x}(k)\|^2 = P$.

In the MIMO-OFDM SDMA system, the time-domain channel impulse response from the j-th transmit antenna to user i is modeled by N_p finite impulse responses $\nu_{i,j}(\tau) = \sum_{i=0}^{N_p-1} \bar{\nu}(i)\delta(\tau - iT_s)$ where T_s denotes the sampling period, the channel coefficients $\bar{\nu}(i)$ are independent Gaussian variables with zero mean, $\delta(\cdot)$ represents the Dirac delta function, and N_p represents the maximum number of channel taps. With the discrete Fourier transform (DFT) at the receiver, the channel frequency response from the j-th transmit antenna to user i at the k-th subcarrier is written by

$$h_{i,j}(k) = \sum_{i=0}^{N_p-1} \bar{\nu}(i) \exp\left(-j\frac{2\pi k i}{N_c}\right). \tag{1}$$

Defining $\mathbf{h}_i(k) = [h_{i,1}(k),\ h_{i,2}(k), \cdots, h_{i,N_T}(k)]$ as a $1 \times N_T$ channel matrix at the k-th subcarrier of user i the received signal at the k-th subcarrier at user i is described as

$$y_i(k) = \mathbf{h}_i(k)\mathbf{x}(k) + n_i(k), \quad i = 1, \cdots, N_s \tag{2}$$

where $n_i(k)$ is independent complex Gaussian noise with variance σ^2. Concatenating channel vectors of the selected N_s users by $\mathbf{H}(k) = [\mathbf{h}_1(k)\mathbf{h}_2(k)\cdots\mathbf{h}_{N_s}(k)]^T$, signal transmission from the BS to the N_s users is equivalently expressed as

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{n}(k). \tag{3}$$

When the perfect CSI is available at the transmitter and the ZF precoder is employed, $\mathbf{x}(k)$ is obtained by

$$\mathbf{x}(k) = \frac{1}{\sqrt{\gamma(k)}}\mathbf{H}^H(k)(\mathbf{H}(k)\mathbf{H}^H(k))^{-1}\mathbf{s} \tag{4}$$

where $\gamma(k)$ normalizes $\mathbf{x}(k)$ such that $\|\mathbf{x}(k)\|^2 = P$ and $(\cdot)^H$ denotes the Hermitian transpose of a matrix. By multiplying $\sqrt{\gamma(k)}$ to the received signal, the estimation of the transmitted symbol for user i is made by

$$r_i(k) = \sqrt{\gamma(k)}y_i(k) = s_i(k) + \sqrt{\gamma}n_i(k).$$

B. Finite-Rate Feedback Model

The precoder construction in the previous subsection assumed that the CSI is perfectly known at the transmitter. In finite-rate feedback systems, the CSI at the receiver is quantized and $N_{FB} \triangleq N_T \times B$ bits are used for the feedback where B denotes feedback bits per antenna. The quantization is accomplished by utilizing codebooks known at both the transmitter and receivers. Each receiver quantizes the measured channel vector into an appropriate codeword in the codebook that consists of $2^{N_{FB}}$ unit norm vectors $\mathcal{C} = \{\mathbf{c}_0\ \mathbf{c}_1 \cdots \mathbf{c}_{2^{N_{FB}-1}}\}$ where each codeword $\mathbf{c}_l$ is N_{FB} bits in length. The receiver sends the index of the selected codeword as the feedback information.

The channel quantization is to find a codeword that has the minimum distance from the channel vector. In this paper, the distance is measured by the *chordal distance* between two vectors defined as [6]

$$d(\mathbf{v}_1, \mathbf{v}_2) = \sin\angle(\mathbf{v}_1^H\mathbf{v}_2) = \sqrt{1 - \frac{|\mathbf{v}_1^H\mathbf{v}_2|^2}{\|\mathbf{v}_1\|^2\|\mathbf{v}_2\|^2}} \tag{5}$$

where $\angle(\mathbf{v})$ is angle of the complex number $\mathbf{v}$. Consequently, the codeword index to be fed back is obtained by

$$\begin{aligned} l^* &= \arg\min_{\mathbf{c}_l \in \{\mathbf{c}_1, \cdots, \mathbf{c}_{2^{N_{FB}}}\}} d(\mathbf{h}_i, \mathbf{c}_l) \\ &= \arg\max_{\mathbf{c}_l \in \{\mathbf{c}_1, \cdots, \mathbf{c}_{2^{N_{FB}}}\}} \cos^2(\angle(\mathbf{h}_i, \mathbf{c}_l)) \end{aligned} \tag{6}$$

To solve the problem in (6), all $2^{N_{FB}}$ codewords in the codebook are to be traversed in general non-structured codebook designs such as [3, 6]. In this case, the search complexity becomes prohibitively huge as N_T and B increases.

After receiving the feedback information from all users, the BS constructs the ZF precoder by treating the quantized versions of CSI as real CSI [2]. Let $\hat{\mathbf{h}}_i(k)$ be the quantized version of the channel vector at the k-th subcarrier of user i and $\hat{\mathbf{H}}(k)$ be $\hat{\mathbf{H}}(k) = [\hat{\mathbf{h}}_1(k)\ \hat{\mathbf{h}}_2(k)\cdots\hat{\mathbf{h}}_{N_s}(k)]^T$. The ZF precoder $\mathbf{W}(k)$ with $\hat{\mathbf{H}}(k)$ is computed with the normalized pseudo-inverse of $\hat{\mathbf{H}}(k)$ given as $\mathbf{W}(k) = \frac{1}{\sqrt{\gamma(k)}}\hat{\mathbf{H}}^H(k)(\hat{\mathbf{H}}(k)\hat{\mathbf{H}}^H(k))^{-1}$. Denoting $\mathbf{w}_i(k)$ as the i-th column of $\mathbf{W}(k)$, the transmitting signal is given as $\mathbf{x}(k) = \sum_{j=1}^{N_s}\mathbf{w}_j(k)s_j(k)$ and the received signal at the k-th subcarrier of user i is expressed as

$$y_i(k) = \sum_{j=1}^{N_s}\mathbf{h}_i(k)\mathbf{w}_j(k)s_j(k) + n_i. \tag{7}$$

In (7), $\mathbf{h}_i^T(k)\mathbf{w}_j(k)$ is no longer zeros for $i \neq j$. The SINR at user i with the finite rate feedback is then computed as

$$\text{SINR}_i^{\text{partial}} = \frac{\frac{P}{M}|\mathbf{h}_i(k)\mathbf{w}_i(k)|^2}{\sigma^2 + \sum_{j\neq i}\frac{P}{M}|\mathbf{h}_i(k)\mathbf{w}_j(k)|^2} \tag{8}$$

III. Multiuser MIMO-OFDM Transmissions

A. M-PSK based Codebook Proposed for Multiuser MIMO with ZF Precoder

In [2], it is shown that the number of feedback bits (i.e., resolution of channel quantizations) must be scaled proportionally to N_T and the received SNR in dB to maintain the rate loss within an expected value.

In this subsection, we propose to use the M-PSK based codebook in [4] to make the codebook used for the channel quantization to be scalable with respect to N_T and SNR, and show that the M-PSK based codebook achieves the diversity order up to N_T.

The M-PSK based codebook $\mathcal{C}$ is formed by normalizing the set of all M^{N_T} possible sequences of M-ary PSK symbols of length N_T to unit magnitude. In other words, the M-PSK based codebook is constructed with all unit normalized vectors of length N_T with alphabet χ, i.e., $\underbrace{\chi \times \chi \times \cdots \times \chi}_{N_T \text{ times}}$ where χ is defined as

$$\chi = \left\{\exp\left(j\frac{2\pi}{M}(i-1)\right)|i = 0, \cdots, M-1\right\}. \tag{9}$$

In doing so, the codebook for the multiuser MIMO system with $N_T + 1$ can be obtained by concatenating an extra alphabet χ without the exhaustive search. We note that the optimal unitary codebook for multiuser MIMO systems in [3] requires off-line design effort to obtain codebooks for all possible antenna configurations and all possible the codebook size.

In the M-PSK based codebook, the codeword that satisfies (6) can be obtained by the $O(N_T \log N_T)$ algorithm in [5] by regarding the problem in (6) as an equivalent non-coherent sequence detection problem of M-PSK constellation.

Now, we show that the M-PSK based codebook in the MIMO-OFDM SDMA achieves the diversity order up to N_T by using the following Lemma. A system is said to achieve diversity order D of the probability of symbol error, P_e, averaged over $\mathbf{H}$ if it satisfies

$$\lim_{P/\sigma^2\to\infty}\frac{\log P_e(P/\sigma^2)}{\log(P/\sigma^2)} = -D. \tag{10}$$

Lemma 3.1: In a multiuser MIMO system with a single receive antenna, the M-PSK based codebook provides the diversity order up to N_T. The diversity order N_T can be achieved if and only if codewords fed back from each user span $\mathbb{C}^{N_T}$.

Proof: According to $Theorem$ 1 in [7], (7) provides the maximum diversity order of N_T if and only if the vectors in the M-PSK codebook can generate $\mathbf{W}(k)$ that spans $\mathbb{C}^{N_T}$. For arbitrary $m \times n$ matrix $\mathbf{H}(k)$, the rank of pseudo-inverse is equal to the rank of $\mathbf{H}(k)$ [8]. Therefore, $\mathbf{W}(k)$ spans $\mathbb{C}^{N_T}$ if the vectors in the M-PSK codebook spans $\mathbb{C}^{N_T}$.

For the M-PSK based codebook, it can be shown that the vectors in the codebook spans $\mathbb{C}^{N_T}$ by constructing matrix $\mathbf{B} = \mathbf{1} + (e^{j\frac{2\pi}{M}} - 1)\mathbf{I}$ for $N_T > 2$ and $\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ for $N_T = 2$ where $\mathbf{1}$ is an $N_T \times N_T$ matrix with all entries equal to one. It can be seen that the row vectors of $\mathbf{B}$ are valid codewords from an M-PSK based codebook and all row vectors are linearly independent. Thus, in the multiuser MIMO system, $\mathbf{H}(k)$ spans $\mathbb{C}^{N_T}$ if and only if received codewords from the selected users are linearly independent. ■

B. Proposed Clustered MIMO-OFDM SDMA

In this subsection, we propose a new channel quantization scheme for the MIMO-OFDM SDMA system that quantizes a group of consecutive subcarriers (cluster) into one codeword in the M-PSK based codebook to reduce the amount of feedback information. This clustered quantization utilizes the fact that the frequency response of adjacent subcarriers are highly correlated and, consequently, precoders for adjacent subcarriers are also highly correlated.

In the proposed clustered MIMO-OFDM SDMA, N_c subcarriers are divided into clusters of size L and N_{FB} bits are used to represent the quantized CSI for each cluster. Consider the l-th cluster that consists of the subcarriers from $lL + 1$ to $(l + 1)L$ for $0 \leq l \leq N_c/L - 1$. Each receiver quantizes the CSI of L subcarriers into a quantization vector $\hat{\mathbf{h}}_i(l) \in \mathcal{C}$ where the superscript 'c' stands for 'cluster'. Denoting $\hat{\mathbf{H}}^c(l) = [\hat{\mathbf{h}}_1^c(l)\ \hat{\mathbf{h}}_2^c(l)\ \cdots\hat{\mathbf{h}}_{N_u}^c(l)]$, the precoder for the l-th cluster is constructed by

$$\mathbf{W}^c(l) = \frac{1}{\sqrt{\gamma(l)}}\hat{\mathbf{H}^c}^H(l)(\hat{\mathbf{H}}^c(l)\hat{\mathbf{H}^c}^H(l))^{-1}$$

and $\gamma(l)$ is a normalization constant and $\mathbf{W}^c(l)$ is used as precoder for the subcarriers from $lL + 1$ to $(l + 1)L$.

The simplest way to compute $\hat{\mathbf{h}}^c(l)$ is $\hat{\mathbf{h}}^c(l) = \hat{\mathbf{h}}(lL + k)$ for fixed $1 \leq k \leq L$ as in [1]. This quantization method yields large quantization errors for subcarriers $lL + k'$ with $1 \leq k' \leq L, k' \neq k$. To reduce the quantization error, we propose a novel quantization scheme that minimizes aggregated quantization errors by solving the following problem:

$$\hat{\mathbf{h}}^c(l) = \arg\max_{\mathbf{c}\in\{\mathbf{c}_1,\cdots,\mathbf{c}_{2^{N_{FB}}}\}}\sum_{k=lL+1}^{(l+1)L}|\mathbf{h}(k)^H\mathbf{c}|^2. \tag{11}$$

Codeword selection in (11) needs to traverse all M^{N_T} codeword since the method in [4] can not be used. To reduce the searching complexity, we propose a new codeword searching algorithm to solve the problem in (11) for the M-PSK based codebook. For given $\mathbf{h}_i \in \mathbb{C}^{N_T}, i = 1, \cdots, L$, we rewrite the problem in (11) as finding $\mathbf{c}^*$ that solves the following problem:

$$\begin{aligned}\mathbf{c}^* &= \arg \max_{\mathbf{c}\in\{\mathbf{c}_1,\cdots,\mathbf{c}_{2^{N_{FB}}}\}} \sum_{i=1}^{L} |\mathbf{h}_i^H \mathbf{c}| \\ &= \arg\min_{\mathbf{g}\in G} \sum_{i=1}^{L} |\mathbf{h}_i^H \eta^{\mathbf{g}}| \triangleq \arg\min_{\mathbf{g}\in G} \mathcal{L}(\mathbf{g}) \qquad (12)\end{aligned}$$

where $\eta = \exp\left(j\frac{2\pi}{M}\right)$. In (12), we note that integer vectors $\mathbf{g}$ that differ by a multiple of all one vector $\mathbf{1}$ are indistinguishable; the solution of (12) is unaffected by the phase of $|\mathbf{h}_i^H \mathbf{c}|$ denoted by φ_i. Thus, without loss of generality, we assume $g_0 = 1$.

When φ_i for $i = 1, \cdots, L$ are known, $\mathbf{g}$ is determined by

$$\Gamma(\mathbf{\Phi}) = \arg\min_{\mathbf{g}\in G} \sum_{i=1}^{L} \|\mathbf{h}_i - \exp(-\varphi_i)\eta^{\mathbf{g}}\| \qquad (13)$$

where $\mathbf{\Phi} = [\varphi_1, \varphi_2, \cdots, \varphi_L]$. The problem in (12) can be solved by scanning all possible $\mathbf{\Phi}$ and picking $\hat{\mathbf{\Phi}}$ with the largest likelihood such that

$$\hat{\mathbf{g}} = \Gamma(\hat{\mathbf{\Phi}}) \quad \text{with } \hat{\mathbf{\Phi}} = \arg\max_{\mathbf{\Phi}\in[0,2\pi)^L} \mathcal{L}(\Gamma(\mathbf{\Phi})) \ . \qquad (14)$$

In (13), a vector differs by all one vector $\mathbf{1}$ is indistinguishable. That is,

$$\Gamma\left(\mathbf{\Phi} + \frac{2\pi}{M}\mathbf{1}\right) = \Gamma(\mathbf{\Phi}) + \mathbf{1}. \qquad (15)$$

According to (15), we reduce the search space into $\Phi \in [0, 2\pi/M)^L$.

Denote the standard basis vector $\mathbf{e}_i$ as the i-th column of identity matrix. Let $S = \{i\}$, $(i = 1, \cdots, N_T)$ and define the set $P(S)$ as a power set of S that is a set taking all subsets of S as elements. The cardinality of $P(S)$ is 2^{N_T}.

Our proposed scheme constructs a new search space $\tilde{G}$ that contain integers $\mathbf{g}$ obtained by $\mathbf{g}^{[0]} + \Delta\mathbf{g}$ where $\Delta\mathbf{g} = \sum_{k\in T_l} \mathbf{e}_k$ for all $T_l \in P(S)$. Hence the size of $\tilde{G}$ is 2^{N_T} and covers the all search spaces by (15) . In doing so, we propose to select $\hat{\mathbf{g}}$ by

$$\hat{\mathbf{g}} = \arg\min_{\mathbf{g}\in\tilde{G}} \sum_{i=1}^{L} |\mathbf{h}_i \eta^{\mathbf{g}}| \qquad (16)$$

where $\mathbf{g}^{[0]}$ is

$$\mathbf{g}^{[0]} = \Gamma(\mathbf{0}) = \arg\max_{\mathbf{g}\in G} \sum_{i=1}^{L} \|\mathbf{h}_i - \eta^{\mathbf{g}}\|. \qquad (17)$$

$g_t^{[0]}$ in (17) can be obtained component-wise such that

$$g_t^{[0]} = \arg\max_{m=1,\cdots,M} \sum_{i=1}^{L} |h_{i,t}\eta^m|. \qquad (18)$$

The complexity of (16) and (18) is $O(2^{N_T})$ and $O(MN_T)$. Thus, overall complexity is dominated by $O(2^{N_T})$ for large N_T such that $M < 2^{N_T}/N_T$.

IV. Quantization Quality

In this section, we compute a new upper bound of the channel quantization errors for the M-PSK based codebook. In (5), the channel quantization error is measured by $\sin^2(\angle(\mathbf{h}, \mathbf{c}_i))$. Therefore, we derive a upper bound of $E[\sin^2(\angle(\mathbf{h}, \mathbf{c}_i))]$.

For each $\mathbf{c}_i \in \mathcal{C}$ and arbitrary channel $\tilde{\mathbf{h}}$, Vornoi region corresponding codeword $\mathbf{c}_i$ and distance γ is given by

$$V_{\mathbf{c}_i}(\gamma) = \{\tilde{\mathbf{h}} \in \Omega_{N_T} | d(\tilde{\mathbf{h}}, \mathbf{c}_i) \le \gamma\} \qquad (19)$$

where $d(\mathbf{a}, \mathbf{b}) = \sqrt{1 - |\mathbf{a}^H \mathbf{b}|^2}$ is the chordal distance defined in (5) and Ω_{N_T} is the space of N_T dimensional unit-norm channel direction vectors. Since the channel direction is isotropically distributed, the probability that a given channel direction $\tilde{\mathbf{h}}$ is included in $V_{\mathbf{c}_i}(\gamma)$ is given as

$$\mathrm{P}(\tilde{\mathbf{h}} \in V_{\mathbf{c}_i}(\gamma)) = \frac{A(V_{\mathbf{c}_i}(\gamma))}{A(\Omega_{N_T})} = \frac{\tan^2\theta_\gamma}{16}$$

where $A(\cdot)$ denotes the area of a given region, $A(\Omega_{N_T}) = 4\pi$ is the surface area of the sphere with radius 1, and $\theta_\gamma = \arcsin(\gamma/2)$. The density of the codebook is defined as [6]

$$\Delta(\mathcal{C}) = \sum_{i=1}^{N} \mathrm{P}(\tilde{\mathbf{h}} \in V_{\mathbf{c}_i}(\gamma_{\min}/2))$$

where N denotes the number of codewords in the codebook and $\gamma_{\min}$ is given as

$$\gamma_{\min} = \min_{1\le k<l\le N} \sqrt{1 - |\mathbf{c}_k^H \mathbf{c}_l|^2} \qquad (20)$$

For the M-PSK based codebook, 2^{N_T-1} neighbors in the codebook are enough to compute the minimum distance from $\mathbf{c}_i$ to any codeword in the codebook by exploiting the property $\mathcal{L}(\mathbf{g}) = \mathcal{L}(\mathbf{g} + \mathbf{1})$. The set of neighbors $\mathcal{N}_i$ is constructed by

$$\mathcal{N}_i = \{\eta^{\mathbf{g}} | \mathbf{g} = \mathbf{g}_i + \Delta\mathbf{g}, \Delta\mathbf{g} = \sum_{k\in T_l} \mathbf{e}_k, \text{ for all } T_l \in P(S)\backslash\{\mathbf{0}\}\} \qquad (21)$$

where $\mathbf{g}_i = [\frac{M}{2\pi}\angle c_{i,1}, \cdots, \frac{M}{2\pi}\angle c_{i,N_T}]$. Therefore, $\gamma_{\min}$ is computed by

$$\gamma_{\min,i} = \min_{\mathbf{v}\in\mathcal{N}_i} d(\mathbf{v}, \mathbf{c}_i) \ . \qquad (22)$$

Due to the π/M rotational symmetry of the M-PSK based codebook, M^{N_T-1} codewords are distinguishable. Consequently, the density of the M-PSK based codebook is computed as

$$\Delta(\mathcal{C}) = \frac{M^{N_T-1}}{16} \tan^2 \sin^{-1}(\gamma_{\min}/2) \qquad (23)$$

Finally, our derived upper bound of quantization errors with the M-PSK based codebook is computed as

$$\begin{aligned}E[\sin^2\theta] &\le P\left(1 - |\mathbf{c}_i^* \tilde{\mathbf{h}}|^2 \le \frac{\gamma_{\min}^2}{4}\right)\frac{\gamma_{\min}/2}{4} \\ &\quad +(1 - P\left(1 - |\mathbf{c}_i^* \tilde{\mathbf{h}}|^2 \le \frac{\gamma_{\min}^2}{4}\right) \\ &= \Delta(\mathcal{C})\frac{\gamma_{\min}^2}{4} + (1 - \Delta(\mathcal{C})) \\ &= 1 + \frac{M^{N_T-1}}{16} \tan^2 \sin^{-1}(\gamma_{\min}/2)\left(\frac{\gamma_{\min}^2}{4} - 1\right) \qquad (24)\end{aligned}$$

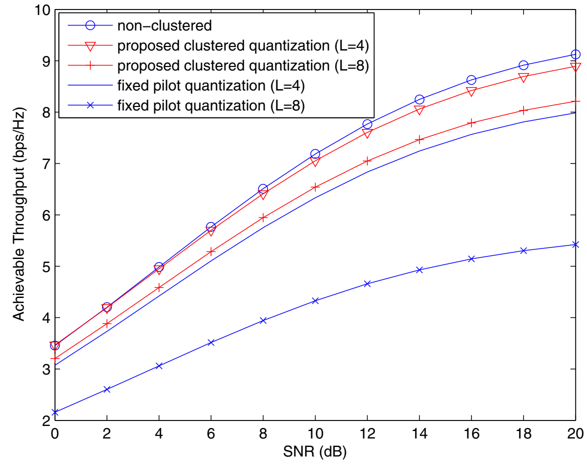

Fig. 1. Achievable throughput of the clustered multiuser MIMO-OFDM with proposed codeword selection scheme and conventional codeword selection scheme at fixed subcarriers.

Lemma 4.1: To ensure that the rate loss of the M-PSK based codebook with a given N_T is no larger than ΔR, M is upper-bounded by

$$M \leq K_{N_T} \log_{N_T-1}\left(1-\frac{2^{\Delta R}-1}{P}\right) \quad (25)$$

where K_{N_T} is defined as $\log_{N_T-1} 64$.

Proof: Since codewords in the M-PSK codebook are independent isotropic, the upper bound of the rate loss is

$$\Delta R \leq \log_2(1+PE[\sin^2\theta]). \quad (26)$$

By applying (24) to (26), ΔR is rewritten as

$$\frac{2^{\Delta R}-1}{P} \leq 1-\frac{M^{N_T-1}}{16}\tan^2\sin^{-1}\frac{\gamma_{\min}}{2}\left(1-\frac{\gamma_{\min}^2}{4}\right). \quad (27)$$

Using the trivial bound $\gamma_{\min} \leq 1$, we obtain

$$M \leq K_{N_T} \log_{N_T-1}\left(1-\frac{2^{\Delta R}-1}{P}\right) \quad (28)$$

where $K_{N_T} = \log_{N_T-1} 64$. ■

V. Simulation Results

In this section, we present a numerical result that shows the achievable throughput of the proposed clustered MIMO-OFDM SDMA system with the M-PSK based codebook through Monte Carlo simulations. We assume that the average received SNR for all users are the same.

The channel is assumed to have exponentially decaying multipath Rayleigh fading. To generate channel impulse responses, $\bar{\nu}(i)$'s in (1) are modeled as a complex Gaussian random variable with $\bar{\nu}(i) \sim \mathcal{CN}(0, \sigma_0 \exp(-iT_s/T_{rms}))$ where T_{rms} denotes root mean square (rms) delay spread and σ_0 is given as $\sigma_0 = 1-\exp(T_s/T_{rms})$. Throughout simulation, we use channels with $T_{rms} = 50$ns.

In Fig. 1, we depict the achievable throughput of the proposed clustered MIMO-OFDM SDMA system with $N_u = 12$ and $N_T = N_s = 4$. For OFDM modulation, N_c=64 subcarriers are used and the cyclic prefix length is set to 16 samples with $T_s = 50$ns. As shown in the figure, the proposed clustered feedback based on MSE criteria shows improved performance compared with the system that performs the channel quantization only on predetermined pilot subcarriers as in [1]. In this figure, it is observed that the performance degradation of the system that performs the channel quantization only on predetermined pilot subcarriers is significant as L grows. Especially, the performance improvement for $L = 8$ in SNR=20dB is almost 54%. Compared with the non-clustered MIMO in which quantized channel information for all subcarriers are fed back to the transmitter, the performance loss of the proposed clustered OFDM is only 3% for $L = 4$ and 10% for $L = 8$. However, the required feedback amount is only 25% for $L = 4$ and 12.5% for $L = 8$ of the non-clustered MIMO-OFDM.

VI. Conclusion

In this paper, we proposed the M-PSK codebook based clustered MIMO-OFDM SDMA system with an efficient codebook search algorithm. We exploited the embedded structure of the M-PSK based book to makes the codebook to be scalable and efficiently searched. Our proposed search algorithm with complexity of $O(2^{N_T})$ enables the MIMO-OFDM SDMA systems with large number of antennas to be deployed in practical systems.

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