

Low Complexity Fast LMMSE-based Channel Estimation for OFDM Systems in Frequency Selective Rayleigh Fading Channels

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Abstract—Channel estimation (CE) is a challenging problem and one of the key technologies in Orthogonal Frequency Division Multiplexing (OFDM) systems. The Linear Minimum Mean Square Error (LMMSE) proposed for OFDM systems has excellent Mean Square Error (MSE) performance. However, the conventional method requires the statistical knowledge of the channel information in advance and the complicated inverse operation of large dimension matrices. In this case, an enhanced fast LMMSE channel estimation using Fast Fourier Transform (FFT) operation and the most significant taps (*MST*) algorithm for OFDM systems is proposed. This method has the advantage of low computational complexity and no requirement on the detailed channel information. In addition, simulation results demonstrate that the proposed algorithm outperforms other LMMSE methods in terms of bit error rate (BER) without the loss of MSE performance.

Keywords—OFDM, pilot symbols, channel estimation, Rayleigh fading channel, *MST* algorithm, channel autocorrelation matrix, MSE.

I. INTRODUCTION

OFDM technology is an efficient high data transmission technique due to its many advantage such as the high spectrum efficiency, mitigation of inter-symbol interference (ISI) by inserting cyclic prefix (CP), simple and efficient implementation by using the Fast Fourier Transform (FFT) operation [1], and its robustness to frequency selective fading channel.

For wideband wireless communication, it is necessary to dynamically estimate the channel before demodulating the signals. There are two kind methods for channel estimation. The first is the pilot assisted estimation, the pilot signals are inserted in certain sub-carriers of each OFDM symbol. At the receiver, the channel components estimated using these pilots are interpolated for estimating the complete channel. Based on the criterion of realization, it can be classified as Least Square (LS) [2], MMSE [3], LMMSE [4], maximum likelihood estimator [5] and so on. The second category is the blind channel estimation [6]. The blind schemes avoid the use of pilots, for achieving high spectral sufficiency. This is achieved at the cost of higher implementation complexity and some amount of performance loss.

Generally, channel estimation methods are based on LS and LMMSE methods in frequency domain. LS estimation [1] can be simply implemented with low computational complexity, however the MSE performance is not satisfactory. LMMSE method is optimum in minimizing the MSE of the channel estimates in terms of of AWGN [4], but requires additional channel information such as the detailed channel power delay profiles. Furthermore, the algorithm contains matrix inverse operation and other complex operations which causes high computational complexity. Therefore, the conventional LMMSE method is not applicable in practical system.

In this paper, we came up a novel estimation method. Instead of researching on the complicated channel statistics, we mainly focus on the estimation of channel autocorrelation matrix. The computational complexity can be significantly reduced by using the *MST* algorithm we proposed, calculating the LMMSE matrix by IFFT operation and making the best use of the cyclic property of the channel autocorrelation matrix.

Immediately following this introduction, Section II describes OFDM system model and sketches of different resource blocks used in our simulations. In Section III, we review the basic LS method and the conventional LMMSE method. The proposed enhanced fast LMMSE algorithm is presented in Section IV in detail. In Section V, we analyze the computational complexity and demonstrate the simulation results. Finally, the conclusion is summarized in Section VI.

Notation: We use bold upper case letters to denote matrices and bold lower case letters to denote vectors. Furthermore, $(\cdot)^{-1}$ reserved for the matrix inverse and $(\cdot)^H$ for Hermitian transposition. The estimated value of a variable ϵ is denoted by $\hat{\epsilon}$. \mathbf{I} denotes an identity matrix, $E(\cdot)$ is defined as the mathematical expectation.

II. SYSTEM MODEL DESCRIPTION

The OFDM system model with pilot symbol used in simulation is shown in Fig 1. The digital binary information is first mapped to encode, then modulation. After signal is converted from serial to parallel form, pilot signals are inserted according to comb pattern from 3GPP LTE protocol R10. Following the conversion, Inverse Discrete Fourier Transform (IDFT) is used to transform the sequence into time domain.

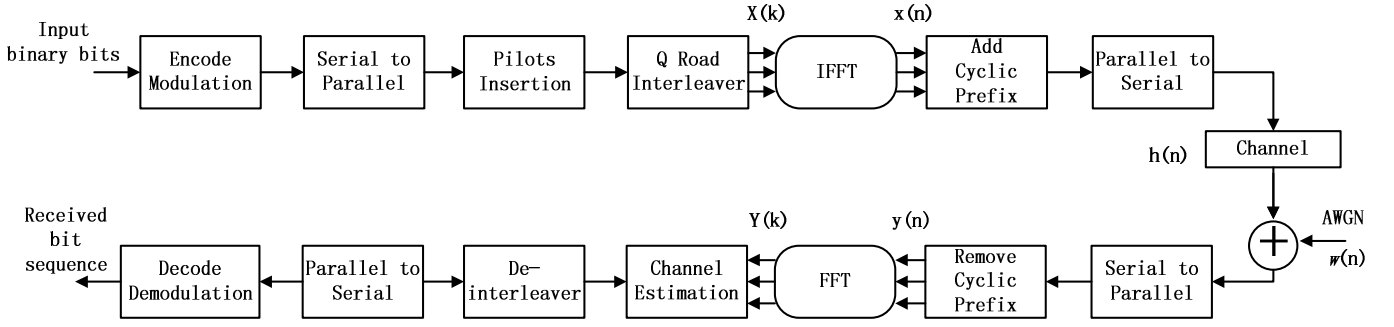


Figure.1: Baseband BICM (Bitinterleaved Coded Modulation)-OFDM system model with pilot symbol insertion

Assuming there are N subcarriers in the OFDM system model, the transmitted signal in time domain is given by

$$\begin{aligned} \mathbf{x}(i, n) &= \text{IFFT}_N\{\mathbf{X}(i, k)\} \quad n = 0, 1, 2, \dots, N-1 \\ &= \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{X}(i, k) \exp\left\{\frac{j2\pi nk}{N}\right\}, \end{aligned} \quad (1)$$

where n is the time domain sample index of an OFDM signal. N also denotes the length of IFFT, $\mathbf{X}(i, k)$ denotes the transmitted signal in frequency domain at the k -th subcarrier in the i -th OFDM symbol. After adding cyclic prefix (CP), the transmitted signal $\mathbf{x}_g(i, n)$ is then sent to a frequency selective multipath fading channel. The received signal can be represented by

$$\mathbf{y}_g(i, n) = \mathbf{x}_g(i, n) \otimes \mathbf{h}(i, n) + \mathbf{w}(i, n) \quad (2)$$

In this paper, all pilot subcarriers exploited as comb pattern are inserted equally spaced into each OFDM symbol, as is shown in Fig 2 and 3 (data : white, pilot : grey).

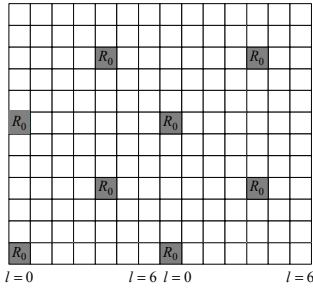


Figure.2: Sketch of a resource block under SISO-OFDM system (normal cyclic prefix, one transmitter and one receiver)

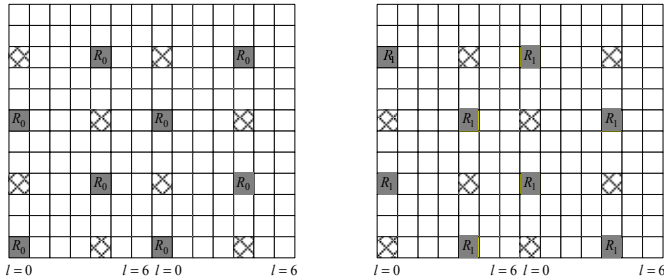


Figure.3: Sketch of a resource block under MIMO-OFDM system (two transmitters and two receivers)

Regarding that N is the total number of pilot subcarriers, the inserting gap denotes by R , and the index of the first pilot

subcarrier is k_0 , hence, we can write the set of the indexes of pilot subcarriers φ as

$$\varphi = \{k | k = mR + k_0, m = 0, 1, \dots, N-1\}, \quad (3)$$

where $k_0 \in [0, R)$.

The received signal $\mathbf{Y}(i, k)$ in frequency domain after FFT block can be written as

$$\mathbf{Y}(i, k) = \mathbf{X}(i, k)\mathbf{H}(i, k) + \mathbf{W}(i, k), \quad (4)$$

where $\mathbf{H}(i, k)$ is the frequency response of the radio channel at the k -th subcarrier of the i -th OFDM symbol, $\mathbf{W}(i, k)$ means the AWGN with zero mean, and variance δ_w^2 .

The channel impulse response in time domain can be expressed as

$$\mathbf{h}(i, n) = \sum_{l=0}^{L-1} h_l(i) \delta(n - \tau_l), \quad (5)$$

$h_l(i)$ is the complex gain of the l -th path in the i -th OFDM symbol period, τ_l is the delay of the l -th path in unit of sample point, and L is the number of resolvable paths. $\mathbf{H}(i, k)$ is the FFT transformation of $\mathbf{h}(i, n)$, denotes the frequency response of the fading channel and is given by

$$\mathbf{H}(i, k) = \text{FFT}_N[\mathbf{h}(i, n)] = \sum_{m=0}^{N-1} h(i, m) \exp\left\{-\frac{j2\pi mk}{N}\right\}. \quad (6)$$

III. RETROSPECT ON THE BASIC CHANNEL ESTIMATION METHOD

LS and LMMSE are the two main channel estimation methods now being used. Further analysis shows that each estimation method is indeed a subset of LMMSE technique. Initially, in our methods, ICI is assumed not to exist and CIR is assumed to be constant for at least one OFDM symbol.

A. LS Channel Estimation

LS estimation algorithm is the most basic method, for it is often needed by many further improved estimation techniques as an initial part. In LS method, it only needs the transmitted pilot signal \mathbf{X}_p and the received pilot signal \mathbf{Y}_p based on the Least Square principle according to

$$\begin{aligned} \tilde{\mathbf{H}}_{LS,p} &= \arg \min_H \{|\mathbf{Y}_p - \mathbf{X}_p \tilde{\mathbf{H}}_{LS,p}|\} = \mathbf{Y}_p \mathbf{X}_p^{-1} \\ &= \mathbf{H}_p + \mathbf{N} \mathbf{X}_p^{-1} = \begin{bmatrix} y_0 & y_1 & \dots & y_{N_p-1} \\ x_0 & x_1 & \dots & x_{N_p-1} \end{bmatrix}^T, \end{aligned} \quad (7)$$

where p poses the position of pilot signal, N_p is the total number of pilot subcarriers.

B. Conventional LMMSE Method

LMMSE channel estimation algorithm, i.e., Wiener filtering [7] [8], is one of the best linear estimation methods. Its essence is to revise the LS algorithm and the effect of the noise on estimation values by using the channel autocorrelation matrix.

The criterion that LMMSE method bases is

$$\tilde{\mathbf{H}}_{LMMSE,p} = \arg \min_{\mathbf{H}} E\{\|\mathbf{Y}_p - \mathbf{X}_p \tilde{\mathbf{H}}_{LMMSE,p}\|^2\} = \mathbf{W} \tilde{\mathbf{H}}_{LS,p} \quad (8)$$

from Equation (5), we can obtain:

$$\begin{aligned} \tilde{\mathbf{H}}_{LMMSE,p}(i) &= [\tilde{H}_{LMMSE,p}(i,0) \tilde{H}_{LMMSE,p}(i,1) \cdots \tilde{H}_{LMMSE,p}(i,N_p-1)] \\ &= \mathbf{R}_{HpHp} \cdot \left(\mathbf{R}_{HpHp} + \frac{\beta}{SNR} \mathbf{I} \right)^{-1} \tilde{\mathbf{H}}_{LS,p}(i). \end{aligned} \quad (9)$$

, i denotes the index of a line. $\beta = E|X_p(k)|^2 E|X_p(k)|^{-2}$ is a constant depending on the signal constellation of different modulation type, shown in Table 1.

Table.1: β -value under various modulation types

| Type | BPSK | QPSK | 16QAM | 64QAM |
|---------|------|------|--------|--------|
| β | 1 | 1 | 1.8889 | 2.6854 |

The channel autocorrelation matrix in frequency domain can be expressed as

$$\begin{aligned} \mathbf{R}_{HpHp}(m,n) &= E[\mathbf{H}(i,m) \mathbf{H}^*(i,n)] \\ &= E \left[\sum_{k=0}^{N_p-1} h(i,k) \exp\left\{-\frac{j2\pi km}{N_p}\right\} \sum_{k=0}^{N_p-1} h^*(i,k) \exp\left\{-\frac{j2\pi kn}{N_p}\right\} \right] \\ &= \sum_{k=0}^{N_p-1} E[|h(i,k)|^2] \exp\left\{-\frac{j2\pi k(m-n)}{N_p}\right\}. \\ &= \sum_{k=0}^{L-1} \delta_l^2 \exp\left\{-\frac{j2\pi \tau_l(m-n)}{N_p}\right\}. \end{aligned} \quad (10)$$

, where L is the number of resolvable paths, τ_l is the delay of the l -th path in the unit of sample point, and δ_l^2 is the amplitude variance of the l -th path. The channel is normalized so that $\sum_l \delta_l^2 = 1$.

Unfortunately, in actual systems, the SNR is often unknown in advance and the \mathbf{R}_{HpHp} matrix is time varying. Therefore, it can be observed that the theoretical LMMSE CE is dependent on channel characteristics and is unavailable in practice.

C. Properties of the Channel Correlation Matrix

It is well known that \mathbf{R}_{HpHp} is a circulant matrix. Therefore, as in [7] [10], the eigenvalues of \mathbf{R}_{HpHp} can be given by

$$[\lambda_0 \lambda_1 \dots \lambda_{N-1}] = [FFT_N\{R_{HpHp}(0,0) R_{HpHp}(0,1) \dots R_{HpHp}(0,N-1)\}], \quad (11)$$

Formula (10) can be equivalently written as

$$\lambda_k = \sum_{n=0}^{N-1} R_{HpHp}(0,n) \exp\left\{-\frac{j2\pi nk}{N}\right\}, k = 0,1, \dots N-1. \quad (12)$$

IV. THE PROPOSED FAST LMMSE ALGORITHM

Regarding our 3GPP LTE OFDM system model, our algorithm is proposed under the limited conditions that, in frequency domain, pilot symbol shall be placed equally-spaced. Furthermore, the numbers of the subcarriers and the intervals between pilot symbols should be 2^n .¹

The proposed algorithm makes full use of the circularity of the channel autocorrelation matrix. Firstly, we utilize the most significant taps (*MST*) algorithm making energy detection on 'taps' and exploiting sparsity to refine our LS channel estimation in time domain and obtain the estimation of SNR. Then, we get the matrix \mathbf{R}_{HpHp} through circle shift and conjugate operation after IFFT operation. Finally, through matrix multiplication we complete the enhanced Fast LMMSE estimation.

The algorithm can be concluded as follows:

- Step1: Obtain the LS channel estimation from (7) at pilot subcarriers in time domain, using

$$\tilde{h}_{LS,p}(i,k) = \frac{1}{N_p} \sum_{n=0}^{N_p-1} \tilde{H}_{LS,p}(i,n) \exp\left\{\frac{j2\pi nk}{N_p}\right\}, k = 0,1, \dots N_p. \quad (13)$$

- Step2: Calculate the average power $P_{LS}(k)$ of each path and reserve the M most significant taps.

The *MST* algorithm we proposed deals with each OFDM symbol by simply reserving certain amount of the most significant paths in terms of power and setting the other taps into zero [9], regarding other taps as noise signal. The method can significantly reduce the influence caused by AWGN and other influences. To avoid the situation that it may pick the wrong paths or neglect the right paths under the influence of AWGN, we process several adjacent OFDM symbols jointly. Calculating the average power of each tap for N_{MST} adjacent OFDM symbols, thus, $P_{LS}(k)$ can be expressed by

$$P_{LS}(k) = \frac{1}{N_{MST}} \sum_{i=0}^{N_{MST}-1} |\tilde{h}_{LS,p}(i,k)|^2, k = 0,1, \dots N_p-1. \quad (14)$$

Then we store the indexes of the M most significant taps from $P_{LS}(k)$ into a set α . Eventually, the revised LS channel estimation $\tilde{h}_{p,MST}(i,k)$ in time domain, is given as

$$\tilde{h}_{p,MST}(i,k) = \begin{cases} \tilde{h}_{LS,p}(i,k) & \text{when } k \in \alpha \\ 0 & \text{when } k \notin \alpha \end{cases}. \quad (15)$$

$$P_{MST}(k) = \begin{cases} P_{LS}(k) & \text{when } k \in \alpha \\ 0 & \text{when } k \notin \alpha \end{cases}. \quad (16)$$

$$k = 0,1, \dots N_p-1, i = 0,1, \dots N_{MST}-1.$$

- Step3: Calculate the estimation of SNR, given by

$$\widetilde{SNR} = \frac{\sum_k P_{MST}(k)}{\sum_k P_{LS}(k) - \sum_k P_{MST}(k)}. \quad (17)$$

¹Such assumption is taken only for analytical convenience for the reason that 2^n point FFT is easy to calculate. Nevertheless, the methodologies proposed herein still apply in other situations.

- Step4: Calculate the matrix $\mathbf{R}_{HpHp} \cdot \left(\mathbf{R}_{HpHp} + \frac{\beta}{SNR} \mathbf{I} \right)^{-1}$.

The first line of matrix \mathbf{R}_{HpHp} is given from (9) by

$$\mathbf{A} = N_p \cdot \text{IFFT}[\mathbf{P}_{MST}], \quad (18)$$

since \mathbf{R}_{HpHp} is a circulant matrix, it can be obtained by circle shift and conjugate operation of \mathbf{A} . Similarly, the product of \mathbf{R}_{HpHp} and $\left(\mathbf{R}_{HpHp} + \frac{\beta}{SNR} \mathbf{I} \right)^{-1}$ is a circulant matrix as well. Therefore, we only need to compute the first line of the matrix by $\boldsymbol{\gamma}$, then the estimated LMMSE matrix can be obtained from circle shift of $\boldsymbol{\gamma}$.

$$\boldsymbol{\gamma} = \text{IFFT}_{N_p} \left[\frac{P_{MST(0)}}{P_{MST(0)} + \frac{\beta}{N_p SNR}} \frac{P_{MST(1)}}{P_{MST(1)} + \frac{\beta}{N_p SNR}} \cdots \frac{P_{MST(N_p-1)}}{P_{MST(N_p-1)} + \frac{\beta}{N_p SNR}} \right]. \quad (19)$$

- Step5: Obtain the final LMMSE channel estimation matrix at pilot symbol $\tilde{\mathbf{H}}_{LMMSE,p}(k)$ through matrix multiplication, given by ²:

$$\tilde{\mathbf{H}}_{LMMSE,p}(k) = \mathbf{R}_{HpHp} \cdot \left(\mathbf{R}_{HpHp} + \frac{\beta}{SNR} \mathbf{I} \right)^{-1} \cdot \tilde{\mathbf{H}}_{LS,p}(k). \quad (20)$$

- Step6: Achieve other regular data symbols using *FFT* interpolation in both time and frequency directions.

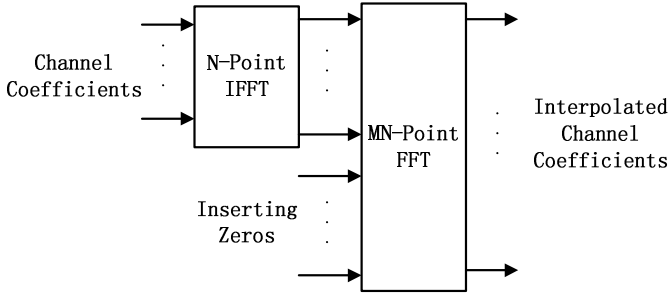


Figure 4: Low complexity FFT based channel estimator

V. ANALYSIS OF THE ENHANCED FAST LMMSE ALGORITHM AND SIMULATION RESULTS

A. Complexity Analysis

The computational complexity reduction ratio (CCRR) of the proposed LMMSE over the conventional scheme is defined as

$$CCRR = \left(1 - \frac{\text{complexity of the proposed scheme}}{\text{complexity of the conventional scheme}} \right) \times 100\% \quad (21)$$

It is well known that the conventional LMMSE has one matrix inversion operation, one matrix multiplication operation and one matrix addition operation, which requires $2N^3$ multiplications and $(N^2 + N^3)$ additions. In the proposed

²We can further lower the computation complexity and improve its performance by calculating \mathbf{R}_{HHp} instead of \mathbf{R}_{HpHp} , eliminating interpolation operation in frequency domain. \mathbf{R}_{HHp} is also a circulant matrix, which can be obtained by circle shift and conjugate operation of its first six lines, since the interval between two adjacent pilot symbols in frequency domain is five subcarriers as is shown in Fig 2 and 3.

algorithm, searching complexity is paid in Step2. However, much of the complexity is lowered by circle shift operation and the inverse operation is unnecessary. Complex multiplication and complex addition only need to be considered in IFFT during Step 5, which requires N^2 multiplications and N^2 additions. When $N=2048$, the CCRR of the proposed LMMSE is given in Table 2.

Table 2: LMMSE computation complexity (when $N=2048$)

| | Conventional | Proposed | CCRR |
|-------------------------|--------------|----------|--------|
| complex multiplications | 17179869184 | 4194304 | 99.96% |
| complex additions | 8594128896 | 4194304 | 99.95% |

B. Performance Analysis

Now, we consider the mean square error (MSE) and BER in both SISO- and MIMO- OFDM systems of the proposed fast LMMSE algorithm. The main system parameters are shown in Table 3. In addition, we use PB channel in Table 4 with the speed of 3km/h in our simulation.

Table 3: System Parameters

| | |
|----------------------------|---------------------|
| Carrier | 2GHz |
| Sampling Rate | 30.72MHz |
| FFT length | 2048 |
| CP length | 512 |
| Number of OFDM symbols | 14 |
| Number of used subcarriers | 1200 |
| Number of pilot | 150 |
| Channel Coding | 5/6 Rate Turbo Code |
| Modulation | QPSK |

Table 4: Rayleigh Fading Channel Parameters (PB channel)

| Path | Power(dB) | Delay(ns) |
|------|-----------|-----------|
| 1 | 0 | 0 |
| 2 | -0.9 | 200 |
| 3 | -4.9 | 800 |
| 4 | -8.0 | 1200 |
| 5 | -7.9 | 2300 |
| 6 | -23.9 | 3700 |

The MSE of the channel estimation is defined by

$$MSE = \frac{\sum_{i=0}^{K-1} \sum_{j=0}^{N_p-1} | \tilde{H}_p(i,j) - H_p(i,j) |^2}{\sum_{i=0}^{K-1} \sum_{j=0}^{N_p-1} | H_p(i,j) |^2} \quad (22)$$

where $\tilde{H}_p(i,j)$ denotes the channel estimate at the j -th pilot subcarrier (total number is N_p) in the i -th OFDM symbol and K denotes the number of OFDM symbols in simulation.

Fig.5 shows the channel estimation method under various algorithms in terms of *MSE* performance with respect to the channel power gain. The analytical results show that the proposed enhanced LMMSE-based method performs the best and has nearly 10dB SNR gain over LS, and 6dB SNR gain over exp LMMSE method when $MSE=10^{-2}$. (exp LMMSE method is the algorithm under the assumption that the channel meets negative exponential distribution).

Over frequency selective Rayleigh fading channel, the proposed technique performs quite close to ideal Wiener

estimation [7]. In PB channel we used, the proposed algorithm shows flattening effect MSE performance in higher SNRs due to model errors probably caused by the effect of estimation errors in the delay parameters. As model errors are unavoidable with PDP approximation, unless one changes the channel estimator length, all one can do is to choose a suitable PDP shape based on knowledge about the channel.

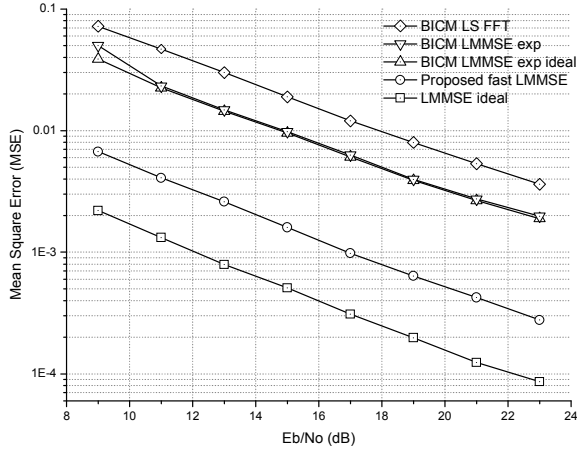


Figure 5: MSE performance of different channel estimation methods

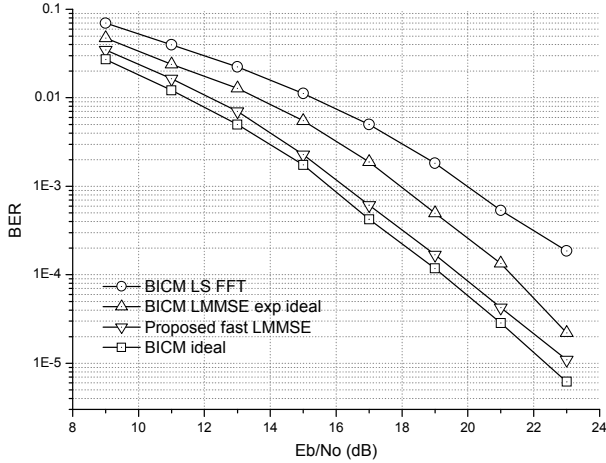


Figure 6: BER performance under SISO-OFDM System

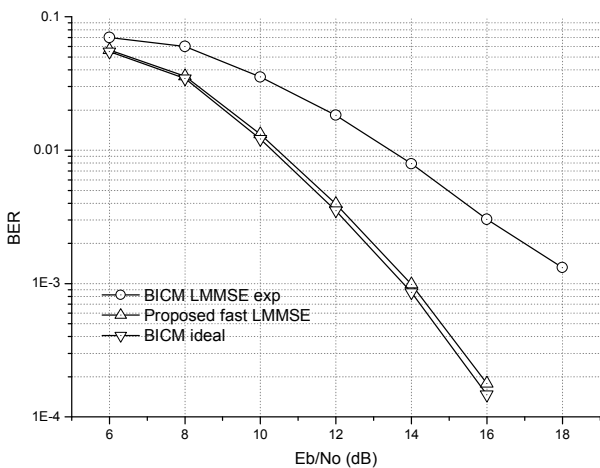


Figure 7: BER performance under MIMO-OFDM System

Fig.6 shows the BER performance of different channel estimation methods under SISO-OFDM system. The proposed LMMSE method achieves very close performance to ideal LMMSE method. When $BER=10^{-4}$, the gap is less than 1dB. Similar with the MSE performance, the proposed method gains much BER performance advantage over FFT-based method when SNR is high. The BER performance in MIMO-OFDM system is shown in Fig.7, which basically accord with BER performance in SISO-OFDM system.

VI. CONCLUSION

In this paper, an enhanced LMMSE-based channel estimation algorithm has been proposed for comb-type pilot aided OFDM system. Since the traditional LMMSE algorithm requires the channel statistics in advance, that is, the channel autocorrelation matrix and SNR, which are always time varying and unavailable in practical systems, its application is limited. Our proposed algorithm can effectively estimate the channel autocorrelation matrix by employing *MST* algorithm and exploiting the circulant property of the channel autocorrelation matrix, in which we can significantly reduce the computational complexity. Moreover, we compare the MSE and BER performance of the proposed method with LS, exp LMMSE and ideal LMMSE under Multipath Rayleigh fading channels. Computer simulations demonstrate that the proposed fast LMMSE algorithm with FFT interpolation outperforms exponential LMMSE estimation and achieves approximate performance to ideal LMMSE method.

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