Efficient Nonlinear Detector of Binary Signals in Rayleigh Fading and Impulsive Interference

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Abstract—The Middleton Class-A (MCA) model is one of the most widely applied models for narrow-band impulsive interference superimposed to additive white Gaussian noise (AWGN). The MCA noise process consists of an infinite number of Gaussian-distributed noise states with different variances. As a result, the optimum detector has irreducible form. Here, our analysis is based on a two-state model, where we further approximate it to a single noise state. Therefore, a log-function reduces the likelihood ratio test (LRT) to a closed-form expression. Since the low-pass equivalent of the noise process can be expressed by in-phase and quadrature (IQ) components. We derive the nonlinear decision rules when the IQ components of noise are independent and identically distributed (i.i.d.). Furthermore, for jointly distributed IQ noise components, we show that the conventional coherent detector over a fading channel with Gaussian noise is still optimum for impulse noise.

Index Terms—Rayleigh fading, Impulse interference, Class-A density, Nonlinear detector.

I. INTRODUCTION

Non-Gaussian distributions are widely used to model impulsive interference in a variety of some practical wireless systems. The interference exists in many channels such as radio frequency interference (RFI) in indoor and outdoor channels [1], [2], RFI generated by computers in embedded wireless data transceivers [3], and co-channel interference in a Poisson field of interferers [4]. The source of interference can be either natural or man-made such as atmospheric noise, power lines, ignition, and closely located wireless systems. There are many distributions for impulse noise such as an MCA density, symmetric-alpha stable distributions, and a generalized Gaussian density. The MCA model [2] appears to be more physically accurate in modeling narrow-band interference. This model has two basic parameters that can be adapted to fitting a wide variety of impulse noise phenomena occurring in practice.

The complex baseband representation of noise has two processes modeling IQ components. Regarding the underlying physical mechanisms that generate interference, there are two assumptions for the IQ components. The assumption of i.i.d. noise components has been considered in [5], which is accurate when the IQ processes are subject to independent fading channels such as co-channel interference [4]. The assumption of spherically symmetric interference [2] is more accurate than the i.i.d. assumption for near-field interference sources

such as power lines and ignition. In [6], [7], the design of the optimum detector over a fading channel with impulse noise is considered, where the noise distribution is assumed to be a spherically invariant random process. The authors show that the conventional coherent and incoherent detectors have the optimum performance for impulse noise. In [8], we show a wide area of nonlinear boundaries for the case of independent MCA noise samples, which results from the impulsive character of noise distribution. So far, there has been no investigation how the optimum IQ combiner should look like for i.i.d. IQ components. Moreover, there are no clear justifications why the conventional detector performs like the optimum detector in a spherically symmetric interference channel. The basic objectives of this paper can be summarized by two contributions. The primarily contribution is to derive a simple non-linear combiner in the case of i.i.d. noise components. The second contribution is to prove and justify why the conventional detector has an optimum performance when the interference is modeled by a spherically symmetric process.

This paper is organized as follows. Section II briefly describes the system model, and it provides a full picture of the noise process at the receiver. In Section III, we derive the nonlinear decision rules in the presence of i.i.d. and spherically symmetric interference. Finally, simulation results and concluding remarks are presented in sections IV and V, respectively.

II. SYSTEM MODEL

We consider a wireless communication channel of binary signal transmission corrupted by MCA interference. For simplicity, we restrict our analysis to binary phase-shift keying (BPSK). However, the generalization to an arbitrary M-ary signal sets is straightforward. We assume that the transmitted signals $s_k(t)$, $k = 0, 1$ use a rectangular pulse over $0 \leq t \leq T_b$. The BPSK signal is transmitted over a Rayleigh flat fading channel. Therefore, the equivalent low-pass received signal in one signaling interval is

$$r(t) = \sqrt{\frac{E_b}{N_0}} h s_k(t) + z(t), \quad k = 0, 1 \tag{1}$$

where $h = \alpha \mathrm{e}^{-j\theta}$ is a complex channel gain with Rayleigh distributed envelope and uniformly distributed phase. E_b is

the transmitted energy per bit and N_0 is the noise variance. The transmitted signals $s_1(t)$ and $s_0(t)$ correspond to a binary symbol $+1$ and -1, respectively and $z(t) = z_I(t) + jz_Q(t)$ denotes a complex noise process of zero mean and unit variance. The noise process as seen by the receiver includes two noise components: a Gaussian component $n(t)$ to represent the AWGN and impulsive component $i(t)$ due to the presence of interference from various sources. Hence, the received noise at the receiver is given by

$$z(t) = n(t) + i(t) , \qquad (2)$$

where $n(t)$ and $i(t)$ are assumed to be statistically independent. Similar to [2], we make the following assumptions: 1) the interference waveforms comprising $i(t)$ have the same form. However, their envelopes, durations, frequencies, and phases are randomly distributed. 2) the locations of interfering sources and their emission times are randomly distributed in space and time according to a homogeneous Poisson point process of rate λ. After matched-filtering and sampling, as illustrated in
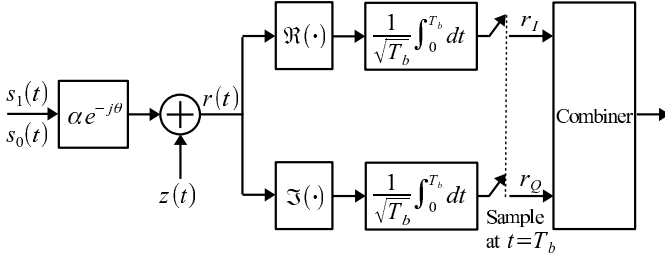

Fig. 1. A baseband model of binary signal transmission over a Rayleigh fading channel

Fig. 1, the samples of the IQ noise processes can be expressed as

$$z_{I,Q} = \frac{1}{\sqrt{T_b}} \int_0^{T_b} z_{I,Q}(t) dt . \qquad (3)$$

When the pulse duration of the interference waveforms comprising $i(t)$ is comparable to the bit duration T_b, the real and imaginary part of noise samples $z = z_I + jz_Q$ can be modeled by an MCA density [2]

$$p(z_{I,Q}) = \sum_{m=0}^{\infty} \frac{\beta_m}{\sqrt{\pi \sigma_m^2}} e^{-\frac{z_{I,Q}^2}{\sigma_m^2}} , \qquad (4)$$

where

$$\beta_m = \frac{e^{-A} A^m}{m!} \qquad (5)$$

and $\sigma_m^2 = \frac{m/A + \Gamma}{1 + \Gamma}$. From (4), we can show that $E[z_I^2] = E[z_Q^2] = \frac{1}{2}$. This model is well defined by two parameters A and Γ. The impulsive index, $A = \lambda T_b$, describes the average number of impulses during the bit interval T_b. The Gaussian factor, Γ, represents the power ratio of a Gaussian to impulsive part of noise. In (4), m can be seen as a noise state, i.e., $m = 0$ and $m \geq 1$ show that there is no impulse and the impulses are present, respectively. The noise state m is a Poisson distributed random variable such that the probability of being in a given state is equal to β_m.

Under the assumption of i.i.d. noise components, the noise vector $[z_I \; z_Q]$ has the following distribution

$$p(z_I, z_Q) = p(z_I)p(z_Q) . \qquad (6)$$

When the receiver is influenced by the same physical process creating the impulse, the i.i.d. assumption of the IQ components may not hold true. Therefore, a more accurate assumption is to assume that the IQ components are statistically dependent. Therefore, the IQ components are spherically symmetric random variables with the following bivariate MCA density

$$p(z_I, z_Q) = \sum_{m=0}^{\infty} \frac{\beta_m}{\pi \sigma_m^2} e^{-\frac{|z|^2}{\sigma_m^2}} , \qquad (7)$$

where $|z|^2 = z_I^2 + z_Q^2$. Throughout this paper, we consider both cases in deriving the proposed combiners of binary signals in the presence of MCA noise.

III. OPTIMUM DETECTOR

In the following analysis, we assume that the receiver has a priori knowledge of the exact impulse noise parameters. This is a reasonable assumption, since it has been shown that reliable estimates can be extracted from noisy samples [3]. We further assume that the channel gain h is known at the receiver. The IQ components of the received signal $r = r_I + jr_Q$ can be expressed as

$$\begin{aligned} r_I &= h_I s + z_I , \\ r_Q &= h_Q s + z_Q , \end{aligned} \qquad (8)$$

where h_I and h_Q represent the real and imaginary parts of h, respectively. $s \in \pm\sqrt{\frac{E_b}{N_0}}$ corresponds to the transmitted antipodal signal. assuming equiprobable transmitted symbols, the optimum detector computes the following likelihood ratio test (LRT):

$$\Lambda = \frac{p(r_I, r_Q | s_1)}{p(r_I, r_Q | s_0)} \mathop{\gtrless}_{s_0}^{s_1} 1 , \qquad (9)$$

where $p(r_I, r_Q | s_{1,0})$ are the conditional probability density functions (pdfs) of the observed samples given $s_{1,0}$. The hypotheses s_1 and s_0 correspond to $+1$ and -1, respectively. Since (6) and (7) contain a sum of exponential functions, the log function cannot be used to simplify (9). As a suboptimum solution, a linear receiver can be used, which is optimum when the interference is Gaussian. In this case, the following single decision variable is computed:

$$\Re(h^* r) = h_I r_I + h_Q r_Q . \qquad (10)$$

This combiner is equivalent to a channel phase compensation (coherent detection) for BPSK over a Rayleigh fading channel.

A. Statistically independent noise observations

Under an independence assumption, the LRT (9) can be expressed as

$$\Lambda = \frac{p(r_I | s_1)p(r_Q | s_1)}{p(r_I | s_0)p(r_Q | s_0)} \mathop{\gtrless}_{s_0}^{s_1} 1 . \qquad (11)$$

From (5), we can show that the noise state probability β_m tends to zero as m approaches infinity. Therefore, the infinite sum may be truncated to a finite sum. It was shown in [9] that the MCA density can be well approximated by a two-term model

$$p(z_{I,Q}) \approx \frac{\beta_0}{\sqrt{\pi\sigma_0^2}}e^{-\frac{z_{I,Q}^2}{\sigma_0^2}} + \frac{\beta_1}{\sqrt{\pi\sigma_1^2}}e^{-\frac{z_{I,Q}^2}{\sigma_1^2}} . \quad (12)$$

In this model, we have two noise states only, i.e., $m = 0$ and $m = 1$ corresponding to a Gaussian and impulsive noise state, respectively. The terms $\beta_0 = e^{-A}$ and $\beta_1 = 1 - e^{-A}$ represent the noise state probabilities. From (12), we note that when the receiver knows the noise state, the MCA density reduces to a single weighted Gaussian distribution. This assumption can be realized by determining the threshold when the densities of noise states are equal, the MCA model can be approximated as

$$p(z_{I,Q}) \approx \begin{cases} \frac{\beta_0}{\sqrt{\pi\sigma_0^2}}e^{-\frac{z_{I,Q}^2}{\sigma_0^2}} , & \text{if } -k_0 \leq z_{I,Q} \leq k_0 , \\ \frac{\beta_1}{\sqrt{\pi\sigma_1^2}}e^{-\frac{z_{I,Q}^2}{\sigma_1^2}} , & \text{otherwise} , \end{cases} \quad (13)$$

where $k_0 = \sqrt{\frac{\sigma_0^2\sigma_1^2}{\sigma_1^2-\sigma_0^2}\ln(\frac{\sigma_1\beta_0}{\sigma_0\beta_1})}$. Therefore, the likelihood functions $p(r_{I,Q}|s_{1,0})$ can be approximated to the corresponding Gaussian or impulsive state density as

$$p(r_{I,Q}|s_{1,0}) \approx$$
$$\begin{cases} \overbrace{\frac{\beta_0}{\sqrt{\pi\sigma_0^2}}e^{-\frac{\left(r_{I,Q}-sh_{I,Q}\right)^2}{\sigma_0^2}}}^{p(r_{I,Q}|s_{1,0},m=0)} , & \text{if } -k_0 \leq r_{I,Q} - sh_{I,Q} \leq k_0 , \\ \overbrace{\frac{\beta_1}{\sqrt{\pi\sigma_1^2}}e^{-\frac{\left(r_{I,Q}-sh_{I,Q}\right)^2}{\sigma_1^2}}}^{p(r_{I,Q}|s_{1,0},m=1)} , & \text{otherwise} , \end{cases}$$
$$(14)$$

where $s = \sqrt{\frac{E_b}{N_0}}$ corresponds to s_1 and $s = -\sqrt{\frac{E_b}{N_0}}$ corresponds to s_0. As we can see in (14), the likelihood functions $p(r_{I,Q}|s_{1,0})$ contain only one exponential function, then a log function would simplify the optimum detector. To derive a closed-form expression for a proposed detector, we start with a decision boundary analysis to determine the decision rules given the received observations $[r_I \; r_Q]$. Figure 2 depicts the regions of likelihood functions according to the noise state. We note that the likelihood functions $p(r_{I,Q}|s_{1,0})$ are centered at (sh_I, sh_Q). According to the values of fading coefficients, the detector will have one of 16 overlapping regions. In the region R_0, the impulsive states are the dominant terms of the likelihood functions for the received samples r_I and r_Q. Then, the decision rule can be calculated as

$$\Lambda_{R_0} = \frac{p(r_I|s_1,m=1)p(r_Q|s_1,m=1)}{p(r_I|s_0,m=1)p(r_Q|s_0,m=1)} \overset{s_1}{\underset{s_0}{\gtrless}} 1 . \quad (15)$$
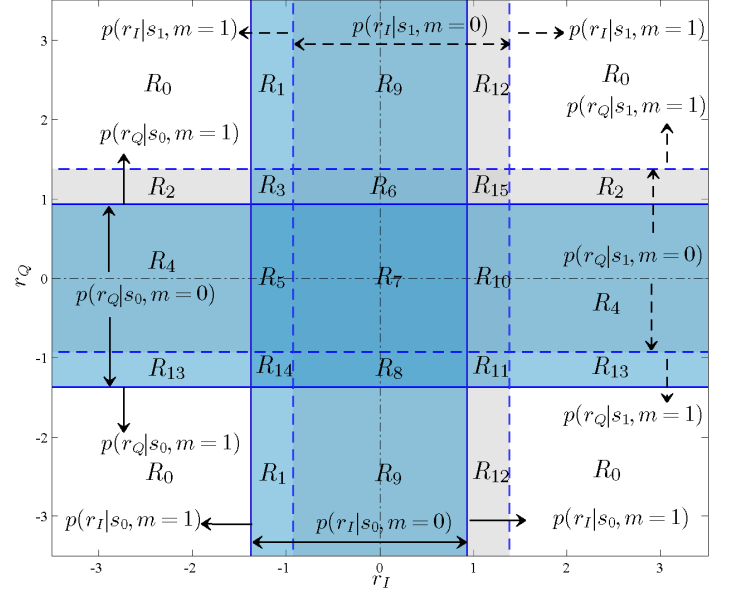

Fig. 2. Overlap regions of binary signals over Rayleigh fading with MCA noise

By taking the natural logarithm of both sides, we obtain

$$h_I r_I + h_Q r_Q \overset{s_1}{\underset{s_0}{\gtrless}} 0 . \quad (16)$$

In the region R_1, the impulsive states are the dominant terms of $p(r_I|s_1)$ and $p(r_Q|s_1)$. The Gaussian and impulsive terms are the states of $p(r_I|s_0)$ and $p(r_Q|s_0)$, respectively. Therefore, the decision rule is

$$\Lambda_{R_1} = \frac{p(r_I|s_1,m=1)p(r_Q|s_1,m=1)}{p(r_I|s_0,m=0)p(r_Q|s_0,m=1)} \overset{s_1}{\underset{s_0}{\gtrless}} 1 , \quad (17)$$

which leads to

$$r_I^2 + \frac{E_b}{N_0}h_I^2 + a_0 h_Q r_Q + b_0 h_I r_I - k_0^2 \overset{s_1}{\underset{s_0}{\gtrless}} 0 , \quad (18)$$

where $a_0 = 4\sqrt{\frac{E_b}{N_0}}\frac{\sigma_0^2}{\sigma_1^2-\sigma_0^2}$ and $b_0 = 2\sqrt{\frac{E_b}{N_0}}\frac{\sigma_1^2+\sigma_0^2}{\sigma_1^2-\sigma_0^2}$. Using the same analytical steps, the decision rules for the remaining regions can be solved as follows:

$$r_Q^2 + \frac{E_b}{N_0}h_Q^2 - b_0 h_Q r_Q - a_0 h_I r_I - k_0^2 , \; \forall r \in R_2 \quad (19)$$

$$r_I^2 + \frac{E_b}{N_0}h_I^2 - r_Q^2 - \frac{E_b}{N_0}h_Q^2 + b_0(h_I r_I + h_Q r_Q) , \; \forall r \in R_3 \quad (20)$$

$$\frac{h_I r_I}{\sigma_1^2} + \frac{h_Q r_Q}{\sigma_0^2} , \; \forall r \in R_4 \quad (21)$$

$$r_I^2 + \frac{E_b}{N_0}h_I^2 + a_1 h_Q r_Q + b_0 h_I r_I - k_0^2 , \; \forall r \in R_5 \quad (22)$$

where $a_1 = 4\sqrt{\frac{E_b}{N_0}}\frac{\sigma_1^2}{\sigma_1^2-\sigma_0^2}$.

$$r_Q^2 + \frac{E_b}{N_0}h_Q^2 - a_1 h_I r_I - b_0 h_Q r_Q - k_0^2 , \; \forall r \in R_6 \quad (23)$$

$$h_I r_I + h_Q r_Q , \; \forall r \in R_7 \quad (24)$$

$$r_Q^2 + \frac{E_b}{N_0}h_Q^2 + a_1 h_I r_I + b_0 h_Q r_Q - k_0^2 , \; \forall r \in R_8 \qquad (25)$$

$$\frac{h_I r_I}{\sigma_0^2} + \frac{h_Q r_Q}{\sigma_1^2} , \; \forall r \in R_9 \qquad (26)$$

$$r_I^2 + \frac{E_b}{N_0}h_I^2 - a_1 h_Q r_Q - b_0 h_I r_I - k_0^2 , \; \forall r \in R_{10} \qquad (27)$$

$$r_I^2 + \frac{E_b}{N_0}h_I^2 - r_Q^2 - \frac{E_b}{N_0}h_Q^2 - b_0(h_I r_I + h_Q r_Q) , \; \forall r \in R_{11} \quad (28)$$

$$r_I^2 + \frac{E_b}{N_0}h_I^2 - a_0 h_Q r_Q - b_0 h_I r_I - k_0^2 , \; \forall r \in R_{12} \qquad (29)$$

$$r_Q^2 + \frac{E_b}{N_0}h_Q^2 + a_0 h_I r_I + b_0 h_Q r_Q - k_0^2 . \; \forall r \in R_{13} \qquad (30)$$

The regions R_{14} and R_{15} are assigned to s_0 and s_1, respectively. As we can see in the above analysis, the proposed detector has nonlinear decision rules. That is, it differs from the conventional detector, which has only one linear decision boundary. Moreover, the proposed combiner approximates the optimum LRT (9) in different overlap regions with a closed-form solution.

B. Spherically symmetric noise observations

In this case, the IQ components of noise obesrvations can be modeled by a bivariate MCA distribution. The two-term model of (7) can be expressed as

$$p(z_I, z_Q) = \frac{\beta_0}{\pi \sigma_0^2} e^{-\frac{z_I^2 + z_Q^2}{\sigma_0^2}} + \frac{\beta_1}{\pi \sigma_1^2} e^{-\frac{z_I^2 + z_Q^2}{\sigma_1^2}} . \qquad (31)$$

The proposed approximation in (13) can be applied directly on (31) as follows:

$$p(z_I, z_Q) \approx \begin{cases} \frac{\beta_0}{\pi \sigma_0^2} e^{-\frac{z_I^2 + z_Q^2}{\sigma_0^2}} , & \text{if } z_I^2 + z_Q^2 \leq k_0^2 , \\ \frac{\beta_1}{\pi \sigma_1^2} e^{-\frac{z_I^2 + z_Q^2}{\sigma_1^2}} , & \text{otherwise} , \end{cases} \qquad (32)$$

where k_0 represents the radius of a circle, which depicts the boundary of equal noise state densities. The likelihood functions $p(r_I, r_Q | s_{1,0})$ reduce to

$$p(r_I, r_Q | s_{1,0}) \approx$$

$$\begin{cases} \overbrace{\frac{\beta_0}{\pi \sigma_0^2} e^{-\frac{(r_I - sh_I)^2 + (r_Q - sh_Q)^2}{\sigma_0^2}}}^{p(r_I, r_Q | s_{1,0}, m=0)} , & \text{if } |r - sh|^2 \leq k_0^2 , \\ \underbrace{\frac{\beta_1}{\pi \sigma_1^2} e^{-\frac{(r_I - sh_I)^2 + (r_Q - sh_Q)^2}{\sigma_1^2}}}_{p(r_I, r_Q | s_{1,0}, m=1)} , & \text{otherwise} , \end{cases} \qquad (33)$$

where $|r - sh|^2 = (r_I - sh_I)^2 + (r_Q - sh_Q)^2$. Figure 3 depicts the overlap regions for jointly distributed received observations. From this figure, each likelihood function $p(r_I, r_Q | s_1)$ and $p(r_I, r_Q | s_0)$ consists of two regions separated by a circle centered at $\left(\sqrt{\frac{E_b}{N_0}} h_I, \sqrt{\frac{E_b}{N_0}} h_Q \right)$ and $\left(-\sqrt{\frac{E_b}{N_0}} h_I, -\sqrt{\frac{E_b}{N_0}} h_Q \right)$, respectively. As illustrated in Fig. 3, there are two overlap
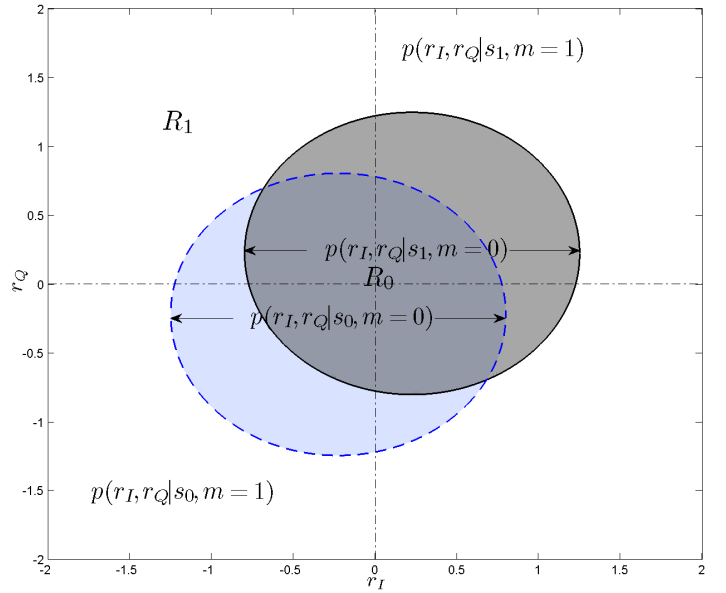

Fig. 3. The overlap regions for jointly distributed IQ components

regions, R_0 and R_1. In the region R_0 the decision boundary can be calculated as

$$p(r_I, r_Q | s_1, m = 0) = p(r_I, r_Q | s_0, m = 0) , \qquad (34)$$

and it can be solved as

$$h_I r_I + h_Q r_Q . \qquad (35)$$

In the region R_1, we have the following solution

$$h_I r_I + h_Q r_Q . \qquad (36)$$

From (35) and (36), the proposed detector has only a single linear decision boundary. Therefore, the optimum detector for spherically distributed IQ observations can be well approximated by a conventional detector. This confirms why both detectors have a similar performance for such a noise distribution.

IV. SIMULATION RESULTS

To validate our analysis, we simulate the bit-error ratio (BER) of BPSK over a Rayleigh fading channel with MCA interference. Our simulation is verified in different impulsive channels: a near Gaussian channel of $(A, \Gamma) = (1, \gg 1)$, a moderate impulsive channel with $(A, \Gamma) = (0.1, 0.01)$, and strong impulse noise with $(A, \Gamma) = (0.01, 10^{-4})$, which are within a practical range of $A \in [10^{-2}, 1]$ and $\Gamma \in [10^{-6}, 1]$ as specified in [10]. In simulating an MCA density, we truncate our model to the first 10 terms, which closely approximates the exact MCA density.

Figure 4 shows the BER performance of the proposed nonlinear detector, the optimum detector, and the conventional coherent detector for i.i.d. IQ components of impulse noise. As we can see in this figure, when the noise is almost Gaussian, all detectors have the same performance. As the channel impulsiveness increases, the performance of the optimum detector
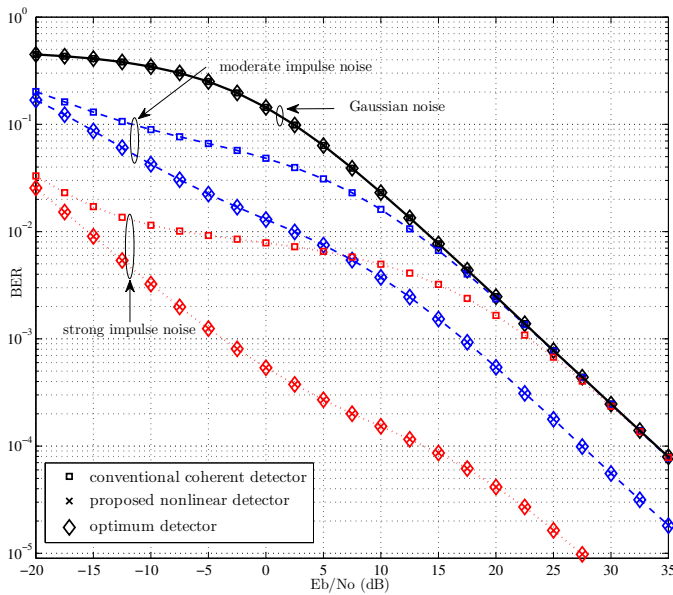
Fig. 4. Performance comparison over Rayleigh fading for i.i.d. noise observations

Fig. 5. Performance comparison over Rayleigh fading for spherically distributed noise observations

and the proposed nonlinear one improve significantly. This improvement exposes an important property of the optimum detection in impulse noise. The optimum detector extracts useful information of received impulses by considering nonlinear decision boundaries, even if the impulses are strong and have an opposite sign to the transmitted signal. Although the proposed detector is designed for a two-term noise model (13), it shows an almost optimum performance.

In Fig. 5, we compare the BER performance of the optimum and conventional coherent detectors when the IQ noise components are spherically distributed. We note that both detectors have the same performance, which confirms our analysis in Section III-B.

V. CONCLUSION

In this paper, we considered the detection of binary signals over Rayleigh fading channel in the presence of Middleton Class-A (MCA) interference. To reduce the analysis of optimum detector, we proposed a further approximation of the MCA density by estimating the noise state at the receiver. Using this approximation, we derived the nonlinear decision rules when the noise observation has independent identically distributed (i.i.d.) IQ components. In the case of spherically distributed IQ components, we approved that the conventional coherent detector is still optimum. Our analysis evaluation is confirmed by simulation and we showed that the proposed detectors provide an almost optimum performance in different impulse noise environments.

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