RACH Collision Probability for Machine-type Communications

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Abstract — **This paper summarizes the two definitions of the collision probability from the perspective of an MTC device and an RAO were presented in 3GPP TR 37.868. We use the results presented in TR 37.868 to show the inconsistency between the two definitions. It worth to note that the researcher needs to clearly specify the definition they used in presenting the results of their proposed radio access network overload control schemes. This paper further presents an analytical model to derive the collision probability, the success probability, and the idle probability based on the two definitions.**

Keywords- collision probability; machine-type communication (MTC); overload control

I. INTRODUCTION

Machine-type communications (MTC), which is also known as machine-to-machine (M2M) communication, is a new service defined by 3GPP to facilitate machines communicating with each other over existing cellular networks [1]. MTC normally involves a large number of MTC devices to support a wide range of applications such as metering, road security, and consumer electronic devices. However, concurrent accesses of a radio network by mass MTC devices may result in intolerable delays, packet loss or even service unavailability to existing human-to-human (H2H) communication services. Hence, proper overload control mechanisms are required to guarantee network availability and quality of H2H services under heavy MTC load [1].

An important technical document summarizing the state-of-the-art standard activities of MTC in 3GPP is TR 37.868 [1]. TR 37.868 is a technical report concluding the output of the study item on radio access network (RAN) improvements for MTC. The study item aims to investigate the traffic characteristics of different MTC applications and define new traffic models based on these findings [1]. Up to now, RAN overload control is identified as the first priority improvement area. The purpose of RAN overload control is to avoid high random-access collision probabilities when mass MTC devices contend for the random-access channel (RACH) at the same time. A number of solutions such as application-level time separation, specific back-off scheme, specific access class barring (ACB) scheme, and separate RACH resources have been discussed and can be applied to throttle the MTC traffic intensity [1].

In [2], the authors studied the random-access intensity generated by smart meters in London. The random-access intensity is the average number of random-access attempts per second and cell [3]. They estimated the number of household in a cell from 2001 census data for London and assumed that each household has 3 smart meters (electric, gas and water) and each meter operates independently as an MTC device and provides their reading as often as every hour [2]. Based on these assumptions, they concluded that the smart meters may generate up to 30 random-access attempts per second. Lots of standard contributions in 3GPP or IEEE 802.16p, the M2M Task Group of IEEE 802.16, estimated the random-access intensity generated by smart meters (e.g., [4]) based on this study.

In [2], the authors further adopted the capacity equation suggested in [3] to estimate the RACH capacity required to support the estimated random-access intensity. According to their analysis, the network needs 400 and 3000 random-access opportunities (RAOs) per second in Central London and the urban area of London, respectively [2]. In Long Term Evolution (LTE), the number of RAOs per second is given by the number of random-access slots per second multiplied by the number of random-access frequency bands in each random-access slot multiplied by the number of random-access preamble signatures [1]. Note that much more RAOs are needed if the smart meters generate their random-access attempts in a synchronous manner. For example, all electric meters may generate their attempts within one minute due to alarms triggered by smart meters. The attempts may also be generated within 10 s due to lack of clock synchronization in smart meters. In these cases, the random-access intensity may up to 595 and 3567 random-access attempts/s, respectively [1]. Both the capacity equation suggested in [3] and the analytical results presented in [2] are included in Annex B of [1].

3GPP TR 37.868 has defined several performance metrics for evaluating the performance of RAN overload control. An important performance metric is the collision probability [1]. As defined in Sec. 6.3 of TR 37.868, the collision probability is the ratio between the number of RAOs when two or more MTC devices send a random-access attempt using exactly the same preamble and the overall number of RAOs (with or without access attempts) in the period. The collision probability depends on the random-access intensity generated by MTC devices and the RACH capacity provided by the network. In [5], we demonstrated that the definitions of the collision probability used in Sec. 6.3 and in Annex B of TR 37.868 are quite different. Hence, we raised the issue in 3GPP RAN2#73bis meeting and request to choose one of the equations as the single definition of collision probability for

TR37.868. Considering the extra effort in re-running the simulations, the conclusion is to "accept discrepancy of the two definitions and realize that 6.3.1 takes the slot perspective rather than MTC device's perspective although some companies agreed to align the definition" [6]. However, it is important to note that different solutions should be compared based on the same definition.

This paper aims to point out the issue mentioned above and remind the researchers to use the proper definition of collision probability used in demonstrating their results. This paper further presents an analytical model for estimating the collision probability of the RAO (i.e., from slot's perspective) as defined in Sec. 6.3 of TR 37.868. The analytical results of the proposed model are then compared with that of Annex B of TR 37.868.

The rest of this work is organized as follows. Sec. II presents the system model and basic assumptions considered in this paper. We use a counterexample presented in [5] to illustrate the inconsistency between the two definitions in TR37.68. In Sec. III, an analytical model is then presented to derive the collision probability, the success probability, and the idle probability for Poisson arrivals. The numerical results are summarized in Sec. IV. Sec. V draws the conclusions.

II. System Model

This paper considers a large number of MTC devices locating in the same cell as illustrated in Fig. 1. The MTC devices use existing LTE radio access network to communicate with the MTC server through the serving gateway. This paper adopts the assumptions used in [3]. It is assumed that the arrival of random-access requests is uniformly distributed over time; a time-domain random-access structure of LTE is used; and a collision occurs only if multiple MTC devices make a random-access in the same random access opportunity (RAO) (i.e., transmit the same preamble signature in the same random-access slot and frequency band).

Two traffic models are assumed by 3GPP to evaluate the performance of RAN overload control schemes under different MTC access intensities. Traffic model 1 assumes that M MTC devices access the network following a uniform distribution over a period of time T. Traffic model 1 is considered as a realistic scenario in which MTC devices access the network uniformly over a period of time, i.e. in a *non-synchronized* manner [3]. Traffic model 2 assumes that M MTC devices access the network following a beta distribution over a distribution period of T. Traffic model 2 is considered as an extreme scenario in which a large amount of MTC devices access the network in a *highly synchronized* manner, for example, after a power outage [3]. T = 10 seconds and T = 60 seconds are both considered.

Two definitions of the collision probability are given in Sec. 6.3 and Annex B of TR 37.868, respectively. In Annex B of TR 37.868, it adopted the capacity equation proposed in [3] to estimate the collision probability experienced by an MTC device which transmits a preamble. This definition is defined based on MTC device's perspective. In Sec. 6.3 of TR 37.868, the collision probability is defined as the ratio between the number of occurrences when two or more MTC devices send a random-access attempt using exactly the same preamble and the overall number of opportunities (with or without access attempts) in the period [1]. In other word, the collision probability is defined based on slot's perspective. In the following, a counterexample is used to illustrate the inconsistency between the two definitions.

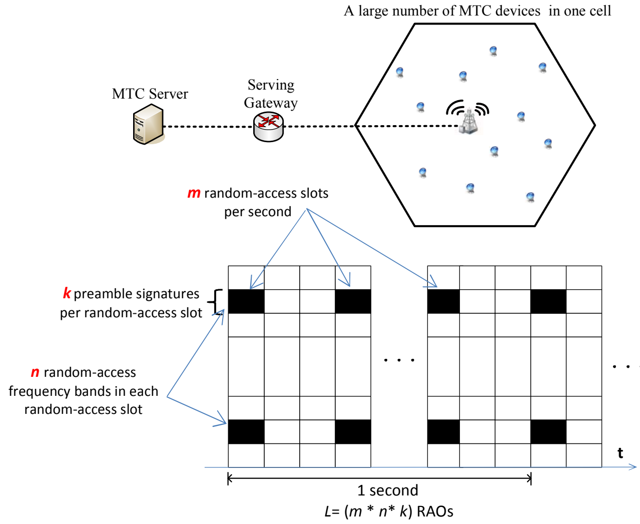

Fig. 1 System model

Table 1 shows the simulation result of the collision probability obtained based on the definition given in Sec. 6.3 of TR 37.868. Fig. 2 shows the analytical result of supported RACH intensity against the number of RACH opportunities per second for a given collision probability 1% based on Eq. (2), which follows the definition of Annex B in TR 37.868. In Table 1, a collision probability of **0.01%** is achieved for $\gamma = M/T$=5000/60=83 random-access attempts per second and cell (5000 MTC devices are uniform distribution over 60 sec. in traffic model 1); and L = 200*54 = 10800 (PRACH Configuration Index 6 with 54 preambles). However, from Eq. (2), the estimated collision probability is **0.78%**. Hence, there is a significant difference between the two definitions.

It can be further found from Fig. 2 that a thousand random-access opportunities per second (L = 1000) can support, on average, ten random-access attempts per second and cell (γ =10) for a collision probability of 0.01. It implies that most of the RAOs are not used. Let $P_{i,RAO}$ be the ratio between the number of idle RAOs and the overall number of RAOs (with or without access attempts) in the period. As we know, the successful random-access attempts cannot exceeds γ, which implies that the average number of idle RAOs per second and cell is no less than L-γ. Hence, the idle probability, $P_{i,RAO}$, can be derived as

$$P_{i,RAO} = \frac{\#\text{ of idle RAOs in } T \text{ sec.}}{\#\text{ of RAOs in } T \text{ sec.}} \geq \frac{L-\gamma}{L} = \frac{990}{1000} = 0.99. \quad (1)$$

It means that 99% of the RAOs are not used.

Table 1: Simulation results for RACH capacity for LTE FDD
(Source: TR37.868, Table 6.4.1.1.1)

Traffic Model	Performance measures	Number of MTC devices per cell (*M*)		
		5000	**10000**	**30000**
1	Collision Probability	0.01%	0.03%	0.22%
	Access Success Probability	100%	100%	100%
2	Collision Probability	0.45%	1.98%	47.76%
	Access Success Probability	100%	100%	29.5%

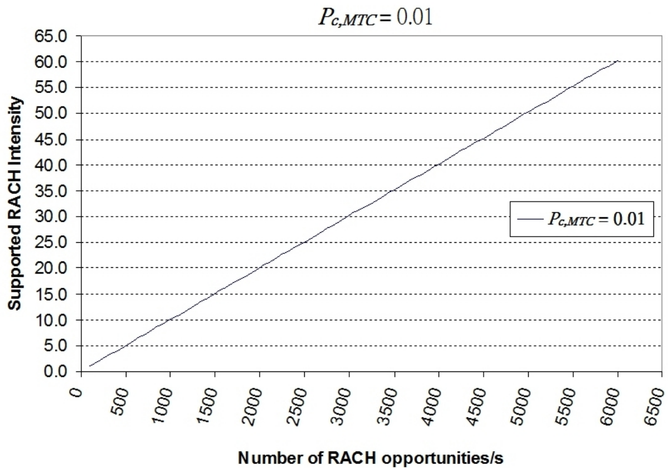

Fig. 2: Supported RACH Intensity against number of RACH opportunities per second for 1% collision probability (Source: TR37.868, Figure B.2)

III. ANALYTICAL MODEL

In this section, an analytical model is presented to estimate the collision probability, the success probability, and the idle probability for Poisson arrivals. We first introduce the analytical model shown in Annex B of TR 37.868 and then describe the proposed analytical model for the collision probability based on the definition given in Sec. 6.3 of TR 37.868.

The capacity equation proposed in [3] and in Annex B of TR 37.868 [1] is a function of the RACH resource and the random-access intensity. In LTE, RACH resource is determined in terms of RAOs, as illustrated in Fig.1. Let m, n, and k be the number of random-access slots in one second; the number of random-access frequency bands in each random-access slot; and the number of random-access preamble signatures per random-access slot. The total number of RAOs per second, L, is given by

$$L = m \times n \times k. \tag{2}$$

Hence, the random-access intensity for a specific RAO is γ/L if the overall random-access intensity is γ per second.

In Annex B of TR 37.868, the collision probability experienced by an MTC device which transmits a preamble, $P_{c,MTC}$, is given by [1,3]

$$P_{c,MTC} = 1 - e^{-\frac{\gamma}{L}}, \tag{3}$$

For Poisson arrival with arrival rate γ/L, $e^{-\frac{\gamma}{L}}$ is the probability that no one arrives in an RAO. Hence, the collision probability in Eq. (3) is the probability of collision under the condition of a preamble is transmitted. The success probability experienced by an MTC device which transmits a preamble, $P_{s,MTC}$, is the complement of $P_{c,MTC}$, and is given by

$$P_{s,MTC} = 1 - P_{c,MTC} = e^{-\frac{\gamma}{L}}. \tag{4}$$

In Sec. 6.3 of TR 37.868, the collision probability is defined as the ratio between the number of occurrences when two or more MTC devices send a random-access attempt using exactly the same preamble and the overall number of opportunities (with or without access attempts) in the period [1]. According to this definition, the collision probability can be derived as the ratio of total number of collided RAOs observed in T seconds and total number of RAOs provided in T seconds. That is, the collision probability of an RAO, $P_{c,RAO}$ is given by

$$\begin{aligned} P_{c,RAO} &= \frac{\#\text{ of collided RAOs in } T \text{ sec.}}{\#\text{ of RAOs in } T \text{ sec.}} \\ &= \frac{\#\text{ of RAOs in } T \text{ sec.} - \#\text{ of idle RAOs in } T \text{ sec.} - \#\text{ of successful RAOs in } T \text{ sec}}{\#\text{ of RAOs in } T \text{ sec.}} \\ &= 1 - \frac{\#\text{ of idle RAOs in } T \text{ sec.}}{\#\text{ of RAOs in } T \text{ sec.}} - \frac{\#\text{ of successful RAOs in } T \text{ sec}}{\#\text{ of RAOs in } T \text{ sec.}} \end{aligned} \tag{5}$$

Different to Eq. (3), Eq. (5) is an unconditional probability. The LTE random-access channel with L RAOs per second can be modeled as a slotted ALOHA system with slot duration of *1/L*. From slotted ALOHA, the average number of success random-access attempts (or successful RAOs) per second is equal to the random-access intensity multiply by the conditional successful transmission probability of the slotted ALOHA system ($\gamma e^{-\gamma\frac{1}{L}}$). Hence, the success probability of an RAO, $P_{s,RAO}$, is given by

$$P_{s,RAO} = \frac{\#\text{ of successful RAOs in } T \text{ sec}}{\#\text{ of RAOs in } T \text{ sec.}} = \frac{\gamma e^{-\gamma\frac{1}{L}} * T}{L * T} \tag{6}$$

The idle probability of an RAO, $P_{i,RAO}$, is the probability that no one arrives in an RAO. Similar to Eq. (3),

$$P_{i,RAO} = e^{-\frac{\gamma}{L}}. \tag{7}$$

Hence, the collision probability of an RAO defined in Eq. (4) can be rewritten as

$$P_{c,RAO} = 1 - e^{-\frac{\gamma}{L}} - \frac{\gamma}{L} e^{-\frac{\gamma}{L}}. \tag{8}$$

Compare Eqs. (3) and (8), it is found that collision probability experienced by an MTC device which transmit a preamble is much higher than the collision probability of an RAO. From Eqs. (4) and (6), it is found that the success probability experienced by an MTC device which transmits a preamble is equal to the idle probability of an RAO. It is because that the preamble transmission of an MTC device is success if the RAO is not used by the other MTC devices.

IV. SIMULATION RESULTS

Computer simulations are conducted to verify the accuracy of the proposed analytical model presented in Eqs. (6), (7), and (8). An error-free wireless channel without capture effect was considered. In the simulation, it is assumed that the network reserves L RAOs per second for MTC devices to transmit their access attempts. Time is divided into fixed-length slots. Some of the slots are reserved as RACH slots for MTC devices to transmit their access attempts. The RACH slots are uniformly distributed in time. In other words, the time intervals between any two successive RACH slots are equal. The MTC device which arrives in the time interval between two RACH slots shall wait and transmit its access attempt at the beginning of the next RACH slot. The access attempt is transmitted through a randomly chosen RAO in the RACH slot. The access attempts are collided if more than one access attempt choose the same RAO in the same RACH slot. The collided access attempts, which are referred as the backlogged access attempts, herein, shall perform random backoff for further re-transmission. In the simulation, it is assumed that the new and backlogged access attempts generated by MTC devices follow a Poisson process with mean γ=10. In the simulations, each point represents the statistics obtained from 10^7 seconds. In the following figures, lines and symbols are used to represent the analytical and simulation results, respectively.

Two configurations were considered to investigate the effect of the RACH slot's distribution on the collision probability. In Configuration 1, one RACH slot per second with L preambles per RACH slot is used. In Configuration 2, L RACH slots with 1 preamble per RACH slot is used. The L RACH slots are uniformly distributed in one second. Fig. 3 shows that the simulation results of $P_{s,RAO}$. $P_{i,RAO}$ and $P_{c,RAO}$ in Configurations 1 and 2 were almost identical. It implies that the collision probability may not be affected by the distribution of the RACH slots.

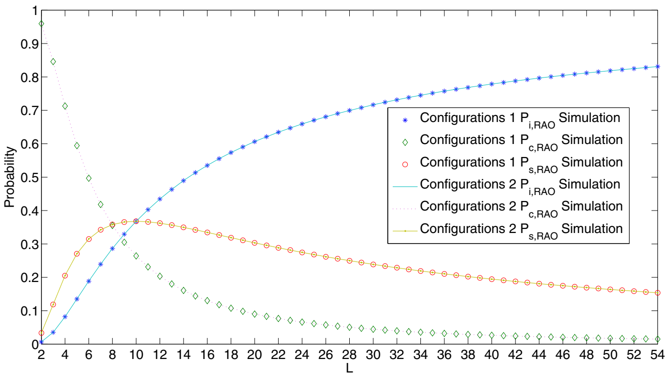

Fig. 3: Simulation results for γ=10 and L = 2 to 54 (statistics obtained based on the definition of Sec. 6.3.1 of TR 37.868).

Fig. 4 shows the simulation results of $P_{c,MTC}$ and $P_{s,MTC}$ for γ =10 and L = 2 to 54. The statistics were obtained based on the definition of Annex B of TR 37.868 as given in Eqs. (3) and (4), respectively. That is, the collision probability experienced by an MTC device which transmits a preamble, $P_{c,MTC}$, is obtained as the ratio between the number of collided access attempts (two or more MTC devices send their random-access attempt using exactly the same preamble) and the overall number of access attempts in the period. Similarly, the success probability experienced by an MTC device which transmits a preamble, $P_{s,MTC}$, is obtained as the ratio between the number of success access attempts (a preamble is used by only one random-access attempt) and the overall number of access attempts in the period. As shown in Fig. 4, the Eqs. (3) and (4) can properly predict the simulation results. It is found $P_{c,MTC}$ is monotonously decreased and $P_{s,MTC}$ is monotonously increased when we increase L. It is because that $P_{c,MTC}$ and $P_{s,MTC}$ are defined from MTC devices' perspective. For a fixed offered load γ, the number of access attempts generated by the MTC devices is fixed. Therefore, the collision probability decreases if we provide more RAOs (i.e., increasing L).

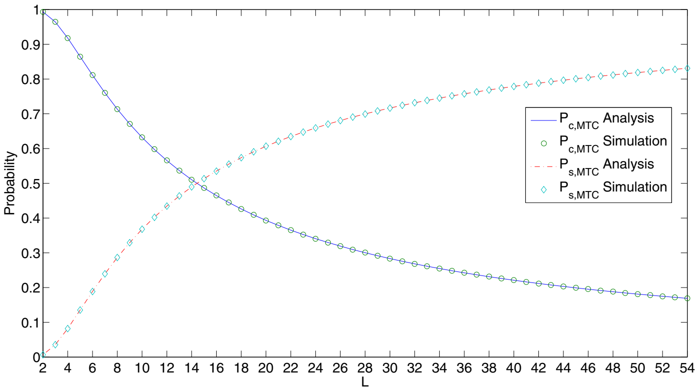

Fig. 4: Simulation results for γ=10 and L = 2 to 54 (statistics obtained based on the definition of Annex B of TR 37.868).

Fig. 5 shows the simulation and analytical results of $P_{s,RAO}$. $P_{i,RAO}$ and $P_{c,RAO}$ for γ=10 and L = 2 to 54. The simulation results of $P_{c,RAO}$ were obtained based on the definition of Sec. 6.3.1 of TR 37.868. The analytical results of $P_{s,RAO}$, $P_{i,RAO}$ and $P_{c,RAO}$ are derived from Eqs. (6) to (8), respectively. It shows that Eqs. (6) to (8) can accurately estimate the simulation results of $P_{s,RAO}$, $P_{i,RAO}$ and $P_{c,RAO}$. It is found that increasing L will result in a lower $P_{c,RAO}$, which is similar to that of $P_{c,MTC}$. However, $P_{s,RAO}$ is first increased and then decreased by increasing L. The maximum value of $P_{s,RAO}$ occurs when γ is equal to L. It is because that $P_{s,RAO}$, $P_{i,RAO}$ and $P_{c,RAO}$ are defined from slot's perspective (i.e., more precisely, it is defined from RAO's perspective). Therefore, offering more RAOs results in more success access attempts and thus, a higher $P_{s,RAO}$, in overloaded situation ($L < \gamma$). However, further increasing L may not increase $P_{s,RAO}$ because most of RAOs are wasted in underused situation ($L > \gamma$). Similarly, increasing L results in a higher $P_{i,RAO}$ because more RAOs are not used.

Fig. 6 demonstrates the analytical results of $P_{c,MTC}$ and $P_{c,RAO}$. It shows that the collision probability given in Eq. (5) (i.e., $P_{c,RAO}$) decreases much faster than that of Eq. (3) (i.e., $P_{c,MTC}$). It implies that, for a given target collision probability, Eq. (3) requires more RAOs than Eq. (5) does.

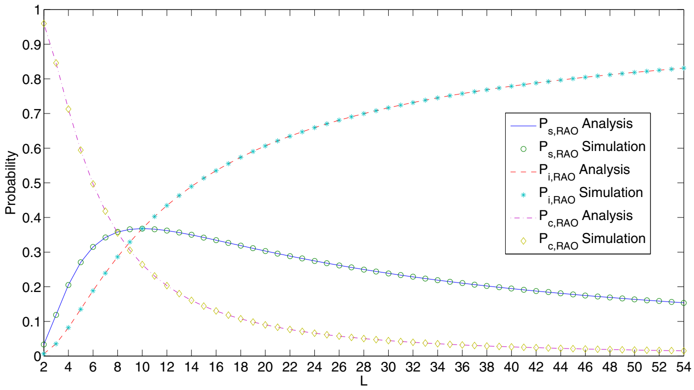

Fig. 5: Simulation results for γ=10 and L = 2 to 54 (statistics obtained based on the definition of Sec. 6.3.1 of TR 37.868).

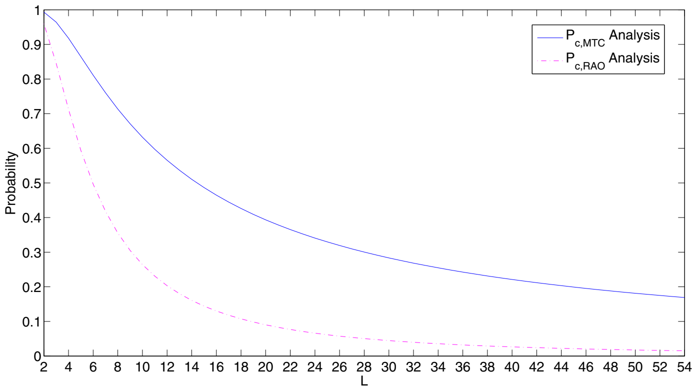

Fig. 6: Analysis results of $P_{c,MTC}$ and $P_{c,RAO}$ for γ=10 and L = 2 to 54

V. Conclusions and Future Works

This paper summarizes the two definitions of the collision probability used in Sec. 6.3 and Annex B of 3GPP TR 37.868. We use an example to illustrate the inconsistency of the two definitions and then demonstrate that the two definitions are derived from the perspective of an MTC device and an RAO. An analytical model is then presented to derive the collision probability, the success probability, and the idle probability based on the two definitions. It is worth to note that different solutions of the RAN overload control should be compared based on the same definition. The researcher needs to clearly specify the definition they used in presenting the results.

It should be noted that the analytical model presented in this paper is valid only if the MTC traffic follows a Poisson process with constant arrival rate. The performance metric of the collision probability is derived based on the steady-state behavior of the system. However, the Poisson assumption is valid for H2H services but may not be applicable for bursty MTC traffic. Normally, the RAN overload may be resulted from unexpected heavy MTC traffic generated in a short period of time, which has been specified in 3GPP TR 37.868 [1]. Therefore, an important future work is to develop a proper analytical model to investigate the transient behavior of the random access system triggered by the bursty MTC traffic with non-Poisson arrivals.

Another important future work is to further extend the analytical model to derive the rest of the performance metrics defined in 3GPP TR 37.868 [1]. The performance metrics include *access success probability*, *statistics of number of preamble transmissions*, and *statistics of access delay*. It should be pointed out that the access success probability is neither $P_{s,MTC}$ nor $P_{s,RAO}$ derived in this paper. The access success probability is defined as the probability to successfully complete the random access procedure within the maximum number of preamble transmissions. To analyze these performance metrics, the analytical model needs to consider the effect of LTE implementation constraints [1] such as the LTE random backoff procedure and the time-varying detection probability resulted from the power ramping effect.

Acknowledgment

The authors would like to thank anonymous reviewers for their valuable comments, which help to improve the quality of the presentation. This work was supported in part by the National Science Council (NSC), Taiwan, under Contract NSC 100-2219-E-002-004.

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