Threshold-Triggered Selective Phase-Forward of Differential PSK in Cooperative Communication*

Huai Tan and Paul Ho

***Abstract*—We study in this paper the performance of a one-way cooperative transmission system using differential PSK (DPSK) modulation with differential detection. As opposed to previous works which consider either a decode-and-forward (DF) or an amplify-and-forward (AF) relay, we adopt in this paper a phase-forward (PF) relay, whereby each forwarded symbol has constant modulus and a phase equals the phase of the corresponding relay's received symbol. The rationale for adopting this relaying strategy is to avoid potential non-linear amplifier distortion in an amplify-and-forward relay, as well as the implicit information loss/quantization in a DF relay. Through analysis and simulation, we found that this PF-DPSK cooperative transmission scheme has a lower bit-error rate (BER) than that of its DF counterpart. Furthermore, by adopting a threshold-based selective forwarding approach, it can attain a BER similar to that of AF. Finally, we anticipate that PF is most useful in two or multi-way relaying, in which any non-linear amplifier distortion on an AF signal will manifest into significant inter-modulation.**

Indexing terms - *cooperative diversity, phase-forward, differential modulation, threshold-triggered forwarding.*

I. Introduction

Coperative transmission [1-3] has received much attention from the wireless research community as it provides means to form a virtual multiple input multiple output (MIMO) system requiring only a single antenna at each node in the system. Commonly adopted relaying protocols for cooperative transmission are amplify-and-forward (AF), compress-and-forward (CF), and decode-and-forward (DF). In DF, the relay makes a decision on the source signal and then regenerates it for transmission to the destination. This process requires substantial processing at the relay, which can be undesirable. In contrast, the AF protocol is relatively easy to implement as it bypasses the signal demodulation and regeneration process altogether. However, one disadvantage of AF is that in a fading channel, the forwarded signal can have large amplitude fluctuation, rendering the forwarded signal very sensitive to amplifier non-linear distortion. Although one can rectify the situation by using a highly linear amplifier, such amplifiers are expensive and power-inefficient.

A common assumption shared by many studies in the cooperative communication literature is that channel state information (CSI) is readily available [4-6] at the relays/destination and coherent demodulation can be performed at the destination. Since in a practical system, the CSI available will not be perfect, the authors of [7,8] study the impact of CSI error on system performance. In any event, whether the CSI is perfect or not, it requires additional transmission overhead, thus reducing the effective data throughput. In [9], Himsoon et. al. consider the use of differential PSK (DPSK) modulation, with differential detection, as a means to eliminate the need to estimate the CSI altogether. In [10], Bhatnagar derives a maximum likelihood detector for the DF relay. Similarly in [11], Yang et. al. adopt non-coherent discriminator detection of a phase-forward (PF) cooperative transmission scheme that employs continuous- phase frequency shift keying (CPFSK) modulation. Not only is channel estimation avoided through non-coherent detection, the adoption of PF in conjunction with CPFSK in [11] means that constant envelope transmission can be achieved at both the source and the relay, thus avoiding the problems associated with amplifier non-linearity and AF. The results in [11] show that for CPFSK modulation, PF provides a lower bit-error rate (BER) than DF. Compared to AF, PF attains the same BER performance in a static fading channel. However, in a time-selective fading channel, PF has a substantially poorer performance unless receive antenna diversity is used at the relay.

In this paper, we study the performance of a one-way cooperative transmission system that employs PF and differential PSK modulation with differential detection. For convenience, we will refer to this configuration as PF-DPSK. Such a PF-DPSK scheme will alleviate the cooperative transmission system the problems associated with AF and non-linear power amplification, especially the amplitude-to-phase (AM-PM) distortion [12]. Moreover, compared to the (nonlinear) CPFSK scheme in [11], the use of (linear) PSK modulation has the potential of a higher spectral efficiency. We will show later in the paper that by adopting a threshold-based selective forwarding approach, PF can indeed attain the same performance as AF but without all the afore-mentioned problems associated with the latter. It should be pointed out while the concept of selective relaying is not new, our approach does not require the relay to make any data decision, as opposed to the case of [13], thus avoiding unnecessary processing. In addition, we use an amplitude threshold over two consecutive intervals for selective forwarding, instead of a signal-to-noise (SNR) threshold [14,15] that may not be readily available.

The remainder of the paper is outlined as following. Section II describes the transmission and system models. Two phase-forward approaches are presented in this section: *continuous forwarding* and *selective forwarding*. The BER of the two PF approaches are derived in Section III, with the end results provided in semi-analytical form. Both analytical and simulation results of the BER of the proposed PF systems are presented in Section IV. Finally, Section V provides a summary and conclusion of this investigation.

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II. PF-DPSK System Model

We consider a 3 node cooperative network consisting of a source node S, a half-duplex relay node R, and a destination node D. Cooperative transmission over this network is organized into cycles, with each cycle consisting of two equal length transmission phases. In Phase 1 of each cycle, S transmits a block of data to both R and D. In Phase 2, R processes the signal it received in the previous phase from S and forwards it to D. In the following, we will use a discrete-time signal model to describe the detail characteristics of this network.

At the physical layer level, each transmission link exhibits time-selective Rayleigh flat fading with additive white Gaussian noise (AWGN). This means if $x_i[k]$ is the symbol transmitted by node i at time, where $i \in \{S, R\}$ depending on the transmission phase, then the corresponding received symbol at node j, $j \in \{R, D\}$ is

$$y_{ij}[k] = g_{ij}[k] x_i[k] + n_{ij}[k] \tag{1}$$

where $g_{ij}[k]$ is the channel fading gain from node i to j, and $n_{ij}[k]$ is the corresponding AWGN at node j. We assume the $g_{ij}[k]$'s in the different links are i.i.d. $CN\left(0, \sigma_{ij}^2\right)$. Similarly, the $n_{ij}[k]$'s are i.i.d. $CN(0, \sigma_N^2)$. The link SNR is defined as $\gamma_{ij} = \sigma_{ij}^2 / \sigma_N^2$. In addition, we allow time-selective fading in our link model. Specifically, the autocorrelation function of $g_{ij}[k]$ is $\lambda_{ij}[m] = \frac{1}{2} E\left[g_{ij}[k] g_{ij}^*[k-m]\right] = \sigma_{ij}^2 J_0(2\pi m f_d T_s)$, where f_d is the Doppler frequency (assumed the same in all the links) and $J_0(.)$ is the zero-th order Bessel function.

Regarding the transmitted signal format, each symbol transmitted by the source is a differentially encoded symbol of the form $x_S[k] = e^{j\theta[k]}$, where $\theta[k]$ is the differentially encoded phase at time k and it is related to the actual data phase $\phi[k]$ of the data symbol $d[k] = e^{j\phi[k]}$ according to $\theta[k] = \theta[k-1] + \phi[k]$. Assuming differential M-ary PSK modulation (DMPSK), then $\phi[k]$ is taken randomly from the set $S \in \{2\pi n/M\,; n = 0, 1, \ldots, M-1\}$ with equal probability.

Based on the link model in (1), the received versions of $x_S[k]$ at the destination and the relays are,

$$\begin{aligned} y_{SR}[k] &= g_{SR}[k] x_S[k] + n_{SR}[k] \\ y_{SD}[k] &= g_{SD}[k] x_S[k] + n_{SD}[k] \end{aligned} \tag{2}$$

Since statistically, $n_{SR}[k]$ and $n_{SR}[k] e^{j\theta[k]}$ are identical, so we can rewrite the received sample at the relay as

$$\begin{aligned} y_{SR}[k] &= g_{SR}[k] e^{j\theta[k]} + n_{SR}[k] e^{j\theta[k]} \\ &= (g_{SR}[k] + n_{SR}[k]) e^{j\theta[k]} \\ &= a[k] e^{j\psi[k]} e^{j\theta[k]} \end{aligned} \tag{3}$$

where $a[k]$ and $\psi[k]$ are respectively the amplitude and phase of $g_{SR}[k] + n_{SR}[k]$. It is evident from this equation that the received phase at the relay is the transmitted phase subjected to an additive disturbance equaling the phase of the combined fading and noise process. As far as a differential detector is concerned, it is this combined phase term $e^{j\psi[k]} e^{j\theta[k]}$ in $y_{SR}[k]$ that is relevant. The amplitude component $a[k]$ carries no information about the transmitted phase and hence need not be included in the forwarded signal. Not including $a[k]$ in the forwarded signal allows the relay to transmit a constant envelope signal and avoid potential AM-PM distortion arising from the use of a non-linear power amplifier. This is the basis of the phase-forward idea described next. Specifically we consider two PF approaches: continuous and selective phase forward. The signal models for the two cases are described in the following subsections.

A. Continuous PF

In Continuous PF, the relay transmits at each time instant k the complex symbol

$$x_R[k] = e^{j\psi[k]} e^{j\theta[k]} \tag{4}$$

which is simply the phase component of the relay's received signal in (3). Using the link model in (1), the corresponding received signal at the destination is

$$\begin{aligned} y_{RD}[k] &= g_{RD}[k] x_R[k] + n_{RD}[k] \\ &= g_{RD}[k] e^{j\psi[k]} e^{j\theta[k]} + n_{RD}[k] \end{aligned} \tag{5}$$

After receiving both $y_{SD}[k]$ and $y_{RD}[k]$ from Phase 1 and 2 respectively, the destination performs differential detection. The differential detected symbols $D_{SD}[k] = y_{SD}^*[k-1] y_{SD}[k]$ and $D_{RD}[k] = y_{RD}^*[k-1] y_{RD}[k]$ are then combined to obtain the decision variable

$$D[k] = 2\,\mathcal{Re}\{D_{SD}[k] + D_{RD}[k]\}. \tag{6}$$

The decision on the data symbol $d[k]$ is

$$\hat{d}[k] = \underset{e^{j\frac{2\pi n}{M}},\ n=0,1,\ldots,M-1}{\operatorname{argmax}} \mathcal{Re}\left\{e^{-j\frac{2\pi n}{M}} D[k]\right\}. \tag{7}$$

B. Selective PF

In continuous PF, the relays always forwards the phase of its received signal to the destination, ignoring the fact that some sections of the signal is of poor signal quality because of deep fades in the S-R link. This indiscriminate approach could create a negative effect on the BER of the decoded data because, when the transmitted signal goes through a deep fade in the S-R link, then the received phase at R may have little resemblance to the transmitted phase. Forwarding such a random phase to the destination will actually deteriorate (instead of boosting) the signal quality of the combined signal $D[k]$ in (6). In these situations, it may actually be more beneficial not to have signal forwarding at all and instead ask the destination to make data decisions based on the signal it receives directly from S. This conjecture brings us to the selective PF approach described below.

In the proposed selective PF protocol, the relay forwards the phase of its received signal only when the S-R link quality, measured by the received signal amplitude at R, exceeds a certain threshold in two consecutive symbol intervals. The rationale behind this is that, with differential encoding and detection, the receiver can only detect the information in the current interval reliably when the quality of the received signal in the current and last intervals are BOTH good. Specifically, we set the value of the k-th symbol emitted by R to

$$x_R[k] = \begin{cases} e^{j\psi[k]} e^{j\theta[k]}, & a[k] \geq T \text{ and } a[k-1] \geq T \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where $a[k]$ is the signal amplitude defined in (3), and T is the threshold that decides if forwarding should take place or not. The corresponding received signal at D is thus

$$y_{RD}[k] = \begin{cases} g_{RD}[k]x_R[k] + n_{RD}[k], & a[k] \geq T \text{ and } a[k-1] \geq T \\ n_{RD}[k], & \text{otherwise} \end{cases} \tag{9}$$

By comparing the amplitude of this received signal against another threshold, the destination can detect if phase-forwarding had taken place or not. If PF is detected, then the destination will perform differential detection and signal combining of the direct and relay link signals, just like that in the continuous PF system; see (6). On the other hand, if PF is not detected, the destination will simply rely on the direct link received signal in making data decisions. In this case, the decision variable is simply

$$D[k] = D_{SD}[k] \tag{10}$$

For simplicity, we assume in this investigation that the destination can always tell if the relay has forwarded a symbol or not.

III. Bit Error Rate Analysis

In this section we provide semi-analytical expressions for the BER of the continuous and selective PF schemes described in the last section. For simplicity, we assume differential BPSK modulation. As it will be shown shortly, the BER of selective PF is that of continuous PF minus a threshold-dependent adjustment term.

A. Continuous PF

To analyze the BER performance of the continuous PF scheme, we start by examining the decision variables $D_{SD}[k]$ and $D_{RD}[k]$ in (6) individually. First, the direct path decision variable $D_{SD}[k]$ can be written as a quadratic form of complex Gaussian random variables as

$$D_{SD} = 2\,\mathcal{Re}\{y_{SD}^*[k-1]y_{SD}[k]\} = \boldsymbol{y}_{SD}^{\dagger}\boldsymbol{F}\boldsymbol{y}_{SD} \tag{11}$$

where

$$\boldsymbol{y}_{SD} = \begin{bmatrix} y_{SD}[k-1] \\ y_{SD}[k] \end{bmatrix} \text{ and } \boldsymbol{F} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The vector $\boldsymbol{y}_{SD}$ has a covariance matrix of

$$\boldsymbol{R}_{SD} = E[\boldsymbol{y}_{SD}\boldsymbol{y}_{SD}^{\dagger}] = \begin{bmatrix} \sigma_{SD}^2 + \sigma_N^2 & \sigma_{SD}^2\rho \\ \sigma_{SD}^2\rho & \sigma_{SD}^2 + \sigma_N^2 \end{bmatrix} \tag{12}$$

where $\rho = J_0(2\pi f_d T_s)$. The characteristic function of $D_{SD}[k]$ is [11]

$$\boldsymbol{\Phi}_{SD}(s) = \frac{p_1 p_2}{(s - p_1)(s - p_2)} \tag{13}$$

where the poles p_1 and p_2 are

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{2[\sigma_{SD}^2(1+\rho) + \sigma_N^2]} \\ \dfrac{1}{2[\sigma_{SD}^2(1-\rho) + \sigma_N^2]} \end{bmatrix} \tag{14}$$

Similarly, the relay path decision variable $D_{RD}[k]$ has the following covariance matrix and poles when conditioned on the phase difference $\Delta\psi = \psi[k] - \psi[k-1]$:

$$\boldsymbol{R}_{RD} = E[\boldsymbol{y}_{RD}\boldsymbol{y}_{RD}^{\dagger}] = \begin{bmatrix} \sigma_{RD}^2 + \sigma_N^2 & \sigma_{RD}^2\rho e^{-j\Delta\psi} \\ \sigma_{RD}^2\rho e^{j\Delta\psi} & \sigma_{RD}^2 + \sigma_N^2 \end{bmatrix}$$

$$\boldsymbol{\Phi}_{RD}(s) = \frac{q_1 q_2}{(s - q_1)(s - q_2)}$$

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\sigma_{RD}^2\rho\cos\Delta\psi - \sqrt{(\sigma_{RD}^2 + \sigma_N^2)^2 - (\sigma_{RD}^2\rho\sin\Delta\psi)^2}}{2[(\sigma_{RD}^2 + \sigma_N^2)^2 - (\sigma_{RD}^2\rho)^2]} \\ \dfrac{\sigma_{RD}^2\rho\cos\Delta\psi + \sqrt{(\sigma_{RD}^2 + \sigma_N^2)^2 - (\sigma_{RD}^2\rho\sin\Delta\psi)^2}}{2[(\sigma_{RD}^2 + \sigma_N^2)^2 - (\sigma_{RD}^2\rho)^2]} \end{bmatrix} \tag{15}$$

When conditioned on the transmitted symbol $d[k]$, the decision variables $D_{SD}[k]$ and $D_{RD}[k]$ are independent. Consequently, the characteristic function of the decision variable $D[k]$ is simply the product $\boldsymbol{\Phi}(s) = \boldsymbol{\Phi}_{SD}(s)\boldsymbol{\Phi}_{RD}(s)$. Assuming $d[k] = 1$, then the conditional BER is the probability that $D[k]$ is less than zero. It can be determined from $\boldsymbol{\Phi}(s)/s$ using the residual theorem. The end result is

$$P_{e|\Delta\psi}^{CPF} = \left(\frac{-p_1}{p_2 - p_1}\right)\frac{q_1 q_2}{(p_2 - q_1)(p_2 - q_2)} + \left(\frac{-q_1}{q_2 - q_1}\right)\frac{p_1 p_2}{(q_2 - p_1)(q_2 - p_2)} \tag{16}$$

Since the probability density function (pdf) of the phase difference of two joint circular symmetric Gaussian random variables is [16,17-6.611]

$$p_{\Delta\psi}(\Delta\psi) = \frac{1-\mu}{2\pi}\left(\frac{1}{1 - \mu^2\cos^2(\Delta\psi)}\right) \times \left[1 + \frac{\mu\cos(\Delta\psi)\cos^{-1}(-\mu\cos(\Delta\psi))}{\sqrt{1 - \mu^2\cos^2(\Delta\psi)}}\right] \tag{17}$$

where $\mu = \sigma_{SR}^2 J_0(2\pi f_d T_s)/(\sigma_{SR}^2 + \sigma_N^2)$, consequently, the overall BER is

$$P_e^{CPF} = \int_{-\pi}^{\pi} P_{e|\Delta\psi}^{CPF} p_{\Delta\psi}(\Delta\psi)\, d\Delta\psi \tag{18}$$

The above error probability can be easily evaluated numerically since it involves a single finite integral.

B. Selective PF

For the selective relaying PF scheme, there are two scenarios to consider in the BER analysis. The first is that the received signal at the relay exceeds the forwarding threshold in two consecutive intervals and the relay forwards. In this case, the conditional BER is the same as that of continuous PF; see (16). On the other hand, when the relay does not forward, the conditional BER is simply that of conventional differential BPSK and is given by [18]

$$P_e^{DPSK} = \frac{1}{2}\left(1 - \frac{\rho\sigma_{SD}^2}{\sigma_{SD}^2 + \sigma_N^2}\right) \tag{19}$$

Let A be the event that both $a[k]$ and $a[k-1]$ exceed the threshold T, and $\mathbb{P}\{\bar{\mathrm{A}}\}$ the probability that this condition is not met. This means the unconditional BER is given by the integral

$$\begin{aligned} P_e^{SPF} &= \int_{-\pi}^{\pi}\iint_{\mathrm{A}} P_{e|\Delta\psi}^{CPF} p(a, a', \Delta\psi)\, da\, da'\, d\Delta\psi \\ &\quad + \int_{-\pi}^{\pi}\iint_{\bar{\mathrm{A}}} P_e^{DPSK} p(a, a', \Delta\psi)\, da\, da'\, d\Delta\psi \\ &= \int_{-\pi}^{\pi}\int_{T}^{\infty}\int_{T}^{\infty} P_{e|\Delta\psi}^{CPF}\, p(a, a', \Delta\psi)\, da\, da'\, d\Delta\psi + P_e^{DPSK}\,\mathbb{P}\{\bar{\mathrm{A}}\} \end{aligned} \tag{20}$$

where a and a' are abbreviated notations for $a[k]$ and $a[k-1]$, respectively. The joint pdf $p(a, a', \Delta\psi)$ is obtained by taking the polar transform [16] of $y_{SR}[k]$ and $y_{SD}[k-1]$ and then averaging over the phase. It assumes the following form:

$$p(a, a', \Delta\psi) = \frac{aa'}{2\pi\sigma_{SR}^4(1-\mu^2)} \exp\left\{-\frac{a^2 + a'^2 - 2\mu a a' \cos(\Delta\psi)}{2\sigma_{SR}^4(1-\mu^2)}\right\} \tag{21}$$

Using [19, eq. (37)], $\mathbb{P}\{\overline{\mathrm{A}}\}$ can be found equal to

$$\mathbb{P}\{\overline{\mathrm{A}}\} = 1 - e^{-\frac{T^2}{2\sigma_{SR}^2}}\left[1 + Q_1\left(\frac{\mu T}{\sqrt{\sigma_{SR}^2(1-\mu^2)}}, \frac{T}{\sqrt{\sigma_{SR}^2(1-\mu^2)}}\right) - Q_1\left(\frac{T}{\sqrt{\sigma_{SR}^2(1-\mu^2)}}, \frac{\mu T}{\sqrt{\sigma_{SR}^2(1-\mu^2)}}\right)\right] \tag{22}$$

where $Q_1(\alpha, \beta)$ is the Marcum's Q function. To gain further insights of the performance of the error probability, we can rewrite (20) as following

$$P_e^{SPF} = P_e^{CPF} - \Delta P_e^{SPF} \tag{23}$$

where P_e^{CPF} is the BER of continuous PF in (18) and

$$\Delta P_e^{SPF} = \int_{-\pi}^{\pi} P_{e|\Delta\psi}^{CPF} \iint_{\overline{\mathrm{A}}} p(a, a', \Delta\psi)\, da\, da'\, d\Delta\psi - P_e^{DPSK}\,\mathbb{P}\{\overline{\mathrm{A}}\} \tag{24}$$

represents an error probability adjustment. Specifically, if $\Delta P_e^{SPF} > 0$, then it means selective relaying outperforms continuous PF. It should be pointed out that using [17-3.326] the inner double integral in (24) can be simplified to

$$\begin{aligned}
&\iint_{\overline{\mathrm{A}}} p(a, a', \Delta\psi)\, da\, da' = \\
&\frac{1-\mu^2}{2\pi}\left[1 - \exp\left\{-\frac{T^2(1-\cos\Delta\psi)}{\sigma_{SR}^2(1-\mu^2)}\right\}\right] + \\
&\frac{T\mu\cos\Delta\psi}{2\pi}\sqrt{\frac{1-\mu^2}{2\sigma_{SR}^2}} \exp\left\{-\frac{T^2(1-\mu^2\cos^2\Delta\psi)}{2\sigma_{SR}^2(1-\mu^2)}\right\} \times \\
&\left[\Gamma\left(\frac{1}{2}, \frac{(T\mu\cos\Delta\psi)^2}{2\sigma_{SR}^2(1-\mu^2)}\right) - \Gamma\left(\frac{1}{2}, \frac{T^2(1-\mu\cos\Delta\psi)^2}{2\sigma_{SR}^2(1-\mu^2)}\right)\right] + \\
&\frac{\mu\cos\Delta\psi}{2\pi\sigma_{SR}^2\sqrt{\sigma_{SR}^2(1-\mu^2)}} \int_0^T x^2 \exp\left\{-\frac{x^2(1-\mu^2\cos^2\Delta\psi)}{2\sigma_{SR}^2(1-\mu^2)}\right\} \\
&\times\left[\Gamma\left(\frac{1}{2}, \frac{(x\mu\cos\Delta\psi)^2}{2\sigma_{SR}^2(1-\mu^2)}\right) + \Gamma\left(\frac{1}{2}, \frac{(T-x\mu\cos\Delta\psi)^2}{2\sigma_{SR}^2(1-\mu^2)}\right)\right] dx
\end{aligned} \tag{25}$$

where $\Gamma(\alpha, \beta)$ is the incomplete Gamma function. The above simplification reduces the semi-infinite triple integral in (24) to a double integral of finite ranges, which can be evaluated numerically.

IV. RESULTS AND DISCUSSION

In this section we present BER of the proposed continuous and selective PF DPSK systems. Also shown are results for DPSK with DF and AF over the same cooperative network. For DF, the relay detects its received signal using differential detection and regenerates a DPSK signal using the data decisions and forward that signal to the destination. On the other hand in AF, the relay simply scales its received signal (the $y_{SR}[k]$'s in (3)) by a constant G and forward that to the destination. The scaling factor G is chosen in such a way that the resultant forwarded signal has the same average power as the proposed PF schemes. For brevity, we do not include the performance analysis for AF and DF in the paper. It should also be pointed out that in the performance evaluation, we assume all the links have the same SNR. Also, the threshold T for selective PF is optimized at each SNR shown.

We first show in Fig. 1 the BER curves for the case of static fading, i.e. $f_d T_s = 0$. We can see that continuous PF is consistently 2 dB more power efficient than DF, but neither schemes provide any diversity effect. This lack of diversity can be attributed to the continuous forwarding nature of both schemes and that all the three links are equally strong. Although not shown, we found that when the S-R link has a larger SNR than the other two links, the performance of both continuous PF and DF do improve. As observed from Fig. 1, a dramatic improvement in BER can be achieved through adopting selective PF. As a matter of fact, selective PF has the same BER as AF but without all those problems like sensitivity to amplifier distortion or the requirement to use an expensive (but power inefficient) linear power amplifier. Both schemes are able to deliver a second order diversity effect. This result confirms our earlier belief that signal forwarding is actual harmful to the overall signal quality at the destination if the forwarded signal is highly corrupted by fading in the S-R link. In this situation, making data decision solely on the direct link's receive signal will, most of the time, yields a better result. Note that when the S-R link is in a fade (while the R-D link is "perfect"), selective PF will only fail if the S-D link also experiences a fade at the same time. This is the reason why we are seeing a second order diversity effect in selective PF.

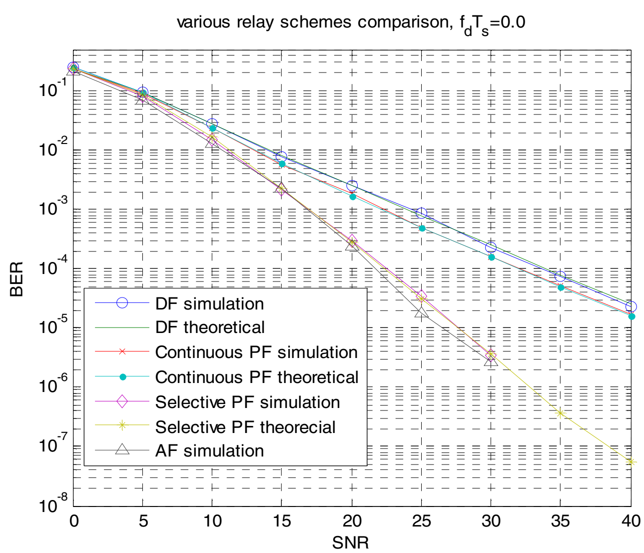

Fig. 1: BER of the proposed PF schemes in a static fading channel.

Fig. 2 is similar to Fig. 1 except that we now have time-selective fading in all the links. The common normalized Doppler frequency is $f_d T_s = 0.01$. For the SNR range between 15 to 30 dB, continuous PF outperforms DF by about 2 dB. It also has a lower irreducible error floor at large SNR. Again, selective PF provides a big improvement in the BER and attains the same performance of AF.

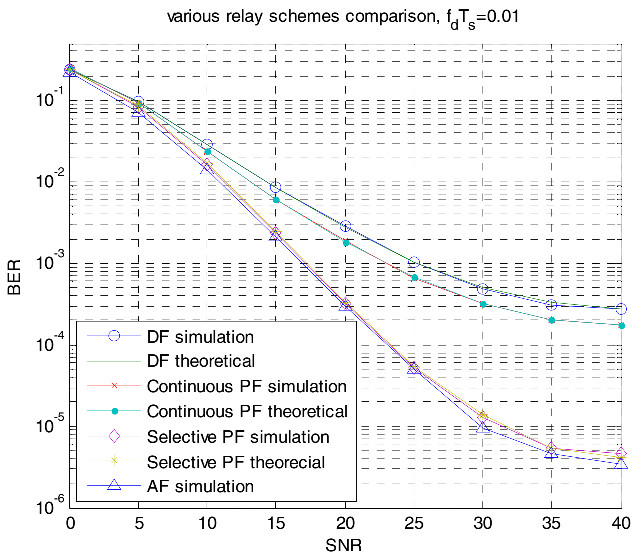

Fig. 2: BER of the proposed PF schemes in a time-selective fading channel with a normalized Doppler frequency of 0.01.

The optimal thresholds obtained numerically are illustrated in Table 1. In the low SNR range, the AWGN is the dominant factor in determining the BER. Thus we can expect the same optimal thresholds for both $f_d T_s$'s. At large SNR, fading becomes dominant, the threshold needs to be set higher for the time-selective fading. This can be attributed to the fact that the phase difference of the fading process typically increases with the Doppler frequency but decreases with the fading amplitude.

Table 1 Optimal threshold T

SNR(dB)	0	10	20	30	40
T ($f_d T_s = 0$)	0.6	1.3	2.1	2.7	3.3
T ($f_d T_s = 0.01$)	0.6	1.3	2.2	3.6	9.0

V. CONCLUSIONS

In this paper, we propose and analyze the performance of phase-forward (PF) based one-way cooperative communication systems that employ differential PSK modulation and differential detection. This transmission methodology requires neither channel estimation nor demodulation at the relay, and it allows inexpensive nonlinear power amplifier to be used, since the forwarded signal is essentially a constant envelope signal. In comparison, amplify-and-forward (AF) can be sensitive to non-linear amplifier distortion (especially the amplitude-to-phase distortion) and hence it requires expensive linear amplifier at the relay. Through analysis and simulation, we found that a threshold-based selective PF approach can practically attain the same performance as AF but without all those aforementioned problems. While this work focuses on one-way relaying, our ultimate goal is to apply PF to two (or multiple) way relaying. Due to the superposition nature of an AF signal in these systems, inter-modulation effect will arise when the signal passes through a nonlinear amplifier, preventing accurate self-interference cancellation. In contrast, PF, with proper signal processing, can prevent this from happening.

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