The Quasi-Uniform Redundant Carrier Placement for UW-OFDM

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Abstract-Unique-word (UW) OFDM is a new multicarrier technique that was recently proposed in [1]. To construct the UW-OFDM signal, some carriers must be sacrificed to transmit redundant information - these carriers are called the redundant carriers. It turns out that the average redundant energy needed strongly depends on the positions of the redundant carriers. In this paper, we look for the redundant carrier placement that minimizes the average redundant energy - this needs an exhaustive search over all possible redundant carrier placements. Hence, the computational complexity to find the optimal redundant carrier distribution is very high. Therefore, we introduce a suboptimal redundant carrier placement - the quasi-uniform (QU) distribution, that is based on the results of the exhaustive search, and determines the positions of the redundant carriers at no computational cost. We show in this paper that the redundant energy needed for the proposed QU distribution is very close to the theoretical minimum average redundant energy.

I. INTRODUCTION

In this paper, we consider the unique-word (UW) OFDM technique. This technique is a variant on the multicarrier technique, where the guard interval is a part of the inverse fast Fourier transform (IFFT) block. This is in contrast with other guard interval techniques, where the length of the multicarrier symbol in the time domain is extended [2]-[3]. In UW-OFDM, the guard interval, that corresponds to the last samples of the IFFT block, contain known samples – the unique word. As this part of the multicarrier time domain symbol does not contain a data contribution, this implies that not all carriers can be used for data transmission: some carriers must transmit information that depends on the data symbols transmitted on the other carriers. These carriers are therefore called the redundant carriers. The construction method that requires the lowest energy is the two-step method from [4]. In this approach, first a block of zero samples in the time domain is generated by properly selecting the information transmitted on the redundant carriers, and then the known samples are added in the time domain.

The UW-OFDM system acts like an error correcting code: the UW-OFDM symbol contains a block of zeros in the time domain by adding redundancy in the frequency domain. In that sense, it can be compared to a Reed-Solomon code [5]. The presence of the redundancy will of course lower the throughput efficiency: UW-OFDM will have a slightly lower throughput efficiency than CP-OFDM. This will lead to a small BER degradation in the case of an AWGN channel [4]. However, in the case of a dispersive channel, it is shown in [1], [4]

and [6] that UW-OFDM outperforms CP-OFDM in terms of BER – a gain that increases when there are more deep fades. This can be explained as the receiver of UW-OFDM acts like a decoder that transforms the redundancy into a coding gain. However, to fully exploit the redundancy, the receiver will be more complex than in CP-OFDM [7]. Note that precoding is also able to mitigate the effect of deep fades in CP-OFDM [8]-[10]. However, this requires channel state information (CSI) to be available at the transmitter side, which is not obvious to obtain in a sufficiently accurate and timely manner. In contrast, UW-OFDM does not need CSI.

The energy needed for the redundant carriers turns out to strongly depend on the positions of the redundant carriers [11]. To keep the energy usage of the UW-OFDM system as low as possible, we look for the (optimal) placement of the redundant carriers that minimizes the average redundant energy. In this paper, we have performed an exhaustive search for this optimum. The computational complexity allows such an exhaustive search for small FFT size only. As based on these results, the optimum positions for the general case, e.g. for larger FFT size, cannot be extracted, we consider in this paper a suboptimal placement of the redundant carriers, i.e. the quasi-uniform distribution, that is based on the results from the exhaustive search. In this paper, we show that the quasi-uniform distribution has a lower average redundant energy than the split distribution from [11].

II. THE UW-OFDM SYSTEM

Considering the two-step approach from [4] for the construction of UW-OFDM, the time-domain structure can be visualized as shown in figure 1. The time-domain signal can be decomposed into blocks of N samples generated at rate 1/T. Each block of N samples consists of two parts: the data part corresponds to the first $N-N_u$ samples and the unique word of prior known samples with the last N_u samples. To construct the signal corresponding to the data part in the first step, we use an IFFT¹. In order to obtain that the last N_u samples of the IFFT output contain zeros, it is necessary to select $N_r \geq N_u$ carriers that will be modulated by a linear combination of the data symbols – we call these carriers the redundant carriers. The resulting time-domain samples after

¹Hence, we restrict our attention to the case that N is a power of 2.

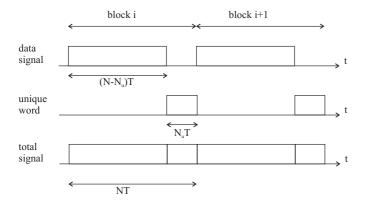


Fig. 1. Time-domain signal of UW-OFDM.

this first step are given by

$$\mathbf{y} = \mathbf{F}_N^{-1} \mathbf{P} \begin{pmatrix} \mathbf{x}_d \\ \mathbf{x}_r \end{pmatrix}, \tag{1}$$

where the $N_d \times 1$ vector $\mathbf{x}_d = [x_d(1) \dots x_d(N_d)]^T$ contains the $N_d = N - N_r$ data symbols and the $N_r \times 1$ vector $\mathbf{x}_r = [x_r(1) \dots x_r(N_r)]^T$ the redundant symbols. The $N \times N$ matrix \mathbf{P} is the permutation matrix: it defines the positions of the data and redundant carriers, and \mathbf{F}_N^{-1} is the $N \times N$ IFFT matrix with $(\mathbf{F}_N)_{k,\ell} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{k\ell}{N}}$. The permutation matrix can be decomposed as $\mathbf{P} = [\mathbf{P}_d \quad \mathbf{P}_r]$ with $\mathbf{P}_d \ (N \times N_d)$ and $\mathbf{P}_r \ (N \times N_r)$ the matrices corresponding to the data and redundant carrier positions, respectively. We define I_d as the set of data carrier positions and I_r as the set of redundant carrier positions. Then, the columns of \mathbf{P}_d and \mathbf{P}_r consist of unit-weight vectors with a '1' at the positions $\tilde{n}_\ell \in I_d$ and $n_\ell \in I_r$, respectively. In the second step, the unique word is added to \mathbf{y} .

Let us concentrate on the construction of the zero samples in the data part of the signal. We define the transform matrix \mathbf{M} as $\mathbf{M} = \mathbf{F}_N^{-1} \mathbf{P}$. We decompose the matrix \mathbf{M} as:

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{pmatrix}, \tag{2}$$

with \mathbf{M}_{11} is $(N-N_u) \times N_d$, \mathbf{M}_{12} is $(N-N_u) \times N_r$, \mathbf{M}_{21} is $N_u \times N_d$ and \mathbf{M}_{22} is $N_u \times N_r$. Imposing that the last N_u samples have to be zero corresponds to the set of linear equations $\mathbf{M}_{21}\mathbf{x}_d + \mathbf{M}_{22}\mathbf{x}_r = \mathbf{0}$. The solution

$$\mathbf{x}_r = -\mathbf{M}_{22}^{\dagger} \mathbf{M}_{21} \mathbf{x}_d = \mathbf{T} \mathbf{x}_d, \tag{3}$$

– where $\mathbf{M}_{22}^{\dagger} = \mathbf{M}_{22}^{H}(\mathbf{M}_{22}\mathbf{M}_{22}^{H})^{-1}$ is the Penrose-Moore pseudo-inverse – is the solution that minimizes the average redundant energy $E[\mathbf{x}_{r}^{H}\mathbf{x}_{r}]$. Taking this into account, the time domain signal (1) can be rewritten as

$$\mathbf{y} = \mathbf{M} \begin{pmatrix} \mathbf{I}_{N_d} \\ \mathbf{T} \end{pmatrix} \mathbf{x}_d = \mathbf{G} \mathbf{x}_d, \tag{4}$$

with I_{N_d} is the $N_d \times N_d$ identity matrix. Further, the last N_u rows in G are zero rows.

The average energy of the time-domain signal y is defined as

$$P_t = E[\mathbf{y}^H \mathbf{y}] = P_d + P_r. \tag{5}$$

Assuming $E[x_d(i)x_d^*(j)] = E_s\delta_{i,j}$ is the energy of a data symbol, the average data energy equals $P_d = N_dE_s$. Further, the average redundant energy can be rewritten as $P_r = E[\mathbf{x}_r^H\mathbf{x}_r] = E_s \mathrm{trace}(\mathbf{T}^H\mathbf{T})$. Taking into account the definition of \mathbf{T} (3), this results in

$$P_r = E_s \operatorname{trace}[\mathbf{M}_{21} \mathbf{M}_{21}^H (\mathbf{M}_{22} \mathbf{M}_{22}^H)^{-1}],$$
 (6)

where we have used the property trace(ABC)=trace(CAB) [12]. The matrices $M_{21}M_{21}^H$ and $M_{22}M_{22}^H$ from (6) are Hermitian Toeplitz matrices with as elements

$$(\mathbf{M}_{21}\mathbf{M}_{21}^{H})_{k,k'} = \frac{1}{N} \sum_{\ell=0}^{N_d-1} e^{j2\pi \frac{(k-k')\bar{n}_{\ell}}{N}}$$
(7)

$$(\mathbf{M}_{22}\mathbf{M}_{22}^{H})_{k,k'} = \frac{1}{N} \sum_{\ell=0}^{N_r-1} e^{j2\pi \frac{(k-k')n_{\ell}}{N}}$$
(8)

where $k, k' = 0, ..., N_u - 1$. Taking into account that $I_r \cup I_d = \{0, ..., N-1\}$, it can easily be verified that $\mathbf{M}_{21}\mathbf{M}_{21}^H + \mathbf{M}_{22}\mathbf{M}_{22}^H = \mathbf{I}_{N_u}$.

Let us take a closer look at the average redundant energy (6). Assume the redundant and data carrier positions are given by n_ℓ and \tilde{n}_ℓ , respectively. A cyclic shift of all carrier positions over ϵ , i.e. $n'_\ell = (n_\ell + \epsilon)_N$ and $\tilde{n}'_\ell = (\tilde{n}_\ell + \epsilon)_N$, where $(x)_N$ is the modulo-N reduction of x, will have no influence on the average redundant energy. This can be verified as follows. Define \mathbf{M}'_{21} and \mathbf{M}'_{22} as the matrices constructed using the redundant and data carrier positions n'_ℓ and \tilde{n}'_ℓ . It can easily be verified that the products $\mathbf{M}'_{21}\mathbf{M}'_{21}$ and $\mathbf{M}'_{22}\mathbf{M}'_{22}$ can be rewritten as

$$\mathbf{M}_{21}^{\prime H} \mathbf{M}_{21}^{\prime} = \mathbf{D}(\mathbf{M}_{21}^{H} \mathbf{M}_{21}) \mathbf{D}^{H}$$

 $\mathbf{M}_{22}^{\prime H} \mathbf{M}_{22}^{\prime} = \mathbf{D}(\mathbf{M}_{22}^{H} \mathbf{M}_{22}) \mathbf{D}^{H}$ (9)

where $\mathbf{D} = \mathrm{diag}\{e^{j2\pi\frac{k\epsilon}{N}}\}$. Using the property trace(\mathbf{ABC})=trace(\mathbf{CAB}), it is straightforward to prove that $\mathrm{trace}[\mathbf{M}'_{21}\mathbf{M}'^H_{21}(\mathbf{M}'_{22}\mathbf{M}'^H_{22})^{-1}] = \mathrm{trace}[\mathbf{M}_{21}\mathbf{M}^H_{21}(\mathbf{M}_{22}\mathbf{M}^H_{22})^{-1}].$

Next, we derive a theoretical lower bound on the average redundant energy. Assume the eigenvalues and eigenvectors of $\mathbf{M}_{22}\mathbf{M}_{22}^H$ are λ_m and \mathbf{c}_m , $m=1,\ldots,N_u$. Taking into account that $\mathbf{M}_{21}\mathbf{M}_{21}^H+\mathbf{M}_{22}\mathbf{M}_{22}^H=\mathbf{I}_{N_u}$, it follows that the eigenvalues and eigenvectors of $\mathbf{M}_{21}\mathbf{M}_{21}^H$ are $1-\lambda_m$ and \mathbf{c}_m , $m=1,\ldots,N_u$. As both $\mathbf{M}_{21}\mathbf{M}_{21}^H$ and $\mathbf{M}_{22}\mathbf{M}_{22}^H$ are semipositive definite matrices (see [13]), the eigenvalues λ_m are restricted to the interval $0 \leq \lambda_m \leq 1$. The average redundant energy (6) can be rewritten as

$$P_r = E_s \sum_{m=1}^{N_u} \frac{1 - \lambda_m}{\lambda_m}.$$
 (10)

Deriving (10) with respect to λ_m and taking into account that the sum of the eigenvalues equals $\sum_{\ell=1}^{N_u} \lambda_m = \operatorname{trace}(\mathbf{M}_{22}\mathbf{M}_{22}^H) = \frac{N_r N_u}{N}$, it follows that the average redundant energy is minimized when all eigenvalues are equal and are given by $\lambda_m = \frac{N_r}{N}$; the corresponding minimum average redundant energy equals

$$P_{r,LB} = E_s \frac{N_u N_d}{N_r}. (11)$$

This implies that the matrices $\mathbf{M}_{21}\mathbf{M}_{21}^H$ and $\mathbf{M}_{22}\mathbf{M}_{22}^H$ have to be diagonal matrices. As it is not certain that this requirement can be achieved by any redundant carrier distribution, it is a theoretical lower bound on the average redundant energy.

III. REDUNDANT CARRIER PLACEMENT

It is important that the average redundant energy is as small as possible so that the power efficiency is high. In the previous section, we have derived a lower bound on the minimum average redundant energy, without having the guarantee that this bound can be reached. In this section, we will search for the optimal placement of the redundant carriers. In [13], it is analytically shown that when N_r is a power of 2, a uniform distribution of the redundant carriers over the bandwidth is optimal, and the lower bound on the average redundant energy from the previous section is reached. The indices of the redundant carriers are in this case given by $n_{\ell} = n_0 + \ell \Delta$, $\ell = 1 \dots N_r - 1$, where $0 \le n_0 < \Delta$ and $\Delta = N/N_r$. However, results in [11] have shown that the uniform distribution with $\Delta = \lfloor \frac{N}{N} \rfloor$ is no longer optimal when N_r is not a power of 2. The average redundant energy in this case can become very large, which can be explained as for this distribution, the matrix $\mathbf{M}_{22}\mathbf{M}_{22}^H$ in (6) can become (close to) singular.

To find the optimal redundant carrier placement that minimizes the average redundant energy for the general case where N_r is not a power of two, an exhaustive search has to be performed. However, the complexity of the exhaustive search becomes prohibitively large at already small values for Nand N_r . Only for $N \leq 32$, we could obtain the optimal redundant carrier placement within a reasonable time. In table I, the optimal carrier distribution is shown for N=32, for different values of N_r and N_u . Further, all cyclic shifts of the carrier positions in the table also yielded the minimum average redundant energy (see also the previous section). Hence, the redundant carrier distribution given in the table is just a representative of a set containing all cyclically shifted versions of this representative. Mostly, only one carrier set gave rise to the minimum average redundant energy, but in some cases, more than one set of carrier placements gave rise to the minimum average redundant energy, e.g. for $N_u = 1$ where all redundant carrier placements give the same average redundant energy. For these cases, the tables shows only one representative carrier placement. It can be observed in the table that the optimal distribution more or less evenly spreads the redundant carriers over the bandwidth. However, it can also be observed that for given N and N_r , the optimal positions change when N_u changes. So, no general rule of thumb could be found to generate the optimal redundant carrier positions for general N, N_r and N_u .

As for general N, N_r and N_u , finding the optimal positions is practically infeasible, we have to resort to suboptimal redundant carrier placements that can be generated at no cost and need a close to minimum average redundant energy. In [11], such a redundant carrier placement is introduced, i.e. the split distribution. This distribution follows a simple analytical expression and depends only on N and N_r . However, the

average redundant energy needed can become quite large when $N_u=N_r$. In the following, we consider a new suboptimal redundant carrier placement that is based on the results of the exhaustive search for the optimal positions. Let us focus on the special case where $N_u=N_r$. In table I, we observe that the optimal distribution for all N_r has the same structure². A (cyclically shifted version of) the optimal positions of the redundant carriers for $N_r=N_u$ can be generated as follows:

$$n_{\ell} = \left[\frac{\ell N}{N_r}\right]_{R} \qquad \ell = 1, \dots, N_r \tag{12}$$

where $[x]_R$ rounds x to the nearest integer. The positions of the redundant carriers only depend on N and N_r , but not on N_u . This algorithm spreads the redundant carriers more or less evenly over the bandwidth. Therefore, we call this redundant carrier placement the quasi-uniform (QU) distribution. Note that when N_r is a power of two, the QU distribution reduces to the (optimal) uniform distribution. Note that this quasi-uniform distribution for the redundant carriers is similar to the suboptimal placement of pilots from [14].

IV. NUMERICAL RESULTS

In this section we evaluate the average redundant energy needed for the QU distribution. For small N, i.e. $N \leq 32$, we compare it with the minimum obtained with the exhaustive search, whereas for larger N, the theoretical lower bound, derived in section II, is used as a benchmark. Further, we compare the results with the average redundant energy for the split distribution. In the following, we assume that the energy per symbol equals $E_s=1$ for the ease of interpretation.

In figure 2, the average redundant energy is shown as function of N_u , for $N_r=7$ and $N_r=9$, respectively. Also, the theoretical lower bound $P_{r,LB}=\frac{N_dN_u}{N_r}$ is shown. Although the optimal distribution obtained with the exhaustive search is close to the lower bound $P_{r,LB}$, there is a small difference that increases with increasing N_u . Hence, for $N_u \approx N_r$, the theoretical lower bound cannot be reached, although the loss is small. Further, the average redundant energy for the QU distribution is very close to the curve of the optimal distribution. The average redundant energy $P_{r,QU}$ increases essentially linearly with N_u and is slightly larger than the theoretical minimum $N_d E_s = (N - N_r) E_s$ when $N_u = N_r$. The average redundant energy for the split distribution, however, is close to the lower bound for small N_u , but strongly increases when $N_u \approx N_r$. This effect has also been shown in [13]. From the figure, it follows that the QU distribution outperforms the split distribution. However, it turns out that in some cases, the split distribution yields a slightly lower average redundant energy. This effect is only observed when $N_r > N/2$ and for some isolated values for N_u . The minimum number of redundant carriers necessary to obtain a zero interval of length N_u in the time domain equals $N_r = N_u$. Hence, it is expected that in practical situations $N_r \gtrsim N_u$. Further, in practice the guard interval length $N_u \ll N$, and thus $N_r \ll N$, so it can be concluded that the QU distribution yields better results than the split distribution in practical situations.

 $^{^{2}}$ For N < 32, the same structure was found.

 $\label{eq:table in the continuous for label in the cont$

N_u	$N_r = 6$	$N_r = 7$	$N_r = 9$
1	(1,6,11,17,22,27)	(1, 5, 10, 14, 19, 23, 28)	(1,4,8,11,15,18,22,25,29)
2	(1,6,11,17,22,27)	(1, 4, 10, 13, 19, 23, 27)	(1, 2, 3, 11, 13, 16, 20, 23, 27)
3	(1, 2, 11, 13, 22, 24)	(1, 3, 10, 13, 18, 23, 26)	(1, 2, 8, 10, 14, 18, 21, 25, 28)
4	(1,6,11,17,22,27)	(1, 5, 10, 14, 19, 23, 28)	(1, 4, 8, 11, 15, 18, 22, 25, 29)
5	(1,6,11,16,21,27)	(1, 5, 9, 14, 19, 23, 28)	(1,4,8,11,15,18,22,25,29)
6	(1,6,11,17,22,27)	(1, 5, 9, 13, 18, 23, 28)	(1,4,7,11,14,18,22,25,29)
7	_	(1, 5, 10, 14, 19, 23, 28)	(1,4,7,11,15,18,22,25,29)
8	_	_	(1,4,7,10,13,17,21,25,29)
9	_	_	(1,4,8,11,15,18,22,25,29)

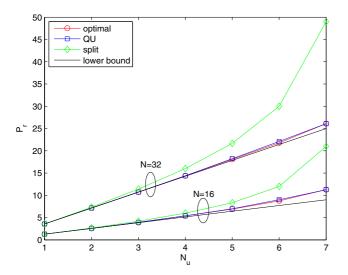


Fig. 2. Average redundant energy as function of N_u , $N_r = 7$.

The QU distribution for the redundant carriers results in an average redundant energy that is close to the optimum from the exhaustive search. To evaluate the increase of the average energy as compared to this minimum, we define the relative difference $100 \times \frac{P_r - P_{r,\min}}{P_{r,\min}}$, expressed in %. This relative difference is shown in figure 3 for the QU distribution and in figure 4 for the split distribution, as function of N_u for different values of N_r . When $N_u = N_r$, the relative difference becomes zero for the QU distribution, as in this case the QU distribution was found to be optimum, whereas for the split distribution the relative difference is quite large (of the order of 80-90 %)³. Further, we observe that when $N_r > N/2 = 8$, the relative difference for the QU distribution shows peaks of 10-20 % for some values of N_u , e.g. for $N_u = 8$. On the other hand, for $N_u = 8$, the relative difference for the split distribution equals zero: for this case, the split distribution was the optimum distribution. However, for most values of N_r and N_u , the relative difference for the QU distribution is smaller than for the split distribution. This confirms the results from figure 2. Moreover, in practical situations where N_u and

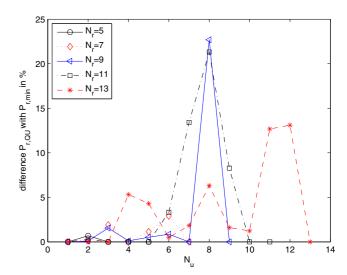


Fig. 3. Relative difference in % between average redundant energy $P_{r,QU}$ for the QU distribution compared to the minimum average redundant energy $P_{r,min}$ from the exhaustive search, N=16.

 $N_r \ll N$, it follows from figure 3 that the relative increase in average redundant energy when using the QU distribution instead of the optimal distribution, is less than 5%.

Until now, we have considered small values of N, where an exhaustive search was able to provide the optimal distribution and the corresponding minimum average redundant energy $P_{r,\min}$. In the following, we consider larger values for N. As $P_{r,\min}$ is no longer available, we compare the results with the theoretical lower bound, which appeared to be a tight lower bound for small N. In figure 5, the average redundant energy is shown for the QU and split distribution, for $N_u = N_r$ and $N_r = [0.15N]$, i.e. approximately 15% of the carriers are redundant carriers. Although the case $N_r = N_u$ yields the worst situation for the split distribution, it is of practical importance as it corresponds to the minimum number of redundant carriers for a given guard interval length N_u . The lower bound on the average redundant energy $P_{r,LB}$ in this case reduces to $P_{r,LB} = N_d E_s = (N - N_r) E_s$ and thus linearly increases with N. As the data energy equals P_d = $N_d E_s$, the total energy in this case equals minimally $2P_d$. The average redundant energy for the QU distribution tightly

³The worst case for the split distribution corresponds to $N_u = N_r$, as the average redundant energy increase is the largest (see [13]).

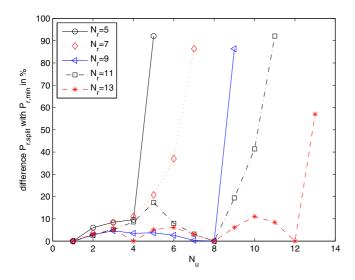


Fig. 4. Relative difference in % between average redundant energy $P_{r,split}$ for the split distribution compared to the minimum average redundant energy $P_{r,min}$ from the exhaustive search, N=16.

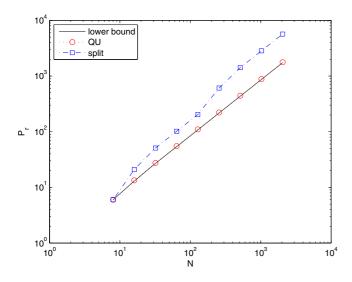


Fig. 5. Average redundant energy as function of N, $N_r = \lceil 0.15N \rceil$, $N_u = \lceil 0.15N \rceil$

follows the lower bound, whereas the difference with the split distribution is much larger. Hence, the QU distribution, which was found to be optimal for $N_r = N_u$ and small N, yields also very good results for large N. For the QU distribution, we need maximally about 2% more redundant energy than the theoretical lower bound.

V. Conclusions

A major concern in UW-OFDM is the average redundant energy usage. To keep the total energy usage in UW-OFDM as low as possible, we search for the redundant carrier placement that minimizes the average redundant energy. In this paper, we have carried out an exhaustive search for the optimal positions for small FFT size. Based on the results of the exhaustive search, we have proposed a suboptimal redundant

carrier distribution: the quasi-uniform (QU) distribution. In this redundant carrier distribution, the positions of the redundant carriers can be generated with very low complexity. Further, for the special case where the number of redundant carriers is a power of two, the QU distribution results in the uniform distribution, which was analytically shown in [13] to be the optimal distribution for this case. In the paper, we have evaluated the redundant energy consumption of the proposed QU distribution. In some cases, the QU distribution was the optimal redundant carrier placement, and for the other cases, the necessary redundant energy was only slightly larger than the minimum. For larger values of N, we have compared the redundant energy with the theoretical minimum redundant energy. Although there is no guarantee that this theoretical minimum can be achieved by any redundant carrier distribution, it is a useful benchmark. Results for large N however show that the necessary redundant energy for the QU distribution is very close to this theoretical minimum. Further, we have compared the QU distribution with the split distribution from [11]. In this paper it is shown that for practical scenarios, the proposed distribution outperforms the split distribution in terms of the needed redundant energy.

ACKNOWLEDGEMENT

The author gratefully acknowledges the financial support from the Flemish Fund for Scientific Research (FWO).

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