Capacity of a Modulo-Sum Arbitrary SISO Relay Network

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Abstract [1] **—This paper shows how to obtain the capacity of modular additive noise of an arbitrary single-input-single-output (SISO) relay network. The arbitrary SISO relay network is a network where the relays are interconnected with each other in various combinations. Its capacity is obtained by using a quantize-and-forward (QAF) strategy at each relay node. This paper considers all channels as discrete memoryless channels. In order to obtain the capacity of an arbitrary SISO relay network, this paper simplifies the network in terms of a parallel and serial relay network and finds the capacity. First, the capacity expression for a parallel relay network is obtained. In a parallel relay network, the source is transmitted to all relays; then each relay, using a QAF strategy, forwards the source to the destination. Second, the capacity expression for a serial relay network where the relays are interconnected serially is obtained. Finally, after expressing the capacities for parallel and serial relay networks, this paper explains how to obtain the capacity for any arbitrary SISO relay network.**

Keywords-channel capacity; relay network; modulo-sum channel; quantize-and-forward; single-input-single-output.

I. INTRODUCTION

The relay channel plays a very important role in the future field of communication systems. Relay channels will overtake multiple-input-multiple-output (MIMO) systems in the near future, since the relay can be another cell phone or small base station node, which assists communication between the source and the destination. By using relay channels, the bandwidth and number of antennas at the source and destination can be reduced. According to the literature, the capacity of general relay networks has not yet been found, thus providing motivation in the current paper to obtain the capacity of an arbitrary relay network. The capacity of a modulo-sum simple relay network is found in [1]. The simple relay network has one source, one receiver, and one relay node. The capacities of a parallel and a serial modulo-sum relay network are found in [2]. However, these capacities are for the case where direct channel noise is transmitted to the relay nodes [2]. This paper first obtains the capacity expression for parallel and serial relay networks where the source input is transmitted to all relay nodes, and then the relay nodes forward this input to the destination in the case of a parallel network. And in the case of a serial network, each relay node forwards the input to the next relay until it reaches the destination. Figure 2 shows a parallel relay network, and Figure 3 shows a serial relay network. The relay node uses a QAF strategy. With that information, this paper explains how to obtain the capacity for any arbitrary SISO relay network.

In the literature, relay channels (or relay networks) have been studied extensively. Cover et al. [4] discuss capacity theorems for the relay channel. Furthermore, the capacity of semi-deterministic relay channels was found in [9]. The capacity of relay channels for orthogonal components was determined in [10]. Later, Kim et al. [11] found the capacity of deterministic relay channels, where the channel from the relay to the destination is deterministic. Furthermore, Aleksic et al. [3] modified the deterministic channel to a modulo-sum channel and obtained the capacity of a class of modulo-sum relay channels. The authors of this paper obtained the capacity of a modulo-sum simple relay network [1], and this paper is an extension of finding the capacity of a modulo-sum arbitrary SISO relay network, which is a combination of parallel and serial relay networks. It is known that an optimum input distribution is $X \sim Ber(1/2)$ for a binary symmetric channel (BSC). In [3], [1], and [2], a uniform input distribution, i.e., $X \sim Ber(1/2)$ is assumed for the proofs of achievability and the converse. This current paper will also assume the same. The quantize-and-forward is an optimal relay strategy for a modulo-sum simple relay network according to the surveys in [3]. Hence, this paper also employs the quantize-and-forward strategy.

Section II describes the parallel relay network and presents the capacity expression for binary symmetric *n*-parallel relay network. Section III also discusses the serial relay network and presents a capacity expression for binary symmetric serial relay network. Section IV extends the binary symmetric network to an *m*-ary modulo-sum relay network for both parallel and serial networks. Section V presents the capacity expression for a given arbitrary SISO relay network, as shown in Figure 5. Section VI provides the analytical results for an arbitrary SISO relay network. Section VII concludes this paper.

Notations: The binary random variable Z with a Bernoulli distributed p denoted as $Z \sim Ber(p)$, i.e., $Z = 1$ with probability p and $Z = 0$ with probability $1 - p$; $H(X)$, $H(X|Y)$, and $I(X;Y)$ are, respectively, the entropy of random

[1] This work was partly sponsored by the Army Research Office under DEPSCoR ARO Grant W911NF-08-1-0256, and by NASA under EPSCoR CAN Grant NNX08AV84A.

variable X, the conditional entropy of X given Y, and the mutual information between X and Y [6]; $\mathcal{H}(\alpha)$ is the binary entropy function written as $\mathcal{H}(\alpha) = -\alpha\log_2\alpha - (1-\alpha)\log_2(1-\alpha)$, and $\mathcal{H}^{-1}(\cdot)$ is the inverse of $\mathcal{H}(\alpha)$ in the domain $\alpha \in [0,0.5]$ (i.e., if $\varepsilon = \mathcal{H}(\alpha)$, then $\mathcal{H}^{-1}(\varepsilon) = \alpha$); $\alpha * \beta = \alpha(1-\beta) + (1-\alpha)\,\beta$ [7]; $\mathcal{U}$ denotes the alphabet set for random variable U; $|\mathcal{U}|$denotes the cardinality of $\mathcal{U}$; $\mathcal{U}^n$ is a codebook, i.e., a set of codewords of length n; and $\oplus$ denotes the binary modulo-sum.

II. Parallel Relay Network

Consider the binary symmetric parallel relay network shown in Figure 1, which has one sender, one receiver, and two relay nodes. The two relay nodes are connected in parallel, as shown. The source sends the input X to both relays, $R1$ and $R2$. The source input X is transmitted simultaneously through the direct BSC to obtain the output $\oplus\, Z$, where $Z \sim Ber(p)$. The received vectors at both the relays are given by $Y_1 = X \oplus N_{11}$ and $Y_2 = X \oplus N_{21}$, where $N_{11} \sim Ber(\delta_1)$ and $N_{21} \sim Ber(\delta_2)$. Both the relay nodes encode using the codebooks $\mathcal{U}_1^n$ and $\mathcal{U}_2^n$ available at relay nodes $R1$ and $R2$ respectively. The encoded codeword from each relay is transmitted to the destination, i.e., the received vectors at destination from relay nodes are $S_1 = X_1 \oplus N_{12}$ and $S_2 = X_2 \oplus N_{22}$, respectively, where $N_{12} \sim Ber(\varepsilon_1)$ and $N_{22} \sim Ber(\varepsilon_2)$.

After receiving codewords from the relay nodes, the destination then decodes them separately to find the codeword $\mathcal{U}^n$. This paper assumes input X as $\boldsymbol{X \sim Ber(1/2)}$.

Lemma 1: Let

$$R_1 \triangleq \max_{p(x_1)} I(X_1; S_1) \tag{1}$$

$$R_2 \triangleq \max_{p(x_2)} I(X_2; S_2). \tag{2}$$

Then, the capacity of the binary symmetric two-parallel relay network shown in Figure 1 can be written as

$$C_{2-parallel} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ p(u_2|y_2):I(U_2;Y_2)\le R_2}} \{1 - H(X|U_1,U_2) + H(Y|U_1,U_2) - H(Z)\}, \tag{3}$$

The closed-form capacity of the binary symmetric two-parallel relay network shown in Figure 1 can be written as

$$C_{2-parallel} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ p(u_2|y_2):I(U_2;Y_2)\le R_2}} \{\mathcal{H}(\mathcal{H}^{-1}(\beta) * p) - \mathcal{H}(p) + 1 - \beta\}, \tag{4}$$

where, $\beta = \mathcal{H}(\delta_1 * \varepsilon_1) + \mathcal{H}(\delta_2 * \varepsilon_2) - \mathcal{H}(\{\delta_1 * \varepsilon_1\} * \{\delta_2 * \varepsilon_2\})$.

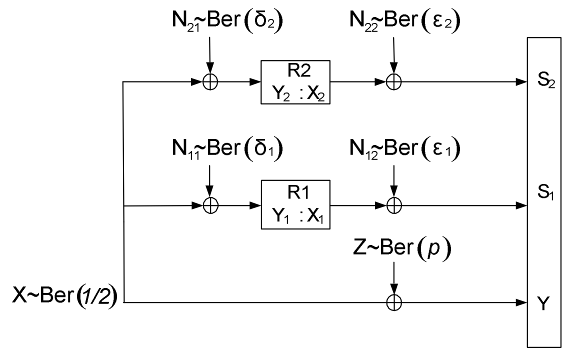

Figure 1. Binary symmetric parallel relay network

Theorem 1: The capacity of the binary symmetric *n*-parallel relay network shown in Figure 2 can be written as

$$C_{n-parallel} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ \vdots \\ p(u_n|y_n):I(U_n;Y_n)\le R_n}} \{H(Y|U_1,U_2,\dots,U_n) - H(Z) + 1 - H(X|U_1,U_2,\dots,U_n)\}, \tag{5}$$

The closed-form capacity of the binary symmetric *n*-parallel relay network shown in Figure 2 can be written as

$$C_{n-parallel} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ \vdots \\ p(u_n|y_n):I(U_n;Y_n)\le R_n}} \{\mathcal{H}(\mathcal{H}^{-1}(\hat{p}_n) * p) - \mathcal{H}(p) + 1 - \hat{p}_n, \tag{6}$$

Proofs of equations (5) and (6) can be achieved by following the same steps in [2 Th1].

where $\hat{p}_n = H(X|U_1,U_2,\dots,U_n)$ and can be simplified by repetitive calculation given as

$$\hat{p}_n = \hat{p}_{n-1} - \mathcal{H}(\mathcal{H}^{-1}(\hat{p}_{n-1}) * \{\delta_1 * \varepsilon_1\}) + \mathcal{H}(\delta_1 * \varepsilon_1) \tag{7}$$

where, $\hat{p}_{n-1} = H(X|U_2,\dots,U_n)$, which can be simplified as follows:

$$\hat{p}_{n-1} = \hat{p}_{n-2} - \mathcal{H}(\mathcal{H}^{-1}(\hat{p}_{n-2}) * \{\delta_2 * \varepsilon_2\}) + \mathcal{H}(\delta_2 * \varepsilon_2) \tag{8}$$

$$\hat{p}_{n-2} = \hat{p}_{n-3} - \mathcal{H}(\mathcal{H}^{-1}(\hat{p}_{n-3}) * \{\delta_3 * \varepsilon_3\}) + \mathcal{H}(\delta_3 * \varepsilon_3) \tag{9}$$

$$\vdots \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\hat{p}_{n-n-2} = \hat{p}_{n-n-1} - \mathcal{H}(\mathcal{H}^{-1}(\hat{p}_{n-n-1}) * \{\delta_{n-1} * \varepsilon_{n-1}\}) + \mathcal{H}(\delta_{n-1} * \varepsilon_{n-1}) \tag{10}$$

where $\hat{p}_{n-n-1} = H(X|U_n)$, which can again be simplified by applying the "Mrs. Gerber's Lemma" on the conditional entropy of binary random variables, i.e.,

$$H(Y_n|U_n) \ge \alpha, \tag{11}$$

then

$$H(X|U_n) \ge \mathcal{H}(\mathcal{H}^{-1}(\alpha) * \delta_n). \tag{12}$$

The equality holds if Y_n given U_n is a random variable of the Ber($\mathcal{H}^{-1}(\alpha)$) distribution. Let $\alpha \triangleq H(Y_n) - R_n$. Then, U_n achieves this equality, i.e., if Y_n given U_n is a Ber($\mathcal{H}^{-1}(H(Y_n) - R_n)$). Then this U_n under a rate constraint R_n in the standard rate distortion theory minimizes the

Hamming distortion of Y_n [3, p.925]. Since $Y_n \sim Ber(1/2)$, $H(Y_n) = 1$. Hence, $H(X|U_n)$ can be written as

$$H(X|U_n) = \mathcal{H}(\mathcal{H}^{-1}(1 - R_n) * \delta_n). \quad (13)$$

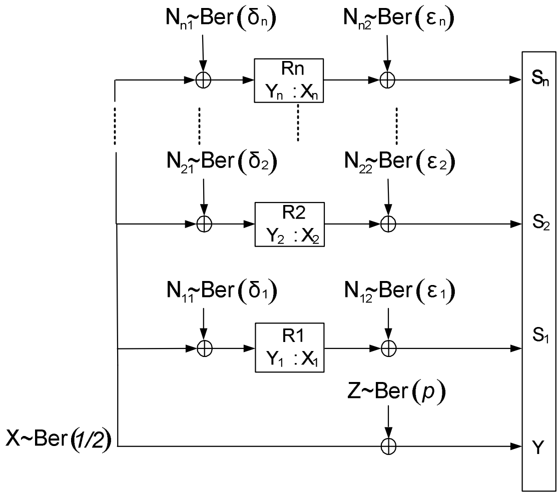

Figure 2. Binary symmetric *n*-parallel relay network

Now, from equations (1) and (2),

$$R_n = 1 - \mathcal{H}(\varepsilon_n) \quad (14)$$

By substituting (14) into (13), equation (13) becomes

$$\hat{p}_{n-n-1} = H(X|U_n) = \mathcal{H}(\varepsilon_n * \delta_n). \quad (15)$$

Hence, the capacity of *n*-parallel relay network is obtained.

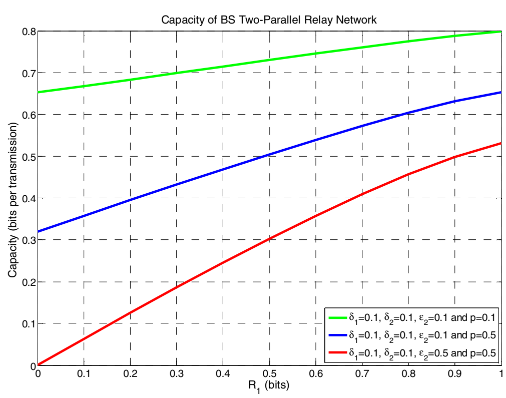

Figure 4(a). Plot of capacity versus R_1 (bits) for two-parallel relay network.

III. SERIAL RELAY NETWORK

Consider the binary symmetric serial relay network shown in Figure 3, which has one sender, one receiver, and a number of relay nodes. The relay nodes are connected serially, i.e., the output of one relay is input to the next relay node, as shown in Figure 3. The input X is transmitted to the relay node $R1$, which uses QAF and forwards it to the next relay node R2, and continues like this until it reaches the destination. During the source transmission to the relay node, the source also transmits simultaneously to the destination through a direct BSC, which can be written as $Y = X \oplus Z$, where X and Z denote the transmitted and noise random variable with distribution $Ber(1/2)$ and $Ber(p)$, respectively, and $\oplus$ denotes the binary modulo-sum,

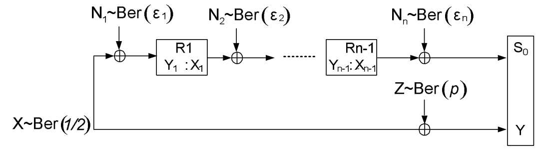

Figure 3. Serial relay network

i.e., $Z = 1$ with probability p, and $Z = 0$ with probability $(1 - p)$.

Theorem 2: The capacity of the binary symmetric *n*-serial relay network shown in Figure 3 can be written as

$$C_{n-serial} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\leq R_1 \\ \vdots \\ p(u_{n-1}|y_{n-1}):I(U_{n-1};Y_{n-1})\leq R_{n-1}}} \{H(Y|U_n) - H(Z) + 1 - H(X|U_n)\}, \quad (16)$$

The closed-form capacity of the binary symmetric *n*-serial relay network shown in Figure 3 can be written as

$$C_{n-serial} = \{\mathcal{H}(\varepsilon_n * \varepsilon_{n-1} * \dots * \varepsilon_1 * p) - \mathcal{H}(p) + 1 - \mathcal{H}(\varepsilon_n * \varepsilon_{n-1} * \dots * \varepsilon_1)\}, \quad (17)$$

Proofs of equations (16) and (17) can be achieved by following the same steps as in [2, Th3].

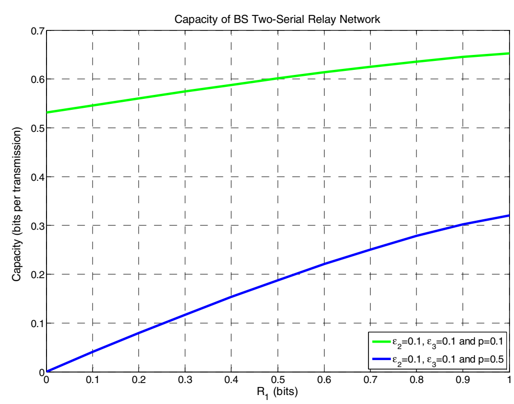

Figure 4(b). Plot of capacity (bits per transmission) versus R_1 (bits) for two-serial relay network

IV. CAPACITY FOR M-ARY MODULO-SUM RELAY NETWORK

In this section, the capacity derived for the parallel and serial relay network is extended for the modular-two to *m*-ary modular additive relay networks. The received signal at the destination can be written as

$$Y = X + Z \bmod (m) \quad (18)$$

For the serial case, this is

$$S_0 = X_{n-1} + N_n \bmod (m) \quad (19)$$

For the parallel case, this is

$$S_0 = X_{n-1} + N_n \, mod(m) \tag{20}$$

Therefore, the capacity obtained for the binary symmetric parallel and serial relay network can be written for the m-ary modulo-sum parallel and serial relay network as

$$C_{n-parallel} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ \vdots \\ p(u_n|y_n):I(U_n;Y_n)\le R_n}} \{H(Y|U_1, U_2, \dots, U_n) - H(Z) + m - H(X|U_1, U_2, \dots, U_n)\}, \tag{21}$$

and,

$$C_{n-serial} = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ \vdots \\ p(u_{n-1}|y_{n-1}):I(U_{n-1};Y_{n-1})\le R_{n-1}}} \{H(Y|U_n) - H(Z) + m - H(X|U_n)\}, \tag{22}$$

V. Capacity For Any Given Arbitrary SISO Relay Network

This section provides a method for obtaining the capacity for any given arbitrary SISO relay network. For example, consider the SISO relay network shown in Figure 5, which has one sender, one receiver, and three relay nodes. The relay nodes $R2$ and $R3$ are connected in series, and together they are parallel to relay node $R1$, as shown in Figure 5. The source transmits the input X through the direct BSC to the destination such that

$$Y = X + Z \, mod(2) \tag{23}$$

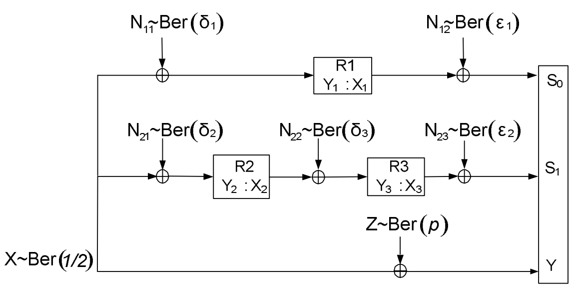

Figure 5. Arbitrary SISO relay network

Relay nodes $R1$ and $R2$ simultaneously receive the corrupted version of X such that

$$Y_1 = X + N_{11} \, mod(2) \tag{24}$$

$$Y_2 = X + N_{21} \, mod(2) \tag{25}$$

According to the channel coding theorem, for any $\epsilon > 0$ (ϵ is the small probability of error fixed) and rate $R < C$, we can transmit at rate R so that the average probability of error $< \epsilon$. A very important result is given in [6]. The converse coding theorem says that if $R > C$, then reliable communication is not possible [6]. So the undetected codeword error probability is less than $2^{-n(I(X;Y,U_1,U_2)-\gamma}$, where $\gamma > 0$ goes to zero as n tends to infinity. Then,

$$R < I(X; Y, U_1, U_2) \tag{26}$$

and

$$I(X; Y, U_1, U_2) = I(X; Y|U_1, U_2) + I(X; U_1, U_2) \tag{27}$$

$$= H(Y|U_1, U_2) - H(Y|U_1, U_2, X) + H(X) - H(X|U_1, U_2) \tag{28}$$

$$= H(Y|U_1, U_2) - H(Z) + 1 - H(X|U_1, U_2)$$

The entropy $H(X|U_1, U_2)$ can be simplified as

$$H(X|U_1, U_2) = 1 - I(X; U_1, U_2) \tag{29}$$

$$= 1 - [I(X; U_1) + I(X; U_2|U_1)] \tag{30}$$

$$= 1 - [H(X) - H(X|U_1) + H(U_2|U_1) - H(U_2|U_1, X)] \tag{31}$$

$$= H(X|U_1) - H(U_2|U_1) + H(U_2|U_1, X)] \tag{32}$$

Now, applying the "Mrs. Gerber's Lemma" on the conditional entropy of binary random variables, i.e.,

$$H(Y_1|U_1) \ge \alpha, \tag{33}$$

then,

$$H(X|U_1) \ge \mathcal{H}(\mathcal{H}^{-1}(\alpha) * \delta_1). \tag{34}$$

The equality holds if Y_1 given U_1 is a random variable of the Ber($\mathcal{H}^{-1}(\alpha)$) distribution. Let $\alpha \triangleq H(Y_1) - R_1$. Then, U_1 achieves this equality, i.e., if Y_1 given U_1 is a Ber($\mathcal{H}^{-1}(H(Y_1) - R_1)$), then this U_1 under a rate constraint R_1 in the standard rate distortion theory minimizes the Hamming distortion of Y_1 [3, p.925]. Since Y_1~Ber(1/2), $H(Y_1) = 1$. Hence, $H(X|U_1)$ can be written as

$$H(X|U_1) = \mathcal{H}(\mathcal{H}^{-1}(1 - R_1) * \delta_1) \tag{35}$$

where R_1 is the rate, i.e., $R_1 \triangleq \max_{p(x_1)} I(X_1; S_0) = 1 - \mathcal{H}(\varepsilon_1)$ Substituting this into (35) yields

$$H(X|U_1) = \mathcal{H}(\varepsilon_1 * \delta_1). \tag{36}$$

Similarly, the entropy $H(U_2|U_1)$ can be simplified by applying the "Mrs. Gerber's Lemma" on the conditional entropy of binary random variables, such as

$$H(X|U_1) \ge \hat{\alpha} \tag{37}$$

and then

$$H(U_2|U_1) \ge \mathcal{H}(\mathcal{H}^{-1}(\hat{\alpha}) * (\delta_2 * \delta_3 * \varepsilon_2)). \tag{38}$$

Since $\hat{\alpha} = \mathcal{H}(\varepsilon_1 * \delta_1)$, substituting this into (38) yields

$$H(U_2|U_1) = \mathcal{H}((\varepsilon_1 * \delta_1) * (\delta_2 * \delta_3 * \varepsilon_2)). \tag{39}$$

Substituting (39) and (36) into (32) yields

$$\eta \triangleq H(X|U_1, U_2) = \mathcal{H}(\varepsilon_1 * \delta_1) - \mathcal{H}\big((\varepsilon_1 * \delta_1) * (\delta_2 * \delta_3 * \varepsilon_2)\big) + \mathcal{H}(\delta_2 * \delta_3 * \varepsilon_2) \tag{40}$$

Also, applying the same "Mrs. Gerber's Lemma" on the conditional entropy of binary random variables gives

$$H(Y|U_1, U_2) = \mathcal{H}(\mathcal{H}^{-1}(\eta) * p) \tag{41}$$

So, the final mutual information in (28) yields

$$I(X; Y, U_1, U_2) = \mathcal{H}(\mathcal{H}^{-1}(\eta) * p) - \mathcal{H}(p) + 1 - \eta \tag{42}$$

Hence, the capacity of the modulo-sum relay network shown in Figure 5 (given arbitrary SISO relay network) is

$$C = \max_{\substack{p(u_1|y_1):I(U_1;Y_1)\le R_1 \\ p(u_2|y_2):I(U_2;Y_2)\le R_2 \\ p(u_3|y_3):I(U_3;Y_3)\le R_3}} \{\mathcal{H}(\mathcal{H}^{-1}(\eta) * p) - \mathcal{H}(p) + 1 - \eta\} \quad (43)$$

where, $\eta = \mathcal{H}(\varepsilon_1 * \delta_1) - \mathcal{H}\big((\varepsilon_1 * \delta_1) * (\delta_2 * \delta_3 * \varepsilon_2)\big) + \mathcal{H}(\delta_2 * \delta_3 * \varepsilon_2)$.

VI. ANALYTICAL RESULTS

Based on the derived result in (43) for Figure 5, this section provides the analytical results. Figures 6(a) and 6(b) show the capacity in bits per transmission versus the rate R_1 for the arbitrary SISO relay network shown in Figure 5. The parameters considered for plotting Figure 6(a) are $\{\delta_1 = 0.1$, $\delta_2 = 0.1$, $\delta_3 = 0.1$ $\varepsilon_2 = 0.1$, and $p = 0.1\}$, and for Figure 6(b) are $\{\delta_1 = 0.1$, $\delta_2 = 0.1$, $\delta_3 = 0.1\}$, whereas ε_2 and p vary for different curves.

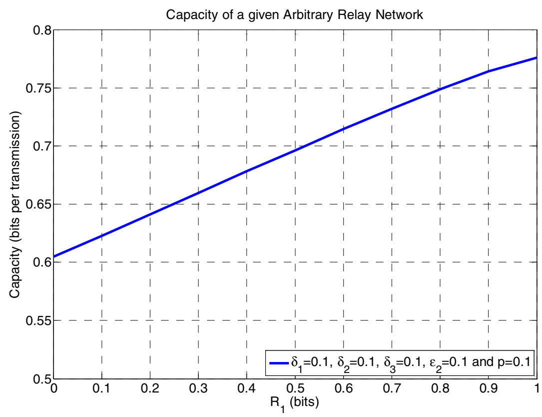

Figure 6(a). Capacity (bits per transmission) versus rate R_1 (bits, function of ε_1) for relay network shown in Figure 5, with parameters $\delta_1 = 0.1$, $\delta_2 = 0.1$, $\delta_3 = 0.1$, $\varepsilon_2 = 0.1$, and $p = 0.1$.

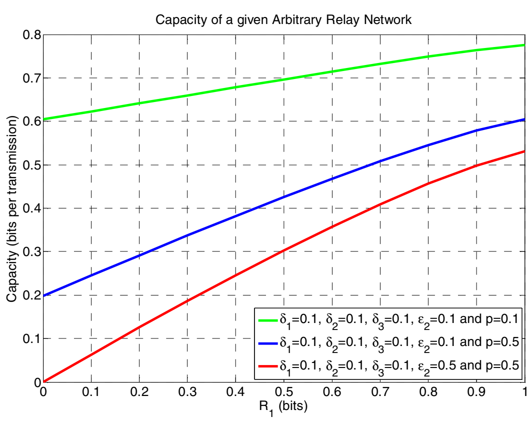

Figure 6(b). Capacity (bits per transmission) versus rate R_1 (bits function of ε_1) for relay network shown in Figure 5, with parameters $\delta_1 = 0.1$, $\delta_2 = 0.1$, $\delta_3 = 0.1$, and ε_2 and p varying.

VII. CONCLUSION

This paper derived the capacity of a modulo-sum two-parallel relay network and generalized the capacity expression for a modulo-sum *n*-parallel relay network. In addition, this paper presented the capacity of a modulo-sum serial relay network. Both parallel and serial relay networks closed-form capacity expressions are obtained. Furthermore, capacities obtained for parallel and serial relay networks for the binary case are extended to a modular additive, i.e., *m*-ary modulo-sum. The capacities are obtained by using a quantize-and-forward strategy at each relay node. Finally, this paper assumes an arbitrary SISO relay network (a network with serial and parallel combinations of relay nodes) and obtains the capacity expression for that given network. With this, we can find the capacity for any given arbitrary SISO relay network.

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