Optimum and Sub-Optimum Receivers for OFDM Signals with Iterative Clipping and Filtering

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Abstract - In this paper we consider the optimum ML (Maximum-Likelihood) detection for OFDM signals (Orthogonal Frequency Division Multiplexing) with iterative clipping and filtering. It is shown that the nonlinear distortion does not necessarily mean significant performance degradation and, in fact, the optimum performance could even better than the performance with ideal, linear transmitters. We also present sub-optimum receivers that allow remarkable performance improvements, being able to reduce significantly the gap between the optimum performance and the performance of typical OFDM receivers.[1]

I. Introduction

OFDM signals (Orthogonal Frequency Division Multiplexing) are known to have envelope fluctuations PAPR (Peak-to-Average Power Ratio), leading to amplification difficulties. However, the simpler and most promising method to reduce the envelope fluctuations of OFDM signals is by employing clipping techniques. Due to the nonlinear nature of the clipping operation, a subsequent filtering procedure is recommendable to avoid out-of-band radiation [1], [2], which is particularly simple and effective if implemented in the frequency domain [3], [4]. However, this filtering operation leads to some regrowth in the envelope fluctuations, limiting the achievable PAPR gains of clipping and filtering techniques. A simple way to overcome this problem is repeating the clipping and filtering procedures several times [4], [5]. However, the overall nonlinear distortion effects increase with the number of clipping and filtering iterations, which means that we can have severe performance degradation if we want to reduce significantly the envelope fluctuations of the transmitted signals.

The transmitted signal when we employ iterative clipping and filtering procedures can be decomposed as the sum of two uncorrelated components: a useful component, which is a mildly filtered version of the original OFDM signal, and nonlinear distortion component [4]. Conventional receiver implementations treat the nonlinear distortion component as an additional noise-like term that leads to performance degradation. To improve the performance we can try to estimate the nonlinear distortion component and cancel its effects [6], [7], [8]. However, these receivers have poor performance, especially for low SNR (Signal-to-Noise Ratio) due to the difficulty in estimating the nonlinear distortion component. Even if we were able to estimate and cancel completely the nonlinear distortion component, the achievable performance would not be necessarily the optimum one because the nonlinear distortion component has information concerning the transmitted signal that could be employed to improve the performance [9]. The optimum performance is obtained by employing an ideal ML (Maximum Likelihood) receiver where the data estimate is the one corresponding to a transmitted signal that is closer (in terms of Euclidean distance) to the received signal, which means that the optimum receiver takes into account the overall nonlinear transmitted signal (i.e., it considers not just the useful component of the transmitted signal but also the nonlinear distortion component) [10].

In this paper we consider the use of iterative clipping and filtering techniques to reduce significantly the envelope fluctuations of OFDM signals and we study the performance of optimum receivers. It is shown that the optimum performance of iterative clipped and filtered OFDM can be better than with an ideal linear transmitter. Since the complexity of an optimum receiver is extremely high, we present and evaluate sub-optimal receivers that try to approach the optimum performance.

II. Nonlinear Effects in Iterative Clipped and Filtered OFDM Signals

Throughout this paper we consider the use of the iterative clipping and filtering techniques of [5], [4] that allow significant reductions in the envelope fluctuations of OFDM signals while maintaining the spectral occupation of conventional OFDM schemes with linear transmitters.

The transmitted signals are generated as follows. The data bits to be transmitted are mapped into the data symbols $\{S_k; k = 0, 1, \ldots, N-1\}$, where S_k is selected from an appropriate constellation (e.g., a QPSK constellation) according to a given mapping rule (usually a Grey maping) and corresponds to the complex amplitude associated to the kth subcarrier (for conventional OFDM schemes $\{S_k; k = 0, 1, \ldots, N-1\}$ would be the frequency-domain block to be transmitted). Next we add $N' - N$ zeros to the block $\{S_k; k = 0, 1, \ldots, N-1\}$,

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leading to the block $\{S'_k; k=0,1,\ldots,N'-1\}$, and compute its IDFT so as to form the time-domain samples $\{s'_n; n=0,1,\ldots,N'-1\}$. Adding $N'-N$ zeros to the block $\{S_k; k=0,1,\ldots,N-1\}$ is formally equivalent to add $(N'-N)/2$ idle subcarriers at each edge of the band, which means that the block $\{s'_n; n=0,1,\ldots,N'-1\}$ can be regarded as an oversampled version of the original OFDM block with the oversampling factor $M=N'/N$. This oversampling is required for an efficient implementation of the clipping and filtering procedure (an oversampling factor $M=4$ is enough for most applications). The oversampled samples $s_n^{CF(0)}=s'_n$ are then submitted to a nonlinear device leading to the samples $s_n^{C(1)}=g^{NL}(|s_n^{CF(0)}|)\exp(j\arg(s_n^{CF(0)}))$. In this paper we consider an ideal envelope clipping with normalized clipping level s_M/σ, i.e., the nonlinear operation is $g^{NL}(R)=R$ for $R\leq s_M$ and $g^{NL}(R)=s_M$ for $R>s_M$, with R denoting the envelope at its input. Clearly, the clipping is employed to reduce the envelope fluctuations on the samples $s_n^{C(0)}$. However, due to its nonlinear nature, it also produces spectral widening, leading to out-of-band radiation. The out-of-band radiation can be reduced or eliminated by employing a suitable filter after the clipping procedure, which can be implemented in the frequency domain. The frequency-domain block associated to the clipped signal is $\{S_k^{C(1)}; k=0,1,\ldots,N'-1\}$ = DFT $\{s_n^{C(1)}; n=0,1,\ldots,N'-1\}$ and the corresponding filtered signal is $\{S_k^{CF(1)}=S_k^{C(1)}G_k; k=0,1,\ldots,N'-1\}$, with G_k denoting the filtering coefficients that can selected to be $G_k=1$ for the N in-band subcarriers and $G_k=0$ for the remaining $N'-N$ out-of-band subcarriers so as to remove completely the out-of-band radiation while maintaining the in-band samples unchanged. However, this filtering procedure has the undesirable side effect of producing some regrowth in the envelope fluctuations of the corresponding time-domain samples $\{s_n^{CF(1)}; n=0,1,\ldots,N'-1\}$ = IDFT $\{S_k^{CF(1)}; k=0,1,\ldots,N'-1\}$. Therefore, we can repeat the clipping and filtering procedures L times, where the input of the ith clipping operation is the block $\{s_n^{CF(i-1)}; n=0,1,\ldots,N'-1\}$ = IDFT $\{S_k^{CF(i-1)}; k=0,1,\ldots,N'-1\}$ associated to the previous filtering procedure (i.e., the $(i-1)$th filtering procedures). If the number of subcarriers is high ($N>>1$ then the frequency-domain samples to be transmitted can be decomposed as the sum of a filtered version of the original OFDM signal with a nonlinear distortion component, i.e., written as

$$S_k^{Tx}=\alpha_k^{Tx}S_k+D_k^{Tx}=S_k^{CF(L)}=\alpha_k^{(L)}S'_k+G_kD_k^{(L)} \quad (1)$$

(see [4]), where the nonlinear distortion component $D_k^{(L)}$ is almost uncorrelated with the data samples. Clearly, this nonlinear distortion component can be regarded as an additional noise component that leads to performance degradation. This degradation can be serious when we have strong nonlinear distortion effects (i.e., small clipping levels combined with several clipping and filtering iterations), especially for large constellations.

III. Performance of ML Receivers

Conventional OFDM receivers treat the nonlinear distortion component as an undesirable noise component and are highly suboptimal. The optimum performance is achieved with an ideal ML receiver. If the duration of the cyclic prefix is longer than the overall channel impulse response (as in typical OFDM implementations), the received frequency-domain block (after removing the samples associated to the cyclic prefix and performing the DFT operation) will be $\{Y_k; k=0,1,\ldots,MN-1\}$, with

$$Y_k=S_k^{Tx}+N_k=\alpha_k^{(L)}S'_k+D_k^{(L)}+N_k, \quad (2)$$

with H_k denoting the channel frequency response for the kth subcarrier and N_k denoting the channel noise component (for the sake of simplicity, we assume the same oversampling factor at the transmitter and the receiver).

Without loss of generality we focus our analysis entirely on the frequency-domain samples. We define the average bit energy as[2]

$$E_b \triangleq \frac{1}{2N}\sum_k E[|S_k^{Tx}|^2] =$$
$$\frac{1}{2N}\sum_k\left(|\alpha_k^{(L)}E[|S_k^{'}|^2]+E[|D_k^{(L)}|^2]\right), \quad (3)$$

where the sums are over the set of N in-band subcarriers. Under this definition $E_b=1$ for conventional OFDM signals with a linear transmitter and normalized QPSK constellations with $S_k=\pm1\pm j$. When we employ iterative clipping and filtering techniques we reduce E_b, which needs to be obtained by simulation (the only case where we can obtain it analytically is when we have a single clipping operation [**?**]).

To understand the impact of iterative clipping and filtering on the performance let us consider two possible transmitted sequences $\{S_k^{Tx(1)}; k=0,1,...,MN-1\}$ and $\S_k^{Tx(2)}; k=0,1,...,MN-1\}$ associated to data sequences that differ in a single bit. Fig. 1 shows the absolute value of the difference between these sequences. Clearly, the signals differ in all in-band frequencies, not only the frequency where we modified the bit (in this example the modified bit is at the center of the band). Moreover, the normalized Euclidean distance between these sequences

$$D^2=\sum_k|S_k^{Tx(1)}-S_k^{Tx(2)}|^2=$$
$$=\sum_{k\in\Psi_G}|\alpha_k^{Tx(1)}S_k^{'(1)}+D_k^{Tx(1)}-\alpha_k^{Tx(2)}S_k^{'(2)}+D_k^{Tx(2)}|^2 \quad (4)$$

is higher than $4E_b$ (the value of the minimum Euclidean distance for conventional OFDM with a QPSK constellation),

[2]Naturally, the average bit energy is related to the time domain samples, but since $\sum_{k=0}^{N-1}|X_k|^2=N^2\sum_{n=0}^{N-1}|x_n|^2$ when $\{X_k; k=0,1,\ldots,N-1\}$ denotes the DFT of the block $\{x_n; n=0,1,\ldots,N-1\}$, the SNR, the ratio between the average bit energy and the squared Euclidean distances (defined in the frequency domain) and other related parameters are identical to the ones defined in the time domain.

especially for $L > 1$. Since this is a typical case, we can assume that the nonlinear operation has two unexpected positive effects on the performance of OFDM schemes: since typically we have $D^2 > 4E_b$, instead of leading to significant performance degradation the nonlinear operation could lead to some asymptotic gain relatively to the linear OFDM case; since different sequences differ in many subcarriers, there is an intrinsic diversity effect that could be employed to further improve the performance in fading channels.

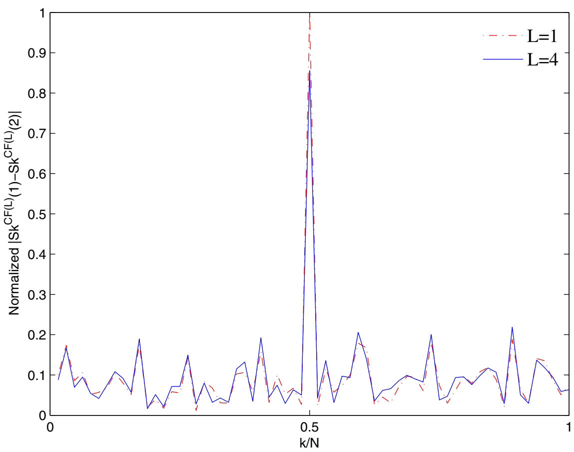

Fig. 1. Normalized absolute value of the difference between two OFDM signals with iterative clipping and filtering with L iterations. The data blocks have $N = 64$ symbols and the normalized clipping level is $s_M/\sigma = 0.5$.

To take advantage of this we need to employ an ideal ML receiver that selects $\{\hat{S}'_k; k = 0, 1, \ldots, MN-1\}$ (and, inherently, $\{\hat{S}_k; k = 0, 1, \ldots, N-1\}$) that minimizes

$$J = \sum_{k\in\Psi_G} |Y_k - S_k^{Tx}|^2 = \sum_k |Y_k - \alpha^{Tx} H_k S'_k - H_k D_k^{Tx}|^2. \tag{5}$$

Clearly, the performance of an ideal ML receiver will be related to the equivalent Euclidean distance between the useful received signals associated to two possible transmitted sequences $\{S_k^{'(i)}; k = 0, 1, ..., MN-1\}$, $i = 1$ or 2, as

$$D^2_{Eq_{1,2}}(\mathbf{H}) \triangleq \sum_k |H_k|^2 |\alpha^{Tx(1)} S_k^{'(1)} + D_k^{(1)} - \alpha^{Tx(2)} S_k^{'(2)} - D_k^{Tx(2)}|^2, \tag{6}$$

where $\mathbf{H} \triangleq [H_0\ H_1\ ...\ H_{MN-1}]$ denotes the overall channel frequency response. Since this distance is dominated by data sequences that differ in a single bit, the BER associated to the channel realization $\mathbf{H}$ will be approximately given by

$$BER(\mathbf{H}) \approx \frac{1}{2N} \sum_{\{S_k^{(2)}\}\in\Phi_1(\{S_k^{(1)}\})} Q\left(\sqrt{\frac{D^2_{Eq_{1,2}}(\mathbf{H})/2}{N_0}}\right), \tag{7}$$

where $\Phi_1(\{S_k\})$ denotes the set of sequences that differ from $\{S_k\}$ in only 1 bit (clearly, the cardinality of $\Phi_1(\{S_k\})$ is $\#\Phi_1(\{S_k\}) = 2N-1$, since there are $2N$ bits in N QPSK symbols). The average BER will be given by the multiple integral

$$BER \approx \int BER(\mathbf{H}) p(\mathbf{H}) d\mathbf{H}, \tag{8}$$

where $p(\mathbf{H})$ denotes the joint probability density function of the overall channel frequency response $\mathbf{H}$. Since this integral is difficult to evaluate, we can estimate (8) by averaging over a large number of independent channel realizations.

For an ideal AWGN channel we have $|H_k| = 1$ for all subcarriers, leading to

$$D^2_{Eq_{1,2}}(\mathbf{H}) = D^2_{1,2} \triangleq \sum_k |\alpha^{Tx(1)} S_k^{'(1)} + D_k^{(1)} - \alpha^{Tx(2)} S_k^{'(2)} - D_k^{Tx(2)}|^2, \tag{9}$$

which means that the BER will be given by

$$BER \approx E_{\{S_k^{(1)}\}}\left[\frac{1}{2N} \sum_{\{S_k^{(2)}\}\in\Phi_1(\{S_k^{(1)}\})} Q\left(\sqrt{\frac{D^2_{1,2}/2}{N_0}}\right)\right]. \tag{10}$$

Naturally, for a conventional linear OFDM transmitter we have $D^2_{1,2} = 4E_b$, leading to the well-known BER expression

$$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{11}$$

The approximate ML performance obtained this way is depicted in figs. 2 and 3 for an ideal AWGN channel and a typical frequency-selective channel, respectively (the ideal performance of conventional linear OFDM was included for the sake of comparisons). In both cases we have $N = 64$ data symbols and L clipping and filtering iterations, with normalized clipping levels s_M/σ. From these figures, it is clear that the ideal ML performance of nonlinear OFDM can be better than the performance of conventional linear OFDM schemes, with gains around 1dB for an ideal AWGN channel and huge diversity gains for the typical frequency-selective channel. These gains are higher when we have stronger non-linear distortion effects (i.e., for lower clipping levels and/or a higher number of clipping and filtering iterations).

The above analysis considers the transmission of "typical" sequences and the conclusions could be different for some sequences such as sequences where S_k is constant. However, since these sequences are very rare (and, in fact, they can be avoided in practice through the use of suitable scrambling procedures), its effect on the overall performance can be neglected.

IV. Suboptimal ML-Based Receiver

The optimum receiver is too complex, even for a moderate number of subcarriers N and small constellations. In this section we present a sub-optimal receiver that tries to approach the ML receiver performance. For this propose we start with the estimated signal associated to a conventional OFDM receiver (which will be denoted "*hard decision sequence*"

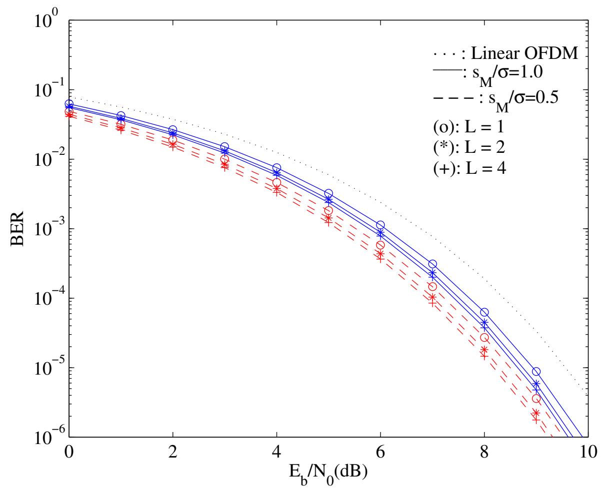

Fig. 2. AWGN BER performance with ML receivers that apply ICF techniques.

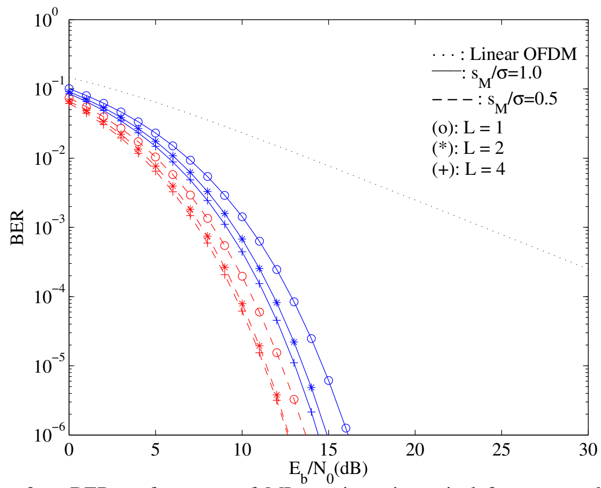

Fig. 3. BER performance of ML receivers in typical frequency-selective channels using ICF techniques.

in the remaining of the paper) and perform variations of their bits, obtain the corresponding nonlinear signal to be transmitted (i.e., the corresponding signal is submitted to the iterative clipping and subsequent frequency-domain filtering operation L times as it was employed in the transmitter), compute the corresponding Euclidean distance relatively to the received sequence and choose the sequence among the original hard decision sequence and all variations of it that were tested that has smaller Euclidean distance to the received signal. The motivation for our techniques is that usually the optimal ML sequence differs from the *hard decision sequence* in a small number of bits. Therefore, we could test only a small fraction of all possible sequences and still have the optimal ML sequence among them. If the ML sequence is among the sequences that we tested than we obtain the optimal performance.

Our receiver start with *hard decision sequence* and switch the P in $2N$ bits with smaller reliability. If the Euclidean distance relatively to the received sequence improves the bit remains changed, if not we return to the original bit. After this we end up with a sequence that has Euclidean distance relatively to the received sequence that is smaller (or at least equal) to the distance from the *hard decision sequence* to the received signal. Since some of the bits might be changed with this procedure we can restart changing the first bit and repeat the procedure K times. The proposed method works similarly for an ideal AWGN channel and a frequency-selective channel. Naturally, in the later case we should take into account the channel frequency response (i.e., the set $\mathbf{H}$) when computing the distances, as in (6).

V. Performance Results

In this section we present several BER performance results for the sub-optimal receiver described in the previous section. Unless otherwise stated, the OFDM signal has $N = 64$ useful subcarriers with QPSK constellations and an oversampling factor $M = 4$. The QPSK symbols are selected from the data signal using a Gray mapping rule. The non-linear device corresponds to an ideal envelope clipping with a normalized clipping level s_M/σ (unless otherwise stated, we assume $s_M/\sigma = 1.0$). The filtering after each clipping operation removes completely the out of band radiation, leaving the in-band subcarriers unchanged. We consider both an ideal AWGN channel and a severely frequency-selective channel with uncorrelated Rayleigh fading on different multipath components. We assume perfect synchronization and channel estimation at the receiver.

To obtain an approximation of the achievable ML performance we could proceed as follows. The optimum ML estimate is likely to be one of the following sequences: the *hard decision sequence* or one of its variations[3] (that could be obtained as described in the previous section); the transmitted sequence or one of its variations. These variations could be obtained using a procedure similar to the one described in the previous section). For the sake of comparisons, this approximation of the ML performance will be included in the following figures, being denoted as ”ML performance”.

Let us start by considering an ideal AWGN channel.

Fig 4 shows the BER for our sub-optimal receiver when we have L ICF iterations and different values of P. We also have $K = 2$, since larger values of K provide only marginal gains. From this figure it is clear that we can improve significantly the performance relatively to conventional OFDM schemes, with results close to the ML performance. We also note that the higher the P the better the performance achieved, although $P = N/2$ yields almost the same performance as $P = 2N$. For larger values of L the performance degrades since this means that the signal have harder nonlinear distortion effects.

Let us now consider a frequency-selective channel. The figure 5 shows the BER performance of the proposed receiver. Although the ML performance is better for higher values of L (see figure 3) this is not the case of our sub-optimum receiver mainly because of its nature (i.e., the receiver does not tests the all possible sequences but only the most likely candidates).

[3]The term ”sequence variation” will be used to define sequences that differ in a small number of bits.

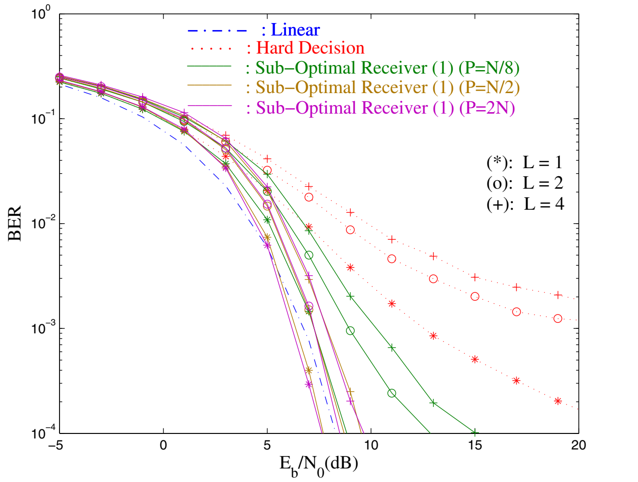

Fig. 4. BER performance for our sub-optimal receiver in an AWGN channel and variable L and P

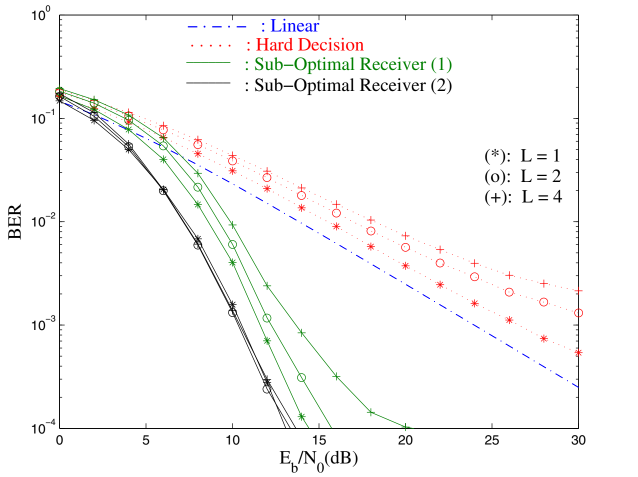

Fig. 5. BER performance for our sub-optimal receiver in an dispersive channel and variable number of ICF iterations

From figure 6 it is clear that when we consider an dispersive channel the larger the value of P the better the performance and contrary to AWGN ideal case, our sub-optimal receiver must evaluate the $P = 2N$ comparisons since there are a significant difference to $P = N/2$ case.

VI. Conclusions

In this paper we considered the use of iterative clipping and filtering procedures to design OFDM signals with very low envelope fluctuations while maintaining the spectral occupation of conventional OFDM schemes. However, instead of the conventional receivers that treat the nonlinear distortion as an undesirable noise-like component, we considered the optimum ML detection.

It was shown that the nonlinear distortion does not necessarily mean significant performance degradation and, in fact, the ML performance could even better than the performance with ideal, linear transmitters. We also presented a sub-optimum ML-based receiver that allows remarkable performance improvements, being able to reduce significantly the gap between

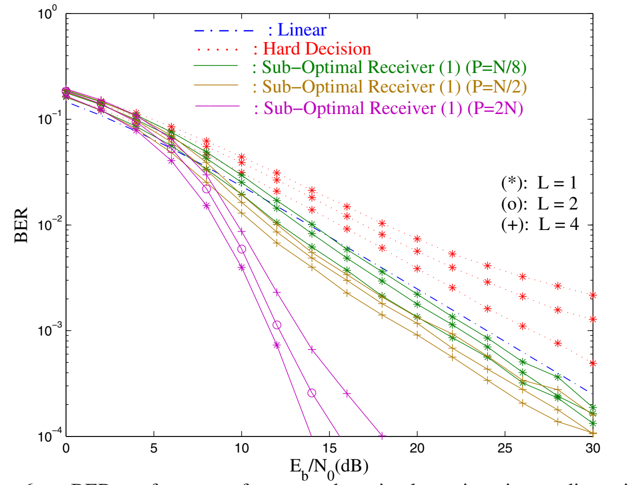

Fig. 6. BER performance for our sub-optimal receiver in an dispersive channel and variable number of ICF iterations and modified bits

the ML performance and the performance of typical OFDM receivers.

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