Iterative Frequency-Domain Channel Estimation and Equalization for Relay-Assisted SFBC Single-Carrier Systems

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***Abstract*—Cooperative diversity is an effective transmission technique allowing multiple users to create a virtual antenna array although each of them is equipped with single antenna. In this paper, an iterative channel estimation and equalization (ICEE) scheme for single-carrier systems is proposed, realizing space-frequency block code (SFBC) with amplify-and-forward (AF) relaying in a distributed fashion. Most existing researchers assume perfect channel information available at the receiver, however, the effect of imperfect channel estimation should be considered for practical applications. In the proposed algorithm, pilot-aided channel estimation (CE) and soft inter-symbol interference (ISI) cancelation with minimum mean squared error (MMSE) filtering are performed iteratively to suppress interferences. An implementation that accounts for imperfect channel estimation is also investigated. The bit error rate (BER) performances and extrinsic information transfer (EXIT) charts are presented for Rayleigh fading channels. Simulation results demonstrate the effectiveness of the proposed algorithm.**

***Index Terms*—channel estimation, equalization, iterative detection, relay, SFBC**

I. INTRODUCTION

Single-carrier frequency-domain equalization (SC-FDE) has similar performance and complexity compared with orthogonal frequency division multiplexing (OFDM) [1]. Recently, SC-FDE has drawn a great attention as an alternative to OFDM. SC-FDE offers simple transmit structure at mobile equipment. Moreover, SC-FDE does not suffer high peak-to-average power ratio (PAPR) as well as sensitivity to frequency and phase offsets, thus alleviating the nonlinear distortion and carrier synchronization problems inherent to OFDM [2].

Transmit diversity is an effective technique to combat the fading effect in mobile wireless communications. Alamouti proposed a simple space-time block code (STBC) for two transmit antennas, guaranteeing full spatial diversity and full rate over frequency-flat channels [3]. Based on the Alamouti's scheme, Al-Dhahir proposed a combination of STBC with SC-FDE [4]. Although the scheme can achieve the maximum possible diversity gain, the performance deteriorates considerably in a time-varying environment. In order to mitigate the fast fading distortion caused by high-speed mobility, Jang et. al. presented SFBC combined with SC-FDE [5]. However, the schemes above have only adopted non-iterative equalization and have not taken channel coding into account. By performing joint equalization and decoding, turbo processing can achieve an impressive performance [6]. A turbo frequency-domain equalization (T-FDE) scheme for SFBC SC-FDE was proposed in [7]. Due to the size and power limitation, the deployment of multiple antennas might not be practical for mobile devices. Recently, it has been demonstrated that ”cooperative diversity” provides an effective means as an alternative to multiple-antenna transmission schemes [8]. Iterative detection for relay-assisted STBC and SFBC SC-FDE were proposed in [9] and [10].The STBC and SFBC SC-FDE schemes above assume perfect channel state information (CSI), which is not available at the receiver. Training sequences aided channel estimations for STBC and SFBC were proposed in [11] and [12], however the training sequences require two additional time slots, which is inefficient in the sense of spectral efficiency. Meanwhile the channels are assumed to be constant over two consecutive time intervals or between two adjacent frequencies. Unfortunately, such realistic assumptions cannot be satisfied in practical applications. Consequently, the interferences, such as intra-SFBC interference and ISI cannot be canceled effectively when used in a time/frequency-varying mobile environment due to the non-quasi-orthogonal channel.

In the proposed algorithm, iterative frequency-domain channel estimation and equalization algorithms are described in a unifying framework. Pilot-aided channel estimation in the frequency-domain is carried out at the receiver first, then diagonalized space-frequency decoding and symbol-wise frequency-domain equalization are performed using the channel frequency responses (CFRs) and MSE obtained from the channel estimator. In the following iterative stage, the channel estimator refine the CFRs using the a-priori information passed by the decoder, then soft ISI cancelation and symbol-wise MMSE filtering are employed iteratively with updated CFRs and MSE. With symbol-oriented processing, the detection can conduct ISI cancelation more effectively while avoiding using the a-priori information about the estimate symbol, which is not expected in turbo principle [13].

The remainder of the paper is organized as follows. The system model is introduced in Section II. Section III describes the proposed iterative frequency-domain channel estimation and equalization algorithms for relay-assisted SFBC single-carrier systems. Simulation results are given in Section IV. The conclusions are drawn in Section V.

II. System Model

A. Protocol

A single relay-assisted cooperative communication scenario is considered for an uplink system, where the source, relay and destination terminals are equipped with single transmit and receive antennas. We assume AF relaying and adopt the user cooperation protocol in [14]. Specifically, the source communicates with the relay during the first signaling interval. In the second signaling interval, both the source and relay communicate with the destination while the relay amplifies and retransmits the signals received from the source in the first signaling interval.

B. Transmit Diversity Scheme

In the relay-assisted cooperation, Alamouti SFBC precoding encodes two components over two adjacent frequencies onto the source and the relay respectively. The space-frequency block-coded transmission matrix may be represented by

$$\begin{bmatrix} X_S(2k) & X_R(2k) \\ X_S(2k+1) & X_R(2k+1) \end{bmatrix} = \begin{bmatrix} X(2k) & X(2k+1) \\ -X^*(2k+1) & X^*(2k) \end{bmatrix} \tag{1}$$

where $X_S(k)$ and $X_R(k)$ are the frequency-domain signals of the source and the relay at the kth frequency.

At the source terminal, the modulated data symbols $\mathbf{s} \triangleq [s(0), s(1), \ldots, s(N_d-1)]^{\mathrm{T}}$ are transformed to the frequency-domain using FFT, $\mathbf{S} = \mathbf{F}_{N_d}\mathbf{s}$. The transmit data sequence in the frequency-domain $\mathbf{X}_d$ can be formed as

$$\mathbf{X}_d = [S(0), -S^*(1), \ldots, S(N_d-2), -S^*(N_d-1)]^{\mathrm{T}} \tag{2}$$

$\mathbf{X}_d$ are multiplexed with the pilot sequence $\mathbf{X}_p$ to form the transmit sequence in the frequency-domain using the pilot and data multiplexing structure in Section II-C. Then $\mathbf{X}_S$ are transformed to the time domain to obtain $\mathbf{x}_S$ using IFFT. $\mathbf{x}_S$ are appended with a cyclic-prefix (CP) and transmitted to the relay.

The channel impulse responses (CIRs) from transmitting node A to receiving node B is $\mathbf{h}_{AB} = [h(0), \ldots, h(L_{AB})]^{\mathrm{T}}$. Subscripts S, R and D stand for the source, relay and destination respectively. Removing the CP, the received signals at the relay are given by

$$\mathbf{r}_R = \sqrt{E_{SR}}\mathbf{H}_{SR}\mathbf{x}_S + \mathbf{n}_R \tag{3}$$

where E_{AB} represents the average energy available at the receiving node B, $\mathbf{H}_{SR}$ is the $N \times N$ circulant channel matrix with entries $[\mathbf{H}_{AB}]_{m,n} = \mathbf{h}_{AB}\left((m-n)_N\right)$, and $\mathbf{n}_R$ is the additive white Gaussian noise (AWGN) sample sequence with variance σ_w^2. To ensure a balance of received power at the destination terminal, $\mathbf{r}_R$ is normalized by a factor γ_R as $\tilde{\mathbf{r}}_R \triangleq \mathbf{r}_R / \sqrt{E_{SR} + \sigma_w^2} \triangleq \gamma_R \mathbf{r}_R$. To have an Alamouti's orthogonal structure in the frequency-domain as in (1), the transmit signals of the relay are processed as

$$\begin{aligned} \mathbf{x}_R &= \mathbf{F}_N^{\mathrm{H}}\mathbf{PS}(\mathbf{F}_N\tilde{\mathbf{r}}_R)^* \\ &= \gamma_R\sqrt{E_{SR}}\mathbf{F}_N^{\mathrm{H}}\mathbf{PS}(\mathbf{F}_N\mathbf{H}_{SR}\mathbf{x}_S)^* + \mathbf{n}'_R \end{aligned} \tag{4}$$

where $\mathbf{S} = \mathbf{I}_{N/2} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\mathbf{P} = \mathbf{I}_{N/2} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\otimes$ denotes the Kronecker product, and $\mathbf{n}'_R = \gamma_R \mathbf{F}_N^{\mathrm{H}}\mathbf{PS}(\mathbf{F}_N\mathbf{n}_R)^*$.

In the second time slot, the source retransmits $\mathbf{x}_S$, while the relay transmits $\mathbf{x}_R$, after appending a CP with length $L = \max(L_{SD}, L_{RD})$. At the destination, removing the CP, the received signals can be written as

$$\mathbf{r}_D = \sqrt{E_{SD}}\mathbf{H}_{SD}\mathbf{x}_S + \sqrt{E_{RD}}\mathbf{H}_{RD}\mathbf{x}_R + \mathbf{n}_D \tag{5}$$

The destination normalizes the received signals by a factor $\gamma_D = 1/\sqrt{1+\gamma_R^2 E_{RD}}$. This does not affect SNR, but simplifies the ensuring presentation [14]. After normalization, we have

$$\mathbf{r}'_D = \gamma_{SD}\mathbf{H}_{SD}\mathbf{x}_S + \gamma_{SRD}\mathbf{H}_{RD}\mathbf{F}_N^{\mathrm{H}}\mathbf{PS}(\mathbf{F}_N\mathbf{H}_{SR}\mathbf{x}_S)^* + \mathbf{n}'_D \tag{6}$$

where $\gamma_{SD} = \gamma_D\sqrt{E_{SD}}$, $\gamma_{SRD} = \gamma_D\gamma_R\sqrt{E_{SR}E_{RD}}$, $\mathbf{n}'_D = \gamma_D(\sqrt{E_{RD}}\mathbf{H}_{RD}\mathbf{n}'_R + \mathbf{n}_D)$. By performing FFT on $\mathbf{r}'_D$, $\mathbf{R}'_D$ can be obtained as

$$\begin{aligned} \mathbf{R}'_D &= \gamma_{SD}\mathbf{\Lambda}_{SD}\mathbf{X}_S + \gamma_{SRD}\mathbf{\Lambda}_{RD}\mathbf{PS}\mathbf{\Lambda}_{SR}^*\mathbf{X}_S^* + \mathbf{N}'_D \\ &= \gamma_{SD}\mathbf{\Lambda}_{SD}\mathbf{X}_S + \gamma_{SRD}\mathbf{\Lambda}_{RD}\mathbf{P}\mathbf{\Lambda}_{SR}^*\mathbf{PPS}\mathbf{X}_S^* + \mathbf{N}'_D \end{aligned} \tag{7}$$

where $\mathbf{\Lambda}_{AB} = \mathbf{F}_N\mathbf{H}_{AB}\mathbf{F}_N^{\mathrm{H}}$ is $N \times N$ diagonal matrix with channel frequency responses (CFRs) along the diagonal, and $\mathbf{N}'_D = \mathbf{F}_N\mathbf{n}'_D$. We define $\mathbf{\Lambda}_{SRD} = \mathbf{\Lambda}_{RD}\mathbf{P}\mathbf{\Lambda}_{SR}^*\mathbf{P}$, which are the equivalent CFRs of the $S \to R \to D$ link, then we have

$$\mathbf{R}'_D = \mathbf{\Lambda}'_{SD}\mathbf{X}_S + \mathbf{\Lambda}'_{SRD}\mathbf{PS}\mathbf{X}_S^* + \mathbf{N}'_D \tag{8}$$

where $\mathbf{\Lambda}'_{SD} = \gamma_{SD}\mathbf{\Lambda}_{SD}$, $\mathbf{\Lambda}'_{SRD} = \gamma_{SRD}\mathbf{\Lambda}_{SRD}$.

C. Pilot and Data Multiplexing Structure

In order to estimate $\mathbf{\Lambda}'_{SD}$ and $\mathbf{\Lambda}'_{SRD}$ at the same time without introducing additional training sequence time slots, the pilot sequences are designed to be orthogonal in the frequency-domain as follows

$$\begin{aligned} \mathbf{X}_p &= [X_p(0), X_p(1), \ldots, X_p(N_p)]^{\mathrm{T}} \\ &= [C(0), 0, C(1), 0, \ldots, C(N_p/2-1), 0]^{\mathrm{T}} \end{aligned} \tag{9}$$

Then the pilot and data signals in the frequency-domain are multiplexed as in (10), which is shown on the top of next page. where D_f is the frequency spacing of the adjacent pilot. The CFRs are estimated by a using *Chu* sequence in [15] as a pilot sequence. Due to the space-frequency structure in (1) and the frequency orthogonal character in (9), the destination can estimate $\mathbf{\Lambda}'_{SD}$ and $\mathbf{\Lambda}'_{SRD}$ separately at the corresponding pilot tones ($\Lambda'_{SD}(k),\ k \bmod D_f = 0$; $\Lambda'_{SRD}(k),\ k \bmod D_f = 1$). The remaining CFRs of $\mathbf{\Lambda}'_{SD}$ and $\mathbf{\Lambda}'_{SRD}$ at the pilot and data position tones are interpolated using the channel estimation method in Section III-A1.

III. Receiver Structure

The receiver consists of two major blocks: the MMSE-based channel estimation block and the turbo-equalization block. The details are explained as follows.

$$\mathbf{X}_S = [X_p(0), X_p(1), X_d(0), \ldots, X_d(D_f-3), X_p(2), X_p(3), X_d(D_f-2), \ldots, X_d(2D_f-5), \ldots, X_p(N_p-2), X_p(N_p-1), X_d(N_p/2(D_f-2)), \ldots, X_d(N_d-1)]^{\mathrm{T}} \tag{10}$$

A. CE within the Out-Loop Iteration

1) Pilot-Based Initial CE: In the initial detection when the estimates of data signals are unavailable, the CFRs at the pilot positions of the $S \to D$ and $S \to R \to D$ link are estimated respectively as

$$\begin{cases} \tilde{\Lambda}'_{SD}(k) = \frac{R'_D(k)}{C(\lfloor k/D_f \rfloor)}, & k \bmod D_f = 0 \\ \tilde{\Lambda}'_{SRD}(k) = \left(\frac{R'_D(k)}{C(\lfloor k/D_f \rfloor)}\right)^*, & k \bmod D_f = 1 \end{cases} \tag{11}$$

The remaining $\tilde{\Lambda}'_{SD}(k)$ and $\tilde{\Lambda}'_{SRD}(k)$ at the pilot and data positions have to be interpolated by time-frequency filtering based on grid by grid. In the following text, we focus on $\tilde{\Lambda}'_{SD}(k)$, $\tilde{\Lambda}'_{SRD}(k)$ can be estimated in a similar method. Stack the $\tilde{\Lambda}'_{SD}(k)$ at the pilot positions of a grid into a vector $\tilde{\mathbf{\Lambda}}'_{SD,p}$, then the CFRs at the pilot and data positions of the grid are estimated as

$$\tilde{\Lambda}'_{SD}(k) = \mathbf{R}_{\Lambda'_{SD,k}\tilde{\Lambda}'_{SD,p}} \mathbf{R}^{-1}_{\tilde{\Lambda}'_{SD,p}\tilde{\Lambda}'_{SD,p}} \tilde{\mathbf{\Lambda}}'_{SD,p} \tag{12}$$

where $\mathbf{R}_{\Lambda'_{SD,k}\tilde{\Lambda}'_{SD,p}}$, $\mathbf{R}_{\tilde{\Lambda}'_{SD,p}\tilde{\Lambda}'_{SD,p}}$ are the covariance matrix, whose element can be represented as $R(\Delta n, \Delta k) = R_t(\Delta n) R_f(\Delta k)$, where Δn, Δk are the time and frequency spacing of different CFRs in the grid. $R_t(\Delta n)$ can be computed according to [16].$R_f(\Delta k) = \sum_{l=0}^{L} \sigma_l^2 e^{-j2\pi \Delta k/N}$. After filtering for all grids, the estimates $\tilde{\Lambda}'_{SD}(k)$ of $\Lambda'_{SD}(k)$ over the whole time-frequency plane can be found. The resulting MSE is given by

$$\begin{aligned} &E\{|\tilde{\Lambda}'_{SD}(k) - \Lambda'_{SD}(k)|^2\} \\ =&1 - \mathbf{R}_{\Lambda'_{SD,k}\tilde{\Lambda}'_{SD,p}} \mathbf{R}^{-1}_{\tilde{\Lambda}'_{SD,p}\tilde{\Lambda}'_{SD,p}} \mathbf{R}^{\mathrm{H}}_{\Lambda'_{SD,k}\tilde{\Lambda}'_{SD,p}} \end{aligned} \tag{13}$$

2) Iterative CE based on combination of known pilot signals and the current soft estimates of the data signals: To find the raw estimates of the channel, we use the frequency replacement method proposed in [17]. Specifically, for $\tilde{\Lambda}'_{SD}(k)$ at the pilot positions

$$\tilde{\Lambda}'_{SD}(k) = \begin{cases} \frac{R'_D(k)}{C(\lfloor k/D_f \rfloor)}, & \text{if } |C(\lfloor k/D_f \rfloor)| > \rho \\ \tilde{\Lambda}'^{prev}_{SD}(k), & \text{if } |C(\lfloor k/D_f \rfloor)| \le \rho \end{cases} \tag{14}$$

For $\tilde{\Lambda}'_{SD}(k)$ at the data positions, the CFRs are given by (15), which is shown on the top of next page,

where $S(2k)$, $S(2k+1)$ and $\tilde{\Lambda}^{prev}_{SD}(k)$ are the current soft estimate of data signals in the frequency-domain and CFR at data position in the previous iteration, respectively. ρ is a value of threshold. The filtering process is similar to the one used for initial channel estimation, while $\tilde{\mathbf{\Lambda}}'_{SD,p}$ comprises $\tilde{\Lambda}'_{SD}(k)$ at all frequency positions of a grid.

B. SFBC Decoder

In order to avoid the intra-SFBC interference, diagonalized SFBC decoder is applied. For the $2k$th and $2k+1$th frequencies, the received signals can be considered jointly as

$$\mathbf{R}'_{D,k} \triangleq \begin{bmatrix} R'_D(2k) \\ R'^*_D(2k+1) \end{bmatrix} \triangleq \mathbf{\Lambda}'^{Eq}_k \mathbf{X}'_{S,k} + \mathbf{N}'_{D,k} \tag{16}$$

where $\mathbf{\Lambda}'^{Eq}_k = \begin{bmatrix} \Lambda'_{SD}(2k) & \Lambda'_{SRD}(2k) \\ \Lambda'^*_{SRD}(2k+1) & -\Lambda'^*_{SD}(2k+1) \end{bmatrix}$, $\mathbf{X}'_{S,k} = \begin{bmatrix} X_S(2k) \\ -X^*_S(2k+1) \end{bmatrix}$, $\mathbf{N}'_{D,k} = \begin{bmatrix} N'_D(2k) \\ N'^*_D(2k+1) \end{bmatrix}$. The diagonalized space-frequency decoding matrix at $2k$th and $2k+1$th frequencies is

$$\mathbf{\Lambda}'^{De}_k = \begin{bmatrix} \Lambda'^*_{SD}(2k+1) & \Lambda'_{SRD}(2k) \\ \Lambda'^*_{SRD}(2k+1) & -\Lambda'_{SD}(2k) \end{bmatrix} \tag{17}$$

The decoded signals can be written as

$$\begin{aligned} \mathbf{Y}_k \triangleq \begin{bmatrix} Y(2k) \\ Y(2k+1) \end{bmatrix} &= \mathbf{\Lambda}'^{De}_k \mathbf{R}'_{D,k} = \hat{\mathbf{\Lambda}}_k \mathbf{X}'_{S,k} + \mathbf{N}_k \\ &\triangleq \begin{bmatrix} \hat{\Lambda}^e(k) & \\ & \hat{\Lambda}^o(k) \end{bmatrix} \begin{bmatrix} X_S(2k) \\ -X^*_S(2k+1) \end{bmatrix} + \mathbf{\Lambda}'^{De}_k \mathbf{N}'_{D,k} \end{aligned} \tag{18}$$

where $\hat{\Lambda}^e(k) = \hat{\Lambda}^o(k) = \Lambda'_{SD}(2k)\Lambda'^*_{SD}(2k+1) + \Lambda'_{SRD}(2k)\Lambda'^*_{SRD}(2k+1)$. Obviously there is no intra-SFBC interference in the decoded signals.

C. FDE within the In-loop Iteration

Stack the received frequency-domain signals at the data frequency tones in a block into a vector, we obtain

$$\mathbf{Y}_d = \mathbf{\Lambda}_d \mathbf{F}_{N_d} \mathbf{s} + \mathbf{N}_d \tag{19}$$

where

$$\begin{aligned} \mathbf{Y}_d =& [Y(k_0), Y(k_1), \ldots, Y(k_{N_d-1})]^{\mathrm{T}}, \\ \mathbf{N}_d =& [N(k_0), N(k_1), \ldots, N(k_{N_d-1})]^{\mathrm{T}}, \\ \mathbf{\Lambda}_d =& \mathrm{diag}(\Lambda(k_0), \Lambda(k_1), \ldots, \Lambda(k_{N_d-1})) \end{aligned} \tag{20}$$

$\{k_i\}_{i=0}^{N_d-1}$ denote the indices of data frequency tones of the block.

1) FDE with perfect channel knowledge: The channel estimation is assumed to be perfect at the equalizer input. The output of the FDE can be expressed as

$$\tilde{\mathbf{s}} = \mathbf{F}^{\mathrm{H}}_{N_d} \mathbf{P} (\mathbf{Y}_d - \mathbf{\Lambda}_d \mathbf{F}_{N_d} \bar{\mathbf{s}}) + \mu \bar{\mathbf{s}} \tag{21}$$

where $\bar{\mathbf{s}}$ denotes the soft feedback from the previous iteration, $\mathbf{P}$ and μ are the frequency-domain equalizer taps and the compensation coefficient respectively, given as

$$\begin{aligned} \mathbf{P} =& \mathrm{diag}(P(0), P(1), \ldots, P(N_d-1)) \\ =& \mathbf{\Lambda}^{\mathrm{H}}_d (\lambda \mathbf{\Lambda}_d \mathbf{\Lambda}^{\mathrm{H}}_d + \sigma^2_N \mathbf{I}_{N_d})^{-1} \end{aligned} \tag{22}$$

$$\begin{cases} \tilde{\Lambda}'_{SD}(2k) = \tilde{\Lambda}'_{SD}(2k+1) = \frac{R'_D(2k)S^*(2k) - R'^*_D(2k+1)S(2k+1)}{|S(2k)|^2 + |S(2k+1)|^2}, & \text{if } S(2k) > \rho \text{ and } S(2k+1) > \rho \\ \tilde{\Lambda}'_{SD}(2k) = \tilde{\Lambda}'^{prev}_{SD}(2k), \tilde{\Lambda}'_{SD}(2k+1) = \tilde{\Lambda}'^{prev}_{SD}(2k+1), & \text{if } S(2k) \le \rho \text{ or } S(2k+1) \le \rho \end{cases} \tag{15}$$

$$\mu = \frac{1}{N_d} \sum_{i=0}^{N_d-1} P_i \Lambda_{k_i} \tag{23}$$

where $\lambda = ((N_d - 1)\nu^2 + \sigma_d^2)/N_d$, and σ_N^2 is the average variance of $\mathbf{N}_d$. To extract information from the equalizer output in the form of extrinsic LLR for each coded bit, the output can be approximated as

$$\tilde{\mathbf{s}} = \mu \mathbf{s} + \boldsymbol{\eta} \tag{24}$$

where $\boldsymbol{\eta}$ denotes the noise vector, whose entries are Gaussian random variables with zero-mean and variance given as

$$\sigma^2 = \frac{1}{N_d} \sum_{i=0}^{N_d-1} (\nu^2 |\Lambda_{k_i}|^2 + \sigma_N^2)|P_i|^2 - \mu^2 \nu^2 \tag{25}$$

where ν^2 is the average variance of data symbols.

2) FDE with CE error: The channel coefficients, $\tilde{\Lambda}'_{SD}(k)$ and $\tilde{\Lambda}'_{SRD}(k)$, treated as random variables with mean and variance given by the channel estimator, can be expressed as

$$\Lambda_k = \tilde{\Lambda}_k + \Delta_k \tag{26}$$

where Δ_k denotes the channel estimate error at the kth frequency tone and is a zero-mean Gaussian variable with variance given by (13). By using (26), the received signals at the data positions in (19) can be re-expressed as

$$\mathbf{Y}_d = (\tilde{\mathbf{\Lambda}}_d + \mathbf{\Delta}_d)\mathbf{F}_{N_d}\mathbf{s} + \mathbf{N}_d \tag{27}$$

where $\tilde{\mathbf{\Lambda}}_d = \text{diag}(\tilde{\Lambda}_{k_0}, \tilde{\Lambda}_{k_1}, \ldots, \tilde{\Lambda}_{k_{(N_d-1)}})$, $\mathbf{\Delta}_d = \text{diag}(\Delta_{k_0}, \Delta_{k_1}, \ldots, \Delta_{k_{(N_d-1)}})$. The frequency-domain interference resulting from channel estimation error is given by

$$\mathbf{I} = \mathbf{\Delta}_d \mathbf{F}_{N_d} \mathbf{s} \tag{28}$$

Remember that the a-priori information about one symbol should not be used in the evaluation of its estimate [13], the covariance matrix of $\mathbf{I}$ when the nth symbol in the block is to be detected can be evaluated as

$$E\left\{\mathbf{I}\mathbf{I}^H\right\} = \text{diag}\left(\rho_n \sigma^2_{\Delta_{k_0}}, \rho_n \sigma^2_{\Delta_{k_1}}, \ldots, \rho_n \sigma^2_{\Delta_{k_{(N_d-1)}}}\right) \tag{29}$$

where

$$\rho_n = \frac{1}{N_d}\left(\sum_{i=0, i\neq n}^{N_d-1} E\left\{|s_i|^2\right\} + \sigma_d^2\right) \tag{30}$$

The FDE coefficients are obtained as

$$\mathbf{P} = \tilde{\mathbf{\Lambda}}_d^{\mathrm{H}} (\lambda \tilde{\mathbf{\Lambda}}_d \tilde{\mathbf{\Lambda}}_d^{\mathrm{H}} + \rho \mathbf{\Sigma} + \sigma_N^2 \mathbf{I}_{N_d})^{-1} \tag{31}$$

where $\mathbf{\Sigma} = \text{diag}\left(\sigma^2_{\Delta_{k_0}}, \sigma^2_{\Delta_{k_1}}, \ldots, \sigma^2_{\Delta_{k_{(N_d-1)}}}\right)$, $\rho = \sum_{n=0}^{N_d-1} \rho_n / N_d$. The variance of equivalent noise given in (25) becomes

$$\sigma^2 = \frac{1}{N_d} \sum_{i=0}^{N_d-1} (\nu^2 |\tilde{\Lambda}_{k_i}|^2 + \rho \sigma^2_{\Delta_{k_i}} + \sigma_N^2)|P_i|^2 - \mu^2 \nu^2 \tag{32}$$

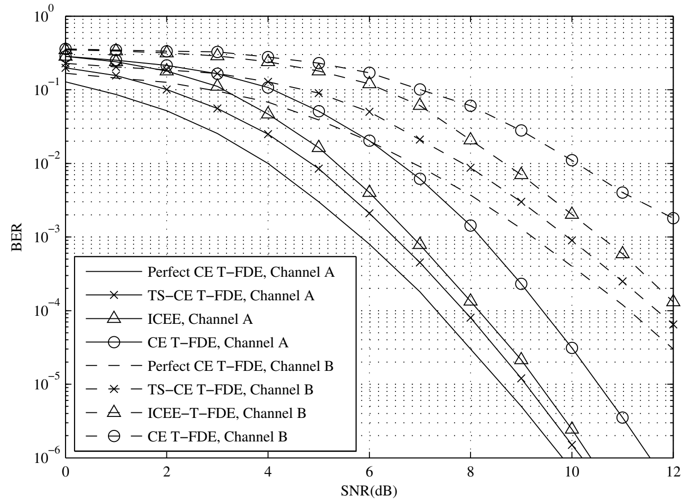

Fig. 1. BER performance for different $\text{SNR}_{S\to D}$, $\text{SNR}_{S\to R} = 20$dB.

IV. Simulation Results

In this section, we investigate the performances of the proposed scheme. We consider a frame-based transmission with 10 information blocks where each block consists of 1024 modulated symbols. $N_p = 64$, $N_d = 960$. The information bit sequence is generated by a rate 1/2 forward error correcting (FEC) code, generator polynomial $(1, 15/13_{octal}, 17/13_{octal})$, and interleaved randomly within a block. The channels are defined as exponentially power-decaying multipath Rayleigh fading channels for all links. The $S \to D$ and $R \to D$ links are assumed to be balanced, i.e., perfect power control.

In Fig. 1, BER versus $\text{SNR}_{S\to D}$ performance is investigated for a slow fading channel, Channel A (normalized Doppler frequency defined as $f_d N T_s = 0.001$, correspond to mobile equipment speed of 4 km/h) and a fast fading channel, Channel B ($f_d N T_s = 0.05$, correspond to the speed of 200 km/h) respectively. For comparison, the results of turbo FDE with perfect CE (Perfect CE T-FDE), Training Sequences aided CE T-FDE (TS-CE T-FDE) in [12] and initial pilot aided CE T-FDE (CE T-FDE) are also presented. As is shown, the proposed ICEE approaches Perfect CE T-FDE and TS-CE T-FDE over slow fading channel for 5 iterations, and significantly outperforms CE T-FDE in both Channel A and Channel B.

Fig. 2 shows the transfer characteristics. The EXIT curves of the proposed scheme yield better output mutual information than CE T-FDE. We also observe that the EXIT curves of

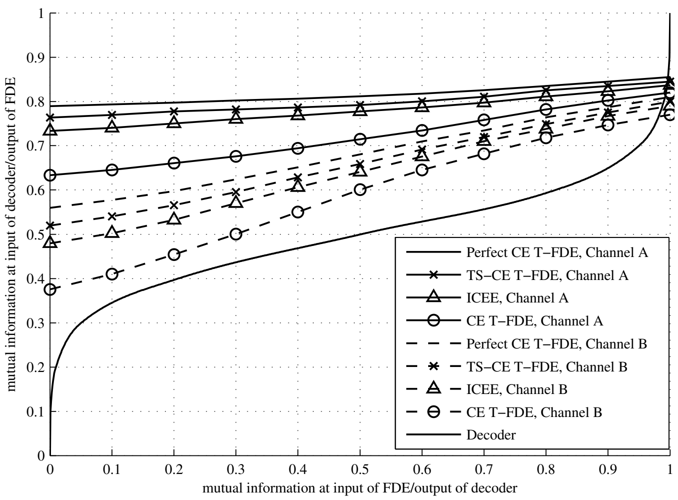

Fig. 2. EXIT chart at $\mathrm{SNR}_{S\rightarrow D} = 8$dB, $\mathrm{SNR}_{S\rightarrow R} = 20$dB.

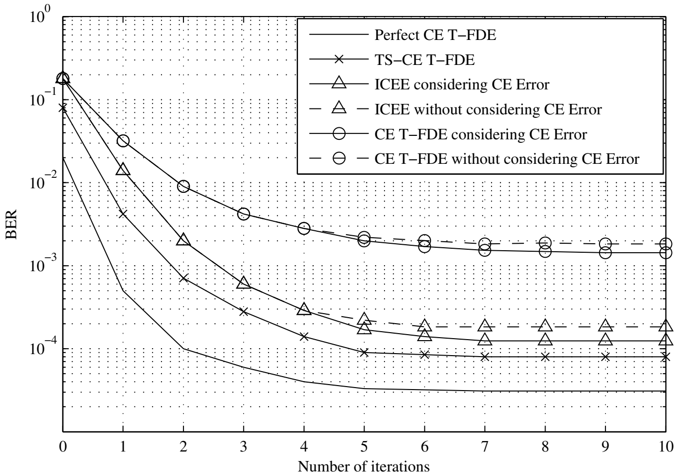

Fig. 3. BER performance versus number of iterations, $\mathrm{SNR}_{S\rightarrow D} = 8$dB, $\mathrm{SNR}_{S\rightarrow R} = 20$dB.

Perfect CE T-FDE, TS-CE T-FDE and ICEE approach when the mutual information at the input of FDE increases, which indicates that the performance of ICEE can gradually improve and eventually be asymptotic with Perfect CE T-FDE and TS-CE by iterative operations. The system trajectories follow the transfer curves, indicating that the EXIT chart analysis is accurate in a given simulation environment.

Fig. 3 show the BER performance as a function of the number of iterations. We observe that there are performance floors for all algorithms and the iteration gains become comparatively small after 5 iterations. The BER performances can be further improved by considering the CE error in each iteration.

V. Conclusions

In this paper, an iterative frequency-domain channel estimation and equalization scheme for relay-assisted SFBC single-carrier transmissions is proposed. Implementations of iterative FDE with both perfect CE and CE error have been derived. With interpolated pilot aided channel estimation, the scheme avoids the additional training sequences time slots, which is more efficient in the sense of spectral efficiency. Simulation results have shown that the performance significantly improves compared to traditional non-iterative ones, which makes the proposed algorithm more suitable for high-speed wireless communications.

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