Optimal Resource Allocation for Multi-Access in Heterogeneous Wireless Networks

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***Abstract*—Multi-access for multi-mode terminals is possible in heterogeneous networks environment which becomes a critical issue for transmission in parallel with meeting user quality of service (QoS) performance requirements and throughput maximization. The previous optimal resource allocation schemes, which do not consider service type, may allocate scarce radio resources inefficiently. To solve this, we propose an optimal resource allocation algorithm with distinguishing the services traffic into two classes: Delay-Constraint (DC) and Best-Effort (BE). In our work, we formulate a mathematical optimal model to support such heterogeneous service requirements in multi-radio access scenario. Then, we develop a optimal radio resource allocation algorithm that achieves the goal of maximizing the total system throughput in heterogeneous networks, while efficiently satisfying the QoS requirement for DC services traffic and fairness for BE services traffic. Simulation results show that the proposed service traffic differentiation based radio resource allocation algorithm significantly outperforms other existing schemes.**

***Index Terms*—multi-access, radio resource allocation, power, bandwidth, heterogeneous networks**

I. Introduction

Heterogeneous wireless networks are leading the overwhelming evolution trend of 4G wireless networks. It is envisaged that various radio access technologies (RATs) such as cellular systems, 802.16 WMANs, 802.11 WLANs, will be deployed in the overlapping coverage area. Meanwhile, Multi-Mode Terminals (MMTs) have the ability to access multiple heterogeneous radio access networks simultaneously, which are refereed to as a multi-access scenario in heterogeneous networks. For such a multi-access case, it is worth designing an adaptive radio resources allocation scheme among multiple radio access in order to achieve multi-access diversity gain.

Achieving spectral efficiency in terms of aggregating throughput is sometimes unfair to those users experiencing poor channel condition. Whereas, absolute fairness may lead to QoS degradation for some users and system efficiency loss. Hence, radio resource allocation presents even more challenging for delay sensitive services with a certain delay constraint. Therefore, radio resource allocation in heterogeneous networks with the mixed traffic nature requires an efficient tradeoff between efficiency and fairness with considering multi-access gains.

The optimal radio resource allocation in heterogeneous networks was investigated in the literature [1–3]. Dimou et al. in [1] proposed a novel greedy rate allocation algorithm in parallel transmission over the generic link layer across heterogeneous networks. In [2], Zhu et al. addressed the problem of rate allocation among realtime services sharing multi-access in heterogeneous networks. Joint radio resource allocation has been studied by Choi et al. in [3], where radio resource is allocated in the heterogeneous networks framework with BE users, aiming at the system capacity maximization. However, existing works only focus on the optimal solution of joint resource allocation for multi-access problem without diverse QoS requirements consideration or assuming only single one type service traffic. It could not reflect the practical cases in heterogeneous networks. In this paper, we exploit the uplink modeling method further to provide an optimal resource allocation algorithm for the heterogeneous networks over software defined radio (SDR) [3]. Joint resource allocation for mixed traffic in multi-access case in heterogeneous networks will be studied. Finite delay tolerance for DC users and proportional fairness for BE users have been considered with the aim to maximize the aggregate system capacity. To the best of our knowledge, joint power and spectral bandwidth allocation has been studied in OFDM-based radio resource management, however, the optimal resource allocation algorithm in multi-access with mixed traffic has not been addressed fully. In the light of multi-access system model which has a different physical interpretation and assumption from former problem, the both above problems are totally distinct in nature.

The remainder of paper is organized as follows. First, we introduce a system model and formulate a joint radio resource allocation optimization problem in section II and III. In section IV, an optimal radio resource allocation scheme for joint power and bandwidth allocation problem is provided. And numerical simulation results and analysis are illustrated in section V. Finally, section VI concludes the full paper.

II. System Model

In this section, we consider a common heterogeneous networks scenario as shown in Fig. 1. It is assumed that both RATs (cellular base stations or wireless access points) and

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MMTs are implemented by the reconfigurable SDR technology. Hence, each MMT is able to access multiple RATs simultaneously, whereas it is also capable of operating in multiple different frequency bandwidth through parallel transmission manner. Under the above system assumption, MMTs can access multiple RATs for their service data transmission simultaneously or select only a part of available RATs with better channel condition. Throughout the paper, we make the following assumptions: heterogeneous radio access systems can operate in different frequency bands, which is reasonable especially in 4G era actually, so that there is no significant interference between them; and perfect synchronization is necessary for no inference happening for simplicity. Channels experience slow fading such that the instantaneous Channel State Information (CSI) is updated periodically with the help of feedback channels to the algorithm decision maker.

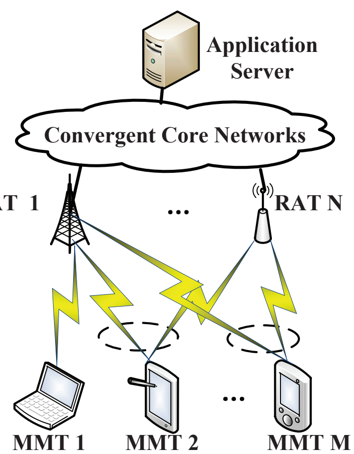

Fig. 1. The multi-access scenario in heterogeneous networks

In this work, we consider a heterogeneous wireless network environment consisting of M active MMTs which are requesting diverse services and N available RATs as depicted in Fig. 1. Assuming that M MMTs are classified by heterogeneous service traffic delay requirements: the first class users who have DC service traffic which requires a minimum constant transmission rate of $R_i^{\min}(i = 1, 2, \dots, K)$ bits constraint, known as **DC** MMTs [4]. Here $R_i^{\min}$ are the mathematical expectation of minimum data rate for DC MMTs. And the remaining $(M - K)$ MMTs transmit BE service traffic, known as **BE** MMTs. In order to transmit service data by multi-access manner, each MMT should obtain the radio resource from the available RATs. After the radio resource allocation, each user service data experience different channel gains on each channel from each RAT. For RAT j $(j = 1, 2, \dots, N)$ and MMT i $(i = 1, 2, \dots, M)$, channel transfer function and total noise power spectral density are denoted as H_{ij} and N_{ij}, respectively. The channel gain to noise ratio function for RAT j and MMT i can be indicated by

$$g_{ij} = \frac{|H_{ij}|^2}{N_{ij}}, \tag{1}$$

where g_{ij} is assumed flat during each transmission time interval which is small enough and N_{ij} is the power spectral density for Additive White Gaussian Noise (AWGN). Therefore, from Shannon capacity formula for Gaussian channel, the achievable data rate r_i of MMT i can be defined as

$$r_i = \sum_{j=1}^{N} \beta_j x_{ij} log_2(1 + \frac{g_{ij} p_{ij}}{x_{ij}}), \tag{2}$$

where x_{ij} is allocated frequency bandwidth to MMT i from RAT j, p_{ij} is the transmission power of MMT i to RAT j, and β_j $(0 \leq \beta_j \leq 1)$ represents the system efficiency which can be guaranteed by RAT j to all MMTs.

From equation (2), β_j defines the validity of each RAT throughput because it can express the offered system efficiency. Hence, RAT j, which has a better coder and decoder scheme, can have a higher β_j value. For instance, β_j could be 0.6 and 0.29 for LTE(1x2) and WiMAX Wave 1, respectively [3]. Therefore, the achievable fraction of Shannon capacity of r_i is determined by (2) with the weight β_j.

III. Problem Formulation

It is assumed that the multi-access in heterogeneous networks are associated with different QoS requirements MMTs. The system consists of M MMTs. The first K MMTs have DC traffic rate $r_i (i = 1, 2, \dots, K)$, which requires a constant minimum transmission rate, respectively. The traffic for the remaining $(M - K)$ BE MMTs has no delay constraint and can be delivered in the best-effort manner. However, the traffic for BE MMTs needs to be guaranteed proportional fairness [5].

Thus, we have

$$r_i \geq R_i^{\min}, i = 1, 2, \dots, K, \tag{3}$$

$$r_{K+1} : r_{K+2} : \dots : r_M = \gamma_{K+1} : \gamma_{K+2} : \dots : \gamma_M, \tag{4}$$

where $R_i^{\min}(i = 1, 2, \dots, K)$ is the minimum rate constraint for DC MMTs and γ_i $(i = K + 1, K + 2, \dots, M)$ is the proportional fairness parameter for BE MMTs.

Therefore, the heterogeneous networks system capacity maximization problem for multi-access with service traffic differentiation can be formulated as follow.

$$\begin{aligned} \max R(\mathbf{x}, \mathbf{p}) &= \max \sum_{i}^{M} r_i \\ &= \max \sum_{i}^{M} \sum_{j}^{N} \beta_j x_{ij} log_2(1 + \frac{g_{ij} p_{ij}}{x_{ij}}), \end{aligned} \tag{5}$$

subject to:

$$\sum_{i=1}^{M} x_{ij} \leq X_j, \forall j, \tag{6}$$

$$\sum_{j=1}^{N} p_{ij} \leq P_i, \forall i, \tag{7}$$

$$\sum_{j=1}^{N} \beta_j x_{ij} \log_2(1 + \frac{g_{ij} p_{ij}}{x_{ij}}) \geq R_i^{\min}, \forall i \in 1, 2, \dots, K, \tag{8}$$

$$L(x_{ij},p_{ij};\lambda_j,\mu_i,\nu_i,\omega_i)=\sum_{i=1}^{M}\sum_{j=1}^{N}\beta_j x_{ij}\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})+\sum_{j=1}^{N}\lambda_j(X_j-\sum_{i=1}^{M}x_{ij})+\sum_{i=1}^{M}\mu_i(P_i-\sum_{j=1}^{N}p_{ij})$$
$$+\sum_{i=1}^{K}\nu_i(\sum_{j=1}^{N}\beta_j x_{ij}\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})-R_i^{\min})+\sum_{i=K+1}^{M}\omega_i(\sum_{j=1}^{N}\beta_j x_{1i}\log_2(1+\frac{g_{1j}p_{1j}}{x_{1j}})-\frac{\gamma_{K+1}}{\gamma_i}\sum_{j=1}^{N}\beta_j x_{ij}\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})). \tag{11}$$

$$\frac{r_i}{r_{K+1}}=\frac{\gamma_i}{\gamma_{K+1}},\forall i\in K+1,K+2,\ldots,M, \tag{9}$$

$$x_{ij},p_{ij}\geq 0,\forall i,j. \tag{10}$$

where X_j is the total system bandwidth of RAT j, and P_i is the maximum available power of MMT i. In the above model, we consider an optimization operation of multiple heterogeneous networks, and propose an efficient decision making method in multi-access. Note that the objective function in (5) is concave with respect to $\{\mathbf{x},\mathbf{p}\}$. This means that an optimal solution can be derived, where a local maximum is also the global maximum [6]. The inequality (6) and (7) denote that there is a total system bandwidth and power constraint due to resource finiteness. The other two inequalities (8) and (9) constraint conditions, which are equivalent to (3) and (4), denote that MMTs with heterogeneous services traffic QoS requirements have different data rate constraints respectively.

IV. Optimal Radio Resource Allocation Algorithm for Multi-access

A. Optimal Radio Resource Allocation Analysis

For the optimal solution of capacity maximum problem, the Lagrangian is given by (11), where shadow prices λ_j, μ_i, ν_i and ω_i are nonnegative Lagrange multipliers for the constraints. By taking derivatives with respect to λ_j, μ_i, ν_i and ω_i respectively, we can get a general differentiation of (11) for both types of service traffic by Karush-Kuhn-Tucker (KKT) conditions [6]:

$$\frac{\partial L}{\partial x_{ij}}=\widetilde{\beta}_j\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})-\frac{\widetilde{\beta}_j g_{ij}p_{ij}}{(x_{ij}+g_{ij}p_{ij})\ln 2}-\lambda_j\leq 0, \tag{12}$$

$$\frac{\partial L}{\partial p_{ij}}=\frac{\widetilde{\beta}_j g_{ij}x_{ij}}{(x_{ij}+g_{ij}p_{ij})\ln 2}-\mu_i\leq 0, \tag{13}$$

For $1\leq i\leq K$, we have:

$$\widetilde{\beta}_j=(1+\nu_i)\beta_j,i=1,\ldots,K. \tag{14}$$

For $K+1\leq i\leq M$, we have such two cases:

$$\widetilde{\beta}_j=(1+\sum_{k=K+2}^{M}\omega_k)\beta_j,i=K+1, \tag{15}$$

$$\widetilde{\beta}_j=(1-\omega_i\frac{\gamma_{K+1}}{\gamma_i})\beta_j,i=K+2,\ldots,M. \tag{16}$$

With inequation (12) and (13), we have

$$x_{ij}\frac{\partial L}{\partial x_{ij}}=0, \tag{17}$$

$$p_{ij}\frac{\partial L}{\partial p_{ij}}=0. \tag{18}$$

Using (13) and (18), the relation between bandwidth and power allocation can be obtained as:

$$p_{ij}=x_{ij}[\frac{\widetilde{\beta}_j}{\mu_i\ln 2}-\frac{1}{g_{ij}}]^+, \tag{19}$$

where $[z]^+=\max\{z,0\}$. From above equation (19), the optimal power and bandwidth scaled by the portion of time is allocated to each channel per MMT, according to water-filling theory. The water level of each channel per MMT may differ from one another. And in order to get the optimal x_{ij} and p_{ij} solution, we need to have one of them.

Based on problem (10), we can express the dual problem as follows:

$$D(\lambda_j,\mu_i,\nu_i,\omega_i)=\max L(x_{ij},p_{ij};\lambda_j,\mu_i,\nu_i,\omega_i). \tag{20}$$

According to the convex analysis [6], strong duality (zero dual gap) holds between the optimum of primal problem (11) and its dual problem (20). Thus the optimal solution for primal problem can always be found by solving (20) without any performance loss. Hence, in the following proposed algorithm, we use the gradient projection method to approach to the optimal solution, which is proved to be feasible if the iterative step sizes are properly chosen. So, we utilize the best-response method to update the bandwidth as follows,

$$x_{ij}^{k+1}=[x_{ij}^k+\alpha\frac{\partial L}{\partial x_{ij}}]^+,\forall i,j, \tag{21}$$

$$x_{ij}^{k+1}=\begin{cases}x_{ij}^k+\alpha[(1+\nu_i)\beta_j\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})\\ -\frac{(1+\nu_i)\beta_j g_{ij}p_{ij}}{\ln 2(x_{ij}+g_{ij}p_{ij})}-\lambda_j]^+,\\ \text{if } i=1,2,\ldots,K,\\ x_{ij}^k+\alpha[(1+\sum_{k=K+2}^{M}\omega_k)\beta_j\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})\\ -\frac{(1+\sum_{k=K+2}^{M}\omega_k)\beta_j g_{ij}p_{ij}}{\ln 2(x_{ij}+g_{ij}p_{ij})}-\lambda_j]^+\\ \text{if } i=K+1,\\ \alpha[(1-\omega_i\frac{\gamma_{K+1}}{\gamma_i})\beta_j\log_2(1+\frac{g_{ij}p_{ij}}{x_{ij}})\\ -\frac{(1-\omega_i\frac{\gamma_{K+1}}{\gamma_i})\beta_j g_{ij}p_{ij}}{\ln 2(x_{ij}+g_{ij}p_{ij})}-\lambda_j]^+\\ \text{if } i=K+2,K+3,\ldots,M,\end{cases} \tag{22}$$

where α is the constant step size for primal variable x_{ij}, which is converge to optimal value as long as the step size α is appropriately chosen. After x_{ij} is obtained, p_{ij} can be determined by using (19). To update the Lagrange multiplier values for the optimal solution, we consider the contiguously differentiable dual function. Using the gradient projection method [6], the updated nonnegative multiplier value for power and bandwidth allocation is given by

$$\lambda_j^{k+1} = [\lambda_j^k + \xi_1 \frac{\partial D}{\partial \lambda_j^k}]^+ = [\lambda_j^k + \xi_1 (\sum_{i=1}^{M} x_{ij}^k - X_j)]^+, \quad (23)$$

$$\mu_i^{k+1} = [\mu_j^k + \xi_2 \frac{\partial D}{\partial \mu_j^k}]^+ = [\mu_i^k + \xi_2 (\sum_{j=1}^{N} p_{ij}^k - P_i)]^+, \quad (24)$$

$$\nu_i^{k+1} = [\nu_j^k + \xi_3 \frac{\partial D}{\partial \nu_j^k}]^+ = [\nu_j^k + \xi_3 (\sum_{j=1}^{N} r_{ij} - R_i^{\min})]^+, \quad (25)$$

$$\begin{aligned} \omega_i^{k+1} &= [\omega_j^k + \xi_4 \frac{\partial D}{\partial \omega_j^k}]^+ \\ &= [\omega_j^k + \xi_4 (\sum_{j=1}^{N} r_{(K+1)j} - \frac{\gamma_{K+1}}{\gamma_i} \sum_{j=1}^{N} r_{ij})]^+, \end{aligned} \quad (26)$$

where $\vec{\xi} = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ is a constant step size vector. From iterations, we can solve the optimal problem of the multi-access in heterogeneous networks with traffic differentiation which maximizes system total capacity.

Algorithm 1 Optimal Radio Resource Allocation Algorithm for Multi-access in Heterogeneous Wireless Networks

```
Initialize α_ij, ξ⃗, k = 0
if k = 0 then
    Initialize x_ij^0, p_ij^0, μ_i^0 ν_i^0 and ω_i^0
else
    Calculate x_ij^{k+1} using gradient projection method.
    x_ij^{k+1} = [x_ij^k + α ∂L/∂x_ij]^+, ∀i, j
    Determine p_ij = x_ij[β̃_j/(μ_i ln 2) − 1/g_ij]^+, ∀i, j
    if Iteration reaches the convergence precision (condition) of p_ij and x_ij or the maximum iteration number then
        Transmit data packet to the RAT(s) using x_ij^{k+1} and p_ij^{k+1}
    else
        Update μ_i^{k+1}, ν_i^{k+1}, ω_i^{k+1} and λ_j^{k+1} using x_ij^{k+1}, p_ij^{k+1} information.
        μ_i^{k+1} = [μ_i^k + ξ_2(Σ_{j=1}^N p_ij − P_i)]^+
        ν_i^{k+1} = [ν_j^k + ξ_3(Σ_{j=1}^N r_ij − R_i^min)]^+
        ω_i^{k+1} = [ω_j^k + ξ_4(Σ_{j=1}^N r_(K+1)j − (γ_{K+1}/γ_i) Σ_{j=1}^N r_ij)]^+
        λ_j^{k+1} = [λ_j^k + ξ_1(Σ_{i=1}^M x_ij^k − X_j)]^+
        k ← k + 1
    end if
end if
```

B. Optimal Radio Resource Allocation Algorithm

Based on the optimality conditions analysis for multi-access in heterogeneous networks, we propose an optimal radio resource allocation algorithm for multi-access in heterogeneous networks as depicted in Algorithm 1. The proposed algorithm for solving problem (5) uses the Projected Gradient approach as a basis for the optimization solution. It can realize the dynamic radio resource allocation for MMTs supporting QoS requirements of heterogeneous service traffic. Note that the r_{ij} value is determined by radio resource owned by both MMTs and RATs. And the proposed algorithm complexity is a step size and initial value related function. Hence, this algorithm gives a distributed decision making manner for the multi-access radio resource allocation problem with the intention of total system capacity maximization.

V. Performance Evaluation

We simulate a multi-access scenario in heterogeneous networks where an LTE base station and an IEEE 802.11b wireless access point offer access to all MMTs in the overlapping coverage area. And it is assumed that both of them have total frequency bandwidth of 10 and 20 MHz, respectively. Random user spatial distribution is considered in simulation with random channel gain for different MMTs.

In the simulation, it is assumed that there are 3 MMTs with maximum power constraint of 20 mW and they are randomly distributed in region. 1 DC ($R_1^{\min} = 1Mbps$) MMT and 2 BE ($\gamma_2 : \gamma_3 = 1 : 1$) ones access multiple wireless networks, simultaneously. As example for how to find the optimal radio resource allocation scheme is provided in Fig.2. The proposed algorithm provide data rate convergence to the an optimal solution of MMTs transmission data rate, which are jointly determined by transmit power and bandwidth optimal values. According to the optimal allocation strategies, it is observed that both BE users obtain the same amount of transmission rate under the proportional fairness resource allocation principle. As a result, radio resource allocation algorithm with QoS support is feasible and can efficiently iterate to the global optimal solution.

In Fig. 3, we provide a comparison of the proposed algorithm and existing algorithm [3]. We simulate all MMTs composed of only DC MMTs ($M = 6$) and compare the perform of QoS guarantee for DC traffic with minimum rate constraints. The reference minimum rate denotes the minimum rate requirements for DC MMTs. It is obvious that there are data rate gaps for some DC users (MMT 2, 3, 6) where the existing algorithm can't differentiate QoS requirement. However, the proposed algorithm can completely meet the each DC MMT QoS requirement, in term of guaranteeing the minimum data rate.

For performance comparison, we also evaluate the total aggregating capacity of multi-RAT radio resource allocation scheme using different methods, and results are shown in Fig. 4. It is observed that the proposed joint radio resource allocation scheme outperforms the other two static ones, which are equal bandwidth and equal power scheme, respectively, even though both of them have considered the service differentiation. This is because that, when the power and bandwidth are variable simultaneously, optimal resource allocation is determined jointly by power and bandwidth. As we can see, dynamic optimal radio resource perform well for multi-access scenario in heterogeneous network.

At last, we set 10 DC ($R_i^{\min} = 1Mbps, i = 1, 2, \ldots, 10$) MMTs and 10 BE MMTs with equal fairness in the simulation and vary the SNR for MMTs. Proportional fairness between MMTs is validated by Fairness Index (FI) function [7] as follows,

$$FI(r_i) = \frac{\left(\sum_{i=K+1}^{M} r_i\right)^2}{(M-K)\sum_{i=K+1}^{M} r_i^2}, \tag{27}$$

where r_i is the transmission rate of MMT i. And we define ADR as Additional Datarate Ratio (ADR) for DC MMTs to weight the degree of satisfaction as follows,

$$ADR(r_i, R_i^{\min}) = \frac{\sum_{i=1}^{K}\left(r_i - R_i^{\min}\right)}{R_i^{\min}}. \tag{28}$$

To compare the degree of satisfaction which the resource allocation scheme affects, both FI for BE MMTs and ADR for DC MMTs are presented in Fig.3, which are improved more than 12% and 3% on average respectively. The simulation result shows that the algorithm outperforms than the existing algorithm [3] in QoS guarantee. The proposed algorithm doesn't only tend to allocate resource to guarantee DC traffic QoS requirements, fairness is also considered among BE MMTs.

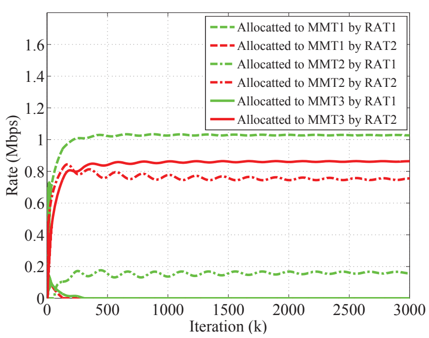

Fig. 2. The convergent rate solution of the proposed joint power and bandwidth allocation algorithm in multi-access heterogeneous networks

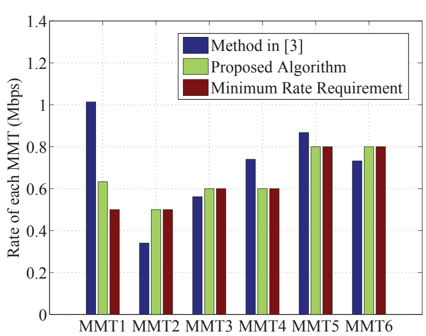

Fig. 3. An illustration of DC MMTs resource allocation results of proposed algorithm and the method in [3]

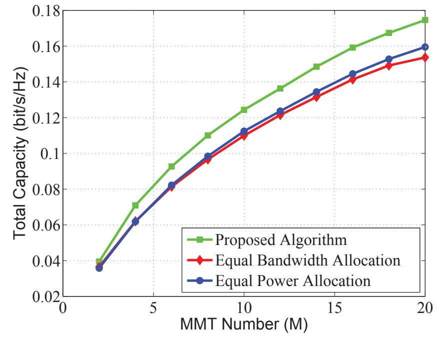

Fig. 4. The capacity comparison results when the proposed algorithm is applied at $N = 5$ and $K : (M - K) = 1 : 1$

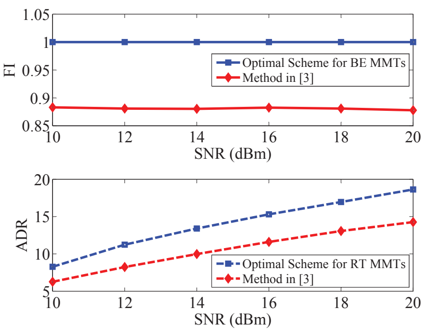

Fig. 5. The FI and ADR comparison between the proposed algorithm and the method in [3]

VI. Conclusion

The optimal resource allocation algorithm for multi-access in heterogeneous networks is proposed in this paper, which supports mixed uplink service traffic. We have investigated the problem of maximizing the total system capacity, while satisfying strict QoS requirements for MMTs with DC service traffic and guaranteeing proportional fairness for the remaining MMTs with BE service traffic. It is demonstrated that proposed algorithm can significantly improve the system throughput and efficiently allocate radio resource to MMTs with QoS guarantee. Joint admission control scheme for multi-access in heterochromous networks is left for future work.

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