Throughput Optimization for MIMO Y Channels with Physical Network Coding and Adaptive Modulation

Keov Kolyan Teav, Zhendong Zhou and Branka Vucetic
School of Electrical and Information Engineering, The University of Sydney, NSW 2006, Australia
Email: keovkolyan.teav@sydney.edu.au

***Abstract*—The multiple-input multiple-output (MIMO) Y channel, where three users simultaneously exchange independent messages with each other via a single relay within two time slots, is considered in this paper. We first propose a cooperative network coding protocol, which is called denoise-demodulate-and-forward (DDF), with the design of transmit beamforming and combining schemes to increase network throughput. More importantly, we formulate an optimization problem by using the newly derived bit error rate (BER) expression of adaptive M-ary quadrature amplitude modulation (M-QAM). As the result, the modulation types for both time slots can be chosen to maximize the total throughput of the proposed system under the BER constraint. Performance evaluations show that the proposed scheme can significantly improve the total throughput by comparing to the existing MIMO Y channel and the solution of the optimization problem is validated.**

I. Introduction

The emergence of network coding has contributed to a huge gain that is foreseen in the cooperative relaying scenarios with two-way [1]-[3] or multi-way [4] traffic. The two-way relaying channel is one of the most important applications of network coding. Recently, a novel MIMO Y channel [5] with its generalized versions [6]-[8], has been proposed. In this channel, the network information flow of a MIMO system with three users, each equipped with m antennas and a single relay with n antennas, has been investigated. Each of the three users intends to send independent messages to the other two users using two time slots, corresponding as a multiple access (MA) and broadcast (BC) stage. The concept of physical layer network coding [1] and interference alignment [9] have been applied. Some specific issues, such as multiuser case, precoding design and diversity gain, have been studied. However, the optimization of the spectral efficiency and network throughput of such a MIMO Y scheme is still an open issue.

In this paper, we formulate an optimization problem to maximize the system throughput of MIMO Y channels by taking the advantage of adaptive modulation under the BER requirement. In order to solve this optimization problem and ensure low-complexity detection at the relay, the combination of a simple exclusive-or denoise-and-forward (XOR-DNF) [2] and demodulation-and-forward (DF) [10] protocol, called a DDF protocol, is proposed. In short, each user first transmits its information symbols in one time slot choosing a modulation type, and then the relay node receives all the messages. Next, the relay uses a simple constellation mapping mechanism to map the received symbols and then demodulates the mapped symbols. Finally, the demodulated information symbols are transmitted in the following slot using another modulation type. In addition, the signal space alignment for network coding and linear transmit beamforming, as in [5], are utilized in the proposed system model in order to efficiently deal with interferences and ensure the reliability of signal transmissions.

The main contributions of this paper are two-folds. First, we design the DDF protocol to enhance the network throughput. Second, we propose an optimization problem to jointly choose the best modulation types for the MA and BC stages. The approximate closed-form expression of the BER for the proposed DDF MIMO Y channel for a square M-QAM is formulated. To our best knowledge, our paper is the first one to attempt to optimize the throughput of this channel by using the BER expression with the relation to adaptive modulation.

II. System Model

A. *MIMO Y channel*

The system model of a MIMO Y channel [5], where basically each user wants to send two independent messages to the other two users via a single relay, is depicted in Fig. 1. Each user and the relay are equipped with m and n antennas, respectively. The channel coefficients are generated using quasi-static Rayleigh fading and the half-duplexing constraint is imposed to all nodes. For simplicity, direct links between users are neglected and each node has the full access to global channel state information (CSI).

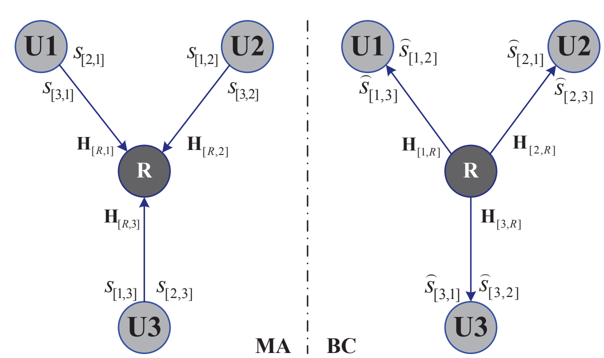

Fig. 1. MIMO Y channel

978-1-4673-0990-5/12/$31.00 ©2012 IEEE

1) MA: All three users simultaneously transmit signals to the relay, and the relay receives

$$\mathbf{r}^{\mathrm{MA}} = \sum_{i=1}^{3} \mathbf{H}_{[R,i]} \mathbf{x}_{[i]} + \mathbf{n}_{[R]}, \tag{1}$$

where $\mathbf{r}^{\mathrm{MA}}$ and $\mathbf{n}_{[R]}$ are the received signal and additive white Gaussian noise (AWGN) vector at the relay, respectively. Without loss of generality, the total transmit power from all sources P_U is assumed to be one and the noises at all receivers have the same power σ_n^2. Hence, the transmit signal-to-noise ratio (SNR) is $\rho = \frac{P_U}{\sigma_n^2} = \frac{1}{\sigma_n^2}$. $\mathbf{H}_{[R,i]}$ is the (*n* x *m*) channel matrix from user *i* to the relay. $\mathbf{x}_{[i]}$ is the transmit vector at user *i* from its *m* antennas.

2) BC: The relay re-encodes the signals obtained during the MA stage and broadcasts the transmit signals to all users. Then, each user receives

$$\mathbf{r}_{[i]}^{\mathrm{BC}} = \mathbf{H}_{[i,R]} \mathbf{x}_{[R]} + \mathbf{n}_{[i]}, \tag{2}$$

where $\mathbf{r}_{[i]}^{\mathrm{BC}}$ and $\mathbf{n}_{[i]}$ are the received signal vector and AWGN vector at user *i*, respectively. $\mathbf{H}_{[i,R]}$ is the (*m* x *n*) channel matrix from relay to user *i*, which is equal to $\mathbf{H}_{[R,i]}^H$ where $(.)^H$ denotes the Hermitian transpose. $\mathbf{x}_{[R]}$ denotes the transmit vector at the relay from its *n* antennas.

B. Beamforming and physical network coding design

To deal with the cochannel interferences from the other users and ensure the network information flow, the concept of linear transmit beamforming and receive combining schemes are used and the design of the DDF protocol is proposed based on the XOR-DNF protocol and the DF strategy.

1) Beamforming and combining vectors: In the MA phase, user *i* transmits $\mathbf{x}_{[i]}$, which can be expressed as

$$\mathbf{x}_{[i]} = \sum_{j=1, j\neq i}^{3} \kappa_{[j,i]} \mathbf{v}_{[j,i]} s_{[j,i]}, \tag{3}$$

where $s_{[j,i]}$ is the modulated message symbols that user *i* sends to user *j*, $\mathbf{v}_{[j,i]}$ denotes the corresponding beamforming vector and $\kappa_{[j,i]}$ is the power scaling factor shown in the Appendix. By applying the design of beamforming vectors, the signal subspaces of each reciprocal messages $s_{[j,i]}$ and $s_{[i,j]}$ are aligned. Based on the proper power allocation, we have

$$\begin{aligned} \mathbf{u}_1 &= \kappa_{[2,1]} \mathbf{H}_{[R,1]} \mathbf{v}_{[2,1]} = \kappa_{[1,2]} \mathbf{H}_{[R,2]} \mathbf{v}_{[1,2]} \\ \mathbf{u}_2 &= \kappa_{[3,2]} \mathbf{H}_{[R,2]} \mathbf{v}_{[3,2]} = \kappa_{[2,3]} \mathbf{H}_{[R,3]} \mathbf{v}_{[2,3]} \\ \mathbf{u}_3 &= \kappa_{[1,3]} \mathbf{H}_{[R,3]} \mathbf{v}_{[1,3]} = \kappa_{[3,1]} \mathbf{H}_{[R,1]} \mathbf{v}_{[3,1]}. \end{aligned} \tag{4}$$

Therefore, the signal model in Eq. (1) can be rewritten as

$$\begin{aligned} \mathbf{r}^{\mathrm{MA}} &= \mathbf{u}_1 (s_{[2,1]} + s_{[1,2]}) + \mathbf{u}_2 (s_{[3,2]} + s_{[2,3]}) \\ &\quad + \mathbf{u}_3 (s_{[1,3]} + s_{[3,1]}) + \mathbf{n}_{[R]} \\ &= \mathbf{u}_1 s_1 + \mathbf{u}_2 s_2 + \mathbf{u}_3 s_3 + \mathbf{n}_{[R]}, \end{aligned} \tag{5}$$

where $s_i \triangleq s_{[j,i]} + s_{[i,j]}$ is the physical layer network coded symbol. Then, the relay extracts s_i by a unit-norm combining vector $\mathbf{w}_i$ (ie. $\mathbf{w}_1$ for s_1) as

$$\mathbf{w}_1^H \mathbf{r}^{\mathrm{MA}} = \mathbf{w}_1^H \mathbf{u}_1 s_1 + \mathbf{w}_1^H \mathbf{u}_2 s_2 + \mathbf{w}_1^H \mathbf{u}_3 s_3 + \mathbf{w}_1^H \mathbf{n}_{[R]}, \tag{6}$$

where $\mathbf{w}_1$ is designed, as shown in the Appendix, such that

$$\begin{aligned} &\mathbf{w}_1^H \mathbf{u}_2 = \mathbf{w}_1^H \mathbf{u}_3 = 0, \\ &\phi_1 = \mathbf{w}_1^H \mathbf{H}_{[R,1]} \mathbf{v}_{[2,1]} \text{ is maximized.} \end{aligned} \tag{7}$$

So, Eq. (6) becomes

$$\begin{aligned} \mathbf{w}_1^H \mathbf{r}^{\mathrm{MA}} &= \mathbf{w}_1^H \mathbf{u}_1 s_1 + \mathbf{w}_1^H \mathbf{n}_{[R]} \\ &= \kappa_{[2,1]} \mathbf{w}_1^H \mathbf{H}_{[R,1]} \mathbf{v}_{[2,1]} s_1 + \mathbf{w}_1^H \mathbf{n}_{[R]} \\ &= \kappa_{[2,1]} \phi_1 s_1 + \mathbf{w}_1^H \mathbf{n}_{[R]}. \end{aligned} \tag{8}$$

Similarly, the relay extracts s_2 and s_3 by using the combining vectors $\mathbf{w}_2$ and $\mathbf{w}_3$, respectively.

Now, it is assumed that the relay has successfully decoded the network coded symbols $\widetilde{s}_i = \mathbf{w}_i^H \mathbf{r}^{\mathrm{MA}}$ and its mapped symbols $\widehat{s}_i$ by using the DDF protocol as in subsection 3.

During the BC stage, the relay transmits $\mathbf{x}_{[R]}$ to all users as

$$\mathbf{x}_{[R]} = \kappa_1 \mathbf{v}_1 \hat{s}_1 + \kappa_2 \mathbf{v}_2 \hat{s}_2 + \kappa_3 \mathbf{v}_3 \hat{s}_3, \tag{9}$$

where $\{\mathbf{v}_i\}$ are the beamforming vectors and $\{\kappa_i\}$ are the power scaling factors at the relay shown in the Appendix.

For user 1, by substituting (9) into (2) and using the combining vectors $\mathbf{w}_{12}^H$ and $\mathbf{w}_{13}^H$, the two desired symbols are

$$\begin{aligned} \mathbf{w}_{12}^H \mathbf{r}_{[1]}^{\mathrm{BC}} &= \kappa_1 \mathbf{w}_{12}^H \mathbf{H}_{[1,R]} \mathbf{v}_1 \widehat{s}_1 + \kappa_2 \mathbf{w}_{12}^H \mathbf{H}_{[1,R]} \mathbf{v}_2 \widehat{s}_2 \\ &\quad + \kappa_3 \mathbf{w}_{12}^H \mathbf{H}_{[1,R]} \mathbf{v}_3 \widehat{s}_3 + \mathbf{w}_{12}^H \mathbf{n}_{[1]} \\ \mathbf{w}_{13}^H \mathbf{r}_{[1]}^{\mathrm{BC}} &= \kappa_1 \mathbf{w}_{13}^H \mathbf{H}_{[1,R]} \mathbf{v}_1 \widehat{s}_1 + \kappa_2 \mathbf{w}_{13}^H \mathbf{H}_{[1,R]} \mathbf{v}_2 \widehat{s}_2 \\ &\quad + \kappa_3 \mathbf{w}_{13}^H \mathbf{H}_{[1,R]} \mathbf{v}_3 \widehat{s}_3 + \mathbf{w}_{13}^H \mathbf{n}_{[1]}, \end{aligned} \tag{10}$$

where the beamforming and combining vectors are designed, as shown in the Appendix, such that

$$\begin{aligned} \mathbf{w}_{12}^H \mathbf{H}_{[1,R]} \mathbf{v}_2 &= \mathbf{w}_{13}^H \mathbf{H}_{[1,R]} \mathbf{v}_2 = 0 \\ \mathbf{w}_{12}^H \mathbf{H}_{[1,R]} \mathbf{v}_3 &= \mathbf{w}_{13}^H \mathbf{H}_{[1,R]} \mathbf{v}_1 = 0. \end{aligned} \tag{11}$$

Hence, the two desired messages of user 1 are obtained as

$$\begin{aligned} \mathbf{w}_{12}^H \mathbf{r}_{[1]}^{\mathrm{BC}} &= \kappa_1 \mathbf{w}_{12}^H \mathbf{H}_{[1,R]} \mathbf{v}_1 \widehat{s}_1 + \mathbf{w}_{12}^H \mathbf{n}_{[1]} \\ \mathbf{w}_{13}^H \mathbf{r}_{[1]}^{\mathrm{BC}} &= \kappa_3 \mathbf{w}_{13}^H \mathbf{H}_{[1,R]} \mathbf{v}_3 \widehat{s}_3 + \mathbf{w}_{13}^H \mathbf{n}_{[1]}. \end{aligned} \tag{12}$$

The process works similarly for users 2 and 3.

2) XOR-DNR protocol: Apparently, the constellation size of received signal $\tilde{s}_i$ at the relay is no longer the same as the original symbols $s_{[j,i]}$. The observed constellation size of the received signal becomes large when the original constellation size is increased. This complicates the receiver design at the relay. Therefore, the relay employs the XOR-DNF to efficiently design the mapping with the ability to condense the number of signal points as shown in Fig. 2. Hence, the computational complexity at the relay is reduced. We consider an XOR-DNF mapping consisting of a denoising mapper *D* as $b_i = D\left(b_{[j,i]}, b_{[i,j]}\right) = \left(b_{[j,i]} \oplus b_{[i,j]}\right)$ and the constellation mapper *C* as $\hat{s}_i = C\left(b_i\right)$, where $\oplus$ denotes the bit-wise XOR, and $b_{[j,i]}$ and $b_{[i,j]}$ are the Gray-coded bit assignments of $s_{[j,i]}$ and $s_{[i,j]}$, respectively. Thus, the mapping function *F* is

$$F : \tilde{s}_i \mapsto \hat{s}_i. \tag{13}$$

During the BC stage, all users can obtain their desirable signals from $\hat{s}_i$ by applying the XOR operation with its own side message, i.e., $b_{[i,j]} = \left(b_{[j,i]} \oplus b_{[i,j]}\right) \oplus b_{[j,i]}$.

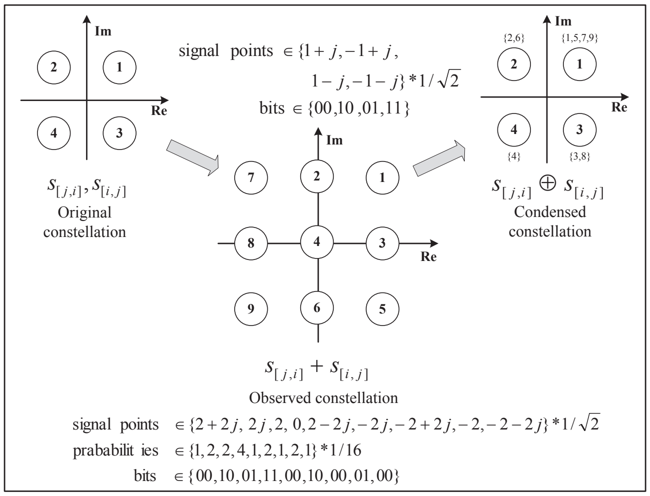

Fig. 2. XOR-DNF mapping

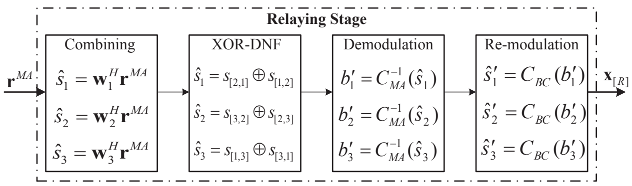

Fig. 3. DDF protocol

3) Denoise-Demodulate-and-Forward (DDF) protocol: Using the DDF protocol, which combines DNF and DF protocols as in Fig. 3, the relay demodulates the mapped symbols as $b_i' = C_{\text{MA}}^{-1}(\hat{s}_i)$. Then, the demodulated symbols are transmitted in the BC stage by using another modulation type as $\hat{s}_i' = C_{\text{BC}}(b_i')$. C_{MA} and C_{BC} are the constellation mappers for the MA and BC stages, respectively. Hence, (9) becomes

$$\mathbf{x}_{[R]} = \kappa_1 \mathbf{v}_1 \hat{s}_1' + \kappa_2 \mathbf{v}_2 \hat{s}_2' + \kappa_3 \mathbf{v}_3 \hat{s}_3', \tag{14}$$

where $\hat{s}_i'$ is the re-modulated symbols. The advantage of this protocol is that each user and the relay choose appropriate modulation types to jointly maximize the throughput, which will be discussed in Section III-B.

III. BER ANALYSIS AND THROUGHPUT OPTIMIZATION

A. Bit error probability evaluation

Here, an approximate closed-form BER expression of the proposed protocol is derived. Since the detection procedure is different at the relay and each user, the BER analysis will be studied separately for the MA and BC stages. Without loss of generality, square M-QAM is considered in this subsection.

1) BER for the MA stage: As the received signals $\{\tilde{s}_i\}$ are no longer from the original constellation, the error detection for the MA stage is focused on the observed constellation. The number of the observed signal points of a square M-QAM is equal to $(2\sqrt{M}-1)^2$. By collecting all the error events as in Fig. 2, the expression of symbol error probability is

$$\text{SER}_{ob} \le \sum_{h=1}^{4} A_h P\left(|real(n)| > \frac{d}{2} | \tilde{s} = \text{SP}_h\right), \tag{15}$$

where $\{\text{SP}_i\}$ are the signal points as the form of

$$\text{SP}_1 = a + bj,\ \ \text{SP}_2 = a,\ \ \text{SP}_3 = bj,\ \ \text{SP}_4 = 0, \tag{16}$$

with the corresponding coefficients $\{A_i\}$ are calculated as

$$A_1 = \frac{4}{M^2}[1 + \alpha(3 + 2\alpha)],\ \ A_2 = A_3 = \frac{1}{M^{3/2}}(3 + 4\alpha)],$$
$$A_4 = \frac{2}{M}, \quad \text{where } \alpha = \frac{M - \sqrt{M} - 2}{2}, \tag{17}$$

and $real(x)$ denotes the real part of a complex number x and $n = (\bar{s} - \tilde{s})$, where $\bar{s}$ is the estimated version of the signal points $\tilde{s}$. d is the Euclidean distance between two adjacent signal points of M-QAM calculated as $d = \sqrt{\frac{3\log_2 M E_b}{2(M-1)}}$, where E_b is the bit energy. Therefore, using the fact that noise is complex Gaussian distributed [11], the symbol error probability of M-QAM is expressed as

$$\begin{aligned} \text{SER}_{ob} &\le A \cdot P\left(|real(n)| > \frac{d}{2}\right) \\ &\approx A \frac{4(\sqrt{M_{MA}} - 1)}{\sqrt{M_{MA}}} Q\left(\sqrt{\frac{3}{M_{MA} - 1}\text{SNR}^r_{MA}}\right), \end{aligned} \tag{18}$$

where the Q-function is defined as $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^{\infty} e^{\left(-\frac{t^2}{2}\right)} dt$ and the coefficient A can be calculated as a function of the constellation size M as $A = A_1 + A_2 + A_3 + A_4 = \frac{2(M-1)}{M}$. Furthermore, the BER depends on the bit to symbol mapping and the approximate conversion between BER and SER based on a Gray-coded assignment when $E_b/N_o \gg 1$ is defined as $\text{BER} \approx \frac{\text{SER}}{\log_2 M}$, where $\frac{E_s}{N_o} = \log_2 M\left(\frac{E_b}{N_o}\right)$. Then, a BER expression for the MA stage is obtained as

$$\begin{aligned} \text{BER}^r_{\text{MA}} &\approx A \frac{4(\sqrt{M_{\text{MA}}} - 1)}{\sqrt{M_{\text{MA}}}\log_2 M_{\text{MA}}} \\ &\quad Q\left(\sqrt{\frac{3\log_2 M_{\text{MA}}}{M_{\text{MA}} - 1}\text{SNR}^r_{\text{MA}}}\right) \\ &\approx A \cdot B_1 Q\left(\sqrt{C_1 \text{SNR}^r_{\text{MA}}}\right), \end{aligned} \tag{19}$$

with $B_1 = \frac{4(\sqrt{M_{\text{MA}}}-1)}{\sqrt{M_{\text{MA}}}\log_2 M_{\text{MA}}}$ and $C_1 = \frac{3\log_2 M_{\text{MA}}}{M_{\text{MA}}-1}$ where $k_{\text{MA}} = \log_2 M_{\text{MA}}$ and $\text{SNR}^r_{\text{MA}} = 2\rho\gamma_k$ denotes the instantanous SNR of the signal received at the relay where the probability density function (pdf) of γ_k is $f(\gamma_k) = \frac{2}{m}e^{\left(-\frac{2}{m}\gamma_k\right)}$ [7]. Hence, the expectation of the BER for the MA stage is

$$\text{BER}_{\text{MA}} \le A \cdot B_1 \int_0^{\infty} Q\left(\sqrt{C_1 2\rho\gamma_k}\right) f(\gamma_k) d\gamma_k. \tag{20}$$

To simplify this integral function, we denote $x^2 = \rho\gamma_k$, $c = \frac{2}{m\rho}$, hence $d\gamma_k = \frac{2x dx}{\rho}$. Then, Eq. (20) becomes

$$\begin{aligned} \text{BER}_{\text{MA}} &\le A \cdot B_1 c \int_0^{\infty} 2x \cdot Q\left(x\sqrt{2C_1}\right) e^{\left(-cx^2\right)} dx \\ &\approx \frac{A \cdot B_1}{2C_1 m\rho\sqrt{1 + \frac{2}{C_1 m\rho}}} \approx \frac{A \cdot B_1}{2C_1 m\rho}, \end{aligned} \tag{21}$$

where the Q-function is approximated at high SNR [11] as $Q(z) \le \frac{1}{z\sqrt{2\pi}}e^{\left(-\frac{z^2}{2}\right)}$. Note that m is the number of antennas at each user and ρ is the transmit SNR.

2) BER for the BC stage: The received signal for each user during the BC stage is from a regular modulation constellation, so a simple bit error rate can be formulated as

$$\begin{aligned}\mathrm{BER}_{\mathrm{BC}}^{i} &\leq \frac{4(\sqrt{M_{\mathrm{BC}}}-1)}{\sqrt{M_{\mathrm{BC}}}\log_2 M_{\mathrm{BC}}} Q\left(\sqrt{\frac{3\log_2 M_{\mathrm{BC}}}{M_{\mathrm{BC}}-1}\mathrm{SNR}_{\mathrm{BC}}^{i}}\right) \\ &\approx B_2 Q\left(\sqrt{C_2 \mathrm{SNR}_{\mathrm{BC}}^{i}}\right), \qquad (22)\end{aligned}$$

where $k_{\mathrm{BC}} = \log_2 M_{\mathrm{BC}}$ and $\mathrm{SNR}_{\mathrm{BC}}^{i} = \frac{\rho}{2m}\gamma_k$ denotes the SNR of the signal received at user *i* [4]. The expectation of the BER for the BC stage, shown in (22), is derived as

$$\begin{aligned}\mathrm{BER}_{\mathrm{BC}} &\leq B_2 \int_0^{\infty} Q\left(\sqrt{C_2 \frac{\rho}{2m}\gamma_k}\right) f(\gamma_k) d\gamma_k \\ &\leq B_2 c \int_0^{\infty} 2x \cdot Q\left(x\sqrt{\frac{C_2}{2m}}\right) e^{(-cx^2)} dx \qquad (23) \\ &\approx \frac{2B_2}{C_2 \rho \sqrt{1+\frac{8}{C_2\rho}}} \approx \frac{2B_2}{C_2\rho}.\end{aligned}$$

B. Optimization design with adaptive modulation

The optimization problem to maximize the total throughput under the constraint of the BER requirements is formulated as

$$\begin{aligned}&\underset{(k_{\mathrm{MA}},k_{\mathrm{BC}})}{\text{maximize}} \quad R_{\text{total}}(k_{\mathrm{MA}}, k_{\mathrm{BC}}) \\ &\text{subject to} \quad \mathrm{BER}_{\text{total}} \leq \mathrm{BER}_{\text{req}}, \qquad (24)\end{aligned}$$

where k_{MA} and k_{BC} are the number of bits per modulation symbol for the MA and BC, respectively and R_{total} is the total throughput. The transmission times of the two stages are $T_{\mathrm{MA}} = \frac{L}{k_{\mathrm{MA}}W}$ and $T_{\mathrm{BC}} = \frac{L}{k_{\mathrm{BC}}W}$, where L is the frame size and W is the symbol rate. Without loss of generality, we assume both L and W are constant. Therefore, the total throughput is

$$R_{\text{total}} = \frac{1}{T_{\mathrm{MA}} + T_{\mathrm{BC}}} = \frac{a}{\frac{1}{k_{\mathrm{MA}}} + \frac{1}{k_{\mathrm{BC}}}}, \quad \text{where } a = \frac{W}{L}. \qquad (25)$$

$\mathrm{BER}_{\text{req}}$ is the required BER constraint and $\mathrm{BER}_{\text{total}}$ is the total BER performance, which is defined as

$$\begin{aligned}\mathrm{BER}_{\text{total}} &= (1-\mathrm{BER}_{\mathrm{BC}})\mathrm{BER}_{\mathrm{MA}} + (1-\mathrm{BER}_{\mathrm{MA}})\mathrm{BER}_{\mathrm{BC}} \\ &\approx \mathrm{BER}_{\mathrm{MA}} + \mathrm{BER}_{\mathrm{BC}}, \qquad (26)\end{aligned}$$

where $\mathrm{BER}_{\mathrm{MA}}$ as in (21) and $\mathrm{BER}_{\mathrm{BC}}$ as in (23) are the BER expression for the MA and BC stages, respectively.

Therefore, the optimization problem in (24) is rewritten as

$$\begin{aligned}&\underset{(k_{\mathrm{MA}},k_{\mathrm{BC}})}{\text{maximize}} \quad \left(\frac{1}{T_{\mathrm{MA}}(k_{\mathrm{MA}}) + T_{\mathrm{BC}}(k_{\mathrm{BC}})}\right) \\ &\text{subject to} \quad \mathrm{BER}_{\mathrm{MA}} + \mathrm{BER}_{\mathrm{BC}} \leq \mathrm{BER}_{\text{req}}. \qquad (27)\end{aligned}$$

As the optimization problem above is non-linear and complex, there is no simple closed-form solution for (27). However, in a practical system, the integer value k of square M-QAM is within a small set, e.g., $\{2, 4, 6\}$. Then, the size of all possible combinations of $\{k_{\mathrm{MA}}, k_{\mathrm{BC}}\}$ is limited to $3^2 = 9$. Therefore, an exhaustive search algorithm is used to solve the above problem in real time.

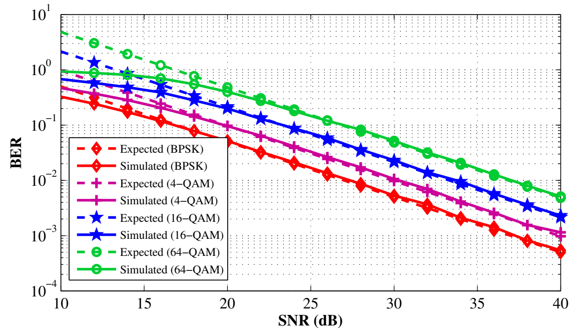

Fig. 4. Comparison of the expected and simulated BER performances

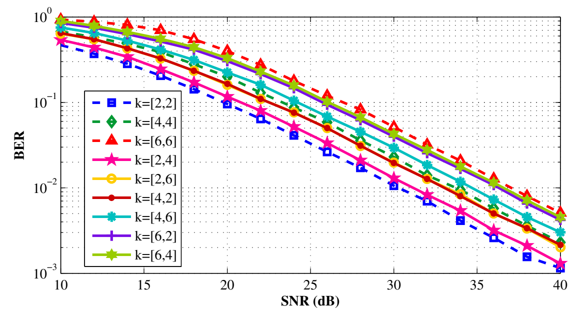

Fig. 5. BER performances of the proposed DDF MIMO Y system

IV. Performance Evaluation

The BER performance of the MIMO Y channel based on the proposed DDF protocol is evaluated. We plot the analytical BER performance by using Eq. (26), where *m*=2 and *n*=3. Moreover, we compare them to the actual BER performance for BPSK and various M-QAMs, as shown in Fig. 4. This result shows that for general M-QAM, the expected BER matches the simulated BER from about SNR of 20dB upwards.

We obtain the optimal modulation schemes for both MA and BC from Eq. (27), with various reference SNRs and BER requirements. For example, with SNR=35dB and $\mathrm{BER}_{\text{req}} = 10^{-2}$, the solution of the optimization algorithm is $[k_{\mathrm{MA}} = 4, k_{\mathrm{BC}} = 6]$. In Fig. 5, we illustrate the BER performances of the proposed system for different fixed modulation schemes. It is shown that at SNR=35 dB, the $k = [4, 6]$ scheme can lead to the BER slightly below 10^{-2} with the maximized throughput that satisfy this BER constraint. Therefore, this simulated result validates the solution obtained from Eq. (27).

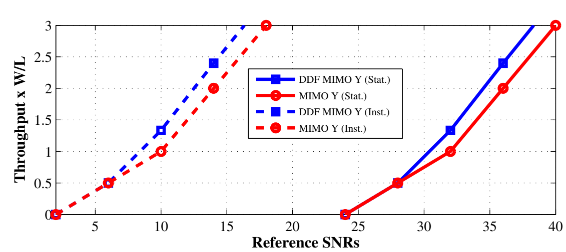

Fig. 6. Throughputs comparison of the proposed system and the MIMO Y

In Fig. 6, the comparison of the throughputs between the proposed DDF system and the existing MIMO Y is presented for various reference SNRs. As shown by the solid curves, the proposed system with jointly optimized modulation schemes can always achieve a higher throughput than the existing one with around 20% average gain. Furthermore, we also simulated the proposed system with the adaptive modulation based on the instantaneous SNR as in (19) and (22) as shown by the dash curves. Similarly, the total throughput of the proposed DDF system is higher than that of the MIMO Y with 20% average gain. Moreover, it is clear that the systems as marked by dash curves achieve the same throughput as those with solid curves, but at much lower SNR. Although the proposed schemes show the gain in an achievable throughput, the BER performance is limited, as there is no diversity gain due to the uniquely determined beamforming vectors. Hence, introducing diversity with adaptive detection algorithms needs to be considered to improve the error performances of the proposed system.

V. Conclusion

In this paper, the network information flow of a three-user MIMO Y channel, where each user sends independent messages to the other two users via a relay by using two time slots, has been investigated. We have formulated the throughput optimization problem to maximize the total throughput of this system with a BER constraint, by using the newly derived closed-form BER expressions with the proposed DDF protocol. In addition, by applying the beamforming scheme and the physical network coding with simple XOR-DNF mapping, the proposed protocol has been able to avoid the interferences among multiple destinations and ensure the low complexity at the relay, respectively. The performance evaluations have demonstrated that the proposed protocol can effectively improve the throughput, by taking advantage of the adaptive modulation with the ability to flexibly choose the modulation type for the MA and BC stages.

Appendix

Here, we focus only on the MA stage as it can be easily proved that the same beamforming and combining vectors from the MA stage can be reversely used in the BC stage.

A. Beamforming vectors

The beamforming and combining vectors are generated for the minimum system $(m = 2, n = 3)$. Therefore, $\mathbf{u}_1$, $\mathbf{v}_{[2,1]}$ and $\mathbf{v}_{[1,2]}$ are uniquely determined. First, a matrix $\mathbf{E}_1$ is constructed and a QR factorization is performed as

$$\mathbf{E}_1 = \begin{bmatrix} \mathbf{I} & -\mathbf{H}_{[R,1]} & 0 \\ \mathbf{I} & 0 & -\mathbf{H}_{[R,2]} \end{bmatrix} \text{ and } \mathbf{E}_1^H = \mathbf{QR}, \quad (28)$$

where $\mathbf{E}_1\mathbf{q}_7 = 0$ and $\mathbf{q}_7$ is the last column of $\mathbf{Q}$. Therefore, with a proper normalization/scaling, we can take the first n elements of $\mathbf{q}_7$ to obtain $\mathbf{u}_1$, the next m elements for $\mathbf{v}_{[2,1]}$ and the last m elements for $\mathbf{v}_{[1,2]}$.

B. Combining vectors

Given all the beamforming vectors, the combining vector is

$$\mathbf{w}_1 = \left\langle \mathbf{M}_1\mathbf{H}_{[R,1]}\mathbf{v}_{[2,1]} \right\rangle, \quad (29)$$

where $\langle \mathbf{x} \rangle = \frac{\mathbf{x}}{|\mathbf{x}|}$ denotes the normalization operation and

$$\begin{aligned} \mathbf{M}_1 &= \mathbf{I} - \mathbf{G}_1(\mathbf{G}_1^H\mathbf{G}_1)^{-1}\mathbf{G}_1^H, \\ \mathbf{G}_1 &= \left[\mathbf{H}_{[R,2]}\mathbf{v}_{[3,2]}, \mathbf{H}_{[R,3]}\mathbf{v}_{[1,3]}\right]. \end{aligned} \quad (30)$$

C. Power allocation

In the MA stage, the total transmit power from all users is

$$\begin{aligned} P_U &= \kappa_{[2,1]}^2 + \kappa_{[1,2]}^2 + \kappa_{[3,2]}^2 + \kappa_{[2,3]}^2 + \kappa_{[1,3]}^2 + \kappa_{[3,1]}^2 \\ &= \kappa_{[2,1]}^2(1+\beta_1^2) + \kappa_{[3,2]}^2(1+\beta_2^2) + \kappa_{[1,3]}^2(1+\beta_3^2), \end{aligned} \quad (31)$$

where $\{\beta_1, \beta_2, \beta_3\}$ are defined as $\beta_1 = \kappa_{[1,2]}/\kappa_{[2,1]}$, $\beta_2 = \kappa_{[2,3]}/\kappa_{[3,2]}$, $\beta_3 = \kappa_{[3,1]}/\kappa_{[1,3]}$. The total transmit power can be allocated among three users and the system adopts an equal effective SNR criterion as

$$\begin{aligned} &\gamma_1 = \gamma_2 = \gamma_3, \\ &\kappa_{[2,1]}^2\phi_1^2 = \kappa_{[3,2]}^2\phi_3^2 = \kappa_{[1,3]}^2\phi_3^2, \end{aligned} \quad (32)$$

where $\phi_1^2 = \mathbf{v}_{[2,1]}^H\mathbf{H}_{[R,1]}^H\mathbf{M}_1\mathbf{H}_{[R,1]}\mathbf{v}_{[2,1]}$ is the maximum effective channel gain. Thus, the common effective SNR is

$$\gamma_1 = \kappa_{[2,1]}^2\phi_1^2 = \frac{P_U}{\frac{1+\beta_1^2}{\phi_1^2} + \frac{1+\beta_2^2}{\phi_2^2} + \frac{1+\beta_3^2}{\phi_3^2}}. \quad (33)$$

Acknowledgment

This work was supported by the University of Sydney International Scholarship (USydIS) and the Australian Research Council (ARC) Discovery Projects DP1095650.

References

[1] P. Popovski and H. Yomo, "Physical network coding in two-way wireless relay channels," *IEEE ICC*, pp. 707-712, Jun. 2007.

[2] T. K. Akino, P. Popovski, and V. Tarokh, "Optmized constellation for two-way wireless relaying with physical network coding," *IEEE Trans. Communication*, vol. 27, pp. 773-787, Jun. 2009.

[3] Z. Zhou and B. Vucetic, "An optimized network coding scheme in two-way relay channels with multiple relay antennas," *IEEE PIMRC*, pp. 1717-1721, Sep. 2009.

[4] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," *IEEE VTC*, pp. 588-593, May 2006.

[5] N. Lee, J-B. Lim, and J. Chun, "Degrees of freedom of the MIMO Y channel: signal space alignment for network coding," *IEEE Trans. Information Theory*, vol.56, pp. 3332-3342, Jul. 2010.

[6] K. Lee, N. Lee, and I. Lee, "Feasibility conditions of signal space alignment for network coding on K-user MIMO Y channels," *IEEE ICC*, pp. 1-5, Jun. 2011.

[7] N. Wang, Z. Ding, X. Dai, and A.V. Vasilakos, "On generalized MIMO Y channels: precoding design, mapping and diversity gain," *IEEE Trans. Vehicular Technology*, vol. 60, pp. 3525-3532, Sep. 2011.

[8] N. Lee and J. Chun, "Signal space alignment for an encryption message and successive network code decoding on the MIMO K-way relay channel," *IEEE ICC*, pp. 1-5, Jul. 2011.

[9] M.A. Maddah-Ali, A.S. Motahari, and A.K. Khandani, "Communication over MIMO X channels: interference alignment, decomposition and performance analysis," *IEEE Trans. Information Theory*, vol.54, pp. 3457-3470, Aug. 2008.

[10] Y. Luo and L. Cai, "Throughput maximization for user cooperative wireless systems with adaptive modulation," *IEEE ICC*, pp. 1-5, May 2010.

[11] A. Goldsmith, *Wireless Communications*, Int. ed., London: Cambridge University Press, 2005.