# Radio Resource Allocation for Single-network and Multi-homing Services in Heterogeneous Wireless Access Medium

Muhammad Ismail†, Weihua Zhuang†, and Ming Yu‡
†Department of Electrical and Computer Engineering, University of Waterloo, Ontario, Canada
Email: {m6ismail, wzhuang}@uwaterloo.ca
‡Com Dev, 155 Sheldon Dr., Cambridge, Ontario, Canada, N1R 7H6

*Abstract*—**In this paper, radio resource allocation for mobile terminals (MTs) in a heterogeneous wireless access medium is investigated. Unlike the existing solutions in literature, two types of services are considered in this paper, namely single-network and multi-homing services. In single-network services, an MT is assigned to the best available wireless network, while in multi-homing services an MT utilizes all available wireless access networks simultaneously. With the presence of both services in the heterogeneous wireless access medium, the radio resource allocation objective is of twofold: We aim to find the optimal assignment of MTs with single-network service to the available wireless access networks and to determine the corresponding optimal bandwidth allocation to the MTs with single-network and multi-homing services. The radio resource allocation problem is formulated to guarantee the service quality for both service types. Numerical results are presented to demonstrate the performance of the proposed radio resource allocation scheme.**

## I. Introduction

Currently, there exist different wireless networks that offer a variety of access options. Such wireless access networks include the cellular networks, the IEEE 802.11 wireless local area networks (WLANs), and the IEEE 802.16 wireless metropolitan area networks (WMANs). In spite of the fierce competition in the wireless service market, these wireless networks will coexist, due to their complementary service capabilities in terms of bandwidth, coverage area, and cost [1]. In such a heterogeneous wireless access medium with overlapped coverage from different networks, the integration of these networks will improve the service quality of the mobile users and enhance the performance of the networks [2]. As a result, new radio resource management mechanisms for bandwidth allocation and call admission control are needed in order to satisfy the required quality-of-service (QoS) by the mobile users via different available wireless access networks and to make efficient utilization of the available resources from these networks.

Radio resource allocation in a heterogeneous wireless access medium has been studied in several works in literature. Two types of services can be distinguished in these works. In the first type, referred to as single-network service, an MT is assigned to the best available wireless access network at its location and obtains its required bandwidth from that network. In the second type, referred to as multi-homing service, the MT obtains its required bandwidth for a certain application from all wireless access networks available at its location using its multi-homing capability. However, these two service types are treated separately in literature. It is expected that these two service types will coexist. Hence, it is necessary to develop a radio resource allocation mechanism with such a consideration.

In this paper, the radio resource allocation problem for MTs in a heterogeneous wireless access medium is investigated. Unlike the existing solutions in literature, we consider the presence of both single-network and multi-homing services. The objective of the radio resource allocation is to find the optimal network assignment with the corresponding bandwidth allocation for MTs with single-network service, and the optimal resource allocation from each available network for MTs with multi-homing service.

The rest of the paper is organized as follows: In Section II, the related work is reviewed. The system model is presented in Section III. The radio resource allocation problem is formulated in Section IV. Numerical results and discussions are presented in Section V. Finally, conclusions are drawn in Section VI.

## II. Related Work

The problem of radio resource allocation in a heterogeneous wireless access medium has been studied in several works in literature. The existing solutions can be classified in two types, namely single-network and multi-homing resource allocation.

In the single-network radio resource allocation, an MT is allocated its required bandwidth from the best wireless access network available at its location. The selection of the best available wireless access network is based on a predefined criterion. One predefined criterion is the received signal strength (RSS) [3], where the MT is assigned to the network with the highest RSS from its base station (BS) or access point (AP) as compared to other available networks' BSs/APs. Another criterion is the available bandwidth [4], where the MT is assigned to the network BS/AP with the largest available bandwidth as compared to other available networks' BSs/APs. Further, different metrics such as RSS, available bandwidth, and monetary cost can be combined in a utility function and the MT network assignment is based on the results of this function for the candidate networks' BSs/APs [5]. One limitation of single-network radio resource allocation is that an incoming call is blocked if no network in the service area can individually satisfy the required bandwidth by that

[1]This work was supported by a research grant from the Natural Science and Engineering Research Council (NSERC) of Canada.

978-1-4673-1881-5/12/$31.00 ©2012 IEEE

call. Hence, the available resources from different networks are not fully exploited.

With multi-homing capability, an MT can maintain multiple simultaneous associations with different networks. Hence, in multi-homing radio resource allocation, the MT obtains its required bandwidth from all networks' BSs/APs available at its location using its multi-homing capability. This has the advantage of supporting applications with high required data rate through aggregating the offered resources from different networks and using multiple threads at the application layer. Also, it allows for mobility support, since at least one of the used radio interfaces will remain active during the call duration. Moreover, the multi-homing concept can reduce the call blocking rate and improve the system capacity. Multi-homing radio resource allocation has been employed in [6] using the concept of utility fairness, and using convex optimization formulation in [7] for constant bit rate (CBR) service and in [8] for both CBR and variable bit rate (VBR) services.

Existing solutions in literature for radio resource allocation in a heterogeneous wireless access medium focus on either a single-network or a multi-homing service. Despite of the advantages of multi-homing radio resource allocation, it is very likely that both single-network and multi-homing services will exist simultaneously. There will be some MTs with multi-homing capabilities, which can utilize a multi-homing service, while other MTs, although equipped with multiple radio interfaces, do not have multi-homing capabilities, which can only utilize a single-network service. Even for an MT with multi-homing capabilities, the utilization of the multi-homing service should depend on the available energy at the MT. With sufficient available energy at the MT, multiple radio interfaces can be used simultaneously with multi-homing service. However, when there is no sufficient energy available at the MT, the MT should switch to the single-network service where the radio interface of the best available network is utilized while all other interfaces are turned off to save energy. Hence, it is required to develop a radio resource allocation mechanism that can support both single-network and multi-homing services.

In the following, radio resource allocation for MTs with single-network and multi-homing services is investigated. The objective of the radio resource allocation is of twofold. For the single-network service, we aim to find the optimal assignment of MTs to the available wireless networks' BSs/APs with the required bandwidth allocation, and for the multi-homing service, we aim to find the optimal bandwidth allocation for the MTs from each available network BS/AP.

## III. System Model

Consider a geographical region where a set $\mathcal{N} = \{1, 2, \cdots, N\}$ of different wireless access networks is available. There exists a central resource manager in the geographical region. Each network, $n \in \mathcal{N}$, is operated by a unique service provider, and has a set $\mathcal{S}_n = \{1, 2, \cdots, S_n\}$ of BSs/APs. Each BS/AP of network $n$ has transmission capacity $C_n$ Mbps and different coverage area from those of other networks. Different networks' BSs/APs have overlapped coverage in some areas. Hence, the geographical region is partitioned to a set $\mathcal{K} = \{1, 2, \cdots, K\}$ of service areas, with each service area $k \in \mathcal{K}$ covered by a unique subset of BSs/APs. The BSs/APs covering service area $k$ are given in the subset $\mathcal{S}_k$ with cardinality $|\mathcal{S}_k|$.

The set of MTs in the geographical region is denoted by $\mathcal{M} = \{1, 2, \cdots, M\}$. The subset of MTs in a given service area, $k$, is denoted by $\mathcal{M}_k$. Each MT, $m \in \mathcal{M}$, has its own home network, but can also get service from other networks available at its location. An MT using its own home network $n$ is referred to as network subscriber, while an MT which is using a network other than its own home network is referred to as network user [8]. A priority parameter $p_{nms}$ is used to represent service priority of each network $n$ in allocating its resources via BS/AP $s$ to mobile user $m$, where $p_{nms} = 1$ for high-priority network subscribers and $p_{nms} \in [0, 1)$ for low-priority network users [8]. The subset of MTs with single-network service in a given service area $k$ is denoted by $\mathcal{M}_{1k}$, while the subset of MTs with multi-homing service is denoted by $\mathcal{M}_{2k}$. An MT, $m \in \mathcal{M}_{1k}$ in a given service area $k$, is assigned to a single network $n$ BS/AP $s \in \mathcal{S}_k$. The assignment criterion is based on the allocated bandwidth to the MT. The network assignment vector in the geographical region for MTs with single-network service is given by $A = [a_1, \cdots, a_m, \cdots, a_{|\mathcal{M}_{1k}|}]$, where $a_m = ns$ is the assignment of MT $m \in \mathcal{M}_{1k}$ to network $n$ BS/AP $s$. For example, $a_1 = 12$ is the assignment of MT 1 to network 1 BS/AP 2. On the other hand, an MT, $m \in \mathcal{M}_{2k}$ in service area $k$, obtains its required bandwidth from all BSs/APs in $\mathcal{S}_k$ using its multi-homing capability. The set of MTs assigned to network $n$ BS/AP $s$, including both multi-homing and single-network MTs, is given by $\mathcal{M}_{ns}$.

All calls in the geographical region for MT $m$ are VBR calls with required bandwidth $B_m \in [B_m^{\min}, B_m^{\max}]$, where $B_m^{\min}$ guarantees a minimum required QoS and $B_m^{\max}$ is enforced to incorporate the MT technical limitation [9]. When there are sufficient resources in service area $k$, a VBR call in the area is allocated its maximum required bandwidth $B_m^{\max}$. However, when all BSs/APs in $\mathcal{S}_k$ reach their capacity limitation $C_n$, the bandwidth allocations are reduced towards $B_m^{\min}$ to support these calls. The allocated bandwidth from network $n$ to MT $m$ through BS/AP $s$ is given by $b_{nms}$.

The objective of the radio resource allocation is to find the optimal network assignment vector $A$ for MTs with single-network service and the corresponding bandwidth allocation matrix $B = [b_{nms}]$ for all $n \in \mathcal{N}$, $m \in \mathcal{M}$, and $s \in \mathcal{S}$. The radio resource allocation is to guarantee the service quality for each service type in terms of the required bandwidth $B_m$.

## IV. Problem Formulation

In this section, the problem of radio resource allocation for MTs with single-network and multi-homing services in the heterogeneous wireless access medium is formulated.

The utility of network $n$ allocating bandwidth $b_{nms}$ to MT $m$ through BS/AP $s$ is given by [8]

$$u_{nms}(b_{nms}) = \log(1 + \eta_1 b_{nms}) - \eta_2(1 - p_{nms})b_{nms} \quad (1)$$

where $\eta_1$ and $\eta_2$ are used for scalability of $b_{nms}$. For a network subscriber, with $p_{nms} = 1$, the utility function of (1) accounts only for the attained network utility by that subscriber, which is represented by the first term in the right hand side (RHS) of (1) [10]. On the other hand, a network user, with $p_{nms} \in [0, 1)$, suffers from a tradeoff between the attained network utility and the cost that the network sets on its own resources. The cost is represented by the second term in the RHS of (1). Hence, each network gives a higher priority in allocating its resources to its own subscribers as compared to other users [8].

For a given network assignment vector $A$, the overall resource allocation objective of all networks in the geographical region is to find the optimal allocation $b_{nms}$, $\forall n \in \mathcal{N}$, $m \in \mathcal{M}_{ns}$, and $s \in \mathcal{S}_n$ that maximizes the total utility in the region, given by

$$U(b_{nms}) = \sum_{n=1}^{N} \sum_{s=1}^{S_n} \sum_{m \in \mathcal{M}_{ns}} u_{nms}(b_{nms}). \tag{2}$$

The allocated resources from network $n$ BS/AP $s$ should satisfy the network BS/AP capacity constraint, that is

$$\sum_{m \in \mathcal{M}_{ns}} b_{nms} \leq C_n, \qquad \forall s \in \mathcal{S}_n, n \in \mathcal{N}. \tag{3}$$

For MTs with single-network service, given a network assignment vector $A$, the allocated resources from the assigned network $n$ BS/AP $s \in \mathcal{S}_k$ to MT $m \in \mathcal{M}_{1k}$ in service area $k$ should satisfy its application required bandwidth, given by

$$B_m^{\min} \leq b_{nms} \leq B_m^{\max}, \qquad \forall m \in \mathcal{M}_{1k}, k \in \mathcal{K}. \tag{4}$$

On the other hand, for MTs with multi-homing service, the total allocated resources from all available wireless access networks BSs/APs in $\mathcal{S}_k$ to a given MT $m \in \mathcal{M}_{2k}$ in service area $k$ should satisfy its application required bandwidth, given by

$$B_m^{\min} \leq \sum_{n=1}^{N} \sum_{s \in \mathcal{S}_k} b_{nms} \leq B_m^{\max}, \qquad \forall m \in \mathcal{M}_{2k}, k \in \mathcal{K}. \tag{5}$$

In order to find the optimal network assignment vector $A$ and the corresponding optimal bandwidth allocation matrix $B$ for MTs with single-network and multi-homing services, the radio resource allocation problem in the heterogeneous wireless access medium is expressed by the following optimization problem

$$\begin{aligned} \max_{A} \{ \max_{B} \quad & U(b_{nms}) \\ s.t. \quad & (3) - (5) \}. \end{aligned} \tag{6}$$

Although the radio resource allocation problem for a given network assignment vector is a convex optimization problem and hence can be solved efficiently using polynomial time algorithms [11], finding the optimal vector $A$ incurs high computational complexity. In a given service area $k$ with a total of $|\mathcal{S}_k|$ available BSs/APs from different networks and $|\mathcal{M}_{1k}|$ MTs with single-network service, there exist $|\mathcal{S}_k|^{|\mathcal{M}_{1k}|}$ distinct assignment vectors. Hence, in the whole geographical region, the total number of distinct assignment vectors is $\prod_k |\mathcal{S}_k|^{|\mathcal{M}_{1k}|}$. That is, the inner maximization problem of (6) needs to be solved $\prod_k |\mathcal{S}_k|^{|\mathcal{M}_{1k}|}$ times in order to find the optimal network assignment $A$ for $m \in \cup_k \mathcal{M}_{1k}$ and bandwidth allocations $b_{nms}$ for $m \in \mathcal{M}$. Consider one service area with 3 BSs/APs with overlapped coverage and 50 MTs with single-network service. There are a total of $3^{50} = 7 * 10^{23}$ distinct network assignments in this service area. For the whole geographical region, it is expected that the inner maximization problem of (6) needs to be solved for a huge number of times in order to find the optimal radio resource allocation. As a result, it is desirable to reformulate (6) to a less complex problem. In order to do so, we introduce a binary assignment variable $x_{nms}$, which is determined from the network assignment vector $A$ for MT $m \in \mathcal{M}_{1k}$ by

$$x_{nms} = \begin{cases} 1, & \text{if } a_m = ns \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

On the other hand, $x_{nms} = 1$ for MTs in service area $k$ with multi-homing service for all $s \in \mathcal{S}_k$. Using this binary assignment variable, the problem given in (6) can be reformulated as

$$\begin{aligned} \max_{x_{nms}, b_{nms}} \quad & \sum_{n=1}^{N} \sum_{s=1}^{S_n} \sum_{m \in \mathcal{M}_{ns}} \{ \log(1 + \eta_1 x_{nms} b_{nms}) \\ & - \eta_2 (1 - p_{nms}) x_{nms} b_{nms} \} \\ s.t. \quad & \sum_{m \in \mathcal{M}_{ns}} x_{nms} b_{nms} \leq C_n, \quad \forall s \in \mathcal{S}_n, n \in \mathcal{N} \\ & B_m^{\min} \leq \sum_{n=1}^{N} \sum_{s \in \mathcal{S}_k} x_{nms} b_{nms} \leq B_m^{\max}, \\ & \qquad \forall m \in \mathcal{M}_k, k \in \mathcal{K} \\ & x_{nms} \in \{0, 1\}, \quad \forall m \in \mathcal{M}_{1k}, k \in \mathcal{K} \\ & \sum_{n=1}^{N} \sum_{s \in \mathcal{S}_k} x_{nms} = 1, \quad \forall m \in \mathcal{M}_{1k}, k \in \mathcal{K} \\ & x_{nms} = 1, \quad \forall m \in \mathcal{M}_{2k}, k \in \mathcal{K}. \end{aligned} \tag{8}$$

The fourth constraint in (8) guarantees that an MT with single-network service is assigned to one and only one network BS/AP available at its location. On the other hand, the last constraint in (8) allows an MT with multi-homing service to obtain its required bandwidth from all networks available at its location.

The problem of (8) is a non-convex mixed integer non-linear programming (MINLP) problem. MINLP problems are difficult to solve, as they combine the difficulty of optimizing over integer variables with the handling of non-linear functions [12]. This is especially true when the objective and/or constraint functions are non-convex, which is the case in (8). Recently, several new methods are proposed for solving MINLP problems [13]. In literature, two classes of algorithms that solve MINLP problems can be distinguished. The first class includes deterministic algorithms such as branch and bound, outer approximation, generalized benders decomposition, and extended cutting plane [12], [13]. Global optimization approaches for addressing non-convexities in MINLP problems are developed using convex envelopes or under-estimators to formulate lower-bounding convex MINLP problems [13].

An example of deterministic global optimization methods for MINLP problems is branch and reduce [14], and other methods can be found in [13]. The second class of algorithms that solve MINLP problems includes stochastic optimization algorithms such as the extended ant colony optimization [15]. The different methods of solving MINLP problems have been available through many solvers [17]. Deterministic solvers that claim to guarantee global optimality for non-convex general MINLP problems include AlphaBB, BARON, COUENNE, and LINDOGLOBAL [17]. Stochastic solvers include MIDACO [16], however there is no guarantee for global optimality [17]. The BARON [18], which is a GAMS [19] solver, has proven to be the most robust one among the currently available global solvers [20]. The BARON implements deterministic global optimization algorithms which integrate conventional branch and bound with a wide variety of range reduction tests [18]. The BARON guarantees to provide global optima under fairly general assumptions which include the availability of finite lower and upper bounds on the variables and their expressions in the MINLP to be solved [18]. Hence, for the solution of the radio resource allocation problem of (8), we use the BARON solver through GAMS.

## V. Numerical Results

This section presents analytical results for problem (8) using the BARON/GAMS solver. Consider a simplified system model with one service area that is covered by two networks, i.e. $\mathcal{N} = \{1, 2\}$. Each network has one BS/AP in the service area, i.e. $\mathcal{S}_1 = \{1\}$ and $\mathcal{S}_2 = \{1\}$. The transmission capacity of each BS/AP for the service area under consideration is $4$ Mbps for network $1$ and $1.248$ Mbps for network $2$. The transmission capacities are chosen such that they can support a total of 12 MTs with VBR calls of required bandwidth $B_m \in [64, 128]$ Kbps of single-network service, and a total of 17 MTs with VBR calls of required bandwidth $B_m \in [256, 512]$ Kbps of multi-homing service. Let $M_{nr}$ denotes the number of subscribers from network $n$ with service $r$, where $r = 1$ represents a single-network service while $r = 2$ represents a multi-homing service. With $M_{11} = 6$, $M_{21} = 6$, $M_{22} = 8$, we vary the number of network 1 subscribers with multi-homing service $M_{12}$, in order to study the performance of the resource allocation mechanism as the call traffic load of the subscribers of the network with the larger capacity varies. The two networks set different costs on their resources using the priority parameter $p_{nms}$. Since network 2 has a smaller transmission capacity than network 1, it sets a higher cost on its resources so that it can devote its resources to its own subscribers [8]. Hence, let $p_{1m1} = 0.8$ and $p_{2m1} = 0.6$ for network users, while $p_{nm1} = 1$ for network subscribers with $n \in \mathcal{N}$. Let $\eta_1$ and $\eta_2$ equal 1. Let $L_{n_1 n_2}$ denote the number of assigned subscribers of network $n_1$, with single-network service, to network $n_2$.

Figure 1 shows the radio resource allocation per call for MTs with single-network service versus the number $M_{12}$ of network 1 subscribers with multi-homing service. As $M_{12}$ increases, the bandwidth allocation for network 2 subscribers is reduced first towards the minimum required bandwidth. This is because network 2 subscribers rely heavily on network 1 resources in addition to their home network to support their high required bandwidth, while network 1 gives a higher priority to its own subscribers on its resources. The resource allocation to network 1 subscribers is then reduced in order to accommodate more multi-homing subscribers from this network. The resource allocation guarantees the desired bandwidth range for the VBR calls.

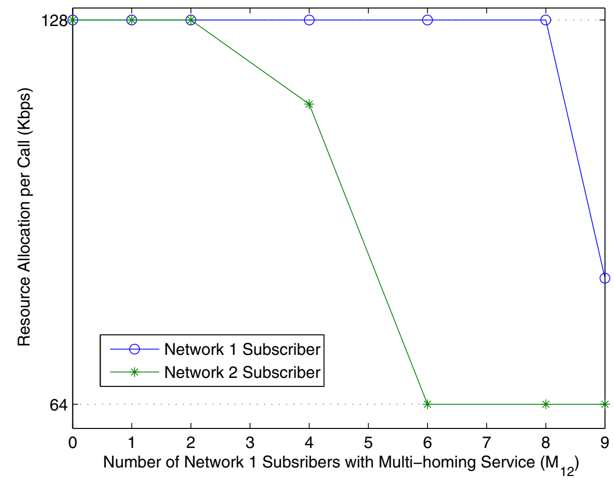


Fig. 1. Radio resource allocation for MTs with single-network service.

TABLE I
Network assignments for network 1 subscribers with single-network service

| $M_{12}$ | $L_{11}$ | $L_{12}$ |
|---|---|---|
| 0 | 6 | 0 |
| 1 | 6 | 0 |
| 2 | 6 | 0 |
| 4 | 6 | 0 |
| 6 | 6 | 0 |
| 8 | 6 | 0 |
| 9 | 6 | 0 |

Tables I and II show the numbers of MTs with single-network services assigned to each network BS/AP for network 1 and network 2 subscribers respectively, versus the number $M_{12}$ of network 1 subscribers with multi-homing service. Due to the larger capacity of network 1, its subscribers are always assigned to their home network which provides them with high allocated bandwidth (refer to Figure 1). For network 2 subscribers, the network assignment varies with $M_{12}$. At a small number of $M_{12}$ (from 0 to 2), all network 2 subscribers with single-network service are assigned to network 1, as it provides them with their maximum required bandwidth (refer to Figure 1). As the call traffic load increases in network 1, more subscribers from network 2 are assigned to their home network, as network 1 gives higher priority to its own subscribers on its resources.

Figure 2 shows the radio resource allocation per call for MTs with multi-homing service from each available network versus the number $M_{12}$ of network 1 subscribers with multi-homing service. The total bandwidth allocation to network 1 subscribers ($N1$) comes from network 1 ($N1-1$). The

TABLE II
NETWORK ASSIGNMENTS FOR NETWORK 2 SUBSCRIBERS WITH SINGLE-NETWORK SERVICE

| $M_{12}$ | $L_{21}$ | $L_{22}$ |
|---|---|---|
| 0 | 6 | 0 |
| 1 | 6 | 0 |
| 2 | 6 | 0 |
| 4 | 4 | 2 |
| 6 | 3 | 3 |
| 8 | 3 | 3 |
| 9 | 2 | 4 |

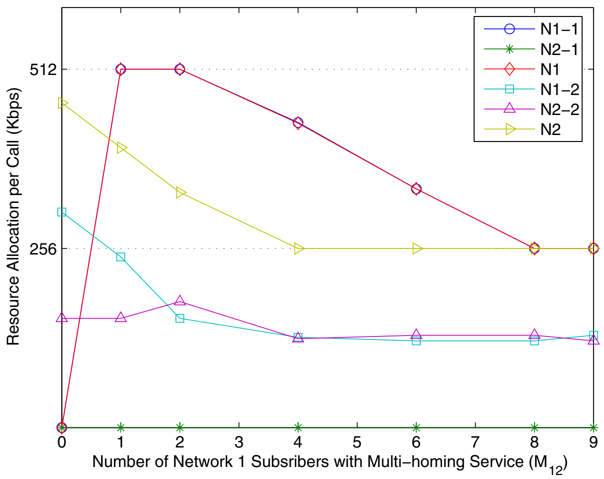


Fig. 2. Radio resource allocation for MTs with multi-homing service.

allocation from network 2 ($N2-1$) is zero, as network 2 devotes its resources to support its own subscribers using the priority parameter $p_{2m1}$. The total resource allocation per call for network 1 subscribers ($N1$) decreases with $M_{12}$ towards the minimum required bandwidth in order to accommodate more subscribers. For network 2 subscribers, the allocation from network 1 ($N1-2$) is decreased as $M_{12}$ increases, as network 1 uses its resources to support its own subscribers. This is compensated by an increase in the allocation from network 2 ($N2-2$) to improve the resource allocation of its own subscribers. However, for $M_{12} > 2$, network 2 decreases its allocation to its subscribers with multi-homing service, as more single-network subscribers are assigned to its BSs/APs (refer to Table II). As a result, the total resource allocation per call for network 2 subscribers ($N2$) decreases with $M_{12}$ towards the minimum required bandwidth. The total resource allocation per call for network 1 and network 2 subscribers with multi-homing services ($N1$ and $N2$ respectively) is within the desired bandwidth range for the VBR calls.

## VI. CONCLUSION

In this paper, radio resource allocation for MTs with single-network and multi-homing services in a heterogeneous wireless access medium is investigated. The proposed resource allocation finds the optimal network assignment for MTs with single-network service and the corresponding optimal bandwidth allocation for MTs with single-network and multi-homing services. Also, each network gives a higher priority in resource allocation to its subscribers as compared to other users. For multi-homing services, all available networks support the bandwidth required by the VBR calls. However, as the number of its subscribers increases, each network reduces its allocation to other users in a way that does not violate their minimum required bandwidth, in order to provide its own subscribers with their maximum possible required bandwidth. Similarly, for MTs with single-network services, their network assignment and the corresponding allocated bandwidth depend on the call traffic load of the subscribers in each network. In this sense, each network supports its own subscribers to ensure that they are satisfied with the maximum possible required bandwidth, while at the same time supporting the subscribers of other networks.

## REFERENCES


[1] M. Kassar, B. Kervella, and G. Pujolle, “An overview of vertical handover strategies in heterogeneous wireless networks,” *Computer Communications*, vol. 31, no. 10, pp. 2607-2620, June 2008.
[2] M. Ismail and W. Zhuang, “Network cooperation for energy saving in green radio communications,” *IEEE Wireless Communications*, vol. 18, no. 5, pp. 76-81, Oct. 2011.
[3] S. Mohanty and I. F. Akylidiz, “A cross-layer (layer 2 + 3) handoff management protocol for next generation wireless systems,” *IEEE Trans. mobile computing*, vol. 5, no. 10, pp. 1347-1360, 2006.
[4] W. Shen and Q. Zeng, “Resource management schemes for multiple traffic in integrated heterogeneous wireless and mobile networks,” *Proc. 17th Int. conf. ICCCN*, pp. 105-110, August 2008.
[5] E. S. Navarro, Y. Lin, and W. S. Wong, “An MDP-based vertical handoff decision algorithm for heterogeneous wireless networks,” *IEEE Trans. Vehicular Technology*, vol. 57, no. 2, pp. 1243-1254, March 2008.
[6] C. Luo, H. Ji, and Y. Li, “Utility based multi-service bandwidth allocation in the 4G heterogeneous wireless access networks,” *Proc. IEEE WCNC*, April 2009.
[7] M. Ismail and W. Zhuang, “A distributed resource allocation algorithm in heterogeneous wireless access medium,” *Proc. IEEE ICC 2011*, June 2011.
[8] M. Ismail and W. Zhuang, “A distributed multi-service resource allocation algorithm in heterogeneous wireless access medium,” *IEEE J. Selected Areas Communications*, vol. 30, no. 2, pp. 425-432, Feb. 2012.
[9] R. Litjens, H. van den Berg, and R. J. Boucherie, “Throughputs in processor sharing models for integrated stream and elastic traffic,” *Performance Evaluation*, vol. 65, no. 2, pp. 152-180, Feb. 2008.
[10] H. Shen and T. Basar, “Differentiated Internet pricing using a hierarchical network game model,” *Proc. 2004 American Control Conference*, pp. 2322-2327, vol.3, 2004.
[11] D. P. Bertsekas, *Non-linear programming*, Athena Scientific, 2003.
[12] P. Bonami, M. Kilinc, and J. Linderoth, “Algorithms and software for convex mixed integer nonlinear programs,” *Technical Report 1664*, Computer Sciences Department, University of Wisconsin-Madison, 2009.
[13] I. E. Grossmann, “Review of nonlinear mixed-integer and disjunctive programming techniques,” *Optimization and Engineering*, vol. 3, no. 3, pp. 227-252, Sept. 2002.
[14] M. Tawarmalani and N. .V. Sahinidis, “Global optimization of mixed integer nonlinear programs: a theoritical and computational study,” *Math. Program.*, vol. 99, no. 3, April 2004.
[15] M. Schluter, J. A. Egea, and J. R. Banga, “Extended ant colony optimization for non-convex mixed integer nonlinear programming,” *Comput. Oper. Res.*, vol. 36, no. 7, pp. 2217-2229, 2009.
[16] M. Schluter, M. Gerdts, and J. J. Ruckmann, “MIDACO: new global optimization software for MINLP,” Sept. 2011.
[17] M. R. Bussieck and S. Vigerske, “MINLP Solver Software,” *Wiley Encyclopedia of Operations Research and Management Science*, Apr. 2011.
[18] N. V. Sahinidis and M. Tawarmalani, “BARON: GAMS solver manual,” May 2011.
[19] www.gams.com.
[20] A. Neumaier, O. Shcherbina, W. Huyer, and T. Vinko, “A comparison of complete global optimization solvers,” *Math. Program.*, vol. 103, pp. 335-356, 2005.