Optimal Amplitude Design for MN-ary Pulse Position Amplitude Modulation

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***Abstract* - The *MN*-ary pulse position amplitude modulation (PPAM) signal is constructed by combining *M*-ary pulse amplitude modulation and *N*-ary pulse position modulation. The PPAM scheme provides improved reciprocity in system performance and computational complexity. In this paper, we investigate the Euclidean distance in a set of *MN*-ary PPAM signals. And moreover, we derive the optimal amplitudes to maximize the minimum Euclidean distance for PPAM signals. Finally, simulation results are conducted to show the performance improvement from the proposed amplitude design.**

***Index Terms*— Pulse position amplitude modulation (PPAM), pulse position modulation (PPM), pulse amplitude modulation (PAM), Euclidean distance.**

I. INTRODUCTION

Pulse position modulation (PPM) is a signal modulation used for both analog and digital signal transmissions. Studies have shown that PPM is more battery power-efficient than on-off keying and frequency shift keying for a wireless sensor network [1,2]. Moreover, PPM is a desirable transmission scheme for various communication systems, e.g., ultra-wideband (UWB) and optical communication systems [3–6].

Recently, MN-ary pulse position amplitude modulation (PPAM) was proposed in [7, 8]. In principle, PPAM directly combines PPM and pulse amplitude modulation (PAM). A set of MN-ary PPAM signals is constructed by combining an M-ary PAM and an N-ary PPM. PPAM provides improved reciprocity between system performance and complexity.

Several studies have investigated PPAM in different scenarios [9–11]. A new distance notion for MN-ary PPAM was introduced in [9], based on the set partitioning, and a $(2,2)$-ary PPAM space-time trellis code is constructed. The UWB system with PPAM using space-time block code was investigated in [10]. The receiver design for PPAM was studied in [11]. However, to our knowledge, no studies have investigated the design for amplitudes in MN-ary PPAM signals.

Therefore, this paper investigates the design of amplitudes for MN-ary PPAM signals. First, we review conventional PPAM signals, which use the amplitudes designed for M-ary PAM signals. We then analyze the minimum Euclidean distance in a set of MN-ary PPAM signals and derive the optimal amplitudes to maximize the minimum Euclidean distance. The symbol error rate (SER) can be minimized by maximizing the minimum Euclidean distance [12]. Finally, simulation results are conducted to verify the performance improvement in terms of SER.

The remainder of this paper is organized as follows. In Section II, the MN-ary PPAM signal using conventional amplitudes is studied. The optimal amplitudes that maximize the minimum Euclidean distance in a set of MN-ary PPAM signals are derived in Section III. Finally, simulation results are presented in Section IV and conclusions are given in Section V.

II. SIGNAL MODEL FOR PPAM

A set of MN-ary PPAM signals is constructed by including an M-ary PAM signal in each dimension of N-ary PPM signals. Notably, $\log_2(N)$ bits are represented by the pulse position and $\log_2(M)$ bits are represented by the pulse amplitude. The bit-rate of an MN-ary PPAM signal is given by $\frac{1}{N}\left[\log_2(M)+\log_2(N)\right]$.

The MN-ary PPAM signal $\mathbf{s}_{m,n}$ can be represented as an N dimensional vector with a nonzero real value α_m in the nth dimension [7, 8], i.e.,

$$\mathbf{s}_{m,n}=\left[0,...,0,\alpha_m,0,...,0\right], \qquad (1)$$

where $n\in\{1,2,...,N\}$, $m\in\{1,2,...,M\}$, and α_m is one of M possible amplitudes. In the literatures, the value of α_m is

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determined according to the amplitudes in PAM signal, i.e.,

$$\alpha_m = \sqrt{E} \cdot (2m - 1 - M), \tag{2}$$

where $E = \frac{3 \cdot N}{M^2 - 1}$ is the normalization factor resulting in the average energy of the MN-ary PPAM signals equaling 1, i.e., $\frac{1}{N} \cdot \left\{ \frac{1}{NM} \cdot \sum_{m=1}^{M} \sum_{n=1}^{N} \left[\mathbf{s}_{m,n} \cdot \left(\mathbf{s}_{m,n} \right)^T \right] \right\} = 1$ .The baseband signals are shown in Fig. 1, with $M = 4$ and $N = 2$ as an example,. In addition, by using $\mathbf{s}_{m,1} \cdot \left(\mathbf{s}_{m,2} \right)^T = 0$, the corresponding signal space diagram is depicted in Fig. 2, which indicates that the minimum Euclidean distance is $\frac{2}{\sqrt{5}}$.

The general expression of the minimum Euclidean distance for MN-ary PPAM signals is derived for further analysis. The Euclidean distance between two MN-ary PPAM signals is given by $\left\| \mathbf{s}_{m,n} - \mathbf{s}_{m',n'} \right\|$, where $n' \in \{1, 2, ..., N\}$, $m' \in \{1, 2, ..., M\}$, and $\|\bullet\|$ stands for the Frobenius norm. In the case of $n' \neq n$, $\min\left[\left\| \mathbf{s}_{m,n} - \mathbf{s}_{m',n'} \right\| \right] = \sqrt{2E} = \sqrt{\frac{6 \cdot N}{M^2 - 1}}$. On the other hand, for $n' = n$ and $m' \neq m$, $\min\left[\left\| \mathbf{s}_{m,n} - \mathbf{s}_{m',n'} \right\| \right] = 2\sqrt{E} = 2\sqrt{\frac{3 \cdot N}{M^2 - 1}}$. Therefore, the minimum Euclidean distance $d_\alpha(M, N) \equiv \min\left[\left\| \mathbf{s}_{m,n} - \mathbf{s}_{m',n'} \right\|^2 \right]$ is given by

$$\begin{aligned} d_\alpha(M, N) &\equiv \min\left[\left\| \mathbf{s}_{m,n} - \mathbf{s}_{m',n'} \right\|^2 \right] \\ &= \sqrt{2E} \\ &= \sqrt{\frac{6 \cdot N}{M^2 - 1}} \end{aligned} \tag{3}$$

Considering the case of $M = 4$ and $N = 2$ again, we can obtain the minimum Euclidean distance from (3), i.e.,

$$d_\alpha(4, 2) = \sqrt{\frac{6 \cdot 2}{4^2 - 1}} = \frac{2}{\sqrt{5}}. \tag{4}$$

The amplitudes $\alpha_m = \sqrt{E} \cdot (2m - 1 - M)$ are known to be optimal for PAM, which maximizes its minimum Euclidean distance. In addition, to our knowledge, the previous studies adopt this amplitude for MN-ary PPAM [1–3]. However, no study has investigated the optimal design for amplitudes in MN-ary PPAM. Accordingly, the amplitude design for MN-ary PPAM is studied in Section III. Moreover, the optimal amplitudes are derived to maximize the minimum Euclidean distance in MN-ary PPAM.

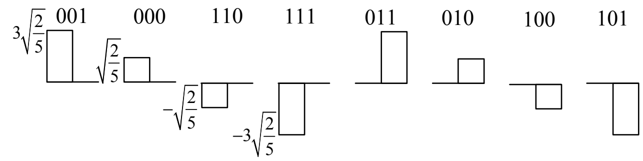

Fig. 1. An illustration for PPAM baseband signals with $M = 4$ and $N = 2$

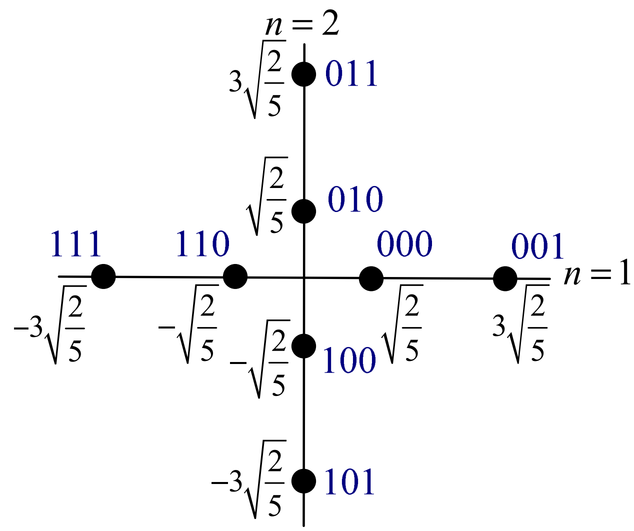

Fig. 2. Signal space of the conventional PPAM signals for $M = 4$ and $N = 2$

III. Analysis on Amplitudes for PPAM

The minimum Euclidean distance must be maximized to minimize SER for MN-ary PPAM signals. In this section, we will derive the amplitudes that maximize the minimum Euclidean distance for a set of MN-ary PPAM signals. To avoid confusion, the designed amplitudes of MN-ary PPAM signals are denoted by β_m in subsequent analysis. Specifically, the MN-ary PPAM signal $\mathbf{s}_{m,n}$ can be represented as an $N \times 1$ vector with the nonzero value, β_m, as the nth element, i.e.,

$$\mathbf{s}_{m,n} = [0, ..., 0, \beta_m, 0, ..., 0]. \tag{5}$$

Note that β_m have M possible values and satisfy $\frac{1}{NM} \cdot \sum_{m=1}^{M} |\beta_m|^2 = 1$, which guarantees that the average energy of the MN-ary PPAM signals is 1, i.e., $\frac{1}{N} \cdot \left\{ \frac{1}{NM} \cdot \sum_{m=1}^{M} \sum_{n=1}^{N} \left[\mathbf{s}_{m,n} \cdot \left(\mathbf{s}_{m,n} \right)^T \right] \right\} = 1$.

We first consider $n \neq n'$. In such a case, any two PPAM signals are mutually orthogonal. The corresponding Euclidean distance between the two PPAM signals, i.e., $\left\| \mathbf{s}_{m,n} - \mathbf{s}_{m',n'} \right\|$, can be represented as

$$\left\|\mathbf{s}_{m,n}-\mathbf{s}_{m',n'}\right\|=\sqrt{\left|\beta_m\right|^2+\left|\beta_{m'}\right|^2}\ . \tag{6}$$

Therefore, we derive the minimum Euclidean distance, d_1, for $n \neq n'$,

$$\begin{aligned} d_1 &\equiv \min_{m,m'}\left(\left\|\mathbf{s}_{m,n}-\mathbf{s}_{m',n'}\right\|\right) \\ &= \min_{m,m'}\left(\sqrt{\left|\beta_m\right|^2+\left|\beta_{m'}\right|^2}\right) \\ &= \sqrt{2}\cdot\min_m\left(\left|\beta_m\right|\right) \end{aligned} \tag{7}$$

Inspecting (7), the minimum amplitude, i.e., $\min_m\left(\left|\beta_m\right|\right)$, determines the minimum Euclidean distance among PPAM signals for $n \neq n'$.

In the case of $n = n'$ and $m \neq m'$, any two PPAM signals must be in the same dimension. Therefore, the resultant Euclidean distance between two PPAM signals can be represented by

$$\left\|\mathbf{s}_{m,n}-\mathbf{s}_{m',n'}\right\|=\left|\beta_m-\beta_{m'}\right|\ . \tag{8}$$

We subsequently obtain the corresponding minimum Euclidean distance, d_2, for $n = n'$ and $m \neq m'$,

$$\begin{aligned} d_2 &\equiv \min_{m,m'}\left(\left\|\mathbf{s}_{m,n}-\mathbf{s}_{m',n'}\right\|\right) \\ &= \min_{m,m'}\left(\left|\beta_m-\beta_{m'}\right|\right) \end{aligned} \tag{9}$$

Equation (9) indicates that the minimum difference between amplitudes, i.e., $\min_{m,m'}\left(\left|\beta_m-\beta_{m'}\right|\right)$, determines the minimum Euclidean distance among PPAM signals in the case of $n = n'$ and $m \neq m'$.

By using the results given in (8) and (9), the minimum Euclidean distance among MN-ary PPAM signals, which is denoted by $d_\beta\left(M,N\right)$, has the following form,

$$\begin{aligned} d_\beta\left(M,N\right) &\equiv \min\left(\left\|\mathbf{s}_{m,n}-\mathbf{s}_{m',n'}\right\|\right) \\ &= \begin{cases} \sqrt{2}\cdot\min\left(\left|\beta_m\right|\right),\ n \neq n' \\ \min\left(\left|\beta_m-\beta_{m'}\right|\right),\ n = n' \text{ and } m \neq m' \end{cases} \end{aligned} \tag{10}$$

The minimum distance shown in (10) can be rewritten as follows,

$$\begin{aligned} d_\beta\left(M,N\right) &\equiv \min_{m,m'}\left[\left\|\mathbf{s}_{m,n}-\mathbf{s}_{m',n'}\right\|\right] \\ &= \min_{m,m'}\left[\sqrt{2}\cdot\left|\beta_m\right|,\left|\beta_m-\beta_{m'}\right|\right] \end{aligned} \tag{11}$$

According to (11), the optimization problem, which maximizes the minimum Euclidean distance, can be represented by

$$\begin{aligned} &\max_{\beta_m}\left\{\min_{m,m'}\left[\sqrt{2}\cdot\left|\beta_m\right|,\left|\beta_m-\beta_{m'}\right|\right]\right\} \\ &\text{subject to } \frac{1}{NM}\sum_{m=1}^{M}\left|\beta_m\right|^2=1 \end{aligned} \tag{12}$$

Subsequently, the amplitudes β_m satisfying (12) are derived.

Given the power constraint, $\frac{1}{NM}\sum_{m=1}^{M}\left|\beta_m\right|^2=1$, the minimum Euclidean distance is maximized when

$$\min\left(\left|\beta_m-\beta_{m'}\right|\right)=\sqrt{2}\cdot\min\left(\left|\beta_m\right|^2\right). \tag{13}$$

Specifically, the minimum Euclidean distances are equivalent in the two cases. Without loss of generality, the minimum amplitude $\min_m\left(\left|\beta_m\right|\right)$ is defined as ρ, i.e.,

$$\rho \equiv \min_m\left(\left|\beta_m\right|\right). \tag{14}$$

Substituting (14) into (13), we observe that

$$\min\left(\left|\beta_m-\beta_{m'}\right|\right)=\sqrt{2}\cdot\min\left(\left|\beta_m\right|^2\right)=\sqrt{2}\cdot\rho \tag{15}$$

Equation (15) indicates that the minimum difference between two amplitudes must be $\sqrt{2}\cdot\rho$. Therefore, we derive the optimal amplitudes β_m, which maximize the minimum Euclidean distance for MN-ary PPAM signals, i.e.,

$$\beta_m=\begin{cases} -\left(\frac{M}{2}-m\right)\cdot\sqrt{2}\rho-\rho,\ m=1,2,\ldots,\frac{M}{2} \\ \rho+\left(m-\frac{M}{2}-1\right)\cdot\sqrt{2}\rho,\ m=\frac{M}{2}+1,\frac{M}{2}+2,\ldots,M \end{cases}, \tag{16}$$

where

$$\rho=\frac{\sqrt{NM}}{\sqrt{2\cdot\sum_{q=0}^{\mathrm{M}/2-1}\left(1+\sqrt{2}\cdot q\right)^2}} \tag{17}$$

fulfills the power constraint $\frac{1}{NM}\sum_{m=1}^{M}\left|\beta_m\right|^2=1$. In addition, for arbitrary M and N, the minimum Euclidean distance $d_\beta\left(M,N\right)$ is given by

$$d_\beta\left(M,N\right)=\sqrt{2}\cdot\rho=\frac{\sqrt{M}}{\sqrt{\sum_{q=0}^{\mathrm{M}/2-1}\left(1+\sqrt{2}\cdot q\right)^2}} \tag{18}$$

Considering the case of $M=4$ and $N=2$, we can observe from (17) that $\rho=\frac{1}{\sqrt{2+\sqrt{2}}}$. Then, substituting $\rho=\frac{1}{\sqrt{2+\sqrt{2}}}$ into (18), we derive that the corresponding minimum Euclidean distance $d_\beta\left(4,2\right)$ is

$$d_\beta\left(4,2\right)=\sqrt{2}\cdot\rho=\frac{\sqrt{2}}{\sqrt{2+\sqrt{2}}}\ . \tag{19}$$

Moreover, Fig. 3 depicts the signal space diagram for PPAM signals with $M=4$ and $N=2$. We can verify from Fig. 3 that the corresponding minimum Euclidean distance is

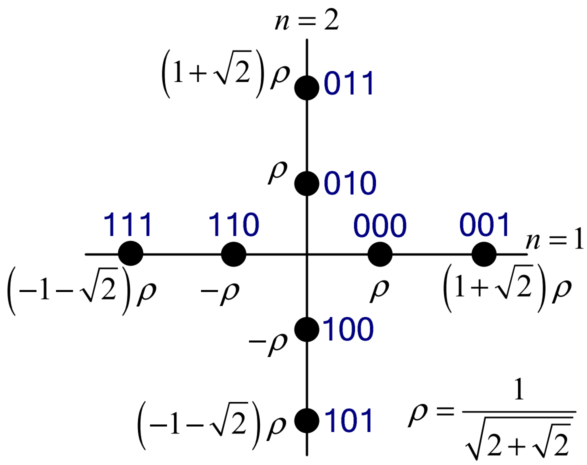

Fig. 3. Signal space of proposed PPAM signals for $M=4$ and $N=2$

$\frac{\sqrt{2}}{\sqrt{2+\sqrt{2}}}$. A comparison of (4) and (19) emphasizes that the proposed PPAM signal has a larger minimum Euclidean distance than conventional PPAM signals in the case of $M=4$ and $N=2$, i.e., $d_{\beta}(4,2) \approx 0.5858 > d_{\alpha}(4,2) = 0.4$. In Section IV, the SER performance for MN-ary PPAM signals is investigated by conducting simulation results.

IV. Simulation Results And Conclusions

In this section, simulation results are conducted to compare the SER performances for PPAM signals using two different amplitude designs. Without loss of generality, the average power of MN-ary PPAM signals is assumed to be P_{av}. At the receiver end, the received MN-ary PPAM signals are demodulated based on the maximum likelihood principle [13].

The simulations commence by considering the additive white Gaussian noise (AWGN) channel. We assume that the noise variance is σ^2 and the signal-to-noise ratio (SNR) is defined as $\frac{P_{\text{av}}}{\sigma^2}$. Fig. 4 shows the SER performances for MN-ary PPAM signals with $M=4$ and $N=\{8,16,32\}$. The simulation results demonstrate that increasing N improves SER performance at the expense of data rate. Moreover, the PPAM signals using proposed amplitudes have better SER than conventional PPAM for various values of N. The SNR is improved by up to 2 dB by applying the proposed amplitudes. A similar conclusion can be obtained from Fig. 5, where we assume that $M=8$ and $N=\{8,16,32\}$.

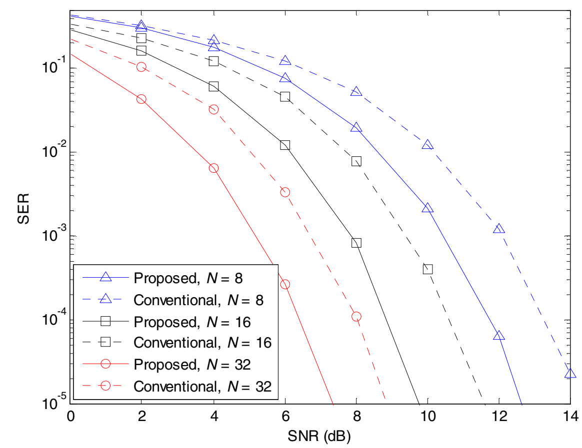

Fig. 4. SER performance for PPAM using different amplitude designs in AWGN channel ($M=4$)

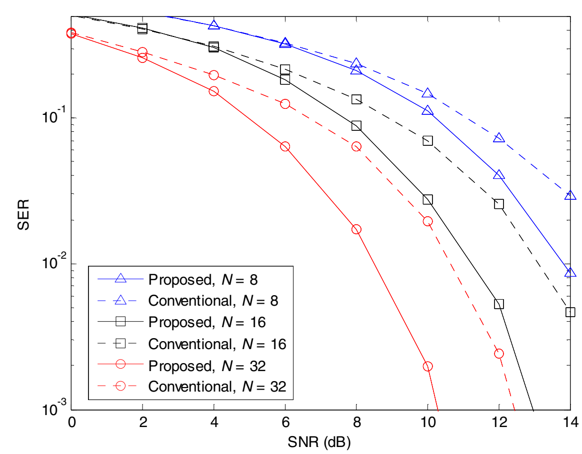

Fig. 5. SER performance for PPAM using different amplitude designs in AWGN channel ($M=8$)

V. Conclusions

The MN-ary PPAM provides good performance and low complexity by combining M-ary PAM and N-ary PPM. In this work, we analyzed the minimum Euclidean distance for MN-ary PPAM signals. And moreover, the optimal amplitudes are derived, which maximize the minimum Euclidean distance in a set of MN-ary PPAM signals. The developed amplitudes can be applied for general M and N. Finally, simulation results are conducted to verify the performance improvement in SER.

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