Low Complexity SNR Estimation for Linear Modulations on AWGN Channel

Chaoxing Yan, Hua Wang, Nan Wu, and Jingming Kuang

Abstract—**In this paper, we propose a novel signal-to-noise ratio (SNR) estimation method for linear modulations on additive white Gaussian noise (AWGN) channel. It estimates noise power directly without estimating the received total power. The estimation mean and mean square error (MSE) are analyzed theoretically and verified by simulations. Results show that the proposed SNR estimator performs better at low SNR than the conventional estimators while it suffers a little degradation at high SNR. Moreover, the proposed estimator can be implemented with less real multiplications than the conventional ones.**

Index Terms—**SNR estimation, AWGN, linear modulations, mean square error.**

I. Introduction

The iterative decoding for turbo codes and link adaptations in communication receivers require accurate and simple signal-to-noise ratio (SNR) or noise power estimation. The powerful channel coding makes the transmission systems perform close to Shannon limit, whereas the estimation techniques at low SNR are challenged in the receivers. There are several categories of conventional SNR estimations [1]: the data-aided (DA) methods are preferred at low SNR; the non-data-aided (NDA) methods with decision-directed (DD) or and moment-based techniques are preferred at high SNR [2, 3]. Recently, some emerging iterative estimations were proposed to improve the performance at low SNRs [4, 5], i.e. code-aided (CA) estimations [5, 6]. They are usually initialized by conventional DA and NDA estimation methods.

In this paper, we propose a novel SNR estimation method without estimation of received total power for *M*-ary linear modulations. It can be designed to be a DA, NDA or CA estimator according to the types of symbols employed to remove modulation in estimation. The mean and mean square error (MSE) of the proposed DA SNR estimator are calculated and verified by simulation. Performance comparisons for different estimation methods are also presented by computer simulations in terms of bias and normalized MSE (NMSE) with *M*-ary phase-shift keying (*M*-PSK) and *M*-ary quadrature amplitude modulation (*M*-QAM) signals. The NMSE results show that the proposed estimator performs better at low SNR than the conventional estimator although it suffers a little degradation at high SNR. The analysis for the computational complexity of the proposed estimation shows that it requires fewer multiplications.

II. System Model and Novel SNR Estimation

In this paper, we consider a complex, discrete, band-limited communication system with an *M*-ary constellation in complex additive white Gaussian noise (AWGN) channel. Perfect carrier and symbol timing recovery are assumed. The received symbols after matched filtering are:

$$r_k = \sqrt{E_S}\, a_k + \sqrt{N_0}\, n_k,\ k = 1, 2, ..., K \tag{1}$$

where a_k are independent and identically distributed (i.i.d.) data symbols from *M*-ary constellations $\mathcal{A} = \{\alpha_0, \alpha_1, ..., \alpha_{M-1}\}$ with $E\{|a_k|^2\} = 1$, E_S and $N_0 = 2\sigma^2$ are the signal and noise power, respectively, and n_k are samples of a complex-valued zero-mean AWGN process with unit variance. The in-phase (I) and quadrature (Q) components of complex signals $n_k = n_{Ik} + jn_{Qk}$ are assumed to be independent. Denote noise $v_k := \sqrt{N_0}\, n_k$, the probability density function is written as $p(v_k) = (\pi N_0)^{-1} e^{-|v_k|^2/N_0}$.

A. Novel SNR Estimation

Firstly, we review one of the conventional DA SNR estimators. After some algebraic manipulations [1], the estimation of E_S is given as,

$$\hat{E}_S := \frac{1}{M_0^2} \left| \frac{1}{K} \sum_{k=1}^{K} \mathrm{Re}\left(r_k a_k^*\right) \right|^2 \tag{2}$$

where $M_0 := \sum_{k=1}^{K} |a_k|^2 \Big/ K$.

Denote $\hat{R} := \sum_{k=1}^{K} |r_k|^2 \Big/ K$ as total received power, the noise power is estimated as $\hat{N}_0 := \hat{R} - \hat{E}_S M_0$. Define SNR as $\gamma := E_S / N_0$, the conventional SNR estimation [1] can be written as

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$$\hat{\gamma} = \frac{\frac{1}{M_0^2}\left|\frac{1}{K}\sum_{k=1}^{K}\mathrm{Re}(z_k)\right|^2}{\frac{1}{K}\sum_{k=1}^{K}|r_k|^2 - \frac{1}{M_0}\left|\frac{1}{K}\sum_{k=1}^{K}\mathrm{Re}(z_k)\right|^2} \tag{3}$$

where $z_k = r_k a_k^*$. Usually, the estimation (3) is modified to reduce its bias: $\hat{\gamma}' = \hat{\gamma}(K-3/2)/K$.

We notice that the SNR estimation (3) employs direct estimations of total power and signal power, whereas the noise power estimation is their difference. Here, we propose the novel noise power estimation expressed as

$$\hat{N}_0 := \frac{2}{K}\sum_{k=1}^{K}\left|\mathrm{Im}(z_k)\right|^2 \tag{4}$$

Therefore, using the same signal power estimation (2), we propose an SNR estimation method presented as,

$$\hat{\gamma} = \frac{\frac{1}{M_0^2}\left|\frac{1}{K}\sum_{k=1}^{K}\mathrm{Re}(z_k)\right|^2}{\frac{2}{K}\sum_{k=1}^{K}\left|\mathrm{Im}(z_k)\right|^2} \tag{5}$$

If the pilot symbols a_k are used to remove modulation in $z_k = r_k a_k^*$, the proposed estimation (5) is a DA SNR estimator $\hat{\gamma}^{(DA)}$, whereas if the decision values $\hat{a}_k (k=1,...,K)$ are employed in $z_k = r_k \hat{a}_k^*$, it is a NDA estimator $\hat{\gamma}^{(NDA)}$. What's more, for coded symbols with *a posterior* values from the output of decoders, it becomes the CA estimator $\hat{\gamma}^{(CA)}$.

B. *Performance of the Novel SNR Estimation*

We will calculate the mean and MSE of the proposed estimator $\hat{\gamma}^{(DA)}$ as follows. It is known that $\mathrm{Re}(z_k)$ is a real Gaussian random variable. The signal power estimation is,

$$\hat{E}_S := \left|\frac{1}{KM_0}\sum_{k=1}^{K}\mathrm{Re}(z_k)\right|^2 = \left|\sqrt{E_S} + \frac{\sqrt{N_0}}{KM_0}\sum_{k=1}^{K}\mathrm{Re}\left(n_k a_k^*\right)\right|^2 \tag{6}$$

The expectation of the constituent Gaussian random variable above is $\sqrt{E_S}$. Considering the variance

$$\mathrm{var}\left\{\sum_{k=1}^{K}\mathrm{Re}\left(n_k a_k^*\right)\right\} = E\left[\sum_{k=1}^{K}\mathrm{Re}\left(n_k a_k^*\right)\sum_{l=1}^{K}\mathrm{Re}\left(n_l a_l^*\right)\right] \tag{7}$$

where the expectation exists only when *k*=*l*. We have the variance of the constituent random variable in (6) as,

$$\mathrm{var}\left\{\frac{1}{KM_0}\sum_{k=1}^{K}\mathrm{Re}(z_k)\right\} = \frac{N_0}{K^2M_0^2}E\left[\sum_{k=1}^{K}\left(\mathrm{Re}\left(n_k a_k^*\right)\right)^2\right] \tag{8}$$

Substitute $\mathrm{Re}(n_k a_k^*) = n_{Ik}a_{Ik} + n_{Qk}a_{Qk}$ into above, we get

$$E\left[\sum_{k=1}^{K}\left(\mathrm{Re}\left(n_k a_k^*\right)\right)^2\right] = \sum_{k=1}^{K}E\left[n_{Ik}^2a_{Ik}^2 + n_{Qk}^2a_{Qk}^2\right] = \frac{KM_0}{2} \tag{9}$$

Therefore, the variance of individual random variable in (6) is $N_0/(2KM_0)$. Hence, we conclude that $\hat{E}_S$ follows the 1 degree of freedom chi-squared distribution with individual random variable mean E_S and variance $N_0/(2KM_0)$, that is $\hat{E}_S \sim \chi^2(1, E_S, N_0/(2KM_0))$, where we denote $\chi^2(n,\mu,\sigma_{rv}^2)$ as a chi-squared distribution with n degrees of freedom, μ non-centrality parameter and σ_{rv}^2 variance of the constituent Gaussian random variables.

Similarly, the variance of Gaussian random variable in noise power (4) is calculated as

$$\mathrm{var}\left\{\mathrm{Im}(z_k)\right\} = E\left[\left(\mathrm{Im}(z_k)\right)^2\right] = \frac{N_0M_0}{2} \tag{10}$$

Therefore, the noise power estimation $\hat{N}_0$ follows the K degrees of freedom chi-squared distribution, that is $\hat{N}_0 \sim \chi^2(K,0,N_0M_0/K)$. We also notice that the modification has $\hat{N}_0/(2M_0^2) \sim \chi^2(K,0,N_0/(2KM_0))$. Then, the SNR estimation (5) follows a non-central F-distribution with the analytical form of $\hat{\gamma}$ being expressed as,

$$2KM_0^2\hat{\gamma} \sim F\left(1,K,2KM_0\gamma\right) \tag{11}$$

where we denote $F(n_1,n_2,\lambda)$ with $n_1 = 1$, $n_2 = K$ degrees of freedom, and non-central parameter $\lambda := 2KM_0\gamma$. Therefore, the expectation of the proposed SNR estimation is (*K*>4):

$$\mu_{\hat{\gamma}} = \frac{E[F]}{2KM_0^2} = \frac{K}{(K-2)M_0}\left(\frac{1}{2KM_0} + \gamma\right). \tag{12}$$

Moreover, the bias in above expectation is reduced with the modification: $\hat{\gamma}' = \hat{\gamma}(K-2)M_0/K$, whose variance is calculated as

$$\sigma_{\hat{\gamma}'} = \frac{(1+2KM_0\gamma)^2 + (1+4KM_0\gamma)(K-2)}{2K^2(K-4)M_0^2} \tag{13}$$

In the similar way, the performance of conventional estimator can also be calculated. The analytical performance of the DA estimator will be verified by simulations.

III. Numerical Results and Discussions

In this section, we will investigate the performance of different SNR estimations in terms of bias and NMSE by simulations. The statistical MSE normalized by γ^2, $E\{(\hat{\gamma}-\gamma)^2\}/\gamma^2$, will be compared with the normalized Cramer-Rao Bound (NCRB) of DA SNR estimation [7, 8],

$$NCRB\{\hat{\gamma}\} = \frac{2}{K\gamma} + \frac{1}{K} \tag{14}$$

A. Simulation for the Novel SNR Estimation

We notice that the calculated performances (12) and (13) for DA SNR estimator are independent of modulations, whereas they are affected only by M_0 term (M_0=1 for M-PSK) in simulations. We will study these estimators with several cases: 1) DA estimation with K=128 8PSK symbols, 2) NDA estimation with K=128 16QAM symbols, 3) DA estimation with K=1024 16QAM symbols, 4) NDA estimation with K=1024 QPSK symbols, 5) CA estimation with K=1024 QPSK symbols. The CA estimation is initialized by NDA estimation and then embedded in maximum 40 decoding iterations of the low-density parity-check (LDPC) decoder with irregular (2048, 1030) code [9] as the framework of iterative receiver in [5].

The normalized biases (with true γ) of cases 1)-4) are given in Fig. 1. It can be found that, the bias of the proposed estimation agrees well with that of conventional estimation except that little difference can be observed at high SNR. Then, we will further investigate the NMSE values of different estimations in Fig. 2 and Fig. 3.

Fig. 2 shows the results of cases 1) and 2). The ratio of the calculated NMSE of the proposed DA estimator to that of conventional one (MSE ratio) is about 0.5 at low SNR and 2 at high SNR. Their calculated values (cal.) agree well with simulation results (sim.). Therefore, the proposed estimator performs better at low SNR although it suffers a little degradation at high SNR. In addition, the performance of NDA estimations can reach that of DA ones at high SNR (about 15dB for 16QAM).

Fig. 3 gives the results of cases 3)-5). It can be found that the simulation performance of case 3) has a little deviation from the calculated value due to the M_0 of limited observations. For QPSK, the NDA and CA estimations can approach the DA ones at 9dB and 1.4dB, respectively. The latter lies in the waterfall region of the LDPC code employed, where the proposed CA estimation performs better than the conventional one.

B. Computation Complexity

The computational complexity comparison is considered in Table I. With accumulated K symbols used for estimation, the $z_k = r_k a_k^*$ term in (3) and (5) is usually implemented with K complex multiplications and the ratio $\gamma = E_S / N_0$ is completed by 1 division for both proposed and conventional estimations. However, compared with the conventional estimation, the proposed estimation can reduce K additions and K multiplications. The multiplication reduction which is critical in practice amounts up to about 50%.

TABLE I
Complexity Comparison for SNR Estimators

Operations	Conventional	Proposed
Addition	3K+1	2K
Multiplication	2K+1	K+1
Division	1	1
Complex multiplication	K	K

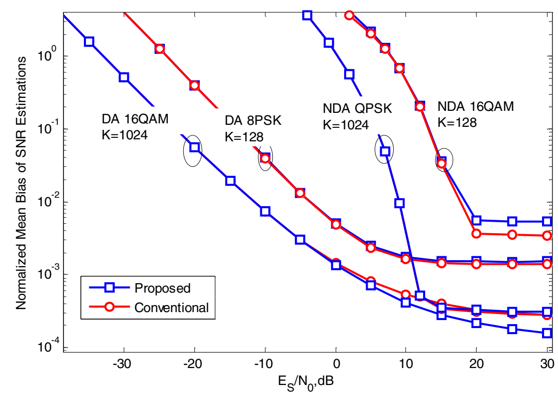

Fig. 1 Normalized bias for estimators with different modulations and observation lengths.

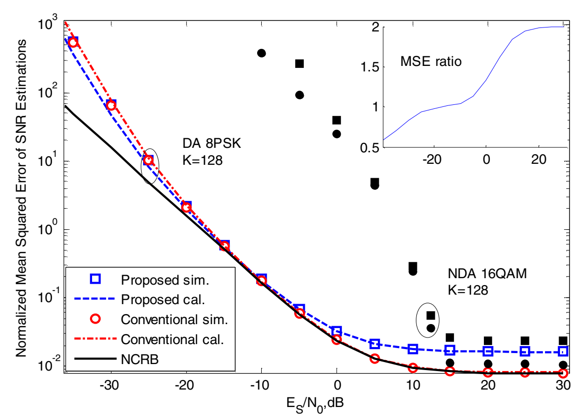

Fig. 2 NMSE for DA estimation with 8PSK, NDA estimation with 16QAM, K=128 symbols.

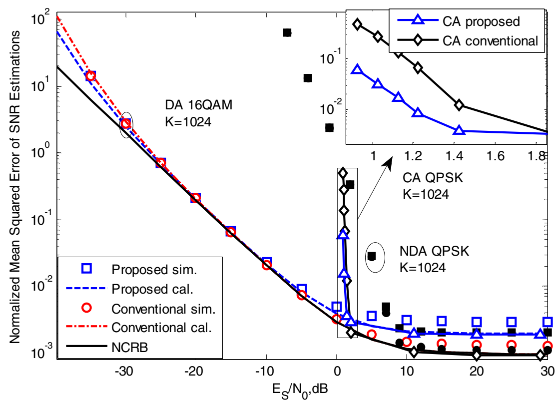

Fig. 3 NMSE for DA estimation with 16QAM, NDA estimation with QPSK, CA estimation with QPSK, K=1024 symbols.

C. Discussions of SNR Estimators

There are also some other DA SNR estimators [10-12]. Furthermore, considering the residual phase offset together, we can estimate the signal power by maximum likelihood (ML) criterion as

$$\hat{E}_S := \left|\frac{1}{KM_0}\sum_{k=1}^{K} r_k a_k^*\right|^2 = \left|\sqrt{E_S} + \frac{\sqrt{N_0}}{KM_0}\sum_{k=1}^{K} n_k a_k^*\right|^2 \qquad (15)$$

where $\sqrt{E_S} + \sqrt{N_0}\sum_{k=1}^{K} n_k a_k^* \Big/ KM_0$ is a complex Gaussian random variable with mean $\sqrt{E_S}$ and variance $N_0/2KM_0$. The deriving process is given as

$$\frac{\partial}{\partial \tilde{\mathbf{b}}}\ln p(\mathbf{r} \mid \tilde{\mathbf{b}}) = \frac{\partial}{\partial \tilde{\mathbf{b}}}\left\{ 2\frac{\sqrt{\tilde{E}_S}}{\tilde{N}_0}\mathrm{Re}\left\{ e^{-j\tilde{\theta}}\sum_{k=1}^{K} r_k a_k^* \right\} - \frac{\tilde{E}_S}{\tilde{N}_0}KM_0 - \frac{1}{\tilde{N}_0}\sum_{k=1}^{K}|r_k|^2 - K\ln(\pi\tilde{N}_0) \right\} \qquad (16)$$

where $\mathbf{r} := [r_1, \ldots, r_K]$, $\mathbf{b} := [E_S, N_0, \theta]$. Therefore, the proposed SNR estimator can be written as

$$\hat{\gamma} = \frac{\hat{E}_S}{\hat{N}_0} = \frac{|S_0|^2}{M_0^2 K \sum_{k=1}^{K}|r_k|^2 - M_0|S_0|^2} \qquad (17)$$

where $S_0 := \sum_{k=1}^{K} r_k a_k^*$. Finally, the analytical form of $\hat{\gamma}$ is expressed as,

$$\frac{2KF-2}{2}\hat{\gamma}M_0 \sim F\left(2, 2K-2, 2KM_0E_S/N_0\right) \qquad (18)$$

where we denote $F(n_1, n_2, \lambda)$ with $n_1 = 2$, $n_2 = 2K-2$ degrees of freedom, and non-central parameter $\lambda := 2KM_0E_S/N_0 = 2KM_0\gamma$. The expectation of SNR estimation is (K>4):

$$\mu_{\hat{\gamma}} = \frac{2E[F]}{(2K-2)M_0} = \frac{K}{K-2}\left(\frac{1}{KM_0} + \gamma\right) \qquad (19)$$

The variance for $\hat{\gamma}$ is,

$$\sigma_{\hat{\gamma}}^2 = \frac{K^2(M_0^2\gamma^2 + 2M_0\gamma) - 2KM_0\gamma + K - 1}{(K^3 - 7K^2 + 16K - 12)M_0^2} \qquad (20)$$

We have the normalized CRB (NCRB) as

$$NCRB\{\hat{\gamma}\} = \frac{2K}{M_0\gamma(K-2)^2} + \frac{K}{(K-2)^2} \qquad (21)$$

After examining the SNR estimation methods (3), (5) and (17), we conclude that, the proposed method (5) holds less complexity than (3) and (17). The ML estimation (17) is independent of phase offset, whereas (3) and (5) are not.

IV. Conclusion

In this paper, we propose an improved SNR estimation method for linear modulations on AWGN channel. It has DA, NDA and CA variations according to the symbols used to remove modulations. The mean and MSE of the proposed DA estimator are calculated and verified by simulations. The simulation bias performance validates the equal accuracy of the proposed and conventional estimations. Results show that the proposed DA and CA estimators perform better at low SNR than the conventional estimators in terms of NMSE although a little degradation can be observed at high SNR. In addition, the proposed SNR estimation can be implemented with less real multiplications and additions than the conventional one.

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