A PMI Feedback Scheme for Downlink Multi-user MIMO Based on Dual-Codebook of LTE-Advanced

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***Abstract*—In order to achieve a better tradeoff between overhead and performance, this paper proposes a separately selected dual-codebook (SSDC) based best companion cluster (BCC) approach. In this scheme, each UE selects its best and worst long-term precoding matrix indicators (PMI1), and feeds them back as the cluster indicators (CIs) for UE pairing. The final precoder W is determined by two precoders indicated by the best PMI1 and the short-term feedback PMI2. Besides, channel quality indicators (CQIs) are calculated for UE scheduling. To compare with SSDC-based BCC, we also discuss and propose a jointly selected dual-codebook (JSDC) based BCC scheme. Simulation results show that SSDC-based BCC has a comparable throughput performance with JSDC-based BCC but with a substantially reduced complexity and feedback overhead for 8Tx.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technologies can provide great increases in capacity of a communication link [1–3], and are widely used in wireless industrial standards these years. Multi-user (MU) MIMO is a promising way to increase downlink cell throughput by enabling multiple users to share the same time and frequency resource. Taking advantage of multiuser diversity and spatial domain, MU-MIMO can also achieve significantly high cell spectrum efficiency [4], [5], which has drawn much attention from both standardization bodies and telecommunication industry.

Although 3GPP Long Term Evolution (LTE) Release 8 (Rel. 8) specifies a simple version of downlink MU-MIMO with the same feedback mechanism as single-user (SU) MIMO [6], more discussions are made for MU-MIMO enhancement in LTE-Advanced (LTE-A) to further improve the system performance. Nowadays codebook based precoding scheme has been a focus during the standardization [7–9]. Due to the limited feedback amount of downlink MU-MIMO system, the codebook structure and feedback scheme designs are very important. [10] presented that the best companion cluster (BCC) scheme gives a reasonable compromise between the feedback overhead and throughput, with each user equipment (UE) feeding back the best precoding matrix indicator (PMI) and a cluster indicator (CI) that includes the least interfering PMI for pairing and precoding. Whereas only 4-bits codebooks such as LTE Rel. 8 codebook for rank 1 were discussed in [10], this proposed BCC will be ineffective under 8Tx assumption.

In order to meet the peak spectral efficiency requirements of up to 30 bit/s/Hz, LTE-A supports up to 8Tx antennas. To fulfill the requirements in terms of different antenna setups and associated channel correlation structure, a dual-codebook design was proposed and finally fixed in [11]. In this approach [12], [13], the precoding matrix $\mathbf{W}$ is obtained as the product of two matrices $\mathbf{W}_1$ and $\mathbf{W}_2$, where $\mathbf{W}_1$ is a block diagonal matrix denoting a cluster composed of over-sampled 4-DFT beams, representing the long-term statistical properties of the channel as well as $\mathbf{W}_2$ captures the short-term variations of the channel by beam selection and co-phasing [14]. Then, for this codebook structure, UE is required to feed back PMIs corresponding to both $\mathbf{W}_1$ and $\mathbf{W}_2$. Generally, to obtain the best precoding matrix, we should use the extensive joint select method, searching over all possible combinations of $\mathbf{W}_1$ and $\mathbf{W}_2$, but the great complexity makes it difficult to realize at UE. Considering the dual-codebook functionality, a low-complexity precoding select approach were proposed by [15], which selects $\mathbf{W}_1$ and $\mathbf{W}_2$ separately. However, only SU-MIMO system was considered in [15].

In this paper, we propose a PMI feedback scheme named separately selected dual-coodbook (SSDC) based BCC for downlink MU-MIMO, to reduce feedback overhead and promise reliability of the transmission. Considering the dual-codebook for 8Tx antennas functionality, the best and worst indicators PMI1 for $\mathbf{W}_1$ are firstly chosen at UE and fed back to the base station (BS) for UE pairing. Then each UE transmits its short-term feedback PMI2 for $\mathbf{W}_2$ and channel quality indicator (CQI) to the BS for obtaining the final precoder and UE scheduling, respectively. Under 8Tx assumption, a jointly selected dual-codebook (JSDC) based BCC scheme is discussed and simulated, comparing with SSDC-based BCC. Numerical results also reveal the performance differences between dual-codebook based BCC and single-codebook (SC) based BCC.

The rest of this paper is organized as follows. In Section II we present the system model and give its system diagram for LTE-A downlink. Afterwards, the proposed JSDC-based BCC and SSDC-based BCC schemes will be described in Section III, followed with the simulation configuration and the simulation results in Section IV. Finally, Section V provides conclusions.

II. SYSTEM MODEL

We consider a downlink MU-MIMO system with N_T transmit antennas and N_R receive antennas. As the system diagram shown in Fig. 1, there are K users, each of which has only one data stream to transmit. Users k and l are scheduled simultaneously, and the corresponding data symbols x_k and x_l are precoded respectively by the selected matrix $\mathbf{W}^{(\mathrm{PMI}_k)}$ and $\mathbf{W}^{(\mathrm{PMI}_l)}$ from the LTE-Advanced dual-codebook. The received signal $\mathbf{y}_k$ at UE k side in the case of MU-MIMO

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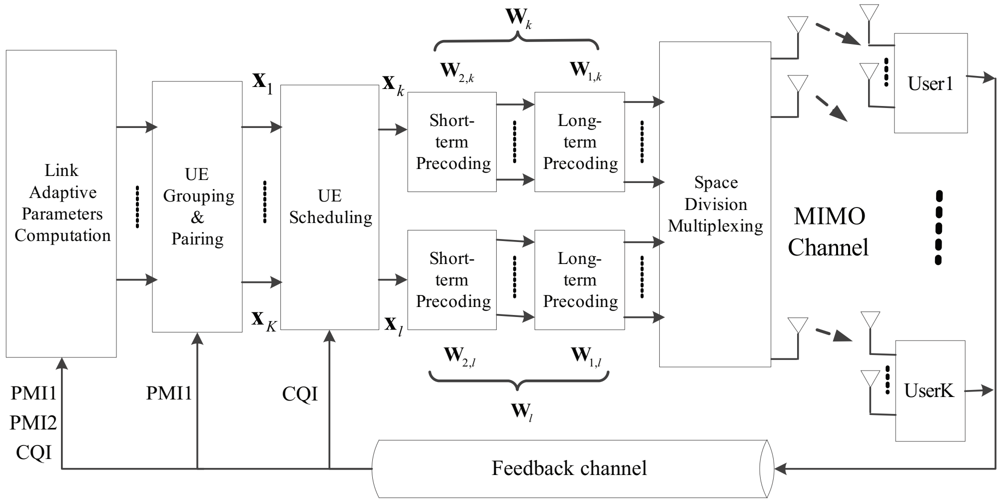

Fig. 1. System diagram for LTE-Advanced downlink.

transmission can be expressed as

$$\mathbf{y}_k = \mathbf{h}_k^H \mathbf{W}^{(\mathrm{PMI}_k)} x_k + \mathbf{h}_k^H \mathbf{W}^{(\mathrm{PMI}_l)} x_l + \mathbf{n}_k,\ l \neq k \quad (1)$$

where $k = 1, \ldots K$, $\mathbf{y}_k \in \mathbb{C}^{N_R \times 1}$ is the received signal vector , $\mathbf{h}_k \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix and $\mathbf{n}_k \sim \mathcal{CN}\left(0,\ \sigma_n^2 \mathbf{I}_{N_R}\right)$ is the complex Gaussian additive noise. $(\mathbf{A})^H$ denotes the conjugate transpose of the matrix $\mathbf{A}$. The precoding matrix $\mathbf{W}$ of user k is given by

$$\mathbf{W}^{(\mathrm{PMI}_k)} = \mathbf{W}_1^{(\mathrm{PMI1}_k)} \mathbf{W}_2^{(\mathrm{PMI2}_k)} \quad (2)$$

where $\mathbf{W}_1^{(\mathrm{PMI1}_k)}$ is the selected block diagonal matrix denoting a cluster composed of over-sampled 4-DFT beams, and $\mathbf{W}_2^{(\mathrm{PMI2}_k)}$ represents the chosen short-term precoding matrix. In this paper, the number for both $\mathbf{W}_1$ and $\mathbf{W}_2$ of rank one is 16, and for each $\mathbf{W}_1$, adjacent overlapping beams are used to reduce edge effect in frequency selective precoding. More details are given in [11].

III. Feedback and Pairing Schemes

In order to maximize signal-to-interference plus noise ratio (SINR) and therefore maximize the cell throughput, additional feedback information of so-called "Best Companion Cluster (BCC)" indicators are provided [10]. In this section, JSDC-based BCC and SSDC-based BCC schemes for downlink MU-MIMO, accompanying corresponding scheduling and pairing schemes at BS, will be described. Using the dual-codebook of 8Tx for rank 1, we will analyze and divide it into several clusters according to its specific structure [12], [13].

A. Jointly Selected Dual-Codebook (JSDC) based BCC

We search over all possible combinations of $\mathbf{W}_1$ and $\mathbf{W}_2$ to find the optimum precoder, and regard the dual-codebook as an equivalent single-codebook. The dual-codebook size is 256, easily calculated by 16×16. Actually, there are only 128 different beams to be selected because each $\mathbf{W}_1$ in the codebook has adjacent overlapping vectors. From [11], we know that $\mathbf{W}_1$ is composed of 32 over-sampled 4-DFT beams. Since $\mathbf{W}_2$ for rank one only consists of beam selecting and co-phasing, we focus on the 32 basic beams that constitute $\mathbf{W}_1$ in this subsection.

1) Scheme Discription: After removing the overlapping beams of $\mathbf{W}_1$, we divide 128 beams of the equivalent single-codebook into 8 clusters, according to its beamforming pattern shown in Fig. 2. In different clusters, UE selects its right beam to minimize the mutual interference. It is noted that just 32 basic beams of interest are given in Fig. 2. To further reduce the feedback overhead, beams can also be grouped into 4 clusters, in which case, 3-bits CI feedback will fall to 2 bits.

Supposing there are 8 clusters stated above and each cluster $C_i (i = 0, 1, \ldots, 7)$ has 16 precoding vectors $\mathbf{W}^{(m)}$ with $m \in \Omega_i = \{16i+1, 16i+2, \ldots, 16i+16\}$, the feedback of BCC scheme [10] can be obtained as $(\mathrm{PMI}_k, \mathrm{CI}_l)$

$$\mathrm{PMI}_k = \arg\max_{i \in [1, \ldots 128]} (\left|\mathbf{h}_k^H \mathbf{W}^{(i)}\right|^2), \mathbf{W}^{(\mathrm{PMI}_k)} \in \mathbf{C}_{\mathrm{CI}_k} \quad (3)$$

$$\mathrm{PMI}_l = \arg\min_{\substack{n \in [1, \ldots 128], \\ n \notin \Omega_k}} (\left|\mathbf{h}_k^H \mathbf{W}^{(n)}\right|^2), \mathbf{W}^{(\mathrm{PMI}_l)} \in \mathbf{C}_{\mathrm{CI}_l} \quad (4)$$

$$\mathbf{W}^{(m)} = \mathbf{W}_1^{(i)} \mathbf{W}_2^{(j)}, \quad i, j \in [0, \ldots 15] \quad (5)$$

where $\mathbf{C}_{\mathrm{CI}_k}$ is the cluster containing the best precoder $\mathbf{W}^{(\mathrm{PMI}_k)}$, selected from the equivalent single-codebook by the rule of maximizing the received signal power, and $\mathbf{C}_{\mathrm{CI}_l}$ is the cluster containing the least interfering $\mathbf{W}^{(\mathrm{PMI}_l)}$. $|\cdot|$ is the magnitude of a scalar.

Owing to the pairing PMI uncertainty, the UE k has to assume the paired user l to have the highest interference in the selected cluster CI_l, and calculate a surely workable CQI

$$\mathrm{CQI}_k = \min_{j \in \Omega_{\mathrm{CI}_l}} \left(\frac{\left|\mathbf{h}_k^H \mathbf{W}^{(\mathrm{PMI}_k)}\right|^2}{\sigma_n^2 + \left|\mathbf{h}_k^H \mathbf{W}^{(j)}\right|^2} \right) \quad (6)$$

2) UE Pairing and Scheduling: Having obtained the best PMIs, interference CIs and CQIs fed back by each UE, BS can easily do the UE pairing and scheduling. The procedure can be described as follows:

Step 1: BS first selects the target user k with the maximum CQI. Here, we assume that the best PMI of user k is PMI_k in the preferred cluster CI_k, and its low interference cluster is CI_l.

Step 2: Users indicating the preferred cluster CI_k and the

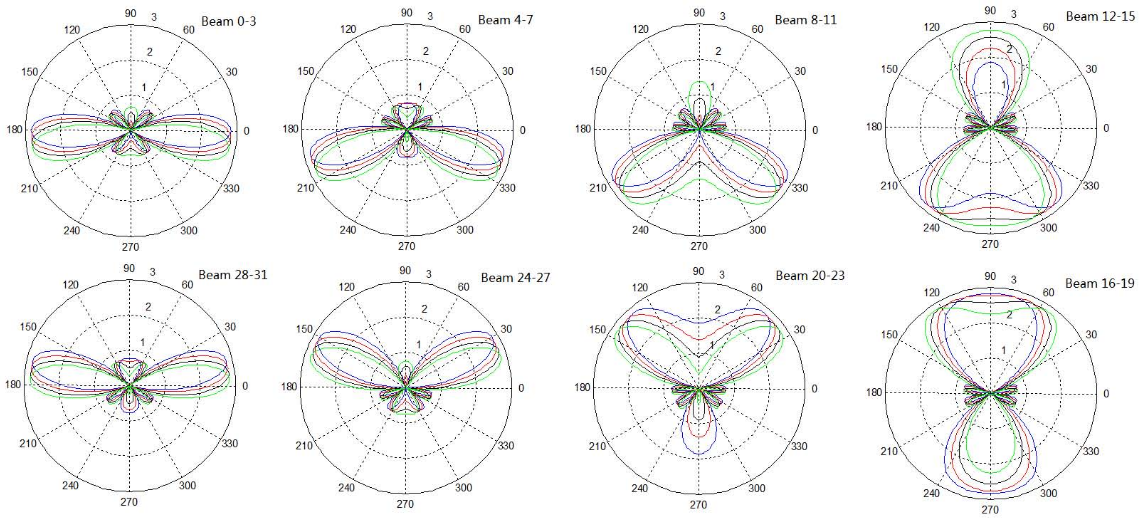

Fig. 2. Beamforming pattern of 8Tx rank1 dual-codebook (8-Clusters).

low interference cluster CI_l are classified into a paired class for the target user k. Then user l with the maximum CQI in this class and the target user k form a UE pair successfully.

Step 3: Finally, BS schedules the UE pair for transmission and allocates corresponding resource.

Similar to 4-bits SC-based BCC [10] for rank 1, JSDC-based BCC only extends the used codebook to a larger size to support 8Tx antennas for downlink MU-MIMO. As there are totally 128 different beams grouped into 8 clusters according to the beamforming pattern, each UE needs 10 bits PMI feedback overhead including 7 bits the best PMI and 3 bits CI.

B. Separately Selected Dual-Codebook (SSDC) based BCC

In the case of JSDC-based BCC stated in Section III-A, high feedback overhead is required and the extensive search is difficult to realize for UE. To explore the specific functionality of the dual-codebook structure, where $\mathbf{W}_1$ and $\mathbf{W}_2$ track long-term and short-term channel properties respectively, we propose SSDC-based BCC scheme to achieve a better tradeoff between overhead and performance.

Since $\mathbf{W}_2$ consists of beam selection vectors only, combined with proper co-phasing terms [13], we focus on $\mathbf{W}_1$, accounting for the principal long-term stable channel direction. To reduce the feedback overhead, PMI1 for $\mathbf{W}_1$ can act as CI effectively for UE pairing which means that CI has become a long-term feedback in this scheme.

1) Scheme Discription: This scheme selects $\mathbf{W}_1$ and $\mathbf{W}_2$ separately to avoid an extensive joint search and too much feedback overhead.

• **Obtain the best and worst PMI1:**

The fundamental idea of this step is to determine $\mathbf{W}_1$ by the principal direction of the long-term stable channel. Assuming that the feedback period of long-term is M times the short-term, transmit covariance matrix accumulated over M short-term periods can be obtained by

$$\mathbf{R}_{TX,k} = \frac{1}{M}\sum_{m=1}^{M}\left(\mathbf{h}_k^{(m)}\right)^H \mathbf{h}_k^{(m)} \tag{7}$$

where $\mathbf{h}_k^{(m)}$ is the channel matrix of user k in the mth short-term period.

Taking into account the block-diagonal structure of $\mathbf{W}_1$ under the cross-polarized (X-Pol) antenna setup, an approximate calculation is done to $\mathbf{R}_{TX,k}$, and its same block matrix on the diagonal is given by

$$\bar{\mathbf{R}}_{TX,k} = \frac{1}{2}\sum_{l=1}^{2}\mathbf{R}_{TX,k}^{(l)} \tag{8}$$

where $\mathbf{R}_{TX,k}^{(l)}$ is the lth block matrix on the diagonal. By eigenvalue decomposition, $\bar{\mathbf{R}}_{TX,k}$ can be represented as

$$\bar{\mathbf{R}}_{TX,k} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H \tag{9}$$

where $\mathbf{\Lambda} = diag\{\lambda_1, \dots \lambda_{N_T/2}\}$, $\lambda_i (i = 1, \dots N_T/2)$ are the eigenvalues in descending order, $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots \mathbf{v}_{N_T/2}]$, and $\mathbf{v}_i (i = 1, \dots N_T/2)$ are the corresponding eigenvectors, indicating different directions of the long-term channel.

As mentioned above, the block matrices in $\mathbf{W}_1$ comprise over-sampled DFT vectors of length four, and each vector captures a direction of the long-term stable channel. To obtain the best PMI1 for user k, we try to find the beam closest to the principal direction of the channel

$$i = \arg\max_{i\in[1,2,\dots 32]} \cos\theta_i \tag{10}$$

$$\cos\theta_i = \frac{\left|\mathbf{v}_1^H \mathbf{b}_i\right|}{\|\mathbf{v}_1\|\,\|\mathbf{b}_i\|} \tag{11}$$

where $\mathbf{b}_i$ represents 32 basic beams and $\mathbf{v}_1$ is the principal direction of the long-term channel. According to the finally obtained beam index, the corresponding PMI1 for user k can be easily determined, and CI is given by $\mathrm{CI}_k = \mathrm{PMI1}_k$, occupying 4 bits as the number for $\mathbf{W}_1$ of rank one is 16.

Similarly, by finding the beam farthest to the principle direction

$$j = \arg \min_{j\in[1,2,\ldots 32]} \cos\theta_j \tag{12}$$

the best companion CI can be easily obtained by $\mathrm{CI}_l = \mathrm{PMI1}_l$, where $\mathrm{PMI1}_l$ is the worst 4-bits PMI1 of user k.

- **Obtain the best PMI2:**

Given the selected PMI1, the best PMI2 for user k can also be obtained from the other codebook, following the rule of maximizing the received signal power

$$\mathrm{PMI2}_k = \arg \max_{i\in[0,\ldots 15]} (\left|\tilde{\mathbf{h}}_k^H \mathbf{W}_2^{(i)}\right|^2) \tag{13}$$

where $\tilde{\mathbf{h}}_k$ is the equivalent channel matrix presented as

$$\tilde{\mathbf{h}}_k^H = \mathbf{h}_k^H \mathbf{W}_1^{(\mathrm{PMI1}_k)} \tag{14}$$

- **Calculate the CQI:**

The corresponding CQI for user k can be given by

$$\mathrm{CQI}_k = \min_{j\neq \mathrm{PMI2}_k} (\frac{\left|\tilde{\mathbf{h}}_k^H \mathbf{W}_2^{(\mathrm{PMI2}_k)}\right|^2}{\sigma_n^2 + \left|\mathbf{h}_k^H \mathbf{W}_1^{(\mathrm{PMI1}_l)} \mathbf{W}_2^{(j)}\right|^2}) \tag{15}$$

2) UE Pairing and Scheduling: When BS gets all feedback information of both long-term and short-term from the whole cell, it will try to pair and schedule users efficiently. The procedure can be described as follows:

Step 1: BS first selects the target user k with the maximum short-term feedback CQI. Here, we assume that the long-term feedback of user k is $(\mathrm{CI}_k, \mathrm{CI}_l)$, where CI_k indicates the preferred cluster, and CI_l is its low interference cluster.

Step 2: Utilizing the same method stated in the above scheme, we can get a paired class. Together with target user k, user l with the maximum CQI in the class can form a UE pair.

Step 3: BS schedules the UE pair obtained by Step 2 for transmission and allocates corresponding resource.

In comparison with JSDC-based BCC, the proposed SSDC-based BCC scheme has low-complexity precoding selection and simple beam grouping methods for 8Tx. Only $8/M$-bits long-term PMI1 and 4-bits short-term PMI2 need to be fed back, which will get smaller as the period M increases. For example, when we set $M = 1000$, the comparison of feedback bits for these two schemes can be seen in Table III.

IV. Numerical Results

In this section, simulations are performed for 4Tx downlink MU-MIMO first to quantify the throughput gains obtained over SC-based BCC by both JSDC-based and SSDC-based BCC, considering total throughput of the selected two users. Then under 8Tx configuration, JSDC-based BCC and SSDC-based BCC are also compared in terms of throughput gain.

To generate highly transmit correlated channels for users, the channel model used for simulation is the spatial channel model (SCM) [16] in an urban macro-cell. The X-Pol antenna deployments with half wave length antenna spacing is considered, and the UE velocity is 3 km/h. The detailed simulation parameters and assumptions are listed in Table I.

TABLE I. Simulation Parameters

Parameters	Setting
Codebook	LTE-A 8Tx/4Tx Rank 1 LTE 4Tx Rank 1
Long-term/Short-term feedback period	1000/1 Time samples
PMI Scheme	Best Companion Cluster
Carrier frequency	2 GHz
UE speed	3 Km/h
Channel model	SCM Urban Macro
Number of UEs	$10 \sim 50$
Antenna configuration	X-Pol 0.5λ $(8\times4/4\times2)$
Signal-to-noise ratio	10 dB
Receiver algorithm	MMSE

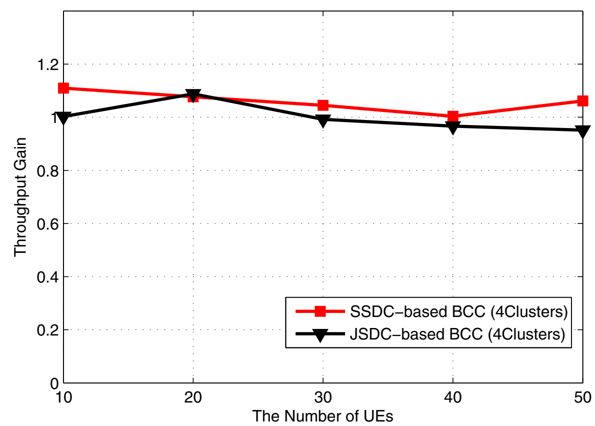

Fig. 3. Throughput gain over SC-based BCC versus the number of UEs for the 4x2 X-Pol setup.

Throughput gain over SC-based BCC versus the number of UEs is illustrated in Fig. 3, considering JSDC-based BCC and SSDC-based BCC scheme, respectively. For 4Tx setup, we adopt LTE Rel. 8 codebook and the dual-codebook stated in [17]. In the case of SSDC-based BCC, nearly up to 11% gains could be obtained when the number of UEs is 10. While in JSDC-based BCC scheme, the gain is changing smoothly in the range of $-5\% \sim 10\%$ with the increase of the UE, indicating the similar performance with SC-based BCC.

A comparison of the feedback requirement and relative throughput gain for different schemes with 4×2 X-Pol setup and 10 UEs is given in Table II. It is noted that the first scheme in Table II requires the least feedback bits, so long as $M \geq 2$. Furthermore, we can see that SSDC-based BCC has the highest success ratio of pairing among these three schemes, and its relative throughput gain over SC-based BCC is 10.97%, even a little higher than that of JSDC-based BCC.

With 8×4 X-Pol antennas setup, similar simulations are provided for the dual-codebook based BCC schemes. Considering both JSDC-based BCC scheme of 8 and 4 clusters, Fig. 4 gives the throughput gain over SSDC-based BCC versus the number of UEs. It is indicated that the variation of throughput gain is quite small and limited to a range of $-4\% \sim 5\%$.

TABLE II. FEEDBACK REQUIREMENT AND RELATIVE THROUGHPUT GAIN COMPARISON OF SCHEMES (4TX,10UES)

Schemes (4Clusters)	No. of Feedback Bits	Relative Gain	Success Ratio of Pairing
SSDC-based BCC	$4/1000+2$	10.97%	100.00%
JSDC-based BCC	$6+2$	0.29%	82.43%
SC-based BCC	$4+2$	_	53.68%

TABLE III. FEEDBACK REQUIREMENT AND RELATIVE THROUGHPUT GAIN COMPARISON OF SCHEMES (8TX,30UES)

Schemes	No. of Feedback Bits	Relative Gain	Success Ratio of Pairing
SSDC-based BCC (16Clusters)	$8/1000+4$	_	100.00%
JSDC-based BCC (8Clusters)	$7+3$	-0.26%	77.90%
JSDC-based BCC (4Clusters)	$7+2$	4.72%	100.00%

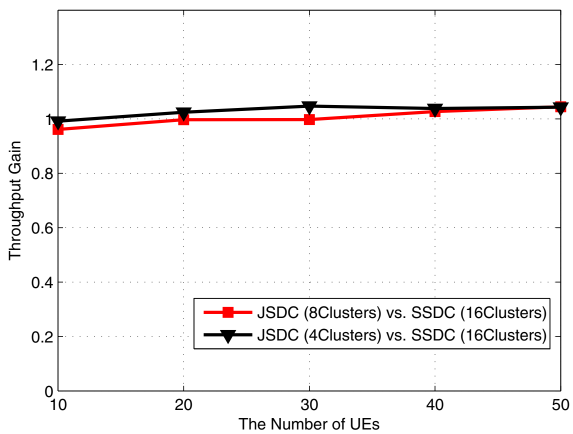

Fig. 4. Throughput gain over SSDC-based BCC versus the number of UEs for the 8x4 X-Pol setup.

Intuitively, the pairing probability for JSDC-based BCC increases with the number of clusters reduced from 8 to 4, since the preferred cluster and the low interference cluster are more likely to be paired in this case. As observed from Table III, the feedback overhead of SSDC-based BCC is much smaller than JSDC-based BCC, approaching 4 bits if the period M is long enough. The proposed SSDC-based BCC scheme provides a reasonable compromise between the feedback overhead and throughput gain.

V. CONCLUSION

We proposed a PMI feedback scheme based on dual-codebook of LTE-A, named SSDC-based BCC for downlink MU-MIMO. In this scheme, each UE first selects its best and worst PMI1s for UE pairing, according to the principal direction of the long-term stable channel. Then the short-term feedback PMI2 and CQI are obtained by BS to determine the final precoder and schedule UEs, respectively. Analytical and simulation results showed that SSDC-based BCC has a comparable throughput performance with JSDC-based BCC but with a substantially reduced complexity and feedback overhead for 8Tx, and that the small feedback overhead also makes it more attractive than others under 4Tx assumption.

VI. ACKNOWLEDGEMENT

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