Weighted MMSE Beamforming Design for Weighted Sum-rate Maximization in Coordinated Multi-Cell MIMO Systems

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***Abstract*—This paper proposes a low-complexity design for the linear weighted MMSE (WMMSE) transmit filters of a coordinated multi-cell system with multiple users per cell. This design is based on a modified WMMSE approach applied to each transmitting base station individually incorporating the signals sent to the cell of interest and the signals leaked to the other cells. This novel design allows for closed-form expressions of the associated Lagrange multipliers and offers a lower computational complexity compared to known methods. In addition, we incorporate this design in an iterative algorithm to find the linear transmit filter maximizing the weighted sum-rate of the multi-cell system. This algorithm is based on WMMSE where the MSE weights are optimally adjusted so that the WMMSE optimum coincides with the WSR optimum.**

I. INTRODUCTION

In conventional cellular systems, each cell operates, to a large extent, independently from the other cells. Cooperation between cells is restricted to frequency planning where neighboring cells are assigned an orthogonal set of frequencies. Because this strategy limits the overall spectral efficiency significantly, some level of frequency reuse among cells is usually allowed, leading to inter-cell interference.

To control inter-cell interference, a new cellular architecture [1] has been proposed where cells are coordinated. This type of coordination targets downlink transmission: the cooperating base stations (BS) are connected via backhaul links to a central processing unit. This unit gathers information from the multiple cells and centralizes multi-cell operations by coordinating the distribution of resources. Designs to improve the performance of the coordinated multi-cell systems is one recent research focus [2]–[4].

In this paper, we assume a multi-cell coordination system where the data is available locally: each BS possesses the data to be sent to multiple users in its own cell. The knowledge of the CSI is global, meaning that all links are known at each BS. The BSs equipped with multiple antennas allowing transmit beamforming techniques, are able to account for and minimize the impact of inter-cell interference.

We propose a low-complexity design of the transmit filters based on a weighted minimum mean squared error (WMMSE) approach. The traditional WMMSE approach includes a step where the Lagrange multiplier corresponding to the transmit power per BS is calculated via solving a polynomial equation [4]. The proposed modified WMMSE approach of the transmit filters avoids the complexity to solve the polynomial equation and results in a lower complexity. Our approach contains two main ingredients. First, the modified WMMSE cost function, named as weighted minimum mean squared error for signal and leakage (WMMSE-SL), considers only the signal and the leakage delivered from one BS: the signals sent to the users in its cell and the signals interfering with users in the other cells. Note that this approach is related to [5], where the signal-to-leakage-and-noise ratio (SLNR) is used as the maximization criterion. However, the SLNR criterion takes simply the suboptimal matched filtering at the receivers. The second ingredient consists in introducing an additional degree of freedom, a scalar, as was done in [6] for the point-to-point MIMO channel. This allows for a scaling of the received signal corresponding to a scaling of the transmit power constraint. This design results in closed-form expressions of the Lagrange multipliers and hence of the transmit filters. Although the proposed method is suboptimal because it is based on a modified version of the WMMSE cost function, simulations shows nearly no performance loss compared to [4].

Furthermore, based on a recent result [7], [8], we use the fact that the mean squared error (MSE) weights can be adjusted so that the WMMSE optimization becomes equivalent to the weighted sum-rate (WSR) optimization. This result requires an iterative process which incorporates WMMSE-SL.

II. SYSTEM MODEL

We consider a multi-cell system (figure 1) with M_C cells. In each cell, one base station with M_T transmit antennas serves M_U users, each with M_R receive antennas. We use superscript c, c' as indexes for the cells (or equivalently the BSs) and subscript u, u' as indexes for the users. The channel of all links is assumed to be perfectly known at the central processing unit.

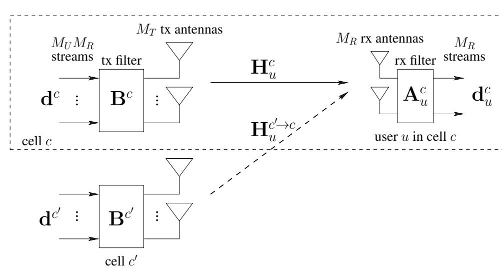

Fig. 1. System model: cell c serving M_U users and interfering cell c'.

1) Intra-cell Signals: Within cell c, the BS proceeds to transmit data simultaneously to the multiple users in the cell using linear beamforming. The data vectors $\mathbf{d}_1^c, \cdots \mathbf{d}_{M_U}^c \in \mathbb{C}^{[d \times 1]}$ are to be delivered to users $1 \cdots M_U$ respectively. The data is pre-processed as $\mathbf{B}_u^c \mathbf{d}_u^c$. $\mathbf{B}_u^c \in \mathbb{C}^{[M_T \times d]}$ is the transmit

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filter for user u. The complex-valued signal $\mathbf{x}^c \in \mathbb{C}^{[M_T\times 1]}$ sent from BS c is the sum of the pre-processed data: $\mathbf{x}^c = \sum_{u=1}^{M_U} \mathbf{B}_u^c \mathbf{d}_u^c = \mathbf{B}^c\mathbf{d}^c$. $\mathbf{B}^c = \left[\mathbf{B}_1^c \cdots \mathbf{B}_{M_U}^c\right] \in \mathbb{C}^{[M_T\times dM_U]}$ groups the transmit filters and $\mathbf{d}^{c\,\mathrm{T}} = \left[\mathbf{d}_1^{c\,\mathrm{T}} \cdots \mathbf{d}_{M_U}^{c\,\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{[dM_U\times 1]}$ groups the data for all users in the cell. Each user receives d independent data streams satisfying $\mathbb{E}\left[\mathbf{d}_u^c \mathbf{d}_u^{c\,\mathrm{H}}\right] = \mathbb{I}$. Notice that d is chosen to fulfill the degrees of freedom requirement in [9]. The transmit vectors $\mathbf{x}^c$ should also fulfill the following power constraint:

$$\mathbb{E}\left[\mathbf{x}^{c\mathrm{H}}\mathbf{x}^c\right] = \sum_{u=1}^{M_U} \mathrm{Tr}\left(\mathbf{B}_u^c \mathbf{B}_u^{c\,\mathrm{H}}\right) \le P_{tx}^c. \quad (1)$$

Within cell c, the MIMO channel between the BS and user u is assumed to be narrowband and is denoted as $\mathbf{H}_u^c \in \mathbb{C}^{[M_R\times M_T]}$. The signal received at user u containing only signals generated within the cell is denoted as the complex vector $\mathbf{y}_u^{c\,\text{intra}} \in \mathbb{C}^{[M_R\times 1]}$: $\mathbf{y}_u^{c\,\text{intra}} = \mathbf{H}_u^c\mathbf{x}^c + \mathbf{n}_u^c = \mathbf{H}_u^c\mathbf{B}^c\mathbf{d}^c + \mathbf{n}_u^c$ where $\mathbf{n}_u^c \in \mathbb{C}^{[M_R\times 1]}$ is a vector containing circularly symmetric Gaussian noise. Without loss of generality, the noise is assumed white with covariance $\mathbf{R}_{\mathbf{n}_u^c} = \mathbb{I}$ (assuming an appropriate whitening transform on the channel matrix).

From the perspective of user u, the signals sent from BS c and intended to the other users within the cell act as intra-cell interference. They are denoted as $\mathbf{I}_u^{c\,\text{intra}}$.

$$\mathbf{y}_u^{c\,\text{intra}} = \mathbf{H}_u^c\mathbf{B}_u^c\mathbf{d}_u^c + \mathbf{I}_u^{c\,\text{intra}} + \mathbf{n}_u^c, \quad \mathbf{I}_u^{c\,\text{intra}} = \mathbf{H}_u^c \sum_{u'=1,u'\neq u}^{M_U} \mathbf{B}_{u'}^c \mathbf{d}_{u'}^c.$$

2) Inter-cell Signals: User u in cell c receives interfering signals from a neighboring cell c'. We denote $\mathbf{H}_u^{c'\to c}$ the narrowband MIMO channel from BS c' to user u located in cell c. The interfering signal coming from cell c' is $\mathbf{y}_u^{c'\to c} = \mathbf{H}_u^{c'\to c}\mathbf{x}^{c'} = \mathbf{H}_u^{c'\to c}\mathbf{B}^{c'}\mathbf{d}^{c'}$.

3) Total Received Signal: The received signal at user u is:

$$\mathbf{y}_u^c = \mathbf{H}_u^c\mathbf{B}_u^c\mathbf{d}_u^c \ + \ \mathbf{I}_u^c, \quad (2)$$

$$\mathbf{I}_u^c = \mathbf{I}_u^{c\,\text{intra}} + \mathbf{I}_u^{c\,\text{inter}} + \mathbf{n}_u^c, \quad \mathbf{I}_u^{c\,\text{inter}} = \sum_{c'=1,c'\neq c}^{M_C} \mathbf{H}_u^{c'\to c}\mathbf{B}^{c'}\mathbf{d}^{c'}. \quad (3)$$

The effective noise covariance matrix at user u is an important quantity accounting for the inter- and intra-cell interference:

$$\mathbf{R}_{\mathbf{I}_u^c} = \sum_{u'\neq u} \mathbf{H}_u^c\mathbf{B}_{u'}^c\mathbf{B}_{u'}^{c\,\mathrm{H}}\mathbf{H}_u^{c\,\mathrm{H}} + \sum_{c'\neq c} \mathbf{H}_u^{c'\to c}\mathbf{B}^{c'}\mathbf{B}^{c'\,\mathrm{H}}\mathbf{H}_u^{c'\to c\,\mathrm{H}} + \mathbb{I}.$$

III. Weighted MMSE for transceiver filters

The WMMSE criterion can be used to jointly design the transceiver filters. Denote $\mathbf{A}_u^c \in \mathbb{C}^{[d\times M_R]}$ a linear filtering at user u belonging to cell c, the WMMSE cost function is:

$$\arg\min_{\mathbf{A}_u^c,\mathbf{B}_u^c} \sum_{c=1}^{M_C}\sum_{u=1}^{M_U} \mathrm{Tr}\left(\mathbf{W}_u^c\mathcal{E}_u^c\right) \ \text{s.t. } \mathrm{Tr}\left(\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right) \le P_{tx}^c \ \ \forall c \quad (4)$$

$$\mathcal{E}_u^c = \mathbb{E}\left(\mathbf{A}_u^c\mathbf{y}_u^c - \mathbf{d}_u^c\right)\left(\mathbf{A}_u^c\mathbf{y}_u^c - \mathbf{d}_u^c\right)^{\mathrm{H}} = \mathbb{E}\left(\mathbf{A}_u^c\mathbf{H}_u^c\mathbf{B}_u^c\mathbf{d}_u^c + \mathbf{A}_u^c\mathbf{I}_u^c - \mathbf{d}_u^c\right)\left(\mathbf{A}_u^c\mathbf{H}_u^c\mathbf{B}_u^c\mathbf{d}_u^c + \mathbf{A}_u^c\mathbf{I}_u^c - \mathbf{d}_u^c\right)^{\mathrm{H}}.$$

One advantage of the weighted MMSE approach is as follows: when the set of transmit filters are fixed, the optimization problem to find the set of received filters becomes quadratic, likewise when $\{\mathbf{A}_u^c\}$ is fixed and the cost function is optimized w.r.t. $\{\mathbf{B}_u^c\}$. This leads to a low complexity alternating minimization procedure.

A. Alternating Minimization

When setting the gradient of the cost function w.r.t. $\{\mathbf{A}_u^c\}$ to zero, we can find the expression of the MMSE receive filter for user u within cell c as:

$$\mathbf{A}_{u,\text{MMSE}}^c = \mathbf{B}_u^{c\,\mathrm{H}}\mathbf{H}_u^{c\,\mathrm{H}}\left(\mathbf{H}_u^c\mathbf{B}_u^c\mathbf{B}_u^{c\,\mathrm{H}}\mathbf{H}_u^{c\,\mathrm{H}} + \mathbf{R}_{\mathbf{I}_u^c}\right)^{-1}. \quad (5)$$

The error covariance $\mathcal{E}_u^c$ after using MMSE receivers is:

$$\mathbf{E}_u^c = \left[\mathbb{I} + \mathbf{B}_u^{c\,\mathrm{H}}\mathbf{H}_u^{c\,\mathrm{H}}\mathbf{R}_{\mathbf{I}_u^c}^{-1}\mathbf{H}_u^c\mathbf{B}_u^c\right]^{-1}. \quad (6)$$

When the receive filters are fixed, only the transmit filters are optimization parameters. Hence, we get a new criterion, WMMSE-B, for the optimization of the transmit filters:

$$\arg\min_{\mathbf{B}_u^c} \sum_{c=1}^{M_C}\sum_{u=1}^{M_U} \mathrm{Tr}\left(\mathbf{W}_u^c\mathbf{E}_u^c\right) \ \text{s.t. } \mathrm{Tr}\left(\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right) \le P_{tx}^c \ \forall c. \quad (7)$$

IV. Computation of WMMSE transmit filters

Different techniques exist to determine the WMMSE transmit filter, e.g. [4]. We first describe the reference method in [4] and then our proposed method.

A. Transmit Filter Design for WMMSE

We extend the transmit filter design for the MIMO interference channel in [4] to the multi-cell scenario first. When the MSE weights are given and not part of the optimization, the WMMSE-B cost function can be written in a more compact way, as the sum of the weighted errors per cell $\sum_{c=1}^{M_C}\mathrm{Tr}\left(\mathbf{W}^c\mathbf{E}^c\right)$ where $\mathbf{E}^c = \left[\mathbf{E}_1^{c\,\mathrm{T}} \cdots \mathbf{E}_{M_U}^{c\,\mathrm{T}}\right]^{\mathrm{T}}$ groups the error vectors from all users in the same cell. $\mathbf{W}^c = \mathrm{Diag}\left(\mathbf{W}_1^c, \ldots, \mathbf{W}_{M_U}^c\right)$ is a block diagonal matrix with blocks equal to the weight matrix corresponding to each individual user. Defining $\mathbf{y}^c = \left[\mathbf{y}_1^{c\,\mathrm{T}} \cdots \mathbf{y}_{M_U}^{c\,\mathrm{T}}\right]^{\mathrm{T}}$ and $\mathbf{n}^c = \left[\mathbf{n}_1^{c\,\mathrm{T}} \cdots \mathbf{n}_{M_U}^{c\,\mathrm{T}}\right]^{\mathrm{T}}$, we have $\mathbf{y}^c = \mathbf{H}^c\mathbf{B}^c\mathbf{d}^c + \mathbf{I}^{c\,\text{inter}} + \mathbf{n}^c$ and $\mathbf{I}^{c\,\text{inter}} = \sum_{c'=1,c'\neq c}^{M_U} \mathbf{H}^{c'\to c}\mathbf{B}^{c'}\mathbf{d}^{c'}$ where $\mathbf{I}^{c\,\text{inter}}$ groups the interference from neighboring cells to all users in cell c. $\mathbf{A}^c = \mathrm{Diag}\left(\mathbf{A}_1^c, \ldots, \mathbf{A}_{M_U}^c\right)$ is a block diagonal matrix and $\mathbf{H}^c = \left[\mathbf{H}_1^{c\,\mathrm{T}} \ \ldots \ \mathbf{H}_{M_U}^{c\,\mathrm{T}}\right]^{\mathrm{T}}$ represents the equivalent MIMO channel matrix from BS c to all users in cell c. From the Karush-Kuhn-Tucker (KKT) conditions, the transmit filter matrix has the following expression:

$$\mathbf{B}^c = \left(\mathbf{H}^{c\mathrm{H}}\mathbf{A}^{c\mathrm{H}}\mathbf{W}^c\mathbf{A}^c\mathbf{H}^c + \mathbf{R}_{\mathbf{I}^c}^{\text{inter}} + \lambda^c\mathbb{I}\right)^{-1}\mathbf{H}^{c\,\mathrm{H}}\mathbf{A}^{c\,\mathrm{H}}\mathbf{W}^c \quad (8)$$

$$\mathbf{R}_{\mathbf{I}^c}^{\text{inter}} = \sum_{c'=1,c'\neq c}^{M_C} \mathbf{H}^{c\to c'\,\mathrm{H}}\mathbf{A}^{c'\,\mathrm{H}}\mathbf{W}^{c'}\mathbf{A}^{c'}\mathbf{H}^{c\to c'}$$

where the Lagrange multiplier λ^c from the KKT conditions can be computed by setting $\mathrm{Tr}\left(\mathbf{B}^c\mathbf{B}^{c\,\mathrm{H}}\right) = P_{tx}^c$, which results in a polynomial equation of degree $2M_T$. When no $\lambda^c \ge 0$ can be found, λ^c is set to zero in [4].

B. WMMSE for Signal and Leakage (WMMSE-SL)

The motivation to look for an alternative solution to (8) is to have closed-form expressions of the Lagrange multipliers and eliminate the complexity to solve the polynomial equation for the Lagrange multiplier. For the MIMO broadcast channel, an answer to this issue was provided in [6]. Unfortunately, this approach cannot be directly transposed mainly because of the inter-cell interference. For the MIMO interference channel, we propose a design in [10]. Here we extend the design to the coordinated multi-cell system with multiple users per cell.

1) Rewriting the WMMSE cost function: Traditionally, the WMMSE approach considers the squared error at each user and takes the sum of the errors over all the users to get the final cost function. Now, we take another view point.

Consider a given BS indexed by c. The channel from BS c to all users in the multi-cell system can be seen as a broadcast channel where some users receive desired data and others receive interference leakage. We distinguish between two types of flows transmitted from the BS: 1) *Intended Signal*: the desired signal to the users in this cell is $\mathbf{d}^c$ and the corresponding observation vector seen by the users is $\mathbf{S}^c = \mathbf{H}^c\mathbf{B}^c\mathbf{d}^c + \mathbf{n}^c$; 2) *Leakage Signal to cell c'*: the leakage observation vector seen by all the users in cell c' is $\mathbf{L}^{c\to c'} = \mathbf{H}^{c\to c'}\mathbf{B}^c\mathbf{d}^c$. We define the corresponding modified MMSE cost function $\mathbf{E}^c_{SL}$ as:

$$\mathbf{E}^c_{SL} = \mathbb{E}\left((\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c)(\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c)^{\mathrm{H}}\right) + \sum_{c'=1,c'\neq c}^{M_C} \mathbb{E}\left(\mathbf{A}^{c'}\mathbf{L}^{c\to c'}\mathbf{L}^{c\to c'\,\mathrm{H}}\mathbf{A}^{c'\,\mathrm{H}}\right). \quad (9)$$

The modified WMSE cost function for each BS is

$$\mathbf{WMSE}^c_{SL} = \mathrm{Tr}\left(\mathbf{W}^c\mathbb{E}\left((\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c)(\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c)^{\mathrm{H}}\right)\right) + \mathrm{Tr}\left(\sum_{c'=1,c'\neq c}^{M_C}\mathbf{W}^{c'}\mathbb{E}\left(\mathbf{A}^{c'}\mathbf{L}^{c\to c'}\mathbf{L}^{c\to c'\,\mathrm{H}}\mathbf{A}^{c'\,\mathrm{H}}\right)\right).$$

2) Modified WMMSE cost function per BS: The technique in [6], [10] can be applied: an additional weighting β^c is introduced into the WMMSE-SL cost function in Eq. (9). The modified cost function for BS c with β^c is reformulated as $\mathcal{E}^c_{SL,\beta} = \mathbb{E}\left((\beta^{c\,-1}\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c)(\beta^{c\,-1}\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c)^{\mathrm{H}}\right) + \beta^{c\,-2}\sum_{c'=1,c'\neq c}^{M_C}\mathbb{E}\left(\mathbf{A}^{c'}\mathbf{L}^{c\to c'}\mathbf{L}^{c\to c'\,\mathrm{H}}\mathbf{A}^{c'\,\mathrm{H}}\right)$. The Lagrangian formulation of the modified WMMSE problem at BS c is:

$$\begin{aligned}&\mathbf{WMSE}^c_{SL,\beta} + \lambda^c\left(\mathrm{Tr}\left(\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right) - P^c_{tx}\right) = \\ &\mathrm{Tr}\left(\mathbf{W}^c\mathbb{E}\left(\left(\beta^{c\,-1}\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c\right)\left(\beta^{c\,-1}\mathbf{A}^c\mathbf{S}^c - \mathbf{d}^c\right)^{\mathrm{H}}\right)\right) \\ &+ \beta^{c\,-2}\mathrm{Tr}\left(\sum_{c'=1,c'\neq c}^{M_C}\mathbf{W}^{c'}\mathbb{E}\left(\mathbf{A}^{c'}\mathbf{L}^{c\to c'}\mathbf{L}^{c\to c'\,\mathrm{H}}\mathbf{A}^{c'\,\mathrm{H}}\right)\right) \\ &+ \lambda^c\left(\mathrm{Tr}\left(\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right) - P^c_{tx}\right).\end{aligned}$$

From the KKT conditions, the solution is:

$$\begin{aligned}\mathbf{B}^c_{SL} &= \beta^c\left(\mathbf{H}^{c\mathrm{H}}\mathbf{A}^{c\mathrm{H}}\mathbf{W}^c\mathbf{A}^c\mathbf{H}^c + \mathbf{R}^{\text{inter}}_{\mathbf{I}^c} + \alpha^c\mathbb{I}\right)^{-1}\mathbf{H}^{c\,\mathrm{H}}\mathbf{A}^{c\,\mathrm{H}}\mathbf{W}^c \\ \alpha^c &= \mathrm{Tr}\left(\mathbf{W}^c\mathbf{A}^c\mathbf{A}^{c\,\mathrm{H}}\right)/P^c_{tx}.\end{aligned} \quad (10)$$

The factor β^c is used to set the transmit power to P^c_{tx} and $\alpha^c = \lambda^c(\beta^c)^2$. Therefore, the Lagrange multiplier is expressed in closed-form. Using (10), all BSs transmit at full power. However, this is not necessarily the optimal strategy in the multi-cell coordination system with multiple antennas per user. As highlighted in section VII, this solution gives approximately the same performance as (8).

V. Relationship between WSR and WMMSE

In this section we will demonstrate that WMMSE is instrumental in optimizing WSR. We start with the description of the WSR criterion.

A. Weighted Sum-rate

The main objective is to find the transmit filters $\mathbf{B}^1 \ldots \mathbf{B}^{M_C}$ maximizing the sum of weighted achievable rates over all users in all cells. Assuming Gaussian signalling, the signal of interest and interference are Gaussian. As the interference is not white, the optimal processing consists in first whitening the interference, i.e. multiplying the received signal $\mathbf{y}^c_u$ by $\mathbf{R}^{-1/2}_{\mathbf{I}^c_u}$ (where $\mathbf{R}^{1/2}_{\mathbf{I}^c_u}$ is the Cholesky factor of $\mathbf{R}_{\mathbf{I}^c_u}$). Hence, the achievable rate for user u is:

$$R^c_u = \log_2\det\left(\mathbb{I} + \mathbf{B}^{c\,\mathrm{H}}_u\mathbf{H}^{c\,\mathrm{H}}_u\mathbf{R}^{-1}_{\mathbf{I}^c_u}\mathbf{H}^c_u\mathbf{B}^c_u\right). \quad (11)$$

WSR can be written as the minimization problem:

$$\arg\min_{\mathbf{B}^c_u}\sum_{c=1}^{M_C}\sum_{u=1}^{M_U} -\mu_{R^c_u}R^c_u \quad \text{s.t.} \quad \mathrm{Tr}\left(\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right) \leq P^c_{tx} \quad \forall c,$$

The weights $\mu_{R_k} \geq 0$ allow for a flexible quality of service among users. Note that, when transmission is performed from a single point, it is optimal to transmit at maximal power. However, for multi-point transmission, inter-cell interference control might dictate a reduction of the transmission power.

We summarize the main result about the relationship between WSR and WMMSE-B in the following lemma.

Lemma 1: The gradients of WSR and WMMSE-B are equal when the MSE weights are selected as follows:

$$\mathbf{W}^c_u = \mu^c_u(\mathbf{E}^c_u)^{-1}. \quad (12)$$

When the MSE weights have the expression (12), WSR and WMMSE-B share the same global and local optimum points.

Proof: The cost functions corresponding to the Lagrangian formulation for WSR and WMMSE-B are denoted (for short) as f and g and are as follows:

$$f = \sum_{c=1}^{M_C}\sum_{u=1}^{M_U} -\mu^c_u\ln\det(\mathbf{E}^c_u)^{-1} + \sum_{c=1}^{M_C}\lambda^c\left(\mathrm{Tr}\left[\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right] - P^c_{tx}\right) \quad (13)$$

$$g = \sum_{c=1}^{M_C}\sum_{u=1}^{M_U}\mathrm{Tr}\left(\mathbf{W}^c_u\mathbf{E}^c_u\right) + \sum_{c=1}^{M_C}\lambda^c\left(\mathrm{Tr}\left[\mathbf{B}^c\mathbf{B}^{c\mathrm{H}}\right] - P^c_{tx}\right) \quad (14)$$

$\ln(\cdot)$ is used in (13) instead of $\log_2(\cdot)$ here as it does not change the optimization. Let $\nabla_{[B_k^j]_{mn}}$ be the complex gradient operator w.r.t. element (m,n) of matrix B_k^j. From the KKT conditions, a local optimum must satisfy for all m,n,k,j: $\nabla_{[B_k^j]_{mn}} f = \mathbf{0}$, $\nabla_\lambda f = 0$ and likewise for g. Noting that $\nabla \ln(\det(\mathbf{X})) = \mathrm{Tr}\left(\mathbf{X}^{-1}\nabla\mathbf{X}\right)$, we have:

$$\nabla_{[B_k^j]_{mn}} f = \sum_{c=1}^{M_C}\sum_{u=1}^{M_U} -\mathrm{Tr}\left(\mu_u^c \mathbf{E}_u^c \left[\nabla_{[B_k^j]_{mn}} (\mathbf{E}_u^c)^{-1}\right]\right) + \lambda^j \mathcal{P}_k^j \tag{15}$$

$$\nabla_{[B_k^j]_{mn}} g = \sum_{c=1}^{M_C}\sum_{u=1}^{M_U} -\mathrm{Tr}\left(\mathbf{E}_u^c \mathbf{W}_u^c \mathbf{E}_u^c \left[\nabla_{[B_k^j]_{mn}} (\mathbf{E}_u^c)^{-1}\right]\right) + \lambda^j \mathcal{P}_k^j \tag{16}$$

$\mathcal{P}_k^j$ is the gradient of the power constraint term and is the same for both cost functions. Comparing (15) and (16), the result shown in *Lemma 1* has been proved. ■

VI. Iterative Adaptive Weighted Algorithms

Based on the relationship between WSR and WMMSE-B, we form two iterative algorithms for WSR maximization in the multi-cell coordination scenario with local data. Using efficiently the relationship between WSR and WMMSE-B is not straightforward [7]. We briefly summarize the main points as they are important to understand how the following algorithm functions.

First, the optimal MSE weights and the optimal transmit filters are dependent on each other. This naturally points towards the use of an iterative algorithm, alternating between WMMSE-B optimization of the transmit filters and the MSE weights update based on (12). When this algorithm converges, it converges to a fixed point (also a stationary point of WSR).

Second, direct WSR maximization w.r.t. the transmit filters of WMMSE-B would require a high complexity solution as the WSR cost function is non-convex. A simplification is to go back to the original MMSE cost function (4). For a fixed set of weight matrices, an alternating minimization w.r.t. the received filters in (5) and the transmit filters (detailed in section IV) insures convergence to a local optimum. The resulting transmit filters can be used to update the weight matrices. However, this method requires an inner loop to perform WMMSE optimization over the transmit filters for fixed MSE weights. The following proposal requires less iterations and guarantees convergence to a WSR local optimum.

At last, the weight matrices can be updated at different stages of the iterative process resulting in a different behavior of the algorithm. The proposed order of the optimization sequence was proven in [7], [8] to lead to a monotonic decrease of a lower bounded auxiliary function. Hence, convergence to a local optimum is guaranteed.

The adaptive weighted MMSE (AW-MMSE) algorithm is:

Algorithm: **AW-MMSE**
set $n = 0$
set $\mathbf{B}_u^{c\,(n)} = \mathbf{B}_u^{c\,(\text{init})}$ $\forall u, \forall c$
iterate
  update $n = n + 1$
  I. *compute* $\mathbf{A}_u^{c\,(n)} | \mathbf{B}_u^{c\,(n-1)}$ $\forall u, c$ using (5)
  II. *compute* $\mathbf{W}_u^{c\,(n)} | \mathbf{B}_u^{c\,(n-1)}$ $\forall u, c$ using (12) and (6)
  III. *compute* $\mathbf{B}^{c(n)} | \mathbf{A}^{c(n)}, \mathbf{W}^{c(n)}$ $\forall c$ using (8)
until WSR convergence

The AW-MMSE algorithm based on [8] is used as a benchmark to evaluate the following proposal. If we substitute (8) with (10) in Step III of the AW-MMSE algorithm, the new algorithm is denoted as AW-MMSE for signal and leakage (AW-MMSE-SL). This proposed AW-MMSE-SL algorithm incorporates the WMMSE-SL transmit filter design and thus requires less computational complexity compared to AW-MMSE as it eliminates the complexity to solve the polynomial equation for each Lagrange multiplier.

The convergence behavior of the heuristic AW-MMSE-SL algorithm is not straightforward to determine as the optimization of the transmit filters comes from WMMSE-SL and the receive filters comes from WMMSE. Therefore, the set of the transmit filters and the set of the receive filters come from two different cost functions. The WMMSE-SL optimization of the transmit filters is valid in a broadcast scenario when considering the weighted MSE minimization of the error on the signal of interest and the interference leakage. However, the WMMSE-SL optimization of the receive filters is invalid. At one user, signals from multiple transmitters correspond to multiple scaling factors β^c $(1 \leq \beta^c \leq M_c)$. But the receiver for this user lacks of a sufficient number of degrees of freedom to compensate for all of them. In order to minimize the weighted MSE, the MMSE receivers are applied for all the users. The corresponding convergence of the AW-MMSE-SL algorithm will be verified by the simulation results.

A. Computational Complexity Analysis

The focus lies in analyzing the complexity brought by matrix multiplication, matrix inversion and determinant operation. For simplicity, we focus our analysis on the MIMO interference channel with one user per cell $M_U = 1$. Then, the complexity comparison between the AW-MMSE algorithm and the AW-MMSE-SL algorithm is basically an extension of the complexity analysis in [10] to include the calculations for all the weights $\mathbf{W}_u^c$ $\forall u, c$. In both AW-MMSE and AW-MMSE-SL, updating $\mathbf{A}_u^c$ and $\mathbf{W}_u^c$ require the same computational complexity. In AW-MMSE, for updating each $\mathbf{B}^c$, the additional polynomial equation related complexity brought by the calculation of λ^c is $O(8M_T^3)$ and obviously non-negligible. This complexity increases dramatically with the increase of the number of transmit antenna M_T. Therefore, there is a clear reduction in complexity from using AW-MMSE-SL especially when M_T is large.

VII. Numerical Evaluations

All simulations are conducted in the 3-user MIMO interference channel (the degenerate scenario with one user per cell) with each BS and user having the same number of antennas $M_T = M_R = 4$. The maximal transmit power P is the same for all BSs. And the number of data streams delivered by each BS is the same $d = 2$. The elements of the channel matrices are generated as i.i.d. complex Gaussian random variables. The average energy of the channel between a transmitter and its desired user is $\sigma_h^{2\,\text{intra}}$; the average energy of the cross links is $\sigma_h^{2\,\text{inter}}$ with the average inter-cell SNR as $\rho^{\text{inter}} = P\sigma_h^{2\,\text{inter}}$ and $\rho^{\text{gap}} = \rho^{\text{intra}}/\rho^{\text{inter}}$.

We test two initialization values for the transmit filters: 1) Random initialization: initialize with i.i.d. Gaussian random variables; 2) Singular initialization: initialize the transmit filter for the BS in cell c with the first d column of the right singular matrix of $\mathbf{H}_1^c$. The transmit filters are then normalized to fulfill the individual power constraints.

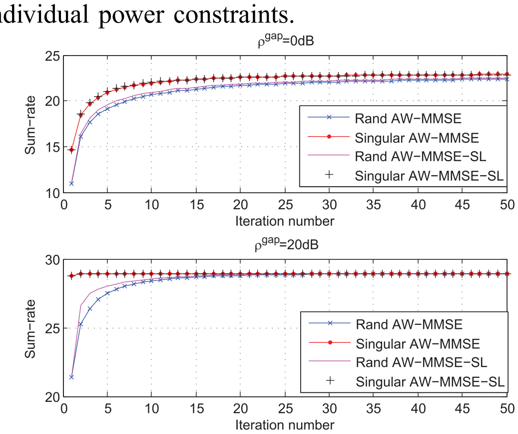

Fig. 2. Sum-rate vs iteration number with $\rho^{\text{intra}} = 10$dB.

In figure 2, we show an example of the convergence behavior for AW-MMSE and AW-MMSE-SL where the sum-rate is plotted against the iteration number. The sum-rate is averaged over many channel realizations. The intra-cell SNR is fixed to $\rho^{\text{intra}} = 10$dB. We compare the two initialization methods. The plots display that the convergence speed is comparable for the AW-MMSE and the AW-MMSE-SL algorithms: 20 iterations appear to be sufficient. Furthermore, convergence becomes slower as ρ^{gap} decreases; singular initialization gives better convergence compared to random initialization. In the following evaluations, we initialize both algorithms with singular initialization and stop at 20 iterations.

In figure 3, we show the sum-rate performance of the different algorithms. The iterative MMSE algorithm in [4] is also included as a benchmark. When $\rho^{gap} = 0$dB, it performs worse compared to the AW-MMSE and AW-MMSE-SL algorithms; when $\rho^{gap} = 20$dB, the AW-MMSE and AW-MMSE-SL algorithms are slightly better than the iterative MMSE algorithm. We can also see that AW-MMSE-SL performs approximately the same as AW-MMSE for both $\rho^{gap} = 0$dB and $\rho^{gap} = 20$dB. In view of those results, we can conclude that the low-complexity AW-MMSE-SL algorithm has no sum-rate performance loss compared to the AW-MMSE algorithm.

VIII. Conclusion

We have proposed a low-complexity design for the linear transmit filters in the coordinated multi-cell system with

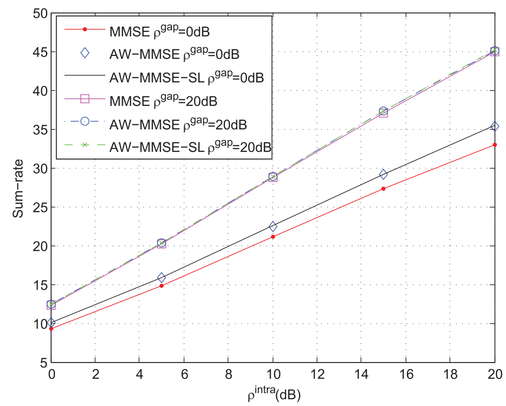

Fig. 3. Sum-rate performance.

multiple user in each cell. WMMSE-SL relies on a modified MMSE cost function which considers the desired signal and the leakage signal from each BS. The novel design allows for closed-form expressions of the associated Lagrange multipliers and offers a comparatively low computational complexity. Furthermore, the proposed AW-MMSE-SL algorithm incorporating the WMMSE-SL transmit filter design maximizes the WSR of the coordinated system. This iterative algorithm is based on the fact that the MSE weights can be adjusted so that the WMMSE optimization becomes equivalent to the optimization of WSR. AW-MMSE-SL guarantees convergence with a few iterations while ensuring no performance loss compared to the more complicated AW-MMSE algorithm.

Acknowledgment

This work is supported by the Danish Research Council for Technology and Production, grant nr. 09-065920.

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