A Simplified LLR-Based Detector for Signals in Class-A Noise

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***Abstract*—The design of a simplified detector for signal in Middleton's class-A noise is considered. The optimal detector is impractical due to the complexity of the probability density function of the noise. The conventional Gaussian detector (known as the matched filter or the correlator) has near-optimal performance only with relatively high SNR values. Different suboptimal detectors have been proposed to give robust performance with different levels of complexity such as the locally optimal Bayesian detector. In this paper, we propose a unified simple approach to design a near-optimal detector with considerably low complexity by linearly approximating the optimal log-likelihood ratios of the received symbols. The resultant detector has near-optimal performance with low complexity.**

Keywords-Non-Gaussian noise, Detectors, Class A noise

I. Introduction

Middleton's Class A noise model [1], [2] is one of the statistical distributions that is used to model the man-made interference and the impulsive noise in different systems such as cognitive radio networks [3] and power line communication systems [4]. The optimal maximum likelihood (ML) signal detector has a complex structure due to the highly non-linear operations it required to calculate the optimal test statistics [5]. The conventional Gaussian detector has been used as a simple suboptimal detector [5]. It gives near optimal performance with relatively higher values of the SNR, however, the performance deteriorates for moderate and low SNR values. The locally optimal Baysian detector has been proposed in [5] which uses the locally optimal criterion assuming weak signal reception [6]. The locally optimal detector depends on the Taylor expansion of the optimal test statistics and neglects the higher terms by using the assumption of weak signals. This detector gives better performance than the Gaussian one, however, it adds more complexity and it is still limited by the weak signal assumption which limits its performance at higher values of SNR. Other approaches use simplified forms of the probability density function (pdf) of the noise such as the use of a truncated series of the pdf as in [7].

In this paper, we propose a simple approach to design low-complexity detector for signals in class-A noise by linearly approximating the optimal log likelihood ratios (LLRs) of the received symbol. Instead of simplifying the pdf of the noise and use the approximated one to calculate the test statistics, we directly approximate the optimal LLRs with a first-order linear approximation. The proposed detector gives robust near-optimal performance with low-complexity which approaches the complexity of the Gaussian detector.

This paper is organized as follows. In Section II, the system model is described and the problem statement is clarified. In Section III, we propose the simplified linear LLR and evaluate the performance with different noise parameters by simulation.

II. System Model and Problem Statement

A BPSK symbol $s \in \{\pm\sqrt{E}\}$ is transmitted, and N independent samples per symbol are collected at the receiver. The received samples are modeled as

$$r_k = s + n_k \qquad k = 1, \ldots, N \tag{1}$$

where $\{n_k\}$ are N independent Middleton's Class A noise samples. The probability density function of the Middleton's Class A noise model is given by

$$f_{A,\Gamma}(n) = \sum_{m=0}^{\infty} \frac{e^{-A}A^m}{m!} \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{n^2}{2\sigma_m^2}\right). \tag{2}$$

where

$$\sigma_m^2 = \sigma^2 \frac{\frac{m}{A} + \Gamma}{1+\Gamma}, \tag{3}$$

and A is the impulsive index. When A is large, the impulsive noise will be continuous which leads the Class A noise to become more likely to be Gaussian. $\Gamma = \sigma_G^2/\sigma_I^2$ is the Gaussian to interference noise power ratio with the Gaussian noise power σ_G^2 and the interference noise power σ_I^2 and σ^2 is the total noise power.

The optimal maximum likelihood (ML) detector has been analyzed in [5] by using the following test statistics for a received symbol:

$$\lambda_{ML} = \log\left\{\frac{\prod_{k=1}^{N} f_{A,\Gamma}(r_k - \sqrt{E})}{\prod_{k=1}^{N} f_{A,\Gamma}(r_k + \sqrt{E})}\right\} \underset{s_0}{\overset{s_1}{\gtrless}} 0 \tag{4}$$

where s_0 is the hypothesis that $s = -\sqrt{E}$ was transmitted, and s_1 is the hypothesis that $s = +\sqrt{E}$ was transmitted. The ML detector is impractical because it requires complex computations. As a simple suboptimal solution, the Gaussian detector, which uses the conventional matched filter or the correlator, has been proposed in [5] with the following test statistics

$$\lambda_{Gauss} = \sum_{k=1}^{N} r_k. \tag{5}$$

It was shown that the Gaussian detector gives near optimal performance at a relatively high SNR values and lower N with low and moderate impulsive noise, otherwise, it gives poor performance compared to the optimal detector.

To improve the performance, another suboptimal detector has been proposed in [5] which depends on the locally optimal criteria for weak signal reception [6]. The locally optimal detector uses the following test statistics

$$\lambda_{LOD} = \sum_{k=1}^{N} (s_1 - s_o) \frac{d}{dr_k} \log f_{A,\Gamma}(r_k) \underset{s_0}{\overset{s_1}{\gtrless}} 0. \tag{6}$$

The locally optimal detector gives better performance than the Gaussian detector at lower values of SNR, however, it adds more complexity, and it is still limited by the weak signal assumption.

In this paper, we propose a different approach to design low complexity detectors with near optimal performance. Instead of simplifying the pdf of the noise, we propose to simplify the optimal log likelihood ratios (LLRs) in (4) directly. In the proposed approach, the optimal LLR is divided into three different regions where linear approximation can be used effectively which results in near optimal performance with different values of the noise parameters at low complexity approaching that of the Gaussian detector.

III. A Simplified Suboptimal LLR

In this section, we will analyze the optimal LLR graphically to understand how each term in the noise pdf in (2) contributes to the optimal LLR. There are different simplifications for the pdf of class A noise, however, they result in a nonlinear detector. In this section, we will propose a simplified approximation of the LLR of the Class A noise which can be used to design a suboptimal detector with linearly approximated test statistic.

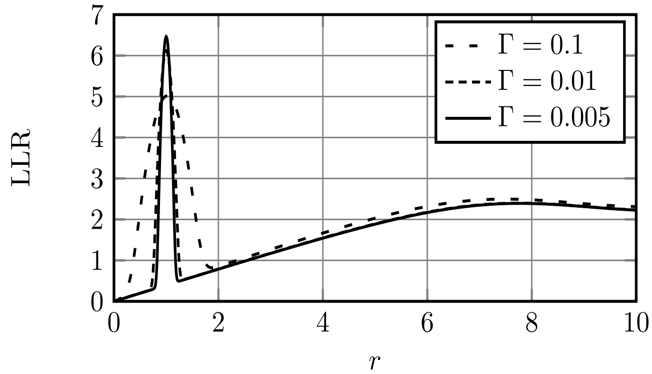
Fig. 1 shows the optimal LLR for different values of A and Γ at 0 dB, which is given by

$$LLR_{Opt} = \log f_{A,\Gamma}(r - s_1) - \log f_{A,\Gamma}(r - s_o) \tag{7}$$

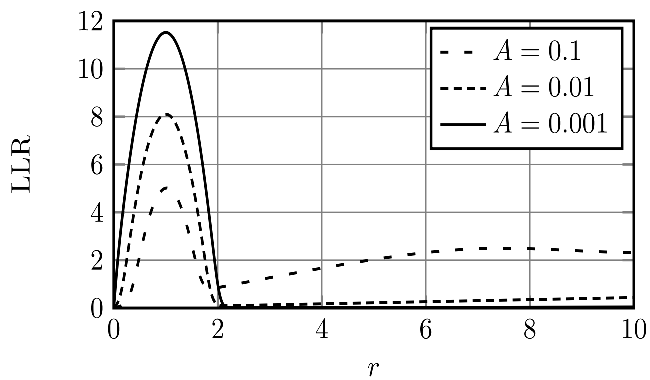
where s_1 and s_o are the possible transmitted BPSK symbols. We can see that the LLR can be divided into three regions, a linear region, a parabolic region and a saturated flat region. We propose to use a piecewise linear approximation of the optimal LLR (PWL-LLR) as shown in Fig. 3. To completely define the PWL-LLR, we need to calculate the boundaries a, b and c.

Due to the anti-symmetry of the optimal LLR, we will consider only the right-hand side ($r \geq 0$). Let us define $g_m(x)$ as the m^{th} term in (2),

$$g_m(x) = \frac{e^{-A} A^m}{m!} \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \tag{8}$$



(a) LLRs with $A = 0.1$ and different Γ



(b) LLRs with $\Gamma = 0.1$ and different A

Fig. 1. The optimal LLR with SNR = 0 dB

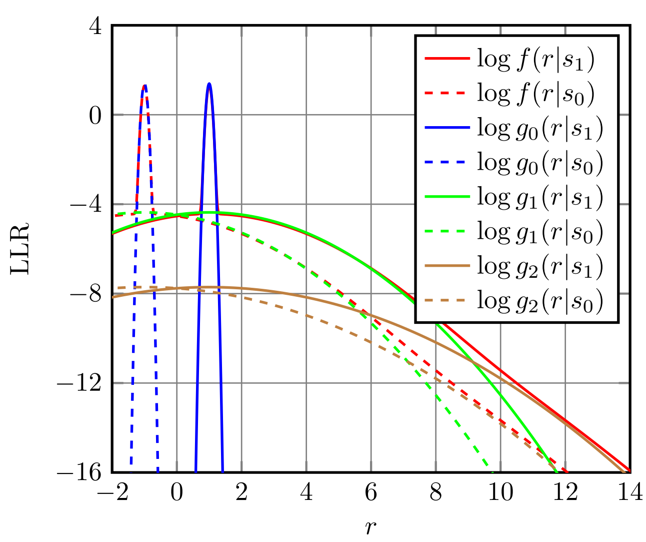


Fig. 2. The conditional pdf, $\log f_{A,\Gamma}(r|s_i)$, and other terms

Fig. 2 shows $\log f_{A,\Gamma}(r - s_0)$ and $\log f_{A,\Gamma}(r - s_1)$ along with $\log g_m(r - s_0)$ and $\log g_m(r - s_1)$ for $m = 0, 1, 2$. From the figure, we can draw some useful conclusions to understand how each term in the conditional pdf contributes to the LLR and also to identify the boundaries of different regions and the values at each boundary.

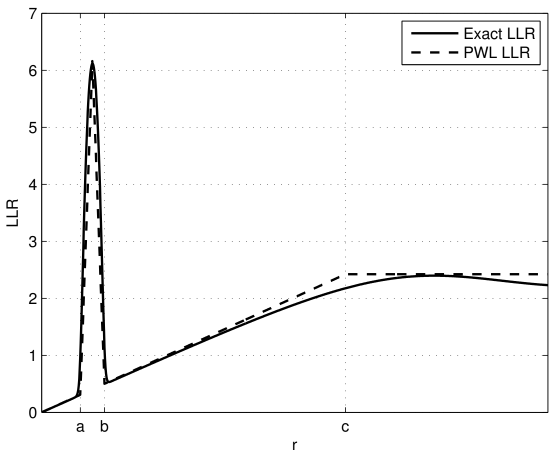


Fig. 3. The piecewise linear (PWL) simplified LLR

In the linear region, where $0 \leq r \leq a$ and $b \leq r \leq c$, Fig. 2 shows that the terms $\log g_1(r - s_0)$ and $\log g_1(r - s_1)$ can be used to approximate $\log f_{A,\Gamma}(r - s_0)$ and $\log f_{A,\Gamma}(r - s_1)$, respectively. The optimal LLR can therefore be approximated by

$$LLR_{lin} \approx \log g_1(r - s_1) - \log g_1(r - s_0) \qquad (9)$$
$$= \frac{2\sqrt{E}r}{\sigma_1^2}.$$

Also, c can be found by finding the intersection between $\log g_1(r - s_1)$ and $\log g_2(r - s_1)$, which is given by

$$c = \sqrt{E} + \sqrt{\frac{2\log(\frac{A\sigma_1}{2\sigma_2})}{\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2}}} \qquad (10)$$

In the parabolic region $a \leq r \leq b$, the optimal LLR can be approximated by

$$LLR_{par} \approx \log g_0(r - s_1) - \log g_1(r - s_0), \qquad (11)$$

which results in a quadratic function of r. The maximum of the parabolic region occurs at $r = r_o$ and is given by

$$\gamma = \log \frac{\sigma_1}{A\sigma_0} + \frac{2E}{\sigma_1^2 - \sigma_0^2}, \qquad (12)$$

where $r_o = \sqrt{E}(1 + 2A\Gamma)$, which can be approximated by $\sqrt{E}$ for moderate and high impulsive noise where $A\Gamma \ll 1$. By equating (9) and (11), we get a and b. To further simplify this region, we will use a piecewise linear approximation to connect $a \rightarrow \gamma \rightarrow b$ as shown in Fig. 3.

In the flat region ($r \geq c$), the LLR is approximated by a flat line as follows:

$$LLR \approx \frac{2c}{\sigma_1^2} \qquad (13)$$

It can be shown in Fig. 3 that the proposed PWL-LLR has a simple implementation using linear segments which has low complexity approaching that of the Gaussian detector in (5).

A. Performance Evaluation and Simulation Results

As a performance comparison, the probability of bit error (P_b) of the optimal and suboptimal detectors is evaluated by simulation for the case when (A, Γ) is (0.1,0.1) and (0.1,0.01). The SNR is defined as $\frac{E_s}{N_o}$, where $E_s = NE$ is the energy per symbol and N_o is the noise spectral density. The receiver collects N independent samples per symbol. To calculate an accurate P_b, up to 10^6 frames with 250 bits each are transmitted, and the simulation is stopped when the number of bit errors reaches 10^3 or $P_b \leq 10^{-4}$ is achieved.

Fig. 4 and 5 show the performance of the proposed PWL-LLR detector when $(A = 0.1, \Gamma = 0.1)$ with $N = 2$ and 5 respectively. It can be shown that the proposed detector gives near-optimal performance compared to the Gaussian one for a wide range of SNR. However, the performance of the PWL-LLR deviates from the optimal at low values of SNR, typically for SNR< -8 dB when $N = 2$ and SNR< -2 dB when $N = 5$. The reason for that degradation is that at low values of SNR, the noise variance per received pulse becomes larger which makes the parabolic region of the LLR wider. In this case, the approximation of the parabolic region with linear segments becomes inaccurate. On the other hand, the Gaussian detector approaches the optimal for relatively high SNR values, as can be seen in Fig. 4 when $N = 2$.

Fig. 6 and 7 show the performance of the PWL-LLR detector compared to the Gaussian one when $(A = 0.1, \Gamma = 0.01)$ with $N = 2$ and 5 respectively. The PWL-LLR gives near-optimal performance for relatively moderate and high SNR values with performance degradation at low SNR, typically when SNR < -12 dB when $N = 5$.

In summary, the PWL-LLR detector has near optimal performance due to the good fit to the optimal LLR when the SNR is large enough to give a narrow parabolic region of the LLR. On the other hand, the proposed detector has low-complexity that is almost equal to that of the Gaussian detector because both have first-order LLRs, although there are multiple regions in case of the proposed detector.

IV. Conclusion

The problem of designing low-complexity suboptimal decoders for signals in class-A noise is considered. The optimal detector requires complex computations while the conventional Gaussian detector gives good performance only with high SNR values. To improve the performance and reduce the complexity, several suboptimal detectors have been proposed with different performance levels compared to the optimal detector. In this paper, a simple approach has been proposed to design a simple suboptimal detector with robust near-optimal performance. The proposed approach uses a simplified linear approximation of the optimal log likelihood ratios. The resultant PWL-LLR detector has near-optimal performance with complexity approaching the Gaussian detector for different noise parameters

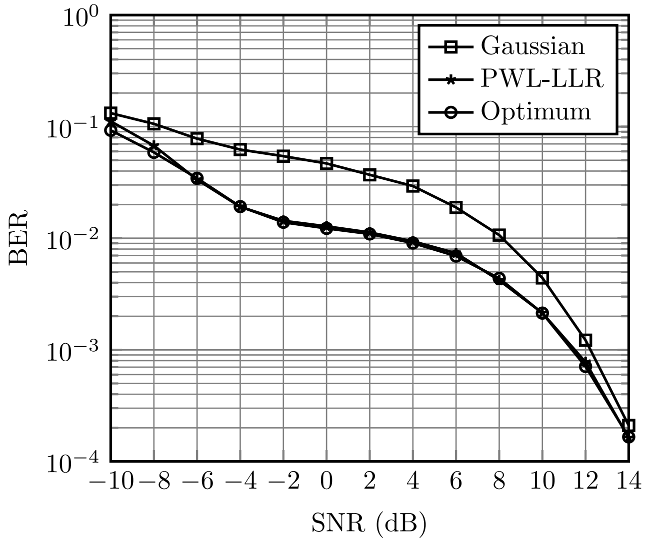


Fig. 4. Performance comparison with $A = 0.1$, $\Gamma = 0.1$ and $N = 2$.

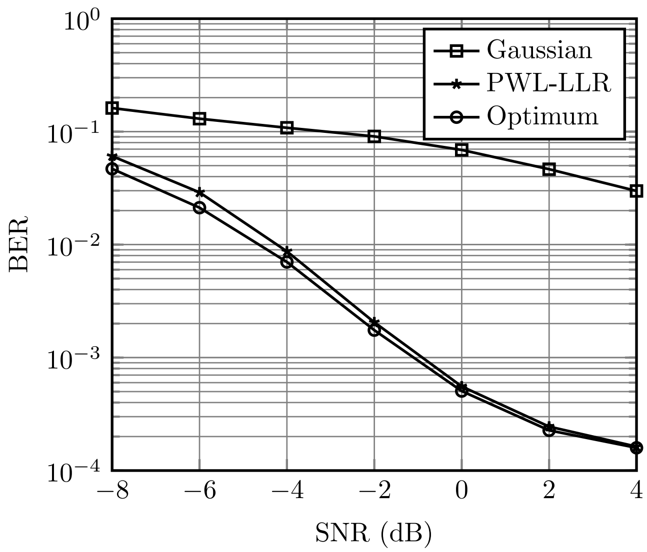


Fig. 5. Performance comparison with $A = 0.1$, $\Gamma = 0.1$ and $N = 5$.

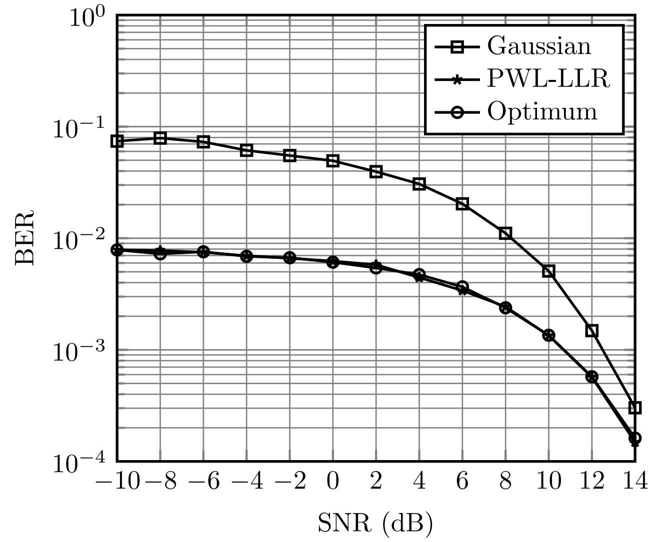


Fig. 6. Performance comparison with $A = 0.1$, $\Gamma = 0.01$ and $N = 2$.

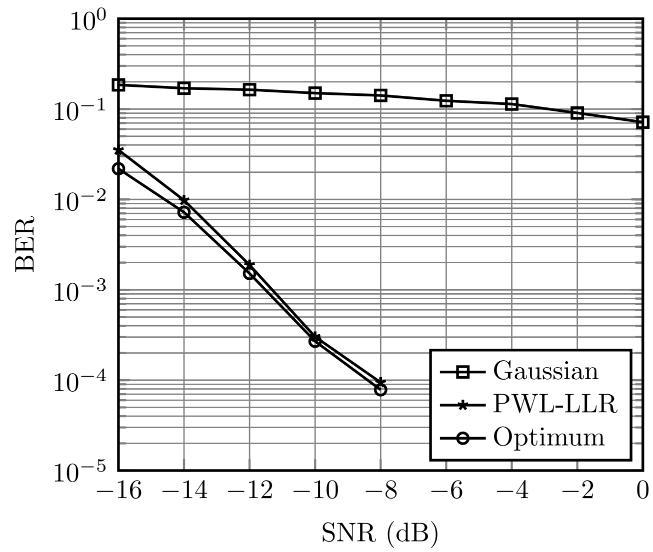


Fig. 7. Performance comparison with $A = 0.1$, $\Gamma = 0.01$ and $N = 5$.

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