Long Step User Selection Algorithm for Unitary Beamforming

Yan Li, Yi Sun, Feng Zhou
Faculty of Electronic Information and Electrical Engineering
Dalian University Of Technology
Dalian, China
liyanfd@hotmail.com

***Abstract*—This paper focuses on the design of user selection algorithm for Unitary Beamforming (UBF) in multi-user multiple-input multiple-output (MIMO) systems. A long step user selection (LSUS) algorithm is proposed, and is verified to be rather robust when combined with Givens rotation-based iterative optimization UBF. For most of existing user selection algorithms, only the best user is chosen at each selection, we regard the step length as one. However, because members of the optimal user set must both have high channel gains and low inter-user interference, if only one user is considered each step, the user that will be selected at next step can not be guaranteed to have both high channel gain and low inter-user interference with the just selected users. Therefore, an algorithm that two users are considered at each selection is proposed in this paper, the step length is regarded as two. It will in some extent help find the favorable user set and finally lead to a higher capacity. Moreover, Gram-Schmidt orthogonalization is used in our proposed algorithm to select the favorable users and at the same time generate the initial unitary beamforming matrix according to the channel state information (CSI). Two other beamforming schemes are simulated as references to demonstrate the good performance of the proposed algorithm even under low signal-to-noise-ratio (SNR) scenario.**

I. Introduction

MIMO technology brings a significant improvement both in spectral efficiency and system reliability. How to increase downlink channel capacity is one of the important research topics in multiuser MIMO systems. Dirty paper coding (D-PC) [1] achieves a capacity up to the capacity region of Gaussian broadcast channel [2, 3], but its practical application is greatly limited because of the high computational complexity. The orthogonal linear beamforming (OLBF) attracts much attention by virtue of simplicity and near optimal performance. As a representative scheme of OLBF, zero-forcing beamforming (ZFBF) achieves the asymptotic sum rate as that of DPC as the number of users grows to infinity [4], but ZFBF can not promise a good performance under low SNR scenario and is sensitive to channel estimation errors [5].

In [5–7], UBF is verified to be superior to ZFBF at low SNR region and to be rather robust with channel estimation errors. However, when the number of users in a system is much larger than that of the transmit antennas, user selection is necessary to focus the limited power on users with favorable channel states in order to achieve higher capacity. In [7], two user selection algorithms incorporated with UBF are proposed for which initial unitary beamforming matrixes are generated according to the CSI of user with the highest channel gain and the rest users are selected each has the strongest correlation with one column of the initial beamforming matrix. In [8], an iterative user selection algorithm is proposed, for which the unitary beamforming matrix is rotated by a rotation matrix each time a favorable user is selected, and the updated beamforming matrix is then as the reference to choose the next user.

Many other algorithms are proposed each trying to find users with high channel gains and low inter-user interference. However, for most of existing algorithms, only the best user is chosen at each selection. But, user with the strongest channel gain can not be guaranteed to be one member of the optimal user set. Therefore, it's necessary to shift the focus onto the aspect of lowering the inter-user interference. Ideally, all the user combinations should be considered, and the optimal user set with the highest sum-rate will be selected as the target users. However, this method is not considered as an excellent user selection algorithm because of high computational complexity. In this paper, a LSUS algorithm is proposed. Its basic idea is that when one user is selected, not only the inter-user interference between current user and the already selected users but also the interference between current user and next favorable user should be considered. In other words, two users are selected each time, thus this algorithm is named as 'Long Step User Selection Algorithm'. It in some extent shifts the focus from high channel gain to low inter-user interference and makes a trade-off between these two elements.

The detailed analysis of the proposed LSUS algorithm is discussed in this paper. The rest of this paper is organized as follows. The system model and Givens rotation-based iterative optimization UBF are introduced in Section II, Section III gives the outline of conventional user selection algorithm, and Section IV elaborates our proposed LSUS algorithm. Afterwards, several simulation results are demonstrated in Section V to verify the robustness of our proposed algorithm compared with two other beamforming schemes. At last, Section VI concludes this paper.

II. System Model and Givens Rotation-Based Iterative Optimization Unitary Beamforming

A. System Model

Consider a MIMO system with one base station equipped with M transmit antennas, $K(K \geq M)$ users each with

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single antenna, *M* users are selected after user scheduling process. The average transmit power constraint of *P* is equally distributed to each antenna. Throughout this paper, perfect CSI is assumed at the transmitter if not specifically stated.

Note that h_i is a $1 \times M-$ dimensional vector which represents channel information of the *i*th user, thus the channel matrix of the selected *M* users can be represented as $\mathrm{H} = \begin{bmatrix} h_1 & h_2 & \cdots & h_M \end{bmatrix}^H$. The received signal vector is

$$\mathrm{Y} = \sqrt{\frac{P}{M}}\mathrm{HV}s + n \tag{1}$$

where $\mathrm{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_M \end{bmatrix}$ denotes the unitary beamforming matrix $\mathrm{V^H V = I_M}$ and $v_i \in C^{M \times 1},\ i = 1, 2, \cdots, M$ denotes the beamforming vector for each user. *s* is the transmitted signal vector, *n* is the noise vector with zero mean and unit variance. The received signal of the *i*th user can be represented as

$$y_i = \sqrt{\frac{P}{M}} h_i v_i s_i + \sqrt{\frac{P}{M}} \cdot \sum_{j=1, j \neq i}^{M} h_i v_j s_j + n_i \tag{2}$$

Thus, the SINR of the *i*th user is

$$SINR_i = \frac{\|h_i\|^2 \cdot t_i}{\|h_i\|^2 \cdot (1 - t_i) + M/P} \tag{3}$$

where $t_i = \frac{|h_i v_i|^2}{\|h_i\|^2}$. Therefore, the sum rate of this system can be derived as

$$SR = \sum_{i=1}^{M} log_2(1 + SINR_i) \tag{4}$$

B. Givens Rotation-Based Iterative Optimization Unitary Beamforming

UBF is a beamforming scheme for which the beamforming matrix is unitary. This characteristic enables UBF a robust beamforming scheme even under low SNR region because the beamforming vectors are orthogonal to each other and that can in some extent reduce the inter-user interference. And Givens rotation is an orthogonal rotation method which makes the beamforming matrix match the wireless channel better and at the same time keep the unitary property of beamforming matrix. The Givens rotation matrix can be expressed as follows

$$G_{mn}(\alpha, \beta) = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cos\alpha & \cdots & \sin\alpha e^{j\beta} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ 0 & -\sin\alpha e^{j\beta} & \cdots & \cos\alpha & \cdots & 0 \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{5}$$

which performs a rotation on the two-dimensional space formed by the *m*th and *n*th columns of the target matrix without changing the other two-dimensional spaces. If the target matrix is a unitary matrix, the unitary characteristic will be kept. In UBF, the two angles α and β are computed following the principle of maximizing the sum-rate of current two users corresponding to the *m*th and *n*th columns. To achieve a higher sum-rate, iterative Givens rotation should be performed on each of the two-dimensional spaces formed by the columns of the beamforming matrix

$$V = V_{initial} \prod_{m=1}^{M-1} \prod_{n=m+1}^{M} G_{mn} \tag{6}$$

The times of rotation can be flexibly controlled to make a trade-off between sum-rate and the computational complexity.

The UBF scheme proposed in [5] is verified to be a flexible and robust beamforming scheme which makes a good match with our proposed user selection algorithm. Outline of the Givens rotation-based iterative optimization UBF scheme is as follows.

Step 1) Initialization: randomly generate a unitary matrix V.

Step 2) Iterative optimization using Givens rotation.

Circularly perform Givens rotation on each of the two-dimensional space formed by two columns of V. The Givens rotation angles are computed to achieve the maximum sum rate of the two users corresponding to the current two columns.

Update V once a Givens rotation is performed.

Step 3) Terminate the beamforming process.

Control the times of rotation to make a trade-off between computational complexity and sum rate.

III. CONVENTIONAL USER SELECTION ALGORITHM

In beamforming technique, user selection algorithm is necessary to concentrate the limited power on the favorable users in order to achieve higher sum rate. In this section, the conventional user selection algorithm is summarized by analyzing the existing user selection algorithms. For most of existing algorithms adopt Gram-Schmidt orthogonalization to select the favorable users, user with the highest channel gain is firstly selected as leader, and the other favorable users are selected one at a time according to the selected users. Outline of conventional algorithm is described as follows.

Step 1) Initialization.

Define two user sets. U_sel denotes the indexes of users that have been selected as members of the favorable set and U_unsel as the unselected user set.

$$U\_sel = \phi(\text{empty set}) \tag{7}$$

$$U\_unsel = \{1, 2, \cdots, K\}, \quad Num\_unsel = K \tag{8}$$

Step 2) Select user with the highest channel gain as the group leader.

$$U\_sel = \{ \underset{i \in U\_unsel}{\arg\max} \|h_i\| \} \tag{9}$$

$$U\_unsel = \{1, 2, \cdots, K\} - U\_sel \tag{10}$$

$$Num\_unsel = K - 1 \tag{11}$$

$$v_1 = h_{U\_sel(1)} \tag{12}$$

Step 3) Select users one at a time according to the selected users using Gram-Schmidt orthogonalization following the principle of achieving the highest sum rate.

Loop 1: i = 2,3,...,M

Loop 2: j = 1,2,...,Num_unsel

$$v_j = h^H_{U\_unsel(j)} - \sum_{k=1}^{i-1} \frac{h_{U\_unsel(j)} v_k}{\|v_k\|^2} \cdot v_k \tag{13}$$

Compute SINRs according to (3), and find out user with the highest SINR.

$$U\_sel = U\_sel + \{U\_unsel(\mathop{\arg\max}_{j=1,2,\ldots,Num\_unsel} \|SINR_j\|)\} \tag{14}$$

$$v_i = v_{U\_sel(i)} \tag{15}$$

$$Num\_unsel = Num\_unsel - 1 \tag{16}$$

$$U\_unsel = \{1, 2...K\} - U\_sel \tag{17}$$

Step 4) Terminate the user selection process.

Normalize V and finally obtain the beamforming matrix.

U_sel: the favorable user set.

IV. LONG STEP USER SELECTION ALGORITHM

A LSUS algorithm is proposed here which outperforms ZFBF-SUS at low SNR region when combined with the above mentioned Givens rotation-based iterative optimization UBF scheme.

In our proposed algorithm, the Gram-Schmidt orthogonalization is used to generate the beamforming matrix according to the CSI. Different from the conventional user selection algorithm, two users are considered at each selection to in some extent shift focus from high channel gains to low inter-user interference. This will help find the favorable users and finally lead to a higher capacity.

Details of this LSUS algorithm are elaborated as follows.

Step 1) Sorting and initialization.

Step 1.1) Sort the K rows of H to make the channel gain descend from the first row to the last.

$$\|h_i\| > \|h_j\| \quad (K > i > j > 0) \tag{18}$$

Step 1.2) Define two user sets. U_sel denotes the indexes of users been selected and U_unsel as the unselected user set.

$$U\_sel = \phi(\text{empty set}), \quad Num\_sel = 0 \tag{19}$$

$$U\_unsel = \{1, 2, \cdots, K\}, \quad Num\_unsel = K \tag{20}$$

Step 2) Select users two at a time (we assume M is an even number for simplicity) .

Loop1: i = 1,3,...,$M-1$, two users are selected each loop.

Step 2.1) The first selected user in this loop has min(5M, Num_unsel) candidates, this threshold can help reduce computational complexity without largely reducing the sum rate.

Loop2: j = 1,2,...,min(5M, Num_unsel)

$$v_i = h^H_{U\_unsel(j)} - \sum_{l=1}^{i-1} \frac{h_{U\_unsel(j)} v_l}{\|v_l\|^2} \cdot v_l \tag{21}$$

Loop3: k = 1,...,$j-1$,$j+1$,...,Num_unsel, find the next favorable user according to the just selected user.

$$v_{i+1} = h^H_{U\_unsel(k)} - \sum_{l=1}^{i} \frac{h_{U\_unsel(k)} v_l}{\|v_l\|^2} \cdot v_l \tag{22}$$

Compute sum rate of the current focused two users.

$$SR\_two = \sum_{l=i}^{i+1} \log_2(1 + SINR_l) \tag{23}$$

Find out the favorable two users with the largest SR_two whose indexes are u_1 and u_2.

Step 2.2) Update U_sel and U_unsel.

$$U\_sel = U\_sel + \{u_1\} + \{u_2\} \tag{24}$$

$$U\_unsel = U\_unsel - \{u_1\} - \{u_2\} \tag{25}$$

Step 2.3) Remove users in U_unsel whose channel information satisfies (26) in order to reduce the computational complexity [4].

$$\frac{|h_l v_{i+1}|}{\|h_l\| \ \|v_{i+1}\|} > threshold \quad l \in U\_unsel, \ threshold = 0.7 \tag{26}$$

Step 3) Terminate user selection process.

Normalize V and finally obtain the beamforming matrix.

U_sel: the favorable user set.

To verify the advantages of our proposed LSUS algorithm in terms of capacity, a simulation is performed with M=4, SNR=10 and user number ranges from 10 to 100.

As demonstrated in Fig. 1, the proposed LSUS algorithm outperforms the typical algorithm which selects users one at a time. This verifies that the proposed algorithm is an effective method to increase system capacity. However, the computational complexity of LSUS is a little higher than algorithm that selects user one at a time as demonstrate in Fig. 2. But, the gap between these two algorithms is not very large because there are several steps in LSUS algorithm to reduce the computational complexity. For example, LSUS removes users that's strongly correlated with the just selected user as described in step 2.3 of our proposed algorithm. Furthermore, computational complexity can be reduced by means of setting a feedback threshold which is commonly adopted in beamforming schemes [9].

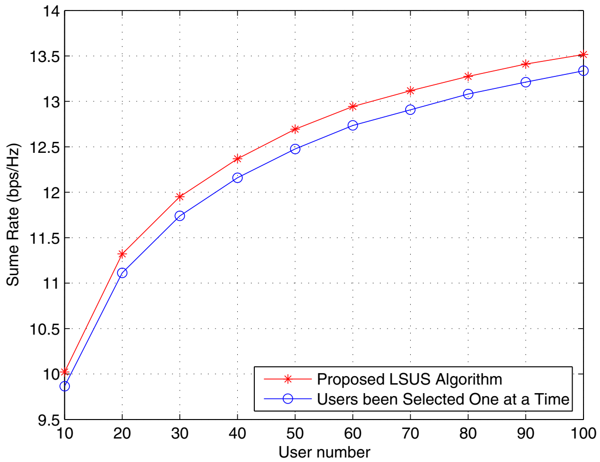

Fig. 1. Sum rate of LSUS and conventional user selection algorithm, with M=4, SNR=10 dB.

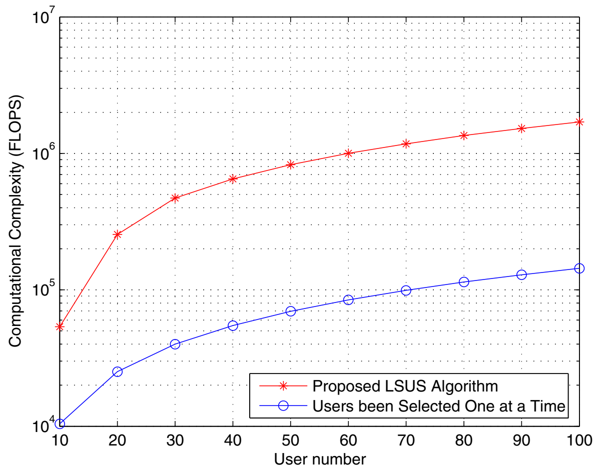

Fig. 2. Computational complexity of LSUS and conventional user selection algorithm, with M=4, SNR=10 dB.

V. Performance Simulation Results

A. Performance in Terms of Sum Rate with Perfect CSI

In this subsection, several simulation results are presented to demonstrate the robustness of our proposed LSUS algorithm with perfect CSI.

Figure 3 shows the performance of three beamforming schemes in terms of sum rate with SNR=10 dB. The blue line denotes the sum rate simulation results of ZFBF-SUS proposed in [4]. For fair comparison, two different schemes both using the Givens rotation-based iterative optimization UBF but two different user selection algorithms are simulated in order to compare the performance of these two scheduling algorithms. The green one denotes the performance of scheme adopts the user selection algorithm proposed in [7] named ‘Algorithm B’, and the red one represents the performance of scheme adopts our proposed LSUS algorithm.

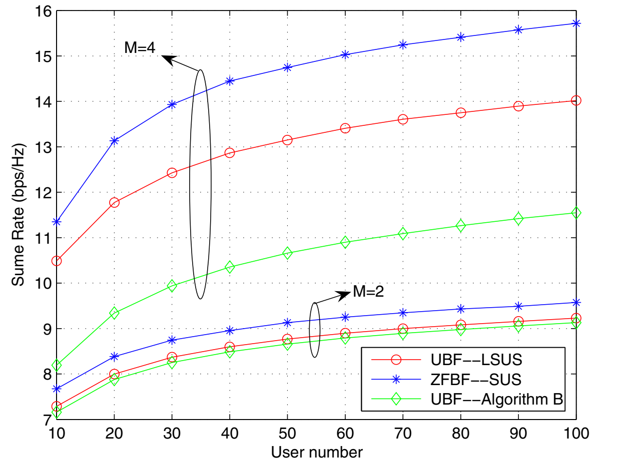

Fig. 3. Sum rate as a function of the user number with SNR=10 dB and M=2, M=4 respectively.

As showed in Fig. 3, when SNR=10 dB, the ZFBF-SUS scheme gives the best performance in these three schemes both with M=2 and M=4. Our proposed scheme gives a suboptimal performance, and the sum rate is much higher than the scheme named ‘UBF-Algorithm B’ especially when M=4.

ZFBF concentrates on perfectly avoiding the inter-user interference at the sacrifice of signal power [6], the beamforming matrix is the Moore-Penrose inverse of channel matrix, therefore it gives a near optimal performance as user number increases and with perfect acknowledge of the CSI. As to the UBF, no matter how many times the Givens rotation is performed, the beamforming matrix is still a unitary matrix, that means the UBF scheme will not eliminate all the inter-user interference unless the channel matrix itself is a unitary matrix. So, it’s not surprising that ZFBF scheme outperforms UBF scheme when SNR is high. However, the UBF is still an excellent beamforming scheme, Fig. 4 illustrates the robustness of UBF scheme by means of performing the same simulation as Fig. 3 did but lower the SNR from 10 dB to 0 dB.

Figure 4 gives us a strong confirmation of the robustness of our proposed LSUS algorithm together with the Givens rotation-based iterative optimization UBF. As showed in Fig. 4, the ZFBF is very sensitive to channel condition, and can’t promise a good performance when noise power is equal to signal power. The robustness of UBF is due to the unitary characteristic of the beamforming matrix, which guarantees the orthogonality between any beamforming vectors. And it will in some extent avoids the inter-user interference although it’s almost impossible to make the beamforming matrix of UBF perfectly match with the channel matrix and entirely eliminate inter-user interference.

When compare the performance of ‘UBF-LSUS’ with that of the ‘UBF-Algorithm B’ scheme, the large gap between these two schemes demonstrates the good property of our proposed LSUS algorithm. The ‘UBF-Algorithm B’ scheme gives a

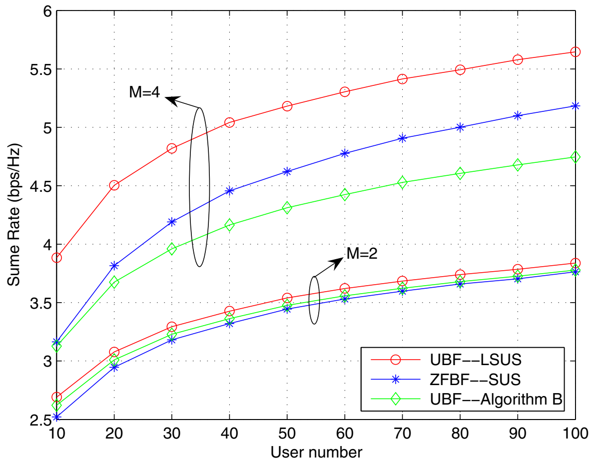

Fig. 4. Sum rate as a function of the user number with SNR=0 dB and M=2, M=4 respectively.

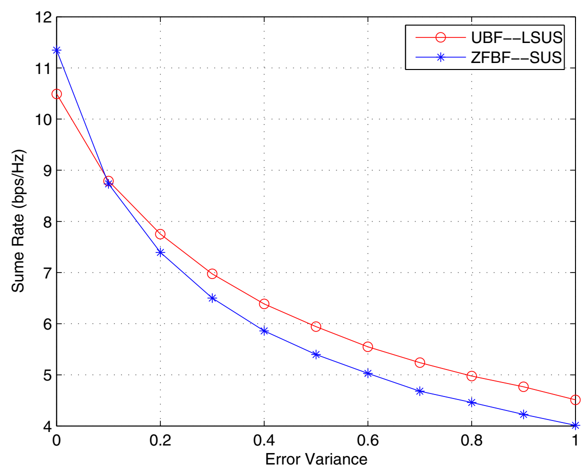

Fig. 5. Sum rate as a function of channel estimation error variance with SNR=10 dB and M=4, K=10.

poor performance especially when M=4, that's because the initial beamforming matrix is generated only refer to the CSI of user with the highest channel gain using Gram-Schmidt orthogonalization, which can't promise to match with the CSI matrix, thus the users selected according to this beamforming matrix can't promise to perform well under current channel state.

B. Performance in Terms of Sum Rate with Imperfect CSI

According to the above simulation results, it can be inferred that the two schemes of UBF-LSUS and ZFBF-SUS give good performance. However, these simulations are performed with perfect CSI which is not coincidence with actual situation. To further evaluate the performance of these two schemes, situation with imperfect CSI is considered below. Figure 5 gives us the performance of these two schemes in terms of sum rate under different channel estimation error variance.

In Fig. 5, it's obvious that as the channel estimation error variance increases, the sum rate of ZFBF-SUS decreases faster than UBF-LSUS. ZFBF-SUS gives good performance when the estimation error is low, but UBF-LSUS gives better performance with high estimation error. For the ZFBF-SUS scheme, because it's beamforming matrix is the Moore-Penrose inverse of the channel matrix, it can eliminate all the inter-user interference with perfect CSI. But when the channel estimation error is high, the beamforming matrix can't promise to match with channel matrix. Thus, the user interference is high and can't get a high sum rate. As to the UBF-LSUS scheme, the beamforming matrix itself is a unitary matrix, it's good performance in some extent benefits from the orthogonality between each of two beamforming vectors. Thus, even the accurate CSI can't be obtained, the beamforming vectors are still orthogonal to each other and it will help reduce inter-user interference. Therefore, UBF-LSUS scheme is not as sensitive as ZFBF-SUS scheme to channel estimation error and can get relatively high sum rate with high channel estimation error.

C. Convergence Rate of Our Proposed LSUS Algorithm

In [5], where the Givens rotation-based iterative optimization UBF scheme is proposed, the initial unitary beamforming matrix is randomly generated. However, the convergence rate of this initial beamforming matrix generating scheme is not favorable. Too many times rotations should be performed in order to get a higher sum rate which significantly increases the computational complexity. Figure 6 shows a comparison of convergence rate between the scheme proposed in [5] and our proposed LSUS algorithm which generate the initial unitary beamforming matrix using Gram-Schmidt orthogonalization according to the CSI. It's obvious that our proposed algorithm performs a much faster convergence rate. For example, when $K = M = 8$, only approximately 30 times rotations need to be performed in order to make the convergence, but if the initial beamforming matrix is randomly generated, more than 100 times rotations is needed.

The good performance of our proposed algorithm on convergence rate benefits from the idea that the initial unitary beamforming matrix is generated according to the CSI of users already selected using Gram-Schmidt orthogonalization. Therefore, the initial beamforming matrix can promise to be strongly match with the channel matrix and at the same a unitary matrix. And not too many times of rotations should be performed in order to get the convergence.

VI. Conclusion

A LSUS algorithm is proposed in this paper, for which the favorable users are selected two at a time following the principle of achieving the highest sum rate. At the same time the initial unitary beamforming matrix is generated using Gram-Schmidt orthogonalization according to the CSI which performs a much faster convergence rate compared with the random initial unitary beamforming scheme. Moreover, the LSUS algorithm makes a good match with Givens rotation-based iterative optimization algorithm, several numerical re-

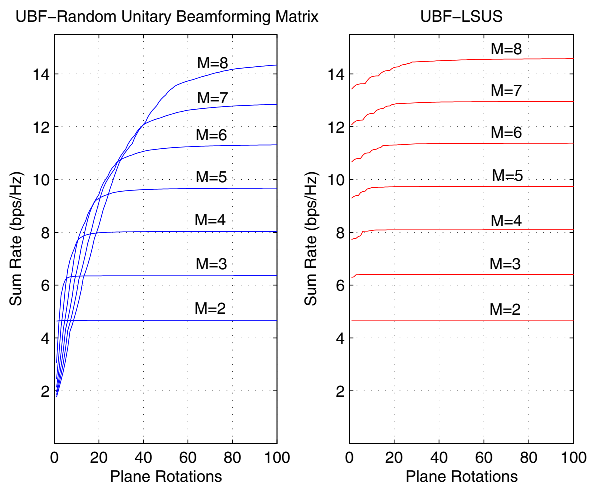

Fig. 6. Comparison of convergence rate between two initial unitary beamforming matrix generating schemes for UBF when K=M and SNR=10 dB.

sults are presented to demonstrate the robustness of this scheme compared with ZFBF-SUS. At the same time, our proposed LSUS algorithm outperforms 'UBF-Algorithm B' in terms of sum rate both in high SNR and low SNR scenarios. Overall, the proposed LSUS algorithm is an effective user selection scheme for unitary beamforming in multi-user MIMO systems.

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