Outage Constrained Power Allocation and Relay Selection for Multi-Hop Cognitive Network

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***Abstract*—In this paper, we consider the power saving issue in the cluster based multi-hop cognitive radio (CR) network with one pair of primary user (PU) in presence. By underlay spectrum sharing, the transmit power of CR nodes are strictly restricted to protect PU from suffering severe interference. Moreover, the end-to-end outage probability is put forward as an essential QoS indicator for multi-hop transmission and has been carefully studied in the article. The objective of this paper is to minimize the total power consumption of CR transmitters along relay path. Both the end-to-end outage requirement and power budget of relays are incorporated as constraints. To solve the formulated problem and obtain the optimal solution, we propose a joint Lagrange dual method based power allocation and objective oriented optimal relay selection algorithm, and give thorough evidences and illustrations as well. Finally, numerical simulations are made and results demonstrate that the proposed algorithm has a good performance in power saving.**

I. INTRODUCTION

Relaying, emerging as a remarkable technology for B3G/4G wireless networks, brings benefits of many aspects. For example, it provides better service for users at cell edge who are generally suffering from severe shadowing or fading. Moreover, it is considered as a preferable tool for extending the coverage of existing cellular network [1]. Motivated by these advantages, relaying has aroused lots of concerns among both the academic circles and the industrial world.

There are plenty of works focusing on resource allocation for dual-hop and multi-hop relay networks [2]-[3]. However, considering both the users' growing demand for better perceived quality and the low utilized spectrum situation, purely relying on relay assisted network is far from enough. Therefore, cognitive radio (CR) based relay networks arise as a preferable solution. Many literatures have already studied CR based relay networks. In [4], subcarrier pairs, relay selection and power allocation are jointly optimized to maximize the total system capacity. In [5], a dynamic time slot allocation scheme is proposed for receiving and transmitting to maximize the throughput. Besides system capacity, outage probability is another crucial parameter that has been widely explored [6]-[9]. In [10] and [11], multi-hop relay scenario is discussed. The end-to-end outage is used as optimization criterion in [10], while [11] maximizes the average end-to-end throughput. To sum up, all the papers mentioned above choose either outage probability or summed system capacity as the evaluation criterion. However, in cases where power is a more limited and precious resource, such as in wireless sensor networks (WSN), power consumption should be paid more attention to, which requires to study from a new point of view. With regard to this concern, in [12], a minimum allocated power is obtained under the constraint of outage probability for conventional dual-hop relays with and without diversity. However, the results cannot be directly applied to CR multi-hop network which is constrained by end-to-end performance requirement. Inspired by this, our paper will focus on minimizing the total power consumption for multi-hop cognitive network under the end-to-end outage requirement, which has not been fully discussed yet in the literature.

In this paper, we investigate the decode-and-forward (DF) relays deployed scenario. Taking CR spectrum sharing and wireless channel state into account, power allocation and relay selection are jointly optimized to minimize total power consumption, which is a cross-layer issue. Compared with existing papers, this paper contributes in the following aspects. 1) *Total power consumption minimized objective*. In contrast to most papers aiming at maximizing system capacity or minimizing outage probability, the paper focuses on minimizing the total power consumption of relays; 2) *End-to-end outage analysis*. For multi-hop scenario, the end-to-end outage is an indicator of great importance, which reflects the overall QoS, and therefore is carefully tackled; 3) *Lagrange dual method based optimal power allocation (OPA) and objective oriented optimal relay selection (ORS) algorithm*. To obtain the optimal solution, OPA and ORS are jointly optimized.

The remainder of the paper is organized as follows. Section II describes the system model and derives the end-to-end outage probability. Based on these, the optimization problem is formulated. Section III proposes and analyzes the optimal power allocation and objective oriented relay selection algorithm. In Section IV, numerical simulations are made and results are presented to validate the theoretic analysis. Section V summarizes the paper.

II. SYSTEM MODEL

We consider a M-hop cognitive network with the coexistence of primary network. The primary network involves a pair of primary transmitter (PT) and primary receiver (PR). PT is assumed to be located far away from CR network. Thus, interference caused by PT to CR receivers is treated as

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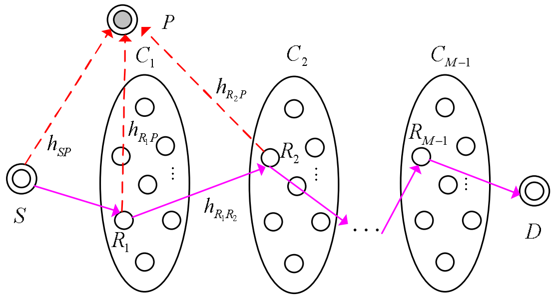

Fig. 1. System Model

white noise. Only PR is depicted in Fig.1, denoted as P. For the cognitive network, one pair of communicating nodes (i.e., S, D) is included. Between S and D, $M-1$ relay clusters are placed based on their geographic position, represented by C_m. Each cluster contains L decode and forward (DF) relays, operating on half-duplex mode. Only one relay in each cluster is selected for multi-hop transmission. Further, time division multiple access (TDMA) protocol is deployed. For each frame duration T_f, the first T_s period is allocated for spectrum sensing and the rest is divided into M equal slots for each hop's transmission.

All links involved in the scenario are assumed to be independent Rayleigh flat fading channels. Without loss of generality, the noises of receivers are supposed to be additive white Gaussian noises (AWGN) with variance σ^2. Let the channel gain between link $i \rightarrow j$ denoted as h_{ij}. Taking both large scale path loss and small scale fading effect into account, we have $h_{ij} = \chi_{ij}(d_0/d_{ij})^{\alpha/2}$ where α represents the path-loss exponent and d_{ij} d_0 denote the distance between two nodes and the reference distance respectively. χ_{ij} is a complex Gaussian random variable with unit variance. Based on these assumptions, ${h_{ij}}^2$ follows an exponential distribution with the mean value $m_{ij} = (d_0/d_{ij})^{\alpha}$. Assume that ${h_{ij}}^2$ remains the same within the period of a frame, but varies from frame to frame. Moreover, in terms of the feedback delay of channel gain, exact values of instant ${h_{ij}}^2$ cannot be acquired. Therefore, only the mean value of channel gain is assumed to be obtained.

A. Analysis of CR Spectrum Sharing

To protect PU from suffering severe interference, the CR users are supposed to share spectrum with PU by either overlay or underlay way. Here in this paper, underlay spectrum sharing is adopted. To avoid exceeding the interference tolerance of PU and guarantee PU's communication priority, a maximum tolerated transmit power should be calculated. Due to the fact that only one relay is selected in each cluster, it means that only the selected relay can possibly cause interference to primary receiver. Therefore, by combining the tolerated interference with the power limit of transmitter itself, the power budget of each CR node is derived:

$$P_{\max}^i = \min\{\frac{Q}{m_{iP}}, P_{\max}\} \tag{1}$$

where $Q, P_{\max}$ denote the maximum tolerated interference of PU and power limit of CR transmitter, respectively. $P_{\max}$ is assumed to be identical for all transmitters. m_{iP} represents the mean value of channel gain between CR node i and PU.

B. Analysis of End-to-End Outage

In wireless network, outage probability is one crucial parameter indicating users' received service quality [7]. Here, we adopt a widely used definition of outage as the probability that mutual information is less than the required data rate. For single link $i \rightarrow j$, outage probability is calculated by:

$$\begin{aligned} P_{ij}^{out} &= \Pr\{\tfrac{1}{M}\log(1+\tfrac{P_i h_{ij}^2}{\sigma^2}) < C_0\} \\ &= 1-\exp(-\tfrac{\sigma^2\gamma_0}{P_i m_{ij}}) \end{aligned} \tag{2}$$

where P_i is the transmit power and C_0 is the minimum rate for relays to correctly decode the received signal below which an outage occurs. And $\gamma_0 = 2^{MC_0} - 1$ is the equivalent SNR threshold for notation simplicity. M in the denominator exists due to the fact that TDMA protocol is incorporated.

However, for multi-hop scenario, the end-to-end outage is a much more significant criterion for performance analysis compared with the single link outage. The indicator reflects the end-to-end QoS of traffic flow. Therefore, multi-hop outage is worthy being thoroughly analyzed and derived. Since the outage is closely related with the received throughput at destination node which is always strictly restricted by the worst link in M hops, the received throughput C at destination is given by:

$$C = \min_{i,j \in \text{Relay}\ Path}\{C_{i,j}\} \tag{3}$$

where i is the transmitter and j is the corresponding receiver during each hop along the selected relay path, which will be determined in later relay selection part. Finally, the end-to-end outage P_{out} is calculated as:

$$\begin{aligned} P_{out} &= \Pr\{\tfrac{1}{M}\min_i\{\log(1+\tfrac{P_i h_{ij}^2}{\sigma^2})\} \le C_0\} \\ &= 1-\exp(-\gamma_0\sum_i \tfrac{\sigma^2}{P_i m_{i,j}}) \end{aligned} \tag{4}$$

C. Problem Formulation

The idea throughout the paper is to minimize the total power consumption of transmitters along multi-hop links, and at the same time guarantee the end-to-end outage requirement. Based on these thoughts, the optimization problem is formulated as:

$$\min \quad (1-\frac{T_s}{T_f})\sum_i P_i \tag{5}$$

subjected to:

$$\begin{aligned} &(\text{C1}): \quad P_{out} \le \rho_0 \\ &(\text{C2}): \quad 0 \le P_i \le P_{\max}^i, \forall i \end{aligned}$$

where P_i is the transmit power of hop i along selected relay path. And ρ_0 is the threshold of tolerated outage. $(1-\frac{T_s}{T_f})$ is used to exclude the spectrum sensing time during which no effective data contributing to end-to-end throughput transfers. (C1) is the maximum tolerated end-to-end outage requirement. (C2) naturally holds in consideration of the power budget as described in former part.

III. Optimal Power Allocation and Objective Oriented Relay Selection Strategy

Considering the problem mentioned above, two main issues are intended to be delicately dealt with in this section. One is how much power should be allocated for transmitters along the relay path. The other is how to select the optimal relay during each hop to minimize the total power consumption.

It is noted that the two problems are correlated and should be jointly optimized to find the optimal solution. Firstly, to focus on power allocation, let us assume that the optimal relay path is established already. Since $(1-\frac{T_s}{T_f})$ is a constant which doesn't make a difference to the final results, it is omitted in the following discussion. By some manipulation, (C1) can be equivalently transformed into another form:

$$\text{(C3)}: \qquad P_{eq}(P_i)=\sum_i \frac{1}{P_i m_{i,j}} \le \varepsilon_0$$

and $\varepsilon_0 = -\frac{\ln(1-\rho_0)}{\gamma_0\sigma^2}$ which denotes the equivalent outage requirement.

A. Power Allocation

Lagrange dual method is a preferable candidate to solve problem with constraints. By relaxing (C3), the Lagrange function is given by:

$$L(P_i,,\pi)=\sum_i P_i + \pi[P_{eq}-\varepsilon_0] \tag{6}$$

Based on Karush-Kuhn-Tucker (KKT) condition, we can easily derive the equations below.

$$\begin{cases} \dfrac{\partial L}{\partial P_i} = 1+\pi\dfrac{\partial P_{eq}}{\partial P_i}=0 & (7)\\ \pi[P_{eq}-\varepsilon_0]=0 & (8)\end{cases}$$

From (7) and (8), it is inferred that $\pi \neq 0$. And the equality of (C3) holds for the optimal solution, i.e., $P_{eq}-\varepsilon_0=0$. By substituting $\partial P_{eq}/\partial P_i$ into (7) and combining with (8), we acquire the optimal solution.

$$P_i^* = \frac{1}{\varepsilon_0\sqrt{m_{i,j}}}\sum_{l\in U}\frac{1}{\sqrt{m_{l,j}}} \tag{9}$$

where U is the relay set along the selected relay path. However, the solution obtained is not taking (C2) into account until now. To obtain the optimal solution that satisfies both constraints, further modifications are still needed. We first illustrate the modification rule and then give our explanations in Proof 1 and Proof 2.

Modification Rule: 1) for relays that $P_i^* > P_{\max}^i$, reset the optimal power by $P_i^* = P_{\max}^i$, and update the relay set U_0 with maximum power by $U_0 = U_0 \cup \{i\}$ (Proof 1); 2) for the rest relays, update left relay set U with no power allocated and the equivalent outage threshold ε_0 by $U = U_{init} - U_0$ and $\varepsilon_0 = \varepsilon_{init} - \sum_{l\in U_0} 1/(P_{\max}^l m_{l,j})$, where $U_{init}, \varepsilon_{init}$ are the entire relays on the multi-hop path initially and the initial value of end-to-end QoS requirement, respectively. Recalculate optimal power in (9) until (C2) is met for all relays (Proof 2).

Proof 1: First let us analyze the differential equation given in KKT condition $\frac{\partial L}{\partial P_i} = 1-\frac{\pi}{P_i m_{i,j}}$. If $P_i^* \le P_{\max}^i$ holds, which means that stationary point is within the feasible region of power constraint, it proves to be optimal. If not, $\frac{\partial L}{\partial P_i} \le 0$ is satisfied in the feasible region, indicating that the objective is monotonously decreasing with regard to parameter P_i. Therefore the minimum value is acquired at $P_i^* = P_{\max}^i$. Thus the relay set U_0 with maximum power is updated by $U_0 = U_0 \cup \{i\}$.

Proof 2: To obtain the optimal solution, it means that $P_{eq} - \varepsilon_0 = 0$ should always be satisfied. After the modification by resetting $P_i^* = P_{\max}^i$ for U_0, to make sure the equation still holds, equivalent manipulation should be made by $\sum_{i\in U_{init}-U_0}\frac{1}{P_i m_{i,j}} = \varepsilon_0$, where $\varepsilon_0 = \varepsilon_{init} - \sum_{l\in U_0}\frac{1}{P_{\max}^l m_{l,j}}$. Moreover, the relay set U without allocated power is updated by $U = U_{init} - U_0$. Thus ε_0 and U are iteratively updated to obtain the optimal power for rest relays in set U.

It can be seen from the mentioned proofs that it is an iterative procedure to find the optimal solution within the feasible region bounded by both (C1) and (C2). The **optimal power allocation (OPA)** algorithm is outlined in details in **Algorithm 1**.

Algorithm 1 Optimal Power Allocation (OPA)

$\varepsilon_{init} = \varepsilon_0$,$U_0 = \emptyset$,$U_{init} = U = \{i \mid i \in \text{Relays on Path}\}$.
repeat
  for $i \in$ U **do**
    obtain P_i^* according to (9).
    if $P_i^* > P_{\max}^i$ **then**
      $U_0 = U_0 \cup \{i\}$
      $P_i^* = P_{\max}^i$
    end if
  end for
  $U = U_{init} - U_0$
  $\varepsilon_0 = \varepsilon_{init} - \sum_{l\in U_0}\frac{1}{P_{\max}^l m_{l,j}}$
until $P_i^* \le P_{\max}^i, \forall i$

B. Relay Selection

Objective Oriented Optimal Relay Selection (ORS): To find the optimal relay path that minimizes the total power consumption, an ideal tool is exhaustive search. By comparing power consumption of all potential relay paths, the minimum value can be acquired within a complexity of $O(L^{M-1})$.

We can see that the proposed **ORS** algorithm has a rather considerable complexity when large M or L are set in the network. For comparison and simplicity purpose, a relay selection scheme with low complexity is also described as below (**CRS**).

Contrast Relay Selection (CRS): Different from the scheme mentioned above, the best relay for each cluster is determined in the reverse order of data transmission, which means relay in cluster $M-1$ is selected first and then cluster $M-2$, and so on. Further, power budget is incorporated as well. The criterion for relay selection during each hop is given as follows:

$$i^* = \max_i \{P^i_{\max} m_{i,j}\} \tag{10}$$

Compared with existing works only taking $m_{i,j}$ into account, this scheme is much more thoughtful and reasonable by taking power budget of transmitter into account. The scheme has a complexity of $O(ML)$.

To sum up, in order to acquire the optimal solution, the optimal power allocation and the objective oriented optimal relay selection should be jointly optimized. Detailed descriptions are given in **Algorithm 2**.

Algorithm 2 Proposed Algorithm (OPA+ORS)

```
obtain the channel gain m_{i,j} and the power budget P^i_max
    for each node i. set Ψ = ∅.
for m ∈ All Relay Paths ( L^{M−1} ) do
    set P_i = P^i_max, ∀i.
    if (C3) is satisfied then
        Ψ = Ψ ∪ {m}
    end if
end for
if Ψ = ∅ then
    no available relay path that meets both constraints.
else
    for m ∈ Ψ do
        execute OPA algorithm for this path.
        record the total power consumption of this path.
    end for
    find the path with minimum power.
end if
where Ψ denotes the set of feasible relay paths that
satisfies the outage constraint
```

IV. Numerical Results and Discussions

In this section, numerical simulations are made to validate the theoretical analysis. Consider a $M = 4$ hop relay network, located within a rectangular region of 2000m $\times$ 400m. Thus 3 clusters are deployed each with $L = 20$ relays randomly distributed. S, D are placed at (0, 0) and (2000, 0) respectively, with P located at (600,-200). And the maximum tolerated interference is set to be $Q = 0.01$W. Each transmitter is supposed to possess the same power limit $P_{\max} = 1$W. The variance of white noise is $\sigma^2 = 10^{-4}$.

Fig.2 depicts the optimal relay path and total consumption of proposed algorithm under diverse values of outage threshold ρ_0. And further Fig.3 gives the transmit power of individual relay along the optimal path. From Fig.2, it is seen that with a tighter demand of outage requirement, i.e., a smaller ρ_0 as in Relay Path1, the relays are allocated with a larger transmit power, i.e., 2.4065W, which is also supported by results of each relay's power allocation in Fig.3. As a result, it is reasonable to reach the conclusion that to receive a better service quality, a higher cost should be paid for. For each hop's transmit power in Fig.3, the transmit power of hop2 and hop3 are larger than the other two hops'. The explanation is that both hop2 and hop3 are located far from their next hop (seen in Fig.2), making them the bottleneck links due to the comparatively worse channel condition. Hence a larger power is required.

Fig.4 investigates the influence of end-to-end outage threshold on the total power consumption and compares the performance of the proposed algorithm (OPA+ORS) and the contrast one (OPA+CRS). From the curves, we can conclude that the proposed algorithm has a better performance in minimizing total power than the contrast one, and saves as much as 16.76% power in $\gamma_0 = -0.5$dB case and 16.61% in $\gamma_0 = 1$dB case. Further, the curves show that total transmit power is a decreasing function of ρ_0, which is easy to understand. An interesting thing here is that there is no optimal power in the range of ρ_0 from 0.1 to 0.2 for OPA+CRS scheme under $\gamma_0 = 1$dB. The reason is that under this condition, a harsh requirement for power is put forward to guarantee outage constraint, and the optimal power calculated exceeds power budget of each relay along the pre-determined relay path. While for ORS, there are still many relay path candidates to search for. Thus for other three schemes, the solutions are still available.

Fig. 5 explores the relationship between maximum tolerated interference power and total power consumption. The figure shows that the proposed algorithm outperforms the contrast one at both acquiring the feasible solutions and also minimizing the total power consumption. As reflected in the

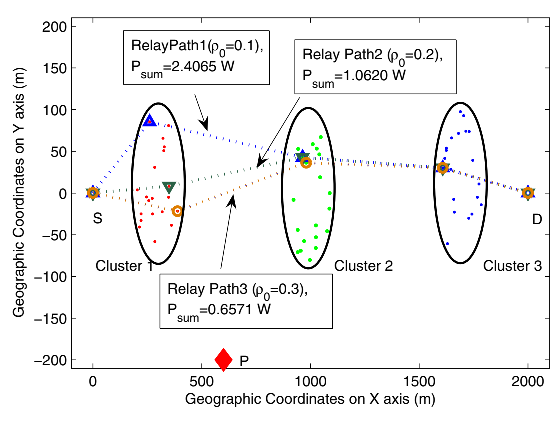

Fig. 2. Relay selection and total transmit power under diverse ρ_0 by proposed algorithm. Here, $\gamma_0 = 0.5$dB and ρ_0 is set to be $0.1, 0.2, 0.3$ respectively and the corresponding minimized power is calculated to be 2.4065W,1.0620W,0.6571W.

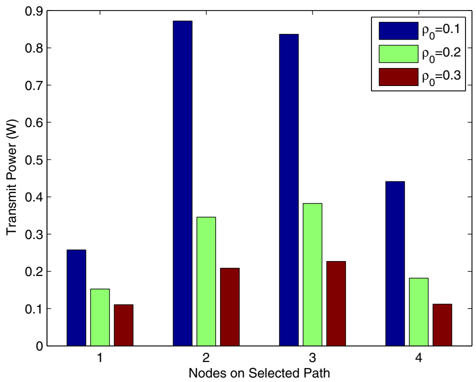

Fig. 3. Transmit power of each selected relay in 4 hops by proposed algorithm under diverse ρ_0.

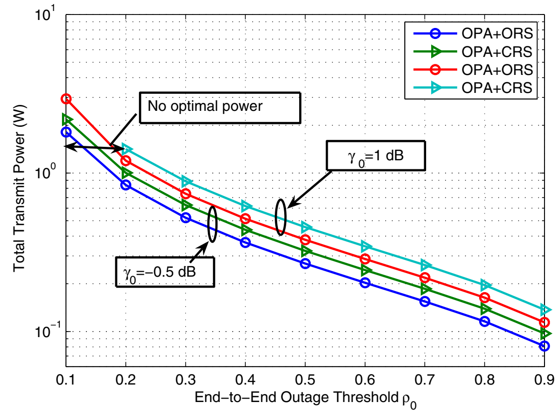

Fig. 4. Outage requirement ρ_0 VS. the total transmit power. ρ_0 ranges from 0.1 to 0.9, and $\gamma_0 = -0.5$, 1dB, respectively. $Q = 0.01$W.

figure, the total transmit power has a decreasing trend at a slow speed and then remains steady with an increasing Q. It is attributed to the reason that with each relay's increased maximum tolerated power, the previous infeasible relay path, which has the minimum total power but violates the power limits constraint, now becomes feasible and turns to be optimal solution. Thus, the total power consumption decreases. Further considering γ_0, the DF relays require a higher power to correctly decode received signal at 1dB than -0.5dB, thus leading to a higher cost of transmit power.

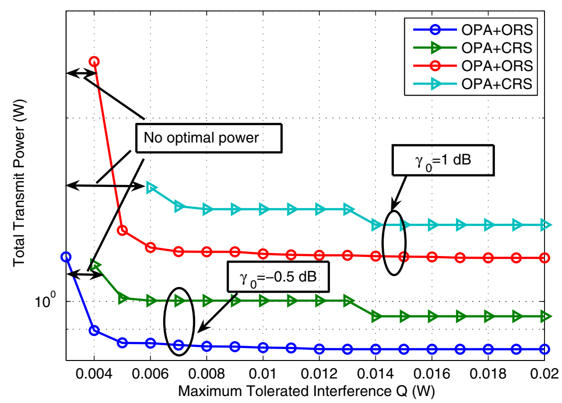

Fig. 5. Maximum tolerated interference Q VS. total transmit power. Q varies from 0.004W to 0.02W. $\gamma_0 = -0.5$, 1dB, respectively. $\rho_0 = 0.2$.

V. Conclusion

In this paper, we focus on power saving issue in the cluster based multi-hop cognitive network. Aiming at minimizing the total power consumption along relay path and combining with the end-to-end outage requirement and power budget constraint of CR transmitters, a joint Lagrange dual method based optimal power allocation and objective oriented optimal relay selection algorithm is proposed. Numerical results show that our scheme performs well in saving the total transmit power. Moreover, a conclusion is drawn that a higher power cost should be paid to guarantee a stringent outage requirement in return.

Acknowledgment

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References

[1] J. Sydir, R. Taori, "An Evolved Cellular System Architecture Incorporating Relay Stations", in *IEEE Commun. Mag.*, vol.47, no.6, pp.115-121, June 2009.

[2] E. Liu, Q. Q. Zhang, K. K. Leung, "Theoretical Analysis of Selective Relaying,Cooperative Multi-Hop Networks with Fairness Constraints", in *IEEE International Conf. on Commun. (ICC)*, June 2009.

[3] D. H. Zhang, Y. Z. Wang, J. H. Lu, "QoS Aware Relay Selection and Subcarrier Allocation in Cooperative OFDMA Systems", in *IEEE Commun. Lett.*, vol.14, no.4, pp.294-296, Apr. 2010.

[4] M. Shaat, F. Bader, "Asymptotically Optimal Resource Allocation in OFDM-Based Cognitive Networks with Multiple Relays", in *IEEE Trans. Wireleess Commun.*, vol.11, no.3, pp.892-897, Mar. 2012.

[5] Q. X. Zhang, Z. Y. Feng, C. He and P. Zhang, "Dynamic Time Slot Allocation for Cooperative Relay in Cognitive Radio Networks", in *Electronic Letters*, vol.46, no.13, pp.953-955, June 2010.

[6] J. B. Si, Z. Li, et al., "On the Performance of Cognitive Relay Networks Under Primary User's Outage Constraint", in *IEEE Commun. Lett.*, vol.15, no.4, pp.422-424, Apr. 2011.

[7] L. P. Luo, P. Zhang, G. C. Zhang, and J. Y. Qin, "Outage Performance for Cognitive Relay Networks with Underlay Spectrum Sharing", in *IEEE Commun. Lett.*, vol.15, no. 7, pp.710-712, July 2011.

[8] Z. Yan, X. Zhang and W. B. Wang, "Exact Outage Performance of Cognitive Relay Networks with Maximum Transmit Power Limits", in *IEEE Commun. Lett.*, vol.15, no.12, pp.1317-1319, Dec. 2011.

[9] H. Y. Lateef, D. McLernon and M. Ghogho, "Performance Analysis of Multi-User, Multi-Hop Cooperative Relay Networks over Nakagami-m Fading Channels", in *IEEE Commun. Lett.*, vol.15, no.7, pp.776-778, July 2011.

[10] R. Babaee and N. C. Beaulieu, "Cross-Layer Design for Multihop Wireless Relaying Networks", in *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp.3522-3531, Nov. 2010.

[11] L. Z. Ruan and V. K. N. Lau, "Decentralized Dynamic Hop Selection and Power Control in Cognitive Multi-Hop Relay Systems", in *IEEE Trans. Wireless Commun.*, vol.9, no.10, pp.3024-3030, Oct. 2010.

[12] Y. R. Tsai and L. C. Lin, "Optimal Power Allocation for Decode-and-Forward Cooperative Diversity Under an Outage Performance Constraint", in *IEEE Commun. Lett.*, vol.14, no.10, pp.945-947, Oct. 2010.