# Error Performance of Opportunistic Relaying with Outdated Channel State Information

Nien-En Wu[1], Po-Lin Yeh[2], and Hsueh-Jyh Li[1]
[1]Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan, ROC
[2]Mstar Semiconductor, Chupei, Taiwan, ROC
Email: f95942033@ntu.edu.tw

***Abstract*—Relay selection for cooperative communications has been proposed as a technique to provide remarkable gains in multi-relay cooperative networks. Opportunistic relaying (OR) is a relay selection scheme used in dual-hop decode-and-forward cooperative networks, shown to be globally outage-optimal under an aggregate power constraint. However, due to channel fluctuations, the channel state information (CSI) used in the selection process may become outdated and differ from the CSI during the actual transmission of data. In this work, we analyze the effect of outdated CSI on the performance of OR over independent but not necessarily identically distributed Rayleigh fading channels. Particularly, we derive an exact expression for the average symbol error probability (SEP) of OR with outdated CSI. An approximate but tight closed-form SEP expression is also provided. We further study the asymptotic behavior of the SEP and show that the diversity order of OR reduces to one due to the outdated CSI. Finally, we perform Monte Carlo simulations to verify the analytical results.**

***Keywords* – Cooperative diversity, decode-and-forward, opportunistic relaying, symbol error probability, outdated CSI.**

## I. Introduction

Cooperative diversity has been proposed as a promising technique to exploit the spatial diversity in a distributed fashion. It is based on the broadcast nature of the wireless medium and utilizes relay nodes to forward an overheard source message to a destination. In this way, a virtual antenna array is formed without the requirement of installing multiple antennas on small terminals [1].

Relay selection scheme, in which only the best relay is used for the data forwarding, is the most favorable among various cooperative transmission protocols. This is because that it avoids inter-node coordination and provides exceeding performance. Specifically, [2] and [3] showed that the relay selection scheme is globally outage-optimal under an aggregate power constraint, i.e., it minimizes the outage probability (OP) and outperforms techniques based on multiple relay transmissions.

Opportunistic relaying (OR) and selection cooperation (SC) are two major relay selection schemes for dual-hop decode-and-forward (DF) cooperative networks [4]. In OR, the relay selection is performed according to the metrics of the dual-hop relaying paths. In SC, on the other hand, the best relay is selected by using the metrics of the second-hop paths. The OP of OR was derived in [5]. In [6], the authors analyzed the average symbol error probability (SEP) for OR. The error probability and channel capacity of SC can be found in [7].

Relay selection technique is shown to provide remarkable gains under the assumption of perfect channel state information (CSI). However, there may exist a delay between the best-relay selection process and the actual transmission of data in practical systems such as LTE-Advanced [8]. This causes that the best relay is chosen based on outdated CSI. As a result, the best relay during the selection process is not necessarily the best relay during the actual data transmission. Several works have analyzed the impact of outdated CSI in various scenarios. For example, [9] studied the relay selection technique in amplify-and-forward (AF) cooperative networks and provided closed-form expressions for the average channel capacity and OP. Moreover, [10] investigated AF networks with beamforming. On the other hand, for DF networks, the OP and SEP of SC with outdated CSI over independent and identically distributed fading channels were analyzed in [11] and [12], respectively. To the best of our knowledge, the error performance analysis for OR in the presence of outdated CSI is not presently available in the literature.

In this work, we investigate the effect of outdated CSI on the performance of OR over independent but not necessarily identically distributed Rayleigh fading channels. Specifically, we derive an exact expression for the SEP of OR with outdated CSI. An accurate approximation for the SEP is also provided in closed form. Finally, simulation results are presented to validate the theoretical analysis.

## II. System Model

Figure 1 shows a relaying network which consists of a source, $S$, a destination, $D$, and $M$ relays, $R_i$, $i \in \{1,2,\ldots,M\}$. We assume that the direct transmission between the source and the destination is deep-faded, and the source transmits data to the destination with the help of the relays. In OR, only the best relay is selected out from the $M$ available relays and assigned for assisting the data transmission. A selection interval prior to the data transmission is assumed for the best-relay selection process. We consider flat fading channels and denote $h_{AB}$ as the $A$-$B$ channel coefficient during the selection interval. Throughout this paper, the pair $(A,B)$ belongs to the set $\{(S,R_i),(R_i,D), i=1,2,...,M\}$. In addition, let $\gamma_{AB}$ be the instantaneous signal-to-noise ratio (SNR) of the $A$-$B$ channel during the selection interval. In OR, the best relay is the one with the highest $\min(\gamma_{SR_i},\gamma_{R_iD})$ [4]. All nodes are assumed to use a single antenna. Moreover, the relays are assumed to follow the DF relaying strategy and operate in the half-duplex mode. Therefore, a relaying transmission consists of two phases. During the first phase, the source transmits the data signal to the best relay. During the second phase, the best relay detects the received signal and then re-modulates the data information and forwards it to the destination.

We model the channel coefficients, $h_{AB}$, as independent zero-mean complex Gaussian random variables with variances

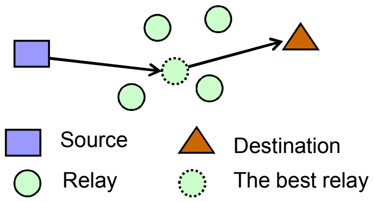


Fig. 1. A relaying network.

$\lambda_{AB}$. Let $\tilde{h}_{AB}$ be the *A-B* channel coefficient during the actual transmission of data. The channel coefficients $h_{AB}$ and $\tilde{h}_{AB}$ are related as follows [13]:

$$\tilde{h}_{AB} = \rho_{AB} h_{AB} + \sqrt{1-\rho_{AB}^2}\, w_{AB}, \tag{1}$$

where $\rho_{AB}$ is the channel correlation coefficient, $w_{AB}$ is a zero-mean complex Gaussian random variable with variance $\lambda_{AB}$ and is independent of $h_{AB}$. Under the assumption of the Jakes' model, the correlation coefficient is given by $\rho_{AB} = J_0(2\pi f_{AB} t_{AB})$ [13] where $f_{AB}$ is the maximum Doppler frequency on the *A-B* link, $t_{AB}$ represents the time difference between the channel coefficient, $\tilde{h}_{AB}$, and its outdated value, $h_{AB}$, and $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind. In addition, the maximum Doppler frequency can be expressed as $f_{AB} = v_{AB} f_c / c$ where $v_{AB}$ is the relative speed between nodes *A* and *B*, $f_c$ is the carrier frequency, and *c* is the speed of light. We assume that the total transmitted power of the relaying network under consideration, *P*, is evenly distributed among the source and the best relay for simplicity. Moreover, the thermal noises at all receivers are assumed to be mutual independent complex additive white Gaussian noises (AWGN) with a mean of zero and a variance of $N_0$. The instantaneous SNR $\gamma_{AB}$ can then be expressed as $\gamma_{AB} = \bar{\gamma} |h_{AB}|^2$ where $\bar{\gamma} = 0.5P/N_0$ is the average SNR. The probability density function (pdf) and moment generating function (MGF) of $\gamma_{AB}$ can be written as $f_{\gamma_{AB}}(u) = \exp(-u/\bar{\gamma}_{AB})/\bar{\gamma}_{AB}$ and

$$\mathcal{M}_{\gamma_{AB}}(s) \equiv \mathbb{E}_{\gamma_{AB}}\{e^{s\gamma_{AB}}\} = 1/(1-s\bar{\gamma}_{AB}), \tag{2}$$

respectively, where $\mathbb{E}_{\gamma_{AB}}\{\cdot\}$ denotes the expectation operator with respect to $\gamma_{AB}$ and $\bar{\gamma}_{AB} \equiv \mathbb{E}\{\gamma_{AB}\} = \bar{\gamma}\lambda_{AB}$. Denote $\tilde{\gamma}_{AB} = \bar{\gamma} |\tilde{h}_{AB}|^2$ as the instantaneous SNR of the *A-B* channel during the actual data transmission. From (1), it can be shown that $\tilde{\gamma}_{AB}$, given $\gamma_{AB}$, follows a non-central chi-square distribution with two degrees of freedom and non-centrality parameter $\rho_{AB}^2\gamma_{AB}$. Accordingly, the conditional MGF of $\tilde{\gamma}_{AB}$, given $\gamma_{AB}$, is expressed as [13]:

$$\begin{aligned}\mathcal{M}_{\tilde{\gamma}_{AB}|\gamma_{AB}}(s) &\equiv \mathbb{E}\{e^{s\tilde{\gamma}_{AB}} | \gamma_{AB}\} \\ &= \mathcal{M}_{\gamma_{AB}}\big(s(1-\rho_{AB}^2)\big) e^{\gamma_{AB}\Psi_{\gamma_{AB}}(s)},\end{aligned} \tag{3}$$

where we define $\Psi_{\gamma_{AB}}(s) \equiv s\rho_{AB}^2 / (1 - s\bar{\gamma}_{AB}(1-\rho_{AB}^2))$.

## III. SEP Analysis

In this section, we analyze the SEP of OR with outdated CSI. We derive an exact expression for the SEP. An approximate closed-form SEP expression is also provided.

Consider *L*-ary phase shift keying (*L*-PSK) modulation scheme. The modulated symbol can be expressed as $x^k = \exp(j2\pi(k-1)/L)$ for $k = 1,\ldots,L$. The SEP for *L*-PSK modulation can be written as [14]

$$\Pr(\mathcal{E}) = \frac{1}{\pi}\mathbb{E}_\gamma\left\{\int_0^{\pi-\Theta} \exp\left(\frac{-b\gamma}{\sin^2\theta}\right) d\theta\right\}, \tag{4}$$

where $b = \sin^2(\Theta)$ with $\Theta = \pi/L$ and $\gamma$ is the end-to-end SNR. Consider an event that an *L*-PSK modulated symbol $x^k$ is transmitted and it is detected as a symbol $x^l$ for $l = 1,\ldots,L$ and $l \neq k$. The probability of this event is given by [14]

$$\begin{aligned}\Pr(\mathcal{E}^{k,l}) = \frac{1}{2\pi}\mathbb{E}_\gamma\Bigg\{&\int_0^{\pi-\Theta_{k,l}} \exp\left(\frac{-b_{k,l}\gamma}{\sin^2\theta}\right) d\theta \\ &- \int_0^{\pi-\Theta_{k,l+1}} \exp\left(\frac{-b_{k,l+1}\gamma}{\sin^2\theta}\right) d\theta\Bigg\},\end{aligned} \tag{5}$$

where $b_{k,l} = \sin^2(\Theta_{k,l})$ with $\Theta_{k,l} = (2\pi(l-k)-\pi)/L$.

*Lemma 1:* Consider a set $\mathcal{S} = \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$ consisting of *N* exponentially distributed random variables with $\mathbb{E}\{\alpha_i\} = \bar{\alpha}_i$. Let the *i*th element, $\alpha_i$, have the largest value among $\mathcal{S}$, i.e., $\alpha_i = \max_{1\le k\le N}\{\alpha_k\}$ and denote $\mathcal{M}^{\mathcal{S}}_{\max,\alpha_i}(s)$ as the MGF of $\alpha_i$. The MGF of $\alpha_i$ is given by

$$\mathcal{M}^{\mathcal{S}}_{\max,\alpha_i}(s) = \frac{1}{1-\bar{\alpha}_i s} + \sum_{l=1}^{N-1} \sum_{\substack{\mathcal{S}_l \subseteq \mathcal{S}^i, \\ |\mathcal{S}_l| = l}} \frac{(-1)^l}{1 + \sum_{k\in\mathcal{S}_l} \bar{\alpha}_i/\bar{\alpha}_k - \bar{\alpha}_i s}, \tag{6}$$

where $|\mathcal{S}_l|$ denotes the cardinality of $\mathcal{S}_l$ and $\mathcal{S}^i$ is the relative complement of $\alpha_i$ in the set $\mathcal{S}$, i.e., $\mathcal{S}^i = \{\alpha_1, \ldots \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_N\}$.

*Proof:* From the definition of MGF, we have

$$\begin{aligned}&\mathcal{M}^{\mathcal{S}}_{\max,\alpha_i}(s) \\ &= \int_0^\infty e^{s\alpha_i} \Pr(\alpha_i > \alpha_k, \forall k, k\neq i \,|\, \alpha_i) f_{\alpha_i}(\alpha_i) d\alpha_i \\ &= \int_0^\infty e^{s\alpha_i} \left[\prod_{k=1,k\neq i}^{N} 1 - e^{-\alpha_i/\bar{\alpha}_k}\right] \frac{1}{\bar{\alpha}_i} e^{-\alpha_i/\bar{\alpha}_i} d\alpha_i,\end{aligned} \tag{7}$$

where $f_{\alpha_i}(\alpha_i)$ is the pdf of $\alpha_i$. To expand the product in (7), we utilize the multinomial expansion identity [4]:

$$\prod_{k=1,k\neq i}^{N} 1 - e^{-\alpha_i/\bar{\alpha}_k} = 1 + \sum_{l=1}^{N-1} \sum_{\substack{\mathcal{S}_l \subseteq \mathcal{S}^i, \\ |\mathcal{S}_l| = l}} (-1)^l \exp\Big(-\alpha_i \sum_{k\in\mathcal{S}_l} \bar{\alpha}_k^{-1}\Big), \tag{8}$$

where the second summation is over all possible subsets of $\mathcal{S}^i$, whose number of elements equals to *l*. A more detailed description of the multinomial expansion identity is provided in [4]. We assume that the parameter *s* in (7) is negative, which is valid throughout this paper. Substituting (8) into (7) and solving the integral, we obtain the result in (6). □

Define a random variable $\gamma_i$ as $\gamma_i \equiv \min(\gamma_{SR_i}, \gamma_{R_iD})$ for $i \in \{1, 2, \ldots, M\}$. It can be shown that $\gamma_i$ follows the exponential distribution with mean $\bar{\gamma}_i = \bar{\gamma}_{SR_i}\bar{\gamma}_{R_iD} / (\bar{\gamma}_{SR_i} + \bar{\gamma}_{R_iD})$. In addition, let $\mathcal{G}$ be a set consisting of all the $\gamma_i$, i.e., $\mathcal{G} = \{\gamma_1, \gamma_2, \ldots, \gamma_M\}$ and $\mathcal{G}^i$ be the relative complement of $\gamma_i$ in the set $\mathcal{G}$, i.e., $\mathcal{G}^i = \{\gamma_1, \ldots, \gamma_{i-1}, \gamma_{i+1}, \ldots, \gamma_M\}$.

Let $\mathcal{E}_{AB}$ be the event that a symbol error occurs on the *A-B* channel link. Moreover, denote $\mathcal{E}^{k,l}_{AB}$ as the event that an *L*-PSK symbol $x^k$ is transmitted from a node *A* while it is detected as a symbol $x^l$ for $l = 1,\ldots,L$ and $l \neq k$ at a node *B*. Without loss of generality, assume that the symbol $x^k$ is transmitted from the source. The SEP of OR can be expressed as [14]

$$\Pr(\mathcal{E}_{SR_i},\xi_{i,1})=\mathcal{I}_1\big(\pi-\Theta,1+\alpha_i,b\bar{\gamma}(\lambda_{SR_i}+\mu_{SR_i}\alpha_i)\big)+\sum_{l=1}^{M-1}\sum_{\mathcal{G}_l^i\subseteq\mathcal{G}^i,|\mathcal{G}_l^i|=l}(-1)^l\mathcal{I}_1\Big(\pi-\Theta,1+\alpha_i+\bar{\gamma}_{SR_i}\sum_{k\in\mathcal{G}_l^i}\bar{\gamma}_k^{-1},b\bar{\gamma}\big(\lambda_{SR_i}+\mu_{SR_i}(\alpha_i+\bar{\gamma}_{SR_i}\sum_{k\in\mathcal{G}_l^i}\bar{\gamma}_k^{-1})\big)\Big),\tag{11}$$

$$\Pr(\mathcal{E}_{SR_i},\xi_{i,2})=\mathcal{I}_2\big(\pi-\Theta,1,b\bar{\gamma}_{SR_i},1+\alpha_i^{-1},b\bar{\gamma}(\lambda_{R_iD}+\mu_{SR_i}),b\mu_{SR_i}\bar{\gamma}\big)+\sum_{l=1}^{M-1}\sum_{\mathcal{G}_l^i\subseteq\mathcal{G}^i,|\mathcal{G}_l^i|=l}(-1)^l\mathcal{I}_2\Big(\pi-\Theta,1,b\bar{\gamma}_{SR_i},1+\alpha_i^{-1}+\bar{\gamma}_{R_iD}\sum_{k\in\mathcal{G}_l^i}\bar{\gamma}_k^{-1},b\bar{\gamma}\big(\lambda_{R_iD}+\mu_{SR_i}(1+\bar{\gamma}_{R_iD}\sum_{k\in\mathcal{G}_l^i}\bar{\gamma}_k^{-1})\big),b\mu_{SR_i}\bar{\gamma}\Big),\tag{13}$$

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$$P_{\mathcal{E}}=\sum_{i=1}^{M}\Big[\Pr(\mathcal{E}_{SR_i},\phi_i)+\Pr(\mathcal{E}_{R_iD},\phi_i)-\Pr(\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\phi_i)-\sum_{l=1,l\neq k}^{L}\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\phi_i)\Big],\tag{9}$$

where $\phi_i$ denotes the event that the $i$th relay is chosen as the best relay. For facilitating the calculation of (9), we further define $\xi_{i,1}$ as the event that the $i$th relay is selected as the best relay and $\gamma_{R_iD}>\gamma_{SR_i}$, i.e., $\xi_{i,1}=\{\gamma_{R_iD}>\gamma_{SR_i}>\gamma_k, \forall k, k\neq i\}$. Moreover, let $\xi_{i,2}$ be the event that the $i$th relay is selected as the best relay and $\gamma_{SR_i}>\gamma_{R_iD}$, that is, $\xi_{i,2}=\{\gamma_{SR_i}>\gamma_{R_iD}>\gamma_k,\forall k,k\neq i\}$. Then, (9) can be rewritten as

$$P_{\mathcal{E}}=\sum_{j=1}^{2}\sum_{i=1}^{M}\Big[\Pr(\mathcal{E}_{SR_i},\xi_{i,j})+\Pr(\mathcal{E}_{R_iD},\xi_{i,j})-\Pr(\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\xi_{i,j})-\sum_{l=1,l\neq k}^{L}\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,j})\Big].\tag{10}$$

In the following, we analyze each term in (10) respectively.

*A. Analysis of* $\Pr(\mathcal{E}_{SR_i},\xi_{i,j})$ *and* $\Pr(\mathcal{E}_{R_iD},\xi_{i,j})$ *for j=1,2:*

First, we show in Appendix A that $\Pr(\mathcal{E}_{SR_i},\xi_{i,1})$ can be expressed as (11) at the top of this page where $\alpha_i=\bar{\gamma}_{SR_i}/\bar{\gamma}_{R_iD}$, $\mu_{SR_i}=\lambda_{SR_i}(1-\rho_{SR_i}^2)$ and

$$\mathcal{I}_1(\phi,c_1,c_2)=\frac{1}{\pi}\int_0^{\phi}\frac{\sin^2\theta}{c_1\sin^2\theta+c_2}d\theta=\frac{1}{\pi c_1}\left[\phi-t_1\left(\frac{\pi}{2}+\arctan t_2\right)\right],\tag{12}$$

with $t_1=\sqrt{c_2/(c_1+c_2)}\,\mathrm{sgn}\,\phi$ and $t_2=-t_1\cot\phi$ [6] in which $\mathrm{sgn}(\cdot)$ denotes the sign function. Furthermore, $\Pr(\mathcal{E}_{SR_i},\xi_{i,2})$ can be expressed as (13) (shown in Appendix A) where

$$\mathcal{I}_2(\phi,c_3,c_4,c_5,c_6,c_7)=\frac{1}{\pi}\int_0^{\phi}[\mathcal{J}_1(\theta)+\mathcal{J}_2(\theta)]d\theta,\tag{14}$$

with

$$\mathcal{J}_1(\theta)=\frac{\sin^2\theta}{c_3\sin^2\theta+c_4}\frac{\sin^2\theta}{c_5\sin^2\theta+c_6},$$

and

$$\mathcal{J}_2(\theta)=\frac{\sin^2\theta}{c_3\sin^2\theta+c_4}\frac{c_7}{c_5\sin^2\theta+c_6}.$$

The closed-form expression for the integral of function $\mathcal{J}_1(\theta)$ in (14) can be found in [6] and is omitted here due to lack of space. On the other hand, the closed-form expression for the integral of $\mathcal{J}_2(\theta)$ is given in Appendix A. Due to the symmetry, we can obtain the closed-form expressions for $\Pr(\mathcal{E}_{R_iD},\xi_{i,2})$ and $\Pr(\mathcal{E}_{R_iD},\xi_{i,1})$ by replacing $\lambda_{SR_i}$, $\lambda_{R_iD}$ and $\rho_{SR_i}$ in (11) and (13) by $\lambda_{R_iD}$, $\lambda_{SR_i}$ and $\rho_{R_iD}$, respectively.

*B. Analysis of* $\Pr(\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\xi_{i,j})$ *and* $\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,j})$ *for j=1,2:*

For $\Pr(\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\xi_{i,1})$, we utilize (3) to yield (15) at the top of next page where $a(\theta)=-b/\sin^2\theta$ and we define

$$\mathcal{F}(\Theta_1,\Theta_2,\varpi(\theta_1,\theta_2))\equiv\frac{1}{\pi^2}\int_0^{\pi-\Theta_2}\left[\int_0^{\pi-\Theta_1}\varpi(\theta_1,\theta_2)d\theta_1\right]d\theta_2,$$

in which $\varpi(\theta_1,\theta_2)$ denotes a function of variables $\theta_1$ and $\theta_2$. To calculate $\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,1})$, we need to use (5). Observing the similarities between (4) and (5), we can obtain $\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,1})$ from the results in (15). Specifically, by using (5), we have (16) where $\beta(\theta,u)=-u/\sin^2\theta$. Finally, $\Pr(\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\xi_{i,2})$ and $\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,2})$ can be obtained by replacing $\bar{\gamma}_{SR_i}$, $\bar{\gamma}_{R_iD}$, $\rho_{SR_i}$ and $\rho_{R_iD}$ in (15) and (16) by $\bar{\gamma}_{R_iD}$, $\bar{\gamma}_{SR_i}$, $\rho_{R_iD}$ and $\rho_{SR_i}$, respectively.

Substituting the above results into (10), the finite-range integral SEP expression can be numerically evaluated more efficiently than via Monte Carlo simulations. Alternatively, note that the event $\{\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\xi_{i,j}\}$ in (10) occurs with significantly low probability since the best relay is assigned for data forwarding. In addition, it is clear that we can approximate the probability of the event $\{\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,j}\}$ in (10) by zero. Accordingly, an approximate closed-form SEP expression is given by

$$P_{\mathcal{E}}\approx\sum_{j=1}^{2}\sum_{i=1}^{M}\Big[\Pr(\mathcal{E}_{SR_i},\xi_{i,j})+\Pr(\mathcal{E}_{R_iD},\xi_{i,j})\Big].\tag{17}$$

*C. Asymptotic SEP*

For more insights, we examine the asymptotic SEP as SNR tends to infinity. Consider BPSK modulation and the case $0<\rho_{AB}<1$. Observing (11) and (13), we have $c_{2n-1}\ll c_{2n}$ for $n=1,2,3$ in the high SNR region where the notations $c_1,\ldots,c_6$ in (12) and (14) are used to represent the corresponding terms in (11) and (13). Substituting $\phi=\pi/2$ into (12), (14), and (23) in Appendix A, and utilizing the fact that $\sqrt{c_{2n}/(c_{2n-1}+c_{2n})}\approx 1-c_{2n-1}/[2(c_{2n-1}+c_{2n})]$ for $n=1,2,3$, we obtain $\mathcal{I}_1(\pi/2,c_1,c_2)=1/(4c_2)$ and $\mathcal{I}_2(\pi/2,c_3,c_4,\ldots,c_7)=c_7/(4c_4c_6)$. Accordingly, from (11) and (13), we yield

$$\Pr(\mathcal{E}_{SR_i},\xi_{i,1})=\frac{1}{4b\bar{\gamma}}\left[\frac{1}{\lambda_{SR_i}+\alpha_i\mu_{SR_i}}+\sum_{l=1}^{M-1}\sum_{\mathcal{G}_l^i\subseteq\mathcal{G}^i,|\mathcal{G}_l^i|=l}\frac{(-1)^l}{\lambda_{SR_i}+\alpha_i\mu_{SR_i}+\mu_{SR_i}\lambda_{SR_i}\sum_{k\in\mathcal{G}_l^i}\lambda_k^{-1}}\right],\tag{18}$$

and

$$
\begin{aligned}
\Pr(\mathcal{E}_{SR_i},\mathcal{E}_{R_iD},\xi_{i,1}) &= \frac{1}{\pi^2}\int_0^{\pi-\Theta}\int_0^{\pi-\Theta}\mathbb{E}_{\gamma_{SR_i}}\Big\{\int_0^{\infty} e^{a(\theta_1)u_1} f_{\tilde{\gamma}_{SR_i}|\gamma_{SR_i}}(u_1\,|\,\gamma_{SR_i})\,du_1 \\
&\quad \times\int_{\gamma_{SR_i}}^{\infty}\int_0^{\infty} e^{a(\theta_2)u_2} f_{\tilde{\gamma}_{R_iD}|\gamma_{R_iD}}(u_2\,|\,u_3) f_{\gamma_{R_iD}}(u_3)\,du_2du_3 \Pr(\gamma_{SR_i}>\gamma_k,\forall k,k\neq i\,|\,\gamma_{SR_i})\Big\}d\theta_1 d\theta_2 \\
&= \mathcal{F}\Big(\Theta,\Theta,\mathcal{M}_{\gamma_{SR_i}}(a(\theta_1)(1-\rho_{SR_i}^2))\mathcal{M}_{\gamma_{R_iD}}(a(\theta_2))\mathcal{M}^{\mathcal{B}_i}_{\max,\gamma_{SR_i}}\big(\Psi_{\gamma_{SR_i}}(a(\theta_1))+\Psi_{\gamma_{R_iD}}(a(\theta_2))-\bar{\gamma}_{R_iD}^{-1}\big)\Big) \\
&\equiv \mathcal{F}\big(\Theta,\Theta,\mathcal{H}_{i,1}(a(\theta_1),a(\theta_2))\big), \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\Pr(\mathcal{E}_{SR_i}^{k,l},\mathcal{E}_{R_iD}^{l,k},\xi_{i,1}) &= \frac{1}{4}\Big[\mathcal{F}\big(\Theta_{k,l},\Theta_{l,k},\mathcal{H}_{i,1}(\beta(\theta_1,b_{k,l}),\beta(\theta_2,b_{l,k}))\big)-\mathcal{F}\big(\Theta_{k,l},\Theta_{l,k+1},\mathcal{H}_{i,1}(\beta(\theta_1,b_{k,l}),\beta(\theta_2,b_{l,k+1}))\big) \\
&\quad -\mathcal{F}\big(\Theta_{k,l+1},\Theta_{l,k},\mathcal{H}_{i,1}(\beta(\theta_1,b_{k,l+1}),\beta(\theta_2,b_{l,k}))\big)+\mathcal{F}\big(\Theta_{k,l+1},\Theta_{l,k+1},\mathcal{H}_{i,1}(\beta(\theta_1,b_{k,l+1}),\beta(\theta_2,b_{l,k+1}))\big)\Big], \qquad (16)
\end{aligned}
$$

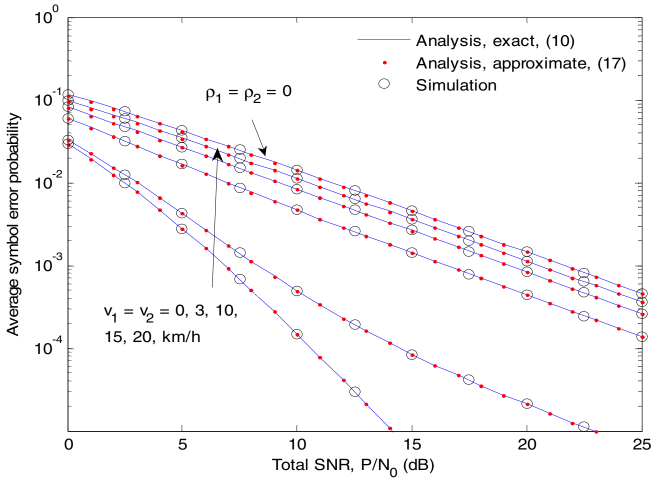


Fig. 2. SEP of OR versus the total SNR, $P/N_0$, for various speed.

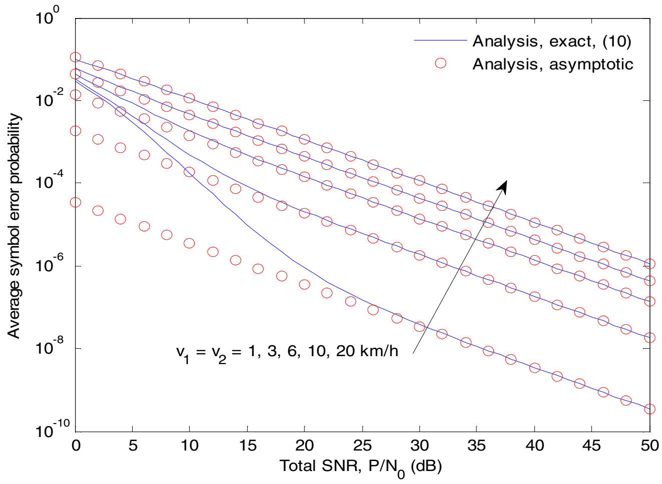


Fig. 3. Asymptotic SEP of OR with outdated CSI.

$$
\Pr(\mathcal{E}_{SR_i},\xi_{i,2}) = \frac{\mu_{SR_i}}{4b\lambda_{SR_i}\bar{\gamma}}\left[\frac{1}{\mu_{SR_i}+\lambda_{R_iD}} + \sum_{l=1}^{M-1}\sum_{\mathcal{G}_l^i\subseteq\mathcal{G}^i,|\mathcal{G}_l^i|=l}\frac{(-1)^l}{\mu_{SR_i}+\lambda_{R_iD}+\mu_{SR_i}\lambda_{R_iD}\sum_{k\in\mathcal{G}_l^i}\lambda_k^{-1}}\right], \qquad (19)
$$

where $\lambda_i = \bar{\gamma}_i/\bar{\gamma}$. From (18), (19) and (17), it is clear that the asymptotic SEP depends on $\bar{\gamma}^{-1}$, which reveals that the diversity order of OR reduces to one under the outdated CSI.

## IV. Simulation Results And Discussions

In this section, simulation results are presented to verify the theoretical analysis. We set $\lambda_{AB} = d_{AB}^{-\alpha}$ to take into account the effect of path loss where $d_{AB}$ denotes the distance between nodes $A$ and $B$ and $\alpha$ is the path loss exponent. In all simulations, we normalize $d_{SD}$ as 1, and assume $d_{SR_i} = 1 - d_{R_iD}, \forall i$ and $\alpha = 3$. In addition, let $\rho_{SR_i} = \rho_1$ and $\rho_{R_iD} = \rho_2$, $\forall i$, (i.e., $v_{SR_i} = v_1$ and $v_{R_iD} = v_2$, $\forall i$) for the sake of simplicity. Finally, we follow the Jakes' model with the parameters $t_{AB} = 6$ msec and $f_c = 2$ GHz, as specified in [8] for the coordinated multipoint transmission/reception techniques in LTE-Advanced systems.

In Fig. 2, we illustrate the theoretical SEP as a function of the total SNR of the network, $P/N_0$, for various speed where an asymmetric network consisting of $M = 3$ relays with $d_{SR_i} = \{0.3, 0.5, 0.6\}$ and BPSK modulation are considered. The analytical results are obtained by using (10) or (17). It is noteworthy that the simulated results match well with the exact SEP expression in (10). In addition, note that the approximate result in (17) is accurate. Fig. 2 shows that the performance of OR is degraded severely as the CSI is outdated even at low speed such as 3 km/h (corresponding to $\rho = 0.99$).

In Fig. 3, we investigate the asymptotic behavior of OR with outdated CSI. The simulation environments in Fig. 2 are considered and the analytical asymptotic SEP is obtained by using (17) with (18) and (19). Fig. 3 shows that the asymptotic results precisely describe the high SNR behavior of OR and even agree well with the exact results at high speed. In addition, it reveals that the diversity order of OR reduces to one even when the correlation coefficients lie in the proximity of one as in the case of 1 km/h (i.e., $\rho = 0.999$).

We study the SEP performance of OR under imbalanced correlation coefficients in Fig. 4. In which we consider two four-relay networks with $d_{SR_i} = 0.5, \forall i$ and $d_{SR_i} = 0.1, \forall i$, respectively, and assume QPSK modulation. Two scenarios are investigated: $(v_1, v_2) = (0, 10)$ and $(v_1, v_2) = (10, 0)$ km/h. We first note that the SEP is degraded severely when either $\rho_1$ or $\rho_2$ deviates from unity. In addition, in the network with $d_{SR_i} = 0.5$, Fig. 4 shows that OR exhibits the same behavior in both scenarios due to the symmetry of network. On the other hand, when the relays are close to the source as in the case of $d_{SR_i} = 0.1$, OR is more sensitive to the outdated $R_i$-$D$ CSI than the outdated $S$-$R_i$ CSI. This is because that the $R_i$-$D$ channels dominate the metrics for the relay selection, i.e., $\min(\gamma_{SR_i}, \gamma_{R_iD})$, in this case.

## V. Conclusions

In this paper, we analyzed the OR scheme in the presence of outdated CSI. We derived an exact analytical expression for

$$\begin{aligned}\Pr(\mathcal{E}_{SR_i},\xi_{i,1}) &= \mathbb{E}_{\gamma_{SR_i}}\left\{\mathbb{E}_{\tilde{\gamma}_{SR_i}}\left\{\Pr(\mathcal{E}_{SR_i},\xi_{i,1}\mid\gamma_{SR_i})\right\}\right\}\\ &= \frac{1}{\pi}\int_0^{\pi-\Theta}\mathbb{E}_{\gamma_{SR_i}}\left\{\int_0^{\infty} e^{a(\theta)u_1} f_{\tilde{\gamma}_{SR_i}|\gamma_{SR_i}}(u_1\mid\gamma_{SR_i})\,du_1\int_{\gamma_{SR_i}}^{\infty} f_{\gamma_{R_iD}}(u_2)\,du_2\,\Pr(\gamma_{SR_i}>\gamma_k,\forall k,k\neq i|\gamma_{SR_i})\right\}d\theta,\end{aligned} \tag{20}$$

$$\Pr(\mathcal{E}_{SR_i},\xi_{i,2}) = \frac{1}{\pi}\int_0^{\pi-\Theta}\mathbb{E}_{\gamma_{R_iD}}\left\{\int_{\gamma_{R_iD}}^{\infty}\int_0^{\infty} e^{a(\theta)u_1} f_{\tilde{\gamma}_{SR_i}|\gamma_{SR_i}}(u_1\mid u_2) f_{\gamma_{SR_i}}(u_2)\,du_1 du_2\,\Pr(\gamma_{R_iD}>\gamma_k,\forall k,k\neq i\mid\gamma_{R_iD})\right\}d\theta. \tag{22}$$

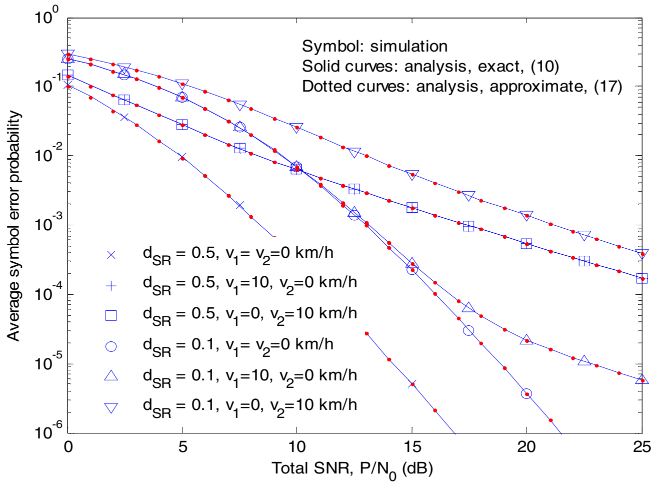


Fig. 4. SEP of OR with imbalanced correlation coefficients.

the SEP which can be numerically evaluated efficiently. A closed-form approximate SEP expression is also provided. Furthermore, we examined the asymptotic behavior of OR with BPSK modulation and showed that the diversity order reduces to one due to the outdated CSI. We verified our analytical results through simulations.

## APPENDIX A

In this appendix, we provide the derivation of (11) and (13). First, for $\Pr(\mathcal{E}_{SR_i},\xi_{i,1})$, we use (4) to obtain (20) where $a(\theta)=-b/\sin^2\theta$ and $f_{\tilde{\gamma}_{AB}|\gamma_{AB}}(u_1\mid u_2)$ denotes the conditional pdf of $\tilde{\gamma}_{AB}$ given $\gamma_{AB}$. In addition, by utilizing (3), we have

$$\begin{aligned}\Pr(\mathcal{E}_{SR_i},\xi_{i,1}) = \frac{1}{\pi}\int_0^{\pi-\Theta}&\mathcal{M}_{\gamma_{SR_i}}(a(\theta)(1-\rho_{SR_i}^2))\\ &\times\mathcal{M}^{\mathcal{B}_i}_{\max,\gamma_{SR_i}}(\Psi_{\gamma_{SR_i}}(a(\theta))-\bar{\gamma}_{R_iD}^{-1})d\theta,\end{aligned} \tag{21}$$

where $\mathcal{B}_i\equiv\{\mathcal{G}^i\cup\gamma_{SR_i}\}$. Substituting (6) into (21), and carrying out some manipulations, (21) is rewritten as (11). Similarly, $\Pr(\mathcal{E}_{SR_i},\xi_{i,2})$ can be expressed as (22). Using (3) and (6), after some manipulations, we yield (13). By applying partial fraction expansion, we can rewrite $\mathcal{J}_2(\theta)$ in (14) as

$$\mathcal{J}_2(\theta)=\frac{c_7}{c_3c_6-c_4c_5}\left(\frac{c_6}{c_5\sin^2\theta+c_6}-\frac{c_4}{c_3\sin^2\theta+c_4}\right),$$

for $c_4/c_3\neq c_6/c_5$ and use [15, eq. 2.562.1] to obtain

$$\begin{aligned}&\frac{1}{\pi}\int_0^{\phi}\mathcal{J}_2(\theta)d\theta\\ &=\frac{c_7}{\pi(c_3c_6-c_4c_5)}\left[\frac{c_6\,\mathrm{sgn}\,c_6}{\sqrt{c_6(c_5+c_6)}}\arctan\left(\sqrt{1+c_5/c_6}\,\tan\phi\right)\right.\\ &\left.-\frac{c_4\,\mathrm{sgn}\,c_4}{\sqrt{c_4(c_3+c_4)}}\arctan\left(\sqrt{1+c_3/c_4}\,\tan\phi\right)\right].\end{aligned} \tag{23}$$

When $c_4/c_3=c_6/c_5=c$ which corresponds to $\rho_{AB}=0$, $\mathcal{J}_2(\theta)$ can be expanded as

$$\mathcal{J}_2(\theta)=\frac{c_7}{c_3c_5}\left(\frac{1}{\sin^2\theta+c}-\frac{c}{(\sin^2\theta+c)^2}\right),$$

and [15, eq. 2.563.1] is used to yield

$$\begin{aligned}\frac{1}{\pi}\int_0^{\phi}\mathcal{J}_2(\theta)d\theta&=\frac{c_7}{2\pi c_3c_5(c+1)}\\ &\times\left\{\frac{\mathrm{sgn}\,c}{\sqrt{c(c+1)}}\arctan\left(\sqrt{\frac{c+1}{c}}\tan\phi\right)-\frac{\sin\phi\cos\phi}{c+\sin^2\phi}\right\}.\end{aligned} \tag{24}$$

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