Predicted Eigenvalue Threshold Based Spectrum Sensing With Correlated Multiple-Antennas

K. Hassan[*§], R. Gautier[†‡], I. Dayoub[§], E. Radoi [†‡] and M. Berbineau[*]
[*]Univ Lille Nord de France, F-59000 Lille, IFSTTAR, LEOST, F-59650 Villeneuve d'Ascq
[†] Université Européenne de Bretagne, France
[‡] Université de Brest ; CNRS, UMR 3192 Lab-STICC, ISSTB
6 avenue Victor Le Gorgeu, CS 93837, 29238 Brest cedex 3, France
[§]Univ Lille Nord de France, F-59000 Lille, IEMN, DOAE, F-59313 Valenciennes

***Abstract*—In this paper, we consider the problem of sensing a primary user in a cognitive radio network by employing multiple-antennas at the secondary user. Among the many spectrum-sensing methods, the predicted eigenvalue threshold (PET) based method is a promising non-parametric blind method that can reliably detect the primary users without any prior information. Also, a simplified PET sensing method, which needs to compare only one eigenvalue to its threshold, is introduced. A performance comparison between the proposed method and other existing methods is provided. Spatial antenna correlation at the secondary user is a crucial factor for practical systems. The effect of the spatial correlation presence on the different sensing methods is investigated.**

***Index Terms*—cognitive radio, spectrum sensing, multiple-antenna, predicted eigenvalue threshold, spatial correlation.**

I. Introduction

Recently, the rapid growth in wireless communications has contributed to a huge demand on the deployment of new wireless services. However, recent research published by the Federal Communications Commission (FCC) [1] shows that the traditional static frequency allocation policy is not efficient and results in poor spectrum utilization. The dramatic increase in the demand for radio spectrum and the actual low spectral efficiency has spurred the development of a next generation wireless technology referred to as cognitive radio (CR). An early work by Mitola introducing the concept of CR is [2].

Ultimately, a CR device (secondary user) must be aware of its radio environment and capable of detecting the licensed users, also known as primary users (PUs), i.e. a CR must identify the unoccupied frequency bands, called *white spaces*. Sensing the spectrum and dynamically accessing the white spaces will significantly improve the spectrum utilization efficiency. This ability to detect the white spaces in the spectrum band of interest by the secondary user (SU) is called *spectrum sensing*. In order to avoid interference with PUs, the SU must quickly and robustly sense which parts of the relevant spectrum are available or not. For instance, the upcoming CR based standard IEEE 802.22, which is a wireless regional network standard (WRAN), requires the detector to sense a PU at a signal-to-noise ratio (SNR) of at least -22dB. At this power level, the detection probability must be higher than 0.9, while the false alarm probability is lower than 0.1 [3]. Obviously, spectrum sensing is the backbone of any autonomous CR.

The performance of energy detector (ED) based sensing is limited since it is very sensitive to noise variations. On the other hand, the energy detector is simple, non coherent and needs no prior knowledge on the primary users signals [4]. Cyclostationarity-based detectors provide higher performance than energy detectors and are more robust to noise variations [5]. But they require prior knowledge on the cyclic frequencies of the PU's signal. Without this prior knowledge, the cyclostationarity-based detectors have a large complexity [6].

Multiple-antenna systems have been widely deployed to improve the transmission reliability in wireless communications. In a cognitive radio network, multiple-antennas at SUs are beneficial not only for a reliable communications but also to improve the sensing performance. The multiple-antenna techniques are employed to exploit the spatial correlations of multiple received signals. The maximum ratio combining, the equal gain combining and the selection combining techniques are applied to better sense the spectrum [7], [8]. These methods are ED based and suffers of the noise uncertainty. To overcome this difficulty, the authors in [9] proposed the optimally combined energy detection (OCED) method based on the principle of maximizing the SNR. The OCED is approximated by the blindly combined energy detection (BCED) [9]. These methods, unlike ED, does not need noise power estimation and overcomes ED's sensibility to noise uncertainty. Some methods that employ the structure of the signal sample covariance matrix exist in the literature. For instance, we mention the covariance absolute value (CAV) and the covariance Frobenius norm (CFN) methods [10]. Recently, some methods based on the eigenvalues analysis of covariance matrix had emerged. The authors in [11] introduced the maximum-minimum eigenvalue (MME) and the energy with minimum eigenvalue (EME) detectors .

It was shown in [12], that the test statistic based on the generalized likelihood ratio test (GLRT), when all parameters are completely unknown, is the arithmetic-to-geometric mean (AGM) of the eigenvalues of the sample covariance matrix. Actually, this is a sphericity test. Even though it is effective, the AGM detector does not fully exploit the received signal structure. Therefore, a new GLRT detector was proposed in [13]. The test statistic of the proposed detector is given by the ratio of the largest eigenvalue to the average of eigenvalues of the sample covariance matrix of the received signal. This GRLT was developed for a single PU signal through memoryless channel. Blind spectrum sensing with no knowledge about the signal and the noise level at the receiver was studied in [14] based on information-theoretic criteria (ITC).

So far, most of the work on multiple-antenna spectrum sensing does not consider the spatial correlation among the antennas at the receiver. For instance, the authors in [15] analyzed the sensing performance of energy detectors when multiple-antennas are correlated. Based on the performance analysis, it was verified that the sensing performance of the energy detector is degraded when the antennas are spatially correlated and the performance degradation is proportional to the antenna correlation.

The predicted eigenvalue threshold (PET) was originally introduced in [16]. This method is employed to detect the number of communications sources. Here, the PET method is applied for spectrum sensing, since it is a special case of source number detection problem. The main contributions of our paper are concluded as follows:

- A non-parametric blind spectrum sensing method based on PET is introduced.
- Thereafter, the PET method is simplified to reduce the complexity with no performance loss. The simplified PET (SPET) generalizes the GLRT based test statistic proposed in [13] to the case of multiple PUs through a channel with memory.
- Finally, the effect of the spatial correlation on the performance of spectrum sensing methods is investigated.

The remainder of the paper is organized as follows: Section II defines the system model and introduces the different assumptions. Section III presents the original and simplified PET detection methods. The results and methods performance evaluation are presented in section IV. Finally, conclusions of the research work are presented in section V.

II. System Model

A. Signal Model

We assume that there are $M \geq 1$ antennas at the SU. There are two hypotheses:

$$\mathcal{H}_0 \text{, absence of signals, and } \mathcal{H}_1 \text{, at least one signal is present.} \tag{1}$$

In the presence of P primary users (PU$_j$, $1 \leq j \leq P$), the $M \times 1$ observation vector at the receiver is expressed as:

$$\mathbf{x}(n) = \sum_{j=1}^{P} \sum_{k=0}^{C_j} \mathbf{h}_j(k) s_j(n-k) + \mathbf{w}(n), \quad n = 1, 2, \cdots \tag{2}$$

where $\mathbf{w}(n) = [w_1(n), ..., w_M(n)]^T$ is the $M \times 1$ received additive white Gaussian noise vector with zero-mean and variance σ_w^2. The order of the channel between PU$_j$ and each antenna is C_j. The vector $\mathbf{h}_j(n) = [h_{1j}(n), ..., h_{Mj}(n)]^T$ represents the channel among PU$_j$ and all the antennas; i.e. $h_{ij}(n)$ is the n^{th} tap of the channel response between PU$_j$ and the i^{th} antenna.

Let us consider L consecutive samples and define the corresponding signal/noise vectors:

$$\begin{aligned} \mathbf{x}_L(n) &= [\mathbf{x}^T(n), \mathbf{x}^T(n-1), ..., \mathbf{x}^T(n-L+1)]^T \\ \mathbf{s}_L(n) &= [\mathbf{s}_1^T(n), \mathbf{s}_2^T(n), ..., \mathbf{s}_P^T(n)]^T \\ \mathbf{w}_L(n) &= [\mathbf{w}^T(n), \mathbf{w}^T(n-1), ..., \mathbf{w}^T(n-L+1)]^T \end{aligned} \tag{3}$$

where $\mathbf{s}_j(n) = [s_j(n), s_j(n-1), ..., s_j(n-C_j-L+1)]^T$ and L is called the smoothing factor. A similar model was used in [11], [17] and the system is expressed in matrix form as

$$\mathbf{x}_L(n) = \mathbf{H}\mathbf{s}_L(n) + \mathbf{w}_L(n) \tag{4}$$

where $\mathbf{H}$ is an $ML \times (C + PL)$ matrix and $C = \sum_{j=1}^{P} C_j$. Defining the $ML \times (C_j + L)$ matrix $\mathbf{H}_j$ as

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{h}_j(0) & \cdots & \cdots & \mathbf{h}_j(C_j) & \cdots & 0 \\ \vdots & \ddots & & & \ddots & \vdots \\ 0 & \cdots & \mathbf{h}_j(0) & \cdots & \cdots & \mathbf{h}_j(C_j) \end{bmatrix}, \tag{5}$$

$\mathbf{H}$ is expressed as

$$\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_P]. \tag{6}$$

The covariance matrix of the received signal, set to as $\mathcal{X} = \mathbf{E}[\mathbf{x}_L {\mathbf{x}_L}^H]$, gives

$$\mathcal{X} = \mathbf{H}\mathcal{S}\mathbf{H}^H + \sigma_w^2 \mathbf{I}_{ML} \tag{7}$$

where $(*)^H$ represents the Hermitian transpose and $\mathcal{S} = \mathbf{E}[\mathbf{s}_L {\mathbf{s}_L}^H]$ is assumed to be of full rank. Let $\lambda_1 \geq \cdots \geq \lambda_{ML}$ denote the eigenvalues of $\mathcal{X}$. The received signal covariance matrix is usually unknown. The sample covariance matrix is employed to overcome this difficulty, and is given by

$$\mathcal{X}_N = \frac{1}{N} \sum_{k=1}^{N} \mathbf{x}_L(k)(\mathbf{x}_L(k))^H \tag{8}$$

where N is the number of observed samples. The estimated eigenvalues are $\ell_1, \cdots, \ell_{ML}$ such as $\ell_1 \geq \cdots \geq \ell_{ML}$.

B. Spatial Correlation Model

Many propagation environments result in spatial correlation. Hence, spatial correlation is a crucial factor for practical systems and its effect on their performance must be evaluated. In the presence of spatial correlation at the multiple-antenna receiver, the $ML \times (C + PL)$ channel matrix introduced in (5) is expresses as:

$$\mathbf{H} = \left(\mathbf{I}_L \otimes \mathbf{R}_r^{1/2}\right) \mathbf{H}_w, \tag{9}$$

where $\mathbf{R}_r$ is the receiver spatial correlation matrix and $\otimes$ is the Kronecker product. $\mathbf{H}_w$ is a $ML \times (C + PL)$ gain matrix which the structure is identical to that defined in (6). Also, the non-equal entries of $\mathbf{H}_w$ are i.i.d and follow a circularly symmetric complex Gaussian distribution with zero-mean and variance σ_h^2. Here, the spatial correlation matrix is presented by the exponential correlation model which was introduced in [18] . This model defines the entries of a correlation matrix

$\mathbf{R}_r$ as:

$$[\mathbf{R}_r]_{ij} = \left\{ \begin{array}{ll} \rho^{j-i}, & i \leq j \\ [\mathbf{R}_r]^*_{ji}, & i > j \end{array} \right. , \quad |\rho| < 1, \tag{10}$$

where ρ is the correlation coefficient of neighboring receive antennas. This model may not be accurate for some real-world scenarios but it allows us to study the effect of spatial correlation in an explicit way. However, this model is physically reasonable in the sense that the correlation decreases with increasing the distance between antennas.

III. Predicted Eigenvalues Threshold (PET)

A. Mathematical Preliminaries

The rank of the part of the covariance matrix that represents the signal (i.e., $\mathbf{H}\mathcal{S}\mathbf{H}^H$) is $C + PL$. Hence, the lowest eigenvalue of $\mathcal{X}$ is equal to σ_w^2 and its multiplicity order is equal to $(M - P)L - C$ [17]. By applying the eigenvalue decomposition, the matrix $\mathcal{X}$ has a diagonal form,

$$\mathbf{U}^H \mathcal{X} \mathbf{U} = \text{diag}(\vartheta_1, \cdots, \vartheta_{C+PL}, 0, \cdots, 0) + \sigma_w^2 \mathbf{I}_{ML} \tag{11}$$

in the basis $\mathbf{U}$, where $\vartheta_1 \geq \vartheta_2 \cdots \geq \vartheta_{C+PL} > 0$. Obviously, $\lambda_k = \vartheta_k + \sigma_w^2$ for $1 \leq k \leq C + PL$. This result requires that the matrix $\mathbf{H}$ is overdetermined, i.e.,

$$L > \frac{C}{M - P}. \tag{12}$$

For simplicity, we define $q = ML$ for the rest of the paper.

Theorem 1: Suppose $N\mathcal{X}_N$ has the complex Wishart distribution $\mathbf{W}_q(N, \mathcal{X})$, and the eigenvalues of $\mathcal{X}_N$ and $\mathcal{X}$ are $l_1 \geq \cdots \geq \ell_q$ and $\lambda_1 \geq \cdots \geq \lambda_k = \cdots = \lambda_q = \lambda$ respectively. The limiting distribution of ℓ_k^{av} the average of the lowest $q - k$ eigenvalue, as $N \to \infty$, is

$$(N(q-k))^{(1/2)}(\ell_k^{av} - \lambda)/\lambda \xrightarrow{dist} \mathcal{N}(0, 1). \tag{13}$$

In the absence of signal, the matrix $N\mathcal{X}_N$ follows a Wishart complex distribution, i.e. $N\mathcal{X}_N \backsim \mathbf{W}_q(N, \mathcal{X})$.

Based on that, the authors in [16] proposed an upper bound for each eigenvalue of the noise subspace. The upper threshold of ℓ_k is predicted as

$$\ell_k^{up} = \underbrace{\left[(m_k + 1) \frac{1 + t[N(m_k + 1)]^{1/2}}{1 - t(N.m_k)^{-1/2}} - m_k \right]}_{\eta_k(t)} \underbrace{\frac{1}{q - k} \sum_{i=k+1}^{q} \ell_i}_{\ell_k^{av}} \tag{14}$$

where ℓ_k^{av} is the average of the $m_k \, (= q - k)$ lowest eigenvalues, $\eta_k(t)$ is the prediction factor and t is a two-direction threshold that represents the confidence interval of the averaged eigenvalue. The eigenvalue ℓ_k is considered in the noise subspace when it satisfies the following condition

$$\ell_k \leq \ell_k^{up}. \tag{15}$$

B. Predicted Eigenvalues Threshold Based Spectrum Sensing

The PET method is employed for detecting the communications sources number. Another well-known approach to solve this problem is the ITC, and in particular minimum description length (MDL) and Akaike information criterion (AIC) [14]. The performance of PET method is superior to the MDL under low SNR and enjoys consistency under high SNR (contrary to the AIC) [16]. The PET method is based on (15) and consists in adaptively modeling the noise eigenvalues increase to determine the dimension of the noise subspace, and hence, the signal subspace dimension. Actually, the PET model is controlled by a single parameter t.

Spectrum sensing problem is a special case of the sources number detection one. In the absence of PUs, the estimated number of source signals should be zero. The original PET (OPET) method, as described above, can be applied directly to conduct spectrum sensing. Based on that, the two hypotheses in (1) are reformulated as

$$\begin{aligned} &\mathcal{H}_0 : \ell_k \leq \ell_k^{up}, \; k = 1, 2, \cdots, q - 1 \\ &\mathcal{H}_1 : \hat{k} \geq 1, \; \hat{k} = \arg \max_{k=1,\cdots,q-1} \ell_k > \ell_k^{up}. \end{aligned} \tag{16}$$

C. Simplified Predicted Eigenvalues Threshold (SPET) Method

In the presence of P primary users, the dimension of signal subspace is $PL + \sum_{j=1}^{P} C_j$ (i.e., the contribution of PU_j is equivalent to $L + C_j$ eigenvalues). The idea is to detect *at least* one PU under $\mathcal{H}_1$. Since C_j is unknown and difficult to be estimated, the presence of PUs is reflected, *at least*, by L signal eigenvalues. It is clear from (11) that when a certain eigenvalue $\lambda_{\hat{k}}$ belongs to the signal subspace, then $\lambda_1, \cdots, \lambda_{\hat{k}}$ are all signal eigenvalues. This leads to the fact that when ℓ_1 is noise eigenvalue then $\mathcal{H}_0$ is detected, i.e., there is no signal when every eigenvalue does not exceed its own predicted threshold.

The above discussion indicates that the PET method could be simplified to employ only one eigenvalue. The question is which eigenvalue $\ell_{\hat{k}}, 1 \leq \hat{k} \leq L$, must be employed for the spectrum sensing problem? It is ℓ_1. In fact, when ℓ_1 corresponds to noise, we are confident that all eigenvalues belong to the noise subspace. On the other hand, if ℓ_1 is a signal eigenvalue, the PU detection must not be missed even though all the other eigenvalues corresponds to noise. This is due to the privilege of protecting PUs over spectrum utilization efficiency. Then, the PET method is simplified to test the largest eigenvalue against its own predicted threshold. Hence, the sensing problem can be expressed as $\ell_1/\ell_1^{av} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_1(t)$, and is altered to:

$$T_{\text{SPET}} = \frac{\ell_1}{\frac{1}{q}\sum_{i=1}^{q} \ell_i} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{17}$$

This decision statistic was found in [13] based on GLRT for a single source through memoryless channel. Our work extends this decision statistic to a more general scenario.

IV. Results and Discussion

Here, we present some simulations results to demonstrate the effectiveness of the proposed sensing methods. These methods are evaluated through the probability of missing at a false alarm probability of 0.1. In order to determine the threshold for this given P_f, the decision statistic is calculated for 10^5 independent random trials at $\mathcal{H}_0$, i. e. when there is no signal. We sort the decision statistic values in descending order to choose the detection threshold such as $P_f \times 10^5$ samples of the generated statistic are above the chosen threshold.

All results are based on 1000 Monte Carlo trials for each method. For each realization, the binary phase-shift keying modulated PU signals and the channel taps are randomly generated. For different values of SNR, a random additive white Gaussian noise is added such that

$$\text{SNR} = \frac{E[\ \|\mathbf{x}(n) - \mathbf{w}(n)\|^2]}{E[\ \|\mathbf{w}(n)\|^2]} \tag{18}$$

We consider in all our simulations a secondary user with four antennas ($M = 4$) and the presence of two primary users ($P = 2$). The spatial correlation is presented by ρ the correlation coefficient as defined in (10).

Fig. 1 shows the adaptive PET model of the sample covariance matrix eigenvalues at SNR $= 0$dB and $L = 10$. It is clear that the noise subspace dimension is $(M - P)L$, while each PU is represented by L eigenvalue (PL in total). Each signal eigenvalue exceeds its own predicted threshold.

Fig. 2 compares the sensing performance of OPET and SPET methods. These simulations are done for different values of N when $\rho = 0.6$, $C_1 = C_2 = 6$ and $L = 1$. Based on simulations, the two-direction threshold t is chosen 1.108 to achieve a false alarm probability of 0.1. It is clear that the SPET method simplifies the original one to reduce the complexity without leading to any performance loss. Comparing the probability of missing for different values of N shows that the performance improves when the number of the observed samples is larger.

The impact of the spatial correlation on SPET performance is evaluated and the results are depicted in Fig. 3. The detection performance is improved when the antennas are spatially correlated and becomes ever more improved as the correlation increases. Obviously, the eigenvalues of the spatial correlation matrix $\mathbf{R}_r$ strengthen the signals eigenvalues and reflect time accumulation of SNR. This reinforce the contribution of

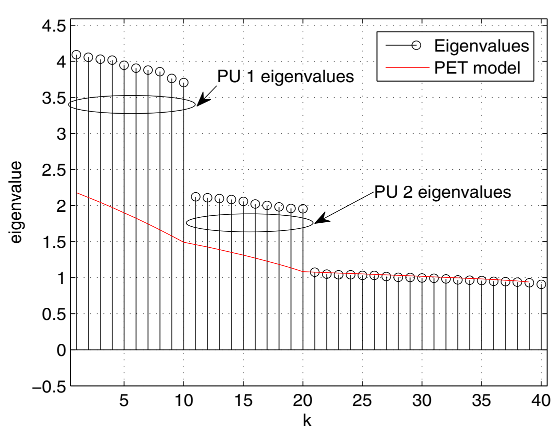

Figure 1. The eigenvalues and the adaptive PET model at SNR $= 0$dB, $N = 1000$, $P = 2$, $M = 4$, $C_1 = C_2 = 0$ and $L = 10$.

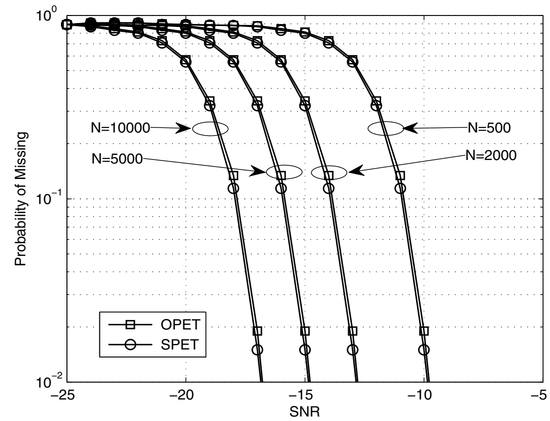

Figure 2. The probability of missing versus SNR for OPET and SPET methods at different N values when $\rho = 0.6$, $P = 2$, $M = 4$, $L = 1$ and $C_1 = C_2 = 6$.

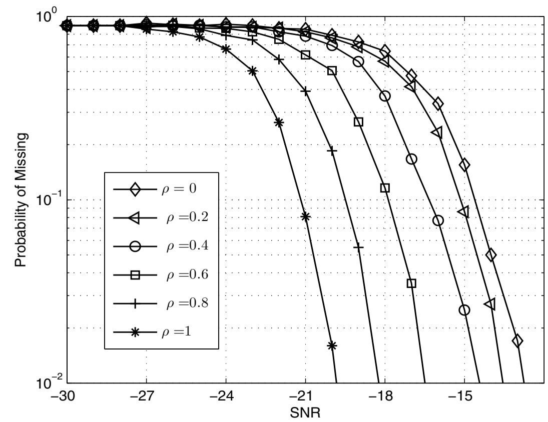

Figure 3. The probability of missing versus SNR for SPET method at different spatial correlation coefficient values when $N = 10000$, $P = 2$, $M = 4$, $L = 1$ and $C_1 = C_2 = 6$.

signals eigenvalues in T_{SPET} compared to the noise eigenvalues and increase the probability of detection. Also, the false alarm probability is not affected by the spatial correlation since it is not reflected in the covariance matrix when there is no signal.

In Fig. 4 the probability of missing versus SNR for SPET method at different smoothing factor values is shown. As can be seen from Fig. 4, increasing the smoothing factor up to 12 does improve the detection performance. But, the probability of missing is almost constant when $L > 12$. In fact, the computational complexity increases when L is larger. Hence, the smoothing factor is chosen relatively low as a compromise between complexity and performance degradation. Moreover, we note that the SPET method still detects the presence of a signal even when L does not satisfy the condition in (12), e.g. the results shown in Fig. 4 for $L = 1, 5$.

A performance comparison among several sensing methods is provided in Fig. 5. This comparison is done for the following parameters: $N = 10000$, $\rho = 0.6$, $C_1 = C_2 = 6$ and

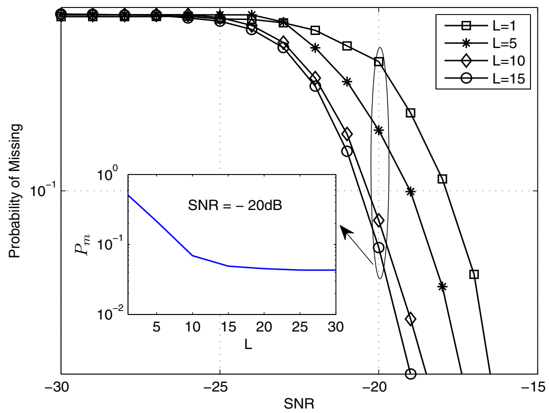

Figure 4. The probability of missing versus SNR for SPET method at different L values when ρ = 0.6, N=10000, M=4, P=2 and C_1=C_2=6. Also, the probability of missing versus L at a fixed SNR $= -16$dB.

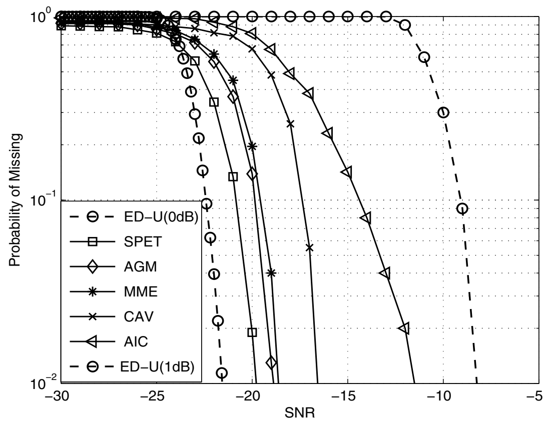

Figure 5. Probability of missing versus SNR for several sensing methods when $N = 10000$, $M = 4$, $P = 2$, $C_1 = C_2 = 6$, $\rho = 0.6$ and $L = 12$.

$L = 12$. The ED suffers of noise uncertainty. The noise power estimation error (in dB) is assumed to be uniformly distributed in the interval $[-B, B]$ [4], i.e., the ED is denoted "ED-U(BdB)". It is clear that the performance significantly degrades when $B = 1$dB. Among the different blind methods the SPET one has the best performance followed by the AGM method [12]. This result is well expected since SPET employs more the signal structure. The MME detection [11] is a bit less effective than the AGM detector but outperforms the CAV detector [10]. The CAV detection has a smaller complexity but it is less representative of the signals. Even though the ITC based methods [14] offer a low false alarm probability, they are not as efficient as the other blind methods.

V. Conclusions

In this paper, the PET method, originally used for the communications sources number detection, is employed for multiple-antenna spectrum sensing. This method is simplified to significantly reduce the computational complexity without any performance loss compared with the original PET. The SPET test statistic generalizes that of the GLRT [13] to a more general scenario. The detection performance improves in the presence of the spatial correlation at the multiple-antenna secondary user. The blind non-parametric SPET detector outperforms the other blind detectors existing in the literature.

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