# Hybrid Access Design for Femtocell Networks With Dynamic User Association and Power Control

Ha Nguyen Vu and Long Bao Le

***Abstract*—In this paper, we propose a universal power control (PC) algorithm that can provide QoS support in minimum signal-to-interference-plus-noise ratios (SINRs) for all users while exploiting differentiated channel conditions to enhance the network throughput. In particular, we design the PC algorithm by using non-cooperative game theory and establish sufficient conditions for its convergence. Then, we apply it to design a hybrid access scheme for two-tier macrocell-femtocell networks. Specifically, we devise a distributed load-award association algorithm for macro users, which enables flexible user association to BSs of either tier. In addition, we develop an efficient mechanism based on which users can steer the equilibrium in such a way that they achieve their desirable performance targets. Numerical results are then presented to validate the theoretical results and demonstrate the desirable performance of the proposed algorithms.**

***Index Terms*—Femtocell networks, interference management, hybrid access, power control, user association.**

## I. Introduction

Deployment of low-power femto BSs (FBSs) can significantly enhance indoor coverage and capacity [1], [2]. However, FBSs operate on the same licensed frequency bands with the existing macrocell network, which can create severe cross-tier interference for MUEs. There are three different access modes for spectrum sharing between the macro and femto tiers, namely closed, open, and hybrid access. In general, open access is more effective in mitigating cross-tier interference compared to the closed access while it may result in uncontrollable performance degradation for macro users [2]. Design of an efficient hybrid access scheme that can balance between advantages and disadvantages of the other two access modes is an interesting and important research topic.

Most existing works on interference management for femtocell networks, however, have assumed the closed access [3], [4], [5], [6]. Moreover, these papers did not consider the scenarios where users of both network tiers may have diverse QoS targets (e.g., voice versus data users) and they did not attempt to enhance the network throughput by exploiting users' differentiated transmission conditions. This paper aims at resolving these limitations. Toward this end, we develop a universal PC algorithm that can support minimum users' SINRs (if possible) while being able to enhance the throughput for users in favorable conditions. Then, we devise a load-award user association algorithm and apply the proposed PC algorithm to design an efficient hybrid access strategy for two-tier femtocell networks.

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The remaining of this paper is organized as follows. We describe the system model in Section II. In Section III, we present the proposed PC algorithm and prove its convergence. Its application to design the hybrid access strategy for femtocell networks is described in Section IV. Numerical results are presented in Section V followed by conclusion in Section VI.

## II. System Model

We consider uplink PC for a general multicell wireless network in which a two-tier macrocell-femtocell wireless network is a special case. Let the set of all users be $\mathcal{M}$. To develop such a PC algorithm, we assume fixed user association where each user connects and communicates with its fixed BS. For the given user association, let $b_i$ denote the receiving BS of user $i \in \mathcal{M}$. Let the transmit power of user $i$ be $p_i$, whose maximum value is $\bar{p}_i$, i.e., $0 \leq p_i \leq \bar{p}_i$.

Let $h_{ij}$ denote the channel gain from the user $j$ to BS $b_i$ (note that $h_{ij}$ becomes $h_{jj}$ if $b_i \equiv b_j$). Let $\mathrm{p} = (p_1, p_2, ..., p_M)$ and $\eta_i$ be the noise power at base station $b_i$. Assuming CDMA technology is employed, the SINR at $b_i$ for the signal transmitted from user $i$ can be written as

$$\Gamma_i(\mathrm{p}) = \frac{g_i h_{ii} p_i}{\sum_{j \neq i} h_{ij} p_j + \eta_i} = \frac{p_i}{R_i(\mathrm{p})} \tag{1}$$

where $g_i$ is the processing gain for user $i$, which is defined as the ratio of spreading bandwidth to the symbol rate; $R_i(\mathrm{p})$ is the effective interference to user $i$ which is defined as

$$R_i(\mathrm{p}) = \frac{\sum_{j \neq i} h_{ij} p_j + \eta_i}{g_i h_{ii}}. \tag{2}$$

The objective of this paper is to design a distributed PC algorithm which can support required quality-of-service (QoS) in minimum SINRs while exploiting the multiuser diversity gain to increase the system throughput. In addition, we are interested in a heterogeneous environment where there are different kinds of users with diverse QoS targets (e.g., voice versus data users).

## III. Proposed Hybrid Power Control Algorithm

### A. Game-Theoretic Formulation

We develop the distributed PC algorithm by using the non-cooperative game theory approach. In particular, we define a PC game as follows.

- Players: The set of mobile users $\mathcal{M}$
- Strategies: Each user $i$ chooses transmit power in the set $[0, \bar{p}_i]$

- Payoffs: User $i$ is interested in maximizing the following payoff function

$$U_i(\mathrm{p}) \triangleq -\alpha_i(p_i R_i(\mathrm{p})^{\frac{x}{1-x}} - \xi_i)^2 - (\Gamma_i(\mathrm{p}) - \widehat{\gamma}_i)^2 \quad (3)$$

where $\widehat{\gamma}_i$ denotes the target SINR for user $i$; $\alpha_i$ and $\xi_i$ are control parameters, which will be adaptively adjusted to achieve our design objectives.

This game-theoretic formulation arises quite naturally in autonomous spectrum access scenarios such as two-tier femtocell networks where mobile users tend to be selfish and only interested in maximizing their own benefits. Using this formulation, we will develop an iterative PC algorithm in which each user maximizes its own payoff in each iteration given the chosen power levels from other users in the previous iteration (i.e., each user plays the *best response* strategy). To devise such an algorithm, each user $i$ chooses the power level, which is obtained by setting the first derivative of the underlying user's payoff function to zero.

In fact, by maximizing the payoff function given in (3) each user $i$ strikes to balance between achieving the SINR target $\widehat{\gamma}_i$ and exploiting its potential favorable channel condition to increase his/her SINR. While it is quite intuitive that maximizing $-(\Gamma_i(\mathrm{p}) - \widehat{\gamma}_i)^2$ enables user $i$ to reach its target SINR $\widehat{\gamma}_i$, the design intuition in optimizing the first term $-\alpha_i\left(p_i R_i(\mathrm{p})^{\frac{x}{1-x}} - \xi_i\right)^2$ may not be very straight-forward. We state our first result in the following lemma, which reveals the engineering intuition behind this design.

**Lemma 1.** *Consider the game formulation described above with infinite power budget (i.e., $\bar{p}_i = \infty$, $\forall i$). Then, best responses due to the following two payoff functions are the same*

$$U_i^{(1)}(\mathrm{p}) \triangleq \Gamma_i^x - \lambda_i p_i \quad (4)$$

$$U_i^{(2)}(\mathrm{p}) \triangleq -\left(p_i R_i(\mathrm{p})^{\frac{x}{1-x}} - \xi_i\right)^2 \quad (5)$$

*if $\xi_i = (\lambda_i/x)^{\frac{1}{x-1}}$ and $0 < x < 1$.*

*Proof:* We can rewrite the payoff function in (4) as

$$U_i^{(1)}(\mathrm{p}) = \Gamma_i^x - \lambda_i p_i = (p_i/R_i(\mathrm{p}))^x - \lambda_i p_i. \quad (6)$$

Taking the first and second derivatives of this payoff function with respect to $p_i$, we have

$$\frac{\partial U_i^{(1)}}{\partial p_i} = \frac{x}{R_i(\mathrm{p})}\left(\frac{p_i}{R_i(\mathrm{p})}\right)^{x-1} - \lambda_i \quad (7)$$

$$\frac{\partial^2 U_i^{(1)}}{\partial p_i^2} = \frac{x(x-1)}{R_i(\mathrm{p})^2}\left(\frac{p_i}{R_i(\mathrm{p})}\right)^{x-2}. \quad (8)$$

From (8), it can be easily verified that $\frac{\partial^2 U_i^{(1)}}{\partial p_i^2} < 0$ for $0 < x < 1$, which implies that $U_i^{(1)}(\mathrm{p})$ is a concave function. Therefore, the best response can be obtained by setting the first derivative in (7) to zero. After some manipulations, we can obtain the best response corresponding to the payoff function $U_i^{(1)}(\mathrm{p})$ as follows:

$$p_i = R_i(\mathrm{p})^{\frac{x}{x-1}}(\lambda_i/x)^{\frac{1}{x-1}} = \xi_i R_i(\mathrm{p})^{\frac{x}{x-1}} \quad (9)$$

where the second relationship in (9) holds for $\xi_i = (\lambda_i/x)^{\frac{1}{x-1}}$. Moreover, it can also be verified that the best response obtained in (9) is exactly the one corresponding to the payoff function $U_i^{(2)}(\mathrm{p})$. Therefore, we have proved the lemma. ■

*Remark* 1. *It has been shown that the opportunistic PC algorithm (OPC) proposed in [9] can be achieved if users iteratively play their corresponding best-response strategies with payoff function $U_i^{(1)}(\mathrm{p})$ for $x = 1/2$ [9], [10]. Moreover, letting users play the best-response strategies using $U_i(\mathrm{p})$ with $\alpha_i = 0$ results in the well-known distributed SINR-tracking PC algorithm (TPC) originally proposed by Foschini and Milijanic in [7]. Therefore, our chosen payoff function in (3) can be used to design a hybrid PC strategy that exploits the advantages of both existing PC algorithms.*

### B. Proposed Hybrid Power Control (HPC) Algorithm

We are now ready to develop a hybrid power control algorithm corresponding to the payoff function in (3). Specifically, we can derive the power update rule for the HPC algorithm according to the best-response strategy of the underlying payoff function. After some manipulations, we can obtain the following best response under payoff function (3)

$$p_i = I_i(\mathrm{p}) \triangleq \frac{\alpha_i \xi_i R_i(\mathrm{p})^{\frac{x}{x-1}} + \hat{\gamma}_i R_i(\mathrm{p})}{\alpha_i + 1}. \quad (10)$$

Considering the maximum power constraints, the HPC algorithm employs the following iterative power update rule

$$p_i(t+1) = I_i^H(\mathrm{p}) = \min\{\bar{p}_i, I_i(\mathrm{p}(t))\} \quad (11)$$

where $I_i(\mathrm{p}(t))$ is given in (10). Here, parameters $\alpha_i$ can be used to control the desirable performance of the proposed HPC algorithm. Specifically, by setting $\alpha_i = 0$ user $i$ actually employs the standard Foschini-Milijanic PC algorithm to achieve its target SINR $\hat{\gamma}_i$ while if $\alpha_i \to \infty$, user $i$ attempts to achieve higher SINR (if it is in favorable condition).

It is worth noting that the proposed PC algorithm given in (11) can be easily implemented in a distributed fashion. This is because each user $i$ only needs to estimate $R_i(\mathrm{p})$ to update its power. Moreover, $R_i(\mathrm{p})$ can be obtained by estimating the total received interference and user channel gain $h_{ii}$ as being implied by (2).

### C. Convergence of HPC Algorithm

In this section, we establish the convergence condition for the proposed HPC algorithm by using the *"two-sided scalable (2.s.s.) function"* approach proposed in [9], which was developed based on the standard-function approach originally proposed by Yates [8]. To proceed, we provide a definition of 2.s.s. functions [9] in the following.

**Definition 1.** *A power-update function (p.u.f.) $\mathrm{J}(\mathrm{p}) = [J_1(\mathrm{p}), \ldots, J_M(\mathrm{p})]^{\mathrm{T}}$ is 2.s.s. if for all $a > 1$ and $\frac{1}{a}\mathrm{p} \le \mathrm{p}' \le a\mathrm{p}$, we have*

$$(1/a)J_i(\mathrm{p}) < J_i(\mathrm{p}') < aJ_i(\mathrm{p}), \ \forall i \in \mathcal{M}. \quad (12)$$

Now, we state a sufficient condition under which the p.u.f. $\mathrm{I}^H(\mathrm{p})$ in (11) is 2.s.s..

**Theorem 1.** *When $x \le 1/2$, the p.u.f. $\mathrm{I}^H(\mathrm{p}) = \left[I_1^H(\mathrm{p}), \ldots, I_M^H(\mathrm{p})\right]$ used in the proposed HPC algorithm in (11) is 2.s.s..*

**Algorithm 1** MUE ASSOCIATION ALGORITHM

1: Each FBS and the MBS estimate the channel gains from nearby MUEs to itself.
2: By assuming all MUEs transmit with their maximum powers, each BS estimates/calculates SINRs achieved by nearby MUEs; it transmits these estimated SINRs to MUEs.
3: Upon receiving estimated SINRs from all potential FBSs and the MBS, each MUE will associate with the BS provided the largest estimated SINR.

*Proof:* The proof is given in Appendix A. ■

We are now ready to state the main result of this paper in the following theorem.

**Theorem 2.**

*1) The p.u.f. of the proposed HPC algorithm* $\mathrm{I}^H(\mathrm{p})$ *has a unique fixed point* $\mathrm{p}^*$ *that satisfies* $\mathrm{p}^* = \mathrm{I}^H(\mathrm{p}^*)$.

*2) For any initial power vector* $\mathrm{p}^{(0)}$*, the HPC algorithm converges to the fixed point* $\mathrm{p}^*$*, which is the Nash equilibrium (NE) of the PC game defined in Section III.A.*

*Proof:* The first property and the convergence of this theorem can be proved by utilizing the properties of 2.s.s. function established in [9]. The fact that the resulting equilibrium is the NE of the underlying PC game is immediate since users play the best-response strategy. ■

## IV. APPLICATION OF HPC ALGORITHM IN TWO-TIER FEMTOCELL NETWORKS

We apply the proposed HPC to design an efficient hybrid spectrum access strategy for two-tier femtocell networks. In particular, we consider a two-tier network where $M_f$ femto user equipments (FUEs) served by $K$ femto base stations (FBSs) are underlaid with one macro cell serving $M_m$ macro users equipments (MUEs). We denote the sets of MUEs and FUEs by $\mathcal{M}_m \triangleq \{1, ..., M_m\}$ and $\mathcal{M}_f \triangleq \{M_m+1, ..., M_m+M_f\}$, respectively; and the set of all users is $\mathcal{M} = \mathcal{M}_m \cup \mathcal{M}_f$.

We assume that FUEs have fixed association with their own FBSs while MUEs can establish connections with either the MBS or nearby FBSs. In addition, users of either types perform PC to achieve their performance objectives.

### *A. Load-aware MUE Association Algorithm*

All MUEs decide their associated BSs by running Algorithm 1. Specifically, each BS estimates and broadcasts the SINR achieved by each MUE in the uplink if all users transmit at their maximum powers. Upon receiving the broadcast SINR values, each MUE establishes its connection with the best BS. It is evident that this algorithm can be easily implemented in a distributed manner since each BS only need to know uplink channel gains to estimate MUEs' SINRs.

**Algorithm 2** HYBRID SPECTRUM ACCESS ALGORITHM

1: Initialization:
- $p_i^{(0)} = 0$ for all user $i$, $i \in \mathcal{M}$.
- $\alpha_i^{(0)} = 0$ for voice user and $\alpha_i^{(0)} = \alpha_0$ ($\alpha_0 \gg 1$) for data user.
- Run HPC algorithm until convergence.

2: Step $k$:
- If $\Gamma_i^{(k)} < \hat{\gamma}_i$ and $\Gamma_i^{(k)} > \Gamma_i^{(k-1)}(1+\sigma_i)$ where $\sigma_i$ is a predetermined coefficient for user $i$
  a. $\alpha_i^{(k)} = (\Gamma_i^{(k)}/\hat{\gamma}_i)\alpha_i^{(k-1)}$.
  b. $b_i$ sends a warning message to other users in the network if $p_i^{(k)} = \bar{p}_i$.
- If $i$ receives the warning message and $\Gamma_i^{(k)} > \hat{\gamma}_i$, set $\alpha_i^{(k)} = (\hat{\gamma}_i/\Gamma_i^{(k)})\alpha_i^{(k-1)}$.
- Run HPC algorithm until convergence.

3: Increase $k$ and go back to step 2 until there is no update request for $\alpha_i$.

### *B. Hybrid Access Design for Femtocell Networks*

Given the user association solution given by Algorithm 1, we develop a hybrid spectrum access mechanism for efficient spectrum sharing and QoS support for users of both network tiers. In fact, the NE achieved by the underlying PC game strongly depends on the values of $\xi_i$ and $\alpha_i$ ($i \in \mathcal{M}$). Therefore, by adaptively varying $\xi_i$ and $\alpha_i$, user $i$ can potentially steer the equilibrium in such a way that it better meets its QoS goals. In addition, adjusting these parameters can provide useful mechanisms for FBSs to control the spectrum access from associated MUEs, which enables hybrid access design for the underlying two-tier network.

Toward this end, if user $i$ is a voice user who is only interested in maintaining its target SINR $\hat{\gamma}_i$ then we can simply set $\alpha_i = 0$ in (3). For each data user $i$, it will fix $\xi_i$ while adaptively updating $\alpha_i$ to achieve the desirable target. To set the value for $\xi_i$, suppose that data user $i$ would need to use its maximum power budget $\bar{p}_i$ to reach the target SINR $\hat{\gamma}_i$. Then, the value of $\xi_i$ can be found using the relation in (9) as

$$\xi_i = (\bar{p}_i/\hat{\gamma}_i^x)^{\frac{1}{1-x}} \tag{13}$$

where we have substituted $\Gamma_i = \hat{\gamma}_i$ to (9). To determine how to update $\alpha_i$, we can rewrite the equilibrium condition from the proposed p.u.f. in (11) assuming that $p_i \neq \bar{p}_i$ as

$$\Gamma_i(\mathrm{p}^*) = \frac{\alpha_i \xi_i R_i(\mathrm{p}^*)^{\frac{1}{x-1}} + \hat{\gamma}_i}{\alpha_i + 1} \tag{14}$$

Taking the derivative of $\Gamma_i(\mathrm{p}^*)$ with respect to $\alpha_i$, we have

$$\frac{\partial \Gamma_i(\mathrm{p}^*)}{\partial \alpha_i} = \frac{\xi_i R_i(\mathrm{p}^*)^{\frac{1}{x-1}} - \hat{\gamma}_i}{(1+\alpha_i)^2}. \tag{15}$$

The result in (15) implies that when $R(\mathrm{p}) \leq R^*(\mathrm{p}) = (\xi_i/\hat{\gamma}_i)^{1-x}$, $\Gamma_i$ decreases if $\alpha_i$ decreases. Inversely, $\Gamma_i$ increases if $\alpha_i$ decreases when $R(\mathrm{p}) \geq R^*(\mathrm{p})$. Moreover, it can be verified that $R(\mathrm{p}) \lesseqgtr R^*(\mathrm{p})$ corresponds to $\Gamma_i \gtreqless \hat{\gamma}_i$. Hence, we will decrease $\alpha_i$ for user $i$ if its target SINRs is not achieved. Based on these results, we propose a hybrid

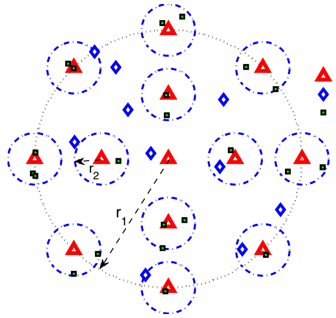


Fig. 1. Simulated two-tier macrocell-femtocell network

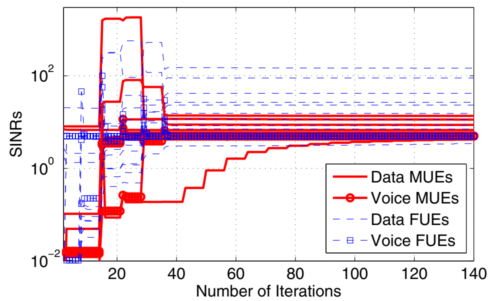


Fig. 2. SINRs of all users when target SINRs are equal to 5.

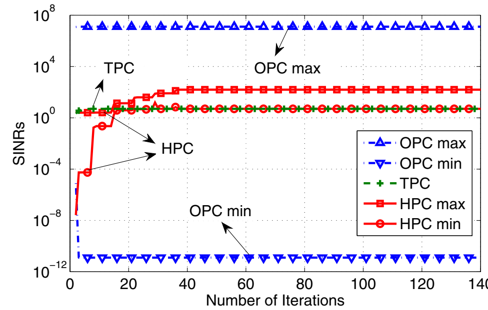


Fig. 3. SINRs of max- and min-users in OPC, TPC and HPC schemes.

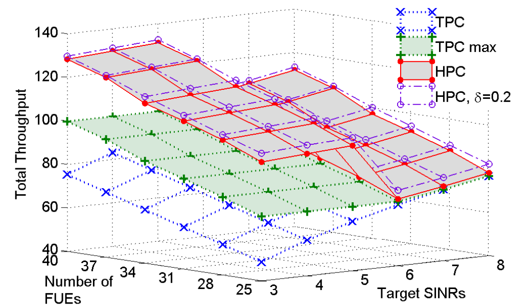


Fig. 4. Total throughput of all users vs. target SINRs and number of FUEs.

access scheme based on the proposed HPC algorithm, which is described in Algorithm 2. In particular, each user $i$ will slowly update $\alpha_i$ based on the achieved equilibrium obtained by running the proposed HPC algorithm. All users $i$ initially set $\alpha_i$ to be a sufficiently large value; therefore, they reach the first equilibrium that favor strong users. Then, user $i$ whose SINR at the equilibrium is lower than the target $\hat{\gamma}_i$ decreases its $\alpha_i$ to enhance its SINR. If user $i$ still could not reach the target $\hat{\gamma}_i$ while using his/her maximum power budget $\hat{\gamma}_i$, it sends a "warning message" through the backhauls to other users $j$ who will decrease their parameters $\alpha_i$ to "help" user $i$. This warning message can also be sent by unhappy FUEs whose FBSs allows connections from nearby MUEs. This enables these unhappy FUEs to enhance their SINRs.

## V. Numerical Results

We present illustrative numerical results to demonstrate the performance of the proposed HPC algorithm and Algorithm 2. The network setting and user placement in these examples are illustrated in Fig. 1, where MUEs and FUEs are randomly located inside circles of radii of $r_1 = 1000\,m$ and $r_2 = 50\,m$, respectively. Assume that bounds of femtocells are heavy walls. We fix $M_m = 10$ and randomly choose the number of FUEs in each femtocell from 1 to 3. Then, eight of users are set as voice users randomly. The channel gain $h_{ij}$ is chosen according to the path loss $L_{ij} = A_i \log_{10}(d_{ij}) + B_i + C\log_{10}(\frac{f_c}{5}) + 12n_{ij}$, where $d_{ij}$ is the distance between $b_i$ and user $j$; $(A_i, B_i)$ are set as $(36, 40)$ and $(35, 35)$ for MBS and FBSs, respectively; $C = 20$, $f_c = 2.5\,GHz$; $n_{ij}$ is the number of heavy walls between $b_i$ and $j$. Let $x = 0.5$, $\bar{p}_i = 0.01\,W, \eta_i = 10^{-13}\,W, g_i = 128, \sigma_i = 0.05$ for $\forall i \in \mathcal{M}$.

Fig. 2 illustrates the convergence of Algorithm 2 where $\hat{\gamma}_i = 5$ for all users. As can be seen, Algorithm 2 converges to an equilibrium for which some users attain the target SINRs while others achieve SINRs higher than their target values. In Fig. 3, the fairness achieved OPC, TPC and HPC algorithms is compared by illustrating SINRs achieved by users with highest and lowest SINRs (indicated as X min and X max in this figure where X represents the corresponding PC strategy). In the TPC scheme, all users reach the same target SINR; hence, the performance of only one user is shown. It is evident that the OPC scheme results in a very unfair equilibrium where the max-user attains the very high SINR compared to that achieved by the min-user. In contrast, our proposed scheme allows the min-user to reach its target SINR while the max-user settles at a higher SINR.

Fig. 4 illustrates the total throughput achieved by different schemes versus target SINRs and the number of FUEs where the throughput of user $i$ is calculated as $\log_2(1 + \Gamma_i)$ (b/s/Hz). Furthermore, a conventional method to achieve higher throughput under the TPC scheme by increasing the target SINRs for all users as high as possible, is also investigated (denoted as TPC max in these figures). As we can see, our algorithm attains higher total throughput than other algorithms. Specifically, when the network load is low ($\hat{\gamma}_i < 6$), our proposed algorithm attains much higher throughput than the others. Moreover, when the network load is higher ($\hat{\gamma}_i \geq 6$), the gaps between our proposed algorithm and others become smaller. This is because Algorithm 2 also attempts to maintain target SINRs for all users as in the TPC schemes. In addition, when all users can tolerate the decrease in their target SINRs by a factor of $1 > \delta > 0$, the total throughput becomes higher.

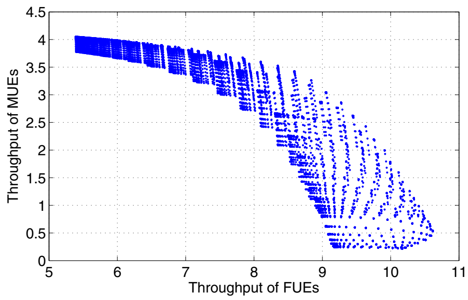


Fig. 5. Hybrid access of MUEs in femtocells.

Fig. 5 illustrates the total throughput of FUEs versus that of MUEs in femtocells these MUEs associate with. Let $\mathcal{S}$ be the set of these MUEs and FUEs. To obtain the results in this figure, we first obtain the results similar to those in Fig. 2 and Fig. 3 to determine the values of $\alpha_j$ for $j \notin \mathcal{S}$; then, we obtain the presented results by varying $\alpha_i$ from $10^{-3}$ to $10^3$ for $i \in \mathcal{S}$. Each point in Fig. 5 presents the total throughputs of MUEs and FUEs in $\mathcal{S}$ with certain values of $\{\alpha_i\}$. This figure confirms that we can achieve flexible capacity sharing between MUEs and FUEs by changing parameters $\alpha_i$. This means that FUEs can control the level of spectrum access from associated MUEs by adaptively changing $\alpha_i$. This would be a very desirable feature of the proposed hybrid spectrum access design.

## VI. Conclusion

We developed a distributed PC algorithm and showed how it can be applied to design an efficient hybrid spectrum access strategy in two-tier femtocell networks. The proposed algorithms can support minimum SINRs of all users if possible while enabling us to achieve high throughput by exploiting users' favorable channel conditions. Numerical results were presented to demonstrate the desirable performance of the proposed algorithms.

## Appendix A
## Proof of Theorem 1

If $\frac{1}{a}\mathrm{p} \leq \mathrm{p}' \leq a\mathrm{p}$ for a given $a > 1$, from the definition of $R_i\left(\mathrm{p}\right)$ in (2), we have

$$(1/a)R_i\left(\mathrm{p}\right) \leq R_i(\mathrm{p}') \leq aR_i(\mathrm{p}),\ \forall i \in \mathcal{M}. \tag{16}$$

Therefore, instead of proving $\mathrm{I}^H(\mathrm{p})$ is 2.s.s. w.r.t. p, we can show that it is 2.s.s. w.r.t. $R_i(\mathrm{p})$. Let $\mathrm{I}^H(R_i(\mathrm{p})) = \mathrm{I}^H(\mathrm{p})$.

Because $\frac{x}{x-1} < 0$, it can be verified that $I_i(\mathrm{p})$ is convex w.r.t. $R_i(\mathrm{p}) > 0$ and $\lim_{R_i(\mathrm{p})\rightarrow\{0,\infty\}} I_i(\mathrm{p}) = \infty$. Hence, there are at most two values of $R_i(\mathrm{p})$ that satisfy $I_i(\mathrm{p}) = \bar{p}_i$.

If there is no or only one such intersection point, we have $I_i(\mathrm{p}) \geq \bar{p}_i$ or $I_i^H(\mathrm{p}) = \bar{p}_i$. For both cases, we have $(1/a)I_i^H(\mathrm{p}) < I_i^H(\mathrm{p}') < aI_i^H(\mathrm{p})$ since $a > 1$.

If there are two intersection points, let $R_i^l$ and $R_i^u$ be the values of $R_i(\mathrm{p})$ at these two points where $R_i^l < R_i^u$. Moreover, the p.u.f. of HPC algorithm in these cases must satisfy

$$I_i^H(\mathrm{p}) = \begin{cases} I_i(\mathrm{p}), & \text{if } R_i^l \leq R_i(\mathrm{p}) \leq R_i^u \\ \bar{p}_i, & \text{otherwise} \end{cases} \tag{17}$$

Let us consider all possible cases in the following.

- If $\{R_i(\mathrm{p}), R_i(\mathrm{p}')\} \not\subset \left[R_i^l, R_i^u\right]$, then $I_i^H(\mathrm{p}) = I_i^H(\mathrm{p}') = \bar{p}_i$. Therefore, it is easy to obtain that $(1/a)I_i^H(\mathrm{p}) < I_i^H(\mathrm{p}') < aI_i^H(\mathrm{p})$.
- If $\{R_i(\mathrm{p}), R_i(\mathrm{p}')\} \subset \left[R_i^l, R_i^u\right]$, we have $I_i^H(\mathrm{p}) = I_i(\mathrm{p})$ and $I_i^H(\mathrm{p}') = I_i(\mathrm{p}')$. Let $r = \frac{R_i(\mathrm{p})}{R_i(\mathrm{p}')}$, we have

$$I_i\left(\mathrm{p}\right) = \frac{(1/r)^{\frac{x}{1-x}}\alpha_i\xi_i R_i(\mathrm{p}')^{\frac{x}{x-1}} + r\hat{\gamma}_i R_i(\mathrm{p}')}{\alpha_i + 1}. \tag{18}$$

  Additionally, from (16), we obtain $\left\{1/r, r, (1/r)^{\frac{x}{1-x}}\right\} \subset [1/a, a]$ because $0 < x \leq 1/2$. Therefore, we have $(1/a)I_i(\mathrm{p}') \leq I_i(\mathrm{p}) \leq aI_i(\mathrm{p}')$. Moreover, these inequations hold if $x = 1/2$ and both $1/r$ and $r$ are equal to $a$ or $1/a$. Evidently, this cannot be satisfied since $a > 1$. Thus, when $0 < x \leq 1/2$, we must have

$$(1/a)I_i^H(\mathrm{p}) < I_i^H(\mathrm{p}') < aI_i^H(\mathrm{p}) \tag{19}$$

- Finally, we have remaining cases where $R_i(\mathrm{p})$ or $R_i(\mathrm{p}')$ $\in \left[R_i^l, R_i^u\right]$. We will show the proof for the case $R_i(\mathrm{p}) \leq R_i^l \leq R_i(\mathrm{p}') \leq R_i^u\ (*)$ because the proofs for the remaining cases can be obtained similarly.
  Due to $(*)$, we have $I_i^H(\mathrm{p}) = \bar{p}_i$, $I_i^H(R_i^l) = \bar{p}_i = I_i(R_i^l)$ and $I_i^H(\mathrm{p}') = I_i(\mathrm{p}')$. By performing some simple mathematical manipulations, we obtain

$$(1/a)R_i^l \leq R_i(\mathrm{p}') \leq aR_i^l. \tag{20}$$

  By applying the results in the previous case, we have $\frac{1}{a}I_i^H(R_i^l) < I_i^H(\mathrm{p}') < aI_i^H(R_i^l)$. Since $I_i^H(R_i^l) = I_i^H(\mathrm{p}) = \bar{p}_i$ we have completed the proof for this case.

In summary, we have proved that our proposed p.u.f. is 2.s.s. when $0 < x \leq 1/2$. In fact, when $x > 1/2$, $(1/r)^{\frac{x}{1-x}}$ may not be in $[1/a, a]$. Hence, the p.u.f. might not be 2.s.s..

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