Interference Alignment with Random Vector Quantization for MIMO Interference Channels

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***Abstract*—In this paper, we propose an interference alignment scheme based on the random vector quantization codebooks for the constant MIMO interference channel. We quantify an upper bound of the rate loss of the proposed scheme and derive the scaling law of the feedback load to maintain the constant rate loss relative to the interference alignment with perfect channel knowledge. Moreover, we compare the proposed scheme with the interference alignment scheme using analog feedback and show the advantage of the proposed scheme under the constraint of the feedback channel uses. From simulation results, we demonstrate that the proposed scheme can provide the constant rate loss by imposing the feedback load according to the derived scaling law.**

***Index Terms*—MIMO interference channel, limited feedback, random vector quantization, multiplexing gain.**

I. Introduction

Interference alignment (IA) is a technique to achieve the maximum degrees of freedom (DOF) of the K-user interference channel (IC) by confining the interference signals to a reduced dimensional subspace of the received signal space such that an interference-free subspace is available for the desired signal [1]. IA can be performed by making use of any available dimensions such as time, frequency, or space. Although some results on DOF for the K-user IC by using IA with symbol extensions have been found [1], IA over time or frequency extensions of the channel is not practical since it requires infinite resource in time or frequency dimensions [2]. Hence, most recent researches on IA have been focusing on designing the precoding matrices and the receive filters for the constant multiple-input multiple-output (MIMO) IC [2]–[4]. However, due to NP-hardness of the problem to compute such matrices for the constant MIMO IC [5], a closed-form solution has been found only for some specific cases in [2] and iterative algorithms have been proposed as alternative approaches [3], [4].

The above IA schemes assume that globally perfect channel state information (CSI) is available for the transmitter. However, in practical scenarios, acquiring perfect CSI at the transmitter (CSIT) is not feasible due to the limited rate feedback link. Thus, in [6], [7], IA with limited feedback have been developed and analyzed to quantize and feedback the channel coefficients with Grassmannian codebooks. Specifically, the required number of feedback bits with respect to the signal-to-noise-ratio (SNR) has been derived to preserve the multiplexing gain in the K-user IC. Although designing a near-optimal codebook, explicitly linked to the Grassmannian line packing problem, can be realized numerically by minimizing the maximum inner product between any codewords, it is challenging to find the optimal codebook except for some special cases [8] so that it incurs a lack of practicality of the approaches in [6], [7].

In this paper, we propose an IA scheme with random vector quantization (RVQ) codebooks for the constant MIMO IC. Assuming that perfect CSI is available at the receiver side, IA solution for the proposed scheme can be achieved at the receiver side. Then, instead of quantizing the channel coefficients as in [6], [7], each receiver quantizes the precoding matrix to its codebook and informs the index to the associated transmitter. Under the assumption that information on the quantized precoding matrix is delivered through the limited rate feedback link, we analyze the rate loss as a function of the number of feedback bits and quantify the scaling law of the feedback load to maintain a constant rate loss relative to IA with perfect CSIT. Furthermore, we show that the proposed scheme outperforms the IA scheme using analog feedback in [9] when the channel uses per element of the quantized stream is larger than 1. Similar to the results in [10], we can see from simulation results that the average sum rate saturates at a constant value if the number of feedback bits is fixed from a certain SNR value. We also confirm that the multiplexing gain of the proposed scheme is preserved by increasing the number of feedback bits according to the derived scaling law.

The following mathematical notations will be used throughout the paper. Upper case and lower case boldfaces are used to denote matrices and vectors, respectively. $(\cdot)^{\mathsf{T}}$, $(\cdot)^{\mathsf{H}}$, $\mathrm{tr}\{\cdot\}$, and $\mathbb{E}[\cdot]$ represent the transpose, conjugate transpose, trace, and expectation operator, respectively.

II. System Model

We consider a K-user constant MIMO IC where each transmitter is equipped with N_t antennas and each receiver with N_r antennas. We assume a flat-fading channel both in time and frequency [3], [4]. Assuming that the j-th transmitter attempts to send the symbol $\mathbf{x}_j \in \mathbb{C}^{d_j \times 1}$ with d_j independent data streams to the j-th receiver, the signal vector at the j-th

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receiver, $\mathbf{y}_j \in \mathbb{C}^{N_r \times 1}$, can be represented as

$$\mathbf{y}_j = \sqrt{\frac{P}{d_j}}\mathbf{H}_{j,j}\mathbf{V}_j\mathbf{x}_j + \sum_{i=1,i\neq j}^{K}\sqrt{\frac{P}{d_i}}\mathbf{H}_{j,i}\mathbf{V}_i\mathbf{x}_i + \mathbf{n}_j, \quad (1)$$

where $\mathbf{H}_{j,i} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix between the i-th transmitter and the j-th receiver, $\mathbf{V}_j \in \mathbb{C}^{N_t \times d_j}$ is the precoding matrix for the j-th transmitter with unit-norm column vectors $\mathbf{v}_{j,m}$ for $m = 1, 2, \cdots, d_j$, and $\mathbf{n}_j \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise (AWGN) with elements distributed as $\mathcal{CN}(0, 1)$. Note that P is the transmission power, $\mathbf{x}_j$ is the data symbol vector for the j-th transmitter with $\mathbb{E}[\|\mathbf{x}_j\|^2] = d_j$, and the total number of streams is denoted as $d_{\text{tot}} = \sum_{j=1}^{K} d_j$. We assume that all the channel coefficients are independent and identically distributed (i.i.d.) Gaussian random variables with $\mathcal{CN}(0, 1)$. Denoting the receive filter for the j-th receiver as $\mathbf{U}_j \in \mathbb{C}^{N_r \times d_j}$ with unit-norm column vectors $\mathbf{u}_{j,m}$ for $m = 1, 2, \cdots, d_j$, we can write the m-th stream of the j-th receiver as the sum of the desired signal, the inter-stream interference (ISI), the inter-user interference (IUI), and the noise term, which is given by

$$\begin{aligned}
&\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{y}_j \\
= \ &\mathbf{u}_{j,m}^{\mathsf{H}}\sqrt{\frac{P}{d_j}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}x_{j,m} + \underbrace{\sum_{\substack{l=1\\ l\neq m}}^{d_j}\mathbf{u}_{j,m}^{\mathsf{H}}\sqrt{\frac{P}{d_j}}\mathbf{H}_{j,j}\mathbf{v}_{j,l}x_{j,l}}_{\text{inter-stream interference (ISI)}} \\
+ \ &\underbrace{\sum_{\substack{i=1\\ i\neq j}}^{K}\sum_{l=1}^{d_i}\mathbf{u}_{j,m}^{\mathsf{H}}\sqrt{\frac{P}{d_i}}\mathbf{H}_{j,i}\mathbf{v}_{i,l}x_{i,l}}_{\text{inter-user interference (IUI)}} + \mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{n}_j \ , \quad (2)
\end{aligned}$$

where $x_{j,m}$ is the m-th data stream of the j-th transmitter.

III. Interference Alignment with Random Vector Quantization

In this section, we briefly introduce an infinite rate feedback system so that perfect CSI is available at the transmitter side. Then, we describe a limited feedback system based on a RVQ codebook and subsequently analyze a rate loss due to the limited rate of the feedback link. Finally, we investigate the scaling law of the number of feedback bits in order to preserve a bounded rate loss.

A. Perfect Feedback System

We assume that the receiver jointly designs the precoding matrices and receive filters with perfect CSI at the receiver side. As the specific designs for the precoding matrices and receive filters, we consider the numerical IA schemes addressed in [3], [4]. Under the assumption that each precoding matrix is fed back to the corresponding transmitter via error-free infinite rate feedback link, the instantaneous sum rate of the system is given by

$$R_{\text{sum}}^{(\text{P})} = \sum_{j=1}^{K}\sum_{m=1}^{d_j} R_{j,m}^{(\text{P})}, \quad (3)$$

where

$$R_{j,m}^{(\text{P})} = \log_2\left(1 + \frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right) \quad (4)$$

is the achievable rate for the m-th stream of the j-th receiver. Note in (4) that both IUI and ISI are eliminated by interference alignment only if perfect CSIT is available [3].

B. Limited Feedback System

After the precoding matrices and receive filters are obtained at the receiver side, we assume that each receiver quantizes the precoding matrix to its codebook and informs the index to the associated transmitter via limited rate feedback link. For 1) analytical tractability and 2) scalability for any antenna configurations, we consider a RVQ codebook with independent unit-norm vectors from the isotropic distribution on the complex unit sphere [10], [11]. The codebook for the m-th stream of the j-th receiver is denoted as $\mathcal{F}_{j,m} = \left\{\mathbf{f}_{j,m,1}, \mathbf{f}_{j,m,2}, \cdots, \mathbf{f}_{j,m,2^{B_{j,m}}}\right\}$, where $B_{j,m}$ is the number of feedback bits for the m-th stream of the j-th receiver. Note that each receiver uses an independently generated codebook per stream to ensure the number of spatial dimensions and the common codebook is accessible to its associated transmitter. By adopting the metric based on the chordal distance in [10] as

$$i_{j,m} = \arg\max_{1\leq k\leq 2^{B_{j,m}}}\left|\mathbf{f}_{j,m,k}^{\mathsf{H}}\mathbf{v}_{j,m}\right|, \quad (5)$$

the quantization index is determined and fed back to the j-th transmitter. As a result, the quantized transmit beamforming vector is obtained as $\hat{\mathbf{v}}_{j,m} = \mathbf{f}_{j,m,i_{j,m}}$.

C. Rate Loss Analysis

To characterize the performance loss of the proposed IA scheme, we quantify an upper bound of the rate loss as a function of the number of feedback bits. For the limited feedback system, both IUI and ISI cannot be eliminated so that the resulting residual interference power degrades the sum rate. The achievable rate for the m-th stream of the j-th receiver with the limited feedback can be written as

$$R_{j,m}^{(\text{L})} = \log_2\left(1 + \frac{\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2}{1 + I_{j,m}}\right), \quad (6)$$

where

$$\begin{aligned}
I_{j,m} \ = \ &\sum_{\substack{l=1\\ l\neq m}}^{d_j}\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,l}\right|^2 \\
+ \ &\sum_{\substack{i=1\\ i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\hat{\mathbf{v}}_{i,l}\right|^2 \quad (7)
\end{aligned}$$

is the residual interference power due to the limited feedback. From (4) and (6), we define the rate loss for the m-th stream of the j-th receiver as

$$\triangle R_{j,m} = \mathbb{E}\left[R_{j,m}^{(\text{P})} - R_{j,m}^{(\text{L})}\right], \quad (8)$$

where the expectation is carried out over the channel distribution and random codebooks. We provide three lemmas used for proving Theorem 1 and then specify the upper bound of the rate loss for the m-th stream of the j-th receiver in the following theorem.

Lemma 1: The equality holds between the following expectation terms with $\mathbf{v}_{j,m}$ and $\hat{\mathbf{v}}_{j,m}$, which is given by

$$\begin{aligned} & \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right] \\ = \; & \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2\right)\right]. \end{aligned} \tag{9}$$

Proof: Considering the solutions to IA precoders such as [3], [4], we have the following properties proven in [9, Lemma 1] as

- $\mathbf{H}_{j,j}\mathbf{v}_{j,m}$ are Gaussian vectors with covariance $\mathbf{I}$.
- $\mathbf{H}_{j,j}\mathbf{v}_{j,m}$ and $\mathbf{H}_{j,j}\mathbf{v}_{j,l}$ are independent for $\forall l \neq m$.
- $\mathbf{u}_{j,m}$ is independent of $\mathbf{H}_{j,j}\mathbf{v}_{j,m}$.

These properties lead that $\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2$ is exponentially distributed with parameter 1. Since $\hat{\mathbf{v}}_{j,m}$ is chosen from a random generated codebook with unit vectors, which can be represented as $\mathbf{v}_{j,m}$ and the quantization error, it can be shown that $\hat{\mathbf{v}}_{j,m}$ also follows the properties by replacing $\mathbf{v}_{j,m}$. Therefore, $\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2$ is also exponentially distributed with parameter 1, which means that the equality holds in (9). ■

Lemma 2: $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$ is beta distributed with parameters $(1, N_t-2)$.

Proof: Noting that for the perfect IA we have $\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\mathbf{v}_{i,l}=0$, $\forall(j,m)\neq(i,l)$, $\bar{\mathbf{e}}_{j,i,m}$ is an isotropic unit vector on the (N_t-1)-dimensional hyperplane orthogonal to $\mathbf{v}_{i,l}$. From (13), $\mathbf{z}_{i,l}$ is also an isotropic unit vector on the (N_t-1)-dimensional hyperplane orthogonal to $\mathbf{v}_{i,l}$. Since $\bar{\mathbf{e}}_{j,i,m}$ and $\mathbf{z}_{i,l}$ are independently and isotropically distributed within the same (N_t-1)-dimensional nullspace of $\mathbf{v}_{i,l}$, the quantity $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$ is beta distributed with parameters $(1, N_t-2)$ [10, Lemma 2]. ■

Lemma 3: The upper bound of $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]$ is given by $\bar{\lambda}_{\max}$.

Proof: From the definition of $\mathbf{e}_{j,i,m}$, we have $\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2 = \mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\mathsf{H}}\mathbf{u}_{j,m}$. Note that we have $\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\mathsf{H}}\mathbf{u}_{j,m} \leq \lambda_{\max}$ according to the Rayleigh-Ritz theorem [12], where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\mathsf{H}}$. By using the empirical result of the mean of the largest eigenvalue in [13, Eq. (28)], we obtain the upper bound of $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]$ as $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right] \leq \mathbb{E}\left[\lambda_{\max}\right] \approx N_rN_t\left(\frac{N_t+N_r}{N_tN_r+1}\right)^{2/3}$ for $N_rN_t \leq 250$. ■

Theorem 1: The upper bound of the rate loss for the m-th stream of the j-th receiver is given by

$$\triangle R_{j,m} < \log_2\left(\begin{array}{c} 1+\sum\limits_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{j,l}}{N_t-1}} \\ +\sum\limits_{\substack{i=1\\i\neq j}}^{K}\sum\limits_{l=1}^{d_i}\frac{P}{d_i}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{i,l}}{N_t-1}} \end{array}\right), \tag{10}$$

where $B_{j,l}$ is the number of feedback bits for the l-th stream of the j-th receiver and $\bar{\lambda}_{\max} \triangleq N_rN_t\left(\frac{N_t+N_r}{N_tN_r+1}\right)^{2/3}$.

Proof: The rate loss in (8) can be rewritten as

$$\begin{aligned} \triangle R_{j,m} &= \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right] \\ &\quad -\mathbb{E}\left[\log_2\left(1+\frac{\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2}{1+I_{j,m}}\right)\right] \\ &= \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right] \\ &\quad -\mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2+I_{j,m}\right)\right] \\ &\quad +\mathbb{E}\left[\log_2\left(1+I_{j,m}\right)\right] \\ &\overset{(a)}{\leq} \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right] \\ &\quad -\mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2\right)\right] \\ &\quad +\mathbb{E}\left[\log_2\left(1+I_{j,m}\right)\right] \\ &\overset{(b)}{\leq} \mathbb{E}\left[\log_2\left(1+I_{j,m}\right)\right] \\ &\overset{(c)}{\leq} \log_2\left(1+\mathbb{E}\left[I_{j,m}\right]\right), \end{aligned} \tag{11}$$

where (a) comes from the fact that $I_{j,m} \geq 0$ and $\log(\cdot)$ is a monotonically increasing function [10], (b) follows from Lemma 1, and (c) follows from the Jensen's inequality.

We can rewrite (7) as

$$\begin{aligned} I_{j,m} &= \sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left\|\mathbf{e}_{j,j,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,j,m}^{\mathsf{H}}\hat{\mathbf{v}}_{j,l}\right|^2 \\ &+ \sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\hat{\mathbf{v}}_{i,l}\right|^2, \end{aligned} \tag{12}$$

where $\mathbf{e}_{j,i,m}^{\mathsf{H}} = \mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}$ and $\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}} = \mathbf{e}_{j,i,m}^{\mathsf{H}}/\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|$. Denoting the angle between $\mathbf{v}_{j,m}$ and $\hat{\mathbf{v}}_{j,m}$ as $\theta_{j,m}$, i.e. $\cos\theta_{j,m} = |\mathbf{v}_{j,m}^{\mathsf{H}}\hat{\mathbf{v}}_{j,m}|$, we decompose $\hat{\mathbf{v}}_{j,m}$ as [8]

$$\hat{\mathbf{v}}_{j,m} = \cos\theta_{j,m}\mathbf{v}_{j,m} + \sin\theta_{j,m}\mathbf{z}_{j,m}, \tag{13}$$

where $\mathbf{z}_{j,m}$ is the error vector due to the quantization. By substituting (13) into (12), we have

$$I_{j,m} = \sum_{\substack{l=1 \\ l\neq m}}^{d_j} \frac{P}{d_j} \left(\sin^2\theta_{j,l}\right) \left\|\mathbf{e}_{j,j,m}^{\mathsf{H}}\right\|^2 \left|\bar{\mathbf{e}}_{j,j,m}^{\mathsf{H}}\mathbf{z}_{j,l}\right|^2 + \sum_{\substack{i=1 \\ i\neq j}}^{K}\sum_{l=1}^{d_i} \frac{P}{d_i} \left(\sin^2\theta_{i,l}\right) \left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2 \left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2. \quad (14)$$

Since the random variables $\sin^2\theta_{j,l}$, $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$, and $\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2$ are all independent, the upper bound of the expected residual interference power can be obtained as

$$\begin{aligned}
&\mathbb{E}\left[I_{j,m}\right] \\
&\leq \sum_{\substack{l=1 \\ l\neq m}}^{d_j} \frac{P}{d_j}\mathbb{E}\left[\left(\sin^2\theta_{j,l}\right)\right]\mathbb{E}\left[\left\|\mathbf{e}_{j,j,m}^{\mathsf{H}}\right\|^2\right]\mathbb{E}\left[\left|\bar{\mathbf{e}}_{j,j,m}^{\mathsf{H}}\mathbf{z}_{j,l}\right|^2\right] \\
&+ \sum_{\substack{i=1 \\ i\neq j}}^{K}\sum_{l=1}^{d_i} \frac{P}{d_i}\mathbb{E}\left[\left(\sin^2\theta_{i,l}\right)\right]\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]\mathbb{E}\left[\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2\right].
\end{aligned} \quad (15)$$

Note that $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$ is beta distributed with parameters $(1, N_t - 2)$ from Lemma 2 and its expectation is equal to $\frac{1}{N_t-1}$ [14]. From [10, Lemma 1], the upper bound of $\mathbb{E}\left[\left(\sin^2\theta_{i,l}\right)\right]$ is given by $2^{-\frac{B_{i,l}}{N_t-1}}$. Using Lemma 3, we can rewrite (15) as

$$\mathbb{E}\left[I_{j,m}\right] < \sum_{\substack{l=1 \\ l\neq m}}^{d_j} \frac{P}{d_j}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{j,l}}{N_t-1}} + \sum_{\substack{i=1 \\ i\neq j}}^{K}\sum_{l=1}^{d_i} \frac{P}{d_i}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{i,l}}{N_t-1}}. \quad (16)$$

Substituting (16) into (11), we derive the upper bound of the rate loss for the m-th stream of the j-th receiver as (10). ■

D. Scaling Law of the Number of Feedback Bits per Stream

If the number of feedback bits per stream is fixed for all the SNR value, the residual interference power will dominate the desired signal power as SNR goes to infinity, which results in zero multiplexing gain [10], [15]. Therefore, the upper bound of the rate loss derived in Theorem 1 can be maintained constant by increasing the number of feedback bits per stream as a function of SNR. Since we consider i.i.d. channel coefficients, the gain from the allocation of the feedback bits per stream can be ignored. For the simplicity of the analysis, we assume that the number of feedback bits per stream is set to B for all users, i.e. $B_{j,m} = B$, $\forall(j,m)$. In the following theorem, we verify the sufficient scaling law of the feedback bits per stream to maintain the constant upper bound of the rate loss.

Theorem 2: The sufficient scaling law of the feedback bits per stream to maintain the constant upper bound of the rate loss is given by

$$\begin{aligned}
B &\geq (N_t-1)\log_2\left(\frac{Kd-1}{d}\frac{P\bar{\lambda}_{\max}}{N_t-1}\right) \\
&- (N_t-1)\log_2(b-1) \\
&\approx \frac{N_t-1}{3}P_{\mathrm{dB}} + (N_t-1)\log_2\left(\frac{Kd-1}{d}\frac{\bar{\lambda}_{\max}}{N_t-1}\right) \\
&- (N_t-1)\log_2(b-1).
\end{aligned} \quad (17)$$

Proof: Under the assumption of the equal feedback over streams, the upper bound in (10) can be simplified as

$$\triangle R_{j,m} < \log_2\left(1 + \left(\frac{Kd-1}{d}\frac{P\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B}{N_t-1}}\right). \quad (18)$$

To exploit the sufficient number of feedback bits for a rate loss of no more than $\log_2 b$, we set (18) to the maximum allowable gap of $\log_2 b$ as

$$\log_2\left(1 + \frac{Kd-1}{d}\frac{P\bar{\lambda}_{\max}}{N_t-1}2^{-\frac{B}{N_t-1}}\right) \leq \log_2 b. \quad (19)$$

From (19), we can solve the number of feedback bits per stream as a function of b and SNR, which is given by (17). ■

If we set $b = 2$, which implies that the rate loss per stream is kept within 1 bps/Hz, the scaling law in (17) can be simplified as

$$B \geq \frac{N_t-1}{3}P_{\mathrm{dB}} + (N_t-1)\log_2\left(\frac{Kd-1}{d}\frac{\bar{\lambda}_{\max}}{N_t-1}\right). \quad (20)$$

E. Comparison with Analog CSI Feedback

We compare our proposed scheme with the IA scheme using analog CSI feedback addressed in [9] under the assumption of perfect CSI at the receiver. Assuming that the feedback link can operate error-free at capacity, it implies that $\log_2(1+P)$ bits per symbol can be transmitted reliably [16], where P is the transmitted power of the forward link. Thus, the number of feedback bits per stream via the error-free feedback link becomes $\beta_{\mathrm{AF}}N_t\log_2(1+P_f)$, where β_{AF} is the number of channel uses per element of the quantized stream via the feedback link and P_f is the transmitted power of the feedback link. By alleviating the upper bound in (19) as $\log_2\left(1 + \left(\frac{Kd-1}{d}\frac{P\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B}{N_t}}\right)$ and assuming $b = 2$, the number of feedback bits per stream for the proposed scheme is given by $N_t\log_2\left(\frac{Kd-1}{d}\frac{P\bar{\lambda}_{\max}}{N_t-1}\right)$. Then, we can obtain the relationship between P_f and P as

$$P_f = \left(\frac{Kd-1}{d}\frac{\bar{\lambda}_{\max}}{N_t-1}P\right)^{\frac{1}{\beta_{\mathrm{AF}}}} - 1. \quad (21)$$

From (21) and [9, Theorem 4], we can confirm that the proposed scheme asymptotically outperforms the IA scheme using analog CSI feedback as long as $\beta_{\mathrm{AF}} > 1$, which can be also found in multi-user MIMO downlink systems [16].

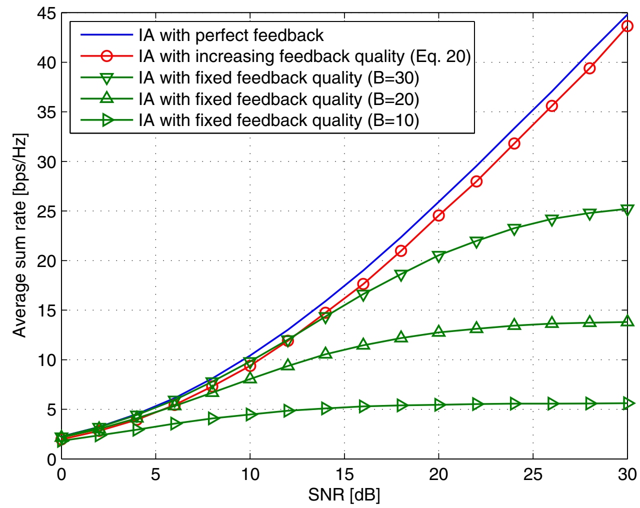

Fig. 1. Average sum rate in a 3-user constant MIMO IC with $N_t = 4$, $N_r = 5$, and $d_{\text{tot}} = 6$ ($d_1 = d_2 = d_3 = 2$).

IV. Numerical Results

In this section, we present simulation results to evaluate the sum rate performance of the proposed IA scheme. In Fig. 1, we illustrate the average sum rate as a function of SNR for the 3-user constant MIMO IC where $N_t = 4$, $N_r = 5$, and $d_{\text{tot}} = 6$. We confirm that the proposed IA scheme can maintain a constant rate loss compared with IA with perfect feedback by increasing the feedback quality according to the relationship in (20) with SNR. If the feedback quality is fixed regardless of SNR, we observe that the multiplexing gain goes to zero in the high SNR regime since the residual interference terms will dominate the rate in (6) as P increases.

V. Conclusion

In this paper, we proposed an IA scheme based on random vector quantization codebooks for the constant MIMO IC. We derived the upper bound of the rate loss and the scaling law of the number of feedback bits in order to preserve the multiplexing gain of IA with perfect CSIT. Using the derived upper bound, we showed that the proposed scheme is superior to the IA scheme with the analog feedback when the channel uses per element of the quantized stream is larger than 1.

References

[1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[2] C. Yetis, T. Gou, S. A. Jafar, and A. Kayran, "On feasibility of interference alignment in MIMO interference networks," *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4771–4782, Sep. 2010.

[3] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, Jun. 2011.

[4] S. W. Peters and R. W. Heath, Jr., "Cooperative algorithms for MIMO interference channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 1, pp. 206–218, Jan. 2011.

[5] M. Razaviyayn, M. S. Boroujeni, and Z.-Q. Luo, "Linear transceiver design for interference alignment: Complexity and computation," in *Proc. IEEE SPAWC*, Marrakech, Morocco, Jun. 2010, pp. 1–5.

[6] J. Thukral and H. Bölcskei, "Interference alignment with limited feedback," in *Proc. IEEE ISIT*, Seoul, Korea, Jun. 2009, pp. 1759–1763.

[7] R. Krishnamachari and M. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," in *Proc. IEEE ISIT*, Austin, TX, USA, Jun. 2010, pp. 619–623.

[8] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.

[9] O. E. Ayach and R. W. Heath, Jr., "Interference alignment with analog channel state feedback," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 626–636, Feb. 2012.

[10] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045–5060, Nov. 2006.

[11] C. K. Au-Yeung and D. J. Love, "On the performance of random vector quantization limited feedback beamforming in a MISO system," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 458–462, Feb. 2007.

[12] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge, UK: Cambridge University Press, 1985.

[13] Y. Li, L. Zhang, L. Cimini, and H. Zhang, "Statistical analysis of MIMO beamforming with co-channel unequal-power MIMO interferers under path-loss and rayleigh fading," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3738–3748, Aug. 2011.

[14] A. Gupta and S. Nadarajah, *Handbook of beta distribution and its applications*. New York, NY, USA: Marcel Dekker, 2004.

[15] N. Ravindran and N. Jindal, "Limited feedback-based block diagonalization for the MIMO broadcast channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1473–1482, Oct. 2008.

[16] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Quantized vs. analog feedback for the MIMO broadcast channel: A comparison between zero-forcing based achievable rates," in *Proc. IEEE ISIT*, Nice, France, Jun. 2007, pp. 2046–2050.