Optimal Design of Probabilistic Slot Allocation for Multiple-Sensor Relaying Networks

Insook Kim and Dongwoo Kim

The Department of Electrical Engineering and Computer Science
Hanyang University, Ansan, 426-791 South Korea
Email: iskim@wnl.hanyang.ac.kr, dkim@hanyang.ac.kr

***Abstract*—The aim of this paper is to design a bandwidth-efficient multiuser network. We consider a wireless sensor network where a relay helps multiple source-destination pairs which require a minimum data rate. The relay helps communication pairs in probabilistic manner during relaying slots. In the proposed scheme, the relay does not need orthogonal channels (time slots) as many as the number of source-destination pairs but the channels are shared by the relay with some probability. We optimize the slot-sharing ratio to maximize the expected sum rate while guaranteeing the minimum rate for a given number of relaying slots. Numerical examples show that the proposed relaying scheme improves the expected data rate even though the number of relaying slots is less than the number of source-destination pairs.**

I. INTRODUCTION

In wireless sensor networks, the sensors usually become sources of sending information. If the sensors are far from the destination nodes responsible for gathering the information data, it is plausible to make the sensors form a virtual antenna array to exploit cooperative diversity [1], [2].

In [3]-[7], so-called *multiuser relay networks* have been investigated for various design goals though an exact solution procedure is not addressed. In [3]-[6] and the references therein, the power allocation is optimized to satisfy various criteria in multiuser relay networks. However, they assume that the data transmission of each source node occurs in two pre-assigned channels that can be either different time slots or different frequencies. All the channels used by sources and relay nodes are distinct and nonoverlapping, which is bandwidth inefficient.

Standard relaying transmission protocols usually demand orthogonal channels (time-division-multiple-access (TDMA) time slots) for the source and then the relays to communicate with the destination [7]. To overcome the multiplexing limitation of standard protocols, they propose repetition coding based protocols and examine the diversity performance of the proposed protocols in [7]. However, they do not consider how to optimize the system performance (for example, by relay selection or power allocation).

The aim of this paper is to design a bandwidth-efficient multiuser network. First, we propose a relaying mechanism for multiple-sensor relaying networks. In the proposed scheme, a relay assists multiple source-destination pairs in a probabilistic manner during relaying slots. In the proposed scheme, the relay does not need orthogonal channels (time slots) as many as the number of source-destination pairs but the channels are shared by the relay with some probability. We formulate slot-sharing ratio (SSR) allocation problem to figure out with what probability the relay helps each source-destination pair.

This work was supported by the Second-Phase Brain Korea 21 Project in 2011.

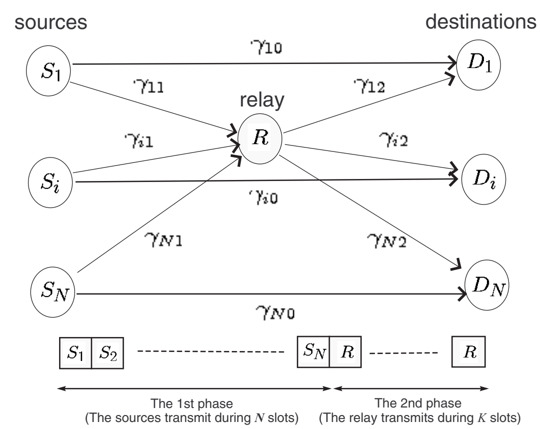

Fig. 1. System model.

II. SYSTEM MODEL

A. Network Model

We consider a wireless sensor network where there are N independent source-destination pairs and a relay. A certain target data rate ζ_i $(i = 1, 2, \cdots, N)$ is required for reliable communication between each pair of source $(\mathcal{S}_i)$ and destination $(\mathcal{D}_i)$. The relay $(\mathcal{R})$ assists N source-destination pairs so as to achieve the target data rate. Time division protocol is assumed and $N + K$ equal-length time slots consist of two phases. In the first phase, each of N sources transmits its own data during the time slot exclusively pre-allocated to the source. The relay as well as the corresponding destination node receives the signal. The subsequent K time slots following the first phase consist of the second phase. During the second phase, the relay amplifies and forwards the signals received

978-1-4673-0990-5/12/$31.00 ©2012 IEEE

from the sources to the destinations. The signals from the source (during the first phase) and the relay (during the second phase) are combined at the destination by using maximal ratio combining (MRC).

Let γ_{i0}, γ_{i1} and γ_{i2} denote the signal-to-noise ratio (SNR) for the link $\mathcal{S}_i - \mathcal{D}_i$, $\mathcal{S}_i - \mathcal{R}$ and $\mathcal{R} - \mathcal{D}_i$, respectively. We assume that the SNRs are known to the relay for optimization and unchanged during the whole transmission period, specifically, at least $N + K$ slots. If the relay is not introduced to the system (i.e., when $K = 0$), each communication pair can achieve data rate

$$\rho_i = \frac{1}{N}\log_2\left(1 + \gamma_{i0}\right), \; i = 1, 2, \cdots, N. \tag{1}$$

If the bandwidth is limited and $\zeta_i > \rho_i$ for some i, then introducing a relay and allocating certain bandwidth to the relay is a possible solution.

When $K(> 0)$ *relaying slots* are used by the relay, the relay assigns each time slot to supporting communication between $\mathcal{S}_i - \mathcal{D}_i$ with probability p_i, which is called *slot-sharing ratio* (SSR) in this paper. We assume that each relaying slot is assigned independently. Let X_i denote the number of relaying slots assigned to the ith communication pair among K relaying slots. X_i has then a binomial distribution whose density function is $\Pr\{X_i = x\} = \binom{K}{x}(p_i)^x(1 - p_i)^{K-x}$, $x = 0, 1, \cdots, K$. For given N and K, the achievable data rate for the ith pair with X_i relaying slots is

$$I_i(X_i; N, K) = \frac{1}{N + K}\log_2(1 + \gamma_{i0} + X_i\gamma_{iR}), \tag{2}$$

where γ_{iR} is relayed end-to-end SNR [8]

$$\gamma_{iR} = \frac{\gamma_{i1}\gamma_{i2}}{\gamma_{i1} + \gamma_{i2} + 1}. \tag{3}$$

In the sequel, we use $I_i(X_i)$ instead of $I_i(X_i; N, K)$ for notational convenience.

B. Problem Formulation

Since $I_i(X_i)$ is a function of random variable X_i binomially-distributed, an expected data rate for the i-th pair is given as the following;

$$r_i(p_i) = \mathbb{E}\left[I_i(X_i)\right] = \sum_{k=0}^{K} I_i(k)\binom{K}{k}(p_i)^k(1 - p_i)^{K-k}. \tag{4}$$

Let ω_i is a positive reward factor for the expected data rate $r_i(p_i)$. We are interested in designing SSR that maximizes the sum of reward-weighted rate for all the pairs while satisfying the given requirement ζ_i. The problem can be formulated as;

$$\textbf{(P1-K)} \qquad \max_{\{p_i\}} \quad z_K = \sum_{i=1}^{N}\omega_i r_i(p_i) \tag{5}$$

$$\text{s.t.} \quad r_i(p_i) \geq \zeta_i, \; \text{for all } i, \tag{6}$$

$$\sum_{i=1}^{N} p_i = 1, \tag{7}$$

$$0 \leq p_i \leq 1, \; \text{for all } i, \tag{8}$$

where $\{p_i\}$ is a set of $\{p_1, p_2, \cdots, p_N\}$.

If we can solve (P1-K), finding an optimal K is not much difficult since it is an one-dimensional combinatorial search on K. In the following sections, we provide an optimal SSR allocation when K is given.

III. Optimal Slot-Sharing Ratio $\{p_i\}$

Given the above described protocol, we now investigate the optimum value of SSR $\{p_i\}$ to maximize the sum of reward-weighted rate at the destinations with satisfying the target data rates, $\{\zeta_i\}$.

The expected data rate for the i-th pair, $r_i(p_i)$ in (4) can be rewritten as

$$r_i(p_i) = a_{i0} + a_{i1}p_i + a_{i2}p_i^2 + \cdots + a_{iK}p_i^K = \sum_{k=0}^{K} a_{ik}{p_i}^k \tag{9}$$

where

$$a_{ik} = \sum_{x=0}^{k} I_i(x)\binom{K}{x}\binom{K - x}{k - x}(-1)^{k-x}. \tag{10}$$

In (9), $r_i(p_i)$ is a K-th order polynomial function of p_i. However, in Proposition 1, we prove that $r_i(p_i)$ is increasing function over $0 \leq p_i \leq 1$.

Proposition 1: The expected data rate $r_i(p_i)$ is a monotonic increasing function for $0 \leq p_i \leq 1$.

Proof: See Appendix. ■

The constant term in (9) represents an expected rate of a link $\mathcal{S}_i - \mathcal{D}_i$ when allocated no relaying slot during the second phase.

$$a_{i0} = \frac{1}{N + K}\log_2(1 + \gamma_{i0}). \tag{11}$$

We may allocate no relaying slots to i such that $a_{i0} \geq \zeta_i$, i.e., $p_i = 0$. However, if $a_{i0} < \zeta_i$, the source-destination pair should be helped by a relay with positive p_i to meet the target data rate. In the following, we present how to find the optimal SSR for a system with N, K and $\{\zeta_i\}$.

Let define $\mathbb{S}$ as an index set of source-destination pairs such that $a_{i0} < \zeta_i$, i.e.,

$$\mathbb{S} = \left\{i | a_{i0} < \zeta_i\right\}. \tag{12}$$

Then a complement of $\mathbb{S}$ is $\mathbb{S}^c = \{i | a_{i0} \geq \zeta_i\}$. Since $r_i(p_i)$ is increasing over p_i, there exists a positive real root of $r_i(p_i) = \zeta_i$ for $i \in \mathbb{S}$, denoted by $\bar{\mathrm{p}}_i$. If $\bar{\mathrm{p}}_i \leq 1$, $\bar{\mathrm{p}}_i$ is the minimum SSR to guarantee the target rate for $i \in \mathbb{S}$. If $\bar{\mathrm{p}}_i \geq 1$ for some $i \in \mathbb{S}$, it means that the target rate cannot be achieved even though the relay fully supports this source-destination pair. Such a pair should be removed. In this work, we assume that $r_i(1) \geq \zeta_i$ for all i. It means that the achievable rate $S_i - D_i$ with $p_i = 1$ is greater than the target rate ζ_i. Under this assumption, we have $\bar{\mathrm{p}}_i \leq 1$ for $i \in \mathbb{S}$. Meanwhile, there is no positive $\bar{\mathrm{p}}_i$ for $i \in \mathbb{S}^c$. For simplicity of description, we assume that $\bar{\mathrm{p}}_i = 0$ for $i \in \mathbb{S}^c$. Notice that, for $i \in \mathbb{S}^c$, the target rate is guaranteed at destination even though $p_i = 0$.

A. Feasibility

From the increasing property of $r_i(p_i)$ over $0 \le p_i \le 1$, an optimal solution $\{p_i^*\}$ can be written as

$$p_i^* = \bar{\mathrm{p}}_i + \Delta_i, \text{ for all } i. \quad (13)$$

where all Δ_i are nonnegative and $\sum_{i=1}^N \Delta_i \le 1 - \sum_{i=1}^N \bar{\mathrm{p}}_i$. However, $\sum_{i=1}^N \bar{\mathrm{p}}_i$ could be greater than unity. We call (P1-K) *feasible* if $\sum_{i=1}^N \bar{\mathrm{p}}_i \le 1$ for a network with N, K and $\{\zeta_i\}$. Otherwise, (P1-K) is called *infeasible*, which means that the total N of communication pairs in the network cannot achieve the target data rate $\{\zeta_i\}$ under the design parameter K. It may happen to a system if there are too many number of communication pairs (too big N) or too large rate required (too high ζ_i). In the rest of this section, we discuss solving the problem when (P1-K) is feasible.

B. Optimal SSR Allocation

Suppose that $\sum_{i=1}^N \bar{\mathrm{p}}_i = 1$. Definitely, $\{\bar{\mathrm{p}}_i\}$ is a feasible solution of (P1-K). If the value of any p_i is increased above $\bar{\mathrm{p}}_i$, p_j for some $j \ne i$ should be reduced below $\bar{\mathrm{p}}_j$ to meet the constraint (7). Any $p_j < \bar{\mathrm{p}}_j$ makes not to meet the constraint (6). Therefore $\{\bar{\mathrm{p}}_i\}$ is an only feasible and hence the optimal solution of (P1-K) if $\sum_{i=1}^N \bar{\mathrm{p}}_i = 1$.

If $\sum_{i=1}^N \bar{\mathrm{p}}_i < 1$, solving (P1-K) is equivalent to solve the following problem

$$\textbf{(P1}'\textbf{-K)} \qquad \max_{\{\Delta_i\}} \quad z_K = \sum_{i=1}^N \omega_i r_i(\bar{\mathrm{p}}_i + \Delta_i) \quad (14)$$

$$\text{s.t.} \quad \sum_{i=1}^N \Delta_i = 1 - \sum_{i=1}^N \bar{\mathrm{p}}_i, \quad (15)$$

$$\Delta_i \ge 0, \text{ for all } i. \quad (16)$$

(P1′-K) is a convex optimization problem. The Lagrangian of (P1′-K) is

$$\mathcal{L} = \sum_{i=1}^N \omega_i r_i(\bar{\mathrm{p}}_i + \Delta_i) - \lambda\left(\sum_{i=1}^N \Delta_i - 1 + \sum_{i=1}^N \bar{\mathrm{p}}_i\right) + \sum_{i=1}^N \mu_i \Delta_i, \quad (17)$$

where λ and $\{\mu_i\}$ are dual variables for the constraints (15) and (16), respectively.

Let $\{\Delta_i^*\}$ denote an optimal solution of (P1′-K). Hereafter, x^* means an optimal value of variable x. Karush-Kuhn-Tucker (KKT) conditions [9] for (P1′-K) are:

$$1) \quad \omega_i r_i'(\bar{\mathrm{p}}_i + \Delta_i^*) - \lambda^* + \mu_i^* = 0 \quad (18)$$

$$2) \quad \sum_{i=1}^N \Delta_i^* = 1 - \sum_{i=1}^N \bar{\mathrm{p}}_i \quad (19)$$

$$3) \quad \mu_i^* \Delta_i^* = 0 \quad (20)$$

$$4) \quad \Delta_i^* \ge 0, \mu_i^* \ge 0. \quad (21)$$

At optimality, some Δ_i^*s are greater than zero and the others are equal to zero. Let Ω denote a set of indices satisfying $\Delta_i^* > 0$ and Ω^c a set of indices satisfying $\Delta_i^* = 0$. For $i \in \Omega$, by substituting $\mu_i^* = 0$ into the KKT condition (18), we have

$$\lambda^* = \omega_i r_i'(\bar{\mathrm{p}}_i + \Delta_i^*) \quad \text{for } i \in \Omega. \quad (22)$$

Suppose that (22) can be solved with respect to Δ_i^*.

$$\Delta_i^* = r_i'^{-1}\left(\frac{\lambda^*}{\omega_i}\right) - \bar{\mathrm{p}}_i, \quad \text{for } i \in \Omega. \quad (23)$$

Then, substituting Δ_i^* into (19), we have the following equation.

$$\sum_{i \in \Omega}\left(r_i'^{-1}\left(\frac{\lambda^*}{\omega_i}\right) - \bar{\mathrm{p}}_i\right) = 1 - \sum_{i=1}^N \bar{\mathrm{p}}_i. \quad (24)$$

If we can solve the equation (24) with respect to λ^*, by substituting λ^* into (23) we can find Δ^* for $i \in \Omega$.

Now, we present how to determine Ω and Ω^c. From $\mu_i^* > 0$, $\Delta_i^* = 0$ for $i \in \Omega^c$ and the KKT condition (18), we have the following condition which is useful to identify the set Ω and Ω^c.

$$\lambda^* > \omega_i r_i'(\bar{\mathrm{p}}_i), \ i \in \Omega^c. \quad (25)$$

Let us assume that the relay nodes are sorted by a descending order of $\omega_i r_i'(\bar{\mathrm{p}}_i)$ without loss of generality. $\omega_i r_i'(\bar{\mathrm{p}}_i)$ can be interpreted as a marginal benefit of $(\mathcal{S}_i, \mathcal{D}_i)$ for which SSR is allocated by $\bar{\mathrm{p}}_i$. Ω and Ω^c are determined by the following procedure.

- step 1. Let $\Omega = \phi$. And let $\Omega^c = \{1, 2, \cdots, N\}$ be an index set in a descending order of $\omega_i r_i'(\bar{\mathrm{p}}_i)$. Start with $j = 1$.
- step 2. Compute λ^* and check if $\lambda^* > \omega_i r_i'(\bar{\mathrm{p}}_i)$ for $i \in \Omega^c$. If the condition is not satisfied, update $\Omega = \Omega \cup \{j\}$ and $\Omega^c = \Omega^c - \{j\}$ and set $j = j + 1$. Repeat step 2 until the condition is satisfied.
- step 3. If the condition is satisfied, stop with $\Omega = \{1, 2, \cdots, j-1\}$ and $\Omega^c = \{j, j+1, \cdots, N\}$.

Finally, with an optimal solution to (P1′-K), Δ_i^*, we obtain an optimal SSR by (13).

In the process described above, first, we need to solve $r_i(p_i)$ with respect to p_i to find $\bar{\mathrm{p}}_i$. And we should solve (22) and (24) in terms of Δ_i^* and λ_i^*, respectively. Remind that $r_i(p_i)$ is K-th order polynomial function. In general, for polynomials of higher degree K, it is not easy to find exact expressions for roots and an even known methods for the exact solution formulae are computationally expensive [10]. In addition, even though (22) can be solved, it is not trivial to solve (24) in terms of λ_i^* with higher K. These restrict us to find an optimal SSR by solving (P1-K) or (P1′-K) for general K. In the following subsection, to appreciate the method described previously, we explain a procedure to find an optimal SSR when $K = 1$ or 2 and present a closed form of the optimal solution.

C. Allocating A Single Relay Slot ($K = 1$)

When $K = 1$, the expected data rate at $\mathcal{D}_i$ is a linear function over p_i;

$$r_i(p_i) = a_{i0} + a_{i1} p_i. \quad (26)$$

It is easily to calculate $\bar{\mathrm{p}}_i$ such that $a_{i0} + a_{i1}\bar{\mathrm{p}}_i = 0$. We have $\bar{\mathrm{p}}_i = \frac{\zeta_i - a_{i0}}{a_{i1}} > 0$ for $i \in \mathbb{S}$. As mentioned before, we set $\bar{\mathrm{p}}_i = 0$ for $i \in \mathbb{S}^c$. Therefore,

$$\bar{\mathrm{p}}_i = \max\left\{\frac{\zeta_i - a_{i0}}{a_{i1}}, 0\right\}. \quad (27)$$

Since $r_i' = \omega_i a_{i1}$, we have

$$\lambda^* = \omega_i a_{i1}, \quad \text{for } i \in \Omega. \tag{28}$$

And for $i \in \Omega^c$, the following inequality should be satisfied:

$$\lambda^* > \omega_i a_{i1}, \quad \text{for } i \in \Omega^c. \tag{29}$$

Without loss of generality, we assume that $\omega_i a_i \neq \omega_j a_j$ if $i \neq j$. Therefore, Ω should contain only one element which has the biggest $\omega_i a_{i1}$. Let $k \triangleq \arg\max_i \{\omega_i a_{i1}\}$. At optimality, we have $\Delta_k^* > 0$ and $\Delta_i^* = 0$ for $i \neq k$. Finally, when $K = 1$ an optimal SSR p_i^* can be written as

$$p_i^* = \begin{cases} 1 - \sum\limits_{i \neq k} \bar{\mathrm{p}}_i, & i = k, \\ \bar{\mathrm{p}}_i, & i \neq k. \end{cases} \tag{30}$$

The above result is intuitively meaningful. Remind that $\bar{\mathrm{p}}_i = \max\left\{\frac{\zeta_i - a_{i0}}{a_{i1}}, 0\right\}$, where a_{i0} is the achievable rate with $p_i = 0$, $a_{i0} + a_{i1}$ is the rate with $p_i = 1$. Hence, a_{i1} can be regarded as the relaying gain additionally attained when the relay supports $(\mathcal{S}_i, \mathcal{D}_i)$ link. With optimal SSR $\{p_i^*\}$, the relay helps the pair $(\mathcal{S}_k, \mathcal{D}_k)$ as much as possible, whose relaying gain is the highest, while the other $N-1$ pairs are minimally supported to meet the target rate.

D. Allocating Two Relaying Slots ($K = 2$)

When $K = 2$, $r_i(p_i)$ is a quadratic function;

$$r_i(p_i) = a_{i0} + a_{i1} p_i + a_{i2} p_i^2. \tag{31}$$

We set $\bar{\mathrm{p}}_i = 0$ for $i \in \mathbb{S}^c$. For $i \in \mathbb{S}$, we can find $\bar{\mathrm{p}}_i$ given by [1]

$$\bar{\mathrm{p}}_i = \frac{-a_{i1} + \sqrt{a_{i1}^2 - 4a_{i2}\left(a_{i0} - \zeta_i\right)}}{2a_{i2}}. \tag{32}$$

Substituting $r_i'(\bar{\mathrm{p}}_i + \Delta_i^*) = a_{i1} + 2a_{i2}(\bar{\mathrm{p}}_i + \Delta_i^*)$, solving (22), we can obtain

$$\Delta_i^* = \frac{1}{2\omega_i a_{i2}} \lambda^* - \frac{a_{i1}}{2a_{i2}} - \bar{\mathrm{p}}_i, \ i \in \Omega. \tag{33}$$

And from $\sum_{i \in \Omega} \Delta_i^* = 1 - \sum_{i=1}^N \bar{\mathrm{p}}_i$, we can obtain

$$\lambda^* = \frac{1 - \sum\limits_{i \in \Omega^c} \bar{\mathrm{p}}_i + \sum\limits_{i \in \Omega} \frac{a_{i1}}{2a_{i2}}}{\sum\limits_{i \in \Omega} \frac{1}{2\omega_i a_{i2}}} \tag{34}$$

where Ω and Ω^c can be obtained by the procedure described earlier. Therefore when $K = 2$, an optimal SSR $\{p_i^*\}$ can be expressed as

$$p_i^* = \begin{cases} \frac{1}{2\omega_i a_{i2}} \lambda^* - \frac{a_{i1}}{2a_{i2}}, & i \in \Omega, \\ \bar{\mathrm{p}}_i, & i \in \Omega^c \end{cases} \tag{35}$$

where λ^* is given as in (34).

[1] A value not greater than unity is taken among two roots of a quadratic equation $a_{i0} + a_{i1} p_i + a_{i2} p_i^2 = \zeta_i$.

IV. Numerical results

In this section, we present some numerical results to demonstrate the performance of the proposed system. We assume that all the nodes $\mathcal{S}_i, \mathcal{D}_i$ and $\mathcal{R}$ are located in a straight line. We fix the location of the relay to $(0, 0)$. We assume that $\mathcal{S}_i$ and $\mathcal{D}_i$ are on the opposite side of the line from $\mathcal{R}_i$. We set $\omega_i = 1$ for all i and the path loss factor to four.

In Fig. 2, we illustrate the sum of reward-weighted rates when $K = 1$ and $K = 2$ with optimal SSR over N from 1 to 4. We set the target data rate $\zeta_i = 0.1$ for all i. We start with case of $N = 1$, where the locations of $\mathcal{S}_1$ and $\mathcal{D}_1$ are $(-0.85, 0)$ and $(0.85, 0)$, respectively. In this example, we have $\rho_1 = 0.1632$ which satisfies the target data rate without the relay's help. Now, suppose that a new source-destination pair (located at $(-0.9, 0)$ and $(0.9, 0)$) joins the system. In this case, without the relay's help, $(\mathcal{S}_1, \mathcal{D}_1)$ and $(\mathcal{S}_2, \mathcal{D}_2)$ achieve data rate 0.0816 and 0.0656, respectively, which are not met the target rate. By introducing a relaying time slot and allocating the optimal SSR ($p_1^* = 0.7224, p_2^* = 0.2776$), the data rates of $(\mathcal{S}_1, \mathcal{D}_1)$ and $(\mathcal{S}_2, \mathcal{D}_2)$, increase upto 0.2343 and 0.1, respectively. All the source-destination pairs satisfy the target rate by allocating a time slot to the relay. Sum of reward-weighted rates (z_1) is 0.3343. If an additional time slot is allocated to the relay, i.e., $K = 2$, the data rates of $(\mathcal{S}_1, \mathcal{D}_1)$ and $(\mathcal{S}_2, \mathcal{D}_2)$ is $0.2794, 0.1113$, respectively, with optimal SSR ($p_1^* = 0.7309, p_2^* = 0.2691$), which yields $z_2 = 0.3907$ which is higher than $z_1 = 0.3343$. However, suppose that the third source-destination pair joins the system, specifically, $N = 3$. Assume that the locations of $\mathcal{S}_3$ and $\mathcal{D}_3$ are $(-0.95, 0)$ and $(0.95, 0)$, respectively. In this case, we have $\rho_1 = 0.0544, \rho_2 = 0.0438$ and $\rho_3 = 0.0356$ which are below the target rate. We observe that the target rate cannot be achieved at all $\mathcal{S}_i - \mathcal{D}_i$ with one relaying time slot, i.e., it is *infeasible*. In Fig.2, the line representing $K = 1$ breaks at $N = 2$. If two time slots are occupied by the relay, all $\mathcal{S}_i - \mathcal{D}_i$ can achieve the data rates satisfying the target rate. In the example, we have $r_1 = 0.1063, r_2 = 0.1$ and $r_3 = 0.1$ by allocating $p_1^* = 0.2578, p_2^* = 0.3185$ and $p_3^* = 0.4237$.

In Fig. 3, we compare sum of reward-weighted rates with optimal allocation to that with even allocation of p_i. In each experiment, the locations for $\mathcal{S}_i$ and $\mathcal{D}_i$ are randomly generated at a distance from $\mathcal{R}$, which ranges from 0.5 to 1. The curves presented hereafter are averages over the feasible cases among $10,000$ random locations for $\mathcal{S}_i$ and $\mathcal{D}_i$. The target rate is set to 0.05. The solid and dashed lines represent z_1 and z_2 with optimal SSR and with SSR under even allocation, respectively. Under even allocation, we have $p_i = \left(1 - \sum_i^N \bar{\mathrm{p}}_i\right)/N$, where $\bar{\mathrm{p}}_i$ is as given in section III. It can be seen that both z_1 and z_2 are improved by optimal allocation. We observe that z_k decreases as N increases, which results from the factor $1/(N+K)$ in r_i. We also observe that $z_1 < z_2$ when $N \geq 2$. It means that we can increase sum of rates of the system with multiple source-destination pairs by allocating two time slots to the relay instead of one time slot in the given scenario.

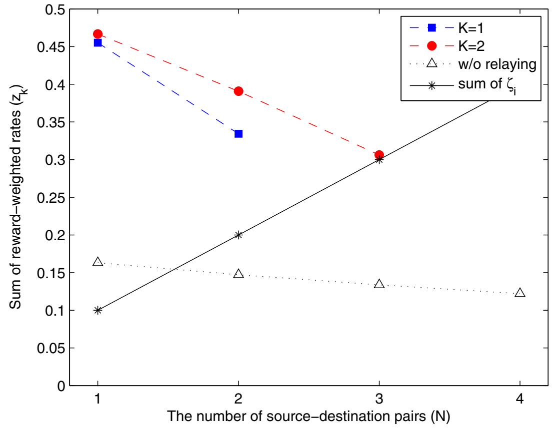

Fig. 2. Verification of optimal solutions when $K = 1$ and $K = 2$, $\zeta_i = 0.1$.

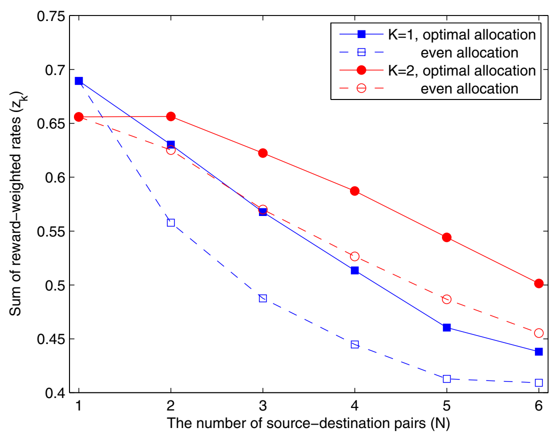

Fig. 3. Performance improvement by optimal allocation compared to even allocation when K=1, 2, $\zeta_i = 0.05$.

As shown in Fig. 2, a system of N source-destination pairs might not be feasible with one relaying slot meet the target rate. Fig. 4 compares *feasibility* of a system of $K = 1$ with that of $K = 2$ with respect to N. The simulation environmental parameters are same as those in Fig. 3. We measure feasibility as the ratio of the number of feasible experiments to total number of experiments. In Fig. 4, for a given ζ_i, feasibility of a system with N source-destination pairs decreases as N increases or as ζ_i increases.

V. Conclusion

We consider a multiple-sensor relaying network. We optimize the slot-sharing ratio to maximize the expected sum rate while guaranteeing the minimum data rate. Numerical examples show that the proposed relaying scheme is helpful to improve the expected data rate and optimal SSR outperforms even allocation in terms of sum of the expected data rate.

Appendix

To show that $r_i(p_i)$ is a monotonic increasing function over $0 \le p_i \le 1$, we take the first derivative of $r_i(p_i)$

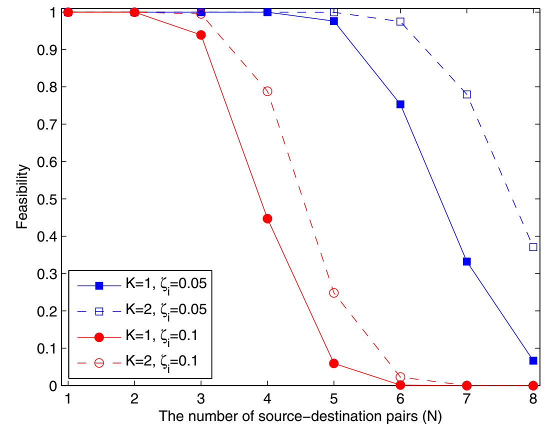

Fig. 4. Comparison of feasibility when $K = 1$ and $K = 2$, $\zeta_i = 0.05, 0.1$.

and show that it is greater than zero for $0 \le p_i \le 1$. The first derivative of $r_i(p_i)$ can be obtained as given in (36). After some manipulations, (36) can be modified as (37). We omit the detailed derivations due to the space limit. Since $I_i(k+1) > I_i(k)$, (37) is greater than zero over $0 \le p_i \le 1$. This proves that $r_i(p_i)$ is a monotonic increasing function over $0 \le p_i \le 1$.

$$r_i'(p_i) = \sum_{k=0}^{K} I_i(k)\binom{K}{k}\left\{kp_i^{k-1}(1-p_i)^{K-k} - (K-k)p_i^k(1-p_i)^{K-k-1}\right\} \tag{36}$$

$$= \sum_{k=0}^{K-1}\left\{I_i(k+1) - I_i(k)\right\}\binom{K}{k}(K-k)p_i^k(1-p_i)^{K-k-1}. \tag{37}$$

References

[1] J. N. Laneman and G. W. Wornell, “Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks,” *IEEE Trans. Inform. Theory*, vol. 49, pp. 2415-2425, Nov. 2003.

[2] J.-B. Kim and D. Kim, “Distributed channel co-phasing is optimal and robust for cooperative parallel-relay networks,” in *Proc. IEEE Radio and Wireless Symposium 2009,* Jan. 2009.

[3] S. Serbetli and A. Yener, “Relay assisted F/TDMA ad hoc networks: Node classification, power allocation and relaying strategies,” *IEEE Trans. Commun.*, vol. 56, no. 6, pp. 937-947, June 2008.

[4] K. T. Phan, T. Le-Ngoc, S. A. Vorobyov and C. Tellambura, “Power allocation in wireless multi-user relay networks,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2535-2545, May. 2009.

[5] X. Gong , S. A. Vorobyov and C. Tellambura, “Joint bandwidth and power allocation with admission control in wireless multi-user networks with and without relaying,” *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1801-1813, Apr. 2011.

[6] I. Kim, D. Kim, “Minimizimg source-sum-power consumption in multi-sensor single-relay networks,” *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2362-2366, Sep. 2011.

[7] C. Wang, Y. Fan, J. Thompson, and H. V. Poor, “A comprehensive study of repetition-coded protocols in multi-user multi-relay networks,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4329-4339, Aug. 2009.

[8] M. O. Hasna and M. -S. Alouini, “End-to-end performance of transmission systems with relays over Rayleigh-fading channels,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126-1131, Nov. 2003.

[9] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, UK: Cambridge University Press, 2004.

[10] V. Y. Pan, “Solving a polynomial equation: Some history and recent progress,” *SIAM Review*, vol. 39, no. 2, pp. 187-220, June 1997.