Time Domain Delay Items Design for Memory Orthogonal Polynomial Predistorter

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***Abstract*—The memory polynomial is commonly used in power amplifier (PA) modeling and predistorter (PD) design. However, the conventional memory polynomial and even memory orthogonal polynomial exhibit condition number increasing (eigenvalue spread) in covariance matrix when time domain delay items are included. In this paper, a method of designing time delay items for memory orthogonal polynomial is introduced. The method alleviates the eigenvalue spread problem effectively. Simulation results show that the new polynomial predistorter achieves much better predistortion performance based on indirect learning architecture (ILA) and normalized least mean square (NLMS) algorithm compared with the conventional memory orthogonal polynomial predistorter.**

***Index Terms*—Power amplifier, Predistortion, Memory orthogonal polynomial, Eigenvalue spread, Time domain, Indirect learning architecture, Normalized least mean square**

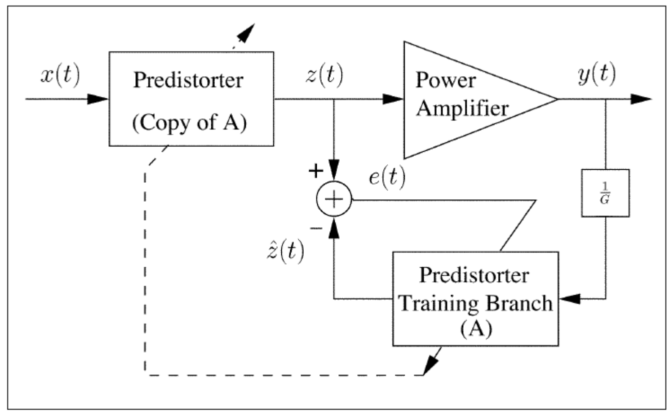

Fig. 1. Indirect Learning Architecture

I. Introduction

Power amplifier (PA) is an important part for many electronic devices and is widely utilized in all kinds of wireless communication systems. In order to meet the capacity requirement, many modern wireless communication systems, such as WCDMA, WiMAX and LTE, not only use high order and nonconstant envelope modulation, but also employ very high data sampling rate. The former increases the peak-to-average power ratio (PAPR) and the latter expands the bandwidth of the transmission signals. The inherent nonlinearity of PA [1] appears with high PAPR and the frequency selective characteristic, i.e. memory effect of PA [2], emerges with wide bandwidth. Both nonlinearity and memory effect would pollute PA output signal and result in its distortion, which needs to be compensated for.

Predistorter based on polynomial [3] is a common method to compensate for PA nonlinearity. However, the covariance matrix becomes ill-condition, i.e. serious eigenvalue spread, with the increasing of polynomial order. In [4][5], a couple of novel sets of orthogonal polynomials are proposed, which consists of some special basis functions to deal with the numerical instability problem associated with the conventional polynomial. To overcome the memory effect, the orthogonal polynomial is extended and repeated in different time delays. Although the orthogonality holds for orthogonal polynomial at each time delay, it is violated for the different delayed elements and the eigenvalue spread occurs again.

In this paper, a new memory orthogonal polynomial predistorter is proposed, in which the memory effect items are not the simple repeat of original polynomial and the basis functions are redesigned at different time delays. By using genetic algorithm (GA) [6], parameters of the new predistorter are designed and optimized, and then the eigenvalue spread is alleviated greatly compared with the conventional memory orthogonal polynomial.

Considering the feasibility in hardware and circuit, the normalized least mean square (NLMS) [7] is adopted in predistorter training for indirect learning architecture (ILA) [8][9]. Comparison of linearisation performance, i.e. adjacent channel leakage ratio (ACLR) and normalized mean square error (NMSE), are implemented with different predistorters based on ILA and NLMS.

The remainder of the paper is organized as follows: The ILA and NLMS are introduced in section II. Then section III describes the new predistorter. The optimization results and simulation comparisons about the predistorters are illustrated in section IV. The conclusion is drawn in section V.

II. Indirect Learning Architecture and Normalized Least Mean Square

A. Indirect Learning Architecture

The indirect learning architecture, as shown in Fig. 1, is used to train predistorter. The baseband predistorter input is $x(t)$, the predistorter output is $z(t)$, and the baseband PA output is $y(t)$. The input of Predistorter Training Branch (block A)

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is $y(t)/G$, where G is the intended PA gain. The output of block A is $\hat{z}(t)$. The actual predistorter is the copy of A and its output is $z(t)$. When $y(t) = Gx(t)$, the error $e(t)$ is 0. In order to reduce the difference between $y(t)$ and $Gx(t)$, the coefficients of predistorter is updated to minimize $e(t)$. We have

$$\hat{z}(t) = \sum_{k=1}^{K} w_k \psi_k(y(t)/G) \tag{1}$$

where $\psi_k(\cdot)$ means the basis function in the predistorter. Based on PA input $\{z(t_n)\}_{n=1}^{N}$ and output $\{y(t_n)\}_{n=1}^{N}$, the predistorter coefficients w_k could be estimated. After the coefficients are plugged into predistorter, we have

$$z(t) = \sum_{k=1}^{K} w_k \psi_k(x(t)) \tag{2}$$

This procedure is repeated to obtain more accurate coefficients estimates.

B. Normalized Least Mean Square

There are several methods to estimate predistorter coefficients w_k, such as least square (LS), recursive least square (RLS) and least mean square (LMS) [10]. By using LS, the data set $\{z(t_n)\}_{n=1}^{N}$ and $\{y(t_n)\}_{n=1}^{N}$ are stored to construct $N \times 1$ data vector $\vec{Z} = [z(t_1), ..., z(t_N)]^T$ and $N \times K$ data matrix

$$\begin{aligned} \Psi &= [\vec{\psi}_1(Y_G), ..., \vec{\psi}_K(Y_G)] \\ &= [\vec{\psi}_{t_1}^T, ..., \vec{\psi}_{t_N}^T]^T \end{aligned} \tag{3}$$

where $N \times 1$ vector

$$\vec{\psi}_k(Y_G) = [\psi_k(y(t_1)/G), ..., \psi_k(y(t_N)/G)]^T \tag{4}$$

and $1 \times K$ vector

$$\vec{\psi}_{t_n} = [\psi_1(y(t_n)/G), ..., \psi_K(y(t_n)/G)] \tag{5}$$

So

$$\vec{Z} = \Psi \vec{W} \tag{6}$$

where $K \times 1$ vector $\vec{W} = [w_1, ..., w_K]^T$. Then the matrix conjugate transpose multiplication is implemented to construct covariance matrix $\Psi^H \Psi$ and matrix inversion is needed to produce:

$$\vec{W} = (\Psi^H \Psi)^{-1} \Psi^H \vec{Z} \tag{7}$$

The huge hardware circuit size is necessary to realize the above mentioned process. To avoid the problem, LMS is preferred. The significant feature of the LMS algorithm is its simplicity [7]. By using it, the complicated matrix calculation is unnecessary and hardware circuit size is saved. In order to improve the convergency performance of LMS, the normalized least mean square (NLMS) is used in our analysis. It is as follows:

$$e(t_n) = z(t_n) - \vec{\psi}_{t_n} \vec{W}(t_n) \tag{8}$$

$$\vec{W}(t_{n+1}) = \vec{W}(t_n) + \frac{u}{\vec{\psi}_{t_n} \vec{\psi}_{t_n}^H} \vec{\psi}_{t_n}^H e(t_n) \tag{9}$$

The eigenvalue spread in the covariance matrix $\Psi^H \Psi$ is the key factor of NLMS convergency [7] and the eigenvalue spread is characterized by the covariance matrix condition number, which is defined as

$$\rho = \frac{\lambda_{max}}{\lambda_{min}} \tag{10}$$

where λ_{max} and λ_{min} are the maximum and minimum eigenvalue of the covariance matrix $\Psi^H \Psi$, respectively. The smaller the ρ, the slighter the eigenvalue spread. Our primary concern is to estimate predistortion parameters, i.e. $\vec{W}$ accurately and fast based on ILA and NLMS. Therefore, small ρ in the covariance matrix $\Psi^H \Psi$ is the requirement on the polynomial basis function designing.

III. New Memory Orthogonal Polynomial

A. New Basis Function

In [4][5], the basis function $\psi_k(\cdot)$ is orthogonalized and the condition number is improved greatly ($\rho \rightarrow 1$). However, when memory effect is taken into account, time delay items are included in the polynomial and the output of the block A is rewritten as

$$\hat{z}(t) = \sum_{p=0}^{P} \sum_{k=1}^{K} w_{pk} \psi_k(y(t-p)/G) \tag{11}$$

Correspondingly, the predistorter output is

$$z(t) = \sum_{p=0}^{P} \sum_{k=1}^{K} w_{pk} \psi_k(x(t-p)) \tag{12}$$

For simplicity, $\psi_k(y(t-p)/G)$ is expressed as ψ_{pk} and the corresponding $N \times 1$ vector in time domain is $\vec{\psi}_{pk}$, and then the data matrix in equation (3) is rewritten as:

$$\Psi = [\vec{\psi}_{01}, ..., \vec{\psi}_{0K}, \vec{\psi}_{11}, ..., \vec{\psi}_{1K}, ..., \vec{\psi}_{PK}] \tag{13}$$

which is a $N \times (P+1)K$ matrix. The elements in the matrix have the character as follows:

$$E\{\psi_{im}^H \psi_{jn}\} \rightarrow \begin{cases} 0, & m \neq n, i = j \\ 1, & m = n, i = j \end{cases} \tag{14}$$

because of the orthogonality for the same time delay. However, for different time delay, we generally have

$$E\{\psi_{im}^H \psi_{jn}\} \neq 0, \quad i \neq j \tag{15}$$

and

$$\rho >> 1 \tag{16}$$

because of the signal correlation in time domain. In other words, the orthogonality is violated in time domain and the condition number increases.

In order to improve the orthogonality of memory orthogonal polynomial basis, the time domain tap items also need to be designed. In this paper, the $\psi_{im} (i > 0)$ is substituted by a new polynomial, which is formulated as:

$$\phi_{im} = \sum_{q=0}^{i} u_{iqm} \psi_{qm}, \quad i = 1, ..., P \quad m = 1, ..., K \tag{17}$$

where the parameters u_{iqm} are introduced to redesign the time domain tap items. The corresponding $N \times 1$ vector in time domain is $\vec{\phi}_{im}$. The new data matrix is

$$\begin{aligned}\hat{\Psi} &= [\vec{\psi}_{01}, ..., \vec{\psi}_{0K}, \vec{\phi}_{11}, ..., \vec{\phi}_{1K}, ..., \vec{\phi}_{PK}] \\ &= [\vec{\psi}_{01}, ..., \vec{\psi}_{0K}, \vec{\psi}_{11}, ..., \vec{\psi}_{1K}, ..., \vec{\psi}_{PK}]U \\ &= \Psi U\end{aligned} \tag{18}$$

where $(P+1)K \times (P+1)K$ matrix

$$\begin{aligned}U &= \begin{pmatrix} \boldsymbol{I} & \bar{u}_{10} & \bar{u}_{20} & \dots & \bar{u}_{P0} \\ \mathbf{0} & \bar{u}_{11} & \bar{u}_{21} & \dots & \bar{u}_{P1} \\ \mathbf{0} & \mathbf{0} & \bar{u}_{22} & \dots & \bar{u}_{P2} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \bar{u}_{PP} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & \dots & u_{101} & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 & u_{102} & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & u_{111} & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & u_{112} & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & u_{PPK} \end{pmatrix}\end{aligned} \tag{19}$$

Here, $\boldsymbol{I}$ and $\mathbf{0}$ are $K \times K$ unity matrix and zeros matrix, respectively. The $K \times K$ sub-matrix $\bar{u}_{iq}$ is defined as:

$$\bar{u}_{iq} = \begin{pmatrix} u_{iq1} & 0 & 0 & \dots & 0 \\ 0 & u_{iq2} & 0 & \dots & 0 \\ 0 & 0 & u_{iq3} & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & u_{iqK} \end{pmatrix} \tag{20}$$

The new covariance matrix is

$$\hat{\Psi}^H \hat{\Psi} = U^H \Psi^H \Psi U \tag{21}$$

and

$$\Psi^H \Psi = \begin{pmatrix} \vec{\psi}_{01}^H \vec{\psi}_{01} & \vec{\psi}_{01}^H \vec{\psi}_{02} & \dots & \vec{\psi}_{01}^H \vec{\psi}_{0K} & \dots & \vec{\psi}_{01}^H \vec{\psi}_{PK} \\ \vec{\psi}_{02}^H \vec{\psi}_{01} & \vec{\psi}_{02}^H \vec{\psi}_{02} & \dots & \vec{\psi}_{02}^H \vec{\psi}_{0K} & \dots & \vec{\psi}_{02}^H \vec{\psi}_{PK} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vec{\psi}_{PK}^H \vec{\psi}_{01} & \vec{\psi}_{PK}^H \vec{\psi}_{02} & \dots & \vec{\psi}_{PK}^H \vec{\psi}_{PK-1} & \dots & \vec{\psi}_{PK}^H \vec{\psi}_{PK} \end{pmatrix} \tag{22}$$

The parameters u_{iqm} need to be designed and optimized to improve the orthogonality of the basis function and condition number of the covariance matrix $\hat{\Psi}^H \hat{\Psi}$.

B. *Parameters Design and GA*

If the matrix $\Psi^H \Psi$ is exactly known, the matrix U could be calculated to orthogonalize the basis function and improve the condition number of $\hat{\Psi}^H \hat{\Psi}$. However, it is difficult to obtain the general solution of U because the statistical property of

$$E\{\psi_{im}^H \psi_{jn}\}, \quad i \neq j \tag{23}$$

is closely related to the character of transmission signal itself. In other words, different methods and processes of generating transmission signal would result in different values of equation (23) and different matrices $\Psi^H \Psi$, and then different u_{iqm} are needed. Fortunately, the method and process are usually constant and prior known in most communication systems. Therefore the parameters u_{iqm} could be designed and stored beforehand for different transmission signals in different communication systems. Moreover, as the eigenvalue spread is what we concern, the condition number of $\hat{\Psi}^H \hat{\Psi}$ is optimized directly instead of minimizing non-diagonal elements in it.

The above mentioned multi-parameter optimization problem is fit to be solved by GA, which is a powerful tool of searching for optimal solution. Based on the GA theory[6], the parameters u_{iqm} construct a chromosome and the condition number ρ of the transmission signal covariance matrix acts as the fitness of each chromosome in the population.

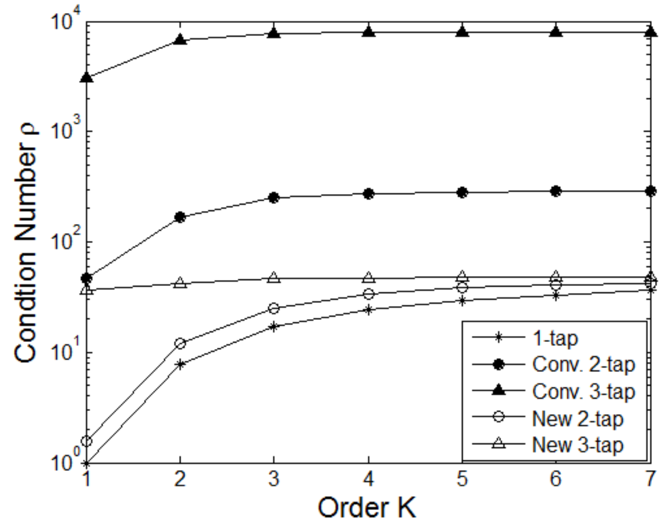

Fig. 2. Comparison of condition number between conventional (Conv.) predistorter and new predistorter

IV. Simulation

The simulation analysis includes two parts. The first one introduces the improving of condition number by designing and optimizing new basis function. The second one compares and analyzes the predistortion performance of different basis functions based on memory effect PA model.

A. *Condition Number Optimization*

In this paper, the conventional orthogonal basis function with odd and even items [4] are employed to act as $\psi_k(\cdot)$. The 200 000 samples of a 4-carrier WCDMA signal are used to serve as signal source for designing u_{iqm} and optimizing condition number ρ.

The maximum number of orders and taps in predistorter are 7 and 3, respectively. The population size of GA is 300. The crossover probability is 0.7 and the mutation probability is 0.001. After optimization of 500 generations, the comparisons of ρ among different basis functions, orders and taps are shown in Fig. 2. From the figure, we can see that the condition number ρ of conventional orthogonal polynomial

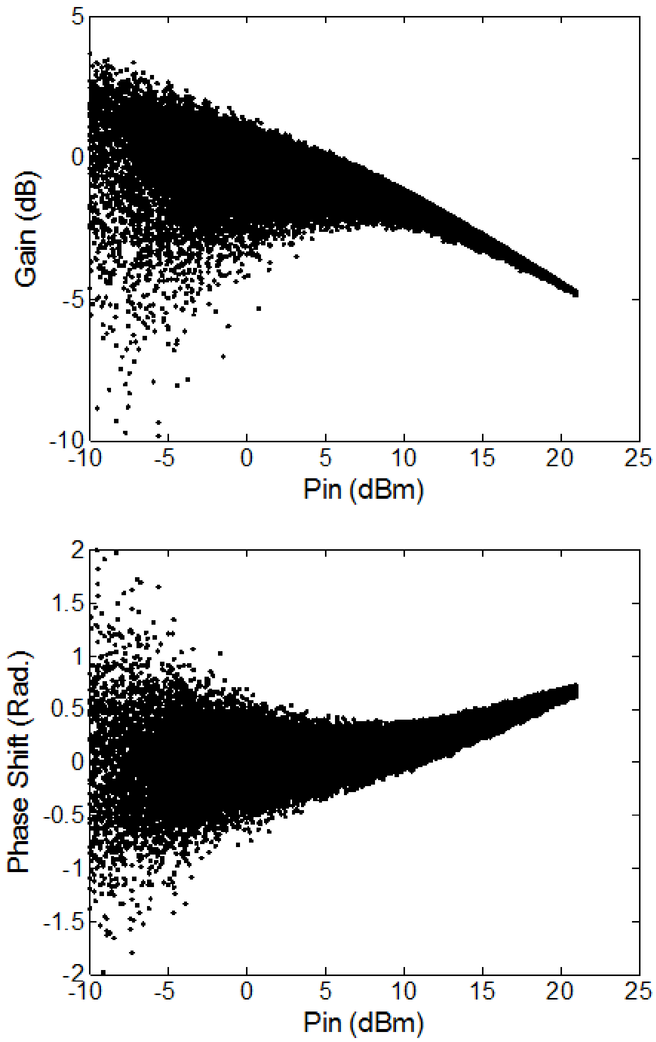

Fig. 3. Characteristic of PA model with memory effect

predistorter increases rapidly with the increasing of time delay (tap number). However, the ρ is improved obviously, which is even similar to the memoryless condition (one tap), by using our new predistorter.

B. Predistortion Performance Analysis

1) Memory effect PA model: The PA input and output are assumed to obey the W-H model, i.e., a linear time-invariant (LTI) system followed by a memoryless nonlinearity, which in turn is followed by another LTI system. The LTI blocks before and after the memoryless nonlinearity are given by

$$H(z) = \frac{1}{1.5}\frac{1+0.25z^{-2}}{1+0.4z^{-1}} \qquad G(z) = \frac{1}{0.52}\frac{1-0.1z^{-1}}{1-0.2z^{-1}} \tag{24}$$

and the memoryless nonlinearity is an ARCTAN model:

$$y(t) = (\gamma_1 \tan^{-1}(\zeta_1|z(t)|)) + \gamma_2 \tan^{-1}(\zeta_2|z(t)|))e^{j\angle z(t)} \tag{25}$$

where $\gamma_1 = 8.00335 - j4.61157$, $\gamma_2 = -3.77167 + j12.03758$, $\zeta_1 = 2.26895$ and $\zeta_2 = 0.8234$ [4]. The PA characteristic is

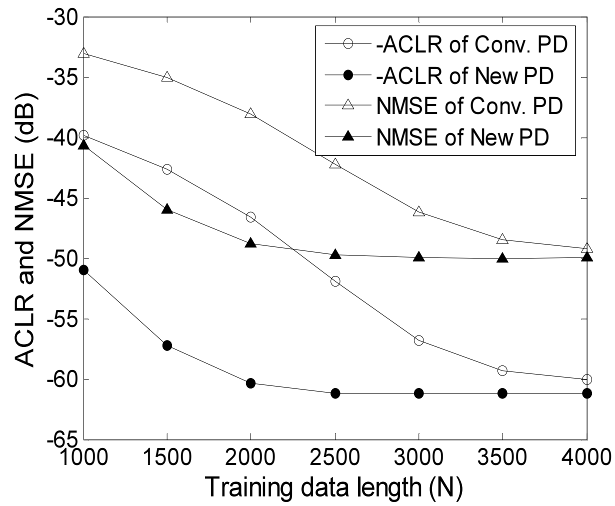

Fig. 4. Comparison of Mean ACLRs and NMSEs between conventional predistorter and new predistorter

shown in Fig. 3, which is based on the PA model and the 4-carrier WCDMA signal source. From Fig. 3, we can see that both the nonlinearity and memory effect are included in this PA model.

2) Measures for predistortion performance: To compare the performance of predistorters fully, both the ACLR and the NMSE are calculated. According to 3GPP standardization [11], the ACLR is calculated by

$$ACLR = 10lg\frac{2 * P_M}{P_L + P_R} \tag{26}$$

where P_M means power in main channel. P_L and P_R mean power in left adjacent channel and right adjacent channel, respectively. The NMSE is defined as:

$$\vec{V} = \vec{X} - \vec{Y}_G \tag{27}$$

$$NMSE = 10lg\frac{\vec{V}^H\vec{V}}{\vec{X}^H\vec{X}} \tag{28}$$

where $\vec{X}$ and $\vec{Y}_G$ mean the vector consisted of $\{x(t_n)\}_{n=1}^N$ and $\{y(t_n)/G\}_{n=1}^N$, respectively.

3) Simulation comparison: The performance of the conventional memory orthogonal polynomial and the new predistorters with 3-tap and 5-order are compared here.

Monte carlo simulation is adopted. 2000 independent realizations are generated based on ILA and NLMS for each data length of training. In every realization, N samples are sent to PA model one by one and every output sample of the PA model would construct data vector to implement NLMS according to the equations (8) and (9).

Fig. 4 shows the mean values of ACLR and NMSE for these realizations. From the figure, it can be seen that the new predistorter obtains much better performance in both ACLR and NMSE. To obtain similar ACLR or NMSE value, the training data lengths of the new predistorter are much shorter than those of the conventional one.

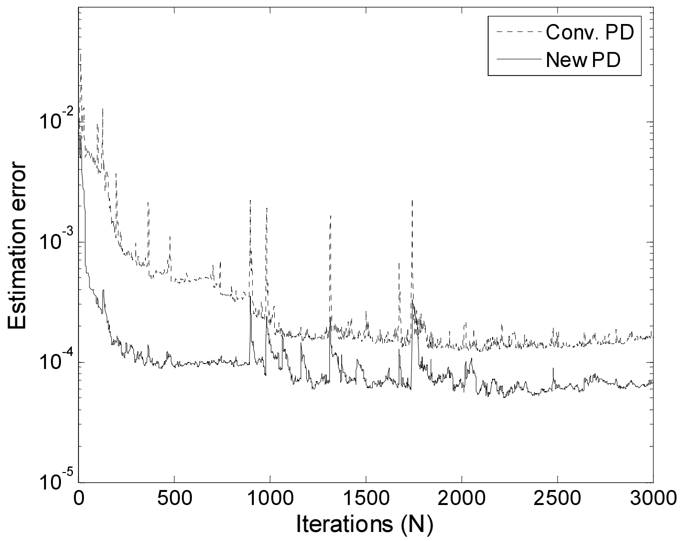

Fig. 5. Comparison of convergency process between conventional predistorter and new predistorter

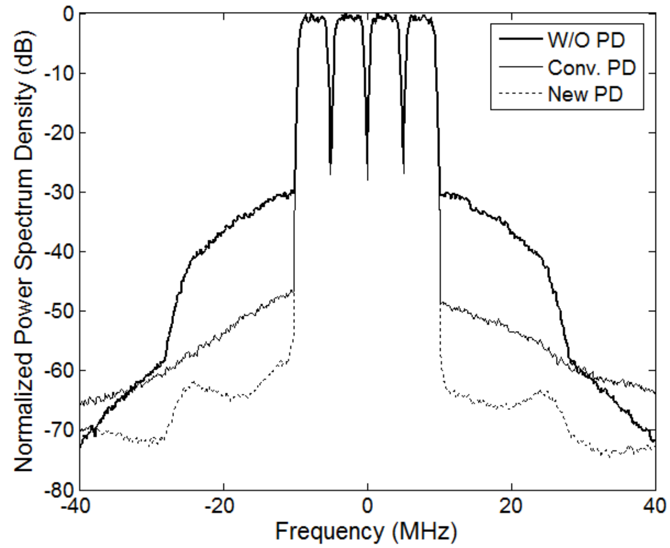

Fig. 6. Comparison of spectrums between conventional predistorter and new predistorter

A realization with same 3000 training data are implemented with the two predistorters respectively to compare convergency process, which is shown in Fig. 5. According to the predistortion coefficients, which is obtained from the realization, the frequency spectrums are compared in Fig. 6. Fig. 5 shows that both the convergency speed and residual estimation error of the new predistorter are much better than those of the conventional predistorter. From Fig. 6, we can see that the sidelobe of spectrum is restrained greatly by using the new predistorter compared with the conventional predistorter.

V. Conclusion

In this paper, the predistortion technology for power amplifier linearization is discussed.

To overcome memory effect, memory orthogonal polynomial, where time domain delay items are included in the conventional orthogonal polynomial, acts as predistorter. However, the condition number of the covariance matrix increases because of the basis function correlation in time domain. In this paper, a method of designing time domain delay basis function is proposed to reduce condition number of the covariance matrix. Simulation results demonstrate the new predistorter achieves better performance in linearizing PA with memory effect by using ILA and NLMS compared with the conventional memory orthogonal polynomial predistorter.

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