Suboptimal Power Allocation for a Two-Path Successive Relay System with Full Interference Cancellation

Chang-Chen Chu‡, Hung-Chin Wang‡, and Chin-Liang Wang†‡
Department of Electrical Engineering† and Institute of Communications Engineering‡
National Tsing Hua University, Hsinchu, Taiwan 30013, Republic of China
dr915603@com.nthu.edu.tw, d919603@oz.nthu.edu.tw, clwang@ee.nthu.edu.tw

***Abstract*—Although power allocation considerably affects the symbol error rate (SER) performance of a two-path successive relay system (TPSRS), it is not yet studied in a full interference cancellation (FIC) scenario. In this paper, we develop a suboptimal power allocation scheme for the TPSRS-FIC through minimizing an SER-based cost function. To obtain a closed-form power allocation solution, the cost function is defined as a part of the Taylor series expansion of the theoretical SER. Simulation results show that the proposed scheme can provide almost the lowest SER under limited power resources.**

Keywords- Amplify-and-forward; cooperative communications; interference cancellation; power allocation; relay transmission

I. Introduction

In wireless communications, the cooperative transmission technique has attracted a lot of interests in the recent years since it can improve the system performance [1], [2]. When the distance between the source (*S*) and the destination (*D*) is too long, the signal transmitted from *S* may not be received by *D*. With the cooperative technique, one may introduce a relay (*R*) to form a three-node (i.e., *S*-*R*-*D*) cooperative system and thus the signal coverage of the transmission from *S* can be enlarged, so that *D* can decode the received signal. To prevent from the self-interference, a transceiver must satisfy the half-duplex restriction. That is, a node can either "speak" or "listen" at any time instant in the same channel. Therefore, a complete transmission of an *S*'s symbol via the help of *R* to *D* needs two symbol times, where the first is that *S* sends data symbols and *R* receives the data information, and the second is that *R* forwards its received messages at the previous symbol time and *D* decodes the desired symbol after reception. As reveal in [3], [4], the spectral efficiency of such a two-step transmission scheme is reduced to one half when compared with that of a direct transmission from *S* to *D*.

The earliest method for overcoming the problem of the halved spectral efficiency in the amplify-and-forward (AF) protocol is studied in [3], where two relays are introduced for alternately relaying the *S*'s signal to *D* at the odd and even symbol times. The transmission using such an alternately relaying scheme by two relays is named the two-path successive relay system (TPSRS) [5]. If the direct channel link between *S* and *D* in the TPSRS is too weak and can be ignored, it is clear that the *n*th symbol transmitted from *S* at the *n*th symbol time will be relayed to *D* at the (*n*+1)th symbol time. Hence, *N* symbols sent from *S* only need *N*+1 symbol times to reach *D*, and thus the TPSRS recovers the spectral efficiency to almost one.

The AF TPSRS may suffer from the inter-relay interference (IRI) problem at *D*. This is because the received data at a relay not only contain the signal sent from *S* but also consist of the interference forward from the other relay, where the interference is essentially the previous data symbols transmitted from *S*. If a relay does not remove the IRI before relaying the received data, it would be passed to *D* during the forwarding from the relay at the next symbol time. To alleviate the IRI at *D*, the approach in [4] uses some previous decoded symbols to reconstruct the IRI and then subtracts it from the received data at *D*. Since such a partial reconstruction cannot completely express the true IRI and the previous decoded symbols may not be all correct, the partial cancellation scheme would still remain some IRI power and the error propagation problem usually occurs in the future symbol detection at *D* [5]. In [5], the authors discovered that the IRI component at *D* at a given symbol time is essentially a transformation of the received data at *D* at the previous symbol time, where the transformation is a function of some channel coefficients and an amplify factor of one of the relays. With this observation, the IRI at *D* at a given symbol time can be completely recovered by the transformation of the received data at *D* at the previous symbol time, and then it can be fully removed. The authors named the scheme "full interference cancellation" (FIC) due to this full cancelling manner. Unlike the partial cancellation scheme in [4] that recovers the IRI by involving some previous decoded symbols whether they are correct or not, the FIC method reconstructs the IRI by the transformation of the nearest previous received data at *D* and thus avoids the error propagation problem.

Another issue for performance improvement of cooperative systems is power allocation under limited power resources. Related works in [6] and [7] present the optimal power allocation for a three-node cooperative system and a multi-hop one, respectively, based on minimizing the corresponding symbol error rate (SER) and outage probability.

This work was supported by the National Science Council of the Republic of China under Grant NSC 100-2221-E-007-059-MY3.

To the best of our knowledge, there is no literature studying the power allocation for the TPSRS with FIC (TPSRS-FIC). In this paper, we propose a suboptimal power allocation scheme for the TPSRS-FIC. First, we formulate the suboptimal power allocation algorithm by minimizing a cost function which is an SER approximation of the TPSRS-FIC. To obtain a solution closed-form, we simplify the cost function by keeping the dominant terms of its Taylor series form. Then, we take the partial derivative of the simplified cost function with respect to S's transmission power and set it to be zero to find the solution. Since the resulting equation is a quadratic form in the variable of S's transmission power under given total power resources and channel variances, the suboptimal power allocation solution can be easily obtained by the quadratic formula. Computer simulations show that the proposed suboptimal power allocation helps the TPSRS-FIC achieve nearly the lowest SER under a given total power limitation.

II. System Model

Consider an AF TPSRS without direct link between S and D shown in Fig. 1 (which is also investigated in [4] and [5]), where R_1 and R_2 alternately forward S's signal to D at even and odd symbol times, respectively, the channel coefficients from S to R_1, S to R_2, R_1 to D, and R_2 to D are defined as h_{S1}, h_{S2}, h_{1D}, and h_{2D}, respectively, and the inter-relay channels from R_1 to R_2 and R_2 to R_1 are denoted by h_{12} and h_{21}, respectively. The channel coefficients are modeled as zero-mean complex Gaussian random variables and are assumed to be unchanged within a symbol time. Their variances are given as follows:

$$E[|h_{S1}|^2]=\sigma_{S1}^2,\ \ E[|h_{S2}|^2]=\sigma_{S2}^2,\ \ E[|h_{1D}|^2]=\sigma_{1D}^2,$$
$$E[|h_{2D}|^2]=\sigma_{2D}^2,\ \text{and}\ \ E[|h_{12}|^2]=E[|h_{21}|^2]=\sigma_{21}^2, \quad (1)$$

where $E[\bullet]$ denotes the expectation operation.

A. The IRI problem at D

Assume that at the nth symbol time R_2 transmits $x_2[n]$ by using an amplifying factor β_2 on its previous received data, $y_2[n-1]$. Hence, the transmitted information $x_2[n]$ can be described by

$$x_2[n]=\beta_2 y_2[n-1], \quad (2)$$

where β_2 will be defined later, and $y_2[n-1]$ is R_2's received data information at the $(n-1)$th symbol time. At the same symbol time, D also receives the data $x_2[n]$ transmitted from R_2 as follows:

$$y_D[n]=h_{2D}[n]x_2[n]+w_D[n], \quad (3)$$

where w_D is the zero-mean complex Gaussian noise at D.

The received data $y_2[n-1]$ at R_2 in (2) is essentially from both S and R_1 so that it can be expressed by

$$y_2[n-1]=h_{S2}[n-1]x_S[n-1]+h_{12}[n-1]x_1[n-1]+w_2[n-1], \quad (4)$$

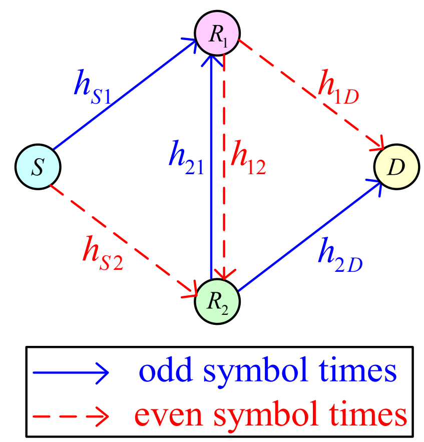

Fig. 1. The transmission flow of the TPSRS.

where $x_S[n-1]$ is the data symbol sent from S at the $(n-1)$th symbol time, $x_1[n-1]$ is the information forwarded by R_1 at the $(n-1)$th symbol time, and w_2 is the zero-mean complex Gaussian noise at R_2. For convenience, we assume that all noise terms at receivers have an identical variance σ^2. Substituting (2) and (4) into (3), we have

$$\begin{aligned} y_D[n]=&\,h_{2D}[n]\beta_2 h_{S2}[n-1]x_S[n-1]\\ &+h_{2D}[n]\beta_2 h_{12}[n-1]x_1[n-1]\\ &+h_{2D}[n]\beta_2 w_2[n-1]+w_D[n]. \end{aligned} \quad (5)$$

It can be found from (5) that the second term on the right hand side is the IRI transmitted from R_1 via R_2 to D. Therefore, some appropriate operations are needed to suppress the IRI before detecting $x_S[n-1]$. As revealed in [4], the IRI term is a weighted sum of the previous data symbols transmitted from S, so that the authors partially recovered the IRI by some previous decoded symbols and then cancelled it to make symbol detection. Although this partial cancellation approach removes most IRI power, it may suffer from the error propagation problem due to the unsuccessfully decoding on the previous symbols. In the next section, we will make a brief review of a FIC scheme proposed in [5], which can perfectly cancel the IRI.

B. The constraint of power resources [5]

In applications, since the transmission power must be constrained, we assume that the total average transmission power and the average transmission of S are given as P_T and P_S, respectively. Consequently, the average transmission power of each relay is

$$P_R=P_T-P_S. \quad (6)$$

With P_S and P_R, we have the following relationships:

$$E[|x_S|^2]=P_S \ \text{ and } \ E[|x_2|^2]=E[|x_1|^2]=P_R \quad (7)$$

and thus can determine the amplifying factor β_2 by sequentially using (2), (4), (7), and (1) as follows:

$$\beta_2 = \sqrt{\frac{E[|x_2|^2]}{E[|y_2|^2]}} = \sqrt{\frac{P_R}{\sigma_{S2}^2 P_S + \sigma_{21}^2 P_R + \sigma^2}}. \tag{8}$$

Also, the amplifying factor of R_1 can be obtained by

$$\beta_1 = \sqrt{\frac{P_R}{\sigma_{S1}^2 P_S + \sigma_{21}^2 P_R + \sigma^2}}. \tag{9}$$

III. THE TPSRS-FIC

This section reviews an improved IRI cancellation technique of [5] for the TPSRS. Due to the feature of the full cancellation for the IRI, the scheme is named FIC by the authors. Then, we introduce an SER approximation closed-form of the FIC approach, which will be used as a cost function for finding the suboptimal power allocation solution in the next section.

A. FIC for TPSRS [5]

The authors observed that $x_1[n-1]$ in (5) is also received at D at the $(n-1)$th symbol time so that $y_D[n-1]$ is written as

$$y_D[n-1] = h_{1D}[n-1]x_1[n-1] + w_D[n-1], \tag{10}$$

which can be rewritten as

$$x_1[n-1] = \frac{y_D[n-1] - w_D[n-1]}{h_{1D}[n-1]}. \tag{11}$$

Substituting (11) into (5), we obtain

$$y_D[n] = h_{2D}[n]\beta_2 h_{S2}[n-1]x_S[n-1] + I_1[n] + w'[n], \tag{12}$$

where

$$I_1[n] = \frac{h_{2D}[n]\beta_2 h_{12}[n-1]}{h_{1D}[n-1]} y_D[n-1] \tag{13}$$

expresses the IRI transmitted by R_1 via R_2 to D at the nth symbol time, and

$$\begin{aligned} w'[n] &= h_{2D}[n]\beta_2 w_2[n-1] \\ &\quad - \frac{h_{2D}[n]\beta_2 h_{12}[n-1]}{h_{1D}[n-1]} w_D[n-1] + w_D[n] \end{aligned} \tag{14}$$

is the accumulated noise at D. We find that the IRI in (13) is a transformation of $y_D[n-1]$, which is the function of some channel coefficients and an amplifying factor. Therefore, once D has the channel information and the amplifying factors of the relays, it can perfectly remove the IRI in (13) as follows:

$$y'_D[n] = y_D[n] - I_1[n] = h_{2D}[n]\beta_2 h_{S2}[n-1]x_S[n-1] + w'[n], \tag{15}$$

where the IRI does not exist and the detection for the desired symbol $x_S[n-1]$ can be made by dividing h_{2D}, β_2, and h_{S2}.

B. SER approximation of the TPSRS-FIC

For simplicity, we omit the timing index n without affecting the following derivation and first discuss the SER whose desired signal component is sent by R_2. From (14) and (15), the signal-to-noise ratio (SNR) can be expressed by

$$\Gamma_2 = \frac{|h_{2D}|^2 \beta_2^2 |h_{S2}|^2 P_S}{|h_{2D}|^2 \beta_2^2 \sigma^2 + \sigma^2 + \dfrac{|h_{2D}|^2 \beta_2^2 |h_{12}|^2 \sigma^2}{|h_{1D}|^2}}. \tag{16}$$

Since it is difficult to get an SER closed-form of the TPSRS-FIC by (16), we simplify the SNR of (16) as

$$\Gamma_2 \approx \frac{|h_{2D}|^2 \beta_2^2 |h_{S2}|^2 P_S}{\sigma^2} = \left(\frac{|h_{S2}|^2 P_S}{\sigma^2}\right)\left(|h_{2D}|^2 \beta_2^2\right) \equiv \Gamma_{2,\mathrm{AP}}, \tag{17}$$

where $\Gamma_{2,\mathrm{AP}}$ denotes the SNR approximation of Γ_2 and can be written as a product of two exponential random variables $|h_{S2}|^2 P_S/\sigma^2$ and $|h_{2D}|^2\beta_2^2$, respectively, described by the probability density functions $f_X(x) = \lambda_2\exp(-\lambda_2 x)$ and $g_Y(y) = \mu_2\exp(-\mu_2 y)$ with $\lambda_2 \equiv \sigma^2/(P_S\sigma_{S2}^2)$ and $\mu_2 \equiv 1/(\beta_2^2\sigma_{2D}^2)$. Referring to [8], the approximated conditional SER using M-phase shift keying (M-PSK) modulations can be given by

$$P_{\mathrm{e},2}^{h_{S2},h_{2D}} \approx \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-b_{\mathrm{PSK}}\Gamma_{2,\mathrm{AP}}/\sin^2\theta\right) d\theta, \tag{18}$$

where $b_{\mathrm{PSK}} = \sin^2(\pi/M)$. Therefore, the approximated SER can be derived by averaging over the Rayleigh fading channels h_{S2} and h_{2D}, and after some mathematical manipulation, the SER whose desired signal contribution forwarded by R_2 at a high SNR is derived as

$$\begin{aligned} P_{\mathrm{e},2} &= E\left[P_{\mathrm{e},2}^{h_{S2},h_{2D}}\right] \\ &= \frac{1}{\pi}\int_0^{(M-1)\pi/M} E\left[\exp\left(-b_{\mathrm{PSK}}\Gamma_{2,\mathrm{AP}}/\sin^2\theta\right)\right] d\theta \\ &= B\lambda_2\mu_2 \ln\left(\lambda_2\mu_2\right), \end{aligned} \tag{19}$$

where

$$B = -\frac{1}{b_{\mathrm{PSK}}}\left[\frac{(M-1)}{2M} - \frac{1}{4\pi}\sin\left(\frac{2(M-1)\pi}{M}\right)\right].$$

Similar to derivation from (16) through (19), the approximated SER whose desired signal component forwarded by R_1 at a high SNR for M-PSK modulations can be obtained by

$$P_{\mathrm{e},1} = B\lambda_1\mu_1 \ln\left(\lambda_1\mu_1\right), \tag{20}$$

where λ_1 and μ_1 are defined as $\lambda_1 \equiv \sigma^2/(P_S\sigma_{S1}^2)$ and $\mu_1 \equiv 1/(\beta_1^2\sigma_{1D}^2)$, respectively. Finally, the approximated SER of the TPSRS-FIC can be computed by combining (19) and (20) as follows:

$$P_{\mathrm{e}} = \frac{1}{2}(P_{\mathrm{e},1} + P_{\mathrm{e},2}). \tag{21}$$

IV. PROPOSED SUBOPTIMAL POWER ALLOCATION FOR THE TPSRS-FIC

With the SER approximation of (21), a suboptimal power allocation algorithm under a given P_T can be described as

$$\begin{aligned} P_S^* &= \arg\min_{P_S} P_e \\ &= \arg\min_{P_S}\left(\frac{1}{2}B\left(\lambda_1\mu_1\ln\left(\lambda_1\mu_1\right)+\lambda_2\mu_2\ln\left(\lambda_2\mu_2\right)\right)\right) \\ &= \arg\max_{P_S}\left(\lambda_1\mu_1\ln\left(\lambda_1\mu_1\right)+\lambda_2\mu_2\ln\left(\lambda_2\mu_2\right)\right), \quad (\because B<0). \end{aligned} \tag{22}$$

At a sufficient high SNR, $\lambda_1\mu_1$ and $\lambda_2\mu_2$ are both very small values. This allows us to apply Taylor series of logarithm in [9] into (22) as follows:

$$\begin{aligned} P_S^* = \arg\max_{P_S}\Bigg(&\lambda_1\mu_1\left[\left(\lambda_1\mu_1-1\right)-\frac{\left(\lambda_1\mu_1-1\right)^2}{2}+\frac{\left(\lambda_1\mu_1-1\right)^3}{3}-\ldots\right] \\ &+\lambda_2\mu_2\left[\left(\lambda_2\mu_2-1\right)-\frac{\left(\lambda_2\mu_2-1\right)^2}{2}+\frac{\left(\lambda_2\mu_2-1\right)^3}{3}-\ldots\right]\Bigg) \end{aligned} \tag{23}$$

Again, because $\lambda_1\mu_1$ and $\lambda_2\mu_2$ are very small, we ignore the second and the higher order terms of $\lambda_1\mu_1$ and $\lambda_2\mu_2$ in (23) and obtain

$$\begin{aligned} P_S^* &\approx \arg\max_{P_S}\left(\left[-\lambda_1\mu_1-\frac{\lambda_1\mu_1}{2}-\frac{\lambda_1\mu_1}{3}-\ldots\right]+\left[-\lambda_2\mu_2-\frac{\lambda_2\mu_2}{2}-\frac{\lambda_2\mu_2}{3}-\ldots\right]\right) \\ &= \arg\max_{P_S}\left(\lambda_1\mu_1\left[-1-\frac{1}{2}-\frac{1}{3}-\ldots\right]+\lambda_2\mu_2\left[-1-\frac{1}{2}-\frac{1}{3}-\ldots\right]\right) \\ &= \arg\min_{P_S}\left(\lambda_1\mu_1+\lambda_2\mu_2\right). \end{aligned} \tag{24}$$

In (24), since $\lambda_1\mu_1$ and $\lambda_2\mu_2$ are functions of P_S, we can find the suboptimal power allocation solution by taking the partial derivative of the resulting cost function of the last equality with respect to P_S and setting it to zero as follows:

$$\frac{\partial\ \left(\lambda_1\mu_1+\lambda_2\mu_2\right)}{\partial\ P_S}=0. \tag{25}$$

After some manipulation, the result of (25) can be expressed by

$$\delta P_S^2+\psi P_S+\rho=0, \tag{26}$$

where

$$\begin{aligned} \delta &= \sigma_{S1}^2\sigma_{1D}^2(\sigma_{S2}^2-\sigma_{21}^2)+\sigma_{S2}^2\sigma_{2D}^2(\sigma_{S1}^2-\sigma_{21}^2), \\ \psi &= 2(P_T\sigma_{21}^2+\sigma^2)(\sigma_{S1}^2\sigma_{1D}^2+\sigma_{S2}^2\sigma_{2D}^2) \\ &\approx 2P_T\sigma_{21}^2(\sigma_{S1}^2\sigma_{1D}^2+\sigma_{S2}^2\sigma_{2D}^2), \ \text{(at high SNR)} \end{aligned}$$

and

$$\begin{aligned} \rho &= -P_T(P_T\sigma_{21}^2+\sigma^2)(\sigma_{S1}^2\sigma_{1D}^2+\sigma_{S2}^2\sigma_{2D}^2) \\ &\approx -P_T^2\sigma_{21}^2(\sigma_{S1}^2\sigma_{1D}^2+\sigma_{S2}^2\sigma_{2D}^2). \ \text{(at high SNR)} \end{aligned} \tag{27}$$

For $\delta\neq 0$, since (26) is a quadratic form in the variable of P_S, the solution can be obtained by

$$P_S^*=\frac{-\psi+\sqrt{\psi^2-4\delta\rho}}{2\delta}. \tag{28}$$

For $\delta=0$, the solution to (26) is

$$P_S^*=\frac{-\rho}{\psi}=\frac{1}{2}P_T. \tag{29}$$

Because at any symbol time there are S and one of the relays transmitting information, the power assigned to a relay is

$$P_R^*=P_T-P_S^*. \tag{30}$$

Summarily, by substituting the given total power resources P_T and all the channel variances of (1) into (28) or (29), we can easily obtain the suboptimal power allocation solution.

V. SIMULATION RESULTS

In this section, we show some simulations to evaluate our proposed suboptimal power allocation scheme, where quadrature PSK (i.e., M=4) is used to modulate data symbols transmitted from S and each noise variance at receivers is set as unity (σ^2=1). Fig. 2 shows the proposed suboptimal power allocation solution: P_S^*/P_T=0.5 by using (29) with P_T/σ^2=25dB and the SER simulations versus various P_S/P_T under different cases of P_T/σ^2, where all the channel variances in (1) are set to be unity. From this figure, we can see that the corresponding SERs of the proposed suboptimal solution are slightly higher than the lowest SERs of each P_T/σ^2 case. Fig. 3 demonstrates the proposed suboptimal solution and the SER simulation results in different P_T/σ^2 under the inter-relay channel variance set by 10, where the other channel variances are still given as unity. It can be found from this figure that the suboptimal solution becomes P_S^*/P_T=0.7597 by using (28) with P_T/σ^2=25dB and also helps the TPSRS-FIC nearly achieve the lowest SERs in each P_T/σ^2 cases.

As compare the two figures, we observe that the proposed suboptimal power allocation solutions are both smaller than the optimal solutions where the lowest SER simulation results occur. This is because the SER approximation in (21) is derived from the noise-reduced SNR approximation in (17), and thus the use of the SER minimization for finding the suboptimal solution leads us to obtain a smaller P_S. Although the proposed suboptimal power allocation scheme does not achieve the lowest SERs, it can be seen that the corresponding SERs of the proposed scheme are very close to the minimum SERs in the various cases of P_T/σ^2. This implies that the proposed power allocation scheme is quite effective.

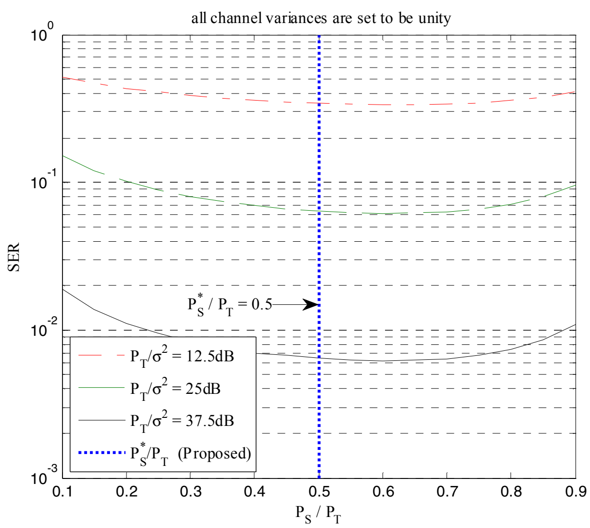

Fig. 2. The suboptimal power allocation solution and the results of SERs versus various power allocation in an environment with moderate inter-relay channel power.

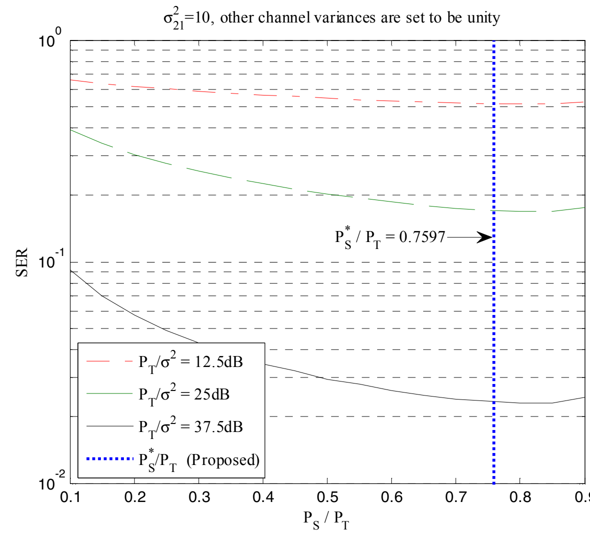

Fig. 3. The suboptimal power allocation solution and the results of SERs versus various power allocation in an environment with high inter-relay channel power.

VI. CONCLUSIONS

In this paper, we have proposed a suboptimal power allocation scheme for the TPSRS-FIC based on minimizing the SER approximation. The proposed approach has a closed-form solution that clearly reveals how much power resources should be distributed between the source and a relay according to the channel statistics. Computer simulation results have shown that the proposed suboptimal power allocation approach achieves virtually the lowest SERs under different total power resources.

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