Outage Performance of Cognitive Relay Networks with Primary User's ISR Constraint

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***Abstract*—In the underlay spectrum sharing systems, secondary users (SUs) are allowed to transmit their data in the licensed spectrum band when primary users(PUs) are also transmitting, as long as the transmission of SUs do not interfere PUs' communications. In cognitive relay networks, the source and relay nodes both need to tune their transmit power to mitigate the interference to PU. In this paper, we investigate the outage performance of cognitive relay networks with PU's interference to signal ratio (ISR) constraint, where both average and peak ISR constraint are considered. Finally, We derive the exact outage probability in the scenario without cooperation and the upper bound of outage probability in the scenario with cooperation.**

***Index Terms*—Cognitive Relay Networks; Outage Probability; Interference to Signal Ratio**

I. Introduction

Cognitive radio (CR) [1] is becoming one of the most promising technology for efficient spectrum utilization. With the flexible and comprehensive uses of the available spectrum [2], CR enables the optimization for radio resource usage. Basically, spectrum sharing in CR has two categories: spectrum underlay and spectrum overlay. In spectrum underlay, secondary users (SUs) are allowed to transmit their data in the licensed spectrum band when primary users (PUs) are also transmitting, while in spectrum overlay, the licensed spectrum is only available to SUs when PUs are not transmitting. In spectrum underlay, to mitigate the interference in primary receiver generated from SUs, the concept of interference temperature (IT) was proposed by FCC [2]. Although to apply IT constraint is a practical method to protect the PU's transmission, more advanced techniques such as PU's outage constraint [11] and PU's interference to signal ratio (ISR) constraint [4] are proposed, which are more related to PU's QoS.

The IT constraint only considers S-P (secondary transmitter - primary receiver) link and S-S (secondary transmitter - secondary receiver) link. The first link is for the evaluation of interference power and the latter one is for the evaluation of secondary user's QoS. However the fact that P-P (primary transmitter - primary receiver) links is not in the consideration. The IT constraint will encounter the problem of determining the interference threshold, i.e. when the P-P links are of good quality, the interference threshold should be high, which means the interference that PU can tolerate is high, but when the P-P links are of bad quality, the interference threshold should be low. Thus, if the P-P links are taken into consideration and the ISR constraint is used in the system optimization. By setting the ISR threshold as a fixed value, the interference threshold derived from ISR threshold will be more reasonable.

We address cognitive relay networks in this paper. Relay networks have been proposed as a way to enhance the total throughput and coverage of wireless networks [5] by reducing the overall transport distance and the path loss eventually. Besides, by applying the diversity, cooperative cognitive relay [6] can mitigate the signal fading arising from multipath propagation and improve the outage performance of wireless networks. The cooperative relay may reduce the throughput of the whole network because the time-slotted transmission. Thus Zhang in [7-8] proposes a dynamic time slot allocation scheme which can exploit the dynamics of the traffic in relay links and improve the throughput of the whole network.

Cognitive relay network (CRN) is a combination of cognitive radio and relay networks and commonly has two approaches: cooperative transmission of primary traffic by secondary users [9] and cooperative transmission between secondary users [10]-[14]. [9] proposes a two-phase protocol based on cooperative decode-and-forward relaying for the cooperative between primary and secondary users, where SUs help PU by cognitive relay and PU shares the licensed spectrum with SUs. In [10], the authors present an exact outage performance analysis for a decode-and-forward cooperative network where a source communicates with its destination using selection cooperation. In [11], under PU's outage constraint, the outage performance of CRN is derived. [12]-[13] address the outage performance of CRN in underlay spectrum sharing environment over Rayleigh fading channel, and derive the outage probability of secondary system. [14] extend the results in [12], by considering the dependence among the received SNRs, the authors derive a tight lower bound on the outage probability, while [12] and [13] both give the upper bounds on the outage probability. In this paper, we address the cooperative transmission between secondary users.

Although the studies above have investigate cognitive relay networks in detail, but by our knowledge, there are no literatures considering the ISR constraint in cognitive relay networks. In this paper, we analyze the outage performance by using ISR constraint. We derive the exact outage probability

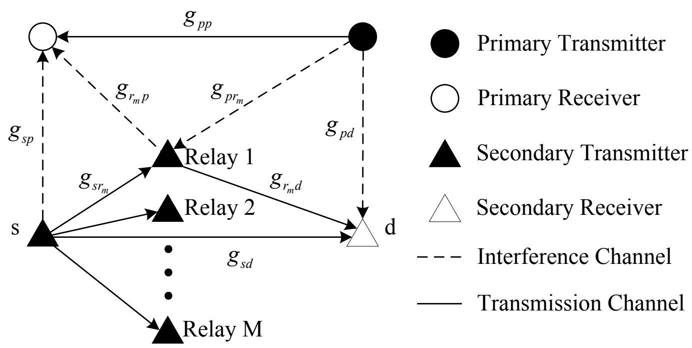

Fig. 1. System model for cognitive relay networks

in the scenario without cooperation and the upper bound of outage probability in the scenario with cooperation.

The rest of this paper is organized as follow. The network model and channel model are introduced in Section II. The ISR constraint is presented in Section III, the scenario without cooperation is addressed in Section IV, and the scenario with cooperation is addressed in Section V. Section VI shows the simulation results and Section VII concludes this paper.

II. System Model

A. Network Model

In the underlay spectrum sharing, a primary transmitter-receiver pair is communicating with fixed power P_p, while several SUs that operate in the same spectrum are communicating as Fig 1, where the source-destination pair is assisted by M potential relays.

B. Channel Model

Two types of channel are considered, namely, transmission channel and interference channel. The transmission channel is for data transmission in P-P (primary transmitter - primary receiver) links and S-S (secondary transmitter - secondary receiver) links. The interference channel is for the interference power transmission in S-P (secondary transmitter - primary receiver) links and P-S (primary transmitter - secondary receiver) links. For clarity of exposition, an interference-limited environment is assumed, which allows us to ignore thermal noise at the receiver [4].

III. ISR Constraints

The secondary transmitters including the source node s and relay nodes, which are denoted as a set $\Omega = \{s, 1, 2, \cdots, M\}$, and the set of secondary receivers is $\Phi = \{1, 2, \cdots, M, d\}$. Consider two SUs $i \in \Omega$ and $j \in \Phi$. The channel power gain g_{ij} is a log-normal distribution random variable (r.v.) with mean u_{ij} and variance σ_{ij}^2, namely, $g_{ij} \sim \mathcal{LN}(u_{ij}, \sigma_{ij}^2)$. The probability density function (PDF) and cumulative distribution function (CDF) are as follow.

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma_{ij}^2}} \exp\left(-\frac{(\ln x - u_{ij})^2}{2\sigma_{ij}^2}\right) \tag{1}$$

$$F(x) = \int_{-\infty}^{\ln x} \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \exp\left(-\frac{(z - u_{ij})^2}{2\sigma_{ij}^2}\right) dz \tag{2}$$

Assume the interference to signal ratio (ISR) threshold of primary receiver is β. And the transmit power of PU is P_p, then the received signal power of primary receiver is $P_p g_{pp}$, and the interference power received by PU from ith secondary transmitter is $P_i g_{ip}, i \in \Omega$, with P_i the transmit power of ith secondary transmitter. Then the ISR of PU is $\frac{P_i g_{ip}}{P_p g_{pp}}$, which is bounded by β. In this paper, both peak ISR constraint and average ISR constraint are considered.

A. Peak ISR constraint

Under the peak ISR constraint, we have

$$\frac{P_i g_{ip}}{P_p g_{pp}} \le \beta \Rightarrow P_i \le \frac{\beta P_p g_{pp}}{g_{ip}} \triangleq P_{i,\max} \tag{3}$$

Where $P_{i,\max}$ is the maximum transmit power of ith secondary transmitter. For simplicity, we don't take the hardware limitations into consideration and let $P_i = P_{i,\max}$.

In the secondary system, for the receiver $j \in \Phi$, the received signal to interference ratio (SIR) is

$$\gamma_{ij} = \frac{P_{i,\max} g_{ij}}{P_p g_{pj}} = \beta \frac{g_{pp} g_{ij}}{g_{ip} g_{pj}} \tag{4}$$

To find the probability density function (PDF) and cumulative distribution function (CDF) of γ_{ij}, let $g_{pp} = e^{X_{pp}}$, $g_{ij} = e^{X_{ij}}$, $g_{ip} = e^{X_{ip}}$, $g_{pj} = e^{X_{pj}}$, and we have

$$\begin{aligned}
g_{pp} &\sim \mathcal{LN}(u_{pp}, \sigma_{pp}^2); \;\; X_{pp} \sim \mathcal{N}(\xi_{pp}, \eta_{pp}^2) \\
g_{ij} &\sim \mathcal{LN}(u_{ij}, \sigma_{ij}^2); \;\; X_{ij} \sim \mathcal{N}(\xi_{ij}, \eta_{ij}^2) \\
g_{ip} &\sim \mathcal{LN}(u_{ip}, \sigma_{ip}^2); \;\; X_{ip} \sim \mathcal{N}(\xi_{ip}, \eta_{ip}^2) \\
g_{pj} &\sim \mathcal{LN}(u_{pj}, \sigma_{pj}^2); \;\; X_{pj} \sim \mathcal{N}(\xi_{pj}, \eta_{pj}^2)
\end{aligned}$$

Where

$$\begin{aligned}
\xi_{pp} &= 2\ln u_{pp} - \frac{1}{2}\ln\left(\sigma_{pp}^2 + u_{pp}^2\right) \\
\xi_{ij} &= 2\ln u_{ij} - \frac{1}{2}\ln\left(\sigma_{ij}^2 + u_{ij}^2\right) \\
\xi_{ip} &= 2\ln u_{ip} - \frac{1}{2}\ln\left(\sigma_{ip}^2 + u_{ip}^2\right) \\
\xi_{pj} &= 2\ln u_{pj} - \frac{1}{2}\ln\left(\sigma_{pj}^2 + u_{pj}^2\right) \\
\eta_{pp}^2 &= -2\ln u_{pp} + \ln(\sigma_{pp}^2 + u_{pp}^2) \\
\eta_{ij}^2 &= -2\ln u_{ij} + \ln(\sigma_{ij}^2 + u_{ij}^2) \\
\eta_{ip}^2 &= -2\ln u_{ip} + \ln(\sigma_{ip}^2 + u_{ip}^2) \\
\eta_{pj}^2 &= -2\ln u_{pj} + \ln(\sigma_{pj}^2 + u_{pj}^2)
\end{aligned}$$

Let $Z = \frac{g_{pp} g_{ij}}{g_{ip} g_{pj}} = \exp\left(X_{pp} + X_{ij} - X_{ip} - X_{pj}\right)$, which still follows log-normal distribution, since $Y = X_{pp} + X_{ij} - X_{ip} -$

X_{pj} follows normal distribution with mean

$$\begin{aligned} E[Y] &= E[X_{pp} + X_{ij} - X_{ip} - X_{pj}] \\ &= E[X_{pp}] + E[X_{ij}] - E[X_{ip}] - E[X_{pj}] \\ &= 2\ln\frac{u_{pp}u_{ij}}{u_{ip}u_{pj}} - \frac{1}{2}\ln\frac{(\sigma_{pp}^2 + u_{pp}^2)(\sigma_{ij}^2 + u_{ij}^2)}{(\sigma_{ip}^2 + u_{ip}^2)(\sigma_{pj}^2 + u_{pj}^2)} \\ &\triangleq u_Y \end{aligned} \tag{5}$$

And the variance of Y is

$$\begin{aligned} D[Y] &= D[X_{pp}] + D[X_{ij}] + D[X_{ip}] + D[X_{pj}] \\ &= \ln\frac{(\sigma_{pp}^2 + u_{pp}^2)(\sigma_{ij}^2 + u_{ij}^2)(\sigma_{ip}^2 + u_{ip}^2)(\sigma_{pj}^2 + u_{pj}^2)}{(u_{pp}u_{ij}u_{ip}u_{pj})^2} \\ &\triangleq \sigma_Y^2 \end{aligned} \tag{6}$$

Thus, Z is a log-normal random variable (r.v.) with mean $u_Z = \exp(u_Y + \frac{1}{2}\sigma_Y^2)$ and variance $\sigma_Z^2 = e^{2u_Y + \sigma_Y^2}(e^{\sigma_Y^2} - 1)$. And $\gamma_{ij} = \beta Z$ is also a log-normal r.v. with mean $u_{\gamma_{ij}} = \exp(u_Y + \frac{1}{2}\sigma_Y^2) + \ln\beta$ and variance $\sigma_{\gamma_{ij}}^2 = \sigma_Z^2$. And the PDF and CDF of which are as follow [16].

$$f_{ij}(x) = \frac{1}{x\sqrt{2\pi\sigma_{\gamma_{ij}}^2}}\exp\left(-\frac{\left(\ln(x) - u_{\gamma_{ij}}\right)^2}{2\sigma_{\gamma_{ij}}^2}\right) \tag{7}$$

$$F_{ij}(x) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\ln(x) - u_{\gamma_{ij}}}{\sigma_{\gamma_{ij}}\sqrt{2}}\right) \tag{8}$$

B. Average ISR constraint

In the average ISR constraint, we have

$$\mathbb{E}\left[\frac{P_i g_{ip}}{P_p g_{pp}}\right] \le \beta \Rightarrow P_i \le \frac{\beta P_p u_{pp}}{u_{ip}}$$

To have the maximum SIR, the transmit power of ith secondary transmitter should be $P_i = \frac{\beta P_p u_{pp}}{u_{ip}}$. Thus, the received SIR of jth secondary transceiver is

$$\gamma_{ij} = \frac{\beta u_{pp}}{u_{ip}}\frac{g_{ij}}{g_{pj}} \tag{9}$$

Let $U = \exp(X_{ij} - X_{pj})$, with $V = X_{ij} - X_{pj}$ a normally distributed random variables whose mean is

$$\begin{aligned} E[V] &= E[X_{ij}] - E[X_{pj}] \\ &= 2\ln\frac{u_{ij}}{u_{pj}} + \frac{1}{2}\ln\frac{\sigma_{pj}^2 + u_{pj}^2}{\sigma_{ij}^2 + u_{ij}^2} \triangleq u_V \end{aligned} \tag{10}$$

and the variance of V is

$$\begin{aligned} D[V] &= D[X_{ij}] + D[X_{pj}] \\ &= -2\ln(u_{ij}u_{pj}) + \ln(\sigma_{ij}^2 + u_{ij}^2)(\sigma_{pj}^2 + u_{pj}^2) \\ &\triangleq \sigma_V^2 \end{aligned} \tag{11}$$

Thus, $\gamma_{ij} = \frac{\beta u_{pp}}{u_{ip}}U$ is a log-normal r.v. with mean and variance as follow

$$u_{\gamma_{ij}} = \exp\left(u_V + \frac{1}{2}\sigma_V^2\right) + \ln\frac{\beta u_{pp}}{u_{ip}}$$

$$\sigma_{\gamma_{ij}}^2 = e^{2u_V + \sigma_V^2}\left(e^{\sigma_V^2} - 1\right)$$

The PDF and CDF of γ_{ij} are in the same form as Equation (7) and (8) but with different expectation and variance.

IV. With a Direct Link

The concept of cognitive relay is to use SUs as potential relays to assist the data transmission of source node, which combines the cooperative diversity with cognitive radio and can improve the performance of communication. In this paper, the decode-and-forward (DF) relay scheme is applied in secondary system. The transmission protocol consists of two orthogonal time slots. In the first time slot, the source broadcasts its data to the destination and potential relays. Potential relays that can decode source's data become decoding relays and participate in relay contention, which can be centralized or distributed. The "best" relay is selected to repeat the source message in the second time slot. The destination combines the received data from the "best" relay and the source for joint decoding [12].

We define the potential relaying set D_l to be the set of relays which can correctly decode the message from the source, where subscript $l = |D_l|$ is the cardinality of D_l. Thus, the ith relay that is selected into D_l needs to satisfy $\frac{1}{2}\log(1 + \gamma_{si}) \ge R$, where all logarithms are base 2 and $\frac{1}{2}$ denotes the transmission from source takes half of the total time slots. The probability that the ith relay selected into D_l is

$$\Pr\{i \in D_l\} = \Pr\{\gamma_{si} \ge 2^{2R} - 1\} = 1 - F_{si}(2^{2R} - 1) \tag{14}$$

Where γ_{si} is the SNR from source to ith relay, and the function $F_{si}(*)$ is the CDF of γ_{si}. We assume $\gamma_{si}, \forall i$ are independent random variables (r.v.) and have

$$\Pr\{D_l\} = \prod_{i \in D_l}\left(1 - F_{si}(2^{2R} - 1)\right)\prod_{i \notin D_l} F_{si}(2^{2R} - 1) \tag{15}$$

The relay selection criterion is

$$c = \arg\max_{i \in D_l}\{g_{id}\}$$

When D_l is not empty, the relay links exist. And the mutual information between s and d is

$$I = \frac{1}{2}\log\left(1 + \gamma_{sd} + \gamma_{cd}\right) \tag{16}$$

Through order statistics, the CDF of γ_{cd} is given by

$$F_{cd}(x) = \prod_{i \in D_l} F_{id}(x) \tag{17}$$

The outage occurs when the mutual information I falls below a certain rate R, which is the same as that in Equation (15), hence the outage probability of in the secondary destination is defined as $\Pr\{I < R\}$. For a given D_l, we have a I and an outage probability can be derived, such that the outage probability can be written in the form of the total probability law as follow.

$$\begin{aligned} p_o = &\Pr\{I < R | D_l = \emptyset\}\Pr\{D_l = \emptyset\} + \\ &\sum_{D_l \ne \emptyset}\Pr\{I < R | D_l\}\Pr\{D_l\} \triangleq p_1 + p_2 \end{aligned} \tag{18}$$

$$p_{o1} \le \left(\tfrac{1}{2}+\tfrac{1}{2}\mathrm{erf}\left(\tfrac{\ln(2^{2R}-1)-u_{\gamma_{sd}}}{\sigma_{\gamma_{sd}}\sqrt{2}}\right)\right)\prod_{i=1}^{M}\left(\tfrac{1}{2}+\tfrac{1}{2}\mathrm{erf}\left(\tfrac{\ln(2^{2R}-1)-u_{\gamma_{si}}}{\sigma_{\gamma_{si}}\sqrt{2}}\right)\right)+\sum_{D_l\neq\emptyset}\left(\prod_{i\in D_l}\left(\tfrac{1}{2}-\tfrac{1}{2}\mathrm{erf}\left(\tfrac{\ln(2^{2R}-1)-u_{\gamma_{si}}}{\sigma_{\gamma_{si}}\sqrt{2}}\right)\right)\times \prod_{i\notin D_l}\left(\tfrac{1}{2}+\tfrac{1}{2}\mathrm{erf}\left(\tfrac{\ln(2^{2R}-1)-u_{\gamma_{si}}}{\sigma_{\gamma_{si}}\sqrt{2}}\right)\right)\left(\tfrac{1}{2}+\tfrac{1}{2}\mathrm{erf}\left(\tfrac{\ln(2^{2R}-1)-u_{\gamma_{sd}}}{\sigma_{\gamma_{sd}}\sqrt{2}}\right)\right)\prod_{i\in D_l}\left(\tfrac{1}{2}+\tfrac{1}{2}\mathrm{erf}\left(\tfrac{\ln(2^{2R}-1)-u_{\gamma_{id}}}{\sigma_{\gamma_{id}}\sqrt{2}}\right)\right)\right) \quad (12)$$

$$p_{o2} \le \left(\frac{1}{2}-\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2R}-1)-u_{\gamma_{sd}}}{\sigma_{\gamma_{sd}}\sqrt{2}})\right)\times \left((\frac{1}{2}-\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2R}-1)-u_{\gamma_{sr}}}{\sigma_{\gamma_{sr}}\sqrt{2}}))(\frac{1}{2}+\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2R}-1)-u_{\gamma_{rd}}}{\sigma_{\gamma_{rd}}\sqrt{2}}))+(\frac{1}{2}+\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2R}-1)-u_{\gamma_{sr}}}{\sigma_{\gamma_{sr}}\sqrt{2}}))\right)^M \quad (13)$$

When the potential relaying set D_l is empty, there is only direct link and the outage probability is p_1. When D_l is not empty, the outage probability is p_2. We have

$$\begin{aligned}&\Pr\{I<R|D_l=\emptyset\}=\Pr\{\frac{1}{2}\log(1+\gamma_{sd})<R\}\\&=\Pr\{\gamma_{sd}<2^{2R}-1\}=F_{sd}(2^{2R}-1)\end{aligned} \quad (19)$$

And

$$\begin{aligned}\Pr\{I<R|D_l\} &= \Pr\{\frac{1}{2}\log(1+\gamma_{sd}+\gamma_{cd})<R\}\\ &= \Pr\{\gamma_{sd}+\gamma_{cd}<2^{2R}-1\}\\ &\le \Pr\{\gamma_{sd}<2^{2R}-1\}\Pr\{\gamma_{cd}<2^{2R}-1\}\\ &= F_{sd}(2^{2R}-1)F_{cd}(2^{2R}-1)\end{aligned} \quad (20)$$

Through Equation (15), (17) and (20), we have the upper bound of outage probability as follow.

$$\begin{aligned}&p_{o1}\le F_{sd}(2^{2R}-1)\prod_{i=1}^{M}F_{si}(2^{2R}-1)+\\&\sum_{D_l\neq\emptyset}\left(\prod_{i\in D_l}\left(1-F_{si}(2^{2R}-1)\right)\prod_{i\notin D_l}F_{si}(2^{2R}-1)\times\right.\\&\left.F_{sd}(2^{2R}-1)\prod_{i\in D_l}F_{id}(2^{2R}-1)\right)\end{aligned} \quad (21)$$

Where $\sum_{D_l}(*)$ is to enumerate all possible set of D_l, namely, the power sets of the total set, which has 2^M subsets.

When $u_{si}=u_{sr}$, $u_{id}=u_{rd}$, $u_{ip}=u_{rp}$, $u_{pi}=u_{pr}$, and $\sigma_{si}^2=\sigma_{sr}^2$, $\sigma_{id}^2=\sigma_{rd}^2$, $\sigma_{ip}^2=\sigma_{rp}^2$, $\sigma_{pi}^2=\sigma_{pr}^2$, we have $F_{si}=F_{sr}$, $F_{id}=F_{rd}$, $F_{ip}=F_{rp}$, and the upper bound of outage probability is

$$\begin{aligned}&p_{o2}\le F_{sd}(2^{2R}-1)\left(F_{sr}(2^{2R}-1)\right)^M+\\&F_{sd}(2^{2R}-1)\sum_{l=1}^{M}\begin{pmatrix}M\\l\end{pmatrix}\left(1-F_{sr}(2^{2R}-1)\right)^l\times\\&\left(F_{sr}(2^{2R}-1)\right)^{M-l}\left(F_{rd}(2^{2R}-1)\right)^l\\&=F_{sd}(2^{2R}-1)\times\\&\left((1-F_{sr}(2^{2R}-1))F_{rd}(2^{2R}-1)+F_{sr}(2^{2R}-1)\right)^M\\&\triangleq A\times B^M\end{aligned} \quad (22)$$

Taking the logarithm on both sides of Equation (22), we have

$$\log p_{o2}=\log A+M\log B \quad (23)$$

Thus there is linear relation between the logarithm form of outage probability $\log p_{o2}$ and the number of relays M. It's obvious that the increase of M can reduce the outage probability, as the coefficient $\log B<0$. The complete forms of p_{o1} and p_{o2} are in Equation (12) and (13) on the top of this page.

V. Without A Direct Link

In the scenario without a direct link, the relay is used to extend the coverage of secondary users. As the cooperative transmission between secondary nodes does not exist, there is no spatial diversity gain. In this case, When D_l is not empty, the relay links exist, and the mutual information between s and d is

$$I=\frac{1}{2}\log(1+r_{cd}) \quad (26)$$

We take the same notation as in Section IV. The outage probability is

$$p_o=\Pr\{D_l=\emptyset\}+\sum_{D_l\neq\emptyset}\Pr\{I<R|D_l\}\Pr\{D_l\} \quad (27)$$

And

$$\begin{aligned}\Pr\{I<R|D_l\} &= \Pr\{\frac{1}{2}\log(1+\gamma_{cd})<R\}\\ &= \Pr\{\gamma_{cd}<2^{2R}-1\}=F_{\gamma_{cd}}(2^{2R}-1)\end{aligned}$$

As the exact value of $\Pr\{I<R|D_l\}$ is obtained, we have the exact value of outage probability

$$\begin{aligned}p_{o3}=&\prod_{i=1}^{M}F_{si}(2^{2R}-1)+\sum_{D_l\neq\emptyset}(\prod_{i\in D_l}(1-F_{si}(2^{2R}-1))\\&\times\prod_{i\notin D_l}F_{si}(2^{2R}-1)\prod_{i\in D_l}F_{id}(2^{2R}-1))\end{aligned} \quad (28)$$

$$p_{o3}=\prod_{i=1}^{M}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{si}}}}{\sigma_{\gamma_{\mathrm{si}}}\sqrt{2}}\right)\right)+\sum_{D_l\neq\emptyset}\left(\prod_{i\in D_l}\left(\frac{1}{2}-\frac{1}{2}\mathrm{erf}\left(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{si}}}}{\sigma_{\gamma_{\mathrm{si}}}\sqrt{2}}\right)\right)\times\right.$$
$$\left.\prod_{i\notin D_l}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{si}}}}{\sigma_{\gamma_{\mathrm{si}}}\sqrt{2}}\right)\right)\prod_{i\in D_l}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{id}}}}{\sigma_{\gamma_{\mathrm{id}}}\sqrt{2}}\right)\right)\right) \tag{24}$$

$$p_{o4}=\left((\frac{1}{2}-\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{sr}}}}{\sigma_{\gamma_{\mathrm{sr}}}\sqrt{2}}))(\frac{1}{2}+\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{rd}}}}{\sigma_{\gamma_{\mathrm{rd}}}\sqrt{2}}))+(\frac{1}{2}+\frac{1}{2}\mathrm{erf}(\frac{\ln(2^{2\mathrm{R}}-1)-\mathrm{u}_{\gamma_{\mathrm{sr}}}}{\sigma_{\gamma_{\mathrm{sr}}}\sqrt{2}}))\right)^M \tag{25}$$

When $u_{si}=u_{sr}$, $u_{id}=u_{rd}$, $u_{ip}=u_{rp}$, $u_{pi}=u_{pr}$, and $\sigma_{si}^2=\sigma_{sr}^2$, $\sigma_{id}^2=\sigma_{rd}^2$, $\sigma_{ip}^2=\sigma_{rp}^2$, $\sigma_{pi}^2=\sigma_{pr}^2$, we have $F_{si}=F_{sr}$, $F_{id}=F_{rd}$, $F_{ip}=F_{rp}$, and the exact value of outage probability is

$$\begin{aligned}p_{o4}&=\left(F_{sr}(2^{2R}-1)\right)^M+\sum_{l=1}^{M}\left(\begin{array}{c}M\\ l\end{array}\right)(1-F_{sr}(2^{2R}-1)^l\\&\times\left(F_{sr}(2^{2R}-1)\right)^{M-l}\left(F_{rd}(2^{2R}-1)\right)^l\\&=\left(\left(1-F_{sr}(2^{2R}-1)\right)F_{rd}(2^{2R}-1)+F_{sr}(2^{2R}-1)\right)^M\\&\triangleq B^M\end{aligned} \tag{29}$$

Taking the logarithm on both sides of Equation (22), we have

$$\log p_{o4}=M\log B \tag{30}$$

There is also linear relation between the logarithm form of outage probability $\log p_{o4}$ and the number of relays M. The complete forms of p_{o3} and p_{o4} are in Equation (24) and (25) on the top of this page.

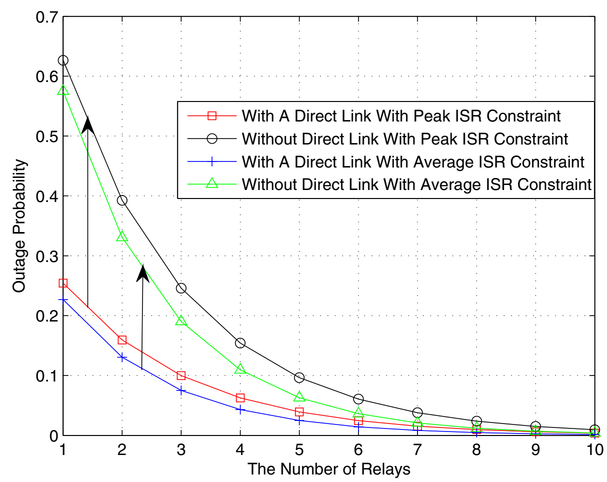

Fig. 2. the relation between the outage probability and the number of relays. For $\beta=0.2$ and $R=0.3$

VI. Simulation Results

We set $u_{pp}=1.2$, $u_{sd}=0.5$, $u_{si}=u_{sr}=0.8$, $u_{id}=u_{rd}=0.6$, $u_{ip}=u_{rp}=0.9$, $u_{pi}=u_{pr}=0.7$, $u_{sp}=u_{rp}$, $u_{pd}=u_{pr}$ and $\sigma_{pp}=0.5$, $\sigma_{sd}=0.3$, $\sigma_{si}^2=\sigma_{sr}^2=0.4$, $\sigma_{id}^2=\sigma_{rd}^2=0.6$, $\sigma_{ip}^2=\sigma_{rp}^2=0.7$, $\sigma_{pi}^2=\sigma_{pr}^2=0.3$, $\sigma_{sp}=\sigma_{rp}$, $\sigma_{pd}=\sigma_{pr}$. And Fig 2 illustrates the relation between the outage probability and the number of relays M for $\beta=0.2$ and $R=0.3$. The larger M, the smaller outage probability, which is obvious, because there is linear relation between the logarithm form of outage probability and the number of relays M according to Equations (23) and (30). For $\beta=0.2$ and $R=0.3$, the outage probability with average ISR constraint is smaller than that with peak ISR constraint. And Fig 2 tells that the cooperation in secondary system can reduce the outage probability, as the outage probability with a direct link is smaller than that without direct link.

Fig 3 illustrates the relation between the outage probability and ISR threshold β. For $R=0.3$ and $M=5$. The larger β, the smaller outage probability, as the larger ISR threshold can tolerate more interference power generated by SUs in primary receiver and SUs can transmit with larger power, which result in smaller outage probability. And we notice that for $R=0.3$ and $M=5$, the outage probability with average ISR constraint is commonly smaller than that with peak ISR constraint. Besides, the cooperation can reduce the outage probability, for that the outage probability with a direct link is smaller than that without direct link.

Fig 4 illustrates the relation between the outage probability and the data rate threshold R for $\beta=0.2$ and $M=5$. The larger R, the larger the outage probability, as the larger R makes the outage event occur more easily. Fig 4 also tells that the outage probability with average ISR constraint is not always smaller than that with peak ISR constraint. When R is small, the outage with peak ISR constraint is smaller than that with average ISR constraint, but when R is large, the outage with peak ISR constraint is larger than that with average ISR constraint. Besides, Fig 4 also tells the cooperation can improve the outage performance.

VII. Conclusion

In this paper, we investigate the outage performance of cognitive relay networks with PU's interference to signal ratio (ISR) constraint, where both average and peak ISR constraint are considered. And we derive the exact outage probability in the scenario without cooperation and the upper bound of

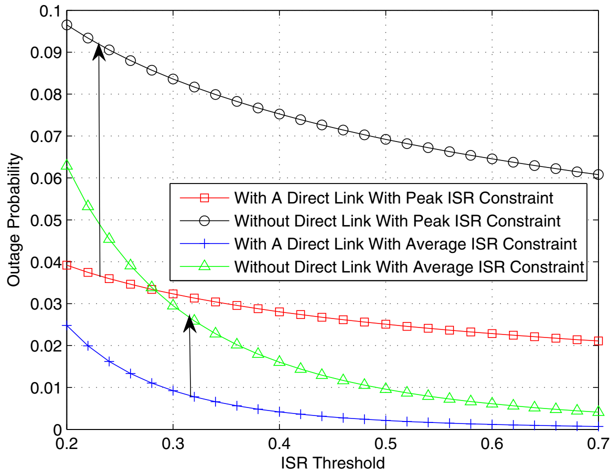

Fig. 3. the relation between the outage probability and ISR threshold β. For $R = 0.3$ and $M = 5$

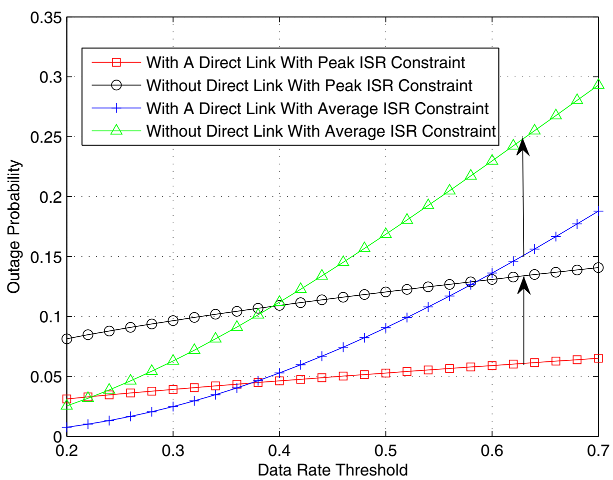

Fig. 4. the relation between the outage probability and the data rate threshold R. For $\beta = 0.2$ and $M = 5$

outage probability in the scenario with cooperation. By the simulation results, we notice that cooperation can improve the outage performance significantly and the system parameters such as ISR threshold β and data rate threshold R also have significant impact on the outage performance.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (60832009), the National Science and Technology Major Project (2010ZX03003-001-01, 2012ZX03006003-003), Sino-Finland ICT Collaboration Program Project on "Future Wireless Access Technologies" (2010DFB10410).

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