



BRANDEIS UNIVERSITY

UNDERGRADUATE THESIS IN COMPUTER SCIENCE

**Graph Isomorphism,
Reconstruction and the Cycles
Invariant**

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