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MODELLING BELT PASSAGE EXIT ALTITUDE

Predicting the altitude for the radiation belt passage at the start (exit) and end (entry) of each revolution has been done in an ad-hoc fashion from the start of the mission until now. It happened recently that the instruments were switched on, (at the start of the revolution), while the radiation environment was still high. This can cause permanent damage, and should therefore be avoided.

To address this problem, I took on the task of constructing a model based on the radiation monitor data from revolutions 1 to 1050 (provided by Mike Walker from MOC). The aim of the model it to allow us to predict more accurately the evolution of the belt exit altitudes at the start of each revolution to ensure a safe switch-on time that is not too early, and also of the belt entry altitude at the end of each revolution, to maximize the science time by not switching off too soon. This document discusses the modelling for the belt exit altitude only.

1. A First Look at the Data

The radiation belt exit altitude, defined as the altitude where the radiation monitor detects 600 cps, as a function of time in seconds from the start of the first revolution (Fig. 1), shows two main features: the yearly periodic modulation, and the underlying trend, that appears more complicated than a simple linear function.

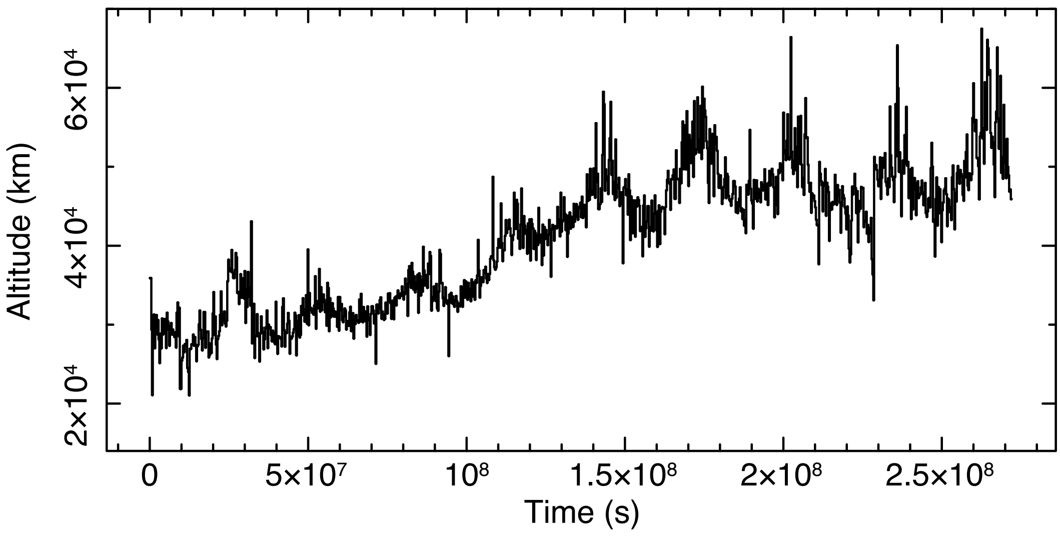


Figure 1. Radiation belt exit altitude from the start of revolution 1. Data gaps were filled using a Monte Carlo technique that takes into account the local trend and variance.

A periodogram of the time series (Fig. 2, left) shows a large rise in power at the lowest frequencies due to the monotonic upward trend in the time series, and another prominent feature centred near 3e-8 Hz (Fig. 2, right). The result of a Gaussian fit on that peak gives a central frequency of 3.34797e-8 Hz, and a width of 1.17417e-9 Hz. These values corresponds to a period of 2.98688e7 s, and if we take half the width as the uncertainty, the period can be expressed as 345.7 ± 18 days.

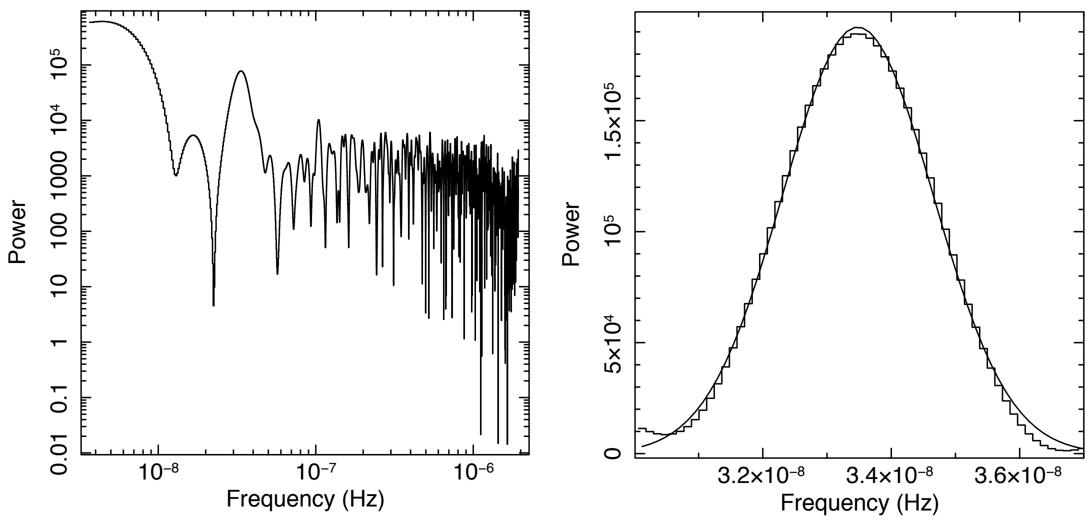


Figure 2. Left panel: Periodogram of times series shown in Fig. 1, made using an oversampling factor of 8, and for which the input data was smoothed using a Blackman window function that broadens peaks, but ensures very low power leakage to neighbouring frequencies. Right panel: zoom on the prominent peak at 3.3e-8 Hz with Gaussian fit on a periodogram made with oversampling factor of 16, and the rectangular window (no smoothing) that suffers from the greatest leakage of power, but ensures the highest frequency resolution.

2. Modelling the Long, Medium and Short Term Trends

To get to the underlying shape as best we can, a Kalman filter was applied to remove the bulk of the statistical noise (Fig. 3), and then a sliding smoothing window spanning 51 bins (Fig. 4).

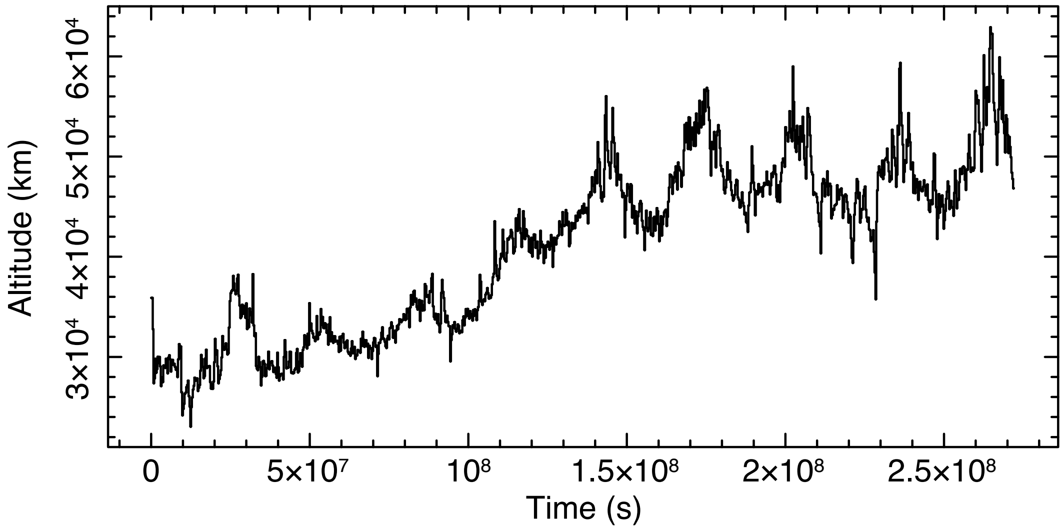


Figure 3. Kalman filtered, gap-filled time series.

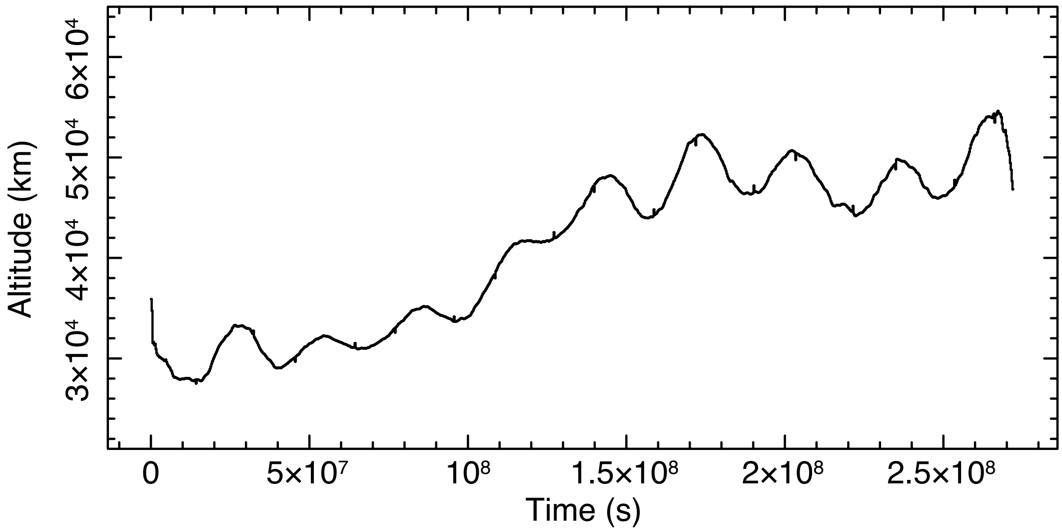


Figure 4. Smoothed Kalman filtered time series using a sliding window spanning 51 bins.

The modelling was done in steps: first the long term trend, then the medium, and finally the short term modulation. The long and medium term trends are not χ^2 fits to the data, but rather fits that have been adjusted by hand to follow the curve as closely as possible *from below*. Once the long term trend (Fig. 5, left) was subtracted from the data, the medium term trend (Fig. 5, middle) was adjusted and also subtracted. The short term trend (Fig. 5, right) was modelled by a fit to the de-trended curve.

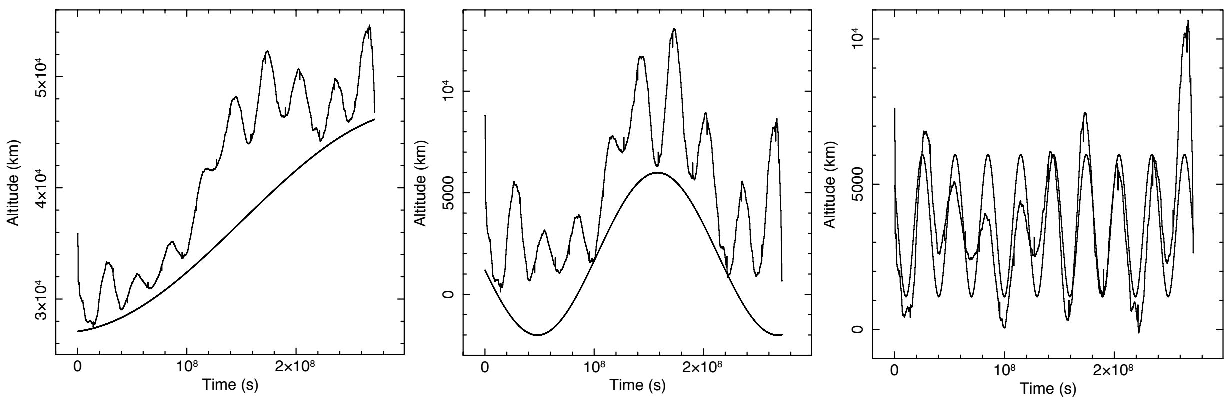


Figure 5. Long term trend (left), medium term (middle) and short trend trend (right). The values of the parameters for these individual model components are shown in Table 1 with those of the combined model in Table 1.

3. The Combined Model

The parameters of the three sinusoidal components for the long, medium and short term trends were used to define the combined model that included an additional constant to account for the vertical offset. The model can be written as: *constant sine sine sine.* The constant model has a single parameter labelled CO. The sine model has three: the period (PE), the phase (PH) and the amplitude or norm (SN). The final fit to the data based on this combined model is shown in Figure 6 and the best fit parameter values for each of the model components are given in Table 1. Albeit expected, it is interesting to note how well the period of the short term modulation derived from the combined fit agrees with the centroid of the periodogram peak shown in Figure 2.

The resolution of the curve, and therefore of the model, is a single revolution. This means that there are about 120 points in a year. We must discuss and agree on a strategy for implementing the periodic updates to the critical belt exit altitude.

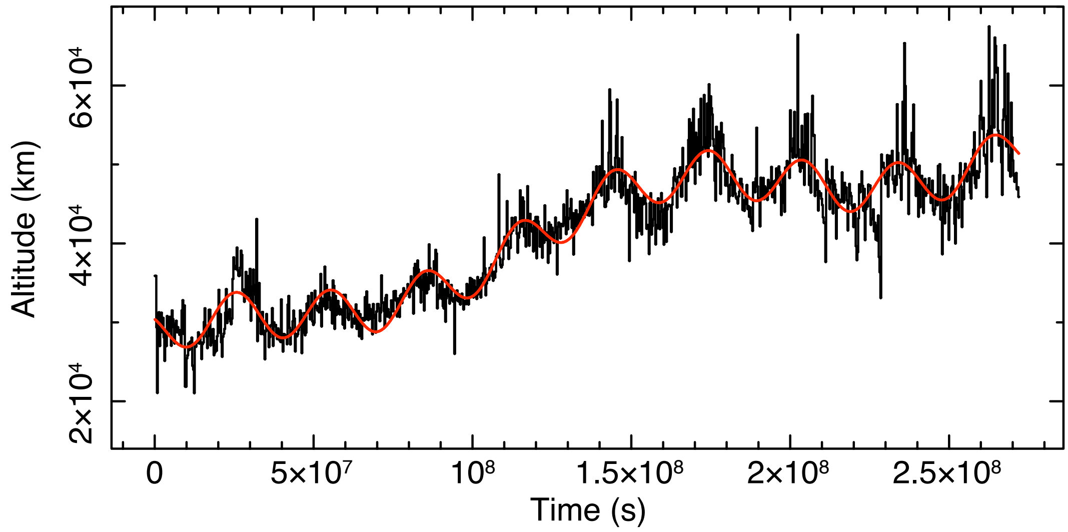


Figure 6. Best-fit to original time series using the combined model.

Table 1. Best-fit Parameter Values of Combined Model

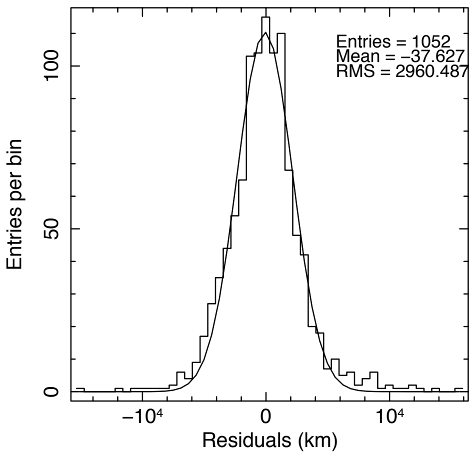
Model Component	Combined Model	Individual Models
PE (long term)	1.4579E+09 s	6.6E+08 s
PH (long term)	-3.8534E+07 s	1.5E+08 s
SN (long term)	33377 km	10000 km
PE (medium term)	1.5462E+08 s	2.2E+08 s
PH (medium term)	1.1862E+08 s	1.03E+08 s
SN (medium term)	3278 km	4000 km
PE (short term)	2.9739E+07 s	2.983E+07 s
PH (short term)	1.0728E+08 s	1.79E+07 s
SN (short term)	2979 km	2443 km
CO	19780 km	37000, 2000, 3575 km (*)

(*) Value of constant model component for the long, medium and short term trend respectively.

5. The Critical Altitude

To estimate the critical altitude that we referred to earlier as the suitable envelope, vertically offset from the best-fit model of the exit altitude, we use the distribution of the residuals, (the difference between the data and the model). This gives us a quantitative measure of the statistical spread of values about the best-fit model, and therefore a means to accurately define a safe altitude within the chosen limit.

The symmetry and tightness of the distribution of residuals give us confidence that the model is a good representation of the data. Fitting a Gaussian function to the distribution yields a standard deviation of 2243 km. Therefore, the one-sigma envelope would be vertically offset from the best-fit model by this amount, the three-sigma envelope would be offset by 6729 km, and the five-sigma envelope would be offset by 11215 km. The level of safety we chose is arbitrary, and should therefore be discussed.



4. Predicting the Evolution of the Exit Altitude

The purpose of the model is to give us the ability to predict, more or less accurately, the evolution of the exit altitude, and use the prediction to adjust the critical altitude at which the instruments are switched on at the start of the revolution, when the satellite emerges from the radiation belts. The model is really quite simple, as it does not include time-dependent periods or amplitudes, (which are indeed seen in the data; see Fig. 5, right panel), because this would increase the complexity excessively.

Based on the combined model derived from 8.75 years of data, the prediction for the evolution of the exit altitude for half as many more is shown in Figure 7, together with the one, three and five sigma envelopes as defined above.

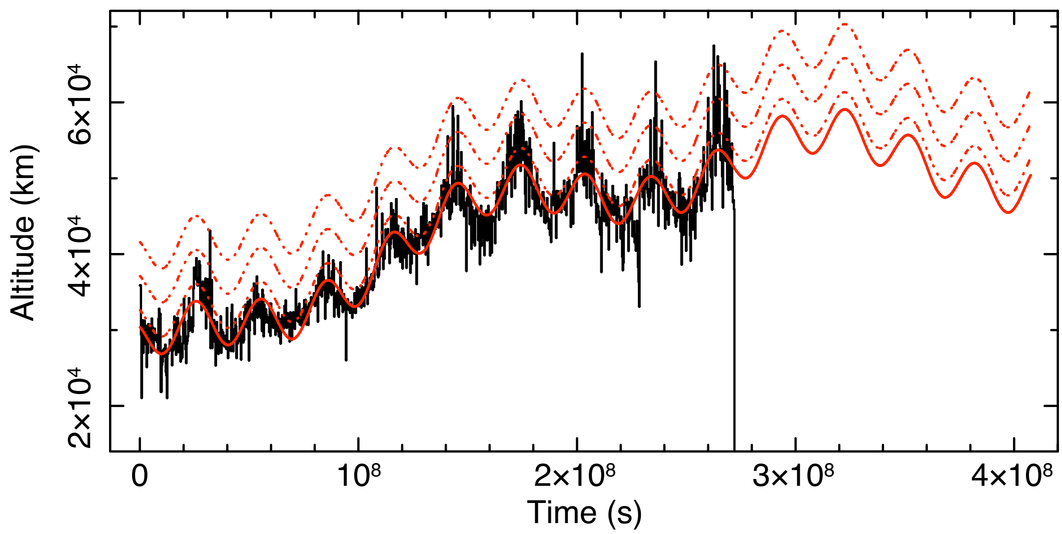


Figure 7. Prediction of the exit altitude for the next 4.5 years (up to rev. 1575). The dotted lines correspond to the 1, 3 and 5 sigma envelopes, with altitude offsets of 2243, 6729 and 11215 km, respectively.