Market Segmentation in Online Platforms

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Abstract

This paper studies ranking policies in a stylized trial-offer marketplace model, in which a single firm offers multiple products and has consumers who express heterogeneous preferences. Consumer trials are influenced by past purchases, the inherent appeal of the products, and the ranking of each product. Consumer purchases conditional on trying the product are dependent on the inherent quality for the given consumer segment. The platform owner needs to devise a ranking policy to display the products to maximize the number of purchases in the long run, and to decide whether to display the number of past purchases. The model proposed attempts to understand the impact of market segmentation in a trial-offer market with position bias and social influence. Under our model, consumer choices are based on a very general choice model known as the mixed multinomial logit model, which embeds product appeal, ranking, and past purchases into the taste parameters. We analyze the long-term dynamics of this highly complex stochastic model and we quantify the expected benefits of market segmentation as well as the value of social influence. When past purchases are displayed, consumer heterogeneity makes buyers try the sub-optimal products, reducing the overall sales rate. We show that consumer heterogeneity makes the ranking problem NP-hard. We then analyze the benefits of market segmentation. We find tight bounds to the expected benefits of offering a distinct ranking to each consumer segment. Finally, we show that the market segmentation strategy always benefits from social influence when the average quality ranking is used. One of the managerial implications is that the firm is better off using an aggregate ranking policy when the variety of consumer preference is limited, but it should perform a market segmentation policy when consumers are highly heterogeneous. We also show that this result is robust to relatively small consumer classification mistakes; when these are large, an aggregate ranking is preferred.

Keywords: revenue management; OR in marketing; social influence; market segmentation; ranking policies.

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1 Introduction

The effects of social influence on consumer behaviour have been observed in a wide range of settings (Salganik et al. 2006, Tucker & Zhang 2011, Viglia et al. 2014). Depending on the market and/or the marketing platform, social influence may be induced by different signals, including the number of past purchases, consumer ratings, and consumer recommendations. Moreover, these popularity signals can be amplified through product visibilities. Indeed, in digital markets, the impact of visibility on consumer behavior has been widely observed, including in internet advertisement where sophisticated mathematical models have been developed to determine the relative importance of the various product positions (Craswell et al. 2008). Positioning effects are also of high significance in online stores such as Expedia, Amazon, and iTunes, as well as physical retail stores (see, e.g., Lim et al. (2004)).

The combination of popularity signals and product visibilities have been extensively studied in the past few years, both theoretically and experimentally: See, for instance, Abeliuk et al. (2016), Krumme et al. (2012), Abeliuk et al. (2015), Maldonado et al. (2018), Van Hentenryck et al. (2016a). This research stream considered trial-offer markets in which consumers have the opportunity to try products or services for free and later decide whether to purchase them. Prominent examples of online trial-offer markets include the following: music markets such as ITUNES where users can listen to songs prior to the purchase decision, phone apps stores such as GOOGLE PLAY* or the APPLE STORE where "apps" have a free trial-version (limited by the functionality or with an expiration deadline), video on demand platforms such as Netflix, where consumer try episodes of series and finish watching them if they like them, and game platforms, such as Steam, where games may be returned for free within 2 hours of usage. Furthermore, other examples of trial-offer markets include online stores offering physical products that have free shipping (sometimes for purchases above a threshold) and free returns, such as Amazon.com or Nike.com. Previous studies have shown that social influence†, in conjunction with the optimization of product visibilities, can have significant benefits on market efficiency. Moreover, simple ranking policies, e.g., giving the most visibility to products with the best estimated qualities, are needed to realize these benefits. These positive results however assume that customer preferences are homogeneous and can be modeled by a multinomial logit.

This paper studies a more realistic model with heterogeneous customers that follow a laten-

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*Google Play's refund policy allows consumers to get full refunds on app purchases if the claim is made within 48 hours.

†Almost all examples of trial-offer markets show some form of popularity signal (rating and/or number of purchases) of its products when consumers browse through them.
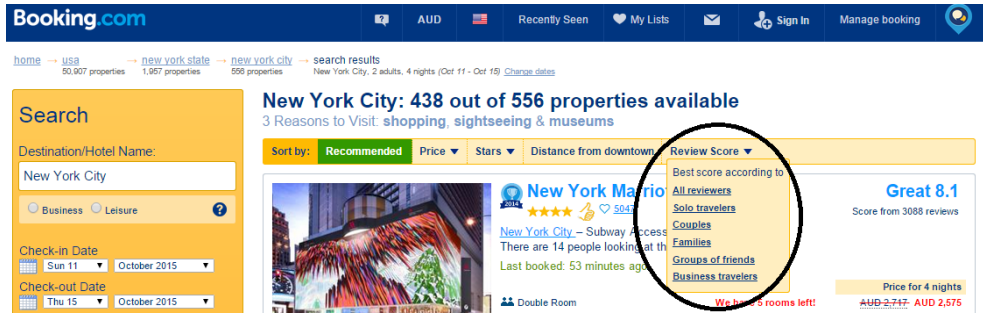
Figure 1: Rankings Averaging Scores of a Consumer Segment at Booking.com.

t class multinomial logit model (McFadden & Train 2000, Rusmevichientong et al. 2014) and attempts to understand whether the results obtained earlier in simple models would still hold under these new setting. It presents both negative and positive results. First, contrary to the homogeneous case, this paper shows that, in mixed Multinomial Logit (MNL) trial-offer markets, computing the ranking of products that maximizes the probability that the next customer will make a purchase is NP-Hard under Turing reductions. Moreover, the paper shows that popularity signals, that are beneficial in the homogeneous case, may become detrimental to market efficiency in mixed MNL trial-offer markets. However, this paper also shows that these negative results can be addressed by a simple segmentation strategy where customers are shown a quality ranking dedicated to their own consumer segment and only observe the popularity signal for their own market segment (i.e., the past purchases of customers of the same segment). This market segmentation strategy can be implemented easily by collecting information on customers and/or by providing customers different rankings based on various demographic features. Indeed, a recent analysis performed by the online travel agent Orbitz has shown that Mac users spend up to about 30% more in hotel bookings than their PC counterparts (Mattioli 2012), suggesting that it is beneficial to show different rankings to customers depending on the computer they use. Moreover, the major online travel agent Booking.com allows users to rank hotels according to the average consumers score of a particular segment such as couples, families, and solo travelers. See Figure 1 for an illustration of this feature. Although hotels are not a trial-offer market, the benefits of market segmentation extend to trial-offer markets, as Netflix, Google Play and Steam, among others, implement targeting strategies, giving different product recommendations based on past consumer behavior (India 2019, Loten 2020).

The contributions of this paper can be summarized as follows:

1. The paper constructs the first trial-offer market model with social influence and position biases in which consumers preferences can follow any finite mixture of multinomial logits.

2. The paper shows that, in mixed MNL trial-offer markets, computing the ranking of products that maximizes the probability that the next customer will make a purchase is NP-Hard under Turing reductions.

3. The paper shows that the popularity signal may, under some circumstances, decrease the expected market efficiency. In other words, the display of past purchases may reduce the number of sales by confusing consumers about which products to try.

4. The paper studies the average quality ranking, which ranks the items in decreasing order of average quality. It shows that the average quality ranking converges to a unique equilibrium when consumers are shown the number of past purchases (the popularity signal). This proof is rather involved as it requires to show that the mixed MNL model can be seen as a special MNL model in which some parameters (appeal and quality) are no longer constants but functions of the past purchases vector, and that these quantities can be upper and lower bounded in order to demonstrate convergence.

5. The paper presents a simple segmentation strategy, where customers are shown a quality ranking dedicated to their own segment and only observe the popularity signal for their own market segment (i.e., the past purchases of customers of the same segment). The paper quantifies the potential benefits in market efficiency of this strategy. Specifically, it proves that the expected purchases can increase up to a factor of K, where K is the number of segments of the mixed MNL model.

6. These theoretical results are complemented by a series of computational experiments which provide several managerial insights about trial-offer markets.

The remaining of this paper is organized as follows. Section 2 reviews the literature most related to this work. Section 3 introduces the model of the dynamic trial-offer market. The most relevant ranking policies for this model are described in Section 4, which also presents the NP-hardness results for performance ranking in mixed multinomial logit models. Section 4.2 describes the convergence and the impact of social influence for the quality ranking in the same setting. Section 5 presents our segmentation strategy and its benefits. Section 6 presents results of computational experiments and Section 7 concludes the paper. The proofs are deferred to the appendix. In the supplementary appendix A, we consider an extension of the model in which the platform owner makes mistakes during the customer classification process. The supplementary appendix B shows that a local search heuristic for solving the ranking optimization problem does not bring much

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benefit with respect to the much less costly average quality ranking policy studied in Section 4.2. Supplementary appendix C analyzes an extension in which the firm's objective is to maximize the expected revenue instead of purchases. Finally, the supplementary appendix D analyzes an extension in which the platform owner may show a subset of products to all consumer segments.

2 Related literature

Our work is related to the MusicLab experiment performed by Salganik et al. (2006). In that experiment, participants were presented a list of unknown songs from unknown bands, each song being described by its name and band. The participants were partitioned into two groups exposed to two different experimental conditions: the *independent* condition and the *social influence* condition. In the independent group, participants were shown the songs in a random order and they were allowed to listen to each of them and then download them if they wish. In the second group (social influence condition), participants were shown the songs in popularity order, i.e., allocating the most popular songs to the most visible positions. Moreover, these participants were also shown a popularity signal, i.e., the number of times each song was downloaded too. In order to investigate the impact of social influence, participants in the second group were distributed in eight "worlds" evolving completely independently. In particular, participants in one world had no visibility about the downloads and the rankings in the other worlds. The MusicLab is an ideal experimental example of a trial-offer market where each song represents a product, and listening and downloading a song represent trying and purchasing a product respectively. The results by Salganik et al. (2006) show that the different worlds evolve significantly differently from one another, providing evidence that social influence may introduce unpredictability in a market.

To explain these results, Krumme et al. (2012) proposed a framework in which consumer choices are captured by a multinomial logit model whose product utilities depend on songs appeal, position bias, and social influence. Abeliuk et al. (2015) provided a theoretical and experimental analysis of such trial-offer markets using different ranking policies following the framework of Krumme et al. (2012). They proved that social influence is beneficial in order to maximize the expected number of purchases when using a greedy heuristic known as *performance ranking*. The *performance ranking* selects the ranking that maximizes the expected number of purchases at the next time period, i.e. it maximizes the short-term market efficiency. Abeliuk et al. (2015) have also illustrated experimentally that the popularity ranking [‡] is outperformed by the performance ranking in a variety of settings. Still based on the model of Krumme et al. (2012), Van Hentenryck

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[‡]The popularity ranking ranks (dynamically) products by the number of purchases in decreasing order.

et al. (2016a) have studied the performance of the *quality ranking* which ranks products by their intrinsic quality (the quality of a product is here defined as the probability that a consumer would purchase/download the product once she has tried the product out). They show that the quality ranking is in fact asymptotically optimal and has a considerably less unpredictability than the popularity ranking.

Maldonado et al. (2018) studied the impact of the popularity signal strength on a market with multiple products and social influence. In their model, the popularity signal strength is a an exogenous parameter $r > 0$. The authors provide a complete characterization of the long-term market share of each of the products and show that the market is completely predictable as long as $r \leq 1$.

The relative importance of different popularity signals have been recently investigated by Engström & Forsell (2018) and Viglia et al. (2014). The first paper focuses on how consumers choose apps in the GOOGLE PLAY platform, and the second one studies how people select hotels. Both experiments arrived to the same conclusion, namely that the popularity signal (i.e., the number of purchases) has a much stronger impact on consumer behavior than the average consumer rating signal.

Our work is related to the recent paper by Hu et al. (2015) who consider a monopolist facing a newsvendor problem with two substitutable products with the same quality in which consumer preferences are affected by past purchases. The authors showed that the market is unpredictable but it can become less so if one of products has an initial sales advantage (such as for example by providing an initial discount). Our model has considerable differences including the incorporation of position biases, highly richer consumer preferences (Mixed MNL), an arbitrary number of products, and the allowance of products to have difference qualities.

Ghose et al. (2012) proposed a ranking system for hotels which takes into account the economic value of different locations and service-based characteristics, as well as consumer heterogeneity. In a follow-up paper, Ghose et al. (2014) studied the effects of three ranking policies on consumer behaviour using archival data analysis and randomized experiments. The general idea of both papers is to build a simultaneous equations model of clickthrough, conversion (purchase decision), ranking (performed by the platform owner), and customer rating. In their model, the demand for the different hotels (i.e., the products) is independent between the different hotels (apart from the fact that each hotel is assigned a different position in the ranking) whereas, the model we study in this paper, the different products are in direct competition to attract the demand.

In another related paper, Gopal et al. (2016) perform a quantitative study on how a firms can

strategically alter malleable networks such as enterprise social networks (ESN) or consumer social networks (CSN) in order to increase the transmission of ideas, innovation, or other information. In our setting, we focus on comparing a complete network (consumers observe all purchases) versus a network where consumers are partitioned in K classes or clusters (and a consumer only observes purchases of individuals from their own cluster).

Vaccari et al. (2018) studied a model consumers where arrive in sequence and estimate the quality of products based on product reviews (likes and dislikes). In their model, consumers like a product if the product's quality exceeds their expectation (which is calculated based on past ratings). The authors provide conditions that allow consumers to learn the true quality of products in the long run. Our work is also related to recent studies of theoretical choice models that incorporate position biases. As in our model, they consider situations in which the probability of selecting a product does not only depend on the offer set but also on the way products are displayed (Abeliuk et al. 2016, Aouad & Segev 2015, Davis et al. 2013, Gallego et al. 2020).

Recently, Golrezaei et al. (2018) considered a similar framework to ours in which the firm needs to decide how to rank a set of products to sell to consumers. Similar to our setting, sorting products by utility in their model is not always optimal §. Moreover, finding the optimal ranking of products to maximize short-term sales (or revenue) can be found in polynomial time when there is a single consumer type, but the problem becomes NP-hard when the number of consumer types is more than one (We note that a first preprint of our paper containing all these results was posted on November, 2015 (ArXiv) (Berbeglia et al. 2015)). In their work, the authors constructed the choice probabilities from a two-stage consumer search model based on a seminal work by Weitzman (1979) on the Pandora's problem. As a result, the resulting mathematical expression for the choice probabilities is different from the one we study. Another difference is that Golrezaei et al. (2018) considers the problem of maximizing welfare (not studied here) whereas a fundamental focus of this work is on segmentation strategies which are presented in Section 5.

Another recent paper that models the dynamics of customers influenced by social influence is due by Chen et al. (2019). An important difference to our work is that social influence signal are product ratings rather than purchases, and that their choice model is based on the MNL model rather than on any finite mixture of MNLs.

Other papers related to our work are Lee & Eun (2020), who uses sales transaction data to estimate the parameters of a Mixed MNL through which are able to identify heterogeneous consumers groups, Zhen et al. (2019), Capuano et al. (2017), and Molinero et al. (2015) who

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§Although the underlying reasons for it under each of the models are different.

studied social influence models, and Lutz & Newlands (2018) and Anderson & Xie (2014) who focused on market segmentation of customers.

3   The Model

Motivation   We consider a firm running a marketplace that sells a set of products. Following Krumme et al. (2012) and Salganik et al. (2006), we focus our attention to trial-offer markets, i.e., markets in which consumers can try the product for free before deciding to make a purchase. Consumers are position-biased in the following sense: the likelihood of trying a specific product is affected by the position of the product, as well as the position of the other products in the market. We also consider that in this marketplace it is possible to display information about product popularity. In particular, we assume that the firm shows the total number of purchases for each product at each point in time.

Unlike Krumme et al. (2012), we consider that there are different segments of consumers. More precisely, the probability that a given consumer tries a product will follows a Mixed Multinomial Logit (MMNL). Since McFadden & Train (2000) proved that every random utility model can be well approximated by a MMNL, our model of consumer preferences is indeed very general.

Formalization   We now formally describe a dynamic model for this marketplace. Let $[N] = \{1, 2, \ldots, N\}$ denote the set of items in the marketplace and S_N denote the set of the permutations of these items. At any point in time, the firm decides how to position the items in the market by selecting a permutation $\sigma \in S_N$ such that $\sigma(i) = j$ implies that item i is placed in position j ($j \in [N]$).

The consumer behavior can be described as follows. There are K different segments of consumers. At any point in time the probability that the next arriving customer belongs to segment $k \in [K]$ is given by w_k, the segment's *weight*[¶]. Consumers from the same segment exhibit different purchase profiles due to idiosyncratic shocks. This is captured with the MNL model.

When consumer t enters the market, she observes all the items and a popularity vector $d^t = (d_1^t, d_2^t, \ldots, d_N^t) \in \mathbb{N}^N$, where d_i^t is the number of times item i has been bought for prior to her arrival at time t. When the consumer arrives, each of the N items have been given a position through a permutation $\sigma \in S_N$. The consumer selects an item to try and then decides whether to buy it. Following Krumme et al. (2012) and extending their model to multiple segments, if the

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[¶]A special case of this is a Poisson arrival process with arrival rate λ_k for each consumer segment $k \in [K]$, such that $w_k = \frac{\lambda_k}{\sum_j \lambda_j}$.

consumer belongs to segment k, the probability that she tries item i is given by

$$p_{i,k}(\sigma, d^t) = \frac{v_{\sigma(i)}(a_{i,k} + d_i^t)}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \tag{1}$$

where $z_k \in \mathbb{R}_{\geq 0}$ for all $k \in [K]$ is fixed to a constant for the duration of the process, $v_j \in \mathbb{R}_{\geq 0}$ represents the visibility of position $j \in [N]$ regardless of the consumer class (the higher the value v_j the more visible the item in that position is), and $a_{i,k} \in \mathbb{R}_{>0}$ captures the intrinsic appeal of item i for consumer segment k for all $i \in [N]$ (higher values correspond to more appealing items)$^{\|}$. The value z_k represents the outside option for those consumers of segment k that enter the market. As the total number of purchases increases, the fraction of consumers choosing the outside option will decrease.

If a consumer from segment k has selected item i for a trial, the probability that she would purchase the item is given by $q_{i,k} \in [0,1]$. Observe that this probability is independent of both the appeal vector $(a_{1,k}, \ldots, a_{N,k})$ and the visibility vector v. Intuitively, this assumption which has been validated in the MusicLab experiment, captures the fact that it is more difficult to influence consumers after they have tested a product than before.

When the consumer decides to purchase item i, the popularity/sales vector d is increased by one in position i. To analyze this process, we divide time into discrete periods such that each new period begins when a new consumer arrives. Hence, the length of each time period is not constant.

The objective of the firm running this market is to maximize the total expected number of purchases. To achieve this, the key managerial decision of the firm is what is known as the ranking policy (Abeliuk et al. 2015), which consists in deciding at each point in time the permutation $\sigma \in S_N$ to display the items. The next section describes a number of relevant ranking policies for this model.

A key aspect of this paper is to study the potential benefits of the popularity signal in terms of the rate of purchases (market efficiency) and compare the ranking policies with and without this signal. In this paper, we always assume that the popularity signal is used as specified in Equation (1). When the popularity signal is not used, the probability of trying (or sampling) a product is obtained as if the popularity signal is simply the vector $\langle 0, \ldots, 0 \rangle$.

We conclude this section with a several comments about the model described above. First, under the trial-offer market model, a customer who tries, but does not like, a product simply walks away without trying any other product. An alternative model would be to allow the customer

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$^{\|}$Note that the expression in Equation (1) is the special case of a MMNL when the consumer utilities are logarithmic.

to try another product with probability c_i every time she tried but did not like a product i. Perhaps surprisingly, one can show that under mild conditions**, the resulting model is equivalent to the original trial-offer market with a single trial per consumer Van Hentenryck et al. (2016b) ††. Second, although there are many different ways to model consumer trial and purchase probabilities as a function of the previous purchases, intrinsic appeal, etc; the way we model the problem in this paper is a natural extension to an MNL model that has been empirically tested (Krumme et al. 2012): it has succeeded to fit the data from a large scale randomized experiment where over 14, 000 users listened and downloaded songs from an assortment 48 song pieces. The participation in the experiment was unpaid and voluntary and therefore, the setup is arguably closer to a genuine internet platform than if individuals were paid to participate (see Salganik et al. (2006) for more details). Third, the model studied here could potentially be useful in markets that do not have the trial-offer structure. Indeed, this model can be captured as a function whose inputs are (1) market observations (current product ranking and past product purchases) and (2) some parameters subject to estimation (product appeals, product qualities, and position visibility values). The function then returns each product purchasing probability. Thus, even for non-trial-offer markets, it is an open empirical question on whether this model is better or worse than other choice models that also consider product rankings and/or popularity signals such as Golrezaei et al. (2018) and Vaccari et al. (2018), in terms of predictive accuracy of the purchasing probabilities. Finally, it is worth observing that our model assumes there is an independence between trying a product and buying it. Although this assumption was not problematic to fit the large-scale experiment carried out in Salganik et al. (2006), a model extension that incorporates a correlation between the two steps would be interesting.

4   Ranking Policies

Consider without loss of generality that the N locations are sorted by their visibility such that $v_1 \geq v_2 \geq \ldots \geq v_N$. A ranking policy is a function $f : \mathbb{N}^N \to S_N$ which, given a vector of past purchases, returns a ranking of the items.

Ranking policies can be partitioned into two groups: *static* and *dynamic.* A ranking policy g is said to be *static* if the output ranking does not depend on the popularity signal, i.e., if $f(d) = f(d')$ for all $d, d' \in \mathbb{N}^N$. On the other hand, a *dynamic* ranking policy is one in which the output ranking depends on this signal.

**The probability c_i is a polynomial function on the product quality q_i.
††The equivalence is based on a redefinition of product appeal and quality.

4.1 Performance ranking

The *performance ranking* is a dynamic policy that greedily selects a ranking that maximizes the expected number of purchases in the following period. This strategy was first proposed by Abeliuk et al. (2015) for the special case with $K = 1$ (where they show it is asymptotically optimal) and we now generalize its definition for the more general model considered in this paper. The probability that the next incoming consumer belongs to segment k is given by w_k, therefore the performance ranking at time period t consists of finding the permutation $\sigma^* \in S_N$ maximizing the probability of a purchase in the next time period, i.e.,

$$\sigma^* = \operatorname*{argmax}_{\sigma \in S_N} \sum_{k=1}^{K} w_k \cdot \sum_{i=1}^{N} p_{i,k}(\sigma, d^t) \cdot q_{i,k}. \tag{2}$$

The probability Π^{PR} of a purchase in the next time period is thus given by

$$\Pi^{PR} = \max_{\sigma \in S_N} \left\{ \sum_{k=1}^{K} \left(w_k \cdot \sum_{i=1}^{N} (p_{i,k}(\sigma, d^t) \cdot q_{i,k}) \right) \right\} \tag{3}$$

$$= \max_{\sigma \in S_N} \left\{ \sum_{k=1}^{K} \left(w_k \cdot \sum_{i=1}^{N} \left(\frac{v_{\sigma(i)}(a_{i,k} + d_i^t)}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \cdot q_{i,k} \right) \right) \right\}. \tag{4}$$

Abeliuk et al. (2015) showed that when $K = 1$, this greedy ranking policy can be computed efficiently, i.e., in strongly polynomial time (Theorem 1 in Abeliuk et al. (2015), they assumed $z = 0$ but their proof can be easily generalized for any $z \in \mathbb{R}_{\geq 0}$). Moreover, despite the myopic focus of the performance ranking, a series of computational experiments performed by Abeliuk et al. (2015) showed that, for the special case of $K = 1$, the performance ranking was superior than the standard popularity ranking both in terms of unpredictability as well as in terms of number of purchases. Unfortunately, the performance ranking cannot be computed efficiently when there are at least two classes of consumers. More precisely, we can show that the assortment problem under a 2-segment Mixed Multinomial Logit choice model which is known to be NP-hard (Rusmevichientong et al. 2014) can be reduced (under Turing reductions) to computing the performance ranking in our setting.

Theorem 1. *Computing the performance ranking is NP-hard under Turing reductions. This is true even when $K = 2$ and the product qualities are the same for all consumer classes.*

In order to deal with this negative result, this paper explores two options. The first is to rank products based on their average quality, a strategy we called *average quality ranking* and is studied in the next section. The second avenue is to explore a greedy local search heuristic which

worked as follows. In the first period, we set the average quality ranking as initial solution, and then, we evaluate possible ways to exchange the position of two products in the ranking (2-swaps) and select the first swap that improves the objective function. The local search process is then repeated until there are no more improvements. In each consecutive period, we use the previous period ranking as the starting solution. Notice that this heuristic will finish in a finite number of steps. This is because the objective function increases with each swap and there exist finitely many assortments of products. Our experimental results have shown that the additional benefit of the local search is minimal (see Appendix C), we thus used the average quality ranking as our approximate method which is several orders of magnitude faster.

4.2 Average quality ranking

The *average quality ranking* (or quality ranking for short) is a simple and natural static policy: it consists in ranking the products by the weighted average quality (among the different customer classes), ignoring the appeals and popularity signal. The quality ranking for the MMNL model thus consists in placing in position j the item with the jth highest weighted average quality, where the weighted average quality of item $i \in [N]$ is

$$\bar{q}_i = \sum_{k=1}^{K} w_k q_{i,k}. \tag{5}$$

For the special case $K = 1$, the quality ranking is optimal asymptotically and always benefits from the popularity signal used in our model (Van Hentenryck et al. 2016a). The next section will study whether this continues to hold in richer contexts when $K > 1$. Note that, in the following, the ranking which orders the products by decreasing values of $\bar{q}_i$ is called the *average quality ranking*.

Before analyzing some fundamental properties of the MMNL it is important to first make the following definition.

Definition 1. The MMNL model *goes to a monopoly* using a ranking policy f if, for each realization of the N random sequences $\{\phi_i^t\}_{t\in\mathbb{N}}$ ($i \in [N]$), there exists a product i^* such that the realized sequence $\{\phi_{i^*}^t\}_{t\in\mathbb{N}}$ converges almost surely to 1 as t goes to infinity. In this case, we also say that item i^* *goes (predictably) to a monopoly.*

We now study some fundamental properties of the quality ranking for the MMNL model. We first show that the average quality ranking converges to a monopoly (under weak conditions). We then study the benefits of displaying the popularity signal for the average quality ranking.

Given a ranking policy f, the random variable

$$\phi_i^t = \frac{d_i^t}{\sum_j d_j^t}$$

is known as the market share of item i at time t: It represents the ratio between the number of times that item i was purchased and the total number of purchases up to time t.

We can now show that the MMNL model goes to a monopoly when using the average quality ranking. The proof is quite technical: Its key idea is to show that the Mixed Multinomial Logit Model (MMNL) can be reduced to a generalized case of the Multinomial Logit Model (MNL) where the appeal and the quality of an item at time t depend on the popularity signal at t (This is shown in Theorem 2). Then this generalized appeals and qualities can be bounded to obtain the result. The proof relies on the following lemma that generalizes the convergence result of the quality ranking for the MNL model by Van Hentenryck et al. (2016a) to the case where the appeal and quality of an item depend on the popularity ranking provided that the resulting functions are bounded by above and below. In Theorem 2 we show that there exists a time period t^* after which the time dependent qualities and appeals of this modified MNL model are bounded as in 6 and that these bounds satisfy 7. Thus, by using Lemma 1, we can show that this modified MNL model goes to a monopoly which implies that the MMNL goes to a monopoly.

Lemma 1. *Consider a different Multinomial Logit Model, i.e., a setting with $K = 1$ where the appeal and quality of each item i are functions of the purchases vector d^t, i.e., $\widetilde{a}_i^t = \widetilde{a}_i(d^t)$ and $\widetilde{q}_i^t = \widetilde{q}_i(d^t)$ respectively. Suppose that there exists a time period t^* such that these two quantities are upper and lower bounded by constants for any period $t > t^*$ independent of the realizations of $\widetilde{a}_i^t$ and $\widetilde{q}_i^t$, i.e.,*

$$q_{i,min} \leq \widetilde{q}_i^t \leq q_{i,max} \ and \ a_{i,min} \leq \widetilde{a}_i^t \leq a_{i,max} \quad \forall i \in [N], t > t^*. \tag{6}$$

Let $\sigma \in S_N$ denote a static ranking policy. If there exists an item i^* and an instant $\hat{t}$ such that

$$v_{\sigma(i^*)} q_{i^*,min} > v_{\sigma(i)} q_{i,max} \ \forall \ i \neq i^* \ and \ \forall \ t > \hat{t}, \tag{7}$$

then item i^* goes to a monopoly when using the ranking policy σ.

The main result of this section is about the convergence to a monopoly of a large class of static ranking policies. For simplicity, we assume a weak condition to break potential ties between items.

Definition 2. A static ranking policy σ is *tie-breaking* for a MMNL model if there exists a unique

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item i^* with the highest product of visibility and weighted average quality, i.e.,

$$\exists\, i^* \in [N] : \bar{q}_{i^*} v_{\sigma(i^*)} > \bar{q}_i v_{\sigma(i)} \quad \forall i \in [N], i \neq i^*. \tag{8}$$

Note that this tie-breaking property is a very mild assumption: If the quality for each pair (consumer class, product) is a realization from a (different or the same) continuous probability distribution over some interval (regardless of how small), the probability that a ranking policy is tie-breaking is indeed 1. We are now ready to prove the main result of this section.

Theorem 2. *Consider a MMNL model $\mathcal{M}$ and a static, tie-breaking ranking policy $\sigma \in S_N$ for $\mathcal{M}$. Model $\mathcal{M}$ goes to a monopoly using σ and the item i^* that goes predictably to a monopoly using σ in $\mathcal{M}$ is given by*

$$i^* = \operatorname*{argmax}_{1 \leq i \leq N} v_{\sigma(i)} \bar{q}_i.$$

The following corollary asserts that the average quality ranking converges to a monopoly for the product of highest average quality. This result is particularly interesting since it shows that the average quality ranking is asymptotically optimal, and that it generalizes the quality ranking from the MNL to the MMNL model. By asymptotically optimal we mean that it is the global ranking with global social influence that maximizes the purchase probability as time goes to infinity, $\lim_{t \to \infty} P^t$, where P^t is the expected purchase probability at period t.

Corollary 1. *(Asymptotic Optimality of the Average Quality Ranking). Whenever the average quality ranking is used, a MMNL model goes to a monopoly for the product with the highest weighted average quality.*

We end this section with some comments about the implications of Theorem 2 and Corollary 1. In general, market monopolies are not a desirable outcome from a consumer perspective. A key aspect of Theorem 2 is that the monopoly outcome does not come as a result of a restriction on the product offer variety (every products is always offered in our model), but because the consumer's utility ratio between the most popular product and the other product utilities tends to infinity. Although such monopoly convergence is somewhat surprising, it naturally applies to the very long run dynamics of these trial-offer markets (i.e., when time tends to infinity). In practice, however, it takes a very long time to even get close to a monopoly. It is not simple to find the convergence rate to a monopoly, even for the case $K = 1$. This model can be seen as an unbalanced and irreducible Pólya Urn Process for which the convergence rate is an open problem except for a few special cases of replacement matrices (see remark 4.7 in Janson (2004), where

they conjecture a convergence rate of $o(n^\gamma)$ with $\gamma = \min\{1/2, 3(1/2 - Re(\lambda_2/\lambda_1))\}$, with λ_1 and λ_2 being the greatest and second greatest eigenvalues of the replacement matrix respectively). As an illustration of the dynamics, we have performed a variety of computational experiments which are reported in Section 6. In all those experiments, the convergence to a monopoly of a single product was not yet achieved at the end of the simulation. From a practical point of view, the main insight obtained from Theorem 2 and Corollary 1 is that, as time goes by, consumers are more likely to purchase the product that has the greatest average quality, which is due to the effect of the popularity signal in the market dynamics. An interesting question that arises is whether the effect of the popularity signal is beneficial to the firm. Specifically, what is the effect of the popularity signal on the expected rate of products purchased? This question is addressed in the following section.

4.2.1 The Impact of the Popularity Signal

In the previous subsection, we have shown that the average quality ranking for the MMNL model inherits the asymptotic convergence of the quality ranking for the MNL. Under the MNL model, the probability (in expectation) that the next individual purchases some product is always increasing if the popularity signal is used (Van Hentenryck et al. 2016a). In other words, the information of past purchases is helping consumers to make better choices on which products to try, meaning that they are more likely to purchase them. Unfortunately, this result does not always hold when consumers follow the more general MMNL model.

Theorem 3. *When using the average quality ranking, the MMNL model can perform (1) up to K times better if the popularity signal is not shown, where K is the number of classes; and (2) can perform arbitrarily worse without showing the popularity signal than by showing it.*

Theorem 3 provides an upper bound on how much the expected sales per period can be reduced due to the impact of the popularity signal (part (1)). In addition, it shows that sometimes, showing the social influence can be extremely beneficial (part (2)). For the latter, it is important to note that not showing the popularity signal can perform arbitrarily worse than showing it even when $z = 0$. This implies that this reduction is not caused by having consumers switching from the outside option to other products, but because consumers under social influence can sometimes try popular products that have low quality for them (relatively to other products). In Proposition 1 we show that the bounds in Theorem 3 are tight, and this happens when we choose $z = 0$ (however, for different values of z, not showing the popularity signal can still perform arbitrarily worse than by showing it).

Proposition 1. *The bounds in Theorem 3 are tight.*

This result shows that, depending on the specific parameters of the trial offer market (i.e. consumer's appeals, product qualities and weights of the different consumer segments), the popularity signal can enhance the number of sales or it can be detrimental to them. Specifically, the expected number of purchases under the average quality ranking policy can decrease by a factor of K if the popularity signal is used (for some settings) but they can also be increased by an arbitrarily large factor in other settings. In Proposition 1, the first tight bound occurs in a trial-offer market where each of the K different classes of consumers have a unique product in which they are interested; Moreover, this product is different for each consumer segment and there is a perfect alignment between product appeal and product quality for each class. In such a setting, the global popularity signal would be detrimental for the firm as well as for the consumers. The reason is that in the long run product 1 will become a monopoly (since its average quality ranking is higher than all the others). This means that in the long run, consumers will tend to try product 1. But this is problematic because, except for one consumer segment, all consumers segments do not like this product. In the limit, only a $1/K$ fraction of the consumers would purchase this product. In short, in this scenario a global popularity signal will persuade consumers to try products they do not like.

On the other hand, the second bound in Proposition 1 was obtained in a similar market setting but in which there is negative correlation between the products appeal and their quality. In that setting, the market converges slowly to a monopoly in the case the popularity signal is used. If no social influence is used, the rate of purchases tends to zero as the products have essentially no appeal. Thus, an heterogeneous set of customers complicates the managerial decisions in the marketplace: Whether or not social influence is beneficial in trial-offer markets depends on the particular structure of preferences among consumers. We recall that this is in sharp contrast to the results in (Van Hentenryck et al. 2016a) where it is shown that the popularity signal is always beneficial in the case consumers share preferences (i.e $K = 1$). We now quantify the benefit (or cost) of showing the popularity signal given the specific model parameters. The asymptotic purchase probability ratio between the average quality ranking with no social influence (AQNSI) and the average quality ranking with social influence (AQGSI) is (see the proof of Theorem 3):

$$\frac{\lim_{t\to\infty} P^t_{AQNSI}}{\lim_{t\to\infty} P^t_{AQGSI}} = \frac{\sum_{k=1}^K w_k \sum_{i=1}^N q_{i,k} \frac{v_{\sigma(i)} a_{i,k}}{\sum_{j=1}^N v_{\sigma(j)} a_{j,k} + z_k}}{\max_{1 \le i \le N} \sum_{k=1}^K w_k q_{i,k}}. \tag{9}$$

If the expression in 9 is smaller (greater) than 1 showing the popularity signal under the

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average quality ranking is beneficial (detrimental). Proposition 1 shows that under different model parameters, the ratio in Equation (9) could be equal to zero, K, or any number between them. This is important for the platform owner, and has to be decided particularly for each market settings, since social influence may hurt or benefit purchases in the long run.

5 Market Segmentation and its Benefits

In the previous section, we have shown a number of negative results for the MMNL model. In particular, we have shown that, in MMNL models, computing the performance ranking is intractable and that displaying the popularity signal to customers may significantly reduce the asymptotic market efficiency (i.e. the expected rate of purchases) of the average quality ranking. In this section, we show that the widely used marketing strategy known as *market segmentation* remedies these limitations, while retaining the original benefits of quality ranking for the Multinomial Logit Model.

The market segmentation considered here assumes that the firm has the ability to know the segment of each arriving consumer. This is a natural assumption in a number of online markets (e.g., Amazon, online retail stores, iTunes, Google Play and Netflix) where firms are able to learn information about their customers over time. Armed with this information, the firm will now propose item rankings dedicated to each customer segment. Moreover, and equally important, the popularity signal will be tailored to each segment. In other words, *the firm will only show the popularity signal derived from purchases of customers of the same segment as the incoming customer*, not the popularity obtained from the entire customer pool. As shown in Figure 1, websites such as `Booking.com` already give customers the option of selecting their peer groups to refine the site recommendations (although hotels bookings are not a trial-offer market, segmentation is an important factor in their revenue maximizing strategies). Under this new strategy where each consumer segment has its own quality ranking and observes the past purchases of its own segment only, the policy is called the *segmented quality ranking*. The firm uses K permutations $\sigma_k \in S_N(k \in [K])$, where σ_k sorts the products in decreasing order according to their quality for consumer segment k. In addition, the probability of trying item i for a customer of segment k is given by

$$p_{i,k}(\sigma, d_k^t)$$

where $d_k^t = (d_{1,k}^t, \ldots, d_{N,k}^t)$ and $d_{i,k}^t$ denotes the number of purchases of item i by customers from segment k up to time t.

We now study the benefits of this market segmentation. Observe first that each market segment can be viewed as evolving independently and hence directly inherits the original benefits identified for the quality ranking under the MNL model: The market share of the highest quality product converges asymptotically to 1. This observation will enable us to quantify the benefits of market segmentation. We begin by providing some key definitions that will be required later.

Definition 3. The segmented quality ranking policy $\sigma_k \in S_N$ ($k \in [K]$) is *tie-breaking* if, for each segment k, there exists a unique item i_k^* with the highest quality:

$$\forall k \in [K] \; \exists i_k^* \in [N] \; \forall j \in [N], j \neq i_k^* : \; q_{i_k^*,k} > q_{j,k}. \tag{10}$$

Assuming a MMNL model for which the average quality ranking and the segmented quality ranking are tie-breaking, we compare the probability of a purchase at time t in both settings. More precisely, we compare two quantities:

- P_{AQGSI}^t: the probability of a purchase at time t when the firm uses the average quality ranking and the "global" popularity signal d^t;

- P_{SQSSI}^t: the probability of a purchase at time t when the firm uses the segmented quality ranking with the consumer segment's popularity signal d_k^t.

The probabilities P_{AQGSI}^t and P_{SQSSI}^t concern the behavior of the consumer arriving to the market at time t independently from the customer class. Comparing P_{AQGSI}^t and P_{SQSSI}^t for any time t is a very challenging task. Instead, we compare both variables in the limit. The following theorem shows that the market segmentation strategy is always beneficial for the firm and the benefit it provides is upper bounded by a factor of K.

Theorem 4. *Assume that the average quality ranking and its segmented version are tie-breaking for a MMNL model. Then,*

$$1 \leq \lim_{t \to \infty} \frac{P_{SQSSI}^t}{P_{AQGSI}^t} \leq K. \tag{11}$$

The following proposition shows that there are settings in which the (multiplicative) benefits of market segmentation are indeed equal to the upper bound provided in Theorem 4.

Proposition 2. *The upper bound of Theorem 4 is tight.*

These results show that the segmented quality ranking always outperforms (or it is equal to) the average quality ranking in expectation, and that the improvement in market efficiency can be up to a factor of K. As it can be seen from the proof of Proposition 2, the market settings in which

the segmentation strategy beats by the most the global ranking is when each consumer segment $k \in \{1, \ldots, K\}$ of consumers have a single product with a non-zero quality and it is pairwise different between any two classes. Thus, the different segments of the market have very distinct preferences, which is the setting where the strategy of market segmentation is generally used in practice (Dickson & Ginter 1987). Proposition 2 means that there are settings under which a segmented ranking performs the same, or up to K times better than a single ranking. This imposes a maximum cost rate to what the platform owner may be willing to pay for information about consumer segments, and deploying a segmented ranking. It is important to remark that these results hold for the very long run (i.e. asymptotic in nature). They don't necessarily imply that the segmented quality ranking is always better, since the popularity signal is weaker early in the market evolution. This will be illustrated in the computational experiments presented in the next section.

It is important to remark that the results of this section can be extended to the case where there are classification mistakes while performing the segmentation policy. This situation is analyzed in the supplementary appendix A. We formulate the problem using a mistake probability matrix and show that the system converges to a monopoly, analogous to Theorem 2 but incorporating the values of the error probability matrix. Assuming that classification mistakes occur with an exogenous probability β_0 evenly distributed among all products, we find an analogous bound as Theorem 4, which becomes $K(1 - \beta_0)$. For an example, if we make classification errors 10% of the time with 5 consumer classes of equal weights, the maximum benefit of segmentation is $5(1 - 0.1) = 4.5$, while its maximum benefit is 5 with perfect classification. This extension generalizes our model to a more realistic setting where we can analyze the trade-off between segmenting the market and taking the risk of making classification errors versus being risk averse and showing an average quality ranking. We performed a numerical simulation to analyze the impact of having different mistake probabilities for the SQSSI policy and we find that the average quality ranking AQGSI outperforms the SQSSI policy when $\beta_0 > 10\%$ for a parameter set (see Figure 13 in Appendix A).

We end this section with two additional comparisons. First, we compare how the average quality ranking without social influence (AQNSI) performs in comparison to the segmented quality segmented social influence ranking (SQSSI). It is no surprise that SQSSI outperforms AQNSI as the number of purchases goes to infinity: this is because under SQSSI, in the limit, each consumer segment will try the product that maximizes its purchase probability. Finally, we analyze how SQSSI performs in comparison to its counterpart without social influence (SQNSI). Again here,

it should be no surprise that SQSSI outpeforms SQNSI. Both of these results are shown in the following theorem.

Theorem 5. *The ratio between the asymptotic purchase probability of the average quality or the segmented quality ranking without social influence (AQNSI or SQNSI), and its segmented quality ranking with segmented social influence (SQSSI) is always less than 1,*

$$\frac{\lim_{t \to \infty} P_{AQNSI}^t}{\lim_{t \to \infty} P_{SQSSI}^t} \leq 1 \ \text{and} \ \frac{\lim_{t \to \infty} P_{SQNSI}^t}{\lim_{t \to \infty} P_{SQSSI}^t} \leq 1.$$

We have shown that SQSSI outperforms AQGSI, AQNSI and SQNSI, and that AQGSI can outperform or underperform AQNSI. Figure 2 shows a pictogram with the ranking policies studied.
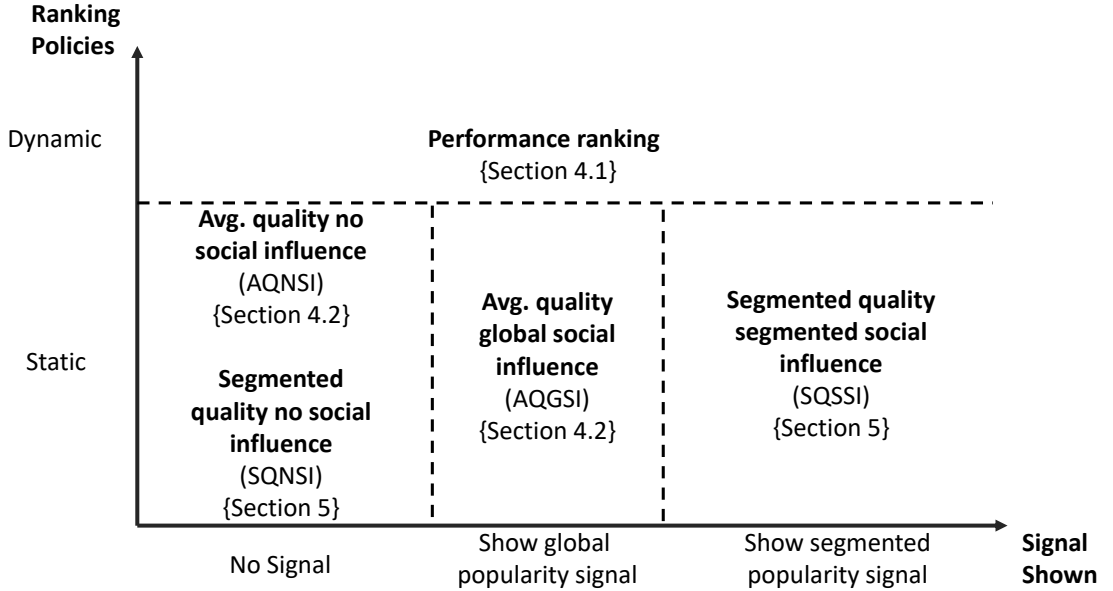
Figure 2: Pictogram of the ranking policies studied and where to find them.

6 Computational Experiments

This section presents the results of computational experiments to illustrate the theoretical results and complement them by depicting how the markets evolve over time for different types of rankings.

6.1 The Experimental Setting

The Agent-Based Simulation The experimental setting uses an agent-based simulation to emulate the MusicLab (Salganik et al. 2006). It generalizes prior results which simulated the MusicLab through the use of a MNL model (e.g., (Krumme et al. 2012, Abeliuk et al. 2015)) to a

MMNL model. Each simulation consists of T iterations and, at each iteration t $(1 \leq t \leq T)$, the simulator

1. randomly selects a customer segment k according to the classes weights w_k;

2. randomly selects an item i for the incoming customer according to the probabilities $p_{i,k}(\sigma, d)$, where σ is the ranking proposed by the policy under evaluation and d is the popularity signal;

3. randomly determines, with probability $q_{i,k}$, whether the selected item i is purchased. In the case of a purchase, the simulator increases the popularity signal for item i, i.e., $d_{i,t+1} = d_{i,t} + 1$. Otherwise, $d_{i,t+1} = d_{i,t}$.

The experimental setting aims at being close to the MusicLab experiments and it considers 50 items and simulations with $T = 200,000$ steps. The reported results in the graphs are the average of 400 simulations. The analysis in Krumme et al. (2012) indicated that participants are more likely to sample products with better ranking positions. More precisely, the visibility decreases with the ranking position, except for a slight increase at the bottom positions. To have a fair comparison between the settings with and without social influence we set $z = 0$ for all simulations, in that way the fraction of customers choosing the outside option does not change over time.

Qualities and Appeals To highlight and complement the theoretical results, we consider four different schemes, the schemes share the following characteristics: They have two customer classes with the same weight and they use 50 products. They differ in how the values for the item appeals and qualities are chosen. The schemes are depicted visually in Figures 3 and 4 and were obtained as follows:

1. Scheme 1: The product qualities for each consumer segment were chosen randomly with a standard uniform distribution ($q_{i,1}$ and $q_{i,2}$ are independent for all $i \in [N]$). Appeals were negatively correlated with quality, i.e., $a_{i,k} = 1 - q_{i,k}$ for all $i \in [N]$.

2. Scheme 2: Product qualities are similar to Scheme 1. Appeal vectors are now correlated with the quality vectors. More precisely, the appeal vector for each consumer segment was set to 0.8 times the quality plus a random uniform vector between -0.4 and 0.4, i.e., $a_{i,k} = q_{i,k}(0.8 + 0.4 * \epsilon_i)$ for all $i \in [N]$, where ϵ_i is a standard uniform random variable.

3. Scheme 3: The product quality for segment 1 is a random vector, while $q_{i,2} = 1 - q_{i,1} + 0.01 * \epsilon_i$ for all $i \in [N]$, where ϵ_i is a standard uniform random variable. Appeals are negatively correlated with quality, i.e., $a_{i,k} = 1 - q_{i,k}$ for all $i \in [N]$.
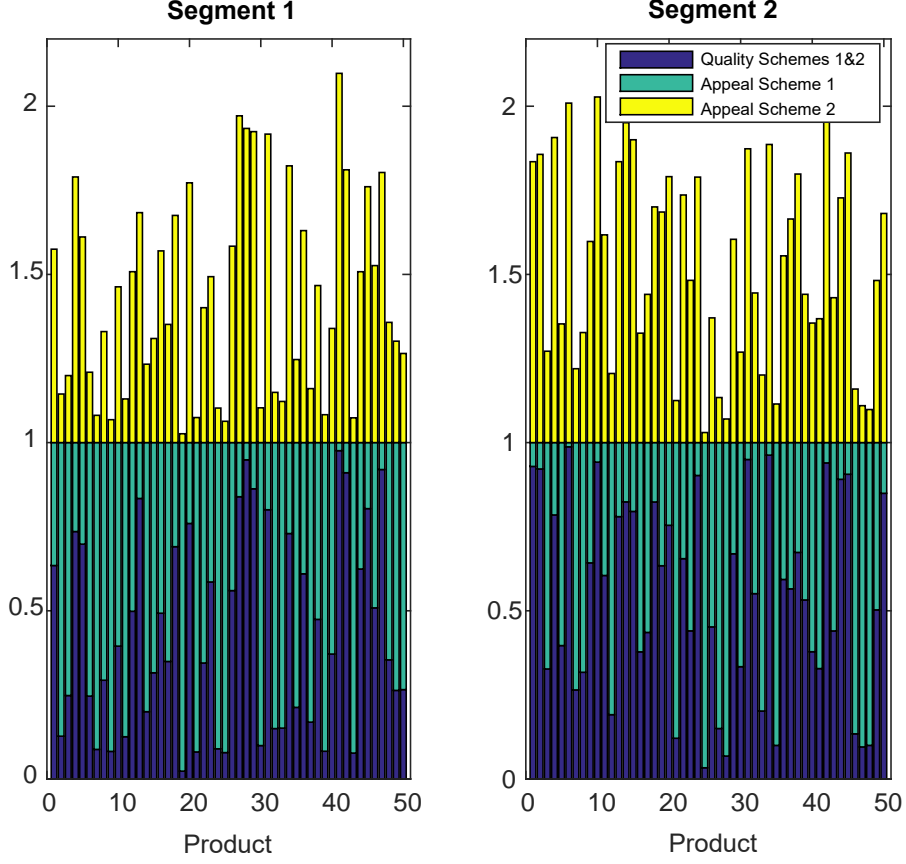
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Figure 3: The quality q_i (blue) and appeal A_i (green and yellow) of product i for settings 1 and 2 for both classes of consumers. The settings only differ in the appeal of items, and not in the quality of items. In Setting 1, the appeals are negatively correlated with quality, so that the sum between them is always 1. In Setting 2, the appeal is correlated to the quality with a small noise.

4. Scheme 4: The product qualities are the same as in Scheme 3 but the appeals are correlated with qualities, $a_{i,k} = q_{i,k}(0.8 + 0.4 \, rand(1, 50))$ for all $i \in [N]$.

Observe that, in Schemes 3 and 4, customers in the two classes associate fundamentally different qualities with the products. For the simulations, the appeals vector were multiplied by a factor of 200.

The Policies The simulations compare the average and segmented quality rankings with and without the popularity signal. We use the following notations:

- SQSSI: Segmented quality ranking with segmented popularity signal;

- SQNSI: Segmented quality ranking without popularity signal;

- AQGSI: Average quality ranking with global popularity signal;

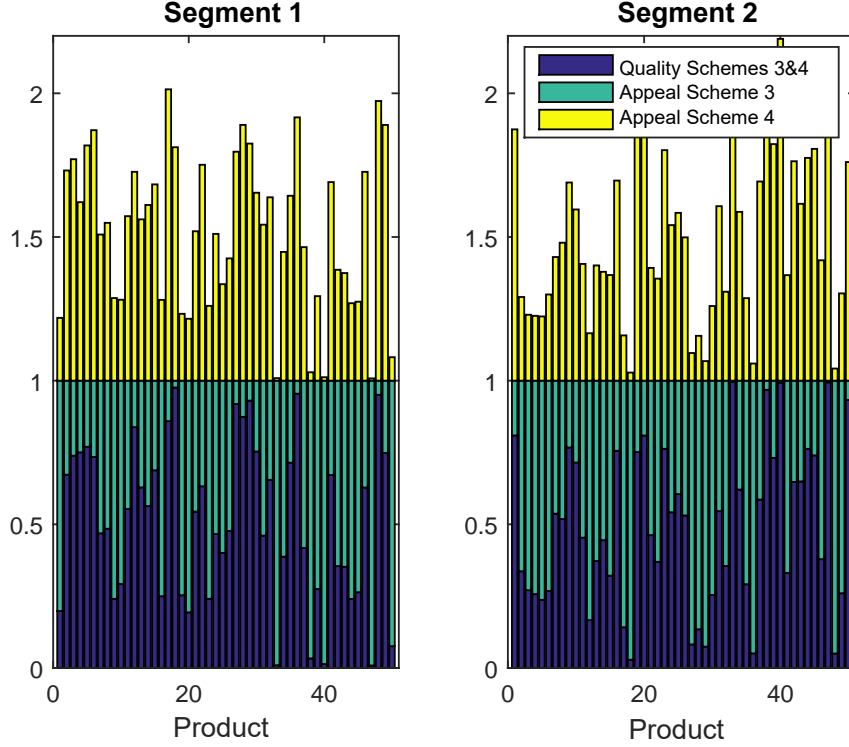- AQNSI: Average quality ranking without popularity signal.

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Figure 4: The quality q_i (blue) and appeal A_i (green and yellow) of product i in settings 3 and 4 for both classes of consumers. The settings only differ in the appeal of items, and not in the quality of items. In Setting 3, the appeals are negatively correlated with quality, so that the sum between them is always 1. In Setting 4, the appeal is correlated to the quality with a small noise. All appeals were then multiplied by a factor of 200 to use in the model.

6.2 Market Efficiency

Figure 5 depicts the results for Schemes 1 and 2. For Scheme 1, the popularity signal is beneficial for both the segmented and average quality rankings. SQSSI is the most efficient ranking policy. It is also interesting to observe that AQGSI outperforms SQSSI early on before being overtaken as highlighted in Figure 6. Scheme 2 exhibits similar results but the benefit of the popularity signal is lower.

Figure 7 depicts the results for Schemes 3 and 4 and they are particularly interesting. Recall that, in Schemes 3 and 4, the two classes of customers have opposite preferences in terms of product qualities. For Scheme 3, the popularity signal is again beneficial for the segmented and average quality rankings. SQSSI is again the best ranking policy, and particularly is almost twice as efficient than AQGSI, nicely illustrating Theorem 4, since the improvement is close to the best possible ratio. Once again, AQNSI performs the worst. For Scheme 4, SQSSI is again the best ranking policy but the second best policy is SQNSI, the segmented quality ranking with no popularity signal. The worst policy is AQGSI, providing a compelling illustration of Theorem 3:
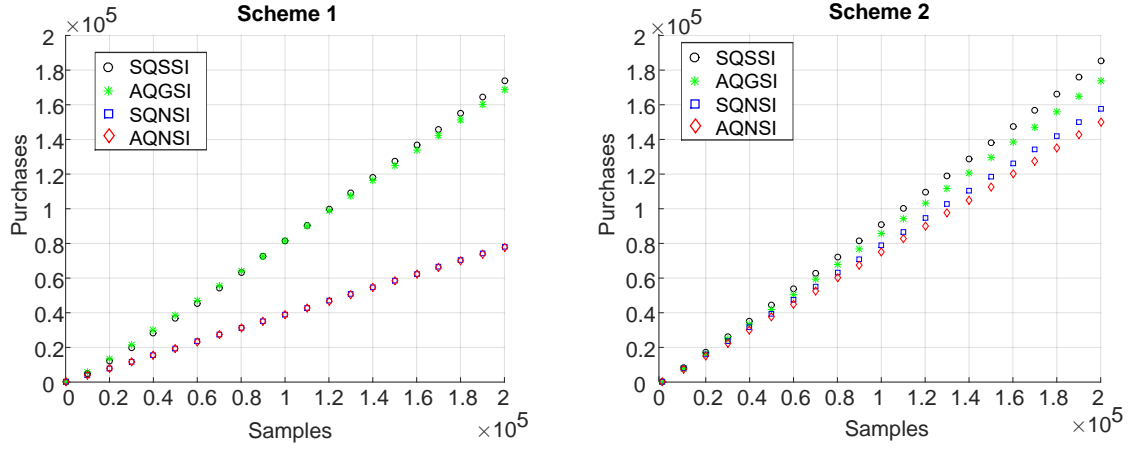
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Figure 5: The Number of Purchases over Time for the Various Rankings. The x-axis represents the number of items tried and the y-axis represents the average number of purchases over all experiments. The left figure depicts the results for Scheme 1 and the right figure for Scheme 2.
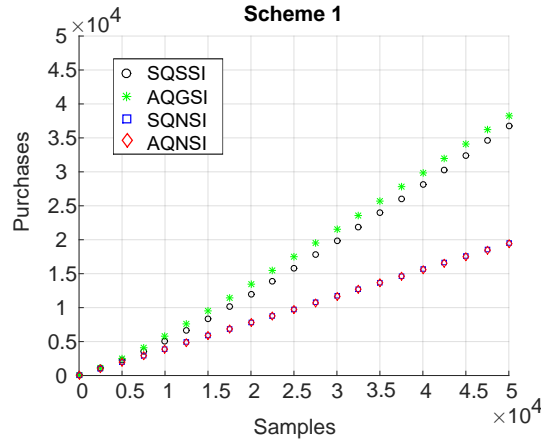

Figure 6: The number of purchases over time for the various rankings. The x-axis represents the number of items tried and the y-axis represents the average number of purchases over all experiments. The figure depicts the results for Scheme 1 in the early part of the simulation.

The popularity signal may be detrimental to the average quality ranking.

These results can be summarized as follows:

1. SQSSI (segmentation with the popularity signal) is clearly the best policy and it dominates all other policies. Market segmentation with the popularity signal is very effective in these trial-offer markets.

2. The global popularity signal may be beneficial or detrimental to the average quality ranking. It is detrimental when the market has customers with very different product preferences.
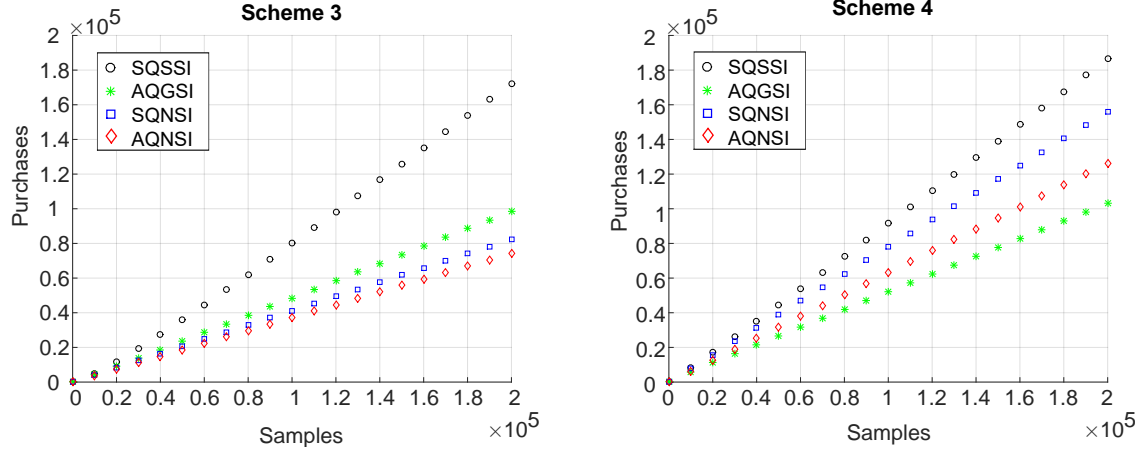
Figure 7: The number of purchases over Time for the various rankings. The x-axis represents the number of items tried and the y-axis represents the average number of purchases over all experiments. The left figure depicts the results for Scheme 3 and the right figure for Scheme 4.
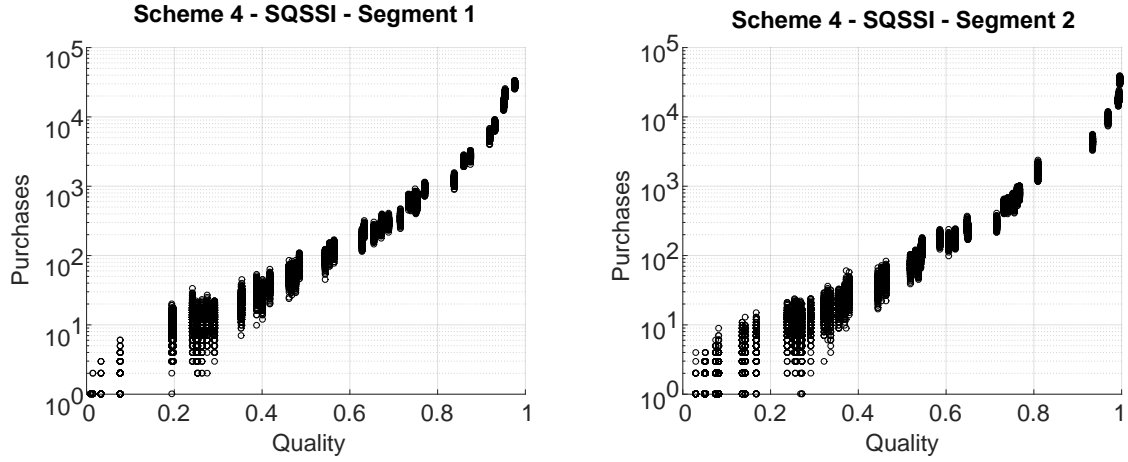

Figure 8: The purchase profiles of SQSSI on Scheme 4 for consumer segments 1 (left) and 2 (right).

6.3 Purchase Profiles

We now illustrate the customer and market behaviors for the SQSSI and AQGSI rankings, which exhibit some significant differences. For Scheme 4, the results are presented in Figures 8, 9, and 10. Figure 8 depicts separately the purchase profiles of customers of segments 1 and 2 for policy SQSSI. The products are sorted by increasing quality for each segment: i.e., the products of highest quality for customers of segment 1 (resp. segment 2) is in the rightmost position in the left (resp. right) picture. Since the market is segmented, the results are not surprising and consistent with past results: The number of purchases is strongly correlated with quality. Figure 9 is more interesting and depicts the same information for policy AQGSI. Here the number of purchases is no longer correlated with quality for a specific customer segment. Figure 10 compares SQSSI and
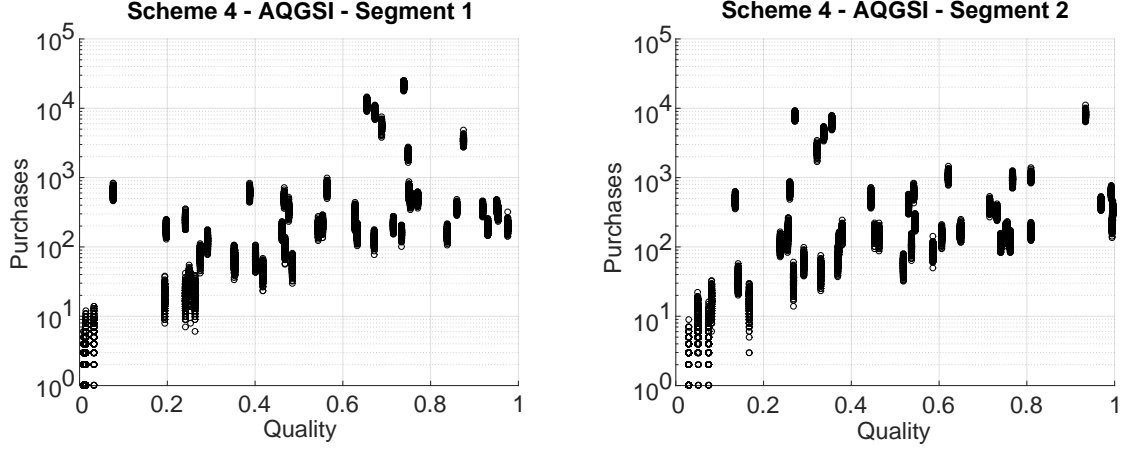
25

Figure 9: The purchase profiles of AQGSI on Scheme 4 for consumer segments 1 (left) and 2 (right).
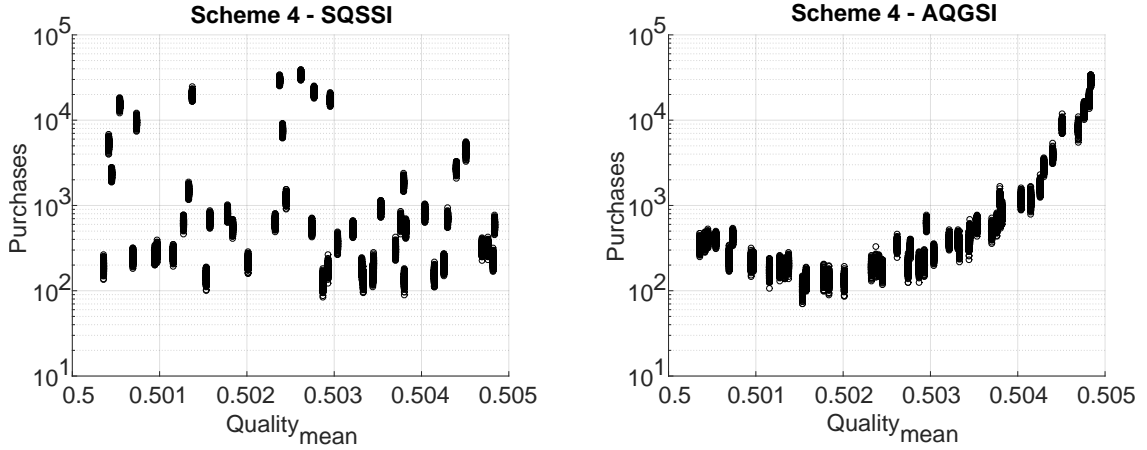

Figure 10: The purchase profiles of SQSSI and AQGSI on Scheme 4 for both segments of customers.

AQGSI over all customers and the products are sorted by average quality. The figure highlights a fundamental difference in market behavior between the two policies, with very different products emerging as the "best sellers".

Schemes 3 and 4 feature customer classes with opposite preferences. It is thus interesting to report the results on Scheme 2 where the product qualities were generated independently for the two classes. Figure 11 depicts these results. We already know from Figure 5 that policy SQSSI outperforms AQGSI but it is interesting to see how different the market behaves under these two policies. For AQGSI, as expected, the products of best average quality receives the most purchases: Asymptotically the market goes to a monopoly for that product. For SQSSI, the purchases at this stage of the market are distributed through a larger number of products, each of which have fewer purchases. Asymptotically, the market will go to a monopoly for two products (one for segment 1
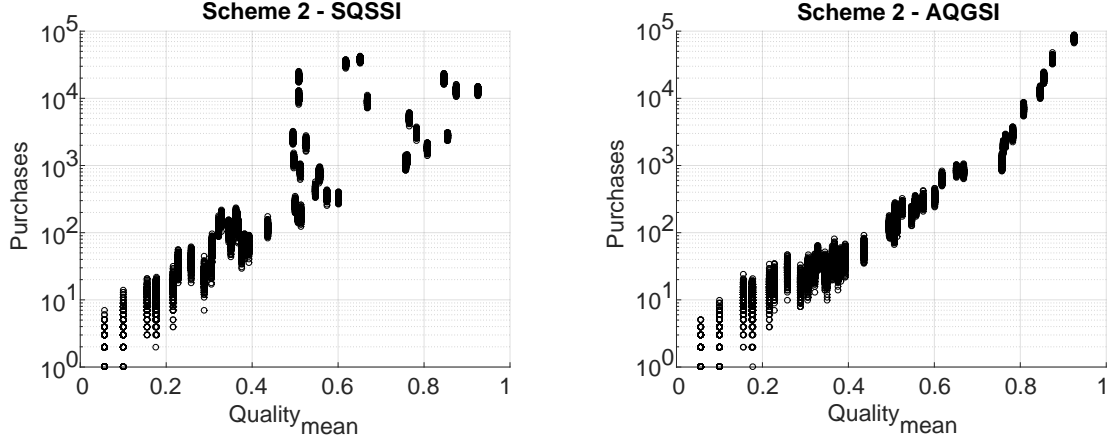
Figure 11: The purchase profiles of SQSSI and AQGSI on Scheme 2 for both Classes of Customers.
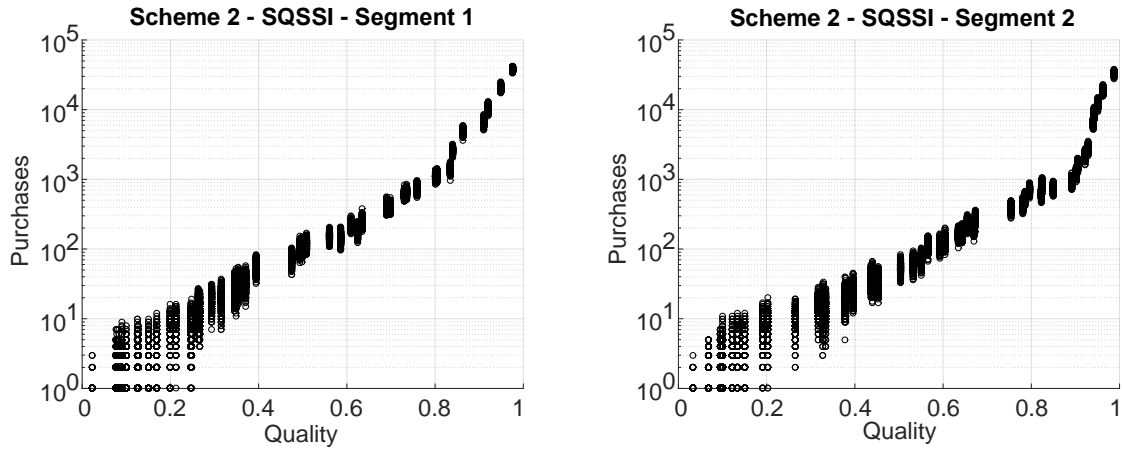

Figure 12: The purchase profiles of SQSSI on Scheme 2 for consumer segments 1 (left) and 2 (right).

and one for segment 2) but the popularity signal is weaker for SQSSI since it is spread across the two classes. It is interesting to observe that the segmentation policy SQSSI is still more efficient than policy AQGSI despite this weaker popularity signal. Figure 12 depicts the profiles for policy SQSSI and nicely highlights that many products are receiving significant purchases.

We finish this section with a set of managerial insights that can be observed from the theoretical results and the computational experiments. First, whenever consumer segment information is available and the goal is to maximize long term purchases, it is optimal to segment the consumers and to rank the products within each segment according to their quality, while using a social signal that is only shown across consumers within the same segment (Theorems 3 and 5). The first theorem shows that a segmented quality ranking with social influence within each segment (SQSSI) always outperforms the unique average quality ranking with the same social signal across all consumer segments (AQGSI). The second theorem shows that using social influence with a

segmented quality ranking (SQSSI) always outperforms doing a segmented quality ranking without social influence (SQNSI). In the computational experiments we observe similar results. However, the platform owner may be better off in the short term using a unique average quality ranking with a global social signal (AQGSI) (Scheme 1, Figure 6). This is because the aggregate ranking AQGSI enhances the social signal instead of distributing it across different segments as seen with the segmented ranking SQSSI. But in the long run the segmented ranking is optimal whereas the aggregate ranking may lead to a monopoly of suboptimal products. Finally, when consumer segment information is not available, it is not always beneficial to show social influence. This is proved in Theorem 3 and nicely illustrated with the experiments, where the average quality ranking with a social signal (AQGSI) outperforms its counter-part without a social signal (AQNSI) in all schemes except for Scheme 4 (when consumer segments exhibit opposite preferences, and product appeals are positively correlated with qualities). On aggregate rankings, the platform owner should use social influence carefully, and mainly when consumer segments have similar product preferences, as otherwise social influence may confuse consumers on which products to try and reduce the rate of purchases.

7 Conclusions and Future Research

In this paper, we studied a trial-offer market where consumers choices about which products to try are affected by the display of past product purchases and by product positions. Specifically, we focused on studying the case in which consumer choices follow a mixed multinomial logit model (MMNL), which generalizes the multinomial logit model proposed by Krumme et al. (2012) to explain the behavior in an online cultural market (Salganik et al. 2006). Unlike the case for the Multinomial Logit, we showed that finding the best way to rank products at every step in order to maximize the purchases is a computationally hard problem.

The paper then studied the performance of a ranking policy (AQGSI) which ranks the products by the average quality (in decreasing order). Under such policy, we proved that the trial-offer market in the long run converges predictably to a monopoly by transforming the MMNL model into a traditional MNL model whose appeals and qualities depends on the popularity signal at each time step but are bounded from below and above. Unfortunately, this average quality ranking policy is no longer guaranteed to benefit from the popularity signal in all cases. In other words, the rate of product purchases can sometimes be reduced as time passes due to the fact that consumers are able to observe (and their decisions are affected by) the number past purchases for each product. This is in sharp contrast to the case where there is a unique consumer segment

($K = 1$) (Van Hentenryck et al. 2016a).

The paper also studied a market segmentation policy for settings in which the firm is capable of detecting the consumer segment in advance. Under the segmentation policy, the products are presented to each consumer according to the quality ranking for their own class. In addition, the popularity signal displayed only aggregates the past purchases associated to the customers of the same class. The resulting policy (SQSSI) is optimal asymptotically in expectation and may improve the market efficiency up to a factor K over AQGSI, where K is the number of customer classes.

Computational experiments have been presented to illustrate our theoretical results and to show the market dynamics in the short term. These experiments, which were carried over four different settings, showed that there are settings in which the popularity signal is indeed detrimental to policy AQGSI and that the segmentation strategy produces the best possible improvement predicted by the theory.

Overall, the paper shows that, in trial offer markets, the decision to display or not the popularity signal to consumers has to be analyzed very carefully. In markets where consumers have very different product preferences, we showed that the display of past purchases can be detrimental to the rate of purchases. Intuitively, this is because consumers watching past purchases become confused about which products to try. On the other hand, in markets where the preferences of the different consumer classes are not very distinct, the display of aggregated past purchases is beneficial. Nevertheless, our theoretical (asymptotic) results and our short-term simulation results indicate that the segmentation policy SQSSI outperforms all other policies. Moreover, these results highlight the fact that AQGSI and SQSSI produce very different market behavior, even in settings where the overall market efficiency is relatively close.

The paper leaves some interesting questions for future research. The first is to study the market-share dynamics of the different ranking policies for more complex consumer choice models. One weakness of the current model is that the consumer choice model only depends on the displayed vector of past purchases and the current ranking (as well as the appeal of the products). However, a more sophisticated choice model could incorporate into the consumer choice, the type of past purchase information displayed: A consumer who observes a past purchase vector d might change the behavior depending on whether she/he knows that the vector d comes from all consumer purchases or if only comes from consumers of it own type or class. The second one is to extend the model to scenarios when the trial probabilities depend non-linearly on the past purchases, this is studied in Maldonado et al. (2018) for the special case of a unique consumer

segment ($K = 1$). Another interesting research direction is to study the firm potential incentives to hide or mis-report some reviews and how it this affects the outcomes in terms of market share dynamics and consumer welfare.

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Appendix: Proofs

Proof of Theorem 1. The proof uses the 2-Class Logit problem which is known to be NP-hard (Rusmevichientong et al. 2014). The inputs to a 2-Class Logit instance are N products, two sequences $V^1 = (V_1^1, V_2^1, \ldots, V_N^1)$ and $V^2 = (V_1^2, V_2^2, \ldots, V_N^2)$ with $V^1, V^2 \in \mathbb{Q}_+^N$, and a number $\alpha \in \mathbb{R}_{[0,1]}$. Each product i has a revenue $r_i \in \mathbb{Z}_+$. Each sequence V^i represents a realization of the product utilities under a multinomial logit model. Sequence V^1 (resp. V^2) has a realization probability of α (resp. $1 - \alpha$). The problem consists in finding a product assortment $S \subseteq [N]$ maximizing the expected revenue Π^{Logit}, i.e.,

$$\Pi^{Logit} = \max_{S \subseteq [N]} \alpha_1 \frac{\sum_{i \in S} r_i V_i^1}{1 + \sum_{i \in S} V_i^1} + (1 - \alpha) \frac{\sum_{i \in S} r_i V_i^2}{1 + \sum_{i \in S} V_i^2}.$$

The proof shows that, if there exists an oracle to compute the performance ranking for the MMNL with two classes of consumers (i.e., Equation (2) with $K = 2$), then the 2-Class logit problem can be solved in polynomial time.

Given an instance of the 2-Class Logit problem, the idea is to create N different instances of the performance-ranking problem in order to capture the various possible assortments. The N instances have a common core. Each of them has the same N items and two classes of consumers (i.e., $K = 2$). For each consumer segment $j \in \{0, 1\}$ and each item $i \in [N]$, we set the appeal of item i for segment j to satisfy $a_{i,j} = V_i^j$. Similarly, for each consumer segment $j \in \{0, 1\}$ and each item $i \in [N]$, we set the quality of i for segment j to satisfy $q_{i,j} = r_i$. Note that the quality of item i is the same for both classes. The weights of classes 0 and 1 are α and $1 - \alpha$ respectively. We also set $z = 1$ and $t = 0$ which implies that $d^t = 0$. The N instances differ in the position visibilities. In instance i ($i \in [N]$), the visibility of position $j \in [N]$ is:

$$
v_j = \begin{cases} 1 & \text{if } j \leq i \\ 0 & \text{otherwise.} \end{cases}
$$

Let Π_i^{PR} denote the short-term optimal value of the performance ranking for problem instance i and let $\mathcal{S}_i$ denote the collection of all possible subsets of products whose size is i, i.e., $\mathcal{S}_i = \{S \subseteq [N] : |S| = i\}$. Define Π_i^{Logit} as the following optimization problem:

$$
\Pi_i^{Logit} = \max_{S \in \mathcal{S}_i} \alpha_1 \frac{\sum_{i \in S} r_i V_i^1}{1 + \sum_{i \in S} V_i^1} + (1 - \alpha) \frac{\sum_{i \in S} r_i V_i^2}{1 + \sum_{i \in S} V_i^2}.
$$

It follows that

$$
\Pi^{Logit} = \max_{i=1,\dots,N} \Pi_i^{Logit}. \tag{12}
$$

We now show that Π_i^{PR} is equal to Π_i^{Logit}.

$$\Pi_i^{PR} = \max_{\sigma \in S_n} \left\{ \sum_{c=1}^{2} \left(w_c \cdot \sum_{\ell=1}^{N} (p_i(\sigma, 0) \cdot q_{\ell,c}) \right) \right\}, \tag{13}$$

$$= \max_{\sigma \in S_n} \left\{ \sum_{c=1}^{2} \left(w_c \cdot \sum_{\ell=1}^{N} \left(\frac{v_{\sigma(\ell)}(a_{\ell,k})}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,c}) + 1} \cdot q_{\ell,k} \right) \right) \right\}. \tag{14}$$

$$= \max_{\sigma \in S_n} \left\{ \alpha \cdot \sum_{\ell=1}^{N} \left(\frac{v_{\sigma(\ell)} V_\ell^1 r_\ell}{\sum_{j=1}^{N} v_{\sigma(j)} V_j^1 + 1} \right) + (1-\alpha) \cdot \sum_{\ell=1}^{N} \left(\frac{v_{\sigma(\ell)} V_\ell^2 r_\ell}{\sum_{j=1}^{N} v_{\sigma(j)} V_j^2 + 1} \right) \right\} \tag{15}$$

$$= \max_{S \in \mathcal{S}_i} \left\{ \alpha \cdot \sum_{\ell \in S} \left(\frac{V_\ell^1 r_\ell}{\sum_{j \in S} V_j^1 + 1} \right) + (1-\alpha) \cdot \sum_{\ell \in S} \left(\frac{V_\ell^2 r_\ell}{\sum_{j \in S} V_j^2 + 1} \right) \right\} \tag{16}$$

$$= \Pi_i^{Logit} \tag{17}$$

where the equivalence between (15) and (16) follows from the fact that the first i positions have visibility of 1 and the remaining ones have a visibility of 0 and therefore selecting a permutation $\sigma \in S_n$ reduces to deciding which i items should be assigned the top i positions. As a consequence, using (12), we have

$$\Pi^{Logit} = \max_{i=1,\ldots,N} \Pi_i^{Logit} = \max_{i=1,\ldots,N} \Pi_i^{PR}. \tag{18}$$

We have shown that, by using an oracle to solve N instances of the performance-ranking problem, it is possible to solve the original 2-class logit problem instance in polynomial time. Hence, the performance ranking is NP-hard under Turing reductions.

$\square$

Proof of Lemma 1. The market share of item i^* at any period of time $t > \hat{t}$ for this system would be underestimated by considering the following set of qualities and appeals:

$$q_{i,new} = \begin{cases} q_{i,min} & \text{if } i = i^* \\ q_{i,max} & \text{if } i \neq i^* \end{cases} \text{ and } a_{i,new} = \begin{cases} a_{i,min} & \text{if } i = i^* \\ a_{i,max} & \text{if } i \neq i^* \end{cases}.$$

If this new set of qualities satisfies that $v_{\sigma(i^*)} q_{i^*,new} > v_{\sigma(i)} q_{i,new}$ for all $i \in [N] \setminus \{i^*\}$, it follows from the convergence result in (Van Hentenryck et al. 2016a) (Theorem 4.3) that the system goes to a monopoly for item i^*. Therefore, the original system also goes to a monopoly for item i^*. $\square$

Proof of Theorem 2. The proof first shows that the MMNL model can be reduced to a Multinomial Logit Model whose item appeals and qualities are functions of the vector of purchases at each time t. It then shows that these functions stay in the bounded range, so that it is possible to apply

Lemma 1.

When the same ranking σ and popularity signals are shown to all consumers, the probability that item i is purchased in time period t is given by

$$P_i(\sigma, d^t) = \sum_{k=1}^{K} \left(w_k \cdot \left(v_{\sigma(i)} \frac{(a_{i,k} + d_i^t)}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \cdot q_{i,k} \right) \right). \tag{19}$$

By rearranging the previous expression, it comes

$$
\begin{aligned}
P_i(\sigma, d^t) &= \sum_{k=1}^{K} \frac{w_k q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} a_{i,k} + \sum_{k=1}^{K} \frac{w_k \ q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} d_i^t \\
&= \sum_{k=1}^{K} \frac{w_k q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} a_{i,k} + d_i^t \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \\
&= \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \left(\frac{\left(\sum_{k=1}^{K} \frac{w_k q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} a_{i,k} \right)}{\left(\sum_{k=1}^{K} \frac{w_k q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right)} + d_i^t \right) \\
&= \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \left(\frac{v_{\sigma(i)} \left(\sum_{k=1}^{K} \frac{w_k q_{i,k} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right)}{v_{\sigma(i)} \left(\sum_{k=1}^{K} \frac{w_k q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right)} + d_i^t \right) \\
&= \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \left(\frac{\left(\sum_{k=1}^{K} \frac{w_k q_{i,k} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right)}{\left(\sum_{k=1}^{K} \frac{w_k q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right)} + d_i^t \right).
\end{aligned}
$$

Now, for each item i and each time period t, define the function

$$\widetilde{a}_i(t) = \left(\sum_{k=1}^{K} \frac{w_k q_{i,k} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \Big/ \left(\sum_{k=1}^{K} \frac{w_k q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \tag{20}$$

which depends on the total number of purchases at time t. Using this definition, we have that:

$$P_i(\sigma, d^t) = \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k} v_{\sigma(i)}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \left(\widetilde{a}_i(t) + d_i^t \right) = \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) v_{\sigma(i)} \left(\widetilde{a}_i(t) + d_i^t \right).$$

By dividing and multiplying by $\sum_{j=1}^{N} v_{\sigma(j)}(\widetilde{a}_j(t) + d_j^t)$, $P_i(\sigma, d^t)$ becomes

$$\left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \right) \left(\sum_{j=1}^{N} v_{\sigma(j)}(\widetilde{a}_j(t) + d_j^t) \right) \left(\frac{v_{\sigma(i)} \left(\widetilde{a}_i(t) + d_i^t \right)}{\sum_{j=1}^{N} v_{\sigma(j)}(\widetilde{a}_j(t) + d_j^t)} \right).$$

Now define the following function for each item i at each time period t:

$$\widetilde{q}_i(t) = \left(\sum_{k=1}^{K} \frac{w_k \ q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k}\right) \left(\sum_{j=1}^{N} v_{\sigma(j)}(\widetilde{a}_j(t) + d_j^t)\right). \tag{21}$$

The probability of purchasing product i in the next iteration becomes:

$$P_i(\sigma, d^t) = \left(\frac{v_{\sigma(i)}(\widetilde{a}_i(t) + d_i^t)}{\sum_{j=1}^{N} v_{\sigma(j)}(\widetilde{a}_j(t) + d_j^t)}\right) \widetilde{q}_i(t).$$

This is almost a multinomial logit model, except that the quality and appeal vectors that depend on time. When the number of iterations t tends to infinity, the total number of purchases $\sum_{j=1}^{N} d_j^t$ also goes to infinity. Moreover, as t goes to infinity, the generalized appeal $(\widetilde{a}_i(t))$ and quality $(\widetilde{q}_i(t))$ for every item converges to

$$\bar{a}_i \doteq \lim_{t\to\infty} \widetilde{a}_i(t) = \frac{\sum_{k=1}^{K} w_k a_{i,k} q_{i,k}}{\sum_{k=1}^{K} w_k q_{i,k}} \quad \text{and} \quad \bar{q}_i \doteq \lim_{t\to\infty} \widetilde{q}_i(t) = \sum_{k=1}^{K} w_k q_{i,k}. \tag{22}$$

In addition, observe that $\widetilde{Q}_i(t) \doteq v_{\sigma(i)}\widetilde{q}_i(t)$ also converges when t goes to infinity:

$$\bar{Q}_i \doteq \lim_{t\to\infty} v_{\sigma(i)}\widetilde{q}_i(t) = v_{\sigma(i)}\bar{q}_i$$

The tie-breaking condition (Equation 8) guarantees that there exists only one item i^* such that $i^* = \text{argmax}_{i \in [N]} \bar{Q}_i$. Let i^{**} be the item with the second highest value $\bar{Q}_i$, i.e., $\bar{Q}_{i^{**}} \geq \bar{Q}_j$ for all $j \in [N], j \neq i^*$. Consider now the following difference $\Delta\bar{Q} = \bar{Q}_{i^*} - \bar{Q}_{i^{**}}$. Equation (20) can be seen as a weighted average on k for $a_{i,k}$ and hence

$$\min_{1\leq k\leq K} a_{i,k} \leq \widetilde{a}_i(t) \leq \max_{1\leq k\leq K} a_{i,k} \quad \forall i \in [N], t \in \mathbb{N} \tag{23}$$

Moreover, by applying this result to Equation (21), we obtain the following bounds for $\widetilde{q}_i$:

$$\widetilde{q}_i(t) \geq \sum_{k=1}^{K} \frac{w_k q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)}(\max_{1 \leq k \leq K} a_{j,k} + d_j^t) + z_k} \left(\sum_{j=1}^{N} v_{\sigma(j)}(-\max_{1 \leq k \leq K} a_{j,k} + \max_{1 \leq k \leq K} a_{j,k} + d_j^t) - z_k + z_k \right)$$

$$= \sum_{k=1}^{K} \frac{w_k q_{i,k}\left(\sum_{j=1}^{N} v_{\sigma(j)}(\max_{1 \leq k \leq K} a_{j,k} + d_j^t) + z_k\right) - w_k q_{i,k} \sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k} - w_k q_{i,k} z_k}{\sum_{j=1}^{N} v_{\sigma(j)}(\max_{1 \leq k \leq K} a_{j,k} + d_j^t) + z_k}$$

$$\geq \left(1 - \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k} + \max_{1 \leq k \leq K} z_k}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^t} \right) \sum_{k=1}^{K} w_k q_{i,k}$$

$$= \left(1 - \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k} + \max_{1 \leq k \leq K} z_k}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^t} \right) \bar{q}_i \text{ and}$$

$$\widetilde{q}_i(t) \leq \sum_{k=1}^{K} \frac{w_k q_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^t} \left(\sum_{j=1}^{N} v_{\sigma(j)}(\max_{1 \leq k \leq K} a_{i,k} + d_j^t) \right) \leq \left(1 + \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k}}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^t} \right) \bar{q}_i.$$

As a result, the bounds for $\widetilde{Q}_i(t)$ ($\forall i \in [1, N], t \in \mathbb{N}$) are given by

$$\left(1 + \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k}}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^t} \right) \bar{Q}_i \geq \widetilde{Q}_i(t) \geq \left(1 - \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k} + \max_{1 \leq k \leq K} z_k}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^t} \right) \bar{Q}_i.$$

Notice that these bounds converge to $\bar{Q}_i$, so they could be arbitrary close to $\bar{Q}_i$ by choosing a sufficiently large d^t vector.

To conclude the proof, we need to estimate the total number of purchases $\hat{d}_{tot}$ that guarantees that

$$\forall t > t^* : \widetilde{Q}_{i^*}(t) > \widetilde{Q}_{i^{**}(t)}$$

where t^* is the time period in which the total number of purchases becomes $\hat{d}_{tot} = \sum_{i \in [N]} d_i^{t^*}$.
The value $\hat{d}_{tot}$ and its associated vector of purchases d^{t^*} must satisfy the following condition

$$\Delta \bar{Q} > \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k}(\bar{Q}_{i^*} + \bar{Q}_{i^{**}}) + \bar{Q}_{i^*} \max_{1 \leq k \leq K} z_k}{\sum_{j=1}^{N} v_{\sigma(j)} d_j^{t^*}}. \tag{24}$$

To verify inequality (24), it suffices to choose $\hat{d}_{tot}$ to satisfy

$$\hat{d}_{tot} > \frac{\sum_{j=1}^{N} v_{\sigma(j)} \max_{1 \leq k \leq K} a_{j,k}(\bar{Q}_{i^*} + \bar{Q}_{i^{**}}) + \bar{Q}_{i^*} \max_{1 \leq k \leq K} z_k}{\max_j v_{\sigma(j)} \Delta \bar{Q}}. \tag{25}$$

Since $\hat{d}_{tot}$ can be as large as desired, with the previous condition we guarantee the validity of Eq. (24).

We have just shown that the conditions of Lemma 1 are satisfied, we can now apply it using ranking policy σ to prove that the model goes to a monopoly for item i^*, which maximizes the product of its visibility and its weighted average quality, i.e., $v_{\sigma(i)}\bar{q}_i$. $\square$

Proof of Corollary 1. From Theorem 2, a MMNL model goes to a monopoly for the item i that maximizes $v_{\sigma(i)}\bar{q}_i$. When the quality ranking is used, the product i^* that goes to a monopoly is

$$i^* = \operatorname*{argmax}_i v_{\sigma(i)}\bar{q}_i = \operatorname*{argmax}_i (\bar{q}_i).$$

$\square$

Proof of Theorem 3. To prove this result, we need the following property. Let $A \in \mathbb{R}_{\geq 0}^{N \times K}$, then

$$\sum_{k=1}^{K} \max_{1 \leq i \leq N} a_{i,k} \leq K \max_{1 \leq i \leq N} \sum_{k=1}^{K} a_{i,k} \tag{26}$$

where $a_{i,k} \in A$. Its proof follows from the following argument; $K \max_{1 \leq i \leq N} \sum_{k=1}^{K} a_{ik} = K \max_{1 \leq i \leq N} \sum_{k=1}^{K} |a_{ik}| = K \|A\|_{\infty} = K \sup_{v \in \mathbb{R}^{K \times 1} : \|v\|_{\infty} = 1} \|Av\|_{\infty} \geq K \left\|A\mathbb{I}^{K \times 1} \frac{1}{K}\right\|_{\infty} = \left\|A\mathbb{I}^{K \times 1}\right\|_{\infty} = \sum_{i=1}^{N} \sum_{k=1}^{K} |a_{ik}| \geq \sum_{k=1}^{K} \max_{1 \leq i \leq N} |a_{ik}| = \sum_{k=1}^{K} \max_{1 \leq i \leq N} a_{ik}$, where $\mathbb{I}^{K \times 1}$ is an all-one vector of dimension $K \times 1$.

At the limit, the probability that an item is purchased under the average quality ranking with the popularity signal is given by

$$P_{AQGSI} = \max_{1 \leq i \leq N} \bar{q}_i. \tag{27}$$

When no popularity signal is shown, this probability becomes

$$P_{AQNSI} = \sum_{k=1}^{K} w_k \sum_{i=1}^{N} q_{i,k} \frac{v_{\sigma(i)} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)} a_{j,k} + z_k}. \tag{28}$$

We can easily bound P_{AQNSI} as follows:

$$0 \leq \sum_{k=1}^{K} \min_{1 \leq i \leq N} (w_k q_{i,k}) \frac{\sum_{i=1}^{N} v_{\sigma(i)} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)} a_{j,k} + z_k} \leq P_{AQNSI} \leq \sum_{k=1}^{K} \max_{1 \leq i \leq N} (w_k q_{i,k}) \tag{29}$$

and hence, by Inequality (26),

$$0 \leq \frac{P_{AQNSI}}{P_{AQGSI}} \leq K. \tag{30}$$

$\square$

Proof of Proposition 1. Consider first the upper bound. Choose a MMNL model where $z = 0$, $K = N$, the quality matrix is diagonal with a value of 1 for the first element and $1 - \epsilon$ for all

others, the appeal matrix is the identity, and the classes have the same weights $w_i = \frac{1}{K}$. Then,

$$P_{AQNSI} = \sum_{1 \leq i \leq N} \frac{1}{K} \left(1 - \epsilon(1 - \delta_{i1})\right) \text{ and } P_{AQGSI} = \frac{1}{K},$$

where δ_{ij} is the Kronecker delta, thus

$$\lim_{\epsilon \to 0} \frac{P_{AQNSI}}{P_{AQGSI}} = \lim_{\epsilon \to 0} \sum_{1 \leq i \leq N} (1 - \epsilon \delta_{i1}) = \lim_{\epsilon \to 0} (K - \epsilon(K-1)) = K. \tag{31}$$

Consider now the lower bound. Choose a MMNL model where $z = 0$, $K = N$ with the same quality matrix as before, the same weights, and an appeal matrix filled with ones except in its diagonal where each element has a value of ϵ_A. Then,

$$P_{AQNSI} = \sum_{1 \leq i \leq N} \frac{1}{K} \left(1 - \epsilon(1 - \delta_{i1})\right) \frac{v_{\sigma(i)} \epsilon_A}{\epsilon_A v_{\sigma(i)} + \sum_{j \neq i} v_{\sigma(j)}} \text{ and } P_{AQGSI} = \frac{1}{K},$$

thus

$$\lim_{\epsilon_A \to 0} \frac{P_{AQNSI}}{P_{AQGSI}} = \lim_{\epsilon_A \to 0} \sum_{1 \leq i \leq N} (1 - \epsilon(1 - \delta_{i1})) \frac{v_{\sigma(i)} \epsilon_A}{\epsilon_A v_{\sigma(i)} + \sum_{j \neq i} v_{\sigma(j)}} = 0. \tag{32}$$

$\square$

Proof of Theorem 4. By Theorem 2, we have

$$P_{AQGSI} \doteq \lim_{t \to \infty} P_{AQGSI}^t = \max\{\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_N\}.$$

As mentioned earlier, for the segmented quality ranking, each segment is independent from each other and all of them will converge to a monopoly for the product with the highest quality in that class. We have that

$$P_{SQSSI} \doteq \lim_{t \to \infty} P_{SQSSI}^t = \sum_{k=1}^{K} w_k \max_i q_{i,k}.$$

As a result,

$$\frac{P_{SQSSI}}{P_{AQGSI}} = \frac{\sum_{k=1}^{K} w_k \max_{1 \leq i \leq N} q_{i,k}}{\max_{1 \leq i \leq N} \sum_{k=1}^{K} w_k q_{i,k}} = \frac{\sum_{k=1}^{K} \max_{1 \leq i \leq N} w_k q_{i,k}}{\max_{1 \leq i \leq N} \sum_{k=1}^{K} w_k q_{i,k}}.$$

The lower bound is obviously valid and the upper bound follows from inequality (26). $\square$

Proof of Proposition 2. Consider a model with K items and K consumer classes. Without loss of generality, let the segment 1 be the segment with the lowest weight, i.e., $w_1 \leq w_k \forall k \in [K]$. Then,

for any set of positive appeals in each class, define the elements $q_{i,k}$ as follows:

$$
q_{i,k} = \begin{cases} \frac{\min_{j \in [K]} w_j}{w_1} & \text{if } i = k = 1 \\ \frac{\min_{j \in [K]} w_j}{w_k} - \epsilon & \text{if } i = k \neq 1 \\ 0 & \text{otherwise} \end{cases}
$$

where ϵ is a positive number ensuring that the model is tie-breaking for the quality rankings. Then

$$
\lim_{\epsilon \to 0} \frac{P_{SQSSI}}{P_{AQGSI}} = \lim_{\epsilon_A \to 0} \frac{K \min_{j \in [K]} w_j - \epsilon \sum_{k=2}^{K} w_k}{\min_{j \in [K]} w_j} = K.
$$

$\square$

Proof of Theorem 5. Using the result from Van Hentenryck et al. (2016a), each segment under the SQSSI ranking will go to a monopoly of the product that maximizes its quality. If this product is not unique, the market will be shared between such products. Thus, the ratio between the asymptotic purchase probability of the average quality ranking without social influence and its segmented version with segmented social influence is given by

$$
\frac{\lim_{t \to \infty} P_{AQNSI}^t}{\lim_{t \to \infty} P_{SQSSI}^t} = \frac{\sum_{k=1}^{K} w_k \sum_{i=1}^{N} q_{i,k} \frac{v_{\sigma(i)} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)} a_{j,k} + z_k}}{\sum_{k=1}^{K} w_k \max_{1 \leq i \leq N} q_{i,k}}.
$$

Since $\sum_{i=1}^{N} q_{i,k} \frac{v_{\sigma(i)} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma(j)} a_{j,k} + z_k} \leq \max_{1 \leq i \leq N} q_{i,k}$, we have that $\frac{\lim_{t \to \infty} P_{AQNSI}^t}{\lim_{t \to \infty} P_{SQSSI}^t} \leq 1$.
A similar argument can be made for the second comparison:

$$
\frac{\lim_{t \to \infty} P_{SQNSI}^t}{\lim_{t \to \infty} P_{SQSSI}^t} = \frac{\sum_{k=1}^{K} w_k \sum_{i=1}^{N} q_{i,k} \frac{v_{\sigma_k(i)} a_{i,k}}{\sum_{j=1}^{N} v_{\sigma_k(j)} a_{j,k} + z_k}}{\sum_{k=1}^{K} w_k \max_{1 \leq i \leq N} q_{i,k}} \leq 1.
$$

$\square$

Supplementary material for "Market Segmentation in Online Platforms"

Supplementary Appendix A: The impact of misclassification

In this appendix we consider scenarios in which the firm is not always able to identify correctly the corresponding consumer segment. Misclassification may occur, for example, when there is no historical data about the incoming consumer and it has to be assigned to one of the classes

solely based on basic information provided by some internet marketing companies (e.g. keywords searched, geographical region, sex, age, etc). We analyze what is the impact of having some classification errors under some mild model assumptions. First, we provide some theoretical results about the convergence under our misclassification model. Finally, using a numerical experiment, we illustrate what is the impact in performance as a function on the misclassification rate.

We begin describing the model extension. Suppose that every time the system has an arriving customer of segment $l \in [K]$ (this occurs with probability w_l), there exists a probability α_{lk} that the consumer is recognized as segment $k \in [K]$. If a consumer is recognized by the system (or classifier) as segment k consumer (even if it is not really from segment k), it is said that the consumer is *observed* in segment k. Similarly to the policy SQSSI let SQSSIM denote the ranking policy of classifying consumers (but now misclassification errors occur) and then providing to the consumers observed as segment k the quality ranking of consumer segment k and updating the popularity signal locally in each segment.

Since SQSSIM has misclassification, the benefits of segmentations might be deteriorated depending on how often segments are mistakenly recognized. Under SQSSIM, which product will become the most popular in each of the segments? To answer this question, we rely on the important result obtained in Theorem 2. Namely, it provides the long term convergence of using a static ranking with global social influence among all consumer segments. The following corollary identifies the product in each segment that will become the most popular in the long term.

Corollary 2. *Suppose the solution to* $\operatorname{argmax}_{1 \leq i \leq N} v_{\sigma_k(i)} \sum_{l=1}^{K} w_l \alpha_{lk} q_{i,l}$ *is unique for segment k where $\sigma_k(\cdot)$ denotes a static ranking associated to segment k. Then, under the SQSSIM policy, the product i_k^* that converges to a monopoly for the observed segment k is:*

$$i_k^* = \operatorname*{argmax}_{1 \leq i \leq N} v_{\sigma_k(i)} \sum_{l=1}^{K} w_l \alpha_{lk} q_{i,l}. \tag{33}$$

Proof. Consumers observed in segment k may belong to K different segments due to classifications mistakes, thus, each observed segment k can be seen as a MMNL. Given a consumer is observed in segment k, the probability that this customer belongs to $l \in [K]$ is given by $w_l \alpha_{lk}$. Then, the purchase probability of product i for a consumer observed in segment k, with ranking σ_k and purchase vector $d^{t,k}$ is given by

$$P_i^k(\sigma_k, d^{t,k}) = \sum_{l=1}^{K} \left(w_l \alpha_{lk} \cdot \left(v_{\sigma_k(i)} \frac{(a_{i,l} + d_i^{t,k})}{\sum_{j=1}^{N} v_{\sigma_k(j)}(a_{j,l} + d_j^{t,k}) + z_l} \cdot q_{i,l} \right) \right).$$

This probability resembles Eq. (19) in the proof of Theorem 2. Thus, we know that each observed segment k of consumers converges to a monopoly for product

$$i_k^* = \operatorname*{argmax}_{1 \leq i \leq N} v_{\sigma_k(i)} \sum_{l=1}^{K} w_l \alpha_{lk} q_{i,l}.$$

$\square$

Now that we have found the long term behavior of the model with classifying errors, we analyze the impact of errors in market efficiency. From now on we assume that the probability of committing a mistake in classifying a segment is the same for every segment, and is equally likely to identify it as any other segment. Then we may define the mistake probabilities with two parameters, α_0 and β_0:

$$\alpha_{lk} = \begin{cases} \alpha_0 & \text{if } l = k \\ \frac{\beta_0}{K-1} & \text{otherwise} \end{cases}. \tag{34}$$

Equation (34) means that every customer has a probability α_0 of being recognized as their correct segment, while a probability β_0 that the consumer's segment is mistaken for another one. Naturally, $\alpha_0 = 1 - \beta_0$.

Similarly to the definition of P_{SQSSI}^t, let P_{SQSSIM}^t denote the probability of a purchase at time t when the firm applies the segmented quality ranking with the local popularity signal d_k^t under a classifier with errors. We are now ready to prove the following:

Theorem 6. *Assume that the average quality ranking and its segmented version are tie-breaking for a MMNL model with mistake probability matrix α given by (34), and that $\alpha_0 > \frac{\beta_0}{K-1}$ (the probability than an observed segment is classified correctly is greater than the probability of misclassifying it with any other segment). Then,*

$$\frac{K}{K-1} \beta_0 \leq \lim_{t \to \infty} \frac{P_{SQSSIM}^t}{P_{AQGSI}^t} \leq K \alpha_0 \tag{35}$$

Proof. As mentioned earlier, when we have segmentation, each segment is independent from each other and every recognized segment k will converge to a monopoly for the product i_k^* given by Eq. (33). If each observed segment k is ranked according to the new qualities $\hat{q}_{i,k} \doteq \sum_{l=1}^{K} w_l \alpha_{lk} q_{i,l}$, then

$$\lim_{t\to\infty} P^t_{SQSSIM} = \sum_{l=1}^{K} \sum_{k=1}^{K} w_l \alpha_{lk} q_{i_k^*,l} = \sum_{k=1}^{K} \max_{1\le i\le N} \sum_{l=1}^{K} w_l \alpha_{lk} q_{i,l}$$

$$\le \sum_{k=1}^{K} [\max_{1\le l\le K} \alpha_{lk}] \max_{1\le i\le N} \sum_{l=1}^{K} w_l q_{i,l} = \sum_{k=1}^{K} \max\{\alpha_0, \frac{\beta_0}{K-1}\} \max_{1\le i\le N} \bar{q}_i$$

$$= K\alpha_0 \max_{1\le i\le N} \bar{q}_i,$$

and

$$\lim_{t\to\infty} P^t_{SQSSIM} = \sum_{l=1}^{K} \sum_{k=1}^{K} w_l \alpha_{lk} q_{i_k^*,l} = \sum_{k=1}^{K} \max_{1\le i\le N} \sum_{l=1}^{K} w_l \alpha_{lk} q_{i,l}$$

$$\ge \sum_{k=1}^{K} [\min_{1\le l\le K} \alpha_{lk}] \max_{1\le i\le N} \sum_{l=1}^{K} w_l q_{i,l} = \sum_{k=1}^{K} \min\{\alpha_0, \frac{\beta_0}{K-1}\} \max_{1\le i\le N} \bar{q}_i$$

$$= K\frac{\beta_0}{K-1} \max_{1\le i\le N} \bar{q}_i.$$

Since $\lim_{t\to\infty} P^t_{AQGSI} = \max_{1\le i\le N} \bar{q}_i$, we finally have that

$$\frac{K}{K-1}\beta_0 \le \lim_{t\to\infty} \frac{P^t_{SQSSIM}}{P^t_{AQGSI}} \le K\alpha_0.$$

$\square$

To illustrate the effects of classifications mistakes in the market, we perform a simulation for Scheme 2 under the ranking policy SQSSIM with different values of α_0. We also plot the ranking policy AQGSI in the same graph, the results are shown in Figure 13. As expected, the performance of the ranking policy SQSSIM decreases as the percentage of correct consumer segment classification (α_0) decreases. Furthermore, it is interesting to see that the AQGSI ranking policy outperforms SQSSIM with an $\alpha = 0.8$ or less. The managerial insight is that segmentation in this setting is better than showing a single ranking, but only as long as the misclassification errors are relatively small. In cases where $\alpha = 0.8$ or lower, market segmentation is harmful.

Supplementary Appendix B: 2-Swap, a Performance Ranking Heuristic

In order to assess the average quality ranking policy (AQGSI), we performed computational experiments and compare it to a substantially more computationally expensive heuristic: the 2-swap heuristic. In all our experiments using the different schemes, the results obtained with the 2-swap heuristic were not significantly better than those obtained with the average quality ranking
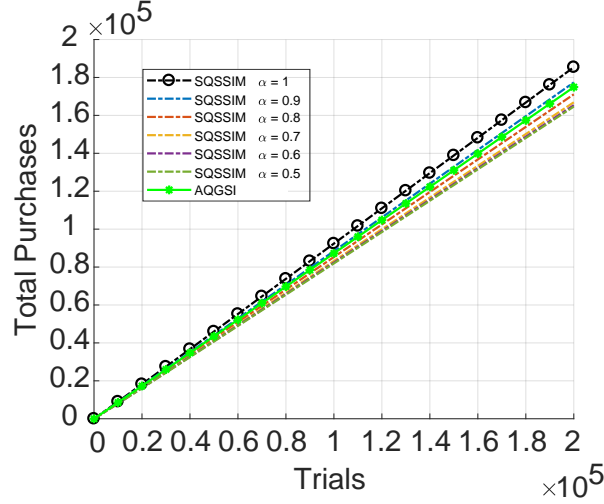
Figure 13: The Number of Purchases over time for various SQSSIM rankings with different α_0 values, and the AQGSI ranking for Scheme 2. The x-axis represents the number of items tried and the y-axis represents the average number of purchases over all experiments. We can see that only when the classifications errors are small ($\alpha \geq 0.8$), the segmentation policy outperforms the global ranking policy (average quality ranking AQGSI).

(AQGSI). Figure 14 shows the average total purchases over 400 simulations as a function of the number of trials using the both methods. For completeness we also included the segmented ranking policy SQSSI.
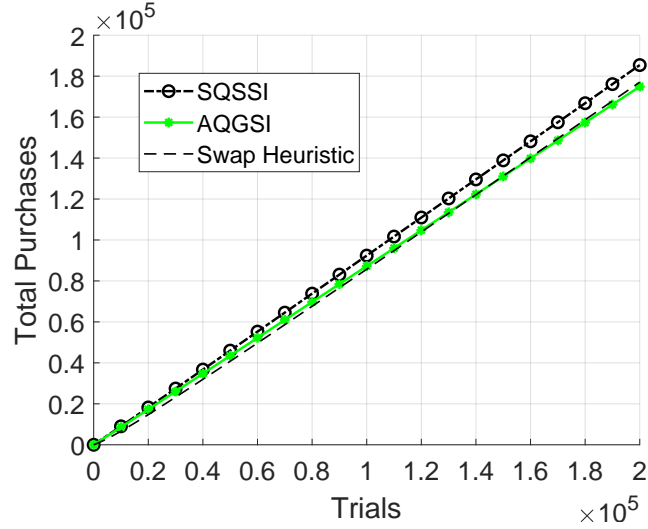

Figure 14: Average total purchases vs trials of 400 simulations under SQSSI, SQSSI and the 2-swap heuristic with global social influence using Scheme 2.

Supplementary Appendix C: Revenue Maximization

In this appendix we analyze an extension of the model in which each product $i \in [N]$ generates a profit r_i to the platform owner. Consumers are aware of the product prices and those are taken into account in the consumer choice probabilities of our model (Equation (1)).

Since now the firm is interested in maximizing profit, we need to modify the previous objective (Equation (2)) to the following one:

$$\Pi^{PR} = \max_{\sigma \in S_N} \left\{ \sum_{k=1}^{K} \left(w_k \cdot \sum_{i=1}^{N} \left(\frac{v_{\sigma(i)}(a_{i,k} + d_i^t)}{\sum_{j=1}^{N} v_{\sigma(j)}(a_{j,k} + d_j^t) + z_k} \cdot r_i q_{i,k} \right) \right) \right\}. \tag{36}$$

Observe that when $r_i = 1$ for all $i = 1, \ldots, N$ the above problem reduces to the original setting in which the firm tries to maximize the expected number of purchases. This means that our NP-hardness result (Theorem 1) for the original model still holds for this new setting. Moreover, Theorem 2 (the monopoly convergence) 1 still hold, since consumer preferences under the Mixed MNL already account for the product's prices/revenues and the market dynamics is only dependent on those mixed MNL preferences (as previously). What is interesting to note is that in this model extension, the average quality ranking may lead to a monopoly for a product that doesn't yield the greatest revenue rate. This is because the product with the largest average quality, may not be the one that generates the largest expected revenue conditional on a trial. This lead us to analyze a new ranking heuristic, the "Average Expected Revenue Ranking", which ranks products based on their expected revenue conditional on a trial, namely $\sum_k w_k q_{i,k} \times r_i$. To obtain analytical results, we analyze this policy under an important assumption: the average quality ordering is the same as the revenue ordering. This assumption is reasonable in multiple settings: it says that higher quality products have a higher price. With this assumption, the average expected revenue ranking is the same as the average quality ranking, and thus, from Theorem 2, the market converges to monopoly for the product with the largest expected revenue conditional on a trial (asymptotic optimality). In addition, for the Average Expected Revenue Ranking, Theorem 3 still holds. The Average Expected Revenue Ranking with the same ordering between average product qualities and product revenues, can perform up to K (the number of consumer segments) times better, or arbitrarily worst, by not showing the popularity signal. The proof is straightforward from Theorem 3's proof, in which $q_{i,k}$ is exchanged by $q_{i,k}r_i$ each time.

For Theorem 4 to hold with the Average Expected Revenue Ranking, we need a stronger assumption. If the average quality ordering is just the same as the revenue ordering, it could happen that for some consumer segments product qualities and revenues are not ordered the same

way. Thus, ranking by expected revenue conditional on a trial for each segment might not achieve asymptotic optimality in each consumer segment. In order to guarantee asymptotic optimality of each consumer segment we need that the quality ordering for each them is the same as the revenue ordering. Thus, the results from Theorem 4 hold only when all consumer segment product qualities follow the same ordering as the product revenue ordering.

Supplementary Appendix D: Assortment Optimization

In this section we analyze the extension in which in each time period the platform owner chooses a subset $S \subseteq [N]$ of products to show to consumers, as well as a ranking σ_S among them. We particularly focus in the case where the platform owner has no information on consumers segments, and thus, needs to display the same assortment of products to all consumers. If there is only one consumer segment, this problem can be solved efficiently (see Abeliuk et al. (2016) and Sumida et al. (2019)). However, when there are at least two segments, the problem becomes a generalization of the classical assortment optimization under the latent class MNL model which is already NP-hard (see Bront et al. (2009) and Rusmevichientong et al. (2014)).

Given the impossibility of finding efficiently an optimal assortment, we study a heuristic which we called the *Average Quality Threshold Heuristic*. This heuristic first ranks products by their average quality, so let product i denote the product with the ith highest average quality. Then, at each time period, it chooses how many products to show with the following condition: if it shows k products, those must be $\{1, 2 \dots, k\}$ and they should be ranked in the same order: higher quality products are placed in positions with higher visibility. This heuristic is computationally more intensive than the standard average quality, since for each time period, we need to evaluate N different scenarios and choose the best one. Nevertheless, it is still computational practical since it takes at most $O(N^3)$ time.

Observe that the number of products shown by the Average Quality Threshold Heuristic is sensitive to the values associated to the outside option z_k's. In one extreme, very large values of the outside option (in comparison to the value associated to the products) means that the products offered by the platform owner face a rather weak cannibalization. In those scenarios, adding an extra product to an assortment is likely to be beneficial since it will probably increase overall sales. On the other extreme, when the outside option values are very small, products offered by the firm face a strong cannibalization: in these scenarios it is likely that offering only a few products is the optimal strategy.

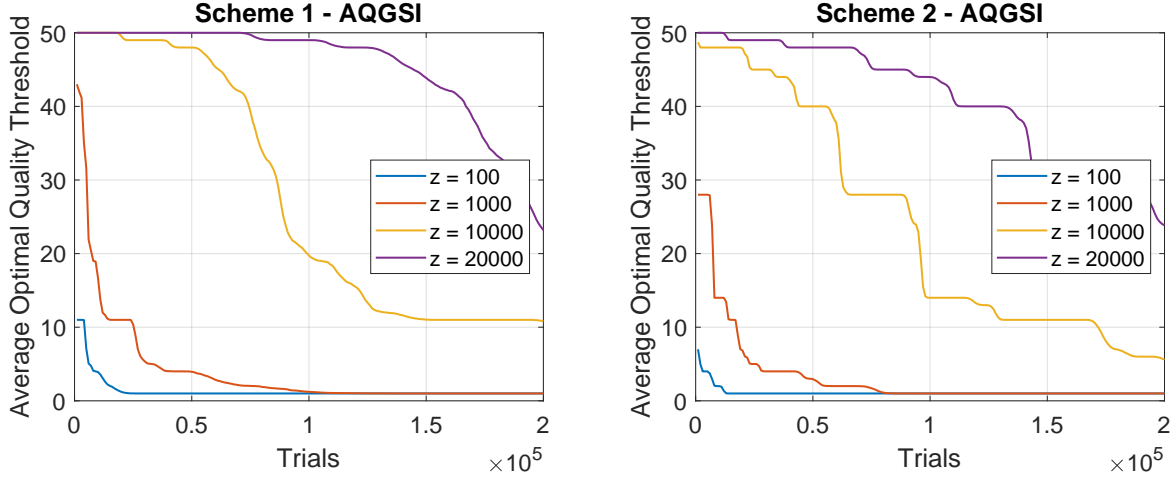We performed the same computational experiments as those in Section 6 on Scheme 1 and

Figure 15: Average threshold for the AQTGSI policy versus trials. On the left (a) we have it for Scheme 1, while on the right (b) we have it for Scheme 2.

Scheme 2, but using the Average Quality Threshold Heuristic and varying the outside option value (z). A large outside option value means that at the early stages a higher number of consumers will decide not to try a product at all. In the long run, the effect of the outside option diminishes as some products become popular and their overall appeal increases. Figures 15(a) and 15(b) show the optimal number of products that are shown at each period (on average) for Schemes 1 and 2 respectively. As expected, observe that the optimal number of products shown increases when the outside option increases. However, as the number of purchases increase (and the social influence signal becomes stronger), the offer sets shown tend to reduce in size to exhibit only the products with the highest average quality.

In all our computational experiments, the offer sets offered by the average quality threshold heuristic are always reduced in size or they stay the same as times goes by. This suggests to us that even under the average quality threshold heuristic, which is a dynamic ranking policy, the market will also converge to a monopoly for the product with the highest weighted average quality. We leave this conjecture below.

Conjecture 1. *(Asymptotic optimality of the Average Quality Threshold Heuristic) Whenever the average quality threshold heuristic is used, the market goes to a monopoly for the product with the highest weighted average quality.*

We conclude this section by assessing the performance of the Average Quality Threshold Heuristic with Global Social Influence (AQTGSI) and we compare against the performance of the Average Quality with Global Social Influence (AQGSI). Figures 16(a) and 16(b) display the average number of purchases under AQTGSI for Schemes 1 and 2 respectively. We can observe that
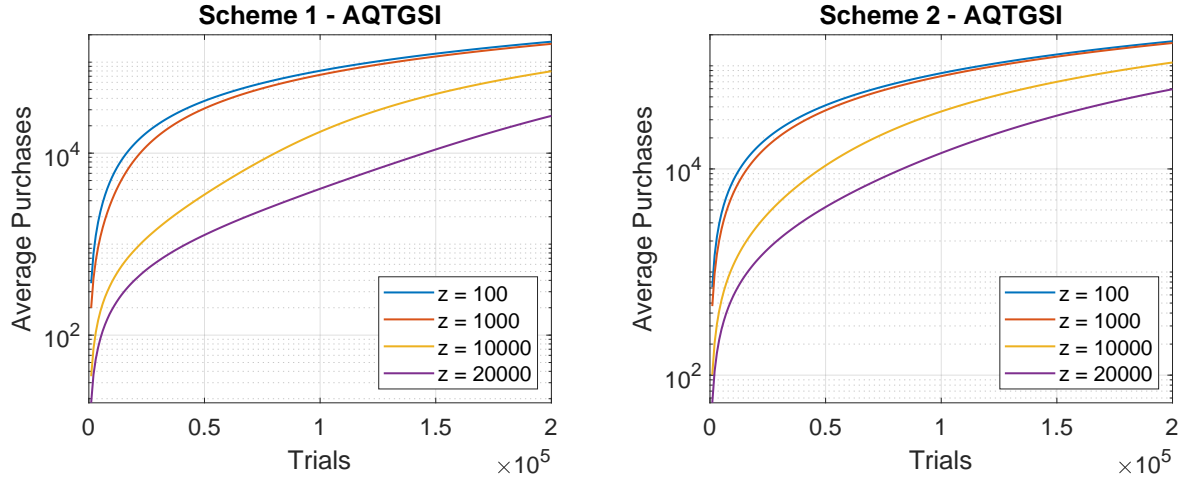
Figure 16: Number of purchases versus trials using the AQTGSI policy for scheme 1 on the left (a) and Scheme 2 on the right (b).
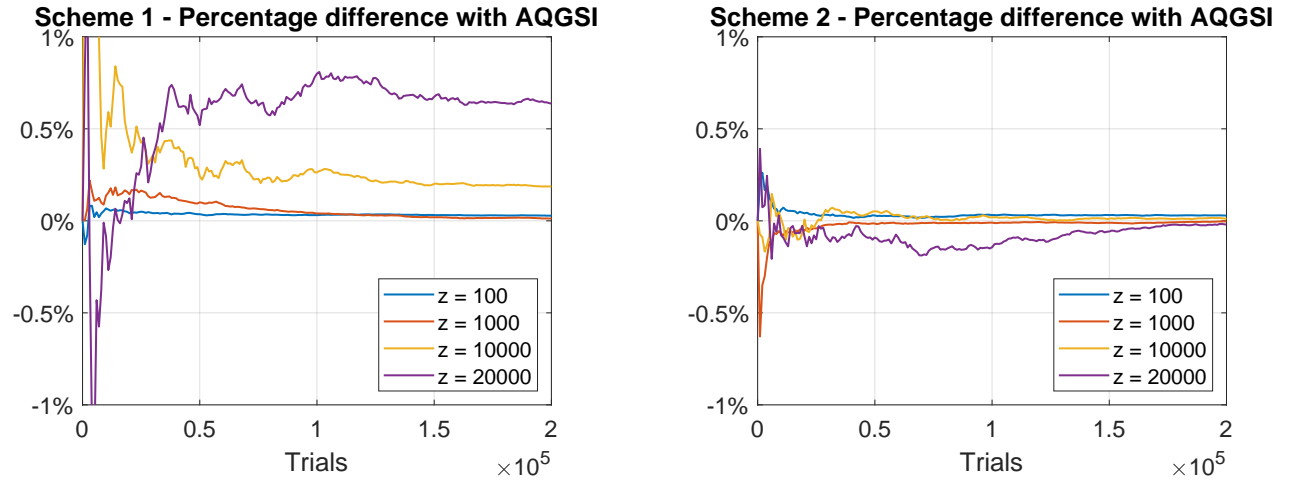

Figure 17: Percentage difference in the number of purchases between AQTGSI and AQGSI for Scheme 1 on the left (a) and Scheme 2 on the right (b).

more sales occur in settings where the outside option value is small, but this difference is reduced as time goes by. Figures 17(a) and 17(b) show the percentage improvement (or deterioration) that AQTGSI has over AQGSI for Schemes 1 and 2 respectively. Our experiments show that these two policies have approximately the same performance for all the outside option values we tried, with less than 0.75% improvement in 200,000 trials.