DEPP - Differential Evolution Parallel Program

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Abstract

DEPP is an open source optimization software, written in Fortran 2008 and based on the Differential Evolution heuristic. It was designed to solve non-linear optimization problems of a real valued function (objective function) constrained to a boxed domain. Additionally, this software is gradient-free, it is not code invasive, i.e., objective function is treated as a "black-box", it is tolerant to failures, to poor precision calculated or noisy objective functions, it uses parallelization, that is particularly useful for handling computationally expensive objective functions and it allows a Response Surface Methodology-Differential Evolution hybridization for convergence acceleration. In this document, the general structure of DEPP is presented with some illustrative examples.

1 Introduction

Optimization (maximization and minimization) is a fundamental mathematical problem with numerous application in science and engineering[1]. This article focus on a particular kind of maximization problem[1], often found in practical applications, i.e.,

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{maximize}} \quad & f(\boldsymbol{x}) \\ \text{subject to} \quad & \\ & \boldsymbol{L} \leq \boldsymbol{x} \leq \boldsymbol{U}, \end{aligned} \tag{1}$$

[1] Indeed, maximization and minimization are equivalent problems[2], so in the following text, only maximization will be discussed.

where f is a real valued function, also known as objective function, depending on a D-dimensional vector $\boldsymbol{x} = (x_1, \cdots, x_D)$ of real variables. The variable $\boldsymbol{x}$ is confined in a box with lower $\boldsymbol{L} = (L_1, \cdots, L_D)$ and upper $\boldsymbol{U} = (U_1, \cdots, U_D)$ bounds.

There are, at least, two main classes of methods for addressing the optimization problem (1), both iterative: the class based on Newton's method[2] and the class of heuristics methods[1, 3, 4, 5]. In general, the former class converge quickly to local maximum, while the latter converge more slowly, but with greater probability, to the global maximum. Currently, there is no general method capable to find the global maximum of an arbitrary function. Despite the (in general) slow convergence of the heuristic methods, they have (in general) some advantages: they probably converge to the global maximum, they handle constraints in a simpler way, they do not require the derivative of the objective function, they are not sensitive to non-smooth or non-accurate objective functions and do not suffer with eventual failures in the calculation of the objective function.

Among the heuristic methods, Differential Evolution (DE)[6] had emerged as a simple and efficient method for finding the global maximum. This method is based on the principles of biological evolution and works, basically, as follow. Given a population (set of discrete points in the domain of search), new individuals (trial points) are generated by "mutation" and "crossing over". Here, "mutation" means a rule for creating new trial individuals while "crossing over" means a rule of exchanging information between a trial individual and an individual of the population. The fitness (objective function) of the new individuals is compared to the fitness of the parent individuals and the worse individuals are eliminated. The procedure is repeated for several generations (iterations) until a criterion of convergence is satisfied within a given tolerance.

In order to accelerate Differential Evolution convergence, it had been coupled with other models, as the Response Surface[7, 8]. In this case, the objective function is approximated by a quadratic polynomial and the maximizer $\boldsymbol{x}^*$ is obtained analytically. For a smooth function $f(\boldsymbol{x})$, the quadratic polynomial is a good representation of the objective function in the vicinity of a local maximum.

Additional computational time reduction can be achieved with parallelization, specially, for computationally expensive objective functions. There are problems whose objective function is the result of a numerical solution of partial differential equations, which take hours of computation. For these problems, the calculation of the objective function can be made in parallel and, hence, the final wall clock time can be reduced substantially.

The coupling of DE, models and parallelization makes the resulting algo-

rithm a powerful tool for solving optimization problems in a wide range of scientific and engineering areas.

This document presents DEPP, the Differential Evolution Parallel Program. DEPP combines DE, the Response Surface methodology and MPI parallelization[10]. The objective function is calculated externally and is treated as a "black box". This is useful because the calculation of the objective function is often performed by a software whose source code is closed. The constraints of the objective function and other input parameters are informed to DEPP through text files. In the following sections, a brief description of DEPP, its supporting theory and some examples are presented.

2 Software description

2.1 Software Architecture

DEPP is written in FORTRAN 2008 standard language[11] with the Message Passing Interface (MPI) directives[10] and takes advantage of the Object-Oriented Paradigm.

The folder structure of DEPP is shown in Figure 1. The directory *depp_input* contains the input (text) files which defines the control parameters of the optimization. Results of DEPP are saved in the *depp_output* directory. The source code is within *depp_src* directory. This directory also contains the Bash[12] script *compile.sh*, which compiles DEPP source code and generates the executable *depp.x* in the root of the file structure. The interface between DEPP and the external program (EP), which calculates the objective function, is defined in the *interface* directory. Finally, the script *run.sh* runs DEPP using MPI.

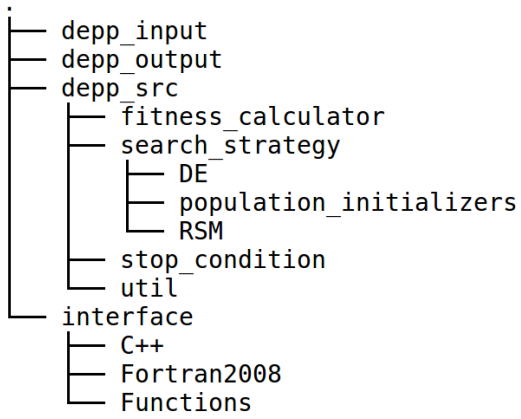


Fig. 1: Folder structure of DEPP.

Thanks to the Object-Oriented Programming, the source code was designed to simplify the implementation of new methods. The algorithm works on abstract classes separated, basically, in (i) population initializers, (ii) search strategies, (iii) fitness calculation and (iv) stop conditions. The concrete classes are generated using the Factory Design Pattern[13]. All MPI commands are encapsulated into proper classes, following the logic of the Adapter Design Pattern. In this way, users interested in the implementation of new methods will not concern about MPI details.

Interfacing DEPP with an external program is exemplified in *interface* directory. This folder contains three examples based on C++ and FORTRAN 2008 languages. Any programming language may be used for interfacing.

The basic algorithm of DEPP is illustrated in Figure 2. DEPP reads the input data from *depp_input* directory (optionally, it may start from some backup). The stop condition is analyzed. In the first iteration, the stop condition is generally not satisfied (it may not be true when starting from a backup). When stop condition is satisfied, the iterative procedure is finished and the output data is saved in the *depp_output* directory. Otherwise, a trial population is generated. DEPP sends the information to a set of threads (T1,$\cdots$, TN) using MPI. Then, each of these threads runs a copy of the external program (EP) for a given individual of the population. After that, DEPP receives the calculated objective function and compares their fitness with the fitness of the current population. Only the best individuals are held. A backup is optionally generated and the iterative cycle is restarted. During the calculation of the objective function, some failures may occur. DEPP handles failures using an error code returned by the external program. If the error code is 0, the calculations were performed correctly; if 1 is returned, then a failure occurred and the trial individual is discarded; if 2 is returned, then a failure occurred, the trial individual is discarded and a new one is generated. All the failures are registered for posterior user's analysis.

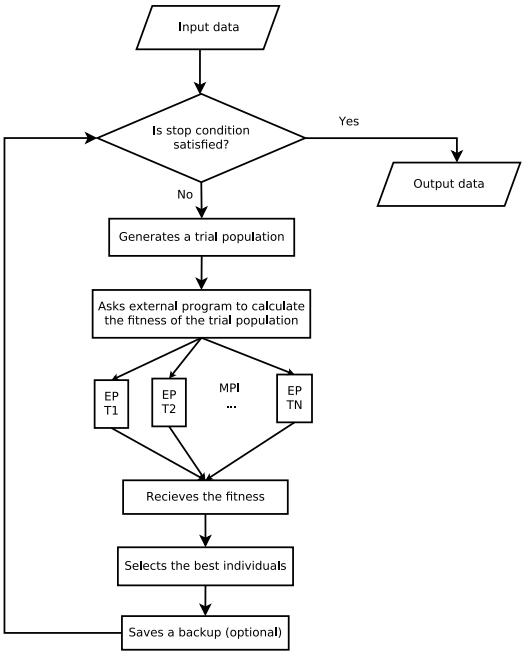


Fig. 2: DEPP basic algorithm.

2.2 Supporting theory

The following sections describe the optimization theory behind the current implementation of DEPP. It must be emphasized that DEPP was designed based on an Object-Oriented Paradigm, in such a way to simplify the implementation of new and better methods.

2.2.1 Differential Evolution

The first step toward solving the optimization problem (1) is the population initialization. For the first generation ($g = 1$), a population (set of points) $\{\boldsymbol{x}_i\}$ $1 \leq i \leq N_p$ is created randomly within the box $\boldsymbol{L} \leq \boldsymbol{x} \leq \boldsymbol{U}$, *e.g.*

$$x_{ij} = L_j + (U_j - L_j)\mathcal{R}, \quad 1 \leq j \leq D, \tag{2}$$

where $\mathcal{R}$ is a random number uniformly distributed in $[0, 1)$ and x_{ij} is the j-th element of the i-th individual.

After the population initialization, the fitness of each individual $\boldsymbol{x}_i$ is calculated.

For the next generations, trial individuals are calculated based on the population of the previous generations. Trial individuals are created by a process of mutation and crossing over as follows. For each individual $\boldsymbol{x}_i$ of the previous population, other three distinct individuals ($\boldsymbol{x}_{i_1}$, $\boldsymbol{x}_{i_2}$ and $\boldsymbol{x}_{i_3}$) of the same population are randomly selected and a mutant individual $\boldsymbol{v}$ is created

$$\boldsymbol{v} = \boldsymbol{x}_{i_1} + F(\boldsymbol{x}_{i_3} - \boldsymbol{x}_{i_2}), \tag{3}$$

where F is the differentiation parameter. Then, the mutant individual suffers a crossing over with $\boldsymbol{x}_i$, that produces the trial individual $\boldsymbol{u}$. The components u_j $(1 \leq j \leq D)$ of $\boldsymbol{u}$ are given by

$$u_j = \begin{cases} v_j & \text{if } \mathcal{R} < C_r \text{ or } j = j^*, \\ x_{ij} & \text{otherwise.} \end{cases} \tag{4}$$

In Eq. (4), C_r is the crossover parameter and j^* is an integer randomly chosen between 1 and D. If the trial individual violates the constraints, i.e. $\boldsymbol{L} \leq \boldsymbol{u} \leq \boldsymbol{U}$, then the process of generating a trial individual is repeated.

The fitness of the trial individual is compared with the fitness of $\boldsymbol{x}_i$. If $f(\boldsymbol{x}_i) \leq f(\boldsymbol{u})$, then $\boldsymbol{u}$ replaces $\boldsymbol{x}_i$. This is the so called selection.

New generations are created until a stopping condition is satisfied. DEPP implements a composition of the following stopping conditions: (i) best fitness stagnation, (ii) maximum number of generations and (iii) population convergence measure. In the first one, the generation cycle is interrupted if no

improvement of the fitness function is achieved within N_A generations. The second condition establishes the maximum number of generations allowed. Finally, in the third condition, the cycle is interrupted if the P-measure $P_m[1]$ is lower or equal to a given tolerance. P-measure is evaluated as

$$P_m = \max_{1 \leq i \leq N_p} |\boldsymbol{x}'_i - \boldsymbol{x}'_m|, \tag{5}$$

where $|\cdot|$ is the L_2 norm, the components of $\boldsymbol{x}'_i$ are given by

$$x'_{ij} = \frac{x_{ij} - L_j}{U_j - L_j}, \quad 1 \leq j \leq D \tag{6}$$

and

$$\boldsymbol{x}'_m = \frac{1}{N_p} \sum_{i=1}^{N_p} \boldsymbol{x}'_i. \tag{7}$$

The approach presented in this section has three parameters: the population size N_p, the differentiation parameter F and the crossover parameter C_r. Feoktistov[1] gives reference values/ranges of these parameters based on optimization studies performed on many test functions.

2.2.2 Differential Evolution and Response Surface Methodology

In order to accelerate the convergence of DE, Vincenzi and Savoia[8] proposed a new mutation operation based on the Response Surface Methodology (RSM)[14, 15]. The basic idea consists of fitting a simple response surface to a selected set of individuals of the population. The mutant vector is, then, obtained by maximizing the response surface, which is done through a matrix inversion. For a given benchmark, the authors showed that the number of generations necessary to reach a given precision was substantially reduced (more than 50%).

Vincenzi and Savoia considered two response surfaces. The first one is a quadratic polynomial $P(\boldsymbol{x})$, given by

$$P(\boldsymbol{x}) = \beta_0 + \sum_{i=1}^{D} \beta_i x_i + \sum_{i=1}^{D} \beta_{ii} x_i^2 + \sum_{i=1}^{D-1} \sum_{j=i+1}^{D} \beta_{ij} x_i x_j, \tag{8}$$

where β are coefficients to be fitted. The quadratic polynomial has $N_{f_m} = (D+1)(D+2)/2$ coefficients and, so, it requires, at least, the same number of points to be fitted. Note that the number of points grows significantly when the number of unknowns D is increased. That is why Vincenzi and Savoia proposed a second model.

The second model is an "Incomplete Quadratic Model", where the terms β_{ij} of Eq. (8) are neglected. In this case, the minimum number of fitting points is reduced to $N_{f_m} = 2D + 1$. On the other hand, the accuracy of the model is also reduced.

The DE-RSM hybridization implemented in DEPP was inspired in the work of Vincenzi and Savoia, but with some modifications that are explained below.

DEPP stores a history list, *i.e.* a list of all individuals calculated since the first generation and their corresponding fitness. This list is employed to generate mutant vectors by RSM hybridization and may be used by other methods to be implemented in the future.

According to DE algorithm, for each individual $\boldsymbol{x}_i$ of the population, a trial individual $\boldsymbol{u}$ is created, resulting from a mutation and a crossing over. When the RSM hybridization is active, the mutant vector may be created by the DE procedure explained in the previous section or by hybridization with RSM. The following conditions must be satisfied simultaneously for applying RSM:

- The number of individuals in the history list must be, at least, twice the number of individuals required to fit the response surface N_f. Notice that $N_{f_m} \leq N_f$;
- A random number $\mathcal{R}$ (within [0,1)) must be less than the probability of hybridization f_h.

The probability of hybridization f_h may be prescribed by the user or may be calculated dynamically during the optimization. The dynamic calculation of f_h considers the probability of success f_s when applying RSM. More precisely,

$$f_h = \begin{cases} f_{h_{min}} & f_s \leq f_{h_{min}}, \\ f_s & f_{h_{min}} < f_s < f_{h_{max}}, \\ f_{h_{max}} & f_{h_{max}} \leq f_s, \end{cases} \tag{9}$$

where $f_{h_{min}}$ and $f_{h_{max}}$ are the minimum and maximum values of f_h allowed.

The probability of success of RSM f_s is the ratio of the number of times that the hybridization contributed to maximize the objective function to the last N_p times RSM hybridization was applied. In the first generations, when the number of samples are less than N_p, f_s is not available. In this case f_h is equal to f_{h0}, an initial value of the fraction of hybridization.

Suppose, now, that a mutant vector, associated to individual $\boldsymbol{x}_i$, will be calculated by DE-RSM hybridization. In order to fit a response surface, it is necessary to select a set of individuals. This selection is made in two steps.

First, the history list is ordered from the best individual to the worst one and the i-th best individual $\hat{\boldsymbol{x}}$ is taken (target individual). Second, the history list is reordered from the closest to the furthermost individual to the target individual. From this list, $N_f - 1$ individuals are selected according to the following rules, starting from the closest individual:

- The distance between $\hat{\boldsymbol{x}}$ and the candidate individual $\boldsymbol{x}_k$ must satisfy

$$\sqrt{\sum_{j=1}^{D} \left(\frac{\hat{x}_j - x_{kj}}{U_j - L_j} \right)^2} \geq \eta_{\text{tol}}, \tag{10}$$

  where η_{tol} is a prescribed tolerance. In other words, there is a minimum distance between the target individual and its neighbors.

- The candidate individual $\boldsymbol{x}_k$ is selected with a probability of 50%.

Once the fitting individuals are selected, the coefficients of the response surface are determined using the weighted least squares method[14].

Two weighting functions are implemented in DEPP: uniform and exponential[8]. The exponential weighting function reads as

$$w(\boldsymbol{x}_k) = \begin{cases} \exp\left(\frac{f(\boldsymbol{x}_k) - f_{\text{best}}}{f_{\text{best}}} \right) & f_{\text{best}} \neq 0 \\ \exp\left(f(\boldsymbol{x}_k) - f_{\text{best}} \right) & f_{\text{best}} = 0, \end{cases} \tag{11}$$

where $\boldsymbol{x}_k$ is any of the selected individuals for fitting and f_{best} is the best fitness among the individuals selected for fitting.

Finally, the mutant vector is obtained by maximization of the response surface.

If the application of RSM fails or the trial individual is out of range, then a pure DE individual is created.

3 Examples

Four test functions, whose maximizers are known analytically, were chosen for illustrating the application of DEPP:

- Step function [16]

$$f(\boldsymbol{x}) = -\sum_{i=1}^{D} \left(\lfloor x_i - 0.5 \rfloor \right)^2, \tag{12}$$

- Rosenbrock function [1]

$$f(\boldsymbol{x}) = -\sum_{i=1}^{D-1} 100\left(x_i^2 - x_{i+1}\right)^2 + (1 - x_i)^2, \tag{13}$$

- Noisy Quartic function [16]

$$f(\boldsymbol{x}) = -\mathcal{R} - \sum_{i=1}^{D} i\ x_i^4, \tag{14}$$

  where $\mathcal{R}$ is a uniform random number belonging to $[0, 1)$,

- Schwefel 2.26 function [16]

$$f(\boldsymbol{x}) = -418.98288727243369\ D + \sum_{i=1}^{D} x_i \sin\left(\sqrt{|x_i|}\right). \tag{15}$$

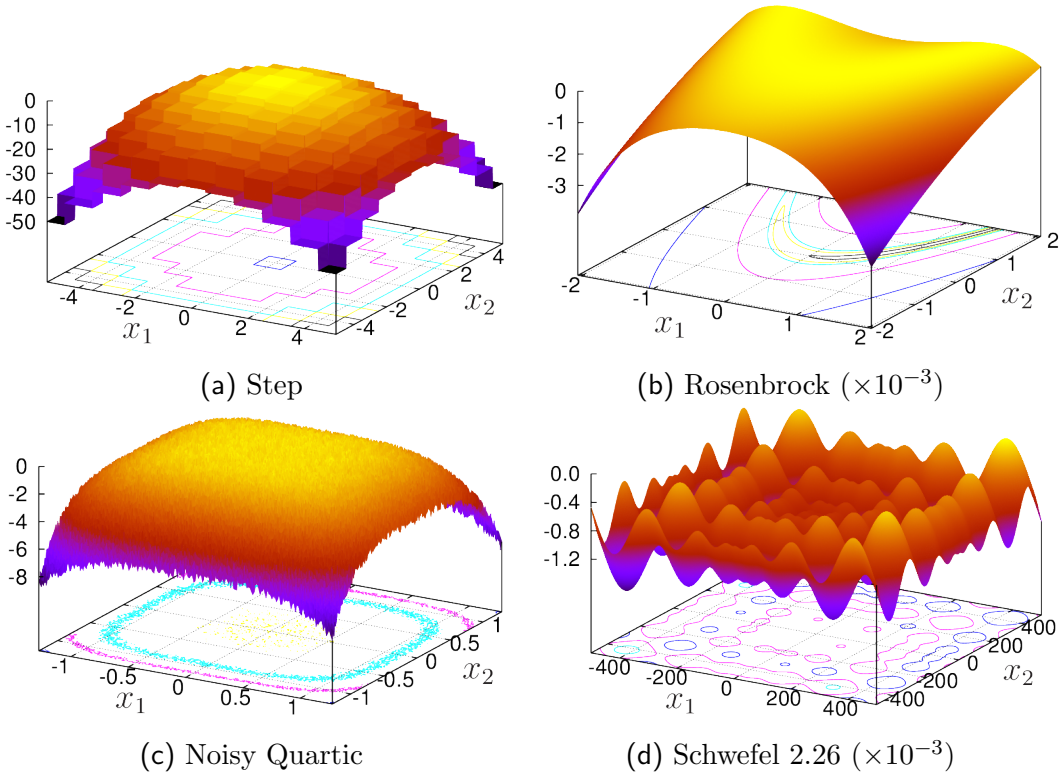
These functions were chosen because they present particular features that make optimization difficult. Step function (Fig. 3a), a set o flat surfaces, pose a hard problem to optimizers relying on functions' derivatives. This is even worst for Noisy Quartic function (Fig. 3c), which has a noisy plateau near its maximum. Rosenbrock function (Fig. 3b), although smooth, has a sharp narrow ridge around a parabola, that makes it a challenge to find an appropriate search direction. Finally, Schwefel 2.26 (Fig. 3d) has several local optima, which may lead to premature convergence.

The maximizers x_i^* and the domain of optimization of the test functions (12)-(15) are presented in Table 1.

Tab. 1: Lower L_i and upper U_i bounds for maximization and the global maximizer x_i^* $(1 \leq i \leq D)$.

Function	L_i	U_i	x_i^*
Step	-100	100	0.5
Rosenbrock	-2	2	1
Noisy Quartic	-1.28	1.28	0
Schwefel 2.26	-500	500	420.968597844358

Additionally, this example aims (i) to show the effect of DE-Response Surface (DE-RSM) coupling on reducing the number of iterations required to achieve convergence and (ii) show the effect of parallelization on reducing



(a) Step

(b) Rosenbrock ($\times 10^{-3}$)

(c) Noisy Quartic

(d) Schwefel 2.26 ($\times 10^{-3}$)

Fig. 3: Test functions for $D = 2$.

the optimization time. In order to accomplish the first task, test functions (12)-(15) were optimized for $D \in \{2, 4, 8\}$ dimensions using DE and DE-RSM. For the second task, the number of dimensions was restricted to $D = 2$, but for each call of the external software that calculates the objective function it was added a delay of 1 s in order to simulate a computationally expensive program. Since DE is a stochastic method, in both tasks, the final results are the average of 50 optmizations.

The parameters applied in the optimization are as follows. The population size is $N_p = 20$ for $D = 2$ and $N_p = 40$ for $D = 4$ and $D = 8$. For pure DE, $F = 0.85$, $C_r = 0.5$. Hybridization uses quadratic polynomial as the response surface. The fraction of hybridization is calculated dynamically using $f_{h0} = 0.35$, $f_{\min} = 0.1$ and $f_{\max} = 0.9$. The crossing over parameter of RSM is $C_r = 1$. The number of individuals selected for fitting the response surface is twice the minimum required, *i.e.* $N_f = 2N_{f_m}$. The tolerance for selecting individuals for fitting is $\eta_{\text{tol}} = 0.0001$. Uniform weighting is applied for fitting. Concerning the stopping conditions, the maximum number of

generations allowed is 5000, the maximum number of generations allowed without improving fitness function is 40, for $D = 2$, and 80, for $D \in \{4; 8\}$, and the tolerance of P-measure is $P_m \leq P_{\text{tol}} = 5 \times 10^{-4}$.

For the test functions considered, numerical results show that DE-Response Surface (DE-RSM) coupling reduces the number of generations necessary to find the global maximizer. Table 2 compares the mean number of generations G necessary to reach stop condition and the probability of success P_s to achieve the global maximum. The number in parenthesis is the standard deviation σ. The poor performance of DE in finding the global maximum of Rosenbrock function is due to stop condition. In this case, the number of generations without improving the fitness function was exceeded and optimization was interrupted.

Tab. 2: Number of generations G to reach stop condition and probability of success P_s.

		$G(\pm\sigma)$		P_s (%)	
Function	D	DE	DE-RSM	DE	DE-RSM
Step	2	59(4)	42(0)	100	100
	4	130(4)	82(0)	100	100
	8	221(9)	85(0)	100	100
Rosenbrock	2	106(10)	35(4)	100	100
	4	636(131)	101(17)	94	100
	8	1526(395)	288(68)	20	100
Noisy Quartic	2	82(30)	80(30)	100	100
	4	178(60)	155(59)	100	100
	8	222(60)	154(72)	100	100
Schwefel 2.26	2	47(4)	20(3)	98	90
	4	107(6)	43(4)	100	100
	8	262(12)	116(11)	100	98

The maximizer $\boldsymbol{x}^*$ of an optimization was considered a success if

$$|f(\boldsymbol{x}^*) - f(\boldsymbol{x}_a^*)| \leq F_{\text{tol}} \quad \text{or} \quad |\boldsymbol{x}^* - \boldsymbol{x}_a^*| \leq P_{\text{tol}}, \tag{16}$$

where $\boldsymbol{x}_a^*$ is the analytical global maximizer and F_{tol} is given by

$$F_{\text{tol}} = \max_{\boldsymbol{x}} |f(\boldsymbol{x}) - f(\boldsymbol{x}_a^*)|, \quad \text{subject to} \quad |\boldsymbol{x} - \boldsymbol{x}_a^*| \leq P_{\text{tol}}. \tag{17}$$

Finally, Tab. 3 presents the mean computational time per generation τ for different number of threads N_t. As can be seen, τ_{N_t} is approximately proportional to τ_1/N_p.

Tab. 3: Mean computational time per generation τ for different numbers of threads N_t. $D = 2$.

		τ (s)	
Function	N_t	DE	DE-RSM
Step	1	20.27	20.27
	2	10.11	10.10
	3	7.07	7.09
	4	5.06	5.06
Rosenbrock	1	20.23	20.28
	2	10.11	10.11
	3	7.10	7.08
	4	5.07	5.07
Noisy Quartic	1	20.23	20.19
	2	10.11	10.12
	3	7.07	7.07
	4	5.08	5.07
Schwefel 2.26	1	20.21	20.26
	2	10.14	10.13
	3	7.09	7.08
	4	5.07	5.09

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