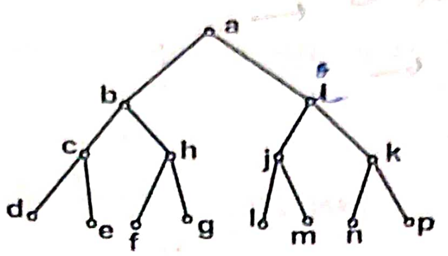
Complete Binary Tree :

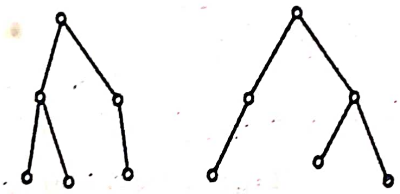
If all the leaves of a full binary tree are at level m, then such a tree is called **complete binary tree** of depth m. For example consider the following tree.



Here d, e, f, g, l, m, n, p are the leaves and they all are at level 3 from the root a. Hence this is a complete binary tree.

Forest:

A set of disjoint trees is called a forest. For example the following figure is a forest.



The following algorithm gives every tree or a forest a unique binary tree representation.

Algorithm :

Step (1): Delete all the edges originating in every vertex except the left most edge.

Step (2): Draw edges from a vertex to the vertex on the right if any of them is at the same level.

Step (3) : Choose the left child and right child of any vertex as follows. The left child is the vertex immediately below the given vertex and the right child is to the immediate right of the given vertex on the same horizontal line.

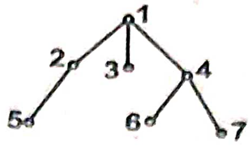
Example (32) : Draw a tree for the relation R = {(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7) on a set A = {1, 2, 3,4, 5, 6,7}. Also give the corresponding binary tree.(S-06,W-07,08)

Solution : Given : R = {(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7)}

A = { 1, 2, 3, 4, 5, 6, 7}.

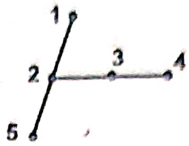
We observe that the vertex with highest number of edges is 1.

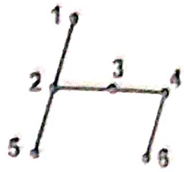
Hence we select 1 as the root. The required tree for the given relation R is drawn below.

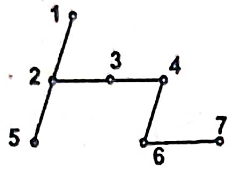


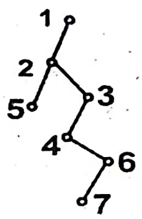
The corresponding binary tree is drawn below :

Step (1) : 

Step (2) : 

Step (3): 

Step (4) : 

Step (5) : 

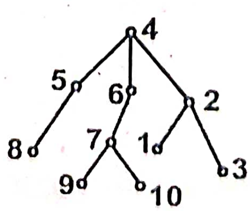
This is the required binary tree.

2→2
4→3
7→2

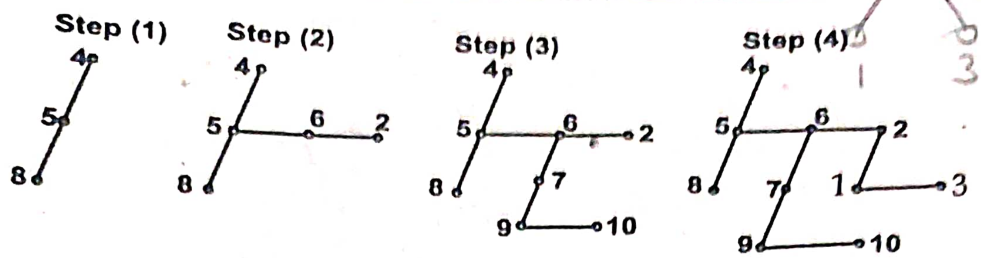
Example (33) : Let A= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} , T = {(2,3), (2, 1), (4, 5), (4, 6), (5, 8), (6, 7), (4, 2), (7, 9), (7,10)}. Indentify the root and show that T is a rooted tree. Also give the corresponding binary tree. (S-06)

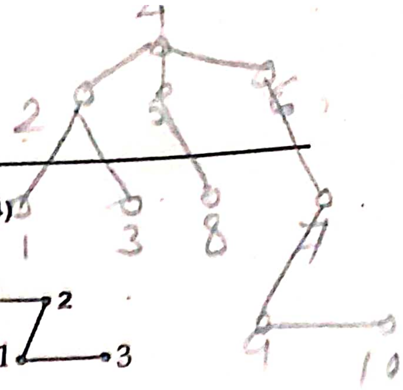
Solution : **Given :** A = {1, 2,3,4,.5, 6, 7, 8, 9, 10} and T = { (2, 3), (2,1), (4, 5), (4,6), (5, 8), (6, 7); (4, 2), (7, 9), (7, 10)}

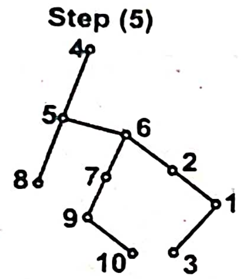
We observe that the vertex with highest number of edges is 4. Hence we select 4 as the root. The required rooted tree is drawn below.



The corresponding binary tree is drawn below:



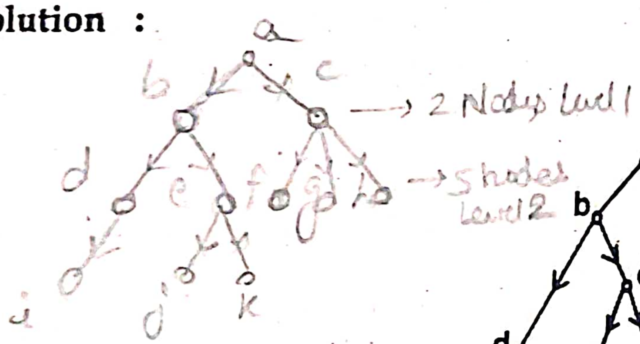


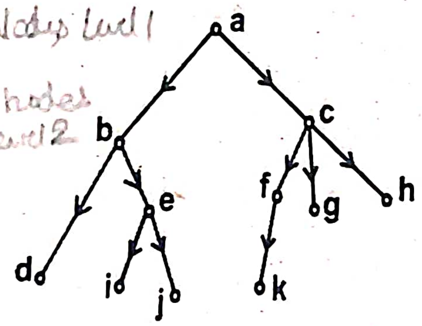


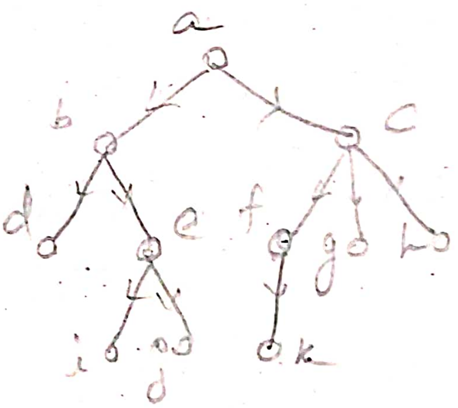
This is the required binary tree.

Example (34) : Draw a directed tree with 2 nodes at level 1, 5 nodes at level 2, 3 nodes at level 3. Obtain the corresponding binary tree. **(W-06,09)**

Solution :

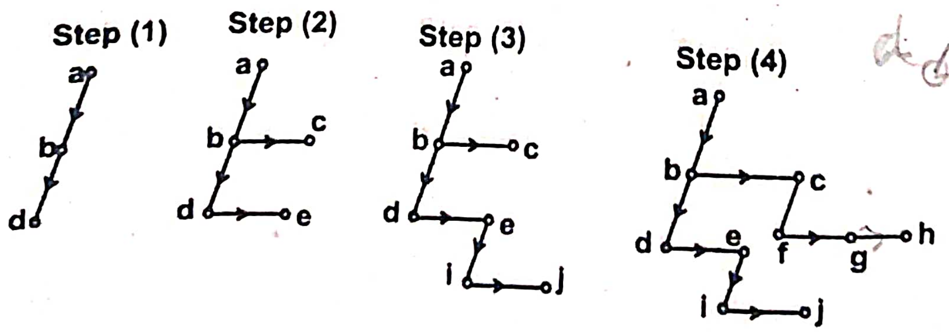


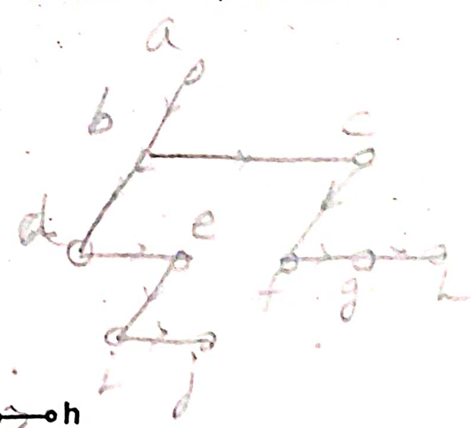


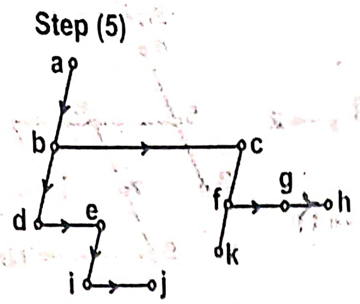


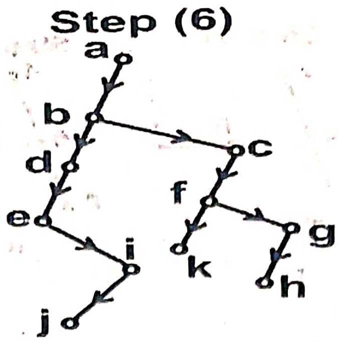
The above tree is a directed tree with root a, 2 nodes b and c at level 1, 5 nodes b, e, f, g and h at level 2, 3 nodes i, j and k at level 3.

The corresponding binary tree is drawn below :







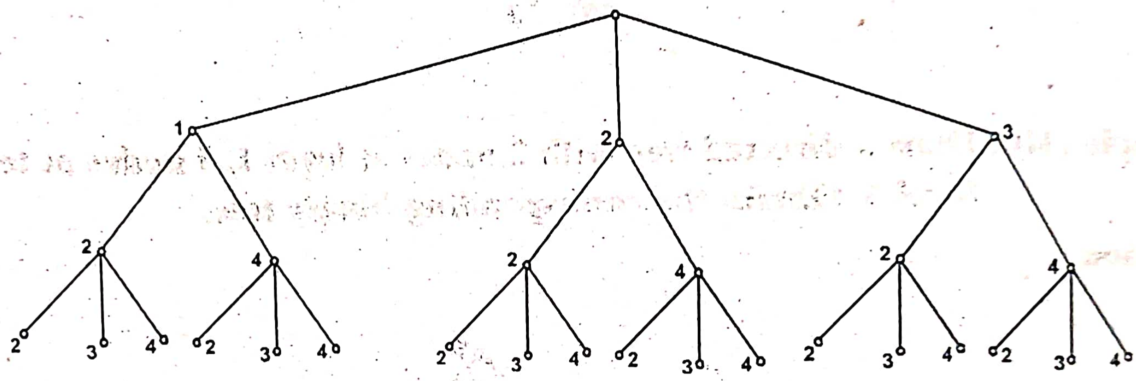


This is the required binary tree.

ES10

Example (35) : Find the product {1,2,3} x {2, 4} × {2,3, 4} by constructing appropriate tree diagram. (S-10)

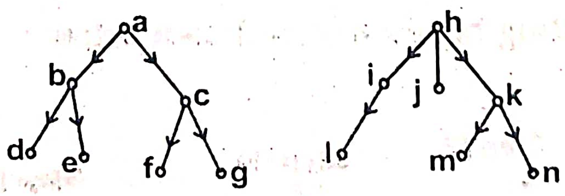
Solution : The required tree diagram is drawn below :



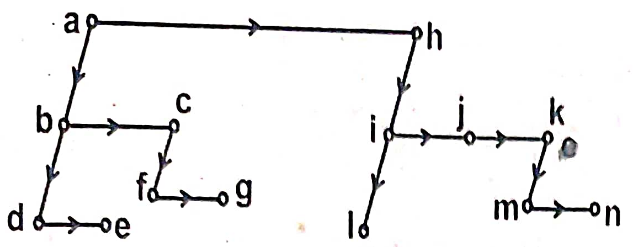
$\therefore \{1, 2, 3\} \times \{2, 4\} \times \{2, 3, 4\} = \{(1, 2, 2), (1, 2,3), (1, 2, 4), (1,4, 2), (1, 4, 3), (1, 4, 4), (2, 2,2), (2,2,3), (2, 2, 4), (2, 4, 2), (2, 4,3), (2, 4, 4), (3, 2, 2), (3, 2, 3), (3, 2, 4), (3, 4, 2), (3, 4, 3), (3, 4,4)\}$ 18

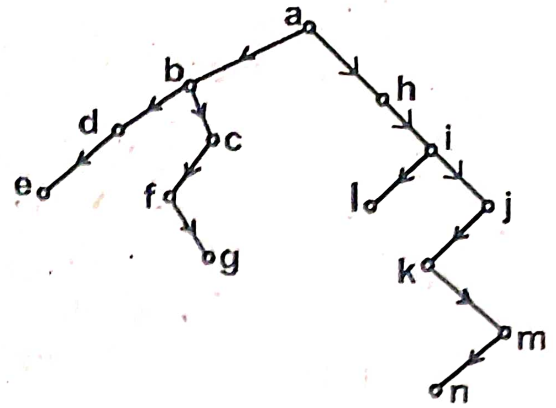
The elements of the above set are all ordered triples.

Example (36) : Give the binary tree representation for the following forest.



Solution : The two trees in the given forest can be combined to form a tree as follows.





This is the required binary tree.

5.19 Rrepresentation of Algebraic Expressions by Binary Trees :

In an algebraic expression, we assume that no operations such +, -, ×, ÷ etc can be performed until both of its arguments have been evaluated.

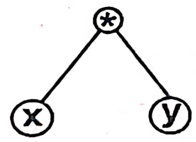
Every expression contains a central operator corresponding to the last computation that can be performed. The Central operator corresponds to the root of the binary tree. The left and right arguments of the central operator are the left and right child of the root respectively. This process continues until the expression is exhausted.

Example (37) : Represent the following algebraic expression using binary tree. Also draw the Venn diagrams.

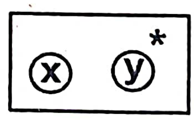
(i) $x * y$ (ii) $(a * b) \mid (c + d)$ (iii) $((a+b)\mid c)+(x+y)$ (iv) $(x + (y + z)) - (a \times (b + c))$

CSE S13 (v) $(3-2 (-(11-(9-4)))) \div (2 + (3+ (4 + 7)))$ (S-08)

Solution : (i) Given algebraic expression is $x * y$. Here $*$ is the Central operator and hence it is the root. The required binary tree and Venn diagram are drawn below.



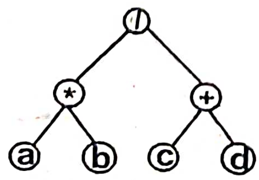
Binary Tree



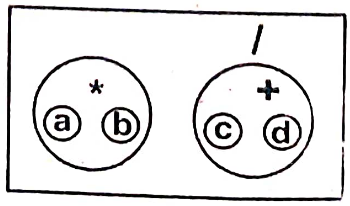
Venn Diagram

+
*

(ii) Given algebraic expression is $(a * b) \mid (c + d)$. Here $\mid$ is the central operator and hence it is the root. The required binary tree and Venn diagram are drawn below.

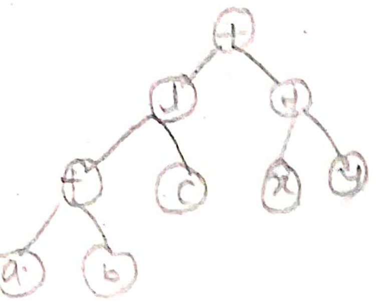


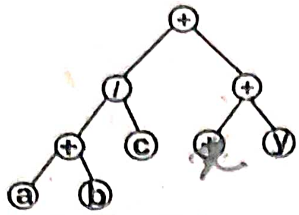
Binary Tree



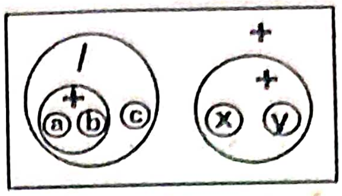
Venn Diagram

(iii) Given algebraic expression is ((a+ b) | c) + (x + y). Here + is the central operator and hence it is the root. The required binary tree and Venn diagram are drawn below.



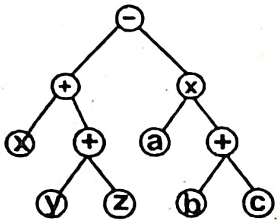


Binary Tree

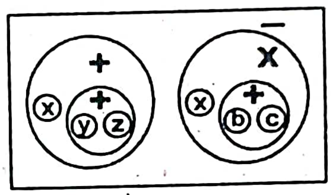


Venn Diagram

(iv) Given algebraic expression is (x + (y + z)) - (a × (b + c)). Here - is the central opeator and hence it is the root. The required binary tree and Venn diagram are drawn below.



Binary Tree



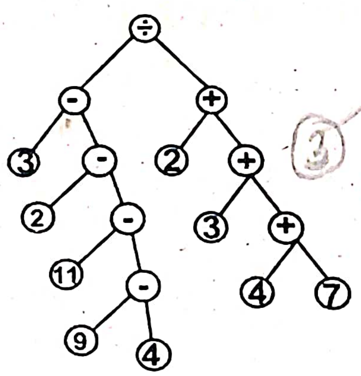
Venn Diagram

(v) Given algebraic expression is (3 - 2 (-(11- (9-4)))) ÷ (2 + (3 + (4 + 7)))

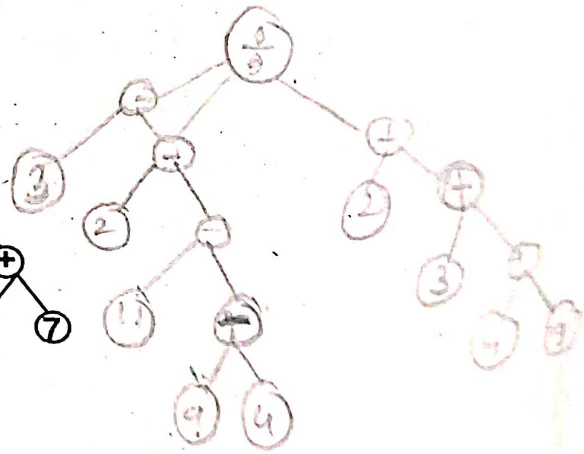
Here ÷ is the central operator and hence it is the root.

The required binary tree and Venn diagram are drawn below.

(3 −(2 −(11−(9−4)))

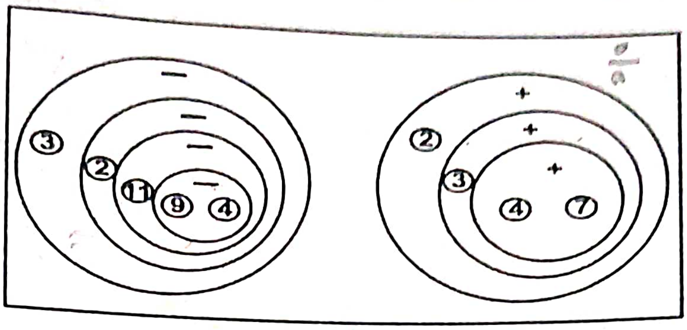


Binary Tree



$\therefore$ (3-2 (-(11- (9 -4)))) ÷ (2 +(3 + (4 + 7)))

$= (3 - 2 (- (11- 5))) \div (2 + (3 + 11)) = (3 - 2- 6) \div (2 + 14) = \dfrac{-5}{16}$



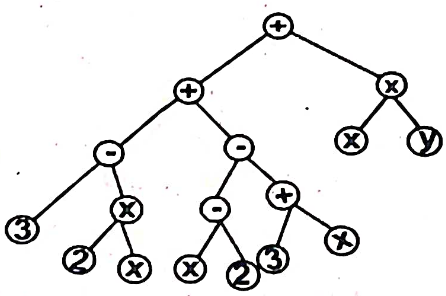
Venn Diagram

Example (38) : Represent the following algebraic expressions by binary tree.

(i) $[(3 - (2x)) + \{ (x-2)- (3 + x)\}] + xy$ (W-06,08)

(ii) $(2x + (3 - 4x)) + (x -(3 \times 11))$ (S-06)

Solution : (i) The given algebraic expression is $\{3 - (2x) + [(x-2) - (3 + x)\} + xy$. Here + is the central operator and hence it is the root. The required binary tree is drawn below.

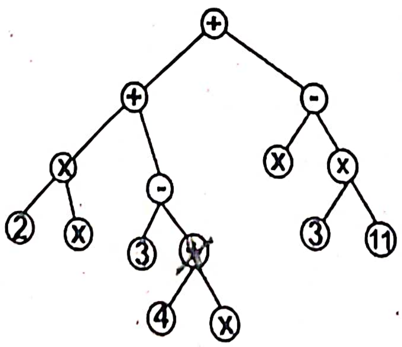


Binary Tree

(ii) The given algebraic expression is $(2x + (3 - 4x)) + (x - (3 \times 11)$.

Here + is the central operator and hence it is the root.

The required binary tree is drawn below.

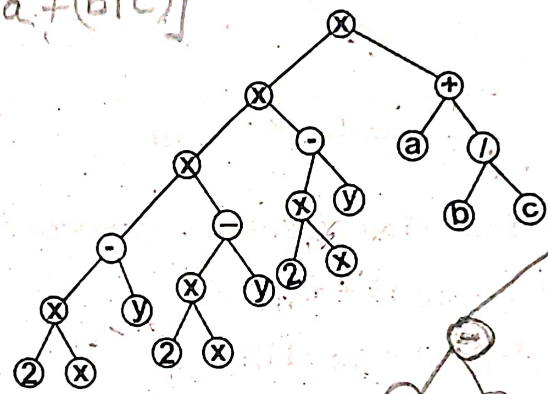


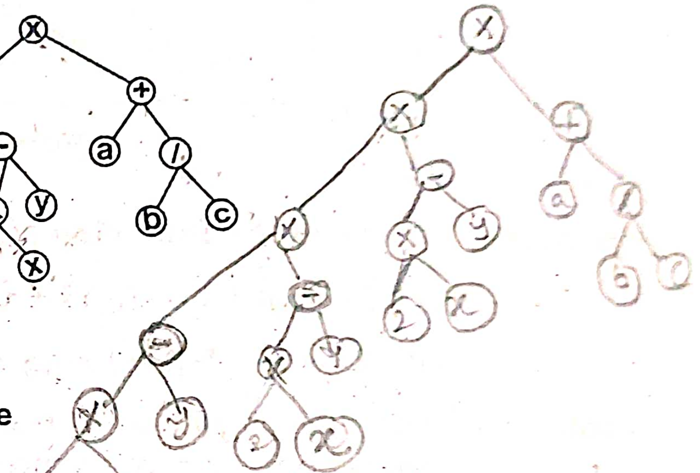
Binary Tree

Example (39) : Draw the binary tree for $(2x - y)^3$ [a + (b/c)] and find the subtree at the exponential operator. **(W-06)**

Solution : The given algebraic expression is $(2x - y)^3$ [a + (b|c)]. Here x is the central operator and hence it is the root. The required binary tree is drawn below.

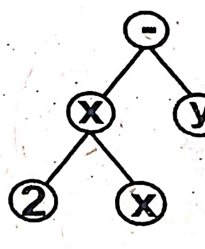
$[(2x-y)\times(2x-y)\times(2x-y)]\times[a+(b|c)]$

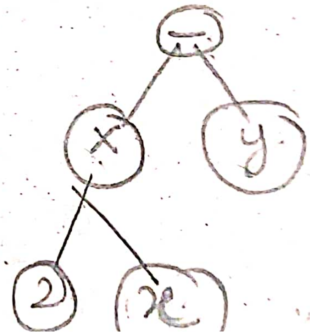




Binary Tree

Here 2x - y is the exponential operator with exponent 3. Its subtree is drawn below.





5.20 Tree Searching :