IT 4TO
CSE 308

Isomorphic graphs :

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs. Then G_1 and G_2 are said to be isomorphic if there exists a function $f: V_1 \to V_2$ such that

i) f is one-one and onto

ii) $(a, b) \in E_1 \Rightarrow (f(a), f(b)) \in E_2$, $\forall a, b \in V_1$

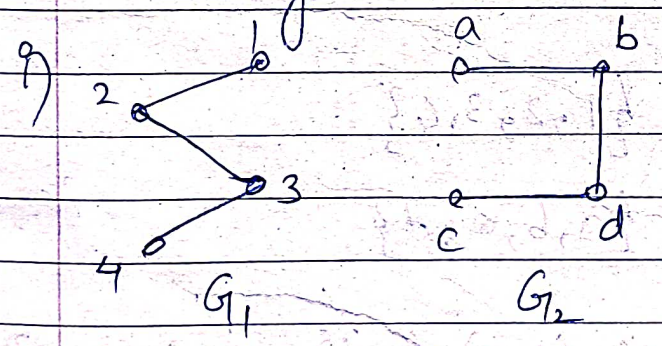
If G_1 is isomorphic to G_2 then it is denoted by $G_1 \simeq G_2$ or $G_1 \cong G_2$

Note:

i) If the number of vertices in G_1 and G_2 are unequal, then G_1 and G_2 cannot be isomorphic.

ii) If the number of edges in G_1 and G_2 are unequal, then G_1 and G_2 cannot be isomorphic.

Q6.(a) Find ~~the~~ which of the following graphs are isomorphic.



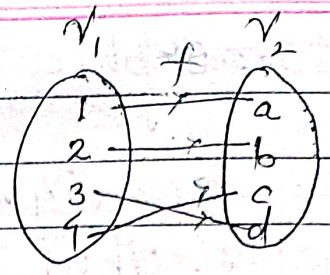
Sol^n Here $V_1 = V(G_1) = \{1, 2, 3, 4\}$

$V_2 = V(G_2) = \{a, b, c, d\}$

$E_1 = E(G_1) = \{(1,2), (2,3), (3,4)\}$

$E_2 = E(G_2) = \{(a,b), (b,d), (d,c)\}$

Let $f: V_1 \to V_2$



$f(1) = a$
$f(2) = b$
$f(3) = d$
$f(4) = c$

from the figure it is clear that $f: V_1 \to V_2$ is both one-one & onto.

Also $(1,2) \in E_1 \Rightarrow (f(1), f(2)) = (a,b) \in E_2$

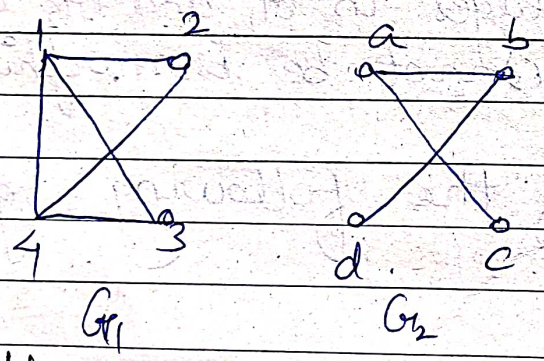
Similarly $(2,3) \in E_1 \Rightarrow (f(2), f(3)) = (b,d) \in E_2$

& $(3,4) \in E_1 \Rightarrow (f(3), f(4)) = (d,c) \in E_2$.

$\therefore (a,b) \in E_1 \Rightarrow (f(a), f(b)) \in E_2$, $\forall a,b \in V_1$

$\therefore G_1 \cong G_2$.

ii)



Sol^n Here $V_1 = V(G_1) = \{1,2,3,4\}$

$V_2 = V(G_2) = \{a,b,c,d\}$

$E_1 = E(G_1) = \{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

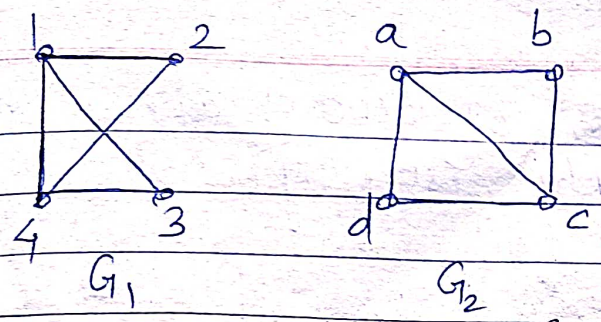
$E_2 = E(G_2) = \{(a,b), (a,c), (b,d)\}$

$n(E_1) = 5$, $n(E_2) = 3$

$\therefore n(E_1) \neq n(E_2)$

$\therefore G_1 \ncong G_2$

(iii)



Sol: Here $V_1 = V(G_1) = \{1, 2, 3, 4\}$

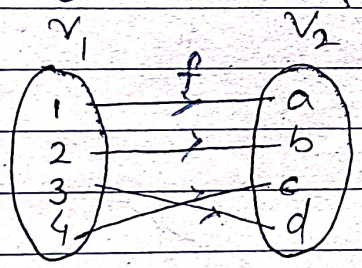
$V_2 = V(G_2) = \{a, b, c, d\}$

$E_1 = E(G_1) = \{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

$E_2 = E(G_2) = \{(a,b), (a,c), (a,d), (b,c), (c,d)\}$.

Let $f : V_1 \to V_2$

From the figure it is clear that $f : V_1 \to V_2$ is both one-one & onto



$f(1) = a$
$f(2) = b$
$f(3) = d$
$f(4) = c$

Also .. $(1,2) \in E_1 \Rightarrow (f(1), f(2)) = (a,b) \in E_2$.

$(1,3) \in E_1 \Rightarrow (f(1), f(3)) = (a,d) \in E_2$.

$(1,4) \in E_1 \Rightarrow (f(1), f(4)) = (a,c) \in E_2$

$(2,4) \in E_1 \Rightarrow (f(2), f(4)) = (b,d) \in E_2$

$(3,4) \in E_1 \Rightarrow (f(3), f(4)) = (d,c) \in E_2$

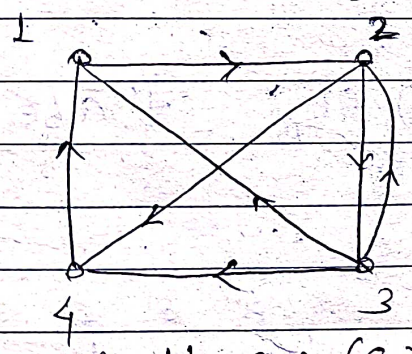
$\therefore (a,b) \in E_1 \Rightarrow (f(a), f(b)) \in E_2 \quad \forall\, a, b \in V_1$

$\therefore G_1 \cong G_2$

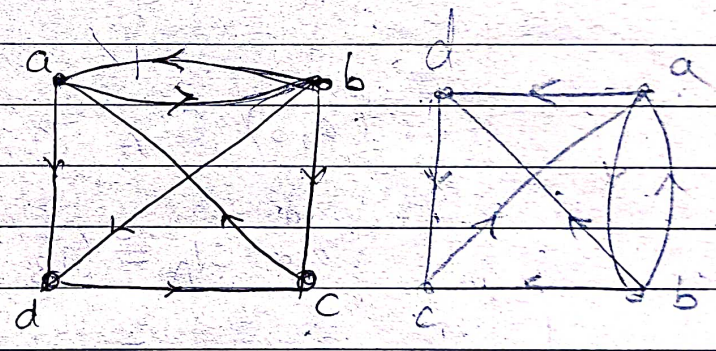
Q7. Draw the digraphs of the following adjacency matrices A and B. Are these graphs isomorphic? Also draw the digraph corresponding to A^T and B^T. Are these graph isomorphic?

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Sol^n Since the order of matrices A and B is 4×4 hence their digraphs will have 4 vertices say 1, 2, 3, 4 & a, b, c, d respectively. The required digraphs are:



graph of A (G_A)



graph of B (G_B)

Here $V_A = V(G_A) = \{1, 2, 3, 4\}$

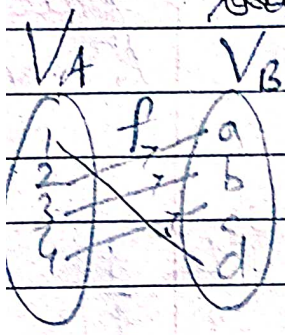
$V_B = V(G_B) = \{a, b, c, d\}$

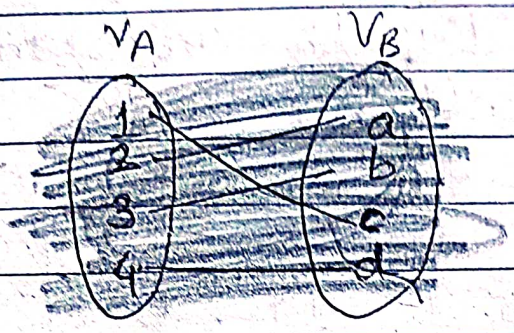
$E_A = E(G_A) = \{(1,2), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1)\}$

$E_B = E(G_B) = \{(a,b), (a,d), (b,a), (b,c), (b,d), (c,a), (d,c)\}$

Let $f : V_A \to V_B$

From the fig. it is clear that $f: V_A \to V_B$ ~~both~~ is both one-one and onto.





$f(1) = d = d$
$f(2) = a = a$
$f(3) = b = b$
$f(4) = c = c$

Also $(2,3) \in E_A \Rightarrow (f(2), f(3)) = (a, b) \in E_B$.
$(3,1) \in E_A \Rightarrow (f(3), f(1)) = (b, d) \in E_B$.
$(3,4) \in E_A \Rightarrow (f(3), f(4)) = (b, c) \in E_B$.
$\vdots$

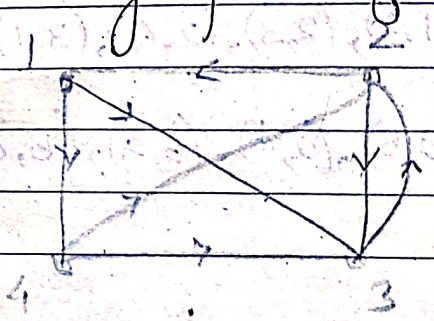
$\therefore (a, b) \in E_A \Rightarrow (f(a), f(b)) \in E_B \quad \forall\, a, b \in E_A$.

$\therefore G_A \cong G_B$.

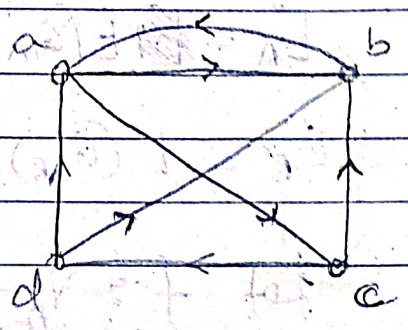
Now

$$A^T = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 \end{array}, \quad B^T = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 1 & 0 & 0 \end{array}$$

ii) The diagraphs of A^T and B^T are drawn below:



G_{A^T}



G_{B^T}

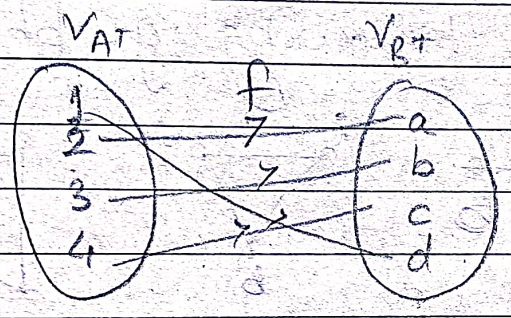
Here, $V_{AT} = V(G_{AT}) = \{1, 2, 3, 4\}$

$V_{BT} = V(G_{BT}) = \{a, b, c, d\}$.

$E_{AT} = E(G_{AT}) = \{(1,3), (1,4), (2,1), (2,3), (3,2), (4,2), (4,3)\}$

$E_{BT} = E(G_{BT}) = \{(a,b), (a,c), (b,a), (c,b), (c,d), (d,a), (d,b)\}$

Let $f : V_{AT} \to V_{BT}$.



$f(1) = d$
$f(2) = a$
$f(3) = b$
$f(4) = c$.

From the fig., it is clear that $f : V_A \to V_B$ is both one-one & onto.

Also, $(1,3) \in E_{AT} \Rightarrow (f(1), f(3)) = (d, b) \in E_{BT}$
$(2,3) \in E_{AT} \Rightarrow (f(2), f(3)) = (a, b) \in E_{BT}$.
$(4,3) \in E_{AT} \Rightarrow (f(4), f(3)) = (c, b) \in E_{BT}$

$\therefore (a,b) \in E_{AT} \Rightarrow (f(a), f(b)) \in E_B \quad \forall a, b \in E_A$

$\therefore G_1 \cong G_B$.

S07, 12, W08
W06, S07

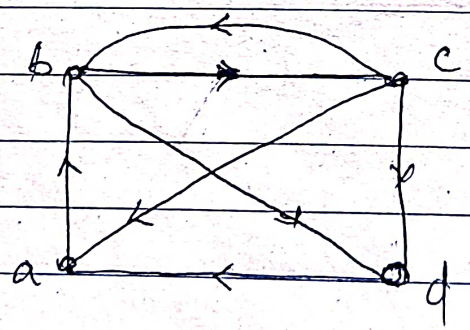
Q8. (a) Draw a digraph corresponding to the following adjacency matrix and interpret the results

S16 AA^T, A^TA, A^2, A^3, A^4.

~~Sol^n~~

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Sol^n The given adjacency matrix is a square matrix of order 4×4. Hence the graph will have 4 vertices say a, b, c, d. The required digraph for the given adjacency matrix is :



Now $A^T = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$$AA^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Interpretation of AA^T: The diagonal entries of AA^T shows the outdegree of the nodes.

Node	Outdegree
a	1
b	2
c	3
d	1

Interpretation of A^TA: The diagonal entries of A^TA shows the indegree of the nodes.

Node	Outdegree
a	2
b	2
c	1
d	2

Now

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Interpretation of A^2: The elements of first row of A^2 show the number of paths of length 2 from vertex a to the vertices a, b, c, d resp.

Edges	No. of paths of length 2
(a, a)	0
(a, b)	0
(a, c)	1
(a, d)	1

Now ~~Finds~~ $A^3 = A^2 \cdot A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 2 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Interpretation of A^3: The elements of first row of A^3 show the number of paths of length 3 from the vertex a to the vertices a, b, c, d resp.

Edge	No. of paths of length 3
(a, a)	2
(a, b)	1
(a, c)	0
(a, d)	1

Now

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 2 & 3 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Interpretation of A^4: The elements of first row of A^4 show the numbers of paths of length 4 from the vertex a to the vertices a, b, c, d resp.

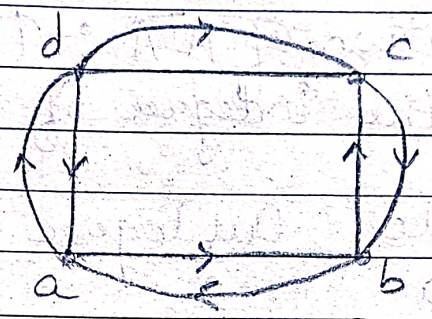
Edge	No. of paths of length 4
(a, a)	1
(a, b)	2
(a, c)	1
(a, d)	1

CSE S13

Q8 (b) Draw the digraph represented by the given adjacency matrix and interpret AA^T, A^TA, A^2.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Sol: The given adjacency matrix is a square matrix of order 4×4. Hence the graph will have four vertices say a, b, c, d. The required digraph for the given adjacency matrix is :



Now A^T = Transpose of $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Now $AA^T = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$

Interpretation of AA^T : The diagonal of AA^T shows the outdegree of nodes.

Node	Outdegree
a	2
b	2
c	2
d	2

$$A^T A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

Interpretation of $A^T A$: The diagonal of $A^T A$ shows the indegree of node.

Node	Outdegree
a	2
b	2
c	2
d	2

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

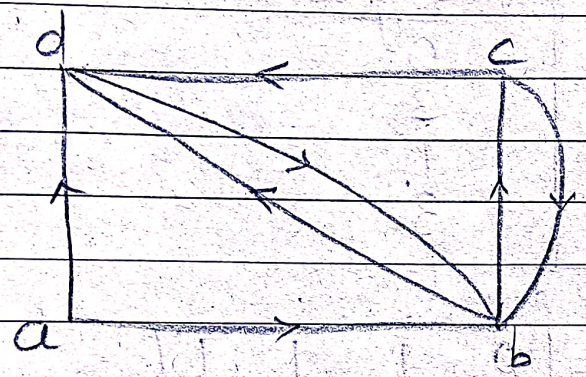
Interpretation of A^2 : The elements of first row of A^2 shows the number of paths of length 2 from vertex a to the vertices a, b, c, d respectively.

Edges	No. of paths of length 2
(a,a)	2
(a,b)	0
(a,c)	2
(a,d)	0

Q8.(c) Draw digraph corresponding to $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

& interpret AA^T, A^TA, ~~A~~, A^2.

Soln The given adjacency matrix is a square matrix of order 4×4. Hence the graph will have four vertices say a, b, c, d. The required digraph for the given adjacency matrix is



$$A^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Now; $$AA^T = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Interpretation of AA^T: The diagonal of AA^T shows the outdegree of nodes.

Node	Outdegree
a	2
b	2
c	2
d	1

$$A^TA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

Interpretation of A^TA : The diagonal of A^TA shows the indegree of node.

Node	Indegree
a	0
b	3
c	1
d	3

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

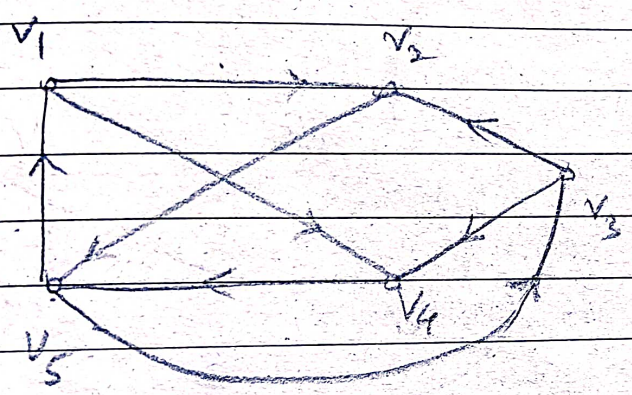
Interpretation of A^2 : The elements of first row of A^2 shows the number of paths of length 2 from vertex a to the vertices a, b, c, d respectively.

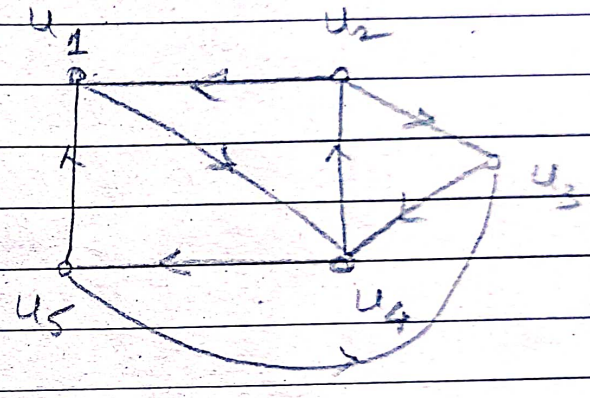
Edges	No. of paths of length 2
(a, a)	0
(a, b)	1
(a, c)	1
(a, d)	1

CE 506 W07, S11
IT 507

Q.13(a) Draw graphs corresponding to adjacency matrix & show that they are isomorphic

$$M_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad M_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$





II WOT

Q 6b) Draw digraphs corresponding to adjacency matrices

(B)

$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 & 0 \end{array}, \quad B = \begin{array}{c|cccc} & u_1 & u_2 & u_3 & u_4 \\ \hline u_1 & 0 & 1 & 0 & 1 \\ u_2 & 1 & 0 & 1 & 0 \\ u_3 & 1 & 1 & 0 & 1 \\ u_4 & 1 & 1 & 1 & 0 \end{array}$$

and

prove that digraphs of A and B are isomorphic.

IT S10, W11
CSE W09, S12

Q. 13 (C) Draw a digraphs corresponding to adjacency matrix
(22) A, B, A^T and B^T where

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

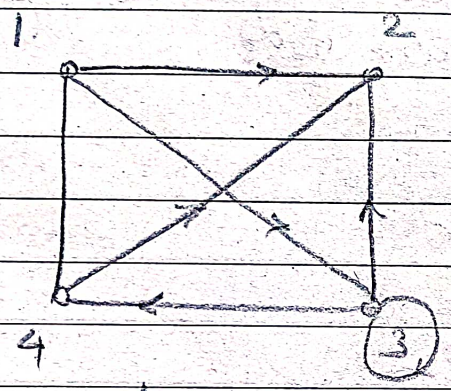
Show that the diagraphs corresponding to A^T and B^T are isomorphic.

IT 2008

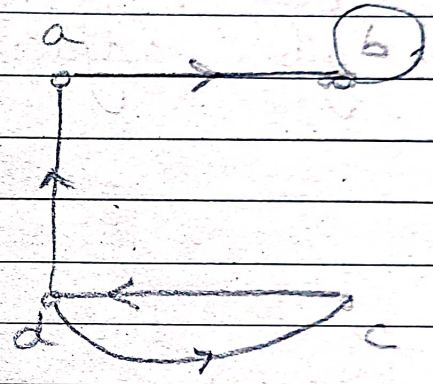
Q13 d) Draw the digraph corresponding to adjacency matrices A and B.

$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 1 & 1 \\ 4 & 0 & 1 & 0 & 0 \end{array} \quad \& \quad B = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

Are these two digraphs isomorphic?



Graph of A



Graph of B

From the digraphs,
Graph of A has 5 edges and 1 Loop.
Graph of B has 4 edges and 1 Loop.

∵ there is one-one correspondence between vertices, but no one-one correspondence between edges.

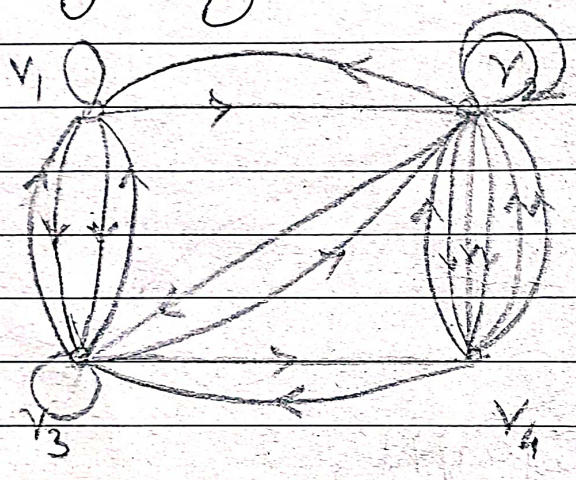
∴ These two digraphs are not isomorphic

CSE 508, IT 409

Q14. Draw a graph corresponding to the adjacency matrix.

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 2 & 0 \\ 1 & 2 & 1 & 3 \\ 2 & 1 & 0 & 1 \\ 0 & 3 & 1 & 0 \end{bmatrix} \end{array}$$

Sol^n: The given matrix is a square matrix of 4×4. Hence the graph will have 4 vertices say v_1, v_2, v_3, v_4. The required graph for given adjacency matrix is:



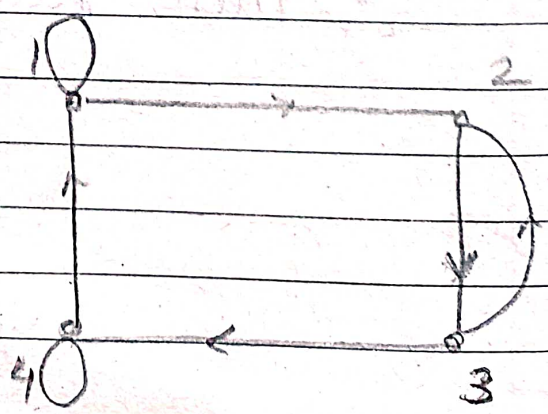
Q15. Draw the digraph corresponding to the adjacency matrix.

CSE 411

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Sol^n: Let $X = \{1, 2, 3, 4\}$

The reqd. digraph is as follows:

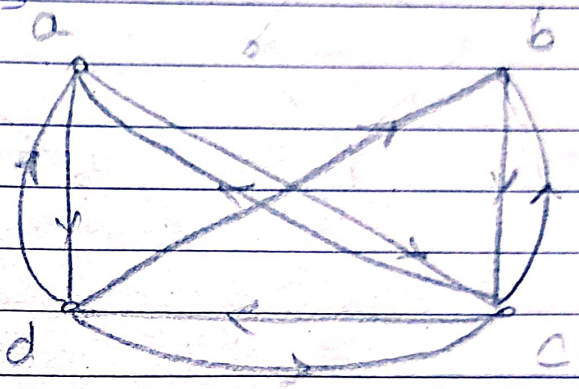


IT011

ii)

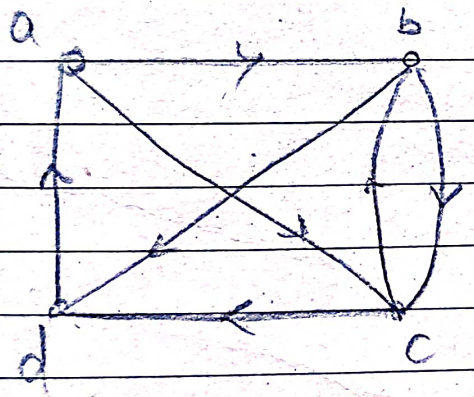
$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 1 & 1 \\ b & 0 & 0 & 1 & 0 \\ c & 1 & 1 & 0 & 1 \\ d & 1 & 1 & 1 & 0 \end{array}$$

Sol^n:

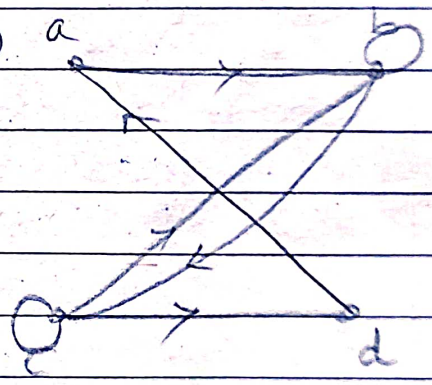


Q16. Find the adjacency matrix of the following graph:

(a)



(b)



Sol^n: The adjacency matrix is drawn below:

$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 1 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 0 & 0 \end{array}$$

Sol^n: The adjacency matrix is drawn below:

$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 1 & 1 \\ d & 1 & 0 & 0 & 0 \end{array}$$

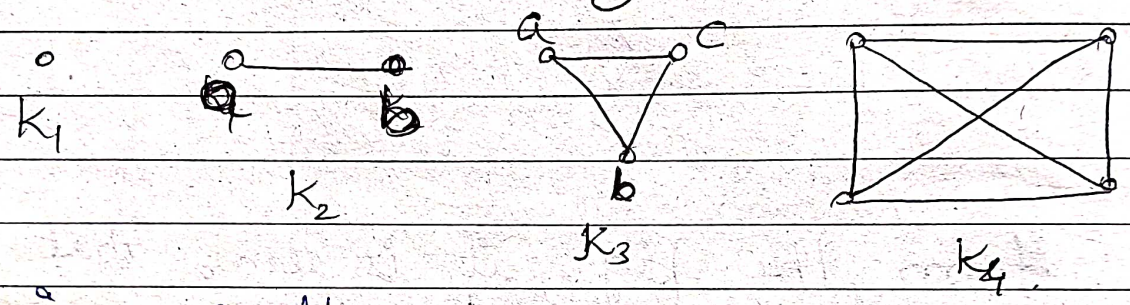
Types of graph :

CSE W10,11

W15 1. **Complete graph :** $K_n = \frac{n(n-1)}{2}$ edges

A graph $G=(V,E)$ is said to be complete if every vertex of G is adjacent to every other vertex. A complete graph with n vertices is denoted by K_n. There are $\frac{n(n-1)}{2}$ edges in K_n.

for eg. Consider the following graph.

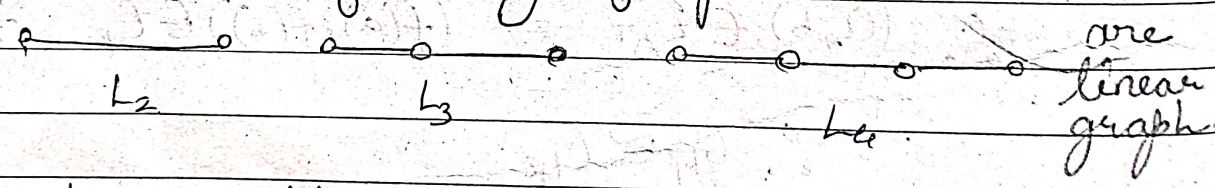


2. **Linear graph :**

A graph $G=(V,E)$ is said to be linear if all the vertices of G appear along a straight line. A linear graph with n vertices is denoted by L_n.

Consider the following graph

for eg.

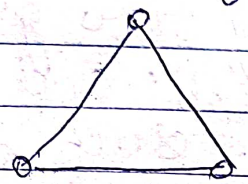


CSE W09, S11

W15 3. **Regular graph :**

A graph $G=(V,E)$ is said to be regular if all the vertices of G have the same degree. If the degree of each vertex is r, then the graph is called a regular graph of degree r.

for eg. Consider the following graph :



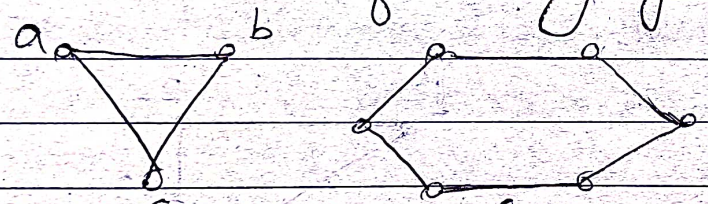
K_3

In K_3, each vertex has degree 2. Hence K_3 is a regular graph.

4. Cycle :

The cycle $C_n (n \geq 3)$ consists of n vertices namely $v_1, v_2, \ldots, v_n$ and edges $(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n), (v_n, v_1)$

eg. Consider the following graph.



C_3 C_6 are cycles.

5. Wheel :

The wheel W_n is obtained when an additional vertex to the cycle C_n is connected with all other vertices.

for eg. Consider the following graph.

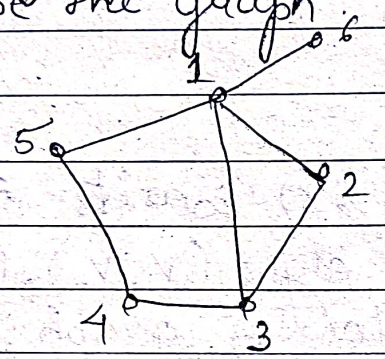
W_4 W_7

CSE 511

6. Subgraph :

If G and H are any two graphs with vertex sets V(G) and V(H) and edge sets E(G) and E(H) respectively such that $V(H) \subseteq V(G)$ & $E(H) \subseteq E(G)$; then H is called the subgroup of G & G is called the subgraph of H.

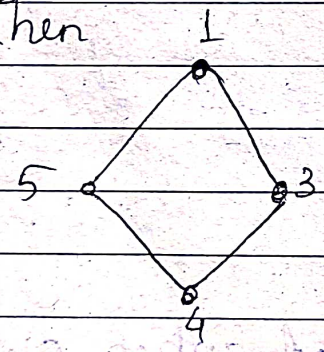
for eg: Let G be the graph.



G

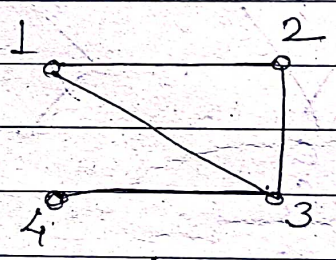
Then

Subgraph →



H_1

&



H_2

If $V(H) \subset V(G)$ and $E(H) \subset E(G)$, then H is called a proper subgraph of G. If $V(H) = V(G)$, then H is called a spanning subgraph of G. A spanning subgraph need not contain all the edges of G.

for eg:

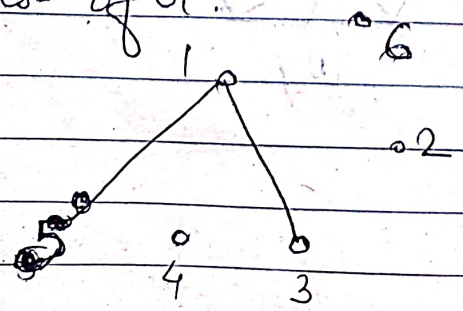


fig: spanning graph

S06,09,11 CSE W06,08,10 IT S06 S10 W07

Path, reachability & connectedness.

W15

i) Path :

A path in a graph $G=(V,E)$ is a sequency of the form $v_1, v_2, v_3, \ldots, v_k$ where each vertex is adjacent to the next. In other words, Starting with a vertex v_1 and travelling along the vertices $(v_1, v_2), (v_2, v_3), \ldots$ One can reach a vertex v_k

ii) Length :

The number of edges appearing in the sequence of a path is called the length of the path. If the length is zero, then the path has no edges. A path which contains only a single vertex is called Trivial path and such a vertex is called Trivial vertex.

iii) Closed path :

A closed path is a path whose beginning and ending points are same, otherwise the path is called an open path.

W16 S16

iv) Trail :

RV ✓
RE ✗

A trail from a vertex v to a vertex u is a path that does not contain any repeated edge. Repeated vertices are allowed in a trail.

IT S06,09,13, W07,09,11 W14

v) Simple path :

RV ✗
RE ✗

A simple path from a vertex v to a vertex u is a path that does not contain any repeated vertex & repeated edge.

CSE W11

(vi) Circuit : { RV ✓ RE ✗

A circuit is a closed path that does not contain any repeated edge.

CSE S13

(vii) Cycle : { first & last only RV

A cycle is a circuit that does not have a repeated vertex except the first & the last.

IT S09, W09

(viii) Elementary path : { RV ✗

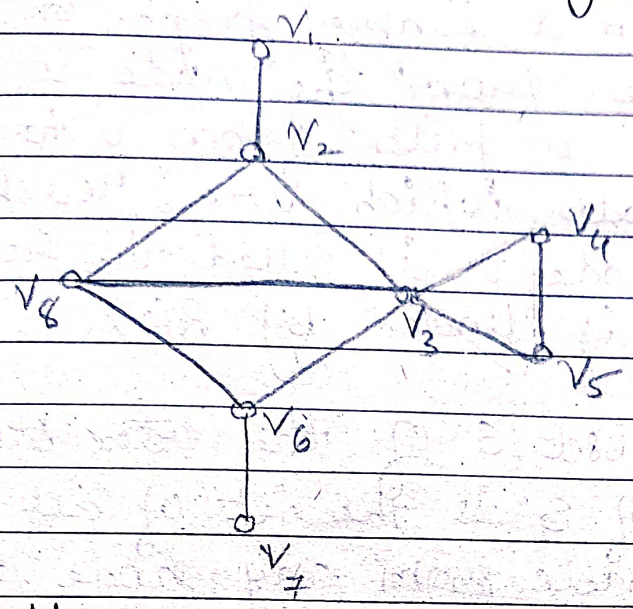
W14

A path in graph $G = (V, E)$ is called elementary if no vertex occurs more than once.

- All the above definitions are arranged in tabular form as follows :

	Repeated vertex	Repeated edge	Same Starting & ending point
Path	allowed	allowed	allowed
Closed path	allowed	allowed	Yes
Trail	allowed	no	allowed
Simple path	no	no	no
Circuit	allowed	no	yes
Cycle	first & last only	no	yes

For eg: Consider the graph.



(1) Path :

i) $v_1 v_2 v_3 v_6 v_8$

ii) $v_8 v_3 v_4 v_5 v_3 v_6 v_7$

(2) Trail : (i) $v_8 v_2 v_3 v_4 v_5 v_3 v_6$ (ii) $v_7 v_6 v_3 v_2 v_8 v_6$

(3) Simple paths : (i) $v_8 v_2 v_3 v_4 v_5$ (ii) $v_2 v_3 v_6 v_7$

(4) Closed paths : (i) $v_8 v_2 v_3 v_6 v_8$ (ii) $v_4 v_5 v_3 v_8 v_2 v_3 v_4$

(5) Circuits : (i) $v_6 v_8 v_2 v_3 v_6$ (ii) $v_5 v_4 v_3 v_2 v_8 v_3 v_5$

(6) Cycles : (i) $v_3 v_4 v_5 v_3$ (ii) $v_2 v_3 v_6 v_8 v_2$.

IT 509
CSE W06,08,09,10

W16
S16

Reachability: Reachable Node

A node v in a simple graph G is said to be reachable from the node u of G if there exists a path from u to v. The sets of nodes which are reachable from a given node v is called the reachable set of v and is denoted by R(v).

For any subset S of the vertex set V, the reachable set of S is the set of all nodes which are reachable from any node of S and this set is denoted by R(S).

IT W06

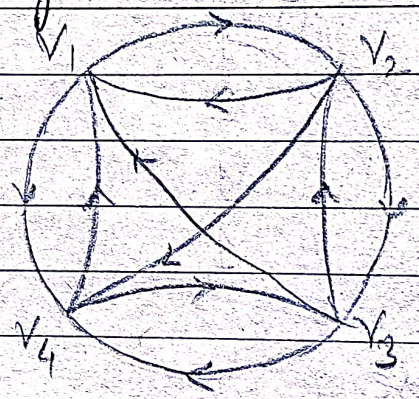
Node Base :

In a digraph $G = (V, E)$, a subset $A \subseteq V$ is called a node base if its reachable set is V and if no proper subset of A has this property.

- Every isolated pt. in a digraph must be present in a node base. Any node whose indegree is zero must be present in a node base.
- No node in the node base is reachable from another node in the node base. From the nodes lying on an elementary cycle any one node could be choosen as an element of a node base.

- Any node that does not have indegree zero and does not lie on a cycle cannot be present in a node base. In an acyclic graph, a node base contains only those nodes whose indegree is zero.
- For a given digraph, we may have more than one node base, however every node base has the same number of nodes.

for eg. Consider the digraph.



Here $R(v_1) = \{v_2, v_3, v_4\}$

$R(v_2) = \{v_1, v_3, v_4\}$

$R(v_3) = \{v_1, v_2, v_4\}$

$R(v_4) = \{v_1, v_2, v_3\}$

$R(\{v_1, v_2\}) = \{v_3, v_4\}$ ... and so on.

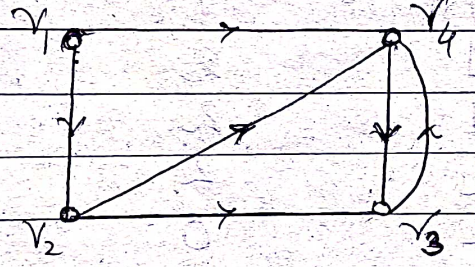
IT W08

Q9. Draw the digraph corresponding to the adjacency matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Find three different elementary paths from v_1 to v_3. What is the shortest distance between v_1 and v_3? Is there a cycle in this graph?

Sol^n Since the given adjacency matrix is of order 4×4, the digraph will have 4 vertices say v_1, v_2, v_3, v_4. The required digraph is

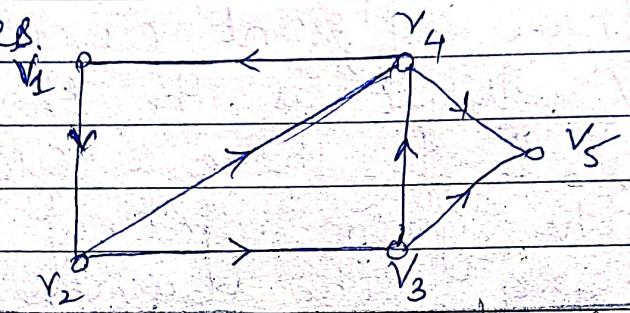


Three different elementary paths from v_1 to v_3 are $P_1 = \{v_1 v_4 v_3\}$, $P_2 = \{v_1 v_2 v_3\}$, $P_3 = \{v_1 v_2 v_3 v_4\}$.

From P_1, P_2, P_3 it is clear that the shortest distance between v_1 and v_3 is 2.

In this digraph the paths $v_3 v_4 v_3$ & $v_4 v_3 v_4$ are cycles.

Q. 10(a) Find all the indegrees and outdegrees of the nodes of the following digraph. Give all the elementary cycles of the digraph. Obtain an acyclic digraph by deleting one edge. List all the nodes which are reachable from other nodes.

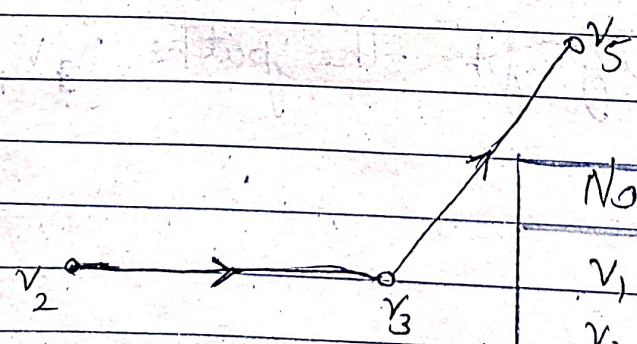


Sol.

Node	Indegree	Outdegree	Total degree
v_1	1	1	2
v_2	1	2	3
v_3	1	2	3
v_4	2	2	4
v_5	2	0	2

The elementary cycles of this digraph are: $v_1 v_2 v_4 v_1$ and $v_1 v_2 v_3 v_4 v_1$.

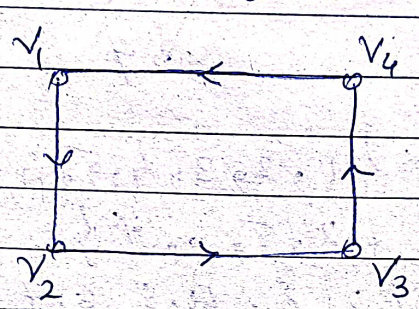
By deleting the edge (v_4, v_1), we can obtain an acyclic digraph as below:



Nodes	Reachable from nodes
v_1	v_2, v_3, v_4
v_2	v_1, v_3, v_4
v_3	v_1, v_2, v_4
v_4	v_1, v_2, v_3

DTS11

Q10(b) Find all the indegrees and outdegrees of the nodes of the following diagram. Give all the elementary cycles of this digraph. Obtain an acyclic digraph by deleting one edge. List all the nodes which are reachable from other nodes.



Sol^n

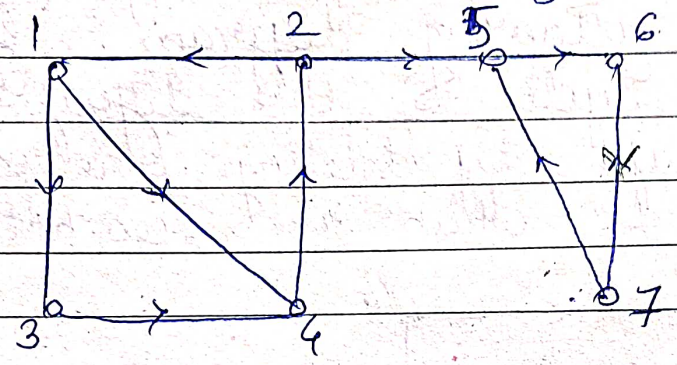
Node	Indegree	Outdegree	Total degree
v_1	1	1	2
v_2	1	1	2
v_3	1	1	2
v_4	1	1	2

The elementary cycles of this digraph are $v_1 v_2 v_3 v_4$.

Node	Reachable from nodes
v_1	v_2, v_3, v_4
v_2	v_1, v_3, v_4
v_3	v_1, v_2, v_4

Q.11 Find node base of the following digraphs:

CSE W06 W08 IT 509 (i)



Soln: Here $R(1) = \{1, 2, 3, 4, 5, 6, 7\}$

$R(2) = \{1, 2, 3, 4, 5, 6, 7\}$

$R(3) = \{1, 2, 3, 4, 5, 6, 7\}$

$R(4) = \{1, 2, 3, 4, 5, 6, 7\}$

$R(5) = \{5, 6, 7\}$

$R(6) = \{5, 6, 7\}$

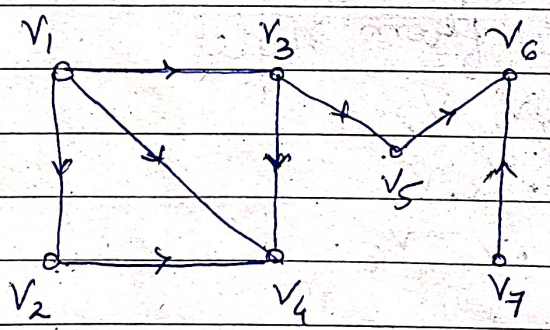
$R(7) = \{5, 6, 7\}$.

Vertex set $= V = \{1, 2, 3, 4, 5, 6, 7\}$

We observe that $R(1) = V$.

$\therefore$ Node Base $= \{1\}$.

(ii)



Soln: Here $R(v_1) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$R(v_2) = \{v_2, v_4\}$

$R(v_3) = \{v_3, v_4, v_5, v_6\}$

$R(v_4) = \{v_4\}$

$R(v_5) = \{v_5, v_6, v_7\}$

$R(v_6) = \{v_6\}$

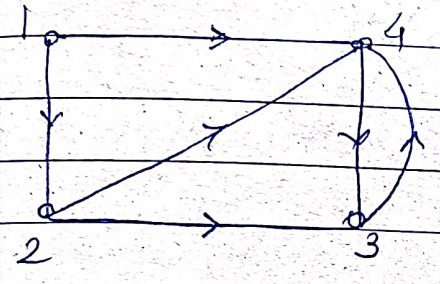
$R(v_7) = \{v_6, v_7\}$

Vertex set $= V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

We observe that none of the reachable sets is equal to the set of vertices V.

$\therefore$ This digraph does not contain node base.

(III)



Sol^n Here ~~$R(v_1) = \{v_1\}$~~ $R(1) = \{1, 2, 3, 4\}$

$R(2) = \{2, 3, 4\}$

$R(3) = \{3, 4\}$

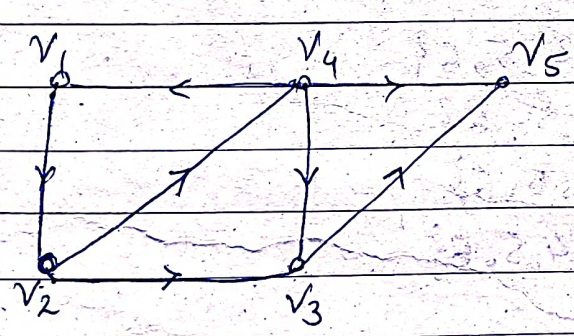
$R(4) = \{3, 4\}$

Vertex set $V = \{1, 2, 3, 4\}$

We observe that $R(1) = V$

$\therefore$ Node base $= \{1\}$.

CSE 513
IT 1920
(IV)



Sol^n Here $R(v_1) = \{v_1, v_2, v_3, v_4, v_5\}$

$R(v_2) = \{v_1, v_2, v_3, v_4, v_5\}$

$R(v_3) = \{v_3, v_5\}$

$R(v_4) = \{v_1, v_2, v_3, v_4, v_5\}$

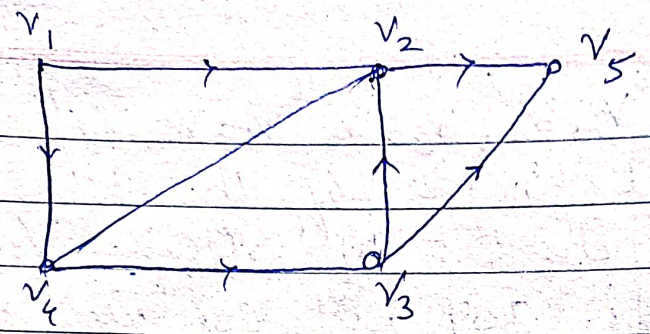
$R(v_5) = \{v_5\}$

Vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$

We observe that $R(v_1) = R(v_2) = R(v_4) = V$.

$\therefore$ Node Base $= \{v_1\}, \{v_2\}, \{v_4\}$

(v)



Solⁿ. Here $R(v_1) = \{v_1, v_2, v_3, v_4, v_5\}$

$R(v_2) = \{v_2, v_5\}$

$R(v_3) = \{v_2, v_3, v_5\}$

$R(v_4) = \{v_2, v_3, v_4, v_5\}$

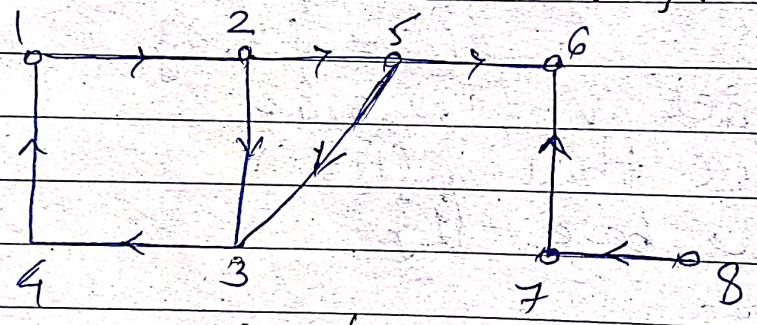
$R(v_5) = \{v_5\}$

Vertex Set $V = \{v_1, v_2, v_3, v_4, v_5\}$

We observe that $R(v_1) = V$

$\therefore$ Node Base $= \{v_1\}$.

vi)



Solⁿ. Here $R(1) = \{1, 2, 3, 4, 5, 6\}$

$R(2) = \{1, 2, 3, 4, 5, 6\}$

$R(3) = \{1, 2, 3, 4, 5, 6\}$

$R(4) = \{1, 2, 3, 4, 5, 6\}$

$R(5) = \{5, 6\}$

$R(6) = \{6\}$

$R(7) = \{6, 7\}$

$R(8) = \{6, 7, 8\}$

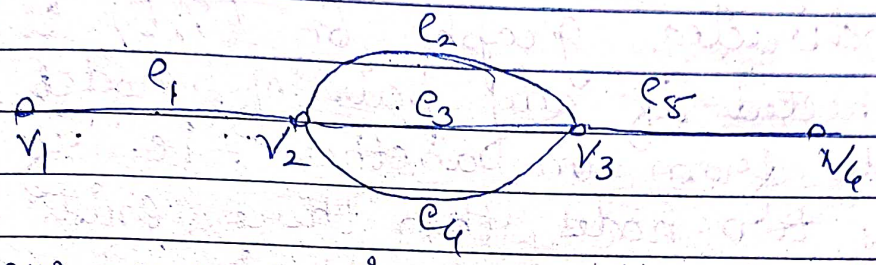
Vertex Set $= V = \{1, 2, 3, 4, 5, 6, 7, 8\}$

We observe that none of the reachable sets is equal to the set of vertices V.

This digraph does not contain a node base.

CSE S10.

Q.17 Consider the following graph :



i) How many simple paths are there from v_1 to v_4?
ii) How many trails are there from v_1 to v_4?
iii) How many paths are there from v_1 to v_4?

Solⁿ (i) Simple paths from v_1 to v_4 are :
$(v_1 e_1 v_2 e_2 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_3 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_4 v_3 e_5 v_4)$.
Hence there are 3 simple paths from v_1 to v_4.

(ii) Trails from v_1 to v_4 are :
$(v_1 e_1 v_2 e_2 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_3 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_4 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_2 v_3 e_3 v_2 e_4 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_3 v_3 e_2 v_2 e_4 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_3 v_3 e_4 v_2 e_2 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_4 v_3 e_3 v_2 e_2 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_4 v_3 e_2 v_2 e_3 v_3 e_5 v_4)$
Hence there are nine trails from v_1 to v_4

(iii) Paths from v_1 to v_4 will be infinite, because in a path a vertex as well as an edge may repeat.
Some paths are $(v_1 e_1 v_2 e_2 v_3 e_5 v_4)$,
$(v_1 e_1 v_2 e_4 v_3 e_3 e_2 v_1 v_2 e_2 v_3 e_5 v_4)$ etc.