Date

- Permutations : arrangement

$$^nP_r = \frac{n!}{(n-r)!}$$ $n \to$ objects, $r \to$ time

- Combinations : Selections

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

Pigeon-Hole principle:

The Pigeon-Hole Principle, also known as Dirichlet Drawer Principle is applicable in many problems where we want to show that a given situation can occur. Many results in combinatorial theory come from this principle. The description of this principle is often given in terms of pigeons & nesting holes (pigeon holes).

Suppose that m pigeons are placed into n nesting holes, where $m > n$. Then atleast one nesting hole contains two or more pigeons.

Generalised Pigeon-Hole principle :

If m pigeons are placed into n nesting holes, then one of the nesting holes must contain atleast $\left(\frac{m-1}{n}\right)+1$ pigeons.

Note :- If m and n are positive integers $\left(\frac{m}{n}\right)$ stands for the largest integer less than or equal to the rational number $\frac{m}{n}$.

Hence $\left(\frac{3}{2}\right) = (1.5) = 1$,

$\left(\frac{9}{4}\right) = (2.25) = 2$,

$\left(\frac{19}{3}\right) = (6.33) = 6$,

$\left(\frac{6}{2}\right) = (3) = 3$ etc.

CSE 509 . 314

Q1. Show that if any 30 people are selected from a group, then we may select a subset of 5 so that all 5 were born on the same day of the week.

Soln: By generalised pigeon hole principle, WKT if m pigeons are placed into n nesting holes, then one of the nesting hole must contain $\left(\frac{m-1}{n}\right)+1$ pigeons.

Here we treat the 30 people as pigeons (m) & 7 days of week as nesting holes

i.e. $m=30$, $n=7$

Hence by generalised pigeon hole principle,

$$\left(\frac{m-1}{n}\right)+1 = \frac{30-1}{7}+1$$

$$= \frac{29}{7}+1$$

$$= (4.14)+1$$

$$= 4+1$$

$$= 5$$

i.e. atleast 5 people have been born on the same day of the week.

2. Find the minimum number of students in a class to be sure that four out of them are born in the same month?

Sol: By generalised Pigeon-Hole principle:
If m pigeons are placed into n nesting holes, then one of the nesting holes must contain atleast $\left[\frac{m-1}{n}\right]+1$ pigeons.

Here we treat the required number of students as pigeons (m) and 12 months of an year as nesting holes (n)

Here we have to find the minimum no. of students such that 4 out of them are born in the same month.

Hence by generalised pigeon hole principle,

$$\left(\frac{m-1}{n}\right)+1=4$$

$$\frac{m-1}{12}+1=4$$

$$m-1=3\times12$$

$$m-1=36$$

$$\boxed{m=37}$$

∴ The required no. of students is 37.

3. If 9 books are to be Kept in 4 shelves, then show that there must be atleast one shelf which contains atleast 3 books.

Sol. By generalised Pigeon-Hole principle:
If m pigeons are placed into n nesting holes, then one of the nesting holes must contain atleast $\left[\frac{m-1}{n}\right]+1$ pigeons.

Here we treat the 9 books as pigeons (m) and 4 shelves as nesting holes (n).
ie. $m=9$, $n=4$

Hence by generalised pigeon hole principle,

$$\left[\frac{m-1}{n}\right]+1 = \left[\frac{9-1}{4}\right]+1$$
$$= \frac{8}{4}+1$$
$$= 2+1$$
$$= 3$$

ie. atleast one shelf contains 3 books atleast.

4. Show that if 8 persons are choosen from a group, then atleast 2 of them are born on the same day of the week.

Sol. By generalised Pigeon-Hole principle:

If m pigeons are placed into n nesting holes, then one of the nesting holes must contain atleast $\left[\frac{m-1}{n}\right]+1$ pigeons.

Here we treat the number of persons as pigeons (m) and 7 days of a week as nesting holes or pigeon holes (n)

i.e. $m=8$, $n=7$

Hence by generalized pigeon hole principle,

$$\left[\frac{m-1}{n}\right]+1=\left[\frac{8-1}{7}\right]+1=\left[\frac{7}{7}\right]+1=1+1=2$$

i.e. atleast 2 persons were born on the same day of the week.

5. Show that if an institute contains 13 professors, then two of them are born in the same month.

Sol. By generalised Pigeon-Hole principle, If m pigeons are placed into n nesting holes, then one of the nesting holes must contain atleast $\left\lfloor \frac{m-1}{n} \right\rfloor + 1$ pigeons.

Here we treat the number of professors as pigeons (m) & 12 months of an year as nesting holes (n)

i.e. $m = 13$, $n = 12$.

Hence by generalized pigeon-Hole principle,

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = \left\lfloor \frac{13-1}{12} \right\rfloor + 1$$

$$= \left\lfloor \frac{12}{12} \right\rfloor + 1$$

$$= 1 + 1$$

$$= 2.$$

i.e. atleast 2 professors were born in the same month.

W14
W15

6. Show that if 5 integers from 1 to 8 are chosen, then two of them will have a sum 9?

Sol^n Here we construct 4 different sets, each containing 2 integers that will give a sum of 9 as follows:

$A_1 = \{1, 8\}$,
$A_2 = \{2, 7\}$,
$A_3 = \{3, 6\}$,
$A_4 = \{4, 5\}$

∴ we have to select any 5 integers, each of them must belong to one of these sets.

Hence we treat the 5 integers as pigeons (m) & the 4 sets as nesting holes (n)

i.e. $m=5$, $n=4$.

By generalised pigeon-hole principle,

$$\left[\frac{m-1}{n}\right]+1 = \left[\frac{5-1}{4}\right]+1 = \left[\frac{4}{4}\right]+1 = 1+1=2,$$

ie. two of the integers chosen must belong to the same set.