Übung "Grundbegriffe der Informatik"

Karlsruher Institut für Technologie

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Abschätzen

for $i \leftarrow 0$ **to** n **do**

$R(i)$

od

.

Abschätzen

for $i \leftarrow 0$ **to** n **do**

$R(i)$

od

Zeitbedarf:

$$\sum_{i=0}^{n} T(R(i))$$

.

Abschätzen

for $i \leftarrow 0$ **to** n **do**

$R(i)$

od

$$T(R(i)) = 4i^3 + 7i + 3 + \log_2 i$$

.

Abschätzen

for $i \leftarrow 0$ **to** n **do**
    $R(i)$
od

$T(R(i)) = 4i^3 + 7i + 3 + \log_2 i$
$T(R(i)) \in \Theta(i^3)$

.

Abschätzen

for $i \leftarrow 0$ **to** n **do**

$R(i)$

od

$T(R(i)) = 4i^3 + 7i + 3 + \log_2 i$
$T(R(i)) \in \Theta(i^3)$
$T(n) \in \Theta(\sum_{i=0}^{n} i^3)$

.

Abschätzen

$T(R(i)) = 4i^3 + 7i + 3 + \log_2 i$

$T(R(i)) \in \Theta(i^3)$

$T(n) \in \Theta(\sum_{i=0}^{n} i^3)$

$T(n) \leq c \cdot (n+1)n^3 \in O(n^4)$

.

Abschätzen

$$T(R(i)) = 4i^3 + 7i + 3 + \log_2 i$$
$$T(R(i)) \in \Theta(i^3)$$
$$T(n) \in \Theta(\sum_{i=0}^{n} i^3)$$
$$T(n) \geq c \cdot \frac{n}{2} \cdot (\frac{n}{2})^3 \in \Omega(n^4)$$

.

Abschätzen

$T(R(i)) = 4i^3 + 7i + 3 + \log_2 i$
$T(n) \in \Theta(n^4)$

.

Abschätzen

Wieso muss man bei $O(\log n)$ keine Basis angeben?

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$\forall a, b > 1 : \log_a n \in \Theta(\log_b(n))$

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Wieso muss man bei $O(\log n)$ keine Basis angeben?

$\forall a, b > 1 : \log_a n \in \Theta(\log_b(n))$

$$\log_a(n) = \frac{\log_2 n}{\log_2 a} = \frac{\log_2 a}{\log_2 b}\frac{\log_2 n}{\log_2 a} = \frac{\log_2 a}{\log_2 b}\log_b n$$

.

Endliche Automaten

Merke:

Mealy Automaten: Ausgabefunktion hat Argumente (Zustand, Symbol)

Moore Automaten: Ausgabefunktion hat Argument (Zustand)

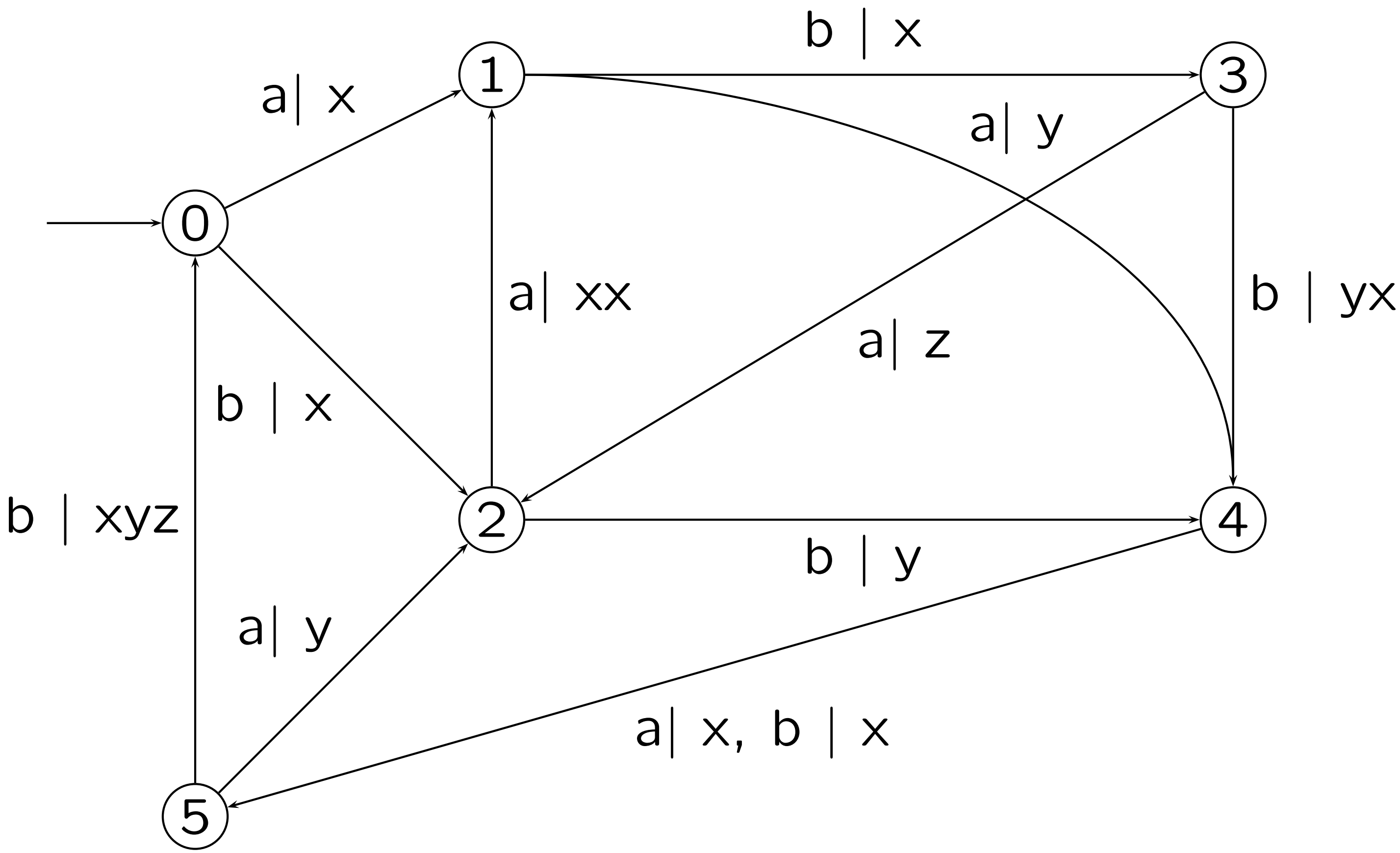
.

Merke:

Mea-ly Automaten: 2 Silben
$\rightarrow$ Ausgabefunktion hat zwei Argumente

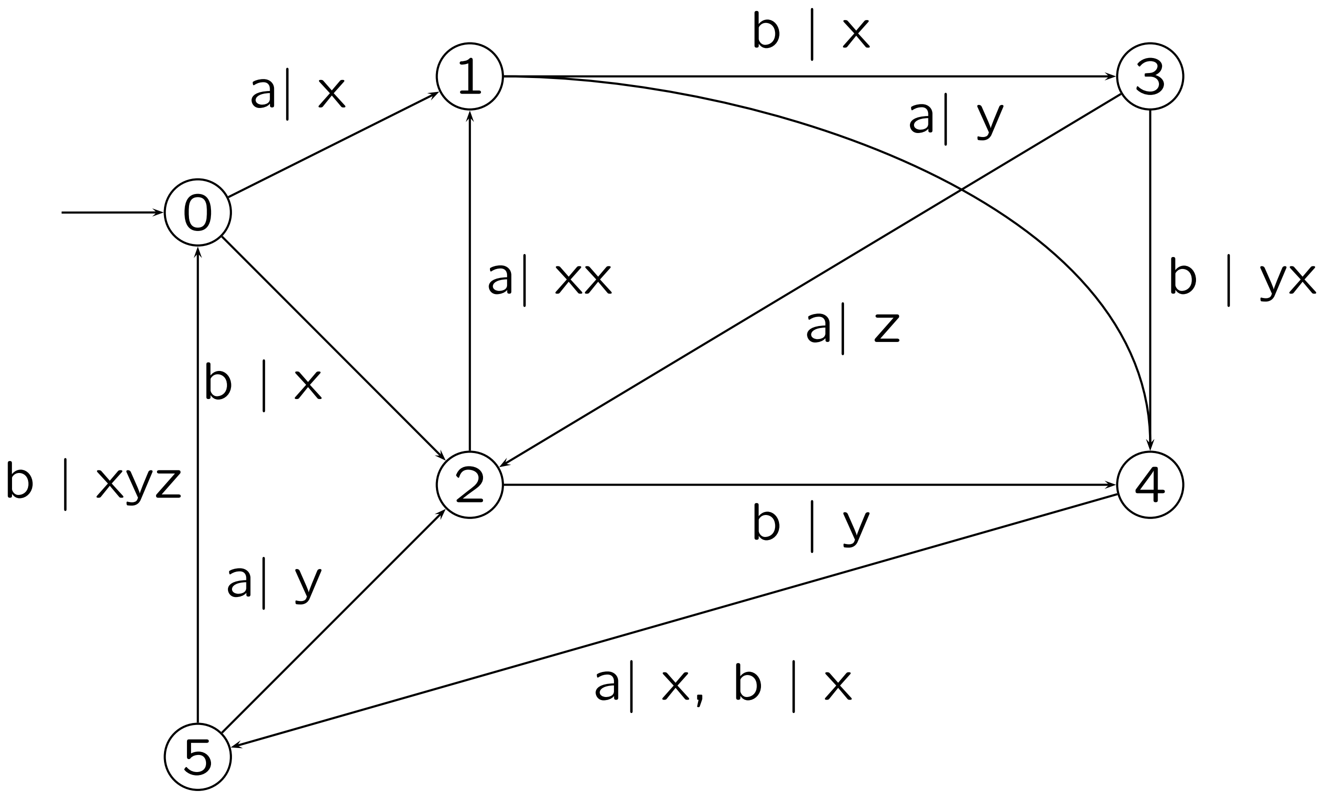
Moore Automaten: 1 Silbe
$\rightarrow$ Ausgabefunktion hat ein Argument

.

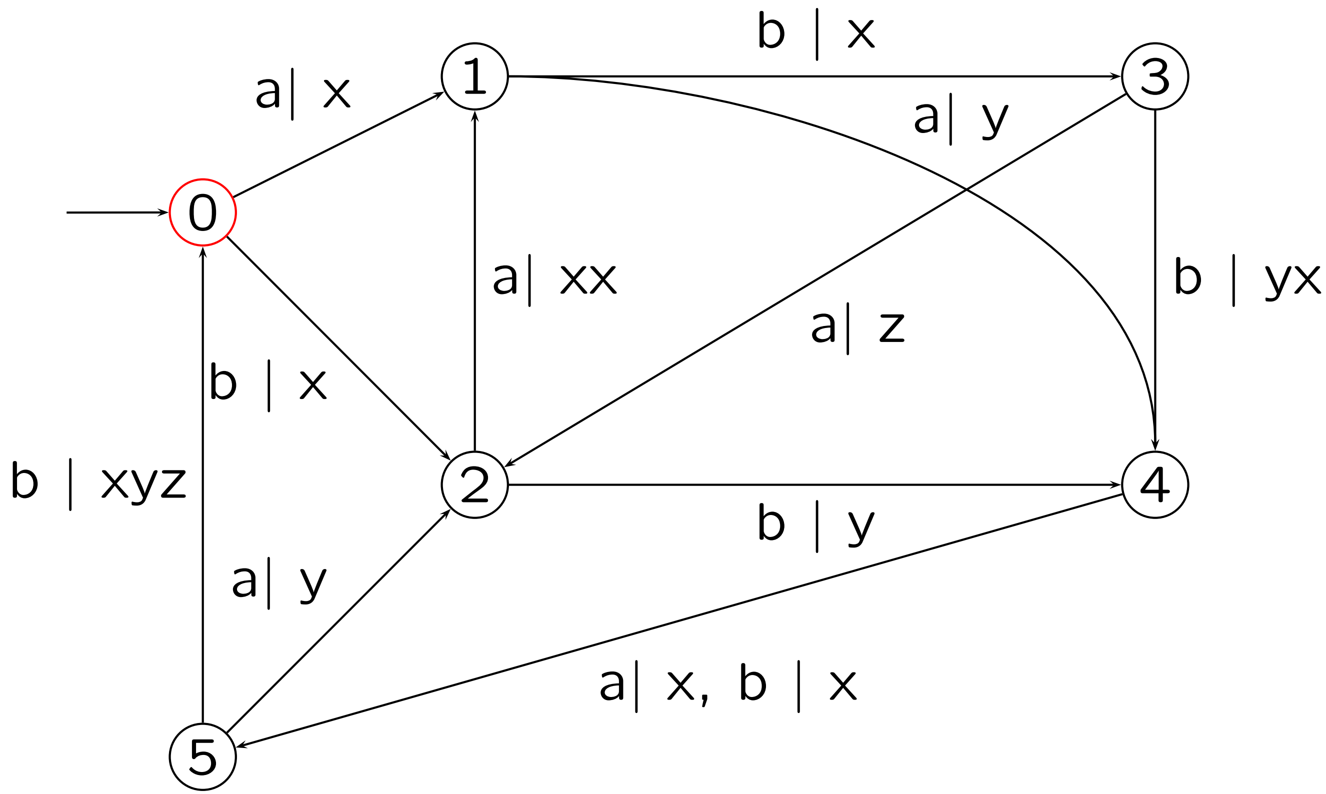


b | x
1
3
a| x
a| y
0
a| xx
b | yx
a| z
b | x
b | xyz
2
4
b | y
a| y
a| x, b | x
5

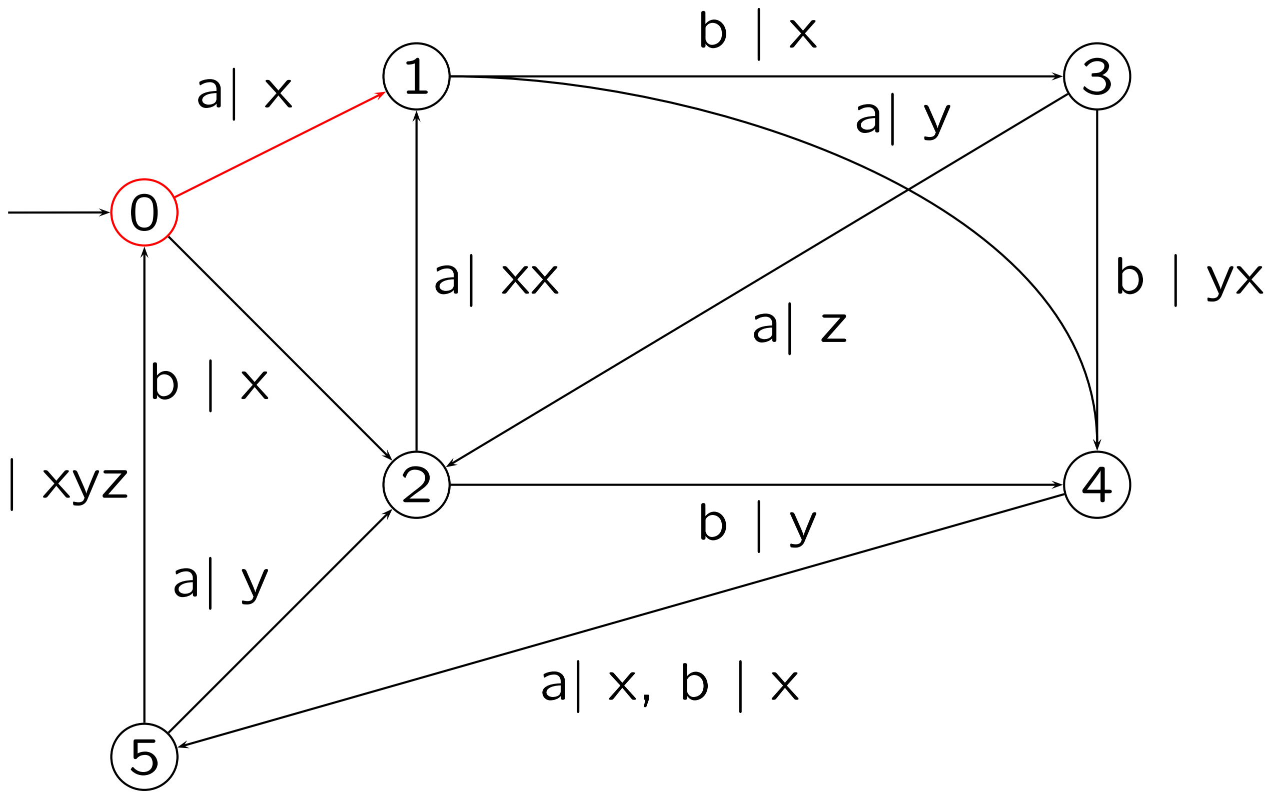
Eingabe: abbababba, Ausgabe:



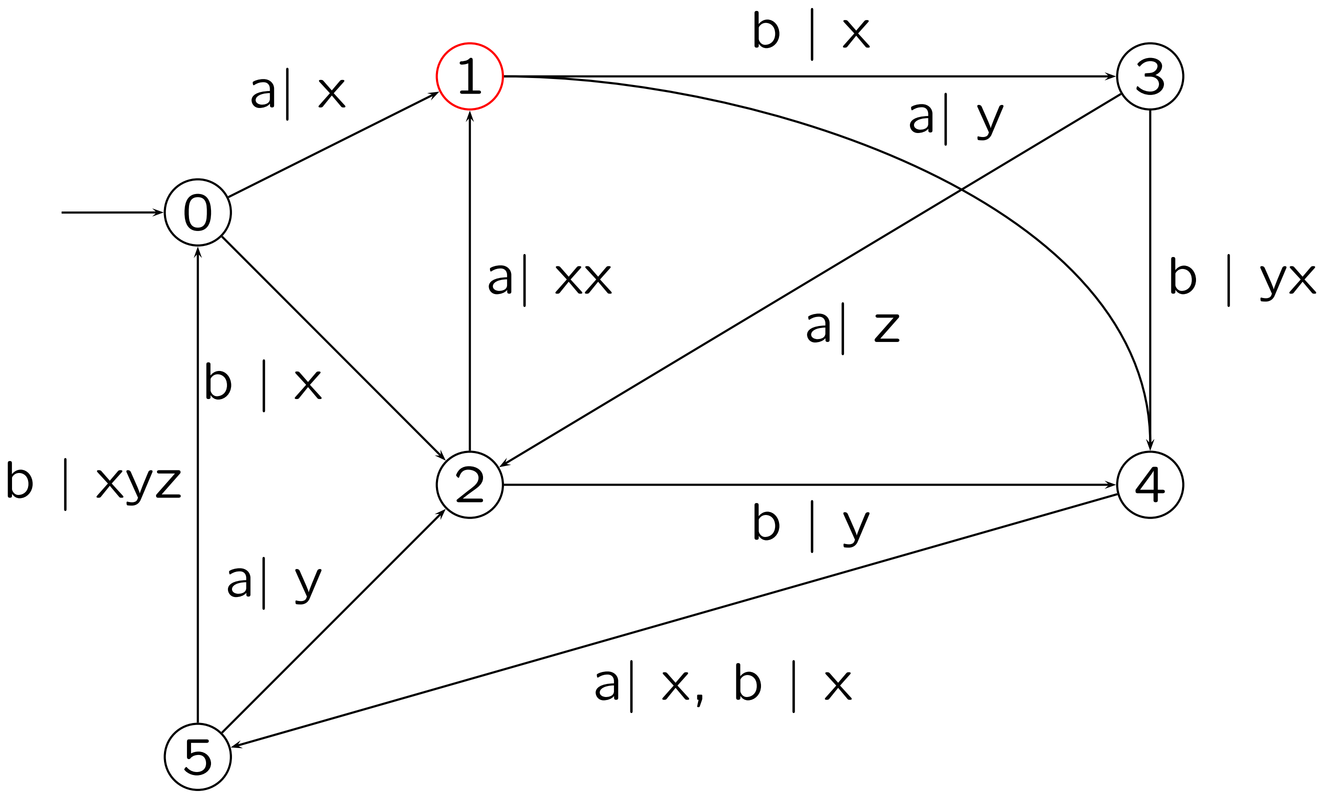
Eingabe: abbababba, Ausgabe:



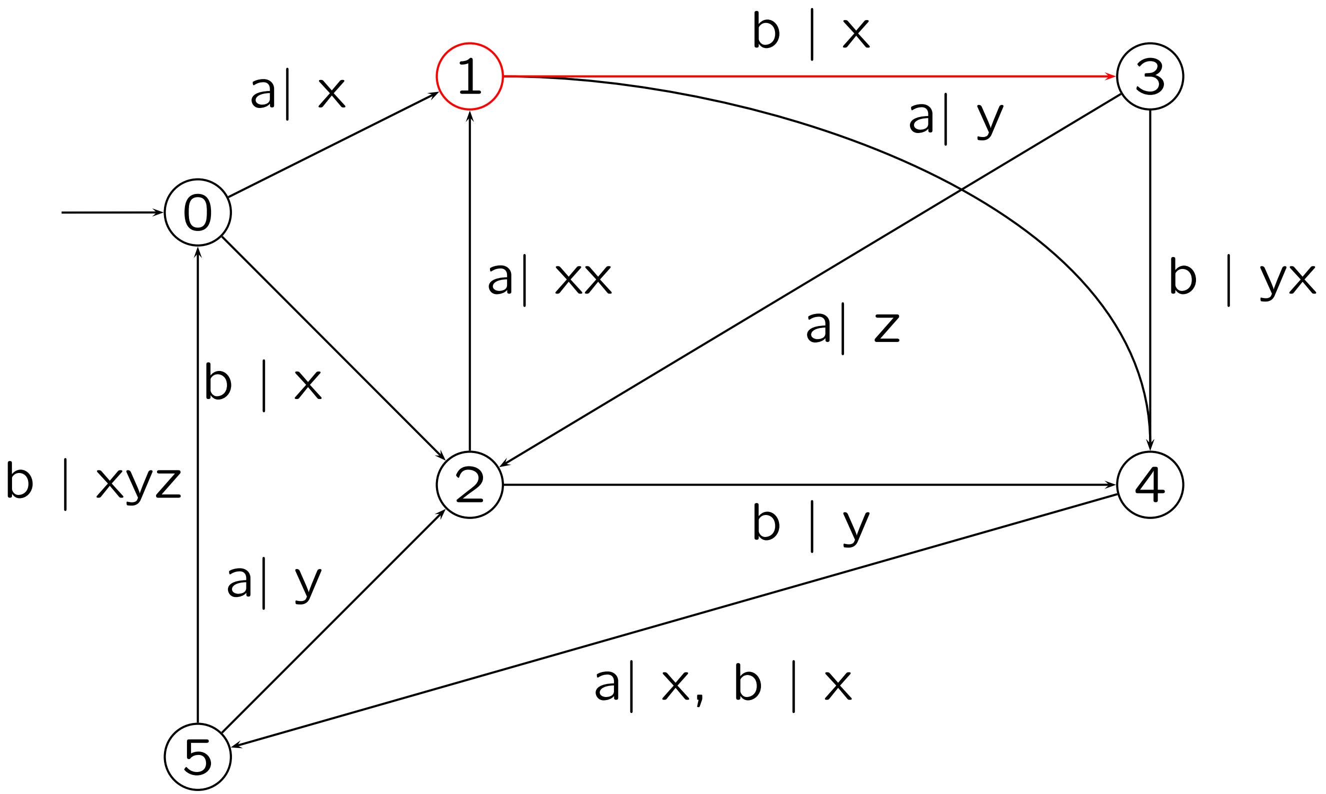
Eingabe: abbababba, Ausgabe: x



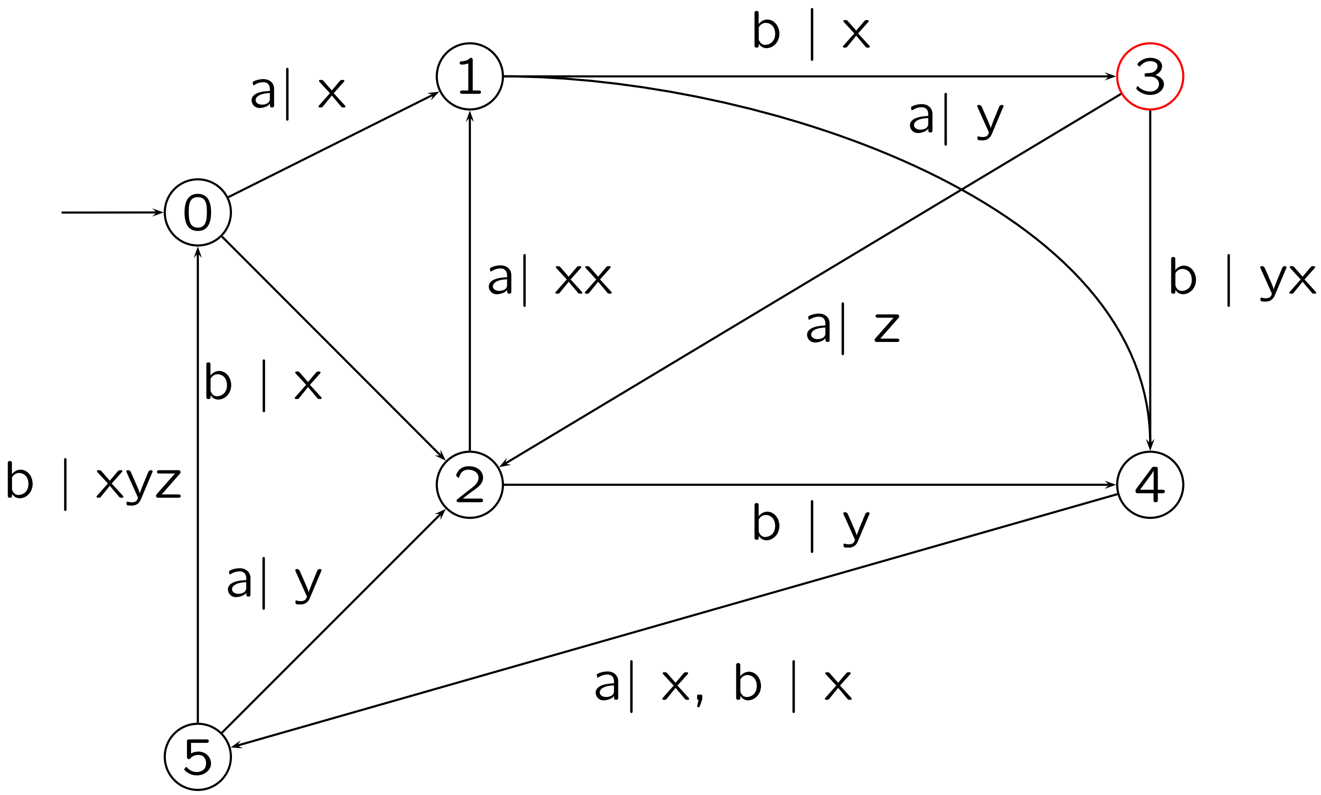
Eingabe: abbababba, Ausgabe: x



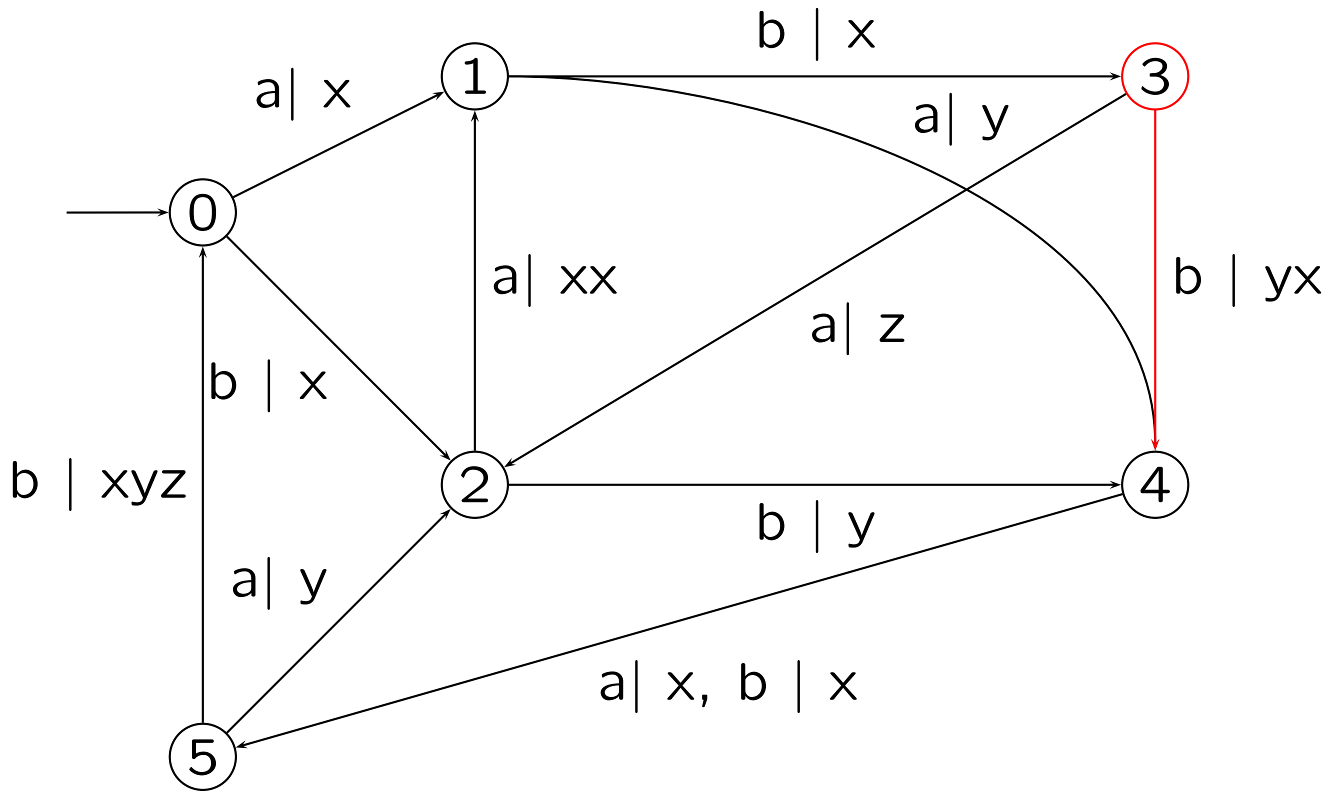
Eingabe: abbababba, Ausgabe: xx



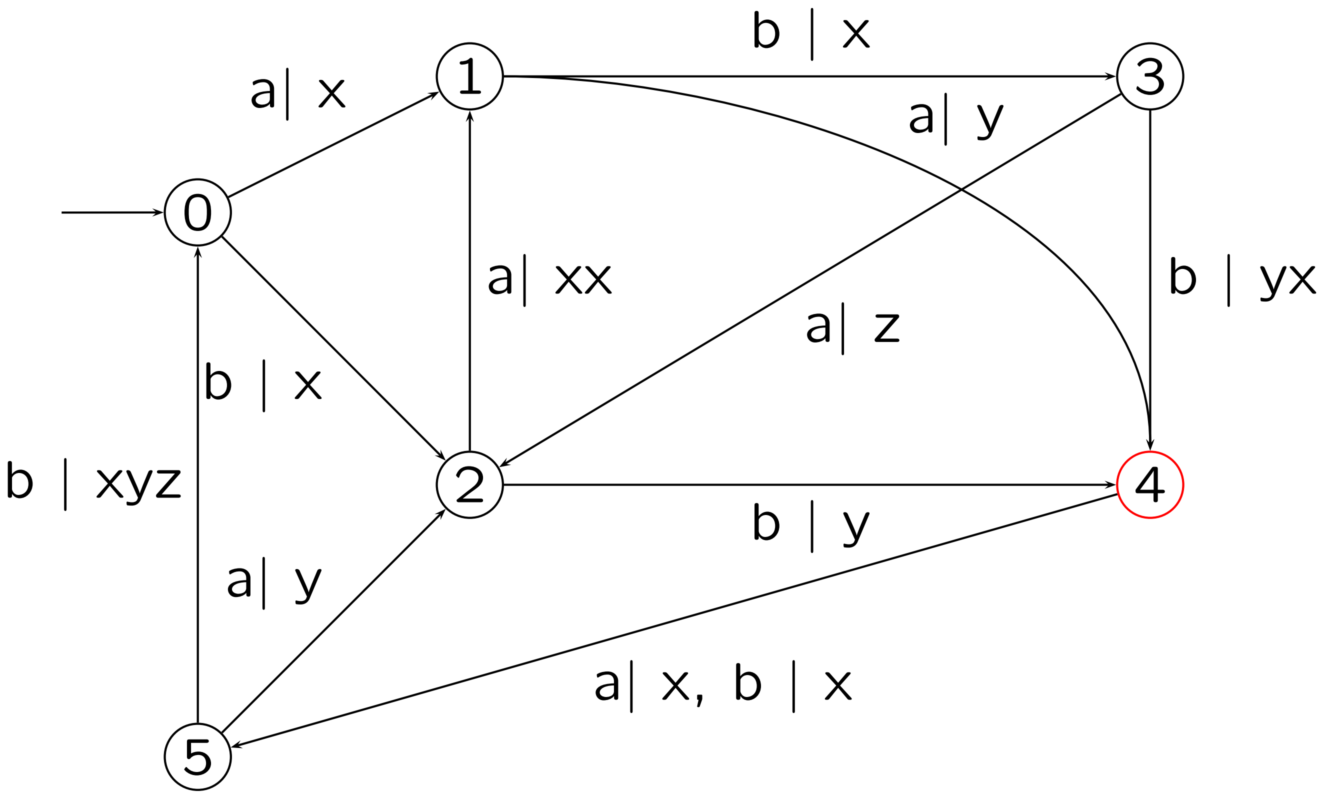
Eingabe: abbababba, Ausgabe: xx



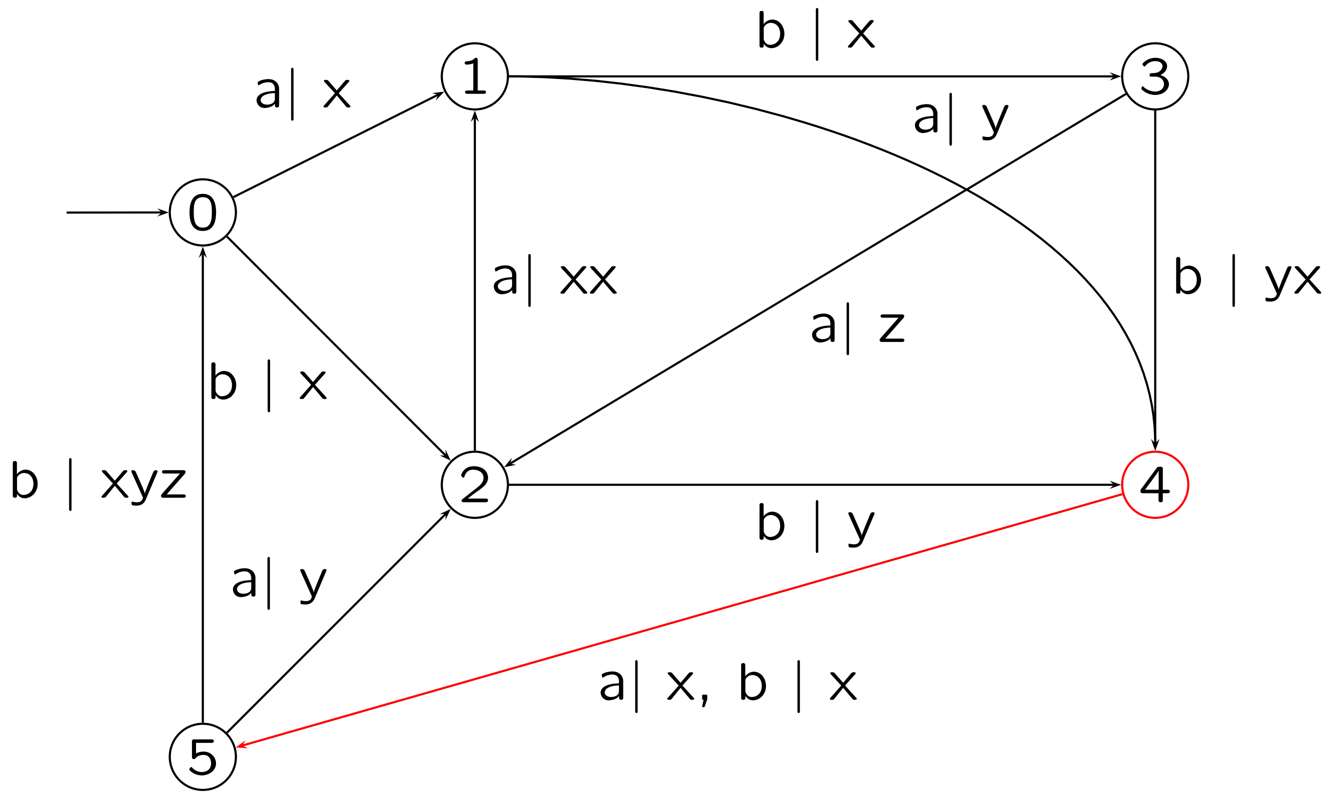
Eingabe: abbababba, Ausgabe: xxyx



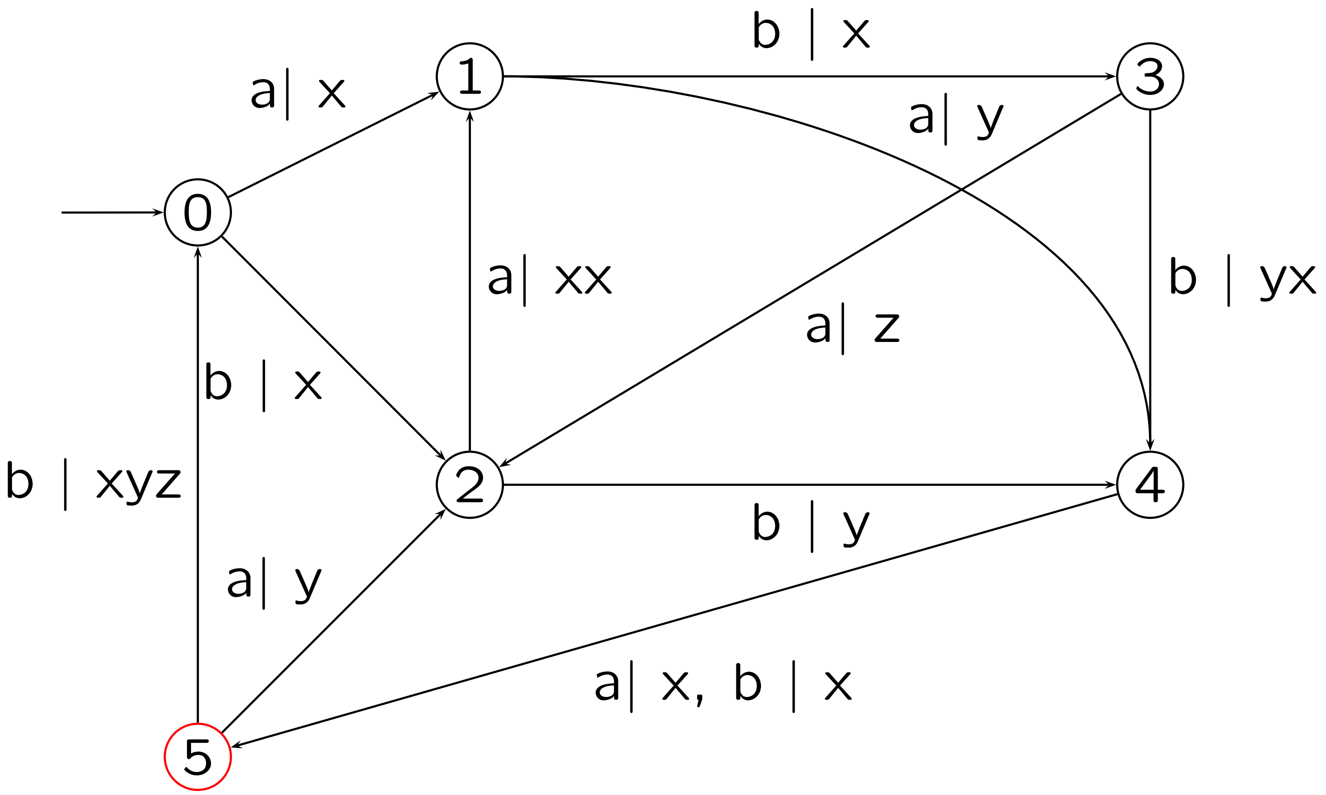
Eingabe: abbababba, Ausgabe: xxyx



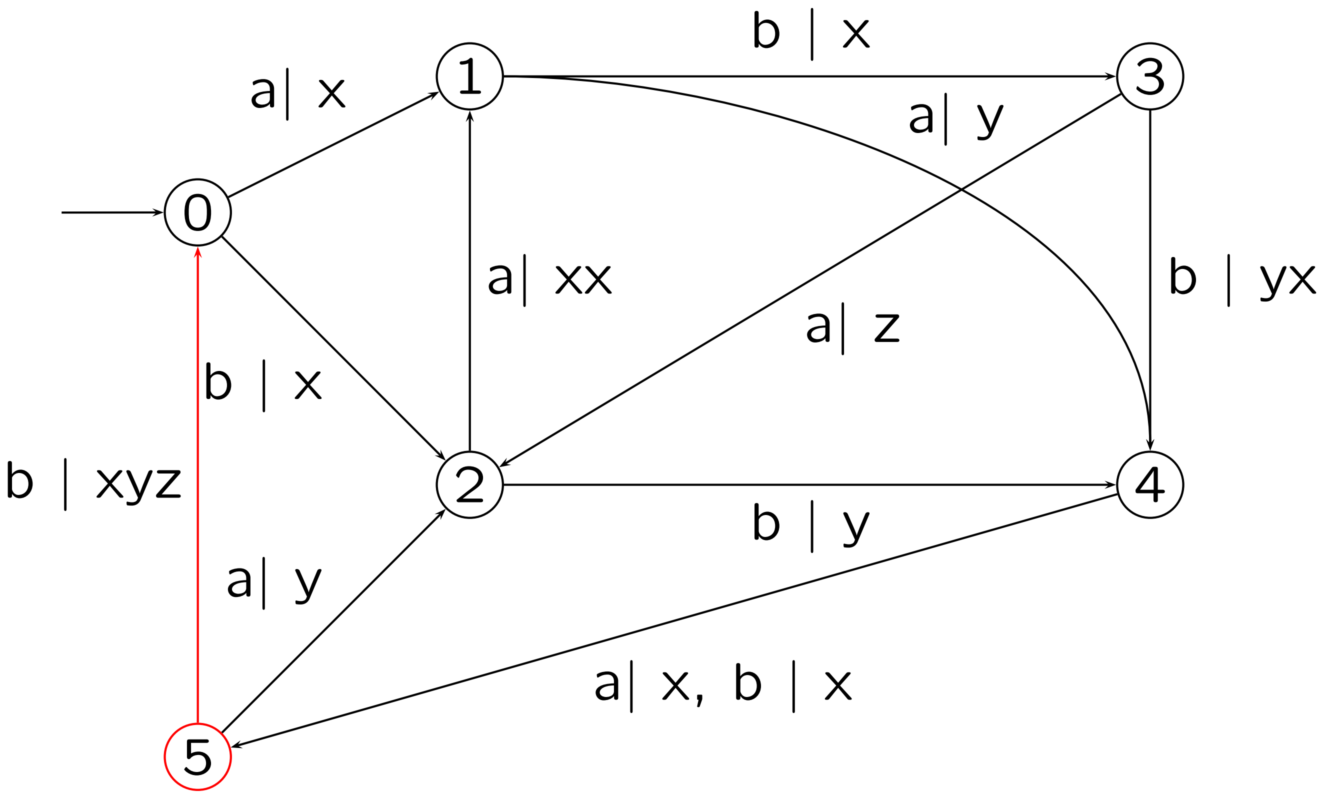
Eingabe: abbababba, Ausgabe: xxyxx



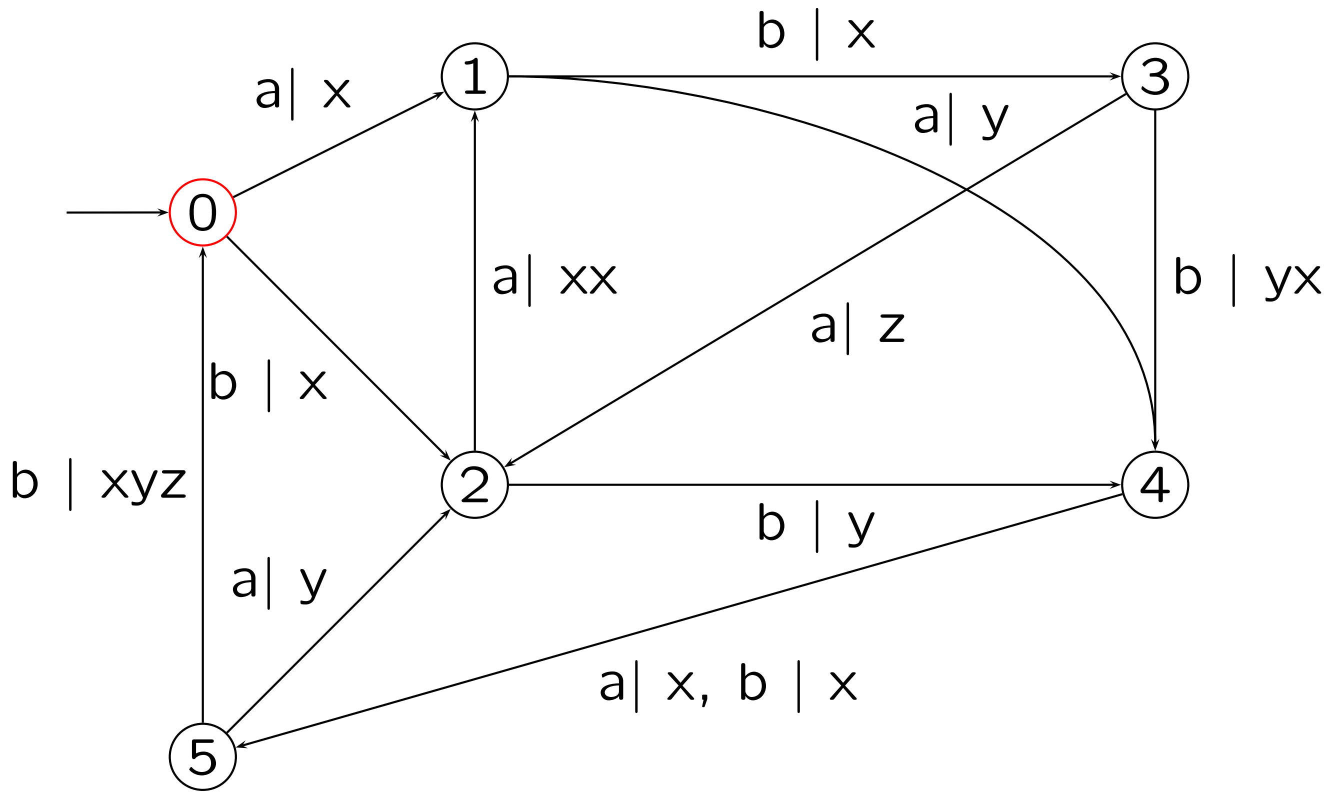
Eingabe: abbababba, Ausgabe: xxyxx



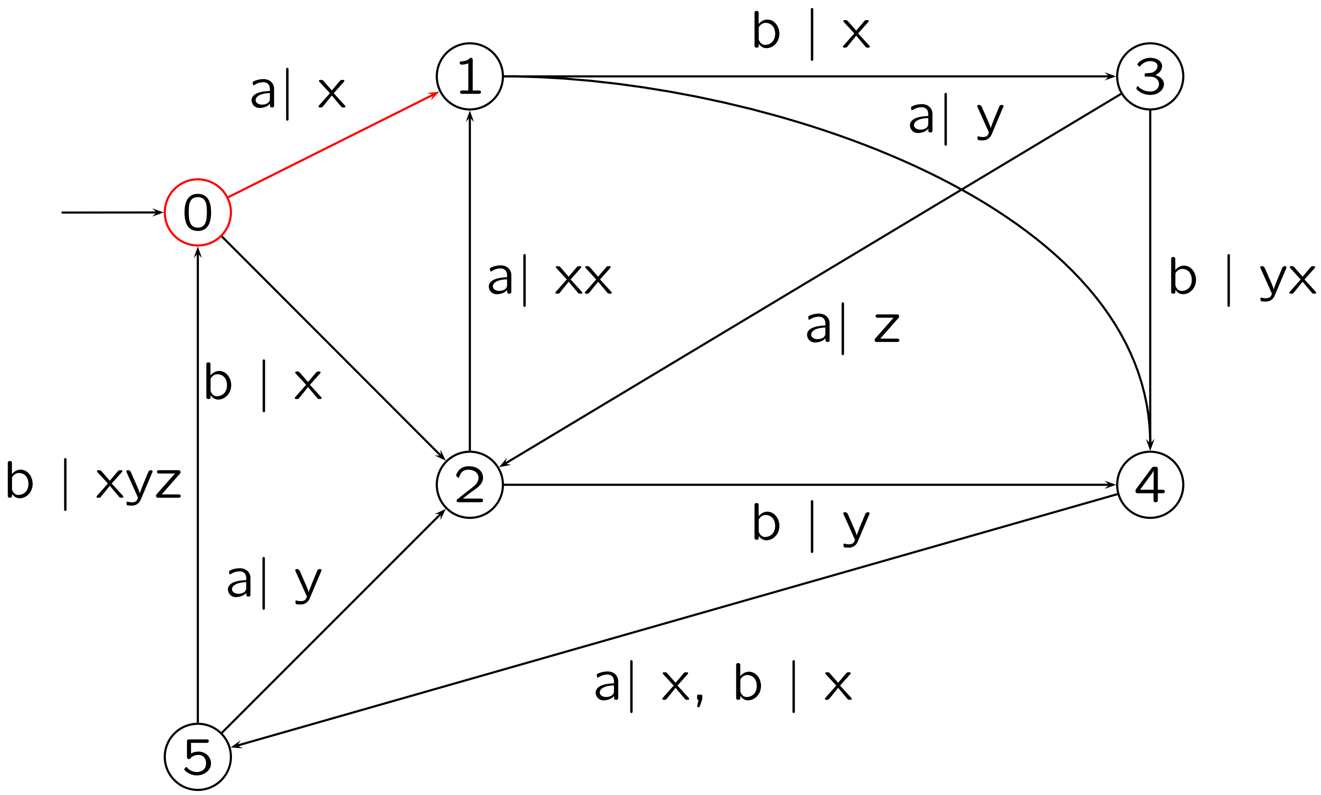
Eingabe: abbababba, Ausgabe: xxyxxxyz



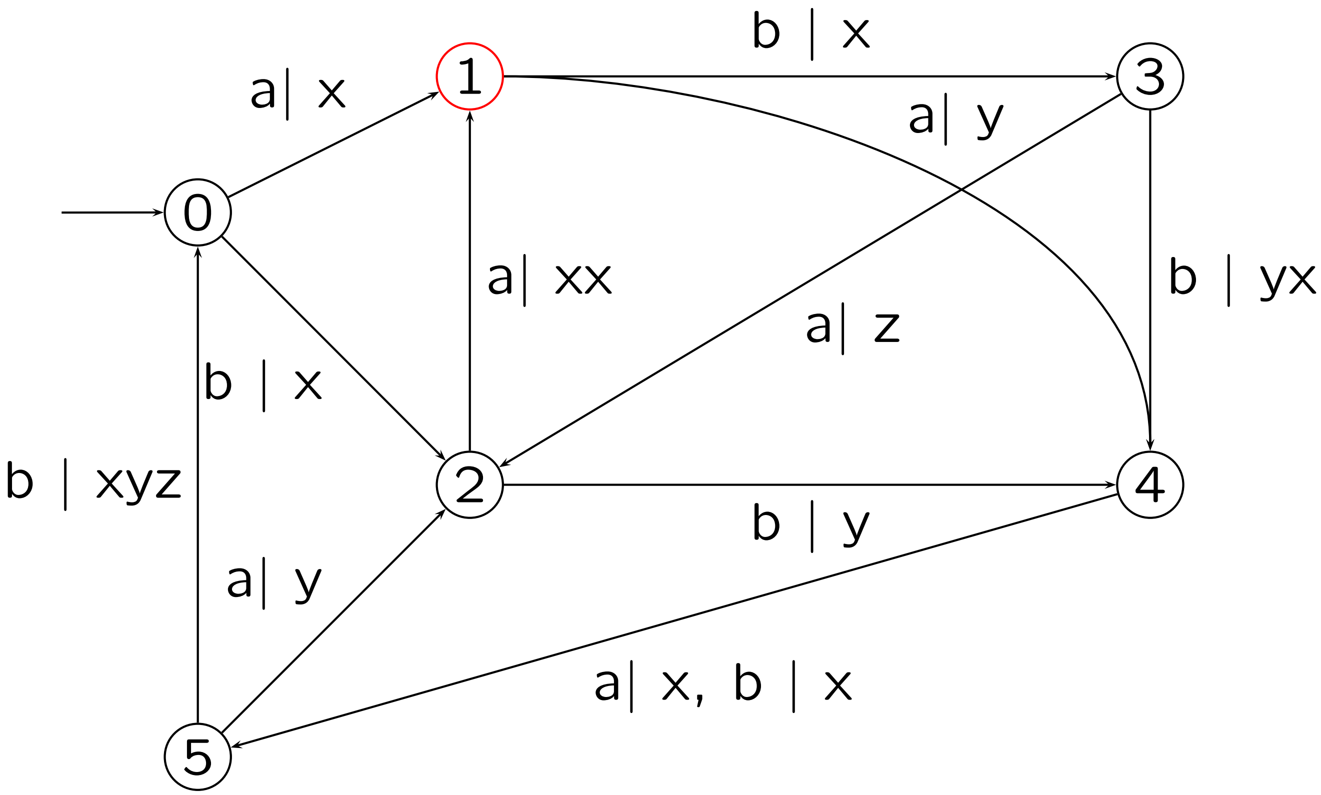
Eingabe: abbababba, Ausgabe: xxyxxxyz



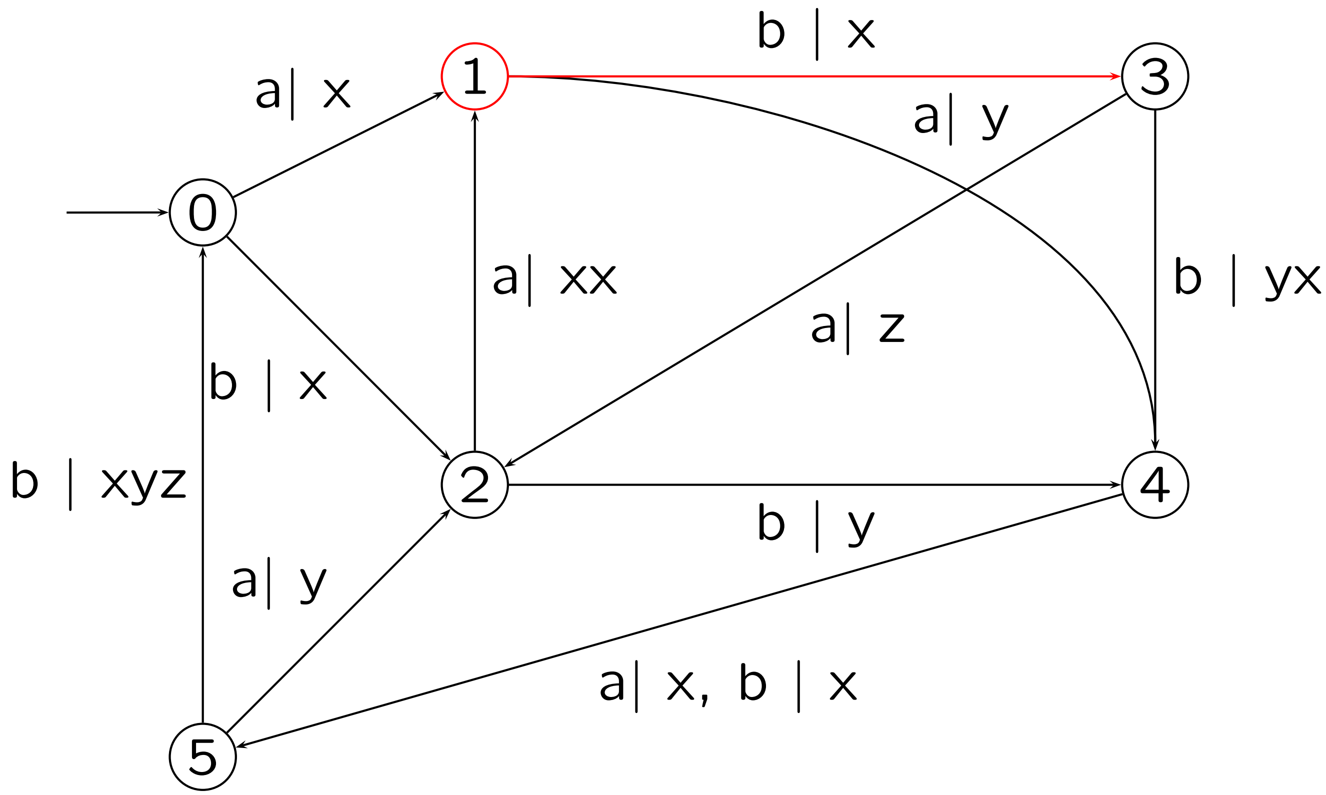
Eingabe: abbababba, Ausgabe: xxyxxxyzx



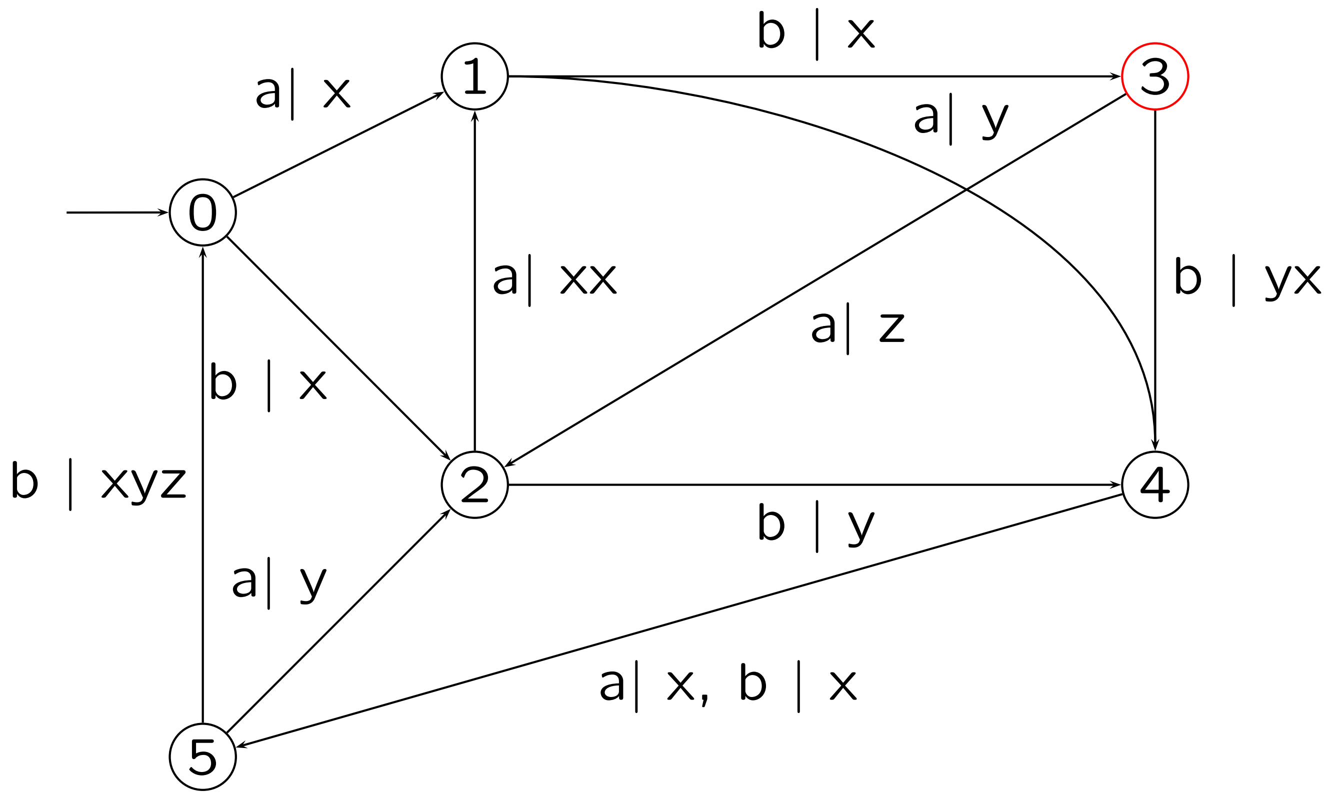
Eingabe: abbababba, Ausgabe: xxyxxxyzx



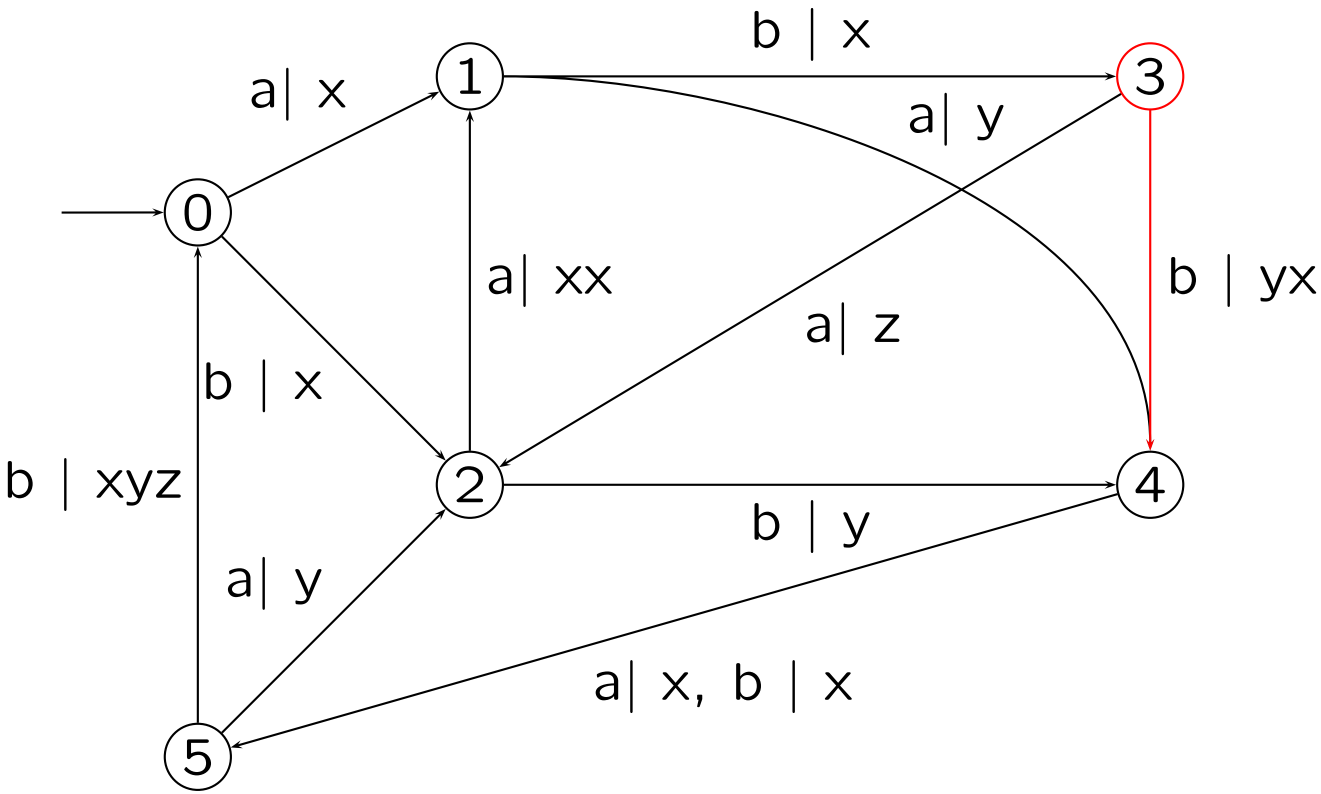
Eingabe: abbababba, Ausgabe: xxyxxxyzxx



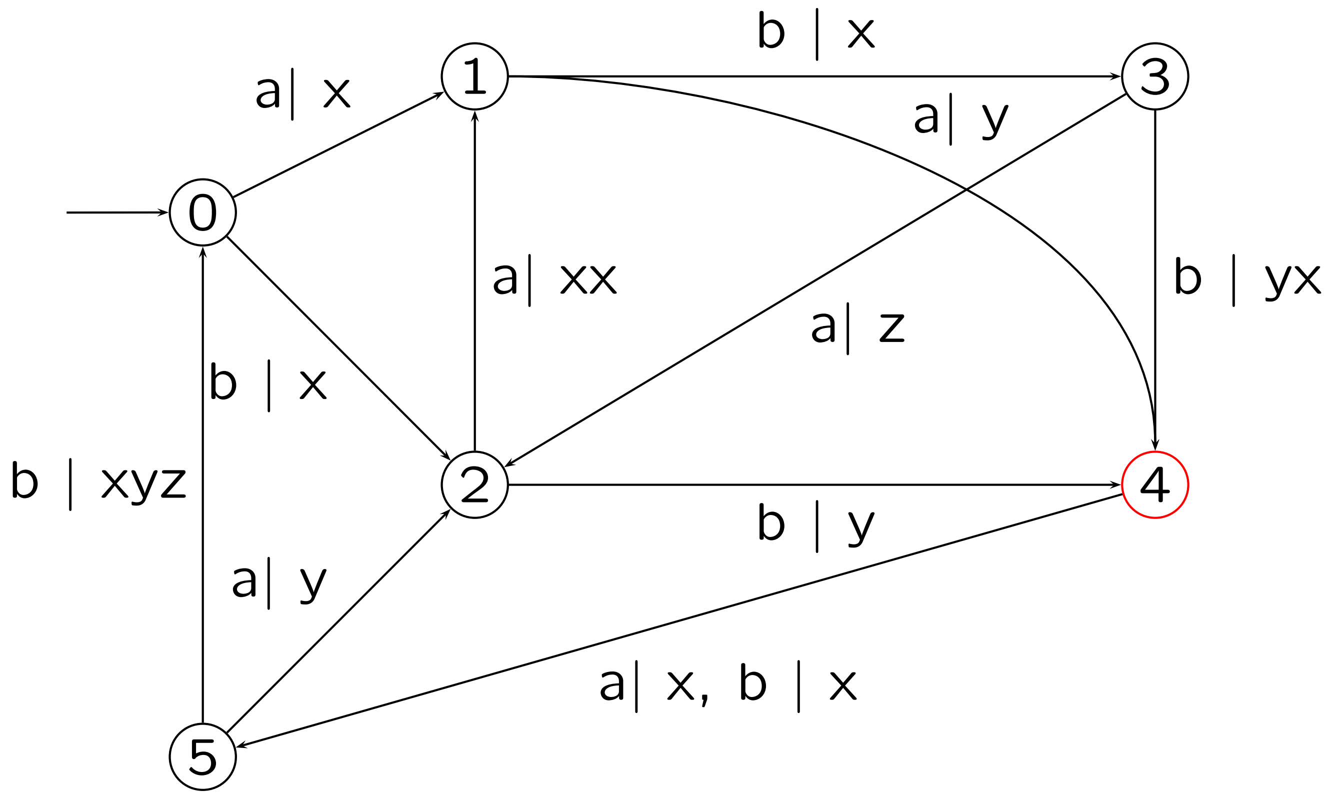
Eingabe: abbababba, Ausgabe: xxyxxxyzxx



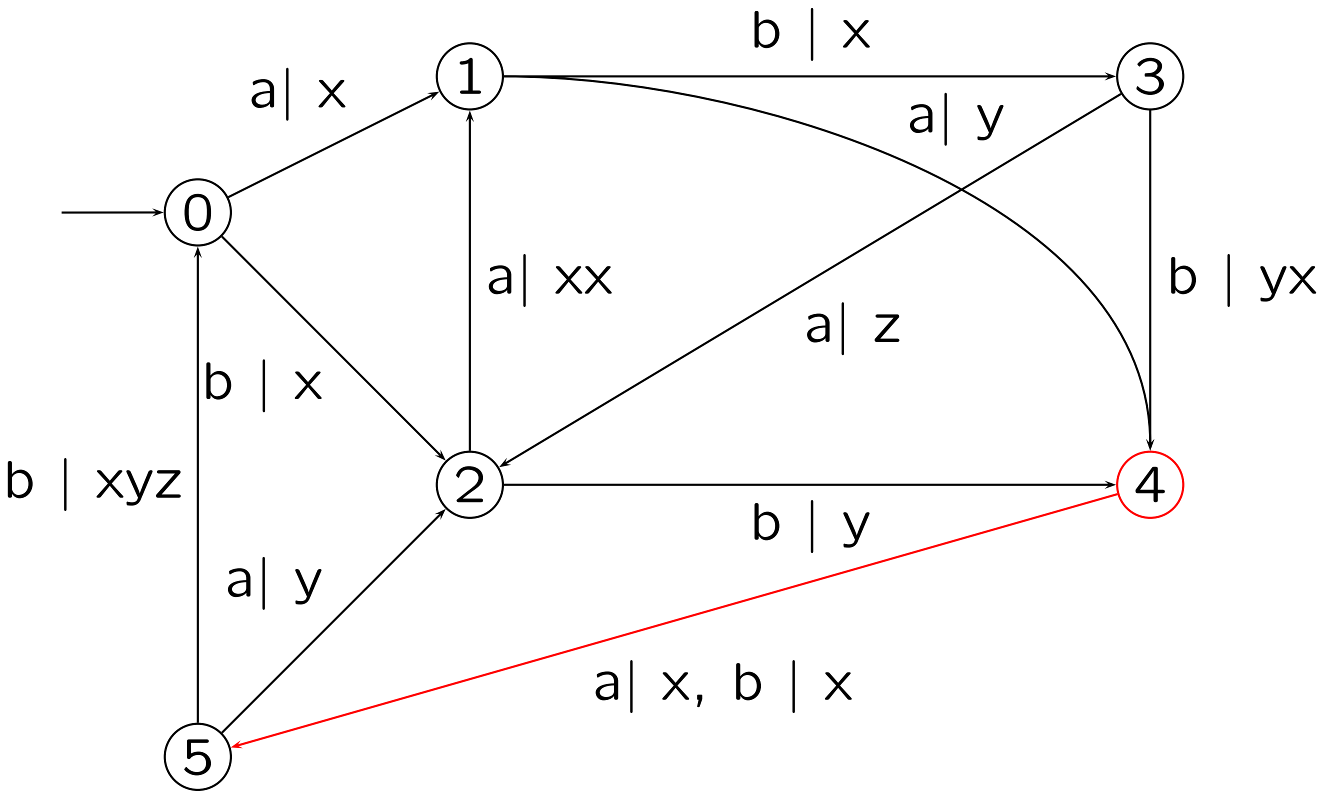
Eingabe: abbababba, Ausgabe: xxyxxxyzxxyx



Eingabe: abbababba, Ausgabe: xxyxxxyzxxyx



Eingabe: abbababba, Ausgabe: xxyxxxyzxxyxx



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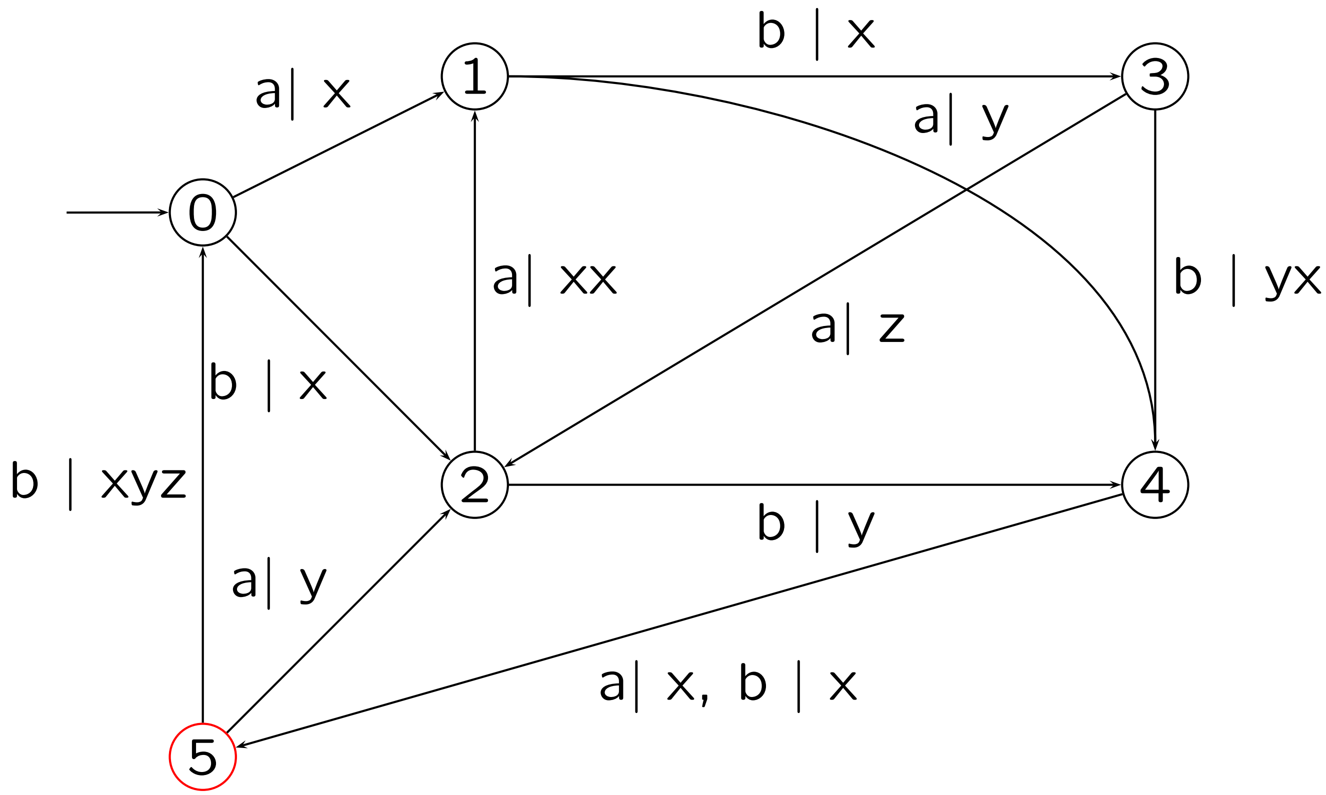
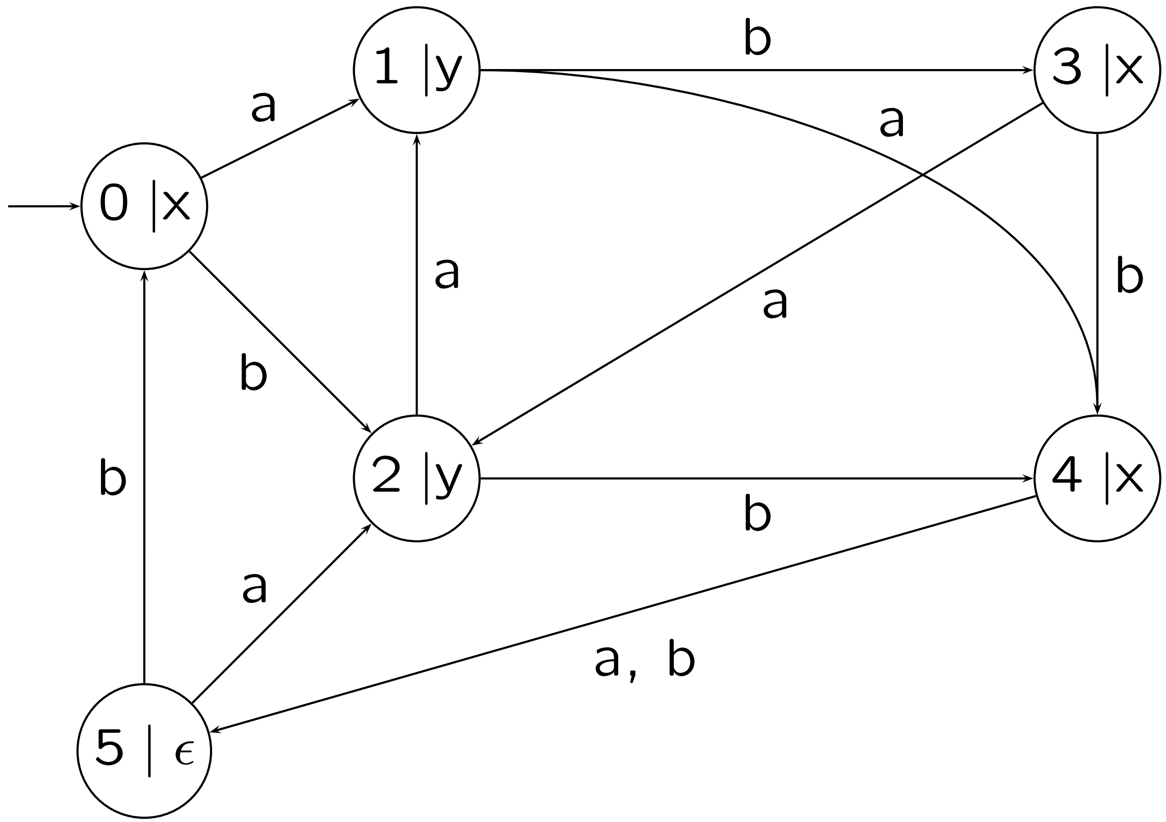
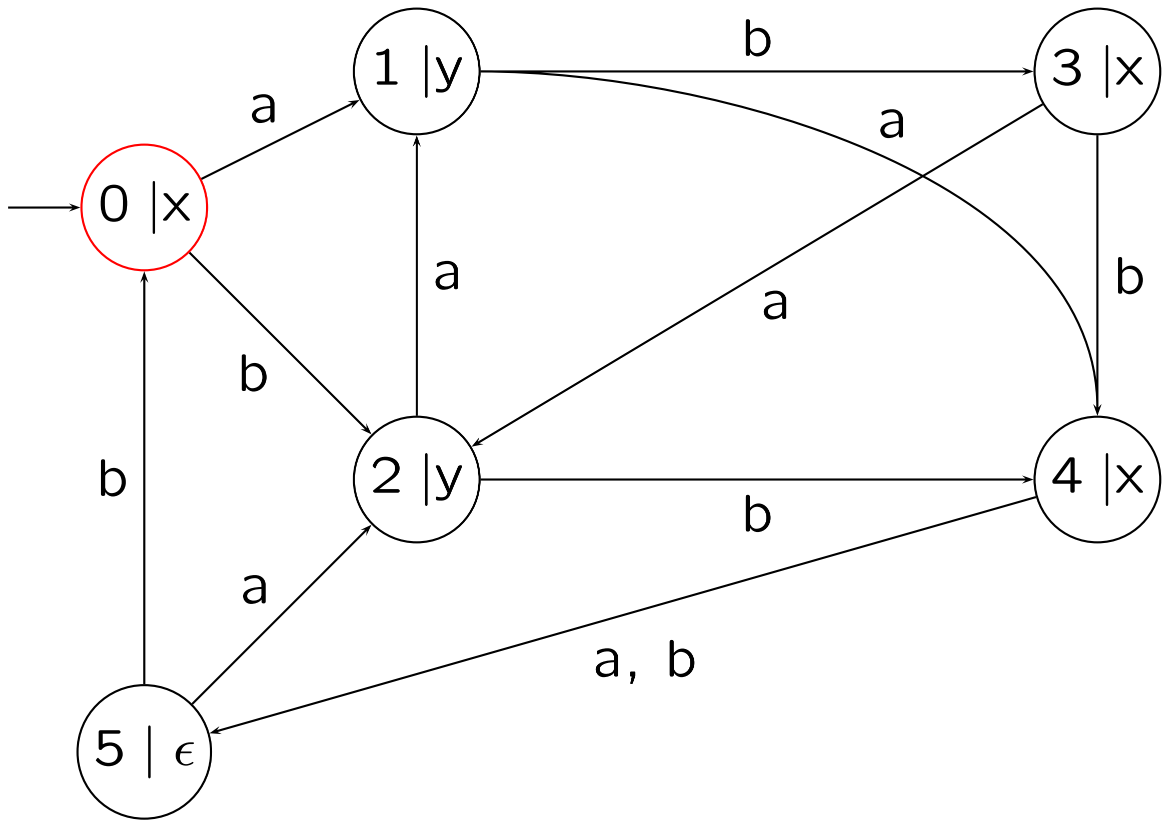


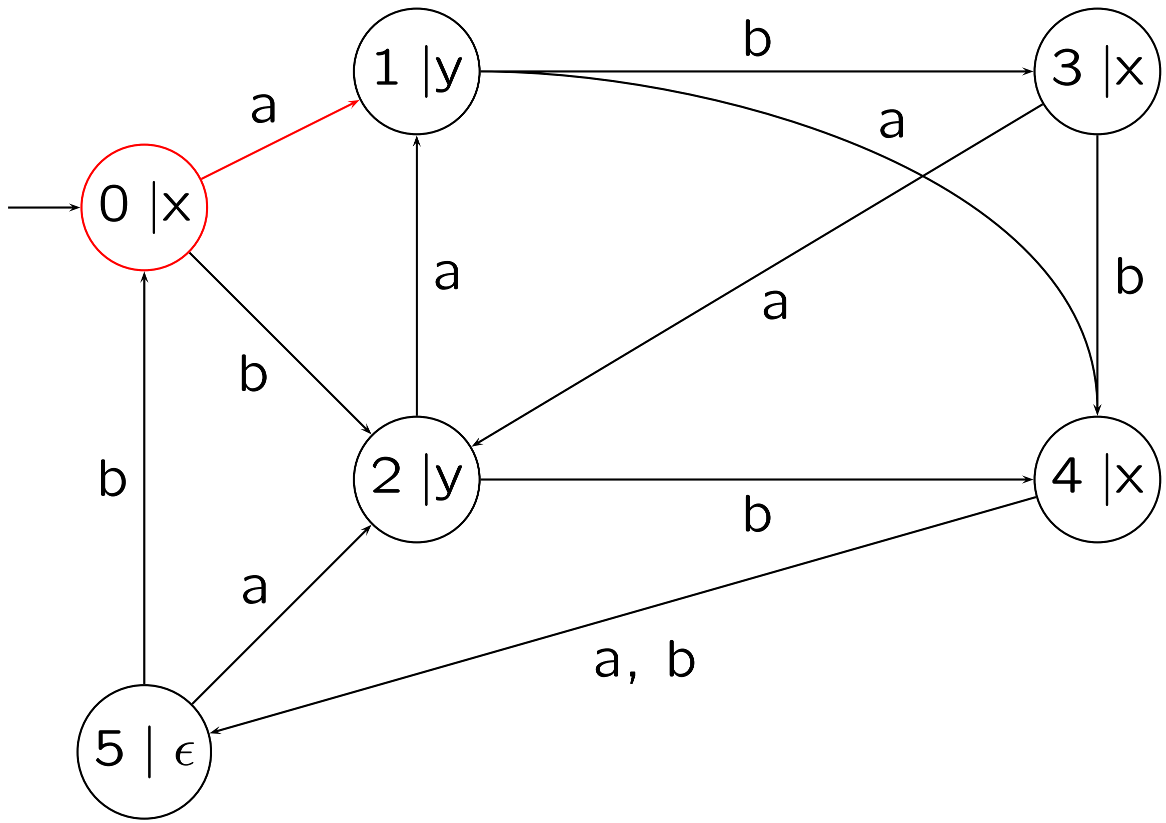


0 |x
1 |y
2 |y
3 |x
4 |x
5 | ϵ
a
b
a
a
a
b
b
b
a
a, b

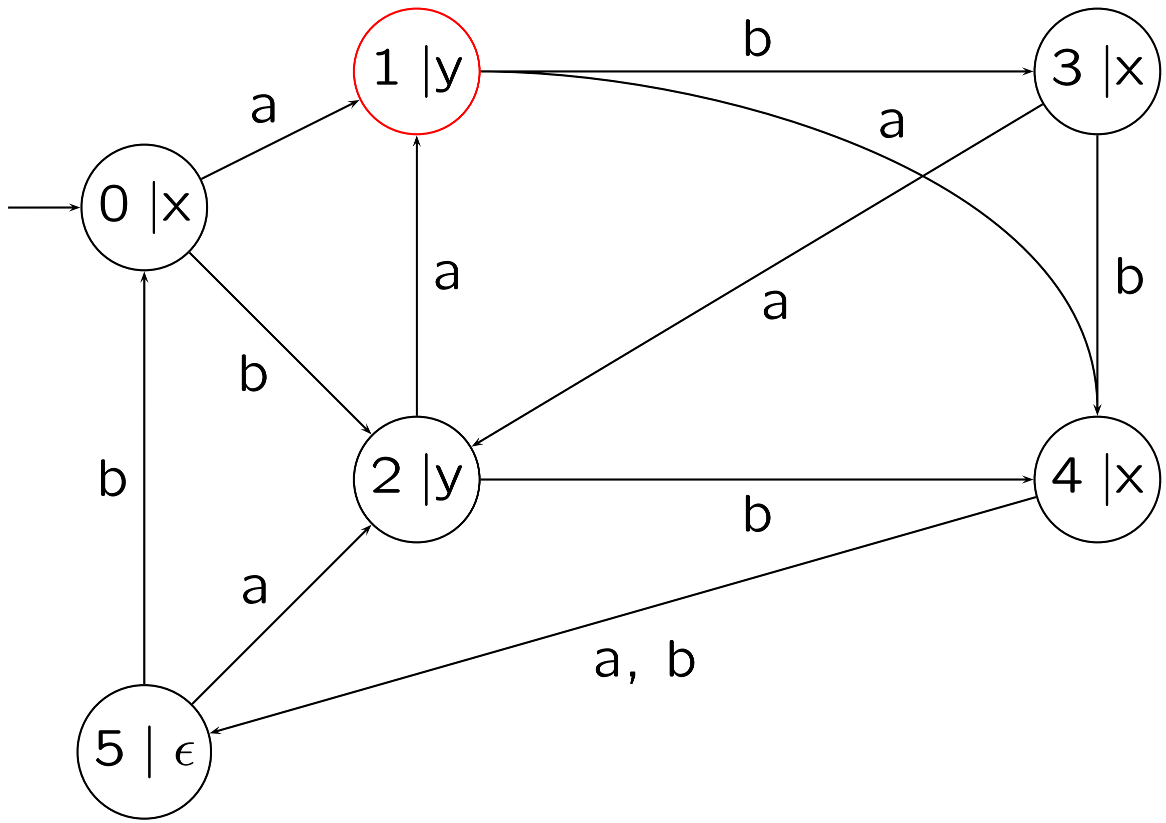
Eingabe: aabab, Ausgabe: x



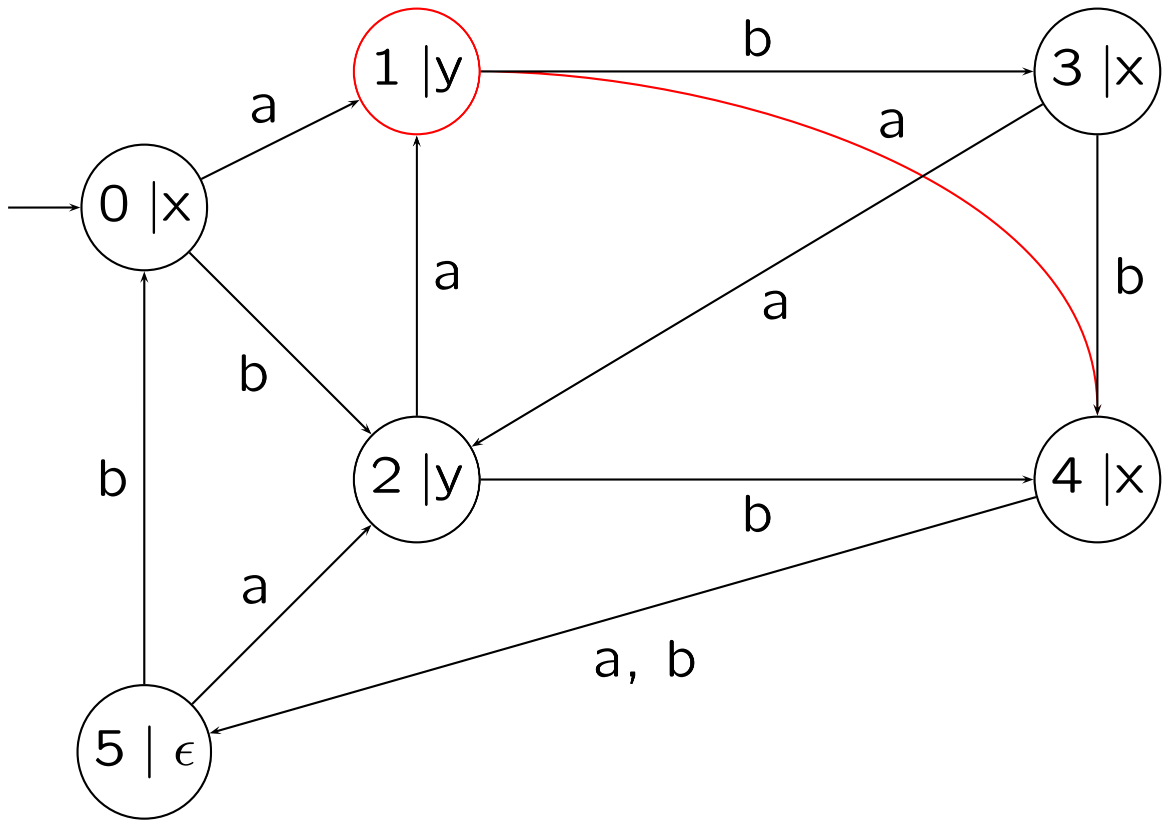
Eingabe: aabab, Ausgabe: x



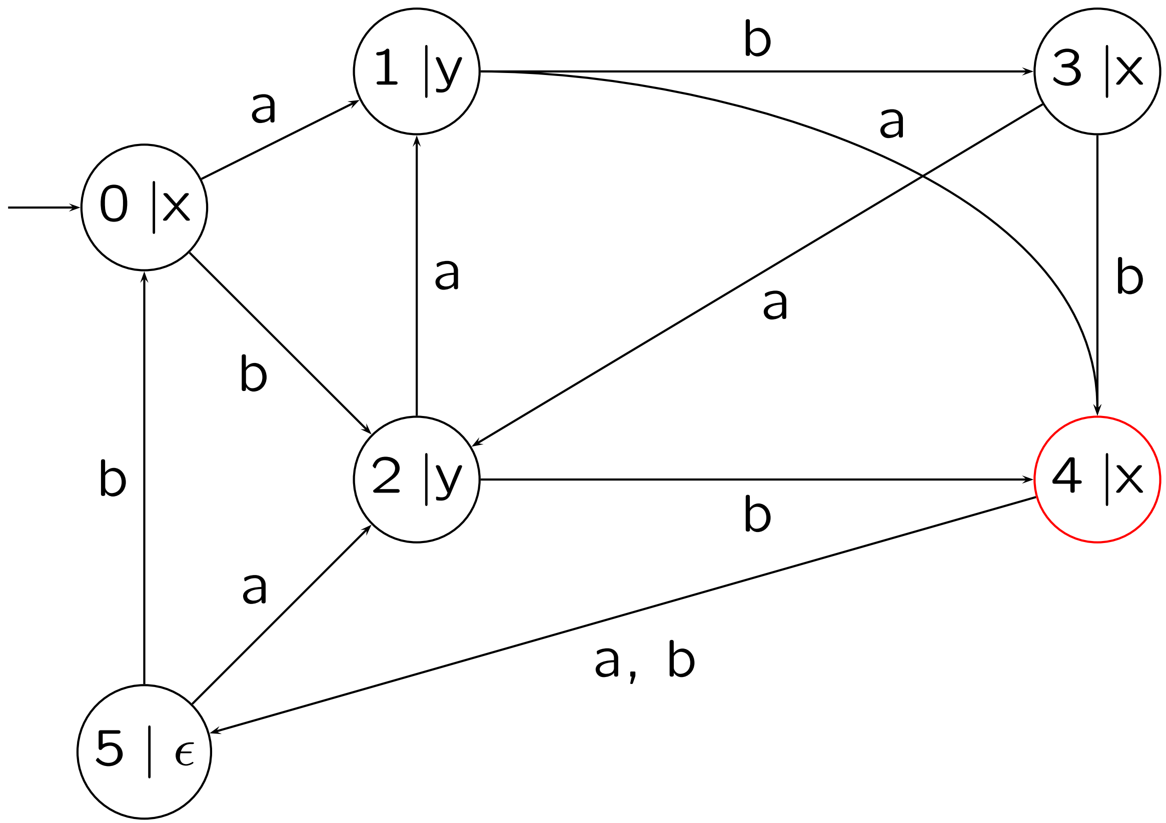
Eingabe: aabab, Ausgabe: xy



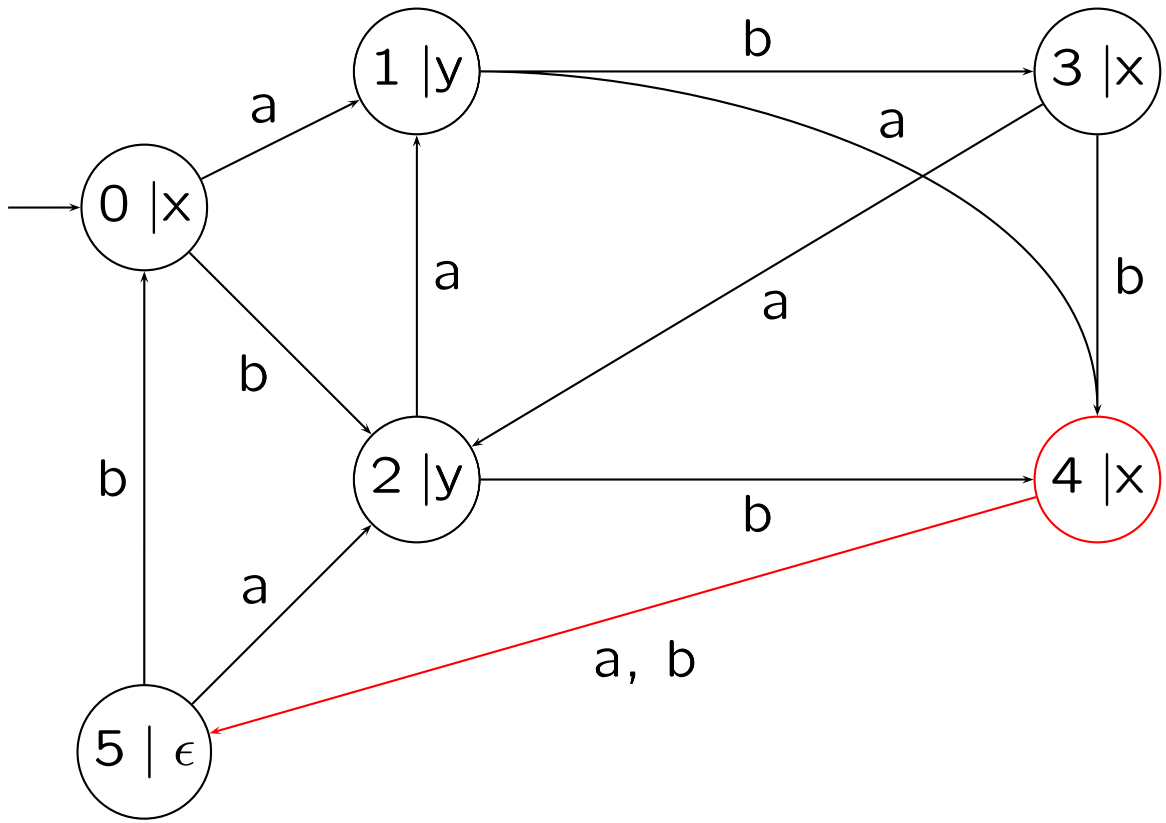
Eingabe: aabab, Ausgabe: xy



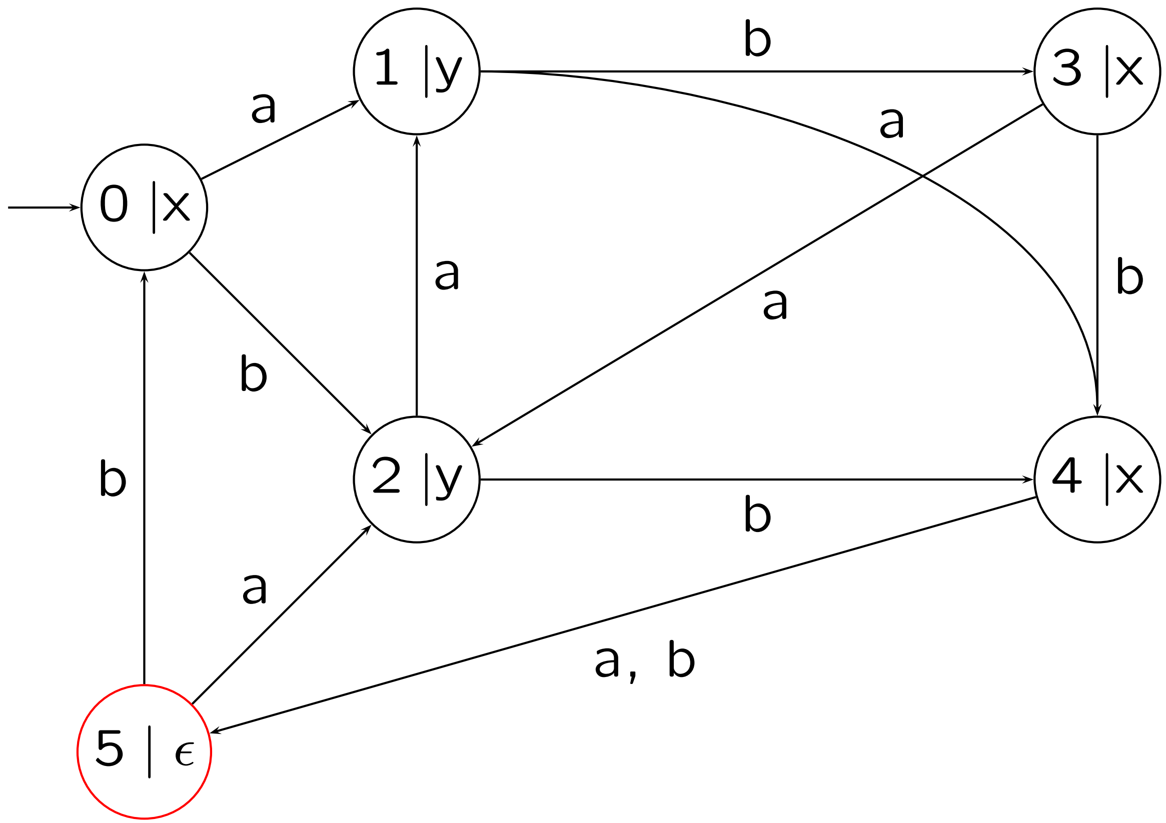
Eingabe: aabab, Ausgabe: xyx



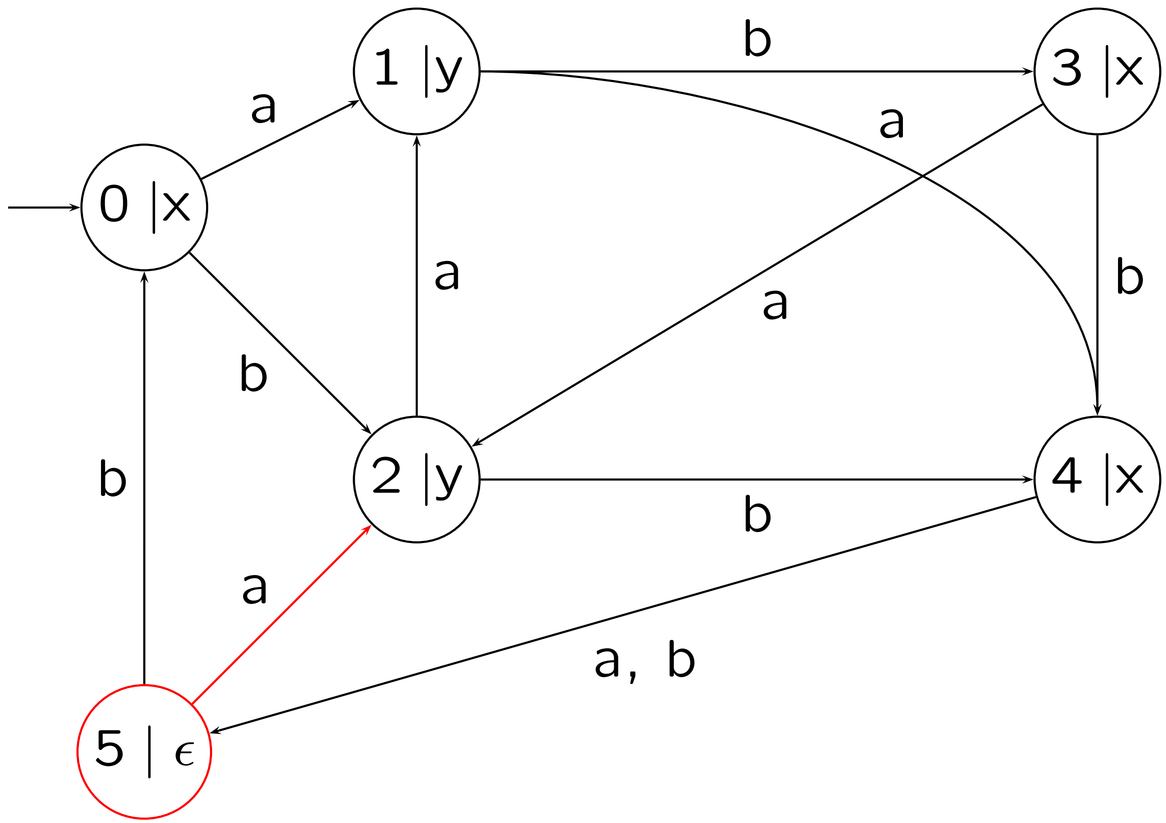
Eingabe: aabab, Ausgabe: xyx



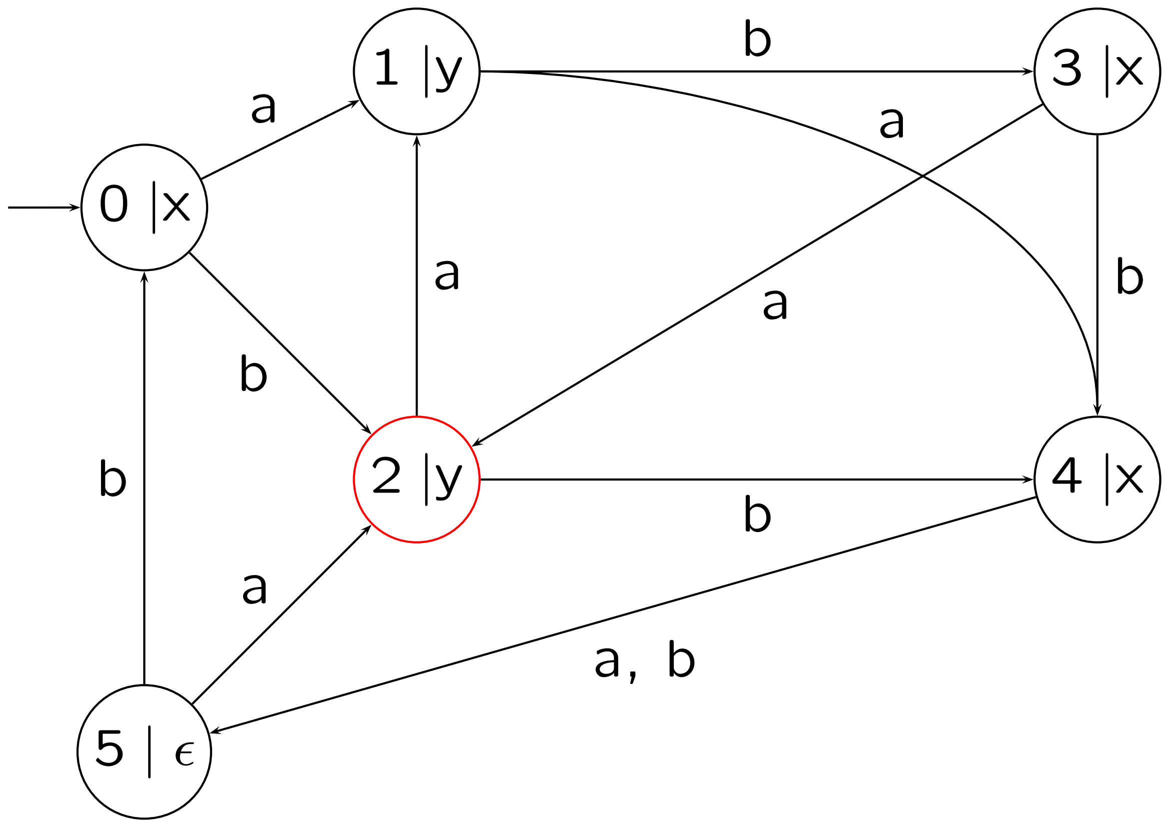
Eingabe: aabab, Ausgabe: xyx



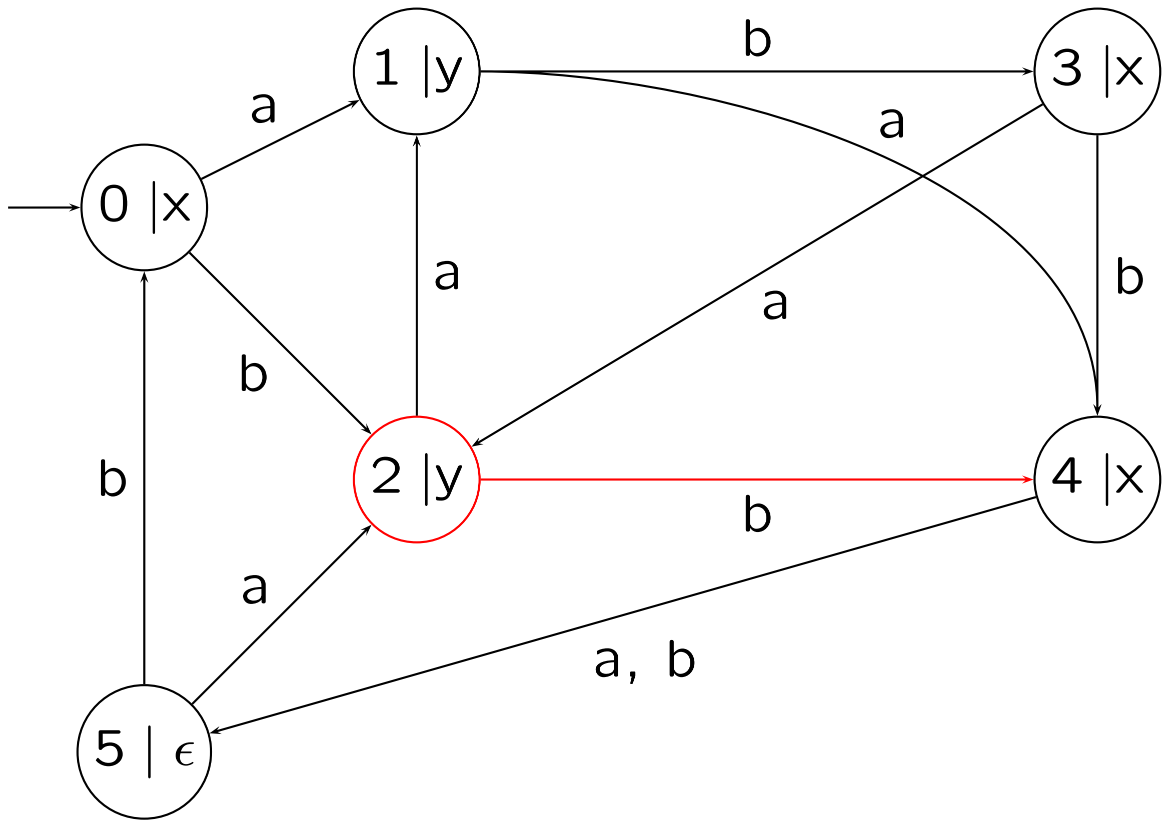
Eingabe: aabab, Ausgabe: xyx



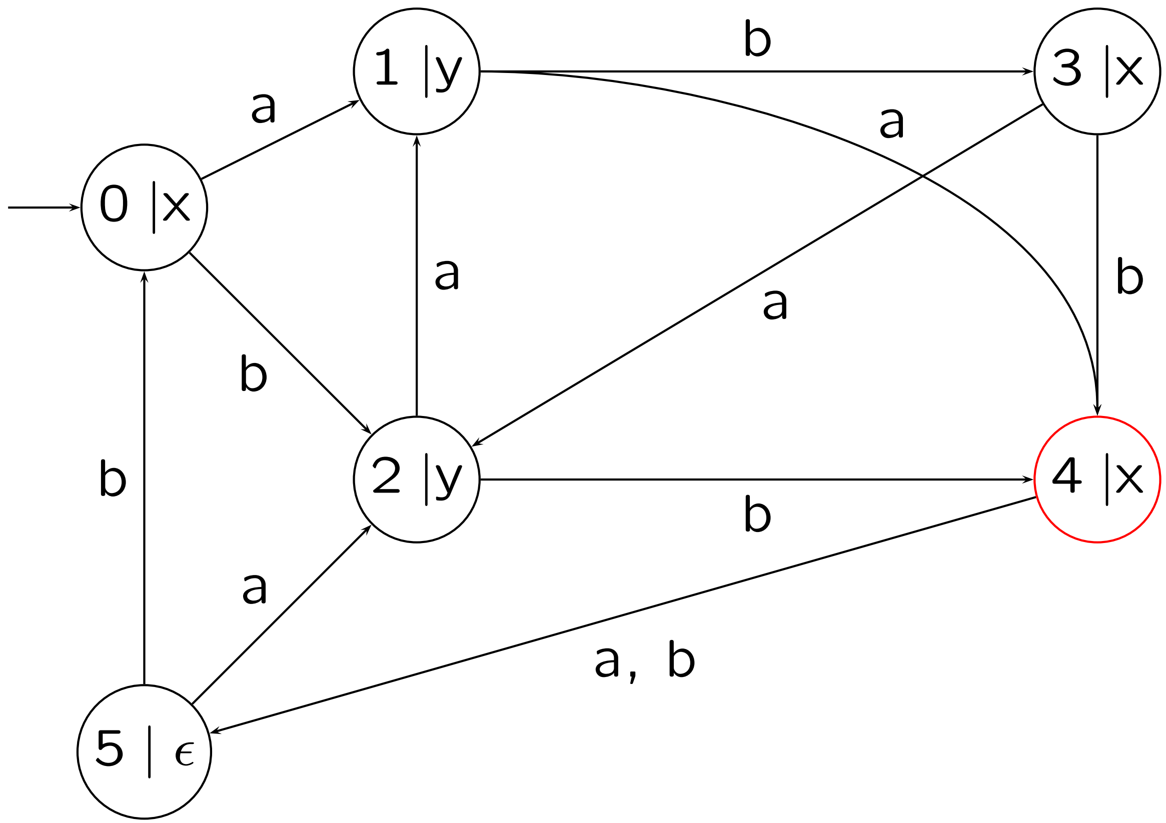
Eingabe: aabab, Ausgabe: xyxy



Eingabe: aabab, Ausgabe: xyxy



Eingabe: aabab, Ausgabe: xyxyx



Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z+x+y) \text{ div } 2$
$g(z,(x,y)) = (z+x+y) \mod 2$

.

Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z+x+y)$ div 2
$g(z,(x,y)) = (z+x+y) \mod 2$

Eingabe $w(0,0)$

.

Sinnvolle Beispiele

$$R(\epsilon) = \epsilon$$
$$R(wx) = xR(w)$$

.

Sinnvolle Beispiele

$$w_1(\epsilon) = \epsilon$$
$$w_1(w(x, y)) = w_1(w)x$$

$$w_2(\epsilon) = \epsilon$$
$$w_2(w(x, y)) = w_2(w)x$$

.

Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z + x + y) \text{ div } 2$
$g(z,(x,y)) = (z + x + y) \mod 2$

Eingabe: $w(0,0)$

Ausgabe: $Num_2(R(w')) = Num_2(R(w_1(w))) + Num_2(R(w_2(w)))$

.

Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z+x+y) \text{ div } 2$
$g(z,(x,y)) = (z+x+y) \mod 2$

Eingabe: $w(0,0)$

Ausgabe: $Num_2(R(w')) = Num_2(R(w_1(w)))+Num_2(R(w_2(w)))$

Beweis: $Num_2(R(g^{**}(0,w)))+f^*(0,w)\cdot 2^{|w|} = Num_2(R(w_1(w)))+ Num_2(R(w_2(w)))$.

.

Sinnvolle Beispiele

Definitionen zum Abkürzen:

$$\bar{g}(w) = g^{**}(0, w)$$
$$\bar{f}(w) = f^{*}(0, w)$$
$$N(w) = Num_2(R(w))$$

.

Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z + x + y)$ div 2
$g(z,(x,y)) = (z + x + y) \mod 2$

Eingabe: $w(0,0)$

Ausgabe: $Num_2(R(w')) = Num_2(R(w_1(w))) + Num_2(R(w_2(w)))$

Beweis: $N(\bar{g}(w)) + \bar{f}(w) \cdot 2^{|w|} = N(w_1(w)) + N(w_2(w))$.

.

Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z+x+y) \text{ div } 2$
$g(z,(x,y)) = (z+x+y) \mod 2$

Eingabe: $w(0,0)$

Ausgabe: $Num_2(R(w')) = Num_2(R(w_1(w)))+Num_2(R(w_2(w)))$

Beweis: $N(\bar{g}(w)) + \bar{f}(w) \cdot 2^{|w|} = N(w_1(w)) + N(w_2(w))$.

Stimmt für $w = \epsilon$.

.

Sinnvolle Beispiele

$X = \{0,1\}^2, Y = \{0,1\}, Z = \{0,1\}, z_0 = 0$
$f(z,(x,y)) = (z + x + y) \text{ div } 2$
$g(z,(x,y)) = (z + x + y) \mod 2$

Eingabe: $w(0,0)$

Ausgabe: $Num_2(R(w')) = Num_2(R(w_1(w))) + Num_2(R(w_2(w)))$

Beweis: $N(\bar{g}(w)) + \bar{f}(w) \cdot 2^{|w|} = N(w_1(w)) + N(w_2(w))$.

Wenn es für w gilt, dann auch für $w(x,y)$.

.

Sinnvolle Beispiele

$$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1}$$

.

Sinnvolle Beispiele

$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$
$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$
$N(\bar{g}(w)(\bar{f}(w)+x+y) \mod 2)+((\bar{f}(w)+x+y) \operatorname{div} 2)\cdot 2^{|w|+1}$

.

Sinnvolle Beispiele

$$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)(\bar{f}(w)+x+y) \mod 2)+((\bar{f}(w)+x+y) \operatorname{div} 2)\cdot 2^{|w|+1}$$
$$= Num_2(((\bar{f}(w)+x+y) \mod 2)R(\bar{g}(w)))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \operatorname{div} 2)$$

.

Sinnvolle Beispiele

$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$
$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$
$N(\bar{g}(w)(\bar{f}(w)+x+y) \bmod 2)+((\bar{f}(w)+x+y) \operatorname{div} 2)\cdot 2^{|w|+1}$
$= Num_2(((\bar{f}(w)+x+y) \bmod 2)R(\bar{g}(w)))$
$+2^{|w|}(2(\bar{f}(w)+x+y) \operatorname{div} 2) =$
$2^{|w|}((\bar{f}(w)+x+y) \bmod 2) + N(\bar{g}(w))$
$+2^{|w|}(2(\bar{f}(w)+x+y) \operatorname{div} 2)$

.

Sinnvolle Beispiele

$$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)(\bar{f}(w)+x+y) \mod 2)+((\bar{f}(w)+x+y) \text{ div } 2)\cdot 2^{|w|+1}$$
$$= Num_2(((\bar{f}(w)+x+y) \mod 2)R(\bar{g}(w)))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$2^{|w|}((\bar{f}(w)+x+y) \mod 2) + N(\bar{g}(w))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$N(\bar{g}(w)) + 2^{|w|}(\bar{f}(w)+x+y)$$

.

Sinnvolle Beispiele

$$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)(\bar{f}(w)+x+y) \mod 2)+((\bar{f}(w)+x+y) \text{ div } 2)\cdot 2^{|w|+1}$$
$$= Num_2(((\bar{f}(w)+x+y) \mod 2)R(\bar{g}(w)))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$2^{|w|}((\bar{f}(w)+x+y) \mod 2) + N(\bar{g}(w))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$N(\bar{g}(w)) + 2^{|w|}(\bar{f}(w)+x+y) =$$
$$N(\bar{g}(w)) + 2^{|w|}\bar{f}(w) + 2^{|w|}x + 2^{|w|}y$$

.

Sinnvolle Beispiele

$$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)(\bar{f}(w)+x+y) \mod 2)+((\bar{f}(w)+x+y) \text{ div } 2)\cdot 2^{|w|+1}$$
$$= Num_2(((\bar{f}(w)+x+y) \mod 2)R(\bar{g}(w)))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$2^{|w|}((\bar{f}(w)+x+y) \mod 2) + N(\bar{g}(w))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$N(\bar{g}(w)) + 2^{|w|}(\bar{f}(w)+x+y) =$$
$$N(\bar{g}(w)) + 2^{|w|}\bar{f}(w) + 2^{|w|}x + 2^{|w|}y \overset{IV}{=}$$
$$N(w_1(w)) + 2^{|w|}x + N(w_2(w)) + 2^{|w|}y$$

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Sinnvolle Beispiele

$$N(\bar{g}(w(x,y))) + \bar{f}(w(x,y)) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)g(\bar{f}(w),(x,y))) + f(\bar{f}(w,(x,y))) \cdot 2^{|w|+1} =$$
$$N(\bar{g}(w)(\bar{f}(w)+x+y) \mod 2)+((\bar{f}(w)+x+y) \text{ div } 2)\cdot 2^{|w|+1}$$
$$= Num_2(((\bar{f}(w)+x+y) \mod 2)R(\bar{g}(w)))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$2^{|w|}((\bar{f}(w)+x+y) \mod 2) + N(\bar{g}(w))$$
$$+2^{|w|}(2(\bar{f}(w)+x+y) \text{ div } 2) =$$
$$N(\bar{g}(w)) + 2^{|w|}(\bar{f}(w)+x+y) =$$
$$N(\bar{g}(w)) + 2^{|w|}\bar{f}(w) + 2^{|w|}x + 2^{|w|}y \overset{IV}{=}$$
$$N(w_1(w))+2^{|w|}x+N(w_2(w))+2^{|w|}y = N(w_1(w)x)+N(w_2(w)y) =$$
$$N(w_1(w(x,y))) + N(w_2(w(x,y)))$$

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