Homework 3 Advanced Analytics and Metaheuristics

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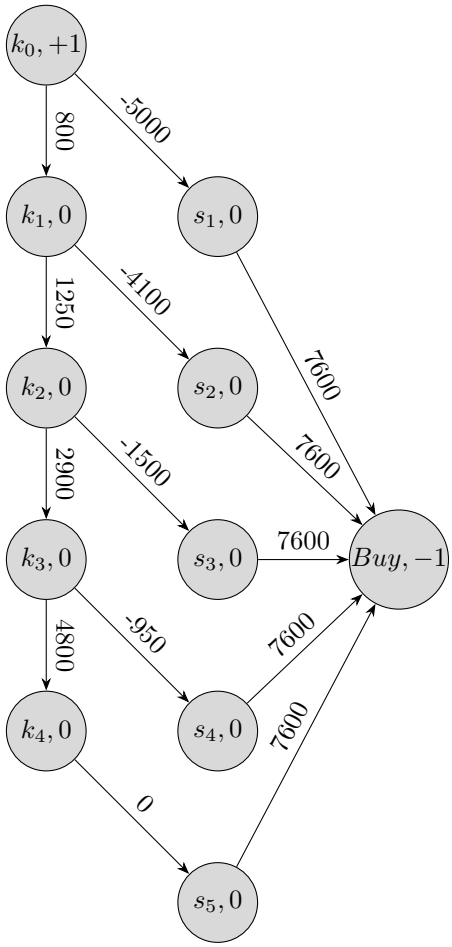
February 21, 2024

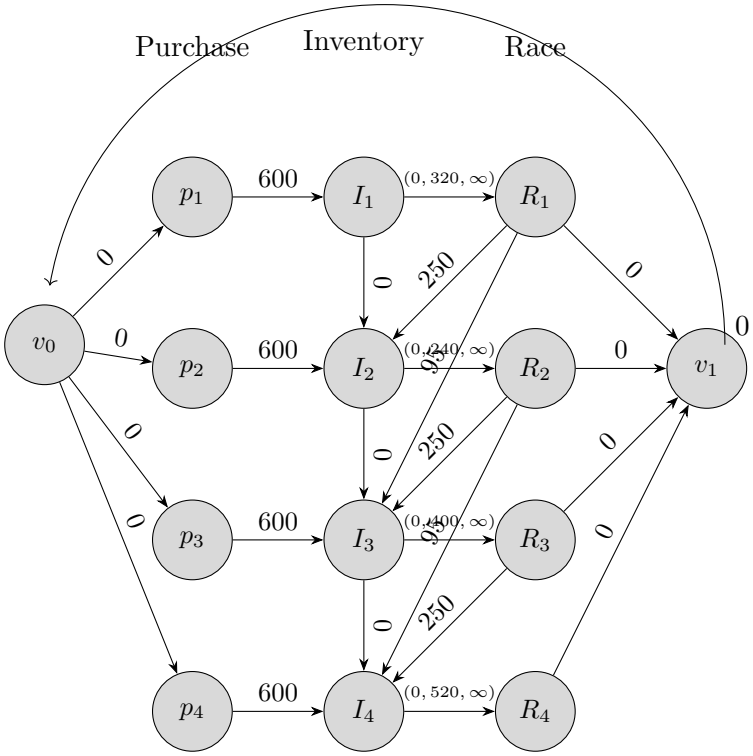
1. Team Building
   (a)
   (b)
2. Outdoor Grilling

Maintain Sell



3. Race Car Tires

   Here is my flow model: All nodes have zero b costs are displayed on arcs. If minimums are needed, the ordered triple represents $(cost, lowerLimit, upperLimit)$. The v nodes are virtual to balance the flow.



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};     # arcs in the network

param b {NODES} default 0;          # supply/demand for node i
param c {ARCS}  default 0;          # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;          # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)

var x {ARCS};                       # flow on arc (i,j)
```

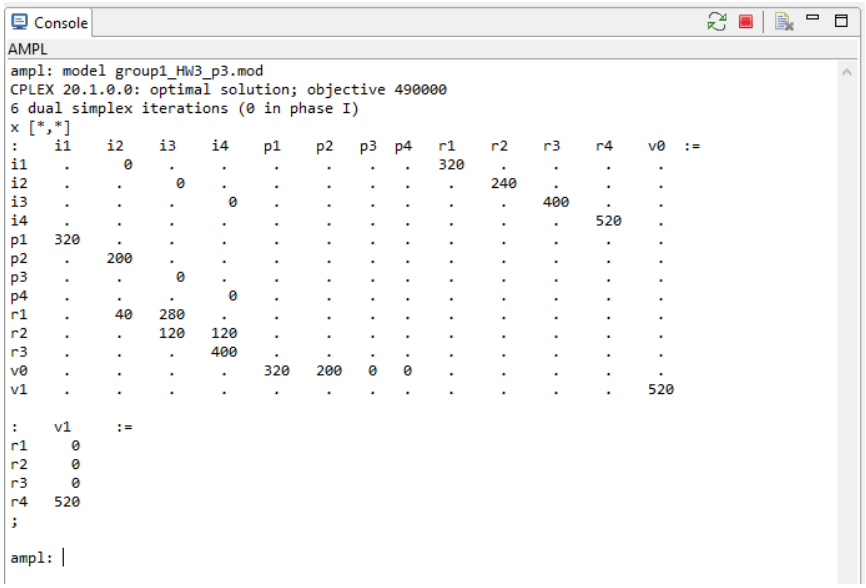
```
minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p3.dat;

solve;

display x;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=          v0, p1, p2,p3,p4, i1,i2,i3,i4,r1,r2,r3,r4,v1 ;

set ARCS := (v0,p1),(v0,p2),(v0,p3),(v0,p4), #start the flow
                        (p1,i1),(p2,i2),(p3,i3),(p4,i4), #purchase new tires each race
                        (i1,r1),(i2,r2),(i3,r3),(i4,r4), #move inventory to race
                        (r1,v1),(r2,v1),(r3,v1),(r4,v1), #move spent tires not fixed to
                        (i1,i2),(i2,i3),(i3,i4), #move unused inventory
                        (r1,i2),(r1,i3), #race 1 quick and slow fix
                        (r2,i3),(r2,i4), #race 2 quick and slow fix
                        (r3,i4), #race 3 quick fix
                        (v1,v0) #move from virtual to virtual to complete flow
                        ;



param:        c  l u :=
                [p1,i1] 600  . . #purchase new tires each race
                [p2,i2] 600  . .
```

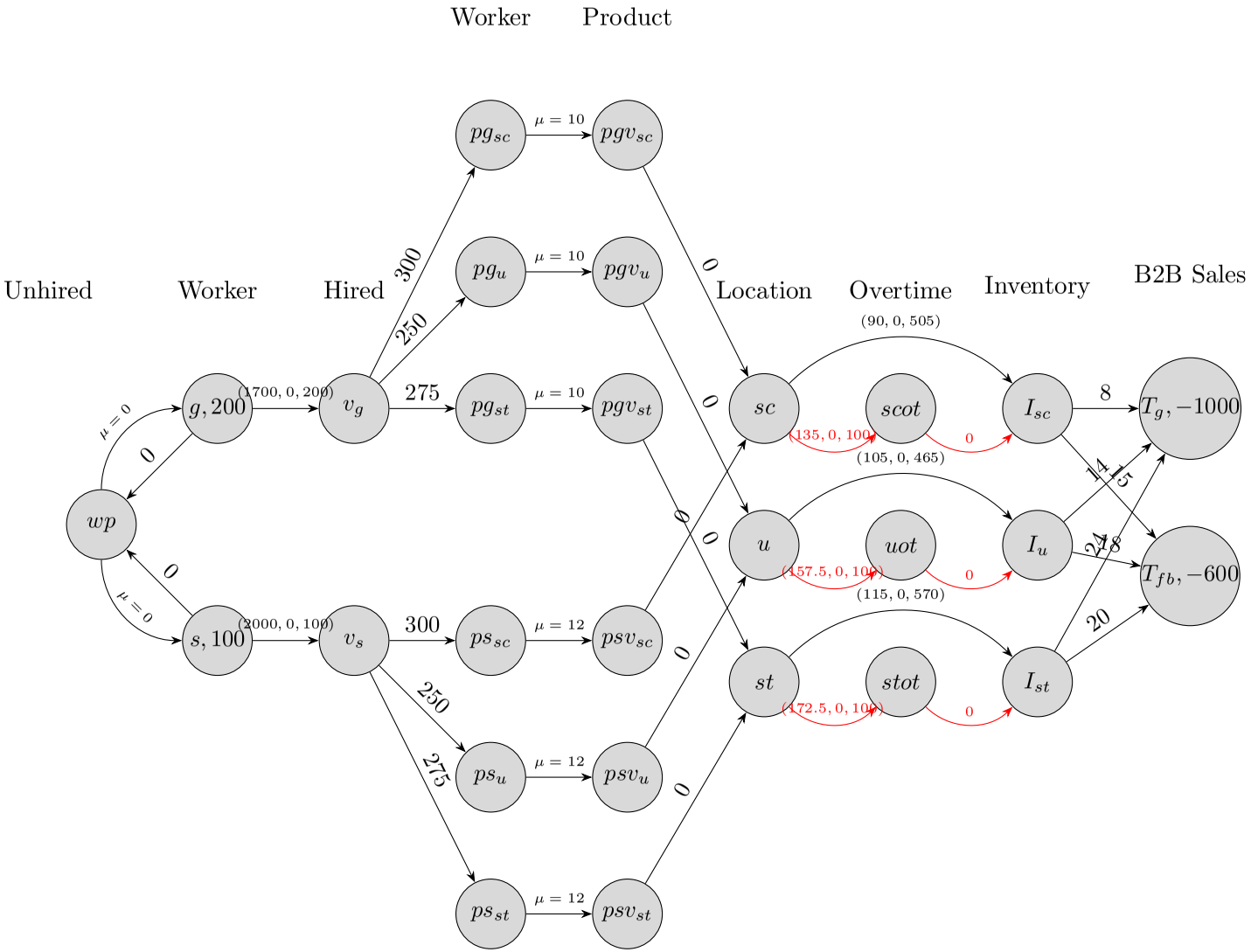
```
[p3,i3] 600   . .
[p4,i4] 600   . .
[i1,r1] .         320 . #minimum tires needed each race
[i2,r2] .         240 .
[i3,r3] .         400 .
[i4,r4] .         520 .
[r1,i2] 250 .          . #quick fix
[r2,i3] 250 .         .
[r3,i4] 250 .         .
[r1,i3] 95  .         .#slowfix
[r2,i4] 95         .         .
;
```

Here is our output:



We look to be purchasing new tires for both the needs of the first two races, 320 and 200 respectively. This is the maximum number of tires needed. We use the normal service on 280 tires from the first race and quick on the other 40. In second race we use the normal service on 120 but quick fix 120. For the third race we quick fix all 400 tires used. We end up with exactly the number of tires needed in the fourth race. Total cost is $490 000.

4. Dunder Mifflin


Worker
Product
Unhired
Worker
Hired
Location
Overtime
Inventory
B2B Sales
wp
$g, 200$
$s, 100$
v_g
v_s
pg_{sc}
pg_u
pg_{st}
ps_{sc}
ps_u
ps_{st}
pgv_{sc}
pgv_u
pgv_{st}
psv_{sc}
psv_u
psv_{st}
sc
u
st
$scot$
uot
$stot$
I_{sc}
I_u
I_{st}
$T_g, -1000$
$T_{fb}, -600$
$\mu = 0$
$\mu = 10$
$\mu = 12$
(1700, 0, 200)
(2000, 0, 100)
300
250
275
0
(90, 0, 505)
(135, 0, 100)
(105, 0, 465)
(157.5, 0, 100)
(115, 0, 570)
(172.5, 0, 100)
8
14
17
24
18
20

Here is our model file:

```
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#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;       # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p4.dat;

solve;

display x;
```
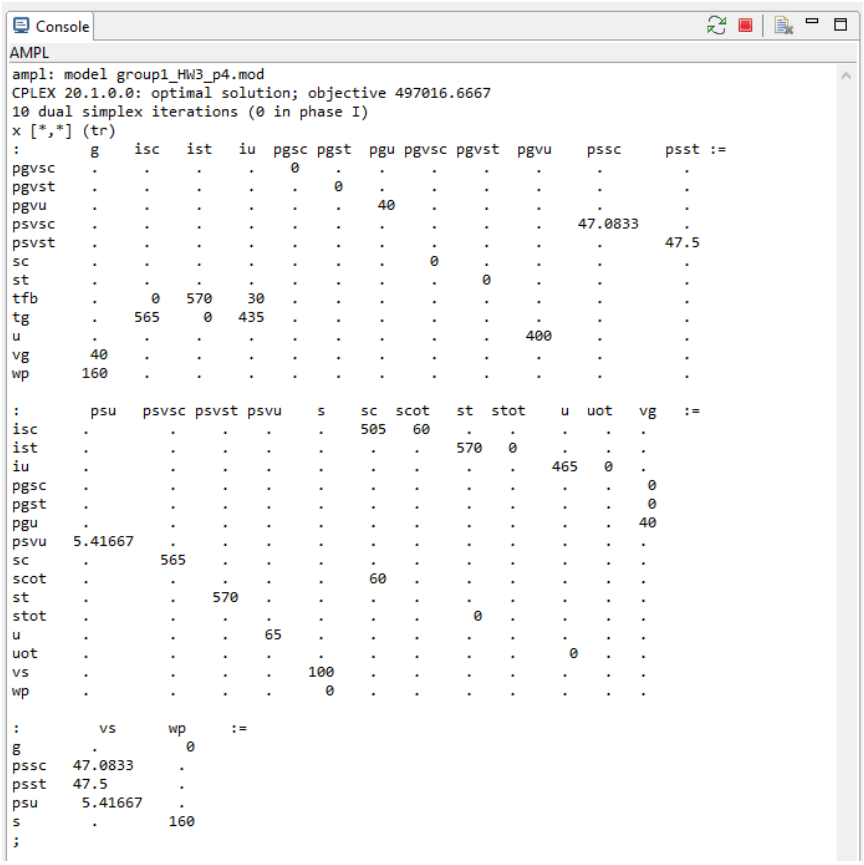

Here is our data file:

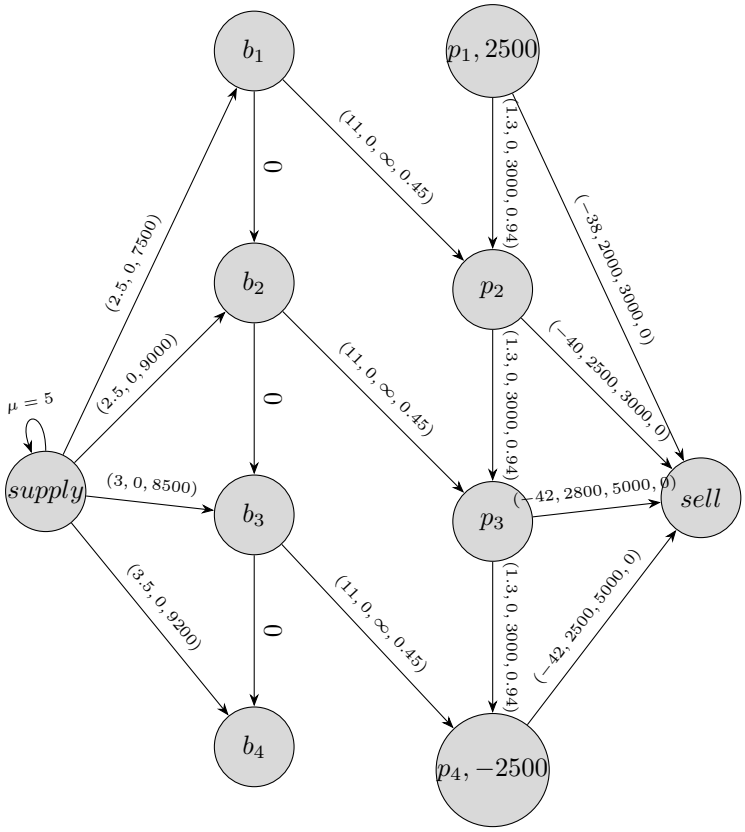
```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=          #v0, v1, #virtual nodes at begining and end to get the flow going
                              g, s, #general and specialist
                              vg, vs, #virtual to get the cost of general and specialist
                              pgsc, pgu, pgst, pssc, psu, psst, #shipping cost of each employee
                              pgvsc, pgvu, pgvst, psvsc, psvu, psvst, #convert each employee to items
                              sc, u, st, #workers (as items) now at the plants
                              scot, uot, stot # overtime possible
                              isc, iu, ist, #inventory at each plant
                              tg, tfb, #transport goods to location
                              wp; #unhired worker pool

set ARCS := (s,vs),(g,vg), #hire the workers
                        (s,wp), (g,wp), #unhired workers
                        (wp,s), (wp,g), #flow the unhired workers back to keep the balance
                        (vg,pgsc),(vg,pgu),(vg,pgst),(vs,pssc),(vs,psu),(vs,psst), #move different workers to factories
                        (pgsc,pgvsc),(pgu,pgvu),(pgst,pgvst),(pssc,psvsc),(psu,psvu),(psst,psvst), #convert the workers into items
                        (pgvsc,sc),(psvsc,sc),(pgvu,u),(psvu,u),(pgvst,st),(psvst,st), #more production capacity to each factory
                        (sc,isc),(u,iu),(st,ist), #create the products
                        (sc, scot), (u, uot), (st,stot), #overtime hours making products
                        (scot, isc), (uot,iu), (stot, ist), #overtime products created go to inventory for free
                        (isc,tg), (isc,tfb), (iu, tg), (iu,tfb), (ist,tg), (ist,tfb), #move the product from inventory to customer
                        ;

param: b:=
        g 200
```

```
       s 100
       tg -1000
       tfb -600;

param:        c  l u mu:=
                [s,vs]                      2000          .         100         . #recruit workers
                [g,vg]                      1700          .         200         .
                [vg,pgsc]                    300          .         .           . #move workers to factories
                [vg,pgu]                     250          .         .           .
                [vg,pgst]                    275          .         .           .
                [vs,pssc]                    300          .         .           .
                [vs,psu]                     250          .         .           .
                [vs,psst]                    275          .         .           .
                [pgsc,pgvsc]         .       .          .         .           10 #convert workers to items
                [pgu,pgvu]                   .          .         .           .          10
                [pgst,pgvst]         .       .          .         .           10
                [pssc,psvsc]         .       .          .         .           12
                [psu,psvu]                   .          .         .           .          12
                [psst,psvst]         .       .          .         .           12
                [sc,isc]                    90          .         505         . #create the products
                [u,iu]                         105       .         465         .
                [st, ist]                    115         .         570         .
                [sc, scot]                    135        .         100         . #overtime possible
                [u,uot]                     157.5        .         100         .
                [st,stot]                    172.5       .         100         .
                [isc, tg]                    8           .         .           . #move product to customer
                [isc, tfb]                    15         .         .           . ################################check me
                [iu, tg]                     14          .         .           .
                [iu, tfb]                    18          .         .           .
                [ist, tg]                    24          .         .           .
                [ist,tfb]                    20          .         .           .
                [wp,s]                       .           .         .           0
                [wp,g]                       .           .         .           0
;
```

Here is my output:


Console
AMPL
ampl: model group1_HW3_p4.mod
CPLEX 20.1.0.0: optimal solution; objective 497016.6667
10 dual simplex iterations (0 in phase I)
x [*,*] (tr)
: g isc ist iu pgsc pgst pgu pgvsc pgvst pgvu pssc psst :=
pgvsc . . . . 0 . . . .
pgvst . . . . . 0 . . . . . .
pgvu . . . . . . 40 . . . . .
psvsc . . . . 47.0833 .
psvst 47.5
sc . . . . 0 . . . .
st . . . . 0 . . .
tfb . 0 570 30 . . . .
tg . 565 0 435 . . . .
u . . . . 400 . .
vg 40 . . . .
wp 160 . . . .
: psu psvsc psvst psvu s sc scot st stot u uot vg :=
isc . . . . . 505 60 . . . . .
ist . . . . 570 0 . . .
iu . . . . 465 0 .
pgsc 0
pgst 0
pgu 40
psvu 5.41667 . . . .
sc . 565 . . . .
scot . . . . . 60 . . . . . .
st . . 570 . . . .
stot . . . . 0 . . . .
u . . . 65 . . . .
uot . . . . 0 . .
vs . . . . 100 . . . .
wp . . . . 0 . . . .
: vs wp :=
g . 0
pssc 47.0833 .
psst 47.5 .
psu 5.41667 .
s . 160
;

5. Mud b Gone



Here is our model file:

```
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#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;
option cplex_options 'sensitivity';

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;       # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];
```
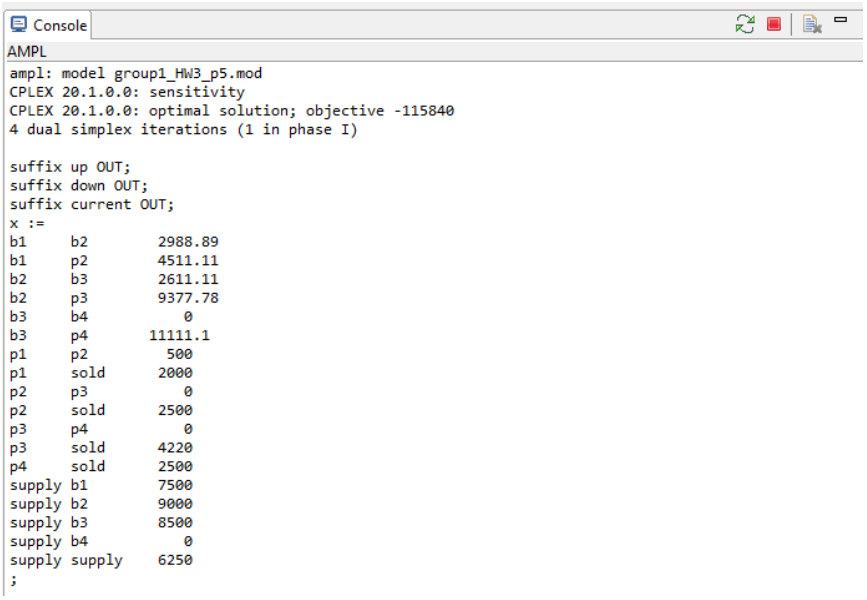

```
subject to upcapacity {(i,j) in ARCS}: x[i,j] <= u[i,j];

subject to lowcapacity {(i,j) in ARCS}: l[i,j] <= x[i,j];

data group1_HW3_p5.dat;

solve;

display x;

display upcapacity,upcapacity.up, upcapacity.down;

display x.current, x.up, x.down;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=  supply, #suppliers
                        b1,b2,b3,b4, #base for production
                        p1, p2, p3, p4, #product
                        sold #sold product
;

set ARCS := (supply,*) b1 b2 b3 b4 #base purchased from supplier
                       (*,sold)   p1 p2 p3 p4 #product sold
                       (b1,p2),(b2,p3),(b3,p4), #base converted to product
                       (b1,b2),(b2,b3),(b3,b4),
                       (p1,p2),(p2,p3),(p3,p4)
                       (supply,supply)
                       ;

param: b:=
        p1 2500
        p4 -2500;

param:        c  l u mu:=
               [supply,b1] 2.5        .        7500        . #buy new base
               [supply,b2] 2.5 .         9000        .
               [supply, b3] 3 .         8500        .
               [supply,b4]        3.5        .        9200        .

               [p1,sold] -38 2000        3000        0 #sell product
               [p2,sold] -40 2500        3000        0
               [p3,sold] -42 2800        5000        0
               [p4,sold] -42 2500        5000        0

               [p1,p2] 1.3        .        3000        .94 #store product till next month
               [p2,p3] 1.3        .        3000        .94
               [p3,p4] 1.3        .        3000        .94

               [b1,p2] 11        .        .        .45 #convert base into product.  Assumption not too worry about max storage
               [b2,p3] 11        .        .        .45
               [b3,p4] 11        .        .        .45

               [supply,supply] .        .        .         5
;
```

(a) We see a solution for our flow. We sell 2000, 2500, 4220 and 2500 in each of the respective periods. We remark on the infinite supply house obtained by creating a loop with a μ factor set to 5 but could be any value greater than 1.



(b) We next look at sensitivity of the capacity. We see a value of -$5.40 in the report, supply to b2. We interpret this as having additional gallons of base will increase (recall minimizing) our income by $5.40. Both the up and the down are reported as 0. We are unsure of how to interpret this as we clearly would use more of the base if it was available but are unsure why these values are all zero. We even broke apart the upper and lower limit thinking this was the issue but did not change the up and down on the shadow price. We do clearly see this as the bottle neck in our model.

```
:             upcapacity upcapacity.up upcapacity.down    :=
b1     b2           0            0              0
b1     p2           0            0              0
b2     b3           0            0              0
b2     p3           0            0              0
b3     b4           0            0              0
b3     p4           0            0              0
p1     p2           0            0              0
p1     sold         0            0              0
p2     p3           0            0              0
p2     sold         0            0              0
p3     p4           0            0              0
p3     sold         0            0              0
p4     sold         0            0              0
supply b1          -5.4          0              0
supply b2          -5.4          0              0
supply b3          -4.9          0              0
supply b4           0            0              0
supply supply       0            0              0
;
```

(c) We see that the contribution to the cost is -$40 on the flow from p2 to Sold. This value can be modified up to -$42 and not change the model at all.

```
:             x.current       x.up            x.down         :=
b1     b2           0          0.0861702   -1e+20
b1     p2          11      1e+20               10.9138
b2     b3           0          1.82872         -3.19744e-15
b2     p3          11         11                9.17128
b3     b4           0          0                0
b3     p4          11         12.8287          11
p1     p2           1.3        1.48        -1e+20
p1     sold       -38      1e+20              -38.18
p2     p3           1.3    1e+20               -2.52
p2     sold       -40      1e+20              -42
p3     p4           1.3    1e+20               -2.52
p3     sold       -42        -42           -1e+20
p4     sold       -42      1e+20              -42
supply b1           2.5        7.9         -1e+20
supply b2           2.5        7.9         -1e+20
supply b3           3          7.9         -1e+20
supply b4           0          0                0
supply supply       0         19.6         -1e+20
;
```