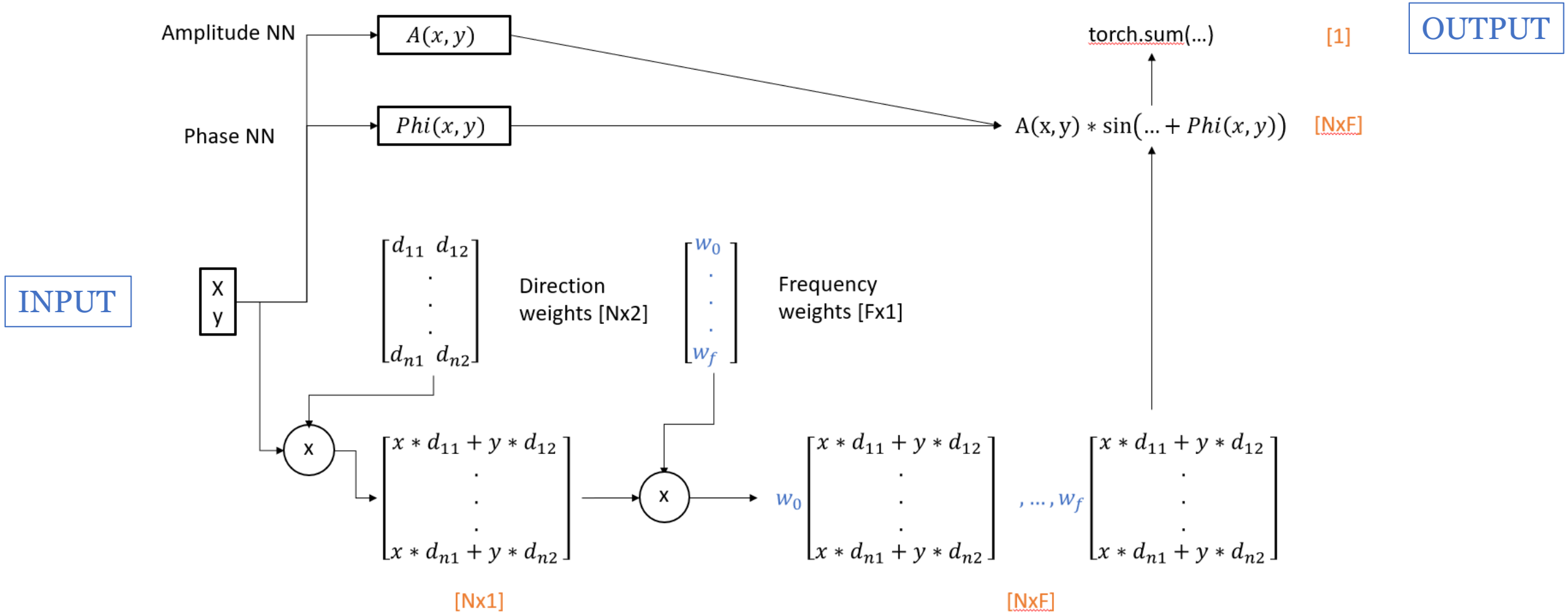
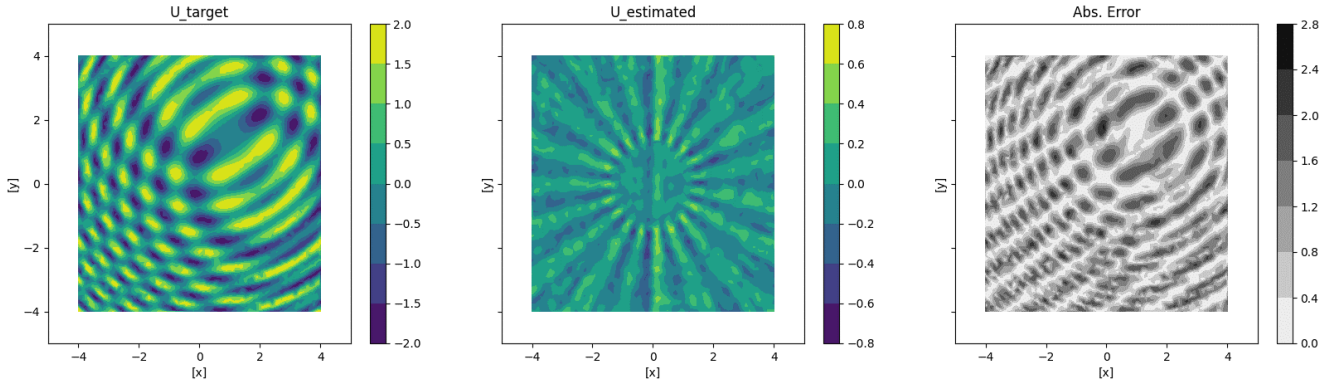
Frequency PINN

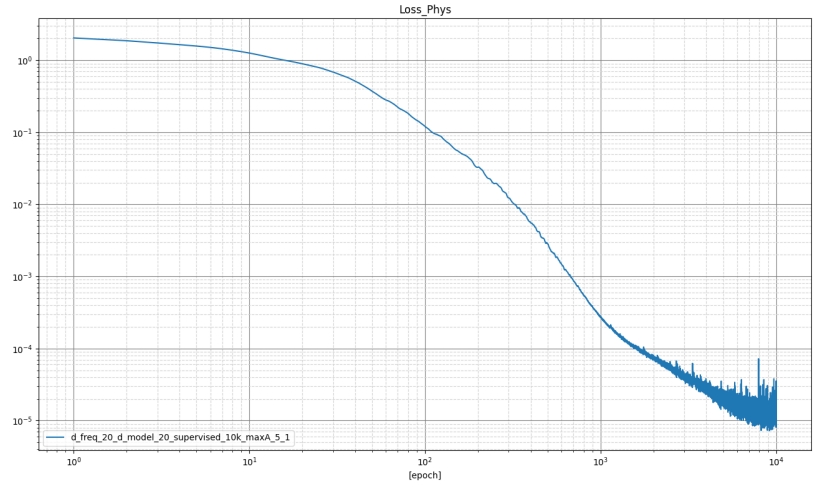


Frequency PINN: sin(x^2)

Supervised

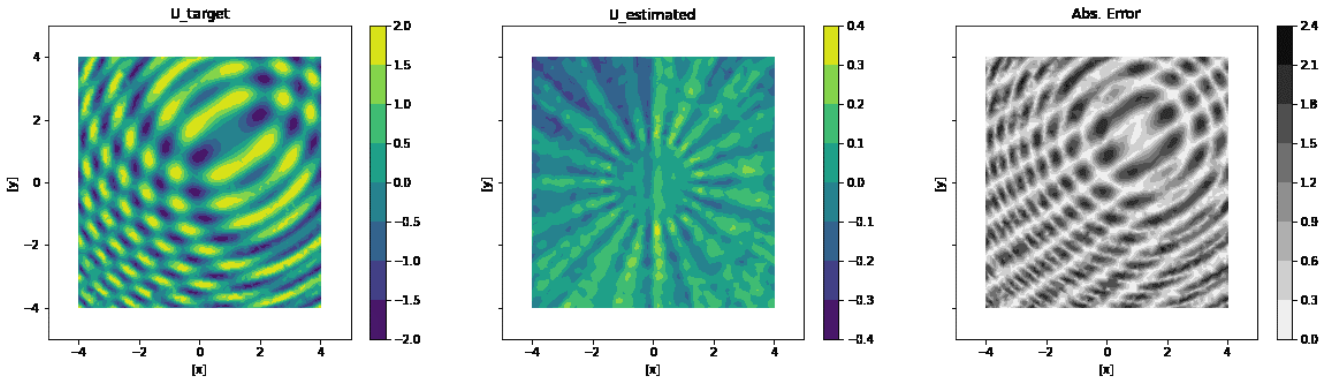
Every 1k it. Max 10k it.

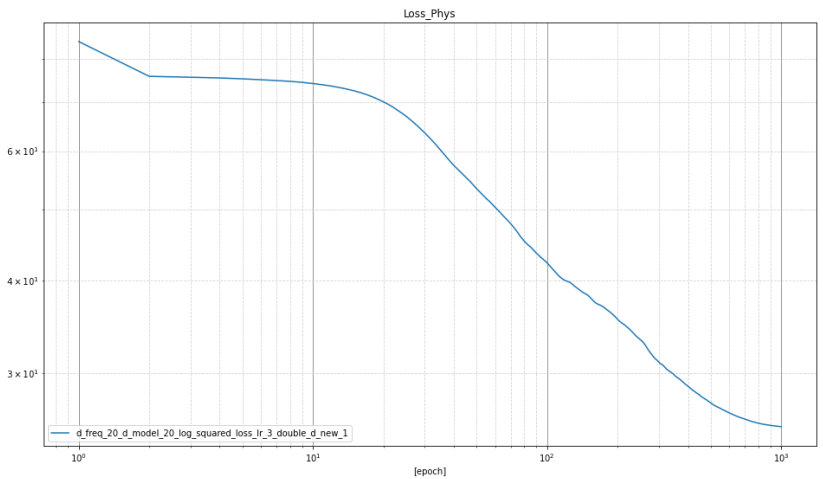




Unsupervised

Every 100 it. Max 1k it.





Error on Amplitude

Solution

$$u_{\{sol\}} = A_{\{sol\}} \sin(\omega x)$$

$$\frac{d^2u}{dx^2} = -A_{\{sol\}}\omega^2 \sin(\omega x)$$

Estimation

$$\tilde{u} = \mathrm{A} \sin(\omega x)$$

$$\frac{d^2\tilde{u}}{dx^2} = -A\omega^2 \sin(\omega x)$$

Error on Amplitude

$$error = \left|A - A_{\{sol\}}\right|$$

$$error\_der = \left|A - A_{\{sol\}}\right| \omega^2$$

Optimize

If signal is composed of several frequencies, then there will be a big difference between errors in amplitude of low frequency components and errors in amplitude of high frequency components…!

Error on Amplitude

$$u_{\{sol\}} = A_{1\_sol}\sin(1x) + A_{2\_sol}\sin(10x)$$

$$error_1 = |A_1 - A_{1\_sol}|$$

$$\tilde{u} = A_1\sin(1x) + A_2\sin(10x)$$

$$error_2 = |A_2 - A_{2\_sol}|$$

$$\frac{d^2u_{\{sol\}}}{dx^2} = -A_{1\_sol}\sin(1x) - A_{2_{sol}}(10^2)\sin(10x)$$

$$error\_der_1 = |A_1 - A_{1\_sol}| * 1^2$$

$$\frac{d^2\tilde{u}}{dx^2} = -A_1\sin(1x) - A_2(10^2)\sin(10x)$$

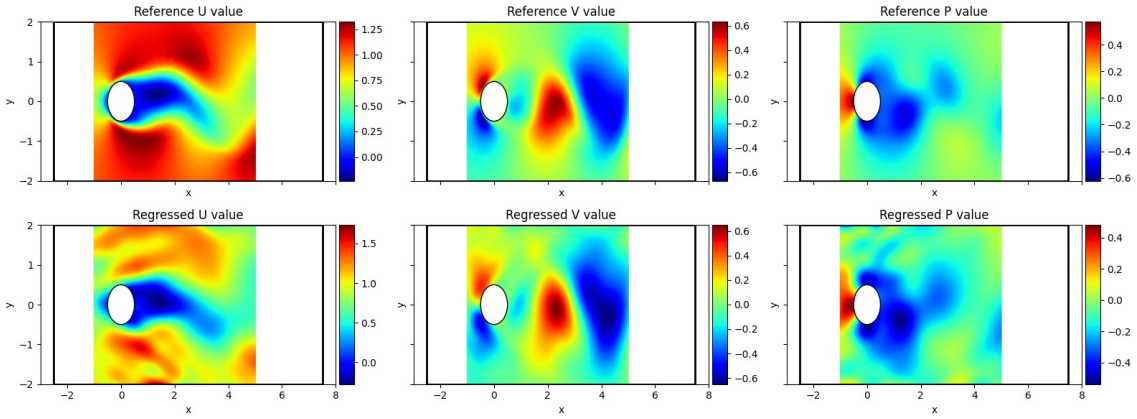
$$error\_der_2 = |A_2 - A_{2\_sol}| * 10^2$$

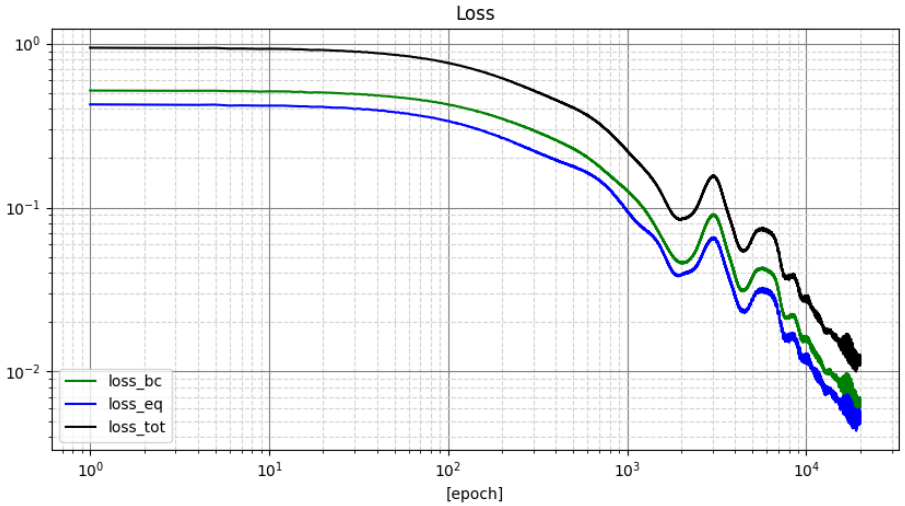
What NN tries to do is reduce where error is large.
Suppose $error\_der_1$ and $error\_der_2$ are equal to 1:

$$error_1 = error\_der_1 = 1 = 10^0$$
$$error_2 = \frac{error\_der_2}{10^2} = \frac{1}{100} = 10^{-2}$$

Frequency PINN: 2D-NS Incompressible Low-Re

Supervised: result at 20k it. Low freq. range init.





Supervised: result at 20k it. High freq. range init.