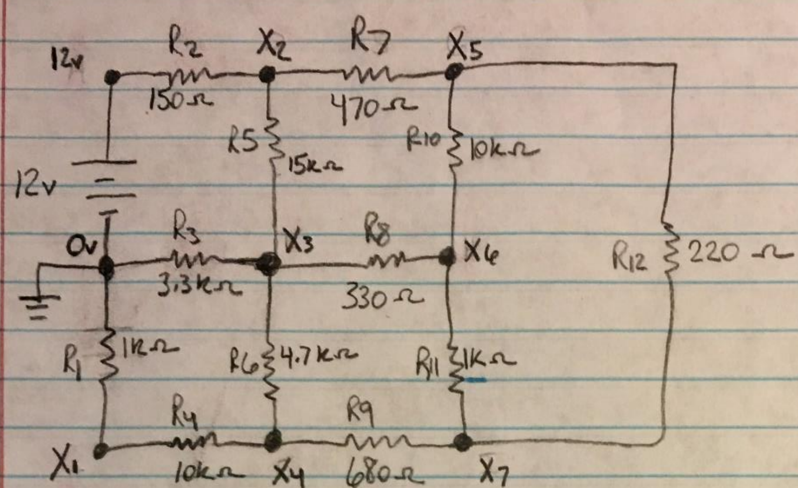
A web search for linear systems related to my field of study in Electrical Engineering resulted in the following links.

https://sites.math.washington.edu/~king/coursedir/m308a01/Projects/m308a01-pdf/taing.pdf

http://faculty.uaeu.ac.ae/w_ahmed/differential/Lectures/Applications%20of%20Systems%20of%20Linear%20Equations.pdf

There are two types of problems discussed in these web pages that would apply to Electrical Engineering. The first is node voltage analysis of resistor circuits. Complex resistive circuits form a linear system of n equations and n unknowns. The following is an example to illustrate using a linear solver to analyze the node voltages in a complex resistor circuit. The circuit has a 12VDC power source and a common reference at the negative side of the power source. This means we know the value for the two nodes the power source is connected to, 12VDC on the positive side and 0VDC on the negative side. The nodes labeled X1 through X7 are unknown and we can form a linear system to compute the unknown voltages at each of the nodes. The system will be seven equations and seven unknowns and can be derived using Kirchhoff's Current Law which states that all currents entering and leaving a node must sum to zero. As a note, the node voltages X1 through X7 are values with respect to the common reference at the negative side of the power source. See figure 1 through 3 for the resistor circuit and derived equations for the linear system to solve. Figure 4 defines equalities between variable names in the circuit for comparison with the results from my linear solving routine and the results from simulating the circuit in LTspice (a computer application for circuit analysis). The derived node voltage equations are put in matrix form and augmented with the known values forming the linear problem Hx=b where x is a vector of unknowns to solve for. The results from my programed linear solver are shown in figure 5, the results of my linear solver can also be viewed in homework problem 10. For comparison, the LTspice simulator results are shown in figure 6.



use current law for node voltage } all currents in and out of a node must sum to 0.

node A: 12V or $A = 12$,

node B: 0V or $B = 0$,

node C: $\dfrac{C-B}{R_1} + \dfrac{C-F}{R_4} = 0$,

node D: $\dfrac{D-A}{R_2} + \dfrac{D-E}{R_5} + \dfrac{D-G}{R_7} = 0$,

node E: $\dfrac{E-B}{R_3} + \dfrac{E-D}{R_5} + \dfrac{E-F}{R_6} + \dfrac{E-H}{R_8} = 0$,

node F: $\dfrac{F-C}{R_4} + \dfrac{F-E}{R_6} + \dfrac{F-I}{R_9} = 0$,

Figure 1: Resistor circuit and node voltage equations.

node G: $\dfrac{G-D}{R_7} + \dfrac{G-H}{R_{10}} + \dfrac{G-I}{R_{12}} = 0,$

node H: $\dfrac{H-G}{R_{10}} + \dfrac{H-I}{R_{11}} + \dfrac{H-E}{R_8} = 0,$

node I: $\dfrac{I-F}{R_9} + \dfrac{I-H}{R_{11}} + \dfrac{I-G}{R_{12}} = 0.$

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node C: $-\frac{1}{R_1} B + \frac{1}{R_1} C + \frac{1}{R_4} C - \frac{1}{R_4} F = 0$

$$\left(-\frac{1}{R_1}\right)B + \left(\frac{1}{R_1} + \frac{1}{R_4}\right)C - \left(\frac{1}{R_4}\right)F = 0$$

$$\boxed{-0.\overset{0}{\cancel{0}}01\,B + 0.0011\,C - 0.0001\,F = 0}$$

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node D: $\left(\frac{1}{R_2}\right)D - \frac{12}{R_2} + \left(\frac{1}{R_5}\right)D - \left(\frac{1}{R_5}\right)E + \left(\frac{1}{R_7}\right)D - \left(\frac{1}{R_7}\right)G = 0$

$$\boxed{0.00886099 2908\,D - 0.0000666666667\,E - 0.002127659574\,G = 0.08}$$

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node E: $\left(\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_6} + \frac{1}{R_8}\right)E - \left(\frac{1}{R_5}\right)D - \left(\frac{1}{R_6}\right)F - \left(\frac{1}{R_8}\right)H = 0$

$$\boxed{0.003612765957\,E - 0.0000666666667\,D - 0.000212765957\,F - 0.00303030303\,H = 0}$$

Figure 2: Node voltage equations.

node F: $\left(\frac{1}{R4} + \frac{1}{R6} + \frac{1}{R9}\right) F - \left(\frac{1}{R4}\right) C - \left(\frac{1}{R6}\right) E - \left(\frac{1}{R9}\right) I = 0$

$$\boxed{\begin{gathered}0.00178335 4193\, F - 0.0001\, C - 0.000212765957\, E \\ - 0.00147058 8235\, I = 0\end{gathered}}$$

node G: $\left(\frac{1}{R7} + \frac{1}{R10} + \frac{1}{R12}\right) G - \left(\frac{1}{R7}\right) D - \left(\frac{1}{R10}\right) H - \left(\frac{1}{R12}\right) I = 0$

$$\boxed{\begin{gathered}0.00677311412\, G - 0.002127659574\, D - 0.0001\, H \\ - 0.004545454545\, I = 0\end{gathered}}$$

node H: $\left(\frac{1}{R10} + \frac{1}{R11} + \frac{1}{R8}\right) H - \left(\frac{1}{R10}\right) G - \left(\frac{1}{R11}\right) I - \left(\frac{1}{R8}\right) E = 0$

$$\boxed{\begin{gathered}0.00413030303\, H - 0.0001\, G - 0.001\, I - \\ 0.00303030303\, E = 0\end{gathered}}$$

node I: $\left(\frac{1}{R9} + \frac{1}{R11} + \frac{1}{R12}\right) I - \left(\frac{1}{R9}\right) F - \left(\frac{1}{R11}\right) H - \left(\frac{1}{R12}\right) G = 0$

$$\boxed{\begin{gathered}0.00701604 2781\, I - 0.001470588235\, F - 0.001\, H \\ - 0.004545454545\, G = 0\end{gathered}}$$

Figure 3: Node voltage equations.

node X_1 = node C = $V(x_1)$

node X_2 = node D = $V(x_2)$

node X_3 = node E = $V(x_3)$

node X_4 = node F = $V(x_4)$

node X_5 = node G = $V(x_5)$

node X_6 = node H = $V(x_6)$

nod X_7 = node I = $V(x_7)$

Figure 4: Equalities between variable names.

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Augmented matrix H =
1.100e-03       0.000e+00       0.000e+00       -1.000e-04      0.000e+00       0.000e+00       0.000e+00       |0.000e+00
0.000e+00       8.861e-03       -6.667e-05      0.000e+00       2.128e-03       0.000e+00       0.000e+00       |8.000e-02
0.000e+00       -6.667e-05      3.613e-03       -2.128e-04      0.000e+00       -3.030e-03      0.000e+00       |0.000e+00
-1.000e-04      0.000e+00       -2.128e-04      1.783e-03       0.000e+00       0.000e+00       -1.471e-03      |0.000e+00
0.000e+00       -2.128e-03      0.000e+00       0.000e+00       6.773e-03       -1.000e-04      -4.545e-03      |0.000e+00
0.000e+00       0.000e+00       -3.030e-03      0.000e+00       -1.000e-04      4.130e-03       -1.000e-03      |0.000e+00
0.000e+00       0.000e+00       0.000e+00       -1.471e-03      -4.545e-03      -1.000e-03      7.016e-03       |0.000e+00


Solution Hx=b, using Cholesky decomposition on matrix H
x1 = 8.080e-01
x2 = 1.154e+01
x3 = 7.539e+00
x4 = 8.888e+00
x5 = 1.021e+01
x6 = 8.111e+00
x7 = 9.632e+00
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Figure 5: Results of my linear solver.

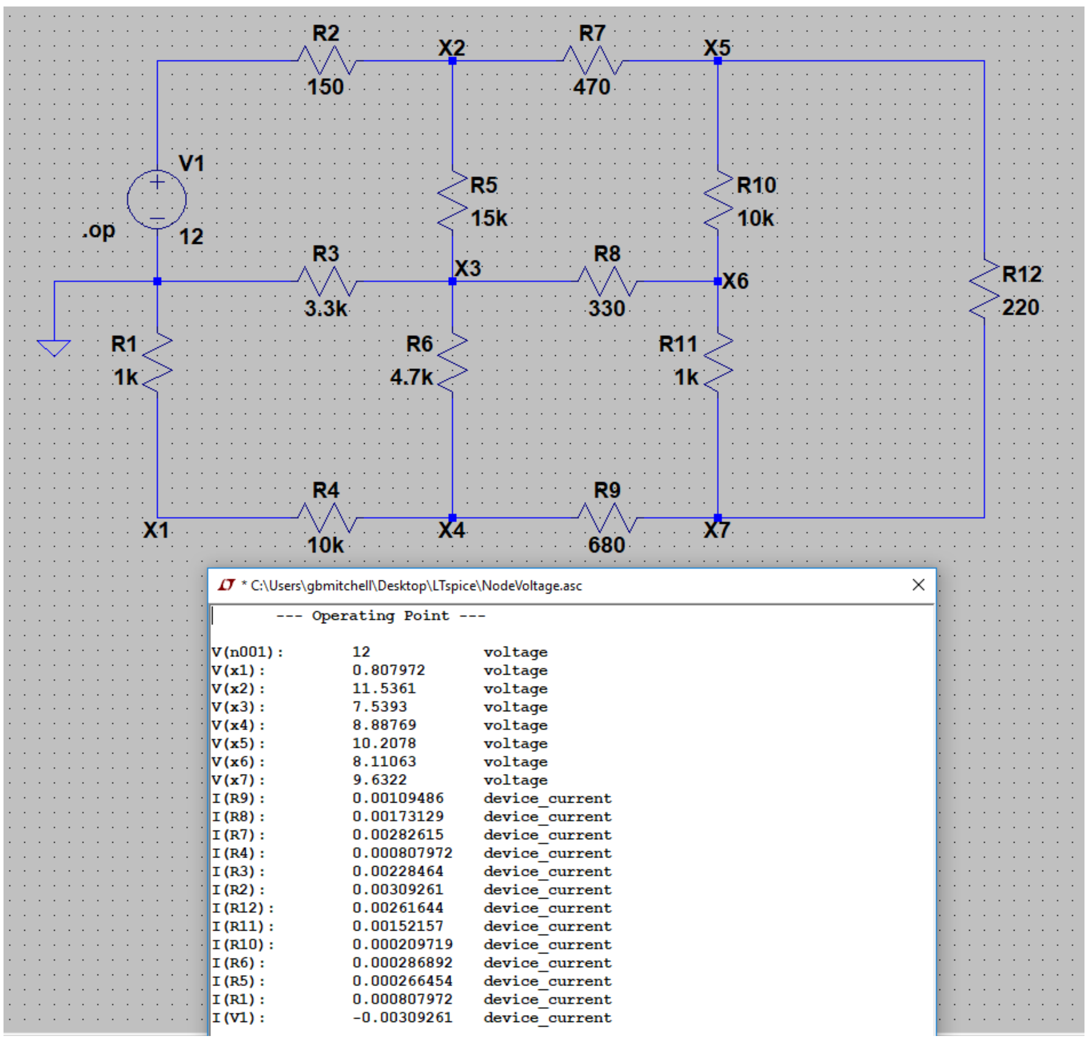
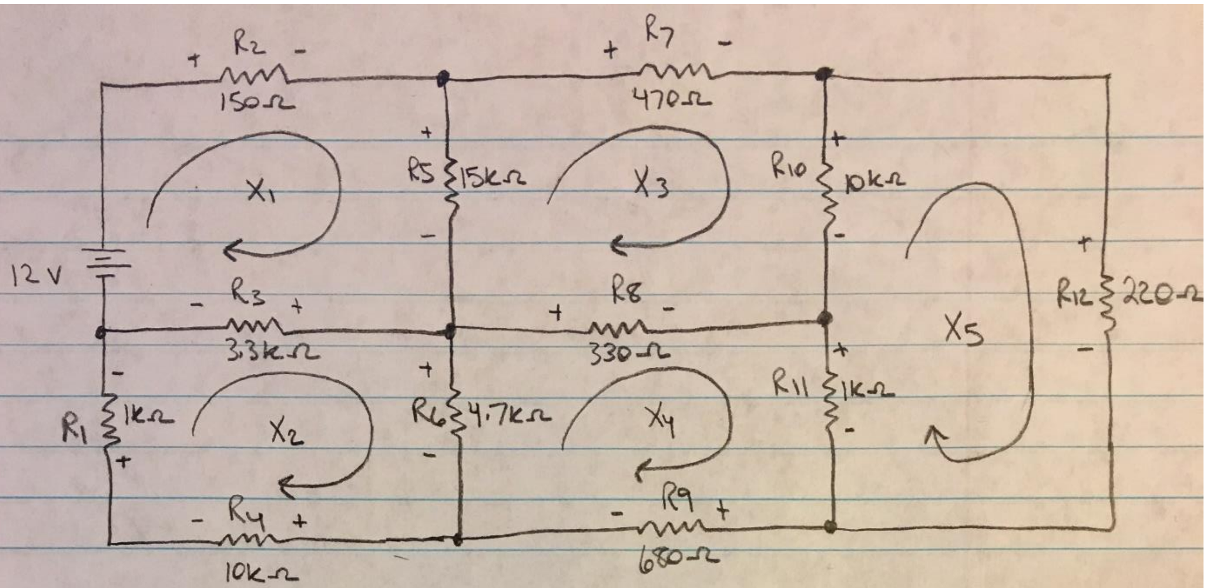


Figure 6: LTspice simulation.

The second example of linear solvers in Electrical Engineering is loop (mesh) current analysis of resistor circuits. Complex resistive circuits form a linear system of n equations and n unknowns. The following is an example to illustrate using a linear solver to analyze the loop currents in a complex resistor circuit. The circuit has a 12VDC power source and a common reference at the negative side of the power source. Each mesh in the circuit, shown in figure 7, will have an associated loop current. The loop currents labeled X1 through X5 are unknown and we can form a linear system to compute the unknown currents. The system will be five equations and five unknowns and can be derived using Kirchhoff's voltage Law which states that all voltages in a mesh loop must sum to zero. See figure 7 and 8 for the resistor circuit and derived equations for the linear system to solve. At the bottom of figure 8 are variable name equalities defined for comparison with the results from my linear solving routine and the results from simulating the circuit in LTspice. The derived loop current equations are put in matrix form and augmented with the known values forming the linear problem Px=b where x is a vector of unknowns to solve for. The results from my programed linear solver are shown in figure 9, the results of my linear solver can also be viewed in homework problem 10. For comparison, the LTspice simulator results are shown in figure 10.



use voltage law for mesh current } all voltages in a loop must sum to 0.

$$-12 + R2(X_1) + R5(X_1 - X_3) + R3(X_1 - X_2) = 0$$

$$R1(X2) + R3(X2 - X_1) + R6(X2 - X4) + R4(X2) = 0$$

$$R5(X3 - X_1) + R7(X3) + R10(X3 - X5) + R8(X3 - X4) = 0$$

$$R6(X4 - X2) + R8(X4 - X3) + R11(X4 - X5) + R9(X4) = 0$$

$$R11(X5 - X4) + R10(X5 - X3) + R12(X5) = 0$$

Figure 7: Resistor circuit and loop current equations.

$$(R_2 + R_5 + R_3)X_1 - (R_3)X_2 - (R_5)X_3 = 12$$

$$(R_1 + R_3 + R_4 + R_6)X_2 - (R_3)X_1 - (R_6)X_4 = 0$$

$$(R_5 + R_7 + R_8 + R_{10})X_3 - (R_5)X_1 - (R_{10})X_5 - (R_8)X_4 = 0$$

$$(R_6 + R_8 + R_{11} + R_9)X_4 - (R_6)X_2 - (R_8)X_3 - (R_{11})X_5 = 0$$

$$(R_{11} + R_{10} + R_{12})X_5 - (R_{11})X_4 - (R_{10})X_3 = 0$$

$$18450X_1 - 3300X_2 - 15000X_3 = 12$$

$$-3300X_1 + 19000X_2 - 4700X_4 = 0$$

$$-15000X_1 + 25800X_3 - 330X_4 - 10000X_5 = 0$$

$$-4700X_2 - 330X_3 + 6710X_4 - 1000X_5 = 0$$

$$-10000X_3 - 1000X_4 + 11220X_5 = 0$$

$$X_1 = I(R_2)$$
$$X_2 = I(R_1) = I(R_4)$$
$$X_3 = I(R_7)$$
$$X_4 = I(R_9)$$
$$X_5 = I(R_{12})$$

Figure 8: Loop current equations and variable equalities.

```
Augmented matrix P =
1.845e+04      -3.300e+03      -1.500e+04      0.000e+00       0.000e+00       |1.200e+01
-3.300e+03     1.900e+04       0.000e+00       -4.700e+03      0.000e+00       |0.000e+00
-1.500e+04     0.000e+00       2.580e+04       -3.300e+02      -1.000e+04      |0.000e+00
0.000e+00      -4.700e+03      -3.300e+02      6.710e+03       -1.000e+03      |0.000e+00
0.000e+00      0.000e+00       -1.000e+04      -1.000e+03      1.122e+04       |0.000e+00


Solution Px=b, using Cholesky decomposition on matrix P
x1 = 3.093e-03
x2 = 8.080e-04
x3 = 2.826e-03
x4 = 1.095e-03
x5 = 2.616e-03
```

Figure 9: Results of my linear solver.

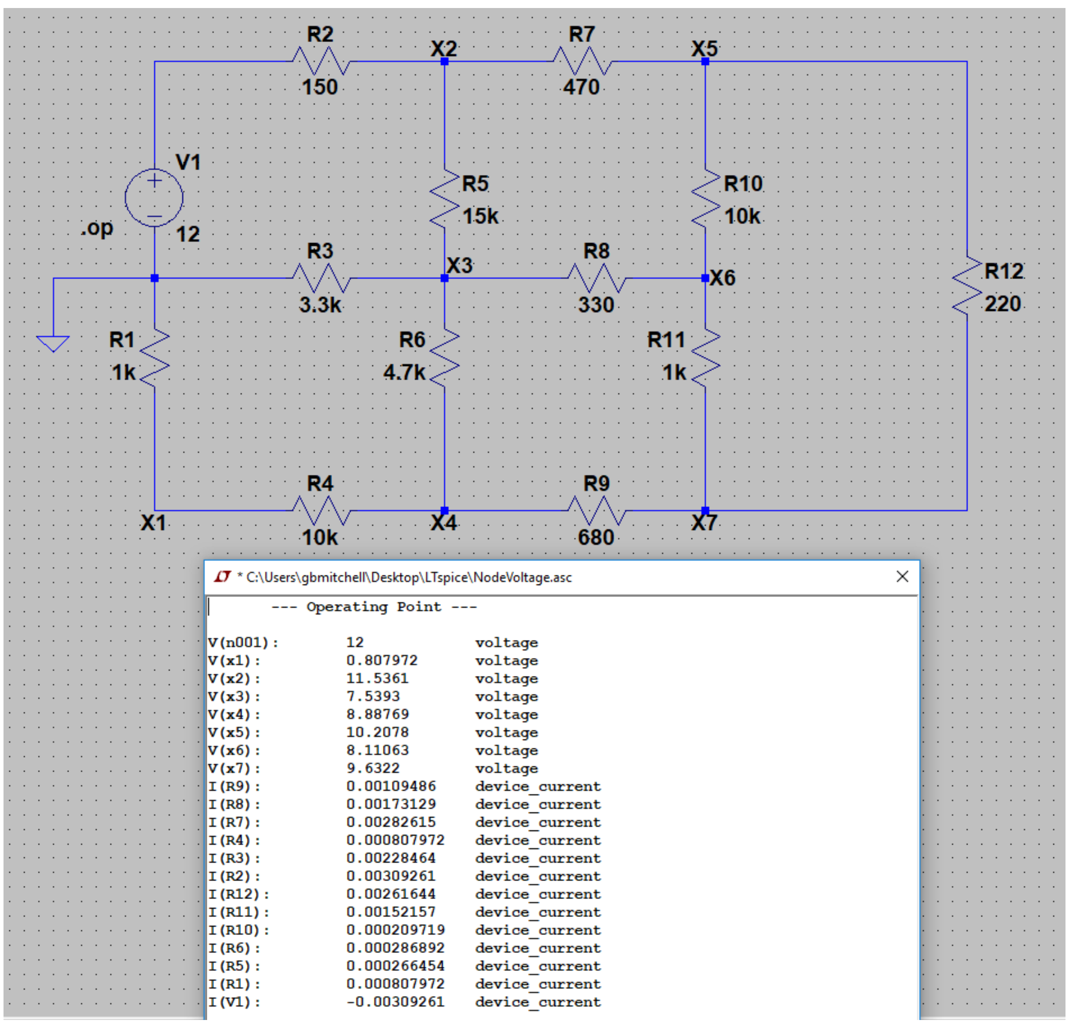


Figure 10: LTspice simulation.