

$$\text{a.) } \int x \sin(2x) dx, \quad u = x, \quad dv = \sin(2x) dx \\ du = 1 dx, \quad v = -\frac{1}{2} \cos(2x),$$

$$\int x \sin(2x) dx = -\frac{1}{2} \cos(2x) x - \int -\frac{1}{2} \cos(2x) dx$$

$$= -\frac{1}{2} \left[x \cos(2x) - \int \cos(2x) dx \right]$$

$$= -\frac{1}{2} \left[x \cos(2x) - \frac{1}{2} \sin(2x) \right]$$

$$= \boxed{\frac{1}{4} \sin(2x) - \frac{1}{2} x \cos(2x) + C.}$$

$$\text{b.) } \int x e^{x^2} dx, \quad u = x^2, \quad du = 2x dx$$

$$\int \sqrt{u} e^u \frac{1}{2\sqrt{u}} du = \frac{1}{2} \int e^u = \frac{1}{2} e^u$$

$$= \boxed{\frac{1}{2} e^{x^2} + C.}$$

$$c.) \int x e^x dx, \quad u = x, \quad dv = e^x dx, \\ du = 1 dx, \quad v = e^x,$$

$$\int x e^x dx = x e^x - \int e^x dx$$

$$= x e^x - e^x = \boxed{e^x (x-1) + C.}$$

$$d.) \int e^{x^2} dx = \boxed{x + \frac{x^3}{3} + \frac{x^5}{10}, \text{ at } x=0.}$$

$$e) \int x \sqrt{1+x} \, dx, \quad u=1+x, \quad du=1 \, dx,$$

$$\int x \sqrt{1+x} \, dx \rightarrow \int (u+1) \sqrt{u} \, du$$

$$= \int u \sqrt{u} - \sqrt{u} \, du =$$

$$\int u \sqrt{u} \, du - \int \sqrt{u} \, du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}} \, du$$

$$= \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}}$$

$$= \boxed{\frac{2}{5} (1+x)^{\frac{5}{2}} - \frac{2}{3} (1+x)^{\frac{3}{2}} + C.}$$

$$f.) \int \sec(\theta) d\theta = \int \sec(\theta) \frac{(\sec(\theta) + \tan(\theta))}{(\sec(\theta) + \tan(\theta))} d\theta$$

$$= \int \frac{\sec^2(\theta) + \sec(\theta)\tan(\theta)}{\sec(\theta) + \tan(\theta)} d\theta,$$

$$u = \sec(\theta) + \tan(\theta), \quad du = [\sec(\theta)\tan(\theta) + \sec^2(\theta)] d\theta$$

$$\int \frac{\sec^2(\theta) + \sec(\theta)\tan(\theta)}{\sec(\theta) + \tan(\theta)} d\theta \rightarrow$$

$$\int \frac{1}{u} du = \ln|u|$$

$$= \boxed{\ln|\sec(\theta) + \tan(\theta)| + C.}$$

$$g.) \int \sec^2(\theta) d\theta, \quad u = \tan(\theta) = \frac{\sin^2(\theta)}{\cos^2(\theta)},$$

$$\frac{du}{d\theta} = \frac{1}{\cos^2(\theta)},$$

$$\int \sec^2(\theta) d\theta = \int \frac{1}{\cos^2(\theta)} d\theta$$

$$= \boxed{\tan(\theta) + C.}$$

$$h.) \int \operatorname{sech}^2(\theta) d\theta = \int \frac{1}{\cosh^2(\theta)} d\theta,$$

$$\cosh(\theta) = \frac{e^\theta + e^{-\theta}}{2}, \quad \int \frac{1}{\cosh^2(\theta)} d\theta =$$

$$\int \frac{1}{\left(\frac{e^\theta + e^{-\theta}}{2}\right)^2} d\theta = \boxed{-\frac{2}{e^{2x} + 1} + C.}$$

$$i.) \int \frac{x^2 + 2}{7 - x^2} dx =$$

$$\frac{-(9(\sqrt{7}) \ln|x - \sqrt{7}| - 9(\sqrt{7}) \ln|x + \sqrt{7}| + 14x)}{14} + C.$$

$$j.) \int \frac{1}{ap - bp^2} dp = \frac{-\ln\left(\frac{|pb - a|}{|p|}\right) + C}{a}$$