a.) $\frac{dx}{dt} = 3x$, $dx = 3x\,dt$, $\frac{1}{3x}\,dx = dt$,

$\int \frac{1}{3x}\,dx = \int dt$, $\frac{1}{3}\int \frac{1}{x}\,dx = \int dt$,

$\frac{1}{3}\ln|x| = t + c$, $\ln|x| = 3t + 3c$,

$|x| = (e^{3t})(e^{3c})$, $|x| = e^{3t}\,c$,

$x = ce^{3t}$, $x(0) = 1.0$,

$1 = ce^{3(0)}$, $1 = c$,

$\boxed{x = e^{3t}.}$

b.) $\frac{dx}{dt} = 3tx$, $x(0) = 1.0$,

$dx = 3tx\,dt$, $\frac{1}{x}\,dx = 3t\,dt$,

$\int \frac{1}{x}\,dx = \int 3t\,dt$, $\ln|x| = \frac{3t^2}{2} + C$,

$|x| = e^{(\frac{3}{2}t^2 + C)} = e^{(\frac{3}{2}t^2)}e^{C} = e^{\frac{3}{2}t^2}C$,

$1 = e^{\frac{3}{2}(0)^2}C$, $C = 1$,

$$x = e^{\frac{3}{2}t^2}.$$

c.) $\frac{dx}{dt} = 0.1x - 0.003x^2$, $x(0) = 4$,

$$\frac{1}{0.1x - 0.003x^2}\,dx = dt,$$

$$\int \frac{1}{0.1x - 0.003x^2}\,dx = \int dt,$$

$$\ln|0.1x - 0.003x^2| = t + C,$$

$$0.1x - 0.003x^2 = Ce^t,$$

$$0.1(4) - 0.003(4) = Ce^{(0)},$$

$$0.352 = C,$$

$$\boxed{0.1x - 0.003x^2 = 0.352e^t.}$$

$$\boxed{x = -18.257\left(\sqrt{0.833 - 0.352e^t} - 0.913\right)}$$

$0.833 - 0.352e^t \geq 0$ for $t \leq 3.164$

d.) from part (c.)

$$0.1x - 0.003x^2 = ce^t$$

$$0.1(400) - 0.003(400^2) = ce^{(0)}$$

$$-440 = c$$

$$0.1x - 0.003x^2 = -440e^t$$

$$\boxed{X = 18.257\left(\sqrt{0.833 + 440e^t} - 0.913\right)}$$

$0.833 + 440e^t \geq 0$ for all t.