Práctica 3 Medición de Riesgos QFB - Cópulas

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In [1]:
```
%load_ext autoreload
%autoreload 2

import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

from utils_copulas import (tri_pdf, tri_rvs, gaussian_copula_uv,
                           F1_ppf, F2_ppf,
                           portfolio_returns, var_left_tail, normal_fit_pdf,
                           clayton_quantile_curve_r2, clayton_returns_from_pdfI
```

Exercise 2

The previous exercise is related to the **Triangular (Tr)** distribution, denoted as $X \sim \mathrm{Tr}(a, b, c)$, where $a < c < b$, with the points:

- $A = (a, 0)$
- $B = (b, 0)$
- $C = \left(c, \dfrac{2}{b-a}\right)$

Answer the following questions:

i)
Obtain $f(x)$ and $F(x)$, corresponding to the **pdf** and **cdf** of $X \sim \mathrm{Tr}(a, b, c)$.

ii)
Compute

$$\mu_X = \mathbb{E}(X) \quad \text{and} \quad \sigma_X^2 = \mathrm{Var}(X).$$

iii)
Find the expression of

$$X = F^{-1}(U),$$

where $U \sim \mathcal{U}(0, 1)$.

Generate **10,000 draws** of $X \sim \mathrm{Tr}(1, 5, 3)$.
Plot the graph of $f(x)$, already obtained in **Exercise 1**, together with the **histogram of the simulated draws**.

...

Para $X \sim \mathrm{Tr}(a,b,c)$ con $a < c < b$, la densidad es triangular con soporte $[a,b]$ y altura máxima en $x = c$. La función de densidad $f(x)$ es lineal creciente en $[a,c]$ y lineal decreciente en $[c,b]$, y vale cero fuera del soporte:

$$f(x) = \begin{cases} 0, & x < a, \\ \dfrac{2(x-a)}{(b-a)(c-a)}, & a \le x \le c, \\ \dfrac{2(b-x)}{(b-a)(b-c)}, & c \le x \le b, \\ 0, & x > b. \end{cases}$$

Integrando por tramos (integrales inmediatas) se obtiene la función de distribución acumulada $F(x)$. Para $x \le a$ es 0 y para $x \ge b$ es 1. En los tramos intermedios queda:

$$F(x) = \begin{cases} 0, & x \le a, \\ \dfrac{(x-a)^2}{(b-a)(c-a)}, & a \le x \le c, \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)}, & c \le x \le b, \\ 1, & x \ge b. \end{cases}$$

A continuación calculamos $\mu_X = \mathbb{E}[X]$ y $\mathbb{E}[X^2]$. Partimos de

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_a^c x\,\frac{2(x-a)}{(b-a)(c-a)}\,dx + \int_c^b x\,\frac{2(b-x)}{(b-a)(b-c)}\,dx.$$

Para el tramo izquierdo:

$$\int_a^c x\,\frac{2(x-a)}{(b-a)(c-a)}\,dx = \frac{2}{(b-a)(c-a)} \int_a^c (x^2 - ax)\,dx$$

$$= \frac{2}{(b-a)(c-a)} \left[\frac{x^3}{3} - a\frac{x^2}{2}\right]_a^c = \frac{2}{(b-a)(c-a)} \left(\frac{c^3-a^3}{3} - a\frac{c^2-a^2}{2}\right).$$

Para el tramo derecho:

$$\int_c^b x\,\frac{2(b-x)}{(b-a)(b-c)}\,dx = \frac{2}{(b-a)(b-c)} \int_c^b (bx - x^2)\,dx$$

$$= \frac{2}{(b-a)(b-c)} \left[b\frac{x^2}{2} - \frac{x^3}{3}\right]_c^b = \frac{2}{(b-a)(b-c)} \left(b\frac{b^2-c^2}{2} - \frac{b^3-c^3}{3}\right).$$

Sumando ambos términos y simplificando algebraicamente se obtiene el resultado conocido:

$$\mu_X = \mathbb{E}[X] = \frac{a+b+c}{3}.$$

De manera análoga, para el segundo momento:

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_a^c x^2\,\frac{2(x-a)}{(b-a)(c-a)}\,dx + \int_c^b x^2\,\frac{2(b-x)}{(b-a)(b-c)}\,dx.$$

Tramo izquierdo:

$$\int_a^c x^2\,\frac{2(x-a)}{(b-a)(c-a)}\,dx = \frac{2}{(b-a)(c-a)}\int_a^c (x^3 - ax^2)\,dx$$

$$= \frac{2}{(b-a)(c-a)}\left[\frac{x^4}{4} - a\frac{x^3}{3}\right]_a^c = \frac{2}{(b-a)(c-a)}\left(\frac{c^4-a^4}{4} - a\frac{c^3-a^3}{3}\right).$$

Tramo derecho:

$$\int_c^b x^2\,\frac{2(b-x)}{(b-a)(b-c)}\,dx = \frac{2}{(b-a)(b-c)}\int_c^b (bx^2 - x^3)\,dx$$

$$= \frac{2}{(b-a)(b-c)}\left[b\frac{x^3}{3} - \frac{x^4}{4}\right]_c^b = \frac{2}{(b-a)(b-c)}\left(b\frac{b^3-c^3}{3} - \frac{b^4-c^4}{4}\right).$$

Sumando y simplificando se obtiene:

$$\mathbb{E}[X^2] = \frac{a^2+b^2+c^2+ab+ac+bc}{6}.$$

Finalmente,

$$\sigma_X^2 = \mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = \frac{a^2+b^2+c^2+ab+ac+bc}{6} - \left(\frac{a+b+c}{3}\right)^2 = \frac{a}{}$$

Para simular por transformada inversa, tomamos $U \sim \mathcal{U}(0,1)$ y resolvemos $U = F(x)$ en cada tramo. Primero, el punto de cambio en el dominio de U es $F(c)$:

$$F(c) = \frac{(c-a)^2}{(b-a)(c-a)} = \frac{c-a}{b-a}.$$

Definiendo $p = \dfrac{c-a}{b-a}$, la inversa $F^{-1}(u)$ queda:

$$F^{-1}(u) = \begin{cases} a + \sqrt{u\,(b-a)(c-a)}, & 0 \le u \le p, \\ b - \sqrt{(1-u)\,(b-a)(b-c)}, & p < u \le 1. \end{cases}$$

En el caso concreto $X \sim \mathrm{Tr}(1,5,3)$ se tiene $a=1$, $b=5$, $c=3$ y por tanto $p = \dfrac{c-a}{b-a} = \dfrac{2}{4} = \dfrac{1}{2}$. La inversa se simplifica a:

$$F^{-1}(u) = \begin{cases} 1 + \sqrt{8u}, & 0 \le u \le \frac{1}{2}, \\ 5 - \sqrt{8(1-u)}, & \frac{1}{2} < u \le 1. \end{cases}$$

Con esto, para generar 10,000 muestras basta tomar $u_1, \ldots, u_{10000} \overset{iid}{\sim} \mathcal{U}(0,1)$ y transformar $x_k = F^{-1}(u_k)$. Para comparar con la teoría, se superpone el histograma de

$\{x_k\}$ con la densidad:

$$f(x) = \begin{cases} \dfrac{x-1}{4}, & 1 \le x \le 3, \\ \dfrac{5-x}{4}, & 3 \le x \le 5, \\ 0, & \text{en otro caso.} \end{cases}$$

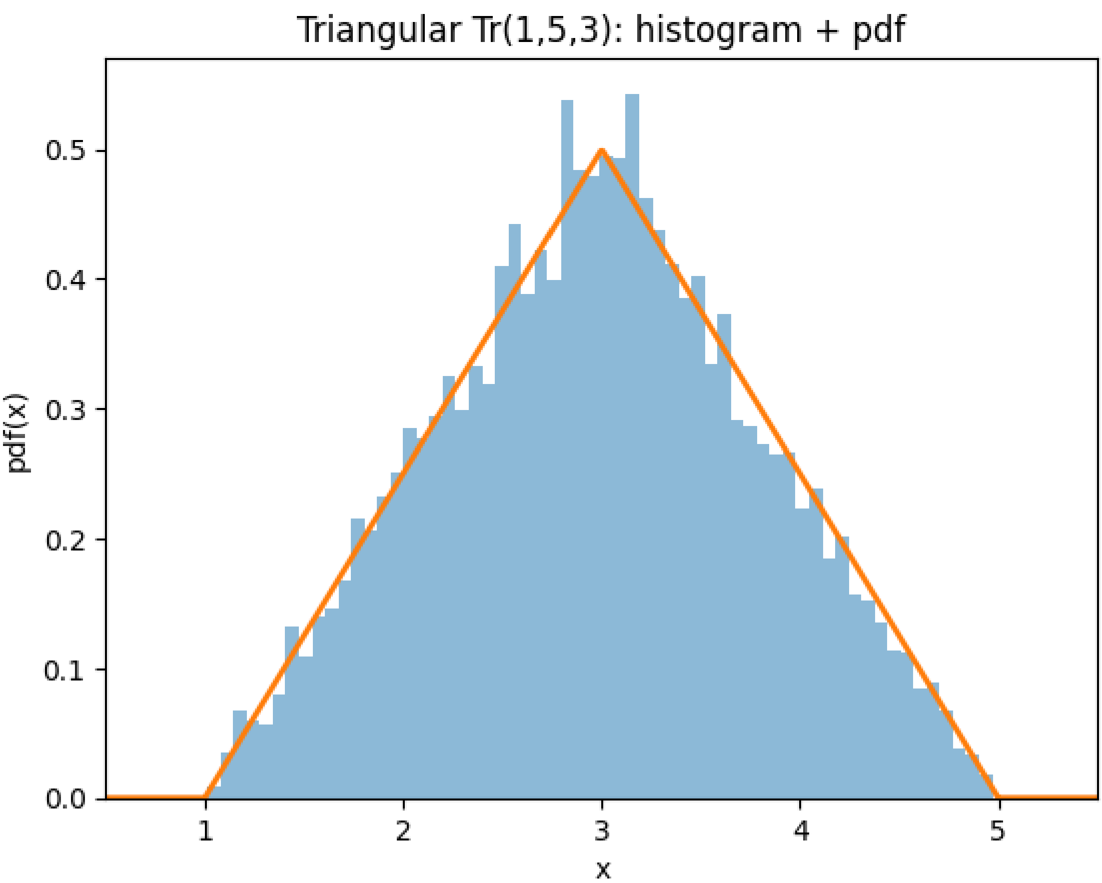
Si el número de muestras es grande, el histograma debería aproximar bien esta forma triangular, con máximo en $x = 3$ y base entre 1 y 5.

In [2]:
```
a, b, c = 1.0, 5.0, 3.0
n = 10_000
seed = 123

x = tri_rvs(a, b, c, size=n, seed=seed)

grid = np.linspace(a - 0.5, b + 0.5, 600)
pdf = tri_pdf(grid, a, b, c)

plt.hist(x, bins=60, density=True, alpha=0.5)
plt.plot(grid, pdf, linewidth=2)
plt.xlim(a - 0.5, b + 0.5)
plt.title("Triangular Tr(1,5,3): histogram + pdf")
plt.xlabel("x")
plt.ylabel("pdf(x)")
plt.show()
```



Exercise 4.

Consider the random variables of stock returns R_1 and R_2 with bivariate pdf given by

$$f(r_1, r_2) = c(F_1(r_1), F_2(r_2))\, f_1(r_1) f_2(r_2),$$

such that the marginal distributions are those from the bivariate pdf (I).

Obtain $f(r_1, r_2)$ under alternative bivariate copulas in the following cases:

i) The bivariate standardized Gaussian (G) copula pdf with ρ as the correlation coefficient is given by

$$c^G(u_1, u_2; \rho) = |\Psi|^{-1/2} \exp\left(-\frac{1}{2}\, \eta'(\Psi^{-1} - I_2)\eta\right),$$

where

$$\eta = \left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right)',$$

Ψ is the correlation matrix, I_2 is the identity matrix of order 2, and $\Phi^{-1}(\cdot)$ is the inverse of the $N(0,1)$ cdf.

ii) The bivariate FGM copula pdf is given by

$$c^{FGM}(u_1, u_2; \lambda) = 1 + \lambda(2u_1 - 1)(2u_2 - 1), \qquad \lambda \in [-1, 1].$$

...

i) Sea (R_1, R_2) un par con marginales continuas F_1, F_2 y densidades f_1, f_2. Definimos las transformaciones

$$U_1 = F_1(R_1), \qquad U_2 = F_2(R_2),$$

de modo que $U_1, U_2 \sim \mathcal{U}(0,1)$ y su densidad conjunta viene dada por una cópula con densidad $c(u_1, u_2)$:

$$f_{U_1,U_2}(u_1, u_2) = c(u_1, u_2).$$

La densidad conjunta de (R_1, R_2) se obtiene por cambio de variable usando que $u_1 = F_1(r_1)$, $u_2 = F_2(r_2)$ y que el jacobiano es

$$\left|\frac{\partial(u_1, u_2)}{\partial(r_1, r_2)}\right| = \begin{vmatrix} \frac{\partial F_1(r_1)}{\partial r_1} & 0 \\ 0 & \frac{\partial F_2(r_2)}{\partial r_2} \end{vmatrix} = f_1(r_1)\, f_2(r_2).$$

Por tanto,

$$f_{R_1,R_2}(r_1, r_2) = f_{U_1,U_2}(F_1(r_1), F_2(r_2)) \left|\frac{\partial(u_1, u_2)}{\partial(r_1, r_2)}\right| = c(F_1(r_1), F_2(r_2))\, f_1(r_1) f_2(r_2).$$

A continuación se particulariza c para cada familia de cópulas pedida.

ii)

En la cópula Gaussiana bivariante, fijamos una matriz de correlación

$$\Psi = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \qquad |\Psi| = 1-\rho^2, \qquad \Psi^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}.$$

Definimos

$$z_1 = \Phi^{-1}(u_1), \qquad z_2 = \Phi^{-1}(u_2), \qquad \eta = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}.$$

La densidad de la cópula Gaussiana está dada por

$$c^G(u_1, u_2; \rho) = |\Psi|^{-1/2} \exp\left(-\frac{1}{2}\,\eta'(\Psi^{-1} - I_2)\eta\right),$$

donde I_2 es la identidad. Sustituyendo Ψ^{-1} e I_2:

$$\Psi^{-1} - I_2 = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{1-\rho^2}\begin{pmatrix} \rho^2 & -\rho \\ -\rho & \rho^2 \end{pmatrix}.$$

Entonces el término cuadrático queda

$$\eta'(\Psi^{-1} - I_2)\eta = \frac{1}{1-\rho^2}\begin{pmatrix} z_1 & z_2 \end{pmatrix}\begin{pmatrix} \rho^2 & -\rho \\ -\rho & \rho^2 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \frac{1}{1-\rho^2}\left(\rho^2 z_1^2 - 2\rho z_1 z_2 + \rho^2 z_2^2\right.$$

Por tanto, como $|\Psi|^{-1/2} = (1-\rho^2)^{-1/2}$,

$$c^G(u_1, u_2; \rho) = (1-\rho^2)^{-1/2}\ \exp\left(-\frac{1}{2}\cdot\frac{\rho^2(z_1^2 + z_2^2) - 2\rho z_1 z_2}{1-\rho^2}\right), \qquad z_i = \Phi^{-1}(u_i).$$

Finalmente, sustituyendo $u_1 = F_1(r_1)$ y $u_2 = F_2(r_2)$ en la fórmula general:

$$f^G_{R_1,R_2}(r_1, r_2; \rho) = c^G(F_1(r_1), F_2(r_2); \rho)\ f_1(r_1) f_2(r_2),$$

es decir,

$$f^G_{R_1,R_2}(r_1, r_2; \rho) = (1-\rho^2)^{-1/2}\ \exp\left(-\frac{1}{2}\cdot\frac{\rho^2(z_1^2 + z_2^2) - 2\rho z_1 z_2}{1-\rho^2}\right)\ f_1(r_1) f_2(r_2),$$

donde

$$z_1 = \Phi^{-1}(F_1(r_1)), \qquad z_2 = \Phi^{-1}(F_2(r_2)).$$

iii)

En la cópula FGM bivariante se da directamente la densidad

$$c^{FGM}(u_1, u_2; \lambda) = 1 + \lambda(2u_1 - 1)(2u_2 - 1), \qquad \lambda \in [-1, 1].$$

Insertando en la descomposición copular:

$$f^{FGM}_{R_1,R_2}(r_1, r_2; \lambda) = c^{FGM}(F_1(r_1), F_2(r_2); \lambda)\ f_1(r_1) f_2(r_2),$$

obtenemos

$$f_{R_1,R_2}^{FGM}(r_1, r_2; \lambda) = [1 + \lambda\,(2F_1(r_1) - 1)\,(2F_2(r_2) - 1)]\, f_1(r_1) f_2(r_2).$$

Estas expresiones dan la densidad conjunta $f(r_1, r_2)$ para cada cópula manteniendo fijas las marginales F_1, F_2.

Exercise 7.

Let (U_1, U_2) be a bivariate uniform generated from the G copula with $\rho = -0.7$ as correlation coefficient (standardized Gaussian copula), see section i) in Exercise 4. Repeat the same analysis as in Exercise 6 by obtaining $N = 10{,}000$ draws of (R_1, R_2) with marginal distributions of R_i from the bivariate pdf (I).

The cdf of U_1 given $U_2 = u_2$ in the G copula (or conditional G copula) is obtained from the cdf of the G copula, $\Phi_2(\cdot)$, then

$$C^G(u_1 \mid u_2) = \frac{\partial}{\partial u_2}\, \Phi_2\big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\big) = \Phi\left(\frac{\Phi^{-1}(u_1) - \rho\, \Phi^{-1}(u_2)}{\sqrt{1-\rho^2}}\right),$$

where $\Phi(\cdot)$ and $\Phi^{-1}(\cdot)$ are, respectively, the $N(0,1)$ cdf and its inverse.

Obtain both the sample mean and standard deviation of the portfolio D returns, containing 25% of stock 1 and 75% of stock 2 under the above procedure, and also the VaR values at 1% and 5%.

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La densidad conjunta (I) es

$$f(r_1, r_2) = \begin{cases} \alpha + \beta r_1 + \gamma r_2, & -1 \le r_1 \le 2,\ -2 \le r_2 \le 2.5, \\ 0, & \text{en otro caso.} \end{cases}$$

Sus marginales son (integrando en el rectángulo):

$$f_1(r_1) = \int_{-2}^{2.5} (\alpha + \beta r_1 + \gamma r_2)\, dr_2 = 4.5(\alpha + \beta r_1) + 1.125\,\gamma, \qquad -1 \le r_1 \le 2,$$

$$f_2(r_2) = \int_{-1}^{2} (\alpha + \beta r_1 + \gamma r_2)\, dr_1 = 3(\alpha + \gamma r_2) + 1.5\,\beta, \qquad -2 \le r_2 \le 2.5.$$

Y sus cdf marginales:

$$F_1(x) = \int_{-1}^{x} f_1(t)\, dt = (4.5\alpha + 1.125\gamma)\,(x+1) + 2.25\,\beta\,(x^2 - 1), \qquad -1 \le x \le 2,$$

$$F_2(y) = \int_{-2}^{y} f_2(t)\, dt = (3\alpha + 1.5\beta)\,(y+2) + 1.5\,\gamma\,(y^2 - 4), \qquad -2 \le y \le 2.5.$$

La normalización de f impone

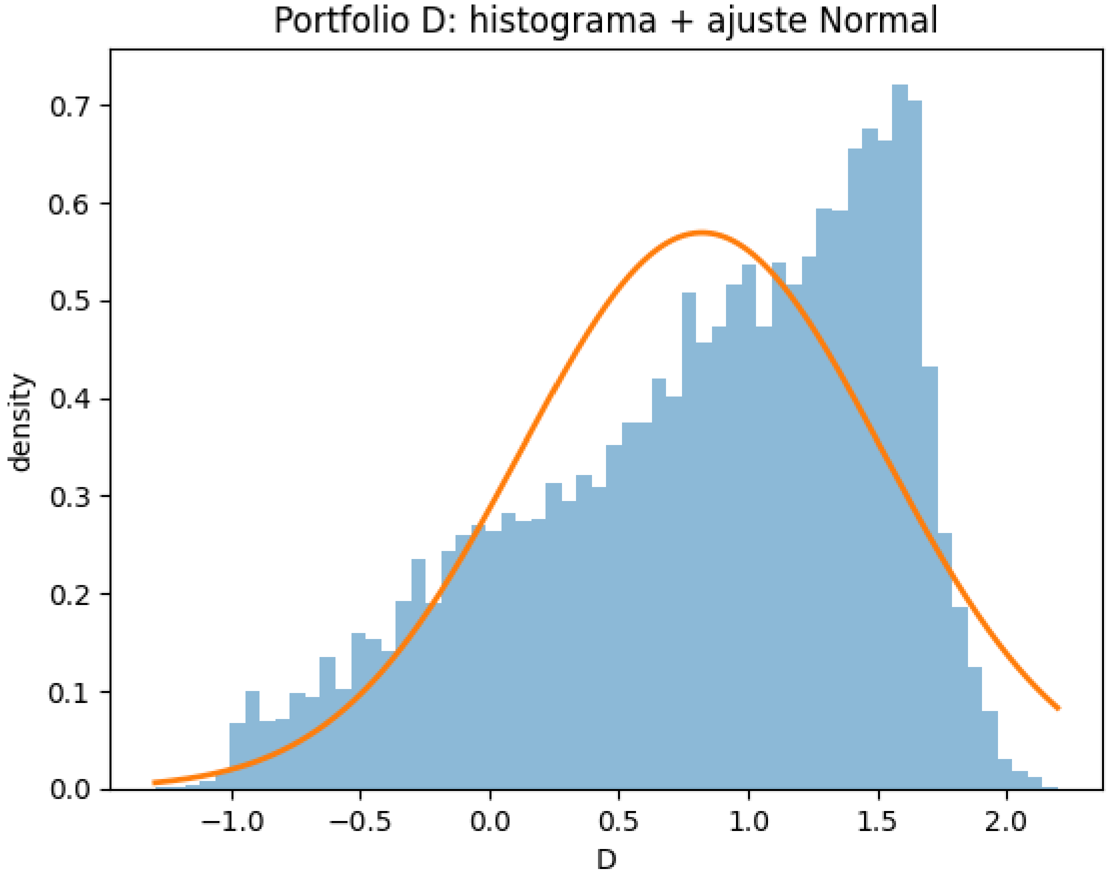
$$\int_{-1}^{2}\int_{-2}^{2.5}(\alpha+\beta r_1+\gamma r_2)\,dr_2\,dr_1 = 13.5\,\alpha + 6.75\,\beta + 3.375\,\gamma = 1.$$

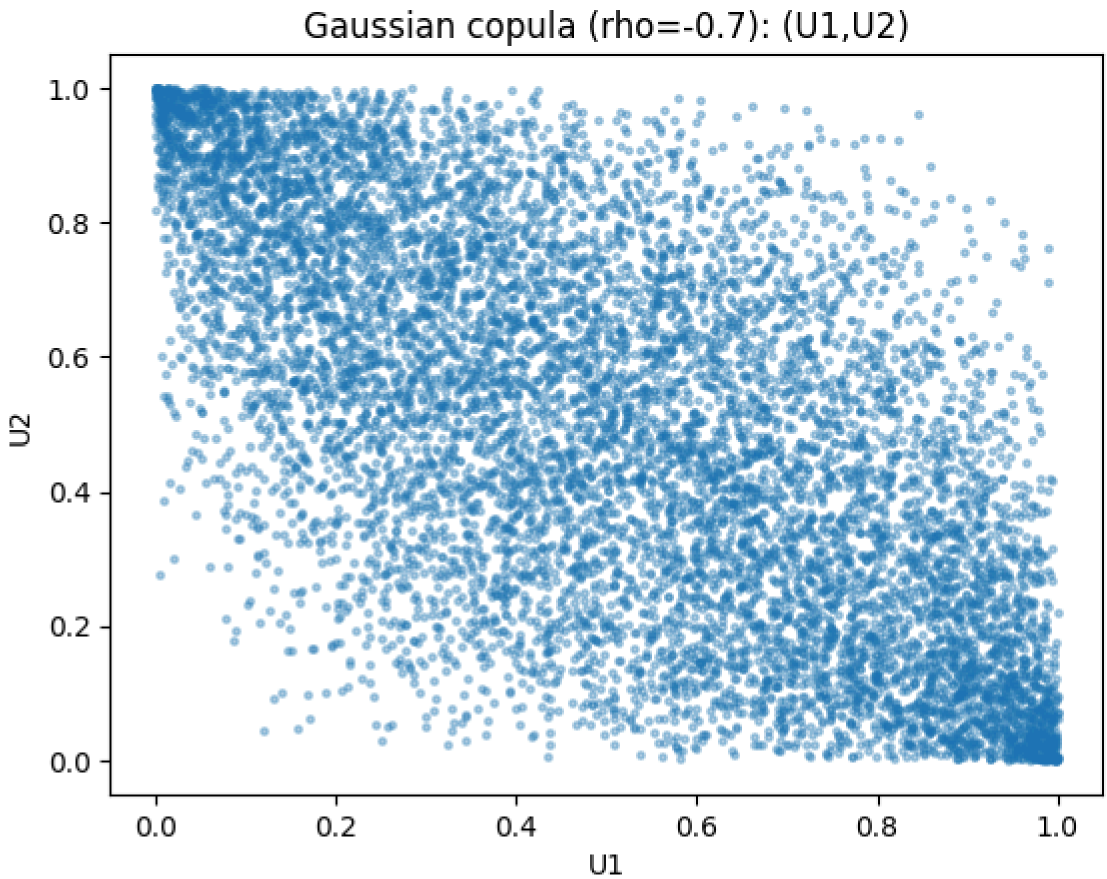
Para el Ejercicio 7 necesitas F_1^{-1} y F_2^{-1} para transformar $R_i = F_i^{-1}(U_i)$ tras simular (U_1, U_2) con cópula Gaussiana ($\rho = -0.7$). Abajo tienes un `utils.py` con:

- simulación de cópula Gaussiana por muestreo condicional,
- marginales f_1, f_2, F_1, F_2 y sus inversas F_1^{-1}, F_2^{-1} (resolviendo la cuadrática),
- portfolio, VaR y ajuste normal para el histograma.

In [3]:
```
N = 10_000
rho = -0.7
seed = 123

alpha = 0.0642
beta  = 0.0049
gamma = 0.0296

# 1) Simular (U1,U2) con cópula Gaussiana
U1, U2 = gaussian_copula_uv(rho=rho, size=N, seed=seed)

# 2) Transformar con marginales de (I)
R1 = F1_ppf(U1, alpha, beta, gamma)
R2 = F2_ppf(U2, alpha, beta, gamma)

# 3) Portfolio D = 0.25 R1 + 0.75 R2
D = portfolio_returns(R1, R2, w1=0.25, w2=0.75)

mean_D = float(np.mean(D))
std_D  = float(np.std(D, ddof=1))
VaR_1  = var_left_tail(D, 0.01)
VaR_5  = var_left_tail(D, 0.05)

print(f"mean(D) = {mean_D:.6g}")
print(f"std(D)  = {std_D:.6g}")
print(f"VaR 1%  = {VaR_1:.6g}   (VaR = -cuantil cola izqda)")
print(f"VaR 5%  = {VaR_5:.6g}")

# Histograma + ajuste Normal
grid = np.linspace(np.min(D), np.max(D), 500)
mu_fit, sigma_fit, pdf_fit = normal_fit_pdf(D, grid)

plt.hist(D, bins=60, density=True, alpha=0.5)
plt.plot(grid, pdf_fit, linewidth=2)
plt.xlabel("D")
plt.ylabel("density")
plt.title("Portfolio D: histograma + ajuste Normal")
plt.show()

# scatter de (U1,U2)
plt.scatter(U1, U2, s=6, alpha=0.35)
plt.xlabel("U1")
plt.ylabel("U2")
plt.title("Gaussian copula (rho=-0.7): (U1,U2)")
plt.show()
```

```
mean(D) = 0.821494
std(D)  = 0.701125
VaR 1%  = 0.90354    (VaR = -cuantil cola izqda)
VaR 5%  = 0.501716
```





Exercise 8.

According to Exercise 7, the q quantile curve under the G copula, when the marginals are arbitrary distributions with cdf's $F_i(\cdot)$, $i = 1, 2$, is given by

$$C^G(u_1 \mid u_2) = q,$$

then

$$u_2 = \Phi\left(\rho\,\Phi^{-1}(u_1) + \sqrt{1-\rho^2}\,\Phi^{-1}(q)\right),$$

where $u_i = F_i(r_i)$.

Definitively,

$$r_2 = F_2^{-1}\left[\Phi\left(\rho\,\Phi^{-1}(F_1(r_1)) + \sqrt{1-\rho^2}\,\Phi^{-1}(q)\right)\right].$$

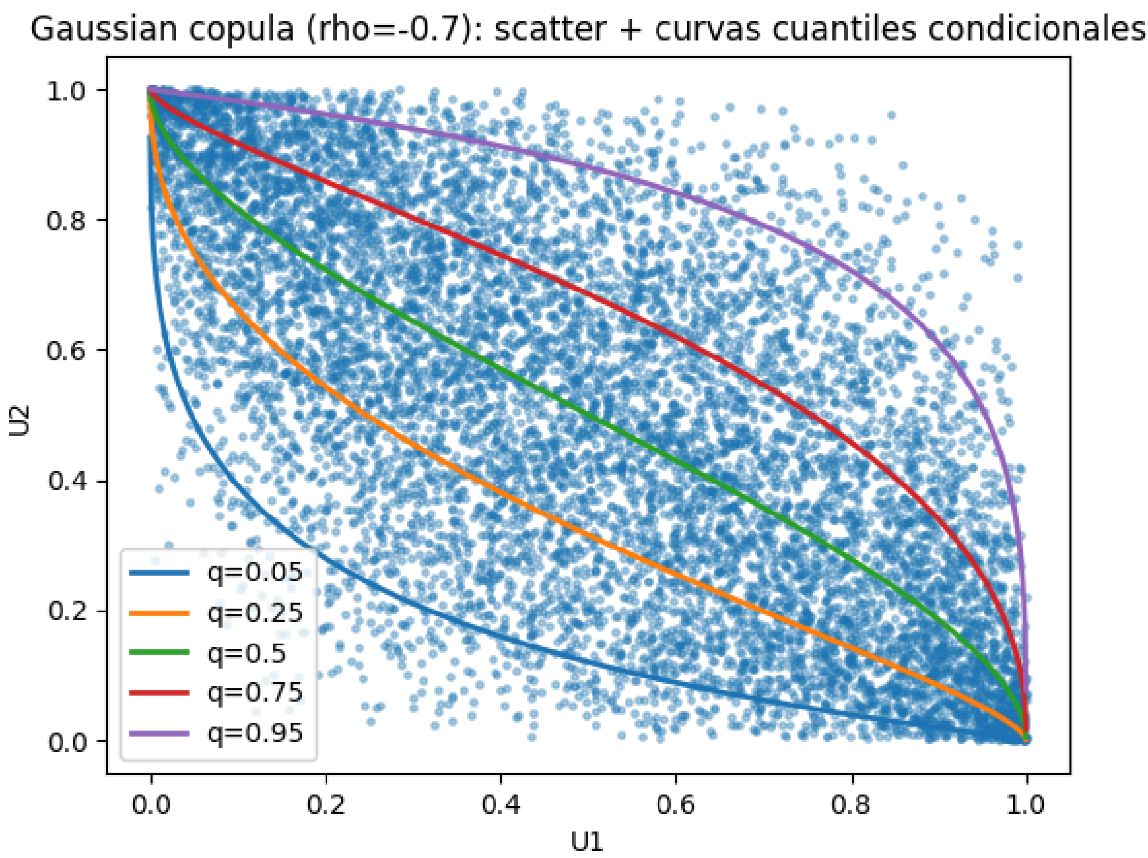
Consider as marginals those obtained from the bivariate pdf (I). Make a plot of the q quantile curves for

$$q \in \{0.05,\, 0.25,\, 0.5,\, 0.75,\, 0.95\}.$$

...

In [4]:

```
# --- notebook cell: scatter (U1,U2) + curvas cuantiles u2(u1|q) ---
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

rho = -0.7
qs = [0.05, 0.25, 0.50, 0.75, 0.95]

# malla en u1 evitando 0 y 1 para no tener inf en Phi^{-1}
u1_grid = np.linspace(1e-4, 1 - 1e-4, 800)
z1 = norm.ppf(u1_grid)

plt.scatter(U1, U2, s=6, alpha=0.35)
for q in qs:
    u2_curve = norm.cdf(rho * z1 +
                        np.sqrt(1 - rho**2) *
                        norm.ppf(q))
    plt.plot(u1_grid,
             u2_curve,
             linewidth=2,
             label=f"q={q}")

plt.xlabel("U1")
plt.ylabel("U2")
plt.title("Gaussian copula (rho=-0.7): " \
"scatter + curvas cuantiles condicionales")
plt.legend()
plt.show()
```



Exercise 10.

The cdf of the conditional Clayton copula is

$$C^{Cla}(u_1 \mid u_2) = \frac{\partial}{\partial u_2}\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta} = u_1^{-(1+\theta)}\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-(1+\theta)/\theta},$$

such that $\theta > 0$. Given the equation

$$C^{Cla}(u_1 \mid u_2) = q,$$

the q quantile for the Clayton copula is obtained as

$$u_2 = \left(1 + u_1^{-\theta}\left(q^{-\theta/(1+\theta)} - 1\right)\right)^{-1/\theta},$$

where $u_i = F_i(r_i)$.

If we set the marginals to be those obtained from the bivariate pdf (I), then

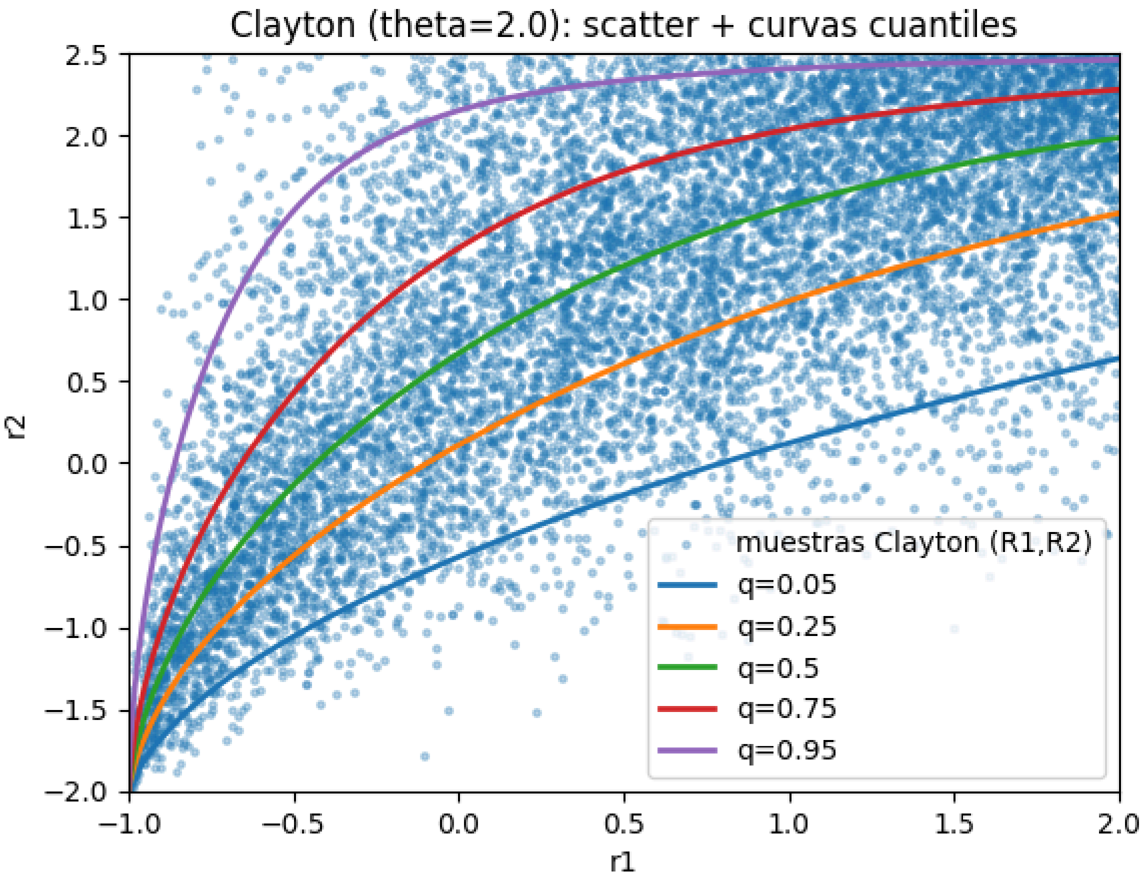
$$r_2 = F_2^{-1}\left[\left(1 + F_1(r_1)^{-\theta}\left(q^{-\theta/(1+\theta)} - 1\right)\right)^{-1/\theta}\right].$$

Draw the q quantile curves for

$$q \in \{0.05,\, 0.25,\, 0.5,\, 0.75,\, 0.95\}.$$

...

In [5]:
```python
N = 10_000
theta = 2.0
qs = [0.05, 0.25, 0.50, 0.75, 0.95]
seed = 123

# 1) scatter con dependencia Clayton(theta)
R1, R2, U1, U2 = clayton_returns_from_pdfI(
    theta=theta,
    size=N,
    alpha=alpha,
    beta=beta,
    gamma=gamma,
    seed=seed
)

plt.scatter(R1, R2, s=6, alpha=0.30, label="muestras Clayton (R1,R2)")

# 2) curvas cuantiles Clayton en (r1,r2)
r1_grid = np.linspace(-1.0 + 1e-6, 2.0 - 1e-6, 800)
for q in qs:
    r2_curve = clayton_quantile_curve_r2(
        r1_grid,
        q=q,
        theta=theta,
        alpha=alpha,
        beta=beta,
        gamma=gamma
    )
    plt.plot(r1_grid, r2_curve, linewidth=2, label=f"q={q}")

plt.xlabel("r1")
plt.ylabel("r2")
plt.title(f"Clayton (theta={theta}): scatter + curvas cuantiles")
plt.legend()
plt.xlim(-1, 2)
plt.ylim(-2, 2.5)
plt.show()
```


Clayton (theta=2.0): scatter + curvas cuantiles
r2
r1
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
muestras Clayton (R1,R2)
q=0.05
q=0.25
q=0.5
q=0.75
q=0.95