

# A primer on calculus of indications

Gonalo Braga

October 28, 2024

## Abstract

A primer on calculus of indications, specifically building towards an understanding of Varela's extended calculus. Very informal summary.

## Calculus of indications

The marked state is presented by  $\top$ , and the unmarked state is presented by the blank  $\square$ . The two-fold meaning of making a distinction with  $\top$  allows for a calculus without the common operand/operator distinction.

## Initials

$$(1) \quad \neg \neg = \neg$$

$$(2) \quad \lceil =$$

## Primary arithmetic

Consequences from initials 1 and 2, with the example:

$$\overline{\Gamma\Gamma}\overline{\Gamma\Gamma}\Gamma = \Gamma\Gamma = \Gamma$$

As for the central axioms of this primary arithmetic, we have:

$$(3) \quad \overline{p|p} =$$

and

$$(4) \quad \overline{\overline{\text{pr}} \mid \overline{\text{qr}}} = \overline{\overline{\text{p}} \mid \overline{\text{q}}}r$$

## Re-entry in the primary algebra

In order to fix situations of infinite regress like the following

$$\overline{\overline{\overline{\dots}}}$$

one can have re-entry in the following manner, much as with the Y-combinator in the  $\lambda$ -calculus:

$$(5) \qquad f = \overline{f}$$

where one gets a fixed and finite expression, for what would otherwise be evaluated onto:

$$(6) \qquad f = \overline{f} = \overline{\overline{f}} = \overline{\overline{\overline{\dots}}}$$