$$\begin{cases} \text{minimizar} & f(x) \\ \text{sa.} & h(x)=0 \end{cases} \Rightarrow \text{ se define } \quad \bar{\ell}(x) = \ell(x,y) + \frac{\rho}{2}\|h(x)\|_2^2$$

$$= f(x) + h'(x)^T \cdot y + \frac{\rho}{2}\|h(x)\|_2^2$$

$y \longrightarrow y_*$ mult. de Lagrange

Ejemplo: $(P) \begin{cases} \text{minimizar} & f(x_1,x_2) = \frac{1}{2}(x_1^2+x_2^2) \\ \text{sa} & x_1 = 1 \quad (x_1 - 1 = 0) \end{cases}$

Como $x_1 = 1$ y $x_2^2 \geq 0 \Rightarrow f((1,0)) \leq f(1,x_2) \quad \forall x_2 \in \mathbb{R}$.

$\Rightarrow$ solución de (P) es $(x,y)^* = (1,0)$

$$\bar{\ell}(x) = \frac{1}{2}(x_1^2+x_2^2) + y\cdot(x_1-1) + \frac{\rho}{2}(x_1-1)^2$$

(P') minimizar $\bar{\ell}(x)$

a) $\nabla_x \bar{\ell}(x) = \begin{bmatrix} x_1 + y + \rho(x_1-1) \\ x_2 \end{bmatrix} \Rightarrow \nabla_x \bar{\ell}(x) = 0 \Leftrightarrow \begin{cases} x_1+y+\rho(x_1-1)=0 \\ x_2=0 \end{cases}$

$$\therefore \quad x_1 + y + \rho(x_1-1) = 0$$

$$x_1(1+\rho) + y - \rho = 0$$

$$x_1 = \frac{\rho - y}{\rho+1} \quad y \quad x_2 = 0$$

$\therefore$ el mínimo de $\bar{\ell}(x)$ debe ser $(x_1(\rho,y),0) = \left(\frac{\rho-y}{\rho+1}, 0\right)$

Si $\rho \to \infty$, $\lim\limits_{\rho\to\infty} x_1(\rho,y) = \lim\limits_{\rho\to\infty} \frac{\rho-y}{\rho+1} = \lim\limits_{\rho\to\infty} \frac{1-y/\rho}{1+1/\rho} = 1$

$$\lim_{\rho\to\infty} x_2(\rho,y)=0$$

$$\therefore \quad \lim_{\rho\to\infty}\underbrace{\left(x_1(\rho,y),\, x_2(\rho,y)\right)}_{\substack{\text{solución del}\\ \text{problema } (P')\\ \text{(irrestricto)}}} = \underbrace{(1,0)}_{\substack{\text{solución del}\\ \text{problema } (P)\\ \text{(restricto)}}}$$

Si y es mult. de Lagrange de $\ell(x,\lambda)=f(x)+\lambda\cdot(x_1-1)$

entonces $\boxed{y=-1}$ verificar esto.

$$\lim_{y\to-1} x_1(y,\rho)=\lim_{y\to-1}\frac{\rho-y}{\rho+1}=\frac{\rho+1}{\rho+1}=1,\quad \lim_{y\to-1}x_2(y,\rho)=0$$

$$\therefore \quad \lim_{y\to-1}\underbrace{\left(x_1(y,\rho),\, x_2(y,\rho)\right)}_{\text{sol de } (P')} = \underbrace{(1,0)}_{\text{sol. de } (P)}$$

b) Veamos el método de L. aumentado

Defino $\quad x^k=\left(\dfrac{\rho^k-y^k}{\rho^k+1},\, 0\right)$

Definimos un ρ^0, y^0 y con la formula calculamos x^1

Luego actualizar (de algún criterio) el ρ^k

$$y^{k+1}=y^k+\rho_k\cdot h(x^k)=y^k+\rho_k\left(\frac{\rho^k-y^k}{\rho^k+1}-1\right)$$

$$=\frac{y^k}{\rho^k+1}-\frac{\rho^k}{\rho^k+1}=\frac{y^k}{\rho^k+1}-\overset{=1=-y}{\frac{\rho^k+1}{\rho^k+1}}-\frac{\overset{-y}{1}}{\rho^k+1}$$

$\Rightarrow$ usando que el mul. de Lagrange es -1

$$\Rightarrow \quad y^{k+1} - y = \frac{y^k - y}{\rho^k + 1}$$

a) Si $y^k \longrightarrow y = 1 \Rightarrow x^k \longrightarrow x^*$

b) Si $\rho^k \longrightarrow \infty \Rightarrow y^{k+1} - y \longrightarrow 0 \Rightarrow y^k \longrightarrow y = -1 \Rightarrow x^k \longrightarrow x^*$