ONLINE APPENDIX FOR:
Attending to Inattention: Identification of Deadweight Loss under Non-Salient Taxes

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A Additional Results and Proofs

A.1 Additional results and proofs from section 2

Proposition 2. *Suppose we can extend u to $\mathbb{R}^2$ such that u is continuous and quasi-concave. Then for any $\bar{p}$, p^{NT}, τ, and W on which $\boldsymbol{q}$ is defined, there exist scalar values p^s and W^s such that:*

$$\boldsymbol{q}(\bar{p}, p^{NT}, \tau, W) \in \underset{p^s q + p^{NT} q^{NT} \leq W^s}{\arg\max} u(q, q^{NT})$$
$$\boldsymbol{q}(\bar{p}, p^{NT}, \tau, W^s) * (\bar{p} + \tau, p^{NT}) = W$$

We demonstrate a generalization of proposition 2, in which multiple goods may be taxed. We consider a general setting with N goods, consumption set $X = X^T \times X^{NT} \subseteq \mathbb{R}^N_+$, with consumption vector $\boldsymbol{q} = (\boldsymbol{q^T}, \boldsymbol{q^{NT}}) \in X$. Here X^T is the consumption set for taxed goods, while X^{NT} is the consumption set for non-taxed goods. We assume that either $X^{NT} \subseteq \mathbb{R}_+$ or X^{NT} is convex.

The agent has preferences $\succeq$ on X. Informally, we want to assume preferences such that agents smoothly prefer moderation. To say that they prefer moderation, one generally assumes convex preferences. However, we do not want to assume a convex consumption set X. We might alternatively assume that preferences are *pseudo-convex*, in that for any $\boldsymbol{q} \in X$ and any finite n:

$$\boldsymbol{q}_k \in X, \boldsymbol{q}_k \succ \boldsymbol{q}, \lambda_k \geq 0 \ \forall k = 1, ..., n, \sum_{k=1}^{n} \lambda_k = 1, \sum_{k=1}^{n} \lambda_k \boldsymbol{q}_k \in X \Rightarrow \sum_{k=1}^{n} \lambda_k \boldsymbol{q}_k \succ \boldsymbol{q}$$

However, we also want some smoothness to preferences. More formally, we want to figure that if $\boldsymbol{q'} \succ \boldsymbol{q}$, then there is an epsilon ball around $\boldsymbol{q'}$ such that the agent would prefer any element in that epsilon ball to $\boldsymbol{q}$ if that element were also in the consumption set. Furthermore, any convex combination of points in these epsilon balls should yield a point that, if contained in X, is also strictly preferred to $\boldsymbol{q}$. We refer to this assumption on preferences as *continuous pseudo-convexity* (CPC).

Assumption 2. *For any $\boldsymbol{q} \in X$, define the set of strictly preferred allocations:*

$$\mathcal{A} \equiv \{\boldsymbol{q'} \in X | \boldsymbol{q'} \succ \boldsymbol{q}\}$$

There exists some function $\epsilon : \mathcal{A} \to \mathbb{R}_{++}$ such that for any $n \in \mathbb{N}$, for any $\lambda_1, ..., \lambda_n \geq 0$ and $\boldsymbol{q}_1, ..., \boldsymbol{q}_n \in \mathcal{A}$, if $\sum_{k=1}^{n} \lambda_k = 1$, then :

$$\exists \boldsymbol{q}'_1, ..., \boldsymbol{q}'_n \in \mathbb{R}^n : ||\boldsymbol{q}'_k - \boldsymbol{q}_k|| < \epsilon(\boldsymbol{q_k}) \ \forall k, \sum_{k=1}^{n} \lambda_k \boldsymbol{q_k}' \in X \Rightarrow \sum_{k=1}^{n} \lambda_k \boldsymbol{q_k}' \succ \boldsymbol{q}$$

We provide this description of CPC preferences to facilitate intuition, but our main result for this section comes from an equivalent, yet more geometric, expression of this description.

Lemma 1. *Preferences $\succeq$ are CPC if and only if for every $\boldsymbol{q} \in X$ with corresponding set of strictly preferred bundles $\mathcal{A}$ there is an open and convex set $\mathcal{O} \subseteq \mathbb{R}^N$ such that $\mathcal{O} \cap X = \mathcal{A}$.*[1]

Proof: For one direction, the convex hull of the union of open $\epsilon(\boldsymbol{q'})$ balls around $\boldsymbol{q'} \in \mathcal{A}$ is open, and by assumption does not contain any elements of $X \setminus \mathcal{A}$. For the other direction, for any $\boldsymbol{q'} \in \mathcal{A}$, define $\epsilon(\boldsymbol{q'})$ as a positive value such that $\boldsymbol{q''} \in \mathbb{R}^N_+ : ||\boldsymbol{q''} - \boldsymbol{q'}|| < \epsilon(\boldsymbol{q'}) \Rightarrow \boldsymbol{q''} \in \mathcal{O}$. We can do so because $\mathcal{O}$ is open. For any such $\boldsymbol{q''}$, if $\boldsymbol{q''} \in X$, then $\boldsymbol{q''} \succ \boldsymbol{q}$. $\square$

Let $\boldsymbol{p} = (\boldsymbol{p^T}, \boldsymbol{p^{NT}}) \in \mathbb{R}^N_+$ denote a generic price vector, where $\boldsymbol{p^T}$ and $\boldsymbol{p^{NT}}$ are price vectors for taxed and non-taxed goods respectively. In particular, let $\bar{\boldsymbol{p}} = (\bar{\boldsymbol{p}}^{\boldsymbol{T}}, \bar{\boldsymbol{p}}^{\boldsymbol{NT}})$ denote the vector of sticker prices.

Let $\boldsymbol{\tau}$ denote the vector of taxes for taxed goods, so that $\boldsymbol{q^T}$, $\boldsymbol{q^{NT}}$, and $\boldsymbol{\tau}$ all have the same number of elements. The consumption vector $\boldsymbol{q}(\bar{\boldsymbol{p}}, \boldsymbol{\tau}) = (\boldsymbol{q}^T(\bar{\boldsymbol{p}}, \boldsymbol{\tau}), \boldsymbol{q}^{NT}(\bar{\boldsymbol{p}}, \boldsymbol{\tau}))$ satisfies the following properties:

$$\begin{aligned}
\bar{p}^{NT} * \tilde{\boldsymbol{q}}^{NT} &\leq W - \bar{\boldsymbol{p}}^{\boldsymbol{T}} * \boldsymbol{q}^T \\
(\boldsymbol{q}^T, \tilde{\boldsymbol{q}}^{NT}) &\succ (\boldsymbol{q}^T, \hat{\boldsymbol{q}}^{NT}) \quad \forall \hat{\boldsymbol{q}}^{NT} \in X^{NT} : \bar{\boldsymbol{p}}^{\boldsymbol{NT}} * \tilde{\boldsymbol{q}}^{NT} \leq W - \bar{\boldsymbol{p}}^{\boldsymbol{T}} * \boldsymbol{q}^T \\
\boldsymbol{q}(\bar{\boldsymbol{p}}, \boldsymbol{0}) &\in \underset{\tilde{\boldsymbol{q}} \in X : \bar{\boldsymbol{p}} * \tilde{\boldsymbol{q}} \leq W}{\arg\max} \succeq
\end{aligned}$$

In words, consumption of the non-taxed goods is always optimally determined upon choosing consumption of the taxed goods, and consumption is optimally determined when the agent correctly perceives prices, i.e. when there are no taxes. We also restrict the domain of sticker prices and taxes so that expenditure on non-taxed goods is positive, i.e.:

$$\bar{\boldsymbol{p}}^{NT} * \boldsymbol{q}^{NT}(\bar{\boldsymbol{p}}, \boldsymbol{\tau}) > 0$$

The claim is that for any $\bar{\boldsymbol{p}}$ and $\boldsymbol{\tau}$ in this domain, there is a (p^s, W^s) that explains $\boldsymbol{q}(\bar{\boldsymbol{p}}, \boldsymbol{\tau})$.

Proof of Generalization of Proposition 2: Define $\boldsymbol{q} = (\boldsymbol{q^T}, \boldsymbol{q^{NT}}) = \boldsymbol{q}(\bar{\boldsymbol{p}}, \boldsymbol{\tau})$ and:

$$\mathcal{A}^e \equiv \{(\boldsymbol{q^{T'}}, e^{NT'}) | \boldsymbol{q^{T'}} \in X^T, \exists \boldsymbol{q^{NT'}} \in X^{NT} : \bar{\boldsymbol{p}}^{NT} * \boldsymbol{q^{NT'}} = e^{NT'}, (\boldsymbol{q}^{T'}, \boldsymbol{q}^{NT'}) \in \mathcal{O}\}$$

Suppose for the sake of contradiction that $(\boldsymbol{q^T}, \bar{\boldsymbol{p}}^{\boldsymbol{NT}} * \boldsymbol{q^{NT}}) \in Co(\mathcal{A}^e)$, i.e. that $\exists n \in \mathbb{N}$, $(\boldsymbol{q}^T_k, e^{NT}_k) \in \mathcal{A}^e$, and $\lambda_k \geq 0\ \forall k = 1, ..., n$ such that $\sum_{k=1}^n \lambda_k = 1$ and:

$$\sum_{k=1}^n \lambda_k (\boldsymbol{q}^T_k, e^{NT}_k) = (\boldsymbol{q^T}, \bar{\boldsymbol{p}}^{\boldsymbol{NT}} * \boldsymbol{q^{NT}})$$

Since $(\boldsymbol{q}^T_k, e^{NT}_k) \in \mathcal{A}^e\ \forall k$, that means that:

$$\forall k\ \exists \boldsymbol{q}^{NT}_k : e^{NT}_k = \bar{\boldsymbol{p}}^{\boldsymbol{NT}} * \boldsymbol{q}^{NT}_k, \boldsymbol{q}_k \equiv (\boldsymbol{q}^T_k, \boldsymbol{q}^{NT}_k) \Rightarrow \boldsymbol{q}_k \in \mathcal{O}$$

If $X^{NT} \subseteq \mathbb{R}_+$, then $\sum_{k=1}^n \lambda_k \bar{\boldsymbol{p}}^{\boldsymbol{NT}} * \boldsymbol{q}^{NT}_k = \bar{\boldsymbol{p}}^{\boldsymbol{NT}} * \boldsymbol{q^{NT}}$ implies that $\sum_{k=1}^n \lambda_k \boldsymbol{q}^{NT}_k = \boldsymbol{q^{NT}}$ because positive non-tax expenditure requires that $\bar{\boldsymbol{p}}^{\boldsymbol{NT}} \neq 0$. In that case:

$$\sum_{k=1}^n \lambda_k \boldsymbol{q}_k = \boldsymbol{q} \Rightarrow\Leftarrow \sum_{k=1}^n \lambda_k \boldsymbol{q}_k \in \mathcal{O}$$

This is a contradiction arising from $\boldsymbol{q} \notin \mathcal{O}$.

If X^{NT} is not a subset of $\mathbb{R}_+$, then X^{NT} is convex. This means $\sum_{k=1}^n \lambda_k \boldsymbol{q}_k \in X$. Pseudo-convexity of preferences implies that:

$$\sum_{k=1}^n \lambda_k \boldsymbol{q}_k \succ \boldsymbol{q}$$

[1] Note that $\mathcal{O}$ is open in $\mathbb{R}^N$.

Yet the weighted average of taxed goods is the desired taxed good consumption bundle, whereas the weighted average of non-taxed goods is affordable:

$$\sum_{k=1}^{n} \lambda_k \boldsymbol{q_k^T} = \boldsymbol{q}^T$$

$$\boldsymbol{\bar{p}^{NT}} * \sum_{k=1}^{n} \boldsymbol{q_k^{NT}} = \sum_{k=1}^{n} e_k^{NT} = \boldsymbol{\bar{p}^{NT}} * \boldsymbol{q}^{NT}$$

Thus, the agent could not have optimally chosen $\boldsymbol{q}^{NT}$, another contradiction. We conclude that $(\boldsymbol{q}^T, \boldsymbol{\bar{p}}^{NT} * \boldsymbol{q}^{NT}) \notin Co(\mathcal{A}^e)$.

Now, we can apply the Separating Hyperplane Theorem to say that there is a vector $(\boldsymbol{p}^{Ts}, 1)$, where $\boldsymbol{p}^{Ts}$ has as many elements as $\boldsymbol{q}^T$, such that:

$$(\boldsymbol{p}^{Ts}, 1) * (\boldsymbol{q}^T, \boldsymbol{\bar{p}}^{NT} * \boldsymbol{q}^{NT}) \leq (\boldsymbol{p}^{Ts}, 1) * (\boldsymbol{q^{T'}}, e^{NT'}) \; \forall (\boldsymbol{q^{T'}}, e^{NT'}) \in Co(\mathcal{A}^e)$$

Defining $\boldsymbol{p^s} \equiv (\boldsymbol{p}^{Ts}, \boldsymbol{\bar{p}}^{NT})$, this implies that for any bundle $\boldsymbol{q'} = (\boldsymbol{q}^{T'}, \boldsymbol{q}^{NT'}) \in \mathcal{O}$:

$$\boldsymbol{p}^s * \boldsymbol{q}^{'} \geq \boldsymbol{p}^s * \boldsymbol{q}$$

Since $\mathcal{O}$ is open, the above expression can never be satisfied with equality. To see this, suppose otherwise, i.e. that $\exists \boldsymbol{q'} \in \mathcal{O}$ such that:

$$\boldsymbol{p}^s * \boldsymbol{q}^{'} = \boldsymbol{p}^s * \boldsymbol{q}$$

Note that $\boldsymbol{\bar{p}}^{NT} > \boldsymbol{0}$ implies that we can choose $\boldsymbol{q''}$ within $\epsilon(\boldsymbol{q'})$ of $\boldsymbol{q'}$ by slightly reducing a component of $\boldsymbol{q'}$ for which the corresponding perceived price is positive. Thus, $\boldsymbol{q''} \in \mathcal{O}$, yet $\boldsymbol{p}^s * \boldsymbol{q''} < \boldsymbol{p}^s * \boldsymbol{q}$. This yields our desired contradiction. Therefore:

$$\boldsymbol{p}^s * \boldsymbol{q'} > \boldsymbol{p}^s * \boldsymbol{q} \; \forall \boldsymbol{q'} \in \mathcal{O}$$

We conclude by defining $W^s \equiv \boldsymbol{p}^s * \boldsymbol{q}$ and noting that $\forall \boldsymbol{q'} \in X$:

$$\boldsymbol{q'} \succ \boldsymbol{q} \Rightarrow \boldsymbol{q'} \in \mathcal{O} \Rightarrow \boldsymbol{p^s} > W^s$$

Therefore, the model has rationalized consumption because no preferred consumption bundle is perceived to be affordable. □

Now that we've gone through the proof, we can make a couple of observations. One, the assumption of CPC preferences is satisfied when preferences are represented by a lower semi-continuous and quasi-concave function u on $\mathbb{R}^N$, so that:

$$\forall x, y \in X : x \succeq y \Leftrightarrow u(x) \geq u(y)$$

This makes it clear that we have, in fact, generalized proposition 2. Also, note that it may be easier in practice to check to see that preferences have such a utility representation than to check that they satisfy continuous pseudo-convexity.

Two, it may appear strange that we needed to assume that X^{NT} is concave specifically if it has dimension greater than one. This is because a discrete grid for consumption of non-taxed goods can create a lumpy evaluation of non-tax expenditure, thwarting the existence of a separating hyperplane. For example, consider a consumption set $\mathbb{R}_+ \times \{0,1\}^2$, where there is one taxed good chosen continuously and two non-taxed goods chosen from $\{0,1\}$. The sticker price vector is $\bar{p} = (1,1,1)$. The consumer have preferences rationalized by the function:

$$u(\boldsymbol{q}) = q_1 + \min\{q_2, q_3\}$$

In words, the taxed good is perfect substitutes with the minimum consumption of the two non-taxed goods, which are perfect complements with each other. Consider the consumption bundle:

$$\boldsymbol{q} = (0, 1, 0)$$

If the agent perceived income $W^s \geq 2$, they could do better by consuming $(0,1,1)$. Supposing otherwise, if the agent perceives a positive tax-inclusive price of the taxed good, then optimally $q_1 > 0$ and $q_2 = q_3 = 0$. Finally, there is no optimal consumption bundle if $p_1^s \leq 0$. Thus, the consumption bundle cannot be rationalized.

Next, we derive our expression for the change in consumer surplus due to the tax:

Proposition 3. *Let $e(p)$ and $h(p)$ denote the expenditure function and compensated demand for the taxed good respectively at price p for the taxed good and price p^{NT} for the other good, so that the agent is minimally compensated so as to achieve utility of at least $u(\boldsymbol{q}(\bar{p}, p^{NT}, 0, W))$; formally, $e(p) = \min\{W' | u(d(p, p^{NT}, W'), d^{NT}(p, p^{NT}, W)) \geq u(\boldsymbol{q}(\bar{p}, p^{NT}, 0, W))\}$, which is well-defined by continuity of u and connectedness of the choice set. Then compensating variation due to the tax satisfies:*

$$\Delta CS = (\bar{p} + \tau - p^s)h(p^s) + e(p^s) - e(\bar{p}) \tag{10}$$

Proof: Letting W^s denote conjectured wealth when facing tax τ, local non-satiation of preferences implies that:

$$(p^s, \bar{p}^{NT}) * \boldsymbol{q}(\bar{p}, p^{NT}, \tau, W + \Delta CS) = W^s = e(p^s)$$

In words, total perceived expenditures equal perceived wealth, which must be exactly the wealth the agent would need under perceived prices to achieve the utility from before the tax. Plugging in and using the fact that $h(p^s) = q(\bar{p}, p^{NT}, \tau, W + \Delta CS)$ yields:

$$\begin{aligned}
(\bar{p} + \tau - p^s)h(p^s) &= [(\bar{p} + \tau, \bar{p}^{NT}) - (p^s, \bar{p}^{NT})] * \boldsymbol{q}(\bar{\boldsymbol{p}}, \tau, W + \Delta CS) \\
(\bar{p} + \tau - p^s)h(p^s) &= W + \Delta CS - e(p^s)
\end{aligned}$$

Rearranging and again using local non-satiation yields:

$$\Delta CS = e(p^s) - W + (\bar{p} + \tau - p^s)h(p^s) = e(p^s) - e(\bar{p}) + (\bar{p} + \tau - p^s)h(p^s)$$

□

The following lemma establishes the Compensated Law of Demand (CLD) in our setting:

Lemma 2. *For any agent i with type θ_i and any two prices p and p':*

$$p < p' \Rightarrow q(p'; \theta_i, h) \leq q(p; \theta_i, l)$$

Proof: Note that there must be values q^{NT} and $q^{NT'}$ such that:

$$(q(p; \theta_i, l), q^{NT}) \sim_{\theta_i} (q(p'; \theta_i, h), q^{NT'})$$

From local non-satiation:

$$p * q(p; \theta_i, l) + p^{NT} * q^{NT} \leq p * q(p'; \theta_i, h) + p^{NT} * q^{NT'}$$

$$p' * q(p'; \theta_i, h) + p^{NT} * q^{NT'} \leq p' * q(p; \theta_i, l) + p^{NT} * q^{NT}$$

Rearranging yields:

$$p * [q(p; \theta_i, l) - q(p'; \theta_i, h)] \leq p^{NT} * [q^{NT'} - q^{NT}] \leq p' * [q(p; \theta_i, l) - q(p'; \theta_i, h)]$$

Thus, $p' > p \Rightarrow q(p; \theta_i, l) \geq q(p'; \theta_i, h)$. □

Proposition 4. *Assume a continuously differentiable and strictly increasing aggregate supply function Q^{supply}, as well as continuously differentiable compensated demand functions h_i and subjective price functions p_i^s $\forall i$. Subjective price functions change one-for-one with sticker prices, so that:*

$$p_i^s(\bar{p}, \tau) = \bar{p} + p_i^s(0, \tau) \;\forall \bar{p} \;\forall \tau \;\forall i$$

Subjective prices also agree with sticker prices when there is no tax:

$$p_i^s(\bar{p}, 0) = \bar{p} \quad \forall \bar{p} \;\forall i$$

We implicitly define the pre-tax sticker price $\bar{p}^{old}$ by:[2]

$$Q^{supply}(\bar{p}^{old}) = \sum_i h_i(\bar{p}^{old}, \nu_i)$$

[2] $\nu_i \equiv u_i(\boldsymbol{d}_i(\boldsymbol{p}, W_i)) \;\forall i$

and the new sticker price $\bar{p}^{new}$ after the imposition of the tax τ when agents are compensated by:

$$Q^{supply}(\bar{p}^{new}) = \sum_i h_i((p_i^s(\bar{p}^{new}, \tau)), \nu_i)$$

Defining deadweight loss by:[3]

$$DWL \equiv \sum_i \Delta CS_i + \int_{\bar{p}^{new}}^{\bar{p}^{old}} Q^{supply}(p)dp - \tau \sum_i q_i^c$$

where

$$\Delta CS_i = (\bar{p}^{new} + \tau - p_i^s(\bar{p}^{new}, \tau))q_i^c + \int_{\bar{p}^{old}}^{p_i^s(\bar{p}^{new}, \tau)} h_i(p, \nu_i)dp \ \forall i$$

$$q_i^c \equiv h_i(p_i^s(\bar{p}^{new}, \tau), \nu_i) \ \forall i$$

then aggregate deadweight loss has second order approximation around $\tau = 0$:

$$DWL \approx -\frac{1}{2}[\sum_i m_i \frac{\partial h_i}{\partial p} - \frac{(\sum_i m_i \frac{\partial h_i}{\partial p})^2}{\sum_i \frac{\partial h_i}{\partial p} - \frac{\partial Q^{supply}}{\partial p}}]\tau^2$$

Proof:

$$DWL = \sum_i \int_{\bar{p}^{old}}^{p_i^s(\bar{p}^{new}, \tau)} h_i(p, \nu_i)dp + \int_{\bar{p}^{new}}^{\bar{p}^{old}} Q^{supply}(p)dp + \sum_i (\bar{p}^{new} - p_i^s(\bar{p}^{new}, \tau))q_i^c$$

Note that $\bar{p}^{new}$ is a function of τ. One can easily confirm that $\bar{p}^{new}|_{\tau=0} = \bar{p}^{old}$, so that deadweight loss is zero when $\tau = 0$. We can find $\frac{\partial \bar{p}^{new}}{\partial \tau}$ from the Inverse Function Theorem:[4]

$$\frac{\partial Q^{supply}}{\partial p}\frac{\partial \bar{p}^{new}}{\partial \tau} = \sum_i \frac{\partial h_i}{\partial p}[\frac{\partial p_i^s}{\partial \bar{p}^{new}}\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}] = \sum_i \frac{\partial h_i}{\partial p}[\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}]$$

$$\frac{\partial \bar{p}^{new}}{\partial \tau} = \frac{\sum_i \frac{\partial h_i}{\partial p}\frac{\partial p_i^s}{\partial \tau}}{\frac{\partial Q^{supply}}{\partial p} - \sum_i \frac{\partial h_i}{\partial p}}$$

We can then take the first derivative of deadweight loss with respect to the tax:

$$\begin{aligned}
\frac{\partial DWL}{\partial \tau} &= \sum_i [\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}]h_i - \frac{\partial \bar{p}^{new}}{\partial \tau}Q^{supply}(\bar{p}^{new}) \\
&- \sum_i \Big[\frac{\partial p_i^s}{\partial \tau}h_i + (p_i^s(\bar{p}^{new}, \tau) - \bar{p}^{new})\frac{\partial h_i}{\partial p}[\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}]\Big] \\
&= \frac{\partial \bar{p}^{new}}{\partial \tau}\sum_i h_i - \frac{\partial \bar{p}^{new}}{\partial \tau}Q^{supply}(\bar{p}^{new}) - \sum_i \Big[(p_i^s(\bar{p}^{new}, \tau) - \bar{p}^{new})\frac{\partial h_i}{\partial p}[\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}]\Big] \\
&= -\sum_i \Big[(p_i^s(\bar{p}^{new}, \tau) - \bar{p}^{new})\frac{\partial h_i}{\partial p}[\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}]\Big]
\end{aligned}$$

Since $p_i^s(\bar{p}^{new}, 0) = \bar{p}^{new}$, it follows that

$$\left.\frac{\partial DWL}{\partial \tau}\right|_{\tau=0} = 0$$

[3]Note that $\bar{p}^{new} \leq \bar{p}^{old} \ \forall \tau \geq 0$ from the Compensated Law of Demand and the fact that supply is strictly increasing in price.

[4]This claim also uses the fact that aggregate supply is strictly increasing while aggregate compensated demand is weakly decreasing, so that there is always a unique value for $\bar{p}^{new}$.

Obtaining the second derivative would be straightforward if $h_i \in \mathbb{C}^2\ \forall i$. Instead, we find the second derivative at $\tau = 0$ from the definition:

$$\frac{\partial^2 DWL}{\partial \tau^2}\bigg|_{\tau=0} = \lim_{\tau \to 0} - \frac{\sum_i \left[(p_i^s(\bar{p}^{new}, \tau) - \bar{p}^{new}) \frac{\partial h_i}{\partial p} [\frac{\partial \bar{p}^{new}}{\partial \tau} + \frac{\partial p_i^s}{\partial \tau}] \right]}{\tau}$$

Note that continuity of $\frac{\partial p_i^s}{\partial \tau}$ with respect to τ for all agents implies that $\frac{\partial \bar{p}^{new}}{\partial \tau}$ is continuous. Since $\frac{\partial Q^{supply}}{\partial p}$ and $\frac{\partial h_i}{\partial p}$ $\forall i$ are also continuous:

$$\begin{aligned}
\frac{\partial^2 DWL}{\partial \tau^2}\bigg|_{\tau=0} &= -\sum_i \frac{\partial h_i}{\partial p}\bigg|_{\tau=0} [\frac{\partial \bar{p}^{new}}{\partial \tau}\bigg|_{\tau=0} + \frac{\partial p_i^s}{\partial \tau}\bigg|_{\tau=0}] \lim_{\tau \to 0} \frac{(p_i^s(\bar{p}^{new}, \tau) - \bar{p}^{new})}{\tau} \\
&= -\sum_i \frac{\partial h_i}{\partial p}\bigg|_{\tau=0} [\frac{\partial \bar{p}^{new}}{\partial \tau}\bigg|_{\tau=0} + \frac{\partial p_i^s}{\partial \tau}\bigg|_{\tau=0}] \frac{\partial p_i^s}{\partial \tau}\bigg|_{\tau=0}
\end{aligned}$$

Using the fact that $m_i \equiv \frac{\partial p_i^s}{\partial \tau}|_{\tau=0}$, we can note that:

$$\frac{\partial \bar{p}^{new}}{\partial \tau}|_{\tau=0} = \frac{\sum_i m_i \frac{\partial h_i}{\partial p}}{\frac{\partial Q^{supply}}{\partial p} - \sum_i \frac{\partial h_i}{\partial p}}$$

and so:

$$\frac{\partial^2 DWL}{\partial \tau^2}|_{\tau=0} = -[\sum_i m_i^2 \frac{\partial h_i}{\partial p} + \frac{(\sum_i m_i \frac{\partial h_i}{\partial p})^2}{\frac{\partial Q^{supply}}{\partial p} - \sum_i \frac{\partial h_i}{\partial p}}]$$

Now we can find the second order approximation for deadweight loss:

$$DWL \approx DWL|_{\tau=0} + \frac{\partial DWL}{\partial \tau}|_{\tau=0}\tau + \frac{1}{2}\frac{\partial^2 DWL}{\partial \tau^2}|_{\tau=0}\tau^2$$

$$DWL \approx -\frac{1}{2}[\sum_i m_i^2 \frac{\partial h_i}{\partial p} + \frac{(\sum_i m_i \frac{\partial h_i}{\partial p})^2}{\frac{\partial Q^{supply}}{\partial p} - \sum_i \frac{\partial h_i}{\partial p}}]\tau^2$$

□

A.2 Additional results and proofs from section 3

The upper and lower bounds use the following lemma:

Lemma 3. *For any agent i with type θ_i and any two pairs (p, ζ_i) and (p', ζ_i'):*

$$dwl(p'; \theta_i, \zeta_i) \geq dwl(p; \theta_i, \zeta_i') - (p - \bar{p})(q(p'; \theta_i, \zeta_i') - q(p; \theta_i, \zeta_i)).$$

Proof: Note from the definition of the expenditure function and optimal compensated consumption vectors $\boldsymbol{q}$ and $\boldsymbol{q'}$ for price vectors (p, p^{NT}) and (p', p^{NT}) respectively:

$$e(p') - e(p) = (p', p^{NT}) * \boldsymbol{q'} - (p, p^{NT}) * \boldsymbol{q} \geq (p', p^{NT}) * \boldsymbol{q'} - (p, p^{NT}) * \boldsymbol{q'} = (p' - p)q(p'; \theta_i, \zeta_i')$$

Plugging in yields:

$$\begin{aligned}
dwl(p'; \theta_i) &= [e(p') - e(\bar{p})] - (p' - \bar{p})q(p'; \theta_i, \zeta_i') \\
&= [e(p') - e(p)] + [e(p) - e(\bar{p})] - [(p' - p) + (p - \bar{p})]q(p'; \theta_i, \zeta_i') \\
&\geq (p' - p)q(p'; \theta_i, \zeta_i') + e(p) - e(\bar{p}) - [(p' - p) + (p - \bar{p})]q(p'; \theta_i, \zeta_i') \\
&= e(p) - e(\bar{p}) - (p - \bar{p})q(p'; \theta_i, \zeta_i') \\
&= dwl(p) + (p - \bar{p})q(p; \theta_i, \zeta_i) - (p - \bar{p})q(p'; \theta_i, \zeta_i') \\
&= dwl(p; \theta_i) - (p - \bar{p})(q(p'; \theta_i, \zeta_i') - q(p; \theta_i, \zeta_i))
\end{aligned}$$

See also appendix figure 5 for a graphical demonstration. □

Proof of Proposition 1: From lemma 2 and prices being bound away from zero, we can always find a value of $\hat{p}^s$ such that:

$$\int_{\theta_i} q(\hat{p}^s;\theta_i,l)dF_\theta(\theta_i) \leq \int_{p_i^s,\theta_i,\zeta_i} q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \leq \int_{\theta_i} q(\hat{p}^s;\theta_i,h)dF_\theta(\theta_i)$$

Pick $\lambda \in [0,1]$ such that:

$$\lambda \int_{\theta_i} q(\hat{p}^s;\theta_i,h)dF_\theta(\theta_i) + (1-\lambda)\int_{\theta_i} q(\hat{p}^s;\theta_i,l)dF_\theta(\theta_i) = \int_{p_i^s,\theta_i,\zeta_i} q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)$$

Define $F'_{\theta,\zeta}$ such that $F'_\theta = F_\theta$ and $\zeta = h$ with probability λ, $\zeta = l$ with probability $1-\lambda$, $\theta \perp \zeta$. Then:

$$\begin{aligned}\int_{\theta_i,\zeta_i} q(\hat{p}^s;\theta_i,\zeta_i)dF_\theta(\theta_i) &= \int_{\theta_i} [\lambda q(\hat{p}^s;\theta_i,h) + (1-\lambda)q(\hat{p}^s;\theta_i,l)]dF_\theta(\theta_i) \\ &= \int_{\theta_i,\zeta_i} q(pl_i^s;\theta_i,\zeta_i)dF'_{p^s,\theta,\zeta}(\theta_i,\zeta_i)\end{aligned}$$

□

Proof of theorem 1: From lemma 3:

$$\begin{aligned}&\int_{p_i^s,\theta_i,\zeta_i} [dwl(p_i^s;\theta_i,\zeta_i) + (\hat{p}^s - \bar{p})q(p_i^s;\theta_i,\zeta_i)]dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\ &\qquad \geq \int_{p_i^s,\theta_i,\zeta_i} [dwl(\hat{p}^s;\theta_i,\zeta_i) + (\hat{p}^s - \bar{p})q(\hat{p}^s;\theta_i,\zeta_i)]dF'_{\theta,\zeta}(\theta_i,\zeta_i)\end{aligned}$$

But note from the rationalizability of the data that:

$$(\hat{p}^s - \bar{p})\int_{p_i^s,\theta_i,\zeta_i} q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) = (\hat{p}^s - \bar{p})\int_{\theta_i,\zeta_i} q(\hat{p}^s;\theta_i,\zeta_i)dF'_{\theta,\zeta}(\theta_i,\zeta_i)$$

We can thus conclude that:

$$\int_{p_i^s,\theta_i,\zeta_i} dwl(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \geq \int_{\theta_i,\zeta_i} dwl(\hat{p}^s;\theta_i,\zeta_i)dF_{\theta,\zeta}(\theta_i,\zeta_i)$$

□

Proof of theorem 2: From lemma 3 and rationalizability of the data:

$$\begin{aligned}&\int_{p_i^s,\theta_i} [dwl(p^b(p_i^s);\theta_i) + (p_i^s - \bar{p})q(p^b(p_i^s);\theta_i,\zeta_i)]dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\ &\qquad \geq \int_{p_i^s,\theta_i,\zeta_i} [dwl(p_i^s;\theta_i) + (p_i^s - \bar{p})q(p_i^s;\theta_i,\zeta_i)]dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)\end{aligned}$$

$$\begin{aligned}&\int_{p_i^s,\theta_i} [dwl(p^b(p_i^s);\theta_i) + p_i^s q(p^b(p_i^s);\theta_i,\zeta_i)]dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\ &\qquad \geq \int_{p_i^s,\theta_i,\zeta_i} [dwl(p_i^s;\theta_i) + p_i^s q(p_i^s;\theta_i,\zeta_i)]dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)\end{aligned}$$

$$\begin{aligned}&\int_{p_i^s,\theta_i} [dwl(p^b(p_i^s);\theta_i,\zeta_i) + (p_i^s - \tilde{p}^s)q(p^b(p_i^s);\theta_i,\zeta_i)]dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\ &\qquad \geq \int_{p_i^s,\theta_i,\zeta_i} [dwl(p_i^s;\theta_i) + (p_i^s - \tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)]dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)\end{aligned}$$

Rearranging yields:

$$\int_{p_i^s,\theta_i,\zeta_i} dwl(p^b(p_i^s);\theta_i,\zeta_i)dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \geq \int_{p_i^s,\theta_i,\zeta_i} dwl(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)$$
$$-\int_{p_i^s,\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p^b(p_i^s);\theta_i,\zeta_i)dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) + \int_{p_i^s,\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)$$

We can show from lemma 2 and $p_i^s \in [\bar{p},\bar{p}+\bar{m}\tau] = \mathcal{P}\ \forall i$ that the term on the second line is non-negative. Formally for any $p_i^s \in (\bar{p},\bar{p}+\bar{m}\tau),\theta_i,\zeta_i,\zeta_i'$:

$$p_i^s > \tilde{p}^s \Rightarrow p^b(p_i^s) > \tilde{p}^s \Rightarrow q(p^b(p_i^s);\theta_i,\zeta_i') \leq q(p_i^s;\theta_i,\zeta_i)$$
$$p_i^s \leq \tilde{p}^s \Rightarrow p^b(p_i^s) < \tilde{p}^s \Rightarrow q(p^b(p_i^s);\theta_i,\zeta_i') \geq q(p_i^s;\theta_i,\zeta_i)$$

Either way:

$$(p_i^s-\tilde{p}^s)q(p^b(p_i^s);\theta_i,\zeta_i') \leq (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)$$

Thus:

$$\begin{aligned}
\int_{p_i^s,\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) &= \int_{p_i^s\in int(\mathcal{P}),\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\
&+ \int_{p_i^s\in\{\bar{p},\bar{p}+\bar{m}\tau\},\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\
&\geq \int_{p_i^s\in int(\mathcal{P}),\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p^b(p_i^s);\theta_i,\zeta_i)dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\
&+ \int\limits_{p_i^s\in\{\bar{p},\bar{p}+\bar{m}\tau\},\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \\
&= \int_{p_i^s,\theta_i,\zeta_i} (p_i^s-\tilde{p}^s)q(p_i^s;\theta_i,\zeta_i)dF''_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)
\end{aligned}$$

$$\int\limits_{p_i^s,\theta_i,\zeta_i} dwl(p^b(p_i^s);\theta_i,\zeta_i)dF_{p^s,\theta}(p_i^s,\theta_i) \geq \int\limits_{p_i^s,\theta_i,\zeta_i} dwl(p_i^s;\theta_i,\zeta_i)dF_{p^s,\theta}(p_i^s,\theta_i,\zeta_i)$$

□

Before stating and proving theorem 3, we note that deadweight loss is bounded by the product of the reduction in demand and $\bar{m}\tau$.

Lemma 4. *If $p_i^s \in [\bar{p},\bar{p}+\bar{m}\tau]$, then $dwl(p_i^s;\theta_i,\zeta_i) \leq [q(\bar{p};\theta_i,\zeta_i) - q(p_i^s;\theta_i,\zeta_i)]\bar{m}\tau\ \forall\theta_i,\zeta_i$.*

Proof: Using lemma 3:

$$0 = dwl(\bar{p};\theta_i,\zeta_i) \geq dwl(p_i^s;\theta_i,\zeta_i) - (p_i^s-\bar{p})[q(\bar{p};\theta_i,\zeta_i) - q(p_i^s;\theta_i,\zeta_i)]$$
$$dwl(p_i^s;\theta_i,\zeta_i) \leq [q(\bar{p};\theta_i,\zeta_i) - q(p_i^s;\theta_i,\zeta_i)](p_i^s-\bar{p}) \leq [q(\bar{p};\theta_i,\zeta_i) - q(p_i^s;\theta_i,\zeta_i)]\bar{m}\tau.$$

□

Next, we state and prove theorem 3. It says that the maximal value of deadweight loss consistent with the data and knowledge of F_θ^* is given by having some agents perceive the highest possible price and some others perceive the lowest possible price. It achieves this by assigning the good there where it will generate the most deadweight loss, while holding aggregate demand constant. The resulting demand function, $\tilde{q}_{\Delta,\gamma}(\theta_i)$, is such that those for whom the ratio of deadweight loss[5] to change in quantity exceeds Δ perceive the highest price, those with such a ratio below Δ perceive the lowest possible price, and those with ratio equal to Δ are split between perceiving the high and low price in a way that rationalizes demand. Those who perceive the high (low) price consume the least (most) possible consistent with their perceptions.

[5]Note that when $p^s = \bar{p}$, the tax is effectively lump-sum and so there is no deadweight loss.

Theorem 3. *There exist values $\Delta \in [0, \bar{m}\tau]$ and $\gamma \in [0,1]$ such that:*

$$\int_{p_i^s,\theta_i} \tilde{q}_{\Delta,\gamma}(\theta_i) dF^*_{p^s,\theta_i} = \int_{p_i^s,\theta_i,\zeta_i} q(p_i^s;\theta_i,\zeta_i) dF^*_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)$$

where:

$$\begin{aligned}\tilde{q}_{\Delta,\gamma}(\theta_i) = &\left[\mathbb{I}\left(\frac{dwl(\bar{p}+\bar{m}\tau;\theta_i,l)}{q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)} > \Delta\right) + \gamma\mathbb{I}\left(\frac{dwl(\bar{p}+\bar{m}\tau;\theta_i,l)}{q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)} = \Delta\right)\right] q(\bar{p}+\bar{m}\tau;\theta_i,l) \\ &+ \left[\mathbb{I}\left(\frac{dwl(\bar{p}+\bar{m}\tau;\theta_i,l)}{q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)} < \Delta\right) + (1-\gamma)\mathbb{I}\left(\frac{dwl(\bar{p}+\bar{m}\tau;\theta_i,l)}{q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)} = \Delta\right)\right] q(\bar{p};\theta_i,h)\end{aligned}$$

Of course, if $q(\bar{p};\theta_i,h) = q(\bar{p}+\bar{m}\tau;\theta_i,l)$, then $\tilde{q}_{\Delta,\gamma}(\theta_i) = q(\bar{p};\theta_i,h)$. Furthermore, under assumption 1, for any $F_{p^s,\theta,\zeta}$ that rationalizes the data such that $F_\theta = F^*_\theta$:

$$\int_{p_i^s,\theta_i} \frac{\tilde{q}_{\Delta,\gamma}(\theta_i) - q(\bar{p};\theta_i,h)}{q(\bar{p}+\bar{m}\tau;\theta_i,l) - q(\bar{p};\theta_i,h)} dwl(\bar{p}+\bar{m}\tau;\theta_i,l) dF^*_{p^s,\theta}(p_i^s,\theta_i) \geq \int_{p_i^s,\theta_i,\zeta_i} dwl(p_i^s;\theta_i,\zeta_i) dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)$$

where the integrand on the left-hand side is defined as zero for any θ_i such that $q(\bar{p}+\bar{m}\tau;\theta_i,l) = q(\bar{p};\theta_i,h)$.

The intuition is for the Δ term is straightforward. The econometrician observes the reduction in aggregate demand due to the tax. In searching for the explanation of that reduction in demand that maximizes deadweight loss, one should assign the reduction in quantity demanded to those for whom that allocation yields the greatest deadweight loss. Following this procedure, there is a cutoff value Δ which describes the amount of deadweight loss obtained relative to the reduction in quantity demanded sufficient to warrant the assignment of subjective tax-inclusive price $p_i^s = \bar{p} + \bar{m}\tau$ to that agent.

The idea behind the tie-breaking provision is that those individuals who perceive the high price should reduce their consumption as much as possible to maximize deadweight loss; those who perceive the sticker price should maximize their consumption to permit even more individuals to perceive the high price.

Proof of Theorem 3: The outline of the proof is as follows. First, we use lemma 4 to show that the maximal deadweight loss consistent with aggregate demand and F^*_θ comes from a data-generating process in which agents perceiving the price $\bar{p}+\bar{m}\tau$ choose the lowest quantity consistent with preference maximization, whereas the other agents choose the largest such quantity. Then, we show that distributions satisfying such a property yield deadweight loss no larger than the proposed distribution, which exists.

First, consider an arbitrary distribution $F_{p^s,\theta,\zeta}$ (yielding well-defined aggregate demand and deadweight loss) such that $F_\theta = F^*_\theta$ and:

$$F_{p^s} = \begin{cases} 0 & p_i^s < \bar{p} \\ F_{p^s}(\bar{p}) & p_i^s \in [\bar{p}, \bar{p}+\bar{m}\tau) \\ 1 & p_i^s \geq \bar{p}+\bar{m}\tau \end{cases}$$

In words, the above expression says that the support of p^s is contained in $\{\bar{p}, \bar{p}+\bar{m}\tau\}$. By theorem 2, the maximal value of deadweight loss consistent with aggregate demand and F^*_θ must satisfy this property. Consider some value $\rho \in [0,1]$ such that:

$$\begin{aligned}&\rho\int_{\theta_i} q(\bar{p}+\bar{m}\tau;\theta_i,l) dF_{\theta|p^s\neq\bar{p}}(\theta_i) + [1-\rho]\int_{\theta_i} q(\bar{p};\theta_i,h) dF_{\theta|p^s=\bar{p}}(\theta_i) \\ &= \int_{p_i^s,\theta_i,\zeta_i} q(p_i^s;\theta_i,\zeta_i) dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i)\end{aligned} \tag{11}$$

Such a value of ρ must exist by the Intermediate Value Theorem, since by the definition of l and h and the CLD as expressed in lemma 2:

$$\int_{\theta_i} q(\bar{p}+\bar{m}\tau;\theta_i,l) dF_{\theta|p^s\neq\bar{p}}(\theta_i) \leq \int_{p_i^s,\theta_i,\zeta_i} q(p_i^s;\theta_i,\zeta_i) dF_{p^s,\theta,\zeta}(p_i^s,\theta_i,\zeta_i) \quad \leq \int_{\theta_i} q(\bar{p};\theta_i,h) dF_{\theta|p^s=\bar{p}}(\theta_i)$$

In words, we are constructing an alternative distribution that rationalizes aggregate demand such that $p^s = \bar{p} + \bar{m}\tau$ and $\zeta = l$ with probability ρ, and otherwise $p^s = \bar{p}$ and $\zeta = h$. We now show that this alternate distribution yields at least as much deadweight loss, thus showing that the maximal value of deadweight loss consistent with aggregate demand and F^*_θ must arise from a distribution in which almost surely $(p^s, \zeta) = (\bar{p}, h)$ or $(p^s, \zeta) = (\bar{p} + \bar{m}\tau, l)$.

From the definition of deadweight loss:

$$\int\limits_{\theta_i,\zeta_i} \bar{m}\tau[q(\bar{p};\theta_i,\zeta_i) - q(\bar{p};\theta_i,l)]dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)$$
$$= \int\limits_{\theta_i,\zeta_i} [dwl(\bar{p}+\bar{m}\tau;\theta_i,l) - dwl(\bar{p};\theta_i,\zeta_i)]dF_{\theta,\zeta|p^s\neq\bar{p}}$$

From here, the definition of l, and using the fact that $dwl(\bar{p};\theta_i,\zeta_i) = 0\ \forall\theta_i,\zeta_i$, we have that $\rho \geq 1 - F_{p^s}(\bar{p})$ implies that:

$$\begin{aligned}
&\rho \int\limits_{\theta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,l)dF_{\theta|p^s\neq\bar{p}}(\theta_i) + (1-\rho)\int_{\theta_i} dwl(\bar{p};\theta_i,h)dF_{\theta|p^s=\bar{p}}(\theta_i) \\
&\quad = \rho\int_{\theta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,l)dF_{\theta|p^s\neq\bar{p}}(\theta_i) \\
&\quad \geq [1-F_{p^s}(\bar{p})]\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) \\
&\quad = [1-F_{p^s}(\bar{p})]\int\limits_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) + F_{p^s}(\bar{p})\int\limits_{\theta_i,\zeta_i} dwl(\bar{p};\theta_i,\zeta_i)dF_{\theta,\zeta|p^s=\bar{p}}(\theta_i,\zeta_i) \\
&\quad = \int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta}(\theta_i,\zeta_i)
\end{aligned}$$

Where the inequality follows from the fact that $\rho \geq 1 - F_{p^s}(\bar{p})$ by assumption, and the fact that $dwl(\bar{p} + \bar{m}\tau;\theta_i,\zeta_i)$ and the definition of l. This shows that whenever $\rho \geq 1 - F_{p^s}(\bar{p})$, the proposed alternative distribution yields at least as much deadweight loss. Now suppose instead $\rho < 1 - F_{p^s}(\bar{p})$. From lemma 4:

$$\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)$$
$$\geq \bar{m}\tau\int_{\theta_i,\zeta_i}[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)]\bar{m}\tau dF_{\theta_i,\zeta_i|p^s\neq\bar{p}}(\theta_i,\zeta_i)$$

In addition, we find it convenient to rewrite the aggregate demand-rationalizing equation as:

$$\begin{aligned}
&\int_{p^s_i,\theta_i,\zeta_i} q(p^s_i;\theta_i,\zeta_i)dF_{p^s,\theta,\zeta}(p^s_i,\theta_i,\zeta_i) \\
&\quad = [1-F_{p^s}(\bar{p})]\int_{\theta_i,\zeta_i} q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta|p^s\neq\bar{p}}(\theta_i,\zeta_i) + F_{p^s}(\bar{p})\int_{\theta_i,\zeta_i} q(\bar{p};\theta_i,\zeta_i)dF_{\theta|p^s=\bar{p}}(\theta_i,\zeta_i)
\end{aligned}$$

And so, using equation 11 and rearranging terms,

$$\begin{aligned}
&\rho\int_{\theta_i,\zeta_i}[q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]dF_{\theta,\zeta}(\theta_i,\zeta_i) \\
&\quad = (1-\rho)\int_{\theta_i} q(\bar{p};\theta_i,h)dF_{\theta|p^s=\bar{p}}(\theta_i) - [1-F_{p^s}(\bar{p})-\rho]\int_{\theta_i,\zeta_i} q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) \\
&\quad - F_{p^s}(\bar{p})\int_{\theta_i,\zeta_i} q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)
\end{aligned}$$

Thus, plugging in and using lemma 4:

$$
\begin{aligned}
&\rho\int\limits_{\theta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,l)dF_{\theta|p^s\neq\bar{p}}(\theta_i) + (1-\rho)\int_{\theta_i} dwl(\bar{p};\theta_i,h)dF_{\theta|p^s=\bar{p}}(\theta_i)\\
&=\rho\int_{\theta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,l)dF_{\theta|p^s\neq\bar{p}}(\theta_i)\\
&=\rho\int_{\theta_i,\zeta_i}\big[dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i) - q(\bar{p}+\bar{m}\tau;\theta_i,l) + q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)\big]dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&=\rho\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&+\rho\int_{\theta_i,\zeta_i}[q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]dF_{\theta,\zeta}(\theta_i,\zeta_i)\\
&=\rho\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) + (1-\rho)\int_{\theta_i} q(\bar{p};\theta_i,h)dF_{\theta|p^s=\bar{p}}(\theta_i)\\
&-[1-F_{p^s}(\bar{p})-\rho]\int_{\theta_i,\zeta_i} q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&-F_{p^s}(\bar{p})\int_{\theta_i,\zeta_i} q(\bar{p};\theta_i,\zeta_i)dF_{\theta,\zeta|p^s=\bar{p}}(\theta_i,\zeta_i)\\
&=\rho\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) + F_{p^s}(\bar{p})\int_{\theta_i} q(\bar{p};\theta_i,h)dF_{\theta|p^s=\bar{p}}(\theta_i)\\
&+[1-F_{p^s}(\bar{p})-\rho]\int_{\theta_i,\zeta_i}\big[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)\big]dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&-F_{p^s}(\bar{p})\int_{\theta_i,\zeta_i} q(\bar{p};\theta_i,\zeta_i)dF_{\theta,\zeta|p^s=\bar{p}}(\theta_i,\zeta_i)\\
&\geq\rho\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) + F_{p^s}(\bar{p})\int_{\theta_i} q(\bar{p};\theta_i,h)dF_{\theta|p^s=\bar{p}}(\theta_i)\\
&+[1-F_{p^s}(\bar{p})-\rho]\bar{m}\tau\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&-F_{p^s}(\bar{p})\int_{\theta_i,\zeta_i} q(\bar{p};\theta_i,\zeta_i)dF_{\theta,\zeta|p^s=\bar{p}}(\theta_i,\zeta_i)\\
&\geq\rho\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&+[1-F_{p^s}(\bar{p})-\rho]\bar{m}\tau\int_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i)\\
&=[1-F_{p^s}(\bar{p})]\int\limits_{\theta_i,\zeta_i} dwl(\bar{p}+\bar{m}\tau;\theta_i,\zeta_i)dF_{\theta,\zeta|p^s\neq\bar{p}}(\theta_i,\zeta_i) + F_{p^s}(\bar{p})\int\limits_{\theta_i,\zeta_i} dwl(\bar{p};\theta_i,\zeta_i)dF_{\theta,\zeta|p^s=\bar{p}}(\theta_i,\zeta_i)
\end{aligned}
$$

Thus, we know that the maximal deadweight loss consistent with aggregate demand and F^*_θ is generated by a distribution in which with probability one either $(p^s,\zeta)=(\bar{p},h)$ or $(p^s,\zeta)=(\bar{p}+\bar{m}\tau,l)$. We refer to distributions of this sort as *binary* distributions.

Now, we show that the proposed distribution maximizes deadweight loss among all binary distributions, and thus among all distributions, that rationalize aggregate demand such that $F_\theta = F^*_\theta$. Towards that end, we first show that the proposed distribution exists. Note by lemma 4 and the CLD as in lemma 2:

$$
\int\limits_{\theta_i}\tilde{q}_{\bar{m}\tau,1}(\theta_i)dF^*_{p^s,\theta}(p^s_i,\theta_i) \leq \int\limits_{p^s_i,\theta_i,\zeta_i} q(p^s_i;\theta_i,\zeta_i)dF^*_{p^s,\theta,\zeta}(p^s_i,\theta_i,\zeta_i) \leq \int\limits_{\theta_i}\tilde{q}_{0,0}(\theta_i)dF^*_{p^s,\theta}(p^s_i,\theta_i)
$$

In words, aggregate demand is contained between when all agents perceive a high price and have type h and when all agents perceive a low price and have type l. Furthermore, one can confirm that for any $\Delta,\Delta',\gamma,\gamma'$

such that $0 \leq \Delta < \Delta' \leq \bar{m}\tau$ and $0 \leq \gamma < \gamma' \leq 1$:

$$\int_{\theta_i} \tilde{q}_{\Delta,\gamma'}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i) \leq \int_{\theta_i} \tilde{q}_{\Delta,\gamma}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i)$$

$$\int_{\theta_i} \tilde{q}_{\Delta',\gamma'}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i) \geq \int_{\theta_i} \tilde{q}_{\Delta,\gamma}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i)$$

Thus, we can pick Δ such that:

$$\int\limits_{\theta_i} \tilde{q}_{\Delta,1}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i) \leq \int\limits_{p^s_i,\theta_i,\zeta_i} q(p^s_i; \theta_i, \zeta_i) dF^*_{p^s,\theta,\zeta}(p^s_i, \theta_i, \zeta_i) \leq \int\limits_{\theta_i} \tilde{q}_{\Delta,0}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i)$$

If both sides hold with equality, we can define γ arbitrarily. Otherwise, we define γ so that the market clears:

$$\gamma \equiv \frac{\int_{p^s_i,\theta_i,\zeta_i} q(p^s_i; \theta_i, \zeta_i) dF^*_{p^s,\theta,\zeta}(p^s_i, \theta_i, \zeta_i) - \int_{\theta_i} \tilde{q}_{\Delta,0}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i)}{\int_{\theta_i} \tilde{q}_{\Delta,1}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i) - \int_{\theta_i} \tilde{q}_{\Delta,0}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i)}$$

We now have the values Δ and γ such that the market clears. Suppressing Δ and γ subscripts from $\tilde{q}$, we can say that:

$$\int_{\theta_i} \tilde{q}(\theta_i) dF^*_{p^s,\theta}(p^s_i, \theta_i) = \int_{p^s_i,\theta_i,\zeta_i} q(p^s_i; \theta_i, \zeta_i) dF^*_{p^s,\theta,\zeta}(p^s_i, \theta_i, \zeta_i)$$

Finally, to show that the proposed distribution maximizes deadweight loss, consider arbitrary binary distribution $F_{p^s,\theta,\zeta}$ that rationalizes aggregate demand. Defining $\mathbb{P}_F(p^s \neq \bar{p}|\theta_i) \equiv 1 - F_{p^s|\theta=\theta_i}(\bar{p} + \bar{m}\tau)$ as the probability that $(p^s, \zeta) = (\bar{p} + \bar{m}\tau, l)$ conditional on θ_i, rationalizing aggregate demand with $F_\theta = F^*_\theta$ means that:

$$\begin{aligned}\int_{\theta_i} \big[\mathbb{P}_F(p^s \neq \bar{p}|\theta_i)\ q(\bar{p} + \bar{m}\tau; \theta_i, l) + F_{p^s|\theta=\theta_i}(\bar{p})\ q(\bar{p}; \theta_i, h)\big] dF^*_\theta(\theta_i) \\ = \int_{p^s_i,\theta_i,\zeta_i} q(p^s_i; \theta_i, \zeta_i) dF^*_{p^s,\theta,\zeta}(p^s_i, \theta_i, \zeta_i)\end{aligned}$$

We can now write the difference in generated values of aggregate deadweight loss as:

$$\begin{aligned}
&\int_{\theta_i} \big[\frac{\tilde{q}(\theta_i) - q(\bar{p}; \theta_i, h)}{q(\bar{p} + \bar{m}\tau; \theta_i, l) - q(\bar{p}; \theta_i, h)} - \mathbb{P}_F(p^s \neq \bar{p}|\theta_i)\big] dwl(\bar{p} + \bar{m}\tau; \theta_i, l) dF^*_\theta(\theta_i) \\
&\quad = \int_{\theta_i : dwl(\bar{p}+\bar{m}\tau) > \Delta[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]} [1 - \mathbb{P}_F(p^s \neq \bar{p}|\theta_i)] dwl(\bar{p} + \bar{m}\tau; \theta_i) dF^*_\theta(\theta_i) \\
&\quad + \int_{\theta_i : dwl(\bar{p}+\bar{m}\tau) = \Delta[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]} [\gamma - \mathbb{P}_F(p^s \neq \bar{p}|\theta_i)] dwl(\bar{p} + \bar{m}\tau; \theta_i) dF^*_\theta(\theta_i) \\
&\quad - \int_{\theta_i : dwl(\bar{p}+\bar{m}\tau) < \Delta[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]} \mathbb{P}_F(p^s \neq \bar{p}|\theta_i) dwl(\bar{p} + \bar{m}\tau; \theta_i) dF^*_\theta(\theta_i) \\
&\quad \geq \Delta \int\limits_{\theta_i : dwl(\bar{p}+\bar{m}\tau) > \Delta[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]} [1 - \mathbb{P}_F(p^s \neq \bar{p}|\theta_i)]\ [q(\bar{p}; \theta_i, h) - q(\bar{p} + \bar{m}\tau; \theta_i, l)] dF^*_\theta(\theta_i) \\
&\quad + \Delta \int\limits_{\theta_i : dwl(\bar{p}+\bar{m}\tau) = \Delta[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]} [\gamma - \mathbb{P}_F(p^s \neq \bar{p}|\theta_i)]\ [q(\bar{p}; \theta_i, h) - q(\bar{p} + \bar{m}\tau; \theta_i, l)] dF^*_\theta(\theta_i) \\
&\quad - \Delta \int\limits_{\theta_i : dwl(\bar{p}+\bar{m}\tau) < \Delta[q(\bar{p};\theta_i,h) - q(\bar{p}+\bar{m}\tau;\theta_i,l)]} \mathbb{P}_F(p^s \neq \bar{p}|\theta_i) [q(\bar{p}; \theta_i, h) - q(\bar{p} + \bar{m}\tau; \theta_i, l)] dF^*_\theta(\theta_i)
\end{aligned}$$

We complete the proof by showing the right-hand side of the last inequality is zero. Since both distributions rationalize the same aggregate demand:

$$\begin{aligned}
&\int_{\theta_i:dwl(\bar{p}+\bar{m}\tau)>\Delta[q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]} q(\bar{p}+\bar{m}\tau;\theta_i,l)dF^*_\theta(\theta_i) \\
&+\int_{\theta_i:dwl(\bar{p}+\bar{m}\tau)=\Delta[q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]} [\gamma q(\bar{p}+\bar{m}\tau;\theta_i,l)+(1-\gamma)q(\bar{p};\theta_i,h)]dF^*_\theta(\theta_i) \\
&+\int_{\theta_i:dwl(\bar{p}+\bar{m}\tau)<\Delta[q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]} q(\bar{p};\theta_i,h)dF^*_\theta(\theta_i) \\
&=\int_{\theta_i} \big[\mathbb{P}_F(p^s \neq p^s|\theta_i)\ [q(\bar{p}+\bar{m}\tau;\theta_i,l)-q(\bar{p};\theta_i,h)]+q(\bar{p};\theta_i,h)\big]dF^*_\theta(\theta_i)
\end{aligned}$$

Subtracting both sides from $\int_{\theta_i} q(\bar{p};\theta_i,h)dF^*_\theta(\theta_i)$ yields:

$$\begin{aligned}
&\int_{\theta_i:dwl(\bar{p}+\bar{m}\tau)>\Delta[q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]} [q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]dF^*_\theta(\theta_i) \\
&+\int_{\theta_i:dwl(\bar{p}+\bar{m}\tau)=\Delta[q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]} \gamma[q(\bar{p};\theta_i,h)-q(\bar{p}+\bar{m}\tau;\theta_i,l)]dF^*_\theta(\theta_i) \\
&=\int_{\theta_i} \mathbb{P}_F(p^s \neq p^s|\theta_i)\ [q(\bar{p}+\bar{m}\tau;\theta_i,l)-q(\bar{p};\theta_i,h)]dF^*_\theta(\theta_i)
\end{aligned}$$

Finally, subtracting the right-hand side from the left-hand size and multiplying by zero yields the desired result. Thus:

$$\int_{\theta_i} \big[\frac{\tilde{q}(\theta_i)-q(\bar{p};\theta_i,h)}{q(\bar{p}+\bar{m}\tau;\theta_i,l)-q(\bar{p};\theta_i,h)} - \mathbb{P}_F(p^s \neq \bar{p}|\theta_i)\big]dwl(\bar{p}+\bar{m}\tau;\theta_i,l)dF^*_\theta(\theta_i)=0$$

In words, deadweight loss from the proposed distribution is at least as great as the deadweight loss from any binary distribution that also rationalizes aggregate demand and with the true distribution of preference types. From the first part of the proof, any distribution that rationalized aggregate demand and had the support of perceived prices contained in $\partial d\mathcal{P}$ yielded deadweight loss no greater than what one could obtain with a binary distribution that rationalized aggregate demand with $F_\theta = F^*_\theta$. Theorem 2 noted that any distribution that rationalized aggregate demand with $F_\theta = F^*_\theta$ yielded deadweight loss no greater than that one could obtain with a distribution that had the support of perceived prices contained in $\partial d\mathcal{P}$, rationalized aggregate demand, and had $F_\theta = F^*_\theta$. Therefore, any distribution that rationalizes aggregate demand and with $F_\theta = F^*_\theta$ yields deadweight loss no greater than the proposed distribution. □

B Details on Application of Linear Model

We use data gathered by CLK (2009) on the aggregate consumption of beer in U.S. states between 1970 and 2003, and cross-sectional data gathered by Goldin and Homonoff (2013) on tobacco consumption between 1984 and 2000. We translate their two models (in logs) to our linear specification. In particular, we are interested in estimating models like the one in equation 8.

In the case of aggregate beer consumption, we follow CLK (2009) in using a specification in first differences, and so we estimate regressions of the type:

$$\Delta y_{st} = \alpha + \beta \Delta \tau^e_{st} + \tilde{\beta} \Delta \tau^s_{st} + \gamma X_{st} + \epsilon_{st}$$

where y_{st} represents per-capita consumption of beer, in gallons, for state s at time t, τ^e represents excise taxes on beer (included in sticker price), τ^s represents sales taxes (non-salient), X is a vector of controls, and ϵ is an i.i.d. error term. All taxes are expressed in dollar amounts.

For each linear specification, we compute $\hat{m} = \frac{\tilde{\beta}}{\beta}$, which gives us the ratio of upper bound of deadweight loss to lower bound of deadweight loss (assuming that maximal attention, $\bar{m} = 1$). Results are presented in table 1. We also estimate a number of other specifications, again following CLK (2009), presented in table 2. These are meant to address concerns for spurious results – in particular, it could be the case that consumers react differently to the two tax rates because while sales taxes affect a variety of goods, excise taxes on beer affect only beer prices. The second last column of table 2 shows estimates for a regression only for those states that exempt food (a likely substitute of beer) from sales tax, demonstrating that even in this restricted sample beer consumption is quite insensitive to sales tax. Finally, the last column addresses the potential concern that people might be substituting toward other alcoholic beverages when they face a beer tax increase, and not when they face a sales tax increase. As we can see, the share of ethanol people consume in the form of beer is quite insensitive to either tax rate.

We repeat the exercise for Goldin and Homonoff (2013), who have a similar set-up with individual-level, cross-sectional data on cigarette consumption. Even though this is not aggregate data, estimating a linear model that only measures average effects effectively leaves the analysis of section 4 unchanged. We again follow the original authors of the paper when we estimate the equation:

$$c_{ist} = \alpha + \beta\tau^e_{st} + \tilde{\beta}\tau^s_{st} + \gamma X_{st} + \delta Z_{ist} + \varepsilon_{ist}$$

where now c_{ist} stands for tobacco consumption, in average cigarettes per day, for individual i from state s in period t, τ^e, τ^s, and X_{st} should be interpreted as before, and Z_{ist} is a vector of individual-level controls. All the details can be found in the original paper. Results in table 3 showcase a number of different specifications, including several sets of fixed-effects, all following Goldin and Homonoff (2013).

	Baseline	Business cycle	Alcohol regulations	Region trends
Δ(excise tax)	-0.966	-0.875	-0.808	-0.715
	(0.4)	(0.393)	(0.394)	(0.394)
Δ(sales tax)	-0.305	-0.113	-0.114	-0.241
	(0.708)	(0.698)	(0.699)	(.7)
Δ(population)	-0.0002	-.0002	-0.0001	-0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Δ(income per cap.)		0.0002	0.0001	0.0002
		(0.00006)	(0.00006)	(0.00006)
Δ(unemployment)		-.094	-0.093	-0.093
		(.026)	(0.026)	(0.026)
Alcohol reg. controls			X	X
Year FE	X	X	X	X
Region FE				X
$\hat{m}$	0.316	0.129	0.141	0.338
	(0.743)	(0.8)	(0.866)	(0.996)
Sample size	1,607	1,487	1,487	1,487

Table 1: Estimating $\hat{m}$ with several sets of controls, following the specifications in CLK (2009) in the context of a linear model. Standard errors in parentheses.

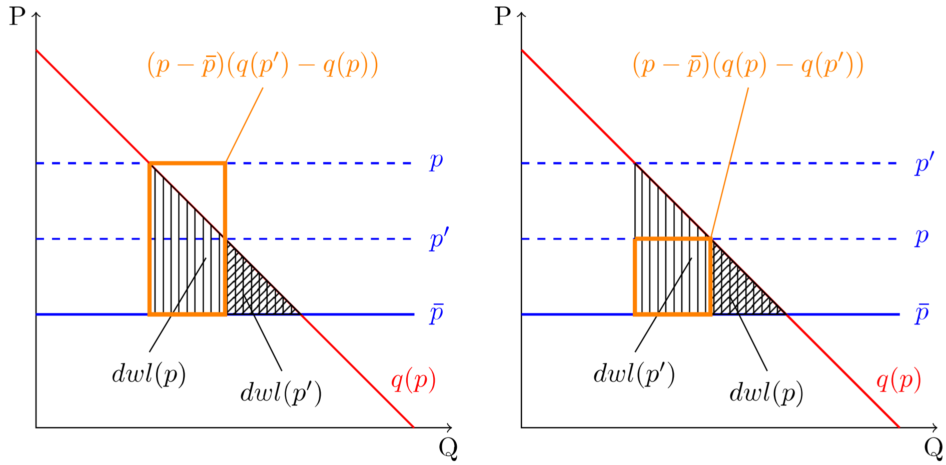
	Policy IV for excise tax	3-Year differences	Food exempt	Dep. var.: share of ethanol from beer
Δ(excise tax)	-0.808	-2.092	-1.114	0.036
	(0.395)	(0.897)	(1.174)	(0.006)
Δ(sales tax)	-0.114	-0.131	-0.449	0.018
	(0.699)	(0.826)	(0.757)	(0.011)
Δ(population)	-0.0001	-0.002	-0.00007	0.0000
	(0.0002)	(0.002)	(.0002)	(0.0000)
Δ(income per cap.)	0.0001	0.0002	0.0001	-0.0000
	(.00006)	(0.00007)	(0.00007)	(0.0000)
Δ(unemployment)	-0.094	-0.03	-0.056	-0.0001
	(.026)	(0.028)	(.032)	(0.0004)
Alcohol reg. controls	X	X	X	X
Year FE	X	X	X	X
$\hat{m}$	0.141	0.062	.403	
	(0.866)	(0.395)	(0.819)	
Sample size	1,487	1,389	937	1,487

Table 2: Estimating $\hat{m}$ following the strategy of CLK (2009) in the context of a linear model. As in CLK, we use the nominal excise tax rate divided by the average price of a case of beer from 1970 to 2003 as an IV for excise tax to eliminate tax-rate variation coming from inflation erosion. Next, we run the same regression in 3-year differences. Next, we run it only for states where food is exempt from sales-tax, to address concerns about whether consumers react differently to changes in the two taxes only because sales taxes apply to a broad set of goods. Finally, the last column addresses the concern that beer taxes may induce substitution with other alcoholic products, biasing the coefficient on excise tax relative to the one on sales tax. While in the log-log specification of CLK (2009) it seems to show that beer excise taxes have no discernable effect on the share of ethanol consumed from beer, we do find a significant effect. Standard errors in parentheses.

	Outcome variable: Number of cigarettes		
Specification	1	2	3
Excise Tax	-0.015	-0.015	-0.016
	(.004)	(.004)	(.004)
Sales Tax	-0.024	-0.02	-0.022
	(0.022)	(0.025)	(0.025)
Demographic controls	X	X	X
Econ. conditions controls		X	X
Income trend controls			X
State,year, and month FE	X	X	X
$\hat{m}$	1.57	1.33	1.37
	(1.65)	(1.83)	(1.82)
Sample size	274,138	274,138	274,138

Table 3: Estimating $\hat{m}$ based on the intensive response of cigarette consumption to sales taxes (not included in sticker price) and excise taxes (included in the sticker price). The specifications are a linearized version of the specifications in Goldin and Homonoff (2013). Standard errors in parentheses.

C Appedix Figures



(a) $p > p'$ (b) $p' > p$

Figure 5: A graphical illustration of Lemma 3. As long as demand is weakly decreasing, $dwl(p')$ cannot be smaller than $dwl(p)$ minus (plus) the orange rectangle.

Data Availability Statement: The data that support the findings of this study are openly available on the American Economic Association's website at http://doi.org/10.1257/aer.99.4.1145, reference number 10.1257/aer.99.4.1145, for the data in CLK (2009), and at http://doi.org/10.1257/pol.5.1.302, reference number 10.1257/pol.5.1.302, for the data in Goldin and Homonoff (2013).