Altruism and Public Good Provision

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PFFLS

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Introduction

- Public good provision old problem in economics
- At first blush, altruism would seem to alleviate the problem
- Q: But does it? And if so, in what sense?
  - A: It depends on how you define social welfare function

Related Literature

- **Public goods** Samuelson (1954), Foley (1970)
- **Altruism and neutrality** Barro (1974), Bergstrom, Blume and Varian (1985), Bagwell and Bernheim (1988), Andreoni (1990)
- **Mech design** Vickrey (1961), Clarke (1971), Groves (1973), Mezzetti (2004)

Does altruism solve PGP?

Suppose

$$V_1 = u^1(x_1, G) + u^2(x_2, G)$$
$$V_2 = u^2(x_2, G)$$

Where G is the public good

$$G = g_1 + g_2$$

and the production techonology is linear

$$x_i + g_i = y_i$$

Does altruism solve PGP?

Assuming interior solutions under private provision, each agent maximizes their own utility, taking g_{-i} as given, resulting in FOC's

$$u^1_{x_1} = u^1_G + u^2_G$$
$$u^2_{x_2} = u^2_G$$

For agents 1 and 2 respectively

Does altruism solve PGP?

Social planner solves:

$$
\begin{aligned}
& max_{x_1, x_2, G} \sum_i V_i \\
\text{s.t. } & x_1 + x_2 + G = y_1 + y_2 \\
& x_1 \geq 0 \\
& x_2 \geq 0 \\
& G \geq 0
\end{aligned}
$$

FOC:

$$
\begin{aligned}
u^1_{x_1} &= u^1_G + 2u^2_G \\
u^2_{x_2} &= 0.5u^1_G + u^2_G
\end{aligned}
$$

Does altruism solve PGP?

Assuming an interior solution:

Social planner FOC:

$$u^1_{x_1} = u^1_G + 2u^2_G$$
$$u^2_{x_2} = 0.5u^1_G + u^2_G$$

Private provision FOC:

$$u^1_{x_1} = u^1_G + u^2_G$$
$$u^2_{x_2} = u^2_G$$

Does altruism solve PGP?

Assuming an interior solution:

Social planner FOC:

$$u^1_{x_1} = u^1_G + 2u^2_G$$
$$u^2_{x_2} = 0.5u^1_G + u^2_G$$

Private provision FOC:

$$u^1_{x_1} = u^1_G + u^2_G$$
$$u^2_{x_2} = u^2_G$$

$\rightarrow$ Even altruistic agent underprovides g_1

"Perfect" Altruism

Glimmer of hope: the perfectly altruistic society *can* have perfect public good provision:

- Players $\{1, 2, ..., n\}$
- Each player picks $x_i \geq 0$, $g_i \geq 0$ to maximize utility

$$v_i = \sum_{j=1}^{n} u_j(G, x_j; \theta_j)$$

- Social welfare: $\sum_{i=1}^{n} \sum_{j=1}^{n} u_j(G, x_j; \theta_j) = nv_i$ for any i
- If everyone is picking efficient contribution g^*_{-i}, then it is optimal to pick efficient contribution g^*_i

Groves-Clarke Mechanisms

- Can generically solve public good provision problems by setting appropriate transfers à la Groves-Clarke
- Problem: generically require massive transfers from the outside (see, e.g. Williams (1999))
- Could it be that altruism alleviates this problem?

"Imperfect" altruism

Restrict attention to the quasilinear environment with linear values:

$$u_i(G, x_i; \theta_i) = \theta_i G + x_i$$
$$G \in [0, 1]$$

Agents' utility:

$$\underbrace{\theta_i G + \sum_{j \neq i} \alpha \theta_j G}_{v_i} + x_i$$

With $0 \leq \alpha \leq 1$. For now, define SWF as sum of utilities from consumption of the public good, so that efficient rule is:

$$G^*(\theta) = \mathbb{I}(\Sigma_i \theta_i \geq 0)$$

Idea is that θ's are private information, and we want to construct a mechanism that implements the efficient rule.

“Imperfect” altruism

Transferring their externality to each individual induces a truth-telling equilibrium in an ex-post sense:

$$
\begin{aligned}
x_i &= G^*(\hat{\theta}_i, \theta_{-i})\Big(\hat{\theta}_i + \sum_{j \neq i} \theta_j\Big) - v_i(\hat{\theta}_i, \theta_{-i}) \\
&= (1-\alpha) G^*(\hat{\theta}_i, \theta_{-i}) \sum_{j \neq i} \theta_j
\end{aligned}
$$

Knowing this, individuals now implicitly optimize the SWF:

$$
v_i = G^*(\hat{\theta}_i, \theta_{-i}) \sum_{j=1}^{n} \theta_j
$$

- Truth-telling no longer dominant strategy
- But, if everyone else is telling the truth, optimal to truthfully report

“Imperfect” altruism

$$x_i = (1-\alpha)G^*(\hat{\theta}_i, \theta_{-i}) \sum_{j \neq i} \theta_j$$

- If $\alpha = 1$ (perfect altruism) then $x_i = 0 \ \forall i$
- Expected transfer decreasing in α as long as there is some positive probability the project is undertaken

Heterogeneous altruism

- Similar story if $\alpha_{ij} \in [0, 1]$ heterogeneous, as long as they are common knowledge
- In the truth-telling equilibrium, the expected average transfer is decreasing in altruism

$$\mathbb{E}_\theta\left[\frac{1}{n}\sum_i x_i\right] = \left((n-1) - \underbrace{\frac{1}{n}\sum_i \sum_{j\neq i} \alpha_{ij}}_{\text{Average altruism}}\right)\mathbb{E}_\theta[G^*(\theta)\theta_j]$$

Altruistic SWF

- Things get more complicated if SWF takes into account altruistic components of utility

$$G^*(\theta) = \mathbb{I}[\Sigma_i \Sigma_j \alpha_{ij} \theta_j \geq 0]$$

- Where $\alpha_{ii} = 1 \ \forall i$, and $\alpha_{ij} \leq 1 \ \forall i, j$
- Clarke-Groves transfers no longer induce truth-telling

No Truth Telling with Altruistic SWF

- With altruistic SWF, an individual i's externality is

$$G^*(\theta) \sum_{k \neq i} \sum_{j=1}^{n} \alpha_{kj} \theta_j$$

- Think of $n = 2$ case, with $\alpha_{12} > 0$

$$x_2(\theta_1, \hat{\theta}_2) = G^*(\theta_1, \hat{\theta}_2)(\theta_1 + \alpha_{12}\hat{\theta}_2)$$

- Want to over-report $\hat{\theta}_2$, as it will result in a bigger transfer
- Groves mechanisms only work in general with private values
- Here values clearly interdependent

Mezzetti (2004)

- Two-stage Groves mechanism can implement efficient rule even in the case of interdependent values
- First stage: report type $\hat{\theta}_i$

$\rightarrow$ use reports to implement rule $G^*(\hat{\theta})$

- Second stage: report your realized utility $\hat{v}_i$

$\rightarrow$ use reports to construct transfers $x_i = \sum_{j \neq i} \hat{v}_j$

- Perfect Bayesian implementation

Altruistic SWF

Now can handle altruistic SWF:

- Second stage: truthfully reporting $\hat{v}_i = v_i$ is optimal as it does not affect your own payment
- First stage: given transfers in second stage, and assuming everyone else is telling the truth, agent i picks $\hat{\theta}_i$ to maximize

$$G^*(\hat{\theta}_i, \theta_{-i})\left(\sum_{j=1}^{n} \alpha_{ij}\theta_j + \sum_{k \neq i}\left(\alpha_{ki}\theta_i + \sum_{j \neq i}\alpha_{kj}\theta_j\right)\right)$$

- Maximized at $\hat{\theta}_i = \theta_i$, since G^* is the efficient rule

Altruism makes things worse?

- Note that now more altruism results in *larger* payments!

$$\mathbb{E}_\theta[x_i] = \sum_{j \neq i} \mathbb{E}_\theta[\hat{v}_j]$$
$$= \mathbb{E}_\theta[\theta_k] \sum_{j \neq i} \sum_{k=1}^{n} \alpha_{jk}$$

- Larger externality the more *other* people are altruistic

Privately altruistic

Can also handle α_{ij} as private information:

- Simply define private "type" as including $\boldsymbol{\alpha}_i = \{\alpha_{ij}\}_{j \neq i}$
- Second stage: report $\hat{v}_i = v_i$
- First stage: report $\hat{\theta}_i, \boldsymbol{\alpha}_i$ to maximize

$$G(\hat{\theta}_i, \hat{\boldsymbol{\alpha}}_i, \theta_{-i}, \boldsymbol{\alpha}_{-i}) \left(\sum_{j=1}^{n} \alpha_{ij}\theta_j + \sum_{k \neq i} \left(\alpha_{ki}\theta_i + \sum_{j \neq i} \alpha_{kj}\theta_j \right) \right)$$

- Again, truthful reporting optimal if everyone else is truthfully reporting
- Again, more altruism increases payments

Higher transfers a "bad" thing?

- Not necessarily! Budget balancing, individually rational mechanism still possible as long as

$$\sum_{i=1}^{n} \bar{v}_i(\underline{\theta}_i, \underline{\boldsymbol{\alpha}}_i) - v_i^0(\underline{\theta}_i, \underline{\boldsymbol{\alpha}}_i) \leq 0$$

- Where $\bar{v}_i(\underline{\theta}_i, \underline{\boldsymbol{\alpha}}_i)$ is the expected utility obtained for the mechanism for an agent of the "unluckiest" type $\underline{\theta}_i, \underline{\boldsymbol{\alpha}}_i$
- $v_i^0(\underline{\theta}_i, \underline{\boldsymbol{\alpha}}_i)$ is the utility from not participating in the mechanism at all
- Note that if individuals "truly" altruistic, value of outside option depends on things that happen *in* the mechanism

Concluding remarks

- Depending on objective, altruism can make things better or worse for public good provision
- Thinking about cases were this is relevant
- Ethnic diversity and public good provision
  - Altruism occasionally advanced as possible explanation
  - But should it? And
- Altruism vs. social pressure?
  - DellaVigna et al. (2012): social pressure important
  - Which one better for public good provision?
  - Welfare effects?