I. Functions

$$\operatorname{rect}(t) = \mathbb{1}\left(|t| < \frac{1}{2}\right) \tag{1}$$

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \tag{2}$$

(3)

II. Identities

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \tag{4}$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \tag{5}$$

$$\cos(a)\sin(b) = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right] \tag{6}$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right] \tag{7}$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \tag{8}$$

(9)

III. Fourier Transform

$$x(t) \qquad \int_{-\infty}^{\infty} x(t) e^{-2\pi j f t}\,\mathrm{d}t \tag{10}$$

$$\operatorname{sinc}\left(\frac{t}{T_s}\right) \qquad \frac{1}{F_s}\operatorname{rect}\left(\frac{f}{F_s}\right) \tag{11}$$

$$\operatorname{rect}\left(\frac{t}{T_s}\right) \qquad \frac{1}{F_s}\operatorname{sinc}\left(\frac{f}{F_s}\right) \tag{12}$$

$$x(\alpha t) \qquad \frac{1}{\alpha} X\left(\frac{f}{\alpha}\right) \tag{13}$$

$$\cos(2\pi f_0 t) \qquad \frac{1}{2}\left[\delta(f+f_0) + \delta(f-f_0)\right] \tag{14}$$

$$\sin(2\pi f_0 t) \qquad \frac{j}{2}\left[\delta(f+f_0) - \delta(f-f_0)\right] \tag{15}$$

IV. z-Transform

A. Upsampling/Downsampling

$$x\left[\left\lfloor \frac{n}{N} \right\rfloor\right] \qquad X(z^N) \tag{16}$$

$$x\left[nN\right] \qquad \sum_{m=0}^{N-1} \frac{1}{N} X(z^{\frac{1}{N}} e^{j2\pi\frac{m}{N}}) \tag{17}$$

V. DTFT

$$\cos(w_0 n + \phi) \tag{18}$$

$$\frac{1}{2}\left[e^{j\phi}\delta(w - w_0) + e^{-j\phi}\delta(w + w_0)\right]$$

$$\sin(w_0 n + \phi)$$

$$\frac{-j}{2}\left[e^{j\phi}\delta(w - w_0) - e^{-j\phi}\delta(w + w_0)\right] \tag{19}$$

(20)

VI. Sampling of Bandlimited Function

$$x[n] = \left\langle \operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right), x(\cdot) \right\rangle = T_s x(nT_s) \tag{21}$$

$$x(t) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} x[n]\operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right) \tag{22}$$

A. Bandpass Sampling

Signal support $|f| \in [f_0, f_1]$. Minimum sampling frequency

$$F_s \geq 2\frac{f_1}{\left\lfloor \frac{f_1}{f_1 - f_0} \right\rfloor} \tag{23}$$

VII. Lagrange Polynomials

$$L_n^{(N)}(t) = \prod_{k=-N, k\neq n}^{N} \frac{t-k}{k-n}, \quad n = -N, \ldots, N \tag{24}$$

Property:

$$L_n^{(N)} = \begin{cases} 1 & t = n \\ 0 & t \in \mathbf{Z} \setminus n \end{cases}$$