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CSC505, Homework 2, Question 1

Thesis: We want to show that $T(n) \leq kn^2$ for some k. Proving this will be sufficient to show that $T(n) \in O(n^2)$ where $T(n) = 5T(n/3) + \Theta(n^2) \in O(n^2)$

Base Case: As a base case, when n = 1, we will assume there would be no recurrence and therefore we can see that $T(1) \leq cn^2 \leq kn^2$ as long as $k \geq c$.
Inductive Case: We need to show that if $T\left(\frac{n}{3}\right) \leq \frac{kn^2}{9}$ then $T(n) \leq kn^2$. Since if we can show this, we can build from the base case we have already proven and show that the relation holds for any arbitrary n. First substitute our assumed relation into the relation we wish to prove.

$$T(n) = 5T\left(\frac{n}{3}\right) + \mathrm{cn}^2 \leq kn^2$$
$$T(n) \leq \frac{5kn^2}{9} + cn^2 \quad \leq kn^2$$

As long as the above result remains below kn^2, then we have a successful proof. Next we will deduce a constant where this relation holds:

$$\frac{5kn^2}{9} + cn^2 \leq kn^2 \Rightarrow \ k \geq \frac{9c}{4}$$

Therefor the relation will hold as long as $k \geq 9c/4$. I will validate with $k = 9c$:

$$T(n) \leq 5T\left(\frac{n}{3}\right) + cn^2$$
$$T(n) \leq \frac{45cn^2}{9} + cn^2$$
$$T(n) \leq 6cn^2 \leq 9cn^2$$
$$T(n) \leq kn^2$$

Thus proof by induction:

$$T(n) \in O(n^2)$$