CSC/ECE/OR 579:
Introduction to Computer Performance Evaluation
Homework #3.

Question 1.

Given the stochastic transition probability matrix

$$P = \begin{pmatrix} 0.2 & 0.0 & 0.005 & 0.795 & 0.0 \\ 0.0 & 0.0 & 0.998 & 0.002 & 0.0 \\ 0.002 & 0.0 & 0.0 & 0.0 & 0.998 \\ 0.8 & 0.001 & 0.0 & 0.198 & 0.001 \\ 0.0 & 0.998 & 0.0 & 0.002 & 0.0 \end{pmatrix},$$

provide a Harwell-Boeing compact form for the corresponding transition-rate matrix, $Q^T = P^T - I$, as it might be used, for example, in the method of Gauss-Seidel.

Question 2.

Consider the following stochastic transition probability matrix, provided in the Harwell-Boeing compact storage format.

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.4  .6  .4  .4  .2  .2  .8
 3   1   2   1   3   3   2
 1   3   6   8
```

Use the Gauss-Seidel iterative method and an initial approximation $x^{(0)} = (0, 1, 0)^T$ to compute $x^{(1)}$ and $x^{(2)}$.

Question 3.

Customers arrive at a service center according to a Poisson process with a mean interarrival time of 15 minutes.

(a) What is the probability that no arrivals occur in the first half hour?

(b) What is the expected time until the tenth arrival occurs?

(c) What is the probability of more than five arrivals occurring in any half-hour period?

(d) If two customers were observed to have arrived in the first hour, what is the probability that both arrived in the last 10 minutes of that hour?

(e) If two customers were observed to have arrived in the first hour, what is the probability that at least one arrived in the last 10 minutes of that hour?

Question 4.

The single server at a service center appears to be busy for four minutes out of every five, on average. Also the mean service time has been observed to be equal to half a minute. If the time spent waiting for service to begin is equal to 2.5 minutes, what is the mean number of customers in the queueing system and the mean response time?

Question 5.

Each evening Bin Peng, a graduate teaching assistant, works the takeaway counter at Goodberry's frozen yogurt parlor. Arrivals to the counter appear to follow a Poisson distribution with mean of ten per hour. Each customer is served one at a time by Bin and the service time appears to follow an exponential distribution with a mean service time of four minutes.

(a) What is the probability of having a queue?

(b) What is the average queue length?

(c) What is the average time a customer spends in the system?

(d) How fast, on average, does Bin need to serve customers in order for the average total time a customer spends in Goodberry's to be less than 7.5 minutes?

(e) If Bin can spend his idle time grading papers and if he can grade, on average, 24 papers an hour, how many papers per hour can he average while working at Goodberry's?

Question 6.

Consider a barber's shop with a single barber who takes on average 15 minutes to cut a client's hair. Model this situation as an $M/M/1$ queue and find the largest number of incoming clients per hour that can be handled so that the average waiting time will be less than 12 minutes.

Question 7.

Customers arrive at Bunkey's car wash service at a rate of one every 20 minutes and the average time it takes for a car to proceed through their single wash station is 8 minutes. Answer the following questions under the assumption of Poisson arrivals and exponential service.

(a) What is the probability that an arriving customer will have to wait?

(b) What is the average number of cars waiting to begin their wash?

(c) What is the probability that there are more than five cars altogether?

(d) What is the probability that a customer will spend more than 12 minutes actually waiting before her car begins to be washed?

(e) Bunkey is looking to expand and he can justify the creation of a second wash station so that two cars can be washed simultaneously if the average time spent waiting prior to service exceeds 8 minutes. How much does the arrival rate have to increase in order for Bunkey to justify installing this second wash station?