CSC505 – HW 1
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1.) $T(n) = 2T(n/4) + \theta(\sqrt{n})$

Method 1 fails since $n^{1/2} = n^{log_4(2)}$, there can be no epsilon such that $n^{1/2} \in O(n^{log_4(2-\varepsilon)})$.
Method 2 holds:

$n^{1/2} \in O(n^{log_4(2)})$ since $n^{1/2} = n^{log_4(2)}$, and $0 \leq f(n) \leq cg(n)$ for all positive n and c > 1

also

$n^{1/2} \in \Omega(n^{log_4(2)})$ since $n^{1/2} = n^{log_4(2)}$, and $cg(n) \leq f(n) \leq 0$ for all positive n and c = 1.

Thus

$$T(n) \in \Theta(\, n^{log_4(2)} * log_2(n)\,)$$

2.) $T(n) = 9T(n/3) + \Theta(\mathrm{n})$
Consider:

$$n^{log_3(9)} = n^2$$

Let $\varepsilon = .5:$ Since $0 \leq n \leq cn^{1.5}$ for all n > 1 and c = 2
therefore $f(n) = n \in O(n^{log_3(9-\varepsilon)})$
and therefore $T(n) = \Theta(n^2)$

3.) $T(n) = 5T(n/2) + \Theta(n^2)$
Consider:

$$n^{log_2(5)} = n^{\left(\frac{\ln(5)}{\ln(2)}\right)} \approx n^{2.321}$$

Let $\varepsilon = .1$: Since $0 \leq n^2 \leq cn^{2.121}$ for all n > 1 and c = 2
Therefore $f(n) = n^2 \in O(n^{log_2(5-\varepsilon)})$
and therefore $T(n) = \Theta(n^{\left(\frac{\ln(5)}{\ln(2)}\right)})$

4.) $T(n) = 2T(n/2) + \Theta(n\sqrt{n})$
Consider

$$n^{log_2(2)} = n$$

Method 1 fails since $n\sqrt{n} \notin O(n)$ and therefore $n\sqrt{n} \notin O(n^{1-\varepsilon})$.
Method 2 fails also for the same reason $n\sqrt{n} \notin O(n)$and therefore $n\sqrt{n} \notin \Theta(n)$

Let $\varepsilon = .25$: Since $0 \leq cn^{1+.25} \leq n^{1.5}$ for all $n > 1$ when c = 1 we know $f(n) = n^{1.5} \in \Omega(n^{1+\varepsilon})$ when ε is .25.

Now consider:

$$2(n/2)^{1.5} = 2\left(\frac{n^{1.5}}{2^{1.5}}\right) = \left(\frac{2}{2\sqrt{2}}\right)n\sqrt{n} \leq 2n\sqrt{n}$$

There for when c = 2, the following is satisfied when c =2 and n > 1.

$$af\left({}^{n}/_{b}\right) \leq cf(n)$$

And therefore $T(n) \in \Theta(n\sqrt{n})$