CSC505 – HW1
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1.) $f(n) \in O\big(g(n)\big)$ if and only if there exists c, k such that $f(n) \leq cg(n)$ for all $n > k$.

Consider the following:

$$\begin{aligned}
&2n^2 + 5n\log_2(n) \\
&\leq 5n^2 + 5n\log_2(n) \\
&\leq 5n(\,n + \log(n)) \\
&\leq 5n(\,n + n)\ ^{(*)} \\
&\leq 10n^2
\end{aligned}$$

**This step true provided that n > 1*

Thus if c = 10 and n = 1 then $f(n) \leq cg(n)$ for all $n > k$ and therefor $f(n) \in O\big(g(n)\big)$

2.) The definition of $f(n) \in \omega\big(g(n)\big)$ then for any positive c there exists a k such that $0 \leq cg(n) < f(n)$ for all $n \geq k$. Since the definition of $f(n) \in \Omega\big(g(n)\big)$ is that there exists at least on c, k such that $0 \leq cg(n) \leq f(n)$ for all $n > k$.

Since $0 \leq cg(n) < f(n) \Rightarrow 0 \leq cg(n) \leq f(n)$ and by the first definition a k must exist for any positive c, there must be at least one c and k pair by the second definition and therefore

$f(n) \in \omega\big(g(n)\big) \Rightarrow f(n) \in \Omega\big(g(n)\big)$.