Homework 2
ENERGY 293

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1. $$I = I_L - I_0\left(e^{\frac{qV}{kT}} - 1\right)$$

$$P = IV = V\left[I_L - I_0\left(e^{\frac{qV}{kT}} - 1\right)\right]$$

$$V_{oc} = \frac{kT}{q}\ln\left(\frac{I_L}{I_0} + 1\right)$$

Maximum power when $\frac{dP}{dV} = 0$

$$\left.\frac{dP}{dV}\right|_{V_m} = V_m\left(-I_0\frac{q}{kT}e^{\frac{qV_m}{kT}}\right) + I_L - I_0\left(e^{\frac{qV_m}{kT}} - 1\right) = 0$$

$$\left(-I_0 V_m\frac{q}{kT} - I_0\right)e^{\frac{qV_m}{kT}} + I_L + I_0 = 0$$

$$-\left(V_m\frac{q}{kT} + 1\right)e^{\frac{qV_m}{kT}} + \frac{I_L}{I_0} + 1 = 0$$

$$\left(V_m\frac{q}{kT} + 1\right)e^{\frac{qV_m}{kT}} = \frac{I_L}{I_0} + 1$$

$$\ln\left(\frac{qV_m}{kT} + 1\right) + \frac{qV_m}{kT} = \ln\left(\frac{I_L}{I_0} + 1\right)$$

$$\frac{kT}{q}\ln\left(\frac{qV_m}{kT} + 1\right) + V_m = V_{oc}$$

This cannot be solved into elementary functions, but math software can easily find a numerical value for V_m. Once this has been calculated, it is easy enough to find the value of I_m:

$$I_m = I_L - I_0\left(e^{\frac{qV_m}{kT}} - 1\right)$$

From these results, it is apparent that there is a complicated relationship between V_m and V_{oc}, but V_m increases as V_{oc} increases. It is not possible to find the value of I_m without knowing the value of I_0.

2. $$V_D = V + IR_S$$

$$I_L = I + I_D$$

$$I = I_L - I_D = I_L - I_0\left(e^{\frac{qV_D}{kT}} - 1\right) = I_L - I_0\left(e^{\frac{q(V+IR_S)}{kT}} - 1\right)$$

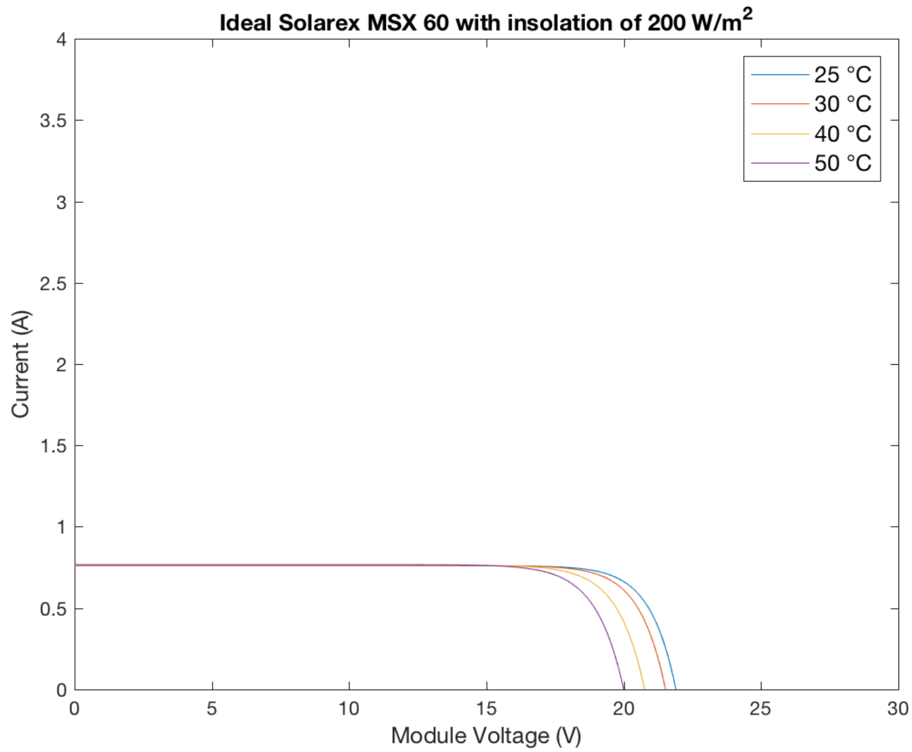
$$e^{\frac{q(V+IR_S)}{kT}} = \frac{I_L - I}{I_0} + 1$$

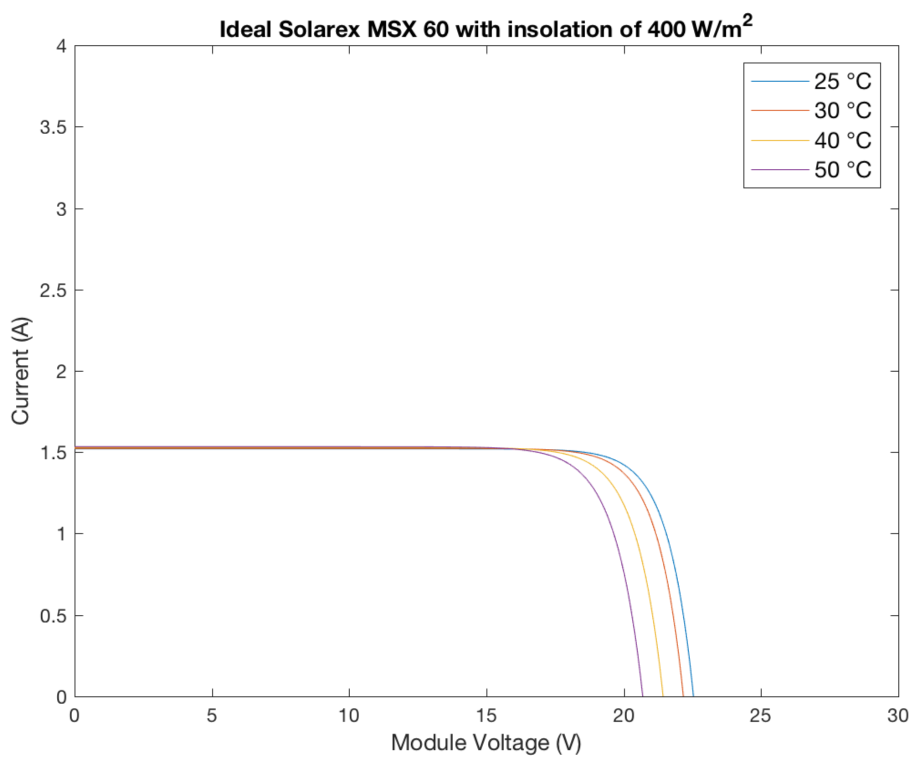
$$V + IR_S = \frac{kT}{q}\ln\left(\frac{I_L - I}{I_0} + 1\right)$$

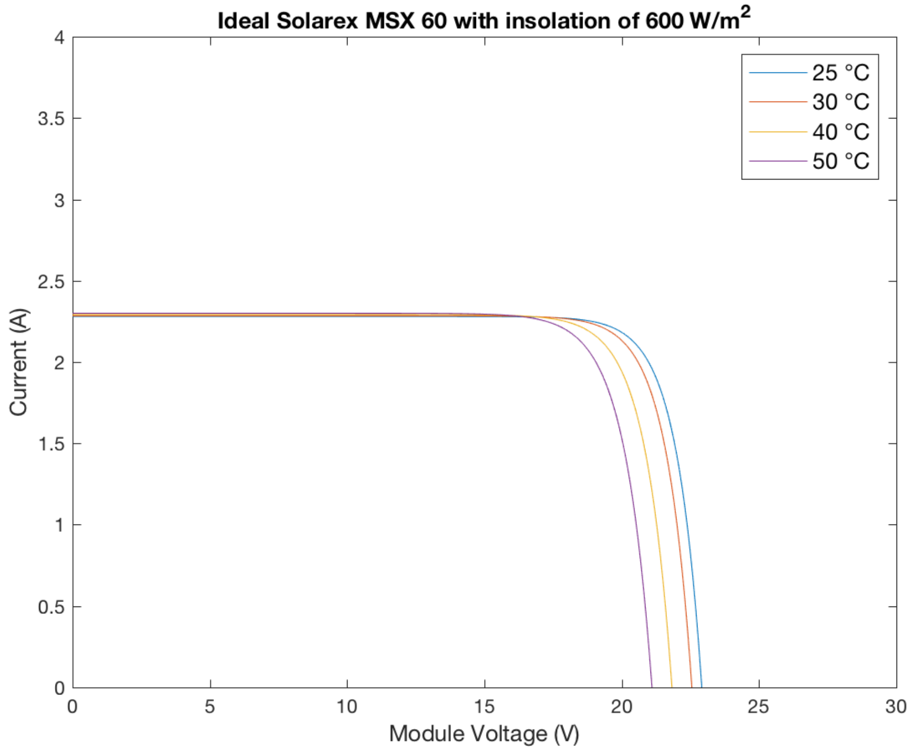
$$R_S = \frac{\frac{kT}{q}\ln\left(\frac{I_L - I}{I_0} + 1\right) - V}{I}$$

$$R_S = \frac{\frac{kT}{q}\ln\left(\frac{I_L - I_{sc}}{I_0} + 1\right)}{I_{sc}}$$

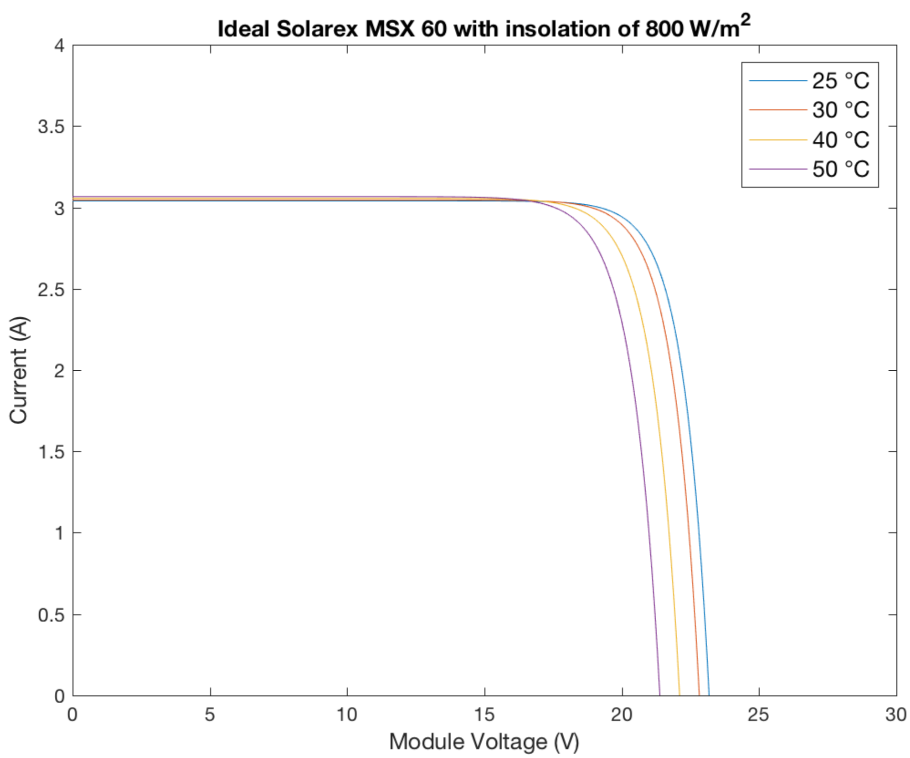
3. I-V plots for ideal solar cells at constant irradiance, varying temperature:



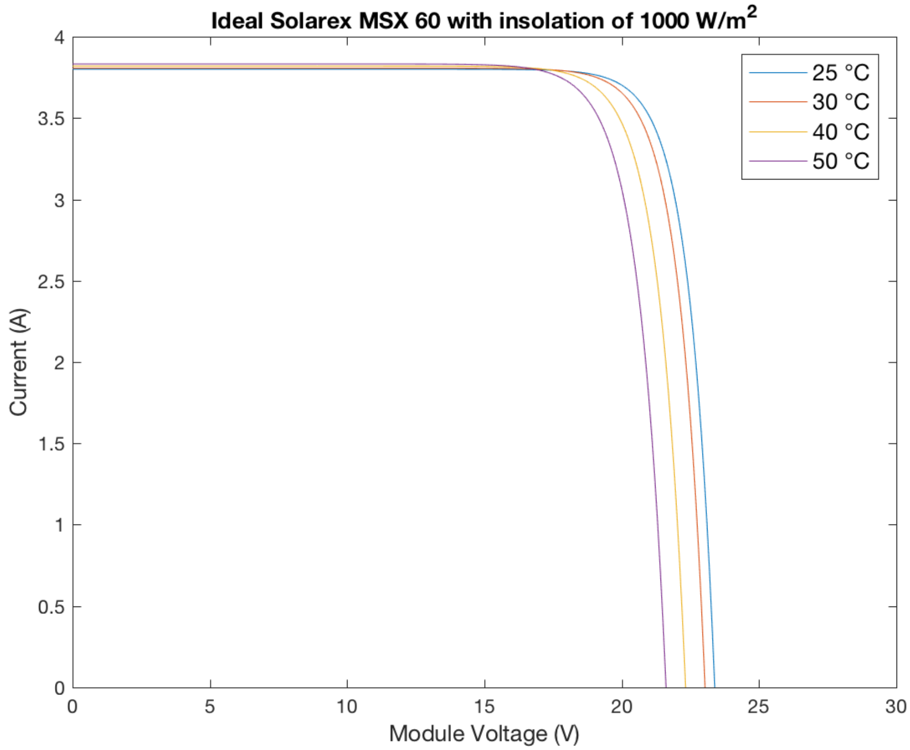




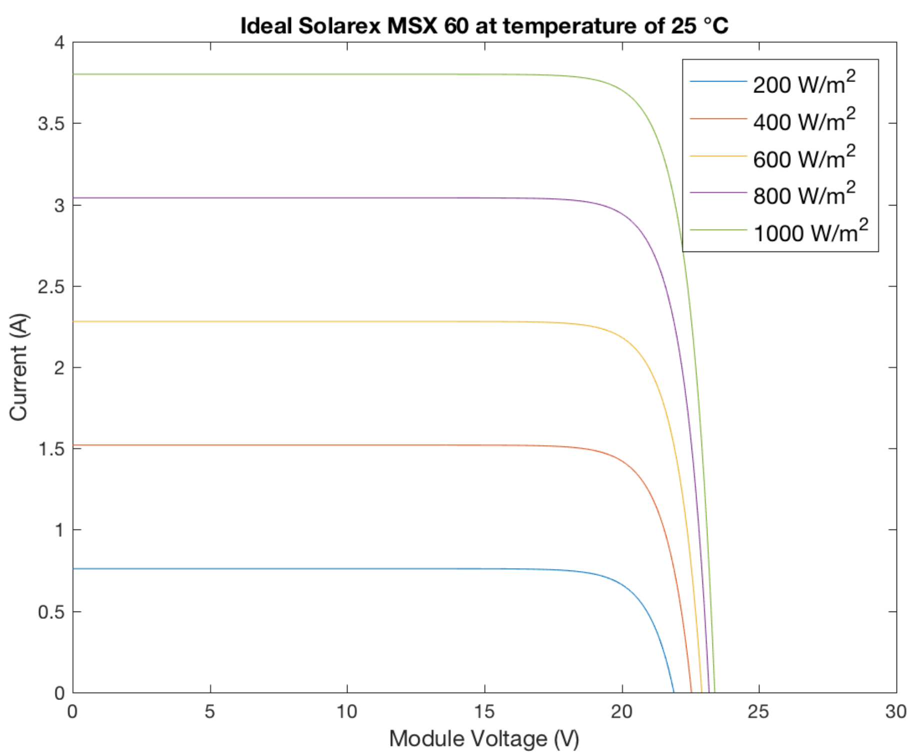
Ideal Solarex MSX 60 with insolation of 600 W/m2
Current (A)
Module Voltage (V)
25 °C
30 °C
40 °C
50 °C

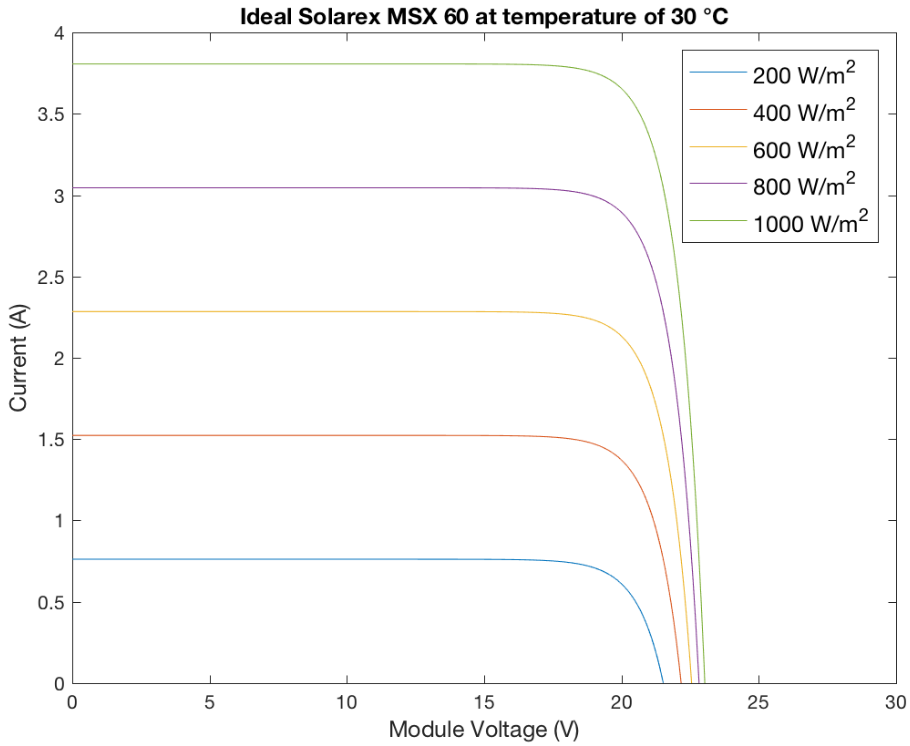


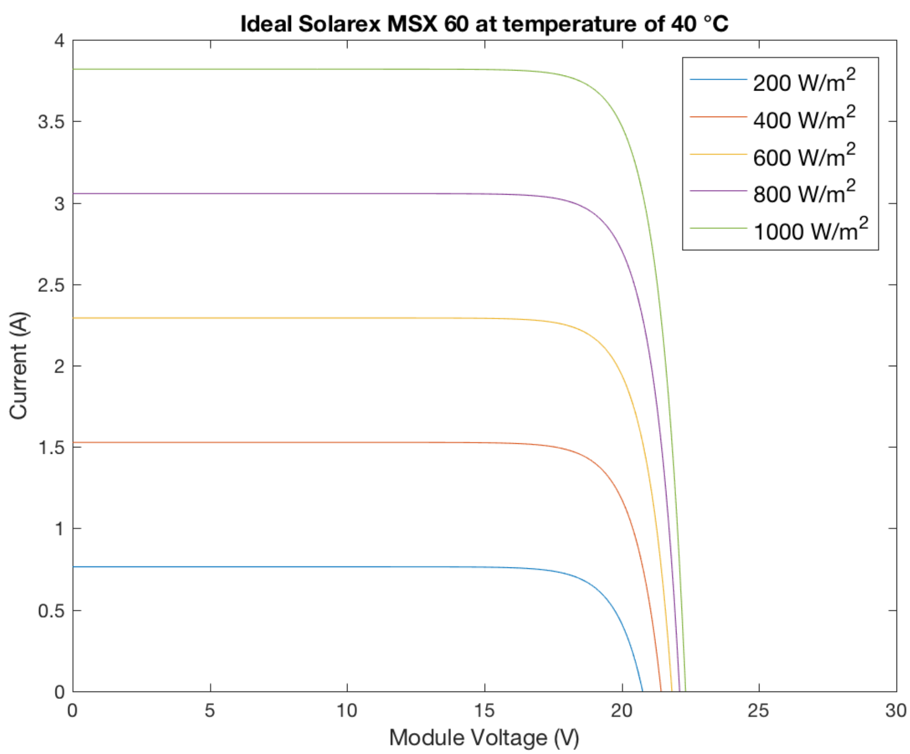
Ideal Solarex MSX 60 with insolation of 800 W/m2
Current (A)
Module Voltage (V)
25 °C
30 °C
40 °C
50 °C

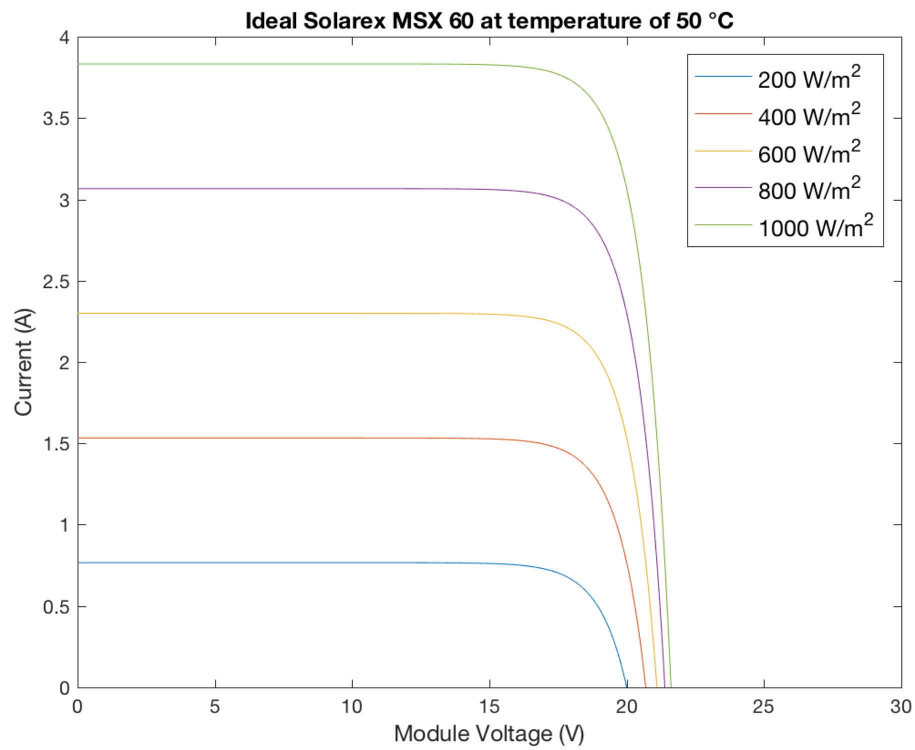


I-V plots for ideal solar cells at constant temperature, varying irradiance:

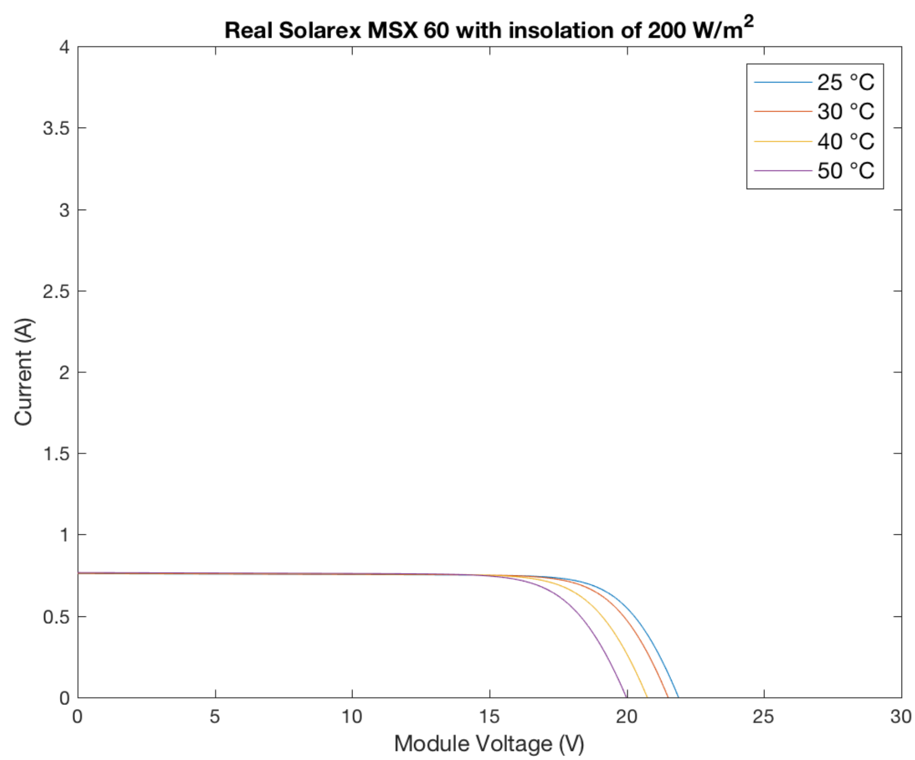


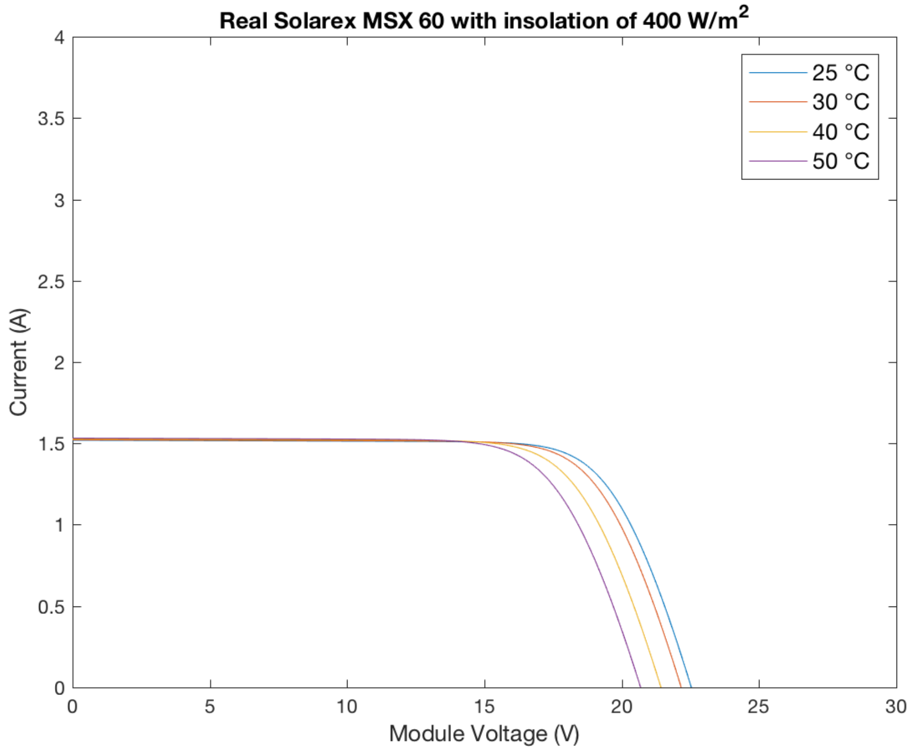

Ideal Solarex MSX 60 at temperature of 30 °C
200 W/m2
400 W/m2
600 W/m2
800 W/m2
1000 W/m2
Current (A)
Module Voltage (V)

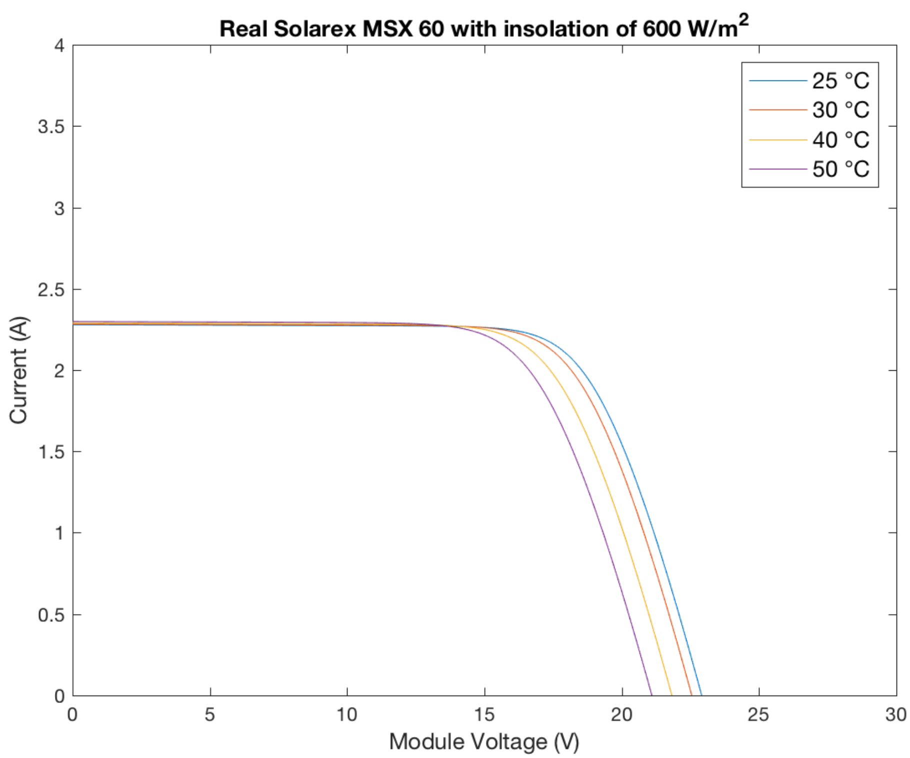

Ideal Solarex MSX 60 at temperature of 40 °C
200 W/m2
400 W/m2
600 W/m2
800 W/m2
1000 W/m2
Current (A)
Module Voltage (V)

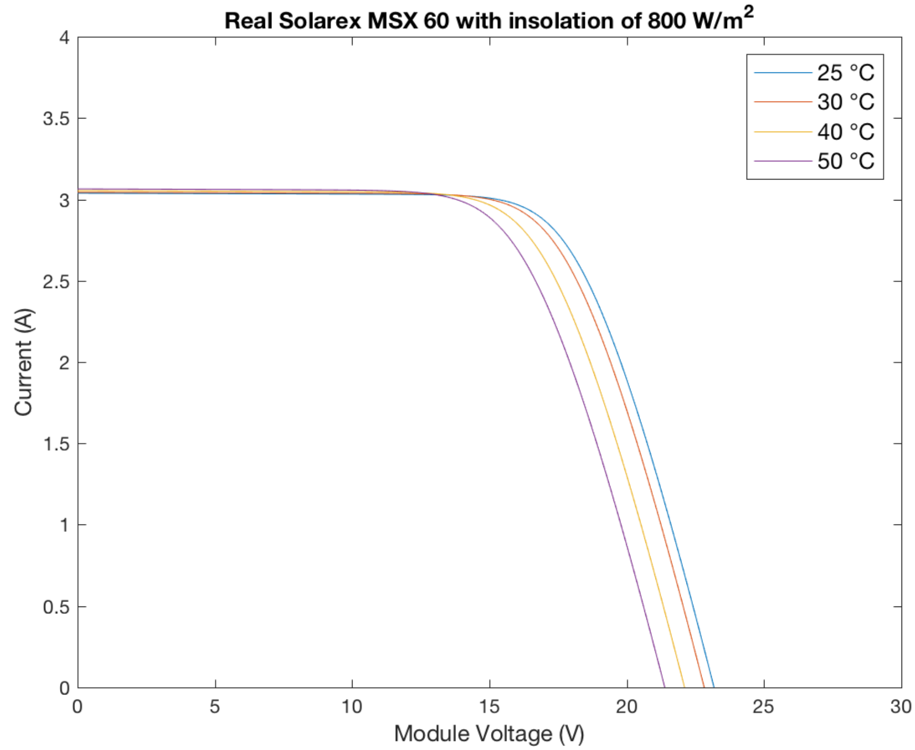


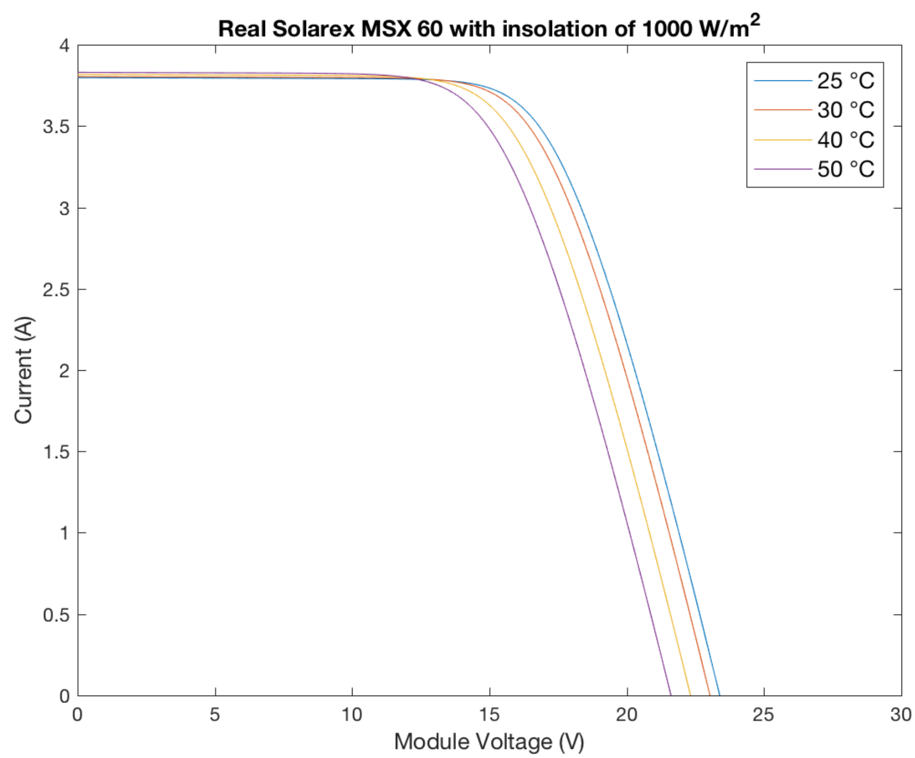
I-V plots for solar cells with R_s and R_{sh} at constant irradiance, varying temperature:



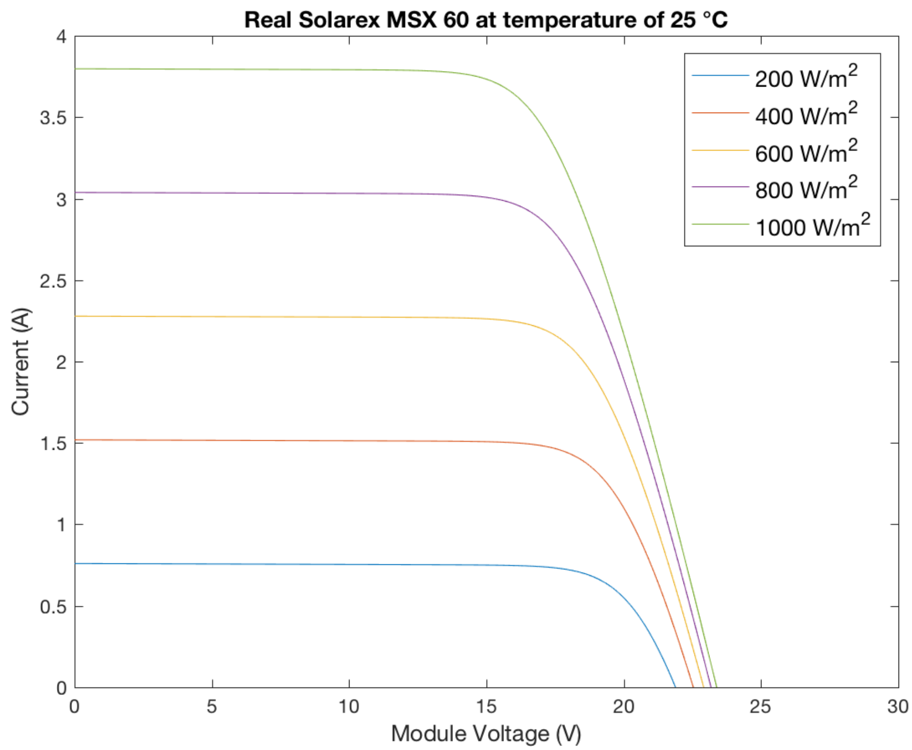

Real Solarex MSX 60 with insolation of 400 W/m2
Current (A)
Module Voltage (V)
25 °C
30 °C
40 °C
50 °C
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20
25
30

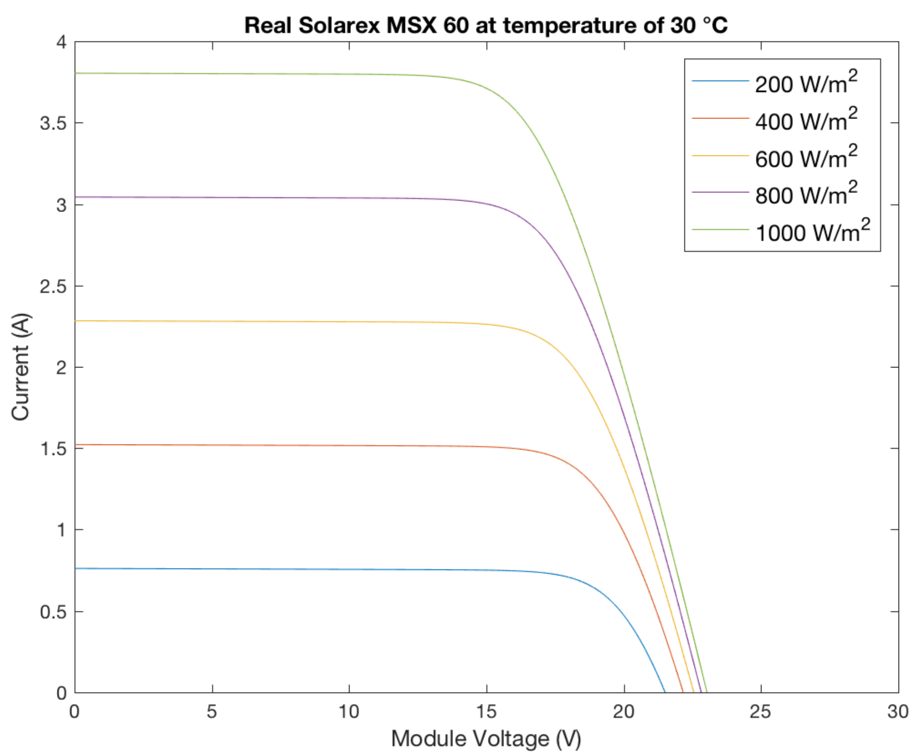

Real Solarex MSX 60 with insolation of 600 W/m2
Current (A)
Module Voltage (V)
25 °C
30 °C
40 °C
50 °C
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20
25
30

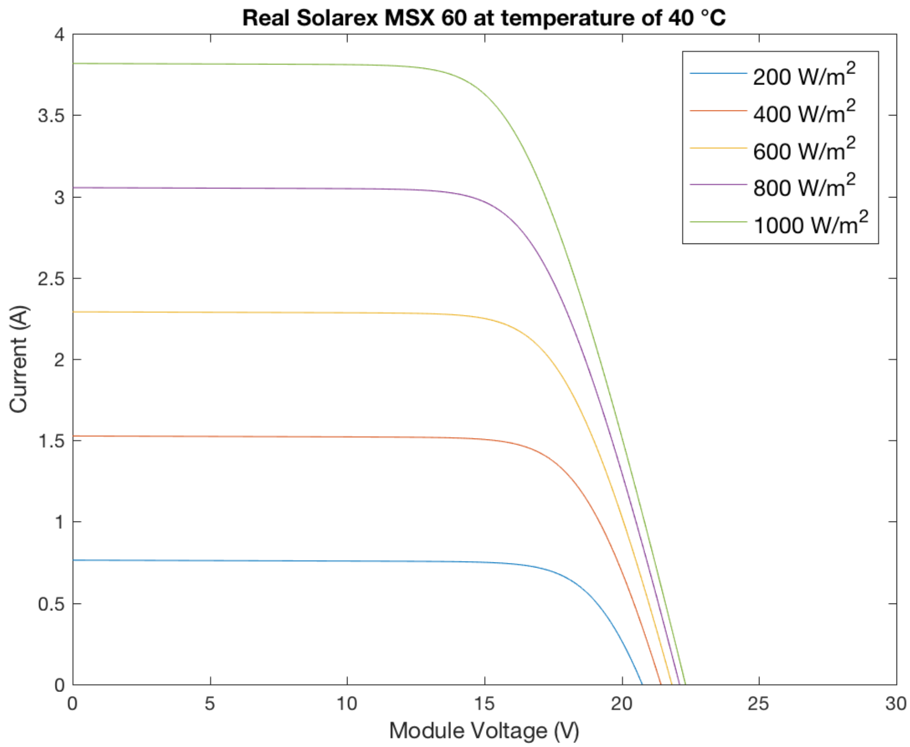

Real Solarex MSX 60 with insolation of 800 W/m2
Current (A)
Module Voltage (V)
25 °C
30 °C
40 °C
50 °C
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20
25
30

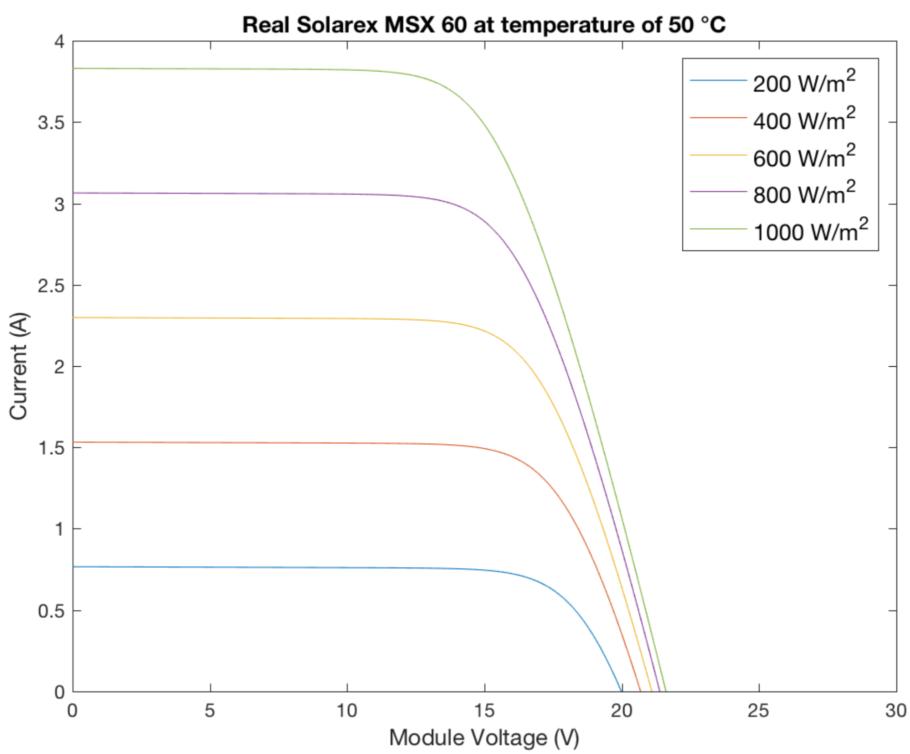

Real Solarex MSX 60 with insolation of 1000 W/m2
Current (A)
Module Voltage (V)
25 °C
30 °C
40 °C
50 °C
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20
25
30

I-V plots for solar cells with R_s and R_{sh} at constant temperature, varying irradiance:






Real Solarex MSX 60 at temperature of 40 °C
Current (A)
Module Voltage (V)
200 W/m2
400 W/m2
600 W/m2
800 W/m2
1000 W/m2


Real Solarex MSX 60 at temperature of 50 °C
Current (A)
Module Voltage (V)
200 W/m2
400 W/m2
600 W/m2
800 W/m2
1000 W/m2

Max power (in Watts) of an ideal solar module under each set of conditions:

		Irradiance (W/m^2)				
		200	400	600	800	1000
Temperature (°C)	25	13.8	28.6	43.6	59.0	74.5
	30	13.5	28.0	42.8	57.9	73.1
	40	13.0	26.9	41.2	55.7	70.4
	50	12.4	25.8	39.5	53.6	67.8

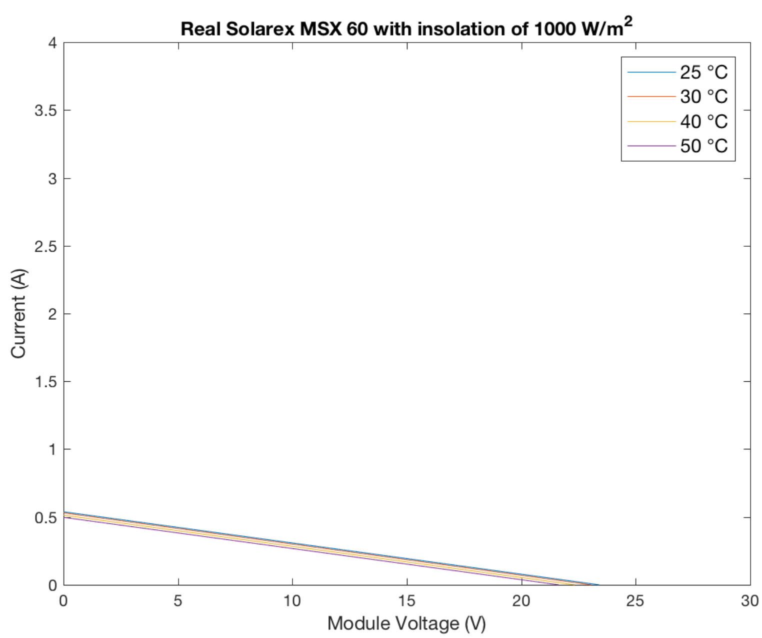
Max power (in Watts) of a solar module with R_s and R_{sh} under each set of conditions:

		Irradiance (W/m^2)				
		200	400	600	800	1000
Temperature (°C)	25	13.0	25.9	37.9	48.9	58.8
	30	12.7	25.3	37.0	47.8	57.5
	40	12.2	24.2	35.4	45.7	54.9
	50	11.6	23.1	33.8	43.5	52.3

From these results, a few observations are clear:

- Increasing temperature results in slightly increased short-circuit current, but decreased open-circuit voltage, and thus a decrease in maximum power.
- Increasing irradiance results in a proportional increase in short-circuit current, as well as a slight increase in open-circuit voltage, resulting in a more-than-proportional increase in maximum power.
- Adding in series and shunt resistances causes only negligible decreases in short-circuit current and open-circuit voltage. However, it can significantly decrease the voltage that maximum power occurs at, resulting in a significant decrease in maximum power. This effect is much more pronounced at higher irradiances than at lower irradiances. In fact, while V_m increases with irradiance for ideal cells, it decreases with irradiance for 'real' cells.

Note: these results assume that the given series resistance of $1.2\,\Omega$ is for the whole modues, i.e. that each individual cell only has an equivalent series resistance of 1/36 that value. If given resistance is the value for each individual cell, the following (clearly ineffective) result occurs:



Appendix: Matlab code

```
% close all plots and delete all existing variables to prevent conflicts
close all
clearvars

% Physical constants
q = 1.6 * 10^-19; % electron charge, C
k = 1.38 * 10^-23; % Boltzmann constant, J/K
Eq = 1.16; % silicon bandgap, eV

% Specifications
Pm = 60; % maximum power, W
Vm = 17.1; % voltage at maximum power, V
Im = 3.5; % current at maximum power, A
Isc = 3.8; % short circuit current, A
Voc = 21.1; % open circuit voltage, V
nSeries = 36; % number of cells in series

% Reference conditions
musc = 1.3 * 10^-3; % change in photocurrent with temperature, A/K
Tnom = 298; % temperature at STC, K
Gref = 1000; % irradiance at STC, W/m^2
I0ref = 4.0 * 10^-11; % reverse saturation current at STC, A
ILref = Isc; % light induced current at STC, A

% Irradiances and temperatures considered
Grange = [200,400,600,800,1000];
Trange = [25,30,40,50];

% Set basic instructions
options = optimoptions('fsolve','Display','none'); % solver options
step = 0.1; % step size in plots
len = ceil(24/step); % number of data points in plots (up to 24 V)
fsize = 12; % font size in plots

% Set up arrays to hold results
% Results at each voltage
Videal = zeros(20,len);
Iideal = zeros(20,len);
Pideal = zeros(20,len);
% Specific module parameters
VocIdeal = zeros(4,5);
IscIdeal = zeros(4,5);
PmIdeal = zeros(4,5);
VmIdeal = zeros(4,5);
ImIdeal = zeros(4,5);

% Make calculations assuming ideal solar cells
for g = 1:5
figure
hold on
G = Grange(g); % choose desired irradiance
    for t = 1:4
```

```
    T = Trange(t) + 273; % choose desired temperature, and conver to K
    % Calculate I0 and IL at given irradiance and temperature
    I0 = I0ref * (T / Tnom)^3 * exp(q * Eg / k * (1 / Tnom - 1 / T));
    IL = G/Gref * (ILref + musc * (T-Tnom));
    % Calculate Vt at given temperature (for clarity and faster
    % calculations)
    Vt = k * T / q;

    % Calculate Voc, and determine number of points to go one point
    % past it
    eqn1 = @(V) IL - I0 * (exp(V/nSeries / Vt)-1);
    VocIdeal(t,g) = fsolve(eqn1,20,options);
    ilim = ceil(VocIdeal(t,g)/step)+1;

    % Cycle through each module voltage, converting to cell voltage and
    % calculating current from that. Power is calculated by multiplying
    % current and module voltage. The method used to store results
    % means that each plot must have the same number of points;
    % however, it is a waste of time to calculate data beyond the Voc.
    % As a result, P and V values beyond the first point above Voc are
    % left as zero, while I values are set to -1 to avoid showing up on
    % the plot.
    for i = 1:len
        if i <= ilim
            Vmodule = (i-1)*step;
            Vc = Vmodule / nSeries; % cell voltage
            Videal(4*(g-1)+t,i) = Vmodule;
            Iideal(4*(g-1)+t,i) = IL - I0 * (exp(Vc / Vt)-1);
            Pideal(4*(g-1)+t,i)=Iideal(4*(g-1)+t,i)*Videal(4*(g-1)+t,i);
        else
            Iideal(4*(g-1)+t,i) = -1;
        end
    end
    % Add results to plot
    plot(Videal(4*(g-1)+t,:),Iideal(4*(g-1)+t,:))
    % Short circuit current is current at V = 0
    IscIdeal(t,g) = Iideal(4*(g-1)+t,1);
    % Max power is found by finding highest value of power in array.
    % Since it is only calculated at every 0.1 V, it is moderately less
    % precise than solving for the maximum, but the added precision is
    % not worth the complexity, particulary for the 'real' module.
    [PmIdeal(t,g),index] = max(Pideal(4*(g-1)+t,:));
    VmIdeal(t,g) = Videal(4*(g-1)+t,index);
    ImIdeal(t,g) = Iideal(4*(g-1)+t,index);
end
% Set up plot. These plots show how temperature affects the IV curve at
% a given irradiance.
xlim([0,30]);
ylim([0,4]);
xlabel('Module Voltage (V)','Fontsize',fsize);
ylabel('Current (A)','Fontsize',fsize);
legend({'25 C','30 C','40 C','50 C'},'Fontsize',fsize)
ttl = ['Ideal Solarex MSX 60 with insolation of ',int2str(G),' W/m^2'];
title(ttl,'Fontsize',fsize)
```

```
    box on
    saveas(gcf,[int2str(G),'W-i.png'])
    hold off
end

% Redo plots to show how irradiance affects the IV curve at a given
% temperature
for t = 1:4
    figure
    hold on
    T = Trange(t);
    for g = 1:5
        plot(Videal(4*(g-1)+t,:),Iideal(4*(g-1)+t,:))
    end
    xlim([0,30]);
    ylim([0,4]);
    xlabel('Module Voltage (V)','Fontsize',fsize);
    ylabel('Current (A)','Fontsize',fsize);
    lgd = {'200 W/m^2','400 W/m^2','600 W/m^2','800 W/m^2','1000 W/m^2'};
    legend(lgd,'Fontsize',fsize)
    ttl = ['Ideal Solarex MSX 60 at temperature of ',int2str(T),' C'];
    title(ttl,'Fontsize',fsize)
    box on
    saveas(gcf,[int2str(T),'C-i.png'])
    hold off
end

% Input the given series and shunt resistance. This assumes that the given
% series resistance applies to the whole module, and hence divides it by
% the number of cells to determine how each individual cell behaves.
Rs = 1.2/nSeries;
Rsh = 50;

% Again, set up arrays to store results
Vreal = zeros(20,len);
Ireal = zeros(20,len);
Preal = zeros(20,len);
VocReal = zeros(4,5);
IscReal = zeros(4,5);
PmReal = zeros(4,5);
VmReal = zeros(4,5);
ImReal = zeros(4,5);

% Perform calculations assuming solar cells have series and shunt
% resistances. This functions the same way as the ideal cells, except for a
% different equation for current.
for g = 1:5
    figure
    hold on
    G = Grange(g);
    for t = 1:4
        T = Trange(t) + 273;
        I0 = I0ref * (T / Tnom)^3 * exp(q * Eq / k * (1 / Tnom - 1 / T));
        IL = G/Gref * (ILref + musc * (T-Tnom));
```

```
        Vt = k * T / q;

        eqn2 = @(V) IL-I0*(exp((V/nSeries)/Vt)-1)-(V/nSeries)/Rsh;
        VocReal(t,g)=fsolve(eqn2,20,options);
        ilim = ceil(VocReal(t,g)/step)+1;

        for i = 1:len
            if i <= ilim
                Vmodule = (i-1)*step;
                Vc = Vmodule / nSeries;
                Vreal(4*(g-1)+t,i) = Vmodule;
                % Use implicit equation solving to find current at given
                % voltage
                eqn3 = @(I) IL-I0*(exp((Vc+I*Rs)/Vt)-1)-(Vc+I*Rs)/Rsh-I;
                Ireal(4*(g-1)+t,i) = fsolve(eqn3,0,options);
                Preal(4*(g-1)+t,i) = Ireal(4*(g-1)+t,i)*Vreal(4*(g-1)+t,i);
            else
                Ireal(4*(g-1)+t,i) = -1;
            end
        end
        plot(Vreal(4*(g-1)+t,:),Ireal(4*(g-1)+t,:))
        IscReal(t,g) = Ireal(4*(g-1)+t,1);
        [PmReal(t,g),index] = max(Preal(4*(g-1)+t,:));
        VmReal(t,g) = Vreal(4*(g-1)+t,index);
        ImReal(t,g) = Ireal(4*(g-1)+t,index);
    end
    xlim([0,30]);
    ylim([0,4]);
    xlabel('Module Voltage (V)','Fontsize',fsize);
    ylabel('Current (A)','Fontsize',fsize);
    legend({'25 C','30 C','40 C','50 C'},'Fontsize',fsize)
    ttl = ['Real Solarex MSX 60 with insolation of ',int2str(G),' W/m^2'];
    title(ttl,'Fontsize',fsize)
    box on
    saveas(gcf,[int2str(G),'W-r.png'])
    hold off
end

for t = 1:4
    figure
    hold on
    T = Trange(t);
    for g = 1:5
        plot(Vreal(4*(g-1)+t,:),Ireal(4*(g-1)+t,:))
    end
    xlim([0,30]);
    ylim([0,4]);
    xlabel('Module Voltage (V)','Fontsize',fsize);
    ylabel('Current (A)','Fontsize',fsize);
    lgd = {'200 W/m^2','400 W/m^2','600 W/m^2','800 W/m^2','1000 W/m^2'};
    legend(lgd,'Fontsize',fsize)
    ttl = ['Real Solarex MSX 60 at temperature of ',int2str(T),' C'];
    title(ttl,'Fontsize',fsize)
    box on
```

```
    saveas(gcf,[int2str(T),'C-r.png'])
    hold off
end

% Print numerical results
VocIdeal
VocReal
IscIdeal
IscReal
PmIdeal
PmReal
VmIdeal
VmReal
ImIdeal
ImReal
```