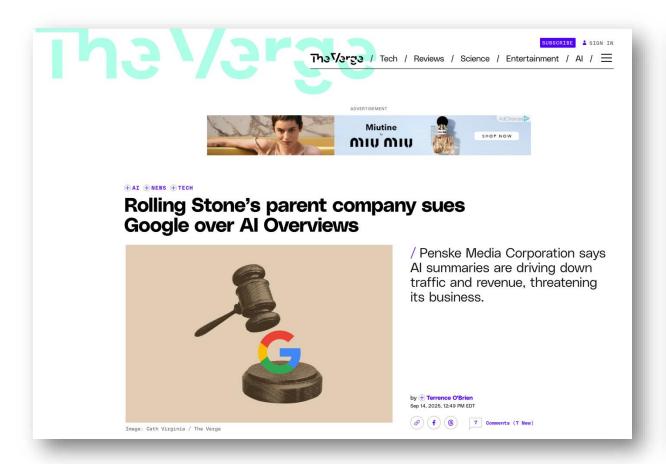
# Intro to Neural Nets

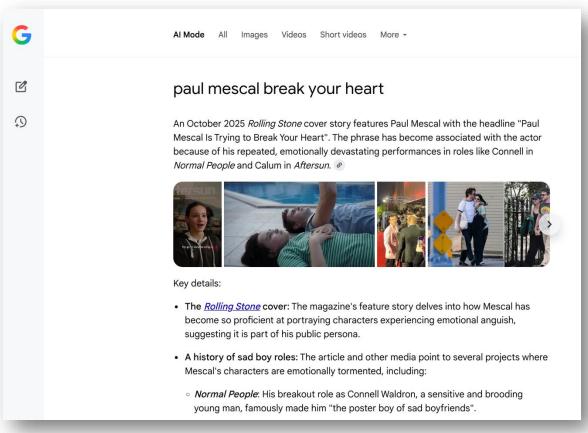
Mathematical Building Blocks & Working with Keras API

## Today's / Next Week's Agenda

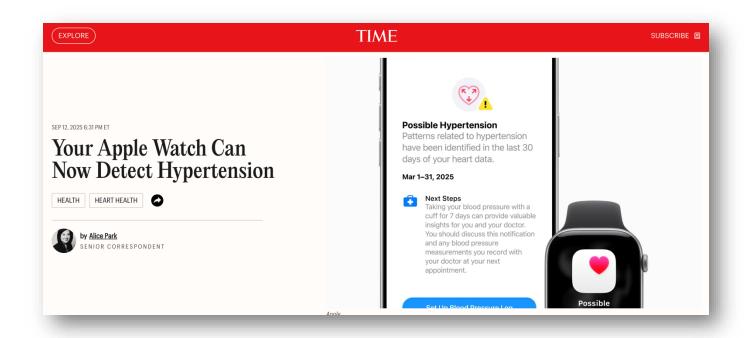
- 1. What's in the News?
- 2. Building Blocks of NNs
  - Tensors (and relevant mathematical operations)
  - Activation Functions
  - Loss Functions
  - Backpropagation: Derivatives, Gradients & the Chain Rule (with examples)
  - Optimizers
- 3. Building a Linear Classifier
  - Overview of Keras and Tensorflow.
  - Implementing a linear classifier in Keras (now that we know the components).

### What's In the News?





### What's In the News?

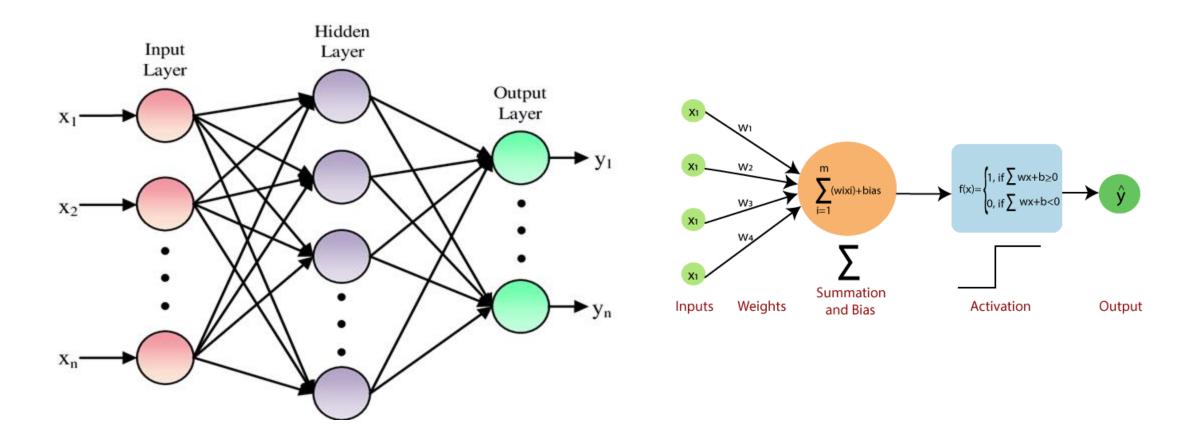


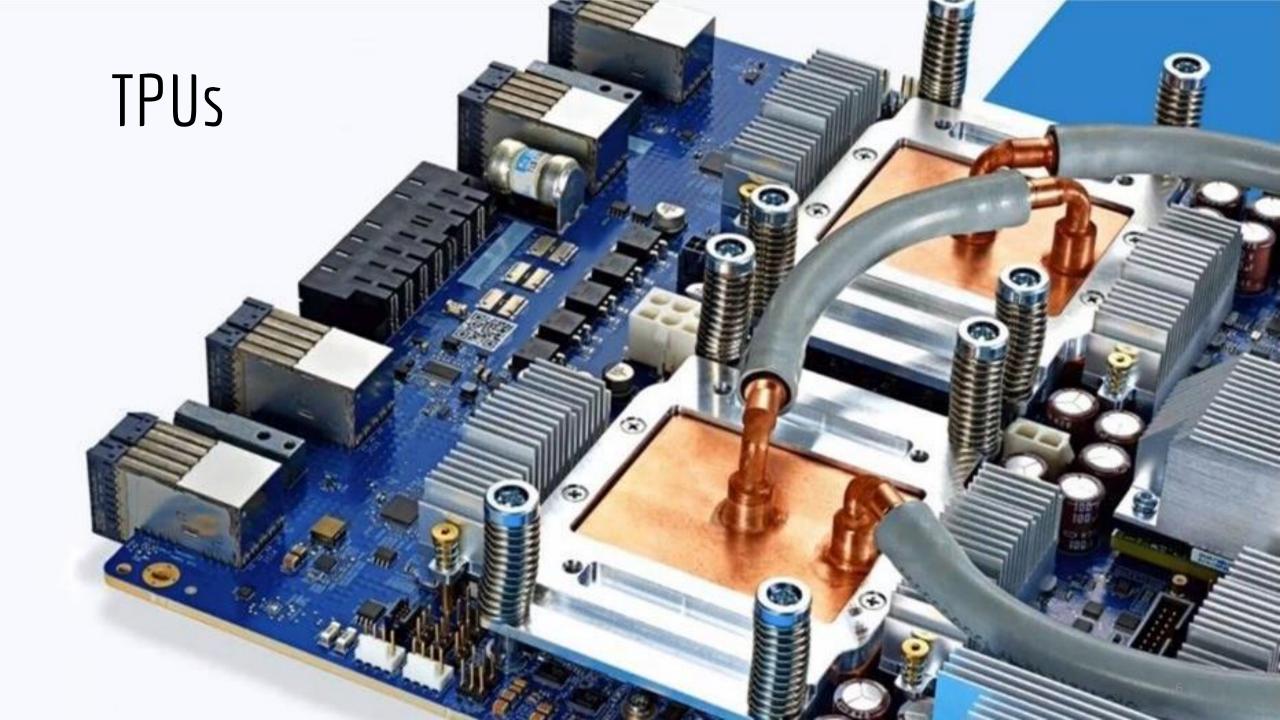
### What are Apple's hypertension notifications?

Apple Watch's optical heart sensor uses photoplethysmography, or PPG, data, which measures blood volume changes through the skin, to analyze how your blood vessels respond to your heartbeats. Working passively, the deep learning algorithm uses 60-second segments of PPG signals as inputs, which are collected about every two hours. This PPG data is also filtered using the Apple Watch's accelerometer data to figure out if the user is sitting still, which is required for the algorithm. It then reviews this data over 30-day periods, and if it notices consistent signs of hypertension, it will alert the user.

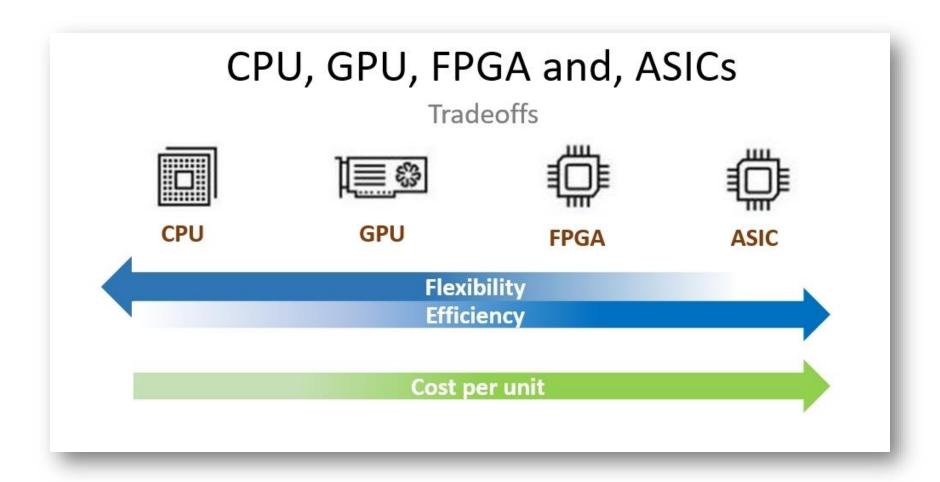
Though this feature won't detect all instances of hypertension, Apple expects these alerts to notify over 1 million people of undiagnosed hypertension within the first year of its launch. This is crucial because there are lifestyle changes and medical interventions that can help people control their high blood pressure, preventing more serious issues down the line. Early intervention and treatment are key. Plus, as we mentioned, people can be unaware that they have hypertension because of no noticeable symptoms, so receiving a hypertension alert on their watch allows them to take action they wouldn't have otherwise done.

### Recall

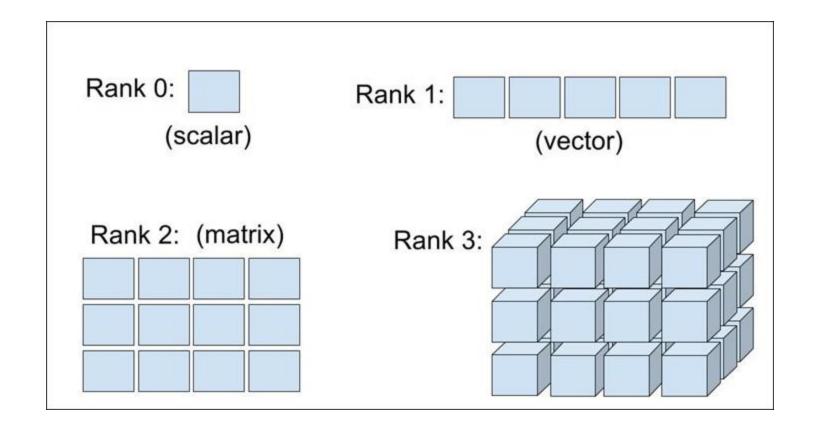




### An Aside: GPU vs. ASIC

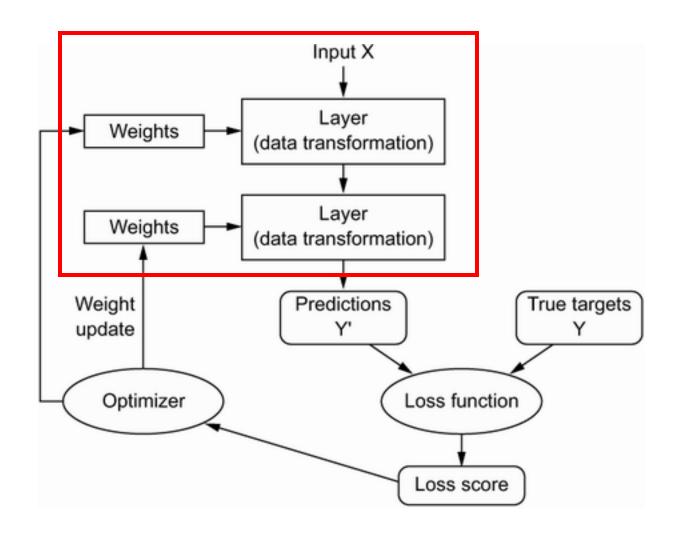


### Tensors



Question: What sort of data (give an example) would be stored in a rank-3 tensor? How about a rank-4 tensor?

### Forward Pass



## The Perceptron



### **NEW NAVY DEVICE** LEARNS BY DOING

of Computer Designed to Read and Grow Wiser

The BROOKLYN SAVINGS BANK Gives You All reproduce itself and be con-scious of its existence, These Banking UNDER ONE ROOF

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### SAVINGS BANKCHECK\* SERVICE ... THE CONVENIENCE O

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today that it expects will be able to walk, talk, see, write,

scious of its existence.

The embryo—the Weather
Bureau's \$2,000,000 "704" computer—learned to differentiate
between right and left after
fifty aftempts in the Navy's
demonstration for newsmen.

demonstration for newsmen.

The service said it would use
this principle to build the first
or its Perceipron thinking maand write. It is expected to be
finished in about a year at a
cost of \$100,000

Frank Resembatt, desize frank Resembatt, desi

Dr. Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechani-cal space explorers. Without Human Controls

Without Human Controls
The Navy said the perceptron
would be the first mon-living
mechanism "capable of receivcapable of receivits surroundings without any
tits surroundings without any
times without any
times and the second of the control
that perceived itself. Ordinary computers remember only
what is fed into them on punch
cards or magnetic tape.

All the control of the control
to recornize people and call out

to recognize people and call out their names and instantly translate speech in one language to speech or writing in another language, it was predicted. Mr. Rosenblatt said in prin-

Psychologist Shows Embryo

WASHINGTON, July 7 (UPI) -The Navy revealed the em-bryo of an electronic computer

plicity, Claude, just Claude.

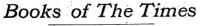
It was her Irish friends and customers who gave her the name of Mrs. O'. A reference to herself, near the end of the book, as one who holds in reserve "the resignation to the inevitable that lingers in the blood of those born in fatalistic East," marks the beginning of a cosmopolitan outlook.

A beau sabreur named Sean soon spotted talent for Gaelic. And Claude tells us she has 'drunk rye with Americans, schmuppe with Dutchmen, beer with Germans, wine the bar.

A rather formally informal romance in with charidates.' The charidates and the duchesses, presumably, carry internations of the duchesses, presumably and the duchesses, presumably a

The problems of running a pub in Cork were often hilarious, seldom businesslike and sometimes tragic. The gamut of life she saw was as various as the life you will encounter on Manhattan Island if you follow Park Avenue down from the street fairs near 125th Street to the local Mayfair within a mile of the Grand Central Terminal.

She liked the Irish and the Irish liked her.



THE NEW YORK TIMES, TUESDAY, JULY 8, 1958

By CHARLES POORE

If this were an entirely accurate account of my life in Cork," the author of "Mrs. O'" tells us, "I should probably be writing it behind bars. So.I should say that it is impressionistically true when not always factually so."

Fair enough. However, when you have finished her entertaining book, you may want to go back to that preface and wonder whether the bit about behind bars is a pun or an Irish bull.

Why? Because she ran a pub in Cork, The idea of doing so came to her in London one afternoon when she found herself rather rich afternoon when she found herself rather rich and completely free. "My decree absolute came through on the same day as my Great Aunt's legacy—not a fortune, but such a sum as I had never dreamed of owning or saving." The fact that she happened to choose for refreshment a place called Mooney's, in London, gave the notion a

proper touch of predestination, Once in Ireland she made forays around the country. It did not take her very long to find the pub she wanted in Cork and buy it from a maiden lady who did not appreciate its seedy elegance. What names she signed to the deed we do not know, although this book is copyrighted by C. M. Forde. As au-thor of it she calls herself, with royal sim-

### Named by Irish Friends

her as French in spite of a quickly acquired awe, saluted her, was that she had, shall we



Claude, author of "Mrs. O'."

witz with any attention, you would think of witz with any attention, you would think or the long arid stretches with no hostelries." Early on, as proprietor of her own pub, she had learned to tap a mighty keg of high stout, after just one lesson from a friendly

rival called foxy:

"Wrapping the barrel end of the tap in
three thicknesses of newspaper as I had
seen Foxy do. I placed the tap against the
bung, raised the mallet, and thinking briefly that I should probably be the first foreigner ever to be killed by Guinness, I hit the tap two fairly light, quick blows."

### Mallet's Force Augmented

It worked fine. The third whack was delivered at full strength. The tap went into place, the newspaper sealed the crack around it. One thing she was too shy to mention when congratulations, offered in say, augmented the force of the mallet with a huge horseshoe she had discovered under

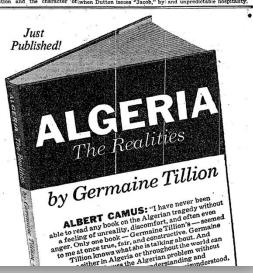
for tragedy. When he meets it, as no reader can doubt he will, Ireland loses magic for Mrs. O'.

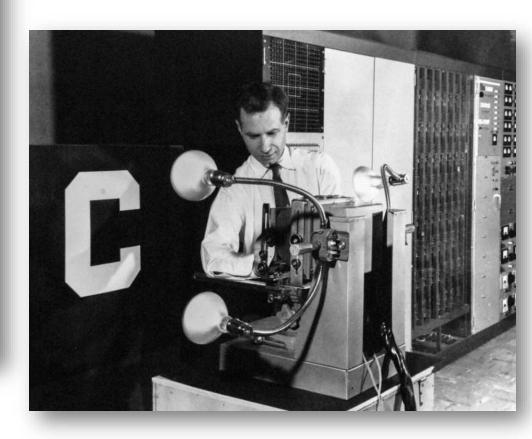
Then there is Phelan, the ambitious burglar. He is a genial sort. But he never owns up to his limitations. In defiance of the best advice he attempts robberies that are beyond the scope of his abilities. The result is that he is often in court, sometimes in fail. The problem of Phelan is solved more happily

Books—Authors will be published Aug. 6.

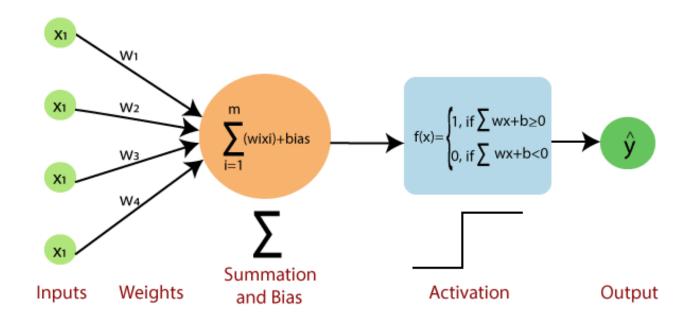
Books—Authors

A month before Joyce Cary's death last year the complete immunerity of his new look from the production and the heart of the complete immunerity of his new look torship in 82 B. G. through from Isaac's house of his previous control of the composition, established a dictal manuscript of his new look torship in 82 B. G. through from Isaac's house to his ree-manuscript in the production of the composition his name became again part on leading a symbol of cold, calculating legentations his name became a symbol of cold, calculating legentations his name became a symbol of cold, calculating legentations his name became a symbol of cold, calculating legentations his name became one, it is the production of the original his production of the artist with the world as it seems to him and to see with a symbol of cold, calculating legentations his name became one, it is the production of the artist with the world as it seems to him and to see with the does with it." He examines the nature of intuition into production and truth in art, the difficulties of translating intuition into production and the character of when Dutton issues "Jacob," by and unpredictable hospitality,"



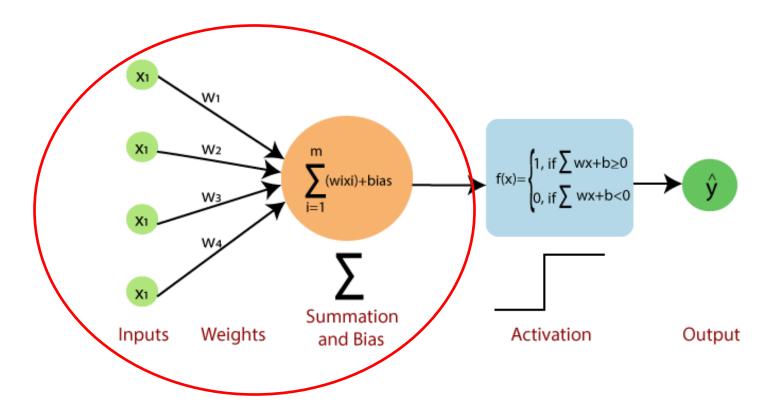


### Neuron / Network Components



Question: What rank tensor are x, w and b here? What will the shape of y be? What is the order of operations in a forward pass?

### Neuron / Network Components

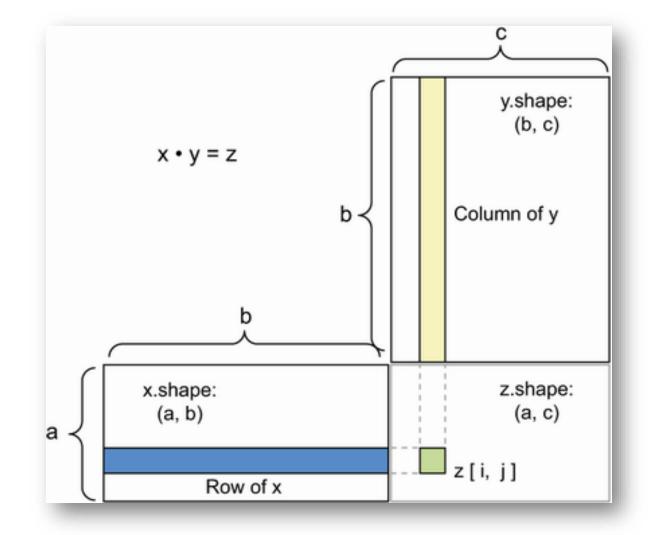


Question: Which of these values are constants? Which are trainable parameters?

### Multiplication

$$y_1 = \varphi \left( \mathbf{x_1} \cdot \mathbf{w_1} + b_1 \right)$$

$$egin{bmatrix} a_1 & a_2 & a_3 \ b_1 & b_2 & b_3 \ c_1 & c_2 & c_3 \end{bmatrix} egin{bmatrix} x \ y \ z \end{bmatrix} = egin{bmatrix} a_1x + a_2y + a_3z \ b_1x + b_2y + b_3z \ c_1x + c_2y + c_3z \end{bmatrix}$$



### Matrix Addition (Broadcast)

$$y_1 = \varphi \left( x_1 \cdot w_1 + b_1 \right)$$

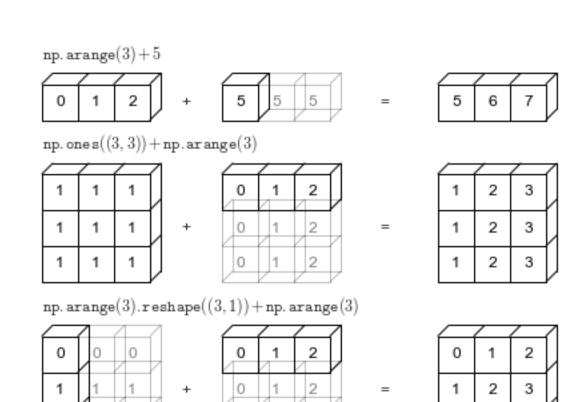
### Shape of the Two Tensors Needs to Conform

 A + B will only work if A is cleanly divisible by B (or vice versa)

### Sum Element-wise

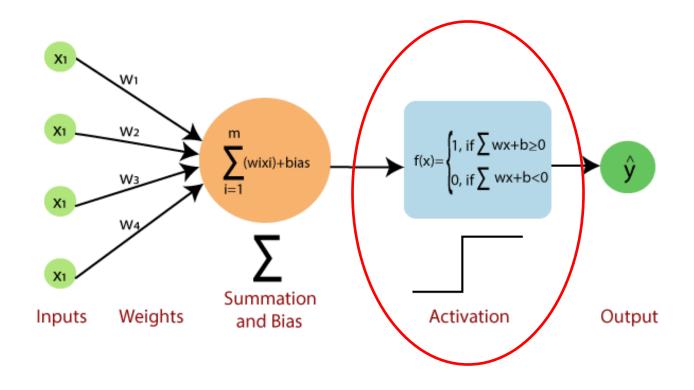
 Replicate B until it matches A's dimensions, then perform elementwise addition.

We Use This for the Addition Step



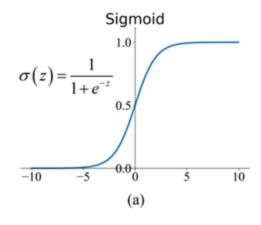
© Gordon Burtch, 2025 Add x\*w and b (bias)

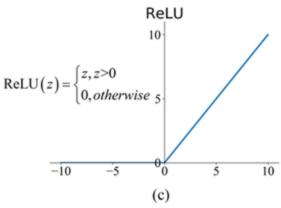
### Neuron / Network Components

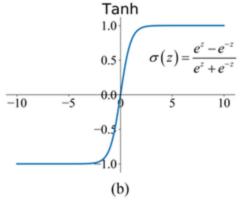


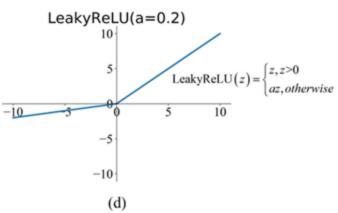
### **Activation Functions**

 $y_1 = \varphi \left( x_1 \cdot w_1 + b_1 \right)$ 









### **Activation Functions**

 $y_1 = \varphi \left( x_1 \cdot w_1 + b_1 \right)$ 



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# CHM RELEASES ALEXNET SOURCE CODE

By Hansen Hsu | March 20, 2025



### Multi-Class, Single-Label

$$y_1 = \varphi \left( x_1 \cdot w_1 + b_1 \right)$$

### **Softmax (MLOGIT):**

We have D inputs (x's). We have k outputs (classes).

So, W is a (D,k) matrix and X is a (D,1) matrix.

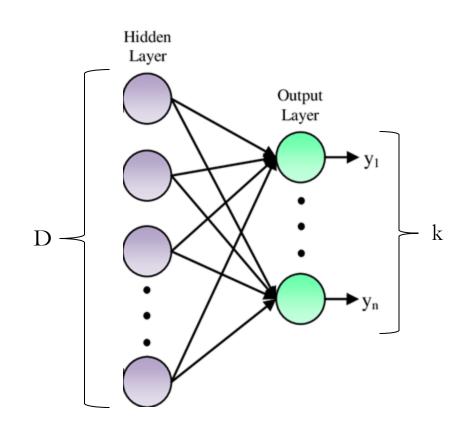
That means, A is a (k,1) matrix.

That means Y is also a (k,1) matrix.

$$A = W^T X,$$

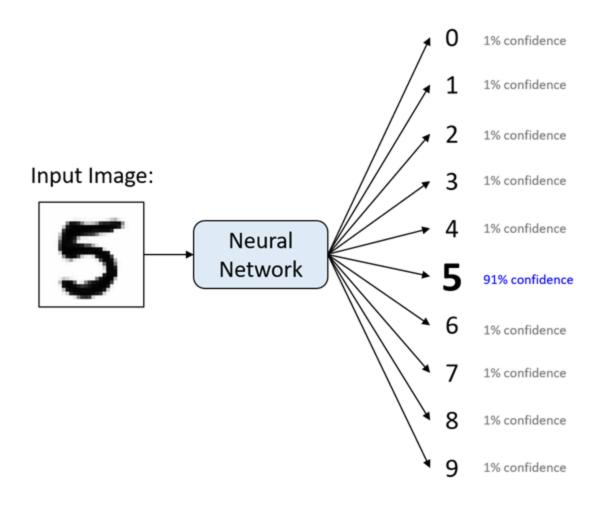
 $Y = \operatorname{softmax}(A),$ 

$$Y_i = \frac{e^{A_i}}{\sum_{j=1}^k e^{A_j}}$$



### Multi-Class, Single-Label

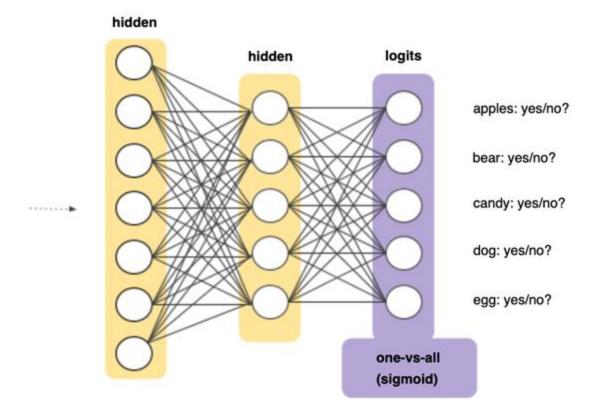
 $y_1 = \varphi \left( x_1 \cdot w_1 + b_1 \right)$ 



### Multi-Class, Multi-Label

### Many Non-Exclusive Labels

- We would create a sigmoid output layer with one output for each class we are predicting.
- Train on all labels together.



### We Now Know Enough for a Forward Pass

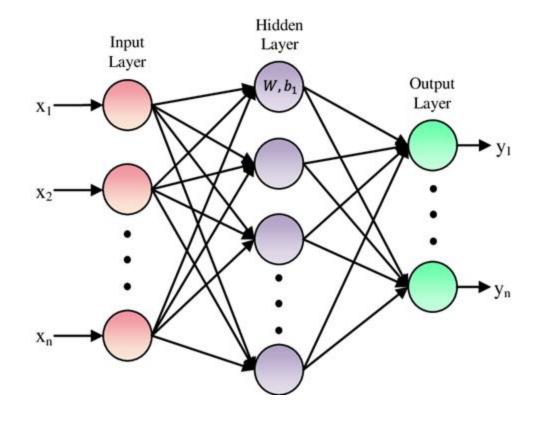
Calculate Output of Each Node Sequentially

$$y_1 = \varphi (x_1 \cdot w_{1,1} + x_2 \cdot w_{1,2} + \dots + b_1)$$
  
$$y_2 = \varphi (x_1 \cdot w_{2,1} + x_2 \cdot w_{2,2} + \dots + b_2)$$

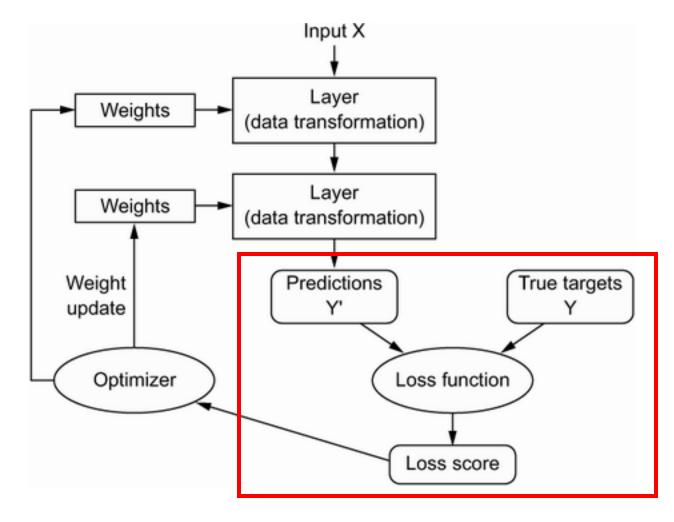
•••

Eventually We Obtain Model's Predictions

Multi-Layer Perceptron (MLP) – Dense, Fully-connected, Feed-forward



### Calculate Loss



### Loss Functions

### Cross-Entropy / Log-Loss

$$H_p(q) = -\frac{1}{N} \sum_{i=1}^{N} y_i \cdot log(p(y_i)) + (1 - y_i) \cdot log(1 - p(y_i))$$

- Typical for binary outcomes.
   Value grows exponentially larger as the predicted probability moves away from the true 0,1 label.
- Multi-category outcomes have an analogous loss function known as categorical cross-entropy.

$$CE = -\sum_{i}^{C} t_{i} log(s_{i})$$

### MAE / L1 Loss

$$MAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}$$

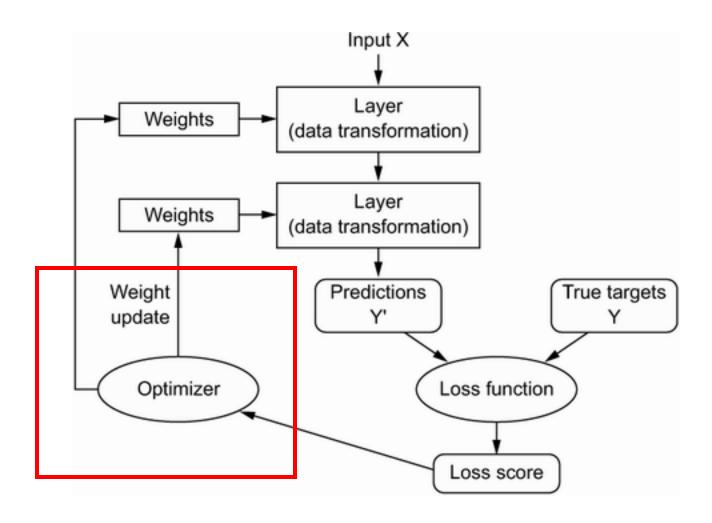
 Typical for continuous outcomes. Errors are penalized homogenously, in magnitude and direction. This should look familiar!

### MSE / Quadratic / L2 Loss

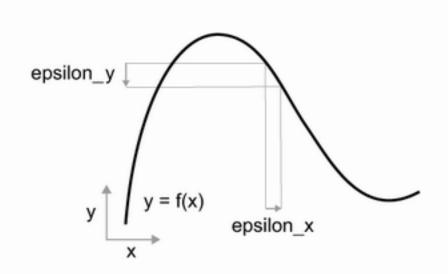
$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$

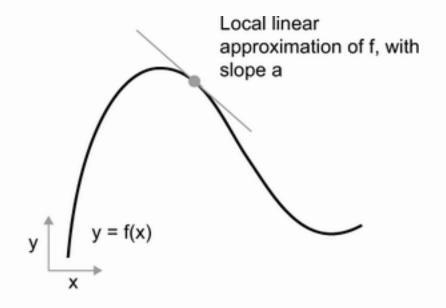
 Typical for continuous outcomes, larger errors penalized exponentially more. This should look familiar!

### Backpropagation



# Derivative = "Rate" of Change

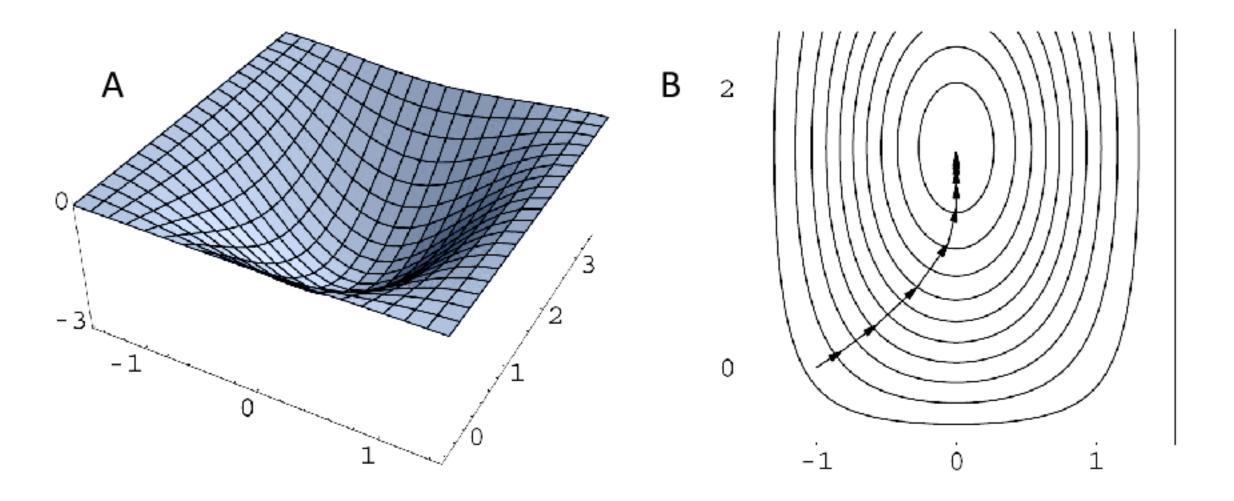




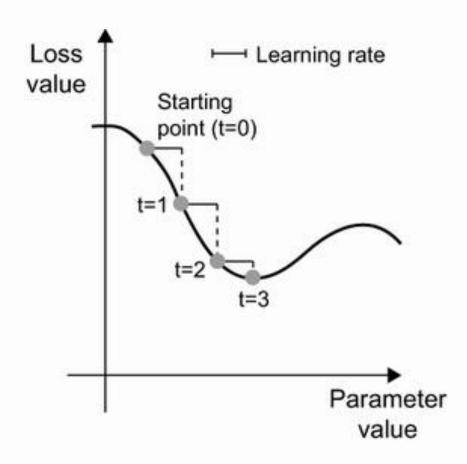
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# Gradient = Derivative in Multiple Dimensions



### Gradient Descent

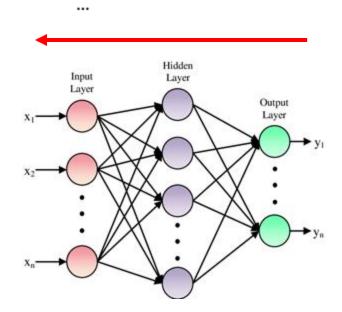


### Derivatives of Loss w.r.t All Parameters

Recall that Each Node's Output Can be Expressed as a Function of the Prior Nodes' Outputs

$$y_1 = \varphi (x_1 \cdot w_{1,1} + x_2 \cdot w_{1,2} + \dots + b_1)$$

$$y_2 = \varphi \left( x_1 \cdot w_{2,1} + x_2 \cdot w_{2,2} + \dots + b_2 \right)$$

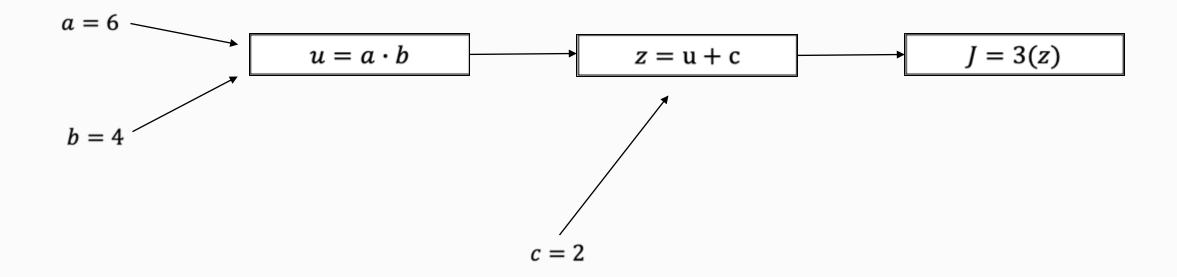


### Start at the final nodes in the network and work backwards

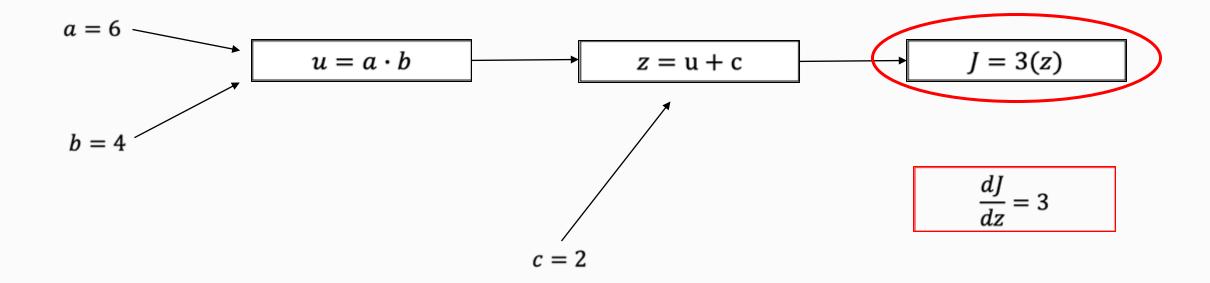
- We calculate partial derivatives w.r.t. their inputs / weights.
- Then, use those partial derivatives and work backward into earlier layers to get partial derivatives w.r.t. their inputs / weights, and so on.

# Simplifying Gradients: Computation Graph

$$J = 3(a \cdot b + c)$$

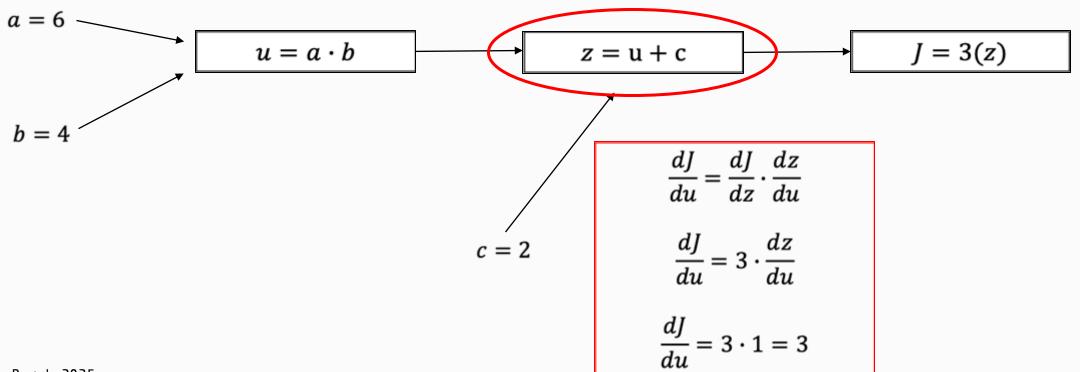


$$J = 3(a \cdot b + c)$$



$$\frac{dJ}{dz} = 3$$

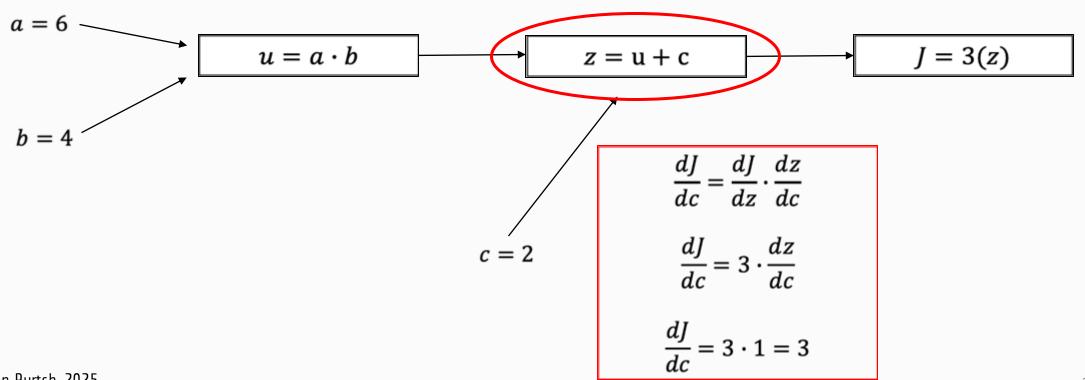
$$J = 3(a \cdot b + c)$$



$$\frac{dJ}{dz} = 3$$

$$\frac{dJ}{du} = 3$$

$$J=3(a\cdot b+c)$$

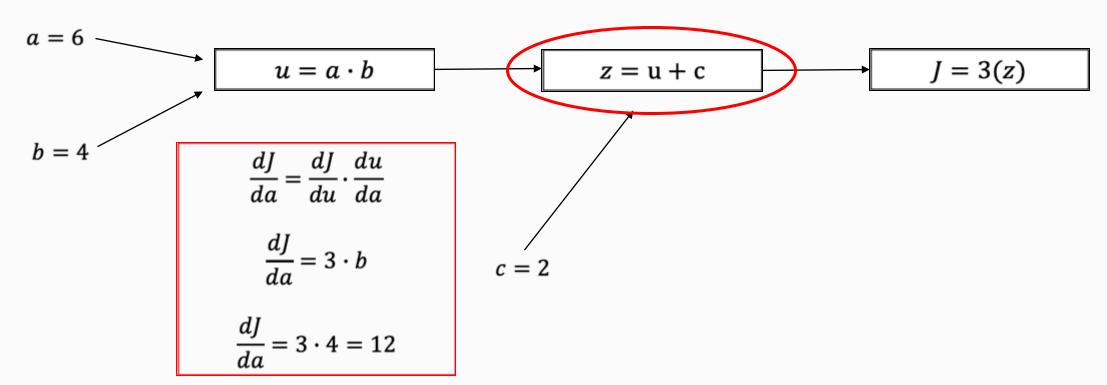


$$\frac{dJ}{dz} = 3$$

$$J = 3(a \cdot b + c)$$

$$\frac{dJ}{du} = 3$$

$$\frac{dJ}{dc} = 3$$



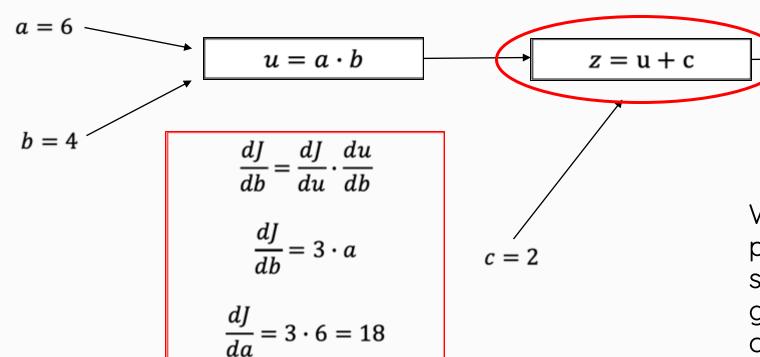
$$\frac{dJ}{dz} = 3$$

$$J=3(a\cdot b+c)$$

$$\frac{dJ}{da} = 12$$

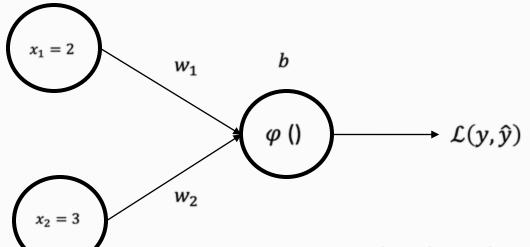
= 3(z)

$$\frac{dJ}{ds} = 3$$



We thus update our parameters, a, b, and c, subtracting each's gradients\*epsilon from its current value. Epsilon is the learning rate.

# Single Node with Sigmoid & Cross-Entropy Loss (i.e., Logistic Regression)



Remember that  $\varphi$  here is just a placeholder for the argument to the loss function. It happens to be a sigmoid transformation of 'something', i.e.,  $\varphi$ (wx+b), but it doesn't really matter. We just represent it with some variable name and calculate an expression for the derivative.

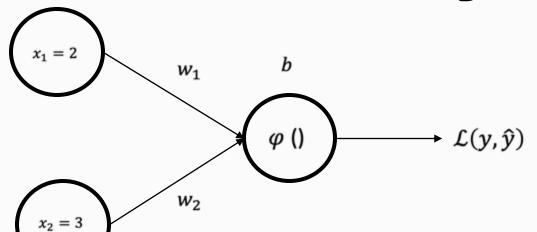
$$\frac{d\mathcal{L}}{d\varphi} = -\frac{y}{\varphi} + \frac{1-y}{1-\varphi}$$

$$\frac{d\mathcal{L}}{d\varphi} = \frac{\varphi(1-y) - y(1-\varphi)}{\varphi(1-\varphi)}$$

$$\frac{d\mathcal{L}}{d\varphi} = \frac{\varphi - \varphi y - y + \varphi y}{\varphi(1-\varphi)}$$

$$\frac{d\mathcal{L}}{d\varphi} = \frac{\varphi - y}{\varphi(1-\varphi)}$$

# Single Node with Sigmoid & Cross-Entropy Loss (i.e., Logistic Regression)



Now we calculate derivative of the sigmoid with respect to its argument, z.

$$\begin{split} \varphi(z) &= (1 + e^{-z})^{-1} \\ \varphi'(z) &= -1 \cdot (1 + e^{-z})^{-2} \cdot (0 + e^{-z} \cdot -1) \\ \varphi'(z) &= (1 + e^{-z})^{-2} \cdot e^{-z} \\ \varphi'(z) &= \varphi(z) \cdot (1 - \varphi(z)) \end{split}$$

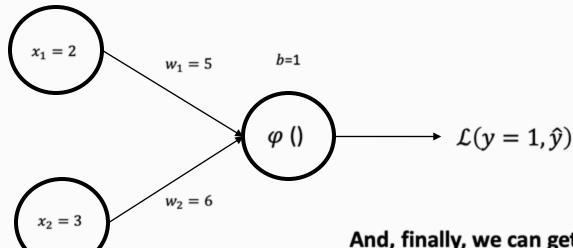
$$\frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}}{d\varphi} \cdot \frac{d\varphi}{dz}$$

$$\frac{d\mathcal{L}}{dz} = \frac{\varphi - y}{\varphi(1 - y)} \cdot \frac{d\varphi}{dz}$$

$$\frac{d\mathcal{L}}{dz} = \frac{\varphi - y}{\varphi(1 - y)} \cdot \varphi(1 - \varphi)$$

$$\frac{d\mathcal{L}}{dz} = \varphi - y$$

# Single Node with Sigmoid & Cross-Entropy Loss (i.e., Logistic Regression)



And, finally, we can get gradient of loss with respect to weights and bias. For example, for the first weight...

Evaluate  $\varphi$  based on current values of parameters and the data.

Finally, update the weights...

$$\frac{d\mathcal{L}}{dw_1} = \frac{d\mathcal{L}}{dz} \cdot \frac{dz}{dw_1}$$

$$\frac{d\mathcal{L}}{dw_1} = (\varphi - y) \cdot x_1$$

$$w_{1,new} = w_{1,old} - (\frac{d\mathcal{L}}{dw_1,old} \cdot \varepsilon)$$

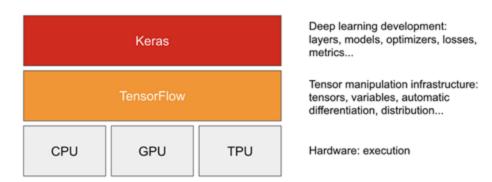
### Keras and Tensorflow

#### 1. Tensorflow

• A Python platform for working with tensors, implementing automatic differentiation, providing access to repositories of (well-known) pre-trained models.

#### 2. Keras

- A higher-level API that wraps common usage patterns with Tensorflow functions, predefined loss functions, optimization algorithms, etc.
- Keras simplifies data scientists' interaction with Tensorflow.



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# Tensorflow GradientTape: AutoDiff

#### 1. Gradient Tape

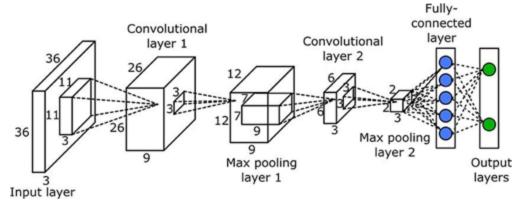
- A Tensorflow function that automates the calculation of derivatives.
- It constructs a computation graph in the background and implements codified rules for calculating derivatives of functions.
- You could technically use gradient tape to implement a gradient descent algorithm for many optimization problems.



# The Layer

#### Layers are the Key Building Block of NNs in Keras

- There are a few subclasses of the Layers class: e.g., Dense is the one we have seen so far layers. Dense(), but we also have convolutional layers, max-pooling layers, recurrent layers, and so on. There are many pre-defined layers in Keras. See: <a href="https://keras.io/api/layers/">https://keras.io/api/layers/</a>.
- These are different architectural components that can be mixed and matched in different ways to create different network topologies.
- It is also possible to construct custom layers.



### Sequential vs. Functional API

#### We Have Only Used Sequential API So Far

• Sequential is easy to work with but is also very inflexible. Can only really handle basic feed-forward networks. It automatically figures out the shape of each layer's output tensor and specifies the next layer's input shape accordingly.

#### Functional API Let's You Construct Any Topology You Want

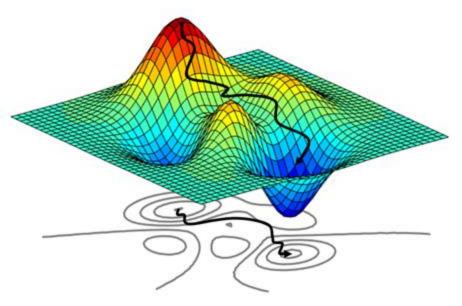
But – we will look at the difference in how each API is used, syntactically.



# **Optimizers**

#### Keras Supports 8 Optimizers

- SGD = Stochastic Gradient Descent
- Momentum
- Ftrl (2010) = Follow the Regularized Leader
- Adagrad and Adadelta (2012) = Adaptive Gradient Des
- RMSprop (~2012) = Root Mean Squared propagation
- Adam (2015) = Adadelta / RMSProp with Momentum.
  - Adamax, Nadam are extensions to Adam.



### SGD: Gradient Descent

#### Types of GD

- Batch GD = Use all the available training data in each pass.
  - Works well if the loss surface is smooth and lacks any saddle points / valleys.
- Stochastic GD = Mini-batch with batch size = 1.
  - If troughs / saddles exist, we move past them as our exploration of gradients for the model will vary withe a given observation that we are considering in an iteration.
  - Computationally quite burdensome but performs well on non-linear problems (eventually).
- Mini-batch GD = What we have been doing so far (randomly split the data in each epoch, into folds, and then cycle over the folds for training).
  - This is a happy-medium between batch and stochastic GD.

#### Role of Batch Size

• Empirically has been observed that smaller batches yield less overfitting (because of implicit noise in the training process – variance of the gradients obtained will go up).

### Batch (All) vs. Stochastic (1)

#### Same Convergence

• If you have a convex surface, either approach will converge to the global optimum (no guarantee your problem is convex of course). Always converges at least to a local minimum.

#### Tradeoff s

• Batch, each step is slower, more computationally burdensome, but convergence with fewer iterations; Need to be able to hold the entire dataset in memory.

• SGD makes noisier updates, and requires more iterations to converge, but a single iteration is quick. Only need one observation in mamory at a time.

### Momentum

#### Getting Past Local Minima

• SGD gets stuck in local minima; the idea of momentum is to make updates be a function of current gradient\*learning rate, as well as some fraction (decay) of the update you made last iteration.

• This reduces updates to parameters where the gradients are flipping sign and amplifies updates to gradients that are going in a consistent direction (steeply

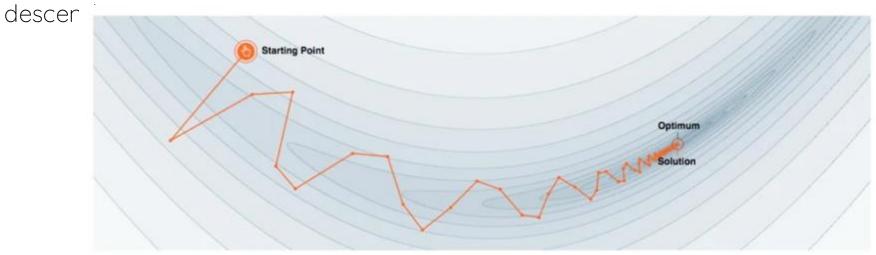


Figure: Optimization with momentum (Source: distill.pub)

### FTRL

#### Google Developed in 2010...

- This is an optimization technique that is used in "online" learning; it's typically used in situations where your model training is happening continuously as new data arrives, and where drift might therefore happen.
- It works well in situations where you have a ton of sparse features.
- Was originally used for predicting conversion in online advertising systems.



# Adagrad & Adadelta (RMS Prop)

#### Adaptive Gradient Descent (Variable Learning Rate)

- We implicitly apply a high learning rate for features we have been updating very little so far (speed up movement through saddle points, for example).
- We implicitly apply a low learning rate for features we have been updating a lot so far.
- Technically learning rate is removed from the process, every update is a function of past updates.

#### Adadelta

- Same idea but we use a sliding window of previous updates to determine magnitude of current updates (rather than all prior updates).
- RMSProp is conceptually very similar but was independently developed (around the same time).

# Recap

#### Building Blocks of NNs

- Tensors and Tensor Operations
- Activation Functions
- Loss Functions
- Backpropagation: Derivatives, Gradients & the Chain Rule

#### Procedure of Minibatch Stochastic Gradient Descent

- Grab a batch of observations (samples)
- Predict their labels using current weights / bias terms.
- Calculate loss value.
- Calculate gradient of loss w.r.t. all weight / bias terms.
- Update each weight by subtracting its gradient\*learning rate
- Cycle over the whole training dataset (each cycle is an epoch) repeatedly, until loss is small.