

# chromosome size

For  $M$  matrix, a  $(c+1) \times (c+1)$  matrix.

① the diagonal of  $M$  is  $D(M) = [T_1 + N, T_2 + N, \dots, T_c + N, N]$ .

$T_i = \frac{N^2}{Me_i}$ ,  $Me_i$  是第  $i$  条染色体的有效标记

② The off-diagonal of  $M$  is  $M_{ij(c+1j)} = N$ .

根据高斯消元法求矩阵  $M$  的逆矩阵  $M^{-1}$

①

$$\left[ \begin{array}{ccc|ccc} T_1 + N & & & & & \\ & T_2 + N & & & & \\ & & \ddots & & & \\ & & & T_c + N & & \\ N & & & & N & \\ & & & & & \ddots \\ & & & & & & 1 \\ & & & & & & & \ddots \\ & & & & & & & & 1 \end{array} \right]$$

② 选最后一行为主元，第  $1 \sim c$  行分别减去第  $c+1$  行，得到

$$\left[ \begin{array}{ccc|ccc} T_1 & & & & & \\ & T_2 & & & & \\ & & \ddots & & & \\ & & & T_c & & \\ N & & & & N & \\ & & & & & \ddots \\ & & & & & & 1 \\ & & & & & & & \ddots \\ & & & & & & & & 1 \end{array} \right] \quad \begin{array}{c} 1 \\ -1 \\ -1 \\ \vdots \\ -1 \\ 1 \end{array}$$

伴随矩阵对角线是 1，最后一列为 -1。

③ 第  $1$  到  $c$  行分别乘以  $\frac{1}{T_i}$ ，最后一行乘以  $\frac{1}{N}$

$$\left[ \begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & & & N & \\ & & & & & \ddots \\ & & & & & & 1 \\ & & & & & & & \ddots \\ & & & & & & & & 1 \end{array} \right] \quad \begin{array}{c} \frac{1}{T_1} \\ \frac{1}{T_2} \\ \vdots \\ \frac{1}{T_c} \\ \frac{1}{N} \end{array}$$

④ 最后一行依次减去第  $1$  到第  $c$  行，完成求逆

$$\left[ \begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & & & N & \\ & & & & & \ddots \\ & & & & & & 1 \\ & & & & & & & \ddots \\ & & & & & & & & 1 \end{array} \right] \quad \begin{array}{c} \frac{1}{T_1} \\ \frac{1}{T_2} \\ \vdots \\ \frac{1}{T_c} \\ -\frac{1}{T_1} - \frac{1}{T_2} - \frac{1}{T_3} \dots \\ \frac{1}{N} + \sum \frac{1}{T_i} \end{array}$$

$M$  的逆矩阵可直接描述如下。

$$M_{ii(c \leq c)}^{-1} = \frac{1}{T_i} = \frac{Me_{ii}}{N^2}$$

$$M_{ii(c=c+1)}^{-1} = \frac{1}{N} + \sum \frac{1}{T_i} = \frac{1}{N} \left( 1 + \sum \frac{Me_{ii}}{N} \right)$$

$$\text{最后一列/行 (除去最后一个元素)} = -\frac{1}{T_i} = -\frac{Me_{ii}}{N^2}$$

因为  $Me_i$  与  $N$  都是常数，求逆矩阵时间复杂度为  $C$