Handwritten Examples of Theorems and Their Proofs in LaTeX

1 Composites

`c1.txt`

Definition. *A number n is said to be a tail if $n = a * b * c$ and $a > 2$, $b > 2$, and $c > 2$.*

Theorem. *The number 957 is a tail.*

Proof. $957 = 29 * 11 * 3 >$. Moreover, $29 > 2$, $11 > 2$, and $3 > 2$. □

`c2.txt`

Definition. *A number n is said to be a triplefactor if there exist integers a, b, c such that $n = a * b * c$.*

Theorem. *The number 30 is a triplefactor.*

Proof. Notice that $2 * 3 * 5 = 30$. Hence, 30 is a triplefactor. □

`c3.txt`

Definition. *A number n is said to be composite if there exist integers a, b greater than 1 such that $n = a * b$.*

Theorem. *The number 30 is composite.*

Proof. Notice that $2 * 15 = 30$. Hence, 30 is composite. □

`c4.txt`

Definition. *A number n is said to be doublycomposite if there exist integers a, b greater than 1 such that $n = a * b$.*

Theorem. *The number 30 is doublycomposite.*

Proof. Notice that $30 = 3 * 10$. Hence, 30 is doublycomposite. □

`c5.txt`

Definition. *A number n is said to be composite if there exist integers a, b greater than 1 such that $n = a * b$.*

Theorem. *The number 300 is composite.*

Proof. Note that $300 = 3 * 100$, and that 3 and 100 are greater than 1. □

c6.txt

Definition. *$w \in \mathbb{N}$ is called a 2-composite number if there are $a, b \in \mathbb{N}$ greater than or equal to 2 such that $a * b = w$.*

Theorem. *The natural number 918 is 2-composite.*

Proof. $918 = 27 * 34$, therefore according to the definition it is a 2-composite number. □

c7.txt

Definition. *A composite number is any $n \in \mathbb{N}$ that is a product $a * b$, where a and b are natural numbers greater than or equal to 2.*

Theorem. *1749 is a composite number.*

Proof. $1749 = 53 * 33$, which is by definition a composite number. □

c8.txt

Theorem. *99 is the product of $a, b \in \mathbb{N}$ greater than or equal to 2.*

Proof. 99 is the product of the factors 3 and 33, thus the theorem is proven. □

c9.txt

Theorem. *A number is called composite if it is equal to $a * b$, where a and b are natural numbers greater than or equal to 2. 15 is a composite number.*

Proof. 15 is a composite number because it is equal to $3 * 5$. □

c10.txt

Theorem. *121 is the product of $a, b \in \mathbb{N}$ that are greater than or equal to 2.*

Proof. $121 = a * b$, where $a = 11$ and $b = 11$. □

c11.txt

Theorem. *Any number x equal to the product of natural numbers a and b greater than or equal to 2 is called a composite number. 100 is composite.*

Proof. 100 is equal to $4 * 25$, which shows that it is composite. □

`c12.txt`

Definition. *A composite number is a natural number x that is the product of natural numbers a and b that are 2 or larger.*

Theorem. *56 is a composite number.*

Proof. Let $a = 7$ and $b = 8$. Since $a \geq 2$ and $b \geq 2$, 56 is a composite number. $\square$

`c13.txt`

Theorem. $12 = ab$ *for a and b greater than or equal to 2.*

Proof. $12 = 4 \times 3$. $\square$

`c14.txt`

Definition. *x is composite if there exist a, $b \in \mathbb{N}$ such that $x = ab$ and a, b are greater than or equal to 2.*

Theorem. *10 is composite.*

Proof. Let $a = 5$ and $b = 2$. $ab = 5 \times 2 = 10$. $\square$

`c15.txt`

Theorem. *The natural number n is said to be a composite given A, d, $m \in \mathbb{N}$ are greater than or equal to 2, and $A \times d \times m = n$.*

Proof. $2 \cdot 3 \cdot 5 = 30$, therefore 30 must be as in the theorem. $\square$

2 Even-odd

`e1.txt`

Theorem. $4 * a + 9$ *is an odd integer for any* $a \in \mathbb{Z}^+$.

Proof. Since 4 is even, $4 * a$ is even. Clearly, 9 is odd. The sum of an even integer with an odd integer is an odd integer. Thus, the expression is odd. $\square$

`e2.txt`

Theorem. $6x + 2$ *is even.*

Proof. The addition of even numbers will be even. $6x$ is clearly even. $\square$

`e3.txt`

Theorem. *For every natural number* n, $6 * n + 7$ *is odd.*

Proof. Observe that 6 is even and hence $6 * n$ is even. Likewise, 7 is odd. Hence, $6 * n + 7$ is odd. $\square$

`e4.txt`

Theorem. *For any* $x, y \in \mathbb{Z}^+$, *the expression* $2 * x + 4 * y + 1$ *is guaranteed to be odd.*

Proof. Since 2 is even, and 4 is even, $2 * x$ and $4 * y$ are even. We know the addition of an even number with an even number is even. Furthermore, 1 is trivially odd. Recall that the addition of an even number with an odd number is odd, thus the summation is an odd integer. $\square$

`e5.txt`

Theorem. *For* $a, b \in \mathbb{N}$, $4a + 10b + 9$ *is an odd number.*

Proof. Since $4a$ and $10b$ are even, the expression is odd. $\square$

`e6.txt`

Theorem. *For any natural numbers* a *and* b, $6 \times a + 14 \times b + 5$ *is odd.*

Proof. 6 is even, therefore $6 \times a$ is even. Similarly, $14 \times b$ is even because 14 is even. Since the sum of even numbers is an even number, $6 \times a + 14 \times b$ must be even. The addition of the odd number 5 with an even number must be odd. Therefore, $6 \times a + 14 \times b + 5$ is odd. $\square$

`e7.txt`

Theorem. *For any natural numbers a, b, and c, $12 \times a + 26 \times b + 4 \times c + 17$ is odd.*

Proof. The multiplication of an even number with a natural number will be an even number. Since 12, 26, and 4 are all even numbers, the products $12 \times a$, $26 \times b$, and $4 \times c$ are even. Also, the sum of even numbers is an even number, consequently the sum of the 3 products $12 \times a + 26 \times b + 4 \times c$ must be even. Furthermore, we know that the addition of an odd number with an even number is an odd number. Since 17 is an odd number, $12 \times a + 26 \times b + 4 \times c + 17$ will be an odd number. $\square$

`e8.txt`

Theorem. *For any natural numbers a and b, the sum $2 \times a + 6 \times b + 4$ must be even.*

Proof. The multiplication of an even number with any number is an even number. Since 2 is an even number, the term $2 \times a$ is an even number. Similarly, since 6 is an even number, $6 \times b$ is an even number. Next, the sum of even numbers is an even number, hence $2 \times a + 6 \times b$ is even. Similarly, the addition of the even number 4 to this even sum will be even, therefore $2 \times a + 6 \times b + 4$ is even. $\square$

`e9.txt`

Theorem. *The sum $22 \times a + 60 \times b + 34 \times c + 10$ is even, for any natural numbers a, b, and c.*

Proof. 22 is even, therefore $22 \times a$ must be even. 60 is even, therefore $60 \times b$ is even too. 34 is even, therefore $34 \times c$ is even too. Since the sum of even numbers is even, and 10 is also even, $22 \times a + 60 \times b + 34 \times c + 10$ is even. $\square$

`e10.txt`

Theorem. *$6 \times a + 14 \times b + 3$ is odd, for any natural numbers a and b.*

Proof. The product of an even number with any natural number is an even number. Since 6 and 14 are even numbers, then $6 \times a$ and $14 \times b$ are even. We also know that the sum of even numbers is itself an even number, therefore $6 \times a + 14 \times b$ must be even. Furthermore, 3 is odd and it is known that the addition of an odd number with an even number is an odd number. Thus $6 \times a + 14 \times b + 3$ is an odd number. $\square$

`e11.txt`

Theorem. *The expression $4 \times a + 10 \times b + 17$ is an odd number, for any integers a and b.*

Proof. It is known that the product of an even number with any integer is an even number. Therefore, the terms $4 \times a$ and $10 \times b$ must be even. Since the sum of even numbers is an even number, then $4 \times a + 10 \times b$ must be even as well. We know that the addition of an even number with an odd number is an odd number. Since 17 is an odd number, the sum $4 \times a + 10 \times b + 17$ must be odd. $\square$

e12.txt

Theorem. $12 \times a + 46 \times b + 34 \times c + 6$ *is even, for any integers a, b, and c.*

Proof. 6 is an even number. Furthermore, the terms $12 \times a$, $46 \times b$, and $34 \times c$ are all even using the fact that multiplying an even number with any integer is an even number. Thus, all terms in the expression are even. Since the sum of even numbers is an even number, the expression must be even. □

e13.txt

Theorem. *For any integers a, b, c, and d the following expression* $72 \times a + 26 \times b + 88 \times c + 18 \times d + 47$ *is odd.*

Proof. The multiplication of an integer with an even number is in an even number. Since 72, 26, 88, and 18 are all even numbers, the terms $72 \times a$, $26 \times b$, $88 \times c$, and $18 \times d$ must be even. Furthermore, their sum $72 \times a + 26 \times b + 88 \times c + 18 \times d$ must be even as well, since the sum of even numbers must be even. Additionally, 47 is odd and it is known that the addition of an odd number with an even number is an odd number. Therefore, $72 \times a + 26 \times b + 88 \times c + 18 \times d + 47$ must be odd. □

e14.txt

Theorem. *For any integers a and b it must be true that* $44 \times a + 62 \times b + 14$ *is even.*

Proof. The terms $44 \times a$ and $62 \times b$ must be even using the fact that the multiplication of an even number with any integer is even. Since the sum of even numbers must be even, the expression $44 \times a + 62 \times b + 14$ is even. □

e15.txt

Theorem. $58 \times a + 38 \times b + 85$ *is odd, for any integers a and b.*

Proof. It can be proven that the product of an even number with any integer is an even number. Since 58 is even, then $58 \times a$ is even. Similarly, since 38 is even, then $38 \times b$ is even as well. It is also known that the sum of even numbers is even, therefore the expression $58 \times a + 38 \times b$ must be even. The addition of an odd number with an even number is an odd number. Since $58 \times a + 38 \times b$ is even and 85 is odd, their sum $58 \times a + 38 \times b + 85$ must be odd. □

3 Powers

`p1.txt`

Definition. *$a \in \mathbb{N}$ is called a cataract if there is a natural number b such that $a = b^{12}$.*

Theorem. *4096 is a cataract.*

Proof. Let $b = 2$. Then $4096 = b^{12}$, which gives that it is a cataract. ☐

`p2.txt`

Definition. *A cadillact is a number a such that $a = b^{11}$, where b is a natural number.*

Theorem. *2048 is a cadillact.*

Proof. 2048 is equal to 2 raised to the power 11, and is therefore a cadillact by definition. ☐

`p3.txt`

Definition. *We say that $a \in \mathbb{N}$ is an artifact if it is equal to b^{10}, for some $b \in \mathbb{N}$.*

Theorem. *1024 is an artifact.*

Proof. By definition, 1024 is an artifact because it is equal to 2^{10}. ☐

`p4.txt`

Definition. *An enthusiast is a natural number $a = b^4$, where $b \in \mathbb{N}$.*

Theorem. *81 is an enthusiast.*

Proof. 81 is 3 raised to the power 4, and therefore is an enthusiast. ☐

`p5.txt`

Definition. *A natural number a is called a megaloblast if it is equal to b raised to the power 10, where b is a natural number.*

Theorem. *1024 is a megaloblast.*

Proof. $1024 = 2^{10}$, therefore it is a megaloblast. ☐

`p6.txt`

Definition. *A notsofast is any natural number x that is equal to a whole number a raised to the power 2.*

Theorem. *4 is notsofast.*

Proof. $4 = 2^2$, therefore 4 is notsofast. ☐

`p7.txt`

Definition. *We say that $a \in \mathbb{N}$ is an artifact of $b \in \mathbb{N}$ if $a = b^{10}$.*

Theorem. *1024 is an artifact of 2.*

Proof. By definition, 1024 is an artifact of 2 because it is equal to 2^{10}. $\square$

`p8.txt`

Definition. *A google is a number n such that there exists a k such that $n = k^{100}$.*

Theorem. *The number 1 is a google.*

Proof. Note that $1^{100} = 1$, and hence that 1 is a google. $\square$

`p9.txt`

Definition. *A google is a number n such that there exists a k such that $n = k^3$.*

Theorem. *The number 8 is a google.*

Proof. Note that $2^3 = 8$. Hence, 8 is a google. $\square$

`p10.txt`

Definition. *A number n is a half-google if there exists an integer k such that $n = k^5$.*

Theorem. *The number 32 is a half-google.*

Proof. Notice that $32 = 2^5$, and hence it is a half-google. $\square$

`p11.txt`

Definition. *A number n is a quasi-half-google if there exists an integer a such that $n = a^5$, where $a > 1$.*

Theorem. *The number 243 is a quasi-half-google.*

Proof. Let $a = 3$. Then, $243 = 3^5$. Hence, it is a quasi-half-google. $\square$

`p12.txt`

Definition. *A number b is called an enneract if there exists an integer a such that $a^9 = b$.*

Theorem. *There exists a such that the number 512 is an enneract.*

Proof. Let $a = 2$. Clearly, $2^9 = 512$. Thus, there exists an integer such that $a^9 = 512$ and therefore, 512 must be an enneract. $\square$

`p13.txt`

Theorem. *There exists some integer x satisfying the condition $243 = x^5$.*

Proof. The number 3 raised to the power of 5 is equal to 243, hence the theorem clearly holds. □

`p14.txt`

Theorem. *There exists $x \in \mathbb{N}$ such that $x^2 = 16$.*

Proof. $4^2 = 16$. □

`p15.txt`

Definition. *A square number is any number y where there is some x such that $x^2 = y$.*

Theorem. *16 is square.*

Proof. Let $x = 4$. Then $4^2 = 16$. Therefore, 16 is square. □