Intro to Analysis, Self-Study

Selected Solutions for Rudin's Principles of Mathematical Analysis (3rd) and Abbott's Understanding Analysis (2nd)

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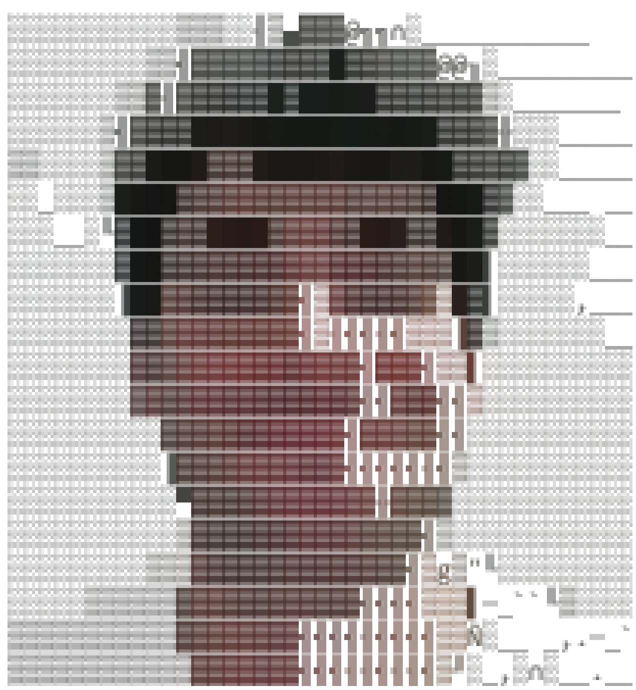

Rudin Problems denoted R#.#
Aboott Problems denoted A#.#

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Preface

TODO Baby Rudin 3rd and Abbot 2nd blah blah

Chapter 1

The Real (and Rudin's Complex) Number Systems

Before continuing, we examine the equations Rudin seems to arbitrarily present. The recurrence formula for the Secant method:

$$x_{n+1} = x_n - \frac{f(x_n) \cdot (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

In this formula, x_n and x_{n-1} represent the current and previous approximations of the root, respectively. $f(x_n)$ and $f(x_{n-1})$ represent the function values at x_n and x_{n-1}, respectively. We can simplify this to

$$x_{n+1} = \frac{x_n \cdot f(x_{n-1}) - x_{n-1} \cdot f(x_n)}{f(x_{n-1}) - f(x_n)}$$

You would need to provide initial approximations x_0 and x_1 close to the root you are trying to find. Then, you can iteratively update the approximation using the above formula until you reach the desired level of accuracy or convergence (if at all).

Now setting $q \equiv x_{n+1}$, $p \equiv x_n$, and $x_{n-1} = 2$ for a function $f(x_n) = x_n^2 - x_{n-1}$, we get

$$\begin{aligned} q &= p - \frac{p^2 - 2}{p + 2} = \frac{2p + 2}{p + 2} = \frac{2(p + 1)}{p + 2} \\ \implies q^2 &= \frac{4(p + 1)^2}{(p + 2)^2} = \frac{4(p^2 + 2p + 1)}{(p + 2)^2} \\ \implies q^2 - 2 &= \frac{4(p^2 + 2p + 1) - 2(p^2 + 4p + 4)}{(p + 2)^2} = \frac{2p^2 - 4}{(p + 2)^2} = \frac{2(p^2 - 2)}{(p + 2)^2} \end{aligned}$$

after some simplification. This arrives at the result given without proof.

1.1 Exercises

Exercise R1
If r is rational ($r \neq 0$) and x is irrational, prove that $r + x$ and rx are irrational.
Exercise R2
Prove that there is no rational number whose square is 12.
Exercise R3
Prove the following:

- If $x \neq 0$ and $xy = xz \implies y = z$.
- If $x \neq 0$ and $xy = x \implies y = 1$.

- If $x \neq 0$ and $xy = 1 \implies y = \frac{1}{x}$.
- If $x \neq 0 \implies \frac{1}{\frac{1}{x}} = x$.

Exercise R4
Let E be a nonempty subset of an ordered set; suppose α is a lower bound of E and β is an upper bound of E. Prove that $\alpha \leq \beta$.

Exercise R5
Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. Prove that $\inf A = -\sup(-A)$.