# Model Learning

MMCi Block 2

Matthew Engelhard

*Learning* (or *training*) our model means:

-> Finding specific values of our model parameters that predict $y$ effectively from $x$ in our training set

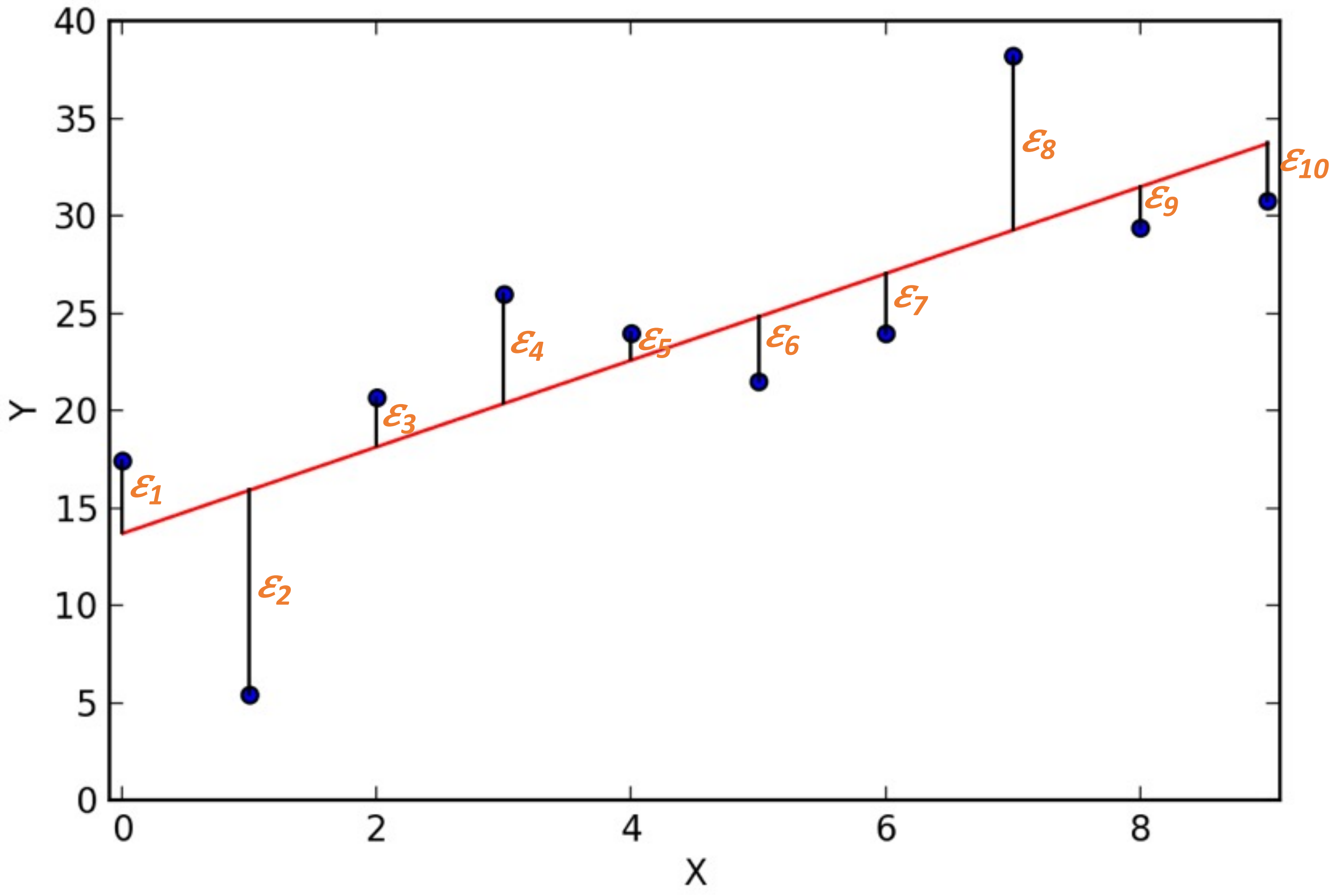
What do we mean by "predict $y$ effectively"?

-> We need a number that measures how well we're predicting $y$

-> Then, we can set our parameters to the specific values that are best, according to this number

-> We call this number the "loss", and try to find parameters that minimize it

# In *linear* regression, the loss is the mean squared error.



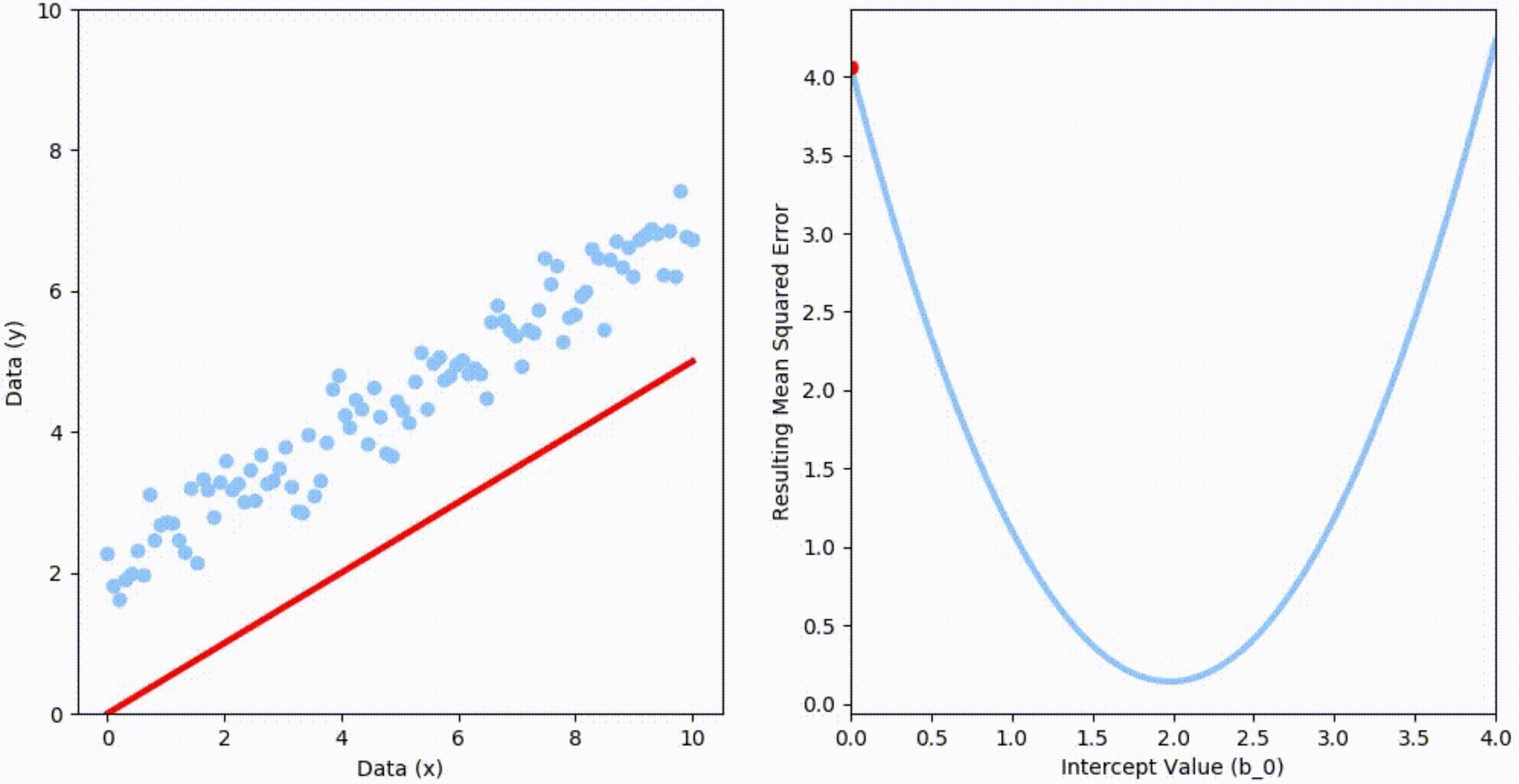

Error:

$$\varepsilon_i = y_i - \hat{y}_i$$

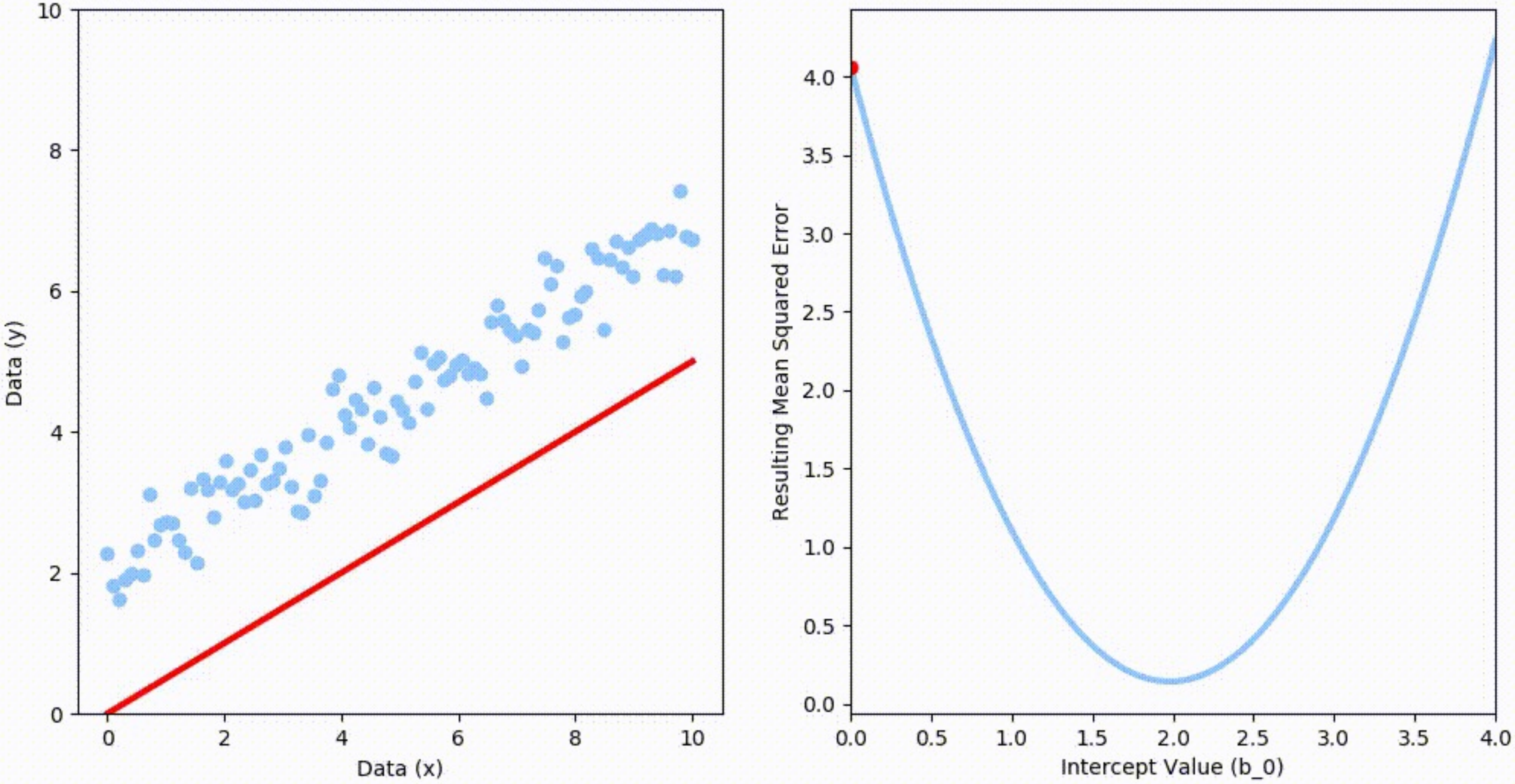
Mean square error (MSE)

$$\frac{1}{N}\sum_{i=1}^{N} \varepsilon_i^2$$

We have two parameters: the slope, and the intercept.
As we change the intercept, we can see that the MSE changes.



# We're looking for the intercept that minimizes the MSE.



# What loss makes sense for *logistic* regression?

- Our model predicts the *probability* of death for each patient.
- If we change the parameters, we change the predicted probabilities for each patient.

$y_1 = 1$ (dies) $\quad$ $y_2 = 0$ (survives) $\quad$ $y_3 = 0$ (survives)

$p_1$? x $p_2$? x $p_3$?

# What loss makes sense for *logistic* regression?

- Suppose we predict:
  - $p_1 = .1$
  - $p_2 = .9$
  - $p_3 = .7$
- Is this a good model? Why or why not?

$y_1 = 1$ (dies) $y_2 = 0$ (survives) $y_3 = 0$ (survives)

$p_1?$ x $p_2?$ x $p_3?$

# What loss makes sense for *logistic* regression?

- Suppose we predict:
  - $p_1 = .8$
  - $p_2 = .3$
  - $p_3 = .1$

- Is this a good model? Why or why not?

$y_1 = 1$ (dies) $\qquad y_2 = 0$ (survives) $\qquad y_3 = 0$ (survives)

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  - $p_1 = .8$
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- Our parameters affect *all* the predictions: changing a parameter to *decrease* $y_2$ may also *increase* $y_3$

$y_1 = 1$ (dies) | $y_2 = 0$ (survives) | $y_3 = 0$ (survives)

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# What loss makes sense for *logistic* regression?

- Suppose we predict:
  - $p_1 = .8$
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- What is the probability of the observed outcomes?

- This is called the likelihood. We want to find parameters that maximize it.

$y_1 = 1$ (dies) | $y_2 = 0$ (survives) | $y_3 = 0$ (survives)

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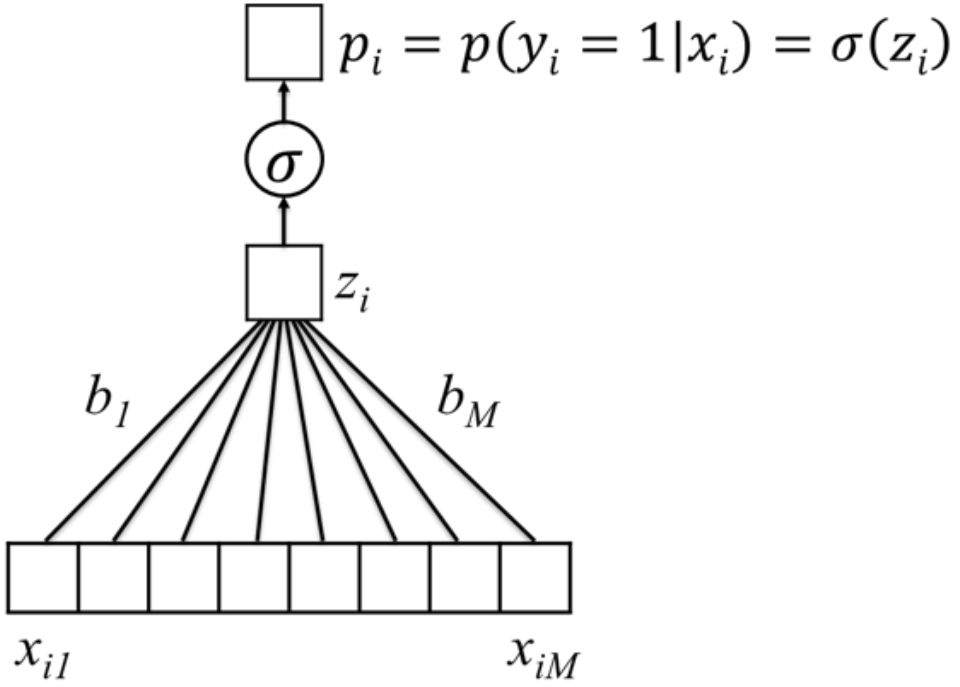
# Logistic Regression Likelihood

Probability of all the events $y_1, \ldots, y_N$ given our current model parameters




The predicted probability of $y_i$ is:

$$\begin{cases} p_i & \text{if } y_i = 1 \\ 1 - p_i & \text{if } y_i = 0 \end{cases}$$

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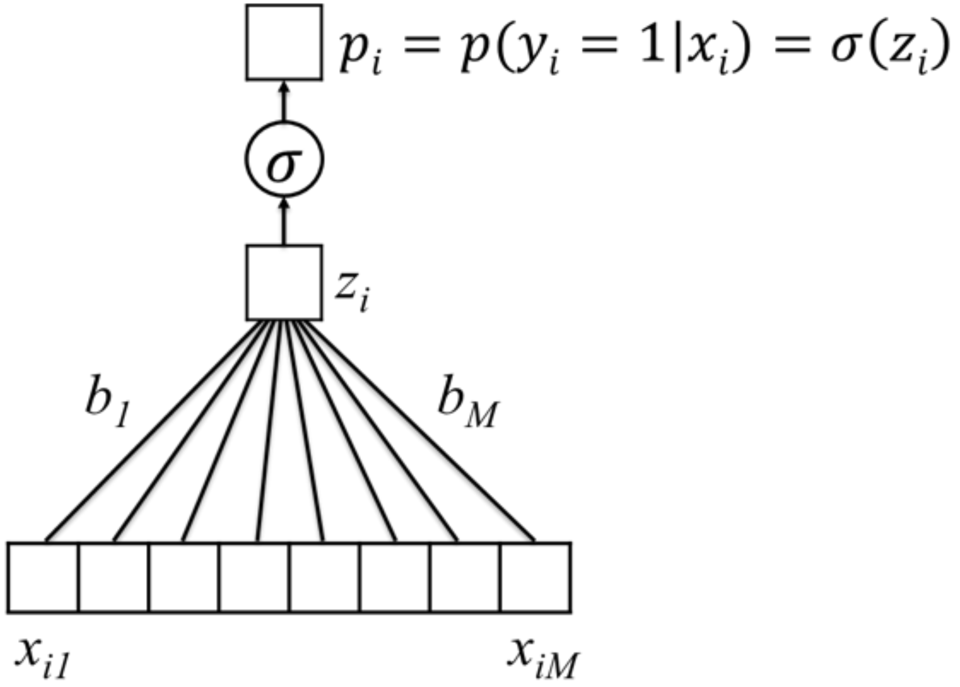


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$$p_i^{y_i}(1 - p_i)^{1-y_i}$$

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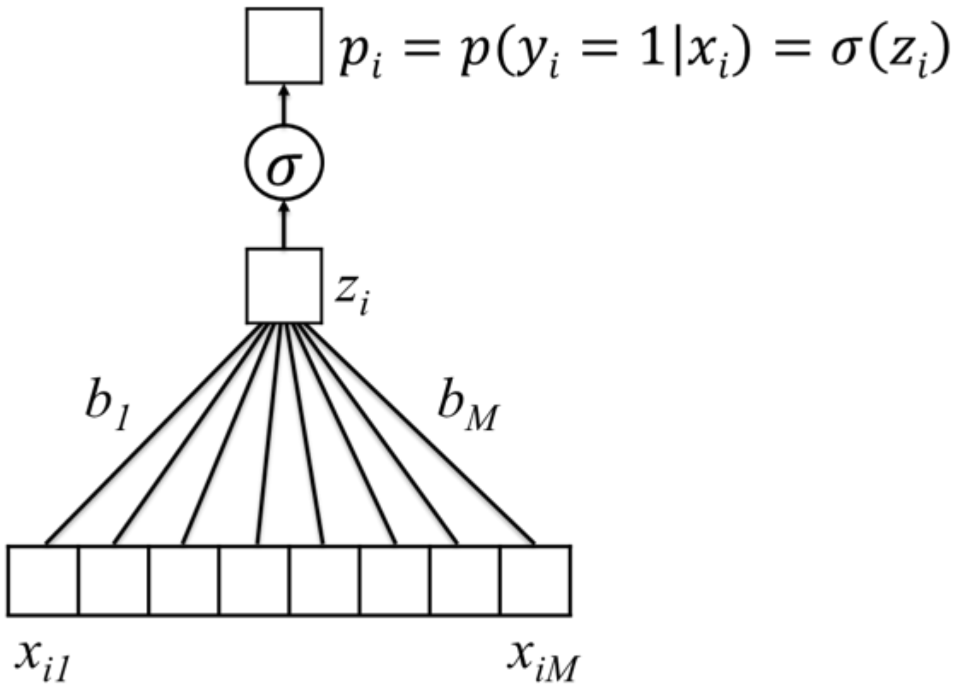


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Now we multiply all the predicted probabilities together:

$$\prod_{i=1}^{N}(p_i)^{y_i}(1 - p_i)^{1-y_i}$$

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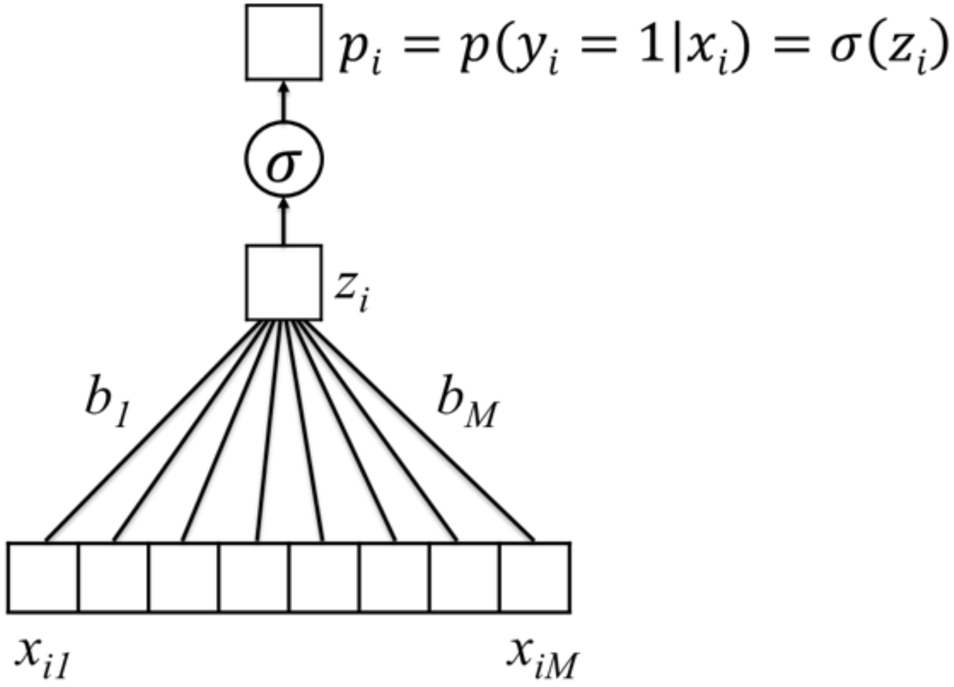


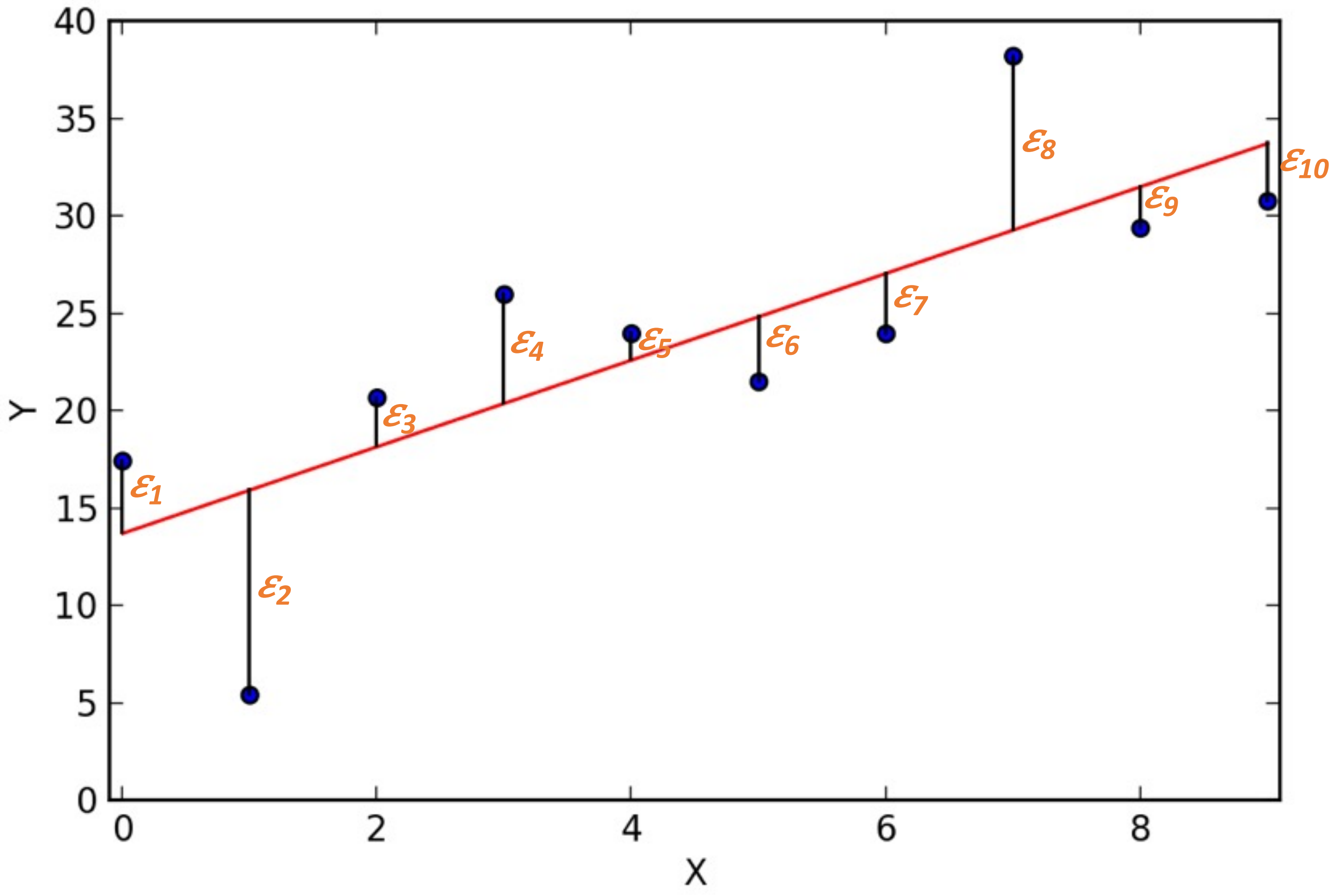

Now we multiply all the predicted probabilities together:

$$\prod_{i=1}^{N} (p_i)^{y_i}(1 - p_i)^{1-y_i}$$

Often we see $\sigma(z_i)$ substituted for $p_i$:

$$\prod_{i=1}^{N} \sigma(z_i)^{y_i}(1 - \sigma(z_i))^{1-y_i}$$

# Remember that for *linear* regression, we want to *minimize* the mean squared error.

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Again, here's the *likelihood*:

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1. For numerical stability, we instead work with the *log-likelihood*:

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2. And by convention / for consistency, we minimize the *negative* log-likelihood rather than maximizing the positive:

$$-\sum_{i=1}^{N} y_i \log \sigma(z_i) + (1-y_i) \log\bigl(1-\sigma(z_i)\bigr)$$

# Cross-Entropy Loss

Again, here's the *likelihood*:

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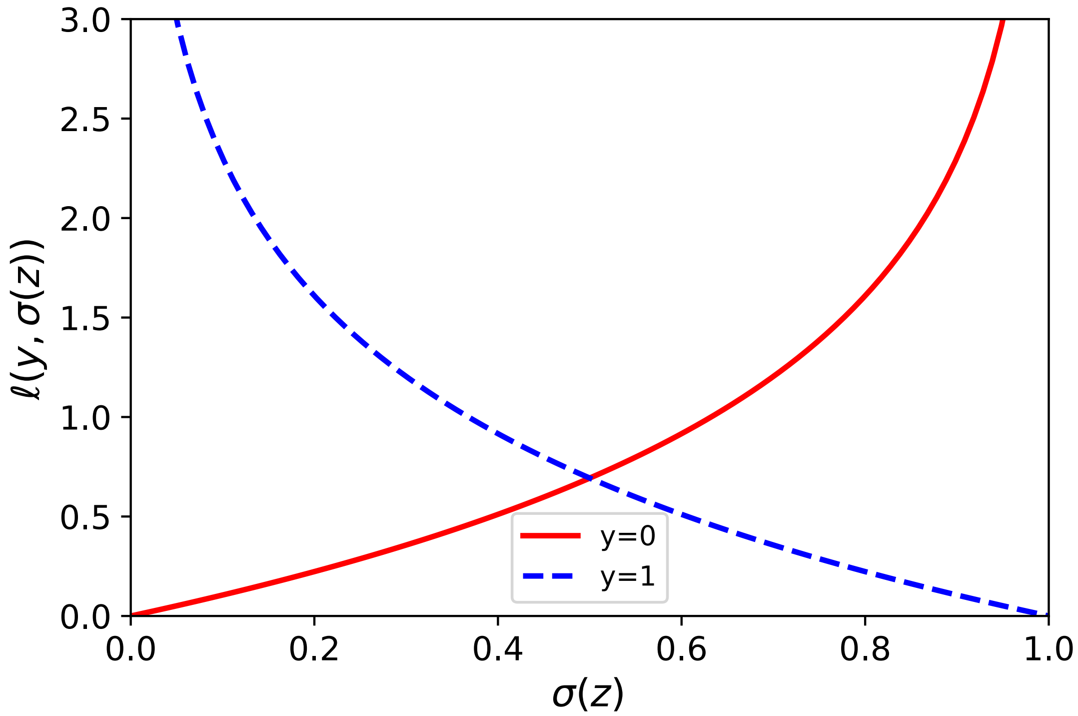
$$-\sum_{i=1}^{N} y_i \log \sigma(z_i) + (1-y_i)\log\big(1-\sigma(z_i)\big)$$

This is called the *cross-entropy loss.*
We are looking for parameters that make it as small as possible.
It is used for *all* the prediction tasks we consider in this course.



# *How* do we minimize the loss?

- The cross-entropy loss just tells us *what* quantity we should be minimizing
- In some cases (e.g. linear regression), we can solve for the minimum directly
- But, we’d like to have an approach that works even for very complex models

# Strategy: determine how small changes in parameters affect the loss

**MORTALITY PREDICTION WORKSHEET**

**COVARIATES**

| patient | age | age_normalized | female | temp | temp_normalized |
|---|---|---|---|---|---|
| 0 | 30.5 | -0.5 | 0 | 105.0 | 2.4 |
| 1 | 74.0 | 1.1 | 1 | 96.7 | -0.8 |
| 2 | 27.4 | -0.6 | 0 | 96.1 | -1.0 |
| 3 | 0.1 | -1.5 | 1 | 98.5 | -0.1 |
| 4 | 0.7 | -1.5 | 1 | 96.5 | -0.9 |
| 5 | 49.9 | 0.2 | 1 | 97.1 | -0.6 |
| 6 | 72.9 | 1.0 | 1 | 100.1 | 0.5 |
| 7 | 29.1 | -0.5 | 1 | 99.6 | 0.3 |
| 8 | 83.5 | 1.4 | 1 | 100.6 | 0.7 |
| 9 | 82.3 | 1.4 | 1 | 95.2 | -1.3 |
| 10 | 23.7 | -0.7 | 0 | 99.4 | 0.2 |
| 11 | 12.9 | -1.1 | 0 | 96.6 | -0.8 |
| 12 | 53.9 | 0.4 | 1 | 100.3 | 0.6 |
| 13 | 18.8 | -0.9 | 0 | 98.6 | 0.0 |
| 14 | 51.8 | 0.3 | 0 | 98.5 | -0.1 |
| 15 | 3.3 | -1.4 | 0 | 94.6 | -1.6 |
| 16 | 69.7 | 0.9 | 0 | 99.1 | 0.1 |
| 17 | 60.4 | 0.6 | 1 | 104.2 | 2.1 |
| 18 | 73.6 | 1.1 | 1 | 99.1 | 0.1 |
| 19 | 53.3 | 0.3 | 1 | 99.1 | 0.1 |

**OUTCOMES AND PREDICTIONS**

| mortality | predicted_log_odds | predicted_prob | prediction | correct? | loss |
|---|---|---|---|---|---|
| 1 | 0.00 | 0.50 | 0 | 0 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 1 | 0.00 | 0.50 | 0 | 0 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 1 | 0.00 | 0.50 | 0 | 0 | 0.3010 |
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| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
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| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |
| 1 | 0.00 | 0.50 | 0 | 0 | 0.3010 |
| 1 | 0.00 | 0.50 | 0 | 0 | 0.3010 |
| 0 | 0.00 | 0.50 | 0 | 1 | 0.3010 |

**PARAMETERS**

| | b_age | b_female | | b_temp | bias |
|---|---|---|---|---|---|
| guess | 0.00 | 0.00 | | 0.00 | 0.00 |
| optimal | [redacted] | | | | |

**PERFORMANCE**

| accuracy | avg_loss |
|---|---|
| 0.65 | 0.3010 |