Reinforcement Learning

August 2, 2019

MMCi Applied Data Science Block 5, Lecture 1

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$$Q^{\pi}(s, a) = R(s, a) + \gamma \sum_{s' \in S} P(s, a, s') \max_{a' \in A} Q^{\pi}(s', a')$$

$$Q(s_t, a_t; \theta^{new}) = Q(s_t, a_t; \theta^{old} + \alpha \cdot [r_t + \gamma \cdot \max_{a'} Q(s'_{t+1}, a'; \theta^{old}) - Q(s_t, a_t; \theta^{old})] \nabla_{\theta} Q(s_t, a_t; \theta)|_{\theta = \theta^{old}})$$

$$\Delta\theta = \alpha \cdot [r_t + \gamma \cdot \max_{a'} Q(s'_{t+1}, a'; \theta^{old}) - Q(s_t, a_t; \theta^{old})] \nabla_\theta Q(s_t, a_t; \theta)|_{\theta = \theta^{old}}$$

$$Q(s_t, a_t; \theta^{old} + \Delta \theta) \approx Q(s_t, a_t; \theta^{old}) + \alpha \cdot \left[r_t + \gamma \cdot \max_{a'} Q(s'_{t+1}, a'; \theta^{old}) - Q(s_t, a_t; \theta^{old}) \right] ||\nabla_{\theta} Q(s_t, a_t; \theta)||_{\theta = \theta^{old}} ||_2^2$$



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$$\Delta\theta = \alpha \left[r_t + \gamma \cdot \log X Q(s_t, a_t; \theta^{old}) - Q(s_t, a_t; \theta^{old}) \right] \nabla_\theta Q(s_t, a_t; \theta)|_{\theta = \theta^{old}}$$

$$Q(s_t, a_t; \mathbf{0}^{old} + \Delta \theta) \approx Q(s_t, a_t; \theta^{old}) + \alpha \cdot \left[r_t + \gamma \cdot \max_{a'} Q(s'_{t+1}, a'; \theta^{old}) - Q(s_t, a_t; \theta^{old}) \right] ||\nabla_{\theta} Q(s_t, a_t; \theta)||_{\theta = \theta^{old}} ||_2^2$$

Sequential Decision-Making

An agent

takes actions

based on the state of a system



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https://www.techradar.com/news/fords-robot-postman-will-deliver-packages-to-your-front-door

An agent

takes actions

based on the state of a system

Consider a clinician seeking to treat patients effectively while minimizing costs
The health of each patient is observed as a set of clinical variables
Assume the clinician has a fixed set of actions he or she can perform
Each action may (or may not) change the health state of the patient.
The clinician seeks a policy that achieves the best outcomes at the lowest average cost
Could apply to particular types of patients (e.g., diabetics) or in particular settings within a health system (e.g., ICU)



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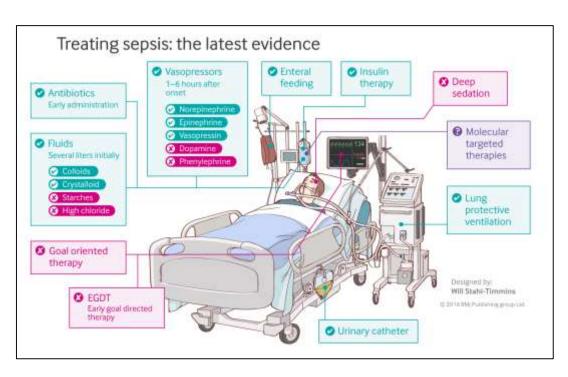
to maximize reward

☐ Consider a clinician seeking to treat patients effectively while minimizing costs ☐ The health of each patient is observed as a set of clinical variables ☐ Assume the clinician has a fixed set of actions he or she can perform ☐ Each action may (or may not) change the health state of the patient. ☐ The clinician seeks a policy that achieves the best outcomes at the lowest average cost ☐ Could apply to particular types of patients (e.g., diabetics) or in particular settings within a health system (e.g., ICU)

An agent

takes actions

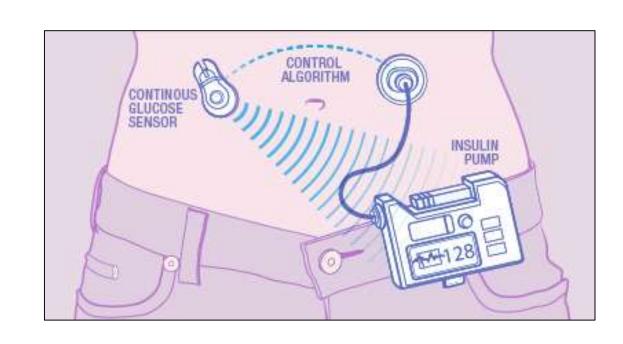
based on the state of a system



An agent

takes actions

based on the state of a system



An agent

A set $\{a^1, ..., a^m\}$ of possible actions takes actions

based on the state A vector s of measurements of a system

A function r(s, a, s') that quantifies the impact to maximize reward of transitioning from s to s' as well as the cost associated with action a

• There is randomness in how a patient in state s will respond to a given action, because the information in s yields an incomplete view of patient health

> • Let P(s, a, s') represent the probability that when a patient is in state s and the MD takes action a, the patient state will change to s'

☐ The clinician interacts with the patient through a series of actions, patient state
changes, and rewards/costs

...
$$S_{t-1}$$
 a_{t-1} r_{t-1} s_t a_t r_t s_{t+1} ...



 \Box Goal: Develop a **policy** that specifies the <u>optimal action</u> a to take when the patient is in state s. This policy might define the *standard of care*

☐ The optimal policy will maximize the average reward over time, with reward accounting for patient outcome and costs

☐ The policy should be non-myopic, in that it thinks ahead, to the long-run impact of actions

☐ The policy will typically weight impacts in the near-term more highly than what happens in the long run

The Challenge



- Unfortunately, we typically do not know P(s, a, s'). In other words, we don't know ahead of time how our actions will affect the state
- So, how can we learn a policy?
- We can just experience/try things, keep a record of outcomes, and adapt and adjust
- Reinforce actions for particular patient states that are rewarding
- <u>Discourage</u> actions that are expensive and yield poor outcomes
- In many ways this is how medicine works (over a long period of time)



- Reinforcement learning is the formalization of this challenge
- Addresses sequential decision making in an uncertain (stochastic) world



Medicine/Health



Monitoring/Maintenance of Factory



Investing



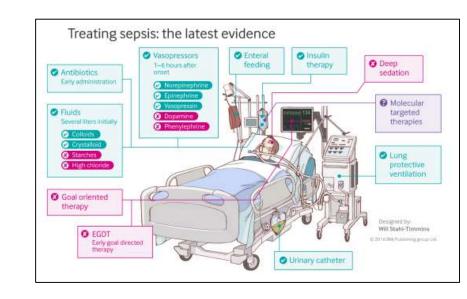






- ☐ The patient **state** *s* is defined by physiologic measures available at the bedside; as well as patient characteristics
 - <u>Discretize</u> the continuous range of the state values into *n* distinct bins
- ☐ The action *a* specifies the amount of fluid and amount of vasopressor given to the patient
 - <u>Discretize</u> the possible actions into m distinct bins

r(s, a, s') is specified by the clinician and/or health system to reward (punish) patient states that are desirable (undesirable)





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r(s, a, s') is specified by the clinician and/or health system to reward (punish) patient states that are desirable (undesirable)

	a^1	a^2	 a^m
s ¹			
s^2		$Q(s^2,a^2)$	
s^3			
s ⁴			
s ⁵			
S^n			



 \square Set initial values of Q(s,a) based on prior medical knowledge, at random, or to a default value

	a^1	a^2	 a^m
s^1	0	0	0
s ²	0	0	0
s^3	0	0	0
s ⁴	0	0	0
s ⁵	0	0	0
			0
s^n	0	0	0



- \square Set initial values of Q(s,a) based on prior medical knowledge, at random, or to a default value
- After initializing Q(s,a), take an action a when patient is in particular state s, and then observe the new state s' $(s,a) \rightarrow s'$, r(s,a,s')
- Assume that the prior/old value for the Q function when taking action a in state s is $Q^{old}(s, a)$
- ☐ We may consider the update rule:

$$Q^{new}(s, a) \leftarrow (1 - \alpha) \cdot Q^{old}(s, a) + \alpha \cdot r(s, a, s')$$
 where $\alpha \in (0,1]$

	a^1	a^2	 a^m
s^1	0	0	0
s^2 s^3	0	0	0
s^3	0	0	0
s^4	0	0	0
s ⁵	0	0	0
			0
s^n	0	0	0



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		a^1	a^2	 a^m
	s^1	0	0	0
	s^2	0	0	0
	s ³	0	0	0
	\$4	0	0	0
	s^5	0	0	0
	•••			0
	s^n	0	0	0



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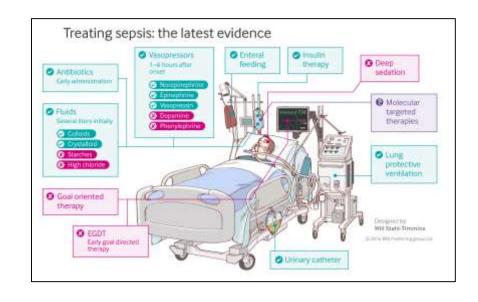
	a^1	a^2	•••	a^m
s^1	0	0		0
s^2	0	0		0
s^3	0	$\alpha \cdot r(s^3, a^2, s')$		0
s ⁴	0	0		0
s^5	0	0		0
•••				0
s^n	0	0		0

Simple, Intuitive Solution



$$Q^{new}(s,a) \leftarrow (1-\alpha) \cdot Q^{old}(s,a) + \alpha \cdot r(s,a,s'), \text{ with } \alpha \in (0,1]$$

- α is called the **learning rate**, and controls the relative balance between our old estimate $Q^{old}(s,a)$ and new information provided by r(s,a,s')
- \Box If α is large, we quickly replace our old estimate with new information
- However, due to possible randomness, we typically want to be conservative in updating $Q^{old}(s, a)$

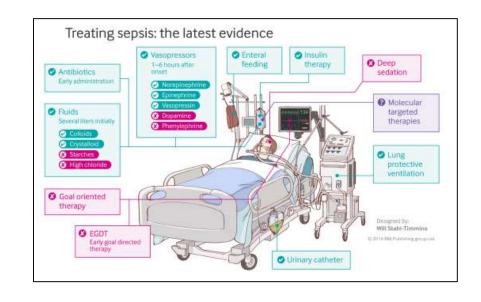


Problem: We're only looking one step ahead!



$$Q^{new}(s,a) \leftarrow (1-\alpha) \cdot Q^{old}(s,a) + \alpha \cdot r(s,a,s'), \text{ with } \alpha \in (0,1]$$

- ☐ This simple solution only accounts for the <u>immediate</u> reward r(s, a, s')
- □ Doesn't account for what may happen subsequently once the patient state changes to s'
- ☐ What if our decisions look good in the short-term, but ultimately cause the patient's condition to deteriorate?
- ☐ This rule leads to a *myopic* policy



Let's augment our reward to consider what happens next

$$Q^{new}(s, a) \leftarrow (1 - \alpha) \cdot Q^{old}(s, a) + \alpha \cdot r(s, a, s')$$
, with $\alpha \in (0, 1]$

- \Box Question: How much reward can we expect to earn going forward now that we're in state s?
- \square Answer: $\max_{a'} Q^{old}(s', a')$
- Our effective reward is how much we earn now, plus how much we expect to earn going forward if we choose the best actions available to us.
- ☐ So, let's consider the following modification to our update rule:

$$Q^{new}(s,a) \leftarrow (1-\alpha) \cdot Q^{old}(s,a) + \alpha \cdot [r(s,a,s') + \gamma \cdot \max_{a'} Q^{old}(s',a')]$$

 \Box The parameter γ controls how much we value immediate reward vs expected future reward

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$$Q^{new}(s, a) \leftarrow (1 - \alpha) \cdot Q^{old}(s, a) + \alpha \cdot r(s, a, s')$$
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 \Box The parameter γ is called the **discount factor**, and controls how much we value immediate reward versus expected future reward

Non-Myopic Update Rule

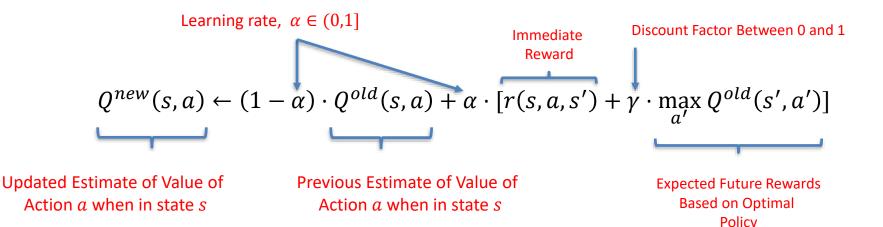
$$Q^{new}(s,a) \leftarrow (1-\alpha) \cdot Q^{old}(s,a) + \alpha \cdot [r(s,a,s') + \gamma \cdot \max_{a'} Q^{old}(s',a')]$$

- We have derived an algorithm for a clinician (or system) to learn based on experience
- ☐ This is **Q Learning**, a widely employed method for reinforcement learning
- \square Based on the learned matrix Q(s, a), we typically employ the following policy:

$$\pi(s) = \operatorname*{argmax}_{a} Q(s, a)$$

Expected Future Rewards
Based on Optimal
Policy

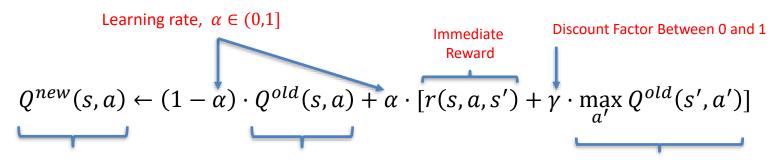
Q Learning



- lacktriangle Note: while we're learning, we won't always follow the action defined by $rgmax Q^{old}(s,a)$
- ☐ If we did, we would never learn more about alternative actions
- ☐ Most of the time, we'd like to advantage of actions we know are rewarding. But sometimes, we should explore alternative actions to see whether they might be better. This tradeoff is known as *exploration versus exploitation*.



Q Learning



Updated Estimate of Value of Action *a* when in state *s*

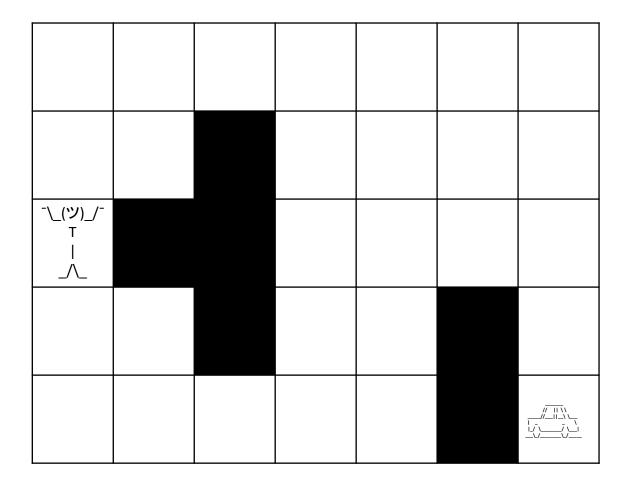
Previous Estimate of Value of Action *a* when in state *s*

Expected Future Rewards
Based on Optimal
Policy

The canonical RL example...

GRIDWORLD

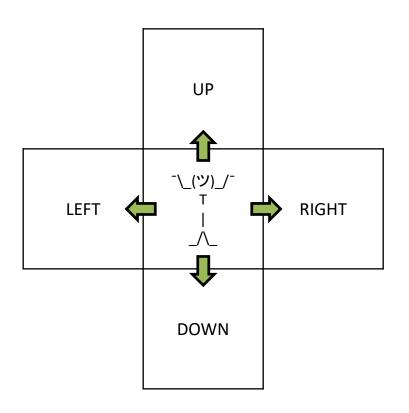




Gridworld: States

The states are the different squares of gridworld.





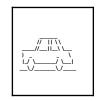
Gridworld: Actions

The actions are:

- Move left
- Move right
- Move up
- Move down



• 1 for reaching

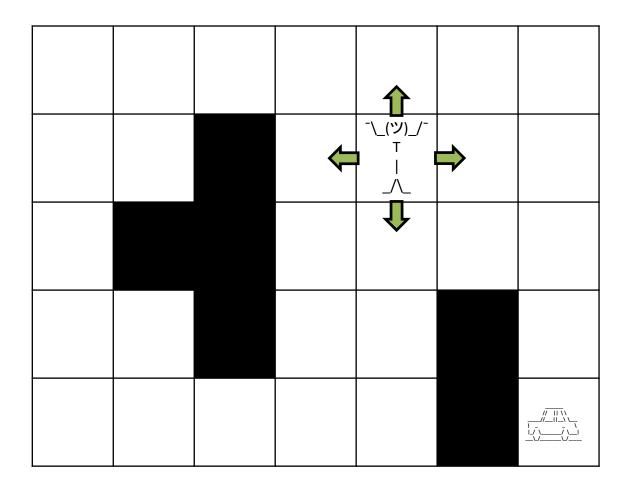


• 0 otherwise

Gridworld: Rewards

The goal is to reach the car. So our reward is:

$$r(s, a, s') = \begin{cases} 1, & s' = \text{car} \\ 0, & \text{otherwise} \end{cases}$$



GOAL:

Learn a policy $\pi: S \to A$

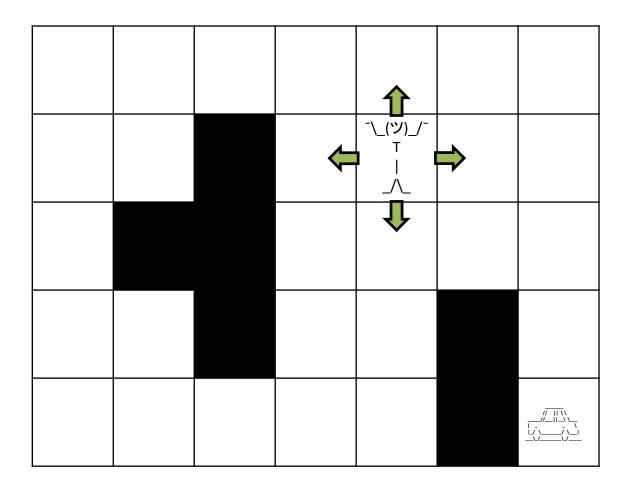
that maximizes expected reward over time

HOW?

Learn the value Q(s, a) of action a when in state s

Q(current square, *down*)





GOAL:

Learn a policy $\pi: S \to A$

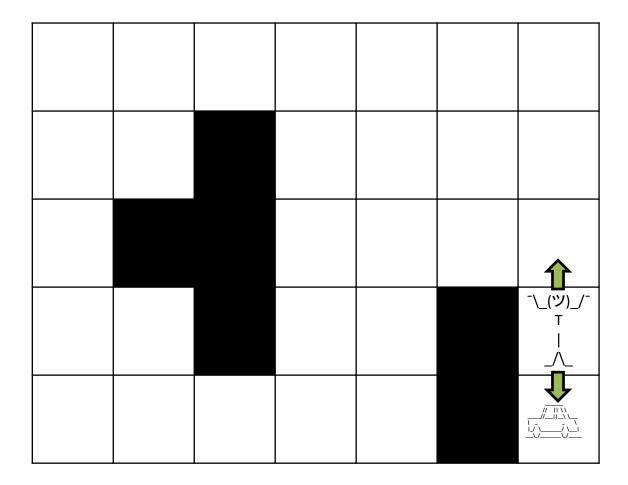
that gets us to the car as quickly as possible

HOW?

Learn the value Q(s, a) of action a when in state s

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GOAL:

Learn a policy $\pi: S \to A$

that gets us to the car as quickly as possible

HOW?

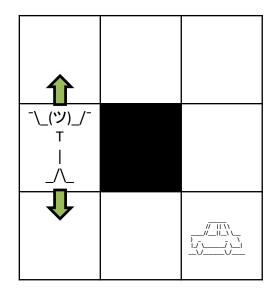
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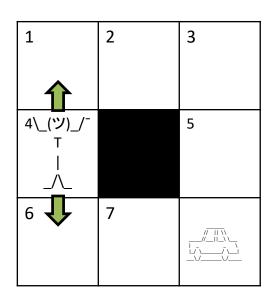
Gridworld Example

$$Q^{new}(s,a) \leftarrow (1-\alpha) \cdot Q^{old}(s,a) + \alpha \cdot [r(s,a,s') + \gamma \cdot \max_{a'} Q^{old}(s',a')]$$



$$Q^{new}(s,a) \leftarrow (1-1) \cdot Q^{old}(s,a) + 1 \cdot [r(s,a,s') + .8 \cdot \max_{a'} Q^{old}(s',a')]$$

$$r(s, a, s') = \begin{cases} 1, & s' = \text{car} \\ 0, & \text{otherwise} \end{cases}$$



$$Q^{new}(s,a) \leftarrow \mathbf{1} \cdot [r(s,a,s') + .8 \cdot \max_{a'} Q^{old}(s',a')]$$

$$r(s, a, s') = \begin{cases} 1, & s' = \text{car} \\ 0, & \text{otherwise} \end{cases}$$

1	2	3
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Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1				
Square 2				
Square 3				
Square 4				
Square 5				
Square 6				
Square 7				

$$Q^{new}(s,a) \leftarrow 1 \cdot [r(s,a,s') + .8 \cdot \max_{a'} Q^{old}(s',a')]$$

$$r(s, a, s') = \begin{cases} 1, & s' = \text{car} \\ 0, & \text{otherwise} \end{cases}$$

Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		0		0
Square 2	0	0		
Square 3	0			0
Square 4			0	0
Square 5			0	0
Square 6		0	0	
Square 7	0	0		

$$Q^{new}(s,a) \leftarrow 1 \cdot [r(s,a,s') + .8 \cdot \max_{a'} Q^{old}(s',a')]$$

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Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		0		0
Square 2	0	0		
Square 3	0			0
Square 4			0	0
Square 5			0	1
Square 6		0	0	
Square 7	0	1		

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$$r(s, a, s') = \begin{cases} 1, & s' = \text{car} \\ 0, & \text{otherwise} \end{cases}$$

Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		0		0
Square 2	0	0		
Square 3	0			.8
Square 4			0	0
Square 5			0	1
Square 6		.8	0	
Square 7	0	1		

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Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		0		0
Square 2	0	.8^2		
Square 3	0			.8
Square 4			0	.8^2
Square 5			.8^2	1
Square 6		.8	0	
Square 7	.8^2	1		

$Q^{new}(s,a) \leftarrow 1$	$\cdot [r(s, a)]$	$(a,s') + .8 \cdot \max_{a'} Q^{old}(s',a')$
$r(s,a,s') = \langle$	(1, (0,	s' = car otherwise

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6	7	/ \ \ //_ __

Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		.8^3		.8^3
Square 2	0	.8^2		
Square 3	.8^3			.8
Square 4			0	.8^2
Square 5			.8^2	1
Square 6		.8	.8^3	
Square 7	.8^2	1		

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Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		.8^3		.8^3
Square 2	.8^4	.8^2		
Square 3	.8^3			.8
Square 4			.8^4	.8^2
Square 5			.8^2	1
Square 6		.8	.8^3	
Square 7	.8^2	1		

 $Q(s,a) = \gamma^N$, where N is the fewest possible moves required to get to the car after taking action a from state s

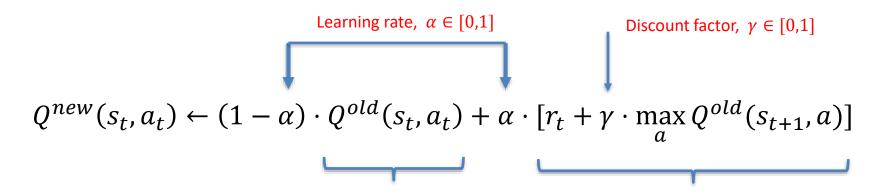
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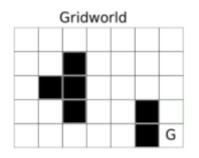
Q(s,a)	LEFT	RIGHT	UP	DOWN
Square 1		.8^3		.8^3
Square 2	.8^4	.8^2		
Square 3	.8^3			.8
Square 4			.8^4	.8^2
Square 5			.8^2	1
Square 6		.8	.8^3	
Square 7	.8^2	1		

Q-Learning: Represent Q Function as a Look-Up Table

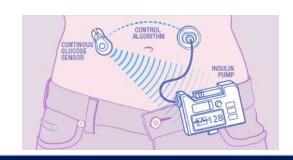


Previous Estimate of Value of Action a_t when in state s_t

Updated Estimate of Value of Action a_t when in state s_t After Observing r_t







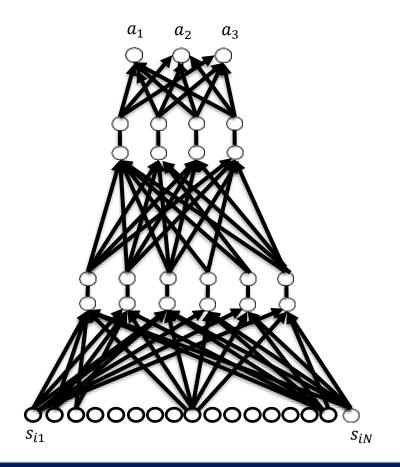
Limitations of Tabular Q Learning

$$Q^{new}(s_t, a_t) \leftarrow (1 - \alpha) \cdot Q^{old}(s_t, a_t) + \alpha \cdot [r_t + \gamma \cdot \max_{a} Q^{old}(s_{t+1}, a)]$$

☐ Simple update rule, but

- The Q function is stored as a table/matrix, which is impractical when considering a large number of states and actions
- No real capacity to generalize across sequence types, because the tabular Q function does not have a functional form

Deep Q Learning: Represent Q Function as a Deep Neural Network



☐ The Q-function modeled via a neural network

 \Box The state is input (e.g., an image with N pixels), and at the output we model the Q-function Q(s,a) for each possible action

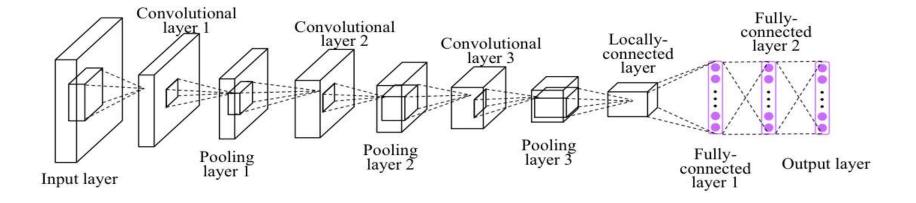
Represent the neural network model as $Q(s, a; \theta)$ where θ represent the neural network parameters we wish to learn

 \Box After we learn θ the policy is

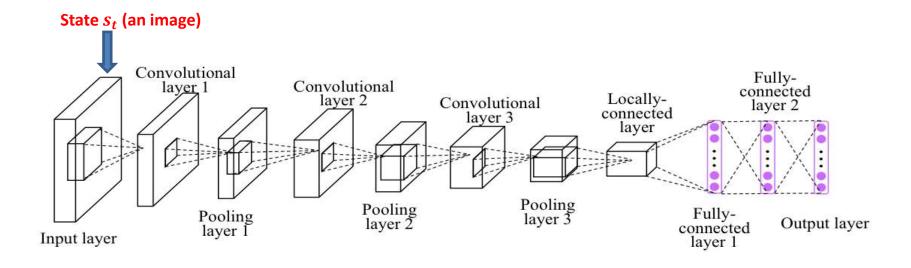
$$\pi(s) = \operatorname*{argmax}_{a} Q(s, a; \theta)$$

Power of Deep Q Networks for RL With Images

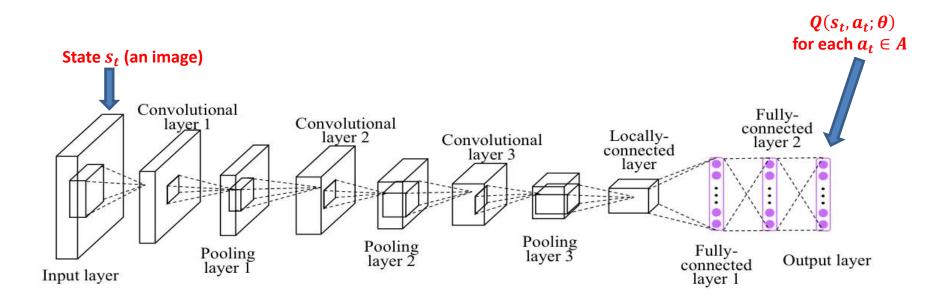
lacksquare Seek parameters heta that best approximate the Q function



Power of Deep Q Networks for RL With Images



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Atari Games





Final Thoughts

☐ Tabular Q Learning employs the update rule

$$Q^{new}(s_t, a_t) \leftarrow (1 - \alpha) \cdot Q^{old}(s_t, a_t) + \alpha \cdot [r_t + \gamma \cdot Q^{old}(s_{t+1}, a_{t+1})]$$

lacktriangle Deep Q Learning uses a deep neural network with parameters heta to approximate the Q function

Reinforcement learning has been studied for decades, and there are many other methods that we have not considered here, and are worth learning about for those interested

☐ There is also a rich theoretical foundation to RL, and therefore much is known about the fundamentals of these methods