9. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (2 pts)

Show that for the dimensionless velocity $\omega^\mu = u^\mu/c$

$$\left(\frac{\mathrm{d}\omega}{\mathrm{d}\tau}\right)^2 = \gamma^6\left[\left(\vec{\beta}\times\frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t}\right)^2 - \left(\frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t}\right)^2\right]$$

Exercise 2 (5 pts)

Consider an electron travelling in a circular orbit with constant angular velocity $\vec{\omega}_0$. (Eg. electron in constant magnetic field.)

i) From the relativistic formula in HW7 Ex. 1 derive $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$ as a function of the angles θ and ϕ. Hint: You can align your coordinate system such that $\vec{\beta} = \beta\hat{z}$, $\frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t} = \frac{\mathrm{d}\beta}{\mathrm{d}t}\hat{x}$, $\vec{\omega} = \omega\hat{y}$, and $\vec{n}\cdot\vec{\beta} = \beta\cos(\theta)$.

ii) What is the total radiated power? Hint: The relativistic, Liénard-power is given by

$$P = -\frac{e^2}{6\pi c}\left(\frac{\mathrm{d}\omega}{\mathrm{d}\tau}\right)^2$$

iii) Consider the extreme relativistic case, when $\beta \longrightarrow 1$. Show that

$$P = \frac{e^2 c}{6\pi}\left(\frac{E}{mc^2}\right)^4\frac{1}{r^2},$$

where r stand for the radius and the particle energy is $E = \gamma mc^2$.