Physics 412-3 Spring 2020

Problem Set 3

Due by 5 pm Friday May 15.

1) (5 pts) Consider a system of N identical particles subject to a potential which consists of an external potential as well as a pairwise interaction term between the particles. Prove that the permutation operator P_{ij} as defined in class commutes with the Hamiltonian.

2) (5 pts) a) N identical spin ½ particles are placed in a one-dimensional harmonic oscillator potential. What is the ground state energy? What is the Fermi energy?

b) Instead suppose we put in N identical non-interacting spin-1 particles, what is ground state energy?

c). N identical spin ½ particles are placed in a **3-dimensional** harmonic oscillator potential. In the limit that N is very large, write an approximation for the Fermi energy.

3.) (5 pts) Two **identical** fermions are occupying the ground state of the 1-d infinite square well potential

$$V(x) = \begin{Bmatrix} 0, 0 < x < a \\ \infty, \text{otherwise} \end{Bmatrix}.$$

a.) Write the state vector of the system (including the spin part).
b.) Now assume that at time t=0, the potential is abruptly modified to have a step:

$$V(x) = \begin{bmatrix} 0, 0 \le x \le \dfrac{a}{2} \\ V_0, \dfrac{a}{2} < x < a \\ \infty, \text{otherwise} \end{bmatrix}$$

for a time T, and then restored to the original potential. Using 1st order time-dependent perturbation theory, find the probability that immediately after these events, one particle occupies the 1st excited state and the other remains in the ground state.

c.) Write the state vector of the system that corresponds to the outcome discussed in b.)

4) (5 pts) Three non-interacting identical particles of spin ½ and mass μ are confined in a 2-d rectangular infinite potential well, with sides of length a and b, respectively, with $V = 0$ for $0 < x < a$, $0 < y < b$, and $V = \infty$ otherwise. You may assume $a < b$.

a) Find the ground state energy.
b) Let S_z be the z-component of total spin $\boldsymbol{S} = \boldsymbol{S_1} + \boldsymbol{S_2} + \boldsymbol{S_3}$. What are the possible values of S_z in the ground state? What is the degeneracy of the ground state?
c) In terms of the x- and y- spatial coordinates of each particle (x_1,y_1), (x_2,y_2), (x_3,y_3), where the subscript label $(_{1,2,3})$ denotes the particle #, write a physically acceptable ground state wave function for this system for one of the possible values of S_z you found in part (b).