HW 3

Monday, February 1, 2021 1:44 PM

2.) i)

17.1: $x_1' = x_1 - vt$

17.2: $x_2' = x_2$

17.3: $x_3' = x_3$

17.4: $t' = t$

Newton's 2nd Law: $F_j = m\ddot{x}_j$

$\dot{x}_1' = \dot{x}_1 - v$ $\quad\quad \ddot{x}_1' = \ddot{x}_1 \longrightarrow \ddot{x}_j' = \ddot{x}_j$

$F_j = m\ddot{x}_j = m\ddot{x}_j' = F_j' \longrightarrow$ invariant

17.6: $\dfrac{\partial}{\partial x_1} = \dfrac{\partial}{\partial x_1'} + \dfrac{1}{v}\dfrac{\partial}{\partial t'} \rightarrow$ incorrect

corrected $\rightarrow \dfrac{\partial \psi}{\partial t'} = \dfrac{\partial \psi}{\partial t} + v\dfrac{\partial \psi}{\partial x_1}$

ii)

17.12: $$\underbrace{\left\{\nabla'^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right\}}_{①}\psi = -\frac{1}{v^2}\frac{\partial^2\psi}{\partial t'^2} - \frac{2}{v}\frac{\partial^2\psi}{\partial x_1'\partial t'}$$

$$\neq 0$$

if $\left\{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}\psi = 0$

then ① $\neq 0$

$$\frac{\partial^2\psi}{\partial t'^2} = \frac{\partial^2\psi}{\partial t^2} + 2v\frac{\partial^2\psi}{\partial x_1\partial t} + v^2\frac{\partial^2\psi}{\partial x_1^2}$$

iii)

17.23: $x_0' = \gamma(x_0 - \beta x_1)$

17.24: $x_1' = \gamma(x_1 - \beta x_0)$

17.25: $x_2' = x_2$

17.26 ...

11.26: $x_3' = x_3$

$$\frac{\partial^2 \psi}{\partial x_i'^2} = \frac{\partial}{\partial x_0}\left\{\left[\gamma(x_0-\beta x_1)\right]\psi\right\} - \frac{\partial}{\partial x_1}\left\{\left[\gamma(x_1-\beta x_0)\right]\psi\right\}$$

$$-\frac{\partial^2 \psi}{\partial x_2'^2} - \frac{\partial^2 \psi}{\partial x_3'^2}$$

$$= \gamma^2 \frac{\partial^2 \psi}{\partial x_0'^2} - \beta\gamma^2 \frac{\partial^2 \psi}{\partial x_0' \partial x_1'} - \beta\gamma^2 \frac{\partial^2 \psi}{\partial x_1' \partial x_0'}$$

$$+\beta^2\gamma^2 \frac{\partial^2 \psi}{\partial x_1'^2} - \beta^2\gamma^2 \frac{\partial^2 \psi}{\partial x_0'^2} + \beta\gamma^2 \frac{\partial^2 \psi}{\partial x_1' \partial x_0'}$$

$$+\beta\gamma^2 \frac{\partial^2 \psi}{\partial x_0' \partial x_1'} - \gamma^2 \frac{\partial^2 \psi}{\partial x_1'^2} - \frac{\partial^2 \psi}{\partial x_2'^2} - \frac{\partial^2 \psi}{\partial x_3'^2}$$

$$= \gamma^2 \partial^2 \psi - \beta^2\gamma^2 \partial^2 \psi - \gamma^2 \partial^2 \psi$$

$$\partial x_0'^2 \quad \partial x_0'^2 \quad \partial x_1'^2$$

$$+\beta^2\gamma^2\frac{\partial^2\psi}{\partial x_1'^2} - \frac{\partial^2\psi}{\partial x_2'^2} - \frac{\partial^2\psi}{\partial x_3'^2} = 0$$

invariant

3.) i)

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \qquad A^\mu = (\phi, \vec{A})$$

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla\phi$$

$$= \left(-\frac{1}{c}\frac{\partial A_x}{\partial t} - \frac{\partial\phi}{\partial x}, -\frac{1}{c}\frac{\partial A_y}{\partial t} - \frac{\partial\phi}{\partial y}, -\frac{1}{c}\frac{\partial A_z}{\partial t} - \frac{\partial\phi}{\partial z}\right)$$

$$= (-\partial^0 A^1 + \partial^1 A^0, -\partial^0 A^2 + \partial^2 A^0,$$

$$\partial^0 A^3 + \partial^3 A^0) = (E \quad E \quad E)$$

$$-\partial A + \partial A) = (E_x, E_y, E_z)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$= (-\partial^2 A^3 + \partial^3 A^2, -\partial^3 A^1 + \partial^1 A^3, -\partial^1 A^2 - \partial^2 A^1)$$

$$= (B_x, B_y, B_z)$$

$$F^{\mu\nu} = \begin{bmatrix} F^{00} & F^{01} & F^{02} & F^{03} \\ F^{10} & F^{11} & F^{12} & F^{13} \\ F^{20} & F^{21} & F^{22} & F^{23} \\ F^{30} & F^{31} & F^{32} & F^{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \end{bmatrix}$$

$$\begin{bmatrix} E_z & -B_y & B_x & 0 \end{bmatrix}$$

ii) $F^*_{\mu\nu} \equiv -\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} = -\varepsilon_{\mu\nu\alpha\beta}\partial^\alpha A^\beta$

$\varepsilon_{0123} = +1 \qquad \varepsilon_{\mu\nu\alpha\beta} = -\varepsilon_{\nu\mu\alpha\beta} = \varepsilon_{\mu\nu\beta\alpha}$

$F^*_{00} = F^*_{11} = F^*_{22} = F^*_{33} = 0$ repeating index

$F^*_{01} = -\varepsilon_{0123}\partial^2 A^3 - \varepsilon_{0132}\partial^3 A^2 = -\partial^2 A^3 + \partial^3 A^2 = B_x$

$F^*_{02} = -\varepsilon_{0213}\partial^1 A^3 - \varepsilon_{0231}\partial^3 A^1 = \partial^1 A^3 - \partial^3 A^1 = B_y$

$F^*_{03} = -\varepsilon_{0312}\partial^1 A^2 - \varepsilon_{0321}\partial^2 A^1 = -\partial^1 A^2 + \partial^2 A^1 = B_z$

$F^*_{12} = -\varepsilon \quad \partial^3 A^0 - \varepsilon \quad \partial^0 A^3 - \partial^0 A^3 \quad \partial^3 A^0$

$$\ldots_{12} \ldots_{1230} \ldots \varepsilon_{1203}\, \partial A - \partial A - \partial A = E_z$$

$$F^*_{13} = -\varepsilon_{1320}\, \partial^2 A^0 - \varepsilon_{1302}\, \partial^0 A^2 = \partial^0 A^2 - \partial^2 A^0 = -E_y$$

$$F^*_{23} = -\varepsilon_{2310}\, \partial^1 A^0 - \varepsilon_{2301}\, \partial^0 A^1 = \partial^1 A^0 - \partial^0 A^1 = E_x$$

We know that F must be anti-symmetric, therefore we don't need to calculate the rest

$$F^*_{\mu\nu} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & E_z & -E_y \\ -B_y & -E_z & 0 & E_x \\ -B_z & E_y & -E_x & 0 \end{bmatrix}$$

1.)

18.87: $[S_i, S_j] = \varepsilon_{ijk} S_k$

18.88: $[S_i, K_j] = \varepsilon_{ijk} K_k$

18.89: $[K_i, K_j] = -\varepsilon_{ijk} S_k$

18.75: $S_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

18.76: $S_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

18.77: $S_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

18.78: $K_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

'8.79: $K_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

'8.80: $K_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

Define $L^{ij} = -L^{ji}$ $\quad i,j = 0,1,2,3$

where $S_i = \frac{-1}{2}\varepsilon_{ijk} L_j{}^k$

and $K_i = L_0{}^i$

$$[L^{ij}, L^{kl}] = \delta^{ik} L^{jl} - \delta^{jk} L^{il} + \delta^{jl} L^{ik} - \delta^{il} L^{jk}$$

$$[S_i, S_j] = \left[-\frac{1}{2}\varepsilon_{ijk} L_j^{\ k}, -\frac{1}{2}\varepsilon_{ijk} L_k^{\ i}\right] = \varepsilon_{ijk} S_k$$

$$[S_i, K_j] = \left[-\frac{1}{2}\varepsilon_{ijk} L_j^{\ k}, L_0^{\ j}\right] = \varepsilon_{ijk} K_k$$

$$[K_i, K_j] = [L_0^{\ i}, L_0^{\ j}] = -\varepsilon_{ijk} S_k$$