Hw 4

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1.) $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

Show $ds^2 = ds'^2$ $\quad y = y' \quad z = z'$

$$x = \gamma(x' - vt') \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$ds^2 = c^2\gamma^2\left(dt' + \frac{v\,dx'}{c^2}\right)^2 - \gamma^2(dx' + v\,dt')^2$$

$$- dy'^2 - dz'^2$$

$$= c^2\gamma^2\left(dt'^2 + \frac{v^2dx'^2}{c^4} + \frac{2v\,dx'dt'}{c^2}\right)$$

$$- \gamma^2(dx'^2 + v^2dt'^2 + 2v\,dx'dt') - dy'^2 - dz'^2$$

$$= \gamma^2\left(c^2dt'^2 + \frac{v^2dx'^2}{c^2} + \cancel{2v\,dx'dt'}\right.$$

$$\left. - dx'^2 - v^2dt'^2 \cancel{- 2v\,dx'dt'}\right) - dy'^2 - dz'^2$$

$$= \gamma^2\left[dt'^2(c^2 - v^2) - dx'^2\left(1 - \frac{v^2}{c^2}\right)\right] - dy'^2 - dz'^2$$

$$= \gamma^2\left[dt'^2\left(c^2 - v^2\right) - dx'^2\left(1 - \frac{v^2}{c^2}\right)\right] - dy'^2 - dz'^2$$

$$= \gamma^2\Big[c^2 dt'^2 \underbrace{\left(1 - \frac{v^2}{c^2}\right)}_{1/\gamma^2} - dx'^2 \underbrace{\left(1 - \frac{v^2}{c^2}\right)}_{1/\gamma^2}\Big] - dy'^2 - dz'^2$$

$$= c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2$$

2.) $x = \gamma(x' - vt')$ $\quad t = \gamma\left(t' - \frac{vx'}{c^2}\right)$

$ct(\sigma) = \lambda \sinh(\sigma)$ $\quad x(\sigma) = \lambda \cosh(\sigma)$

i.) $ds^2 = c^2 dt^2 - dx^2$

$$dct(\sigma) = \lambda \sinh(\sigma)\, d\sigma$$

$$dx(\sigma) = \lambda \cosh(\sigma)\, d\sigma$$

$$\int ds = \int dct - \int dx$$

$$= \int_0^\sigma \lambda \sinh(\sigma')\,d\sigma' - \int_0^\sigma \lambda \cosh(\sigma')\,d\sigma'$$

$$= \frac{\lambda}{\sigma}\cosh(\sigma) - \frac{\lambda}{\sigma}\sinh(\sigma)$$

ii.) $\frac{dx}{d\sigma} = \lambda \sinh(\sigma)$

3.) $\vec{a} = \vec{g}$

Rindler Coordinates:

$$t' = x \sinh(at)$$

$$x' = x \cosh(at)$$

$$x = \sqrt{x'^2 - t'^2}$$

$$t = \frac{1}{a} \tanh^{-1}\left(\frac{t'}{x'}\right)$$

$$dx = \gamma(dx' - v\,dt')$$

$$dt = \gamma(dt' - \frac{v}{c^2}dx')$$

$$u = \frac{u' - v}{1 - \frac{u'v}{\ldots}}$$

u' = velocity of rest frame

$1 - \frac{\ }{c^2}$

u = vel. of moving ref. frame

$$du = \frac{du'\left(1 - \frac{u'v}{c^2}\right) + (u' - v)\frac{v\,du'}{c^2}}{1 - \frac{u'v}{c^2}}$$

$$= \frac{du'}{\gamma^2\left(1 - \frac{u'v}{c^2}\right)^2}$$

$$\frac{du}{dt} = a = \frac{\frac{du'}{dt}}{\gamma^3\left(1 - \frac{u'v}{c^2}\right)^3} = \frac{a'}{\gamma^3\left(1 - \frac{u'v}{c^2}\right)^3}$$

in 1-D motion: $u' = v$

$$a = \gamma^3 a' = \frac{d(\gamma v)}{dt'} = v\frac{d\gamma}{dt'} + \gamma\frac{dv}{dt'}$$

$$= \gamma a'\left(1 + \frac{\gamma^2 v^2}{c^2}\right)$$

$$\int d(\gamma v) = \int a\,dt'$$

$$\boxed{\gamma v = a t'}$$

$$\gamma = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}} = \sqrt{1+\frac{a^2 t'^2}{c^2}}$$

$$t' = \frac{\gamma v}{a} = \frac{v}{g}\sqrt{\frac{1}{1-\frac{v^2}{c^2}}} \qquad \boxed{i}$$

$$\boxed{ii} \quad t = \gamma\left(t' - \frac{xv}{c^2}\right) \qquad ?$$

$$\underline{iii} \quad t' = \frac{c}{4g}\sqrt{\frac{1}{1-\frac{\cancel{c^2}/4}{\cancel{c^2}}}} = \frac{c}{4g}\sqrt{\frac{1}{3/4}}$$

$$= \frac{c}{4g}\sqrt{4/3}$$