

Random walker

(1)



random walker: confined to 1D line, randomly steps to the left or to the right

step length = a

prob to step right = p

walker takes n steps right, n' steps left

$n + n' = N$ total steps

What is the displacement, x , of the walker?

$$x = na - n'a$$

$$= a(n - n')$$

$$= a(n - (N - n))$$

$$= \boxed{a(2n - N)}$$

What is the average distance covered after N steps? (2)

This is just the expected value of the displacement

$$\langle x \rangle = \langle a(2n - N) \rangle$$

$$= \langle 2an \rangle - \langle aN \rangle$$

← this is a constant so

$$= 2a\langle n \rangle - aN$$

$$\langle aN \rangle = aN$$

↑ this is a binomial variable so $\langle n \rangle = Np$

$$= 2aNp - aN$$

$$\boxed{\langle x \rangle = aN(2p - 1)}$$

What is the variance of the displacement?

$$\sigma^2 \equiv \langle (x - \langle x \rangle)^2 \rangle = \langle (a(2n - N) - aN(2p - 1))^2 \rangle$$

definition ↪

$$= \langle (2na - aN - 2pNa + aN)^2 \rangle$$

$$= \langle (2a(n - Np))^2 \rangle$$

binomial, $Np = \langle n \rangle$

$$= 4a^2 \langle (n - \langle n \rangle)^2 \rangle$$

this is the variance of the binomial variable n , $\sigma_n^2 = Np(1-p)$

$$\boxed{\langle (x - \langle x \rangle)^2 \rangle = 4a^2 Np(1-p)}$$

(3)

Note :

$$\textcircled{1} \quad \frac{\sigma_x}{\langle x \rangle} = \frac{(aN(2p-1))^{\frac{1}{2}}}{4a^2 p(1-p)N} \sim \frac{1}{\sqrt{N}} \quad \text{we will see this again } \frac{1}{\sqrt{N}} \text{ again!}$$

② If left & right steps are equally probable, $p = \frac{1}{2}$

$$\langle x \rangle = aN(2 \cdot \frac{1}{2} - 1) = 0$$

$$\langle (x - \langle x \rangle)^2 \rangle = 4a^2 N \frac{1}{2} (1 - \frac{1}{2}) = Na^2$$

- On average you don't go anywhere, but you are still travelling