Physics 412-2 (Spring 2020) **Problem Set 5** Professor Gerald Gabrielse

Posted: 9 Feb. 2020 Due: 9 am Monday, 17 Feb. 2020

The problem sets are a serious part of the learning experience in this class. The problem sets will sometimes deliberately range away from what can be covered in the lectures in some cases. The goal to expose you to basic concepts and important examples of quantum mechanics. Problems will often be divided into many small parts to guide your through a solution. Corrections to the assignments, if needed, will be posted to the class web page. Your solutions should be placed in the box outside my office.

1. **Gauge Invariance:** Start with the generalized Hamiltonian

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\Phi \tag{1}$$

where $\vec{A} = \vec{A}(\vec{x}, t)$.

(a) Evaluate $m\partial < \vec{x} > /\partial t$ and express your answer in terms of the operator $\vec{\pi} = \vec{p} - q\vec{A}$.

(b) Evaluate $m\partial^2 < \vec{x} > /\partial t^2$, expressing your result in terms of the invariants that we labeled as $\vec{E}$ and $\vec{B}$. Compare with the appropriate, corresponding classical equation, thereby identifying the electric and magnetic field operators.

2. **Uniform Magnetic Field:**

(a) Show that $\vec{A} = \frac{1}{2}\vec{B}\, x\, \vec{r}$ is the vector potential for a spatially uniform magnetic field $\vec{B}$.

(b) Show that this vector potential is in the Coulomb gauge, with $\vec{\nabla} \cdot \vec{A} = 0$.

3. **A Trivial Perturbation to the Harmonic Oscillator:**

(a) Consider the trivial oscillator perturbation $V = cz^2$ that is added to the Hamiltonian of a harmonic oscillator. What is the condition for this to be considered a small perturbation?

(b) Compare the exact solution for this V (i.e. no use of perturbation theory) to your perturbation solution.

4. **The Anharmonic Oscillator:**

(a) Adding a small term $V = az^4$ to the Hamiltonian for a harmonic oscillator produces an anharmonic oscillator. What is the required condition for non-degenerate state perturbation theory to be appropriate.

(b) How do the levels of the oscillator then shift due to this perturbation?

(c) How do the levels of the harmonic oscillator shift if the perturbation is instead $V = bz^3$?

(d) Reflect upon the importance of party symmetry for the two versions of the anharmonic oscillator considered in this problem.