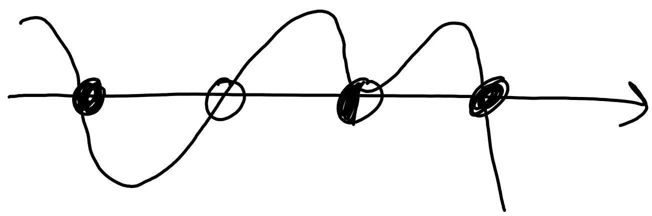
HW 1

Sunday, April 10, 2022 11:05 PM

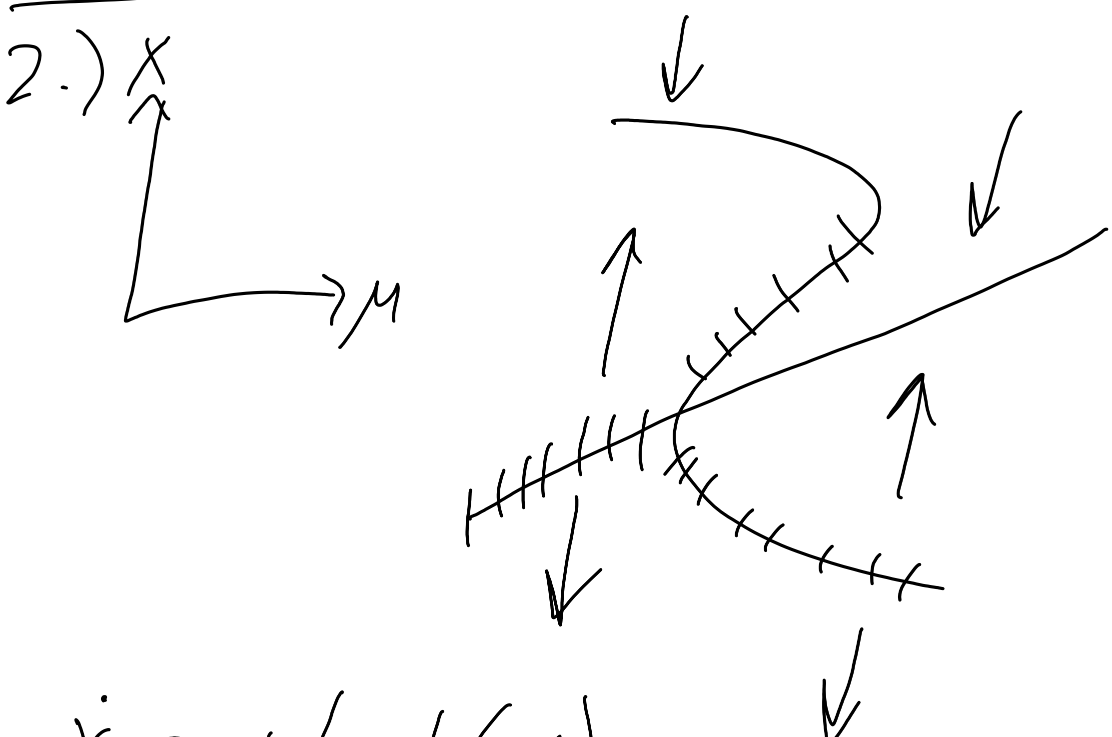
1.) a.)



$$\dot{x} = -x(x^2-1)(x-2)(x-1)$$

b.) NO

2.)



$$\dot{x} = \mu + \tanh(\mu)$$

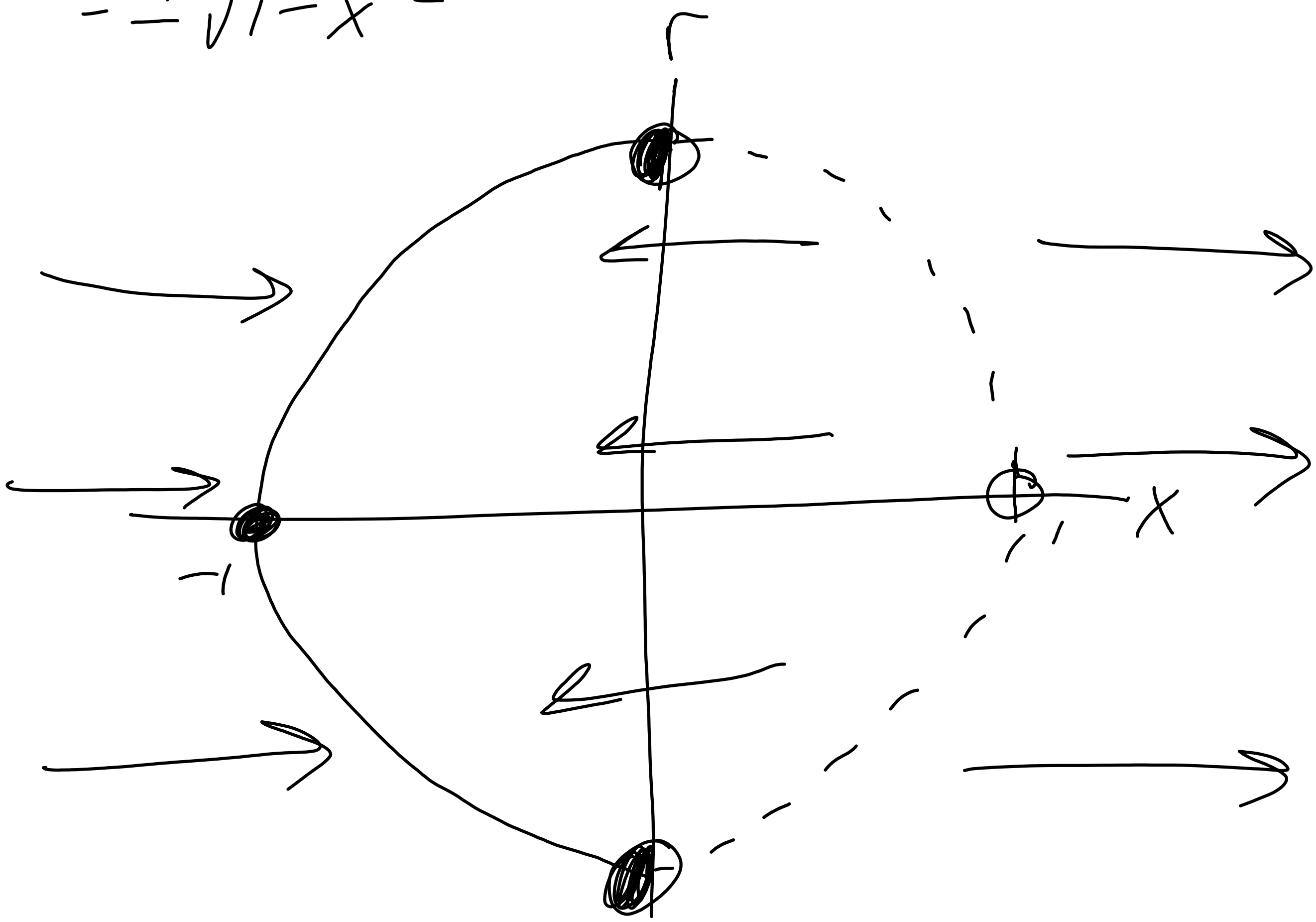
Saddle-Node Bifurcation

3.) a.) $\dot{x} = -x(x^2-1+r^2) = f(x)$

$f(x) = 0 \qquad -x(x^2 - 1 + r^2) = 0$

$x^2 = 1 - r^2 \qquad x = \pm\sqrt{1-r^2},\ 0$

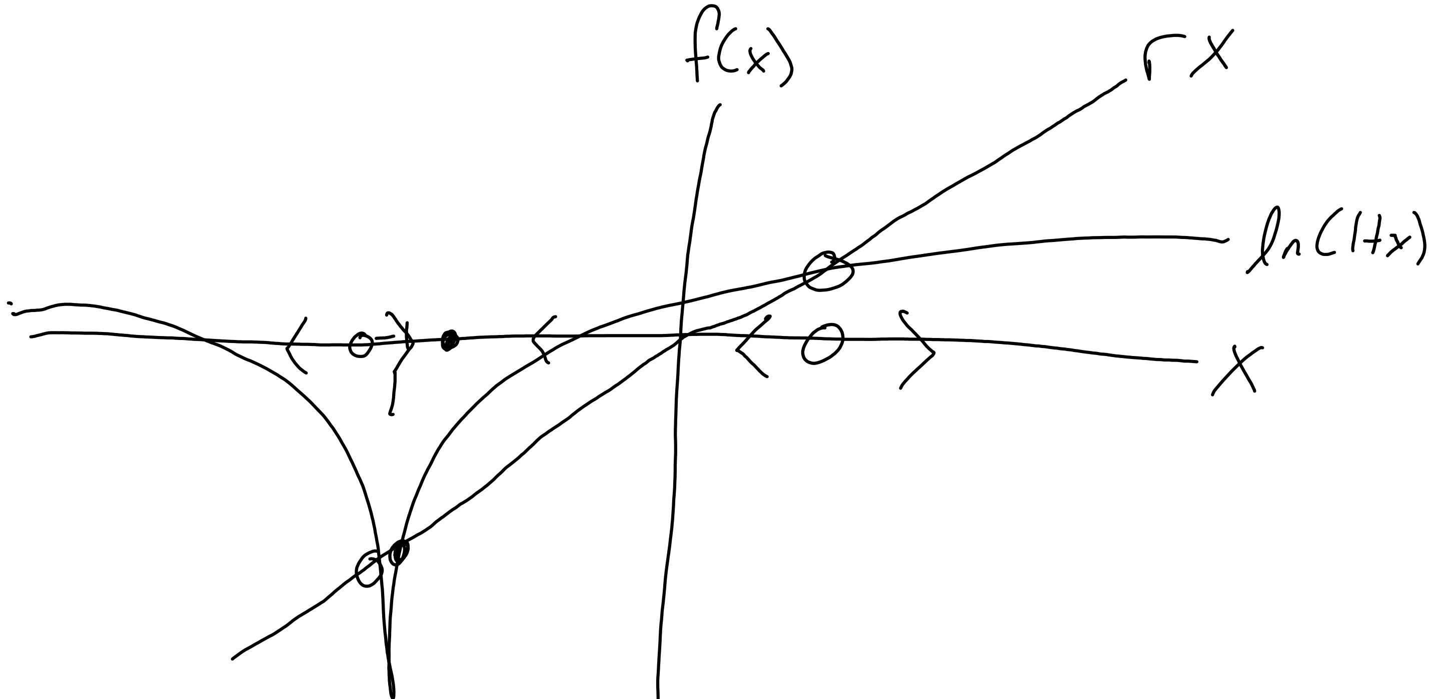
$r = \pm\sqrt{1-x^2}$

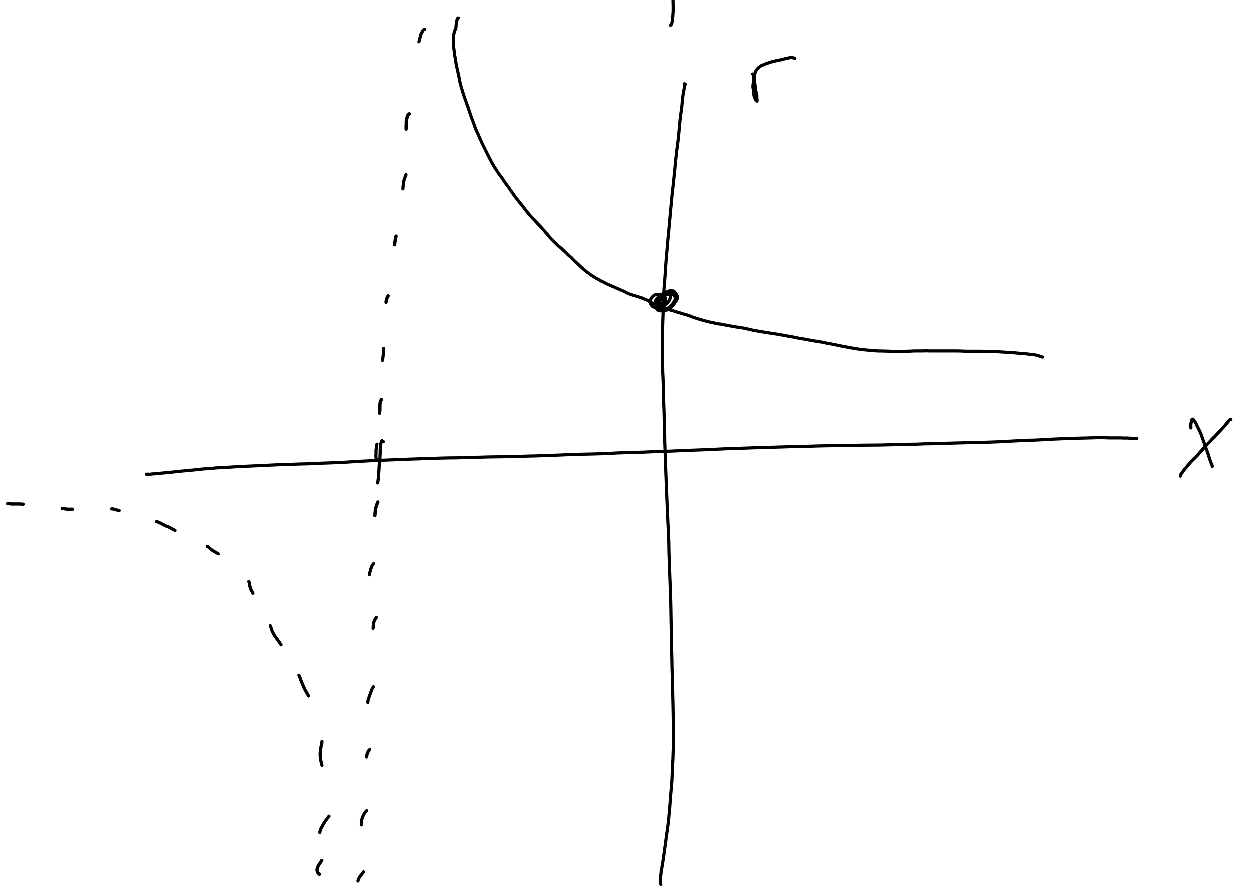


Saddle Node Bifurcation

b.) $\dot{x} = rx - \ln(1+x) = f(x)$

$f(x) = 0 \qquad rx = \ln(1+x) \qquad x^* = 0$





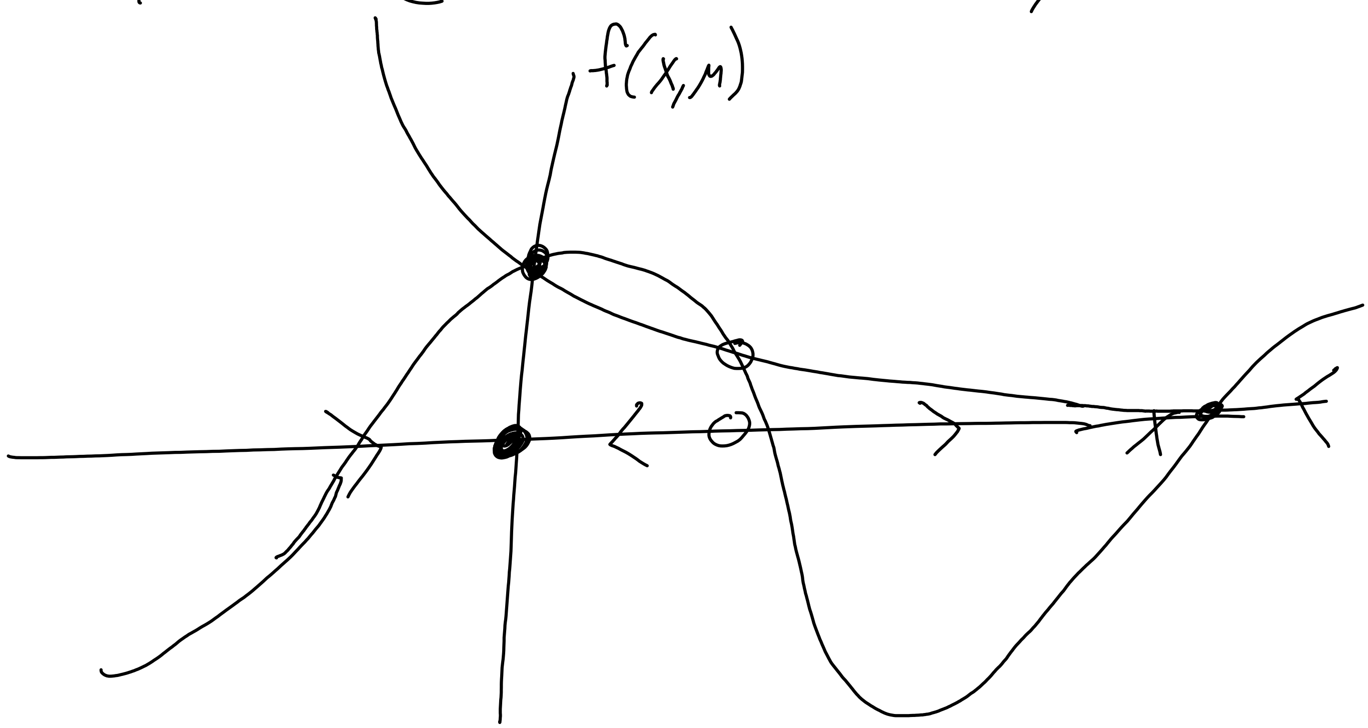
$$f'(x) = r - \frac{1}{1+x} = 0 \qquad r = \frac{1}{1+x}$$

$$\frac{1}{r} = 1+x \qquad x = \frac{1}{r} - 1 \qquad r_c = 1$$

4.) $\dot{x} = f(x,\mu) = e^{-x} - \cos(x-\mu)$

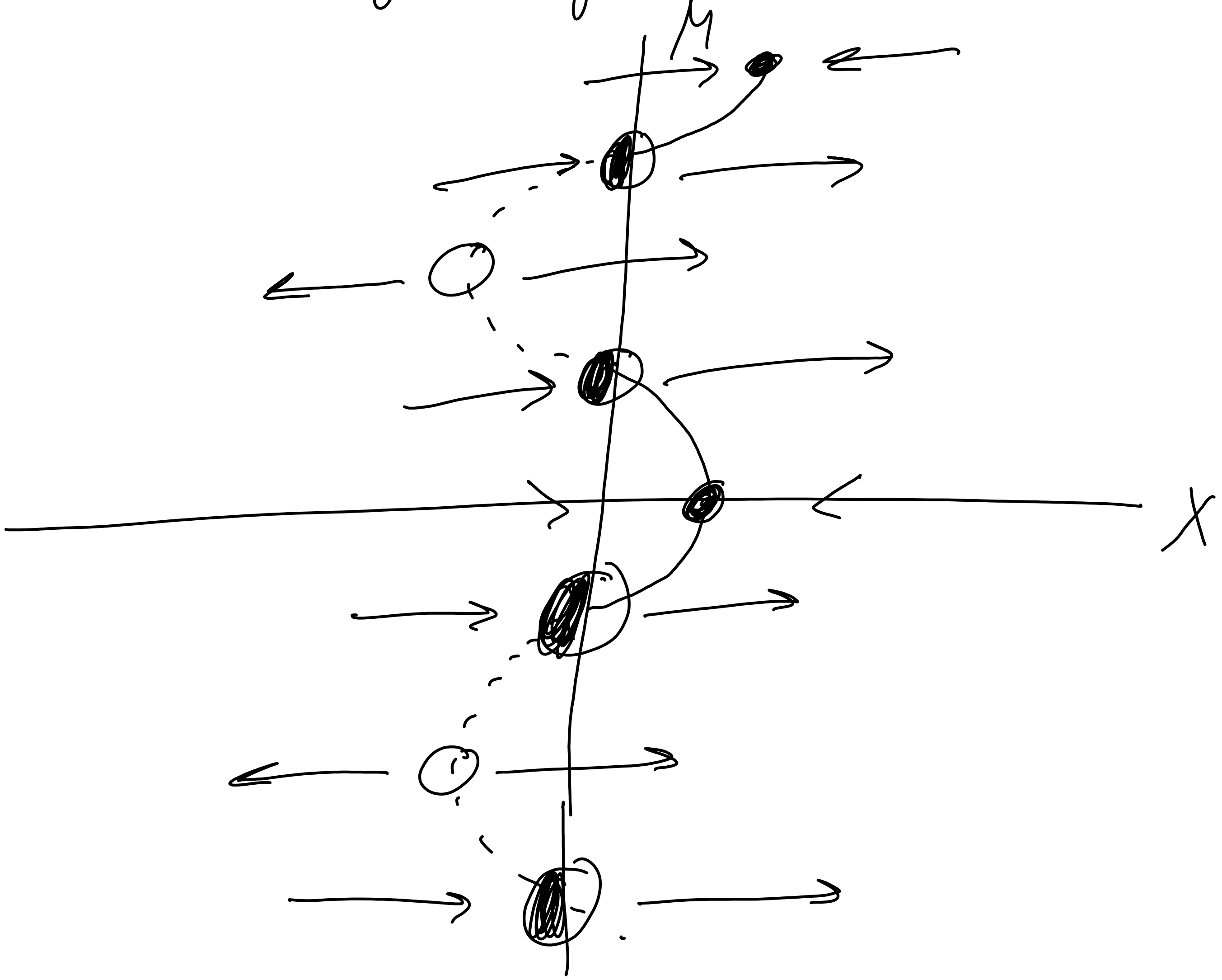
a.) $f(x,\mu) = 0 \qquad e^{-x} = \cos(x-\mu)$

$$f(x,0) \Rightarrow e^{-x} = \cos(x) \qquad x^* = 0$$



b.) μ varying from $[0, 2\pi]$ causes a phase shift, where ... There are an infinite

$2\pi \to 0$. There are an infinite amount of bifurcations.



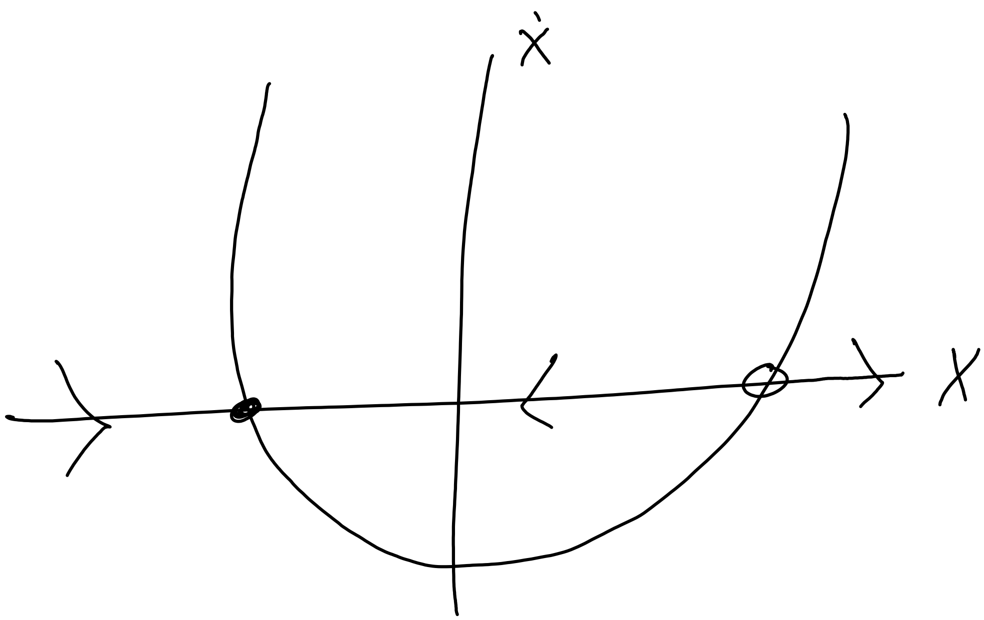
As μ increases, fixed points X^* decreases. When μ value reaches outside of $[0, 2\pi]$, the cycle repeats. This is a stable node bifurcation.

5.) a.) $\dot{x} = x^2 - 1$ $\quad f(x) = 0$ $\quad x^* = \pm 1$

$f'(x^*) = 2x$

$f'(1) = 2 > 0$, unstable

$f'(-1) = -2 < 0$, stable



b.) $\dot{x} = x^4$ $\quad f(x) = 0$ $\quad x^* = 0$

$f'(0) = 0$