3. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (2 pts)

Calculate the commutators [Brown Eqs. (18.87)-(18.89)] of the Lorentz boost and rotation generator matrices **K** and **S** [Brown Eqs. (18.75)-(18.80)].

Exercise 2 (4 pts)

Even the Brown Lecture note has some mistakes. Go ahead and correct and extend Chapter 17.2 by using the chain rule!

- i) Show that Newton's law is invariant w.r.t the Galilean transformation in Eqs. (17.1)-(17.4)! Is Eq. (17.6) correct?
- ii) Show that the wave equation is *not* invariant w.r.t the Galilean transformation and present the correct version of (17.12)!
- iii) Similarly, show that the wave equation is invariant w.r.t the Lorentz boost given in Eqs. (17.23)-(17.26) ($x_0 = ct$)!

Exercise 3 (5 pts)

Write out the explicit 4×4 matrix form of the following relativistic tensors, using the components of E and B.

i) The field strength tensor: defined as

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \ ,$$

where $A^\mu = (\phi, \vec{A})$ is the contravariant vector potential and $\partial^\mu = (\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla})$.

ii) The dual field strength tensor: defined as

$$F^\star{}_{\mu\nu} \equiv -\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} = -\varepsilon_{\mu\nu\alpha\beta}\partial^\alpha A^\beta \ ,$$

where $\varepsilon_{\mu\nu\alpha\beta}$ is the completely anti-symmetric 4-tensor with the sign convention $\varepsilon_{0123} = +1$. So $\varepsilon_{\mu\nu\alpha\beta} = -\varepsilon_{\nu\mu\alpha\beta} = \varepsilon_{\nu\mu\beta\alpha}$, etc.