38

$$dN_p = \frac{N}{V}\left(\frac{\beta}{2\pi m}\right)^{3/2} e^{-\frac{\beta}{2m}(p_x^2+p_y^2+p_z^2)} d^3p$$

Maxwell distribution

$$dN_p = \cdots$$

$u(\vec{r}) \neq 0$; case of gravity

$\uparrow z$ $\qquad \epsilon = \frac{p^2}{2m} - mgz$

one molecule m

do integral $\int d^3p \to$ const

$$n(z) = n_0 e^{-\beta mgz}$$

Law of Atmospheres

F related to Z

$$Z = \sum_n e^{-\beta E_n} = \sum_{k_1}\sum_{k_2}\cdots\sum_{k_N} e^{-\beta(\epsilon_1+\epsilon_2+\cdots+\epsilon_N)}$$

$$= \left(\sum_k e^{\beta\epsilon_k}\right)^N$$ part. Function

particle identity issue

$$Z \longrightarrow \frac{1}{N!} Z$$

$$Z \longrightarrow F$$

$$F = -\frac{N}{\beta}\ln\sum_k e^{-\beta\epsilon_k} + \frac{1}{\beta}\ln N!$$

Stirling's approx

$$\ln N! = N\ln N - N$$
$$= N\ln\frac{N}{e}$$

$$\boxed{F = -\frac{N}{\beta}\ln\left(\frac{e}{N}\sum_k e^{-\beta\epsilon_k}\right)}$$

quasi classi (tras.)

$$F = -\frac{N}{\beta}\ln\left(\frac{e}{N}\int\frac{d^3p\,d^3z}{(2\pi\hbar)^3}e^{-\beta\epsilon(p,z)}\right)$$

Include internal deg of freedom

$\epsilon \to \epsilon_{total}$ (for a single part.)

$$\epsilon_{total} = \epsilon_p + \epsilon_{int}$$

$$\epsilon_p = p^2/2m \qquad p^2 = p_x^2+p_y^2+p_z^2$$

$$\epsilon_{int} = \sum_k \epsilon_k$$

$$F = -\frac{N}{\beta}\ln\frac{eV}{N}\int\frac{d^3p}{(2\pi\hbar)^3}e^{-\beta p^2/2m} \times \sum e^{-\beta\epsilon_k}$$