Physics 412-3 Final Exam -- Spring 2020 (Take-home exam)

June 11, 2020

Available: 12:00 noon June 11, 2020

Due: 12:00 noon June 12, 2020

Please work on this exam individually. Do not consult your classmates or any other persons outside of this class. For questions please contact the instructor or TA by email, and we will try to answer as promptly as possible. Also the instructor will hold a 1 hour Zoom office hours from 12noon-1pm on June 11, 2020 in order to answer any initial questions. Although there is a 24 hour period given to complete the exam and you are free to arrange your schedule, my intention is that this exam should take approximately 2-3 hours to complete.

1.) A spin-0 particle of mass *m* and energy $E = \hbar^2 k^2 / 2m$ scatters elastically from a central potential of the form $V(r) = \left\{ \begin{matrix} \infty, r < R \\ 0, r \geq R \end{matrix} \right\}$ where *R* is a positive real constant.

a.) Evaluate the s-wave phase shift $\delta_0(k)$.

b.) Evaluate the s-wave contribution to the differential scattering cross section. Hint: the scattering amplitude $f_k(\theta) = \frac{1}{k}\sum_{l=0}^{\infty}\sqrt{4\pi(2l+1)}\exp[i\delta_l]\sin(\delta_l)\, Y_l^0(\theta)$

c.) For what values of *k* is the result you found in part (a) valid? Under what conditions do we expect that the s-wave contribution will be the dominant contribution to scattering?

d.) Discuss the physical significance of the sign (i.e. positive or negative) of your result in part (a).

e.) Does the first-Born approximation give a reasonable estimate for this type of scattering? Why or why not?

2.) A spinless particle of mass m and energy $E = \hbar^2 k^2 / 2m$ scatters elastically from an aspherical δ-function shell potential $V(r,\theta) = \frac{\hbar^2 \gamma}{2m}\delta(r-R)\cos^2[\theta]$ where γ is a positive real constant.

Determine the scattering amplitude $f_k^{(1)}(\theta,\phi)$ using the first-order Born Approximation. Evaluate all integrals to the extent possible. For which angle θ is the amplitude maximized?

3.) Consider a system of two **distinguishable** spin-1/2 particles. Let the total spin $\vec{S} = \vec{S}_1 + \vec{S}_2$. Let $|m_1, m_2>$ be the eigenstates common to S_{1z} and S_{2z} with eigenvalues $\hbar m_1$ and $\hbar m_2$ respectively. Let $|s, m>$ be eigenstates of S^2 and S_z with eigenvalues $\hbar^2 s(s+1)$ and $m\hbar$, respectively.

a.) What are the dimension of the state space and the allowed values of *s*?

b.) Now assume the particles are **indistinguishable.** Does your answer to part (a) change? If so why and how does it change? Write any acceptable state vector for the system in terms of one of the bases provided above.

c.) Repeat parts (a) and (b) now assuming they are spin-1particles instead of spin-1/2.

4.) Two **identical** non-interacting spin-1/2 particles occupy the lowest energy state of a harmonic oscillator potential $V(x) = \frac{1}{2} m\omega^2 x^2$.

a.) In terms of the basis $\{|n_1, m_{s1}; n_2, m_{s2}>\}$ write the normalized ket which describes this state. Here n_i is the vibrational quantum number of each particle, and m_{s1}, m_{s2} are the spin quantum numbers, e.g. $S_{1z}|n_1, m_{s1}; n_2, m_{s2}> = m_{s1}\hbar|n_1, m_{s1}; n_2, m_{s2}>$. What is the total energy?

b.) Now assume the particles occupy a spin-triplet state. Write the normalized state ket corresponding to the lowest energy state. What is the total energy?

c.) Now add a small interaction perturbation $A\vec{S}_1 \cdot \vec{S}_2$ where A is a small positive real constant. Does this operator commute with P_{12}? How do the energies of the states you found in (a) and (b) shift?

5.) Consider a non-relativistic free particle (ignore spin).

a.) Write down an explicit expression for the propagator in momentum space

$K(\mathbf{p},t;\mathbf{p}_0,t_0) = <\mathbf{p},t \mid \mathbf{p}_0,t_0>$.

b.) Now write down the expression in position space $K(\mathbf{x},t;\mathbf{x}_0,t_0)$. Discuss the relation between your answer and the answer to part (a). Hint: $\int e^{(-ax^2+bx)}dx = \sqrt{\frac{\pi}{a}}e^{b^2/4a}$

c.) Use the Feynman path integral method to determine the amplitude for the particle to appear at position *x* at time Δt, given that it starts at *x*=0 and *t*=0. Discuss the relation between your answer and the answers above.