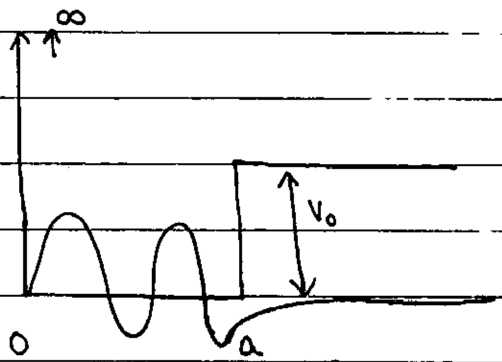
(1)



solution must satisfy the boundary conditions:

$\psi(0) = 0$

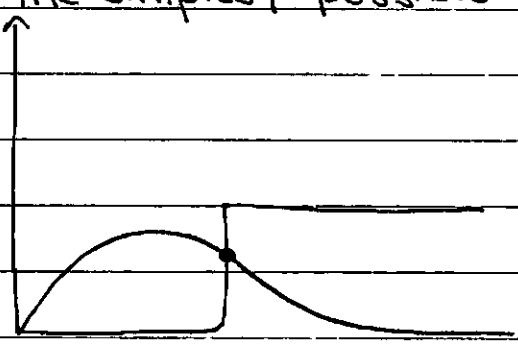
$\psi(x)$ continuous at a

$\frac{\partial \psi}{\partial x}$ continuous at a

bound state solution will have $E < 0$, so $E = -|E|$.

solution will propagate inside the well & evanescently decay outside of it (since $E < V = 0$), as generically sketched above.

the simplest possible bound state looks like:



$\sim \sin(kx)$ $\sim e^{-gx}$

in order to match B.C.'s, $\partial\psi/\partial x\,|_{x=a^+}$ is negative $(-g\, e^{-ga})$ so ~~[illegible]~~ $\partial\psi/\partial x\,|_{x=a^-}$ must have the same negative value. for ψ to be negative at all, $\sin(kx)$ must be past its maximum, so:

$ka > \pi/2$ $k^2a^2 > \pi^2/4$

where from the Schrödinger equation,

$$+\frac{\hbar^2}{2m}k^2 - V_0 = E$$

$$k^2 = \frac{2m}{\hbar^2}(V_0 - |E|)$$

$$\frac{2m}{\hbar^2}a^2(V_0 - |E|) > \pi^2/4$$

$$V_0 - |E| > \frac{\hbar^2\pi^2}{8ma^2}$$

$$V_0 > |E| + \frac{\hbar^2\pi^2}{8ma^2}$$

so the shallowest the well can possibly be is:

$$\boxed{V_0 > \frac{\hbar^2\pi^2}{8ma^2}}$$

if $V_0 < \hbar^2\pi^2/8ma^2$, no value of E corresponds to a bound state solution.