Resonant tidal forcing in close binaries: I.Implications for Cataclysmic Variables

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ABSTRACT
Resonant tidal forcing occurs when the tidal forcing frequency of a binary matches a quadrupolar oscillation mode of one of the binary members and energy is transferred from the orbit of the binary to the mode. Tidal locking permits ongoing resonant driving of modes even as binary orbital parameters change. At small binary separations during tidal lock, a significant fraction of binary orbital energy can be deposited quickly into a resonant mode and the binary decays faster than via the emission of gravitational radiation alone. Here we discuss some of the implications of resonant tidal forcing for the class of binaries known as Cataclysmic Variable (CV) stars. We show that resonant tidal forcing of the donor's Roche lobe could explain the observed $2-3$hr period gap in CVs, assuming modest orbital eccentricities are allowed ($e_b \sim 0.03$), and can be complementary or an alternative to, existing models. Sudden collapse of the companion orbit, yielding a Type Ia supernova is disfavoured, since Hydrogen is not observed in Type Ia supernova spectra. Therefore, resonance must generally be truncated, probably via mass loss from the Roche lobe or orbital perturbation, yielding a red giant stage for the WD, to reveal a short period CV containing an 'overheated' white dwarf.

Key words: stars:interiors–stars:oscillations–stars: white dwarfs – supernovae:general – binaries:general

1 INTRODUCTION

Resonant tidal forcing occurs in a binary when the tidal forcing frequency resonates with the frequency of a quadrupolar oscillation of one of the binary members. Resonant forcing is most efficient at: small binary separations, particularly during tidal locking (Witte & Savonije 1999) and for low-order quadrupolar modes (Rathore et al. 2005). Resonant forcing extracts energy from the binary orbit and converts it into increased amplitude oscillations in the binary member. As a result, the binary orbit shrinks faster than expected from the emission of gravitational waves (GW) alone (Fuller & Lai 2011). In white dwarf (WD) binaries, enough orbital energy can be extracted during resonance to excite novae (Fuller & Lai 2011) and possibly a Type Ia supernova (McKernan & Ford 2016). Here we discuss the consequences of resonant tidal forcing of in Cataclysmic Variable (CV) binaries and observational implications.

2 RESONANT TIDAL FORCING OF MODES

A binary shrinks over time as it loses orbital energy due to gravitational wave (GW) emission. Treating the binary members as point particles and ignoring spins, the binary merger timescale is (Peters 1964)

$$t_{\rm GW} \approx \frac{5}{128}\frac{c^5}{G^3}\frac{a_{\rm b}^4}{M_{\rm b}^2\mu_{\rm b}}(1-e_{\rm b}^2)^{7/2} \tag{1}$$

where $M_{\rm b} = M_1+M_2$ is the binary mass, $\mu_{\rm b} = M_1M_2/M_{\rm b}$ is the binary reduced mass and $(a_{\rm b}, e_{\rm b})$ are the initial binary semi-major axis and eccentricity respectively.

As a binary shrinks, tidal forcing increases in amplitude and can resonate with the jth quadrupolar $\ell = 2$ eigenmode of one of the binary members at angular frequency ω_j (with usual associated mode numbers n, ℓ, m). A tidal torque $T_n^{\ell m}$ acts on the resonant mode, where n is the mode radial order (number of nodes) and $|m| \leqslant \ell$, but we shall restrict ourselves to $\ell, |m| = 2$ (Rathore et al. 2005). Harmonics of the tidal potential can drive modes at forcing frequencies

$$\omega_{\rm F} = n\Omega_{\rm orb} - m\omega_{\rm spin} \tag{2}$$

where $\Omega_{\rm orb} = \sqrt{{\rm GM_b/a_b^3}}$ is the binary orbital frequency and $\omega_{\rm spin} = f_{\rm spin}\omega_2$ is the spin frequency of the forced binary member, with $f_{\rm spin} = [-1, +1]$ the fraction of the break-up spin frequency $\omega_2 = \sqrt{{\rm GM_2/R_2^3}}$ (McKernan & Ford 2016). During tidal locking,

$\Omega_{\rm orb} \approx \omega_{\rm spin}$ so that

$$\omega_F \approx (n-m)\Omega_{\rm orb}. \quad (3)$$

If $\dot{\Omega}_{\rm orb} \approx \dot{\omega}_{\rm spin}$ then ω_F remains approximately constant even as the orbit changes (Witte & Savonije 1999). The forcing frequency can be multiples of $\omega_{\rm F}$ so that $\omega_{\rm F} = (n-m)\Omega_{\rm orb}/k$ where $k = 1, 2, 3, \ldots$ is an integer.

The energy deposited into a resonant mode is taken from the binary orbital energy $E_{\rm orb} = -GM_b^2/a_b$ instead of being emitted as gravitational waves. If enough energy is taken from the orbit and deposited into a resonant mode, a_b can change significantly during the resonant forcing time ($t_{\rm res} \ll t_{\rm GW}$), decreasing the binary period (Fuller & Lai 2011). Such a orbit-skipping effect changes the predicted gravitational wave emission profile expected from close binaries and will cause binaries to merge faster than predicted by eqn. (1). The largest energy transfer occurs during resonance with low-order modes (McKernan & Ford 2016). We discuss energy deposition into the mode, and the subsequent behaviour of the mode below.

2.1 Mode heating

Following McKernan & Ford (2016) the equation of motion for the resonating jth mode of mass M_j is:

$$\ddot{x_j} + \frac{\dot{x_j}}{\tau_{\rm d}} + \omega_j^2 x_j = \frac{F_j}{M_j} \quad (4)$$

where x_j is the displacement of the jth mode, F_j is the overlap integral beween the mode and the driver and $\tau_d = 1/\Delta\omega_j$ is the decay time of the mode. For a constant (tidal) forcing frequency $\mathrm{F} = |\mathrm{F}|\mathrm{e}^{\mathrm{i}\omega_{\rm F}t}$, the maximum steady-state displacement $x_{\rm max}$ of the mode is

$$x_{\rm max} = \frac{|F|}{\sqrt{(\omega_F^2 - \omega_j^2)^2 + (\omega_F/\tau_d)^2}} \quad (5)$$

which yields $x_{\rm max} = |F|\tau_d/\omega_j$ in the resonant limit ($\omega_j = \omega_{\rm F}$). The effective forcing time (t) experienced by a mode is $t = [\tau_d, 1/\sqrt{\omega_{\rm F}}]$ in the [saturated, unsaturated] case (McKernan et al. 2014). The mode is saturated if the actual forcing time $t_{\rm F} > \tau_d$ and unsaturated if $t_{\rm F} < \tau_d$. Whether a mode is saturated or unsaturated makes a significant difference to the energy deposited into the mode (see McKernan et al. (2014) for discussion).

The tidal forcing of a star or stellar remnant mode can be written as (McKernan & Ford 2016)

$$\langle|F|\rangle \approx \left(\frac{12\pi}{5}\right)\left(\frac{E_2}{R_2}\right)\epsilon^2\chi_j^2 k^{-1}\left(\frac{\omega_j/\tau_d}{\delta\omega^2 + (1/\tau_d^2)}\right) \quad (6)$$

where k is the harmonic of the mode and ϵ is the tidal factor

$$\epsilon = \left(\frac{M_1}{M_2}\right)\left(\frac{R_2}{a_b}\right)^3, \quad (7)$$

with $\delta\omega = \omega_j - \omega_{\rm F}$ the detuning frequency, and χ_j is the overlap integral between the driving quadrupolar tidal force and the quadrupolar oscillation mode. The expression in eqn. (6) was derived for the specific example of white dwarfs, but its form should be similar for any stellar object. Here we assume that χ_j for the kth harmonic of a mode is unchanged, but the amplitude of the forcing is diminished by factor $1/k$.

As the resonant driving frequency drifts across resonance with a mode, the mode driven for time t acquires total energy $E_{tot} = M_j|F/M_j|^2t^2/2$ (McKernan et al. 2014), where $M_j = \chi_j M_2$ is the mass in the jth mode. Assuming the resonance is not exact and that $\delta\omega/(2r\tau_d)$ is some multiple ($r = 1, 2, 3, \ldots$) of the HWHM of the resonance, and we ignore non-linear effects, we can write (McKernan & Ford 2016)

$$E_{tot} \approx \left(\frac{6\pi^2}{5}\right)^2 E_2\omega_2^2 q_j^2 \epsilon^4 k^{-2} r^{-1}\chi_j^3 t^2 \quad (8)$$

where $q_j = \omega_j\tau_d/\pi$ is the quality factor for the mode, and the mode is driven for time $t = t_{\rm F}(\tau_d)$ in the unsaturated (saturated) case. From (McKernan & Ford 2016) the total energy deposited in a WD can be a large fraction of the binding energy (E_2). Resonance can be truncated by a number of processes including: wave breaking, changes in the structure of the excited star and skipping sufficient orbits (Fuller & Lai 2011).

2.2 Mode cooling

Mode cooling can occur via thermodynamics or gravitational wave emission (McKernan & Ford 2016). GW emission is an efficient cooling mechanism for dense stellar remnants (including WDs), but inefficient for stars, so we shall ignore GW emission as a channel for cooling modes in the CV secondary. For thermal cooling, in the limit of large numbers of daughter modes, energy dissipated to daughter modes is effectively 'thermalized' to the atoms involved in the oscillation. Mode coupling tends to occur near boundaries where the turning points of multiple nodes overlap, so the thermal heating tends to dissipate at these boundaries (Kumar & Goodman 1996).

At modest E_j/E_2, excited quadrupolar f-modes or low-order p-modes couple to $\ell = 0, 2, 4$ p-modes leading to decay timescales of (Kumar & Goodman 1996)

$$\tau_d \approx \frac{2}{E_j\sum(\kappa^2/\tau_p)} \approx 2\frac{\overline{\tau}_p}{N_p}\frac{E_2}{E_j} \quad (9)$$

where the sum is over the number (N_p) of the low radial order $\ell = 0, 2, 4$ p-modes with typical decay time $\overline{\tau}_p$ that couple to the quadrupolar f-mode or low-order p-modes.

At modest E_j/E_2, low order g-modes decay efficiently via mode coupling with an energy dependence (Kumar & Goodman 1996)

$$\tau_d \approx 4\pi\omega_{\rm j}^{-1}\kappa^{-1}\mathrm{E}_{\rm j}^{-1/2} \approx \frac{4\pi}{\omega_{\rm j}}\left(\frac{\mathrm{E}_2}{\mathrm{E}_{\rm j}}\right)^{1/2} \quad (10)$$

where the mode coupling coefficient is $\kappa \approx \mathrm{G}^{-1/2}\mathrm{R}_2^{1/2}\mathrm{M}_2^{-1} \approx \mathrm{E}_2^{-1/2}$ where E_2 is the binding energy of the forced binary member.

3 APPLICATION TO CATACLYSMIC VARIABLES

It has long been known that white dwarfs (WD) in binaries with donor stars (the Cataclysmic Variable stars, hereafter CVs) display a clear period gap between 2-3hours. This gap is only believed to apply to non-magnetic CVs and is typically explained in terms of a rapid rate of change of angular momentum loss due to the switching off of magnetic braking in the donor star (e.g. Rappaport et al. 1982). More recent observations suggest a more complicated model is required, with phenomenological elements on top of the standard model of angular momentum loss (Knigge et al. 2011). Recent binary population synthesis (BPS) modelling suggests additional consequential angular momentum loss is required from CV systems (Schreiber, Zorotovic & Wijnen 2016). In this section we discuss resonant tidal forcing of the (low mass star or brown dwarf)

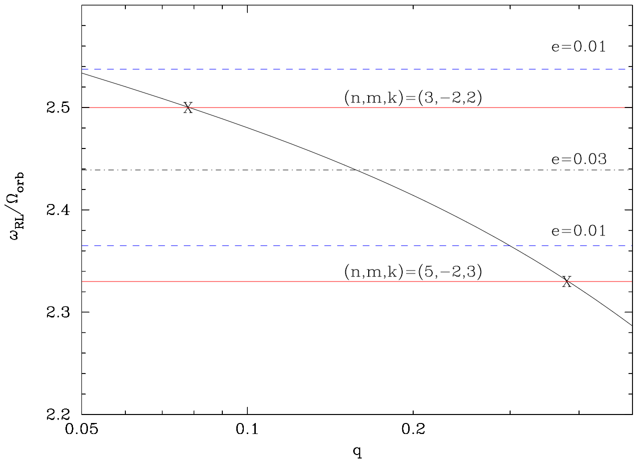

Figure 1. $\omega_{RL}/\Omega_{\rm orb}$ as a function of binary mass ratio ($q = M_2/M_1$) from eqn. (12). CV mass ratio spans the expected range $q = [0.05, 0.5]$ (Knigge et al. 2011). Black solid curve corresponds to a single-value $e = 0$ in eqn. (12). Horizontal red solid lines correspond to resonances at $(n, m, k) = (3, -2, 2)$ and $(n, m, k) = (5, -2, 3)$ respectively (see test) and intersect the black solid curve at only two values of q (marked by X) when $e = 0$. Blue dashed lines correspond to the shift in resonance at $e = 0.01$ and black dash-dot horizontal line corresponds to the shift in the resonance at $e = 0.03$. If the binary eccentricity is small but non-zero in most CVs, resonance can occur at any q.

companion to the WD in CVs, and the implications for the CV period gap. First we shall consider resonant tidal forcing of the Roche lobe of the donor star.

3.1 Resonant tidal forcing of a Roche lobe

The volume-averaged Roche lobe radius is generally well-approximated as (Eggleton 1983)

$$\frac{R_L}{a_b(1-e_b)} \approx \frac{0.49q^{2/3}}{0.6q^{2/3}+\ln(1+\mathrm{q}^{1/3})} \tag{11}$$

for a binary of semi-major axis a_b, eccentricity e_b and mass ratio $q = M_2/M_1$ where $M_b = M_1 + M_2$. Cataclysmic variable stars (CVs) have mass ratios spanning $q \sim [0.05, 0.5]$ (Knigge et al. 2011), so from eqn. (11) we expect $R_L/a_b \sim 0.2 - 0.3$. The fundamental frequency associated with a Roche-lobe is $\omega_{RL} = (GM_2/R_L^3)^{1/2}$ which can be written as

$$\omega_{RL} \approx \Omega_{\rm orb}\left(\frac{q}{1+q}\right)^{1/2}\frac{0.465q+(\ln(1+\mathrm{q}^{1/3}))^{3/2}}{0.343q}\frac{1}{(1-e_b)^{3/2}}. \tag{12}$$

From §2, the ratio $\omega_{RL}/\Omega_{\rm orb}$ corresponds to a unique combination of integers, $\omega_{RL}/\Omega_{\rm orb} = (n-m)/k$ as well as unique combination of (q, e_b). If e_b is identically zero, $\omega_{RL}/\Omega_{\rm orb}$ is single-valued in q, and resonance can only occur at one mass ratio value. However, if e is not identically zero, there is a range of values of q for which resonance can occur.

Fig. 1 shows the ratio $(\omega_{RL}/\Omega_{\rm orb})$ as a function of binary mass ratio. The black solid curve is single-valued and assumes $e_b = 0$. The ratio $\omega_{RL}/\Omega_{\rm orb}$ spans the range $\sim [2.3, 2.5]$ in all CVs over the range $q \sim [0.05, 0.5]$. In order for resonance to occur, we require $\omega_F \approx \omega_{RL}$, so from eqn. (3) we require $(n-m)/k \sim [2.3, 2.5]$. For quadrupolar modes $m = \pm 2$ (Rathore et al. 2005), therefore for resonance to occur requires $(n, m, k) = (3, -2, 2)$ for $\omega_{RL}/\Omega_{\rm orb} = 2.5$ and $(n, m, k) = (5, -2, 3)$ for $\omega_{RL}/\Omega_{\rm orb} = 2.33$, depicted as red, horizontal solid lines in Fig. 1, which intersect the black solid curve at locations marked by Xs. If we allow for non-zero eccentricity, the blue dashed lines correspond to $e = 0.01$ in each case and the black dash-dot line corresponds to $e = 0.03$. An eccentricity of $e < 0.05$ allows resonance to occur over the full range of mass ratios in CVs. Eccentricity pumping via occasional novae may occur, allowing an average rate of eccentricity decay due to gravitational radiation emission alone as (Peters 1964)

$$\langle\frac{de_b}{dt}\rangle \approx \frac{-304}{15}\frac{G^3}{c^5}\frac{e_b}{a^4(1-e^2)^{5/2}}M_1M_2M_b \tag{13}$$

where we ignore terms O(e^2). Reparameterizing we find

$$\langle\frac{de_b}{dt}\rangle \approx 10^{-10}\mathrm{yr}^{-1}\frac{\mathrm{e}}{(1-\mathrm{e}^2)^{5/2}}\left(\frac{\mathrm{M}_1}{1\mathrm{M}_\odot}\right)\left(\frac{\mathrm{M}_2}{0.1\mathrm{M}_\odot}\right)\left(\frac{\mathrm{a_b}}{\mathrm{R}_\odot}\right)^4 \tag{14}$$

so e_b is likely to drift across low-order resonances for a given value of q.

The total energy deposited in the resonant Roche lobe over forcing time t is given by eqn. (8). If we parameterize energy $E_2 = GM_2^2/R_L$ in terms of the orbital energy $E_{\rm orb} = G(M_1 + M_2)^2/2a_b$ as

$$E_2 \approx 2E_{\rm orb}\left(\frac{q}{1+q}\right)^2\left(\frac{a_b}{R_L}\right) \tag{15}$$

and we write $\Omega_{\rm orb} \sim 0.6(T_{\rm orb}/3\mathrm{hr})^{-1}$mHz, eqn. (8) becomes

$$\begin{aligned} E_{tot} \approx\; & 0.11E_{\rm orb}\left(\frac{\Omega_{RL}}{0.6\mathrm{mHz}}\right)^{4/5}\left(\frac{q}{0.1}\right)^{8/5}\left(\frac{R_L/a_b}{0.2}\right)^{1/5} \\ \times & \left(\frac{1+q}{1.1}\right)^{-12/5}\left(\frac{k}{2}\right)^{-2/5}\left(\frac{r}{1}\right)^{-1/5}\left(\frac{\chi_j}{10^{-2}}\right)^{3/5} \\ \times & \left(\frac{\overline{\tau}_p}{10^6\mathrm{s}}\right)^{4/5}\left(\frac{N_p}{10}\right)^{-4/5}. \end{aligned} \tag{16}$$

Eqn. (16) shows that if the donor Roche lobe is resonantly forced at saturation ($t_F = \tau_d$), a mode with overlap intergral $\chi_j \sim 10^{-2}$ could in principle extract $O(10\%)$ binary orbital energy in a very short time. If the tidal forcing is unsaturated, the energy taken from the orbit decreases from eqn. (16) by a factor $(t_F/\tau_d)^2$ where $t_F/\tau_d < 1$ is the ratio of the forcing time to the mode decay time. Note however that wave-breaking and related non-linear effects will tend to limit mode oscillations, once the amplitude becomes sufficiently large (Fuller & Lai 2011). The excited f-mode mode will decay on a timescale $\tau_d \sim 3 \times 10^9\mathrm{s}(\mathrm{E_{tot}}/10^{42}\mathrm{erg})^{-1}$ (Kumar & Goodman 1996). At energies $E_{tot} \sim 10^{45}$erg $\sim 2 \times 10^{-4}E_{\rm orb}(M_b/1.1M_\odot)^2(a_b/R_\odot)^{-1}$, we expect $\tau_d \sim 10^6$s.

The CV binary period can be parameterized as

$$T_{\rm orb} \approx 3\mathrm{hr}\left(\frac{\mathrm{a_b}}{\mathrm{R}_\odot}\right)^{3/2}\left(\frac{1+1/\mathrm{q}}{11}\right)^{1/2}\left(\frac{\mathrm{M}_2}{0.1\mathrm{M}_\odot}\right)^{-1/2} \tag{17}$$

Extracting $E_{tot} \geqslant 0.25E_{\rm orb}$ from the orbit and placing it into the excited mode reduces the CV period to $T_{\rm orb} \leqslant 2$hr. Fig. 2 shows the ratio of $E_{tot}/E_{\rm orb}$ from eqn. 16 as a function of q. The dashed horizontal line indicates $E_{tot}/E_{\rm orb} \sim 0.26$. Solid curves show $E_{tot}/E_{\rm orb}$ for $\chi_j = 0.005, 0.01, 0.05$ from bottom to top. Curves above $E_{tot}/E_{\rm orb}$ indicate an orbit that has decreased in time $t_F = \tau_d$ from 3hr to $\leqslant$ 2hr.

3.2 Catastrophic orbit collapse: Type Ia Supernovae not favoured

Eqn. 16 shows that under some circumstances all of the CV orbital energy could be extracted at resonance. The hydrodynamic details

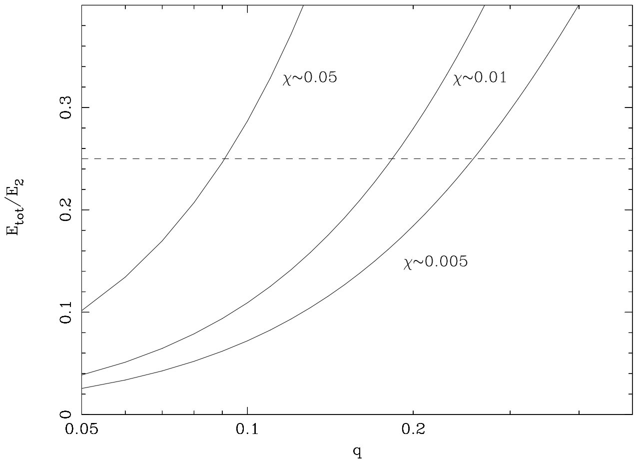

Figure 2. $(E_{\rm tot}/E_2)$ as a function of binary mass ratio $q = M_2/M_1$ and choice of $\chi_j \sim 0.005, 0.01, 0.05$ in eqn. (16). Dashed line indicates a ratio of $E_{tot}/E_2 \approx 0.26$ required to reduce a CV period from $T_{orb} \sim$ 3hrs to $T_{orb} \sim$ 2hrs.

of a sudden orbital collapse and impact between a WD and a companion are far beyond the scope of this paper, but it is instructive to simply equate the orbital energy with the average temperature per nucleus in some fraction of the WD. In this case

$$E_{\rm orb} = \frac{3}{2} f_N N_N k_B T_N \quad (18)$$

where f_N is the fraction of nuclei of the number of nuclei N_N in the WD heated by the interaction, k_B is the Boltzmann constant and T_N is the average temperature per nucleus. Writing $N_N \approx M_1/m_C$ where m_C is the mass of a Carbon atom we can parameterize the average temperature per nucleus as

$$T_N \approx 6 \times 10^8 {\rm K} \left(\frac{\rm M_2}{0.1{\rm M}_\odot}\right)\left(\frac{\rm a}{\rm R_\odot}\right)^{-1}\left(\frac{\rm f_N}{0.05}\right). \quad (19)$$

If the initial energy of impact is smeared impulsively over a small fraction f_N of the mass of the WD, T_N reaches the Carbon fusion threshold, whereupon a runaway detonation of the entire WD can occur. However, the result of such an explosion should contain Hydrogen due to contamination from M_2. Since Hydrogen is by definition not present in Type Ia SN, or very weakly at best (Maoz et al. 2014), it seems that catastrophic collapse of the orbit during resonance and subsequent detonation is unlikely. While Type Ia SN could occur from mass transfer to a near Chandrasekhar mass WD, a collision and Type Ia SN seems unlikely, so either $\chi_j \leqslant 0.05$ typically, or resonance gets shut down.

3.3 Shutting off resonance

Resonance can be terminated if a change perturbs either the forced frequency (ω_{RL}) or forcing frequency (ω_F) by more than the resonance width ($\Delta\omega_{RL} \approx 1/\tau_d \sim 1\mu{\rm Hz}(\tau_d/10^6{\rm s})$). Since ω_F and ω_{RL} both depend directly on $\Omega_{\rm orb}$, a change in a_b alone in a resonant system will not drive the system away from resonance. From eqn. (12), ω_{RL} depends only on $(\Omega_{\rm orb}, q, e_b)$. Since $de_b/dt \approx 10^{-10}$/yr, from eqn. (14), only a change in e_b due to an orbital perturbation from a WD nova, or a change in q can perturb ω_{RL} away from resonance.

Resonant excitation of the fundamental mode of the Roche lobe, will drive mass loss from M_2. Assuming $\sim 1/2$ of the mass lost from M_2 is accreted onto M_1 we find the change in ω_{RL} due to mass exchange is

$$\Delta\omega_{RL} \approx 0.5\mu{\rm Hz}\left(\frac{q}{0.1}\right)^{-1}\frac{\Delta M_1}{10^{-3}M_\odot}\frac{M_1}{1M_\odot}\frac{\Omega_{\rm orb}}{0.6{\rm mHz}}. \quad (20)$$

If mass $\Delta M \sim 10^{-3}M_\odot(M_1/1M_\odot)$ is lost from M_2, the binary mass ratio changes by $\delta q/q \sim 0.1(q/0.1)^{-1}$ and ω_{RL} changes by more than the resonance width $\Delta\omega_{RL}$, so resonance is truncated. Since $M_j = \chi_j M_2$ is the mass in the f-mode, a mass loss of $\Delta M \sim 10^{-3}M_\odot(M_1/1M_\odot)$ from M_2 corresponds to a large fraction of the initial mass in the f-mode. The dynamical timescale for M_2 to readjust is $t_2 \sim 1/\omega_{RL} = 5 \times 10^3{\rm s}(\omega_{\rm RL}/0.6{\rm mHz})^{-1}$. As M_2 loses mass it will contract adiabatically on timescale t_2, and if M_2 is a star, it will readjust on the thermal timescale $t_{th} = GM_2^2/R_2L_2$ where L_2 is the stellar luminosity.

Feedback from nova-like eruptions, if they occur, can truncate resonance by changing both M_1 and M_2 and/or (a_b, e_b). Mass loss from the secondary in a short-timescale initiates a burst of accretion onto the WD. Depending on the actual accretion rate, a super-luminous nova, or red giant atmosphere could result. If a nova occurs, at least initially, the luminosity will limit the accretion flow and drive most of ΔM away from the system, rather than accrete onto the WD. The shockwave from such a nova can also drive momentum into the Roche lobe of the secondary and cause further mass loss from M_2. The precise details of resonance truncations are far beyond the scope of this letter, but suggest the importance of modelling feedback of novae in CV systems where resonant tidal interaction can occur.

3.4 Observational implications

Observationally, WDs on the short-period side of the CV period gap are generally hotter than expected from the standard model of CV production (Knigge et al. 2011). The net additional heating corresponds to ∼few 10^3K in effective temperature. In the model we outline here, if the fundamental mode of the Roche lobe is excited significantly during resonance, it will substantially overflow leading to an elevated accretion rate onto the WD, heating the surface. If $\Delta M \sim 10^{-3}M_\odot$ is lost from M_2 and some small fraction f_{acc} of this actually accretes onto the WD on a timescale $t_F \sim 10^6$s, then the associated accretion rate is $\sim (f_{acc}/10^{-3})10^{-5}M_\odot$/yr. However the inferred accretion rate exceeds that onto super-soft X-ray sources (Kahabka & van den Heuvel 1997) by orders of magnitude. Even if a nova commences, it will quickly cease at such high accretion rates since Hydrogen shell burning is stable (Paczynski & Zytkow 1978; Kato & Hachisu 1988) and soft X-rays luminosity from accretion $L_X \sim 10^{37.5}{\rm erg/s}({\rm R}/10^4{\rm km})^2({\rm T_{WD}}/40{\rm eV})^4$ should be obscured by the inflowing optically thick flow. Thus, the end stage of resonance should correspond observationally to a red giant stage (Paczynski & Zytkow 1978), with a central WD and a modest envelope of mass $\Delta M \sim 10^{-3}$. This red giant phase should last $\sim 10^4$ yrs at Hydrogen burning rates of $\sim 10^{-7}M_\odot$/yr. Emission lines from the photosphere of the red giant should display an oscillation around the line centroid energy, on the binary period of $\lesssim$ 2hrs, as the binary orbits its center of mass inside the red giant envelope. Constraints on CV eccentricity (e_b) can rule out this model; if $e_b < 0.01$ in most CVs, then the resonances will only apply across a relatively small range of CV mass ratios.

4 CONCLUSIONS

We show that resonant tidal forcing of the fundamental frequency of the Roche lobe of the donor star in CVs can extract enough energy from the CV binary orbit to explain the observed period-gap in CVs, provided that there is a small binary eccentricity ($e_b \sim 0.03$). This model can complement or act as an alternative to existing models of the CV period-gap. Resonance begins around an orbital period of $\sim$ 3hours when binary separation becomes small enough that sufficient energy is deposited into the resonant mode. A catastrophic merger yielding a Type Ia supernova is disfavoured since Hydrogen signatures are not observed in Type Ia supernovae, so resonance must be generally truncated, or there must be little ablation of the donor in any explosion. The CV system is driven away from resonance via mass loss from the secondary in the CV binary, or orbital perturbation, or by non-linear wave-breaking effects. Unstable overflow accretion onto the white dwarf at the end of resonance, will yield a red-giant phase for the WD, followed by a short-period CV with hotter than expected WDs.

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