HW 7 P. 1

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$E < B$ $\boxed{i}$

$$\vec{u} = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{E\hat{y}\times B\hat{z}}{B^2} = \frac{E}{B}\hat{x}$$

boosted frame S':

$$\vec{E}' = 0 \qquad \vec{B}' = \frac{B}{\gamma}\hat{z}$$

$$x'(t') = a\sin(\omega_B t') \qquad \omega_B = \frac{qB'}{\gamma m c}$$

$$y'(t') = a\cos(\omega_B t')$$

$$z'(t') = v_{\parallel} t'$$

$S' \longrightarrow S$

$$t(t') = \gamma\left(t' + \frac{u\,x'(t')}{\cdots}\right) = \gamma\left(t' + \frac{u}{\cdots} a\sin(\ldots)\right)$$

$$c^2 \quad) \quad c^2 a\sin(\omega_B t')$$

$$x(t') = \gamma(x'(t') + ut')$$

$$= \gamma(a\sin(\omega_B t') + ut')$$

$$y(t') = y'(t') = a\cos(\omega_B t')$$

$$z(t') = z'(t') = v_{\parallel} t'$$

$E > B$

$$\vec{u} = \frac{B}{E}\hat{x} \qquad B' = 0 \qquad E' = \frac{E}{\gamma}\hat{y}$$

$$\frac{d}{dt}(m\gamma v) \begin{cases} m\frac{d}{dt}(\gamma(t') v_{\perp}(t') = 0 \\ m\frac{d}{dt}(\gamma(t') v_{\parallel}(t') = qE' \end{cases} \Rightarrow$$

pick origin in S' such that,

$$V_\perp(t'=0) = V_0 \qquad \gamma_0 = \frac{1}{\sqrt{1-\left(\frac{V_0}{c}\right)^2}}$$

$$\Rightarrow \begin{cases} m\gamma_0 V_0 \\ qE't' \end{cases}$$

$$V_\perp(t') = \frac{m\gamma_0 V_0}{m\gamma} = \frac{\gamma_0 V_0}{\gamma}$$

$$V_\parallel(t') = \frac{qE't'}{m\gamma}$$

$$V^2(t') = V_\perp^2 + V_\parallel^2 = c^2\left[\frac{(\gamma_0 V_0)^2 + \left(\frac{qE't'}{m}\right)^2}{(\gamma_0 c)^2 + \left(\frac{q^2E't'}{m}\right)^2}\right]$$

$$\gamma^2(t') = \gamma_0^2 + \frac{q^2E'^2t'^2}{m^2c^2}$$

$$V(t') = \gamma V$$

$$v_\perp(t') = \frac{\gamma_0 v_0}{\sqrt{\gamma_0^2 + \frac{q^2 E'^2 t'^2}{m^2 c^2}}}$$

$$V_\parallel(t') = \frac{qE't'}{m\sqrt{\gamma_0^2 + \left(\frac{qE't'}{mc}\right)^2}}$$

$$Y_\perp(t') = \int v_\perp(t')\,dt' \qquad \text{(Wolfram)}$$

$$= \frac{\gamma_0 v_0 mc}{qE'} \operatorname{arcsinh}\left(\frac{qE't'}{\gamma_0 mc}\right)$$

$$Y_\parallel(t') = \int V_\parallel(t')\,dt'$$

$$= \frac{\gamma_0 mc^2}{qE'}\left(\sqrt{1 + \left(\frac{qE't'}{\gamma_0 mc}\right)^2} - 1\right)$$

$$S \longrightarrow S'$$

$$y'(t') = \frac{\gamma_0 mc}{qE'}\left(\sqrt{1+\left(\frac{qE't'}{\gamma_0 mc}\right)^2} - 1\right)$$

$$z'(t') = \cos\left(\alpha \frac{\gamma_0 v_0 mc}{qE'}\right)\operatorname{arcsinh}\left(\frac{qE't'}{\gamma_0 mc}\right)$$

$$x'(t') = \sin\left(\alpha \frac{\gamma_0 v_0 mc}{qE's}\right)\operatorname{arcsinh}\left(\frac{qE't'}{\gamma_0 mc}\right)$$

trajectory for particle is
a cycloid

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ii) $|\vec{E}_0| = |\vec{B}_0|$

$$\vec{u} = \frac{\vec{E}\times\vec{B}}{B^2} =$$