Physics 412-2 (Spring 2020) **Problem Set 2** Professor Gerald Gabrielse
Posted: 19 Jan. 2020 Due: 9 am on Mon. 27 Jan. 2020

The problem sets are a serious part of the learning experience in this class. The problem sets will sometimes deliberately range away from what can be covered in the lectures in some cases. The goal to expose you to basic concepts and important examples of quantum mechanics. Problems will often be divided into many small parts to guide your through a solution. Corrections to the assignments, if needed, will be posted to the class web page. Your solutions should be placed in the box outside my office.

1. **Spin 1/2 Rotation Operators:** Use the explicit spin 1/2 rotation operator to examine and demonstrate the following:

   (a) Show that rotation operators for arbitrary rotations about arbitrary axes are unitary.

   (b) Show that these rotation operators are "unimodular" by showing that they all have a determinant of equal to 1. This is what is referred to when the group of these rotations is called the "special unitary group in 2 dimensions, SU(2).

   (c) Use the explicit rotation operator that rotates a spin 1/2 system by a finite angle ϕ about the z axis to describe how the average value of σ_x depends on the angle.

   (d) Use the explicit rotation operator that rotates a spin 1/2 system by a finite angle ϕ about the z axis to describe how the average value of σ_y depends on the angle.

   (e) Use the explicit rotation operator that rotates a spin 1/2 system by a finite angle ϕ about the z axis to describe how the average value of σ_z depends on the angle.

   (f) Compare to what happens when the rotation angle is an integer times 2π, and compare to such a rotation of a wavefunction. Explain.

2. **Explicit Rotation Matrices:** The rotation matrices $D^{(j)}_{m',m}(\alpha,\beta,\gamma)$ can be written in terms of

$$d^{(j)}_{m',m}(\beta) = \langle jm'|e^{-i\beta J_y/\hbar}|jm\rangle. \tag{1}$$

   Evaluate and display the matrix $D^{(j)}_{m',m}(\alpha,\beta,\gamma)$ and $d^{(j)}_{m',m}(\beta)$ directly starting with an explicit matrix representation for J_y in the following cases:

   (a) j=1/2

   (b) j=1

   Use the Wigner formula for a general rotation matrix element to determine the matrices $D^{(j)}_{m',m}(\alpha,\beta,\gamma)$ and $d^{(j)}_{m',m}(\beta)$ for the following cases.

   (c) $j = 1/2$

   (d) $j = 1$

   Check your answers using Mathematica. Hand in the code that you use. Do this for the following cases:

   (e) $j = 1/2$

(f) $j = 1$

3. **Vector Operators.** For each of the following familiar operators do two things. (i) Prove they are vector operators. (ii) Explicitly express their spherical tensor components for each in terms of the Cartesian operator components.

   (a) $\mathbf{x}$

   (b) $\mathbf{p}$

   (c) $\mathbf{L}$

   (d) $\mathbf{S}$

   (e) $\mathbf{J} = \mathbf{L} + \mathbf{S}$