2. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (3 pts)

Invent your own gauge! Specify a condition and show that it is always possible (in principle) to make a gauge transformation (χ) so that $\vec{A}'$ and ϕ' satisfy the condition.

Exercise 2 (5 pts)

The angular momentum of a distribution of electromagnetic fields in vacuum (in Lorentz-Heaviside units) is given by

$$\vec{L} = \frac{1}{c} \int d^3 r \vec{r} \times \left(\vec{E} \times \vec{B} \right)$$

where the integration is over all space. For fields produced a finite time in the past (and so localized to a finite region of space) show that the angular momentum can be written in the form

$$\vec{L} = \frac{1}{c} \int d^3 r \left[\vec{E} \times \vec{A} + \sum_{l=1}^{3} E_l \left(\vec{r} \times \vec{\nabla} \right) A_l \right]$$

The first term is sometimes identified as the "spin" of the photon and the second with the "orbital" angular momentum because of the presence of the angular momentum operator $\vec{L}_{op} = -i \left(\vec{r} \times \vec{\nabla} \right)$

Exercise 3 (3 pts)

In the classical theory of the electron, it is often modelled by a sphere of charge $-e$ and radius $r_0 = \frac{e^2}{6\pi m c^2 \varepsilon_0}$ (in SI units). According to experiments, the electron has a magnetic moment $|\vec{m}| = \frac{e\hbar}{2mc}$. Calculate the angular moment of the electron's electromagnetic field

$$N = \varepsilon_0 \int d^3 r \left[\vec{r} \times \left(\vec{E} \times \vec{B} \right) \right]$$

Hints: For $r > r_0$, $\vec{E}$ and $\vec{B}$ are given by

$$\vec{E} = -\frac{e\vec{r}}{4\pi \varepsilon_0 r^3}$$

and

$$\vec{B} = \frac{\mu_0}{4\pi} \left[3\frac{(\vec{m}\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right]$$