Physics 412-2 (Spring 2020) **Problem Set 6** Professor Gerald Gabrielse

Posted: 16 Feb. 2020 Due: 9 am Monday, 24 Feb. 2020

**Take home exam reminder: See course syllabus and web page**

*The problem sets are a serious part of the learning experience in this class. The problem sets will sometimes deliberately range away from what can be covered in the lectures in some cases. The goal to expose you to basic concepts and important examples of quantum mechanics. Problems will often be divided into many small parts to guide your through a solution. Corrections to the assignments, if needed, will be posted to the class web page. Your solutions should be placed in the box outside my office.*

1. **Spin-orbit Coupling:** The spin-orbit coupling in hydrogen comes from the interaction of the magnetic moments associated with the spin and orbital angular momenta. The form is $V = -a\mathbf{L}\cdot\mathbf{S}$.

   (a) Does this perturbation couple the states made by coupling orbital and spin angular momenta to make states with $j$ and $m_j$? Use the Wigner-Eckhart theorem to make your answer.

   (b) Use perturbation theory to evaluate how each of the coupled states with $j$ and $m_j$ shift.

2. **Nuclear Size:**

   (a) Assuming the proton is a uniformly charged sphere, determine the interaction V that describes the difference between the Coulomb potential of the nucleus (used to solve the hydrogen atom in undergrad physics) and the actual potential of the uniformly charged sphere of radius $a_p$.

   (b) How big is the the proton compared to the radius of the ground state hydrogen atom? Use this ratio to justify the assertion that the atomic wavefunction does not change over the size of the nucleus.

   (c) Neglecting spin and nuclear spin, what it the shift of the ground state energy level due to the finite size of the proton.

   *The next problem is a partial redo given that it gave trouble to some in Problem Set 3. An efficient approach will be discussed in class, and additional questions below provide more guidance. If your previous solution was a good one, then you can simply resubmit a copy of it and get credit for a second time. :)*

3. **Hydrogen matrix elements:** Using the Wigner-Eckhart Theorem and Mathematica (as much as possible) to automate and minimize your labor, compute the matrix elements of the operators $x$, $y$ and $z$ for each of the following cases.

   A: Matrix elements between the coupled $n = 1$ states of hydrogen formed by coupling orbital angular momentum to electron spin, assuming no nuclear spin.

B: Matrix elements between the coupled $n = 2$ states of hydrogen formed by coupling orbital angular momentum to electron spin, assuming no nuclear spin.

(a) Write down the operators x, y and z in terms of the spherical tensors $r_{1,1}$, $r_{1,0}$ and $r_{1,-1}$.

(b) What is the parity of the operators? What consequence does this have on the matrix elements you are asked to compute.

(c) What is the parity consequence for the case A matrix elements? Explain.

(d) For case B matrix elements:

   i. What matrix elements vanish because of parity symmetry?
   ii. For the remaining matrix elements, how can you take advantage of the fact that the operators are Hermitian?
   iii. List the reduced matrix elements that you must compute to determine the rest of the matrix elements.
   iv. List your choice of the simplest matrix element that you must compute to determine the reduced matrix element.
   v. Use Mathematica to compute the radial integrals needed.
   vi. Compute the simple matrix elements.
   vii. Use the above to compute the value of the needed reduced matrix elements.
   viii. Use the Wigner-Eckart theorem to calculate the remaining matrix elements for the $r_{kq}$.
   ix. From these matrix elements, deduce the matrix elements of x, y, and z.

4. **Zeeman and Stark Shifts:**

(a) Evaluate the energy levels of the states of the n=2 hydrogen atom when a magnetic field is applied?

(b) Evaluate the energy levels of the states of the n=2 hydrogen atom when an electric field is applied.

(c) Evaluate the first and second order Stark shifts for a ground state hydrogen atom. If you encounter an infinite sum, use Mathematica to see if your result converges with increasing numbers of terms. (There are more elegant methods.)