

Ideal Gases (non-interacting part.)

based on  $w_n$  Petit

→  $w_{NN}$  Grand

criterion for "classical" behavior

$n \equiv \frac{N}{V} \rightarrow$  separation  $a_0 \sim \frac{1}{n^{1/3}}$

for class. behavior

$$\bar{n}_k \ll 1$$

assign volume to quantum state

$$\frac{\Delta q \Delta p}{2\pi\hbar} \sim \bar{n}_k \ll 1$$

$\Delta q: a_0$ ;  $\Delta p$  thermal momentum

$$a_0 \gg \lambda_{dB} \rightarrow kT \gg \frac{\hbar^2}{2ma_0^2}$$

$$\sum_k \bar{n}_k = N$$

if part don't then each part can be regarded as separate subsystem

$$\text{then } \bar{n}_k = w_k = e^{-\alpha + \beta \epsilon_k}$$

$$\bar{n}_k = a e^{-\beta \epsilon_k} \quad \text{Gibbs petit}$$

$$\text{normalized } \sum_k \bar{n}_k = N$$

Grand case

$$\text{many body } E_{NN} \rightarrow n e_{kk} \quad w_{NN} \rightarrow w_{nk}$$

$$\sum_{nN} w_N = 1; \quad \sum w_{nk}(\epsilon_k)$$

Gibbs Grand ensemble

$$W = e^{\beta(\Omega + \sum_k n_k \mu - \sum_k n_k \epsilon_k)}$$

$$n_k = \text{integers: } 0, 1, 2$$

$$w_0 = e^{\beta\Omega}; \quad w_1 = e^{\beta(\Omega + \mu - \epsilon_1)}; \quad w_2 = e^{\beta(\Omega + 2\mu - 2\epsilon_1)}$$

Norm

$$\sum_{n_k} w_{n_k} = 1 = w_0 + w_1 + \dots$$

$$\bar{n}_k = \sum_{n_k} n_k w_{n_k} \quad \text{converges fast}$$

$$= 0 \cdot w_0 + 1 \cdot w_1 + \dots \quad \text{stop}$$

$$\bar{n}_k = w_1(\epsilon_k) / w_0 = e^{\beta(\mu - \epsilon_k)}$$

$$\frac{\bar{n}_k}{n_k} = e^{\beta(\mu - \epsilon_k)} \quad \text{Gibbs Grand}$$

$$\Rightarrow a = e^{\beta\mu}$$

Classical particle energies

$$\epsilon_k = \frac{p^2}{2m} + U(q)$$

so (gently)

$$n \rightarrow n(\vec{p}, \vec{q})$$

$$\text{normalize } \int n(\vec{p}, \vec{q}) d\vec{p} d\vec{q} = N$$

$$= V \left( \frac{\int d^3p}{(2\pi\hbar)^3} \right) e^{\beta(\mu - (\frac{p^2}{2m} + U(q)))}$$

home work do integral

$$= N$$