# Quantum Mechanics 412-1 Discussion

Tuesday, 29 October 2019

1. **Commutator games.**
   (a) Using $[x, p] = i\hbar$, compute $[x, p^2]$. (Do not assume a position space representation, meaning, the momentum operator does not necessarily take the form $p = -i\hbar\frac{d}{dx}$ that it does in position space.)
   (b) Compute $[x, p^3]$.
   (c) Using those two results, can you guess a general form for $[x, p^n]$? *Bonus:* prove this form by induction.

2. **Hamilton's equations of motion in quantum mechanics.**
   Consider a system describing a single particle moving in a position-dependent potential $V(x)$ with Hamiltonian, $H = \frac{p^2}{2m} + V(x)$.
   (a) Calculate $\frac{d\langle x\rangle}{dt}$ using Ehrenfest's theorem,
   $$\frac{d\langle A\rangle}{dt} = \frac{1}{i\hbar}\langle[A, H]\rangle + \frac{\partial A}{\partial t} \qquad (1)$$
   (b) Assuming that a power series expansion for $V(x)$ exists and using the result $[p, x^n] = -i\hbar n x^{n-1}$, calculate $\frac{d\langle p\rangle}{dt}$.
   (c) Show your answers are equivalent to Hamilton's equations of motion:
   $$\frac{d\langle x\rangle}{dt} = \left\langle \frac{\partial H}{\partial p} \right\rangle \qquad \frac{d\langle p\rangle}{dt} = -\left\langle \frac{\partial H}{\partial x} \right\rangle \qquad (2)$$