HW 6.3

Monday, February 22, 2021 8:22 PM

$\boxed{i}$

$$L(\phi, \phi^*) = |\partial_\mu \phi|^2 - m^2 |\phi|^2$$

$$= \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

$$\frac{\partial L}{\partial(\partial_\mu \phi^*)} = \frac{\partial L}{\partial \phi^*}$$

$$\phi = \frac{1}{2}(a(x) + ib(x))$$

$$\phi^* = \frac{1}{2}(a(x) - ib(x))$$

$$\partial_\mu \partial^\mu \phi = -m^2 \phi$$

$$(\partial^2 + m^2)\phi = 0 \quad \boxed{E.O.M\ 1}$$

$$(\partial^2 + m^2)\phi^* = 0 \quad \boxed{E.O.M.\ 2}$$

$\boxed{ii}$ $\phi \rightarrow e^{i\alpha}\phi$

$$\delta S = \delta\left(\int L\, d^4x\right)$$

$$= \int (\delta L + \partial_\mu \delta x^\mu)\, d^4x \Rightarrow$$

$$\boxed{d^4x' = \left|\frac{\partial x'}{\partial x}\right| d^4x = (1 + \partial_\mu \delta x^\mu) d^4x}$$

$$\Rightarrow \int \left\{ \partial_\mu L \, \delta x^\mu + \partial_\nu \left[\frac{\partial L}{\partial(\partial_\nu \phi)} \delta_\phi \phi \right] + L \partial_\mu \delta x^\mu \right\} d^4x$$

$$= \int \partial_\mu \left[\frac{\partial L}{\partial(\partial_\nu \phi)} \delta_\phi \phi + L \, \delta x^\mu \right] d^4x$$

$$= \int \partial_\mu \left[\frac{\partial L}{\partial(\partial_\nu \phi)} (\delta \phi - \partial_\mu \phi \, \partial x^\mu) + L \delta x^\mu \right] d^4x$$

current $j^\mu{}_\nu = \dfrac{-\partial L}{\partial(\partial_\nu \phi)} (\partial \phi - \partial_\mu \phi \, \partial x^\mu) - L \delta x^\mu$

$\delta S = -\int \partial_\mu j^\mu{}_\nu \, d^4x$ (divergence of current)