8. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (2 pts)

In the relativistic case the power radiated by a particle into solid angle Ω around $\vec{\hat{n}}$ is given by

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{e^2}{16\pi^2 c} \frac{1}{\left(1 - \vec{\hat{n}} \cdot \vec{\beta}\right)^5} \left| \vec{\hat{n}} \times \left((\vec{\hat{n}} - \vec{\beta}) \times \frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t} \right) \right|^2 \Bigg|_{\mathrm{ret}}$$

i) Derive the corresponding non-relativistic "Larmor" formula as a function of θ instead of $\vec{\hat{n}}$, where θ is the angle between $\vec{\hat{n}}$ and $\frac{\mathrm{d}\beta}{\mathrm{d}t}$.

Hint: You can use $1 - \vec{\hat{n}} \cdot \vec{\beta} \approx 1$ and $(\vec{\hat{n}} - \vec{\beta}) \times \frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t} \approx \vec{\hat{n}} \times \frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t}$.

ii) What is the total "Larmor" power radiated in the non-relativistic case?

Exercise 2 (3 pts)

Use the Larmor formula to obtain the angular distribution of the radiation $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$ and the time-averaged total radiated power i) for a linear harmonic motion $x(t) = a\cos(\omega_0 t)$ and ii) for a circular motion with radius R and constant angular frequency ω_0 in the $x - y$ plane.

Exercise 3 (3 pts)

Consider the relativistic formula in Ex. 1 and i) derive $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$ as a function of θ for a linear, relativistic acceleration, where $\vec{\beta} \parallel \frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t}$. ii) What is the total radiated power?

Exercise 4 (3 pts)

Consider a relativistic particle that enters a medium with speed v_0 (linear problem), slowed down by a 4-force proportional to its 3-velocity: $F^i = -\eta v^i$. i) Calculate the 3-acceleration of the particle. ii) Using the result of Ex. 3, calculate the total radiated power since the particle entered the medium until it stops, i.e. the total energy emitted. In your calculation you can assume that the power is as if it is emitted in vacuum.