PS4

Wednesday, April 27, 2022 11:50 PM

1.) $P(x) = \begin{cases} Ae^{-\lambda x} & 0 \leq x \leq \infty \\ 0 & x < 0 \end{cases}$

a.) $\int_0^\infty Ae^{-\lambda x}\,dx = A\int_0^\infty e^{-\lambda x}\,dx = \frac{-A}{\lambda}\left(e^{-\lambda x}\right)\Big|_0^\infty$

$= \frac{-A}{\lambda}\left(e^{-\infty} - e^{0}\right) = \frac{-A}{\lambda}(0-1) = \frac{A}{\lambda} = 1$

$A = \lambda$

b.) $\langle x \rangle = \int_0^\infty x P(x)\,dx = \int_0^\infty x\lambda e^{-\lambda x}\,dx$

$= \lambda\int_0^\infty xe^{-\lambda x}\,dx = \lambda\left[\frac{e^{-\lambda x}}{\lambda}\left(x - \frac{1}{\lambda}\right)\right]\Big|_0^\infty$

$= \left(xe^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda}\right)\Big|_0^\infty = 0 - \frac{1}{\lambda} = \frac{-1}{\lambda}$

$\frac{d[P(x)]}{dx} = 0 \quad \frac{d}{dx}\left[\lambda e^{-\lambda x}\right] = -\lambda^2 e^{-\lambda x} = 0$

$-\lambda x$

$e^{\cdots} = 0 \quad \ln|e^{-\lambda x}| = \ln|0|$

$-\lambda x = 1 \qquad x = \frac{-1}{\lambda}$

c.) $P_{12}(x) = \int_1^2 \lambda e^{-\lambda x} \, dx = \int_1^2 e^{-x} \, dx$ ($\lambda = 1$)

$= -(e^{-2} - e^{-1}) = 0.23 = 23\%$

2.) a.) $P(b) = 0.2\,N$

b.) $P(\frac{1}{5}) = (0.2)^5 = 3.2 \times 10^{-4}$

c.) $P(\frac{2}{10}) = 2 \cdot (0.2)^{10} = 2.05 \times 10^{-7}$

d.) Having a 20% chance of success per turn means that over large N (turns), chances of success are exponentially less.

3.) a.) ideal gas: $\Omega \sim V^N$

$\Omega \sim V^{-N}$

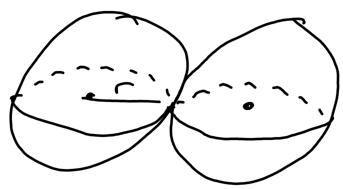
hard-sphere gas.

b.) $PV = NkT$

when $Nv_0 << V$, then $V \rightarrow V - jv_0$

let $b = jv_0$, then $V \rightarrow V - b$

c.) $V = \frac{4\pi r^3}{3}$



$d = 2r$

If you have two particles, volume per particle $V = \frac{4\pi r^3}{3}$. The excluded volume $V' = \frac{4\pi d^3}{3} = \frac{32\pi r^3}{3}$

$$b = \frac{V'}{2} = \frac{1}{2}\frac{32\pi r^3}{3} = \frac{16\pi r^3}{3} = 4V$$

4.) $\epsilon(n) = nh\nu$, $n = 0, 1, 2, \ldots$

$$\Omega(n, N) = \frac{N!}{n!(N-n)!}$$

$$\Omega(N, E) = \frac{(E+N-1)!}{E!(N-1)!}$$

b.) $S = k\log(\Omega) = k\ln|\Omega|$

$$= k\ln\left[\frac{(E+N-1)!}{E!(N-1)!}\right]$$

$$= k\{\ln[(E+N-1)!] - \ln[E!] - \ln[(N-1)!]\}$$

$$= k\{(E+N-1)\ln(E+N-1) - (E+N-1)$$
$$- E\ln(E) + E - (N-1)\ln(N-1) + (N-1)\}$$

$$= k\{(E+N-1)\ln(E+N-1) - E\ln(E)$$
$$- (N-1)\ln(N-1)\}$$

c.) $E = \frac{3}{2}NkT$

$$T = \frac{2}{3}\frac{E}{Nk} \quad ?$$