Quantum Mechanics 412-1 Discussion (Week One)

1. A certain observable C in quantum mechanics has the 3-by-3 matrix representation:

$$C = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{1}$$

   find the eigenvalues and eigenvectors of this operator, and give a physical example of an operator that might take this matrix form.

2. Consider two operators, A and B, represented in a three-dimensional ket-space by:

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \quad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix} \tag{2}$$

   (a) A is clearly degenerate. Is B also degenerate?
   (b) Show that A and B commute.
   (c) Find a simultaneous eigenbasis for both A and B.

3. Prove the Schwarz inequality by considering ket $|v\rangle$ for some choice of parameter λ:

$$|v\rangle = |\alpha\rangle + \lambda\,|\beta\rangle \tag{3}$$

4. Find the 3-vector $\vec{j}$ such that $\frac{\partial|\psi|^2}{\partial t} = -\vec{\nabla}\cdot\vec{j}$. What is the physical interpretation of $\vec{j}$? Calculate $\vec{j}$ for the wavefunction $\psi(\vec{r}) = Ae^{i\vec{p}\cdot\vec{r}/\hbar} + Be^{-i\vec{p}\cdot\vec{r}/\hbar}$

5. A particle is in the ground state of a box of length L. Suddenly the box expands (symmetrically) to twice its size, leaving the wavefunction undisturbed. Show that the probability of finding the particle in the ground state of the new box is $(8/3\pi)^2$.

6. Two observables J and Q do not commute, but both commute with a system's Hamiltonian, H. Show that the energy eigenstates of the system are, in general, degenerate. Come up with a physical example of such a system and two such observables J and Q.