Physics 412-3 Spring 2020

Problem Set 2

Due by 5 pm Friday May 1.

1) (5 pts) A particle of mass m is scattered from a potential $V(r) = -\frac{\varepsilon}{r^3} exp[-r/\lambda]$ where ε, λ are positive real constants. Determine the scattering amplitude $f_k^{(1)}(\theta, \phi)$ using the first order Born Approximation. Evaluate all integrals to the extent possible. Is the scattering amplitude isotropic? Why or why not do you expect it to be isotropic?

2) (5 pts) A particle of mass m scatters from a target potential $V(r)$. Show that, using the first Born Approximation, that the total cross section σ approximately equals $\frac{m^2}{\pi\hbar^4}\int d^3r'\int d^3r V(r)V(r')\frac{\sin^2[k|\vec{r}-\vec{r}'|]}{k^2|\vec{r}-\vec{r}'|^2}$ by integrating the differential cross section.

3) (5 pts) Consider the p-wave (l=1) phase shift $\delta_1(k)$ produced during scattering by a hard-sphere potential: $V = \infty, r < r_0, and\ V = 0, r > r_0$.

a.) Show that the solution of the radial equation for the function $u_{k,1}(r)$ for $r > r0$ is of the form $u_{k,1}(r) = C\left[\frac{sin\,kr}{kr} - cos\,kr + a\left(\frac{cos\,kr}{kr} + sin\,kr\right)\right]$ where C and a are constants.
b.) Show that $a = tan\,\delta_1(k)$ and determine the value of a given the boundary conditions at $r = r_0$.
c.) Show that as $k \rightarrow 0$, $\delta_1(k)$ becomes negligible compared with $\delta_0(k)$ (derived in class).

4) (5 pts) Consider s-wave scattering from a central potential $V(r) = -V_0, r < r_0, V = 0$ otherwise. Assume V_0 is positive, and the incident energy E > 0.

a.) Write the radial equation and show that the solution is of the form $u_{k,0}(r) = A\,sin(kr + \delta_0)\,, r > r_0$, and $u_{k,0}(r) = B\,sin(Kr)\,, r < r_0$, where $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $K = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$, and A and B are constants.

b.) Assuming A=1, using the boundary conditions at $r = r_0$, show that B and δ_0 are given by $B^2 = \frac{1}{1+\frac{V_0}{E}cos^2 Kr_0}$ and $\delta_0 = -kr_0 + tan^{-1}(\frac{k}{K}tan\,Kr_0)$.

c.) Show that B^2 exhibits maxima as a function of k and determine the values of k associated with these maxima as well as the value of δ_0 at these scattering resonances in the limit of small energy $kr_0 \ll 1$.