HW 1

Friday, April 16, 2021 7:25 PM

1.) a.)

$$E_1 = E_0 e^{i(k_1 x - \omega_1 t)} \hat{z} \qquad k_i = \frac{\omega_i}{c}$$

$$E_2 = E_0 e^{i(k_2 x - \omega_2 t)} \hat{z}$$

$$I(x,t) = \frac{c}{8\pi} \vec{E} \cdot \vec{E}^* \qquad E = E_1 + E_2$$

$$E \cdot E^* = (E_1 + E_2) \cdot (E_1^* + E_2^*)$$

$$= E_1 \cdot E_1^* + E_1 \cdot E_2^* + E_2 \cdot E_1^* + E_2 \cdot E_2^*$$

$$= E_0^2 \left[e^{i(k_1 x - \omega_1 t)} \cdot e^{-i(k_1 x - \omega_1 t)} \right]$$

$$+ E_0^2 \left[e^{i(k_1 x - \omega_1 t)} \cdot e^{-i(k_2 x - \omega_2 t)} \right]$$

$$+ E_0^2 \left[e^{i(-k_2 x - \omega_2 t)} \cdot e^{-i(k_1 x - \omega_1 t)} \right]$$

$$+ E_0^2 \left[e^{i(-k_2 x - \omega_2 t)} \cdot e^{-i(-k_2 - \omega_2 t)} \right]$$

$$= E_0^2 \left\{ 1 + e^{i(k_1 x - \omega_1 t - k_2 x + \omega_2 t)} \right.$$

$$\left. + e^{i(-k_2 x - \omega_2 t - k_1 x + \omega_1 t)} + 1 \right\}$$

$$= E_0^2 \left\{ 2 + e^{i[x(k_1 - k_2) - t(\omega_1 - \omega_2)]} \right.$$

$$\left. + e^{i[x(-k_1 - k_2) + t(\omega_1 - \omega_2)]} \right\}$$

b.) $\Delta\omega = \omega_1 - \omega_2$ $\qquad \omega_1 = ck + \frac{\Delta\omega}{2}$

$$\omega_2 = ck - \frac{\Delta\omega}{2}$$

$$E_0^2 \left\{ 2 + e^{i\left[x\left(\frac{\omega_1}{c} - \frac{\omega_2}{c}\right) - t(\omega_1 - \omega_2)\right]} \right.$$

$$\left. + e^{i\left[x\left(-\frac{\omega_1}{c} - \frac{\omega_2}{c}\right) + t(\omega_1 - \omega_2)\right]} \right\}$$

$$= E_0^2 \left\{ 2 + e^{i[cx\Delta\omega - t\Delta\omega]} \right.$$

$$\left. + e^{i\left[x\left(-\frac{\omega_1 - \omega_2}{c}\right) + t\Delta\omega\right]} \right\}$$

c.) $v = c$

2.a

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} + \cdots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\right)$$

$$= \cos\theta + i\sin\theta$$

b. $a = a_0 e^{-i\omega t}$ $\quad b = b_0 e^{-i\omega t}$

$$a = a_0\cos(\omega t) - i a_0 \sin(\omega t) = Re(a) + Im(a)$$

$$b = b_0\cos(\omega t) - i b_0 \sin(\omega t) = Re(b) + Im(b)$$

$$a \cdot b^* = a_0 b_0 \cos^2(\omega t) + a_0 b_0 \sin^2(\omega t)$$

$$+ i a_0 b_0 \cos(\omega t) - i a_0 b_0 \cos(\omega t)\sin(\omega t)$$

$$= a_0 b_0$$

$$Re(a)\cdot Re(b) = ab\cos^2(\omega t)$$

$Re(a)\, Re(b) = a_0 b_0 \cos$

$$\langle \cos^2(\omega t) \rangle = \frac{1}{2}$$

$$\langle Re(a) \cdot Re(b) \rangle = \frac{1}{2} a_0 b_0 = \frac{1}{2} Re\left[a \cdot b^*\right]$$