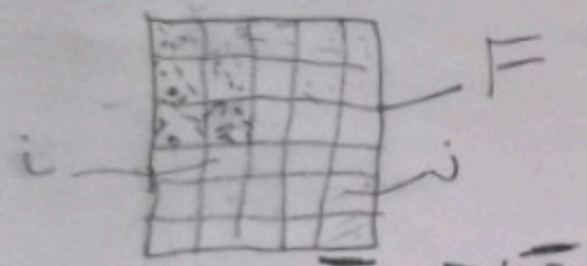
Law of Large Numbers

(fluctations)

f_i little part



$F = \sum f_i$; $\bar{F} = \sum_i \bar{f}$

$\bar{f}_i$ = averag

$\Delta f_i = \bar{f}_i - f_i$

$\Delta F = F - \bar{F} = \sum_i^N (f_i - \bar{f}) = \sum_i \Delta f_i = 0$

$(\Delta F)^2 = \left(\sum_{i=1}^N \Delta f_i\right) = \sum_i \Delta f_i \sum_j \Delta f_j$

terms w $i \neq j$ uncorrelated

therefore vanish

$(\Delta F)^2 = \sum_{i=1}^N (\Delta f_i)^2$

make all parts (approx.)

(2)

$\sum_{i=1}^N (\Delta f_i)^2 = N(\overline{\Delta f})^2$

$\frac{(\Delta F)^2}{F^2} \rightarrow \frac{[(\Delta F)^2]^{1/2}}{F}$

$= \frac{N^{1/2}\,\Delta f}{N\bar{f}}$

$= \frac{\Delta F}{N^{1/2}\bar{f}} = \frac{[(\Delta F)^2]^{1/2}}{F}$

fluc in f fluc in F

drop of water

$1\ cm^3 = 10^{22}$ mol.

$\frac{1}{(10^{22})^{1/2}} \sim 10^{-11}$ N