Applied Nonlinear Dynamics 322

Spring 2022

Hermann Riecke

Problem Set 1 **Saturday, April 2, 2022** **due Friday April 8, 2022**

1. Is it possible to find a function $f(x)$ such that the equation $\dot{x} = f(x)$ has the fixed points indicated in Fig.1a,b below, which shows the phase line of that system? If so give a function that will lead to these dynamics, if not, explain why not.
   Stable fixed points are indicated by solid circles, unstable ones by open circles, and semistable fixed points as half-open and half-solid circles.

a) b)

Figure 1: a) Phase line. b) Phase line.

2. In the bifurcation diagram shown in Fig.2 add the arrows indicating the flow along the phase lines (i.e. along vertical lines at fixed μ). Hashing indicates branches of unstable fixed points, solid lines indicate branches of stable fixed points. Give a function $f(x, \mu)$ that leads to such a bifurcation diagram. What kind of bifurcations arise in this system?

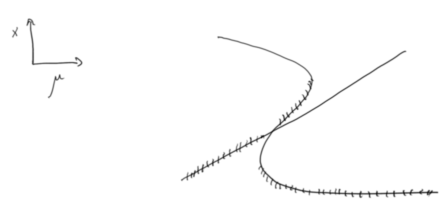

Figure 2: A bifurcation diagram.

3. For each of the following dynamical systems sketch the bifurcation diagram of the fixed points as a function of r and add in the bifurcation diagrams all qualitatively different vector fields that occur as r is varied. Identify the type of bifurcation that occurs and the value of r at that bifurcation point.

   (a) $\dot{x} = -x\left(x^2 - 1 + r^2\right)$

   (b) $\dot{x} = rx - \ln\left(1 + x\right)$

4. Consider

$$\dot{x} = f(x, \mu) = e^{-x} - \cos(x - \mu) \ .$$

(a) Sketch $f(x, \mu)$ for a representative value of μ and indicate the corresponding flow on the phase line.

(b) Analyze and discuss what happens when μ is varied in $[0, 2\pi]$. How many bifurcations occur? What type are they? Sketch a bifurcation diagram. What happens for μ outside $[0, 2\pi]$?

5. Determine all fixed points and their stability for

(a) $\dot{x} = x^2 - 1$

(b) $\dot{x} = x^4$.

Analyze the stability of the fixed points in two ways: i) plot the vector field on the phase line and ii) perform a linear stability analysis. Do both approaches yield equivalent information? In part b) also calculate the exact solution for an initial condition $x(0) \neq 0$. Compare this time dependence with that obtained in the linear stability analysis.