

Quantum Prob. $-\alpha^{(a)} - \beta E_n^{(a)}$

$$W_n^{(a)} = e$$

$$\sum_n W_n^{(a)} = 1 \quad \text{Petit Ensemble}$$

$$W_{nN}^{(a)} = e^{-\alpha^{(a)} - \beta E_{nN}^{(a)} - \gamma N^{(a)}} \quad \text{Grand Ensemble}$$

$$\sum_n \sum_N W_{nN}^{(a)} = 1$$

$$W_n = e^{\frac{F - E_n}{k_B T}} \quad \text{Gibbs Petit}$$

apply Norm.

$$F = \frac{1}{\beta} \ln \sum_n e^{-\beta E_n} \equiv Z; Z = \sum_n e^{-\beta E_n}$$

$$F = -\frac{1}{\beta} \ln Z \quad F = F(V, T) \\ Z = Z(V, T)$$

$$S = \left(\frac{\partial F}{\partial T} \right)_V; \bar{E} = F + TS$$

$$\bar{E} = k_B T^2 \frac{\partial \ln Z}{\partial T}$$

$$\bar{E} = \frac{\sum_n E_n e^{-\beta E_n}}{\sum_n e^{-\beta E_n}}$$

$$W_{nN} = e^{\beta(\Omega + \mu N - E_n)} \quad \text{Grand Ensemble}$$

$$d\Omega = -SdT + PdV - Nd\mu$$

$$W_n = A e^{-\beta E_n} \quad \text{Petit}$$

$$W_{nN} = A_G e^{\beta(\mu N - E_n)}$$

$$A_P = e^{\beta F} \quad A_G = e^{\beta \Omega}$$

Classical Limits

$$dw = A_c e^{-\beta E(p, q)} dp dq \quad \int dw = \int dp dq = \text{phase space volume}$$

$$\int dw = 1 \Rightarrow A_c = \frac{1}{\int dp dq e^{-\beta E(p, q)}}$$

Classical Partition Function

$$Z = \int dp dq e^{-\beta E(p, q)}$$

Average of some $f(p, q)$

$$\bar{f} = \frac{\int f(p, q) e^{-\beta E(p, q)} dp dq}{\int e^{-\beta E(p, q)} dp dq}$$

$$\bar{f} = \frac{\int dp dq f(p, q) e^{-\beta E(p, q)}}{Z}$$

$$E(p, q) = K(p) + U(q)$$

$$dw = dw_p dw_q \quad \int dw_p = 1 \quad \int dw_q = 1$$