7. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (4 pts)

A relativistic particle with mass m and charge e moves in a uniform, static, non-vanishing electric and magnetic field such that $\vec{E}_0 \perp \vec{B}_0$. You can assume that $\vec{E}_0 = (0, E, 0)$ and $\vec{B}_0 = (0, 0, B)$.

i) Describe the particle trajectory for $|\vec{E}_0| \neq |\vec{B}_0|$.

ii) In the special case when $|\vec{E}_0| = |\vec{B}_0|$, show that $t(\tau)$ and $x(\tau)$ are generally cubic polynomials in τ, while $y(\tau)$ is a quadratic polynomial, while $z(\tau)$ is linear.

Exercise 2 (8 pts)

In the video lectures, we derived the Lienard-Wiechert field for a charged particle moving with constant velocity ($\dot{\omega}^\mu = 0$):

$$F^{\mu\nu} = \frac{e}{4\pi} \frac{1}{(R \cdot \omega)^2} \left(k^\mu \omega^\nu - \omega^\mu k^\nu\right)\Big|_{\text{ret}}$$

Earlier, we also calculated the corresponding $\vec{E}$ and $\vec{B}$ fields by a Lorentz transformation of the fields of a charged particle. In particular, the electric field was found to be (we called $\vec{x}$ as $\vec{r}$ back then and e was called q):

$$\vec{E} = \frac{e}{4\pi\gamma^2 \left(1 - \beta^2 \sin^2\theta\right)^{3/2}} \frac{\vec{x}}{x^3}$$

i) Show that the L.-W. formula recovers this expression of $\vec{E}$.

ii) Using the L.-W. field show that

$$\vec{B} = \vec{n} \times \vec{E}$$

iii) Calculate the total radiated power by integrating the Poynting vector over a spherical surface.