Physics 332, Spring 2022
Problem Set 2
Due Wednesday April 13th - either bring to class or e-mail to Prof. Driscoll by 1:00 pm.

1. Why are thermometers small devices in comparison to the system of interest?

2. Why do we use the triple point of water to calibrate thermometers? Why not use the melting point or the boiling point?

3. Consider a system described by the van der Waals equation of state which expands at constant temperature from volume V_1 to volume V_2. Assume that the density is very low over the range of volumes of interest, e.g. $\rho = N/V \ll 1$.

   (a) Calculate the work W_{vdw} done on the gas to the lowest relevant order in ρ.

   (b) Now calculate the work W_{ideal} done on the gas under the same conditions, assuming that the gas is an ideal gas (and thus has a different equation of state).

   (c) Find the difference $W_{\text{vdw}} - W_{\text{ideal}}$, and discuss the reason this difference is positive or negative as a function of the temperature.

4. Thermodynamics is a powerful tool which we can use to calculate the properties of all kinds of systems, even quite exotic ones such as black holes. A black hole is created from the collapse of a massive object into one so dense that nothing, including light, can escape beyond a certain radius. Here we will use what we have learned about thermodynamics to estimate the entropy and temperature of a charge neutral non-rotating black hole.

   (a) A black hole is a consequence of the general theory of relativity, and thus we expect that its radius R depends only on its mass M, the gravitational constant G, and and the speed of light c. Use dimensional analysis to estimate the radius R of a black hole in terms of M, G, and c.

   (b) Now write an expression for the entropy of a black hole in terms of G, M, h, c, and k. Start with the assumption that entropy is of order Nk. *Hint: The maximum entropy occurs when the black hole is made of photons whose wavelength is the size of the black hole ($\lambda = 2R$).*

   (c) Does the entropy increase or decrease when two black holes coalesce into one?

   (d) Express the entropy in terms of the surface area A of the black hole instead of M. Note that the area is a direct measure of the entropy.

   (e) Express S in terms of the total energy E instead of M, and determine the temperature for a one solar mass black hole.