HW 5

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1.) 5.81 $dw(q) = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} \exp\left(\frac{-q^2\omega}{\hbar}\right) dq$

$$dw(p) = \left(\frac{1}{\pi\hbar\omega} \tanh\left(\frac{\beta\hbar\omega}{2}\right)\right)^{1/2} \exp\left[\frac{-p^2}{\hbar\omega} \tanh\left(\frac{\beta\hbar\omega}{2}\right)\right] dp$$

5.55 $dw_{p'_\alpha} = \sqrt{\frac{\beta}{2\pi}}\, e^{-\frac{1}{2}\beta p'^2_\alpha}$

$\overline{p'^2_\alpha} = \frac{1}{\beta}$ $\rho_{nn} = W_n$

$\rho_{pp} = W_p$

$$dw_{q'_\alpha} = \sqrt{\frac{\beta\omega_\alpha^2}{2\pi}}\, e^{-\frac{1}{2}\beta\omega_\alpha^2 q'^2_\alpha}$$

$$\overline{q'^2_\alpha} = \frac{1}{\beta\omega_\alpha^2}$$

$W_n^{(\alpha)} = e^{-\alpha^{(a)} - \beta E_n^{(a)}}$ $\boxed{5.1}$

$$\rho(p,p',t) = \sum_{m,n} \rho_{mn} \psi_n^*(p',t)\psi_m(p,t)$$

$$\rho(p,p,t) = \sum_n w_n \psi_n^2(q) = A\sum_n e^{-\beta\varepsilon_n}\psi_n^2(p)$$

$$dw_p = \rho(p,p)dp$$

$$dw_p = \sum_n A e^{-\beta\varepsilon_n}\psi_n^2 dp$$

$$\frac{d\rho(p)}{dp} = 2A\sum_n e^{-\beta\varepsilon_n}\psi_n(p)d\frac{d\psi_n}{dp}$$

$$\frac{d\psi_n(p)}{dp} = \frac{i}{\hbar}\hat{p}\psi_n(q)$$

$$= \frac{i}{\hbar}\left[(\hat{p})_{n-1,n}\psi_{n-1} + (\hat{p})_{n+1,n}\psi_{n+1}\right]$$

$$\frac{d\rho(p)}{dp} = \frac{2Ai}{\hbar}\left[\sum_{n=0}^{\infty}(\hat{p})_{n-1,n}\psi_n\psi_{n-1}e^{-\beta\varepsilon_n}\right.$$

$$-\sum_{n=0}^{\infty}(\hat{p})_{n+1,n}\psi_n\psi_{n+1}e^{-\beta\varepsilon_n}\Big]$$

$$=-\frac{2Ai}{\hbar}\left(1-e^{-\beta\varepsilon_n\omega}\right)\sum_{n=0}^{\infty}(\hat{p})_{n,n+1}\psi_n\psi_{n+1}e^{-\beta\varepsilon_n}$$

$$p\rho(p)=A\left(1+e^{-\beta\hbar\omega}\right)\sum_{n=0}^{\infty}p_{n,n+1}\psi_n\psi_{n+1}e^{-\beta\varepsilon_n}$$

$$\frac{d\rho(p)}{dp}=\frac{-2i}{\hbar}\frac{\left(1-e^{-\beta\hbar\omega}\right)}{\left(1+e^{-\beta\hbar\omega}\right)}p\rho(p)$$

$$=\frac{-2i}{\hbar}\tanh\left(\frac{\beta\hbar\omega}{2}\right)p\rho(p)$$

$$\rho(p)=(\mathrm{Const})\,e^{\frac{-p^2}{\hbar\omega}\tanh\left(\frac{\beta\hbar\omega}{2}\right)}.$$

$$\int dw(p)=\int dp\,\rho(p)=1$$

$$\left[\;\cdots\left(\beta\hbar\omega\right)\right]^{1/2}e^{\frac{-p^2}{\hbar\omega}\tanh\left(\frac{\beta\hbar\omega}{2}\right)}$$

$$dW(p) = \left[\frac{1}{\pi\hbar\omega}\tan\left(\frac{\beta\hbar\omega}{2}\right)\right] e \qquad dp$$

2.] mean square deviation

$$\overline{(\Delta v^2)^2} = \overline{(v^2 - \overline{v^2})^2} \qquad \frac{\sqrt{\overline{(\Delta v^2)^2}}}{\overline{v}^2}$$

$$\overline{(\Delta v_x^2)^2} = \int (\Delta v_x^2)^2 \, dW_{\Delta v_x}$$

$$= \left(\frac{\beta m}{2\pi}\right)^{3/2} \int (\Delta v_x^2)^2 e^{\frac{\Delta\beta m}{2}(\Delta \vec{v}^{\,2})} d\vec{v}$$

$$= \frac{1}{\Delta\beta m} = \frac{k_B \Delta T}{m}$$

$$\frac{\sqrt{\overline{(\Delta v^2)^2}}}{\overline{v^2}}$$ is independent of volume

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4.]

a.) $Z = \int e^{-\beta U} dp\, dq$

$$= \int e^{+\beta \mu_E \mathcal{E} \cos\theta} \sin\theta \, d\theta \, d\phi$$

$$= \frac{4\pi}{\beta \mu \mathcal{E}} \sinh(\beta \mu \mathcal{E})$$

b.)

5.) $E_{tot} = K + \Delta E_{\varepsilon} = K + \frac{1}{2} \chi V \mathcal{E}^2$

$$E_{tot} = Nk_B T = K + \frac{1}{2}\chi V \mathcal{E}^2$$

$$N(V) = N_0 e^{\frac{K + \frac{1}{2}\chi V \mathcal{E}^2}{k_B T}}$$

$$\frac{dN(V)}{dV} = \frac{N_0 \chi \mathcal{E}^2}{2 k_B T} e^{\frac{K + \frac{1}{2}\chi V \mathcal{E}^2}{k_B T}}$$