Local Phase and Gauge Transformations

Global phase invariance [ind. of
$$\vec{x}$$
, t
 $\psi(\vec{x},t) \rightarrow \hat{\psi}(\vec{x},t) = e^{-i\vec{x}}\psi(\vec{x},t)$

Observables: $\langle \vec{\nu} | \vec{0} | \vec{\psi} \rangle = \langle \psi | e^{i\vec{x}} \sigma e^{-i\vec{x}} | \psi \rangle$
 $= \langle \psi | o | \psi \rangle$
 $= \langle \psi | o | \psi \rangle$

Time evolution: use $|\psi\rangle = e^{i\vec{x}}|\vec{\psi}\rangle$
 $ih \frac{\partial}{\partial t}|\psi\rangle = H\psi$
 $ih \frac{\partial}{\partial t}|\psi\rangle = H\psi$
 $ih \frac{\partial}{\partial t}|\psi\rangle = H\psi$

Local phase transformation
$$\psi(\vec{x},t) \longrightarrow \psi(\vec{x},t) = e^{-iX(\vec{x},t)} \psi(\vec{x},t)$$

depends on local location in spacertime

Observables: Use
$$U(\vec{x},t) = e^{-(X(\vec{x},t))}$$

$$< \tilde{\varphi} | \tilde{\partial} | \tilde{\psi} > = < \psi | \partial | \psi >$$

$$\tilde{\theta} = U \partial U^{\dagger} = e^{i \chi(\vec{x}, t)} \partial e^{-i \chi(\vec{x}, t)}$$

Not all operators are invariant x is clearly inv. ガ=-itが is not inv.

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Time evolution after a local phase transformation $H = \overrightarrow{p}^{2} + 2 \overrightarrow{b} \qquad \Rightarrow \text{ complicated because } \overrightarrow{p} \text{ operates on } \mathcal{T}(\vec{x},t)$ $\overrightarrow{p} = u \overrightarrow{p} \qquad = u(i \overrightarrow{h} \overrightarrow{\sigma} i x) u^{+} + u u^{+} \overrightarrow{p}$ $= \overrightarrow{p} + h(\overrightarrow{\sigma} x)$ $U = e^{i x}$ $U = e^{i x}$ $Consider : U(\overrightarrow{p} - 2\overrightarrow{A}) u^{+} = \overrightarrow{p} + h(\overrightarrow{\sigma} x) - 2\overrightarrow{A}$ $\overrightarrow{A} = \overrightarrow{A} - \frac{h(\overrightarrow{\sigma} x)}{2}$

Need generalize H and look for fisuch that it ? IF) = HIF) Try generalization H= (p-9A) + 2 To Thus $\frac{\partial}{\partial t} = \left[\left(\overrightarrow{p} - g \overrightarrow{P} \right)^2 + g \overrightarrow{E} \right] |\psi\rangle$ -/ u+14> Uはまれず>= リレダータデン・ルナ リタ車はナイダ> istat nis $H = (\vec{p} - g\vec{A})^2 + g\vec{\Xi}$ with 第二月一节 To and 第二里+东部

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So for: Generalize to $H = (\vec{p} - 9\vec{A})^2 + 2\vec{E}$

for wavefunction 14>

Make a local phase transformation: $|\psi\rangle \rightarrow |\tilde{\psi}\rangle = e^{-i\mathcal{X}(\vec{x},t)}|\psi\rangle$ The \hat{H} that governs time evolution of $|\tilde{\psi}\rangle$ has $\hat{A} = \hat{A} - \frac{\pi}{2}(\vec{\tau}\mathcal{X})$ transformation $\hat{A} = \Phi + \frac{\pi}{2}\frac{\partial \mathcal{X}}{\partial t}$

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Covariant Derivative - hiding in the generalized Schrödinger equation

$$ih\frac{\partial}{\partial t}\psi = \frac{(p-qh)^2}{2m}\psi + q \oplus \psi$$

$$respected indices$$

$$(ih\frac{\partial}{\partial t} - 2\Phi)\psi = \frac{1}{2m}(-ih\frac{\partial}{\partial x_i} - 2H_i)(-ih\frac{\partial}{\partial x_i} - 2H_i)\psi$$

$$ihc(\frac{\partial}{\partial (ch)} + \frac{iq\Phi}{hc})\psi = -\frac{h^2}{2m}(\frac{\partial}{\partial x_i} + \frac{iq}{h}H_i)(\frac{\partial}{\partial x_i} + \frac{iq}{h}H_i)\psi$$

$$0 = \frac{\partial}{\partial (ch)} + \frac{iq\Phi}{hc}$$

$$p'' = (p', p')$$

$$= (\frac{2}{3c} + \frac{1}{3c}, -\frac{2}{3c}; +\frac{1}{3c}, -\frac{2}{3c}; +\frac{1}{3c}, -\frac{1}{3c})$$

$$= (\frac{2}{3c}, -\overline{7}) + \frac{1}{3c}(\frac{1}{2}\overline{B}, \overline{A})$$

4-vector

smells like a familiar 4-vector $A^{\mu}=(\pm \overline{\mathbf{P}},\widehat{\mathbf{A}})$

electric I vector de potential

S.E.
$$(hc p^{\circ} \psi = -\frac{h^{2}}{2m} p^{j} p^{j} \psi$$
 $(hc up^{\circ} u^{+} \psi = -\frac{h^{2}}{2m} (up^{j} u^{+}) (up^{j} u^{+}) \psi$
 $\tilde{p}^{\circ} = up^{\circ} u^{+} \psi$

eg. $\vec{D}^{j} = -\frac{2}{3x_{j}} + \frac{ig}{\pi}A_{j}$ $ih \vec{D}^{j} = -ih \frac{2}{3x_{j}} + 2A_{j}$ $ih \vec{D} = \vec{p} - 2\vec{A}$ er covariant derivative $i.e. \vec{p} - 2\vec{A}$ transforms like ψ

$$\psi \rightarrow \vec{p} = e^{-iX}\psi$$

$$\vec{p} - g\vec{h} \rightarrow \vec{p} - g\vec{h} = e^{-iX}\vec{p} - g\vec{h}$$

$$\vec{\pi} \rightarrow \vec{\pi} = e^{-iX}\vec{\pi}$$

under gauge transformation (includes a local phase transformation) Invariants $-\tilde{\Xi} = \bar{\Xi} + \frac{\hbar}{9} \frac{\partial x}{\partial t}$ $\widehat{A} = \widehat{A} - \frac{1}{9} \overrightarrow{\nabla} \chi$ OFE = PE+ \$ PEX Same for well-behaved X $\frac{\partial \hat{A}}{\partial t} = \frac{\partial \hat{A}}{\partial t} - \frac{1}{9} \left(\frac{\partial}{\partial t} \nabla X \right)$: [= - PD - DA jauge transformations any scaler

magnétie : B = PXA invariant

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Uniform Magnetic Field and Magnetic Moment) (1)

Consider $\vec{A} = \frac{1}{3}\vec{B} \times \vec{r}$ with \vec{B} independent of time and \vec{l}

 $H = \frac{1}{2m} - \frac{2}{2} \overrightarrow{L} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} - \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$ $= \frac{1}{2m} \cdot \overrightarrow{B} + \frac{9^2 \cancel{B}}{8m} (x^2 + y^2) / 2 \text{ assuming}$



Orbital moment is not so surprising q=-e p=IA er sIunite w=eB 12= mUp $\vec{p} = \frac{-e}{2\pi} \pi e^{2} \hat{z}$ = m (wp)p = mw_p7 = meBp2 - - en 6 3 3 = eBp2 = - \(\frac{1}{2} \) \(\frac{1}{m} \) \(\frac{1}{2} \) \($\widehat{\mathcal{D}} = \frac{9}{2m} \widehat{L}$ Orbital and spin moments orbital D= 2 T -> comes out naturally postulate similar moment for spin ang, mom. $N_s = 9\frac{2}{am}$ 1 dimensionless fudge factor (g-value or g-factor) with g=2 Spin moment dues come naturally from Dirac equation A. A is often neglected 2 eg. for atoms $\frac{3h^{2}}{2m} = \frac{2}{4}3^{2} = \frac{28a_{B}}{4h} = 10^{-5} \frac{8}{107esla}$ i.e. small even for very large field