Postulates of QM, Part III

Postulate 2 covers case where system is prepared in an eigenstate of measurement operator $\hat{O}$. But what if it is not in an eigenstate of $\hat{O}$?

Postulate 3:

The only possible results of a measurement of $\hat{O}$ is one of the eigenvalues of $\hat{O}$.

Postulate 4:

The probability that, in a normalized state $|\psi\rangle$, we measure a particular eigenvalue λ_i, is given by

$$P_i = |\langle\psi_i|\psi\rangle|^2, \text{ where } \hat{O}|\psi_i\rangle = \lambda_i|\psi_i\rangle$$

Postulate 5:

The state of the system immediately after the measurement of the value λ_i is $|\psi_i\rangle$

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- In other words, the measurement of $\mathcal{O}$ is a catastrophic operation on $|\psi\rangle$. "Wave function collapse."

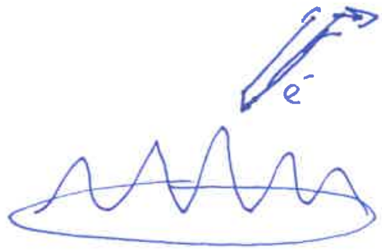
- We can expand $|\psi\rangle$ in basis of eigenstates of $\mathcal{O}$:

$$|\psi\rangle = \sum_i \alpha_i |\psi_i\rangle \quad , \text{where } \alpha_i = \langle \psi_i | \psi \rangle$$

From here, take $|\psi\rangle$ to be normalized & $|\psi_i\rangle$ the basis to be orthonormal,

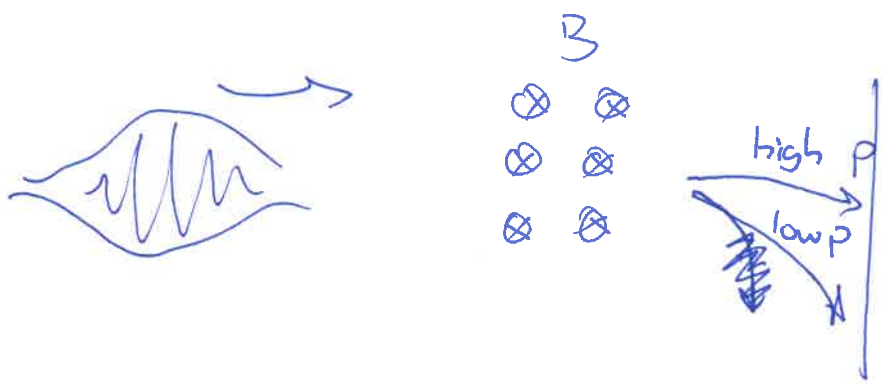
$$1 = \langle \psi | \psi \rangle = \sum_i |\alpha_i|^2$$

- Postulates 3-5 say that the result of measuring $\mathcal{O}$ is to force $|\psi\rangle$ into one of the eigenstates $|\psi_i\rangle$, and that occurs with a probability $|\alpha_i|^2$
- At first this makes some sense. Measurements of microscopic things generally are very invasive to those delicate systems.
- Consider measuring the position of an electron in a "quantum corral", using a scanning tunneling microscope



The electron gets removed in order to measure its position. Certainly seems like a big change in $|\psi\rangle$ is necessary there to measure.

- Or measuring the momentum of an electron using deflection in a $\vec{B}$ field.



Position it strikes on screen tells you momentum, but again we have done something catastrophic to the state.

- In some sense, the measurement postulates reflect the fact that there is no way to measure the properties of an atomic-scale system without changing them in some essential way

- We will return to this question in a moment, to look at it from a updated perspective.
- This idea of wave fuction ~~is this~~ collapse, is built into the Copenhagen intepretation of QM.
- Regardless of whether we believe that interpretation of what is really going on when a measurement is made, Postulates 3-5 are agreed upon.
- And we need to distinguish between a given measurement (always one of the λ_i) and the average result of many repeated measurements. The latter is called the expectation value $\langle O \rangle$ & it is what we get for averaging over many experiments where the system was prepared in the same way at the beginning of each.

Average value of many measurements:

$$\langle \cancel{} \rangle = \sum_i \lambda_i P_i$$

$$= \sum_i \lambda_i |\alpha_i|^2$$

$$= \sum_i \lambda_i \, \cancel{} \, |\langle \psi_i | \psi \rangle|^2$$

$$= \sum_i \lambda_i \langle \psi | \psi_i \rangle \langle \psi_i | \psi \rangle$$

$$= \sum_i \langle \psi | \psi_i \rangle \langle \psi_i | O | \psi \rangle$$

(Now using the completeness of $|\psi_i\rangle$:)

$$\mathbb{1} = \sum_i |\psi_i\rangle\langle\psi_i|$$

$$= \langle \psi | O | \psi \rangle \text{ or } \langle \psi | O \psi \rangle$$

$\langle O \rangle$, the "expectation value of O in ψ"

$= \langle \psi | O | \psi \rangle$

Another quantity $\langle \psi | O | \chi \rangle$ is "the matrix element of O between ψ + χ

QND Measurements

Quantum non-demolition

Ex. 1 Atomic level structure (AMO state readout technique,

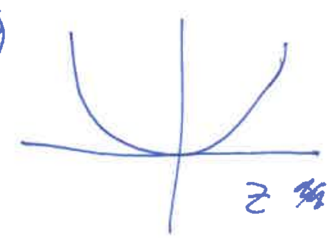
"Shelf" state - metastable, long lifetime

Short lifetime
⇓
Many photons quickly

- Quantum jumps in atomic fluorescence.
- We detect single atom state without changing it, or at least its eigenvalue of an energy measurement.
- If electron was in shelf state, we seemingly didn't even disturb its phase.

Ex. 2 Readout of electron spin in Penning trap

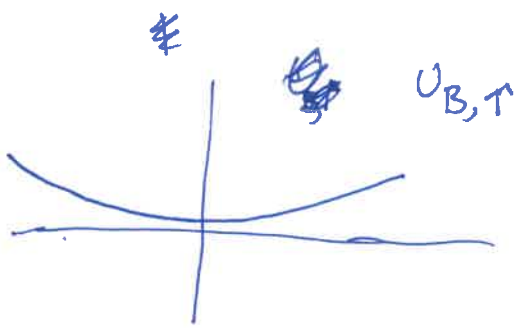
$V(x)$ ~~...~~

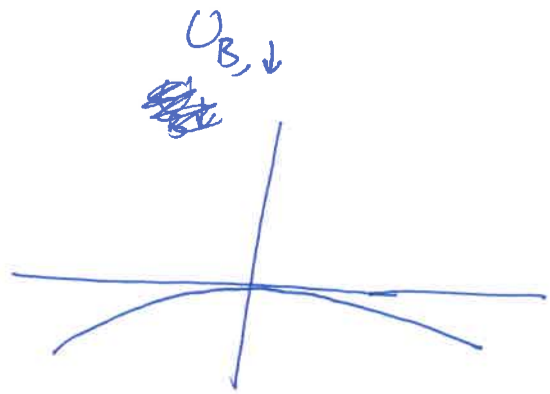


Add to this

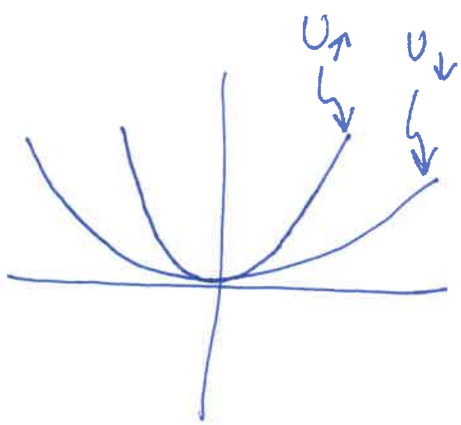
$$B(z) = \beta z^2 \quad , \quad U_B = -\vec{\mu}\cdot\vec{B}$$

Then if spin is $\uparrow$, the electron sees a different quadratic potential than if it is down:





Now overall potential looks like



If we detect harmonic oscillations of electron in trap, without destroying it or changing its spin orientation, we can read out its quantum state!

- These modern AMO measurement techniques are much more gentle than what (most?) people thought was possible in the early days of quantum mechanics.

- And ~~yet~~ the picture of wave function collapse still seems to give the right results.

- Signatures for electron spin.

① Say $|\psi(t=0)\rangle = |\rightarrow\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$

$= |\uparrow\rangle_x$

$T = \frac{h}{E_\uparrow - E_\downarrow}$

$P_\rightarrow(t)$

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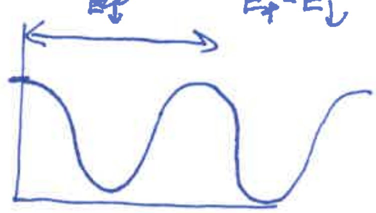

But if at $t = t'$, we measure the spin along the z axis, we get $P_\rightarrow(t) = \frac{1}{2}$ for all subsequent times. ~~(That's true whether we found $|\uparrow\rangle$ or $|\downarrow\rangle$ in our measurement)~~

(If we measured $|\uparrow\rangle$, we prepared the system in that energy eigenstate, so there is no time dependence for any subsequent measurement)

And $|\uparrow\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle$,

thus the $P_{\rightarrow} = \frac{1}{2}$.

② At all subsequent times after t', we measured the spin along z, we would get with 100% confidence the same ~~thing~~ result we got at t'.

$\Rightarrow$ Evidently even the act of "looking" as gently as we can really does fundamentally disturb the system.

~~Many Worlds~~

~~What if~~