HW 7 P. 2

Wednesday, March 3, 2021 11:04 AM

i) $$\vec{E} = -\nabla \vec{V} - \frac{\partial \vec{A}}{\partial t} = -\nabla \vec{V} - \frac{v}{c^2}\frac{\partial V}{\partial t}$$

$$t' = t - \underline{|\vec{r} - \vec{r}_0(t')|}$$

$$t - t') = |\vec{r} - \vec{r}_0(t')|$$

$r_0(t') \to r_0$ (simplicity)

$$(t - t')^2 = |\vec{r} - \vec{r}_0|^2 = r^2 + v^2 t'^2 - 2\vec{r}\cdot\vec{v}\,t'$$

$$\Rightarrow t' = \frac{(t - \vec{r}\cdot\vec{v}) \pm \sqrt{(1 - -\vec{r}\cdot\vec{v})^2 - (t - r^2)(1 - v^2)}}{1 - v^2}$$

$$V(\vec{r}, t) = \frac{e}{4\pi|\vec{r} - \vec{r}_0|(1 - \vec{n}\cdot\vec{v})} \Rightarrow$$

$$\hat{n} = \overrightarrow{r - r_0}$$

$$\boxed{|\vec{r}-\vec{r}_0|}$$

$$\Rightarrow \frac{e}{4\pi\sqrt{(\vec{r}-\vec{v}t)^2+(\vec{r}\cdot\vec{v})^2-r^2v^2}}$$

$$\nabla V = e\,\frac{\left[\vec{r}-\vec{v}t+\vec{v}(\vec{r}\cdot\vec{v})-v^2\vec{r}\right]}{4\pi\left[(\vec{r}-\vec{v}t)^2+(\vec{r}\cdot\vec{v})^2-r^2v^2\right]^{3/2}}$$

$$\frac{\partial V}{\partial t} = -\frac{\left[(\vec{r}-\vec{v}t)\cdot\vec{v}\right]e}{4\pi\left[(\vec{r}-\vec{v}t)^2+(\vec{r}\cdot\vec{v})^2-r^2v^2\right]^{3/2}}$$

$$\vec{E}(\vec{r},t) = \frac{(\vec{r}-\vec{v}t)\left(1-\frac{v^2}{c^2}\right)e}{4\pi\left[(\vec{r}-\vec{v}t)^2+(\vec{r}\cdot\vec{v})^2-r^2v^2\right]^{3/2}}$$

$$= \frac{e}{4\pi\gamma^2(1-\beta^2\sin^2\theta)^{3/2}}\,\frac{\vec{x}}{x^3} \qquad \boxed{\vec{x}\rightarrow\vec{r}}$$

ii) $\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \left(\frac{\vec{v}}{c^2} V \right) = \vec{\nabla} V \times \frac{\vec{v}}{c^2}$

$$= -\left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) \times \frac{\vec{v}}{c^2} = -\vec{E} \times \frac{\vec{v}}{c^2} - \cancelto{0}{\frac{\partial}{\partial t}\left(\frac{\vec{v}}{c^2} V \right)} \times \frac{\vec{v}}{c^2}$$

$$= +\frac{\vec{v}}{c^2} \times \vec{E} = \hat{n} \times \vec{E}$$

iii) $\vec{S} = \vec{E} \times \vec{B} = \vec{E} \times (\hat{n} \times \vec{E})$

$$= -(\vec{E} \cdot \hat{n})\vec{E} + (\vec{E} \cdot \vec{E})\hat{n}$$

$$\oint_A S \, dA = 0$$