Physics 414-2 Problem Set 6

May 6, 2022

Due: Friday, May 13 at 4 pm

1. Line charge in a dielectric medium. An infinite line charge with charge density λ is oriented parallel to the x-axis, at z coordinate $z = d$. The line charge is embedded in a dielectric medium characterized by dielectric constant ϵ_1, a distance d from the boundary with a dielectric medium characterized by ϵ_2. The boundary surface is the entire x-y plane, so that the ϵ_2 dielectric fills the region $z < 0$, and the ϵ_1 dielectric fills the region $z > 0$.

Find the force per unit length acting on the line charge. Under what conditions is this force repulsive versus attractive?

2. Line charge near a dielectric cylinder. An infinite line charge with charge density λ is embedded in a dielectric medium characterized by dielectric constant ϵ_1, next to an infinite cylinder made of a dielectric medium characterized by ϵ_2. The cylinder is aligned along the z direction and is centered at $x = 0$, $y = 0$ as shown in Fig. 1.

(a) Calculate the potential at all points in space. (Hint: see if you can make educated guesses as to where the image charges should be. If you have trouble with this, feel free to ask me.)

(b) Find the force per unit length acting on the line charge. Under what conditions is this force repulsive versus attractive?

3. Conducting spheres with dielectric filling. Two concentric conducting spheres of inner and outer radii a and b, respectively, carry respective charges $+Q$ and $-Q$ as shown in Fig. 2. The empty space between the spheres is half filled by a hemispherical shell of dielectric, with dielectric constant ϵ (illustrated by the green region in Fig. 2). The other half is vacuum.

(a) Find the electric field everywhere between the spheres.

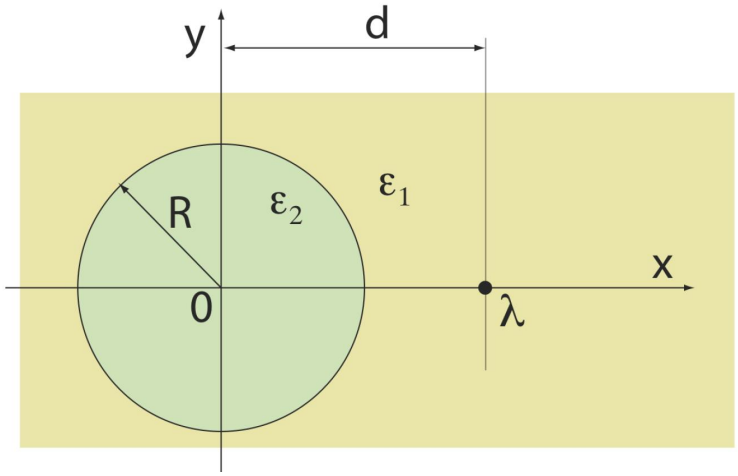

Figure 1: Illustration of the setup in problem 2.

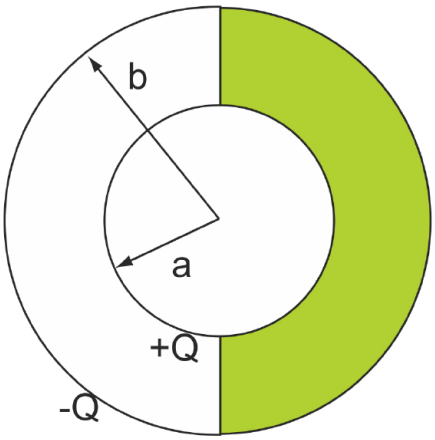

Figure 2: Illustration of the setup in problem 3.

(b) What is the surface charge density on the inner conducting sphere (not including the porlarization surface charge density on the inner surface of the dielectric)?

(c) As we saw in lecture, on the surface of a dielectric medium, there will be a polarization surface charge density σ_P. On the inner surface of the dielectric at $r = a$, what is this polarization surface charge density σ_P? Note that this

polarization surface charge density is distinct from the charge on the surface of the inner conducting sphere that you found in part (b).

4. Magnetic shielding.

First, review the section 6 problems and solutions on magnetic materials. These problems show that there are many similarities between solving electrostatics problems involving dielectrics and solving magnetostatics problems involving magnetic materials. It will also be useful to review the section 5 notes, problems, and solutions on solving boundary value problems by expressing the solution for the potential as a linear combination of spherical harmonics.

Materials with high magnetic permeability can be used to form magnetic shields. In this problem, we will consider a spherical shell of magnetic material in an external magnetic field. We will see that in certain situations, the shell magnetically shields the region inside it. A spherical shell of inner radius r_1 and outer radius r_2 is placed in a uniform magnetic field B_0 that points in the z direction. The spherical shell has magnetic permeability μ, defined so the $\mathbf{B} = \mu\mathbf{H}$. What is the magnetic field inside the shell? How does it depend on μ and on the thickness $r_2 - r_1$ of the shell? (Hint: solve for the magnetic scalar potential ϕ_M by expressing your solution as a linear combination of spherical harmonics).