5. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (4 pts)

The Maxwell *stress tensor* is given by

$$T_{ij} = E_i E_j + B_i B_j - \delta_{ij} \frac{1}{2} \left(\vec{E}^2 + \vec{B}^2 \right)$$

With this show that the Lorentz force density satisfies

$$\vec{f} = \nabla \cdot T - \frac{1}{c^2} \frac{\partial \vec{S}}{\partial t}$$

where $\vec{S} = c\vec{E} \times \vec{B}$ is the Poynting vector.

Exercise 2 (3 pts)

Use $F'^{\mu\nu} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta F^{\alpha\beta}$ to derive the transformation of $\vec{E}$ and $\vec{B}$ fields under a Lorentz boost in the x-direction.

Exercise 3 (4 pts)

A relativistic particle with mass m and charge e moves in a uniform, static, electric field $\vec{E}_0$ (pointing in the x-direction).

i) Find the velocity and position of the particle as explicit functions of time, assuming that the initial velocity $\vec{v}_0$ is perpendicular to the field.

ii) Eliminate the time to obtain the particle trajectory in space. Discuss the shape of the path for short and long times (define "short" and "long" times).

Exercise 4 (2 pts)

i) Show that the following relativistic equation incorporates the Lorentz-force, by calculating the temporal and spatial components for a charged particle:

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \frac{1}{c} F^{\mu\nu} J_\nu$$

ii) What is the interpretation of the temporal component? Is there any difference in the spatial component compared to what we had classically?