→ mirror inversion & parity inversion are not the same! parity is defined by the inversion of all spatial coordinates,

$$x \rightarrow -x \qquad y \rightarrow -y \qquad z \rightarrow -z$$

whereas, for mirror inversion, only direction normal to the plane of the mirror is inverted (so "mirror symmetry" depends on your choice of mirror; the idea of parity is more intrinsic)

→ the action of an operator, like parity Π, on any other operator Θ, is given by:

$$\Pi^{+}\Theta\,\Pi \qquad \text{parity-inverted } \Theta$$

<u>not</u> by $\Pi\Theta$. so we may write for the position operator:

$$\Pi^{+}\vec{r}\,\Pi = -\vec{r}$$

$$\Pi^{+}\vec{p}\,\Pi = -\vec{p}$$

→ vectors go to minus themselves under parity. angular momentum is a pseudovector, defined as an object that behaves like a vector under rotation, but is not affected by a parity transformation. can see this:

$$\vec{L} = \vec{r}\times\vec{p}$$

$$\begin{aligned}
\Pi^{+}\vec{L}\,\Pi &= \Pi^{+}(\vec{r}\times\vec{p})\Pi \qquad \text{use } \Pi\Pi^{+} = I \\
&= (\Pi^{+}\vec{r}\,\Pi)\times(\Pi^{+}\vec{p}\,\Pi) \\
&= (-\vec{r})\times(-\vec{p}) \\
&= +\vec{L}
\end{aligned}$$

true of any type of angular momentum, including spin:

$$\Pi^{+}\vec{S}\,\Pi = +\vec{S}$$

in this problem, an experiment (A) is performed and so is a parity-inverted experiment (B). this means that the initial states (set-up) are related:

$$|\psi_B(0)\rangle = \Pi|\psi_A(0)\rangle$$

both experiments proceed; the relevant physics of the reaction encoded in the system's Hamiltonian and thus the effects in the time-evolution operator $U(t)$. the helicity h of the final state is then measured, $\langle h\rangle$. helicity has the following property under parity:

$$h \equiv \frac{\vec{p}\cdot\vec{S}}{|\vec{p}||\vec{S}|}$$

$$\Pi^+ h\Pi = \Pi^+\left(\frac{\vec{p}}{|\vec{p}|}\cdot\frac{\vec{S}}{|\vec{S}|}\right)\Pi = \underbrace{\left(\Pi^+\frac{\vec{p}}{|\vec{p}|}\Pi\right)}_{-\frac{\vec{p}}{|\vec{p}|}}\cdot\underbrace{\left(\Pi^+\frac{\vec{S}}{|\vec{S}|}\Pi\right)}_{+\frac{\vec{S}}{|\vec{S}|}}$$

$$\Pi^+ h\Pi = -\frac{\vec{p}\cdot\vec{S}}{|\vec{p}||\vec{S}|} \quad \text{so} \quad \boxed{\Pi^+ h\Pi = -h}$$

this is <u>always true for the helicity operator</u>, whether parity is a symmetry of the system or not.

to determine whether or not parity is violated, let's assume that it isn't (so assume it is a symmetry $[H,\Pi]=0$) and use that to make a prediction about $\langle h\rangle_A$ & $\langle h\rangle_B$; we can then compare this prediction to observation.

$$\langle h\rangle_A = \langle\psi_A(t_f)|h|\psi_A(t_f)\rangle = \langle\psi_A(0)|U^+(t_f)hU(t_f)|\psi_A(0)\rangle$$

$$\langle h\rangle_B = \langle\psi_B(t_f)|h|\psi_B(t_f)\rangle$$

$$= \langle\psi_B(0)|U^+(t_f)hU(t_f)|\psi_B(0)\rangle \qquad |\psi_B(0)\rangle = \Pi|\psi_A(0)\rangle$$

$$= \langle\psi_A(0)|\Pi^+U^+(t_f)hU(t_f)\Pi|\psi_A(0)\rangle$$

if Π is a symmetry, $[\Pi,H]=0$ & thus $[\Pi,U]=0$

$$\langle h\rangle_B = \langle \psi_A(0)|\, U^\dagger(t_f)\, \underbrace{\Pi^\dagger h \Pi}_{-h}\, U(t_f)\, |\psi_A(0)\rangle$$

$$\langle h\rangle_B = -\langle \psi_A(0)|\, U^\dagger(t_f)\, h\, U(t_f) |\psi_A(0)\rangle$$

$$\langle h\rangle_B = -\langle \psi_A(t_f)| h |\psi_A(t_f)\rangle$$

$$\boxed{\langle h\rangle_B = -\langle h\rangle_A \quad \text{if } [H,\Pi] = 0}$$

so, if parity is a symmetry of this system, if experiment A obsreves $\vec{p} \parallel \vec{S}$, so $\langle h\rangle_A = +1$, B should obseve $\langle h\rangle_B = -1$. however, observation tells us that $\langle h\rangle_A = \langle h\rangle_B = +1$, contradicting this result, so we must conclude that parity is violated/ parity is not a symmetry of this system, $[H,\Pi] \neq 0$. (process involves the weak interaction).