

Hw 4

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$$1.) \quad ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$\text{Show } ds^2 = ds'^2 \quad y = y' \quad z = z'$$

$$x = \gamma(x' - vt') \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$ds^2 = c^2 \gamma^2 \left(dt' + \frac{v dx'}{c^2}\right)^2 - \gamma^2 (dx' + v dt')^2 - dy'^2 - dz'^2$$

$$= c^2 \gamma^2 \left(dt'^2 + \frac{v^2 dx'^2}{c^4} + \frac{2v dx' dt'}{c^2}\right)$$

$$- \gamma^2 (dx'^2 + v^2 dt'^2 + 2v dx' dt') - dy'^2 - dz'^2$$

$$= \gamma^2 \left(c^2 dt'^2 + \frac{v^2 dx'^2}{c^2} + \cancel{2v dx' dt'} \right)$$

$$- dx'^2 - v^2 dt'^2 - \cancel{2v dx' dt'} - dy'^2 - dz'^2$$

$$= \gamma^2 \left[dt'^2 \left(c^2 - v^2 \right) - \frac{v^2 dx'^2}{c^2} \right] - dy'^2 - dz'^2$$

$$= 0 \quad \left[\frac{1}{\gamma^2} \left(1 - \frac{v^2}{c^2} \right) \right] - dy'^2 - dz'^2$$

$$= \gamma^2 \left[\underbrace{c^2 dt'^2}_{1/\gamma^2} \left(1 - \frac{v^2}{c^2} \right) - \underbrace{dx'^2}_{1/\gamma^2} \right] - dy'^2 - dz'^2$$

$$= c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2$$

$$2.) \quad x = \gamma(x' - vt') \quad t = \gamma\left(t' - \frac{vx'}{c^2}\right)$$

$$ct(\sigma) = \lambda \sinh(\sigma) \quad x(\sigma) = \lambda \cosh(\sigma)$$

$$i.) \quad ds^2 = c^2 dt^2 - dx^2$$

$$dct(\sigma) = \lambda \cosh(\sigma) d\sigma$$

$$dx(\sigma) = \lambda \sinh(\sigma) d\sigma$$

$$\int ds = \int dct - \int dx$$

$$= \int_0^{\sigma} \lambda \sinh(\sigma') d\sigma' - \int_0^{\sigma} \lambda \cosh(\sigma') d\sigma'$$

$$= \frac{\lambda}{\sigma} \cosh(\sigma) - \frac{\lambda}{\sigma} \sinh(\sigma)$$

$$\text{ii.) } \frac{dx}{d\sigma} = \lambda \sinh(\sigma)$$

$$3.) \vec{a} = \vec{g}$$

Rindler Coordinates:

$$t' = x \sinh(at)$$

$$x' = x \cosh(at)$$

$$x = \sqrt{x'^2 - t'^2}$$

$$t = \frac{1}{a} \tanh^{-1}\left(\frac{t'}{x'}\right)$$

$$dx = \gamma(dx' - v dt')$$

$$dt = \gamma(dt' - \frac{v}{c^2} dx')$$

$$u = \frac{u' - v}{1 - \frac{u'v}{c^2}}$$

$u' =$ velocity of rest frame

$$1 - \frac{u'^2}{c^2}$$

$u = \text{vel. of moving ref. frame}$

$$du = \frac{du' \left(1 - \frac{u'v}{c^2} \right) + (u' - v) \frac{v du'}{c^2}}{1 - \frac{u'v}{c^2}}$$

$$= \frac{du'}{\gamma^2 \left(1 - \frac{u'v}{c^2} \right)^2}$$

$$\frac{du}{dt} = a = \frac{\frac{du'}{dt}}{\gamma^3 \left(1 - \frac{u'v}{c^2} \right)^3} = \frac{a'}{\gamma^3 \left(1 - \frac{u'v}{c^2} \right)^3}$$

in 1-D motion: $u' = v$

$$a = \gamma^3 a' = \frac{d(\gamma v)}{dt'} = v \frac{d\gamma}{dt'} + \gamma \frac{dv}{dt'}$$

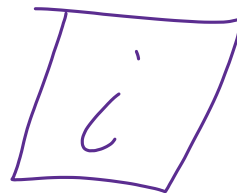
$$= \gamma a' \left(1 + \frac{\gamma^2 v^2}{c^2} \right)$$

$$\int c d(\gamma v) = \int a dt'$$

$$\boxed{\gamma v = at'}$$

$$\gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = \sqrt{1 + \frac{a^2 t'^2}{c^2}}$$

$$t' = \frac{\gamma v}{a} = \frac{v}{g} \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}}$$



ii $t = \gamma \left(t' - \frac{xv}{c^2} \right) \quad ?$

iii $t' = \frac{c}{4g} \sqrt{\frac{1}{1 - \frac{c^2/4}{c^2}}} = \frac{c}{4g} \sqrt{\frac{1}{3/4}}$

$$= \frac{c}{4g} \sqrt{\frac{4}{3}}$$