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Stars as resonant absorbers of gravitational waves

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ABSTRACT

Quadrupole oscillation modes in stars can resonate with incident gravitational waves (GWs), and grow non-linear at the expense of GW energy. Stars near massive black hole binaries (MBHBs) can act as GW-charged batteries, discharging radiatively. Mass-loss from these stars can prompt MBHB accretion at near-Eddington rates. GW opacity is independent of amplitude, so distant resonating stars can eclipse GW sources. Absorption by the Sun of GWs from Galactic white dwarf binaries may be detectable with second-generation space-based GW detectors as a shadow within a complex diffraction pattern.

Key words: gravitational waves – opacity – stars: interiors – stars: oscillations – galaxies: active.

1 INTRODUCTION

Supermassive black holes (SMBH) with masses in the range $\sim 10^6$–$10^9\,\mathrm{M}_\odot$ are present in the nuclei of most, perhaps all, nearby galaxies (see e.g. the recent review by Kormendy & Ho 2013). Mergers between galaxies should result in SMBH binaries; indeed active SMBH binaries have been directly resolved at 0.1–1 kpc separations in X-rays (Komossa et al. 2003; Fabbiano et al. 2011), from sub-kpc to a few kpc separations in the optical band (Comerford et al. 2013; Comerford & Greene 2014; Woo et al. 2014), and at $\sim$10 pc separation in the radio (Rodriguez et al. 2006). Most of the binding energy of a merging massive binary is radiated as gravitational waves (Thorne & Braginsky 1976). As the binary approaches merger, the gravitational wave (GW) frequency ($\nu_{\rm GW}$) increases in a chirp, passing through quadrupolar ($\ell = 2$) oscillation frequencies (ν_*) of stars and stellar remnants, resonating whenever $\nu_{\rm GW} \sim \nu_*$. The interaction of GWs with matter has been considered in various contexts (e.g. Hawking 1966; Kocsis & Loeb 2008; Li, Kocsis & Loeb 2012), the latter suggesting that viscous heating of Sun-like stars by GW from a nearby merging massive black hole binary (MBHB) can reach $\sim \mathrm{L}_\odot$. However, resonant interactions of GWs with normal, as opposed to compact stars (similar to a bar detector), have not received very much attention (Misner, Thorne & Wheeler 1973; Chandrasekhar & Ferrari 1991, 1992; Siegel & Roth 2010, 2011). It has been shown that GWs can do work on stellar oscillations leading to potential observable effects on the oscillations (e.g. Fabian & Gough 1984; Kojima & Tanimoto 2005). After this manuscript was submitted, a pre-print appeared on arXiv.org by Lopes & Silk (2014), considering the resonant interaction of GWs with stars, as well as assessing the feasibility of detecting the induced stellar oscillations through astroseismological measurements. In this Letter, we discuss the possibility of GW absorption lines at resonant frequencies in stars, eclipses of GW sources by foreground stars (including the Sun) and the possible use of stars in galactic nuclei as electromagnetic detectors of resonating GW from nearby MBHBs. In the latter case, we show that *resonant* heating of a single mode in a Sun-like star can be up to $\sim$11 orders of magnitude larger than the viscous heating in Li et al. (2012).

2 GWS FROM A BINARY RESONATING WITH STELLAR OSCILLATIONS

A circularized binary with individual BH masses M_1 and M_2 and physical separation $a_{\rm bin}$ emits GWs at frequency

$$\nu_{\rm GW} = \frac{2}{t_{\rm orb}} = \frac{G^{1/2} M_{\rm bin}^{1/2}}{\pi a_{\rm bin}^{3/2}} = 2\, M_6^{-1} a_1^{-3/2}\, 1\,\mathrm{mHz}, \qquad (1)$$

for a characteristic duration

$$t_{\rm GW} = \frac{a_{\rm bin}}{|\dot{a}_{\rm bin}|} = 0.8\, \eta_{-3}^{-1} M_6^{-5/3} \nu_{\rm GW,1}^{-8/3}\ \mathrm{yr}, \qquad (2)$$

where $t_{\rm orb}$ is the orbital period, $M_{\rm bin} = M_1 + M_2$ is total binary mass, $r_{\rm g} = GM_{\rm bin}/c^2$ is the gravitational radius of the binary, $a_1 \equiv a_{\rm bin}/(10 r_{\rm g})$, $M_6 \equiv M_{\rm bin}/10^6\,\mathrm{M}_\odot$, $\eta \equiv M_1 M_2/(M_1 + M_2)^2$ is the symmetric mass ratio, $\eta_{-3} \equiv \eta/10^{-3}$, $\nu_{\rm GW,1} \equiv \nu_{\rm GW}/1\,\mathrm{mHz}$ and

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where the orbital decay $\dot{a}_{\rm bin}$ is driven by the (quadrupolar) GW emission (Peters & Mathews 1963). The resulting GW strain amplitude averaged over directions is given by

$$h = \sqrt{\frac{32}{5}}\frac{G^2}{c^4}\frac{M_{\rm bin}\mu_{\rm bin}}{D_* a_{\rm bin}} = 1.6\times10^{-7}\nu_{\rm GW,1}^{2/3}M_6^{-1/3}\mu_3 D_{*,3}^{-1}, \quad (3)$$

where $\mu_{\rm bin}$ is the reduced mass, D_* is the resonating star's distance from the binary, $\mu_3 \equiv \mu_{\rm bin}/10^3\,{\rm M}_\odot$ and $D_{*,3} \equiv D_*/10^3 r_{\rm g}$. Sun-like stars have $\ell = 2$ oscillation modes with frequencies $\omega_* = 2\pi\nu_*$ spanning 10 μHz−0.1 Hz (Aerts, Christensen-Dalsgaard & Kurtz 2010),[1] which can match the frequency of GWs from a binary source. Tens of low-radial-order f, g, and p modes with overlap integrals $\gtrsim 10^{-3}M_*$ span ∼0.1−1 mHz in solar models (Aerts et al. 2010).

We follow the approach and definitions of Rathore, Blandford & Broderick (2005) in representing GW-driven oscillations of a stellar mode by a driven damped harmonic oscillator, whose displacement $x(t)$ is the solution to

$$\ddot{x} + \frac{\dot{x}}{\tau_{\rm d}} + \omega_*^2 x = F(t), \quad (4)$$

where $\tau_{\rm d}$ is the damping time of the stellar mode and $F(t) = F_{\rm GW}(t)$ is the driving force. Low-order g modes in the linear regime damp radiatively on the time-scale, $\tau_{\rm d} \sim 10^6$ yr (Kumar & Goodman 1996). However, in the non-linear regime (with energy in the mode $E_{\rm m} \gtrsim 10^{37}$ erg), coupling to high-degree g modes reduces this time-scale to $\tau_{\rm d} \approx 50E_{42}^{-1/2}$ d (here $E_{42} = E_{\rm m}/10^{42}$ erg). For f modes in convective stars, linear dissipation through turbulent viscosity takes $\sim 10^4$ yr (Ray, Kembhavi & Antia 1987), but in the non-linear regime, dissipation via high-order p modes (of degree $\ell = 0$, 2, 4) occurs on time-scale $\tau_{\rm d} = 3\times10^4 E_{42}^{-1}$ d (Kumar & Goodman 1996). Multiplying equation (4) by $\dot{x}$ and integrating over time yields the familiar form, expressing conservation of energy

$$\dot{E}_{\rm m} + \dot{Q} = \dot{W}, \quad (5)$$

where $E_{\rm m}$ is the mechanical energy, Q is the energy lost via dissipation, and W is the work done by the driving force,

$$E_{\rm m} = \frac{\dot{x}^2}{2} + \frac{\omega_*^2 x^2}{2}, \quad Q = \int_{t0}^{t}\frac{\dot{x}^2}{\tau_{\rm d}}\,{\rm d}t, \quad W = \int_{t0}^{t}F(t)\dot{x}\,{\rm d}t, \quad (6)$$

respectively. All three quantities are per unit mass in a single mode.

Excitation of non-radial oscillations in stars and stellar remnants due to tidal capture is relatively well studied (e.g. Press & Teukolsky 1977; Reisenegger & Goldreich 1994; Rathore et al. 2005), compared to oscillation excitation due to incident GWs. In the latter case, the effective driving force per unit mass due to GWs is nearly sinusoidal, with a characteristic amplitude $|F_{\rm GW}| = \omega_{\rm GW}^2|h|R_*$ (Misner et al. 1973; Khosroshahi & Sobouti 1997; Siegel & Roth 2010, 2011). The frequency of $|F_{\rm GW}|$ during the inspiral phase of a binary evolves slowly (i.e. the number of orbits at $\nu_{\rm GW}$ is $N = \nu_{\rm GW}^2/\dot{\nu}_{\rm GW} \gg 1$). In this limit, $|F_{\rm GW}|$ is a sinusoid of nearly constant amplitude but slowly increasing frequency $\nu_{\rm GW,0} + \dot{\nu}_{\rm GW,0}t$. It can be shown that in the absence of damping ($\tau_{\rm d} \to \infty$), the effective duration of the resonant forcing, while the source drifts across a resonance, is $t_{\rm F} \approx 1/\sqrt{4\dot{\nu}_{\rm GW}}$ (e.g. Rathore et al. 2005), yielding

$$t_{\rm F} = 6.6\left(\frac{M_{\rm ch}}{{\rm M}_\odot}\right)^{-5/6}\left(\frac{\nu_{\rm GW}}{1\,{\rm mHz}}\right)^{-11/6}\ {\rm yr}. \quad (7)$$

[1] For the solar model in fig. 3.20 in Aerts et al. (2010), $\nu_{\rm n} \approx 0.14\,(n+2)$ and $1.5\,(n+3)^{-1}$ mHz approximately for p and g modes, respectively, for $n \le 30$.

Here, $M_{\rm ch} = \eta^{3/5}M_{\rm bin}$ is the chirp mass. Analytic solutions to equation (4) can be found in two limiting cases: the saturated/steady-state case with constant forcing frequency ($t_{\rm F} \gg \tau_{\rm d}$) (Misner et al. 1973) and the undamped case ($t_{\rm F} \ll \tau_{\rm d}$) (Rathore et al. 2005). Expressing the damping time of a given stellar oscillation mode in terms of the 'quality factor' $q_{\rm f} = \omega_*\tau_{\rm d}/\pi$, the saturation condition $t_{\rm F} = \tau_{\rm d}$ implies that steady state is reached approximately for

$$M_{\rm ch} \lesssim 0.35\left(\frac{\nu_{\rm GW}}{1\,{\rm mHz}}\right)^{-1}\left(\frac{q_{\rm f}}{10^6}\right)^{-6/5}\ {\rm M}_\odot. \quad (8)$$

Steady state will not be reached for $q_{\rm f} \gg 10^6$. However, if the star is close to the GW source such that $|F_{\rm GW}|$ is sufficiently large, the oscillations can grow non-linear before reaching the steady-state limit. If so, the mode coupling to higher order modes prohibits further growth, and the effective quality factor is greatly decreased.

3 SATURATED/STEADY-STATE LIMIT ($t_{\rm F} \gg \tau_{\rm d}$)

Assuming stationary GW forcing at a constant frequency ($F = |F|{\rm e}^{{\rm i}\omega_* t}$), the maximum steady-state displacement $x_{\rm max}$ is

$$x_{\rm max} = \frac{|F|}{\sqrt{(\omega_*^2 - \omega_{\rm GW}^2)^2 + (\omega_{\rm GW}/\tau_{\rm d})^2}} \quad (9)$$

or $x_{\rm max} \approx |F|\tau_{\rm d}/\omega_* = \pi R_* q_{\rm f} h$ in the limit $(\omega_*^2 - \omega^2)^2 \ll (\omega/\tau_{\rm d})^2$. In the steady-state solution, $\dot{E}_{\rm m} = 0$ and the cycle-averaged power of the external forcing $\langle\dot{W}\rangle = \langle F\dot{x}\rangle$ equals the rate of heating $\langle\dot{Q}\rangle = \langle\dot{x}^2\rangle/\tau_{\rm d}$. Taking the limit $\omega_{\rm GW} \approx \omega_*$, the rate of work done in the steady-state case is $\dot{W}_{\rm s} \approx \langle F^2\rangle\tau_{\rm d}$ or

$$\dot{W}_{\rm s} = \dot{Q}_{\rm s} \approx \frac{\pi}{2}R_*^2 h^2 q_{\rm f}\omega_*^3. \quad (10)$$

The cross-section for absorbing GWs is given by

$$\sigma_{\rm GW} = \frac{M_{\rm m}\langle\dot{W}_{\rm s}\rangle}{\Phi_{\rm GW}} \approx \frac{8\pi G}{c^3}M_{\rm m}R_*^2\omega_*^2\tau_{\rm d}, \quad (11)$$

where $\Phi_{\rm GW} = (c^3/16\pi G)\dot{h}^2$ is the GW flux incident on the star and $M_{\rm m}$ is the overlap with the normal mode expressed as a measure of the mass involved in the mode such that (Khosroshahi & Sobouti 1997)

$$M_{\rm m}x \equiv \left(\int \xi\rho h\nabla V\,{\rm d}^3x\right)\Big/\int\rho|\xi|^2{\rm d}^3x, \quad (12)$$

where $\xi = \xi_{nlm}(r)$ is the displacement for a normal mode and $V = \frac{1}{2}(x^2 - y^2)$. The fractional energy flux removed from the incident GW corresponds to a resonant 'optical depth' (${\rm e}^{-\tau}$). The 'effective opacity' seen by the GWs in the steady-state limit is $\tau_{\rm eff,s} \equiv \sigma_{\rm GW}/\pi R_*^2$ or

$$\tau_{\rm eff,s} \approx \frac{8\pi G}{c^3}M_{\rm m}q_{\rm f}\omega_* = 0.8\left(\frac{\nu_*}{1\,{\rm mHz}}\right)\left(\frac{q_{\rm f}}{10^6}\right)\left(\frac{M_{\rm m}}{{\rm M}_\odot}\right). \quad (13)$$

In general, computing overlap integrals ($M_{\rm m}$) between stellar modes and the GW forcing for realistic stellar structure models will be difficult, and will also be very sensitive to the details of stellar structure. A full investigation is beyond the scope of this Letter. However, overlap calculations exist for the somewhat similar case of Newtonian tidal forcing by a nearby point source, both for simplified polytropes (e.g. Press & Teukolsky 1977; Reisenegger & Goldreich 1994) and for more realistic stellar models (Aerts et al. 2010). In general, these show that the lowest order modes have large overlap integrals, between O(0.1) and O(1) for polytropes (see e.g. table 1 in Press & Teukolsky 1977), but also that simple polytrope models are insufficient to estimate the excitation of g modes in Sun-like stars (e.g. Weinberg et al. 2012). Khosroshahi & Sobouti (1997) calculate overlap integrals for GW forcing of polytropes, showing that the fundamental mode has an overlap integral between 20 and 40 per cent for simple polytropic fluid models, with polytrope index $1.5 < n < 2.5$ (see their table 1, where the fundamental f mode is labelled as p_1). We find this result intuitively unsurprising, since the angular part of the overall integral, for $\ell = 2$ modes, matches the quadrupolar pattern of the GWs, and the radial integral is over the product of a non-oscillatory eigenmode and a slowly varying GW forcing function. We conclude that the overlap integral for a number of low-radial-order $\ell = 2$ modes is likely to be significant, i.e. close to O(0.1)–O(1) in at least a few cases, depending on the details of stellar structure. More sophisticated stellar modelling is needed to compute the overlap for g modes, and also for non-solar-type giant stars, where GW wavelength is closer to the stellar radius and where higher order modes may have substantially greater overlap integrals. Lopes & Silk (2014) calculate solar models beyond a simple polytrope and find that values of $M_{\rm m}$ ($\Xi_{\rm n}$ from their equation 9) for p modes and f modes are suppressed by one to two orders of magnitude relative to the values in Khosroshahi & Sobouti (1997) because of the rapid change in density profile. Stellar structures closer to simple polytropic models (e.g. red giants or fully convective low-mass stars) may therefore be the most efficient at GW absorption.

In the linear regime, low-order $\ell = 2$ modes with $q_{\rm f} \sim$ few $\times\, 10^7$, $\nu_* \sim$ few $\times$ 0.1 mHz, driven by a long-lived $\sim {\rm M}_{\odot}$ binary, can approach saturation and attenuate GWs significantly. Work done by GWs on resonant stellar oscillation modes can extract a significant fraction of incident GW energy, far exceeding non-resonant viscous GW dissipation (Li et al. 2012). At higher ν (and $q_{\rm f}$), the mode will not saturate (equation 8), whereas for much smaller $q_{\rm f}$ (e.g. for stars near MBHBs, whose mode amplitudes are driven non-linear), the saturated opacity is small.

4 UNDAMPED LIMIT ($t_{\rm F} \ll \tau_{\rm D}$)

Undamped ($\tau_{\rm d} \to \infty$) stellar oscillations driven by a slowly varying frequency ($\dot{\omega} \ll \omega^2$) acquire energy equal to $E_{\rm m} = |F|^2 t_{\rm F}^2/2$ (Rathore et al. 2005). This energy is then dissipated on the longer time-scale $\tau_{\rm d} \gg t_{\rm F}$, so that $Q_{\rm u} = E_{\rm m}$. The average rate of work done during the forcing is $\dot{W}_{\rm u} = W_{\rm u}/t_{\rm F} = E_{\rm m}/t_{\rm F} \approx |F|^2 t_{\rm F}/2$. Thus, $\dot{W}_{\rm u} \approx \dot{W}_{\rm s}(t_{\rm F}/\tau_{\rm d})$, and the average effective opacity to GWs, while the modes are being resonantly driven, is $\tau_{\rm eff,u} = \tau_{\rm eff,s}(t_{\rm F}/\tau_{\rm d})$:

$$\tau_{\rm eff,u} = 0.3 \left(\frac{M_{\rm ch}}{{\rm M}_{\odot}}\right)^{-5/6} \left(\frac{\nu_*}{1\,{\rm mHz}}\right)^{1/6} \left(\frac{M_{\rm m}}{{\rm M}_{\odot}}\right), \tag{14}$$

independent of $q_{\rm f}$. The mean dissipation rate is $\dot{Q}_{\rm u} = \dot{Q}_{\rm s}(t_{\rm F}/\tau_{\rm d})$. For large but finite $\tau_{\rm d}$, both $\tau_{\rm eff,\,u}$ and $\dot{Q}_{\rm u}$ approach the maximal saturated case as $t_{\rm F} \to \tau_{\rm d}$. Equation (14) implies that GWs from stellar-mass binaries can be strongly attenuated by resonant low-order stellar f modes.

5 ECLIPSES OF GW SOURCES BY THE SUN OR STARS IN THE MBHB HOST GALAXY

The opacities $\tau_{\rm eff,u(s)}$ are independent of D_*, and g-, f-, and p-mode frequencies for the Sun are coincidentally in the sensitivity band of the proposed *eLISA* instrument (Cutler 1998). The Sun could therefore annually eclipse *eLISA* GW sources located in the ecliptic plane – in particular, white dwarf binaries (WDBs; Crowder & Cornish 2007). The effect would be a ‘shadow’ within a complex GW diffraction pattern near the resonant frequency (since the GW wavelength exceeds 1 au). Using the Monte Carlo simulations of Timpano, Rubbo & Cornish (2006), we estimate (from their fig. 12) that between 20 (SNR $\geq$ 5) and 5 (SNR $\geq$ 10) individually resolvable WDBs with *LISA* in a 1 yr observation will lie in a 3.3 μHz bin (corresponding to width $d\nu/\nu 1/q_{\rm f} 1/100$) around log(−3.5) Hz, i.e. near prominent solar modes. Given that the plane of the ecliptic is 1/360th of the sky, this gives an ∼2–6 per cent chance that one such WDB will be occulted annually by a low-order p mode of the Sun annually. Using the three largest mass solar p modes (listed in table 1 in Cutler & Lindblom 1996), each with $q_{\rm f} \sim 100-400$, we find odds of ∼1–6 per cent for each of the modes that a WDB could lie in the ecliptic plane at that frequency in a 3.3-μHz-wide bin. The overwhelming majority of WDBs will lie off the ecliptic, but the orbits of future space-based GW detectors may be chosen to allow the most promising eclipses to be observed. We estimate that *eLISA* will have an O(10 per cent) chance of identifying a WDB near a solar resonance with a deep O(0.1)–O(1) transit depth that a future space-based GW detector could observe. The chances increase significantly for higher order modes with much broader resonances (Stix et al. 1993) but owing to their low overlap integrals and/or low $q_{\rm f}$, these modes will likely produce much shallower transits.

GW absorption could also be detected as a result of transits by bloated stars in the ∼35 000 galactic nuclei within 50 Mpc or extreme mass-ratio inspirals (EMRI) around Sgr A* (Amaro-Seoane et al. 2007). Conservatively assuming a 1 per cent AGN rate, ∼350 active galaxies lie within the *LISA* search window for transits. From McKernan et al. (2012, 2014) most of these AGN host binaries consisting of the SMBH and an intermediate mass black hole (IMBH) or stellar mass black hole. For ∼10 such binaries in the *LISA* frequency window (∼0.1 to few mHz), we estimate a probability of 0.01−1 that we would see one transit in a 10 yr mission, assuming 0.01–1 per cent chance of a transit/AGN/yr (Beky & Kocsis 2013). Electromagnetic (EM) study of such transits would be challenging but potentially detectable (e.g. McKernan & Yaqoob 1998; Turner & Miller 2009). However, the resonant driving by these systems lasts for only $t_{\rm F} \lesssim$ d (equation 7) so the chance of GW absorption coinciding with EM transits will be negligible.

6 RESONANT GW HEATING OF STARS

A star orbiting near a merging MBHB (within ∼1 pc) can absorb a significant amount of resonant GW energy. The average undamped heating rate of a single mode during the passage through resonance is $M_{\rm m}\dot{Q}_{\rm u} = (1/2)M_{\rm m}|F_{\rm GW}|^2 t_{\rm F}$. As an illustrative example, we consider an $M_2 = 10^4\,{\rm M}_{\odot}$ IMBH separated by $\approx 15 r_{\rm g}$ from Sgr A* ($M_1 = 4 \times 10^6\,{\rm M}_{\odot}$) and a Sun-like star $10^3 r_{\rm g}$ away, resonating with the GWs (at $\nu \approx 0.3$ mHz):

$$M_{\rm m}\dot{Q}_{\rm u} = 400\,{\rm L}_{\odot} \left(\frac{M_{\rm m}}{{\rm M}_{\odot}}\right) \left(\frac{R_*}{{\rm R}_{\odot}}\right)^2 \left(\frac{D_*}{10^3 r_{\rm g}}\right)^{-2} \times \left(\frac{M_{\rm bin}}{4\times10^6\,{\rm M}_{\odot}}\right)^{-1} \left(\frac{\mu}{10^4\,{\rm M}_{\odot}}\right)^{3/2} \left(\frac{\nu}{0.3\,{\rm mHz}}\right)^{7/2}. \tag{15}$$

The heating rate is high, but lasts only for $t_{\rm F} \sim 1.5$ d for the fiducial parameters for a single mode. The corresponding energy dumped into the mode is

$$E_{\rm m} = M_{\rm m} Q_{\rm u} = 10^{41}\,{\rm erg} \left(\frac{M_{\rm m}}{{\rm M}_{\odot}}\right) \left(\frac{R_*}{{\rm R}_{\odot}}\right)^2 \left(\frac{D_*}{10^3 r_{\rm g}}\right)^{-2} \times \left(\frac{M_{\rm bin}}{4\times10^6\,{\rm M}_{\odot}}\right)^{-4/3} \left(\frac{\mu}{10^4\,{\rm M}_{\odot}}\right) \left(\frac{\nu}{0.3\,{\rm mHz}}\right)^{5/3}. \tag{16}$$

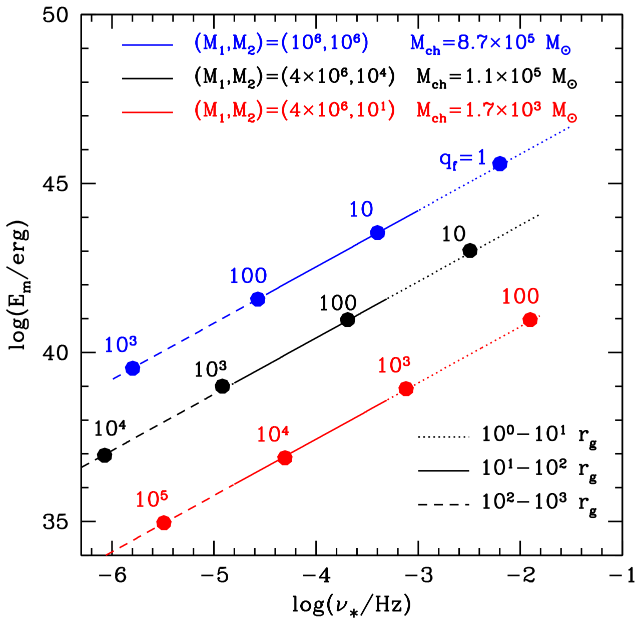

Figure 1. Total energy ($E_{\rm m}$) deposited in a single resonant mode at frequency ν_* of a Sun-like star, when an inspiralling binary GW source sweeps across this frequency. The star is located at $D_* = 10^3 r_{\rm g}$ from a merging MBHB. Three different BH mass combinations are shown, and we assume $M_{\rm m} \sim M_*$. Values of $q_{\rm f} = 2\nu t_{\rm F}$ are indicated for saturation at the corresponding frequency.

Fig. 1 shows the total energy $E_{\rm m}$, deposited in a single resonant mode of a star for the fiducial values of $M_{\rm m} = \mathrm{M}_\odot$, $R_* = \mathrm{R}_\odot$, $D_* = 10^3 r_{\rm g}$ for three different MBHBs. From Fig. 1, for a large overlap integral, up to 10^{45}erg can be deposited into a single mode of a star near an equal mass ($10^6\,\mathrm{M}_\odot$, $10^6\,\mathrm{M}_\odot$) MBHB. This is ~11 orders of magnitude larger than the expected viscous heating of stars (Li et al. 2012). If this much energy can emerge on short time-scales, the resonating star can act as a prompt electromagnetic signpost of incident GWs.

7 RATE OF DISCHARGE OF GW-CHARGED BATTERIES

Stars can release $E_{\rm m}$ either electromagnetically (EM) or via GW emission (equivalent to elastic scattering of incident GWs). The GW time-scale $\approx 5c^5/(GE_{\rm m}\omega_{\rm GW}^2) \gg$ EM time-scales. Energy thermalized in radiative zones emerges on the thermal time-scale $\tau_{\rm th}$ ($\approx 10^7$ yr for Sun-like stars), implying that massive stars will brighten on this long time-scale. The fractional luminosity increase is limited to $E_{\rm m}/E_* \ll 1$. However, energy deposited in the convection zone emerges on time-scale $t_{\rm conv} \sim 10^6$ s, which may cause significant brightening (see below).

7.1 Resonant destruction of stars by GWs

A star is completely disrupted when the total energy dumped into the star (equation 16) becomes greater than the binding energy of the star $E_* \sim GM_*^2/R_*$. In the limit $M_{\rm m} \sim M_*$, this happens at a radius $D_{\rm rd} \gg r_{\rm g}$ where

$$\frac{D_{\rm rd}}{D_{\rm td}} = 0.03 \left(\frac{M_*}{\mathrm{M}_\odot}\right)^{-1/6} \left(\frac{R_*}{\mathrm{R}_\odot}\right)^{1/2} \left(\frac{\nu_*}{1\,\mathrm{mHz}}\right)^{5/6} \left(\frac{\mu}{10^4\,\mathrm{M}_\odot}\right)^{1/2} \quad (17)$$

with $D_{\rm td} = R_*(M/M_*)^{1/3}$ being the tidal disruption radius. Solar-type stars near MBHBs are thus disrupted by Newtonian tides well before destruction due to resonant GW absorption.

7.2 Near-field destructive effects

Far from MBHBs ($r \gg a_{\rm bin}$), $F(t)$ in equation (4) is dominated by GWs. Close to the MBHB, tidal forcing at frequency $\nu_{\rm GW} = 2\nu_{\rm bin}$ is added to $F(t)$ via the Newtonian quadrupole potential ($F_{\rm NQ} \sim G\mu_{\rm bin}a_{\rm bin}^2 R_* D_*^{-5}$) and relativistic current dipole force [$F_{\rm CD} \sim (G/c)\mu_{\rm bin}\nu_{\rm bin}a_{\rm bin}R_*D_*^{-4}$; Misner et al. 1973 – see equation 2.15 in Alvi (2000) and equation 6.4 in Johnson-McDaniel et al. (2009)]. Compared to GW forcing,

$$\frac{F_{\rm NQ}}{F_{\rm GW}} \approx \left(\frac{\nu_{\rm GW}}{0.02\,\mathrm{mHz}}\right)^{-4} \left(\frac{D_*}{10^3 r_{\rm g}}\right)^{-4} \quad (18)$$

and

$$\frac{F_{\rm CD}}{F_{\rm GW}} \approx \left(\frac{\nu_{\rm GW}}{0.02\,\mathrm{mHz}}\right)^{-3} \left(\frac{D_*}{10^3 r_{\rm g}}\right)^{-3}. \quad (19)$$

Tidal forcing on a star $10^3 D_{*,3} r_{\rm g}$ from an MBHB is dominated by near-field effects at $\lesssim 0.02 D_{*,3}^{-1}$ mHz. The heating from equation (15) scales in the near field as $|F_{\rm NQ}|^2/|F_{\rm GW}|^2$ and $|F_{\rm CD}|^2/|F_{\rm GW}|^2$.

8 ELECTROMAGNETIC OBSERVABLES

GW heating of stars with a large radiative core causes modest structural changes and increase in luminosity, since $E_{\rm m} \ll E_*$. For fully (or mostly) convective stars (e.g. M stars), $E_{\rm m}$ is transferred to high-degree modes concentrated in the outer convective skin, with small mass $M_{\rm out} \ll M_*$ (Kumar & Goodman 1996). If $E_{\rm m} > (M_{\rm out}/M_*)E_*$, the binding energy of the surface skin, the skin can expand (Podsiadlowski 1996), provided $E_{\rm m}$ is thermalized faster than $t_{\rm conv}$, i.e. for $E_{\rm m} \gtrsim 10^{45}$erg. If $10^{-3}\,\mathrm{M}_\odot$ is shed from a Sun-like star and subsequently accreted on to a $10^6\mathrm{M}_\odot$ MBHB over an ~year (or ≈10 stellar orbits at $10^3 r_{\rm g}$), the MBHB is fuelled at 0.01–0.1 its Eddington rate (for 10 per cent radiative efficiency; Dai, Escala & Coppi 2013; Hayasaki, Stone & Loeb 2013). During accretion, the MBHB period appears as see-saw variability in the wings of broad emission lines (McKernan et al. 2013), possibly preceding tidal disruption events by MBHBs.

If $E_{\rm m} \lesssim 10^{45}$ erg is thermalized, the star may not bloat but the luminosity $L'_* = E_{\rm m}/\tau_{\rm d}$ can be large. For the $M_1 = M_2 = 10^6\mathrm{M}_\odot$ MBHB in Fig. 1, at $\nu \approx 0.3$ mHz, a star at $D_* = 10^3\, r_{\rm g}$ is heated for $t_{\rm F} \approx 6$ h, and $E_{\rm m} = 2 \times 10^{43}$ erg emerges over the non-linear dissipation time-scale $\tau_{\rm d} \approx 4$ yr. During this period $L'_* \approx 45(M_{\rm m}/M_*)^2\,\mathrm{L}_\odot$. Moreover, $\dot{\omega}_{\rm GW}$ sweeps through a large number of resonant modes $N_{\rm m}$ between 40 μHz and 12 mHz in the final 4 yr before merger (Aerts et al. 2010). If $N_{\rm m} \sim 10$ modes can be driven resonantly within a dissipation time-scale, then $L'_* \sim$ few $\times\, 10^{2-3}(N_{\rm m}/10)(M_{\rm m}/M_*)^2\,\mathrm{L}_\odot$.

9 CONCLUSIONS

Quadrupolar oscillation modes in stars can resonate with incident GWs, reaching non-linear amplitudes at the expense of GW energy. The opacity to GWs is distance independent, so the Sun can eclipse GW sources (e.g. WDBs) in the ecliptic plane, imprinting absorption lines in GW spectra. Stars near MBHBs act as GW-charged batteries, discharging via a brief, significant luminosity increase in convective stars. Mass-loss from the outer skin of stars yields bursts of near-Eddington accretion on to a nearby MBHB. Detailed numerical studies (including models of stellar structure, effects of rotation) are needed for more quantitative predictions.

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