the CNO cycles wins out and the pp-chains lose their dominance. Note that the temperature exponent for the CNO cycles is

$$\nu(\mathrm{CNO}) = \frac{50.8}{T_6^{1/3}} - \tfrac{2}{3} \tag{6.78}$$

which is about 18 for $T_6 = 20$.

6.5 Helium-Burning Reactions

This section will deal with the triple-α and subsequent reactions in helium-burning. For the most part, the stellar environment is assumed to correspond to that of the cores of normal post-main sequence stars where temperatures do not greatly exceed 10^8 K. The primary reaction sequence considered here is

$$\alpha + \alpha \longrightarrow {}^8\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}\,.$$

For an excellent historical review of the subject see Fowler (1986).

Helium-burning begins with the inverse of the ${}^8\mathrm{Be} \rightarrow 2\,{}^4\mathrm{He}$ decay that terminates the PP–III chain; that is, the first reaction is ${}^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^8\mathrm{Be}$, which is endothermic (energy absorbing) by 91.78 keV. We remarked earlier that ${}^8\mathrm{Be}$ has a lifetime of only 10^{-16} s. Thus to produce ${}^8\mathrm{Be}$ in any quantity whatsoever, the α-particles must have sufficient energy to gain access to the ground state of ${}^8\mathrm{Be}$ and the formation rate of ${}^8\mathrm{Be}$ must be sufficiently rapid to make up for its short lifetime. Since the ground state of ${}^8\mathrm{Be}$ has a finite width ($\Gamma_{\alpha\alpha} \sim 7$ eV), we may ask at what temperature the Gamow peak begins to encroach upon that resonance. In other words, if the reaction does not begin to look resonant, then the reaction rate for production may not catch up with the inverse decay. From (6.40), the location of the peak is at $\mathcal{E}_0 = 3.9T_6^{2/3}$ keV ($Z_\alpha = 2$, $\mu = 2$) and it equals 92 keV when $T = 1.2 \times 10^8$ K. If the effects of electron screening in high-density situations are ignored (for the present) then this roughly sets the minimum temperature for helium-burning.

Assume then that temperatures exceed 10^8 K and the ${}^8\mathrm{Be}$ producing reaction proceeds rapidly. If rapid enough, the formation rate of ${}^8\mathrm{Be}$ should begin to match the rate at which it is destroyed by decays; that is, the concentration of ${}^8\mathrm{Be}$ should approach equilibrium. (This should, and can, be justified—as it is in the references.) One way to find the equilibrium concentration is to compute the rate of production by equating $n_\alpha^2\langle\sigma v\rangle_{\alpha\alpha}/2$ (remember the factor of two for like particles) to $\lambda n({}^8\mathrm{Be})$, where λ is the decay constant for ${}^8\mathrm{Be}$. This unfortunately requires knowing $\langle\sigma v\rangle_{\alpha\alpha}$. But there is an easier and more illuminating way to go about it. We may assume chemical equilibrium and use the Saha equation (3.35) except that we now have nuclei and not atoms, ions, and electrons as was the case for the hydrogen ionization reaction (3.31).

The equilibrium reaction we are talking about is

$$\alpha + \alpha \Longleftrightarrow {}^{8}\mathrm{Be}$$

and several easy modifications must be made to the Saha equation of 3.5. The first is to replace the number densities by $(n^+, n_e) \rightarrow n_\alpha$ and $n^+ \rightarrow n(^8\mathrm{Be})$. The statistical factors, g, are unity for both ^{4}He and the ground state of ^{8}Be because both have zero spin. Instead of an ionization potential χ_H we now have the Q-value, which is –91.78 keV. Finally, the mass m_e is replaced by $m_\alpha^2/m(^8\mathrm{Be}) \approx m_\alpha/2$ as the reduced mass. The "nuclear" Saha equation is then

$$\frac{n_\alpha^2}{n(^8\mathrm{Be})} = \left(\frac{\pi m_\alpha kT}{h^2}\right)^{3/2} e^{-Q/kT}$$
$$= 1.69 \times 10^{34} T_9^{3/2}\, e^{1.065/T_9}. \tag{6.79}$$

For typical conditions at, say, the start of the helium flash in lower mass stars where $\rho \approx 10^6$ g cm^{-3} ($n_\alpha \approx 1.5 \times 10^{29}$ cm^{-3} if the flash starts with pure helium) and $T_9 \approx 0.1$, find that the equilibrium concentration of ^{8}Be is about 10^{21} cm^{-3} or $n(^8\mathrm{Be})/n_\alpha$ is only 7×10^{-9}.

With a seed of ^{8}Be nuclei now in place, however, the second stage of the triple-α reaction may now continue with the capture reaction $^8\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}$. This is an exothermic resonant reaction, with Q=7.367 MeV, which proceeds through an excited state ^{12}C* with zero spin at 7.654 MeV. The emission of a γ-ray photon by ^{12}C* does not come easily because once the compound excited state is formed it almost always decays right back to ^{8}Be and an α-particle. Yet, as in the first step of the triple-α described above, the forward reaction is sufficiently rapid (assuming a high enough temperature) that a small pool of ^{12}C nuclei in the excited state is built up and, again, the nuclear Saha equation may be used to find the concentration in the pool. It is not difficult to do this and it should be obvious that it finally results in an expression for $n(^{12}\mathrm{C}^*)/n_\alpha^3$ as a function of temperature after (6.79) is applied.

Having found $n(^{12}\mathrm{C}^*)$, we can then determine the net rate of decay of ^{12}C* by γ-ray cascade (or electron–positron pair emission) rather than by an α-particle: $n(^{12}\mathrm{C}^*) \times \Gamma_\mathrm{rad}/\hbar$, where the combination $\Gamma_\mathrm{rad}/\hbar$ is the decay rate, λ_rad, through the uncertainty relation. (The value of Γ_rad is only 3.67 meV.) The overall sequence of the triple-α is illustrated in Fig. 6.11. (The entries to the right of the ^{12}C levels are the spins and parities of the levels.) It should be clear that the overall rate of the triple-α reaction is the same as the formation rate of the ground state of ^{12}C.

The above contains all the elements for computing the energy generation rate of the triple-α sequence. The final result we quote is taken from Harris et al. (1983, in their Table 1) where the quantity $N_\mathrm{A}^2 \langle\alpha\alpha\alpha\rangle$ is to be found. This is multiplied by $\rho^2 Y^3 N_\mathrm{A} Q/6A_\alpha^3$ and Q=7.367–0.0918=7.275 MeV to yield

$$\varepsilon_{\alpha\alpha\alpha} = \varepsilon_{3\alpha} = \frac{5.1 \times 10^8 \rho^2 Y^3}{T_9^3}\, e^{-4.4027/T_9} \quad \mathrm{erg\ g^{-1}\ s^{-1}}. \tag{6.80}$$

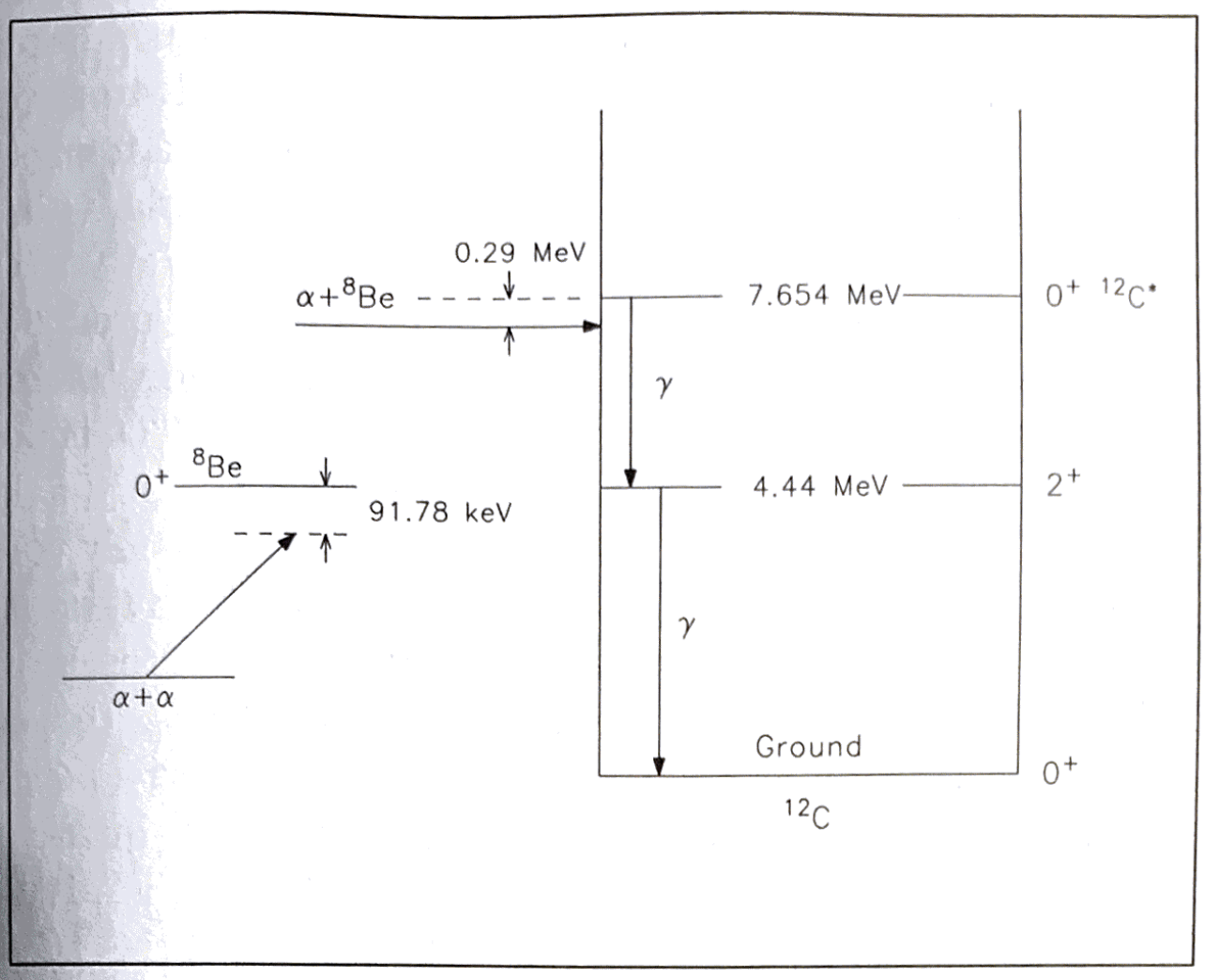

Fig. 6.11. The level diagrams and energetics of the two reactions composing the triple-α reaction (not to scale). The final result is the nucleus ^{12}C.

Here, $A_\alpha = 4$ (as part of n_α), and the division by 6 in the multiplying factor comes about because of triple counting of α-particles (as in dividing by 2 for double counting of protons in the pp chains). To verify (6.80) requires searching through some of the papers already referenced and we suggest you try to reproduce it to gain experience in how to use these references.[8] If intermediate rates are fast enough to satisfy the Saha equation, then the uncertainty in (6.80) is estimated to be only 15% (Fowler, 1986).

It is easy to show from (6.80) that the temperature and density exponents for the triple-α reaction are

$$\lambda_{3\alpha} = 2, \quad \text{and} \quad \nu_{3\alpha} = \frac{4.4}{T_9} - 3\,. \tag{6.81}$$

For $T_8 = 1$, $\nu_{3\alpha} \approx 40$, which is considerably larger than the corresponding exponent for hydrogen-burning. This means that the helium fuel is potentially more explosive than hydrogen—a fact of considerable interest for the helium flash, as discussed in §2.5.

The effects of screening are difficult to assess for a reaction such as the triple-α and we shall not attempt to do so here. In addition, the above analysis is inappropriate for temperatures much below 10^8 K because ^{4}He$(\alpha)^8$Be no longer samples the resonance in ^{8}Be strongly. For an attempt to combine these

[8] In particular, you will need Fowler et al. (1967, 1975) and Harris et al. (1983). High-temperature correction factors and individual rates for the two parts of the triple-α may be found in Caughlan and Fowler (1988).

various elements see Fushiki and Lamb (1987), who give general expressions for the energy generation rate, including effects of weak and strong screening and, in the very-high-density limit, "pycnonuclear" effects (a term coined by Cameron, 1959; see also Ichimaru et al., 1992). These corrections can be very important.

The next step in helium-burning is α capture on ^{12}C to form ^{16}O. In the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction, the $^{12}\mathrm{C}+\alpha$ pair, at zero initial energy, enters ^{16}O at 7.12 MeV (which is the Q-value). The nearest resonance in ^{16}O, however, lies some 45 keV below that energy. Hence the reaction proceeds only in the upper tail of the resonance at temperatures near 10^8 K. Unfortunately the nuclear parameters for this resonance and the detailed behavior of the resonance tail are hard to come by experimentally: a direct measurement of the cross section fails by a few orders of magnitude with present capabilities. To complicate matters, there are two (at least) levels well above the entry point of ^{4}He that can contribute (either constructively or by distructive interference) to the rate. A (hopefully) outdated quote by Fowler (1985) reads: "It pays for that their results in a given study are sensitive to the rate of the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction, then they should repeat their calculations with 0.5 times and 2 times the values recommended here." That is, give a factor of two either way. (And see Imbriani et al., 2001, for an example of this philosophy applied to evolutionary models.)

That situation may have been remedied by Kunz et al. (2002), who have used older experimental data plus new results of their own to calculate (using **R**–matrix theory) a new rate that they claim should be accurate to ±30%. Whether this is an optimistic appraisal or not, their rate is (are you ready?)

$$\begin{aligned} N_{\mathrm{A}}\langle\sigma v\rangle(\alpha,{}^{12}\mathrm{C}) \;=\; & \frac{a_0}{T_9^2\left(1+a_1T_9^{-2/3}\right)^2}\exp\left[-a_2T_9^{-1/3}-(T_9/a_3)^2\right]+ \\ & +\;\frac{a_4}{T_9^2\left(1+a_5T_9^{-2/3}\right)^2}\exp\left[-a_2T_9^{-1/3}\right]+ \\ & +\;\frac{\tilde{a}_9}{T_9^{1/3}}\exp\left[-a_{11}T_9^{-1/3}\right] \end{aligned} \tag{6.82}$$

in the units of cm^3 s^{-1} mole^{-1}. The various constants (in Kunz et al. notation) are $a_0 = 1.21\times10^8$, $a_1 = 6.06\times10^{-2}$, $a_2 = 32.12$, $a_3 = 1.7$, $a_4 = 7.4\times10^8$, $a_5 = 0.47$, $a_{11} = 38.534$, and $\tilde{a}_9 = 3.06\times10^{10}$. The temperature range is $0.02 \lesssim T_9 \lesssim 10$. This may all seem picayune, but nature has found a way to produce a ratio $^{12}\mathrm{C}/^{16}\mathrm{O}$ that seems to fit our needs, so pay attention! Furthermore, the amounts of ^{16}O made also control to a large extent the amounts of heavier elements made in later burning stages.

The next reaction in the helium-burning sequence, $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$, is rather slow at normal helium-burning temperatures because no appropriate resonance in ^{20}Ne is available nearby where the α enters ^{20}Ne. (^{20}Ne is one of

those even-even nuclei that have a very low density of levels.) Thus the competition between how fast ^{12}C is produced by the triple-α and how quickly it is converted to ^{16}O primarily determines the final relative abundances of these two nuclei. For the later evolutionary stages of lower-mass stars, this may determine whether the final core, as in a white dwarf, is mostly carbon or oxygen. For your reference, the energy generation rate for the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ reaction (Q=4.734 MeV) is, from Caughlan and Fowler (1988),

$$\varepsilon(\alpha,{}^{16}\mathrm{O}) = \frac{6.69\times10^{26} Y\, X_{16}\,\rho}{T_9^{2/3}} \times \\ \times \exp\left[-39.757 T_9^{-1/3} - (0.631 T_9)^2\right] \ \mathrm{erg\ g^{-1}\ s^{-1}} \quad (6.83)$$

for not overly high temperatures.

Other capture reactions using α-particles that are of some importance to nucleosynthesis are those on various C, N, and O isotopes, where one of the exit channel particles is a neutron—and we have discussed these briefly before.

6.6 Carbon, Neon, and Oxygen Burning

Once α-particles have been used up in helium-burning and if temperatures can rise to $T_9 \sim 0.5$–1, carbon burning commences and, at yet higher temperatures ($T_9 \gtrsim 1$), oxygen burning. Intermediate between these two burning stages is neon burning, which uses high-energy photons to break down ^{20}Ne by "photodisintegration" (see below) via $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$.

The important branches of the reactions $^{12}\mathrm{C}+^{12}\mathrm{C}$ and $^{16}\mathrm{O}+^{16}\mathrm{O}$ are given in Table 6.3, where "yield" is the percentage of time the reaction results in the particular products on the right-hand side. The yield depends weakly on temperature and we ignore minor branches.

Table 6.3. Carbon- and Oxygen-Burning Reactions

Reaction	Yield	Q (MeV)
$^{12}\mathrm{C}+{}^{12}\mathrm{C}\rightarrow{}^{20}\mathrm{Ne}+\alpha$	44%	4.621
$^{12}\mathrm{C}+{}^{12}\mathrm{C}\rightarrow{}^{23}\mathrm{Na}+\mathrm{p}$	56%	2.242
$^{16}\mathrm{O}+{}^{16}\mathrm{O}\rightarrow{}^{28}\mathrm{Si}+\alpha$	21%	9.593
$^{16}\mathrm{O}+{}^{16}\mathrm{O}\rightarrow{}^{31}\mathrm{P}+\mathrm{p}$	61%	7.678
$^{16}\mathrm{O}+{}^{16}\mathrm{O}\rightarrow{}^{31}\mathrm{S}+\mathrm{n}$	18%	1.500

The $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ reactions are followed by $^{23}\mathrm{Na}(\mathrm{p},\alpha)^{20}\mathrm{Ne}$ (Q=2.379 MeV), and $^{23}\mathrm{Na}(\mathrm{p},\gamma)^{24}\mathrm{Mg}$ (Q=11.691 MeV) using the protons released from the second reaction in Table 6.3. The α-particles can then be used on ^{16}O to