HW2

Sunday, April 17, 2022 11:28 PM

1.) $\frac{dx}{dt} = f(x, \mu) \qquad -\infty < x < \infty$

$f(x, \mu)$

$f(0,0) = 0$?

No

(0,0) is not a fixed point

yes

(0,0) is a fixed point

Qualitative changed at fixed point?
(Created, destroyed, or stability change)

yes

bifurcation

No

No bifurcation

Bifurca...

stability change

created/destroyed

Saddle-Node Bifurcation

Transcritical

symmetric branches

Pitchfork

invariant under $x \to x$?
stable?

yes

No

Supercritical

Subcritical

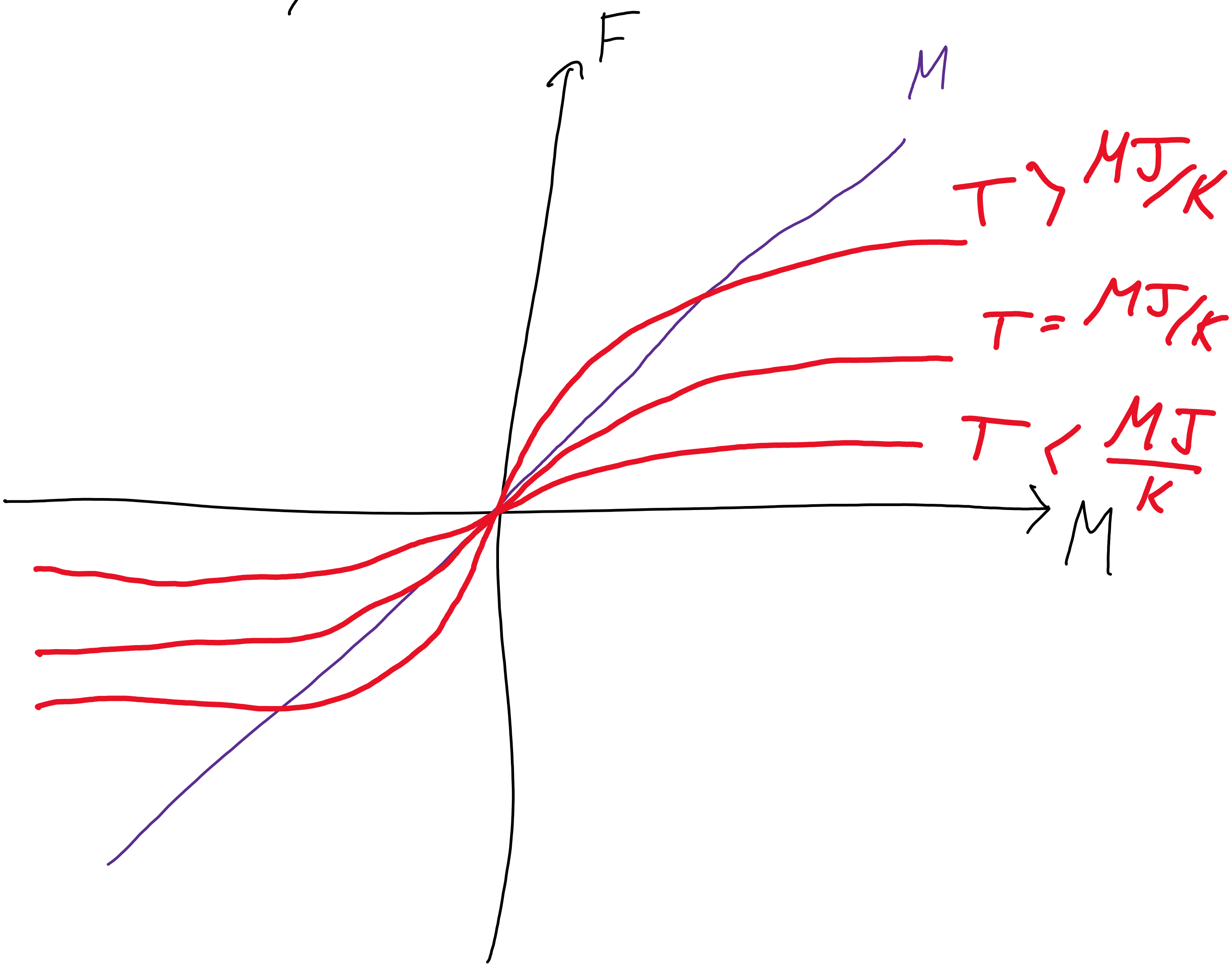
2.) $F(M,T,H) = \tanh\left(\frac{H+MJ}{kT}\right) - M = 0$

a.) $H = 0$

$$f(M,T,0)=0 \Rightarrow \tanh\left(\frac{MJ}{kT}\right)=M$$

let $\alpha = \dfrac{H}{kT}$ $\beta = \dfrac{J}{kT}$

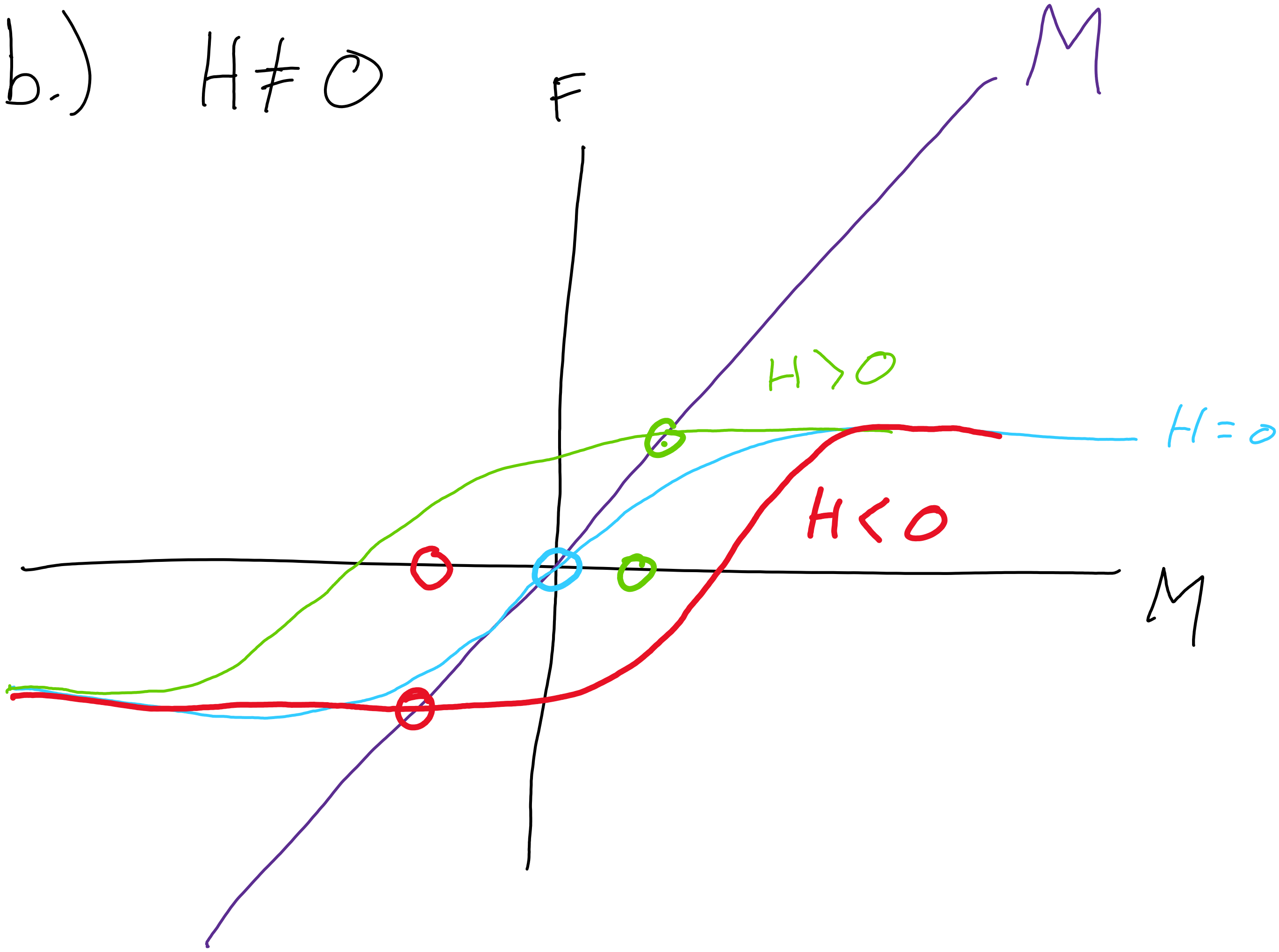
$$\tanh(\beta M) = M$$

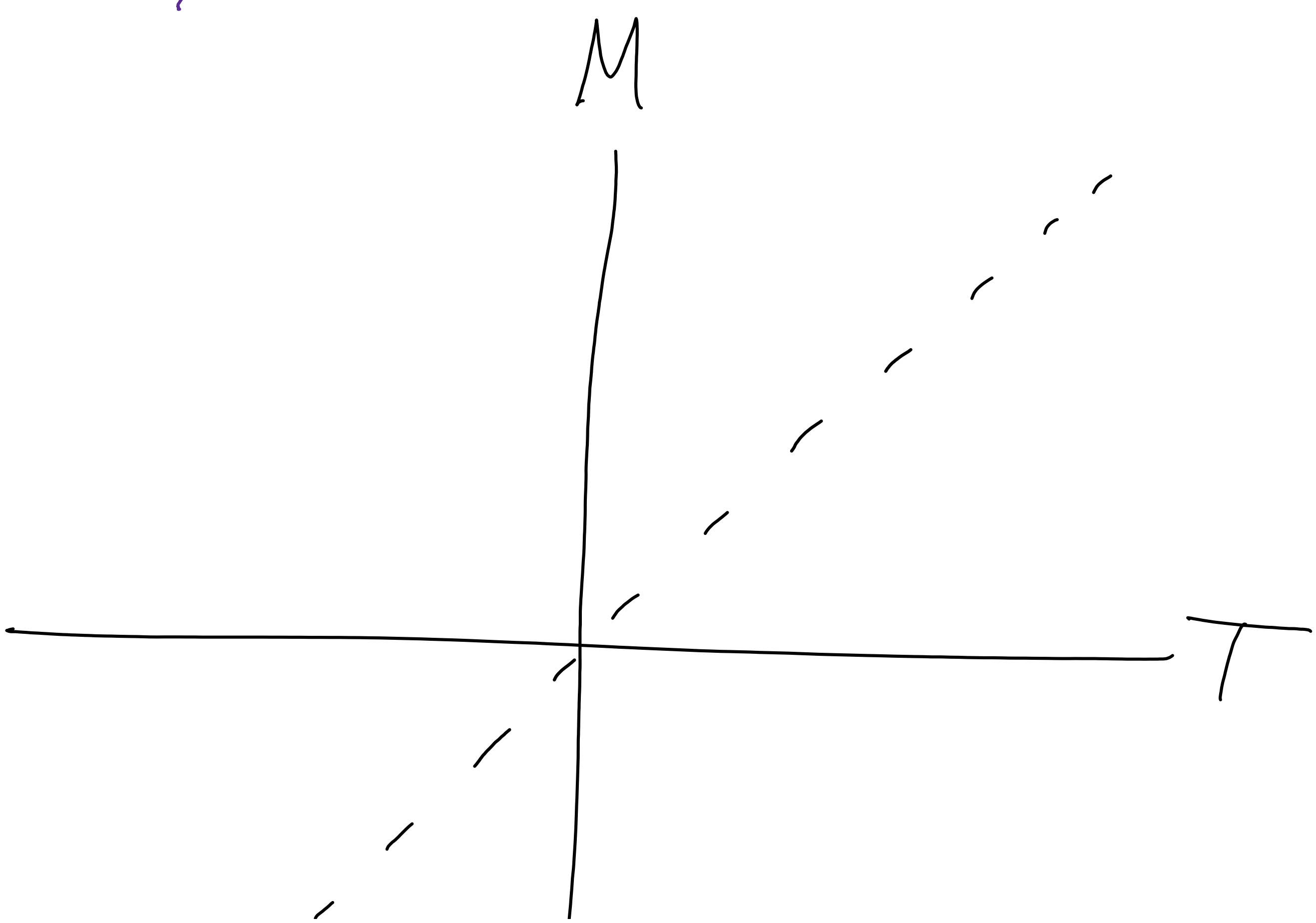


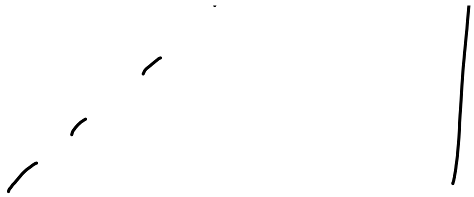
Curie temperature $T_c = \dfrac{MJ}{K}$

Saddle-Node Bifurcation

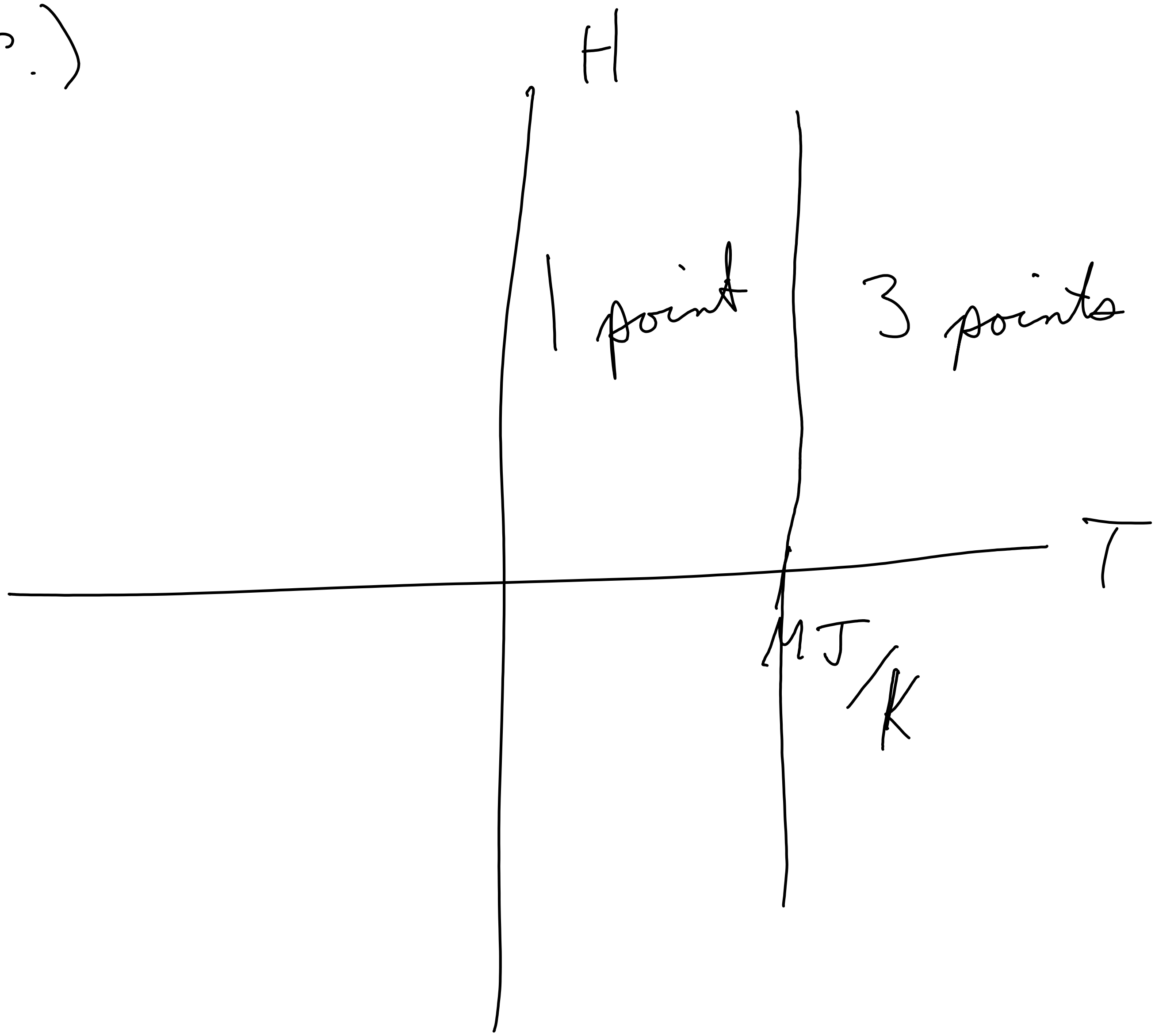
b.) $H \neq 0$







c.)



3.) $\frac{dg}{dt} = \dot{g} = k_1 s_0 - k_2 g + \frac{k_3 g^2}{k_4^2 + g^2}$

$= k_1 s_0 - k_2 g + k_3 g^2$

$$\frac{k_3 g}{k_4^2\left(1+\frac{g^2}{k_4^2}\right)}$$

$$\left(\frac{dg}{dt}\right)\frac{k_4}{k_4 k_3} = \frac{k_1 k_4}{k_4 k_3}S_0 - \frac{k_2 k_4}{k_3}\frac{g}{k_4} + \frac{k_3\, g^2/k_4^2}{\left(1+\frac{g^2}{k_4^2}\right)}\frac{k_4}{k_4 k_3}$$

!

$$\left(\frac{d\left(\frac{g}{k_4}\right)}{dt}\right)\frac{k_4}{k_4 k_3} = \frac{k_1}{k_3}S_0 - \frac{k_2 k_4}{k_3}\frac{g}{k_4} + \frac{g^2/k_4^2}{1+g^2/k_4^2}$$

$$\frac{g}{k_4} = x \qquad \frac{k_2 k_4}{k_3} = r \qquad \frac{k_1 S_0}{k_3} = s \qquad \frac{k_4 k_3}{k_4}t = \tau$$

$$\frac{dx}{d\tau} = s - rx + \frac{x^2}{1+x^2}$$

b.) $s = 0$ $\quad \frac{dx}{d\tau} = 0 - rx + \frac{x^2}{1+x^2} = 0$

$$rx = \frac{x^2}{1+x^2} \qquad rx + rx^3 = x^2$$

$1 + x$

$$x(rx^2 - x + r) = 0$$

$$x = 0, \frac{1 \pm \sqrt{1-4r^2}}{2r}$$

c.) $g(0) = 0 \longrightarrow x(0) = 0$

For high values of r, $g(t)$ will revert to the $x^* = 0$ stable point. Lower r values will make the $g(t)$ go towards one of the other nonzero stable points.

d.) $\frac{dx}{d\tau} = 0 \Rightarrow s - rx + \frac{x^2}{1+x^2}$

$$\frac{d}{dx}\left(\frac{dx}{d\tau}\right) = 0 \qquad x^2(1+x^2)^{-1}$$

$$0 - r + \frac{2x}{1+x^2} + \frac{-x^2}{(1+x^2)^2} = 0$$

$$r = \frac{2x}{(1+x^2)^2}$$

$$s - \frac{2x^2}{(1+x^2)^2} + \left[\frac{x^2}{1+x^2}\right] \times \boxed{\frac{1+x^2}{1+x^2}} = 0$$

$$s - \frac{2x^2}{(1+x^2)^2} + \frac{x^2(1+x^2)}{(1+x^2)^2} = 0$$

$$s = \frac{x^2(1-x^2)}{(x^2-1)^2}$$

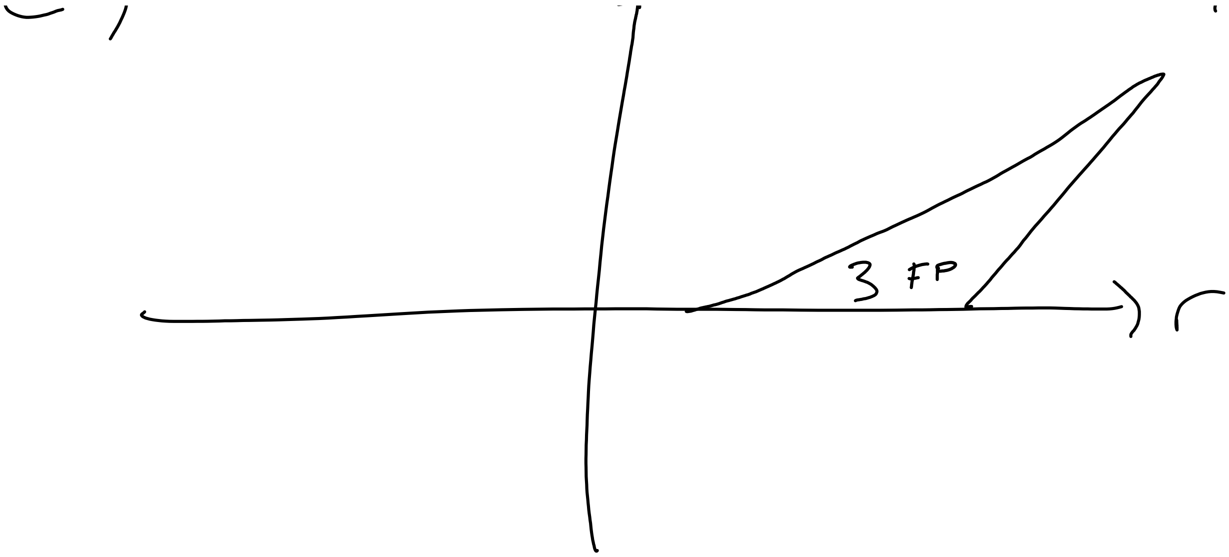
Saddle Node Bifurcations

e.)

s

$\uparrow$

IFP


3 FP
r