Phys 411 Problem Set 6 (Goldstein Chapters 4–5)

Due at the beginning of class, 10 am, Monday, November 18th, 2019

Offices: Tech F273/1800 Sherman 8055

1. Problem 15 from Goldstein Chapter 5:
   Find the principal moments of inertia about the center of mass of a flat rigid body in the shape of a 45° right triangle with uniform mass density. What are the principal axes?
   It will help to use symmetry properties; take triangle to have sides 1, 1, $\sqrt{2}$.

2. Problem 15 from Goldstein Chapter 4:
   Show that the components of the angular velocity along the space set of axes are given in terms of the Euler angles by

$$\omega_x = \dot{\theta}\cos\phi + \dot{\psi}\sin\theta\sin\phi, \tag{1}$$

$$\omega_y = \dot{\theta}\sin\phi - \dot{\psi}\sin\theta\cos\phi, \tag{2}$$

$$\omega_z = \dot{\psi}\cos\theta + \dot{\phi}. \tag{3}$$

3. Problem 6 from Goldstein Chapter 5:
   (a) Show that the angular momentum of the torque-free symmetrical top rotates in the body coordinates about the symmetry axis with an anglar frequency Ω. Show also that the symmetry axis rotates in space about the fixed direction of the angular momentum with the angular frequency

$$\dot{\phi} = \frac{I_3\omega_3}{I_1\cos\theta}, \tag{4}$$

   where ϕ is the Euler angle of the line of nodes with respect to the angular momentum as the space z axis.
   (b) Using the results of Problem 2 above, show that $\boldsymbol{\omega}$ rotates in space about the angular momentum with the same angular frequency $\dot{\phi}$, but that the angle θ' between $\boldsymbol{\omega}$ and $\boldsymbol{L}$ is given by

$$\sin\theta' = \frac{\Omega}{\dot{\phi}}\sin\theta'', \tag{5}$$

   where θ'' is the inclination of $\boldsymbol{\omega}$ to the symmetry axis. Using the data given in Section 5.6, show therefore that Earth's rotation axis and the axis of angular momentum are never more than 1.5 cm apart on Earth's surface.
   (c) Show from parts (a) and (b) that the motion of the force-free symmetrical top can be described in terms of the rotation of a cone fixed in the body whose axis is the symmetry axis, rolling on a fixed cone in space whose axis is along the angular momentum. The angular velocity vector is along the line of contact of the two cones. Show that the same description follows immediately from the Poinsot construction of the inertia ellipsoid. You can find the description of the Poinsot construction in Goldstein.