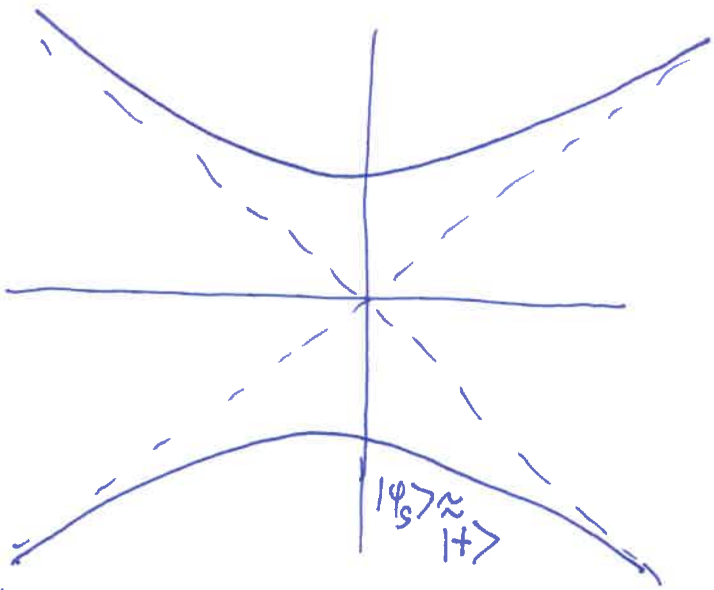
Two-Level Systems, Part IV & Parity



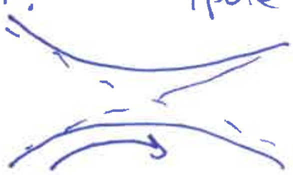
$|\psi_g\rangle \approx |2\rangle \approx |R\rangle$ $\qquad$ $|\psi_g\rangle \approx |1\rangle \approx |L\rangle$

$\epsilon = d E_z$

- Suppose we start w/ ammonia in low-energy state, with dipole oriented along left-pointing strong $\vec{E}$ field, so $|\psi\rangle \approx |R\rangle$.

Q - If we slowly ramp up the field toward right ($\epsilon \to 0 \to$ large value), how will state evolve?

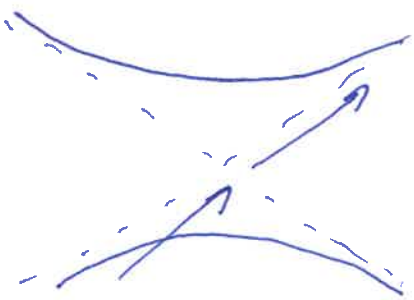
A Will "adiabatically" follow $|\psi_g\rangle$ curve & pass thru $|+\rangle$ & then become oriented so $\vec{d} \parallel \vec{E}$ pointing right. Dipole follows field. Much, since we never provide enough energy in short time for state to jump gap.



$|R\rangle \to |+\rangle \to |L\rangle$

- But what if we ramp field quickly?
- To prove, need to use time-dep. eqn & not easy.
  But result is that a "diabatic transition" is possible, This is the described by the "Landau-Zener" formula.

- In fact, if fast enough, $P_D \to 1$.



$|R\rangle \to |R\rangle$
(aligned) (antialigned)

⇒ Probability of making diabatic transition is
~~- Condition turns out to be~~ $P_D = e^{-2\pi\Gamma}$

~~$\frac{\dot{E}}{E} \gg \frac{2\Delta}{\hbar}$ to get $P_D \to 1$~~

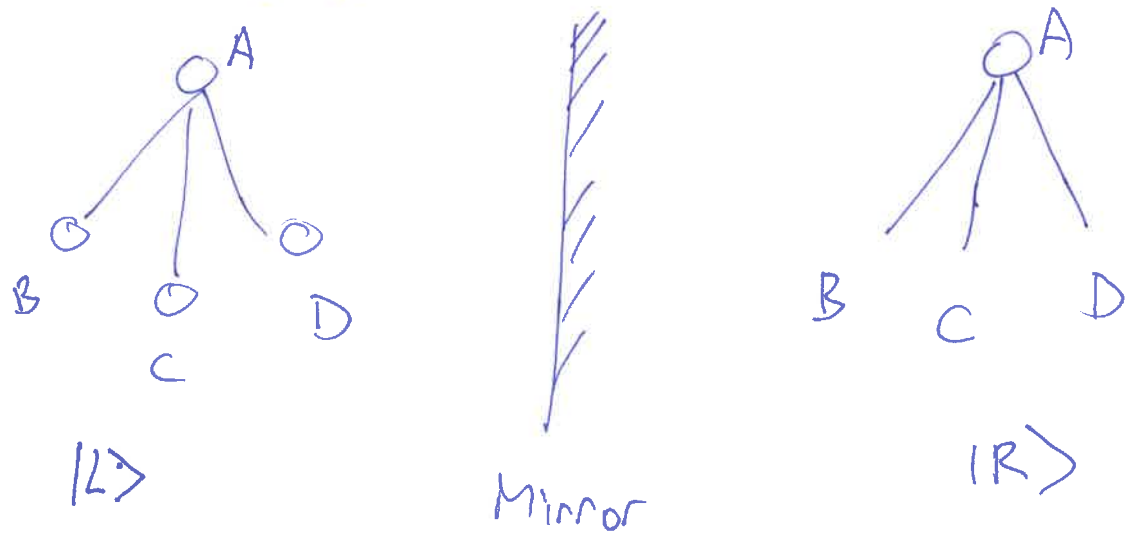
$$\Gamma = \frac{\Delta^2/\hbar}{\left|\frac{\partial}{\partial t}(E_2 - E_1)\right|}$$

(unperturbed crossing state energies)

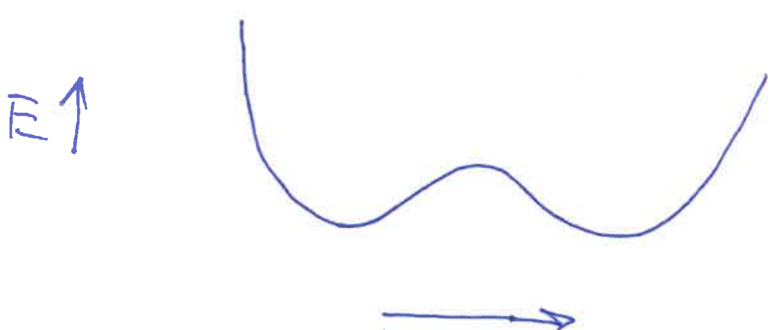
- Avoided crossings very common
  (See molecular potential from TeH^+ paper)

  - Diabatic or adiabatic transition possible
    ($\Gamma=0$) ($\Gamma=\infty$)

~~Similar System~~ Chiral Molecule



Cannot rotate $|L\rangle \to |R\rangle$ by spatial rotation. But there is a tunnelling interaction

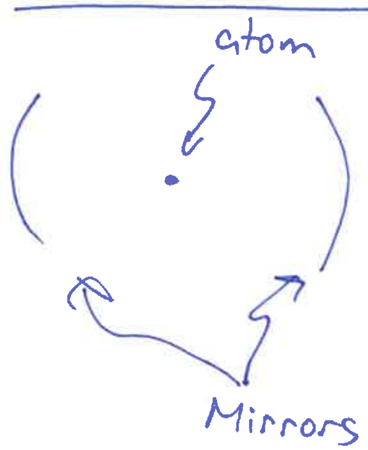


"reaction coordinate" ← atoms rearranging - can do it if they climb over an energy barrier

Implications:

- Prepare molecule in $|L\rangle$. At some time later can be in $|R\rangle$. Times depend on molecule - can be from ps to age of universe.

Cavity QED



(Now a coupling of two 2-level systems)

Atom states : $|g\rangle$ or $|e\rangle$ $\quad \varepsilon = E_e - E_g$

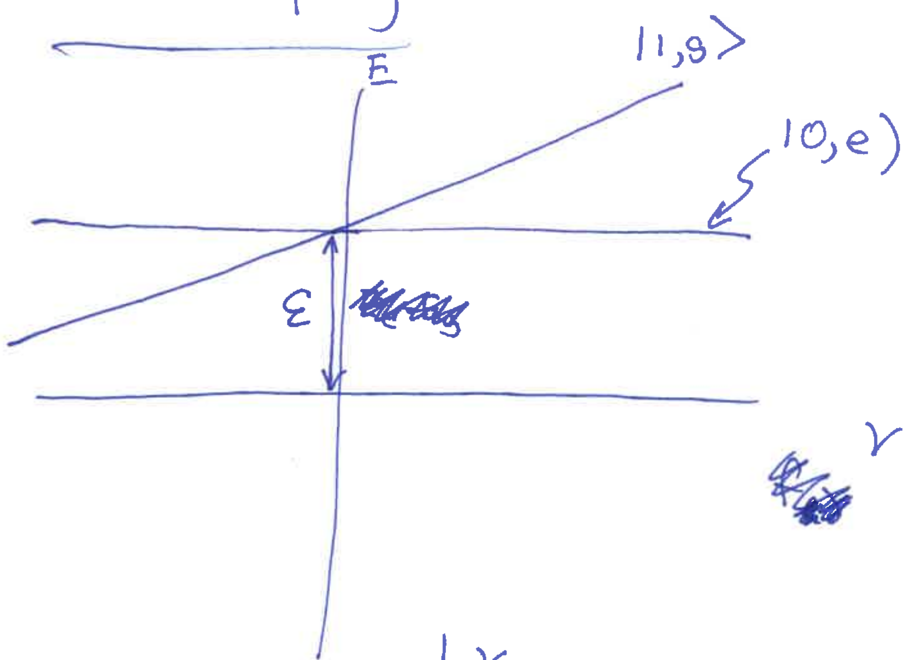
Photon states : $|0\rangle, |1\rangle, |2\rangle, \ldots$

- Say we have inject 1 photon into cavity, and that photon might get absorbed by atom if it is on resonance. $|1,g\rangle$ or $|0,e\rangle$

- ~~With weak coupling~~

For $|1\rangle$, $E = h\nu$

Weak Coupling:



They cross @ $h\nu = \varepsilon$, on resonance where photon energy matches excitation energy.

~~Strong coupling~~

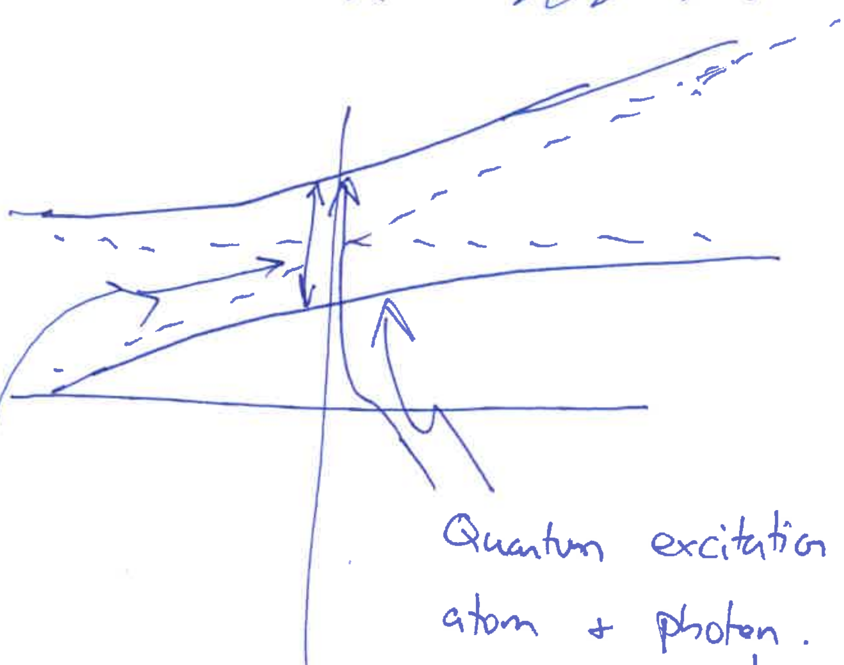
In $|0,e\rangle$, $|1,g\rangle$ basis,

$$H = \begin{pmatrix} \varepsilon & \\ & h\nu \end{pmatrix}$$

Strong coupling:

If coupling is large, photon can be absorbed & re-emitted. This happens when (stationary) cavity is small & light field is tightly focused on atom location.

That case is like ammonia being able to tunnel between basis states $\Rightarrow$ Off-diagonal terms in H.



Quantum excitation is shared between atom + photon. Each eigenstate is neither purely atom excitation nor light excitation.

On resonance,

$$|\varphi_g\rangle = \frac{1}{\sqrt{2}}\left(|0,e\rangle + |1,g\rangle\right)$$

~~neither~~ combination of electronic & light excitation, similar to $|\varphi_g\rangle \cong \frac{1}{\sqrt{2}}\left(|L\rangle + |R\rangle\right)$ for ammonia.

"Vacuum Rabi splitting".

Fields & Parity

$$V(z_N) = V_0(z_N) + U(z_N)$$

$\Pi\Pi^\dagger V_0 \Pi = \Pi V_0$

$V_0 \Pi = \Pi V_0$

$[\Pi, V_0] = 0$

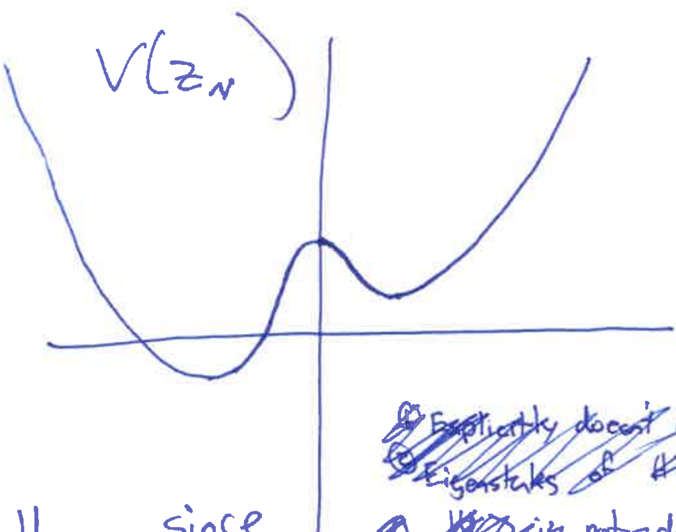
$\Pi^\dagger V_0 \Pi = V_0$

$\Pi^\dagger U \Pi = -U$

$\Rightarrow [V, \Pi] \neq 0$ &

$[H, \Pi] \neq 0$

Plotting $V(z_N)$ we can see it's obviously not symmetric



(The ground state should be more $|L\rangle$ than $|R\rangle$)

~~Explicitly doesn't commute~~

~~Eigenstates of H not Π eigenstates~~

- Formally, since $[H, \Pi] \neq 0$ for $\epsilon \neq 0$ (with external potential on)
  $\rightarrow$ (cannot simult. diagonalize)

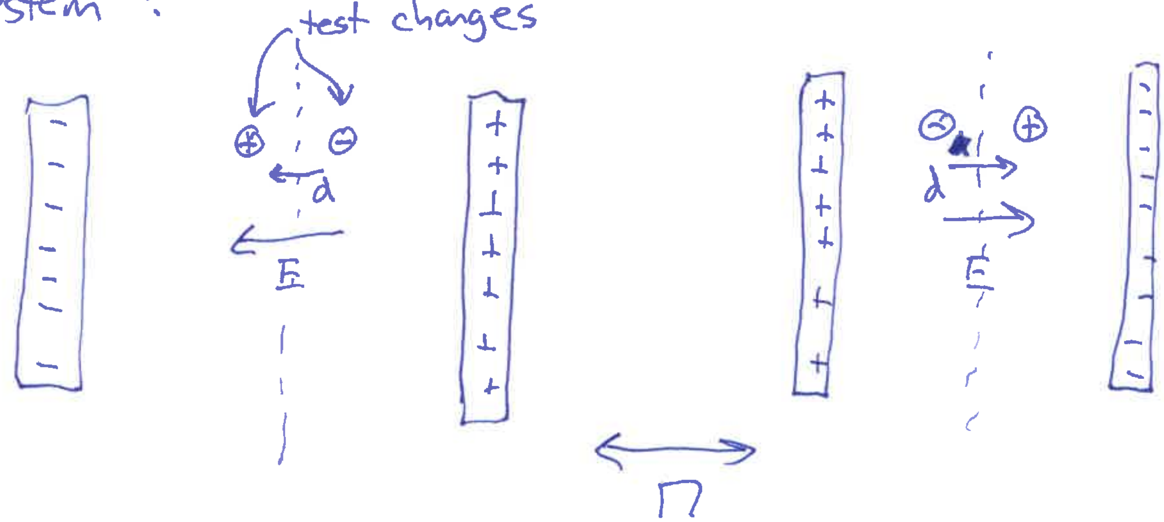
$$\Rightarrow \quad \Pi|\varphi_g\rangle \neq |\varphi_g\rangle$$ as we could easily verify

- If Π were a symmetry, it would leave M.E.s of H unchanged. For example

$$\langle \Pi 1 | H | \Pi 1 \rangle \simeq \underset{(\text{larger})}{\langle 2 | H | 2 \rangle} \neq \underset{(\text{smaller})}{\langle 1 | H | 1 \rangle}$$

$$\Rightarrow \quad \Pi^\dagger H \Pi \neq H$$ if field is external

But what if we consider the sources of the field to be part of the system?



So, if we treat E as an internal field rather than an external one, i.e. we parity transform the sources too, then

(we generally say $\vec{E}$ is odd under parity) + so is electric dipole:

$$\Pi^\dagger \vec{E}\, \Pi = -\vec{E} \qquad \Pi^\dagger \vec{d}\, \Pi = -\vec{d}$$

$$\Rightarrow \Pi^\dagger(-\vec{d}\cdot\vec{E})\Pi = \Pi^\dagger (-\vec{d})\Pi \cdot \Pi^\dagger \vec{E}\, \Pi = +\vec{d}\cdot -\vec{E} = -\vec{d}\cdot\vec{E}$$

$$\Rightarrow [H, \Pi] = 0$$ if we consider sources of field too

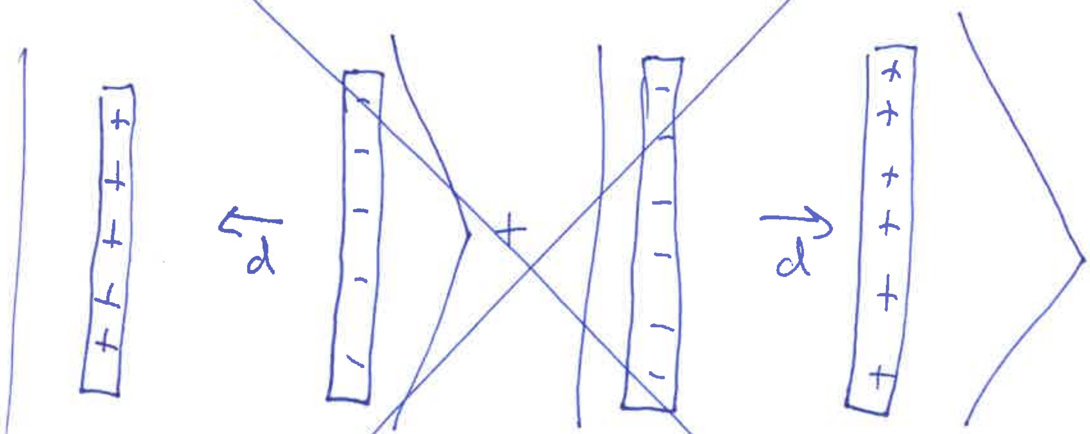
"Parity symmetry is respected by electrostatics."

but "External field breaks Parity symmetry"

- First equation above i.e. $[\Pi, H]=0$ says that physics as viewed in mirror looks like it is obeying the same laws of nature.

- ~~Note that eigenstates should be able to be expressed as states of definite parity if~~ ~~$[H,\Pi]=0$~~

  ~~Those states look like~~



  ~~If charges are treated as part of the system, we need to describe eigenstates as superposition of macroscopic charged plates.~~

- An external $\vec{B}$ field does not break Π symmetry.

$$H = -\vec{\mu}\cdot\vec{B}$$

$$\Pi^{\dagger} H \Pi = -\Pi^{\dagger}\vec{\mu}\,\Pi\,\vec{B} = -\vec{\mu}\cdot\vec{B}$$

Selection Rules

- Operators can be classified based on how they transform.

- e.g. $\left.\begin{array}{l} \Pi^\dagger \vec{p}\, \Pi = -\vec{p} \\ \Pi^\dagger \vec{x}\, \Pi = -\vec{x} \end{array}\right\}$ "Odd parity operators"

$\Pi^\dagger \vec{L}\, \Pi = \vec{L}$ "Even parity operator"

Consider

$\Pi|\psi^+\rangle = +|\psi^+\rangle$ + $\Pi|\psi^-\rangle = -|\psi^-\rangle$

And $\Pi^\dagger O^+ \Pi = +O^+$, $\Pi^\dagger O^- \Pi = -O^-$

Then $\langle \psi^+ | O^- | \psi^+ \rangle = \langle \psi^- | O^- | \psi^- \rangle = 0$

Proof: $\langle \psi^+ | O^- | \psi^+ \rangle =$

$$= -\langle \psi^+ | (\Pi^\dagger O^- \Pi) | \psi^+ \rangle$$

$$= -\langle \Pi\psi^+ | O | \Pi\psi^+ \rangle$$

$$= -\langle \psi^+ | O | \psi^+ \rangle$$

$\Rightarrow$ Negative-parity operators cannot connect states of the same parity

$\Rightarrow$ Similarly, positive-parity operators cannot connect states of different parity

So, what is parity of Hamiltonian? If

$\Pi^+ H \Pi = H$, then it is positive. And

Π is conserved.

(We are considering all sources of fields as part of system – no external fields)

E.g. $\Pi^+ H_{EM} \Pi = H_{EM}$ + same for strong force. But

Hot

$H = H_W + H_S + H_{EM}$

(Start in state of def. parity)

+ or −

$\Pi |\psi_i(t=0)\rangle = \pi_i |\psi(t=0)\rangle$

$(\Pi^+ O^- \Pi = -O^-)$ If $[H, \Pi] = 0 \Rightarrow \Pi|\psi_i(t)\rangle = \pi_i |\psi_i(t)\rangle$

$\langle O^-(t)\rangle = \langle \psi(t)|\Pi\Pi^+ O^- \Pi\Pi^+|\psi(t)\rangle$
$= \langle \psi(t)|\Pi - O^- \Pi^+|\psi(t)\rangle = -\langle O^-(t)\rangle$

$\Rightarrow \langle O^-(t)\rangle = 0$

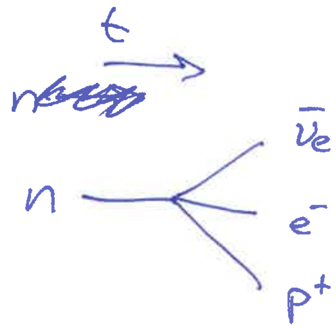
So, start in ~~positive~~ definite parity state

$\Pi|\psi(t=0)\rangle = +|\psi(t=0)\rangle$

or $= -|\psi(t=0)\rangle$

$\Rightarrow$ Now all odd parity operators must have $\langle O^-\rangle = 0$ for all time, if $[\Pi, H] = 0$. "Odd-parity operators always have zero expectation value if Π is a symmetry & we started in definite parity state"

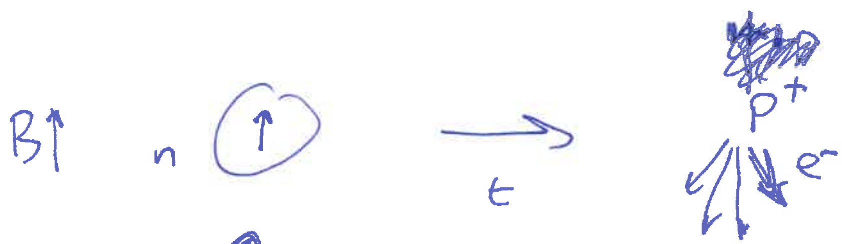
But neutrons decay by weak force



In atom, like ^{60}Co, we find that in β decay, electron momentum comes off anti-aligned to neutron spin. Prepare system in ground state of

Consider external B field. Ground state is non-degenerate. Then $\Pi|\psi_g\rangle = \pi_i|\psi_g\rangle$ if $[H, \Pi] = 0$.

~~One way to say it: if $|\psi\rangle$ has well defined energy & Π is a symmetry, Π cannot change energy of state.~~



here $\langle \vec{S} \cdot \vec{p} \rangle \neq 0$

neutron spin   electron momentum

$H = H_{EM} + H_S + H_G + H_W$

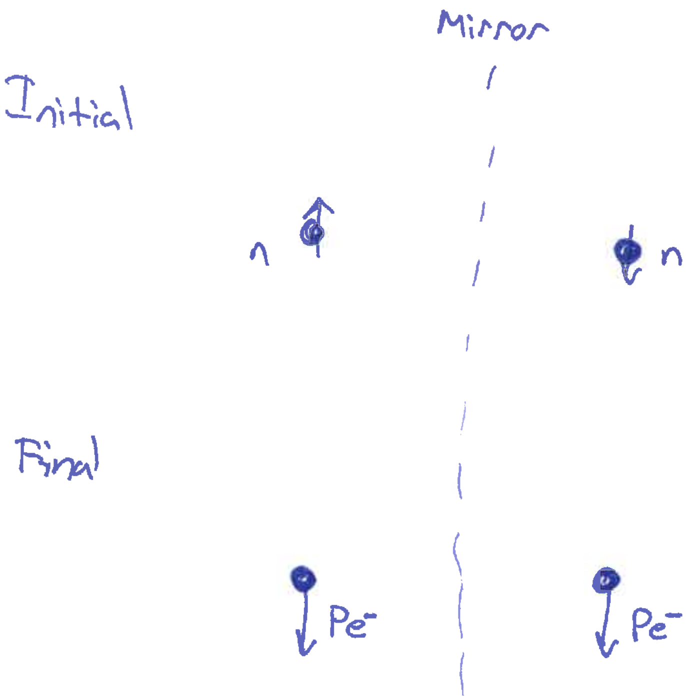
$[H_W, \Pi] \neq 0$

But $\Pi^\dagger \vec{S} \cdot \vec{p}\, \Pi = -\vec{S} \cdot \vec{p}$

Consequences $\Rightarrow$ (1) H must not conserve parity, $\Rightarrow [H, \Pi] = 0$ $\Updownarrow$ occurs if

(2) Ground state of H is not a Π eigen-state

Provocative to consider watching it in a mirror:



Mirror: Right hand → left

↳ $\Pi R_{\hat{x}}(180^\circ)$

In our world, electrons fly off opposite to direction of n spin. The experiment we see in the mirror has the electron going off in the wrong direction! It appears to be ~~[illegible]~~ obeying different laws of physics! ⇒ The weak force does not respect mirror symmetry