ES_APPM 312-0 "Complex Variables"

Homework 1 (DUE TUESDAY, 4/13/2021)

Exercise 1-1 (8 pts). Express the following quantities in the form $a + bi$, where a and b are real:

$$(1-i)^3, \quad \frac{2+3i}{3-2i}, \quad (1+i)^{25}, \quad \frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}} + \frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}.$$

Exercise 1-2 (6 pts). Write in the form $a + ib$ if $z = x + iy$:

$$z^3, \quad \frac{\bar{z}}{z}, \quad \frac{z-i}{1-i\bar{z}}.$$

Exercise 1-3 (3 pts). Find all of the sixth roots of one, $(1)^{1/6}$, and plot them in the complex plane.

Exercise 1-4 (8 pts). Find all of the roots $(-1)^{1/3}$, $(-16)^{1/4}$, plot them in the complex plane, and write them in the form $a + ib$.

Exercise 1-5 (4 pts). Consider the polynomial $P(z) = z^n + z^{n-1} + \ldots + z^2 + z + 1$. Prove that all the zeroes of $P(z)$ lie on the unit circle. **Hint:** first prove that

$$z^n + z^{n-1} + \ldots + z^2 + z + 1 = \frac{1 - z^{n+1}}{1 - z}, \quad z \neq 1.$$

Exercise 1-6 (4 pts). Find the set of points in the complex plane for which

$$\mathrm{Im}\left(z + \frac{1}{z}\right) = 0.$$

Exercise 1-7 (3 pts). Find the set of points in the complex plane for which

$$|z - 1| = |z + i|.$$

Exercise 1-8 (4 pts). Prove that

$$|z| \leq |\mathrm{Re}\ z| + |\mathrm{Im}\ z| \leq \sqrt{2}|z|.$$