Blackbody Radiation $(u \sim T^4)$

Boltzmann 1884

Two assumptions

(1) $P = \frac{1}{3} u(T)$ ← u is only function of T

Start w/ fundamental relation

$$dE = TdS - PdV \qquad (\text{find } N)$$

$$dS = \frac{dE}{T} + \frac{P}{T} dV$$

$$ds = \frac{V}{T} du + \frac{V}{T} dV + \frac{1}{3} \frac{u}{T} dV$$

$u \equiv E/V$, $E = Vu$

$\Rightarrow dE = V du + u dV$

$$dS = \frac{V}{T} \frac{du}{dT} dT + \frac{4}{3} \frac{u}{T} dV$$

$$\Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \frac{4}{3} \frac{u}{T}$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T} \frac{du}{dT}$$

$$\frac{\partial^2 S}{\partial V \partial T} = \frac{\partial^2 S}{\partial T \partial V}$$

$$\frac{4}{3}\left(\frac{\partial}{\partial T} \frac{u}{T}\right) = \frac{\partial}{\partial V} \frac{V}{T} \frac{du}{dT}$$

recall: $u = f(T)$

$$\frac{4}{3}\left(\frac{1}{T}\frac{du}{dT} - \frac{u}{T^2}\right) = \frac{1}{T}\frac{du}{dT}$$

$$\frac{4}{3}\frac{du}{dT} - \frac{4}{3}\frac{u}{T} = \frac{du}{dT}$$

$$\frac{1}{3}\frac{du}{dT} = \frac{4}{3}\frac{u}{T}$$

$$\frac{du}{dT} = 4\frac{u}{T}$$

(2) Assumption : $u(T) \sim a T^n$

$$n a T^{n-1} = 4 a T^{n-1}$$

$$n = 4$$

$$u(T) = a T^4$$