Physics 332, Spring 2022
Problem Set 4
Due Wednesday April 27th - either bring to class or e-mail to Prof. Driscoll by 1:00 pm.

1. The exponential probability density plays an important role in statistical mechanics. This probability density is given by
$$\begin{cases} Ae^{-\lambda x} & 0 \leq x \leq \infty \\ 0 & x < 0 \end{cases}$$
   (a) Determine the normalization constant A in terms of λ.
   (b) What is the mean value of x? What is the most probable value of x?
   (c) Determine the probability that a measurement of x yields a value between 1 and 2.

2. A person playing darts hits a bullseye 20% of the time on the average.
   (a) What is the probability of the person throwing b bullseyes in N attempts?
   (b) Find the probability that the person hits a bullseye once in five throws.
   (c) Find the probability that the person hits a bullseye twice in ten throws.
   (d) Why are the probabilities in (b) and (c) not identical?

3. Many systems have strong particle-particle interactions only when the particles are very close together. A classic model for these types of systems is a *hard sphere gas*; e.g. we assume the particles are non-interacting except that they cannot occupy the same volume (no overlaps allowed). In a hard sphere gas, the spatial distribution of particles is no longer uncorrelated. Roughly speaking, each additional particle removes a volume v_0 from the volume available to other particles, e.g. the presence of j particles in the system leaves only a volume $(V - jv_0)$ available to the $(j+1)th$ particle.
   (a) In a non-interacting gas, the number of microstates is proportional V^N, $\Omega \sim V^N$. Determine the dependence of Ω on V for a hard sphere gas.
   (b) In the limit of low density, e.g. assuming that $Nv_0 \ll V$, show that the ideal gas law is now of the form
$$P(V - b) = NkT.$$
   (c) Show explicitly that b is four times the actual physical volume occupied by N particles. (Hint: v_0 was not the physical volume, it was the 'space' taken up by a particle – how close can two particles be?)

4. Consider a system of particles whose energy quantized (they can only have discrete energies), and is given by
$$\epsilon(n) = nh\nu, n = 0, 1, 2, ...$$
   (a) Obtain an expression for $\Omega(N, E)$, given the number of particles, N, and a given total energy, E. *Hint: Think about the number of quanta the energy can be partitioned into.*
   (b) Obtain an asymptotic expression for the entropy (assuming N is large).
   (c) Find an expression for T as a function of E/N and $h\nu$. What happens when $(E/N)/(h\nu) \gg 1$?