$$F = -\frac{1}{\beta} \ln \sum_n e^{-\beta E_n}$$

$$Z = \sum_n e^{-\beta E_n}$$ quantum partition function

$$F = -\frac{1}{\beta} \ln Z$$

$$\boxed{F = -k_B T \ln Z}$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_N$$

$$E = F - TS$$

$$= k_B T^2 \left(\frac{\partial \ln Z}{\partial T}\right)_V$$

$$\boxed{E = \frac{\sum_n E_n e^{-\beta E_n}}{\sum_n e^{-\beta E_n}}}$$

Grand ensemble

$$S = -k_B \overline{\ln w_{nN}}$$

$$w_n = e^{-\alpha - \beta E_n - \gamma N}$$

$$\rightarrow S = k_B (\alpha + \beta \bar{E}_n + \gamma \bar{N})$$

Def. $\Phi \Leftrightarrow$

$$\Phi = \bar{E} + PV - TS = \mu \bar{N}$$

$$S = -\frac{\mu \bar{N}}{T} + \frac{E + PV}{T} = \Omega$$

$$\alpha = -\frac{\Omega}{k_B T}; \quad \beta = \frac{1}{k_B T}; \quad \gamma = \frac{\mu}{k_B T}$$

$$\boxed{w_{nN} = e^{\beta(\Omega + \mu N - E_{nN})}}$$

$$\sum_{nN} w_{nN} = 1$$

$$\boxed{\Omega = \frac{1}{\beta} \ln \left( \sum_N e^{\beta \mu N} \sum_n e^{-\beta E_{nN}} \right)}$$

$$d\Omega = -S dT + P dV - \bar{N} d\mu$$

$$S = \left(\frac{\partial \Omega}{\partial T}\right)_{\mu, V}; \quad \bar{N} = \left(\frac{\partial \Omega}{\partial \mu}\right)_{T, V}$$