1. Homework Assignment - 414-1 Electrodynamics

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Exercise 1 (2 pts)

Solve

$$\vec{x} = \vec{a} \times (\vec{x} + \vec{b})$$

for three-dimensional vector $\vec{x}$. $\vec{a}$ and $\vec{b}$ are known vectors.

Exercise 2 (2 pts)

We know that the vector field $\vec{v}$ has a zero curl $\vec{\nabla} \times \vec{v} = 0$:

$$v = \begin{pmatrix} 2xy - z \\ x^2 + y^2 + z^2 \\ v_z \end{pmatrix},$$

i) what is v_z?

ii) what is the potential that satisfies $\vec{v} = \nabla\phi$?

Exercise 3 (5 pts)

Using the properties of the Levi-Civita symbol, prove that

i)

$$\vec{\nabla} \times (\nabla\phi) = 0$$

ii)

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) = 0$$

iii)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \nabla(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}$$

iv)

$$\vec{\nabla} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b})$$