4. Homework Assignment - 414-1 Electrodynamics

I. Kovács

February 1, 2021

Exercise 1 (1 pt)

Show that the differential space-time interval $c^2dt^2 - dx^2 - dy^2 - dz^2$ is invariant under a Lorentz transformation, by using an explicit Lorentz transformation in the x direction.

Exercise 2 (3 pts)

A particle moves along the x-axis, given by

$$ct(\sigma) = \lambda \sinh(\sigma)$$
$$x(\sigma) = \lambda \cosh(\sigma)$$

Find the i) trajectory $x(\tau)$, ii) proper velocity $u(\tau)$, and iii) proper acceleration $a(\tau)$. iv) Is this a valid trajectory for a physical particle?

Exercise 3 (4 pts)

A space ship is accelerating in such a way that in it the acceleration is constant (g), corresponding to the gravity on Earth. How long does it take to get to velocity v from $v_0 = 0$, measured in i) Earth frame and ii) in the space ship? iii) What are the numerical values when $v = c/4$?

Exercise 4 (4 pts)

Express in terms of $\vec{E}$ and $\vec{B}$ the following invariants:

i) $F_{\mu\nu}F^{\mu\nu}$

ii) $F^{\star}_{\mu\nu}F^{\mu\nu}$

Having found these invariants, what do they tell us about the transformation of EM fields? In particular, answer the following questions:

iii) Can a purely electric field in one inertial frame seen as a purely magnetic field in another?

iv) Can a progressive EM wave ($\vec{E}\perp\vec{B}$) be seen as a purely electric or magnetic field in an inertial system?