HW 2.2

Sunday, January 24, 2021 6:36 PM

$$\vec{L} = \frac{1}{c}\int d^3r\, \vec{r}\times(\vec{E}\times\vec{B})$$

show: $$\vec{L} = \frac{1}{c}\int d^3r\left[\vec{E}\times\vec{A} + \sum_{l=1}^{3} E_l(\vec{r}\times\vec{\nabla})A_l\right]$$

$$\vec{L} = \frac{1}{c}\int d^3r\, \vec{r}\times(\vec{E}\times\vec{B})$$

$$= \frac{1}{c}\int d^3r\, \vec{r}\times\left[\vec{E}\times(\vec{\nabla}\times\vec{A})\right]$$

$$= \frac{1}{c}\int d^3r\, \vec{r}\times\left[\vec{\nabla}(\vec{E}\cdot\vec{A}) - \vec{A}(E\cdot\vec{\nabla})\right]$$

$$= \frac{1}{c}\int d^3r\left[\vec{r}\times\vec{\nabla}(\vec{E}\cdot\vec{A}) - \vec{r}\times\vec{A}(\vec{E}\cdot\vec{\nabla})\right]$$

$$= \frac{1}{c}\int d^3r\left[\underbrace{\sum_{l=1}^{3} E_l(\vec{r}\times\vec{\nabla})A_l}_{1} - \underbrace{\vec{r}\times(\vec{E}\cdot\vec{\nabla})\vec{A}}_{2}\right] \Rightarrow$$

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2 $\vec{r}\times(\vec{E}\cdot\vec{\nabla})\vec{A} = \epsilon_{\dots}\hat{e}_i x_j E_l \partial A_k$

$$2):\ \vec r \times (\vec E \cdot \vec\nabla)\vec A = \varepsilon_{ijk}\, \hat e_i\, x_j\, E_n \frac{\partial A_k}{\partial x_n}$$

$$= \varepsilon_{ijk}\, \hat e_i \left[\cancelto{0}{\frac{\partial}{\partial x_n}(x_j E_n A_k)} - E_n A_k \frac{\partial x_j}{\partial x_n} - \cancelto{0}{x_j A_k \frac{\partial E_n}{\partial x_n}} \right]$$

$$= -\varepsilon_{ijk}\, \hat e_i\, \delta_{jn} E_n A_k = -\vec E \times \vec A$$

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$$\Rightarrow \vec L = \frac{1}{c}\int d^3r \left[\vec E \times \vec A + \sum_{l=1}^{3} E_l (\vec r \times \vec\nabla) A_l \right]$$