Quantum Mechanics 412-1 Discussion

Tuesday, 15 October 2019

1. **Coupled time evolution.**
   Consider a system with a pair of observable quantities A and B, whose commutation relations with the Hamiltonian take the form $[H, A] = i\hbar\omega B$ and $[H, B] = -i\hbar\omega A$, where ω is some real constant. Suppose that the expectation values of A and B are known at time $t = 0$ as $\langle A\rangle(t = 0) = A_0$ and $\langle B\rangle(t = 0) = B_0$. Find formulas for the expectation values of A and B as a function of time, $\langle A\rangle(t)$ and $\langle B\rangle(t)$, assuming the operators themselves do not explicitly depend on time.

2. **Operator methods and time dependence.**
   The Hamiltonian H has two normalized eigenstates $|\psi_1\rangle$ and $|\psi_2\rangle$ which correspond to distinct eigenvalues E_1 and E_2.
   (a) Show that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal.
   (b) Suppose there is some observable A for which $A\,|\psi_1\rangle = |\psi_2\rangle$ and $A\,|\psi_2\rangle = |\psi_1\rangle$. Calculate the eigenvalues and eigenvectors of A.
   (c) Assuming that at $t = 0$ a system is in the state $|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle)$, show that the probability of the system returning to its initial state is given as a function of time as $P(t) = \cos^2[(E_1 - E_2)t/2\hbar]$

3. **Expected momentum.**
   (a) Show that, for a real, normalized wavefunction $\psi(x)$, the expectation value of momentum vanishes, $\langle P\rangle = 0$.
   (b) Show that if $\psi(x)$ has a mean momentum $\langle P\rangle$, the wavefunction $e^{ip_0x/\hbar}\psi(x)$ has a mean momentum $\langle P\rangle + p_0$.

4. **Operators and eigenbases**
   Consider an operator Q characterized in the basis of energy eigenkets $|1\rangle$ and $|2\rangle$ as:

$$Q = a(|1\rangle\langle 1| + |1\rangle\langle 2| - |2\rangle\langle 2|) \tag{1}$$

   Find the eigenvalues and eigenvectors of Q in terms of a and $|1\rangle$ & $|2\rangle$. Are the eigenkets orthogonal? Did you expect them to be? If there's disagreement between these two answers, try to reconcile it.