Quantum Mechanics 412-1 Discussion

Tuesday, 22 October 2019

1. **Half-infinite well and boundary conditions.**
   A particle of mass m is subject to the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ -V_0 & 0 < x < a \\ 0 & x > a \end{cases}$$

   By considering the boundary conditions of the wavefunction at $x = 0$ and $x = a$, find that the condition for the well to support at least one bound state is:

$$V_0 > \frac{\pi^2 \hbar^2}{8ma^2} \tag{1}$$

   Hint: you do not need to explicitly solve the continuity conditions; rather, it helps to sketch out the shape of the simplest possible bound state, and make general statements about when the boundary conditions are satisfied.

2. **Expected momentum *(from last week)***
   (a) Show that, for a real, normalized wavefunction $\psi(x)$, the expectation value of momentum vanishes, $\langle P \rangle = 0$.
   (b) Show that if $\psi(x)$ has a mean momentum $\langle P \rangle$, the wavefunction $e^{ip_0 x/\hbar}\psi(x)$ has a mean momentum $\langle P \rangle + p_0$.

3. **Operators and eigenbases *(from last week)***
   Consider an operator Q characterized in the basis of energy eigenkets $|1\rangle$ and $|2\rangle$ as:

$$Q = a(|1\rangle \langle 1| + |1\rangle \langle 2| - |2\rangle \langle 2|) \tag{2}$$

   Find the eigenvalues and eigenvectors of Q in terms of a and $|1\rangle$ & $|2\rangle$. Are the eigenkets orthogonal? Did you expect them to be? If there's disagreement between these two answers, try to reconcile it.