Astro 411 Problem Set 1 (Goldstein Chapters 1-2)

Due at 10 am, Friday, October 4th, 2019

Instructor: Sasha Tchekhovskoy
E-mail: atchekho@northwestern.edu
Offices: (i) Tech F273 and (ii) office 55 on the 8th floor of 1800 Sherman
Teaching assistant: Wei-Ting Lin
E-mail: WeiTingLin2014@u.northwestern.edu
Section hours: TBD; Location: TBD

1. What is the Lagrangian for a projectile launched radially outwards from the surface of Earth, and what is the equation of motion? Show that energy is conserved. Deduce Earth's escape speed, i.e. the minimum speed required to escape Earth's gravity from its surface. Give both the symbolic expression for the escape speed and how much faster it is than a Boeing 747.

2. Determine the Lagrangian and equation of motion for a pendulum whose pivot point (i.e. the point at the top) is shaken horizontally so that its position is $x_{\rm piv}(t) = A\sin(\omega t)$?

3. (from Goldstein chapter 1): Obtain the Lagrange equations of motion for a spherical pendulum, i.e. a mass suspended by a rigid weightless rod (or a string) that can swing left- right and back-forth.

4. (from Goldstein chapter 1): Two mass points of mass m_1 and m_2 are connected by a string passing through a hole in a smooth table so that m_1 rests on the table surface and m_2 hangs suspended. Assuming m_2 moves only in a vertical line, what are the generalized coordinates for the system? Write down the Lagrange equations for the system and, if possible, discuss the physical significance any of them might have. Reduce the problem to a single second-order differential equation and obtain a first integral of the equation. What is its physical significance? (Consider the motion only so long as neither m_1 nor m_2 passes through the hole.) Notes: (i) m_1 can move in two dimensions. (ii) A "first integral" means: the equation d/dt(first integral) = 0 is equivalent to the equation of motion.