Phys-411
Final Exam

Due at 5pm on Wednesday, December 11th 2019
(at the P&A office in the Tech building, ask for Bud Robinson)

Instructions:

This is a closed-book take-home exam, but you can use a calculator and a double-sided Letter-size sheet of handwritten notes.

You can freely pick the time you want to work on this exam so long as you turn it in on time (5pm on Wednesday, December 11th 2019) to Bud Robinson in P&A office in the Tech building. You have to do the exam on your own.

Do not look at the exam until you are ready to start. Once you start, you need to finish the exam in a single seating of **3 hours = 180 minutes** (except for short breaks).

You cannot do any additional work after the time has elapsed.

Tips for taking the exam:

1. Go over previous exams I posted on canvas, through the midterm, quiz, and problem sets.
2. Try to get a good night's sleep the night before you take the exam.
3. If you get stuck on a problem, put on paper all the thoughts/ideas you have (and hand them in), and move on to the next problem. If you end up having time left at the end, return to the problem and try to finish it.

If you have any questions about these rules or anything else, please reach out to atchekho@northwestern.edu

Good luck!
Sasha

Phys 411, Fall 2019

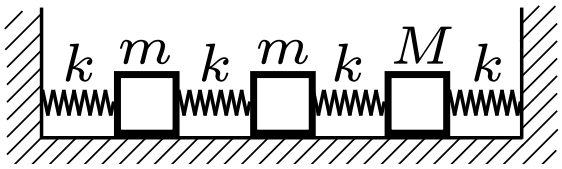
Final Exam

Due Wednesday, December 11th, 5 pm

1. [20 points] Consider the following system of masses and springs. The masses are confined to slide without friction on the floor in one dimension, along the horizontal axis, as shown. All springs have the same spring constant k and equal length in equilibrium. Neglect gravity.

   (a) Determine the Lagrangian L

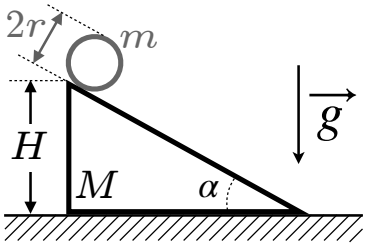
   (b) Find the $\boldsymbol{T}$ and $\boldsymbol{V}$ matrices for small perturbations about equilibrium. (You do not need to solve the system.)



2. [40 points] A ring of mass m and radius r can roll without resistance and without slipping on the surface of a prism of height H and mass M. The prism can slide on the floor horizontally without friction. The prism's surface makes an angle α with the floor. All motion is confined to the vertical plane shown. Gravity acts downwards. Initially, the ring and prism are held stationary, with the ring at its highest position: at height H above the floor. Assume that that the ring is small in comparison to the prism, i.e., that $r \ll H$.

   (a) Determine the Lagrangian.

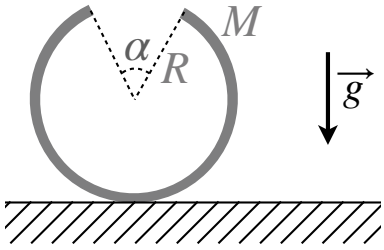
   (b) After the ring and prism are released, what is the speed of the prism just before the ring hits the floor?



3. [40 points] A cookie monster bit off a piece of a uniform ring of radius R. As a result, the ring has a cut-out that subtends an angle α, as viewed from the center of the original ring. Whatever remains of the ring has mass M and can roll without resistance and without slipping on the horizontal floor. The motion is confined to the vertical plane shown. Gravity acts downwards.

   (a) Determine the Lagrangian.

   (b) What is the period of small oscillations about the equilibrium point?



   Hints: (i) The center of mass of the ring with a cut-out is shifted from the center of the original ring. (ii) You can compute the moment of inertia of the ring with a cut-out relative to the point of contact with the floor by applying the parallel axis theorem *twice*. (iii) If you get stuck computing the moment of inertia, leave it as symbol I, but explain relative to what axis you would compute the moment of inertia. If you have time left at the end, you can come back to computing I.