Physics 412-2 (Spring 2019) **Problem Set 1** Professor Gerald Gabrielse
Posted: 11 Jan. 2020 Due: 9:00 A.M. on Monday, 19 Jan. 2020

The problem sets are a serious part of the learning experience in this class. The problem sets will sometimes deliberately range away from what can be covered in the lectures in some cases. The goal to expose you to basic concepts and important examples of quantum mechanics. Problems will often be divided into many small parts to guide your through a solution. Corrections to the assignments, if needed, will be posted to the class web page. Your solutions should be placed in the box outside my office.

13 Jan 2019 – fixed typos in problems 1, 2c, 2d.

1. **Rotation Operator.** Derive the 3 by 3 rotation matrix that rotates a simple vector by an infinitesimal angle ϵ about the y axis.

2. **Pauli matrices**

   (a) Derive the commutation relation for the Pauli matrices by explicitly multiplying them together as needed.

   (b) Show explicitly that an arbitrary, well behaved 2 x 2 operator A can be expanded in terms of the Pauli matrices and the identity matrix.

   (c) Write down and identify the function of the operator $(\sigma_x + i\sigma_y)/2$.

   (d) Write down and identify the function of the operator $(\sigma_x - i\sigma_y)/2$.

3. **Spin 1/2**

   (a) Derive the rotation operator for a spin 1/2 wavefunction for a rotation by a finite angle α about the y axis.

   (b) Derive the rotation operator for a spin 1/2 wavefunction for a rotation by a finite angle β about the x axis.

   (c) The spin $\vec{\mathbf{S}}$ is a vector operator whose component operators rotate like the components of an ordinary vector. Rotate the spin operator by a finite angle α around the y axis.

4. A magnetic moment $\vec{\mu}$ of a simple spin 1/2 system is proportional to its spin operator $\vec{\mathbf{S}}$ divided by the angular momentum for a spin 1/2,

$$\vec{\mu} = \mu \frac{\vec{\mathbf{S}}}{\hbar/2}. \tag{1}$$

   (a) What is the Hamiltonian for this system in a spatial uniform magnetic field $B\hat{\mathbf{z}}$

   (b) What is the time evolution operator for this system?

   (c) Write the time evolution operator in terms of the rotation operator $D_z(\alpha)$.

   (d) How does the angle α depend upon time? What is the angular frequency?

A slowly moving neutron is a spin 1/2 system. An appropriate thin crystal, perpendicular to the neutron's initial velocity, splits the neutron wavefunction into two parts that are spatially separated and in different directions but otherwise identical and phase coherent, each can be written as ψ_0 as they leave leave the crystal. One part only is subject to a magnetic field for a distance d. A second thin crystal, also perpendicular to the initial neutron velocity, brings the two parts back together so that where they initially overlap the wavefunction is the sum of the wavefunctions for the two separate parts. Consider how the wavefunctions of the two parts come back together on a detector plane that is parallel to the two crystals. More details of such experiments are in Phys. Lett. **54A**, 425 (1975) and Phys. Rev. Lett. **35**, 1053 (1975).

(e) How do the wavefunctions for the two parts differ just where the two parts of the wavefunction overlap?

(f) How does the probability of observing the neutron vary along the detector plane?

(g) Identify the points where the magnetic field causes $n\pi$ rotations where n is integer. What special happens at these points?