Use Gibbs dist to calculate

$\overline{v_x} = 0$; $\overline{v_x^2} = \frac{k_B T}{m}$

K.E. $\frac{1}{2} m \overline{v^2} = \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)$

$= \frac{3}{2} k_B T$

Interaction vanishes U

$E = K.E + U$

$E = \frac{3}{2} k_B T$

"Equipartition of Energy

$\frac{1}{2} k_B T$ per degree of freedom

1 part in 3d 3 degrees

for N part 3N deg. of freedom

$E^{total} = \frac{3N}{2} kT$

$C_V = \left(\frac{\partial E}{\partial T}\right)_N = \frac{3}{2} N k_B T$

Assembly of Harmonic Osc.

$H(p,q) = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2} \sum k_{ij} q_i q_j ; k_{ii} = 0$

normal modes p_i', q_i' ; $p_i = \sqrt{m} p_i'$

$E(p', q') = \frac{1}{2} \sum_\alpha (p_\alpha'^2 + \omega_\alpha^2 q_\alpha'^2)$

$= \frac{1}{2} \sum_\alpha \epsilon_\alpha$

$dw = dw_p \, dw_q$

$dw_{p_\alpha'} = \sqrt{\frac{\beta}{2\pi}} e^{-\frac{1}{2}\beta p'^2} dp'$

$dw_{q_\alpha'} = \sqrt{\frac{\beta \omega_\alpha^2}{2\pi}} e^{-\frac{1}{2}\beta q_\alpha'^2} dq'$

$\overline{p_\alpha'^2} = \frac{1}{\beta} = k_B T$

$\overline{q_\alpha'^2} = \frac{1}{\beta \omega_\alpha^2}$

$\overline{E_\alpha} = \frac{k_B T}{2} + \frac{k_B T}{2} = k_B T$

equipartition for harmonic oscillators

$k_B T$ per degree of freedom

in 3d 3 x, y, z ; $3N = N$

total energy of N harmonic osc

$\overline{E} = 3N k_B T$

$C_N = \frac{\partial \overline{E}}{\partial T} = 3N$

law of Dulong & Petit

valid if not too cold or hot

Quantum Mech. Osc.

$\hat{H} = \frac{1}{2} \sum_\alpha (\hat{p}_\alpha'^2 + \omega_\alpha^2 q_\alpha'^2)$

$= \sum_\alpha \epsilon_{n_\alpha}$