Physics 411 Problem Set 4

Due at the beginning of class, 10 am, ~~Wednesday, October 23rd~~ **Friday, October 25th**, 2019

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1. Solve problem 18 from Goldstein Chapter 2 (third edition): A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω.

   (a) Obtain the Lagrange equations of motion assuming the only external forces arise from gravity.

   (b) Show that the energy of the mass, given by the **naive expression**, $\boldsymbol{E = T + V}$, is not conserved. Give a qualitative explanation why.

   (c) What are the constants of motion?

   (d) Show that if ω is greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the hoop at a point other than the bottom, but that if $\omega < \omega_0$, the only stationary point for the particle is at the bottom of the hoop.

   (e) What is the value of ω_0?

2. There are two systems sketched below. For each system, do the following:

   (a) Find the matrices $\boldsymbol{T}$ and $\boldsymbol{V}$ such that $L = 0.5T_{ij}\dot{\eta}_i\dot{\eta}_j - 0.5V_{ij}\eta_i\eta_j$. Be sure to say what η_i represents, and also what the equilibrium position is.

   (b) Find the eigenfrequencies.

   (c) Find the matrix of eigenvectors $\boldsymbol{A}$, properly orthogonalized and normalized (i.e., such that $\tilde{\boldsymbol{A}}\boldsymbol{T}\boldsymbol{A} = \mathbf{1}$).

   (d) Describe physically the behavior of each of the modes, with sketches if necessary.