Problem Set 4

Due Thursday October 24, 9:30 AM.
Submit in class or in TA's mailbox in the Physics office.

1. Consider the Schrödinger equation in one dimension with an attractive delta-function potential $V(x) = -V_0\delta(x)$. Show that this system has only one bound state. Find explicit expressions for its wavefunction and energy eigenvalue.

2. This problem is about a purely quantum phenomenon called 'over-barrier reflection.' In classical physics, a particle moving in one dimension which encounters an attractive potential well is not reflected; it just speeds up a bit when it crosses the potential well. To investigate what happens quantum mechanically, consider a quantum particle of mass m and the potential:

$$V(x) = \begin{cases} -V_0 & 0 < x < a \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

   (a) For an energy $E > 0$, construct the wavefunction with boundary conditions incoming from the left, that is, the function which behaves as $(e^{ikx} + Re^{-ikx})$ as $x \to -\infty$ and as (Te^{ikx}) as $x \to +\infty$. Find the reflection and transmission coefficients R and T.

   (b) Show that, as you would naively expect, $R \to 0$ as $E \to \infty$.

   (c) Work out the behavior of R as $V_0 \to \infty$ with E and a fixed. Explain in words what is going on.