HW 9.2

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1) $$\frac{dP}{d\Omega} = \frac{e^2}{16\pi^2 c}\frac{1}{(1-\hat{n}\cdot\vec{\beta})^5}\left|\hat{n}\times\left[(\hat{n}-\vec{\beta})\times\frac{d\vec{\beta}}{dt}\right]\right|^2\Bigg|_{ret}$$

$$\vec{\beta} = \beta\hat{z} \qquad \frac{d\vec{\beta}}{dt} = \frac{d\beta}{dt}\hat{x} \qquad \vec{\omega} = \omega\hat{y} \qquad \hat{n}\cdot\vec{\beta} = \beta\cos(\theta)$$

$$\hat{n}\cdot\dot{\vec{\beta}} = \dot{\beta}\cos\phi\sin\theta$$

$$\hat{n} = \cos\phi\sin\theta\,\hat{x} + \sin\phi\sin\theta\,\hat{y} + \cos\theta\,\hat{z}$$

$$\frac{dP}{d\Omega} = \frac{e^2}{16\pi^2 c}\frac{1}{(1-\beta\cos(\theta))^5}\underbrace{\left|\hat{n}\times\left[(\hat{n}-\beta\hat{z})\times\frac{d\beta}{dt}\hat{x}\right]\right|^2}_{A}\Bigg|_{ret}$$

$$A = (\hat{n}\cdot\vec{\beta})\cdot(\hat{n}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}\cdot(\hat{n}\cdot(\hat{n}-\vec{\beta}))$$

$$= (\hat{n}-\vec{\beta})(\dot{\beta}\sin\theta\cos\phi) - \dot{\vec{\beta}}(1-\beta\cos\theta)$$

$$|A|^2 = (\dot{\beta}\sin\theta\cos\phi)^2(\hat{n}\cdot\hat{n} + \vec{\beta}\cdot\vec{\beta} - 2\hat{n}\cdot\vec{\beta})$$

$$\cdot 2(1-\beta\cos\theta)^2$$

$$+\beta^{-}(1-\beta\cos\theta)$$

$$=(\dot{\beta}\sin\theta\cos\phi)^2(\cos^2\phi\sin^2\theta+\sin^2\phi\sin^2($$

$$+\dot{\beta}^2(1-\beta\cos\theta)^2$$

$$+(1-\beta\cos\theta)(\hat{n}\cdot\vec{\dot{\beta}})(\beta\sin\theta\cos\phi)$$

$$-\vec{\beta}\cdot\vec{\dot{\beta}}(1-\beta\cos\theta)(\sin\theta\cos\phi)$$

$$=(\dot{\beta}\sin\theta\cos\theta)^2(\cos^2\phi\sin^2\theta$$

$$+\cos^2\theta)+\dot{\beta}^2(1-\beta\cos\theta)^2$$

$$+(1-\beta\cos\theta)(\dot{\beta}\cos\phi\sin\theta)(\sin\theta($$

$$=\dot{\beta}^2[(1-\beta\cos\theta)^2-(1-\beta^2)\sin^2\theta($$

$$\frac{dP}{d\Omega} = \frac{e^2}{16\pi^2 c}\frac{1}{(1-\beta\cos\theta)^5}|A|^2$$

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ii]

$$P = \frac{-e^2}{6\pi c}\left(\frac{d\omega}{d\tau}\right)^2 = \frac{-e^2}{6\pi c}\gamma^6\left[(\vec{\beta}\times\dot{\vec{\beta}})^2 -\right.$$

$$\left. = \frac{-e^2\gamma^6}{6\pi c}\left[\sin\left(\frac{\theta}{2}\right) - 1\right]^2\right.$$

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iii] when $\beta \to 1$, $v \to c$

$$\left(\frac{d\omega}{}\right)^2 = \gamma^6\left[(\vec{R}\times\dot{\vec{\beta}})^2 - \dot{\vec{\beta}}^2\right]$$