Physics 412-2 (Spring 2020) **Problem Set 4** Professor Gerald Gabrielse
Posted: 2 Feb. 2020 Due: 9 AM on Monday 10 Feb. 2020

The problem sets are a serious part of the learning experience in this class. The problem sets will sometimes deliberately range away from what can be covered in the lectures in some cases. The goal to expose you to basic concepts and important examples of quantum mechanics. Problems will often be divided into many small parts to guide your through a solution. Corrections to the assignments, if needed, will be posted to the class web page. Your solutions should be placed in the box outside my office.

1. **Hydrogen selection rules (with nuclear spin):**

   (a) Including nuclear spin, list all the coupled $n = 1$ states of hydrogen formed by first coupling orbital angular momentum to electron spin, and then the resulting states to nuclear spin.

   (b) Including nuclear spin, list all the coupled $n = 3$ states of hydrogen formed by first coupling orbital angular momentum to electron spin, and then the resulting states to nuclear spin.

   (c) Use the electric dipole spherical tensor components Q_{kq} to deduce the electric dipole selection rules determined by rotational symmetry.

   (d) What additional selection rules, if any, come from parity?

   (e) Use the magnetic dipole spherical tensor components M_{kq} to deduce the magnetic dipole selection rules determined by rotational symmetry.

   (f) What additional selection rules, if any, come from parity?

2. **Dark states:** In our ACME electron electric dipole moment (edm) measurement we shine a laser on a $j = 1$, $m = 1$ state and $j = 1$, $m = -1$ state, which are nearly degenerate states. The laser resonantly couples both of these states to a $j = 0$ state, and any population transferred to this state then decays to states other than those mentioned. The transition amplitude for coupling from the $j = 1$, m states to the $j = 0$ state goes as the matrix element of the operator $\hat{\epsilon} \cdot \mathbf{r}$ where $\hat{\epsilon}$ is the fixed linear polarization direction of the laser light, and $\mathbf{r}$ is the position operator that causes transitions.

   (a) What is $\hat{\epsilon} \cdot \mathbf{r}$ in terms of the spherical tensor components of the polarization?

   (b) Before we turn on the laser, the $j = 1$, $m = 1$ state and the $j = 1$, $m = -1$ states are equally and randomly populated by letting a higher state decay into these states. What is the wavefunction describing this initial state, if you can write on down? What is the relative phase of the two states in the superposition?

   (c) When the laser is turned on, some population will be excited and then lost from the system, as described above. However, find a superposition state of the $j = 1$, $m = 1$ state and the $j = 1$, $m = -1$ that will not couple to the resonant $j = 0$ state.

   (d) This state is called a "dark state." Why? What is the relative phase difference of the two states in the superposition?

(e) Suppose that the $j = 1$, $m = 1$ state and the $j = 1$, $m = -1$ states superimposed to make the dark state differ in energy by a tiny amount ΔE that is much too small to resolve directly. After a time evolution for a coherence time T, what is the relative phase difference of the two states in the superposition.

3. **Helium atom (first pass)**

(a) What is the electronic wavefunction for an electron in the helium atom assuming it does not interact at all with the second electron?

(b) Assuming still that the two electrons are not interacting, what is the direct product wave function for the two electrons? What is the binding energy for the two electrons?

(c) What does zero energy mean?