Quantum Mechanics 412-1 Discussion

Tuesday, 8 October 2019

1. **Ehrenfest's theorem.**
   Derive Ehrenfest's theorem, which states that the time dependence of the expectation value of an operator A that does not explicitly depend on time is given by:
   $$\frac{d\langle A\rangle}{dt} = \frac{1}{i\hbar}\langle [A, H]\rangle \tag{1}$$

2. **Fourier decompositions.**
   The function $f(x)$, defined piecewise as:
   $$f(x) = \begin{cases} \left(1 - \frac{x}{L}\right) & 0 \leq x \leq L \\ 0 & otherwise \end{cases}$$
   has a Fourier series decomposition,
   $$f(x) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right) \tag{2}$$
   Show that the coefficients of the Fourier series are given by $c_n = \frac{2}{n\pi}$.

3. **Particle in a changing box.**
   (from last week) A particle is in the ground state of a box (infinite potential well) of length L. Suddenly the box expands (symmetrically) to twice its size, leaving the wavefunction undisturbed. Show that the probability of finding the particle in the ground state of the new box is $(8/3\pi)^2$.

4. **Time dependence of probability density.**
   (from last week) Find the 3-vector $\vec{j}$ such that $\frac{\partial|\psi|^2}{\partial t} = -\vec{\nabla}\cdot\vec{j}$. What is the physical interpretation of $\vec{j}$? Calculate $\vec{j}$ for the wavefunction $\psi(\vec{r}) = Ae^{i\vec{p}\cdot\vec{r}/\hbar} + Be^{-i\vec{p}\cdot\vec{r}/\hbar}$

5. **Incompatible observables & degeneracy.**
   (from last week) Two observables J and Q do not commute, but both commute with a system's Hamiltonian, H. Show that the energy eigenstates of the system are, in general, degenerate. Come up with a physical example of such a system and two such observables J and Q.