HW 1

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1.) a.)

$$E_1 = E_0 e^{i(k_1 x - \omega_1 t)} \hat{z} \qquad k_i = \frac{\omega_i}{c}$$

$$E_2 = E_0 e^{i(-k_2 x - \omega_2 t)} \hat{z}$$

$$I(x,t) = \frac{c}{8\pi} \vec{E} \cdot \vec{E}^* \qquad E = E_1 + E_2$$

$$E \cdot E^* = (E_1 + E_2) \cdot (E_1^* + E_2^*)$$

$$= E_1 \cdot E_1^* + E_1 \cdot E_2^* + E_2 \cdot E_1^* + E_2 \cdot E_2^*$$

$$= E_0^2 \left[e^{i(k_1 x - \omega_1 t)} \cdot e^{-i(k_1 x - \omega_1 t)} \right]$$

$$+ E_0^2 \left[e^{i(k_1 x - \omega_1 t)} \cdot e^{-i(-k_2 x - \omega_2 t)} \right]$$

$$+ E_0^2 \left[e^{i(-k_2 x - \omega_2 t)} \cdot e^{-i(k_1 x - \omega_1 t)} \right]$$

$$+ E_0^2 \left[e^{i(-k_2 x - \omega_2 t)} \cdot e^{-i(-k_2 - \omega_2 t)} \right]$$

$$= E_0^2 \left\{ 1 + e^{i(k_1 x - \omega_1 t - k_2 x + \omega_2 t)} \right.$$

$$\left. + e^{i(-k_2 x - \omega_2 t - k_1 x + \omega_1 t)} + 1 \right\}$$

$$= E_0^2 \left\{ 2 + e^{i[x(k_1 - k_2) - t(\omega_1 - \omega_2)]} \right.$$

$$= E_0 \{ 2 + e$$

$$+ e^{i[x(-k_1 - k_2) + t(\omega_1 - \omega_2)]} \}$$

b.) $\Delta\omega = \omega_1 - \omega_2 \qquad \omega_1 = ck + \frac{\Delta\omega}{2}$

$$\omega_2 = ck - \frac{\Delta\omega}{2}$$

$$E_0^2 \{ 2 + e^{i[x(\frac{\omega_1}{c} - \frac{\omega_2}{c}) - t(\omega_1 - \omega_2)]}$$

$$+ e^{i[x(-\frac{\omega_1}{c} - \frac{\omega_2}{c}) + t(\omega_1 - \omega_2)]} \}$$

$$= E_0^2 \{ 2 + e^{i[cx\Delta\omega - t\Delta\omega]}$$

$$+ e^{i[x(-\frac{\omega_1 - \omega_2}{c}) + t\Delta\omega]} \}$$

c.) $v = c$

2.a)

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} + \dots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \dots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \dots\right)$$

$$= \cos\theta + i\sin\theta$$

b.) $a = a_0 e^{-i\omega t} \qquad b = b_0 e^{-i\omega t}$

$$a = a_0\cos(\omega t) - i a_0 \sin(\omega t) = Re(a) + Im(a)$$

$$b = b_0\cos(\omega t) - i b_0 \sin(\omega t) = Re(b) + Im(b)$$

$$a \cdot b^* = a_0 b_0 \cos^2(\omega t) + a_0 b_0 \sin^2(\omega t)$$
$$+ i a_0 b_0 \cos(\omega t) - i a_0 b_0 \cos(\omega t)\sin(\omega t)$$
$$= a_0 b_0$$

$$Re(a) \cdot Re(b) = a_0 b_0 \cos^2(\omega t)$$

$$\langle \cos^2(\omega t) \rangle = \frac{1}{2}$$

$$\langle Re(a) \cdot Re(b) \rangle = \frac{1}{2} a_0 b_0 = \frac{1}{2} Re[a \cdot b^*]$$

3.) a.) $A = L^2$ path $= 4L$ $\omega = \frac{v}{\lambda}$

time for one circuit: $t_\pm = \dfrac{4L \pm L\Omega t_\pm}{v}$

$$t_\pm \left(\frac{v \pm \frac{L}{2}\Omega}{v} \right) = \frac{4L}{v}$$

$$t_\pm = \frac{4L}{v \mp \frac{L}{2}\Omega}$$

$$\delta t = t_+ - t_- = L^2 \Omega$$

$$\delta t = t_+ - t_- = \frac{2L\Omega}{V^2 - \left(\frac{L}{2}\Omega\right)^2}$$

$$\Delta\phi = \frac{v\,\delta t}{\lambda} \qquad V^2 \gg \left(\frac{L\Omega}{2}\right)^2$$

$$= \frac{4\vec{A}\cdot\vec{\Omega}}{\lambda v} = \frac{4\vec{A}\cdot\vec{\Omega}\,\omega}{v^2}$$

b.) No, because of the fact that $v^2 \gg \sim\Omega^2$, the rotation arm cancels out.

c.) $r = 5\text{cm} = 5\times10^{-2}\,\text{m}$

$L = 1\text{km} = 10^3\,\text{m}$

$\Omega = 7\times10^{-5}\,\frac{\text{rad}}{\text{s}}$

$\lambda = 500\,\text{nm} = 500\times10^{-9}\,\text{m}$

$A_0 = \pi r^2 = 0.16\,\text{m}^2$

$C_0 = 2\pi r = 0.31\,\text{m}$

$\frac{L}{C} = \frac{1000\,\text{m}}{0.31\,\text{m}} = 3225.81$ loops total area $A_T = \text{loops} \times A_0 = (3225.81)(0.16\,m^2)$

$= 516.13\ m^2$

$$\Delta\phi = \frac{4(516.13\,m^2)(7\times10^{-5}\,\frac{rad}{s})}{(500\times10^{-9}\,m)(3\times10^{8}\,\frac{m}{s})}$$

$= 9.6\times10^{-4}\ rad = 0.06\ deg.$