HW 2.3

Sunday, January 24, 2021 8:17 PM

Worked with Nia Burnell

$$r_0 = \frac{e^2}{6\pi m c^2 \varepsilon_0} \qquad |\vec{m}| = \frac{e\hbar}{2mc}$$

$$N = \varepsilon_0 \int d^3r \left[\vec{r} \times (\vec{E} \times \vec{B})\right]$$

$\underline{r > r_0}$: $\vec{E} = -\frac{e\vec{r}}{4\pi\varepsilon_0 r^3}$ $\quad \vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right]$

$$N = \varepsilon_0 \int d^3r \left\{\vec{r} \times \left[\frac{-e\vec{r}}{4\pi\varepsilon_0 r^3} \times \frac{\mu_0}{4\pi}\left(\frac{3(\vec{m}\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right)\right]\right\}$$

$$= \frac{-\cancel{\varepsilon_0} e \mu_0}{16\pi^2 \cancel{\varepsilon_0}} \int d^3r \left\{\vec{r} \times \left[\frac{\vec{r}}{r^3} \times \left(\frac{3(\vec{m}\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right)\right]\right\}$$

$$= \frac{-e\mu_0}{16\pi^2} \int d^3r \left\{\vec{r} \times \left[\frac{3(\vec{m}\vec{r})}{r^8}(\vec{r} \times \vec{r}) - \frac{\vec{r} \times \vec{m}}{r^6}\right]\right\}$$

$$= -\frac{e\mu_0}{16\pi^2}\int d^3r \left\{ \frac{3(\vec{m}\,\vec{r})}{r^8}\cancel{(\vec{r}\times\vec{r}\times\vec{r})} - \frac{\vec{r}}{r^6}\times(\vec{r}\times\vec{m}) \right\}$$

$$0$$

$$= -\frac{e\mu_0}{16\pi^2}\int \frac{d^3r}{r^6}\left[\vec{r}(\vec{r}\cdot\vec{m}) - \vec{m}(\vec{r}\cdot\vec{r})\right]$$

$$= -\frac{e\mu_0}{16\pi^2}\int \frac{d^3r}{r^6}\left[r\hat{r}\left(r\hat{r}\cdot\frac{e\hbar}{2mc}\left\{\cos(\theta)\hat{r} - \sin(\theta)\hat{\theta}\right\}\right) - r^2\frac{e\hbar}{2mc}\left(\cos(\theta)\hat{r} - \sin(\theta)\hat{\theta}\right)\right]$$

$$= -\frac{e^2\mu_0\hbar}{32\pi^2 mc}\int \frac{d^3r}{r^6}\left[\cancel{r^2\cos(\theta)\hat{r}} - \cancel{r^2\cos(\theta)\hat{r}} - r^2\sin(\theta)\hat{\theta}\right]$$

$$= +\frac{e^2\mu_0\hbar}{32\pi^2 mc}\int \frac{d^3r}{r^4}\sin(\theta)\hat{\theta}$$

$$= \frac{e^2 \mu_0 \hbar}{32\pi^2 mc} \int_0^{2\pi} \int_0^{\pi} \int_{r_0}^{\infty} \frac{dr}{r^4} r^2 \sin(\theta)\, \hat{\theta}\, d\theta\, d\phi$$

$$= \frac{e^2 \mu_0 \hbar}{32\pi^2 mc} \left\{ \left(\frac{-1}{r} \Big|_{r_0}^{\infty} \right) (2)(2\pi) \right.$$

$$= \frac{e^2 \mu_0 \hbar}{8\pi mc r_0}$$