Physics 412-3 Spring 2020

Problem Set 4

Due by 5pm Friday May 29.

1) (5 pts) Consider a particle of charge q in a background magnetic field. Evaluate the commutator for the vector components $[\Pi_i, \Pi_j]$ of the mechanical momentum $\vec{\Pi} = \vec{p} - q\vec{A}$. Is your result gauge-invariant? Why or why not?

2) (5 pts) Consider the Landau levels we discussed in class in the symmetric gauge where $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}$ for a uniform magnetic field $\vec{B} = B\hat{z}$. Assume the motion of the electrons is confined in the x-y plane in a square slab of material with side-length L. For the lowest landau level, show that the wave functions look like concentric rings. Show that they are eigenstates of angular momentum. What is determining the radius of the ring? Show that the degeneracy is the same as that derived in the other gauge we used in class where the vector potential was in the y-direction only.

3) (5 pts) In class we derived the expression for the propagator $K(x, t; x', t_0) = \Sigma_n < x|\phi_n >< \phi_n|x' > exp\left[\frac{-i}{\hbar} E_n(t - t_0)\right]$ for position space. Derive the analogous expression for the propagator $K(p, t; p', t_0)$ in momentum space.

4) (5 pts) Evaluate $< \mathrm{x}, \Delta\mathrm{t} \mid \mathrm{x} = 0, \mathrm{t} = 0 >$ for a 1-dimensional simple harmonic oscillator using the Feynman path integral approach in the limit that Δt is small. Keeping terms to order Δt^2 show that it agrees with the position-space propagator shown above in Problem 3 in the limit that (t-t_0) goes to zero.