Physics 411 Problem Set 5

Due at the beginning of class, 10 am, Monday, November 11th, 2019

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1. Consider a particle of mass $m = 1$ moving in the x–y plane. The potential energy is of the form $V = f(x - ay - bt)$ for some function f, where a and b are constant. Determine two independent conserved quantities. Your answers should be expressed in terms of the particle's position and speeds $(x, \dot{x}, y, \dot{y})$.

2. (a) You are sitting on a merry-go-round that goes round at angular speed Ω about the z-axis. A small mass on the floor experiences the harmonic potential $V = 0.5k((x - x_0)^2 + y^2)$, where x and y are measured in your (rotating) frame, and x_0 is constant. You can think of this as a mass attached to the floor by a spring. Assume that Ω is much slower than the natural frequency of the spring and neglect motions in the z dimension. Write down the Lagrangian for the mass in the rotating frame; give the equations of motion; and solve for the linear oscillations (eigenvalues and eigenvectors, always working in the rotating frame). Describe in words how the oscillations proceed when the mass is initially oscillating along the x-axis; be specific (e.g. about clockwise vs. counter-clockwise).

   (b) How do the results change for a Foucault pendulum (i.e., when the merry-go round is Earth, with you standing at latitude θ swinging a pendulum)? You should only need a line or two of math for this part of the problem, and a sentence or two of explanation.

3. You are sitting in a spaceship that is on a circular orbit about the Sun, orbiting at angular speed Ω. You put a small mass on the frictionless floor of the ship. Give the linearized equations of motion for the mass about its equilibrium position, and solve them (as before, give the eigenvalues and eigenvectors, always working in the rotating frame). Describe your result, being specific; also relate your result to PS2, question 4.