6. Homework Assignment - 414-1 Electrodynamics

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February 16, 2021

Exercise 1 (2 pts)

Derive the equation of motion of $g(\lambda)$ and $r(\lambda)$ for the following action:

$$S = \int d\lambda \left[-g(\lambda)^{-1} \frac{\mathrm{d}r}{\mathrm{d}\lambda} \cdot \frac{\mathrm{d}r}{\mathrm{d}\lambda} - \left(\frac{1}{2}mc\right)^2 g(\lambda) \right]$$

Exercise 2 (2 pts)

Derive the equation of motion through the Euler-Lagrange equations of the following EM Lagrange density:

$$\mathcal{L}(A^\mu, \partial^\mu A^\nu) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{c} J^\mu A_\mu$$

Exercise 3 (3 pts)

Consider the following Lagrange density:

$$\mathcal{L}(\phi, \phi^\star) = \left|\partial_\mu \phi\right|^2 - m^2 |\phi|^2 \ ,$$

where $\phi = \frac{1}{\sqrt{2}} (a(x) + ib(x))$ is a complex scalar. i) Derive the two equations of motion. ii) $\mathcal{L}$ is invariant under $\phi \longrightarrow e^{i\alpha}\phi$. Construct the Noether current corresponding to this symmetry!

Exercise 4 (3 pts)

Consider the following Lagrange density:

$$\mathcal{L}(A^\mu, \partial^\mu A^\nu) = F^\star_{\mu\nu} F^{\mu\nu}$$

Explain why we didn't consider a term proportional to this when discussing the equation of motion for EM fields.

Exercise 5 (4 pts)

In the reference frame a static uniform E_0 field is parallel to the x=axis and a static uniform magnetic field $B_0 = 2E_0$ lies in the x-y plane, making an angle θ with the x-axis. i) Determine the velocity of a reference frame in which the electric and magnetic fields are parallel. What are the fields in this frame for ii) $\theta \ll 1$ and iii) $\theta \longrightarrow \pi/2$?