FORTRAN 90

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Interpolation

Interpolation is important concept in numerical analysis. Quite often functions may not be available explicitly but only the values of the function at a set of points, called nodes, tabular points or pivotal points. Then finding the value of the function at any non-tabular point, is called interpolation.

❖ Lagrange's Interpolation Formula

Linear interpolation uses a line segment that passes through two points(x_o,y_o) and (x_1,y_1), then Lagrange's linear interpolation equation will be

$$y = y_0 \frac{(x - x_1)}{(x_0 - x_1)} + y_1 \frac{(x - x_0)}{(x_1 - x_0)}$$

The Lagrange quadratic interpolating polynomial through the three points (x_o,y_o) , (x_1,y_1) and(x_2,y_2) is:

$$P(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

The generalization of this equation, is the cons traction of polynomial $p_n(x)$ of degree at most n that passes through the n+ 1 points (x_o,y_o) , (x_1,y_1) ,(x_2,y_2) ,…… ,(x_n,y_n) has the form:

$$P_n(x) = \sum_{k=0}^{n} \left[\left(\prod_{\substack{j=0 \\ j \neq k}}^{n} (x - x_j) \right) / \left(\prod_{\substack{j=0 \\ j \neq k}}^{n} (x_k - x_j) \right) y_k \right]$$

Advantage of Lagrange interpolation is that the method does not need evenly spaced values in *x*. However, it is usually preferable to search for the nearest value in the table and then use the lowest-order interpolation consistent with the functional form of the data.

The Lagrange polynomial can be used for both unequally spaced data and equally spaced data.

Example 1

A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a $15'' \times 10''$ rectangular plate. The centers of the holes in the plate describe the path the arm needs to take, and the hole centers are located on a Cartesian coordinate system (with the origin at the bottom left corner of the plate) given by the specifications in Table 1.

Table 1 *The coordinates of the holes on the plate.*

x **(in.)**	y **(in.)**
2.00	7.2
4.25	7.1
5.25	6.0
7.81	5.0
9.20	3.5
10.60	5.0

If the laser is traversing from $x = 2$ to $x = 4.25$ in a linear path, what is the value of y at $x = 4.00$ using the Lagrange method and a first order polynomial?

Solution

For first order Lagrange polynomial interpolation (also called linear interpolation), we want to find the value of y at $x = 4.00$, using the two points $x_0 = 2.00$ and $x_1 = 4.25$, then

$$x_0 = 2.00,\ y(x_0) = 7.2 \qquad x_1 = 4.25,\ y(x_1) = 7.1$$

$$y(x) = \frac{x - x_1}{x_0 - x_1} y(x_0) + \frac{x - x_0}{x_1 - x_0} y(x_1)$$

$$= \frac{x - 4.25}{2.00 - 4.25}(7.2) + \frac{x - 2.00}{4.25 - 2.00}(7.1), \quad 2.00 \le x \le 4.25$$

$$y(4.00) = \frac{4.00 - 4.25}{2.00 - 4.25}(7.2) + \frac{4.00 - 2.00}{4.25 - 2.00}(7.1)$$

$$= 0.11111(7.2) + 0.88889(7.1)$$

$$= 7.1111 \text{ in.}$$

Ex :

Write a fortran 90 program to find the interpolated value for (x=3.5), using lagrangian polynomial, from the following data.

X	3.2	2.7	1	4.8
F(x)	22	17.8	14.2	38.3

```
program Iterpolation_by_lagrange
implicit none
real,dimension(4)::x,y
integer::n ,i,j
real::xp ,sum,p
data x /3.2,2.7,1,4.8/
data y/22,17.8,14.2,38.3/
n=4
xp=3.5
do i=1,n        ;        p=1
do j=1,n
if (i.eq.j) cycle
p=p*(xp-x(j))/(x(i)-x(j))        ;        enddo
sum=sum+p*y(i)        ;        enddo
write(*,1)"value of f(x) for point",xp,"=",sum
1format(2x,a,f5.3,a,1x,f10.5)
end
```