OPTIMITZACIO

Fall 2023

Exercises: Line search and trust region methods

Due: **18.10.2023 , 23:59h**, in the virtual campus

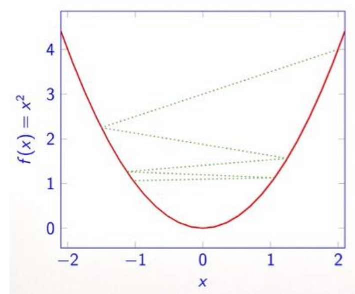
Exercise 2.1: Consider the function $f(x) = x^2$ on $[-2,2]$. Consider the one-dimensional gradient descent method starting at $\mathrm{x}_0 = 2$ in the direction

$$p_k = -sign(\mathrm{x}_k)$$

with step

$$\alpha_k = 2 + 3(2^{-k-1}).$$

1) Verify that p_k is indeed a descent direction, that is, $f(\mathrm{x}_{k+1}) < f(\mathrm{x}_k)$.
2) Perform 5 steps of the descent algorithm.
3) Does this descent converge? (*Hint: see picture on the right.*) Justify your argument. What Wolfe conditions are violated?



Exercise 2.2: Consider the minimization problem

$$m_k(p_k) = f_k + G_k \cdot p_k + \frac{1}{2} p_k^T \cdot B_k \cdot p_k \to min \qquad \text{subject to} \qquad \|p_k\| < \delta,$$

(see the lecture about trust regions). Let $p_k^C = \tau_k\, p_k^\ell$ be the Cauchy point, where

$$p_k^\ell = \arg \min_{\substack{p \in \mathbb{R}^n, \\ \|p\| < \delta}} (f_k + G_k \cdot p), \qquad \tau_k = \arg \min_{\substack{\tau \in \mathbb{R} \\ \|\tau\, p_k^\ell\| < \delta}} m_k(\tau\, p_k^\ell).$$

Show that:

1) $p_k^\ell = -\frac{\delta}{\|G_k\|} G_k$
2) $\tau_k = \begin{cases} 1, & \text{if} \quad G_k \cdot B_k \cdot G_k^T \le 0 \\ \min\{1, \hat{\tau}_k\}, & \text{otherwise} \end{cases}$, where $\hat{\tau}_k = \frac{\|G_k\|^3}{\delta\ G_k \cdot B_k \cdot G_k^T}$

   (*Hint:* The minimization problem for τ_k is one-dimensional and is a minimization of a convex function)

Exercise 2.3: Implement 2 steps of Cauchy point search for the Rosenbrock function $f(x_1, x_2) = (1 - x_1)^2 + 5(x_2 - x_1^2)^2$ starting at $(-2, -2)$ and with the trust regions being balls of radius 0.5 for the both steps.