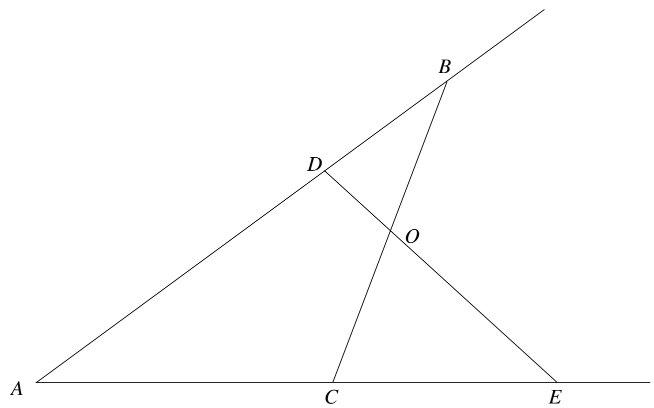
Exercise 1. (Smallest area problem)

Given an angle with vertex A and a point O in its interior. Pass a line BC through the point O that cuts off from the angle a triangle of minimal area



Hint: proof that for a triangle of minimal area the segments OB and OC should be equal.

Solution

Let us see that for a triangle of minimal area the segments OB and OC should be equal.

Suppose that the line BC is such that $OB = CO$, and consider another line - FG - going through O.

Overlap the triangles OFB and CGO, then we have

$$\widehat{COG} = \widehat{FOB}, \quad \widehat{OCG} = \widehat{CAB} + \widehat{ABC} > \widehat{ABC}$$

$$\triangle OFB \subset \triangle OGC \quad \Rightarrow \quad \text{Area}(OFB) < \text{Area}(OGC)$$

$$\text{Area}(AFG) = \text{Area}(AFOC) + \text{Area}(OGC)$$

$$\text{Area}(ABC) = \text{Area}(AFOC) + \text{Area}(OFB)$$

$$\text{Area}(ABC) < \text{Area}(AFG)$$

The case of line DE is considered similarly.