Exercise 2: Concrete Mixing

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Find the best possible approximation $x = (x_1; ...; x_m)$ of the ideal mixture, $c = (c_1; ...; c_n)$, by using the material from the m mines. Show that the optimal mixture will be the point x such that: $min(Cx - c)^T(Cx - c)$, s.t. $\sum_{j=1}^{m} x_j = 1$ and $x_j \geq 0$ where the matrix $C = (C_1; ...; C_m)$ has C_j as columns, and $c = (c_1, ..., c_n)^T$.

We will first interpret what $Cx - c$ represents.

$$\begin{pmatrix} c_1^1 & ... & c_1^m \\ ... & ... & ... \\ c_n^1 & ... & c_n^m \end{pmatrix} \begin{pmatrix} x_1 \\ ... \\ x_m \end{pmatrix} \tag{1}$$

This multiplication will result in a n-dimensional vector, which will work as a approximation of $c = (c_1, ..., c_n)^T$. In terms of data, we are multiplying each row of C (the fraction of a specific gravel size s_i in each one of the mines) by the vector x (which holds how much of each mine mixture is present in our approximation to the ideal mixture). The operation is similar to a weighted average for each gravel size. In the end, Cx will represent the fraction of each one of the gravel sizes in our final mixture.

Therefore, we would obtain the optimal mixture when Cx equals c. It is worth noting that:

$$(Cx - c)^T(Cx - c) = ||Cx - c||^2 \tag{2}$$

Minimizing $||Cx - c||^2$ is equivalent to minimizing $||Cx - c||$ as the norms are always positive, and therefore, we are applying the least squares solution to $Cx = c$. All in all, we can state that the approach to the Concrete Mixing problem given in the exercise is accurate.