OPTIMIZACIÓ

Fall 2023

Exercises: Introduction to optimization

Due: 24.09.2023 , 23:59h, in the virtual campus.

Exercise 1.1. (*The Fermat point of a set of points*) Given a set of points $\boldsymbol{y}_1$,...,$\boldsymbol{y}_m$ in the plane, find a point $\boldsymbol{x}^*$ whose sum of weighted distances to the given set of points is minimized. Mathematically, the problem is

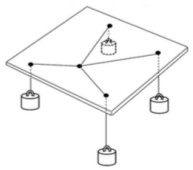
$$\min \sum_{i=1}^{m} w_i \|\boldsymbol{x}^* - \boldsymbol{y}_i\|, \quad \text{subject to } \boldsymbol{x}^* \in \mathbb{R}^2,$$

where $w_1, ..., w_m$ are given positive real numbers.

(i) Show that there exists a global minimum for this problem and find the point at which it is achieved.

(ii) Is the optimal solution always unique?

(iii) (*) Give a physical interpretation of the solution to the problem by means of the mechanical model shown in the following figure:



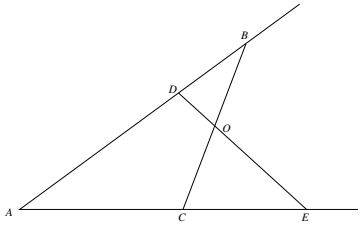
(iv) (*) Show that an optimal solution minimizes the potential energy of the mechanical model defined as

$$\sum_{i=1}^{m} w_i h_i,$$

where h_i is the height of the i-th weight measured from some reference level.

Exercises marked with (*) will give you an extra point

Exercise 1.2. (*Smallest area problem*) Given an angle with vertex A and a point O in its interior. Pass a line BC through the point O that cuts off from the angle a triangle of minimal area



Hint: Prove that for a triangle of minimal area the segments OB and OC should be equal.