X	Y	$\Phi_1(X,Y)$
1	1	0.7
1	2	0.1
2	1	0.4
2	2	0.1

Y	Z	$\Phi_2(Y,Z)$
1	1	0.2
1	2	0.8
1	3	0.5
2	1	0.0
2	2	0.9
2	3	0.3

Table 1: Two factors

X	Y	Z	$\Phi_1(X,Y,Z)$
1	1	1	14
1	1	2	60
1	2	1	40
1	2	2	27
1	3	1	42
1	3	2	85
2	1	1	4
2	1	2	59
2	2	1	54
2	2	2	3
2	3	1	96
2	3	2	30

Table 2: A large factor

1 Exercises

Exercise 1.1 *Factor product: Let X, Y, and Z be random variables with 2, 2 and 3 possible values, respectively. Consider the factors from Table 1. Obtain the table product of $\Phi_1 \times \Phi_2$.*

Exercise 1.2 *Factor reduction: Let X, Y, and Z be random variables with 2, 3 and 2 possible values, respectively. Consider the factor from Table 2. Obtain the reduced factor of Φ_1 given that $Y = 1$.*

Exercise 1.3 *Properties of independent variables: Assume that A and B are independent random variables. Which of the following options are always true? You may select 1 or more options, or none of them.*

a) $P(A,B) = P(A) \times P(B)$

b) $P(A,B) = P(A) + P(B)$

c) $P(A) + P(B) = 1$

d) $P(B|A) = P(B)$

Exercise 1.4 *Independencies in a graph. Which pairs of variables are independent in the graphical model of Figure 1, given that none of them have been observed? You may select 1 or more options, or none of them.*

a) $C \perp\!\!\!\perp D$

b) $A \perp\!\!\!\perp B$

c) $B \perp\!\!\!\perp E$

d) $A \perp\!\!\!\perp C$

Exercise 1.5 *Independencies in a graph. Now assume that the value of E is observed (A, B, C, and D are not). Which pairs of variables (not including E) are independent in the same graphical model of Figure 1, given E? You may select 1 or more options, or none of them.*

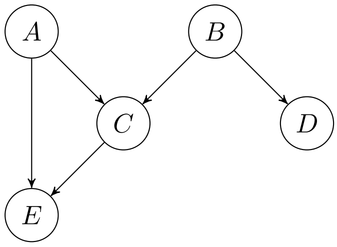


Figure 1: Bayesian network

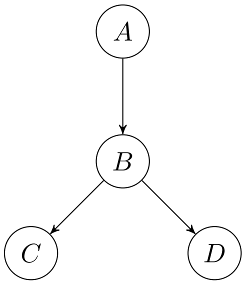


Figure 2: Bayesian network (2)

a) $A \perp\!\!\!\perp D|E$

b) $A \perp\!\!\!\perp C|E$

c) $B \perp\!\!\!\perp C|E$

d) $A \perp\!\!\!\perp B|E$

e) $D \perp\!\!\!\perp C|E$

f) $B \perp\!\!\!\perp D|E$

Exercise 1.6 *Factorization. Given the same model of Figure 1, which of these is an appropriate decomposition of the joint distribution?*

a) $P(A, B, C, D, E) = P(A)P(B)P(A, B|C)P(B|D)P(A, C|E)$

b) $P(A, B, C, D, E) = P(A)P(B)P(C|A)P(C|B)P(D|B)P(E|A)P(E|C)$

c) $P(A, B, C, D, E) = P(A)P(B)P(C|A, B)P(D|B)P(E|A, C)$

d) $P(A, B, C, D, E) = P(A)P(B)P(C)P(D)P(E)$

Exercise 1.7 *Independent parameters. How many independent parameters are required to uniquely define the CPD of E (the conditional probability distribution associated with the variable E) in the same graphical model of Figure 1, if A, B, and D are binary, and C and E have three values each?*

a) 8

b) 11

c) 17

d) 3

e) 18

f) 12

g) 6

Exercise 1.8 *Independencies in a graph. Which of these conditional independence statements are fulfilled in the graphical model of Figure 2? You may select 1 or more options, or none of them.*

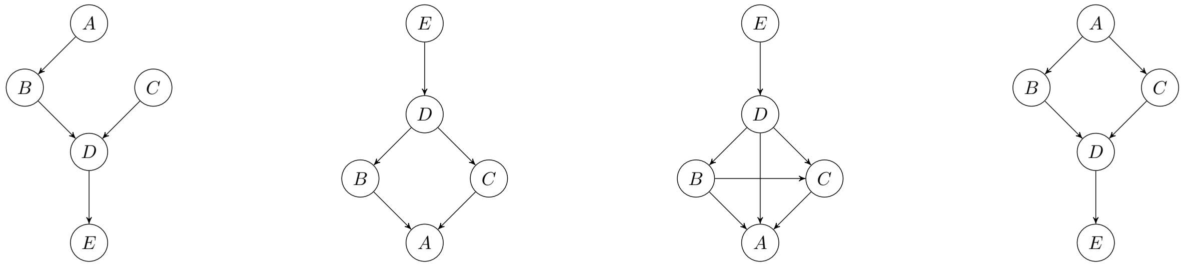
a) $A \perp\!\!\!\perp B|C, D$

b) $C \perp\!\!\!\perp D|A$

c) $A \perp\!\!\!\perp D|C$

d) $A \perp\!\!\!\perp D|B$

Exercise 1.9 *I-maps. Suppose that $(A \perp\!\!\!\perp B) \in \mathcal{I}(P)$, and G is an I-map of P, where G is a Bayesian network and P is a probability distribution. Is it necessarily true that $(A \perp\!\!\!\perp B) \in \mathcal{I}(G)$?*



(a) BN model a (b) BN model b (c) BN model c (d) G

Figure 3: Different BN models (3) and G

- *Yes*
- *No*

Exercise 1.10 *I-maps. Which of the following statements about I?-maps are true? You may select 1 or more options, or none of them.*

a) *The graph K that is the same as the graph G, except that all of the edges are oriented in the opposite direction as the corresponding edges in G, is always an I-map for G, regardless of the structure of G.*

b) *A graph K is an I-map for a graph G if and only if K encodes all of the independences that G has, and more.*

c) *An I-map is a function that maps a graph G to itself, i.e., $f(G) = G$.*

d) *A graph K is an I-map for a graph G if and only if all of the independencies encoded by K are also encoded by G.*

Exercise 1.11 *I-equivalence. Let Bayesian network's structure G be a simple directed chair $X_1 \rightarrow X_2 \rightarrow \cdots \rightarrow X_n$ for some number n. How many Bayesian networks are I-equivalent to including itself?*

a) $2n - 1$

b) n

c) 2^{n-1}

d) $n - 1$

Exercise 1.12 *I-Maps. Graph G (Fig. 3d) is a perfect I-map for distribution P, i.e., $\mathcal{I}(G) = \mathcal{I}(P)$. Which of the other graphs in Figure 3 is a perfect I-map for P? You may select 1 or more options, or none of them.*

a) *Fig. 3a*

b) *Fig. 3b*

c) *Fig. 3c*

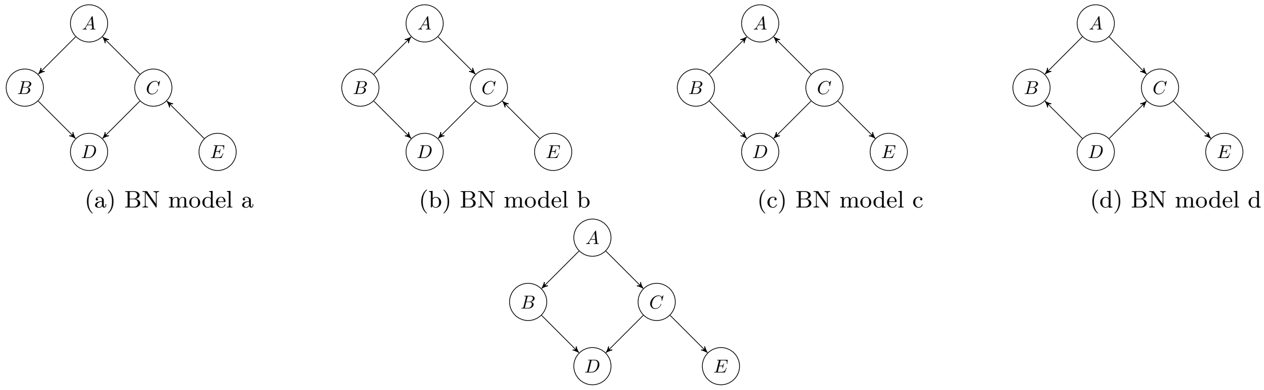
Exercise 1.13 *I-Equivalence. In Figure 4, graph G (Fig. 4e) is I-equivalent to which other graph(s)? You may select 1 or more options, or none of them.*

a) *Fig. 4a*

b) *Fig. 4b*

c) *Fig. 4c*

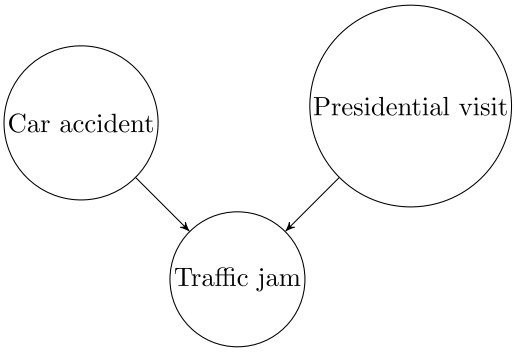
d) *Fig. 4d*



(a) BN model a (b) BN model b (c) BN model c (d) BN model d

(e) G

Figure 4: Different BN models (4) and G



P(PV = True) = 0.01
P(CA = True) = 0.1
P(TJ = True | PV = False, CA = False) = 0.1 P(TJ = True | PV = False, CA = True) = 0.5 P(TJ = True | PV = True, CA = False) = 0.6 P(TJ = True | PV = True, CA = True) = 0.9

Figure 5: Toy Bayesian network for traffic jams

Exercise 1.14 *Inter-causal reasoning. Consider the model of Figure 5 for traffic jams in a small town, which we assume can be caused by a car accident, or by a visit from the president (and the accompanying security motorcade).*

- *Calculate* $P(CA = True|TJ = True)$
- *Calculate* $P(CA = True|TJ = True, PV = True)$

  where TJ, CA and PV stand for Traffic Jam, Car Accident and Presidential Visit respectively.

Exercise 1.15 *Consider the Naive Bayes model of Figure 6 for flu diagnosis. Which of the following statements are true in this model? You may select 1 or more options, or none of them.*

a) Say we observe that 1000 people have a headache (and possibly other symptoms), out of which 500 people have the flu (and possibly other symptoms), and 500 people have a fever (and possibly other symptoms). We would expect that approximately 250 people with a headache also have both the flu and a fever.

b) Say we observe that 500 people have a headache (and possibly other symptoms) and 500 people have a fever (and possibly other symptoms). We would expect that approximately 250 people have both a headache and fever.

c) Say we observe that 1000 people have the flu, out of which 500 people have a headache (and possibly other symptoms) and 500 have a fever (and possibly other symptoms). We would expect that approximately 250 people with the flu also have both a headache and fever.

d) Say we observe that 1000 people have the flu, out of which 500 people have a headache (and possibly other symptoms) and 500 have a fever (and possibly other symptoms). We can conclude that exactly 250 people with the flu also have both a headache and fever.

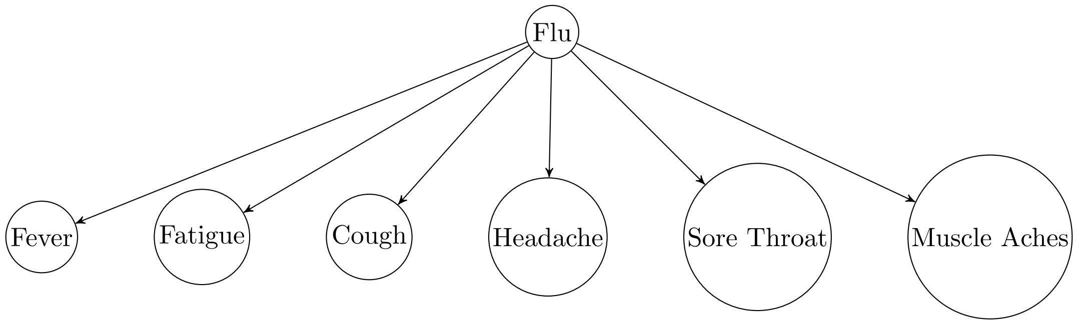


Figure 6: Toy Bayesian network for flu diagnosis

X	Y	Z	$\Phi_p(X,Y,Z)$
1	1	1	0.14
1	1	2	0.56
1	1	3	0.35
1	2	1	0.00
1	2	2	0.09
1	2	3	0.03
2	1	1	0.08
2	1	2	0.32
2	1	3	0.20
2	2	1	0.00
2	2	2	0.09
2	2	3	0.03

Table 3: Product factor

Answers

Ex. 1.1: See Table 3.

Ex. 1.2: See Table 4

Ex. 1.3: a,d

Ex. 1.4: b

Ex. 1.5: None

Ex. 1.6: c

Ex. 1.7: f

Ex. 1.8: d

Ex. 1.9: No

Ex. 1.10: d

Ex. 1.11: b

Ex. 1.12: None

Ex. 1.13: a

Ex. 1.14: $P(CA = True|TJ = True) = 0.35$
$P(CA = True|TJ = True, PV = True) = 0.14$

Ex. 1.15: c

X	Z	$\Phi_r(X,Z)$
1	1	14
1	2	60
2	1	4
2	2	59

Table 4: Reduced factor