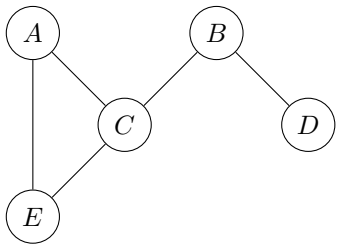


Figure 1: Markov network 1

# 1 Exercises

**Exercise 1.1** *Factor Scope. Let $\phi(c, e)$ be a factor in a graphical model, where $c$ is a value of $C$ and $e$ is a value of $E$. Which is the scope of $\phi$?*

*a)* $C, E$

*b)* $A, C, E$

*c)* $A, B, C, E$

*d)* $C$

**Exercise 1.2** *Independence in Markov Networks. Consider the graphical model in Figure 1. Which pairs of variables are independent in this network? You may select 1 or more options, or none of them.*

*a)* $A, C$

*b)* $D, E$

*c)* $A, D$

**Exercise 1.3** *Consider the following set of factors: $\boldsymbol{\phi} = \{\phi_1(A, B), \phi_2(B, C, D), \phi_3(D), \phi_4(C, E, F)\}$. Now, consider a Markov Network $M$ such that $P_\phi$ factorizes over $M$. Which independence statements hold in the network? You may select 1 or more options, or none of them.*

*a)* $A \perp\!\!\!\perp E|B$

*b)* $C \perp\!\!\!\perp E|B$

*c)* $A \perp\!\!\!\perp F|C$

*d)* $B \perp\!\!\!\perp E|C$

*e)* $B \perp\!\!\!\perp E|A$

*f)* $C \perp\!\!\!\perp D|A$

**Exercise 1.4** *Partition Function. Which of the following is a use of the partition function? You may select 1 or more options, or none of them.*

*a) The partition function is the probability of each variable in the graph taking on a specific value.*

*b) One can divide factor products by the partition function in order to convert them into probabilities.*

*c) The partition function is useless and should be ignored*

*d) The partition function is used only in the context of Bayesian networks, not Markov networks.*

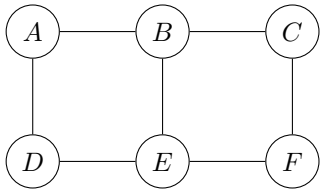


Figure 2: Markov network 2

**Exercise 1.5** *Factorization. Which of the following sets of factors could factorize over the undirected graph of Figure 2? You may select 1 or more options, or none of them.*

*a)* $\phi(A), \phi(B), \phi(C), \phi(D), \phi(E), \phi(F)$

*b)* $\phi(A, B, D), \phi(A, B), \phi(C, D, E), \phi(E, F), \phi(F)$

*c)* $\phi(A, B, D, C), \phi(C, D, E, F)$

*d)* $\phi(A, B), \phi(C, D), \phi(E, F)$

**Exercise 1.6** *Factorization. Which of the following is a valid distribution over the graph of Figure 2? You may select 1 or more options, or none of them.*

*a)* $\phi(A, B, C, D, E, F)$

*b)* $\phi(A, B, D, E) \cdot \phi_2(B, C, E, F)$

*c) There exists no distribution for this graph*

*d)* $\frac{\phi_1(A) \cdot \phi_2(B) \cdot \phi_3(C) \cdot \phi_4(D) \cdot \phi_5(E) \cdot \phi_6(F)}{Z}$*, where* $Z$ *is the partition function*

*e)* $\phi_1(A, B, D) \cdot \phi_2(C, E, F)$

**Exercise 1.7** *Factors in Markov Network. Let* $\phi_1(A, B)$*,* $\phi_2(B, C)$*, and* $\phi_3(A, C)$ *be all of the factors in a particular undirected graphical model. Then what is* $\sum_{A,B,C} \phi_1(A, B) \times \phi_2(B, C) \times \phi_3(A, C)$ *? You may select 1 or more options, or none of them.*

*a) Always less than or equal to* $\phi_1(a, b) \times \phi_2(b, c) \times \phi_3(a, c)$*, where* $a$ *is a value of* $A$*,* $b$ *is a value of* $B$*, and* $c$ *is a value of* $C$*.*

*b) Always greater than or equal to* $\phi_1(a, b) \times \phi_2(b, c) \times \phi_3(a, c)$*, where* $a$ *is a value of* $A$*,* $b$ *is a value of* $B$*, and* $c$ *is a value of* $C$*.*

*c) Always greater than or equal to 0*

*d) Always greater than or equal to 1*

*e) Always equal to the partition function,* $Z$

*f) Always equal to 1*

**Exercise 1.8** *Factorization of probability distributions. Consider a directed graph* $G$*. We construct a new graph* $G'$ *by removing one edge from* $G$*. Which of the following is always true? You may select 1 or more options, or none of them.*

*a) Any probability distribution* $P$ *that factorizes over* $G$ *also factorizes over* $G'$ *.*

*b) Any probability distribution* $P$ *that factorizes over* $G'$ *also factorizes over* $G$ *.*

*c) No probability distribution* $P$ *that factorizes over* $G$ *factorizes over* $G'$*.*

*d) If* $G$ *and* $G'$ *were undirected graphs, the answers to the other options would not change.*

## Answers

***Ex. 1.1***: a

***Ex. 1.2***: None

***Ex. 1.3***: a,c,d

***Ex. 1.4***: b

***Ex. 1.5***: a

***Ex. 1.6***: d

***Ex. 1.7***: b,c,e

***Ex. 1.8***: b,d