Master's Degree on Foundations of Data Science

Probabilistic Graphical Models (PGM)

May 15th, 2020

Universitat de Barcelona

Full name: _____

- Include the reasoning steps followed to reach to the result.
- Each question scores up to 1 point. The final score is the weighted sum according to:

	Q#1	Q#2	Q#3	Q#4	Q#5	Q#6	Q#7	Q#8	Q#9	Q#10	Q#11	Q#12	Q#13	**Total**
Value	$0.0\dot{6}$	$0.0\dot{6}$	$0.0\dot{6}$	$0.0\dot{6}$	0.10	0.10	$0.0\dot{6}$	0.10	$0.0\dot{6}$	$0.0\dot{6}$	$0.0\dot{6}$	$0.0\dot{6}$	0.10	(to 10)
Score														

Question #1: Basic probability.

Calculate the probability $p(A = +|B = -)$ given the following marginal and conditional probability distributions,

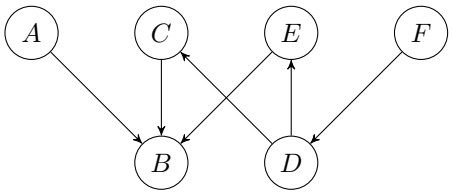
A	$p(A)$
-	0.85
+	0.15

B	A	$p(B\|A)$
-	-	0.40
+	-	0.60
-	+	0.75
+	+	0.25

(____/2)

Question #2: Independencies in Bayesian networks.

Which of the following conditional independence statements are true? (**Multiple choice**)



[ ] $A \perp\!\!\!\perp C|B$

[ ] $E \perp\!\!\!\perp F|D$

[ ] $B \perp\!\!\!\perp F|E$

[ ] $A \perp\!\!\!\perp F|B$

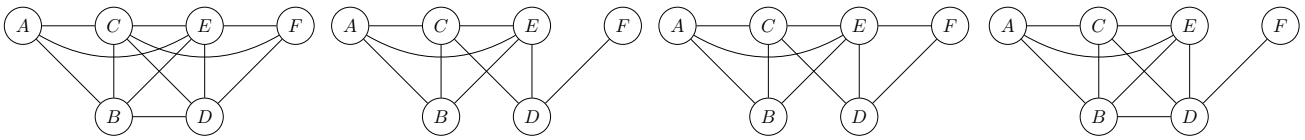
[ ] $A \perp\!\!\!\perp D|C$

[ ] $F \perp\!\!\!\perp B|D$

(____/6)

Question #3: Variable Elimination.

Considering the previous structure G, which of the following structures is the induced graph resulting from following the ordering (A, F, C, D, B, E)? (**Single choice**)

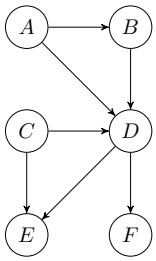


[ ] G_1 [ ] G_2 [ ] G_3 [ ] G_4

(____/1)

Question #4: Factorization in Bayesian networks (I-map).

Consider the following structure G and the set of probability distributions, P, which factorize as displayed. Which distribution P is guaranteed to factorize according to G $[I(G) \subseteq I(P)]$? (**Multiple choice**)



[ ] $P(A, B, C, D, E, F) = P(A)P(B)P(C)P(D)P(E)P(F)$

[ ] $P(A, B, C, D, E, F) = P(A)P(B|A)P(C)P(D|C, B, A)P(E|C, D)P(F|D)$

[ ] $P(A, B, C, D, E, F) = P(A)P(B|A)P(C)P(D|C, B, A)P(E|C, D, F)P(F|D)$

[ ] $P(A, B, C, D, E, F) = P(A, B, D)P(C)P(E|C, D)P(F|D)$

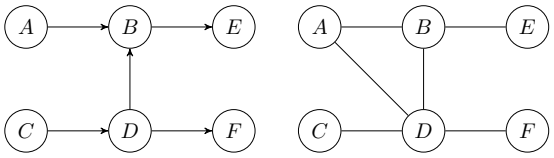
[ ] $P(A, B, C, D, E, F) = P(A)P(B|A)P(D|A, B)P(C|A, B, D)P(E|A, B, C, D)P(F|A, B, C, D, E)$

[ ] $P(A, B, C, D, E, F) = P(A, B)P(D|C)P(C)P(E|C, D)P(F|D)$

(____/4)

Question #5: Independencies in Markov networks.

Identify the independence statement satisfied by the Bayesian network on the left which is not satisfied by the Markov network on the right. (**Single choice**)

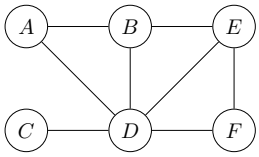


[ ] $A \not\perp\!\!\!\perp D|E$
[ ] $A \perp\!\!\!\perp C$
[ ] $A \perp\!\!\!\perp F|D$
[ ] $C \not\perp\!\!\!\perp A|B$

(____/1)

Question #6: Variable elimination.

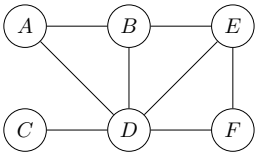
Consider the following graph, H, and the elimination order C, E, D, A, B, F. Which is the size (in terms of no. variables) of the largest intermediate factor found.



(____/1)

Question #7: Factorization in Markov networks.

Consider the following graph, H, and the set of probability distributions, P, which factorize as displayed. Which distribution P is guaranteed to factorize according to H? (**Multiple choice**)

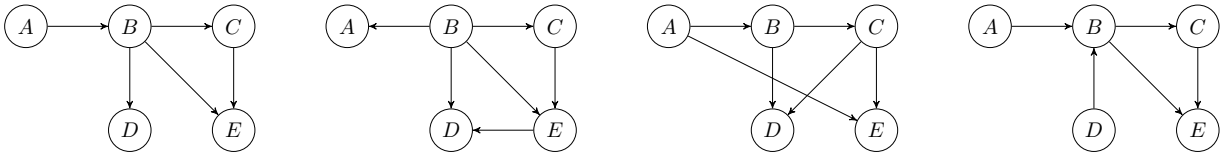


[ ] $P(A,B,C,D,E,F) \propto \phi_1(A,B) \times \phi_2(C,D) \times \phi_3(E,F)$
[ ] $P(A,B,C,D,E,F) \propto \phi_1(A,B,D,E) \times \phi_2(C,D) \times \phi_3(D,F)$
[ ] $P(A,B,C,D,E,F) \propto \phi_1(A,B,D) \times \phi_2(C,D) \times \phi_3(B,D,E,F)$
[ ] $P(A,B,C,D,E,F) \propto \phi_1(A,B,D) \times \phi_2(C,D) \times \phi_3(B,D,E) \times \phi_4(D,E,F)$
[ ] $P(A,B,C,D,E,F) \propto \phi_1(D,E,F) \times \phi_2(A,B) \times \phi_3(C,D)$

(____/4)

Question #8: Structural learning: Likelihood score.

Given these four Bayesian network structures, which of the following statements about the likelihood scores of the different graphs is true? (**Multiple choice**)



G_1 G_2 G_3 G_4

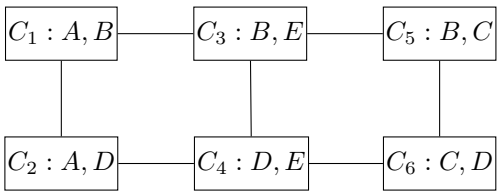
[ ] $Score_L(G_2 : D) \geq Score_L(G_1 : D)$ for every dataset D.
[ ] $Score_L(G_3 : D) \geq Score_L(G_2 : D)$ for every dataset D.
[ ] $Score_L(G_3 : D) \leq Score_L(G_1 : D)$ for every dataset D.
[ ] $Score_L(G_4 : D) \neq Score_L(G_1 : D)$ for every dataset D.

(____/4)

Question #9: Message Passing in a Cluster Graph.

Suppose we wish to perform inference in this Cluster Graph. Which expression correctly represents the message $\delta_{4\to3}$ that cluster C_4 sends to cluster C_3 in belief propagation? (**Single choice**)

Assume that variables in the sepsets = intersection of the variables in adjacent cliques.



[ ] $\delta_{4\to3}(E) = \sum_D \phi_4(D,E) \cdot \delta_{2\to4}(D) \cdot \delta_{6\to4}(D) \cdot \delta_{3\to4}(E)$

[ ] $\delta_{4\to3}(E) = \sum_D \phi_4(D,E) \cdot \delta_{2\to4}(D) \cdot \delta_{6\to4}(D)$

[ ] $\delta_{4\to3}(B,E) = \phi_4(D,E) \cdot \delta_{2\to4}(D) \cdot \delta_{6\to4}(D)$

[ ] $\delta_{4\to3}(E) = \sum_D \delta_{2\to4}(D) \cdot \delta_{6\to4}(D)$ **(____/1)**

Question #10: Parametric learning.

Estimate the distribution of $P(Cough|Fever = yes, Headache = no)$ from the following dataset with 30 instances.

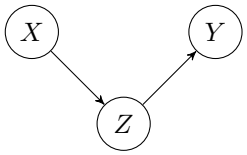
If we are trying to save space, which is the minimum number of parameters to encode CPD $P(Cough|Fever, Headache)$ estimated from this dataset?

Cough	*Fever*	*Headache*	*Cold*	*Flu*
no	yes	no	no	yes
dry	yes	yes	no	no
prod	no	yes	yes	no
dry	yes	yes	no	no
no	yes	no	yes	no
no	no	no	yes	no
prod	no	yes	yes	yes
no	yes	yes	no	yes
no	yes	no	no	yes
prod	yes	no	yes	yes
prod	no	no	no	yes
dry	no	yes	no	yes
dry	yes	no	no	yes
no	yes	yes	yes	yes
no	no	no	yes	no
prod	yes	yes	yes	no
no	no	yes	no	no
dry	yes	no	yes	yes
dry	yes	yes	yes	no
no	yes	yes	yes	yes
prod	no	yes	yes	yes
no	no	yes	no	yes
prod	no	yes	yes	no
dry	yes	no	yes	yes
no	yes	no	no	yes
dry	yes	yes	yes	no
prod	no	yes	yes	no
prod	yes	yes	no	no
no	yes	no	no	no
no	yes	yes	no	yes

(____/4)

Question #11: Computing Sufficient Statistics.

Given this BN and this dataset, which is the value of the sufficient statistic $\tilde{M}[x_0, y_1, z_0]$?



X	$P(X)$
x_0	0.7

Z	X	$P(Z\|X)$
z_0	x_0	0.6
z_0	x_1	0.25

Y	Z	$P(Y\|Z)$
y_0	z_0	0.45
y_0	z_1	0.3

dataset
$(x_1, ?, z_0)$
$(x_0, y_1, ?)$
(x_1, y_0, z_0)
$(x_1, y_1, ?)$
$(?, y_1, z_0)$
(x_0, y_1, z_0)
$(x_1, ?, z_0)$
$(x_0, y_1, ?)$
(x_1, y_0, z_1)
$(x_1, y_0, ?)$
$(?, y_0, z_0)$
(x_0, y_1, z_0)
$(x_0, ?, z_0)$
$(x_1, y_1, ?)$
(x_0, y_0, z_0)

(____/4)