Master on Foundations of Data Science

Recommender Systems

Collaborative Recommender Systems: Factorization Models

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Model-Based CF methods: Factorization Models

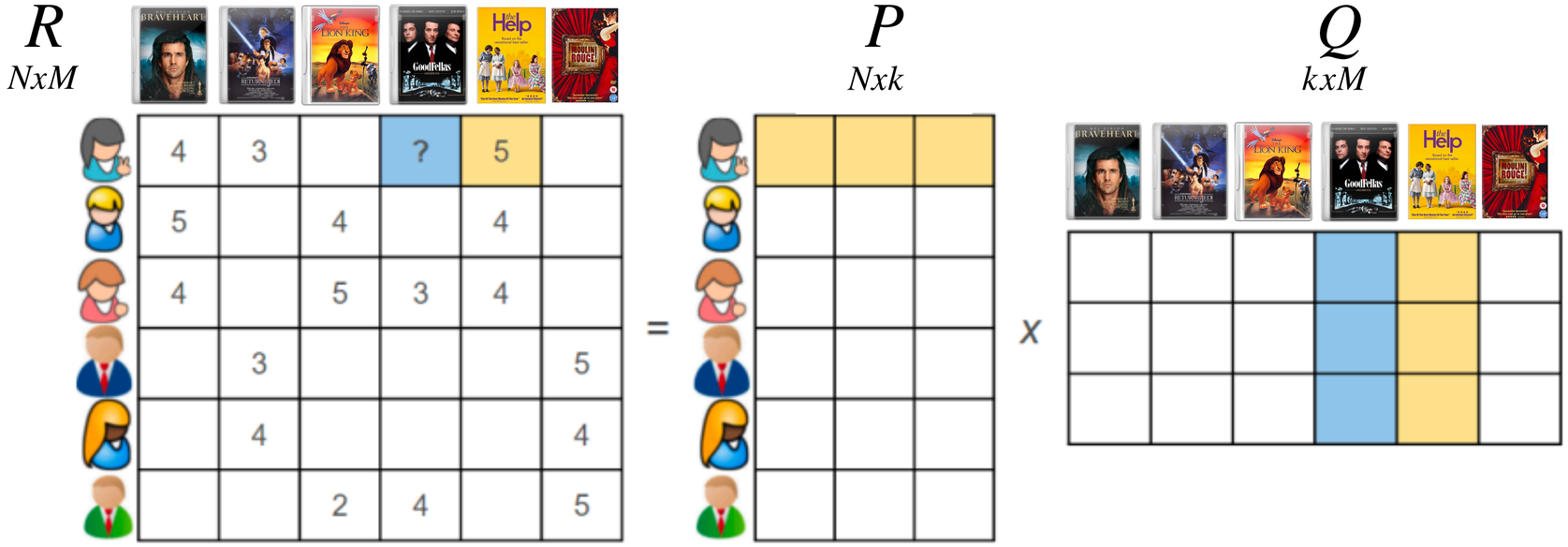
Model-Based methods

Although **Neighborhood-based** models are very popular because its **simplicity**, today are not necessarily the most accurate ones.

Latent Factor Models pose the recommendation problem as an **optimization problem**

- **GOAL:** Make good Recommendations
  - Quantify **goodness** using the **RMSE**.
  - **Lower the RMSE better** is our **recommender** system.
- **THE MOST COMMON BASELINE METHOD**

Factorization Models



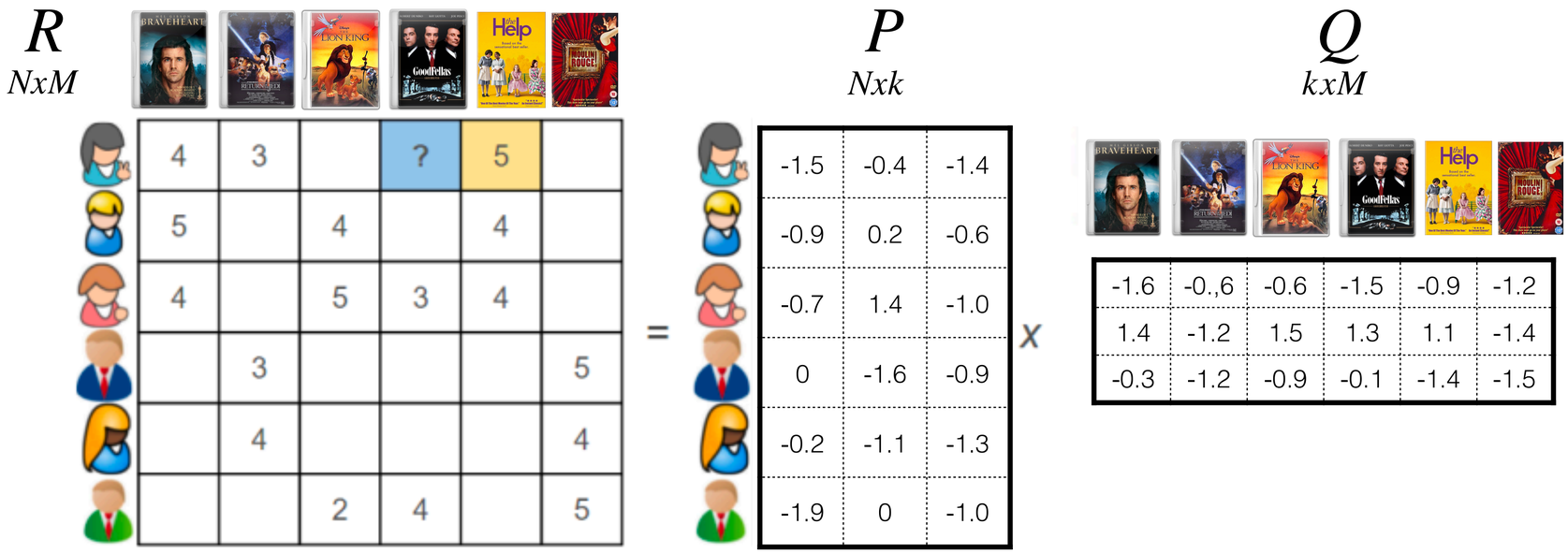
user latent vector

item latent vector

predicted rating

$$\hat{r}_{u,i} = P_u Q_i^T$$

Factorization Models



user latent vector

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$$\hat{r}_{ui} = P_u Q_i^T$$

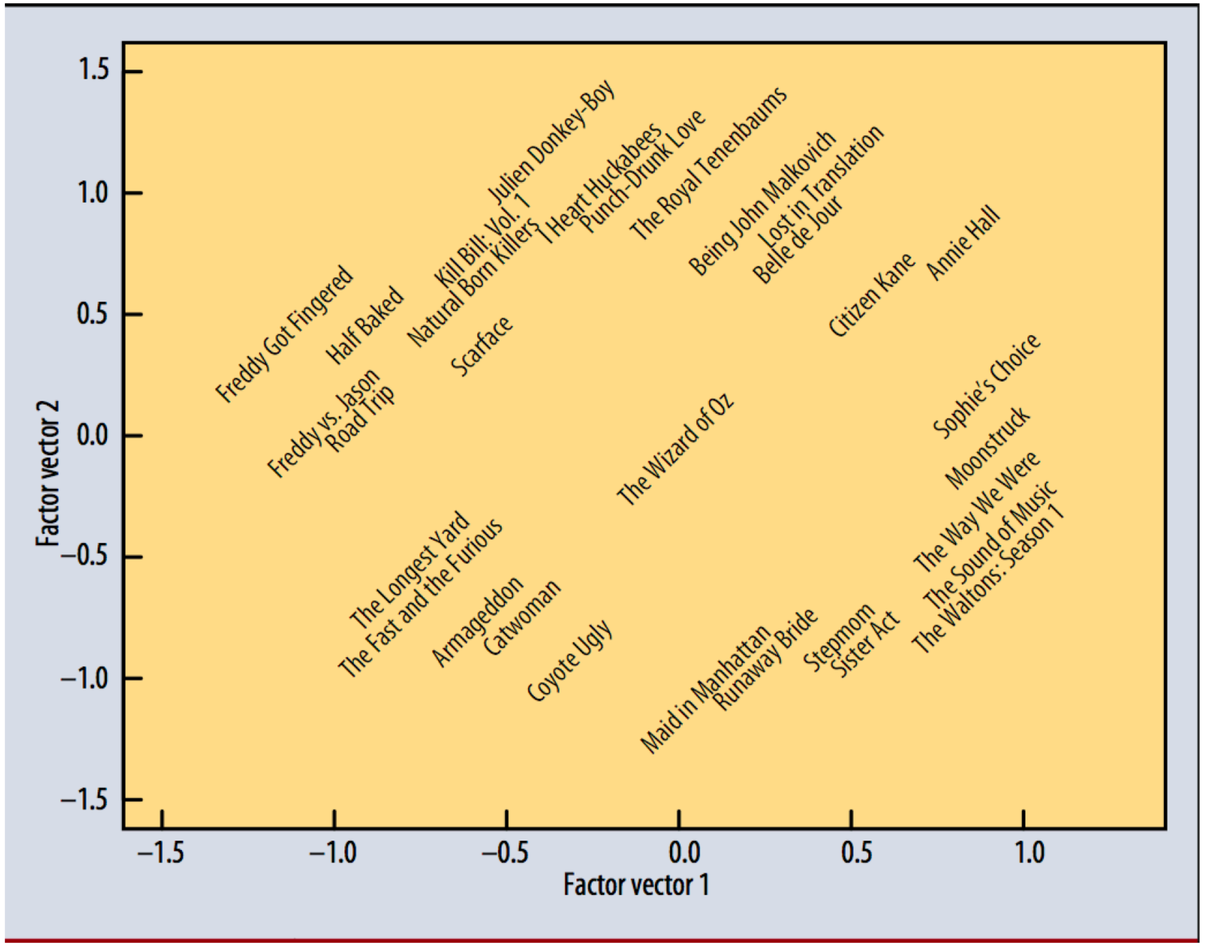
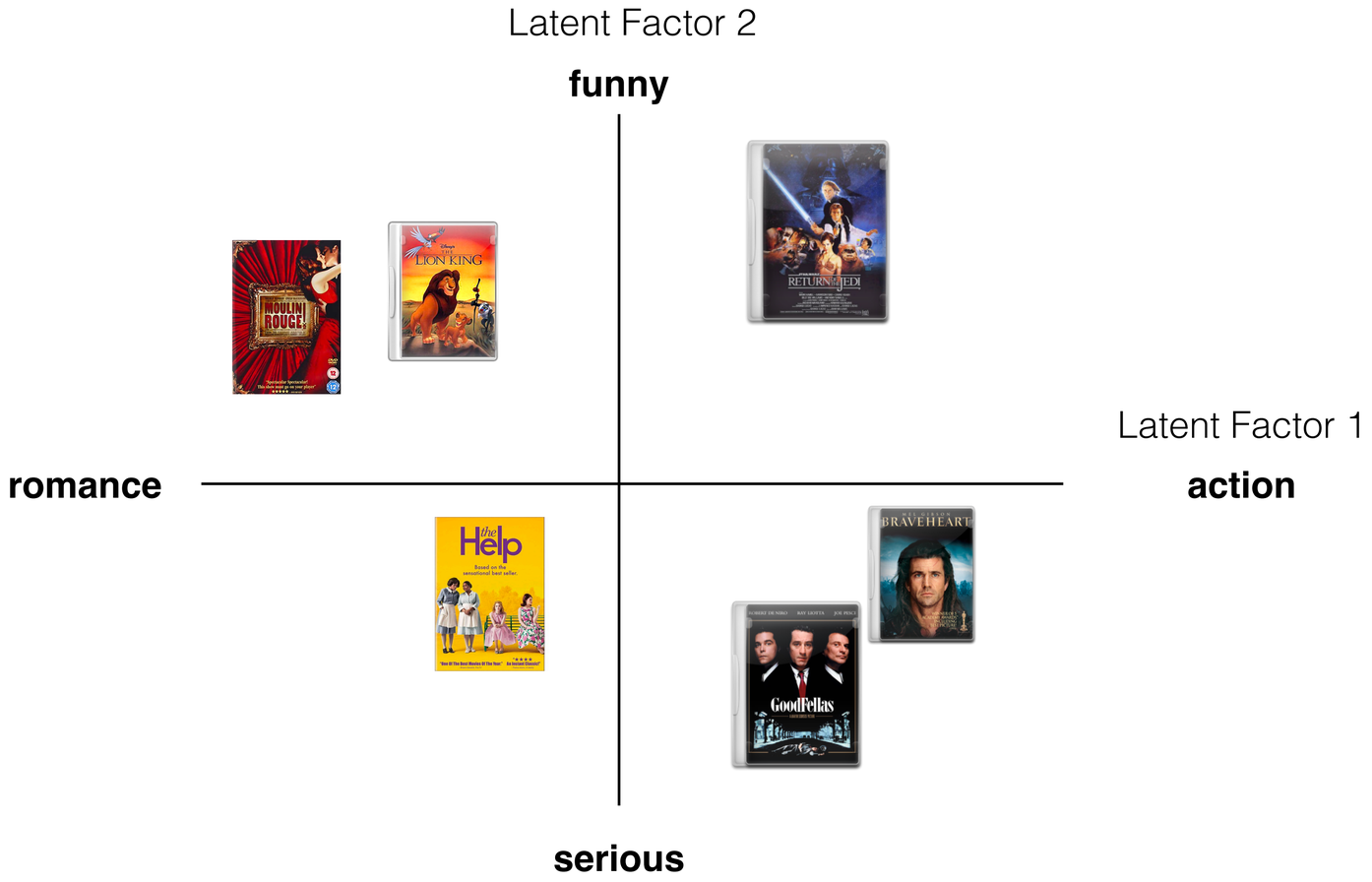


Figure 1: A mapping of movies based on two latent factors


Latent Factor 2
funny
Latent Factor 1
romance
action
serious
MOULIN ROUGE!
LION KING
RETURN OF THE JEDI
the Help
Based on the sensational best seller.
BRAVEHEART
GoodFellas

Factorization Models

- Latent factor models approach tries to explain the ratings by characterizing both items and users to a small number of factors inferred from the rating patterns.

- **PROBLEM**: Hard to optimize due to the huge amount of missing values

Factorization Models

- Several variants of matrix factorization:
  - Singular Value Decomposition (SVD)
  - Vanilla Matrix Factorization
  - Matrix Factorization with biases
  - Non-Negative Matrix Factorization
  - SVD++
  - …

Singular Values Decomposition (SVD)

- Singular Value Decomposition (SVD) is a well established technique for identifying latent semantic factors. Done by factorizing the user-item rating matrix.
- Columns of U and V are constrained to be mutually orthogonal.
- Mutual orthogonality has the advantage that the concepts can be completely independent of one another. Can be interpreted in scatterplots

$$\underset{m \times n}{\hat{X}} \approx \underset{m \times r}{U} \; \underset{r \times r}{S} \; \underset{r \times n}{V^{\mathsf{T}}}$$

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & \\ \vdots & \vdots & \ddots & \\ x_{m1} & & & x_{mn} \end{pmatrix} \approx \begin{pmatrix} u_{11} & \cdots & u_{1r} \\ \vdots & \ddots & \\ u_{m1} & & u_{mr} \end{pmatrix} \begin{pmatrix} s_{11} & 0 & \cdots \\ 0 & \ddots & \\ \vdots & & s_{rr} \end{pmatrix} \begin{pmatrix} v_{11} & \cdots & v_{1n} \\ \vdots & \ddots & \\ v_{r1} & & v_{rn} \end{pmatrix}$$

PROBLEM: Hard to optimize due to the huge amount of missing values

Solution

- Fill missing values with 0
- Fill missing values with some statistics (e.g. user mean rating)
- Recursive methods

Example of Singular Value Decomposition

$$R_f = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & ? & -1 & -1 & -1 \\ ? & 1 & 1 & -1 & -1 & ? \\ 1 & -1 & -1 & 1 & 1 & 1 \\ 1 & ? & -1 & 1 & 1 & 1 \end{pmatrix}$$

The original Matrix

Example of Singular Value Decomposition

$$R_f = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -0.2 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 0.2 & -1 & 1 & 1 & 1 \end{pmatrix}$$

Step 1: Fill missing values with the mean value of the row

Example of Singular Value Decomposition

$$R_f = \begin{pmatrix} 1.0592 & -1.1604 & 0.9716 & -0.8515 & 0.8040 & -1.0592 \\ 0.6636 & 0.9039 & 0.5881 & -0.9242 & -1.1244 & -0.6636 \\ 0.4300 & 0.9623 & 0.3746 & -0.6891 & -1.1045 & -0.4300 \\ -0.9425 & -0.8181 & -0.8412 & 1.2010 & 1.1320 & 0.9425 \\ -1.0290 & -0.2095 & -0.9270 & 1.1475 & 0.5535 & 1.0290 \end{pmatrix}$$

Step 2: Apply SVD to the matrix

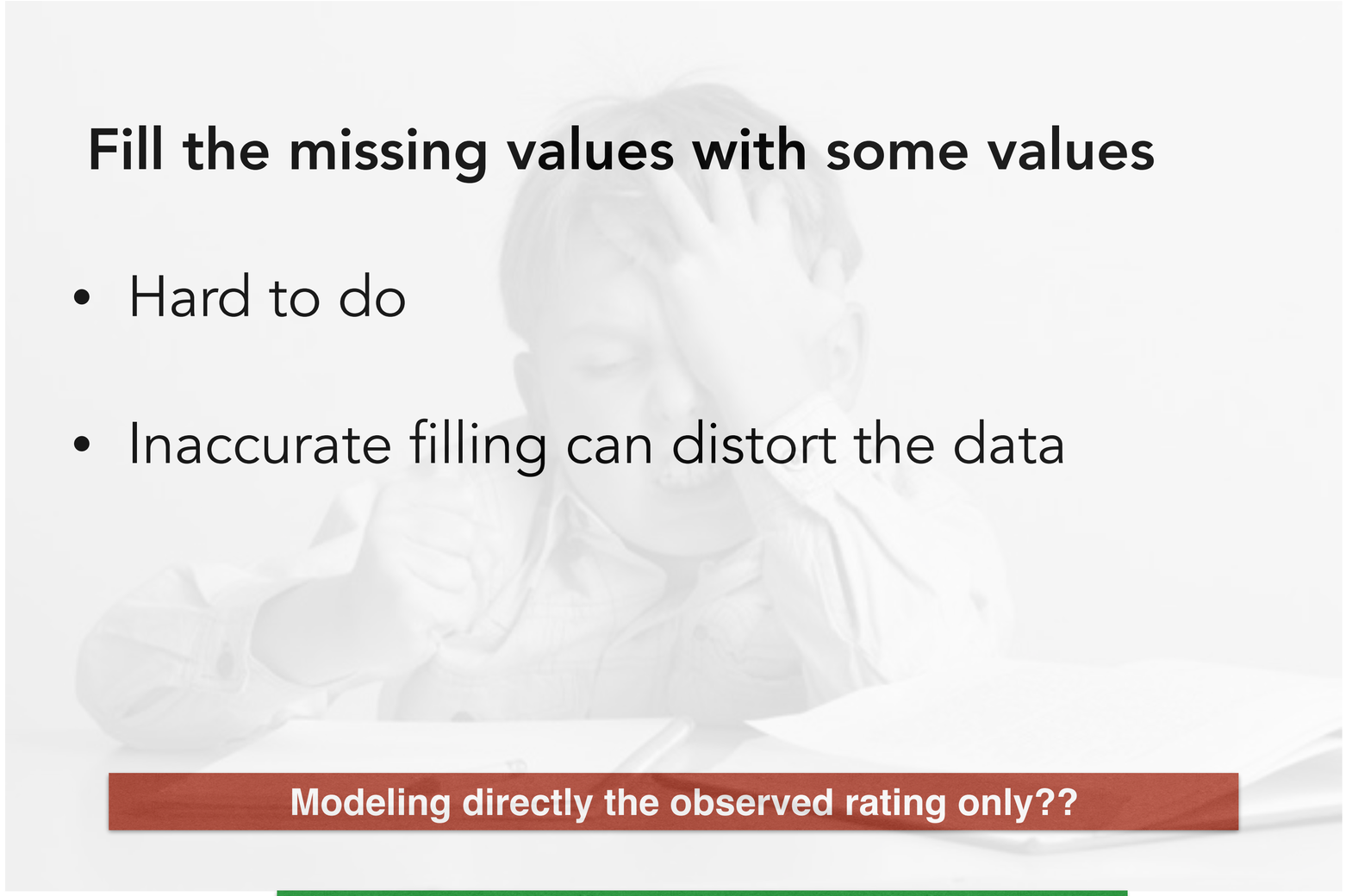
Example of Singular Value Decomposition

$$R_f = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 0.5881 & -1 & -1 & -1 \\ 0.4300 & 1 & 1 & -1 & -1 & -0.43 \\ 1 & -1 & -1 & 1 & 1 & 1 \\ 1 & -0.2095 & -1 & 1 & 1 & 1 \end{pmatrix}$$

Step 3. Modify the target matrix setting in the missing values the learn values and iterate until convergence

Movie Recommender System using SVD

Factorization Models



Fill the missing values with some values

- Hard to do
- Inaccurate filling can distort the data

Modeling directly the observed rating only??

Model-based methods

Modeling directly the observed rating only??

$$\min_{P \in \mathbb{R}^{m \times k}, Q \in \mathbb{R}^{n \times k}} \sum_{(i,j) \in obs} (r_{u,i} - p_u q_i^T)^2$$

Modeling directly the observed rating only?

- We can minimize the objective function with **Stochastic gradient descent:**
  - For each given training set, the system predicts $r_{u,i,}$ and computes the associated prediction error

$$e_{ui} \overset{def}{=} r_{ui} - q_i^T p_u.$$

  - Then it modifies the parameters by a magnitude proportional to ɣ in the opposite direction of the gradient, yielding:

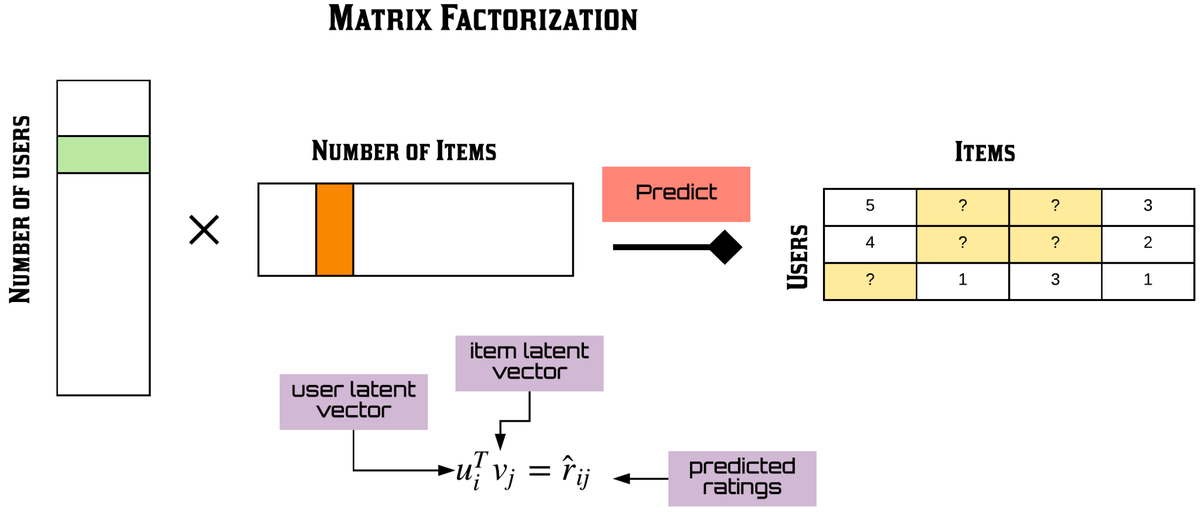
$$p_u = p_u - \gamma \frac{\partial f}{\partial p_u} = p_u + \gamma(e_{ui} q_i)$$

$$q_i = q_i - \gamma \frac{\partial f}{\partial q_i} = q_i + \gamma(e_{ui} p_u)$$

1) Vanilla Matrix Factorization SVD funk

A straightforward matrix factorization model maps both users and items to a joint latent factor space of dimensionality D — such that user-item interactions are modeled as inner products in that space.

- Each item i is associated with a vector q_i, and each user u is associated with a vector p_u
- For a given item i, q_i measures the interest that a user has for the latent factors.
- The resulting dot product ($q_i \cdot p_u$) captures the interaction between user u and item i, which is the user's overall interest in the item's characteristics.
    - $\hat{r}_{ij} = q_i \cdot p_u$



Movie Recommender System using Vanilla MF

2) Regularized Vanilla Matrix Factorization

$$\sum_{(i,j)\in obs} (r_{u,i} - q_i \cdot p_u)^2 + Regularize(q_i + p_i)$$

$$\sum_{(i,j)\in obs} (r_{u,i} - q_i \cdot p_u)^2 + \lambda(\|q_i\|^2 + \|p_i\|^2)$$

where λ controls the extent of regularization

- now, update rules are:

$$q_i \leftarrow q_i + \gamma \cdot (e_{ui} \cdot p_u - \lambda \cdot q_i)$$
$$p_u \leftarrow p_u + \gamma \cdot (e_{ui} \cdot q_i - \lambda \cdot p_u)$$

3) Regularized Matrix Factorization with biases

Adding **Bias**

$$P'_{ui} = b_u + b_i + (p_u^T q_i)$$

b_u is observed deviation of user u;
b_i is observed deviation of item i.

Improving regularized singular value decomposition for collaborative filtering A Paterek Proceedings of KDD Cup and Workshop 2007, 5-8	713	2007

4) SVD++

- An extension of previous method which also uses **implicit information**
  - **Explicit** (e.g. numerical rantings) + **Implicit** information (e.g. likes, purchases, skipped, bookmarked,…)

$$\hat{r}_{ui} = b_{ui} + q_i^T \left(p_u + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} y_j \right)$$

  - *N(u)* is the **set of items for which the user *u* has implicit information**
  - The user u is modeled as $p_u + |N(u)|^{-1/2} \sum_{j \in N(u)} y_j$
  - y_j is a **new item factor** term that captures the implicit ratings (e.g. if a user u has rated a item j, regardless the rating value)

Factorization meets the neighborhood: a multifaceted collaborative filtering model 2098 2008
Y Koren
Proceedings of the 14th ACM SIGKDD international conference on Knowledge …

Model-based methods

Movie Recommender System

5) Non-Negative Matrix Factorization

- Can be used for ratings matrices that are non negative

$$\begin{aligned} &\text{Minimize} && J = \frac{1}{2}\|R - UV^T\| \\ &\text{subject to:} && U \geq 0, V \geq 0 \end{aligned}$$

- The major advantage of this approach is not the accuracy, but that of the **high level of interpretability** it provides in understanding the user-item interactions.

6) Sparse Linear Models (SLIM)

- Computes the item-item relations, by estimating an **item x item** sparse aggregation coefficient **matrix** S.

- The recommendation score of an unrated item i for a user u is:

$$\hat{r}_{u,i} = r_u^T S_i$$

$$\begin{aligned} \underset{S}{\text{minimize}} \quad & \tfrac{1}{2}\textstyle\sum_{u,i}(r_{ui} - \hat{r}_{ui})^2 + \tfrac{\beta}{2}||S||_F^2 + \lambda||S||_1, \\ \text{subject to} \quad & S \geq 0, \text{and} \\ & \text{diag}(S) = 0. \end{aligned}$$

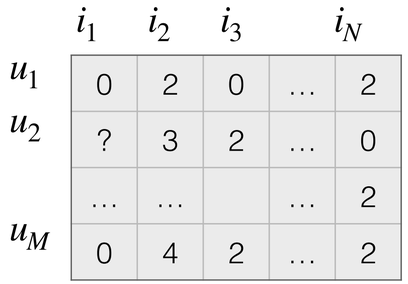
SLIM: Sparse linear methods for top-n recommender systems
X Ning, G Karypis
Data Mining (ICDM), 2011 IEEE 11th International Conference on, 497-506

115 2011

Model-based methods

6) Sparse Linear Models (SLIM)

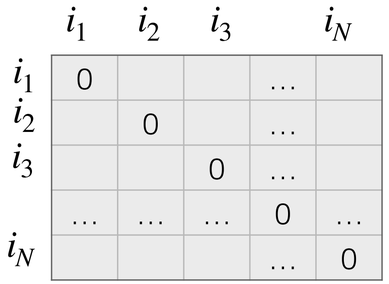
$\hat{R} =$



	i_1	i_2	i_3		i_N
u_1	0	2	0	…	2
u_2	?	3	2	…	0
	…	…		…	2
u_M	0	4	2	…	2

Matrix

$\times$



	i_1	i_2	i_3		i_N
i_1	0			…	
i_2		0		…	
i_3			0	…	
	…	…	…	0	…
i_N				…	0

Matrix

$$\hat{r}_{u,i} = r_u^T S_i$$

$$\begin{aligned} \underset{S}{\text{minimize}} \quad & \tfrac{1}{2}\textstyle\sum_{u,i}(r_{ui} - \hat{r}_{ui})^2 + \tfrac{\beta}{2}||S||_F^2 + \lambda||S||_1, \\ \text{subject to} \quad & S \geq 0, \text{and} \\ & \text{diag}(S) = 0. \end{aligned}$$

6) Sparse Linear Models (SLIM)

- Although the SLIM method proposes a prediction model for the rating, the final use of the ratings is for **ranking** the items
- Since, the weights are restricted to be positive, the impact of each weight on the score are **highly interpretable**
- Generally used for data sets with unary ratings (clicks, buy,..) from **implicit feedback**
  - **Outperforms MF**

Model-Based methods

- Advantages over neighborhood methods:
  - **Space-efficency.** Usually, the learned model is much smaller than the original rating matrix
  - **Training speed and prediction speed.** Neighborhood-based models takes quadratic complexity in either number of users or number of items
  - **Avoiding overfitting.** The summarized model use to help in avoiding overfitting. Regularization methods can be used to make these models robust

Minimizing the Objective Function

- Common algorithms to minimize the objective function include:
  - **Stochastic gradient descent (SGD)** is a generic method to minimize loss functions.
  - **Weighted Alternating Least Squares** (**WALS**) is specialized method to this particular objective.
    - WALS works by initializing the embeddings randomly, then alternating between:
      - Fixing P and solving for Q
      - Fixing Q and solving for P
    - This technique is guaranteed to converge because each step is guaranteed to decrease the loss.

Miniminzing the Objective Function

SGD and WALS have advantages and disadvantages

SGD

- **Very flexible—can use other loss functions.**
- **Can be parallelized.**
- **Slower—does not converge as quickly.**
- **Harder to handle the unobserved entries (need to use negative sampling or gravity).**

WALS

- **Reliant on Loss Squares only.**
- **Can be parallelized.**
- **Converges faster than SGD.**
- **Easier to handle unobserved entries.**