Adaptive Digital Notch Filter Design on the Unit Circle for the Removal of Powerline Noise from Biomedical Signals

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Abstract—**This paper investigates adaptive digital notch filters for the elimination of powerline noise from biomedical signals. Since the distribution of the frequency variation of the powerline noise may or may not be centered at 60 Hz, three different adaptive digital notch filters are considered. For the first case, an adaptive FIR second-order digital notch filter is designed to track the center frequency variation. For the second case, the zeroes of an adaptive IIR second-order digital notch filter are fixed on the unit circle and the poles are adapted to find an optimum bandwidth to eliminate the noise to a pre-defined attenuation level. In the third case, both the poles and zeroes of the adaptive IIR second-order filter are adapted to track the center frequency variation within an optimum bandwidth. The adaptive process is considerably simplified by designing the notch filters by pole-zero placement on the unit circle using some suggested rules. A constrained least mean-squared (CLMS) algorithm is used for the adaptive process. To evaluate their performance, the three adaptive notch filters are applied to a powerline noise sample and to a noisy EEG as an illustration of a biomedical signal.**

I. Introduction

THE CONVENIENT elimination of powerline noise from biomedical signals has been the focus of researchers for some time [1], [2], [6], [8]. While proper grounding and electrical shielding is paramount in analog recordings, the modern use of computers has suggested the investigation of digital approaches to solve the powerline interference problem. In early applications, powerline noise was removed by either a low-pass or band-stop notch filter with proper cutoff frequencies. While this approach may be acceptable for some applications, it is not a universal solution for electrophysiological monitoring. A low-pass filter is adequate only when the true signal has all frequency content below the cutoff frequency. This is usually not the case for general biomedical signal processing. When the frequency of the powerline is stable, a fixed notch filter is sufficient. However, the frequency of the powerline noise is not always stable at exactly 60 Hz. This frequency variation is often assumed to be Gaussian [15]. This suggests that the distribution may or may not be centered at 60 Hz and that the drifting of the frequency may or may not be skewed in one direction. A fixed notch filter may eliminate the noise when the distribution is centered exactly at the frequency for which the filter was designed. However, if the distribution is not centered at the frequency of the fixed notch filter, it may be ineffective.

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Based on incremental estimation, a local self-corrective method was proposed [13] for powerline noise elimination in biomedical signal processing. The method was simple, functional in real time, and applicable for microprocessors [1]. It was later pointed out that the method is actually equivalent to a fixed notch filter where only the bandwidth is being adapted [11].

In the following paper, adaptive second-order digital notch filters are designed by pole-zero placement on the unit circle. A constrained least mean squared (CLMS) algorithm is applied to the coefficients of the filter to track and reduce the noise. The CLMS algorithm operates to constrain the poles to be inside the unit circle and to impose a penalty function when the poles drift toward the origin of the unit circle. Three adaptive notch filters are designed and considered for cases where the noise is assumed to be centered at exactly 60 Hz and where the noise varies about 60 Hz.

When the noise is centered at 60 Hz, an adaptive IIR digital notch filter is designed to adapt the width of the filter to minimize the bandwidth of the notch. Such a filter is called a fixed-zero adaptive IIR digital notch filter. For the case where the frequency of the noise is varying around 60 Hz, two second-order adaptive notch filters can be designed. An adaptive FIR digital notch filter is designed to track the frequency variation with a fixed bandwidth. An adaptive IIR digital filter is designed to track the frequency variation with a bandwidth which is kept as narrow as possible. In this case, neither the poles nor the zeroes of the IIR filter are fixed.

While there is no concern for the stability of the adaptive FIR filter, the stability of the adaptive IIR filters has to be satisfied during each iteration of the adaptive process. The IIR filter is considered unstable when its poles are outside the unit circle [4]. To prevent instability in the filter, the reflection method is used for any poles found outside the unit circle with some suggested rules to eliminate the possibility of any memory lock-up [12].

In general, a second-order FIR digital notch filter has three coefficients and a second-order IIR digital notch filter has five coefficients that have to be adapted during the adaptive process [5]. However, the pole-zero placement method on the unit circle reduces the required coefficients of such classes of filters. Namely, the adaptive process requires that only one coefficient needs to be adapted for the adaptive FIR notch filter and the fixed-zero adaptive IIR filter, and two coefficients that need to be adapted for the non-fixed pole-zero adaptive IIR notch filter. This reduces the requirement for real-time calculations and memory storage during the adaptation process, which is a specific aim of this investigation.

II. Adaptive Digital Notch Filter

Adaptive digital filters are necessary when the characteristics of a signal are changing with time. Such filters have time variant coefficients which are updated continuously by an optimization criterion. The generalized adaptive filter process consists of a noisy input signal $x(n)$ and an output signal $y(n)$ which may be different from the desired signal $d(n)$ (Fig. 1(a)). The error $e(n)$ can be minimized by many different methods. The optimizing algorithm is generally applied to the following set of equations for the general digital filter

$$y(n) = \sum_{i=0}^{N} a_i x(n-i) - \sum_{j=1}^{M} b_j y(n-j) \tag{1a}$$

$$e(n) = d(n) - y(n). \tag{1b}$$

One earlier attempt at the adaptive IIR filter was the use of the recursive LMS algorithm [7]. Its updating equations are a straightforward extension to those derived for the adaptive FIR filter. The coefficients a_i, and b_j are updated as follows [12]

$$a_i(n+1) = a_i(n) + \mu e(n) x(n-i) \tag{2a}$$

$$b_j(n+1) = b_j(n) + \eta e(n) y(n-j). \tag{2b}$$

However, because the gradient of the instantaneous error contains $y(n-j)$'s which also depend on the coefficients of the filter, the recursive LMS algorithm dose not converge in general. Instead, when the recursive LMS algorithm is applied to an IIR adaptive notch filter, the notch filter behaves like an adaptive FIR filter. The zeroes converge to their optimum locations, but the poles tend to segregate radially toward the center of the unit circle away form their optimum locations. This behavior was observed when the recursive maximum likelihood (RML) was applied to IIR adaptive notch filters (ANF) [9]. The recursive maximum likelihood is based on a general Gauss-Newton algorithm for minimizing the squared predicted error. To overcome this problem, a penalty function on the predicted error has been adopted to force the poles to converge toward their optimum locations. The transfer function of an IIR notch filter designed by the pole zero placement on the unit circle can be written as

$$H(z) = \frac{\prod_{i=0}^{N}(z-z_i)}{\prod_{i=0}^{N}(z-p_i)} = \frac{a_0 + a_1 z^{-1} + \cdots + a_n z^{-n}}{1 + b_1 z^{-1} + \cdots + b_n z^{-n}}. \tag{3}$$

The backward coefficients (bj) can be related to the forward coefficients (aj) through a scaling parameter as follows

$$b_j = \lambda_j a_j, \quad j = 1, \ldots, N, \quad 0 < \lambda_j < 1, \quad \text{with} \quad b_0 = 1. \tag{4}$$

Substituting (4) into (2a), the recursive LMS algorithm can be written for the backward coefficients as

$$\lambda_j a_j(n+1) = \lambda_j a_j(n) + \lambda_j \mu e(n) x(n-j) \tag{5a}$$

$$b_j(n+1) = b_j(n) + \eta e(n) x(n-j). \tag{5b}$$

Although (5b) can be used to update the backward coefficients, it was observed that the convergence of the filter is very slow. By comparing (5b) with (2b), the following new equation can be developed

$$b_j(n+1) = b_j(n) + \eta e(n) y(n-j) + \eta e(n)(x(n-j) - y(n-j)). \tag{6}$$

The difference between the unfiltered $x(n)$ and the filtered $y(n)$ signals, $(x(n) - y(n))$, represents the noise distortion in the input signal $x(n)$, and hence the term $\eta e(n)(x(n-j) - y(n-j))$ in (6) is upper bounded. Thus (6) suggests applying a penalty function on the estimated instantaneous error for the backward coefficients.

$$e'(n) = e(n) + \varepsilon(a_j(n-j) - b_j(n-j)). \tag{7}$$

The new error $e'(n)$ is obtained from the error $e(n)$ by adding the correction term $\varepsilon(a_j(n-j) - b_j(n-j))$. To see the effect of this correction term, the gradient can be derived from (7) and can be written as

$$\frac{\partial e'(n)}{\partial b_j} = \frac{\partial e(n)}{\partial b_j} + \varepsilon\left(\frac{1}{\lambda_j} - 1\right). \tag{8}$$

Consequently, as the poles converge toward the zeroes on the unit circle, λ_j converges to one, and the term $(\frac{1}{\lambda_j} - 1)$ approaches zero. The new error $e'(n)$ asymptotically approaches the error $e(n)$. The updating equation for the backward coefficients can therefore be written as

$$b_j(n+1) = b_j(n) + \eta e'(n) y(n-j). \tag{9}$$

Equations (2a), (7), and (9) constitute the constrained least mean squared (CLMS) algorithm. The adaptation coefficients μ and η are chosen small enough to insure the convergence of the CLMS algorithm. The mean square error function formulates the performance of the filter and is defined as

$$\xi = E[(e(n))^2]. \tag{10}$$

However, because of the random noise in the forward and the backward coefficients (2), the mean square error does not converge to its minimum, but rather to an excess mean square. This generally causes a misadjustment [18] which is defined by

$$M = \frac{\xi \text{excess}}{\xi \text{min}}. \tag{11}$$

Since the desired signal $d(n)$ is often not available, an adaptive noise canceling (ANC) system may be appropriate (Fig. 1(b)). An ANC is an adaptive process which has two inputs with synchronized samplers [10], [17]. The primary input consists of the noisy signal $x(n)$. The desired signal is denoted by $s(n)$. The reference input $v_2(n)$ is estimated by $z(n)$ to match the noise $v_1(n)$ in the primary input. The noise in both inputs is assumed to be correlated. Using complex notation, the noise in both inputs can be written in compact forms as

$$v_1(n) = V_{01} e^{j(\omega_0 n \Delta T + \phi_0)}, \quad \text{and} \quad v_2(n) = V_{02} e^{j\omega_0 n \Delta T} \tag{12}$$

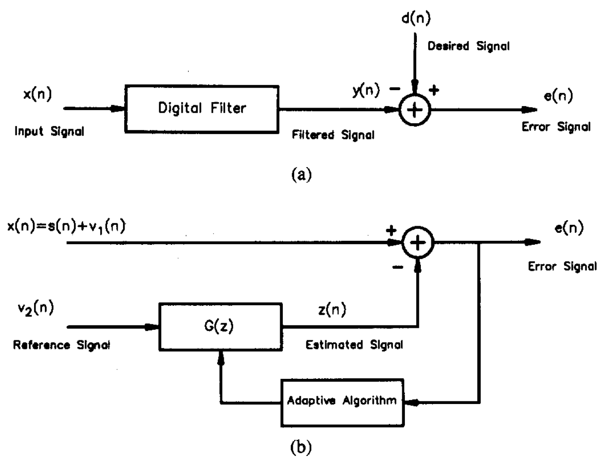

Fig. 1. (a) General digital filter; (b) Adaptive noise canceling system.

where ω_0 is the frequency of the noise and ϕ_0 is an unknown phase shift. The signal input $x(n)$ and the error signal $e(n)$ are given by

$$e(n) = x(n) - z(n) \tag{13a}$$
$$x(n) = s(n) + v_1(n) \tag{13b}$$
$$v_1(n) = g_0 v_2(n). \tag{13c}$$

The filter estimation $G(z)$ is derived from the digital notch filters designed in the next sections.

III. Design of an Adaptive Second-Order FIR Digital Notch Filter

The simplicity of the pole-zero placement on the unit circle method is its major advantage [2], [4] (Fig. 2). For an FIR design, the zeroes of the filter are placed on the unit circle at the position equivalent to the rejected frequency ω_0. For a signal sampled at frequency ω_s (sampling interval $\Delta T = 2\pi/\omega_s$), the zeroes are determined as follows

$$z_{1,2} = \cos(\beta_0) \pm j\sin(\beta_0) \tag{14}$$

where the angle β_0 is given by

$$\beta_0 = \left[\frac{\omega_0}{\omega_s}\right] 2\pi = \omega_0 \Delta T. \tag{15}$$

The transfer function of the 2^{nd}-order FIR filter is written as

$$H(z) = (z - z_1)(z - z_2). \tag{16}$$

Substituting the expressions for the z_i's and dividing by z^2 to make a causal filter, the transfer function reduces to

$$H(z) = (1 - 2\cos(\beta_0)z^{-1} + z^{-2}). \tag{17}$$

By comparison to the canonical form for a second-order FIR filter [2], the different coefficients of the filter can be identified as follows

$$a_0 = 1, \quad a_1 = -2\cos(\beta_0), \quad a_2 = 1. \tag{18}$$

The non-recursive formula in the time domain can be easily deduced as follows

$$y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2). \tag{19}$$

As the frequency ω approaches the noise frequency ω_0 (the rejected frequency), the transfer function magnitude of the filter at this point can be approximated by (Appendix A)

$$|\, H(\omega)\, | \approx 2\sqrt{2}\,|\sin(\beta_0)|\,[1 - \cos(\beta_0 - \omega\Delta T)]^{\frac{1}{2}}. \tag{20}$$

The bandwidth and the quality factor, which determine the sharpness of the notch filter, are calculated analytically by finding the frequency for which the magnitude of the transfer function is reduced to 3-dB and is given by (Appendix A)

$$\mathrm{BW} = \frac{1}{\sqrt{2}\,|\sin(\beta_0)|}\text{rad}, \quad \text{and}$$
$$Q = \frac{\text{center} - \text{frequency}}{\text{bandwidth}} = \omega_0\sqrt{2}\,|\sin(\beta_0)|\,. \tag{21}$$

If the noise is not centered around its nominal frequency, that is, if β is not fixed, there will be a $\Delta\omega$ that needs to be added to the nominal frequency. Replacing ω_0 by $\omega_0 \pm \Delta\omega$ it follows that

$$\beta = \left[\frac{\omega_0 \pm \Delta\omega}{\omega_s}\right] 2\pi = \left[\frac{\omega_0}{\omega_s}\right] 2\pi \pm \left[\frac{\Delta\omega}{\omega_s}\right] 2\pi = \beta_0 \pm \Delta\beta. \tag{22}$$

The coefficients of the filter are as follows

$$a_0 = 1, \quad a_1 = -2\cos(\beta_0 \pm \Delta\beta), \quad a_2 = 1. \tag{23}$$

The coefficient a_1 is the only coefficient which is directly related to the frequency variation of the rejected frequency. Hence, an adaptive 2^{nd}-order FIR notch filter can be designed to adapt the coefficient a_1 (Fig. 3(a)). This filter is actually a fixed bandwidth filter where the zeroes are changing to accommodate the variation of the noise. For the ANC, the transfer function of the filter estimator $G(z)$ can be expressed as follows

$$G(z) = g_0(-a_1 z^{-1} - z^{-2}). \tag{24}$$

The factor g_o is actually the gain of the notch filter which will be incorporated into the coefficients of the filter and updated using the same updating equations.

IV. Design of Two Adaptive Second-Order IIR Digital Notch Filters

Using the pole-zero placement on the unit circle method, the zeroes are located on the unit circle while the poles are located inside the circle at a radial distance from the zeroes. The zeroes and the poles of a second-order IIR filter are determined as follows

$$z_{1,2} = \cos(\beta_0) \pm j\sin(\beta_0) \tag{25a}$$
$$p_{1,2} = \alpha(\cos(\beta_0) \pm j\sin(\beta_0)) \tag{25b}$$

where $\alpha \leq 1$ for filter stability, and $(1 - \alpha)$ is the distance between the poles and the zeroes.

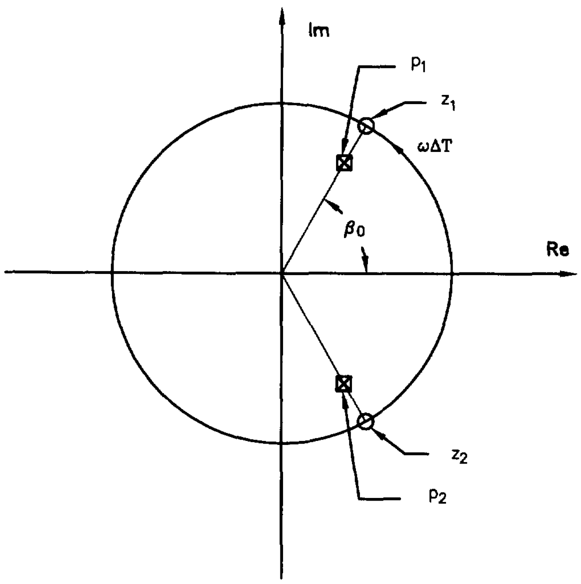

Fig. 2. Illustration of the pole-zero placement method on the unit circle. The zeroes are located on the unit circle and the poles are located inside the unit circle at a distance radial to the position of the zeroes.

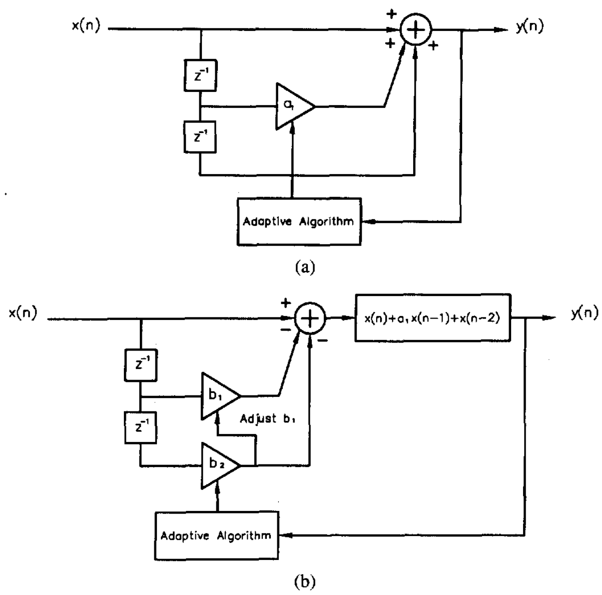

Fig. 3. (a) Block diagram of an adaptive second-order FIR digital notch filter. (b) Block diagram of an adaptive second-order IIR (fixed zeroes) digital notch filter.

The transfer function of a second-order IIR filter is written as

$$H(z) = \frac{(z-z_1)(z-z_2)}{(z-p_1)(z-p_2)}. \tag{26}$$

Substituting the expressions for z_i and p_i, and dividing by z^2, the transfer function of the filter reduces to

$$H(z) = \frac{(1-2\cos(\beta_0)z^{-1}+z^{-2})}{(1-2\alpha\cos(\beta_0)z^{-1}+\alpha^2z^{-2})}. \tag{27}$$

By comparison to the canonical form for a 2nd-order IIR filter [2], the different coefficients of the filter can be identified as follows

$$a_0 = 1, \quad a_1 = -2\cos(\beta_0), \quad a_2 = 1 \tag{28a}$$
$$b_1 = -2\alpha\cos(\beta_0), \quad b_2 = \alpha^2. \tag{28b}$$

The recursive formula in the time domain can be readily deduced as follows

$$y(n) = a_0x(n)+a_1x(n-1)+a_2x(n-2)-b_1y(n-1)-b_2y(n-2). \tag{29}$$

By examining the preceding expressions for the filter coefficients the following alternate relationships for b_1 are observed

$$b_1 = \alpha a_1 = a_1\sqrt{b_2}. \tag{30}$$

These relationships indicate that only a_1 and b_2, need to be adapted by varying β and α. As the frequency ω approaches the frequency ω_0 (the rejected frequency) the transfer function magnitude of the filter can be approximated by (Appendix B)

$$\mid H(\omega)\mid = \frac{2}{(1+\alpha)}\frac{\mid e^{j\omega\Delta T}-\cos(\beta_0)-j\sin(\beta_0)\mid}{\mid e^{j\omega\Delta T}-\alpha(\cos(\beta_0)+j\sin(\beta_0))\mid}. \tag{31}$$

The bandwidth and the quality factor of the filter are calculated analytically and are given by (Appendix B)

$$\mathrm{BW} = \frac{2\sqrt{2}(1-\alpha^2)}{[16-2\alpha(1+\alpha)^2]^{1/2}}\mathrm{rad}, \quad \text{and}$$
$$Q = \omega_0\frac{[16-2\alpha(1+\alpha)^2]^{1/2}}{2\sqrt{2}(1-\alpha^2)}. \tag{32}$$

The bandwidth, which is a function of the distance of the poles to zeroes (see Appendix B), narrows when α approaches unity. Therefore, if the noise frequency is stable, an adaptive second-order IIR filter can be designed where only the bandwidth is changing to accommodate the bandwidth of the noise (Fig. 3(b)). The poles are adapted to track the bandwidth of the noise. However, if the noise is varying around the nominal frequency noise, the coefficients of the filter become

$$a_0 = 1, \quad a_1 = -2\cos(\beta_0 \pm \Delta\beta), \quad a_2 = 1 \tag{33a}$$
$$b_1 = -2\alpha\cos(\beta_0 \pm \Delta\beta), \quad b_2 = \alpha^2. \tag{33b}$$

Since the frequency drift affects the coefficients of the filter and hence its bandwidth, the second adaptive second-order IIR digital notch filter is designed to track the frequency variation within an optimum bandwidth. Both the zeroes and the poles are adapted (Fig. 4). The transfer function of the filter estimator for this case can be written as

$$G(z) = g_0\frac{-(a_1-b_1)z^{-1}-(1-b_2)z^{-2}}{(1+b_1z^{-1}+b_2z^{-2})}. \tag{34}$$

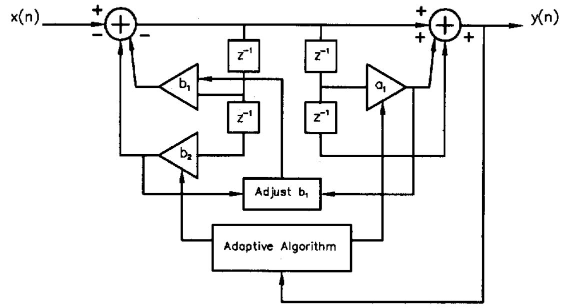

Fig. 4. Block diagram of an adaptive second-order IIR digital notch filter.

The stability of these two second-order IIR filters is critical. A check for stability before each iteration is required throughout the process. The IIR filter is considered unstable when its poles are outside the unit circle. The denominator of a second-order IIR filter

$$D(z) = 1 + b_1 z^{-1} + b_2 z^{-2} \tag{35}$$

has two roots denoted as p_1 and p_2, where

$$p_1, p_2 = \frac{1}{2}(-b_1 \pm \sqrt{b_1^2 - 4b_2}). \tag{36}$$

For both adaptive second-order IIR filters, the stability condition requires that the magnitude of p_1 and p_2 must be less than 1. In terms of the filter coefficients, this stability requirement reduces to the simultaneous stability conditions

$$b_2 = \alpha^2 < 1, \quad b_1^2 < 4. \tag{37}$$

To prevent instability in the filter, the reflection method is used for any poles found outside the unit circle. If the pole was equal to unity, it is multiplied by a number slightly less than one to eliminate the possibility of any memory lock-up [12].

V. Application

The three adaptive notch filters were first applied to a powerline noise sample which was added to a Gaussian noise with a zero mean and a variance equal to 0.5 (ξmin). The adaptive second-order FIR digital notch filter (Fig. 5) converges approximately after 400 iterations and is able to track the frequency variation primarily because of its wider notch. However, its wide bandwidth is its major limitation, since some of the desired signal will be attenuated. The coefficient a_1, which is related to the frequency variation, converges towards its nominal value.

The fixed-zero adaptive second-order IIR digital notch filter (Fig. 6) adapts the bandwidth to the range of frequency variation. This filter coefficient b_2, does not converge to an optimum value, but rather fluctuates about a nominal value. If the frequency spectrum of the powerline noise is very wide, suggesting instability of the powerline source, the filter may exhibit wide fluctuations. For the sample noise here the filter converges after approximately 500 iterations.

In the second adaptive 2^{nd}-order IIR digital notch filter (Fig. 7) the zeroes as well as the poles are adapted to the frequency variation of the powerline noise. The filter tracks the

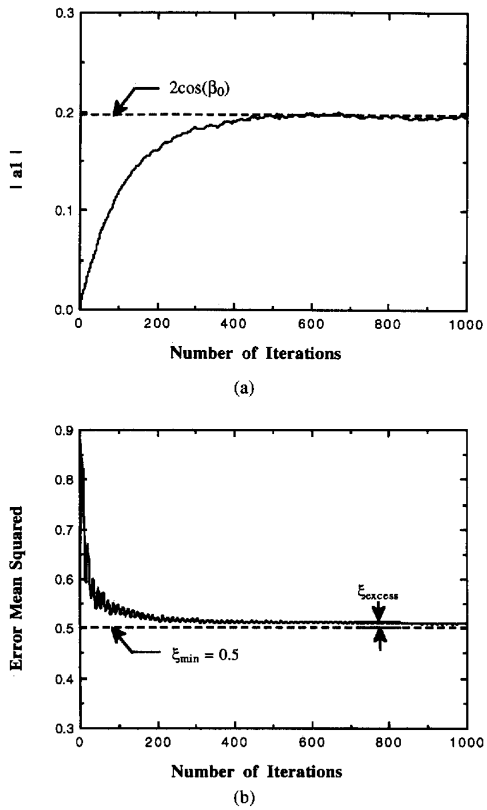

Fig. 5. (a) Adaptive process of a_1 of the adaptive second-order FIR digital notch filter: $\mu = 0.001$. (b) Error mean squared of the adaptive second-order FIR digital notch filter: $\xi_{\text{excess}} = 0.01$, $M = 2\%$.

noise and eliminates it after 400 iterations. The coefficient a_1 converges rapidly toward its nominal value and the coefficient b_2 approaches a number close to unity with less fluctuations than the fixed-zero IIR.

Next, the three adaptive notch filters are applied to a noisy EEG signal (Fig. 8) as adaptive noise canceling systems. The adaptive FIR notch filter eliminates the 60 Hz noise, but also attenuates the nearby frequencies because of its wide bandwidth. The fixed-zero adaptive IIR notch filter may also have a wide bandwidth due to the large distance between the poles and the zeroes when the powerline noise is not stable. Despite the noisy effect of the CLMS process, the non-fixed pole-zero IIR notch filter tracks the frequency variation and eliminates the noise. Since its bandwidth is very narrow, the nearby frequencies are less affected.

VI. Conclusion

The digital adaptive notch filter presents a better alternative over the fixed notch filter in the elimination of the noise. The adaptive second-order FIR notch filter operates with a very wide bandwidth to track and reduce the noise frequency. The fixed-zero adaptive second-order IIR notch eliminates the arbitrariness in the choice of the width of the filter, which may be narrower or wider depending on the nature of the noise. Since it does not adapt to frequency variation of the noise, it

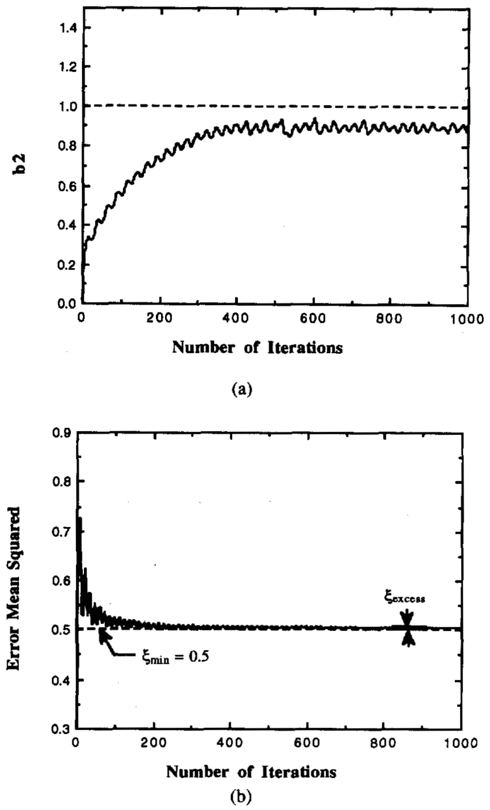

Fig. 6. (a) Adaptive process of b_2 of the adaptive second-order IIR (fixed zeroes) digital notch filter: $\eta = 0.0025$, $\lambda = 0.9$. (b) Error mean squared of the adaptive second-order IIR (fixed zeroes) digital notch filter: $\xi_{\text{excess}} = 0.005$, $M = 1\%$.

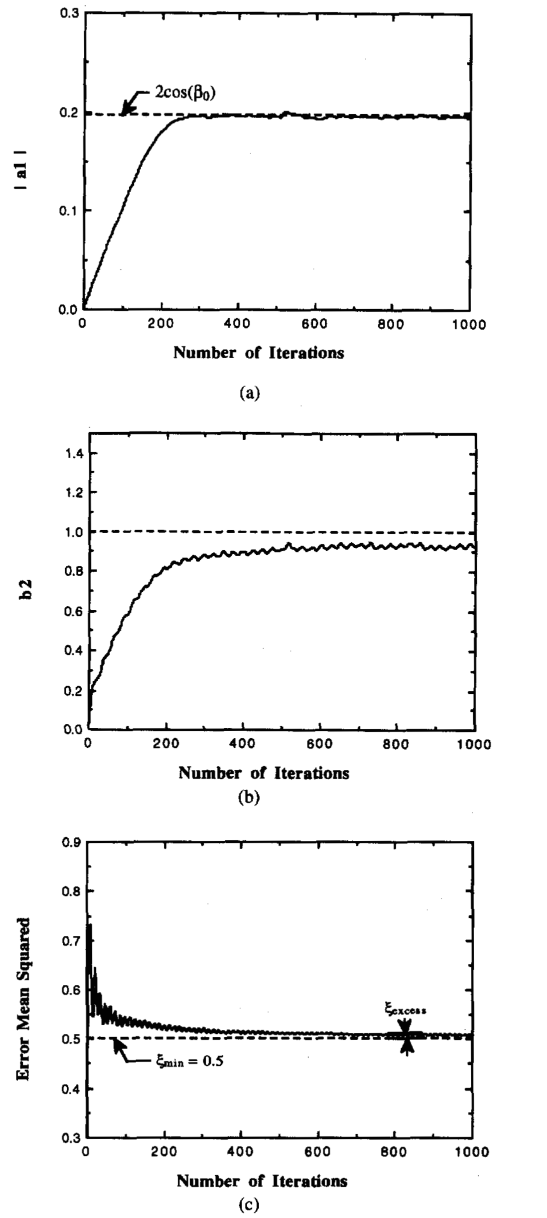

Fig. 7. (a) Adaptive process of a_1 of the adaptive second-order IIR digital notch filter: $\mu = 0.001$. (b) Adaptive process of b_2 of the adaptive second-order IIR digital notch filter: $\eta = 0.005$, $\lambda = 0.9$. (c) Error mean squared of the adaptive second-order IIR digital notch filter: $\xi_{\text{excess}} = 0.008$, $M = 1.6\%$.

reduces the noise source by a variable bandwidth which results in attenuation of surrounding frequencies. Both adaptive FIR and fixed-zero adaptive IIR filters are not recommended when the frequency variation of the noise is very large. The non-fixed pole-zero adaptive second-order IIR notch is the most versatile of the three adaptive filters. This filter tracks the frequency variation by changing the coefficients of the filter that are affected by such a variation (a_1, b_2, and then b_1). The width of the filter is therefore minimized.

The pole-zero placement on the unit circle presents a valuable tool for the design of adaptive digital notch filters. For the IIR filter, the unconstrained LMS algorithm was found not to converge to an optimum state [7], [12]. The constrained least mean squared (CLMS) presented in this study offers a new algorithm to insure stability and convergence of this special class of IIR notch filters. For both IIR filters, the poles were observed to converge to an optimum location.

Appendix A
Adaptive Digital Second-Order FIR Notch Filter

The magnitude of the transfer function of the second-order FIR digital notch filter is written as

$$|H(z)| = |z - z_1| \, |z - z_2| . \tag{A1.1}$$

At the positive frequency in the vicinity of the fixed zero z_1, letting $z = \exp(j\omega\Delta T)$ and assuming that $\omega\Delta T \approx \beta_0$, the expression (A1.1) reduces to

$$|H(\omega)| \approx |2\sin(\beta_0)| \, |e^{j\omega\Delta T} - \cos(\beta_0) - j\sin(\beta_0)| \tag{A1.2a}$$

$$|H(\omega)| \approx |2\sin(\beta_0)| \, [2 - 2\cos(\beta_0 - \omega\Delta T)]^{1/2}. \tag{A1.2b}$$

The bandwidth of the filter is evaluated analytically at −3dB,

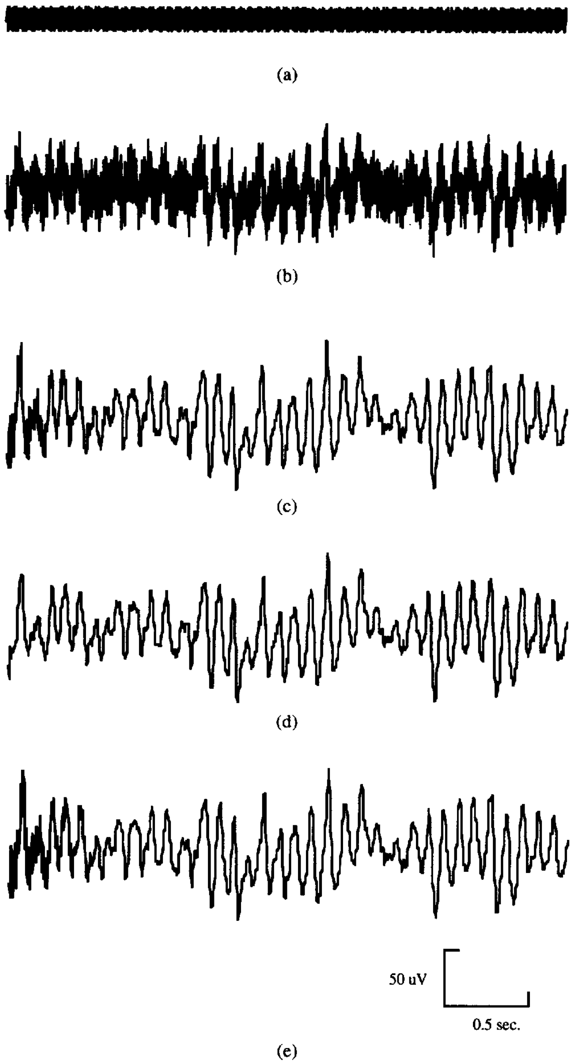

Fig. 8. (a) Reference noise signal. (b) Noisy EEG. (c) Filtered EEG data by the adaptive second-order FIR digital notch filter: $\mu = 0.001$. (d) Filtered EEG data by the adaptive second-order IIR (fixed zeroes) digital notch filter: $\eta = 0.025$, $\lambda = 0.9$. (e) Filtered EEG data by the adaptive 2^{nd}-order IIR digital notch filter: $\mu = 0.001$, $\eta = 0.025$, $\lambda = 0.9$.

which corresponds to the position where

$$| H(\omega) |^2 = 4\sin^2(\beta_0)(2 - 2\cos(\beta_0 - \omega\Delta T)) = \frac{1}{2} \quad \text{(A1.3)}$$

expression (A1.3) reduces to

$$\cos(\beta_0 - \omega\Delta T) = \frac{16\sin^2(\beta_0) - 1}{16\sin^2(\beta_0)}. \quad \text{(A1.4)}$$

Since $(\beta_0 - \omega\Delta T)$ is very small, a Taylor expansion to the first order of the $\cos(\beta_0 - \omega\Delta T)$ gives

$$\cos(\beta_0 - \omega\Delta T) \cong 1 - \frac{(\beta_0 - \omega\Delta T)^2}{2!}. \quad \text{(A1.5)}$$

The bandwidth of the filter can be approximated by twice the distance between the notched frequency and the frequency $\omega\Delta T$ at -3dB

$$\mathrm{BW} = 2(\beta_0 - \omega\Delta T). \quad \text{(A1.6)}$$

After substitution and simplification, the bandwidth of the filter can be expressed as

$$\mathrm{BW} = \frac{1}{| \sin(\beta_0) | \sqrt{2}} \mathrm{rad}. \quad \text{(A1.7)}$$

Appendix B
Adaptive Digital Second-Order IIR Notch Filter

The magnitude of the transfer function of the second-order IIR digital notch filter is written as

$$| H(z) | = \frac{| z - z_1 || z - z_2 |}{| z - p_1 || z - p_2 |}. \quad \text{(B2.1)}$$

At the positive frequency in the vicinity of zero z_1 and pole p_1, and using approximations of Appendix A, expression (B2.1) reduces to

$$| H(\omega) | \cong \frac{2}{(1+\alpha)} \frac{| e^{j\omega\Delta T} - \cos(\beta_0) - j\sin(\beta_0) |}{| e^{j\omega\Delta T} - \alpha(\cos(\beta_0) + j\alpha\sin(\beta_0)) |}. \quad \text{(B2.2)}$$

The bandwidth of the filter is evaluated analytically at -3dB. The following expression is obtained

$$| H(\omega) | \cong \frac{4}{(1+\alpha)^2} \frac{| 2 - 2\cos(\beta_0 - \omega\Delta T) |}{| (1+\alpha)^2 - 2\alpha\cos(\beta_0 - \omega\Delta T) |} \cong \frac{1}{2} \quad \text{(B2.3)}$$

which reduces to

$$\cos(\beta_0 - \omega\Delta T) \cong \frac{16 - (1+\alpha^2)(1+\alpha)^2}{16 - 2\alpha(1+\alpha)^2}. \quad \text{(B2.4)}$$

Expanding to the first order Taylor series of the cosine expression and using the same approximation of the bandwidth (A1.6) we obtain

$$\mathrm{BW} = \frac{2\sqrt{2}(1-\alpha^2)}{[16 - 2\alpha(1+\alpha)^2]^{\frac{1}{2}}} \mathrm{rad}. \quad \text{(B2.5)}$$

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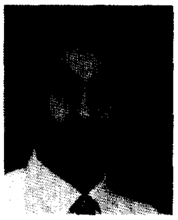

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