Synthesis on Atom Computation

Dingchao Gao

Institute of Software Chinese Academy of Sciences

May 23, 2024

Quick Review of Atom Computation

- **Control Mechanisms:**
  - Neutral atoms controlled by laser lattice (stationary)
  - Laser tweezers (for movement)
- **Single Qubit Gate:**
  - $U3 : (\Omega\tau \cos\varphi, \Omega\tau \sin\varphi, \delta\tau)$
- **Multi-Qubit Gate:**
  - $C\mathcal{Z} : 2|gg\rangle\langle gg| - I$

Outline

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1. Decomposing and Routing Quantum Circuits Under Constraints for Neutral Atom Architectures

- Decomposition Strategy
- Atom Movement Strategy

2. Compiling Quantum Circuits for Dynamically Field-Programmable Neutral Atoms Array Processors
FPQA-C: A Compilation Framework for Field Programmable Qubit Array

- Tan
- Wang

3. Discussion

Decomposition Strategy

- **Native Gate Set**
  - Local CZ gates.
  - Local $R_z(\lambda) : (0, 0, \frac{\lambda}{2})$ gates.
  - Global $GR(\theta, \phi) : (\frac{\theta}{2}\cos\phi, \frac{\theta}{2}\sin\phi, 0)$ gates.
- **Euler-Angle Decomposition**
  - Decompose any single-qubit gate $U3(\theta, \phi, \lambda)$ into Euler angles:

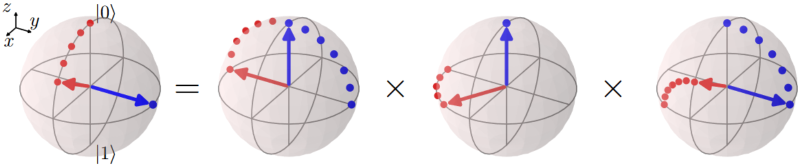
$$U3(\theta, \phi, \lambda) = R_z(\phi)R_y(\theta)R_z(\lambda)$$

Axial Decomposition

- **Decomposition of R_y Gate**
  - Decompose $R_y(\theta)$ gate into global and local gates:

$$\prod_j R_{y_j}(\theta_j) = GR\left(-\frac{\pi}{2}, 0\right)\left[\prod_j R_{z_j}(\theta_j)\right] GR\left(\frac{\pi}{2}, 0\right)$$

  - This involves a net global rotation of π.



$\mathtt{Ry}_j(\theta)$ $\mathtt{GlobalR}(\frac{\pi}{2},0)$ $\mathtt{Rz}_j(\theta)$ $\mathtt{GlobalR}(-\frac{\pi}{2},0)$

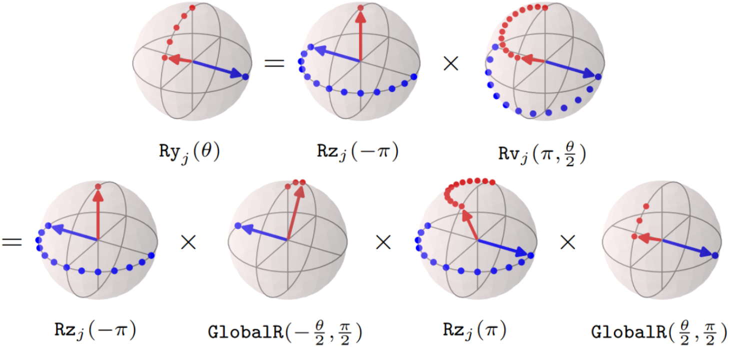
Transverse Decomposition

- **Optimized Decomposition of R_y Gate**
  - Decompose $R_y(\theta)$ gate using minimal global rotation angle:

$$R_y(\theta) = R_v\left(\pi, \frac{\theta}{2}\right) R_z(-\pi)$$

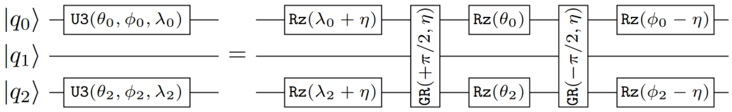
  - Further decompose R_v gate into global and local gates:

$$R_v(\xi, \omega) = GR(\omega, \frac{\pi}{2}) R_z(\xi) GR(-\omega, \frac{\pi}{2})$$

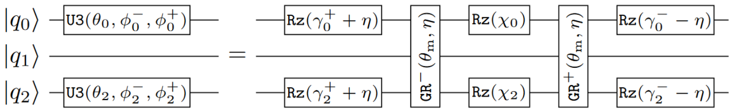


Addition step: Post-Processing and Optimizations

- **Axis of Rotation Adjustment**
  - Change the axis of rotation of global gates to eliminate redundant R_z gates.
- **Gate Merging**
  - Merge consecutive R_z gates to further reduce the gate count.



(a)



(b)

Comparison of Decomposition Methods

- **Axial vs. Transverse Decomposition**
  - Axial decomposition results in a net global rotation of π.
  - Transverse decomposition minimizes global rotation angle to $|\theta|$.
- **Advantages of Transverse Decomposition**
  - Up to 3.5x reduction in global gate pulse duration.
  - Up to 2.9x reduction in single-qubit gate execution time.

Summary

- Efficient decomposition of quantum circuits into native gate sets for neutral atom hardware.
- Transverse decomposition minimizes global rotation angles, leading to significant speedup.
- Post-processing optimizations further reduce gate count and improve execution time.

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Introduction to Atom Movement

- **Goal**
  - Utilize physical atom movement to optimize routing in quantum circuits.
  - Reduce overhead costs associated with traditional SWAP gate decompositions.
- **Challenges**
  - Maintain atom fidelity and avoid interference.
  - Optimize the movement paths to minimize execution time.

Atom Movement Constraints

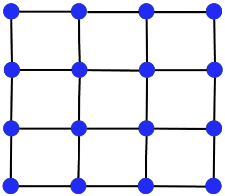
- **Threshold Distance**
  - When moving, an atom must stay at least d_{thr} away from any other atom to avoid interference.
- **Parallel Movement Constraints**
  - Atoms with the same initial x-coordinate can move horizontally together only if they end up with the same final x-coordinate.
  - Atoms with the same initial y-coordinate can move vertically together only if they end up with the same final y-coordinate.
- **Movement Speed**
  - Atoms must be moved at speeds not exceeding **0.55 μm/μs** to maintain qubit fidelity and entanglement.

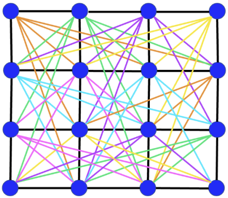
Atom Array Configuration

- **Initial Mapping**
  - Each program qubit is assigned to a unique hardware qubit to minimize routing operations.
- **Displacement**
  - Atoms can be displaced from grid points by d_{thr} to facilitate SWAP operations without violating distance constraints.

Movement Graph

- **Graph Definition**
  - Nodes represent atom sites.
  - Edges indicate direct movement paths between sites without violating d_{thr}.
  - Edge weights represent the distance between atom sites.





CZ Gate Execution Work Flow

- Use movement graph to determine qubit permutation when a CZ gate (qa, qb) can not be executed with current mapping.
- Consider all program qubit pairs (u, v) that:
  - $u \in \{qa, ab\}$ or $v \in \{qa, ab\}$.
  - Have an edge $(m_{t-1}(u), m_{t-1}(v))$ in the movement graph.
  - Bring qa and qb closer after moving u to $m_{t-1}(v)$ or move v to $m_{t-1}(u)$.
- Choose pair (u', v') minimizing distance between qa and qb after permutation.

Comparing SWAP-based and Movement-based Routing

- **SWAP-based Routing**
  - SWAP gate decomposition results in high gate count and execution time.
- **Movement-based Routing**
  - Atom movement reduces routing overhead and improves fidelity.
  - Achieves up to 10x speedup and 2x improvement in fidelity.

Summary

- Utilize physical atom movement to optimize routing in neutral atom quantum computers.
- Maintain fidelity and avoid interference with strategic movement constraints and optimizations.
- Significant improvements in execution speed and circuit fidelity compared to traditional SWAP-based routing.

Outline

DPQA/FPQA Architecture

- **Qubit Traps**
  - Qubits are held in optical traps generated by Acousto-Optic Deflectors (AODs) and Spatial Light Modulators (SLMs).
- **Reconfigurability**
  - AOD traps can move in rows and columns, allowing dynamic reconfiguration.
  - AOD rows/columns cannot cross over or overlap during movement.
- **Entangling Gates**
  - Entangling gates are applied using a Rydberg laser, effective when qubits are within a blockade range r_b.

Outline

Overview of the Method

- **Goal**
  - Develop a compiler for Dynamically Field-Programmable Qubit Arrays (DPQA).
  - Optimize the placement and routing of qubits to minimize circuit depth.
- **Approach**
  - Discretize the state space and formulate the problem as a Satisfiability Modulo Theories (SMT) problem.
  - Use an SMT solver to find optimal solutions.

Discretization of State Space

- **Space Domain**
  - Discretized into interaction sites to ensure qubits are either paired for gates or idle and well-separated.
- **Time Domain**
  - Discretized into stages where qubit positions are adjusted and gates are executed.

SMT Model Variables

- **stage** s: Represents the stage or step in the quantum circuit execution process.
- **qubit** q_i: Denotes the i-th qubit in the quantum circuit.
- **gate** g_j: Represents the j-th gate in the quantum circuit.
- **site indices** $x_{i,s}, y_{i,s}$: Coordinates (x, y) of the i-th qubit at stage s.
- **array index** $a_{i,s}$: Indicates whether the i-th qubit at stage s is in a stationary SLM trap ($a_{i,s} = 0$) or in a mobile AOD trap ($a_{i,s} = 1$).
- **AOD indices** $c_{i,s}, r_{i,s}$: Column c and row r indices for the i-th qubit at stage s in the AOD grid.
- **time coordinate** t_j: The time coordinate when the j-th gate is applied.

SMT Model Constraints

- **Spatial Constraints**
  - Qubits must be within the defined grid bounds:

$$0 \leq x_{i,s} < X, \quad 0 \leq y_{i,s} < Y \quad \forall i \in [0, N), s \in [0, S)$$

  - AOD moves:

$$(a_{i,s} = 1) \implies (c_{i,s+1} = c_{i,s} \wedge r_{i,s+1} = r_{i,s})$$

  - Site order implying row/column order enforing:

$$(x_{i,s} < x_{i',s}) \implies (c_{i,s} < c_{i',s}), \quad (y_{i,s} < y_{i',s}) \implies (r_{i,s} < r_{i',s})$$

  - AOD rows/columns cannot cross each other:

$$((a_{i,s} = 1) \wedge (a_{i',s} = 1) \wedge (c_{i,s} < c_{i',s})) \implies (x_{i,s+1} \leq x_{i',s+1}),$$
$$((a_{i,s} = 1) \wedge (a_{i',s} = 1) \wedge (r_{i,s} < r_{i',s})) \implies (y_{i,s+1} \leq y_{i',s+1}).$$

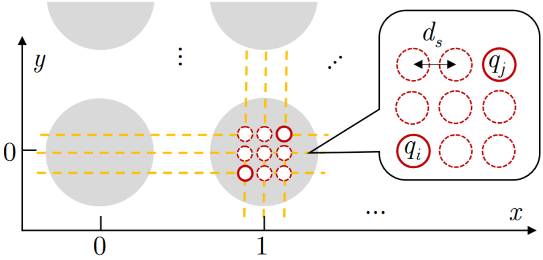
SMT Model Constraints

- **Interaction Constraints**
  - Qubits must be within r_b for entangling gates:

$$((a_{i,s-1} = 1) \wedge (a_{i',s-1} = 1) \wedge (c_{i,s-1} - c_{i',s-1} \geq C_{\text{STK}})) \implies (x_{i,s} > x_{i',s}),$$
$$((a_{i,s-1} = 1) \wedge (a_{i',s-1} = 1) \wedge (r_{i,s-1} - r_{i',s-1} \geq R_{\text{STK}})) \implies (y_{i,s} > y_{i',s}).$$

- **Circuit-Dependent Constraints**
  - Connectivity ensures:

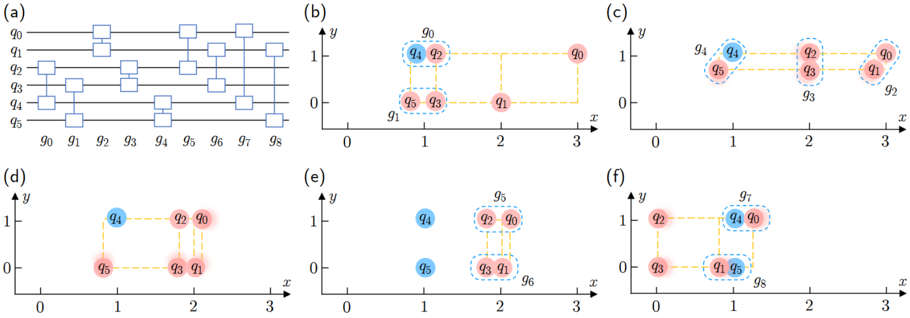
$$(t_j = s) \implies (x_{i,s} = x_{i',s} \wedge y_{i,s} = y_{i',s})$$



Scalability of the Model

- **Variables:** $5NS + G$
- **Constraints:** $O(G^2 + GS + N^2 S)$
- **Bits to Represent Variables:** $NS \log(2XYRC) + G \log(S)$
- **Worst-Case Runtime:** $O((N_{SLM} N_{AOD})^{NS} \cdot S^G)$
- **Shallow Circuit Regime and sparse graphs ($G = O(n)$):**
  - Bits required: $O(N \log(N_{SLM} N_{AOD}))$
  - Constraints: $O(N^2)$

Example of Compiled Quantum Circuit



Greedy Heuristic Method

- **Steps:**
  - Construct a ”single-step” SMT model with two stages.
  - Optimize the number of gates executed in the second stage.
  - Append the solution to the full solution and remove executed gates.
  - Repeat until fewer than 5% of gates remain.
- **Benefit:** Significantly faster than the optimal compiler with some sacrifice in optimality.

Experimental Results

- **Benchmarks:**
  - Evaluated on random graphs.
  - Circuits with 10 to 90 qubits.
- **Main Findings:**
  - For 90-qubit circuits: 5.1x fewer two-qubit gates compared to fixed planar architecture.
- **Error Sources:**
  - AOD movements cause 27x less infidelity.
- **Compiler Performance:**
  - Hybrid approach faster than optimal compiler.
  - Compiles up to 90-qubit circuits within a day.

Summary

- Utilize reconfigurability and non-local connectivity of DPQA for efficient quantum circuit compilation.
- Formulate constraints as an SMT problem for optimal solutions.
- Implement hybrid approach for scalability.

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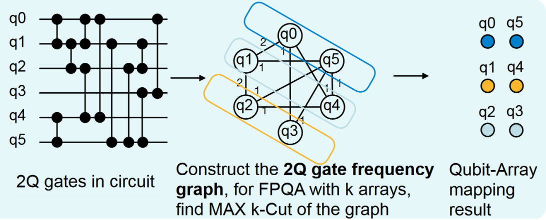
Compilation Framework

- **Qubit-Array Mapper**
  - Uses MAX k-Cut on a gate frequency graph to minimize SWAP overhead.
  - Coarse-grained mapping of qubits to arrays.
- **Qubit-Atom Mapper**
  - Fine-grained mapping of qubits to specific atoms in the array.
  - Load balance to prevent hardware constraint violations.
- **High-Parallelism Router**
  - Iteratively identifies parallelizable 2Q gates.
  - Decides atom movements and gate executions to maximize parallelism.

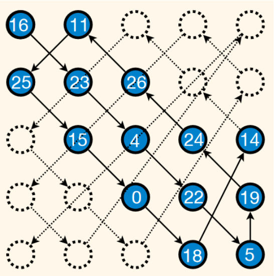
Qubit-Array Mapping Method

- **MAX k-Cut**
  - Finds mapping that maximizes inter-array 2Q gates.
  - Greedy algorithm used for approximation.
- **Gate Frequency Graph**
  - Vertices represent qubits, edges represent gates.
  - Edge weights determined by gate frequency.



Qubit-Atom Mapping Method

- **Load Balance Mapping**
  - Ensures balanced distribution of qubits across rows and columns.
  - Avoids constraint violations from dense qubit clusters.
- **Alignment Mapping**
  - Maps qubit pairs with frequent 2Q gates to the same positions in different arrays.
  - Enhances parallelism by aligning high-frequency gates.



High-Parallelism Router

- **Non-Dependent Frontier Gates**
  - Identifies and schedules gates with no dependencies.
- **Constraint Checks**
  - Ensures movements do not violate spatial or interaction constraints.
  - Greedily adds gates to the parallel set, checking each for legality.

Unique Experimental Approach

- **Comprehensive Simulations**
  - Evaluates logical error rates, execution times, and physical qubit requirements.
  - Includes realistic modeling of movement overheads (heating, cooling, decoherence, atom loss).
- **Scalability and Performance**
  - Demonstrates significant reductions in 2Q gate count and circuit depth.
  - Achieves up to 1000x faster compilation speed compared to solver-based methods.

Summary

- FPQA-C effectively addresses qubit mapping, atom movement, and gate scheduling in FPQA.
- Incorporates innovative methods to handle hardware constraints and optimize performance.
- Experimental results highlight the advantages of FPQA-C over traditional approaches in terms of gate count, depth, and fidelity.

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map

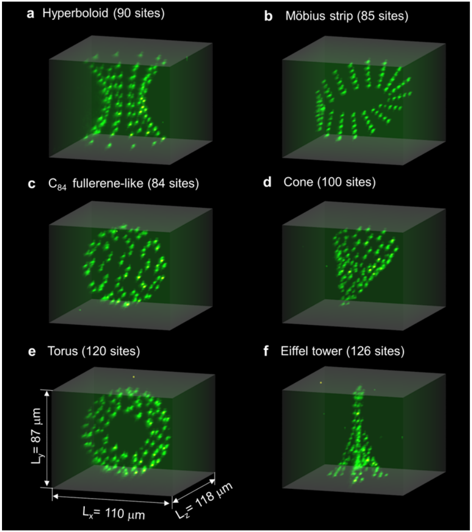

Figure: 3D map from DB17. Q1: Differences between 2D and 3D mapping.

CZ ? $C\mathcal{Z}$

- $C\mathcal{Z}$: $2|gg\rangle\langle gg| - I$, while CZ : $I - 2|11\rangle\langle 11|$
- in gererally:

$$|gg\rangle \to |gg\rangle$$
$$|ge\rangle \to |ge\rangle e^{i\phi_1}$$
$$|eg\rangle \to |eg\rangle e^{i\phi_1}$$
$$|ee\rangle \to |ee\rangle e^{i(2\phi_1+\pi)}$$

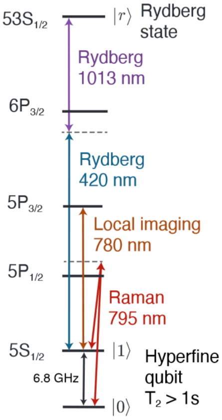

Figure: Level structure for ^{87}Rb atoms, with the relevant atomic transitions employed in *Quera paper*.

How to use mcz

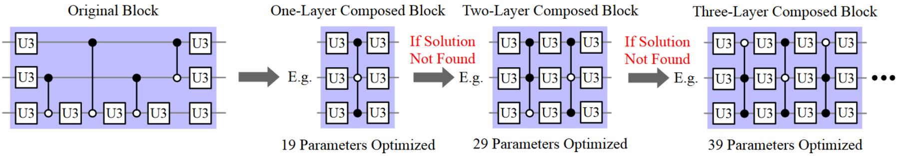

Figure: Main idea in Gyser paper. Q2: Applying some rules to gap the CZ gate circuit and MCZ gate circuits.

How to use mcz

- Q3: How to use mcz to construct circuit directly.

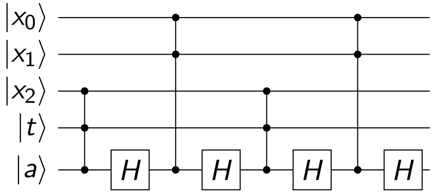

Figure: use ccz and Hadmard gate to construct cccz gate.