

Synthesis on Atom Computation

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Quick Review of Atom Computation

- **Control Mechanisms:**

- Neutral atoms controlled by laser lattice (stationary)
- Laser tweezers (for movement)

- **Single Qubit Gate:**

- $U3 : (\Omega\tau \cos \varphi, \Omega\tau \sin \varphi, \delta\tau)$

- **Multi-Qubit Gate:**

- $CZ : 2|gg\rangle\langle gg| - I$

- 1. Decomposing and Routing Quantum Circuits Under Constraints for Neutral Atom Architectures**
2. Compiling Quantum Circuits for Dynamically Field-Programmable Neutral Atoms Array Processors
FPQA-C: A Compilation Framework for Field Programmable Qubit Array
3. Discussion

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- Decomposition Strategy
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Decomposition Strategy

- **Native Gate Set**

- Local CZ gates.
- Local $R_z(\lambda) : (0, 0, \frac{\lambda}{2})$ gates.
- Global $GR(\theta, \phi) : (\frac{\theta}{2} \cos \phi, \frac{\theta}{2} \sin \phi, 0)$ gates.

- **Euler-Angle Decomposition**

- Decompose any single-qubit gate $U3(\theta, \phi, \lambda)$ into Euler angles:

$$U3(\theta, \phi, \lambda) = R_z(\phi)R_y(\theta)R_z(\lambda)$$

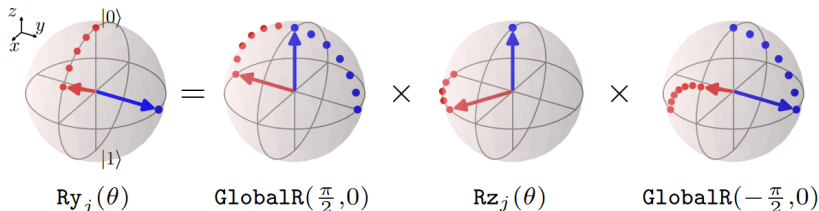
Axial Decomposition

- **Decomposition of R_y Gate**

- Decompose $R_y(\theta)$ gate into global and local gates:

$$\prod_j R_{y_j}(\theta_j) = GR\left(-\frac{\pi}{2}, 0\right) \left[\prod_j R_{z_j}(\theta_j) \right] GR\left(\frac{\pi}{2}, 0\right)$$

- This involves a net global rotation of π .



Transverse Decomposition

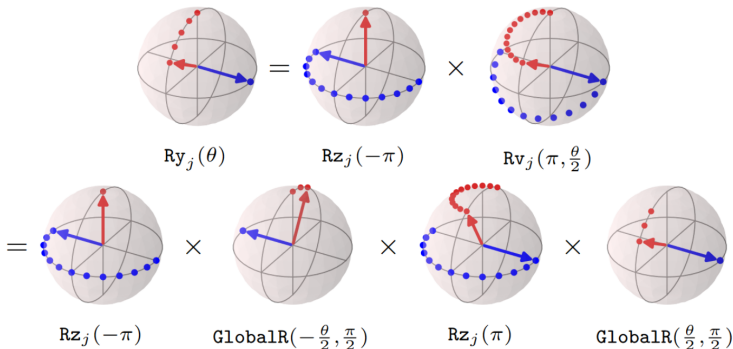
Optimized Decomposition of R_y Gate

- Decompose $R_y(\theta)$ gate using minimal global rotation angle:

$$R_y(\theta) = R_v\left(\pi, \frac{\theta}{2}\right) R_z(-\pi)$$

- Further decompose R_v gate into global and local gates:

$$R_v(\xi, \omega) = GR(\omega, \frac{\pi}{2}) R_z(\xi) GR(-\omega, \frac{\pi}{2})$$



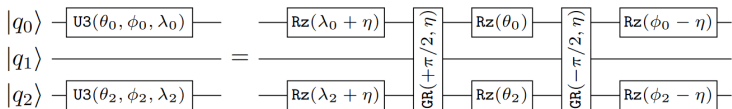
Addition step: Post-Processing and Optimizations

■ Axis of Rotation Adjustment

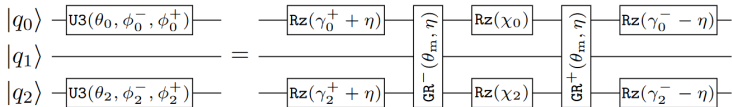
- Change the axis of rotation of global gates to eliminate redundant R_z gates.

■ Gate Merging

- Merge consecutive R_z gates to further reduce the gate count.



(a)



(b)

Comparison of Decomposition Methods

- **Axial vs. Transverse Decomposition**
 - Axial decomposition results in a net global rotation of π .
 - Transverse decomposition minimizes global rotation angle to $|\theta|$.
- **Advantages of Transverse Decomposition**
 - Up to 3.5x reduction in global gate pulse duration.
 - Up to 2.9x reduction in single-qubit gate execution time.

Summary

- Efficient decomposition of quantum circuits into native gate sets for neutral atom hardware.
- Transverse decomposition minimizes global rotation angles, leading to significant speedup.
- Post-processing optimizations further reduce gate count and improve execution time.

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Introduction to Atom Movement

- **Goal**

- Utilize physical atom movement to optimize routing in quantum circuits.
- Reduce overhead costs associated with traditional SWAP gate decompositions.

- **Challenges**

- Maintain atom fidelity and avoid interference.
- Optimize the movement paths to minimize execution time.

Atom Movement Constraints

- **Threshold Distance**

- When moving, an atom must stay at least d_{thr} away from any other atom to avoid interference.

- **Parallel Movement Constraints**

- Atoms with the same initial x -coordinate can move horizontally together only if they end up with the same final x -coordinate.
- Atoms with the same initial y -coordinate can move vertically together only if they end up with the same final y -coordinate.

- **Movement Speed**

- Atoms must be moved at speeds not exceeding **$0.55 \mu\text{m}/\mu\text{s}$** to maintain qubit fidelity and entanglement.

Atom Array Configuration

- **Initial Mapping**

- Each program qubit is assigned to a unique hardware qubit to minimize routing operations.

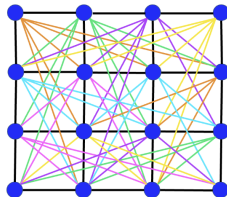
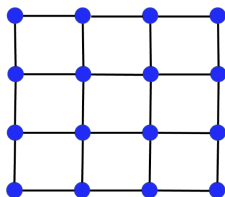
- **Displacement**

- Atoms can be displaced from grid points by d_{thr} to facilitate SWAP operations without violating distance constraints.

Movement Graph

- **Graph Definition**

- Nodes represent atom sites.
- Edges indicate direct movement paths between sites without violating d_{thr} .
- Edge weights represent the distance between atom sites.



CZ Gate Execution Work Flow

- Use movement graph to determine qubit permutation when a CZ gate (qa, qb) can not be executed with current mapping.
- Consider all program qubit pairs (u, v) that:
 - $u \in \{qa, ab\}$ or $v \in \{qa, ab\}$.
 - Have an edge $(m_{t-1}(u), m_{t-1}(v))$ in the movement graph.
 - Bring qa and qb closer after moving u to $m_{t-1}(v)$ or move v to $m_{t-1}(u)$.
- Choose pair (u', v') minimizing distance between qa and qb after permutation.

Comparing SWAP-based and Movement-based Routing

- **SWAP-based Routing**

- SWAP gate decomposition results in high gate count and execution time.

- **Movement-based Routing**

- Atom movement reduces routing overhead and improves fidelity.
- Achieves up to 10x speedup and 2x improvement in fidelity.

Summary

- Utilize physical atom movement to optimize routing in neutral atom quantum computers.
- Maintain fidelity and avoid interference with strategic movement constraints and optimizations.
- Significant improvements in execution speed and circuit fidelity compared to traditional SWAP-based routing.

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- **Qubit Traps**

- Qubits are held in optical traps generated by Acousto-Optic Deflectors (AODs) and Spatial Light Modulators (SLMs).

- **Reconfigurability**

- AOD traps can move in rows and columns, allowing dynamic reconfiguration.
- AOD rows/columns cannot cross over or overlap during movement.

- **Entangling Gates**

- Entangling gates are applied using a Rydberg laser, effective when qubits are within a blockade range r_b .

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Overview of the Method

- **Goal**

- Develop a compiler for Dynamically Field-Programmable Qubit Arrays (DPQA).
- Optimize the placement and routing of qubits to minimize circuit depth.

- **Approach**

- Discretize the state space and formulate the problem as a Satisfiability Modulo Theories (SMT) problem.
- Use an SMT solver to find optimal solutions.

SMT Model Variables

- **stage** s : Represents the stage or step in the quantum circuit execution process.
- **qubit** q_i : Denotes the i -th qubit in the quantum circuit.
- **gate** g_j : Represents the j -th gate in the quantum circuit.
- **site indices** $x_{i,s}, y_{i,s}$: Coordinates (x, y) of the i -th qubit at stage s .
- **array index** $a_{i,s}$: Indicates whether the i -th qubit at stage s is in a stationary SLM trap ($a_{i,s} = 0$) or in a mobile AOD trap ($a_{i,s} = 1$).
- **AOD indices** $c_{i,s}, r_{i,s}$: Column c and row r indices for the i -th qubit at stage s in the AOD grid.
- **time coordinate** t_j : The time coordinate when the j -th gate is applied.

Spatial Constraints

- Qubits must be within the defined grid bounds:

$$0 \leq x_{i,s} < X, \quad 0 \leq y_{i,s} < Y \quad \forall i \in [0, N), s \in [0, S)$$

- AOD moves:

$$(a_{i,s} = 1) \implies (c_{i,s+1} = c_{i,s} \wedge r_{i,s+1} = r_{i,s})$$

- Site order implying row/column order enforcing:

$$(x_{i,s} < x_{i',s}) \implies (c_{i,s} < c_{i',s}), \quad (y_{i,s} < y_{i',s}) \implies (r_{i,s} < r_{i',s})$$

- AOD rows/columns cannot cross each other:

$$\begin{aligned} ((a_{i,s} = 1) \wedge (a_{i',s} = 1) \wedge (c_{i,s} < c_{i',s})) &\implies (x_{i,s+1} \leq x_{i',s+1}), \\ ((a_{i,s} = 1) \wedge (a_{i',s} = 1) \wedge (r_{i,s} < r_{i',s})) &\implies (y_{i,s+1} \leq y_{i',s+1}). \end{aligned}$$

SMT Model Constraints

■ Interaction Constraints

- Qubits must be within r_b for entangling gates:

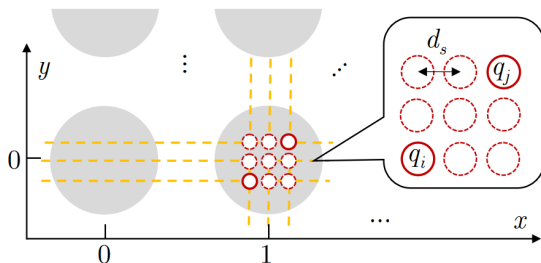
$$((a_{i,s-1} = 1) \wedge (a_{i',s-1} = 1) \wedge (c_{i,s-1} - c_{i',s-1} \geq C_{\text{STK}})) \implies (x_{i,s} > x_{i',s}),$$

$$((a_{i,s-1} = 1) \wedge (a_{i',s-1} = 1) \wedge (r_{i,s-1} - r_{i',s-1} \geq R_{\text{STK}})) \implies (y_{i,s} > y_{i',s}).$$

■ Circuit-Dependent Constraints

- Connectivity ensures:

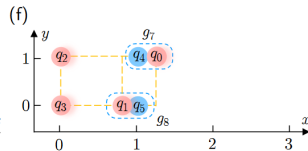
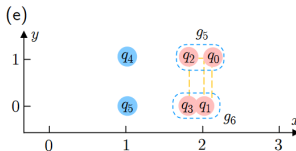
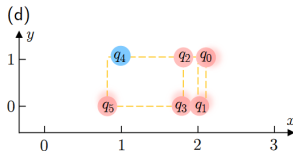
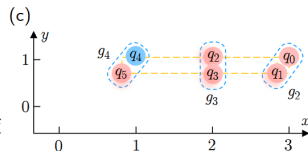
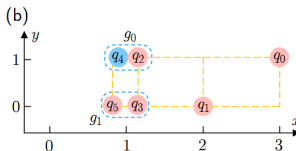
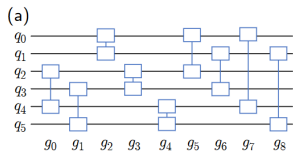
$$(t_j = s) \implies (x_{i,s} = x_{i',s} \wedge y_{i,s} = y_{i',s})$$



Scalability of the Model

- **Variables:** $5NS + G$
- **Constraints:** $O(G^2 + GS + N^2S)$
- **Bits to Represent Variables:** $NS \log(2XYRC) + G \log(S)$
- **Worst-Case Runtime:** $O((N_{SLM}N_{AOD})^{NS} \cdot S^G)$
- **Shallow Circuit Regime and sparse graphs ($G = O(n)$):**
 - Bits required: $O(N \log(N_{SLM}N_{AOD}))$
 - Constraints: $O(N^2)$

Example of Compiled Quantum Circuit



Greedy Heuristic Method

- **Steps:**
 - Construct a "single-step" SMT model with two stages.
 - Optimize the number of gates executed in the second stage.
 - Append the solution to the full solution and remove executed gates.
 - Repeat until fewer than 5% of gates remain.
- **Benefit:** Significantly faster than the optimal compiler with some sacrifice in optimality.

Experimental Results

- **Benchmarks:**

- Evaluated on random graphs.
- Circuits with 10 to 90 qubits.

- **Main Findings:**

- For 90-qubit circuits: 5.1x fewer two-qubit gates compared to fixed planar architecture.

- **Error Sources:**

- AOD movements cause 27x less infidelity.

- **Compiler Performance:**

- Hybrid approach faster than optimal compiler.
- Compiles up to 90-qubit circuits within a day.

Summary

- Utilize reconfigurability and non-local connectivity of DPQA for efficient quantum circuit compilation.
- Formulate constraints as an SMT problem for optimal solutions.
- Implement hybrid approach for scalability.

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Compilation Framework

- **Qubit-Array Mapper**

- Uses MAX k-Cut on a gate frequency graph to minimize SWAP overhead.
- Coarse-grained mapping of qubits to arrays.

- **Qubit-Atom Mapper**

- Fine-grained mapping of qubits to specific atoms in the array.
- Load balance to prevent hardware constraint violations.

- **High-Parallelism Router**

- Iteratively identifies parallelizable 2Q gates.
- Decides atom movements and gate executions to maximize parallelism.

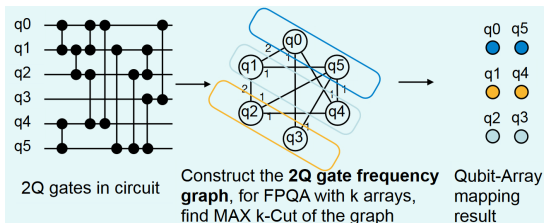
Qubit-Array Mapping Method

- **MAX k-Cut**

- Finds mapping that maximizes inter-array 2Q gates.
- Greedy algorithm used for approximation.

- **Gate Frequency Graph**

- Vertices represent qubits, edges represent gates.
- Edge weights determined by gate frequency.



High-Parallelism Router

- **Non-Dependent Frontier Gates**
 - Identifies and schedules gates with no dependencies.
- **Constraint Checks**
 - Ensures movements do not violate spatial or interaction constraints.
 - Greedily adds gates to the parallel set, checking each for legality.

Unique Experimental Approach

- **Comprehensive Simulations**

- Evaluates logical error rates, execution times, and physical qubit requirements.
- Includes realistic modeling of movement overheads (heating, cooling, decoherence, atom loss).

- **Scalability and Performance**

- Demonstrates significant reductions in 2Q gate count and circuit depth.
- Achieves up to 1000x faster compilation speed compared to solver-based methods.

Summary

- FPQA-C effectively addresses qubit mapping, atom movement, and gate scheduling in FPQA.
- Incorporates innovative methods to handle hardware constraints and optimize performance.
- Experimental results highlight the advantages of FPQA-C over traditional approaches in terms of gate count, depth, and fidelity.

Outline

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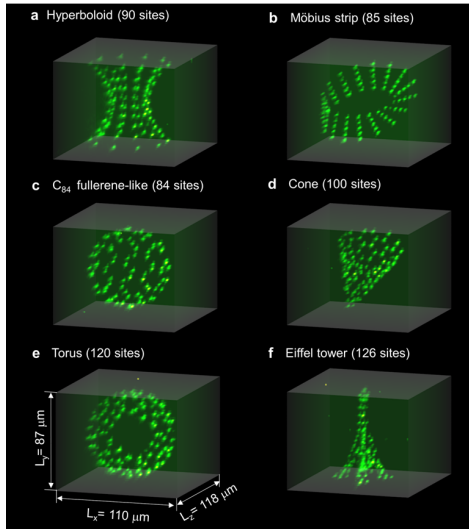


Figure: 3D map from DB17. Q1: Differences between 2D and 3D mapping.

- $CZ : 2|gg\rangle\langle gg| - I$, while
 $CZ : I - 2|11\rangle\langle 11|$
- in gererally:

$$\begin{aligned}
 |gg\rangle &\rightarrow |gg\rangle \\
 |ge\rangle &\rightarrow |ge\rangle e^{i\phi_1} \\
 |eg\rangle &\rightarrow |eg\rangle e^{i\phi_1} \\
 |ee\rangle &\rightarrow |ee\rangle e^{i(2\phi_1 + \pi)}
 \end{aligned}$$

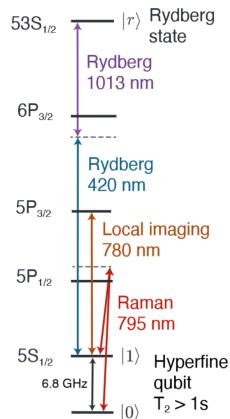


Figure: Level structure for ^{87}Rb atoms, with the relevant atomic transitions employed in *Quera paper*.

How to use mcz

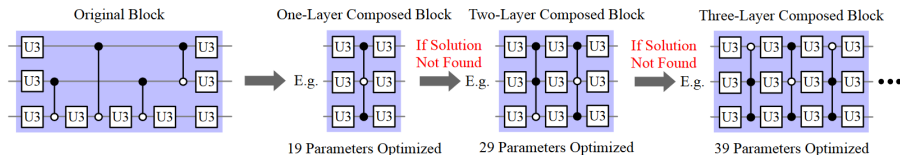


Figure: Main idea in Gyser paper. Q2: Applying some rules to gap the CZ gate circuit and MCZ gate circuits.

How to use mcz

- Q3: How to use mcz to construct circuit directly.

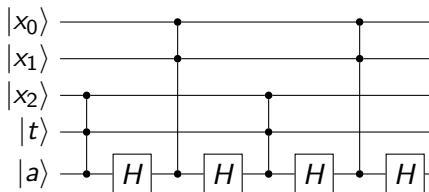


Figure: use ccz and Hadmard gate to construct cccz gate.