Quantum circuit optimizations for NISQ architectures

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outline I

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sparse qubit connectivity

- controlled operations
- NISQ architectures:

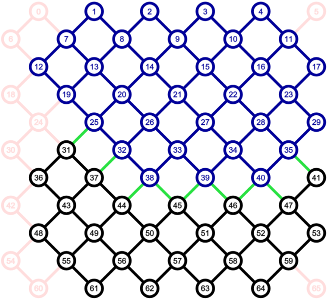
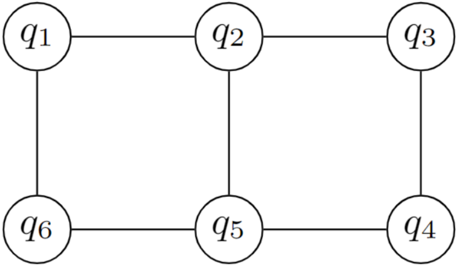


Figure: Qubit ordering and optimal cut for 56-qubit circuit with 20 cycles in [**Nash˙2020**]

different sequence of operations



$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

linear reversible circuit synthesis

- input: the qubit connectivity graph and a binary matrix
- $O\left(\frac{n^3}{\log n}\right)$ VS $O\left(n^2\right)$
- universal set

contents

steiner tree

- terminals and steiner nodes
- minima steiner nodes to connect terminals

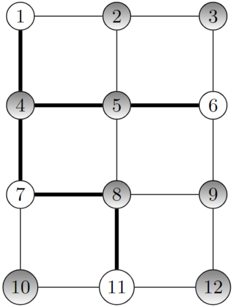


Figure: a solution to steiner tree problem[**Nash˙2020**]

generate tree

- root c
- breadth first tree order
- sub-tree rooted at terminal c_i

sequence of row generations

- start with last sub-tree
- sequence R: traverse the tree in reverse depth first order
- sequence $R^{'} = reverse(R - R[j])$
- sequence $R^{*} = R + R^{'} - R_S$

work flow

- start with column $i = 1$
- judge $(i, i) = 0?$
- find steiner tree
- perform row operations and compute resulting matrix
- repeat 2-5
- transpose

result for CNOT

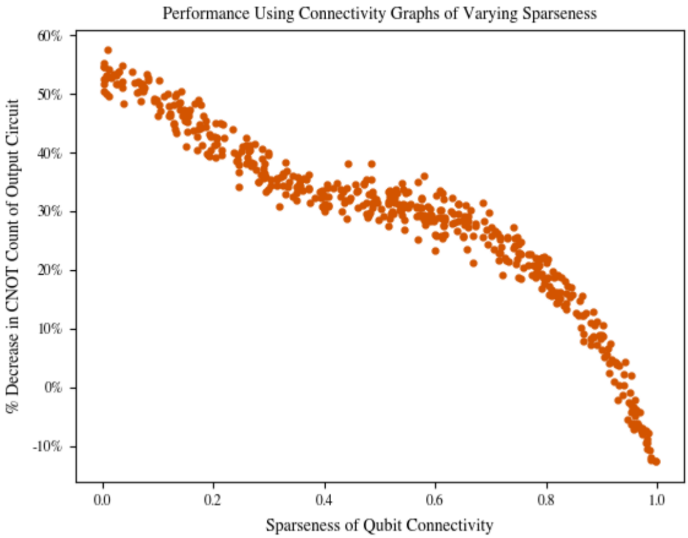


Figure: result for the synthesis of CNOT circuits on 20 qubits

contents

sum-over-paths form

- $\{CNOT, R_Z\}$:phase polynomial f and matrix A
- $U_C = \sum_{\mathbf{x} \in \mathbb{F}_2^n} e^{2\pi i f(\mathbf{x})} |A\mathbf{x}\rangle\langle\mathbf{x}|$
- $f(\mathbf{x}) = \sum_{\mathbf{y} \in \mathbb{F}_2^n} \hat{f}(\mathbf{y}) \left(x_1 y_1 \oplus x_2 y_2 \oplus \ldots \oplus x_n y_n\right)$

algorithm[Patel˙2008]

- compute a minimal parity network and compute the linear transformation C
- compute the linear transformation AC^{-1}

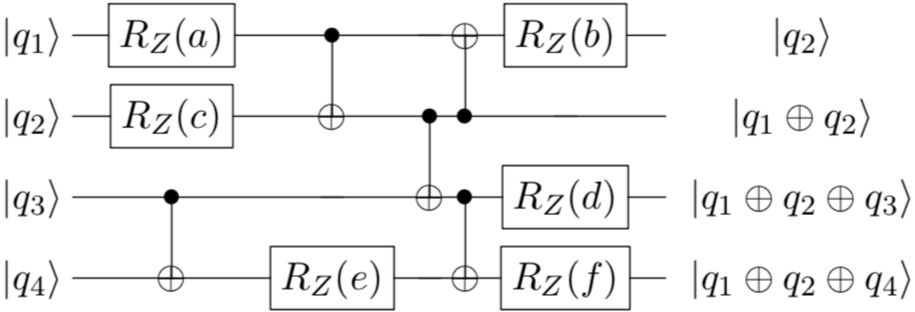


Figure: CNOT + phase circuit example

universal set

- $\{CNOT, S, T, S^\dagger, T^\dagger, H\}$
- for a circuit C, $S_{k,CNOT} S_{k,H} \ldots S_{1,CNOT} S_{1,H} = C$

results for CNOT + R_Z

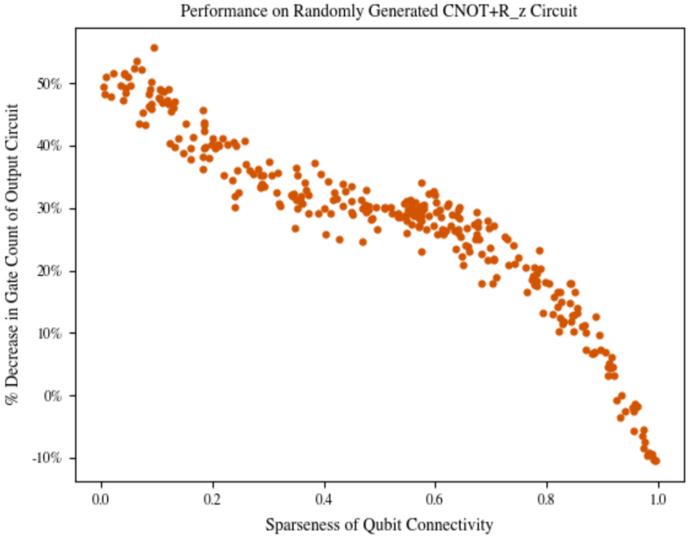


Figure: results for the synthesis of $CNOT + R_Z$ circuits on 20 qubits

results for universal sets

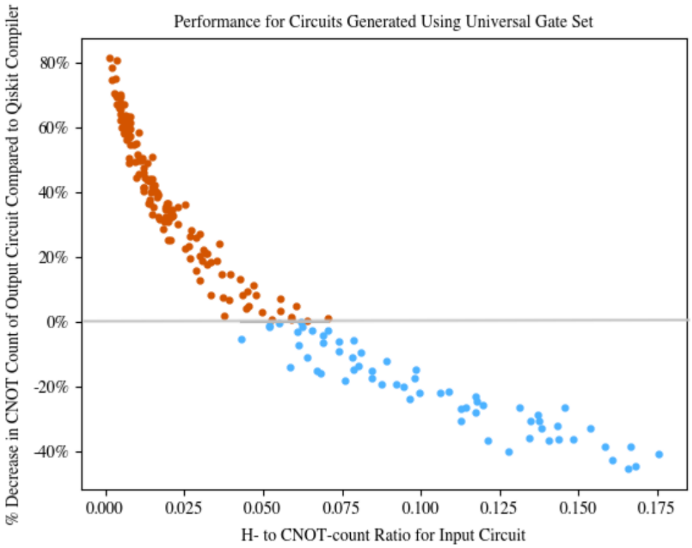


Figure: generate circuits from universal set $\{CNOT, S, T, S^\dagger, T^\dagger, H\}$ compared with IBM's compiler

summary

- tip: not necessarily starting form row i
- disadvantage: partition

END

Happy new year