Circuit Partitioning and Transmission Cost Optimization in Distributed Quantum Circuits (IEEE TCAD)

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Motivation: Challenges in Distributed Quantum Computing

- NISQ devices have limited qubits, preventing large-scale quantum computation.
- Distributed quantum computing partitions circuits across multiple QPUs.
- Excessive inter-QPU communication increases error rates and latency.

Background: state transmission

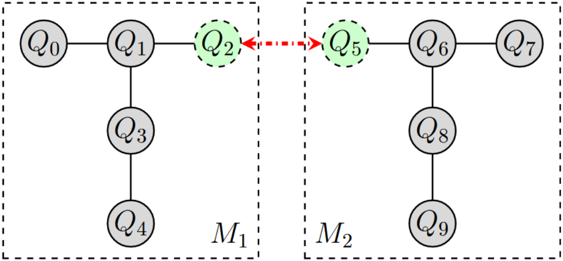


Figure: An example of a distributed quantum architecture which consists of two IBMQ quito architectures. This work focuses on direct quantum state transmission as the technology for transferring quantum states in the distributed quantum circuits.

Methodology

- Circuit Partitioning as a Graph Cut Problem
- Dynamic Lookahead Transmission Optimization

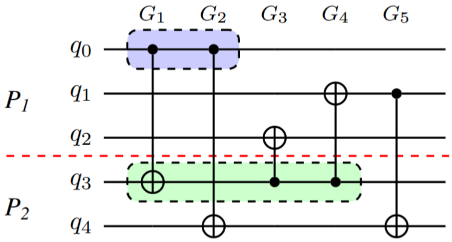
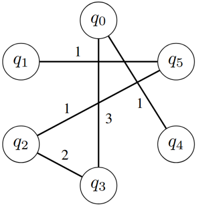


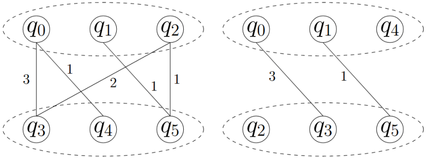
Figure: Example of the impact of transmission qubits on transmission cost. The final transmission cost of using q0 as the transmission qubit is 6, while the final transmission cost of using q3 is 4.

Circuit Partitioning as a Graph Cut Problem

- Quantum circuits are represented as a qubit-weighted graph.
- Partitioning aims to minimize inter-QPU quantum gates.



(a)



(b)

Figure: (a): Qubit-weighted graph with nodes as qubits, edges as quantum gates acting between qubits, and weights as the number of quantum gates. (b): Global gate dispersion relation graph.

Introduction to QUBO

- **Quadratic Unconstrained Binary Optimization (QUBO)** is a mathematical model used for solving combinatorial optimization problems.
- It involves minimizing a quadratic function of binary variables, where each variable can be either 0 or 1.

QUBO Objective Function (from wiki)

The general form of the QUBO objective function is:

$$f(x) = x^T Q x = \sum_{i=1}^{n} \sum_{j=i}^{n} Q_{ij} x_i x_j$$

where:

- $x = (x_1, x_2, \ldots, x_n)$ is a vector of binary variables.
- Q is an upper-triangular matrix of real coefficients defining interactions between variables.

Example of a QUBO Problem

Consider a simple QUBO problem with three binary variables:

$$f(x) = Q_{11}x_1^2 + Q_{22}x_2^2 + Q_{33}x_3^2 + Q_{12}x_1x_2 + Q_{13}x_1x_3 + Q_{23}x_2x_3$$

Represented in matrix form:

$$Q = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ 0 & Q_{22} & Q_{23} \\ 0 & 0 & Q_{33} \end{pmatrix}$$

Including Constraints in QUBO

- Constraints in QUBO are introduced as penalties.
- Penalty functions penalize infeasible solutions.
- General form with constraints:

$$f(x) = \sum_{i=1}^{n} \sum_{j=i}^{n} Q_{ij} x_i x_j + \sum_{k} P_k \cdot \text{penalty}_k(x)$$

where:

  - P_k are penalty coefficients.
  - $\text{penalty}_k(x)$ measures constraint violations.

Natural QUBO Formulations (from A Tutorial)

- The Number Partitioning Problem: For $S = \{3, 1, 1, 2, 2, 1\}$, one possible partition is $S_1 = \{1, 1, 1, 2\}$ and $S_2 = \{2, 3\}$, both summing to 5.
- The Max Cut Problem: In a triangle graph with vertices $\{A, B, C\}$ and edges $\{(A, B), (B, C), (C, A)\}$, one maximum cut is $S = \{A\}$ and $\bar{S} = \{B, C\}$, resulting in two edges crossing the cut.
- Various algorithms can solve QUBO problems, including **Quantum annealing methods using quantum computers**. (Ref. [1811.11538,s41598-019-53585-5])

Comparison with Existing Methods

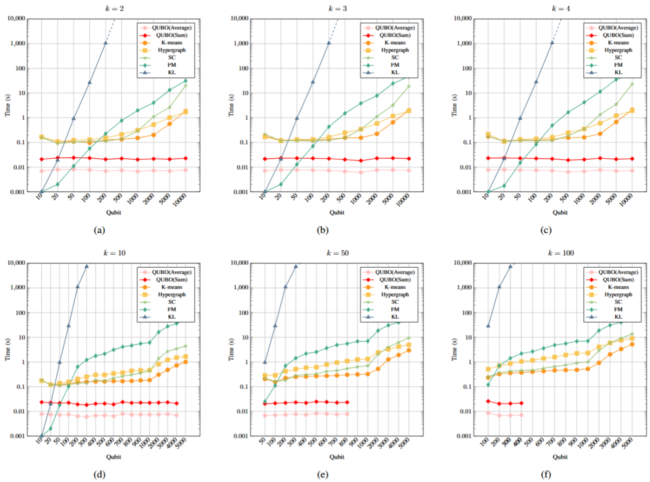


Figure: Comparison of runtime performance between the QUBO model-based and other circuit partitioning methods under different numbers of qubits and partition counts k.

Dynamic Lookahead Transmission Optimization

- Looks ahead dynamically to evaluate impact of transmission choices. Prioritizes merging quantum gate executions to minimize state transfers.

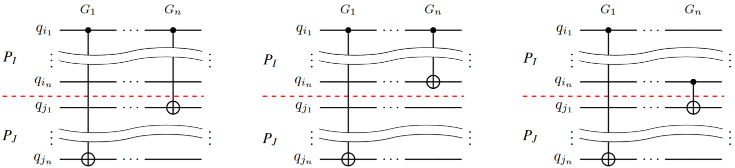


Figure: The different impacts of transferring qubit q_{i1} on subsequent gates.

Performance Gains

- Achieving an average optimization rate of 18.12% and a peak rate of 73.85% in transmission cost compare with meta-heuristic methods.
- Achieving an average optimization rate of 32.27% and a peak rate of 55.56% with greedy strategies.

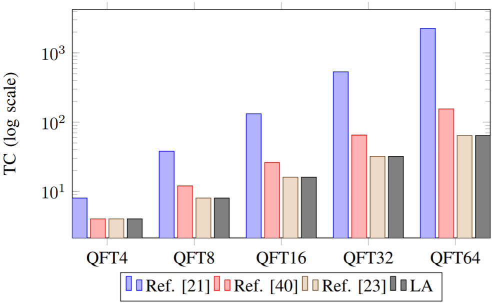


Figure: Bar chart comparing the transmission cost results in QFT circuits.

Conclusion

- Proposed QUBO-based partitioning reduces inter-QPU gates.
- Dynamic lookahead scheduling significantly lowers transmission cost.
- Approach enables more efficient execution of distributed quantum algorithms.