realization of three qubit-gates and synthesis Algorithm

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Two-Qubit Quantum Gates Recap

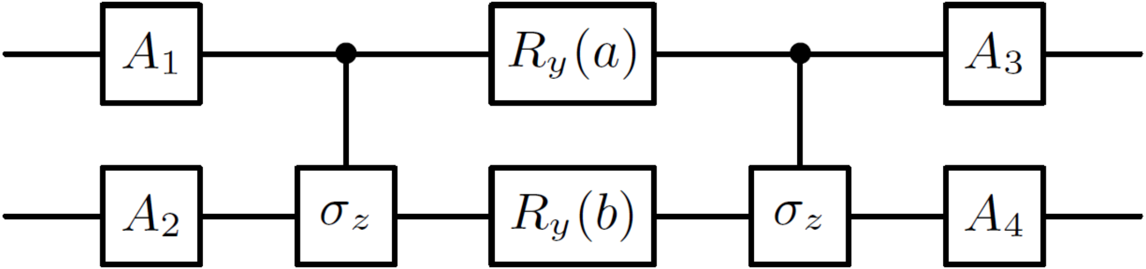


Figure: Optimal Quantum Circuits for General Two-Qubit Gates are 15 single qubit gates and 3 CNOT gates[2]

Realization of Three-Qubit Quantum Gates

- universal gate family: $\{R_z, R_y, CNOT\}$
- basic idea: KAK Algorithm[1], block-diagonal matrix
- results:98 single qubits and 40 CNOT gates

KAK Decomposition[1]

any $SU(8)$ can be decomposed as

$$U = K_1 \exp\left(-i\left(\beta_1 XXX + \beta_2 YYX + \beta_3 ZZX + \beta_4 IIX\right)\right) K_2 \tag{1}$$

$$K_1 = P_1 \exp\left(-i\left(\alpha_1 XXX + \alpha_2 YYZ + \alpha_3 ZZZ\right)\right) P_2, \tag{2}$$

$$K_1 = P_3 \exp\left(-i\left(\gamma_1 XXX + \gamma_2 YYZ + \gamma_3 ZZZ\right)\right) P_4 \tag{3}$$

KAK Decomposition[1]

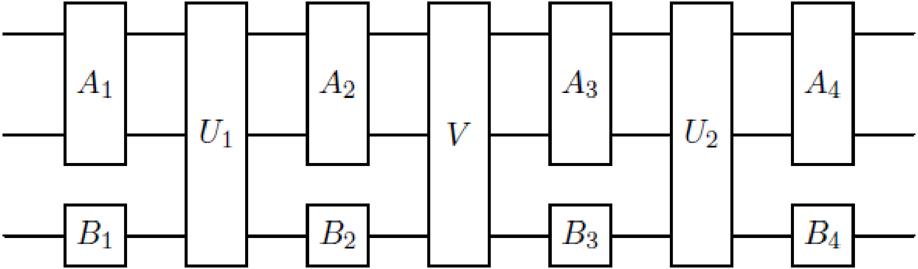


Figure: KAK decomposition of a three-qubit unitary operation.

Diagonalization

- target:

$$P = \begin{bmatrix} P_1 & & & \\ & P_1 & & \\ & & P_2 & \\ & & & P_2 \end{bmatrix}, \tag{4}$$

$$P_1 = \begin{bmatrix} \cos(a-b) & i\sin(a-b) \\ i\sin(a-b) & \cos(a-b) \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} \cos(a+b) & i\sin(a+b) \\ i\sin(a+b) & \cos(a+b) \end{bmatrix} \tag{5}$$

- using $\{CNOT, CZ, SWAP\}$

Block-Diagonalization

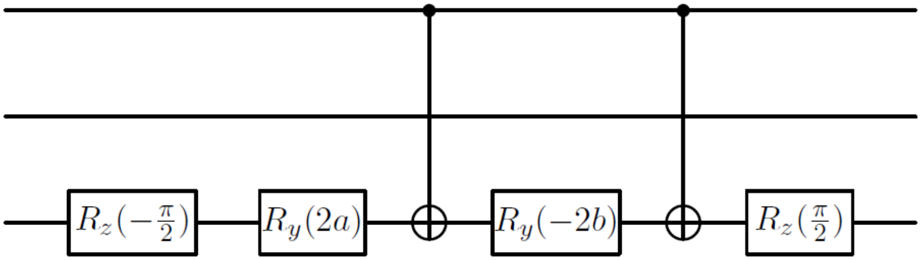


Figure: a realization of Block-Diagonalization P

Diagonalization[3]

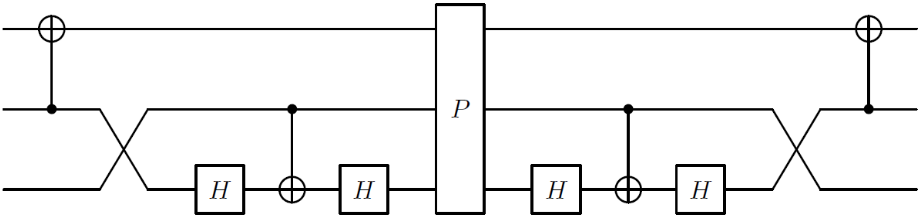


Figure: Decomposition of the unitary operationexp$(i(aXXZ + bYYZ))$

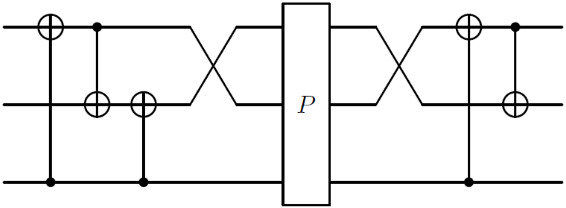


Figure: Decomposition of the unitary operation $\exp(i(\alpha_1 XXX + \alpha_2 YYX))$

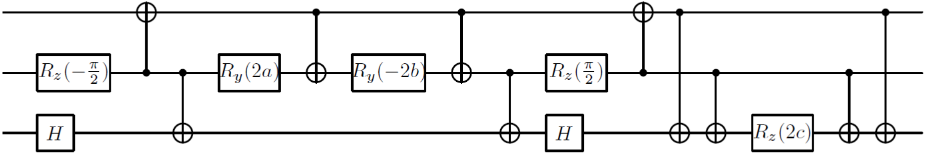


Figure: a realization of U

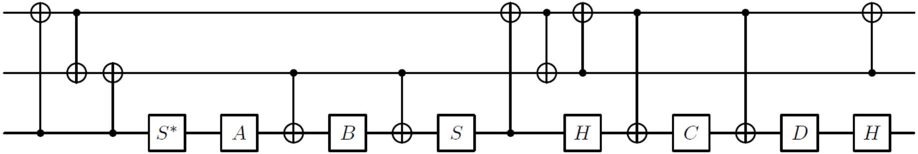
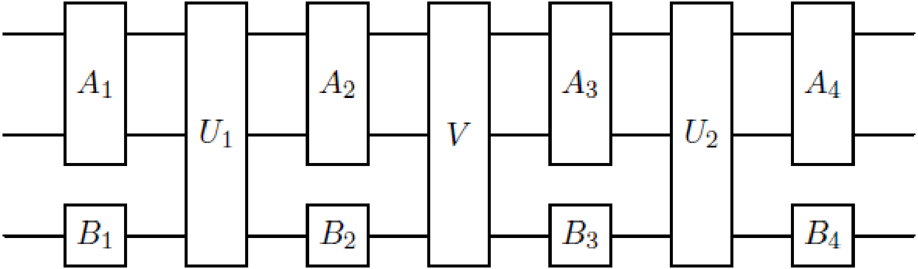


Figure: a realization of V

Count



$$9 \times 2 + 10 + 3 \times 4 = 40 \tag{6}$$

$$5 \times 2 + 6 + 15 \times 4 + 3 \times 4 = 88 \tag{7}$$

Top-Down vs Bottom-Up Synthesizers

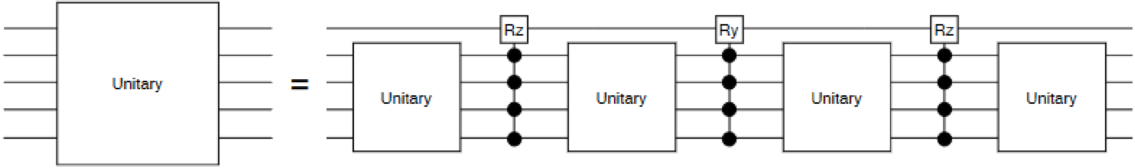


Figure: Top-down synthesizers,decompose large unitaries into smaller ones while maintaining equality

Top-Down vs Bottom-Up Synthesizers

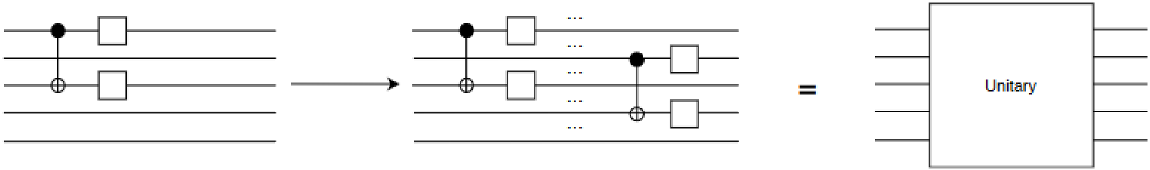


Figure: Bottom-up synthesizers start with an empty circuit and build up to equality.

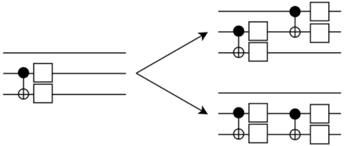


Figure: QSearch uses native gates in synthesis and searches for structure in their circuit space. github repository: `https://github.com/BQSKit/qsearch`

Basic idea to QFAST Algorithm[4]

- use function to represent gates and circuits
- replaces expensive searches over circuit structures with numerical optimization
- github repository: `https://github.com/BQSKit/qfast`

Gate Representation

$$F(Q, \vec{\alpha}) = P_Q(G(\vec{\alpha}) \otimes I)P_Q^T \tag{8}$$

$$V(\vec{Q}, \vec{\alpha}, \vec{I}) = \left(\sum_{Q \in \vec{Q}} I_Q \cdot P_Q\right)(G(\vec{\alpha}) \otimes I)\left(\sum_{Q \in \vec{Q}} I_Q \cdot P_Q^T\right) \tag{9}$$

where

$$G(\vec{\alpha}) = e^{\mathrm{i}\left(\vec{\alpha} \cdot \sigma^{\vec{\otimes} n}\right)} \tag{10}$$

Cost Function for Optimization

derive form Frobenius norm

$$\Delta\left(U_C, U_T\right) = 1 - \frac{\left|\operatorname{Tr}\left(U_T^{\dagger} U_C\right)\right|}{d} \tag{11}$$

- decomposition
- instantiation
- recombination

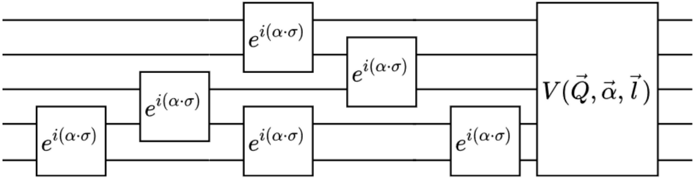


Figure: Decomposition step in QFAST

Comparison with Other Algorithms

- Evaluation Metrics:
  - total CNOT gate count
  - total count of single qubit gates
  - critical path length
  - average gate parallelism
  - scalability and execution time
- Benchmark: Transverse Field Ising Models
- 3-7 qubits

Comparison results

compare with Qiskit on average:

- $10\times$ fewer CNOT gates
- $5.2\times$ fewer U3 gates
- $5.7\times$ decrease of the circuit critical
- $1.03\times$ better parallelism
- $15\times$ slower than IBM Qiskit
- Qiskit:10^{-14},QFAST: 10^{-6}

Comparison results

tfim-7-20	Qiskit mapped	Qiskit synthesized	QFAST
singal gate	260	19360	89
CNOT	240	18653	41
critical patth	152	89478	55
pararllelism	3.29	1.03	2.77
time	0.61	13222.11	307.57

Table: In terms of ALL-to-ALL synthesis results. QFAST time out when running the tfim-7-40 and tfim-7-100 benchmark examples

Comparison results

tfim-7-100	Qiskit mapped	Qiskit synthesized	QFAST
ssingal gate	1300	89483	87
CNOT	1200	58316	40
critical patth	712	82915	49
pararllelism	3.51	1.78	2.59
time	6.09	705.01	8208.44

Table: In terms of Linear synthesis results.

Summary

- general three-qubit gate:40 CNOT and 88 single qubits gates
- a bottom-up synthesis algorithm:QFAST

references I

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END

Thank you