



Computation techniques for encrypted data using Python

Gajendra Deshpande

KLS Gogte Institute of Technology, India

<https://gcdeshpande.github.io>

Contents

- ▣ Homomorphic Encryption
- ▣ Properties of Homomorphic Encryption
- ▣ Summary of Homomorphic Properties
- ▣ The Example
- ▣ Experiment with RSA Algorithm
- ▣ Results
- ▣ Machine Learning and Homomorphic Encryption
- ▣ Conclusion

Homomorphic Encryption

- Imagine taking all of your credit card statements and locking them into a safe, to which you have the only key. Your statements are now protected from prying eyes. This is what encryption does.
- But what if you wanted to analyse your expenditure on groceries in the last 12 months? First you would have to unlock the safe and retrieve the statements. So now the documents are out in the open and they can be read by anyone. This is what decryption does.
- The difference with Homomorphic Encryption is that you can create your report without taking the documents out of the safe.

Properties of Homomorphic Encryption

▣ Additive Homomorphic Encryption:

A Homomorphic encryption is additive, if

$$E_k (PT1 \oplus PT2) = E_k (PT1) \oplus E_k (PT2)$$

As the encryption function is additively homomorphic, the following identities can be described:

The product of two cipher texts will decrypt to the sum of their corresponding plaintexts,

$$D (E (m1) \cdot E (m2) \bmod n) = m1 + m2 \bmod n.$$

The product of a cipher text with a plaintext raising g will decrypt to the sum of the corresponding plaintexts,

$$D (E (m1) \cdot g^{m2} \bmod n) = m1 + m2 \bmod n.$$

Properties of Homomorphic Encryption

- **Multiplicative Homomorphic Encryption:** Homomorphic encryption is multiplicative, if

$$E_k (PT1 \otimes PT2) = E_k (PT1) \otimes E_k (PT2)$$

- The homomorphic property of the RSA.

Suppose there are two cipher texts, CT1 and CT2.

$$CT1 = m1^e \bmod n$$

$$CT2 = m2^e \bmod n$$

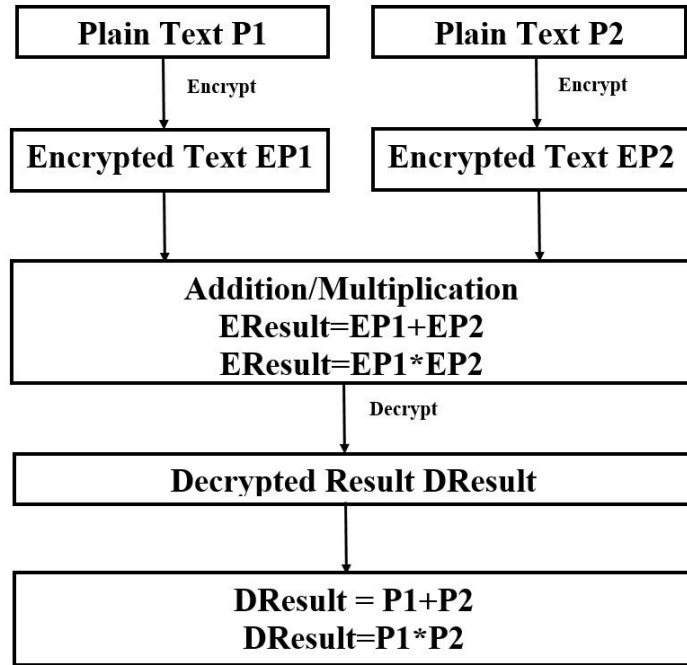
$$CT1 \cdot CT2 = m1^e \cdot m2^e \bmod n$$

So, multiplicative property: $(m1 \cdot m2)^e \bmod n$

Summary of Homomorphic Properties

Algorithm	Additive	Multiplicative	Applications
RSA	No.	Yes	To secure Internet Banking and credit card transactions
Paillier	Yes	No	E-voting system
ElGamal	No.	Yes	In Hybrid Systems

The Example



- $P1 = 5; P2 = 10$
- $EP1 = 50 \ EP2 = 100$
- $EResult1 = 50 + 100 = 150;$
 $Eresult2 = 50 * 100 = 5000$
- $DResult1 = 15;$
 $DResult2 = 50$

Experiment with RSA Algorithm

- Selecting two large primes at random: p, q
- Computing their system modulus $n=p.q$
- Note $\phi(n)=(p-1)(q-1)$
- Selecting at random the encryption key e where $1 < e < \phi(n)$, $\gcd(e, \phi(n))=1$
- Solve following equation to find decryption key d
- $e.d=1 \bmod \phi(n)$ and $0 \leq d \leq n$
- Publish their public encryption key: $pu=\{e,n\}$
- Keep secret private decryption key: $pr=\{d,n\}$

Experiment with RSA Algorithm

- To encrypt a message M the sender:
 - obtains public key of recipient $pu=\{e,n\}$
 - computes: $C = m^e \bmod n$, where $0 \leq m < n$
- To decrypt the ciphertext C the owner:
 - uses their private key $pr=\{d,n\}$
 - computes: $M = c^d \bmod n$

Experiment with RSA Algorithm

Grade School Method	Karatsuba Method	Fast Fourier Transform
$O(n^2)$	$O(n^{\log_2 3}) = O(n^{1.58})$	$\Theta(n \log(n) \log(\log(n)))$

- ❑ The multiplication algorithms were implemented in two steps
- ❑ Multiplication algorithms in RSA algorithm were replaced by Karatsuba and FFT methods one after another.
- ❑ Multiplication operations were performed on encrypted numbers by Karatsuba and FFT Methods.

ActivitiesTerminalNov 2 07:14gajendra@gajendra-HP-Laptop-15-da0xxx: ~/Desktop/scipy 2019/Scipy

```
(base) gajendra@gajendra-HP-Laptop-15-da0xxx:~/Desktop/scipy 2019/Scipy$ python2 rsakar.py
First list of random numbers
[22]

Second list of random numbers
[87]

0.421537876129
0.391266107559
0.39028096199
0.389554977417
0.389764070511
0.389742851257
('M = ', 3125743, '\ne = ', 5)
Encryption of First list of Random numbers
[2027889]

Encryption of Second list of Random Numbers
[1774865]

Decryption of First list of Random Numbers
[22]

Decryption of Second list of Random Numbers
[87]

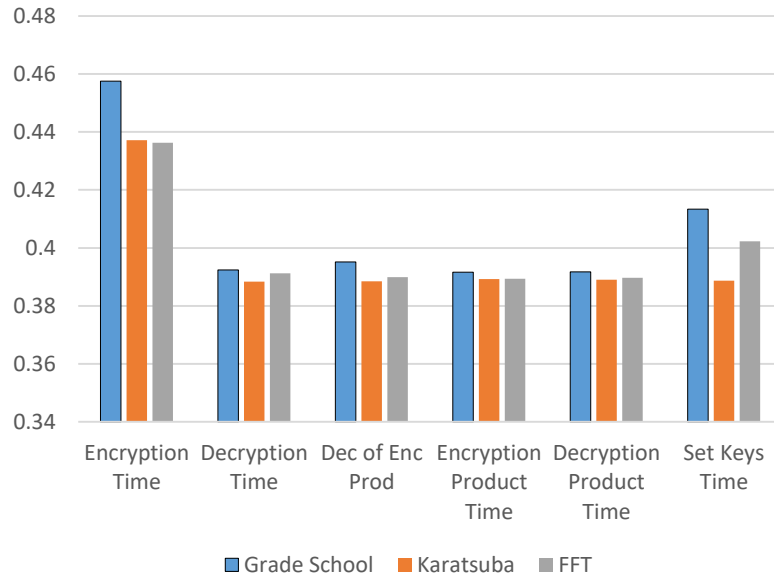
Product of Two Encrypted Lists
[[3599229209985]]

Product of Two Decrypted Lists
[[1914]]

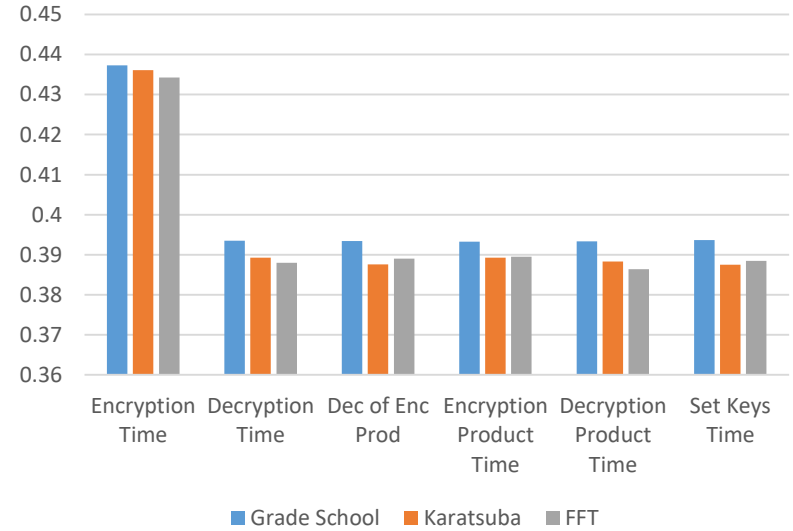
Decryption of Product of Encrypted Lists
[1914L]
```

Results-High Resource

Comparison of Multiplication in RSA

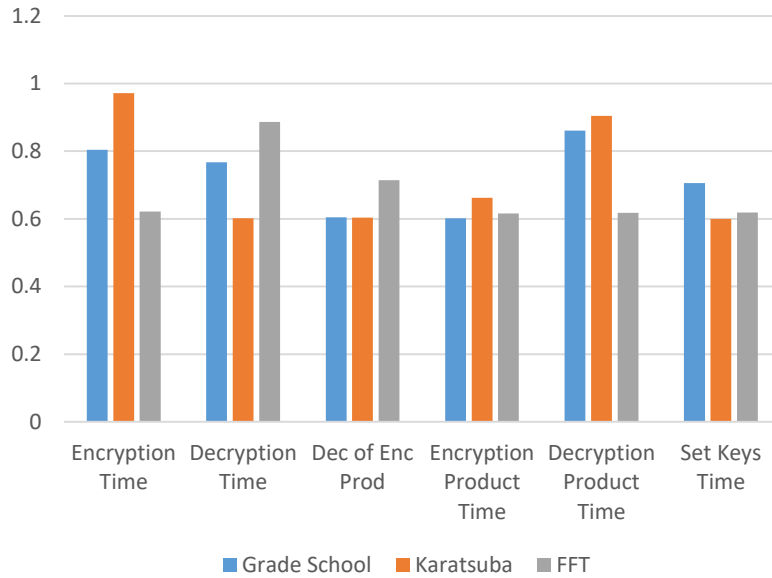


Comparison of Multiplication on Encrypted Text

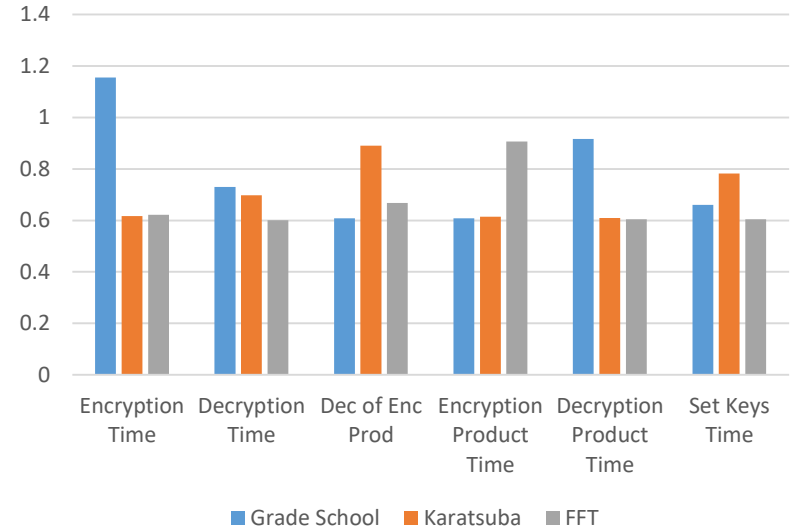


Results-Low Resource

Comparison of Multiplication in RSA



Comparison of Multiplication on Encrypted Text



Machine Learning and Homomorphic Encryption

- ❑ Pre-processing: Map the numbers in the dataset to random numbers
- ❑ Encrypt the data set using cryptographic algorithms such as RSA, paillier or any other cryptosystem
- ❑ Perform the computations on encrypted data
- ❑ Decrypt the results
- ❑ Post-processing: rounding up/down, remap random numbers to original numbers

```
#!/usr/bin/env python
__author__ = 'Tom Schaul, tom@idsia.ch'

from pybrain.datasets import SupervisedDataSet, ImportanceDataSet

class XORDataSet(SupervisedDataSet):
    """ A dataset for the XOR function."""
    def __init__(self):
        SupervisedDataSet.__init__(self, 2, 1)
        self.addSample([0,0],[0])
        self.addSample([0,1],[1])
        self.addSample([1,0],[1])
        self.addSample([1,1],[0])

class SequentialXORDataSet(ImportanceDataSet):
    """ same thing, but sequential, and having no importance on a second output"""
    def __init__(self):
        ImportanceDataSet.__init__(self, 2, 2)
        self.addSample([0,0],[0, 1], [1,0])
        self.addSample([0,1],[1, 10], [1,0])
        self.addSample([1,0],[1, -1], [1,0])
        self.addSample([1,1],[0, 0], [1,0])

~
~
"xorco.py" 24 lines, 801 characters
```

```
Activities Terminal Nov 2 14:05
gajendra@gajendra-HP-Laptop-15-da0xxx: ~/Desktop/scipy 2019
gajendra@gajendra-HP-Laptop-15-da0xxx: ~/pybrain/examples/supervised/backprop
osbrain-master osbrain-master.zip RSA rsa1.py rsa2.py RSA-Library-master rsa.py rsa.pyc Scipy spade-master spade
-master.zip
(base) gajendra@gajendra-HP-Laptop-15-da0xxx:~/Desktop/scipy 2019$ python rsa2.py

1. Set Public Key
2. Encode
3. Decode
0. Quit
Your choice? 1
p: 17
q: 19
N = 323
e = 5

1. Set Public Key
2. Encode
3. Decode
0. Quit
Your choice? 2
Number to encode: 10
193
Number to encode: 20
39
Number to encode: 0

1. Set Public Key
2. Encode
3. Decode
0. Quit
Your choice? 0
(base) gajendra@gajendra-HP-Laptop-15-da0xxx:~/Desktop/scipy 2019$
```



```
#!/usr/bin/env python
__author__ = 'Tom Schaul, tom@idsia.ch'

from pybrain.datasets import SupervisedDataSet, ImportanceDataSet

class XORDataSet(SupervisedDataSet):
    """ A dataset for the XOR function."""
    def __init__(self):
        SupervisedDataSet.__init__(self, 2, 1)
        self.addSample([193,193],[193])
        self.addSample([193,39],[39])
        self.addSample([39,193],[39])
        self.addSample([39,39],[193])

class SequentialXORDataSet(ImportanceDataSet):
    """ same thing, but sequential, and having no importance on a second output"""
    def __init__(self):
        ImportanceDataSet.__init__(self, 2, 2)
        self.addSample([0,0],[0, 1], [1,0])
        self.addSample([0,1],[1, 10], [1,0])
        self.addSample([1,0],[1, -1], [1,0])
        self.addSample([1,1],[0, 0], [1,0])
```

```
#!/usr/bin/env python
```

```
# A simple feedforward neural network that learns XOR.
```

```
__author__ = 'Tom Schaul, tom@idsia.ch'
```

```
from datasets import XORDataSet #@UnresolvedImport
```

```
from pybrain.tools.shortcuts import buildNetwork
```

```
from pybrain.supervised import BackpropTrainer
```

```
def testTraining():
```

```
    d = XORDataSet()
```

```
    n = buildNetwork(d.indim, 4, d.outdim, recurrent=True)
```

```
    t = BackpropTrainer(n, learningrate = 0.01, momentum = 0.99, verbose = True)
```

```
    t.trainOnDataset(d, 1000)
```

```
    t.testOnData(verbose= True)
```

```
if __name__ == '__main__':
```

```
    testTraining()
```

```
~
```

```
~
```

```
~
```

```
~
```

```
~
```

```
~
```

```
~
```

```
❏ "hexor.py" 20 lines, 559 characters
```

Total error: 2182.41045911

Total error: 2119.30936398

Testing on data:

('out: ', '[156.163]')

('correct:', '[193]')

error: 678.49269375

('out: ', '[156.163]')

('correct:', '[39]')

error: 6863.55117722

('out: ', '[52.638]')

('correct:', '[39]')

error: 92.99464931

('out: ', '[156.163]')

('correct:', '[193]')

error: 678.49269375

('All errors:', [678.4926937498457, 6863.551177222227, 92.99464931148584, 678.4926937498603])

('Average error:', 2078.3828035083548)

(base) gajendra@gajendra-HP-Laptop-15-da0xxx:~/pybrain/examples/supervised/backprop

(base) gajendra@gajendra-HP-Laptop-15-da0xxx:~/pybrain/examples/supervised/backp

rop\$

Conclusion



- Homomorphic Encryption enables computation on untrusted resource. The Computation time over cipher text can be reduced by using Karatsuba or FFT techniques.
- Training and testing machine learning model may involve additional steps such as pre-processing and post-processing and results into additional computational complexity.