

Homework 1

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Problem 1

$$p(\theta) = \frac{1}{21} \text{ for } \theta = 0, 0.05, 0.1, \dots, 1$$
$$Y|\theta \sim \text{Binom}(n, \theta)$$

$$P(Y = 5|\theta = .05) = \binom{n}{5} \theta^5 (1 - \theta)^{n-5} = \binom{100}{5} .05^5 (1 - .05)^{100-5} \approx .180$$

$$P(Y = 5|\theta = .5) = \binom{n}{5} \theta^5 (1 - \theta)^{n-5} = \binom{100}{5} .5^5 (1 - .5)^{100-5} \approx 5.94 \times 10^{-23} \approx 0$$

$$P(\theta = .05|Y = 5) = \frac{P(Y = 5|\theta = .05)P(\theta = .05)}{P(Y = 5)}$$

If we denote $\Theta = \{0, 1, 2, \dots, 20\}$ then $\theta \in \frac{\Theta}{20}$

$$P(Y = 5) = \sum_{\theta' \in \Theta} P(Y, \theta') = \sum_{\theta' \in \Theta} P(Y|\theta')P(\theta') = \sum_{i=0}^{20} P(Y = 5|\theta' = \frac{i}{20})P(\theta' = \frac{i}{20})$$

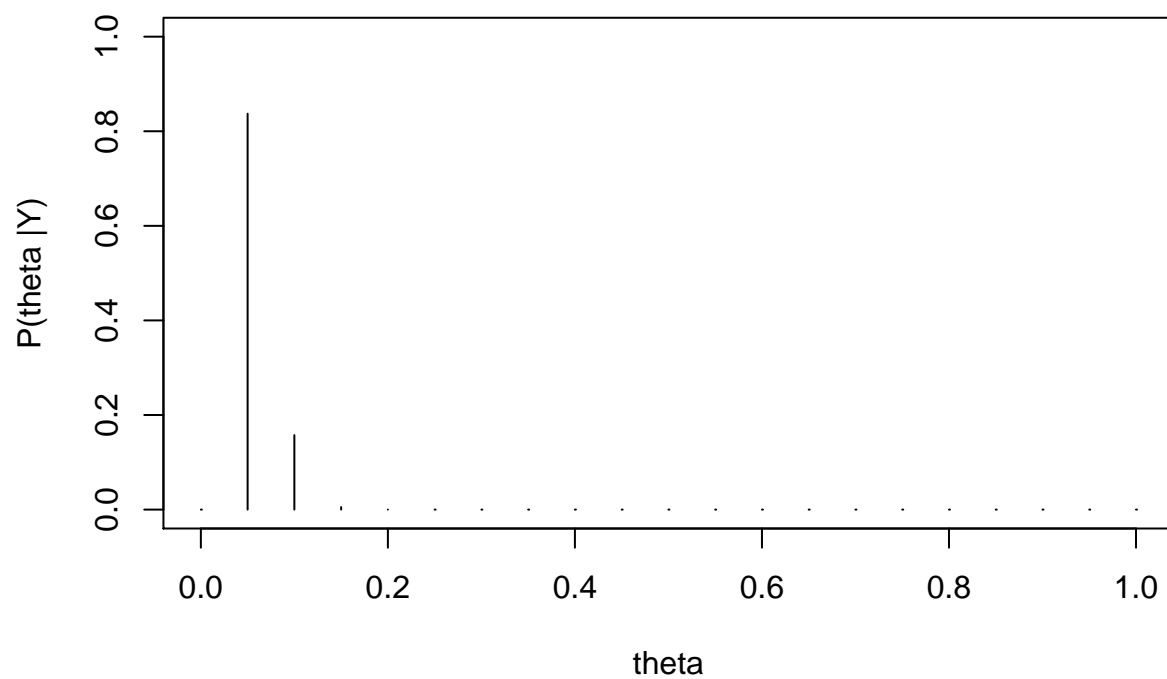
$$= \sum_{i=0}^{20} \binom{100}{5} \left(\frac{i}{20}\right)^5 \left(1 - \left(\frac{i}{20}\right)\right)^{100-5} \frac{1}{21} \approx .010$$

$$P(\theta = .05|Y = 5) = \frac{.18 * \frac{1}{21}}{.010} \approx .857$$

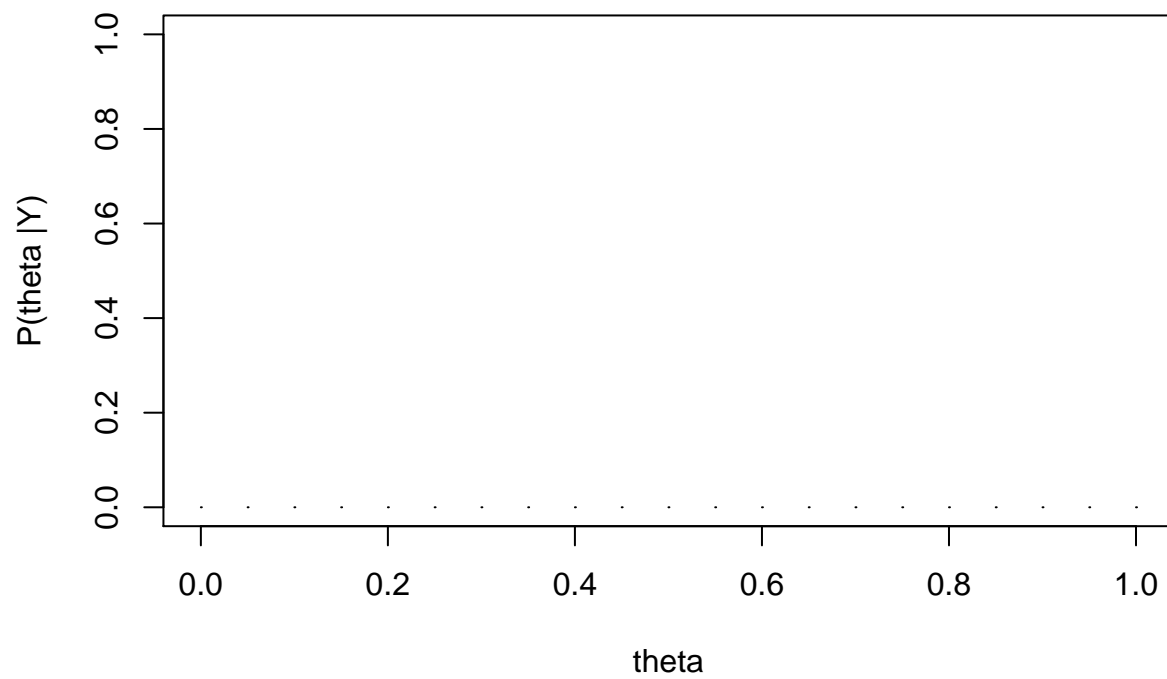
$$P(\theta = .5|Y = 5) = \frac{0 \frac{1}{21}}{.010} \approx 0$$

Problem 2

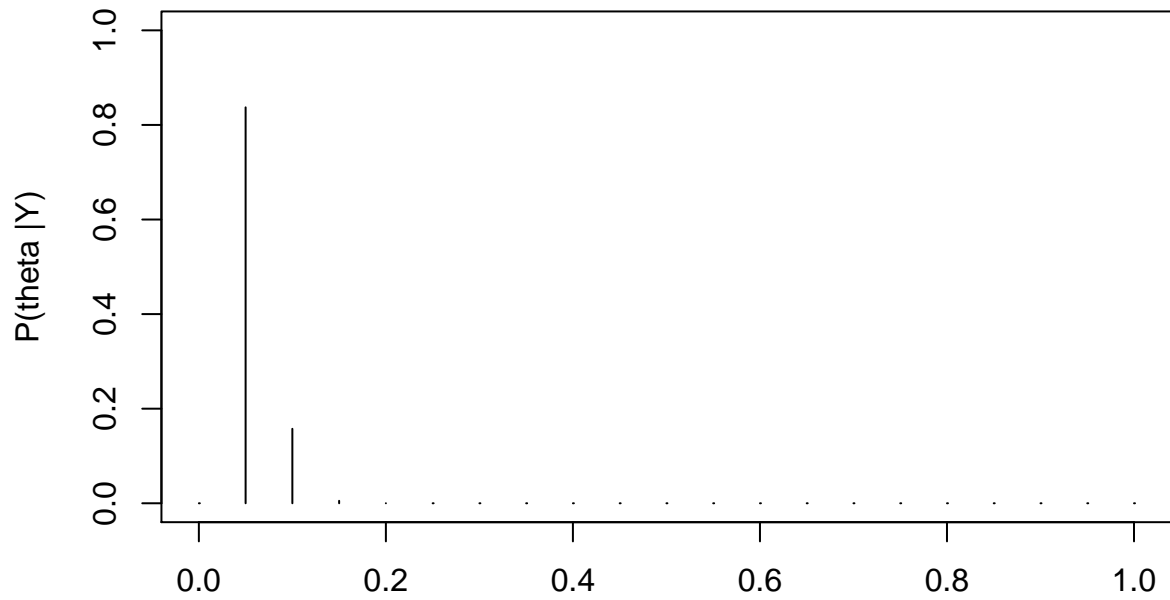
Posterior under prior 1



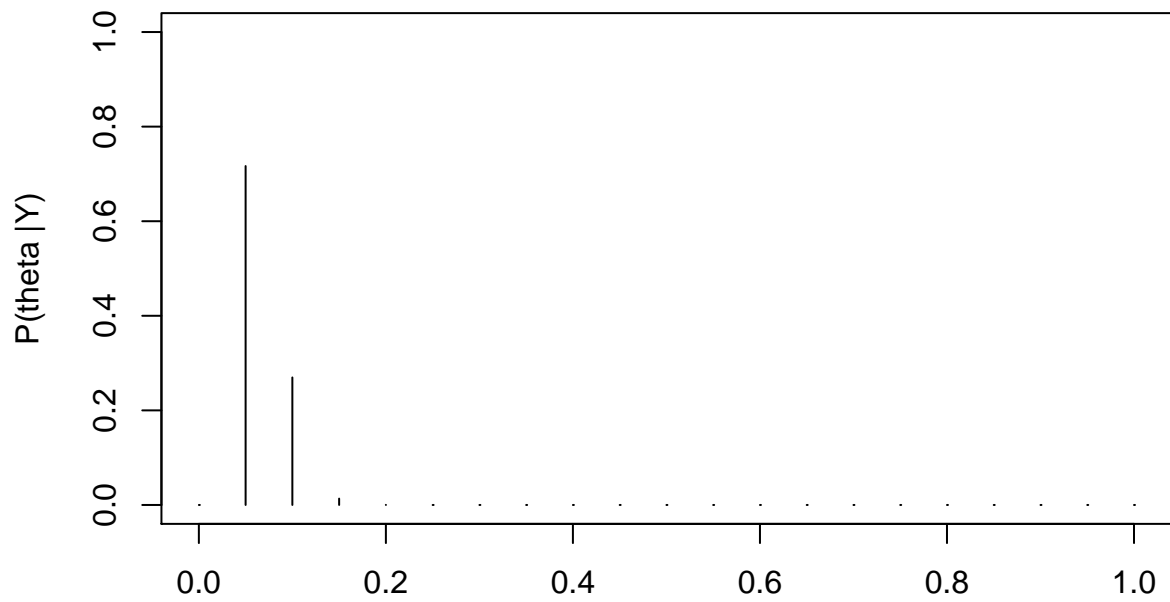
Posterior under prior 2



Posterior under prior 3



Posterior under prior 4



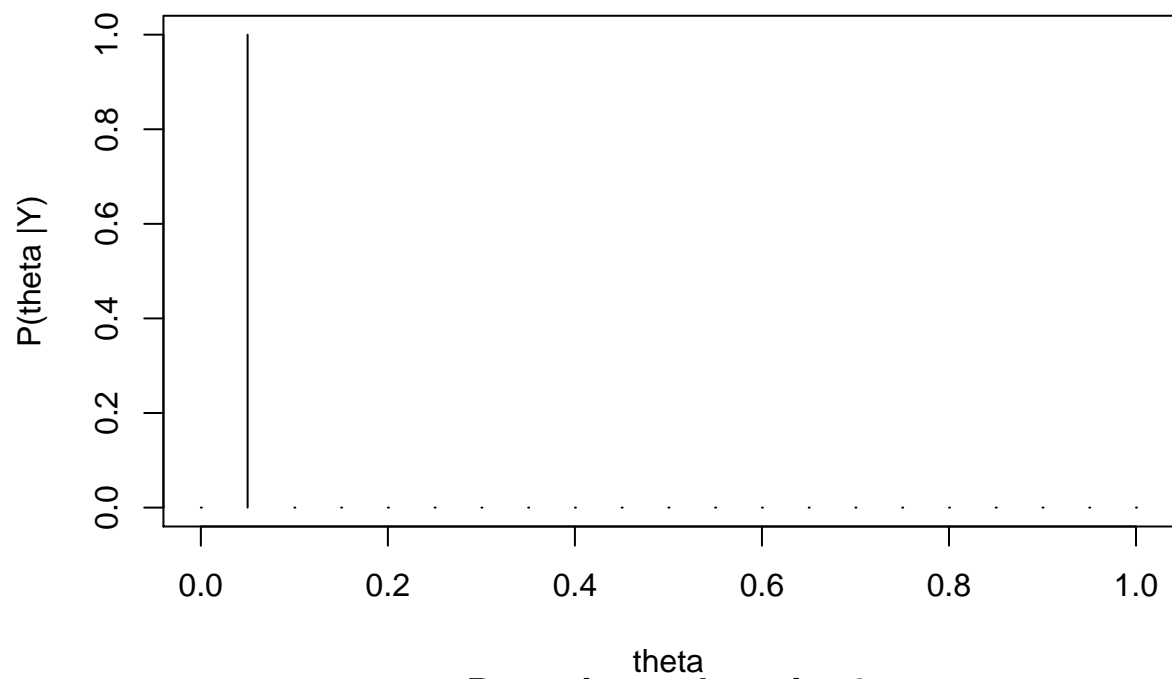
## -----		
## Expected Value Prior 1	Expected Value Prior 2	Expected Value Prior 3
## -----		
## 0.05841	0.5	0.05841
## -----		

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##
## Table: Table continues below
##
## -----
## Expected Value Prior 4
## -----
##          0.06486
## -----
```

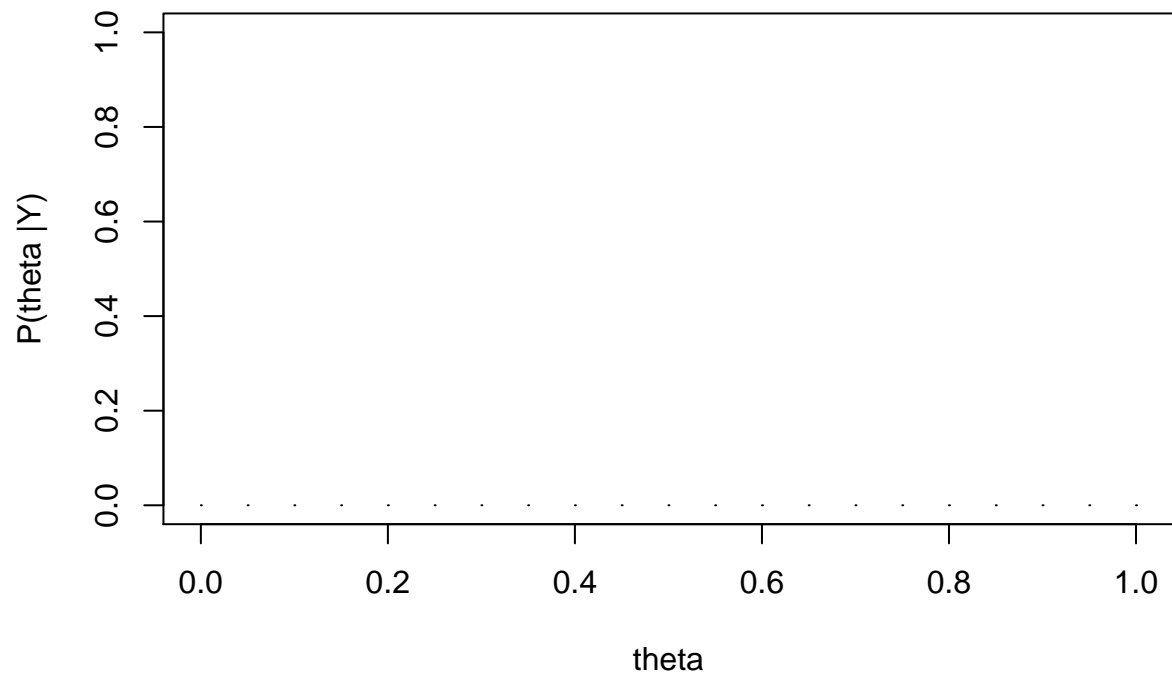
Prior 2 clearly pulls the posterior towards 1 because it forces all thetas less than .5 to have 0 probability. Prior 3 yeilds essentially the same results, which makes sense because the likelihood is so skewed towards the range 0, .5, so excluding .5, 1 does not change the posterior much. Prior 4 pushes the posterior slightly upwards, which again makes sense because the prior assigns higher probabilities to higher values of theta, pushing the posterior towards 1.

Problem 3

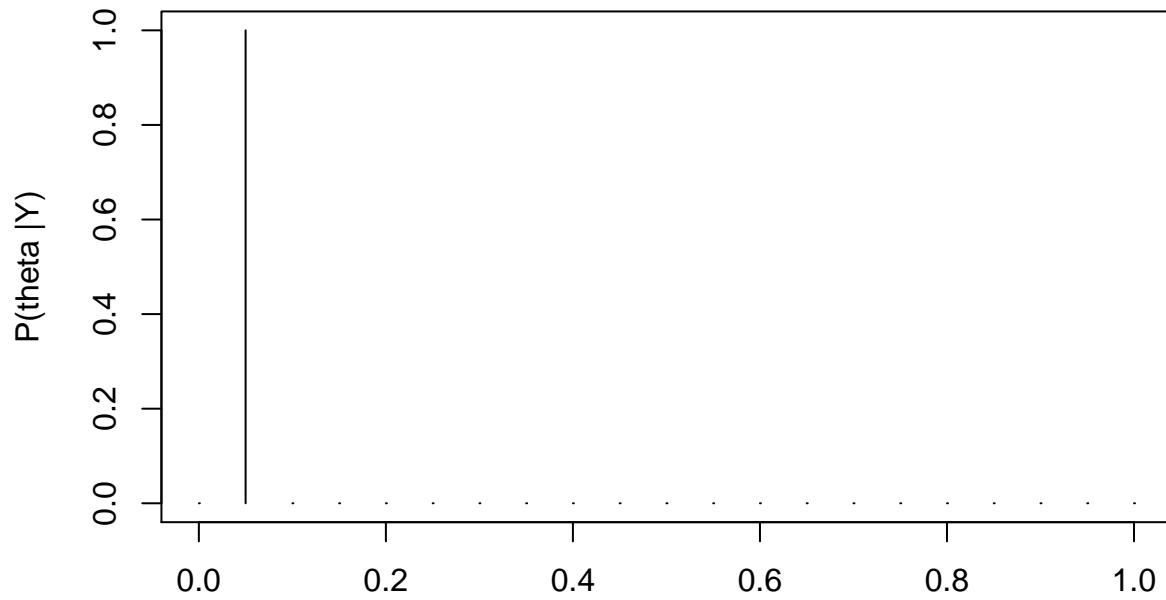
Posterior under prior 1



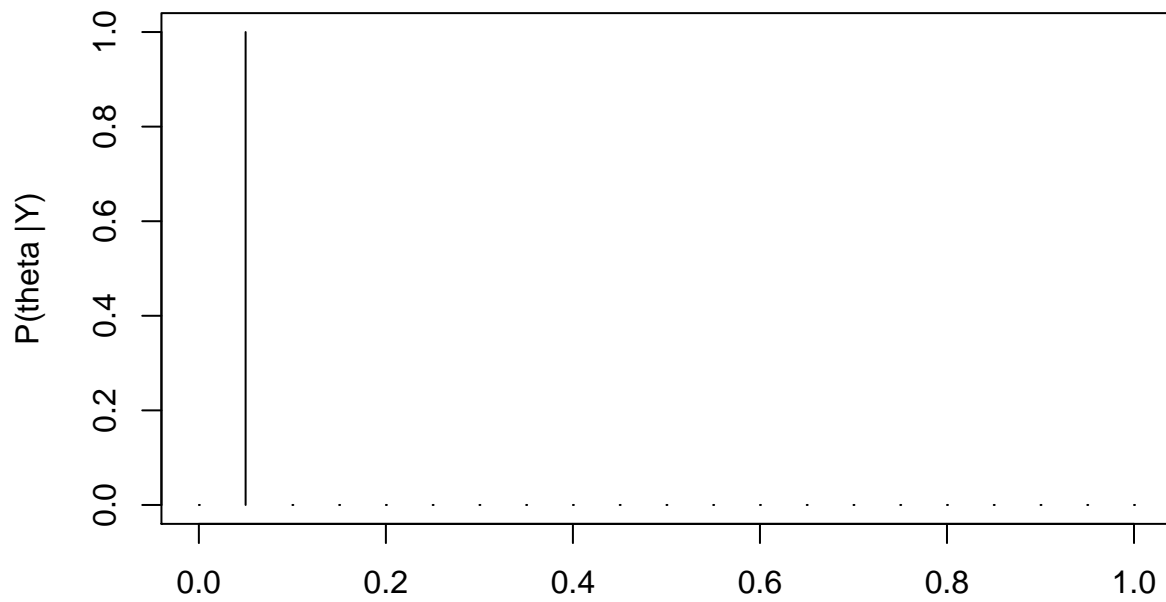
Posterior under prior 2



Posterior under prior 3



Posterior under prior 4



##			
##	-----		
##	Expected Value Prior 1	Expected Value Prior 2	Expected Value Prior 3
##	-----		
##	0.05	0.5	0.05
##	-----		

```
##
## Table: Table continues below
##
## -----
## Expected Value Prior 4
## -----
##          0.05
## -----
```

As we get more data the effect of the prior diminishes (which is called “swamping” I think?). This makes more sense, to me at least, if we consider the log posterior.

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

taking logs we see

$$\log p(\theta|y) \propto \log p(y|\theta) + \log p(\theta)$$

$$\log p(\theta|y) \propto \log \prod_{i=1}^n L(\theta|y) + \log p(\theta)$$

where $L(\theta|y)$ is the likelihood function.

This becomes

$$\log p(\theta|y) \propto \sum_{i=1}^n \log L(\theta|y) + \log p(\theta)$$

Now we can clearly see, as n increases the effect of the $\log p(\theta)$ diminishes. It is harder for me to see this clearly in the non-log space because we are dealing with multiplication of small numbers.