Homework 2

Casey Gibson

Problem 1

We can write $\theta \sim U(0,1)$ prior as a $\theta \sim Beta(1,1)$ prior. Under a binomial likelihood this becomes

$$\theta|y \sim Beta(1+y, 1+n-y)$$

So when $y = 10$ and $n = 100$ this becomes

$$\theta|y \sim Beta(11, 91)$$

We know that the expected value of a beta distribution is

$$E(\theta|y) = \frac{11}{102} = .108$$

We can find the quantiles using *qbeta* in r

```
print (signif(qbeta(.025,11,91),3))
```

```
## [1] 0.0556
```

```
print (signif(qbeta(.975,11,91),3))
```

```
## [1] 0.175
```

Problem 2

Under prior 1 we have

$$\theta \sim Beta(.2, .8)$$

so

$$\theta|y \sim Beta(.2+10, .8+90)$$

$$E(\theta|y) = .101$$

```
#library(pander)
print (signif(qbeta(.025,.2+10,.8+90),3))
```

```
## [1] 0.0504
```

```
print (signif(qbeta(.975,.2+10,.8+90),3))
```

```
## [1] 0.166
```

Under prior 2 we have

$$\theta \sim Beta(20, 80)$$

so

$$\theta|y \sim Beta(20+10, 80+90)$$

$$E(\theta|y) = .15$$

```
print (signif(qbeta(.025,20+10,80+90),3))
```

```
## [1] 0.104
```

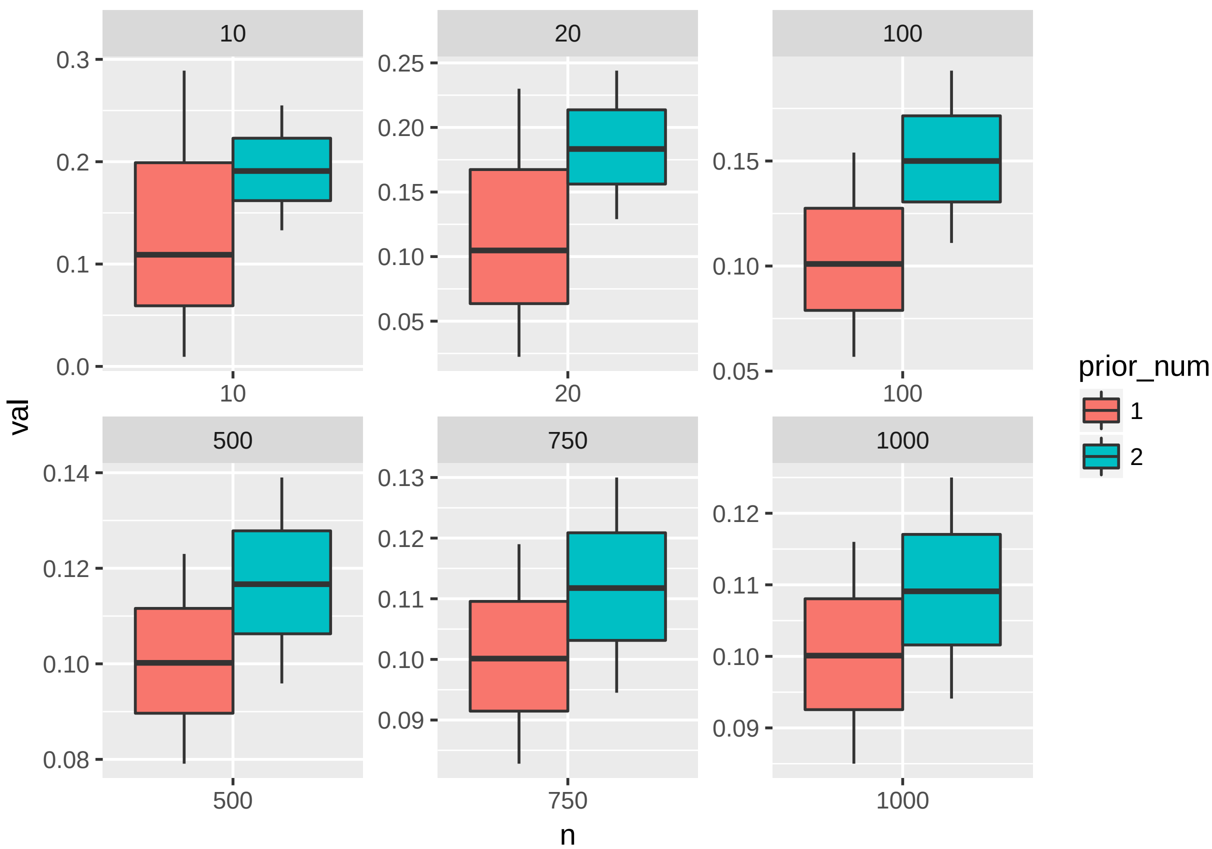
```
print (signif(qbeta(.975,20+10,80+90),3))
```

```
## [1] 0.203
```

We can see that as n_0 increases, the effect of the prior becomes stronger. It pulls the posterior expected value and quantiles closer to the original study (20% incorrect).

Problem 3

We now investigate the effect of the sample size.



We can see from the plots above that as n increases three important properties of the model emerge.

1. The 95% confidence interval for both priors shrinks (which is maybe difficult to see in the plots unless you look at the scale on the y-axis)
2. Both priors are pulled closer and closer to the maximum likelihood estimate of .10

3. Under prior 1, the posterior becomes more symmetric about the mean , the box-plots of $n = 10, 20$ seem skewed relative to those of $n = 500, 1000$.

Problem 4

We can use a discrete uniform prior on θ of the form

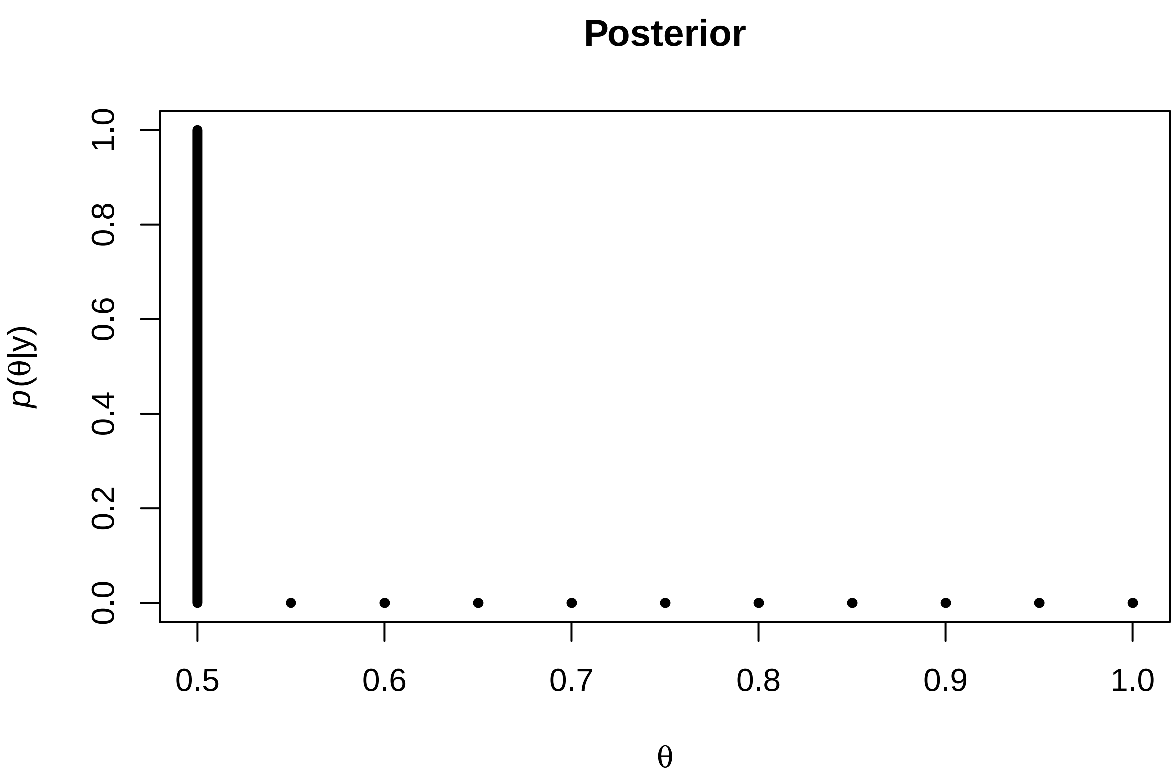
$$p(\theta) = \frac{1}{11} \text{ for } \theta = .5, .55, .6 ... 1$$

```
theta <- seq(.5,1,by=.05)
p_of_theta <- rep(1/11,length(theta))
posterior <- function(theta,p_of_theta){
  posterior_vals <- c()
 # Z <- 0
  for (i in 1:length(theta)){
    posterior_vals <- c(posterior_vals, dbinom(10,100,theta[i])*p_of_theta[i])

  }
  return (posterior_vals/sum(dbinom(10,100,theta)*p_of_theta))
}

post <- posterior(theta,p_of_theta)

ylab <- expression(paste(italic("p"),"(",theta,"|y)", sep=""))
xlab <- expression(theta)
plot(post~theta, type = "h", lwd = 5, main = "Posterior", ylim = c(0, 1), ylab = ylab, xlab = xlab)
```



```r
print (sum(theta*posterior(theta,p_of_theta)))
```

```
## [1] 0.5000099
```

The mean is pushed up towards .5 because all values of $\theta < .5$ are assigned probability 0.