Numerical Methods, PROJECT C No. 10

I For the following experimental measurements (samples):

x_i	y_i
–5	–4.6643
–4	–5.5445
–3	–4.0378
–2	–2.0868
–1	–1.0393
0	0.6916
1	–0.0237
2	0.4107
3	–3.6761
4	–9.8466
5	–18.8868

determine a polynomial function *y=f(x)* that best fits the experimental data by using the least-squares approximation (test polynomials of various degrees). Present the graphs of obtained functions along with the experimental data. To solve the least-squares problem use the system of normal equations with QR factorization of a matrix **A**. For each solution calculate the error defined as the Euclidean norm of the vector of residuum and the condition number of the Gram's matrix. Compare the results in terms of solutions' errors.

II A motion of a point is given by equations:

$$dx_1/dt = x_2 + x_1 (0.5 - x_1^2 - x_2^2),$$
$$dx_2/dt = -x_1 + x_2 (0.5 - x_1^2 - x_2^2).$$

Determine the trajectory of the motion on the interval [0, 20] for the following initial conditions: $x_1(0) = 0.002$, $x_2(0) = 0.02$. Evaluate the solution using:

a) Runge-Kutta method of 4^{th} order (RK4) and Adams PC (P_5EC_5E) – each method a few times, with different constant step-sizes until an „optimal" constant step size is found, i.e., when its decrease does not influence the solution significantly but its increase does,

b) Runge-Kutta method of 4^{th} order (RK4) with a variable step size automatically adjusted by the algorithm, making error estimation according to the step-doubling rule.

Compare the results with the ones obtained using an ODE solver, e.g. ode45.

The report should contain printouts of programs (with comments explaining what subsequent parts of every program do) and explanation of what has been done and calculated, in particular:

Ad IIa) A discussion of constant step size selection illustrated by: two solution curves x_2 versus x_1 on one plot: first for the optimal constant step size and second for the larger step-size (for which the solution visibly differs from the first one); plots of problem solution versus time, obtained for the same optimal and larger step-sizes,

Ad IIb) A discussion of the chosen value of h_{min} (minimal step size), absolute and relative tolerances, and the following plots:

1) x_2 versus x_1, 2) problem solution versus time, 3) step size versus time, 4) error estimate versus time.

A flow diagram of the algorithm in IIb) should be also attached.

The report is to be uploaded in the PDF format to the 'Reports' module on the 'Studia' server by Jan. 6, 2 p.m. Project C carries 20 points. Each day of delay results in deduction of 1 point.