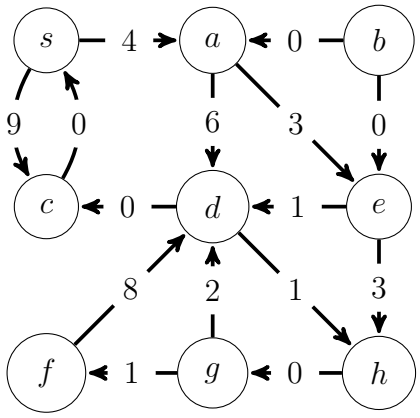
Problem Set 7

All parts are due on April 12, 2019 at 6PM. Please write your solutions in the LaTeX and Python templates provided. Aim for concise solutions; convoluted and obtuse descriptions might receive low marks, even when they are correct. Solutions should be submitted on the course website, and any code should be submitted for automated checking on `alg.mit.edu`.

Problem 7-1. [20 points] **Dijkstra Practice**

(a) [16 points] Run Dijkstra on the following graph from vertex s to every vertex in $V = \{a, b, c, d, e, f, g, h, s\}$. Write down (1) the minimum-weight path weight $\delta(s, v)$ for each vertex $v \in V$, and (2) the order that vertices are removed from Dijkstra's queue.



(b) [4 points] Change the weight of edge (f, d) to -3. Identify a vertex v for which running Dijkstra from s as in part (a) would result in an incorrect output for $\delta(s, v)$.

Problem 7-2. [20 points] **Color Limit**

A graph is **two-colored** if each vertex is assigned a color either red or blue. Given a two-colored weighted directed graph $G = (V, E)$ having only positive weights and positive integer k, describe an efficient algorithm to return a path of minimum weight from a blue vertex s to a blue vertex t which passes through red vertices at most k times, or return that no such path exists.

Problem 7-3. [20 points] **Duper Dario**

Duper Dario is a video game in which plumber Dario explores time and space by traversing g game levels. Game levels are connected via w **warp pipes**. Each warp pipe is one-way, where a warp pipe from level a to level b allows Dario to travel from level a to level b, and every level is reachable from every other level by traveling a sequence of warp pipes. Each warp pipe (a, b) is labeled with an integer t_{ab}: if Dario travels in warp pipe (a, b), the game clock will change by time t_{ab}, meaning Dario will go forward in time if t_{ab} is positive, and back in time if t_{ab} is negative (the game clock does not change while Dario is in a level). The warp pipes in the game are designed so that it is impossible for Dario to go back in time to the same location, i.e., if Dario warps away from any level a when the game clock shows time t, the game clock will always be at a higher time than t if he ever returns to level a. Describe an efficient algorithm to determine for **every** game level, the set of levels that Dario can reach from that level while going **strictly** back in time.

Problem 7-4. [20 points] **Egg Rage**

Wester Rabbit is upset about last Halloween: apparently many neighbors thought dressing as a human was not a good Halloween costume and refused to give Wester candy. Now, whenever Wester passes one of these neighbors' houses, Wester descends into an uncontrollable fit of rage and throws an egg at the house. If Wester passes such a house without an egg, Wester will turn violent, which is most distressing. Wester plans to attend an egg festival across town and wants to avoid violence. Wester has a map showing the m two-way roads connecting all n intersections in town, including the intersection where the festival will be held. Wester marks each road with the number of houses on the road that refused candy. Wester also marks some intersections as having grocery stores where Wester can buy eggs, including the intersection where Wester starts. Wester's backpack has limited space, so Wester can only carry at most k eggs at any given time. Assuming that Wester always has enough money to buy eggs when needed, describe an efficient algorithm to determine the minimum number of eggs Wester must throw and still reach the festival carrying at least $12 < k$ eggs, without turning violent along the way (or return that violence is inevitable).

Problem 7-5. [20 points] **Hiking Hibernation**

Steryl Chrayed is planning a winter hike through Canyonmaze National Park, a large network of deep canyons. Steryl has a map of the park depicting all the canyons and clearings in Canyonmaze, where a single canyon is a trail with high-walls that directly connects two clearings, where each clearing connects at most six canyons. Each canyon on the map is marked with its length. Steryl leaves her car at one clearing and wants to hike across the park to her favorite clearing, but she does not want to disturb the wildlife along the way. In particular, some clearings are home to groundhogs that hibernate in the winter. Hikers will disturb groundhogs if they hike too close. Steryl knows which clearings are home to groundhogs, and has marked each groundhog-occupied clearing c_i on the map with a (possibly different) positive **safety distance** d_i, meaning if Steryl is ever within hiking distance d_i of clearing c_i, she would disturb the groundhogs that live at c_i. Given the lengths of the n canyons in Canyonmaze, along with the g groundhog-occupied clearings and their associated safety distances, describe an efficient algorithm to determine whether Steryl can hike from her car to her favorite clearing without disturbing any groundhogs.