

# Micro Teaching Presentation on Minimum Spanning Tree

By

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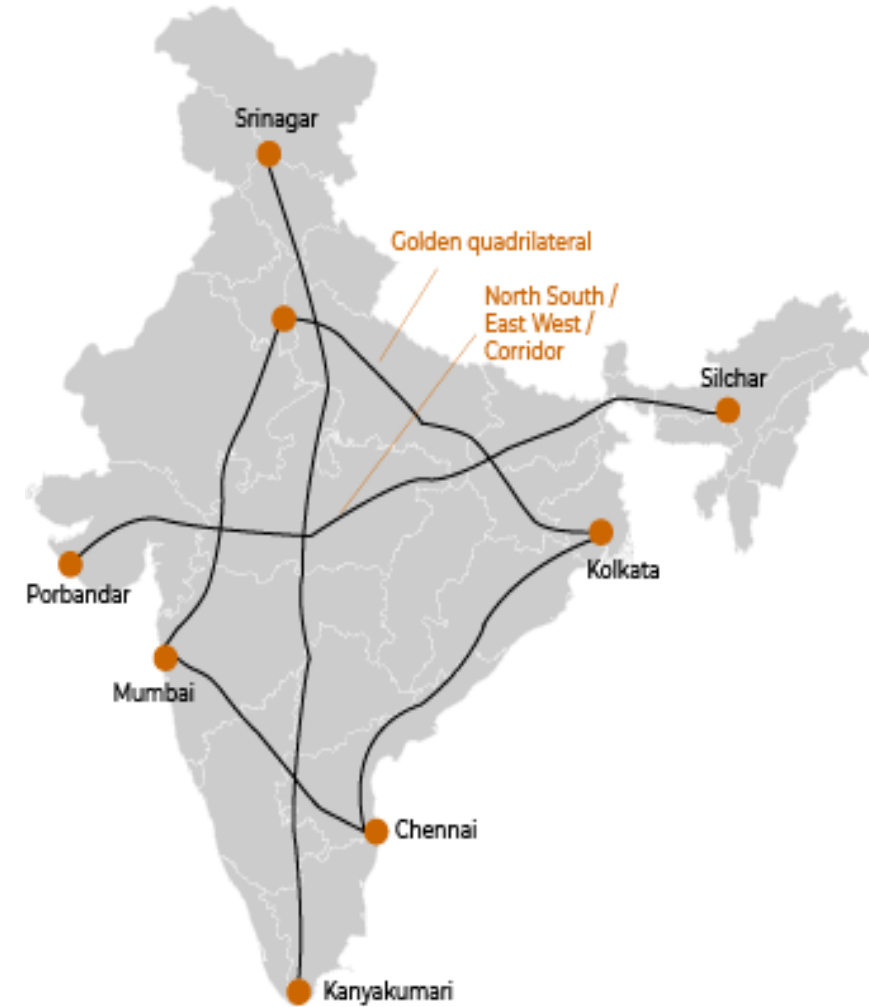
Date: 17/12/2024

**Aim of the Session:** Introduction to **Minimum-cost Spanning Tree (MST)** with **undirected connected weighted graph** and **Finding the Minimum-cost Spanning Tree Using Prim's Algorithm**.

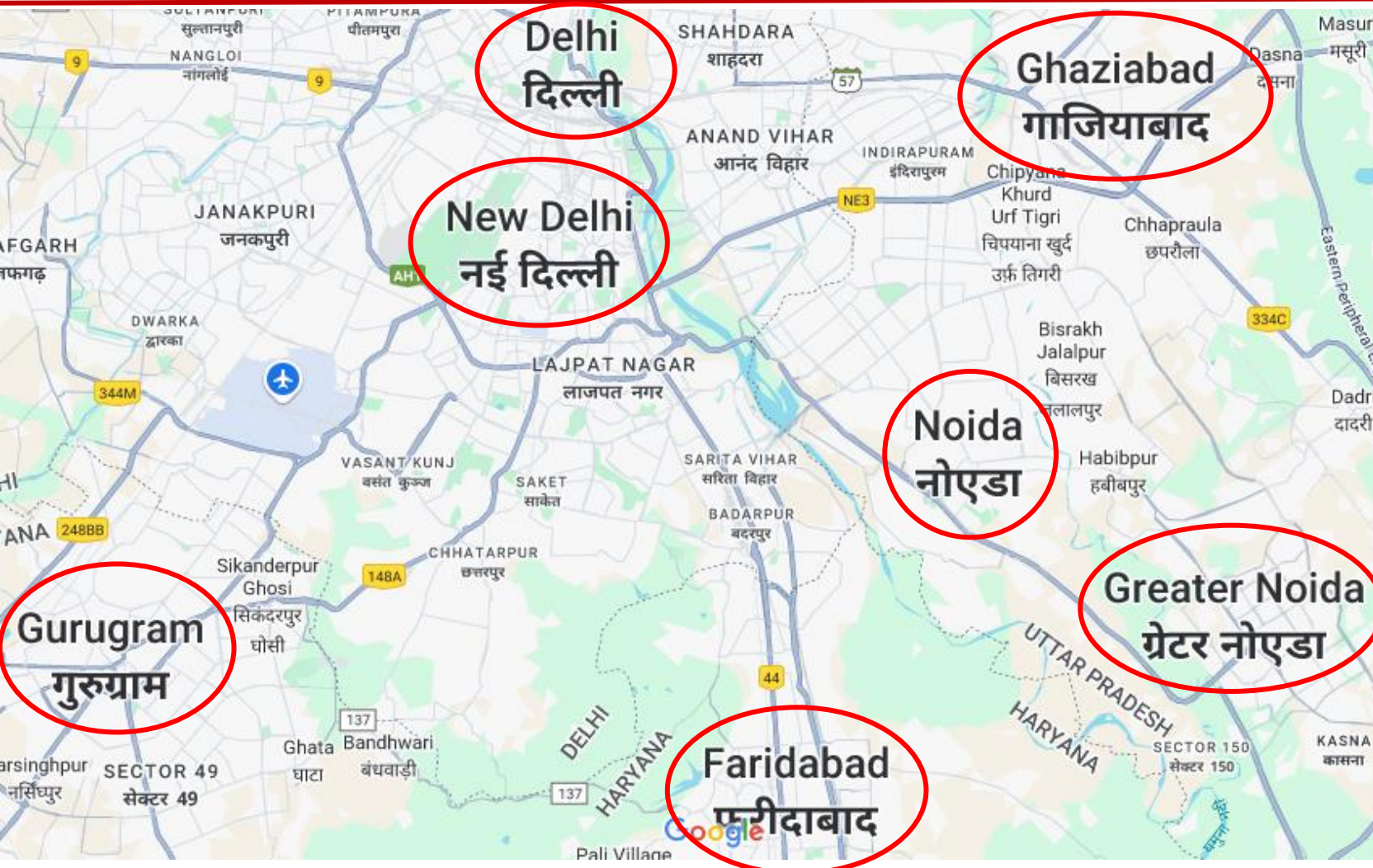
**Learning Outcomes:**

By the end of this lesson, you should be able to:

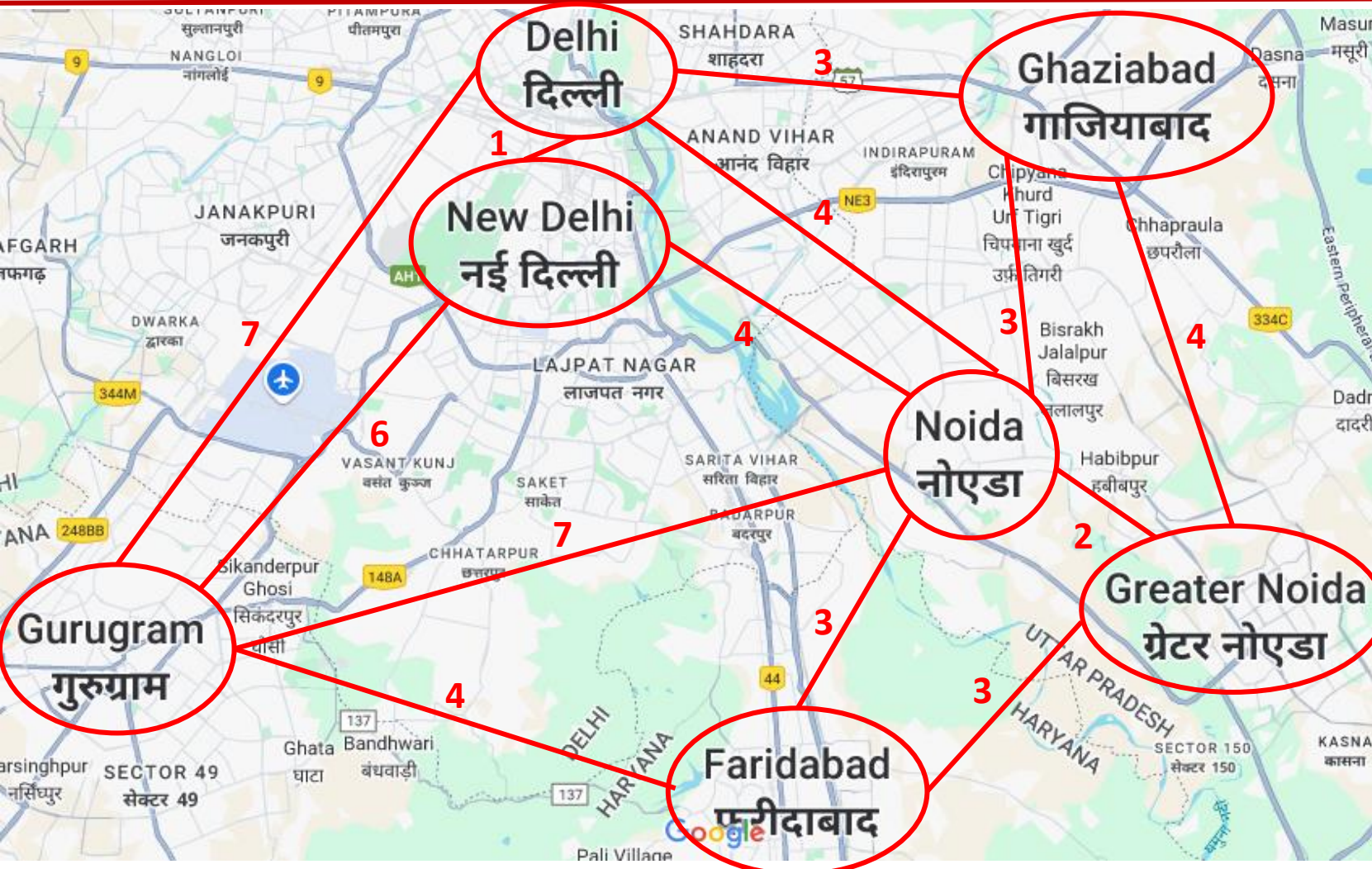
1. Compute all possible spanning trees and the Minimum Spanning Tree (MST) for a given undirected, connected, and weighted graph.
2. Apply and evaluate Prim's algorithm to find a given graph's Minimum Spanning Tree (MST), optimizing connectivity with minimal cost.
3. Evaluate the relevance of the Minimum Spanning Tree (MST), Prim's algorithm, in solving real-world problems involved in cost-effective design.





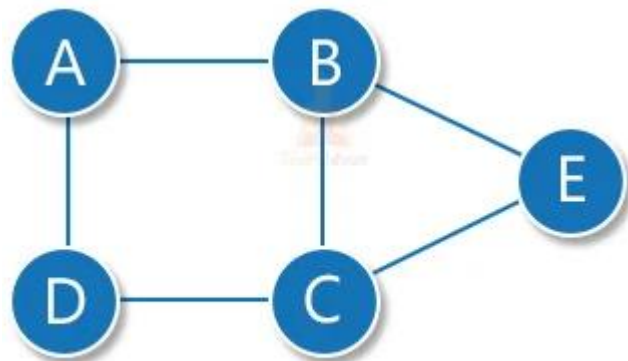
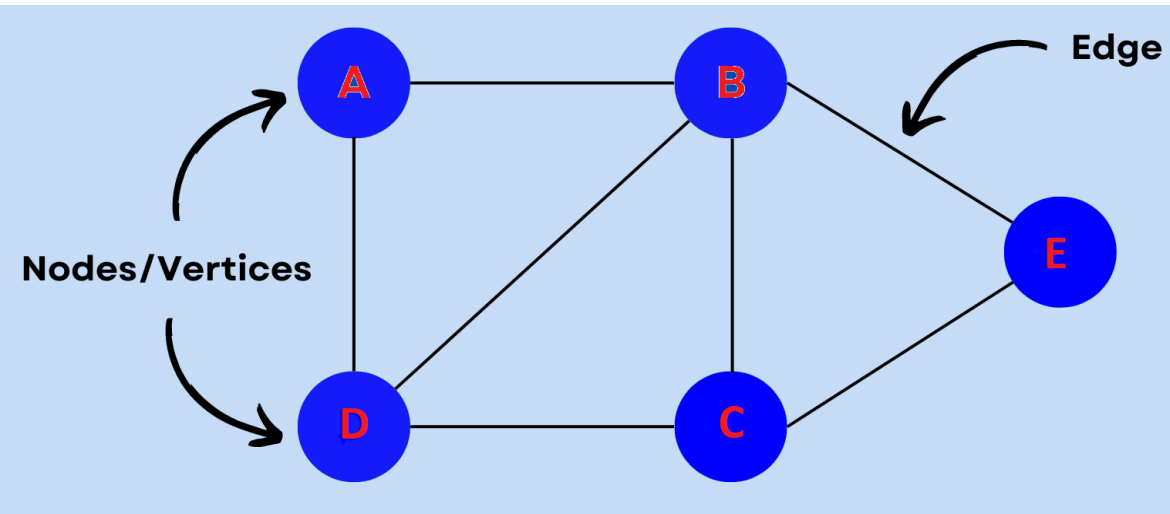




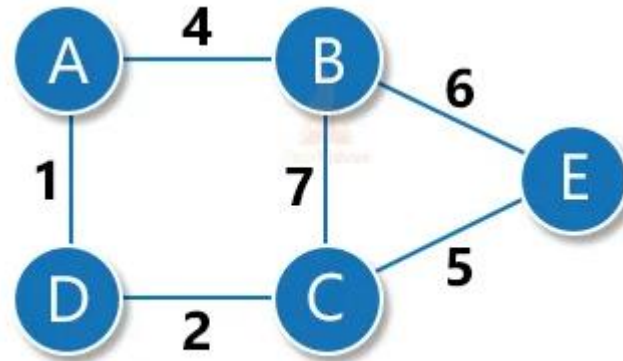


**Scenario: Designing a Road Network**  
Imagine planning a highway system to connect seven major Indian cities. To minimize construction costs, the goal is to determine the minimum total distance of roads required to connect all cities without creating redundant routes.

- > Each city is represented as a node.
- > The possible roads between cities are represented as weighted edges, where the weight corresponds to the road length or construction cost.



Non-Weighted graph



Weighted graph

Logical Representation of Graph Data Structure

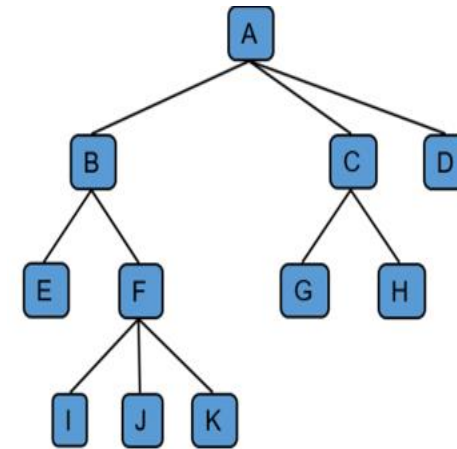
Graph =>  $G = \{V, E\}$

\*Can have cycles



Tree =>  $T = \{\text{Nodes, Edges}\}$

\*have no cycles



Logical Representation of Tree Data Structure



## Minimum Cost **Spanning Tree** (MST)

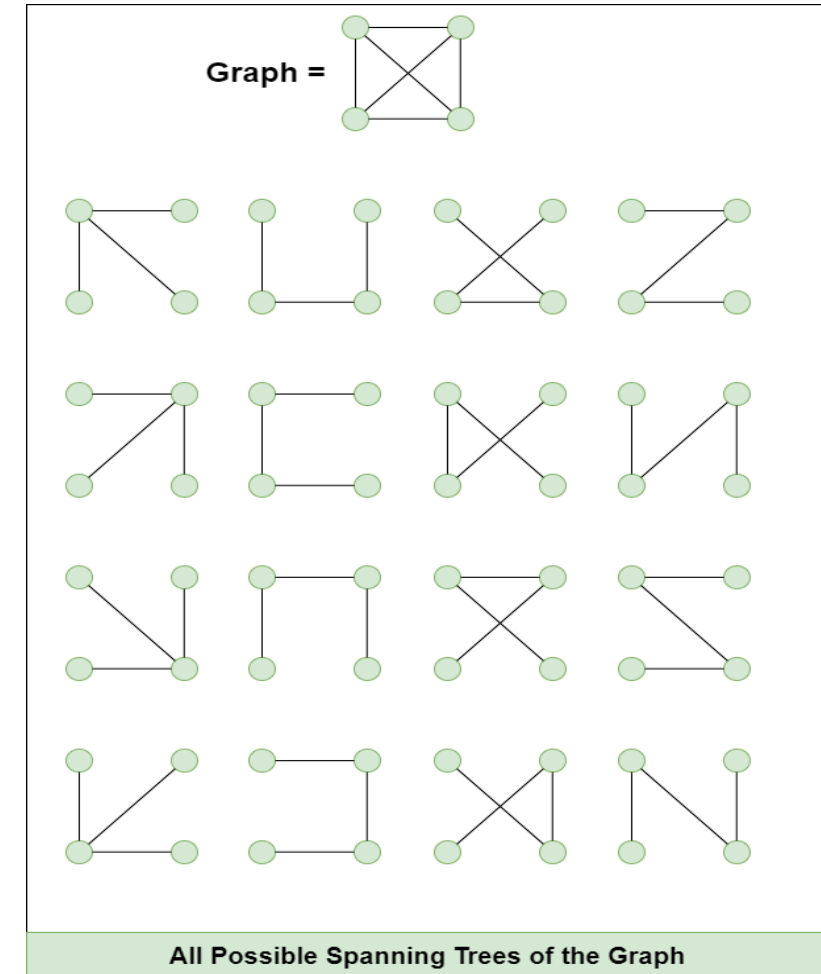
### What is a Spanning Tree?

A spanning tree is a subset of Graph G, such that **all the vertices are connected using the minimum possible number of edges.**

Hence, a **spanning tree does not have cycles**, and a **graph may have more than one spanning tree.**

It states that the number of spanning trees in a complete graph with **N vertices** is  **$N^{N-2}$** .

For example:  **$N=4$** , then maximum number of spanning tree possible =  **$4^{4-2} = 16$**  (shown in the above image).



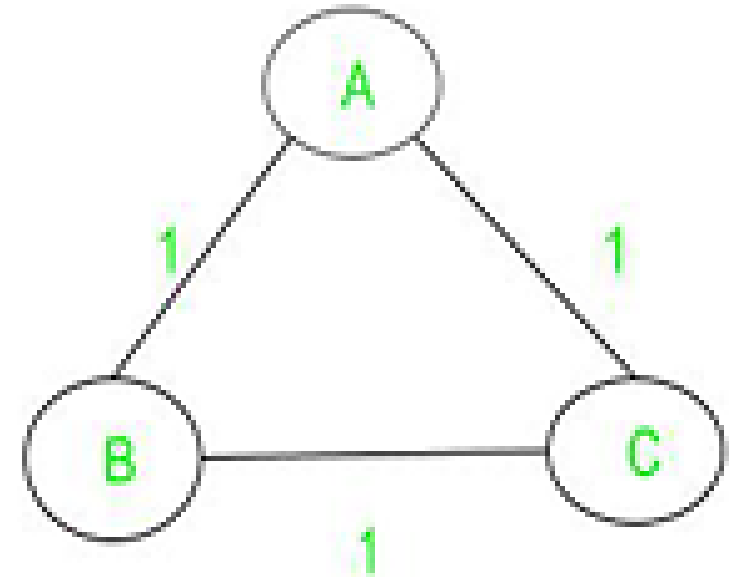


**Activity-1:** Compute the number of possible spanning trees from the given graph.



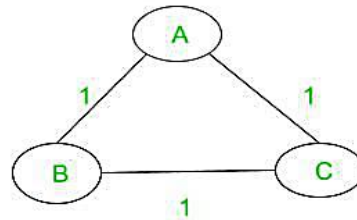
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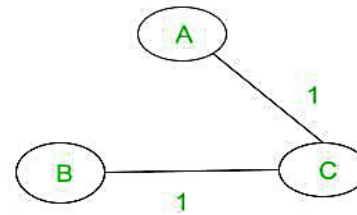
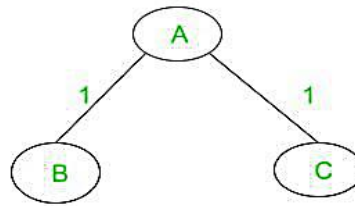
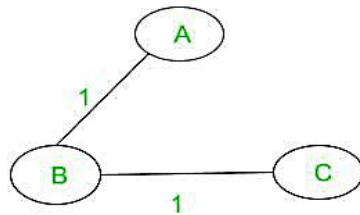




**Answer for Activity 1:** The number of possible spanning trees is **3**.



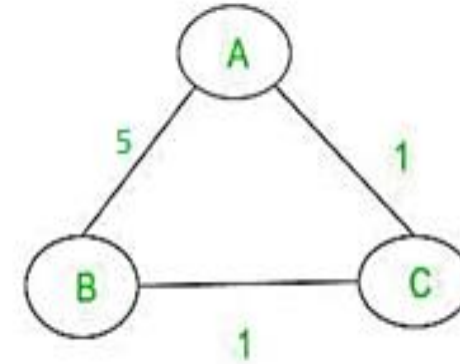
All possible minimal spanning trees are



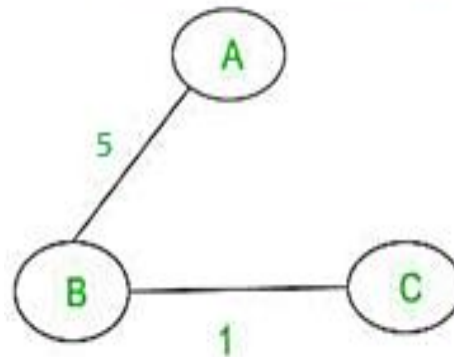
Each of the spanning trees has the same weight equal to 2.

## Minimum Cost Spanning Tree (MST)

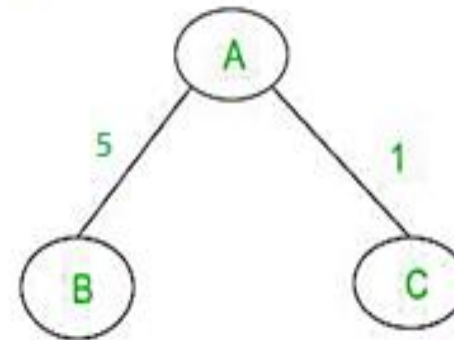
Considering the edges (A, C) and (C, B), we will have a minimum cost-spanning Tree for the given graph.



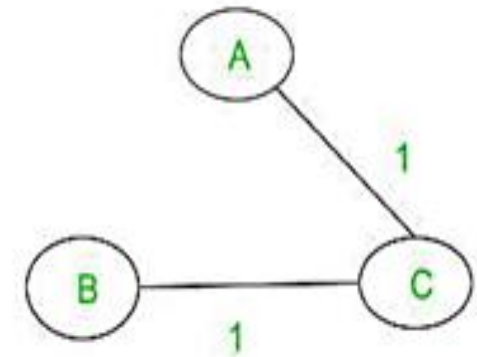
All possible minimal spanning trees are



Cost = 6

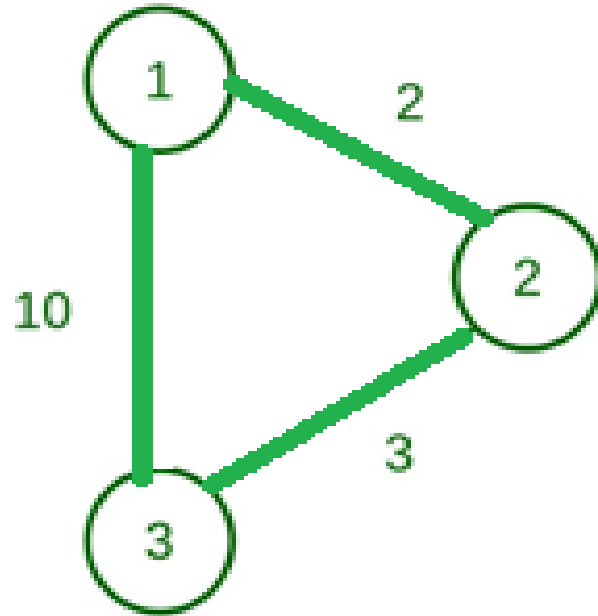


Cost = 6



Cost = 2

**Activity 2:** Compute the minimum-cost spanning trees from the given graph.

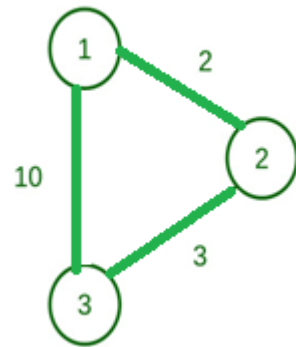
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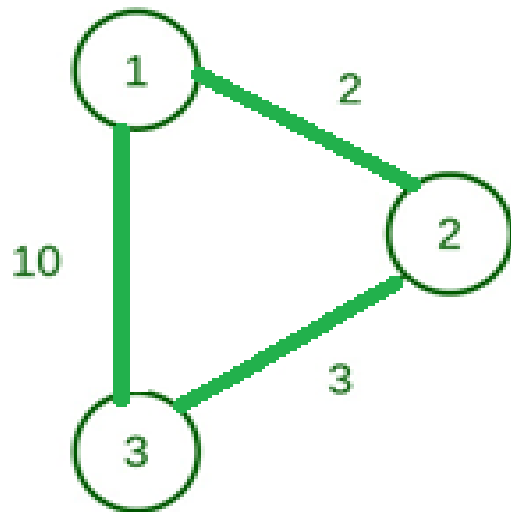
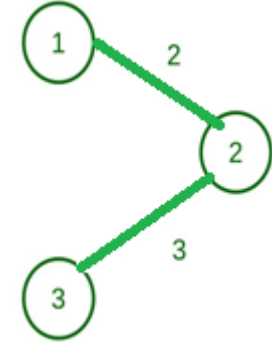
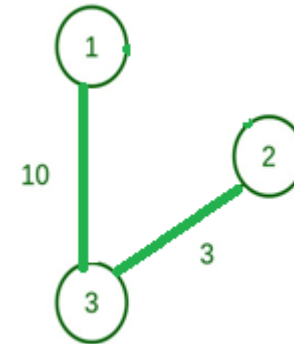
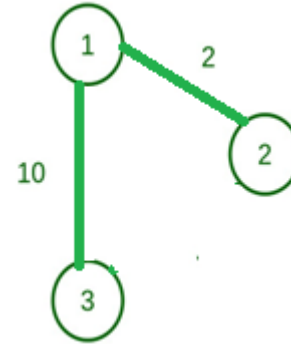
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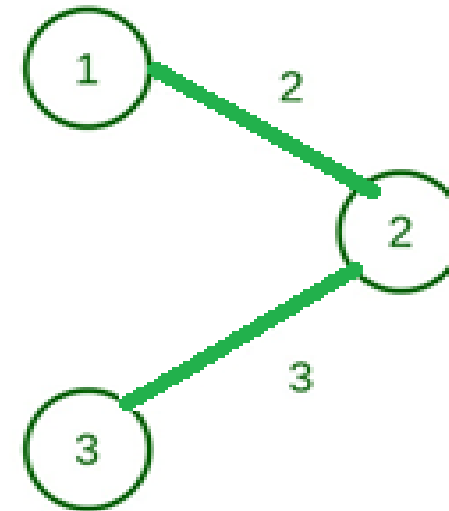
Answer for Activity 2:



Possible  
Spanning =>  
Trees



MST



What will be the  
computing  
approach when  
the graph size is  
large (i.e., input  
data size is  
large) ?

### Prim's Algorithm for Minimum Spanning Tree (MST):

#### How does Prim's Algorithm Work?

The working of Prim's algorithm can be described by using the following steps:

**Step 1:** Determine an arbitrary vertex as the starting vertex of the MST.

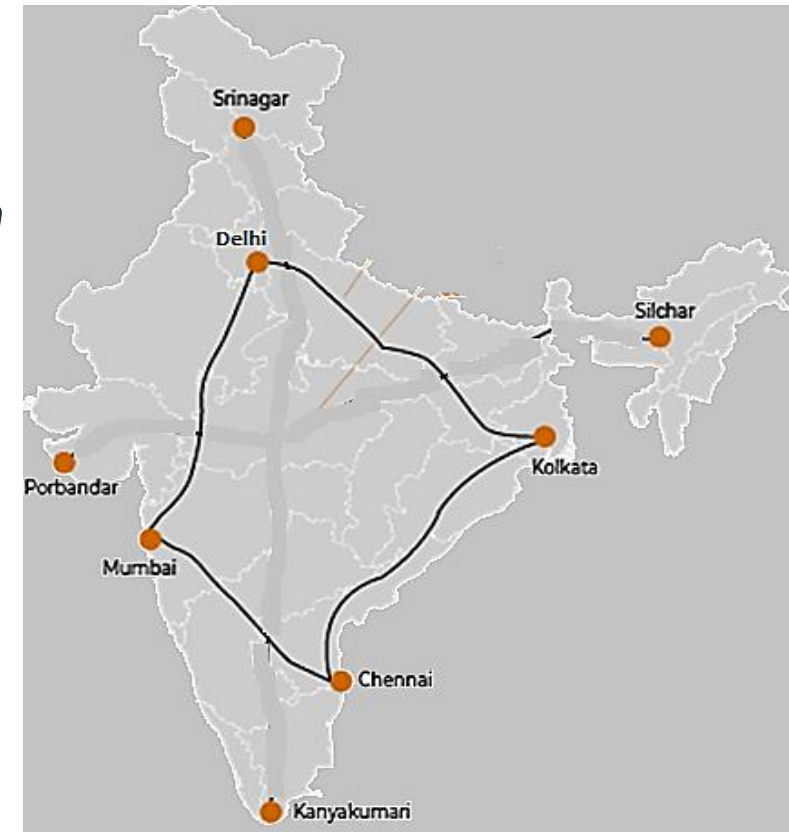
**Step 2:** Follow steps 3 to 5 till there are vertices that are not included in the MST (known as fringe vertex).

**Step 3:** Find edges connecting any tree vertex with the fringe vertices.

**Step 4:** Find the minimum among these edges.

**Step 5:** Add the chosen edge to the MST if it does not form any cycle.

**Step 6:** Return the MST and exit

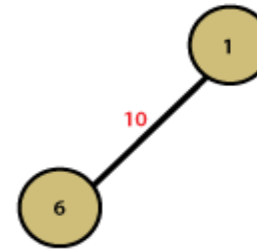
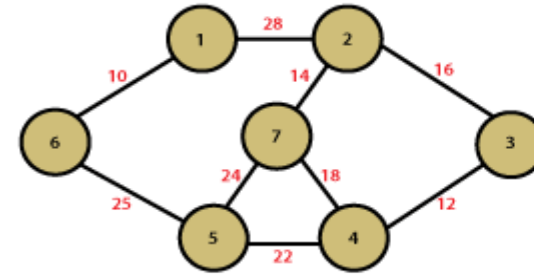


1. Select **Start Vertex**, for eg. 1
2. Find **all edges connected** with the start vertex and remaining vertex.
3. **Consider the minimum edge** in the MST if it is **not forming a cycle**.

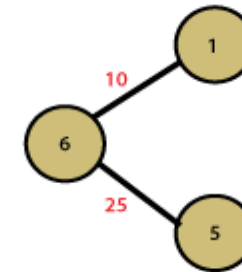
(1,6), (6,5), (5,4), (4,3), (3,2), and (2,7)

total cost= 99

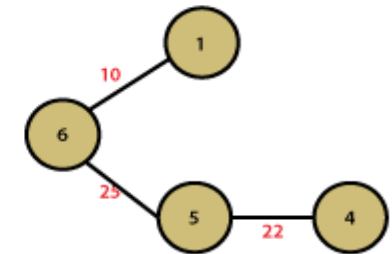
Prim's Algorithm



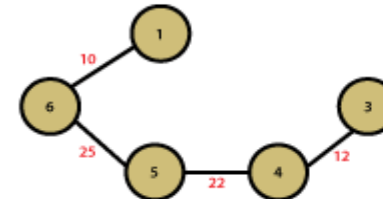
Step 1



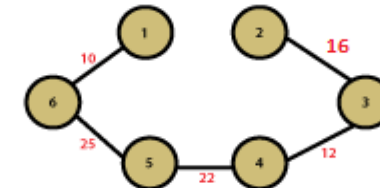
Step 2



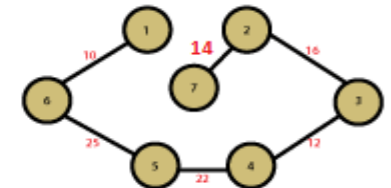
Step 3



Step 4



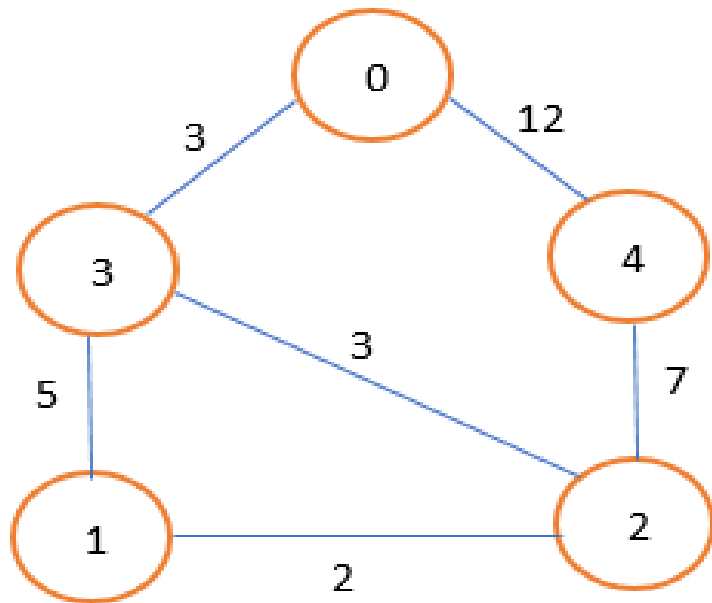
Step 5



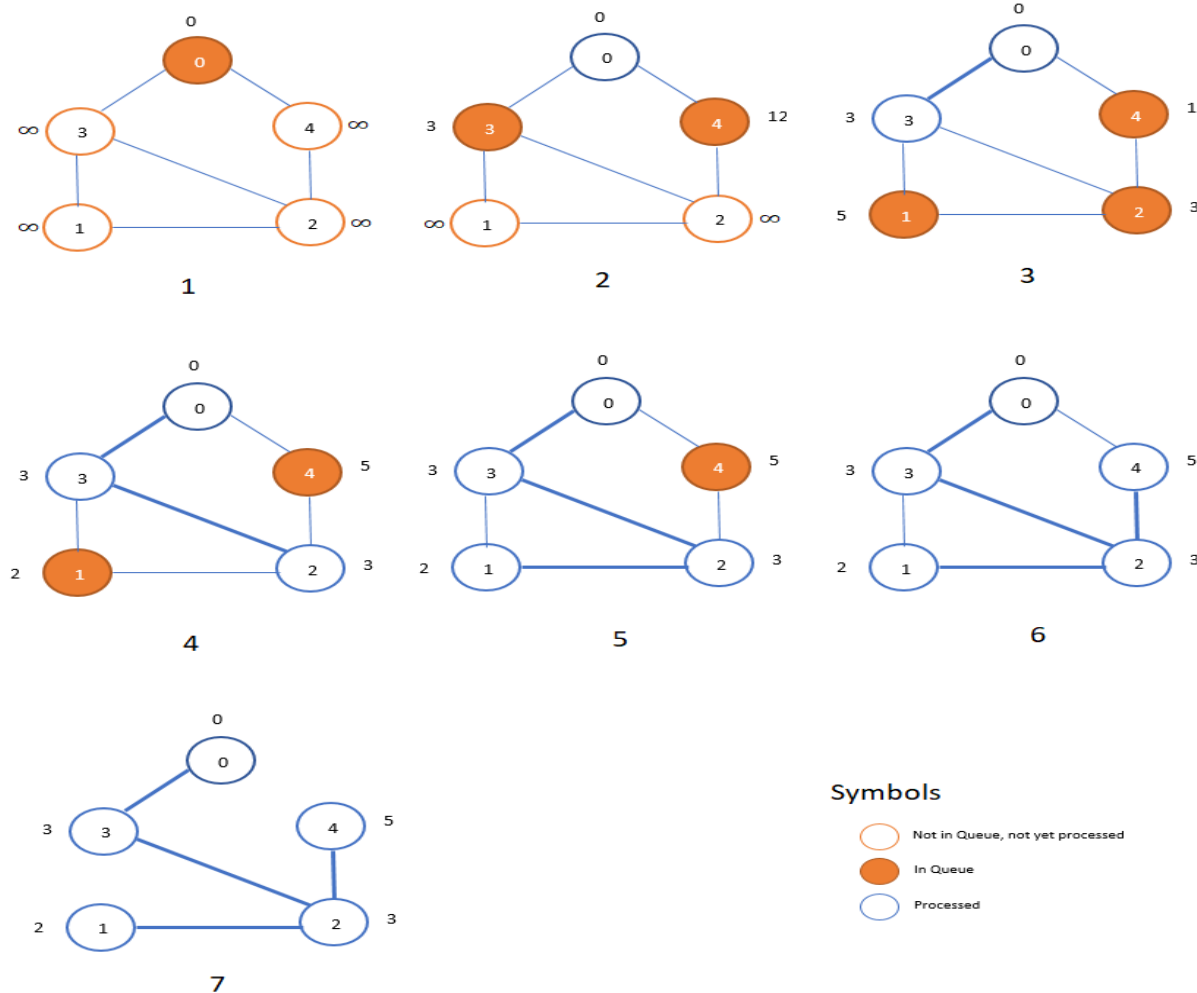
Step 6



**Activity 3:** Apply the Prim's Algorithm and evaluate the minimum spanning tree (MST) for the given graph.

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## Answer for activity 3:

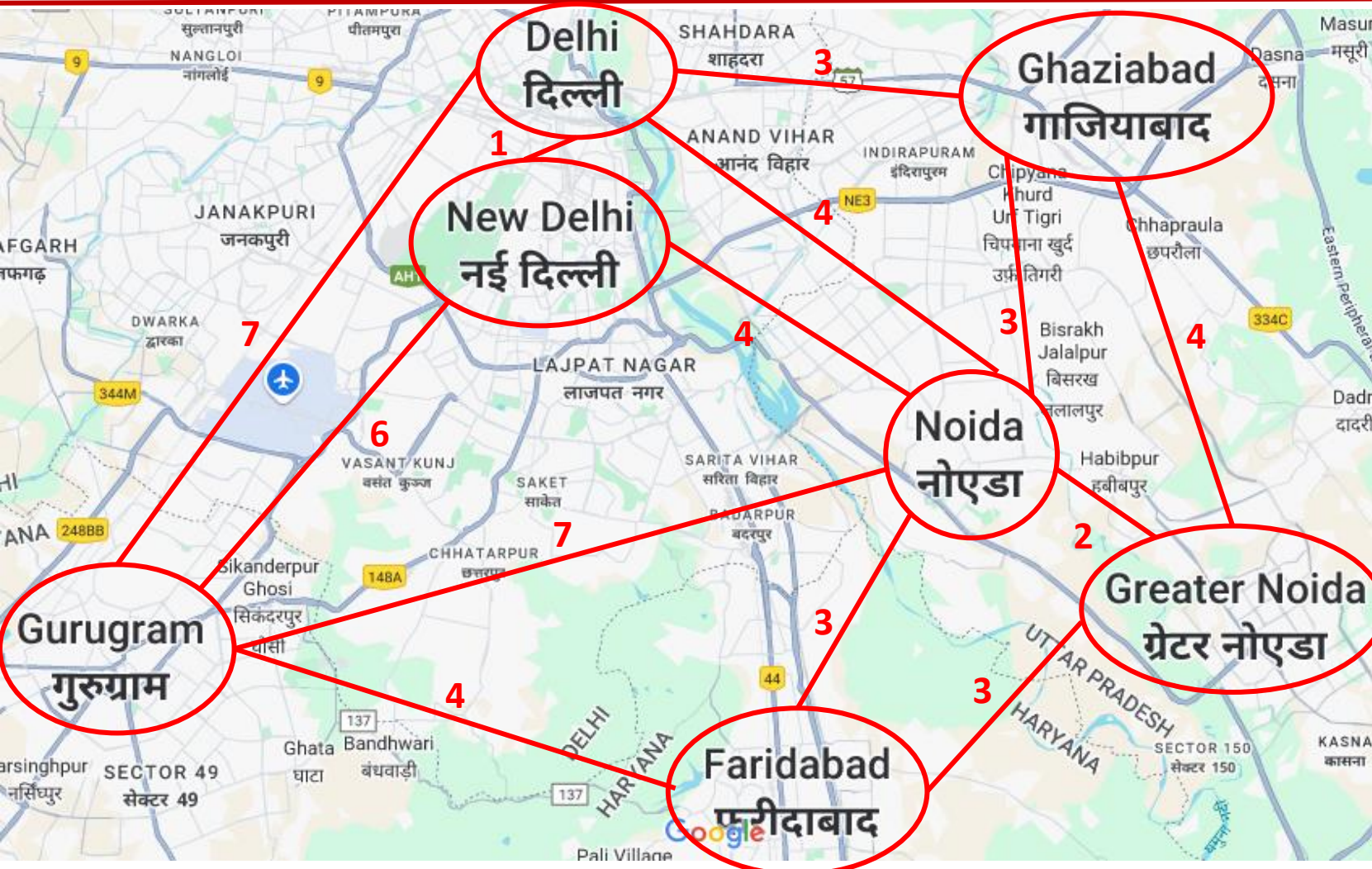


## Real-Life Applications of MST:

**Road Networks:** Planning highways to minimize construction costs.

**Telecommunications:** Laying out cable networks between cities.

**Electrical Grids:** Connecting power substations.



**Take Home Assignment:**  
**Scenario: Designing a Road Network**  
Imagine planning a highway system to connect seven major Indian cities. To minimize construction costs, the goal is to determine the minimum total distance of roads required to connect all cities without creating redundant routes.

- > Each city is represented as a node.
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**Summary of the Session:** We have discussed

- ✓ About the basics of Graph and Tree data structures
- ✓ About Spanning Tree and its computation in an activity
- ✓ About Minimum Cost Spanning Tree and its computation in an activity
- ✓ Described Prim's Algorithm to evaluate the Minimum cost spanning tree and its computation in an activity.

**Session Feedback is more  
valuable to me.**



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