Scientific Computation of Two–Phase Ferrofluid Flows

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Solving PDEs using deal.II

Each program in deal.II can be broken down into 5 main steps:

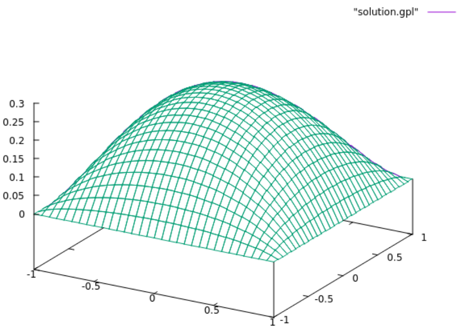
1) Generate the mesh.
2) Distribute degrees of freedom and set up the matrices for the associated system.
3) Compute the left and right hand sides of the system.
4) Solve the system numerically.
5) Output the solution in a specific format for visualization or post-processing.

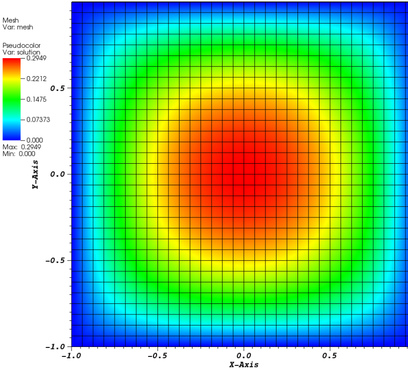
Example 1

The first PDE solved was Poisson's equation:

$$\begin{cases} -\Delta u = f(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $f(x) = 1$ and $\Omega = [0,1]^2$.





Example 2

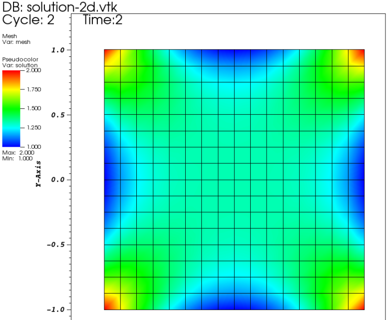
The second PDE solved was Poisson's equation:

$$\begin{cases} -\Delta u = f(x) & \text{in } \Omega \\ u = g(x) & \text{on } \partial\Omega, \end{cases}$$

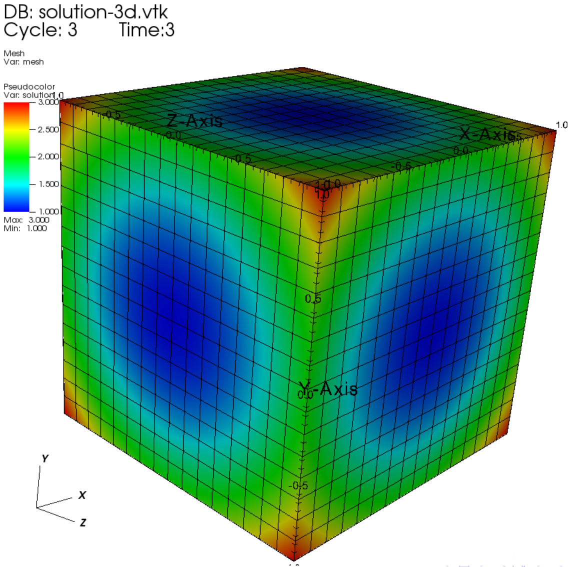
where

$$f(x) = \begin{cases} 4(x^4 + y^4) & \text{if } \Omega \subset \mathbb{R}^2 \\ 4(x^4 + y^4 + z^4) & \text{if } \Omega \subset \mathbb{R}^3 \end{cases}, \quad g(x) = \begin{cases} x^2 + y^2 & \text{if } \Omega \subset \mathbb{R}^2 \\ x^2 + y^2 + z^2 & \text{if } \Omega \subset \mathbb{R}^3 \end{cases},$$

and Ω is the unit square or cube.



3D Plot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Cycle: 3 Time:3
Mesh
Var: mesh
Pseudocolor
Var: solution
3.000
2.500
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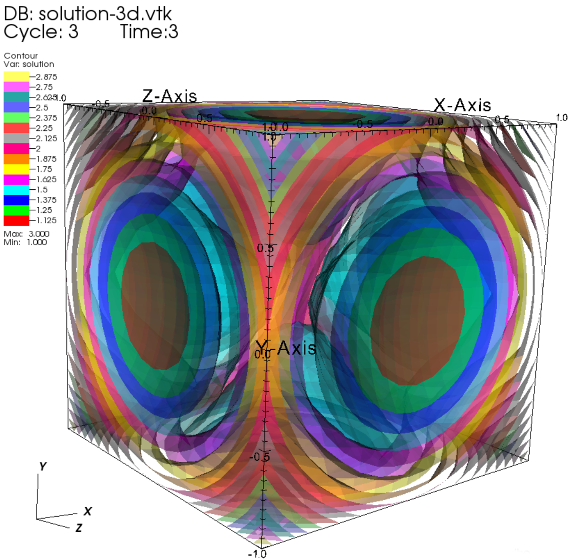
1.000
Max: 3.000
Min: 1.000
Z-Axis
X-Axis
Y-Axis
Y
X
Z

3D Contour Plot



Example 3

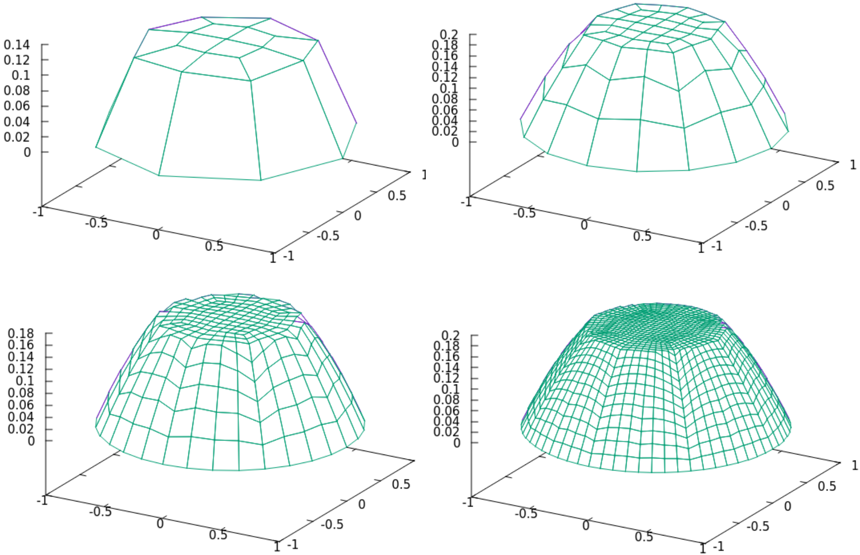
The final PDE solved was Poisson's equation:

$$\begin{cases} -\nabla \cdot (a(x) \nabla u(x)) = 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega = [0, 1]^2$ and

$$a(x) = \begin{cases} 20 & \text{if } |x| < 0.5 \\ 1 & \text{otherwise} \end{cases}.$$

The problem was solved multiple times on increasing global refinements of the mesh.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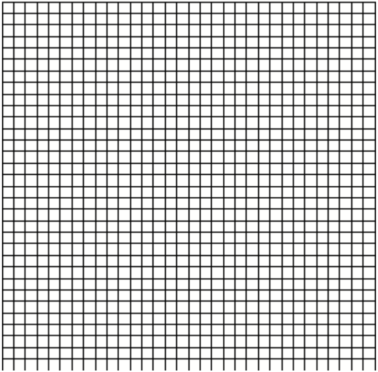
Milestone Updates

1) Successfully generated and verified input mesh.



2) Implement adaptive mesh refinement/coarsening throughout the project, instead of at the end.

3) Implement and verify the Navier–Stokes solver first, instead of the Cahn–Hilliard solver.

Next Steps

1) Learn and implement an adaptive local mesh refinement algorithm for stationary problems.

2) Learn to solve time dependent problems, starting with the heat equation.

3) Learn to solve the Navier–Stokes problem.