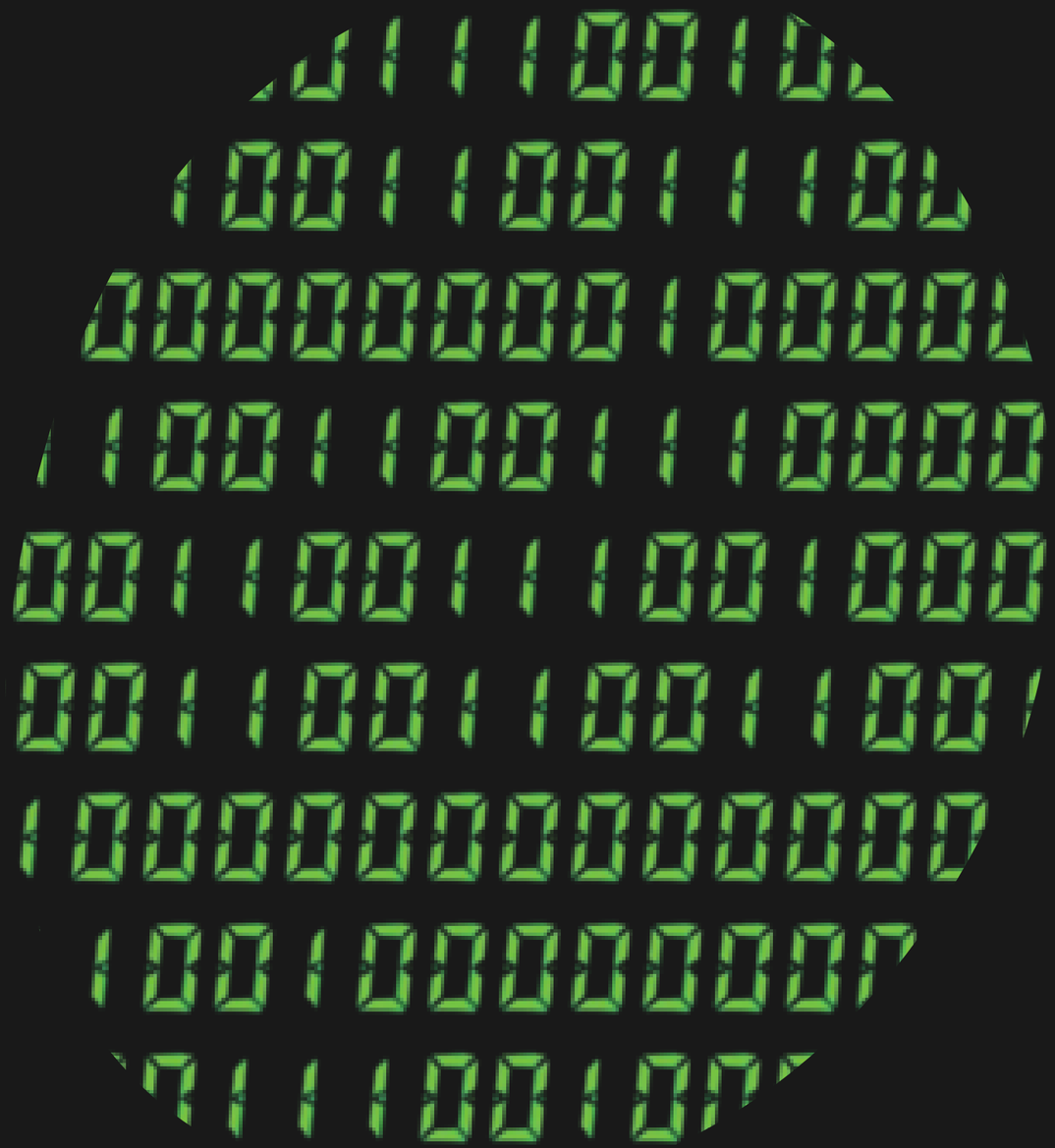
Algoritmos Aproximativos

Gustavo Cunha, Antônio Brum

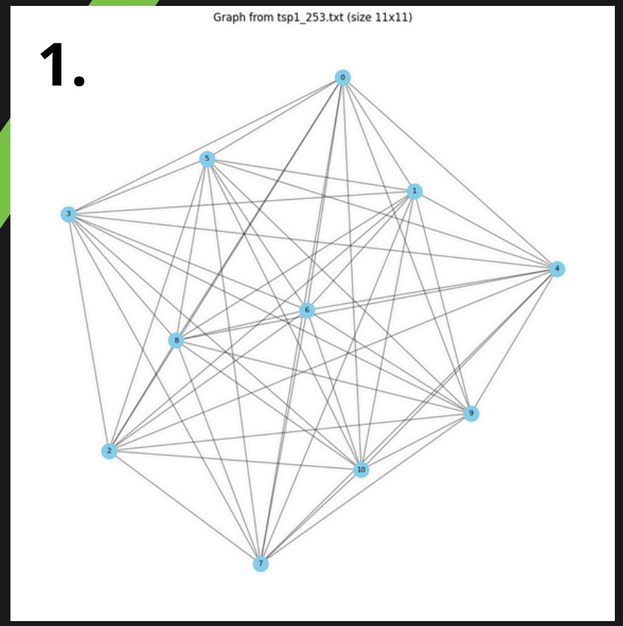


Introdução

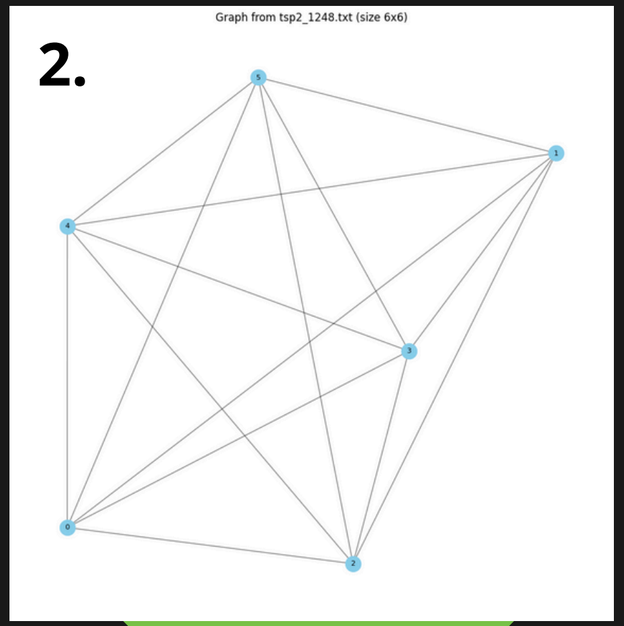
Arquivo	Número de nodos	Menor custo
tsp1_253.txt	11	253
tsp2_1248.txt	6	1248
tsp3_1194.txt	15	1194
tsp4_7013.txt	44	7013
tsp5_27603.txt	29	27603

Tabela 1: Características principais dos TSPs

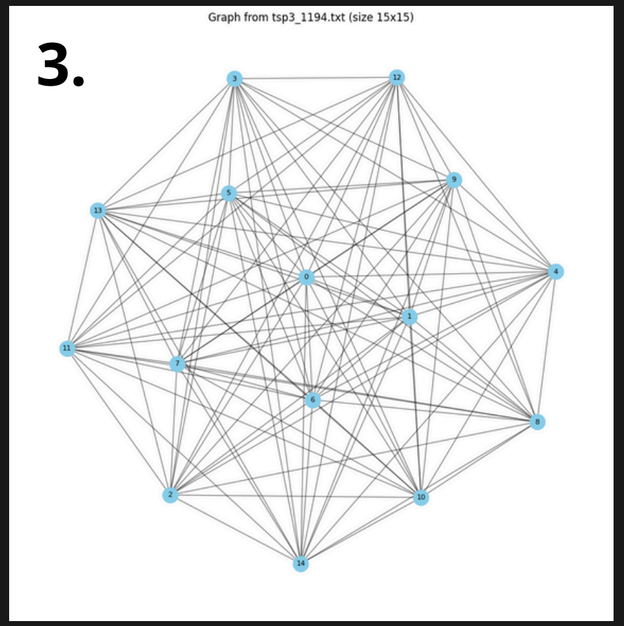
1.


Graph from tsp1_253.txt (size 11x11)

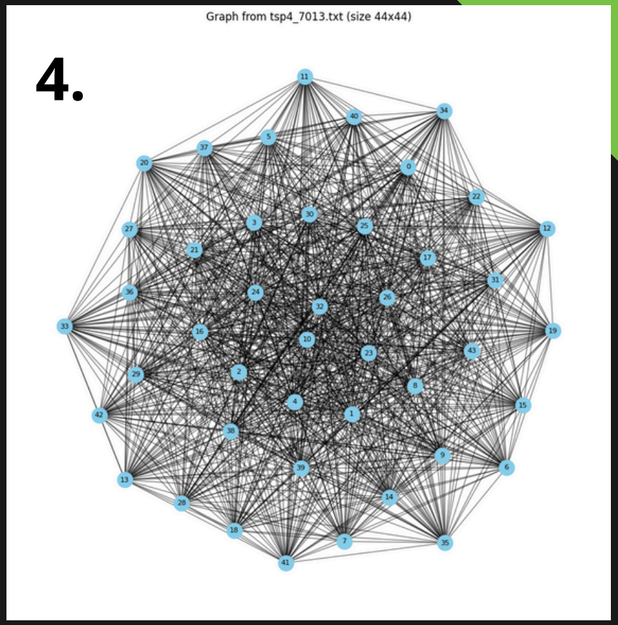
2.


Graph from tsp2_1248.txt (size 6x6)

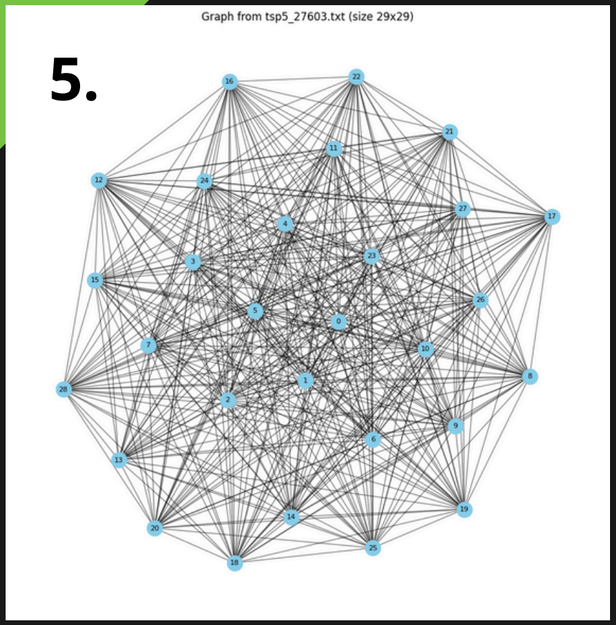
3.


Graph from tsp3_1194.txt (size 15x15)

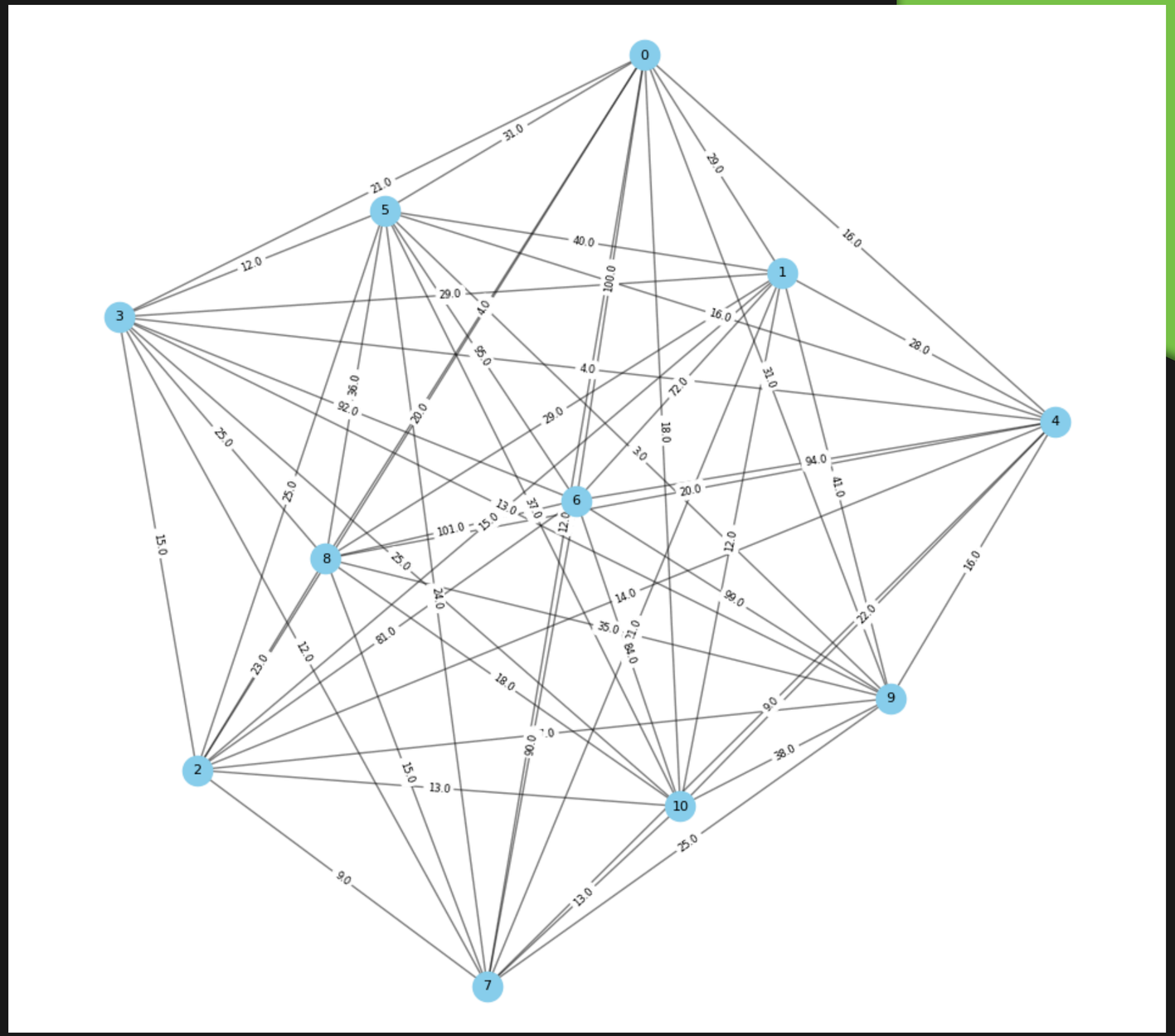
4.


Graph from tsp4_7013.txt (size 44x44)

5.

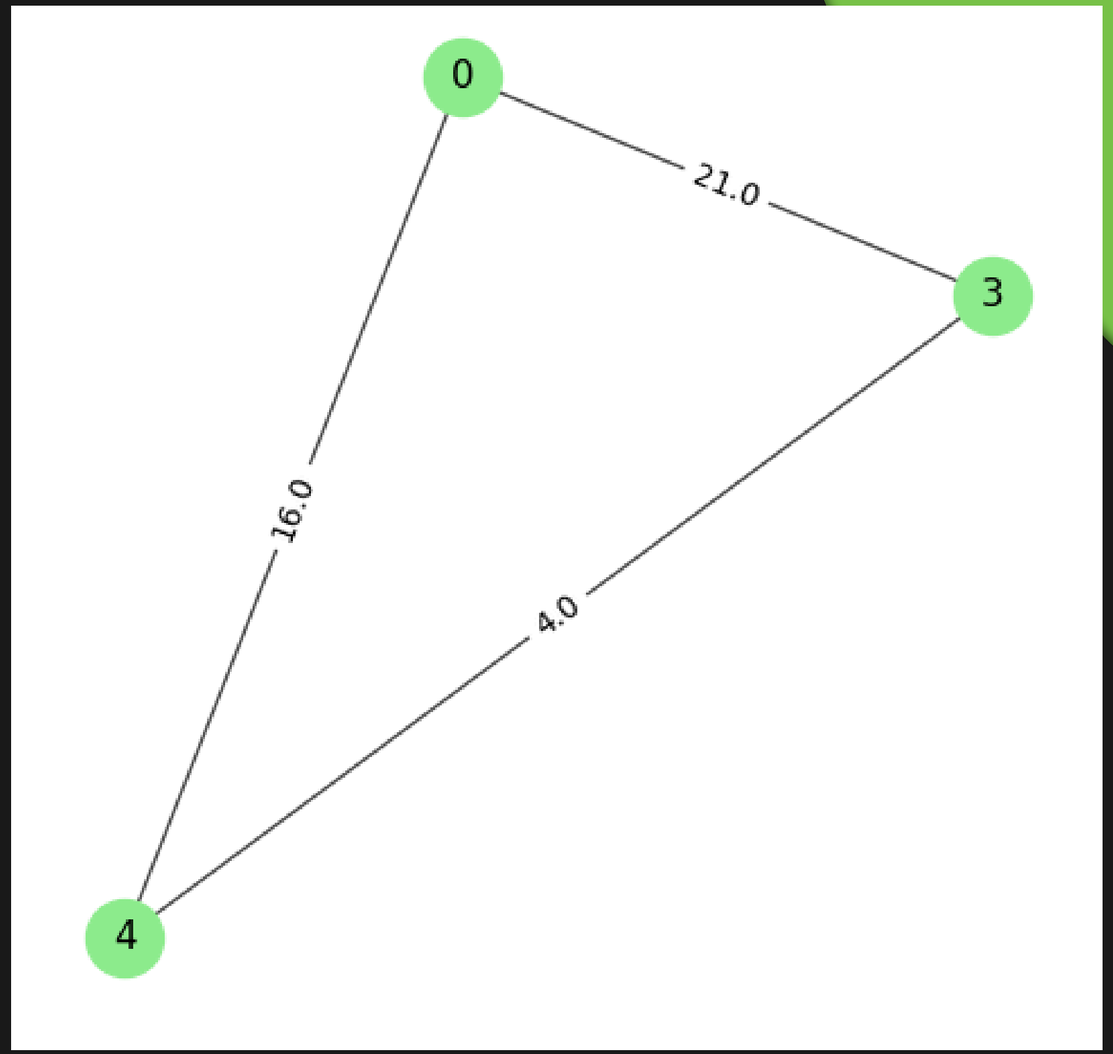

Graph from tsp5_27603.txt (size 29x29)

tsp1_253



tsp1_253

d(0,3) = 21 > d(0,4) + d(4,3) =
16 + 4 =
20



tsp1_253

Algoritmos / Thomas
H. Cormen... [et al.]

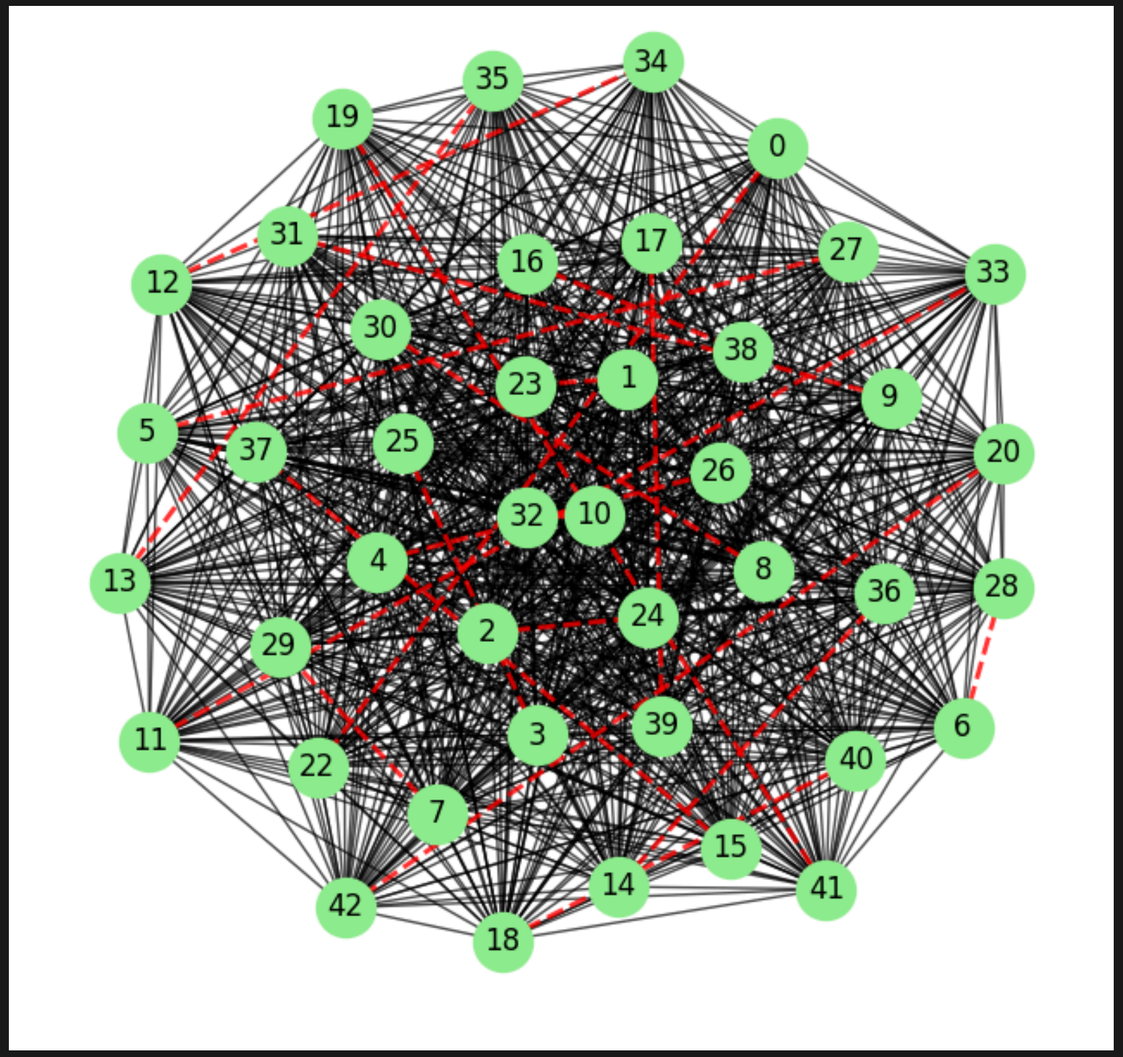
35.2.2 O PROBLEMA GERAL DO CAIXEIRO-VIAJANTE

Se eliminarmos a hipótese de que a função custo c satisfaz a desigualdade triangular, então não podemos encontrar bons passeios aproximados em tempo polinomial, a menos que P = NP.

NÃO RESPEITA

a desigualdade triangular

tsp4_7013

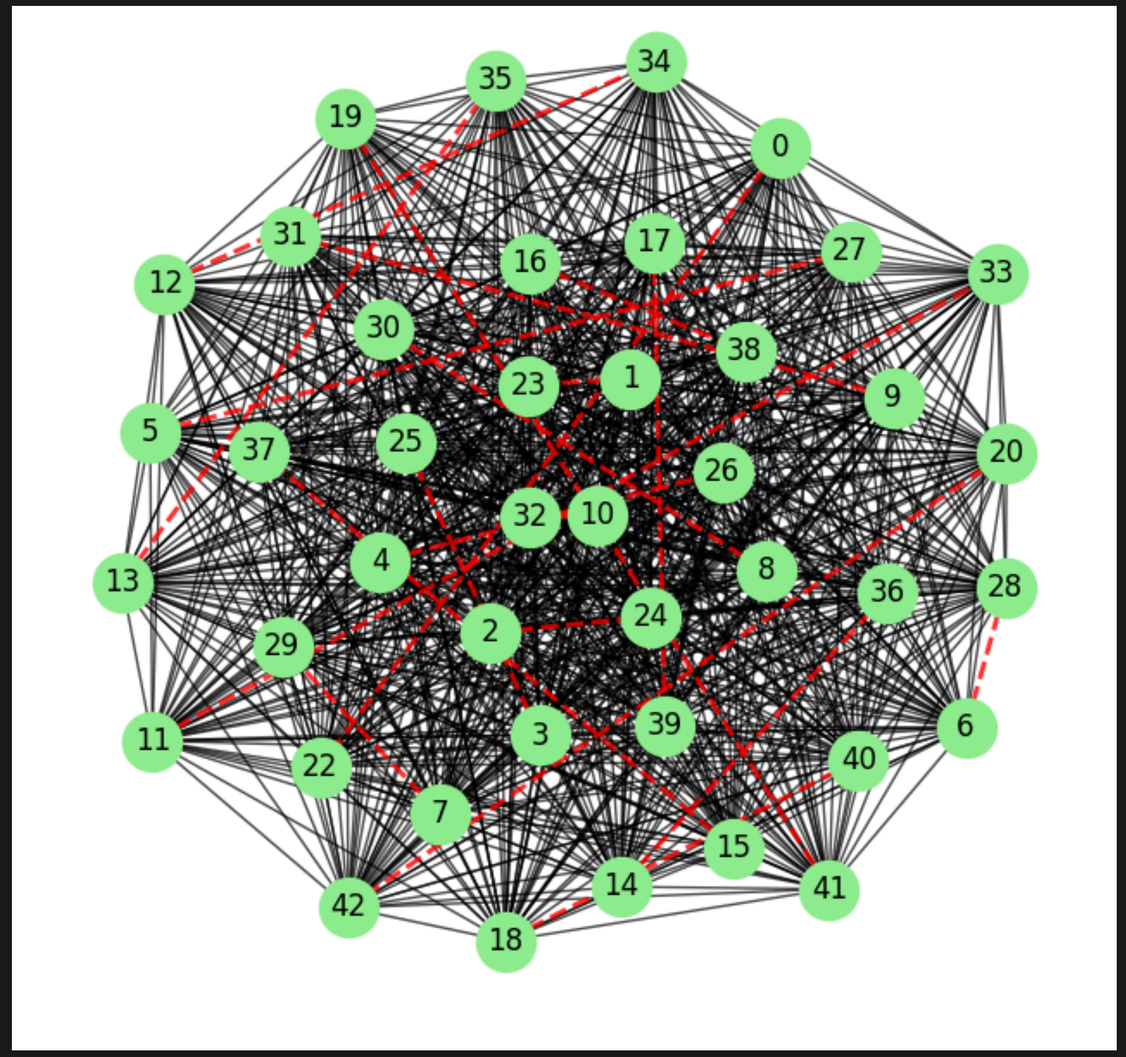

34
35
19
0
31
17
27
16
33
12
30
38
23
1
9
5
37
25
20
26
32
10
4
8
13
36
28
24
2
29
39
6
11
3
40
22
7
15
14
41
42
18

tsp4_7013

```
Path: 0, 7, 21, 16, 1, 2, 23, 24, 38, 43, 3, 17, 25, 39, 22, 29, 15, 12, 11,
 6, 5, 28, 27, 35, 13, 14, 4, 26, 36, 34, 33, 19, 18, 9, 8, 10, 32, 30, 31,
40, 20, 41, 42, 37, 0,
```

tsp4_7013

NÃO É COMPLETO



Implementação

Permutação

Time complexity: $O((n-1)!)$

Time complexity: $O(n^2)$

1 2 3 4 5

1 2 3 5 4

1 2 4 3 5

1 2 4 5 3

. . .

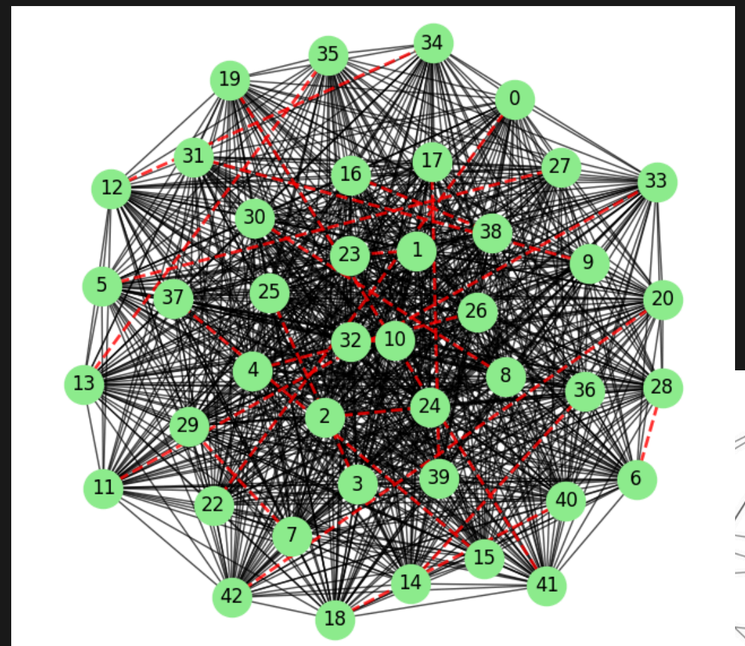
Floyd Warshall

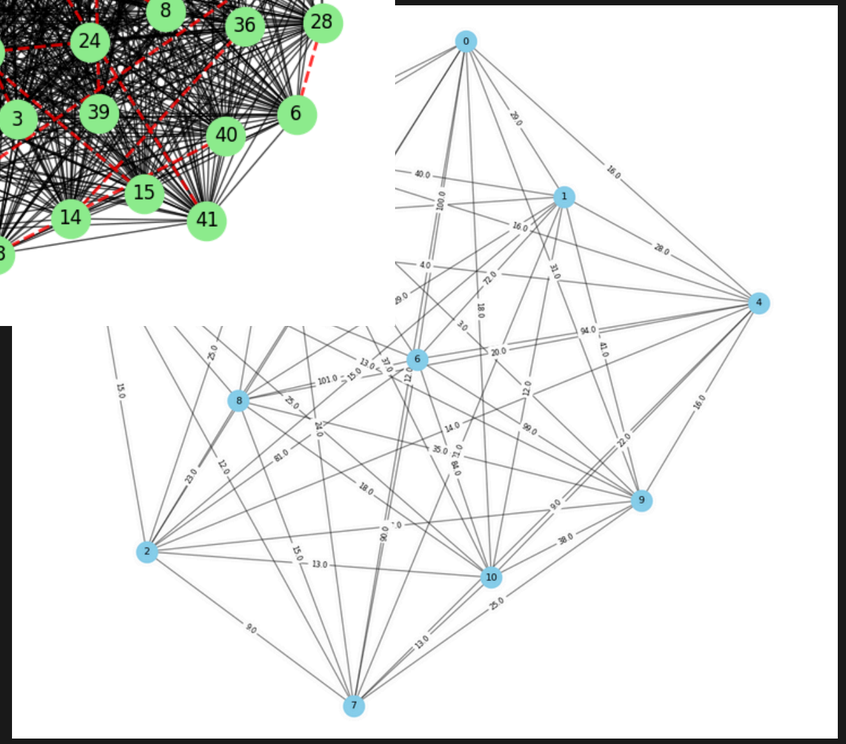
Completude de grafos

- Transforma grafos não completos em completos

Resolve a desigualdade triangular

- Faz com que grafos que não respeitam a desigualdade triangular passem a respeita-la





Held-Karp

Crescimento assintótico:

- Memória: O(n*2^n)
- Tempo: O(n² * 2^n)

- Divide to conquer
- Máscara binária
- Floyd Warshall

(x,y,z)

(z) → 001
(y,z) → 011
...
(x,y,z) → 111

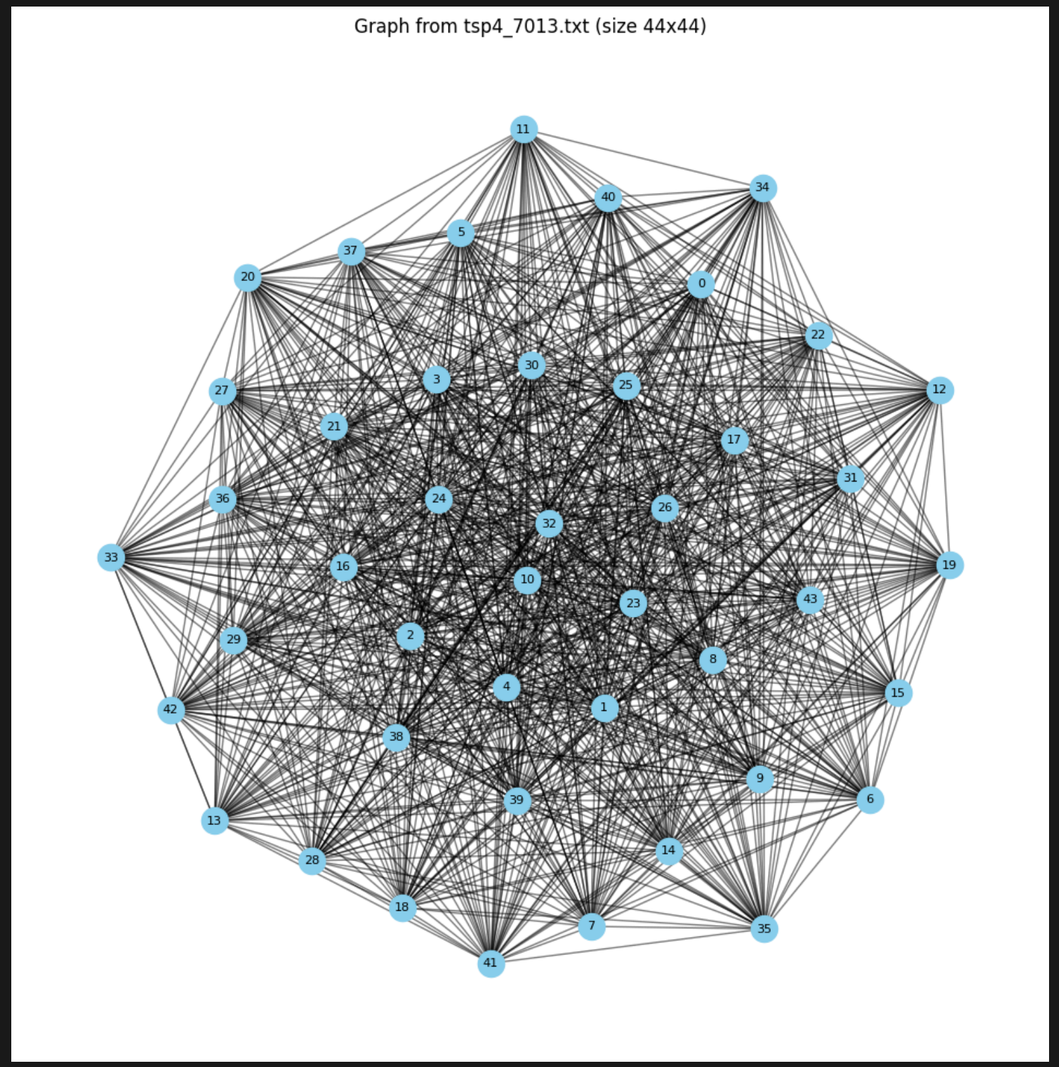
} mask

dp[011][y] = menor custo para visitar
y e z, terminando em y

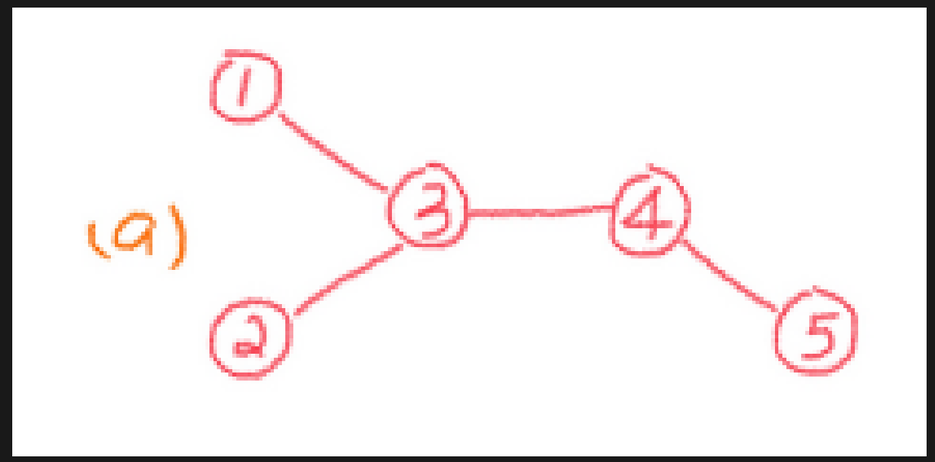
Christofides

Time complexity: $O(n^3)$

Time complexity: $O(n^2)$

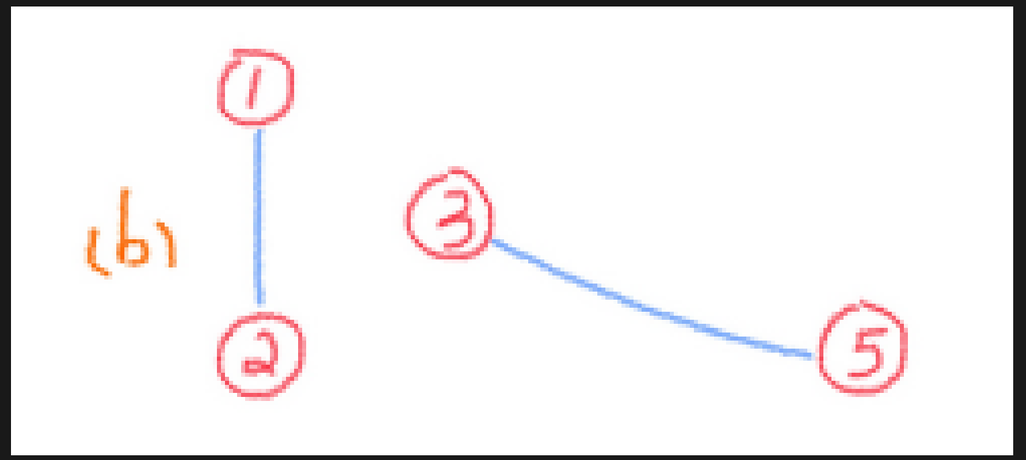


Christofides

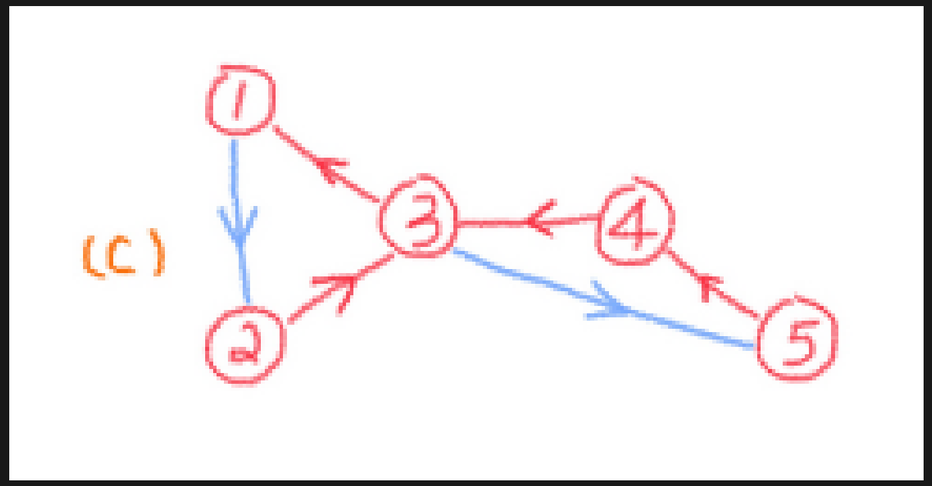


MST

Blossom

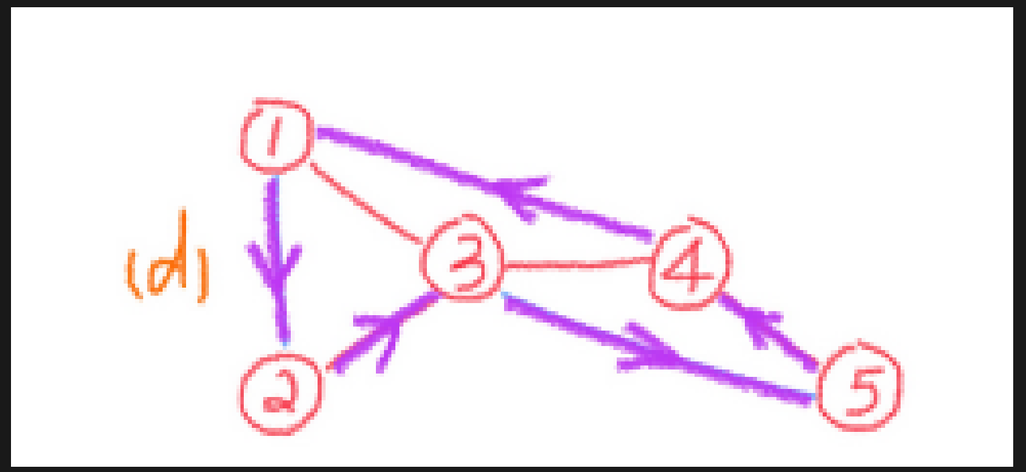


Christofides



Eulerian Path

Hamiltonian Path



Resultados

Força Bruta

Instância	Custo Obtido	Tempo (s)	Dif. p/ Ótimo
tsp1_253.txt	253	12.9678	0
tsp2_1248.txt	1.248	0.000706583	0
tsp3_1194.txt	1287	7200	1,078
tsp4_7013.txt	22.304	36000	3,18
tsp5_27603.txt	42.065	36000	1,52

Held-Karp

Instância	Custo Obtido	Tempo (s)
tsp1_253.txt	253	0.00282435
tsp2_1248.txt	1248	6.9771e-05
tsp3_1194.txt	1194	0.0537157
tsp4_7013.txt	–	–
tsp5_27603.txt	–	–

Christofide

Instância	Custo Obtido	Tempo (s)	Dif. p/ Ótimo
tsp1_253.txt	259	0,0016	1,02
tsp2_1248.txt	1.272	0,000779	1,019
tsp3_1194.txt	1.411	0,001159	1,18
tsp4_7013.txt	13.512	0,01	1,92
tsp5_27603.txt	30.652	0,005	1,11

Conclusões

Referências :

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- Held, M. and Karp, R. M. A dynamic programming approach to sequencing problems. Journal of the Society for Industrial and Applied Mathematics, 1962.
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- Christofides, N. Worst-case analysis of a new heuristic for the travelling salesman problem. Technical Report 388, Carnegie Mellon University, 1976.
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- Kolmogorov, V. Blossom V: A New Implementation of a Minimum Cost Perfect Matching Algorithm. Mathematical Programming Computation, v. 1, n. 1, p. 43–67, 2009.