5 Pre-lab Worksheet (5 points)

To be turned in before the start of the lab. This is an individual assignment.

1. Which do you think would have a higher value of K: an old, leaky house or a new, well-sealed house of the same size?

$$K = \sum_j U_j A_j$$

2. Which do you think would have a higher value of K: a small house or a large house with similar construction?

3. Calculate the current flowing through a 10Ω resistive element if the voltage applied across the terminals is 5V. Calculate the power that is dissipated under the same conditions.

$$I = \frac{5V}{10k\Omega} = \boxed{0.5\ mA} \qquad P = IV = I^2R = \frac{V^2}{R} = \frac{(5V)^2}{10k\Omega} = \boxed{2.5\ mW}$$

4. A 5Ω resistive element is rated for 100W. What is the largest voltage differential that can be applied to it without exceeding the power rating? How many amps will flow through it at that voltage?

$$P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR} = \sqrt{100W \cdot 5\Omega} = \boxed{22.4V}$$

$$I = \sqrt{\frac{P}{R}} = \boxed{4.47A} \quad \left(\text{or } I = \frac{V}{R} = \frac{22.4}{5} = 4.47A\right)$$

5. During the dreaded "Polar Vortex", the heater in my house could not bring the indoor temperature to the desired set-point. After running long enough to reach steady-state, the inside only got up to 19C. Assuming a constant outside temperature of -18C (it was pretty cold for a long time), determine K for my house if my heater puts out 15,000W.

$$Q_{furnace} = K\Delta T \Rightarrow K = \frac{Q}{\Delta T} = \frac{15kW}{37C} = \boxed{405.4\ \frac{W}{C}}$$

6. 10kg of water at 50C is placed in an insulated box with an overall heat transfer coefficient of 1W/C. The box is in a room at 20C. How long will it take for the temperature of the water to drop to 30C? Ignore the mass of the insulation – use only the mass of water to find C for the system.

$$T_{in} = T_{in}(0)e^{-\frac{K}{C}t} + \left[T_{out} + \cancel{\frac{Q_i}{K}}^{\,0}\right]\left[1 - e^{-\frac{K}{C}t}\right]$$

$$\Rightarrow -\frac{C}{K}\ln\left[\frac{T_{in}(t) - T_{out}}{T_{in}(0) - T_{out}}\right] = t \quad \text{where } C = 10kg \cdot 4.16\frac{kJ}{C \cdot kg} = 41.6\frac{kJ}{C}$$

$$t = -41.6\times10^3 \cdot \ln\left[\frac{30-20}{50-20}\right]$$

$$\boxed{= 45{,}700 \text{ seconds} = 12.7 \text{ hours}}$$

7. Substitute Eq. 2 into Eq. 1 to show that it is a solution to the governing equation when T_o and Q_i are constant. Do not solve Eq. 1 "from scratch." Just prove that Eq. 2 is a solution.

l.h.s. $C_{in}\frac{d}{dt}\left[T_{in}\right] + KT_{in} = C_{in}\left[T_{in}(0)\left(-\frac{K}{C_{in}}\right)e^{-\frac{K}{C}t} + \left(T_{out}+\frac{Q_{int}}{K}\right)\left(\frac{K}{C_{in}}\right)e^{-\frac{K}{C_{in}}t}\right] +$

all that's left $\rightarrow$ $K\left[\cancel{T_{in}(0)e^{-K/C\,t}} + \left(T_{out}+\frac{Q_{int}}{K}\right)\left(1-\cancel{e^{-K/C\,t}}\right)\right] = KT_{out} + Q_{int}$

$\checkmark$ r.h.s.

8. Derive the steady-state solution for Eq. 2, as determined by taking the limit of Eq. 2 as $t \to \infty$. Assume T_o and Q_i are constant.

$$T_{in} = T_{in}(0)\underbrace{e^{-\frac{K}{C}t}}_{0} + \left[T_{out}+\frac{Q_{int}}{K}\right]\left[1-\underbrace{e^{-\frac{K}{C}t}}_{0}\right] \Rightarrow \boxed{T_{in} = T_{out}+\frac{Q_{int}}{K}} \text{ or } \underline{Q_{int} = K(T_{in}-T_{out})}$$

9. Derive the steady-state solution from Eq. 1, as determined by setting the derivative to zero and solving the remaining algebraic expression. Assume T_o and Q_i are constant. Same

$$C\underbrace{\frac{dT_{in}}{dt}}_{0} + KT_{in} = KT_{out} + Q_{int} \Rightarrow \boxed{T_{in} = T_{out}+\frac{Q_{int}}{K}}$$

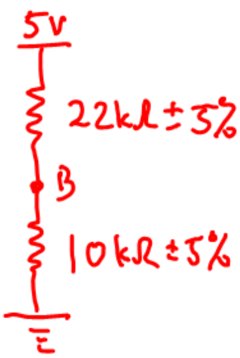
10. Given the voltage divider in Figure 11, calculate the voltage, V_B, for the nominal resistor values. Calculate the range of possible values using the specified tolerances of the resistors.

See next page

11. How much current can the voltage regulator on a SparkFun RedBoard reliably supply?

800 mA (from their website)

Figure 11: Voltage divider for the pre-lab.



Nominal: $V_B = 5V \cdot \frac{10}{10+22} = \boxed{1.5625V}$

Worst-case high (22kΩ − 5%; 10kΩ + 5%) ⇒

$$V_B = 5V \cdot \frac{10.5}{10.5 + 20.9} = \boxed{1.6720\ V}$$

Worst-case low $= 5V \cdot \frac{9.5}{9.5 + 23.1} = \boxed{1.4571\ V}$