Experimental analysis of heat transfer properties of an enclosure

1 Introduction

In this lab, you will develop a thermal model of an enclosure, which is an important early step in designing a climate control system. Indeed, one of the most important tasks of an engineer is to create system models that faithfully represent the physics with as little complication as necessary. Such models can then be used to test control algorithms, estimate performance, and determine important control parameters. Though we're focusing on building a greenhouse, the theory here has many applications: a home heating system, climate control in a car, or heat removal in a server room packed with computers.

You'll start by performing experiments on a small enclosure – in this case a small plastic container (about as low-tech as we can get). As part of the lab, you will use an oscilloscope and apply basic electrical theory to collect experimental data from a temperature sensor. You will then use the data to parameterize a thermodynamic model. In the next few of weeks, you'll apply these techniques to the design of your greenhouse.

Though we're focusing on a physical prototype, thermal systems are usually characterized using theoretical models. When designing a large building (or even a greenhouse), it would not make sense to build the structure, test a bunch of heating systems, and then pick one to install. Because the procedure is so common, there are entire building simulation programs dedicated to modelling heating and cooling. Indeed, you're asked to make an estimate of the thermal characteristics of your system from published sources in the pre-lab. But since this is a class in electromechanical systems, we'll use the physical prototype to motivate the introduction to some common electronic concepts.

Objectives

Upon successful completion of this module, the student should be able to:

- Apply basic principles of electricity to calculate current and power,

- Demonstrate understanding of the principles of a voltage divider,

- Demonstrate the use of a thermistor for measuring temperature,

- Use an oscilloscope to read voltages,

- Use an Arduino to gather time-series voltage data, and

- Use energy principles to develop and parameterize a first-order thermodynamic model.

2   Preparation

There are a number of topics to cover before coming to lab. In addition to the background material in this handout, be sure to review the following before coming to lab:

Basic laws of circuits and resistive elements. There's a decent overview in this SparkFun tutorial.

Thermistors. Read Sections 13 - 13.3 in the reading posted on collab.

Use of the Digilent Analog Discovery. Watch the introductory videos and *install the Waveforms software.* You will not be using the Arbitrary Waveform Generator in this lab, so you may skip Video 4 for now. **I cannot understate the usefulness of the oscilloscope!**

If you've never used a multimeter before, this SparkFun tutorial is very comprehensive. Concentrate on the parts that explain how to do measurements.

You must complete the pre-lab worksheet in Section 5 and turn it in at the beginning of lab. The pre-lab is an individual assignment.

2.1   Thermodynamic model

Your goal is to develop a model of your greenhouse that can be used to design an appropriate heating system. Specifically, you will want to size the heating element(s) so that a minimum temperature can be maintained for all expected weather conditions. Here, you will start with a simplified, first-order model of the heat transfer process. It is called first-order because the resulting ordinary differential equation, the *governing equation*,[1] will have only first derivatives in it. First-order o.d.e.'s are the simplest form of o.d.e. and are easy to solve, even if you haven't completed a course in differential equations.

Imagine an enclosure at some internal temperature, T_i, that is placed in an environment at another temperature, which we'll call T_o. In general, energy in the form of heat will flow either into or out of the enclosure, depending on the relative temperatures. The rate at which energy is transferred is dependent on many factors, including the temperature difference, the insulation around the enclosure, and the leakiness of the enclosure. In addition, the enclosure may have some internal addition of energy, for example from a heater.

If we draw a *control volume* around the enclosure as shown in Figure 1, then we can perform an *energy balance* in that control volume. As you remember from your physics classes (you do remember them, yes?), conservation of energy requires that the change in energy, E, in the control volume is equal to the rate at which energy flows across the boundary as heat, Q_s, plus the rate at which heat is added internally, Q_i.[2] Mathematically, this is expressed as:

$$\frac{dE}{dt} = Q_s + Q_i$$

Note that we count heat *entering* the system as a positive quantity; when heat is leaving the system, energy flows are negative.

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[1]The governing equation is the most basic equation, usually a differential equation, that describes the physics of a system.

[2]To be clear, Q is formally a rate of energy flow, or heat, which has units of Watts.
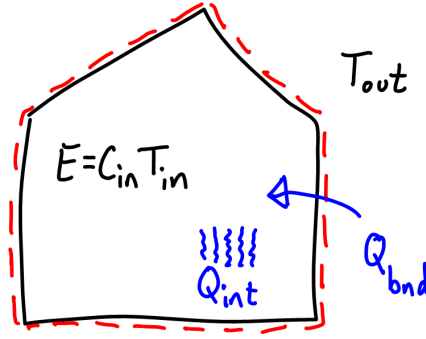
Figure 1: Control volume (shown in dashed red) for our thermodynamic analysis.

The heat capacity relates the temperature of a mass to the amount of thermal energy it contains. In reality, it is a complex quantity related to the masses of the individual components (e.g., air, metal, glass, plastic) and their thermodynamic properties. For our purposes, we'll just lump everything together and stipulate:

$$E = C_i T_i$$

where C_i is the lumped heat capacitance.

To calculate the heat moving across the boundary, Q_s, one can sum up the contribution of each type of boundary material by multiplying its thermal conductance, typically denoted by U in structural applications, by the area of that material and the temperature difference. That is,

$$Q_s = \sum_j U_j A_j \left(T_o - T_i\right)$$

Again, note that positive heat flow is into the system: if it's warmer outside than in, then $Q_s > 0$. The greater the differential, the greater the rate at which energy flows. We'll simplify things further by defining an overall heat transfer coefficient for a given structure, K_s, as:

$$K_s \equiv \sum_j U_j A_j$$

thus, Q_s can be simplified to,

$$Q_s = K_s \left(T_o - T_i\right)$$

We can substitute the expressions above into our energy balance equation to get:

$$\frac{dE}{dt} = C_i \frac{dT_i}{dt} = K_s \left(T_o - T_i\right) + Q_i$$

which can be rearranged slightly to produce:

$$C_i \frac{dT_i}{dt} + K_s T_i = K_s T_o + Q_i \tag{1}$$

If you've had a class in differential equations, you should recognize this as a first-order, linear, non-homogeneous, ordinary differential equation with constant coefficients. If you haven't, some of those terms might not mean much to you, but just note that under certain conditions, the equation

is easily solved. Specifically, if we stipulate that both the outside temperature and internal heat are constant, we can readily solve the equation to get:

$$T_i(t) = T_i(0)e^{-\frac{K_s}{C_i}t} + \left[T_o + \frac{Q_i}{K_s}\right]\left[1 - e^{-\frac{K_s}{C_i}t}\right] \tag{2}$$

Physically, we can interpret the solution as follows: the first term on the right hand side (r.h.s) is the effect of the initial temperature, $T_i(0)$, whose influence diminishes as time goes on (as seen from the exponential). When $t = 0$, all of the exponential terms are 1, and so the only non-zero term on the r.h.s. is the initial temperature. As time goes on, the effect of the initial temperature approaches zero. The second term on the r.h.s. captures the effects of "forcing", whether from a temperature differential between the inside and outside or from internal heating. As time goes to infinity, the exponential terms will go to zero, and the only remaining terms will be constants related to T_o and Q_i (remember, we assumed that T_o and Q_i are constant). Because the temperature will not change any further under those conditions, the resulting temperature is called the *steady-state* solution.

Before continuing, substitute Equation 2 into the governing equation to verify that the above equation is indeed a solution to the governing equation. Determine the steady-state solution by taking the limit of Eq. 2 as $t \to \infty$. Separately, note that if the temperature isn't changing, the derivative $dT/dt \equiv 0$ in Equation 1; if you set the derivative in the governing equation to 0, you should be able to generate the same steady-state solution. The pre-lab exercises ask you to do just that.

There are two unknown physical parameters that you need to determine for your model: C_i and K_s. They both show up in the exponential (time) term, but only K_s appears in the steady-state solution. Thus, you can determine K_s by adding energy with a heater and observing the interior temperature over a long period. Once K_s is known, you can use the transient response over short periods of time to determine C_i.

2.2 The Voltage Divider

To read the temperature of your system, you'll make use of a thermistor – a device that changes resistance based on temperature – in something called a *voltage divider*, which will translate changes in resistance to changes in voltage, which are easily read by microcontrollers. The voltage divider is easily the most common circuit that you will encounter in electronics. Luckily, it is also a fairly simple circuit, and you should get to the point where analyzing one is practically automatic. Consider the circuit in Figure 2, where a voltage potential is applied to two resistors connected in series. We wish to find the voltage in between them, at point B. If no current flows in or out at point B – a practical occurrence in many situations – the current through each resistor must be equal to the current through the entire circuit,

$$I_{A \to B} = I_{B \to C} = I_{A \to C}$$

which from Ohm's Law gives,

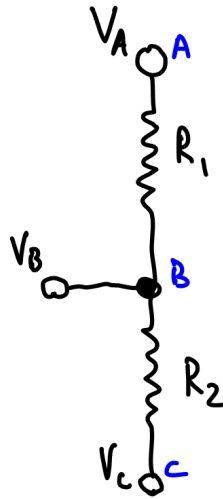$$\frac{V_A - V_B}{R_1} = \frac{V_B - V_C}{R_2} = \frac{V_A - V_C}{R_1 + R_2}$$

Figure 2: A voltage divider. Please, control your excitement.

A little rearrangement of the second equality leads to

$$\frac{V_B - V_C}{V_A - V_C} = \frac{R_2}{R_1 + R_2}$$

or

$$V_B = V_C + (V_A - V_C)\frac{R_2}{R_1 + R_2}$$

which is just a line fit to the points as shown in Figure 3. Usually, *but not always*[3], V_C is ground, which if we take to be 0V yields,

$$V_B = V_A\frac{R_2}{R_1 + R_2}$$

That is, the voltage at point B is just the input voltage, V_A, times the ratio of R_2 to the total resistance.

Care must be taken to ensure that no current leaves junction B through anything we might connect to it; otherwise, the voltage at point B will change. For example, if V_B is a reference voltage for another purpose, we'll have to make sure that the current is negligible compared to that flowing through the resistors. The inputs on our oscilloscopes have extremely high resistances, so this is not a concern in this lab. In other cases, you can construct a buffer circuit to prevent changes in voltage.

2.3   The thermistor

The resistance of any resistor changes as a function of temperature. For many applications, the exact size of the resistor isn't terribly important and so the temperature effects are safely ignored. When the exact resistance is important, the changes can be accounted for in the circuit design or compensated for afterwards in software. A *thermistor*, on the other hand, is specifically designed to

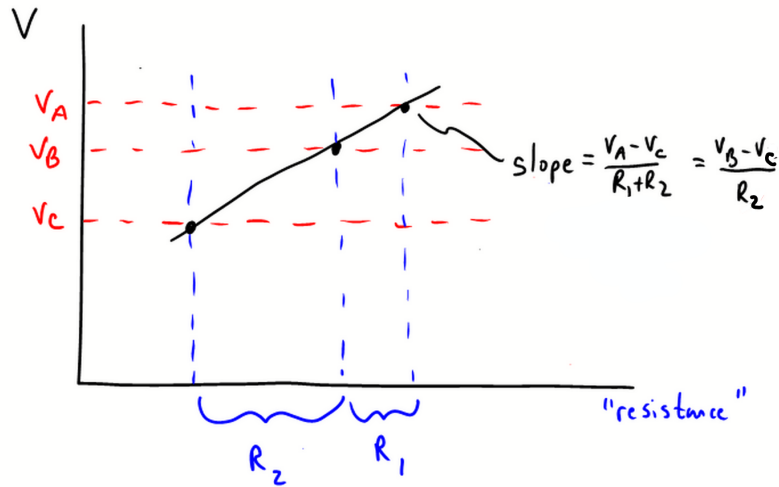---

[3]In particular, on quizzes...

Figure 3: A graphical interpretation of a voltage divider.

take advantage of the changes in resistance so that it can be used to *measure* temperature. Thermistors are optimized to have predictable changes in resistance over a given range of temperatures. By putting the thermistor in one leg of a voltage divider, a change in temperature will ultimately result in a change in voltage, which is easily measured.

Over the limited range of temperatures we'll see in this lab, the resistance of the thermistor can be approximated as,

$$R = R_0 e^{-B\left(\frac{1}{T_0} - \frac{1}{T}\right)} \tag{3}$$

where B is a constant specified by the manufacturer, and R_0 is the resistance measured at some specified temperature, T_0, in Kelvin. Manufacturers typically calibrate thermistors at 25C, but we'll calibrate ours independently at room temperature, which will be slightly lower than that.

2.4 The potentiometer

A potentiometer is a common electrical component that allows the user to easily change the relative resistances between its contacts. A potentiometer (often simply called a "pot") can either be used as a variable resistor (sometimes called a rheostat) or as a voltage divider. The resistance between the end terminals is fixed to some specified resistance ($10k\Omega$ is a popular size), while a third terminal, the *wiper*, is allowed to slide along the resistive material between the ends; for a linear potentiometer, the resistance between the wiper and a given end is proportional to the distance between the contacts. By connecting one end terminal and the wiper to a circuit, one creates a variable resistor, which might be used, for example, for volume control of an amplifier. By applying a voltage differential across the end terminals, the wiper can be used as a variable voltage divider. Figure 4 shows a potentiometer configured as a variable resistor. Figure 5 shows a variable voltage divider.
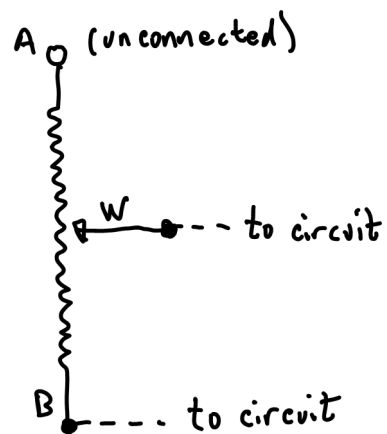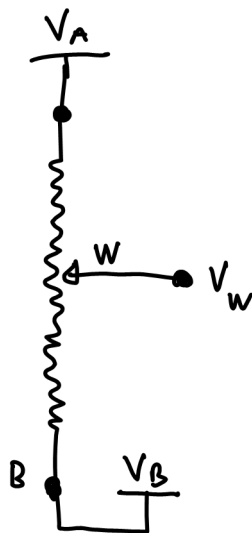
Figure 4: A variable resistor, or rheostat.

Figure 5: A variable voltage divider.

3   In lab

Your goal in this lab is to create a thermal model for an enclosure and determine the model parameters from experimental data. You'll do this for a small enclosure and later apply the techniques to your actual greenhouse. You'll measure the temperature using a thermistor configured as part of a voltage divider, which will allow you to calculate the resistance of the thermistor and thereby the temperature.

Using an oscilloscope and multimeter

Before getting started on the temperature measurements, you'll be introduced to two important tools for designing and testing circuits: the oscilloscope and the multimeter. The multimeter you'll use in lab can measure many things, including voltage, resistance, and capacitance. Because it is digital, we will refer to it as a digital multimeter, or DMM. The Digilent Analog Discovery includes many tools: oscilloscope, arbitrary wave generator, power supply, voltmeter, and a bunch of digital tools. Here, we'll use a potentiometer to demonstrate the concept of a voltage divider and you'll become familiar with some of the functionality of the Analog Discovery.

Procedure

1. Insert the leads from the $10k\Omega$ potentiometer on your workstation into the breadboard and connect the oscilloscope as shown in Figure 6. For this and many of the following labs, your life will be *much* easier if you plug the oscilloscope leads into a central location and then use jumper wires to "probe" your circuit. It's kind of a pain to keep the oscilloscope wires connected otherwise, and it's much quicker to just move a jumper wire around.
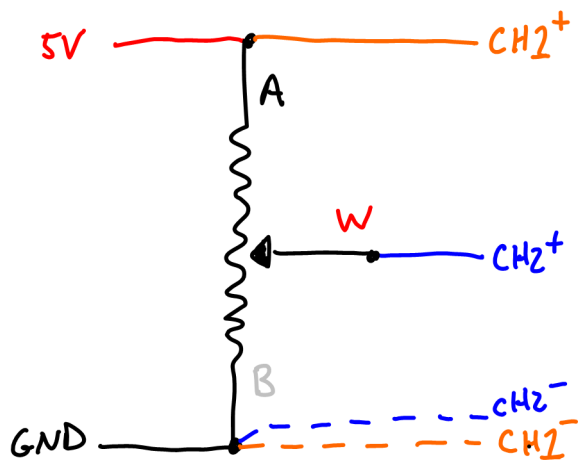

Figure 6: Connections for the potentiometer, with the terminals labelled in the same convention as on your workstation.

2. Open the oscilloscope software and start the oscilloscope. Since the 5V power line is not yet energized, you should see readings of 0V. Set the scale to 1V per division and the offset to -2V for both channels. Set the Trigger to "None".

3. From the main screen of the oscilloscope, turn on the +5V supply. You should see CH1 jump to somewhere around 5V. Note that the source is a nominal 5V, so it is important to measure the actual value.

4. Turn the potentiometer and observe CH2. What is happening?

5. Turn the potentiometer such that CH2 reads 3.5V. Pull the potentiometer leads out from the breadboard and, using the DMM, measure the resistance between each terminal and the wiper. **It is imperative that whenever you measure resistance that the component in question be isolated from the circuit.** Otherwise, you certainly won't get a meaningful reading and you might damage the multimeter.

6. Given the measured resistances, calculate the expected voltage at the wiper. How does it compare to the measured value?

7. Plug the potentiometer back into the breadboard and measure the voltage using the DMM. Note that for constant or slowly changing voltages, it is generally easier to use the DMM than an oscilloscope, but it is important that you know how to use both.

Reading voltages with a microcontroller

Here, you'll use an Arduino Uno to read voltages instead of the oscilloscope. The oscilloscope is a critical tool for exploring and debugging, but using the Uno will make collecting the data significantly easier.[4] We'll talk much about how the Uno reads voltages in the future, but the short version is that the Uno has a built-in *analog-to-digital converter*, or ADC. The ADC takes a voltage and turns it into a number using the formula:

$$result = floor\left(\frac{V}{V_{ref}} \times 1024\right) \tag{4}$$

where V is the voltage that you're reading and V_{ref} is the reference voltage that the microcontroller uses to compare to V. In this case $V_{ref} \approx 5V$. Why the formula has this form and how the ADC works is a subject for another day. Using the Uno, however, will allow you to easily collect data for later processing. Since we have yet to cover the details of the Uno, your instructor has provided a basic program for data collection.

Procedure

- **Disconnect the Analog Discovery from the circuit entirely** and build the circuit shown in Figure 7. Note that the Uno will be providing power to the circuit from the 5V pin.

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[4]The oscilloscope can save data, but you can't interact with it while it's collecting data, so there's too much risk something will go wrong and you'll have to do the data collection all over.
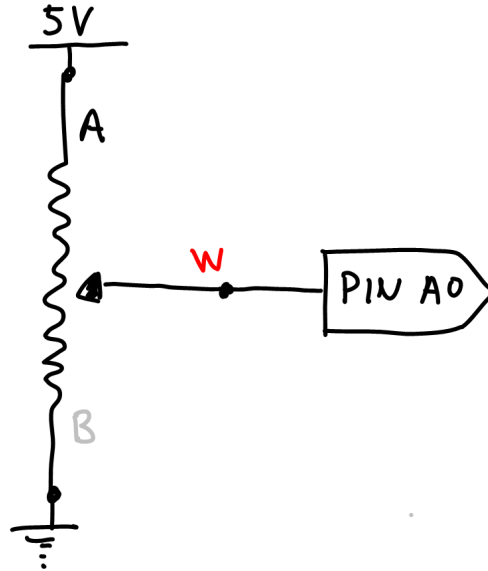
Figure 7: Circuit for exploring the Uno's ADC.

- Upload the program provided. When finished, open the Serial Monitor by clicking the icon in the upper right of the Arduino IDE. Two columns should start scrolling down the screen: the first is the time in seconds since the program has started, and the second the reading from the ADC (Equation 4).

- Using a DMM to measure voltage, adjust the potentiometer the wiper terminal measures 1.0V and note the ADC reading. Fill in the appropriate line in Table 1.

- Repeat the process for each of the values in Table 1. How well do the actual and theoretical values compare?

Wiper voltage	Theoretical ADC reading	Actual ADC reading
0V		
1V		
2V		
3V		
4V		
5V		

Table 1: Record your ADC calculations here.

Heat transfer experiment

Here, you'll apply the tools, techniques, and theory from above to build a thermodynamic model of a small enclosure.

To collect data for the model, you will be provided with the following supplies:
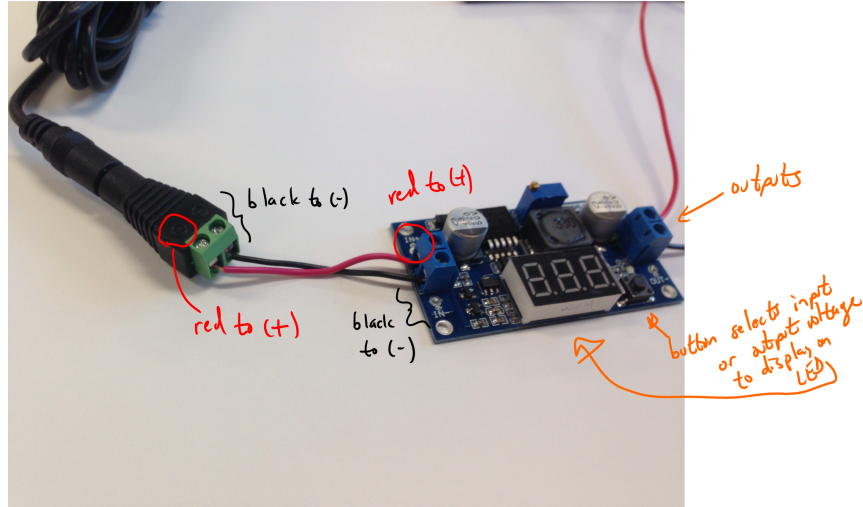
Figure 8: Image showing power supply connections.

- A small plastic container (not your greenhouse – that comes later),

- A 10W, 6.2Ω power resistor,

- A 12V "wall wart" power supply (2A max[5]),

- A $10k\Omega$ thermistor, model B57164K0103J, and

- Basic electronic components to complete your circuits.

Procedure

Read these directions thoroughly before you start the experiment. Timing is important!

1. Measure the actual resistance of your power resistor and calculate the voltage needed to generate 8W of heat from it. Verify that you are not exceeding the 2A limit on your power supply.

2. If it hasn't been done already, connect the 5.5mm/2.1mm output terminal of the power supply to the voltage regulator using the supplied adaptor. Use the standard convention of red for positive voltage and black for ground (negative). See Figure 8.

3. Plug in your power supply and the LED on the voltage regulator should light up. The LED can be toggled between input and output voltages. The input voltage should be around 12V (the rating on the supply). Adjust the output voltage to the value determined previously using the small screw on the face of the regulator.

4. Unplug the power supply and connect the power resistor to the output of the voltage regulator. Use a small breadboard, as shown in Figure 9 to do this. Be careful not to short anything

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[5]It is a common misconception that the amperage rating on a power supply means it will put out that many amps. In actuality, the voltage supply will put out a (nominal) voltage and as much current as the load requires to maintain that voltage *up to the rated current*. Exceeding the rated current may damage the power supply.
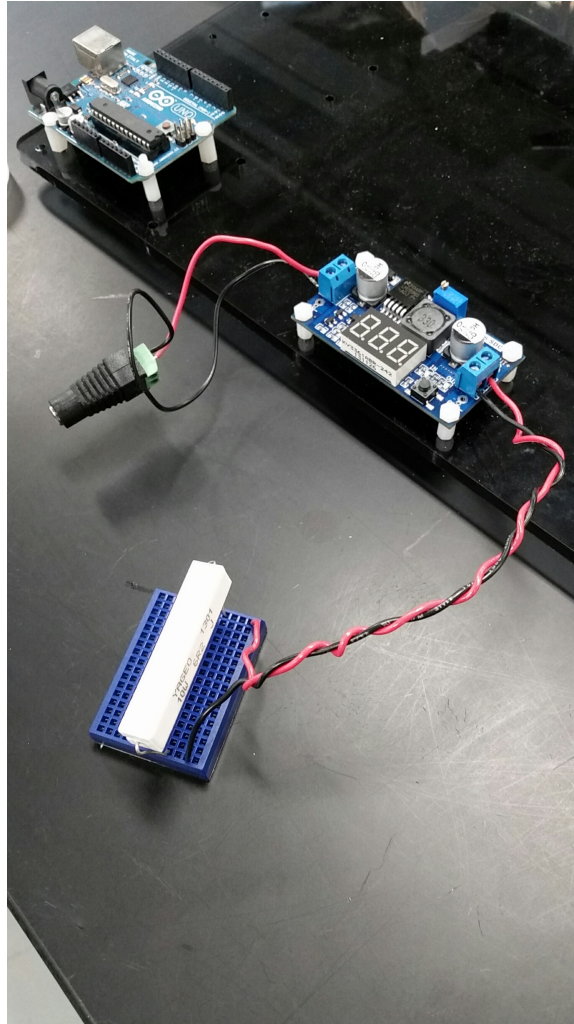
Figure 9: Image showing power resistor connections.

together!

5. Grab a $10k\Omega$ resistor and use the DMM to measure its exact resistance. $10k\Omega$ is only a nominal value: How can you tell the tolerance on it?

6. Using a breadboard, wire your thermistor according to the schematic provided in Figure 10. You'll need to connect a ground, provide 5V power, and read voltages of your voltage divider.[6] You'll want the thermistor to be easily moved, so put it at the end of a two-wire ribbon cable.

7. Plan how you will arrange your heater and temperature sensor in you plastic container. Note that arranging the heater and sensor is not an exact science, but you probably don't want the temperature sensor too near the heater, neither do you want it too close to a wall. Also, creating a short circuit in any of the components would be decidedly bad. **The power to the temperature sensor and the heating resistor must be kept separate at all times or you risk frying the Uno!** The thermistors aren't free, either.

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[6]Because the 5V power is only nominal, you need to measure the actual value so you can calculate the resistance of the thermistor accurately.
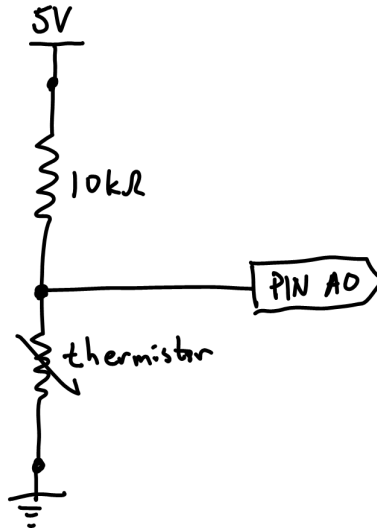
Figure 10: Schematic for temperature sensor circuit.

8. Plug in your power supply and let your power resistor heat up, *but do not put it in your enclosure yet.* **Be careful! It will get hot!** Not third-degree burns hot, but drop it if you grab it hot. Let the heater warm up for a good 5-10 minutes.

9. While the heater is warming, calibrate your thermistor. Though the *change* in resistance is guaranteed by the manufacture, the nominal resistance is not. Therefore, you must find R_0 in Equation 3 at a known value of T_0. Use a DMM to measure the output voltage of your circuit and find the resistance of the thermistor. We'll assume the exterior temperature to be constant for the rest of the experiment.

10. When ready, reset the Uno by pressing the rest button and place your sensor in the enclosure. The experiment will take on the order of 20 - 30 minutes, so you may want to call over your instructor to confirm that you're properly set up before you start. The data collection process is a bit long, so if you proceed without your professor's blessing, you risk wasting time repeating the experiment.

11. Sit back and relax while the Uno does all the work for you. Now might be a good time to go buy your TA a fancy coffee product. You can also get started on the post-processing in Section 4.

12. Periodically check to see that your system has reached steady-state, since this is important for determining K_s. If it hasn't, let it run a bit longer. Note that if you disable the "Autoscroll" check box in the Serial Monitor, you can scroll up to see if the ADC reading is still changing.

    Steady-state temperature, T_{in}: _____

13. When the system has reached steady-state, grab the data from the Serial Monitor for post-processing. You can do this by disabling the Autoscroll in the Serial Monitor and cutting and

pasting the data to a file on your computer – not particularly high-tech, but it'll work.

4   Post processing

Due Monday, Feb. 5. Submit as a hard-copy, one per team. **Answers without proper units will be marked incorrect.**

1. Import your data file into the data processing software of your choice: a spreadsheet, MAT-LAB, JAVA program, or similar. Convert the ADC readings to voltages and use those to find resistance of the thermistor at each timestep. Finally, convert the resistance to temperatures. If you reset the Uno when you started the experiment, then your timestamps should line up. If not, you'll have to manually adjust them so that $t = 0$ when you first put the power resistor in the container.

2. Using the calculated value of Q_i and the steady-state value for T_i, find K_s.

   K_s: ————————————————

3. Calculate the time it took for the temperature differential between inside and outside to reach 63.2% of the steady-state difference. As we'll discuss more next week, this is one way to calculate the *rise-time* of the system (why 63.2%?). Use the rise time to find C_{in} for your model.

   Rise-time: ————————————————

   C_i: ————————————————

4. Create another data set that corresponds to your parameterized model. That is, put C_i, K_s, and Q_i into the solution of the governing equation (Equation 2) and plot the resulting curve on the same graph as your experimental data. How well does it compare to the collected data? Can you adjust the parameters to qualitatively improve the fit?

5. Submit a graph comparing your actual data to your parameterized model. Fill in the table below with both the calculated values of the coefficients and the "best-fit' values after adjustment.

Parameter	Calculated	Best-fit
K_s		
C_i		
Q_i		

Table 2: Model parameters.

6. For each parameter in our model, C_i, K_s, and Q_i, how would raising them affect the steady-state temperature in a heating scenario, all else being equal? Fill in Table 3.

Parameter	Effect on steady-state temperature
$K_s \uparrow$	
$C_i \uparrow$	
$Q_i \uparrow$	

Table 3: Effect of raising model parameters.

7. The greenhouse from IKEA are substantially larger than the ones we used for the in-lab experiment. It is not always easy to apply a small-scale experiment to a larger one, but it is often possible to approximate K by scaling according to surface area (assuming the thermal conductance of the material per unit area doesn't change, which is reasonable in this case). Estimate the surface area of the IKEA greenhouse and estimate K_s for the full-scale model. How big of a heater would be needed to maintain a 10C temperature differential above the outside temperature?

8. Find the resistance tolerance, $\Delta R/R$, on the <u>datasheet for the thermistor</u>. (we're using the 'J' model). Calculate the error bounds for the steady-state temperature, in Celsius. Calculate the resultant error bounds on K. Ignore all other sources of error.

5 Pre-lab Worksheet (5 points)

To be turned in before the start of the lab. This is an individual assignment.

1. Which do you think would have a higher value of K: an old, leaky house or a new, well-sealed house of the same size?

2. Which do you think would have a higher value of K: a small house or a large house with similar construction?

3. Calculate the current flowing through a 10Ω resistive element if the voltage applied across the terminals is 5V. Calculate the power that is dissipated under the same conditions.

4. A 5Ω resistive element is rated for 100W. What is the largest voltage differential that can be applied to it without exceeding the power rating? How many amps will flow through it at that voltage?

5. During the dreaded "Polar Vortex", the heater in my house could not bring the indoor temperature to the desired set-point. After running long enough to reach steady-state, the inside only got up to 19C. Assuming a constant outside temperature of -18C (it was pretty cold for a long time), determine K for my house if my heater puts out 15,000W.

6. 10kg of water at 50C is placed in an insulated box with an overall heat transfer coefficient of 1W/C. The box is in a room at 20C. How long will it take for the temperature of the water to drop to 30C? Ignore the mass of the insulation – use only the mass of water to find C for the system.

7. Substitute Eq. 2 into Eq. 1 to show that it is a solution to the governing equation when T_o and Q_i are constant. Do not solve Eq. 1 "from scratch." Just prove that Eq. 2 is a solution.

8. Derive the steady-state solution for Eq. 2, as determined by taking the limit of Eq. 2 as $t \to \infty$. Assume T_o and Q_i are constant.

9. Derive the steady-state solution from Eq. 1, as determined by setting the derivative to zero and solving the remaining algebraic expression. Assume T_o and Q_i are constant.

10. Given the voltage divider in Figure 11, calculate the voltage, V_B, for the nominal resistor values. Calculate the range of possible values using the specified tolerances of the resistors.

11. How much current can the voltage regulator on a SparkFun RedBoard reliably supply?

Figure 11: Voltage divider for the pre-lab.