AP Computer Science
Chapter 8 Notes – Recursion

Name Key Per ____

re·cur·sion [ri-**kur**-zh*uh* n]

–noun *See* recursion.

8.1 Recursive Thinking

Recursion is a programming technique in which a method can call itself to solve a problem. Every recursive method has two distinct parts:

- A base case or termination condition that causes the method to end.
- A non-base case whose actions move the algorithm towards the base case and termination.

```
public void drawLine (int n)
{
   if (n == 0)
      System.out.println("All done!");
   else
   {
      for (int i = 1; i <= n; i++)
         System.out.print ("*");
      System.out.println();
      drawLine(n-1);
   }
}
```

base case.

non-base case

- What is the output for the call `drawLine(3)` ?

***
**
*

d(3)
↓
d(2)
↓
d(1)
↓
d(0) ← base case

~~d(0)~~
~~d(1)~~
~~d(2)~~
~~d(3)~~

- What is the base case in the `drawLine` method?

n==0

Infinite Recursion

All recursive definitions must have a non-recursive part or base case. If they don't, there is no way to terminate the recursive path. A definition without a non-recursive part causes infinte recursion. This problem is similar to an infinite loop with the definition itself causing the infinite "loop".

```
public void catastrophic (int n)
{
   System.out.println(n);
   catastrophic(n);
}
```

c(3)
↓
c(3)
↓
c(3)
⋮

3
3
3

Recursive Definition

Mathematical formulas often are expressed recursively. A good strategy for writing recursive methods is to first state the algorithm recursively in words.

Write a method that returns $n!$ (n factorial). $n!$, for any positive integer n, is defined to be the product of all integers between 1 and n inclusive. This definition can be expressed recursively as:

$$n! \begin{cases} 1! = 1 & n = 1 \quad // \text{ base case} \\ n * (n-1)! = n! & n > 1 \end{cases}$$

The concept of the factorial is defined in terms of another factorial until the base case of 1! is reached

```
public static int factorial(int n)
{
    if (n==1)
        return 1;
    else
        return n * factorial (n-1);

}
```

5! ← 60
↓
5 * 4! ← 60
↓
4 · 3! ← 24
↓
3 · 2! ← 6
↓
2 · 1! ← → 2
↓
1

Write a recursive method `revDigs` that outputs its integer parameter with the digits reversed. For example,

`revDigs(147)`	outputs	741
`revDigs(4)`	outputs	4

First, describe the process recursively:

if n < 10 print n
else print right digit, truncate Right digit, call again

```
public static void revDigs( int n )
{
    if (n<10)
        S.O.P (n)
    else {
        S.O.P (n%10);
        revDigs (n/10);
}   }
```

8.2 Recursive Programming

A method in Java can invoke itself; if set up that way, it is called a *recursive* method. The code of a recursive method must be structured to handle both the base case and the recursive case. Each call to the method sets up a new execution environment, with new *parameters* and new *local variables.* As always, when the method execution completes, control returns to the method that invoked it (which may be an earlier invocation of itself).

push on stack
pop off stack

Example: For the method below, what does `result(5)` return? Draw the recursive call tree. If $n > 0$, how many times will `result` be called to evaluate `result(n)` (including the initial call)?

```
public int result(int n)
{
   if (n == 1)
      return 2;
   else
      return 2 * result(n - 1);
}
```

r(5) = 32
16
2* r(4) = 32
8
2 * r(3) = 16
4
2 * r(2) = 8
2
2 * r(1) = 4
2

result(n) will call method n times

Example: What would be returned by `t(5)` using the following method:

```
//Precondition: n >= 1
public int t(int n)
{
   if (n == 1 || n == 2)
      return 2 * n;
   else
      return t(n - 1) - t(n - 2);
}
```

-4
t(5)
-2 2
t(4) - t(3)
2 4 4 2
t(3) - t(2) t(2) - t(1)
4 2
t(2) - t(1) 2*2 2*2 2*1
2*2 2*1

-4

Example: Consider the problem of computing the sum of all the numbers between 1 and any positive integer *n*, inclusive. What will be the result of `sum(4)`?

```
public int sum (int num)
{
      int result = 0;
      if (num == 1)
         result = 1;
      else
         result = num + sum (num - 1);
      return result;
}
```

S(4) 10
10 6
result = 4 + S(3)
6 3
result = 3 + S(2)
3 1
result = 2 + S(1)
result = 1

4 + 3 + 2 + 1 = 10

Recursion vs Iteration

Just because we can use recursion to solve a problem, doesn't mean we should. For instance, we usually would not use recursion to solve the sum of 1 to *n* problem, because the *iterative* version is easier to understand. Write the iterative version of the method below:

```
public int sum (int num)
{
    int result = 0;
    for (int i = num; i > 0; i--)
        result += i;
    return result;
}
```

You must be able to determine when recursion is the correct technique to use. Every recursive solution has a corresponding *iterative* solution. Recursion has the overhead of multiple method invocations. Nevertheless, *recursive* solutions often are more simple and elegant than iterative solutions.

Rewrite the following iterative method as a recursive method that computes the same thing. NOTE: your recursive method will require an extra parameter.

```
public int nums(int x)
{
    int count = 0, factor = 2;
    while(factor < x)
    {
        if (x % factor = = 0) count++;
        factor++;
    }
    return count;
}
```

```
public int nums (int x, int y) {
  if (y==x)
     return 0;
  else
     if (x%y==0)
        return 1 + nums(x, y+1);
  else
     return nums(x, y+1);
```

factor	count	x
~~2~~	~~0~~	6
~~3~~	~~1~~	
4	2	
5		

returns number factors in x, not including 1 and x.

2

n(6,2) 2

1 + n(6,3) 1

1 + n(6,4) 0

n(6,5) 0

n(6,6)

0

Size

- Write a *recursive* method to compute the power of x^n for non-negative n.

$x^n = x_1 \cdot x_2 \cdots x_n$

```
public int power (int x, int n) {
    if (n==1)
        return x;
    else
        return x * power (x,n-1);
}
```

- Write an *iterative* method to compute the power of x^n for non-negative n.

```
public int power (int x, int n) {
    int result = 1;
    for (int i = 1; i <= n; i++)
        result *= x;
    return result;
}
```

```
while (n > 0) {
    result *= x;
    n--;
}
```

Indirect Recursion

A method invoking itself is considered to be *direct* recursion. A method could invoke another method, which invokes another, etc., until eventually the original method is invoked again.

For example, method `m1` could invoke `m2`, which invokes `m3`, which in turn invokes `m1` again until a base case is reached. This is called *indirect* recursion, and requires all the same care as direct recursion. It is often more difficult to trace and debug.

m1 → m2 → m3

8.3 Using Recursion

Strings

int

Example: Write a recursive method, `len`, which accepts a string and returns the number of characters in the string .

The length of a string is:

- 0 if the string is the empty string (""). // base case.
- 1 more than the length of the rest of the string beyond the first character .

```
public int len(String s) {
    if (s.equals(""))
        return 0;
    else
        return 1 + s.substring(1);
}
```

5
len("hello")
↓ 4
1 + len("ello")
↓ 3
1 + len("llo")
↓ 2
1 + len("lo")
↓ 1
1 + len("o")
↓ 0
1 + len("")
↓
0

Example: Write a recursive method named `makeStarBucks` which receives a non-negative integer `n` and returns a `String` consisting of `n` asterisks followed by `n` dollars signs.

`makeStarBucks(5)` -> `*****$$$$$`

`makeStarBucks(3)` -> `***$$$`

```
public String makeStarBucks(int n) {
    if (n==1)
        return "*$";
    else
        return "*" + makeStarBucks(n-1) + "$";
}
```

*****$$$$$
m(5)
****$$$$
* m(4) $
↓***$$$
* m(3) $
↓**$$
* m(2) $
↓*$
* m(1) $
↓
*$

Arrays

Example: Write a `void` method named `clear` which accepts an `integer` array, and the number of elements in the array and sets the elements of the array to 0. The items can be cleared recursively as follows:

- An array of size 0 is already cleared;
- Otherwise, set the first element of the array to 0, and clear the rest of the array

```
 0 1 2 3 4
a [ | | | | ]   n=5
```

```
public void clear(int [] a, int n) {
    if (n==0)
        return;
    a[a.length - n] = 0;
    clear(a, n-1);
}
```

c(a,5) cell 0
c(a,4) cell 1
c(a,3) cell 2
c(a,2) cell 3
c(a,1) cell 4
c(a,0) - return

Example: Write a `void` method named `init` which accepts an integer array, and the number of elements in the array and recursively initializes the array so that `a[i] == i`. The elements can be initialized recursively as follows:

- An array of size 0 is already initialized;
- Otherwise
  - set the last element of the array to n-1 (where n is the number of elements in the array, for example, an array of size 3 will have its last element (index 2) set to 2; and
  - initialize the portion of the array consisting of the first n-1 elements (i.e., the other elements of the array)

```
public void init (int[] a, int n) {
    if (n==0)
        return;
    a[n-1] = n-1;
    init (a, n-1);
}
```

remember, return is here
flow returned to wherever method called from.

```
 0 1 2 3 4
[0|1|2|3|4]   n=5
```

init (a,5) cell 4 = 4
init (a,4) cell 3 = 3
init (a,3) cell 2 = 2
init (a,2) cell 1 = 1
init (a,1) cell 0 = 0
init(a,0) - return

8.4 Recursion and Sorting

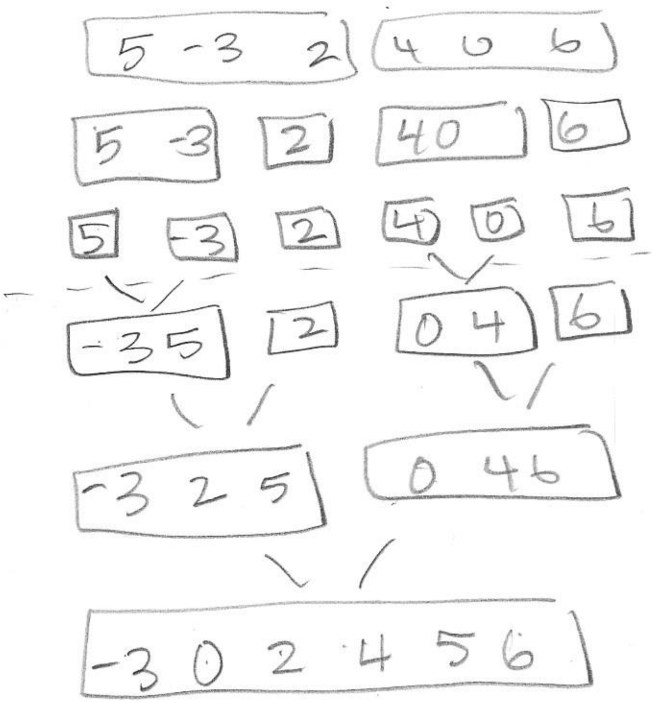
Mergesort divides a list in half; recursively sorts each half, and then combines the two lists. At the deepest level of recursion, one-element lists are reached. A one-element list is already sorted. The work of the sort comes in when the sorted sublists are merged together.

Here is how the mergesort works:

- Break the array into two halves.
- Mergesort the left half.
- Mergesort the right half.
- Merge the two subarrays into a sorted array.

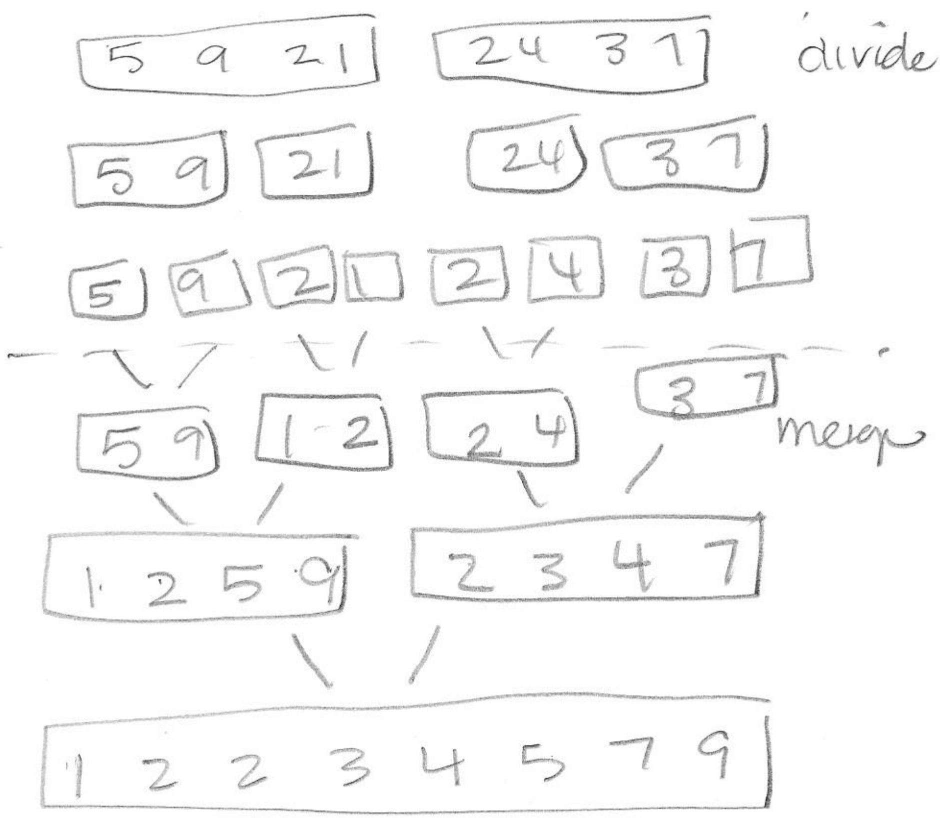
Example a)

5	−3	2	4	0	6



Example b)

5	9	2	1	2	4	3	7



- merge 2 arrays at a time

- index for each array.

* show sorting simulator