

# Problem 1 - Multiples of 3 and 5

Gautam Manohar

9 June 2018

*This document originally appeared as a blog post on my website. Find it at [gautammanohar.com/euler/1](http://gautammanohar.com/euler/1).*

## 1 Problem Statement

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below  $n$ .

## 2 My Algorithm

For the Project Euler problem,  $n = 1000$ .

In general, the sum of the natural numbers up to  $n$  is the  $n$ -th *triangular number* (see [here](#)). Let's call this  $T(n)$ . A well known formula for this is

$$T(n) = \sum_{i=1}^n i = \frac{n(n+1)}{2}. \quad (1)$$

The sum of all multiples of 3 below  $n$  looks like this:

$$1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \dots + \left\lfloor \frac{n-1}{3} \right\rfloor \cdot 3. \quad (2)$$

We can factor 3 out and write (2) as  $3T\left[\frac{n-1}{3}\right]$ , which we know how to find with (1). We can do the same thing with 5.

But now we've overcounted! Each multiple of 15 under  $n$  has been counted twice: once as a multiple of 3, then again as a multiple of 5. We can fix this by subtracting the sum of all multiples of 15 under  $n$ . And so our desired answer is

$$3T\left[\frac{n-1}{3}\right] + 5T\left[\frac{n-1}{5}\right] - 15T\left[\frac{n-1}{15}\right]. \quad (3)$$

The complexity of this solution is  $O(1)$ , because our answer is just a computation.

## 2.1 Other Solutions

A brute-force solution that adds each number  $i$  from 1 to  $n - 1$  to a count if  $i$  is divisible by 3 *or* 5 would have time complexity  $O(n)$ . With the large input sizes of the Hackerrank problem ( $n \leq 10^9$ ) this solution is too slow, but it easily passes the original Project Euler problem.