

Problem 46 - Goldbach's Other Conjecture

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This document originally appeared as a blog post on my website. Find it at gautammanohar.com/euler/46.

1 Problem Statement

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square. It turns out that the conjecture was false. How many ways can a given N be represented as the sum of a prime and twice a square?

2 My Algorithm

Suppose an odd composite number n and a prime p provide a match. Then $n = p + 2k^2$ for some positive integer k . And so

$$\begin{aligned} n - p &= 2k^2 \\ k &= \sqrt{\frac{n - p}{2}}. \end{aligned} \tag{1}$$

The numbers n, p give a valid match if and only if (1) is a positive integer.

We generate a list of primes up to N_{\max} . Then, we test every prime $p < N$ to see if (1) is a positive integer; if so, we have found a valid way of representing N . This solution has time complexity $O(N \log \log N + \frac{TN}{\log N})$, where T is the number of queries.

2.1 Project Euler

To solve the Project Euler problem, we write a **while** loop using the function **ways**. We increment our index i , which is initialized at 3, until i is composite. Then, if **ways(i) == 0**, we break the loop; we have found the smallest number

that cannot be represented as conjectured. Otherwise, we increment i by 2, to the next odd number.