Problem 63 - Powerful Digit Counts

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1 Problem Statement

The five-digit number $16807 = 7^5$ is also a fifth power. Given N, find the N-digit positive integers that are also an N-th power.

2 My Algorithm

An n-digit n-th power has n-th root at most 9, because 10^n is an $(n+1)$-digit number. If a^n is an n-digit number, then unless $a = 9$, we also have that $(a+1)^n$ is an n-digit number. We must find the lowest a such that a^n has n digits. We have

$$\begin{aligned} 10^{n-1} &\leq a^n < 10^n \\ 10^{1-\frac{1}{n}} &\leq a < 10 \\ a &= \lceil 10^{1-\frac{1}{n}} \rceil. \end{aligned} \tag{1}$$

And so k^n is an n-digit number for all $k \in [\lceil 10^{1-\frac{1}{n}} \rceil, 9]$. We enumerate these in $O(1)$ time.