Problem 10 - Summation of Primes

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1 Problem Statement

The sum of all primes below 10 is $2 + 3 + 5 + 7 = 17$. Find the sum of all primes at most N.

2 My Algorithm

We use a Sieve of Eratosthenes to compute the primes up to the maximum possible value of N. We then construct a prefix sum array in $O(\frac{N}{\log N})$ time. Using a binary search, we find the index of largest prime less than N in $O(\log N - \log\log N)$ time. In $O(1)$ time, we look up the corresponding partial sum of prime numbers. Our solution has time complexity $O(n \log\log n + \frac{n}{\log n} + T \log n)$, where T is the number of queries.