Problem 53 - Combinatoric Selections

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This document originally appeared as a blog post on my website. Find it at `gautammanohar.com/euler/53`.

1 Problem Statement

How many, not necessarily distinct, values of $\binom{n}{r}$, for $n \leq N$, are greater than K?

2 My Algorithm

A brute-force search is too slow. And so we exploit the symmetry of Pascal's triangle, which contains the binomial coefficients. Because $\binom{n}{k} = \binom{n}{n-k}$, we need only check up to $\frac{n}{2}$. Furthermore, if $\binom{n}{k}$ is the first value greater than K, then because the entries of Pascal's triangle strictly increase until the central term, all the entries between k and $n-k$, inclusive, will be greater than K. This amounts to $n - 2k + 1$.

We also make use of the recurrence relation

$$\binom{a}{b+1} = \binom{a}{b}\frac{a-b}{b+1}, \tag{1}$$

as described in my solution to Project Euler 15.

We iterate through the binomial coefficients in row n until we find one greater than K, say $\binom{n}{i}$. Then the answer is the sum of $n + 1 - 2i$ for each $n \leq N$. This solution has time complexity $O(N^2)$.