Problem 34 - Digit Factorials

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This document originally appeared as a blog post on my website. Find it at gautammanohar.com/euler/34.

1 Problem Statement

145 is a curious number, as $1! + 4! + 5! = 145$. Find the sum of all numbers that are the sum of the factorials of their digits. Note that $1! = 1$ and $2! = 2$ are not sums, as they contain only one digit.

2 My Algorithm

We use a combinatorial approach similar to Problem 30. The sum of the factorials of the digits of a number with d digits is $d \cdot 9!$. So the upper bound on the number we need to check is $7 \cdot 9!$, because $8 \cdot 9!$ only has 7 digits. This means we can test valid candidate sets of size $2 \leq k \leq 7$ chosen with replacement from $0!, \ldots, 9!$.

Like in Problem 30, we use multisets to check whether a combination is valid. And so we perform

$$\sum_{k=2}^{7} \binom{9+k}{k} = 19347 \tag{1}$$

operations, as in Problem 30.

2.1 HackerRank

The HackerRank problem is, for once, much easier than the original Project Euler problem. It asks us to find the sum of all $n < N$ such that n divides its factorial digit sum. Because $N \leq 10^5$, we can run a brute force search, checking whether each $n < N$ is a valid solution.