

Problem 42 - Coded Triangle Numbers

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This document originally appeared as a blog post on my website. Find it at gautammanohar.com/euler/42.

1 Problem Statement

The n -th term of the sequence of triangle numbers is given by $t_n = \frac{n(n+1)}{2}$; so the first ten triangle numbers are

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \dots$$

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is $19 + 11 + 25 = 55 = t_{10}$.

You are given a file `words.txt`. Of the English words it contains, how many are triangle words?

2 My Algorithm

Let us suppose we have some number $t = \frac{n(n+1)}{2}$. Then

$$\begin{aligned} 2t &= n(n+1) \\ 2t &= n^2 + n \\ n^2 + n - 2t &= 0 \\ n &= \frac{-1 \pm \sqrt{8t+1}}{2}. \end{aligned} \tag{1}$$

Because we are solving for positive n , we have

$$n = \frac{-1 + \sqrt{8t+1}}{2}. \tag{2}$$

We know that t is a triangle number if and only if (2) is an integer.

To compute the name scores, we use the same method as in [Problem 22](#). To finish the problem, we read in the names, and count them if their score is a triangle number. The time complexity of this solution is $O(n)$, where n is the number of names.

2.1 HackerRank

The HackerRank version of this problem is simpler. It does not involve words. Instead, it asks whether a number $n \leq 10^{18}$ is triangular. We have an $O(1)$ method of answering this question.