Problem 58 - Spiral Primes

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This document originally appeared as a blog post on my website. Find it at gautammanohar.com/euler/58.

1 Problem Statement

Starting with 1 and spiralling clockwise in the following way, a square spiral with side length 7 is formed.

43	44	45	46	47	48	49
42	21	22	23	24	25	26
41	20	**7**	8	9	10	27
40	19	6	1	2	11	28
39	18	**5**	4	**3**	12	29
38	**17**	16	15	14	**13**	30
37	36	35	34	33	32	**31**

Figure 1. A 7-by-7 number spiral.

It is interesting to note that 8 out of the 13 numbers lying along both diagonals are prime; that is, a ratio of $\frac{8}{13} \approx 62\%$.

If one complete new layer is wrapped around the spiral above, a square spiral with side length 9 will be formed. If this process is continued, what is the side length of the square spiral for which the ratio of primes along both diagonals first falls below $N\%$?

2 My Algorithm

As shown in my solution to Project Euler 28, the corners of a number spiral of size n are of the form $n^2 - i(n-1)$, for $i \in \{1, 2, 3, 4\}$. We discount the case where $i = 4$, because this is $(n+2)^2$. We use the Miller-Rabin primality test

to search such numbers and count the primes until the desired ratio is reached. This solution has time complexity $O(N \log^3 N)$, where N is the maximum side length we must search.