

Problem 32 - Pandigital Products

Gautam Manohar

24 June 2018

This document originally appeared as a blog post on my website. Find it at gautammanohar.com/euler/32.

1 Problem Statement

We shall say that an N -digit number is pandigital if it makes use of all the digits 1 to N exactly once; for example, the 5-digit number 15234 is 1 through 5 pandigital. The product 7254 is unusual, as the identity $39 \cdot 186 = 7254$ containing multiplicand, multiplier, and product is 1 through 9 pandigital. Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through N pandigital.

2 My Algorithm

We use a brute-force check on all ways of splitting up a pandigital number into three non-empty parts. We can generate each pandigital number by finding the permutations of a string with digits from 1 to N . There are $N!$ of these. It is then simple to verify whether the product of the first two parts is equal to the third part.

Using the combinatorial technique of [stars and bars](#), there are $\binom{n-1}{k-1}$ ways to distribute n indistinguishable objects among k distinguishable groups. Our groups—the three parts—are distinguishable, as their order matters. For a given pandigital number, the digits cannot move, so they are thought of as indistinguishable. And so there are $\binom{N-1}{2}$ ways of splitting up each pandigital number.

We know

$$N! \binom{N-1}{2} = \frac{N!(N-1)!}{2(N-3)!} = \frac{N!(N-1)(N-2)}{2} \leq \frac{N^2 N!}{2}. \quad (1)$$

And so our solution has time complexity $O(N^2 N!)$.