Assignment #1

Computational Physics I (**Phys381**)
R. Ouyed
Due February 16, 2014 (by Midnight)
[Total: 100 marks including 20 marks for the report]

(i) Use Latex to prepare your report; Include your codes in appendices (using the *verbatim* **command); (ii) When I refer to** *lecture notes* **it means the Ouyed&Dobler notes; (iii) If applicable, animations should be shown to the teacher and/or TA before handing in the report. The animation should be well documented and contains necessary information (student name, assignment number, run time etc ...). Basically, the information should be included in each frame before they are put together.**

→The Latexed report is worth **20%** of the total mark. Only well documented and neatly presented reports will be worth that much!

- It is not sufficient to give only numerical results and show plots, you should also discuss them. Include complete figure captions, introduction and conclusion sections.
- Your report must be in a *two-column format.*
- Your report should include your *Fortran codes* and *Gnuplot scripts* in an Appendix using the *verbatim* command.

  Make sure your codes compile and run properly. You might be asked to use them during the exams (midterm and/or final).
- You must name your report using your first and last name: *firstnamelastname.pdf.*
- email your report (pdf file) to:

  rouyed@ucalgary.ca and Cc it to:

  amraghoo@ucalgary.ca & zachary.a.shand@gmail.com

Assignments are due (i.e. should be emailed) on the date and time stated above.

Late submissions will be penalized.

1 Gnuplot: visualizing matrices (i.e arrays) as 2-dimensional color maps

[20 marks]

First, study the gnuplot script in Appendix A below which shows you how to display a matrix of length N (i.e. an $N \times N$ array) containing only "1s" and "0s" as a 2-dimensional color map.

- **[5 marks]** Run the gnuplot script and check what it does exactly and view the output file. Explain clearly what is performed by each line.

- **[10 marks]** Write a Fortran 90 code that creates a matrix of length 20 (i.e. a 20×20, 2-dimensional array) containing only "1s" and "0s". The matrix should be an output of (i.e. created by calling) a subroutine internal to the code – *name your subroutine "buildmatrix"*. Run your code to generate the following four matrices:

  (i) An array containing "1s" along the two diagonals and "0s" elsewhere. It means the "1s" should be in an "X" configuration.

  (ii) An array containing "1s" along the centered row and centred column and "0s" elsewhere. It means the "1s" should be in an centered "+" configuration. *An even $N \times N$ array has 2 center indexes.*

  (iii) An array containing "1s" along the outermost border and "0s" elsewhere. It means the "1s" should be in an "squared" configuration along the perimeter.

  (iv) An array containing "1s" in a configuration of your choice and "0s" elsewhere.

- **[5 marks]** Write a gnuplot script to display (in a multiplot setting) the four matrices. It means each matrix should be displayed (as a 2-dimensional color map; see appendix A) in a separate panel. Save your plot and attach it as a figure to your report and comment it.

  Hint: The few lines below show you how to save an $N \times N$ matrix MAT into a file (Unit=14). This should be added to your Fortran code, after you have generated the matrix MAT.

  ```
  Do i=1,N
  write(14,*)  MAT(i,:)
  End Do
  ```

2 Astrophysics: Ultra-High Energy Cosmic Rays

[60 marks]

Ultra-high energy cosmic rays, (UHECR), are thought to be particles (protons and heavy nuclei) with very high energies, E, of up to 3×10^{20} eV. The passage of these single hydrogen nuclei or protons is recorded by particle detectors, and their incidence or flux ($Flux$) is defined as the number of arriving particles per unit area, per solid angle, per unit time.

(i) [**10 marks**] The highest energy protons that can currently be produced in any particle accelerator on Earth is 10^{12} eV. How does this compare to the energy of a typical 10^{20} eV UHECR? Can you think of an astrophysical source and a physical mechanism that could accelerate a proton to such energies?

(ii) [**10 marks**] Shown in **Appendix B** below is the UHECRs data you will be handling in this problem. The first column is the particle energy ($E(eV)$, in electron-Volts) and the second one is the flux ($Flux$, in units of *particles/m^2/s/ster*). The energy spectra of these particles is the variation of the flux (right column) with energy (left column).

Copy and paste the data from the pdf file and save it into a file called *phys381-UHECR.data*. Write a Fortran 90 code that reads in data from *phys381-UHECR.data* and writes out a file (name it *phys381-UHECR-out.data*) which contains the following five columns:

```
E (eV),   Flux,  log10[E],  log10[Flux],  log10[(E^3*Flux)/1e24]
```

The last column is created by multiplying the flux by E^3 then dividing by 10^{24}. (note we are dealing with log base 10 not the natural base).

(iii) [**15 marks**] Using gnuplot's multiplot capabilities, graph:

- the flux versus energy
- the log10(flux) versus log10(energy)
- the log10($E^3\times$ Flux/10^{24}) versus log10(energy)

Use a different point style (for example open circles; dagger; solid diamonds) per plot when displaying your data.

Which of the three plots show the three natural energy ranges of UHECRs the best? Explain why and discuss your result. In this plot only (i.e. the one that best show the three natural energy ranges), use gnuplot to display arrows pointing at the transition energy between the regimes. Since there are three regimes you should have two arrows pointing at two transitions energies. You need to look up the "*set arrow*" command in gnuplot.

(iv) **[15 marks]** The energy spectra of these particles appears to follow a power law of the form

$$F(E) = C \times \left(\frac{E}{E_o}\right)^{-p} , \quad (1)$$

where the constants C, E_o and p are fit parameters given in the table below.

regime	Energy range	C	E_o (eV)	p
regime 1	10^{17} eV $< E < 3 \times 10^{18}$ eV	5×10^{-27}	10^{17}	-3.35
regime 2	**to be completed**	5.5×10^{-32}	3×10^{18}	-2.79
regime 3	2.5×10^{19} eV $< E < 10^{21}$ eV	1.5×10^{-34}	2.5×10^{19}	-3.49

To complete the table you will need to find the energy range for regime 2. The energy value at the intersection between two regimes (say, regime i and regime j) is given by the equation

$$E_{\text{intersection}} = \left(\frac{C_j}{C_i}\right)^{\frac{1}{p_i - p_j}} \times \frac{E_{o,i}^{\frac{p_i}{p_i - p_j}}}{E_{o,j}^{\frac{p_j}{p_i - p_j}}} \quad (2)$$

Write a simple Fortran 90 code that reads in (from the screen) the parameters of the two regimes and gives out the value of $E_{\text{intersection}}$ where the regimes cross. What is the $E_{\text{intersection}}$ when regimes 1 and 2 cross? What is the $E_{\text{intersection}}$ when regimes 2 and 3 cross? What is the error message you get when you run your code with $p_i = p_j$? Explain the error message.

(v) **[10 marks]** First, in your report, reproduce the table above exactly as shown. Now complete the table by:

- specifying the energy range for regime 2 (i.e. replace "to be completed" with the corresponding energy range).
- adding a caption to the figure.

FYI: The first energy group of UHECR has $p = -3.35$ and correspond to cosmic rays trapped within our galaxy by magnetic fields. The second energy group of UHECR has $p = -2.79$ and correspond to cosmic rays from other galaxies. The third energy group of UHECR has $p = -3.49$ and correspond to cosmic rays from the intergalactic space.

A Gnuplot: displaying matrices as 2-dimensional color maps

Suppose we have the following 5×5 data matrix stored in a file called "phys381-example.txt":

```
1 0 1 1 1
0 1 1 0 1
0 1 0 0 0
1 1 0 0 1
0 0 1 0 1
```

The following Gnuplot script plots (i.e. displays) the matrix as a 2-dimensional color map:

```
reset
set terminal gif
set output "2Dmap.gif"
set palette maxcolors 2
set palette defined (0 'blue', 1 'red')
plot 'phys381-example.txt'  matrix with image
!convert "2Dmap.gif" "2Dmap.pdf" && rm "2Dmap.gif"
```

B UHECRs data

# energy (in eV),	Flux (in particles per m^2 per s per ster)
1.81E+17	8.34430E-28
2.26E+17	2.75658E-28
2.84E+17	1.69333E-28
3.55E+17	6.98795E-29
4.52E+17	3.16394E-29
5.58E+17	1.48661E-29
7.19E+17	6.99711E-30
9.01E+17	3.13088E-30
1.13E+18	1.58169E-30
1.41E+18	6.88898E-31
1.77E+18	3.36254E-31
2.25E+18	1.65076E-31
2.78E+18	6.73229E-32
3.53E+18	3.96494E-32
3.58E+18	3.65068E-32
4.49E+18	1.85676E-32
4.62E+18	1.54257E-32
5.62E+18	1.20379E-32
5.78E+18	7.84560E-33
7.05E+18	5.38639E-33
8.00E+18	3.91198E-33
8.83E+18	2.44287E-33
1.12E+19	1.28292E-33
1.26E+19	1.03263E-33
1.42E+19	6.69216E-34
1.79E+19	4.42736E-34
2.00E+19	2.99062E-34
2.24E+19	1.92699E-34
2.80E+19	7.43377E-35
3.14E+19	7.23176E-35
3.56E+19	5.89008E-35
4.46E+19	1.88132E-35
4.99E+19	1.64304E-35
5.67E+19	1.46080E-35
7.10E+19	5.09327E-36
7.95E+19	1.58554E-36
8.90E+19	1.30229E-36
1.28E+20	4.11261E-37