Think Bayesian

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Bayesian Data analysis and Probabilistic Programming

Credits

- Chap. 1 of *Bayes Rules! An Introduction to Applied Bayesian Modeling*
  - https://www.bayesrulesbook.com/chapter-1.html

The big picture

- We continuously update our knowledge about the world as we accumulate experience or collect data.
- The Bayesian approach rigorously models the knowledge-building process, in which you update your knowledge on the basis of new data.
- Knowledge (or ignorance) is expressed by means of a probability distribution on the variables of interest.

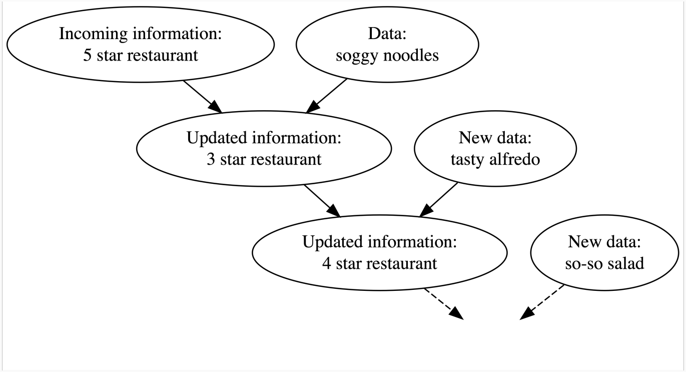
Changing opinion based on experience

- Suppose there’s a new restaurant with 5-star online rating.
- Prior to enter the restaurant, you expect that it will be delicious.
- On your first visit, the pasta is poorly cooked.
- You weigh the high online rating against your poor meal (which might have just been a fluke), and you update your opinion knowledge: this is a medium, not an excellent restaurant.

Changing opinion based on experience

- In your second meal at the restaurant you’re pleased with your dinner and increase your personal restaurant’s rating to good.
- You continue to visit the restaurant, collecting edible data and updating your knowledge each time.
- After enough visits, you have your own informed opinion.

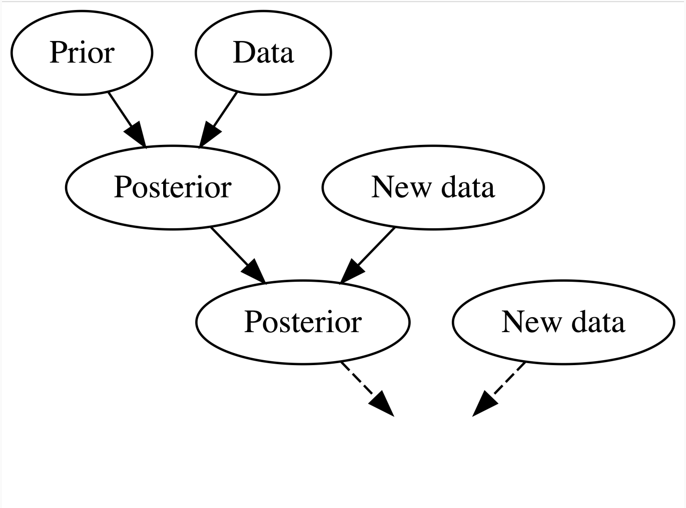
Building knowledge



Building knowledge

- An environmental scientist analyses of the human role in climate change.
- He carries a degree of incoming or prior information based on previous research and experience.
- In light of this information he interprets new data, considering both to develop an updated understanding (posterior information).
- He continues to refine this information as he gathers new evidence.

Building knowledge as a Bayesian process



Your turn

- Think of a recent situation in which you changed your mind.
- As with the Italian restaurant example, make a diagram that includes your prior information, your new data that helped you change your mind, and your posterior conclusion.

Bayesian interpretation of probability

- The probability of a coin landing Heads is 0.5.
  - How do you interpret this probability?
- If I flip this coin over and over, roughly 50% will be Heads (*frequentist* interpretation).
- Heads and Tails are equally plausible in the next flip (*Bayesian* interpretation).

Bayesian vs frequentist probability

- According to both Bayesian and frequentist probability, the probability of observing Heads on a fair coin flip is 1/2. The difference is in their interpretation:
- frequentist: probability is the long-run relative frequency of a repeatable event.
- Bayesian: probability measures the relative plausibility of an event.

Bayesian vs frequentist intepretation

- An election is coming up and a pollster claims that candidate A has a 0.9 probability of winning. How do you interpret this probability?
  - If we observe the election over and over, candidate A will win roughly 90% of the time.
  - Candidate A is much more likely to win than to lose.
- This event is non-repeatable and frequentist probability does not really apply.

Your turn

- Identify a topic that you know about (e.g., a sport, a school subject, music).
- Identify a hypothesis about this subject (e.g.: with probability 80% rider X will become world champion)
- Describe how this probability is informed by your expertise and you updated it over time based on the observed data (e.g., race results).

Testing hypothesis: frequentist vs Bayesian

- Imagine that you tested positive for a disease; you can make a single question to the doctor.

1) what's the probability that I have the disease?

2) if I do not have the disease, what's the chance that I would've gotten the positive result?

Asking questions

- Bayesian analysis would answer the first question, by assessing the probability of the hypothesis H given the data D, $P(H|D)$.
- Frequentist analysis answers the second question: the probability of the data D given of an assumed hypothesis H ($P(D|H)$).

Probability of having the disease, given a positive test

	test pos	test neg	total
disease	3	1	4
no disease	9	87	96
total	12	88	100

- Only 3 of the 12 people that tested positive have the disease; there's only a 25% chance that you have the disease.
- This is the Bayesian answer: it measures the probability of a hypothesis, given the observed data.

Probability of testing positive, given that I do not have the disease

	test pos	test neg	total
disease	3	1	4
no disease	9	87	96
total	12	88	100

- From the frequentist standpoint, since disease status isn't repeatable, the probability you have the disease is either 1 or 0 – you have it or you don't.
- Medical testing is repeatable. You can get tested for the disease over and over and over.
- Thus, a frequentist analysis asks: If I don't actually have the disease, what's the chance that I would've tested positive?

Probability of testing positive, given that I do not have the disease

- Since only 9 of the 96 people without the disease tested positive, there’s a roughly 10% (9/96) chance that you would’ve tested positive even if you didn’t have the disease.
- This computation is similar in spirit to the p-value of traditional hypothesis testing.
- In general the answer you look for is $P(H|D)$ rather than $P(D|H)$.