Reporting the results of a Bayesian analysis

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Analysis of the results - 1

- Assume you want to make inference about parameter θ.
- The result of the Bayesian analysis is the posterior distribution of θ, **not** a single value.
- The dispersion of the posterior distribution (posterior variance) quantifies our uncertainty.
- Uncertainty decreases when we have more data.

The impact of the prior

- If many data are available, the posterior is the same regardless the prior.
- But how many data are needed for the likelihood to *overwhelm* the prior? That is not know in advance.

The impact of the prior

- With few data, the posterior might be different using different priors; it makes sense to repeat the analysis with different priors (analysis of *sensitivity* to the prior).

- The priors impact our results. This makes sense, since the prior encodes previous knowledge and different experts have different opinions.

Discussion

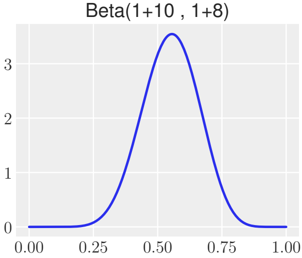
- Priors and likelihood are based on assumptions which should be justified.
- There is no single right prior, but many reasonable priors.
- *Slightly informative* priors are recommended, for instance limiting the parameter values only to the positive range, or to a certain order or magnitude, etc.
- A Beta(1,1) prior is flat but limits the values of θ between 0 and 1.
- Sensitivity analysis: evaluate if the results significantly change with the priors, or instead the data are strong enough to overwhelm the prior.

Summarizing the posterior

- Different measures can be used to summarize the a priori:
- the mean (or the mode, or the median) of the posterior distribution
- the probability of θ belonging to a certain interval
- the HPD, also called the *credibility* interval.

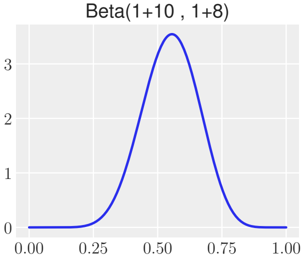
Example: inference on the bias θ of the coin

- Let us consider a uniform prior, Beta(1,1)
- Let us assume the data are y=8 heads out of n=18 tosses.



- The posterior mean is $E_{\text{post}} = \frac{1+8}{1+18} = 0.474$

Probability of the coin being almost fair



$$p_{\text{post}} = 0.49 \leq \theta \leq 0.51 = \int_{0.49}^{0.51} p_{\text{post}}(\theta) d\theta$$

```
from scipy.stats import beta
beta.cdf(0.51, 9, 19) - beta.cdf(0.49, 9, 11)
```

```
## 0.3356900915680642
```

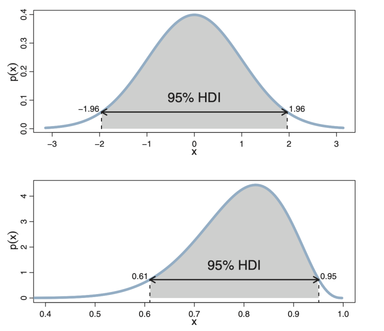
Highest Density Interval (HDI)

- The HDI indicates which points of a distribution are most credible.
- The HDI is the shortest interval that contains a chosen portion of the probability density, usually 95% (although other values such as 90% or 50% are common).
- Any point within this interval has a higher density than any point outside the interval. For a unimodal distribution, the HDI 95 is the interval between the 2.5th and 97.5th percentiles.
- The HDI is also referred to as HPD (high posterior density) in some books.

HDI of unimodal distributions

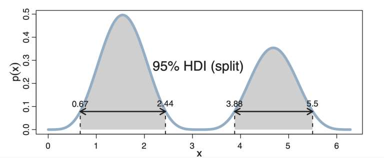
- (1-α) is the coverage of the HDI.
- If the distribution is unimodal, the HDI lies between the quantile $\alpha/2$ and $1 - \alpha/2$.
- For instance, the 95% HDI lies between quantiles 0.025 and 0.975

HDI of unimodal distributions



- The height of the horizontal arrow marks the minimal density exceeded by all x values inside the 95% HDI

HDI of bimodal distribution



- The HDI could (not always) be split into two sub-intervals, one for each mode of the distribution.
- The characteristics are as before:
  - The shaded area has total area of 0.95.
  - Any x within such limits has higher probability density than any x outside the limits.
- In this case you can use the numerical procedure of arviz in order to compute the HDI.