AFFINE AND METRIC RECTIFICATION OF AN IMAGE

IMPLEMENTATION:

1.AFFINE RECTIFICATION

Under a projective transformation ideal points may mapped to finite points and **l** at infinity is mapped to a finite line. However, if the transformation is affine, then **l** at infinity is not mapped to a finite line, but remains at infinity. we can recover the affine properties from images by the transformation matrix H_1 that maps the vanishing line back into the line at infinity.
Given two pairs of image lines l_1, l_2, m_1 and m_2, where $l_1 \parallel l_2$ and $m_1 \parallel m_2$. In a perspective distorted image, these two sets of parallel lines will intersect at points p1 and p2. The line formed by connecting p1 and p2 is the vanishing line $\mathbf{l} = (l_1, l_2, l_3)^T$. We have in homogeneous coordinates:

$$\begin{aligned} \vec{p}^{(1)} &= \vec{l}^{(1)} \times \vec{l}^{(2)}, \\ \vec{p}^{(2)} &= \vec{m}^{(1)} \times \vec{m}^{(2)}, \\ \vec{l} &= \vec{p}^{(1)} \times \vec{p}^{(2)}. \end{aligned}$$

Therefore, by applying transformation H_1, where

$$H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{pmatrix}$$

the vanishing line will be mapped into the line at infinity $\mathbf{l} = (0, 0, 1)^T$. This can be verified as,

$$H_1^{-T} = \begin{pmatrix} 1 & 0 & -l_1/l_3 \\ 0 & 1 & -l_2/l_3 \\ 0 & 0 & 1/l_3 \end{pmatrix}$$ and $$H_1^{-T}\vec{l} = (0,0,1)^T.$$

Algorithm:

STEP1: choose two sets of image lines which are physically parallel.
STEP2: find the vanishing line $\mathbf{l} = (l_1, l_2, l_3)^T$ using the above two set of lines.
STEP3: form matrix H1 and apply H1 to camera images Xc, i.e., $Xa = H_1Xc$, where Xa is the affinely rectified image.

I used MATLAB for the implementation of algorithm. And used functions like:
<u>Cross():</u> to find cross product of two vectors.
<u>Maketform('Projective', H):</u> creates a TFORM struct for an N-dimensional projective transformation.
<u>Imtransform(Input, H):</u> to transform the image according to Homography H.
After transforming the image, I got the correct affine rectification. And Now parallel lines are seems to be parallel in the modified image.

2.METRIC RECTIFICATION

Like affine properties are recovered by specifying line at infinity, metric properties to be recovered from an image of a plane by transforming the circular points to their canonical positions. The transformation between the world plane and rectified image is a similarity. Suppose an image has been affinely rectified then I required two constraints to specify the 2 degrees of freedom of the circular points in order to determine a metric rectification. Now I get the affinely rectified image Xa, Now I want to find the affine transform matrix:

$$H_2 = \begin{pmatrix} A & \vec{t} \\ \vec{0} & 1 \end{pmatrix}$$

such that Xa = H_2Xs, where Xs is the scene image in the real world.
Suppose we have a pair of physically orthogonal lines, l ⊥ m. Let l′ , m′ be the transformed lines under affine transformation H_2 (l′ = H_2^{-T} l), i.e., lines l′, m′ are from the affinely rectified image Xa. By orthogonality, we know that

$$(l_1/l_3, l_2/l_3)(m_1/m_3, m_2/m_3)^T = 0,$$

Then,

$$l_1m_1 + l_2m_2 = \vec{l}^T C^*_\infty \vec{m} = 0,$$ Where,

$$C^*_\infty = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ is the dual degenerate conic. Since $C^{*\prime}_\infty = H_2 C^*_\infty H_2^T$, we have:

$$\vec{l}^T C^*_\infty \vec{m} = \vec{l}'^T H_2 H_2^{-1} C^{*\prime}_\infty H_2^{-T} H_2^T \vec{m}' = \vec{l}'^T C'^*_\infty \vec{m}' = 0.$$ Therefore,

$$\begin{aligned} \vec{l}'^T C'^*_\infty \vec{m}' &= \vec{l}'^T H_2 C^*_\infty H_2^T \vec{m} \\ &= \vec{l}'^T \begin{pmatrix} A & \vec{t} \\ \vec{0} & 1 \end{pmatrix} \begin{pmatrix} I & \vec{0} \\ \vec{0} & 1 \end{pmatrix} \begin{pmatrix} A^T & \vec{0} \\ \vec{t}^T & 1 \end{pmatrix} \vec{m}' \\ &= \vec{l}'^T \begin{pmatrix} AA^T & \vec{0} \\ \vec{0} & 0 \end{pmatrix} \vec{m}'. \end{aligned}$$

We have,

$$(l'_1, l'_2) AA^T (m'_1, m'_2)^T = 0.$$ In order to get A, let S = AA^T, where

$$S = \begin{pmatrix} s_{11} & s_{12} \\ s_{12} & 1 \end{pmatrix}$$ is symmetric matrix. Now, the last element of S is set as 1 considering the scale problem. Thus, finally I need to solve the equations:

$$(l'_1 m'_1, l'_1 m'_2 + l'_2 m'_1) \begin{pmatrix} s_{11} \\ s_{12} \end{pmatrix} = -l'_2 m'_2$$

Therefore, a pair of orthogonal lines provides a equation. I took two pairs of orthogonal lines $l_1 \perp m_1$ and $l_2 \perp m_2$ to solve the matrix S which has two unknown parameters. Now S is symmetric, so we can do its Singular value decomposition as S = UDU^T. Thus, I got $A = U\sqrt{D}U^T$. Now, H_2 is available and the restored scene image is Xs = H_2^{-1} Xa.

I used MATLAB to implement the metric rectification. I used functions:
<u>Linsolve():</u> to solve linear system of equations.
<u>Svd(A):</u> To get Singular value decomposition of matrix A.
<u>Maketform('projective', H):</u> creates a TFORM struct for an N-dimensional projective transformation.
<u>Imtransform(Input, H):</u> to transform the image according to Homography H.
After transforming the rectified image, I got the correct metric rectifications.

EXPERIMENTAL ANALYSIS:

I have experimented on three sets of different images and got the following results:

IMAGE1:

IMAGE AFTER AFFINE_RECTICIFATION:

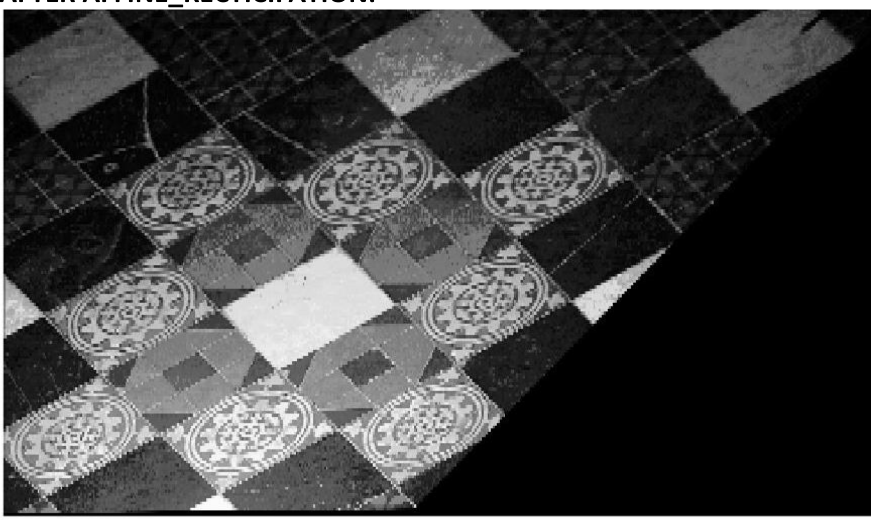

IMAGE AFTER METRIC RECTIFICATION:

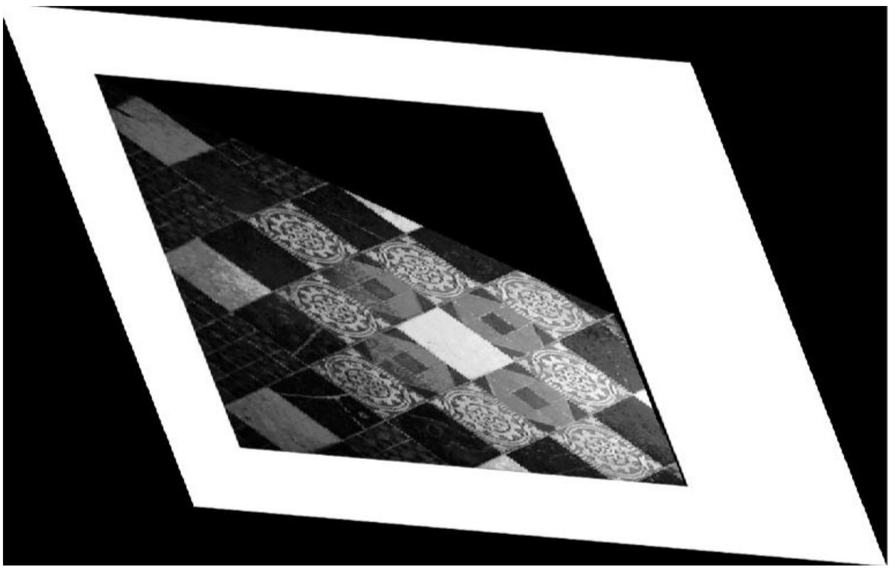

IMAGE2:

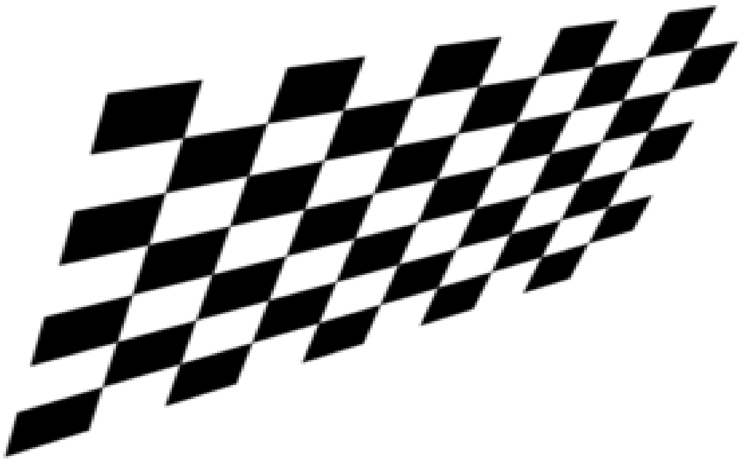

IMAGE AFTER AFFINE_RECTIFICATION:

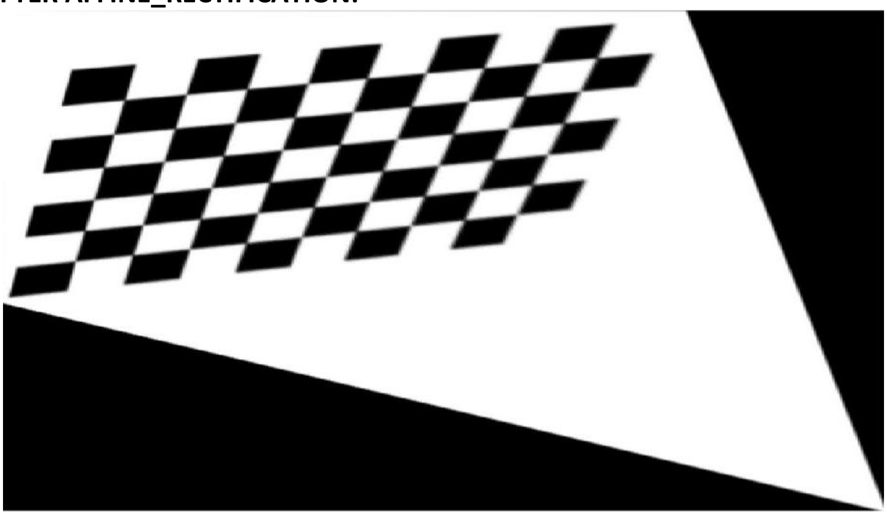

IMAGE AFTER METRIC_RECTIFICATION:

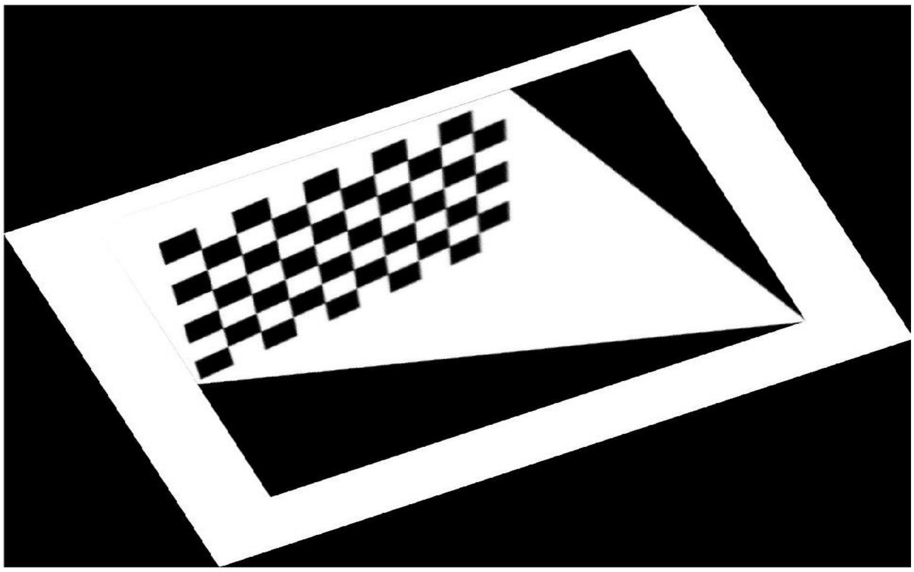

IMAGE3:

IMAGE AFTER AFFINE_RECTIFICATION:

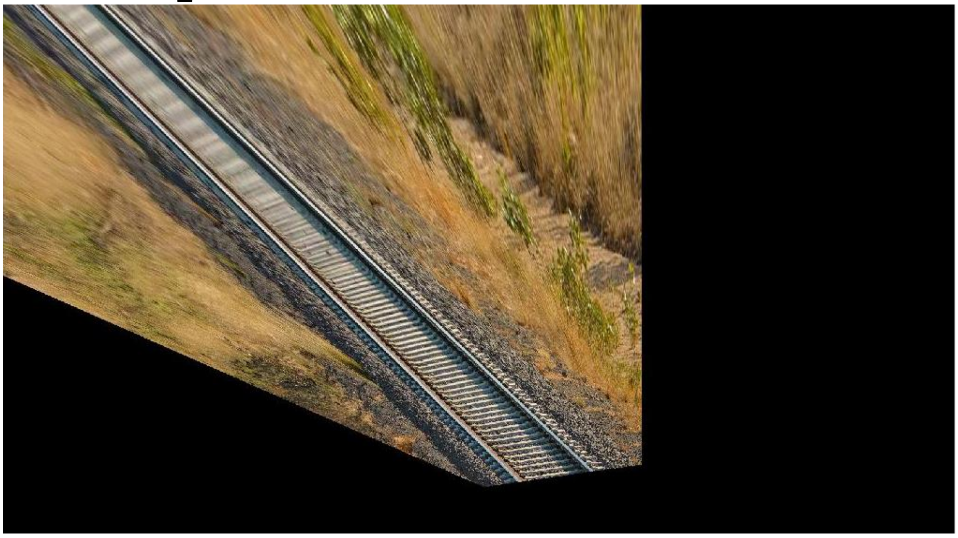

IMAGE AFTER METRIC_RECTIFICATION:

OBSERVATION:

I have observed several important facts about the projective and affine distortions which lead to irregularity in the geometry in the images. Some of the important points are:

1.Under the projective geometry, ideals points get mapped to finite points and line at infinity also mapped to finite lines. This distortion can be corrected by using Affine transformation concept. Affine transformation mapped the vanishing line to line at infinity.

2. I got important observation that is: The line at infinity is a fixed line under the projective transformation H if and only if H is an affinity.

3.Metric properties is recovered from image of a plane by transforming the circular points to their canonical positions. If circular points is identified in an image, and the image is then rectified by a projective transformation H that maps the image circular points to their canonical positions on line at infinity. Then the transformation between the word plane and the rectified image is similarity.

4.I also got another important observation that is: The circular points I,J are fixed points under the projective transformation H if and only if H is a similarity.

5.A pair of orthogonal lines provides a equation to solve for the unknown parameters and hence we get H_2 matrix and then we will able to get scene image in the real world from affinely rectified image.