The MVGC Toolbox

Multivariate Granger Causality

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Introduction: Granger causality

Emerging theories of consciousness highlight complex patterns of causal interactions among neuronal elements, for example in *causal density* and *integrated information* [1]. *Multivariate Granger causality* (MVGC), a statistical measure of functional (causal) connectivity in neural systems, may be inferred from recorded neurophysiological time series (e.g. LFP, EEG, MEG, fMRI, . . .)

- Given multivariate time series $\boldsymbol{X}_t, \boldsymbol{Y}_t$ ("variables"), the MVGC statistic $\mathcal{F}_{\boldsymbol{Y}\to\boldsymbol{X}}$ quantifies the degree to which

  $\boldsymbol{Y}$ helps predict the future of $\boldsymbol{X}$ over and above the degree to which $\boldsymbol{X}$ predicts its own future.

  Prediction is operationalised by linear vector auto-regressive (VAR) modelling [2]. The statistic $\mathcal{F}_{\boldsymbol{Y}\to\boldsymbol{X}}$ has a known χ^2 sampling distribution, useful for significance testing and construction of confidence intervals. It also has a quantitative interpretation as time-directed information flow—*transfer entropy*—naturally measured in bits-per-unit-time [3].
- Common influence of a third variable $\boldsymbol{Z}$ may be accounted for by *conditional* MVGC, written $\mathcal{F}_{\boldsymbol{Y}\to\boldsymbol{X}\,|\,\boldsymbol{Z}}$. MVGC may also be decomposed by frequency: the *spectral* MVGC, $f_{\boldsymbol{Y}\to\boldsymbol{X}}(\lambda)$ integrates over all frequencies to time-domain MVGC. Finally, MVGC is invariant under a wide group of transformations of variables, including rescaling and (almost) arbitrary stable, invertible filters [4].

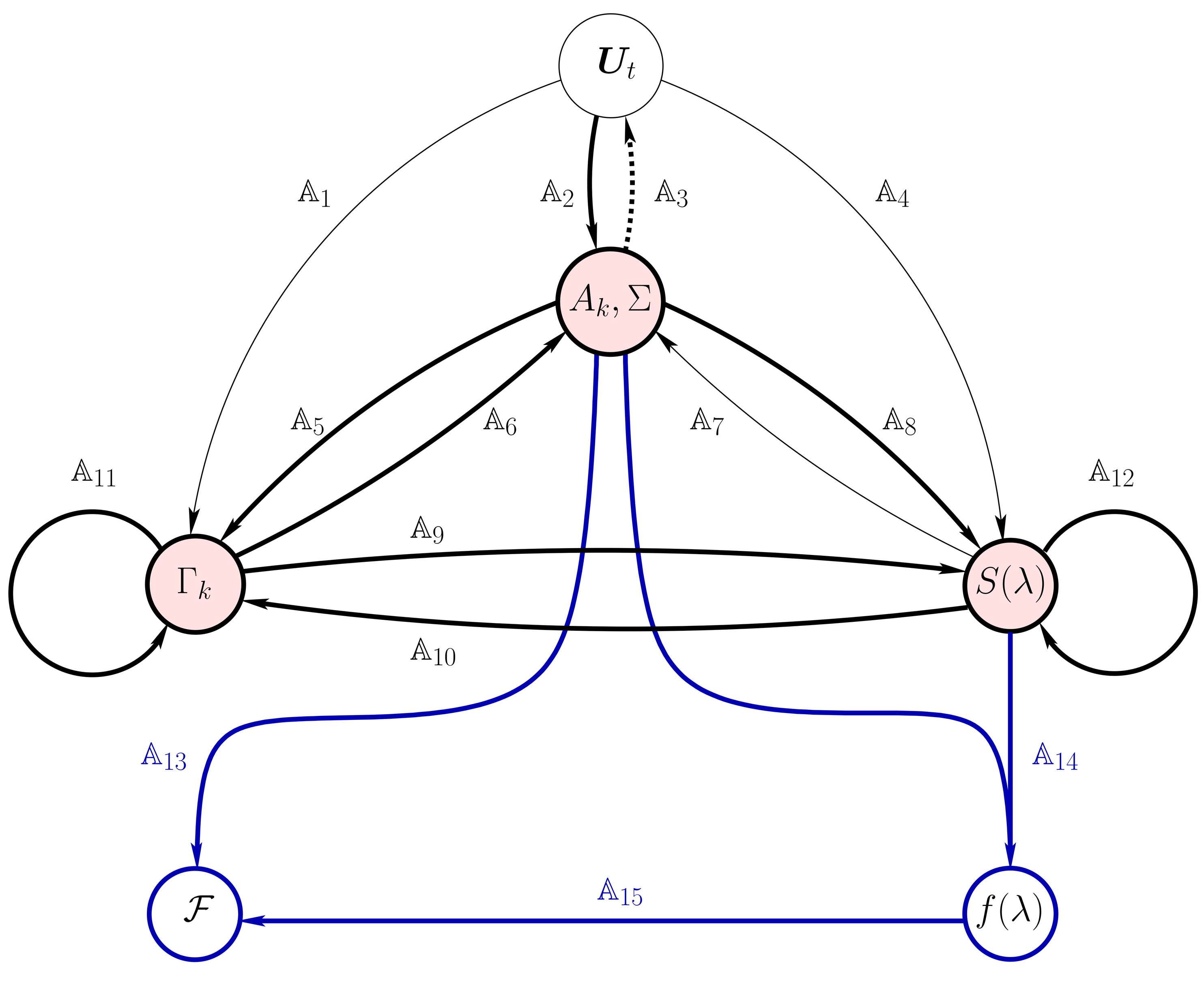
The MVGC Toolbox: computational approach

The MVGC Matlab© Toolbox [∗] facilitates accurate and computationally efficient estimation and inference of MVGC—conditional and unconditional, in both time and frequency domains—from single- or multi-trial multivariate neural time series data.

The toolbox exploits equivalent representations of VAR model:

$$\begin{array}{lcl} A_k, \Sigma & : & \text{VAR parameters} \\ \Gamma_k & : & \text{Autocovariance sequence} \\ S(\lambda) & : & \text{Cross-power spectral density} \end{array}$$

A single regression of the "universe" of variables $\boldsymbol{U}_t$ required; the toolbox then enables selection of the most efficient, stable and numerically accurate computational pathways.

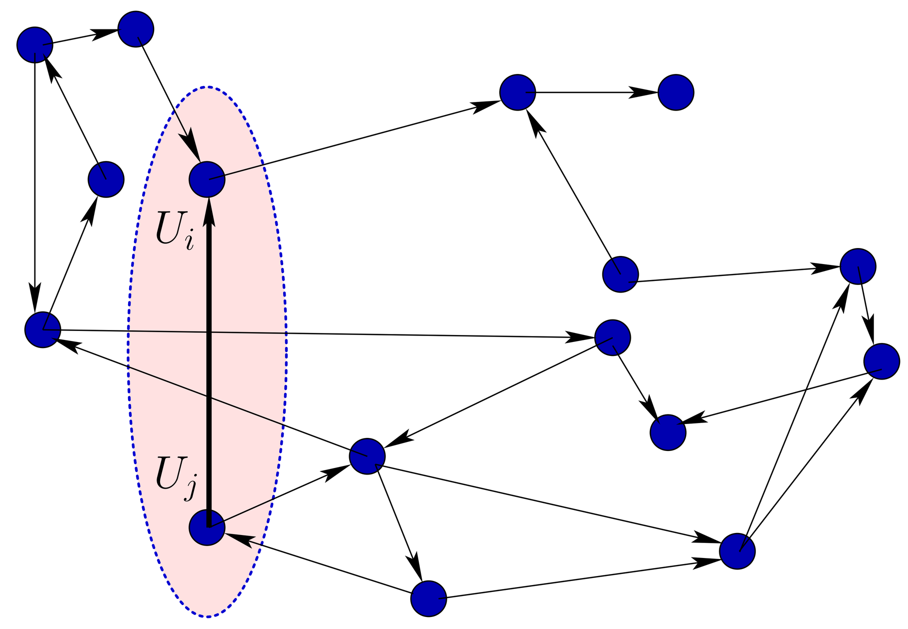


MVGC computational pathways

- $\mathbb{A}_1$ Sample autocovariance estimation
- $\mathbb{A}_2$ VAR parameter estimation: OLS or Morf's LWR algorithm
- $\mathbb{A}_3$ VAR simulation (for testing)
- $\mathbb{A}_4$ Sample spectral estimation: Welch method or multi-taper
- $\mathbb{A}_5$ Reverse-solve Yule-Walker equations: discrete Lyapunov equation solver
- $\mathbb{A}_6$ Solve Yule-Walker equations: Whittle's LWR algorithm
- $\mathbb{A}_7$ VAR spectral factorisation: solve for $H(\lambda), \Sigma$: Wilson's algorithm
- $\mathbb{A}_8$ VAR spectral calculation: $S(\lambda) = H(\lambda)\Sigma H^*(\lambda)$
- $\mathbb{A}_9$ Fourier transform (FFT) of autocovariance sequence
- $\mathbb{A}_{10}$ Inverse Fourier transform (IFFT) of cross-power spectral density
- $\mathbb{A}_{11}$ Autocovariance transform for reduced regression
- $\mathbb{A}_{12}$ Spectral transform for reduced regression
- $\mathbb{A}_{13}$ Time-domain MVGC calculation
- $\mathbb{A}_{14}$ Frequency-domain MVGC calculation
- $\mathbb{A}_{15}$ Integration of spectral MVGC: $\mathcal{F} = \frac{1}{2\pi}\int_0^{2\pi} f(\lambda)\,d\lambda$

Pairwise-conditional Granger causality and causal density

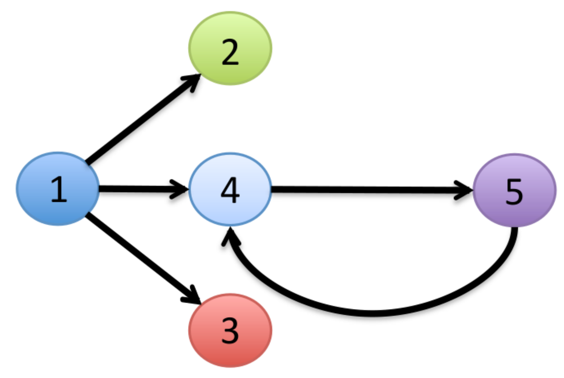
Given a system $\boldsymbol{U}_t$ of (accessible, recorded) variables, we may calculate all Granger-causal interactions between pairs of variables, conditioning out the joint effect of all other variables in the system. For the pair of variables labelled i, j, this interaction is captured by the *pairwise-conditional* Granger causality $\mathcal{G}_{ij}(\boldsymbol{U}) \equiv \mathcal{F}_{U_j \to U_i \,|\, \boldsymbol{U}_{[ij]}}$, where $\boldsymbol{U}_{[ij]}$ denotes omission of the variables U_i, U_j:



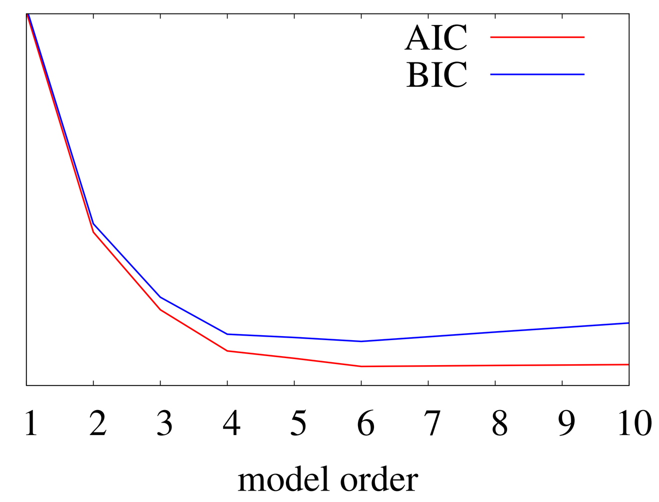
The $\mathcal{G}_{ij}(\boldsymbol{U})$ may be considered as defining a weighted, directed graph - the *causal graph* [3]. Averaged over all potential pairwise connections $i \neq j$, we obtain the *causal density* (cd), which reflects the balance between functional *integration* and *segregation* of the system. Causal density has been proposed as a metric for "level of consciousness" [1].

MVGC usage: 5-variable example

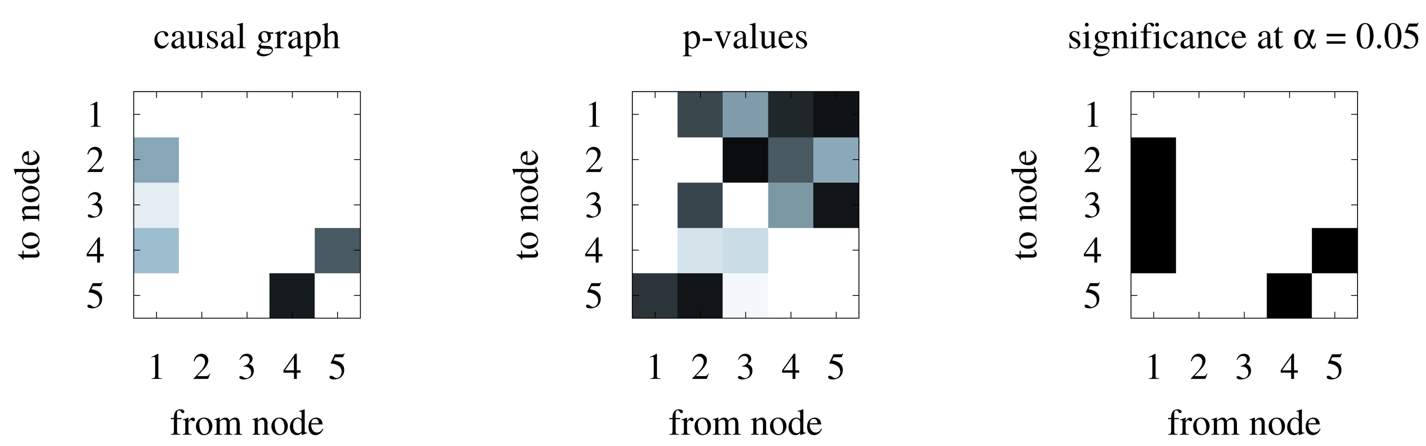
```
>> mvgc_test

*** var_to_tsdata... 0.336832 secs
*** tsdata_to_infocrit... 0.448252 secs

best model order (AIC) = 6
best model order (BIC) = 6
actual model order     = 6

*** tsdata_to_var... 0.075969 secs
*** var_to_autocov... 0.302044 secs

VAR check:
          rho: 0.9516
        actol: 8.8563e-15
       aclags: 372

Durbin-Watson statistics =
    1.9998    2.0001    2.0034    1.9991    1.9998
residuals white by Durbin-Watson test at p-val = 0.05

RSQ (adjusted) =
   -0.0133    0.3159    0.0799    0.8085    0.7745
WARNING: low adjusted RSQ values (< 0.3) for vars 1,3

model consistency = 84%
consistency OK: > 80%

*** autocov_to_cpsd... 0.277283 secs
*** tsdata_to_cpsd... 0.345370 secs
*** autocov_to_mvgc_pwc... 0.856700 secs
*** autocov_to_smvgc_pwc... 1.993274 secs

checking that spectral GC integrates to time-domain GC ...
maximum absolute difference OK: = 1.26e-14 (< 1.00e-05)

causal density = 0.0862
```



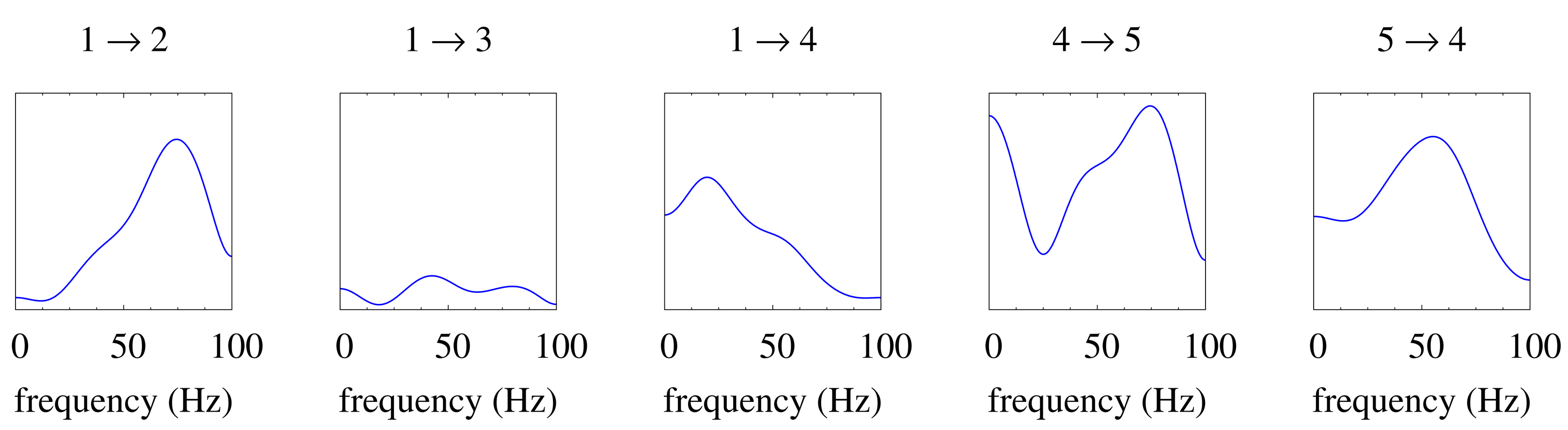
Causal structure



Model order selection:
Akaike & Bayesian information criteria



Granger causality: time domain



Granger causality: frequency domain

Conclusions

Granger causality represents a compelling technique for data-driven functional/causal inference in neural systems, with promising application to developing theories of consciousness. The MVGC Toolbox furnishes a theoretically correct, numerically accurate and computationally efficient means of estimation of Granger causality from empirical neural time series data.

[∗] The MVGC Toolbox will be released shortly under a GPL license. It is intended to supersede the GCCA (Granger Causal Connectivity Analysis) Matlab© toolbox: A. K. Seth, *A MATLAB toolbox for Granger causal connectivity analysis*, J. Neurosci. Methods **186**(2), 2010.

[1] A. K. Seth, A. B. Barrett and L. Barnett, *Causal density and integrated information as measures of conscious level*, Phil. Trans. R. Soc. A **369**, 2011.

[2] A. B. Barrett, L. Barnett and A. K. Seth, *Multivariate Granger causality and generalized variance*, Phys. Rev. E **81**(4), 2010.

[3] L. Barnett, A. B. Barrett and A. K. Seth, *Granger causality and transfer entropy are equivalent for Gaussian variables*, Phys. Rev. Lett. **103**(23), 2009.

[4] L. Barnett and A. K. Seth, *Behaviour of Granger causality under filtering: Theoretical invariance and practical application*, J. Neurosci. Meth. **201**(2), 2011.