BOOLEAN ALGEBRA

Logic Gates

Truth table: a table that shows the result of applying the logical function to all possible combination of inputs. Each row represents one of the possible combination of inputs and the last column the corresponding outputs.

Input X	Input Y	Output Q
0	0	1
0	1	0
1	0	0
1	1	0

On the right example of a truth table for a NOR gate

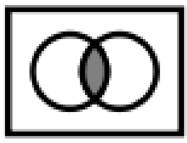
Boolean Variable: has two discrete possible values: True (1) and False (0).

Boolean equation: an equation that expresses a Boolean output Q in terms of Boolean inputs X, Y, Z, etc. to which one or more Boolean functions are applied.

Logic symbols: instead of drawing switch circuits for each gate we can use symbols that are associated with each function.

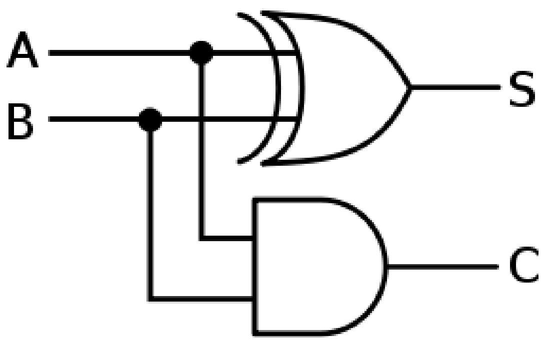
Venn diagrams: you can represent Boolean Algebra with Venn Diagrams

A AND B



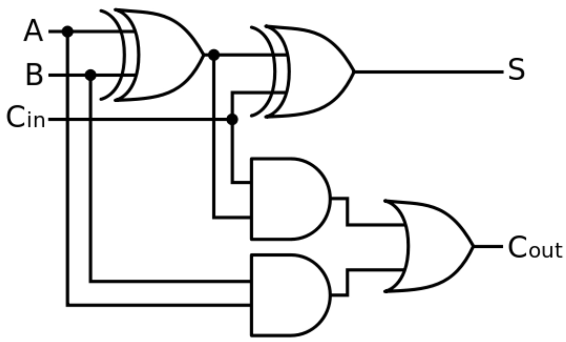
Half Adder

Is a circuit that perform the addition of two bits, it doesn't consider a possible carry coming in therefore it can't be used to perform a multi bits addition.



Full adder

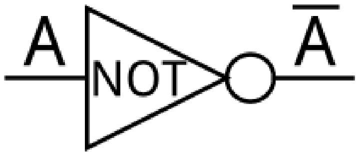
Circuit that perform the addition of two bits considering also a possible carry in. Placing different full adders in row you get a multi bits adder.



NOT

the output is the inverse of the input

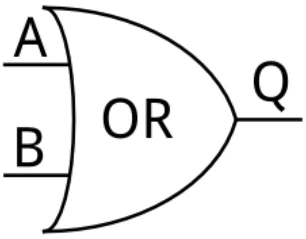
$$Q = \overline{X}$$



OR

the output is True if either or both inputs are True

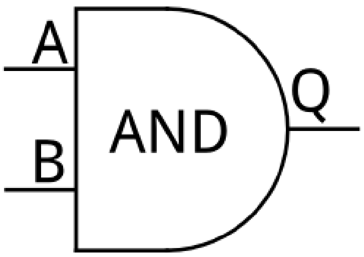
$$Q = X + Y$$



AND

the output is True if all inputs are True

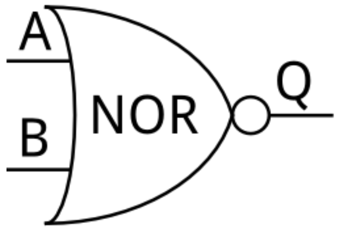
$$Q = X \cdot Y$$



NOR

the output is true only when all the inputs are false

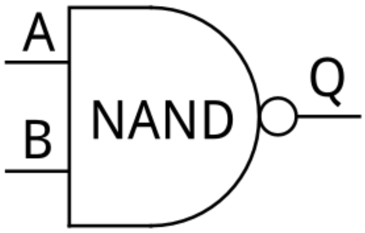
$$Q = \overline{X + Y}$$



NAND

the output is false if any input is false

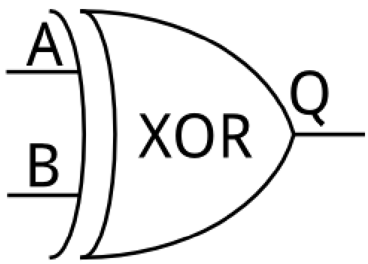
$$Q = \overline{X \cdot Y}$$



Exclusive OR (XOR)

the output is true if either input is true but not if both inputs are true

$$Q = X \oplus Y$$



Boolean Identities:

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0+x = x$
Null (or Dominance) Law	$0x = 0$	$1+x = 1$
Idempotent Law	$xx = x$	$x+x = x$
Inverse Law	$x\bar{x} = 0$	$x+\bar{x} = 1$
Commutative Law	$xy = yx$	$x+y = y+x$
Associative Law	$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$
Distributive Law	$x+yz = (x+y)(x+z)$	$x(y+z) = xy+xz$
Absorption Law	$x(x+y) = x$	$x+xy = x$
DeMorgan's Law	$(\overline{xy}) = \bar{x}+\bar{y}$	$(\overline{x+y}) = \bar{x}\bar{y}$
Double Complement Law	$\bar{\bar{x}} = x$	

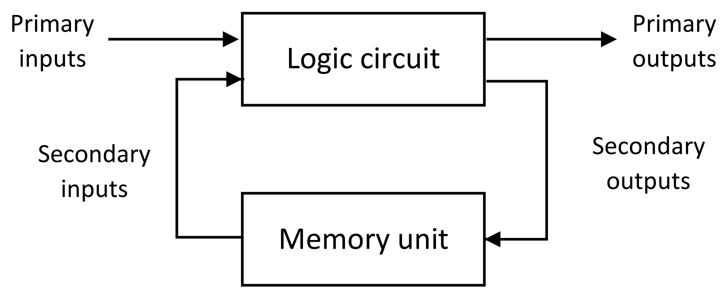
De Morgan's Law

A Boolean statement can be simplified by inverting all the variables, changing ANDs to ORs and ORs to ANDs and the inverting the whole expression.

$$\overline{A.B} = \overline{A} + \overline{B}$$

$$\overline{A+B} = \overline{A} . \overline{B}$$

Edge-triggered D-type flip-flop



In logic gates and circuit data is passed around but as soon as the next inputs are fed in the previous inputs are lost. Often some form of memory is needed and this is provided by a flip-flop, which is capable of storing one bit. It uses the system clock to synchronise the two inputs. The edge-triggered D-type flip-flop is a memory unit that changes state with each pulse of the clock.

The D-type flip-flop has two inputs:

- data signal
- clock, output is updated to reflect current status of signal and bit stored