Matériaux et structures composites

Modélisation des composites stratifiés

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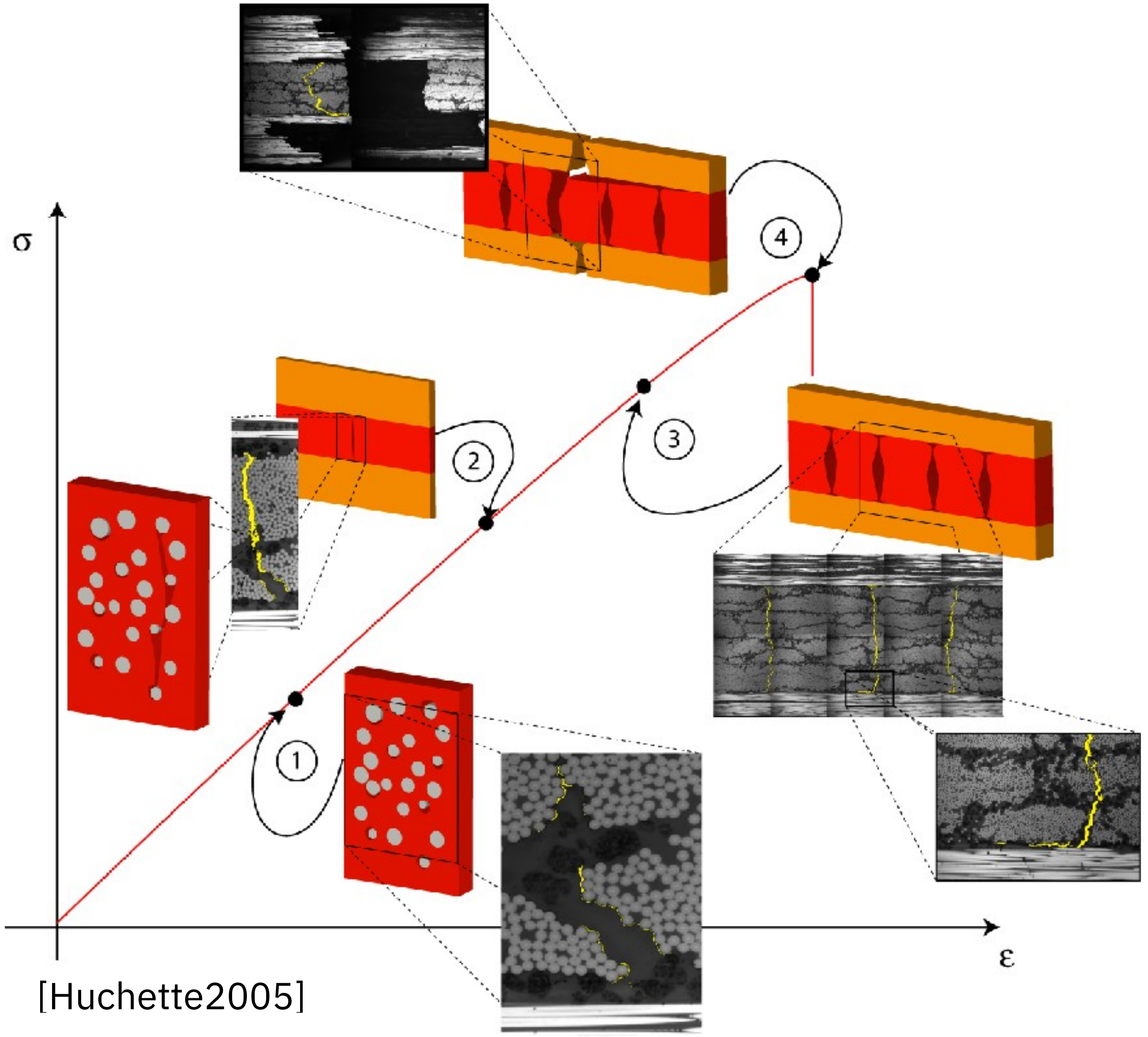
Endommagement des stratifiés

- Endommagement progressif
- Reports de charges entre plis
- Séquence d'endommagement dépend de l'empilement (orientation, épaisseur)

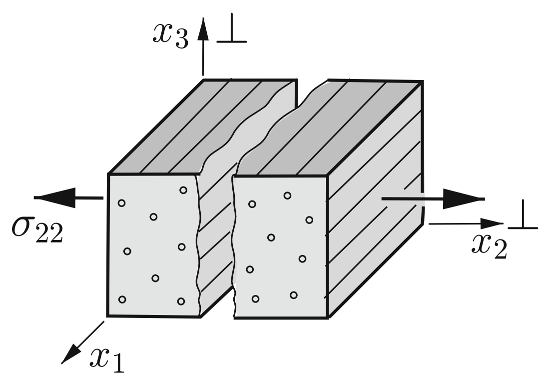


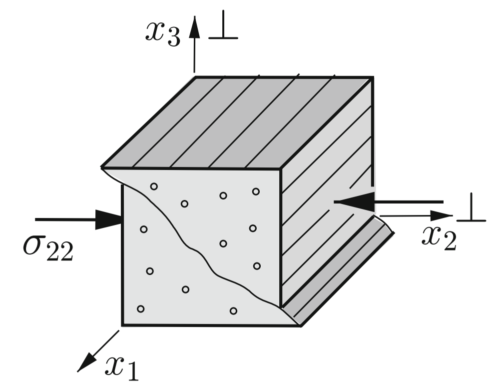
[Huchette2005]

Endommagement des stratifiés

- Chargement transverse au pli

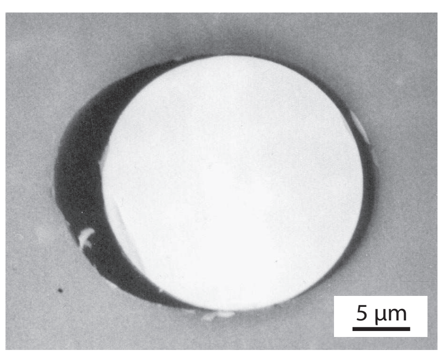
Traction : décohésions F/M, fissures transverses

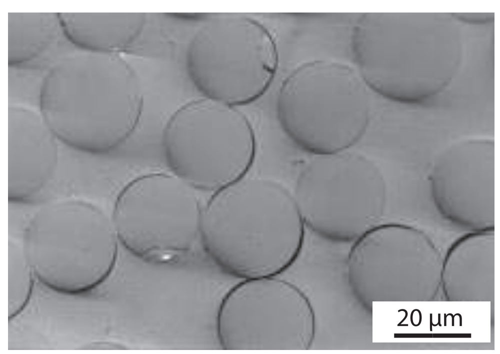


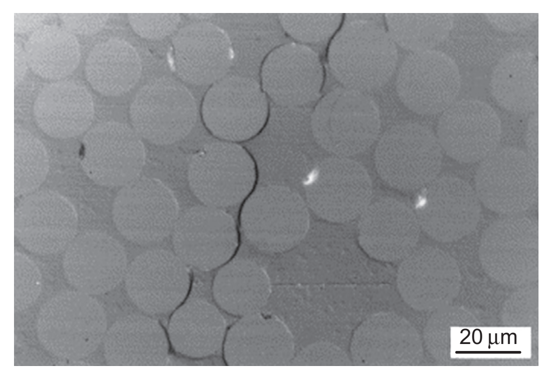


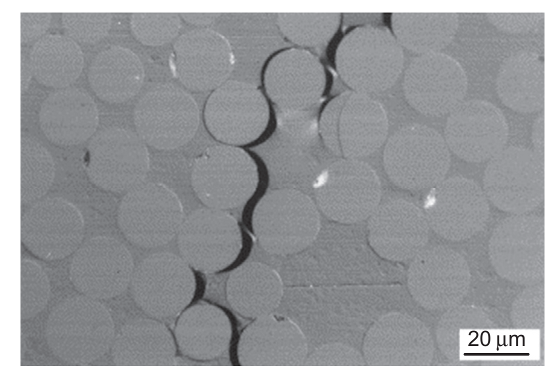
Compression : cisaillement hors-plan de la matrice + décohésions F/M ?

Décohésion F/M → *Multiplication des décohésions* → *Percolation* → *Fissure*

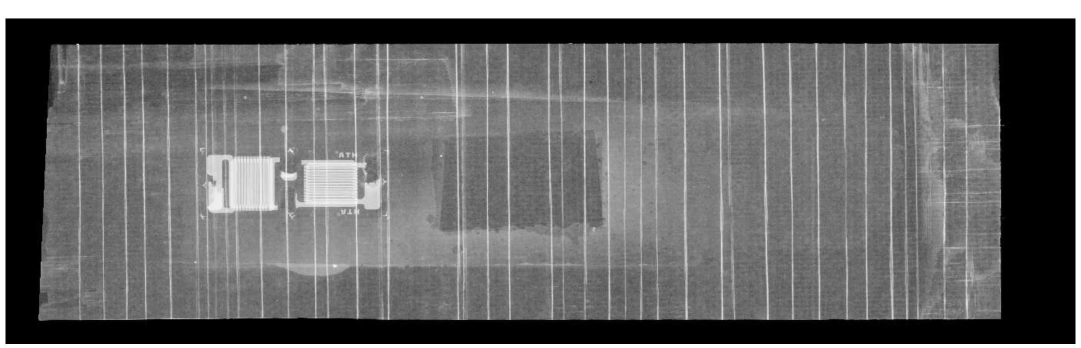


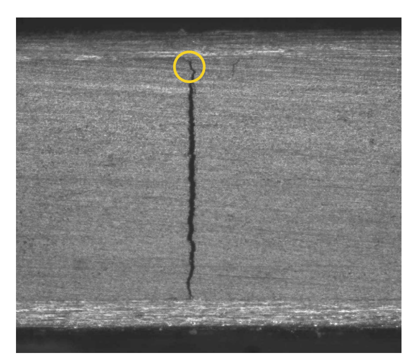




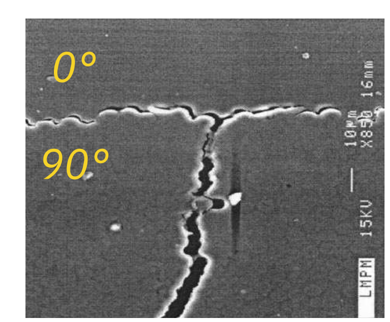


[Gamstedt1999]





⚠ *Plis épais : possible micro-délaminage en pointe de fissure*

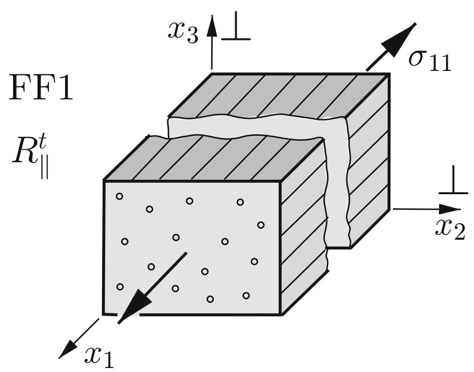


[Lubineau2002]

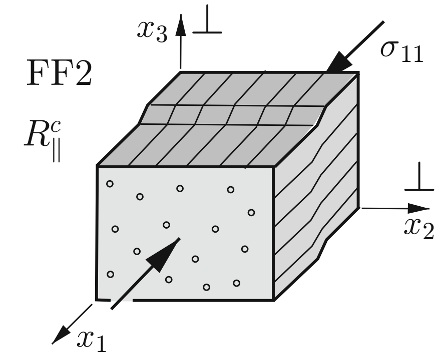
[Marsal2005]

Endommagement des stratifiés

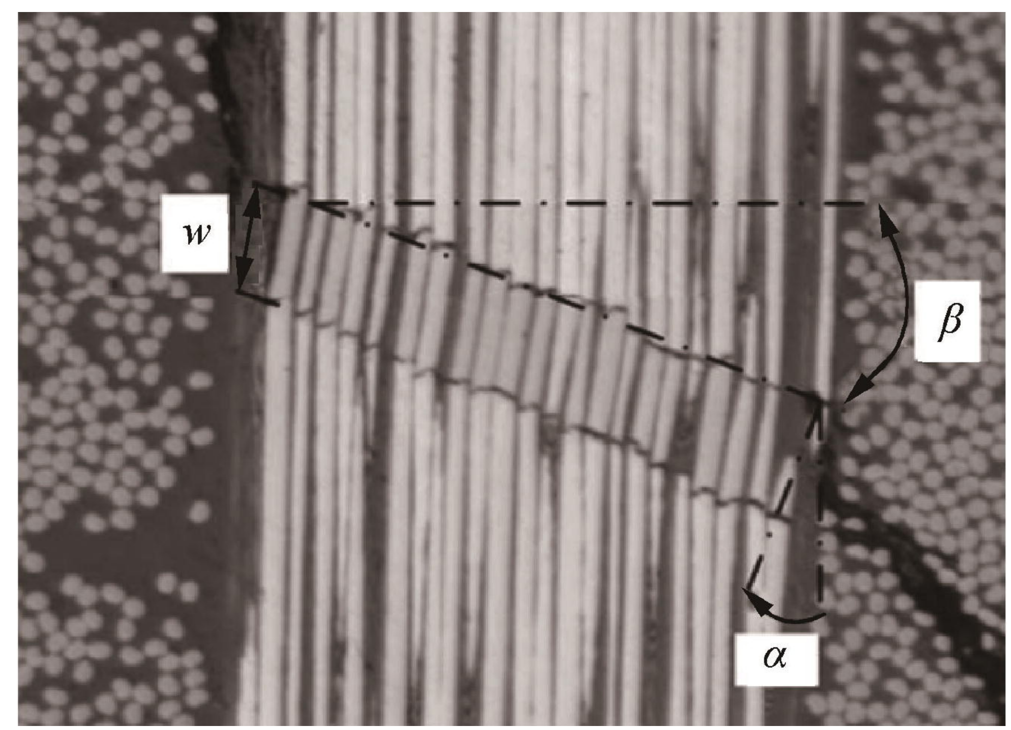
- Chargement dans le sens des fibres



Traction : rupture progressive des fibres



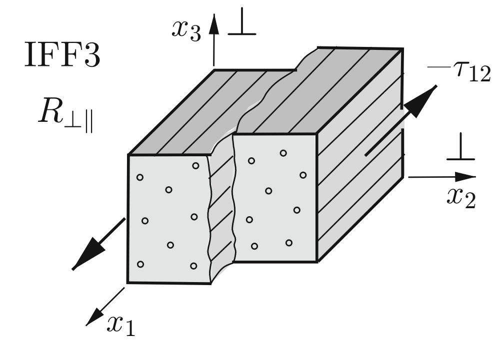
Compression : micro-flambage (kinking)



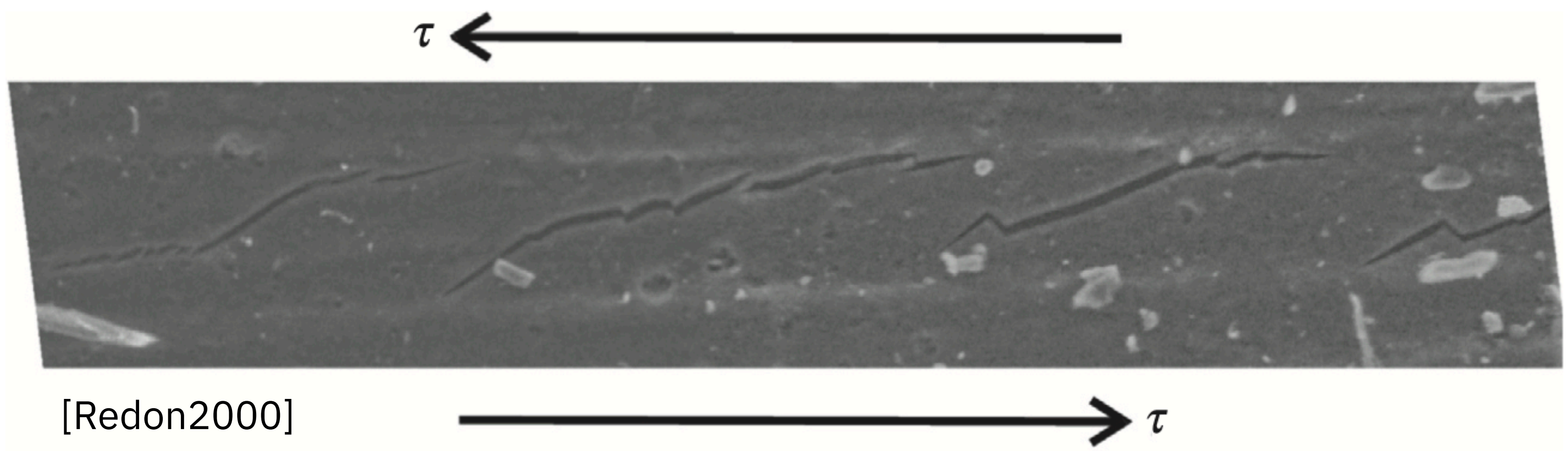
[Gutkin2010]

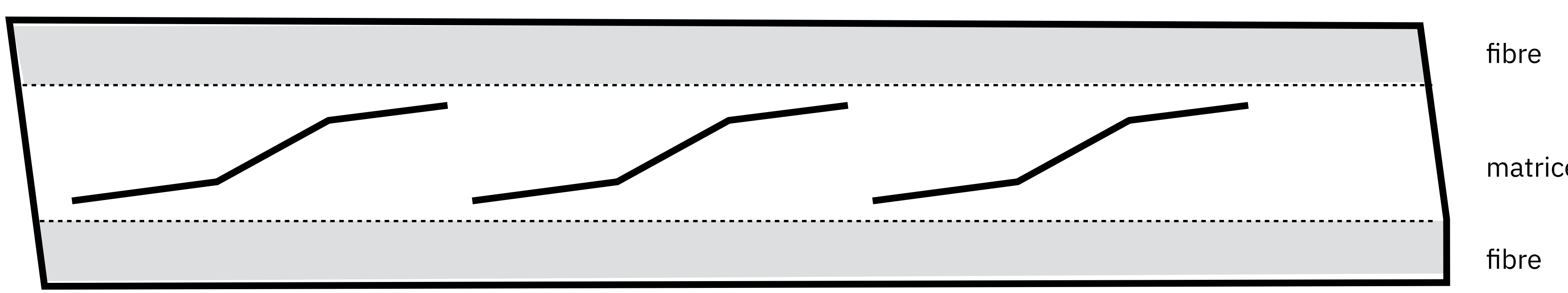
Endommagement des stratifiés

- Cisaillement plan



Rupture en cisaillement de la matrice





Endommagement des stratifiés

- Endommagement progressif
- Reports de charges entre plis
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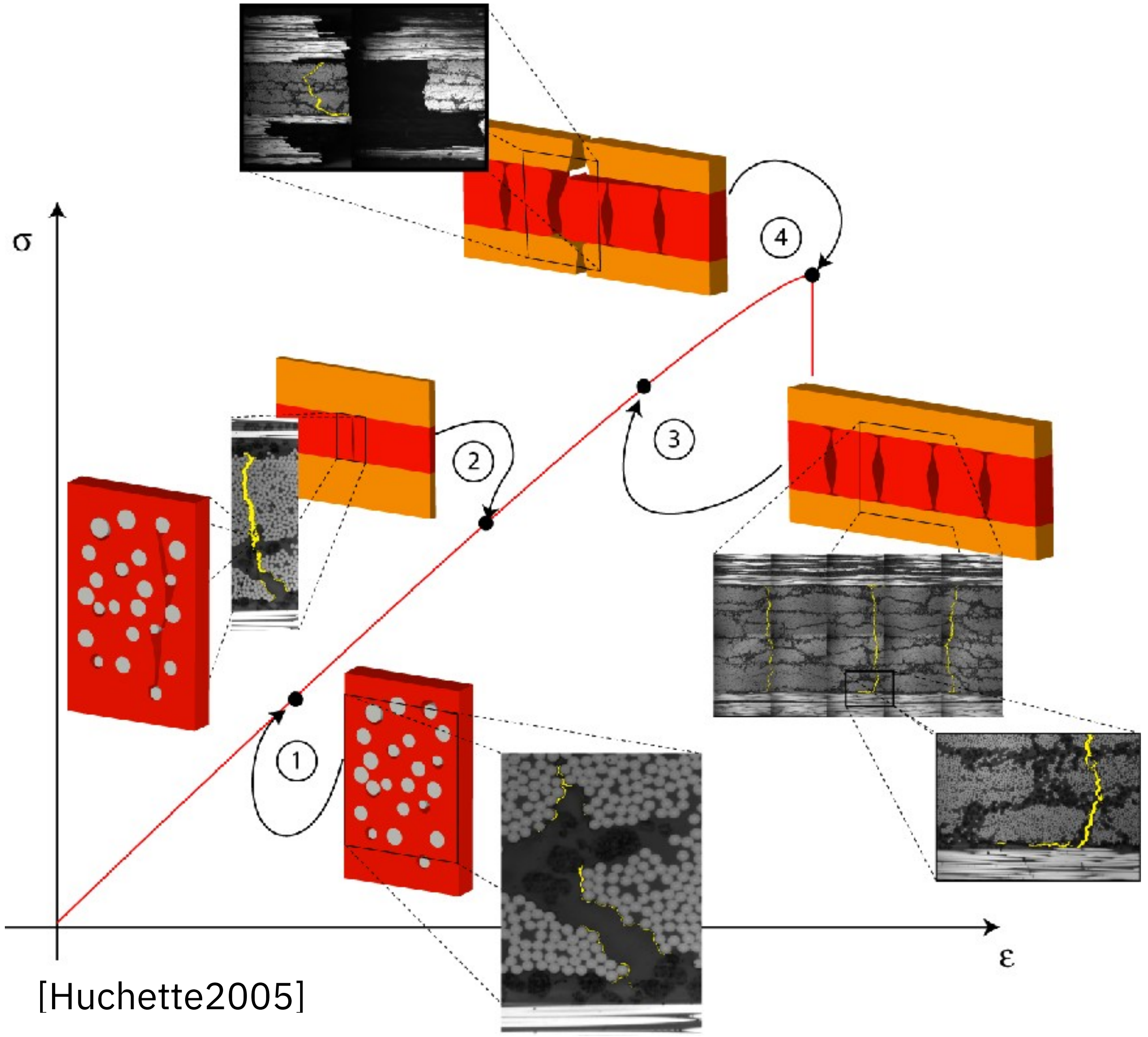


[Huchette2005]

Cadre générale d’un modèle d’endommagement

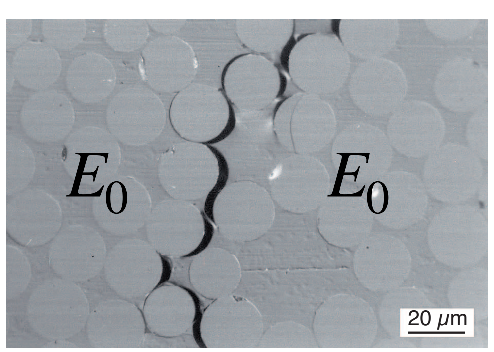
- *Approche (macro) phénoménologique*

  On ne décrit pas directement les mécanismes élémentaires (décohésion, fissures, rupture de fibre) mais seulement leur effet sur le comportement

- *Continuum damage mechanics*

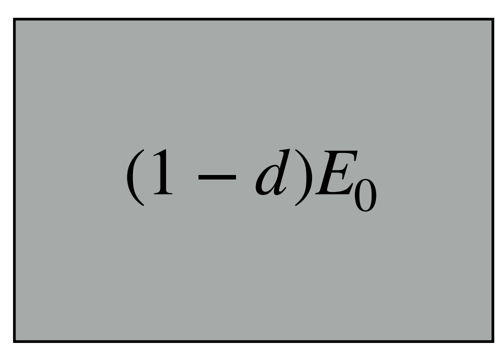
  - Variable(s) d’endommagement d_i = “mesure” indirecte de l’endommagement, par exemple, dégradation de la rigidité $\tilde{E} = f(d_i)$)

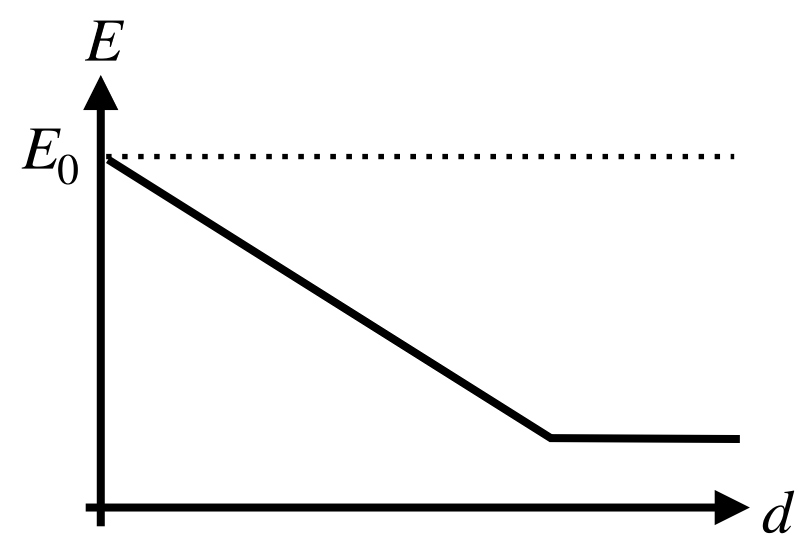
  - Un variable d_i peut être reliée à un ou plusieurs mécanismes sous-jacents



E_0 + fissure

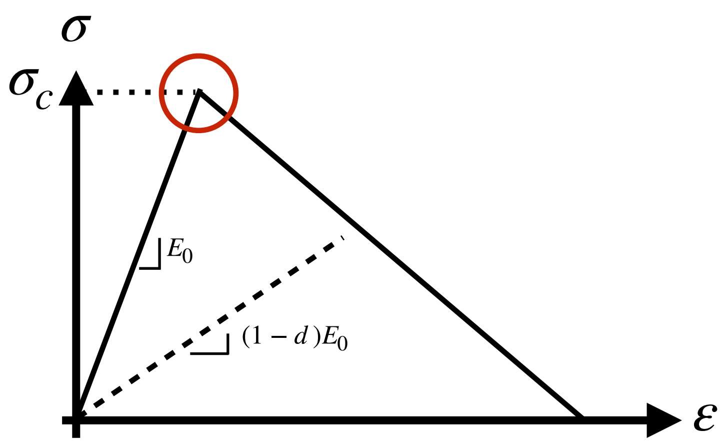
$\approx$

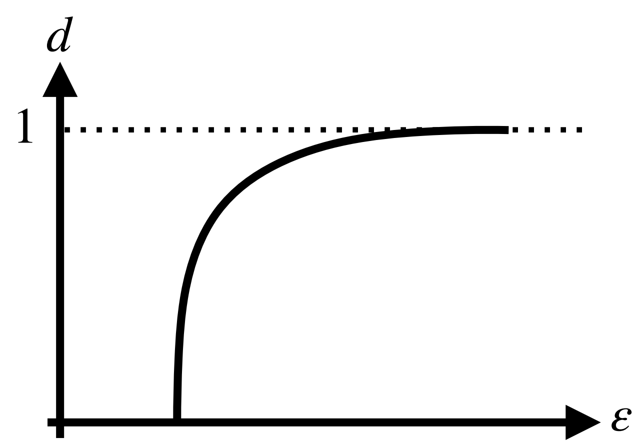


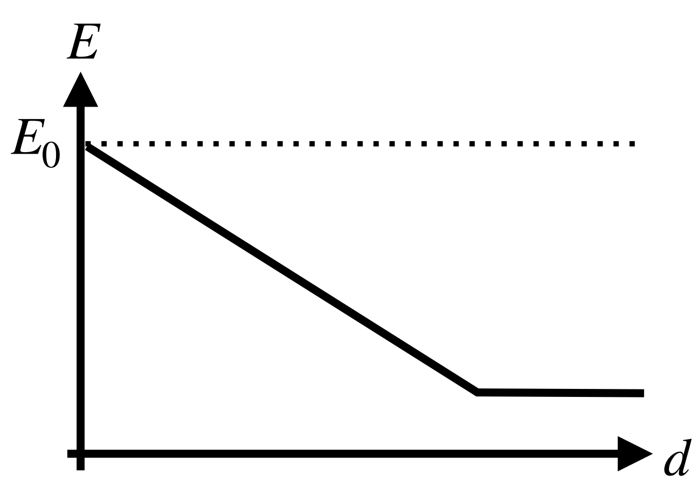


Cadre générale d’un modèle d’endommagement

- Ingrédients nécessaires







- **Initiation** : à partir de quel chargement commence l’endommagement ?

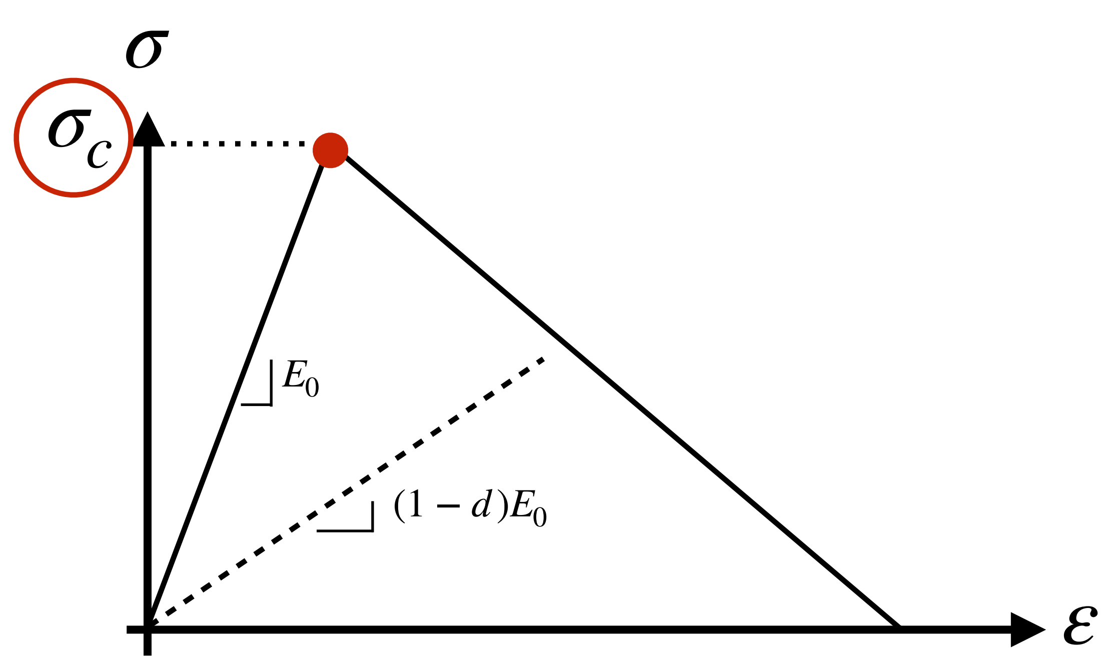
- **Propagation** : comment se développe l’endommagement en fonction du chargement ?

- **Effet de l’endommagement** : pour un état d’endommagement donné, comment sont affectées les propriétés ?

Modèle d'endommagement

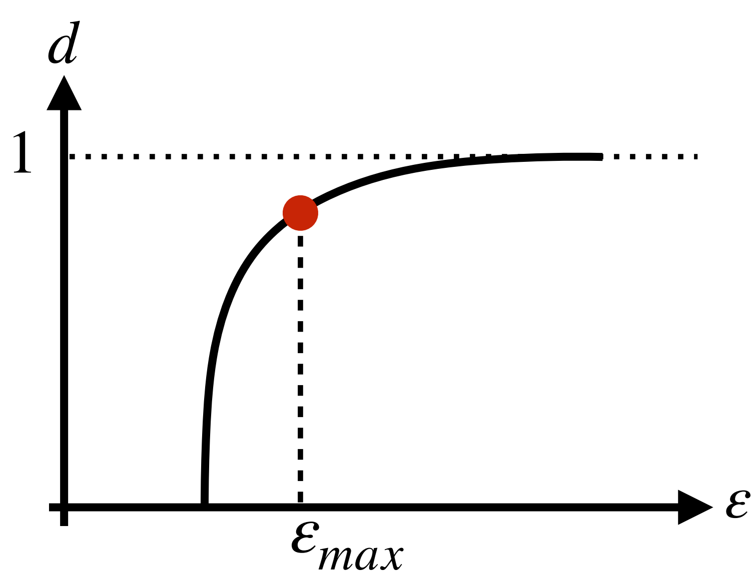
- **Critère d'amorçage**
  - A partir de "quand" l'endommagement est actif ?
  - Généralement, critère en contraintes $F(\sigma)$

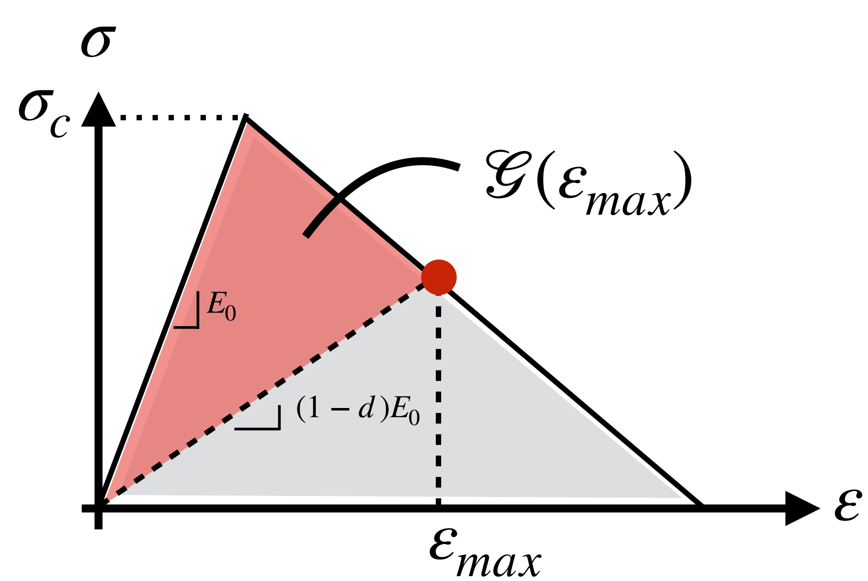
$$\begin{cases} F(\sigma) < 1 & \text{Domaine élastique } (d = 0) \\ F(\sigma) \geq 1 & \text{Endommagement actif} \end{cases}$$



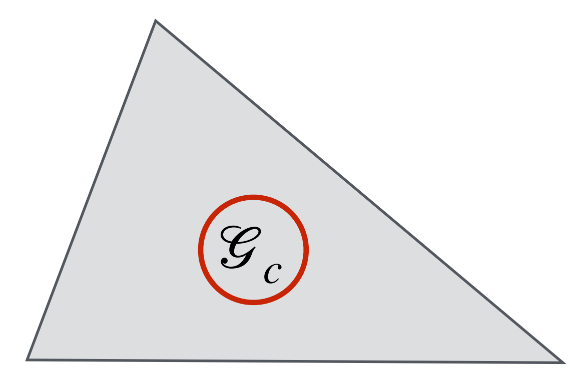
Modèle d’endommagement

- **Critère de propagation**
  - Comment se développe l’endommagement en fonction du chargement
  - Généralement, critère en taux d’énergie dissipée (justification thermo.)





$\mathscr{G}(\varepsilon_{max})$: énergie dissipée par l’endommagement à ε_{max}



$\mathscr{G}_c$: énergie dissipée à rupture

Modèle d'endommagement

- **Critère de propagation**
  - Comment se développe l'endommagement en fonction du chargement
  - Généralement, critère en taux d'énergie dissipée (justification thermo.)

Matériaux standards généralisés (GSM)

variables d'état

Potentiel d'état : $\Phi(\varepsilon, z_i)$

$$\sigma = \frac{\partial \Phi}{\partial \varepsilon} \qquad y_i = -\frac{\partial \Phi}{\partial z_i}$$

forces thermodynamiques associées

Potentiel (dual) de dissipation : $\pi^*(\sigma, y_i)$

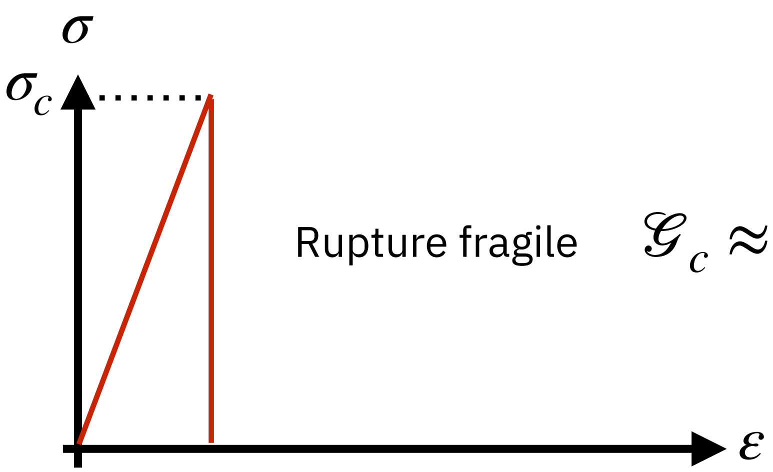
$$\dot{z}_i = \frac{\partial \pi^*}{\partial y_i}$$

$$\mathscr{D} = \sigma : \dot{\varepsilon} + y_i \cdot \dot{z}_i \geq 0$$

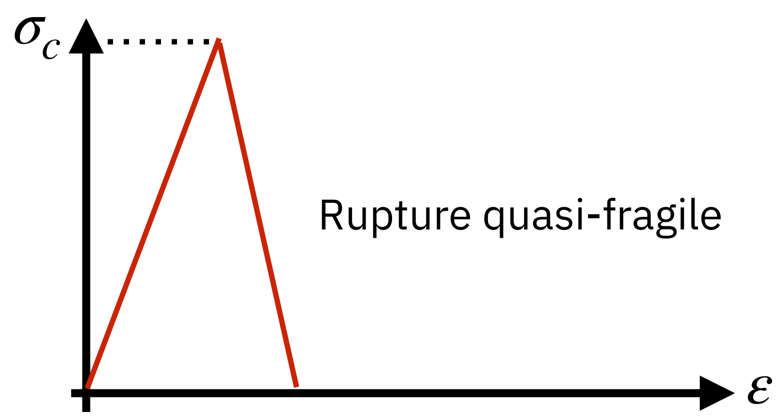
dissipation intrinsèque

Modèle d’endommagement

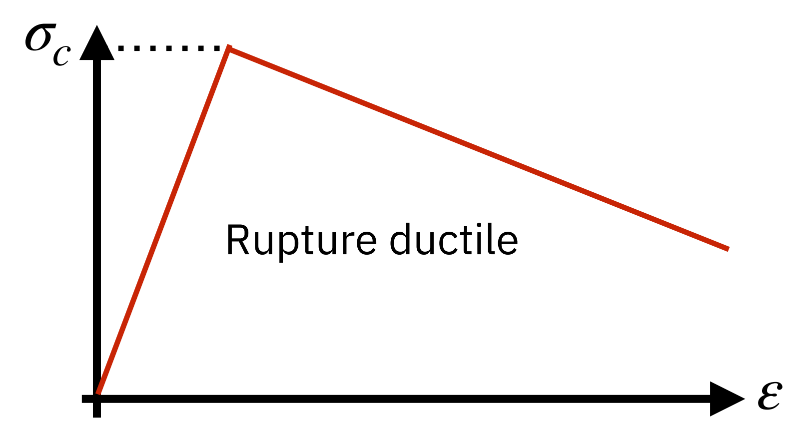
- Rupture fragile vs. rupture ductile



Rupture fragile $\mathscr{G}_c \approx W_{elas}$



Rupture quasi-fragile $\mathscr{G}_c \gtrsim W_{elas}$

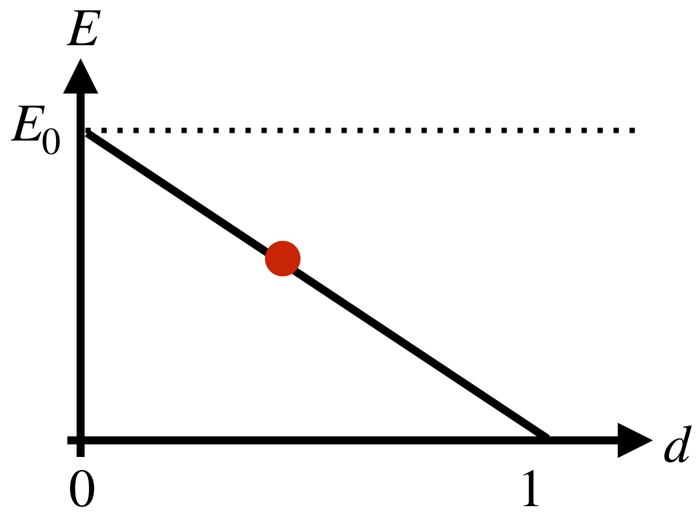


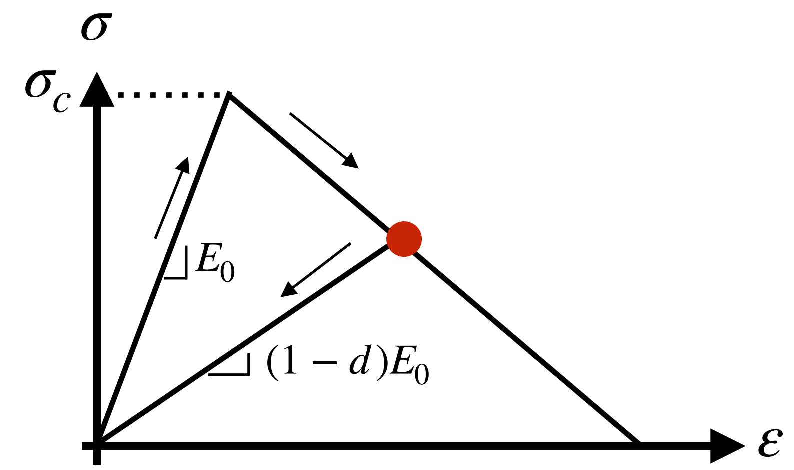
Rupture ductile $\mathscr{G}_c \gg W_{elas}$

Modèle d’endommagement

- **Effet de l’endommagement**

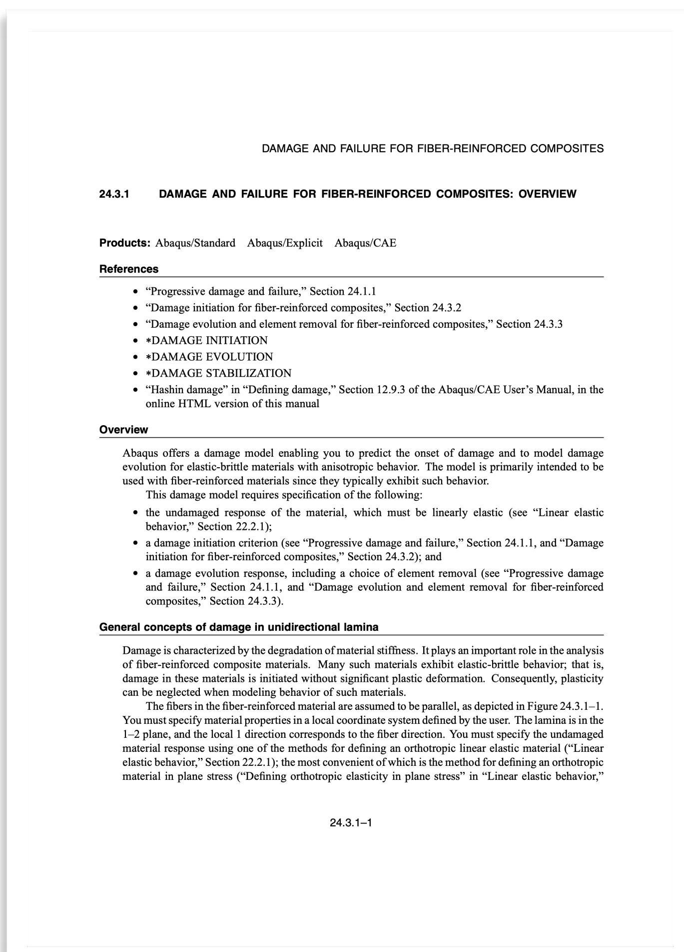
  - Quelles sont les propriétés résiduelles du matériau pour un état d’endommagement donné ? $E = f(d)$





Modèle de “Hashin”

- Modèle endommagement pour composites fibres longues
- Modèle phénoménologique, cadre *continuum damage mechanics*
- *Hashin (1980) + Matzenmiller (1995) + Camanho (2002)*
- ⚠Hypothèse **contraintes planes**



DAMAGE AND FAILURE FOR FIBER-REINFORCED COMPOSITES

24.3.1 DAMAGE AND FAILURE FOR FIBER-REINFORCED COMPOSITES: OVERVIEW

Products: Abaqus/Standard Abaqus/Explicit Abaqus/CAE

References

- “Progressive damage and failure,” Section 24.1.1
- “Damage initiation for fiber-reinforced composites,” Section 24.3.2
- “Damage evolution and element removal for fiber-reinforced composites,” Section 24.3.3
- *DAMAGE INITIATION
- *DAMAGE EVOLUTION
- *DAMAGE STABILIZATION
- “Hashin damage” in “Defining damage,” Section 12.9.3 of the Abaqus/CAE User’s Manual, in the online HTML version of this manual

Overview

Abaqus offers a damage model enabling you to predict the onset of damage and to model damage evolution for elastic-brittle materials with anisotropic behavior. The model is primarily intended to be used with fiber-reinforced materials since they typically exhibit such behavior.

This damage model requires specification of the following:

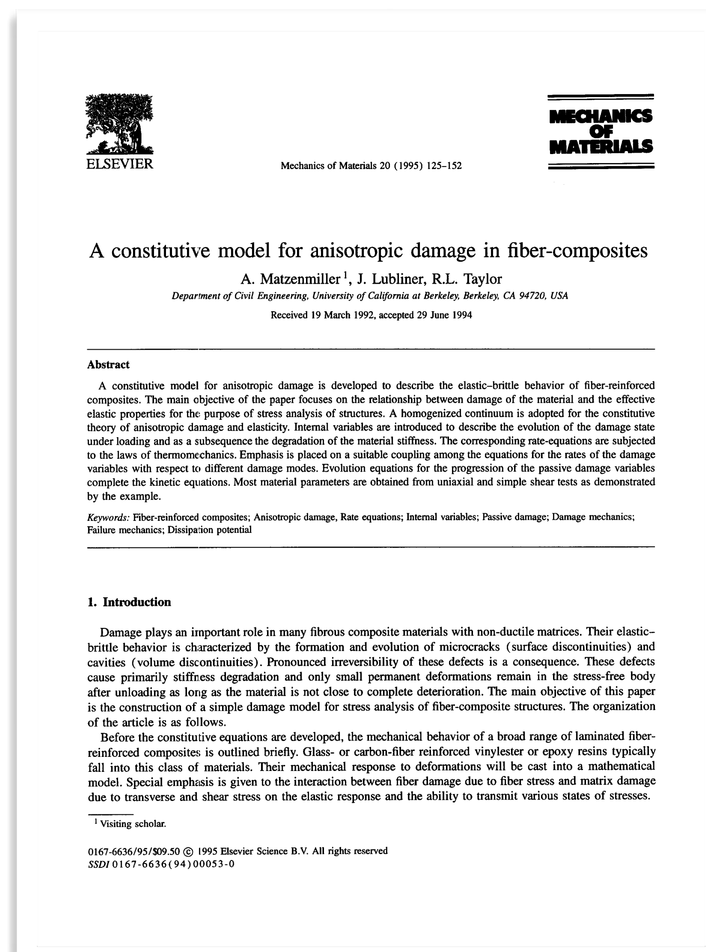
- the undamaged response of the material, which must be linearly elastic (see “Linear elastic behavior,” Section 22.2.1);
- a damage initiation criterion (see “Progressive damage and failure,” Section 24.1.1, and “Damage initiation for fiber-reinforced composites,” Section 24.3.2); and
- a damage evolution response, including a choice of element removal (see “Progressive damage and failure,” Section 24.1.1, and “Damage evolution and element removal for fiber-reinforced composites,” Section 24.3.3).

General concepts of damage in unidirectional lamina

Damage is characterized by the degradation of material stiffness. It plays an important role in the analysis of fiber-reinforced composite materials. Many such materials exhibit elastic-brittle behavior; that is, damage in these materials is initiated without significant plastic deformation. Consequently, plasticity can be neglected when modeling behavior of such materials.

The fibers in the fiber-reinforced material are assumed to be parallel, as depicted in Figure 24.3.1–1. You must specify material properties in a local coordinate system defined by the user. The lamina is in the 1–2 plane, and the local 1 direction corresponds to the fiber direction. You must specify the undamaged material response using one of the methods for defining an orthotropic linear elastic material (“Linear elastic behavior,” Section 22.2.1); the most convenient of which is the method for defining an orthotropic material in plane stress (“Defining orthotropic elasticity in plane stress” in “Linear elastic behavior,”

24.3.1–1



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Mechanics of Materials 20 (1995) 125–152

MECHANICS OF MATERIALS

A constitutive model for anisotropic damage in fiber-composites

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Received 19 March 1992, accepted 29 June 1994

Abstract

A constitutive model for anisotropic damage is developed to describe the elastic–brittle behavior of fiber-reinforced composites. The main objective of the paper focuses on the relationship between damage of the material and the effective elastic properties for the purpose of stress analysis of structures. A homogenized continuum is adopted for the constitutive theory of anisotropic damage and elasticity. Internal variables are introduced to describe the evolution of the damage state under loading and as a subsequence the degradation of the material stiffness. The corresponding rate-equations are subjected to the laws of thermomechanics. Emphasis is placed on a suitable coupling among the equations for the rates of the damage variables with respect to different damage modes. Evolution equations for the progression of the passive damage variables complete the kinetic equations. Most material parameters are obtained from uniaxial and simple shear tests as demonstrated by the example.

Keywords: Fiber-reinforced composites; Anisotropic damage, Rate equations; Internal variables; Passive damage; Damage mechanics; Failure mechanics; Dissipation potential

1. Introduction

Damage plays an important role in many fibrous composite materials with non-ductile matrices. Their elastic–brittle behavior is characterized by the formation and evolution of microcracks (surface discontinuities) and cavities (volume discontinuities). Pronounced irreversibility of these defects is a consequence. These defects cause primarily stiffness degradation and only small permanent deformations remain in the stress-free body after unloading as long as the material is not close to complete deterioration. The main objective of this paper is the construction of a simple damage model for stress analysis of fiber-composite structures. The organization of the article is as follows.

Before the constitutive equations are developed, the mechanical behavior of a broad range of laminated fiber-reinforced composites is outlined briefly. Glass- or carbon-fiber reinforced vinylester or epoxy resins typically fall into this class of materials. Their mechanical response to deformations will be cast into a mathematical model. Special emphasis is given to the interaction between fiber damage due to fiber stress and matrix damage due to transverse and shear stress on the elastic response and the ability to transmit various states of stresses.

[1] Visiting scholar.

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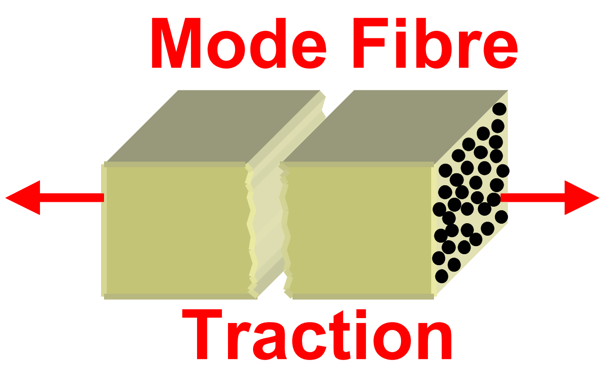
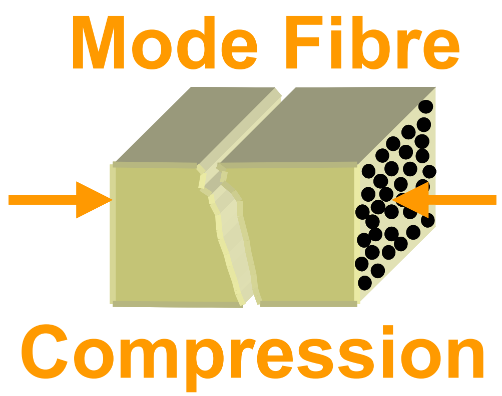
SSDI 0167-6636(94)00053-0

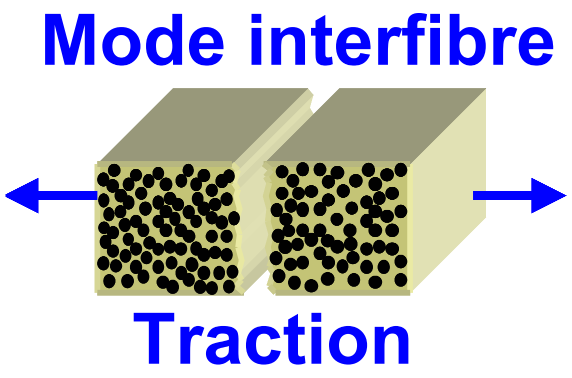
Modèle de Hashin

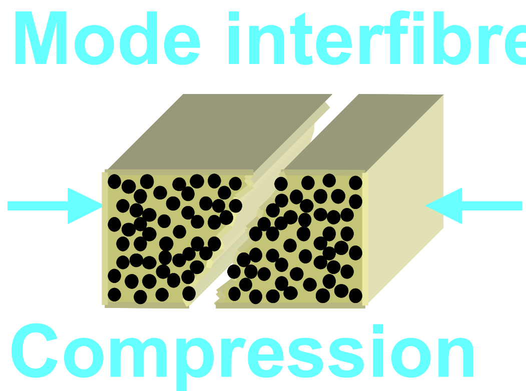
- **4 modes** d’endommagement
  - 2 modes “fibre" : en traction (FT) et compression (FC)





  - 2 modes “matrice” : en traction (MT) et compression (MC)

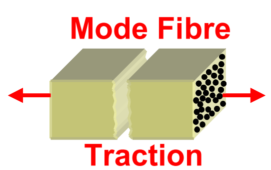




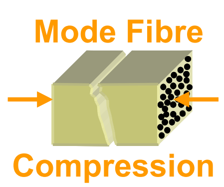
Modèle de Hashin

- Pour chacun des 4 modes d'endommagement, **critère d'amorçage** quadratique en contraintes

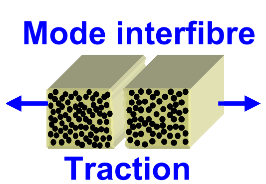
- $F(\sigma) < 1$: endommagement inactif, $F(\sigma) \geq 1$: endommagement actif



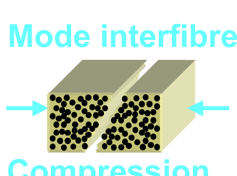
$$F_{FT} = \left(\frac{\sigma_{11}}{X_T}\right)^2 + \alpha\left(\frac{\sigma_{12}}{S_L}\right)^2$$



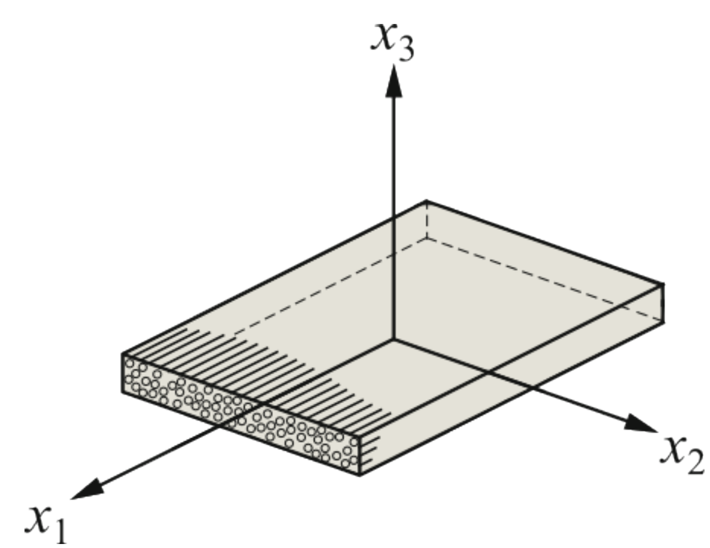
$$F_{FC} = \left(\frac{\sigma_{11}}{X_C}\right)^2$$



$$F_{MT} = \left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\sigma_{12}}{S_L}\right)^2$$



$$F_{MC} = \left(\frac{\sigma_{22}}{2S_T}\right)^2 + \left[\left(\left(\frac{Y_C}{2S_T}\right)^2 - 1\right)\right]\frac{\sigma_{22}}{Y_C} + \left(\frac{\sigma_{12}}{S_L}\right)^2$$



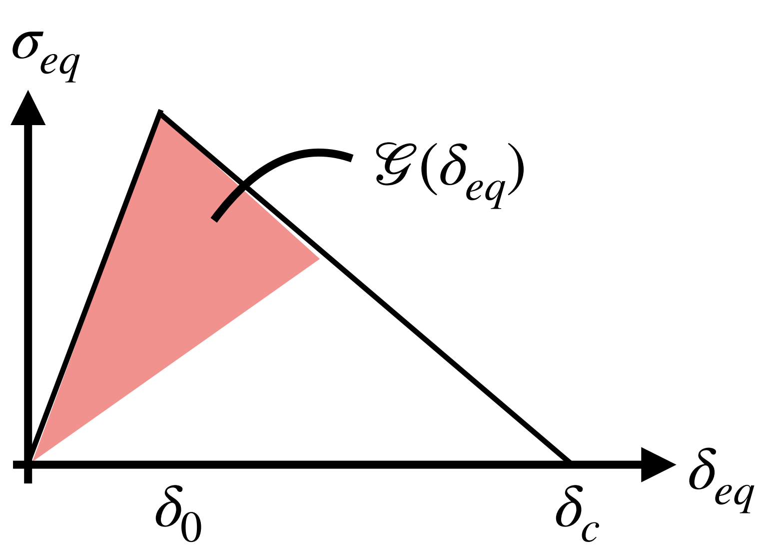
Paramètres matériaux : X_T, X_C, Y_T, Y_C, S_L, S_T

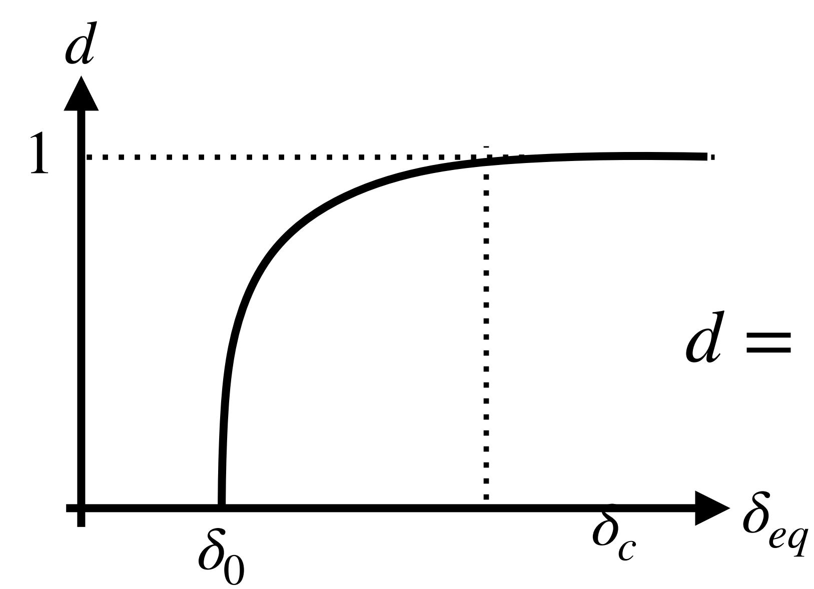
Modèle de Hashin

- Une variable d'endommagement par mode : $d_{FT}, d_{FC}, d_{MT}, d_{MC}$
- Activée dès que le critère d'amorçage correspondant $F_{XX} \geq 1$

$$F_{XX} < 1 \quad d_{XX} = 0 \qquad\qquad F_{XX} \geq 1 \quad d_{XX} \geq 0$$

- Critère de **propagation** en **énergie** dissipée : $d = f(\mathscr{G}, \mathscr{G}_c) = \mathscr{G}/\mathscr{G}_c$





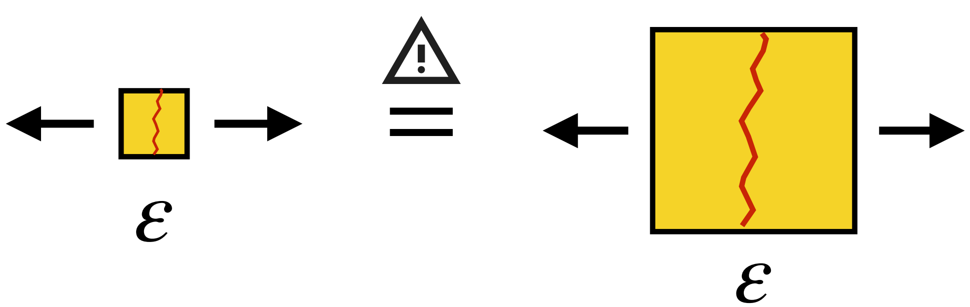
$$d = \frac{\delta_c(\delta_{eq} - \delta_0)}{\delta_{eq}(\delta_c - \delta_0)}$$

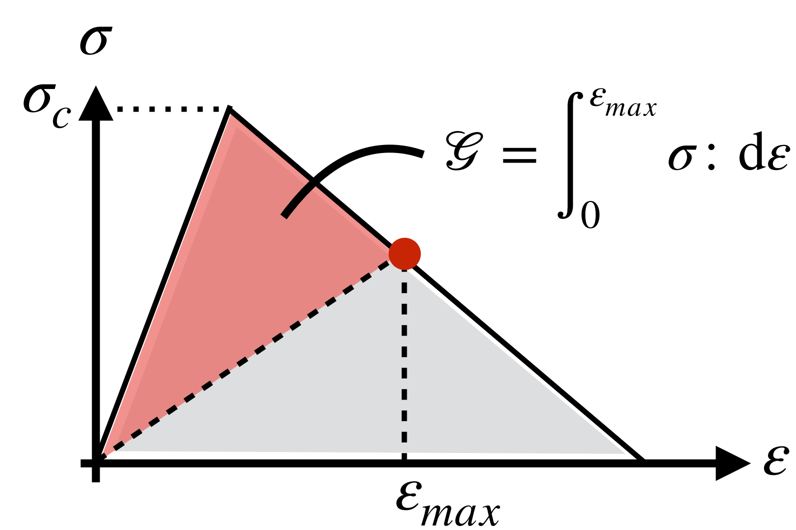
Paramètres matériaux : $G_c^{FT}, G_c^{FC}, G_c^{MT}, G_c^{MC}$

Modèle de Hashin

- **Energie** dissipée (grandeur extensive)
- Une surface 2 x plus grande doit dissiper 2 x plus d'énergie à rupture
- **Longueur interne** “cachée”

- Mais, formulation en contrainte/déformation, tout est normalisé.
- L'énergie dissipée est la même quelle que soit la taille de l'élément (*celle d'un élément de taille 1*)

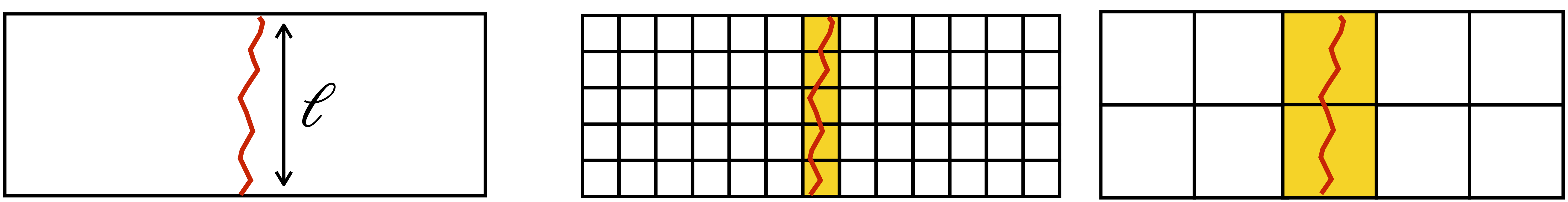




Modèle de Hashin

- **Energie** dissipée (grandeur extensive)
- Une surface 2 x plus grande doit dissiper 2 x plus d'énergie à rupture
- **Longueur interne** "cachée"

- **Problème** : l'énergie dissipée à rupture dépend du maillage !



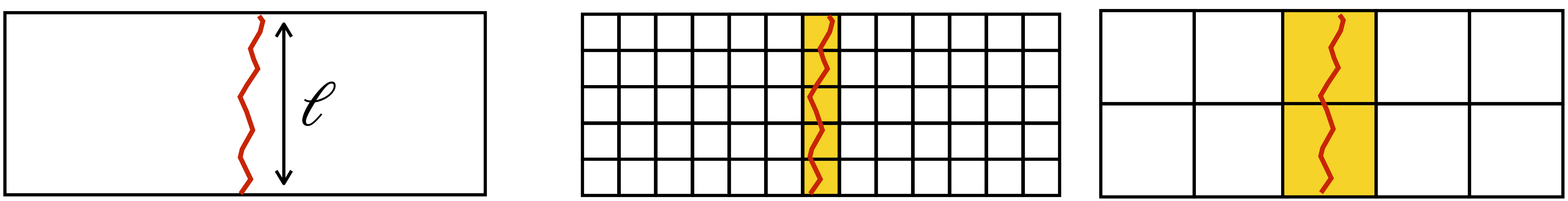
$\mathscr{E}_{rupt} = \mathscr{G}_c \ell$ ⚠ $\mathscr{E}_{rupt} = 5 \times \mathscr{G}_c$ ⚠ $\mathscr{E}_{rupt} = 2 \times \mathscr{G}_c$

Modèle de Hashin

- Remède : introduction d'une **longueur de régularisation L_c**
- Méthode *crack band* : **L_c = taille élément**
- On passe d'une déformation à un déplacement équivalent

$$\varepsilon \rightarrow \delta_{eq} = \varepsilon L_c \quad \Longrightarrow \quad \mathscr{G}(\delta_{eq}) = \mathscr{G}(\varepsilon) L_c$$

- Energie dissipée est **indépendante de la taille** caractéristique du maillage



$$\mathscr{E}_{rupt} = \mathscr{G}_c \ell \qquad \mathscr{E}_{rupt} = 5 \times \mathscr{G}_c \frac{\ell}{5} \qquad \mathscr{E}_{rupt} = 2 \times \mathscr{G}_c \frac{\ell}{2}$$

Modèle de Hashin

- Effet de l'endommagement : pour un état d'endommagement caractérisé par d_{FT}, d_{FC}, d_{MT}, d_{MC}, quelle est la rigidité résiduelle ?

$$C_D = f(d_{FT}, d_{FC}, d_{MT}, d_{MC})$$

$$C_D = \frac{1}{D}\begin{bmatrix} (1-d_F)E_1 & (1-d_F)(1-d_M)\nu_{21}E_1 & 0 \\ (1-d_F)(1-d_M)\nu_{12}E_2 & (1-d_M)E_2 & 0 \\ 0 & 0 & D(1-d_S)G_{12} \end{bmatrix}$$

avec : $d_F = \begin{cases} d_{FT} & \sigma_{11} > 0 \\ d_{FC} & \sigma_{11} < 0 \end{cases}$ $\quad d_M = \begin{cases} d_{MT} & \sigma_{22} > 0 \\ d_{MC} & \sigma_{22} < 0 \end{cases}$

$$d_S = 1 - (1-d_{FT})(1-d_{FC})(1-d_{MT})(1-d_{MC})$$

$$D = 1 - (1-d_F)(1-d_M)\nu_{12}\nu_{21}$$

Modèle de Hashin

- Une variable d'endo. peut affecter un ou plusieurs termes du tenseur de rigidité
- La dégradation d'un terme du tenseur peut dépendre de une ou plusieurs variables

$$C_D = \frac{1}{D}\begin{bmatrix} (1-d_F)E_1 & (1-d_F)(1-d_M)\nu_{21}E_1 & 0 \\ (1-d_F)(1-d_M)\nu_{12}E_2 & (1-d_M)E_2 & 0 \\ 0 & 0 & D(1-d_S)G_{12} \end{bmatrix}$$

avec : $$d_F = \begin{cases} d_{FT} & \sigma_{11} > 0 \\ d_{FC} & \sigma_{11} < 0 \end{cases} \qquad d_M = \begin{cases} d_{MT} & \sigma_{22} > 0 \\ d_{MC} & \sigma_{22} < 0 \end{cases}$$

$$d_S = 1 - (1-d_{FT})(1-d_{FC})(1-d_{MT})(1-d_{MC})$$

$$D = 1 - (1-d_F)(1-d_M)\nu_{12}\nu_{21}$$