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SEMI-LAGRANGIAN INTERPOLATION FOR NUMERICAL SIMULATIONS OF ENVIRONMENTAL FLUID FLOWS THROUGH Z-TYPE FINITE ELEMENTS

Advective transport (or simply *advection*) of immersed quantities into a portion of a fluid is a phenomenon widely studied in Fluid Dynamics, being, hence, some notions about particles on movement, a necessary issue. Several types of fluid flows exhibit a specificity concerning to the description of particle trajectories that compound them. In this set, geophysical fluid flows highlight and, in turn, so the environmental ones, at which wide variations of relevant physical quantities to their behavior on the continuum media are present, like density, momentum, pressure and velocity, to cite some possibilities. In practical aspects, fluid flows at which the transport of pollutants and chemical substances either through the atmospheric air or the fluvial vias, for instance, are included into that previous class. Scientific importance of studying fluid flows such as the mentioned above may be perceived bilaterally: on the one hand, it conducts to the maturation of the research related to the theme; on the other hand, it brings benefits that spread among distinct fields, from economy to public safety, because convey enginnering applications that result in efficient solutions to the observed problems, begetting predictability of natural disasters, geological impacts, as well as causing immediate actions and justified decision-makings that assuage or extinguish big environmental damages.

Correlated to the concept of advective transport, it is the Lagrangian interpretation of particles movement, that concerns to register, along the time, the greatest possible amount of informations about their trajectories. In this movement interpretation, the referential is set up on the particle itself, so that to accompany it, dynamically, during each instant of movement that is performed [1]. Vantages in the utilization of this approach are reported by the literature and were checked in past decades with the development of numerical models for meteorological predictions. Since then, numerical improvements have been investigated by several researchers.

Main motivation to this work come from the need to obtain qualitative numerical methods that approximate satisfactorily the temporal evolution of the real fluid flows observed. For this purpose, the formulation of discrete mathematical models suitable to the research interests becomes essential. Inserted into this context, we introduce here an approach used in our numerical experiments builded not only for numerical simulations of general fluid flows, but also for that ones under demand of environmental scope: the Semi-Lagrangian Method.

The Semi-Lagrangian Method originates from the scientific community linked to Meteorology, as cited in [2], [3] and [4]. Its name suggests a combination between Lagrangian and Eulerian movement descriptions. In this latter, by elucidating, the referential keeps fixed and the observer is concentrated on the phenomena ocurring at a given spatial region. Interested reader can proceed to [5] to acquaint with this theory of the movement descriptions.

In the semi-lagrangian approach, the discretization mesh by which the computations are carried out keeps fixed along all the time and the particles trajectories are privileged. Computational algorithm determines that a particle at a current step time have necessarily to occupy, at the next step time, the same position as the a fixed mesh point is occupying. This way, as the time marchs step by step, the particles go being transported directly to the position of mesh point. However, to find the departure point of each trajectory, a backward-in-time integration is performed along the curve drawn up by the particle. Finally, the outtermost points of the trajectories match upstream to the departure point of the curve and downstream to the mesh point.

One of the main traits of the Semi-Lagrangian Method is interpolation. Better interpolation schemes produce more accurate results. The Finite Element Method used to discretize the differential equations invokes consistent interpolation operators. By seeking more accuracy for our purposes, we have looked for interpolation schemes using internal iterations by step time [6] and gradient evaluations to recovery further data about the coupled fields (in the sense that we are simulating equations mixing scalar and vectorial quantities) as Zienkiewicz-Type (or Z-Type, only) finite elements described by [7], [8], [9] and [10]. Figure (1) shows us a comparative result between two semi-lagrangian interpolation methods both developed in our research group and tested on simulations of a Lamb-Oseen vortex with a CFL number equal to 3. A tetrahedrical mesh made up by the “MINI 3D“ element was used. Then, a scalar field of a chemical species was placed inside the kernel of vortex and allowed to disperse itself freely just by action of the vorticity. Upper snapshot is a simulation using the method by [11], whereas the inferior was developed by [12]. Differences are observed through the regions with more or less color incidences.

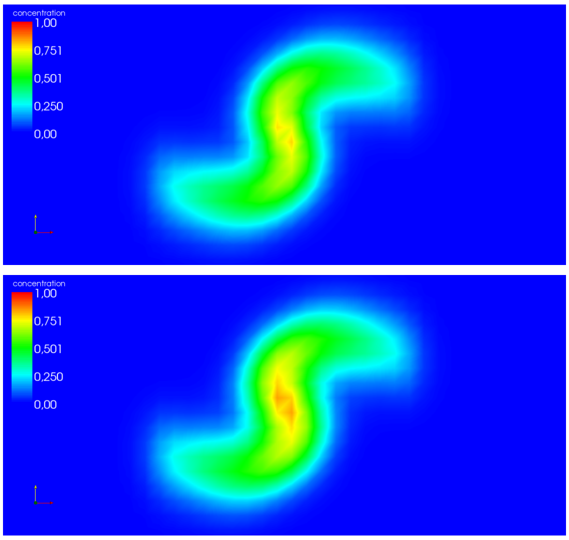

Figura 1: Lamb-Oseen Vortex simulation: CFL = 3, Reynolds = 100

Referências

[1] Samelson, R. E and Wiggins, S. *Lagrangian Transport in Geophysical Jets and Waves*, 1st. ed., New York, Springer-Verlag, 2006.

[2] Bonaventura, L. “An Introduction to Semi-Lagrangian Methods for Geophysical Scale Flows”. In: Ercoftac Leonhard Euler lectures, Swiss Federal Institute of Technology Zurich, Zurich, Swiss, 2004.

[3] Staniforth, A., Côté, J. "Semi-Lagrangian Integration Schemes for Atmospheric Models-A Review. Monthly Weather Review", v. 119, pp. 2206–2223, 1991.

[4] Smolarkiewicz P. K., Pudykiewicz, J. A. "A class of Semi-Lagrangian Approximations for Fluids. Journal of the Atmospheric Sciences", v. 49, n. 22, pp. 2082–2096, 1992.

[5] Malvern, L. E. *Introduction to the Mechanics of a Continuous Medium.* 1st. ed. New Jersey, Prentice-Hall, 1969.

[6] Christov, C. I., Pontes, J. "Numerical Scheme for Swift-Hohenberg Equation with Strict Implementation of Lyapunov Functional". Math. and Comp. Modelling, v. 35, n. 1-2, pp. 87–99, 2002.

[7] Wang, M., Xu, J. "Some Tetrahedron Nonconforming Elements for Fourth Order Elliptic Equations. Technical Report AM 290, University of PennState, Pennsylvania, United States, 2006.

[8] Wang, M., Xu, J. "A New Class of Zienkiewicz-Type Nonconforming Element in Any Dimensions". Technical Report AM 297, University of PennState, Pennsylvania, United States, 2006.

[9] Andreev, A. B., Racheva, M. R. "A Zienkiewicz-Type Finite Element Applied to Fourth-Order Problems". Journal of Comput. and Appl. Math.,v. 235, pp. 348–357, 2010.

[10] Silva, R. C. C., Loula, A. F., Guerreiro, J. N. C. "Local Gradient and Stress Recovery for Triangular Elements". Computers & Structures, v. 82, n. 23-26, pp. 2083–2092, 2004

[11] dos Anjos, G. R., *Solução do Campo Hidrodinâmico em Células Eletroquímicas pelo Método dos Elementos Finitos.* COPPE/UFRJ, Dissertação de M.Sc.,Rio de Janeiro, RJ, Brasil, 2007.

[12] Oliveira, G. C. P. *Estabilidade Hidrodinâmica em Células Eletroquímicas pelo Método de Elementos Finitos.* COPPE/UFRJ, Dissertação de M.Sc.,Rio de Janeiro, RJ, Brasil, 2011.