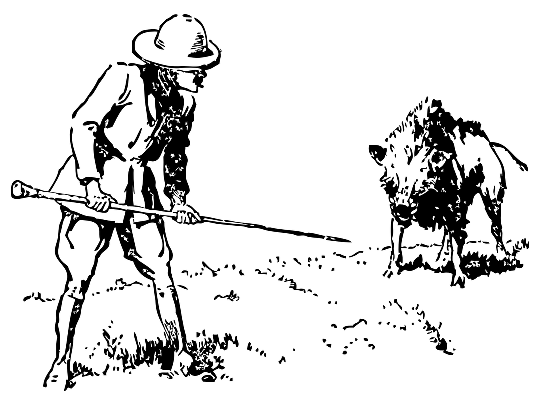
A computational quest for quantum codes

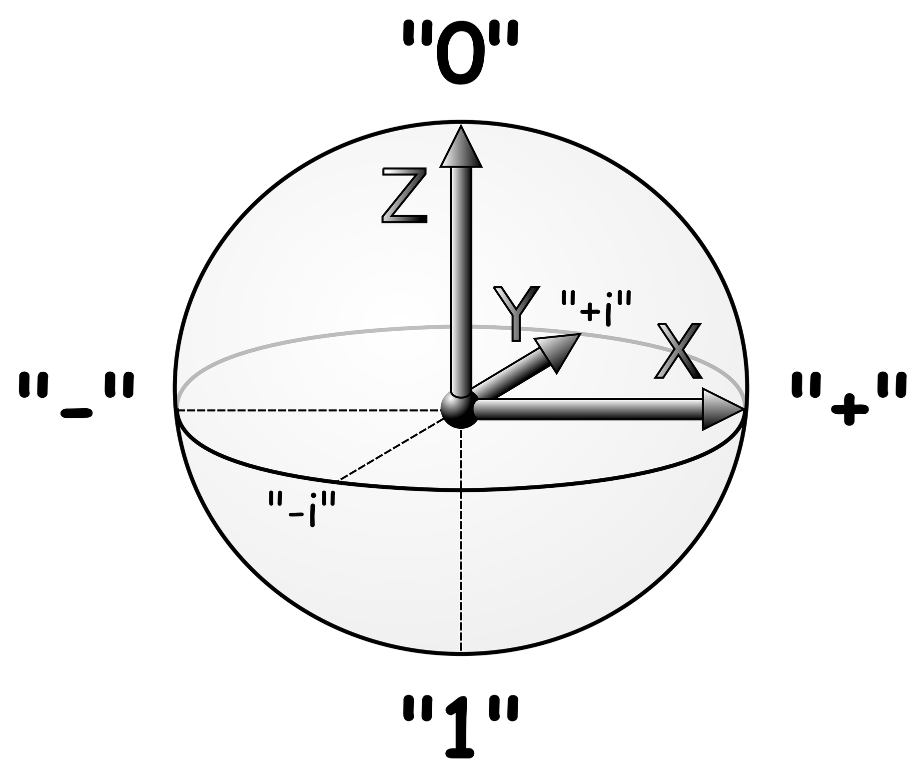
Gregory Crosswhite and Dave Bacon

The Prey



A qubit...

- ...is an abstract unit of quantum information, analagous to how a bit is an abstract unit of classical information.
- ...has a continuous degrees of freedom lying on the surface of a sphere.
- ...corresponds to a classical "0" at the North pole and a classical "1" at the South pole.
- *...unlike a classical bit, cannot be copied!*



WHY SHOULD ANYONE CARE ABOUT QUANTUM INFORMATION?

- When the physical features of our computers become small enough, we cannot avoid entering the quantum regime, so we may as well understand how we can make this work for us rather than against us!
- Understanding how to harness quantum information gives us a better understanding of harnessing quantum effects in general, which contributes to the quest for new materials with useful exotic properties!

This last point is why robustly storing and manipulating quantum information is harder than doing the same for classical information! Which leads us to...

THE CHALLENGE:

What physical systems exist that allow us to robustly store and manipulate quantum information as easily as we can classical information?

Such physical systems are said to manifest a quantum code because they provide a way to encode abstract quantum information into the state of the physical system.

The Hunt

$$H = \begin{matrix} X & X \\ I & I \end{matrix} + \begin{matrix} I & I \\ X & X \end{matrix} + \begin{matrix} Z & I \\ Z & I \end{matrix} + \begin{matrix} I & Z \\ I & Z \end{matrix}$$

Input: Description of the physical system as a list of operators appearing inside the Hamiltonian.

Output: Lists of operators defining how the quantum information is encoded inside the system.

Stabilizers

$$\begin{matrix} X & X \\ X & X \end{matrix} \qquad \begin{matrix} Z & Z \\ Z & Z \end{matrix}$$

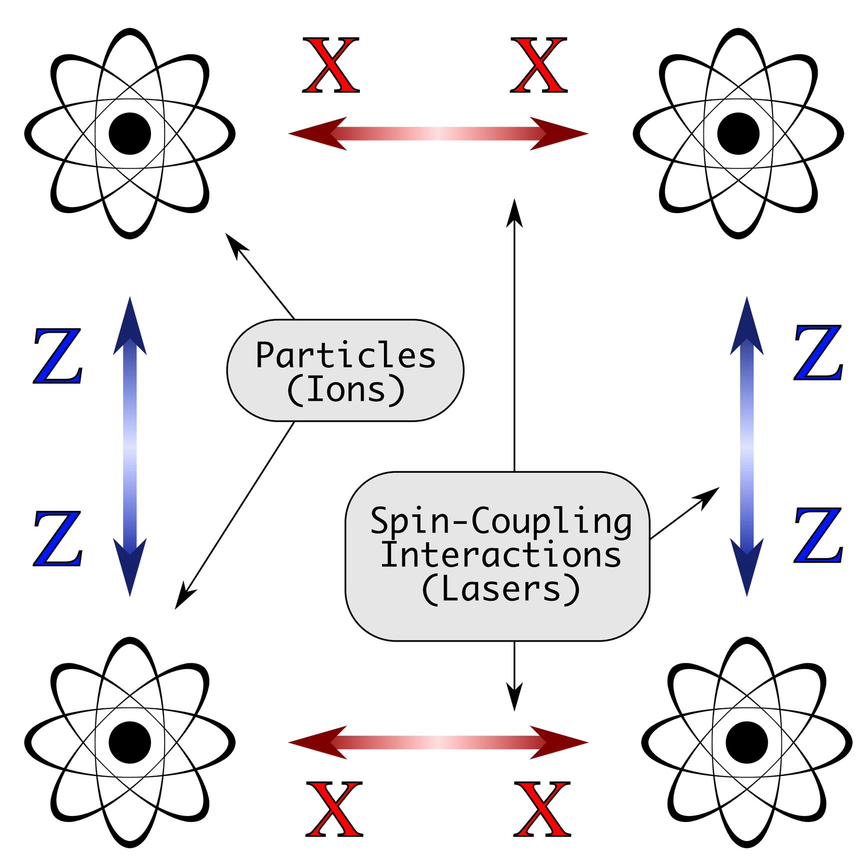
Gauge Qubit

$$\underset{\overline{X}_G}{\begin{matrix} I & I \\ X & X \end{matrix}} \qquad \underset{\overline{Z}_G}{\begin{matrix} Z & I \\ Z & I \end{matrix}}$$

Logical Qubit

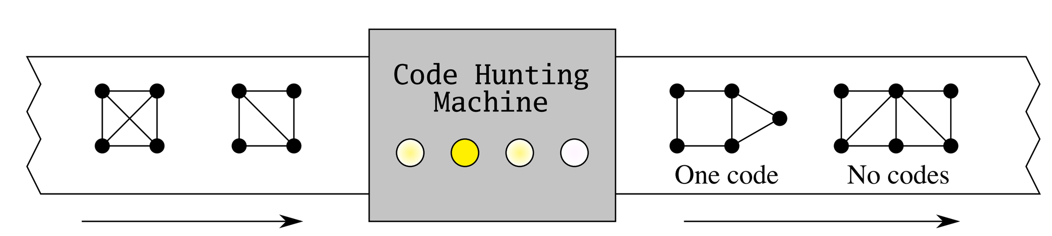
$$\underset{\overline{X}_L}{\begin{matrix} X & I \\ X & I \end{matrix}} \qquad \underset{\overline{Z}_L}{\begin{matrix} I & I \\ Z & Z \end{matrix}}$$

Our approach is unique for this field in that we do not attempt to use cleverness to find physical systems that are useful for storing and manipulating quantum information, but instead we employ numerical brute-force searches through classes of systems. The keystone of this approach is an algorithm which takes as input a physical system and outputs the quantum code manifested by that system. All systems manifest a code, but in most cases the code stores no information and hence is useless.



The particular focus of our hunt is on systems of particles with pairwise couplings between the magnetic moments of particles along a particular axis. Given a graph which tells us where the particles are located and which particles may interact, there are many possibilities for the axes that could be chosen; for example, the system on the left has ZZ and XX interactions (i.e., interactions between respectively the Z- and X-axes of the dipole), but another system with this graph structure might replace the top XX interaction with a YY interaction or even an XY or XZ interaction.

Building on our "keystone" algorithm, we have also written a code that enumerates all of the possible interactions with a given graph structure and outputs those with useful properties. Importantly, redundancies such as graph automorphisms are automatically filtered out.



The Prize

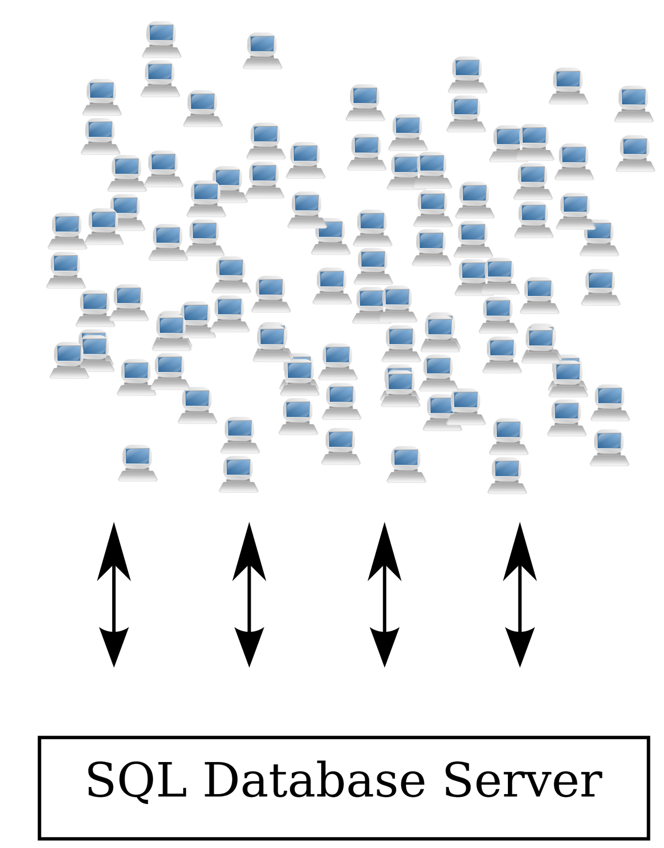
Using a Condor cluster with a hundred nodes, we were able to scan through thousands of graphs. In particular, we were able to perform complete scans through all graphs with 4 through 6 particles, through all planar graphs with 7 particles. Thus, we have enough information to conclude that:

- there is only a single useful 4-particle system

- there is only a single useful 5-particle system

- there are exactly 222 non-equivalent encodings in exactly 25 six-qubit systems

- there are at least 13,759 non-equivalent encoding in at least 216 seven-qubit systems; we suspect that it is unlikely that there are more than these since the graphs which were too combinatorically difficult to scan are probably too strongly coupled to usefully store quantum information



In addition to obtaining some useful results, this research primarly demonstrates the effectiveness of brute-force scanning through classes of systems, and the formalism and techniques we have introduced should prove useful in helping to guide experimentalists in determining what systems they should focus on building.